

# Computer algebra independent integration tests

1\_Algebraic\_functions/1.2\_Trinomial\_products/1.2.1Quadratic/1.2.1.4(d+ex)^m(f+gx)^n

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# 1 Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from Albert Rich Rubi web site.

## 1.1 Listing of CAS systems tested

The following systems were tested at this time.

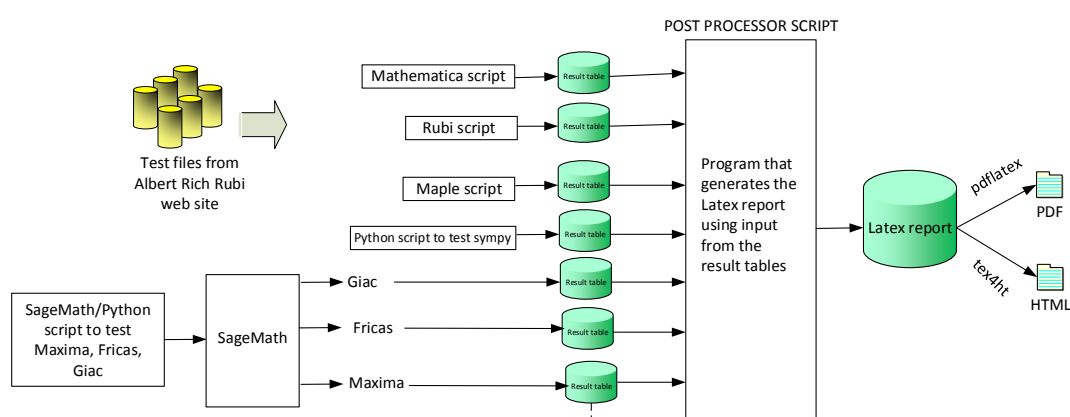
1. Mathematica 11.3 (64 bit).
2. Rubi 4.15.2 in Mathematica 11.3.
3. Rubi in Sympy (Version 1.3) under Python 3.7.0 using Anaconda distribution.
4. Maple 2018.1 (64 bit).
5. Maxima 5.41 Using Lisp ECL 16.1.2.
6. Fricas 1.3.4.
7. Sympy 1.3 under Python 3.7.0 using Anaconda distribution.
8. Giac/Xcas 1.4.9.

Maxima, Fricas and Giac/Xcas were called from inside SageMath version 8.3. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python. Rubi in Sympy was also called directly using sympy 1.3 in python.

## 1.2 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is comma delimited. It contains 12 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"

**High level overview of the CAS independent integration test build system**

## 1.3 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int', int(expr, x)), output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

## 1.4 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

## 1.5 Important notes about some of the results

Important note about Maxima results Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

Important note about FriCAS and Giac/XCAS results There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

Important note about finding leaf size of antiderivative For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-express>

```
def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

When these cas systems implement a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

## 1.6 Grading of results

The table below summarizes the grading of each CAS system.

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. ( 958 )	% 0. ( 0 )
Rubi in Sympy	% 82.99 ( 795 )	% 17.01 ( 163 )
Mathematica	% 95.41 ( 914 )	% 4.59 ( 44 )
Maple	% 75.99 ( 728 )	% 24.01 ( 230 )
Maxima	% 19.42 ( 186 )	% 80.58 ( 772 )
Fricas	% 61.38 ( 588 )	% 38.62 ( 370 )
Sympy	% 24.63 ( 236 )	% 75.37 ( 722 )
Giac	% 35.7 ( 342 )	% 64.3 ( 616 )

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

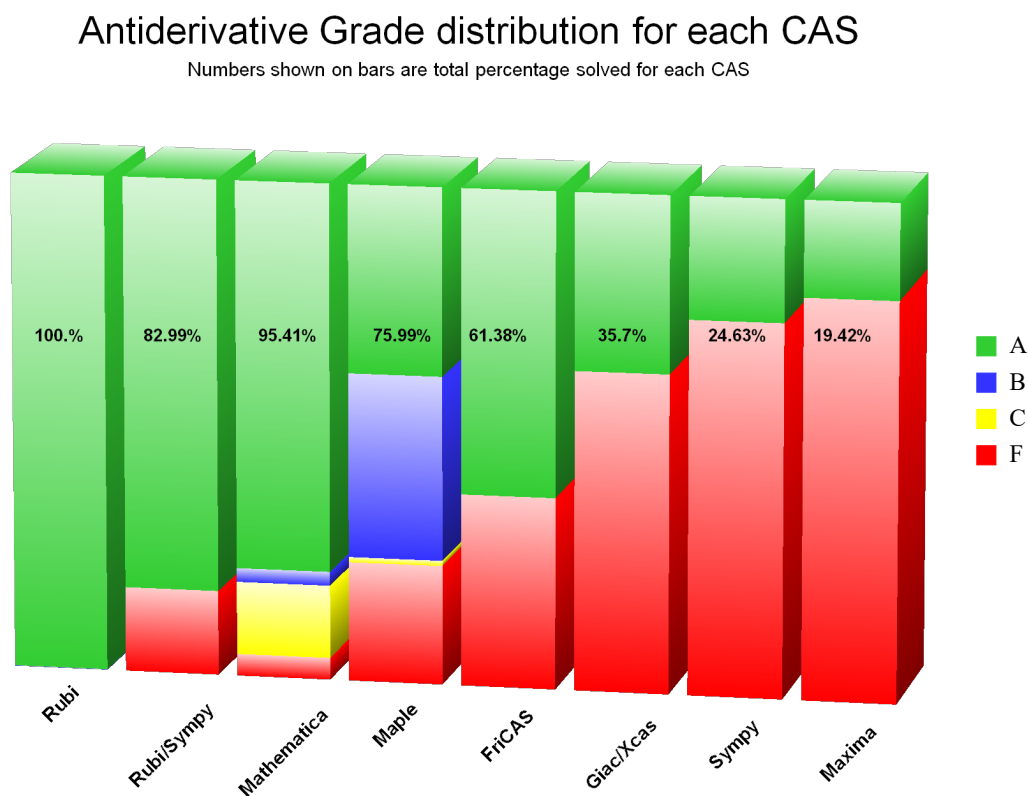
grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ul style="list-style-type: none"> <li>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.</li> <li>2. antiderivative contains a special function and the optimal antiderivative does not.</li> <li>3. antiderivative contains the imaginary unit and the optimal antiderivative does not.</li> </ul>
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

Grading is currently implemented only for for Mathematica, Rubi and Maple results. For all other CAS systems (Maxima, Fricas, Sympy, Giac, Rubi in sympy), the grading function is not yet implemented. For these systems, a grade of A is assigned if the integrate command completes successfully and a grade of F otherwise.

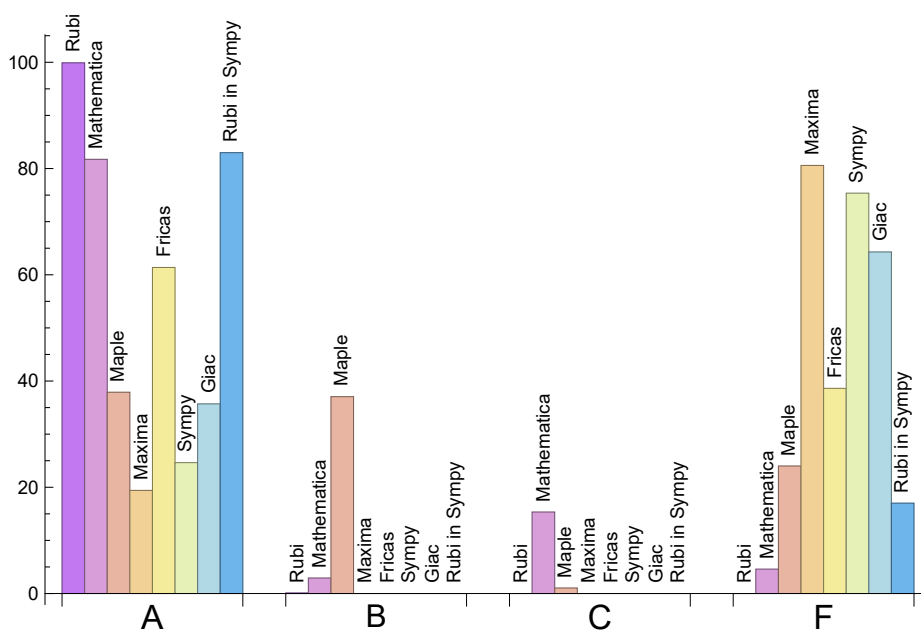
Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.9	0.1	0.	0.
Rubi in Sympy	82.99	0.	0.	17.01
Mathematica	81.73	2.92	15.34	4.59
Maple	37.89	37.06	1.04	24.01
Maxima	19.42	0.	0.	80.58
Fricas	61.38	0.	0.	38.62
Sympy	24.63	0.	0.	75.37
Giac	35.7	0.	0.	64.3

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



## 1.7 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	1.09	234.01	1.	175.	1.
Rubi in Sympy	56.04	165.98	0.94	146.	0.93
Mathematica	1.36	767.64	1.58	161.	0.94
Maple	0.04	1944.09	3.97	288.	1.76
Maxima	0.75	270.32	1.79	186.	1.58
Fricas	2.31	635.56	3.16	276.	2.97
Sympy	36.34	964.06	6.29	386.	3.38
Giac	1.55	160.37	1.27	95.5	1.

## 1.8 list of integrals that has no closed form antiderivative

{948, 952, 957}

## 1.9 list of integrals not solved by each system

**Not solved by Rubi** {}

**Not solved by Rubi in Sympy** {65, 66, 67, 68, 370, 375, 434, 435, 436, 445, 446, 455, 456, 457, 461, 464, 465, 468, 469, 476, 477, 478, 484, 485, 486, 525, 526, 527, 528, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 548, 549, 551, 552, 557, 558, 559, 560, 561, 562, 569, 570, 571, 572, 587, 588, 589, 590, 596, 600, 602, 605, 606, 608, 609, 610, 614, 615, 617, 622, 623, 626, 627, 628, 629, 630, 633, 634, 635, 636, 637, 641, 642, 643, 645, 650, 651, 653, 699, 710, 711, 751, 768, 783, 796, 797, 800, 801, 814, 815, 816, 817, 818, 819, 820, 825, 826, 827, 830, 831, 832, 850, 852, 853, 859, 860, 861, 862, 863, 864, 866, 867, 868, 869, 877, 878, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 897, 898, 899, 900, 901, 902, 905, 906, 907, 908, 909, 910, 914, 915, 916, 917, 918, 919, 925, 926, 927, 945, 949, 954, 958}

**Not solved by Mathematica** {370, 371, 372, 373, 374, 375, 376, 379, 380, 408, 416, 422, 423, 424, 428, 429, 434, 435, 436, 541, 773, 774, 803, 804, 810, 811, 812, 923, 924, 928, 929, 937, 938, 939, 940, 941, 942, 943, 945, 946, 949, 950, 954, 955}

**Not solved by Maple** {136, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 354, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 541, 542, 543, 544, 545, 546, 547, 761, 762, 763, 764, 765, 766, 767, 772, 773, 774, 775, 776, 777, 778, 779, 781, 782, 802, 803, 804, 809, 810, 811, 812, 813, 922, 923, 924, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956}

**Not solved by Maxima** {7, 8, 9, 10, 11, 12, 13, 14, 15, 27, 28, 29, 38, 39, 40, 41, 42, 43, 50, 51, 52, 53, 64, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 155, 156, 157, 158, 159, 160, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 212, 213, 214, 215, 216, 217, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 353, 354, 357, 358, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395,

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**Not solved by Fricas** {226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 354, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 455, 456, 466, 467, 468, 469, 486, 488, 489, 491, 492, 494, 495, 496, 498, 499, 501, 502, 503, 505, 506, 508, 509, 510, 512, 513, 515, 516, 517, 519, 520, 522, 523, 540, 541, 542, 543, 544, 545, 546, 547, 605, 606, 609, 610, 614, 617, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 761, 762, 763, 764, 765, 766, 767, 772, 773, 774, 775, 776, 777, 778, 779, 782, 796, 800, 801, 802, 803, 804, 809, 810, 811, 812, 813, 817, 818, 850, 852, 853, 854, 855, 856, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 877, 878, 879, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 922, 923, 924, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956, 958}

**Not solved by Sympy** {44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 65, 66, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 221, 223, 225, 226, 227, 230, 231, 232, 233, 234, 235, 237, 238, 239, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 311, 312, 314, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 359, 360, 361, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 399, 400, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 525, 526, 527, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 594, 595, 596, 597, 600, 601, 602, 603, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 665, 666, 667, 668, 669, 670, 671, 672, 673, 674, 675, 676, 677, 678, 679, 680, 681, 682, 683, 684,



685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 720, 721, 722, 723, 724, 725, 726, 727, 728, 729, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 761, 762, 763, 764, 765, 766, 767, 768, 769, 770, 771, 772, 773, 774, 775, 776, 777, 778, 779, 780, 781, 782, 783, 784, 785, 786, 787, 788, 789, 790, 791, 792, 793, 795, 796, 797, 798, 800, 801, 802, 803, 804, 805, 809, 810, 811, 812, 813, 816, 817, 818, 824, 825, 826, 827, 830, 831, 832, 833, 834, 835, 836, 837, 838, 839, 840, 841, 842, 843, 844, 845, 846, 847, 848, 849, 850, 851, 852, 853, 854, 855, 856, 857, 858, 859, 860, 861, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 874, 875, 876, 877, 878, 879, 880, 881, 882, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 922, 923, 924, 925, 926, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956, 957, 958}

**Not solved by Giac** {99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 179, 180, 181, 182, 183, 184, 185, 212, 213, 214, 215, 216, 217, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 321, 331, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 354, 358, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 442, 446, 447, 448, 449, 451, 452, 454, 457, 458, 459, 460, 462, 463, 464, 465, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 491, 492, 493, 494, 495, 496, 498, 499, 500, 501, 502, 503, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 553, 554, 555, 565, 566, 577, 586, 607, 611, 612, 613, 614, 615, 616, 618, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634, 635, 636, 637, 638, 639, 640, 641, 642, 643, 644, 645, 646, 647, 648, 649, 650, 651, 652, 653, 654, 655, 656, 657, 658, 659, 660, 661, 662, 663, 664, 679, 680, 681, 682, 683, 684, 685, 686, 687, 688, 689, 690, 691, 692, 693, 694, 695, 696, 697, 698, 699, 700, 701, 702, 703, 704, 705, 706, 707, 708, 709, 710, 711, 712, 713, 714, 715, 716, 717, 718, 719, 723, 724, 725, 726, 730, 731, 732, 733, 734, 735, 736, 737, 738, 739, 740, 741, 742, 743, 744, 745, 746, 747, 748, 749, 750, 751, 752, 753, 754, 755, 756, 757, 758, 759, 760, 763, 764, 765, 766, 767, 772, 773, 774, 775, 776, 777, 778, 779, 781, 782, 785, 786, 787, 788, 789, 790, 791, 795, 796, 800, 801, 802, 803, 804, 809, 810, 811, 812, 813, 814, 815, 816, 818, 833, 851, 852, 853, 854, 855, 856, 857, 858, 859, 862, 863, 864, 865, 866, 867, 868, 869, 870, 871, 872, 873, 875, 876, 878, 879, 883, 884, 885, 886, 887, 888, 889, 890, 891, 892, 893, 894, 895, 896, 897, 898, 899, 900, 901, 902, 903, 904, 905, 906, 907, 908, 909, 910, 911, 912, 913, 914, 915, 916, 917, 918, 919, 922, 923, 924, 927, 928, 929, 930, 931, 932, 933, 934, 935, 936, 937, 938, 939, 940, 941, 942, 943, 944, 945, 946, 947, 949, 950, 951, 953, 954, 955, 956, 958}

## 1.10 list of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Rubi in Sympy** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Ericas** {}

**Sympy** {}

**Giac** {}

## 1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

**Rubi** {420, 628, 635, 642, 651, 653, 655, 801, 886, 887, 892, 893, 899, 906, 915, 917, 919, 945, 946, 949, 950, 954, 955}

**Mathematica** {232, 237, 238, 239, 267, 275, 276, 282, 286, 287, 291, 292, 296, 297, 301, 302, 310, 311, 312, 313, 314, 409, 410, 411, 412, 413, 414, 415, 417, 418, 419, 420, 421, 425, 426, 427, 498, 501, 542, 656, 775, 801, 802, 813, 884, 885, 886, 887, 890, 891, 892, 893, 894, 897, 898, 899, 900, 905, 906, 907, 908, 914, 915, 917, 918, 927, 947, 951, 953, 956}

**Maple** Verification phase not implemented yet.

**Maxima** Verification phase not implemented yet.

**Fricas** Verification phase not implemented yet.

**Sympy** Verification phase not implemented yet.

**Giac** Verification phase not implemented yet.

**Rubi in Sympy** Verification phase not implemented yet.

## 2 detailed summary tables of results

### 2.1 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as  $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	91	125	158	479	279	100	112
normalized size	1	1.	0.69	0.95	1.2	3.63	2.11	0.76	0.85
time (sec)	N/A	0.2	0.086	0.057	0.817	0.279	14.634	0.334	31.879

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	136	198	257	806	830	158	178
normalized size	1	1.	0.68	0.99	1.28	4.01	4.13	0.79	0.89
time (sec)	N/A	0.444	0.132	0.036	0.801	0.283	55.043	0.316	57.003

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	125	173	223	744	775	143	151
normalized size	1	1.	0.73	1.01	1.3	4.33	4.51	0.83	0.88
time (sec)	N/A	0.317	0.115	0.022	0.814	0.282	52.306	0.322	43.081

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	114	148	189	657	653	130	136
normalized size	1	1.	0.72	0.93	1.19	4.13	4.11	0.82	0.86
time (sec)	N/A	0.242	0.102	0.11	0.809	0.278	34.314	0.316	36.98

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	103	123	155	598	580	113	114
normalized size	1	1.	0.89	1.06	1.34	5.16	5.	0.97	0.98
time (sec)	N/A	0.103	0.092	0.018	0.82	0.283	32.332	0.321	21.708

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	103	123	155	598	580	113	114
normalized size	1	1.	0.89	1.06	1.34	5.16	5.	0.97	0.98
time (sec)	N/A	0.098	0.023	0.	0.801	0.285	32.953	0.32	21.714

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	108	151	0	524	469	134	95
normalized size	1	1.	0.96	1.34	0.	4.64	4.15	1.19	0.84
time (sec)	N/A	0.288	0.102	0.019	0.	0.288	25.965	0.298	43.786

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	114	182	0	532	386	212	99
normalized size	1	1.	0.97	1.56	0.	4.55	3.3	1.81	0.85
time (sec)	N/A	0.293	0.161	0.033	0.	0.288	18.101	0.32	41.369

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	118	212	0	576	461	293	109
normalized size	1	1.	0.98	1.75	0.	4.76	3.81	2.42	0.9
time (sec)	N/A	0.289	0.16	0.018	0.	0.291	22.857	0.307	41.85

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	115	235	0	572	457	352	104
normalized size	1	1.	0.96	1.96	0.	4.77	3.81	2.93	0.87
time (sec)	N/A	0.29	0.174	0.023	0.	0.289	19.646	0.304	43.441

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	111	260	0	564	541	401	102
normalized size	1	1.	0.94	2.2	0.	4.78	4.58	3.4	0.86
time (sec)	N/A	0.282	0.226	0.026	0.	0.303	27.81	0.317	43.09

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	106	158	0	531	774	497	88
normalized size	1	1.	0.98	1.46	0.	4.92	7.17	4.6	0.81
time (sec)	N/A	0.171	0.1	0.03	0.	0.293	26.174	0.295	19.765

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	117	186	0	624	918	1	116
normalized size	1	1.	0.82	1.3	0.	4.36	6.42	0.01	0.81
time (sec)	N/A	0.257	0.213	0.037	0.	0.349	40.625	0.295	31.996

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	128	211	0	711	1037	1	146
normalized size	1	1.	0.74	1.23	0.	4.13	6.03	0.01	0.85
time (sec)	N/A	0.345	0.178	0.053	0.	0.372	43.526	0.294	45.4

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	139	236	0	799	1159	1	175
normalized size	1	1.	0.69	1.17	0.	3.98	5.77	0.	0.87
time (sec)	N/A	0.443	0.206	0.075	0.	0.523	63.517	0.299	59.217

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	70	102	127	301	177	73	87
normalized size	1	1.	0.68	0.99	1.23	2.92	1.72	0.71	0.84
time (sec)	N/A	0.159	0.069	0.02	0.805	0.277	11.376	0.291	27.506

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	63	99	123	243	163	89	63
normalized size	1	1.	0.86	1.36	1.68	3.33	2.23	1.22	0.86
time (sec)	N/A	0.134	0.146	0.021	0.802	0.277	11.742	0.297	22.712

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	59	55	119	159	231	69	46
normalized size	1	1.	1.02	0.95	2.05	2.74	3.98	1.19	0.79
time (sec)	N/A	0.095	0.053	0.013	0.72	0.271	12.803	0.298	10.796

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	128	227	439	965	2004	162	146
normalized size	1	1.	0.8	1.41	2.73	5.99	12.45	1.01	0.91
time (sec)	N/A	0.423	0.194	0.102	0.804	0.313	56.171	0.298	47.408

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	115	195	393	841	1821	147	133
normalized size	1	1.	0.78	1.33	2.67	5.72	12.39	1.	0.9
time (sec)	N/A	0.362	0.172	0.034	0.804	0.301	40.659	0.301	45.83

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	103	166	355	729	1739	131	109
normalized size	1	1.	0.84	1.36	2.91	5.98	14.25	1.07	0.89
time (sec)	N/A	0.266	0.151	0.03	0.798	0.294	39.83	0.297	34.295

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	82	77	215	336	418	86	70
normalized size	1	1.	0.98	0.92	2.56	4.	4.98	1.02	0.83
time (sec)	N/A	0.164	0.062	0.012	0.721	0.283	24.287	0.294	17.997

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	82	77	181	369	337	78	80
normalized size	1	1.	0.91	0.86	2.01	4.1	3.74	0.87	0.89
time (sec)	N/A	0.166	0.062	0.013	0.72	0.281	23.65	0.301	20.305

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	82	77	151	398	513	86	80
normalized size	1	1.	0.87	0.82	1.61	4.23	5.46	0.91	0.85
time (sec)	N/A	0.171	0.058	0.012	0.726	0.283	24.242	0.296	16.97

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	82	77	117	429	432	77	68
normalized size	1	1.	0.99	0.93	1.41	5.17	5.2	0.93	0.82
time (sec)	N/A	0.086	0.051	0.013	0.715	0.284	23.619	0.3	8.344

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	82	77	108	452	604	88	66
normalized size	1	1.	1.02	0.96	1.35	5.65	7.55	1.1	0.82
time (sec)	N/A	0.053	0.046	0.012	0.714	0.286	24.096	0.299	7.794

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	106	163	0	718	2378	165	97
normalized size	1	1.	0.91	1.39	0.	6.14	20.32	1.41	0.83
time (sec)	N/A	0.314	0.102	0.018	0.	0.294	36.523	0.292	44.755

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	123	195	0	875	2404	255	126
normalized size	1	1.	0.8	1.27	0.	5.72	15.71	1.67	0.82
time (sec)	N/A	0.403	0.118	0.02	0.	0.296	43.266	0.3	52.798

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	138	227	0	1013	2691	351	162
normalized size	1	1.	0.75	1.23	0.	5.51	14.62	1.91	0.88
time (sec)	N/A	0.5	0.133	0.023	0.	0.32	62.719	0.3	66.641

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	104	99	182	637	903	104	105
normalized size	1	1.	0.86	0.82	1.5	5.26	7.46	0.86	0.87
time (sec)	N/A	0.188	0.086	0.015	0.711	0.331	48.956	0.291	20.17

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	126	121	213	875	1401	122	131
normalized size	1	1.	0.85	0.82	1.44	5.91	9.47	0.82	0.89
time (sec)	N/A	0.206	0.106	0.015	0.708	0.45	160.059	0.295	23.556

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	37	85	101	197	102	95	44
normalized size	1	1.	0.69	1.57	1.87	3.65	1.89	1.76	0.81
time (sec)	N/A	0.128	0.092	0.022	0.789	0.279	10.165	0.29	14.337

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	103	174	224	539	558	113	167
normalized size	1	1.	0.6	1.01	1.29	3.12	3.23	0.65	0.97
time (sec)	N/A	0.573	0.115	0.036	0.787	0.28	37.018	0.291	56.541

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	92	149	190	479	357	99	136
normalized size	1	1.	0.64	1.03	1.32	3.33	2.48	0.69	0.94
time (sec)	N/A	0.446	0.098	0.014	0.792	0.277	19.082	0.294	55.668

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	81	124	157	387	386	85	116
normalized size	1	1.	0.7	1.08	1.37	3.37	3.36	0.74	1.01
time (sec)	N/A	0.319	0.087	0.013	0.786	0.277	22.608	0.288	41.051

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	70	98	122	313	218	66	95
normalized size	1	1.	0.84	1.18	1.47	3.77	2.63	0.8	1.14
time (sec)	N/A	0.174	0.073	0.012	0.792	0.281	11.352	0.289	22.062

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	60	71	85	227	274	54	68
normalized size	1	1.	0.72	0.86	1.02	2.73	3.3	0.65	0.82
time (sec)	N/A	0.077	0.052	0.011	0.792	0.274	9.962	0.292	16.601

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	68	91	0	171	189	88	53
normalized size	1	1.	1.03	1.38	0.	2.59	2.86	1.33	0.8
time (sec)	N/A	0.236	0.05	0.012	0.	0.282	6.549	0.292	23.274

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	71	93	0	209	212	144	54
normalized size	1	1.	1.04	1.37	0.	3.07	3.12	2.12	0.79
time (sec)	N/A	0.236	0.067	0.015	0.	0.282	7.555	0.291	24.262

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	71	86	0	255	214	230	65
normalized size	1	1.	0.89	1.08	0.	3.19	2.68	2.88	0.81
time (sec)	N/A	0.229	0.086	0.015	0.	0.276	13.709	0.297	25.245



Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	76	114	0	355	303	323	88
normalized size	1	1.	0.71	1.07	0.	3.32	2.83	3.02	0.82
time (sec)	N/A	0.315	0.164	0.018	0.	0.274	13.721	0.295	28.177

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	95	139	0	443	449	412	122
normalized size	1	1.	0.68	0.99	0.	3.16	3.21	2.94	0.87
time (sec)	N/A	0.403	0.133	0.019	0.	0.289	24.96	0.293	37.307

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	106	164	0	533	510	493	148
normalized size	1	1.	0.63	0.97	0.	3.15	3.02	2.92	0.88
time (sec)	N/A	0.5	0.172	0.022	0.	0.295	22.097	0.299	42.136

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	118	193	390	635	0	143	144
normalized size	1	1.	0.83	1.35	2.73	4.44	0.	1.	1.01
time (sec)	N/A	0.416	0.138	0.013	0.795	0.28	0.	0.3	75.275

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	106	236	420	548	0	128	121
normalized size	1	1.	0.88	1.95	3.47	4.53	0.	1.06	1.
time (sec)	N/A	0.316	0.118	0.023	0.8	0.28	0.	0.292	58.075

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	70	65	209	247	0	85	97
normalized size	1	1.	0.72	0.67	2.15	2.55	0.	0.88	1.
time (sec)	N/A	0.278	0.055	0.011	0.715	0.27	0.	0.296	44.034

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	70	66	177	277	0	82	75
normalized size	1	1.	0.8	0.76	2.03	3.18	0.	0.94	0.86
time (sec)	N/A	0.222	0.057	0.011	0.722	0.274	0.	0.309	39.841

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	69	64	147	306	0	86	76
normalized size	1	1.	0.78	0.72	1.65	3.44	0.	0.97	0.85
time (sec)	N/A	0.122	0.05	0.013	0.71	0.276	0.	0.294	12.292

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	70	65	105	333	0	82	66
normalized size	1	1.	0.91	0.84	1.36	4.32	0.	1.06	0.86
time (sec)	N/A	0.056	0.048	0.011	0.717	0.271	0.	0.288	7.701

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	95	160	0	540	0	159	114
normalized size	1	1.	0.81	1.37	0.	4.62	0.	1.36	0.97
time (sec)	N/A	0.339	0.091	0.013	0.	0.28	0.	0.297	41.597

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	112	193	0	670	0	254	143
normalized size	1	1.	0.77	1.33	0.	4.62	0.	1.75	0.99
time (sec)	N/A	0.429	0.109	0.016	0.	0.284	0.	0.296	52.551

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	127	224	0	832	0	351	199
normalized size	1	1.	0.7	1.23	0.	4.57	0.	1.93	1.09
time (sec)	N/A	0.56	0.121	0.018	0.	0.281	0.	0.297	68.711

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	138	249	0	953	0	439	226
normalized size	1	1.	0.66	1.19	0.	4.56	0.	2.1	1.08
time (sec)	N/A	0.722	0.127	0.025	0.	0.285	0.	0.303	84.274

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	71	95	239	73	46	61
normalized size	1	1.	0.95	0.88	1.17	2.95	0.9	0.57	0.75
time (sec)	N/A	0.216	0.048	0.027	0.788	0.272	3.483	0.274	15.074

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	72	57	76	193	60	41	54
normalized size	1	1.	1.14	0.9	1.21	3.06	0.95	0.65	0.86
time (sec)	N/A	0.161	0.043	0.008	0.782	0.27	1.985	0.284	14.289

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	26	41	54	159	37	28	39
normalized size	1	1.	0.63	1.	1.32	3.88	0.9	0.68	0.95
time (sec)	N/A	0.09	0.024	0.007	0.783	0.27	1.006	0.276	7.117

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	60	29	38	123	27	26	31
normalized size	1	1.	1.5	0.72	0.95	3.08	0.68	0.65	0.78
time (sec)	N/A	0.036	0.023	0.007	0.781	0.271	0.531	0.278	5.05

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	36	29	55	101	31	46	22
normalized size	1	1.	1.12	0.91	1.72	3.16	0.97	1.44	0.69
time (sec)	N/A	0.118	0.025	0.009	0.78	0.279	6.235	0.277	7.611

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	39	30	57	132	51	74	24
normalized size	1	1.	1.18	0.91	1.73	4.	1.55	2.24	0.73
time (sec)	N/A	0.12	0.032	0.011	0.786	0.279	9.281	0.284	8.457

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	46	42	73	144	116	123	39
normalized size	1	1.	0.9	0.82	1.43	2.82	2.27	2.41	0.76
time (sec)	N/A	0.127	0.044	0.01	0.794	0.271	16.931	0.296	10.817

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	47	56	92	194	128	169	49
normalized size	1	1.	0.7	0.84	1.37	2.9	1.91	2.52	0.73
time (sec)	N/A	0.167	0.05	0.012	0.793	0.273	18.227	0.295	12.548

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	58	70	111	231	223	220	71
normalized size	1	1.	0.65	0.79	1.25	2.6	2.51	2.47	0.8
time (sec)	N/A	0.209	0.056	0.011	0.793	0.269	33.115	0.293	13.55

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	63	84	130	275	201	269	85
normalized size	1	1.	0.59	0.79	1.21	2.57	1.88	2.51	0.79
time (sec)	N/A	0.252	0.062	0.013	0.792	0.271	34.984	0.292	15.419

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	102	212	0	541	544	398	129
normalized size	1	1.	0.76	1.58	0.	4.04	4.06	2.97	0.96
time (sec)	N/A	0.41	0.169	0.03	0.	0.288	26.077	0.305	46.753

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	216	291	382	1251	0	230	0
normalized size	1	1.	0.7	0.94	1.23	4.04	0.	0.74	0.
time (sec)	N/A	0.99	0.257	0.145	0.799	0.289	0.	0.289	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	203	266	348	1162	0	216	0
normalized size	1	1.	0.72	0.95	1.24	4.14	0.	0.77	0.
time (sec)	N/A	0.808	0.237	0.024	0.804	0.287	0.	0.288	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	190	241	315	1103	1919	201	0
normalized size	1	1.	0.75	0.96	1.25	4.38	7.62	0.8	0.
time (sec)	N/A	0.671	0.223	0.021	0.799	0.291	169.554	0.286	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	177	216	281	1014	1681	188	0
normalized size	1	1.	0.79	0.97	1.26	4.55	7.54	0.84	0.
time (sec)	N/A	0.536	0.197	0.017	0.798	0.288	115.11	0.289	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	164	191	247	954	1554	173	204
normalized size	1	1.	0.71	0.83	1.07	4.15	6.76	0.75	0.89
time (sec)	N/A	0.296	0.187	0.013	0.796	0.291	107.362	0.301	41.092

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	135	154	197	860	1284	158	163
normalized size	1	1.	0.72	0.82	1.05	4.57	6.83	0.84	0.87
time (sec)	N/A	0.177	0.136	0.012	0.794	0.288	69.764	0.284	31.605

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	165	231	0	910	1263	193	218
normalized size	1	1.	0.87	1.22	0.	4.79	6.65	1.02	1.15
time (sec)	N/A	0.62	0.142	0.016	0.	0.297	76.375	0.286	123.452

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	156	243	0	948	1057	269	223
normalized size	1	1.	0.81	1.26	0.	4.91	5.48	1.39	1.16
time (sec)	N/A	0.627	0.304	0.019	0.	0.296	53.927	0.29	104.758

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	166	252	0	994	1059	354	238
normalized size	1	1.	0.8	1.22	0.	4.8	5.12	1.71	1.15
time (sec)	N/A	0.625	0.297	0.021	0.	0.298	56.919	0.29	101.537

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	167	277	0	979	911	429	243
normalized size	1	1.	0.8	1.32	0.	4.66	4.34	2.04	1.16
time (sec)	N/A	0.628	0.46	0.022	0.	0.299	41.279	0.292	89.774

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	164	302	0	994	1028	505	238
normalized size	1	1.	0.78	1.44	0.	4.76	4.92	2.42	1.14
time (sec)	N/A	0.622	0.322	0.029	0.	0.301	50.926	0.295	83.786

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	167	327	0	994	1178	1	245
normalized size	1	1.	0.77	1.51	0.	4.6	5.45	0.	1.13
time (sec)	N/A	0.619	0.487	0.035	0.	0.303	43.576	0.288	90.714

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	167	352	0	994	1397	1	248
normalized size	1	1.	0.78	1.64	0.	4.64	6.53	0.	1.16
time (sec)	N/A	0.62	0.366	0.045	0.	0.301	61.438	0.296	91.814

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	161	377	0	988	1513	1	245
normalized size	1	1.	0.78	1.83	0.	4.8	7.34	0.	1.19
time (sec)	N/A	0.621	0.273	0.062	0.	0.303	60.842	0.299	97.313

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	158	402	0	980	1719	1	248
normalized size	1	1.	0.77	1.97	0.	4.8	8.43	0.	1.22
time (sec)	N/A	0.606	0.288	0.094	0.	0.553	89.102	0.306	109.416

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	150	250	0	887	1889	1	253
normalized size	1	1.	0.8	1.34	0.	4.74	10.1	0.01	1.35
time (sec)	N/A	0.437	0.305	0.134	0.	0.579	90.895	0.3	122.167

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	161	278	0	980	2159	1	280
normalized size	1	1.	0.72	1.24	0.	4.36	9.6	0.	1.24
time (sec)	N/A	0.536	0.266	0.209	0.	0.983	137.099	0.316	138.234

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	172	303	0	1068	2397	1	309
normalized size	1	1.	0.68	1.19	0.	4.2	9.44	0.	1.22
time (sec)	N/A	0.638	0.252	0.327	0.	1.131	146.641	0.318	164.825

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	108	222	429	729	0	159	156
normalized size	1	1.	0.62	1.28	2.47	4.19	0.	0.91	0.9
time (sec)	N/A	0.595	0.135	0.014	0.805	0.3	0.	0.304	68.435

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	86	262	455	603	0	144	128
normalized size	1	1.	0.61	1.85	3.2	4.25	0.	1.01	0.9
time (sec)	N/A	0.481	0.139	0.013	0.795	0.297	0.	0.292	56.469

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	83	234	417	454	0	128	104
normalized size	1	1.	0.7	1.98	3.53	3.85	0.	1.08	0.88
time (sec)	N/A	0.369	0.098	0.02	0.81	0.294	0.	0.293	53.724

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	53	55	208	212	0	97	80
normalized size	1	1.	0.57	0.59	2.24	2.28	0.	1.04	0.86
time (sec)	N/A	0.274	0.046	0.01	0.726	0.282	0.	0.298	46.344

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	50	52	173	236	0	81	73
normalized size	1	1.	0.58	0.6	2.01	2.74	0.	0.94	0.85
time (sec)	N/A	0.134	0.042	0.011	0.727	0.277	0.	0.296	13.186

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	53	55	136	271	0	95	80
normalized size	1	1.	0.51	0.53	1.32	2.63	0.	0.92	0.78
time (sec)	N/A	0.136	0.036	0.01	0.726	0.279	0.	0.29	23.791

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	77	158	0	447	0	158	97
normalized size	1	1.	0.68	1.39	0.	3.92	0.	1.39	0.85
time (sec)	N/A	0.347	0.146	0.017	0.	0.289	0.	0.293	46.677

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	94	190	0	620	0	250	124
normalized size	1	1.	0.65	1.31	0.	4.28	0.	1.72	0.86
time (sec)	N/A	0.448	0.152	0.015	0.	0.295	0.	0.3	55.928

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	109	222	0	765	0	350	158
normalized size	1	1.	0.6	1.22	0.	4.2	0.	1.92	0.87
time (sec)	N/A	0.563	0.182	0.018	0.	0.302	0.	0.31	65.807

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	91	208	0	450	0	104	129
normalized size	1	1.	0.62	1.41	0.	3.06	0.	0.71	0.88
time (sec)	N/A	0.473	0.101	0.028	0.	0.292	0.	0.297	52.024

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	80	185	0	387	0	89	102
normalized size	1	1.	0.68	1.57	0.	3.28	0.	0.75	0.86
time (sec)	N/A	0.352	0.079	0.015	0.	0.289	0.	0.294	40.469

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	69	160	0	301	0	73	87
normalized size	1	1.	0.8	1.86	0.	3.5	0.	0.85	1.01
time (sec)	N/A	0.253	0.067	0.016	0.	0.284	0.	0.289	29.529

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	57	140	0	232	0	58	71
normalized size	1	1.	0.92	2.26	0.	3.74	0.	0.94	1.15
time (sec)	N/A	0.136	0.053	0.011	0.	0.289	0.	0.293	16.88

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	77	0	111	0	42	36
normalized size	1	1.	0.93	1.67	0.	2.41	0.	0.91	0.78
time (sec)	N/A	0.045	0.027	0.005	0.	0.283	0.	0.285	12.308



Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	137	0	73	0	65	36
normalized size	1	1.	1.	2.98	0.	1.59	0.	1.41	0.78
time (sec)	N/A	0.191	0.024	0.016	0.	0.29	0.	0.288	24.114

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	53	222	0	142	0	138	36
normalized size	1	1.	1.04	4.35	0.	2.78	0.	2.71	0.71
time (sec)	N/A	0.186	0.044	0.02	0.	0.284	0.	0.3	16.363

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	70	254	0	259	0	0	65
normalized size	1	1.	0.85	3.1	0.	3.16	0.	0.	0.79
time (sec)	N/A	0.265	0.081	0.016	0.	0.28	0.	0.	25.952

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	84	280	0	355	0	0	94
normalized size	1	1.	0.74	2.46	0.	3.11	0.	0.	0.82
time (sec)	N/A	0.365	0.076	0.017	0.	0.285	0.	0.	38.338

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	95	304	0	428	0	0	122
normalized size	1	1.	0.66	2.13	0.	2.99	0.	0.	0.85
time (sec)	N/A	0.455	0.095	0.018	0.	0.289	0.	0.	50.656

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	91	222	0	479	279	0	112
normalized size	1	1.	0.81	1.96	0.	4.24	2.47	0.	0.99
time (sec)	N/A	0.27	0.085	0.018	0.	0.288	16.788	0.	33.436

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	135	330	0	806	830	0	178
normalized size	1	1.	0.67	1.64	0.	4.01	4.13	0.	0.89
time (sec)	N/A	0.513	0.14	0.019	0.	0.293	62.595	0.	63.692

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	124	305	0	744	775	0	151
normalized size	1	1.	0.72	1.77	0.	4.33	4.51	0.	0.88
time (sec)	N/A	0.388	0.122	0.016	0.	0.296	82.499	0.	47.258

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	113	282	0	657	653	0	136
normalized size	1	1.	0.81	2.01	0.	4.69	4.66	0.	0.97
time (sec)	N/A	0.303	0.109	0.017	0.	0.287	40.521	0.	38.587

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	102	260	0	598	580	0	116
normalized size	1	1.	0.88	2.24	0.	5.16	5.	0.	1.
time (sec)	N/A	0.173	0.095	0.013	0.	0.291	35.514	0.	22.063

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	91	147	147	504	435	0	83
normalized size	1	1.	0.91	1.47	1.47	5.04	4.35	0.	0.83
time (sec)	N/A	0.074	0.092	0.009	0.79	0.291	21.917	0.	16.728

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	108	245	0	524	469	0	95
normalized size	1	1.	0.96	2.17	0.	4.64	4.15	0.	0.84
time (sec)	N/A	0.358	0.102	0.015	0.	0.292	26.739	0.	49.932

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	114	380	0	532	386	0	99
normalized size	1	1.	0.99	3.3	0.	4.63	3.36	0.	0.86
time (sec)	N/A	0.359	0.151	0.016	0.	0.298	21.063	0.	47.867

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	119	411	0	576	461	0	112
normalized size	1	1.	0.98	3.4	0.	4.76	3.81	0.	0.93
time (sec)	N/A	0.369	0.2	0.018	0.	0.299	25.719	0.	50.646

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	116	439	0	572	457	0	104
normalized size	1	1.	0.97	3.66	0.	4.77	3.81	0.	0.87
time (sec)	N/A	0.37	0.162	0.018	0.	0.297	22.567	0.	51.163

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	111	463	0	564	541	0	102
normalized size	1	1.	0.93	3.89	0.	4.74	4.55	0.	0.86
time (sec)	N/A	0.363	0.223	0.019	0.	0.31	30.928	0.	51.723

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	106	493	0	531	774	1	88
normalized size	1	1.	0.98	4.56	0.	4.92	7.17	0.01	0.81
time (sec)	N/A	0.264	0.109	0.021	0.	0.307	30.536	0.295	25.898

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	117	521	0	624	918	1	116
normalized size	1	1.	0.82	3.64	0.	4.36	6.42	0.01	0.81
time (sec)	N/A	0.349	0.118	0.021	0.	0.353	79.674	0.315	37.897

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	128	546	0	711	1037	1	146
normalized size	1	1.	0.74	3.17	0.	4.13	6.03	0.01	0.85
time (sec)	N/A	0.44	0.128	0.022	0.	0.382	49.101	0.314	53.765

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	139	571	0	799	1159	1	175
normalized size	1	1.	0.69	2.84	0.	3.98	5.77	0.	0.87
time (sec)	N/A	0.534	0.146	0.024	0.	0.541	72.919	0.326	63.86

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	26	34	38	123	29	26	29
normalized size	1	1.	0.96	1.26	1.41	4.56	1.07	0.96	1.07
time (sec)	N/A	0.068	0.026	0.009	0.785	0.286	5.62	0.283	5.266

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	54	238	111	242	170	169	41
normalized size	1	1.	1.06	4.67	2.18	4.75	3.33	3.31	0.8
time (sec)	N/A	0.241	0.064	0.03	0.793	0.297	12.007	0.29	21.462

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	93	147	0	491	0	0	116
normalized size	1	1.	0.79	1.25	0.	4.16	0.	0.	0.98
time (sec)	N/A	0.372	0.099	0.018	0.	0.288	0.	0.	44.152

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	70	120	0	370	0	0	92
normalized size	1	1.	0.77	1.32	0.	4.07	0.	0.	1.01
time (sec)	N/A	0.248	0.116	0.015	0.	0.284	0.	0.	30.465

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	59	97	0	243	0	0	65
normalized size	1	1.	0.77	1.26	0.	3.16	0.	0.	0.84
time (sec)	N/A	0.198	0.116	0.014	0.	0.293	0.	0.	22.186

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	49	74	0	119	0	0	42
normalized size	1	1.	0.94	1.42	0.	2.29	0.	0.	0.81
time (sec)	N/A	0.082	0.054	0.013	0.	0.285	0.	0.	13.196

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	29	0	41	0	0	22
normalized size	1	1.	1.	0.94	0.	1.32	0.	0.	0.71
time (sec)	N/A	0.034	0.024	0.009	0.	0.276	0.	0.	5.373

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	52	88	0	112	0	0	41
normalized size	1	1.	0.96	1.63	0.	2.07	0.	0.	0.76
time (sec)	N/A	0.144	0.064	0.016	0.	0.283	0.	0.	15.783

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	65	108	0	271	0	0	65
normalized size	1	1.	0.8	1.33	0.	3.35	0.	0.	0.8
time (sec)	N/A	0.215	0.105	0.016	0.	0.284	0.	0.	21.351

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	84	133	0	410	0	0	97
normalized size	1	1.	0.74	1.18	0.	3.63	0.	0.	0.86
time (sec)	N/A	0.305	0.142	0.017	0.	0.287	0.	0.	33.853

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	106	208	0	605	0	0	163
normalized size	1	1.	0.83	1.62	0.	4.73	0.	0.	1.27
time (sec)	N/A	0.368	0.117	0.026	0.	0.283	0.	0.	59.401

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	93	179	0	481	0	0	112
normalized size	1	1.	0.82	1.58	0.	4.26	0.	0.	0.99
time (sec)	N/A	0.317	0.13	0.014	0.	0.292	0.	0.	44.367

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	80	153	0	369	0	0	88
normalized size	1	1.	0.9	1.72	0.	4.15	0.	0.	0.99
time (sec)	N/A	0.239	0.133	0.014	0.	0.291	0.	0.	30.598

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	60	48	0	158	0	1	46
normalized size	1	1.	1.	0.8	0.	2.63	0.	0.02	0.77
time (sec)	N/A	0.115	0.043	0.011	0.	0.28	0.	0.289	9.208

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	56	44	0	186	0	0	46
normalized size	1	1.	0.97	0.76	0.	3.21	0.	0.	0.79
time (sec)	N/A	0.082	0.038	0.011	0.	0.284	0.	0.	6.721

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	58	46	0	213	0	0	46
normalized size	1	1.	1.	0.79	0.	3.67	0.	0.	0.79
time (sec)	N/A	0.047	0.04	0.009	0.	0.284	0.	0.	6.195

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	83	142	0	335	0	0	70
normalized size	1	1.	0.94	1.61	0.	3.81	0.	0.	0.8
time (sec)	N/A	0.239	0.132	0.017	0.	0.289	0.	0.	27.868

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	101	188	0	514	0	1	97
normalized size	1	1.	0.84	1.57	0.	4.28	0.	0.01	0.81
time (sec)	N/A	0.327	0.144	0.021	0.	0.288	0.	0.293	35.517

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	115	216	0	653	0	0	131
normalized size	1	1.	0.76	1.42	0.	4.3	0.	0.	0.86
time (sec)	N/A	0.412	0.107	0.021	0.	0.289	0.	0.	47.397

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	128	0	0	963	0	0	260
normalized size	1	1.	0.79	0.	0.	5.94	0.	0.	1.6
time (sec)	N/A	0.511	0.15	180.	0.	0.327	0.	0.	98.34

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	115	288	0	838	0	0	182
normalized size	1	1.	0.78	1.95	0.	5.66	0.	0.	1.23
time (sec)	N/A	0.454	0.161	0.045	0.	0.307	0.	0.	77.206

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	103	259	0	726	0	0	160
normalized size	1	1.	0.84	2.12	0.	5.95	0.	0.	1.31
time (sec)	N/A	0.351	0.117	0.015	0.	0.306	0.	0.	58.691

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	82	70	0	336	0	0	70
normalized size	1	1.	0.96	0.82	0.	3.95	0.	0.	0.82
time (sec)	N/A	0.254	0.066	0.011	0.	0.292	0.	0.	21.061

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	82	70	0	367	0	0	80
normalized size	1	1.	0.9	0.77	0.	4.03	0.	0.	0.88
time (sec)	N/A	0.25	0.058	0.011	0.	0.285	0.	0.	15.426

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	82	70	0	397	0	0	80
normalized size	1	1.	0.86	0.74	0.	4.18	0.	0.	0.84
time (sec)	N/A	0.177	0.054	0.01	0.	0.289	0.	0.	13.887

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	82	70	0	427	0	0	70
normalized size	1	1.	0.96	0.82	0.	5.02	0.	0.	0.82
time (sec)	N/A	0.097	0.054	0.011	0.	0.296	0.	0.	8.704

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	82	70	0	452	0	0	68
normalized size	1	1.	1.	0.85	0.	5.51	0.	0.	0.83
time (sec)	N/A	0.059	0.052	0.011	0.	0.295	0.	0.	8.01

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	106	196	0	717	0	0	97
normalized size	1	1.	0.89	1.65	0.	6.03	0.	0.	0.82
time (sec)	N/A	0.331	0.11	0.019	0.	0.307	0.	0.	40.386

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	122	268	0	871	0	0	126
normalized size	1	1.	0.79	1.74	0.	5.66	0.	0.	0.82
time (sec)	N/A	0.431	0.149	0.02	0.	0.307	0.	0.	50.928

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	137	298	0	1010	0	0	162
normalized size	1	1.	0.74	1.6	0.	5.43	0.	0.	0.87
time (sec)	N/A	0.524	0.151	0.023	0.	0.33	0.	0.	65.164

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	148	326	0	1125	0	0	189
normalized size	1	1.	0.69	1.52	0.	5.23	0.	0.	0.88
time (sec)	N/A	0.641	0.181	0.025	0.	0.33	0.	0.	79.088

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	104	92	0	605	0	0	116
normalized size	1	1.	0.88	0.78	0.	5.13	0.	0.	0.98
time (sec)	N/A	0.268	0.083	0.012	0.	0.341	0.	0.	26.151

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	104	92	0	636	0	0	105
normalized size	1	1.	0.85	0.75	0.	5.17	0.	0.	0.85
time (sec)	N/A	0.201	0.07	0.012	0.	0.344	0.	0.	17.186

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	44	100	92	286	0	105	65
normalized size	1	1.	0.67	1.52	1.39	4.33	0.	1.59	0.98
time (sec)	N/A	0.222	0.091	0.018	0.79	0.293	0.	0.284	17.931

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	37	84	70	197	0	95	44
normalized size	1	1.	0.67	1.53	1.27	3.58	0.	1.73	0.8
time (sec)	N/A	0.182	0.081	0.011	0.789	0.293	0.	0.288	13.062

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	31	65	45	104	0	70	27
normalized size	1	1.	0.91	1.91	1.32	3.06	0.	2.06	0.79
time (sec)	N/A	0.074	0.037	0.011	0.788	0.283	0.	0.312	7.218



Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	23	22	31	32	0	46	19
normalized size	1	1.	0.88	0.85	1.19	1.23	0.	1.77	0.73
time (sec)	N/A	0.032	0.017	0.007	0.782	0.267	0.	0.3	4.627

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	45	58	78	84	0	100	29
normalized size	1	1.	1.1	1.41	1.9	2.05	0.	2.44	0.71
time (sec)	N/A	0.132	0.062	0.016	0.899	0.285	0.	0.301	11.071

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	54	73	0	211	0	203	48
normalized size	1	1.	0.84	1.14	0.	3.3	0.	3.17	0.75
time (sec)	N/A	0.2	0.077	0.016	0.	0.285	0.	0.289	15.802

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	74	94	0	317	0	288	75
normalized size	1	1.	0.82	1.04	0.	3.52	0.	3.2	0.83
time (sec)	N/A	0.275	0.143	0.017	0.	0.288	0.	0.287	22.958

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	135	375	0	806	0	0	212
normalized size	1	1.	0.59	1.64	0.	3.52	0.	0.	0.93
time (sec)	N/A	0.815	0.135	0.027	0.	0.294	0.	0.	89.496

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	124	350	0	714	0	0	196
normalized size	1	1.	0.62	1.75	0.	3.57	0.	0.	0.98
time (sec)	N/A	0.676	0.121	0.02	0.	0.294	0.	0.	75.302

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	113	327	0	657	0	0	162
normalized size	1	1.	0.66	1.91	0.	3.84	0.	0.	0.95
time (sec)	N/A	0.532	0.106	0.02	0.	0.29	0.	0.	69.483

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	102	303	0	568	0	0	144
normalized size	1	1.	0.72	2.13	0.	4.	0.	0.	1.01
time (sec)	N/A	0.412	0.099	0.017	0.	0.288	0.	0.	57.52

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	91	198	225	509	321	0	116
normalized size	1	1.	0.67	1.46	1.65	3.74	2.36	0.	0.85
time (sec)	N/A	0.151	0.086	0.015	0.792	0.295	21.002	0.	22.197

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	80	194	161	412	350	0	90
normalized size	1	1.	0.74	1.8	1.49	3.81	3.24	0.	0.83
time (sec)	N/A	0.112	0.069	0.009	0.792	0.288	23.473	0.	24.351

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	96	290	0	392	267	1	92
normalized size	1	1.	1.	3.02	0.	4.08	2.78	0.01	0.96
time (sec)	N/A	0.391	0.086	0.017	0.	0.301	19.063	4.916	34.922

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	100	425	0	429	347	0	105
normalized size	1	1.	0.95	4.05	0.	4.09	3.3	0.	1.
time (sec)	N/A	0.383	0.115	0.019	0.	0.295	22.738	0.	36.702

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	102	456	0	470	347	0	107
normalized size	1	1.	0.93	4.15	0.	4.27	3.15	0.	0.97
time (sec)	N/A	0.398	0.127	0.018	0.	0.294	23.305	0.	39.837

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	96	479	0	463	338	0	99
normalized size	1	1.	0.94	4.7	0.	4.54	3.31	0.	0.97
time (sec)	N/A	0.39	0.165	0.02	0.	0.293	26.17	0.	37.372

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	95	513	0	425	0	0	94
normalized size	1	1.	0.88	4.75	0.	3.94	0.	0.	0.87
time (sec)	N/A	0.328	0.122	0.02	0.	0.291	0.	0.	38.566

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	106	541	0	533	0	0	116
normalized size	1	1.	0.76	3.86	0.	3.81	0.	0.	0.83
time (sec)	N/A	0.416	0.125	0.022	0.	0.305	0.	0.	40.298

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	117	566	0	624	0	0	150
normalized size	1	1.	0.69	3.35	0.	3.69	0.	0.	0.89
time (sec)	N/A	0.499	0.168	0.025	0.	0.36	0.	0.	53.525

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	128	591	0	711	0	0	168
normalized size	1	1.	0.65	2.98	0.	3.59	0.	0.	0.85
time (sec)	N/A	0.606	0.134	0.025	0.	0.389	0.	0.	54.169

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	106	198	0	547	0	4	121
normalized size	1	1.	0.86	1.61	0.	4.45	0.	0.03	0.98
time (sec)	N/A	0.402	0.116	0.02	0.	0.293	0.	0.673	43.096

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	70	65	0	246	0	4	97
normalized size	1	1.	0.71	0.66	0.	2.48	0.	0.04	0.98
time (sec)	N/A	0.375	0.052	0.011	0.	0.281	0.	0.665	30.928

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	70	65	0	278	0	4	75
normalized size	1	1.	0.79	0.73	0.	3.12	0.	0.04	0.84
time (sec)	N/A	0.311	0.048	0.012	0.	0.283	0.	0.633	25.727

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	69	64	0	308	0	4	76
normalized size	1	1.	0.76	0.7	0.	3.38	0.	0.04	0.84
time (sec)	N/A	0.122	0.046	0.011	0.	0.284	0.	0.647	11.099

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	70	66	0	332	0	4	75
normalized size	1	1.	0.77	0.73	0.	3.65	0.	0.04	0.82
time (sec)	N/A	0.084	0.053	0.01	0.	0.282	0.	0.64	10.395

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	95	187	0	539	0	4	114
normalized size	1	1.	0.81	1.58	0.	4.57	0.	0.03	0.97
time (sec)	N/A	0.425	0.09	0.017	0.	0.291	0.	0.679	33.434

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	112	234	0	670	0	4	143
normalized size	1	1.	0.77	1.6	0.	4.59	0.	0.03	0.98
time (sec)	N/A	0.518	0.109	0.018	0.	0.287	0.	0.665	44.439

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	127	259	0	830	0	4	199
normalized size	1	1.	0.69	1.42	0.	4.54	0.	0.02	1.09
time (sec)	N/A	0.638	0.115	0.021	0.	0.292	0.	0.654	61.46

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	98	212	0	730	0	0	156
normalized size	1	1.	0.55	1.2	0.	4.12	0.	0.	0.88
time (sec)	N/A	0.724	0.149	0.025	0.	0.305	0.	0.	55.229

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	85	187	0	605	0	0	128
normalized size	1	1.	0.58	1.28	0.	4.14	0.	0.	0.88
time (sec)	N/A	0.594	0.165	0.018	0.	0.287	0.	0.	45.03

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	73	163	0	456	0	0	104
normalized size	1	1.	0.61	1.36	0.	3.8	0.	0.	0.87
time (sec)	N/A	0.483	0.11	0.016	0.	0.291	0.	0.	40.507

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	52	55	0	213	0	0	80
normalized size	1	1.	0.55	0.58	0.	2.24	0.	0.	0.84
time (sec)	N/A	0.275	0.044	0.011	0.	0.283	0.	0.	30.167

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	49	52	0	238	0	0	78
normalized size	1	1.	0.51	0.54	0.	2.45	0.	0.	0.8
time (sec)	N/A	0.145	0.039	0.011	0.	0.281	0.	0.	13.655

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	52	55	0	273	0	0	82
normalized size	1	1.	0.52	0.55	0.	2.73	0.	0.	0.82
time (sec)	N/A	0.111	0.035	0.01	0.	0.281	0.	0.	13.122

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	76	179	0	450	0	0	97
normalized size	1	1.	0.66	1.56	0.	3.91	0.	0.	0.84
time (sec)	N/A	0.426	0.136	0.017	0.	0.287	0.	0.	35.924

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	92	199	0	620	0	1	124
normalized size	1	1.	0.63	1.36	0.	4.25	0.	0.01	0.85
time (sec)	N/A	0.531	0.217	0.018	0.	0.293	0.	0.31	44.3

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	107	222	0	767	0	1	158
normalized size	1	1.	0.58	1.21	0.	4.19	0.	0.01	0.86
time (sec)	N/A	0.649	0.187	0.019	0.	0.3	0.	0.32	57.712

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	109	297	0	845	0	1	175
normalized size	1	1.	0.53	1.46	0.	4.14	0.	0.	0.86
time (sec)	N/A	0.93	0.115	0.032	0.	0.312	0.	0.324	55.582

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	98	273	0	730	0	1	155
normalized size	1	1.	0.61	1.71	0.	4.56	0.	0.01	0.97
time (sec)	N/A	0.714	0.183	0.018	0.	0.302	0.	0.323	48.679

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	85	212	0	605	0	1	124
normalized size	1	1.	0.57	1.43	0.	4.09	0.	0.01	0.84
time (sec)	N/A	0.424	0.121	0.015	0.	0.297	0.	0.297	40.913

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	73	214	0	455	0	1	99
normalized size	1	1.	0.63	1.86	0.	3.96	0.	0.01	0.86
time (sec)	N/A	0.279	0.135	0.016	0.	0.294	0.	0.296	31.405

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	50	42	0	242	0	1	53
normalized size	1	1.	0.78	0.66	0.	3.78	0.	0.02	0.83
time (sec)	N/A	0.101	0.034	0.01	0.	0.285	0.	0.288	8.867

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	51	43	0	271	0	1	53
normalized size	1	1.	0.76	0.64	0.	4.04	0.	0.01	0.79
time (sec)	N/A	0.067	0.031	0.007	0.	0.273	0.	0.29	8.532

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	76	196	0	448	0	1	95
normalized size	1	1.	0.69	1.78	0.	4.07	0.	0.01	0.86
time (sec)	N/A	0.425	0.123	0.017	0.	0.288	0.	0.301	48.237

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	92	361	0	629	0	1	122
normalized size	1	1.	0.64	2.52	0.	4.4	0.	0.01	0.85
time (sec)	N/A	0.527	0.215	0.018	0.	0.292	0.	0.293	60.206

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	107	389	0	767	0	1	156
normalized size	1	1.	0.58	2.13	0.	4.19	0.	0.01	0.85
time (sec)	N/A	0.645	0.176	0.02	0.	0.302	0.	0.296	73.908

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	118	412	0	882	0	1	180
normalized size	1	1.	0.56	1.96	0.	4.2	0.	0.	0.86
time (sec)	N/A	0.795	0.248	0.02	0.	0.311	0.	0.337	82.119

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	131	416	0	967	0	1	218
normalized size	1	1.	0.52	1.65	0.	3.84	0.	0.	0.87
time (sec)	N/A	0.974	0.169	0.031	0.	0.303	0.	0.302	120.445

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	125	393	0	849	0	1	194
normalized size	1	1.	0.56	1.75	0.	3.79	0.	0.	0.87
time (sec)	N/A	0.804	0.138	0.021	0.	0.301	0.	0.307	93.203

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	109	285	0	729	0	1	163
normalized size	1	1.	0.57	1.48	0.	3.8	0.	0.01	0.85
time (sec)	N/A	0.666	0.134	0.019	0.	0.296	0.	0.335	88.979

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	103	288	0	612	0	1	143
normalized size	1	1.	0.57	1.58	0.	3.36	0.	0.01	0.79
time (sec)	N/A	0.482	0.144	0.017	0.	0.292	0.	0.331	62.56

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	83	290	0	489	0	1	116
normalized size	1	1.	0.64	2.23	0.	3.76	0.	0.01	0.89
time (sec)	N/A	0.166	0.132	0.016	0.	0.292	0.	0.31	25.263

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	75	284	0	366	0	1	95
normalized size	1	1.	0.66	2.51	0.	3.24	0.	0.01	0.84
time (sec)	N/A	0.116	0.1	0.008	0.	0.289	0.	0.302	23.023

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	79	378	0	328	0	1	75
normalized size	1	1.	0.89	4.25	0.	3.69	0.	0.01	0.84
time (sec)	N/A	0.393	0.118	0.017	0.	0.293	0.	0.306	31.794

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	84	515	0	367	0	1	76
normalized size	1	1.	0.89	5.48	0.	3.9	0.	0.01	0.81
time (sec)	N/A	0.394	0.164	0.019	0.	0.293	0.	0.341	32.571

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	85	504	0	405	0	1	88
normalized size	1	1.	0.77	4.58	0.	3.68	0.	0.01	0.8
time (sec)	N/A	0.413	0.2	0.021	0.	0.282	0.	0.338	33.363

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	94	575	0	528	0	1	117
normalized size	1	1.	0.69	4.2	0.	3.85	0.	0.01	0.85
time (sec)	N/A	0.526	0.218	0.024	0.	0.276	0.	0.392	41.996

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	107	600	0	648	0	1	148
normalized size	1	1.	0.63	3.53	0.	3.81	0.	0.01	0.87
time (sec)	N/A	0.648	0.193	0.023	0.	0.286	0.	0.353	54.624



Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	118	628	0	765	0	1	172
normalized size	1	1.	0.6	3.2	0.	3.9	0.	0.01	0.88
time (sec)	N/A	0.797	0.299	0.024	0.	0.285	0.	0.348	67.506

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	50	200	0	352	0	217	78
normalized size	1	1.	0.53	2.11	0.	3.71	0.	2.28	0.82
time (sec)	N/A	0.25	0.085	0.025	0.	0.289	0.	0.276	24.474

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	50	44	207	251	0	274	73
normalized size	1	1.	0.57	0.5	2.35	2.85	0.	3.11	0.83
time (sec)	N/A	0.252	0.038	0.009	0.722	0.281	0.	0.283	25.761

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	137	132	0	961	0	0	187
normalized size	1	1.	0.66	0.63	0.	4.6	0.	0.	0.89
time (sec)	N/A	0.571	0.111	0.015	0.	0.521	0.	0.	73.401

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	137	132	0	992	0	0	187
normalized size	1	1.	0.66	0.63	0.	4.75	0.	0.	0.89
time (sec)	N/A	0.416	0.097	0.015	0.	0.514	0.	0.	62.179

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	137	132	0	1021	0	0	185
normalized size	1	1.	0.65	0.63	0.	4.84	0.	0.	0.88
time (sec)	N/A	0.261	0.101	0.014	0.	0.517	0.	0.	29.765

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	137	132	0	1045	0	0	177
normalized size	1	1.	0.67	0.64	0.	5.1	0.	0.	0.86
time (sec)	N/A	0.218	0.099	0.014	0.	0.518	0.	0.	29.219

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	161	385	0	1608	0	0	257
normalized size	1	1.	0.69	1.65	0.	6.87	0.	0.	1.1
time (sec)	N/A	0.855	0.178	0.034	0.	0.656	0.	0.	87.156

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	183	484	0	1769	0	0	296
normalized size	1	1.	0.68	1.79	0.	6.53	0.	0.	1.09
time (sec)	N/A	1.037	0.181	0.021	0.	0.947	0.	0.	117.476

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	92	95	72	1	0	154	87
normalized size	1	1.	0.9	0.93	0.71	0.01	0.	1.51	0.85
time (sec)	N/A	0.375	0.174	0.04	0.796	0.294	0.	0.303	22.908

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	47	58	38	1	0	69	36
normalized size	1	1.	1.21	1.49	0.97	0.03	0.	1.77	0.92
time (sec)	N/A	0.135	0.032	0.017	0.769	0.291	0.	0.296	10.139

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	62	0	1	83	0	29
normalized size	1	1.	1.	1.77	0.	0.03	2.37	0.	0.83
time (sec)	N/A	0.033	0.027	0.026	0.	0.288	5.87	0.	5.926

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	80	76	0	1	0	0	29
normalized size	1	1.	2.29	2.17	0.	0.03	0.	0.	0.83
time (sec)	N/A	0.109	0.107	0.019	0.	0.312	0.	0.	9.753

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	34	57	0	1	29	0	29
normalized size	1	1.	1.	1.68	0.	0.03	0.85	0.	0.85
time (sec)	N/A	0.028	0.022	0.008	0.	0.288	5.784	0.	5.088

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	100	86	0	1	0	0	29
normalized size	1	1.	2.94	2.53	0.	0.03	0.	0.	0.85
time (sec)	N/A	0.112	0.142	0.017	0.	0.318	0.	0.	9.624

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	49	79	0	1	148	0	51
normalized size	1	1.	0.78	1.25	0.	0.02	2.35	0.	0.81
time (sec)	N/A	0.049	0.041	0.007	0.	0.287	10.235	0.	8.209

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	94	92	0	1	0	0	49
normalized size	1	1.	1.49	1.46	0.	0.02	0.	0.	0.78
time (sec)	N/A	0.12	0.141	0.016	0.	0.319	0.	0.	11.966

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	377	0	0	0	0	0	287
normalized size	1	1.	1.51	0.	0.	0.	0.	0.	1.15
time (sec)	N/A	0.676	0.716	0.099	0.	0.	0.	0.	72.827

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	335	0	0	0	0	0	214
normalized size	1	1.	1.63	0.	0.	0.	0.	0.	1.04
time (sec)	N/A	0.424	0.555	0.038	0.	0.	0.	0.	52.655

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	289	0	0	0	374	0	138
normalized size	1	1.	1.78	0.	0.	0.	2.31	0.	0.85
time (sec)	N/A	0.225	0.396	0.03	0.	0.	176.499	0.	26.583

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	183	0	0	0	61	0	68
normalized size	1	1.	2.29	0.	0.	0.	0.76	0.	0.85
time (sec)	N/A	0.072	0.166	0.028	0.	0.	134.14	0.	11.674

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	203	0	0	0	0	0	138
normalized size	1	1.	1.25	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.433	0.227	0.049	0.	0.	0.	0.	50.895

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	175	0	0	0	0	0	209
normalized size	1	1.	0.86	0.	0.	0.	0.	0.	1.02
time (sec)	N/A	0.486	0.165	0.042	0.	0.	0.	0.	77.745

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	272	0	0	0	0	0	274
normalized size	1	1.	1.09	0.	0.	0.	0.	0.	1.1
time (sec)	N/A	0.737	1.271	0.041	0.	0.	0.	0.	113.819

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	240	0	0	0	0	0	280
normalized size	1	1.	1.13	0.	0.	0.	0.	0.	1.31
time (sec)	N/A	0.43	0.357	0.035	0.	0.	0.	0.	71.134

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	178	0	0	0	0	0	209
normalized size	1	1.	0.82	0.	0.	0.	0.	0.	0.97
time (sec)	N/A	0.427	0.216	0.035	0.	0.	0.	0.	52.269

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	162	121	0	0	0	0	0	134
normalized size	1	1.31	0.98	0.	0.	0.	0.	0.	1.08
time (sec)	N/A	0.238	0.145	0.031	0.	0.	0.	0.	26.533

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	76	0	0	0	60	0	66
normalized size	1	1.	0.95	0.	0.	0.	0.75	0.	0.82
time (sec)	N/A	0.073	0.042	0.029	0.	0.	155.627	0.	11.578

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	161	0	0	0	0	0	134
normalized size	1	1.	0.99	0.	0.	0.	0.	0.	0.82
time (sec)	N/A	0.443	1.009	0.041	0.	0.	0.	0.	51.397

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	A	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	157	0	0	0	0	4	209
normalized size	1	1.	0.72	0.	0.	0.	0.	0.02	0.96
time (sec)	N/A	0.505	1.052	0.041	0.	0.	0.	0.654	86.44

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	157	0	0	0	0	0	280
normalized size	1	1.	0.73	0.	0.	0.	0.	0.	1.31
time (sec)	N/A	0.506	1.042	0.043	0.	0.	0.	0.	115.693

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	208	0	0	0	972	0	121
normalized size	1	1.	1.41	0.	0.	0.	6.57	0.	0.82
time (sec)	N/A	0.226	0.245	0.127	0.	0.	19.067	0.	38.172

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	208	0	0	0	972	0	119
normalized size	1	1.	1.41	0.	0.	0.	6.61	0.	0.81
time (sec)	N/A	0.214	0.21	0.073	0.	0.	16.924	0.	38.649

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	161	0	0	0	382	0	97
normalized size	1	1.	1.34	0.	0.	0.	3.18	0.	0.81
time (sec)	N/A	0.171	0.15	0.066	0.	0.	12.05	0.	33.398

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	161	0	0	0	382	0	95
normalized size	1	1.	1.35	0.	0.	0.	3.21	0.	0.8
time (sec)	N/A	0.17	0.12	0.057	0.	0.	10.801	0.	34.88

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	121	0	0	0	85	0	71
normalized size	1	1.	1.36	0.	0.	0.	0.96	0.	0.8
time (sec)	N/A	0.094	0.108	0.052	0.	0.	6.985	0.	17.814

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	116	0	0	0	82	0	66
normalized size	1	1.	1.4	0.	0.	0.	0.99	0.	0.8
time (sec)	N/A	0.057	0.072	0.046	0.	0.	6.315	0.	25.107

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	104	0	0	0	78	0	82
normalized size	1	1.	1.	0.	0.	0.	0.75	0.	0.79
time (sec)	N/A	0.126	0.076	0.038	0.	0.	9.956	0.	30.47

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	106	0	0	0	82	0	88
normalized size	1	1.	0.98	0.	0.	0.	0.76	0.	0.81
time (sec)	N/A	0.139	0.07	0.046	0.	0.	10.062	0.	22.181

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	111	0	0	0	85	0	90
normalized size	1	1.	1.01	0.	0.	0.	0.77	0.	0.82
time (sec)	N/A	0.146	0.103	0.054	0.	0.	12.138	0.	23.611

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	268	0	0	0	2924	0	150
normalized size	1	1.	1.51	0.	0.	0.	16.43	0.	0.84
time (sec)	N/A	0.316	0.297	0.093	0.	0.	36.13	0.	82.62

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	252	0	0	0	1015	0	172
normalized size	1	1.	1.36	0.	0.	0.	5.49	0.	0.93
time (sec)	N/A	0.329	0.394	0.086	0.	0.	22.542	0.	64.092

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	211	0	0	0	1328	0	124
normalized size	1	1.	1.42	0.	0.	0.	8.91	0.	0.83
time (sec)	N/A	0.266	0.239	0.073	0.	0.	21.345	0.	64.773

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	202	0	0	0	425	0	146
normalized size	1	1.	1.3	0.	0.	0.	2.74	0.	0.94
time (sec)	N/A	0.275	0.246	0.066	0.	0.	14.377	0.	50.103

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	163	0	0	0	440	0	102
normalized size	1	1.	1.38	0.	0.	0.	3.73	0.	0.86
time (sec)	N/A	0.182	0.157	0.058	0.	0.	11.705	0.	34.077

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	150	0	0	0	124	0	68
normalized size	1	1.	2.11	0.	0.	0.	1.75	0.	0.96
time (sec)	N/A	0.078	0.183	0.053	0.	0.	8.68	0.	26.708

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	147	0	0	0	136	0	102
normalized size	1	1.	1.15	0.	0.	0.	1.06	0.	0.8
time (sec)	N/A	0.186	0.419	0.041	0.	0.	10.287	0.	37.863

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	148	0	0	0	116	0	133
normalized size	1	1.	1.16	0.	0.	0.	0.91	0.	1.04
time (sec)	N/A	0.218	0.206	0.046	0.	0.	12.699	0.	44.874

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	152	0	0	0	139	0	136
normalized size	1	1.	1.09	0.	0.	0.	1.	0.	0.98
time (sec)	N/A	0.249	0.233	0.057	0.	0.	15.252	0.	35.023

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	320	0	0	0	2966	0	207
normalized size	1	1.	1.44	0.	0.	0.	13.36	0.	0.93
time (sec)	N/A	0.411	0.473	0.083	0.	0.	44.325	0.	101.881

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	323	0	0	0	2966	0	204
normalized size	1	1.	1.48	0.	0.	0.	13.61	0.	0.94
time (sec)	N/A	0.4	0.447	0.092	0.	0.	39.709	0.	94.658

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	262	0	0	0	1370	0	182
normalized size	1	1.	1.36	0.	0.	0.	7.1	0.	0.94
time (sec)	N/A	0.367	0.36	0.084	0.	0.	26.954	0.	82.278

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	269	0	0	0	1370	0	178
normalized size	1	1.	1.42	0.	0.	0.	7.25	0.	0.94
time (sec)	N/A	0.362	0.344	0.076	0.	0.	23.915	0.	77.581

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	207	0	0	0	479	0	104
normalized size	1	1.	1.78	0.	0.	0.	4.13	0.	0.9
time (sec)	N/A	0.17	0.253	0.079	0.	0.	15.443	0.	33.336

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	271	0	0	0	476	0	68
normalized size	1	1.	3.71	0.	0.	0.	6.52	0.	0.93
time (sec)	N/A	0.073	0.23	0.059	0.	0.	13.511	0.	26.102

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	194	0	0	0	178	0	160
normalized size	1	1.	1.13	0.	0.	0.	1.04	0.	0.94
time (sec)	N/A	0.257	0.733	0.041	0.	0.	13.926	0.	51.846



Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	264	0	0	0	177	0	158
normalized size	1	1.	1.66	0.	0.	0.	1.11	0.	0.99
time (sec)	N/A	0.323	0.363	0.047	0.	0.	14.068	0.	51.883

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	201	0	0	0	177	0	182
normalized size	1	1.	1.21	0.	0.	0.	1.07	0.	1.1
time (sec)	N/A	0.367	0.29	0.058	0.	0.	18.313	0.	58.705

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	225	0	0	0	4442	0	116
normalized size	1	1.	1.52	0.	0.	0.	30.01	0.	0.78
time (sec)	N/A	0.296	0.738	0.085	0.	0.	57.215	0.	56.836

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	245	0	0	0	5090	0	97
normalized size	1	1.	2.02	0.	0.	0.	42.07	0.	0.8
time (sec)	N/A	0.241	0.464	0.074	0.	0.	36.686	0.	44.932

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	198	0	0	0	4124	0	92
normalized size	1	1.	1.66	0.	0.	0.	34.66	0.	0.77
time (sec)	N/A	0.236	0.427	0.065	0.	0.	23.279	0.	44.708

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	147	0	0	0	427	0	73
normalized size	1	1.	1.63	0.	0.	0.	4.74	0.	0.81
time (sec)	N/A	0.166	0.145	0.059	0.	0.	15.096	0.	29.389

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	75	0	0	0	321	0	41
normalized size	1	1.	1.03	0.	0.	0.	4.4	0.	0.56
time (sec)	N/A	0.081	0.046	0.063	0.	0.	13.389	0.	24.409

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	151	0	0	0	355	0	82
normalized size	1	1.	1.45	0.	0.	0.	3.41	0.	0.79
time (sec)	N/A	0.186	0.146	0.065	0.	0.	14.841	0.	41.543

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	167	0	0	0	450	0	82
normalized size	1	1.	1.58	0.	0.	0.	4.25	0.	0.77
time (sec)	N/A	0.212	0.256	0.077	0.	0.	18.218	0.	33.471

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	219	0	0	0	498	0	87
normalized size	1	1.	2.03	0.	0.	0.	4.61	0.	0.81
time (sec)	N/A	0.218	1.234	0.091	0.	0.	22.469	0.	35.484

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	140	0	0	0	0	0	148
normalized size	1	1.	0.78	0.	0.	0.	0.	0.	0.83
time (sec)	N/A	0.408	0.455	0.107	0.	0.	0.	0.	111.53

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	140	0	0	0	0	0	172
normalized size	1	1.	0.76	0.	0.	0.	0.	0.	0.93
time (sec)	N/A	0.431	0.425	0.116	0.	0.	0.	0.	87.252

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	332	0	0	0	0	0	122
normalized size	1	1.	2.21	0.	0.	0.	0.	0.	0.81
time (sec)	N/A	0.369	0.436	0.107	0.	0.	0.	0.	88.395

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	177	0	0	0	0	0	146
normalized size	1	1.	1.13	0.	0.	0.	0.	0.	0.94
time (sec)	N/A	0.383	0.196	0.079	0.	0.	0.	0.	73.442

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	102	0	0	0	0	0	75
normalized size	1	1.	0.89	0.	0.	0.	0.	0.	0.65
time (sec)	N/A	0.154	0.088	0.088	0.	0.	0.	0.	48.432

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	75	0	0	0	0	0	66
normalized size	1	1.	1.03	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.082	0.053	0.075	0.	0.	0.	0.	26.723

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	201	0	0	0	0	0	102
normalized size	1	1.	1.57	0.	0.	0.	0.	0.	0.8
time (sec)	N/A	0.275	0.223	0.095	0.	0.	0.	0.	60.831

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	195	0	0	0	0	0	134
normalized size	1	1.	1.42	0.	0.	0.	0.	0.	0.98
time (sec)	N/A	0.34	0.476	0.08	0.	0.	0.	0.	68.737

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	283	0	0	0	0	0	133
normalized size	1	1.	1.98	0.	0.	0.	0.	0.	0.93
time (sec)	N/A	0.356	1.185	0.089	0.	0.	0.	0.	58.894

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	334	0	0	0	0	0	146
normalized size	1	1.	2.3	0.	0.	0.	0.	0.	1.01
time (sec)	N/A	0.38	0.603	0.109	0.	0.	0.	0.	62.504

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	389	0	0	0	0	0	138
normalized size	1	1.	2.68	0.	0.	0.	0.	0.	0.95
time (sec)	N/A	0.366	1.004	0.125	0.	0.	0.	0.	64.33

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	140	0	0	0	0	0	202
normalized size	1	1.	0.64	0.	0.	0.	0.	0.	0.92
time (sec)	N/A	0.532	0.436	0.153	0.	0.	0.	0.	132.337

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	140	0	0	0	0	0	182
normalized size	1	1.	0.72	0.	0.	0.	0.	0.	0.94
time (sec)	N/A	0.492	0.445	0.118	0.	0.	0.	0.	109.922

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	130	0	0	0	0	0	202
normalized size	1	1.	0.83	0.	0.	0.	0.	0.	1.29
time (sec)	N/A	0.361	0.128	0.117	0.	0.	0.	0.	125.777

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	102	0	0	0	0	0	114
normalized size	1	1.	0.86	0.	0.	0.	0.	0.	0.97
time (sec)	N/A	0.151	0.092	0.111	0.	0.	0.	0.	60.684

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	75	0	0	0	0	0	66
normalized size	1	1.	1.03	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.075	0.053	0.108	0.	0.	0.	0.	24.303

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	196	0	0	0	0	0	163
normalized size	1	1.	1.12	0.	0.	0.	0.	0.	0.93
time (sec)	N/A	0.359	0.474	0.072	0.	0.	0.	0.	75.934

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	198	0	0	0	0	0	160
normalized size	1	1.	1.19	0.	0.	0.	0.	0.	0.96
time (sec)	N/A	0.431	0.482	0.105	0.	0.	0.	0.	74.235

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	341	0	0	0	0	0	185
normalized size	1	1.	1.97	0.	0.	0.	0.	0.	1.07
time (sec)	N/A	0.486	1.588	0.128	0.	0.	0.	0.	82.997

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	393	0	0	0	0	0	196
normalized size	1	1.	2.2	0.	0.	0.	0.	0.	1.09
time (sec)	N/A	0.517	0.75	0.137	0.	0.	0.	0.	72.26

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	446	0	0	0	0	0	192
normalized size	1	1.	2.56	0.	0.	0.	0.	0.	1.1
time (sec)	N/A	0.519	1.013	0.109	0.	0.	0.	0.	74.204

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	140	0	0	0	0	0	257
normalized size	1	1.	0.53	0.	0.	0.	0.	0.	0.97
time (sec)	N/A	0.743	0.47	0.184	0.	0.	0.	0.	143.648

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	140	0	0	0	0	0	206
normalized size	1	1.	0.66	0.	0.	0.	0.	0.	0.98
time (sec)	N/A	0.678	0.458	0.163	0.	0.	0.	0.	141.427

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	130	0	0	0	0	0	202
normalized size	1	1.	0.8	0.	0.	0.	0.	0.	1.24
time (sec)	N/A	0.397	0.129	0.151	0.	0.	0.	0.	102.308

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	102	0	0	0	0	0	112
normalized size	1	1.	0.86	0.	0.	0.	0.	0.	0.95
time (sec)	N/A	0.157	0.099	0.146	0.	0.	0.	0.	27.968

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	75	0	0	0	0	0	66
normalized size	1	1.	1.03	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.076	0.052	0.14	0.	0.	0.	0.	11.831

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	199	0	0	0	0	0	187
normalized size	1	1.	0.98	0.	0.	0.	0.	0.	0.92
time (sec)	N/A	0.463	0.499	0.071	0.	0.	0.	0.	48.252

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	195	0	0	0	0	0	216
normalized size	1	1.	0.94	0.	0.	0.	0.	0.	1.04
time (sec)	N/A	0.528	0.534	0.108	0.	0.	0.	0.	73.686

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	399	0	0	0	0	0	204
normalized size	1	1.	1.89	0.	0.	0.	0.	0.	0.97
time (sec)	N/A	0.634	2.031	0.127	0.	0.	0.	0.	88.193

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	452	0	0	0	0	0	240
normalized size	1	1.	2.15	0.	0.	0.	0.	0.	1.14
time (sec)	N/A	0.665	1.013	0.092	0.	0.	0.	0.	96.312

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	505	0	0	0	0	0	238
normalized size	1	1.	2.34	0.	0.	0.	0.	0.	1.1
time (sec)	N/A	0.696	1.422	0.104	0.	0.	0.	0.	87.492

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	198	0	0	0	0	0	262
normalized size	1	1.	0.75	0.	0.	0.	0.	0.	0.99
time (sec)	N/A	0.655	0.24	0.19	0.	0.	0.	0.	68.317

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	173	0	0	0	192	0	194
normalized size	1	1.	0.84	0.	0.	0.	0.93	0.	0.94
time (sec)	N/A	0.35	0.168	0.082	0.	0.	100.798	0.	50.277

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	116	0	0	0	122	0	122
normalized size	1	1.	0.76	0.	0.	0.	0.8	0.	0.8
time (sec)	N/A	0.167	0.109	0.083	0.	0.	52.719	0.	25.86

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	73	0	0	0	61	0	60
normalized size	1	1.	0.97	0.	0.	0.	0.81	0.	0.8
time (sec)	N/A	0.053	0.041	0.099	0.	0.	15.295	0.	11.417

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	168	0	0	0	337	0	126
normalized size	1	1.	1.03	0.	0.	0.	2.07	0.	0.77
time (sec)	N/A	0.375	0.474	0.099	0.	0.	44.235	0.	43.5

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	166	0	0	0	0	0	199
normalized size	1	1.	0.78	0.	0.	0.	0.	0.	0.93
time (sec)	N/A	0.483	0.48	0.089	0.	0.	0.	0.	83.472

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	166	0	0	0	0	0	267
normalized size	1	1.	0.6	0.	0.	0.	0.	0.	0.97
time (sec)	N/A	0.81	0.529	0.088	0.	0.	0.	0.	108.302

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	A	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	145	0	0	0	308	0	63
normalized size	1	1.	1.63	0.	0.	0.	3.46	0.	0.71
time (sec)	N/A	0.212	0.277	0.405	0.	0.	42.96	0.	21.66

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	175	0	0	0	0	0	78
normalized size	1	1.	1.82	0.	0.	0.	0.	0.	0.81
time (sec)	N/A	0.218	0.577	0.224	0.	0.	0.	0.	36.067

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	60	240	0	711	68	0	323
normalized size	1	1.	0.28	1.12	0.	3.32	0.32	0.	1.51
time (sec)	N/A	0.527	0.03	0.121	0.	0.304	11.302	0.	51.191

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	250	560	0	1	0	340	287
normalized size	1	1.	0.98	2.2	0.	0.	0.	1.33	1.13
time (sec)	N/A	1.102	0.414	0.034	0.	8.804	0.	0.299	65.151

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	219	515	0	1	0	271	240
normalized size	1	1.	1.04	2.44	0.	0.	0.	1.28	1.14
time (sec)	N/A	0.739	0.483	0.014	0.	6.771	0.	0.305	54.558

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	179	448	0	1	0	212	167
normalized size	1	1.	1.17	2.93	0.	0.01	0.	1.39	1.09
time (sec)	N/A	0.443	0.185	0.014	0.	0.571	0.	0.278	44.1

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	158	423	0	1	0	182	114
normalized size	1	1.	1.24	3.33	0.	0.01	0.	1.43	0.9
time (sec)	N/A	0.285	0.173	0.009	0.	0.535	0.	0.276	34.475

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	124	381	0	1	0	147	90
normalized size	1	1.	1.2	3.7	0.	0.01	0.	1.43	0.87
time (sec)	N/A	0.187	0.065	0.006	0.	0.366	0.	0.275	26.059



Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	150	420	0	1	0	0	99
normalized size	1	1.	1.29	3.62	0.	0.01	0.	0.	0.85
time (sec)	N/A	0.285	0.085	0.015	0.	1.03	0.	0.	28.421

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	139	486	0	1	0	196	90
normalized size	1	1.	1.32	4.63	0.	0.01	0.	1.87	0.86
time (sec)	N/A	0.411	0.079	0.017	0.	0.328	0.	0.275	49.429

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	170	567	0	1	0	311	141
normalized size	1	1.	1.06	3.54	0.	0.01	0.	1.94	0.88
time (sec)	N/A	0.505	0.264	0.017	0.	0.342	0.	0.28	57.765

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	191	216	600	0	1	0	417	170
normalized size	1	1.	1.13	3.14	0.	0.01	0.	2.18	0.89
time (sec)	N/A	0.549	0.283	0.017	0.	0.342	0.	0.298	62.951

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	248	703	0	1	0	805	243
normalized size	1	1.	0.91	2.57	0.	0.	0.	2.94	0.89
time (sec)	N/A	0.702	0.645	0.02	0.	0.419	0.	0.288	73.7

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	178	260	0	1	0	220	189
normalized size	1	1.	0.91	1.33	0.	0.01	0.	1.13	0.97
time (sec)	N/A	0.833	0.798	0.021	0.	2.921	0.	0.278	38.403

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	155	217	0	1	0	174	146
normalized size	1	1.	1.02	1.43	0.	0.01	0.	1.14	0.96
time (sec)	N/A	0.503	0.403	0.013	0.	2.992	0.	0.282	28.27

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	133	172	0	1	0	142	95
normalized size	1	1.	1.22	1.58	0.	0.01	0.	1.3	0.87
time (sec)	N/A	0.269	0.158	0.013	0.	0.402	0.	0.276	21.915

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	111	151	0	1	0	119	75
normalized size	1	1.	1.29	1.76	0.	0.01	0.	1.38	0.87
time (sec)	N/A	0.126	0.101	0.009	0.	0.35	0.	0.277	15.974

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	62	127	0	1	0	80	48
normalized size	1	1.	1.15	2.35	0.	0.02	0.	1.48	0.89
time (sec)	N/A	0.045	0.038	0.007	0.	0.299	0.	0.271	8.779

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	118	158	0	1	0	0	73
normalized size	1	1.	1.37	1.84	0.	0.01	0.	0.	0.85
time (sec)	N/A	0.216	0.128	0.014	0.	0.314	0.	0.	21.133

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	144	180	0	1	0	192	95
normalized size	1	1.	1.3	1.62	0.	0.01	0.	1.73	0.86
time (sec)	N/A	0.26	0.171	0.017	0.	0.337	0.	0.28	25.392

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	177	236	0	1	0	323	146
normalized size	1	1.	1.05	1.4	0.	0.01	0.	1.92	0.87
time (sec)	N/A	0.356	0.405	0.021	0.	0.358	0.	0.286	32.529

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	173	396	0	1	0	404	209
normalized size	1	1.	1.18	2.71	0.	0.01	0.	2.77	1.43
time (sec)	N/A	0.57	0.504	0.024	0.	5.067	0.	0.282	54.972

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	157	354	0	1	0	296	167
normalized size	1	1.	1.28	2.88	0.	0.01	0.	2.41	1.36
time (sec)	N/A	0.329	0.441	0.012	0.	4.938	0.	0.281	44.645

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	121	311	0	1	0	235	121
normalized size	1	1.	1.27	3.27	0.	0.01	0.	2.47	1.27
time (sec)	N/A	0.18	0.249	0.014	0.	0.336	0.	0.272	36.388

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	113	283	0	1	0	219	75
normalized size	1	1.	1.28	3.22	0.	0.01	0.	2.49	0.85
time (sec)	N/A	0.142	0.209	0.018	0.	0.341	0.	0.275	23.86

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	119	260	0	1	0	232	80
normalized size	1	1.	1.27	2.77	0.	0.01	0.	2.47	0.85
time (sec)	N/A	0.106	0.189	0.01	0.	0.335	0.	0.3	19.674

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	167	318	0	1	0	0	124
normalized size	1	1.	1.14	2.16	0.	0.01	0.	0.	0.84
time (sec)	N/A	0.325	0.461	0.016	0.	0.543	0.	0.	39.331

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	188	363	0	1	0	0	172
normalized size	1	1.	0.97	1.87	0.	0.01	0.	0.	0.89
time (sec)	N/A	0.401	0.59	0.017	0.	0.535	0.	0.	46.805

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	226	439	0	1	0	0	250
normalized size	1	1.	0.82	1.6	0.	0.	0.	0.	0.91
time (sec)	N/A	0.54	0.874	0.019	0.	0.887	0.	0.	57.685

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	230	474	0	1	0	0	282
normalized size	1	1.	0.94	1.94	0.	0.	0.	0.	1.16
time (sec)	N/A	1.486	0.774	0.025	0.	48.969	0.	0.	56.738

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	208	435	0	1	0	0	243
normalized size	1	1.	1.02	2.13	0.	0.	0.	0.	1.19
time (sec)	N/A	0.906	0.636	0.018	0.	66.849	0.	0.	44.761

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	184	386	0	1	0	0	189
normalized size	1	1.	1.15	2.41	0.	0.01	0.	0.	1.18
time (sec)	N/A	0.595	0.471	0.016	0.	6.605	0.	0.	38.546

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	172	368	0	1	0	0	167
normalized size	1	1.	1.26	2.69	0.	0.01	0.	0.	1.22
time (sec)	N/A	0.349	0.608	0.015	0.	7.647	0.	0.	33.805

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	114	340	0	1	0	0	76
normalized size	1	1.	1.27	3.78	0.	0.01	0.	0.	0.84
time (sec)	N/A	0.118	0.123	0.012	0.	0.322	0.	0.	14.339

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	115	210	0	1	0	439	78
normalized size	1	1.	1.26	2.31	0.	0.01	0.	4.82	0.86
time (sec)	N/A	0.089	0.113	0.007	0.	0.318	0.	1.44	13.804

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	178	364	0	1	0	0	158
normalized size	1	1.	0.99	2.03	0.	0.01	0.	0.	0.88
time (sec)	N/A	0.345	0.4	0.016	0.	0.622	0.	0.	33.46

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	197	395	0	1	0	0	187
normalized size	1	1.	0.93	1.86	0.	0.	0.	0.	0.88
time (sec)	N/A	0.412	0.563	0.016	0.	0.59	0.	0.	41.116

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	229	452	0	1	0	0	243
normalized size	1	1.	0.85	1.69	0.	0.	0.	0.	0.91
time (sec)	N/A	0.522	0.779	0.019	0.	1.105	0.	0.	49.438

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	158	328	284	497	4078	926	122
normalized size	1	1.	1.17	2.43	2.1	3.68	30.21	6.86	0.9
time (sec)	N/A	0.176	0.135	0.01	0.71	0.281	12.81	0.297	31.664

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	109	195	197	338	2241	610	90
normalized size	1	1.	1.07	1.91	1.93	3.31	21.97	5.98	0.88
time (sec)	N/A	0.118	0.114	0.008	0.713	0.279	7.682	0.28	23.191

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	65	100	0	200	978	355	61
normalized size	1	1.	0.93	1.43	0.	2.86	13.97	5.07	0.87
time (sec)	N/A	0.068	0.062	0.006	0.	0.279	4.313	0.265	16.149

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	98	0	0	0	347	0	61
normalized size	1	1.	1.27	0.	0.	0.	4.51	0.	0.79
time (sec)	N/A	0.121	0.292	0.058	0.	0.	9.021	0.	14.27

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	378	1000	603	1386	14300	1	218
normalized size	1	1.	1.63	4.31	2.6	5.97	61.64	0.	0.94
time (sec)	N/A	0.301	0.384	0.017	0.724	0.285	39.925	0.275	61.937

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	279	677	452	1022	9083	1	170
normalized size	1	1.	1.51	3.66	2.44	5.52	49.1	0.01	0.92
time (sec)	N/A	0.231	0.288	0.015	0.723	0.283	25.674	0.294	47.599

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	184	420	0	701	5044	1260	128
normalized size	1	1.	1.31	3.	0.	5.01	36.03	9.	0.91
time (sec)	N/A	0.15	0.16	0.013	0.	0.282	15.762	0.301	35.876

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	259	0	0	0	1681	0	129
normalized size	1	1.	1.75	0.	0.	0.	11.36	0.	0.87
time (sec)	N/A	0.179	0.539	0.051	0.	0.	15.558	0.	32.373

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	746	2232	1073	2923	0	1	328
normalized size	1	1.	2.17	6.51	3.13	8.52	0.	0.	0.96
time (sec)	N/A	0.431	1.199	0.024	0.726	0.294	0.	0.308	97.512

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	578	1639	844	2261	0	1	265
normalized size	1	1.	2.05	5.81	2.99	8.02	0.	0.	0.94
time (sec)	N/A	0.369	0.829	0.019	0.715	0.294	0.	0.28	77.004

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	413	1140	0	1679	0	1	207
normalized size	1	1.	1.85	5.11	0.	7.53	0.	0.	0.93
time (sec)	N/A	0.264	0.409	0.016	0.	0.286	0.	0.269	60.525

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	546	0	0	0	5702	0	226
normalized size	1	1.	2.22	0.	0.	0.	23.18	0.	0.92
time (sec)	N/A	0.287	0.906	0.044	0.	0.	33.966	0.	61.343

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	354	0	0	0	0	0	204
normalized size	1	1.	1.42	0.	0.	0.	0.	0.	0.82
time (sec)	N/A	0.696	0.916	0.091	0.	0.	0.	0.	89.963

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	275	0	0	0	0	0	167
normalized size	1	1.	1.32	0.	0.	0.	0.	0.	0.8
time (sec)	N/A	0.409	1.114	0.079	0.	0.	0.	0.	59.841

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	233	0	0	0	0	0	150
normalized size	1	1.	1.2	0.	0.	0.	0.	0.	0.77
time (sec)	N/A	0.47	0.443	0.073	0.	0.	0.	0.	77.38

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	200	0	0	0	0	0	129
normalized size	1	1.	1.23	0.	0.	0.	0.	0.	0.79
time (sec)	N/A	0.198	0.149	0.059	0.	0.	0.	0.	38.364

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	210	0	0	0	0	0	131
normalized size	1	1.	1.26	0.	0.	0.	0.	0.	0.78
time (sec)	N/A	0.237	0.123	0.06	0.	0.	0.	0.	42.593

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	239	0	0	0	0	0	158
normalized size	1	1.	1.15	0.	0.	0.	0.	0.	0.76
time (sec)	N/A	0.379	0.482	0.053	0.	0.	0.	0.	55.566

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	263	0	0	0	0	0	165
normalized size	1	1.	1.27	0.	0.	0.	0.	0.	0.8
time (sec)	N/A	0.468	0.585	0.081	0.	0.	0.	0.	72.926

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	332	332	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.797	0.167	0.091	0.	0.	0.	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	0	0	0	0	0	0	354
normalized size	1	1.	0.	0.	0.	0.	0.	0.	1.19
time (sec)	N/A	0.77	0.141	0.087	0.	0.	0.	0.	165.105

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	0	0	0	0	0	0	382
normalized size	1	1.	0.	0.	0.	0.	0.	0.	1.25
time (sec)	N/A	1.059	0.128	0.082	0.	0.	0.	0.	161.51

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	0	0	0	0	0	0	224
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.8
time (sec)	N/A	0.664	0.097	0.08	0.	0.	0.	0.	104.559

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	0	0	0	0	0	0	246
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.81
time (sec)	N/A	0.838	0.049	0.078	0.	0.	0.	0.	91.36

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	489	489	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.25	0.098	0.068	0.	0.	0.	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	513	513	0	0	0	0	0	0	422
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.82
time (sec)	N/A	1.503	0.121	0.106	0.	0.	0.	0.	175.627



Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	377	128	0	0	0	0	0	289
normalized size	1	0.94	0.32	0.	0.	0.	0.	0.	0.72
time (sec)	N/A	1.452	0.231	0.111	0.	0.	0.	0.	88.001

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	150	84	0	0	0	0	0	156
normalized size	1	0.91	0.51	0.	0.	0.	0.	0.	0.95
time (sec)	N/A	0.303	0.099	0.072	0.	0.	0.	0.	41.398

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	0	0	0	0	0	0	112
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.76
time (sec)	N/A	0.412	0.085	0.078	0.	0.	0.	0.	61.068

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	0	0	0	0	0	0	233
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.79
time (sec)	N/A	0.962	0.109	0.084	0.	0.	0.	0.	152.239

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	183	0	0	0	1012	0	104
normalized size	1	1.	1.46	0.	0.	0.	8.1	0.	0.83
time (sec)	N/A	0.2	0.241	0.077	0.	0.	117.074	0.	28.917

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	183	0	0	0	1012	0	104
normalized size	1	1.	1.46	0.	0.	0.	8.1	0.	0.83
time (sec)	N/A	0.196	0.204	0.079	0.	0.	83.123	0.	28.769

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	141	0	0	0	394	0	82
normalized size	1	1.	1.41	0.	0.	0.	3.94	0.	0.82
time (sec)	N/A	0.152	0.149	0.06	0.	0.	62.189	0.	20.886

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	141	0	0	0	394	0	82
normalized size	1	1.	1.41	0.	0.	0.	3.94	0.	0.82
time (sec)	N/A	0.148	0.133	0.053	0.	0.	42.36	0.	21.473

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	102	0	0	0	65	0	58
normalized size	1	1.	1.36	0.	0.	0.	0.87	0.	0.77
time (sec)	N/A	0.086	0.089	0.047	0.	0.	32.282	0.	12.234

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	98	0	0	0	61	0	54
normalized size	1	1.	1.4	0.	0.	0.	0.87	0.	0.77
time (sec)	N/A	0.049	0.075	0.046	0.	0.	20.749	0.	8.764

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	91	0	0	0	65	0	68
normalized size	1	1.	1.03	0.	0.	0.	0.74	0.	0.77
time (sec)	N/A	0.106	0.074	0.038	0.	0.	31.237	0.	13.219

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	93	0	0	0	68	0	71
normalized size	1	1.	1.02	0.	0.	0.	0.75	0.	0.78
time (sec)	N/A	0.121	0.068	0.044	0.	0.	42.522	0.	15.142

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	98	0	0	0	71	0	73
normalized size	1	1.	1.07	0.	0.	0.	0.77	0.	0.79
time (sec)	N/A	0.125	0.096	0.056	0.	0.	62.284	0.	15.593

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	303	0	0	0	3046	0	212
normalized size	1	1.	1.61	0.	0.	0.	16.2	0.	1.13
time (sec)	N/A	0.353	0.478	0.096	0.	0.	170.363	0.	62.96

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	169	229	0	0	0	1047	0	151
normalized size	1	0.95	1.29	0.	0.	0.	5.92	0.	0.85
time (sec)	N/A	0.324	0.291	0.085	0.	0.	151.911	0.	45.133

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	241	0	0	0	1386	0	158
normalized size	1	1.	1.62	0.	0.	0.	9.3	0.	1.06
time (sec)	N/A	0.288	0.301	0.075	0.	0.	93.305	0.	45.412

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	144	182	0	0	0	430	0	126
normalized size	1	0.95	1.2	0.	0.	0.	2.83	0.	0.83
time (sec)	N/A	0.279	0.261	0.067	0.	0.	81.628	0.	34.362

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	184	0	0	0	439	0	126
normalized size	1	1.	1.63	0.	0.	0.	3.88	0.	1.12
time (sec)	N/A	0.19	0.274	0.06	0.	0.	45.521	0.	27.323

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	125	133	0	0	0	97	0	107
normalized size	1	0.94	1.	0.	0.	0.	0.73	0.	0.8
time (sec)	N/A	0.172	0.154	0.055	0.	0.	40.854	0.	28.055

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	130	0	0	0	109	0	94
normalized size	1	1.	1.1	0.	0.	0.	0.92	0.	0.8
time (sec)	N/A	0.181	0.563	0.059	0.	0.	37.749	0.	19.987

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	131	0	0	0	95	0	110
normalized size	1	1.	1.03	0.	0.	0.	0.75	0.	0.87
time (sec)	N/A	0.229	0.187	0.058	0.	0.	57.633	0.	24.922

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	138	0	0	0	119	0	112
normalized size	1	1.	1.09	0.	0.	0.	0.94	0.	0.88
time (sec)	N/A	0.239	0.275	0.142	0.	0.	74.213	0.	25.648

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	241	356	0	0	0	0	0	267
normalized size	1	0.98	1.44	0.	0.	0.	0.	0.	1.08
time (sec)	N/A	0.503	0.702	0.096	0.	0.	0.	0.	78.585

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	241	357	0	0	0	0	0	267
normalized size	1	0.97	1.43	0.	0.	0.	0.	0.	1.07
time (sec)	N/A	0.502	0.698	0.101	0.	0.	0.	0.	75.331

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	201	289	0	0	0	1421	0	212
normalized size	1	0.97	1.4	0.	0.	0.	6.86	0.	1.02
time (sec)	N/A	0.419	0.487	0.095	0.	0.	162.661	0.	58.335

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	202	291	0	0	0	1421	0	209
normalized size	1	0.96	1.39	0.	0.	0.	6.77	0.	1.
time (sec)	N/A	0.421	0.474	0.082	0.	0.	114.196	0.	55.472

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F(-2)	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	159	228	0	0	0	471	0	212
normalized size	1	0.95	1.37	0.	0.	0.	2.82	0.	1.27
time (sec)	N/A	0.325	0.368	0.074	0.	0.	85.644	0.	64.947

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	169	223	0	0	0	468	0	160
normalized size	1	0.96	1.27	0.	0.	0.	2.66	0.	0.91
time (sec)	N/A	0.331	0.353	0.064	0.	0.	55.654	0.	45.189

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	165	175	0	0	0	144	0	141
normalized size	1	0.96	1.02	0.	0.	0.	0.84	0.	0.82
time (sec)	N/A	0.271	0.838	0.055	0.	0.	63.791	0.	29.722

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	174	0	0	0	143	0	139
normalized size	1	1.	1.09	0.	0.	0.	0.9	0.	0.87
time (sec)	N/A	0.326	0.956	0.087	0.	0.	69.643	0.	29.524

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	176	0	0	0	150	0	156
normalized size	1	1.	1.05	0.	0.	0.	0.89	0.	0.93
time (sec)	N/A	0.382	0.663	0.063	0.	0.	94.446	0.	33.104

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	0	0	0	0	0	0	277
normalized size	1	1.	0.	0.	0.	0.	0.	0.	1.39
time (sec)	N/A	0.496	0.97	0.082	0.	0.	0.	0.	62.353

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	260	0	0	0	0	0	230
normalized size	1	1.	1.6	0.	0.	0.	0.	0.	1.41
time (sec)	N/A	0.371	0.532	0.095	0.	0.	0.	0.	46.638

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	227	0	0	0	0	0	184
normalized size	1	1.	1.41	0.	0.	0.	0.	0.	1.14
time (sec)	N/A	0.366	0.397	0.076	0.	0.	0.	0.	37.051

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	172	0	0	0	0	0	160
normalized size	1	1.	0.99	0.	0.	0.	0.	0.	0.92
time (sec)	N/A	0.318	0.35	0.131	0.	0.	0.	0.	28.529

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	131	0	0	0	0	0	119
normalized size	1	1.	1.05	0.	0.	0.	0.	0.	0.95
time (sec)	N/A	0.225	0.103	0.057	0.	0.	0.	0.	17.595

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	170	0	0	0	0	0	153
normalized size	1	1.	0.97	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.346	0.378	0.06	0.	0.	0.	0.	32.932

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	214	0	0	0	0	0	197
normalized size	1	1.	1.2	0.	0.	0.	0.	0.	1.11
time (sec)	N/A	0.413	1.016	0.152	0.	0.	0.	0.	42.883

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	256	0	0	0	0	0	240
normalized size	1	1.	1.2	0.	0.	0.	0.	0.	1.13
time (sec)	N/A	0.583	0.52	0.092	0.	0.	0.	0.	51.658

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	392	392	0	0	0	0	0	0	381
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.97
time (sec)	N/A	1.544	1.059	0.1	0.	0.	0.	0.	75.046

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	343	0	0	0	0	0	338
normalized size	1	1.	1.07	0.	0.	0.	0.	0.	1.05
time (sec)	N/A	1.	1.477	0.088	0.	0.	0.	0.	64.721

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	281	277	300	0	0	0	0	0	309
normalized size	1	0.99	1.07	0.	0.	0.	0.	0.	1.1
time (sec)	N/A	0.656	0.542	0.086	0.	0.	0.	0.	59.098

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	223	0	0	0	0	0	230
normalized size	1	1.	0.82	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.509	0.23	0.077	0.	0.	0.	0.	48.739

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	244	191	141	0	0	0	0	0	144
normalized size	1	0.78	0.58	0.	0.	0.	0.	0.	0.59
time (sec)	N/A	0.412	0.131	0.069	0.	0.	0.	0.	18.23

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	303	0	0	0	0	0	299
normalized size	1	1.	0.82	0.	0.	0.	0.	0.	0.81
time (sec)	N/A	0.901	0.761	0.067	0.	0.	0.	0.	55.097

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	0	0	0	0	0	0	343
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.81
time (sec)	N/A	0.968	0.098	0.091	0.	0.	0.	0.	68.196

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	0	0	0	0	0	0	488
normalized size	1	1.	0.	0.	0.	0.	0.	0.	1.09
time (sec)	N/A	1.631	1.374	0.116	0.	0.	0.	0.	91.257

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	416	416	0	0	0	0	0	0	464
normalized size	1	1.	0.	0.	0.	0.	0.	0.	1.12
time (sec)	N/A	1.082	1.069	0.117	0.	0.	0.	0.	86.089

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	290	0	0	0	0	0	420
normalized size	1	1.	0.73	0.	0.	0.	0.	0.	1.06
time (sec)	N/A	1.1	0.392	0.106	0.	0.	0.	0.	73.939

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	229	0	0	0	0	0	292
normalized size	1	1.	0.68	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.833	0.291	0.1	0.	0.	0.	0.	87.011

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	142	0	0	0	0	0	144
normalized size	1	1.	0.44	0.	0.	0.	0.	0.	0.45
time (sec)	N/A	0.724	0.181	0.087	0.	0.	0.	0.	18.418

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	700	700	0	0	0	0	0	0	445
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.64
time (sec)	N/A	1.759	0.069	0.084	0.	0.	0.	0.	77.761

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	754	754	0	0	0	0	0	0	500
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.66
time (sec)	N/A	1.827	0.156	0.14	0.	0.	0.	0.	96.075

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	254	186	0	0	0	0	0	233
normalized size	1	0.92	0.67	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.851	0.253	0.086	0.	0.	0.	0.	65.769

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	194	162	0	0	0	0	0	167
normalized size	1	0.95	0.79	0.	0.	0.	0.	0.	0.81
time (sec)	N/A	0.401	0.168	0.079	0.	0.	0.	0.	35.394

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	106	0	0	0	0	0	105
normalized size	1	1.	0.79	0.	0.	0.	0.	0.	0.78
time (sec)	N/A	0.151	0.1	0.067	0.	0.	0.	0.	20.831



Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	64	0	0	0	54	0	51
normalized size	1	1.	0.97	0.	0.	0.	0.82	0.	0.77
time (sec)	N/A	0.048	0.036	0.08	0.	0.	107.458	0.	8.981

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.364	0.094	0.076	0.	0.	0.	0.	0.

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.655	0.11	0.081	0.	0.	0.	0.	0.

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.876	0.268	0.098	0.	0.	0.	0.	0.

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	276	946	0	1	0	0	348
normalized size	1	1.	0.8	2.74	0.	0.	0.	0.	1.01
time (sec)	N/A	1.352	0.498	0.031	0.	0.375	0.	0.	144.696

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	207	713	0	1	0	0	245
normalized size	1	1.	0.82	2.84	0.	0.	0.	0.	0.98
time (sec)	N/A	0.83	0.366	0.019	0.	0.333	0.	0.	85.915

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	159	516	0	1	0	0	189
normalized size	1	1.	0.77	2.49	0.	0.	0.	0.	0.91
time (sec)	N/A	0.485	0.382	0.013	0.	0.308	0.	0.	44.345

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	133	205	0	1	0	0	122
normalized size	1	1.	1.02	1.56	0.	0.01	0.	0.	0.93
time (sec)	N/A	0.165	0.25	0.008	0.	0.296	0.	0.	27.027

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	185	439	0	1	0	4	162
normalized size	1	1.	1.1	2.61	0.	0.01	0.	0.02	0.96
time (sec)	N/A	0.554	0.226	0.017	0.	0.499	0.	2.255	49.885

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	187	594	0	1	0	0	124
normalized size	1	1.	1.36	4.34	0.	0.01	0.	0.	0.91
time (sec)	N/A	0.516	0.302	0.018	0.	0.337	0.	0.	41.423

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	238	882	0	1	0	1	185
normalized size	1	1.	1.18	4.37	0.	0.	0.	0.	0.92
time (sec)	N/A	0.833	0.418	0.022	0.	0.403	0.	0.525	80.796

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	299	1165	0	1	0	1	270
normalized size	1	1.	1.05	4.07	0.	0.	0.	0.	0.94
time (sec)	N/A	1.183	0.574	0.024	0.	0.811	0.	1.178	147.703

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	378	1494	0	1	0	1	0
normalized size	1	1.	0.97	3.84	0.	0.	0.	0.	0.
time (sec)	N/A	1.643	0.826	0.033	0.	1.995	0.	3.42	0.

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	449	422	1883	0	1	0	0	0
normalized size	1	1.	0.94	4.19	0.	0.	0.	0.	0.
time (sec)	N/A	1.525	0.805	0.034	0.	0.39	0.	0.	0.

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	320	1560	0	1	0	0	354
normalized size	1	1.	0.91	4.43	0.	0.	0.	0.	1.01
time (sec)	N/A	0.959	0.589	0.022	0.	0.33	0.	0.	109.759

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	240	1279	0	1	0	0	275
normalized size	1	1.	0.81	4.34	0.	0.	0.	0.	0.93
time (sec)	N/A	0.656	0.462	0.016	0.	0.323	0.	0.	60.847

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	178	566	0	1	0	0	187
normalized size	1	1.	0.89	2.82	0.	0.	0.	0.	0.93
time (sec)	N/A	0.264	0.309	0.009	0.	0.303	0.	0.	39.137

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	285	1130	0	1	0	4	245
normalized size	1	1.	1.14	4.5	0.	0.	0.	0.02	0.98
time (sec)	N/A	0.86	0.681	0.02	0.	4.755	0.	0.711	104.539

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	356	1310	0	1	0	0	228
normalized size	1	1.	1.48	5.46	0.	0.	0.	0.	0.95
time (sec)	N/A	0.857	0.561	0.023	0.	1.704	0.	0.	83.914

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	310	1604	0	1	0	0	246
normalized size	1	1.	1.21	6.27	0.	0.	0.	0.	0.96
time (sec)	N/A	0.875	0.633	0.025	0.	2.001	0.	0.	105.661

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	251	1945	0	1	0	1	194
normalized size	1	1.	1.19	9.22	0.	0.	0.	0.	0.92
time (sec)	N/A	0.709	0.362	0.029	0.	1.261	0.	2.141	61.181

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	323	2427	0	1	0	0	274
normalized size	1	1.	1.09	8.23	0.	0.	0.	0.	0.93
time (sec)	N/A	1.074	0.469	0.036	0.	1.937	0.	0.	106.171

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	395	407	2888	0	0	0	1	0
normalized size	1	1.	1.03	7.31	0.	0.	0.	0.	0.
time (sec)	N/A	1.396	0.593	0.043	0.	0.	0.	17.961	0.

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	498	498	506	3387	0	0	0	1	0
normalized size	1	1.	1.02	6.8	0.	0.	0.	0.	0.
time (sec)	N/A	1.888	0.872	0.058	0.	0.	0.	50.919	0.

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	574	574	579	3178	0	1	0	0	0
normalized size	1	1.	1.01	5.54	0.	0.	0.	0.	0.
time (sec)	N/A	1.742	1.629	0.034	0.	0.426	0.	0.	0.

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	452	452	478	2731	0	1	0	0	454
normalized size	1	1.	1.06	6.04	0.	0.	0.	0.	1.
time (sec)	N/A	1.125	1.115	0.024	0.	0.386	0.	0.	139.745

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	386	2411	0	1	0	0	360
normalized size	1	1.	1.01	6.33	0.	0.	0.	0.	0.94
time (sec)	N/A	0.852	0.922	0.017	0.	0.356	0.	0.	86.287

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	309	1123	0	1	0	0	262
normalized size	1	1.	1.13	4.1	0.	0.	0.	0.	0.96
time (sec)	N/A	0.377	0.704	0.011	0.	0.321	0.	0.	59.811

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	397	2180	0	1	0	4	0
normalized size	1	1.	1.01	5.53	0.	0.	0.	0.01	0.
time (sec)	N/A	1.275	1.446	0.024	0.	48.265	0.	0.736	0.

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	376	2364	0	1	0	0	345
normalized size	1	1.	1.07	6.72	0.	0.	0.	0.	0.98
time (sec)	N/A	1.224	1.175	0.025	0.	18.817	0.	0.	144.324

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	361	2688	0	1	0	0	338
normalized size	1	1.	1.06	7.93	0.	0.	0.	0.	1.
time (sec)	N/A	1.15	0.97	0.029	0.	7.655	0.	0.	127.867

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	399	3144	0	1	0	0	0
normalized size	1	1.	1.08	8.47	0.	0.	0.	0.	0.
time (sec)	N/A	1.295	1.116	0.034	0.	9.086	0.	0.	0.

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	404	404	454	3646	0	1	0	0	0
normalized size	1	1.	1.12	9.02	0.	0.	0.	0.	0.
time (sec)	N/A	1.312	1.352	0.043	0.	18.422	0.	0.	0.

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	354	3991	0	0	0	1	272
normalized size	1	1.	1.22	13.81	0.	0.	0.	0.	0.94
time (sec)	N/A	0.914	0.787	0.057	0.	0.	0.	29.534	91.441

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	386	386	450	4735	0	0	0	1	362
normalized size	1	1.	1.17	12.27	0.	0.	0.	0.	0.94
time (sec)	N/A	1.316	0.898	0.081	0.	0.	0.	77.519	141.543

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	500	500	557	5353	0	0	0	1	0
normalized size	1	1.	1.11	10.71	0.	0.	0.	0.	0.
time (sec)	N/A	1.683	1.238	0.107	0.	0.	0.	176.329	0.

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-2)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	628	628	676	6030	0	0	0	0	0
normalized size	1	1.	1.08	9.6	0.	0.	0.	0.	0.
time (sec)	N/A	2.249	1.403	0.154	0.	0.	0.	0.	0.

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	298	211	391	0	1	0	0	461
normalized size	1	1.1	0.78	1.44	0.	0.	0.	0.	1.7
time (sec)	N/A	0.999	0.455	0.028	0.	0.604	0.	0.	118.094

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	178	241	0	1	0	0	262
normalized size	1	1.	0.91	1.24	0.	0.01	0.	0.	1.34
time (sec)	N/A	0.683	0.328	0.017	0.	0.433	0.	0.	67.371

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	145	131	0	1	0	0	129
normalized size	1	1.	1.04	0.94	0.	0.01	0.	0.	0.93
time (sec)	N/A	0.305	0.338	0.014	0.	0.384	0.	0.	23.267

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	42	51	0	80	0	0	46
normalized size	1	1.	0.81	0.98	0.	1.54	0.	0.	0.88
time (sec)	N/A	0.077	0.057	0.01	0.	0.307	0.	0.	18.411

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	200	136	0	1	0	0	133
normalized size	1	1.	1.4	0.95	0.	0.01	0.	0.	0.93
time (sec)	N/A	0.574	0.359	0.018	0.	0.372	0.	0.	55.199

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	276	253	0	1	0	0	207
normalized size	1	1.	1.21	1.1	0.	0.	0.	0.	0.9
time (sec)	N/A	0.895	0.741	0.02	0.	0.433	0.	0.	105.608

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	279	414	0	1	0	0	0
normalized size	1	1.	0.85	1.26	0.	0.	0.	0.	0.
time (sec)	N/A	1.428	0.638	0.024	0.	0.629	0.	0.	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	515	515	296	1680	0	1	0	0	0
normalized size	1	1.	0.57	3.26	0.	0.	0.	0.	0.
time (sec)	N/A	1.661	1.083	0.035	0.	3.472	0.	0.	0.

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	261	1266	0	1	0	0	0
normalized size	1	1.	0.6	2.89	0.	0.	0.	0.	0.
time (sec)	N/A	1.462	1.16	0.018	0.	1.622	0.	0.	0.

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	219	977	0	1	0	0	471
normalized size	1	1.	0.74	3.29	0.	0.	0.	0.	1.59
time (sec)	N/A	0.898	0.597	0.017	0.	0.894	0.	0.	127.145

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	99	145	0	416	0	0	122
normalized size	1	1.	0.79	1.15	0.	3.3	0.	0.	0.97
time (sec)	N/A	0.356	0.207	0.013	0.	0.58	0.	0.	38.583

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	100	149	0	424	0	0	128
normalized size	1	1.	0.72	1.08	0.	3.07	0.	0.	0.93
time (sec)	N/A	0.286	0.159	0.015	0.	0.586	0.	0.	21.611

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	95	138	0	413	0	0	116
normalized size	1	1.	0.79	1.14	0.	3.41	0.	0.	0.96
time (sec)	N/A	0.123	0.14	0.013	0.	0.585	0.	0.	23.403

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	254	682	0	1	0	0	260
normalized size	1	1.	0.94	2.52	0.	0.	0.	0.	0.96
time (sec)	N/A	1.003	0.81	0.02	0.	0.722	0.	0.	138.43

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	394	394	312	912	0	1	0	0	0
normalized size	1	1.	0.79	2.31	0.	0.	0.	0.	0.
time (sec)	N/A	1.502	1.502	0.022	0.	1.43	0.	0.	0.

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	522	522	362	1319	0	1	0	0	0
normalized size	1	1.	0.69	2.53	0.	0.	0.	0.	0.
time (sec)	N/A	2.039	1.939	0.025	0.	3.661	0.	0.	0.

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	664	664	429	1705	0	0	0	0	0
normalized size	1	1.	0.65	2.57	0.	0.	0.	0.	0.
time (sec)	N/A	2.799	2.536	0.03	0.	0.	0.	0.	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	210	366	0	1107	0	0	252
normalized size	1	1.	0.81	1.41	0.	4.27	0.	0.	0.97
time (sec)	N/A	0.67	1.273	0.019	0.	5.85	0.	0.	76.052

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	317	663	0	0	0	0	340
normalized size	1	1.	0.93	1.94	0.	0.	0.	0.	1.
time (sec)	N/A	0.766	2.433	0.025	0.	0.	0.	0.	94.537



Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	221	257	0	0	0	0	151
normalized size	1	1.	1.3	1.51	0.	0.	0.	0.	0.89
time (sec)	N/A	0.158	1.434	0.225	0.	0.	0.	0.	12.489

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	30	30	0	36	24
normalized size	1	1.	1.	0.78	1.3	1.3	0.	1.57	1.04
time (sec)	N/A	0.017	0.028	0.006	0.776	0.281	0.	0.269	7.239

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	347	361	0	0	0	0	262
normalized size	1	1.	1.18	1.23	0.	0.	0.	0.	0.89
time (sec)	N/A	0.2	0.843	0.033	0.	0.	0.	0.	17.191

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	169	252	0	0	0	0	128
normalized size	1	1.	1.17	1.75	0.	0.	0.	0.	0.89
time (sec)	N/A	0.082	0.84	0.031	0.	0.	0.	0.	7.173

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	197	43	0	81	0	0	58
normalized size	1	1.	2.98	0.65	0.	1.23	0.	0.	0.88
time (sec)	N/A	0.089	0.626	0.031	0.	0.287	0.	0.	9.789

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	349	363	0	0	0	0	250
normalized size	1	1.	1.22	1.26	0.	0.	0.	0.	0.87
time (sec)	N/A	0.21	0.684	0.036	0.	0.	0.	0.	18.385

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	185	259	0	0	0	0	126
normalized size	1	1.	1.27	1.77	0.	0.	0.	0.	0.86
time (sec)	N/A	0.125	0.675	0.036	0.	0.	0.	0.	10.034

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	235	262	0	0	0	0	180
normalized size	1	1.	1.17	1.3	0.	0.	0.	0.	0.9
time (sec)	N/A	0.176	1.425	0.035	0.	0.	0.	0.	14.824

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	36	36	0	100	26
normalized size	1	1.	1.	0.78	1.57	1.57	0.	4.35	1.13
time (sec)	N/A	0.017	0.036	0.006	0.779	0.282	0.	0.294	7.316

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	244	366	0	0	0	0	291
normalized size	1	1.	0.75	1.13	0.	0.	0.	0.	0.9
time (sec)	N/A	0.23	0.833	0.034	0.	0.	0.	0.	19.782

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	176	257	0	0	0	0	155
normalized size	1	1.	1.02	1.49	0.	0.	0.	0.	0.9
time (sec)	N/A	0.102	1.079	0.016	0.	0.	0.	0.	8.08

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	201	57	0	88	0	0	83
normalized size	1	1.	2.14	0.61	0.	0.94	0.	0.	0.88
time (sec)	N/A	0.105	0.467	0.012	0.	0.29	0.	0.	10.714

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	244	368	0	0	0	0	286
normalized size	1	1.	0.76	1.14	0.	0.	0.	0.	0.89
time (sec)	N/A	0.248	0.846	0.022	0.	0.	0.	0.	20.934

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	192	264	0	0	0	0	155
normalized size	1	1.	1.1	1.51	0.	0.	0.	0.	0.89
time (sec)	N/A	0.144	0.647	0.019	0.	0.	0.	0.	10.941

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	169	248	0	0	0	0	128
normalized size	1	1.	1.19	1.75	0.	0.	0.	0.	0.9
time (sec)	N/A	0.13	0.949	0.05	0.	0.	0.	0.	10.437

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	30	23	0	24	19
normalized size	1	1.	1.	0.78	1.3	1.	0.	1.04	0.83
time (sec)	N/A	0.019	0.029	0.006	0.774	0.287	0.	0.274	7.399

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	375	275	0	0	0	0	231
normalized size	1	1.	1.48	1.09	0.	0.	0.	0.	0.91
time (sec)	N/A	0.161	1.732	0.037	0.	0.	0.	0.	14.709

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	148	137	0	0	0	0	105
normalized size	1	1.	1.35	1.25	0.	0.	0.	0.	0.95
time (sec)	N/A	0.063	0.248	0.02	0.	0.	0.	0.	6.371

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	68	33	0	58	0	0	39
normalized size	1	1.	1.62	0.79	0.	1.38	0.	0.	0.93
time (sec)	N/A	0.085	0.405	0.033	0.	0.286	0.	0.	8.973

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	400	363	0	0	0	0	248
normalized size	1	1.	1.42	1.29	0.	0.	0.	0.	0.88
time (sec)	N/A	0.227	1.15	0.025	0.	0.	0.	0.	18.191

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	171	259	0	0	0	0	128
normalized size	1	1.	1.19	1.8	0.	0.	0.	0.	0.89
time (sec)	N/A	0.131	1.151	0.022	0.	0.	0.	0.	10.374

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	161	245	0	0	0	0	133
normalized size	1	1.	1.18	1.79	0.	0.	0.	0.	0.97
time (sec)	N/A	0.13	0.975	0.075	0.	0.	0.	0.	10.279

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	23	23	0	0	26
normalized size	1	1.	1.	0.78	1.	1.	0.	0.	1.13
time (sec)	N/A	0.019	0.034	0.006	0.772	0.28	0.	0.	7.291

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	402	356	0	0	0	0	265
normalized size	1	1.	1.43	1.26	0.	0.	0.	0.	0.94
time (sec)	N/A	0.195	1.456	0.058	0.	0.	0.	0.	17.256

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	216	247	0	0	0	0	133
normalized size	1	1.	1.58	1.8	0.	0.	0.	0.	0.97
time (sec)	N/A	0.083	0.628	0.042	0.	0.	0.	0.	7.205

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	88	43	0	122	0	0	63
normalized size	1	1.	1.33	0.65	0.	1.85	0.	0.	0.95
time (sec)	N/A	0.096	0.344	0.05	0.	0.287	0.	0.	9.794

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	409	363	0	0	0	0	287
normalized size	1	1.	1.29	1.15	0.	0.	0.	0.	0.91
time (sec)	N/A	0.252	1.447	0.046	0.	0.	0.	0.	20.817

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	170	259	0	0	0	0	158
normalized size	1	1.	1.	1.52	0.	0.	0.	0.	0.93
time (sec)	N/A	0.159	0.675	0.045	0.	0.	0.	0.	12.287

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	178	467	0	0	0	0	162
normalized size	1	1.	1.06	2.78	0.	0.	0.	0.	0.96
time (sec)	N/A	0.149	0.761	0.084	0.	0.	0.	0.	11.171

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	18	32	32	0	0	27
normalized size	1	1.	1.	0.78	1.39	1.39	0.	0.	1.17
time (sec)	N/A	0.019	0.044	0.006	0.771	0.272	0.	0.	7.21

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	318	318	409	688	0	0	0	0	298
normalized size	1	1.	1.29	2.16	0.	0.	0.	0.	0.94
time (sec)	N/A	0.234	1.361	0.066	0.	0.	0.	0.	19.913

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	178	469	0	0	0	0	162
normalized size	1	1.	1.06	2.79	0.	0.	0.	0.	0.96
time (sec)	N/A	0.102	0.755	0.061	0.	0.	0.	0.	8.158

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	98	69	0	153	0	0	90
normalized size	1	1.	1.02	0.72	0.	1.59	0.	0.	0.94
time (sec)	N/A	0.113	0.369	0.071	0.	0.278	0.	0.	10.854

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	414	695	0	0	0	0	316
normalized size	1	1.	1.19	1.99	0.	0.	0.	0.	0.91
time (sec)	N/A	0.295	1.697	0.067	0.	0.	0.	0.	23.858

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	183	481	0	0	0	0	189
normalized size	1	1.	0.9	2.37	0.	0.	0.	0.	0.93
time (sec)	N/A	0.184	1.082	0.063	0.	0.	0.	0.	14.448

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	78	68	101	197	88	96	83
normalized size	1	1.	0.8	0.7	1.04	2.03	0.91	0.99	0.86
time (sec)	N/A	0.152	0.07	0.017	0.774	0.272	0.606	0.267	21.373

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	490	490	568	2218	0	7434	0	0	0
normalized size	1	1.	1.16	4.53	0.	15.17	0.	0.	0.
time (sec)	N/A	21.707	1.241	0.114	0.	0.842	0.	0.	0.

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	397	466	1764	0	5731	0	0	0
normalized size	1	1.22	1.43	5.41	0.	17.58	0.	0.	0.
time (sec)	N/A	14.129	0.877	0.061	0.	0.535	0.	0.	0.

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	316	316	375	1329	0	4004	0	0	0
normalized size	1	1.	1.19	4.21	0.	12.67	0.	0.	0.
time (sec)	N/A	5.982	0.684	0.051	0.	0.375	0.	0.	0.

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	301	926	0	2323	1435	0	0
normalized size	1	1.	1.05	3.23	0.	8.09	5.	0.	0.
time (sec)	N/A	5.732	0.543	0.043	0.	0.314	138.83	0.	0.

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	175	545	0	965	155	0	185
normalized size	1	1.	0.88	2.75	0.	4.87	0.78	0.	0.93
time (sec)	N/A	0.677	0.751	0.032	0.	0.294	21.968	0.	82.62

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	267	581	0	1	1352	0	0
normalized size	1	1.	0.97	2.11	0.	0.	4.92	0.	0.
time (sec)	N/A	2.234	0.433	0.045	0.	0.647	125.775	0.	0.

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	356	340	999	0	1	0	0	0
normalized size	1	0.97	0.92	2.71	0.	0.	0.	0.	0.
time (sec)	N/A	6.814	0.694	0.052	0.	10.377	0.	0.	0.

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	531	531	435	1486	0	1	0	0	0
normalized size	1	1.	0.82	2.8	0.	0.	0.	0.	0.
time (sec)	N/A	7.129	1.395	0.057	0.	162.592	0.	0.	0.

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	650	650	808	3685	0	19359	0	0	0
normalized size	1	1.	1.24	5.67	0.	29.78	0.	0.	0.
time (sec)	N/A	8.429	2.829	0.091	0.	10.856	0.	0.	0.

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	581	581	680	2988	0	15470	0	0	0
normalized size	1	1.	1.17	5.14	0.	26.63	0.	0.	0.
time (sec)	N/A	27.446	1.811	0.076	0.	5.805	0.	0.	0.

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	441	441	538	2358	0	11516	0	0	0
normalized size	1	1.	1.22	5.35	0.	26.11	0.	0.	0.
time (sec)	N/A	7.097	1.224	0.063	0.	2.259	0.	0.	0.

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	453	453	441	1714	0	7522	0	0	0
normalized size	1	1.	0.97	3.78	0.	16.6	0.	0.	0.
time (sec)	N/A	8.746	0.892	0.053	0.	0.941	0.	0.	0.

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	317	1138	0	3740	0	0	0
normalized size	1	1.	0.98	3.53	0.	11.61	0.	0.	0.
time (sec)	N/A	2.471	0.729	0.041	0.	0.401	0.	0.	0.

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	331	944	0	1	0	0	0
normalized size	1	1.	0.97	2.78	0.	0.	0.	0.	0.
time (sec)	N/A	3.166	0.629	0.046	0.	8.622	0.	0.	0.

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	402	369	1215	0	1	0	0	0
normalized size	1	1.	0.92	3.01	0.	0.	0.	0.	0.
time (sec)	N/A	6.006	0.897	0.055	0.	39.07	0.	0.	0.

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	607	607	499	1880	0	0	0	0	0
normalized size	1	1.	0.82	3.1	0.	0.	0.	0.	0.
time (sec)	N/A	7.431	1.728	0.067	0.	0.	0.	0.	0.

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	0	0	0	0	0	0	168
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.84
time (sec)	N/A	0.786	0.104	0.223	0.	0.	0.	0.	70.77

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	353	0	0	0	0	0	272
normalized size	1	1.	1.22	0.	0.	0.	0.	0.	0.94
time (sec)	N/A	1.243	1.189	0.135	0.	0.	0.	0.	72.374

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	346	0	0	0	0	0	228
normalized size	1	1.	1.47	0.	0.	0.	0.	0.	0.97
time (sec)	N/A	0.708	1.906	0.136	0.	0.	0.	0.	66.577

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	289	0	0	0	0	0	196
normalized size	1	1.	1.47	0.	0.	0.	0.	0.	0.99
time (sec)	N/A	0.399	0.696	0.123	0.	0.	0.	0.	47.964



Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	245	0	0	0	0	0	172
normalized size	1	1.	1.29	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	0.478	0.647	0.154	0.	0.	0.	0.	58.557

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	331	0	0	0	0	0	228
normalized size	1	1.	1.37	0.	0.	0.	0.	0.	0.95
time (sec)	N/A	0.797	4.255	0.113	0.	0.	0.	0.	86.39

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	431	0	0	0	0	0	282
normalized size	1	1.	1.46	0.	0.	0.	0.	0.	0.96
time (sec)	N/A	1.004	3.736	0.143	0.	0.	0.	0.	116.988

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	134	186	236	238	156	336	0
normalized size	1	1.	0.95	1.32	1.67	1.69	1.11	2.38	0.
time (sec)	N/A	0.322	0.13	0.007	0.715	0.28	2.601	0.276	0.

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	103	145	186	188	116	285	0
normalized size	1	1.	0.94	1.33	1.71	1.72	1.06	2.61	0.
time (sec)	N/A	0.243	0.095	0.006	0.708	0.282	2.224	0.273	0.

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	73	110	131	132	75	232	60
normalized size	1	1.	1.12	1.69	2.02	2.03	1.15	3.57	0.92
time (sec)	N/A	0.111	0.066	0.005	0.692	0.278	1.916	0.271	18.784

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	43	82	85	86	46	181	0
normalized size	1	1.	0.86	1.64	1.7	1.72	0.92	3.62	0.
time (sec)	N/A	0.068	0.033	0.005	0.691	0.273	1.571	0.276	0.

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	55	107	111	103	112	109	0
normalized size	1	1.	0.89	1.73	1.79	1.66	1.81	1.76	0.
time (sec)	N/A	0.158	0.038	0.01	0.69	0.278	2.681	0.273	0.

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	82	149	153	223	182	0	75
normalized size	1	1.	0.95	1.73	1.78	2.59	2.12	0.	0.87
time (sec)	N/A	0.204	0.08	0.017	0.693	0.279	4.106	0.	31.999

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	206	201	366	185	0	73
normalized size	1	1.	1.	2.37	2.31	4.21	2.13	0.	0.84
time (sec)	N/A	0.218	0.128	0.015	0.692	0.281	4.061	0.	43.514

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	122	259	278	540	248	0	97
normalized size	1	1.	1.08	2.29	2.46	4.78	2.19	0.	0.86
time (sec)	N/A	0.257	0.095	0.016	0.696	0.283	5.234	0.	50.864

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	142	312	319	690	282	1	121
normalized size	1	1.	1.02	2.24	2.29	4.96	2.03	0.01	0.87
time (sec)	N/A	0.293	0.15	0.016	0.701	0.282	6.099	0.282	58.506

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	226	286	348	443	255	1	0
normalized size	1	1.	1.04	1.31	1.6	2.03	1.17	0.	0.
time (sec)	N/A	0.539	0.25	0.014	0.697	0.27	4.571	0.308	0.

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	185	245	294	389	204	441	0
normalized size	1	1.	1.05	1.38	1.66	2.2	1.15	2.49	0.
time (sec)	N/A	0.459	0.279	0.013	0.702	0.275	4.094	0.303	0.

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	154	204	246	339	167	393	0
normalized size	1	1.	1.05	1.4	1.68	2.32	1.14	2.69	0.
time (sec)	N/A	0.381	0.177	0.014	0.689	0.276	3.691	0.287	0.

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	115	167	190	278	122	338	0
normalized size	1	1.	1.07	1.56	1.78	2.6	1.14	3.16	0.
time (sec)	N/A	0.291	0.157	0.012	0.691	0.277	3.141	0.281	0.

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	83	138	140	212	92	286	0
normalized size	1	1.	1.06	1.77	1.79	2.72	1.18	3.67	0.
time (sec)	N/A	0.208	0.117	0.012	0.687	0.276	2.766	0.292	0.

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	46	96	93	128	60	216	0
normalized size	1	1.	0.92	1.92	1.86	2.56	1.2	4.32	0.
time (sec)	N/A	0.12	0.07	0.01	0.692	0.267	2.125	0.279	0.

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	91	156	154	227	180	215	73
normalized size	1	1.	1.06	1.81	1.79	2.64	2.09	2.5	0.85
time (sec)	N/A	0.175	0.08	0.016	0.688	0.279	4.133	0.277	29.927

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	85	180	150	209	155	136	63
normalized size	1	1.	1.15	2.43	2.03	2.82	2.09	1.84	0.85
time (sec)	N/A	0.074	0.063	0.017	0.692	0.278	2.963	0.301	13.473

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	139	253	286	563	279	0	133
normalized size	1	1.	1.15	2.09	2.36	4.65	2.31	0.	1.1
time (sec)	N/A	0.295	0.182	0.019	0.704	0.277	5.127	0.	47.862

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	171	270	266	455	236	0	150
normalized size	1	1.	1.17	1.85	1.82	3.12	1.62	0.	1.03
time (sec)	N/A	0.339	0.164	0.02	0.709	0.277	5.847	0.	53.241

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	195	341	402	875	371	1	187
normalized size	1	1.	1.1	1.92	2.26	4.92	2.08	0.01	1.05
time (sec)	N/A	0.43	0.247	0.022	0.706	0.28	7.463	0.272	64.384

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	229	394	462	936	420	1	218
normalized size	1	1.	1.09	1.88	2.2	4.46	2.	0.	1.04
time (sec)	N/A	0.522	0.348	0.026	0.719	0.284	8.986	0.283	75.957

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	193	263	306	454	223	1	0
normalized size	1	1.	1.08	1.47	1.71	2.54	1.25	0.01	0.
time (sec)	N/A	0.499	0.161	0.014	0.699	0.274	6.188	0.269	0.

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	157	228	254	397	180	1	0
normalized size	1	1.	1.05	1.53	1.7	2.66	1.21	0.01	0.
time (sec)	N/A	0.408	0.144	0.015	0.699	0.276	5.5	0.266	0.

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	118	198	201	325	148	369	0
normalized size	1	1.	1.	1.68	1.7	2.75	1.25	3.13	0.
time (sec)	N/A	0.317	0.16	0.013	0.698	0.281	4.954	0.275	0.

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	93	151	142	215	99	306	0
normalized size	1	1.	1.15	1.86	1.75	2.65	1.22	3.78	0.
time (sec)	N/A	0.219	0.066	0.012	0.703	0.275	3.941	0.27	0.

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	49	105	109	135	80	263	51
normalized size	1	1.	0.8	1.72	1.79	2.21	1.31	4.31	0.84
time (sec)	N/A	0.112	0.049	0.01	0.694	0.272	2.729	0.271	20.718

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	90	218	203	366	185	266	73
normalized size	1	1.	1.02	2.48	2.31	4.16	2.1	3.02	0.83
time (sec)	N/A	0.214	0.133	0.015	0.71	0.279	4.243	0.276	43.458

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	140	257	285	563	277	258	133
normalized size	1	1.	1.15	2.11	2.34	4.61	2.27	2.11	1.09
time (sec)	N/A	0.264	0.212	0.019	0.704	0.278	5.289	0.27	46.652

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	110	298	205	340	143	171	110
normalized size	1	1.	0.87	2.35	1.61	2.68	1.13	1.35	0.87
time (sec)	N/A	0.134	0.073	0.019	0.698	0.274	4.071	0.275	30.491

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	197	348	416	894	320	0	206
normalized size	1	1.	1.05	1.85	2.21	4.76	1.7	0.	1.1
time (sec)	N/A	0.458	0.291	0.021	0.718	0.285	7.61	0.	68.384

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	244	421	485	1071	371	1	252
normalized size	1	1.	1.04	1.79	2.06	4.56	1.58	0.	1.07
time (sec)	N/A	0.591	0.302	0.022	0.71	0.286	8.927	0.344	86.451

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	193	1308	2167	2519	0	725	411
normalized size	1	1.	0.72	4.86	8.06	9.36	0.	2.7	1.53
time (sec)	N/A	1.429	0.571	0.021	0.788	0.318	0.	0.296	117.876

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	168	1030	1608	1616	0	555	282
normalized size	1	1.	0.78	4.79	7.48	7.52	0.	2.58	1.31
time (sec)	N/A	0.982	0.34	0.017	0.784	0.31	0.	0.294	88.542

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	145	713	1220	887	0	417	243
normalized size	1	1.	0.79	3.9	6.67	4.85	0.	2.28	1.33
time (sec)	N/A	0.635	0.307	0.016	0.782	0.31	0.	0.295	77.108

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	105	131	787	456	0	267	212
normalized size	1	1.	0.72	0.9	5.43	3.14	0.	1.84	1.46
time (sec)	N/A	0.436	0.124	0.012	0.701	0.286	0.	0.29	59.149

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	77	85	504	363	0	188	102
normalized size	1	1.	0.66	0.73	4.31	3.1	0.	1.61	0.87
time (sec)	N/A	0.185	0.085	0.011	0.694	0.283	0.	0.294	15.656

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	53	55	136	271	0	95	80
normalized size	1	1.	0.51	0.53	1.32	2.63	0.	0.92	0.78
time (sec)	N/A	0.137	0.043	0.	0.694	0.281	0.	0.298	21.214

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	219	3961	0	1	0	1	267
normalized size	1	1.	0.9	16.37	0.	0.	0.	0.	1.1
time (sec)	N/A	1.19	0.356	0.048	0.	0.329	0.	0.307	82.672

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	308	6760	0	1	0	0	405
normalized size	1	1.	0.99	21.74	0.	0.	0.	0.	1.3
time (sec)	N/A	2.419	1.01	0.038	0.	0.472	0.	0.	129.43

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	398	387	9593	0	1	0	1	0
normalized size	1	1.	0.97	24.1	0.	0.	0.	0.	0.
time (sec)	N/A	5.427	1.652	0.046	0.	0.906	0.	1.159	0.

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	108	114	0	1	0	157	0
normalized size	1	1.	0.96	1.02	0.	0.01	0.	1.4	0.
time (sec)	N/A	0.456	0.333	0.022	0.	0.291	0.	0.288	0.

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	282	365	440	437	1040	612	0
normalized size	1	1.	1.18	1.52	1.83	1.82	4.33	2.55	0.
time (sec)	N/A	0.684	0.486	0.01	0.7	0.277	76.102	0.271	0.

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	177	215	266	266	673	389	0
normalized size	1	1.	1.01	1.23	1.52	1.52	3.85	2.22	0.
time (sec)	N/A	0.491	0.186	0.01	0.705	0.272	44.974	0.27	0.

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	94	101	140	135	374	209	112
normalized size	1	1.	0.83	0.89	1.24	1.19	3.31	1.85	0.99
time (sec)	N/A	0.177	0.099	0.007	0.695	0.273	23.586	0.266	23.033

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	44	41	72	54	150	84	58
normalized size	1	1.	0.72	0.67	1.18	0.89	2.46	1.38	0.95
time (sec)	N/A	0.062	0.035	0.005	0.689	0.274	6.209	0.3	10.929

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	92	132	0	1	245	144	95
normalized size	1	1.	0.88	1.27	0.	0.01	2.36	1.38	0.91
time (sec)	N/A	0.263	0.228	0.02	0.	0.291	17.442	0.301	63.446

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	115	237	0	1	0	200	124
normalized size	1	1.	0.94	1.94	0.	0.01	0.	1.64	1.02
time (sec)	N/A	0.445	0.349	0.022	0.	0.29	0.	0.319	83.965

Problem 595	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	166	384	0	1	0	375	194
normalized size	1	1.	0.93	2.16	0.	0.01	0.	2.11	1.09
time (sec)	N/A	0.635	0.409	0.026	0.	0.297	0.	0.287	106.987

Problem 596	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	278	365	451	436	0	612	0
normalized size	1	1.	1.17	1.53	1.89	1.83	0.	2.57	0.
time (sec)	N/A	0.683	0.328	0.011	0.698	0.277	0.	0.286	0.

Problem 597	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	177	215	277	265	0	371	173
normalized size	1	1.	1.02	1.24	1.6	1.53	0.	2.14	1.
time (sec)	N/A	0.493	0.28	0.01	0.701	0.275	0.	0.282	79.74

Problem 598	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	92	101	151	135	2139	193	110
normalized size	1	1.	0.83	0.91	1.36	1.22	19.27	1.74	0.99
time (sec)	N/A	0.171	0.126	0.006	0.701	0.273	20.203	0.27	23.201

Problem 599	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	43	41	73	53	549	76	58
normalized size	1	1.	0.73	0.69	1.24	0.9	9.31	1.29	0.98
time (sec)	N/A	0.062	0.039	0.006	0.693	0.276	9.192	0.27	11.324

Problem 600	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	108	165	0	1	0	136	0
normalized size	1	1.	0.96	1.47	0.	0.01	0.	1.21	0.
time (sec)	N/A	0.371	0.341	0.017	0.	0.29	0.	0.278	0.



Problem 601	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	135	269	0	1	0	304	141
normalized size	1	1.	0.94	1.87	0.	0.01	0.	2.11	0.98
time (sec)	N/A	0.61	0.572	0.03	0.	0.296	0.	0.275	174.434

Problem 602	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	190	546	0	1	0	487	0
normalized size	1	1.	0.89	2.55	0.	0.	0.	2.28	0.
time (sec)	N/A	1.008	1.376	0.033	0.	0.312	0.	0.282	0.

Problem 603	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	136	306	0	1	0	209	163
normalized size	1	1.	0.93	2.08	0.	0.01	0.	1.42	1.11
time (sec)	N/A	0.324	0.165	0.044	0.	0.609	0.	0.288	30.3

Problem 604	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	13	12	74	129	16	14
normalized size	1	1.	1.	0.81	0.75	4.62	8.06	1.	0.88
time (sec)	N/A	0.011	0.024	0.005	0.685	0.277	26.293	0.267	5.632

Problem 605	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	1076	2497	0	0	0	4	0
normalized size	1	1.	2.62	6.08	0.	0.	0.	0.01	0.
time (sec)	N/A	4.648	8.307	0.153	0.	0.	0.	3.551	0.

Problem 606	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	524	1569	0	0	0	4	0
normalized size	1	1.	1.53	4.59	0.	0.	0.	0.01	0.
time (sec)	N/A	3.869	3.726	0.036	0.	0.	0.	3.647	0.

Problem 607	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	496	1383	0	2593	0	0	209
normalized size	1	1.	2.07	5.76	0.	10.8	0.	0.	0.87
time (sec)	N/A	0.918	2.743	0.045	0.	9.807	0.	0.	77.383

Problem 608	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	531	5383	0	7852	0	4	0
normalized size	1	1.	1.51	15.34	0.	22.37	0.	0.01	0.
time (sec)	N/A	4.068	4.479	0.1	0.	54.561	0.	6.674	0.

Problem 609	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	613	613	600	14861	0	0	0	4	0
normalized size	1	1.	0.98	24.24	0.	0.	0.	0.01	0.
time (sec)	N/A	6.538	5.725	0.108	0.	0.	0.	6.513	0.

Problem 610	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	527	2336	0	0	0	4	0
normalized size	1	1.	1.56	6.93	0.	0.	0.	0.01	0.
time (sec)	N/A	4.707	3.309	0.05	0.	0.	0.	0.644	0.

Problem 611	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	496	1383	0	2583	0	0	209
normalized size	1	1.	2.07	5.76	0.	10.76	0.	0.	0.87
time (sec)	N/A	0.846	2.526	0.045	0.	10.48	0.	0.	80.022

Problem 612	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	451	1415	0	5839	0	0	199
normalized size	1	1.	1.96	6.15	0.	25.39	0.	0.	0.87
time (sec)	N/A	0.645	3.857	0.055	0.	24.556	0.	0.	77.067

Problem 613	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	555	10977	0	15992	0	0	304
normalized size	1	1.	1.57	31.01	0.	45.18	0.	0.	0.86
time (sec)	N/A	1.597	3.416	0.11	0.	84.327	0.	0.	143.349

Problem 614	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	625	625	558	8264	0	0	0	0	0
normalized size	1	1.	0.89	13.22	0.	0.	0.	0.	0.
time (sec)	N/A	4.865	3.91	0.108	0.	0.	0.	0.	0.

Problem 615	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	531	5383	0	7889	0	0	0
normalized size	1	1.	1.51	15.34	0.	22.48	0.	0.	0.
time (sec)	N/A	3.756	4.344	0.07	0.	74.985	0.	0.	0.

Problem 616	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	555	10977	0	16238	0	0	304
normalized size	1	1.	1.57	31.01	0.	45.87	0.	0.	0.86
time (sec)	N/A	1.715	4.363	0.112	0.	147.838	0.	0.	142.261

Problem 617	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	549	543	1619	30648	0	0	0	4	0
normalized size	1	0.99	2.95	55.83	0.	0.	0.	0.01	0.
time (sec)	N/A	3.447	9.174	0.231	0.	0.	0.	0.706	0.

Problem 618	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	65	305	0	1146	0	0	121
normalized size	1	1.	1.	4.69	0.	17.63	0.	0.	1.86
time (sec)	N/A	0.126	0.126	0.19	0.	0.321	0.	0.	19.299

Problem 619	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	104	125	0	416	0	359	85
normalized size	1	1.	1.3	1.56	0.	5.2	0.	4.49	1.06
time (sec)	N/A	0.273	0.157	0.022	0.	0.289	0.	0.296	21.777

Problem 620	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	148	1178	0	1	0	281	92
normalized size	1	1.	1.38	11.01	0.	0.01	0.	2.63	0.86
time (sec)	N/A	0.499	0.262	0.053	0.	0.416	0.	0.325	43.414

Problem 621	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	148	524	0	1	0	177	92
normalized size	1	1.	1.38	4.9	0.	0.01	0.	1.65	0.86
time (sec)	N/A	0.353	0.078	0.016	0.	0.419	0.	0.311	36.604

Problem 622	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	851	851	6884	6457	0	0	0	0	0
normalized size	1	1.	8.09	7.59	0.	0.	0.	0.	0.
time (sec)	N/A	4.75	13.726	0.166	0.	0.	0.	0.	0.

Problem 623	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	635	635	910	4351	0	0	0	0	0
normalized size	1	1.	1.43	6.85	0.	0.	0.	0.	0.
time (sec)	N/A	2.95	12.715	0.041	0.	0.	0.	0.	0.

Problem 624	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	434	434	610	2549	0	0	0	0	437
normalized size	1	1.	1.41	5.87	0.	0.	0.	0.	1.01
time (sec)	N/A	1.202	7.982	0.03	0.	0.	0.	0.	174.839

Problem 625	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	362	536	1162	0	0	0	0	340
normalized size	1	1.	1.48	3.21	0.	0.	0.	0.	0.94
time (sec)	N/A	0.85	4.601	0.025	0.	0.	0.	0.	144.348

Problem 626	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	683	683	1216	2496	0	0	0	0	0
normalized size	1	1.	1.78	3.65	0.	0.	0.	0.	0.
time (sec)	N/A	4.653	12.67	0.05	0.	0.	0.	0.	0.

Problem 627	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	650	650	7969	6044	0	0	0	0	0
normalized size	1	1.	12.26	9.3	0.	0.	0.	0.	0.
time (sec)	N/A	4.104	13.539	0.064	0.	0.	0.	0.	0.

Problem 628	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1205	1205	12364	19181	0	0	0	0	0
normalized size	1	1.	10.26	15.92	0.	0.	0.	0.	0.
time (sec)	N/A	9.715	14.932	0.099	0.	0.	0.	0.	0.

Problem 629	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	666	666	5379	5079	0	0	0	0	0
normalized size	1	1.	8.08	7.63	0.	0.	0.	0.	0.
time (sec)	N/A	3.012	13.558	0.063	0.	0.	0.	0.	0.

Problem 630	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	508	503	712	3278	0	0	0	0	0
normalized size	1	0.99	1.4	6.45	0.	0.	0.	0.	0.
time (sec)	N/A	1.98	9.455	0.039	0.	0.	0.	0.	0.

Problem 631	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	364	364	545	1828	0	0	0	0	359
normalized size	1	1.	1.5	5.02	0.	0.	0.	0.	0.99
time (sec)	N/A	0.947	7.751	0.03	0.	0.	0.	0.	131.463

Problem 632	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	456	688	0	0	0	0	303
normalized size	1	1.	1.42	2.14	0.	0.	0.	0.	0.94
time (sec)	N/A	0.635	3.727	0.024	0.	0.	0.	0.	112.412

Problem 633	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	473	473	1096	1216	0	0	0	0	0
normalized size	1	1.	2.32	2.57	0.	0.	0.	0.	0.
time (sec)	N/A	2.233	7.469	0.036	0.	0.	0.	0.	0.

Problem 634	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	694	694	1336	6034	0	0	0	0	0
normalized size	1	1.	1.93	8.69	0.	0.	0.	0.	0.
time (sec)	N/A	4.181	10.359	0.046	0.	0.	0.	0.	0.

Problem 635	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-2)	F(-2)	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1241	1241	12365	19187	0	0	0	0	0
normalized size	1	1.	9.96	15.46	0.	0.	0.	0.	0.
time (sec)	N/A	9.705	14.815	0.079	0.	0.	0.	0.	0.

Problem 636	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	531	527	747	3922	0	0	0	0	0
normalized size	1	0.99	1.41	7.39	0.	0.	0.	0.	0.
time (sec)	N/A	2.19	10.99	0.059	0.	0.	0.	0.	0.

Problem 637	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	596	2470	0	0	0	0	0
normalized size	1	1.	1.45	6.02	0.	0.	0.	0.	0.
time (sec)	N/A	1.229	7.588	0.054	0.	0.	0.	0.	0.

Problem 638	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	464	1286	0	0	0	0	309
normalized size	1	1.	1.4	3.89	0.	0.	0.	0.	0.93
time (sec)	N/A	0.736	5.398	0.042	0.	0.	0.	0.	123.818

Problem 639	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	294	396	0	0	0	0	129
normalized size	1	1.	2.16	2.91	0.	0.	0.	0.	0.95
time (sec)	N/A	0.209	0.656	0.04	0.	0.	0.	0.	37.252

Problem 640	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	300	439	0	0	0	0	330
normalized size	1	1.	0.94	1.38	0.	0.	0.	0.	1.03
time (sec)	N/A	1.809	0.903	0.051	0.	0.	0.	0.	95.722

Problem 641	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	698	698	1330	5743	0	0	0	0	0
normalized size	1	1.	1.91	8.23	0.	0.	0.	0.	0.
time (sec)	N/A	4.727	10.356	0.063	0.	0.	0.	0.	0.

Problem 642	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1246	1246	12363	20364	0	0	0	0	0
normalized size	1	1.	9.92	16.34	0.	0.	0.	0.	0.
time (sec)	N/A	10.169	15.141	0.094	0.	0.	0.	0.	0.

Problem 643	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	600	808	1440	3164	0	0	0	0	0
normalized size	1	1.35	2.4	5.27	0.	0.	0.	0.	0.
time (sec)	N/A	2.9	13.776	0.057	0.	0.	0.	0.	0.

Problem 644	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	469	469	927	959	0	0	0	0	478
normalized size	1	1.	1.98	2.04	0.	0.	0.	0.	1.02
time (sec)	N/A	2.14	1.686	0.052	0.	0.	0.	0.	150.141

Problem 645	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	457	457	625	2949	0	0	0	0	0
normalized size	1	1.	1.37	6.45	0.	0.	0.	0.	0.
time (sec)	N/A	1.405	7.704	0.061	0.	0.	0.	0.	0.

Problem 646	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	473	1769	0	0	0	0	602
normalized size	1	1.	1.33	4.97	0.	0.	0.	0.	1.69
time (sec)	N/A	1.032	5.853	0.054	0.	0.	0.	0.	174.557

Problem 647	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	439	520	0	0	0	0	270
normalized size	1	1.	1.52	1.81	0.	0.	0.	0.	0.94
time (sec)	N/A	0.543	2.662	0.048	0.	0.	0.	0.	93.134

Problem 648	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	186	200	0	0	0	0	129
normalized size	1	1.	1.37	1.47	0.	0.	0.	0.	0.95
time (sec)	N/A	0.214	0.443	0.044	0.	0.	0.	0.	44.969

Problem 649	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	311	235	0	0	0	0	201
normalized size	1	1.	1.86	1.41	0.	0.	0.	0.	1.2
time (sec)	N/A	1.416	0.903	0.052	0.	0.	0.	0.	18.542

Problem 650	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	746	746	1491	5738	0	0	0	0	0
normalized size	1	1.	2.	7.69	0.	0.	0.	0.	0.
time (sec)	N/A	5.036	11.102	0.072	0.	0.	0.	0.	0.

Problem 651	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-2)	F	F
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1257	1257	15233	20365	0	0	0	0	0
normalized size	1	1.	12.12	16.2	0.	0.	0.	0.	0.
time (sec)	N/A	10.234	15.733	0.128	0.	0.	0.	0.	0.

Problem 652	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	468	2011	0	0	0	0	394
normalized size	1	1.	1.21	5.2	0.	0.	0.	0.	1.02
time (sec)	N/A	2.034	6.358	0.093	0.	0.	0.	0.	115.104

Problem 653	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	818	818	2069	9409	0	0	0	0	0
normalized size	1	1.	2.53	11.5	0.	0.	0.	0.	0.
time (sec)	N/A	3.086	14.235	0.129	0.	0.	0.	0.	0.

Problem 654	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	261	215	0	0	0	0	162
normalized size	1	1.	2.37	1.95	0.	0.	0.	0.	1.47
time (sec)	N/A	1.021	0.804	0.128	0.	0.	0.	0.	14.279

Problem 655	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	454	454	344	433	0	0	0	0	400
normalized size	1	1.	0.76	0.95	0.	0.	0.	0.	0.88
time (sec)	N/A	1.216	1.951	0.221	0.	0.	0.	0.	170.923

Problem 656	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	107	58	0	0	0	0	26
normalized size	1	1.	2.06	1.12	0.	0.	0.	0.	0.5
time (sec)	N/A	0.135	0.239	0.151	0.	0.	0.	0.	15.29



Problem 657	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	136	188	294	605	0	0	262
normalized size	1	1.	0.51	0.7	1.09	2.25	0.	0.	0.97
time (sec)	N/A	1.18	0.194	0.012	0.78	0.291	0.	0.	95.108

Problem 658	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	89	116	180	378	0	0	194
normalized size	1	1.	0.44	0.58	0.9	1.89	0.	0.	0.97
time (sec)	N/A	0.707	0.126	0.011	0.764	0.288	0.	0.	61.815

Problem 659	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	53	67	88	198	0	0	119
normalized size	1	1.	0.42	0.54	0.7	1.58	0.	0.	0.95
time (sec)	N/A	0.335	0.06	0.008	0.762	0.293	0.	0.	35.652

Problem 660	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	35	50	24	57	0	0	41
normalized size	1	1.	0.76	1.09	0.52	1.24	0.	0.	0.89
time (sec)	N/A	0.072	0.029	0.007	0.752	0.288	0.	0.	17.18

Problem 661	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	93	87	0	1	0	0	78
normalized size	1	1.	1.16	1.09	0.	0.01	0.	0.	0.98
time (sec)	N/A	0.378	0.09	0.036	0.	0.3	0.	0.	32.82

Problem 662	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	138	168	0	1	0	0	131
normalized size	1	1.	0.99	1.2	0.	0.01	0.	0.	0.94
time (sec)	N/A	0.64	0.264	0.029	0.	0.304	0.	0.	56.404

Problem 663	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	163	285	0	1	0	0	202
normalized size	1	1.	0.77	1.34	0.	0.	0.	0.	0.95
time (sec)	N/A	1.015	0.368	0.03	0.	0.311	0.	0.	84.507

Problem 664	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	188	450	0	1	0	0	269
normalized size	1	1.	0.67	1.61	0.	0.	0.	0.	0.96
time (sec)	N/A	1.375	0.667	0.031	0.	0.321	0.	0.	116.729

Problem 665	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	134	187	223	424	0	4	252
normalized size	1	1.	0.52	0.73	0.87	1.65	0.	0.02	0.98
time (sec)	N/A	1.013	0.187	0.012	0.786	0.282	0.	0.739	90.559

Problem 666	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	88	116	132	247	0	4	175
normalized size	1	1.	0.49	0.64	0.73	1.36	0.	0.02	0.97
time (sec)	N/A	0.606	0.118	0.012	0.775	0.27	0.	0.715	61.106

Problem 667	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	51	66	65	119	0	4	139
normalized size	1	1.	0.34	0.44	0.43	0.79	0.	0.03	0.93
time (sec)	N/A	0.429	0.064	0.007	0.763	0.27	0.	0.654	46.142

Problem 668	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	35	50	24	100	0	4	42
normalized size	1	1.	0.76	1.09	0.52	2.17	0.	0.09	0.91
time (sec)	N/A	0.07	0.021	0.004	0.713	0.265	0.	0.682	17.114

Problem 669	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	110	128	0	1	0	4	126
normalized size	1	1.	0.83	0.96	0.	0.01	0.	0.03	0.95
time (sec)	N/A	0.605	0.172	0.03	0.	0.284	0.	0.781	57.608

Problem 670	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	141	225	0	1	0	4	192
normalized size	1	1.	0.7	1.11	0.	0.	0.	0.02	0.95
time (sec)	N/A	0.862	0.439	0.042	0.	0.29	0.	0.768	83.445

Problem 671	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	185	379	0	1	0	4	265
normalized size	1	1.	0.68	1.38	0.	0.	0.	0.01	0.97
time (sec)	N/A	1.25	0.491	0.043	0.	0.302	0.	0.908	116.395

Problem 672	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	131	187	296	447	0	4	233
normalized size	1	1.	0.55	0.78	1.24	1.87	0.	0.02	0.97
time (sec)	N/A	0.9	0.21	0.012	0.764	0.274	0.	0.908	91.85

Problem 673	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	87	116	186	271	0	4	201
normalized size	1	1.	0.41	0.55	0.88	1.28	0.	0.02	0.95
time (sec)	N/A	0.682	0.143	0.012	0.744	0.271	0.	0.832	76.615

Problem 674	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	52	66	99	174	0	4	146
normalized size	1	1.	0.34	0.43	0.64	1.13	0.	0.03	0.95
time (sec)	N/A	0.411	0.07	0.008	0.73	0.27	0.	0.835	46.166

Problem 675	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	37	50	38	144	0	4	44
normalized size	1	1.	0.77	1.04	0.79	3.	0.	0.08	0.92
time (sec)	N/A	0.07	0.047	0.004	0.716	0.268	0.	0.755	17.058

Problem 676	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	128	219	0	1	0	4	180
normalized size	1	1.	0.68	1.16	0.	0.01	0.	0.02	0.96
time (sec)	N/A	0.897	0.341	0.033	0.	0.289	0.	1.008	87.122

Problem 677	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	180	424	0	1	0	4	255
normalized size	1	1.	0.67	1.58	0.	0.	0.	0.01	0.95
time (sec)	N/A	1.184	0.534	0.042	0.	0.299	0.	1.169	115.781

Problem 678	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	212	670	0	1	0	4	332
normalized size	1	1.	0.62	1.96	0.	0.	0.	0.01	0.97
time (sec)	N/A	1.841	0.985	0.047	0.	0.312	0.	13.296	153.982

Problem 679	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	195	283	432	1127	0	0	330
normalized size	1	1.	0.58	0.84	1.29	3.35	0.	0.	0.98
time (sec)	N/A	1.633	0.255	0.012	0.76	0.275	0.	0.	132.075

Problem 680	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	136	188	294	799	0	0	264
normalized size	1	1.	0.51	0.7	1.09	2.97	0.	0.	0.98
time (sec)	N/A	1.109	0.214	0.012	0.748	0.276	0.	0.	95.065

Problem 681	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	90	116	180	522	0	0	196
normalized size	1	1.	0.45	0.58	0.9	2.61	0.	0.	0.98
time (sec)	N/A	0.683	0.135	0.012	0.734	0.271	0.	0.	61.623

Problem 682	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	54	67	88	293	0	0	121
normalized size	1	1.	0.43	0.54	0.7	2.34	0.	0.	0.97
time (sec)	N/A	0.316	0.069	0.008	0.72	0.27	0.	0.	35.572

Problem 683	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	37	50	24	135	0	0	42
normalized size	1	1.	0.77	1.04	0.5	2.81	0.	0.	0.88
time (sec)	N/A	0.07	0.037	0.003	0.715	0.266	0.	0.	17.418

Problem 684	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	114	153	0	1	0	0	117
normalized size	1	1.	0.92	1.23	0.	0.01	0.	0.	0.94
time (sec)	N/A	0.595	0.132	0.026	0.	0.282	0.	0.	56.444

Problem 685	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	111	161	0	1	0	0	124
normalized size	1	1.	0.84	1.22	0.	0.01	0.	0.	0.94
time (sec)	N/A	0.547	0.221	0.032	0.	0.285	0.	0.	56.226

Problem 686	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	164	285	0	1	0	0	190
normalized size	1	1.	0.79	1.38	0.	0.	0.	0.	0.92
time (sec)	N/A	0.938	0.387	0.035	0.	0.29	0.	0.	83.209

Problem 687	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	168	453	0	1	0	0	257
normalized size	1	1.	0.61	1.64	0.	0.	0.	0.	0.93
time (sec)	N/A	1.154	0.704	0.042	0.	0.303	0.	0.	116.688

Problem 688	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	195	696	0	1	0	0	328
normalized size	1	1.	0.56	2.01	0.	0.	0.	0.	0.95
time (sec)	N/A	1.609	1.047	0.041	0.	0.314	0.	0.	152.208

Problem 689	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	195	283	558	1368	0	0	330
normalized size	1	1.	0.58	0.84	1.66	4.07	0.	0.	0.98
time (sec)	N/A	1.612	0.335	0.013	0.767	0.275	0.	0.	129.65

Problem 690	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	137	188	397	990	0	0	264
normalized size	1	1.	0.51	0.7	1.48	3.68	0.	0.	0.98
time (sec)	N/A	1.109	0.243	0.012	0.744	0.276	0.	0.	93.757

Problem 691	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	90	116	259	662	0	0	196
normalized size	1	1.	0.45	0.58	1.3	3.31	0.	0.	0.98
time (sec)	N/A	0.71	0.19	0.011	0.73	0.271	0.	0.	60.388

Problem 692	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	54	67	144	382	0	0	121
normalized size	1	1.	0.43	0.54	1.15	3.06	0.	0.	0.97
time (sec)	N/A	0.327	0.099	0.007	0.723	0.268	0.	0.	34.955

Problem 693	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	37	50	58	178	0	0	42
normalized size	1	1.	0.77	1.04	1.21	3.71	0.	0.	0.88
time (sec)	N/A	0.071	0.046	0.006	0.714	0.269	0.	0.	16.909

Problem 694	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	132	263	0	1	0	0	172
normalized size	1	1.	0.74	1.47	0.	0.01	0.	0.	0.96
time (sec)	N/A	0.977	0.306	0.027	0.	0.283	0.	0.	82.996

Problem 695	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	127	306	0	1	0	0	170
normalized size	1	1.	0.71	1.72	0.	0.01	0.	0.	0.96
time (sec)	N/A	0.879	0.742	0.035	0.	0.308	0.	0.	82.046

Problem 696	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	135	276	0	1	0	0	187
normalized size	1	1.	0.69	1.42	0.	0.01	0.	0.	0.96
time (sec)	N/A	0.876	0.389	0.035	0.	0.29	0.	0.	82.338

Problem 697	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	190	453	0	1	0	0	248
normalized size	1	1.	0.72	1.71	0.	0.	0.	0.	0.94
time (sec)	N/A	1.226	0.715	0.037	0.	0.298	0.	0.	114.174

Problem 698	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	201	665	0	1	0	0	320
normalized size	1	1.	0.6	1.99	0.	0.	0.	0.	0.96
time (sec)	N/A	1.504	1.012	0.045	0.	0.307	0.	0.	150.577

Problem 699	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	405	405	231	955	0	1	0	0	0
normalized size	1	1.	0.57	2.36	0.	0.	0.	0.	0.
time (sec)	N/A	2.087	1.362	0.048	0.	0.324	0.	0.	0.

Problem 700	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	205	283	672	1615	0	0	330
normalized size	1	1.	0.61	0.84	2.	4.81	0.	0.	0.98
time (sec)	N/A	1.624	0.354	0.013	0.768	0.287	0.	0.	129.653

Problem 701	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	147	188	489	1187	0	0	264
normalized size	1	1.	0.55	0.7	1.82	4.41	0.	0.	0.98
time (sec)	N/A	1.145	0.28	0.012	0.746	0.275	0.	0.	94.204

Problem 702	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	100	116	328	809	0	0	196
normalized size	1	1.	0.5	0.58	1.64	4.04	0.	0.	0.98
time (sec)	N/A	0.709	0.209	0.011	0.736	0.279	0.	0.	60.569

Problem 703	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	64	67	190	479	0	0	121
normalized size	1	1.	0.51	0.54	1.52	3.83	0.	0.	0.97
time (sec)	N/A	0.33	0.114	0.007	0.718	0.27	0.	0.	35.143

Problem 704	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	37	50	81	227	0	0	42
normalized size	1	1.	0.77	1.04	1.69	4.73	0.	0.	0.88
time (sec)	N/A	0.072	0.065	0.007	0.711	0.269	0.	0.	17.404

Problem 705	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	168	431	0	1	0	0	228
normalized size	1	1.	0.71	1.83	0.	0.	0.	0.	0.97
time (sec)	N/A	1.397	0.446	0.029	0.	0.293	0.	0.	110.871

Problem 706	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	161	523	0	1	0	0	228
normalized size	1	1.	0.69	2.23	0.	0.	0.	0.	0.97
time (sec)	N/A	1.224	0.695	0.039	0.	0.316	0.	0.	109.327

Problem 707	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	178	526	0	1	0	0	236
normalized size	1	1.	0.72	2.14	0.	0.	0.	0.	0.96
time (sec)	N/A	1.141	0.738	0.04	0.	0.569	0.	0.	108.741

Problem 708	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	181	441	0	1	0	0	245
normalized size	1	1.	0.72	1.74	0.	0.	0.	0.	0.97
time (sec)	N/A	1.132	0.744	0.038	0.	0.297	0.	0.	107.933

Problem 709	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	204	665	0	1	0	0	311
normalized size	1	1.	0.63	2.06	0.	0.	0.	0.	0.96
time (sec)	N/A	1.629	1.043	0.041	0.	0.304	0.	0.	144.115

Problem 710	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	231	924	0	1	0	0	0
normalized size	1	1.	0.59	2.35	0.	0.	0.	0.	0.
time (sec)	N/A	1.837	1.428	0.049	0.	0.319	0.	0.	0.

Problem 711	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	463	258	1261	0	1	0	0	0
normalized size	1	1.	0.56	2.72	0.	0.	0.	0.	0.
time (sec)	N/A	2.416	1.795	0.049	0.	0.341	0.	0.	0.

Problem 712	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	205	511	0	1	0	0	301
normalized size	1	1.	0.65	1.63	0.	0.	0.	0.	0.96
time (sec)	N/A	1.467	0.387	0.052	0.	1.057	0.	0.	121.9



Problem 713	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	168	328	0	1	0	0	233
normalized size	1	1.	0.69	1.34	0.	0.	0.	0.	0.95
time (sec)	N/A	1.035	0.247	0.035	0.	0.951	0.	0.	89.139

Problem 714	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	157	201	0	1	0	0	160
normalized size	1	1.	0.93	1.19	0.	0.01	0.	0.	0.95
time (sec)	N/A	0.655	0.229	0.029	0.	0.92	0.	0.	60.783

Problem 715	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	109	112	0	1	0	0	102
normalized size	1	1.	1.04	1.07	0.	0.01	0.	0.	0.97
time (sec)	N/A	0.323	0.091	0.038	0.	0.895	0.	0.	38.173

Problem 716	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	63	0	154	0	0	58
normalized size	1	1.	0.82	1.03	0.	2.52	0.	0.	0.95
time (sec)	N/A	0.254	0.092	0.01	0.	0.282	0.	0.	22.125

Problem 717	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	69	98	0	389	0	0	122
normalized size	1	1.	0.53	0.76	0.	3.02	0.	0.	0.95
time (sec)	N/A	0.516	0.146	0.011	0.	0.285	0.	0.	43.697

Problem 718	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	105	169	0	772	0	0	190
normalized size	1	1.	0.53	0.85	0.	3.9	0.	0.	0.96
time (sec)	N/A	0.796	0.255	0.013	0.	0.29	0.	0.	70.024

Problem 719	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	128	260	0	1287	0	0	258
normalized size	1	1.	0.48	0.97	0.	4.82	0.	0.	0.97
time (sec)	N/A	1.1	0.367	0.017	0.	0.298	0.	0.	99.956

Problem 720	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	199	648	0	1	0	4	292
normalized size	1	1.	0.66	2.15	0.	0.	0.	0.01	0.97
time (sec)	N/A	1.322	0.447	0.048	0.	0.959	0.	0.756	120.841

Problem 721	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	161	396	0	1	0	4	219
normalized size	1	1.	0.71	1.74	0.	0.	0.	0.02	0.96
time (sec)	N/A	0.963	0.289	0.039	0.	0.936	0.	0.694	88.049

Problem 722	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	131	210	0	1	0	4	153
normalized size	1	1.	0.81	1.3	0.	0.01	0.	0.02	0.95
time (sec)	N/A	0.617	0.189	0.034	0.	0.898	0.	0.633	62.465

Problem 723	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	50	63	0	169	0	0	56
normalized size	1	1.	0.82	1.03	0.	2.77	0.	0.	0.92
time (sec)	N/A	0.252	0.078	0.01	0.	0.284	0.	0.	21.753

Problem 724	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	64	97	0	439	0	0	117
normalized size	1	1.	0.52	0.78	0.	3.54	0.	0.	0.94
time (sec)	N/A	0.522	0.143	0.009	0.	0.29	0.	0.	44.345

Problem 725	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	105	168	0	876	0	0	185
normalized size	1	1.	0.55	0.88	0.	4.56	0.	0.	0.96
time (sec)	N/A	0.793	0.232	0.014	0.	0.299	0.	0.	69.304

Problem 726	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	150	259	0	1434	0	0	255
normalized size	1	1.	0.57	0.99	0.	5.47	0.	0.	0.97
time (sec)	N/A	1.093	0.34	0.017	0.	0.318	0.	0.	98.606

Problem 727	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	202	652	0	1	0	4	280
normalized size	1	1.	0.7	2.26	0.	0.	0.	0.01	0.97
time (sec)	N/A	1.25	0.55	0.043	0.	0.964	0.	1.026	118.211

Problem 728	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	150	343	0	1	0	4	211
normalized size	1	1.	0.68	1.57	0.	0.	0.	0.02	0.96
time (sec)	N/A	0.887	0.363	0.042	0.	0.924	0.	0.856	89.258

Problem 729	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	63	0	261	0	4	58
normalized size	1	1.	0.83	1.	0.	4.14	0.	0.06	0.92
time (sec)	N/A	0.252	0.096	0.01	0.	0.289	0.	0.736	21.743

Problem 730	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	68	99	0	429	0	0	121
normalized size	1	1.	0.53	0.77	0.	3.35	0.	0.	0.95
time (sec)	N/A	0.532	0.144	0.011	0.	0.292	0.	0.	44.197

Problem 731	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	103	169	0	900	0	0	185
normalized size	1	1.	0.53	0.87	0.	4.64	0.	0.	0.95
time (sec)	N/A	0.794	0.233	0.013	0.	0.305	0.	0.	70.272

Problem 732	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	260	152	258	0	1438	0	0	252
normalized size	1	1.	0.58	0.99	0.	5.53	0.	0.	0.97
time (sec)	N/A	1.098	0.32	0.015	0.	0.322	0.	0.	99.327

Problem 733	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	247	870	0	1	0	0	364
normalized size	1	1.	0.64	2.26	0.	0.	0.	0.	0.95
time (sec)	N/A	1.904	0.41	0.032	0.	1.242	0.	0.	157.767

Problem 734	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	200	602	0	1	0	0	291
normalized size	1	1.	0.64	1.92	0.	0.	0.	0.	0.93
time (sec)	N/A	1.431	0.291	0.027	0.	1.049	0.	0.	120.556

Problem 735	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	178	385	0	1	0	0	223
normalized size	1	1.	0.74	1.6	0.	0.	0.	0.	0.93
time (sec)	N/A	1.006	0.271	0.023	0.	0.945	0.	0.	88.292

Problem 736	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	163	198	0	1	0	0	158
normalized size	1	1.	0.98	1.19	0.	0.01	0.	0.	0.95
time (sec)	N/A	0.642	0.194	0.027	0.	0.931	0.	0.	61.303

Problem 737	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	147	197	0	1	0	0	151
normalized size	1	1.	0.93	1.25	0.	0.01	0.	0.	0.96
time (sec)	N/A	0.61	0.189	0.037	0.	0.902	0.	0.	60.859

Problem 738	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	63	0	228	0	0	60
normalized size	1	1.	0.83	1.	0.	3.62	0.	0.	0.95
time (sec)	N/A	0.255	0.091	0.01	0.	0.286	0.	0.	21.854

Problem 739	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	69	99	0	543	0	0	122
normalized size	1	1.	0.53	0.77	0.	4.21	0.	0.	0.95
time (sec)	N/A	0.528	0.145	0.011	0.	0.289	0.	0.	43.631

Problem 740	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	105	169	0	1010	0	0	190
normalized size	1	1.	0.53	0.85	0.	5.1	0.	0.	0.96
time (sec)	N/A	0.816	0.212	0.013	0.	0.297	0.	0.	69.286

Problem 741	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	152	260	0	1592	0	0	258
normalized size	1	1.	0.57	0.97	0.	5.96	0.	0.	0.97
time (sec)	N/A	1.1	0.356	0.017	0.	0.305	0.	0.	99.016

Problem 742	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	382	254	870	0	1	0	0	364
normalized size	1	1.	0.66	2.28	0.	0.	0.	0.	0.95
time (sec)	N/A	1.873	0.65	0.028	0.	1.278	0.	0.	155.836

Problem 743	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	210	602	0	1	0	0	291
normalized size	1	1.	0.68	1.94	0.	0.	0.	0.	0.94
time (sec)	N/A	1.41	0.464	0.028	0.	1.05	0.	0.	120.398

Problem 744	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	181	325	0	1	0	0	228
normalized size	1	1.	0.76	1.37	0.	0.	0.	0.	0.96
time (sec)	N/A	1.011	0.415	0.033	0.	0.952	0.	0.	89.268

Problem 745	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	164	383	0	1	0	0	216
normalized size	1	1.	0.74	1.73	0.	0.	0.	0.	0.97
time (sec)	N/A	0.935	0.579	0.037	0.	0.941	0.	0.	87.702

Problem 746	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	167	331	0	1	0	0	207
normalized size	1	1.	0.78	1.55	0.	0.	0.	0.	0.97
time (sec)	N/A	0.881	0.588	0.042	0.	0.909	0.	0.	84.913

Problem 747	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	52	63	308	313	0	0	60
normalized size	1	1.	0.83	1.	4.89	4.97	0.	0.	0.95
time (sec)	N/A	0.255	0.17	0.01	1.067	0.289	0.	0.	21.662

Problem 748	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	69	99	707	710	0	0	122
normalized size	1	1.	0.53	0.77	5.48	5.5	0.	0.	0.95
time (sec)	N/A	0.526	0.206	0.012	1.205	0.296	0.	0.	43.176

Problem 749	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	105	169	1245	1239	0	0	190
normalized size	1	1.	0.53	0.85	6.29	6.26	0.	0.	0.96
time (sec)	N/A	0.814	0.284	0.014	1.402	0.307	0.	0.	68.861

Problem 750	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	152	260	1928	1917	0	0	258
normalized size	1	1.	0.57	0.97	7.22	7.18	0.	0.	0.97
time (sec)	N/A	1.113	0.387	0.017	1.689	0.318	0.	0.	98.057

Problem 751	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	448	448	315	1191	0	1	0	0	0
normalized size	1	1.	0.7	2.66	0.	0.	0.	0.	0.
time (sec)	N/A	2.335	1.113	0.035	0.	1.708	0.	0.	0.

Problem 752	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	256	870	0	1	0	0	360
normalized size	1	1.	0.68	2.31	0.	0.	0.	0.	0.96
time (sec)	N/A	1.841	0.686	0.03	0.	1.247	0.	0.	153.646

Problem 753	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	304	304	205	508	0	1	0	0	292
normalized size	1	1.	0.67	1.67	0.	0.	0.	0.	0.96
time (sec)	N/A	1.399	0.475	0.038	0.	1.057	0.	0.	118.3

Problem 754	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	202	635	0	1	0	0	287
normalized size	1	1.	0.69	2.16	0.	0.	0.	0.	0.98
time (sec)	N/A	1.305	0.596	0.042	0.	0.954	0.	0.	117.598

Problem 755	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	204	638	0	1	0	0	277
normalized size	1	1.	0.72	2.25	0.	0.	0.	0.	0.98
time (sec)	N/A	1.228	0.668	0.039	0.	0.941	0.	0.	112.763

Problem 756	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	185	511	0	1	0	0	267
normalized size	1	1.	0.68	1.86	0.	0.	0.	0.	0.97
time (sec)	N/A	1.181	0.695	0.046	0.	0.916	0.	0.	109.725

Problem 757	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	62	63	0	404	0	0	60
normalized size	1	1.	0.98	1.	0.	6.41	0.	0.	0.95
time (sec)	N/A	0.253	0.212	0.01	0.	0.295	0.	0.	21.825

Problem 758	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	79	99	0	863	0	0	122
normalized size	1	1.	0.61	0.77	0.	6.69	0.	0.	0.95
time (sec)	N/A	0.529	0.265	0.011	0.	0.303	0.	0.	43.164

Problem 759	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	115	169	0	1486	0	0	190
normalized size	1	1.	0.58	0.85	0.	7.51	0.	0.	0.96
time (sec)	N/A	0.81	0.338	0.013	0.	0.314	0.	0.	68.835

Problem 760	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	162	260	0	2225	0	0	258
normalized size	1	1.	0.61	0.97	0.	8.33	0.	0.	0.97
time (sec)	N/A	1.122	0.405	0.017	0.	0.338	0.	0.	98.549

Problem 761	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	122	100	0	0	0	0	4	114
normalized size	1	1.17	0.96	0.	0.	0.	0.	0.04	1.1
time (sec)	N/A	0.37	0.209	0.157	0.	0.	0.	0.806	61.664

Problem 762	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	110	98	0	0	0	0	4	109
normalized size	1	1.06	0.94	0.	0.	0.	0.	0.04	1.05
time (sec)	N/A	0.353	0.098	0.126	0.	0.	0.	1.064	61.517

Problem 763	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	118	98	0	0	0	0	0	97
normalized size	1	1.13	0.94	0.	0.	0.	0.	0.	0.93
time (sec)	N/A	0.345	0.095	0.11	0.	0.	0.	0.	61.876

Problem 764	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	120	100	0	0	0	0	0	107
normalized size	1	1.15	0.96	0.	0.	0.	0.	0.	1.03
time (sec)	N/A	0.339	0.125	0.112	0.	0.	0.	0.	62.032

Problem 765	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	122	100	0	0	0	0	0	109
normalized size	1	1.17	0.96	0.	0.	0.	0.	0.	1.05
time (sec)	N/A	0.342	0.201	0.172	0.	0.	0.	0.	61.446

Problem 766	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	122	110	0	0	0	0	0	109
normalized size	1	1.17	1.06	0.	0.	0.	0.	0.	1.05
time (sec)	N/A	0.348	0.152	0.107	0.	0.	0.	0.	61.519

Problem 767	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	107	95	0	0	0	0	0	94
normalized size	1	1.04	0.92	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	0.2	0.196	0.24	0.	0.	0.	0.	47.769

Problem 768	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	233	527	447	952	0	1	0
normalized size	1	1.	0.68	1.54	1.3	2.78	0.	0.	0.
time (sec)	N/A	0.932	0.302	0.015	0.757	0.288	0.	0.277	0.



Problem 769	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	131	235	261	473	0	1	223
normalized size	1	1.	0.53	0.96	1.06	1.92	0.	0.	0.91
time (sec)	N/A	0.502	0.173	0.013	0.743	0.283	0.	0.275	111.687

Problem 770	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	67	89	127	196	0	298	136
normalized size	1	1.	0.45	0.59	0.85	1.31	0.	1.99	0.91
time (sec)	N/A	0.232	0.091	0.007	0.742	0.285	0.	0.278	59.749

Problem 771	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	42	57	45	77	0	68	42
normalized size	1	1.	0.78	1.06	0.83	1.43	0.	1.26	0.78
time (sec)	N/A	0.042	0.031	0.002	0.737	0.281	0.	0.276	18.313

Problem 772	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	87	0	0	0	0	0	95
normalized size	1	1.	0.88	0.	0.	0.	0.	0.	0.96
time (sec)	N/A	0.168	0.095	0.134	0.	0.	0.	0.	35.377

Problem 773	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	99
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.98
time (sec)	N/A	0.158	0.079	0.139	0.	0.	0.	0.	35.475

Problem 774	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	0	0	0	0	0	0	104
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.99
time (sec)	N/A	0.173	0.097	0.156	0.	0.	0.	0.	39.023

Problem 775	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	234	0	0	0	0	0	94
normalized size	1	1.	2.23	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.214	0.621	0.112	0.	0.	0.	0.	46.422

Problem 776	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	93	0	0	0	0	0	94
normalized size	1	1.	0.89	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.208	0.104	0.114	0.	0.	0.	0.	46.532

Problem 777	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	91	0	0	0	0	0	92
normalized size	1	1.	0.88	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.204	0.092	0.115	0.	0.	0.	0.	47.076

Problem 778	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	91	0	0	0	0	0	95
normalized size	1	1.	0.88	0.	0.	0.	0.	0.	0.92
time (sec)	N/A	0.204	0.087	0.114	0.	0.	0.	0.	46.442

Problem 779	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	93	0	0	0	0	0	99
normalized size	1	1.	0.89	0.	0.	0.	0.	0.	0.94
time (sec)	N/A	0.207	0.105	0.115	0.	0.	0.	0.	46.551

Problem 780	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	53	64	66	89	0	108	54
normalized size	1	1.	0.82	0.98	1.02	1.37	0.	1.66	0.83
time (sec)	N/A	0.123	0.046	0.006	0.709	0.28	0.	0.278	40.436

Problem 781	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	64	0	43	47	0	0	73
normalized size	1	1.	0.82	0.	0.55	0.6	0.	0.	0.94
time (sec)	N/A	0.326	0.055	0.311	0.721	0.274	0.	0.	174.401

Problem 782	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	222	157	0	0	0	0	0	202
normalized size	1	1.04	0.74	0.	0.	0.	0.	0.	0.95
time (sec)	N/A	0.774	0.288	0.112	0.	0.	0.	0.	89.994

Problem 783	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	501	501	380	641	936	1504	0	1	0
normalized size	1	1.	0.76	1.28	1.87	3.	0.	0.	0.
time (sec)	N/A	2.277	0.588	0.014	0.773	0.285	0.	0.415	0.

Problem 784	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	412	412	264	425	653	1061	0	1	418
normalized size	1	1.	0.64	1.03	1.58	2.58	0.	0.	1.01
time (sec)	N/A	1.682	0.43	0.013	0.756	0.276	0.	0.391	139.16

Problem 785	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	169	255	417	689	0	0	325
normalized size	1	1.	0.53	0.79	1.3	2.15	0.	0.	1.01
time (sec)	N/A	1.2	0.271	0.013	0.737	0.271	0.	0.	95.142

Problem 786	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	96	131	227	383	0	0	202
normalized size	1	1.	0.46	0.63	1.09	1.83	0.	0.	0.97
time (sec)	N/A	0.544	0.13	0.006	0.725	0.269	0.	0.	54.741

Problem 787	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	54	69	88	167	0	0	100
normalized size	1	1.	0.5	0.63	0.81	1.53	0.	0.	0.92
time (sec)	N/A	0.174	0.056	0.005	0.721	0.264	0.	0.	32.362

Problem 788	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	141	163	0	1	0	0	129
normalized size	1	1.	1.01	1.17	0.	0.01	0.	0.	0.93
time (sec)	N/A	0.639	0.33	0.029	0.	0.285	0.	0.	58.189

Problem 789	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	153	347	0	1	0	0	160
normalized size	1	1.	0.9	2.04	0.	0.01	0.	0.	0.94
time (sec)	N/A	0.726	0.487	0.036	0.	0.29	0.	0.	65.465

Problem 790	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	193	673	0	1	0	0	250
normalized size	1	1.	0.74	2.58	0.	0.	0.	0.	0.96
time (sec)	N/A	1.071	0.979	0.05	0.	0.297	0.	0.	97.937

Problem 791	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	351	245	1142	0	1	0	0	340
normalized size	1	1.	0.7	3.25	0.	0.	0.	0.	0.97
time (sec)	N/A	1.652	1.7	0.051	0.	0.311	0.	0.	136.285

Problem 792	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	229	602	554	1593	0	518	309
normalized size	1	1.	0.71	1.86	1.71	4.92	0.	1.6	0.95
time (sec)	N/A	1.42	0.365	0.084	0.794	0.29	0.	0.318	169.637

Problem 793	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	114	291	275	717	0	243	128
normalized size	1	1.	0.69	1.75	1.66	4.32	0.	1.46	0.77
time (sec)	N/A	0.535	0.172	0.033	0.78	0.281	0.	0.292	44.772

Problem 794	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	45	117	105	244	282	97	41
normalized size	1	1.	0.71	1.86	1.67	3.87	4.48	1.54	0.65
time (sec)	N/A	0.132	0.061	0.023	0.768	0.273	55.151	0.286	16.306

Problem 795	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	A	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	455	1759	0	5823	0	0	284
normalized size	1	1.	1.61	6.24	0.	20.65	0.	0.	1.01
time (sec)	N/A	0.993	1.156	0.14	0.	0.425	0.	0.	78.975

Problem 796	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	571	571	800	41837	0	0	0	0	0
normalized size	1	1.	1.4	73.27	0.	0.	0.	0.	0.
time (sec)	N/A	10.481	4.032	0.77	0.	0.	0.	0.	0.

Problem 797	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	226	755	549	1362	0	961	0
normalized size	1	1.	0.82	2.74	1.99	4.93	0.	3.48	0.
time (sec)	N/A	1.022	0.977	0.053	0.775	0.301	0.	0.343	0.

Problem 798	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	114	380	270	593	0	509	155
normalized size	1	1.	0.83	2.75	1.96	4.3	0.	3.69	1.12
time (sec)	N/A	0.327	0.262	0.036	0.773	0.286	0.	0.304	97.21

Problem 799	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	39	147	99	188	255	246	34
normalized size	1	1.	0.98	3.68	2.48	4.7	6.38	6.15	0.85
time (sec)	N/A	0.101	0.123	0.026	0.776	0.282	91.982	0.293	15.844

Problem 800	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	443	443	658	11142	0	0	0	0	0
normalized size	1	1.	1.49	25.15	0.	0.	0.	0.	0.
time (sec)	N/A	3.214	1.605	0.13	0.	0.	0.	0.	0.

Problem 801	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	C	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	939	938	1656	108974	0	0	0	0	0
normalized size	1	1.	1.76	116.05	0.	0.	0.	0.	0.
time (sec)	N/A	18.134	6.663	3.458	0.	0.	0.	0.	0.

Problem 802	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	167	0	0	0	0	0	41
normalized size	1	1.	3.09	0.	0.	0.	0.	0.	0.76
time (sec)	N/A	0.114	0.356	0.226	0.	0.	0.	0.	23.403

Problem 803	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	0	0	0	0	0	0	71
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.8
time (sec)	N/A	0.178	0.138	0.219	0.	0.	0.	0.	41.34

Problem 804	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0	75
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.82
time (sec)	N/A	0.193	0.117	0.23	0.	0.	0.	0.	48.801

Problem 805	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	577	2017	0	2743	0	1	274
normalized size	1	1.	2.1	7.33	0.	9.97	0.	0.	1.
time (sec)	N/A	0.569	1.364	0.019	0.	0.313	0.	0.278	118.36

Problem 806	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	348	1048	0	1515	11924	1	206
normalized size	1	1.	1.67	5.04	0.	7.28	57.33	0.	0.99
time (sec)	N/A	0.408	0.563	0.015	0.	0.307	34.964	0.27	87.53

Problem 807	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	190	449	0	741	4935	1	141
normalized size	1	1.	1.3	3.08	0.	5.08	33.8	0.01	0.97
time (sec)	N/A	0.258	0.307	0.009	0.	0.301	14.981	0.267	54.519

Problem 808	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	92	147	0	294	1530	558	78
normalized size	1	1.	1.1	1.75	0.	3.5	18.21	6.64	0.93
time (sec)	N/A	0.121	0.115	0.007	0.	0.294	6.115	0.267	25.739

Problem 809	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	125	0	0	0	0	0	90
normalized size	1	1.	1.1	0.	0.	0.	0.	0.	0.79
time (sec)	N/A	0.302	0.558	0.065	0.	0.	0.	0.	35.314

Problem 810	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	0	0	0	0	0	0	68
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.77
time (sec)	N/A	0.195	0.18	0.076	0.	0.	0.	0.	29.123

Problem 811	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	193	193	0	0	0	0	0	0	95
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.49
time (sec)	N/A	0.468	0.233	0.109	0.	0.	0.	0.	37.296

Problem 812	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	0	0	0	0	0	0	99
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.5
time (sec)	N/A	0.514	0.283	0.161	0.	0.	0.	0.	38.834

Problem 813	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	231	227	328	0	0	0	0	0	231
normalized size	1	0.98	1.42	0.	0.	0.	0.	0.	1.
time (sec)	N/A	0.606	1.301	0.089	0.	0.	0.	0.	104.94

Problem 814	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	85	142	117	134	420	0	0
normalized size	1	1.	1.02	1.71	1.41	1.61	5.06	0.	0.
time (sec)	N/A	0.205	0.096	0.011	0.693	0.295	15.426	0.	0.

Problem 815	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	177	444	344	423	989	0	0
normalized size	1	1.	0.96	2.41	1.87	2.3	5.38	0.	0.
time (sec)	N/A	0.603	0.272	0.015	0.696	0.6	117.993	0.	0.

Problem 816	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	531	531	476	1232	973	994	0	0	0
normalized size	1	1.	0.9	2.32	1.83	1.87	0.	0.	0.
time (sec)	N/A	2.85	1.138	0.024	0.703	4.418	0.	0.	0.

Problem 817	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	246	606	0	0	0	529	0
normalized size	1	1.	1.	2.46	0.	0.	0.	2.15	0.
time (sec)	N/A	0.924	0.653	0.015	0.	0.	0.	0.268	0.

Problem 818	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	644	644	710	11490	0	0	0	0	0
normalized size	1	1.	1.1	17.84	0.	0.	0.	0.	0.
time (sec)	N/A	5.938	5.479	0.082	0.	0.	0.	0.	0.

Problem 819	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	A	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	412	540	579	579	1544	1	0
normalized size	1	1.	1.44	1.88	2.02	2.02	5.38	0.	0.
time (sec)	N/A	1.021	1.041	0.011	0.699	0.293	110.025	0.268	0.

Problem 820	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	256	315	352	351	1001	579	0
normalized size	1	1.	1.21	1.49	1.66	1.66	4.72	2.73	0.
time (sec)	N/A	0.693	0.331	0.01	0.697	0.277	64.999	0.266	0.

Problem 821	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	131	144	174	169	549	309	139
normalized size	1	1.	0.96	1.05	1.27	1.23	4.01	2.26	1.01
time (sec)	N/A	0.25	0.192	0.006	0.691	0.271	32.741	0.263	38.118

Problem 822	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	54	53	104	73	223	116	68
normalized size	1	1.	0.74	0.73	1.42	1.	3.05	1.59	0.93
time (sec)	N/A	0.089	0.049	0.006	0.689	0.269	7.707	0.265	14.437

Problem 823	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	104	189	0	1	257	173	107
normalized size	1	1.	0.9	1.63	0.	0.01	2.22	1.49	0.92
time (sec)	N/A	0.338	0.158	0.014	0.	0.285	30.853	0.265	80.066

Problem 824	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-2)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	133	371	0	1	0	236	144
normalized size	1	1.	0.95	2.65	0.	0.01	0.	1.69	1.03
time (sec)	N/A	0.603	0.448	0.022	0.	0.294	0.	0.267	128.764



Problem 825	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	191	538	0	1	0	504	0
normalized size	1	1.	0.93	2.61	0.	0.	0.	2.45	0.
time (sec)	N/A	0.884	0.639	0.026	0.	0.294	0.	0.274	0.

Problem 826	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	406	540	590	578	0	903	0
normalized size	1	1.	1.42	1.89	2.07	2.03	0.	3.17	0.
time (sec)	N/A	1.008	0.752	0.01	0.697	0.272	0.	0.277	0.

Problem 827	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	252	315	363	350	0	545	0
normalized size	1	1.	1.2	1.5	1.73	1.67	0.	2.6	0.
time (sec)	N/A	0.709	0.342	0.011	0.699	0.272	0.	0.272	0.

Problem 828	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	128	144	185	169	2720	275	138
normalized size	1	1.	0.95	1.07	1.37	1.25	20.15	2.04	1.02
time (sec)	N/A	0.239	0.16	0.008	0.696	0.269	26.13	0.271	38.081

Problem 829	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	54	53	89	72	590	100	70
normalized size	1	1.	0.76	0.75	1.25	1.01	8.31	1.41	0.99
time (sec)	N/A	0.088	0.054	0.005	0.684	0.269	11.085	0.264	14.531

Problem 830	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	118	237	0	1	0	151	0
normalized size	1	1.	0.97	1.94	0.	0.01	0.	1.24	0.
time (sec)	N/A	0.482	0.406	0.019	0.	0.284	0.	0.269	0.

Problem 831	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	155	418	0	1	0	381	0
normalized size	1	1.	0.94	2.53	0.	0.01	0.	2.31	0.
time (sec)	N/A	0.773	0.877	0.03	0.	0.292	0.	0.277	0.

Problem 832	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	223	847	0	1	0	624	0
normalized size	1	1.	0.9	3.43	0.	0.	0.	2.53	0.
time (sec)	N/A	1.343	2.162	0.034	0.	0.301	0.	0.279	0.

Problem 833	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	B	A	B	F	A	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	191	143	226	0	308	0	0	187
normalized size	1	2.1	1.57	2.48	0.	3.38	0.	0.	2.05
time (sec)	N/A	0.298	0.776	0.094	0.	0.297	0.	0.	33.414

Problem 834	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	154	425	0	1	0	242	199
normalized size	1	1.	0.94	2.59	0.	0.01	0.	1.48	1.21
time (sec)	N/A	0.413	0.208	0.032	0.	1.474	0.	0.288	37.912

Problem 835	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	306	1207	0	1	0	605	434
normalized size	1	1.	0.92	3.62	0.	0.	0.	1.82	1.3
time (sec)	N/A	0.788	0.573	0.041	0.	1.805	0.	0.3	74.448

Problem 836	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	212	763	0	1	0	393	318
normalized size	1	1.	0.86	3.1	0.	0.	0.	1.6	1.29
time (sec)	N/A	0.585	0.297	0.03	0.	1.371	0.	0.291	51.914

Problem 837	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	154	425	0	1	0	242	199
normalized size	1	1.	0.94	2.59	0.	0.01	0.	1.48	1.21
time (sec)	N/A	0.377	0.174	0.	0.	1.497	0.	0.287	38.962

Problem 838	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	136	697	0	1	0	4	182
normalized size	1	1.	1.05	5.4	0.	0.01	0.	0.03	1.41
time (sec)	N/A	0.333	0.342	0.037	0.	1.633	0.	0.592	40.844

Problem 839	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	157	773	0	1	0	4	207
normalized size	1	1.	0.98	4.83	0.	0.01	0.	0.02	1.29
time (sec)	N/A	0.426	0.347	0.038	0.	1.621	0.	0.601	48.201

Problem 840	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	178	238	0	477	0	1	275
normalized size	1	1.	0.9	1.2	0.	2.41	0.	0.01	1.39
time (sec)	N/A	0.513	0.474	0.012	0.	1.591	0.	0.387	62.5

Problem 841	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	230	468	0	865	0	1	389
normalized size	1	1.	0.82	1.67	0.	3.08	0.	0.	1.38
time (sec)	N/A	0.7	1.697	0.014	0.	2.46	0.	0.482	88.082

Problem 842	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	180	834	0	1	0	320	311
normalized size	1	1.	0.72	3.35	0.	0.	0.	1.29	1.25
time (sec)	N/A	0.607	0.264	0.048	0.	0.715	0.	0.301	69.546

Problem 843	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	202	571	0	1	0	1	396
normalized size	1	1.	0.84	2.38	0.	0.	0.	0.	1.65
time (sec)	N/A	0.486	0.223	0.049	0.	0.344	0.	0.381	75.985

Problem 844	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	159	392	0	1	0	602	287
normalized size	1	1.	0.9	2.23	0.	0.01	0.	3.42	1.63
time (sec)	N/A	0.373	0.149	0.03	0.	0.314	0.	0.332	54.633

Problem 845	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	115	247	0	1	0	196	184
normalized size	1	1.	0.94	2.02	0.	0.01	0.	1.61	1.51
time (sec)	N/A	0.287	0.109	0.035	0.	0.318	0.	0.285	43.061

Problem 846	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	112	438	0	1	0	261	139
normalized size	1	1.	1.04	4.06	0.	0.01	0.	2.42	1.29
time (sec)	N/A	0.273	0.206	0.049	0.	0.391	0.	0.299	45.332

Problem 847	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	111	601	0	1	0	294	150
normalized size	1	1.	0.96	5.18	0.	0.01	0.	2.53	1.29
time (sec)	N/A	0.278	0.241	0.04	0.	0.451	0.	0.313	49.17

Problem 848	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	110	150	0	396	0	455	207
normalized size	1	1.	0.83	1.13	0.	2.98	0.	3.42	1.56
time (sec)	N/A	0.322	0.171	0.014	0.	0.408	0.	0.319	58.137

Problem 849	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	148	248	0	657	0	707	308
normalized size	1	1.	0.78	1.31	0.	3.48	0.	3.74	1.63
time (sec)	N/A	0.448	0.366	0.017	0.	0.782	0.	0.362	80.907

Problem 850	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	1319	11688	0	0	0	4	0
normalized size	1	1.	3.16	28.03	0.	0.	0.	0.01	0.
time (sec)	N/A	5.753	6.212	0.174	0.	0.	0.	0.637	0.

Problem 851	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	A	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	925	5484	0	6036	0	0	267
normalized size	1	1.	3.25	19.24	0.	21.18	0.	0.	0.94
time (sec)	N/A	1.197	4.752	0.06	0.	40.961	0.	0.	178.437

Problem 852	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	1000	5507	0	0	0	0	0
normalized size	1	1.	3.48	19.19	0.	0.	0.	0.	0.
time (sec)	N/A	1.055	6.239	0.072	0.	0.	0.	0.	0.

Problem 853	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	F(-1)	F(-1)	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	1011	47349	0	0	0	0	0
normalized size	1	1.	2.36	110.37	0.	0.	0.	0.	0.
time (sec)	N/A	3.354	2.845	0.266	0.	0.	0.	0.	0.

Problem 854	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	532	532	553	3941	0	0	0	0	578
normalized size	1	1.	1.04	7.41	0.	0.	0.	0.	1.09
time (sec)	N/A	3.341	1.923	0.048	0.	0.	0.	0.	153.488

Problem 855	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	356	2602	0	0	0	0	366
normalized size	1	1.	1.1	8.01	0.	0.	0.	0.	1.13
time (sec)	N/A	1.472	0.795	0.022	0.	0.	0.	0.	113.029

Problem 856	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	253	1559	0	0	0	0	211
normalized size	1	1.	1.16	7.12	0.	0.	0.	0.	0.96
time (sec)	N/A	0.712	0.85	0.012	0.	0.	0.	0.	101.168

Problem 857	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	169	715	0	1	0	0	138
normalized size	1	1.	1.11	4.7	0.	0.01	0.	0.	0.91
time (sec)	N/A	0.337	0.556	0.007	0.	1.345	0.	0.	54.487

Problem 858	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	299	1529	0	0	0	0	199
normalized size	1	1.	1.31	6.71	0.	0.	0.	0.	0.87
time (sec)	N/A	0.683	0.653	0.037	0.	0.	0.	0.	96.734

Problem 859	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	490	490	294	3162	0	0	0	0	0
normalized size	1	1.	0.6	6.45	0.	0.	0.	0.	0.
time (sec)	N/A	1.51	0.553	0.029	0.	0.	0.	0.	0.

Problem 860	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	673	673	523	6714	0	0	0	4	0
normalized size	1	1.	0.78	9.98	0.	0.	0.	0.01	0.
time (sec)	N/A	2.082	2.994	0.032	0.	0.	0.	1.601	0.

Problem 861	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-2)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	933	933	1421	11995	0	0	0	4	0
normalized size	1	1.	1.52	12.86	0.	0.	0.	0.	0.
time (sec)	N/A	2.996	6.935	0.039	0.	0.	0.	14.254	0.

Problem 862	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1098	1098	1968	10058	0	0	0	0	0
normalized size	1	1.	1.79	9.16	0.	0.	0.	0.	0.
time (sec)	N/A	8.09	6.708	0.042	0.	0.	0.	0.	0.

Problem 863	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	662	662	740	6860	0	0	0	0	0
normalized size	1	1.	1.12	10.36	0.	0.	0.	0.	0.
time (sec)	N/A	3.461	3.615	0.026	0.	0.	0.	0.	0.

Problem 864	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	441	441	449	4188	0	0	0	0	0
normalized size	1	1.	1.02	9.5	0.	0.	0.	0.	0.
time (sec)	N/A	1.688	1.422	0.014	0.	0.	0.	0.	0.

Problem 865	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	255	1946	0	0	0	0	240
normalized size	1	1.	1.01	7.72	0.	0.	0.	0.	0.95
time (sec)	N/A	0.76	0.412	0.008	0.	0.	0.	0.	119.964

Problem 866	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	491	491	410	4226	0	0	0	0	0
normalized size	1	1.	0.84	8.61	0.	0.	0.	0.	0.
time (sec)	N/A	1.76	0.85	0.025	0.	0.	0.	0.	0.

Problem 867	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	787	787	441	7959	0	0	0	0	0
normalized size	1	1.	0.56	10.11	0.	0.	0.	0.	0.
time (sec)	N/A	3.085	1.76	0.032	0.	0.	0.	0.	0.

Problem 868	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1066	1066	690	15927	0	0	0	0	0
normalized size	1	1.	0.65	14.94	0.	0.	0.	0.	0.
time (sec)	N/A	4.02	2.116	0.04	0.	0.	0.	0.	0.

Problem 869	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	886	886	749	9052	0	0	0	0	0
normalized size	1	1.	0.85	10.22	0.	0.	0.	0.	0.
time (sec)	N/A	3.888	3.081	0.034	0.	0.	0.	0.	0.

Problem 870	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	402	1597	0	0	0	0	524
normalized size	1	1.	0.93	3.71	0.	0.	0.	0.	1.22
time (sec)	N/A	2.304	1.619	0.035	0.	0.	0.	0.	135.431

Problem 871	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	269	1007	0	0	0	0	332
normalized size	1	1.	1.	3.73	0.	0.	0.	0.	1.23
time (sec)	N/A	1.239	0.633	0.022	0.	0.	0.	0.	94.193

Problem 872	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	204	613	0	0	0	0	194
normalized size	1	1.	1.16	3.48	0.	0.	0.	0.	1.1
time (sec)	N/A	0.591	0.349	0.019	0.	0.	0.	0.	59.664

Problem 873	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	167	349	0	1	0	0	117
normalized size	1	1.	1.27	2.66	0.	0.01	0.	0.	0.89
time (sec)	N/A	0.236	0.319	0.011	0.	28.557	0.	0.	36.423

Problem 874	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	84	157	0	1	0	97	73
normalized size	1	1.	1.06	1.99	0.	0.01	0.	1.23	0.92
time (sec)	N/A	0.09	0.076	0.007	0.	0.356	0.	0.275	17.316

Problem 875	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F(-2)	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	222	327	0	1	0	0	162
normalized size	1	1.	1.22	1.8	0.	0.01	0.	0.	0.89
time (sec)	N/A	0.493	0.868	0.022	0.	124.009	0.	0.	64.146

Problem 876	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	A	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	343	788	0	1	0	0	304
normalized size	1	1.	1.01	2.32	0.	0.	0.	0.	0.89
time (sec)	N/A	0.903	1.585	0.026	0.	9.735	0.	0.	101.24

Problem 877	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	587	587	835	1817	0	0	0	4	0
normalized size	1	1.	1.42	3.1	0.	0.	0.	0.01	0.
time (sec)	N/A	1.856	6.929	0.03	0.	0.	0.	1.475	0.

Problem 878	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	496	496	587	4453	0	0	0	0	0
normalized size	1	1.	1.18	8.98	0.	0.	0.	0.	0.
time (sec)	N/A	2.328	5.971	0.035	0.	0.	0.	0.	0.

Problem 879	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	373	3127	0	0	0	0	376
normalized size	1	1.	1.04	8.76	0.	0.	0.	0.	1.05
time (sec)	N/A	1.179	1.586	0.022	0.	0.	0.	0.	143.946

Problem 880	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	265	2123	0	1	0	1022	240
normalized size	1	1.	1.1	8.85	0.	0.	0.	4.26	1.
time (sec)	N/A	0.655	0.987	0.021	0.	2.033	0.	0.283	103.567



Problem 881	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	217	1261	0	1	0	767	180
normalized size	1	1.	1.16	6.74	0.	0.01	0.	4.1	0.96
time (sec)	N/A	0.326	0.546	0.014	0.	2.01	0.	0.282	55.551

Problem 882	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	207	603	0	1	0	603	146
normalized size	1	1.	1.34	3.89	0.	0.01	0.	3.89	0.94
time (sec)	N/A	0.239	0.699	0.009	0.	0.663	0.	0.281	45.981

Problem 883	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F	F(-2)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	352	352	393	1343	0	0	0	0	318
normalized size	1	1.	1.12	3.82	0.	0.	0.	0.	0.9
time (sec)	N/A	1.011	2.755	0.026	0.	0.	0.	0.	152.818

Problem 884	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-2)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	642	642	575	2807	0	0	0	0	0
normalized size	1	1.	0.9	4.37	0.	0.	0.	0.	0.
time (sec)	N/A	2.029	9.43	0.034	0.	0.	0.	0.	0.

Problem 885	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1064	1064	1288	5459	0	0	0	0	0
normalized size	1	1.	1.21	5.13	0.	0.	0.	0.	0.
time (sec)	N/A	5.066	14.466	0.054	0.	0.	0.	0.	0.

Problem 886	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-1)	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1551	1551	32331	32647	0	0	0	0	0
normalized size	1	1.	20.85	21.05	0.	0.	0.	0.	0.
time (sec)	N/A	17.555	22.323	0.239	0.	0.	0.	0.	0.

Problem 887	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-1)	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1015	1015	15781	20224	0	0	0	0	0
normalized size	1	1.	15.55	19.93	0.	0.	0.	0.	0.
time (sec)	N/A	8.277	18.219	0.078	0.	0.	0.	0.	0.

Problem 888	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	652	652	8432	10711	0	0	0	0	0
normalized size	1	1.	12.93	16.43	0.	0.	0.	0.	0.
time (sec)	N/A	2.476	15.324	0.051	0.	0.	0.	0.	0.

Problem 889	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	513	513	3384	4356	0	0	0	0	0
normalized size	1	1.	6.6	8.49	0.	0.	0.	0.	0.
time (sec)	N/A	1.337	13.303	0.035	0.	0.	0.	0.	0.

Problem 890	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	764	969	35245	6812	0	0	0	0	0
normalized size	1	1.27	46.13	8.92	0.	0.	0.	0.	0.
time (sec)	N/A	8.971	17.472	0.06	0.	0.	0.	0.	0.

Problem 891	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	743	934	16573	16696	0	0	0	0	0
normalized size	1	1.26	22.31	22.47	0.	0.	0.	0.	0.
time (sec)	N/A	7.751	15.388	0.08	0.	0.	0.	0.	0.

Problem 892	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1034	1705	33765	55360	0	0	0	0	0
normalized size	1	1.65	32.65	53.54	0.	0.	0.	0.	0.
time (sec)	N/A	18.667	19.89	0.146	0.	0.	0.	0.	0.

Problem 893	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F(-1)	F(-1)	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1098	1098	17771	22215	0	0	0	0	0
normalized size	1	1.	16.18	20.23	0.	0.	0.	0.	0.
time (sec)	N/A	10.509	18.618	0.102	0.	0.	0.	0.	0.

Problem 894	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	755	755	10030	12923	0	0	0	0	0
normalized size	1	1.	13.28	17.12	0.	0.	0.	0.	0.
time (sec)	N/A	3.974	15.982	0.063	0.	0.	0.	0.	0.

Problem 895	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	519	519	4921	6207	0	0	0	0	0
normalized size	1	1.	9.48	11.96	0.	0.	0.	0.	0.
time (sec)	N/A	1.348	13.973	0.045	0.	0.	0.	0.	0.

Problem 896	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	444	444	591	1854	0	0	0	0	418
normalized size	1	1.	1.33	4.18	0.	0.	0.	0.	0.94
time (sec)	N/A	0.963	11.859	0.032	0.	0.	0.	0.	157.082

Problem 897	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	700	700	16471	3126	0	0	0	0	0
normalized size	1	1.	23.53	4.47	0.	0.	0.	0.	0.
time (sec)	N/A	5.397	14.235	0.047	0.	0.	0.	0.	0.

Problem 898	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	736	957	6911	13872	0	0	0	0	0
normalized size	1	1.3	9.39	18.85	0.	0.	0.	0.	0.
time (sec)	N/A	7.707	14.2	0.062	0.	0.	0.	0.	0.

Problem 899	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F(-1)	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1049	1747	36617	57835	0	0	0	0	0
normalized size	1	1.67	34.91	55.13	0.	0.	0.	0.	0.
time (sec)	N/A	18.686	19.813	0.142	0.	0.	0.	0.	0.

Problem 900	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	774	774	10649	14978	0	0	0	0	0
normalized size	1	1.	13.76	19.35	0.	0.	0.	0.	0.
time (sec)	N/A	4.465	16.258	0.086	0.	0.	0.	0.	0.

Problem 901	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	567	567	5536	8248	0	0	0	0	0
normalized size	1	1.	9.76	14.55	0.	0.	0.	0.	0.
time (sec)	N/A	2.047	14.333	0.065	0.	0.	0.	0.	0.

Problem 902	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	452	452	766	3805	0	0	0	0	0
normalized size	1	1.	1.69	8.42	0.	0.	0.	0.	0.
time (sec)	N/A	1.106	10.584	0.051	0.	0.	0.	0.	0.

Problem 903	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	365	747	0	0	0	0	177
normalized size	1	1.	1.94	3.97	0.	0.	0.	0.	0.94
time (sec)	N/A	0.247	1.291	0.044	0.	0.	0.	0.	42.495

Problem 904	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	467	467	379	834	0	0	0	0	440
normalized size	1	1.	0.81	1.79	0.	0.	0.	0.	0.94
time (sec)	N/A	4.52	1.515	0.057	0.	0.	0.	0.	148.005

Problem 905	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	994	994	18563	13017	0	0	0	0	0
normalized size	1	1.	18.68	13.1	0.	0.	0.	0.	0.
time (sec)	N/A	8.64	14.944	0.074	0.	0.	0.	0.	0.

Problem 906	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1786	1786	36634	59522	0	0	0	0	0
normalized size	1	1.	20.51	33.33	0.	0.	0.	0.	0.
time (sec)	N/A	19.5	20.658	0.193	0.	0.	0.	0.	0.

Problem 907	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	675	675	2358	1879	0	0	0	0	0
normalized size	1	1.	3.49	2.78	0.	0.	0.	0.	0.
time (sec)	N/A	4.725	6.364	0.057	0.	0.	0.	0.	0.

Problem 908	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1138	1138	37137	7464	0	0	0	0	0
normalized size	1	1.	32.63	6.56	0.	0.	0.	0.	0.
time (sec)	N/A	6.032	18.56	0.068	0.	0.	0.	0.	0.

Problem 909	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	631	631	12746	8755	0	0	0	0	0
normalized size	1	1.	20.2	13.87	0.	0.	0.	0.	0.
time (sec)	N/A	2.35	14.664	0.079	0.	0.	0.	0.	0.

Problem 910	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	6194	4295	0	0	0	0	0
normalized size	1	1.	12.93	8.97	0.	0.	0.	0.	0.
time (sec)	N/A	1.543	13.558	0.064	0.	0.	0.	0.	0.

Problem 911	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F	F	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	814	1014	0	0	0	0	369
normalized size	1	1.	2.07	2.58	0.	0.	0.	0.	0.94
time (sec)	N/A	0.791	10.247	0.054	0.	0.	0.	0.	116.989

Problem 912	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	308	287	0	0	0	0	178
normalized size	1	1.	1.63	1.52	0.	0.	0.	0.	0.94
time (sec)	N/A	0.279	1.392	0.048	0.	0.	0.	0.	50.613

Problem 913	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	499	330	0	0	0	0	262
normalized size	1	1.	1.78	1.18	0.	0.	0.	0.	0.94
time (sec)	N/A	4.031	2.282	0.057	0.	0.	0.	0.	32.887

Problem 914	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1037	1037	10881	14048	0	0	0	0	0
normalized size	1	1.	10.49	13.55	0.	0.	0.	0.	0.
time (sec)	N/A	9.417	15.258	0.096	0.	0.	0.	0.	0.

Problem 915	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F(-1)	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1114	1762	40396	64947	0	0	0	0	0
normalized size	1	1.58	36.26	58.3	0.	0.	0.	0.	0.
time (sec)	N/A	19.821	21.904	0.314	0.	0.	0.	0.	0.

Problem 916	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	553	1061	4757	0	0	0	0	0
normalized size	1	1.	1.92	8.6	0.	0.	0.	0.	0.
time (sec)	N/A	4.9	11.431	0.106	0.	0.	0.	0.	0.

Problem 917	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	B	F	F(-1)	F	F	F(-1)
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	1125	1125	14762	27601	0	0	0	0	0
normalized size	1	1.	13.12	24.53	0.	0.	0.	0.	0.
time (sec)	N/A	7.123	18.266	0.202	0.	0.	0.	0.	0.

Problem 918	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	A	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	475	475	2493	645	0	0	0	0	0
normalized size	1	1.	5.25	1.36	0.	0.	0.	0.	0.
time (sec)	N/A	0.883	13.57	0.192	0.	0.	0.	0.	0.

Problem 919	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	A	F	F	F	F	F(-1)
verified	N/A	NO	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	588	589	375	605	0	0	0	0	0
normalized size	1	1.	0.64	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	2.448	6.055	0.114	0.	0.	0.	0.	0.

Problem 920	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	440	1347	0	1864	15734	1	230
normalized size	1	1.	2.	6.12	0.	8.47	71.52	0.	1.05
time (sec)	N/A	0.509	0.886	0.017	0.	0.314	52.675	0.269	122.246

Problem 921	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	190	503	0	828	5846	1	141
normalized size	1	1.	1.32	3.49	0.	5.75	40.6	0.01	0.98
time (sec)	N/A	0.29	0.364	0.009	0.	0.312	16.98	0.266	57.313

Problem 922	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	166	0	0	0	0	0	107
normalized size	1	1.	1.29	0.	0.	0.	0.	0.	0.83
time (sec)	N/A	0.4	1.159	0.062	0.	0.	0.	0.	41.764

Problem 923	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	0	0	0	0	0	0	126
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.8
time (sec)	N/A	0.533	0.21	0.083	0.	0.	0.	0.	42.921

Problem 924	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	245	243	0	0	0	0	0	0	155
normalized size	1	0.99	0.	0.	0.	0.	0.	0.	0.63
time (sec)	N/A	0.847	0.287	0.11	0.	0.	0.	0.	51.278

Problem 925	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	525	525	1263	5890	0	6408	0	1	0
normalized size	1	1.	2.41	11.22	0.	12.21	0.	0.	0.
time (sec)	N/A	1.939	4.335	0.026	0.	0.349	0.	0.299	0.

Problem 926	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	B	F(-2)	A	F(-1)	A	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	311	593	2563	0	3197	0	1	0
normalized size	1	1.	1.91	8.24	0.	10.28	0.	0.	0.
time (sec)	N/A	0.984	1.701	0.019	0.	0.32	0.	0.28	0.

Problem 927	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	F(-1)
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	733	0	0	0	0	0	0
normalized size	1	1.	2.55	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.926	6.934	0.125	0.	0.	0.	0.	0.

Problem 928	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	0	0	0	0	0	0	277
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.93
time (sec)	N/A	2.338	0.402	0.14	0.	0.	0.	0.	148.022

Problem 929	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	461	461	0	0	0	0	0	0	265
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.57
time (sec)	N/A	4.351	0.693	0.182	0.	0.	0.	0.	112.77

Problem 930	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	568	0	0	0	0	0	144
normalized size	1	1.	3.1	0.	0.	0.	0.	0.	0.79
time (sec)	N/A	0.434	1.438	0.269	0.	0.	0.	0.	26.481

Problem 931	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	B	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	410	0	0	0	0	0	129
normalized size	1	1.	2.48	0.	0.	0.	0.	0.	0.78
time (sec)	N/A	0.305	0.506	0.163	0.	0.	0.	0.	24.17

Problem 932	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	188	0	0	0	0	0	114
normalized size	1	1.	1.28	0.	0.	0.	0.	0.	0.78
time (sec)	N/A	0.296	0.834	0.15	0.	0.	0.	0.	22.476

Problem 933	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	162	0	0	0	0	0	100
normalized size	1	1.	1.26	0.	0.	0.	0.	0.	0.78
time (sec)	N/A	0.239	0.442	0.144	0.	0.	0.	0.	19.208

Problem 934	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	144	0	0	0	0	0	95
normalized size	1	1.	1.23	0.	0.	0.	0.	0.	0.81
time (sec)	N/A	0.245	0.237	0.144	0.	0.	0.	0.	16.844

Problem 935	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	233	0	0	0	0	0	133
normalized size	1	1.	1.42	0.	0.	0.	0.	0.	0.81
time (sec)	N/A	0.461	0.95	0.171	0.	0.	0.	0.	33.157

Problem 936	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	284	0	0	0	0	0	163
normalized size	1	1.	1.43	0.	0.	0.	0.	0.	0.82
time (sec)	N/A	0.489	3.969	0.171	0.	0.	0.	0.	35.517



Problem 937	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	0	0	0	0	0	0	255
normalized size	1	1.	0.	0.	0.	0.	0.	0.	1.26
time (sec)	N/A	0.554	0.209	0.17	0.	0.	0.	0.	58.545

Problem 938	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	0	0	0	0	0	0	240
normalized size	1	1.	0.	0.	0.	0.	0.	0.	1.33
time (sec)	N/A	0.553	0.174	0.176	0.	0.	0.	0.	56.655

Problem 939	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	0	0	0	0	0	0	233
normalized size	1	1.	0.	0.	0.	0.	0.	0.	1.3
time (sec)	N/A	0.508	0.16	0.165	0.	0.	0.	0.	53.805

Problem 940	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	0	0	0	0	0	0	136
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.76
time (sec)	N/A	0.468	0.095	0.157	0.	0.	0.	0.	34.595

Problem 941	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	0	0	0	0	0	0	136
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.77
time (sec)	N/A	0.413	0.037	0.155	0.	0.	0.	0.	31.833

Problem 942	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	0	0	0	0	0	0	274
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.81
time (sec)	N/A	1.01	0.14	0.177	0.	0.	0.	0.	62.158

Problem 943	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	0	0	0	0	0	0	304
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.81
time (sec)	N/A	1.079	0.143	0.18	0.	0.	0.	0.	64.426

Problem 944	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	230	171	0	0	0	0	0	201
normalized size	1	0.97	0.72	0.	0.	0.	0.	0.	0.85
time (sec)	N/A	0.838	0.283	0.048	0.	0.	0.	0.	70.895

Problem 945	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	509	506	0	0	0	0	0	0	0
normalized size	1	0.99	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.363	2.126	0.129	0.	0.	0.	0.	0.

Problem 946	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	0	0	0	0	0	0	364
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.94
time (sec)	N/A	1.403	1.578	0.099	0.	0.	0.	0.	93.153

Problem 947	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	207	0	0	0	0	0	175
normalized size	1	1.	1.1	0.	0.	0.	0.	0.	0.93
time (sec)	N/A	0.629	0.02	0.096	0.	0.	0.	0.	37.118

Problem 948	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.12	0.09	0.113	0.	0.	0.	0.	0.

Problem 949	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	F(-1)
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	502	500	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	2.107	2.139	0.111	0.	0.	0.	0.	0.

Problem 950	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	0	0	0	0	0	0	357
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.92
time (sec)	N/A	1.416	1.388	0.101	0.	0.	0.	0.	95.757

Problem 951	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	207	0	0	0	0	0	172
normalized size	1	1.	1.1	0.	0.	0.	0.	0.	0.91
time (sec)	N/A	0.644	0.027	0.176	0.	0.	0.	0.	37.725

Problem 952	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	0.102	0.125	0.	0.	0.	0.	0.

Problem 953	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	265	263	327	0	0	0	0	0	230
normalized size	1	0.99	1.23	0.	0.	0.	0.	0.	0.87
time (sec)	N/A	0.853	0.927	0.089	0.	0.	0.	0.	97.681

Problem 954	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	F(-1)
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	525	523	0	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	1.713	2.708	0.156	0.	0.	0.	0.	0.

Problem 955	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	NO	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	0	0	0	0	0	0	347
normalized size	1	1.	0.	0.	0.	0.	0.	0.	0.9
time (sec)	N/A	0.884	1.993	0.135	0.	0.	0.	0.	99.026

Problem 956	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	A	F	F	F	F(-1)	F	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	205	0	0	0	0	0	167
normalized size	1	1.	1.1	0.	0.	0.	0.	0.	0.89
time (sec)	N/A	0.38	0.082	0.193	0.	0.	0.	0.	38.225

Problem 957	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	30	0	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	0.164	0.148	0.	0.	0.	0.	0.

Problem 958	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Rubi in Sympy
grade	A	A	C	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	188	148	0	0	0	0	0
normalized size	1	1.	2.11	1.66	0.	0.	0.	0.	0.
time (sec)	N/A	0.877	0.574	0.165	0.	0.	0.	0.	0.

## 2.2 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio  $\frac{\text{number of rules}}{\text{integrand size}}$  is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [429] had the largest ratio of [ 0.75 ]

Table 1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	10	5	1.	25	0.2
2	A	8	5	1.	25	0.2
3	A	7	5	1.	25	0.2
4	A	12	5	1.	25	0.2
5	A	5	4	1.	23	0.174
6	A	5	4	1.	23	0.174
7	A	8	7	1.	25	0.28
8	A	8	8	1.	25	0.32
9	A	8	7	1.	25	0.28
10	A	8	8	1.	25	0.32
11	A	8	7	1.	25	0.28
12	A	6	5	1.	25	0.2
13	A	7	6	1.	25	0.24
14	A	8	6	1.	25	0.24
15	A	9	6	1.	25	0.24
16	A	8	5	1.	25	0.2
17	A	5	5	1.	25	0.2
18	A	3	3	1.	25	0.12
19	A	6	4	1.	25	0.16
20	A	6	4	1.	25	0.16
21	A	5	4	1.	25	0.16
22	A	4	3	1.	25	0.12
23	A	3	3	1.	25	0.12
24	A	3	3	1.	25	0.12
25	A	3	3	1.	23	0.13
26	A	3	3	1.	22	0.136
27	A	7	5	1.	25	0.2
28	A	7	5	1.	25	0.2
29	A	8	6	1.	25	0.24
30	A	4	4	1.	25	0.16
31	A	5	4	1.	25	0.16

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
32	A	4	4	1.	24	0.167
33	A	7	5	1.	27	0.185
34	A	6	5	1.	27	0.185
35	A	5	5	1.	27	0.185
36	A	4	4	1.	25	0.16
37	A	4	4	1.	24	0.167
38	A	7	7	1.	27	0.259
39	A	7	7	1.	27	0.259
40	A	5	5	1.	27	0.185
41	A	6	6	1.	27	0.222
42	A	7	6	1.	27	0.222
43	A	8	6	1.	27	0.222
44	A	6	5	1.	27	0.185
45	A	6	5	1.	27	0.185
46	A	3	2	1.	27	0.074
47	A	3	3	1.	27	0.111
48	A	3	3	1.	25	0.12
49	A	3	3	1.	24	0.125
50	A	7	6	1.	27	0.222
51	A	7	5	1.	27	0.185
52	A	8	6	1.	27	0.222
53	A	9	6	1.	27	0.222
54	A	5	4	1.	20	0.2
55	A	4	4	1.	20	0.2
56	A	3	3	1.	18	0.167
57	A	3	3	1.	17	0.176
58	A	6	6	1.	20	0.3
59	A	6	6	1.	20	0.3
60	A	5	5	1.	20	0.25
61	A	6	6	1.	20	0.3
62	A	7	6	1.	20	0.3
63	A	8	6	1.	20	0.3
64	A	9	8	1.	27	0.296
65	A	12	6	1.	27	0.222
66	A	11	6	1.	27	0.222
67	A	10	6	1.	27	0.222
68	A	9	6	1.	27	0.222
69	A	9	6	1.	25	0.24
70	A	8	5	1.	24	0.208
71	A	11	8	1.	27	0.296
72	A	11	9	1.	27	0.333
73	A	11	8	1.	27	0.296
74	A	11	9	1.	27	0.333
75	A	11	9	1.	27	0.333
76	A	11	8	1.	27	0.296
77	A	11	9	1.	27	0.333
78	A	11	9	1.	27	0.333

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
79	A	11	8	1.	27	0.296
80	A	9	6	1.	27	0.222
81	A	10	7	1.	27	0.259
82	A	11	7	1.	27	0.259
83	A	7	5	1.	27	0.185
84	A	6	4	1.	27	0.148
85	A	5	4	1.	27	0.148
86	A	3	3	1.	27	0.111
87	A	3	3	1.	25	0.12
88	A	4	3	1.	24	0.125
89	A	7	6	1.	27	0.222
90	A	7	5	1.	27	0.185
91	A	8	6	1.	27	0.222
92	A	7	5	1.	27	0.185
93	A	6	5	1.	27	0.185
94	A	6	6	1.	27	0.222
95	A	4	4	1.	25	0.16
96	A	3	3	1.	24	0.125
97	A	7	7	1.	27	0.259
98	A	5	5	1.	27	0.185
99	A	6	6	1.	27	0.222
100	A	7	6	1.	27	0.222
101	A	8	6	1.	27	0.222
102	A	7	7	1.	27	0.259
103	A	9	6	1.	27	0.222
104	A	8	6	1.	27	0.222
105	A	8	7	1.	27	0.259
106	A	6	5	1.	25	0.2
107	A	5	4	1.	24	0.167
108	A	9	8	1.	27	0.296
109	A	9	9	1.	27	0.333
110	A	9	8	1.	27	0.296
111	A	9	9	1.	27	0.333
112	A	9	8	1.	27	0.296
113	A	7	6	1.	27	0.222
114	A	8	7	1.	27	0.259
115	A	9	7	1.	27	0.259
116	A	10	7	1.	27	0.259
117	A	3	3	1.	18	0.167
118	A	7	7	1.	26	0.269
119	A	6	6	1.	27	0.222
120	A	5	5	1.	27	0.185
121	A	5	5	1.	27	0.185
122	A	3	3	1.	25	0.12
123	A	1	1	1.	24	0.042
124	A	5	5	1.	27	0.185
125	A	5	5	1.	27	0.185

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
126	A	6	6	1.	27	0.222
127	A	6	5	1.	27	0.185
128	A	6	5	1.	27	0.185
129	A	5	5	1.	27	0.185
130	A	3	3	1.	27	0.111
131	A	2	2	1.	25	0.08
132	A	2	2	1.	24	0.083
133	A	6	6	1.	27	0.222
134	A	6	6	1.	27	0.222
135	A	7	7	1.	27	0.259
136	A	7	5	1.	27	0.185
137	A	7	5	1.	27	0.185
138	A	6	5	1.	27	0.185
139	A	5	4	1.	27	0.148
140	A	4	4	1.	27	0.148
141	A	3	3	1.	27	0.111
142	A	3	3	1.	25	0.12
143	A	3	3	1.	24	0.125
144	A	7	6	1.	27	0.222
145	A	7	6	1.	27	0.222
146	A	8	7	1.	27	0.259
147	A	9	7	1.	27	0.259
148	A	5	5	1.	27	0.185
149	A	4	4	1.	27	0.148
150	A	4	4	1.	25	0.16
151	A	4	4	1.	25	0.16
152	A	2	2	1.	23	0.087
153	A	1	1	1.	22	0.045
154	A	5	5	1.	26	0.192
155	A	5	5	1.	26	0.192
156	A	6	6	1.	26	0.231
157	A	10	7	1.	27	0.259
158	A	9	7	1.	27	0.259
159	A	8	7	1.	27	0.259
160	A	7	7	1.	27	0.259
161	A	6	5	1.	25	0.2
162	A	6	6	1.	24	0.25
163	A	9	9	1.	27	0.333
164	A	9	9	1.	27	0.333
165	A	9	9	1.	27	0.333
166	A	9	9	1.	27	0.333
167	A	7	7	1.	27	0.259
168	A	8	8	1.	27	0.296
169	A	9	8	1.	27	0.296
170	A	10	8	1.	27	0.296
171	A	7	6	1.	27	0.222
172	A	4	3	1.	27	0.111

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
173	A	4	4	1.	27	0.148
174	A	3	3	1.	25	0.12
175	A	3	2	1.	24	0.083
176	A	8	7	1.	27	0.259
177	A	8	6	1.	27	0.222
178	A	9	7	1.	27	0.259
179	A	8	6	1.	27	0.222
180	A	7	5	1.	27	0.185
181	A	6	5	1.	27	0.185
182	A	4	4	1.	27	0.148
183	A	3	3	1.	25	0.12
184	A	3	2	1.	24	0.083
185	A	8	7	1.	27	0.259
186	A	8	6	1.	27	0.222
187	A	9	7	1.	27	0.259
188	A	9	6	1.	27	0.222
189	A	7	5	1.	27	0.185
190	A	9	7	1.	27	0.259
191	A	8	6	1.	27	0.222
192	A	2	2	1.	25	0.08
193	A	2	2	1.	24	0.083
194	A	8	7	1.	27	0.259
195	A	8	6	1.	27	0.222
196	A	9	7	1.	27	0.259
197	A	10	7	1.	27	0.259
198	A	11	6	1.	27	0.222
199	A	10	6	1.	27	0.222
200	A	9	6	1.	27	0.222
201	A	8	7	1.	27	0.259
202	A	6	6	1.	25	0.24
203	A	5	4	1.	24	0.167
204	A	9	9	1.	27	0.333
205	A	9	9	1.	27	0.333
206	A	7	7	1.	27	0.259
207	A	8	7	1.	27	0.259
208	A	9	7	1.	27	0.259
209	A	10	7	1.	27	0.259
210	A	7	5	1.	26	0.192
211	A	4	4	1.	26	0.154
212	A	9	5	1.	27	0.185
213	A	8	5	1.	27	0.185
214	A	7	4	1.	25	0.16
215	A	7	3	1.	24	0.125
216	A	12	7	1.	27	0.259
217	A	12	6	1.	27	0.222
218	A	4	4	1.	29	0.138
219	A	2	2	1.	29	0.069

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
220	A	3	3	1.	16	0.188
221	A	4	4	1.	29	0.138
222	A	3	3	1.	15	0.2
223	A	4	4	1.	30	0.133
224	A	4	3	1.	16	0.188
225	A	5	4	1.	29	0.138
226	A	7	4	1.	29	0.138
227	A	6	4	1.	29	0.138
228	A	5	3	1.	27	0.111
229	A	2	2	1.	22	0.091
230	A	8	5	1.	29	0.172
231	A	7	5	1.	29	0.172
232	A	8	5	1.	29	0.172
233	A	6	4	1.	29	0.138
234	A	6	4	1.	29	0.138
235	A	5	3	1.31	27	0.111
236	A	2	2	1.	22	0.091
237	A	8	5	1.	29	0.172
238	A	7	5	1.	29	0.172
239	A	7	5	1.	29	0.172
240	A	6	5	1.	23	0.217
241	A	6	5	1.	23	0.217
242	A	6	5	1.	23	0.217
243	A	6	5	1.	23	0.217
244	A	4	4	1.	21	0.19
245	A	3	3	1.	20	0.15
246	A	5	5	1.	23	0.217
247	A	5	5	1.	23	0.217
248	A	5	5	1.	23	0.217
249	A	7	6	1.	25	0.24
250	A	8	7	1.	25	0.28
251	A	7	6	1.	25	0.24
252	A	8	7	1.	25	0.28
253	A	7	6	1.	23	0.261
254	A	2	2	1.	22	0.091
255	A	7	7	1.	25	0.28
256	A	6	6	1.	25	0.24
257	A	6	6	1.	25	0.24
258	A	7	6	1.	25	0.24
259	A	7	6	1.	25	0.24
260	A	7	6	1.	25	0.24
261	A	7	6	1.	25	0.24
262	A	3	3	1.	23	0.13
263	A	2	2	1.	22	0.091
264	A	7	7	1.	25	0.28
265	A	8	8	1.	25	0.32
266	A	7	6	1.	25	0.24

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
267	A	7	6	1.	25	0.24
268	A	7	6	1.	25	0.24
269	A	7	6	1.	25	0.24
270	A	5	5	1.	23	0.217
271	A	2	2	1.	22	0.091
272	A	6	6	1.	25	0.24
273	A	6	6	1.	25	0.24
274	A	6	6	1.	25	0.24
275	A	8	7	1.	25	0.28
276	A	9	8	1.	25	0.32
277	A	8	7	1.	25	0.28
278	A	9	8	1.	25	0.32
279	A	3	3	1.	23	0.13
280	A	2	2	1.	22	0.091
281	A	8	8	1.	25	0.32
282	A	7	7	1.	25	0.28
283	A	7	7	1.	25	0.28
284	A	7	7	1.	25	0.28
285	A	7	7	1.	25	0.28
286	A	8	7	1.	25	0.28
287	A	8	7	1.	25	0.28
288	A	4	4	1.	25	0.16
289	A	3	3	1.	23	0.13
290	A	2	2	1.	22	0.091
291	A	8	8	1.	25	0.32
292	A	9	9	1.	25	0.36
293	A	8	7	1.	25	0.28
294	A	8	7	1.	25	0.28
295	A	8	7	1.	25	0.28
296	A	10	9	1.	25	0.36
297	A	5	4	1.	25	0.16
298	A	4	4	1.	25	0.16
299	A	3	3	1.	23	0.13
300	A	2	2	1.	22	0.091
301	A	9	9	1.	25	0.36
302	A	9	9	1.	25	0.36
303	A	10	9	1.	25	0.36
304	A	9	7	1.	25	0.28
305	A	9	7	1.	25	0.28
306	A	7	4	1.	27	0.148
307	A	6	4	1.	27	0.148
308	A	5	3	1.	25	0.12
309	A	2	2	1.	20	0.1
310	A	8	5	1.	27	0.185
311	A	7	5	1.	27	0.185
312	A	8	5	1.	27	0.185
313	A	6	4	1.	25	0.16

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
314	A	4	3	1.	27	0.111
315	A	11	7	1.	16	0.438
316	A	9	6	1.	22	0.273
317	A	8	6	1.	22	0.273
318	A	8	7	1.	22	0.318
319	A	6	5	1.	20	0.25
320	A	6	5	1.	19	0.263
321	A	9	8	1.	22	0.364
322	A	15	11	1.	22	0.5
323	A	19	12	1.	22	0.546
324	A	20	13	1.	22	0.591
325	A	25	14	1.	22	0.636
326	A	8	5	1.	22	0.227
327	A	7	5	1.	22	0.227
328	A	7	6	1.	22	0.273
329	A	5	4	1.	20	0.2
330	A	2	2	1.	19	0.105
331	A	7	6	1.	22	0.273
332	A	8	7	1.	22	0.318
333	A	12	8	1.	22	0.364
334	A	7	6	1.	22	0.273
335	A	6	5	1.	22	0.227
336	A	4	4	1.	22	0.182
337	A	4	4	1.	20	0.2
338	A	4	4	1.	19	0.21
339	A	10	9	1.	22	0.409
340	A	12	11	1.	22	0.5
341	A	17	12	1.	22	0.546
342	A	9	6	1.	22	0.273
343	A	8	6	1.	22	0.273
344	A	7	6	1.	22	0.273
345	A	6	5	1.	22	0.227
346	A	3	3	1.	20	0.15
347	A	3	3	1.	19	0.158
348	A	10	7	1.	22	0.318
349	A	11	8	1.	22	0.364
350	A	15	9	1.	22	0.409
351	A	2	1	1.	18	0.056
352	A	2	1	1.	16	0.062
353	A	2	1	1.	15	0.067
354	A	3	3	1.	18	0.167
355	A	2	1	1.	20	0.05
356	A	2	1	1.	18	0.056
357	A	2	1	1.	17	0.059
358	A	3	2	1.	20	0.1
359	A	2	1	1.	20	0.05
360	A	2	1	1.	18	0.056

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
361	A	2	1	1.	17	0.059
362	A	3	2	1.	20	0.1
363	A	6	3	1.	20	0.15
364	A	6	3	1.	20	0.15
365	A	6	3	1.	20	0.15
366	A	4	2	1.	18	0.111
367	A	4	2	1.	17	0.118
368	A	7	4	1.	20	0.2
369	A	7	4	1.	20	0.2
370	A	5	3	1.	20	0.15
371	A	5	3	1.	20	0.15
372	A	5	3	1.	20	0.15
373	A	5	3	1.	18	0.167
374	A	5	3	1.	17	0.176
375	A	12	5	1.	20	0.25
376	A	12	6	1.	20	0.3
377	A	6	5	0.94	22	0.227
378	A	4	4	0.91	20	0.2
379	A	6	3	1.	22	0.136
380	A	12	4	1.	22	0.182
381	A	6	5	1.	18	0.278
382	A	6	5	1.	18	0.278
383	A	6	5	1.	18	0.278
384	A	6	5	1.	18	0.278
385	A	4	4	1.	16	0.25
386	A	3	3	1.	15	0.2
387	A	5	5	1.	18	0.278
388	A	5	5	1.	18	0.278
389	A	5	5	1.	18	0.278
390	A	7	6	1.	20	0.3
391	A	8	7	0.95	20	0.35
392	A	7	6	1.	20	0.3
393	A	8	7	0.95	20	0.35
394	A	7	6	1.	18	0.333
395	A	4	4	0.94	17	0.235
396	A	7	7	1.	20	0.35
397	A	6	6	1.	20	0.3
398	A	6	6	1.	20	0.3
399	A	7	6	0.98	20	0.3
400	A	7	6	0.97	20	0.3
401	A	7	6	0.97	20	0.3
402	A	7	6	0.96	20	0.3
403	A	7	6	0.95	18	0.333
404	A	4	4	0.96	17	0.235
405	A	7	7	0.96	20	0.35
406	A	8	8	1.	20	0.4
407	A	7	6	1.	20	0.3

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
408	A	7	6	1.	20	0.3
409	A	6	6	1.	20	0.3
410	A	6	6	1.	20	0.3
411	A	9	8	1.	18	0.444
412	A	6	5	1.	17	0.294
413	A	7	7	1.	20	0.35
414	A	7	7	1.	20	0.35
415	A	8	8	1.	20	0.4
416	A	12	10	1.	20	0.5
417	A	11	10	1.	20	0.5
418	A	10	9	0.99	20	0.45
419	A	10	9	1.	18	0.5
420	A	8	7	0.78	17	0.412
421	A	18	11	1.	20	0.55
422	A	20	13	1.	20	0.65
423	A	12	10	1.	20	0.5
424	A	11	9	1.	20	0.45
425	A	11	10	1.	20	0.5
426	A	11	9	1.	18	0.5
427	A	11	9	1.	17	0.529
428	A	29	13	1.	20	0.65
429	A	31	15	1.	20	0.75
430	A	7	4	0.92	22	0.182
431	A	6	4	0.95	22	0.182
432	A	5	3	1.	20	0.15
433	A	2	2	1.	15	0.133
434	A	5	3	1.	22	0.136
435	A	8	3	1.	22	0.136
436	A	10	3	1.	22	0.136
437	A	6	5	1.	40	0.125
438	A	5	5	1.	40	0.125
439	A	4	4	1.	38	0.105
440	A	3	3	1.	37	0.081
441	A	6	5	1.	40	0.125
442	A	4	4	1.	40	0.1
443	A	5	5	1.	40	0.125
444	A	6	5	1.	40	0.125
445	A	7	5	1.	40	0.125
446	A	7	6	1.	40	0.15
447	A	6	6	1.	40	0.15
448	A	5	5	1.	38	0.132
449	A	4	4	1.	37	0.108
450	A	7	6	1.	40	0.15
451	A	7	6	1.	40	0.15
452	A	7	6	1.	40	0.15
453	A	5	5	1.	40	0.125
454	A	6	6	1.	40	0.15

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
455	A	7	6	1.	40	0.15
456	A	8	6	1.	40	0.15
457	A	8	6	1.	40	0.15
458	A	7	6	1.	40	0.15
459	A	6	5	1.	38	0.132
460	A	5	4	1.	37	0.108
461	A	8	6	1.	40	0.15
462	A	8	7	1.	40	0.175
463	A	8	6	1.	40	0.15
464	A	8	7	1.	40	0.175
465	A	8	6	1.	40	0.15
466	A	6	5	1.	40	0.125
467	A	7	6	1.	40	0.15
468	A	8	6	1.	40	0.15
469	A	9	6	1.	40	0.15
470	A	5	5	1.1	40	0.125
471	A	4	4	1.	40	0.1
472	A	3	3	1.	38	0.079
473	A	1	1	1.	37	0.027
474	A	5	5	1.	40	0.125
475	A	5	5	1.	40	0.125
476	A	6	6	1.	40	0.15
477	A	6	5	1.	40	0.125
478	A	6	5	1.	40	0.125
479	A	5	5	1.	40	0.125
480	A	3	3	1.	40	0.075
481	A	2	2	1.	38	0.053
482	A	2	2	1.	37	0.054
483	A	6	5	1.	40	0.125
484	A	6	5	1.	40	0.125
485	A	7	6	1.	40	0.15
486	A	8	6	1.	40	0.15
487	A	3	3	1.	40	0.075
488	A	4	4	1.	40	0.1
489	A	4	4	1.	23	0.174
490	A	1	1	1.	23	0.043
491	A	5	5	1.	21	0.238
492	A	3	3	1.	20	0.15
493	A	5	5	1.	23	0.217
494	A	5	5	1.	23	0.217
495	A	3	3	1.	23	0.13
496	A	5	4	1.	23	0.174
497	A	1	1	1.	23	0.043
498	A	6	5	1.	21	0.238
499	A	4	3	1.	20	0.15
500	A	6	5	1.	23	0.217
501	A	6	6	1.	23	0.261

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
502	A	4	4	1.	23	0.174
503	A	3	3	1.	23	0.13
504	A	1	1	1.	23	0.043
505	A	4	4	1.	21	0.19
506	A	2	2	1.	20	0.1
507	A	4	4	1.	23	0.174
508	A	5	5	1.	23	0.217
509	A	3	3	1.	23	0.13
510	A	3	3	1.	23	0.13
511	A	1	1	1.	23	0.043
512	A	5	5	1.	21	0.238
513	A	3	3	1.	20	0.15
514	A	5	5	1.	23	0.217
515	A	6	6	1.	23	0.261
516	A	4	4	1.	23	0.174
517	A	4	4	1.	23	0.174
518	A	1	1	1.	23	0.043
519	A	6	5	1.	21	0.238
520	A	4	3	1.	20	0.15
521	A	6	5	1.	23	0.217
522	A	7	6	1.	23	0.261
523	A	5	4	1.	23	0.174
524	A	7	6	1.	19	0.316
525	A	6	4	1.	25	0.16
526	A	6	4	1.22	25	0.16
527	A	6	4	1.	25	0.16
528	A	5	4	1.	23	0.174
529	A	4	3	1.	22	0.136
530	A	7	5	1.	25	0.2
531	A	9	6	0.97	25	0.24
532	A	12	6	1.	25	0.24
533	A	6	4	1.	25	0.16
534	A	6	4	1.	25	0.16
535	A	6	4	1.	25	0.16
536	A	6	4	1.	23	0.174
537	A	5	4	1.	22	0.182
538	A	7	5	1.	25	0.2
539	A	9	6	1.	25	0.24
540	A	12	6	1.	25	0.24
541	A	6	3	1.	23	0.13
542	A	4	2	1.	23	0.087
543	A	4	2	1.	23	0.087
544	A	4	2	1.	21	0.095
545	A	4	2	1.	20	0.1
546	A	7	4	1.	23	0.174
547	A	8	5	1.	23	0.217
548	A	3	2	1.	29	0.069

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
549	A	3	2	1.	29	0.069
550	A	3	2	1.	29	0.069
551	A	3	2	1.	27	0.074
552	A	5	3	1.	22	0.136
553	A	3	2	1.	29	0.069
554	A	4	3	1.	29	0.103
555	A	4	3	1.	29	0.103
556	A	4	3	1.	29	0.103
557	A	3	2	1.	29	0.069
558	A	3	2	1.	29	0.069
559	A	3	2	1.	29	0.069
560	A	3	2	1.	29	0.069
561	A	3	2	1.	29	0.069
562	A	3	2	1.	29	0.069
563	A	3	2	1.	27	0.074
564	A	2	2	1.	22	0.091
565	A	4	3	1.	29	0.103
566	A	4	3	1.	29	0.103
567	A	4	3	1.	29	0.103
568	A	4	3	1.	29	0.103
569	A	3	2	1.	29	0.069
570	A	3	2	1.	29	0.069
571	A	3	2	1.	29	0.069
572	A	3	2	1.	29	0.069
573	A	3	2	1.	29	0.069
574	A	4	3	1.	29	0.103
575	A	4	3	1.	27	0.111
576	A	3	3	1.	22	0.136
577	A	4	3	1.	29	0.103
578	A	4	3	1.	29	0.103
579	A	7	5	1.	31	0.161
580	A	6	4	1.	31	0.129
581	A	5	4	1.	31	0.129
582	A	3	3	1.	31	0.097
583	A	3	3	1.	29	0.103
584	A	4	3	1.	24	0.125
585	A	6	5	1.	31	0.161
586	A	6	4	1.	31	0.129
587	A	7	5	1.	31	0.161
588	A	4	3	1.	24	0.125
589	A	3	2	1.	24	0.083
590	A	3	2	1.	24	0.083
591	A	2	1	1.	22	0.045
592	A	2	1	1.	17	0.059
593	A	4	3	1.	24	0.125
594	A	4	4	1.	24	0.167
595	A	4	4	1.	24	0.167

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
596	A	3	2	1.	24	0.083
597	A	3	2	1.	24	0.083
598	A	2	1	1.	22	0.045
599	A	2	1	1.	17	0.059
600	A	4	3	1.	24	0.125
601	A	4	4	1.	24	0.167
602	A	5	5	1.	24	0.208
603	A	4	4	1.	26	0.154
604	A	1	1	1.	22	0.045
605	A	10	6	1.	28	0.214
606	A	9	5	1.	28	0.179
607	A	6	3	1.	28	0.107
608	A	8	5	1.	28	0.179
609	A	11	7	1.	28	0.25
610	A	10	5	1.	28	0.179
611	A	6	3	1.	28	0.107
612	A	6	3	1.	28	0.107
613	A	8	4	1.	28	0.143
614	A	16	7	1.	28	0.25
615	A	8	5	1.	28	0.179
616	A	8	4	1.	28	0.143
617	A	12	6	0.99	28	0.214
618	A	6	3	1.	20	0.15
619	A	5	5	1.	26	0.192
620	A	6	6	1.	30	0.2
621	A	5	5	1.	29	0.172
622	A	10	6	1.	28	0.214
623	A	9	6	1.	28	0.214
624	A	7	6	1.	26	0.231
625	A	7	6	1.	21	0.286
626	A	14	10	1.	28	0.357
627	A	14	10	1.	28	0.357
628	A	23	11	1.	28	0.393
629	A	9	6	1.	28	0.214
630	A	8	6	0.99	28	0.214
631	A	6	5	1.	26	0.192
632	A	6	5	1.	21	0.238
633	A	10	9	1.	28	0.321
634	A	14	10	1.	28	0.357
635	A	23	11	1.	28	0.393
636	A	8	6	0.99	28	0.214
637	A	7	6	1.	28	0.214
638	A	6	5	1.	26	0.192
639	A	2	2	1.	21	0.095
640	A	7	7	1.	28	0.25
641	A	14	10	1.	28	0.357
642	A	23	11	1.	28	0.393

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
643	A	16	10	1.35	28	0.357
644	A	10	8	1.	28	0.286
645	A	7	6	1.	28	0.214
646	A	7	6	1.	28	0.214
647	A	5	4	1.	26	0.154
648	A	2	2	1.	21	0.095
649	A	4	4	1.	28	0.143
650	A	14	10	1.	28	0.357
651	A	23	10	1.	28	0.357
652	A	10	9	1.	28	0.321
653	A	17	12	1.	28	0.429
654	A	4	4	1.	28	0.143
655	A	2	2	1.	30	0.067
656	A	4	3	1.	26	0.115
657	A	4	3	1.	46	0.065
658	A	3	3	1.	46	0.065
659	A	2	2	1.	44	0.045
660	A	1	1	1.	39	0.026
661	A	2	2	1.	46	0.043
662	A	3	3	1.	46	0.065
663	A	4	3	1.	46	0.065
664	A	5	3	1.	46	0.065
665	A	4	4	1.	46	0.087
666	A	3	3	1.	46	0.065
667	A	2	2	1.	44	0.045
668	A	1	1	1.	39	0.026
669	A	3	3	1.	46	0.065
670	A	4	4	1.	46	0.087
671	A	5	4	1.	46	0.087
672	A	4	3	1.	46	0.065
673	A	3	3	1.	46	0.065
674	A	2	2	1.	44	0.045
675	A	1	1	1.	39	0.026
676	A	4	3	1.	46	0.065
677	A	5	4	1.	46	0.087
678	A	6	4	1.	46	0.087
679	A	5	3	1.	46	0.065
680	A	4	3	1.	46	0.065
681	A	3	3	1.	46	0.065
682	A	2	2	1.	44	0.045
683	A	1	1	1.	39	0.026
684	A	3	3	1.	46	0.065
685	A	3	3	1.	46	0.065
686	A	4	4	1.	46	0.087
687	A	5	4	1.	46	0.087
688	A	6	4	1.	46	0.087
689	A	5	3	1.	46	0.065

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
690	A	4	3	1.	46	0.065
691	A	3	3	1.	46	0.065
692	A	2	2	1.	44	0.045
693	A	1	1	1.	39	0.026
694	A	4	3	1.	46	0.065
695	A	4	4	1.	46	0.087
696	A	4	3	1.	46	0.065
697	A	5	4	1.	46	0.087
698	A	6	4	1.	46	0.087
699	A	7	4	1.	46	0.087
700	A	5	3	1.	46	0.065
701	A	4	3	1.	46	0.065
702	A	3	3	1.	46	0.065
703	A	2	2	1.	44	0.045
704	A	1	1	1.	39	0.026
705	A	5	3	1.	46	0.065
706	A	5	4	1.	46	0.087
707	A	5	4	1.	46	0.087
708	A	5	3	1.	46	0.065
709	A	6	4	1.	46	0.087
710	A	7	4	1.	46	0.087
711	A	8	4	1.	46	0.087
712	A	6	4	1.	48	0.083
713	A	5	4	1.	48	0.083
714	A	4	4	1.	48	0.083
715	A	3	3	1.	48	0.062
716	A	1	1	1.	48	0.021
717	A	2	2	1.	48	0.042
718	A	3	2	1.	48	0.042
719	A	4	2	1.	48	0.042
720	A	6	5	1.	48	0.104
721	A	5	5	1.	48	0.104
722	A	4	4	1.	48	0.083
723	A	1	1	1.	48	0.021
724	A	2	2	1.	48	0.042
725	A	3	3	1.	48	0.062
726	A	4	3	1.	48	0.062
727	A	6	5	1.	48	0.104
728	A	5	4	1.	48	0.083
729	A	1	1	1.	48	0.021
730	A	2	2	1.	48	0.042
731	A	3	2	1.	48	0.042
732	A	4	3	1.	48	0.062
733	A	7	5	1.	48	0.104
734	A	6	5	1.	48	0.104
735	A	5	5	1.	48	0.104
736	A	4	4	1.	48	0.083

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
737	A	4	4	1.	48	0.083
738	A	1	1	1.	48	0.021
739	A	2	2	1.	48	0.042
740	A	3	2	1.	48	0.042
741	A	4	2	1.	48	0.042
742	A	7	5	1.	48	0.104
743	A	6	5	1.	48	0.104
744	A	5	4	1.	48	0.083
745	A	5	5	1.	48	0.104
746	A	5	4	1.	48	0.083
747	A	1	1	1.	48	0.021
748	A	2	2	1.	48	0.042
749	A	3	2	1.	48	0.042
750	A	4	2	1.	48	0.042
751	A	8	5	1.	48	0.104
752	A	7	5	1.	48	0.104
753	A	6	4	1.	48	0.083
754	A	6	5	1.	48	0.104
755	A	6	5	1.	48	0.104
756	A	6	4	1.	48	0.083
757	A	1	1	1.	48	0.021
758	A	2	2	1.	48	0.042
759	A	3	2	1.	48	0.042
760	A	4	2	1.	48	0.042
761	A	3	3	1.17	46	0.065
762	A	3	3	1.06	46	0.065
763	A	3	3	1.13	46	0.065
764	A	3	3	1.15	46	0.065
765	A	3	3	1.17	46	0.065
766	A	3	3	1.17	46	0.065
767	A	3	3	1.04	44	0.068
768	A	4	3	1.	44	0.068
769	A	3	3	1.	44	0.068
770	A	2	2	1.	42	0.048
771	A	1	1	1.	37	0.027
772	A	2	2	1.	44	0.045
773	A	2	2	1.	44	0.045
774	A	2	2	1.	44	0.045
775	A	3	3	1.	46	0.065
776	A	3	3	1.	46	0.065
777	A	3	3	1.	46	0.065
778	A	3	3	1.	46	0.065
779	A	3	3	1.	46	0.065
780	A	1	1	1.	47	0.021
781	A	3	3	1.	73	0.041
782	A	4	4	1.04	46	0.087
783	A	6	4	1.	46	0.087

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
784	A	5	4	1.	46	0.087
785	A	4	4	1.	46	0.087
786	A	3	3	1.	44	0.068
787	A	2	2	1.	39	0.051
788	A	3	3	1.	46	0.065
789	A	3	3	1.	46	0.065
790	A	4	4	1.	46	0.087
791	A	5	4	1.	46	0.087
792	A	8	4	1.	32	0.125
793	A	6	4	1.	32	0.125
794	A	4	4	1.	30	0.133
795	A	6	4	1.	32	0.125
796	A	7	5	1.	32	0.156
797	A	7	5	1.	32	0.156
798	A	5	5	1.	32	0.156
799	A	4	4	1.	30	0.133
800	A	7	5	1.	32	0.156
801	A	8	6	1.	32	0.188
802	A	3	3	1.	25	0.12
803	A	4	3	1.	25	0.12
804	A	4	3	1.	28	0.107
805	A	2	1	1.	28	0.036
806	A	2	1	1.	28	0.036
807	A	2	1	1.	26	0.038
808	A	2	1	1.	21	0.048
809	A	3	3	1.	28	0.107
810	A	3	2	1.	28	0.071
811	A	3	3	1.	28	0.107
812	A	3	3	1.	28	0.107
813	A	4	4	0.98	28	0.143
814	A	2	1	1.	25	0.04
815	A	2	1	1.	27	0.037
816	A	2	1	1.	27	0.037
817	A	6	5	1.	27	0.185
818	A	9	6	1.	27	0.222
819	A	3	2	1.	27	0.074
820	A	3	2	1.	27	0.074
821	A	2	1	1.	25	0.04
822	A	2	1	1.	20	0.05
823	A	4	3	1.	27	0.111
824	A	4	4	1.	27	0.148
825	A	4	4	1.	27	0.148
826	A	3	2	1.	27	0.074
827	A	3	2	1.	27	0.074
828	A	2	1	1.	25	0.04
829	A	2	1	1.	20	0.05
830	A	4	3	1.	27	0.111

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
831	A	4	4	1.	27	0.148
832	A	5	5	1.	27	0.185
833	B	9	7	2.1	25	0.28
834	A	4	4	1.	29	0.138
835	A	6	5	1.	29	0.172
836	A	5	5	1.	29	0.172
837	A	4	4	1.	29	0.138
838	A	4	4	1.	29	0.138
839	A	4	4	1.	29	0.138
840	A	3	3	1.	29	0.103
841	A	4	4	1.	29	0.138
842	A	5	5	1.	29	0.172
843	A	6	5	1.	38	0.132
844	A	5	5	1.	38	0.132
845	A	4	4	1.	38	0.105
846	A	4	4	1.	38	0.105
847	A	4	4	1.	38	0.105
848	A	3	3	1.	38	0.079
849	A	4	4	1.	38	0.105
850	A	10	5	1.	31	0.161
851	A	6	3	1.	31	0.097
852	A	6	3	1.	31	0.097
853	A	8	4	1.	31	0.129
854	A	8	6	1.	29	0.207
855	A	7	6	1.	29	0.207
856	A	6	5	1.	27	0.185
857	A	6	5	1.	22	0.227
858	A	8	5	1.	29	0.172
859	A	20	7	1.	29	0.241
860	A	23	8	1.	29	0.276
861	A	27	9	1.	29	0.31
862	A	9	6	1.	29	0.207
863	A	8	6	1.	29	0.207
864	A	7	5	1.	27	0.185
865	A	7	6	1.	22	0.273
866	A	13	7	1.	29	0.241
867	A	23	8	1.	29	0.276
868	A	30	9	1.	29	0.31
869	A	15	7	1.	29	0.241
870	A	8	5	1.	29	0.172
871	A	7	5	1.	29	0.172
872	A	6	5	1.	29	0.172
873	A	5	4	1.	27	0.148
874	A	2	2	1.	22	0.091
875	A	6	3	1.	29	0.103
876	A	9	4	1.	29	0.138
877	A	13	6	1.	29	0.207

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
878	A	7	6	1.	29	0.207
879	A	6	5	1.	29	0.172
880	A	4	4	1.	29	0.138
881	A	4	4	1.	27	0.148
882	A	4	4	1.	22	0.182
883	A	10	5	1.	29	0.172
884	A	14	6	1.	29	0.207
885	A	19	7	1.	29	0.241
886	A	10	6	1.	31	0.194
887	A	9	6	1.	31	0.194
888	A	7	6	1.	29	0.207
889	A	7	6	1.	24	0.25
890	A	15	10	1.27	31	0.323
891	A	15	10	1.26	31	0.323
892	A	25	11	1.65	31	0.355
893	A	9	6	1.	31	0.194
894	A	8	6	1.	31	0.194
895	A	6	5	1.	29	0.172
896	A	6	5	1.	24	0.208
897	A	11	9	1.	31	0.29
898	A	15	10	1.3	31	0.323
899	A	25	11	1.67	31	0.355
900	A	8	6	1.	31	0.194
901	A	7	6	1.	31	0.194
902	A	6	5	1.	29	0.172
903	A	2	2	1.	24	0.083
904	A	8	7	1.	31	0.226
905	A	15	10	1.	31	0.323
906	A	25	11	1.	31	0.355
907	A	11	8	1.	31	0.258
908	A	17	10	1.	31	0.323
909	A	7	6	1.	31	0.194
910	A	7	6	1.	31	0.194
911	A	5	4	1.	29	0.138
912	A	2	2	1.	24	0.083
913	A	5	4	1.	31	0.129
914	A	15	10	1.	31	0.323
915	A	25	10	1.58	31	0.323
916	A	11	9	1.	31	0.29
917	A	18	12	1.	31	0.387
918	A	1	1	1.	33	0.03
919	A	2	2	1.	33	0.061
920	A	2	1	1.	25	0.04
921	A	2	1	1.	23	0.043
922	A	3	3	1.	25	0.12
923	A	3	3	1.	25	0.12
924	A	3	3	0.99	25	0.12

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Table 1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
925	A	2	1	1.	27	0.037
926	A	2	1	1.	25	0.04
927	A	3	2	1.	27	0.074
928	A	4	3	1.	27	0.111
929	A	5	4	1.	27	0.148
930	A	4	2	1.	27	0.074
931	A	4	2	1.	27	0.074
932	A	4	2	1.	27	0.074
933	A	4	2	1.	25	0.08
934	A	4	2	1.	20	0.1
935	A	7	3	1.	27	0.111
936	A	8	4	1.	27	0.148
937	A	5	3	1.	27	0.111
938	A	5	3	1.	27	0.111
939	A	5	3	1.	27	0.111
940	A	5	3	1.	25	0.12
941	A	5	3	1.	20	0.15
942	A	12	4	1.	27	0.148
943	A	13	5	1.	27	0.185
944	A	4	4	0.97	27	0.148
945	A	6	4	0.99	29	0.138
946	A	5	3	1.	27	0.111
947	A	2	2	1.	22	0.091
948	A	0	0	0.	0	0.
949	A	6	4	1.	29	0.138
950	A	5	3	1.	27	0.111
951	A	2	2	1.	22	0.091
952	A	0	0	0.	0	0.
953	A	4	4	0.99	25	0.16
954	A	6	4	1.	27	0.148
955	A	5	3	1.	25	0.12
956	A	2	2	1.	20	0.1
957	A	0	0	0.	0	0.
958	A	5	5	1.	27	0.185



### 3 Listing of integrals

#### 3.1 $\int x^2(d + ex)\sqrt{d^2 - e^2x^2} dx$

**Optimal.** Leaf size=132

$$-\frac{dx(d^2 - e^2x^2)^{3/2}}{4e^2} - \frac{d^2(d^2 - e^2x^2)^{3/2}}{3e^3} + \frac{(d^2 - e^2x^2)^{5/2}}{5e^3} + \frac{d^5 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{8e^3} + \frac{d^3x\sqrt{d^2 - e^2x^2}}{8e^2}$$

[Out]  $(d^3x\sqrt{d^2 - e^2x^2})/(8e^2) - (d^2(d^2 - e^2x^2)^{3/2})/(3e^3) - (d^5 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right))/(8e^3) + (d^2 - e^2x^2)^{5/2}/(5e^3) + (d^5 \text{ArcTan}[(ex)/\sqrt{d^2 - e^2x^2}])/(8e^3)$

**Rubi [A]** time = 0.20001, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{dx(d^2 - e^2x^2)^{3/2}}{4e^2} - \frac{d^2(d^2 - e^2x^2)^{3/2}}{3e^3} + \frac{(d^2 - e^2x^2)^{5/2}}{5e^3} + \frac{d^5 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{8e^3} + \frac{d^3x\sqrt{d^2 - e^2x^2}}{8e^2}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(d + e\*x)\*Sqrt[d^2 - e^2\*x^2], x]

[Out]  $(d^3x\sqrt{d^2 - e^2x^2})/(8e^2) - (d^2(d^2 - e^2x^2)^{3/2})/(3e^3) - (d^5 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right))/(8e^3) + (d^2 - e^2x^2)^{5/2}/(5e^3) + (d^5 \text{ArcTan}[(ex)/\sqrt{d^2 - e^2x^2}])/(8e^3)$

**Rubi in Sympy [A]** time = 31.8788, size = 112, normalized size = 0.85

$$\frac{d^5 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{8e^3} + \frac{d^3x\sqrt{d^2 - e^2x^2}}{8e^2} - \frac{d^2(d^2 - e^2x^2)^{3/2}}{3e^3} - \frac{dx(d^2 - e^2x^2)^{3/2}}{4e^2} + \frac{(d^2 - e^2x^2)^{5/2}}{5e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2), x)

[Out]  $d^5 \operatorname{atan}(ex/\sqrt{d^2 - e^2x^2})/(8e^3) + d^3x\sqrt{d^2 - e^2x^2}/(8e^2) - d^2(d^2 - e^2x^2)^{3/2}/(3e^3) - dx(d^2 - e^2x^2)^{3/2}/(4e^2) + (d^2 - e^2x^2)^{5/2}/(5e^3)$

**Mathematica [A]** time = 0.0862242, size = 91, normalized size = 0.69

$$\frac{15d^5 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) + \sqrt{d^2 - e^2x^2}(-16d^4 - 15d^3ex - 8d^2e^2x^2 + 30de^3x^3 + 24e^4x^4)}{120e^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(d + e\*x)\*Sqrt[d^2 - e^2\*x^2], x]

[Out]  $(\sqrt{d^2 - e^2x^2}(-16d^4 - 15d^3ex - 8d^2e^2x^2 + 30de^3x^3 + 24e^4x^4) + 15d^5 \text{ArcTan}[(ex)/\sqrt{d^2 - e^2x^2}])$

)/(120\*e^3)

**Maple [A]** time = 0.057, size = 125, normalized size = 1.

$$-\frac{dx}{4e^2}(-e^2x^2+d^2)^{\frac{3}{2}} + \frac{d^3x}{8e^2}\sqrt{-e^2x^2+d^2} + \frac{d^5}{8e^2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}} - \frac{x^2}{5e}(-e^2x^2+d^2)^{\frac{3}{2}} - \frac{2d^2}{15e^3}(-e^2x^2+d^2)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(e\*x+d)\*(-e^2\*x^2+d^2)^(1/2),x)

[Out] -1/4\*d\*x\*(-e^2\*x^2+d^2)^(3/2)/e^2+1/8\*d^3\*x\*(-e^2\*x^2+d^2)^(1/2)/e^2+1/8\*d^5/e^2/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))-1/5\*x^2\*(-e^2\*x^2+d^2)^(3/2)/e-2/15\*d^2\*(-e^2\*x^2+d^2)^(3/2)/e^3

**Maxima [A]** time = 0.81739, size = 158, normalized size = 1.2

$$\frac{d^5 \arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{8\sqrt{e^2}e^2} + \frac{\sqrt{-e^2x^2+d^2}d^3x}{8e^2} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}x^2}{5e} - \frac{(-e^2x^2+d^2)^{\frac{3}{2}}dx}{4e^2} - \frac{2(-e^2x^2+d^2)^{\frac{3}{2}}d^2}{15e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)\*(e\*x + d)\*x^2,x, algorithm="maxima")

[Out] 1/8\*d^5\*arcsin(e^2\*x/sqrt(d^2\*e^2))/(sqrt(e^2)\*e^2) + 1/8\*sqrt(-e^2\*x^2 + d^2)\*d^3\*x/e^2 - 1/5\*(-e^2\*x^2 + d^2)^(3/2)\*x^2/e - 1/4\*(-e^2\*x^2 + d^2)^(3/2)\*d\*x/e^2 - 2/15\*(-e^2\*x^2 + d^2)^(3/2)\*d^2/e^3

**Fricas [A]** time = 0.278723, size = 479, normalized size = 3.63

$$24e^{10}x^{10} + 30de^9x^9 - 320d^2e^8x^8 - 405d^3e^7x^7 + 760d^4e^6x^6 + 1035d^5e^5x^5 - 480d^6e^4x^4 - 900d^7e^3x^3 + 240d^9ex - 30(5d^2e^2x^2 + 16d^4e^4x^4 + 156d^6e^6x^6 + 1035d^8e^8x^8 + 480d^{10}e^{10}x^{10})\sqrt{-e^2x^2 + d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)\*(e\*x + d)\*x^2,x, algorithm="fricas")

[Out] 1/120\*(24\*e^10\*x^10 + 30\*d\*e^9\*x^9 - 320\*d^2\*e^8\*x^8 - 405\*d^3\*e^7\*x^7 + 760\*d^4\*e^6\*x^6 + 1035\*d^5\*e^5\*x^5 - 480\*d^6\*e^4\*x^4 - 900\*d^7\*e^3\*x^3 + 240\*d^9\*e\*x - 30\*(5\*d^2\*e^2\*x^2 + 16\*d^4\*e^4\*x^4 + 156\*d^6\*e^6\*x^6 + 1035\*d^8\*e^8\*x^8 + 480\*d^10\*e^10\*x^10)\*sqrt(-e^2\*x^2 + d^2))\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) + 5\*(24\*d^2\*e^8\*x^8 + 30\*d^2\*e^7\*x^7 - 104\*d^3\*e^6\*x^6 - 135\*d^4\*e^5\*x^5 + 96\*d^5\*e^4\*x^4 + 156\*d^6\*e^3\*x^3 - 48\*d^8\*e\*x)\*sqrt(-e^2\*x^2 + d^2))/(5\*d^2\*e^7\*x^4 - 20\*d^3\*e^5\*x^2 + 16\*d^5\*e^3 - (e^7\*x^4 - 12\*d^2\*e^5\*x^2 + 16\*d^4\*e^3)\*sqrt(-e^2\*x^2 + d^2))

**Sympy [A]** time = 14.6335, size = 279, normalized size = 2.11

$$d \left( \begin{array}{l} \left( -\frac{id^4 \operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{id^3 x}{8e^2 \sqrt{-1 + \frac{e^2 x^2}{d^2}}} - \frac{3idx^3}{8\sqrt{-1 + \frac{e^2 x^2}{d^2}}} + \frac{ie^2 x^5}{4d\sqrt{-1 + \frac{e^2 x^2}{d^2}}} \quad \text{for } \left| \frac{e^2 x^2}{d^2} \right| > 1 \right) \\ \left( \frac{d^4 \operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3 x}{8e^2 \sqrt{1 - \frac{e^2 x^2}{d^2}}} + \frac{3dx^3}{8\sqrt{1 - \frac{e^2 x^2}{d^2}}} - \frac{e^2 x^5}{4d\sqrt{1 - \frac{e^2 x^2}{d^2}}} \quad \text{otherwise} \right) \end{array} \right) \\ + e \left( \begin{array}{l} \left( -\frac{2d^4 \sqrt{d^2 - e^2 x^2}}{15e^4} - \frac{d^2 x^2 \sqrt{d^2 - e^2 x^2}}{15e^2} + \frac{x^4 \sqrt{d^2 - e^2 x^2}}{5} \quad \text{for } e \neq 0 \right) \\ \left( \frac{x^4 \sqrt{d^2}}{4} \quad \text{otherwise} \right) \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2), x)

[Out] d\*Piecewise((-I\*d\*\*4\*acosh(e\*x/d)/(8\*e\*\*3) + I\*d\*\*3\*x/(8\*e\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 3\*I\*d\*x\*\*3/(8\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*5/(4\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*4\*asin(e\*x/d)/(8\*e\*\*3) - d\*\*3\*x/(8\*e\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 3\*d\*x\*\*3/(8\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*\*2\*x\*\*5/(4\*d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True)) + e\*Piecewise((-2\*d\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*e\*\*4) - d\*\*2\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*e\*\*2) + x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/5, Ne(e, 0)), (x\*\*4\*sqrt(d\*\*2)/4, True))

**GIAC/XCAS [A]** time = 0.333623, size = 100, normalized size = 0.76

$$\frac{1}{8} d^5 \arcsin\left(\frac{xe}{d}\right) e^{(-3)} \operatorname{sign}(d) - \frac{1}{120} \left( 16 d^4 e^{(-3)} + \left( 15 d^3 e^{(-2)} + 2 \left( 4 d^2 e^{(-1)} - 3(4xe + 5d)x \right) x \right) \sqrt{-x^2 e^2 + d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)\*(e\*x + d)\*x^2, x, algorithm="giac")

[Out] 1/8\*d^5\*arcsin(x\*e/d)\*e^(-3)\*sign(d) - 1/120\*(16\*d^4\*e^(-3) + (15\*d^3\*e^(-2) + 2\*(4\*d^2\*e^(-1) - 3\*(4\*x\*e + 5\*d)\*x)\*x)\*sqrt(-x^2\*e^2 + d^2)

### 3.2 $\int x^4(d+ex)(d^2-e^2x^2)^{3/2} dx$

**Optimal.** Leaf size=201

$$\begin{aligned} & -\frac{x^4(d^2-e^2x^2)^{5/2}}{9e} - \frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2} - \frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3} + \frac{3d^9 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^5} \\ & + \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^5x(d^2-e^2x^2)^{3/2}}{64e^4} - \frac{d^3(128d+315ex)(d^2-e^2x^2)^{5/2}}{5040e^5} \end{aligned}$$

[Out]  $(3*d^7*x*\text{Sqrt}[d^2 - e^2*x^2])/(128*e^4) + (d^5*x*(d^2 - e^2*x^2)^(3/2))/(64*e^4) - (4*d^2*x^2*(d^2 - e^2*x^2)^(5/2))/(63*e^3) - (d*x^3*(d^2 - e^2*x^2)^(5/2))/(8*e^2) - (x^4*(d^2 - e^2*x^2)^(5/2))/(9*e) - (d^3*(128*d + 315*e*x)*(d^2 - e^2*x^2)^(5/2))/(5040*e^5) + (3*d^9*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(128*e^5)$

**Rubi [A]** time = 0.443924, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\begin{aligned} & -\frac{x^4(d^2-e^2x^2)^{5/2}}{9e} - \frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2} - \frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3} + \frac{3d^9 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^5} \\ & + \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^5x(d^2-e^2x^2)^{3/2}}{64e^4} - \frac{d^3(128d+315ex)(d^2-e^2x^2)^{5/2}}{5040e^5} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^4*(d+e*x)*(d^2-e^2*x^2)^(3/2),x]$

[Out]  $(3*d^7*x*\text{Sqrt}[d^2 - e^2*x^2])/(128*e^4) + (d^5*x*(d^2 - e^2*x^2)^(3/2))/(64*e^4) - (4*d^2*x^2*(d^2 - e^2*x^2)^(5/2))/(63*e^3) - (d*x^3*(d^2 - e^2*x^2)^(5/2))/(8*e^2) - (x^4*(d^2 - e^2*x^2)^(5/2))/(9*e) - (d^3*(128*d + 315*e*x)*(d^2 - e^2*x^2)^(5/2))/(5040*e^5) + (3*d^9*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(128*e^5)$

**Rubi in Sympy [A]** time = 57.0033, size = 178, normalized size = 0.89

$$\begin{aligned} & \frac{3d^9 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^5} + \frac{3d^7x\sqrt{d^2-e^2x^2}}{128e^4} + \frac{d^5x(d^2-e^2x^2)^{3/2}}{64e^4} - \frac{d^3(384d+945ex)(d^2-e^2x^2)^{5/2}}{15120e^5} \\ & - \frac{4d^2x^2(d^2-e^2x^2)^{5/2}}{63e^3} - \frac{dx^3(d^2-e^2x^2)^{5/2}}{8e^2} - \frac{x^4(d^2-e^2x^2)^{5/2}}{9e} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**4*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)$

[Out]  $3*d**9*\operatorname{atan}(e*x/\text{sqrt}(d**2 - e**2*x**2))/(128*e**5) + 3*d**7*x*\text{sqrt}(d**2 - e**2*x**2)/(128*e**4) + d**5*x*(d**2 - e**2*x**2)**(3/2)/(64*e**4) - d**3*(384*d + 945*e*x)*(d**2 - e**2*x**2)**(5/2)/(15120*e**5) - 4*d**2*x**2*(d**2 - e**2*x**2)**(5/2)/(63*e**3) - d*x**3*(d**2 - e**2*x**2)**(5/2)/(8*e**2) - x**4*(d**2 - e**2*x**2)**(5/2)/(9*e)$

**Mathematica [A]** time = 0.132318, size = 136, normalized size = 0.68

$$945d^9 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \sqrt{d^2-e^2x^2} (1024d^8 + 945d^7ex + 512d^6e^2x^2 + 630d^5e^3x^3 + 384d^4e^4x^4 - 7560d^3e^5x^5 - 6400d^2e^6x^6$$

40320e^5

Antiderivative was successfully verified.

[In] Integrate[x^4\*(d + e\*x)\*(d^2 - e^2\*x^2)^(3/2), x]

[Out]  $(-\text{Sqrt}[d^2 - e^2 x^2] * (1024 d^8 + 945 d^7 e x + 512 d^6 e^2 x^2 + 630 d^5 e^3 x^3 + 384 d^4 e^4 x^4 - 7560 d^3 e^5 x^5 - 6400 d^2 e^6 x^6 + 5040 d e^7 x^7 + 4480 e^8 x^8)) + 945 d^9 \text{ArcTan}[(e x) / \text{Sqrt}[d^2 - e^2 x^2]]) / (40320 e^5)$

**Maple [A]** time = 0.036, size = 198, normalized size = 1.

$$\begin{aligned} & -\frac{dx^3}{8e^2} (-e^2x^2 + d^2)^{\frac{5}{2}} - \frac{d^3x}{16e^4} (-e^2x^2 + d^2)^{\frac{5}{2}} + \frac{d^5x}{64e^4} (-e^2x^2 + d^2)^{\frac{3}{2}} \\ & + \frac{3d^7x}{128e^4} \sqrt{-e^2x^2 + d^2} + \frac{3d^9}{128e^4} \arctan\left(x\sqrt{e^2} \frac{1}{\sqrt{-e^2x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} \\ & - \frac{x^4}{9e} (-e^2x^2 + d^2)^{\frac{5}{2}} - \frac{4d^2x^2}{63e^3} (-e^2x^2 + d^2)^{\frac{5}{2}} - \frac{8d^4}{315e^5} (-e^2x^2 + d^2)^{\frac{5}{2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(e\*x+d)\*(-e^2\*x^2+d^2)^(3/2), x)

[Out]  $-1/8*d*x^3*(-e^2*x^2+d^2)^(5/2)/e^2-1/16*d^3/e^4*x*(-e^2*x^2+d^2)^(5/2)+1/64*d^5*x*(-e^2*x^2+d^2)^(3/2)/e^4+3/128*d^7*x*(-e^2*x^2+d^2)^(1/2)/e^4+3/128*d^9/e^4/(e^2)^(1/2)*\arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/9*x^4*(-e^2*x^2+d^2)^(5/2)/e-4/63*d^2*x^2*(-e^2*x^2+d^2)^(5/2)/e^3-8/315*d^4/e^5*(-e^2*x^2+d^2)^(5/2)$

**Maxima [A]** time = 0.801193, size = 257, normalized size = 1.28

$$\begin{aligned} & -\frac{(-e^2x^2 + d^2)^{\frac{5}{2}}x^4}{9e} + \frac{3d^9 \arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{128\sqrt{e^2}e^4} + \frac{3\sqrt{-e^2x^2 + d^2}d^7x}{128e^4} - \frac{(-e^2x^2 + d^2)^{\frac{5}{2}}dx^3}{8e^2} \\ & + \frac{(-e^2x^2 + d^2)^{\frac{3}{2}}d^5x}{64e^4} - \frac{4(-e^2x^2 + d^2)^{\frac{5}{2}}d^2x^2}{63e^3} - \frac{(-e^2x^2 + d^2)^{\frac{5}{2}}d^3x}{16e^4} - \frac{8(-e^2x^2 + d^2)^{\frac{5}{2}}d^4}{315e^5} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)\*x^4,x, algorithm="maxima")

[Out]  $-1/9*(-e^2*x^2 + d^2)^(5/2)*x^4/e + 3/128*d^9*\arcsin(e^2*x/\text{sqrt}(d^2*e^2))/(\text{sqrt}(e^2)*e^4) + 3/128*\text{sqrt}(-e^2*x^2 + d^2)*d^7*x/e^4 - 1/8*(-e^2*x^2 + d^2)^(5/2)*d*x^3/e^2 + 1/64*(-e^2*x^2 + d^2)^(3/2)*d^5*x/e^4 - 4/63*(-e^2*x^2 + d^2)^(5/2)*d^2*x^2/e^3 - 1/16*(-e^2*x^2 + d^2)^(5/2)*d^3*x/e^4 - 8/315*(-e^2*x^2 + d^2)^(5/2)*d^4/e^5$

**Fricas [A]** time = 0.282974, size = 806, normalized size = 4.01

$$4480 e^{18} x^{18} + 5040 d e^{17} x^{17} - 190080 d^2 e^{16} x^{16} - 214200 d^3 e^{15} x^{15} + 1517184 d^4 e^{14} x^{14} + 1721790 d^5 e^{13} x^{13} - 4889472 d^6 e^{12} x^{12} - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)\*x^4,x, algorithm="fricas")

```
[Out] -1/40320*(4480*e^18*x^18 + 5040*d*e^17*x^17 - 190080*d^2*e^16*x^16 - 214200*d^3*e^15*x^15 + 1517184*d^4*e^14*x^14 + 1721790*d^5*e^13*x^13 - 4889472*d^6*e^12*x^12 - 5609205*d^7*e^11*x^11 + 7644672*d^8*e^10*x^10 + 8887095*d^9*e^9*x^9 - 5806080*d^10*e^8*x^8 - 6781320*d^11*e^7*x^7 + 1720320*d^12*e^6*x^6 + 1728720*d^13*e^5*x^5 + 504000*d^15*e^3*x^3 - 241920*d^17*e*x + 1890*(9*d^10*e^8*x^8 - 120*d^12*e^6*x^6 + 432*d^14*e^4*x^4 - 576*d^16*e^2*x^2 + 256*d^18 - (d^9*e^8*x^8 - 40*d^11*e^6*x^6 + 240*d^13*e^4*x^4 - 448*d^15*e^2*x^2 + 256*d^17)*sqrt(-e^2*x^2 + d^2))*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 3*(13440*d*e^16*x^16 + 15120*d^2*e^15*x^15 - 198400*d^3*e^14*x^14 - 224280*d^4*e^13*x^13 + 902272*d^5*e^12*x^12 + 1030050*d^6*e^11*x^11 - 1795584*d^7*e^10*x^10 - 2078685*d^8*e^9*x^9 + 1648640*d^9*e^8*x^8 + 1934520*d^10*e^7*x^7 - 573440*d^11*e^6*x^6 - 630000*d^12*e^5*x^5 - 127680*d^14*e^3*x^3 + 80640*d^16*e*x)*sqrt(-e^2*x^2 + d^2))/(9*d*e^13*x^8 - 120*d^3*e^11*x^6 + 432*d^5*e^9*x^4 - 576*d^7*e^7*x^2 + 256*d^9*e^5 - (e^13*x^8 - 40*d^2*e^11*x^6 + 240*d^4*e^9*x^4 - 448*d^6*e^7*x^2 + 256*d^8*e^5)*sqrt(-e^2*x^2 + d^2))
```

---

**Sympy [A]** time = 55.0427, size = 830, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)
```

```
[Out] d**3*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) + d**2*e*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True)) - d*e**2*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**8*asin(e*x/d)/(128*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d**2)), True)) - e**3*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8) - 8*d**6*x**2*sqrt(d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*sqrt(d**2 - e**2*x**2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True))
```

---

**GIAC/XCAS [A]** time = 0.315756, size = 158, normalized size = 0.79

$$\frac{3}{128} d^9 \arcsin\left(\frac{xe}{d}\right) e^{(-5)} \text{sign}(d) - \frac{1}{40320} \left( 1024 d^8 e^{(-5)} + \left( 945 d^7 e^{(-4)} + 2 \left( 256 d^6 e^{(-3)} + \left( 315 d^5 e^{(-2)} + 4 \left( 48 d^4 e^{(-1)} - 5 \left( 189 d^3 + 2 \left( 80 d^2 e - 7 \left( 8 x e^3 + 9 d e^2 \right) \right) \right) \right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x^4,x, algorithm="giac")
```

```
[Out] 3/128*d^9*arcsin(x*e/d)*e^(-5)*sign(d) - 1/40320*(1024*d^8*e^(-5)
+ (945*d^7*e^(-4) + 2*(256*d^6*e^(-3) + (315*d^5*e^(-2) + 4*(48*
d^4*e^(-1) - 5*(189*d^3 + 2*(80*d^2*e - 7*(8*x*e^3 + 9*d*e^2)*x)*
x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

### 3.3 $\int x^3(d+ex)(d^2-e^2x^2)^{3/2} dx$

**Optimal.** Leaf size=172

$$\frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2} - \frac{x^3(d^2-e^2x^2)^{5/2}}{8e} - \frac{d^2(32d+35ex)(d^2-e^2x^2)^{5/2}}{560e^4} + \frac{3d^8 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^4} + \frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} + \frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3}$$

[Out]  $(3*d^6*x*\text{Sqrt}[d^2 - e^2*x^2])/(128*e^3) + (d^4*x*(d^2 - e^2*x^2)^(3/2))/(64*e^3) - (d*x^2*(d^2 - e^2*x^2)^(5/2))/(7*e^2) - (x^3*(d^2 - e^2*x^2)^(5/2))/(8*e) - (d^2*(32*d + 35*e*x)*(d^2 - e^2*x^2)^(5/2))/(560*e^4) + (3*d^8*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(128*e^4)$

**Rubi [A]** time = 0.317049, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{dx^2(d^2-e^2x^2)^{5/2}}{7e^2} - \frac{x^3(d^2-e^2x^2)^{5/2}}{8e} - \frac{d^2(32d+35ex)(d^2-e^2x^2)^{5/2}}{560e^4} + \frac{3d^8 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^4} + \frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} + \frac{d^4x(d^2-e^2x^2)^{3/2}}{64e^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3*(d+e*x)*(d^2-e^2*x^2)^(3/2),x]$

[Out]  $(3*d^6*x*\text{Sqrt}[d^2 - e^2*x^2])/(128*e^3) + (d^4*x*(d^2 - e^2*x^2)^(3/2))/(64*e^3) - (d*x^2*(d^2 - e^2*x^2)^(5/2))/(7*e^2) - (x^3*(d^2 - e^2*x^2)^(5/2))/(8*e) - (d^2*(32*d + 35*e*x)*(d^2 - e^2*x^2)^(5/2))/(560*e^4) + (3*d^8*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(128*e^4)$

**Rubi in Sympy [A]** time = 43.0809, size = 151, normalized size = 0.88

$$\frac{3d^8 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{128e^4} + \frac{3d^6x\sqrt{d^2-e^2x^2}}{128e^3} + \frac{d^4x(d^2-e^2x^2)^{\frac{3}{2}}}{64e^3} - \frac{d^2(96d+105ex)(d^2-e^2x^2)^{\frac{5}{2}}}{1680e^4} - \frac{dx^2(d^2-e^2x^2)^{\frac{5}{2}}}{7e^2} - \frac{x^3(d^2-e^2x^2)^{\frac{5}{2}}}{8e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**3*(e*x+d)*(-e**2*x**2+d**2)**(3/2),x)$

[Out]  $3*d**8*\operatorname{atan}(e*x/\text{sqrt}(d**2 - e**2*x**2))/(128*e**4) + 3*d**6*x*\text{sqrt}(d**2 - e**2*x**2)/(128*e**3) + d**4*x*(d**2 - e**2*x**2)**(3/2)/(64*e**3) - d**2*(96*d + 105*e*x)*(d**2 - e**2*x**2)**(5/2)/(1680*e**4) - d*x**2*(d**2 - e**2*x**2)**(5/2)/(7*e**2) - x**3*(d**2 - e**2*x**2)**(5/2)/(8*e)$

**Mathematica [A]** time = 0.114811, size = 125, normalized size = 0.73

$$105d^8 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \sqrt{d^2-e^2x^2} (256d^7 + 105d^6ex + 128d^5e^2x^2 + 70d^4e^3x^3 - 1024d^3e^4x^4 - 840d^2e^5x^5 + 640de^6x^6 + 5$$

4480e^4



Antiderivative was successfully verified.

[In] Integrate[x^3\*(d + e\*x)\*(d^2 - e^2\*x^2)^(3/2), x]

[Out]  $(-\text{Sqrt}[d^2 - e^2 x^2] * (256 d^7 + 105 d^6 e x + 128 d^5 e^2 x^2 + 70 d^4 e^3 x^3 - 1024 d^3 e^4 x^4 - 840 d^2 e^5 x^5 + 640 d e^6 x^6 + 560 e^7 x^7)) + 105 d^8 \text{ArcTan}[(e x) / \text{Sqrt}[d^2 - e^2 x^2]]) / (4480 e^4)$

**Maple [A]** time = 0.022, size = 173, normalized size = 1.

$$-\frac{dx^2}{7e^2} (-e^2 x^2 + d^2)^{\frac{5}{2}} - \frac{2d^3}{35e^4} (-e^2 x^2 + d^2)^{\frac{5}{2}} - \frac{x^3}{8e} (-e^2 x^2 + d^2)^{\frac{5}{2}} - \frac{d^2 x}{16e^3} (-e^2 x^2 + d^2)^{\frac{5}{2}} + \frac{d^4 x}{64e^3} (-e^2 x^2 + d^2)^{\frac{3}{2}} + \frac{3d^6 x}{128e^3} \sqrt{-e^2 x^2 + d^2} + \frac{3d^8}{128e^3} \arctan\left(x \sqrt{e^2} \frac{1}{\sqrt{-e^2 x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(e\*x+d)\*(-e^2\*x^2+d^2)^(3/2), x)

[Out]  $-1/7*d*x^2*(-e^2*x^2+d^2)^{(5/2)}/e^2-2/35*d^3/e^4*(-e^2*x^2+d^2)^{(5/2)}-1/8*x^3*(-e^2*x^2+d^2)^{(5/2)}/e-1/16*d^2/e^3*x*(-e^2*x^2+d^2)^{(5/2)}+1/64*d^4*x*(-e^2*x^2+d^2)^{(3/2)}/e^3+3/128*d^6*x*(-e^2*x^2+d^2)^{(1/2)}/e^3+3/128*d^8/e^3/(e^2)^{(1/2)}*\arctan((e^2)^{(1/2)}*x/(-e^2*x^2+d^2)^{(1/2)})$

**Maxima [A]** time = 0.814438, size = 223, normalized size = 1.3

$$\frac{3d^8 \arcsin\left(\frac{e^2 x}{\sqrt{d^2 e^2}}\right)}{128 \sqrt{e^2} e^3} + \frac{3 \sqrt{-e^2 x^2 + d^2} d^6 x}{128 e^3} - \frac{(-e^2 x^2 + d^2)^{\frac{5}{2}} x^3}{8 e} + \frac{(-e^2 x^2 + d^2)^{\frac{3}{2}} d^4 x}{64 e^3} - \frac{(-e^2 x^2 + d^2)^{\frac{5}{2}} dx^2}{7 e^2} - \frac{(-e^2 x^2 + d^2)^{\frac{5}{2}} d^2 x}{16 e^3} - \frac{2(-e^2 x^2 + d^2)^{\frac{5}{2}} d^3}{35 e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)\*x^3, x, algorithm="maxima")

[Out]  $3/128*d^8*\arcsin(e^2*x/\text{sqrt}(d^2*e^2))/(\text{sqrt}(e^2)*e^3) + 3/128*\text{sqrt}(-e^2*x^2 + d^2)*d^6*x/e^3 - 1/8*(-e^2*x^2 + d^2)^{(5/2)}*x^3/e + 1/64*(-e^2*x^2 + d^2)^{(3/2)}*d^4*x/e^3 - 1/7*(-e^2*x^2 + d^2)^{(5/2)}*d*x^2/e^2 - 1/16*(-e^2*x^2 + d^2)^{(5/2)}*d^2*x/e^3 - 2/35*(-e^2*x^2 + d^2)^{(5/2)}*d^3/e^4$

**Fricas [A]** time = 0.281755, size = 744, normalized size = 4.33

$$4480 d e^{15} x^{15} + 5120 d^2 e^{14} x^{14} - 56000 d^3 e^{13} x^{13} - 64512 d^4 e^{12} x^{12} + 226800 d^5 e^{11} x^{11} + 265216 d^6 e^{10} x^{10} - 413000 d^7 e^9 x^9 - 4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)\*x^3, x, algorithm="fricas")

[Out]  $1/4480*(4480*d*e^{15}*x^{15} + 5120*d^2*e^{14}*x^{14} - 56000*d^3*e^{13}*x^{13} - 64512*d^4*e^{12}*x^{12} + 226800*d^5*e^{11}*x^{11} + 265216*d^6*e^{10}*x^{10} - 413000*d^7*e^9*x^9 - 4$

```
*x^10 - 413000*d^7*e^9*x^9 - 492800*d^8*e^8*x^8 + 350280*d^9*e^7*
x^7 + 430080*d^10*e^6*x^6 - 101360*d^11*e^5*x^5 - 143360*d^12*e^4
*x^4 - 24640*d^13*e^3*x^3 + 13440*d^15*e*x - 210*(d^8*e^8*x^8 - 3
2*d^10*e^6*x^6 + 160*d^12*e^4*x^4 - 256*d^14*e^2*x^2 + 128*d^16 +
8*(d^9*e^6*x^6 - 10*d^11*e^4*x^4 + 24*d^13*e^2*x^2 - 16*d^15)*sq
rt(-e^2*x^2 + d^2))*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (
560*e^15*x^15 + 640*d*e^14*x^14 - 18760*d^2*e^13*x^13 - 21504*d^3
*e^12*x^12 + 116550*d^4*e^11*x^11 + 135296*d^5*e^10*x^10 - 279895
*d^6*e^9*x^9 - 331520*d^7*e^8*x^8 + 294560*d^8*e^7*x^7 + 358400*d
^9*e^6*x^6 - 108640*d^10*e^5*x^5 - 143360*d^11*e^4*x^4 - 17920*d^
12*e^3*x^3 + 13440*d^14*e*x)*sqrt(-e^2*x^2 + d^2))/(e^12*x^8 - 32
*d^2*e^10*x^6 + 160*d^4*e^8*x^4 - 256*d^6*e^6*x^2 + 128*d^8*e^4 +
8*(d*e^10*x^6 - 10*d^3*e^8*x^4 + 24*d^5*e^6*x^2 - 16*d^7*e^4)*sq
rt(-e^2*x^2 + d^2))
```

**Sympy [A]** time = 52.3057, size = 775, normalized size = 4.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2), x)

[Out] d\*\*3\*Piecewise((-2\*d\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*e\*\*4) - d\*\*2\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*e\*\*2) + x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/5, Ne(e, 0)), (x\*\*4\*sqrt(d\*\*2)/4, True)) + d\*\*2\*e\*Piecewise((-I\*d\*\*6\*acosh(e\*x/d)/(16\*e\*\*5) + I\*d\*\*5\*x/(16\*e\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - I\*d\*\*3\*x\*\*3/(48\*e\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 5\*I\*d\*x\*\*5/(24\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*7/(6\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*6\*asin(e\*x/d)/(16\*e\*\*5) - d\*\*5\*x/(16\*e\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + d\*\*3\*x\*\*3/(48\*e\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 5\*d\*x\*\*5/(24\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*\*2\*x\*\*7/(6\*d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True)) - d\*e\*\*2\*Piecewise((-8\*d\*\*6\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(105\*e\*\*6) - 4\*d\*\*4\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(105\*e\*\*4) - d\*\*2\*x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(35\*e\*\*2) + x\*\*6\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/7, Ne(e, 0)), (x\*\*6\*sqrt(d\*\*2)/6, True)) - e\*\*3\*Piecewise((-5\*I\*d\*\*8\*acosh(e\*x/d)/(128\*e\*\*7) + 5\*I\*d\*\*7\*x/(128\*e\*\*6\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 5\*I\*d\*\*5\*x\*\*3/(384\*e\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - I\*d\*\*3\*x\*\*5/(192\*e\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 7\*I\*d\*x\*\*7/(48\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*9/(8\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (5\*d\*\*8\*asin(e\*x/d)/(128\*e\*\*7) - 5\*d\*\*7\*x/(128\*e\*\*6\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 5\*d\*\*5\*x\*\*3/(384\*e\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + d\*\*3\*x\*\*5/(192\*e\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 7\*d\*x\*\*7/(48\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*\*2\*x\*\*9/(8\*d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True))

**GIAC/XCAS [A]** time = 0.32173, size = 143, normalized size = 0.83

$$\frac{3}{128} d^8 \arcsin\left(\frac{xe}{d}\right) e^{(-4)} \text{sign}(d) - \frac{1}{4480} \left( 256 d^7 e^{(-4)} + \left( 105 d^6 e^{(-3)} + 2 \left( 64 d^5 e^{(-2)} + \left( 35 d^4 e^{(-1)} - 4 \left( 128 d^3 + 5 \left( 21 d^2 e - 2 \left( 7 x e^3 + 8 d e^2 \right) x \right) x \right) x \right) x \right) x \right) \sqrt{-2e^2 + d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)\*x^3,x, algorithm="giac")

[Out] 3/128\*d^8\*arcsin(x\*e/d)\*e^(-4)\*sign(d) - 1/4480\*(256\*d^7\*e^(-4) + (105\*d^6\*e^(-3) + 2\*(64\*d^5\*e^(-2) + (35\*d^4\*e^(-1) - 4\*(128\*d^3 + 5\*(21\*d^2\*e - 2\*(7\*x\*e^3 + 8\*d\*e^2)\*x)\*x)\*x)\*x)\*x)\*sqrt(-x^2\*e^2 + d^2)

### 3.4 $\int x^2(d + ex)(d^2 - e^2x^2)^{3/2} dx$

**Optimal.** Leaf size=159

$$\begin{aligned} & -\frac{dx(d^2 - e^2x^2)^{5/2}}{6e^2} - \frac{d^2(d^2 - e^2x^2)^{5/2}}{5e^3} + \frac{(d^2 - e^2x^2)^{7/2}}{7e^3} \\ & + \frac{d^7 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{16e^3} + \frac{d^5x\sqrt{d^2 - e^2x^2}}{16e^2} + \frac{d^3x(d^2 - e^2x^2)^{3/2}}{24e^2} \end{aligned}$$

[Out]  $(d^5x\sqrt{d^2 - e^2x^2})/(16e^2) + (d^3x(d^2 - e^2x^2)^{3/2})/(24e^2) - (d^2(d^2 - e^2x^2)^{5/2})/(5e^3) - (d^7 \arctan(e x / \sqrt{d^2 - e^2x^2})) / (16e^3) + (d^2 - e^2x^2)^{7/2} / (7e^3) + (d^5x\sqrt{d^2 - e^2x^2}) / (16e^2) + (d^3x(d^2 - e^2x^2)^{3/2}) / (24e^2)$

**Rubi [A]** time = 0.241565, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\begin{aligned} & -\frac{dx(d^2 - e^2x^2)^{5/2}}{6e^2} - \frac{d^2(d^2 - e^2x^2)^{5/2}}{5e^3} + \frac{(d^2 - e^2x^2)^{7/2}}{7e^3} \\ & + \frac{d^7 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{16e^3} + \frac{d^5x\sqrt{d^2 - e^2x^2}}{16e^2} + \frac{d^3x(d^2 - e^2x^2)^{3/2}}{24e^2} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(d + e\*x)\*(d^2 - e^2\*x^2)^(3/2), x]

[Out]  $(d^5x\sqrt{d^2 - e^2x^2})/(16e^2) + (d^3x(d^2 - e^2x^2)^{3/2})/(24e^2) - (d^2(d^2 - e^2x^2)^{5/2})/(5e^3) - (d^7 \arctan(e x / \sqrt{d^2 - e^2x^2})) / (16e^3) + (d^2 - e^2x^2)^{7/2} / (7e^3) + (d^5x\sqrt{d^2 - e^2x^2}) / (16e^2) + (d^3x(d^2 - e^2x^2)^{3/2}) / (24e^2)$

**Rubi in Sympy [A]** time = 36.98, size = 136, normalized size = 0.86

$$\begin{aligned} & \frac{d^7 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{16e^3} + \frac{d^5x\sqrt{d^2 - e^2x^2}}{16e^2} + \frac{d^3x(d^2 - e^2x^2)^{3/2}}{24e^2} \\ & - \frac{d^2(d^2 - e^2x^2)^{5/2}}{5e^3} - \frac{dx(d^2 - e^2x^2)^{5/2}}{6e^2} + \frac{(d^2 - e^2x^2)^{7/2}}{7e^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2), x)

[Out]  $d^7 \operatorname{atan}(e x / \sqrt{d^2 - e^2x^2}) / (16e^3) + d^5x\sqrt{d^2 - e^2x^2} / (16e^2) + d^3x(d^2 - e^2x^2)^{3/2} / (24e^2) - d^2(d^2 - e^2x^2)^{5/2} / (5e^3) - dx(d^2 - e^2x^2)^{5/2} / (6e^2) + (d^2 - e^2x^2)^{7/2} / (7e^3)$

**Mathematica [A]** time = 0.102159, size = 114, normalized size = 0.72

$$\frac{105d^7 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) - \sqrt{d^2 - e^2x^2} (96d^6 + 105d^5ex + 48d^4e^2x^2 - 490d^3e^3x^3 - 384d^2e^4x^4 + 280de^5x^5 + 240e^6x^6)}{1680e^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(d + e\*x)\*(d^2 - e^2\*x^2)^(3/2), x]

[Out]  $(-\text{Sqrt}[d^2 - e^2 x^2] * (96 d^6 + 105 d^5 e x + 48 d^4 e^2 x^2 - 490 d^3 e^3 x^3 - 384 d^2 e^4 x^4 + 280 d e^5 x^5 + 240 e^6 x^6)) + 105 d^7 \text{ArcTan}[(e x) / \text{Sqrt}[d^2 - e^2 x^2]]) / (1680 e^3)$

**Maple [A]** time = 0.11, size = 148, normalized size = 0.9

$$-\frac{dx}{6e^2}(-e^2x^2 + d^2)^{\frac{5}{2}} + \frac{d^3x}{24e^2}(-e^2x^2 + d^2)^{\frac{3}{2}} + \frac{d^5x}{16e^2}\sqrt{-e^2x^2 + d^2} + \frac{d^7}{16e^2} \arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} - \frac{x^2}{7e}(-e^2x^2 + d^2)^{\frac{5}{2}} - \frac{2d^2}{35e^3}(-e^2x^2 + d^2)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(e\*x+d)\*(-e^2\*x^2+d^2)^(3/2), x)

[Out]  $-1/6*d*x*(-e^2*x^2+d^2)^(5/2)/e^2+1/24*d^3*x*(-e^2*x^2+d^2)^(3/2)/e^2+1/16*d^5*x*(-e^2*x^2+d^2)^(1/2)/e^2+1/16*d^7/e^2/(e^2)^(1/2)*\arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/7*x^2*(-e^2*x^2+d^2)^(5/2)/e-2/35*d^2*(-e^2*x^2+d^2)^(5/2)/e^3$

**Maxima [A]** time = 0.809449, size = 189, normalized size = 1.19

$$\frac{d^7 \arcsin\left(\frac{e^2 x}{\sqrt{d^2 e^2}}\right)}{16 \sqrt{e^2} e^2} + \frac{\sqrt{-e^2 x^2 + d^2} d^5 x}{16 e^2} + \frac{(-e^2 x^2 + d^2)^{\frac{3}{2}} d^3 x}{24 e^2} - \frac{(-e^2 x^2 + d^2)^{\frac{5}{2}} x^2}{7 e} - \frac{(-e^2 x^2 + d^2)^{\frac{5}{2}} dx}{6 e^2} - \frac{2(-e^2 x^2 + d^2)^{\frac{5}{2}} d^2}{35 e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)\*x^2, x, algorithm="maxima")

[Out]  $1/16*d^7*\arcsin(e^2*x/\text{sqrt}(d^2*e^2))/(\text{sqrt}(e^2)*e^2) + 1/16*\text{sqrt}(-e^2*x^2 + d^2)*d^5*x/e^2 + 1/24*(-e^2*x^2 + d^2)^(3/2)*d^3*x/e^2 - 1/7*(-e^2*x^2 + d^2)^(5/2)*x^2/e - 1/6*(-e^2*x^2 + d^2)^(5/2)*d*x/e^2 - 2/35*(-e^2*x^2 + d^2)^(5/2)*d^2/e^3$

**Fricas [A]** time = 0.27809, size = 657, normalized size = 4.13

$$240 e^{14} x^{14} + 280 d e^{13} x^{13} - 6384 d^2 e^{12} x^{12} - 7490 d^3 e^{11} x^{11} + 34608 d^4 e^{10} x^{10} + 41475 d^5 e^9 x^9 - 75600 d^6 e^8 x^8 - 93905 d^7 e^7 x^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)\*x^2, x, algorithm="fricas")

[Out]  $-1/1680*(240*e^{14}*x^{14} + 280*d*e^{13}*x^{13} - 6384*d^2*e^{12}*x^{12} - 7490*d^3*e^{11}*x^{11} + 34608*d^4*e^{10}*x^{10} + 41475*d^5*e^9*x^9 - 75600*d^6*e^8*x^8 - 93905*d^7*e^7*x^7 + 73920*d^8*e^6*x^6 + 99400*d^9*e^5*x^5 - 26880*d^{10}*e^4*x^4 - 46480*d^{11}*e^3*x^3 + 6720*d^{13}*e*x + 210*(7*d^8*e^6*x^6 - 56*d^{10}*e^4*x^4 + 112*d^{12}*e^2*x^2 - 64*d^{14} - (d^7*e^6*x^6 - 24*d^9*e^4*x^4 + 80*d^{11}*e^2*x^2 - 64*d^{13}$

```
) * sqrt(-e^2*x^2 + d^2) * arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x))
+ 7*(240*d*e^12*x^12 + 280*d^2*e^11*x^11 - 2304*d^3*e^10*x^10 -
2730*d^4*e^9*x^9 + 6960*d^5*e^8*x^8 + 8505*d^6*e^7*x^7 - 8640*d^7
*e^6*x^6 - 11240*d^8*e^5*x^5 + 3840*d^9*e^4*x^4 + 6160*d^10*e^3*x
^3 - 960*d^12*e*x) * sqrt(-e^2*x^2 + d^2) / (7*d*e^9*x^6 - 56*d^3*e^
7*x^4 + 112*d^5*e^5*x^2 - 64*d^7*e^3 - (e^9*x^6 - 24*d^2*e^7*x^4
+ 80*d^4*e^5*x^2 - 64*d^6*e^3) * sqrt(-e^2*x^2 + d^2))
```

**Sympy [A]** time = 34.3137, size = 653, normalized size = 4.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2), x)

[Out] d\*\*3\*Piecewise((-I\*d\*\*4\*acosh(e\*x/d)/(8\*e\*\*3) + I\*d\*\*3\*x/(8\*e\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 3\*I\*d\*x\*\*3/(8\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*5/(4\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*4\*asin(e\*x/d)/(8\*e\*\*3) - d\*\*3\*x/(8\*e\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 3\*d\*x\*\*3/(8\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*\*2\*x\*\*5/(4\*d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True)) + d\*\*2\*e\*Piecewise((-2\*d\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*e\*\*4) - d\*\*2\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*e\*\*2) + x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/5, Ne(e, 0)), (x\*\*4\*sqrt(d\*\*2)/4, True)) - d\*\*2\*Piecewise((-I\*d\*\*6\*acosh(e\*x/d)/(16\*e\*\*5) + I\*d\*\*5\*x/(16\*e\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - I\*d\*\*3\*x\*\*3/(48\*e\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 5\*I\*d\*x\*\*5/(24\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*7/(6\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*6\*asin(e\*x/d)/(16\*e\*\*5) - d\*\*5\*x/(16\*e\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + d\*\*3\*x\*\*3/(48\*e\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 5\*d\*x\*\*5/(24\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*\*2\*x\*\*7/(6\*d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True)) - e\*\*3\*Piecewise((-8\*d\*\*6\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(105\*e\*\*6) - 4\*d\*\*4\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(105\*e\*\*4) - d\*\*2\*x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(35\*e\*\*2) + x\*\*6\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/7, Ne(e, 0)), (x\*\*6\*sqrt(d\*\*2)/6, True))

**GIAC/XCAS [A]** time = 0.315728, size = 130, normalized size = 0.82

$$\frac{1}{16} d^7 \arcsin\left(\frac{xe}{d}\right) e^{(-3)} \text{sign}(d) - \frac{1}{1680} \left( 96 d^6 e^{(-3)} + \left( 105 d^5 e^{(-2)} + 2 \left( 24 d^4 e^{(-1)} - (245 d^3 + 4(48 d^2 e - 5(6 x e^3 + 7 d e^2) x) x) x \right) x \right) \sqrt{-x^2 e^2 + d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)\*x^2,x, algorithm="giac")

[Out] 1/16\*d^7\*arcsin(x\*e/d)\*e^(-3)\*sign(d) - 1/1680\*(96\*d^6\*e^(-3) + (105\*d^5\*e^(-2) + 2\*(24\*d^4\*e^(-1) - (245\*d^3 + 4\*(48\*d^2\*e - 5\*(6\*x\*e^3 + 7\*d\*e^2)\*x)\*x)\*x)\*x)\*sqrt(-x^2\*e^2 + d^2)

### 3.5 $\int x(d + ex) (d^2 - e^2x^2)^{3/2} dx$

**Optimal.** Leaf size=116

$$\frac{d^2x (d^2 - e^2x^2)^{3/2}}{24e} - \frac{(6d + 5ex) (d^2 - e^2x^2)^{5/2}}{30e^2} + \frac{d^6 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{16e^2} + \frac{d^4x\sqrt{d^2 - e^2x^2}}{16e}$$

[Out]  $(d^4*x*\text{Sqrt}[d^2 - e^2*x^2])/(16*e) + (d^2*x*(d^2 - e^2*x^2)^{(3/2)})/(24*e) - ((6*d + 5*e*x)*(d^2 - e^2*x^2)^{(5/2)})/(30*e^2) + (d^6*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(16*e^2)$

**Rubi [A]** time = 0.103243, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$\frac{d^2x (d^2 - e^2x^2)^{3/2}}{24e} - \frac{(6d + 5ex) (d^2 - e^2x^2)^{5/2}}{30e^2} + \frac{d^6 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{16e^2} + \frac{d^4x\sqrt{d^2 - e^2x^2}}{16e}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*(d + e*x)*(d^2 - e^2*x^2)^{(3/2)}, x]$

[Out]  $(d^4*x*\text{Sqrt}[d^2 - e^2*x^2])/(16*e) + (d^2*x*(d^2 - e^2*x^2)^{(3/2)})/(24*e) - ((6*d + 5*e*x)*(d^2 - e^2*x^2)^{(5/2)})/(30*e^2) + (d^6*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(16*e^2)$

**Rubi in Sympy [A]** time = 21.708, size = 114, normalized size = 0.98

$$\frac{d^6 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{16e^2} + \frac{d^4x\sqrt{d^2 - e^2x^2}}{16e} + \frac{d^2x (d^2 - e^2x^2)^{\frac{3}{2}}}{24e} - \frac{d (d^2 - e^2x^2)^{\frac{5}{2}}}{30e^2} - \frac{(d + ex) (d^2 - e^2x^2)^{\frac{5}{2}}}{6e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x*(e*x+d)*(-e**2*x**2+d**2)**(3/2), x)$

[Out]  $d**6*\operatorname{atan}(e*x/\text{sqrt}(d**2 - e**2*x**2))/(16*e**2) + d**4*x*\text{sqrt}(d**2 - e**2*x**2)/(16*e) + d**2*x*(d**2 - e**2*x**2)**(3/2)/(24*e) - d*(d**2 - e**2*x**2)**(5/2)/(30*e**2) - (d + e*x)*(d**2 - e**2*x**2)**(5/2)/(6*e**2)$

**Mathematica [A]** time = 0.0923253, size = 103, normalized size = 0.89

$$\frac{15d^6 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) - \sqrt{d^2 - e^2x^2} (48d^5 + 15d^4ex - 96d^3e^2x^2 - 70d^2e^3x^3 + 48de^4x^4 + 40e^5x^5)}{240e^2}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x*(d + e*x)*(d^2 - e^2*x^2)^{(3/2)}, x]$

[Out]  $(-\text{Sqrt}[d^2 - e^2*x^2]*(48*d^5 + 15*d^4*e*x - 96*d^3*e^2*x^2 - 70*d^2*e^3*x^3 + 48*d*e^4*x^4 + 40*e^5*x^5) + 15*d^6*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(240*e^2)$

---

**Maple [A]** time = 0.018, size = 123, normalized size = 1.1

$$-\frac{d}{5e^2}(-e^2x^2+d^2)^{\frac{5}{2}} - \frac{x}{6e}(-e^2x^2+d^2)^{\frac{5}{2}} + \frac{d^2x}{24e}(-e^2x^2+d^2)^{\frac{3}{2}} \\ + \frac{d^4x}{16e}\sqrt{-e^2x^2+d^2} + \frac{d^6}{16e}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(e\*x+d)\*(-e^2\*x^2+d^2)^(3/2),x)

[Out] -1/5\*d/e^2\*(-e^2\*x^2+d^2)^(5/2)-1/6\*x\*(-e^2\*x^2+d^2)^(5/2)/e+1/24\*d^2\*x\*(-e^2\*x^2+d^2)^(3/2)/e+1/16\*d^4\*x\*(-e^2\*x^2+d^2)^(1/2)/e+1/16\*d^6/e/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))

---

**Maxima [A]** time = 0.820194, size = 155, normalized size = 1.34

$$\frac{d^6 \arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{16\sqrt{e^2e}} + \frac{\sqrt{-e^2x^2+d^2}d^4x}{16e} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d^2x}{24e} - \frac{(-e^2x^2+d^2)^{\frac{5}{2}}x}{6e} - \frac{(-e^2x^2+d^2)^{\frac{5}{2}}d}{5e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)\*x,x, algorithm="maxima")

[Out] 1/16\*d^6\*arcsin(e^2\*x/sqrt(d^2\*e^2))/(sqrt(e^2)\*e) + 1/16\*sqrt(-e^2\*x^2 + d^2)\*d^4\*x/e + 1/24\*(-e^2\*x^2 + d^2)^(3/2)\*d^2\*x/e - 1/6\*(-e^2\*x^2 + d^2)^(5/2)\*x/e - 1/5\*(-e^2\*x^2 + d^2)^(5/2)\*d/e^2

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**Fricas [A]** time = 0.282978, size = 598, normalized size = 5.16

$$240de^{11}x^{11} + 288d^2e^{10}x^{10} - 1940d^3e^9x^9 - 2400d^4e^8x^8 + 5310d^5e^7x^7 + 6960d^6e^6x^6 - 6330d^7e^5x^5 - 8640d^8e^4x^4 + 3200d^9e^3x^3 + 3840d^{10}e^2x^2 - 480d^{11}e^1x - 30(d^6e^6x^6 - 18d^8e^4x^4 + 48d^{10}e^2x^2 - 32d^{12})\sqrt{-e^2x^2 + d^2} + 2(3d^7e^4x^4 - 16d^9e^2x^2 + 16d^{11})\arctan\left(\frac{-d - \sqrt{-e^2x^2 + d^2}}{e^2x}\right) - (40e^{11}x^{11} + 48d^2e^{10}x^{10} - 790d^3e^9x^9 - 960d^4e^8x^8 + 3195d^5e^7x^7 + 4080d^6e^6x^6 - 4910d^7e^5x^5 - 6720d^8e^4x^4 + 2960d^9e^3x^3 + 3840d^{10}e^2x^2 - 480d^{11}e^1x)\sqrt{-e^2x^2 + d^2} + 2(3d^7e^4x^4 - 16d^9e^2x^2 + 16d^{11})\arctan\left(\frac{-d - \sqrt{-e^2x^2 + d^2}}{e^2x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)\*x,x, algorithm="fricas")

[Out] 1/240\*(240\*d\*e^11\*x^11 + 288\*d^2\*e^10\*x^10 - 1940\*d^3\*e^9\*x^9 - 2400\*d^4\*e^8\*x^8 + 5310\*d^5\*e^7\*x^7 + 6960\*d^6\*e^6\*x^6 - 6330\*d^7\*e^5\*x^5 - 8640\*d^8\*e^4\*x^4 + 3200\*d^9\*e^3\*x^3 + 3840\*d^10\*e^2\*x^2 - 480\*d^11\*e\*x - 30\*(d^6\*e^6\*x^6 - 18\*d^8\*e^4\*x^4 + 48\*d^10\*e^2\*x^2 - 32\*d^12)\*sqrt(-e^2\*x^2 + d^2))\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) - (40\*e^11\*x^11 + 48\*d^2\*e^10\*x^10 - 790\*d^3\*e^9\*x^9 - 960\*d^4\*e^8\*x^8 + 3195\*d^5\*e^7\*x^7 + 4080\*d^6\*e^6\*x^6 - 4910\*d^7\*e^5\*x^5 - 6720\*d^8\*e^4\*x^4 + 2960\*d^9\*e^3\*x^3 + 3840\*d^10\*e^2\*x^2 - 480\*d^11\*e\*x)\*sqrt(-e^2\*x^2 + d^2)/(e^8\*x^6 - 18\*d^2\*e^6\*x^4 + 48\*d^4\*e^4\*x^2 - 32\*d^6\*e^2 + 2\*(3\*d^7\*e^4\*x^4 - 16\*d^9\*e^2\*x^2 + 16\*d^11\*e^2)\*sqrt(-e^2\*x^2 + d^2))

---

**Sympy [A]** time = 32.3319, size = 580, normalized size = 5.

$$\begin{aligned}
 & d^3 \left( \begin{cases} \frac{x^2 \sqrt{d^2}}{2} & \text{for } e^2 = 0 \\ -\frac{(d^2 - e^2 x^2)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases} \right) \\
 & + d^2 e \left( \begin{cases} -\frac{id^4 \operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{id^3 x}{8e^2 \sqrt{-1 + \frac{e^2 x^2}{d^2}}} - \frac{3idx^3}{8\sqrt{-1 + \frac{e^2 x^2}{d^2}}} + \frac{ie^2 x^5}{4d\sqrt{-1 + \frac{e^2 x^2}{d^2}}} & \text{for } \left| \frac{e^2 x^2}{d^2} \right| > 1 \\ \frac{d^4 \operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3 x}{8e^2 \sqrt{1 - \frac{e^2 x^2}{d^2}}} + \frac{3dx^3}{8\sqrt{1 - \frac{e^2 x^2}{d^2}}} - \frac{e^2 x^5}{4d\sqrt{1 - \frac{e^2 x^2}{d^2}}} & \text{otherwise} \end{cases} \right) \\
 & - de^2 \left( \begin{cases} -\frac{2d^4 \sqrt{d^2 - e^2 x^2}}{15e^4} - \frac{d^2 x^2 \sqrt{d^2 - e^2 x^2}}{15e^2} + \frac{x^4 \sqrt{d^2 - e^2 x^2}}{5} & \text{for } e \neq 0 \\ \frac{x^4 \sqrt{d^2}}{4} & \text{otherwise} \end{cases} \right) \\
 & - e^3 \left( \begin{cases} -\frac{id^6 \operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^5} + \frac{id^5 x}{16e^4 \sqrt{-1 + \frac{e^2 x^2}{d^2}}} - \frac{id^3 x^3}{48e^2 \sqrt{-1 + \frac{e^2 x^2}{d^2}}} - \frac{5idx^5}{24\sqrt{-1 + \frac{e^2 x^2}{d^2}}} + \frac{ie^2 x^7}{6d\sqrt{-1 + \frac{e^2 x^2}{d^2}}} & \text{for } \left| \frac{e^2 x^2}{d^2} \right| > 1 \\ \frac{d^6 \operatorname{asin}\left(\frac{ex}{d}\right)}{16e^5} - \frac{d^5 x}{16e^4 \sqrt{1 - \frac{e^2 x^2}{d^2}}} + \frac{d^3 x^3}{48e^2 \sqrt{1 - \frac{e^2 x^2}{d^2}}} + \frac{5dx^5}{24\sqrt{1 - \frac{e^2 x^2}{d^2}}} - \frac{e^2 x^7}{6d\sqrt{1 - \frac{e^2 x^2}{d^2}}} & \text{otherwise} \end{cases} \right)
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2),x)

[Out] d\*\*3\*Piecewise((x\*\*2\*sqrt(d\*\*2)/2, Eq(e\*\*2, 0)), (- (d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(3\*e\*\*2), True)) + d\*\*2\*e\*Piecewise((-I\*d\*\*4\*acosh(e\*x/d)/(8\*e\*\*3) + I\*d\*\*3\*x/(8\*e\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 3\*I\*d\*x\*\*3/(8\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*5/(4\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*4\*asin(e\*x/d)/(8\*e\*\*3) - d\*\*3\*x/(8\*e\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 3\*d\*x\*\*3/(8\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*\*2\*x\*\*5/(4\*d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True)) - d\*e\*\*2\*Piecewise((-2\*d\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*e\*\*4) - d\*\*2\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*e\*\*2) + x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/5, Ne(e, 0)), (x\*\*4\*sqrt(d\*\*2)/4, True)) - e\*\*3\*Piecewise((-I\*d\*\*6\*acosh(e\*x/d)/(16\*e\*\*5) + I\*d\*\*5\*x/(16\*e\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - I\*d\*\*3\*x\*\*3/(48\*e\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 5\*I\*d\*x\*\*5/(24\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*7/(6\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*6\*asin(e\*x/d)/(16\*e\*\*5) - d\*\*5\*x/(16\*e\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + d\*\*3\*x\*\*3/(48\*e\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 5\*d\*x\*\*5/(24\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*\*2\*x\*\*7/(6\*d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True))

**GIAC/XCAS [A]** time = 0.320762, size = 113, normalized size = 0.97

$$\begin{aligned}
 & \frac{1}{16} d^6 \arcsin\left(\frac{xe}{d}\right) e^{(-2)\operatorname{sign}(d)} \\
 & - \frac{1}{240} \left( 48 d^5 e^{(-2)} + \left( 15 d^4 e^{(-1)} - 2 (48 d^3 + (35 d^2 e - 4 (5 x e^3 + 6 d e^2) x) x) x \right) \sqrt{-x^2 e^2 + d^2} \right)
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)\*x,x, algorithm="giac")

[Out] 1/16\*d^6\*arcsin(x\*e/d)\*e^(-2)\*sign(d) - 1/240\*(48\*d^5\*e^(-2) + (15\*d^4\*e^(-1) - 2\*(48\*d^3 + (35\*d^2\*e - 4\*(5\*x\*e^3 + 6\*d\*e^2)\*x)\*x)\*x)\*sqrt(-x^2\*e^2 + d^2)



### 3.6 $\int x(d + ex) (d^2 - e^2x^2)^{3/2} dx$

**Optimal.** Leaf size=116

$$\frac{d^2x (d^2 - e^2x^2)^{3/2}}{24e} - \frac{(6d + 5ex) (d^2 - e^2x^2)^{5/2}}{30e^2} + \frac{d^6 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{16e^2} + \frac{d^4x\sqrt{d^2 - e^2x^2}}{16e}$$

[Out]  $(d^4x\sqrt{d^2 - e^2x^2})/(16e) + (d^2x(d^2 - e^2x^2)^{3/2})/(24e) - ((6d + 5ex)(d^2 - e^2x^2)^{5/2})/(30e^2) + (d^6 \text{ArcTan}[(ex)/\sqrt{d^2 - e^2x^2}])/(16e^2)$

**Rubi [A]** time = 0.0975382, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$\frac{d^2x (d^2 - e^2x^2)^{3/2}}{24e} - \frac{(6d + 5ex) (d^2 - e^2x^2)^{5/2}}{30e^2} + \frac{d^6 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{16e^2} + \frac{d^4x\sqrt{d^2 - e^2x^2}}{16e}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2), x]$

[Out]  $(d^4x\sqrt{d^2 - e^2x^2})/(16e) + (d^2x(d^2 - e^2x^2)^{3/2})/(24e) - ((6d + 5ex)(d^2 - e^2x^2)^{5/2})/(30e^2) + (d^6 \text{ArcTan}[(ex)/\sqrt{d^2 - e^2x^2}])/(16e^2)$

**Rubi in Sympy [A]** time = 21.7141, size = 114, normalized size = 0.98

$$\frac{d^6 \text{atan}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{16e^2} + \frac{d^4x\sqrt{d^2 - e^2x^2}}{16e} + \frac{d^2x (d^2 - e^2x^2)^{3/2}}{24e} - \frac{d (d^2 - e^2x^2)^{5/2}}{30e^2} - \frac{(d + ex) (d^2 - e^2x^2)^{5/2}}{6e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x*(e*x+d)*(-e**2*x**2+d**2)**(3/2), x)$

[Out]  $d**6*\text{atan}(e*x/\text{sqrt}(d**2 - e**2*x**2))/(16*e**2) + d**4*x*\text{sqrt}(d**2 - e**2*x**2)/(16*e) + d**2*x*(d**2 - e**2*x**2)**(3/2)/(24*e) - d*(d**2 - e**2*x**2)**(5/2)/(30*e**2) - (d + e*x)*(d**2 - e**2*x**2)**(5/2)/(6*e**2)$

**Mathematica [A]** time = 0.0226689, size = 103, normalized size = 0.89

$$\frac{15d^6 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) - \sqrt{d^2 - e^2x^2} (48d^5 + 15d^4ex - 96d^3e^2x^2 - 70d^2e^3x^3 + 48de^4x^4 + 40e^5x^5)}{240e^2}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x*(d + e*x)*(d^2 - e^2*x^2)^(3/2), x]$

[Out]  $(-\text{sqrt}(d^2 - e^2x^2)*(48d^5 + 15d^4ex - 96d^3e^2x^2 - 70d^2e^3x^3 + 48de^4x^4 + 40e^5x^5) + 15d^6 \text{ArcTan}[(ex)/\text{sqrt}(d^2 - e^2x^2)])/(240e^2)$

---

**Maple [A]** time = 0., size = 123, normalized size = 1.1

$$-\frac{d}{5e^2}(-e^2x^2+d^2)^{\frac{5}{2}} - \frac{x}{6e}(-e^2x^2+d^2)^{\frac{5}{2}} + \frac{d^2x}{24e}(-e^2x^2+d^2)^{\frac{3}{2}} \\ + \frac{d^4x}{16e}\sqrt{-e^2x^2+d^2} + \frac{d^6}{16e}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(e\*x+d)\*(-e^2\*x^2+d^2)^(3/2),x)

[Out] -1/5\*d/e^2\*(-e^2\*x^2+d^2)^(5/2)-1/6\*x\*(-e^2\*x^2+d^2)^(5/2)/e+1/24\*d^2\*x\*(-e^2\*x^2+d^2)^(3/2)/e+1/16\*d^4\*x\*(-e^2\*x^2+d^2)^(1/2)/e+1/16\*d^6/e/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))

---

**Maxima [A]** time = 0.800865, size = 155, normalized size = 1.34

$$\frac{d^6 \arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{16\sqrt{e^2e}} + \frac{\sqrt{-e^2x^2+d^2}d^4x}{16e} + \frac{(-e^2x^2+d^2)^{\frac{3}{2}}d^2x}{24e} - \frac{(-e^2x^2+d^2)^{\frac{5}{2}}x}{6e} - \frac{(-e^2x^2+d^2)^{\frac{5}{2}}d}{5e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)\*x,x, algorithm="maxima")

[Out] 1/16\*d^6\*arcsin(e^2\*x/sqrt(d^2\*e^2))/(sqrt(e^2)\*e) + 1/16\*sqrt(-e^2\*x^2 + d^2)\*d^4\*x/e + 1/24\*(-e^2\*x^2 + d^2)^(3/2)\*d^2\*x/e - 1/6\*(-e^2\*x^2 + d^2)^(5/2)\*x/e - 1/5\*(-e^2\*x^2 + d^2)^(5/2)\*d/e^2

---

**Fricas [A]** time = 0.285485, size = 598, normalized size = 5.16

$$240de^{11}x^{11} + 288d^2e^{10}x^{10} - 1940d^3e^9x^9 - 2400d^4e^8x^8 + 5310d^5e^7x^7 + 6960d^6e^6x^6 - 6330d^7e^5x^5 - 8640d^8e^4x^4 + 3200d^9e^3x^3 + 3840d^{10}e^2x^2 - 480d^{11}e^1x - 30(d^6e^6x^6 - 18d^8e^4x^4 + 48d^{10}e^2x^2 - 32d^{12}) + 2(3d^7e^4x^4 - 16d^9e^2x^2 + 16d^{11})\sqrt{(-e^2x^2 + d^2)}\arctan\left(\frac{-d - \sqrt{-e^2x^2 + d^2}}{e^2x}\right) - (40e^{11}x^{11} + 48d^2e^{10}x^{10} - 790d^3e^9x^9 - 960d^4e^8x^8 + 3195d^5e^7x^7 + 4080d^6e^6x^6 - 4910d^7e^5x^5 - 6720d^8e^4x^4 + 2960d^9e^3x^3 + 3840d^{10}e^2x^2 - 480d^{11}e^1x)\sqrt{(-e^2x^2 + d^2)}/(e^8x^6 - 18d^2e^6x^4 + 48d^4e^4x^2 - 32d^6e^2 + 2(3d^7e^4x^4 - 16d^9e^2x^2 + 16d^{11}e^1))\sqrt{(-e^2x^2 + d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)\*x,x, algorithm="fricas")

[Out] 1/240\*(240\*d\*e^11\*x^11 + 288\*d^2\*e^10\*x^10 - 1940\*d^3\*e^9\*x^9 - 2400\*d^4\*e^8\*x^8 + 5310\*d^5\*e^7\*x^7 + 6960\*d^6\*e^6\*x^6 - 6330\*d^7\*e^5\*x^5 - 8640\*d^8\*e^4\*x^4 + 3200\*d^9\*e^3\*x^3 + 3840\*d^10\*e^2\*x^2 - 480\*d^11\*e\*x - 30\*(d^6\*e^6\*x^6 - 18\*d^8\*e^4\*x^4 + 48\*d^10\*e^2\*x^2 - 32\*d^12) + 2\*(3\*d^7\*e^4\*x^4 - 16\*d^9\*e^2\*x^2 + 16\*d^11)\*sqrt((-e^2\*x^2 + d^2))\*arctan((-d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) - (40\*e^11\*x^11 + 48\*d^2\*e^10\*x^10 - 790\*d^3\*e^9\*x^9 - 960\*d^4\*e^8\*x^8 + 3195\*d^5\*e^7\*x^7 + 4080\*d^6\*e^6\*x^6 - 4910\*d^7\*e^5\*x^5 - 6720\*d^8\*e^4\*x^4 + 2960\*d^9\*e^3\*x^3 + 3840\*d^10\*e^2\*x^2 - 480\*d^11\*e\*x)\*sqrt((-e^2\*x^2 + d^2))/(e^8\*x^6 - 18\*d^2\*e^6\*x^4 + 48\*d^4\*e^4\*x^2 - 32\*d^6\*e^2 + 2\*(3\*d^7\*e^4\*x^4 - 16\*d^9\*e^2\*x^2 + 16\*d^11\*e^1))\*sqrt((-e^2\*x^2 + d^2))

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**Sympy [A]** time = 32.9528, size = 580, normalized size = 5.

$$\begin{aligned}
 & d^3 \left( \begin{cases} \frac{x^2 \sqrt{d^2}}{2} & \text{for } e^2 = 0 \\ -\frac{(d^2 - e^2 x^2)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases} \right) \\
 & + d^2 e \left( \begin{cases} -\frac{id^4 \operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{id^3 x}{8e^2 \sqrt{-1 + \frac{e^2 x^2}{d^2}}} - \frac{3idx^3}{8\sqrt{-1 + \frac{e^2 x^2}{d^2}}} + \frac{ie^2 x^5}{4d\sqrt{-1 + \frac{e^2 x^2}{d^2}}} & \text{for } \left| \frac{e^2 x^2}{d^2} \right| > 1 \\ \frac{d^4 \operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3 x}{8e^2 \sqrt{1 - \frac{e^2 x^2}{d^2}}} + \frac{3dx^3}{8\sqrt{1 - \frac{e^2 x^2}{d^2}}} - \frac{e^2 x^5}{4d\sqrt{1 - \frac{e^2 x^2}{d^2}}} & \text{otherwise} \end{cases} \right) \\
 & - de^2 \left( \begin{cases} -\frac{2d^4 \sqrt{d^2 - e^2 x^2}}{15e^4} - \frac{d^2 x^2 \sqrt{d^2 - e^2 x^2}}{15e^2} + \frac{x^4 \sqrt{d^2 - e^2 x^2}}{5} & \text{for } e \neq 0 \\ \frac{x^4 \sqrt{d^2}}{4} & \text{otherwise} \end{cases} \right) \\
 & - e^3 \left( \begin{cases} -\frac{id^6 \operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^5} + \frac{id^5 x}{16e^4 \sqrt{-1 + \frac{e^2 x^2}{d^2}}} - \frac{id^3 x^3}{48e^2 \sqrt{-1 + \frac{e^2 x^2}{d^2}}} - \frac{5idx^5}{24\sqrt{-1 + \frac{e^2 x^2}{d^2}}} + \frac{ie^2 x^7}{6d\sqrt{-1 + \frac{e^2 x^2}{d^2}}} & \text{for } \left| \frac{e^2 x^2}{d^2} \right| > 1 \\ \frac{d^6 \operatorname{asin}\left(\frac{ex}{d}\right)}{16e^5} - \frac{d^5 x}{16e^4 \sqrt{1 - \frac{e^2 x^2}{d^2}}} + \frac{d^3 x^3}{48e^2 \sqrt{1 - \frac{e^2 x^2}{d^2}}} + \frac{5dx^5}{24\sqrt{1 - \frac{e^2 x^2}{d^2}}} - \frac{e^2 x^7}{6d\sqrt{1 - \frac{e^2 x^2}{d^2}}} & \text{otherwise} \end{cases} \right)
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2),x)

[Out] d\*\*3\*Piecewise((x\*\*2\*sqrt(d\*\*2)/2, Eq(e\*\*2, 0)), (- (d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(3\*e\*\*2), True)) + d\*\*2\*e\*Piecewise((-I\*d\*\*4\*acosh(e\*x/d)/(8\*e\*\*3) + I\*d\*\*3\*x/(8\*e\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 3\*I\*d\*x\*\*3/(8\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*5/(4\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*4\*asin(e\*x/d)/(8\*e\*\*3) - d\*\*3\*x/(8\*e\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 3\*d\*x\*\*3/(8\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*\*2\*x\*\*5/(4\*d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True)) - d\*e\*\*2\*Piecewise((-2\*d\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*e\*\*4) - d\*\*2\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*e\*\*2) + x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/5, Ne(e, 0)), (x\*\*4\*sqrt(d\*\*2)/4, True)) - e\*\*3\*Piecewise((-I\*d\*\*6\*acosh(e\*x/d)/(16\*e\*\*5) + I\*d\*\*5\*x/(16\*e\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - I\*d\*\*3\*x\*\*3/(48\*e\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 5\*I\*d\*x\*\*5/(24\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*7/(6\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*6\*asin(e\*x/d)/(16\*e\*\*5) - d\*\*5\*x/(16\*e\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + d\*\*3\*x\*\*3/(48\*e\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 5\*d\*x\*\*5/(24\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*\*2\*x\*\*7/(6\*d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True))

**GIAC/XCAS [A]** time = 0.320321, size = 113, normalized size = 0.97

$$\begin{aligned}
 & \frac{1}{16} d^6 \arcsin\left(\frac{xe}{d}\right) e^{(-2)} \operatorname{sign}(d) \\
 & - \frac{1}{240} \left( 48 d^5 e^{(-2)} + \left( 15 d^4 e^{(-1)} - 2 (48 d^3 + (35 d^2 e - 4 (5 x e^3 + 6 d e^2) x) x) x \right) \sqrt{-x^2 e^2 + d^2} \right)
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)\*x,x, algorithm="giac")

[Out] 1/16\*d^6\*arcsin(x\*e/d)\*e^(-2)\*sign(d) - 1/240\*(48\*d^5\*e^(-2) + (15\*d^4\*e^(-1) - 2\*(48\*d^3 + (35\*d^2\*e - 4\*(5\*x\*e^3 + 6\*d\*e^2)\*x)\*x)\*x)\*sqrt(-x^2\*e^2 + d^2)

$$3.7 \quad \int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x} dx$$

**Optimal.** Leaf size=113

$$\frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)(d^2-e^2x^2)^{3/2} + \frac{3}{8}d^4 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d^4 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

[Out]  $(d^2*(8*d + 3*e*x)*\text{Sqrt}[d^2 - e^2*x^2])/8 + ((4*d + 3*e*x)*(d^2 - e^2*x^2)^{(3/2)})/12 + (3*d^4*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/8 - d^4*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d]$

**Rubi [A]** time = 0.287951, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$\frac{1}{8}d^2(8d+3ex)\sqrt{d^2-e^2x^2} + \frac{1}{12}(4d+3ex)(d^2-e^2x^2)^{3/2} + \frac{3}{8}d^4 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d^4 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*(d^2 - e^2\*x^2)^(3/2))/x, x]

[Out]  $(d^2*(8*d + 3*e*x)*\text{Sqrt}[d^2 - e^2*x^2])/8 + ((4*d + 3*e*x)*(d^2 - e^2*x^2)^{(3/2)})/12 + (3*d^4*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/8 - d^4*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d]$

**Rubi in Sympy [A]** time = 43.7862, size = 95, normalized size = 0.84

$$\frac{3d^4 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d^4 \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) + \frac{d^2(8d+3ex)\sqrt{d^2-e^2x^2}}{8} + \frac{(4d+3ex)(d^2-e^2x^2)^{3/2}}{12}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2)/x, x)

[Out]  $3*d**4*\operatorname{atan}(e*x/\operatorname{sqrt}(d**2 - e**2*x**2))/8 - d**4*\operatorname{atanh}(\operatorname{sqrt}(d**2 - e**2*x**2)/d) + d**2*(8*d + 3*e*x)*\operatorname{sqrt}(d**2 - e**2*x**2)/8 + (4*d + 3*e*x)*(d**2 - e**2*x**2)**(3/2)/12$

**Mathematica [A]** time = 0.101565, size = 108, normalized size = 0.96

$$d^4 \log(x) - d^4 \log\left(\sqrt{d^2 - e^2x^2} + d\right) + \frac{3}{8}d^4 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) + \frac{1}{24}\sqrt{d^2 - e^2x^2}(32d^3 + 15d^2ex - 8de^2x^2 - 6e^3x^3)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(d^2 - e^2\*x^2)^(3/2))/x, x]

[Out]  $(\text{Sqrt}[d^2 - e^2*x^2]*(32*d^3 + 15*d^2*e*x - 8*d*e^2*x^2 - 6*e^3*x^3))/24 + (3*d^4*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/8 + d^4*\text{Log}[x] - d^4*\text{Log}[d + \text{Sqrt}[d^2 - e^2*x^2]]$

---

**Maple [A]** time = 0.019, size = 151, normalized size = 1.3

$$\frac{ex}{4} (-e^2x^2 + d^2)^{\frac{3}{2}} + \frac{3ed^2x}{8} \sqrt{-e^2x^2 + d^2} + \frac{3ed^4}{8} \arctan\left(x\sqrt{e^2} \frac{1}{\sqrt{-e^2x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}}$$

$$+ \frac{d}{3} (-e^2x^2 + d^2)^{\frac{3}{2}} + d^3 \sqrt{-e^2x^2 + d^2} - d^5 \ln\left(\frac{1}{x} \left(2d^2 + 2\sqrt{d^2} \sqrt{-e^2x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)\*(-e^2\*x^2+d^2)^(3/2)/x,x)

[Out] 1/4\*e\*x\*(-e^2\*x^2+d^2)^(3/2)+3/8\*e\*d^2\*x\*(-e^2\*x^2+d^2)^(1/2)+3/8\*e\*d^4/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))+1/3\*d\*(-e^2\*x^2+d^2)^(3/2)+d^3\*(-e^2\*x^2+d^2)^(1/2)-d^5/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)/x,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.288134, size = 524, normalized size = 4.64

$$24de^7x^7 + 32d^2e^6x^6 - 132d^3e^5x^5 - 192d^4e^4x^4 + 228d^5e^3x^3 + 192d^6e^2x^2 - 120d^7ex - 18(d^4e^4x^4 - 8d^6e^2x^2 + 8d^8 + 4(d$$


---

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)/x,x, algorithm="fricas")

[Out] 1/24\*(24\*d\*e^7\*x^7 + 32\*d^2\*e^6\*x^6 - 132\*d^3\*e^5\*x^5 - 192\*d^4\*e^4\*x^4 + 228\*d^5\*e^3\*x^3 + 192\*d^6\*e^2\*x^2 - 120\*d^7\*e\*x - 18\*(d^4\*e^4\*x^4 - 8\*d^6\*e^2\*x^2 + 8\*d^8 + 4\*(d^5\*e^2\*x^2 - 2\*d^7)\*sqrt(-e^2\*x^2 + d^2))\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) + 24\*(d^4\*e^4\*x^4 - 8\*d^6\*e^2\*x^2 + 8\*d^8 + 4\*(d^5\*e^2\*x^2 - 2\*d^7)\*sqrt(-e^2\*x^2 + d^2))\*log(-(d - sqrt(-e^2\*x^2 + d^2))/x) - (6\*e^7\*x^7 + 8\*d\*e^6\*x^6 - 63\*d^2\*e^5\*x^5 - 96\*d^3\*e^4\*x^4 + 168\*d^4\*e^3\*x^3 + 192\*d^5\*e^2\*x^2 - 120\*d^6\*e\*x)\*sqrt(-e^2\*x^2 + d^2)/(e^4\*x^4 - 8\*d^2\*e^2\*x^2 + 8\*d^4 + 4\*(d\*e^2\*x^2 - 2\*d^3)\*sqrt(-e^2\*x^2 + d^2))

---

**Sympy [A]** time = 25.965, size = 469, normalized size = 4.15

$$\begin{aligned}
 & d^3 \left( \begin{array}{l} \left( \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}} - d \operatorname{acosh}\left(\frac{d}{ex}\right) - \frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} \quad \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \right) \\ \left( -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}} + id \operatorname{asin}\left(\frac{d}{ex}\right) + \frac{iox}{\sqrt{-\frac{d^2}{e^2x^2}+1}} \quad \text{otherwise} \right) \end{array} \right) \\
 & + d^2 e \left( \begin{array}{l} \left( -\frac{id^2 \operatorname{acosh}\left(\frac{ex}{d}\right)}{2e} - \frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} \quad \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \right) \\ \left( \frac{d^2 \operatorname{asin}\left(\frac{ex}{d}\right)}{2e} + \frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} \quad \text{otherwise} \right) \end{array} \right) \\
 & - de^2 \left( \begin{array}{l} \left( \frac{x^2\sqrt{d^2}}{2} \quad \text{for } e^2 = 0 \right) \\ \left( -\frac{(d^2-e^2x^2)^{\frac{3}{2}}}{3e^2} \quad \text{otherwise} \right) \end{array} \right) \\
 & - e^3 \left( \begin{array}{l} \left( -\frac{id^4 \operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} \quad \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \right) \\ \left( \frac{d^4 \operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} \quad \text{otherwise} \right) \end{array} \right)
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2)/x,x)

[Out] d\*\*3\*Piecewise((d\*\*2/(e\*x\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) - d\*acosh(d/(e\*x)) - e\*x/sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (-I\*d\*\*2/(e\*x\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) + I\*d\*asin(d/(e\*x)) + I\*e\*x/sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1), True)) + d\*\*2\*e\*Piecewise((-I\*d\*\*2\*acosh(e\*x/d)/(2\*e) - I\*d\*x/(2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*3/(2\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2))), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*2\*asin(e\*x/d)/(2\*e) + d\*x\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)/2, True)) - d\*e\*\*2\*Piecewise((x\*\*2\*sqrt(d\*\*2)/2, Eq(e\*\*2, 0)), (- (d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(3\*e\*\*2), True)) - e\*\*3\*Piecewise((-I\*d\*\*4\*acosh(e\*x/d)/(8\*e\*\*3) + I\*d\*\*3\*x/(8\*e\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 3\*I\*d\*x\*\*3/(8\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*5/(4\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2))), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*4\*asin(e\*x/d)/(8\*e\*\*3) - d\*\*3\*x/(8\*e\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 3\*d\*x\*\*3/(8\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*\*2\*x\*\*5/(4\*d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2))), True))

**GIAC/XCAS [A]** time = 0.298387, size = 134, normalized size = 1.19

$$\begin{aligned}
 & \frac{3}{8} d^4 \arcsin\left(\frac{xe}{d}\right) \operatorname{sign}(d) - d^4 \ln\left(\frac{\left|-2de - 2\sqrt{-x^2e^2 + d^2}e\right|e^{(-2)}}{2|x|}\right) \\
 & + \frac{1}{24} (32d^3 + (15d^2e - 2(3xe^3 + 4de^2)x)x)\sqrt{-x^2e^2 + d^2}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)/x,x, algorithm="giac")

[Out] 3/8\*d^4\*arcsin(x\*e/d)\*sign(d) - d^4\*ln(1/2\*abs(-2\*d\*e - 2\*sqrt(-x^2\*e^2 + d^2)\*e)\*e^(-2)/abs(x)) + 1/24\*(32\*d^3 + (15\*d^2\*e - 2\*(3\*x\*e^3 + 4\*d\*e^2)\*x)\*x)\*sqrt(-x^2\*e^2 + d^2)

$$3.8 \quad \int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^2} dx$$

**Optimal.** Leaf size=117

$$\frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2} - \frac{(3d-ex)(d^2-e^2x^2)^{3/2}}{3x} - \frac{3}{2}d^3e \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d^3e \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

[Out] (d\*e\*(2\*d - 3\*e\*x)\*Sqrt[d^2 - e^2\*x^2])/2 - ((3\*d - e\*x)\*(d^2 - e^2\*x^2)^(3/2))/(3\*x) - (3\*d^3\*e\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/2 - d^3\*e\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]

**Rubi [A]** time = 0.293066, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.32$

$$\frac{1}{2}de(2d-3ex)\sqrt{d^2-e^2x^2} - \frac{(3d-ex)(d^2-e^2x^2)^{3/2}}{3x} - \frac{3}{2}d^3e \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - d^3e \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*(d^2 - e^2\*x^2)^(3/2))/x^2, x]

[Out] (d\*e\*(2\*d - 3\*e\*x)\*Sqrt[d^2 - e^2\*x^2])/2 - ((3\*d - e\*x)\*(d^2 - e^2\*x^2)^(3/2))/(3\*x) - (3\*d^3\*e\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/2 - d^3\*e\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]

**Rubi in Sympy [A]** time = 41.3694, size = 99, normalized size = 0.85

$$-\frac{3d^3e \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2} - d^3e \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) + \frac{de(4d-6ex)\sqrt{d^2-e^2x^2}}{4} - \frac{(3d-ex)(d^2-e^2x^2)^{3/2}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2)/x\*\*2, x)

[Out] -3\*d\*\*3\*e\*atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2))/2 - d\*\*3\*e\*atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d) + d\*e\*(4\*d - 6\*e\*x)\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/4 - (3\*d - e\*x)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(3\*x)

**Mathematica [A]** time = 0.161032, size = 114, normalized size = 0.97

$$d^3e \log(x) - d^3e \log\left(\sqrt{d^2-e^2x^2} + d\right) - \frac{3}{2}d^3e \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \sqrt{d^2-e^2x^2}\left(-\frac{d^3}{x} + \frac{4d^2e}{3} - \frac{1}{2}de^2x - \frac{e^3x^2}{3}\right)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(d^2 - e^2\*x^2)^(3/2))/x^2, x]

[Out] Sqrt[d^2 - e^2\*x^2]\*((4\*d^2\*e)/3 - d^3/x - (d\*e^2\*x)/2 - (e^3\*x^2)/3) - (3\*d^3\*e\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/2 + d^3\*e\*Log[

$x] - d^3 e \text{Log}[d + \text{Sqrt}[d^2 - e^2 x^2]]$

**Maple [A]** time = 0.033, size = 182, normalized size = 1.6

$$-\frac{1}{dx} (-e^2 x^2 + d^2)^{\frac{5}{2}} - \frac{e^2 x}{d} (-e^2 x^2 + d^2)^{\frac{3}{2}} - \frac{3 e^2 dx}{2} \sqrt{-e^2 x^2 + d^2} - \frac{3 e^2 d^3}{2} \arctan\left(x \sqrt{e^2} \frac{1}{\sqrt{-e^2 x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} + \frac{e}{3} (-e^2 x^2 + d^2)^{\frac{3}{2}} + ed^2 \sqrt{-e^2 x^2 + d^2} - ed^4 \ln\left(\frac{1}{x} \left(2 d^2 + 2 \sqrt{d^2} \sqrt{-e^2 x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^2,x)`

[Out] `-1/d/x*(-e^2*x^2+d^2)^(5/2)-e^2/d*x*(-e^2*x^2+d^2)^(3/2)-3/2*d*e^2*x*(-e^2*x^2+d^2)^(1/2)-3/2*e^2*d^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+1/3*e*(-e^2*x^2+d^2)^(3/2)+e*d^2*(-e^2*x^2+d^2)^(1/2)-e*d^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(3/2)*(e*x + d)/x^2,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.288147, size = 532, normalized size = 4.55

$$8 d e^7 x^7 + 12 d^2 e^6 x^6 - 48 d^3 e^5 x^5 - 12 d^4 e^4 x^4 + 48 d^5 e^3 x^3 - 48 d^6 e^2 x^2 + 48 d^8 + 18 \left( d^3 e^5 x^5 - 8 d^5 e^3 x^3 + 8 d^7 e x + 4 (d^4 e^3 x^3 - \dots) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(3/2)*(e*x + d)/x^2,x, algorithm="fricas")`

[Out] `1/6*(8*d*e^7*x^7 + 12*d^2*e^6*x^6 - 48*d^3*e^5*x^5 - 12*d^4*e^4*x^4 + 48*d^5*e^3*x^3 - 48*d^6*e^2*x^2 + 48*d^8 + 18*(d^3*e^5*x^5 - 8*d^5*e^3*x^3 + 8*d^7*e*x + 4*(d^4*e^3*x^3 - 2*d^6*e*x)*sqrt(-e^2*x^2 + d^2))*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 6*(d^3*e^5*x^5 - 8*d^5*e^3*x^3 + 8*d^7*e*x + 4*(d^4*e^3*x^3 - 2*d^6*e*x)*sqrt(-e^2*x^2 + d^2))*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (2*e^7*x^7 + 3*d*e^6*x^6 - 24*d^2*e^5*x^5 - 18*d^3*e^4*x^4 + 48*d^4*e^3*x^3 - 24*d^5*e^2*x^2 + 48*d^7)*sqrt(-e^2*x^2 + d^2))/(e^4*x^5 - 8*d^2*e^2*x^3 + 8*d^4*x + 4*(d*e^2*x^3 - 2*d^3*x)*sqrt(-e^2*x^2 + d^2))`



**Sympy [A]** time = 18.1014, size = 386, normalized size = 3.3

$$\begin{aligned}
 & d^3 \left( \begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}} + ie \operatorname{acosh}\left(\frac{ex}{d}\right) - \frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}} - e \operatorname{asin}\left(\frac{ex}{d}\right) + \frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases} \right) \\
 & + d^2 e \left( \begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}} - d \operatorname{acosh}\left(\frac{d}{ex}\right) - \frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}} + id \operatorname{asin}\left(\frac{d}{ex}\right) + \frac{iox}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases} \right) \\
 & - de^2 \left( \begin{cases} -\frac{id^2 \operatorname{acosh}\left(\frac{ex}{d}\right)}{2e} - \frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^2 \operatorname{asin}\left(\frac{ex}{d}\right)}{2e} + \frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise} \end{cases} \right) \\
 & - e^3 \left( \begin{cases} \frac{x^2\sqrt{d^2}}{2} & \text{for } e^2 = 0 \\ -\frac{(d^2-e^2x^2)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases} \right)
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2)/x\*\*2,x)

[Out] d\*\*3\*Piecewise((I\*d/(x\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*acosh(e\*x/d) - I\*e\*\*2\*x/(d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (-d/(x\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*asin(e\*x/d) + e\*\*2\*x/(d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True)) + d\*\*2\*e\*Piecewise((d\*\*2/(e\*x\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) - d\*acosh(d/(e\*x)) - e\*x/sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (-I\*d\*\*2/(e\*x\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) + I\*d\*asin(d/(e\*x)) + I\*e\*x/sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1), True)) - d\*e\*\*2\*Piecewise((-I\*d\*\*2\*acosh(e\*x/d)/(2\*e) - I\*d\*x/(2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*3/(2\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*2\*asin(e\*x/d)/(2\*e) + d\*x\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)/2, True)) - e\*\*3\*Piecewise((x\*\*2\*sqrt(d\*\*2)/2, Eq(e\*\*2, 0)), (-d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(3\*e\*\*2), True))

**GIAC/XCAS [A]** time = 0.319975, size = 212, normalized size = 1.81

$$\begin{aligned}
 & -\frac{3}{2}d^3 \arcsin\left(\frac{xe}{d}\right) \operatorname{esign}(d) - d^3 \operatorname{eln}\left(\frac{\left|-2de - 2\sqrt{-x^2e^2 + d^2}e\right|e^{(-2)}}{2|x|}\right) + \frac{d^3xe^3}{2\left(de + \sqrt{-x^2e^2 + d^2}e\right)} \\
 & - \frac{\left(de + \sqrt{-x^2e^2 + d^2}e\right)d^3e^{(-1)}}{2x} + \frac{1}{6}\sqrt{-x^2e^2 + d^2}(8d^2e - (2xe^3 + 3de^2)x)
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)/x^2,x, algorithm="giac")

[Out] -3/2\*d^3\*arcsin(x\*e/d)\*e\*sign(d) - d^3\*e\*ln(1/2\*abs(-2\*d\*e - 2\*sqrt(-x^2\*e^2 + d^2)\*e)\*e^(-2)/abs(x)) + 1/2\*d^3\*x\*e^3/(d\*e + sqrt(-x^2\*e^2 + d^2)\*e) - 1/2\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*d^3\*e^(-1)/x + 1/6\*sqrt(-x^2\*e^2 + d^2)\*(8\*d^2\*e - (2\*x\*e^3 + 3\*d\*e^2)\*x)

$$3.9 \quad \int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^3} dx$$

**Optimal.** Leaf size=121

$$\begin{aligned} & -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d-ex)(d^2-e^2x^2)^{3/2}}{2x^2} \\ & -\frac{3}{2}d^2e^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{3}{2}d^2e^2 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) \end{aligned}$$

[Out]  $(-3*d*e*(d+e*x)*\text{Sqrt}[d^2-e^2*x^2])/(2*x) - ((d-e*x)*(d^2-e^2*x^2)^{(3/2)})/(2*x^2) - (3*d^2*e^2*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2-e^2*x^2]])/2 + (3*d^2*e^2*\text{ArcTanh}[\text{Sqrt}[d^2-e^2*x^2]/d])/2$

**Rubi [A]** time = 0.28859, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$\begin{aligned} & -\frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d-ex)(d^2-e^2x^2)^{3/2}}{2x^2} \\ & -\frac{3}{2}d^2e^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{3}{2}d^2e^2 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right) \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d+e*x)*(d^2-e^2*x^2)^{(3/2)}/x^3,x]$

[Out]  $(-3*d*e*(d+e*x)*\text{Sqrt}[d^2-e^2*x^2])/(2*x) - ((d-e*x)*(d^2-e^2*x^2)^{(3/2)})/(2*x^2) - (3*d^2*e^2*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2-e^2*x^2]])/2 + (3*d^2*e^2*\text{ArcTanh}[\text{Sqrt}[d^2-e^2*x^2]/d])/2$

**Rubi in Sympy [A]** time = 41.85, size = 109, normalized size = 0.9

$$-\frac{3d^2e^2 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2} + \frac{3d^2e^2 \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2} - \frac{3de(d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d-2ex)(d^2-e^2x^2)^{\frac{3}{2}}}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**3,x)$

[Out]  $-3*d**2*e**2*\operatorname{atan}(e*x/\text{sqrt}(d**2-e**2*x**2))/2 + 3*d**2*e**2*\operatorname{atanh}(\text{sqrt}(d**2-e**2*x**2)/d)/2 - 3*d*e*(d+e*x)*\text{sqrt}(d**2-e**2*x**2)/(2*x) - (2*d-2*e*x)*(d**2-e**2*x**2)**(3/2)/(4*x**2)$

**Mathematica [A]** time = 0.159951, size = 118, normalized size = 0.98

$$\begin{aligned} & \frac{1}{2}\left(3d^2e^2 \log\left(\sqrt{d^2-e^2x^2}+d\right) - 3d^2e^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) \right. \\ & \left. - 3d^2e^2 \log(x) - \frac{\sqrt{d^2-e^2x^2}(d^3+2d^2ex+2de^2x^2+e^3x^3)}{x^2}\right) \end{aligned}$$

Antiderivative was successfully verified.



**Sympy [A]** time = 22.8571, size = 461, normalized size = 3.81

$$\begin{aligned}
 & d^3 \left( \begin{cases} -\frac{d^2}{2ex^3\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e}{2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^2 \operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{2x} - \frac{ie^2 \operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases} \right) \\
 & + d^2 e \left( \begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}} + ie \operatorname{acosh}\left(\frac{ex}{d}\right) - \frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}} - e \operatorname{asin}\left(\frac{ex}{d}\right) + \frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases} \right) \\
 & - de^2 \left( \begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}} - d \operatorname{acosh}\left(\frac{d}{ex}\right) - \frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}} + id \operatorname{asin}\left(\frac{d}{ex}\right) + \frac{ie^2x}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases} \right) \\
 & - e^3 \left( \begin{cases} -\frac{id^2 \operatorname{acosh}\left(\frac{ex}{d}\right)}{2e} - \frac{id^2x}{2\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^2 \operatorname{asin}\left(\frac{ex}{d}\right)}{2e} + \frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise} \end{cases} \right)
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2)/x\*\*3,x)

[Out] d\*\*3\*Piecewise((-d\*\*2/(2\*e\*x\*\*3\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) + e/(2\*x\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) + e\*\*2\*acosh(d/(e\*x))/(2\*d), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (-I\*e\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)/(2\*x) - I\*e\*\*2\*asin(d/(e\*x))/(2\*d), True)) + d\*\*2\*e\*Piecewise((I\*d/(x\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*acosh(e\*x/d) - I\*e\*\*2\*x/(d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2))), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (-d/(x\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*asin(e\*x/d) + e\*\*2\*x/(d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2))), True)) - d\*e\*\*2\*Piecewise((d\*\*2/(e\*x\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) - d\*acosh(d/(e\*x)) - e\*x/sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (-I\*d\*\*2/(e\*x\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) + I\*d\*asin(d/(e\*x)) + I\*e\*x/sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1), True)) - e\*\*3\*Piecewise((-I\*d\*\*2\*acosh(e\*x/d)/(2\*e) - I\*d\*x/(2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*3/(2\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2))), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*2\*asin(e\*x/d)/(2\*e) + d\*x\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)/2, True))

**GIAC/XCAS [A]** time = 0.30692, size = 293, normalized size = 2.42

$$\begin{aligned}
 & -\frac{3}{2} d^2 \arcsin\left(\frac{xe}{d}\right) e^2 \operatorname{sign}(d) + \frac{3}{2} d^2 e^2 \ln\left(\frac{\left|-2de - 2\sqrt{-x^2e^2 + d^2}e\right|e^{(-2)}}{2|x|}\right) \\
 & - \frac{1}{8} \left( \frac{4\left(de + \sqrt{-x^2e^2 + d^2}e\right)d^2e^8}{x} + \frac{\left(de + \sqrt{-x^2e^2 + d^2}e\right)^2 d^2e^6}{x^2} \right) e^{(-8)} \\
 & - \frac{1}{2} \sqrt{-x^2e^2 + d^2} (xe^3 + 2de^2) + \frac{\left(d^2e^6 + \frac{4\left(de + \sqrt{-x^2e^2 + d^2}e\right)d^2e^4}{x}\right)x^2}{8\left(de + \sqrt{-x^2e^2 + d^2}e\right)^2}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)/x^3,x, algorithm="giac")

[Out] -3/2\*d^2\*arcsin(x\*e/d)\*e^2\*sign(d) + 3/2\*d^2\*e^2\*ln(1/2\*abs(-2\*d\*e - 2\*sqrt(-x^2\*e^2 + d^2)\*e)\*e^(-2)/abs(x)) - 1/8\*(4\*(d\*e + sqrt

$$\begin{aligned}
& (-x^2 e^2 + d^2)^* e)^* d^2 * e^8 / x + (d^* e + \text{sqrt}(-x^2 * e^2 + d^2)^* e)^{\wedge 2} * \\
& d^{\wedge 2} * e^6 / x^{\wedge 2}) * e^{\wedge (-8)} - 1/2 * \text{sqrt}(-x^2 * e^2 + d^2)^* (x^* e^3 + 2 * d^* e^2) \\
& + 1/8 * (d^{\wedge 2} * e^6 + 4 * (d^* e + \text{sqrt}(-x^2 * e^2 + d^2)^* e)^* d^{\wedge 2} * e^4 / x) * x^{\wedge 2} / \\
& (d^* e + \text{sqrt}(-x^2 * e^2 + d^2)^* e)^{\wedge 2}
\end{aligned}$$

$$3.10 \quad \int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^4} dx$$

**Optimal.** Leaf size=120

$$\frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d+3ex)(d^2-e^2x^2)^{3/2}}{6x^3} + de^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{3}{2}de^3 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

[Out] (e^2\*(2\*d - 3\*e\*x)\*Sqrt[d^2 - e^2\*x^2])/(2\*x) - ((2\*d + 3\*e\*x)\*(d^2 - e^2\*x^2)^(3/2))/(6\*x^3) + d\*e^3\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] + (3\*d\*e^3\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/2

**Rubi [A]** time = 0.290109, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.32$

$$\frac{e^2(2d-3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d+3ex)(d^2-e^2x^2)^{3/2}}{6x^3} + de^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{3}{2}de^3 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*(d^2 - e^2\*x^2)^(3/2))/x^4, x]

[Out] (e^2\*(2\*d - 3\*e\*x)\*Sqrt[d^2 - e^2\*x^2])/(2\*x) - ((2\*d + 3\*e\*x)\*(d^2 - e^2\*x^2)^(3/2))/(6\*x^3) + d\*e^3\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] + (3\*d\*e^3\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/2

**Rubi in Sympy [A]** time = 43.4413, size = 104, normalized size = 0.87

$$de^3 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{3de^3 \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2} + \frac{e^2(4d-6ex)\sqrt{d^2-e^2x^2}}{4x} - \frac{(2d+3ex)(d^2-e^2x^2)^{3/2}}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2)/x\*\*4, x)

[Out] d\*e\*\*3\*atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2)) + 3\*d\*e\*\*3\*atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d)/2 + e\*\*2\*(4\*d - 6\*e\*x)\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(4\*x) - (2\*d + 3\*e\*x)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(6\*x\*\*3)

**Mathematica [A]** time = 0.174266, size = 115, normalized size = 0.96

$$\frac{1}{6} \left( 9de^3 \log\left(\sqrt{d^2-e^2x^2}+d\right) + 6de^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{\sqrt{d^2-e^2x^2}(2d^3+3d^2ex-8de^2x^2+6e^3x^3)}{x^3} - 9de^3 \log(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(d^2 - e^2\*x^2)^(3/2))/x^4, x]

[Out] (-((Sqrt[d^2 - e^2\*x^2]\*(2\*d^3 + 3\*d^2\*e\*x - 8\*d\*e^2\*x^2 + 6\*e^3\*x^3))/x^3) + 6\*d\*e^3\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] - 9\*d\*e^3\*

$\text{Log}[x] + 9*d*e^3*\text{Log}[d + \text{Sqrt}[d^2 - e^2*x^2]]/6$

**Maple [B]** time = 0.023, size = 235, normalized size = 2.

$$-\frac{1}{3dx^3}(-e^2x^2+d^2)^{\frac{5}{2}} + \frac{2e^2}{3d^3x}(-e^2x^2+d^2)^{\frac{5}{2}} + \frac{2e^4x}{3d^3}(-e^2x^2+d^2)^{\frac{3}{2}} + \frac{e^4x}{d}\sqrt{-e^2x^2+d^2} \\ + de^4 \arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right) \frac{1}{\sqrt{e^2}} - \frac{e}{2d^2x^2}(-e^2x^2+d^2)^{\frac{5}{2}} - \frac{e^3}{2d^2}(-e^2x^2+d^2)^{\frac{3}{2}} \\ - \frac{3e^3}{2}\sqrt{-e^2x^2+d^2} + \frac{3e^3d^2}{2} \ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}\right)\right) \frac{1}{\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^4,x)`

[Out] `-1/3/d/x^3*(-e^2*x^2+d^2)^(5/2)+2/3*e^2/d^3/x*(-e^2*x^2+d^2)^(5/2)+2/3*e^4/d^3*x*(-e^2*x^2+d^2)^(3/2)+e^4/d*x*(-e^2*x^2+d^2)^(1/2)+d*e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/2*e/d^2/x^2*(-e^2*x^2+d^2)^(5/2)-1/2*e^3/d^2*(-e^2*x^2+d^2)^(3/2)-3/2*e^3*(-e^2*x^2+d^2)^(1/2)+3/2*e^3*d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2+d^2)^(3/2)*(e*x+d)/x^4,x,algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.288723, size = 572, normalized size = 4.77

$18de^7x^7 - 32d^2e^6x^6 - 12d^3e^5x^5 + 104d^4e^4x^4 - 36d^5e^3x^3 - 88d^6e^2x^2 + 24d^7ex + 16d^8 - 12\left(de^7x^7 - 8d^3e^5x^5 + 8d^5e^3x^3\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2+d^2)^(3/2)*(e*x+d)/x^4,x,algorithm="fricas")`

[Out] `1/6*(18*d*e^7*x^7 - 32*d^2*e^6*x^6 - 12*d^3*e^5*x^5 + 104*d^4*e^4*x^4 - 36*d^5*e^3*x^3 - 88*d^6*e^2*x^2 + 24*d^7*e*x + 16*d^8 - 12*(d*e^7*x^7 - 8*d^3*e^5*x^5 + 8*d^5*e^3*x^3 + 4*(d^2*e^5*x^5 - 2*d^4*e^3*x^3)*sqrt(-e^2*x^2+d^2))*arctan(-(d-sqrt(-e^2*x^2+d^2))/(e*x)) - 9*(d*e^7*x^7 - 8*d^3*e^5*x^5 + 8*d^5*e^3*x^3 + 4*(d^2*e^5*x^5 - 2*d^4*e^3*x^3)*sqrt(-e^2*x^2+d^2))*log(-(d-sqrt(-e^2*x^2+d^2))/x) - (6*e^7*x^7 - 8*d*e^6*x^6 - 21*d^2*e^5*x^5 + 66*d^3*e^4*x^4 - 24*d^4*e^3*x^3 - 80*d^5*e^2*x^2 + 24*d^6*e*x + 16*d^7)*sqrt(-e^2*x^2+d^2)/(e^4*x^7 - 8*d^2*e^2*x^5 + 8*d^4*x^3 + 4*(d*e^2*x^5 - 2*d^3*x^3)*sqrt(-e^2*x^2+d^2))`

**Sympy [A]** time = 19.6465, size = 457, normalized size = 3.81

$$\begin{aligned}
 & d^3 \left( \begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2} + \frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for } \left| \frac{d^2}{e^2x^2} \right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2} + \frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases} \right) \\
 & + d^2 e \left( \begin{cases} -\frac{d^2}{2ex^3\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e}{2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^2 \operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left| \frac{d^2}{e^2x^2} \right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{2x} - \frac{ie^2 \operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases} \right) \\
 & - de^2 \left( \begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}} + ie \operatorname{acosh}\left(\frac{ex}{d}\right) - \frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left| \frac{e^2x^2}{d^2} \right| > 1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}} - e \operatorname{asin}\left(\frac{ex}{d}\right) + \frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases} \right) \\
 & - e^3 \left( \begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}} - d \operatorname{acosh}\left(\frac{d}{ex}\right) - \frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left| \frac{d^2}{e^2x^2} \right| > 1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}} + id \operatorname{asin}\left(\frac{d}{ex}\right) + \frac{ie^2x}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases} \right)
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2)/x\*\*4,x)

[Out] d\*\*3\*Piecewise((-e\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)/(3\*x\*\*2) + e\*\*3\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)/(3\*d\*\*2), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (-I\*e\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)/(3\*x\*\*2) + I\*e\*\*3\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)/(3\*d\*\*2), True)) + d\*\*2\*e\*Piecewise((-d\*\*2/(2\*e\*x\*\*3\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) + e/(2\*x\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) + e\*\*2\*acosh(d/(e\*x))/(2\*d), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (-I\*e\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)/(2\*x) - I\*e\*\*2\*asin(d/(e\*x))/(2\*d), True)) - d\*e\*\*2\*Piecewise((I\*d/(x\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*acosh(e\*x/d) - I\*e\*\*2\*x/(d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (-d/(x\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*asin(e\*x/d) + e\*\*2\*x/(d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True)) - e\*\*3\*Piecewise((d\*\*2/(e\*x\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) - d\*acosh(d/(e\*x)) - e\*x/sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (-I\*d\*\*2/(e\*x\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) + I\*d\*asin(d/(e\*x)) + I\*e\*x/sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1), True))

**GIAC/XCAS [A]** time = 0.303846, size = 352, normalized size = 2.93

$$\begin{aligned}
 & d \arcsin\left(\frac{xe}{d}\right) e^3 \operatorname{sign}(d) + \frac{3}{2} de^3 \ln\left(\frac{\left|-2de - 2\sqrt{-x^2e^2 + d^2}e\right|e^{(-2)}}{2|x|}\right) \\
 & + \frac{\left(de^8 + \frac{3(de + \sqrt{-x^2e^2 + d^2}e)de^6}{x} - \frac{15(de + \sqrt{-x^2e^2 + d^2}e)^2de^4}{x^2}\right)x^3e}{24(de + \sqrt{-x^2e^2 + d^2}e)^3} \\
 & + \frac{1}{24}\left(\frac{15(de + \sqrt{-x^2e^2 + d^2}e)de^{16}}{x} - \frac{3(de + \sqrt{-x^2e^2 + d^2}e)^2de^{14}}{x^2} - \frac{(de + \sqrt{-x^2e^2 + d^2}e)^3de^{12}}{x^3}\right)e^{(-15)} \\
 & - \sqrt{-x^2e^2 + d^2}e^3
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)/x^4,x, algorithm="giac")

[Out] d\*arcsin(x\*e/d)\*e^3\*sign(d) + 3/2\*d\*e^3\*ln(1/2\*abs(-2\*d\*e - 2\*sqrt(-x^2\*e^2 + d^2)\*e)\*e^(-2)/abs(x)) + 1/24\*(d\*e^8 + 3\*(d\*e + sqrt



$$\begin{aligned}
& (-x^2 e^2 + d^2)^* e)^* d^* e^6/x - 15^*(d^* e + \text{sqrt}(-x^2 e^2 + d^2)^* e)^2 \\
& * d^* e^4/x^2)^* x^3 e/(d^* e + \text{sqrt}(-x^2 e^2 + d^2)^* e)^3 + 1/24^*(15^*(d^* \\
& e + \text{sqrt}(-x^2 e^2 + d^2)^* e)^* d^* e^16/x - 3^*(d^* e + \text{sqrt}(-x^2 e^2 + d \\
& ^2)^* e)^2 * d^* e^14/x^2 - (d^* e + \text{sqrt}(-x^2 e^2 + d^2)^* e)^3 * d^* e^12/x^3 \\
& )^* e^{(-15)} - \text{sqrt}(-x^2 e^2 + d^2)^* e^3
\end{aligned}$$

$$3.11 \quad \int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^5} dx$$

**Optimal.** Leaf size=118

$$\frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d+4ex)(d^2-e^2x^2)^{3/2}}{12x^4} + e^4 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{3}{8}e^4 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

[Out] (e^2\*(3\*d + 8\*e\*x)\*Sqrt[d^2 - e^2\*x^2])/(8\*x^2) - ((3\*d + 4\*e\*x)\*(d^2 - e^2\*x^2)^(3/2))/(12\*x^4) + e^4\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] - (3\*e^4\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/8

**Rubi [A]** time = 0.282199, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$\frac{e^2(3d+8ex)\sqrt{d^2-e^2x^2}}{8x^2} - \frac{(3d+4ex)(d^2-e^2x^2)^{3/2}}{12x^4} + e^4 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{3}{8}e^4 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*(d^2 - e^2\*x^2)^(3/2))/x^5, x]

[Out] (e^2\*(3\*d + 8\*e\*x)\*Sqrt[d^2 - e^2\*x^2])/(8\*x^2) - ((3\*d + 4\*e\*x)\*(d^2 - e^2\*x^2)^(3/2))/(12\*x^4) + e^4\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] - (3\*e^4\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/8

**Rubi in Sympy [A]** time = 43.0902, size = 102, normalized size = 0.86

$$e^4 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{3e^4 \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8} + \frac{e^2(6d+16ex)\sqrt{d^2-e^2x^2}}{16x^2} - \frac{(3d+4ex)(d^2-e^2x^2)^{3/2}}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2)/x\*\*5, x)

[Out] e\*\*4\*atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2)) - 3\*e\*\*4\*atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d)/8 + e\*\*2\*(6\*d + 16\*e\*x)\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(16\*x\*\*2) - (3\*d + 4\*e\*x)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(12\*x\*\*4)

**Mathematica [A]** time = 0.225609, size = 111, normalized size = 0.94

$$\frac{1}{24} \left( -9e^4 \log\left(\sqrt{d^2-e^2x^2}+d\right) + 24e^4 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{\sqrt{d^2-e^2x^2}(-6d^3-8d^2ex+15de^2x^2+32e^3x^3)}{x^4} + 9e^4 \log(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(d^2 - e^2\*x^2)^(3/2))/x^5, x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(-6\*d^3 - 8\*d^2\*e\*x + 15\*d\*e^2\*x^2 + 32\*e^3\*x^3))/x^4 + 24\*e^4\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] + 9\*e^4\*Log

[x] - 9\*e^4\*Log[d + Sqrt[d^2 - e^2\*x^2]]/24

**Maple [B]** time = 0.026, size = 260, normalized size = 2.2

$$-\frac{1}{4dx^4}(-e^2x^2+d^2)^{\frac{5}{2}}+\frac{e^2}{8d^3x^2}(-e^2x^2+d^2)^{\frac{5}{2}}+\frac{e^4}{8d^3}(-e^2x^2+d^2)^{\frac{3}{2}}+\frac{3e^4}{8d}\sqrt{-e^2x^2+d^2}$$

$$-\frac{3de^4}{8}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}-\frac{e}{3d^2x^3}(-e^2x^2+d^2)^{\frac{5}{2}}+\frac{2e^3}{3d^4x}(-e^2x^2+d^2)^{\frac{5}{2}}$$

$$+\frac{2e^5x}{3d^4}(-e^2x^2+d^2)^{\frac{3}{2}}+\frac{e^5x}{d^2}\sqrt{-e^2x^2+d^2}+e^5\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)\*(-e^2\*x^2+d^2)^(3/2)/x^5,x)

[Out] -1/4/d/x^4\*(-e^2\*x^2+d^2)^(5/2)+1/8\*e^2/d^3/x^2\*(-e^2\*x^2+d^2)^(5/2)+1/8\*e^4/d^3\*(-e^2\*x^2+d^2)^(3/2)+3/8\*e^4/d\*(-e^2\*x^2+d^2)^(1/2)-3/8\*d\*e^4/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)-1/3\*e/d^2/x^3\*(-e^2\*x^2+d^2)^(5/2)+2/3\*e^3/d^4/x\*(-e^2\*x^2+d^2)^(5/2)+2/3\*e^5/d^4\*x\*(-e^2\*x^2+d^2)^(3/2)+e^5/d^2\*x\*(-e^2\*x^2+d^2)^(1/2)+e^5/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)/x^5,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.303354, size = 564, normalized size = 4.78

$$128de^7x^7 + 60d^2e^6x^6 - 416d^3e^5x^5 - 204d^4e^4x^4 + 352d^5e^3x^3 + 192d^6e^2x^2 - 64d^7ex - 48d^8 + 48\left(e^8x^8 - 8d^2e^6x^6 + 8d^4e^4x^4 - 8d^6e^2x^2 + 8d^8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)/x^5,x, algorithm="fricas")

[Out] -1/24\*(128\*d\*e^7\*x^7 + 60\*d^2\*e^6\*x^6 - 416\*d^3\*e^5\*x^5 - 204\*d^4\*e^4\*x^4 + 352\*d^5\*e^3\*x^3 + 192\*d^6\*e^2\*x^2 - 64\*d^7\*e\*x - 48\*d^8 + 48\*(e^8\*x^8 - 8\*d^2\*e^6\*x^6 + 8\*d^4\*e^4\*x^4 + 4\*(d^2\*e^6\*x^6 - 2\*d^3\*e^4\*x^4)\*sqrt(-e^2\*x^2 + d^2))\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) - 9\*(e^8\*x^8 - 8\*d^2\*e^6\*x^6 + 8\*d^4\*e^4\*x^4 + 4\*(d^2\*e^6\*x^6 - 2\*d^3\*e^4\*x^4)\*sqrt(-e^2\*x^2 + d^2))\*log(-(d - sqrt(-e^2\*x^2 + d^2))/x) - (32\*e^7\*x^7 + 15\*d\*e^6\*x^6 - 264\*d^2\*e^5\*x^5 - 126\*d^3\*e^4\*x^4 + 320\*d^4\*e^3\*x^3 + 168\*d^5\*e^2\*x^2 - 64\*d^6\*e\*x - 48\*d^7)\*sqrt(-e^2\*x^2 + d^2)/(e^4\*x^8 - 8\*d^2\*e^2\*x^6 + 8\*d^4\*x^4 + 4\*(d^2\*e^2\*x^6 - 2\*d^3\*x^4)\*sqrt(-e^2\*x^2 + d^2))

**Sympy [A]** time = 27.8105, size = 541, normalized size = 4.58

$$\begin{aligned}
 & d^3 \left( \begin{cases} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^4 \operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^4 \operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} & \text{otherwise} \end{cases} \right) \\
 & + d^2 e \left( \begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2} + \frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2} + \frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases} \right) \\
 & - de^2 \left( \begin{cases} -\frac{d^2}{2ex^3\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e}{2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^2 \operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{2x} - \frac{ie^2 \operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases} \right) \\
 & - e^3 \left( \begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}} + ie \operatorname{acosh}\left(\frac{ex}{d}\right) - \frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}} - e \operatorname{asin}\left(\frac{ex}{d}\right) + \frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases} \right)
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2)/x\*\*5,x)

[Out] d\*\*3\*Piecewise((-d\*\*2/(4\*e\*x\*\*5\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) + 3\*e/(8\*x\*\*3\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) - e\*\*3/(8\*d\*\*2\*x\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) + e\*\*4\*acosh(d/(e\*x))/(8\*d\*\*3), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (I\*d\*\*2/(4\*e\*x\*\*5\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) - 3\*I\*e/(8\*x\*\*3\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) + I\*e\*\*3/(8\*d\*\*2\*x\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) - I\*e\*\*4\*asin(d/(e\*x))/(8\*d\*\*3), True)) + d\*\*2\*e\*Piecewise((-e\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)/(3\*x\*\*2) + e\*\*3\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)/(3\*d\*\*2), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (-I\*e\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)/(3\*x\*\*2) + I\*e\*\*3\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)/(3\*d\*\*2), True)) - d\*\*2\*e\*\*2\*Piecewise((-d\*\*2/(2\*e\*x\*\*3\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) + e/(2\*x\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) + e\*\*2\*acosh(d/(e\*x))/(2\*d), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (-I\*e\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)/(2\*x) - I\*e\*\*2\*asin(d/(e\*x))/(2\*d), True)) - e\*\*3\*Piecewise((I\*d/(x\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*acosh(e\*x/d) - I\*e\*\*2\*x/(d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (-d/(x\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*asin(e\*x/d) + e\*\*2\*x/(d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True))

**GIAC/XCAS [A]** time = 0.31666, size = 401, normalized size = 3.4

$$\begin{aligned}
 & \arcsin\left(\frac{xe}{d}\right) e^4 \operatorname{sign}(d) + \frac{x^4 \left( \frac{8(de + \sqrt{-x^2e^2 + d^2}e)e^8}{x} - \frac{24(de + \sqrt{-x^2e^2 + d^2}e)^2e^6}{x^2} - \frac{120(de + \sqrt{-x^2e^2 + d^2}e)^3e^4}{x^3} + 3e^{10} \right) e^2}{192(de + \sqrt{-x^2e^2 + d^2}e)^4} \\
 & + \frac{1}{192} \left( \frac{120(de + \sqrt{-x^2e^2 + d^2}e)e^{26}}{x} + \frac{24(de + \sqrt{-x^2e^2 + d^2}e)^2e^{24}}{x^2} - \frac{8(de + \sqrt{-x^2e^2 + d^2}e)^3e^{22}}{x^3} - \frac{3(de + \sqrt{-x^2e^2 + d^2}e)}{x^4} \right) \\
 & - \frac{3}{8} e^4 \ln \left( \frac{|-2de - 2\sqrt{-x^2e^2 + d^2}e| e^{(-2)}}{2|x|} \right)
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)/x^5,x, algorithm="giac")

[Out] arcsin(x\*e/d)\*e^4\*sign(d) + 1/192\*x^4\*(8\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*e^8/x - 24\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^2\*e^6/x^2 - 120\*

$$\begin{aligned}
& (d^*e + \text{sqrt}(-x^2*e^2 + d^2)*e)^3*e^4/x^3 + 3*e^{10})*e^2/(d^*e + \text{sqrt} \\
& (-x^2*e^2 + d^2)*e)^4 + 1/192*(120*(d^*e + \text{sqrt}(-x^2*e^2 + d^2)*e \\
& )*e^{26}/x + 24*(d^*e + \text{sqrt}(-x^2*e^2 + d^2)*e)^2*e^{24}/x^2 - 8*(d^*e \\
& + \text{sqrt}(-x^2*e^2 + d^2)*e)^3*e^{22}/x^3 - 3*(d^*e + \text{sqrt}(-x^2*e^2 + d \\
& ^2)*e)^4*e^{20}/x^4)*e^{(-24)} - 3/8*e^4*\ln(1/2*\text{abs}(-2*d^*e - 2*\text{sqrt}(- \\
& x^2*e^2 + d^2)*e)*e^{(-2)}/\text{abs}(x))
\end{aligned}$$

$$3.12 \quad \int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^6} dx$$

**Optimal.** Leaf size=108

$$-\frac{(d^2-e^2x^2)^{5/2}}{5dx^5} - \frac{e(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{3e^5 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d} + \frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}$$

[Out]  $(3e^3\sqrt{d^2-e^2x^2})/(8x^2) - (e(d^2-e^2x^2)^{3/2})/(4x^4) - (d^2-e^2x^2)^{5/2}/(5dx^5) - (3e^5 \operatorname{ArcTanh}[\sqrt{d^2-e^2x^2}/d])/(8d)$

**Rubi [A]** time = 0.170625, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{(d^2-e^2x^2)^{5/2}}{5dx^5} - \frac{e(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{3e^5 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d} + \frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*(d^2 - e^2\*x^2)^(3/2))/x^6, x]

[Out]  $(3e^3\sqrt{d^2-e^2x^2})/(8x^2) - (e(d^2-e^2x^2)^{3/2})/(4x^4) - (d^2-e^2x^2)^{5/2}/(5dx^5) - (3e^5 \operatorname{ArcTanh}[\sqrt{d^2-e^2x^2}/d])/(8d)$

**Rubi in Sympy [A]** time = 19.7645, size = 88, normalized size = 0.81

$$\frac{3e^3\sqrt{d^2-e^2x^2}}{8x^2} - \frac{e(d^2-e^2x^2)^{3/2}}{4x^4} - \frac{3e^5 \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{8d} - \frac{(d^2-e^2x^2)^{5/2}}{5dx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2)/x\*\*6, x)

[Out]  $3e^3\sqrt{d^2-e^2x^2}/(8x^2) - e(d^2-e^2x^2)^{3/2}/(4x^4) - 3e^5 \operatorname{atanh}(\sqrt{d^2-e^2x^2}/d)/(8d) - (d^2-e^2x^2)^{5/2}/(5dx^5)$

**Mathematica [A]** time = 0.0998187, size = 106, normalized size = 0.98

$$\frac{-15e^5x^5 \log\left(\sqrt{d^2-e^2x^2}+d\right) + \sqrt{d^2-e^2x^2}(-8d^4-10d^3ex+16d^2e^2x^2+25de^3x^3-8e^4x^4) + 15e^5x^5 \log(x)}{40dx^5}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(d^2 - e^2\*x^2)^(3/2))/x^6, x]

[Out]  $(\sqrt{d^2-e^2x^2}(-8d^4-10d^3ex+16d^2e^2x^2+25de^3x^3-8e^4x^4) + 15e^5x^5 \operatorname{Log}[x] - 15e^5x^5 \operatorname{Log}[d + \sqrt{d^2-e^2x^2}])/(40dx^5)$

**Maple [A]** time = 0.03, size = 158, normalized size = 1.5

$$-\frac{1}{5dx^5}(-e^2x^2+d^2)^{\frac{5}{2}} - \frac{e}{4d^2x^4}(-e^2x^2+d^2)^{\frac{5}{2}} + \frac{e^3}{8d^4x^2}(-e^2x^2+d^2)^{\frac{5}{2}} \\ + \frac{e^5}{8d^4}(-e^2x^2+d^2)^{\frac{3}{2}} + \frac{3e^5}{8d^2}\sqrt{-e^2x^2+d^2} - \frac{3e^5}{8}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)\*(-e^2\*x^2+d^2)^(3/2)/x^6,x)

[Out] -1/5\*(-e^2\*x^2+d^2)^(5/2)/d/x^5-1/4\*e/d^2/x^4\*(-e^2\*x^2+d^2)^(5/2)+1/8\*e^3/d^4/x^2\*(-e^2\*x^2+d^2)^(5/2)+1/8\*e^5/d^4\*(-e^2\*x^2+d^2)^(3/2)+3/8\*e^5/d^2\*(-e^2\*x^2+d^2)^(1/2)-3/8\*e^5/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2+d^2)^(3/2)\*(e\*x+d)/x^6,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.292921, size = 531, normalized size = 4.92

$$8e^{10}x^{10} - 25de^9x^9 - 120d^2e^8x^8 + 335d^3e^7x^7 + 440d^4e^6x^6 - 830d^5e^5x^5 - 680d^6e^4x^4 + 680d^7e^3x^3 + 480d^8e^2x^2 - 160d^9e^1x^1 - 160d^{10}e^0x^0$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2+d^2)^(3/2)\*(e\*x+d)/x^6,x, algorithm="fricas")

[Out] -1/40\*(8\*e^10\*x^10 - 25\*d\*e^9\*x^9 - 120\*d^2\*e^8\*x^8 + 335\*d^3\*e^7\*x^7 + 440\*d^4\*e^6\*x^6 - 830\*d^5\*e^5\*x^5 - 680\*d^6\*e^4\*x^4 + 680\*d^7\*e^3\*x^3 + 480\*d^8\*e^2\*x^2 - 160\*d^9\*e^1\*x^1 - 160\*d^10\*e^0\*x^0)\*sqrt(-e^2\*x^2+d^2)\*log(-(d-sqrt(-e^2\*x^2+d^2))/x) + (40\*d^5\*e^8\*x^8 - 125\*d^2\*e^7\*x^7 - 240\*d^3\*e^6\*x^6 + 550\*d^4\*e^5\*x^5 + 488\*d^5\*e^4\*x^4 - 600\*d^6\*e^3\*x^3 - 416\*d^7\*e^2\*x^2 + 160\*d^8\*e^1\*x^1 + 128\*d^9)\*sqrt(-e^2\*x^2+d^2)/(5\*d^2\*e^4\*x^9 - 20\*d^4\*e^2\*x^7 + 16\*d^6\*x^5 - (d^5\*e^4\*x^9 - 12\*d^3\*e^2\*x^7 + 16\*d^5\*x^5)\*sqrt(-e^2\*x^2+d^2))

**Sympy [A]** time = 26.174, size = 774, normalized size = 7.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2)/x\*\*6,x)

```
[Out] d**3*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5
+ 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15
*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d*
**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e
**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d*
**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e*
**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5
+ 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*
x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-
15*d**3*x**5 + 15*d*e**2*x**7), True)) + d**2*e*Piecewise((-d**2/
(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e*
**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**
4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4
*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(
e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1))
- I*e**4*asin(d/(e*x))/(8*d**3), True)) - d*e**2*Piecewise((-e*sq
rt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) -
1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x
**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2),
True)) - e**3*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) -
1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(
2*d), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) +
1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d), True))
```

**GIAC/XCAS [A]** time = 0.295298, size = 497, normalized size = 4.6

$$\frac{x^5 \left( \frac{5(de + \sqrt{-x^2e^2 + d^2}e)e^{10}}{x} - \frac{10(de + \sqrt{-x^2e^2 + d^2}e)^2e^8}{x^2} - \frac{40(de + \sqrt{-x^2e^2 + d^2}e)^3e^6}{x^3} + \frac{20(de + \sqrt{-x^2e^2 + d^2}e)^4e^4}{x^4} + 2e^{12} \right) e^3}{320(de + \sqrt{-x^2e^2 + d^2}e)^5 d}$$

$$- \frac{3e^5 \ln \left( \frac{|-2de - 2\sqrt{-x^2e^2 + d^2}e|e^{(-2)}}{2|x|} \right)}{8d}$$

$$- \frac{\left( \frac{20(de + \sqrt{-x^2e^2 + d^2}e)d^4e^{38}}{x} - \frac{40(de + \sqrt{-x^2e^2 + d^2}e)^2d^4e^{36}}{x^2} - \frac{10(de + \sqrt{-x^2e^2 + d^2}e)^3d^4e^{34}}{x^3} + \frac{5(de + \sqrt{-x^2e^2 + d^2}e)^4d^4e^{32}}{x^4} + \frac{2(de + \sqrt{-x^2e^2 + d^2}e)^5d^4}{x^5} \right)}{320d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^(3/2)*(e*x + d)/x^6,x, algorithm="giac")
```

```
[Out] 1/320*x^5*(5*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^10/x - 10*(d*e + sq
rt(-x^2*e^2 + d^2)*e)^2*e^8/x^2 - 40*(d*e + sqrt(-x^2*e^2 + d^2)*
e)^3*e^6/x^3 + 20*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*e^4/x^4 + 2*e^1
2)*e^3/((d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d) - 3/8*e^5*ln(1/2*abs
(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d - 1/320*(20*
(d*e + sqrt(-x^2*e^2 + d^2)*e)*d^4*e^38/x - 40*(d*e + sqrt(-x^2*e
^2 + d^2)*e)^2*d^4*e^36/x^2 - 10*(d*e + sqrt(-x^2*e^2 + d^2)*e)^3
*d^4*e^34/x^3 + 5*(d*e + sqrt(-x^2*e^2 + d^2)*e)^4*d^4*e^32/x^4 +
2*(d*e + sqrt(-x^2*e^2 + d^2)*e)^5*d^4*e^30/x^5)*e^(-35)/d^5
```



$$3.13 \quad \int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^7} dx$$

**Optimal.** Leaf size=143

$$-\frac{(d^2-e^2x^2)^{5/2}}{6dx^6} - \frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5} - \frac{e^2(d^2-e^2x^2)^{3/2}}{24dx^4} - \frac{e^6 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^2} + \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}$$

[Out]  $(e^4 \sqrt{d^2 - e^2 x^2}) / (16 d^2 x^2) - (e^2 (d^2 - e^2 x^2)^{3/2}) / (24 d x^4) - (d^2 - e^2 x^2)^{5/2} / (6 d^2 x^5) - (e^6 \operatorname{ArcTanh}[\sqrt{d^2 - e^2 x^2} / d]) / (16 d^2)$

**Rubi [A]** time = 0.256613, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$-\frac{(d^2-e^2x^2)^{5/2}}{6dx^6} - \frac{e(d^2-e^2x^2)^{5/2}}{5d^2x^5} - \frac{e^2(d^2-e^2x^2)^{3/2}}{24dx^4} - \frac{e^6 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^2} + \frac{e^4\sqrt{d^2-e^2x^2}}{16dx^2}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*(d^2 - e^2\*x^2)^(3/2))/x^7, x]

[Out]  $(e^4 \sqrt{d^2 - e^2 x^2}) / (16 d^2 x^2) - (e^2 (d^2 - e^2 x^2)^{3/2}) / (24 d x^4) - (d^2 - e^2 x^2)^{5/2} / (6 d^2 x^5) - (e^6 \operatorname{ArcTanh}[\sqrt{d^2 - e^2 x^2} / d]) / (16 d^2)$

**Rubi in Sympy [A]** time = 31.9964, size = 116, normalized size = 0.81

$$\frac{e^4 \sqrt{d^2 - e^2 x^2}}{16 d x^2} - \frac{e^2 (d^2 - e^2 x^2)^{3/2}}{24 d x^4} - \frac{(d^2 - e^2 x^2)^{5/2}}{6 d x^6} - \frac{e^6 \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{16 d^2} - \frac{e (d^2 - e^2 x^2)^{5/2}}{5 d^2 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2)/x\*\*7, x)

[Out]  $e^4 \sqrt{d^2 - e^2 x^2} / (16 d^2 x^2) - e^2 (d^2 - e^2 x^2)^{3/2} / (24 d x^4) - (d^2 - e^2 x^2)^{5/2} / (6 d^2 x^6) - e^6 \operatorname{atanh}(\sqrt{d^2 - e^2 x^2} / d) / (16 d^2) - e (d^2 - e^2 x^2)^{5/2} / (5 d^2 x^5)$

**Mathematica [A]** time = 0.21337, size = 117, normalized size = 0.82

$$\frac{15e^6x^6 \log\left(\sqrt{d^2 - e^2x^2} + d\right) + \sqrt{d^2 - e^2x^2} (40d^5 + 48d^4ex - 70d^3e^2x^2 - 96d^2e^3x^3 + 15de^4x^4 + 48e^5x^5) - 15e^6x^6 \log(x)}{240d^2x^6}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(d^2 - e^2\*x^2)^(3/2))/x^7, x]

[Out]  $-(\sqrt{d^2 - e^2 x^2}) (40 d^5 + 48 d^4 e x - 70 d^3 e^2 x^2 - 96 d^2 e^3 x^3 + 15 d e^4 x^4 + 48 e^5 x^5) - 15 e^6 x^6 \operatorname{Log}[x] + 15 e^4 \sqrt{d^2 - e^2 x^2} x^6$

$*e^6*x^6*\text{Log}[d + \text{Sqrt}[d^2 - e^2*x^2]]/(240*d^2*x^6)$

**Maple [A]** time = 0.037, size = 186, normalized size = 1.3

$$-\frac{1}{6dx^6}(-e^2x^2+d^2)^{\frac{5}{2}} - \frac{e^2}{24d^3x^4}(-e^2x^2+d^2)^{\frac{5}{2}} + \frac{e^4}{48d^5x^2}(-e^2x^2+d^2)^{\frac{5}{2}} + \frac{e^6}{48d^5}(-e^2x^2+d^2)^{\frac{3}{2}} + \frac{e^6}{16d^3}\sqrt{-e^2x^2+d^2} - \frac{e^6}{16d}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}} - \frac{e}{5d^2x^5}(-e^2x^2+d^2)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)*(-e^2*x^2+d^2)^(3/2)/x^7,x)`

[Out] `-1/6*(-e^2*x^2+d^2)^(5/2)/d/x^6-1/24*e^2/d^3/x^4*(-e^2*x^2+d^2)^(5/2)+1/48*e^4/d^5/x^2*(-e^2*x^2+d^2)^(5/2)+1/48*e^6/d^5*(-e^2*x^2+d^2)^(3/2)+1/16*e^6/d^3*(-e^2*x^2+d^2)^(1/2)-1/16*e^6/d/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/5*e*(-e^2*x^2+d^2)^(5/2)/d^2/x^5`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2+d^2)^(3/2)*(e*x+d)/x^7,x,algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.348645, size = 624, normalized size = 4.36

$288de^{11}x^{11} + 90d^2e^{10}x^{10} - 2400d^3e^9x^9 - 990d^4e^8x^8 + 7008d^5e^7x^7 + 3860d^6e^6x^6 - 9504d^7e^5x^5 - 6480d^8e^4x^4 + 6144d^9e^3x^3 - 4800d^{10}e^2x^2 - 1536d^{11}ex - 1280d^{12} + 15(e^{12}x^{12} - 18d^2e^{10}x^{10} + 48d^4e^8x^8 - 32d^6e^6x^6 + 2(3d^8e^4x^4 - 16d^{10}e^2x^2 + 16d^{12}))\sqrt{-e^2x^2+d^2})\log(-(d - \sqrt{-e^2x^2+d^2})/x) - (48e^{11}x^{11} + 15d^2e^{10}x^{10} - 960d^4e^8x^8 - 340d^6e^6x^6 + 4080d^8e^4x^4 + 2020d^{10}e^2x^2 - 7008d^{12})\sqrt{-e^2x^2+d^2})/(d^2e^6x^{12} - 18d^4e^4x^{10} + 48d^6e^2x^8 - 32d^8x^6 + 2(3d^{10}e^2x^2 - 16d^{12}))\sqrt{-e^2x^2+d^2})$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2+d^2)^(3/2)*(e*x+d)/x^7,x,algorithm="fricas")`

[Out] `1/240*(288*d*e^11*x^11 + 90*d^2*e^10*x^10 - 2400*d^3*e^9*x^9 - 990*d^4*e^8*x^8 + 7008*d^5*e^7*x^7 + 3860*d^6*e^6*x^6 - 9504*d^7*e^5*x^5 - 6480*d^8*e^4*x^4 + 6144*d^9*e^3*x^3 + 4800*d^10*e^2*x^2 - 1536*d^11*e*x - 1280*d^12 + 15*(e^12*x^12 - 18*d^2*e^10*x^10 + 48*d^4*e^8*x^8 - 32*d^6*e^6*x^6 + 2*(3*d^8*e^4*x^4 - 16*d^10*e^2*x^2 + 16*d^12))*sqrt(-e^2*x^2+d^2))*log(-(d - sqrt(-e^2*x^2+d^2))/x) - (48*e^11*x^11 + 15*d^2*e^10*x^10 - 960*d^4*e^8*x^8 - 340*d^6*e^6*x^6 + 4080*d^8*e^4*x^4 + 2020*d^10*e^2*x^2 - 7008*d^12)*sqrt(-e^2*x^2+d^2))/(d^2*e^6*x^12 - 18*d^4*e^4*x^10 + 48*d^6*e^2*x^8 - 32*d^8*x^6 + 2*(3*d^10*e^2*x^2 - 16*d^12))*sqrt(-e^2*x^2+d^2)`

**Sympy [A]** time = 40.6252, size = 918, normalized size = 6.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**7,x)`

[Out]  $d^{*3} \text{Piecewise}((-d^{*2}/(6*e*x^{*7} \sqrt{d^{*2}/(e^{*2}*x^{*2}) - 1})) + 5*e/(24*x^{*5} \sqrt{d^{*2}/(e^{*2}*x^{*2}) - 1})) + e^{*3}/(48*d^{*2}*x^{*3} \sqrt{d^{*2}/(e^{*2}*x^{*2}) - 1})) - e^{*5}/(16*d^{*4}*x \sqrt{d^{*2}/(e^{*2}*x^{*2}) - 1})) + e^{*6} \operatorname{acosh}(d/(e*x))/(16*d^{*5}), \operatorname{Abs}(d^{*2}/(e^{*2}*x^{*2})) > 1), (I*d^{*2}/(6*e*x^{*7} \sqrt{-d^{*2}/(e^{*2}*x^{*2}) + 1})) - 5*I*e/(24*x^{*5} \sqrt{-d^{*2}/(e^{*2}*x^{*2}) + 1})) - I*e^{*3}/(48*d^{*2}*x^{*3} \sqrt{-d^{*2}/(e^{*2}*x^{*2}) + 1})) + I*e^{*5}/(16*d^{*4}*x \sqrt{-d^{*2}/(e^{*2}*x^{*2}) + 1})) - I*e^{*6} \operatorname{asin}(d/(e*x))/(16*d^{*5}), \operatorname{True})) + d^{*2}*e \text{Piecewise}((3*I*d^{*3} \sqrt{-1 + e^{*2}*x^{*2}/d^{*2}})/(-15*d^{*2}*x^{*5} + 15*e^{*2}*x^{*7}) - 4*I*d*e^{*2}*x^{*2} \sqrt{-1 + e^{*2}*x^{*2}/d^{*2}})/(-15*d^{*2}*x^{*5} + 15*e^{*2}*x^{*7}) + 2*I*e^{*6}*x^{*6} \sqrt{-1 + e^{*2}*x^{*2}/d^{*2}})/(-15*d^{*5}*x^{*5} + 15*d^{*3}*e^{*2}*x^{*7}) - I*e^{*4}*x^{*4} \sqrt{-1 + e^{*2}*x^{*2}/d^{*2}})/(-15*d^{*3}*x^{*5} + 15*d*e^{*2}*x^{*7}), \operatorname{Abs}(e^{*2}*x^{*2}/d^{*2}) > 1), (3*d^{*3} \sqrt{1 - e^{*2}*x^{*2}/d^{*2}})/(-15*d^{*2}*x^{*5} + 15*e^{*2}*x^{*7}) - 4*d*e^{*2}*x^{*2} \sqrt{1 - e^{*2}*x^{*2}/d^{*2}})/(-15*d^{*2}*x^{*5} + 15*e^{*2}*x^{*7}) + 2*e^{*6}*x^{*6} \sqrt{1 - e^{*2}*x^{*2}/d^{*2}})/(-15*d^{*5}*x^{*5} + 15*d^{*3}*e^{*2}*x^{*7}) - e^{*4}*x^{*4} \sqrt{1 - e^{*2}*x^{*2}/d^{*2}})/(-15*d^{*3}*x^{*5} + 15*d*e^{*2}*x^{*7}), \operatorname{True})) - d*e^{*2} \text{Piecewise}((-d^{*2}/(4*e*x^{*5} \sqrt{d^{*2}/(e^{*2}*x^{*2}) - 1})) + 3*e/(8*x^{*3} \sqrt{d^{*2}/(e^{*2}*x^{*2}) - 1})) - e^{*3}/(8*d^{*2}*x \sqrt{d^{*2}/(e^{*2}*x^{*2}) - 1})) + e^{*4} \operatorname{acosh}(d/(e*x))/(8*d^{*3}), \operatorname{Abs}(d^{*2}/(e^{*2}*x^{*2})) > 1), (I*d^{*2}/(4*e*x^{*5} \sqrt{-d^{*2}/(e^{*2}*x^{*2}) + 1})) - 3*I*e/(8*x^{*3} \sqrt{-d^{*2}/(e^{*2}*x^{*2}) + 1})) + I*e^{*3}/(8*d^{*2}*x \sqrt{-d^{*2}/(e^{*2}*x^{*2}) + 1})) - I*e^{*4} \operatorname{asin}(d/(e*x))/(8*d^{*3}), \operatorname{True})) - e^{*3} \text{Piecewise}((-e \sqrt{d^{*2}/(e^{*2}*x^{*2}) - 1})/(3*x^{*2}) + e^{*3} \sqrt{d^{*2}/(e^{*2}*x^{*2}) - 1})/(3*d^{*2}), \operatorname{Abs}(d^{*2}/(e^{*2}*x^{*2})) > 1), (-I*e \sqrt{-d^{*2}/(e^{*2}*x^{*2}) + 1})/(3*x^{*2}) + I*e^{*3} \sqrt{-d^{*2}/(e^{*2}*x^{*2}) + 1})/(3*d^{*2}), \operatorname{True}))$

---

**GIAC/XCAS [A]** time = 0.29495, size = 1, normalized size = 0.01

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(3/2)*(e*x + d)/x^7,x, algorithm="giac")`

[Out] Done

$$3.14 \quad \int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^8} dx$$

**Optimal.** Leaf size=172

$$\begin{aligned} & -\frac{(d^2-e^2x^2)^{5/2}}{7dx^7} - \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} + \frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} \\ & - \frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4} - \frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5} - \frac{e^7 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3} \end{aligned}$$

[Out] (e^5\*sqrt[d^2 - e^2\*x^2])/(16\*d^2\*x^2) - (e^3\*(d^2 - e^2\*x^2)^(3/2))/(24\*d^2\*x^4) - (d^2 - e^2\*x^2)^(5/2)/(7\*d\*x^7) - (e\*(d^2 - e^2\*x^2)^(5/2))/(6\*d^2\*x^6) - (2\*e^2\*(d^2 - e^2\*x^2)^(5/2))/(35\*d^3\*x^5) - (e^7\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/(16\*d^3)

**Rubi [A]** time = 0.344622, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\begin{aligned} & -\frac{(d^2-e^2x^2)^{5/2}}{7dx^7} - \frac{e(d^2-e^2x^2)^{5/2}}{6d^2x^6} + \frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} \\ & - \frac{e^3(d^2-e^2x^2)^{3/2}}{24d^2x^4} - \frac{2e^2(d^2-e^2x^2)^{5/2}}{35d^3x^5} - \frac{e^7 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*(d^2 - e^2\*x^2)^(3/2))/x^8, x]

[Out] (e^5\*sqrt[d^2 - e^2\*x^2])/(16\*d^2\*x^2) - (e^3\*(d^2 - e^2\*x^2)^(3/2))/(24\*d^2\*x^4) - (d^2 - e^2\*x^2)^(5/2)/(7\*d\*x^7) - (e\*(d^2 - e^2\*x^2)^(5/2))/(6\*d^2\*x^6) - (2\*e^2\*(d^2 - e^2\*x^2)^(5/2))/(35\*d^3\*x^5) - (e^7\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/(16\*d^3)

**Rubi in Sympy [A]** time = 45.3999, size = 146, normalized size = 0.85

$$\begin{aligned} & -\frac{(d^2-e^2x^2)^{\frac{5}{2}}}{7dx^7} + \frac{e^5\sqrt{d^2-e^2x^2}}{16d^2x^2} - \frac{e^3(d^2-e^2x^2)^{\frac{3}{2}}}{24d^2x^4} \\ & - \frac{e(d^2-e^2x^2)^{\frac{5}{2}}}{6d^2x^6} - \frac{e^7 \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{16d^3} - \frac{2e^2(d^2-e^2x^2)^{\frac{5}{2}}}{35d^3x^5} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2)/x\*\*8, x)

[Out] -(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)/(7\*d\*x\*\*7) + e\*\*5\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(16\*d\*\*2\*x\*\*2) - e\*\*3\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(24\*d\*\*2\*x\*\*4) - e\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)/(6\*d\*\*2\*x\*\*6) - e\*\*7\*atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d)/(16\*d\*\*3) - 2\*e\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)/(35\*d\*\*3\*x\*\*5)

**Mathematica [A]** time = 0.17817, size = 128, normalized size = 0.74

$$\frac{105e^7x^7 \log\left(\sqrt{d^2-e^2x^2}+d\right) + \sqrt{d^2-e^2x^2}\left(240d^6+280d^5ex-384d^4e^2x^2-490d^3e^3x^3+48d^2e^4x^4+105de^5x^5+96e^6x^6\right)}{1680d^3x^7}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(d^2 - e^2\*x^2)^(3/2))/x^8, x]

[Out]  $-(\text{Sqrt}[d^2 - e^2 x^2] * (240 d^6 + 280 d^5 e x - 384 d^4 e^2 x^2 - 490 d^3 e^3 x^3 + 48 d^2 e^4 x^4 + 105 d e^5 x^5 + 96 e^6 x^6) - 105 e^7 x^7 \text{Log}[x] + 105 e^7 x^7 \text{Log}[d + \text{Sqrt}[d^2 - e^2 x^2]]) / (1680 d^3 x^7)$

**Maple [A]** time = 0.053, size = 211, normalized size = 1.2

$$-\frac{1}{7 dx^7} (-e^2 x^2 + d^2)^{\frac{5}{2}} - \frac{2 e^2}{35 d^3 x^5} (-e^2 x^2 + d^2)^{\frac{5}{2}} - \frac{e}{6 d^2 x^6} (-e^2 x^2 + d^2)^{\frac{5}{2}} - \frac{e^3}{24 d^4 x^4} (-e^2 x^2 + d^2)^{\frac{5}{2}} + \frac{e^5}{48 d^6 x^2} (-e^2 x^2 + d^2)^{\frac{5}{2}} + \frac{e^7}{48 d^6} (-e^2 x^2 + d^2)^{\frac{3}{2}} + \frac{e^7}{16 d^4} \sqrt{-e^2 x^2 + d^2} - \frac{e^7}{16 d^2} \ln\left(\frac{1}{x} \left(2 d^2 + 2 \sqrt{d^2} \sqrt{-e^2 x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)\*(-e^2\*x^2+d^2)^(3/2)/x^8, x)

[Out]  $-1/7 * (-e^2 x^2 + d^2)^{(5/2)} / d / x^7 - 2/35 * e^2 * (-e^2 x^2 + d^2)^{(5/2)} / d^3 / x^5 - 1/6 * e * (-e^2 x^2 + d^2)^{(5/2)} / d^2 / x^6 - 1/24 * e^3 / d^4 / x^4 * (-e^2 x^2 + d^2)^{(5/2)} + 1/48 * e^5 / d^6 / x^2 * (-e^2 x^2 + d^2)^{(5/2)} + 1/48 * e^7 / d^6 * (-e^2 x^2 + d^2)^{(3/2)} + 1/16 * e^7 / d^4 * (-e^2 x^2 + d^2)^{(1/2)} - 1/16 * e^7 / d^2 / (d^2)^{(1/2)} * \ln((2 * d^2 + 2 * (d^2)^{(1/2)} * (-e^2 x^2 + d^2)^{(1/2)}) / x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)/x^8, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.37202, size = 711, normalized size = 4.13

$$96 e^{14} x^{14} + 105 d e^{13} x^{13} - 2352 d^2 e^{12} x^{12} - 3115 d^3 e^{11} x^{11} + 8400 d^4 e^{10} x^{10} + 23450 d^5 e^9 x^9 + 1008 d^6 e^8 x^8 - 73080 d^7 e^7 x^7 - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)/x^8, x, algorithm="fricas")

[Out]  $-1/1680 * (96 e^{14} x^{14} + 105 d e^{13} x^{13} - 2352 d^2 e^{12} x^{12} - 3115 d^3 e^{11} x^{11} + 8400 d^4 e^{10} x^{10} + 23450 d^5 e^9 x^9 + 1008 d^6 e^8 x^8 - 73080 d^7 e^7 x^7 - 46704 d^8 e^6 x^6 + 106400 d^9 e^5 x^5 + 83328 d^{10} e^4 x^4 - 71680 d^{11} e^3 x^3 - 59136 d^{12} e^2 x^2 + 17920 d^{13} e x + 15360 d^{14}) - 105 * (7 d e^{13} x^{13} - 56 d^3 e^{11} x^{11} + 112 d^5 e^9 x^9 - 64 d^7 e^7 x^7 - (e^{13} x^{13} - 24 d^2 e^{11} x^{11} + 80 d^4 e^9 x^9 - 64 d^6 e^7 x^7) * \text{sqrt}(-e^2 x^2 + d^2)) / (1680 d^3 x^7)$

$$\begin{aligned} &^2)) * \log(-(d - \sqrt{-e^2 x^2 + d^2})/x) + (672*d*e^{12}x^{12} + 735* \\ &d^2*e^{11}x^{11} - 5040*d^3*e^{10}x^{10} - 9310*d^4*e^9x^9 + 5376*d^5* \\ &e^8x^8 + 41160*d^6*e^7x^7 + 22416*d^7*e^6x^6 - 77280*d^8*e^5x^5 \\ &- 59520*d^9*e^4x^4 + 62720*d^{10}*e^3x^3 + 51456*d^{11}*e^2x^2 \\ &- 17920*d^{12}*e*x - 15360*d^{13}) * \sqrt{-e^2x^2 + d^2}) / (7*d^4*e^6*x \\ &^{13} - 56*d^6*e^4x^{11} + 112*d^8*e^2x^9 - 64*d^{10}x^7 - (d^3*e^6* \\ &x^{13} - 24*d^5*e^4x^{11} + 80*d^7*e^2x^9 - 64*d^9x^7) * \sqrt{-e^2*x \\ &^2 + d^2}) \end{aligned}$$

**Sympy [A]** time = 43.5264, size = 1037, normalized size = 6.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2)/x\*\*8,x)

[Out]  $d^3 \text{Piecewise}((-e \sqrt{d^2/(e^2 x^2)} - 1)/(7 x^6) + e^3 \sqrt{d^2/(e^2 x^2)} - 1)/(35 d^2 x^4) + 4 e^5 \sqrt{d^2/(e^2 x^2)} - 1)/(105 d^4 x^2) + 8 e^7 \sqrt{d^2/(e^2 x^2)} - 1)/(105 d^6), \text{Abs}(d^2/(e^2 x^2)) > 1), (-I e \sqrt{-d^2/(e^2 x^2)} + 1)/(7 x^6) + I e^3 \sqrt{-d^2/(e^2 x^2)} + 1)/(35 d^2 x^4) + 4 I e^5 \sqrt{-d^2/(e^2 x^2)} + 1)/(105 d^4 x^2) + 8 I e^7 \sqrt{-d^2/(e^2 x^2)} + 1)/(105 d^6), \text{True})) + d^2 e \text{Piecewise}((-d^2/(6 e x^7 \sqrt{d^2/(e^2 x^2)} - 1)) + 5 e/(24 x^5 \sqrt{d^2/(e^2 x^2)} - 1)) + e^3/(48 d^2 x^3 \sqrt{d^2/(e^2 x^2)} - 1)) - e^5/(16 d^4 x \sqrt{d^2/(e^2 x^2)} - 1)) + e^6 \text{acosh}(d/(e x))/(16 d^5), \text{Abs}(d^2/(e^2 x^2)) > 1), (I d^2/(6 e x^7 \sqrt{-d^2/(e^2 x^2)} + 1)) - 5 I e/(24 x^5 \sqrt{-d^2/(e^2 x^2)} + 1)) - I e^3/(48 d^2 x^3 \sqrt{-d^2/(e^2 x^2)} + 1)) + I e^5/(16 d^4 x \sqrt{-d^2/(e^2 x^2)} + 1)) - I e^6 \text{asin}(d/(e x))/(16 d^5), \text{True})) - d e^2 \text{Piecewise}((3 I d^3 \sqrt{-1 + e^2 x^2/d^2})/(-15 d^2 x^5 + 15 e^2 x^7) - 4 I d e^2 x^2 \sqrt{-1 + e^2 x^2/d^2})/(-15 d^2 x^5 + 15 e^2 x^7) + 2 I e^6 x^6 \sqrt{-1 + e^2 x^2/d^2})/(-15 d^5 x^5 + 15 d^3 e^2 x^7) - I e^4 x^4 \sqrt{-1 + e^2 x^2/d^2})/(-15 d^3 x^5 + 15 d e^2 x^7), \text{Abs}(e^2 x^2/d^2) > 1), (3 d^3 \sqrt{1 - e^2 x^2/d^2})/(-15 d^2 x^5 + 15 e^2 x^7) - 4 d e^2 x^2 \sqrt{1 - e^2 x^2/d^2})/(-15 d^2 x^5 + 15 d^3 e^2 x^7) - e^4 x^4 \sqrt{1 - e^2 x^2/d^2})/(-15 d^3 x^5 + 15 d e^2 x^7), \text{True})) - e^3 \text{Piecewise}((-d^2/(4 e x^5 \sqrt{d^2/(e^2 x^2)} - 1)) + 3 e/(8 x^3 \sqrt{d^2/(e^2 x^2)} - 1)) - e^3/(8 d^2 x \sqrt{d^2/(e^2 x^2)} - 1)) + e^4 \text{acosh}(d/(e x))/(8 d^3), \text{Abs}(d^2/(e^2 x^2)) > 1), (I d^2/(4 e x^5 \sqrt{-d^2/(e^2 x^2)} + 1)) - 3 I e/(8 x^3 \sqrt{-d^2/(e^2 x^2)} + 1)) + I e^3/(8 d^2 x \sqrt{-d^2/(e^2 x^2)} + 1)) - I e^4 \text{asin}(d/(e x))/(8 d^3), \text{True}))$

**GIAC/XCAS [A]** time = 0.293522, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)/x^8,x, algorithm="giac")

[Out] Done

$$3.15 \quad \int \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{x^9} dx$$

**Optimal.** Leaf size=201

$$\begin{aligned} & -\frac{(d^2-e^2x^2)^{5/2}}{8dx^8} - \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{3e^8 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d^4} \\ & - \frac{2e^3(d^2-e^2x^2)^{5/2}}{35d^4x^5} - \frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6} + \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4(d^2-e^2x^2)^{3/2}}{64d^3x^4} \end{aligned}$$

[Out]  $(3e^6\sqrt{d^2-e^2x^2})/(128d^3x^2) - (e^4(d^2-e^2x^2)^{3/2})/(64d^3x^4) - (d^2-e^2x^2)^{5/2}/(8d^2x^7) - (e(d^2-e^2x^2)^{5/2})/(7d^2x^7) - (e^2(d^2-e^2x^2)^{5/2})/(16d^3x^6) - (2e^3(d^2-e^2x^2)^{5/2})/(35d^4x^5) - (3e^8 \operatorname{ArcTanh}[\sqrt{d^2-e^2x^2}/d])/(128d^4)$

**Rubi [A]** time = 0.442919, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\begin{aligned} & -\frac{(d^2-e^2x^2)^{5/2}}{8dx^8} - \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} - \frac{3e^8 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d^4} \\ & - \frac{2e^3(d^2-e^2x^2)^{5/2}}{35d^4x^5} - \frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6} + \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4(d^2-e^2x^2)^{3/2}}{64d^3x^4} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d+e*x)*(d^2-e^2*x^2)^(3/2)/x^9,x]$

[Out]  $(3e^6\sqrt{d^2-e^2x^2})/(128d^3x^2) - (e^4(d^2-e^2x^2)^{3/2})/(64d^3x^4) - (d^2-e^2x^2)^{5/2}/(8d^2x^7) - (e(d^2-e^2x^2)^{5/2})/(7d^2x^7) - (e^2(d^2-e^2x^2)^{5/2})/(16d^3x^6) - (2e^3(d^2-e^2x^2)^{5/2})/(35d^4x^5) - (3e^8 \operatorname{ArcTanh}[\sqrt{d^2-e^2x^2}/d])/(128d^4)$

**Rubi in Sympy [A]** time = 59.2166, size = 175, normalized size = 0.87

$$\begin{aligned} & -\frac{(d^2-e^2x^2)^{5/2}}{8dx^8} - \frac{e(d^2-e^2x^2)^{5/2}}{7d^2x^7} + \frac{3e^6\sqrt{d^2-e^2x^2}}{128d^3x^2} - \frac{e^4(d^2-e^2x^2)^{3/2}}{64d^3x^4} \\ & - \frac{e^2(d^2-e^2x^2)^{5/2}}{16d^3x^6} - \frac{3e^8 \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{128d^4} - \frac{2e^3(d^2-e^2x^2)^{5/2}}{35d^4x^5} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)*(-e**2*x**2+d**2)**(3/2)/x**9,x)$

[Out]  $-(d**2-e**2*x**2)**(5/2)/(8*d*x**8) - e*(d**2-e**2*x**2)**(5/2)/(7*d**2*x**7) + 3*e**6*sqrt(d**2-e**2*x**2)/(128*d**3*x**2) - e**4*(d**2-e**2*x**2)**(3/2)/(64*d**3*x**4) - e**2*(d**2-e**2*x**2)**(5/2)/(16*d**3*x**6) - 3*e**8*atanh(sqrt(d**2-e**2*x**2)/d)/(128*d**4) - 2*e**3*(d**2-e**2*x**2)**(5/2)/(35*d**4*x**5)$

**Mathematica [A]** time = 0.20593, size = 139, normalized size = 0.69

$$\frac{105e^8x^8 \log\left(\sqrt{d^2-e^2x^2}+d\right) + \sqrt{d^2-e^2x^2}\left(560d^7+640d^6ex-840d^5e^2x^2-1024d^4e^3x^3+70d^3e^4x^4+128d^2e^5x^5+105d\right)}{4480d^4x^8}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(d^2 - e^2\*x^2)^(3/2))/x^9, x]

[Out]  $-(\text{Sqrt}[d^2 - e^2*x^2]*(560*d^7 + 640*d^6*e*x - 840*d^5*e^2*x^2 - 1024*d^4*e^3*x^3 + 70*d^3*e^4*x^4 + 128*d^2*e^5*x^5 + 105*d*e^6*x^6 + 256*e^7*x^7) - 105*e^8*x^8*\text{Log}[x] + 105*e^8*x^8*\text{Log}[d + \text{Sqrt}[d^2 - e^2*x^2]])/(4480*d^4*x^8)$

**Maple [A]** time = 0.075, size = 236, normalized size = 1.2

$$\begin{aligned} & -\frac{1}{8d^8}(-e^2x^2 + d^2)^{\frac{5}{2}} - \frac{e^2}{16d^3x^6}(-e^2x^2 + d^2)^{\frac{5}{2}} - \frac{e^4}{64d^5x^4}(-e^2x^2 + d^2)^{\frac{5}{2}} \\ & + \frac{e^6}{128d^7x^2}(-e^2x^2 + d^2)^{\frac{5}{2}} + \frac{e^8}{128d^7}(-e^2x^2 + d^2)^{\frac{3}{2}} + \frac{3e^8}{128d^5}\sqrt{-e^2x^2 + d^2} \\ & - \frac{3e^8}{128d^3}\ln\left(\frac{1}{x}\left(2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}} - \frac{e}{7d^2x^7}(-e^2x^2 + d^2)^{\frac{5}{2}} - \frac{2e^3}{35d^4x^5}(-e^2x^2 + d^2)^{\frac{5}{2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)\*(-e^2\*x^2+d^2)^(3/2)/x^9, x)

[Out]  $-1/8*(-e^2*x^2+d^2)^(5/2)/d/x^8-1/16*e^2*(-e^2*x^2+d^2)^(5/2)/d^3/x^6-1/64*e^4/d^5/x^4*(-e^2*x^2+d^2)^(5/2)+1/128*e^6/d^7/x^2*(-e^2*x^2+d^2)^(5/2)+1/128*e^8/d^7*(-e^2*x^2+d^2)^(3/2)+3/128*e^8/d^5*(-e^2*x^2+d^2)^(1/2)-3/128*e^8/d^3/(d^2)^(1/2)*\ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/7*e*(-e^2*x^2+d^2)^(5/2)/d^2/x^7-2/35*e^3*(-e^2*x^2+d^2)^(5/2)/d^4/x^5$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)/x^9, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.523407, size = 799, normalized size = 3.98

$$2048de^{15}x^{15} + 840d^2e^{14}x^{14} - 21504d^3e^{13}x^{13} - 8680d^4e^{12}x^{12} + 50176d^5e^{11}x^{11} + 15680d^6e^{10}x^{10} + 48128d^7e^9x^9 + 63840d^8e^8x^8 - 343040d^9e^7x^7 - 286720d^{10}e^6x^6 + 518144d^{11}e^5x^5 + 430080d^{12}e^4x^4 - 335872d^{13}e^3x^3 - 286720d^{14}e^2x^2 + 81920d^{15}e^1x + 71680d^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)/x^9, x, algorithm="fricas")

[Out]  $1/4480*(2048*d*e^{15}*x^{15} + 840*d^2*e^{14}*x^{14} - 21504*d^3*e^{13}*x^{13} - 8680*d^4*e^{12}*x^{12} + 50176*d^5*e^{11}*x^{11} + 15680*d^6*e^{10}*x^{10} + 48128*d^7*e^9*x^9 + 63840*d^8*e^8*x^8 - 343040*d^9*e^7*x^7 - 286720*d^{10}*e^6*x^6 + 518144*d^{11}*e^5*x^5 + 430080*d^{12}*e^4*x^4 - 335872*d^{13}*e^3*x^3 - 286720*d^{14}*e^2*x^2 + 81920*d^{15}*e^1*x + 71680*d^{16})$



$$256*d^6*e^{10}*x^{10} + 128*d^8*e^8*x^8 + 8*(d^e^{14}*x^{14} - 10*d^3*e^{12}*x^{12} + 24*d^5*e^{10}*x^{10} - 16*d^7*e^8*x^8)*\sqrt{-e^2*x^2 + d^2})*\log(-(d - \sqrt{-e^2*x^2 + d^2})/x) - (256*e^{15}*x^{15} + 105*d^e^{14}*x^{14} - 8064*d^2*e^{13}*x^{13} - 3290*d^3*e^{12}*x^{12} + 35840*d^4*e^{11}*x^{11} + 13720*d^5*e^{10}*x^{10} - 11648*d^6*e^9*x^9 + 11760*d^7*e^8*x^8 - 184320*d^8*e^7*x^7 - 156800*d^9*e^6*x^6 + 380928*d^{10}*e^5*x^5 + 313600*d^{11}*e^4*x^4 - 294912*d^{12}*e^3*x^3 - 250880*d^{13}*e^2*x^2 + 81920*d^{14}*e*x + 71680*d^{15})*\sqrt{-e^2*x^2 + d^2})/(d^4*e^8*x^{16} - 32*d^6*e^6*x^{14} + 160*d^8*e^4*x^{12} - 256*d^{10}*e^2*x^{10} + 128*d^{12}*x^8 + 8*(d^5*e^6*x^{14} - 10*d^7*e^4*x^{12} + 24*d^9*e^2*x^{10} - 16*d^{11}*x^8)*\sqrt{-e^2*x^2 + d^2})$$

**Sympy [A]** time = 63.5172, size = 1159, normalized size = 5.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2)/x\*\*9,x)

[Out]  $d^{*3} \text{Piecewise}((-d^{*2}/(8^*e^*x^{*9} \sqrt{d^{*2}/(e^{*2}*x^{*2}) - 1})) + 7^*e/(48^*x^{*7} \sqrt{d^{*2}/(e^{*2}*x^{*2}) - 1})) + e^{*3}/(192^*d^{*2}*x^{*5} \sqrt{d^{*2}/(e^{*2}*x^{*2}) - 1})) + 5^*e^{*5}/(384^*d^{*4}*x^{*3} \sqrt{d^{*2}/(e^{*2}*x^{*2}) - 1})) - 5^*e^{*7}/(128^*d^{*6}*x \sqrt{d^{*2}/(e^{*2}*x^{*2}) - 1})) + 5^*e^{*8} \operatorname{acosh}(d/(e^*x))/(128^*d^{*7}), \operatorname{Abs}(d^{*2}/(e^{*2}*x^{*2})) > 1), (I^*d^{*2}/(8^*e^*x^{*9} \sqrt{-d^{*2}/(e^{*2}*x^{*2}) + 1})) - 7^*I^*e/(48^*x^{*7} \sqrt{-d^{*2}/(e^{*2}*x^{*2}) + 1})) - I^*e^{*3}/(192^*d^{*2}*x^{*5} \sqrt{-d^{*2}/(e^{*2}*x^{*2}) + 1})) - 5^*I^*e^{*5}/(384^*d^{*4}*x^{*3} \sqrt{-d^{*2}/(e^{*2}*x^{*2}) + 1})) + 5^*I^*e^{*7}/(128^*d^{*6}*x \sqrt{-d^{*2}/(e^{*2}*x^{*2}) + 1})) - 5^*I^*e^{*8} \operatorname{asin}(d/(e^*x))/(128^*d^{*7}), \operatorname{True})) + d^{*2}*e \text{Piecewise}((-e^* \sqrt{d^{*2}/(e^{*2}*x^{*2}) - 1})/(7^*x^{*6}) + e^{*3} \sqrt{d^{*2}/(e^{*2}*x^{*2}) - 1})/(35^*d^{*2}*x^{*4}) + 4^*e^{*5} \sqrt{d^{*2}/(e^{*2}*x^{*2}) - 1})/(105^*d^{*4}*x^{*2}) + 8^*e^{*7} \sqrt{d^{*2}/(e^{*2}*x^{*2}) - 1})/(105^*d^{*6}), \operatorname{Abs}(d^{*2}/(e^{*2}*x^{*2})) > 1), (-I^*e^* \sqrt{-d^{*2}/(e^{*2}*x^{*2}) + 1})/(7^*x^{*6}) + I^*e^{*3} \sqrt{-d^{*2}/(e^{*2}*x^{*2}) + 1})/(35^*d^{*2}*x^{*4}) + 4^*I^*e^{*5} \sqrt{-d^{*2}/(e^{*2}*x^{*2}) + 1})/(105^*d^{*4}*x^{*2}) + 8^*I^*e^{*7} \sqrt{-d^{*2}/(e^{*2}*x^{*2}) + 1})/(105^*d^{*6}), \operatorname{True})) - d^*e^{*2} \text{Piecewise}((-d^{*2}/(6^*e^*x^{*7} \sqrt{d^{*2}/(e^{*2}*x^{*2}) - 1})) + 5^*e/(24^*x^{*5} \sqrt{d^{*2}/(e^{*2}*x^{*2}) - 1})) + e^{*3}/(48^*d^{*2}*x^{*3} \sqrt{d^{*2}/(e^{*2}*x^{*2}) - 1})) - e^{*5}/(16^*d^{*4}*x \sqrt{d^{*2}/(e^{*2}*x^{*2}) - 1})) + e^{*6} \operatorname{acosh}(d/(e^*x))/(16^*d^{*5}), \operatorname{Abs}(d^{*2}/(e^{*2}*x^{*2})) > 1), (I^*d^{*2}/(6^*e^*x^{*7} \sqrt{-d^{*2}/(e^{*2}*x^{*2}) + 1})) - 5^*I^*e/(24^*x^{*5} \sqrt{-d^{*2}/(e^{*2}*x^{*2}) + 1})) - I^*e^{*3}/(48^*d^{*2}*x^{*3} \sqrt{-d^{*2}/(e^{*2}*x^{*2}) + 1})) + I^*e^{*5}/(16^*d^{*4}*x \sqrt{-d^{*2}/(e^{*2}*x^{*2}) + 1})) - I^*e^{*6} \operatorname{asin}(d/(e^*x))/(16^*d^{*5}), \operatorname{True})) - e^{*3} \text{Piecewise}((3^*I^*d^{*3} \sqrt{-1 + e^{*2}*x^{*2}/d^{*2}})/(-15^*d^{*2}*x^{*5} + 15^*e^{*2}*x^{*7}) - 4^*I^*d^*e^{*2}*x^{*2} \sqrt{-1 + e^{*2}*x^{*2}/d^{*2}})/(-15^*d^{*2}*x^{*5} + 15^*e^{*2}*x^{*7}) + 2^*I^*e^{*6}*x^{*6} \sqrt{-1 + e^{*2}*x^{*2}/d^{*2}})/(-15^*d^{*5}*x^{*5} + 15^*d^{*3}*e^{*2}*x^{*7}) - I^*e^{*4}*x^{*4} \sqrt{-1 + e^{*2}*x^{*2}/d^{*2}})/(-15^*d^{*3}*x^{*5} + 15^*d^*e^{*2}*x^{*7}), \operatorname{Abs}(e^{*2}*x^{*2}/d^{*2}) > 1), (3^*d^{*3} \sqrt{1 - e^{*2}*x^{*2}/d^{*2}})/(-15^*d^{*2}*x^{*5} + 15^*e^{*2}*x^{*7}) - 4^*d^*e^{*2}*x^{*2} \sqrt{1 - e^{*2}*x^{*2}/d^{*2}})/(-15^*d^{*2}*x^{*5} + 15^*e^{*2}*x^{*7}) + 2^*e^{*6}*x^{*6} \sqrt{1 - e^{*2}*x^{*2}/d^{*2}})/(-15^*d^{*5}*x^{*5} + 15^*d^{*3}*e^{*2}*x^{*7}) - e^{*4}*x^{*4} \sqrt{1 - e^{*2}*x^{*2}/d^{*2}})/(-15^*d^{*3}*x^{*5} + 15^*d^*e^{*2}*x^{*7}), \operatorname{True}))$

**GIAC/XCAS [A]** time = 0.298833, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)/x^9,x, algorithm="giac")

[Out] Done

$$3.16 \quad \int \frac{x^2(d+ex)}{\sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=103

$$-\frac{dx\sqrt{d^2-e^2x^2}}{2e^2} - \frac{d^2\sqrt{d^2-e^2x^2}}{e^3} + \frac{(d^2-e^2x^2)^{3/2}}{3e^3} + \frac{d^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

[Out]  $-\left(\frac{d^2 \sqrt{d^2 - e^2 x^2}}{e^3}\right) - \left(\frac{d x \sqrt{d^2 - e^2 x^2}}{2 e^2}\right) + \frac{(d^2 - e^2 x^2)^{3/2}}{3 e^3} + \frac{d^3 \operatorname{ArcTan}\left[\frac{e x}{\sqrt{d^2 - e^2 x^2}}\right]}{2 e^3}$

**Rubi [A]** time = 0.159127, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{dx\sqrt{d^2-e^2x^2}}{2e^2} - \frac{d^2\sqrt{d^2-e^2x^2}}{e^3} + \frac{(d^2-e^2x^2)^{3/2}}{3e^3} + \frac{d^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3}$$

Antiderivative was successfully verified.

[In] `Int[(x^2*(d + e*x))/Sqrt[d^2 - e^2*x^2],x]`

[Out]  $-\left(\frac{d^2 \sqrt{d^2 - e^2 x^2}}{e^3}\right) - \left(\frac{d x \sqrt{d^2 - e^2 x^2}}{2 e^2}\right) + \frac{(d^2 - e^2 x^2)^{3/2}}{3 e^3} + \frac{d^3 \operatorname{ArcTan}\left[\frac{e x}{\sqrt{d^2 - e^2 x^2}}\right]}{2 e^3}$

**Rubi in Sympy [A]** time = 27.5057, size = 87, normalized size = 0.84

$$\frac{d^3 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^3} - \frac{d^2\sqrt{d^2-e^2x^2}}{e^3} - \frac{dx\sqrt{d^2-e^2x^2}}{2e^2} + \frac{(d^2-e^2x^2)^{3/2}}{3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)`

[Out]  $d^3 \operatorname{atan}(e x / \sqrt{d^2 - e^2 x^2}) / (2 e^3) - d^2 \sqrt{d^2 - e^2 x^2} / e^3 - d x \sqrt{d^2 - e^2 x^2} / (2 e^2) + (d^2 - e^2 x^2)^{3/2} / (3 e^3)$

**Mathematica [A]** time = 0.0692683, size = 70, normalized size = 0.68

$$\frac{3d^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \sqrt{d^2-e^2x^2}(4d^2 + 3dex + 2e^2x^2)}{6e^3}$$

Antiderivative was successfully verified.

[In] `Integrate[(x^2*(d + e*x))/Sqrt[d^2 - e^2*x^2],x]`

[Out]  $-\left(\sqrt{d^2 - e^2 x^2}\right) \left(4 d^2 + 3 d e x + 2 e^2 x^2\right) + 3 d^3 \operatorname{ArcTan}\left[\frac{e x}{\sqrt{d^2 - e^2 x^2}}\right] / (6 e^3)$

**Maple [A]** time = 0.02, size = 102, normalized size = 1.

$$-\frac{dx}{2e^2}\sqrt{-e^2x^2+d^2} + \frac{d^3}{2e^2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}} - \frac{x^2}{3e}\sqrt{-e^2x^2+d^2} - \frac{2d^2}{3e^3}\sqrt{-e^2x^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)`

[Out] `-1/2*d*x*(-e^2*x^2+d^2)^(1/2)/e^2+1/2*d^3/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/3*x^2/e*(-e^2*x^2+d^2)^(1/2)-2/3*d^2*(-e^2*x^2+d^2)^(1/2)/e^3`

**Maxima [A]** time = 0.804686, size = 127, normalized size = 1.23

$$-\frac{\sqrt{-e^2x^2+d^2}x^2}{3e} + \frac{d^3\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{2\sqrt{e^2e^2}} - \frac{\sqrt{-e^2x^2+d^2}dx}{2e^2} - \frac{2\sqrt{-e^2x^2+d^2}d^2}{3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*x^2/sqrt(-e^2*x^2+d^2),x,algorithm="maxima")`

[Out] `-1/3*sqrt(-e^2*x^2+d^2)*x^2/e+1/2*d^3*arcsin(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^2)-1/2*sqrt(-e^2*x^2+d^2)*d*x/e^2-2/3*sqrt(-e^2*x^2+d^2)*d^2/e^3`

**Fricas [A]** time = 0.276598, size = 301, normalized size = 2.92

$$\frac{2e^6x^6 + 3de^5x^5 - 6d^2e^4x^4 - 15d^3e^3x^3 + 12d^5ex + 6\left(3d^4e^2x^2 - 4d^6 - (d^3e^2x^2 - 4d^5)\sqrt{-e^2x^2+d^2}\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)}{6\left(3de^5x^2 - 4d^3e^3 - (e^5x^2 - 4d^2e^3)\sqrt{-e^2x^2+d^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*x^2/sqrt(-e^2*x^2+d^2),x,algorithm="fricas")`

[Out] `-1/6*(2*e^6*x^6+3*d*e^5*x^5-6*d^2*e^4*x^4-15*d^3*e^3*x^3+12*d^5*e*x+6*(3*d^4*e^2*x^2-4*d^6-(d^3*e^2*x^2-4*d^5)*sqrt(-e^2*x^2+d^2))*arctan(-(d-sqrt(-e^2*x^2+d^2))/(e*x))+3*(2*d*e^4*x^4+3*d^2*e^3*x^3-4*d^4*e*x)*sqrt(-e^2*x^2+d^2))/(3*d*e^5*x^2-4*d^3*e^3-(e^5*x^2-4*d^2*e^3)*sqrt(-e^2*x^2+d^2))`

**Sympy [A]** time = 11.3763, size = 177, normalized size = 1.72

$$d\left(\left(\begin{array}{l} \frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3} - \frac{idx\sqrt{-1+\frac{e^2x^2}{d^2}}}{2e^2} \\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3} - \frac{dx}{2e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{x^3}{2d\sqrt{1-\frac{e^2x^2}{d^2}}} \end{array}\right)\begin{array}{l} \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \text{otherwise} \end{array}\right) + e\left(\left(\begin{array}{l} \frac{-2d^2\sqrt{d^2-e^2x^2}}{3e^4} - \frac{x^2\sqrt{d^2-e^2x^2}}{3e^2} \\ \frac{x^4}{4\sqrt{d^2}} \end{array}\right)\begin{array}{l} \text{for } e \neq 0 \\ \text{otherwise} \end{array}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)`

```
[Out] d*Piecewise((-I*d**2*acosh(e*x/d)/(2*e**3) - I*d*x*sqrt(-1 + e**2
*x**2/d**2)/(2*e**2), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)
/(2*e**3) - d*x/(2*e**2*sqrt(1 - e**2*x**2/d**2)) + x**3/(2*d*sqrt
(1 - e**2*x**2/d**2)), True)) + e*Piecewise((-2*d**2*sqrt(d**2 -
e**2*x**2)/(3*e**4) - x**2*sqrt(d**2 - e**2*x**2)/(3*e**2), Ne(e
, 0)), (x**4/(4*sqrt(d**2)), True))
```

**GIAC/XCAS [A]** time = 0.291206, size = 73, normalized size = 0.71

$$\frac{1}{2} d^3 \arcsin\left(\frac{xe}{d}\right) e^{(-3)} \text{sign}(d) - \frac{1}{6} \sqrt{-x^2 e^2 + d^2} \left(4 d^2 e^{(-3)} + \left(2 x e^{(-1)} + 3 d e^{(-2)}\right) x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)*x^2/sqrt(-e^2*x^2 + d^2),x, algorithm="giac")
```

```
[Out] 1/2*d^3*arcsin(x*e/d)*e^(-3)*sign(d) - 1/6*sqrt(-x^2*e^2 + d^2)*(
4*d^2*e^(-3) + (2*x*e^(-1) + 3*d*e^(-2))*x)
```

$$3.17 \quad \int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{3/2}} dx$$

**Optimal.** Leaf size=73

$$\frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^3} - \frac{d \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

[Out] (d\*(d + e\*x))/(e^3\*Sqrt[d^2 - e^2\*x^2]) + Sqrt[d^2 - e^2\*x^2]/e^3 - (d\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/e^3

**Rubi [A]** time = 0.133938, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{d(d+ex)}{e^3\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^3} - \frac{d \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(d + e\*x))/(d^2 - e^2\*x^2)^(3/2), x]

[Out] (d\*(d + e\*x))/(e^3\*Sqrt[d^2 - e^2\*x^2]) + Sqrt[d^2 - e^2\*x^2]/e^3 - (d\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/e^3

**Rubi in Sympy [A]** time = 22.712, size = 63, normalized size = 0.86

$$-\frac{d \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3} + \frac{d\sqrt{d^2-e^2x^2}}{e^3(d-ex)} + \frac{\sqrt{d^2-e^2x^2}}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2), x)

[Out] -d\*atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2))/e\*\*3 + d\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(e\*\*3\*(d - e\*x)) + sqrt(d\*\*2 - e\*\*2\*x\*\*2)/e\*\*3

**Mathematica [A]** time = 0.146113, size = 63, normalized size = 0.86

$$\sqrt{d^2-e^2x^2} \left( \frac{1}{e^3} - \frac{d}{e^3(ex-d)} \right) - \frac{d \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(d + e\*x))/(d^2 - e^2\*x^2)^(3/2), x]

[Out] Sqrt[d^2 - e^2\*x^2]\*(e^(-3) - d/(e^3\*(-d + e\*x))) - (d\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/e^3

**Maple [A]** time = 0.021, size = 99, normalized size = 1.4

$$\frac{dx}{e^2} \frac{1}{\sqrt{-e^2x^2 + d^2}} - \frac{d}{e^2} \arctan\left(x\sqrt{e^2} \frac{1}{\sqrt{-e^2x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} - \frac{x^2}{e} \frac{1}{\sqrt{-e^2x^2 + d^2}} + 2 \frac{d^2}{e^3\sqrt{-e^2x^2 + d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)`

[Out]  $d*x/e^2/(-e^2*x^2+d^2)^{(1/2)}-d/e^2/(e^2)^{(1/2)}*\arctan((e^2)^{(1/2)}*x/(-e^2*x^2+d^2)^{(1/2)})-x^2/e/(-e^2*x^2+d^2)^{(1/2)}+2*d^2/e^3/(-e^2*x^2+d^2)^{(1/2)}$

**Maxima [A]** time = 0.802032, size = 123, normalized size = 1.68

$$-\frac{x^2}{\sqrt{-e^2x^2+d^2}e} + \frac{dx}{\sqrt{-e^2x^2+d^2}e^2} - \frac{d \arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{\sqrt{e^2}e^2} + \frac{2d^2}{\sqrt{-e^2x^2+d^2}e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*x^2/(-e^2*x^2+d^2)^(3/2),x,algorithm="maxima")`

[Out]  $-x^2/(\sqrt{-e^2*x^2+d^2}*e) + d*x/(\sqrt{-e^2*x^2+d^2}*e^2) - d*\arcsin(e^2*x/\sqrt{d^2*e^2})/(\sqrt{e^2}*e^2) + 2*d^2/(\sqrt{-e^2*x^2+d^2}*e^3)$

**Fricas [A]** time = 0.277382, size = 243, normalized size = 3.33

$$\frac{e^3x^3 - de^2x^2 + 2d^2ex + 2\left(de^2x^2 + d^2ex - 2d^3 - \sqrt{-e^2x^2+d^2}(dex - 2d^2)\right) \arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) + (e^2x^2 - 2dex)\sqrt{-e^2x^2+d^2}}{e^5x^2 + de^4x - 2d^2e^3 - (e^4x - 2de^3)\sqrt{-e^2x^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*x^2/(-e^2*x^2+d^2)^(3/2),x,algorithm="fricas")`

[Out]  $(e^3*x^3 - d*e^2*x^2 + 2*d^2*e*x + 2*(d*e^2*x^2 + d^2*e*x - 2*d^3 - \sqrt{-e^2*x^2 + d^2}*(d*e*x - 2*d^2)))/e^5 + (e^2*x^2 - 2*d*e*x)*\sqrt{-e^2*x^2 + d^2}/(e^4*x^2 + d*e^4*x - 2*d^2*e^3 - (e^4*x - 2*d*e^3)*\sqrt{-e^2*x^2 + d^2})$

**Sympy [A]** time = 11.7418, size = 163, normalized size = 2.23

$$d \left( \begin{cases} \frac{i \operatorname{acosh}\left(\frac{ex}{d}\right)}{e^3} - \frac{ix}{de^2\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ -\frac{\operatorname{asin}\left(\frac{ex}{d}\right)}{e^3} + \frac{x}{de^2\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases} \right) + e \left( \begin{cases} \tilde{\infty}x^4 & \text{for } d = 0 \wedge e = 0 \\ \frac{x^4}{4(d^2)^{\frac{3}{2}}} & \text{for } e = 0 \\ \tilde{\infty}x^4 & \text{for } d = -\sqrt{e^2x^2} \vee d = \sqrt{e^2x^2} \\ \frac{2d^2}{e^4\sqrt{d^2-e^2x^2}} - \frac{x^2}{e^2\sqrt{d^2-e^2x^2}} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)`

[Out]  $d*\operatorname{Piecewise}((I*\operatorname{acosh}(e*x/d)/e**3 - I*x/(d*e**2*\sqrt{-1 + e**2*x**2/d**2})), \operatorname{Abs}(e**2*x**2/d**2) > 1), (-\operatorname{asin}(e*x/d)/e**3 + x/(d*e**2*\sqrt{1 - e**2*x**2/d**2})), \operatorname{True})) + e*\operatorname{Piecewise}((zoo*x**4, \operatorname{Eq}(d, 0) \& \operatorname{Eq}(e, 0)), (x**4/(4*(d**2)**(3/2))), \operatorname{Eq}(e, 0)), (zoo*x**4, \operatorname{Eq}(d, \sqrt{e**2*x**2}) | \operatorname{Eq}(d, -\sqrt{e**2*x**2})), (2*d**2/(e**4*\sqrt{d**2 - e**2*x**2})) - x**2/(e**2*\sqrt{d**2 - e**2*x**2})), \operatorname{True}))$

e))

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**GIAC/XCAS [A]** time = 0.296545, size = 89, normalized size = 1.22

$$-d \arcsin\left(\frac{xe}{d}\right) e^{(-3)} \operatorname{sign}(d) - \frac{\sqrt{-x^2 e^2 + d^2} \left(2 d^2 e^{(-3)} - (xe^{(-1)} - de^{(-2)})x\right)}{x^2 e^2 - d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*x^2/(-e^2\*x^2 + d^2)^(3/2),x, algorithm="giac")

[Out] -d\*arcsin(x\*e/d)\*e^(-3)\*sign(d) - sqrt(-x^2\*e^2 + d^2)\*(2\*d^2\*e^(-3) - (x\*e^(-1) - d\*e^(-2))\*x)/(x^2\*e^2 - d^2)



$$3.18 \quad \int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{5/2}} dx$$

**Optimal.** Leaf size=58

$$\frac{x^2(d+ex)}{3de(d^2-e^2x^2)^{3/2}} - \frac{2}{3e^3\sqrt{d^2-e^2x^2}}$$

[Out] (x^2\*(d + e\*x))/(3\*d\*e\*(d^2 - e^2\*x^2)^(3/2)) - 2/(3\*e^3\*Sqrt[d^2 - e^2\*x^2])

**Rubi [A]** time = 0.094754, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$\frac{x^2(d+ex)}{3de(d^2-e^2x^2)^{3/2}} - \frac{2}{3e^3\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(d + e\*x))/(d^2 - e^2\*x^2)^(5/2), x]

[Out] (x^2\*(d + e\*x))/(3\*d\*e\*(d^2 - e^2\*x^2)^(3/2)) - 2/(3\*e^3\*Sqrt[d^2 - e^2\*x^2])

**Rubi in Sympy [A]** time = 10.7957, size = 46, normalized size = 0.79

$$-\frac{2}{3e^3\sqrt{d^2-e^2x^2}} + \frac{x^2(d+ex)}{3de(d^2-e^2x^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2), x)

[Out] -2/(3\*e\*\*3\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) + x\*\*2\*(d + e\*x)/(3\*d\*e\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2))

**Mathematica [A]** time = 0.0527396, size = 59, normalized size = 1.02

$$\frac{\sqrt{d^2-e^2x^2}(-2d^2+2dex+e^2x^2)}{3de^3(d-ex)^2(d+ex)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(d + e\*x))/(d^2 - e^2\*x^2)^(5/2), x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(-2\*d^2 + 2\*d\*e\*x + e^2\*x^2))/(3\*d\*e^3\*(d - e\*x)^2\*(d + e\*x))

**Maple [A]** time = 0.013, size = 55, normalized size = 1.

$$-\frac{(-ex+d)(ex+d)^2(-e^2x^2-2dex+2d^2)}{3de^3}(-e^2x^2+d^2)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)`

[Out]  $-1/3*(-e*x+d)*(e*x+d)^2*(-e^2*x^2-2*d*e*x+2*d^2)/d/e^3/(-e^2*x^2+d^2)^(5/2)$

**Maxima [A]** time = 0.720443, size = 119, normalized size = 2.05

$$\frac{x^2}{(-e^2x^2 + d^2)^{\frac{3}{2}}e} + \frac{dx}{3(-e^2x^2 + d^2)^{\frac{3}{2}}e^2} - \frac{2d^2}{3(-e^2x^2 + d^2)^{\frac{3}{2}}e^3} - \frac{x}{3\sqrt{-e^2x^2 + d^2}de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)*x^2/(-e^2*x^2 + d^2)^(5/2),x, algorithm="maxima")`

[Out]  $x^2/((-e^2*x^2 + d^2)^(3/2)*e) + 1/3*d*x/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2/3*d^2/((-e^2*x^2 + d^2)^(3/2)*e^3) - 1/3*x/(sqrt(-e^2*x^2 + d^2)*d*e^2)$

**Fricas [A]** time = 0.271443, size = 159, normalized size = 2.74

$$\frac{ex^4 - 2dx^3 + 2\sqrt{-e^2x^2 + d^2}x^3}{3\left(2d^2e^3x^3 - 2d^3e^2x^2 - 2d^4ex + 2d^5 - (de^3x^3 - d^2e^2x^2 - 2d^3ex + 2d^4)\sqrt{-e^2x^2 + d^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)*x^2/(-e^2*x^2 + d^2)^(5/2),x, algorithm="fricas")`

[Out]  $1/3*(e*x^4 - 2*d*x^3 + 2*sqrt(-e^2*x^2 + d^2)*x^3)/(2*d^2*e^3*x^3 - 2*d^3*e^2*x^2 - 2*d^4*e*x + 2*d^5 - (d*e^3*x^3 - d^2*e^2*x^2 - 2*d^3*e*x + 2*d^4)*sqrt(-e^2*x^2 + d^2))$

**Sympy [A]** time = 12.8027, size = 231, normalized size = 3.98

$$d \left( \begin{cases} \frac{ix^3}{-3d^5\sqrt{-1+\frac{e^2x^2}{d^2}}+3d^3e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{x^3}{-3d^5\sqrt{1-\frac{e^2x^2}{d^2}}+3d^3e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases} \right) + e \left( \begin{cases} \frac{2d^2}{-3d^2e^4\sqrt{d^2-e^2x^2}+3e^6x^2\sqrt{d^2-e^2x^2}} - \frac{3e^2x^2}{-3d^2e^4\sqrt{d^2-e^2x^2}+3e^6x^2\sqrt{d^2-e^2x^2}} & \text{for } e \neq 0 \\ \frac{x^4}{4(d^2)^{\frac{5}{2}}} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)`

[Out]  $d*\text{Piecewise}((I*x**3/(-3*d**5*sqrt(-1 + e**2*x**2/d**2)) + 3*d**3*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)), \text{Abs}(e**2*x**2/d**2) > 1), (-x**3/(-3*d**5*sqrt(1 - e**2*x**2/d**2)) + 3*d**3*e**2*x**2*sqrt(1 - e**2*x**2/d**2)), \text{True})) + e*\text{Piecewise}((2*d**2/(-3*d**2*e**4*sqrt(d**2 - e**2*x**2)) + 3*e**6*x**2*sqrt(d**2 - e**2*x**2)) - 3*e**2*x**2/(-3*d**2*e**4*sqrt(d**2 - e**2*x**2)) + 3*e**6*x**2*sqrt(d**2 - e**2*x**2)), \text{Ne}(e, 0)), (x**4/(4*(d**2)**(5/2)), \text{True}))$

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**GIAC/XCAS [A]** time = 0.297559, size = 69, normalized size = 1.19

$$\frac{\left(x^2\left(\frac{x}{d} + 3e^{-1}\right) - 2d^2e^{-3}\right)\sqrt{-x^2e^2 + d^2}}{3(x^2e^2 - d^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*x^2/(-e^2\*x^2 + d^2)^(5/2),x, algorithm="giac")

[Out] 1/3\*(x^2\*(x/d + 3\*e^(-1)) - 2\*d^2\*e^(-3))\*sqrt(-x^2\*e^2 + d^2)/(x^2\*e^2 - d^2)^2

$$3.19 \quad \int \frac{x^7(d+ex)}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=161

$$\frac{x^6(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} + \frac{(32d+35ex)\sqrt{d^2-e^2x^2}}{10e^8} - \frac{7d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8} + \frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{x^4(6d+7ex)}{15e^4(d^2-e^2x^2)^{3/2}}$$

[Out]  $(x^6*(d+e*x))/(5*e^2*(d^2-e^2*x^2)^(5/2)) - (x^4*(6*d+7*e*x))/(15*e^4*(d^2-e^2*x^2)^(3/2)) + (x^2*(24*d+35*e*x))/(15*e^6*\text{Sqrt}[d^2-e^2*x^2]) + ((32*d+35*e*x)*\text{Sqrt}[d^2-e^2*x^2])/(10*e^8) - (7*d^2*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2-e^2*x^2]])/(2*e^8)$

**Rubi [A]** time = 0.423463, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{x^6(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} + \frac{(32d+35ex)\sqrt{d^2-e^2x^2}}{10e^8} - \frac{7d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8} + \frac{x^2(24d+35ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{x^4(6d+7ex)}{15e^4(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^7*(d+e*x))/(d^2-e^2*x^2)^(7/2), x]$

[Out]  $(x^6*(d+e*x))/(5*e^2*(d^2-e^2*x^2)^(5/2)) - (x^4*(6*d+7*e*x))/(15*e^4*(d^2-e^2*x^2)^(3/2)) + (x^2*(24*d+35*e*x))/(15*e^6*\text{Sqrt}[d^2-e^2*x^2]) + ((32*d+35*e*x)*\text{Sqrt}[d^2-e^2*x^2])/(10*e^8) - (7*d^2*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2-e^2*x^2]])/(2*e^8)$

**Rubi in Sympy [A]** time = 47.4078, size = 146, normalized size = 0.91

$$-\frac{7d^2 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8} + \frac{x^6(2d+2ex)}{10e^2(d^2-e^2x^2)^{5/2}} - \frac{x^4(24d+28ex)}{60e^4(d^2-e^2x^2)^{3/2}} + \frac{x^2(192d+280ex)}{120e^6\sqrt{d^2-e^2x^2}} + \frac{(768d+840ex)\sqrt{d^2-e^2x^2}}{240e^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**7}*(e*x+d)/(-e^{**2}*x^{**2}+d^{**2})^{**}(7/2), x)$

[Out]  $-7*d^{**2}*\operatorname{atan}(e*x/\operatorname{sqrt}(d^{**2}-e^{**2}*x^{**2}))/ (2*e^{**8}) + x^{**6}*(2*d+2*e*x)/(10*e^{**2}*(d^{**2}-e^{**2}*x^{**2})^{**}(5/2)) - x^{**4}*(24*d+28*e*x)/(60*e^{**4}*(d^{**2}-e^{**2}*x^{**2})^{**}(3/2)) + x^{**2}*(192*d+280*e*x)/(120*e^{**6}*\operatorname{sqrt}(d^{**2}-e^{**2}*x^{**2})) + (768*d+840*e*x)*\operatorname{sqrt}(d^{**2}-e^{**2}*x^{**2})/(240*e^{**8})$

**Mathematica [A]** time = 0.193679, size = 128, normalized size = 0.8

$$\frac{\sqrt{d^2-e^2x^2}(96d^6+9d^5ex-249d^4e^2x^2+4d^3e^3x^3+176d^2e^4x^4-15de^5x^5-15e^6x^6)}{(d-ex)^3(d+ex)^2} - 105d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)$$

$30e^8$

Antiderivative was successfully verified.

[In] Integrate[(x^7\*(d + e\*x))/(d^2 - e^2\*x^2)^(7/2), x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(96\*d^6 + 9\*d^5\*e\*x - 249\*d^4\*e^2\*x^2 + 4\*d^3\*e^3\*x^3 + 176\*d^2\*e^4\*x^4 - 15\*d\*e^5\*x^5 - 15\*e^6\*x^6))/((d - e\*x)^3\*(d + e\*x)^2) - 105\*d^2\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(30\*e^8)

**Maple [A]** time = 0.102, size = 227, normalized size = 1.4

$$\begin{aligned} & -\frac{dx^6}{e^2} (-e^2x^2 + d^2)^{-\frac{5}{2}} + 6 \frac{d^3x^4}{e^4(-e^2x^2 + d^2)^{5/2}} - 8 \frac{d^5x^2}{e^6(-e^2x^2 + d^2)^{5/2}} \\ & + \frac{16d^7}{5e^8} (-e^2x^2 + d^2)^{-\frac{5}{2}} - \frac{x^7}{2e} (-e^2x^2 + d^2)^{-\frac{5}{2}} + \frac{7d^2x^5}{10e^3} (-e^2x^2 + d^2)^{-\frac{5}{2}} \\ & - \frac{7d^2x^3}{6e^5} (-e^2x^2 + d^2)^{-\frac{3}{2}} + \frac{7d^2x}{2e^7} \frac{1}{\sqrt{-e^2x^2 + d^2}} - \frac{7d^2}{2e^7} \arctan\left(x\sqrt{e^2} \frac{1}{\sqrt{-e^2x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7\*(e\*x+d)/(-e^2\*x^2+d^2)^(7/2), x)

[Out] -d\*x^6/e^2/(-e^2\*x^2+d^2)^(5/2)+6\*d^3/e^4\*x^4/(-e^2\*x^2+d^2)^(5/2)-8\*d^5/e^6\*x^2/(-e^2\*x^2+d^2)^(5/2)+16/5\*d^7/e^8/(-e^2\*x^2+d^2)^(5/2)-1/2\*x^7/e/(-e^2\*x^2+d^2)^(5/2)+7/10\*d^2/e^3\*x^5/(-e^2\*x^2+d^2)^(5/2)-7/6\*d^2/e^5\*x^3/(-e^2\*x^2+d^2)^(3/2)+7/2\*d^2/e^7\*x/(-e^2\*x^2+d^2)^(1/2)-7/2\*d^2/e^7/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))

**Maxima [A]** time = 0.804259, size = 439, normalized size = 2.73

$$\begin{aligned} & -\frac{x^7}{2(-e^2x^2 + d^2)^{\frac{5}{2}}e} + \frac{7d^2x\left(\frac{15x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{20d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{8d^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^6}\right)}{30e} - \frac{dx^6}{(-e^2x^2 + d^2)^{\frac{5}{2}}e^2} \\ & - \frac{7d^2x\left(\frac{3x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^2} - \frac{2d^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^4}\right)}{6e^3} + \frac{6d^3x^4}{(-e^2x^2 + d^2)^{\frac{5}{2}}e^4} - \frac{8d^5x^2}{(-e^2x^2 + d^2)^{\frac{5}{2}}e^6} \\ & + \frac{16d^7}{5(-e^2x^2 + d^2)^{\frac{5}{2}}e^8} + \frac{14d^4x}{15(-e^2x^2 + d^2)^{\frac{3}{2}}e^7} - \frac{49d^2x}{30\sqrt{-e^2x^2 + d^2}e^7} - \frac{7d^2 \arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{2\sqrt{e^2}e^7} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*x^7/(-e^2\*x^2 + d^2)^(7/2), x, algorithm="maxima")

[Out] -1/2\*x^7/((-e^2\*x^2 + d^2)^(5/2)\*e) + 7/30\*d^2\*x\*(15\*x^4/((-e^2\*x^2 + d^2)^(5/2)\*e^2) - 20\*d^2\*x^2/((-e^2\*x^2 + d^2)^(5/2)\*e^4) + 8\*d^4/((-e^2\*x^2 + d^2)^(5/2)\*e^6))/e - d\*x^6/((-e^2\*x^2 + d^2)^(5/2)\*e^2) - 7/6\*d^2\*x\*(3\*x^2/((-e^2\*x^2 + d^2)^(3/2)\*e^2) - 2\*d^2/((-e^2\*x^2 + d^2)^(3/2)\*e^4))/e^3 + 6\*d^3\*x^4/((-e^2\*x^2 + d^2)^(5/2)\*e^4) - 8\*d^5\*x^2/((-e^2\*x^2 + d^2)^(5/2)\*e^6) + 16/5\*d^7/((-e^2\*x^2 + d^2)^(5/2)\*e^8) + 14/15\*d^4\*x/((-e^2\*x^2 + d^2)^(3/2)\*e^7) - 49/30\*d^2\*x/(sqrt(-e^2\*x^2 + d^2)\*e^7) - 7/2\*d^2\*arcsin(e^2\*x/sqrt(d^2\*e^2))/sqrt(e^2)\*e^7

**Fricas [A]** time = 0.312542, size = 965, normalized size = 5.99

$$15e^{12}x^{12} + 15de^{11}x^{11} - 446d^2e^{10}x^{10} + 302d^3e^9x^9 + 3561d^4e^8x^8 - 3441d^5e^7x^7 - 9282d^6e^6x^6 + 9282d^7e^5x^5 + 9520d^8e^4x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*x^7/(-e^2\*x^2 + d^2)^(7/2), x, algorithm="fricas")

[Out]  $\frac{1}{30} \cdot (15e^{12}x^{12} + 15d^2e^{11}x^{11} - 446d^2e^{10}x^{10} + 302d^3e^9x^9 + 3561d^4e^8x^8 - 3441d^5e^7x^7 - 9282d^6e^6x^6 + 9282d^7e^5x^5 + 9520d^8e^4x^4 - 9520d^9e^3x^3 - 3360d^{10}e^2x^2 + 3360d^{11}e^1x + 210 \cdot (6d^3e^9x^9 - 6d^4e^8x^8 - 44d^5e^7x^7 + 44d^6e^6x^6 + 102d^7e^5x^5 - 102d^8e^4x^4 - 96d^9e^3x^3 + 96d^{10}e^2x^2 + 32d^{11}e^1x - 32d^{12} - (d^2e^9x^9 - d^3e^8x^8 - 19d^4e^7x^7 + 19d^5e^6x^6 + 66d^6e^5x^5 - 66d^7e^4x^4 - 80d^8e^3x^3 + 80d^9e^2x^2 + 32d^{10}e^1x - 32d^{11}) \cdot \sqrt{-e^2x^2 + d^2}) \cdot \arctan\left(\frac{-d - \sqrt{-e^2x^2 + d^2}}{e \cdot x}\right) + 2 \cdot (45d^2e^{10}x^{10} - 3d^2e^9x^9 - 720d^3e^8x^8 + 660d^4e^7x^7 + 2891d^5e^6x^6 - 2891d^6e^5x^5 - 3920d^7e^4x^4 + 3920d^8e^3x^3 + 1680d^9e^2x^2 - 1680d^{10}e^1x) \cdot \sqrt{-e^2x^2 + d^2}) / (6d^2e^{17}x^9 - 6d^2e^{16}x^8 - 44d^3e^{15}x^7 + 44d^4e^{14}x^6 + 102d^5e^{13}x^5 - 102d^6e^{12}x^4 - 96d^7e^{11}x^3 + 96d^8e^{10}x^2 + 32d^9e^9x - 32d^{10}e^8 - (e^{17}x^9 - d^2e^{16}x^8 - 19d^2e^{15}x^7 + 19d^3e^{14}x^6 + 66d^4e^{13}x^5 - 66d^5e^{12}x^4 - 80d^6e^{11}x^3 + 80d^7e^{10}x^2 + 32d^8e^9x - 32d^9e^8) \cdot \sqrt{-e^2x^2 + d^2})$

**Sympy [A]** time = 56.1707, size = 2004, normalized size = 12.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*7\*(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out]  $d \cdot \text{Piecewise}\left(\left(\frac{16d^6}{5d^4e^8\sqrt{d^2 - e^2x^2}} - 10d^2e^{10}x^2\sqrt{d^2 - e^2x^2} + 5e^{12}x^4\sqrt{d^2 - e^2x^2}\right) - 40d^4e^2x^2/(5d^4e^8\sqrt{d^2 - e^2x^2}) - 10d^2e^{10}x^2\sqrt{d^2 - e^2x^2} + 5e^{12}x^4\sqrt{d^2 - e^2x^2}\right) + 30d^2e^4x^4/(5d^4e^8\sqrt{d^2 - e^2x^2}) - 10d^2e^{10}x^2\sqrt{d^2 - e^2x^2} + 5e^{12}x^4\sqrt{d^2 - e^2x^2}) - 5e^6x^6/(5d^4e^8\sqrt{d^2 - e^2x^2}) - 10d^2e^{10}x^2\sqrt{d^2 - e^2x^2} + 5e^{12}x^4\sqrt{d^2 - e^2x^2})$ ,  $\text{Ne}(e, 0)$ ,  $(x^8/(8(d^2)^{7/2}), \text{True})$  +  $e \cdot \text{Piecewise}\left(\left(\frac{210I^d \sqrt{-1 + e^2x^2/d^2}}{60d^5e^9\sqrt{-1 + e^2x^2/d^2}} - \frac{120d^3e^{11}x^2\sqrt{-1 + e^2x^2/d^2}}{60d^5e^9\sqrt{-1 + e^2x^2/d^2}} + \frac{60d^2e^{13}x^4\sqrt{-1 + e^2x^2/d^2}}{60d^5e^9\sqrt{-1 + e^2x^2/d^2}} - \frac{105\pi d^7\sqrt{-1 + e^2x^2/d^2}}{60d^5e^9\sqrt{-1 + e^2x^2/d^2}} - \frac{120d^3e^{11}x^2\sqrt{-1 + e^2x^2/d^2}}{60d^5e^9\sqrt{-1 + e^2x^2/d^2}} + \frac{60d^2e^{13}x^4\sqrt{-1 + e^2x^2/d^2}}{60d^5e^9\sqrt{-1 + e^2x^2/d^2}} - \frac{210I^d e^6x}{60d^5e^9\sqrt{-1 + e^2x^2/d^2}} - \frac{120d^3e^{11}x^2\sqrt{-1 + e^2x^2/d^2}}{60d^5e^9\sqrt{-1 + e^2x^2/d^2}} + \frac{60d^2e^{13}x^4\sqrt{-1 + e^2x^2/d^2}}{60d^5e^9\sqrt{-1 + e^2x^2/d^2}} - \frac{420I^d e^5e^2x^2\sqrt{-1 + e^2x^2/d^2}}{60d^5e^9\sqrt{-1 + e^2x^2/d^2}} \cdot \text{acosh}\left(\frac{e \cdot x}{d}\right) / (60d^5e^9\sqrt{-1 + e^2x^2/d^2}) - \frac{120d^3e^{11}x^2\sqrt{-1 + e^2x^2/d^2}}{60d^5e^9\sqrt{-1 + e^2x^2/d^2}} + \frac{60d^2e^{13}x^4\sqrt{-1 + e^2x^2/d^2}}{60d^5e^9\sqrt{-1 + e^2x^2/d^2}} + \frac{210\pi d^5e^2x^2\sqrt{-1 + e^2x^2/d^2}}{60d^5e^9\sqrt{-1 + e^2x^2/d^2}} - \frac{120d^3e^{11}x^2\sqrt{-1 + e^2x^2/d^2}}{60d^5e^9\sqrt{-1 + e^2x^2/d^2}} + \frac{60d^2e^{13}x^4\sqrt{-1 + e^2x^2/d^2}}{60d^5e^9\sqrt{-1 + e^2x^2/d^2}}\right) + \frac{490I^d e^4e^3x^3}{60d^5e^9\sqrt{-1 + e^2x^2/d^2}} - \frac{120d^3e^{11}x^2\sqrt{-1 + e^2x^2/d^2}}{60d^5e^9\sqrt{-1 + e^2x^2/d^2}} + \frac{60d^2e^{13}x^4\sqrt{-1 + e^2x^2/d^2}}{60d^5e^9\sqrt{-1 + e^2x^2/d^2}} + \frac{210I^d e^3e^4x^4\sqrt{-1 + e^2x^2/d^2}}{60d^5e^9\sqrt{-1 + e^2x^2/d^2}} \cdot \text{acosh}\left(\frac{e \cdot x}{d}\right) / (60d^5e^9\sqrt{-1 + e^2x^2/d^2}) - \frac{120d^3e^{11}x^2\sqrt{-1 + e^2x^2/d^2}}{60d^5e^9\sqrt{-1 + e^2x^2/d^2}} + \frac{60d^2e^{13}x^4\sqrt{-1 + e^2x^2/d^2}}{60d^5e^9\sqrt{-1 + e^2x^2/d^2}}$

```

rt(-1 + e**2*x**2/d**2)) - 105*pi*d**3*e**4*x**4*sqrt(-1 + e**2*x
**2/d**2)/(60*d**5*e**9*sqrt(-1 + e**2*x**2/d**2) - 120*d**3*e**1
1*x**2*sqrt(-1 + e**2*x**2/d**2) + 60*d*e**13*x**4*sqrt(-1 + e**2
*x**2/d**2)) - 322*I*d**2*e**5*x**5/(60*d**5*e**9*sqrt(-1 + e**2*
x**2/d**2) - 120*d**3*e**11*x**2*sqrt(-1 + e**2*x**2/d**2) + 60*d
*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)) + 30*I*e**7*x**7/(60*d**5*
e**9*sqrt(-1 + e**2*x**2/d**2) - 120*d**3*e**11*x**2*sqrt(-1 + e
**2*x**2/d**2) + 60*d*e**13*x**4*sqrt(-1 + e**2*x**2/d**2)), Abs(e
**2*x**2/d**2) > 1), (-105*d**7*sqrt(1 - e**2*x**2/d**2)*asin(e*x
/d)/(30*d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*x**2*s
qrt(1 - e**2*x**2/d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2
)) + 105*d**6*e*x/(30*d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**
3*e**11*x**2*sqrt(1 - e**2*x**2/d**2) + 30*d*e**13*x**4*sqrt(1 -
e**2*x**2/d**2)) + 210*d**5*e**2*x**2*sqrt(1 - e**2*x**2/d**2)*as
in(e*x/d)/(30*d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*
x**2*sqrt(1 - e**2*x**2/d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**
2/d**2)) - 245*d**4*e**3*x**3/(30*d**5*e**9*sqrt(1 - e**2*x**2/d*
**2) - 60*d**3*e**11*x**2*sqrt(1 - e**2*x**2/d**2) + 30*d*e**13*x
**4*sqrt(1 - e**2*x**2/d**2)) - 105*d**3*e**4*x**4*sqrt(1 - e**2*x
**2/d**2)*asin(e*x/d)/(30*d**5*e**9*sqrt(1 - e**2*x**2/d**2) - 60
*d**3*e**11*x**2*sqrt(1 - e**2*x**2/d**2) + 30*d*e**13*x**4*sqrt(
1 - e**2*x**2/d**2)) + 161*d**2*e**5*x**5/(30*d**5*e**9*sqrt(1 -
e**2*x**2/d**2) - 60*d**3*e**11*x**2*sqrt(1 - e**2*x**2/d**2) + 3
0*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2)) - 15*e**7*x**7/(30*d**5*
e**9*sqrt(1 - e**2*x**2/d**2) - 60*d**3*e**11*x**2*sqrt(1 - e**2*
x**2/d**2) + 30*d*e**13*x**4*sqrt(1 - e**2*x**2/d**2)), True))

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**GIAC/XCAS [A]** time = 0.298067, size = 162, normalized size = 1.01

$$\frac{-\frac{7}{2}d^2 \arcsin\left(\frac{xe}{d}\right) e^{(-8)} \text{sign}(d) + (96d^7e^{(-8)} + (105d^6e^{(-7)} - (240d^5e^{(-6)} + (245d^4e^{(-5)} - (180d^3e^{(-4)} + (161d^2e^{(-3)} - 15(xe^{(-1)} + 2de^{(-2)})x)x)x)x)x)}{30(x^2e^2 - d^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*x^7/(-e^2\*x^2 + d^2)^(7/2),x, algorithm="giac")

[Out] -7/2\*d^2\*arcsin(x\*e/d)\*e^(-8)\*sign(d) - 1/30\*(96\*d^7\*e^(-8) + (105\*d^6\*e^(-7) - (240\*d^5\*e^(-6) + (245\*d^4\*e^(-5) - (180\*d^3\*e^(-4) + (161\*d^2\*e^(-3) - 15\*(x\*e^(-1) + 2\*d\*e^(-2))\*x)\*x)\*x)\*x)\*x)\*sqrt(-x^2\*e^2 + d^2)/(x^2\*e^2 - d^2)^3

$$3.20 \quad \int \frac{x^6(d+ex)}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=147

$$\frac{x^5(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} + \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7} + \frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{x^3(5d+6ex)}{15e^4(d^2-e^2x^2)^{3/2}}$$

[Out] (x^5\*(d + e\*x))/(5\*e^2\*(d^2 - e^2\*x^2)^(5/2)) - (x^3\*(5\*d + 6\*e\*x))/(15\*e^4\*(d^2 - e^2\*x^2)^(3/2)) + (x\*(5\*d + 8\*e\*x))/(5\*e^6\*Sqrt[d^2 - e^2\*x^2]) + (16\*Sqrt[d^2 - e^2\*x^2])/(5\*e^7) - (d\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/e^7

**Rubi [A]** time = 0.362028, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{x^5(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} + \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7} + \frac{x(5d+8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{x^3(5d+6ex)}{15e^4(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^6\*(d + e\*x))/(d^2 - e^2\*x^2)^(7/2), x]

[Out] (x^5\*(d + e\*x))/(5\*e^2\*(d^2 - e^2\*x^2)^(5/2)) - (x^3\*(5\*d + 6\*e\*x))/(15\*e^4\*(d^2 - e^2\*x^2)^(3/2)) + (x\*(5\*d + 8\*e\*x))/(5\*e^6\*Sqrt[d^2 - e^2\*x^2]) + (16\*Sqrt[d^2 - e^2\*x^2])/(5\*e^7) - (d\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/e^7

**Rubi in Sympy [A]** time = 45.8304, size = 133, normalized size = 0.9

$$-\frac{d \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7} + \frac{x^5(2d+2ex)}{10e^2(d^2-e^2x^2)^{5/2}} - \frac{x^3(20d+24ex)}{60e^4(d^2-e^2x^2)^{3/2}} + \frac{x(120d+192ex)}{120e^6\sqrt{d^2-e^2x^2}} + \frac{16\sqrt{d^2-e^2x^2}}{5e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*6\*(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out] -d\*atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2))/e\*\*7 + x\*\*5\*(2\*d + 2\*e\*x)/(10\*e\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)) - x\*\*3\*(20\*d + 24\*e\*x)/(60\*e\*\*4\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)) + x\*(120\*d + 192\*e\*x)/(120\*e\*\*6\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) + 16\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(5\*e\*\*7)

**Mathematica [A]** time = 0.172396, size = 115, normalized size = 0.78

$$\frac{\sqrt{d^2-e^2x^2}(48d^5-33d^4ex-87d^3e^2x^2+52d^2e^3x^3+38de^4x^4-15e^5x^5)}{(d-ex)^3(d+ex)^2} - 15d \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)$$

$$15e^7$$

Antiderivative was successfully verified.

[In] Integrate[(x^6\*(d + e\*x))/(d^2 - e^2\*x^2)^(7/2), x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(48\*d^5 - 33\*d^4\*e\*x - 87\*d^3\*e^2\*x^2 + 52\*d^2\*e^3\*x^3 + 38\*d\*e^4\*x^4 - 15\*e^5\*x^5))/((d - e\*x)^3\*(d + e\*x)^2)



$$2) - 15*d*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]]/(15*e^7)$$

**Maple [A]** time = 0.034, size = 195, normalized size = 1.3

$$\frac{dx^5}{5e^2}(-e^2x^2+d^2)^{-\frac{5}{2}} - \frac{dx^3}{3e^4}(-e^2x^2+d^2)^{-\frac{3}{2}} + \frac{dx}{e^6} \frac{1}{\sqrt{-e^2x^2+d^2}} - \frac{d}{e^6} \arctan\left(x\sqrt{e^2} \frac{1}{\sqrt{-e^2x^2+d^2}}\right) \frac{1}{\sqrt{e^2}}$$

$$- \frac{x^6}{e}(-e^2x^2+d^2)^{-\frac{5}{2}} + 6 \frac{d^2x^4}{e^3(-e^2x^2+d^2)^{5/2}} - 8 \frac{d^4x^2}{e^5(-e^2x^2+d^2)^{5/2}} + \frac{16d^6}{5e^7}(-e^2x^2+d^2)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6\*(e\*x+d)/(-e^2\*x^2+d^2)^(7/2),x)

[Out] 1/5\*d\*x^5/e^2/(-e^2\*x^2+d^2)^(5/2)-1/3\*d/e^4\*x^3/(-e^2\*x^2+d^2)^(3/2)+d/e^6\*x/(-e^2\*x^2+d^2)^(1/2)-d/e^6/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))-x^6/e/(-e^2\*x^2+d^2)^(5/2)+6\*d^2/e^4\*x^4/(-e^2\*x^2+d^2)^(5/2)-8\*d^4/e^5\*x^2/(-e^2\*x^2+d^2)^(5/2)+16/5\*d^6/e^7/(-e^2\*x^2+d^2)^(5/2)

**Maxima [A]** time = 0.804294, size = 393, normalized size = 2.67

$$\frac{1}{15} dx \left( \frac{15x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{20d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{8d^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^6} \right) - \frac{x^6}{(-e^2x^2+d^2)^{\frac{5}{2}}e}$$

$$- \frac{dx \left( \frac{3x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^2} - \frac{2d^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^4} \right)}{3e^2} + \frac{6d^2x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^3} - \frac{8d^4x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^5}$$

$$+ \frac{16d^6}{5(-e^2x^2+d^2)^{\frac{5}{2}}e^7} + \frac{4d^3x}{15(-e^2x^2+d^2)^{\frac{3}{2}}e^6} - \frac{7dx}{15\sqrt{-e^2x^2+d^2}e^6} - \frac{d \arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{\sqrt{e^2}e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*x^6/(-e^2\*x^2 + d^2)^(7/2),x, algorithm="maxima")

[Out] 1/15\*d\*x\*(15\*x^4/((-e^2\*x^2 + d^2)^(5/2)\*e^2) - 20\*d^2\*x^2/((-e^2\*x^2 + d^2)^(5/2)\*e^4) + 8\*d^4/((-e^2\*x^2 + d^2)^(5/2)\*e^6)) - x^6/((-e^2\*x^2 + d^2)^(5/2)\*e) - 1/3\*d\*x\*(3\*x^2/((-e^2\*x^2 + d^2)^(3/2)\*e^2) - 2\*d^2/((-e^2\*x^2 + d^2)^(3/2)\*e^4))/e^2 + 6\*d^2\*x^4/((-e^2\*x^2 + d^2)^(5/2)\*e^3) - 8\*d^4\*x^2/((-e^2\*x^2 + d^2)^(5/2)\*e^5) + 16/5\*d^6/((-e^2\*x^2 + d^2)^(5/2)\*e^7) + 4/15\*d^3\*x/((-e^2\*x^2 + d^2)^(3/2)\*e^6) - 7/15\*d\*x/(sqrt(-e^2\*x^2 + d^2)\*e^6) - d\*arcsin(e^2\*x/sqrt(d^2\*e^2))/(sqrt(e^2)\*e^6)

**Fricas [A]** time = 0.301228, size = 841, normalized size = 5.72

$$27de^9x^9 - 142d^2e^8x^8 + 112d^3e^7x^7 + 523d^4e^6x^6 - 523d^5e^5x^5 - 620d^6e^4x^4 + 620d^7e^3x^3 + 240d^8e^2x^2 - 240d^9ex - 30 \left( \frac{d^2x^4}{(-e^2x^2+d^2)^{5/2}} - \frac{d^4x^2}{(-e^2x^2+d^2)^{5/2}} + \frac{d^6}{(-e^2x^2+d^2)^{5/2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*x^6/(-e^2\*x^2 + d^2)^(7/2),x, algorithm="fricas")

[Out] -1/15\*(27\*d\*e^9\*x^9 - 142\*d^2\*e^8\*x^8 + 112\*d^3\*e^7\*x^7 + 523\*d^4\*e^6\*x^6 - 523\*d^5\*e^5\*x^5 - 620\*d^6\*e^4\*x^4 + 620\*d^7\*e^3\*x^3 + 240\*d^8\*e^2\*x^2 - 240\*d^9\*e\*x - 30 \left( \frac{d^2x^4}{(-e^2x^2+d^2)^{5/2}} - \frac{d^4x^2}{(-e^2x^2+d^2)^{5/2}} + \frac{d^6}{(-e^2x^2+d^2)^{5/2}} \right))

$$240*d^8*e^2*x^2 - 240*d^9*e*x - 30*(d*e^9*x^9 - d^2*e^8*x^8 - 14*d^3*e^7*x^7 + 14*d^4*e^6*x^6 + 41*d^5*e^5*x^5 - 41*d^6*e^4*x^4 - 44*d^7*e^3*x^3 + 44*d^8*e^2*x^2 + 16*d^9*e*x - 16*d^{10} + (5*d^2*e^7*x^7 - 5*d^3*e^6*x^6 - 25*d^4*e^5*x^5 + 25*d^5*e^4*x^4 + 36*d^6*e^3*x^3 - 36*d^7*e^2*x^2 - 16*d^8*e*x + 16*d^9)*\sqrt{-e^2*x^2 + d^2})*\arctan(-(d - \sqrt{-e^2*x^2 + d^2})/(e*x)) - (15*e^9*x^9 - 38*d*e^8*x^8 + 8*d^2*e^7*x^7 + 303*d^3*e^6*x^6 - 303*d^4*e^5*x^5 - 500*d^5*e^4*x^4 + 500*d^6*e^3*x^3 + 240*d^7*e^2*x^2 - 240*d^8*e*x)*\sqrt{-e^2*x^2 + d^2})/(e^{16}*x^9 - d*e^{15}*x^8 - 14*d^2*e^{14}*x^7 + 14*d^3*e^{13}*x^6 + 41*d^4*e^{12}*x^5 - 41*d^5*e^{11}*x^4 - 44*d^6*e^{10}*x^3 + 44*d^7*e^9*x^2 + 16*d^8*e^8*x - 16*d^9*e^7 + (5*d*e^{14}*x^7 - 5*d^2*e^{13}*x^6 - 25*d^3*e^{12}*x^5 + 25*d^4*e^{11}*x^4 + 36*d^5*e^{10}*x^3 - 36*d^6*e^9*x^2 - 16*d^7*e^8*x + 16*d^8*e^7)*\sqrt{-e^2*x^2 + d^2}))$$

**Sympy [A]** time = 40.6592, size = 1821, normalized size = 12.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*6\*(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2),x)

[Out] d\*Piecewise((30\*I\*d\*\*5\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)\*acosh(e\*x/d)/(30\*d\*\*5\*e\*\*7\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 60\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 30\*d\*e\*\*11\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 15\*pi\*d\*\*5\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)/(30\*d\*\*5\*e\*\*7\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 60\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 30\*d\*e\*\*11\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 30\*I\*d\*\*4\*e\*x/(30\*d\*\*5\*e\*\*7\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 60\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 30\*d\*e\*\*11\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 60\*I\*d\*\*3\*e\*\*2\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)\*acosh(e\*x/d)/(30\*d\*\*5\*e\*\*7\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 60\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 30\*d\*e\*\*11\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + 30\*pi\*d\*\*3\*e\*\*2\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)/(30\*d\*\*5\*e\*\*7\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 60\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 30\*d\*e\*\*11\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + 70\*I\*d\*\*2\*e\*\*3\*x\*\*3/(30\*d\*\*5\*e\*\*7\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 60\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 30\*d\*e\*\*11\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + 30\*I\*d\*\*4\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)\*acosh(e\*x/d)/(30\*d\*\*5\*e\*\*7\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 60\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 30\*d\*e\*\*11\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 15\*pi\*d\*e\*\*4\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)/(30\*d\*\*5\*e\*\*7\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 60\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 30\*d\*e\*\*11\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 46\*I\*e\*\*5\*x\*\*5/(30\*d\*\*5\*e\*\*7\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 60\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 30\*d\*e\*\*11\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (-15\*d\*\*5\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)\*asin(e\*x/d)/(15\*d\*\*5\*e\*\*7\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 30\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 15\*d\*e\*\*11\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 15\*d\*\*4\*e\*x/(15\*d\*\*5\*e\*\*7\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 30\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 15\*d\*e\*\*11\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 30\*d\*\*3\*e\*\*2\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)\*asin(e\*x/d)/(15\*d\*\*5\*e\*\*7\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 30\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 15\*d\*e\*\*11\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - 35\*d\*\*2\*e\*\*3\*x\*\*3/(15\*d\*\*5\*e\*\*7\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 30\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 15\*d\*e\*\*11\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - 15\*d\*e\*\*4\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)\*asin(e\*x/d)/(15\*d\*\*5\*e\*\*7\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 30\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 15\*d\*e\*\*11\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 23\*e\*\*5\*x\*\*5/(15\*d\*\*5\*e\*\*7\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 30\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 15\*d\*e\*\*11\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True)) + e\*Piecewise((16\*d\*\*6/(5\*d\*\*4\*e\*\*8\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) - 10\*d\*\*2\*e\*\*10\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) + 5\*e\*\*12\*x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) - 40\*d\*\*4\*e\*\*2\*x\*\*2/(5\*d\*\*4\*e\*\*8\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) - 10\*d\*\*2\*e\*\*10\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) + 5\*e\*\*12\*x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) + 30\*d\*\*2\*e\*\*4\*x\*\*4/

```
(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqrt(d**
2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)) - 5*e**6*x*
*6/(5*d**4*e**8*sqrt(d**2 - e**2*x**2) - 10*d**2*e**10*x**2*sqrt(
d**2 - e**2*x**2) + 5*e**12*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0
)), (x**8/(8*(d**2)**(7/2)), True))
```

**GIAC/XCAS [A]** time = 0.300842, size = 147, normalized size = 1.

$$-d \arcsin\left(\frac{xe}{d}\right) e^{(-7)} \operatorname{sign}(d) - \frac{\left(48 d^6 e^{(-7)} + \left(15 d^5 e^{(-6)} - \left(120 d^4 e^{(-5)} + \left(35 d^3 e^{(-4)} - \left(90 d^2 e^{(-3)} - \left(15 x e^{(-1)} - 23 d e^{(-2)}\right) x\right) x\right) x\right) x\right) \sqrt{-x^2 e^2 + d^2}}{15(x^2 e^2 - d^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)*x^6/(-e^2*x^2 + d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -d*arcsin(x*e/d)*e^(-7)*sign(d) - 1/15*(48*d^6*e^(-7) + (15*d^5*e
^(-6) - (120*d^4*e^(-5) + (35*d^3*e^(-4) - (90*d^2*e^(-3) - (15*x
*e^(-1) - 23*d*e^(-2))*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)/(x^2*e
^2 - d^2)^3
```

$$3.21 \quad \int \frac{x^5(d+ex)}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=122

$$\frac{x^4(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} + \frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6} - \frac{x^2(4d+5ex)}{15e^4(d^2-e^2x^2)^{3/2}}$$

[Out]  $(x^4*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^2*(4*d + 5*e*x))/(15*e^4*(d^2 - e^2*x^2)^(3/2)) + (8*d + 15*e*x)/(15*e^6*\text{Sqrt}[d^2 - e^2*x^2]) - \text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]]/e^6$

**Rubi [A]** time = 0.265515, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{x^4(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} + \frac{8d+15ex}{15e^6\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6} - \frac{x^2(4d+5ex)}{15e^4(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^5*(d + e*x))/(d^2 - e^2*x^2)^(7/2), x]$

[Out]  $(x^4*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^2*(4*d + 5*e*x))/(15*e^4*(d^2 - e^2*x^2)^(3/2)) + (8*d + 15*e*x)/(15*e^6*\text{Sqrt}[d^2 - e^2*x^2]) - \text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]]/e^6$

**Rubi in Sympy [A]** time = 34.2947, size = 109, normalized size = 0.89

$$\frac{x^4(2d+2ex)}{10e^2(d^2-e^2x^2)^{5/2}} - \frac{x^2(16d+20ex)}{60e^4(d^2-e^2x^2)^{3/2}} + \frac{32d+60ex}{60e^6\sqrt{d^2-e^2x^2}} - \frac{\text{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**5*(e*x+d)/(-e**2*x**2+d**2)**(7/2), x)$

[Out]  $x**4*(2*d + 2*e*x)/(10*e**2*(d**2 - e**2*x**2)**(5/2)) - x**2*(16*d + 20*e*x)/(60*e**4*(d**2 - e**2*x**2)**(3/2)) + (32*d + 60*e*x)/(60*e**6*\text{sqrt}(d**2 - e**2*x**2)) - \text{atan}(e*x/\text{sqrt}(d**2 - e**2*x**2))/e**6$

**Mathematica [A]** time = 0.150678, size = 103, normalized size = 0.84

$$\frac{\sqrt{d^2-e^2x^2}(8d^4+7d^3ex-27d^2e^2x^2-8de^3x^3+23e^4x^4)}{(d-ex)^3(d+ex)^2} - 15 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)$$

$$15e^6$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(x^5*(d + e*x))/(d^2 - e^2*x^2)^(7/2), x]$

[Out]  $((\text{Sqrt}[d^2 - e^2*x^2]*(8*d^4 + 7*d^3*e*x - 27*d^2*e^2*x^2 - 8*d*e^3*x^3 + 23*e^4*x^4))/((d - e*x)^3*(d + e*x)^2) - 15*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(15*e^6)$

**Maple [A]** time = 0.03, size = 166, normalized size = 1.4

$$\frac{dx^4}{e^2} (-e^2x^2 + d^2)^{-\frac{5}{2}} - \frac{4d^3x^2}{3e^4} (-e^2x^2 + d^2)^{-\frac{5}{2}} + \frac{8d^5}{15e^6} (-e^2x^2 + d^2)^{-\frac{5}{2}} + \frac{x^5}{5e} (-e^2x^2 + d^2)^{-\frac{5}{2}} - \frac{x^3}{3e^3} (-e^2x^2 + d^2)^{-\frac{3}{2}} + \frac{x}{e^5} \frac{1}{\sqrt{-e^2x^2 + d^2}} - \frac{1}{e^5} \arctan\left(x\sqrt{e^2} \frac{1}{\sqrt{-e^2x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(e\*x+d)/(-e^2\*x^2+d^2)^(7/2),x)

[Out] d\*x^4/e^2/(-e^2\*x^2+d^2)^(5/2)-4/3\*d^3/e^4\*x^2/(-e^2\*x^2+d^2)^(5/2)+8/15\*d^5/e^6/(-e^2\*x^2+d^2)^(5/2)+1/5\*x^5/e/(-e^2\*x^2+d^2)^(5/2)-1/3/e^3\*x^3/(-e^2\*x^2+d^2)^(3/2)+1/e^5\*x/(-e^2\*x^2+d^2)^(1/2)-1/e^5/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))

**Maxima [A]** time = 0.798007, size = 355, normalized size = 2.91

$$\frac{1}{15} ex \left( \frac{15x^4}{(-e^2x^2 + d^2)^{\frac{5}{2}} e^2} - \frac{20d^2x^2}{(-e^2x^2 + d^2)^{\frac{5}{2}} e^4} + \frac{8d^4}{(-e^2x^2 + d^2)^{\frac{5}{2}} e^6} \right) - \frac{x \left( \frac{3x^2}{(-e^2x^2 + d^2)^{\frac{3}{2}} e^2} - \frac{2d^2}{(-e^2x^2 + d^2)^{\frac{3}{2}} e^4} \right)}{3e} + \frac{dx^4}{(-e^2x^2 + d^2)^{\frac{5}{2}} e^2} - \frac{4d^3x^2}{3(-e^2x^2 + d^2)^{\frac{5}{2}} e^4} + \frac{8d^5}{15(-e^2x^2 + d^2)^{\frac{5}{2}} e^6} + \frac{4d^2x}{15(-e^2x^2 + d^2)^{\frac{3}{2}} e^5} - \frac{7x}{15\sqrt{-e^2x^2 + d^2} e^5} - \frac{\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{\sqrt{e^2} e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*x^5/(-e^2\*x^2 + d^2)^(7/2),x, algorithm="maxima")

[Out] 1/15\*e\*x\*(15\*x^4/((-e^2\*x^2 + d^2)^(5/2)\*e^2) - 20\*d^2\*x^2/((-e^2\*x^2 + d^2)^(5/2)\*e^4) + 8\*d^4/((-e^2\*x^2 + d^2)^(5/2)\*e^6)) - 1/3\*x\*(3\*x^2/((-e^2\*x^2 + d^2)^(3/2)\*e^2) - 2\*d^2/((-e^2\*x^2 + d^2)^(3/2)\*e^4))/e + d\*x^4/((-e^2\*x^2 + d^2)^(5/2)\*e^2) - 4/3\*d^3\*x^2/((-e^2\*x^2 + d^2)^(5/2)\*e^4) + 8/15\*d^5/((-e^2\*x^2 + d^2)^(5/2)\*e^6) + 4/15\*d^2\*x/((-e^2\*x^2 + d^2)^(3/2)\*e^5) - 7/15\*x/(sqrt(-e^2\*x^2 + d^2)\*e^5) - arcsin(e^2\*x/sqrt(d^2\*e^2))/(sqrt(e^2)\*e^5)

**Fricas [A]** time = 0.294262, size = 729, normalized size = 5.98

$$\frac{23e^8x^8 - 40de^7x^7 - 179d^2e^6x^6 + 199d^3e^5x^5 + 280d^4e^4x^4 - 280d^5e^3x^3 - 120d^6e^2x^2 + 120d^7ex - 30(4de^7x^7 - 4d^2e^6x^6)}{15(4de^{13}x^7 - 4d^2e^6x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*x^5/(-e^2\*x^2 + d^2)^(7/2),x, algorithm="fricas")

[Out] -1/15\*(23\*e^8\*x^8 - 40\*d\*e^7\*x^7 - 179\*d^2\*e^6\*x^6 + 199\*d^3\*e^5\*x^5 + 280\*d^4\*e^4\*x^4 - 280\*d^5\*e^3\*x^3 - 120\*d^6\*e^2\*x^2 + 120\*d^7\*e\*x - 30\*(4\*d\*e^7\*x^7 - 4\*d^2\*e^6\*x^6 + 20\*d^5\*e^3\*x^3 - 20\*d^6\*e^2\*x^2 - 8\*d^7\*e\*x + 8\*d^8 - (e^7\*x^7 - d\*e^6\*x^6 - 9\*d^2\*e^5\*x^5 + 9\*d^3\*e^4\*x^4 + 16\*d^4\*e^3\*x^3 - 16\*d^5\*e^2\*x^2 - 8\*d^6\*e\*x + 8\*d^7)\*sqrt(-e^2\*x^2 + d^2))

$$\begin{aligned} & \arctan\left(\frac{-d - \sqrt{-e^2x^2 + d^2}}{ex}\right) + 4\left(\frac{2e^7x^7 + 21d^8e^6x^6 - 26d^2e^5x^5 - 55d^3e^4x^4 + 55d^4e^3x^3 + 30d^5e^2x^2 - 30d^6ex}{4d^2e^{12}x^6 - 16d^3e^{11}x^5 + 16d^4e^{10}x^4 + 20d^5e^9x^3 - 20d^6e^8x^2 - 8d^7e^7x + 8d^8e^6 - (e^{13}x^7 - d^8e^{12}x^6 - 9d^2e^{11}x^5 + 9d^3e^{10}x^4 + 16d^4e^9x^3 - 16d^5e^8x^2 - 8d^6e^7x + 8d^7e^6)}\sqrt{-e^2x^2 + d^2}\right) \end{aligned}$$

**Sympy [A]** time = 39.8303, size = 1739, normalized size = 14.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out] d\*Piecewise((8\*d\*\*4/(15\*d\*\*4\*e\*\*6\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) - 30\*d\*\*2\*e\*\*8\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) + 15\*e\*\*10\*x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) - 20\*d\*\*2\*e\*\*2\*x\*\*2/(15\*d\*\*4\*e\*\*6\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) - 30\*d\*\*2\*e\*\*8\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) + 15\*e\*\*10\*x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) + 15\*e\*\*4\*x\*\*4/(15\*d\*\*4\*e\*\*6\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) - 30\*d\*\*2\*e\*\*8\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) + 15\*e\*\*10\*x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2))), Ne(e, 0)), (x\*\*6/(6\*(d\*\*2)\*\*(7/2)), True)) + e\*Piecewise((30\*I\*d\*\*5\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)\*acosh(e\*x/d)/(30\*d\*\*5\*e\*\*7\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 60\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 30\*d\*e\*\*11\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 15\*pi\*d\*\*5\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)/(30\*d\*\*5\*e\*\*7\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 60\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 30\*d\*e\*\*11\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 30\*I\*d\*\*4\*e\*x/(30\*d\*\*5\*e\*\*7\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 60\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 30\*d\*e\*\*11\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 60\*I\*d\*\*3\*e\*\*2\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)\*acosh(e\*x/d)/(30\*d\*\*5\*e\*\*7\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 60\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 30\*d\*e\*\*11\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + 30\*pi\*d\*\*3\*e\*\*2\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)/(30\*d\*\*5\*e\*\*7\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 60\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 30\*d\*e\*\*11\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + 70\*I\*d\*\*2\*e\*\*3\*x\*\*3/(30\*d\*\*5\*e\*\*7\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 60\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 30\*d\*e\*\*11\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + 30\*I\*d\*e\*\*4\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)\*acosh(e\*x/d)/(30\*d\*\*5\*e\*\*7\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 60\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 30\*d\*e\*\*11\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 15\*pi\*d\*\*4\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)/(30\*d\*\*5\*e\*\*7\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 60\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 30\*d\*e\*\*11\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 46\*I\*e\*\*5\*x\*\*5/(30\*d\*\*5\*e\*\*7\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 60\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 30\*d\*e\*\*11\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2))), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (-15\*d\*\*5\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)\*asin(e\*x/d)/(15\*d\*\*5\*e\*\*7\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 30\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 15\*d\*e\*\*11\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 15\*d\*\*4\*e\*x/(15\*d\*\*5\*e\*\*7\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 30\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 15\*d\*e\*\*11\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 30\*d\*\*3\*e\*\*2\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)\*asin(e\*x/d)/(15\*d\*\*5\*e\*\*7\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 30\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 15\*d\*e\*\*11\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - 35\*d\*\*2\*e\*\*3\*x\*\*3/(15\*d\*\*5\*e\*\*7\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 30\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 15\*d\*e\*\*11\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - 15\*d\*e\*\*4\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)\*asin(e\*x/d)/(15\*d\*\*5\*e\*\*7\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 30\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 15\*d\*e\*\*11\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 23\*e\*\*5\*x\*\*5/(15\*d\*\*5\*e\*\*7\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 30\*d\*\*3\*e\*\*9\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 15\*d\*e\*\*11\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2))), True))

**GIAC/XCAS [A]** time = 0.296982, size = 131, normalized size = 1.07

$$-\arcsin\left(\frac{xe}{d}\right)e^{(-6)}\text{sign}(d) - \frac{\left(8d^5e^{(-6)} + \left(15d^4e^{(-5)} - \left(20d^3e^{(-4)} + \left(35d^2e^{(-3)} - \left(23xe^{(-1)} + 15de^{(-2)}\right)x\right)x\right)x\right)\sqrt{-x^2e^2 + d^2}}{15(x^2e^2 - d^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*x^5/(-e^2\*x^2 + d^2)^(7/2),x, algorithm="giac")

[Out] -arcsin(x\*e/d)\*e^(-6)\*sign(d) - 1/15\*(8\*d^5\*e^(-6) + (15\*d^4\*e^(-5) - (20\*d^3\*e^(-4) + (35\*d^2\*e^(-3) - (23\*x\*e^(-1) + 15\*d\*e^(-2))\*x)\*x)\*x)\*sqrt(-x^2\*e^2 + d^2)/(x^2\*e^2 - d^2)^3

$$3.22 \quad \int \frac{x^4(d+ex)}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=84

$$\frac{x^4(d+ex)}{5de(d^2-e^2x^2)^{5/2}} + \frac{4}{5e^5\sqrt{d^2-e^2x^2}} - \frac{4d^2}{15e^5(d^2-e^2x^2)^{3/2}}$$

[Out]  $(x^4*(d+e*x))/(5*d*e*(d^2-e^2*x^2)^(5/2)) - (4*d^2)/(15*e^5*(d^2-e^2*x^2)^(3/2)) + 4/(5*e^5*sqrt[d^2-e^2*x^2])$

**Rubi [A]** time = 0.164312, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$\frac{x^4(d+ex)}{5de(d^2-e^2x^2)^{5/2}} + \frac{4}{5e^5\sqrt{d^2-e^2x^2}} - \frac{4d^2}{15e^5(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^4*(d+e*x))/(d^2-e^2*x^2)^(7/2), x]$

[Out]  $(x^4*(d+e*x))/(5*d*e*(d^2-e^2*x^2)^(5/2)) - (4*d^2)/(15*e^5*(d^2-e^2*x^2)^(3/2)) + 4/(5*e^5*sqrt[d^2-e^2*x^2])$

**Rubi in Sympy [A]** time = 17.9974, size = 70, normalized size = 0.83

$$-\frac{4d^2}{15e^5(d^2-e^2x^2)^{3/2}} + \frac{4}{5e^5\sqrt{d^2-e^2x^2}} + \frac{x^4(d+ex)}{5de(d^2-e^2x^2)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**4}*(e*x+d)/(-e^{**2}*x^{**2}+d^{**2})^{**}(7/2), x)$

[Out]  $-4*d^{**2}/(15*e^{**5}*(d^{**2}-e^{**2}*x^{**2})^{**}(3/2)) + 4/(5*e^{**5}*sqrt(d^{**2}-e^{**2}*x^{**2})) + x^{**4}*(d+e*x)/(5*d*e*(d^{**2}-e^{**2}*x^{**2})^{**}(5/2))$

**Mathematica [A]** time = 0.061653, size = 82, normalized size = 0.98

$$\frac{\sqrt{d^2-e^2x^2}(8d^4-8d^3ex-12d^2e^2x^2+12de^3x^3+3e^4x^4)}{15de^5(d-ex)^3(d+ex)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(x^4*(d+e*x))/(d^2-e^2*x^2)^(7/2), x]$

[Out]  $(\text{Sqrt}[d^2-e^2*x^2]*(8*d^4-8*d^3*e*x-12*d^2*e^2*x^2+12*d*e^3*x^3+3*e^4*x^4))/(15*d*e^5*(d-e*x)^3*(d+e*x)^2)$

**Maple [A]** time = 0.012, size = 77, normalized size = 0.9

$$\frac{(-ex+d)(ex+d)^2(3x^4e^4+12x^3de^3-12d^2x^2e^2-8d^3xe+8d^4)}{15de^5}(-e^2x^2+d^2)^{-7/2}$$



Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^4 * (e * x + d) / (-e^2 * x^2 + d^2)^{(7/2)}, x)$

[Out]  $1/15 * (-e * x + d) * (e * x + d)^2 * (3 * e^4 * x^4 + 12 * d * e^3 * x^3 - 12 * d^2 * e^2 * x^2 - 8 * d^3 * e * x + 8 * d^4) / d / e^5 / (-e^2 * x^2 + d^2)^{(7/2)}$

**Maxima [A]** time = 0.720575, size = 215, normalized size = 2.56

$$\begin{aligned} & \frac{x^4}{(-e^2x^2 + d^2)^{\frac{5}{2}}e} + \frac{dx^3}{2(-e^2x^2 + d^2)^{\frac{5}{2}}e^2} - \frac{4d^2x^2}{3(-e^2x^2 + d^2)^{\frac{5}{2}}e^3} - \frac{3d^3x}{10(-e^2x^2 + d^2)^{\frac{5}{2}}e^4} \\ & + \frac{8d^4}{15(-e^2x^2 + d^2)^{\frac{5}{2}}e^5} + \frac{dx}{10(-e^2x^2 + d^2)^{\frac{3}{2}}e^4} + \frac{x}{5\sqrt{-e^2x^2 + d^2}de^4} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e * x + d) * x^4 / (-e^2 * x^2 + d^2)^{(7/2)}, x, \text{algorithm} = \text{"maxima"})$

[Out]  $x^4 / ((-e^2 * x^2 + d^2)^{(5/2)} * e) + 1/2 * d * x^3 / ((-e^2 * x^2 + d^2)^{(5/2)} * e^2) - 4/3 * d^2 * x^2 / ((-e^2 * x^2 + d^2)^{(5/2)} * e^3) - 3/10 * d^3 * x / ((-e^2 * x^2 + d^2)^{(5/2)} * e^4) + 8/15 * d^4 / ((-e^2 * x^2 + d^2)^{(5/2)} * e^5) + 1/10 * d * x / ((-e^2 * x^2 + d^2)^{(3/2)} * e^4) + 1/5 * x / (\text{sqrt}(-e^2 * x^2 + d^2) * d * e^4)$

**Fricas [A]** time = 0.282955, size = 336, normalized size = 4.

$$\frac{3e^3x^8 - 20de^2x^7 - 4d^2ex^6 + 24d^3x^5 + 4(2e^2x^7 + dex^6 - 6d^2x^5)\sqrt{-e^2x^2 + d^2}}{15(4d^2e^7x^7 - 4d^3e^6x^6 - 16d^4e^5x^5 + 16d^5e^4x^4 + 20d^6e^3x^3 - 20d^7e^2x^2 - 8d^8ex + 8d^9 - (de^7x^7 - d^2e^6x^6 - 9d^3e^5x^5 + 9d^4e^4x^4 - 15d^5e^3x^3 - 10d^6e^2x^2 - 5d^7ex + 5d^8)\sqrt{-e^2x^2 + d^2})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e * x + d) * x^4 / (-e^2 * x^2 + d^2)^{(7/2)}, x, \text{algorithm} = \text{"fricas"})$

[Out]  $-1/15 * (3 * e^3 * x^8 - 20 * d * e^2 * x^7 - 4 * d^2 * e * x^6 + 24 * d^3 * x^5 + 4 * (2 * e^2 * x^7 + d * e * x^6 - 6 * d^2 * x^5) * \text{sqrt}(-e^2 * x^2 + d^2)) / (4 * d^2 * e^7 * x^7 - 4 * d^3 * e^6 * x^6 - 16 * d^4 * e^5 * x^5 + 16 * d^5 * e^4 * x^4 + 20 * d^6 * e^3 * x^3 - 20 * d^7 * e^2 * x^2 - 8 * d^8 * e * x + 8 * d^9 - (d * e^7 * x^7 - d^2 * e^6 * x^6 - 9 * d^3 * e^5 * x^5 + 9 * d^4 * e^4 * x^4 + 16 * d^5 * e^3 * x^3 - 16 * d^6 * e^2 * x^2 - 8 * d^7 * e * x + 8 * d^8) * \text{sqrt}(-e^2 * x^2 + d^2))$

**Sympy [A]** time = 24.287, size = 418, normalized size = 4.98

$$d \left( \left\{ \begin{array}{ll} \frac{-\frac{ix^5}{5d^7\sqrt{-1+\frac{e^2x^2}{d^2}}-10d^5e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+5d^3e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left| \frac{e^2x^2}{d^2} \right| > 1 \\ \frac{x^5}{5d^7\sqrt{1-\frac{e^2x^2}{d^2}}-10d^5e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+5d^3e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{array} \right\} \right) + e \left( \frac{8d^4}{15d^4e^6\sqrt{d^2-e^2x^2}-30d^2e^8x^2\sqrt{d^2-e^2x^2}+15e^{10}x^4\sqrt{d^2-e^2x^2}} - \frac{20d^2e^2x^2}{15d^4e^6\sqrt{d^2-e^2x^2}-30d^2e^8x^2\sqrt{d^2-e^2x^2}+15e^{10}x^4\sqrt{d^2-e^2x^2}} + \frac{x^6}{6(d^2)^{\frac{7}{2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(x ** 4 * (e * x + d) / (-e ** 2 * x ** 2 + d ** 2) ** (7/2), x)$

```
[Out] d*Piecewise((-I*x**5/(5*d**7*sqrt(-1 + e**2*x**2/d**2) - 10*d**5*
e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 5*d**3*e**4*x**4*sqrt(-1 +
e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (x**5/(5*d**7*sqrt(1
- e**2*x**2/d**2) - 10*d**5*e**2*x**2*sqrt(1 - e**2*x**2/d**2) +
5*d**3*e**4*x**4*sqrt(1 - e**2*x**2/d**2)), True)) + e*Piecewise(
(8*d**4/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*e**8*x**2*
sqrt(d**2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2*x**2)) -
20*d**2*e**2*x**2/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*d**2*
e**8*x**2*sqrt(d**2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 - e**2
*x**2)) + 15*e**4*x**4/(15*d**4*e**6*sqrt(d**2 - e**2*x**2) - 30*
d**2*e**8*x**2*sqrt(d**2 - e**2*x**2) + 15*e**10*x**4*sqrt(d**2 -
e**2*x**2)), Ne(e, 0)), (x**6/(6*(d**2)**(7/2)), True))
```

**GIAC/XCAS [A]** time = 0.293953, size = 86, normalized size = 1.02

$$\frac{\left(8d^4e^{(-5)} + \left(3x^2\left(\frac{x}{d} + 5e^{(-1)}\right) - 20d^2e^{(-3)}\right)x^2\right)\sqrt{-x^2e^2 + d^2}}{15(x^2e^2 - d^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)*x^4/(-e^2*x^2 + d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/15*(8*d^4*e^(-5) + (3*x^2*(x/d + 5*e^(-1)) - 20*d^2*e^(-3))*x^
2)*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

$$3.23 \quad \int \frac{x^3(d+ex)}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=90

$$\frac{x^2(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{2d+3ex}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}$$

[Out]  $(x^2*(d+e*x))/(5*e^2*(d^2-e^2*x^2)^(5/2)) - (2*d+3*e*x)/(15*e^4*(d^2-e^2*x^2)^(3/2)) + x/(5*d^2*e^3*sqrt[d^2-e^2*x^2])$

**Rubi [A]** time = 0.165658, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$\frac{x^2(d+ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{2d+3ex}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^3*(d+e*x))/(d^2-e^2*x^2)^(7/2), x]$

[Out]  $(x^2*(d+e*x))/(5*e^2*(d^2-e^2*x^2)^(5/2)) - (2*d+3*e*x)/(15*e^4*(d^2-e^2*x^2)^(3/2)) + x/(5*d^2*e^3*sqrt[d^2-e^2*x^2])$

**Rubi in Sympy [A]** time = 20.3052, size = 80, normalized size = 0.89

$$\frac{x^3(d+ex)}{5de(d^2-e^2x^2)^{5/2}} - \frac{2}{15de^4\sqrt{d^2-e^2x^2}} + \frac{x^2(d-3ex)}{15d^2e^2(d^2-e^2x^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**3}*(e*x+d)/(-e^{**2}*x^{**2}+d^{**2})^{**}(7/2), x)$

[Out]  $x^{**3}*(d+e*x)/(5*d*e*(d^{**2}-e^{**2}*x^{**2})^{**}(5/2)) - 2/(15*d*e^{**4}*sqrt(d^{**2}-e^{**2}*x^{**2})) + x^{**2}*(d-3*e*x)/(15*d^{**2}*e^{**2}*(d^{**2}-e^{**2}*x^{**2})^{**}(3/2))$

**Mathematica [A]** time = 0.0618098, size = 82, normalized size = 0.91

$$\frac{\sqrt{d^2-e^2x^2}(-2d^4+2d^3ex+3d^2e^2x^2-3de^3x^3+3e^4x^4)}{15d^2e^4(d-ex)^3(d+ex)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(x^3*(d+e*x))/(d^2-e^2*x^2)^(7/2), x]$

[Out]  $(sqrt[d^2-e^2*x^2]*(-2*d^4+2*d^3*e*x+3*d^2*e^2*x^2-3*d*e^3*x^3+3*e^4*x^4))/(15*d^2*e^4*(d-e*x)^3*(d+e*x)^2)$

**Maple [A]** time = 0.013, size = 77, normalized size = 0.9

$$\frac{(-ex+d)(ex+d)^2(-3x^4e^4+3x^3de^3-3x^2d^2e^2-2d^3xe+2d^4)}{15d^2e^4}(-e^2x^2+d^2)^{-7/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)`

[Out]  $-1/15*(-e*x+d)*(e*x+d)^2*(-3*e^4*x^4+3*d*e^3*x^3-3*d^2*e^2*x^2-2*d^3*e*x+2*d^4)/d^2/e^4/(-e^2*x^2+d^2)^(7/2)$

**Maxima [A]** time = 0.719988, size = 181, normalized size = 2.01

$$\frac{x^3}{2(-e^2x^2 + d^2)^{\frac{5}{2}}e} + \frac{dx^2}{3(-e^2x^2 + d^2)^{\frac{5}{2}}e^2} - \frac{3d^2x}{10(-e^2x^2 + d^2)^{\frac{5}{2}}e^3} - \frac{2d^3}{15(-e^2x^2 + d^2)^{\frac{5}{2}}e^4} + \frac{x}{10(-e^2x^2 + d^2)^{\frac{3}{2}}e^3} + \frac{x}{5\sqrt{-e^2x^2 + d^2}d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)*x^3/(-e^2*x^2 + d^2)^(7/2),x, algorithm="maxima")`

[Out]  $1/2*x^3/((-e^2*x^2 + d^2)^(5/2)*e) + 1/3*d*x^2/((-e^2*x^2 + d^2)^(5/2)*e^2) - 3/10*d^2*x/((-e^2*x^2 + d^2)^(5/2)*e^3) - 2/15*d^3/((-e^2*x^2 + d^2)^(5/2)*e^4) + 1/10*x/((-e^2*x^2 + d^2)^(3/2)*e^3) + 1/5*x/(sqrt(-e^2*x^2 + d^2)*d^2*e^3)$

**Fricas [A]** time = 0.280792, size = 369, normalized size = 4.1

$$\frac{3e^4x^8 + 5de^3x^7 - 29d^2e^2x^6 - 6d^3ex^5 + 30d^4x^4 - 2(e^3x^7 - 7de^2x^6 - 3d^2ex^5 - 15(4d^3e^7x^7 - 4d^4e^6x^6 - 16d^5e^5x^5 + 16d^6e^4x^4 + 20d^7e^3x^3 - 20d^8e^2x^2 - 8d^9ex + 8d^{10} - (d^2e^7x^7 - d^3e^6x^6 - 9d^4e^5x^5 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)*x^3/(-e^2*x^2 + d^2)^(7/2),x, algorithm="fricas")`

[Out]  $-1/15*(3*e^4*x^8 + 5*d*e^3*x^7 - 29*d^2*e^2*x^6 - 6*d^3*e*x^5 + 30*d^4*x^4 - 2*(e^3*x^7 - 7*d*e^2*x^6 - 3*d^2*e*x^5 + 15*d^3*x^4)*sqrt(-e^2*x^2 + d^2))/(4*d^3*e^7*x^7 - 4*d^4*e^6*x^6 - 16*d^5*e^5*x^5 + 16*d^6*e^4*x^4 + 20*d^7*e^3*x^3 - 20*d^8*e^2*x^2 - 8*d^9*e*x + 8*d^{10} - (d^2*e^7*x^7 - d^3*e^6*x^6 - 9*d^4*e^5*x^5 + 9*d^5*e^4*x^4 + 16*d^6*e^3*x^3 - 16*d^7*e^2*x^2 - 8*d^8*e*x + 8*d^9)*sqrt(-e^2*x^2 + d^2))$

**Sympy [A]** time = 23.6504, size = 337, normalized size = 3.74

$$d \left( \left\{ \begin{array}{l} \frac{2d^2}{15d^4e^4\sqrt{d^2-e^2x^2}-30d^2e^6x^2\sqrt{d^2-e^2x^2}+15e^8x^4\sqrt{d^2-e^2x^2}} + \frac{5e^2x^2}{15d^4e^4\sqrt{d^2-e^2x^2}-30d^2e^6x^2\sqrt{d^2-e^2x^2}+15e^8x^4\sqrt{d^2-e^2x^2}} \text{ for } e \neq 0 \\ \frac{x^4}{4(d^2)^{\frac{7}{2}}} \text{ otherwise} \end{array} \right. \right) + e \left( \left\{ \begin{array}{l} -\frac{ix^5}{5d^7\sqrt{-1+\frac{e^2x^2}{d^2}}-10d^5e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+5d^3e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} \text{ for } \left| \frac{e^2x^2}{d^2} \right| > 1 \\ \frac{x^5}{5d^7\sqrt{1-\frac{e^2x^2}{d^2}}-10d^5e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+5d^3e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} \text{ otherwise} \end{array} \right. \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)`

```
[Out] d*Piecewise((-2*d**2/(15*d**4*e**4*sqrt(d**2 - e**2*x**2) - 30*d**2*e**6*x**2*sqrt(d**2 - e**2*x**2) + 15*e**8*x**4*sqrt(d**2 - e**2*x**2)) + 5*e**2*x**2/(15*d**4*e**4*sqrt(d**2 - e**2*x**2) - 30*d**2*e**6*x**2*sqrt(d**2 - e**2*x**2) + 15*e**8*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**4/(4*(d**2)**(7/2)), True)) + e*Piecewise((-I*x**5/(5*d**7*sqrt(-1 + e**2*x**2/d**2) - 10*d**5*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 5*d**3*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (x**5/(5*d**7*sqrt(1 - e**2*x**2/d**2) - 10*d**5*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 5*d**3*e**4*x**4*sqrt(1 - e**2*x**2/d**2)), True))
```

**GIAC/XCAS [A]** time = 0.301329, size = 78, normalized size = 0.87

$$\frac{\left(2d^3e^{(-4)} - \left(\frac{3x^3e}{d^2} + 5de^{(-2)}\right)x^2\right)\sqrt{-x^2e^2 + d^2}}{15(x^2e^2 - d^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)*x^3/(-e^2*x^2 + d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(2*d^3*e^(-4) - (3*x^3*e/d^2 + 5*d*e^(-2))*x^2)*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

$$3.24 \quad \int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=94

$$\frac{x^2(d+ex)}{5de(d^2-e^2x^2)^{5/2}} - \frac{2(d-ex)}{15de^3(d^2-e^2x^2)^{3/2}} - \frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}$$

[Out] (x^2\*(d + e\*x))/(5\*d\*e\*(d^2 - e^2\*x^2)^(5/2)) - (2\*(d - e\*x))/(15\*d\*e^3\*(d^2 - e^2\*x^2)^(3/2)) - (2\*x)/(15\*d^3\*e^2\*sqrt[d^2 - e^2\*x^2])

**Rubi [A]** time = 0.170861, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$\frac{x^2(d+ex)}{5de(d^2-e^2x^2)^{5/2}} - \frac{2(d-ex)}{15de^3(d^2-e^2x^2)^{3/2}} - \frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(d + e\*x))/(d^2 - e^2\*x^2)^(7/2), x]

[Out] (x^2\*(d + e\*x))/(5\*d\*e\*(d^2 - e^2\*x^2)^(5/2)) - (2\*(d - e\*x))/(15\*d\*e^3\*(d^2 - e^2\*x^2)^(3/2)) - (2\*x)/(15\*d^3\*e^2\*sqrt[d^2 - e^2\*x^2])

**Rubi in Sympy [A]** time = 16.9699, size = 80, normalized size = 0.85

$$\frac{x^2(d+ex)}{5de(d^2-e^2x^2)^{5/2}} - \frac{2d-2ex}{15de^3(d^2-e^2x^2)^{3/2}} - \frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out] x\*\*2\*(d + e\*x)/(5\*d\*e\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)) - (2\*d - 2\*e\*x)/(15\*d\*e\*\*3\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)) - 2\*x/(15\*d\*\*3\*e\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2))

**Mathematica [A]** time = 0.058219, size = 82, normalized size = 0.87

$$\frac{\sqrt{d^2 - e^2x^2} (-2d^4 + 2d^3ex + 3d^2e^2x^2 + 2de^3x^3 - 2e^4x^4)}{15d^3e^3(d - ex)^3(d + ex)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(d + e\*x))/(d^2 - e^2\*x^2)^(7/2), x]

[Out] (sqrt[d^2 - e^2\*x^2]\*(-2\*d^4 + 2\*d^3\*e\*x + 3\*d^2\*e^2\*x^2 + 2\*d\*e^3\*x^3 - 2\*e^4\*x^4))/(15\*d^3\*e^3\*(d - e\*x)^3\*(d + e\*x)^2)

**Maple [A]** time = 0.012, size = 77, normalized size = 0.8

$$-\frac{(-ex+d)(ex+d)^2(2e^4x^4-2x^3de^3-3x^2d^2e^2-2xd^3e+2d^4)}{15d^3e^3}(-e^2x^2+d^2)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)`

[Out] `-1/15*(-e*x+d)*(e*x+d)^2*(2*e^4*x^4-2*d*e^3*x^3-3*d^2*e^2*x^2-2*d^3*e*x+2*d^4)/d^3/e^3/(-e^2*x^2+d^2)^(7/2)`

**Maxima [A]** time = 0.725597, size = 151, normalized size = 1.61

$$\frac{x^2}{3(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{dx}{5(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{2d^2}{15(-e^2x^2+d^2)^{\frac{5}{2}}e^3} - \frac{x}{15(-e^2x^2+d^2)^{\frac{3}{2}}de^2} - \frac{2x}{15\sqrt{-e^2x^2+d^2}d^3e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*x^2/(-e^2*x^2+d^2)^(7/2),x,algorithm="maxima")`

[Out] `1/3*x^2/((-e^2*x^2+d^2)^(5/2)*e)+1/5*d*x/((-e^2*x^2+d^2)^(5/2)*e^2)-2/15*d^2/((-e^2*x^2+d^2)^(5/2)*e^3)-1/15*x/((-e^2*x^2+d^2)^(3/2)*d*e^2)-2/15*x/(sqrt(-e^2*x^2+d^2)*d^3*e^2)`

**Fricas [A]** time = 0.283243, size = 398, normalized size = 4.23

$$\frac{2e^5x^8-10de^4x^7-11d^2e^3x^6+46d^3e^2x^5+10d^4ex^4-40d^5x^3+2(e^4x^7+3de^3x^6-13d^2e^2x^5+4d^4e^7x^7-4d^5e^6x^6-16d^6e^5x^5+16d^7e^4x^4+20d^8e^3x^3-20d^9e^2x^2-8d^{10}ex+8d^{11}-(d^3e^7x^7-d^4e^6x^6-9d^5e^5x^5+15d^6e^4x^4+16d^7e^3x^3-20d^8e^2x^2-8d^9ex+8d^{10}))\sqrt{-e^2x^2+d^2}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*x^2/(-e^2*x^2+d^2)^(7/2),x,algorithm="fricas")`

[Out] `1/15*(2*e^5*x^8-10*d*e^4*x^7-11*d^2*e^3*x^6+46*d^3*e^2*x^5+10*d^4*e*x^4-40*d^5*x^3+2*(e^4*x^7+3*d*e^3*x^6-13*d^2*e^2*x^5+4*d^4*e^7*x^7-4*d^5*e^6*x^6-16*d^6*e^5*x^5+16*d^7*e^4*x^4+20*d^8*e^3*x^3-20*d^9*e^2*x^2-8*d^10*e*x+8*d^11-(d^3*e^7*x^7-d^4*e^6*x^6-9*d^5*e^5*x^5+15*d^6*e^4*x^4+16*d^7*e^3*x^3-20*d^8*e^2*x^2-8*d^9*e*x+8*d^10))*sqrt(-e^2*x^2+d^2))/(4*d^4*e^7*x^7-4*d^5*e^6*x^6-16*d^6*e^5*x^5+16*d^7*e^4*x^4+20*d^8*e^3*x^3-20*d^9*e^2*x^2-8*d^10*e*x+8*d^11-(d^3*e^7*x^7-d^4*e^6*x^6-9*d^5*e^5*x^5+15*d^6*e^4*x^4+16*d^7*e^3*x^3-20*d^8*e^2*x^2-8*d^9*e*x+8*d^10))*sqrt(-e^2*x^2+d^2)`

**Sympy [A]** time = 24.2424, size = 513, normalized size = 5.46

$$d \left( \begin{cases} -\frac{5id^2x^3}{15d^9\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{2ie^2x^5}{15d^9\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left| \frac{e^2x^2}{d^2} \right| > 1 \\ \frac{5d^2x^3}{15d^9\sqrt{1-\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{2e^2x^5}{15d^9\sqrt{1-\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases} \right) + e \left( \begin{cases} -\frac{2d^2}{15d^4e^4\sqrt{d^2-e^2x^2}-30d^2e^6x^2\sqrt{d^2-e^2x^2}+15e^8x^4\sqrt{d^2-e^2x^2}} + \frac{5e^2x^2}{15d^4e^4\sqrt{d^2-e^2x^2}-30d^2e^6x^2\sqrt{d^2-e^2x^2}+15e^8x^4\sqrt{d^2-e^2x^2}} & \text{for } e \neq 0 \\ \frac{x^4}{4(d^2)^{\frac{7}{2}}} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2),x)

[Out] d\*Piecewise((-5\*I\*d\*\*2\*x\*\*3/(15\*d\*\*9\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 30\*d\*\*7\*e\*\*2\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 15\*d\*\*5\*e\*\*4\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + 2\*I\*e\*\*2\*x\*\*5/(15\*d\*\*9\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 30\*d\*\*7\*e\*\*2\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 15\*d\*\*5\*e\*\*4\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (5\*d\*\*2\*x\*\*3/(15\*d\*\*9\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 30\*d\*\*7\*e\*\*2\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 15\*d\*\*5\*e\*\*4\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - 2\*e\*\*2\*x\*\*5/(15\*d\*\*9\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 30\*d\*\*7\*e\*\*2\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 15\*d\*\*5\*e\*\*4\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True)) + e\*Piecewise((-2\*d\*\*2/(15\*d\*\*4\*e\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) - 30\*d\*\*2\*e\*\*6\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) + 15\*e\*\*8\*x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) + 5\*e\*\*2\*x\*\*2/(15\*d\*\*4\*e\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) - 30\*d\*\*2\*e\*\*6\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) + 15\*e\*\*8\*x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2))), Ne(e, 0)), (x\*\*4/(4\*(d\*\*2)\*\*(7/2)), True))

---

**GIAC/XCAS [A]** time = 0.295831, size = 86, normalized size = 0.91

$$\frac{\left(\left(x\left(\frac{2x^2e^2}{d^3} - \frac{5}{d}\right) - 5e^{(-1)}\right)x^2 + 2d^2e^{(-3)}\right)\sqrt{-x^2e^2 + d^2}}{15(x^2e^2 - d^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*x^2/(-e^2\*x^2 + d^2)^(7/2),x, algorithm="giac")

[Out] 1/15\*((x\*(2\*x^2\*e^2/d^3 - 5/d) - 5\*e^(-1))\*x^2 + 2\*d^2\*e^(-3))\*sqrt(-x^2\*e^2 + d^2)/(x^2\*e^2 - d^2)^3



$$3.25 \quad \int \frac{x(d+ex)}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=83

$$-\frac{x}{15d^2e(d^2-e^2x^2)^{3/2}} + \frac{d+ex}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}$$

[Out]  $(d + e*x)/(5*e^2*(d^2 - e^2*x^2)^{(5/2)}) - x/(15*d^2*e*(d^2 - e^2*x^2)^{(3/2)}) - (2*x)/(15*d^4*e*\text{Sqrt}[d^2 - e^2*x^2])$

**Rubi [A]** time = 0.0864293, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$

$$-\frac{x}{15d^2e(d^2-e^2x^2)^{3/2}} + \frac{d+ex}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(x\*(d + e\*x))/(d^2 - e^2\*x^2)^(7/2), x]

[Out]  $(d + e*x)/(5*e^2*(d^2 - e^2*x^2)^{(5/2)}) - x/(15*d^2*e*(d^2 - e^2*x^2)^{(3/2)}) - (2*x)/(15*d^4*e*\text{Sqrt}[d^2 - e^2*x^2])$

**Rubi in Sympy [A]** time = 8.34353, size = 68, normalized size = 0.82

$$\frac{d+ex}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{x}{15d^2e(d^2-e^2x^2)^{3/2}} - \frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out]  $(d + e*x)/(5*e**2*(d**2 - e**2*x**2)**(5/2)) - x/(15*d**2*e*(d**2 - e**2*x**2)**(3/2)) - 2*x/(15*d**4*e*\text{sqrt}(d**2 - e**2*x**2))$

**Mathematica [A]** time = 0.0512878, size = 82, normalized size = 0.99

$$\frac{\sqrt{d^2 - e^2x^2} (3d^4 - 3d^3ex + 3d^2e^2x^2 + 2de^3x^3 - 2e^4x^4)}{15d^4e^2(d - ex)^3(d + ex)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(d + e\*x))/(d^2 - e^2\*x^2)^(7/2), x]

[Out]  $(\text{Sqrt}[d^2 - e^2*x^2]*(3*d^4 - 3*d^3*e*x + 3*d^2*e^2*x^2 + 2*d*e^3*x^3 - 2*e^4*x^4))/(15*d^4*e^2*(d - e*x)^3*(d + e*x)^2)$

**Maple [A]** time = 0.013, size = 77, normalized size = 0.9

$$\frac{(-ex + d)(ex + d)^2 (-2e^4x^4 + 2e^3x^3d + 3x^2d^2e^2 - 3xd^3e + 3d^4)}{15d^4e^2} (-e^2x^2 + d^2)^{-7/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x+d)/(-e^2*x^2+d^2)^(7/2),x)
```

```
[Out] 1/15*(-e*x+d)*(e*x+d)^2*(-2*e^4*x^4+2*d*e^3*x^3+3*d^2*e^2*x^2-3*d^3*e*x+3*d^4)/d^4/e^2/(-e^2*x^2+d^2)^(7/2)
```

**Maxima [A]** time = 0.71483, size = 117, normalized size = 1.41

$$\frac{x}{5(-e^2x^2 + d^2)^{\frac{5}{2}}e} + \frac{d}{5(-e^2x^2 + d^2)^{\frac{5}{2}}e^2} - \frac{x}{15(-e^2x^2 + d^2)^{\frac{3}{2}}d^2e} - \frac{2x}{15\sqrt{-e^2x^2 + d^2}d^4e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)*x/(-e^2*x^2 + d^2)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*x/((-e^2*x^2 + d^2)^(5/2)*e) + 1/5*d/((-e^2*x^2 + d^2)^(5/2)*e^2) - 1/15*x/((-e^2*x^2 + d^2)^(3/2)*d^2*e) - 2/15*x/(sqrt(-e^2*x^2 + d^2)*d^4*e)
```

**Fricas [A]** time = 0.284016, size = 429, normalized size = 5.17

$$\frac{2e^6x^8 + 10de^5x^7 - 31d^2e^4x^6 - 29d^3e^3x^5 + 85d^4e^2x^4 + 20d^5ex^3 - 60d^6x^2 - (3e^5x^7 - 11de^4x^6 - 19d^2e^3x^5 + 15(4d^5e^7x^7 - 4d^6e^6x^6 - 16d^7e^5x^5 + 16d^8e^4x^4 + 20d^9e^3x^3 - 20d^{10}e^2x^2 - 8d^{11}ex + 8d^{12} - (d^4e^7x^7 - d^5e^6x^6 - 9d^6e^5x^5 + 8d^7e^4x^4 + 16d^8e^3x^3 - 16d^9e^2x^2 - 8d^{10}ex + 8d^{11}))\sqrt{-e^2x^2 + d^2}}{15(4d^5e^7x^7 - 4d^6e^6x^6 - 16d^7e^5x^5 + 16d^8e^4x^4 + 20d^9e^3x^3 - 20d^{10}e^2x^2 - 8d^{11}ex + 8d^{12} - (d^4e^7x^7 - d^5e^6x^6 - 9d^6e^5x^5 + 8d^7e^4x^4 + 16d^8e^3x^3 - 16d^9e^2x^2 - 8d^{10}ex + 8d^{11}))\sqrt{-e^2x^2 + d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)*x/(-e^2*x^2 + d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*e^6*x^8 + 10*d*e^5*x^7 - 31*d^2*e^4*x^6 - 29*d^3*e^3*x^5 + 85*d^4*e^2*x^4 + 20*d^5*e*x^3 - 60*d^6*x^2 - (3*e^5*x^7 - 11*d*e^4*x^6 - 19*d^2*e^3*x^5 + 55*d^3*e^2*x^4 + 20*d^4*e*x^3 - 60*d^5*x^2)*sqrt(-e^2*x^2 + d^2))/(4*d^5*e^7*x^7 - 4*d^6*e^6*x^6 - 16*d^7*e^5*x^5 + 16*d^8*e^4*x^4 + 20*d^9*e^3*x^3 - 20*d^10*e^2*x^2 - 8*d^11*e*x + 8*d^12 - (d^4*e^7*x^7 - d^5*e^6*x^6 - 9*d^6*e^5*x^5 + 9*d^7*e^4*x^4 + 16*d^8*e^3*x^3 - 16*d^9*e^2*x^2 - 8*d^10*e*x + 8*d^11)*sqrt(-e^2*x^2 + d^2))
```

**Sympy [A]** time = 23.6193, size = 432, normalized size = 5.2

$$d \left( \begin{cases} \frac{1}{5d^4e^2\sqrt{d^2-e^2x^2}-10d^2e^4x^2\sqrt{d^2-e^2x^2}+5e^6x^4\sqrt{d^2-e^2x^2}} & \text{for } e \neq 0 \\ \frac{x^2}{2(d^2)^{\frac{7}{2}}} & \text{otherwise} \end{cases} \right) + e \left( \begin{cases} -\frac{5id^2x^3}{15d^9\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{2ie^2x^5}{15d^9\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left| \frac{e^2x^2}{d^2} \right| > 1 \\ \frac{5d^2x^3}{15d^9\sqrt{1-\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{2e^2x^5}{15d^9\sqrt{1-\frac{e^2x^2}{d^2}}-30d^7e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^5e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] d*Piecewise((1/(5*d**4*e**2*sqrt(d**2 - e**2*x**2) - 10*d**2*e**4*x**2*sqrt(d**2 - e**2*x**2) + 5*e**6*x**4*sqrt(d**2 - e**2*x**2)), Ne(e, 0)), (x**2/(2*(d**2)**(7/2)), True)) + e*Piecewise((-5*I
```

```

*d**2*x**3/(15*d**9*sqrt(-1 + e**2*x**2/d**2) - 30*d**7*e**2*x**2
*sqrt(-1 + e**2*x**2/d**2) + 15*d**5*e**4*x**4*sqrt(-1 + e**2*x**
2/d**2)) + 2*I*e**2*x**5/(15*d**9*sqrt(-1 + e**2*x**2/d**2) - 30*
d**7*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**5*e**4*x**4*sqrt
(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**2*x**3/(1
5*d**9*sqrt(1 - e**2*x**2/d**2) - 30*d**7*e**2*x**2*sqrt(1 - e**2
*x**2/d**2) + 15*d**5*e**4*x**4*sqrt(1 - e**2*x**2/d**2)) - 2*e**
2*x**5/(15*d**9*sqrt(1 - e**2*x**2/d**2) - 30*d**7*e**2*x**2*sqrt
(1 - e**2*x**2/d**2) + 15*d**5*e**4*x**4*sqrt(1 - e**2*x**2/d**2)
), True))

```

---

**GIAC/XCAS [A]** time = 0.300139, size = 77, normalized size = 0.93

$$\frac{\left(x^3\left(\frac{2x^2e^3}{d^4} - \frac{5e}{d^2}\right) - 3de^{(-2)}\right)\sqrt{-x^2e^2 + d^2}}{15(x^2e^2 - d^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*x/(-e^2\*x^2 + d^2)^(7/2),x, algorithm="giac")

[Out] 1/15\*(x^3\*(2\*x^2\*e^3/d^4 - 5\*e/d^2) - 3\*d\*e^(-2))\*sqrt(-x^2\*e^2 + d^2)/(x^2\*e^2 - d^2)^3

$$3.26 \quad \int \frac{d+ex}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=80

$$\frac{d+ex}{5de(d^2-e^2x^2)^{5/2}} + \frac{8x}{15d^5\sqrt{d^2-e^2x^2}} + \frac{4x}{15d^3(d^2-e^2x^2)^{3/2}}$$

[Out]  $(d + e*x)/(5*d*e*(d^2 - e^2*x^2)^{(5/2)}) + (4*x)/(15*d^3*(d^2 - e^2*x^2)^{(3/2)}) + (8*x)/(15*d^5*\text{Sqrt}[d^2 - e^2*x^2])$

**Rubi [A]** time = 0.0530993, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$\frac{d+ex}{5de(d^2-e^2x^2)^{5/2}} + \frac{8x}{15d^5\sqrt{d^2-e^2x^2}} + \frac{4x}{15d^3(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)/(d^2 - e^2\*x^2)^(7/2), x]

[Out]  $(d + e*x)/(5*d*e*(d^2 - e^2*x^2)^{(5/2)}) + (4*x)/(15*d^3*(d^2 - e^2*x^2)^{(3/2)}) + (8*x)/(15*d^5*\text{Sqrt}[d^2 - e^2*x^2])$

**Rubi in Sympy [A]** time = 7.79355, size = 66, normalized size = 0.82

$$\frac{d+ex}{5de(d^2-e^2x^2)^{5/2}} + \frac{4x}{15d^3(d^2-e^2x^2)^{3/2}} + \frac{8x}{15d^5\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out]  $(d + e*x)/(5*d*e*(d**2 - e**2*x**2)**(5/2)) + 4*x/(15*d**3*(d**2 - e**2*x**2)**(3/2)) + 8*x/(15*d**5*\text{sqrt}(d**2 - e**2*x**2))$

**Mathematica [A]** time = 0.0456302, size = 82, normalized size = 1.02

$$\frac{\sqrt{d^2 - e^2x^2} (3d^4 + 12d^3ex - 12d^2e^2x^2 - 8de^3x^3 + 8e^4x^4)}{15d^5e(d - ex)^3(d + ex)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)/(d^2 - e^2\*x^2)^(7/2), x]

[Out]  $(\text{Sqrt}[d^2 - e^2*x^2]*(3*d^4 + 12*d^3*e*x - 12*d^2*e^2*x^2 - 8*d*e^3*x^3 + 8*e^4*x^4))/(15*d^5*e*(d - e*x)^3*(d + e*x)^2)$

**Maple [A]** time = 0.012, size = 77, normalized size = 1.

$$\frac{(-ex + d)(ex + d)^2 (8e^4x^4 - 8e^3x^3d - 12e^2x^2d^2 + 12xd^3e + 3d^4)}{15d^5e} (-e^2x^2 + d^2)^{-7/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)/(-e^2*x^2+d^2)^(7/2),x)`

[Out]  $1/15*(-e*x+d)*(e*x+d)^2*(8*e^4*x^4-8*d*e^3*x^3-12*d^2*e^2*x^2+12*d^3*e*x+3*d^4)/d^5/e/(-e^2*x^2+d^2)^(7/2)$

**Maxima [A]** time = 0.713501, size = 108, normalized size = 1.35

$$\frac{x}{5(-e^2x^2 + d^2)^{\frac{5}{2}}d} + \frac{1}{5(-e^2x^2 + d^2)^{\frac{5}{2}}e} + \frac{4x}{15(-e^2x^2 + d^2)^{\frac{3}{2}}d^3} + \frac{8x}{15\sqrt{-e^2x^2 + d^2}d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)/(-e^2*x^2 + d^2)^(7/2),x, algorithm="maxima")`

[Out]  $1/5*x/((-e^2*x^2 + d^2)^(5/2)*d) + 1/5/((-e^2*x^2 + d^2)^(5/2)*e) + 4/15*x/((-e^2*x^2 + d^2)^(3/2)*d^3) + 8/15*x/(sqrt(-e^2*x^2 + d^2)*d^5)$

**Fricas [A]** time = 0.286452, size = 452, normalized size = 5.65

$$\frac{8e^7x^8 - 20de^6x^7 - 64d^2e^5x^6 + 124d^3e^4x^5 + 115d^4e^3x^4 - 220d^5e^2x^3 - 60d^6ex^2 + 120d^7x + (3e^6x^7 + 29de^5x^6 - 59d^2e^4x^5 - 85d^3e^3x^4 + 160d^4e^2x^3 + 60d^5ex^2 - 120d^6x)}{15(4d^6e^7x^7 - 4d^7e^6x^6 - 16d^8e^5x^5 + 16d^9e^4x^4 + 20d^{10}e^3x^3 - 20d^{11}e^2x^2 - 8d^{12}ex + 8d^{13} - (d^5e^7x^7 - d^6e^6x^6 - 9d^7e^5x^5 - 85d^8e^4x^4 + 160d^9e^3x^3 + 60d^{10}e^2x^2 - 120d^{11}ex + 120d^{12}))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)/(-e^2*x^2 + d^2)^(7/2),x, algorithm="fricas")`

[Out]  $-1/15*(8*e^7*x^8 - 20*d*e^6*x^7 - 64*d^2*e^5*x^6 + 124*d^3*e^4*x^5 + 115*d^4*e^3*x^4 - 220*d^5*e^2*x^3 - 60*d^6*e*x^2 + 120*d^7*x + (3*e^6*x^7 + 29*d*e^5*x^6 - 59*d^2*e^4*x^5 - 85*d^3*e^3*x^4 + 160*d^4*e^2*x^3 + 60*d^5*e*x^2 - 120*d^6*x)*sqrt(-e^2*x^2 + d^2))/((4*d^6*e^7*x^7 - 4*d^7*e^6*x^6 - 16*d^8*e^5*x^5 + 16*d^9*e^4*x^4 + 20*d^{10}*e^3*x^3 - 20*d^{11}*e^2*x^2 - 8*d^{12}*e*x + 8*d^{13} - (d^5*e^7*x^7 - d^6*e^6*x^6 - 9*d^7*e^5*x^5 + 9*d^8*e^4*x^4 + 16*d^9*e^3*x^3 - 16*d^{10}*e^2*x^2 - 8*d^{11}*e*x + 8*d^{12})*sqrt(-e^2*x^2 + d^2))$

**Sympy [A]** time = 24.0965, size = 604, normalized size = 7.55

$$d \left( \begin{aligned} & -\frac{15id^4x}{15d^{11}\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^7e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{20id^2e^2x^3}{15d^{11}\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^7e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{1}{15d^{11}\sqrt{-1+\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{-1+\frac{e^2x^2}{d^2}}+15d^7e^4x^4\sqrt{-1+\frac{e^2x^2}{d^2}}} \\ & \frac{15d^4x}{15d^{11}\sqrt{1-\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^7e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{20d^2e^2x^3}{15d^{11}\sqrt{1-\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^7e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{8e^4x^3}{15d^{11}\sqrt{1-\frac{e^2x^2}{d^2}}-30d^9e^2x^2\sqrt{1-\frac{e^2x^2}{d^2}}+15d^7e^4x^4\sqrt{1-\frac{e^2x^2}{d^2}}} \end{aligned} \right) + e \left( \begin{aligned} & \frac{1}{5d^4e^2\sqrt{d^2-e^2x^2}-10d^2e^4x^2\sqrt{d^2-e^2x^2}+5e^6x^4\sqrt{d^2-e^2x^2}} \quad \text{for } e \neq 0 \\ & \frac{x^2}{2(d^2)^{\frac{7}{2}}} \quad \text{otherwise} \end{aligned} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)/(-e**2*x**2+d**2)**(7/2),x)`

[Out]  $d*\text{Piecewise}((-15*I*d**4*x/(15*d**11*sqrt(-1 + e**2*x**2/d**2)) - 30*d**9*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) + 15*d**7*e**4*x**4*sq$

```

rt(-1 + e**2*x**2/d**2)) + 20*I*d**2*e**2*x**3/(15*d**11*sqrt(-1
+ e**2*x**2/d**2) - 30*d**9*e**2*x**2*sqrt(-1 + e**2*x**2/d**2) +
15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)) - 8*I*e**4*x**5/(15
*d**11*sqrt(-1 + e**2*x**2/d**2) - 30*d**9*e**2*x**2*sqrt(-1 + e
**2*x**2/d**2) + 15*d**7*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)), Abs
(e**2*x**2/d**2) > 1), (15*d**4*x/(15*d**11*sqrt(1 - e**2*x**2/d
**2) - 30*d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2) + 15*d**7*e**4*x
**4*sqrt(1 - e**2*x**2/d**2)) - 20*d**2*e**2*x**3/(15*d**11*sqrt(
1 - e**2*x**2/d**2) - 30*d**9*e**2*x**2*sqrt(1 - e**2*x**2/d**2)
+ 15*d**7*e**4*x**4*sqrt(1 - e**2*x**2/d**2)) + 8*e**4*x**5/(15*d
**11*sqrt(1 - e**2*x**2/d**2) - 30*d**9*e**2*x**2*sqrt(1 - e**2*x
**2/d**2) + 15*d**7*e**4*x**4*sqrt(1 - e**2*x**2/d**2)), True)) +
e*Piecewise((1/(5*d**4*e**2*sqrt(d**2 - e**2*x**2) - 10*d**2*e**
4*x**2*sqrt(d**2 - e**2*x**2) + 5*e**6*x**4*sqrt(d**2 - e**2*x**2
)), Ne(e, 0)), (x**2/(2*(d**2)**(7/2)), True))

```

**GIAC/XCAS [A]** time = 0.298916, size = 88, normalized size = 1.1

$$\frac{\sqrt{-x^2e^2 + d^2} \left( 4x^2 \left( \frac{2x^2e^4}{d^5} - \frac{5e^2}{d^3} \right) + \frac{15}{d} \right) x + 3e^{(-1)}}{15(x^2e^2 - d^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)/(-e^2\*x^2 + d^2)^(7/2),x, algorithm="giac")

[Out] -1/15\*sqrt(-x^2\*e^2 + d^2)\*((4\*x^2\*(2\*x^2\*e^4/d^5 - 5\*e^2/d^3) + 15/d)\*x + 3\*e^(-1))/(x^2\*e^2 - d^2)^3

$$3.27 \quad \int \frac{d+ex}{x(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=117

$$\frac{d+ex}{5d^2(d^2-e^2x^2)^{5/2}} + \frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6} + \frac{5d+4ex}{15d^4(d^2-e^2x^2)^{3/2}}$$

[Out] (d + e\*x)/(5\*d^2\*(d^2 - e^2\*x^2)^(5/2)) + (5\*d + 4\*e\*x)/(15\*d^4\*(d^2 - e^2\*x^2)^(3/2)) + (15\*d + 8\*e\*x)/(15\*d^6\*sqrt[d^2 - e^2\*x^2]) - ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]/d^6

**Rubi [A]** time = 0.314073, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{d+ex}{5d^2(d^2-e^2x^2)^{5/2}} + \frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6} + \frac{5d+4ex}{15d^4(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)/(x\*(d^2 - e^2\*x^2)^(7/2)),x]

[Out] (d + e\*x)/(5\*d^2\*(d^2 - e^2\*x^2)^(5/2)) + (5\*d + 4\*e\*x)/(15\*d^4\*(d^2 - e^2\*x^2)^(3/2)) + (15\*d + 8\*e\*x)/(15\*d^6\*sqrt[d^2 - e^2\*x^2]) - ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]/d^6

**Rubi in Sympy [A]** time = 44.7554, size = 97, normalized size = 0.83

$$\frac{d+ex}{5d^2(d^2-e^2x^2)^{5/2}} + \frac{5d+4ex}{15d^4(d^2-e^2x^2)^{3/2}} + \frac{15d+8ex}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)/x/((-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2)),x)

[Out] (d + e\*x)/(5\*d\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)) + (5\*d + 4\*e\*x)/(15\*d\*\*4\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)) + (15\*d + 8\*e\*x)/(15\*d\*\*6\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) - atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d)/d\*\*6

**Mathematica [A]** time = 0.101851, size = 106, normalized size = 0.91

$$\frac{-15 \log\left(\sqrt{d^2 - e^2x^2} + d\right) + \frac{\sqrt{d^2 - e^2x^2}(23d^4 - 8d^3ex - 27d^2e^2x^2 + 7de^3x^3 + 8e^4x^4)}{(d-ex)^3(d+ex)^2} + 15 \log(x)}{15d^6}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)/(x\*(d^2 - e^2\*x^2)^(7/2)),x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(23\*d^4 - 8\*d^3\*e\*x - 27\*d^2\*e^2\*x^2 + 7\*d\*e^3\*x^3 + 8\*e^4\*x^4))/((d - e\*x)^3\*(d + e\*x)^2) + 15\*Log[x] - 15\*Log[d + Sqrt[d^2 - e^2\*x^2]])/(15\*d^6)

---

**Maple [A]** time = 0.018, size = 163, normalized size = 1.4

$$\frac{ex}{5d^2}(-e^2x^2 + d^2)^{-\frac{5}{2}} + \frac{4ex}{15d^4}(-e^2x^2 + d^2)^{-\frac{3}{2}} + \frac{8ex}{15d^6} \frac{1}{\sqrt{-e^2x^2 + d^2}} + \frac{1}{5d}(-e^2x^2 + d^2)^{-\frac{5}{2}} \\ + \frac{1}{3d^3}(-e^2x^2 + d^2)^{-\frac{3}{2}} + \frac{1}{d^5} \frac{1}{\sqrt{-e^2x^2 + d^2}} - \frac{1}{d^5} \ln\left(\frac{1}{x} \left(2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)/x/(-e^2\*x^2+d^2)^(7/2), x)

[Out] 1/5\*e\*x/d^2/(-e^2\*x^2+d^2)^(5/2)+4/15\*e/d^4\*x/(-e^2\*x^2+d^2)^(3/2)+8/15\*e/d^6\*x/(-e^2\*x^2+d^2)^(1/2)+1/5/d/(-e^2\*x^2+d^2)^(5/2)+1/3/d^3/(-e^2\*x^2+d^2)^(3/2)+1/d^5/(-e^2\*x^2+d^2)^(1/2)-1/d^5/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)/((-e^2\*x^2 + d^2)^(7/2)\*x), x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.294215, size = 718, normalized size = 6.14

$$\frac{8e^8x^8 - 85de^7x^7 + d^2e^6x^6 + 304d^3e^5x^5 - 65d^4e^4x^4 - 340d^5e^3x^3 + 60d^6e^2x^2 + 120d^7ex - 15(4de^7x^7 - 4d^2e^6x^6 - 16d^3e^5x^5 + 16d^4e^4x^4 - 16d^5e^3x^3 + 16d^6e^2x^2 + 120d^7ex - 15(4d^7e^7x^7 - 4d^8e^6x^6 - 16d^9e^5x^5 + 16d^{10}e^4x^4 + 20d^{11}e^3x^3 - 20d^{12}e^2x^2 - 8d^{13}ex + 8d^{14} - (d^6e^7x^7 - d^7e^6x^6 - 9d^8e^5x^5 + 9d^9e^4x^4 + 16d^{10}e^3x^3 - 16d^{11}e^2x^2 - 8d^{12}ex + 8d^{13}))\sqrt{-e^2x^2 + d^2})}{15(4d^7e^7x^7 - 4d^8e^6x^6 - 16d^9e^5x^5 + 16d^{10}e^4x^4 + 20d^{11}e^3x^3 - 20d^{12}e^2x^2 - 8d^{13}ex + 8d^{14} - (d^6e^7x^7 - d^7e^6x^6 - 9d^8e^5x^5 + 9d^9e^4x^4 + 16d^{10}e^3x^3 - 16d^{11}e^2x^2 - 8d^{12}ex + 8d^{13}))\sqrt{-e^2x^2 + d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)/((-e^2\*x^2 + d^2)^(7/2)\*x), x, algorithm="fricas")

[Out] -1/15\*(8\*e^8\*x^8 - 85\*d\*e^7\*x^7 + d^2\*e^6\*x^6 + 304\*d^3\*e^5\*x^5 - 65\*d^4\*e^4\*x^4 - 340\*d^5\*e^3\*x^3 + 60\*d^6\*e^2\*x^2 + 120\*d^7\*e\*x - 15\*(4\*d^7\*e^7\*x^7 - 4\*d^8\*e^6\*x^6 - 16\*d^9\*e^5\*x^5 + 16\*d^10\*e^4\*x^4 + 20\*d^11\*e^3\*x^3 - 20\*d^12\*e^2\*x^2 - 8\*d^13\*e\*x + 8\*d^14 - (e^7\*x^7 - d\*e^6\*x^6 - 9\*d^2\*e^5\*x^5 + 9\*d^3\*e^4\*x^4 + 16\*d^4\*e^3\*x^3 - 16\*d^5\*e^2\*x^2 - 8\*d^6\*e\*x + 8\*d^7)\*sqrt(-e^2\*x^2 + d^2)))\*log(-(d - sqrt(-e^2\*x^2 + d^2))/x) + (23\*e^7\*x^7 + 9\*d\*e^6\*x^6 - 179\*d^2\*e^5\*x^5 + 35\*d^3\*e^4\*x^4 + 280\*d^4\*e^3\*x^3 - 60\*d^5\*e^2\*x^2 - 120\*d^6\*e\*x)\*sqrt(-e^2\*x^2 + d^2))/(4\*d^7\*e^7\*x^7 - 4\*d^8\*e^6\*x^6 - 16\*d^9\*e^5\*x^5 + 16\*d^10\*e^4\*x^4 + 20\*d^11\*e^3\*x^3 - 20\*d^12\*e^2\*x^2 - 8\*d^13\*e\*x + 8\*d^14 - (d^6\*e^7\*x^7 - d^7\*e^6\*x^6 - 9\*d^8\*e^5\*x^5 + 9\*d^9\*e^4\*x^4 + 16\*d^10\*e^3\*x^3 - 16\*d^11\*e^2\*x^2 - 8\*d^12\*e\*x + 8\*d^13)\*sqrt(-e^2\*x^2 + d^2))

---

**Sympy [A]** time = 36.5228, size = 2378, normalized size = 20.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((e\*x+d)/x/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2),x)

[Out] d\*Piecewise((-46\*I\*d\*\*6\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) - 15\*d\*\*6\*log(e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 30\*d\*\*6\*log(e\*x/d)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) - 30\*I\*d\*\*6\*asin(d/(e\*x))/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 70\*I\*d\*\*4\*e\*\*2\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 45\*d\*\*4\*e\*\*2\*x\*\*2\*log(e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) - 90\*d\*\*4\*e\*\*2\*x\*\*2\*log(e\*x/d)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 90\*I\*d\*\*4\*e\*\*2\*x\*\*2\*asin(d/(e\*x))/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) - 30\*I\*d\*\*2\*e\*\*4\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) - 45\*d\*\*2\*e\*\*4\*x\*\*4\*log(e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 90\*d\*\*2\*e\*\*4\*x\*\*4\*log(e\*x/d)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) - 90\*I\*d\*\*2\*e\*\*4\*x\*\*4\*asin(d/(e\*x))/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 15\*e\*\*6\*x\*\*6\*log(e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 30\*I\*e\*\*6\*x\*\*6\*asin(d/(e\*x))/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (-46\*d\*\*6\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) - 15\*d\*\*6\*log(e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 30\*d\*\*6\*log(sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 1)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) - 15\*I\*pi\*d\*\*6/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 70\*d\*\*4\*e\*\*2\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 45\*d\*\*4\*e\*\*2\*x\*\*2\*log(e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) - 90\*d\*\*4\*e\*\*2\*x\*\*2\*log(sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 1)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 45\*I\*pi\*d\*\*4\*e\*\*2\*x\*\*2/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) - 30\*d\*\*2\*e\*\*4\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) - 45\*d\*\*2\*e\*\*4\*x\*\*4\*log(e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 90\*d\*\*2\*e\*\*4\*x\*\*4\*log(sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 1)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) - 45\*I\*pi\*d\*\*2\*e\*\*4\*x\*\*4/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 15\*e\*\*6\*x\*\*6\*log(e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) - 30\*e\*\*6\*x\*\*6\*log(sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 1)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 15\*I\*pi\*e\*\*6\*x\*\*6/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6), True)) + e\*Piecewise((-15\*I\*d\*\*4\*x/(15\*d\*\*11\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 30\*d\*\*9\*e\*\*2\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 15\*d\*\*7\*e\*\*4\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + 20\*I\*d\*\*2\*e\*\*2\*x\*\*3/(15\*d\*\*11\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 30\*d\*\*9\*e\*\*2\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 15\*d\*\*7\*e\*\*4\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 8\*I\*e\*\*4\*x\*\*5/(15\*d\*\*11\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 30\*d\*\*9\*e\*\*2\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 15\*d\*\*7\*e\*\*4\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (15\*d\*\*4\*x/(15\*d\*\*11\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 30\*d\*\*9\*e\*\*2\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 15\*d\*\*7\*e\*\*4\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - 20\*d\*\*2\*e\*\*2\*x\*\*3/(15\*d\*\*11\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 30\*d\*\*9\*e\*\*2\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 15\*d\*\*7\*e\*\*4\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 8\*e\*\*4\*x\*\*5/(15\*d\*\*11\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 30\*d\*\*9\*e\*\*2\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 15\*d\*\*7\*e\*\*4\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True))

**GIAC/XCAS [A]** time = 0.292394, size = 165, normalized size = 1.41

$$\frac{\sqrt{-x^2e^2 + d^2} \left( \left( \left( x \left( \frac{8xe^5}{d^6} + \frac{15e^4}{d^5} \right) - \frac{20e^3}{d^4} \right) x - \frac{35e^2}{d^3} \right) x + \frac{15e}{d^2} \right) x + \frac{23}{d}}{15(x^2e^2 - d^2)^3} - \frac{\ln \left( \frac{|-2de^{-2}\sqrt{-x^2e^2+d^2}e|e^{(-2)}}{2|x|} \right)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)/((-e^2\*x^2 + d^2)^(7/2)\*x),x, algorithm="giac")

[Out] -1/15\*sqrt(-x^2\*e^2 + d^2)\*(((x\*(8\*x\*e^5/d^6 + 15\*e^4/d^5) - 20\*e^3/d^4)\*x - 35\*e^2/d^3)\*x + 15\*e/d^2)/((x^2\*e^2 - d^2)^3) - ln(1/2\*abs(-2\*d\*e - 2\*sqrt(-x^2\*e^2 + d^2)\*e)\*e^(-2)/abs(x))/d^6

$$3.28 \quad \int \frac{d+ex}{x^2(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=153

$$\frac{d+ex}{5d^2x(d^2-e^2x^2)^{5/2}} - \frac{16\sqrt{d^2-e^2x^2}}{5d^7x} - \frac{e \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7} + \frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}} + \frac{6d+5ex}{15d^4x(d^2-e^2x^2)^{3/2}}$$

[Out] (d + e\*x)/(5\*d^2\*x\*(d^2 - e^2\*x^2)^(5/2)) + (6\*d + 5\*e\*x)/(15\*d^4\*x\*(d^2 - e^2\*x^2)^(3/2)) + (8\*d + 5\*e\*x)/(5\*d^6\*x\*Sqrt[d^2 - e^2\*x^2]) - (16\*Sqrt[d^2 - e^2\*x^2])/(5\*d^7\*x) - (e\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/d^7

**Rubi [A]** time = 0.402593, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{d+ex}{5d^2x(d^2-e^2x^2)^{5/2}} - \frac{16\sqrt{d^2-e^2x^2}}{5d^7x} - \frac{e \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7} + \frac{8d+5ex}{5d^6x\sqrt{d^2-e^2x^2}} + \frac{6d+5ex}{15d^4x(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)/(x^2\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out] (d + e\*x)/(5\*d^2\*x\*(d^2 - e^2\*x^2)^(5/2)) + (6\*d + 5\*e\*x)/(15\*d^4\*x\*(d^2 - e^2\*x^2)^(3/2)) + (8\*d + 5\*e\*x)/(5\*d^6\*x\*Sqrt[d^2 - e^2\*x^2]) - (16\*Sqrt[d^2 - e^2\*x^2])/(5\*d^7\*x) - (e\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/d^7

**Rubi in Sympy [A]** time = 52.7982, size = 126, normalized size = 0.82

$$\frac{d+ex}{5d^2x(d^2-e^2x^2)^{5/2}} + \frac{6d+5ex}{15d^4x(d^2-e^2x^2)^{3/2}} + \frac{24d+15ex}{15d^6x\sqrt{d^2-e^2x^2}} - \frac{e \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7} - \frac{16\sqrt{d^2-e^2x^2}}{5d^7x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)/x\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out] (d + e\*x)/(5\*d\*\*2\*x\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)) + (6\*d + 5\*e\*x)/(15\*d\*\*4\*x\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)) + (24\*d + 15\*e\*x)/(15\*d\*\*6\*x\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) - e\*atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d)/d\*\*7 - 16\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(5\*d\*\*7\*x)

**Mathematica [A]** time = 0.11847, size = 123, normalized size = 0.8

$$\frac{-15e \log\left(\sqrt{d^2-e^2x^2}+d\right) + \frac{\sqrt{d^2-e^2x^2}(15d^5-38d^4ex-52d^3e^2x^2+87d^2e^3x^3+33de^4x^4-48e^5x^5)}{x(ex-d)^3(d+ex)^2} + 15e \log(x)}{15d^7}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)/(x^2\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(15\*d^5 - 38\*d^4\*e\*x - 52\*d^3\*e^2\*x^2 + 87\*d^2\*e^3\*x^3 + 33\*d\*e^4\*x^4 - 48\*e^5\*x^5))/(x\*(-d + e\*x)^3\*(d + e

$$x)^2) + 15 \cdot e \cdot \text{Log}[x] - 15 \cdot e \cdot \text{Log}[d + \text{Sqrt}[d^2 - e^2 \cdot x^2]] / (15 \cdot d^7)$$

**Maple [A]** time = 0.02, size = 195, normalized size = 1.3

$$\begin{aligned} & -\frac{1}{dx} (-e^2 x^2 + d^2)^{-\frac{5}{2}} + \frac{6 e^2 x}{5 d^3} (-e^2 x^2 + d^2)^{-\frac{5}{2}} + \frac{8 e^2 x}{5 d^5} (-e^2 x^2 + d^2)^{-\frac{3}{2}} \\ & + \frac{16 e^2 x}{5 d^7} \frac{1}{\sqrt{-e^2 x^2 + d^2}} + \frac{e}{5 d^2} (-e^2 x^2 + d^2)^{-\frac{5}{2}} + \frac{e}{3 d^4} (-e^2 x^2 + d^2)^{-\frac{3}{2}} \\ & + \frac{e}{d^6} \frac{1}{\sqrt{-e^2 x^2 + d^2}} - \frac{e}{d^6} \ln\left(\frac{1}{x} \left(2 d^2 + 2 \sqrt{d^2} \sqrt{-e^2 x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)/x^2/(-e^2*x^2+d^2)^(7/2),x)`

[Out] `-1/d/x/(-e^2*x^2+d^2)^(5/2)+6/5*e^2/d^3*x/(-e^2*x^2+d^2)^(5/2)+8/5*e^2/d^5*x/(-e^2*x^2+d^2)^(3/2)+16/5*e^2/d^7*x/(-e^2*x^2+d^2)^(1/2)+1/5*e/d^2/(-e^2*x^2+d^2)^(5/2)+1/3*e/d^4/(-e^2*x^2+d^2)^(3/2)+e/d^6/(-e^2*x^2+d^2)^(1/2)-e/d^6/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)/((-e^2*x^2+d^2)^(7/2)*x^2),x,algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.29642, size = 875, normalized size = 5.72

$$23 e^{10} x^{10} + 217 d e^9 x^9 - 487 d^2 e^8 x^8 - 1073 d^3 e^7 x^7 + 1863 d^4 e^6 x^6 + 1755 d^5 e^5 x^5 - 2655 d^6 e^4 x^4 - 1140 d^7 e^3 x^3 + 1500 d^8 e^2 x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)/((-e^2*x^2+d^2)^(7/2)*x^2),x,algorithm="fricas")`

[Out] `1/15*(23*e^10*x^10 + 217*d*e^9*x^9 - 487*d^2*e^8*x^8 - 1073*d^3*e^7*x^7 + 1863*d^4*e^6*x^6 + 1755*d^5*e^5*x^5 - 2655*d^6*e^4*x^4 - 1140*d^7*e^3*x^3 + 1500*d^8*e^2*x^2 + 240*d^9*e*x - 240*d^10 + 15*(e^10*x^10 - d*e^9*x^9 - 14*d^2*e^8*x^8 + 14*d^3*e^7*x^7 + 41*d^4*e^6*x^6 - 41*d^5*e^5*x^5 - 44*d^6*e^4*x^4 + 44*d^7*e^3*x^3 + 16*d^8*e^2*x^2 - 16*d^9*e*x + (5*d*e^8*x^8 - 5*d^2*e^7*x^7 - 25*d^3*e^6*x^6 + 25*d^4*e^5*x^5 + 36*d^5*e^4*x^4 - 36*d^6*e^3*x^3 - 16*d^7*e^2*x^2 + 16*d^8*e*x)*sqrt(-e^2*x^2+d^2))*log(-(d-sqrt(-e^2*x^2+d^2))/x) - (48*e^9*x^9 - 148*d*e^8*x^8 - 548*d^2*e^7*x^7 + 1023*d^3*e^6*x^6 + 1275*d^4*e^5*x^5 - 1995*d^5*e^4*x^4 - 1020*d^6*e^3*x^3 + 1380*d^7*e^2*x^2 + 240*d^8*e*x - 240*d^9)*sqrt(-e^2*x^2+d^2))/(d^7*e^9*x^10 - d^8*e^8*x^9 - 14*d^9*e^7*x^8 + 14*d^10*e^6*x^7 + 41*d^11*e^5*x^6 - 41*d^12*e^4*x^5 - 44*d^13*e^3*x^4 + 44*d^14*e^2*x^3 + 16*d^15*e*x^2 - 16*d^16*x + (5*d^8*e^7*x^8 - 5*d^9*e^6*x^7 - 25*d^10*e^5*x^6 + 25*d^11*e^4*x^5 + 36*d^12*e^3*x^4`

$$x^4 - 36*d^{13}*e^2*x^3 - 16*d^{14}*e*x^2 + 16*d^{15}*x)*\text{sqrt}(-e^2*x^2 + d^2))$$

**Sympy [A]** time = 43.2663, size = 2404, normalized size = 15.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)/x\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2),x)

[Out] d\*Piecewise((5\*d\*\*6\*e\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)/(-5\*d\*\*14 + 15\*d\*\*12\*e\*\*2\*x\*\*2 - 15\*d\*\*10\*e\*\*4\*x\*\*4 + 5\*d\*\*8\*e\*\*6\*x\*\*6) - 30\*d\*\*4\*e\*\*3\*x\*\*2\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)/(-5\*d\*\*14 + 15\*d\*\*12\*e\*\*2\*x\*\*2 - 15\*d\*\*10\*e\*\*4\*x\*\*4 + 5\*d\*\*8\*e\*\*6\*x\*\*6) + 40\*d\*\*2\*e\*\*5\*x\*\*4\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)/(-5\*d\*\*14 + 15\*d\*\*12\*e\*\*2\*x\*\*2 - 15\*d\*\*10\*e\*\*4\*x\*\*4 + 5\*d\*\*8\*e\*\*6\*x\*\*6) - 16\*e\*\*7\*x\*\*6\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)/(-5\*d\*\*14 + 15\*d\*\*12\*e\*\*2\*x\*\*2 - 15\*d\*\*10\*e\*\*4\*x\*\*4 + 5\*d\*\*8\*e\*\*6\*x\*\*6), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (5\*I\*d\*\*6\*e\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)/(-5\*d\*\*14 + 15\*d\*\*12\*e\*\*2\*x\*\*2 - 15\*d\*\*10\*e\*\*4\*x\*\*4 + 5\*d\*\*8\*e\*\*6\*x\*\*6) - 30\*I\*d\*\*4\*e\*\*3\*x\*\*2\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)/(-5\*d\*\*14 + 15\*d\*\*12\*e\*\*2\*x\*\*2 - 15\*d\*\*10\*e\*\*4\*x\*\*4 + 5\*d\*\*8\*e\*\*6\*x\*\*6) + 40\*I\*d\*\*2\*e\*\*5\*x\*\*4\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)/(-5\*d\*\*14 + 15\*d\*\*12\*e\*\*2\*x\*\*2 - 15\*d\*\*10\*e\*\*4\*x\*\*4 + 5\*d\*\*8\*e\*\*6\*x\*\*6) - 16\*I\*e\*\*7\*x\*\*6\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)/(-5\*d\*\*14 + 15\*d\*\*12\*e\*\*2\*x\*\*2 - 15\*d\*\*10\*e\*\*4\*x\*\*4 + 5\*d\*\*8\*e\*\*6\*x\*\*6), True)) + e\*Piecewise((-46\*I\*d\*\*6\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) - 15\*d\*\*6\*log(e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 30\*d\*\*6\*log(e\*x/d)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) - 30\*I\*d\*\*6\*asin(d/(e\*x))/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 70\*I\*d\*\*4\*e\*\*2\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 45\*d\*\*4\*e\*\*2\*x\*\*2\*log(e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) - 90\*d\*\*4\*e\*\*2\*x\*\*2\*log(e\*x/d)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 90\*I\*d\*\*4\*e\*\*2\*x\*\*2\*asin(d/(e\*x))/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) - 30\*I\*d\*\*2\*e\*\*4\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) - 45\*d\*\*2\*e\*\*4\*x\*\*4\*log(e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 90\*d\*\*2\*e\*\*4\*x\*\*4\*log(e\*x/d)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) - 90\*I\*d\*\*2\*e\*\*4\*x\*\*4\*asin(d/(e\*x))/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 15\*e\*\*6\*x\*\*6\*log(e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) - 30\*e\*\*6\*x\*\*6\*log(e\*x/d)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 30\*I\*e\*\*6\*x\*\*6\*asin(d/(e\*x))/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (-46\*d\*\*6\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) - 15\*d\*\*6\*log(e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 30\*d\*\*6\*log(sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 1)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) - 15\*I\*pi\*d\*\*6/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 70\*d\*\*4\*e\*\*2\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 45\*d\*\*4\*e\*\*2\*x\*\*2\*log(e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) - 90\*d\*\*4\*e\*\*2\*x\*\*2\*log(sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 1)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 45\*I\*pi\*d\*\*4\*e\*\*2\*x\*\*2/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) - 30\*d\*\*2\*e\*\*4\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) - 45\*d\*\*2\*e\*\*4

\*x\*\*4\*log(e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 90\*d\*\*2\*e\*\*4\*x\*\*4\*log(sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 1)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) - 45\*I\*pi\*d\*\*2\*e\*\*4\*x\*\*4/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 15\*e\*\*6\*x\*\*6\*log(e\*\*2\*x\*\*2/d\*\*2)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) - 30\*e\*\*6\*x\*\*6\*log(sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 1)/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6) + 15\*I\*pi\*e\*\*6\*x\*\*6/(-30\*d\*\*13 + 90\*d\*\*11\*e\*\*2\*x\*\*2 - 90\*d\*\*9\*e\*\*4\*x\*\*4 + 30\*d\*\*7\*e\*\*6\*x\*\*6), True))

**GIAC/XCAS [A]** time = 0.300276, size = 255, normalized size = 1.67

$$\frac{\sqrt{-x^2e^2 + d^2} \left( \left( \left( 3 \left( x \left( \frac{11xe^6}{d^7} + \frac{5e^5}{d^6} \right) - \frac{25e^4}{d^5} \right) x - \frac{35e^3}{d^4} \right) x + \frac{45e^2}{d^3} \right) x + \frac{23e}{d^2} \right)}{15(x^2e^2 - d^2)^3} - \frac{e \ln \left( \frac{|-2de - 2\sqrt{-x^2e^2 + d^2}e| e^{(-2)}}{2|x|} \right)}{d^7} + \frac{xe^3}{2(de + \sqrt{-x^2e^2 + d^2}e)d^7} - \frac{(de + \sqrt{-x^2e^2 + d^2}e)e^{(-1)}}{2d^7x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)/((-e^2\*x^2 + d^2)^(7/2)\*x^2), x, algorithm="giac")

[Out] -1/15\*sqrt(-x^2\*e^2 + d^2)\*(((3\*(x\*(11\*x\*e^6/d^7 + 5\*e^5/d^6) - 25\*e^4/d^5)\*x - 35\*e^3/d^4)\*x + 45\*e^2/d^3)\*x + 23\*e/d^2)/(x^2\*e^2 - d^2)^3 - e\*ln(1/2\*abs(-2\*d\*e - 2\*sqrt(-x^2\*e^2 + d^2)\*e)\*e^(-2)/abs(x))/d^7 + 1/2\*x\*e^3/((d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*d^7) - 1/2\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*e^(-1)/(d^7\*x)

$$3.29 \quad \int \frac{d+ex}{x^3(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=184

$$\begin{aligned} & \frac{d+ex}{5d^2x^2(d^2-e^2x^2)^{5/2}} - \frac{16e\sqrt{d^2-e^2x^2}}{5d^8x} - \frac{7e^2 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^8} \\ & - \frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2} + \frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} + \frac{7d+6ex}{15d^4x^2(d^2-e^2x^2)^{3/2}} \end{aligned}$$

[Out] (d + e\*x)/(5\*d^2\*x^2\*(d^2 - e^2\*x^2)^(5/2)) + (7\*d + 6\*e\*x)/(15\*d^4\*x^2\*(d^2 - e^2\*x^2)^(3/2)) + (35\*d + 24\*e\*x)/(15\*d^6\*x^2\*Sqrt[d^2 - e^2\*x^2]) - (7\*Sqrt[d^2 - e^2\*x^2])/(2\*d^7\*x^2) - (16\*e\*Sqrt[d^2 - e^2\*x^2])/(5\*d^8\*x) - (7\*e^2\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/(2\*d^8)

**Rubi [A]** time = 0.500497, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\begin{aligned} & \frac{d+ex}{5d^2x^2(d^2-e^2x^2)^{5/2}} - \frac{16e\sqrt{d^2-e^2x^2}}{5d^8x} - \frac{7e^2 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^8} \\ & - \frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2} + \frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} + \frac{7d+6ex}{15d^4x^2(d^2-e^2x^2)^{3/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)/(x^3\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out] (d + e\*x)/(5\*d^2\*x^2\*(d^2 - e^2\*x^2)^(5/2)) + (7\*d + 6\*e\*x)/(15\*d^4\*x^2\*(d^2 - e^2\*x^2)^(3/2)) + (35\*d + 24\*e\*x)/(15\*d^6\*x^2\*Sqrt[d^2 - e^2\*x^2]) - (7\*Sqrt[d^2 - e^2\*x^2])/(2\*d^7\*x^2) - (16\*e\*Sqrt[d^2 - e^2\*x^2])/(5\*d^8\*x) - (7\*e^2\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/(2\*d^8)

**Rubi in Sympy [A]** time = 66.6412, size = 162, normalized size = 0.88

$$\begin{aligned} & \frac{d+ex}{5d^2x^2(d^2-e^2x^2)^{5/2}} + \frac{7d+6ex}{15d^4x^2(d^2-e^2x^2)^{3/2}} + \frac{35d+24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} \\ & - \frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2} - \frac{7e^2 \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^8} - \frac{16e\sqrt{d^2-e^2x^2}}{5d^8x} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)/x\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out] (d + e\*x)/(5\*d\*\*2\*x\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)) + (7\*d + 6\*e\*x)/(15\*d\*\*4\*x\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)) + (35\*d + 24\*e\*x)/(15\*d\*\*6\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) - 7\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(2\*d\*\*7\*x\*\*2) - 7\*e\*\*2\*atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d)/(2\*d\*\*8) - 16\*e\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(5\*d\*\*8\*x)

**Mathematica [A]** time = 0.132576, size = 138, normalized size = 0.75

$$-105e^2 \log\left(\sqrt{d^2 - e^2x^2} + d\right) + \frac{\sqrt{d^2 - e^2x^2}(15d^6 + 15d^5ex - 176d^4e^2x^2 - 4d^3e^3x^3 + 249d^2e^4x^4 - 9de^5x^5 - 96e^6x^6)}{x^2(ex-d)^3(d+ex)^2} + 105e^2 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)/(x^3\*(d^2 - e^2\*x^2)^(7/2)),x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(15\*d^6 + 15\*d^5\*e\*x - 176\*d^4\*e^2\*x^2 - 4\*d^3\*e^3\*x^3 + 249\*d^2\*e^4\*x^4 - 9\*d\*e^5\*x^5 - 96\*e^6\*x^6))/(x^2\*(-d + e\*x)^3\*(d + e\*x)^2) + 105\*e^2\*Log[x] - 105\*e^2\*Log[d + Sqrt[d^2 - e^2\*x^2]])/(30\*d^8)

**Maple [A]** time = 0.023, size = 227, normalized size = 1.2

$$-\frac{1}{2dx^2}(-e^2x^2 + d^2)^{-\frac{5}{2}} + \frac{7e^2}{10d^3}(-e^2x^2 + d^2)^{-\frac{5}{2}} + \frac{7e^2}{6d^5}(-e^2x^2 + d^2)^{-\frac{3}{2}} + \frac{7e^2}{2d^7} \frac{1}{\sqrt{-e^2x^2 + d^2}}$$

$$- \frac{7e^2}{2d^7} \ln\left(\frac{1}{x} \left(2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}} - \frac{e}{d^2x}(-e^2x^2 + d^2)^{-\frac{5}{2}}$$

$$+ \frac{6e^3x}{5d^4}(-e^2x^2 + d^2)^{-\frac{5}{2}} + \frac{8e^3x}{5d^6}(-e^2x^2 + d^2)^{-\frac{3}{2}} + \frac{16e^3x}{5d^8} \frac{1}{\sqrt{-e^2x^2 + d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)/x^3/(-e^2\*x^2+d^2)^(7/2),x)

[Out] -1/2/d/x^2/(-e^2\*x^2+d^2)^(5/2)+7/10\*e^2/d^3/(-e^2\*x^2+d^2)^(5/2)+7/6\*e^2/d^5/(-e^2\*x^2+d^2)^(3/2)+7/2\*e^2/d^7/(-e^2\*x^2+d^2)^(1/2)-7/2\*e^2/d^7/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)-e/d^2/x/(-e^2\*x^2+d^2)^(5/2)+6/5\*e^3/d^4\*x/(-e^2\*x^2+d^2)^(5/2)+8/5\*e^3/d^6\*x/(-e^2\*x^2+d^2)^(3/2)+16/5\*e^3/d^8\*x/(-e^2\*x^2+d^2)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)/((-e^2\*x^2 + d^2)^(7/2)\*x^3),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.319681, size = 1013, normalized size = 5.51

$$96e^{12}x^{12} - 687de^{11}x^{11} - 1281d^2e^{10}x^{10} + 4946d^3e^9x^9 + 4162d^4e^8x^8 - 11487d^5e^7x^7 - 6375d^6e^6x^6 + 11310d^7e^5x^5 + 5550d^8e^4x^4 - 4560d^9e^3x^3 - 2640d^{10}e^2x^2 + 480d^{11}e^1x + 480d^{12} - 105(6*d*e^{11}*x^{11} - 6*d^2*e^{10}*x^{10} - 44*d^3*e^9*x^9 + 44*d^4*e^8*x^8 + 102*d^5*e^7*x^7 - 102*d^6*e^6*x^6 - 96*d^7*e^5*x^5 + 96*d^8*e^4*x^4 + 32*d^9*e^3*x^3 - 2640*d^{10}e^2*x^2 + 480*d^{11}e^1*x + 480*d^{12})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)/((-e^2\*x^2 + d^2)^(7/2)\*x^3),x, algorithm="fricas")

[Out] -1/30\*(96\*e^12\*x^12 - 687\*d\*e^11\*x^11 - 1281\*d^2\*e^10\*x^10 + 4946\*d^3\*e^9\*x^9 + 4162\*d^4\*e^8\*x^8 - 11487\*d^5\*e^7\*x^7 - 6375\*d^6\*e^6\*x^6 + 11310\*d^7\*e^5\*x^5 + 5550\*d^8\*e^4\*x^4 - 4560\*d^9\*e^3\*x^3 - 2640\*d^10\*e^2\*x^2 + 480\*d^11\*e^1\*x + 480\*d^12 - 105\*(6\*d\*e^11\*x^11 - 6\*d^2\*e^10\*x^10 - 44\*d^3\*e^9\*x^9 + 44\*d^4\*e^8\*x^8 + 102\*d^5\*e^7\*x^7 - 102\*d^6\*e^6\*x^6 - 96\*d^7\*e^5\*x^5 + 96\*d^8\*e^4\*x^4 + 32\*d^9\*e^3\*x^3 - 2640\*d^10\*e^2\*x^2 + 480\*d^11\*e^1\*x + 480\*d^12))



$$9e^3x^3 - 32d^{10}e^2x^2 - (e^{11}x^{11} - d^{10}e^2x^{10} - 19d^2e^9x^9 + 19d^3e^8x^8 + 66d^4e^7x^7 - 66d^5e^6x^6 - 80d^6e^5x^5 + 80d^7e^4x^4 + 32d^8e^3x^3 - 32d^9e^2x^2) \sqrt{-e^2x^2 + d^2} \log(-d - \sqrt{-e^2x^2 + d^2})/x + 2(58e^{11}x^{11} + 230d^{10}e^2x^{10} - 1075d^2e^9x^9 - 1181d^3e^8x^8 + 3696d^4e^7x^7 + 2220d^5e^6x^6 - 4605d^6e^5x^5 - 2205d^7e^4x^4 + 2160d^8e^3x^3 + 1200d^9e^2x^2 - 240d^{10}e^2x - 240d^{11}) \sqrt{-e^2x^2 + d^2} / (6d^9e^9x^{11} - 6d^{10}e^8x^{10} - 44d^{11}e^7x^9 + 44d^{12}e^6x^8 + 102d^{13}e^5x^7 - 102d^{14}e^4x^6 - 96d^{15}e^3x^5 + 96d^{16}e^2x^4 + 32d^{17}e^2x^3 - 32d^{18}x^2 - (d^8e^9x^{11} - d^9e^8x^{10} - 19d^{10}e^7x^9 + 19d^{11}e^6x^8 + 66d^{12}e^5x^7 - 66d^{13}e^4x^6 - 80d^{14}e^3x^5 + 80d^{15}e^2x^4 + 32d^{16}e^2x^3 - 32d^{17}x^2) \sqrt{-e^2x^2 + d^2})$$

**Sympy [A]** time = 62.7193, size = 2691, normalized size = 14.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)/x\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out]  $d \cdot \text{Piecewise}((30 \cdot I \cdot d^{**8} \sqrt{-1 + e^{**2}x^{**2}/d^{**2}})/(-60 \cdot d^{**15}x^{**2} + 180 \cdot d^{**13}e^{**2}x^{**4} - 180 \cdot d^{**11}e^{**4}x^{**6} + 60 \cdot d^{**9}e^{**6}x^{**8}) - 322 \cdot I \cdot d^{**6}e^{**2}x^{**2} \sqrt{-1 + e^{**2}x^{**2}/d^{**2}})/(-60 \cdot d^{**15}x^{**2} + 180 \cdot d^{**13}e^{**2}x^{**4} - 180 \cdot d^{**11}e^{**4}x^{**6} + 60 \cdot d^{**9}e^{**6}x^{**8}) - 105 \cdot d^{**6}e^{**2}x^{**2} \log(e^{**2}x^{**2}/d^{**2})/(-60 \cdot d^{**15}x^{**2} + 180 \cdot d^{**13}e^{**2}x^{**4} - 180 \cdot d^{**11}e^{**4}x^{**6} + 60 \cdot d^{**9}e^{**6}x^{**8}) + 210 \cdot d^{**6}e^{**2}x^{**2} \log(e^x/d)/(-60 \cdot d^{**15}x^{**2} + 180 \cdot d^{**13}e^{**2}x^{**4} - 180 \cdot d^{**11}e^{**4}x^{**6} + 60 \cdot d^{**9}e^{**6}x^{**8}) - 210 \cdot I \cdot d^{**6}e^{**2}x^{**2} \text{asin}(d/(e^x))/(-60 \cdot d^{**15}x^{**2} + 180 \cdot d^{**13}e^{**2}x^{**4} - 180 \cdot d^{**11}e^{**4}x^{**6} + 60 \cdot d^{**9}e^{**6}x^{**8}) + 490 \cdot I \cdot d^{**4}e^{**4}x^{**4} \sqrt{-1 + e^{**2}x^{**2}/d^{**2}})/(-60 \cdot d^{**15}x^{**2} + 180 \cdot d^{**13}e^{**2}x^{**4} - 180 \cdot d^{**11}e^{**4}x^{**6} + 60 \cdot d^{**9}e^{**6}x^{**8}) + 315 \cdot d^{**4}e^{**4}x^{**4} \log(e^{**2}x^{**2}/d^{**2})/(-60 \cdot d^{**15}x^{**2} + 180 \cdot d^{**13}e^{**2}x^{**4} - 180 \cdot d^{**11}e^{**4}x^{**6} + 60 \cdot d^{**9}e^{**6}x^{**8}) - 630 \cdot d^{**4}e^{**4}x^{**4} \log(e^x/d)/(-60 \cdot d^{**15}x^{**2} + 180 \cdot d^{**13}e^{**2}x^{**4} - 180 \cdot d^{**11}e^{**4}x^{**6} + 60 \cdot d^{**9}e^{**6}x^{**8}) + 630 \cdot I \cdot d^{**4}e^{**4}x^{**4} \text{asin}(d/(e^x))/(-60 \cdot d^{**15}x^{**2} + 180 \cdot d^{**13}e^{**2}x^{**4} - 180 \cdot d^{**11}e^{**4}x^{**6} + 60 \cdot d^{**9}e^{**6}x^{**8}) - 210 \cdot I \cdot d^{**2}e^{**6}x^{**6} \sqrt{-1 + e^{**2}x^{**2}/d^{**2}})/(-60 \cdot d^{**15}x^{**2} + 180 \cdot d^{**13}e^{**2}x^{**4} - 180 \cdot d^{**11}e^{**4}x^{**6} + 60 \cdot d^{**9}e^{**6}x^{**8}) - 315 \cdot d^{**2}e^{**6}x^{**6} \log(e^{**2}x^{**2}/d^{**2})/(-60 \cdot d^{**15}x^{**2} + 180 \cdot d^{**13}e^{**2}x^{**4} - 180 \cdot d^{**11}e^{**4}x^{**6} + 60 \cdot d^{**9}e^{**6}x^{**8}) + 630 \cdot d^{**2}e^{**6}x^{**6} \log(e^x/d)/(-60 \cdot d^{**15}x^{**2} + 180 \cdot d^{**13}e^{**2}x^{**4} - 180 \cdot d^{**11}e^{**4}x^{**6} + 60 \cdot d^{**9}e^{**6}x^{**8}) - 630 \cdot I \cdot d^{**2}e^{**6}x^{**6} \text{asin}(d/(e^x))/(-60 \cdot d^{**15}x^{**2} + 180 \cdot d^{**13}e^{**2}x^{**4} - 180 \cdot d^{**11}e^{**4}x^{**6} + 60 \cdot d^{**9}e^{**6}x^{**8}) + 105 \cdot e^{**8}x^{**8} \log(e^{**2}x^{**2}/d^{**2})/(-60 \cdot d^{**15}x^{**2} + 180 \cdot d^{**13}e^{**2}x^{**4} - 180 \cdot d^{**11}e^{**4}x^{**6} + 60 \cdot d^{**9}e^{**6}x^{**8}) - 210 \cdot e^{**8}x^{**8} \log(e^x/d)/(-60 \cdot d^{**15}x^{**2} + 180 \cdot d^{**13}e^{**2}x^{**4} - 180 \cdot d^{**11}e^{**4}x^{**6} + 60 \cdot d^{**9}e^{**6}x^{**8}) + 210 \cdot I \cdot e^{**8}x^{**8} \text{asin}(d/(e^x))/(-60 \cdot d^{**15}x^{**2} + 180 \cdot d^{**13}e^{**2}x^{**4} - 180 \cdot d^{**11}e^{**4}x^{**6} + 60 \cdot d^{**9}e^{**6}x^{**8}), \text{Abs}(e^{**2}x^{**2}/d^{**2}) > 1), (30 \cdot d^{**8} \sqrt{1 - e^{**2}x^{**2}/d^{**2}})/(-60 \cdot d^{**15}x^{**2} + 180 \cdot d^{**13}e^{**2}x^{**4} - 180 \cdot d^{**11}e^{**4}x^{**6} + 60 \cdot d^{**9}e^{**6}x^{**8}) - 322 \cdot d^{**6}e^{**2}x^{**2} \sqrt{1 - e^{**2}x^{**2}/d^{**2}})/(-60 \cdot d^{**15}x^{**2} + 180 \cdot d^{**13}e^{**2}x^{**4} - 180 \cdot d^{**11}e^{**4}x^{**6} + 60 \cdot d^{**9}e^{**6}x^{**8}) - 105 \cdot d^{**6}e^{**2}x^{**2} \log(e^{**2}x^{**2}/d^{**2})/(-60 \cdot d^{**15}x^{**2} + 180 \cdot d^{**13}e^{**2}x^{**4} - 180 \cdot d^{**11}e^{**4}x^{**6} + 60 \cdot d^{**9}e^{**6}x^{**8}) + 210 \cdot d^{**6}e^{**2}x^{**2} \log(\sqrt{1 - e^{**2}x^{**2}/d^{**2}} + 1)/(-60 \cdot d^{**15}x^{**2} + 180 \cdot d^{**13}e^{**2}x^{**4} - 180 \cdot d^{**11}e^{**4}x^{**6} + 60 \cdot d^{**9}e^{**6}x^{**8}) - 105 \cdot I \cdot \pi \cdot d^{**6}e^{**2}x^{**2} / (-60 \cdot d^{**15}x^{**2} + 180 \cdot d^{**13}e^{**2}x^{**4} - 180 \cdot d^{**11}e^{**4}x^{**6} + 60 \cdot d^{**9}e^{**6}x^{**8}) + 490 \cdot d^{**4}e^{**4}x^{**4} \sqrt{1 - e^{**2}x^{**2}/d^{**2}})/(-60 \cdot d^{**15}x^{**2} + 180 \cdot d^{**13}e^{**2}x^{**4} - 180 \cdot d^{**11}e^{**4}x^{**6} + 60 \cdot d^{**9}e^{**6}x^{**8}) + 315 \cdot d^{**4}e^{**4}x^{**4} \log(e^{**2}x^{**2}/d^{**2})/(-60 \cdot d^{**15}x^{**2} + 180 \cdot d^{**13}e^{**2}x^{**4} - 180 \cdot d^{**11}e^{**4}x^{**6} + 60 \cdot d^{**9}e^{**6}x^{**8}) - 630 \cdot d^{**4}e^{**4}x^{**4} \log(\sqrt{1 - e^{**2}x^{**2}/d^{**2}} + 1)/(-60 \cdot$

```

d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*
e**6*x**8) + 315*I*pi*d**4*e**4*x**4/(-60*d**15*x**2 + 180*d**13*
e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 210*d**2*e
**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**
2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 315*d**2*e**6
*x**6*log(e**2*x**2/d**2)/(-60*d**15*x**2 + 180*d**13*e**2*x**4 -
180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) + 630*d**2*e**6*x**6*lo
g(sqrt(1 - e**2*x**2/d**2) + 1)/(-60*d**15*x**2 + 180*d**13*e**2*
x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8) - 315*I*pi*d**2*e
**6*x**6/(-60*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x
**6 + 60*d**9*e**6*x**8) + 105*e**8*x**8*log(e**2*x**2/d**2)/(-60
*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**9
*e**6*x**8) - 210*e**8*x**8*log(sqrt(1 - e**2*x**2/d**2) + 1)/(-6
0*d**15*x**2 + 180*d**13*e**2*x**4 - 180*d**11*e**4*x**6 + 60*d**
9*e**6*x**8) + 105*I*pi*e**8*x**8/(-60*d**15*x**2 + 180*d**13*e**
2*x**4 - 180*d**11*e**4*x**6 + 60*d**9*e**6*x**8), True)) + e*Pie
cewise((5*d**6*e*sqrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*
e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) - 30*d**4*e**3
*x**2*sqrt(d**2/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*x**2 -
15*d**10*e**4*x**4 + 5*d**8*e**6*x**6) + 40*d**2*e**5*x**4*sqrt(
d**2/(e**2*x**2) - 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e
**4*x**4 + 5*d**8*e**6*x**6) - 16*e**7*x**6*sqrt(d**2/(e**2*x**2)
- 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**
8*e**6*x**6), Abs(d**2/(e**2*x**2)) > 1), (5*I*d**6*e*sqrt(-d**2/
(e**2*x**2) + 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x
**4 + 5*d**8*e**6*x**6) - 30*I*d**4*e**3*x**2*sqrt(-d**2/(e**2*x*
**2) + 1)/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*
d**8*e**6*x**6) + 40*I*d**2*e**5*x**4*sqrt(-d**2/(e**2*x**2) + 1)
/(-5*d**14 + 15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**
6*x**6) - 16*I*e**7*x**6*sqrt(-d**2/(e**2*x**2) + 1)/(-5*d**14 +
15*d**12*e**2*x**2 - 15*d**10*e**4*x**4 + 5*d**8*e**6*x**6), True
))

```

**GIAC/XCAS [A]** time = 0.300302, size = 351, normalized size = 1.91

$$\frac{\sqrt{-x^2e^2 + d^2} \left( \left( \left( 3 \left( x \left( \frac{11xe^7}{d^8} + \frac{15e^6}{d^7} \right) - \frac{25e^5}{d^6} \right) x - \frac{100e^4}{d^5} \right) x + \frac{45e^3}{d^4} \right) x + \frac{58e^2}{d^3} \right)}{15(x^2e^2 - d^2)^3}$$

$$- \frac{7e^2 \ln \left( \frac{|-2de - 2\sqrt{-x^2e^2 + d^2}e|e^{(-2)}}{2|x|} \right)}{2d^8} + \frac{x^2 \left( \frac{4(de + \sqrt{-x^2e^2 + d^2}e)e^4}{x} + e^6 \right)}{8(de + \sqrt{-x^2e^2 + d^2}e)^2 d^8}$$

$$- \frac{\left( \frac{4(de + \sqrt{-x^2e^2 + d^2}e)d^8e^8}{x} + \frac{(de + \sqrt{-x^2e^2 + d^2}e)^2 d^8e^6}{x^2} \right) e^{(-8)}}{8d^{16}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)/((-e^2\*x^2 + d^2)^(7/2)\*x^3), x, algorithm="giac")

[Out] -1/15\*sqrt(-x^2\*e^2 + d^2)\*(((3\*(x\*(11\*x\*e^7/d^8 + 15\*e^6/d^7) - 25\*e^5/d^6)\*x - 100\*e^4/d^5)\*x + 45\*e^3/d^4)\*x + 58\*e^2/d^3)/(x^2\*e^2 - d^2)^3 - 7/2\*e^2\*ln(1/2\*abs(-2\*d\*e - 2\*sqrt(-x^2\*e^2 + d^2)\*e)\*e^(-2)/abs(x))/d^8 + 1/8\*x^2\*(4\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*e^4/x + e^6)/((d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^2\*d^8) - 1/8\*(4\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*d^8\*e^8/x + (d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^2\*d^8\*e^6/x^2)\*e^(-8)/d^16

$$3.30 \quad \int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{9/2}} dx$$

**Optimal.** Leaf size=121

$$\frac{x^2(d+ex)}{7de(d^2-e^2x^2)^{7/2}} - \frac{2(d-2ex)}{35de^3(d^2-e^2x^2)^{5/2}} - \frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}} - \frac{4x}{105d^3e^2(d^2-e^2x^2)^{3/2}}$$

[Out]  $(x^2*(d + e*x))/(7*d*e*(d^2 - e^2*x^2)^(7/2)) - (2*(d - 2*e*x))/(35*d*e^3*(d^2 - e^2*x^2)^(5/2)) - (4*x)/(105*d^3*e^2*(d^2 - e^2*x^2)^(3/2)) - (8*x)/(105*d^5*e^2*sqrt[d^2 - e^2*x^2])$

**Rubi [A]** time = 0.188055, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{x^2(d+ex)}{7de(d^2-e^2x^2)^{7/2}} - \frac{2(d-2ex)}{35de^3(d^2-e^2x^2)^{5/2}} - \frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}} - \frac{4x}{105d^3e^2(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(d + e\*x))/(d^2 - e^2\*x^2)^(9/2), x]

[Out]  $(x^2*(d + e*x))/(7*d*e*(d^2 - e^2*x^2)^(7/2)) - (2*(d - 2*e*x))/(35*d*e^3*(d^2 - e^2*x^2)^(5/2)) - (4*x)/(105*d^3*e^2*(d^2 - e^2*x^2)^(3/2)) - (8*x)/(105*d^5*e^2*sqrt[d^2 - e^2*x^2])$

**Rubi in Sympy [A]** time = 20.1704, size = 105, normalized size = 0.87

$$\frac{x^2(d+ex)}{7de(d^2-e^2x^2)^{7/2}} - \frac{2d-4ex}{35de^3(d^2-e^2x^2)^{5/2}} - \frac{4x}{105d^3e^2(d^2-e^2x^2)^{3/2}} - \frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(9/2), x)

[Out]  $x**2*(d + e*x)/(7*d*e*(d**2 - e**2*x**2)**(7/2)) - (2*d - 4*e*x)/(35*d*e**3*(d**2 - e**2*x**2)**(5/2)) - 4*x/(105*d**3*e**2*(d**2 - e**2*x**2)**(3/2)) - 8*x/(105*d**5*e**2*sqrt(d**2 - e**2*x**2))$

**Mathematica [A]** time = 0.0863196, size = 104, normalized size = 0.86

$$\frac{\sqrt{d^2 - e^2x^2}(-6d^6 + 6d^5ex + 15d^4e^2x^2 + 20d^3e^3x^3 - 20d^2e^4x^4 - 8de^5x^5 + 8e^6x^6)}{105d^5e^3(d - ex)^4(d + ex)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(d + e\*x))/(d^2 - e^2\*x^2)^(9/2), x]

[Out]  $(sqrt[d^2 - e^2*x^2]*(-6*d^6 + 6*d^5*e*x + 15*d^4*e^2*x^2 + 20*d^3*e^3*x^3 - 20*d^2*e^4*x^4 - 8*d*e^5*x^5 + 8*e^6*x^6))/(105*d^5*e^3*(d - e*x)^4*(d + e*x)^3)$

**Maple [A]** time = 0.015, size = 99, normalized size = 0.8

$$\frac{(-ex + d)(ex + d)^2 (-8e^6x^6 + 8e^5x^5d + 20e^4x^4d^2 - 20x^3d^3e^3 - 15x^2d^4e^2 - 6xd^5e + 6d^6)}{105d^5e^3} (-e^2x^2 + d^2)^{-\frac{9}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(e*x+d)/(-e^2*x^2+d^2)^(9/2),x)`

[Out] `-1/105*(-e*x+d)*(e*x+d)^2*(-8*e^6*x^6+8*d*e^5*x^5+20*d^2*e^4*x^4-20*d^3*e^3*x^3-15*d^4*e^2*x^2-6*d^5*e*x+6*d^6)/d^5/e^3/(-e^2*x^2+d^2)^(9/2)`

**Maxima [A]** time = 0.710858, size = 182, normalized size = 1.5

$$\frac{x^2}{5(-e^2x^2 + d^2)^{\frac{7}{2}}e} + \frac{dx}{7(-e^2x^2 + d^2)^{\frac{7}{2}}e^2} - \frac{2d^2}{35(-e^2x^2 + d^2)^{\frac{7}{2}}e^3} - \frac{x}{35(-e^2x^2 + d^2)^{\frac{5}{2}}de^2} - \frac{4x}{105(-e^2x^2 + d^2)^{\frac{3}{2}}d^3e^2} - \frac{8x}{105\sqrt{-e^2x^2 + d^2}d^5e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)*x^2/(-e^2*x^2 + d^2)^(9/2),x, algorithm="maxima")`

[Out] `1/5*x^2/((-e^2*x^2 + d^2)^(7/2)*e) + 1/7*d*x/((-e^2*x^2 + d^2)^(7/2)*e^2) - 2/35*d^2/((-e^2*x^2 + d^2)^(7/2)*e^3) - 1/35*x/((-e^2*x^2 + d^2)^(5/2)*d*e^2) - 4/105*x/((-e^2*x^2 + d^2)^(3/2)*d^3*e^2) - 8/105*x/(sqrt(-e^2*x^2 + d^2)*d^5*e^2)`

**Fricas [A]** time = 0.330521, size = 637, normalized size = 5.26

$$\frac{8e^9x^{12} - 44de^8x^{11} - 128d^2e^7x^{10} + 464d^3e^6x^9 + 459d^4e^5x^8 - 1614d^5e^4x^7 - 616d^6e^3x^6 - 105(6d^6e^{11}x^{11} - 6d^7e^{10}x^{10} - 50d^8e^9x^9 + 50d^9e^8x^8 + 146d^{10}e^7x^7 - 146d^{11}e^6x^6 - 198d^{12}e^5x^5 + 198d^{13}e^4x^4 + 128d^{14}e^3x^3 - 112d^{14}e^2x^2 - 32d^{15}e^2x + 32d^{16})\sqrt{-e^2x^2 + d^2}}{(6d^6e^{11}x^{11} - 6d^7e^{10}x^{10} - 50d^8e^9x^9 + 50d^9e^8x^8 + 146d^{10}e^7x^7 - 146d^{11}e^6x^6 - 198d^{12}e^5x^5 + 198d^{13}e^4x^4 + 128d^{14}e^3x^3 - 128d^{15}e^2x^2 - 32d^{16}e^2x + 32d^{17} - (d^5e^{11}x^{11} - d^6e^{10}x^{10} - 20d^7e^9x^9 + 20d^8e^8x^8 + 85d^9e^7x^7 - 85d^{10}e^6x^6 - 146d^{11}e^5x^5 + 146d^{12}e^4x^4 + 112d^{13}e^3x^3 - 112d^{14}e^2x^2 - 32d^{15}e^2x + 32d^{16})\sqrt{-e^2x^2 + d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)*x^2/(-e^2*x^2 + d^2)^(9/2),x, algorithm="fricas")`

[Out] `1/105*(8*e^9*x^12 - 44*d*e^8*x^11 - 128*d^2*e^7*x^10 + 464*d^3*e^6*x^9 + 459*d^4*e^5*x^8 - 1614*d^5*e^4*x^7 - 616*d^6*e^3*x^6 + 22*96*d^7*e^2*x^5 + 280*d^8*e*x^4 - 1120*d^9*x^3 + 2*(3*e^8*x^11 + 21*d*e^7*x^10 - 84*d^2*e^6*x^9 - 128*d^3*e^5*x^8 + 443*d^4*e^4*x^7 + 238*d^5*e^3*x^6 - 868*d^6*e^2*x^5 - 140*d^7*e*x^4 + 560*d^8*x^3)*sqrt(-e^2*x^2 + d^2))/(6*d^6*e^11*x^11 - 6*d^7*e^10*x^10 - 50*d^8*e^9*x^9 + 50*d^9*e^8*x^8 + 146*d^10*e^7*x^7 - 146*d^11*e^6*x^6 - 198*d^12*e^5*x^5 + 198*d^13*e^4*x^4 + 128*d^14*e^3*x^3 - 128*d^15*e^2*x^2 - 32*d^16*e*x + 32*d^17 - (d^5*e^11*x^11 - d^6*e^10*x^10 - 20*d^7*e^9*x^9 + 20*d^8*e^8*x^8 + 85*d^9*e^7*x^7 - 85*d^10*e^6*x^6 - 146*d^11*e^5*x^5 + 146*d^12*e^4*x^4 + 112*d^13*e^3*x^3 - 112*d^14*e^2*x^2 - 32*d^15*e*x + 32*d^16)*sqrt(-e^2*x^2 + d^2))`

**Sympy [A]** time = 48.9556, size = 903, normalized size = 7.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(9/2),x)

[Out] d\*Piecewise((35\*I\*d\*\*4\*x\*\*3/(-105\*d\*\*13\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 315\*d\*\*11\*e\*\*2\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 315\*d\*\*9\*e\*\*4\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 105\*d\*\*7\*e\*\*6\*x\*\*6\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 28\*I\*d\*\*2\*e\*\*2\*x\*\*5/(-105\*d\*\*13\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 315\*d\*\*11\*e\*\*2\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 315\*d\*\*9\*e\*\*4\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 105\*d\*\*7\*e\*\*6\*x\*\*6\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + 8\*I\*e\*\*4\*x\*\*7/(-105\*d\*\*13\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 315\*d\*\*11\*e\*\*2\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 315\*d\*\*9\*e\*\*4\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 105\*d\*\*7\*e\*\*6\*x\*\*6\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (-35\*d\*\*4\*x\*\*3/(-105\*d\*\*13\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 315\*d\*\*11\*e\*\*2\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 315\*d\*\*9\*e\*\*4\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 105\*d\*\*7\*e\*\*6\*x\*\*6\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 28\*d\*\*2\*e\*\*2\*x\*\*5/(-105\*d\*\*13\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 315\*d\*\*11\*e\*\*2\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 315\*d\*\*9\*e\*\*4\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 105\*d\*\*7\*e\*\*6\*x\*\*6\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - 8\*e\*\*4\*x\*\*7/(-105\*d\*\*13\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 315\*d\*\*11\*e\*\*2\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 315\*d\*\*9\*e\*\*4\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 105\*d\*\*7\*e\*\*6\*x\*\*6\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True)) + e\*Piecewise((2\*d\*\*2/(-35\*d\*\*6\*e\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) + 105\*d\*\*4\*e\*\*6\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) - 105\*d\*\*2\*e\*\*8\*x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) + 35\*e\*\*10\*x\*\*6\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) - 7\*e\*\*2\*x\*\*2/(-35\*d\*\*6\*e\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) + 105\*d\*\*4\*e\*\*6\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) - 105\*d\*\*2\*e\*\*8\*x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) + 35\*e\*\*10\*x\*\*6\*sqrt(d\*\*2 - e\*\*2\*x\*\*2))), Ne(e, 0)), (x\*\*4/(4\*(d\*\*2)\*\*(9/2)), True))

**GIAC/XCAS [A]** time = 0.291156, size = 104, normalized size = 0.86

$$\frac{\left(\left(4x^2\left(\frac{2x^2e^4}{d^5} - \frac{7e^2}{d^3}\right) + \frac{35}{d}\right)x + 21e^{(-1)}\right)x^2 - 6d^2e^{(-3)}}{105(x^2e^2 - d^2)^4}\sqrt{-x^2e^2 + d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*x^2/(-e^2\*x^2 + d^2)^(9/2),x, algorithm="giac")

[Out] 1/105\*((4\*x^2\*(2\*x^2\*e^4/d^5 - 7\*e^2/d^3) + 35/d)\*x + 21\*e^(-1))\*x^2 - 6\*d^2\*e^(-3))\*sqrt(-x^2\*e^2 + d^2)/(x^2\*e^2 - d^2)^4

$$3.31 \quad \int \frac{x^2(d+ex)}{(d^2-e^2x^2)^{11/2}} dx$$

**Optimal.** Leaf size=148

$$\frac{x^2(d+ex)}{9de(d^2-e^2x^2)^{9/2}} - \frac{2(d-3ex)}{63de^3(d^2-e^2x^2)^{7/2}} - \frac{16x}{315d^7e^2\sqrt{d^2-e^2x^2}}$$

$$- \frac{8x}{315d^5e^2(d^2-e^2x^2)^{3/2}} - \frac{2x}{105d^3e^2(d^2-e^2x^2)^{5/2}}$$

[Out]  $(x^2*(d + e*x))/(9*d*e*(d^2 - e^2*x^2)^(9/2)) - (2*(d - 3*e*x))/(63*d*e^3*(d^2 - e^2*x^2)^(7/2)) - (2*x)/(105*d^3*e^2*(d^2 - e^2*x^2)^(5/2)) - (8*x)/(315*d^5*e^2*(d^2 - e^2*x^2)^(3/2)) - (16*x)/(315*d^7*e^2*Sqrt[d^2 - e^2*x^2])$

**Rubi [A]** time = 0.206091, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{x^2(d+ex)}{9de(d^2-e^2x^2)^{9/2}} - \frac{2(d-3ex)}{63de^3(d^2-e^2x^2)^{7/2}} - \frac{16x}{315d^7e^2\sqrt{d^2-e^2x^2}}$$

$$- \frac{8x}{315d^5e^2(d^2-e^2x^2)^{3/2}} - \frac{2x}{105d^3e^2(d^2-e^2x^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(d + e\*x))/(d^2 - e^2\*x^2)^(11/2), x]

[Out]  $(x^2*(d + e*x))/(9*d*e*(d^2 - e^2*x^2)^(9/2)) - (2*(d - 3*e*x))/(63*d*e^3*(d^2 - e^2*x^2)^(7/2)) - (2*x)/(105*d^3*e^2*(d^2 - e^2*x^2)^(5/2)) - (8*x)/(315*d^5*e^2*(d^2 - e^2*x^2)^(3/2)) - (16*x)/(315*d^7*e^2*Sqrt[d^2 - e^2*x^2])$

**Rubi in Sympy [A]** time = 23.5564, size = 131, normalized size = 0.89

$$\frac{x^2(d+ex)}{9de(d^2-e^2x^2)^{9/2}} - \frac{2d-6ex}{63de^3(d^2-e^2x^2)^{7/2}} - \frac{2x}{105d^3e^2(d^2-e^2x^2)^{5/2}} - \frac{8x}{315d^5e^2(d^2-e^2x^2)^{3/2}} - \frac{16x}{315d^7e^2\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(11/2), x)

[Out]  $x**2*(d + e*x)/(9*d*e*(d**2 - e**2*x**2)**(9/2)) - (2*d - 6*e*x)/(63*d*e**3*(d**2 - e**2*x**2)**(7/2)) - 2*x/(105*d**3*e**2*(d**2 - e**2*x**2)**(5/2)) - 8*x/(315*d**5*e**2*(d**2 - e**2*x**2)**(3/2)) - 16*x/(315*d**7*e**2*sqrt(d**2 - e**2*x**2))$

**Mathematica [A]** time = 0.105592, size = 126, normalized size = 0.85

$$\frac{\sqrt{d^2 - e^2x^2}(-10d^8 + 10d^7ex + 35d^6e^2x^2 + 70d^5e^3x^3 - 70d^4e^4x^4 - 56d^3e^5x^5 + 56d^2e^6x^6 + 16de^7x^7 - 16e^8x^8)}{315d^7e^3(d-ex)^5(d+ex)^4}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(d + e\*x))/(d^2 - e^2\*x^2)^(11/2), x]

[Out]  $(\text{Sqrt}[d^2 - e^2 x^2] * (-10 d^8 + 10 d^7 e x + 35 d^6 e^2 x^2 + 70 d^5 e^3 x^3 - 70 d^4 e^4 x^4 - 56 d^3 e^5 x^5 + 56 d^2 e^6 x^6 + 16 d e^7 x^7 - 16 e^8 x^8)) / (315 d^7 e^3 (d - e x)^5 (d + e x)^4)$

**Maple [A]** time = 0.015, size = 121, normalized size = 0.8

$$\frac{(-ex + d)(ex + d)^2 (16 e^8 x^8 - 16 e^7 x^7 d - 56 e^6 x^6 d^2 + 56 e^5 x^5 d^3 + 70 e^4 x^4 d^4 - 70 x^3 d^5 e^3 - 35 x^2 d^6 e^2 - 10 x d^7 e + 10 d^8)}{315 d^7 e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^2 * (e^x + d) / (-e^2 x^2 + d^2)^{(11/2)}, x)$

[Out]  $-1/315 * (-e^x + d) * (e^x + d)^2 * (16 e^8 x^8 - 16 d e^7 x^7 - 56 d^2 e^6 x^6 + 56 d^3 e^5 x^5 + 70 d^4 e^4 x^4 - 70 d^5 e^3 x^3 - 35 d^6 e^2 x^2 - 10 d^7 e x + 10 d^8) / d^7 / e^3 / (-e^2 x^2 + d^2)^{(11/2)}$

**Maxima [A]** time = 0.707668, size = 213, normalized size = 1.44

$$\frac{\frac{x^2}{7(-e^2 x^2 + d^2)^{\frac{9}{2}} e} + \frac{dx}{9(-e^2 x^2 + d^2)^{\frac{9}{2}} e^2} - \frac{2 d^2}{63(-e^2 x^2 + d^2)^{\frac{9}{2}} e^3} - \frac{x}{63(-e^2 x^2 + d^2)^{\frac{7}{2}} d e^2}}{\frac{2 x}{105(-e^2 x^2 + d^2)^{\frac{5}{2}} d^3 e^2} - \frac{8 x}{315(-e^2 x^2 + d^2)^{\frac{3}{2}} d^5 e^2} - \frac{16 x}{315 \sqrt{-e^2 x^2 + d^2} d^7 e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^x + d) * x^2 / (-e^2 x^2 + d^2)^{(11/2)}, x, \text{algorithm}="maxima")$

[Out]  $1/7 * x^2 / ((-e^2 x^2 + d^2)^{(9/2)} * e) + 1/9 * d * x / ((-e^2 x^2 + d^2)^{(9/2)} * e^2) - 2/63 * d^2 / ((-e^2 x^2 + d^2)^{(9/2)} * e^3) - 1/63 * x / ((-e^2 x^2 + d^2)^{(7/2)} * d * e^2) - 2/105 * x / ((-e^2 x^2 + d^2)^{(5/2)} * d^3 * e^2) - 8/315 * x / ((-e^2 x^2 + d^2)^{(3/2)} * d^5 * e^2) - 16/315 * x / (\text{sqrt}(-e^2 x^2 + d^2) * d^7 * e^2)$

**Fricas [A]** time = 0.450222, size = 875, normalized size = 5.91

$$\frac{16 e^{13} x^{16} - 96 d e^{12} x^{15} - 488 d^2 e^{11} x^{14} + 1688 d^3 e^{10} x^{13} + 3302 d^4 e^9 x^{12} - 10022 d^5 e^8 x^{11} - 9731 d^6 e^7 x^{10} + 29366 d^7 e^6 x^9 + 14634 d^8 e^5 x^8 - 47184 d^9 e^4 x^7 - 11088 d^{10} e^3 x^6 + 39648 d^{11} e^2 x^5 + 3360 d^{12} e x^4 - 13440 d^{13} x^3 + 2 * (5 e^{12} x^{15} + 59 d e^{11} x^{14} - 239 d^2 e^{10} x^{13} - 689 d^3 e^9 x^{12} + 2159 d^4 e^8 x^{11} + 2761 d^5 e^7 x^{10} - 8221 d^6 e^6 x^9 - 5175 d^7 e^5 x^8 + 16200 d^8 e^4 x^7 + 4704 d^9 e^3 x^6 - 16464 d^{10} e^2 x^5 - 1680 d^{11} e x^4 + 6720 d^{12} x^3) * \text{sqrt}(-e^2 x^2 + d^2)}{(8 d^8 e^{15} x^{15} - 8 d^9 e^{14} x^{14} - 112 d^{10} e^{13} x^{13} + 112 d^{11} e^{12} x^{12} + 560 d^{12} e^{11} x^{11} - 560 d^{13} e^{10} x^{10} - 1408 d^{14} e^9 x^9 + 1408 d^{15} e^8 x^8 - 112 d^{16} e^7 x^7 - 1992 d^{17} e^6 x^6 - 1616 d^{18} e^5 x^5 + 1616 d^{19} e^4 x^4 + 704 d^{20} e^3 x^3 - 704 d^{21} e^2 x^2 - 128 d^{22} e x + 128 d^{23})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^x + d) * x^2 / (-e^2 x^2 + d^2)^{(11/2)}, x, \text{algorithm}="fricas")$

[Out]  $1/315 * (16 e^{13} x^{16} - 96 d e^{12} x^{15} - 488 d^2 e^{11} x^{14} + 1688 d^3 e^{10} x^{13} + 3302 d^4 e^9 x^{12} - 10022 d^5 e^8 x^{11} - 9731 d^6 e^7 x^{10} + 29366 d^7 e^6 x^9 + 14634 d^8 e^5 x^8 - 47184 d^9 e^4 x^7 - 11088 d^{10} e^3 x^6 + 39648 d^{11} e^2 x^5 + 3360 d^{12} e x^4 - 13440 d^{13} x^3 + 2 * (5 e^{12} x^{15} + 59 d e^{11} x^{14} - 239 d^2 e^{10} x^{13} - 689 d^3 e^9 x^{12} + 2159 d^4 e^8 x^{11} + 2761 d^5 e^7 x^{10} - 8221 d^6 e^6 x^9 - 5175 d^7 e^5 x^8 + 16200 d^8 e^4 x^7 + 4704 d^9 e^3 x^6 - 16464 d^{10} e^2 x^5 - 1680 d^{11} e x^4 + 6720 d^{12} x^3) * \text{sqrt}(-e^2 x^2 + d^2)) / (8 d^8 e^{15} x^{15} - 8 d^9 e^{14} x^{14} - 112 d^{10} e^{13} x^{13} + 112 d^{11} e^{12} x^{12} + 560 d^{12} e^{11} x^{11} - 560 d^{13} e^{10} x^{10} - 1408 d^{14} e^9 x^9 + 1408 d^{15} e^8 x^8 - 112 d^{16} e^7 x^7 - 1992 d^{17} e^6 x^6 - 1616 d^{18} e^5 x^5 + 1616 d^{19} e^4 x^4 + 704 d^{20} e^3 x^3 - 704 d^{21} e^2 x^2 - 128 d^{22} e x + 128 d^{23})$

$$3 - (d^7 e^{15} x^{15} - d^8 e^{14} x^{14} - 35 d^9 e^{13} x^{13} + 35 d^{10} e^{12} x^{12} + 259 d^{11} e^{11} x^{11} - 259 d^{12} e^{10} x^{10} - 833 d^{13} e^9 x^9 + 833 d^{14} e^8 x^8 + 1408 d^{15} e^7 x^7 - 1408 d^{16} e^6 x^6 - 1312 d^{17} e^5 x^5 + 1312 d^{18} e^4 x^4 + 640 d^{19} e^3 x^3 - 640 d^{20} e^2 x^2 - 128 d^{21} e x + 128 d^{22}) \sqrt{-e^2 x^2 + d^2})$$

**Sympy [A]** time = 160.059, size = 1401, normalized size = 9.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(11/2),x)

[Out] d\*Piecewise((-105\*I\*d\*\*6\*x\*\*3/(315\*d\*\*17\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 1260\*d\*\*15\*e\*\*2\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 1890\*d\*\*13\*e\*\*4\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 1260\*d\*\*11\*e\*\*6\*x\*\*6\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 315\*d\*\*9\*e\*\*8\*x\*\*8\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + 126\*I\*d\*\*4\*e\*\*2\*x\*\*5/(315\*d\*\*17\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 1260\*d\*\*15\*e\*\*2\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 1890\*d\*\*13\*e\*\*4\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 1260\*d\*\*11\*e\*\*6\*x\*\*6\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 315\*d\*\*9\*e\*\*8\*x\*\*8\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 72\*I\*d\*\*2\*e\*\*4\*x\*\*7/(315\*d\*\*17\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 1260\*d\*\*15\*e\*\*2\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 1890\*d\*\*13\*e\*\*4\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 1260\*d\*\*11\*e\*\*6\*x\*\*6\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 315\*d\*\*9\*e\*\*8\*x\*\*8\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + 16\*I\*e\*\*6\*x\*\*9/(315\*d\*\*17\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 1260\*d\*\*15\*e\*\*2\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 1890\*d\*\*13\*e\*\*4\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) - 1260\*d\*\*11\*e\*\*6\*x\*\*6\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2) + 315\*d\*\*9\*e\*\*8\*x\*\*8\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (105\*d\*\*6\*x\*\*3/(315\*d\*\*17\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 1260\*d\*\*15\*e\*\*2\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 1890\*d\*\*13\*e\*\*4\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 1260\*d\*\*11\*e\*\*6\*x\*\*6\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 315\*d\*\*9\*e\*\*8\*x\*\*8\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - 126\*d\*\*4\*e\*\*2\*x\*\*5/(315\*d\*\*17\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 1260\*d\*\*15\*e\*\*2\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 1890\*d\*\*13\*e\*\*4\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 1260\*d\*\*11\*e\*\*6\*x\*\*6\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 315\*d\*\*9\*e\*\*8\*x\*\*8\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 72\*d\*\*2\*e\*\*4\*x\*\*7/(315\*d\*\*17\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 1260\*d\*\*15\*e\*\*2\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 1890\*d\*\*13\*e\*\*4\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 1260\*d\*\*11\*e\*\*6\*x\*\*6\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 315\*d\*\*9\*e\*\*8\*x\*\*8\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - 16\*e\*\*6\*x\*\*9/(315\*d\*\*17\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 1260\*d\*\*15\*e\*\*2\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 1890\*d\*\*13\*e\*\*4\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) - 1260\*d\*\*11\*e\*\*6\*x\*\*6\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2) + 315\*d\*\*9\*e\*\*8\*x\*\*8\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True)) + e\*Piecewise((-2\*d\*\*2/(63\*d\*\*8\*e\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) - 252\*d\*\*6\*e\*\*6\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) + 378\*d\*\*4\*e\*\*8\*x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) - 252\*d\*\*2\*e\*\*10\*x\*\*6\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) + 63\*e\*\*12\*x\*\*8\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) + 9\*e\*\*2\*x\*\*2/(63\*d\*\*8\*e\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) - 252\*d\*\*6\*e\*\*6\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) + 378\*d\*\*4\*e\*\*8\*x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) - 252\*d\*\*2\*e\*\*10\*x\*\*6\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) + 63\*e\*\*12\*x\*\*8\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)), Ne(e, 0)), (x\*\*4/(4\*(d\*\*2)\*\*(11/2)), True))

**GIAC/XCAS [A]** time = 0.295169, size = 122, normalized size = 0.82

$$\frac{\left(\left(\left(2x^2\left(\frac{2x^2e^6}{d^7} - \frac{9e^4}{d^5}\right) + \frac{63e^2}{d^3}\right)x^2 - \frac{105}{d}\right)x - 45e^{(-1)}\right)x^2 + 10d^2e^{(-3)}\sqrt{-x^2e^2 + d^2}}{315(x^2e^2 - d^2)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*x^2/(-e^2\*x^2 + d^2)^(11/2),x, algorithm="giac")



```
[Out] 1/315*(((2*(4*x^2*(2*x^2*e^6/d^7 - 9*e^4/d^5) + 63*e^2/d^3)*x^2 -  
105/d)*x - 45*e^(-1))*x^2 + 10*d^2*e^(-3))*sqrt(-x^2*e^2 + d^2)/  
(x^2*e^2 - d^2)^5
```

$$3.32 \quad \int \frac{x^2(1-ax)}{(1-a^2x^2)^{3/2}} dx$$

**Optimal.** Leaf size=54

$$-\frac{\sin^{-1}(ax)}{a^3} - \frac{1-ax}{a^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{a^3}$$

[Out]  $-\left(\frac{1-ax}{a^3\sqrt{1-a^2x^2}}\right) - \sqrt{1-a^2x^2}/a^3 - \text{ArcSin}[ax]/a^3$

**Rubi [A]** time = 0.127679, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{\sin^{-1}(ax)}{a^3} - \frac{1-ax}{a^3\sqrt{1-a^2x^2}} - \frac{\sqrt{1-a^2x^2}}{a^3}$$

Antiderivative was successfully verified.

[In] `Int[(x^2*(1-a*x))/(1-a^2*x^2)^(3/2),x]`

[Out]  $-\left(\frac{1-ax}{a^3\sqrt{1-a^2x^2}}\right) - \sqrt{1-a^2x^2}/a^3 - \text{ArcSin}[ax]/a^3$

**Rubi in Sympy [A]** time = 14.3374, size = 44, normalized size = 0.81

$$-\frac{\sqrt{-a^2x^2+1}}{a^3} - \frac{\text{asin}(ax)}{a^3} - \frac{\sqrt{-a^2x^2+1}}{a^3(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**2*(-a*x+1)/(-a**2*x**2+1)**(3/2),x)`

[Out]  $-\sqrt{-a^2x^2+1}/a^3 - \text{asin}(ax)/a^3 - \sqrt{-a^2x^2+1}/(a^3(ax+1))$

**Mathematica [A]** time = 0.0923865, size = 37, normalized size = 0.69

$$-\frac{\frac{\sqrt{1-a^2x^2}(ax+2)}{ax+1} + \sin^{-1}(ax)}{a^3}$$

Antiderivative was successfully verified.

[In] `Integrate[(x^2*(1-a*x))/(1-a^2*x^2)^(3/2),x]`

[Out]  $-\left(\frac{(2+ax)\sqrt{1-a^2x^2}}{(1+ax)} + \text{ArcSin}[ax]\right)/a^3$

**Maple [A]** time = 0.022, size = 85, normalized size = 1.6

$$\frac{x^2}{a} \frac{1}{\sqrt{-a^2x^2+1}} - 2 \frac{1}{a^3\sqrt{-a^2x^2+1}} + \frac{x}{a^2} \frac{1}{\sqrt{-a^2x^2+1}} - \frac{1}{a^2} \arctan\left(x\sqrt{a^2} \frac{1}{\sqrt{-a^2x^2+1}}\right) \frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(-a*x+1)/(-a^2*x^2+1)^(3/2),x)`

[Out]  $x^2/a/(-a^2x^2+1)^{1/2}-2/a^3/(-a^2x^2+1)^{1/2}+x/a^2/(-a^2x^2+1)^{1/2}-1/a^2/(a^2)^{1/2}*\arctan((a^2)^{1/2}*x/(-a^2x^2+1)^{1/2})$

**Maxima [A]** time = 0.789037, size = 101, normalized size = 1.87

$$\frac{x^2}{\sqrt{-a^2x^2+1}a} + \frac{x}{\sqrt{-a^2x^2+1}a^2} - \frac{\arcsin\left(\frac{a^2x}{\sqrt{a^2}}\right)}{\sqrt{a^2}a^2} - \frac{2}{\sqrt{-a^2x^2+1}a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(a*x-1)*x^2/(-a^2*x^2+1)^(3/2),x,algorithm="maxima")`

[Out]  $x^2/(\sqrt{-a^2x^2+1}*a) + x/(\sqrt{-a^2x^2+1}*a^2) - \arcsin(a^2x/\sqrt{a^2})/(\sqrt{a^2}*a^2) - 2/(\sqrt{-a^2x^2+1}*a^3)$

**Fricas [A]** time = 0.279162, size = 197, normalized size = 3.65

$$\frac{a^3x^3 + a^2x^2 + 2ax + 2\left(a^2x^2 - ax + \sqrt{-a^2x^2+1}(ax+2) - 2\right)\arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) - (a^2x^2 + 2ax)\sqrt{-a^2x^2+1}}{a^5x^2 - a^4x - 2a^3 + (a^4x + 2a^3)\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(a*x-1)*x^2/(-a^2*x^2+1)^(3/2),x,algorithm="fricas")`

[Out]  $(a^3x^3 + a^2x^2 + 2ax + 2(a^2x^2 - ax + \sqrt{-a^2x^2+1})*(ax+2) - 2)*\arctan((\sqrt{-a^2x^2+1}-1)/(ax)) - (a^2x^2 + 2ax)*\sqrt{-a^2x^2+1}/(a^5x^2 - a^4x - 2a^3 + (a^4x + 2a^3)*\sqrt{-a^2x^2+1})$

**Sympy [A]** time = 10.1649, size = 102, normalized size = 1.89

$$-a \left( \begin{cases} -\frac{x^2}{a^2\sqrt{-a^2x^2+1}} + \frac{2}{a^4\sqrt{-a^2x^2+1}} & \text{for } a \neq 0 \\ \frac{x^4}{4} & \text{otherwise} \end{cases} \right) + \begin{cases} -\frac{ix}{a^2\sqrt{a^2x^2-1}} + \frac{i\operatorname{acosh}(ax)}{a^3} & \text{for } |a^2x^2| > 1 \\ \frac{x}{a^2\sqrt{-a^2x^2+1}} - \frac{\operatorname{asin}(ax)}{a^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(-a*x+1)/(-a**2*x**2+1)**(3/2),x)`

[Out]  $-a*\operatorname{Piecewise}((-x**2/(a**2*\sqrt{-a**2*x**2+1}) + 2/(a**4*\sqrt{-a**2*x**2+1})), \operatorname{Ne}(a, 0)), (x**4/4, \operatorname{True})) + \operatorname{Piecewise}((-I*x/(a**2*\sqrt{a**2*x**2-1}) + I*\operatorname{acosh}(a*x)/a**3, \operatorname{Abs}(a**2*x**2) > 1), (x/(a**2*\sqrt{-a**2*x**2+1}) - \operatorname{asin}(a*x)/a**3, \operatorname{True}))$

**GIAC/XCAS [A]** time = 0.289859, size = 95, normalized size = 1.76

$$-\frac{\arcsin(ax)\operatorname{sign}(a)}{a^2|a|} - \frac{\sqrt{-a^2x^2+1}}{a^3} + \frac{2}{a^2\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} + 1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(a*x - 1)*x^2/(-a^2*x^2 + 1)^(3/2),x, algorithm="giac")
```

```
[Out] -arcsin(a*x)*sign(a)/(a^2*abs(a)) - sqrt(-a^2*x^2 + 1)/a^3 + 2/(a  
^2*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

$$3.33 \quad \int \frac{x^4(d+ex)^2}{\sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=173

$$-\frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} + \frac{11d^6 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^5} - \frac{d^4(256d+165ex)\sqrt{d^2-e^2x^2}}{240e^5} - \frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3}$$

[Out]  $(-8*d^3*x^2*\text{Sqrt}[d^2 - e^2*x^2])/(15*e^3) - (11*d^2*x^3*\text{Sqrt}[d^2 - e^2*x^2])/(24*e^2) - (2*d*x^4*\text{Sqrt}[d^2 - e^2*x^2])/(5*e) - (x^5*\text{Sqrt}[d^2 - e^2*x^2])/6 - (d^4*(256*d + 165*e*x)*\text{Sqrt}[d^2 - e^2*x^2])/(240*e^5) + (11*d^6*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(16*e^5)$

**Rubi [A]** time = 0.573259, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$-\frac{1}{6}x^5\sqrt{d^2-e^2x^2} - \frac{2dx^4\sqrt{d^2-e^2x^2}}{5e} - \frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} + \frac{11d^6 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^5} - \frac{d^4(256d+165ex)\sqrt{d^2-e^2x^2}}{240e^5} - \frac{8d^3x^2\sqrt{d^2-e^2x^2}}{15e^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^4*(d + e*x)^2)/\text{Sqrt}[d^2 - e^2*x^2], x]$

[Out]  $(-8*d^3*x^2*\text{Sqrt}[d^2 - e^2*x^2])/(15*e^3) - (11*d^2*x^3*\text{Sqrt}[d^2 - e^2*x^2])/(24*e^2) - (2*d*x^4*\text{Sqrt}[d^2 - e^2*x^2])/(5*e) - (x^5*\text{Sqrt}[d^2 - e^2*x^2])/6 - (d^4*(256*d + 165*e*x)*\text{Sqrt}[d^2 - e^2*x^2])/(240*e^5) + (11*d^6*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(16*e^5)$

**Rubi in Sympy [A]** time = 56.5409, size = 167, normalized size = 0.97

$$\frac{11d^6 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{16e^5} - \frac{2d^5\sqrt{d^2-e^2x^2}}{e^5} - \frac{11d^4x\sqrt{d^2-e^2x^2}}{16e^4} + \frac{4d^3(d^2-e^2x^2)^{\frac{3}{2}}}{3e^5} - \frac{11d^2x^3\sqrt{d^2-e^2x^2}}{24e^2} - \frac{2d(d^2-e^2x^2)^{\frac{5}{2}}}{5e^5} - \frac{x^5\sqrt{d^2-e^2x^2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**4*(e*x+d)**2/(-e**2*x**2+d**2)**(1/2), x)$

[Out]  $11*d**6*\operatorname{atan}(e*x/\text{sqrt}(d**2 - e**2*x**2))/(16*e**5) - 2*d**5*\text{sqrt}(d**2 - e**2*x**2)/e**5 - 11*d**4*x*\text{sqrt}(d**2 - e**2*x**2)/(16*e**4) + 4*d**3*(d**2 - e**2*x**2)**(3/2)/(3*e**5) - 11*d**2*x**3*\text{sqrt}(d**2 - e**2*x**2)/(24*e**2) - 2*d*(d**2 - e**2*x**2)**(5/2)/(5*e**5) - x**5*\text{sqrt}(d**2 - e**2*x**2)/6$

**Mathematica [A]** time = 0.115227, size = 103, normalized size = 0.6

$$\frac{165d^6 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \sqrt{d^2-e^2x^2} (256d^5 + 165d^4ex + 128d^3e^2x^2 + 110d^2e^3x^3 + 96de^4x^4 + 40e^5x^5)}{240e^5}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4\*(d + e\*x)^2)/Sqrt[d^2 - e^2\*x^2], x]

[Out]  $(-\text{Sqrt}[d^2 - e^2 x^2] * (256 d^5 + 165 d^4 e x + 128 d^3 e^2 x^2 + 110 d^2 e^3 x^3 + 96 d e^4 x^4 + 40 e^5 x^5)) + 165 d^6 \text{ArcTan}[(e x) / \text{Sqrt}[d^2 - e^2 x^2]] / (240 e^5)$

**Maple [A]** time = 0.036, size = 174, normalized size = 1.

$$-\frac{11 d^2 x^3}{24 e^2} \sqrt{-e^2 x^2 + d^2} - \frac{11 d^4 x}{16 e^4} \sqrt{-e^2 x^2 + d^2} + \frac{11 d^6}{16 e^4} \arctan\left(x \sqrt{e^2} \frac{1}{\sqrt{-e^2 x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} - \frac{x^5}{6} \sqrt{-e^2 x^2 + d^2} - \frac{2 d x^4}{5 e} \sqrt{-e^2 x^2 + d^2} - \frac{8 d^3 x^2}{15 e^3} \sqrt{-e^2 x^2 + d^2} - \frac{16 d^5}{15 e^5} \sqrt{-e^2 x^2 + d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(e\*x+d)^2/(-e^2\*x^2+d^2)^(1/2), x)

[Out]  $-11/24 * d^2 * x^3 * (-e^2 * x^2 + d^2)^{(1/2)} / e^2 - 11/16 * d^4 * x * (-e^2 * x^2 + d^2)^{(1/2)} / e^4 + 11/16 * d^6 / e^4 / (e^2)^{(1/2)} * \arctan((e^2)^{(1/2)} * x / (-e^2 * x^2 + d^2)^{(1/2)}) - 1/6 * x^5 * (-e^2 * x^2 + d^2)^{(1/2)} - 2/5 * d * x^4 * (-e^2 * x^2 + d^2)^{(1/2)} / e - 8/15 * d^3 * x^2 * (-e^2 * x^2 + d^2)^{(1/2)} / e^3 - 16/15 * d^5 * (-e^2 * x^2 + d^2)^{(1/2)} / e^5$

**Maxima [A]** time = 0.786545, size = 224, normalized size = 1.29

$$-\frac{1}{6} \sqrt{-e^2 x^2 + d^2} x^5 - \frac{2 \sqrt{-e^2 x^2 + d^2} d x^4}{5 e} - \frac{11 \sqrt{-e^2 x^2 + d^2} d^2 x^3}{24 e^2} - \frac{8 \sqrt{-e^2 x^2 + d^2} d^3 x^2}{15 e^3} + \frac{11 d^6 \arcsin\left(\frac{e^2 x}{\sqrt{d^2 e^2}}\right)}{16 \sqrt{e^2} e^4} - \frac{11 \sqrt{-e^2 x^2 + d^2} d^4 x}{16 e^4} - \frac{16 \sqrt{-e^2 x^2 + d^2} d^5}{15 e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*x^4/sqrt(-e^2\*x^2 + d^2), x, algorithm="maxima")

[Out]  $-1/6 * \text{sqrt}(-e^2 * x^2 + d^2) * x^5 - 2/5 * \text{sqrt}(-e^2 * x^2 + d^2) * d * x^4 / e - 11/24 * \text{sqrt}(-e^2 * x^2 + d^2) * d^2 * x^3 / e^2 - 8/15 * \text{sqrt}(-e^2 * x^2 + d^2) * d^3 * x^2 / e^3 + 11/16 * d^6 * \arcsin(e^2 * x / \text{sqrt}(d^2 * e^2)) / (\text{sqrt}(e^2) * e^4) - 11/16 * \text{sqrt}(-e^2 * x^2 + d^2) * d^4 * x / e^4 - 16/15 * \text{sqrt}(-e^2 * x^2 + d^2) * d^5 / e^5$

**Fricas [A]** time = 0.279516, size = 539, normalized size = 3.12

$$240 d e^{11} x^{11} + 576 d^2 e^{10} x^{10} - 860 d^3 e^9 x^9 - 2880 d^4 e^8 x^8 - 630 d^5 e^7 x^7 + 2560 d^6 e^6 x^6 - 510 d^7 e^5 x^5 + 7040 d^9 e^3 x^3 - 5280 d^{11} e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*x^4/sqrt(-e^2\*x^2 + d^2), x, algorithm="fricas")

[Out]  $1/240 * (240 * d * e^{11} * x^{11} + 576 * d^2 * e^{10} * x^{10} - 860 * d^3 * e^9 * x^9 - 2880 * d^4 * e^8 * x^8 - 630 * d^5 * e^7 * x^7 + 2560 * d^6 * e^6 * x^6 - 510 * d^7 * e^5 * x^5$

$$\begin{aligned}
 & x^5 + 7040d^9e^3x^3 - 5280d^{11}e^5x - 330(d^6e^6x^6 - 18d^8e^4x^4 + 48d^{10}e^2x^2 - 32d^{12} + 2(3d^7e^4x^4 - 16d^9e^2x^2 + 16d^{11}))\sqrt{-e^2x^2 + d^2})\arctan(-(d - \sqrt{-e^2x^2 + d^2})/(e^2x)) - (40e^{11}x^{11} + 96d^2e^{10}x^{10} - 610d^4e^9x^9 - 1600d^6e^8x^8 + 105d^8e^7x^7 + 2560d^{10}e^6x^6 + 1030d^{12}e^5x^5 + 4400d^{14}e^4x^4 - 5280d^{16}e^3x^3 - 5280d^{18}e^2x^2 + d^{20})\sqrt{-e^2x^2 + d^2})/(e^{11}x^6 - 18d^2e^9x^4 + 48d^4e^7x^2 - 32d^6e^5 + 2(3d^7e^4x^4 - 16d^9e^2x^2 + 16d^{11}))\sqrt{-e^2x^2 + d^2})
 \end{aligned}$$

**Sympy [A]** time = 37.0185, size = 558, normalized size = 3.23

$$\begin{aligned}
 & d^2 \left( \begin{aligned} & \left( \begin{aligned} & -\frac{3id^4 \operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^5} + \frac{3id^3x}{8e^4\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{id^2x^3}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{ix^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} \end{aligned} \right) \text{ for } \left| \frac{e^2x^2}{d^2} \right| > 1 \\ & \left( \begin{aligned} & \frac{3d^4 \operatorname{asin}\left(\frac{ex}{d}\right)}{8e^5} - \frac{3d^3x}{8e^4\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{dx^3}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} \end{aligned} \right) \text{ otherwise} \end{aligned} \right) \\
 & + 2de \left( \begin{aligned} & \left( \begin{aligned} & -\frac{8d^4\sqrt{d^2-e^2x^2}}{15e^6} - \frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^4} - \frac{x^4\sqrt{d^2-e^2x^2}}{5e^2} \end{aligned} \right) \text{ for } e \neq 0 \\ & \left( \begin{aligned} & \frac{x^6}{6\sqrt{d^2}} \end{aligned} \right) \text{ otherwise} \end{aligned} \right) \\
 & + e^2 \left( \begin{aligned} & \left( \begin{aligned} & -\frac{5id^6 \operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^7} + \frac{5id^5x}{16e^6\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{5id^3x^3}{48e^4\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{id^2x^5}{24e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{ix^7}{6d\sqrt{-1+\frac{e^2x^2}{d^2}}} \end{aligned} \right) \text{ for } \left| \frac{e^2x^2}{d^2} \right| > 1 \\ & \left( \begin{aligned} & \frac{5d^6 \operatorname{asin}\left(\frac{ex}{d}\right)}{16e^7} - \frac{5d^5x}{16e^6\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{5d^3x^3}{48e^4\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{dx^5}{24e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{x^7}{6d\sqrt{1-\frac{e^2x^2}{d^2}}} \end{aligned} \right) \text{ otherwise} \end{aligned} \right)
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(e\*x+d)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2),x)

[Out] d\*\*2\*Piecewise((-3\*I\*d\*\*4\*acosh(e\*x/d)/(8\*e\*\*5) + 3\*I\*d\*\*3\*x/(8\*e\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - I\*d\*x\*\*3/(8\*e\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - I\*x\*\*5/(4\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (3\*d\*\*4\*asin(e\*x/d)/(8\*e\*\*5) - 3\*d\*\*3\*x/(8\*e\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + d\*x\*\*3/(8\*e\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + x\*\*5/(4\*d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True)) + 2\*d\*e\*Piecewise((-8\*d\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*e\*\*6) - 4\*d\*\*2\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*e\*\*4) - x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(5\*e\*\*2), Ne(e, 0)), (x\*\*6/(6\*sqrt(d\*\*2)), True)) + e\*\*2\*Piecewise((-5\*I\*d\*\*6\*acosh(e\*x/d)/(16\*e\*\*7) + 5\*I\*d\*\*5\*x/(16\*e\*\*6\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 5\*I\*d\*\*3\*x\*\*3/(48\*e\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - I\*d\*x\*\*5/(24\*e\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - I\*x\*\*7/(6\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (5\*d\*\*6\*asin(e\*x/d)/(16\*e\*\*7) - 5\*d\*\*5\*x/(16\*e\*\*6\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 5\*d\*\*3\*x\*\*3/(48\*e\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + d\*x\*\*5/(24\*e\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + x\*\*7/(6\*d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True))

**GIAC/XCAS [A]** time = 0.291465, size = 113, normalized size = 0.65

$$\begin{aligned}
 & \frac{11}{16}d^6 \arcsin\left(\frac{xe}{d}\right) e^{(-5)} \operatorname{sign}(d) \\
 & - \frac{1}{240} \left( 256d^5e^{(-5)} + \left( 165d^4e^{(-4)} + 2 \left( 64d^3e^{(-3)} + \left( 55d^2e^{(-2)} + 4 \left( 12de^{(-1)} + 5x \right) x \right) x \right) x \right) \sqrt{-x^2e^2 + d^2}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*x^4/sqrt(-e^2\*x^2 + d^2),x, algorithm="giac")

[Out] 11/16\*d^6\*arcsin(x\*e/d)\*e^(-5)\*sign(d) - 1/240\*(256\*d^5\*e^(-5) + (165\*d^4\*e^(-4) + 2\*(64\*d^3\*e^(-3) + (55\*d^2\*e^(-2) + 4\*(12\*d\*e^(-1) + 5\*x)\*x)\*x)\*x)\*sqrt(-x^2\*e^2 + d^2)

$$3.34 \quad \int \frac{x^3(d+ex)^2}{\sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=144

$$-\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{2e} + \frac{3d^5 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^4} - \frac{3d^3(8d+5ex)\sqrt{d^2-e^2x^2}}{20e^4}$$

[Out]  $(-3*d^2*x^2*sqrt[d^2 - e^2*x^2])/(5*e^2) - (d*x^3*sqrt[d^2 - e^2*x^2])/(2*e) - (x^4*sqrt[d^2 - e^2*x^2])/5 - (3*d^3*(8*d + 5*e*x)*sqrt[d^2 - e^2*x^2])/(20*e^4) + (3*d^5*ArcTan[(e*x)/sqrt[d^2 - e^2*x^2]])/(4*e^4)$

**Rubi [A]** time = 0.446325, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$-\frac{3d^2x^2\sqrt{d^2-e^2x^2}}{5e^2} - \frac{1}{5}x^4\sqrt{d^2-e^2x^2} - \frac{dx^3\sqrt{d^2-e^2x^2}}{2e} + \frac{3d^5 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^4} - \frac{3d^3(8d+5ex)\sqrt{d^2-e^2x^2}}{20e^4}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(d + e\*x)^2)/Sqrt[d^2 - e^2\*x^2], x]

[Out]  $(-3*d^2*x^2*sqrt[d^2 - e^2*x^2])/(5*e^2) - (d*x^3*sqrt[d^2 - e^2*x^2])/(2*e) - (x^4*sqrt[d^2 - e^2*x^2])/5 - (3*d^3*(8*d + 5*e*x)*sqrt[d^2 - e^2*x^2])/(20*e^4) + (3*d^5*ArcTan[(e*x)/sqrt[d^2 - e^2*x^2]])/(4*e^4)$

**Rubi in Sympy [A]** time = 55.6679, size = 136, normalized size = 0.94

$$\frac{3d^5 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{4e^4} - \frac{2d^4\sqrt{d^2-e^2x^2}}{e^4} - \frac{3d^3x\sqrt{d^2-e^2x^2}}{4e^3} + \frac{d^2(d^2-e^2x^2)^{\frac{3}{2}}}{e^4} - \frac{dx^3\sqrt{d^2-e^2x^2}}{2e} - \frac{(d^2-e^2x^2)^{\frac{5}{2}}}{5e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(e\*x+d)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2), x)

[Out]  $3*d**5*atan(e*x/sqrt(d**2 - e**2*x**2))/(4*e**4) - 2*d**4*sqrt(d**2 - e**2*x**2)/e**4 - 3*d**3*x*sqrt(d**2 - e**2*x**2)/(4*e**3) + d**2*(d**2 - e**2*x**2)**(3/2)/e**4 - d*x**3*sqrt(d**2 - e**2*x**2)/(2*e) - (d**2 - e**2*x**2)**(5/2)/(5*e**4)$

**Mathematica [A]** time = 0.0984249, size = 92, normalized size = 0.64

$$\frac{15d^5 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \sqrt{d^2-e^2x^2} (24d^4 + 15d^3ex + 12d^2e^2x^2 + 10de^3x^3 + 4e^4x^4)}{20e^4}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(d + e\*x)^2)/Sqrt[d^2 - e^2\*x^2], x]



[Out]  $(-\text{Sqrt}[d^2 - e^2 x^2] * (24 d^4 + 15 d^3 e x + 12 d^2 e^2 x^2 + 10 d e^3 x^3 + 4 e^4 x^4)) + 15 d^5 \text{ArcTan}[(e x) / \text{Sqrt}[d^2 - e^2 x^2]] / (20 e^4)$

**Maple [A]** time = 0.014, size = 149, normalized size = 1.

$$-\frac{3 d^2 x^2}{5 e^2} \sqrt{-e^2 x^2 + d^2} - \frac{6 d^4}{5 e^4} \sqrt{-e^2 x^2 + d^2} - \frac{x^4}{5} \sqrt{-e^2 x^2 + d^2} - \frac{d x^3}{2 e} \sqrt{-e^2 x^2 + d^2} - \frac{3 d^3 x}{4 e^3} \sqrt{-e^2 x^2 + d^2} + \frac{3 d^5}{4 e^3} \arctan\left(x \sqrt{e^2} \frac{1}{\sqrt{-e^2 x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x)`

[Out]  $-3/5 d^2 x^2 (-e^2 x^2 + d^2)^{1/2} / e^2 - 6/5 d^4 (-e^2 x^2 + d^2)^{1/2} / e^4 - 1/5 x^4 (-e^2 x^2 + d^2)^{1/2} - 1/2 d x^3 (-e^2 x^2 + d^2)^{1/2} / e - 3/4 d^3 x (-e^2 x^2 + d^2)^{1/2} / e^3 + 3/4 d^5 / e^3 / (e^2)^{1/2} * \arctan((e^2)^{1/2} x / (-e^2 x^2 + d^2)^{1/2})$

**Maxima [A]** time = 0.791558, size = 190, normalized size = 1.32

$$-\frac{1}{5} \sqrt{-e^2 x^2 + d^2} x^4 - \frac{\sqrt{-e^2 x^2 + d^2} d x^3}{2 e} - \frac{3 \sqrt{-e^2 x^2 + d^2} d^2 x^2}{5 e^2} + \frac{3 d^5 \arcsin\left(\frac{e^2 x}{\sqrt{d^2 e^2}}\right)}{4 \sqrt{e^2} e^3} - \frac{3 \sqrt{-e^2 x^2 + d^2} d^3 x}{4 e^3} - \frac{6 \sqrt{-e^2 x^2 + d^2} d^4}{5 e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^2*x^3/sqrt(-e^2*x^2 + d^2),x, algorithm="maxima")`

[Out]  $-1/5 \sqrt{-e^2 x^2 + d^2} x^4 - 1/2 \sqrt{-e^2 x^2 + d^2} d x^3 / e - 3/5 \sqrt{-e^2 x^2 + d^2} d^2 x^2 / e^2 + 3/4 d^5 \arcsin(e^2 x / \sqrt{d^2 e^2}) / (\sqrt{e^2} e^3) - 3/4 \sqrt{-e^2 x^2 + d^2} d^3 x / e^3 - 6/5 \sqrt{-e^2 x^2 + d^2} d^4 / e^4$

**Fricas [A]** time = 0.277345, size = 479, normalized size = 3.33

$$\frac{4 e^{10} x^{10} + 10 d e^9 x^9 - 40 d^2 e^8 x^8 - 115 d^3 e^7 x^7 - 20 d^4 e^6 x^6 + 85 d^5 e^5 x^5 + 80 d^6 e^4 x^4 + 260 d^7 e^3 x^3 - 240 d^9 e x + 30 (5 d^6 e^4 x^4)}{20 (5 d^6 e^4 x^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^2*x^3/sqrt(-e^2*x^2 + d^2),x, algorithm="fricas")`

[Out]  $-1/20 (4 e^{10} x^{10} + 10 d e^9 x^9 - 40 d^2 e^8 x^8 - 115 d^3 e^7 x^7 - 20 d^4 e^6 x^6 + 85 d^5 e^5 x^5 + 80 d^6 e^4 x^4 + 260 d^7 e^3 x^3 - 240 d^9 e x + 30 (5 d^6 e^4 x^4 - 20 d^8 e^2 x^2 + 16 d^{10} - (d^5 e^4 x^4 - 12 d^7 e^2 x^2 + 16 d^9) \sqrt{-e^2 x^2 + d^2})) \arctan(- (d - \sqrt{-e^2 x^2 + d^2}) / (e x)) + 5 (4 d^6 e^4 x^4 + 10 d^2 e^7 x^7 - 4 d^3 e^6 x^6 - 25 d^4 e^5 x^5 - 16 d^5 e^4 x^4 - 28 d^6 e^3 x^3 + 48 d^8 e x) \sqrt{-e^2 x^2 + d^2} / (5 d^6 e^4 x^4 - 20 d^3 e^6 x^2 + 16 d^5 e^4 - (e^8 x^4 - 12 d^2 e^6 x^2 + 16 d^4 e^4) \sqrt{-e^2 x^2 + d^2})$

**Sympy [A]** time = 19.0817, size = 357, normalized size = 2.48

$$d^2 \left( \begin{cases} -\frac{2d^2\sqrt{d^2-e^2x^2}}{3e^4} - \frac{x^2\sqrt{d^2-e^2x^2}}{3e^2} & \text{for } e \neq 0 \\ \frac{x^4}{4\sqrt{d^2}} & \text{otherwise} \end{cases} \right) + 2de \left( \begin{cases} -\frac{3id^4 \operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^5} + \frac{3id^3x}{8e^4\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{id^2x^3}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{ix^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{3d^4 \operatorname{asin}\left(\frac{ex}{d}\right)}{8e^5} - \frac{3d^3x}{8e^4\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{dx^3}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases} \right) + e^2 \left( \begin{cases} -\frac{8d^4\sqrt{d^2-e^2x^2}}{15e^6} - \frac{4d^2x^2\sqrt{d^2-e^2x^2}}{15e^4} - \frac{x^4\sqrt{d^2-e^2x^2}}{5e^2} & \text{for } e \neq 0 \\ \frac{x^6}{6\sqrt{d^2}} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(e\*x+d)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2),x)

[Out] d\*\*2\*Piecewise((-2\*d\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(3\*e\*\*4) - x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(3\*e\*\*2), Ne(e, 0)), (x\*\*4/(4\*sqrt(d\*\*2)), True)) + 2\*d\*e\*Piecewise((-3\*I\*d\*\*4\*acosh(e\*x/d)/(8\*e\*\*5) + 3\*I\*d\*\*3\*x/(8\*e\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - I\*d\*x\*\*3/(8\*e\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - I\*x\*\*5/(4\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (3\*d\*\*4\*asin(e\*x/d)/(8\*e\*\*5) - 3\*d\*\*3\*x/(8\*e\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + d\*x\*\*3/(8\*e\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + x\*\*5/(4\*d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True)) + e\*\*2\*Piecewise((-8\*d\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*e\*\*6) - 4\*d\*\*2\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*e\*\*4) - x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(5\*e\*\*2), Ne(e, 0)), (x\*\*6/(6\*sqrt(d\*\*2)), True))

**GIAC/XCAS [A]** time = 0.294023, size = 99, normalized size = 0.69

$$\frac{3}{4}d^5 \arcsin\left(\frac{xe}{d}\right) e^{(-4)} \operatorname{sign}(d) - \frac{1}{20} \left( 24d^4e^{(-4)} + \left( 15d^3e^{(-3)} + 2 \left( 6d^2e^{(-2)} + \left( 5de^{(-1)} + 2x \right) x \right) x \right) \sqrt{-x^2e^2 + d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*x^3/sqrt(-e^2\*x^2 + d^2),x, algorithm="giac")

[Out] 3/4\*d^5\*arcsin(x\*e/d)\*e^(-4)\*sign(d) - 1/20\*(24\*d^4\*e^(-4) + (15\*d^3\*e^(-3) + 2\*(6\*d^2\*e^(-2) + (5\*d\*e^(-1) + 2\*x)\*x)\*x)\*sqrt(-x^2\*e^2 + d^2))

$$3.35 \quad \int \frac{x^2(d+ex)^2}{\sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=115

$$-\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e} - \frac{1}{4}x^3\sqrt{d^2-e^2x^2} - \frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3} + \frac{7d^4 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

[Out]  $(-2*d*x^2*\text{Sqrt}[d^2 - e^2*x^2])/(3*e) - (x^3*\text{Sqrt}[d^2 - e^2*x^2])/4 - (d^2*(32*d + 21*e*x)*\text{Sqrt}[d^2 - e^2*x^2])/(24*e^3) + (7*d^4*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(8*e^3)$

**Rubi [A]** time = 0.318736, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$-\frac{2dx^2\sqrt{d^2-e^2x^2}}{3e} - \frac{1}{4}x^3\sqrt{d^2-e^2x^2} - \frac{d^2(32d+21ex)\sqrt{d^2-e^2x^2}}{24e^3} + \frac{7d^4 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^2*(d + e*x)^2)/\text{Sqrt}[d^2 - e^2*x^2], x]$

[Out]  $(-2*d*x^2*\text{Sqrt}[d^2 - e^2*x^2])/(3*e) - (x^3*\text{Sqrt}[d^2 - e^2*x^2])/4 - (d^2*(32*d + 21*e*x)*\text{Sqrt}[d^2 - e^2*x^2])/(24*e^3) + (7*d^4*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(8*e^3)$

**Rubi in Sympy [A]** time = 41.0509, size = 116, normalized size = 1.01

$$\frac{7d^4 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3} - \frac{2d^3\sqrt{d^2-e^2x^2}}{e^3} - \frac{7d^2x\sqrt{d^2-e^2x^2}}{8e^2} + \frac{2d(d^2-e^2x^2)^{\frac{3}{2}}}{3e^3} - \frac{x^3\sqrt{d^2-e^2x^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**2}*(e*x+d)^{**2}/(-e^{**2}*x^{**2}+d^{**2})^{**}(1/2), x)$

[Out]  $7*d^{**4}*\operatorname{atan}(e*x/\text{sqrt}(d^{**2} - e^{**2}*x^{**2}))/ (8*e^{**3}) - 2*d^{**3}*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/e^{**3} - 7*d^{**2}*x*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/ (8*e^{**2}) + 2*d*(d^{**2} - e^{**2}*x^{**2})^{**}(3/2)/ (3*e^{**3}) - x^{**3}*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/4$

**Mathematica [A]** time = 0.0866911, size = 81, normalized size = 0.7

$$\frac{21d^4 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \sqrt{d^2-e^2x^2} (32d^3 + 21d^2ex + 16de^2x^2 + 6e^3x^3)}{24e^3}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(x^2*(d + e*x)^2)/\text{Sqrt}[d^2 - e^2*x^2], x]$

[Out]  $(-\text{Sqrt}[d^2 - e^2*x^2]*(32*d^3 + 21*d^2*e*x + 16*d*e^2*x^2 + 6*e^3*x^3)) + 21*d^4*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]]/(24*e^3)$

**Maple [A]** time = 0.013, size = 124, normalized size = 1.1

$$-\frac{7d^2x}{8e^2}\sqrt{-e^2x^2+d^2} + \frac{7d^4}{8e^2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}} - \frac{x^3}{4}\sqrt{-e^2x^2+d^2} - \frac{2dx^2}{3e}\sqrt{-e^2x^2+d^2} - \frac{4d^3}{3e^3}\sqrt{-e^2x^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(e\*x+d)^2/(-e^2\*x^2+d^2)^(1/2),x)

[Out] -7/8\*d^2\*x/e^2\*(-e^2\*x^2+d^2)^(1/2)+7/8\*d^4/e^2/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))-1/4\*x^3\*(-e^2\*x^2+d^2)^(1/2)-2/3\*d\*x^2\*(-e^2\*x^2+d^2)^(1/2)/e-4/3\*d^3\*(-e^2\*x^2+d^2)^(1/2)/e^3

**Maxima [A]** time = 0.786122, size = 157, normalized size = 1.37

$$-\frac{1}{4}\sqrt{-e^2x^2+d^2}x^3 - \frac{2\sqrt{-e^2x^2+d^2}dx^2}{3e} + \frac{7d^4\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{8\sqrt{e^2e^2}} - \frac{7\sqrt{-e^2x^2+d^2}d^2x}{8e^2} - \frac{4\sqrt{-e^2x^2+d^2}d^3}{3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*x^2/sqrt(-e^2\*x^2 + d^2),x, algorithm="maxima")

[Out] -1/4\*sqrt(-e^2\*x^2 + d^2)\*x^3 - 2/3\*sqrt(-e^2\*x^2 + d^2)\*d\*x^2/e + 7/8\*d^4\*arcsin(e^2\*x/sqrt(d^2\*e^2))/(sqrt(e^2)\*e^2) - 7/8\*sqrt(-e^2\*x^2 + d^2)\*d^2\*x/e^2 - 4/3\*sqrt(-e^2\*x^2 + d^2)\*d^3/e^3

**Fricas [A]** time = 0.276942, size = 387, normalized size = 3.37

$$\frac{24de^7x^7 + 64d^2e^6x^6 + 12d^3e^5x^5 - 96d^4e^4x^4 - 204d^5e^3x^3 + 168d^7ex - 42(d^4e^4x^4 - 8d^6e^2x^2 + 8d^8 + 4(d^5e^2x^2 - 2d^7))\sqrt{-e^2x^2 + d^2}}{24(e^7x^4 - 8d^2e^5x^2 + 8d^4e^3 + 4d^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*x^2/sqrt(-e^2\*x^2 + d^2),x, algorithm="fricas")

[Out] 1/24\*(24\*d\*e^7\*x^7 + 64\*d^2\*e^6\*x^6 + 12\*d^3\*e^5\*x^5 - 96\*d^4\*e^4\*x^4 - 204\*d^5\*e^3\*x^3 + 168\*d^7\*e\*x - 42\*(d^4\*e^4\*x^4 - 8\*d^6\*e^2\*x^2 + 8\*d^8 + 4\*(d^5\*e^2\*x^2 - 2\*d^7))\*sqrt(-e^2\*x^2 + d^2))\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) - (6\*e^7\*x^7 + 16\*d\*e^6\*x^6 - 27\*d^2\*e^5\*x^5 - 96\*d^3\*e^4\*x^4 - 120\*d^4\*e^3\*x^3 + 168\*d^6\*e\*x)\*sqrt(-e^2\*x^2 + d^2)/(e^7\*x^4 - 8\*d^2\*e^5\*x^2 + 8\*d^4\*e^3 + 4\*(d\*e^5\*x^2 - 2\*d^3\*e^3))\*sqrt(-e^2\*x^2 + d^2)

**Sympy [A]** time = 22.6082, size = 386, normalized size = 3.36

$$d^2 \left( \begin{array}{l} \left( \begin{array}{l} -\frac{id^2 \operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3} - \frac{idx\sqrt{-1+\frac{e^2x^2}{d^2}}}{2e^2} \\ \frac{d^2 \operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3} - \frac{dx}{2e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{x^3}{2d\sqrt{1-\frac{e^2x^2}{d^2}}} \end{array} \right) \text{ for } \left| \frac{e^2x^2}{d^2} \right| > 1 \\ \text{otherwise} \end{array} \right) \\ + 2de \left( \begin{array}{l} \left( \begin{array}{l} -\frac{2d^2\sqrt{d^2-e^2x^2}}{3e^4} - \frac{x^2\sqrt{d^2-e^2x^2}}{3e^2} \\ \frac{x^4}{4\sqrt{d^2}} \end{array} \right) \text{ for } e \neq 0 \\ \text{otherwise} \end{array} \right) \\ + e^2 \left( \begin{array}{l} \left( \begin{array}{l} -\frac{3id^4 \operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^5} + \frac{3id^3x}{8e^4\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{idx^3}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{ix^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} \\ \frac{3d^4 \operatorname{asin}\left(\frac{ex}{d}\right)}{8e^5} - \frac{3d^3x}{8e^4\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{dx^3}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} \end{array} \right) \text{ for } \left| \frac{e^2x^2}{d^2} \right| > 1 \\ \text{otherwise} \end{array} \right) \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(e\*x+d)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2),x)

[Out] d\*\*2\*Piecewise((-I\*d\*\*2\*acosh(e\*x/d)/(2\*e\*\*3) - I\*d\*x\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)/(2\*e\*\*2), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*2\*asin(e\*x/d)/(2\*e\*\*3) - d\*x/(2\*e\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + x\*\*3/(2\*d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True)) + 2\*d\*e\*Piecewise((-2\*d\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(3\*e\*\*4) - x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(3\*e\*\*2), Ne(e, 0)), (x\*\*4/(4\*sqrt(d\*\*2)), True)) + e\*\*2\*Piecewise((-3\*I\*d\*\*4\*acosh(e\*x/d)/(8\*e\*\*5) + 3\*I\*d\*\*3\*x/(8\*e\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - I\*d\*x\*\*3/(8\*e\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - I\*x\*\*5/(4\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (3\*d\*\*4\*asin(e\*x/d)/(8\*e\*\*5) - 3\*d\*\*3\*x/(8\*e\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + d\*x\*\*3/(8\*e\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + x\*\*5/(4\*d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True))

**GIAC/XCAS [A]** time = 0.288324, size = 85, normalized size = 0.74

$$\frac{7}{8}d^4 \arcsin\left(\frac{xe}{d}\right) e^{(-3)} \operatorname{sign}(d) - \frac{1}{24} \left( 32d^3 e^{(-3)} + \left( 21d^2 e^{(-2)} + 2 \left( 8de^{(-1)} + 3x \right) x \right) \sqrt{-x^2 e^2 + d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*x^2/sqrt(-e^2\*x^2 + d^2),x, algorithm="giac")

[Out] 7/8\*d^4\*arcsin(x\*e/d)\*e^(-3)\*sign(d) - 1/24\*(32\*d^3\*e^(-3) + (21\*d^2\*e^(-2) + 2\*(8\*d\*e^(-1) + 3\*x)\*x)\*sqrt(-x^2\*e^2 + d^2))

$$3.36 \quad \int \frac{x(d+ex)^2}{\sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=83

$$-\frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2} - \frac{1}{3}x^2\sqrt{d^2-e^2x^2} + \frac{d^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

[Out]  $-(x^2 \sqrt{d^2 - e^2 x^2})/3 - (d(5d + 3ex) \sqrt{d^2 - e^2 x^2})/(3e^2) + (d^3 \text{ArcTan}[(ex)/\sqrt{d^2 - e^2 x^2}])/e^2$

**Rubi [A]** time = 0.174011, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$-\frac{d(5d+3ex)\sqrt{d^2-e^2x^2}}{3e^2} - \frac{1}{3}x^2\sqrt{d^2-e^2x^2} + \frac{d^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2}$$

Antiderivative was successfully verified.

[In] Int[(x\*(d + e\*x)^2)/Sqrt[d^2 - e^2\*x^2], x]

[Out]  $-(x^2 \sqrt{d^2 - e^2 x^2})/3 - (d(5d + 3ex) \sqrt{d^2 - e^2 x^2})/(3e^2) + (d^3 \text{ArcTan}[(ex)/\sqrt{d^2 - e^2 x^2}])/e^2$

**Rubi in Sympy [A]** time = 22.0623, size = 95, normalized size = 1.14

$$\frac{d^3 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^2} - \frac{d^2 \sqrt{d^2-e^2x^2}}{e^2} - \frac{d(d+ex)\sqrt{d^2-e^2x^2}}{3e^2} - \frac{(d+ex)^2 \sqrt{d^2-e^2x^2}}{3e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(e\*x+d)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2), x)

[Out]  $d^3 \operatorname{atan}(ex/\sqrt{d^2 - e^2 x^2})/e^2 - d^2 \sqrt{d^2 - e^2 x^2}/e^2 - d(d + ex) \sqrt{d^2 - e^2 x^2}/(3e^2) - (d + ex)^2 \sqrt{d^2 - e^2 x^2}/(3e^2)$

**Mathematica [A]** time = 0.0726329, size = 70, normalized size = 0.84

$$\left(-\frac{5d^2}{3e^2} - \frac{dx}{e} - \frac{x^2}{3}\right) \sqrt{d^2 - e^2 x^2} + \frac{d^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(d + e\*x)^2)/Sqrt[d^2 - e^2\*x^2], x]

[Out]  $((-5d^2)/(3e^2) - (d*x)/e - x^2/3) \sqrt{d^2 - e^2 x^2} + (d^3 \operatorname{ArcTan}[(ex)/\sqrt{d^2 - e^2 x^2}])/e^2$

**Maple [A]** time = 0.012, size = 98, normalized size = 1.2

$$-\frac{5d^2}{3e^2} \sqrt{-e^2 x^2 + d^2} - \frac{x^2}{3} \sqrt{-e^2 x^2 + d^2} - \frac{dx}{e} \sqrt{-e^2 x^2 + d^2} + \frac{d^3}{e} \arctan\left(x \sqrt{e^2} \frac{1}{\sqrt{-e^2 x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x)`

[Out] 
$$-5/3*d^2/e^2*(-e^2*x^2+d^2)^(1/2)-1/3*x^2*(-e^2*x^2+d^2)^(1/2)-d*x*(-e^2*x^2+d^2)^(1/2)/e+d^3/e/(e^2)^(1/2)*\arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))$$

**Maxima [A]** time = 0.791504, size = 122, normalized size = 1.47

$$-\frac{1}{3}\sqrt{-e^2x^2+d^2}x^2+\frac{d^3\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{\sqrt{e^2}e}-\frac{\sqrt{-e^2x^2+d^2}dx}{e}-\frac{5\sqrt{-e^2x^2+d^2}d^2}{3e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^2*x/sqrt(-e^2*x^2+d^2),x,algorithm="maxima")`

[Out] 
$$-1/3*\sqrt{-e^2*x^2+d^2}*x^2+d^3*\arcsin(e^2*x/\sqrt{d^2*e^2})/(\sqrt{e^2}*e)-\sqrt{-e^2*x^2+d^2}*d*x/e-5/3*\sqrt{-e^2*x^2+d^2}*d^2/e^2$$

**Fricas [A]** time = 0.280954, size = 313, normalized size = 3.77

$$\frac{e^6x^6+3de^5x^5-15d^3e^3x^3-6d^4e^2x^2+12d^5ex+6\left(3d^4e^2x^2-4d^6-(d^3e^2x^2-4d^5)\sqrt{-e^2x^2+d^2}\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)}{3\left(3de^4x^2-4d^3e^2-(e^4x^2-4d^2e^2)\sqrt{-e^2x^2+d^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^2*x/sqrt(-e^2*x^2+d^2),x,algorithm="fricas")`

[Out] 
$$-1/3*(e^6*x^6+3*d*e^5*x^5-15*d^3*e^3*x^3-6*d^4*e^2*x^2+12*d^5*e*x+6*(3*d^4*e^2*x^2-4*d^6-(d^3*e^2*x^2-4*d^5)*\sqrt{-e^2*x^2+d^2})*\arctan(-(d-\sqrt{-e^2*x^2+d^2})/(e*x))+3*(d*e^4*x^4+3*d^2*e^3*x^3+2*d^3*e^2*x^2-4*d^4*e*x)*\sqrt{-e^2*x^2+d^2})/(3*d*e^4*x^2-4*d^3*e^2-(e^4*x^2-4*d^2*e^2)*\sqrt{-e^2*x^2+d^2})$$

**Sympy [A]** time = 11.352, size = 218, normalized size = 2.63

$$d^2\left(\begin{cases} \frac{x^2}{2\sqrt{d^2}} & \text{for } e^2 = 0 \\ -\frac{\sqrt{d^2-e^2x^2}}{e^2} & \text{otherwise} \end{cases}\right) + 2de\left(\begin{cases} \frac{id^2\operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3} - \frac{idx\sqrt{-1+\frac{e^2x^2}{d^2}}}{2e^2} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^2\operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3} - \frac{dx}{2e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{x^3}{2d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases}\right) + e^2\left(\begin{cases} -\frac{2d^2\sqrt{d^2-e^2x^2}}{3e^4} - \frac{x^2\sqrt{d^2-e^2x^2}}{3e^2} & \text{for } e \neq 0 \\ \frac{x^4}{4\sqrt{d^2}} & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)`

[Out] 
$$d^{**2}*\operatorname{Piecewise}((x^{**2}/(2*\sqrt{d^{**2}})), \operatorname{Eq}(e^{**2}, 0)), (-\sqrt{d^{**2}-e^{**2}*x^{**2}}/e^{**2}, \operatorname{True})) + 2*d*e*\operatorname{Piecewise}((-I*d^{**2}*\operatorname{acosh}(e*x/d)/(\sqrt{1-\frac{e^2x^2}{d^2}})), \operatorname{Eq}(e^2x^2/d^2, 1)), (d^2*\operatorname{asin}(e*x/d)/\sqrt{1-\frac{e^2x^2}{d^2}} - \frac{dx}{\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{x^3}{2d\sqrt{1-\frac{e^2x^2}{d^2}}}, \operatorname{True})) + e^2*\operatorname{Piecewise}(-\frac{2d^2\sqrt{d^2-e^2x^2}}{3e^4} - \frac{x^2\sqrt{d^2-e^2x^2}}{3e^2}, \operatorname{Eq}(e^2, 0)), (\frac{x^4}{4\sqrt{d^2}}, \operatorname{True}))$$

```

2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2)/(2*e**2), Abs(e**2*x**2
/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e**2*sqrt(1 - e*
**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), True)) + e*
**2*Piecewise((-2*d**2*sqrt(d**2 - e**2*x**2)/(3*e**4) - x**2*sqrt
(d**2 - e**2*x**2)/(3*e**2), Ne(e, 0)), (x**4/(4*sqrt(d**2)), Tru
e))

```

**GIAC/XCAS [A]** time = 0.288799, size = 66, normalized size = 0.8

$$d^3 \arcsin\left(\frac{xe}{d}\right) e^{(-2)} \operatorname{sign}(d) - \frac{1}{3} \sqrt{-x^2 e^2 + d^2} \left(5 d^2 e^{(-2)} + (3 d e^{(-1)} + x) x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^2*x/sqrt(-e^2*x^2 + d^2),x, algorithm="giac")
```

```
[Out] d^3*arcsin(x*e/d)*e^(-2)*sign(d) - 1/3*sqrt(-x^2*e^2 + d^2)*(5*d^
2*e^(-2) + (3*d*e^(-1) + x)*x)
```



$$3.37 \quad \int \frac{(d+ex)^2}{\sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=83

$$-\frac{3d\sqrt{d^2-e^2x^2}}{2e} - \frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e} + \frac{3d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

[Out]  $(-3*d*\text{Sqrt}[d^2 - e^2*x^2])/(2*e) - ((d + e*x)*\text{Sqrt}[d^2 - e^2*x^2])/(2*e) + (3*d^2*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(2*e)$

**Rubi [A]** time = 0.0767847, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{3d\sqrt{d^2-e^2x^2}}{2e} - \frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e} + \frac{3d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^2/Sqrt[d^2 - e^2\*x^2], x]

[Out]  $(-3*d*\text{Sqrt}[d^2 - e^2*x^2])/(2*e) - ((d + e*x)*\text{Sqrt}[d^2 - e^2*x^2])/(2*e) + (3*d^2*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(2*e)$

**Rubi in Sympy [A]** time = 16.6011, size = 68, normalized size = 0.82

$$\frac{3d^2 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e} - \frac{3d\sqrt{d^2-e^2x^2}}{2e} - \frac{(d+ex)\sqrt{d^2-e^2x^2}}{2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2), x)

[Out]  $3*d**2*\operatorname{atan}(e*x/\text{sqrt}(d**2 - e**2*x**2))/(2*e) - 3*d*\text{sqrt}(d**2 - e**2*x**2)/(2*e) - (d + e*x)*\text{sqrt}(d**2 - e**2*x**2)/(2*e)$

**Mathematica [A]** time = 0.0518996, size = 60, normalized size = 0.72

$$\left(-\frac{2d}{e} - \frac{x}{2}\right)\sqrt{d^2-e^2x^2} + \frac{3d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^2/Sqrt[d^2 - e^2\*x^2], x]

[Out]  $((-2*d)/e - x/2)*\text{Sqrt}[d^2 - e^2*x^2] + (3*d^2*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(2*e)$

**Maple [A]** time = 0.011, size = 71, normalized size = 0.9

$$\frac{3d^2}{2} \arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right) \frac{1}{\sqrt{e^2}} - \frac{x}{2}\sqrt{-e^2x^2+d^2} - 2\frac{d\sqrt{-e^2x^2+d^2}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^2/(-e^2*x^2+d^2)^(1/2),x)`

[Out]  $\frac{3}{2}d^2/(e^2)^{(1/2)}*\arctan((e^2)^{(1/2)}*x/(-e^2*x^2+d^2)^{(1/2)})-1/2*x*(-e^2*x^2+d^2)^{(1/2)}-2*d*(-e^2*x^2+d^2)^{(1/2)}/e$

**Maxima [A]** time = 0.792153, size = 85, normalized size = 1.02

$$\frac{3d^2 \arcsin\left(\frac{e^2 x}{\sqrt{d^2 e^2}}\right)}{2\sqrt{e^2}} - \frac{1}{2} \sqrt{-e^2 x^2 + d^2} x - \frac{2\sqrt{-e^2 x^2 + d^2} d}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^2/sqrt(-e^2*x^2 + d^2),x, algorithm="maxima")`

[Out]  $\frac{3}{2}d^2*\arcsin(e^2*x/\sqrt{d^2*e^2})/\sqrt{e^2} - \frac{1}{2}*\sqrt{-e^2*x^2 + d^2}*x - 2*\sqrt{-e^2*x^2 + d^2}*d/e$

**Fricas [A]** time = 0.274417, size = 227, normalized size = 2.73

$$\frac{2de^3x^3 + 4d^2e^2x^2 - 2d^3ex - 6\left(d^2e^2x^2 - 2d^4 + 2\sqrt{-e^2x^2 + d^2}d^3\right) \arctan\left(\frac{d - \sqrt{-e^2x^2 + d^2}}{ex}\right) - (e^3x^3 + 4de^2x^2 - 2d^2ex)\sqrt{-e^2x^2 + d^2}}{2\left(e^3x^2 - 2d^2e + 2\sqrt{-e^2x^2 + d^2}de\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^2/sqrt(-e^2*x^2 + d^2),x, algorithm="fricas")`

[Out]  $\frac{1}{2}*(2*d*e^3*x^3 + 4*d^2*e^2*x^2 - 2*d^3*e*x - 6*(d^2*e^2*x^2 - 2*d^4 + 2*\sqrt{-e^2*x^2 + d^2}*d^3)*\arctan(-(d - \sqrt{-e^2*x^2 + d^2})/(e*x)) - (e^3*x^3 + 4*d^2*e^2*x^2 - 2*d^2*e*x)*\sqrt{-e^2*x^2 + d^2})/(e^3*x^2 - 2*d^2*e + 2*\sqrt{-e^2*x^2 + d^2}*d*e)$

**Sympy [A]** time = 9.96247, size = 274, normalized size = 3.3

$$d^2 \left( \begin{array}{l} \frac{\sqrt{\frac{d^2}{e^2}} \operatorname{asin}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} \quad \text{for } d^2 > 0 \wedge -e^2 < 0 \\ \frac{\sqrt{-\frac{d^2}{e^2}} \operatorname{asinh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} \quad \text{for } d^2 > 0 \wedge -e^2 > 0 \\ \frac{\sqrt{\frac{d^2}{e^2}} \operatorname{acosh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{-d^2}} \quad \text{for } -e^2 > 0 \wedge d^2 < 0 \end{array} \right) + 2de \left( \begin{array}{l} \frac{x^2}{2\sqrt{d^2}} \quad \text{for } e^2 = 0 \\ -\frac{\sqrt{d^2 - e^2 x^2}}{e^2} \quad \text{otherwise} \end{array} \right) + e^2 \left( \begin{array}{l} -\frac{id^2 \operatorname{acosh}\left(\frac{ex}{d}\right)}{2e^3} - \frac{idx\sqrt{-1 + \frac{e^2 x^2}{d^2}}}{2e^2} \quad \text{for } \left|\frac{e^2 x^2}{d^2}\right| > 1 \\ \frac{d^2 \operatorname{asin}\left(\frac{ex}{d}\right)}{2e^3} - \frac{dx}{2e^2\sqrt{1 - \frac{e^2 x^2}{d^2}}} + \frac{x^3}{2d\sqrt{1 - \frac{e^2 x^2}{d^2}}} \quad \text{otherwise} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**2/(-e**2*x**2+d**2)**(1/2),x)`

[Out]  $d^{**2}*\operatorname{Piecewise}((\sqrt{d^{**2}/e^{**2}})*\operatorname{asin}(x*\sqrt{e^{**2}/d^{**2}})/\sqrt{d^{**2}}, (d^{**2} > 0) \& (-e^{**2} < 0)), (\sqrt{-d^{**2}/e^{**2}})*\operatorname{asinh}(x*\sqrt{-e^{**2}/d^{**2}})/\sqrt{-d^{**2}}, (d^{**2} > 0) \& (-e^{**2} > 0)), (\sqrt{d^{**2}/e^{**2}})*\operatorname{acosh}(x*\sqrt{e^{**2}/d^{**2}})/\sqrt{-d^{**2}}, (-e^{**2} > 0) \& (d^{**2} < 0))$

```

2/d**2))/sqrt(d**2), (d**2 > 0) & (-e**2 > 0)), (sqrt(d**2/e**2)*
acosh(x*sqrt(e**2/d**2))/sqrt(-d**2), (d**2 < 0) & (-e**2 > 0)))
+ 2*d*e*Piecewise((x**2/(2*sqrt(d**2)), Eq(e**2, 0)), (-sqrt(d**2
- e**2*x**2)/e**2, True)) + e**2*Piecewise((-I*d**2*acosh(e*x/d)
/(2*e**3) - I*d*x*sqrt(-1 + e**2*x**2/d**2)/(2*e**2), Abs(e**2*x
**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e**3) - d*x/(2*e**2*sqrt(1 -
e**2*x**2/d**2)) + x**3/(2*d*sqrt(1 - e**2*x**2/d**2)), True))

```

**GIAC/XCAS [A]** time = 0.292262, size = 54, normalized size = 0.65

$$\frac{3}{2} d^2 \arcsin\left(\frac{xe}{d}\right) e^{(-1)} \text{sign}(d) - \frac{1}{2} \sqrt{-x^2 e^2 + d^2} (4 d e^{(-1)} + x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^2/sqrt(-e^2*x^2 + d^2),x, algorithm="giac")
```

```
[Out] 3/2*d^2*arcsin(x*e/d)*e^(-1)*sign(d) - 1/2*sqrt(-x^2*e^2 + d^2)*(
4*d*e^(-1) + x)
```

$$3.38 \quad \int \frac{(d+ex)^2}{x\sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=66

$$-\sqrt{d^2 - e^2x^2} + 2d \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2x^2}} \right) - d \tanh^{-1} \left( \frac{\sqrt{d^2 - e^2x^2}}{d} \right)$$

[Out] -Sqrt[d^2 - e^2\*x^2] + 2\*d\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] - d\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]

**Rubi [A]** time = 0.235696, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$-\sqrt{d^2 - e^2x^2} + 2d \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2x^2}} \right) - d \tanh^{-1} \left( \frac{\sqrt{d^2 - e^2x^2}}{d} \right)$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^2/(x\*Sqrt[d^2 - e^2\*x^2]),x]

[Out] -Sqrt[d^2 - e^2\*x^2] + 2\*d\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] - d\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]

**Rubi in Sympy [A]** time = 23.2742, size = 53, normalized size = 0.8

$$2d \operatorname{atan} \left( \frac{ex}{\sqrt{d^2 - e^2x^2}} \right) - d \operatorname{atanh} \left( \frac{\sqrt{d^2 - e^2x^2}}{d} \right) - \sqrt{d^2 - e^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*2/x/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2),x)

[Out] 2\*d\*atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2)) - d\*atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d) - sqrt(d\*\*2 - e\*\*2\*x\*\*2)

**Mathematica [A]** time = 0.049789, size = 68, normalized size = 1.03

$$-\sqrt{d^2 - e^2x^2} - d \log \left( \sqrt{d^2 - e^2x^2} + d \right) + 2d \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2x^2}} \right) + d \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^2/(x\*Sqrt[d^2 - e^2\*x^2]),x]

[Out] -Sqrt[d^2 - e^2\*x^2] + 2\*d\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] + d\*Log[x] - d\*Log[d + Sqrt[d^2 - e^2\*x^2]]

**Maple [A]** time = 0.012, size = 91, normalized size = 1.4

$$-d^2 \ln \left( \frac{1}{x} \left( 2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2} \right) \right) \frac{1}{\sqrt{d^2}} - \sqrt{-e^2x^2 + d^2} + 2 \frac{ed}{\sqrt{e^2}} \arctan \left( \frac{\sqrt{e^2}x}{\sqrt{-e^2x^2 + d^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^2/x/(-e^2*x^2+d^2)^(1/2),x)`

[Out]  $-d^2/(d^2)^{1/2} * \ln((2*d^2+2*(d^2)^{1/2}*(-e^2*x^2+d^2)^{1/2})/x) - (-e^2*x^2+d^2)^{1/2} + 2*e*d/(e^2)^{1/2} * \arctan((e^2)^{1/2}*x/(-e^2*x^2+d^2)^{1/2})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^2/(sqrt(-e^2*x^2+d^2)*x),x,algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.28202, size = 171, normalized size = 2.59

$$\frac{e^2x^2 + 4 \left( d^2 - \sqrt{-e^2x^2 + d^2}d \right) \arctan \left( -\frac{d - \sqrt{-e^2x^2 + d^2}}{ex} \right) - \left( d^2 - \sqrt{-e^2x^2 + d^2}d \right) \log \left( -\frac{d - \sqrt{-e^2x^2 + d^2}}{x} \right)}{d - \sqrt{-e^2x^2 + d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^2/(sqrt(-e^2*x^2+d^2)*x),x,algorithm="fricas")`

[Out]  $-(e^2*x^2 + 4*(d^2 - \sqrt{-e^2*x^2 + d^2}*d)*\arctan(-(d - \sqrt{-e^2*x^2 + d^2})/(e*x)) - (d^2 - \sqrt{-e^2*x^2 + d^2}*d)*\log(-(d - \sqrt{-e^2*x^2 + d^2})/x))/(d - \sqrt{-e^2*x^2 + d^2})$

**Sympy [A]** time = 6.54943, size = 189, normalized size = 2.86

$$d^2 \left( \begin{cases} -\frac{\operatorname{acosh}\left(\frac{d}{ex}\right)}{d} & \text{for } \left| \frac{d^2}{e^2x^2} \right| > 1 \\ \frac{i \operatorname{asin}\left(\frac{d}{ex}\right)}{d} & \text{otherwise} \end{cases} \right) + 2de \left( \begin{cases} \frac{\sqrt{\frac{d^2}{e^2}} \operatorname{asin}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2 > 0 \wedge -e^2 < 0 \\ \frac{\sqrt{-\frac{d^2}{e^2}} \operatorname{asinh}\left(x\sqrt{-\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2 > 0 \wedge -e^2 > 0 \\ \frac{\sqrt{\frac{d^2}{e^2}} \operatorname{acosh}\left(x\sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{-d^2}} & \text{for } -e^2 > 0 \wedge d^2 < 0 \end{cases} \right) + e^2 \left( \begin{cases} \frac{x^2}{2\sqrt{d^2}} & \text{for } e^2 = 0 \\ -\frac{\sqrt{d^2 - e^2x^2}}{e^2} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**2/x/(-e**2*x**2+d**2)**(1/2),x)`

[Out]  $d^{**2} * \operatorname{Piecewise}((- \operatorname{acosh}(d/(e*x))/d, \operatorname{Abs}(d^{**2}/(e^{**2}*x^{**2})) > 1), (i * \operatorname{asin}(d/(e*x))/d, \operatorname{True})) + 2*d*e * \operatorname{Piecewise}((\sqrt{d^{**2}/e^{**2}}) * \operatorname{asin}(x * \sqrt{e^{**2}/d^{**2}}) / \sqrt{d^{**2}}, (d^{**2} > 0) \& (-e^{**2} < 0)), (\sqrt{-d^{**2}/e^{**2}}) * \operatorname{asinh}(x * \sqrt{-e^{**2}/d^{**2}}) / \sqrt{d^{**2}}, (d^{**2} > 0) \& (-e^{**2} > 0)), (\sqrt{d^{**2}/e^{**2}}) * \operatorname{acosh}(x * \sqrt{e^{**2}/d^{**2}}) / \sqrt{-d^{**2}}, (d^{**2} < 0) \& (-e^{**2} > 0))) + e^{**2} * \operatorname{Piecewise}((x^{**2}/(2 * \sqrt{d^{**2}})), \operatorname{Eq}(e^{**2}, 0)), (-\sqrt{d^{**2} - e^{**2}*x^{**2}}/e^{**2}, \operatorname{True}))$

---

**GIAC/XCAS [A]** time = 0.291511, size = 88, normalized size = 1.33

$$2 d \arcsin\left(\frac{x e}{d}\right) \operatorname{sign}(d) - d \ln\left(\frac{\left|-2 d e - 2 \sqrt{-x^2 e^2 + d^2} e\right| e^{(-2)}}{2 |x|}\right) - \sqrt{-x^2 e^2 + d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2/(sqrt(-e^2\*x^2 + d^2)\*x),x, algorithm="giac")

[Out] 2\*d\*arcsin(x\*e/d)\*sign(d) - d\*ln(1/2\*abs(-2\*d\*e - 2\*sqrt(-x^2\*e^2 + d^2)\*e)\*e^(-2)/abs(x)) - sqrt(-x^2\*e^2 + d^2)

$$3.39 \quad \int \frac{(d+ex)^2}{x^2 \sqrt{d^2 - e^2 x^2}} dx$$

**Optimal.** Leaf size=68

$$-\frac{\sqrt{d^2 - e^2 x^2}}{x} + e \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) - 2e \tanh^{-1} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right)$$

[Out]  $-(\text{Sqrt}[d^2 - e^2 x^2]/x) + e \text{ArcTan}[(e x)/\text{Sqrt}[d^2 - e^2 x^2]] - 2 e \text{ArcTanh}[\text{Sqrt}[d^2 - e^2 x^2]/d]$

**Rubi [A]** time = 0.235571, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$-\frac{\sqrt{d^2 - e^2 x^2}}{x} + e \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) - 2e \tanh^{-1} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e x)^2/(x^2 \text{Sqrt}[d^2 - e^2 x^2]), x]$

[Out]  $-(\text{Sqrt}[d^2 - e^2 x^2]/x) + e \text{ArcTan}[(e x)/\text{Sqrt}[d^2 - e^2 x^2]] - 2 e \text{ArcTanh}[\text{Sqrt}[d^2 - e^2 x^2]/d]$

**Rubi in Sympy [A]** time = 24.2617, size = 54, normalized size = 0.79

$$e \operatorname{atan} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) - 2e \operatorname{atanh} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right) - \frac{\sqrt{d^2 - e^2 x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)**2/x**2/(-e**2*x**2+d**2)**(1/2), x)$

[Out]  $e \operatorname{atan}(e x/\text{sqrt}(d^2 - e^2 x^2)) - 2 e \operatorname{atanh}(\text{sqrt}(d^2 - e^2 x^2)/d) - \text{sqrt}(d^2 - e^2 x^2)/x$

**Mathematica [A]** time = 0.0671542, size = 71, normalized size = 1.04

$$-\frac{\sqrt{d^2 - e^2 x^2}}{x} - 2e \log \left( \sqrt{d^2 - e^2 x^2} + d \right) + e \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) + 2e \log(x)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(d + e x)^2/(x^2 \text{Sqrt}[d^2 - e^2 x^2]), x]$

[Out]  $-(\text{Sqrt}[d^2 - e^2 x^2]/x) + e \text{ArcTan}[(e x)/\text{Sqrt}[d^2 - e^2 x^2]] + 2 e \text{Log}[x] - 2 e \text{Log}[d + \text{Sqrt}[d^2 - e^2 x^2]]$

**Maple [A]** time = 0.015, size = 93, normalized size = 1.4

$$e^2 \arctan \left( x \sqrt{e^2} \frac{1}{\sqrt{-e^2 x^2 + d^2}} \right) \frac{1}{\sqrt{e^2}} - \frac{1}{x} \sqrt{-e^2 x^2 + d^2} - 2 \frac{de}{\sqrt{d^2}} \ln \left( \frac{2 d^2 + 2 \sqrt{d^2} \sqrt{-e^2 x^2 + d^2}}{x} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(1/2),x)`

[Out]  $e^2/(e^2)^{(1/2)} * \arctan((e^2)^{(1/2)} * x / (-e^2 * x^2 + d^2)^{(1/2)}) - (-e^2 * x^2 + d^2)^{(1/2)} / x - 2 * d * e / (d^2)^{(1/2)} * \ln((2 * d^2 + 2 * (d^2)^{(1/2)} * (-e^2 * x^2 + d^2)^{(1/2)}) / x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^2/(sqrt(-e^2*x^2 + d^2)*x^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.281517, size = 209, normalized size = 3.07

$$\frac{e^2 x^2 - d^2 + 2 \left( dex - \sqrt{-e^2 x^2 + d^2} ex \right) \arctan \left( -\frac{d - \sqrt{-e^2 x^2 + d^2}}{ex} \right) - 2 \left( dex - \sqrt{-e^2 x^2 + d^2} ex \right) \log \left( -\frac{d - \sqrt{-e^2 x^2 + d^2}}{x} \right) + \sqrt{-e^2 x^2 + d^2}}{dx - \sqrt{-e^2 x^2 + d^2} x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^2/(sqrt(-e^2*x^2 + d^2)*x^2),x, algorithm="fricas")`

[Out]  $-(e^2 * x^2 - d^2 + 2 * (d * e * x - \sqrt{-e^2 * x^2 + d^2} * e * x) * \arctan(-(d - \sqrt{-e^2 * x^2 + d^2}) / (e * x)) - 2 * (d * e * x - \sqrt{-e^2 * x^2 + d^2} * e * x) * \log(-(d - \sqrt{-e^2 * x^2 + d^2}) / x) + \sqrt{-e^2 * x^2 + d^2} * d) / (d * x - \sqrt{-e^2 * x^2 + d^2} * x)$

**Sympy [A]** time = 7.55546, size = 212, normalized size = 3.12

$$d^2 \left( \begin{cases} -\frac{e \sqrt{\frac{d^2}{e^2 x^2} - 1}}{d^2} & \text{for } \left| \frac{d^2}{e^2 x^2} \right| > 1 \\ -\frac{i e \sqrt{-\frac{d^2}{e^2 x^2} + 1}}{d^2} & \text{otherwise} \end{cases} \right) + 2 d e \left( \begin{cases} -\frac{\operatorname{acosh}\left(\frac{d}{e x}\right)}{d} & \text{for } \left| \frac{d^2}{e^2 x^2} \right| > 1 \\ \frac{i \operatorname{asin}\left(\frac{d}{e x}\right)}{d} & \text{otherwise} \end{cases} \right) + e^2 \left( \begin{cases} \frac{\sqrt{\frac{d^2}{e^2}} \operatorname{asin}\left(x \sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2 > 0 \wedge -e^2 < 0 \\ \frac{\sqrt{-\frac{d^2}{e^2}} \operatorname{asinh}\left(x \sqrt{-\frac{e^2}{d^2}}\right)}{\sqrt{d^2}} & \text{for } d^2 > 0 \wedge -e^2 > 0 \\ \frac{\sqrt{\frac{d^2}{e^2}} \operatorname{acosh}\left(x \sqrt{\frac{e^2}{d^2}}\right)}{\sqrt{-d^2}} & \text{for } -e^2 > 0 \wedge d^2 < 0 \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**2/x**2/(-e**2*x**2+d**2)**(1/2),x)`

[Out]  $d^{**2} * \operatorname{Piecewise}((-e * \sqrt{d^{**2} / (e^{**2} * x^{**2})} - 1) / d^{**2}, \operatorname{Abs}(d^{**2} / (e^{**2} * x^{**2})) > 1), (-I * e * \sqrt{-d^{**2} / (e^{**2} * x^{**2})} + 1) / d^{**2}, \operatorname{True})) + 2 * d * e * \operatorname{Piecewise}(-\operatorname{acosh}(d / (e * x)) / d, \operatorname{Abs}(d^{**2} / (e^{**2} * x^{**2})) > 1), (I * \operatorname{asin}(d / (e * x)) / d, \operatorname{True})) + e^{**2} * \operatorname{Piecewise}((\sqrt{d^{**2} / e^{**2}} * \operatorname{asin}(x * \sqrt{e^{**2} / d^{**2}})) / \sqrt{d^{**2}}, (d^{**2} > 0) \& (-e^{**2} < 0)), (\sqrt{-d^{**2} / e^{**2}} * \operatorname{asinh}(x * \sqrt{-e^{**2} / d^{**2}})) / \sqrt{d^{**2}}, (d^{**2} > 0) \& (-e^{**2} > 0)), (\sqrt{d^{**2} / e^{**2}} * \operatorname{acosh}(x * \sqrt{e^{**2} / d^{**2}})) / \sqrt{-d^{**2}}, (-e^{**2} > 0) \& (d^{**2} < 0))$



```

**2/e**2)*asinh(x*sqrt(-e**2/d**2))/sqrt(d**2), (d**2 > 0) & (-e*
**2 > 0)), (sqrt(d**2/e**2)*acosh(x*sqrt(e**2/d**2))/sqrt(-d**2),
(d**2 < 0) & (-e**2 > 0))

```

---

**GIAC/XCAS [A]** time = 0.290523, size = 144, normalized size = 2.12

$$\begin{aligned}
 & \arcsin\left(\frac{xe}{d}\right) \operatorname{sign}(d) - 2e \ln\left(\frac{\left|-2de - 2\sqrt{-x^2e^2 + d^2}e\right|e^{(-2)}}{2|x|}\right) \\
 & + \frac{xe^3}{2\left(de + \sqrt{-x^2e^2 + d^2}e\right)} - \frac{\left(de + \sqrt{-x^2e^2 + d^2}e\right)e^{(-1)}}{2x}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((e*x + d)^2/(sqrt(-e^2*x^2 + d^2)*x^2),x, algorithm="giac")

```

```

[Out] arcsin(x*e/d)*e*sign(d) - 2*e*ln(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2
+ d^2)*e)*e^(-2)/abs(x)) + 1/2*x*e^3/(d*e + sqrt(-x^2*e^2 + d^2)
*e) - 1/2*(d*e + sqrt(-x^2*e^2 + d^2)*e)*e^(-1)/x

```

$$3.40 \quad \int \frac{(d+ex)^2}{x^3 \sqrt{d^2 - e^2 x^2}} dx$$

**Optimal.** Leaf size=80

$$-\frac{2e\sqrt{d^2 - e^2 x^2}}{dx} - \frac{\sqrt{d^2 - e^2 x^2}}{2x^2} - \frac{3e^2 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{2d}$$

[Out]  $-\text{Sqrt}[d^2 - e^2 x^2]/(2 x^2) - (2 e \text{Sqrt}[d^2 - e^2 x^2])/(d x) - (3 e^2 \text{ArcTanh}[\text{Sqrt}[d^2 - e^2 x^2]/d])/(2 d)$

**Rubi [A]** time = 0.228933, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$-\frac{2e\sqrt{d^2 - e^2 x^2}}{dx} - \frac{\sqrt{d^2 - e^2 x^2}}{2x^2} - \frac{3e^2 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{2d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e x)^2/(x^3 \text{Sqrt}[d^2 - e^2 x^2]), x]$

[Out]  $-\text{Sqrt}[d^2 - e^2 x^2]/(2 x^2) - (2 e \text{Sqrt}[d^2 - e^2 x^2])/(d x) - (3 e^2 \text{ArcTanh}[\text{Sqrt}[d^2 - e^2 x^2]/d])/(2 d)$

**Rubi in Sympy [A]** time = 25.2446, size = 65, normalized size = 0.81

$$-\frac{\sqrt{d^2 - e^2 x^2}}{2x^2} - \frac{3e^2 \text{atanh}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{2d} - \frac{2e\sqrt{d^2 - e^2 x^2}}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e x+d)^{**2}/x^{**3}/(-e^{**2} x^{**2}+d^{**2})^{**(1/2)}, x)$

[Out]  $-\text{sqrt}(d^{**2} - e^{**2} x^{**2})/(2 x^{**2}) - 3 e^{**2} \text{atanh}(\text{sqrt}(d^{**2} - e^{**2} x^{**2})/d)/(2 d) - 2 e \text{sqrt}(d^{**2} - e^{**2} x^{**2})/(d x)$

**Mathematica [A]** time = 0.0858217, size = 71, normalized size = 0.89

$$-\frac{(d + 4ex)\sqrt{d^2 - e^2 x^2} + 3e^2 x^2 \log\left(\sqrt{d^2 - e^2 x^2} + d\right) - 3e^2 x^2 \log(x)}{2dx^2}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(d + e x)^2/(x^3 \text{Sqrt}[d^2 - e^2 x^2]), x]$

[Out]  $-\left((d + 4 e x) \text{Sqrt}[d^2 - e^2 x^2] - 3 e^2 x^2 \text{Log}[x] + 3 e^2 x^2 \text{Log}[d + \text{Sqrt}[d^2 - e^2 x^2]]\right)/(2 d x^2)$

**Maple [A]** time = 0.015, size = 86, normalized size = 1.1

$$-\frac{1}{2x^2} \sqrt{-e^2 x^2 + d^2} - \frac{3e^2}{2} \ln\left(\frac{1}{x} \left(2d^2 + 2\sqrt{d^2} \sqrt{-e^2 x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}} - 2 \frac{e \sqrt{-e^2 x^2 + d^2}}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^2/x^3/(-e^2*x^2+d^2)^(1/2),x)`

[Out]  $-1/2*(-e^2*x^2+d^2)^{1/2}/x^2-3/2*e^2/(d^2)^{1/2}*ln((2*d^2+2*(d^2)^{1/2}*(-e^2*x^2+d^2)^{1/2})/x)-2*e*(-e^2*x^2+d^2)^{1/2}/d/x$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^2/(sqrt(-e^2*x^2+d^2)*x^3),x,algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.275803, size = 255, normalized size = 3.19

$$\frac{8de^3x^3 + 2d^2e^2x^2 - 8d^3ex - 2d^4 + 3\left(e^4x^4 - 2d^2e^2x^2 + 2\sqrt{-e^2x^2 + d^2}de^2x^2\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right) - (4e^3x^3 + de^2x^2 - 8d^3ex - 2d^4)}{2\left(de^2x^4 - 2d^3x^2 + 2\sqrt{-e^2x^2 + d^2}d^2x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^2/(sqrt(-e^2*x^2+d^2)*x^3),x,algorithm="fricas")`

[Out]  $1/2*(8*d*e^3*x^3 + 2*d^2*e^2*x^2 - 8*d^3*e*x - 2*d^4 + 3*(e^4*x^4 - 2*d^2*e^2*x^2 + 2*sqrt(-e^2*x^2 + d^2)*d*e^2*x^2)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (4*e^3*x^3 + d*e^2*x^2 - 8*d^3*e*x - 2*d^4)*sqrt(-e^2*x^2 + d^2))/(d*e^2*x^4 - 2*d^3*x^2 + 2*sqrt(-e^2*x^2 + d^2)*d^2*x^2)$

**Sympy [A]** time = 13.7089, size = 214, normalized size = 2.68

$$d^2 \left( \begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2d^2x} - \frac{e^2 \operatorname{acosh}\left(\frac{d}{ex}\right)}{2d^3} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{i}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie}{2d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^2 \operatorname{asin}\left(\frac{d}{ex}\right)}{2d^3} & \text{otherwise} \end{cases} \right) + 2de \left( \begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{d^2} & \text{otherwise} \end{cases} \right) + e^2 \left( \begin{cases} -\frac{\operatorname{acosh}\left(\frac{d}{ex}\right)}{d} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{i \operatorname{asin}\left(\frac{d}{ex}\right)}{d} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**2/x**3/(-e**2*x**2+d**2)**(1/2),x)`

[Out]  $d^{**2} \operatorname{Piecewise}((-e*\sqrt{d^{**2}/(e^{**2}*x^{**2})} - 1)/(2*d^{**2}*x) - e^{**2}*a \operatorname{cosh}(d/(e*x))/(2*d^{**3}), \operatorname{Abs}(d^{**2}/(e^{**2}*x^{**2})) > 1), (I/(2*e*x^{**3}*\sqrt{-d^{**2}/(e^{**2}*x^{**2})} + 1)) - I*e/(2*d^{**2}*x*\sqrt{-d^{**2}/(e^{**2}*x^{**2})} + 1)) + I*e^{**2}*\operatorname{asin}(d/(e*x))/(2*d^{**3}), \operatorname{True})) + 2*d*e*\operatorname{Piecewise}(e((-e*\sqrt{d^{**2}/(e^{**2}*x^{**2})} - 1)/d^{**2}, \operatorname{Abs}(d^{**2}/(e^{**2}*x^{**2})) > 1), (-I*e*\sqrt{-d^{**2}/(e^{**2}*x^{**2})} + 1)/d^{**2}, \operatorname{True})) + e^{**2}*\operatorname{Piecewise}((-a*\operatorname{cosh}(d/(e*x))/d, \operatorname{Abs}(d^{**2}/(e^{**2}*x^{**2})) > 1), (I*\operatorname{asin}(d/(e*x)))$

/d, True))

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**GIAC/XCAS [A]** time = 0.297176, size = 230, normalized size = 2.88

$$\frac{3 e^2 \ln\left(\frac{\left| -2 d e - 2 \sqrt{-x^2 e^2 + d^2} e \right| e^{(-2)}}{2 |x|}\right)}{2 d} + \frac{x^2 \left( \frac{8 (d e + \sqrt{-x^2 e^2 + d^2} e) e^4}{x} + e^6 \right)}{8 (d e + \sqrt{-x^2 e^2 + d^2} e)^2 d} - \frac{\left( \frac{8 (d e + \sqrt{-x^2 e^2 + d^2} e) d e^8}{x} + \frac{(d e + \sqrt{-x^2 e^2 + d^2} e)^2 d e^6}{x^2} \right) e^{(-8)}}{8 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2/(sqrt(-e^2\*x^2 + d^2)\*x^3),x, algorithm="giac")

[Out] -3/2\*e^2\*ln(1/2\*abs(-2\*d\*e - 2\*sqrt(-x^2\*e^2 + d^2)\*e)\*e^(-2)/abs(x))/d + 1/8\*x^2\*(8\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*e^4/x + e^6)/(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^2\*d - 1/8\*(8\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*d\*e^8/x + (d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^2\*d\*e^6/x^2)\*e^(-8)/d^2

$$3.41 \quad \int \frac{(d+ex)^2}{x^4\sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=107

$$-\frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x} - \frac{e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{\sqrt{d^2-e^2x^2}}{3x^3} - \frac{e^3 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

[Out]  $-\text{Sqrt}[d^2 - e^2x^2]/(3x^3) - (e\text{Sqrt}[d^2 - e^2x^2])/(d^2x^2) - (5e^2\text{Sqrt}[d^2 - e^2x^2])/(3d^2x) - (e^3\text{ArcTanh}[\text{Sqrt}[d^2 - e^2x^2]/d])/d^2$

**Rubi [A]** time = 0.315298, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-\frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x} - \frac{e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{\sqrt{d^2-e^2x^2}}{3x^3} - \frac{e^3 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^2/(x^4*\text{Sqrt}[d^2 - e^2*x^2]), x]$

[Out]  $-\text{Sqrt}[d^2 - e^2x^2]/(3x^3) - (e\text{Sqrt}[d^2 - e^2x^2])/(d^2x^2) - (5e^2\text{Sqrt}[d^2 - e^2x^2])/(3d^2x) - (e^3\text{ArcTanh}[\text{Sqrt}[d^2 - e^2x^2]/d])/d^2$

**Rubi in Sympy [A]** time = 28.1774, size = 88, normalized size = 0.82

$$-\frac{\sqrt{d^2-e^2x^2}}{3x^3} - \frac{e\sqrt{d^2-e^2x^2}}{dx^2} - \frac{e^3 \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^2} - \frac{5e^2\sqrt{d^2-e^2x^2}}{3d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)**2/x**4/(-e**2*x**2+d**2)**(1/2), x)$

[Out]  $-\text{sqrt}(d^2 - e^2x^2)/(3x^3) - e*\text{sqrt}(d^2 - e^2x^2)/(d^2x^2) - e^3*\text{atanh}(\text{sqrt}(d^2 - e^2x^2)/d)/d^2 - 5e^2*\text{sqrt}(d^2 - e^2x^2)/(3d^2x)$

**Mathematica [A]** time = 0.164305, size = 76, normalized size = 0.71

$$-\frac{\sqrt{d^2-e^2x^2}(d^2+3dex+5e^2x^2)}{x^3} + \frac{3e^3 \log\left(\sqrt{d^2-e^2x^2} + d\right) - 3e^3 \log(x)}{3d^2}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(d + e*x)^2/(x^4*\text{Sqrt}[d^2 - e^2*x^2]), x]$

[Out]  $-\left(\frac{\text{Sqrt}[d^2 - e^2x^2]*(d^2 + 3*d*e*x + 5*e^2*x^2)}{x^3} - 3*e^3*\text{Log}[x] + 3*e^3*\text{Log}[d + \text{Sqrt}[d^2 - e^2*x^2]]\right)/(3*d^2)$

**Maple [A]** time = 0.018, size = 114, normalized size = 1.1

$$-\frac{1}{3x^3}\sqrt{-e^2x^2+d^2}-\frac{5e^2}{3d^2x}\sqrt{-e^2x^2+d^2}-\frac{e}{dx^2}\sqrt{-e^2x^2+d^2}-\frac{e^3}{d}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^2/x^4/(-e^2\*x^2+d^2)^(1/2),x)

[Out] -1/3\*(-e^2\*x^2+d^2)^(1/2)/x^3-5/3\*e^2\*(-e^2\*x^2+d^2)^(1/2)/d^2/x-e\*(-e^2\*x^2+d^2)^(1/2)/d/x^2-1/d\*e^3/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2/(sqrt(-e^2\*x^2+d^2)\*x^4),x,algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.274398, size = 355, normalized size = 3.32

$$\frac{5e^6x^6+3de^5x^5-24d^2e^4x^4-15d^3e^3x^3+15d^4e^2x^2+12d^5ex+4d^6-3\left(3de^5x^5-4d^3e^3x^3-(e^5x^5-4d^2e^3x^3)\sqrt{-e^2x^2+d^2}\right)}{3\left(3d^3e^2x^5-4d^5x^3-(d^2e^2x^5-4d^4x^3)\sqrt{-e^2x^2+d^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2/(sqrt(-e^2\*x^2+d^2)\*x^4),x,algorithm="fricas")

[Out] -1/3\*(5\*e^6\*x^6+3\*d\*e^5\*x^5-24\*d^2\*e^4\*x^4-15\*d^3\*e^3\*x^3+15\*d^4\*e^2\*x^2+12\*d^5\*e\*x+4\*d^6-3\*(3\*d\*e^5\*x^5-4\*d^3\*e^3\*x^3-(e^5\*x^5-4\*d^2\*e^3\*x^3)\*sqrt(-e^2\*x^2+d^2))\*log(-(d-sqrt(-e^2\*x^2+d^2))/x)+(15\*d\*e^4\*x^4+9\*d^2\*e^3\*x^3-17\*d^3\*e^2\*x^2-12\*d^4\*e\*x-4\*d^5)\*sqrt(-e^2\*x^2+d^2))/(3\*d^3\*e^2\*x^5-4\*d^5\*x^3-(d^2\*e^2\*x^5-4\*d^4\*x^3)\*sqrt(-e^2\*x^2+d^2))

**Sympy [A]** time = 13.7215, size = 303, normalized size = 2.83

$$d^2 \left( \begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2x^2} - \frac{2e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^4} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2x^2} - \frac{2ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^4} & \text{otherwise} \end{cases} \right) + 2de \left( \begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2d^2x} - \frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d^3} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ \frac{i}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie}{2d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d^3} & \text{otherwise} \end{cases} \right) + e^2 \left( \begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{d^2} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{d^2} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2/x\*\*4/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2),x)

[Out] d\*\*2\*Piecewise((-e\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)/(3\*d\*\*2\*x\*\*2) - 2\*e\*\*3\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)/(3\*d\*\*4), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (-I\*e\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)/(3\*d\*\*2\*x\*\*2) - 2\*I\*e\*\*3\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)/(3\*d\*\*4), True)) + 2\*d\*e\*Piecewise((-e\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)/(2\*d\*\*2\*x) - e\*\*2\*acosh(d/(e\*x))/(2\*d\*\*3), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (I/(2\*e\*x\*\*3\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) - I\*e/(2\*d\*\*2\*x\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) + I\*e\*\*2\*asin(d/(e\*x))/(2\*d\*\*3), True)) + e\*\*2\*Piecewise((-e\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)/d\*\*2, Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (-I\*e\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)/d\*\*2, True))

**GIAC/XCAS [A]** time = 0.295299, size = 323, normalized size = 3.02

$$\frac{x^3 \left( \frac{6(de + \sqrt{-x^2e^2 + d^2})e^6}{x} + \frac{21(de + \sqrt{-x^2e^2 + d^2})^2e^4}{x^2} + e^8 \right) e - e^3 \ln \left( \frac{|-2de - 2\sqrt{-x^2e^2 + d^2}e|e^{(-2)}}{2|x|} \right)}{24 \left( de + \sqrt{-x^2e^2 + d^2} \right)^3 d^2} - \frac{21 \left( de + \sqrt{-x^2e^2 + d^2} \right) d^4 e^{16}}{x} + \frac{6 \left( de + \sqrt{-x^2e^2 + d^2} \right)^2 d^4 e^{14}}{x^2} + \frac{\left( de + \sqrt{-x^2e^2 + d^2} \right)^3 d^4 e^{12}}{x^3} \Big) e^{(-15)}}{24 d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2/(sqrt(-e^2\*x^2 + d^2)\*x^4),x, algorithm="giac")

[Out] 1/24\*x^3\*(6\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*e^6/x + 21\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^2\*e^4/x^2 + e^8)\*e/((d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^3\*d^2) - e^3\*ln(1/2\*abs(-2\*d\*e - 2\*sqrt(-x^2\*e^2 + d^2)\*e)\*e^(-2)/abs(x))/d^2 - 1/24\*(21\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*d^4\*e^16/x + 6\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^2\*d^4\*e^14/x^2 + (d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^3\*d^4\*e^12/x^3)\*e^(-15)/d^6

$$3.42 \quad \int \frac{(d+ex)^2}{x^5 \sqrt{d^2 - e^2 x^2}} dx$$

**Optimal.** Leaf size=140

$$-\frac{7e^2 \sqrt{d^2 - e^2 x^2}}{8d^2 x^2} - \frac{\sqrt{d^2 - e^2 x^2}}{4x^4} - \frac{2e \sqrt{d^2 - e^2 x^2}}{3dx^3} - \frac{7e^4 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{8d^3} - \frac{4e^3 \sqrt{d^2 - e^2 x^2}}{3d^3 x}$$

[Out]  $-\text{Sqrt}[d^2 - e^2 x^2]/(4 x^4) - (2 e \text{Sqrt}[d^2 - e^2 x^2])/(3 d x^3) - (7 e^2 \text{Sqrt}[d^2 - e^2 x^2])/(8 d^2 x^2) - (4 e^3 \text{Sqrt}[d^2 - e^2 x^2])/(3 d^3 x) - (7 e^4 \text{ArcTanh}[\text{Sqrt}[d^2 - e^2 x^2]/d])/(8 d^3)$

**Rubi [A]** time = 0.402565, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-\frac{7e^2 \sqrt{d^2 - e^2 x^2}}{8d^2 x^2} - \frac{\sqrt{d^2 - e^2 x^2}}{4x^4} - \frac{2e \sqrt{d^2 - e^2 x^2}}{3dx^3} - \frac{7e^4 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{8d^3} - \frac{4e^3 \sqrt{d^2 - e^2 x^2}}{3d^3 x}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e x)^2/(x^5 \text{Sqrt}[d^2 - e^2 x^2]), x]$

[Out]  $-\text{Sqrt}[d^2 - e^2 x^2]/(4 x^4) - (2 e \text{Sqrt}[d^2 - e^2 x^2])/(3 d x^3) - (7 e^2 \text{Sqrt}[d^2 - e^2 x^2])/(8 d^2 x^2) - (4 e^3 \text{Sqrt}[d^2 - e^2 x^2])/(3 d^3 x) - (7 e^4 \text{ArcTanh}[\text{Sqrt}[d^2 - e^2 x^2]/d])/(8 d^3)$

**Rubi in Sympy [A]** time = 37.3071, size = 122, normalized size = 0.87

$$-\frac{\sqrt{d^2 - e^2 x^2}}{4x^4} - \frac{2e \sqrt{d^2 - e^2 x^2}}{3dx^3} - \frac{7e^2 \sqrt{d^2 - e^2 x^2}}{8d^2 x^2} - \frac{7e^4 \text{atanh}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{8d^3} - \frac{4e^3 \sqrt{d^2 - e^2 x^2}}{3d^3 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e x + d)^2/x^5/(-e^2 x^2 + d^2)^{(1/2)}, x)$

[Out]  $-\text{sqrt}(d^2 - e^2 x^2)/(4 x^4) - 2 e \text{sqrt}(d^2 - e^2 x^2)/(3 d x^3) - 7 e^2 \text{sqrt}(d^2 - e^2 x^2)/(8 d^2 x^2) - 7 e^4 \text{atanh}(\text{sqrt}(d^2 - e^2 x^2)/d)/(8 d^3) - 4 e^3 \text{sqrt}(d^2 - e^2 x^2)/(3 d^3 x)$

**Mathematica [A]** time = 0.133011, size = 95, normalized size = 0.68

$$\frac{21e^4 x^4 \log\left(\sqrt{d^2 - e^2 x^2} + d\right) + \sqrt{d^2 - e^2 x^2} (6d^3 + 16d^2 e x + 21de^2 x^2 + 32e^3 x^3) - 21e^4 x^4 \log(x)}{24d^3 x^4}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(d + e x)^2/(x^5 \text{Sqrt}[d^2 - e^2 x^2]), x]$

[Out]  $-(\text{Sqrt}[d^2 - e^2 x^2])*(6 d^3 + 16 d^2 e x + 21 d e^2 x^2 + 32 e^3 x^3) - 21 e^4 x^4 \text{Log}[x] + 21 e^4 x^4 \text{Log}[d + \text{Sqrt}[d^2 - e^2 x^2]]$



]])/(24\*d^3\*x^4)

**Maple [A]** time = 0.019, size = 139, normalized size = 1.

$$-\frac{1}{4x^4}\sqrt{-e^2x^2+d^2}-\frac{7e^2}{8d^2x^2}\sqrt{-e^2x^2+d^2}-\frac{7e^4}{8d^2}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}$$

$$-\frac{2e}{3dx^3}\sqrt{-e^2x^2+d^2}-\frac{4e^3}{3d^3x}\sqrt{-e^2x^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^2/x^5/(-e^2\*x^2+d^2)^(1/2),x)

[Out] -1/4\*(-e^2\*x^2+d^2)^(1/2)/x^4-7/8\*e^2\*(-e^2\*x^2+d^2)^(1/2)/d^2/x^2-7/8/d^2\*e^4/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)-2/3\*e\*(-e^2\*x^2+d^2)^(1/2)/d/x^3-4/3\*e^3\*(-e^2\*x^2+d^2)^(1/2)/d^3/x

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2/(sqrt(-e^2\*x^2+d^2)\*x^5),x,algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.289214, size = 443, normalized size = 3.16

$$\frac{128de^7x^7+84d^2e^6x^6-320d^3e^5x^5-228d^4e^4x^4+64d^5e^3x^3+96d^6e^2x^2+128d^7ex+48d^8+21\left(e^8x^8-8d^2e^6x^6+8d^4e^4x^4-8d^6e^2x^2+8d^8\right)}{24\left(d^3e^4x^8-8d^5e^2x^6+8d^7x^4-8d^9\right)\sqrt{-e^2x^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^2/(sqrt(-e^2\*x^2+d^2)\*x^5),x,algorithm="fricas")

[Out] 1/24\*(128\*d\*e^7\*x^7+84\*d^2\*e^6\*x^6-320\*d^3\*e^5\*x^5-228\*d^4\*e^4\*x^4+64\*d^5\*e^3\*x^3+96\*d^6\*e^2\*x^2+128\*d^7\*e\*x+48\*d^8+21\*(e^8\*x^8-8\*d^2\*e^6\*x^6+8\*d^4\*e^4\*x^4-8\*d^6\*e^2\*x^2+8\*d^8)\*sqrt(-e^2\*x^2+d^2))\*log(-(d-sqrt(-e^2\*x^2+d^2))/x)-(32\*e^7\*x^7+21\*d\*e^6\*x^6-240\*d^2\*e^5\*x^5-162\*d^3\*e^4\*x^4+128\*d^4\*e^3\*x^3+120\*d^5\*e^2\*x^2+128\*d^6\*e\*x+48\*d^7)\*sqrt(-e^2\*x^2+d^2)/(d^3\*e^4\*x^8-8\*d^5\*e^2\*x^6+8\*d^7\*x^4+4\*(d^4\*e^2\*x^6-2\*d^6\*x^4)\*sqrt(-e^2\*x^2+d^2))

**Sympy [A]** time = 24.9604, size = 449, normalized size = 3.21

$$d^2 \left( \begin{array}{l} \left( -\frac{1}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e}{8d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{3e^3}{8d^4x\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{3e^4 \operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^5} \right. \\ \left. \frac{i}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie}{8d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{3ie^3}{8d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{3ie^4 \operatorname{asin}\left(\frac{d}{ex}\right)}{8d^5} \right) \text{ for } \left| \frac{d^2}{e^2x^2} \right| > 1 \\ \text{otherwise} \end{array} \right) \\ + 2de \left( \begin{array}{l} \left( -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2x^2} - \frac{2e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^4} \right. \\ \left. \frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2x^2} - \frac{2ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^4} \right) \text{ for } \left| \frac{d^2}{e^2x^2} \right| > 1 \\ \text{otherwise} \end{array} \right) \\ + e^2 \left( \begin{array}{l} \left( -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{2d^2x} - \frac{e^2 \operatorname{acosh}\left(\frac{d}{ex}\right)}{2d^3} \right. \\ \left. \frac{i}{2ex^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie}{2d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^2 \operatorname{asin}\left(\frac{d}{ex}\right)}{2d^3} \right) \text{ for } \left| \frac{d^2}{e^2x^2} \right| > 1 \\ \text{otherwise} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2/x\*\*5/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2),x)

[Out] d\*\*2\*Piecewise((-1/(4\*e\*x\*\*5\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) - e/(8\*d\*\*2\*x\*\*3\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) + 3\*e\*\*3/(8\*d\*\*4\*x\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) - 3\*e\*\*4\*acosh(d/(e\*x))/(8\*d\*\*5), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (I/(4\*e\*x\*\*5\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) + I\*e/(8\*d\*\*2\*x\*\*3\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) - 3\*I\*e\*\*3/(8\*d\*\*4\*x\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) + 3\*I\*e\*\*4\*asin(d/(e\*x))/(8\*d\*\*5), True)) + 2\*d\*e\*Piecewise((-e\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)/(3\*d\*\*2\*x\*\*2) - 2\*e\*\*3\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)/(3\*d\*\*4), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (-I\*e\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)/(3\*d\*\*2\*x\*\*2) - 2\*I\*e\*\*3\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)/(3\*d\*\*4), True)) + e\*\*2\*Piecewise((-e\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)/(2\*d\*\*2\*x) - e\*\*2\*acosh(d/(e\*x))/(2\*d\*\*3), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (I/(2\*e\*x\*\*3\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) - I\*e/(2\*d\*\*2\*x\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) + I\*e\*\*2\*asin(d/(e\*x))/(2\*d\*\*3), True))

**GIAC/XCAS [A]** time = 0.293015, size = 412, normalized size = 2.94

$$\frac{x^4 \left( \frac{16 (de + \sqrt{-x^2 e^2 + d^2} e) e^8}{x} + \frac{48 (de + \sqrt{-x^2 e^2 + d^2} e)^2 e^6}{x^2} + \frac{144 (de + \sqrt{-x^2 e^2 + d^2} e)^3 e^4}{x^3} + 3 e^{10} \right) e^2}{192 (de + \sqrt{-x^2 e^2 + d^2} e)^4 d^3} \\ - \frac{7 e^4 \ln \left( \frac{|-2 de - 2 \sqrt{-x^2 e^2 + d^2} e| e^{(-2)}}{2 |x|} \right)}{8 d^3} \\ - \frac{\left( \frac{144 (de + \sqrt{-x^2 e^2 + d^2} e) d^9 e^{26}}{x} + \frac{48 (de + \sqrt{-x^2 e^2 + d^2} e)^2 d^9 e^{24}}{x^2} + \frac{16 (de + \sqrt{-x^2 e^2 + d^2} e)^3 d^9 e^{22}}{x^3} + \frac{3 (de + \sqrt{-x^2 e^2 + d^2} e)^4 d^9 e^{20}}{x^4} \right) e^{(-24)}}{192 d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2/(sqrt(-e^2\*x^2 + d^2)\*x^5),x, algorithm="giac")

[Out] 1/192\*x^4\*(16\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*e^8/x + 48\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^2\*e^6/x^2 + 144\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^3\*e^4/x^3 + 3\*e^10)\*e^2/((d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^4\*d^3) - 7/8\*e^4\*ln(1/2\*abs(-2\*d\*e - 2\*sqrt(-x^2\*e^2 + d^2)\*e)\*e^(-2)/abs(x))/d^3 - 1/192\*(144\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*d^9\*e^26/x + 48\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^2\*d^9\*e^24/x^2 + 16\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^3\*d^9\*e^22/x^3 + 3\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^4\*d^9\*e^20/x^4)\*e^(-24)/d^12

$$3.43 \quad \int \frac{(d+ex)^2}{x^6 \sqrt{d^2 - e^2 x^2}} dx$$

**Optimal.** Leaf size=169

$$\frac{\sqrt{d^2 - e^2 x^2}}{5x^5} - \frac{e\sqrt{d^2 - e^2 x^2}}{2dx^4} - \frac{3e^2\sqrt{d^2 - e^2 x^2}}{5d^2x^3} - \frac{3e^5 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{4d^4} - \frac{6e^4\sqrt{d^2 - e^2 x^2}}{5d^4x} - \frac{3e^3\sqrt{d^2 - e^2 x^2}}{4d^3x^2}$$

[Out] -Sqrt[d^2 - e^2\*x^2]/(5\*x^5) - (e\*Sqrt[d^2 - e^2\*x^2])/(2\*d\*x^4) - (3\*e^2\*Sqrt[d^2 - e^2\*x^2])/(5\*d^2\*x^3) - (3\*e^3\*Sqrt[d^2 - e^2\*x^2])/(4\*d^3\*x^2) - (6\*e^4\*Sqrt[d^2 - e^2\*x^2])/(5\*d^4\*x) - (3\*e^5\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/(4\*d^4)

**Rubi [A]** time = 0.499527, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{\sqrt{d^2 - e^2 x^2}}{5x^5} - \frac{e\sqrt{d^2 - e^2 x^2}}{2dx^4} - \frac{3e^2\sqrt{d^2 - e^2 x^2}}{5d^2x^3} - \frac{3e^5 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{4d^4} - \frac{6e^4\sqrt{d^2 - e^2 x^2}}{5d^4x} - \frac{3e^3\sqrt{d^2 - e^2 x^2}}{4d^3x^2}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^2/(x^6\*Sqrt[d^2 - e^2\*x^2]),x]

[Out] -Sqrt[d^2 - e^2\*x^2]/(5\*x^5) - (e\*Sqrt[d^2 - e^2\*x^2])/(2\*d\*x^4) - (3\*e^2\*Sqrt[d^2 - e^2\*x^2])/(5\*d^2\*x^3) - (3\*e^3\*Sqrt[d^2 - e^2\*x^2])/(4\*d^3\*x^2) - (6\*e^4\*Sqrt[d^2 - e^2\*x^2])/(5\*d^4\*x) - (3\*e^5\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/(4\*d^4)

**Rubi in Sympy [A]** time = 42.136, size = 148, normalized size = 0.88

$$\frac{\sqrt{d^2 - e^2 x^2}}{5x^5} - \frac{e\sqrt{d^2 - e^2 x^2}}{2dx^4} - \frac{3e^2\sqrt{d^2 - e^2 x^2}}{5d^2x^3} - \frac{3e^3\sqrt{d^2 - e^2 x^2}}{4d^3x^2} - \frac{3e^5 \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{4d^4} - \frac{6e^4\sqrt{d^2 - e^2 x^2}}{5d^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*2/x\*\*6/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2),x)

[Out] -sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(5\*x\*\*5) - e\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(2\*d\*x\*\*4) - 3\*e\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(5\*d\*\*2\*x\*\*3) - 3\*e\*\*3\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(4\*d\*\*3\*x\*\*2) - 3\*e\*\*5\*atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d)/(4\*d\*\*4) - 6\*e\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(5\*d\*\*4\*x)

**Mathematica [A]** time = 0.171842, size = 106, normalized size = 0.63

$$\frac{15e^5x^5 \log\left(\sqrt{d^2 - e^2x^2} + d\right) + \sqrt{d^2 - e^2x^2} (4d^4 + 10d^3ex + 12d^2e^2x^2 + 15de^3x^3 + 24e^4x^4) - 15e^5x^5 \log(x)}{20d^4x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^2/(x^6\*Sqrt[d^2 - e^2\*x^2]),x]

[Out] -(Sqrt[d^2 - e^2\*x^2]\*(4\*d^4 + 10\*d^3\*e\*x + 12\*d^2\*e^2\*x^2 + 15\*d^3\*e^3\*x^3 + 24\*e^4\*x^4) - 15\*e^5\*x^5\*Log[x] + 15\*e^5\*x^5\*Log[d + S

$\text{qrt}[d^2 - e^2 x^2]]/(20 d^4 x^5)$

**Maple [A]** time = 0.022, size = 164, normalized size = 1.

$$-\frac{1}{5x^5}\sqrt{-e^2x^2+d^2}-\frac{3e^2}{5d^2x^3}\sqrt{-e^2x^2+d^2}-\frac{6e^4}{5d^4x}\sqrt{-e^2x^2+d^2}-\frac{e}{2dx^4}\sqrt{-e^2x^2+d^2}$$

$$-\frac{3e^3}{4d^3x^2}\sqrt{-e^2x^2+d^2}-\frac{3e^5}{4d^3}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^2/x^6/(-e^2*x^2+d^2)^(1/2),x)`

[Out] `-1/5*(-e^2*x^2+d^2)^(1/2)/x^5-3/5*e^2*(-e^2*x^2+d^2)^(1/2)/d^2/x^3-6/5*e^4*(-e^2*x^2+d^2)^(1/2)/d^4/x-1/2*e*(-e^2*x^2+d^2)^(1/2)/x^4-3/4*e^3*(-e^2*x^2+d^2)^(1/2)/d^3/x^2-3/4/d^3*e^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^2/(sqrt(-e^2*x^2+d^2)*x^6),x,algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.295339, size = 533, normalized size = 3.15

$$24e^{10}x^{10} + 15de^9x^9 - 300d^2e^8x^8 - 185d^3e^7x^7 + 520d^4e^6x^6 + 290d^5e^5x^5 - 100d^6e^4x^4 + 40d^7e^3x^3 - 80d^8e^2x^2 - 160d^9e^1x^1 + 20d^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^2/(sqrt(-e^2*x^2+d^2)*x^6),x,algorithm="fricas")`

[Out] `-1/20*(24*e^10*x^10+15*d*e^9*x^9-300*d^2*e^8*x^8-185*d^3*e^7*x^7+520*d^4*e^6*x^6+290*d^5*e^5*x^5-100*d^6*e^4*x^4+40*d^7*e^3*x^3-80*d^8*e^2*x^2-160*d^9*e^1*x^1+20*d^10)/sqrt(-e^2*x^2+d^2)+log(-d+sqrt(-e^2*x^2+d^2))/x+(120*d^8*e^8*x^8+75*d^2*e^7*x^7-420*d^3*e^6*x^6-250*d^4*e^5*x^5+164*d^5*e^4*x^4+40*d^6*e^3*x^3+112*d^7*e^2*x^2+160*d^8*e^1*x^1+64*d^9)/sqrt(-e^2*x^2+d^2)/(5*d^5*e^4*x^4-20*d^7*e^2*x^2+16*d^9*x^1-d^4*e^4*x^4-12*d^6*e^2*x^2+16*d^8*x^5)*sqrt(-e^2*x^2+d^2)`

**Sympy [A]** time = 22.0968, size = 510, normalized size = 3.02

$$d^2 \left( \begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{5d^2x^4} - \frac{4e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{15d^4x^2} - \frac{8e^5\sqrt{\frac{d^2}{e^2x^2}-1}}{15d^6} & \text{for } \left| \frac{d^2}{e^2x^2} \right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{5d^2x^4} - \frac{4ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{15d^4x^2} - \frac{8ie^5\sqrt{-\frac{d^2}{e^2x^2}+1}}{15d^6} & \text{otherwise} \end{cases} \right) + 2de \left( \begin{cases} -\frac{1}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e}{8d^2x^3\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{3e^3}{8d^4x\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{3e^4\operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^5} & \text{for } \left| \frac{d^2}{e^2x^2} \right| > 1 \\ \frac{i}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie}{8d^2x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{3ie^3}{8d^4x\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{3ie^4\operatorname{asin}\left(\frac{d}{ex}\right)}{8d^5} & \text{otherwise} \end{cases} \right) + e^2 \left( \begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2x^2} - \frac{2e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^4} & \text{for } \left| \frac{d^2}{e^2x^2} \right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2x^2} - \frac{2ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^4} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2/x\*\*6/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2),x)

[Out] d\*\*2\*Piecewise((-e\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)/(5\*d\*\*2\*x\*\*4) - 4\*e\*\*3\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)/(15\*d\*\*4\*x\*\*2) - 8\*e\*\*5\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)/(15\*d\*\*6), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (-I\*e\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)/(5\*d\*\*2\*x\*\*4) - 4\*I\*e\*\*3\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)/(15\*d\*\*6), True)) + 2\*d\*e\*Piecewise((-1/(4\*e\*x\*\*5\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) - e/(8\*d\*\*2\*x\*\*3\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) + 3\*e\*\*3/(8\*d\*\*4\*x\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) - 3\*e\*\*4\*acosh(d/(e\*x)))/(8\*d\*\*5), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (I/(4\*e\*x\*\*5\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) + I\*e/(8\*d\*\*2\*x\*\*3\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) - 3\*I\*e\*\*3/(8\*d\*\*4\*x\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) + 3\*I\*e\*\*4\*asin(d/(e\*x)))/(8\*d\*\*5), True)) + e\*\*2\*Piecewise((-e\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)/(3\*d\*\*2\*x\*\*2) - 2\*e\*\*3\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)/(3\*d\*\*4), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (-I\*e\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)/(3\*d\*\*2\*x\*\*2) - 2\*I\*e\*\*3\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)/(3\*d\*\*4), True))

**GIAC/XCAS [A]** time = 0.298571, size = 493, normalized size = 2.92

$$\frac{x^5 \left( \frac{5(de + \sqrt{-x^2e^2 + d^2}e)e^{10}}{x} + \frac{15(de + \sqrt{-x^2e^2 + d^2}e)^2e^8}{x^2} + \frac{40(de + \sqrt{-x^2e^2 + d^2}e)^3e^6}{x^3} + \frac{110(de + \sqrt{-x^2e^2 + d^2}e)^4e^4}{x^4} + e^{12} \right) e^3}{160(de + \sqrt{-x^2e^2 + d^2}e)^5d^4} - \frac{3e^5 \ln \left( \frac{|-2de - 2\sqrt{-x^2e^2 + d^2}e|e^{-2}}{2|x|} \right)}{4d^4} + \frac{\left( \frac{110(de + \sqrt{-x^2e^2 + d^2}e)d^{16}e^{38}}{x} + \frac{40(de + \sqrt{-x^2e^2 + d^2}e)^2d^{16}e^{36}}{x^2} + \frac{15(de + \sqrt{-x^2e^2 + d^2}e)^3d^{16}e^{34}}{x^3} + \frac{5(de + \sqrt{-x^2e^2 + d^2}e)^4d^{16}e^{32}}{x^4} + \frac{(de + \sqrt{-x^2e^2 + d^2}e)^5d^{16}e^{30}}{x^5} \right)}{160d^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2/(sqrt(-e^2\*x^2 + d^2)\*x^6),x, algorithm="giac")

[Out] 1/160\*x^5\*(5\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*e^10/x + 15\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^2\*e^8/x^2 + 40\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^3\*e^6/x^3 + 110\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^4\*e^4/x^4 + e^12)\*e^3/((d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^5\*d^4) - 3/4\*e^5\*ln(1/2\*abs(-2\*d\*e - 2\*sqrt(-x^2\*e^2 + d^2)\*e)\*e^(-2)/abs(x))/d^4 - 1/160\*(110\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*d^16\*e^38/x + 40\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^2\*d^16\*e^36/x^2 + 15\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^3\*d^16\*e^34/x^3 + 5\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^4\*d^16\*e^32/x^4 + (d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^5\*d^16\*e^30/x^5)

$$\begin{aligned}
 & )^3 d^{16} e^{34} / x^3 + 5 (d e + \sqrt{-x^2 e^2 + d^2})^4 d^{16} e^{32} / x^4 \\
 & + (d e + \sqrt{-x^2 e^2 + d^2})^5 d^{16} e^{30} / x^5) e^{-35} / d^{20}
 \end{aligned}$$

$$3.44 \quad \int \frac{x^5(d+ex)^2}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=143

$$\frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^6} - \frac{2d \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6} + \frac{d^4(d+ex)^2}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{22d^3(d+ex)}{15e^6(d^2-e^2x^2)^{3/2}}$$

[Out]  $(d^4*(d+e*x)^2)/(5*e^6*(d^2-e^2*x^2)^{(5/2)}) - (22*d^3*(d+e*x))/(15*e^6*(d^2-e^2*x^2)^{(3/2)}) + (2*d*(30*d+23*e*x))/(15*e^6*\text{Sqrt}[d^2-e^2*x^2]) + \text{Sqrt}[d^2-e^2*x^2]/e^6 - (2*d*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2-e^2*x^2]])/e^6$

**Rubi [A]** time = 0.415825, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\frac{2d(30d+23ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^6} - \frac{2d \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6} + \frac{d^4(d+ex)^2}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{22d^3(d+ex)}{15e^6(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^5\*(d+e\*x)^2)/(d^2-e^2\*x^2)^(7/2),x]

[Out]  $(d^4*(d+e*x)^2)/(5*e^6*(d^2-e^2*x^2)^{(5/2)}) - (22*d^3*(d+e*x))/(15*e^6*(d^2-e^2*x^2)^{(3/2)}) + (2*d*(30*d+23*e*x))/(15*e^6*\text{Sqrt}[d^2-e^2*x^2]) + \text{Sqrt}[d^2-e^2*x^2]/e^6 - (2*d*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2-e^2*x^2]])/e^6$

**Rubi in Sympy [A]** time = 75.2747, size = 144, normalized size = 1.01

$$\frac{d^4}{5e^6(d-ex)^2\sqrt{d^2-e^2x^2}} - \frac{22d^3}{15e^6(d-ex)\sqrt{d^2-e^2x^2}} + \frac{4d^2}{e^6\sqrt{d^2-e^2x^2}} + \frac{46dx}{15e^5\sqrt{d^2-e^2x^2}} - \frac{2d \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6} + \frac{\sqrt{d^2-e^2x^2}}{e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5\*(e\*x+d)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2),x)

[Out]  $d^4/(5*e^6*(d-e*x)**2*\text{sqrt}(d^2-e^2*x^2)) - 22*d^3/(15*e^6*(d-e*x)*\text{sqrt}(d^2-e^2*x^2)) + 4*d^2/(e^6*\text{sqrt}(d^2-e^2*x^2)) + 46*d*x/(15*e^5*\text{sqrt}(d^2-e^2*x^2)) - 2*d*\operatorname{atan}(e*x/\text{sqrt}(d^2-e^2*x^2))/e^6 + \text{sqrt}(d^2-e^2*x^2)/e^6$

**Mathematica [A]** time = 0.137542, size = 118, normalized size = 0.83

$$\sqrt{d^2-e^2x^2} \left( -\frac{d^3}{10e^6(ex-d)^3} - \frac{41d^2}{60e^6(ex-d)^2} - \frac{383d}{120e^6(ex-d)} + \frac{d}{8e^6(d+ex)} + \frac{1}{e^6} \right) - \frac{2d \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5\*(d+e\*x)^2)/(d^2-e^2\*x^2)^(7/2),x]

[Out]  $\text{Sqrt}[d^2 - e^2 x^2] \cdot (e^{-6} - d^3 / (10 e^6 (-d + e x)^3) - (41 d^2) / (60 e^6 (-d + e x)^2) - (383 d) / (120 e^6 (-d + e x)) + d / (8 e^6 (d + e x))) - (2 d \text{ArcTan}[(e x) / \text{Sqrt}[d^2 - e^2 x^2]]) / e^6$

**Maple [A]** time = 0.013, size = 193, normalized size = 1.4

$$7 \frac{d^2 x^4}{e^2 (-e^2 x^2 + d^2)^{5/2}} - \frac{28 d^4 x^2}{3 e^4} (-e^2 x^2 + d^2)^{-5/2} + \frac{56 d^6}{15 e^6} (-e^2 x^2 + d^2)^{-5/2} - x^6 (-e^2 x^2 + d^2)^{-5/2} \\ + \frac{2 dx^5}{5 e} (-e^2 x^2 + d^2)^{-5/2} - \frac{2 dx^3}{3 e^3} (-e^2 x^2 + d^2)^{-3/2} + 2 \frac{dx}{e^5 \sqrt{-e^2 x^2 + d^2}} - 2 \frac{d}{e^5 \sqrt{e^2}} \arctan\left(\frac{\sqrt{e^2} x}{\sqrt{-e^2 x^2 + d^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^5 (e x + d)^2 / (-e^2 x^2 + d^2)^{7/2}, x)$

[Out]  $7 d^2 x^4 / e^2 / (-e^2 x^2 + d^2)^{5/2} - 28/3 d^4 / e^4 x^2 / (-e^2 x^2 + d^2)^{5/2} + 56/15 d^6 / e^6 / (-e^2 x^2 + d^2)^{5/2} - x^6 / (-e^2 x^2 + d^2)^{5/2} + 2/5 e d x^5 / (-e^2 x^2 + d^2)^{5/2} - 2/3 e^3 d x^3 / (-e^2 x^2 + d^2)^{3/2} + 2/e^5 d x / (-e^2 x^2 + d^2)^{1/2} - 2/e^5 d / (e^2)^{1/2} \arctan((e^2)^{1/2} x / (-e^2 x^2 + d^2)^{1/2})$

**Maxima [A]** time = 0.794713, size = 390, normalized size = 2.73

$$\frac{2}{15} dex \left( \frac{15 x^4}{(-e^2 x^2 + d^2)^{5/2} e^2} - \frac{20 d^2 x^2}{(-e^2 x^2 + d^2)^{5/2} e^4} + \frac{8 d^4}{(-e^2 x^2 + d^2)^{5/2} e^6} \right) - \frac{x^6}{(-e^2 x^2 + d^2)^{5/2}} \\ - \frac{2 dx \left( \frac{3 x^2}{(-e^2 x^2 + d^2)^{3/2} e^2} - \frac{2 d^2}{(-e^2 x^2 + d^2)^{3/2} e^4} \right)}{3 e} + \frac{7 d^2 x^4}{(-e^2 x^2 + d^2)^{5/2} e^2} - \frac{28 d^4 x^2}{3 (-e^2 x^2 + d^2)^{5/2} e^4} \\ + \frac{56 d^6}{15 (-e^2 x^2 + d^2)^{5/2} e^6} + \frac{8 d^3 x}{15 (-e^2 x^2 + d^2)^{3/2} e^5} - \frac{14 dx}{15 \sqrt{-e^2 x^2 + d^2} e^5} - \frac{2 d \arcsin\left(\frac{e x}{\sqrt{d^2 e^2}}\right)}{\sqrt{e^2} e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e x + d)^2 x^5 / (-e^2 x^2 + d^2)^{7/2}, x, \text{algorithm}="maxima")$

[Out]  $2/15 d e x (15 x^4 / ((-e^2 x^2 + d^2)^{5/2} e^2) - 20 d^2 x^2 / ((-e^2 x^2 + d^2)^{5/2} e^4) + 8 d^4 / ((-e^2 x^2 + d^2)^{5/2} e^6)) - x^6 / (-e^2 x^2 + d^2)^{5/2} - 2/3 d x (3 x^2 / ((-e^2 x^2 + d^2)^{3/2} e^2) - 2 d^2 / ((-e^2 x^2 + d^2)^{3/2} e^4)) / e + 7 d^2 x^4 / ((-e^2 x^2 + d^2)^{5/2} e^2) - 28/3 d^4 x^2 / ((-e^2 x^2 + d^2)^{5/2} e^4) + 56/15 d^6 / ((-e^2 x^2 + d^2)^{5/2} e^6) + 8/15 d^3 x / ((-e^2 x^2 + d^2)^{3/2} e^5) - 14/15 d x / (\text{sqrt}(-e^2 x^2 + d^2) e^5) - 2 d \arcsin(e x / \text{sqrt}(d^2 e^2)) / (\text{sqrt}(e^2) e^5)$

**Fricas [A]** time = 0.280377, size = 635, normalized size = 4.44

$$\frac{15 e^8 x^8 - 76 d e^7 x^7 + 136 d^2 e^6 x^6 + 242 d^3 e^5 x^5 - 640 d^4 e^4 x^4 + 80 d^5 e^3 x^3 + 480 d^6 e^2 x^2 - 240 d^7 e x + 60 (4 d^2 e^6 x^6 - 8 d^3 e^5 x^5 - 15 (4 d e^{12} x^6 - 8 d^2 e^{11} x^5))}{15 (4 d e^{12} x^6 - 8 d^2 e^{11} x^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e x + d)^2 x^5 / (-e^2 x^2 + d^2)^{7/2}, x, \text{algorithm}="fricas")$



[Out]  $\frac{1}{15} \cdot (15 \cdot e^8 \cdot x^8 - 76 \cdot d \cdot e^7 \cdot x^7 + 136 \cdot d^2 \cdot e^6 \cdot x^6 + 242 \cdot d^3 \cdot e^5 \cdot x^5 - 640 \cdot d^4 \cdot e^4 \cdot x^4 + 80 \cdot d^5 \cdot e^3 \cdot x^3 + 480 \cdot d^6 \cdot e^2 \cdot x^2 - 240 \cdot d^7 \cdot e \cdot x + 60 \cdot (4 \cdot d^2 \cdot e^6 \cdot x^6 - 8 \cdot d^3 \cdot e^5 \cdot x^5 - 8 \cdot d^4 \cdot e^4 \cdot x^4 + 24 \cdot d^5 \cdot e^3 \cdot x^3 - 4 \cdot d^6 \cdot e^2 \cdot x^2 - 16 \cdot d^7 \cdot e \cdot x + 8 \cdot d^8 - (d \cdot e^6 \cdot x^6 - 2 \cdot d^2 \cdot e^5 \cdot x^5 - 7 \cdot d^3 \cdot e^4 \cdot x^4 + 16 \cdot d^4 \cdot e^3 \cdot x^3 - 16 \cdot d^6 \cdot e \cdot x + 8 \cdot d^7) \cdot \sqrt{-e^2 \cdot x^2 + d^2}) \cdot \arctan\left(\frac{-d - \sqrt{-e^2 \cdot x^2 + d^2}}{e \cdot x}\right) + 4 \cdot (d \cdot e^6 \cdot x^6 - 48 \cdot d^2 \cdot e^5 \cdot x^5 + 100 \cdot d^3 \cdot e^4 \cdot x^4 + 10 \cdot d^4 \cdot e^3 \cdot x^3 - 120 \cdot d^5 \cdot e^2 \cdot x^2 + 60 \cdot d^6 \cdot e \cdot x) \cdot \sqrt{-e^2 \cdot x^2 + d^2}) / (4 \cdot d \cdot e^{12} \cdot x^6 - 8 \cdot d^2 \cdot e^{11} \cdot x^5 - 8 \cdot d^3 \cdot e^{10} \cdot x^4 + 24 \cdot d^4 \cdot e^9 \cdot x^3 - 4 \cdot d^5 \cdot e^8 \cdot x^2 - 16 \cdot d^6 \cdot e^7 \cdot x + 8 \cdot d^7 \cdot e^6 - (e^{12} \cdot x^6 - 2 \cdot d \cdot e^{11} \cdot x^5 - 7 \cdot d^2 \cdot e^{10} \cdot x^4 + 16 \cdot d^3 \cdot e^9 \cdot x^3 - 16 \cdot d^5 \cdot e^7 \cdot x + 8 \cdot d^6 \cdot e^6) \cdot \sqrt{-e^2 \cdot x^2 + d^2})$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 (d + ex)^2}{(-(-d + ex)(d + ex))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)`

[Out] `Integral(x**5*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)`

**GIAC/XCAS [A]** time = 0.300413, size = 143, normalized size = 1.

$$\frac{-2d \arcsin\left(\frac{xe}{d}\right) e^{(-6)} \operatorname{sign}(d) + (56d^6e^{(-6)} + (30d^5e^{(-5)} - (140d^4e^{(-4)} + (70d^3e^{(-3)} - (105d^2e^{(-2)} + (46de^{(-1)} - 15x)x)x)x)x) \sqrt{-x^2e^2 + d^2}}{15(x^2e^2 - d^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^2*x^5/(-e^2*x^2 + d^2)^(7/2),x, algorithm="giac")`

[Out]  $-2 \cdot d \cdot \arcsin(x \cdot e / d) \cdot e^{(-6)} \cdot \operatorname{sign}(d) - \frac{1}{15} \cdot (56 \cdot d^6 \cdot e^{(-6)} + (30 \cdot d^5 \cdot e^{(-5)} - (140 \cdot d^4 \cdot e^{(-4)} + (70 \cdot d^3 \cdot e^{(-3)} - (105 \cdot d^2 \cdot e^{(-2)} + (46 \cdot d \cdot e^{(-1)} - 15 \cdot x) \cdot x) \cdot x) \cdot x) \cdot x) \cdot \sqrt{-x^2 \cdot e^2 + d^2}) / (x^2 \cdot e^2 - d^2)^3$

$$3.45 \quad \int \frac{x^4(d+ex)^2}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=121

$$-\frac{17d^2(d+ex)}{15e^5(d^2-e^2x^2)^{3/2}} + \frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5} + \frac{d^3(d+ex)^2}{5e^5(d^2-e^2x^2)^{5/2}}$$

[Out]  $(d^3*(d+e*x)^2)/(5*e^5*(d^2-e^2*x^2)^{(5/2)}) - (17*d^2*(d+e*x))/(15*e^5*(d^2-e^2*x^2)^{(3/2)}) + (2*(15*d+13*e*x))/(15*e^5*\text{Sqrt}[d^2-e^2*x^2]) - \text{ArcTan}[(e*x)/\text{Sqrt}[d^2-e^2*x^2]]/e^5$

**Rubi [A]** time = 0.316319, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$-\frac{17d^2(d+ex)}{15e^5(d^2-e^2x^2)^{3/2}} + \frac{2(15d+13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5} + \frac{d^3(d+ex)^2}{5e^5(d^2-e^2x^2)^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^4*(d+e*x)^2)/(d^2-e^2*x^2)^{(7/2)}, x]$

[Out]  $(d^3*(d+e*x)^2)/(5*e^5*(d^2-e^2*x^2)^{(5/2)}) - (17*d^2*(d+e*x))/(15*e^5*(d^2-e^2*x^2)^{(3/2)}) + (2*(15*d+13*e*x))/(15*e^5*\text{Sqrt}[d^2-e^2*x^2]) - \text{ArcTan}[(e*x)/\text{Sqrt}[d^2-e^2*x^2]]/e^5$

**Rubi in Sympy [A]** time = 58.0747, size = 121, normalized size = 1.

$$\frac{d^3}{5e^5(d-ex)^2\sqrt{d^2-e^2x^2}} - \frac{17d^2}{15e^5(d-ex)\sqrt{d^2-e^2x^2}} + \frac{2d}{e^5\sqrt{d^2-e^2x^2}} + \frac{26x}{15e^4\sqrt{d^2-e^2x^2}} - \frac{\text{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**4}*(e*x+d)^{**2}/(-e^{**2}*x^{**2}+d^{**2})^{**}(7/2), x)$

[Out]  $d^{**3}/(5*e^{**5}*(d-e*x)^{**2}*\text{sqrt}(d^{**2}-e^{**2}*x^{**2})) - 17*d^{**2}/(15*e^{**5}*(d-e*x)*\text{sqrt}(d^{**2}-e^{**2}*x^{**2})) + 2*d/(e^{**5}*\text{sqrt}(d^{**2}-e^{**2}*x^{**2})) + 26*x/(15*e^{**4}*\text{sqrt}(d^{**2}-e^{**2}*x^{**2})) - \text{atan}(e*x/\text{sqrt}(d^{**2}-e^{**2}*x^{**2}))/e^{**5}$

**Mathematica [A]** time = 0.118366, size = 106, normalized size = 0.88

$$\frac{15(d+ex)(d-ex)^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \sqrt{d^2-e^2x^2}(-16d^3+17d^2ex+22de^2x^2-26e^3x^3)}{15e^5(ex-d)^3(d+ex)}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(x^4*(d+e*x)^2)/(d^2-e^2*x^2)^{(7/2)}, x]$

[Out]  $(\text{Sqrt}[d^2-e^2*x^2]*(-16*d^3+17*d^2*e*x+22*d*e^2*x^2-26*e^3*x^3)+15*(d-e*x)^3*(d+e*x)*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2-e^2*x^2]])/(15*e^5*(-d+e*x)^3*(d+e*x))$

**Maple [B]** time = 0.023, size = 236, normalized size = 2.

$$\begin{aligned} & \frac{d^2 x^3}{2 e^2} (-e^2 x^2 + d^2)^{-\frac{5}{2}} - \frac{3 d^4 x}{10 e^4} (-e^2 x^2 + d^2)^{-\frac{5}{2}} + \frac{d^2 x}{10 e^4} (-e^2 x^2 + d^2)^{-\frac{3}{2}} + \frac{6 x}{5 e^4} \frac{1}{\sqrt{-e^2 x^2 + d^2}} \\ & + \frac{x^5}{5} (-e^2 x^2 + d^2)^{-\frac{5}{2}} - \frac{x^3}{3 e^2} (-e^2 x^2 + d^2)^{-\frac{3}{2}} - \frac{1}{e^4} \arctan\left(x \sqrt{e^2} \frac{1}{\sqrt{-e^2 x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} \\ & + 2 \frac{d x^4}{e (-e^2 x^2 + d^2)^{5/2}} - \frac{8 d^3 x^2}{3 e^3} (-e^2 x^2 + d^2)^{-\frac{5}{2}} + \frac{16 d^5}{15 e^5} (-e^2 x^2 + d^2)^{-\frac{5}{2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(e\*x+d)^2/(-e^2\*x^2+d^2)^(7/2),x)

[Out] 1/2\*d^2\*x^3/e^2/(-e^2\*x^2+d^2)^(5/2)-3/10\*d^4/e^4\*x/(-e^2\*x^2+d^2)^(5/2)+1/10\*d^2/e^4\*x/(-e^2\*x^2+d^2)^(3/2)+6/5/e^4\*x/(-e^2\*x^2+d^2)^(1/2)+1/5\*x^5/(-e^2\*x^2+d^2)^(5/2)-1/3\*x^3/e^2/(-e^2\*x^2+d^2)^(3/2)-1/e^4/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))+2\*d/e\*x^4/(-e^2\*x^2+d^2)^(5/2)-8/3\*d^3/e^3\*x^2/(-e^2\*x^2+d^2)^(5/2)+16/15\*d^5/e^5/(-e^2\*x^2+d^2)^(5/2)

**Maxima [A]** time = 0.80027, size = 420, normalized size = 3.47

$$\begin{aligned} & \frac{1}{15} e^2 x \left( \frac{15 x^4}{(-e^2 x^2 + d^2)^{\frac{5}{2}} e^2} - \frac{20 d^2 x^2}{(-e^2 x^2 + d^2)^{\frac{5}{2}} e^4} + \frac{8 d^4}{(-e^2 x^2 + d^2)^{\frac{5}{2}} e^6} \right) \\ & - \frac{1}{3} x \left( \frac{3 x^2}{(-e^2 x^2 + d^2)^{\frac{3}{2}} e^2} - \frac{2 d^2}{(-e^2 x^2 + d^2)^{\frac{3}{2}} e^4} \right) + \frac{2 d x^4}{(-e^2 x^2 + d^2)^{\frac{5}{2}} e} \\ & + \frac{d^2 x^3}{2 (-e^2 x^2 + d^2)^{\frac{5}{2}} e^2} - \frac{8 d^3 x^2}{3 (-e^2 x^2 + d^2)^{\frac{5}{2}} e^3} - \frac{3 d^4 x}{10 (-e^2 x^2 + d^2)^{\frac{5}{2}} e^4} \\ & + \frac{16 d^5}{15 (-e^2 x^2 + d^2)^{\frac{5}{2}} e^5} + \frac{11 d^2 x}{30 (-e^2 x^2 + d^2)^{\frac{3}{2}} e^4} - \frac{4 x}{15 \sqrt{-e^2 x^2 + d^2} e^4} - \frac{\arcsin\left(\frac{e^2 x}{\sqrt{d^2 e^2}}\right)}{\sqrt{e^2} e^4} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*x^4/(-e^2\*x^2 + d^2)^(7/2),x, algorithm="maxima")

[Out] 1/15\*e^2\*x\*(15\*x^4/((-e^2\*x^2 + d^2)^(5/2)\*e^2) - 20\*d^2\*x^2/((-e^2\*x^2 + d^2)^(5/2)\*e^4) + 8\*d^4/((-e^2\*x^2 + d^2)^(5/2)\*e^6)) - 1/3\*x\*(3\*x^2/((-e^2\*x^2 + d^2)^(3/2)\*e^2) - 2\*d^2/((-e^2\*x^2 + d^2)^(3/2)\*e^4)) + 2\*d\*x^4/((-e^2\*x^2 + d^2)^(5/2)\*e) + 1/2\*d^2\*x^3/((-e^2\*x^2 + d^2)^(5/2)\*e^2) - 8/3\*d^3\*x^2/((-e^2\*x^2 + d^2)^(5/2)\*e^3) - 3/10\*d^4\*x/((-e^2\*x^2 + d^2)^(5/2)\*e^4) + 16/15\*d^5/((-e^2\*x^2 + d^2)^(5/2)\*e^5) + 11/30\*d^2\*x/((-e^2\*x^2 + d^2)^(3/2)\*e^4) - 4/15\*x/(sqrt(-e^2\*x^2 + d^2)\*e^4) - arcsin(e^2\*x/sqrt(d^2\*e^2))/sqrt(e^2)\*e^4

**Fricas [A]** time = 0.280343, size = 548, normalized size = 4.53

$$\frac{16 e^6 x^6 + 46 d e^5 x^5 - 130 d^2 e^4 x^4 + 5 d^3 e^3 x^3 + 120 d^4 e^2 x^2 - 60 d^5 e x + 30 \left( e^6 x^6 - 2 d e^5 x^5 - 4 d^2 e^4 x^4 + 10 d^3 e^3 x^3 - d^4 e^2 x^2 - 15 \left( e^{11} x^6 - 2 d e^{10} x^5 - 4 d^2 e^9 x^4 + 10 d^3 e^8 x^3 \right) \right)}{15 \left( e^{11} x^6 - 2 d e^{10} x^5 - 4 d^2 e^9 x^4 + 10 d^3 e^8 x^3 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*x^4/(-e^2\*x^2 + d^2)^(7/2),x, algorithm="fricas")

```
[Out] 1/15*(16*e^6*x^6 + 46*d*e^5*x^5 - 130*d^2*e^4*x^4 + 5*d^3*e^3*x^3
+ 120*d^4*e^2*x^2 - 60*d^5*e*x + 30*(e^6*x^6 - 2*d*e^5*x^5 - 4*d
^2*e^4*x^4 + 10*d^3*e^3*x^3 - d^4*e^2*x^2 - 8*d^5*e*x + 4*d^6 + (
3*d*e^4*x^4 - 6*d^2*e^3*x^3 - d^3*e^2*x^2 + 8*d^4*e*x - 4*d^5)*sq
rt(-e^2*x^2 + d^2))*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (
26*e^5*x^5 - 70*d*e^4*x^4 - 25*d^2*e^3*x^3 + 120*d^3*e^2*x^2 - 60
*d^4*e*x)*sqrt(-e^2*x^2 + d^2))/(e^11*x^6 - 2*d*e^10*x^5 - 4*d^2*
e^9*x^4 + 10*d^3*e^8*x^3 - d^4*e^7*x^2 - 8*d^5*e^6*x + 4*d^6*e^5
+ (3*d*e^9*x^4 - 6*d^2*e^8*x^3 - d^3*e^7*x^2 + 8*d^4*e^6*x - 4*d
5*e^5)*sqrt(-e^2*x^2 + d^2))
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 (d + ex)^2}{(-(-d + ex)(d + ex))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral(x**4*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)
```

---

**GIAC/XCAS [A]** time = 0.292406, size = 128, normalized size = 1.06

$$-\arcsin\left(\frac{xe}{d}\right) e^{(-5)} \operatorname{sign}(d) - \frac{\left(16d^5e^{(-5)} + \left(15d^4e^{(-4)} - \left(40d^3e^{(-3)} + \left(35d^2e^{(-2)} - 2\left(15de^{(-1)} + 13x\right)x\right)x\right)x\right)\sqrt{-x^2e^2 + d^2}}{15(x^2e^2 - d^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^2*x^4/(-e^2*x^2 + d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -arcsin(x*e/d)*e^(-5)*sign(d) - 1/15*(16*d^5*e^(-5) + (15*d^4*e^(-
-4) - (40*d^3*e^(-3) + (35*d^2*e^(-2) - 2*(15*d*e^(-1) + 13*x)*x)
*x)*x)*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

$$3.46 \quad \int \frac{x^3(d+ex)^2}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=97

$$\frac{d^2(d+ex)^2}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{4d(d+ex)}{5e^4(d^2-e^2x^2)^{3/2}} + \frac{5d+2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

[Out] (d^2\*(d + e\*x)^2)/(5\*e^4\*(d^2 - e^2\*x^2)^(5/2)) - (4\*d\*(d + e\*x))/(5\*e^4\*(d^2 - e^2\*x^2)^(3/2)) + (5\*d + 2\*e\*x)/(5\*d\*e^4\*Sqrt[d^2 - e^2\*x^2])

**Rubi [A]** time = 0.278389, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$

$$\frac{d^2(d+ex)^2}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{4d(d+ex)}{5e^4(d^2-e^2x^2)^{3/2}} + \frac{5d+2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(d + e\*x)^2)/(d^2 - e^2\*x^2)^(7/2), x]

[Out] (d^2\*(d + e\*x)^2)/(5\*e^4\*(d^2 - e^2\*x^2)^(5/2)) - (4\*d\*(d + e\*x))/(5\*e^4\*(d^2 - e^2\*x^2)^(3/2)) + (5\*d + 2\*e\*x)/(5\*d\*e^4\*Sqrt[d^2 - e^2\*x^2])

**Rubi in Sympy [A]** time = 44.0344, size = 97, normalized size = 1.

$$\frac{d^2}{5e^4(d-ex)^2\sqrt{d^2-e^2x^2}} - \frac{4d}{5e^4(d-ex)\sqrt{d^2-e^2x^2}} + \frac{1}{e^4\sqrt{d^2-e^2x^2}} + \frac{2x}{5de^3\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(e\*x+d)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out] d\*\*2/(5\*e\*\*4\*(d - e\*x)\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) - 4\*d/(5\*e\*\*4\*(d - e\*x)\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) + 1/(e\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) + 2\*x/(5\*d\*e\*\*3\*sqrt(d\*\*2 - e\*\*2\*x\*\*2))

**Mathematica [A]** time = 0.0549273, size = 70, normalized size = 0.72

$$\frac{\sqrt{d^2 - e^2x^2} (2d^3 - 4d^2ex + de^2x^2 + 2e^3x^3)}{5de^4(d - ex)^3(d + ex)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(d + e\*x)^2)/(d^2 - e^2\*x^2)^(7/2), x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(2\*d^3 - 4\*d^2\*e\*x + d\*e^2\*x^2 + 2\*e^3\*x^3))/(5\*d\*e^4\*(d - e\*x)^3\*(d + e\*x))

**Maple [A]** time = 0.011, size = 65, normalized size = 0.7

$$\frac{(-ex + d)(ex + d)^3 (2e^3x^3 + de^2x^2 - 4d^2ex + 2d^3)}{5de^4} (-e^2x^2 + d^2)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)`

[Out] `1/5*(-e*x+d)*(e*x+d)^3*(2*e^3*x^3+d*e^2*x^2-4*d^2*e*x+2*d^3)/d/e^4/(-e^2*x^2+d^2)^(7/2)`

**Maxima [A]** time = 0.714582, size = 209, normalized size = 2.15

$$\frac{x^4}{(-e^2x^2 + d^2)^{\frac{5}{2}}} + \frac{dx^3}{(-e^2x^2 + d^2)^{\frac{5}{2}}e} - \frac{d^2x^2}{(-e^2x^2 + d^2)^{\frac{5}{2}}e^2} - \frac{3d^3x}{5(-e^2x^2 + d^2)^{\frac{5}{2}}e^3} + \frac{2d^4}{5(-e^2x^2 + d^2)^{\frac{5}{2}}e^4} + \frac{dx}{5(-e^2x^2 + d^2)^{\frac{3}{2}}e^3} + \frac{2x}{5\sqrt{-e^2x^2 + d^2}de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^2*x^3/(-e^2*x^2 + d^2)^(7/2),x, algorithm="maxima")`

[Out] `x^4/(-e^2*x^2 + d^2)^(5/2) + d*x^3/((-e^2*x^2 + d^2)^(5/2)*e) - d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^2) - 3/5*d^3*x/((-e^2*x^2 + d^2)^(5/2)*e^3) + 2/5*d^4/((-e^2*x^2 + d^2)^(5/2)*e^4) + 1/5*d*x/((-e^2*x^2 + d^2)^(3/2)*e^3) + 2/5*x/(sqrt(-e^2*x^2 + d^2)*d*e^3)`

**Fricas [A]** time = 0.269581, size = 247, normalized size = 2.55

$$\frac{2e^2x^6 + 2dex^5 - 5d^2x^4 - (2ex^5 - 5dx^4)\sqrt{-e^2x^2 + d^2}}{5(d^6x^6 - 2d^2e^5x^5 - 4d^3e^4x^4 + 10d^4e^3x^3 - d^5e^2x^2 - 8d^6ex + 4d^7 + (3d^2e^4x^4 - 6d^3e^3x^3 - d^4e^2x^2 + 8d^5ex - 4d^6)\sqrt{-e^2x^2 + d^2})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^2*x^3/(-e^2*x^2 + d^2)^(7/2),x, algorithm="fricas")`

[Out] `1/5*(2*e^2*x^6 + 2*d*e*x^5 - 5*d^2*x^4 - (2*e*x^5 - 5*d*x^4)*sqrt(-e^2*x^2 + d^2))/(d*e^6*x^6 - 2*d^2*e^5*x^5 - 4*d^3*e^4*x^4 + 10*d^4*e^3*x^3 - d^5*e^2*x^2 - 8*d^6*e*x + 4*d^7 + (3*d^2*e^4*x^4 - 6*d^3*e^3*x^3 - d^4*e^2*x^2 + 8*d^5*e*x - 4*d^6)*sqrt(-e^2*x^2 + d^2))`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3(d + ex)^2}{(-(-d + ex)(d + ex))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)`

[Out] `Integral(x**3*(d + e*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)`

---

**GIAC/XCAS [A]** time = 0.296317, size = 85, normalized size = 0.88

$$-\frac{\left(2d^4e^{(-4)} + \left(x^2\left(\frac{2xe}{d} + 5\right) - 5d^2e^{(-2)}\right)x^2\right)\sqrt{-x^2e^2 + d^2}}{5(x^2e^2 - d^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*x^3/(-e^2\*x^2 + d^2)^(7/2),x, algorithm="giac")

[Out] -1/5\*(2\*d^4\*e^(-4) + (x^2\*(2\*x\*e/d + 5) - 5\*d^2\*e^(-2))\*x^2)\*sqrt(-x^2\*e^2 + d^2)/(x^2\*e^2 - d^2)^3

$$3.47 \quad \int \frac{x^2(d+ex)^2}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=87

$$\frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}} + \frac{d(d+ex)^2}{5e^3(d^2-e^2x^2)^{5/2}} - \frac{7(d+ex)}{15e^3(d^2-e^2x^2)^{3/2}}$$

[Out]  $(d*(d+e*x)^2)/(5*e^3*(d^2-e^2*x^2)^{(5/2)}) - (7*(d+e*x))/(15*e^3*(d^2-e^2*x^2)^{(3/2)}) + x/(15*d^2*e^2*\text{Sqrt}[d^2-e^2*x^2])$

**Rubi [A]** time = 0.221954, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}} + \frac{d(d+ex)^2}{5e^3(d^2-e^2x^2)^{5/2}} - \frac{7(d+ex)}{15e^3(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^2*(d+e*x)^2)/(d^2-e^2*x^2)^{(7/2)}, x]$

[Out]  $(d*(d+e*x)^2)/(5*e^3*(d^2-e^2*x^2)^{(5/2)}) - (7*(d+e*x))/(15*e^3*(d^2-e^2*x^2)^{(3/2)}) + x/(15*d^2*e^2*\text{Sqrt}[d^2-e^2*x^2])$

**Rubi in Sympy [A]** time = 39.8406, size = 75, normalized size = 0.86

$$\frac{d}{5e^3(d-ex)^2\sqrt{d^2-e^2x^2}} - \frac{7}{15e^3(d-ex)\sqrt{d^2-e^2x^2}} + \frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**2}*(e*x+d)^{**2}/(-e^{**2}*x^{**2}+d^{**2})^{**}(7/2), x)$

[Out]  $d/(5*e^{**3}*(d-e*x)^{**2}*\text{sqrt}(d^{**2}-e^{**2}*x^{**2})) - 7/(15*e^{**3}*(d-e*x)*\text{sqrt}(d^{**2}-e^{**2}*x^{**2})) + x/(15*d^{**2}*e^{**2}*\text{sqrt}(d^{**2}-e^{**2}*x^{**2}))$

**Mathematica [A]** time = 0.0574321, size = 70, normalized size = 0.8

$$\frac{\sqrt{d^2-e^2x^2}(-4d^3+8d^2ex-2de^2x^2+e^3x^3)}{15d^2e^3(d-ex)^3(d+ex)}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(x^2*(d+e*x)^2)/(d^2-e^2*x^2)^{(7/2)}, x]$

[Out]  $(\text{Sqrt}[d^2-e^2*x^2]*(-4*d^3+8*d^2*e*x-2*d*e^2*x^2+e^3*x^3))/(15*d^2*e^3*(d-e*x)^3*(d+e*x))$

**Maple [A]** time = 0.011, size = 66, normalized size = 0.8

$$-\frac{(-ex+d)(ex+d)^3(-e^3x^3+2de^2x^2-8d^2ex+4d^3)}{15d^2e^3}(-e^2x^2+d^2)^{-\frac{7}{2}}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)`

[Out]  $-1/15*(-e*x+d)*(e*x+d)^3*(-e^3*x^3+2*d*e^2*x^2-8*d^2*e*x+4*d^3)/d^2/e^3/(-e^2*x^2+d^2)^(7/2)$

**Maxima [A]** time = 0.722242, size = 177, normalized size = 2.03

$$\frac{x^3}{2(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{2dx^2}{3(-e^2x^2+d^2)^{\frac{5}{2}}e} - \frac{d^2x}{10(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{4d^3}{15(-e^2x^2+d^2)^{\frac{5}{2}}e^3} + \frac{x}{30(-e^2x^2+d^2)^{\frac{3}{2}}e^2} + \frac{x}{15\sqrt{-e^2x^2+d^2}d^2e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^2*x^2/(-e^2*x^2+d^2)^(7/2),x,algorithm="maxima")`

[Out]  $1/2*x^3/(-e^2*x^2+d^2)^(5/2) + 2/3*d*x^2/((-e^2*x^2+d^2)^(5/2)*e) - 1/10*d^2*x/((-e^2*x^2+d^2)^(5/2)*e^2) - 4/15*d^3/((-e^2*x^2+d^2)^(5/2)*e^3) + 1/30*x/((-e^2*x^2+d^2)^(3/2)*e^2) + 1/15*x/(sqrt(-e^2*x^2+d^2)*d^2*e^2)$

**Fricas [A]** time = 0.274306, size = 277, normalized size = 3.18

$$\frac{4e^3x^6 - 11de^2x^5 - 10d^2ex^4 + 20d^3x^3 + (e^2x^5 + 10dex^4 - 20d^2x^3)\sqrt{-e^2x^2+d^2}}{15(d^2e^6x^6 - 2d^3e^5x^5 - 4d^4e^4x^4 + 10d^5e^3x^3 - d^6e^2x^2 - 8d^7ex + 4d^8 + (3d^3e^4x^4 - 6d^4e^3x^3 - d^5e^2x^2 + 8d^6ex - 4d^7)\sqrt{-e^2x^2+d^2})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^2*x^2/(-e^2*x^2+d^2)^(7/2),x,algorithm="fricas")`

[Out]  $-1/15*(4*e^3*x^6 - 11*d*e^2*x^5 - 10*d^2*e*x^4 + 20*d^3*x^3 + (e^2*x^5 + 10*dex^4 - 20*d^2*x^3)*sqrt(-e^2*x^2+d^2))/(d^2*e^6*x^6 - 2*d^3*e^5*x^5 - 4*d^4*e^4*x^4 + 10*d^5*e^3*x^3 - d^6*e^2*x^2 - 8*d^7*e*x + 4*d^8 + (3*d^3*e^4*x^4 - 6*d^4*e^3*x^3 - d^5*e^2*x^2 + 8*d^6*ex - 4*d^7)*sqrt(-e^2*x^2+d^2))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2(d+ex)^2}{(-(-d+ex)(d+ex))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)`

[Out] `Integral(x**2*(d+e*x)**2/(-(-d+e*x)*(d+e*x))**(7/2),x)`

**GIAC/XCAS [A]** time = 0.30857, size = 82, normalized size = 0.94

$$\frac{(4d^3e^{(-3)} - (x(\frac{x^2e^2}{d^2} + 5) + 10de^{(-1)})x^2)\sqrt{-x^2e^2+d^2}}{15(x^2e^2-d^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^2*x^2/(-e^2*x^2 + d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*(4*d^3*e^(-3) - (x*(x^2*e^2/d^2 + 5) + 10*d*e^(-1))*x^2)*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

$$3.48 \quad \int \frac{x(d+ex)^2}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=89

$$\frac{(d+ex)^2}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{2(d+ex)}{15de^2(d^2-e^2x^2)^{3/2}} - \frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}$$

[Out]  $(d + e*x)^2/(5*e^2*(d^2 - e^2*x^2)^{(5/2)}) - (2*(d + e*x))/(15*d*e^2*(d^2 - e^2*x^2)^{(3/2)}) - (4*x)/(15*d^3*e*\text{Sqrt}[d^2 - e^2*x^2])$

**Rubi [A]** time = 0.122402, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$\frac{(d+ex)^2}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{2(d+ex)}{15de^2(d^2-e^2x^2)^{3/2}} - \frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x*(d + e*x)^2)/(d^2 - e^2*x^2)^{(7/2)}, x]$

[Out]  $(d + e*x)^2/(5*e^2*(d^2 - e^2*x^2)^{(5/2)}) - (2*(d + e*x))/(15*d*e^2*(d^2 - e^2*x^2)^{(3/2)}) - (4*x)/(15*d^3*e*\text{Sqrt}[d^2 - e^2*x^2])$

**Rubi in Sympy [A]** time = 12.2917, size = 76, normalized size = 0.85

$$\frac{(d+ex)^2}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{2(d+ex)}{15de^2(d^2-e^2x^2)^{3/2}} - \frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2), x)$

[Out]  $(d + e*x)**2/(5*e**2*(d**2 - e**2*x**2)**(5/2)) - 2*(d + e*x)/(15*d*e**2*(d**2 - e**2*x**2)**(3/2)) - 4*x/(15*d**3*e*\text{sqrt}(d**2 - e**2*x**2))$

**Mathematica [A]** time = 0.0504812, size = 69, normalized size = 0.78

$$\frac{\sqrt{d^2 - e^2x^2} (d^3 - 2d^2ex + 8de^2x^2 - 4e^3x^3)}{15d^3e^2(d - ex)^3(d + ex)}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(x*(d + e*x)^2)/(d^2 - e^2*x^2)^{(7/2)}, x]$

[Out]  $(\text{Sqrt}[d^2 - e^2*x^2]*(d^3 - 2*d^2*e*x + 8*d*e^2*x^2 - 4*e^3*x^3))/(15*d^3*e^2*(d - e*x)^3*(d + e*x))$

**Maple [A]** time = 0.013, size = 64, normalized size = 0.7

$$\frac{(-ex + d)(ex + d)^3 (-4e^3x^3 + 8de^2x^2 - 2d^2ex + d^3)}{15d^3e^2} (-e^2x^2 + d^2)^{-7/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)`

[Out]  $1/15*(-e*x+d)*(e*x+d)^3*(-4*e^3*x^3+8*d*e^2*x^2-2*d^2*e*x+d^3)/d^3/e^2/(-e^2*x^2+d^2)^(7/2)$

**Maxima [A]** time = 0.709691, size = 147, normalized size = 1.65

$$\frac{x^2}{3(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{2dx}{5(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{d^2}{15(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{2x}{15(-e^2x^2+d^2)^{\frac{3}{2}}de} - \frac{4x}{15\sqrt{-e^2x^2+d^2}d^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^2*x/(-e^2*x^2+d^2)^(7/2),x,algorithm="maxima")`

[Out]  $1/3*x^2/(-e^2*x^2+d^2)^(5/2) + 2/5*d*x/((-e^2*x^2+d^2)^(5/2)*e) + 1/15*d^2/((-e^2*x^2+d^2)^(5/2)*e^2) - 2/15*x/((-e^2*x^2+d^2)^(3/2)*d*e) - 4/15*x/(sqrt(-e^2*x^2+d^2)*d^3*e)$

**Fricas [A]** time = 0.276486, size = 306, normalized size = 3.44

$$\frac{e^4x^6 - 14de^3x^5 + 20d^2e^2x^4 + 20d^3ex^3 - 30d^4x^2 + (4e^3x^5 - 5de^2x^4 - 20d^2ex^3 + 30d^3x^2)\sqrt{-e^2x^2+d^2}}{15(d^3e^6x^6 - 2d^4e^5x^5 - 4d^5e^4x^4 + 10d^6e^3x^3 - d^7e^2x^2 - 8d^8ex + 4d^9 + (3d^4e^4x^4 - 6d^5e^3x^3 - d^6e^2x^2 + 8d^7ex - 4d^8)\sqrt{-e^2x^2+d^2})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^2*x/(-e^2*x^2+d^2)^(7/2),x,algorithm="fricas")`

[Out]  $1/15*(e^4*x^6 - 14*d*e^3*x^5 + 20*d^2*e^2*x^4 + 20*d^3*e*x^3 - 30*d^4*x^2 + (4*e^3*x^5 - 5*d*e^2*x^4 - 20*d^2*e*x^3 + 30*d^3*x^2)*sqrt(-e^2*x^2+d^2))/(d^3*e^6*x^6 - 2*d^4*e^5*x^5 - 4*d^5*e^4*x^4 + 10*d^6*e^3*x^3 - d^7*e^2*x^2 - 8*d^8*e*x + 4*d^9 + (3*d^4*e^4*x^4 - 6*d^5*e^3*x^3 - d^6*e^2*x^2 + 8*d^7*e*x - 4*d^8)*sqrt(-e^2*x^2+d^2))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x(d+ex)^2}{(-(-d+ex)(d+ex))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)`

[Out] `Integral(x*(d+e*x)**2/(-(-d+e*x)*(d+e*x))**(7/2),x)`

**GIAC/XCAS [A]** time = 0.293586, size = 86, normalized size = 0.97

$$\frac{\left(\left(2x\left(\frac{2x^2e^3}{d^3} - \frac{5e}{d}\right) - 5\right)x^2 - d^2e^{(-2)}\right)\sqrt{-x^2e^2+d^2}}{15(x^2e^2-d^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^2*x/(-e^2*x^2 + d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/15*((2*x*(2*x^2*e^3/d^3 - 5*e/d) - 5)*x^2 - d^2*e^(-2))*sqrt(-x  
^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

$$3.49 \quad \int \frac{(d+ex)^2}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=77

$$\frac{x}{5d^2(d^2-e^2x^2)^{3/2}} + \frac{2(d+ex)}{5e(d^2-e^2x^2)^{5/2}} + \frac{2x}{5d^4\sqrt{d^2-e^2x^2}}$$

[Out] (2\*(d + e\*x))/(5\*e\*(d^2 - e^2\*x^2)^(5/2)) + x/(5\*d^2\*(d^2 - e^2\*x^2)^(3/2)) + (2\*x)/(5\*d^4\*Sqrt[d^2 - e^2\*x^2])

**Rubi [A]** time = 0.0559055, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{x}{5d^2(d^2-e^2x^2)^{3/2}} + \frac{2(d+ex)}{5e(d^2-e^2x^2)^{5/2}} + \frac{2x}{5d^4\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^2/(d^2 - e^2\*x^2)^(7/2), x]

[Out] (2\*(d + e\*x))/(5\*e\*(d^2 - e^2\*x^2)^(5/2)) + x/(5\*d^2\*(d^2 - e^2\*x^2)^(3/2)) + (2\*x)/(5\*d^4\*Sqrt[d^2 - e^2\*x^2])

**Rubi in Sympy [A]** time = 7.70146, size = 66, normalized size = 0.86

$$\frac{2(d+ex)}{5e(d^2-e^2x^2)^{5/2}} + \frac{x}{5d^2(d^2-e^2x^2)^{3/2}} + \frac{2x}{5d^4\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out] 2\*(d + e\*x)/(5\*e\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)) + x/(5\*d\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)) + 2\*x/(5\*d\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2))

**Mathematica [A]** time = 0.0475341, size = 70, normalized size = 0.91

$$\frac{\sqrt{d^2 - e^2x^2} (2d^3 + d^2ex - 4de^2x^2 + 2e^3x^3)}{5d^4e(d - ex)^3(d + ex)}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^2/(d^2 - e^2\*x^2)^(7/2), x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(2\*d^3 + d^2\*e\*x - 4\*d\*e^2\*x^2 + 2\*e^3\*x^3))/(5\*d^4\*e\*(d - e\*x)^3\*(d + e\*x))

**Maple [A]** time = 0.011, size = 65, normalized size = 0.8

$$\frac{(-ex + d)(ex + d)^3 (2e^3x^3 - 4e^2x^2d + xd^2e + 2d^3)}{5d^4e} (-e^2x^2 + d^2)^{-7/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^2/(-e^2*x^2+d^2)^(7/2),x)`

[Out]  $\frac{1}{5} * (-e * x + d) * (e * x + d)^3 * (2 * e^3 * x^3 - 4 * d * e^2 * x^2 + d^2 * e * x + 2 * d^3) / d^4 / e / (-e^2 * x^2 + d^2)^(7/2)$

**Maxima [A]** time = 0.716504, size = 105, normalized size = 1.36

$$\frac{2x}{5(-e^2x^2 + d^2)^{\frac{5}{2}}} + \frac{2d}{5(-e^2x^2 + d^2)^{\frac{5}{2}}e} + \frac{x}{5(-e^2x^2 + d^2)^{\frac{3}{2}}d^2} + \frac{2x}{5\sqrt{-e^2x^2 + d^2}d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^2/(-e^2*x^2 + d^2)^(7/2),x, algorithm="maxima")`

[Out]  $\frac{2}{5} * x / (-e^2 * x^2 + d^2)^(5/2) + \frac{2}{5} * d / ((-e^2 * x^2 + d^2)^(5/2) * e) + \frac{1}{5} * x / ((-e^2 * x^2 + d^2)^(3/2) * d^2) + \frac{2}{5} * x / (\text{sqrt}(-e^2 * x^2 + d^2) * d^4)$

**Fricas [A]** time = 0.270689, size = 333, normalized size = 4.32

$$\frac{2e^5x^6 + 2de^4x^5 - 20d^2e^3x^4 + 15d^3e^2x^3 + 20d^4ex^2 - 20d^5x - (2e^4x^5 - 10de^3x^4 + 5d^2e^2x^3 + 20d^3ex^2 - 20d^4x)\sqrt{-e^2}}{5(d^4e^6x^6 - 2d^5e^5x^5 - 4d^6e^4x^4 + 10d^7e^3x^3 - d^8e^2x^2 - 8d^9ex + 4d^{10} + (3d^5e^4x^4 - 6d^6e^3x^3 - d^7e^2x^2 + 8d^8ex - 4d^9)\sqrt{-e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^2/(-e^2*x^2 + d^2)^(7/2),x, algorithm="fricas")`

[Out]  $\frac{1}{5} * (2 * e^5 * x^6 + 2 * d * e^4 * x^5 - 20 * d^2 * e^3 * x^4 + 15 * d^3 * e^2 * x^3 + 20 * d^4 * e * x^2 - 20 * d^5 * x - (2 * e^4 * x^5 - 10 * d * e^3 * x^4 + 5 * d^2 * e^2 * x^3 + 20 * d^3 * e * x^2 - 20 * d^4 * x) * \text{sqrt}(-e^2 * x^2 + d^2)) / (d^4 * e^6 * x^6 - 2 * d^5 * e^5 * x^5 - 4 * d^6 * e^4 * x^4 + 10 * d^7 * e^3 * x^3 - d^8 * e^2 * x^2 - 8 * d^9 * e * x + 4 * d^{10} + (3 * d^5 * e^4 * x^4 - 6 * d^6 * e^3 * x^3 - d^7 * e^2 * x^2 + 8 * d^8 * e * x - 4 * d^9) * \text{sqrt}(-e^2 * x^2 + d^2))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^2}{(-(-d + ex)(d + ex))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)`

[Out] `Integral((d + e*x)**2/((-d + e*x)*(d + e*x))**(7/2), x)`

**GIAC/XCAS [A]** time = 0.288109, size = 82, normalized size = 1.06

$$\frac{\sqrt{-x^2e^2 + d^2} \left( x^2 \left( \frac{2x^2e^4}{d^4} - \frac{5e^2}{d^2} \right) + 5 \right) x + 2de^{(-1)}}{5(x^2e^2 - d^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^2/(-e^2*x^2 + d^2)^(7/2),x, algorithm="giac")
```

```
[Out] -1/5*sqrt(-x^2*e^2 + d^2)*((x^2*(2*x^2*e^4/d^4 - 5*e^2/d^2) + 5)*  
x + 2*d*e^(-1))/(x^2*e^2 - d^2)^3
```



$$3.50 \quad \int \frac{(d+ex)^2}{x(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=117

$$\frac{2(d+ex)}{5d(d^2-e^2x^2)^{5/2}} + \frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5} + \frac{5d+8ex}{15d^3(d^2-e^2x^2)^{3/2}}$$

[Out] (2\*(d + e\*x))/(5\*d\*(d^2 - e^2\*x^2)^(5/2)) + (5\*d + 8\*e\*x)/(15\*d^3\*(d^2 - e^2\*x^2)^(3/2)) + (15\*d + 16\*e\*x)/(15\*d^5\*Sqrt[d^2 - e^2\*x^2]) - ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]/d^5

**Rubi [A]** time = 0.338625, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{2(d+ex)}{5d(d^2-e^2x^2)^{5/2}} + \frac{15d+16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5} + \frac{5d+8ex}{15d^3(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^2/(x\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out] (2\*(d + e\*x))/(5\*d\*(d^2 - e^2\*x^2)^(5/2)) + (5\*d + 8\*e\*x)/(15\*d^3\*(d^2 - e^2\*x^2)^(3/2)) + (15\*d + 16\*e\*x)/(15\*d^5\*Sqrt[d^2 - e^2\*x^2]) - ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]/d^5

**Rubi in Sympy [A]** time = 41.5973, size = 114, normalized size = 0.97

$$\frac{1}{5d^2(d-ex)^2\sqrt{d^2-e^2x^2}} + \frac{8}{15d^3(d-ex)\sqrt{d^2-e^2x^2}} + \frac{1}{d^4\sqrt{d^2-e^2x^2}} + \frac{16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*2/x/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out] 1/(5\*d\*\*2\*(d - e\*x)\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) + 8/(15\*d\*\*3\*(d - e\*x)\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) + 1/(d\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) + 16\*e\*x/(15\*d\*\*5\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) - atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d)/d\*\*5

**Mathematica [A]** time = 0.0913727, size = 95, normalized size = 0.81

$$\frac{-15 \log\left(\sqrt{d^2 - e^2x^2} + d\right) + \frac{\sqrt{d^2 - e^2x^2}(26d^3 - 22d^2ex - 17de^2x^2 + 16e^3x^3)}{(d-ex)^3(d+ex)} + 15 \log(x)}{15d^5}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^2/(x\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(26\*d^3 - 22\*d^2\*e\*x - 17\*d\*e^2\*x^2 + 16\*e^3\*x^3))/((d - e\*x)^3\*(d + e\*x)) + 15\*Log[x] - 15\*Log[d + Sqrt[d^2 - e^2\*x^2]])/(15\*d^5)

---

**Maple [A]** time = 0.013, size = 160, normalized size = 1.4

$$\begin{aligned} & \frac{2}{5} (-e^2x^2 + d^2)^{-\frac{5}{2}} + \frac{1}{3d^2} (-e^2x^2 + d^2)^{-\frac{3}{2}} + \frac{1}{d^4} \frac{1}{\sqrt{-e^2x^2 + d^2}} \\ & - \frac{1}{d^4} \ln\left(\frac{1}{x} \left(2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}} \\ & + \frac{2ex}{5d} (-e^2x^2 + d^2)^{-\frac{5}{2}} + \frac{8ex}{15d^3} (-e^2x^2 + d^2)^{-\frac{3}{2}} + \frac{16ex}{15d^5} \frac{1}{\sqrt{-e^2x^2 + d^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^2/x/(-e^2*x^2+d^2)^(7/2),x)`

[Out] `2/5/(-e^2*x^2+d^2)^(5/2)+1/3/d^2/(-e^2*x^2+d^2)^(3/2)+1/d^4/(-e^2*x^2+d^2)^(1/2)-1/d^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+2/5/d*e*x/(-e^2*x^2+d^2)^(5/2)+8/15/d^3*e*x/(-e^2*x^2+d^2)^(3/2)+16/15/d^5*e*x/(-e^2*x^2+d^2)^(1/2)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^2/((-e^2*x^2 + d^2)^(7/2)*x),x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.279835, size = 540, normalized size = 4.62

$$\frac{26e^6x^6 - 4de^5x^5 - 155d^2e^4x^4 + 130d^3e^3x^3 + 120d^4e^2x^2 - 120d^5ex + 15(e^6x^6 - 2de^5x^5 - 4d^2e^4x^4 + 10d^3e^3x^3 - d^4e^2x^2 - 120d^5ex + 15(d^5e^6x^6 - 2d^6e^5x^5 - 4d^7e^4x^4 + 10d^8e^3x^3 - d^9e^2x^2 - 120d^{10}ex + 4d^{11} + (3d^6e^4x^4 - 6d^7e^3x^3 - d^8e^2x^2 + 8d^9ex - 4d^{10}))\sqrt{-e^2x^2 + d^2})}{15(d^5e^6x^6 - 2d^6e^5x^5 - 4d^7e^4x^4 + 10d^8e^3x^3 - d^9e^2x^2 - 120d^{10}ex + 4d^{11} + (3d^6e^4x^4 - 6d^7e^3x^3 - d^8e^2x^2 + 8d^9ex - 4d^{10}))\sqrt{-e^2x^2 + d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^2/((-e^2*x^2 + d^2)^(7/2)*x),x, algorithm="fricas")`

[Out] `1/15*(26*e^6*x^6 - 4*d*e^5*x^5 - 155*d^2*e^4*x^4 + 130*d^3*e^3*x^3 + 120*d^4*e^2*x^2 - 120*d^5*e*x + 15*(e^6*x^6 - 2*d*e^5*x^5 - 4*d^2*e^4*x^4 + 10*d^3*e^3*x^3 - d^4*e^2*x^2 - 8*d^5*e*x + 4*d^6 + (3*d*e^4*x^4 - 6*d^2*e^3*x^3 - d^3*e^2*x^2 + 8*d^4*e*x - 4*d^5)*sqrt(-e^2*x^2 + d^2))*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (16*e^5*x^5 - 95*d*e^4*x^4 + 70*d^2*e^3*x^3 + 120*d^3*e^2*x^2 - 120*d^4*e*x)*sqrt(-e^2*x^2 + d^2))/(d^5*e^6*x^6 - 2*d^6*e^5*x^5 - 4*d^7*e^4*x^4 + 10*d^8*e^3*x^3 - d^9*e^2*x^2 - 8*d^10*e*x + 4*d^11 + (3*d^6*e^4*x^4 - 6*d^7*e^3*x^3 - d^8*e^2*x^2 + 8*d^9*e*x - 4*d^10)*sqrt(-e^2*x^2 + d^2))`

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**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^2}{x(-(-d + ex)(d + ex))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**2/x/(-e**2*x**2+d**2)**(7/2),x)`

[Out] `Integral((d + e*x)**2/(x*(-(-d + e*x)*(d + e*x))**(7/2)), x)`

**GIAC/XCAS [A]** time = 0.297339, size = 159, normalized size = 1.36

$$\frac{\sqrt{-x^2e^2 + d^2} \left( \left( \left( x \left( \frac{16xe^5}{d^5} + \frac{15e^4}{d^4} \right) - \frac{40e^3}{d^3} \right) x - \frac{35e^2}{d^2} \right) x + \frac{30e}{d} \right) x + 26}{15(x^2e^2 - d^2)^3} - \frac{\ln \left( \frac{|-2de^{-2}\sqrt{-x^2e^2+d^2}e|e^{(-2)}}{2|x|} \right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^2/((-e^2*x^2 + d^2)^(7/2)*x),x, algorithm="giac")`

[Out] `-1/15*sqrt(-x^2*e^2 + d^2)*(((x*(16*x*e^5/d^5 + 15*e^4/d^4) - 40*e^3/d^3)*x - 35*e^2/d^2)*x + 30*e/d)*x + 26)/(x^2*e^2 - d^2)^3 - ln(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x))/d^5`

$$3.51 \quad \int \frac{(d+ex)^2}{x^2(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=145

$$\frac{2e(d+ex)}{5d^2(d^2-e^2x^2)^{5/2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} + \frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{2e \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6} + \frac{e(10d+13ex)}{15d^4(d^2-e^2x^2)^{3/2}}$$

[Out] (2\*e\*(d + e\*x))/(5\*d^2\*(d^2 - e^2\*x^2)^(5/2)) + (e\*(10\*d + 13\*e\*x))/(15\*d^4\*(d^2 - e^2\*x^2)^(3/2)) + (e\*(30\*d + 41\*e\*x))/(15\*d^6\*Sqrt[d^2 - e^2\*x^2]) - Sqrt[d^2 - e^2\*x^2]/(d^6\*x) - (2\*e\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/d^6

**Rubi [A]** time = 0.429258, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\frac{2e(d+ex)}{5d^2(d^2-e^2x^2)^{5/2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} + \frac{e(30d+41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{2e \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6} + \frac{e(10d+13ex)}{15d^4(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^2/(x^2\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out] (2\*e\*(d + e\*x))/(5\*d^2\*(d^2 - e^2\*x^2)^(5/2)) + (e\*(10\*d + 13\*e\*x))/(15\*d^4\*(d^2 - e^2\*x^2)^(3/2)) + (e\*(30\*d + 41\*e\*x))/(15\*d^6\*Sqrt[d^2 - e^2\*x^2]) - Sqrt[d^2 - e^2\*x^2]/(d^6\*x) - (2\*e\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/d^6

**Rubi in Sympy [A]** time = 52.5511, size = 143, normalized size = 0.99

$$\frac{e}{5d^3(d-ex)^2\sqrt{d^2-e^2x^2}} + \frac{13e}{15d^4(d-ex)\sqrt{d^2-e^2x^2}} - \frac{1}{d^4x\sqrt{d^2-e^2x^2}} + \frac{2e}{d^5\sqrt{d^2-e^2x^2}} + \frac{56e^2x}{15d^6\sqrt{d^2-e^2x^2}} - \frac{2e \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*2/x\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out] e/(5\*d\*\*3\*(d - e\*x)\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) + 13\*e/(15\*d\*\*4\*(d - e\*x)\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) - 1/(d\*\*4\*x\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) + 2\*e/(d\*\*5\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) + 56\*e\*\*2\*x/(15\*d\*\*6\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) - 2\*e\*atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d)/d\*\*6

**Mathematica [A]** time = 0.109448, size = 112, normalized size = 0.77

$$\frac{-30e \log\left(\sqrt{d^2-e^2x^2}+d\right) + \frac{\sqrt{d^2-e^2x^2}(15d^4-76d^3ex+32d^2e^2x^2+82de^3x^3-56e^4x^4)}{x(ex-d)^3(d+ex)}}{15d^6} + 30e \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^2/(x^2\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out]  $((\text{Sqrt}[d^2 - e^2 x^2]) * (15 d^4 - 76 d^3 e x + 32 d^2 e^2 x^2 + 82 d e^3 x^3 - 56 e^4 x^4)) / (x (-d + e x)^3 (d + e x)) + 30 e \text{Log}[x] - 30 e \text{Log}[d + \text{Sqrt}[d^2 - e^2 x^2]]) / (15 d^6)$

**Maple [A]** time = 0.016, size = 193, normalized size = 1.3

$$\begin{aligned} & \frac{7 e^2 x}{5 d^2} (-e^2 x^2 + d^2)^{-\frac{5}{2}} + \frac{28 e^2 x}{15 d^4} (-e^2 x^2 + d^2)^{-\frac{3}{2}} + \frac{56 e^2 x}{15 d^6} \frac{1}{\sqrt{-e^2 x^2 + d^2}} \\ & - \frac{1}{x} (-e^2 x^2 + d^2)^{-\frac{5}{2}} + \frac{2 e}{5 d} (-e^2 x^2 + d^2)^{-\frac{5}{2}} + \frac{2 e}{3 d^3} (-e^2 x^2 + d^2)^{-\frac{3}{2}} \\ & + 2 \frac{e}{d^5 \sqrt{-e^2 x^2 + d^2}} - 2 \frac{e}{d^5 \sqrt{d^2}} \ln \left( \frac{2 d^2 + 2 \sqrt{d^2} \sqrt{-e^2 x^2 + d^2}}{x} \right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^2/x^2/(-e^2*x^2+d^2)^(7/2),x)`

[Out]  $7/5 * e^2 * x / d^2 / (-e^2 * x^2 + d^2)^{(5/2)} + 28/15 * e^2 / d^4 * x / (-e^2 * x^2 + d^2)^{(3/2)} + 56/15 * e^2 / d^6 * x / (-e^2 * x^2 + d^2)^{(1/2)} - 1/x / (-e^2 * x^2 + d^2)^{(5/2)} + 2/5/d * e / (-e^2 * x^2 + d^2)^{(5/2)} + 2/3/d^3 * e / (-e^2 * x^2 + d^2)^{(3/2)} + 2/d^5 * e / (-e^2 * x^2 + d^2)^{(1/2)} - 2/d^5 * e / (d^2)^{(1/2)} * \ln((2 * d^2 + 2 * (d^2)^{(1/2)} * (-e^2 * x^2 + d^2)^{(1/2)})/x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^2/((-e^2*x^2 + d^2)^(7/2)*x^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.283614, size = 670, normalized size = 4.62

$$56 e^8 x^8 - 266 d e^7 x^7 - 112 d^2 e^6 x^6 + 1100 d^3 e^5 x^5 - 415 d^4 e^4 x^4 - 1080 d^5 e^3 x^3 + 600 d^6 e^2 x^2 + 240 d^7 e x - 120 d^8 - 30 (4 d e^7$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^2/((-e^2*x^2 + d^2)^(7/2)*x^2),x, algorithm="fricas")`

[Out]  $-1/15 * (56 * e^8 * x^8 - 266 * d * e^7 * x^7 - 112 * d^2 * e^6 * x^6 + 1100 * d^3 * e^5 * x^5 - 415 * d^4 * e^4 * x^4 - 1080 * d^5 * e^3 * x^3 + 600 * d^6 * e^2 * x^2 + 240 * d^7 * e * x - 120 * d^8 - 30 * (4 * d * e^7 * x^7 - 8 * d^2 * e^6 * x^6 - 8 * d^3 * e^5 * x^5 + 24 * d^4 * e^4 * x^4 - 4 * d^5 * e^3 * x^3 - 16 * d^6 * e^2 * x^2 + 8 * d^7 * e * x - (e^7 * x^7 - 2 * d * e^6 * x^6 - 7 * d^2 * e^5 * x^5 + 16 * d^3 * e^4 * x^4 - 16 * d^4 * e^3 * x^3 + 8 * d^5 * e^2 * x^2 + 8 * d^6 * e * x) * \text{sqrt}(-e^2 * x^2 + d^2)) * \text{log}(-(d - \text{sqrt}(-e^2 * x^2 + d^2))/x) + 2 * (23 * e^7 * x^7 + 66 * d * e^6 * x^6 - 325 * d^2 * e^5 * x^5 + 80 * d^3 * e^4 * x^4 + 480 * d^4 * e^3 * x^3 - 270 * d^5 * e^2 * x^2 - 120 * d^6 * e * x + 60 * d^7) * \text{sqrt}(-e^2 * x^2 + d^2)) / (4 * d^7 * e^6 * x^7 - 8 * d^8 * e^5 * x^6 - 8 * d^9 * e^4 * x^5 + 24 * d^{10} * e^3 * x^4 - 4 * d^{11} * e^2 * x^3 - 16 * d^{12} * e * x^2 + 8 * d^{13} * x - (d^6 * e^6 * x^7 - 2 * d^7 * e^5 * x^6 - 7 * d^8 * e^4 * x^5 + 16 * d^9 * e^3 * x^4 - 16 * d^{11} * e * x^2 + 8 * d^{12} * x) * \text{sqrt}(-e^2 * x^2 + d^2))$

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**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^2}{x^2 (-(-d + ex)(d + ex))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2/x\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2),x)

[Out] Integral((d + e\*x)\*\*2/(x\*\*2\*(-(-d + e\*x)\*(d + e\*x))\*\*(7/2)), x)

---

**GIAC/XCAS [A]** time = 0.295941, size = 254, normalized size = 1.75

$$\frac{\sqrt{-x^2e^2 + d^2} \left( \left( \left( x \left( \frac{41xe^6}{d^6} + \frac{30e^5}{d^5} \right) - \frac{95e^4}{d^4} \right) x - \frac{70e^3}{d^3} \right) x + \frac{60e^2}{d^2} \right) x + \frac{46e}{d}}{15(x^2e^2 - d^2)^3} - \frac{2e \ln \left( \frac{|-2de - 2\sqrt{-x^2e^2 + d^2}e|e^{(-2)}}{2|x|} \right)}{d^6} + \frac{xe^3}{2(de + \sqrt{-x^2e^2 + d^2}e)d^6} - \frac{(de + \sqrt{-x^2e^2 + d^2}e)e^{(-1)}}{2d^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2/((-e^2\*x^2 + d^2)^(7/2)\*x^2),x, algorithm="giac")

[Out] -1/15\*sqrt(-x^2\*e^2 + d^2)\*(((x\*(41\*x\*e^6/d^6 + 30\*e^5/d^5) - 95\*e^4/d^4)\*x - 70\*e^3/d^3)\*x + 60\*e^2/d^2)\*x + 46\*e/d)/(x^2\*e^2 - d^2)^3 - 2\*e\*ln(1/2\*abs(-2\*d\*e - 2\*sqrt(-x^2\*e^2 + d^2)\*e)\*e^(-2)/abs(x))/d^6 + 1/2\*x\*e^3/((d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*d^6) - 1/2\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*e^(-1)/(d^6\*x)

$$3.52 \quad \int \frac{(d+ex)^2}{x^3(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=182

$$\frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{2e\sqrt{d^2-e^2x^2}}{d^7x} - \frac{9e^2 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7} \\ - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} + \frac{e^2(5d+6ex)}{5d^5(d^2-e^2x^2)^{3/2}} + \frac{2e^2(d+ex)}{5d^3(d^2-e^2x^2)^{5/2}}$$

[Out]  $(2e^2(d+ex))/(5d^3(d^2-e^2x^2)^{5/2}) + (e^2(5d+6ex))/(5d^5(d^2-e^2x^2)^{3/2}) + (2e^2(10d+11ex))/(5d^7\sqrt{d^2-e^2x^2}) - \sqrt{d^2-e^2x^2}/(2d^6x^2) - (2e^2\sqrt{d^2-e^2x^2})/(d^7x) - (9e^2\text{ArcTanh}[\sqrt{d^2-e^2x^2}/d])/2d^7$

**Rubi [A]** time = 0.559863, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{2e^2(10d+11ex)}{5d^7\sqrt{d^2-e^2x^2}} - \frac{2e\sqrt{d^2-e^2x^2}}{d^7x} - \frac{9e^2 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7} \\ - \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} + \frac{e^2(5d+6ex)}{5d^5(d^2-e^2x^2)^{3/2}} + \frac{2e^2(d+ex)}{5d^3(d^2-e^2x^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^2/(x^3\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out]  $(2e^2(d+ex))/(5d^3(d^2-e^2x^2)^{5/2}) + (e^2(5d+6ex))/(5d^5(d^2-e^2x^2)^{3/2}) + (2e^2(10d+11ex))/(5d^7\sqrt{d^2-e^2x^2}) - \sqrt{d^2-e^2x^2}/(2d^6x^2) - (2e^2\sqrt{d^2-e^2x^2})/(d^7x) - (9e^2\text{ArcTanh}[\sqrt{d^2-e^2x^2}/d])/2d^7$

**Rubi in Sympy [A]** time = 68.7112, size = 199, normalized size = 1.09

$$\frac{e^2}{5d^4(d-ex)^2\sqrt{d^2-e^2x^2}} + \frac{1}{d^4x^2\sqrt{d^2-e^2x^2}} + \frac{6e^2}{5d^5(d-ex)\sqrt{d^2-e^2x^2}} - \frac{2e}{d^5x\sqrt{d^2-e^2x^2}} \\ + \frac{3e^2}{d^6\sqrt{d^2-e^2x^2}} - \frac{3\sqrt{d^2-e^2x^2}}{2d^6x^2} + \frac{32e^3x}{5d^7\sqrt{d^2-e^2x^2}} - \frac{9e^2 \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*2/x\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out]  $e^2/(5d^4(d-ex)^2\sqrt{d^2-e^2x^2}) + 1/(d^4x^2\sqrt{d^2-e^2x^2}) + 6e^2/(5d^5(d-ex)\sqrt{d^2-e^2x^2}) - 2e/(d^5x\sqrt{d^2-e^2x^2}) + 3e^2/(d^6\sqrt{d^2-e^2x^2}) - 3\sqrt{d^2-e^2x^2}/(2d^6x^2) + 32e^3x/(5d^7\sqrt{d^2-e^2x^2}) - 9e^2\operatorname{atanh}(\sqrt{d^2-e^2x^2}/d)/(2d^7)$

**Mathematica [A]** time = 0.120784, size = 127, normalized size = 0.7

$$\frac{-45e^2 \log\left(\sqrt{d^2 - e^2x^2} + d\right) + \frac{\sqrt{d^2 - e^2x^2}(5d^5 + 10d^4ex - 94d^3e^2x^2 + 58d^2e^3x^3 + 83de^4x^4 - 64e^5x^5)}{x^2(ex-d)^3(d+ex)} + 45e^2 \log(x)}{10d^7}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^2/(x^3\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(5\*d^5 + 10\*d^4\*e\*x - 94\*d^3\*e^2\*x^2 + 58\*d^2\*e^3\*x^3 + 83\*d\*e^4\*x^4 - 64\*e^5\*x^5))/(x^2\*(-d + e\*x)^3\*(d + e\*x)) + 45\*e^2\*Log[x] - 45\*e^2\*Log[d + Sqrt[d^2 - e^2\*x^2]])/(10\*d^7)

**Maple [A]** time = 0.018, size = 224, normalized size = 1.2

$$\begin{aligned} & -\frac{1}{2x^2}(-e^2x^2 + d^2)^{-\frac{5}{2}} + \frac{9e^2}{10d^2}(-e^2x^2 + d^2)^{-\frac{5}{2}} + \frac{3e^2}{2d^4}(-e^2x^2 + d^2)^{-\frac{3}{2}} + \frac{9e^2}{2d^6} \frac{1}{\sqrt{-e^2x^2 + d^2}} \\ & - \frac{9e^2}{2d^6} \ln\left(\frac{1}{x}\left(2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}} - 2 \frac{e}{dx(-e^2x^2 + d^2)^{5/2}} \\ & + \frac{12e^3x}{5d^3}(-e^2x^2 + d^2)^{-\frac{5}{2}} + \frac{16e^3x}{5d^5}(-e^2x^2 + d^2)^{-\frac{3}{2}} + \frac{32e^3x}{5d^7} \frac{1}{\sqrt{-e^2x^2 + d^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^2/x^3/(-e^2\*x^2+d^2)^(7/2), x)

[Out] -1/2/x^2/(-e^2\*x^2+d^2)^(5/2)+9/10/d^2\*e^2/(-e^2\*x^2+d^2)^(5/2)+3/2/d^4\*e^2/(-e^2\*x^2+d^2)^(3/2)+9/2/d^6\*e^2/(-e^2\*x^2+d^2)^(1/2)-9/2/d^6\*e^2/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)-2/d\*e/x/(-e^2\*x^2+d^2)^(5/2)+12/5/d^3\*e^3\*x/(-e^2\*x^2+d^2)^(5/2)+16/5/d^5\*e^3\*x/(-e^2\*x^2+d^2)^(3/2)+32/5/d^7\*e^3\*x/(-e^2\*x^2+d^2)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2/((-e^2\*x^2 + d^2)^(7/2)\*x^3), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.280547, size = 832, normalized size = 4.57

$$54e^{10}x^{10} + 212de^9x^9 - 1063d^2e^8x^8 - 166d^3e^7x^7 + 2940d^4e^6x^6 - 890d^5e^5x^5 - 2585d^6e^4x^4 + 1000d^7e^3x^3 + 740d^8e^2x^2 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2/((-e^2\*x^2 + d^2)^(7/2)\*x^3), x, algorithm="fricas")



[Out]  $1/10*(54*e^{10}*x^{10} + 212*d*e^9*x^9 - 1063*d^2*e^8*x^8 - 166*d^3*e^7*x^7 + 2940*d^4*e^6*x^6 - 890*d^5*e^5*x^5 - 2585*d^6*e^4*x^4 + 1000*d^7*e^3*x^3 + 740*d^8*e^2*x^2 - 160*d^9*e*x - 80*d^{10} + 45*(e^{10}*x^{10} - 2*d*e^9*x^9 - 12*d^2*e^8*x^8 + 26*d^3*e^7*x^7 + 15*d^4*e^6*x^6 - 56*d^5*e^5*x^5 + 12*d^6*e^4*x^4 + 32*d^7*e^3*x^3 - 16*d^8*e^2*x^2 + (5*d*e^8*x^8 - 10*d^2*e^7*x^7 - 15*d^3*e^6*x^6 + 40*d^4*e^5*x^5 - 4*d^5*e^4*x^4 - 32*d^6*e^3*x^3 + 16*d^7*e^2*x^2)*\sqrt{-e^2*x^2 + d^2})*\log(-(d - \sqrt{-e^2*x^2 + d^2})/x) - (64*e^9*x^9 - 353*d*e^8*x^8 - 286*d^2*e^7*x^7 + 1900*d^3*e^6*x^6 - 450*d^4*e^5*x^5 - 2245*d^5*e^4*x^4 + 920*d^6*e^3*x^3 + 700*d^7*e^2*x^2 - 160*d^8*e*x - 80*d^9)*\sqrt{-e^2*x^2 + d^2})/(d^7*e^8*x^{10} - 2*d^8*e^7*x^9 - 12*d^9*e^6*x^8 + 26*d^{10}*e^5*x^7 + 15*d^{11}*e^4*x^6 - 56*d^{12}*e^3*x^5 + 12*d^{13}*e^2*x^4 + 32*d^{14}*e*x^3 - 16*d^{15}*x^2 + (5*d^8*e^6*x^8 - 10*d^9*e^5*x^7 - 15*d^{10}*e^4*x^6 + 40*d^{11}*e^3*x^5 - 4*d^{12}*e^2*x^4 - 32*d^{13}*e*x^3 + 16*d^{14}*x^2)*\sqrt{-e^2*x^2 + d^2}))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^2}{x^3 (-(-d + ex)(d + ex))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2/x\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2),x)

[Out] Integral((d + e\*x)\*\*2/(x\*\*3\*(-(-d + e\*x)\*(d + e\*x))\*\*(7/2)), x)

**GIAC/XCAS [A]** time = 0.297245, size = 351, normalized size = 1.93

$$\frac{\sqrt{-x^2e^2 + d^2} \left( \left( 2 \left( x \left( \frac{11xe^7}{d^7} + \frac{10e^6}{d^6} \right) - \frac{25e^5}{d^5} \right) x - \frac{45e^4}{d^4} \right) x + \frac{30e^3}{d^3} \right) x + \frac{27e^2}{d^2}}{5(x^2e^2 - d^2)^3} - \frac{9e^2 \ln \left( \frac{|-2de - 2\sqrt{-x^2e^2 + d^2}e|e^{(-2)}}{2|x|} \right)}{2d^7} + \frac{x^2 \left( \frac{8(de + \sqrt{-x^2e^2 + d^2}e)e^4}{x} + e^6 \right)}{8(de + \sqrt{-x^2e^2 + d^2}e)^2 d^7} - \frac{\left( \frac{8(de + \sqrt{-x^2e^2 + d^2}e)d^7e^8}{x} + \frac{(de + \sqrt{-x^2e^2 + d^2}e)^2 d^7e^6}{x^2} \right) e^{(-8)}}{8d^{14}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2/((-e^2\*x^2 + d^2)^(7/2)\*x^3),x, algorithm="giac")

[Out]  $-1/5*\sqrt{-x^2*e^2 + d^2}*((2*(x*(11*x*e^7/d^7 + 10*e^6/d^6) - 25*e^5/d^5)*x - 45*e^4/d^4)*x + 30*e^3/d^3)*x + 27*e^2/d^2)/(x^2*e^2 - d^2)^3 - 9/2*e^2*\ln(1/2*abs(-2*d*e - 2*\sqrt{-x^2*e^2 + d^2})*e)^2/abs(x))/d^7 + 1/8*x^2*(8*(d*e + \sqrt{-x^2*e^2 + d^2})*e)*e^4/x + e^6)/((d*e + \sqrt{-x^2*e^2 + d^2})*e)^2*d^7 - 1/8*(8*(d*e + \sqrt{-x^2*e^2 + d^2})*e)*d^7*e^8/x + (d*e + \sqrt{-x^2*e^2 + d^2})*e^2*d^7*e^6/x^2)*e^{(-8)}/d^{14}$

$$3.53 \quad \int \frac{(d+ex)^2}{x^4(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=209

$$\begin{aligned} & -\frac{14e^2\sqrt{d^2-e^2x^2}}{3d^8x} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{7e^3 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^8} \\ & -\frac{e\sqrt{d^2-e^2x^2}}{d^7x^2} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} + \frac{e^3(20d+23ex)}{15d^6(d^2-e^2x^2)^{3/2}} + \frac{2e^3(d+ex)}{5d^4(d^2-e^2x^2)^{5/2}} \end{aligned}$$

[Out]  $(2e^3(d+ex))/(5d^4(d^2-e^2x^2)^{5/2}) + (e^3(20d+23e^3x))/(15d^6(d^2-e^2x^2)^{3/2}) + (2e^3(45d+53e^3x))/(15d^8\sqrt{d^2-e^2x^2}) - \sqrt{d^2-e^2x^2}/(3d^6x^3) - (e\sqrt{d^2-e^2x^2})/(d^7x^2) - (14e^2\sqrt{d^2-e^2x^2})/(3d^8x) - (7e^3\text{ArcTanh}[\sqrt{d^2-e^2x^2}/d])/d^8$

**Rubi [A]** time = 0.722024, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\begin{aligned} & -\frac{14e^2\sqrt{d^2-e^2x^2}}{3d^8x} + \frac{2e^3(45d+53ex)}{15d^8\sqrt{d^2-e^2x^2}} - \frac{7e^3 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^8} \\ & -\frac{e\sqrt{d^2-e^2x^2}}{d^7x^2} - \frac{\sqrt{d^2-e^2x^2}}{3d^6x^3} + \frac{e^3(20d+23ex)}{15d^6(d^2-e^2x^2)^{3/2}} + \frac{2e^3(d+ex)}{5d^4(d^2-e^2x^2)^{5/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^2/(x^4\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out]  $(2e^3(d+ex))/(5d^4(d^2-e^2x^2)^{5/2}) + (e^3(20d+23e^3x))/(15d^6(d^2-e^2x^2)^{3/2}) + (2e^3(45d+53e^3x))/(15d^8\sqrt{d^2-e^2x^2}) - \sqrt{d^2-e^2x^2}/(3d^6x^3) - (e\sqrt{d^2-e^2x^2})/(d^7x^2) - (14e^2\sqrt{d^2-e^2x^2})/(3d^8x) - (7e^3\text{ArcTanh}[\sqrt{d^2-e^2x^2}/d])/d^8$

**Rubi in Sympy [A]** time = 84.2737, size = 226, normalized size = 1.08

$$\begin{aligned} & -\frac{1}{3d^4x^3\sqrt{d^2-e^2x^2}} + \frac{e^3}{5d^5(d-ex)^2\sqrt{d^2-e^2x^2}} + \frac{2e}{d^5x^2\sqrt{d^2-e^2x^2}} + \frac{23e^3}{15d^6(d-ex)\sqrt{d^2-e^2x^2}} \\ & -\frac{13e^2}{3d^6x\sqrt{d^2-e^2x^2}} + \frac{4e^3}{d^7\sqrt{d^2-e^2x^2}} - \frac{3e\sqrt{d^2-e^2x^2}}{d^7x^2} + \frac{176e^4x}{15d^8\sqrt{d^2-e^2x^2}} - \frac{7e^3 \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^8} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*2/x\*\*4/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out]  $-1/(3d^4x^3\sqrt{d^2-e^2x^2}) + e^3/(5d^5(d-ex)^2\sqrt{d^2-e^2x^2}) + 2e/(d^5x^2\sqrt{d^2-e^2x^2}) + 23e^3/(15d^6(d-ex)\sqrt{d^2-e^2x^2}) - 13e^2/(3d^6x\sqrt{d^2-e^2x^2}) + 4e^3/(d^7\sqrt{d^2-e^2x^2}) - 3e\sqrt{d^2-e^2x^2}/(d^7x^2) + 176e^4x/(15d^8\sqrt{d^2-e^2x^2}) - 7e^3\operatorname{atanh}(\sqrt{d^2-e^2x^2}/d)/d^8$

**Mathematica [A]** time = 0.126912, size = 138, normalized size = 0.66

$$\frac{-105e^3 \log\left(\sqrt{d^2 - e^2x^2} + d\right) + \frac{\sqrt{d^2 - e^2x^2}(5d^6 + 5d^5ex + 40d^4e^2x^2 - 246d^3e^3x^3 + 122d^2e^4x^4 + 247de^5x^5 - 176e^6x^6)}{x^3(ex-d)^3(d+ex)} + 105e^3 \log(x)}{15d^8}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^2/(x^4\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(5\*d^6 + 5\*d^5\*e\*x + 40\*d^4\*e^2\*x^2 - 246\*d^3\*e^3\*x^3 + 122\*d^2\*e^4\*x^4 + 247\*d\*e^5\*x^5 - 176\*e^6\*x^6))/(x^3\*(-d + e\*x)^3\*(d + e\*x)) + 105\*e^3\*Log[x] - 105\*e^3\*Log[d + Sqrt[d^2 - e^2\*x^2]])/(15\*d^8)

**Maple [A]** time = 0.025, size = 249, normalized size = 1.2

$$\begin{aligned} & -\frac{1}{3x^3}(-e^2x^2 + d^2)^{-\frac{5}{2}} - \frac{11e^2}{3d^2x}(-e^2x^2 + d^2)^{-\frac{5}{2}} + \frac{22e^4x}{5d^4}(-e^2x^2 + d^2)^{-\frac{5}{2}} \\ & + \frac{88e^4x}{15d^6}(-e^2x^2 + d^2)^{-\frac{3}{2}} + \frac{176e^4x}{15d^8} \frac{1}{\sqrt{-e^2x^2 + d^2}} - \frac{e}{dx^2}(-e^2x^2 + d^2)^{-\frac{5}{2}} + \frac{7e^3}{5d^3}(-e^2x^2 + d^2)^{-\frac{5}{2}} \\ & + \frac{7e^3}{3d^5}(-e^2x^2 + d^2)^{-\frac{3}{2}} + 7 \frac{e^3}{d^7\sqrt{-e^2x^2 + d^2}} - 7 \frac{e^3}{d^7\sqrt{d^2}} \ln\left(\frac{2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2}}{x}\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^2/x^4/(-e^2\*x^2+d^2)^(7/2), x)

[Out] -1/3/x^3/(-e^2\*x^2+d^2)^(5/2)-11/3/d^2\*e^2/x/(-e^2\*x^2+d^2)^(5/2)+22/5/d^4\*e^4\*x/(-e^2\*x^2+d^2)^(5/2)+88/15/d^6\*e^4\*x/(-e^2\*x^2+d^2)^(3/2)+176/15/d^8\*e^4\*x/(-e^2\*x^2+d^2)^(1/2)-1/d\*e/x^2/(-e^2\*x^2+d^2)^(5/2)+7/5/d^3\*e^3/(-e^2\*x^2+d^2)^(5/2)+7/3/d^5\*e^3/(-e^2\*x^2+d^2)^(3/2)+7/d^7\*e^3/(-e^2\*x^2+d^2)^(1/2)-7/d^7\*e^3/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2/((-e^2\*x^2 + d^2)^(7/2)\*x^4), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.284712, size = 953, normalized size = 4.56

$$176e^{12}x^{12} - 943de^{11}x^{11} - 1898d^2e^{10}x^{10} + 8404d^3e^9x^9 + 1788d^4e^8x^8 - 19305d^5e^7x^7 + 4075d^6e^6x^6 + 16090d^7e^5x^5 - 53$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2/((-e^2\*x^2 + d^2)^(7/2)\*x^4), x, algorithm="fricas")

```
[Out] -1/15*(176*e^12*x^12 - 943*d*e^11*x^11 - 1898*d^2*e^10*x^10 + 840
4*d^3*e^9*x^9 + 1788*d^4*e^8*x^8 - 19305*d^5*e^7*x^7 + 4075*d^6*e
^6*x^6 + 16090*d^7*e^5*x^5 - 5350*d^8*e^4*x^4 - 4400*d^9*e^3*x^3
+ 1040*d^10*e^2*x^2 + 160*d^11*e*x + 160*d^12 - 105*(6*d*e^11*x^1
1 - 12*d^2*e^10*x^10 - 32*d^3*e^9*x^9 + 76*d^4*e^8*x^8 + 26*d^5*e
^7*x^7 - 128*d^6*e^6*x^6 + 32*d^7*e^5*x^5 + 64*d^8*e^4*x^4 - 32*d
^9*e^3*x^3 - (e^11*x^11 - 2*d*e^10*x^10 - 17*d^2*e^9*x^9 + 36*d^3
*e^8*x^8 + 30*d^4*e^7*x^7 - 96*d^5*e^6*x^6 + 16*d^6*e^5*x^5 + 64*
d^7*e^4*x^4 - 32*d^8*e^3*x^3)*sqrt(-e^2*x^2 + d^2))*log(-(d - sqr
t(-e^2*x^2 + d^2))/x) + 2*(58*e^11*x^11 + 412*d*e^10*x^10 - 1727*
d^2*e^9*x^9 - 1094*d^3*e^8*x^8 + 6430*d^4*e^7*x^7 - 920*d^5*e^6*x
^6 - 6975*d^6*e^5*x^5 + 2385*d^7*e^4*x^4 + 2160*d^8*e^3*x^3 - 560
*d^9*e^2*x^2 - 80*d^10*e*x - 80*d^11)*sqrt(-e^2*x^2 + d^2))/(6*d^
9*e^8*x^11 - 12*d^10*e^7*x^10 - 32*d^11*e^6*x^9 + 76*d^12*e^5*x^8
+ 26*d^13*e^4*x^7 - 128*d^14*e^3*x^6 + 32*d^15*e^2*x^5 + 64*d^16
*e*x^4 - 32*d^17*x^3 - (d^8*e^8*x^11 - 2*d^9*e^7*x^10 - 17*d^10*e
^6*x^9 + 36*d^11*e^5*x^8 + 30*d^12*e^4*x^7 - 96*d^13*e^3*x^6 + 16
*d^14*e^2*x^5 + 64*d^15*e*x^4 - 32*d^16*x^3)*sqrt(-e^2*x^2 + d^2)
)
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^2}{x^4(-(-d+ex)(d+ex))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2/x**4/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Integral((d + e*x)**2/(x**4*(-(d + e*x)*(d + e*x))**(7/2)), x)
```

---

**GIAC/XCAS [A]** time = 0.303236, size = 439, normalized size = 2.1

$$\frac{\sqrt{-x^2e^2 + d^2} \left( \left( \left( 2x \left( \frac{53xe^8}{d^8} + \frac{45e^7}{d^7} \right) - \frac{235e^6}{d^6} \right) x - \frac{200e^5}{d^5} \right) x + \frac{135e^4}{d^4} \right) x + \frac{116e^3}{d^3} \right)}{15(x^2e^2 - d^2)^3} + \frac{x^3 \left( \frac{6(de + \sqrt{-x^2e^2 + d^2})e^6}{x} + \frac{57(de + \sqrt{-x^2e^2 + d^2})^2e^4}{x^2} + e^8 \right) e}{24(de + \sqrt{-x^2e^2 + d^2})^3 d^8} - \frac{7e^3 \ln \left( \frac{|-2de - 2\sqrt{-x^2e^2 + d^2}e|e^{(-2)}}{2|x|} \right)}{d^8} + \frac{\left( \frac{57(de + \sqrt{-x^2e^2 + d^2})d^{16}e^{16}}{x} + \frac{6(de + \sqrt{-x^2e^2 + d^2})^2d^{16}e^{14}}{x^2} + \frac{(de + \sqrt{-x^2e^2 + d^2})^3d^{16}e^{12}}{x^3} \right) e^{(-15)}}{24d^{24}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^2/((-e^2*x^2 + d^2)^(7/2)*x^4),x, algorithm="giac")
```

```
[Out] -1/15*sqrt(-x^2*e^2 + d^2)*(((2*x*(53*x*e^8/d^8 + 45*e^7/d^7) -
235*e^6/d^6)*x - 200*e^5/d^5)*x + 135*e^4/d^4)*x + 116*e^3/d^3)/
(x^2*e^2 - d^2)^3 + 1/24*x^3*(6*(d*e + sqrt(-x^2*e^2 + d^2))*e)^e^6
/x + 57*(d*e + sqrt(-x^2*e^2 + d^2))*e)^2*e^4/x^2 + e^8)*e/((d*e +
sqrt(-x^2*e^2 + d^2))*e)^3*d^8) - 7*e^3*ln(1/2*abs(-2*d*e - 2*sqrt
(-x^2*e^2 + d^2))*e)*e^(-2)/abs(x))/d^8 - 1/24*(57*(d*e + sqrt(-x
^2*e^2 + d^2))*e)*d^16*e^16/x + 6*(d*e + sqrt(-x^2*e^2 + d^2))*e)^2
*d^16*e^14/x^2 + (d*e + sqrt(-x^2*e^2 + d^2))*e)^3*d^16*e^12/x^3)*
e^(-15)/d^24
```

$$3.54 \quad \int \frac{x^3(1+x)^2}{\sqrt{1-x^2}} dx$$

**Optimal.** Leaf size=81

$$-\frac{3}{5}\sqrt{1-x^2}x^2 - \frac{3}{20}(5x+8)\sqrt{1-x^2} - \frac{1}{5}\sqrt{1-x^2}x^4 - \frac{1}{2}\sqrt{1-x^2}x^3 + \frac{3}{4}\sin^{-1}(x)$$

[Out]  $(-3*x^2*\text{Sqrt}[1-x^2])/5 - (x^3*\text{Sqrt}[1-x^2])/2 - (x^4*\text{Sqrt}[1-x^2])/5 - (3*(8+5*x)*\text{Sqrt}[1-x^2])/20 + (3*\text{ArcSin}[x])/4$

**Rubi [A]** time = 0.215799, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{3}{5}\sqrt{1-x^2}x^2 - \frac{3}{20}(5x+8)\sqrt{1-x^2} - \frac{1}{5}\sqrt{1-x^2}x^4 - \frac{1}{2}\sqrt{1-x^2}x^3 + \frac{3}{4}\sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(1+x)^2)/Sqrt[1-x^2],x]

[Out]  $(-3*x^2*\text{Sqrt}[1-x^2])/5 - (x^3*\text{Sqrt}[1-x^2])/2 - (x^4*\text{Sqrt}[1-x^2])/5 - (3*(8+5*x)*\text{Sqrt}[1-x^2])/20 + (3*\text{ArcSin}[x])/4$

**Rubi in Sympy [A]** time = 15.0738, size = 61, normalized size = 0.75

$$-\frac{x^3\sqrt{-x^2+1}}{2} - \frac{3x\sqrt{-x^2+1}}{4} - \frac{(-x^2+1)^{\frac{5}{2}}}{5} + (-x^2+1)^{\frac{3}{2}} - 2\sqrt{-x^2+1} + \frac{3\text{asin}(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(1+x)\*\*2/(-x\*\*2+1)\*\*(1/2),x)

[Out]  $-x**3*\text{sqrt}(-x**2+1)/2 - 3*x*\text{sqrt}(-x**2+1)/4 - (-x**2+1)**(5/2)/5 + (-x**2+1)**(3/2) - 2*\text{sqrt}(-x**2+1) + 3*\text{asin}(x)/4$

**Mathematica [A]** time = 0.0483942, size = 77, normalized size = 0.95

$$\frac{4x^6 + 10x^5 + 8x^4 + 5x^3 + 12x^2 - 15x + 30\sqrt{x-1}\sqrt{x+1}\log(\sqrt{x-1} + \sqrt{x+1}) - 24}{20\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(1+x)^2)/Sqrt[1-x^2],x]

[Out]  $(-24 - 15*x + 12*x^2 + 5*x^3 + 8*x^4 + 10*x^5 + 4*x^6 + 30*\text{Sqrt}[-1+x]*\text{Sqrt}[1+x]*\text{Log}[\text{Sqrt}[-1+x] + \text{Sqrt}[1+x]])/(20*\text{Sqrt}[1-x^2])$

**Maple [A]** time = 0.027, size = 71, normalized size = 0.9

$$-\frac{3x^2}{5}\sqrt{-x^2+1} - \frac{6}{5}\sqrt{-x^2+1} - \frac{x^4}{5}\sqrt{-x^2+1} - \frac{x^3}{2}\sqrt{-x^2+1} - \frac{3x}{4}\sqrt{-x^2+1} + \frac{3\arcsin(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(1+x)^2/(-x^2+1)^(1/2),x)`

[Out]  $-3/5*x^2*(-x^2+1)^{(1/2)}-6/5*(-x^2+1)^{(1/2)}-1/5*x^4*(-x^2+1)^{(1/2)}-1/2*x^3*(-x^2+1)^{(1/2)}-3/4*x*(-x^2+1)^{(1/2)}+3/4*\arcsin(x)$

**Maxima [A]** time = 0.787851, size = 95, normalized size = 1.17

$$-\frac{1}{5}\sqrt{-x^2+1}x^4 - \frac{1}{2}\sqrt{-x^2+1}x^3 - \frac{3}{5}\sqrt{-x^2+1}x^2 - \frac{3}{4}\sqrt{-x^2+1}x - \frac{6}{5}\sqrt{-x^2+1} + \frac{3}{4}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+1)^2*x^3/sqrt(-x^2+1),x,algorithm="maxima")`

[Out]  $-1/5*\sqrt{-x^2+1}*x^4 - 1/2*\sqrt{-x^2+1}*x^3 - 3/5*\sqrt{-x^2+1}*x^2 - 3/4*\sqrt{-x^2+1}*x - 6/5*\sqrt{-x^2+1} + 3/4*\arcsin(x)$

**Fricas [A]** time = 0.271925, size = 239, normalized size = 2.95

$$\frac{4x^{10} + 10x^9 - 40x^8 - 115x^7 - 20x^6 + 85x^5 + 80x^4 + 260x^3 + 30(5x^4 - 20x^2 - (x^4 - 12x^2 + 16)\sqrt{-x^2+1} + 16)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) + 5(4x^8 + 10x^7 - 4x^6 - 25x^5 - 16x^4 - 28x^3 + 48x)\sqrt{-x^2+1} - 240x}{20(5x^4 - 20x^2 - (x^4 - 12x^2 + 16)\sqrt{-x^2+1} + 16)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+1)^2*x^3/sqrt(-x^2+1),x,algorithm="fricas")`

[Out]  $-1/20*(4*x^{10} + 10*x^9 - 40*x^8 - 115*x^7 - 20*x^6 + 85*x^5 + 80*x^4 + 260*x^3 + 30*(5*x^4 - 20*x^2 - (x^4 - 12*x^2 + 16)*\sqrt{-x^2+1} + 16)*\arctan((\sqrt{-x^2+1}-1)/x) + 5*(4*x^8 + 10*x^7 - 4*x^6 - 25*x^5 - 16*x^4 - 28*x^3 + 48*x)*\sqrt{-x^2+1} - 240*x)/(5*x^4 - 20*x^2 - (x^4 - 12*x^2 + 16)*\sqrt{-x^2+1} + 16)$

**Sympy [A]** time = 3.48297, size = 73, normalized size = 0.9

$$\frac{x^4\sqrt{-x^2+1}}{5} - \frac{x^3\sqrt{-x^2+1}}{2} - \frac{3x^2\sqrt{-x^2+1}}{5} - \frac{3x\sqrt{-x^2+1}}{4} - \frac{6\sqrt{-x^2+1}}{5} + \frac{3\arcsin(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(1+x)**2/(-x**2+1)**(1/2),x)`

[Out]  $-x**4*\sqrt{-x**2+1}/5 - x**3*\sqrt{-x**2+1}/2 - 3*x**2*\sqrt{-x**2+1}/5 - 3*x*\sqrt{-x**2+1}/4 - 6*\sqrt{-x**2+1}/5 + 3*\arcsin(x)/4$

**GIAC/XCAS [A]** time = 0.274286, size = 46, normalized size = 0.57

$$-\frac{1}{20}((2((2x+5)x+6)x+15)x+24)\sqrt{-x^2+1} + \frac{3}{4}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + 1)^2*x^3/sqrt(-x^2 + 1),x, algorithm="giac")
```

```
[Out] -1/20*((2*((2*x + 5)*x + 6)*x + 15)*x + 24)*sqrt(-x^2 + 1) + 3/4*arcsin(x)
```

$$3.55 \quad \int \frac{x^2(1+x)^2}{\sqrt{1-x^2}} dx$$

**Optimal.** Leaf size=63

$$-\frac{2}{3}\sqrt{1-x^2}x^2 - \frac{1}{24}(21x+32)\sqrt{1-x^2} - \frac{1}{4}\sqrt{1-x^2}x^3 + \frac{7}{8}\sin^{-1}(x)$$

[Out]  $(-2*x^2*\text{Sqrt}[1-x^2])/3 - (x^3*\text{Sqrt}[1-x^2])/4 - ((32+21*x)*\text{Sqrt}[1-x^2])/24 + (7*\text{ArcSin}[x])/8$

**Rubi [A]** time = 0.161405, antiderivative size = 63, normalized size of antiderivative = 1., number of rules used = 4, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{2}{3}\sqrt{1-x^2}x^2 - \frac{1}{24}(21x+32)\sqrt{1-x^2} - \frac{1}{4}\sqrt{1-x^2}x^3 + \frac{7}{8}\sin^{-1}(x)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^2*(1+x)^2)/\text{Sqrt}[1-x^2], x]$

[Out]  $(-2*x^2*\text{Sqrt}[1-x^2])/3 - (x^3*\text{Sqrt}[1-x^2])/4 - ((32+21*x)*\text{Sqrt}[1-x^2])/24 + (7*\text{ArcSin}[x])/8$

**Rubi in Sympy [A]** time = 14.2894, size = 54, normalized size = 0.86

$$-\frac{x^3\sqrt{-x^2+1}}{4} - \frac{7x\sqrt{-x^2+1}}{8} + \frac{2(-x^2+1)^{\frac{3}{2}}}{3} - 2\sqrt{-x^2+1} + \frac{7\text{asin}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**2*(1+x)**2/(-x**2+1)**(1/2), x)$

[Out]  $-x**3*\text{sqrt}(-x**2+1)/4 - 7*x*\text{sqrt}(-x**2+1)/8 + 2*(-x**2+1)**(3/2)/3 - 2*\text{sqrt}(-x**2+1) + 7*\text{asin}(x)/8$

**Mathematica [A]** time = 0.0428156, size = 72, normalized size = 1.14

$$\frac{6x^5 + 16x^4 + 15x^3 + 16x^2 - 21x + 42\sqrt{x-1}\sqrt{x+1} \log(\sqrt{x-1} + \sqrt{x+1}) - 32}{24\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(x^2*(1+x)^2)/\text{Sqrt}[1-x^2], x]$

[Out]  $(-32 - 21*x + 16*x^2 + 15*x^3 + 16*x^4 + 6*x^5 + 42*\text{Sqrt}[-1+x]*\text{Sqrt}[1+x]*\text{Log}[\text{Sqrt}[-1+x] + \text{Sqrt}[1+x]])/(24*\text{Sqrt}[1-x^2])$

**Maple [A]** time = 0.008, size = 57, normalized size = 0.9

$$-\frac{7x}{8}\sqrt{-x^2+1} + \frac{7\arcsin(x)}{8} - \frac{x^3}{4}\sqrt{-x^2+1} - \frac{2x^2}{3}\sqrt{-x^2+1} - \frac{4}{3}\sqrt{-x^2+1}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(1+x)^2/(-x^2+1)^(1/2),x)`

[Out]  $-7/8*x*(-x^2+1)^{(1/2)}+7/8*\arcsin(x)-1/4*x^3*(-x^2+1)^{(1/2)}-2/3*x^2*(-x^2+1)^{(1/2)}-4/3*(-x^2+1)^{(1/2)}$

**Maxima [A]** time = 0.782387, size = 76, normalized size = 1.21

$$-\frac{1}{4}\sqrt{-x^2+1}x^3 - \frac{2}{3}\sqrt{-x^2+1}x^2 - \frac{7}{8}\sqrt{-x^2+1}x - \frac{4}{3}\sqrt{-x^2+1} + \frac{7}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+1)^2*x^2/sqrt(-x^2+1),x,algorithm="maxima")`

[Out]  $-1/4*\sqrt{-x^2+1}*x^3 - 2/3*\sqrt{-x^2+1}*x^2 - 7/8*\sqrt{-x^2+1}*x - 4/3*\sqrt{-x^2+1} + 7/8*\arcsin(x)$

**Fricas [A]** time = 0.269787, size = 193, normalized size = 3.06

$$\frac{24x^7 + 64x^6 + 12x^5 - 96x^4 - 204x^3 - 42(x^4 - 8x^2 + 4(x^2 - 2)\sqrt{-x^2 + 1} + 8)\arctan\left(\frac{\sqrt{-x^2 + 1} - 1}{x}\right) - (6x^7 + 16x^6 - 27x^5 - 96x^4 - 120x^3 + 168x^2 - 27x + 16)\sqrt{-x^2 + 1}}{24(x^4 - 8x^2 + 4(x^2 - 2)\sqrt{-x^2 + 1} + 8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+1)^2*x^2/sqrt(-x^2+1),x,algorithm="fricas")`

[Out]  $1/24*(24*x^7 + 64*x^6 + 12*x^5 - 96*x^4 - 204*x^3 - 42*(x^4 - 8*x^2 + 4*(x^2 - 2)*\sqrt{-x^2 + 1} + 8)*\arctan((\sqrt{-x^2 + 1} - 1)/x) - (6*x^7 + 16*x^6 - 27*x^5 - 96*x^4 - 120*x^3 + 168*x^2 - 27*x + 16)*\sqrt{-x^2 + 1})/(x^4 - 8*x^2 + 4*(x^2 - 2)*\sqrt{-x^2 + 1} + 8)$

**Sympy [A]** time = 1.98461, size = 60, normalized size = 0.95

$$-\frac{x^3\sqrt{-x^2+1}}{4} - \frac{2x^2\sqrt{-x^2+1}}{3} - \frac{7x\sqrt{-x^2+1}}{8} - \frac{4\sqrt{-x^2+1}}{3} + \frac{7\operatorname{asin}(x)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(1+x)**2/(-x**2+1)**(1/2),x)`

[Out]  $-x**3*\sqrt{-x**2+1}/4 - 2*x**2*\sqrt{-x**2+1}/3 - 7*x*\sqrt{-x**2+1}/8 - 4*\sqrt{-x**2+1}/3 + 7*\operatorname{asin}(x)/8$

**GIAC/XCAS [A]** time = 0.283878, size = 41, normalized size = 0.65

$$-\frac{1}{24}((2(3x+8)x+21)x+32)\sqrt{-x^2+1} + \frac{7}{8}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + 1)^2*x^2/sqrt(-x^2 + 1),x, algorithm="giac")
```

```
[Out] -1/24*((2*(3*x + 8)*x + 21)*x + 32)*sqrt(-x^2 + 1) + 7/8*arcsin(x)
```

$$3.56 \quad \int \frac{x(1+x)^2}{\sqrt{1-x^2}} dx$$

**Optimal.** Leaf size=41

$$-\frac{1}{3}\sqrt{1-x^2}x^2 - \frac{1}{3}(3x+5)\sqrt{1-x^2} + \sin^{-1}(x)$$

[Out]  $-(x^2 \text{Sqrt}[1 - x^2])/3 - ((5 + 3*x) \text{Sqrt}[1 - x^2])/3 + \text{ArcSin}[x]$

**Rubi [A]** time = 0.0900167, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{1}{3}\sqrt{1-x^2}x^2 - \frac{1}{3}(3x+5)\sqrt{1-x^2} + \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x*(1+x)^2)/\text{Sqrt}[1-x^2], x]$

[Out]  $-(x^2 \text{Sqrt}[1 - x^2])/3 - ((5 + 3*x) \text{Sqrt}[1 - x^2])/3 + \text{ArcSin}[x]$

**Rubi in Sympy [A]** time = 7.1169, size = 39, normalized size = 0.95

$$-\frac{(x+1)^2 \sqrt{-x^2+1}}{3} - \frac{(x+1) \sqrt{-x^2+1}}{3} - \sqrt{-x^2+1} + \text{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x*(1+x)**2/(-x**2+1)**(1/2), x)$

[Out]  $-(x+1)**2*\text{sqrt}(-x**2+1)/3 - (x+1)*\text{sqrt}(-x**2+1)/3 - \text{sqrt}(-x**2+1) + \text{asin}(x)$

**Mathematica [A]** time = 0.0243609, size = 26, normalized size = 0.63

$$\sin^{-1}(x) - \frac{1}{3}\sqrt{1-x^2}(x^2+3x+5)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(x*(1+x)^2)/\text{Sqrt}[1-x^2], x]$

[Out]  $-(\text{Sqrt}[1 - x^2]*(5 + 3*x + x^2))/3 + \text{ArcSin}[x]$

**Maple [A]** time = 0.007, size = 41, normalized size = 1.

$$-\frac{5}{3}\sqrt{-x^2+1} - \frac{x^2}{3}\sqrt{-x^2+1} - x\sqrt{-x^2+1} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x*(1+x)^2/(-x^2+1)^(1/2), x)$

[Out]  $-5/3 * (-x^2+1)^{(1/2)} - 1/3 * x^2 * (-x^2+1)^{(1/2)} - x * (-x^2+1)^{(1/2)} + \arcsin(x)$

**Maxima [A]** time = 0.783173, size = 54, normalized size = 1.32

$$-\frac{1}{3} \sqrt{-x^2+1} x^2 - \sqrt{-x^2+1} x - \frac{5}{3} \sqrt{-x^2+1} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x + 1)^2*x/sqrt(-x^2 + 1),x, algorithm="maxima")`

[Out]  $-1/3 * \sqrt{-x^2 + 1} * x^2 - \sqrt{-x^2 + 1} * x - 5/3 * \sqrt{-x^2 + 1} + \arcsin(x)$

**Fricas [A]** time = 0.270433, size = 159, normalized size = 3.88

$$\frac{x^6 + 3x^5 - 15x^3 - 6x^2 + 6 \left( 3x^2 - (x^2 - 4)\sqrt{-x^2 + 1} - 4 \right) \arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) + 3(x^4 + 3x^3 + 2x^2 - 4x)\sqrt{-x^2 + 1} + 12x}{3 \left( 3x^2 - (x^2 - 4)\sqrt{-x^2 + 1} - 4 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x + 1)^2*x/sqrt(-x^2 + 1),x, algorithm="fricas")`

[Out]  $-1/3 * (x^6 + 3 * x^5 - 15 * x^3 - 6 * x^2 + 6 * (3 * x^2 - (x^2 - 4) * \sqrt{-x^2 + 1} - 4) * \arctan((\sqrt{-x^2 + 1} - 1) / x) + 3 * (x^4 + 3 * x^3 + 2 * x^2 - 4 * x) * \sqrt{-x^2 + 1} + 12 * x) / (3 * x^2 - (x^2 - 4) * \sqrt{-x^2 + 1} - 4)$

**Sympy [A]** time = 1.00585, size = 37, normalized size = 0.9

$$-\frac{x^2 \sqrt{-x^2+1}}{3} - x \sqrt{-x^2+1} - \frac{5 \sqrt{-x^2+1}}{3} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(1+x)**2/(-x**2+1)**(1/2),x)`

[Out]  $-x**2 * \sqrt{-x**2 + 1} / 3 - x * \sqrt{-x**2 + 1} - 5 * \sqrt{-x**2 + 1} / 3 + \arcsin(x)$

**GIAC/XCAS [A]** time = 0.276381, size = 28, normalized size = 0.68

$$-\frac{1}{3} ((x + 3)x + 5) \sqrt{-x^2 + 1} + \arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x + 1)^2*x/sqrt(-x^2 + 1),x, algorithm="giac")`

[Out]  $-1/3 * ((x + 3) * x + 5) * \sqrt{-x^2 + 1} + \arcsin(x)$

$$3.57 \quad \int \frac{(1+x)^2}{\sqrt{1-x^2}} dx$$

**Optimal.** Leaf size=40

$$-\frac{1}{2}\sqrt{1-x^2}(x+1) - \frac{3\sqrt{1-x^2}}{2} + \frac{3}{2}\sin^{-1}(x)$$

[Out]  $(-3*\text{Sqrt}[1 - x^2])/2 - ((1 + x)*\text{Sqrt}[1 - x^2])/2 + (3*\text{ArcSin}[x])/2$

**Rubi [A]** time = 0.0363325, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$-\frac{1}{2}\sqrt{1-x^2}(x+1) - \frac{3\sqrt{1-x^2}}{2} + \frac{3}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(1 + x)^2/\text{Sqrt}[1 - x^2], x]$

[Out]  $(-3*\text{Sqrt}[1 - x^2])/2 - ((1 + x)*\text{Sqrt}[1 - x^2])/2 + (3*\text{ArcSin}[x])/2$

**Rubi in Sympy [A]** time = 5.0503, size = 31, normalized size = 0.78

$$-\frac{(x+1)\sqrt{-x^2+1}}{2} - \frac{3\sqrt{-x^2+1}}{2} + \frac{3\text{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((1+x)**2/(-x**2+1)**(1/2), x)$

[Out]  $-(x + 1)*\text{sqrt}(-x**2 + 1)/2 - 3*\text{sqrt}(-x**2 + 1)/2 + 3*\text{asin}(x)/2$

**Mathematica [A]** time = 0.0234871, size = 60, normalized size = 1.5

$$\frac{x^3 + 4x^2 - x + 6\sqrt{x-1}\sqrt{x+1} \log(\sqrt{x-1} + \sqrt{x+1}) - 4}{2\sqrt{1-x^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(1 + x)^2/\text{Sqrt}[1 - x^2], x]$

[Out]  $(-4 - x + 4*x^2 + x^3 + 6*\text{Sqrt}[-1 + x]*\text{Sqrt}[1 + x]*\text{Log}[\text{Sqrt}[-1 + x] + \text{Sqrt}[1 + x]])/(2*\text{Sqrt}[1 - x^2])$

**Maple [A]** time = 0.007, size = 29, normalized size = 0.7

$$\frac{3 \arcsin(x)}{2} - \frac{x}{2}\sqrt{-x^2+1} - 2\sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+x)^2/(-x^2+1)^(1/2),x)`

[Out]  $3/2*\arcsin(x)-1/2*x*(-x^2+1)^(1/2)-2*(-x^2+1)^(1/2)$

**Maxima [A]** time = 0.781014, size = 38, normalized size = 0.95

$$-\frac{1}{2}\sqrt{-x^2+1}x - 2\sqrt{-x^2+1} + \frac{3}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+1)^2/sqrt(-x^2+1),x,algorithm="maxima")`

[Out]  $-1/2*\sqrt{-x^2+1}*x - 2*\sqrt{-x^2+1} + 3/2*\arcsin(x)$

**Fricas [A]** time = 0.271122, size = 123, normalized size = 3.08

$$\frac{2x^3 + 4x^2 - 6\left(x^2 + 2\sqrt{-x^2+1} - 2\right)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) - (x^3 + 4x^2 - 2x)\sqrt{-x^2+1} - 2x}{2\left(x^2 + 2\sqrt{-x^2+1} - 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+1)^2/sqrt(-x^2+1),x,algorithm="fricas")`

[Out]  $1/2*(2*x^3 + 4*x^2 - 6*(x^2 + 2*\sqrt{-x^2+1} - 2)*\arctan((\sqrt{-x^2+1} - 1)/x) - (x^3 + 4*x^2 - 2*x)*\sqrt{-x^2+1} - 2*x)/(x^2 + 2*\sqrt{-x^2+1} - 2)$

**Sympy [A]** time = 0.530935, size = 27, normalized size = 0.68

$$-\frac{x\sqrt{-x^2+1}}{2} - 2\sqrt{-x^2+1} + \frac{3\operatorname{asin}(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x)**2/(-x**2+1)**(1/2),x)`

[Out]  $-x*\sqrt{-x^2+1}/2 - 2*\sqrt{-x^2+1} + 3*\operatorname{asin}(x)/2$

**GIAC/XCAS [A]** time = 0.277599, size = 26, normalized size = 0.65

$$-\frac{1}{2}\sqrt{-x^2+1}(x+4) + \frac{3}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+1)^2/sqrt(-x^2+1),x,algorithm="giac")`

[Out]  $-1/2*\sqrt{-x^2+1}*(x+4) + 3/2*\arcsin(x)$

$$3.58 \quad \int \frac{(1+x)^2}{x\sqrt{1-x^2}} dx$$

**Optimal.** Leaf size=32

$$-\sqrt{1-x^2} - \tanh^{-1}\left(\sqrt{1-x^2}\right) + 2 \sin^{-1}(x)$$

[Out] -Sqrt[1 - x^2] + 2\*ArcSin[x] - ArcTanh[Sqrt[1 - x^2]]

**Rubi [A]** time = 0.117987, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\sqrt{1-x^2} - \tanh^{-1}\left(\sqrt{1-x^2}\right) + 2 \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x)^2/(x\*Sqrt[1 - x^2]), x]

[Out] -Sqrt[1 - x^2] + 2\*ArcSin[x] - ArcTanh[Sqrt[1 - x^2]]

**Rubi in Sympy [A]** time = 7.6114, size = 22, normalized size = 0.69

$$-\sqrt{-x^2+1} + 2 \operatorname{asin}(x) - \operatorname{atanh}\left(\sqrt{-x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((1+x)\*\*2/x/(-x\*\*2+1)\*\*(1/2), x)

[Out] -sqrt(-x\*\*2 + 1) + 2\*asin(x) - atanh(sqrt(-x\*\*2 + 1))

**Mathematica [A]** time = 0.0245261, size = 36, normalized size = 1.12

$$-\sqrt{1-x^2} - \log\left(\sqrt{1-x^2} + 1\right) + \log(x) + 2 \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x)^2/(x\*Sqrt[1 - x^2]), x]

[Out] -Sqrt[1 - x^2] + 2\*ArcSin[x] + Log[x] - Log[1 + Sqrt[1 - x^2]]

**Maple [A]** time = 0.009, size = 29, normalized size = 0.9

$$-\operatorname{Artanh}\left(\frac{1}{\sqrt{-x^2+1}}\right) - \sqrt{-x^2+1} + 2 \operatorname{arcsin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x)^2/x/(-x^2+1)^(1/2), x)

[Out]  $-\operatorname{arctanh}(1/(-x^2+1)^{(1/2)}) - (-x^2+1)^{(1/2)} + 2 \operatorname{arcsin}(x)$

**Maxima [A]** time = 0.780407, size = 55, normalized size = 1.72

$$-\sqrt{-x^2+1} + 2 \operatorname{arcsin}(x) - \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x + 1)^2/(sqrt(-x^2 + 1)*x),x, algorithm="maxima")`

[Out]  $-\sqrt{-x^2+1} + 2 \operatorname{arcsin}(x) - \log(2 \operatorname{sqrt}(-x^2+1)/\operatorname{abs}(x) + 2/\operatorname{abs}(x))$

**Fricas [A]** time = 0.278993, size = 101, normalized size = 3.16

$$\frac{x^2 - 4 \left( \sqrt{-x^2+1} - 1 \right) \operatorname{arctan}\left(\frac{\sqrt{-x^2+1}-1}{x}\right) + \left( \sqrt{-x^2+1} - 1 \right) \log\left(\frac{\sqrt{-x^2+1}-1}{x}\right)}{\sqrt{-x^2+1} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x + 1)^2/(sqrt(-x^2 + 1)*x),x, algorithm="fricas")`

[Out]  $(x^2 - 4 * (\operatorname{sqrt}(-x^2 + 1) - 1) * \operatorname{arctan}((\operatorname{sqrt}(-x^2 + 1) - 1)/x) + (\operatorname{sqrt}(-x^2 + 1) - 1) * \log((\operatorname{sqrt}(-x^2 + 1) - 1)/x)) / (\operatorname{sqrt}(-x^2 + 1) - 1)$

**Sympy [A]** time = 6.23519, size = 31, normalized size = 0.97

$$-\sqrt{-x^2+1} + \begin{cases} -\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for } \left|\frac{1}{x^2}\right| > 1 \\ i \operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise} \end{cases} + 2 \operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x)**2/x/(-x**2+1)**(1/2),x)`

[Out]  $-\operatorname{sqrt}(-x^2 + 1) + \operatorname{Piecewise}((- \operatorname{acosh}(1/x), \operatorname{Abs}(x^{(-2)}) > 1), (i * \operatorname{asin}(1/x), \operatorname{True})) + 2 * \operatorname{asin}(x)$

**GIAC/XCAS [A]** time = 0.277231, size = 46, normalized size = 1.44

$$-\sqrt{-x^2+1} + 2 \operatorname{arcsin}(x) + \ln\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x + 1)^2/(sqrt(-x^2 + 1)*x),x, algorithm="giac")`

[Out]  $-\operatorname{sqrt}(-x^2 + 1) + 2 * \operatorname{arcsin}(x) + \ln(-(\operatorname{sqrt}(-x^2 + 1) - 1)/\operatorname{abs}(x))$



$$3.59 \quad \int \frac{(1+x)^2}{x^2 \sqrt{1-x^2}} dx$$

**Optimal.** Leaf size=33

$$-\frac{\sqrt{1-x^2}}{x} - 2 \tanh^{-1}(\sqrt{1-x^2}) + \sin^{-1}(x)$$

[Out] -(Sqrt[1 - x^2]/x) + ArcSin[x] - 2\*ArcTanh[Sqrt[1 - x^2]]

**Rubi [A]** time = 0.119516, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{\sqrt{1-x^2}}{x} - 2 \tanh^{-1}(\sqrt{1-x^2}) + \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + x)^2/(x^2\*Sqrt[1 - x^2]), x]

[Out] -(Sqrt[1 - x^2]/x) + ArcSin[x] - 2\*ArcTanh[Sqrt[1 - x^2]]

**Rubi in Sympy [A]** time = 8.45711, size = 24, normalized size = 0.73

$$\text{asin}(x) - 2 \text{atanh}(\sqrt{-x^2 + 1}) - \frac{\sqrt{-x^2 + 1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((1+x)\*\*2/x\*\*2/(-x\*\*2+1)\*\*(1/2), x)

[Out] asin(x) - 2\*atanh(sqrt(-x\*\*2 + 1)) - sqrt(-x\*\*2 + 1)/x

**Mathematica [A]** time = 0.0317497, size = 39, normalized size = 1.18

$$-\frac{\sqrt{1-x^2}}{x} - 2 \log(\sqrt{1-x^2} + 1) + 2 \log(x) + \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x)^2/(x^2\*Sqrt[1 - x^2]), x]

[Out] -(Sqrt[1 - x^2]/x) + ArcSin[x] + 2\*Log[x] - 2\*Log[1 + Sqrt[1 - x^2]]

**Maple [A]** time = 0.011, size = 30, normalized size = 0.9

$$\arcsin(x) - \frac{1}{x} \sqrt{-x^2 + 1} - 2 \text{Artanh}\left(\frac{1}{\sqrt{-x^2 + 1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+x)^2/x^2/(-x^2+1)^(1/2),x)`

[Out] `arcsin(x)-(-x^2+1)^(1/2)/x-2*arctanh(1/(-x^2+1)^(1/2))`

**Maxima [A]** time = 0.786493, size = 57, normalized size = 1.73

$$-\frac{\sqrt{-x^2+1}}{x} + \arcsin(x) - 2 \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+1)^2/(sqrt(-x^2+1)*x^2),x,algorithm="maxima")`

[Out] `-sqrt(-x^2+1)/x + arcsin(x) - 2*log(2*sqrt(-x^2+1)/abs(x) + 2/abs(x))`

**Fricas [A]** time = 0.279256, size = 132, normalized size = 4.

$$\frac{x^2 - 2\left(\sqrt{-x^2+1}x - x\right) \arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) + 2\left(\sqrt{-x^2+1}x - x\right) \log\left(\frac{\sqrt{-x^2+1}-1}{x}\right) + \sqrt{-x^2+1} - 1}{\sqrt{-x^2+1}x - x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+1)^2/(sqrt(-x^2+1)*x^2),x,algorithm="fricas")`

[Out] `(x^2 - 2*(sqrt(-x^2+1)*x - x)*arctan((sqrt(-x^2+1) - 1)/x) + 2*(sqrt(-x^2+1)*x - x)*log((sqrt(-x^2+1) - 1)/x) + sqrt(-x^2+1) - 1)/(sqrt(-x^2+1)*x - x)`

**Sympy [A]** time = 9.28067, size = 51, normalized size = 1.55

$$\begin{cases} -\frac{i\sqrt{x^2-1}}{x} & \text{for } |x^2| > 1 \\ -\frac{\sqrt{-x^2+1}}{x} & \text{otherwise} \end{cases} + 2 \left( \begin{cases} -\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for } \left|\frac{1}{x^2}\right| > 1 \\ i \operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise} \end{cases} \right) + \operatorname{asin}(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x)**2/x**2/(-x**2+1)**(1/2),x)`

[Out] `Piecewise((-I*sqrt(x**2-1)/x, Abs(x**2) > 1), (-sqrt(-x**2+1)/x, True)) + 2*Piecewise((-acosh(1/x), Abs(x**(-2)) > 1), (I*asin(1/x), True)) + asin(x)`

**GIAC/XCAS [A]** time = 0.28351, size = 74, normalized size = 2.24

$$\frac{x}{2\left(\sqrt{-x^2+1}-1\right)} - \frac{\sqrt{-x^2+1}-1}{2x} + \arcsin(x) + 2 \ln\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+1)^2/(sqrt(-x^2+1)*x^2),x,algorithm="giac")`

```
[Out] 1/2*x/(sqrt(-x^2 + 1) - 1) - 1/2*(sqrt(-x^2 + 1) - 1)/x + arcsin(x) + 2*ln(-(sqrt(-x^2 + 1) - 1)/abs(x))
```

$$3.60 \quad \int \frac{(1+x)^2}{x^3 \sqrt{1-x^2}} dx$$

**Optimal.** Leaf size=51

$$-\frac{2\sqrt{1-x^2}}{x} - \frac{\sqrt{1-x^2}}{2x^2} - \frac{3}{2} \tanh^{-1}(\sqrt{1-x^2})$$

[Out] -Sqrt[1 - x^2]/(2\*x^2) - (2\*Sqrt[1 - x^2])/x - (3\*ArcTanh[Sqrt[1 - x^2]])/2

**Rubi [A]** time = 0.127075, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$-\frac{2\sqrt{1-x^2}}{x} - \frac{\sqrt{1-x^2}}{2x^2} - \frac{3}{2} \tanh^{-1}(\sqrt{1-x^2})$$

Antiderivative was successfully verified.

[In] Int[(1 + x)^2/(x^3\*Sqrt[1 - x^2]), x]

[Out] -Sqrt[1 - x^2]/(2\*x^2) - (2\*Sqrt[1 - x^2])/x - (3\*ArcTanh[Sqrt[1 - x^2]])/2

**Rubi in Sympy [A]** time = 10.8174, size = 39, normalized size = 0.76

$$-\frac{3 \operatorname{atanh}(\sqrt{-x^2+1})}{2} - \frac{2\sqrt{-x^2+1}}{x} - \frac{\sqrt{-x^2+1}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((1+x)\*\*2/x\*\*3/(-x\*\*2+1)\*\*(1/2), x)

[Out] -3\*atanh(sqrt(-x\*\*2 + 1))/2 - 2\*sqrt(-x\*\*2 + 1)/x - sqrt(-x\*\*2 + 1)/(2\*x\*\*2)

**Mathematica [A]** time = 0.0442393, size = 46, normalized size = 0.9

$$\frac{1}{2} \left( -\frac{\sqrt{1-x^2}(4x+1)}{x^2} - 3 \log(\sqrt{1-x^2}+1) + 3 \log(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x)^2/(x^3\*Sqrt[1 - x^2]), x]

[Out] (-(((1 + 4\*x)\*Sqrt[1 - x^2])/x^2) + 3\*Log[x] - 3\*Log[1 + Sqrt[1 - x^2]])/2

**Maple [A]** time = 0.01, size = 42, normalized size = 0.8

$$-\frac{1}{2x^2} \sqrt{-x^2+1} - \frac{3}{2} \operatorname{Artanh}\left(\frac{1}{\sqrt{-x^2+1}}\right) - 2 \frac{\sqrt{-x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+x)^2/x^3/(-x^2+1)^(1/2),x)`

[Out]  $-1/2*(-x^2+1)^{(1/2)}/x^2-3/2*\operatorname{arctanh}(1/(-x^2+1)^{(1/2)})-2*(-x^2+1)^{(1/2)}/x$

**Maxima [A]** time = 0.794028, size = 73, normalized size = 1.43

$$-\frac{2\sqrt{-x^2+1}}{x} - \frac{\sqrt{-x^2+1}}{2x^2} - \frac{3}{2} \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+1)^2/(sqrt(-x^2+1)*x^3),x,algorithm="maxima")`

[Out]  $-2*\sqrt{-x^2+1}/x - 1/2*\sqrt{-x^2+1}/x^2 - 3/2*\log(2*\sqrt{-x^2+1}/\operatorname{abs}(x) + 2/\operatorname{abs}(x))$

**Fricas [A]** time = 0.271067, size = 144, normalized size = 2.82

$$\frac{8x^3 + 2x^2 + 3(x^4 + 2\sqrt{-x^2+1}x^2 - 2x^2) \log\left(\frac{\sqrt{-x^2+1}-1}{x}\right) - (4x^3 + x^2 - 8x - 2)\sqrt{-x^2+1} - 8x - 2}{2(x^4 + 2\sqrt{-x^2+1}x^2 - 2x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+1)^2/(sqrt(-x^2+1)*x^3),x,algorithm="fricas")`

[Out]  $1/2*(8*x^3 + 2*x^2 + 3*(x^4 + 2*\sqrt{-x^2+1}*x^2 - 2*x^2)*\log((\sqrt{-x^2+1}-1)/x) - (4*x^3 + x^2 - 8*x - 2)*\sqrt{-x^2+1} - 8*x - 2)/(x^4 + 2*\sqrt{-x^2+1}*x^2 - 2*x^2)$

**Sympy [A]** time = 16.9314, size = 116, normalized size = 2.27

$$2 \left( \begin{cases} -\frac{i\sqrt{x^2-1}}{x} & \text{for } |x^2| > 1 \\ -\frac{\sqrt{-x^2+1}}{x} & \text{otherwise} \end{cases} \right) + \begin{cases} -\frac{\operatorname{acosh}\left(\frac{1}{x}\right)}{2} - \frac{\sqrt{-1+\frac{1}{x^2}}}{2x} & \text{for } \left|\frac{1}{x^2}\right| > 1 \\ \frac{i \operatorname{asin}\left(\frac{1}{x}\right)}{2} - \frac{i}{2x\sqrt{1-\frac{1}{x^2}}} + \frac{i}{2x^3\sqrt{1-\frac{1}{x^2}}} & \text{otherwise} \end{cases} + \begin{cases} -\operatorname{acosh}\left(\frac{1}{x}\right) & \text{for } \left|\frac{1}{x^2}\right| > 1 \\ i \operatorname{asin}\left(\frac{1}{x}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x)**2/x**3/(-x**2+1)**(1/2),x)`

[Out]  $2*\operatorname{Piecewise}((-I*\sqrt{x^2-1}/x, \operatorname{Abs}(x^2) > 1), (-\sqrt{-x^2+1}/x, \operatorname{True})) + \operatorname{Piecewise}((- \operatorname{acosh}(1/x)/2 - \sqrt{-1+x^*(-2)})/(2*x), \operatorname{Abs}(x^*(-2)) > 1), (I*\operatorname{asin}(1/x)/2 - I/(2*x*\sqrt{1-1/x^2})) + I/(2*x^3*\sqrt{1-1/x^2}), \operatorname{True})) + \operatorname{Piecewise}((- \operatorname{acosh}(1/x), \operatorname{Abs}(x^*(-2)) > 1), (I*\operatorname{asin}(1/x), \operatorname{True}))$

**GIAC/XCAS [A]** time = 0.295918, size = 123, normalized size = 2.41

$$\frac{x^2 \left( \frac{8(\sqrt{-x^2+1}-1)}{x} - 1 \right)}{8(\sqrt{-x^2+1}-1)^2} - \frac{\sqrt{-x^2+1}-1}{x} + \frac{(\sqrt{-x^2+1}-1)^2}{8x^2} + \frac{3}{2} \ln \left( -\frac{\sqrt{-x^2+1}-1}{|x|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x + 1)^2/(sqrt(-x^2 + 1)\*x^3),x, algorithm="giac")

[Out] 1/8\*x^2\*(8\*(sqrt(-x^2 + 1) - 1)/x - 1)/(sqrt(-x^2 + 1) - 1)^2 - (sqrt(-x^2 + 1) - 1)/x + 1/8\*(sqrt(-x^2 + 1) - 1)^2/x^2 + 3/2\*ln(-(sqrt(-x^2 + 1) - 1)/abs(x))

$$3.61 \quad \int \frac{(1+x)^2}{x^4\sqrt{1-x^2}} dx$$

**Optimal.** Leaf size=67

$$-\frac{5\sqrt{1-x^2}}{3x} - \frac{\sqrt{1-x^2}}{x^2} - \tanh^{-1}\left(\sqrt{1-x^2}\right) - \frac{\sqrt{1-x^2}}{3x^3}$$

[Out]  $-\text{Sqrt}[1-x^2]/(3*x^3) - \text{Sqrt}[1-x^2]/x^2 - (5*\text{Sqrt}[1-x^2])/(3*x) - \text{ArcTanh}[\text{Sqrt}[1-x^2]]$

**Rubi [A]** time = 0.167301, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{5\sqrt{1-x^2}}{3x} - \frac{\sqrt{1-x^2}}{x^2} - \tanh^{-1}\left(\sqrt{1-x^2}\right) - \frac{\sqrt{1-x^2}}{3x^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(1+x)^2/(x^4*\text{Sqrt}[1-x^2]),x]$

[Out]  $-\text{Sqrt}[1-x^2]/(3*x^3) - \text{Sqrt}[1-x^2]/x^2 - (5*\text{Sqrt}[1-x^2])/(3*x) - \text{ArcTanh}[\text{Sqrt}[1-x^2]]$

**Rubi in Sympy [A]** time = 12.5477, size = 49, normalized size = 0.73

$$-\text{atanh}\left(\sqrt{-x^2+1}\right) - \frac{5\sqrt{-x^2+1}}{3x} - \frac{\sqrt{-x^2+1}}{x^2} - \frac{\sqrt{-x^2+1}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((1+x)**2/x**4/(-x**2+1)**(1/2),x)$

[Out]  $-\text{atanh}(\text{sqrt}(-x**2+1)) - 5*\text{sqrt}(-x**2+1)/(3*x) - \text{sqrt}(-x**2+1)/x**2 - \text{sqrt}(-x**2+1)/(3*x**3)$

**Mathematica [A]** time = 0.0498194, size = 47, normalized size = 0.7

$$-\log\left(\sqrt{1-x^2}+1\right) - \frac{\sqrt{1-x^2}(5x^2+3x+1)}{3x^3} + \log(x)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(1+x)^2/(x^4*\text{Sqrt}[1-x^2]),x]$

[Out]  $-(\text{Sqrt}[1-x^2]*(1+3*x+5*x^2))/(3*x^3) + \text{Log}[x] - \text{Log}[1+\text{Sqrt}[1-x^2]]$

**Maple [A]** time = 0.012, size = 56, normalized size = 0.8

$$-\frac{1}{3x^3}\sqrt{-x^2+1} - \frac{5}{3x}\sqrt{-x^2+1} - \frac{1}{x^2}\sqrt{-x^2+1} - \text{Artanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+x)^2/x^4/(-x^2+1)^(1/2),x)`

[Out]  $-1/3*(-x^2+1)^{(1/2)}/x^3-5/3*(-x^2+1)^{(1/2)}/x-(-x^2+1)^{(1/2)}/x^2-\operatorname{arctanh}(1/(-x^2+1)^{(1/2)})$

**Maxima [A]** time = 0.792782, size = 92, normalized size = 1.37

$$-\frac{5\sqrt{-x^2+1}}{3x} - \frac{\sqrt{-x^2+1}}{x^2} - \frac{\sqrt{-x^2+1}}{3x^3} - \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+1)^2/(sqrt(-x^2+1)*x^4),x,algorithm="maxima")`

[Out]  $-5/3*\sqrt{-x^2+1}/x - \sqrt{-x^2+1}/x^2 - 1/3*\sqrt{-x^2+1}/x^3 - \log(2*\sqrt{-x^2+1}/\operatorname{abs}(x) + 2/\operatorname{abs}(x))$

**Fricas [A]** time = 0.272534, size = 194, normalized size = 2.9

$$\frac{5x^6 + 3x^5 - 24x^4 - 15x^3 + 15x^2 - 3\left(3x^5 - 4x^3 - (x^5 - 4x^3)\sqrt{-x^2+1}\right)\log\left(\frac{\sqrt{-x^2+1}-1}{x}\right) + (15x^4 + 9x^3 - 17x^2 - 12x - 4)\sqrt{-x^2+1}}{3\left(3x^5 - 4x^3 - (x^5 - 4x^3)\sqrt{-x^2+1}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+1)^2/(sqrt(-x^2+1)*x^4),x,algorithm="fricas")`

[Out]  $-1/3*(5*x^6 + 3*x^5 - 24*x^4 - 15*x^3 + 15*x^2 - 3*(3*x^5 - 4*x^3 - (x^5 - 4*x^3)*\sqrt{-x^2+1})*\log((\sqrt{-x^2+1}-1)/x) + (15*x^4 + 9*x^3 - 17*x^2 - 12*x - 4)*\sqrt{-x^2+1} + 12*x + 4)/(3*x^5 - 4*x^3 - (x^5 - 4*x^3)*\sqrt{-x^2+1})$

**Sympy [A]** time = 18.2272, size = 128, normalized size = 1.91

$$\begin{aligned} & \left\{ -\frac{\sqrt{-x^2+1}}{x} - \frac{(-x^2+1)^{\frac{3}{2}}}{3x^3} \quad \text{for } x > -1 \wedge x < 1 + \begin{cases} -\frac{i\sqrt{x^2-1}}{x} & \text{for } |x^2| > 1 \\ -\frac{\sqrt{-x^2+1}}{x} & \text{otherwise} \end{cases} \right. \\ & \left. + 2 \left( \begin{cases} -\frac{\operatorname{acosh}(\frac{1}{x})}{2} - \frac{\sqrt{-1+\frac{1}{x^2}}}{2x} & \text{for } |\frac{1}{x^2}| > 1 \\ \frac{i \operatorname{asin}(\frac{1}{x})}{2} - \frac{i}{2x\sqrt{1-\frac{1}{x^2}}} + \frac{i}{2x^3\sqrt{1-\frac{1}{x^2}}} & \text{otherwise} \end{cases} \right) \right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x)**2/x**4/(-x**2+1)**(1/2),x)`

[Out]  $\operatorname{Piecewise}((- \sqrt{-x^2+1}/x - (-x^2+1)^{(3/2)}/(3*x^3), (x > -1) \& (x < 1)) + \operatorname{Piecewise}((-I*\sqrt{x^2-1}/x, \operatorname{Abs}(x^2) > 1), (-\sqrt{-x^2+1}/x, \operatorname{True})) + 2*\operatorname{Piecewise}((- \operatorname{acosh}(1/x)/2 - \sqrt{-1+x^{(-2)}}/(2*x), \operatorname{Abs}(x^{(-2)}) > 1), (I*\operatorname{asin}(1/x)/2 - I/(2*x*\sqrt{1-1/x^2})), \operatorname{True}))$



**GIAC/XCAS [A]** time = 0.295301, size = 169, normalized size = 2.52

$$\frac{x^3 \left( \frac{6(\sqrt{-x^2+1}-1)}{x} - \frac{21(\sqrt{-x^2+1}-1)^2}{x^2} - 1 \right)}{24(\sqrt{-x^2+1}-1)^3} - \frac{7(\sqrt{-x^2+1}-1)}{8x} + \frac{(\sqrt{-x^2+1}-1)^2}{4x^2} - \frac{(\sqrt{-x^2+1}-1)^3}{24x^3} + \ln\left(-\frac{\sqrt{-x^2+1}-1}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x + 1)^2/(sqrt(-x^2 + 1)\*x^4),x, algorithm="giac")

[Out] -1/24\*x^3\*(6\*(sqrt(-x^2 + 1) - 1)/x - 21\*(sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1)^3 - 7/8\*(sqrt(-x^2 + 1) - 1)/x + 1/4\*(sqrt(-x^2 + 1) - 1)^2/x^2 - 1/24\*(sqrt(-x^2 + 1) - 1)^3/x^3 + ln(-(sqrt(-x^2 + 1) - 1)/abs(x))

$$3.62 \quad \int \frac{(1+x)^2}{x^5 \sqrt{1-x^2}} dx$$

**Optimal.** Leaf size=89

$$-\frac{4\sqrt{1-x^2}}{3x} - \frac{7\sqrt{1-x^2}}{8x^2} - \frac{7}{8} \tanh^{-1}(\sqrt{1-x^2}) - \frac{\sqrt{1-x^2}}{4x^4} - \frac{2\sqrt{1-x^2}}{3x^3}$$

[Out] -Sqrt[1 - x^2]/(4\*x^4) - (2\*Sqrt[1 - x^2])/(3\*x^3) - (7\*Sqrt[1 - x^2])/(8\*x^2) - (4\*Sqrt[1 - x^2])/(3\*x) - (7\*ArcTanh[Sqrt[1 - x^2]])/8

**Rubi [A]** time = 0.20857, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{4\sqrt{1-x^2}}{3x} - \frac{7\sqrt{1-x^2}}{8x^2} - \frac{7}{8} \tanh^{-1}(\sqrt{1-x^2}) - \frac{\sqrt{1-x^2}}{4x^4} - \frac{2\sqrt{1-x^2}}{3x^3}$$

Antiderivative was successfully verified.

[In] Int[(1 + x)^2/(x^5\*Sqrt[1 - x^2]), x]

[Out] -Sqrt[1 - x^2]/(4\*x^4) - (2\*Sqrt[1 - x^2])/(3\*x^3) - (7\*Sqrt[1 - x^2])/(8\*x^2) - (4\*Sqrt[1 - x^2])/(3\*x) - (7\*ArcTanh[Sqrt[1 - x^2]])/8

**Rubi in Sympy [A]** time = 13.5497, size = 71, normalized size = 0.8

$$-\frac{7 \operatorname{atanh}(\sqrt{-x^2+1})}{8} - \frac{4\sqrt{-x^2+1}}{3x} - \frac{7\sqrt{-x^2+1}}{8x^2} - \frac{2\sqrt{-x^2+1}}{3x^3} - \frac{\sqrt{-x^2+1}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((1+x)\*\*2/x\*\*5/(-x\*\*2+1)\*\*(1/2), x)

[Out] -7\*atanh(sqrt(-x\*\*2 + 1))/8 - 4\*sqrt(-x\*\*2 + 1)/(3\*x) - 7\*sqrt(-x\*\*2 + 1)/(8\*x\*\*2) - 2\*sqrt(-x\*\*2 + 1)/(3\*x\*\*3) - sqrt(-x\*\*2 + 1)/(4\*x\*\*4)

**Mathematica [A]** time = 0.0561333, size = 58, normalized size = 0.65

$$-\frac{7}{8} \log(\sqrt{1-x^2}+1) - \frac{\sqrt{1-x^2}(32x^3+21x^2+16x+6)}{24x^4} + \frac{7 \log(x)}{8}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x)^2/(x^5\*Sqrt[1 - x^2]), x]

[Out] -(Sqrt[1 - x^2]\*(6 + 16\*x + 21\*x^2 + 32\*x^3))/(24\*x^4) + (7\*Log[x])/8 - (7\*Log[1 + Sqrt[1 - x^2]])/8

**Maple [A]** time = 0.011, size = 70, normalized size = 0.8

$$-\frac{1}{4x^4} \sqrt{-x^2+1} - \frac{7}{8x^2} \sqrt{-x^2+1} - \frac{7}{8} \operatorname{Artanh}\left(\frac{1}{\sqrt{-x^2+1}}\right) - \frac{2}{3x^3} \sqrt{-x^2+1} - \frac{4}{3x} \sqrt{-x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+x)^2/x^5/(-x^2+1)^(1/2),x)`

[Out]  $-1/4*(-x^2+1)^{(1/2)}/x^4-7/8*(-x^2+1)^{(1/2)}/x^2-7/8*\operatorname{arctanh}(1/(-x^2+1)^{(1/2)})-2/3*(-x^2+1)^{(1/2)}/x^3-4/3*(-x^2+1)^{(1/2)}/x$

**Maxima [A]** time = 0.792891, size = 111, normalized size = 1.25

$$-\frac{4\sqrt{-x^2+1}}{3x} - \frac{7\sqrt{-x^2+1}}{8x^2} - \frac{2\sqrt{-x^2+1}}{3x^3} - \frac{\sqrt{-x^2+1}}{4x^4} - \frac{7}{8} \log\left(\frac{2\sqrt{-x^2+1}}{|x|} + \frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+1)^2/(sqrt(-x^2+1)*x^5),x,algorithm="maxima")`

[Out]  $-4/3*\sqrt{-x^2+1}/x - 7/8*\sqrt{-x^2+1}/x^2 - 2/3*\sqrt{-x^2+1}/x^3 - 1/4*\sqrt{-x^2+1}/x^4 - 7/8*\log(2*\sqrt{-x^2+1}/\operatorname{abs}(x) + 2/\operatorname{abs}(x))$

**Fricas [A]** time = 0.268864, size = 231, normalized size = 2.6

$$\frac{128x^7 + 84x^6 - 320x^5 - 228x^4 + 64x^3 + 96x^2 + 21(x^8 - 8x^6 + 8x^4 + 4(x^6 - 2x^4)\sqrt{-x^2+1}) \log\left(\frac{\sqrt{-x^2+1}-1}{x}\right) - (32x^7 + 24(x^8 - 8x^6 + 8x^4 + 4(x^6 - 2x^4)\sqrt{-x^2+1}))}{24(x^8 - 8x^6 + 8x^4 + 4(x^6 - 2x^4)\sqrt{-x^2+1})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+1)^2/(sqrt(-x^2+1)*x^5),x,algorithm="fricas")`

[Out]  $1/24*(128*x^7 + 84*x^6 - 320*x^5 - 228*x^4 + 64*x^3 + 96*x^2 + 21*(x^8 - 8*x^6 + 8*x^4 + 4*(x^6 - 2*x^4)*\sqrt{-x^2+1})*\log((\sqrt{-x^2+1}-1)/x) - (32*x^7 + 21*x^6 - 240*x^5 - 162*x^4 + 128*x^3 + 120*x^2 + 128*x + 48)*\sqrt{-x^2+1} + 128*x + 48)/(x^8 - 8*x^6 + 8*x^4 + 4*(x^6 - 2*x^4)*\sqrt{-x^2+1})$

**Sympy [A]** time = 33.1151, size = 223, normalized size = 2.51

$$2 \left( \begin{cases} -\frac{\sqrt{-x^2+1}}{x} - \frac{(-x^2+1)^{3/2}}{3x^3} & \text{for } x > -1 \wedge x < 1 \end{cases} \right) + \begin{cases} -\frac{\operatorname{acosh}\left(\frac{1}{x}\right)}{2} - \frac{\sqrt{-1+\frac{1}{x^2}}}{2x} & \text{for } \left|\frac{1}{x^2}\right| > 1 \\ \frac{i \operatorname{asin}\left(\frac{1}{x}\right)}{2} - \frac{i}{2x\sqrt{1-\frac{1}{x^2}}} + \frac{i}{2x^3\sqrt{1-\frac{1}{x^2}}} & \text{otherwise} \end{cases}$$

$$+ \begin{cases} -\frac{3\operatorname{acosh}\left(\frac{1}{x}\right)}{8} + \frac{3}{8x\sqrt{-1+\frac{1}{x^2}}} - \frac{1}{8x^3\sqrt{-1+\frac{1}{x^2}}} - \frac{1}{4x^5\sqrt{-1+\frac{1}{x^2}}} & \text{for } \left|\frac{1}{x^2}\right| > 1 \\ \frac{3i \operatorname{asin}\left(\frac{1}{x}\right)}{8} - \frac{3i}{8x\sqrt{1-\frac{1}{x^2}}} + \frac{i}{8x^3\sqrt{1-\frac{1}{x^2}}} + \frac{i}{4x^5\sqrt{1-\frac{1}{x^2}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x)**2/x**5/(-x**2+1)**(1/2),x)`

[Out]  $2*\operatorname{Piecewise}((-sqrt(-x**2+1)/x - (-x**2+1)**(3/2)/(3*x**3), (x > -1) \& (x < 1))) + \operatorname{Piecewise}((-acosh(1/x)/2 - sqrt(-1+x**(-2)))/(2*x), \operatorname{Abs}(x**(-2)) > 1), (I*asin(1/x)/2 - I/(2*x*sqrt(1-1/x**2)) + I/(2*x**3*sqrt(1-1/x**2)), \operatorname{True})) + \operatorname{Piecewise}((-3*acosh($

```

1/x)/8 + 3/(8*x*sqrt(-1 + x**(-2))) - 1/(8*x**3*sqrt(-1 + x**(-2)
)) - 1/(4*x**5*sqrt(-1 + x**(-2))), Abs(x**(-2)) > 1), (3*I*asin(
1/x)/8 - 3*I/(8*x*sqrt(1 - 1/x**2)) + I/(8*x**3*sqrt(1 - 1/x**2))
+ I/(4*x**5*sqrt(1 - 1/x**2))), True))

```

**GIAC/XCAS [A]** time = 0.29309, size = 220, normalized size = 2.47

$$\frac{x^4 \left( \frac{16(\sqrt{-x^2+1})}{x} - \frac{48(\sqrt{-x^2+1})^2}{x^2} + \frac{144(\sqrt{-x^2+1})^3}{x^3} - 3 \right)}{192(\sqrt{-x^2+1}-1)^4} - \frac{3(\sqrt{-x^2+1}-1)}{4x}$$

$$+ \frac{(\sqrt{-x^2+1}-1)^2}{4x^2} - \frac{(\sqrt{-x^2+1}-1)^3}{12x^3} + \frac{(\sqrt{-x^2+1}-1)^4}{64x^4} + \frac{7}{8} \ln \left( -\frac{\sqrt{-x^2+1}-1}{|x|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x + 1)^2/(sqrt(-x^2 + 1)*x^5),x, algorithm="giac")
```

```
[Out] 1/192*x^4*(16*(sqrt(-x^2 + 1) - 1)/x - 48*(sqrt(-x^2 + 1) - 1)^2/
x^2 + 144*(sqrt(-x^2 + 1) - 1)^3/x^3 - 3)/(sqrt(-x^2 + 1) - 1)^4
- 3/4*(sqrt(-x^2 + 1) - 1)/x + 1/4*(sqrt(-x^2 + 1) - 1)^2/x^2 - 1
/12*(sqrt(-x^2 + 1) - 1)^3/x^3 + 1/64*(sqrt(-x^2 + 1) - 1)^4/x^4
+ 7/8*ln(-(sqrt(-x^2 + 1) - 1)/abs(x))
```

$$3.63 \quad \int \frac{(1+x)^2}{x^6 \sqrt{1-x^2}} dx$$

**Optimal.** Leaf size=107

$$-\frac{6\sqrt{1-x^2}}{5x} - \frac{3\sqrt{1-x^2}}{4x^2} - \frac{3}{4} \tanh^{-1}(\sqrt{1-x^2}) - \frac{\sqrt{1-x^2}}{5x^5} - \frac{\sqrt{1-x^2}}{2x^4} - \frac{3\sqrt{1-x^2}}{5x^3}$$

[Out]  $-\text{Sqrt}[1 - x^2]/(5*x^5) - \text{Sqrt}[1 - x^2]/(2*x^4) - (3*\text{Sqrt}[1 - x^2])/(5*x^3) - (3*\text{Sqrt}[1 - x^2])/(4*x^2) - (6*\text{Sqrt}[1 - x^2])/(5*x) - (3*\text{ArcTanh}[\text{Sqrt}[1 - x^2]])/4$

**Rubi [A]** time = 0.252263, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$-\frac{6\sqrt{1-x^2}}{5x} - \frac{3\sqrt{1-x^2}}{4x^2} - \frac{3}{4} \tanh^{-1}(\sqrt{1-x^2}) - \frac{\sqrt{1-x^2}}{5x^5} - \frac{\sqrt{1-x^2}}{2x^4} - \frac{3\sqrt{1-x^2}}{5x^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(1+x)^2/(x^6*\text{Sqrt}[1-x^2]),x]$

[Out]  $-\text{Sqrt}[1 - x^2]/(5*x^5) - \text{Sqrt}[1 - x^2]/(2*x^4) - (3*\text{Sqrt}[1 - x^2])/(5*x^3) - (3*\text{Sqrt}[1 - x^2])/(4*x^2) - (6*\text{Sqrt}[1 - x^2])/(5*x) - (3*\text{ArcTanh}[\text{Sqrt}[1 - x^2]])/4$

**Rubi in Sympy [A]** time = 15.4195, size = 85, normalized size = 0.79

$$\frac{3 \operatorname{atanh}(\sqrt{-x^2+1})}{4} - \frac{6\sqrt{-x^2+1}}{5x} - \frac{3\sqrt{-x^2+1}}{4x^2} - \frac{3\sqrt{-x^2+1}}{5x^3} - \frac{\sqrt{-x^2+1}}{2x^4} - \frac{\sqrt{-x^2+1}}{5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((1+x)**2/x**6/(-x**2+1)**(1/2),x)$

[Out]  $-3*\operatorname{atanh}(\text{sqrt}(-x**2+1))/4 - 6*\text{sqrt}(-x**2+1)/(5*x) - 3*\text{sqrt}(-x**2+1)/(4*x**2) - 3*\text{sqrt}(-x**2+1)/(5*x**3) - \text{sqrt}(-x**2+1)/(2*x**4) - \text{sqrt}(-x**2+1)/(5*x**5)$

**Mathematica [A]** time = 0.0624434, size = 63, normalized size = 0.59

$$-\frac{3}{4} \log(\sqrt{1-x^2}+1) - \frac{\sqrt{1-x^2}(24x^4+15x^3+12x^2+10x+4)}{20x^5} + \frac{3 \log(x)}{4}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(1+x)^2/(x^6*\text{Sqrt}[1-x^2]),x]$

[Out]  $-(\text{Sqrt}[1-x^2]*(4+10*x+12*x^2+15*x^3+24*x^4))/(20*x^5) + (3*\text{Log}[x])/4 - (3*\text{Log}[1+\text{Sqrt}[1-x^2]])/4$

**Maple [A]** time = 0.013, size = 84, normalized size = 0.8

$$-\frac{1}{5x^5} \sqrt{-x^2+1} - \frac{3}{5x^3} \sqrt{-x^2+1} - \frac{6}{5x} \sqrt{-x^2+1} - \frac{1}{2x^4} \sqrt{-x^2+1} - \frac{3}{4x^2} \sqrt{-x^2+1} - \frac{3}{4} \operatorname{Artanh}\left(\frac{1}{\sqrt{-x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+x)^2/x^6/(-x^2+1)^(1/2),x)`

[Out]  $-1/5*(-x^2+1)^{(1/2)}/x^5-3/5*(-x^2+1)^{(1/2)}/x^3-6/5*(-x^2+1)^{(1/2)}/x-1/2*(-x^2+1)^{(1/2)}/x^4-3/4*(-x^2+1)^{(1/2)}/x^2-3/4*\operatorname{arctanh}(1/(-x^2+1)^{(1/2)})$

**Maxima [A]** time = 0.791741, size = 130, normalized size = 1.21

$$-\frac{6\sqrt{-x^2+1}}{5x}-\frac{3\sqrt{-x^2+1}}{4x^2}-\frac{3\sqrt{-x^2+1}}{5x^3}-\frac{\sqrt{-x^2+1}}{2x^4}-\frac{\sqrt{-x^2+1}}{5x^5}-\frac{3}{4}\log\left(\frac{2\sqrt{-x^2+1}}{|x|}+\frac{2}{|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+1)^2/(sqrt(-x^2+1)*x^6),x,algorithm="maxima")`

[Out]  $-6/5*\sqrt{-x^2+1}/x-3/4*\sqrt{-x^2+1}/x^2-3/5*\sqrt{-x^2+1}/x^3-1/2*\sqrt{-x^2+1}/x^4-1/5*\sqrt{-x^2+1}/x^5-3/4*\log(2*\sqrt{-x^2+1}/\operatorname{abs}(x)+2/\operatorname{abs}(x))$

**Fricas [A]** time = 0.27126, size = 275, normalized size = 2.57

$$\frac{24x^{10}+15x^9-300x^8-185x^7+520x^6+290x^5-100x^4+40x^3-80x^2-15(5x^9-20x^7+16x^5-(x^9-12x^7+16x^5-\dots))}{20(5x^9-20x^7+16x^5-\dots)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x+1)^2/(sqrt(-x^2+1)*x^6),x,algorithm="fricas")`

[Out]  $-1/20*(24*x^{10}+15*x^9-300*x^8-185*x^7+520*x^6+290*x^5-100*x^4+40*x^3-80*x^2-15*(5*x^9-20*x^7+16*x^5-(x^9-12*x^7+16*x^5)*\sqrt{-x^2+1})*\log((\sqrt{-x^2+1}-1)/x)+(120*x^8+75*x^7-420*x^6-250*x^5+164*x^4+40*x^3+112*x^2+160*x+64)*\sqrt{-x^2+1}-160*x-64)/(5*x^9-20*x^7+16*x^5-(x^9-12*x^7+16*x^5)*\sqrt{-x^2+1})$

**Sympy [A]** time = 34.9843, size = 201, normalized size = 1.88

$$\begin{cases} -\frac{\sqrt{-x^2+1}}{x}-\frac{(-x^2+1)^{\frac{3}{2}}}{3x^3} & \text{for } x > -1 \wedge x < 1 \\ -\frac{\sqrt{-x^2+1}}{x}-\frac{2(-x^2+1)^{\frac{3}{2}}}{3x^3}-\frac{(-x^2+1)^{\frac{5}{2}}}{5x^5} & \text{for } x > -1 \wedge x < 1 \\ +2\left(\begin{cases} -\frac{3\operatorname{acosh}(\frac{1}{x})}{8}+\frac{3}{8x\sqrt{-1+\frac{1}{x^2}}}-\frac{1}{8x^3\sqrt{-1+\frac{1}{x^2}}}-\frac{1}{4x^5\sqrt{-1+\frac{1}{x^2}}} & \text{for } \left|\frac{1}{x^2}\right| > 1 \\ \frac{3i\operatorname{asin}(\frac{1}{x})}{8}-\frac{3i}{8x\sqrt{1-\frac{1}{x^2}}}+\frac{i}{8x^3\sqrt{1-\frac{1}{x^2}}}+\frac{i}{4x^5\sqrt{1-\frac{1}{x^2}}} & \text{otherwise} \end{cases}\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x)**2/x**6/(-x**2+1)**(1/2),x)`

[Out]  $\operatorname{Piecewise}((-\sqrt{-x^2+1}/x-(x^2+1)^{(3/2)}/(3*x^3), (x > -1) \& (x < 1))) + \operatorname{Piecewise}((-\sqrt{-x^2+1}/x-2*(-x^2+1)^*$

```

*(3/2)/(3*x**3) - (-x**2 + 1)**(5/2)/(5*x**5), (x > -1) & (x < 1)
)) + 2*Piecewise((-3*acosh(1/x)/8 + 3/(8*x*sqrt(-1 + x**(-2)))) -
1/(8*x**3*sqrt(-1 + x**(-2)))) - 1/(4*x**5*sqrt(-1 + x**(-2))), Ab
s(x**(-2)) > 1), (3*I*asin(1/x)/8 - 3*I/(8*x*sqrt(1 - 1/x**2)) +
I/(8*x**3*sqrt(1 - 1/x**2)) + I/(4*x**5*sqrt(1 - 1/x**2))), True))

```

**GIAC/XCAS [A]** time = 0.292346, size = 269, normalized size = 2.51

$$\frac{x^5 \left( \frac{5(\sqrt{-x^2+1}-1)}{x} - \frac{15(\sqrt{-x^2+1}-1)^2}{x^2} + \frac{40(\sqrt{-x^2+1}-1)^3}{x^3} - \frac{110(\sqrt{-x^2+1}-1)^4}{x^4} - 1 \right)}{160(\sqrt{-x^2+1}-1)^5} - \frac{11(\sqrt{-x^2+1}-1)}{16x} + \frac{(\sqrt{-x^2+1}-1)^2}{4x^2} - \frac{3(\sqrt{-x^2+1}-1)^3}{32x^3} + \frac{(\sqrt{-x^2+1}-1)^4}{32x^4} - \frac{(\sqrt{-x^2+1}-1)^5}{160x^5} + \frac{3}{4} \ln \left( -\frac{\sqrt{-x^2+1}-1}{|x|} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x + 1)^2/(sqrt(-x^2 + 1)\*x^6),x, algorithm="giac")

[Out] -1/160\*x^5\*(5\*(sqrt(-x^2 + 1) - 1)/x - 15\*(sqrt(-x^2 + 1) - 1)^2/x^2 + 40\*(sqrt(-x^2 + 1) - 1)^3/x^3 - 110\*(sqrt(-x^2 + 1) - 1)^4/x^4 - 1)/(sqrt(-x^2 + 1) - 1)^5 - 11/16\*(sqrt(-x^2 + 1) - 1)/x + 1/4\*(sqrt(-x^2 + 1) - 1)^2/x^2 - 3/32\*(sqrt(-x^2 + 1) - 1)^3/x^3 + 1/32\*(sqrt(-x^2 + 1) - 1)^4/x^4 - 1/160\*(sqrt(-x^2 + 1) - 1)^5/x^5 + 3/4\*ln(-(sqrt(-x^2 + 1) - 1)/abs(x))

$$3.64 \quad \int \frac{(d+ex)^3 \sqrt{d^2 - e^2 x^2}}{x^5} dx$$

**Optimal.** Leaf size=134

$$\begin{aligned} & -\frac{e^2(13d + 8ex)\sqrt{d^2 - e^2x^2}}{8x^2} - \frac{d(d^2 - e^2x^2)^{3/2}}{4x^4} - \frac{e(d^2 - e^2x^2)^{3/2}}{x^3} \\ & + e^4 \left( -\tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2x^2}} \right) \right) + \frac{13}{8} e^4 \tanh^{-1} \left( \frac{\sqrt{d^2 - e^2x^2}}{d} \right) \end{aligned}$$

[Out]  $-(e^2*(13*d + 8*e*x)*\text{Sqrt}[d^2 - e^2*x^2])/(8*x^2) - (d*(d^2 - e^2*x^2)^{(3/2)})/(4*x^4) - (e*(d^2 - e^2*x^2)^{(3/2)})/x^3 - e^4*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]] + (13*e^4*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/8$

**Rubi [A]** time = 0.409761, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$

$$\begin{aligned} & -\frac{e^2(13d + 8ex)\sqrt{d^2 - e^2x^2}}{8x^2} - \frac{d(d^2 - e^2x^2)^{3/2}}{4x^4} - \frac{e(d^2 - e^2x^2)^{3/2}}{x^3} \\ & + e^4 \left( -\tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2x^2}} \right) \right) + \frac{13}{8} e^4 \tanh^{-1} \left( \frac{\sqrt{d^2 - e^2x^2}}{d} \right) \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^3*\text{Sqrt}[d^2 - e^2*x^2])/x^5, x]$

[Out]  $-(e^2*(13*d + 8*e*x)*\text{Sqrt}[d^2 - e^2*x^2])/(8*x^2) - (d*(d^2 - e^2*x^2)^{(3/2)})/(4*x^4) - (e*(d^2 - e^2*x^2)^{(3/2)})/x^3 - e^4*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]] + (13*e^4*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/8$

**Rubi in Sympy [A]** time = 46.7529, size = 129, normalized size = 0.96

$$\begin{aligned} & -\frac{d^3 \sqrt{d^2 - e^2 x^2}}{4x^4} - \frac{11de^2 \sqrt{d^2 - e^2 x^2}}{8x^2} - e^4 \operatorname{atan} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) \\ & + \frac{13e^4 \operatorname{atanh} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right)}{8} - \frac{e^3 \sqrt{d^2 - e^2 x^2}}{x} - \frac{e(d^2 - e^2 x^2)^{\frac{3}{2}}}{x^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)**3*(-e**2*x**2+d**2)**(1/2)/x**5, x)$

[Out]  $-d**3*\text{sqrt}(d**2 - e**2*x**2)/(4*x**4) - 11*d*e**2*\text{sqrt}(d**2 - e**2*x**2)/(8*x**2) - e**4*\text{atan}(e*x/\text{sqrt}(d**2 - e**2*x**2)) + 13*e**4*\text{atanh}(\text{sqrt}(d**2 - e**2*x**2)/d)/8 - e**3*\text{sqrt}(d**2 - e**2*x**2)/x - e*(d**2 - e**2*x**2)**(3/2)/x**3$

**Mathematica [A]** time = 0.168626, size = 102, normalized size = 0.76

$$\begin{aligned} & \frac{1}{8} \left( -\frac{d\sqrt{d^2 - e^2x^2}(2d^2 + 8dex + 11e^2x^2)}{x^4} + 13e^4 \log(\sqrt{d^2 - e^2x^2} + d) \right. \\ & \left. - 8e^4 \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2x^2}} \right) - 13e^4 \log(x) \right) \end{aligned}$$



Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*Sqrt[d^2 - e^2\*x^2])/x^5,x]

[Out]  $-\left(\frac{d\sqrt{d^2 - e^2x^2}}{x^4} + \frac{2d^2 + 8d^2e^2x + 11e^2x^2}{x^4}\right) - 8e^4\text{ArcTan}\left[\frac{e^2x}{\sqrt{d^2 - e^2x^2}}\right] - 13e^4\text{Log}[x] + 13e^4\text{Log}[d + \sqrt{d^2 - e^2x^2}]/8$

**Maple [A]** time = 0.03, size = 212, normalized size = 1.6

$$-\frac{d}{4x^4}(-e^2x^2 + d^2)^{\frac{3}{2}} - \frac{13e^2}{8dx^2}(-e^2x^2 + d^2)^{\frac{3}{2}} - \frac{13e^4}{8d}\sqrt{-e^2x^2 + d^2} + \frac{13de^4}{8}\ln\left(\frac{1}{x}\left(2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}} - \frac{e^3}{d^2x}(-e^2x^2 + d^2)^{\frac{3}{2}} - \frac{e^5x}{d^2}\sqrt{-e^2x^2 + d^2} - e^5\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} - \frac{e}{x^3}(-e^2x^2 + d^2)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(-e^2\*x^2+d^2)^(1/2)/x^5,x)

[Out]  $-\frac{1}{4}d^3(-e^2x^2+d^2)^{3/2}/x^4 - \frac{13}{8}d^2e^2/x^2(-e^2x^2+d^2)^{3/2} - \frac{13}{8}d^2e^4/d(-e^2x^2+d^2)^{1/2} + \frac{13}{8}d^2e^4/(d^2)^{1/2}\ln\left(\frac{2d^2+2(-e^2x^2+d^2)^{1/2}}{x}\right) - \frac{e^3}{d^2x}(-e^2x^2+d^2)^{3/2} - \frac{e^5}{d^2}x\sqrt{-e^2x^2+d^2} - \frac{e^5}{(e^2)^{1/2}}\arctan\left(\frac{e^2x}{\sqrt{-e^2x^2+d^2}}\right) - \frac{e}{x^3}(-e^2x^2+d^2)^{3/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)\*(e\*x + d)^3/x^5,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.287954, size = 541, normalized size = 4.04

$$44d^2e^6x^6 + 32d^3e^5x^5 - 124d^4e^4x^4 - 96d^5e^3x^3 + 64d^6e^2x^2 + 64d^7ex + 16d^8 + 16\left(e^8x^8 - 8d^2e^6x^6 + 8d^4e^4x^4 + 4(de^6x^6 - \dots)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)\*(e\*x + d)^3/x^5,x, algorithm="fricas")

[Out]  $\frac{1}{8}(44d^2e^6x^6 + 32d^3e^5x^5 - 124d^4e^4x^4 - 96d^5e^3x^3 + 64d^6e^2x^2 + 64d^7ex + 16d^8 + 16(e^8x^8 - 8d^2e^6x^6 + 8d^4e^4x^4 + 4(de^6x^6 - \dots)))\sqrt{-e^2x^2 + d^2} - 13(e^8x^8 - 8d^2e^6x^6 + 8d^4e^4x^4 + 4(de^6x^6 - \dots))\sqrt{-e^2x^2 + d^2} \log\left(\frac{-d - \sqrt{-e^2x^2 + d^2}}{x}\right) - (11d^2e^6x^6 + 8d^3e^5x^5 - 86d^4e^4x^4 - 64d^5e^3x^3 + 72d^6e^2x^2 + 64d^7ex + 16d^8)\sqrt{-e^2x^2 + d^2}/(e^4)$

$$*x^8 - 8*d^2*e^2*x^6 + 8*d^4*x^4 + 4*(d*e^2*x^6 - 2*d^3*x^4)*\sqrt{-e^2*x^2 + d^2}$$

**Sympy [A]** time = 26.0766, size = 544, normalized size = 4.06

$$d^3 \left( \begin{array}{l} \left\{ \begin{array}{l} -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^4 \operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} \\ \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^4 \operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} \end{array} \right. \text{for } \left| \frac{d^2}{e^2x^2} \right| > 1 \\ \text{otherwise} \end{array} \right) \\ + 3d^2e \left( \begin{array}{l} \left\{ \begin{array}{l} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2} + \frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2} + \frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} \end{array} \right. \text{for } \left| \frac{d^2}{e^2x^2} \right| > 1 \\ \text{otherwise} \end{array} \right) \\ + 3de^2 \left( \begin{array}{l} \left\{ \begin{array}{l} -\frac{d^2}{2ex^3\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e}{2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^2 \operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} \\ \frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{2x} - \frac{ie^2 \operatorname{asin}\left(\frac{d}{ex}\right)}{2d} \end{array} \right. \text{for } \left| \frac{d^2}{e^2x^2} \right| > 1 \\ \text{otherwise} \end{array} \right) \\ + e^3 \left( \begin{array}{l} \left\{ \begin{array}{l} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}} + ie \operatorname{acosh}\left(\frac{ex}{d}\right) - \frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}} - e \operatorname{asin}\left(\frac{ex}{d}\right) + \frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} \end{array} \right. \text{for } \left| \frac{e^2x^2}{d^2} \right| > 1 \\ \text{otherwise} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2)/x\*\*5,x)

[Out]  $d^{**3} \operatorname{Piecewise}\left(\left(-\frac{d^{**2}}{4*e*x^{**5}*\sqrt{d^{**2}/(e^{**2}*x^{**2})-1}}\right) + 3*e / (8*x^{**3}*\sqrt{d^{**2}/(e^{**2}*x^{**2})-1}) - e^{**3}/(8*d^{**2}*x*\sqrt{d^{**2}/(e^{**2}*x^{**2})-1}) + e^{**4}*\operatorname{acosh}(d/(e*x))/(8*d^{**3}), \operatorname{Abs}(d^{**2}/(e^{**2}*x^{**2})) > 1\right), \left(I*d^{**2}/(4*e*x^{**5}*\sqrt{-d^{**2}/(e^{**2}*x^{**2})+1}) - 3*I*e/(8*x^{**3}*\sqrt{-d^{**2}/(e^{**2}*x^{**2})+1}) + I*e^{**3}/(8*d^{**2}*x*\sqrt{-d^{**2}/(e^{**2}*x^{**2})+1}) - I*e^{**4}*\operatorname{asin}(d/(e*x))/(8*d^{**3}), \operatorname{True}\right) + 3*d^{**2}*e*\operatorname{Piecewise}\left(\left(-e*\sqrt{d^{**2}/(e^{**2}*x^{**2})-1}/(3*x^{**2}) + e^{**3}*\sqrt{d^{**2}/(e^{**2}*x^{**2})-1}/(3*d^{**2}), \operatorname{Abs}(d^{**2}/(e^{**2}*x^{**2})) > 1\right), \left(-I*e*\sqrt{-d^{**2}/(e^{**2}*x^{**2})+1}/(3*x^{**2}) + I*e^{**3}*\sqrt{-d^{**2}/(e^{**2}*x^{**2})+1}/(3*d^{**2}), \operatorname{True}\right) + 3*d*e^{**2}*\operatorname{Piecewise}\left(\left(-\frac{d^{**2}}{2*x^3*\sqrt{\frac{d^{**2}}{e^{**2}*x^{**2}}-1}} + \frac{e}{2*x*\sqrt{\frac{d^{**2}}{e^{**2}*x^{**2}}-1}} + \frac{e^2*\operatorname{acosh}(d/(e*x))}{2*d}, \operatorname{Abs}(d^{**2}/(e^{**2}*x^{**2})) > 1\right), \left(-I*e*\sqrt{-d^{**2}/(e^{**2}*x^{**2})+1}/(2*x) - I*e^{**2}*\operatorname{asin}(d/(e*x))/(2*d), \operatorname{True}\right) + e^{**3}*\operatorname{Piecewise}\left(\left(\frac{I*d}{x*\sqrt{-1+e^{**2}*x^{**2}/d^{**2}}}\right) + I*e*\operatorname{acosh}(e*x/d) - I*e^{**2}*x/(d*\sqrt{-1+e^{**2}*x^{**2}/d^{**2}}), \operatorname{Abs}(e^{**2}*x^{**2}/d^{**2}) > 1\right), \left(-\frac{d}{x*\sqrt{1-e^{**2}*x^{**2}/d^{**2}}}\right) - e*\operatorname{asin}(e*x/d) + e^{**2}*x/(d*\sqrt{1-e^{**2}*x^{**2}/d^{**2}}), \operatorname{True}\right)$

**GIAC/XCAS [A]** time = 0.30513, size = 398, normalized size = 2.97

$$- \arcsin\left(\frac{xe}{d}\right) e^4 \operatorname{sign}(d) + \frac{x^4 \left( \frac{8(de + \sqrt{-x^2e^2 + d^2}e)e^8}{x} + \frac{24(de + \sqrt{-x^2e^2 + d^2}e)^2e^6}{x^2} + \frac{8(de + \sqrt{-x^2e^2 + d^2}e)^3e^4}{x^3} + e^{10} \right) e^2}{64(de + \sqrt{-x^2e^2 + d^2}e)^4} \\ - \frac{1}{64} \left( \frac{8(de + \sqrt{-x^2e^2 + d^2}e)e^{26}}{x} + \frac{24(de + \sqrt{-x^2e^2 + d^2}e)^2e^{24}}{x^2} + \frac{8(de + \sqrt{-x^2e^2 + d^2}e)^3e^{22}}{x^3} + \frac{(de + \sqrt{-x^2e^2 + d^2}e)^4e^{20}}{x^4} \right) \\ + \frac{13}{8} e^4 \ln\left(\frac{|-2de - 2\sqrt{-x^2e^2 + d^2}e|e^{(-2)}}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)\*(e\*x + d)^3/x^5,x, algorithm="giac")

[Out] -arcsin(x\*e/d)\*e^4\*sign(d) + 1/64\*x^4\*(8\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*e^8/x + 24\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^2\*e^6/x^2 + 8\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^3\*e^4/x^3 + e^10)\*e^2/(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^4 - 1/64\*(8\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*e^26/x + 24\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^2\*e^24/x^2 + 8\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^3\*e^22/x^3 + (d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^4\*e^20/x^4)\*e^(-24) + 13/8\*e^4\*ln(1/2\*abs(-2\*d\*e - 2\*sqrt(-x^2\*e^2 + d^2)\*e)\*e^(-2)/abs(x))

### 3.65 $\int x^5(d+ex)^3(d^2-e^2x^2)^{5/2} dx$

**Optimal.** Leaf size=310

$$\begin{aligned}
 & -\frac{1}{14}ex^7(d^2-e^2x^2)^{7/2} \\
 & -\frac{3}{13}dx^6(d^2-e^2x^2)^{7/2} - \frac{7d^2x^5(d^2-e^2x^2)^{7/2}}{24e} + \frac{35d^{14}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2048e^6} + \frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5} \\
 & + \frac{35d^{10}x(d^2-e^2x^2)^{3/2}}{3072e^5} + \frac{7d^8x(d^2-e^2x^2)^{5/2}}{768e^5} - \frac{d^6(31744d+63063ex)(d^2-e^2x^2)^{7/2}}{1153152e^6} \\
 & - \frac{124d^5x^2(d^2-e^2x^2)^{7/2}}{1287e^4} - \frac{7d^4x^3(d^2-e^2x^2)^{7/2}}{48e^3} - \frac{31d^3x^4(d^2-e^2x^2)^{7/2}}{143e^2}
 \end{aligned}$$

[Out]  $(35*d^{12}*x*\text{Sqrt}[d^2 - e^2*x^2])/(2048*e^5) + (35*d^{10}*x*(d^2 - e^2*x^2)^{(3/2)})/(3072*e^5) + (7*d^8*x*(d^2 - e^2*x^2)^{(5/2)})/(768*e^5) - (124*d^5*x^2*(d^2 - e^2*x^2)^{(7/2)})/(1287*e^4) - (7*d^4*x^3*(d^2 - e^2*x^2)^{(7/2)})/(48*e^3) - (31*d^3*x^4*(d^2 - e^2*x^2)^{(7/2)})/(143*e^2) - (7*d^2*x^5*(d^2 - e^2*x^2)^{(7/2)})/(24*e) - (3*d*x^6*(d^2 - e^2*x^2)^{(7/2)})/13 - (e*x^7*(d^2 - e^2*x^2)^{(7/2)})/14 - (d^6*(31744*d + 63063*e*x)*(d^2 - e^2*x^2)^{(7/2)})/(1153152*e^6) + (35*d^{14}*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2048*e^6)$

**Rubi [A]** time = 0.989663, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\begin{aligned}
 & -\frac{1}{14}ex^7(d^2-e^2x^2)^{7/2} \\
 & -\frac{3}{13}dx^6(d^2-e^2x^2)^{7/2} - \frac{7d^2x^5(d^2-e^2x^2)^{7/2}}{24e} + \frac{35d^{14}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2048e^6} + \frac{35d^{12}x\sqrt{d^2-e^2x^2}}{2048e^5} \\
 & + \frac{35d^{10}x(d^2-e^2x^2)^{3/2}}{3072e^5} + \frac{7d^8x(d^2-e^2x^2)^{5/2}}{768e^5} - \frac{d^6(31744d+63063ex)(d^2-e^2x^2)^{7/2}}{1153152e^6} \\
 & - \frac{124d^5x^2(d^2-e^2x^2)^{7/2}}{1287e^4} - \frac{7d^4x^3(d^2-e^2x^2)^{7/2}}{48e^3} - \frac{31d^3x^4(d^2-e^2x^2)^{7/2}}{143e^2}
 \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^5*(d+e*x)^3*(d^2-e^2*x^2)^{(5/2)},x]$

[Out]  $(35*d^{12}*x*\text{Sqrt}[d^2 - e^2*x^2])/(2048*e^5) + (35*d^{10}*x*(d^2 - e^2*x^2)^{(3/2)})/(3072*e^5) + (7*d^8*x*(d^2 - e^2*x^2)^{(5/2)})/(768*e^5) - (124*d^5*x^2*(d^2 - e^2*x^2)^{(7/2)})/(1287*e^4) - (7*d^4*x^3*(d^2 - e^2*x^2)^{(7/2)})/(48*e^3) - (31*d^3*x^4*(d^2 - e^2*x^2)^{(7/2)})/(143*e^2) - (7*d^2*x^5*(d^2 - e^2*x^2)^{(7/2)})/(24*e) - (3*d*x^6*(d^2 - e^2*x^2)^{(7/2)})/13 - (e*x^7*(d^2 - e^2*x^2)^{(7/2)})/14 - (d^6*(31744*d + 63063*e*x)*(d^2 - e^2*x^2)^{(7/2)})/(1153152*e^6) + (35*d^{14}*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(2048*e^6)$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**5*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)$

[Out] Timed out

**Mathematica [A]** time = 0.257064, size = 216, normalized size = 0.7

$$\frac{35d^{14} \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{2048e^6} + \sqrt{d^2 - e^2x^2} \left( -\frac{248d^{13}}{9009e^6} - \frac{35d^{12}x}{2048e^5} - \frac{124d^{11}x^2}{9009e^4} - \frac{35d^{10}x^3}{3072e^3} - \frac{31d^9x^4}{3003e^2} - \frac{7d^8x^5}{768e} \right. \\ \left. + \frac{1424d^7x^6}{9009} + \frac{377}{896}d^6ex^7 + \frac{178d^5e^2x^8}{1287} - \frac{173}{336}d^4e^3x^9 - \frac{68}{143}d^3e^4x^{10} + \frac{13}{168}d^2e^5x^{11} + \frac{3}{13}de^6x^{12} + \frac{e^7x^{13}}{14} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2), x]

[Out] Sqrt[d^2 - e^2\*x^2]\*((-248\*d^13)/(9009\*e^6) - (35\*d^12\*x)/(2048\*e^5) - (124\*d^11\*x^2)/(9009\*e^4) - (35\*d^10\*x^3)/(3072\*e^3) - (31\*d^9\*x^4)/(3003\*e^2) - (7\*d^8\*x^5)/(768\*e) + (1424\*d^7\*x^6)/9009 + (377\*d^6\*e\*x^7)/896 + (178\*d^5\*e^2\*x^8)/1287 - (173\*d^4\*e^3\*x^9)/336 - (68\*d^3\*e^4\*x^10)/143 + (13\*d^2\*e^5\*x^11)/168 + (3\*d\*e^6\*x^12)/13 + (e^7\*x^13)/14) + (35\*d^14\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(2048\*e^6)

**Maple [A]** time = 0.145, size = 291, normalized size = 0.9

$$-\frac{31d^3x^4}{143e^2}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{124d^5x^2}{1287e^4}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{248d^7}{9009e^6}(-e^2x^2 + d^2)^{\frac{7}{2}} \\ - \frac{ex^7}{14}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{7d^2x^5}{24e}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{7d^4x^3}{48e^3}(-e^2x^2 + d^2)^{\frac{7}{2}} \\ - \frac{7d^6x}{128e^5}(-e^2x^2 + d^2)^{\frac{7}{2}} + \frac{7d^8x}{768e^5}(-e^2x^2 + d^2)^{\frac{5}{2}} + \frac{35d^{10}x}{3072e^5}(-e^2x^2 + d^2)^{\frac{3}{2}} \\ + \frac{35d^{12}x}{2048e^5}\sqrt{-e^2x^2 + d^2} + \frac{35d^{14}}{2048e^5}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2 + d^2}}\right)\frac{1}{\sqrt{e^2}} - \frac{3dx^6}{13}(-e^2x^2 + d^2)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(e\*x+d)^3\*(-e^2\*x^2+d^2)^(5/2), x)

[Out] -31/143\*d^3\*x^4\*(-e^2\*x^2+d^2)^(7/2)/e^2-124/1287\*d^5\*x^2\*(-e^2\*x^2+d^2)^(7/2)/e^4-248/9009\*d^7/e^6\*(-e^2\*x^2+d^2)^(7/2)-1/14\*e\*x^7\*(-e^2\*x^2+d^2)^(7/2)-7/24\*d^2\*x^5\*(-e^2\*x^2+d^2)^(7/2)/e-7/48\*d^4\*x^3\*(-e^2\*x^2+d^2)^(7/2)/e^3-7/128/e^5\*d^6\*x\*(-e^2\*x^2+d^2)^(7/2)+7/768\*d^8\*x\*(-e^2\*x^2+d^2)^(5/2)/e^5+35/3072\*d^10\*x\*(-e^2\*x^2+d^2)^(3/2)/e^5+35/2048\*d^12\*x\*(-e^2\*x^2+d^2)^(1/2)/e^5+35/2048/e^5\*d^14/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))-3/13\*d\*x^6\*(-e^2\*x^2+d^2)^(7/2)

**Maxima [A]** time = 0.799427, size = 382, normalized size = 1.23

$$-\frac{1}{14}(-e^2x^2 + d^2)^{\frac{7}{2}}ex^7 - \frac{3}{13}(-e^2x^2 + d^2)^{\frac{7}{2}}dx^6 - \frac{7(-e^2x^2 + d^2)^{\frac{7}{2}}d^2x^5}{24e} + \frac{35d^{14} \arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{2048\sqrt{e^2}e^5} \\ + \frac{35\sqrt{-e^2x^2 + d^2}d^{12}x}{2048e^5} - \frac{31(-e^2x^2 + d^2)^{\frac{7}{2}}d^3x^4}{143e^2} + \frac{35(-e^2x^2 + d^2)^{\frac{3}{2}}d^{10}x}{3072e^5} - \frac{7(-e^2x^2 + d^2)^{\frac{7}{2}}d^4x^3}{48e^3} \\ + \frac{7(-e^2x^2 + d^2)^{\frac{5}{2}}d^8x}{768e^5} - \frac{124(-e^2x^2 + d^2)^{\frac{7}{2}}d^5x^2}{1287e^4} - \frac{7(-e^2x^2 + d^2)^{\frac{7}{2}}d^6x}{128e^5} - \frac{248(-e^2x^2 + d^2)^{\frac{7}{2}}d^7}{9009e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3\*x^5, x, algorithm="maxima")

```
[Out] -1/14*(-e^2*x^2 + d^2)^(7/2)*e*x^7 - 3/13*(-e^2*x^2 + d^2)^(7/2)*
d*x^6 - 7/24*(-e^2*x^2 + d^2)^(7/2)*d^2*x^5/e + 35/2048*d^14*arcs
in(e^2*x/sqrt(d^2*e^2))/(sqrt(e^2)*e^5) + 35/2048*sqrt(-e^2*x^2 +
d^2)*d^12*x/e^5 - 31/143*(-e^2*x^2 + d^2)^(7/2)*d^3*x^4/e^2 + 35
/3072*(-e^2*x^2 + d^2)^(3/2)*d^10*x/e^5 - 7/48*(-e^2*x^2 + d^2)^(
7/2)*d^4*x^3/e^3 + 7/768*(-e^2*x^2 + d^2)^(5/2)*d^8*x/e^5 - 124/1
287*(-e^2*x^2 + d^2)^(7/2)*d^5*x^2/e^4 - 7/128*(-e^2*x^2 + d^2)^(
7/2)*d^6*x/e^5 - 248/9009*(-e^2*x^2 + d^2)^(7/2)*d^7/e^6
```

**Fricas [A]** time = 0.289002, size = 1251, normalized size = 4.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3*x^5,x, algorithm="fricas")
```

```
[Out] -1/18450432*(18450432*d*e^27*x^27 + 59609088*d^2*e^26*x^26 - 5888
76288*d^3*e^25*x^25 - 2089930752*d^4*e^24*x^24 + 5111538432*d^5*e
^23*x^23 + 23164051456*d^6*e^22*x^22 - 14662753248*d^7*e^21*x^21
- 123011080192*d^8*e^20*x^20 - 16479647184*d^9*e^19*x^19 + 365519
458304*d^10*e^18*x^18 + 211707355860*d^11*e^17*x^17 - 64270234828
8*d^12*e^16*x^16 - 596741871726*d^13*e^15*x^15 + 655963815936*d^1
4*e^14*x^14 + 877286131722*d^15*e^13*x^13 - 327286063104*d^16*e^1
2*x^12 - 737038638336*d^17*e^11*x^11 - 3148873728*d^18*e^10*x^10
+ 338040935232*d^19*e^9*x^9 + 78721843200*d^20*e^8*x^8 - 71743406
592*d^21*e^7*x^7 - 25190989824*d^22*e^6*x^6 + 11117922816*d^23*e^
5*x^5 - 8610201600*d^25*e^3*x^3 + 2583060480*d^27*e*x + 630630*(d
^14*e^14*x^14 - 98*d^16*e^12*x^12 + 1568*d^18*e^10*x^10 - 9408*d^
20*e^8*x^8 + 26880*d^22*e^6*x^6 - 39424*d^24*e^4*x^4 + 28672*d^26
*e^2*x^2 - 8192*d^28 + 2*(7*d^15*e^12*x^12 - 224*d^17*e^10*x^10 +
2016*d^19*e^8*x^8 - 7680*d^21*e^6*x^6 + 14080*d^23*e^4*x^4 - 122
88*d^25*e^2*x^2 + 4096*d^27)*sqrt(-e^2*x^2 + d^2))*arctan(-(d - s
qrt(-e^2*x^2 + d^2))/(e*x)) - (1317888*e^27*x^27 + 4257792*d*e^26
*x^26 - 127725312*d^2*e^25*x^25 - 426037248*d^3*e^24*x^24 + 19170
32832*d^4*e^23*x^23 + 7538585600*d^5*e^22*x^22 - 9221296656*d^6*e
^21*x^21 - 54061522944*d^7*e^20*x^20 + 6336305976*d^8*e^19*x^19 +
200707020800*d^9*e^18*x^18 + 87983317422*d^10*e^17*x^17 - 423110
578176*d^11*e^16*x^16 - 347133049263*d^12*e^15*x^15 + 50885237145
6*d^13*e^14*x^14 + 614616191046*d^14*e^13*x^13 - 307211993088*d^1
5*e^12*x^12 - 593166274272*d^16*e^11*x^11 + 26765426688*d^17*e^10
*x^10 + 304354070592*d^18*e^9*x^9 + 66126348288*d^19*e^8*x^8 - 68
606064384*d^20*e^7*x^7 - 25190989824*d^21*e^6*x^6 + 7781469696*d^
22*e^5*x^5 - 7318671360*d^24*e^3*x^3 + 2583060480*d^26*e*x)*sqrt(
-e^2*x^2 + d^2))/(e^20*x^14 - 98*d^2*e^18*x^12 + 1568*d^4*e^16*x^
10 - 9408*d^6*e^14*x^8 + 26880*d^8*e^12*x^6 - 39424*d^10*e^10*x^4
+ 28672*d^12*e^8*x^2 - 8192*d^14*e^6 + 2*(7*d^15*e^12*x^12 - 224*d^
17*e^10*x^10 + 2016*d^19*e^8*x^8 - 7680*d^21*e^6*x^6 + 14080*d^23*
e^4*x^4 - 12288*d^25*e^2*x^2 + 4096*d^27)*sqrt(-e^2*x^2 + d^2
))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] Timed out
```

**GIAC/XCAS [A]** time = 0.288883, size = 230, normalized size = 0.74

$$\frac{35}{2048} d^{14} \arcsin\left(\frac{xe}{d}\right) e^{(-6)} \text{sign}(d) - \frac{1}{18450432} \left( 507904 d^{13} e^{(-6)} + \left( 315315 d^{12} e^{(-5)} + 2 \left( 126976 d^{11} e^{(-4)} + \left( 105105 d^{10} e^{(-3)} + 4 \left( 23808 d^9 e^{(-2)} + \left( 21021 d^8 e^{(-1)} \right. \right. \right. \right. \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3\*x^5,x, algorithm="giac")

[Out] 35/2048\*d^14\*arcsin(x\*e/d)\*e^(-6)\*sign(d) - 1/18450432\*(507904\*d^13\*e^(-6) + (315315\*d^12\*e^(-5) + 2\*(126976\*d^11\*e^(-4) + (105105\*d^10\*e^(-3) + 4\*(23808\*d^9\*e^(-2) + (21021\*d^8\*e^(-1) - 2\*(18227\*2\*d^7 + (485199\*d^6\*e + 8\*(19936\*d^5\*e^2 - 3\*(24739\*d^4\*e^3 + 2\*(11424\*d^3\*e^4 - 11\*(169\*d^2\*e^5 + 12\*(13\*x\*e^7 + 42\*d\*e^6)\*x)\*x)\*x)\*x)\*x)\*x)\*x)\*x)\*sqrt(-x^2\*e^2 + d^2)

### 3.66 $\int x^4(d+ex)^3(d^2-e^2x^2)^{5/2} dx$

**Optimal.** Leaf size=281

$$-\frac{1}{13}ex^6(d^2-e^2x^2)^{7/2}-\frac{1}{4}dx^5(d^2-e^2x^2)^{7/2}-\frac{45d^2x^4(d^2-e^2x^2)^{7/2}}{143e} \\ +\frac{27d^{13}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{1024e^5}+\frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4}+\frac{9d^9x(d^2-e^2x^2)^{3/2}}{512e^4}+\frac{9d^7x(d^2-e^2x^2)^{5/2}}{640e^4} \\ -\frac{d^5(12800d+27027ex)(d^2-e^2x^2)^{7/2}}{320320e^5}-\frac{20d^4x^2(d^2-e^2x^2)^{7/2}}{143e^3}-\frac{9d^3x^3(d^2-e^2x^2)^{7/2}}{40e^2}$$

[Out] (27\*d^11\*x\*Sqrt[d^2 - e^2\*x^2])/(1024\*e^4) + (9\*d^9\*x\*(d^2 - e^2\*x^2)^(3/2))/(512\*e^4) + (9\*d^7\*x\*(d^2 - e^2\*x^2)^(5/2))/(640\*e^4) - (20\*d^4\*x^2\*(d^2 - e^2\*x^2)^(7/2))/(143\*e^3) - (9\*d^3\*x^3\*(d^2 - e^2\*x^2)^(7/2))/(40\*e^2) - (45\*d^2\*x^4\*(d^2 - e^2\*x^2)^(7/2))/(143\*e) - (d\*x^5\*(d^2 - e^2\*x^2)^(7/2))/4 - (e\*x^6\*(d^2 - e^2\*x^2)^(7/2))/13 - (d^5\*(12800\*d + 27027\*e\*x)\*(d^2 - e^2\*x^2)^(7/2))/(320320\*e^5) + (27\*d^13\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(1024\*e^5)

**Rubi [A]** time = 0.807803, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-\frac{1}{13}ex^6(d^2-e^2x^2)^{7/2}-\frac{1}{4}dx^5(d^2-e^2x^2)^{7/2}-\frac{45d^2x^4(d^2-e^2x^2)^{7/2}}{143e} \\ +\frac{27d^{13}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{1024e^5}+\frac{27d^{11}x\sqrt{d^2-e^2x^2}}{1024e^4}+\frac{9d^9x(d^2-e^2x^2)^{3/2}}{512e^4}+\frac{9d^7x(d^2-e^2x^2)^{5/2}}{640e^4} \\ -\frac{d^5(12800d+27027ex)(d^2-e^2x^2)^{7/2}}{320320e^5}-\frac{20d^4x^2(d^2-e^2x^2)^{7/2}}{143e^3}-\frac{9d^3x^3(d^2-e^2x^2)^{7/2}}{40e^2}$$

Antiderivative was successfully verified.

[In] Int[x^4\*(d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2), x]

[Out] (27\*d^11\*x\*Sqrt[d^2 - e^2\*x^2])/(1024\*e^4) + (9\*d^9\*x\*(d^2 - e^2\*x^2)^(3/2))/(512\*e^4) + (9\*d^7\*x\*(d^2 - e^2\*x^2)^(5/2))/(640\*e^4) - (20\*d^4\*x^2\*(d^2 - e^2\*x^2)^(7/2))/(143\*e^3) - (9\*d^3\*x^3\*(d^2 - e^2\*x^2)^(7/2))/(40\*e^2) - (45\*d^2\*x^4\*(d^2 - e^2\*x^2)^(7/2))/(143\*e) - (d\*x^5\*(d^2 - e^2\*x^2)^(7/2))/4 - (e\*x^6\*(d^2 - e^2\*x^2)^(7/2))/13 - (d^5\*(12800\*d + 27027\*e\*x)\*(d^2 - e^2\*x^2)^(7/2))/(320320\*e^5) + (27\*d^13\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(1024\*e^5)

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2), x)

[Out] Timed out



**Mathematica [A]** time = 0.23723, size = 203, normalized size = 0.72

$$\frac{27d^{13} \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{1024e^5} + \sqrt{d^2 - e^2x^2} \left( -\frac{40d^{12}}{1001e^5} - \frac{27d^{11}x}{1024e^4} - \frac{20d^{10}x^2}{1001e^3} - \frac{9d^9x^3}{512e^2} - \frac{15d^8x^4}{1001e} \right. \\ \left. + \frac{119d^7x^5}{640} + \frac{488d^6ex^6}{1001} + \frac{51}{320}d^5e^2x^7 - \frac{82}{143}d^4e^3x^8 - \frac{21}{40}d^3e^4x^9 + \frac{12}{143}d^2e^5x^{10} + \frac{1}{4}de^6x^{11} + \frac{e^7x^{12}}{13} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2), x]

[Out] Sqrt[d^2 - e^2\*x^2]\*((-40\*d^12)/(1001\*e^5) - (27\*d^11\*x)/(1024\*e^4) - (20\*d^10\*x^2)/(1001\*e^3) - (9\*d^9\*x^3)/(512\*e^2) - (15\*d^8\*x^4)/(1001\*e) + (119\*d^7\*x^5)/640 + (488\*d^6\*e\*x^6)/1001 + (51\*d^5\*e^2\*x^7)/320 - (82\*d^4\*e^3\*x^8)/143 - (21\*d^3\*e^4\*x^9)/40 + (12\*d^2\*e^5\*x^10)/143 + (d\*e^6\*x^11)/4 + (e^7\*x^12)/13) + (27\*d^13\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(1024\*e^5)

**Maple [A]** time = 0.024, size = 266, normalized size = 1.

$$-\frac{9d^3x^3}{40e^2}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{27d^5x}{320e^4}(-e^2x^2 + d^2)^{\frac{7}{2}} + \frac{9d^7x}{640e^4}(-e^2x^2 + d^2)^{\frac{5}{2}} + \frac{9d^9x}{512e^4}(-e^2x^2 + d^2)^{\frac{3}{2}} \\ + \frac{27d^{11}x}{1024e^4}\sqrt{-e^2x^2 + d^2} + \frac{27d^{13}}{1024e^4} \arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} - \frac{ex^6}{13}(-e^2x^2 + d^2)^{\frac{7}{2}} \\ - \frac{45d^2x^4}{143e}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{20d^4x^2}{143e^3}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{40d^6}{1001e^5}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{dx^5}{4}(-e^2x^2 + d^2)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(e\*x+d)^3\*(-e^2\*x^2+d^2)^(5/2), x)

[Out] -9/40\*d^3\*x^3\*(-e^2\*x^2+d^2)^(7/2)/e^2-27/320\*d^5/e^4\*x\*(-e^2\*x^2+d^2)^(7/2)+9/640\*d^7\*x\*(-e^2\*x^2+d^2)^(5/2)/e^4+9/512\*d^9\*x\*(-e^2\*x^2+d^2)^(3/2)/e^4+27/1024\*d^11\*x\*(-e^2\*x^2+d^2)^(1/2)/e^4+27/1024\*d^13/e^4/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))-1/13\*e\*x^6\*(-e^2\*x^2+d^2)^(7/2)-45/143\*d^2\*x^4\*(-e^2\*x^2+d^2)^(7/2)/e-20/143\*d^4\*x^2\*(-e^2\*x^2+d^2)^(7/2)/e^3-40/1001/e^5\*d^6\*(-e^2\*x^2+d^2)^(7/2)-1/4\*d\*x^5\*(-e^2\*x^2+d^2)^(7/2)

**Maxima [A]** time = 0.80355, size = 348, normalized size = 1.24

$$-\frac{1}{13}(-e^2x^2 + d^2)^{\frac{7}{2}}ex^6 - \frac{1}{4}(-e^2x^2 + d^2)^{\frac{7}{2}}dx^5 + \frac{27d^{13} \arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{1024\sqrt{e^2}e^4} + \frac{27\sqrt{-e^2x^2 + d^2}d^{11}x}{1024e^4} \\ - \frac{45(-e^2x^2 + d^2)^{\frac{7}{2}}d^2x^4}{143e} + \frac{9(-e^2x^2 + d^2)^{\frac{3}{2}}d^9x}{512e^4} - \frac{9(-e^2x^2 + d^2)^{\frac{7}{2}}d^3x^3}{40e^2} + \frac{9(-e^2x^2 + d^2)^{\frac{5}{2}}d^7x}{640e^4} \\ - \frac{20(-e^2x^2 + d^2)^{\frac{7}{2}}d^4x^2}{143e^3} - \frac{27(-e^2x^2 + d^2)^{\frac{7}{2}}d^5x}{320e^4} - \frac{40(-e^2x^2 + d^2)^{\frac{7}{2}}d^6}{1001e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3\*x^4, x, algorithm="maxima")

[Out] -1/13\*(-e^2\*x^2 + d^2)^(7/2)\*e\*x^6 - 1/4\*(-e^2\*x^2 + d^2)^(7/2)\*d\*x^5 + 27/1024\*d^13\*arcsin(e^2\*x/sqrt(d^2\*e^2))/(sqrt(e^2)\*e^4) + 27/1024\*sqrt(-e^2\*x^2 + d^2)\*d^11\*x/e^4 - 45/143\*(-e^2\*x^2 + d^2)^(7/2)\*d^2\*x^4/e + 9/512\*(-e^2\*x^2 + d^2)^(3/2)\*d^9\*x/e^4 - 9/40

$$\begin{aligned} & *(-e^2*x^2 + d^2)^{(7/2)}*d^3*x^3/e^2 + 9/640*(-e^2*x^2 + d^2)^{(5/2)} \\ & )*d^7*x/e^4 - 20/143*(-e^2*x^2 + d^2)^{(7/2)}*d^4*x^2/e^3 - 27/320* \\ & (-e^2*x^2 + d^2)^{(7/2)}*d^5*x/e^4 - 40/1001*(-e^2*x^2 + d^2)^{(7/2)} \\ & *d^6/e^5 \end{aligned}$$

**Fricas [A]** time = 0.287067, size = 1162, normalized size = 4.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3\*x^4,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & 1/5125120*(394240*e^{26}*x^{26} + 1281280*d*e^{25}*x^{25} - 33080320*d^2* \\ & e^{24}*x^{24} - 111599488*d^3*e^{23}*x^{23} + 435169280*d^4*e^{22}*x^{22} + 1 \\ & 772186416*d^5*e^{21}*x^{21} - 1790863360*d^6*e^{20}*x^{20} - 11631259640* \\ & d^7*e^{19}*x^{19} + 116413440*d^8*e^{18}*x^{18} + 40029571582*d^9*e^{17}*x^{17} \\ & + 20389923840*d^{10}*e^{16}*x^{16} - 78805707981*d^{11}*e^{15}*x^{15} - 69 \\ & 713346560*d^{12}*e^{14}*x^{14} + 88498661251*d^{13}*e^{13}*x^{13} + 114556682 \\ & 240*d^{14}*e^{12}*x^{12} - 48663827212*d^{15}*e^{11}*x^{11} - 104634449920*d^{16} \\ & e^{10}*x^{10} + 1210969760*d^{17}*e^9*x^9 + 51169198080*d^{18}*e^8*x^8 \\ & + 11061033984*d^{19}*e^7*x^7 - 10496245760*d^{20}*e^6*x^6 - 22081579 \\ & 52*d^{21}*e^5*x^5 - 1706664960*d^{23}*e^3*x^3 + 553512960*d^{25}*e*x - \\ & 270270*(13*d^{14}*e^{12}*x^{12} - 364*d^{16}*e^{10}*x^{10} + 2912*d^{18}*e^8*x^8 \\ & - 9984*d^{20}*e^6*x^6 + 16640*d^{22}*e^4*x^4 - 13312*d^{24}*e^2*x^2 + \\ & 4096*d^{26} - (d^{13}*e^{12}*x^{12} - 84*d^{15}*e^{10}*x^{10} + 1120*d^{17}*e^8* \\ & x^8 - 5376*d^{19}*e^6*x^6 + 11520*d^{21}*e^4*x^4 - 11264*d^{23}*e^2*x^2 \\ & + 4096*d^{25})*sqrt(-e^2*x^2 + d^2))*arctan(-(d - sqrt(-e^2*x^2 + \\ & d^2))/(e*x)) + 13*(394240*d*e^{24}*x^{24} + 1281280*d^2*e^{23}*x^{23} - 1 \\ & 0608640*d^3*e^{22}*x^{22} - 38566528*d^4*e^{21}*x^{21} + 73328640*d^5*e^2 \\ & 0*x^{20} + 363162800*d^6*e^{19}*x^{19} - 121651200*d^7*e^{18}*x^{18} - 1608 \\ & 655048*d^8*e^{17}*x^{17} - 554019840*d^9*e^{16}*x^{16} + 3862680822*d^{10}* \\ & e^{15}*x^{15} + 2965585920*d^{11}*e^{14}*x^{14} - 5167577415*d^{12}*e^{13}*x^{13} \\ & - 6011371520*d^{13}*e^{12}*x^{12} + 3456227236*d^{14}*e^{11}*x^{11} + 638353 \\ & 4080*d^{15}*e^{10}*x^{10} - 425495840*d^{16}*e^9*x^9 - 3532390400*d^{17}*e^8 \\ & *x^8 - 729994496*d^{18}*e^7*x^7 + 807403520*d^{19}*e^6*x^6 + 2195325 \\ & 44*d^{20}*e^5*x^5 + 109992960*d^{22}*e^3*x^3 - 42577920*d^{24}*e*x)*sqrt \\ & (-e^2*x^2 + d^2))/(13*d*e^{17}*x^{12} - 364*d^3*e^{15}*x^{10} + 2912*d^5 \\ & *e^{13}*x^8 - 9984*d^7*e^{11}*x^6 + 16640*d^9*e^9*x^4 - 13312*d^{11}*e^7 \\ & *x^2 + 4096*d^{13}*e^5 - (e^{17}*x^{12} - 84*d^2*e^{15}*x^{10} + 1120*d^4* \\ & e^{13}*x^8 - 5376*d^6*e^{11}*x^6 + 11520*d^8*e^9*x^4 - 11264*d^{10}*e^7 \\ & *x^2 + 4096*d^{12}*e^5)*sqrt(-e^2*x^2 + d^2)) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2),x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.287962, size = 216, normalized size = 0.77

$$\begin{aligned} & \frac{27}{1024}d^{13}\arcsin\left(\frac{xe}{d}\right)e^{(-5)}\text{sign}(d) \\ & - \frac{1}{5125120}\left(204800d^{12}e^{(-5)} + \left(135135d^{11}e^{(-4)} + 2\left(51200d^{10}e^{(-3)} + \left(45045d^9e^{(-2)} + 4\left(9600d^8e^{(-1)} - (119119d^7 + 2(1561 \right. \right. \right. \right. \right. \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3*x^4,x, algorithm="giac")
```

```
[Out] 27/1024*d^13*arcsin(x*e/d)*e^(-5)*sign(d) - 1/5125120*(204800*d^12*e^(-5) + (135135*d^11*e^(-4) + 2*(51200*d^10*e^(-3) + (45045*d^9*e^(-2) + 4*(9600*d^8*e^(-1) - (119119*d^7 + 2*(156160*d^6*e + 7*(7293*d^5*e^2 - 8*(3280*d^4*e^3 + (3003*d^3*e^4 - 10*(48*d^2*e^5 + 11*(4*x*e^7 + 13*d*e^6)*x)*x)*x)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)
```

### 3.67 $\int x^3(d+ex)^3(d^2-e^2x^2)^{5/2} dx$

**Optimal.** Leaf size=252

$$\begin{aligned}
 & -\frac{1}{12}ex^5(d^2-e^2x^2)^{7/2} - \frac{3}{11}dx^4(d^2-e^2x^2)^{7/2} - \frac{41d^2x^3(d^2-e^2x^2)^{7/2}}{120e} \\
 & + \frac{41d^{12}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{1024e^4} + \frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3} + \frac{41d^8x(d^2-e^2x^2)^{3/2}}{1536e^3} \\
 & + \frac{41d^6x(d^2-e^2x^2)^{5/2}}{1920e^3} - \frac{d^4(14720d+28413ex)(d^2-e^2x^2)^{7/2}}{221760e^4} - \frac{23d^3x^2(d^2-e^2x^2)^{7/2}}{99e^2}
 \end{aligned}$$

[Out]  $(41*d^{10}*x*\text{Sqrt}[d^2 - e^2*x^2])/(1024*e^3) + (41*d^8*x*(d^2 - e^2*x^2)^{(3/2)})/(1536*e^3) + (41*d^6*x*(d^2 - e^2*x^2)^{(5/2)})/(1920*e^3) - (23*d^3*x^2*(d^2 - e^2*x^2)^{(7/2)})/(99*e^2) - (41*d^2*x^3*(d^2 - e^2*x^2)^{(7/2)})/(120*e) - (3*d*x^4*(d^2 - e^2*x^2)^{(7/2)})/11 - (e*x^5*(d^2 - e^2*x^2)^{(7/2)})/12 - (d^4*(14720*d + 28413*e*x)*(d^2 - e^2*x^2)^{(7/2)})/(221760*e^4) + (41*d^{12}*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(1024*e^4)$

**Rubi [A]** time = 0.670631, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\begin{aligned}
 & -\frac{1}{12}ex^5(d^2-e^2x^2)^{7/2} - \frac{3}{11}dx^4(d^2-e^2x^2)^{7/2} - \frac{41d^2x^3(d^2-e^2x^2)^{7/2}}{120e} \\
 & + \frac{41d^{12}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{1024e^4} + \frac{41d^{10}x\sqrt{d^2-e^2x^2}}{1024e^3} + \frac{41d^8x(d^2-e^2x^2)^{3/2}}{1536e^3} \\
 & + \frac{41d^6x(d^2-e^2x^2)^{5/2}}{1920e^3} - \frac{d^4(14720d+28413ex)(d^2-e^2x^2)^{7/2}}{221760e^4} - \frac{23d^3x^2(d^2-e^2x^2)^{7/2}}{99e^2}
 \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3*(d+e*x)^3*(d^2-e^2*x^2)^{(5/2)},x]$

[Out]  $(41*d^{10}*x*\text{Sqrt}[d^2 - e^2*x^2])/(1024*e^3) + (41*d^8*x*(d^2 - e^2*x^2)^{(3/2)})/(1536*e^3) + (41*d^6*x*(d^2 - e^2*x^2)^{(5/2)})/(1920*e^3) - (23*d^3*x^2*(d^2 - e^2*x^2)^{(7/2)})/(99*e^2) - (41*d^2*x^3*(d^2 - e^2*x^2)^{(7/2)})/(120*e) - (3*d*x^4*(d^2 - e^2*x^2)^{(7/2)})/11 - (e*x^5*(d^2 - e^2*x^2)^{(7/2)})/12 - (d^4*(14720*d + 28413*e*x)*(d^2 - e^2*x^2)^{(7/2)})/(221760*e^4) + (41*d^{12}*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(1024*e^4)$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**3*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)$

[Out] Timed out

**Mathematica [A]** time = 0.223069, size = 190, normalized size = 0.75

$$\frac{41d^{12} \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{1024e^4} + \sqrt{d^2 - e^2x^2} \left( -\frac{46d^{11}}{693e^4} - \frac{41d^{10}x}{1024e^3} - \frac{23d^9x^2}{693e^2} - \frac{41d^8x^3}{1536e} + \frac{52d^7x^4}{231} \right. \\ \left. + \frac{1111d^6ex^5}{1920} + \frac{130}{693}d^5e^2x^6 - \frac{207}{320}d^4e^3x^7 - \frac{58}{99}d^3e^4x^8 + \frac{11}{120}d^2e^5x^9 + \frac{3}{11}de^6x^{10} + \frac{e^7x^{11}}{12} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2), x]

[Out] Sqrt[d^2 - e^2\*x^2]\*((-46\*d^11)/(693\*e^4) - (41\*d^10\*x)/(1024\*e^3) - (23\*d^9\*x^2)/(693\*e^2) - (41\*d^8\*x^3)/(1536\*e) + (52\*d^7\*x^4)/231 + (1111\*d^6\*e\*x^5)/1920 + (130\*d^5\*e^2\*x^6)/693 - (207\*d^4\*e^3\*x^7)/320 - (58\*d^3\*e^4\*x^8)/99 + (11\*d^2\*e^5\*x^9)/120 + (3\*d\*e^6\*x^10)/11 + (e^7\*x^11)/12) + (41\*d^12\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(1024\*e^4)

**Maple [A]** time = 0.021, size = 241, normalized size = 1.

$$-\frac{23d^3x^2}{99e^2}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{46d^5}{693e^4}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{ex^5}{12}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{41d^2x^3}{120e}(-e^2x^2 + d^2)^{\frac{7}{2}} \\ - \frac{41d^4x}{320e^3}(-e^2x^2 + d^2)^{\frac{7}{2}} + \frac{41d^6x}{1920e^3}(-e^2x^2 + d^2)^{\frac{5}{2}} + \frac{41d^8x}{1536e^3}(-e^2x^2 + d^2)^{\frac{3}{2}} \\ + \frac{41d^{10}x}{1024e^3}\sqrt{-e^2x^2 + d^2} + \frac{41d^{12}}{1024e^3}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2 + d^2}}\right)\frac{1}{\sqrt{e^2}} - \frac{3dx^4}{11}(-e^2x^2 + d^2)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(e\*x+d)^3\*(-e^2\*x^2+d^2)^(5/2), x)

[Out] -23/99\*d^3\*x^2\*(-e^2\*x^2+d^2)^(7/2)/e^2-46/693\*d^5/e^4\*(-e^2\*x^2+d^2)^(7/2)-1/12\*e\*x^5\*(-e^2\*x^2+d^2)^(7/2)-41/120\*d^2\*x^3\*(-e^2\*x^2+d^2)^(7/2)/e-41/320/e^3\*d^4\*x\*(-e^2\*x^2+d^2)^(7/2)+41/1920\*d^6\*x\*(-e^2\*x^2+d^2)^(5/2)/e^3+41/1536\*d^8\*x\*(-e^2\*x^2+d^2)^(3/2)/e^3+41/1024\*d^10\*x\*(-e^2\*x^2+d^2)^(1/2)/e^3+41/1024/e^3\*d^12/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))-3/11\*d\*x^4\*(-e^2\*x^2+d^2)^(7/2)

**Maxima [A]** time = 0.799463, size = 315, normalized size = 1.25

$$-\frac{1}{12}(-e^2x^2 + d^2)^{\frac{7}{2}}ex^5 + \frac{41d^{12} \arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{1024\sqrt{e^2}e^3} + \frac{41\sqrt{-e^2x^2 + d^2}d^{10}x}{1024e^3} - \frac{3}{11}(-e^2x^2 + d^2)^{\frac{7}{2}}dx^4 \\ + \frac{41(-e^2x^2 + d^2)^{\frac{3}{2}}d^8x}{1536e^3} - \frac{41(-e^2x^2 + d^2)^{\frac{7}{2}}d^2x^3}{120e} + \frac{41(-e^2x^2 + d^2)^{\frac{5}{2}}d^6x}{1920e^3} \\ - \frac{23(-e^2x^2 + d^2)^{\frac{7}{2}}d^3x^2}{99e^2} - \frac{41(-e^2x^2 + d^2)^{\frac{7}{2}}d^4x}{320e^3} - \frac{46(-e^2x^2 + d^2)^{\frac{7}{2}}d^5}{693e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3\*x^3, x, algorithm="maxima")

[Out] -1/12\*(-e^2\*x^2 + d^2)^(7/2)\*e\*x^5 + 41/1024\*d^12\*arcsin(e^2\*x/sqrt(d^2\*e^2))/(sqrt(e^2)\*e^3) + 41/1024\*sqrt(-e^2\*x^2 + d^2)\*d^10\*x/e^3 - 3/11\*(-e^2\*x^2 + d^2)^(7/2)\*d\*x^4 + 41/1536\*(-e^2\*x^2 + d^2)^(3/2)\*d^8\*x/e^3 - 41/120\*(-e^2\*x^2 + d^2)^(7/2)\*d^2\*x^3/e + 4

$$\frac{1}{1920}(-e^2x^2 + d^2)^{5/2}d^6x/e^3 - \frac{23}{99}(-e^2x^2 + d^2)^{7/2}d^3x^2/e^2 - \frac{41}{320}(-e^2x^2 + d^2)^{7/2}d^4x/e^3 - \frac{46}{693}(-e^2x^2 + d^2)^{7/2}d^5/e^4$$

**Fricas [A]** time = 0.291455, size = 1103, normalized size = 4.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3\*x^3,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/3548160*(3548160*d^e^{23}x^{23} + 11612160*d^2e^{22}x^{22} - 824355 \\ & 84*d^3e^{21}x^{21} - 307507200*d^4e^{20}x^{20} + 490133952*d^5e^{19}x \\ & ^{19} + 2620006400*d^6e^{18}x^{18} - 523597536*d^7e^{17}x^{17} - 106850 \\ & 30400*d^8e^{16}x^{16} - 4561549608*d^9e^{15}x^{15} + 23861882880*d^{10} \\ & *e^{14}x^{14} + 20013992460*d^{11}e^{13}x^{13} - 29731215360*d^{12}e^{12}x \\ & ^{12} - 37348461612*d^{13}e^{11}x^{11} + 18053038080*d^{14}e^{10}x^{10} + 3 \\ & 7402830696*d^{15}e^9x^9 - 794787840*d^{16}e^8x^8 - 19429546752*d^ \\ & ^{17}e^7x^7 - 4844421120*d^{18}e^6x^6 + 3501679104*d^{19}e^5x^5 + \\ & 1816657920*d^{20}e^4x^4 + 824355840*d^{21}e^3x^3 - 290949120*d^{23} \\ & *e*x + 284130*(d^{12}e^{12}x^{12} - 72*d^{14}e^{10}x^{10} + 840*d^{16}e^8x^ \\ & ^8 - 3584*d^{18}e^6x^6 + 6912*d^{20}e^4x^4 - 6144*d^{22}e^2x^2 + \\ & 2048*d^{24} + 4*(3*d^{13}e^{10}x^{10} - 70*d^{15}e^8x^8 + 448*d^{17}e^6 \\ & *x^6 - 1152*d^{19}e^4x^4 + 1280*d^{21}e^2x^2 - 512*d^{23})*\text{sqrt}(-e^ \\ & ^2x^2 + d^2))*\text{arctan}(-(d - \text{sqrt}(-e^2x^2 + d^2))/(e*x)) - (295680 \\ & *e^{23}x^{23} + 967680*d^e^{22}x^{22} - 20963712*d^2e^{21}x^{21} - 717516 \\ & 80*d^3e^{20}x^{20} + 222658128*d^4e^{19}x^{19} + 963184640*d^5e^{18}x \\ & ^{18} - 619200120*d^6e^{17}x^{17} - 5261414400*d^7e^{16}x^{16} - 119785 \\ & 0038*d^8e^{15}x^{15} + 14640215040*d^9e^{14}x^{14} + 10388814975*d^{10} \\ & *e^{13}x^{13} - 22019880960*d^{11}e^{12}x^{12} - 24685042536*d^{12}e^{11}x \\ & ^{11} + 16406691840*d^{13}e^{10}x^{10} + 29179241784*d^{14}e^9x^9 - 253 \\ & 5751680*d^{15}e^8x^8 - 17460495360*d^{16}e^7x^7 - 3936092160*d^{17} \\ & *e^6x^6 + 3804751104*d^{18}e^5x^5 + 1816657920*d^{19}e^4x^4 + 67 \\ & 8881280*d^{20}e^3x^3 - 290949120*d^{22}e*x)*\text{sqrt}(-e^2x^2 + d^2))/ \\ & (e^{16}x^{12} - 72*d^2e^{14}x^{10} + 840*d^4e^{12}x^8 - 3584*d^6e^{10}x^ \\ & ^6 + 6912*d^8e^8x^4 - 6144*d^{10}e^6x^2 + 2048*d^{12}e^4 + 4*(3 \\ & *d^e^{14}x^{10} - 70*d^3e^{12}x^8 + 448*d^5e^{10}x^6 - 1152*d^7e^8x^ \\ & ^4 + 1280*d^9e^6x^2 - 512*d^{11}e^4)*\text{sqrt}(-e^2x^2 + d^2)) \end{aligned}$$

**Sympy [A]** time = 169.554, size = 1919, normalized size = 7.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2),x)

[Out] 
$$\begin{aligned} & d^{**7}\text{Piecewise}((-2*d^{**4}\text{sqrt}(d^{**2} - e^{**2}x^{**2})/(15*e^{**4}) - d^{**2}x \\ & **2*\text{sqrt}(d^{**2} - e^{**2}x^{**2})/(15*e^{**2}) + x^{**4}*\text{sqrt}(d^{**2} - e^{**2}x^{**2} \\ & )/5, \text{Ne}(e, 0)), (x^{**4}*\text{sqrt}(d^{**2})/4, \text{True})) + 3*d^{**6}*e*\text{Piecewise}(( \\ & -I*d^{**6}*\text{acosh}(e*x/d)/(16*e^{**5}) + I*d^{**5}x/(16*e^{**4}*\text{sqrt}(-1 + e^{**2} \\ & *x^{**2}/d^{**2})) - I*d^{**3}x^{**3}/(48*e^{**2}*\text{sqrt}(-1 + e^{**2}x^{**2}/d^{**2})) - \\ & 5*I*d*x^{**5}/(24*\text{sqrt}(-1 + e^{**2}x^{**2}/d^{**2})) + I*e^{**2}x^{**7}/(6*d*\text{sqrt} \\ & (-1 + e^{**2}x^{**2}/d^{**2})), \text{Abs}(e^{**2}x^{**2}/d^{**2}) > 1), (d^{**6}*\text{asin}(e*x/ \\ & d)/(16*e^{**5}) - d^{**5}x/(16*e^{**4}*\text{sqrt}(1 - e^{**2}x^{**2}/d^{**2})) + d^{**3}x \\ & **3/(48*e^{**2}*\text{sqrt}(1 - e^{**2}x^{**2}/d^{**2})) + 5*d*x^{**5}/(24*\text{sqrt}(1 - e^ \\ & *2*x^{**2}/d^{**2})) - e^{**2}x^{**7}/(6*d*\text{sqrt}(1 - e^{**2}x^{**2}/d^{**2})), \text{True})) \\ & + d^{**5}*e^{**2}*\text{Piecewise}((-8*d^{**6}*\text{sqrt}(d^{**2} - e^{**2}x^{**2})/(105*e^{**6}) \\ & - 4*d^{**4}x^{**2}*\text{sqrt}(d^{**2} - e^{**2}x^{**2})/(105*e^{**4}) - d^{**2}x^{**4}*\text{sqrt} \\ & (d^{**2} - e^{**2}x^{**2})/(35*e^{**2}) + x^{**6}*\text{sqrt}(d^{**2} - e^{**2}x^{**2})/7, \text{Ne}( \\ & e, 0)), (x^{**6}*\text{sqrt}(d^{**2})/6, \text{True})) - 5*d^{**4}*e^{**3}*\text{Piecewise}((-5*I \\ & d^{**8}*\text{acosh}(e*x/d)/(128*e^{**7}) + 5*I*d^{**7}x/(128*e^{**6}*\text{sqrt}(-1 + e^{**2} \\ & *x^{**2}/d^{**2})), \text{Abs}(e^{**2}x^{**2}/d^{**2}) > 1), (d^{**6}*\text{asin}(e*x/d)/(128*e^{**7}) \\ & - 5*d^{**5}x/(128*e^{**6}*\text{sqrt}(1 - e^{**2}x^{**2}/d^{**2})) + d^{**3}x^{**3}/(128*e^{**4} \\ & *\text{sqrt}(1 - e^{**2}x^{**2}/d^{**2})) + 5*d*x^{**5}/(128*e^{**3}*\text{sqrt}(1 - e^{**2}x^{**2}/d^{**2})), \\ & \text{True})) \end{aligned}$$

```

2*x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/d**2))
) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7
/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2
*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (5*d**8*asin(e*x/d)/(128*
e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**
3/(384*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(
1 - e**2*x**2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e
**2*x**9/(8*d*sqrt(1 - e**2*x**2/d**2)), True)) - 5*d**3*e**4*Pie
cewise((-16*d**8*sqrt(d**2 - e**2*x**2)/(315*e**8) - 8*d**6*x**2*
sqrt(d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 - e**2*
x**2)/(105*e**4) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x
**8*sqrt(d**2 - e**2*x**2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True
)) + d**2*e**5*Piecewise((-7*I*d**10*acosh(e*x/d)/(256*e**9) + 7*
I*d**9*x/(256*e**8*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**7*x**3/(76
8*e**6*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**5*x**5/(1920*e**4*sqrt
(-1 + e**2*x**2/d**2)) - I*d**3*x**7/(480*e**2*sqrt(-1 + e**2*x**
2/d**2)) - 9*I*d*x**9/(80*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**
11/(10*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (7
*d**10*asin(e*x/d)/(256*e**9) - 7*d**9*x/(256*e**8*sqrt(1 - e**2*
x**2/d**2)) + 7*d**7*x**3/(768*e**6*sqrt(1 - e**2*x**2/d**2)) + 7
*d**5*x**5/(1920*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**7/(480*
e**2*sqrt(1 - e**2*x**2/d**2)) + 9*d*x**9/(80*sqrt(1 - e**2*x**2/
d**2)) - e**2*x**11/(10*d*sqrt(1 - e**2*x**2/d**2)), True)) + 3*d
**6*Piecewise((-128*d**10*sqrt(d**2 - e**2*x**2)/(3465*e**10) -
64*d**8*x**2*sqrt(d**2 - e**2*x**2)/(3465*e**8) - 16*d**6*x**4*s
qrt(d**2 - e**2*x**2)/(1155*e**6) - 8*d**4*x**6*sqrt(d**2 - e**2*
x**2)/(693*e**4) - d**2*x**8*sqrt(d**2 - e**2*x**2)/(99*e**2) + x
**10*sqrt(d**2 - e**2*x**2)/11, Ne(e, 0)), (x**10*sqrt(d**2)/10,
True)) + e**7*Piecewise((-21*I*d**12*acosh(e*x/d)/(1024*e**11) +
21*I*d**11*x/(1024*e**10*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**9*x**
3/(1024*e**8*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**7*x**5/(2560*e
**6*sqrt(-1 + e**2*x**2/d**2)) - I*d**5*x**7/(640*e**4*sqrt(-1 + e
**2*x**2/d**2)) - I*d**3*x**9/(960*e**2*sqrt(-1 + e**2*x**2/d**2)
) - 11*I*d*x**11/(120*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**13/(
12*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (21*d
**12*asin(e*x/d)/(1024*e**11) - 21*d**11*x/(1024*e**10*sqrt(1 - e
**2*x**2/d**2)) + 7*d**9*x**3/(1024*e**8*sqrt(1 - e**2*x**2/d**2))
+ 7*d**7*x**5/(2560*e**6*sqrt(1 - e**2*x**2/d**2)) + d**5*x**7/(
640*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**9/(960*e**2*sqrt(1 -
e**2*x**2/d**2)) + 11*d*x**11/(120*sqrt(1 - e**2*x**2/d**2)) - e
**2*x**13/(12*d*sqrt(1 - e**2*x**2/d**2)), True))

```

**GIAC/XCAS [A]** time = 0.286089, size = 201, normalized size = 0.8

$$\frac{41}{1024} d^{12} \arcsin\left(\frac{xe}{d}\right) e^{(-4)} \text{sign}(d) - \frac{1}{3548160} \left( 235520 d^{11} e^{(-4)} + \left( 142065 d^{10} e^{(-3)} + 2 \left( 58880 d^9 e^{(-2)} + \left( 47355 d^8 e^{(-1)} - 4 \left( 99840 d^7 + (256641 d^6 e + 2 (41600 d^5 e^2 - 7* (20493 d^4 e^3 + 8* (2320 d^3 e^4 - 3* (121 d^2 e^5 + 10* (11 x e^7 + 36 d e^6) x) x) x) x) x) x) x) x) x) x) \right) \sqrt{-x^2 e^2 + d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3\*x^3,x, algorithm="giac")

[Out] 41/1024\*d^12\*arcsin(x\*e/d)\*e^(-4)\*sign(d) - 1/3548160\*(235520\*d^11\*e^(-4) + (142065\*d^10\*e^(-3) + 2\*(58880\*d^9\*e^(-2) + (47355\*d^8\*e^(-1) - 4\*(99840\*d^7 + (256641\*d^6\*e + 2\*(41600\*d^5\*e^2 - 7\*(20493\*d^4\*e^3 + 8\*(2320\*d^3\*e^4 - 3\*(121\*d^2\*e^5 + 10\*(11\*x\*e^7 + 36\*d\*e^6)\*x)\*x)\*x)\*x)\*x)\*x)\*x)\*x)\*sqrt(-x^2\*e^2 + d^2)

### 3.68 $\int x^2(d+ex)^3(d^2-e^2x^2)^{5/2} dx$

**Optimal.** Leaf size=223

$$\begin{aligned} & -\frac{37d^2x^2(d^2-e^2x^2)^{7/2}}{99e} \\ & -\frac{1}{11}ex^4(d^2-e^2x^2)^{7/2} - \frac{3}{10}dx^3(d^2-e^2x^2)^{7/2} + \frac{19d^{11}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{256e^3} + \frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2} \\ & + \frac{19d^7x(d^2-e^2x^2)^{3/2}}{384e^2} + \frac{19d^5x(d^2-e^2x^2)^{5/2}}{480e^2} - \frac{d^3(5920d+13167ex)(d^2-e^2x^2)^{7/2}}{55440e^3} \end{aligned}$$

[Out]  $(19*d^9*x*\text{Sqrt}[d^2 - e^2*x^2])/(256*e^2) + (19*d^7*x*(d^2 - e^2*x^2)^{(3/2)})/(384*e^2) + (19*d^5*x*(d^2 - e^2*x^2)^{(5/2)})/(480*e^2) - (37*d^2*x^2*(d^2 - e^2*x^2)^{(7/2)})/(99*e) - (3*d*x^3*(d^2 - e^2*x^2)^{(7/2)})/10 - (e*x^4*(d^2 - e^2*x^2)^{(7/2)})/11 - (d^3*(5920*d + 13167*e*x)*(d^2 - e^2*x^2)^{(7/2)})/(55440*e^3) + (19*d^{11}*\text{ArcT an}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(256*e^3)$

**Rubi [A]** time = 0.53622, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\begin{aligned} & -\frac{37d^2x^2(d^2-e^2x^2)^{7/2}}{99e} \\ & -\frac{1}{11}ex^4(d^2-e^2x^2)^{7/2} - \frac{3}{10}dx^3(d^2-e^2x^2)^{7/2} + \frac{19d^{11}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{256e^3} + \frac{19d^9x\sqrt{d^2-e^2x^2}}{256e^2} \\ & + \frac{19d^7x(d^2-e^2x^2)^{3/2}}{384e^2} + \frac{19d^5x(d^2-e^2x^2)^{5/2}}{480e^2} - \frac{d^3(5920d+13167ex)(d^2-e^2x^2)^{7/2}}{55440e^3} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*(d+e*x)^3*(d^2-e^2*x^2)^{(5/2)},x]$

[Out]  $(19*d^9*x*\text{Sqrt}[d^2 - e^2*x^2])/(256*e^2) + (19*d^7*x*(d^2 - e^2*x^2)^{(3/2)})/(384*e^2) + (19*d^5*x*(d^2 - e^2*x^2)^{(5/2)})/(480*e^2) - (37*d^2*x^2*(d^2 - e^2*x^2)^{(7/2)})/(99*e) - (3*d*x^3*(d^2 - e^2*x^2)^{(7/2)})/10 - (e*x^4*(d^2 - e^2*x^2)^{(7/2)})/11 - (d^3*(5920*d + 13167*e*x)*(d^2 - e^2*x^2)^{(7/2)})/(55440*e^3) + (19*d^{11}*\text{ArcT an}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(256*e^3)$

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^2*(e*x+d)^3*(-e^2*x^2+d^2)^{(5/2)},x)$

[Out] Timed out

**Mathematica [A]** time = 0.197481, size = 177, normalized size = 0.79

$$\begin{aligned} & \frac{19d^{11}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{256e^3} + \sqrt{d^2-e^2x^2}\left(-\frac{74d^{10}}{693e^3} - \frac{19d^9x}{256e^2} - \frac{37d^8x^2}{693e} + \frac{109d^7x^3}{384}\right. \\ & \left. + \frac{164}{231}d^6ex^4 + \frac{109}{480}d^5e^2x^5 - \frac{514}{693}d^4e^3x^6 - \frac{53}{80}d^3e^4x^7 + \frac{10}{99}d^2e^5x^8 + \frac{3}{10}de^6x^9 + \frac{e^7x^{10}}{11}\right) \end{aligned}$$



Antiderivative was successfully verified.

[In] Integrate[x^2\*(d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2), x]

[Out] Sqrt[d^2 - e^2\*x^2]\*((-74\*d^10)/(693\*e^3) - (19\*d^9\*x)/(256\*e^2) - (37\*d^8\*x^2)/(693\*e) + (109\*d^7\*x^3)/384 + (164\*d^6\*e\*x^4)/231 + (109\*d^5\*e^2\*x^5)/480 - (514\*d^4\*e^3\*x^6)/693 - (53\*d^3\*e^4\*x^7)/80 + (10\*d^2\*e^5\*x^8)/99 + (3\*d\*e^6\*x^9)/10 + (e^7\*x^10)/11) + (19\*d^11\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(256\*e^3)

**Maple [A]** time = 0.017, size = 216, normalized size = 1.

$$\begin{aligned} & -\frac{19d^3x}{80e^2}(-e^2x^2 + d^2)^{\frac{7}{2}} + \frac{19d^5x}{480e^2}(-e^2x^2 + d^2)^{\frac{5}{2}} + \frac{19d^7x}{384e^2}(-e^2x^2 + d^2)^{\frac{3}{2}} \\ & + \frac{19d^9x}{256e^2}\sqrt{-e^2x^2 + d^2} + \frac{19d^{11}}{256e^2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2 + d^2}}\right)\frac{1}{\sqrt{e^2}} - \frac{ex^4}{11}(-e^2x^2 + d^2)^{\frac{7}{2}} \\ & - \frac{37d^2x^2}{99e}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{74d^4}{693e^3}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{3dx^3}{10}(-e^2x^2 + d^2)^{\frac{7}{2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(e\*x+d)^3\*(-e^2\*x^2+d^2)^(5/2), x)

[Out] -19/80\*d^3\*x\*(-e^2\*x^2+d^2)^(7/2)/e^2+19/480\*d^5\*x\*(-e^2\*x^2+d^2)^(5/2)/e^2+19/384\*d^7\*x\*(-e^2\*x^2+d^2)^(3/2)/e^2+19/256\*d^9\*x\*(-e^2\*x^2+d^2)^(1/2)/e^2+19/256\*d^11/e^2/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))-1/11\*e\*x^4\*(-e^2\*x^2+d^2)^(7/2)-37/99\*d^2\*x^2\*(-e^2\*x^2+d^2)^(7/2)/e-74/693/e^3\*d^4\*(-e^2\*x^2+d^2)^(7/2)-3/10\*d\*x^3\*(-e^2\*x^2+d^2)^(7/2)

**Maxima [A]** time = 0.798492, size = 281, normalized size = 1.26

$$\begin{aligned} & \frac{19d^{11}\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{256\sqrt{e^2}e^2} + \frac{19\sqrt{-e^2x^2 + d^2}d^9x}{256e^2} - \frac{1}{11}(-e^2x^2 + d^2)^{\frac{7}{2}}ex^4 \\ & + \frac{19(-e^2x^2 + d^2)^{\frac{3}{2}}d^7x}{384e^2} - \frac{3}{10}(-e^2x^2 + d^2)^{\frac{7}{2}}dx^3 + \frac{19(-e^2x^2 + d^2)^{\frac{5}{2}}d^5x}{480e^2} \\ & - \frac{37(-e^2x^2 + d^2)^{\frac{7}{2}}d^2x^2}{99e} - \frac{19(-e^2x^2 + d^2)^{\frac{7}{2}}d^3x}{80e^2} - \frac{74(-e^2x^2 + d^2)^{\frac{7}{2}}d^4}{693e^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3\*x^2, x, algorithm="maxima")

[Out] 19/256\*d^11\*arcsin(e^2\*x/sqrt(d^2\*e^2))/(sqrt(e^2)\*e^2) + 19/256\*sqrt(-e^2\*x^2 + d^2)\*d^9\*x/e^2 - 1/11\*(-e^2\*x^2 + d^2)^(7/2)\*e\*x^4 + 19/384\*(-e^2\*x^2 + d^2)^(3/2)\*d^7\*x/e^2 - 3/10\*(-e^2\*x^2 + d^2)^(7/2)\*d\*x^3 + 19/480\*(-e^2\*x^2 + d^2)^(5/2)\*d^5\*x/e^2 - 37/99\*(-e^2\*x^2 + d^2)^(7/2)\*d^2\*x^2/e - 19/80\*(-e^2\*x^2 + d^2)^(7/2)\*d^3\*x/e^2 - 74/693\*(-e^2\*x^2 + d^2)^(7/2)\*d^4/e^3

**Fricas [A]** time = 0.288256, size = 1014, normalized size = 4.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3*x^2,x, algorithm="fricas")
```

```
[Out] 1/887040*(80640*e^22*x^22 + 266112*d*e^21*x^21 - 4829440*d^2*e^20
*x^20 - 16820496*d^3*e^19*x^19 + 43873280*d^4*e^18*x^18 + 2010383
76*d^5*e^17*x^17 - 93350400*d^6*e^16*x^16 - 1002282666*d^7*e^15*x
^15 - 326810880*d^8*e^14*x^14 + 2581643295*d^9*e^13*x^13 + 203930
4960*d^10*e^12*x^12 - 3606334809*d^11*e^11*x^11 - 4416276480*d^12
*e^10*x^10 + 2420282172*d^13*e^9*x^9 + 4934307840*d^14*e^8*x^8 -
85957872*d^15*e^7*x^7 - 2857451520*d^16*e^6*x^6 - 901350912*d^17*
e^5*x^5 + 681246720*d^18*e^4*x^4 + 476931840*d^19*e^3*x^3 - 67415
040*d^21*e*x - 131670*(11*d^12*e^10*x^10 - 220*d^14*e^8*x^8 + 123
2*d^16*e^6*x^6 - 2816*d^18*e^4*x^4 + 2816*d^20*e^2*x^2 - 1024*d^2
2 - (d^11*e^10*x^10 - 60*d^13*e^8*x^8 + 560*d^15*e^6*x^6 - 1792*d
^17*e^4*x^4 + 2304*d^19*e^2*x^2 - 1024*d^21)*sqrt(-e^2*x^2 + d^2)
)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 11*(80640*d^20*x^
20 + 266112*d^2*e^19*x^19 - 1523200*d^3*e^18*x^18 - 5909904*d^4*e
^17*x^17 + 6581760*d^5*e^16*x^16 + 41759256*d^6*e^15*x^15 + 31795
20*d^7*e^14*x^14 - 137719890*d^8*e^13*x^13 - 88623360*d^9*e^12*x^
12 + 236025405*d^10*e^11*x^11 + 255252480*d^11*e^10*x^10 - 197264
004*d^12*e^9*x^9 - 341913600*d^13*e^8*x^8 + 34441008*d^14*e^7*x^7
+ 228802560*d^15*e^6*x^6 + 62560512*d^16*e^5*x^5 - 61931520*d^17
*e^4*x^4 - 40293120*d^18*e^3*x^3 + 6128640*d^20*e*x)*sqrt(-e^2*x^
2 + d^2))/(11*d^13*x^10 - 220*d^3*e^11*x^8 + 1232*d^5*e^9*x^6 -
2816*d^7*e^7*x^4 + 2816*d^9*e^5*x^2 - 1024*d^11*e^3 - (e^13*x^10
- 60*d^2*e^11*x^8 + 560*d^4*e^9*x^6 - 1792*d^6*e^7*x^4 + 2304*d^
8*e^5*x^2 - 1024*d^10*e^3)*sqrt(-e^2*x^2 + d^2))
```

---

**Sympy [A]** time = 115.11, size = 1681, normalized size = 7.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**7*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*
sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d*
**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2
/d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 -
e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*
x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) + 3*d**6*e*Piecewise(
(-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 -
e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)),
(x**4*sqrt(d**2)/4, True)) + d**5*e**2*Piecewise((-I*d**6*acosh(
e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) -
I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24
*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**
2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x/d)/(16*e**5) -
d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*e**2*s
qrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2))
- e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) - 5*d**4*e**3
*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**4*x*
**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d**2 - e**2
*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**
6*sqrt(d**2)/6, True)) - 5*d**3*e**4*Piecewise((-5*I*d**8*acosh(e
*x/d)/(128*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2)
) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x
**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1
+ e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2*x**2/d**2))
, Abs(e**2*x**2/d**2) > 1), (5*d**8*asin(e*x/d)/(128*e**7) - 5*d*
**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4*
sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x**
2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8*
d*sqrt(1 - e**2*x**2/d**2)), True)) + d**2*e**5*Piecewise((-16*d*
**8*sqrt(d**2 - e**2*x**2)/(315*e**8) - 8*d**6*x**2*sqrt(d**2 - e
**2*x**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 - e**2*x**2)/(105*e**
4) - d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*sqrt(d**2
```

```

- e**2*x**2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True)) + 3*d*e**6*
Piecewise((-7*I*d**10*acosh(e*x/d)/(256*e**9) + 7*I*d**9*x/(256*e
**8*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**7*x**3/(768*e**6*sqrt(-1
+ e**2*x**2/d**2)) - 7*I*d**5*x**5/(1920*e**4*sqrt(-1 + e**2*x**2
/d**2)) - I*d**3*x**7/(480*e**2*sqrt(-1 + e**2*x**2/d**2)) - 9*I*
d*x**9/(80*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**11/(10*d*sqrt(-
1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (7*d**10*asin(e*x
/d)/(256*e**9) - 7*d**9*x/(256*e**8*sqrt(1 - e**2*x**2/d**2)) + 7
*d**7*x**3/(768*e**6*sqrt(1 - e**2*x**2/d**2)) + 7*d**5*x**5/(192
0*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**7/(480*e**2*sqrt(1 - e
**2*x**2/d**2)) + 9*d*x**9/(80*sqrt(1 - e**2*x**2/d**2)) - e**2*x
**11/(10*d*sqrt(1 - e**2*x**2/d**2)), True)) + e**7*Piecewise((-1
28*d**10*sqrt(d**2 - e**2*x**2)/(3465*e**10) - 64*d**8*x**2*sqrt(
d**2 - e**2*x**2)/(3465*e**8) - 16*d**6*x**4*sqrt(d**2 - e**2*x**
2)/(1155*e**6) - 8*d**4*x**6*sqrt(d**2 - e**2*x**2)/(693*e**4) -
d**2*x**8*sqrt(d**2 - e**2*x**2)/(99*e**2) + x**10*sqrt(d**2 - e
**2*x**2)/11, Ne(e, 0)), (x**10*sqrt(d**2)/10, True))

```

**GIAC/XCAS [A]** time = 0.289068, size = 188, normalized size = 0.84

$$\frac{19}{256} d^{11} \arcsin\left(\frac{xe}{d}\right) e^{(-3)} \text{sign}(d) - \frac{1}{887040} \left( 94720 d^{10} e^{(-3)} + (65835 d^9 e^{(-2)} + 2(23680 d^8 e^{(-1)} - (125895 d^7 + 4(78720 d^6 e + (25179 d^5 e^2 - 2(41120 d^4 e^3 + 7(100 d^3 e^4 - 8(10 d^2 e^5 + 9(10 x e^7 + 33 d e^6) x) x) x) x) x) x) \sqrt{-x^2 e^2 + d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3*x^2,x, algorithm="giac")
```

```
[Out] 19/256*d^11*arcsin(x*e/d)*e^(-3)*sign(d) - 1/887040*(94720*d^10*e
^(-3) + (65835*d^9*e^(-2) + 2*(23680*d^8*e^(-1) - (125895*d^7 + 4
*(78720*d^6*e + (25179*d^5*e^2 - 2*(41120*d^4*e^3 + 7*(5247*d^3*e
^4 - 8*(100*d^2*e^5 + 9*(10*x*e^7 + 33*d*e^6)*x)*x)*x)*x)*x)*x)
*sqrt(-x^2*e^2 + d^2)
```

### 3.69 $\int x(d + ex)^3 (d^2 - e^2x^2)^{5/2} dx$

**Optimal.** Leaf size=230

$$\begin{aligned} & -\frac{11d^2(d+ex)(d^2-e^2x^2)^{7/2}}{240e^2} - \frac{d(d+ex)^2(d^2-e^2x^2)^{7/2}}{30e^2} \\ & - \frac{(d+ex)^3(d^2-e^2x^2)^{7/2}}{10e^2} + \frac{33d^{10}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{256e^2} + \frac{33d^8x\sqrt{d^2-e^2x^2}}{256e} \\ & + \frac{11d^6x(d^2-e^2x^2)^{3/2}}{128e} + \frac{11d^4x(d^2-e^2x^2)^{5/2}}{160e} - \frac{33d^3(d^2-e^2x^2)^{7/2}}{560e^2} \end{aligned}$$

[Out]  $(33*d^8*x*\text{Sqrt}[d^2 - e^2*x^2])/(256*e) + (11*d^6*x*(d^2 - e^2*x^2)^{(3/2)})/(128*e) + (11*d^4*x*(d^2 - e^2*x^2)^{(5/2)})/(160*e) - (33*d^3*(d^2 - e^2*x^2)^{(7/2)})/(560*e^2) - (11*d^2*(d + e*x)*(d^2 - e^2*x^2)^{(7/2)})/(240*e^2) - (d*(d + e*x)^2*(d^2 - e^2*x^2)^{(7/2)})/(30*e^2) - ((d + e*x)^3*(d^2 - e^2*x^2)^{(7/2)})/(10*e^2) + (33*d^8*x*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(256*e^2)$

**Rubi [A]** time = 0.296486, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\begin{aligned} & -\frac{11d^2(d+ex)(d^2-e^2x^2)^{7/2}}{240e^2} - \frac{d(d+ex)^2(d^2-e^2x^2)^{7/2}}{30e^2} \\ & - \frac{(d+ex)^3(d^2-e^2x^2)^{7/2}}{10e^2} + \frac{33d^{10}\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{256e^2} + \frac{33d^8x\sqrt{d^2-e^2x^2}}{256e} \\ & + \frac{11d^6x(d^2-e^2x^2)^{3/2}}{128e} + \frac{11d^4x(d^2-e^2x^2)^{5/2}}{160e} - \frac{33d^3(d^2-e^2x^2)^{7/2}}{560e^2} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*(d + e*x)^3*(d^2 - e^2*x^2)^{(5/2)}, x]$

[Out]  $(33*d^8*x*\text{Sqrt}[d^2 - e^2*x^2])/(256*e) + (11*d^6*x*(d^2 - e^2*x^2)^{(3/2)})/(128*e) + (11*d^4*x*(d^2 - e^2*x^2)^{(5/2)})/(160*e) - (33*d^3*(d^2 - e^2*x^2)^{(7/2)})/(560*e^2) - (11*d^2*(d + e*x)*(d^2 - e^2*x^2)^{(7/2)})/(240*e^2) - (d*(d + e*x)^2*(d^2 - e^2*x^2)^{(7/2)})/(30*e^2) - ((d + e*x)^3*(d^2 - e^2*x^2)^{(7/2)})/(10*e^2) + (33*d^8*x*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(256*e^2)$

**Rubi in Sympy [A]** time = 41.0922, size = 204, normalized size = 0.89

$$\begin{aligned} & \frac{33d^{10}\text{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{256e^2} + \frac{33d^8x\sqrt{d^2-e^2x^2}}{256e} + \frac{11d^6x(d^2-e^2x^2)^{3/2}}{128e} \\ & + \frac{11d^4x(d^2-e^2x^2)^{5/2}}{160e} - \frac{33d^3(d^2-e^2x^2)^{7/2}}{560e^2} - \frac{11d^2(d+ex)(d^2-e^2x^2)^{7/2}}{240e^2} \\ & - \frac{d(d+ex)^2(d^2-e^2x^2)^{7/2}}{30e^2} - \frac{(d+ex)^3(d^2-e^2x^2)^{7/2}}{10e^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2), x)$

[Out]  $33*d**10*\text{atan}(e*x/\text{sqrt}(d**2 - e**2*x**2))/(256*e**2) + 33*d**8*x*\text{sqrt}(d**2 - e**2*x**2)/(256*e) + 11*d**6*x*(d**2 - e**2*x**2)**(3/2)/(128*e) + 11*d**4*x*(d**2 - e**2*x**2)**(5/2)/(160*e) - 33*d**3*(d**2 - e**2*x**2)**(7/2)/(560*e**2) - 11*d**2*(d + e*x)*(d**2 - e**2*x**2)**(7/2)/(240*e**2) - (d*(d + e*x)**2*(d**2 - e**2*x**2)**(7/2))/(30*e**2) - ((d + e*x)**3*(d**2 - e**2*x**2)**(7/2))/(10*e**2) + (33*d**8*x*\text{ArcTan}[(e*x)/\text{sqrt}(d**2 - e**2*x**2)])/(256*e**2)$

$$*3*(d^{**2} - e^{**2}*x^{**2})^{**}(7/2)/(560*e^{**2}) - 11*d^{**2}*(d + e*x)*(d^{**2} - e^{**2}*x^{**2})^{**}(7/2)/(240*e^{**2}) - d*(d + e*x)^{**2}*(d^{**2} - e^{**2}*x^{**2})^{**}(7/2)/(30*e^{**2}) - (d + e*x)^{**3}*(d^{**2} - e^{**2}*x^{**2})^{**}(7/2)/(10*e^{**2})$$

**Mathematica [A]** time = 0.186953, size = 164, normalized size = 0.71

$$\frac{33d^{10} \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{256e^2} + \sqrt{d^2 - e^2x^2} \left( -\frac{5d^9}{21e^2} - \frac{33d^8x}{256e} + \frac{8d^7x^2}{21} + \frac{117}{128}d^6ex^3 \right. \\ \left. + \frac{2}{7}d^5e^2x^4 - \frac{139}{160}d^4e^3x^5 - \frac{16}{21}d^3e^4x^6 + \frac{9}{80}d^2e^5x^7 + \frac{1}{3}de^6x^8 + \frac{e^7x^9}{10} \right)$$

Antiderivative was successfully verified.

[In] Integrate[x\*(d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2), x]

[Out] Sqrt[d^2 - e^2\*x^2]\*((-5\*d^9)/(21\*e^2) - (33\*d^8\*x)/(256\*e) + (8\*d^7\*x^2)/21 + (117\*d^6\*e\*x^3)/128 + (2\*d^5\*e^2\*x^4)/7 - (139\*d^4\*e^3\*x^5)/160 - (16\*d^3\*e^4\*x^6)/21 + (9\*d^2\*e^5\*x^7)/80 + (d\*e^6\*x^8)/3 + (e^7\*x^9)/10) + (33\*d^10\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(256\*e^2)

**Maple [A]** time = 0.013, size = 191, normalized size = 0.8

$$-\frac{5d^3}{21e^2}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{ex^3}{10}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{33d^2x}{80e}(-e^2x^2 + d^2)^{\frac{7}{2}} \\ + \frac{11d^4x}{160e}(-e^2x^2 + d^2)^{\frac{5}{2}} + \frac{11d^6x}{128e}(-e^2x^2 + d^2)^{\frac{3}{2}} + \frac{33d^8x}{256e}\sqrt{-e^2x^2 + d^2} \\ + \frac{33d^{10}}{256e} \arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} - \frac{dx^2}{3}(-e^2x^2 + d^2)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(e\*x+d)^3\*(-e^2\*x^2+d^2)^(5/2), x)

[Out] -5/21\*d^3\*(-e^2\*x^2+d^2)^(7/2)/e^2-1/10\*e\*x^3\*(-e^2\*x^2+d^2)^(7/2)-33/80/e\*d^2\*x\*(-e^2\*x^2+d^2)^(7/2)+11/160\*d^4\*x\*(-e^2\*x^2+d^2)^(5/2)/e+11/128\*d^6\*x\*(-e^2\*x^2+d^2)^(3/2)/e+33/256\*d^8\*x\*(-e^2\*x^2+d^2)^(1/2)/e+33/256/e\*d^10/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))-1/3\*d\*x^2\*(-e^2\*x^2+d^2)^(7/2)

**Maxima [A]** time = 0.795565, size = 247, normalized size = 1.07

$$\frac{33d^{10} \arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{256\sqrt{e^2}} + \frac{33\sqrt{-e^2x^2 + d^2}d^8x}{256e} + \frac{11(-e^2x^2 + d^2)^{\frac{3}{2}}d^6x}{128e} - \frac{1}{10}(-e^2x^2 + d^2)^{\frac{7}{2}}ex^3 \\ + \frac{11(-e^2x^2 + d^2)^{\frac{5}{2}}d^4x}{160e} - \frac{1}{3}(-e^2x^2 + d^2)^{\frac{7}{2}}dx^2 - \frac{33(-e^2x^2 + d^2)^{\frac{7}{2}}d^2x}{80e} - \frac{5(-e^2x^2 + d^2)^{\frac{7}{2}}d^3}{21e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3\*x, x, algorithm="maxima")

[Out] 33/256\*d^10\*arcsin(e^2\*x/sqrt(d^2\*e^2))/(sqrt(e^2)\*e) + 33/256\*sqrt(-e^2\*x^2 + d^2)\*d^8\*x/e + 11/128\*(-e^2\*x^2 + d^2)^(3/2)\*d^6\*x/

$$e - 1/10*(-e^2*x^2 + d^2)^{(7/2)}*e*x^3 + 11/160*(-e^2*x^2 + d^2)^{(5/2)}*d^4*x/e - 1/3*(-e^2*x^2 + d^2)^{(7/2)}*d*x^2 - 33/80*(-e^2*x^2 + d^2)^{(7/2)}*d^2*x/e - 5/21*(-e^2*x^2 + d^2)^{(7/2)}*d^3/e^2$$

**Fricas [A]** time = 0.291118, size = 954, normalized size = 4.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3\*x,x, algorithm="fricas")

[Out] -1/26880\*(26880\*d^e^19\*x^19 + 89600\*d^2\*e^18\*x^18 - 426720\*d^3\*e^17\*x^17 - 1728000\*d^4\*e^16\*x^16 + 1488816\*d^5\*e^15\*x^15 + 11013120\*d^6\*e^14\*x^14 + 2172660\*d^7\*e^13\*x^13 - 33438720\*d^8\*e^12\*x^12 - 24640350\*d^9\*e^11\*x^11 + 53088000\*d^10\*e^10\*x^10 + 63904890\*d^11\*e^9\*x^9 - 39782400\*d^12\*e^8\*x^8 - 81970560\*d^13\*e^7\*x^7 + 2150400\*d^14\*e^6\*x^6 + 55572384\*d^15\*e^5\*x^5 + 15482880\*d^16\*e^4\*x^4 - 17902080\*d^17\*e^3\*x^3 - 6881280\*d^18\*e^2\*x^2 + 1774080\*d^19\*e\*x + 6930\*(d^10\*e^10\*x^10 - 50\*d^12\*e^8\*x^8 + 400\*d^14\*e^6\*x^6 - 1120\*d^16\*e^4\*x^4 + 1280\*d^18\*e^2\*x^2 - 512\*d^20 + 2\*(5\*d^11\*e^8\*x^8 - 80\*d^13\*e^6\*x^6 + 336\*d^15\*e^4\*x^4 - 512\*d^17\*e^2\*x^2 + 256\*d^19)\*sqrt(-e^2\*x^2 + d^2))\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) - (2688\*e^19\*x^19 + 8960\*d^e^18\*x^18 - 131376\*d^2\*e^17\*x^17 - 468480\*d^3\*e^16\*x^16 + 900648\*d^4\*e^15\*x^15 + 4615680\*d^5\*e^14\*x^14 - 608790\*d^6\*e^13\*x^13 - 18600960\*d^7\*e^12\*x^12 - 10519005\*d^8\*e^11\*x^11 + 36960000\*d^9\*e^10\*x^10 + 38649954\*d^10\*e^9\*x^9 - 35051520\*d^11\*e^8\*x^8 - 60343248\*d^12\*e^7\*x^7 + 7311360\*d^13\*e^6\*x^6 + 47286624\*d^14\*e^5\*x^5 + 12042240\*d^15\*e^4\*x^4 - 17015040\*d^16\*e^3\*x^3 - 6881280\*d^17\*e^2\*x^2 + 1774080\*d^18\*e\*x)\*sqrt(-e^2\*x^2 + d^2))/(e^12\*x^10 - 50\*d^2\*e^10\*x^8 + 400\*d^4\*e^8\*x^6 - 1120\*d^6\*e^6\*x^4 + 1280\*d^8\*e^4\*x^2 - 512\*d^10\*e^2 + 2\*(5\*d^e^10\*x^8 - 80\*d^3\*e^8\*x^6 + 336\*d^5\*e^6\*x^4 - 512\*d^7\*e^4\*x^2 + 256\*d^9\*e^2)\*sqrt(-e^2\*x^2 + d^2))

**Sympy [A]** time = 107.362, size = 1554, normalized size = 6.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2),x)

[Out] d\*\*7\*Piecewise((x\*\*2\*sqrt(d\*\*2)/2, Eq(e\*\*2, 0)), (- (d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(3\*e\*\*2), True)) + 3\*d\*\*6\*e\*Piecewise((-I\*d\*\*4\*acosh(e\*x/d)/(8\*e\*\*3) + I\*d\*\*3\*x/(8\*e\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 3\*I\*d\*x\*\*3/(8\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*5/(4\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*4\*asin(e\*x/d)/(8\*e\*\*3) - d\*\*3\*x/(8\*e\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 3\*d\*x\*\*3/(8\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*\*2\*x\*\*5/(4\*d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2))), True)) + d\*\*5\*e\*\*2\*Piecewise((-2\*d\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*e\*\*4) - d\*\*2\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*e\*\*2) + x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/5, Ne(e, 0)), (x\*\*4\*sqrt(d\*\*2)/4, True)) - 5\*d\*\*4\*e\*\*3\*Piecewise((-I\*d\*\*6\*acosh(e\*x/d)/(16\*e\*\*5) + I\*d\*\*5\*x/(16\*e\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - I\*d\*\*3\*x\*\*3/(48\*e\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 5\*I\*d\*x\*\*5/(24\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*7/(6\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*6\*asin(e\*x/d)/(16\*e\*\*5) - d\*\*5\*x/(16\*e\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + d\*\*3\*x\*\*3/(48\*e\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 5\*d\*x\*\*5/(24\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*\*2\*x\*\*7/(6\*d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2))), True)) - 5\*d\*\*3\*e\*\*4\*Piecewise((-8\*d\*\*6\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(105\*e\*\*6) - 4\*d\*\*4\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(105\*e\*\*4) - d\*\*2\*x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(35\*e\*\*2) + x\*\*6\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/5, Ne(e, 0)), (x\*\*4\*sqrt(d\*\*2)/4, True)) - 5\*d\*\*4\*e\*\*3\*Piecewise((-I\*d\*\*6\*acosh(e\*x/d)/(16\*e\*\*5) + I\*d\*\*5\*x/(16\*e\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - I\*d\*\*3\*x\*\*3/(48\*e\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 5\*I\*d\*x\*\*5/(24\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*7/(6\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*6\*asin(e\*x/d)/(16\*e\*\*5) - d\*\*5\*x/(16\*e\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + d\*\*3\*x\*\*3/(48\*e\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 5\*d\*x\*\*5/(24\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*\*2\*x\*\*7/(6\*d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2))), True)) - 5\*d\*\*3\*e\*\*4\*Piecewise((-8\*d\*\*6\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(105\*e\*\*6) - 4\*d\*\*4\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(105\*e\*\*4) - d\*\*2\*x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(35\*e\*\*2) + x\*\*6\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/5, Ne(e, 0)), (x\*\*4\*sqrt(d\*\*2)/4, True))

```

t(d**2 - e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True)) + d
*2*e**5*Piecewise((-5*I*d**8*acosh(e*x/d)/(128*e**7) + 5*I*d**7*x
/(128*e**6*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d**5*x**3/(384*e**4*s
qrt(-1 + e**2*x**2/d**2)) - I*d**3*x**5/(192*e**2*sqrt(-1 + e**2*
x**2/d**2)) - 7*I*d*x**7/(48*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*
x**9/(8*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (
5*d**8*asin(e*x/d)/(128*e**7) - 5*d**7*x/(128*e**6*sqrt(1 - e**2*
x**2/d**2)) + 5*d**5*x**3/(384*e**4*sqrt(1 - e**2*x**2/d**2)) + d
**3*x**5/(192*e**2*sqrt(1 - e**2*x**2/d**2)) + 7*d*x**7/(48*sqrt(
1 - e**2*x**2/d**2)) - e**2*x**9/(8*d*sqrt(1 - e**2*x**2/d**2)),
True)) + 3*d*e**6*Piecewise((-16*d**8*sqrt(d**2 - e**2*x**2)/(315
*e**8) - 8*d**6*x**2*sqrt(d**2 - e**2*x**2)/(315*e**6) - 2*d**4*x
**4*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**6*sqrt(d**2 - e**
2*x**2)/(63*e**2) + x**8*sqrt(d**2 - e**2*x**2)/9, Ne(e, 0)), (x*
**8*sqrt(d**2)/8, True)) + e**7*Piecewise((-7*I*d**10*acosh(e*x/d)
/(256*e**9) + 7*I*d**9*x/(256*e**8*sqrt(-1 + e**2*x**2/d**2)) - 7
*I*d**7*x**3/(768*e**6*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d**5*x**5
/(1920*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**7/(480*e**2*sq
rt(-1 + e**2*x**2/d**2)) - 9*I*d*x**9/(80*sqrt(-1 + e**2*x**2/d**
2)) + I*e**2*x**11/(10*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**
2/d**2) > 1), (7*d**10*asin(e*x/d)/(256*e**9) - 7*d**9*x/(256*e**
8*sqrt(1 - e**2*x**2/d**2)) + 7*d**7*x**3/(768*e**6*sqrt(1 - e**2
*x**2/d**2)) + 7*d**5*x**5/(1920*e**4*sqrt(1 - e**2*x**2/d**2)) +
d**3*x**7/(480*e**2*sqrt(1 - e**2*x**2/d**2)) + 9*d*x**9/(80*sq
rt(1 - e**2*x**2/d**2)) - e**2*x**11/(10*d*sqrt(1 - e**2*x**2/d**2
)), True))

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**GIAC/XCAS [A]** time = 0.300621, size = 173, normalized size = 0.75

$$\frac{33}{256} d^{10} \arcsin\left(\frac{xe}{d}\right) e^{(-2)} \text{sign}(d) - \frac{1}{26880} \left(6400 d^9 e^{(-2)} + \left(3465 d^8 e^{(-1)} - 2(5120 d^7 + (12285 d^6 e + 4(960 d^5 e^2 - (2919 d^4 e^3 + 2(1280 d^3 e^4 - 7(27 d^2 e^5 + 8(3 d e^6 + 10 d^2 e^6) * x) * x) * x) * x) * x) * x) * x) * \sqrt{-x^2 e^2 + d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3\*x,x, algorithm="giac")

[Out] 33/256\*d^10\*arcsin(x\*e/d)\*e^(-2)\*sign(d) - 1/26880\*(6400\*d^9\*e^(-2) + (3465\*d^8\*e^(-1) - 2\*(5120\*d^7 + (12285\*d^6\*e + 4\*(960\*d^5\*e^2 - (2919\*d^4\*e^3 + 2\*(1280\*d^3\*e^4 - 7\*(27\*d^2\*e^5 + 8\*(3\*x\*e^6 + 10\*d^2\*e^6)\*x)\*x)\*x)\*x)\*x)\*x)\*sqrt(-x^2\*e^2 + d^2)

### 3.70 $\int (d + ex)^3 (d^2 - e^2x^2)^{5/2} dx$

**Optimal.** Leaf size=188

$$\frac{11d^2 (d^2 - e^2x^2)^{7/2}}{56e} - \frac{11d(d + ex) (d^2 - e^2x^2)^{7/2}}{72e} - \frac{(d + ex)^2 (d^2 - e^2x^2)^{7/2}}{9e} + \frac{55d^9 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{128e} + \frac{55}{128}d^7x\sqrt{d^2 - e^2x^2} + \frac{55}{192}d^5x (d^2 - e^2x^2)^{3/2} + \frac{11}{48}d^3x (d^2 - e^2x^2)^{5/2}$$

[Out] (55\*d^7\*x\*Sqrt[d^2 - e^2\*x^2])/128 + (55\*d^5\*x\*(d^2 - e^2\*x^2)^(3/2))/192 + (11\*d^3\*x\*(d^2 - e^2\*x^2)^(5/2))/48 - (11\*d^2\*(d^2 - e^2\*x^2)^(7/2))/(56\*e) - (11\*d\*(d + e\*x)\*(d^2 - e^2\*x^2)^(7/2))/(72\*e) - ((d + e\*x)^2\*(d^2 - e^2\*x^2)^(7/2))/(9\*e) + (55\*d^9\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(128\*e)

**Rubi [A]** time = 0.177185, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$\frac{11d^2 (d^2 - e^2x^2)^{7/2}}{56e} - \frac{11d(d + ex) (d^2 - e^2x^2)^{7/2}}{72e} - \frac{(d + ex)^2 (d^2 - e^2x^2)^{7/2}}{9e} + \frac{55d^9 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{128e} + \frac{55}{128}d^7x\sqrt{d^2 - e^2x^2} + \frac{55}{192}d^5x (d^2 - e^2x^2)^{3/2} + \frac{11}{48}d^3x (d^2 - e^2x^2)^{5/2}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2), x]

[Out] (55\*d^7\*x\*Sqrt[d^2 - e^2\*x^2])/128 + (55\*d^5\*x\*(d^2 - e^2\*x^2)^(3/2))/192 + (11\*d^3\*x\*(d^2 - e^2\*x^2)^(5/2))/48 - (11\*d^2\*(d^2 - e^2\*x^2)^(7/2))/(56\*e) - (11\*d\*(d + e\*x)\*(d^2 - e^2\*x^2)^(7/2))/(72\*e) - ((d + e\*x)^2\*(d^2 - e^2\*x^2)^(7/2))/(9\*e) + (55\*d^9\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(128\*e)

**Rubi in Sympy [A]** time = 31.6048, size = 163, normalized size = 0.87

$$\frac{55d^9 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{128e} + \frac{55d^7x\sqrt{d^2 - e^2x^2}}{128} + \frac{55d^5x (d^2 - e^2x^2)^{3/2}}{192} + \frac{11d^3x (d^2 - e^2x^2)^{5/2}}{48} - \frac{11d^2 (d^2 - e^2x^2)^{7/2}}{56e} - \frac{11d(d + ex) (d^2 - e^2x^2)^{7/2}}{72e} - \frac{(d + ex)^2 (d^2 - e^2x^2)^{7/2}}{9e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2), x)

[Out] 55\*d\*\*9\*atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2))/(128\*e) + 55\*d\*\*7\*x\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/128 + 55\*d\*\*5\*x\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/192 + 11\*d\*\*3\*x\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)/48 - 11\*d\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(7/2)/(56\*e) - 11\*d\*(d + e\*x)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(7/2)/(72\*e) - (d + e\*x)\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(7/2)/(9\*e)

**Mathematica [A]** time = 0.136007, size = 135, normalized size = 0.72

$$3465d^9 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) + \sqrt{d^2 - e^2x^2} (-3712d^8 + 4599d^7ex + 10240d^6e^2x^2 + 3066d^5e^3x^3 - 8448d^4e^4x^4 - 7224d^3e^5x^5 + 1080d^2e^6x^6 - 8064e^7x^7 + 256e^8x^8)$$



Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2), x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(-3712\*d^8 + 4599\*d^7\*e\*x + 10240\*d^6\*e^2\*x^2 + 3066\*d^5\*e^3\*x^3 - 8448\*d^4\*e^4\*x^4 - 7224\*d^3\*e^5\*x^5 + 1024\*d^2\*e^6\*x^6 + 3024\*d\*e^7\*x^7 + 896\*e^8\*x^8) + 3465\*d^9\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(8064\*e)

**Maple [A]** time = 0.012, size = 154, normalized size = 0.8

$$\begin{aligned} & \frac{11d^3x}{48}(-e^2x^2 + d^2)^{\frac{5}{2}} + \frac{55d^5x}{192}(-e^2x^2 + d^2)^{\frac{3}{2}} + \frac{55d^7x}{128}\sqrt{-e^2x^2 + d^2} \\ & + \frac{55d^9}{128}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2 + d^2}}\right)\frac{1}{\sqrt{e^2}} - \frac{ex^2}{9}(-e^2x^2 + d^2)^{\frac{7}{2}} \\ & - \frac{29d^2}{63e}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{3dx}{8}(-e^2x^2 + d^2)^{\frac{7}{2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(-e^2\*x^2+d^2)^(5/2), x)

[Out] 11/48\*d^3\*x\*(-e^2\*x^2+d^2)^(5/2)+55/192\*d^5\*x\*(-e^2\*x^2+d^2)^(3/2)+55/128\*d^7\*x\*(-e^2\*x^2+d^2)^(1/2)+55/128\*d^9/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))-1/9\*e\*x^2\*(-e^2\*x^2+d^2)^(7/2)-29/63\*d^2\*(-e^2\*x^2+d^2)^(7/2)/e-3/8\*d\*x\*(-e^2\*x^2+d^2)^(7/2)

**Maxima [A]** time = 0.794114, size = 197, normalized size = 1.05

$$\begin{aligned} & \frac{55d^9\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{128\sqrt{e^2}} + \frac{55}{128}\sqrt{-e^2x^2 + d^2}d^7x + \frac{55}{192}(-e^2x^2 + d^2)^{\frac{3}{2}}d^5x + \frac{11}{48}(-e^2x^2 + d^2)^{\frac{5}{2}}d^3x \\ & - \frac{1}{9}(-e^2x^2 + d^2)^{\frac{7}{2}}ex^2 - \frac{3}{8}(-e^2x^2 + d^2)^{\frac{7}{2}}dx - \frac{29(-e^2x^2 + d^2)^{\frac{7}{2}}d^2}{63e} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3, x, algorithm="maxima")

[Out] 55/128\*d^9\*arcsin(e^2\*x/sqrt(d^2\*e^2))/sqrt(e^2) + 55/128\*sqrt(-e^2\*x^2 + d^2)\*d^7\*x + 55/192\*(-e^2\*x^2 + d^2)^(3/2)\*d^5\*x + 11/48\*(-e^2\*x^2 + d^2)^(5/2)\*d^3\*x - 1/9\*(-e^2\*x^2 + d^2)^(7/2)\*e\*x^2 - 3/8\*(-e^2\*x^2 + d^2)^(7/2)\*d\*x - 29/63\*(-e^2\*x^2 + d^2)^(7/2)\*d^2/e

**Fricas [A]** time = 0.287759, size = 860, normalized size = 4.57

$$896e^{18}x^{18} + 3024de^{17}x^{17} - 35712d^2e^{16}x^{16} - 131208d^3e^{15}x^{15} + 200448d^4e^{14}x^{14} + 1145970d^5e^{13}x^{13} + 26880d^6e^{12}x^{12} - 42$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3, x, algorithm="fricas")

```
[Out] 1/8064*(896*e^18*x^18 + 3024*d*e^17*x^17 - 35712*d^2*e^16*x^16 -
131208*d^3*e^15*x^15 + 200448*d^4*e^14*x^14 + 1145970*d^5*e^13*x^
13 + 26880*d^6*e^12*x^12 - 4224339*d^7*e^11*x^11 - 2862720*d^8*e^
10*x^10 + 7768929*d^9*e^9*x^9 + 9289728*d^10*e^8*x^8 - 6681528*d^
11*e^7*x^7 - 13848576*d^12*e^6*x^6 + 843696*d^13*e^5*x^5 + 103219
20*d^14*e^4*x^4 + 2452800*d^15*e^3*x^3 - 3096576*d^16*e^2*x^2 - 1
177344*d^17*e*x - 6930*(9*d^10*e^8*x^8 - 120*d^12*e^6*x^6 + 432*d
^14*e^4*x^4 - 576*d^16*e^2*x^2 + 256*d^18 - (d^9*e^8*x^8 - 40*d^1
1*e^6*x^6 + 240*d^13*e^4*x^4 - 448*d^15*e^2*x^2 + 256*d^17)*sqrt(
-e^2*x^2 + d^2))*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 3*(2
688*d*e^16*x^16 + 9072*d^2*e^15*x^15 - 32768*d^3*e^14*x^14 - 1426
32*d^4*e^13*x^13 + 62720*d^5*e^12*x^12 + 733614*d^6*e^11*x^11 + 3
44064*d^7*e^10*x^10 - 1729707*d^8*e^9*x^9 - 1756160*d^9*e^8*x^8 +
1902600*d^10*e^7*x^7 + 3282944*d^11*e^6*x^6 - 542864*d^12*e^5*x^
5 - 2924544*d^13*e^4*x^4 - 621376*d^14*e^3*x^3 + 1032192*d^15*e^2
*x^2 + 392448*d^16*e*x)*sqrt(-e^2*x^2 + d^2))/(9*d^9*x^8 - 120*
d^3*e^7*x^6 + 432*d^5*e^5*x^4 - 576*d^7*e^3*x^2 + 256*d^9*e - (e^
9*x^8 - 40*d^2*e^7*x^6 + 240*d^4*e^5*x^4 - 448*d^6*e^3*x^2 + 256*
d^8*e)*sqrt(-e^2*x^2 + d^2))
```

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**Sympy [A]** time = 69.7639, size = 1284, normalized size = 6.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)
```

```
[Out] d**7*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e
**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Abs
(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e*
**2*x**2/d**2)/2, True)) + 3*d**6*e*Piecewise((x**2*sqrt(d**2)/2,
Eq(e**2, 0)), (-(d**2 - e**2*x**2)**(3/2)/(3*e**2), True)) + d**5
*e**2*Piecewise((-I*d**4*acosh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2
*sqrt(-1 + e**2*x**2/d**2)) - 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d
**2)) + I*e**2*x**5/(4*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**
2/d**2) > 1), (d**4*asin(e*x/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1
- e**2*x**2/d**2)) + 3*d*x**3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2
*x**5/(4*d*sqrt(1 - e**2*x**2/d**2)), True)) - 5*d**4*e**3*Piecw
ise((-2*d**4*sqrt(d**2 - e**2*x**2)/(15*e**4) - d**2*x**2*sqrt(d*
**2 - e**2*x**2)/(15*e**2) + x**4*sqrt(d**2 - e**2*x**2)/5, Ne(e,
0)), (x**4*sqrt(d**2)/4, True)) - 5*d**3*e**4*Piecewise((-I*d**6*
acosh(e*x/d)/(16*e**5) + I*d**5*x/(16*e**4*sqrt(-1 + e**2*x**2/d*
**2)) - I*d**3*x**3/(48*e**2*sqrt(-1 + e**2*x**2/d**2)) - 5*I*d*x*
**5/(24*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**7/(6*d*sqrt(-1 + e
**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**6*asin(e*x/d)/(16*e
**5) - d**5*x/(16*e**4*sqrt(1 - e**2*x**2/d**2)) + d**3*x**3/(48*
e**2*sqrt(1 - e**2*x**2/d**2)) + 5*d*x**5/(24*sqrt(1 - e**2*x**2/
d**2)) - e**2*x**7/(6*d*sqrt(1 - e**2*x**2/d**2)), True)) + d**2*
e**5*Piecewise((-8*d**6*sqrt(d**2 - e**2*x**2)/(105*e**6) - 4*d**
4*x**2*sqrt(d**2 - e**2*x**2)/(105*e**4) - d**2*x**4*sqrt(d**2 -
e**2*x**2)/(35*e**2) + x**6*sqrt(d**2 - e**2*x**2)/7, Ne(e, 0)),
(x**6*sqrt(d**2)/6, True)) + 3*d*e**6*Piecewise((-5*I*d**8*acosh(
e*x/d)/(128*e**7) + 5*I*d**7*x/(128*e**6*sqrt(-1 + e**2*x**2/d**2
)) - 5*I*d**5*x**3/(384*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*
x**5/(192*e**2*sqrt(-1 + e**2*x**2/d**2)) - 7*I*d*x**7/(48*sqrt(-
1 + e**2*x**2/d**2)) + I*e**2*x**9/(8*d*sqrt(-1 + e**2*x**2/d**2)
), Abs(e**2*x**2/d**2) > 1), (5*d**8*asin(e*x/d)/(128*e**7) - 5*d
**7*x/(128*e**6*sqrt(1 - e**2*x**2/d**2)) + 5*d**5*x**3/(384*e**4
*sqrt(1 - e**2*x**2/d**2)) + d**3*x**5/(192*e**2*sqrt(1 - e**2*x*
**2/d**2)) + 7*d*x**7/(48*sqrt(1 - e**2*x**2/d**2)) - e**2*x**9/(8
*d*sqrt(1 - e**2*x**2/d**2)), True)) + e**7*Piecewise((-16*d**8*s
qrt(d**2 - e**2*x**2)/(315*e**8) - 8*d**6*x**2*sqrt(d**2 - e**2*x
**2)/(315*e**6) - 2*d**4*x**4*sqrt(d**2 - e**2*x**2)/(105*e**4) -
d**2*x**6*sqrt(d**2 - e**2*x**2)/(63*e**2) + x**8*sqrt(d**2 - e
**2*x**2)/9, Ne(e, 0)), (x**8*sqrt(d**2)/8, True))
```

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**GIAC/XCAS [A]** time = 0.283636, size = 158, normalized size = 0.84

$$\frac{55}{128} d^9 \arcsin\left(\frac{xe}{d}\right) e^{(-1)} \text{sign}(d) - \frac{1}{8064} \left( 3712 d^8 e^{(-1)} - (4599 d^7 + 2 (5120 d^6 e + (1533 d^5 e^2 - 4 (1056 d^4 e^3 + (903 d^3 e^4 - 2 (64 d^2 e^5 + 7 (8 x e^7 + 27 d e^6) x) x) x) x) x) \sqrt{-x^2 e^2 + d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3,x, algorithm="giac")

[Out] 55/128\*d^9\*arcsin(x\*e/d)\*e^(-1)\*sign(d) - 1/8064\*(3712\*d^8\*e^(-1) - (4599\*d^7 + 2\*(5120\*d^6\*e + (1533\*d^5\*e^2 - 4\*(1056\*d^4\*e^3 + (903\*d^3\*e^4 - 2\*(64\*d^2\*e^5 + 7\*(8\*x\*e^7 + 27\*d\*e^6)\*x)\*x)\*x)\*x)\*x)\*sqrt(-x^2\*e^2 + d^2)

$$3.71 \quad \int \frac{(d+ex)^3 (d^2 - e^2 x^2)^{5/2}}{x} dx$$

**Optimal.** Leaf size=190

$$\frac{1}{240} d^2 (48d + 125ex) (d^2 - e^2 x^2)^{5/2} - \frac{3}{7} d (d^2 - e^2 x^2)^{7/2} - \frac{1}{8} ex (d^2 - e^2 x^2)^{7/2} + \frac{125}{128} d^8 \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) - d^8 \tanh^{-1} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right) + \frac{1}{128} d^6 (128d + 125ex) \sqrt{d^2 - e^2 x^2} + \frac{1}{192} d^4 (64d + 125ex) (d^2 - e^2 x^2)^{3/2}$$

[Out]  $(d^6 * (128 * d + 125 * e * x) * \text{Sqrt}[d^2 - e^2 * x^2]) / 128 + (d^4 * (64 * d + 125 * e * x) * (d^2 - e^2 * x^2)^{(3/2)}) / 192 + (d^2 * (48 * d + 125 * e * x) * (d^2 - e^2 * x^2)^{(5/2)}) / 240 - (3 * d * (d^2 - e^2 * x^2)^{(7/2)}) / 7 - (e * x * (d^2 - e^2 * x^2)^{(7/2)}) / 8 + (125 * d^8 * \text{ArcTan}[(e * x) / \text{Sqrt}[d^2 - e^2 * x^2]]) / 128 - d^8 * \text{ArcTanh}[\text{Sqrt}[d^2 - e^2 * x^2] / d]$

**Rubi [A]** time = 0.620345, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$

$$\frac{1}{240} d^2 (48d + 125ex) (d^2 - e^2 x^2)^{5/2} - \frac{3}{7} d (d^2 - e^2 x^2)^{7/2} - \frac{1}{8} ex (d^2 - e^2 x^2)^{7/2} + \frac{125}{128} d^8 \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) - d^8 \tanh^{-1} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right) + \frac{1}{128} d^6 (128d + 125ex) \sqrt{d^2 - e^2 x^2} + \frac{1}{192} d^4 (64d + 125ex) (d^2 - e^2 x^2)^{3/2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e * x)^3 * (d^2 - e^2 * x^2)^{(5/2)} / x, x]$

[Out]  $(d^6 * (128 * d + 125 * e * x) * \text{Sqrt}[d^2 - e^2 * x^2]) / 128 + (d^4 * (64 * d + 125 * e * x) * (d^2 - e^2 * x^2)^{(3/2)}) / 192 + (d^2 * (48 * d + 125 * e * x) * (d^2 - e^2 * x^2)^{(5/2)}) / 240 - (3 * d * (d^2 - e^2 * x^2)^{(7/2)}) / 7 - (e * x * (d^2 - e^2 * x^2)^{(7/2)}) / 8 + (125 * d^8 * \text{ArcTan}[(e * x) / \text{Sqrt}[d^2 - e^2 * x^2]]) / 128 - d^8 * \text{ArcTanh}[\text{Sqrt}[d^2 - e^2 * x^2] / d]$

**Rubi in Sympy [A]** time = 123.452, size = 218, normalized size = 1.15

$$\frac{125d^8 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{128} - d^8 \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right) + d^7 \sqrt{d^2 - e^2 x^2} + \frac{259d^6 ex \sqrt{d^2 - e^2 x^2}}{128} + \frac{d^5 (d^2 - e^2 x^2)^{3/2}}{3} - \frac{253d^4 e^3 x^3 \sqrt{d^2 - e^2 x^2}}{192} + \frac{d^3 (d^2 - e^2 x^2)^{5/2}}{5} + \frac{7d^2 e^5 x^5 \sqrt{d^2 - e^2 x^2}}{48} - \frac{3d (d^2 - e^2 x^2)^{7/2}}{7} + \frac{e^7 x^7 \sqrt{d^2 - e^2 x^2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e * x + d)^3 * (-e^2 * x^2 + d^2)^{(5/2)} / x, x)$

[Out]  $125 * d^8 * \operatorname{atan}(e * x / \sqrt{d^2 - e^2 * x^2}) / 128 - d^8 * \operatorname{atanh}(\sqrt{d^2 - e^2 * x^2} / d) + d^7 * \sqrt{d^2 - e^2 * x^2} + 259 * d^6 * e * x * \sqrt{d^2 - e^2 * x^2} / 128 + d^5 * (d^2 - e^2 * x^2)^{(3/2)} / 3 - 253 * d^4 * e^3 * x^3 * \sqrt{d^2 - e^2 * x^2} / 192 + d^3 * (d^2 - e^2 * x^2)^{(5/2)} / 5 + 7 * d^2 * e^5 * x^5 * \sqrt{d^2 - e^2 * x^2} / 48 - 3 * d * (d^2 - e^2 * x^2)^{(7/2)} / 7 + e^7 * x^7 * \sqrt{d^2 - e^2 * x^2} / 8$

**Mathematica [A]** time = 0.142425, size = 165, normalized size = 0.87

$$d^8 \log(x) - d^8 \log\left(\sqrt{d^2 - e^2 x^2} + d\right) + \frac{125}{128} d^8 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right) + \sqrt{d^2 - e^2 x^2} \left(\frac{116d^7}{105} + \frac{259}{128} d^6 ex + \frac{58}{105} d^5 e^2 x^2 - \frac{253}{192} d^4 e^3 x^3 - \frac{38}{35} d^3 e^4 x^4 + \frac{7}{48} d^2 e^5 x^5 + \frac{3}{7} d e^6 x^6 + \frac{e^7 x^7}{8}\right)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2))/x,x]

[Out] Sqrt[d^2 - e^2\*x^2]\*((116\*d^7)/105 + (259\*d^6\*e\*x)/128 + (58\*d^5\*e^2\*x^2)/105 - (253\*d^4\*e^3\*x^3)/192 - (38\*d^3\*e^4\*x^4)/35 + (7\*d^2\*e^5\*x^5)/48 + (3\*d\*e^6\*x^6)/7 + (e^7\*x^7)/8) + (125\*d^8\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/128 + d^8\*Log[x] - d^8\*Log[d + Sqrt[d^2 - e^2\*x^2]]

**Maple [A]** time = 0.016, size = 231, normalized size = 1.2

$$\frac{d^3}{5} (-e^2 x^2 + d^2)^{\frac{5}{2}} + \frac{d^5}{3} (-e^2 x^2 + d^2)^{\frac{3}{2}} + d^7 \sqrt{-e^2 x^2 + d^2} - d^9 \ln\left(\frac{1}{x} (2d^2 + 2\sqrt{d^2} \sqrt{-e^2 x^2 + d^2})\right) \frac{1}{\sqrt{d^2}} - \frac{ex}{8} (-e^2 x^2 + d^2)^{\frac{7}{2}} + \frac{25 d^2 ex}{48} (-e^2 x^2 + d^2)^{\frac{5}{2}} + \frac{125 ed^4 x}{192} (-e^2 x^2 + d^2)^{\frac{3}{2}} + \frac{125 ed^6 x}{128} \sqrt{-e^2 x^2 + d^2} + \frac{125 ed^8}{128} \arctan\left(x \sqrt{e^2} \frac{1}{\sqrt{-e^2 x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} - \frac{3d}{7} (-e^2 x^2 + d^2)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(-e^2\*x^2+d^2)^(5/2)/x,x)

[Out] 1/5\*d^3\*(-e^2\*x^2+d^2)^(5/2)+1/3\*d^5\*(-e^2\*x^2+d^2)^(3/2)+d^7\*(-e^2\*x^2+d^2)^(1/2)-d^9/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)-1/8\*e\*x\*(-e^2\*x^2+d^2)^(7/2)+25/48\*d^2\*e\*x\*(-e^2\*x^2+d^2)^(5/2)+125/192\*e\*d^4\*x\*(-e^2\*x^2+d^2)^(3/2)+125/128\*e\*d^6\*x\*(-e^2\*x^2+d^2)^(1/2)+125/128\*e\*d^8/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))-3/7\*d\*(-e^2\*x^2+d^2)^(7/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3/x,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.297359, size = 910, normalized size = 4.79

$$13440 de^{15}x^{15} + 46080 d^2 e^{14}x^{14} - 132160 d^3 e^{13}x^{13} - 623616 d^4 e^{12}x^{12} + 142800 d^5 e^{11}x^{11} + 2910208 d^6 e^{10}x^{10} + 1771560 d^7 e^9 x^9 - 13440 d^8 e^8 x^8 + 46080 d^9 e^7 x^7 - 132160 d^{10} e^6 x^6 + 623616 d^{11} e^5 x^5 - 142800 d^{12} e^4 x^4 + 2910208 d^{13} e^3 x^3 - 1771560 d^{14} e^2 x^2 + 13440 d^{15} e x - 13440 d^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3/x,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/13440*(13440*d*e^{15}*x^{15} + 46080*d^2*e^{14}*x^{14} - 132160*d^3*e^{13}*x^{13} - 623616*d^4*e^{12}*x^{12} + 142800*d^5*e^{11}*x^{11} + 2910208*d^6*e^{10}*x^{10} + 1771560*d^7*e^9*x^9 - 6361600*d^8*e^8*x^8 - 7622440*d^9*e^7*x^7 + 6594560*d^{10}*e^6*x^6 + 13315120*d^{11}*e^5*x^5 - 2580480*d^{12}*e^4*x^4 - 10969280*d^{13}*e^3*x^3 + 3480960*d^{15}*e*x + 26250*(d^8*e^8*x^8 - 32*d^{10}*e^6*x^6 + 160*d^{12}*e^4*x^4 - 256*d^{14}*e^2*x^2 + 128*d^{16} + 8*(d^9*e^6*x^6 - 10*d^{11}*e^4*x^4 + 24*d^{13}*e^2*x^2 - 16*d^{15})*\sqrt{-e^2*x^2 + d^2})*\arctan(-(d - \sqrt{-e^2*x^2 + d^2}))/e*x) - 13440*(d^8*e^8*x^8 - 32*d^{10}*e^6*x^6 + 160*d^{12}*e^4*x^4 - 256*d^{14}*e^2*x^2 + 128*d^{16} + 8*(d^9*e^6*x^6 - 10*d^{11}*e^4*x^4 + 24*d^{13}*e^2*x^2 - 16*d^{15})*\sqrt{-e^2*x^2 + d^2})*\log(-(d - \sqrt{-e^2*x^2 + d^2})/x) - (1680*e^{15}*x^{15} + 5760*d*e^{14}*x^{14} - 51800*d^2*e^{13}*x^{13} - 198912*d^3*e^{12}*x^{12} + 188370*d^4*e^{11}*x^{11} + 1395968*d^5*e^{10}*x^{10} + 477435*d^6*e^9*x^9 - 4032000*d^7*e^8*x^8 - 3990560*d^8*e^7*x^7 + 5304320*d^9*e^6*x^6 + 9135840*d^{10}*e^5*x^5 - 2580480*d^{11}*e^4*x^4 - 9228800*d^{12}*e^3*x^3 + 3480960*d^{14}*e*x)*\sqrt{-e^2*x^2 + d^2})/(e^8*x^8 - 32*d^2*e^6*x^6 + 160*d^4*e^4*x^4 - 256*d^6*e^2*x^2 + 128*d^8 + 8*(d*e^6*x^6 - 10*d^3*e^4*x^4 + 24*d^5*e^2*x^2 - 16*d^7)*\sqrt{-e^2*x^2 + d^2}) \end{aligned}$$

**Sympy [A]** time = 76.3748, size = 1263, normalized size = 6.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x,x)

[Out] 
$$\begin{aligned} & d^{**7} \text{Piecewise}((d^{**2}/(e*x*\sqrt{d^{**2}/(e^{**2}*x^{**2}) - 1)}) - d*\text{acosh}(d/(e*x)) - e*x/\sqrt{d^{**2}/(e^{**2}*x^{**2}) - 1}, \text{Abs}(d^{**2}/(e^{**2}*x^{**2})) > 1), (-I*d^{**2}/(e*x*\sqrt{-d^{**2}/(e^{**2}*x^{**2}) + 1})) + I*d*\text{asin}(d/(e*x)) + I*e*x/\sqrt{-d^{**2}/(e^{**2}*x^{**2}) + 1}, \text{True})) + 3*d^{**6}*e*\text{Piecewise}((-I*d^{**2}*\text{acosh}(e*x/d)/(2*e) - I*d*x/(2*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}})) + I*e^{**2}*x^{**3}/(2*d*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}})), \text{Abs}(e^{**2}*x^{**2}/d^{**2}) > 1), (d^{**2}*\text{asin}(e*x/d)/(2*e) + d*x*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}})/2, \text{True})) + d^{**5}*e^{**2}*\text{Piecewise}((x^{**2}*\sqrt{d^{**2}})/2, \text{Eq}(e^{**2}, 0)), (-d^{**2} - e^{**2}*x^{**2})^{**}(3/2)/(3*e^{**2}), \text{True})) - 5*d^{**4}*e^{**3}*\text{Piecewise}((-I*d^{**4}*\text{acosh}(e*x/d)/(8*e^{**3}) + I*d^{**3}*x/(8*e^{**2}*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}})) - 3*I*d*x^{**3}/(8*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}})) + I*e^{**2}*x^{**5}/(4*d*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}})), \text{Abs}(e^{**2}*x^{**2}/d^{**2}) > 1), (d^{**4}*\text{asin}(e*x/d)/(8*e^{**3}) - d^{**3}*x/(8*e^{**2}*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}})) + 3*d*x^{**3}/(8*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}})) - e^{**2}*x^{**5}/(4*d*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}})), \text{True})) - 5*d^{**3}*e^{**4}*\text{Piecewise}((-2*d^{**4}*\sqrt{d^{**2} - e^{**2}*x^{**2}})/(15*e^{**4}) - d^{**2}*x^{**2}*\sqrt{d^{**2} - e^{**2}*x^{**2}})/(15*e^{**2}) + x^{**4}*\sqrt{d^{**2} - e^{**2}*x^{**2}}/5, \text{Ne}(e, 0)), (x^{**4}*\sqrt{d^{**2}}/4, \text{True})) + d^{**2}*e^{**5}*\text{Piecewise}((-I*d^{**6}*\text{acosh}(e*x/d)/(16*e^{**5}) + I*d^{**5}*x/(16*e^{**4}*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}})) - I*d^{**3}*x^{**3}/(48*e^{**2}*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}})) - 5*I*d*x^{**5}/(24*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}})) + I*e^{**2}*x^{**7}/(6*d*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}})), \text{Abs}(e^{**2}*x^{**2}/d^{**2}) > 1), (d^{**6}*\text{asin}(e*x/d)/(16*e^{**5}) - d^{**5}*x/(16*e^{**4}*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}})) + d^{**3}*x^{**3}/(48*e^{**2}*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}})) + 5*d*x^{**5}/(24*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}})) - e^{**2}*x^{**7}/(6*d*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}})), \text{True})) + 3*d*e^{**6}*\text{Piecewise}((-8*d^{**6}*\sqrt{d^{**2} - e^{**2}*x^{**2}})/(105*e^{**6}) - 4*d^{**4}*x^{**2}*\sqrt{d^{**2} - e^{**2}*x^{**2}})/(105*e^{**4}) - d^{**2}*x^{**4}*\sqrt{d^{**2} - e^{**2}*x^{**2}})/(35*e^{**2}) + x^{**6}*\sqrt{d^{**2} - e^{**2}*x^{**2}}/7, \text{Ne}(e, 0)), (x^{**6}*\sqrt{d^{**2}}/6, \text{True})) + e^{**7}*\text{Piecewise}((-5*I*d^{**8}*\text{acosh}(e*x/d)/(128*e^{**7}) + 5*I*d^{**7}*x/(128*e^{**6}*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}})) - 5*I*d^{**5}*x^{**3}/(384*e^{**4}*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}})) - I*d^{**3}*x^{**5}/(192*e^{**2}*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}})) - 7*I*d*x^{**7}/(48*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}})) + I*e^{**2}*x^{**9}/(8*d*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}})), \text{Abs}(e^{**2}*x^{**2}/d^{**2}) > 1), (5*d^{**8}*\text{asin}(e*x/d)/(128*e^{**7}) - 5*d^{**7}*x/(128*e^{**6}*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}})) + 5*d^{**5}*x^{**3}/(384*e^{**4}*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}})) + d^{**3}*x^{**5}/(192*e^{**2}*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}})) + 7*d*x^{**7}/(48*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}})) - e^{**2}*x^{**9}/(8*d*\sqrt{1 - e^{**2} \end{aligned}$$

`*x**2/d**2)), True))`

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**GIAC/XCAS [A]** time = 0.285748, size = 193, normalized size = 1.02

$$\frac{125}{128} d^8 \arcsin\left(\frac{xe}{d}\right) \operatorname{sign}(d) - d^8 \ln\left(\frac{\left|-2de - 2\sqrt{-x^2e^2 + d^2}e\right|e^{(-2)}}{2|x|}\right) + \frac{1}{13440} (14848 d^7 + (27195 d^6 e + 2 (3712 d^5 e^2 - (8855 d^4 e^3 + 4 (1824 d^3 e^4 - 5 (49 d^2 e^5 + 6 (7 x e^7 + 24 d e^6) x) x) x) x) x) x) x)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3/x,x, algorithm="giac")`

`[Out] 125/128*d^8*arcsin(x*e/d)*sign(d) - d^8*ln(1/2*abs(-2*d*e - 2*sqrt(-x^2*e^2 + d^2)*e)*e^(-2)/abs(x)) + 1/13440*(14848*d^7 + (27195*d^6*e + 2*(3712*d^5*e^2 - (8855*d^4*e^3 + 4*(1824*d^3*e^4 - 5*(49*d^2*e^5 + 6*(7*x*e^7 + 24*d*e^6)*x)*x)*x)*x)*x)*x)*sqrt(-x^2*e^2 + d^2)`

$$3.72 \quad \int \frac{(d+ex)^3 (d^2 - e^2 x^2)^{5/2}}{x^2} dx$$

**Optimal.** Leaf size=193

$$-\frac{d(d^2 - e^2 x^2)^{7/2}}{x} + \frac{1}{10} de(6d - 5ex)(d^2 - e^2 x^2)^{5/2} - \frac{1}{7} e (d^2 - e^2 x^2)^{7/2} - \frac{15}{16} d^7 e \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) - 3d^7 e \tanh^{-1} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right) + \frac{3}{16} d^5 e(16d - 5ex)\sqrt{d^2 - e^2 x^2} + \frac{1}{8} d^3 e(8d - 5ex)(d^2 - e^2 x^2)^{3/2}$$

[Out] (3\*d^5\*e\*(16\*d - 5\*e\*x)\*Sqrt[d^2 - e^2\*x^2])/16 + (d^3\*e\*(8\*d - 5\*e\*x)\*(d^2 - e^2\*x^2)^(3/2))/8 + (d\*e\*(6\*d - 5\*e\*x)\*(d^2 - e^2\*x^2)^(5/2))/10 - (e\*(d^2 - e^2\*x^2)^(7/2))/7 - (d\*(d^2 - e^2\*x^2)^(7/2))/x - (15\*d^7\*e\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/16 - 3\*d^7\*e\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]

**Rubi [A]** time = 0.627019, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{d(d^2 - e^2 x^2)^{7/2}}{x} + \frac{1}{10} de(6d - 5ex)(d^2 - e^2 x^2)^{5/2} - \frac{1}{7} e (d^2 - e^2 x^2)^{7/2} - \frac{15}{16} d^7 e \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) - 3d^7 e \tanh^{-1} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right) + \frac{3}{16} d^5 e(16d - 5ex)\sqrt{d^2 - e^2 x^2} + \frac{1}{8} d^3 e(8d - 5ex)(d^2 - e^2 x^2)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2))/x^2, x]

[Out] (3\*d^5\*e\*(16\*d - 5\*e\*x)\*Sqrt[d^2 - e^2\*x^2])/16 + (d^3\*e\*(8\*d - 5\*e\*x)\*(d^2 - e^2\*x^2)^(3/2))/8 + (d\*e\*(6\*d - 5\*e\*x)\*(d^2 - e^2\*x^2)^(5/2))/10 - (e\*(d^2 - e^2\*x^2)^(7/2))/7 - (d\*(d^2 - e^2\*x^2)^(7/2))/x - (15\*d^7\*e\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/16 - 3\*d^7\*e\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]

**Rubi in Sympy [A]** time = 104.758, size = 223, normalized size = 1.16

$$-\frac{15d^7 e \operatorname{atan} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right)}{16} - 3d^7 e \operatorname{atanh} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right) - \frac{d^7 \sqrt{d^2 - e^2 x^2}}{x} + 3d^6 e \sqrt{d^2 - e^2 x^2} + \frac{15d^5 e^2 x \sqrt{d^2 - e^2 x^2}}{16} + d^4 e (d^2 - e^2 x^2)^{\frac{3}{2}} - \frac{11d^3 e^4 x^3 \sqrt{d^2 - e^2 x^2}}{8} + \frac{3d^2 e (d^2 - e^2 x^2)^{\frac{5}{2}}}{5} + \frac{de^6 x^5 \sqrt{d^2 - e^2 x^2}}{2} - \frac{e (d^2 - e^2 x^2)^{\frac{7}{2}}}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*2, x)

[Out] -15\*d\*\*7\*e\*atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2))/16 - 3\*d\*\*7\*e\*atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d) - d\*\*7\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/x + 3\*d\*\*6\*e\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) + 15\*d\*\*5\*e\*\*2\*x\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/16 + d\*\*4\*e\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2) - 11\*d\*\*3\*e\*\*4\*x\*\*3\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/8 + 3\*d\*\*2\*e\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)/5 + d\*\*6\*x\*\*5\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/2 - e\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(7/2)/7



**Mathematica [A]** time = 0.303671, size = 156, normalized size = 0.81

$$3d^7e \log(x) - 3d^7e \log\left(\sqrt{d^2 - e^2x^2} + d\right) - \frac{15}{16}d^7e \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) + \frac{1}{560}\sqrt{d^2 - e^2x^2}\left(-\frac{560d^7}{x} + 2496d^6e + 525d^5e^2x - 992d^4e^3x^2 - 770d^3e^4x^3 + 96d^2e^5x^4 + 280de^6x^5 + 80e^7x^6\right)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2))/x^2, x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(2496\*d^6\*e - (560\*d^7)/x + 525\*d^5\*e^2\*x - 992\*d^4\*e^3\*x^2 - 770\*d^3\*e^4\*x^3 + 96\*d^2\*e^5\*x^4 + 280\*d\*e^6\*x^5 + 80\*e^7\*x^6))/560 - (15\*d^7\*e\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/16 + 3\*d^7\*e\*Log[x] - 3\*d^7\*e\*Log[d + Sqrt[d^2 - e^2\*x^2]]

**Maple [A]** time = 0.019, size = 243, normalized size = 1.3

$$-\frac{d}{x}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{de^2x}{2}(-e^2x^2 + d^2)^{\frac{5}{2}} - \frac{5d^3e^2x}{8}(-e^2x^2 + d^2)^{\frac{3}{2}} - \frac{15d^5e^2x}{16}\sqrt{-e^2x^2 + d^2} - \frac{15d^7e^2}{16}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2 + d^2}}\right) - \frac{1}{\sqrt{e^2}} - \frac{e}{7}(-e^2x^2 + d^2)^{\frac{7}{2}} + \frac{3d^2e}{5}(-e^2x^2 + d^2)^{\frac{5}{2}} + d^4e(-e^2x^2 + d^2)^{\frac{3}{2}} + 3d^6e\sqrt{-e^2x^2 + d^2} - 3\frac{d^8e}{\sqrt{d^2}}\ln\left(\frac{2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(-e^2\*x^2+d^2)^(5/2)/x^2, x)

[Out] -d\*(-e^2\*x^2+d^2)^(7/2)/x-1/2\*d\*e^2\*x\*(-e^2\*x^2+d^2)^(5/2)-5/8\*d^3\*e^2\*x\*(-e^2\*x^2+d^2)^(3/2)-15/16\*d^5\*e^2\*x\*(-e^2\*x^2+d^2)^(1/2)-15/16\*d^7\*e^2/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))-1/7\*e\*(-e^2\*x^2+d^2)^(7/2)+3/5\*d^2\*e\*(-e^2\*x^2+d^2)^(5/2)+d^4\*e\*(-e^2\*x^2+d^2)^(3/2)+3\*d^6\*e\*(-e^2\*x^2+d^2)^(1/2)-3\*d^8\*e/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3/x^2, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Ericas [A]** time = 0.296064, size = 948, normalized size = 4.91

$$640de^{15}x^{15} + 2240d^2e^{14}x^{14} - 6272d^3e^{13}x^{13} - 30800d^4e^{12}x^{12} + 5376d^5e^{11}x^{11} + 148120d^6e^{10}x^{10} + 105280d^7e^9x^9 - 3497$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3/x^2,x, algorithm="fricas")
```

```
[Out] -1/560*(640*d*e^15*x^15 + 2240*d^2*e^14*x^14 - 6272*d^3*e^13*x^13
- 30800*d^4*e^12*x^12 + 5376*d^5*e^11*x^11 + 148120*d^6*e^10*x^10
+ 105280*d^7*e^9*x^9 - 349720*d^8*e^8*x^8 - 430080*d^9*e^7*x^7
+ 474320*d^10*e^6*x^6 + 609280*d^11*e^5*x^5 - 418880*d^12*e^4*x^4
- 286720*d^13*e^3*x^3 + 246400*d^14*e^2*x^2 - 71680*d^16 - 1050*
(d^7*e^9*x^9 - 32*d^9*e^7*x^7 + 160*d^11*e^5*x^5 - 256*d^13*e^3*x
^3 + 128*d^15*e*x + 8*(d^8*e^7*x^7 - 10*d^10*e^5*x^5 + 24*d^12*e^
3*x^3 - 16*d^14*e*x)*sqrt(-e^2*x^2 + d^2))*arctan(-(d - sqrt(-e^2
*x^2 + d^2))/(e*x)) - 1680*(d^7*e^9*x^9 - 32*d^9*e^7*x^7 + 160*d^
11*e^5*x^5 - 256*d^13*e^3*x^3 + 128*d^15*e*x + 8*(d^8*e^7*x^7 - 1
0*d^10*e^5*x^5 + 24*d^12*e^3*x^3 - 16*d^14*e*x)*sqrt(-e^2*x^2 + d
^2))*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (80*e^15*x^15 + 280*d*e
^14*x^14 - 2464*d^2*e^13*x^13 - 9730*d^3*e^12*x^12 + 8736*d^4*e^1
1*x^11 + 69965*d^5*e^10*x^10 + 29120*d^6*e^9*x^9 - 212240*d^7*e^8
*x^8 - 232960*d^8*e^7*x^7 + 334880*d^9*e^6*x^6 + 465920*d^10*e^5*
x^5 - 322560*d^11*e^4*x^4 - 286720*d^12*e^3*x^3 + 210560*d^13*e^2
*x^2 - 71680*d^15)*sqrt(-e^2*x^2 + d^2))/(e^8*x^9 - 32*d^2*e^6*x^
7 + 160*d^4*e^4*x^5 - 256*d^6*e^2*x^3 + 128*d^8*x + 8*(d*e^6*x^7
- 10*d^3*e^4*x^5 + 24*d^5*e^2*x^3 - 16*d^7*x)*sqrt(-e^2*x^2 + d^2
))
```

---

**Sympy [A]** time = 53.9275, size = 1057, normalized size = 5.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**2,x)
```

```
[Out] d**7*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x
/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2)
> 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/
(d*sqrt(1 - e**2*x**2/d**2)), True)) + 3*d**6*e*Piecewise((d**2/(
e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**
2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*sq
rt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/
(e**2*x**2) + 1), True)) + d**5*e**2*Piecewise((-I*d**2*acosh(e*x
/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*
d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asi
n(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) - 5*d**4*
e**3*Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), (-d**2 - e**2*x
**2)**(3/2)/(3*e**2), True)) - 5*d**3*e**4*Piecewise((-I*d**4*aco
sh(e*x/d)/(8*e**3) + I*d**3*x/(8*e**2*sqrt(-1 + e**2*x**2/d**2))
- 3*I*d*x**3/(8*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**5/(4*d*sq
rt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**4*asin(e*x
/d)/(8*e**3) - d**3*x/(8*e**2*sqrt(1 - e**2*x**2/d**2)) + 3*d*x**
3/(8*sqrt(1 - e**2*x**2/d**2)) - e**2*x**5/(4*d*sqrt(1 - e**2*x**
2/d**2)), True)) + d**2*e**5*Piecewise((-2*d**4*sqrt(d**2 - e**2*
x**2)/(15*e**4) - d**2*x**2*sqrt(d**2 - e**2*x**2)/(15*e**2) + x*
**4*sqrt(d**2 - e**2*x**2)/5, Ne(e, 0)), (x**4*sqrt(d**2)/4, True)
) + 3*d*e**6*Piecewise((-I*d**6*acosh(e*x/d)/(16*e**5) + I*d**5*x
/(16*e**4*sqrt(-1 + e**2*x**2/d**2)) - I*d**3*x**3/(48*e**2*sqrt(
-1 + e**2*x**2/d**2)) - 5*I*d*x**5/(24*sqrt(-1 + e**2*x**2/d**2))
+ I*e**2*x**7/(6*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**
2) > 1), (d**6*asin(e*x/d)/(16*e**5) - d**5*x/(16*e**4*sqrt(1 - e
**2*x**2/d**2)) + d**3*x**3/(48*e**2*sqrt(1 - e**2*x**2/d**2)) +
5*d*x**5/(24*sqrt(1 - e**2*x**2/d**2)) - e**2*x**7/(6*d*sqrt(1 -
e**2*x**2/d**2)), True)) + e**7*Piecewise((-8*d**6*sqrt(d**2 - e*
**2*x**2)/(105*e**6) - 4*d**4*x**2*sqrt(d**2 - e**2*x**2)/(105*e*
**4) - d**2*x**4*sqrt(d**2 - e**2*x**2)/(35*e**2) + x**6*sqrt(d**2
- e**2*x**2)/7, Ne(e, 0)), (x**6*sqrt(d**2)/6, True))
```

---

**GIAC/XCAS [A]** time = 0.2901, size = 269, normalized size = 1.39

$$\begin{aligned}
 & -\frac{15}{16} d^7 \arcsin\left(\frac{xe}{d}\right) \operatorname{esign}(d) - 3 d^7 \operatorname{eln}\left(\frac{\left|-2 de - 2 \sqrt{-x^2 e^2 + d^2} e\right| e^{(-2)}}{2|x|}\right) \\
 & + \frac{d^7 x e^3}{2 \left(de + \sqrt{-x^2 e^2 + d^2} e\right)} - \frac{\left(de + \sqrt{-x^2 e^2 + d^2} e\right) d^7 e^{(-1)}}{2x} \\
 & + \frac{1}{560} \left(2496 d^6 e + (525 d^5 e^2 - 2(496 d^4 e^3 + (385 d^3 e^4 - 4(12 d^2 e^5 + 5(2 x e^7 + 7 d e^6) x) x) x) x) \sqrt{-x^2 e^2 + d^2}\right)
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3/x^2,x, algorithm="giac")

[Out] -15/16\*d^7\*arcsin(x\*e/d)\*e\*sign(d) - 3\*d^7\*e\*ln(1/2\*abs(-2\*d\*e - 2\*sqrt(-x^2\*e^2 + d^2)\*e)\*e^(-2)/abs(x)) + 1/2\*d^7\*x\*e^3/(d\*e + sqrt(-x^2\*e^2 + d^2)\*e) - 1/2\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*d^7\*e^(-1)/x + 1/560\*(2496\*d^6\*e + (525\*d^5\*e^2 - 2\*(496\*d^4\*e^3 + (385\*d^3\*e^4 - 4\*(12\*d^2\*e^5 + 5\*(2\*x\*e^7 + 7\*d\*e^6)\*x)\*x)\*x)\*x)\*sqrt(-x^2\*e^2 + d^2)

$$3.73 \quad \int \frac{(d+ex)^3 (d^2 - e^2 x^2)^{5/2}}{x^3} dx$$

**Optimal.** Leaf size=207

$$\frac{1}{24} d^2 e^2 (4d - 85ex) (d^2 - e^2 x^2)^{3/2} - \frac{d (d^2 - e^2 x^2)^{7/2}}{2x^2} - \frac{3e (d^2 - e^2 x^2)^{7/2}}{x} + \frac{1}{30} e^2 (3d - 85ex) (d^2 - e^2 x^2)^{5/2} - \frac{85}{16} d^6 e^2 \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) - \frac{1}{2} d^6 e^2 \tanh^{-1} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right) + \frac{1}{16} d^4 e^2 (8d - 85ex) \sqrt{d^2 - e^2 x^2}$$

[Out]  $(d^4 e^2 (8d - 85ex) \sqrt{d^2 - e^2 x^2})/16 + (d^2 e^2 (4d - 85ex) (d^2 - e^2 x^2)^{3/2})/24 + (e^2 (3d - 85ex) (d^2 - e^2 x^2)^{5/2})/30 - (d (d^2 - e^2 x^2)^{7/2})/(2x^2) - (3e (d^2 - e^2 x^2)^{7/2})/x - (85 d^6 e^2 \text{ArcTan}[(ex)/\sqrt{d^2 - e^2 x^2}]) /16 - (d^6 e^2 \text{ArcTanh}[\sqrt{d^2 - e^2 x^2}/d])/2$

**Rubi [A]** time = 0.624537, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$

$$\frac{1}{24} d^2 e^2 (4d - 85ex) (d^2 - e^2 x^2)^{3/2} - \frac{d (d^2 - e^2 x^2)^{7/2}}{2x^2} - \frac{3e (d^2 - e^2 x^2)^{7/2}}{x} + \frac{1}{30} e^2 (3d - 85ex) (d^2 - e^2 x^2)^{5/2} - \frac{85}{16} d^6 e^2 \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) - \frac{1}{2} d^6 e^2 \tanh^{-1} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right) + \frac{1}{16} d^4 e^2 (8d - 85ex) \sqrt{d^2 - e^2 x^2}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2))/x^3, x]

[Out]  $(d^4 e^2 (8d - 85ex) \sqrt{d^2 - e^2 x^2})/16 + (d^2 e^2 (4d - 85ex) (d^2 - e^2 x^2)^{3/2})/24 + (e^2 (3d - 85ex) (d^2 - e^2 x^2)^{5/2})/30 - (d (d^2 - e^2 x^2)^{7/2})/(2x^2) - (3e (d^2 - e^2 x^2)^{7/2})/x - (85 d^6 e^2 \text{ArcTan}[(ex)/\sqrt{d^2 - e^2 x^2}]) /16 - (d^6 e^2 \text{ArcTanh}[\sqrt{d^2 - e^2 x^2}/d])/2$

**Rubi in Sympy [A]** time = 101.537, size = 238, normalized size = 1.15

$$\frac{d^7 \sqrt{d^2 - e^2 x^2}}{2x^2} - \frac{85d^6 e^2 \operatorname{atan} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right)}{16} - \frac{d^6 e^2 \operatorname{atanh} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right)}{2} - \frac{3d^6 e \sqrt{d^2 - e^2 x^2}}{x} + d^5 e^2 \sqrt{d^2 - e^2 x^2} - \frac{43d^4 e^3 x \sqrt{d^2 - e^2 x^2}}{16} + \frac{2d^3 e^2 (d^2 - e^2 x^2)^{3/2}}{3} + \frac{5d^2 e^5 x^3 \sqrt{d^2 - e^2 x^2}}{24} + \frac{3de^2 (d^2 - e^2 x^2)^{5/2}}{5} + \frac{e^7 x^5 \sqrt{d^2 - e^2 x^2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*3, x)

[Out]  $-d^{**7} \sqrt{d^{**2} - e^{**2} x^{**2}} / (2 * x^{**2}) - 85 * d^{**6} * e^{**2} * \operatorname{atan}(ex / \sqrt{d^{**2} - e^{**2} x^{**2}}) / 16 - d^{**6} * e^{**2} * \operatorname{atanh}(\sqrt{d^{**2} - e^{**2} x^{**2}} / d) / 2 - 3 * d^{**6} * e * \sqrt{d^{**2} - e^{**2} x^{**2}} / x + d^{**5} * e^{**2} * \sqrt{d^{**2} - e^{**2} x^{**2}} - 43 * d^{**4} * e^{**3} * x * \sqrt{d^{**2} - e^{**2} x^{**2}} / 16 + 2 * d^{**3} * e^{**2} * (d^{**2} - e^{**2} x^{**2})^{**3/2} / 3 + 5 * d^{**2} * e^{**5} * x^3 * \sqrt{d^{**2} - e^{**2} x^{**2}} / 24 + 3 * d * e^2 * (d^{**2} - e^{**2} x^{**2})^{**5/2} / 5 + e^{**7} * x^5 * \sqrt{d^{**2} - e^{**2} x^{**2}} / 6$

**Mathematica [A]** time = 0.296579, size = 166, normalized size = 0.8

$$\frac{1}{2}d^6e^2\log(x) - \frac{1}{2}d^6e^2\log\left(\sqrt{d^2 - e^2x^2} + d\right) - \frac{85}{16}d^6e^2\tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) + \frac{1}{240}\sqrt{d^2 - e^2x^2}\left(-\frac{120d^7}{x^2} - \frac{720d^6e}{x} + 544d^5e^2 - 645d^4e^3x - 448d^3e^4x^2 + 50d^2e^5x^3 + 144de^6x^4 + 40e^7x^5\right)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2))/x^3, x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(544\*d^5\*e^2 - (120\*d^7)/x^2 - (720\*d^6\*e)/x - 645\*d^4\*e^3\*x - 448\*d^3\*e^4\*x^2 + 50\*d^2\*e^5\*x^3 + 144\*d\*e^6\*x^4 + 40\*e^7\*x^5))/240 - (85\*d^6\*e^2\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/16 + (d^6\*e^2\*Log[x])/2 - (d^6\*e^2\*Log[d + Sqrt[d^2 - e^2\*x^2]])/2

**Maple [A]** time = 0.021, size = 252, normalized size = 1.2

$$-\frac{17e^3x}{6}(-e^2x^2 + d^2)^{\frac{5}{2}} - \frac{85e^3d^2x}{24}(-e^2x^2 + d^2)^{\frac{3}{2}} - \frac{85e^3d^4x}{16}\sqrt{-e^2x^2 + d^2} - \frac{85e^3d^6}{16}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2 + d^2}}\right)\frac{1}{\sqrt{e^2}} - \frac{d}{2x^2}(-e^2x^2 + d^2)^{\frac{7}{2}} + \frac{de^2}{10}(-e^2x^2 + d^2)^{\frac{5}{2}} + \frac{d^3e^2}{6}(-e^2x^2 + d^2)^{\frac{3}{2}} + \frac{d^5e^2}{2}\sqrt{-e^2x^2 + d^2} - \frac{d^7e^2}{2}\ln\left(\frac{1}{x}\left(2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2}\right)\right)\frac{1}{\sqrt{d^2}} - 3\frac{e(-e^2x^2 + d^2)^{7/2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(-e^2\*x^2+d^2)^(5/2)/x^3, x)

[Out] -17/6\*e^3\*x\*(-e^2\*x^2+d^2)^(5/2)-85/24\*e^3\*d^2\*x\*(-e^2\*x^2+d^2)^(3/2)-85/16\*e^3\*d^4\*x\*(-e^2\*x^2+d^2)^(1/2)-85/16\*e^3\*d^6/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))-1/2\*d\*(-e^2\*x^2+d^2)^(7/2)/x^2+1/10\*d\*e^2\*(-e^2\*x^2+d^2)^(5/2)+1/6\*d^3\*e^2\*(-e^2\*x^2+d^2)^(3/2)+1/2\*d^5\*e^2\*(-e^2\*x^2+d^2)^(1/2)-1/2\*d^7\*e^2/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)-3\*e\*(-e^2\*x^2+d^2)^(7/2)/x

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3/x^3, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.297993, size = 994, normalized size = 4.8

$$320de^{15}x^{15} + 1152d^2e^{14}x^{14} - 3120d^3e^{13}x^{13} - 16256d^4e^{12}x^{12} + 1320d^5e^{11}x^{11} + 82400d^6e^{10}x^{10} + 51800d^7e^9x^9 - 199360$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3/x^3,x, algorithm="fricas")`

[Out] 
$$\begin{aligned} & -1/240*(320*d*e^{15}*x^{15} + 1152*d^2*e^{14}*x^{14} - 3120*d^3*e^{13}*x^{13} \\ & - 16256*d^4*e^{12}*x^{12} + 1320*d^5*e^{11}*x^{11} + 82400*d^6*e^{10}*x^{10} \\ & + 51800*d^7*e^9*x^9 - 199360*d^8*e^8*x^8 - 122960*d^9*e^7*x^7 + \\ & 233280*d^{10}*e^6*x^6 + 16960*d^{11}*e^5*x^5 - 124800*d^{12}*e^4*x^4 + \\ & 147840*d^{13}*e^3*x^3 + 38400*d^{14}*e^2*x^2 - 92160*d^{15}*e*x - 15360 \\ & *d^{16} - 2550*(d^6*e^{10}*x^{10} - 32*d^8*e^8*x^8 + 160*d^{10}*e^6*x^6 - \\ & 256*d^{12}*e^4*x^4 + 128*d^{14}*e^2*x^2 + 8*(d^7*e^8*x^8 - 10*d^9*e^6 \\ & *x^6 + 24*d^{11}*e^4*x^4 - 16*d^{13}*e^2*x^2)*\sqrt{-e^2*x^2 + d^2}) \\ & * \arctan(-(d - \sqrt{-e^2*x^2 + d^2})/(e*x)) - 120*(d^6*e^{10}*x^{10} - \\ & 32*d^8*e^8*x^8 + 160*d^{10}*e^6*x^6 - 256*d^{12}*e^4*x^4 + 128*d^{14}*e^2 \\ & *x^2 + 8*(d^7*e^8*x^8 - 10*d^9*e^6*x^6 + 24*d^{11}*e^4*x^4 - 16*d^{13} \\ & *e^2*x^2)*\sqrt{-e^2*x^2 + d^2}) * \log(-(d - \sqrt{-e^2*x^2 + d^2}) \\ & )/x) - (40*e^{15}*x^{15} + 144*d*e^{14}*x^{14} - 1230*d^2*e^{13}*x^{13} - 505 \\ & 6*d^3*e^{12}*x^{12} + 4155*d^4*e^{11}*x^{11} + 37920*d^5*e^{10}*x^{10} + 1768 \\ & 0*d^6*e^9*x^9 - 121720*d^7*e^8*x^8 - 87840*d^8*e^7*x^7 + 180480*d^9 \\ & *e^6*x^6 + 56320*d^{10}*e^5*x^5 - 111360*d^{11}*e^4*x^4 + 101760*d^{12} \\ & *e^3*x^3 + 30720*d^{13}*e^2*x^2 - 92160*d^{14}*e*x - 15360*d^{15}) * \sqrt{-e^2*x^2 + d^2}) \\ & / (e^8*x^{10} - 32*d^2*e^6*x^8 + 160*d^4*e^4*x^6 - 256*d^6*e^2*x^4 + 128*d^8*x^2 + 8*(d^7 \\ & *e^6*x^8 - 10*d^3*e^4*x^6 + 24*d^5*e^2*x^4 - 16*d^7*x^2)*\sqrt{-e^2*x^2 + d^2}) \end{aligned}$$

**Sympy [A]** time = 56.9192, size = 1059, normalized size = 5.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**3,x)`

[Out] 
$$\begin{aligned} & d^{**7} \text{Piecewise}((-d^{**2}/(2*e*x^{**3}*\sqrt{d^{**2}/(e^{**2}*x^{**2}) - 1)}) + e/( \\ & 2*x*\sqrt{d^{**2}/(e^{**2}*x^{**2}) - 1}) + e^{**2}*\text{acosh}(d/(e*x))/(2*d), \text{Abs}( \\ & d^{**2}/(e^{**2}*x^{**2})) > 1), (-I*e*\sqrt{-d^{**2}/(e^{**2}*x^{**2}) + 1})/(2*x) - \\ & I*e^{**2}*\text{asin}(d/(e*x))/(2*d), \text{True})) + 3*d^{**6}*e*\text{Piecewise}((I*d/(x* \\ & \sqrt{-1 + e^{**2}*x^{**2}/d^{**2}}) + I*e*\text{acosh}(e*x/d) - I*e^{**2}*x/(d*\sqrt{ \\ & -1 + e^{**2}*x^{**2}/d^{**2}}), \text{Abs}(e^{**2}*x^{**2}/d^{**2}) > 1), (-d/(x*\sqrt{1 - \\ & e^{**2}*x^{**2}/d^{**2}}) - e*\text{asin}(e*x/d) + e^{**2}*x/(d*\sqrt{1 - e^{**2}*x^{**2}/d \\ & **2}), \text{True})) + d^{**5}*e^{**2}*\text{Piecewise}((d^{**2}/(e*x*\sqrt{d^{**2}/(e^{**2}*x* \\ & **2) - 1}) - d*\text{acosh}(d/(e*x)) - e*x/\sqrt{d^{**2}/(e^{**2}*x^{**2}) - 1}), \text{Ab} \\ & s(d^{**2}/(e^{**2}*x^{**2})) > 1), (-I*d^{**2}/(e*x*\sqrt{-d^{**2}/(e^{**2}*x^{**2}) + \\ & 1}) + I*d*\text{asin}(d/(e*x)) + I*e*x/\sqrt{-d^{**2}/(e^{**2}*x^{**2}) + 1}), \text{True} \\ & )) - 5*d^{**4}*e^{**3}*\text{Piecewise}((-I*d^{**2}*\text{acosh}(e*x/d)/(2*e) - I*d*x/(2 \\ & *\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}}) + I*e^{**2}*x^{**3}/(2*d*\sqrt{-1 + e^{**2}*x^{** \\ & 2}/d^{**2}}), \text{Abs}(e^{**2}*x^{**2}/d^{**2}) > 1), (d^{**2}*\text{asin}(e*x/d)/(2*e) + d*x \\ & *\sqrt{1 - e^{**2}*x^{**2}/d^{**2}})/2, \text{True})) - 5*d^{**3}*e^{**4}*\text{Piecewise}((x^{**2} \\ & *\sqrt{d^{**2}}/2, \text{Eq}(e^{**2}, 0)), (-d^{**2} - e^{**2}*x^{**2})^{**}(3/2)/(3*e^{**2} \\ & ), \text{True})) + d^{**2}*e^{**5}*\text{Piecewise}((-I*d^{**4}*\text{acosh}(e*x/d)/(8*e^{**3}) + I \\ & *d^{**3}*x/(8*e^{**2}*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}}) - 3*I*d*x^{**3}/(8*\sqrt{-1 \\ & + e^{**2}*x^{**2}/d^{**2}}) + I*e^{**2}*x^{**5}/(4*d*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2} \\ & )), \text{Abs}(e^{**2}*x^{**2}/d^{**2}) > 1), (d^{**4}*\text{asin}(e*x/d)/(8*e^{**3}) - d^{**3}*x/ \\ & (8*e^{**2}*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}}) + 3*d*x^{**3}/(8*\sqrt{1 - e^{**2}*x^{** \\ & 2}/d^{**2}}) - e^{**2}*x^{**5}/(4*d*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}}), \text{True})) + 3*d \\ & *e^{**6}*\text{Piecewise}((-2*d^{**4}*\sqrt{d^{**2} - e^{**2}*x^{**2}})/(15*e^{**4}) - d^{**2}* \\ & x^{**2}*\sqrt{d^{**2} - e^{**2}*x^{**2}})/(15*e^{**2}) + x^{**4}*\sqrt{d^{**2} - e^{**2}*x^{** \\ & 2}}/5, \text{Ne}(e, 0)), (x^{**4}*\sqrt{d^{**2}}/4, \text{True})) + e^{**7}*\text{Piecewise}((-I* \\ & d^{**6}*\text{acosh}(e*x/d)/(16*e^{**5}) + I*d^{**5}*x/(16*e^{**4}*\sqrt{-1 + e^{**2}*x* \\ & **2}/d^{**2})) - I*d^{**3}*x^{**3}/(48*e^{**2}*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}}) - 5*I \\ & *d*x^{**5}/(24*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}}) + I*e^{**2}*x^{**7}/(6*d*\sqrt{-1 \\ & + e^{**2}*x^{**2}/d^{**2}}), \text{Abs}(e^{**2}*x^{**2}/d^{**2}) > 1), (d^{**6}*\text{asin}(e*x/d)/ \\ & (16*e^{**5}) - d^{**5}*x/(16*e^{**4}*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}}) + d^{**3}*x^{**3} \\ & / (48*e^{**2}*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}}) + 5*d*x^{**5}/(24*\sqrt{1 - e^{**2}* \\ & x^{**2}/d^{**2}}) - e^{**2}*x^{**7}/(6*d*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}}), \text{True})) \end{aligned}$$

**GIAC/XCAS [A]** time = 0.290056, size = 354, normalized size = 1.71

$$\begin{aligned}
 & -\frac{85}{16} d^6 \arcsin\left(\frac{xe}{d}\right) e^2 \operatorname{sign}(d) - \frac{1}{2} d^6 e^2 \ln\left(\frac{\left|-2de - 2\sqrt{-x^2e^2 + d^2}e\right|e^{(-2)}}{2|x|}\right) \\
 & - \frac{1}{8} \left( \frac{12 \left( de + \sqrt{-x^2e^2 + d^2}e \right) d^6 e^8}{x} + \frac{\left( de + \sqrt{-x^2e^2 + d^2}e \right)^2 d^6 e^6}{x^2} \right) e^{(-8)} \\
 & + \frac{1}{240} \left( 544 d^5 e^2 - (645 d^4 e^3 + 2(224 d^3 e^4 - (25 d^2 e^5 + 4(5 x e^7 + 18 d e^6) x) x) x) \sqrt{-x^2 e^2 + d^2} \right. \\
 & \left. \left( d^6 e^6 + \frac{12 \left( de + \sqrt{-x^2 e^2 + d^2} e \right) d^6 e^4}{x} \right) x^2 \right. \\
 & \left. + \frac{\left( d^6 e^6 + \frac{12 \left( de + \sqrt{-x^2 e^2 + d^2} e \right) d^6 e^4}{x} \right) x^2}{8 \left( de + \sqrt{-x^2 e^2 + d^2} e \right)^2} \right)
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3/x^3,x, algorithm="giac")

[Out] -85/16\*d^6\*arcsin(x\*e/d)\*e^2\*sign(d) - 1/2\*d^6\*e^2\*ln(1/2\*abs(-2\*d\*e - 2\*sqrt(-x^2\*e^2 + d^2)\*e)\*e^(-2)/abs(x)) - 1/8\*(12\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*d^6\*e^8/x + (d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^2\*d^6\*e^6/x^2)\*e^(-8) + 1/240\*(544\*d^5\*e^2 - (645\*d^4\*e^3 + 2\*(224\*d^3\*e^4 - (25\*d^2\*e^5 + 4\*(5\*x\*e^7 + 18\*d\*e^6)\*x)\*x)\*x)\*sqrt(-x^2\*e^2 + d^2) + 1/8\*(d^6\*e^6 + 12\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*d^6\*e^4/x)\*x^2/(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^2

$$3.74 \quad \int \frac{(d+ex)^3 (d^2 - e^2 x^2)^{5/2}}{x^4} dx$$

**Optimal.** Leaf size=210

$$\frac{3e (d^2 - e^2 x^2)^{7/2}}{2x^2} - \frac{e^2(50d + 39ex) (d^2 - e^2 x^2)^{5/2}}{30x} - \frac{d (d^2 - e^2 x^2)^{7/2}}{3x^3} - \frac{1}{12} d e^3 (26d + 25ex) (d^2 - e^2 x^2)^{3/2} - \frac{25}{8} d^5 e^3 \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) + \frac{13}{2} d^5 e^3 \tanh^{-1} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right) - \frac{1}{8} d^3 e^3 (52d + 25ex) \sqrt{d^2 - e^2 x^2}$$

[Out]  $-(d^3 e^3 (52d + 25ex) \sqrt{d^2 - e^2 x^2})/8 - (d^5 e^3 (26d + 25ex) (d^2 - e^2 x^2)^{3/2})/12 - (e^2 (50d + 39ex) (d^2 - e^2 x^2)^{5/2})/(30x) - (d (d^2 - e^2 x^2)^{7/2})/(3x^3) - (3e^3 (d^2 - e^2 x^2)^{7/2})/(2x^2) - (25d^5 e^3 \text{ArcTan}[(ex)/\sqrt{d^2 - e^2 x^2}])/8 + (13d^5 e^3 \text{ArcTanh}[\sqrt{d^2 - e^2 x^2}/d])/2$

**Rubi [A]** time = 0.628499, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{3e (d^2 - e^2 x^2)^{7/2}}{2x^2} - \frac{e^2(50d + 39ex) (d^2 - e^2 x^2)^{5/2}}{30x} - \frac{d (d^2 - e^2 x^2)^{7/2}}{3x^3} - \frac{1}{12} d e^3 (26d + 25ex) (d^2 - e^2 x^2)^{3/2} - \frac{25}{8} d^5 e^3 \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) + \frac{13}{2} d^5 e^3 \tanh^{-1} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right) - \frac{1}{8} d^3 e^3 (52d + 25ex) \sqrt{d^2 - e^2 x^2}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2))/x^4, x]

[Out]  $-(d^3 e^3 (52d + 25ex) \sqrt{d^2 - e^2 x^2})/8 - (d^5 e^3 (26d + 25ex) (d^2 - e^2 x^2)^{3/2})/12 - (e^2 (50d + 39ex) (d^2 - e^2 x^2)^{5/2})/(30x) - (d (d^2 - e^2 x^2)^{7/2})/(3x^3) - (3e^3 (d^2 - e^2 x^2)^{7/2})/(2x^2) - (25d^5 e^3 \text{ArcTan}[(ex)/\sqrt{d^2 - e^2 x^2}])/8 + (13d^5 e^3 \text{ArcTanh}[\sqrt{d^2 - e^2 x^2}/d])/2$

**Rubi in Sympy [A]** time = 89.7743, size = 243, normalized size = 1.16

$$\frac{d^7 \sqrt{d^2 - e^2 x^2}}{3x^3} - \frac{3d^6 e \sqrt{d^2 - e^2 x^2}}{2x^2} - \frac{25d^5 e^3 \operatorname{atan} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right)}{8} + \frac{13d^5 e^3 \operatorname{atanh} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right)}{2} - \frac{2d^5 e^2 \sqrt{d^2 - e^2 x^2}}{3x} - 5d^4 e^3 \sqrt{d^2 - e^2 x^2} - \frac{23d^3 e^4 x \sqrt{d^2 - e^2 x^2}}{8} - \frac{2d^2 e^3 (d^2 - e^2 x^2)^{3/2}}{3} + \frac{3de^6 x^3 \sqrt{d^2 - e^2 x^2}}{4} + \frac{e^3 (d^2 - e^2 x^2)^{5/2}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*4, x)

[Out]  $-d^7 \sqrt{d^2 - e^2 x^2}/(3x^3) - 3d^6 e \sqrt{d^2 - e^2 x^2}/(2x^2) - 25d^5 e^3 \operatorname{atan}(ex/\sqrt{d^2 - e^2 x^2})/8 + 13d^5 e^3 \operatorname{atanh}(\sqrt{d^2 - e^2 x^2}/d)/2 - 2d^5 e^2 \sqrt{d^2 - e^2 x^2}/(3x) - 5d^4 e^3 \sqrt{d^2 - e^2 x^2} - 23d^3 e^4 x \sqrt{d^2 - e^2 x^2}/8 - 2d^2 e^3 (d^2 - e^2 x^2)^{3/2}/3 + 3de^6 x^3 \sqrt{d^2 - e^2 x^2}/4 + e^3 (d^2 - e^2 x^2)^{5/2}/5$



**Mathematica [A]** time = 0.459654, size = 167, normalized size = 0.8

$$\frac{1}{8} \left( -52d^5 e^3 \log(x) + 52d^5 e^3 \log\left(\sqrt{d^2 - e^2 x^2} + d\right) - 25d^5 e^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right) - \frac{\sqrt{d^2 - e^2 x^2} (40d^7 + 180d^6 ex + 80d^5 e^2 x^2 + 656d^4 e^3 x^3 + 345d^3 e^4 x^4 - 32d^2 e^5 x^5 - 90de^6 x^6 - 24e^7 x^7)}{15x^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2))/x^4, x]

[Out]  $(-\text{Sqrt}[d^2 - e^2*x^2]*(40*d^7 + 180*d^6*e*x + 80*d^5*e^2*x^2 + 656*d^4*e^3*x^3 + 345*d^3*e^4*x^4 - 32*d^2*e^5*x^5 - 90*d*e^6*x^6 - 24*e^7*x^7))/(15*x^3) - 25*d^5*e^3*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]] - 52*d^5*e^3*\text{Log}[x] + 52*d^5*e^3*\text{Log}[d + \text{Sqrt}[d^2 - e^2*x^2]])/8$

**Maple [A]** time = 0.022, size = 277, normalized size = 1.3

$$\begin{aligned} & -\frac{d}{3x^3} (-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{5e^2}{3dx} (-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{5e^4x}{3d} (-e^2x^2 + d^2)^{\frac{5}{2}} - \frac{25de^4x}{12} (-e^2x^2 + d^2)^{\frac{3}{2}} \\ & - \frac{25d^3e^4x}{8} \sqrt{-e^2x^2 + d^2} - \frac{25d^5e^4}{8} \arctan\left(x\sqrt{e^2} \frac{1}{\sqrt{-e^2x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} \\ & - \frac{13e^3}{10} (-e^2x^2 + d^2)^{\frac{5}{2}} - \frac{13e^3d^2}{6} (-e^2x^2 + d^2)^{\frac{3}{2}} - \frac{13e^3d^4}{2} \sqrt{-e^2x^2 + d^2} \\ & + \frac{13e^3d^6}{2} \ln\left(\frac{1}{x} (2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2})\right) \frac{1}{\sqrt{d^2}} - \frac{3e}{2x^2} (-e^2x^2 + d^2)^{\frac{7}{2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(-e^2\*x^2+d^2)^(5/2)/x^4, x)

[Out]  $-1/3*d*(-e^2*x^2+d^2)^(7/2)/x^3-5/3/d*e^2/x*(-e^2*x^2+d^2)^(7/2)-5/3/d*e^4*x*(-e^2*x^2+d^2)^(5/2)-25/12*d*e^4*x*(-e^2*x^2+d^2)^(3/2)-25/8*d^3*e^4*x*(-e^2*x^2+d^2)^(1/2)-25/8*d^5*e^4/(e^2)^(1/2)*\text{arctan}((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-13/10*e^3*(-e^2*x^2+d^2)^(5/2)-13/6*e^3*d^2*(-e^2*x^2+d^2)^(3/2)-13/2*e^3*d^4*(-e^2*x^2+d^2)^(1/2)+13/2*e^3*d^6/(d^2)^(1/2)*\ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-3/2*e*(-e^2*x^2+d^2)^(7/2)/x^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3/x^4, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.299202, size = 979, normalized size = 4.66

$$192de^{15}x^{15} + 720d^2e^{14}x^{14} - 1856d^3e^{13}x^{13} - 10680d^4e^{12}x^{12} - 880d^5e^{11}x^{11} + 54200d^6e^{10}x^{10} + 36320d^7e^9x^9 - 115920d^8e^8x^8 - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3/x^4,x, algorithm="fricas")`

[Out] 
$$\begin{aligned} & -1/120*(192*d*e^{15}*x^{15} + 720*d^2*e^{14}*x^{14} - 1856*d^3*e^{13}*x^{13} \\ & - 10680*d^4*e^{12}*x^{12} - 880*d^5*e^{11}*x^{11} + 54200*d^6*e^{10}*x^{10} + \\ & 36320*d^7*e^9*x^9 - 115920*d^8*e^8*x^8 - 64800*d^9*e^7*x^7 + 103 \\ & 680*d^{10}*e^6*x^6 - 2880*d^{11}*e^5*x^5 - 29440*d^{12}*e^4*x^4 + 57600 \\ & *d^{13}*e^3*x^3 + 2560*d^{14}*e^2*x^2 - 23040*d^{15}*e*x - 5120*d^{16} - \\ & 750*(d^5*e^{11}*x^{11} - 32*d^7*e^9*x^9 + 160*d^9*e^7*x^7 - 256*d^{11}* \\ & e^5*x^5 + 128*d^{13}*e^3*x^3 + 8*(d^6*e^9*x^9 - 10*d^8*e^7*x^7 + 24 \\ & *d^{10}*e^5*x^5 - 16*d^{12}*e^3*x^3)*\sqrt{-e^2*x^2 + d^2})*\arctan(-(d \\ & - \sqrt{-e^2*x^2 + d^2})/(e*x)) + 780*(d^5*e^{11}*x^{11} - 32*d^7*e^9 \\ & *x^9 + 160*d^9*e^7*x^7 - 256*d^{11}*e^5*x^5 + 128*d^{13}*e^3*x^3 + 8* \\ & (d^6*e^9*x^9 - 10*d^8*e^7*x^7 + 24*d^{10}*e^5*x^5 - 16*d^{12}*e^3*x^3 \\ & )*\sqrt{-e^2*x^2 + d^2})*\log(-(d - \sqrt{-e^2*x^2 + d^2})/x) - (24* \\ & e^{15}*x^{15} + 90*d*e^{14}*x^{14} - 736*d^2*e^{13}*x^{13} - 3225*d^3*e^{12}*x^{12} \\ & + 2160*d^4*e^{11}*x^{11} + 25360*d^5*e^{10}*x^{10} + 14540*d^6*e^9*x^9 \\ & - 75720*d^7*e^8*x^8 - 51840*d^8*e^7*x^7 + 88320*d^9*e^6*x^6 + 17 \\ & 280*d^{10}*e^5*x^5 - 30080*d^{11}*e^4*x^4 + 46080*d^{12}*e^3*x^3 - 2304 \\ & 0*d^{14}*e*x - 5120*d^{15})*\sqrt{-e^2*x^2 + d^2})/(e^8*x^{11} - 32*d^2* \\ & e^6*x^9 + 160*d^4*e^4*x^7 - 256*d^6*e^2*x^5 + 128*d^8*x^3 + 8*(d* \\ & e^6*x^9 - 10*d^3*e^4*x^7 + 24*d^5*e^2*x^5 - 16*d^7*x^3)*\sqrt{-e^2 \\ & *x^2 + d^2}) \end{aligned}$$

**Sympy [A]** time = 41.2787, size = 911, normalized size = 4.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**4,x)`

[Out] 
$$\begin{aligned} & d^{**7}*Piecewise((-e*\sqrt{d^{**2}/(e^{**2}*x^{**2})} - 1)/(3*x^{**2}) + e^{**3}*\sqrt{ \\ & t(d^{**2}/(e^{**2}*x^{**2}) - 1)/(3*d^{**2}), Abs(d^{**2}/(e^{**2}*x^{**2})) > 1), (-I \\ & *e*\sqrt{-d^{**2}/(e^{**2}*x^{**2})} + 1)/(3*x^{**2}) + I*e^{**3}*\sqrt{-d^{**2}/(e^{**2} \\ & *x^{**2}) + 1)/(3*d^{**2}), True)) + 3*d^{**6}*e*Piecewise((-d^{**2}/(2*e*x^{** \\ & 3}*\sqrt{d^{**2}/(e^{**2}*x^{**2})} - 1)) + e/(2*x*\sqrt{d^{**2}/(e^{**2}*x^{**2})} - 1) \\ & ) + e^{**2}*acosh(d/(e*x))/(2*d), Abs(d^{**2}/(e^{**2}*x^{**2})) > 1), (-I*e* \\ & \sqrt{-d^{**2}/(e^{**2}*x^{**2})} + 1)/(2*x) - I*e^{**2}*asin(d/(e*x))/(2*d), T \\ & rue)) + d^{**5}*e^{**2}*Piecewise((I*d/(x*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}}) + \\ & I*e*acosh(e*x/d) - I*e^{**2}*x/(d*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}}), Abs(e* \\ & **2*x^{**2}/d^{**2}) > 1), (-d/(x*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}}) - e*asin(e*x \\ & /d) + e^{**2}*x/(d*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}}), True)) - 5*d^{**4}*e^{**3}*P \\ & iecewise((d^{**2}/(e*x*\sqrt{d^{**2}/(e^{**2}*x^{**2})} - 1)) - d*acosh(d/(e*x) \\ & ) - e*x/\sqrt{d^{**2}/(e^{**2}*x^{**2})} - 1), Abs(d^{**2}/(e^{**2}*x^{**2})) > 1), ( \\ & -I*d^{**2}/(e*x*\sqrt{-d^{**2}/(e^{**2}*x^{**2})} + 1)) + I*d*asin(d/(e*x)) + I \\ & *e*x/\sqrt{-d^{**2}/(e^{**2}*x^{**2})} + 1), True)) - 5*d^{**3}*e^{**4}*Piecewise( \\ & (-I*d^{**2}*acosh(e*x/d)/(2*e) - I*d*x/(2*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}}) \\ & + I*e^{**2}*x^{**3}/(2*d*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}}), Abs(e^{**2}*x^{**2}/d^{** \\ & 2}) > 1), (d^{**2}*asin(e*x/d)/(2*e) + d*x*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}}/2 \\ & , True)) + d^{**2}*e^{**5}*Piecewise((x^{**2}*\sqrt{d^{**2}}/2, Eq(e^{**2}, 0)), \\ & (-d^{**2} - e^{**2}*x^{**2})^{**3/2}/(3*e^{**2}), True)) + 3*d*e^{**6}*Piecewise \\ & ((-I*d^{**4}*acosh(e*x/d)/(8*e^{**3}) + I*d^{**3}*x/(8*e^{**2}*\sqrt{-1 + e^{**2} \\ & *x^{**2}/d^{**2}}) - 3*I*d*x^{**3}/(8*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}}) + I*e^{**2} \\ & *x^{**5}/(4*d*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}}), Abs(e^{**2}*x^{**2}/d^{**2}) > 1), ( \\ & d^{**4}*asin(e*x/d)/(8*e^{**3}) - d^{**3}*x/(8*e^{**2}*\sqrt{1 - e^{**2}*x^{**2}/d^{** \\ & 2}}) + 3*d*x^{**3}/(8*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}}) - e^{**2}*x^{**5}/(4*d*\sqrt{ \\ & 1 - e^{**2}*x^{**2}/d^{**2}}), True)) + e^{**7}*Piecewise((-2*d^{**4}*\sqrt{d^{**2} \\ & - e^{**2}*x^{**2}})/(15*e^{**4}) - d^{**2}*x^{**2}*\sqrt{d^{**2} - e^{**2}*x^{**2}})/(15*e \\ & **2) + x^{**4}*\sqrt{d^{**2} - e^{**2}*x^{**2}}/5, Ne(e, 0)), (x^{**4}*\sqrt{d^{**2}}/ \\ & 4, True)) \end{aligned}$$

**GIAC/XCAS [A]** time = 0.291921, size = 429, normalized size = 2.04

$$\begin{aligned}
 & -\frac{25}{8} d^5 \arcsin\left(\frac{xe}{d}\right) e^3 \operatorname{sign}(d) + \frac{13}{2} d^5 e^3 \ln\left(\frac{\left|-2de - 2\sqrt{-x^2e^2 + d^2}e\right|e^{(-2)}}{2|x|}\right) \\
 & + \frac{\left(d^5 e^8 + \frac{9(de + \sqrt{-x^2e^2 + d^2}e)d^5 e^6}{x} + \frac{9(de + \sqrt{-x^2e^2 + d^2}e)^2 d^5 e^4}{x^2}\right) x^3 e}{24 (de + \sqrt{-x^2e^2 + d^2}e)^3} \\
 & - \frac{1}{24} \left(\frac{9(de + \sqrt{-x^2e^2 + d^2}e)d^5 e^{16}}{x} + \frac{9(de + \sqrt{-x^2e^2 + d^2}e)^2 d^5 e^{14}}{x^2} + \frac{(de + \sqrt{-x^2e^2 + d^2}e)^3 d^5 e^{12}}{x^3}\right) e^{(-15)} \\
 & - \frac{1}{120} (656 d^4 e^3 + (345 d^3 e^4 - 2(16 d^2 e^5 + 3(4xe^7 + 15de^6)x)x)\sqrt{-x^2e^2 + d^2}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3/x^4,x, algorithm="giac")

[Out] -25/8\*d^5\*arcsin(x\*e/d)\*e^3\*sign(d) + 13/2\*d^5\*e^3\*ln(1/2\*abs(-2\*d\*e - 2\*sqrt(-x^2\*e^2 + d^2)\*e)\*e^(-2)/abs(x)) + 1/24\*(d^5\*e^8 + 9\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*d^5\*e^6/x + 9\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^2\*d^5\*e^4/x^2)\*x^3\*e/(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^3 - 1/24\*(9\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*d^5\*e^16/x + 9\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^2\*d^5\*e^14/x^2 + (d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^3\*d^5\*e^12/x^3)\*e^(-15) - 1/120\*(656\*d^4\*e^3 + (345\*d^3\*e^4 - 2\*(16\*d^2\*e^5 + 3\*(4\*x\*e^7 + 15\*d\*e^6)\*x)\*x)\*sqrt(-x^2\*e^2 + d^2)

$$3.75 \quad \int \frac{(d+ex)^3 (d^2 - e^2 x^2)^{5/2}}{x^5} dx$$

**Optimal.** Leaf size=209

$$\frac{3e^2(3d+2ex)(d^2 - e^2 x^2)^{5/2}}{8x^2} - \frac{d(d^2 - e^2 x^2)^{7/2}}{4x^4} - \frac{e(d^2 - e^2 x^2)^{7/2}}{x^3} - \frac{45}{8}d^2 e^4 (d-ex)\sqrt{d^2 - e^2 x^2} + \frac{15de^3(2d-ex)(d^2 - e^2 x^2)^{3/2}}{8x} + \frac{45}{8}d^4 e^4 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right) + \frac{45}{8}d^4 e^4 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)$$

[Out]  $(-45*d^2*e^4*(d - e*x)*\text{Sqrt}[d^2 - e^2*x^2])/8 + (15*d*e^3*(2*d - e*x)*(d^2 - e^2*x^2)^{(3/2)})/(8*x) - (3*e^2*(3*d + 2*e*x)*(d^2 - e^2*x^2)^{(5/2)})/(8*x^2) - (d*(d^2 - e^2*x^2)^{(7/2)})/(4*x^4) - (e*(d^2 - e^2*x^2)^{(7/2)})/x^3 + (45*d^4*e^4*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/8 + (45*d^4*e^4*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/8$

**Rubi [A]** time = 0.621992, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{3e^2(3d+2ex)(d^2 - e^2 x^2)^{5/2}}{8x^2} - \frac{d(d^2 - e^2 x^2)^{7/2}}{4x^4} - \frac{e(d^2 - e^2 x^2)^{7/2}}{x^3} - \frac{45}{8}d^2 e^4 (d-ex)\sqrt{d^2 - e^2 x^2} + \frac{15de^3(2d-ex)(d^2 - e^2 x^2)^{3/2}}{8x} + \frac{45}{8}d^4 e^4 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right) + \frac{45}{8}d^4 e^4 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2))/x^5, x]

[Out]  $(-45*d^2*e^4*(d - e*x)*\text{Sqrt}[d^2 - e^2*x^2])/8 + (15*d*e^3*(2*d - e*x)*(d^2 - e^2*x^2)^{(3/2)})/(8*x) - (3*e^2*(3*d + 2*e*x)*(d^2 - e^2*x^2)^{(5/2)})/(8*x^2) - (d*(d^2 - e^2*x^2)^{(7/2)})/(4*x^4) - (e*(d^2 - e^2*x^2)^{(7/2)})/x^3 + (45*d^4*e^4*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/8 + (45*d^4*e^4*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/8$

**Rubi in Sympy [A]** time = 83.7857, size = 238, normalized size = 1.14

$$-\frac{d^7\sqrt{d^2 - e^2 x^2}}{4x^4} - \frac{d^6 e\sqrt{d^2 - e^2 x^2}}{x^3} - \frac{3d^5 e^2\sqrt{d^2 - e^2 x^2}}{8x^2} + \frac{45d^4 e^4 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{8} + \frac{45d^4 e^4 \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{8} + \frac{6d^4 e^3\sqrt{d^2 - e^2 x^2}}{x} - \frac{5d^3 e^4\sqrt{d^2 - e^2 x^2}}{8} + \frac{3d^2 e^5 x\sqrt{d^2 - e^2 x^2}}{8} - de^4 (d^2 - e^2 x^2)^{\frac{3}{2}} + \frac{e^7 x^3\sqrt{d^2 - e^2 x^2}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*5, x)

[Out]  $-d**7*\text{sqrt}(d**2 - e**2*x**2)/(4*x**4) - d**6*e*\text{sqrt}(d**2 - e**2*x**2)/x**3 - 3*d**5*e**2*\text{sqrt}(d**2 - e**2*x**2)/(8*x**2) + 45*d**4*e**4*\text{atan}(e*x/\text{sqrt}(d**2 - e**2*x**2))/8 + 45*d**4*e**4*\text{atanh}(\text{sqrt}(d**2 - e**2*x**2)/d)/8 + 6*d**4*e**3*\text{sqrt}(d**2 - e**2*x**2)/x - 5*d**3*e**4*\text{sqrt}(d**2 - e**2*x**2) + 3*d**2*e**5*x*\text{sqrt}(d**2 - e**2*x**2)/8 - d*e**4*(d**2 - e**2*x**2)**(3/2) + e**7*x**3*\text{sqrt}(d**2 - e**2*x**2)/4$

**Mathematica [A]** time = 0.321692, size = 164, normalized size = 0.78

$$\frac{1}{8} \left( -45d^4 e^4 \log(x) + 45d^4 e^4 \log\left(\sqrt{d^2 - e^2 x^2} + d\right) + 45d^4 e^4 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right) + \frac{\sqrt{d^2 - e^2 x^2} (-2d^7 - 8d^6 ex - 3d^5 e^2 x^2 + 48d^4 e^3 x^3 - 48d^3 e^4 x^4 + 3d^2 e^5 x^5 + 8de^6 x^6 + 2e^7 x^7)}{x^4} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2))/x^5, x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(-2\*d^7 - 8\*d^6\*e\*x - 3\*d^5\*e^2\*x^2 + 48\*d^4\*e^3\*x^3 - 48\*d^3\*e^4\*x^4 + 3\*d^2\*e^5\*x^5 + 8\*d\*e^6\*x^6 + 2\*e^7\*x^7))/x^4 + 45\*d^4\*e^4\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] - 45\*d^4\*e^4\*Log[x] + 45\*d^4\*e^4\*Log[d + Sqrt[d^2 - e^2\*x^2]])/8

**Maple [A]** time = 0.029, size = 302, normalized size = 1.4

$$\begin{aligned} & -\frac{d}{4x^4} (-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{9e^2}{8dx^2} (-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{9e^4}{8d} (-e^2x^2 + d^2)^{\frac{5}{2}} - \frac{15de^4}{8} (-e^2x^2 + d^2)^{\frac{3}{2}} \\ & - \frac{45d^3e^4}{8} \sqrt{-e^2x^2 + d^2} + \frac{45d^5e^4}{8} \ln\left(\frac{1}{x} \left(2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}} \\ & + 3 \frac{e^3(-e^2x^2 + d^2)^{7/2}}{d^2x} + 3 \frac{e^5x(-e^2x^2 + d^2)^{5/2}}{d^2} + \frac{15e^5x}{4} (-e^2x^2 + d^2)^{\frac{3}{2}} \\ & + \frac{45e^5d^2x}{8} \sqrt{-e^2x^2 + d^2} + \frac{45e^5d^4}{8} \arctan\left(x\sqrt{e^2} \frac{1}{\sqrt{-e^2x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} - \frac{e}{x^3} (-e^2x^2 + d^2)^{\frac{7}{2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(-e^2\*x^2+d^2)^(5/2)/x^5, x)

[Out] -1/4\*d\*(-e^2\*x^2+d^2)^(7/2)/x^4-9/8/d\*e^2/x^2\*(-e^2\*x^2+d^2)^(7/2)-9/8/d\*e^4\*(-e^2\*x^2+d^2)^(5/2)-15/8\*d\*e^4\*(-e^2\*x^2+d^2)^(3/2)-45/8\*d^3\*e^4\*(-e^2\*x^2+d^2)^(1/2)+45/8\*d^5\*e^4/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)+3\*e^3/d^2/x\*(-e^2\*x^2+d^2)^(7/2)+3\*e^5/d^2\*x\*(-e^2\*x^2+d^2)^(5/2)+15/4\*e^5\*x\*(-e^2\*x^2+d^2)^(3/2)+45/8\*e^5\*d^2\*x\*(-e^2\*x^2+d^2)^(1/2)+45/8\*e^5\*d^4/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))-e\*(-e^2\*x^2+d^2)^(7/2)/x^3

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3/x^5, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.300563, size = 994, normalized size = 4.76

$$16de^{15}x^{15} + 64d^2e^{14}x^{14} - 152d^3e^{13}x^{13} - 1040d^4e^{12}x^{12} + 664d^5e^{11}x^{11} + 4840d^6e^{10}x^{10} - 4112d^7e^9x^9 - 7688d^8e^8x^8 + 13$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3/x^5,x, algorithm="fricas")`

[Out] 
$$\begin{aligned} & -1/8*(16*d^15*x^15 + 64*d^12*e^14*x^14 - 152*d^13*e^13*x^13 - 104 \\ & 0*d^14*e^12*x^12 + 664*d^15*e^11*x^11 + 4840*d^16*e^10*x^10 - 4112*d \\ & ^7*e^9*x^9 - 7688*d^8*e^8*x^8 + 13056*d^9*e^7*x^7 + 3456*d^10*e^6 \\ & *x^6 - 17152*d^11*e^5*x^5 + 416*d^12*e^4*x^4 + 8704*d^13*e^3*x^3 \\ & + 256*d^14*e^2*x^2 - 1024*d^15*e*x - 256*d^16 + 90*(d^4*e^12*x^12 \\ & - 32*d^6*e^10*x^10 + 160*d^8*e^8*x^8 - 256*d^10*e^6*x^6 + 128*d^ \\ & 12*e^4*x^4 + 8*(d^5*e^10*x^10 - 10*d^7*e^8*x^8 + 24*d^9*e^6*x^6 - \\ & 16*d^11*e^4*x^4)*\sqrt{-e^2*x^2 + d^2})*\arctan(-(d - \sqrt{-e^2*x^ \\ & 2 + d^2}))/e^x) + 45*(d^4*e^12*x^12 - 32*d^6*e^10*x^10 + 160*d^8 \\ & *e^8*x^8 - 256*d^10*e^6*x^6 + 128*d^12*e^4*x^4 + 8*(d^5*e^10*x^10 \\ & - 10*d^7*e^8*x^8 + 24*d^9*e^6*x^6 - 16*d^11*e^4*x^4)*\sqrt{-e^2*x \\ & ^2 + d^2})*\log(-(d - \sqrt{-e^2*x^2 + d^2})/x) - (2*e^15*x^15 + 8* \\ & d^14*x^14 - 61*d^12*e^13*x^13 - 304*d^13*e^12*x^12 + 272*d^14*e^11 \\ & *x^11 + 2429*d^15*e^10*x^10 - 1576*d^16*e^9*x^9 - 5794*d^17*e^8*x^8 \\ & + 7424*d^18*e^7*x^7 + 3680*d^19*e^6*x^6 - 13184*d^20*e^5*x^5 + 448* \\ & d^21*e^4*x^4 + 8192*d^22*e^3*x^3 + 128*d^23*e^2*x^2 - 1024*d^24*e \\ & *x - 256*d^25)*\sqrt{-e^2*x^2 + d^2})/(e^8*x^12 - 32*d^2*e^6*x^10 \\ & + 160*d^4*e^4*x^8 - 256*d^6*e^2*x^6 + 128*d^8*x^4 + 8*(d^6*x^6 \\ & - 10*d^3*e^4*x^8 + 24*d^5*e^2*x^6 - 16*d^7*x^4)*\sqrt{-e^2*x^2 + \\ & d^2}) \end{aligned}$$

**Sympy [A]** time = 50.9258, size = 1028, normalized size = 4.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**5,x)`

[Out] 
$$\begin{aligned} & d^{**7} \text{Piecewise}((-d^{**2}/(4*e*x^{**5}*\sqrt{d^{**2}/(e^{**2}*x^{**2})} - 1)) + 3*e \\ & /(8*x^{**3}*\sqrt{d^{**2}/(e^{**2}*x^{**2})} - 1)) - e^{**3}/(8*d^{**2}*x*\sqrt{d^{**2}/( \\ & e^{**2}*x^{**2})} - 1)) + e^{**4}*\text{acosh}(d/(e*x))/(8*d^{**3}), \text{Abs}(d^{**2}/(e^{**2}*x \\ & **2)) > 1), (I*d^{**2}/(4*e*x^{**5}*\sqrt{-d^{**2}/(e^{**2}*x^{**2})} + 1)) - 3*I \\ & e/(8*x^{**3}*\sqrt{-d^{**2}/(e^{**2}*x^{**2})} + 1)) + I*e^{**3}/(8*d^{**2}*x*\sqrt{-d \\ & **2}/(e^{**2}*x^{**2}) + 1)) - I*e^{**4}*\text{asin}(d/(e*x))/(8*d^{**3}), \text{True})) + 3 \\ & *d^{**6}*e*\text{Piecewise}((-e*\sqrt{d^{**2}/(e^{**2}*x^{**2})} - 1)/(3*x^{**2}) + e^{**3} \\ & *\sqrt{d^{**2}/(e^{**2}*x^{**2})} - 1)/(3*d^{**2}), \text{Abs}(d^{**2}/(e^{**2}*x^{**2})) > 1), \\ & (-I*e*\sqrt{-d^{**2}/(e^{**2}*x^{**2})} + 1)/(3*x^{**2}) + I*e^{**3}*\sqrt{-d^{**2}/(e \\ & **2}*x^{**2}) + 1)/(3*d^{**2}), \text{True})) + d^{**5}*e^{**2}*\text{Piecewise}((-d^{**2}/(2*e \\ & *x^{**3}*\sqrt{d^{**2}/(e^{**2}*x^{**2})} - 1)) + e/(2*x*\sqrt{d^{**2}/(e^{**2}*x^{**2})} \\ & - 1)) + e^{**2}*\text{acosh}(d/(e*x))/(2*d), \text{Abs}(d^{**2}/(e^{**2}*x^{**2})) > 1), (- \\ & I*e*\sqrt{-d^{**2}/(e^{**2}*x^{**2})} + 1)/(2*x) - I*e^{**2}*\text{asin}(d/(e*x))/(2*d \\ & ), \text{True})) - 5*d^{**4}*e^{**3}*\text{Piecewise}((I*d/(x*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}} \\ & )) + I*e*\text{acosh}(e*x/d) - I*e^{**2}*x/(d*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}}), \\ & \text{Abs}(e^{**2}*x^{**2}/d^{**2}) > 1), (-d/(x*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}})) - e*\text{as} \\ & \text{in}(e*x/d) + e^{**2}*x/(d*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}}), \text{True})) - 5*d^{**3} \\ & *e^{**4}*\text{Piecewise}((d^{**2}/(e*x*\sqrt{d^{**2}/(e^{**2}*x^{**2})} - 1)) - d*\text{acosh}(d \\ & /(e*x)) - e*x/\sqrt{d^{**2}/(e^{**2}*x^{**2})} - 1), \text{Abs}(d^{**2}/(e^{**2}*x^{**2})) > \\ & 1), (-I*d^{**2}/(e*x*\sqrt{-d^{**2}/(e^{**2}*x^{**2})} + 1)) + I*d*\text{asin}(d/(e*x \\ & )) + I*e*x/\sqrt{-d^{**2}/(e^{**2}*x^{**2})} + 1), \text{True})) + d^{**2}*e^{**5}*\text{Piecew} \\ & \text{ise}((-I*d^{**2}*\text{acosh}(e*x/d)/(2*e) - I*d*x/(2*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}} \\ & **2)) + I*e^{**2}*x^{**3}/(2*d*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}}), \text{Abs}(e^{**2}*x^{**2} \\ & /d^{**2}) > 1), (d^{**2}*\text{asin}(e*x/d)/(2*e) + d*x*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}} \\ & **2)/2, \text{True})) + 3*d^{**6}*\text{Piecewise}((x^{**2}*\sqrt{d^{**2}}/2, \text{Eq}(e^{**2}, 0) \\ & ), (-d^{**2} - e^{**2}*x^{**2})*(3/2)/(3*e^{**2}), \text{True})) + e^{**7}*\text{Piecewise}( \\ & (-I*d^{**4}*\text{acosh}(e*x/d)/(8*e^{**3}) + I*d^{**3}*x/(8*e^{**2}*\sqrt{-1 + e^{**2} \\ & *x^{**2}/d^{**2}})) - 3*I*d*x^{**3}/(8*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}})) + I*e^{**2}*x \\ & **5/(4*d*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}}), \text{Abs}(e^{**2}*x^{**2}/d^{**2}) > 1), (d \\ & **4*\text{asin}(e*x/d)/(8*e^{**3}) - d^{**3}*x/(8*e^{**2}*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}} \\ & )) + 3*d*x^{**3}/(8*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}})) - e^{**2}*x^{**5}/(4*d*\sqrt{1 - \\ & e^{**2}*x^{**2}/d^{**2}}), \text{True})) \end{aligned}$$

**GIAC/XCAS [A]** time = 0.295445, size = 505, normalized size = 2.42

$$\begin{aligned} & \frac{45}{8} d^4 \arcsin\left(\frac{xe}{d}\right) e^4 \operatorname{sign}(d) + \frac{45}{8} d^4 e^4 \ln\left(\frac{|-2de - 2\sqrt{-x^2e^2 + d^2}e|e^{(-2)}}{2|x|}\right) \\ & + \frac{\left(d^4 e^{10} + \frac{8(de + \sqrt{-x^2e^2 + d^2}e)d^4 e^8}{x} + \frac{8(de + \sqrt{-x^2e^2 + d^2}e)^2 d^4 e^6}{x^2} - \frac{184(de + \sqrt{-x^2e^2 + d^2}e)^3 d^4 e^4}{x^3}\right) x^4 e^2}{64(de + \sqrt{-x^2e^2 + d^2}e)^4} \\ & + \frac{1}{64} \left( \frac{184(de + \sqrt{-x^2e^2 + d^2}e)d^4 e^{26}}{x} - \frac{8(de + \sqrt{-x^2e^2 + d^2}e)^2 d^4 e^{24}}{x^2} - \frac{8(de + \sqrt{-x^2e^2 + d^2}e)^3 d^4 e^{22}}{x^3} - \frac{(de + \sqrt{-x^2e^2 + d^2}e)^4 d^4 e^{20}}{x^4} \right) \\ & - \frac{1}{8} (48d^3 e^4 - (3d^2 e^5 + 2(xe^7 + 4de^6)x)x) \sqrt{-x^2e^2 + d^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3/x^5,x, algorithm="giac")

[Out] 45/8\*d^4\*arcsin(x\*e/d)\*e^4\*sign(d) + 45/8\*d^4\*e^4\*ln(1/2\*abs(-2\*d\*e - 2\*sqrt(-x^2\*e^2 + d^2)\*e)\*e^(-2)/abs(x)) + 1/64\*(d^4\*e^10 + 8\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*d^4\*e^8/x + 8\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^2\*d^4\*e^6/x^2 - 184\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^3\*d^4\*e^4/x^3)\*x^4\*e^2/(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^4 + 1/64\*(184\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*d^4\*e^26/x - 8\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^2\*d^4\*e^24/x^2 - 8\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^3\*d^4\*e^22/x^3 - (d\*e + sqrt(-x^2\*e^2 + d^2)\*e)^4\*d^4\*e^20/x^4)\*e^(-24) - 1/8\*(48\*d^3\*e^4 - (3\*d^2\*e^5 + 2\*(x\*e^7 + 4\*d\*e^6)\*x)\*x)\*sqrt(-x^2\*e^2 + d^2)

$$3.76 \quad \int \frac{(d+ex)^3 (d^2 - e^2 x^2)^{5/2}}{x^6} dx$$

**Optimal.** Leaf size=216

$$\begin{aligned} & -\frac{d(d^2 - e^2 x^2)^{7/2}}{5x^5} - \frac{3e(d^2 - e^2 x^2)^{7/2}}{4x^4} - \frac{e^2(52d + 25ex)(d^2 - e^2 x^2)^{5/2}}{60x^3} \\ & + \frac{d^2 e^4 (52d + 25ex) \sqrt{d^2 - e^2 x^2}}{8x} + \frac{de^3 (25d - 52ex)(d^2 - e^2 x^2)^{3/2}}{24x^2} \\ & + \frac{13}{2} d^3 e^5 \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) - \frac{25}{8} d^3 e^5 \tanh^{-1} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right) \end{aligned}$$

[Out] (d^2\*e^4\*(52\*d + 25\*e\*x)\*Sqrt[d^2 - e^2\*x^2])/(8\*x) + (d\*e^3\*(25\*d - 52\*e\*x)\*(d^2 - e^2\*x^2)^(3/2))/(24\*x^2) - (e^2\*(52\*d + 25\*e\*x)\*(d^2 - e^2\*x^2)^(5/2))/(60\*x^3) - (d\*(d^2 - e^2\*x^2)^(7/2))/(5\*x^4) - (3\*e\*(d^2 - e^2\*x^2)^(7/2))/(4\*x^4) + (13\*d^3\*e^5\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/2 - (25\*d^3\*e^5\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/8

**Rubi [A]** time = 0.618965, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$

$$\begin{aligned} & -\frac{d(d^2 - e^2 x^2)^{7/2}}{5x^5} - \frac{3e(d^2 - e^2 x^2)^{7/2}}{4x^4} - \frac{e^2(52d + 25ex)(d^2 - e^2 x^2)^{5/2}}{60x^3} \\ & + \frac{d^2 e^4 (52d + 25ex) \sqrt{d^2 - e^2 x^2}}{8x} + \frac{de^3 (25d - 52ex)(d^2 - e^2 x^2)^{3/2}}{24x^2} \\ & + \frac{13}{2} d^3 e^5 \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) - \frac{25}{8} d^3 e^5 \tanh^{-1} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right) \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2))/x^6, x]

[Out] (d^2\*e^4\*(52\*d + 25\*e\*x)\*Sqrt[d^2 - e^2\*x^2])/(8\*x) + (d\*e^3\*(25\*d - 52\*e\*x)\*(d^2 - e^2\*x^2)^(3/2))/(24\*x^2) - (e^2\*(52\*d + 25\*e\*x)\*(d^2 - e^2\*x^2)^(5/2))/(60\*x^3) - (d\*(d^2 - e^2\*x^2)^(7/2))/(5\*x^4) - (3\*e\*(d^2 - e^2\*x^2)^(7/2))/(4\*x^4) + (13\*d^3\*e^5\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/2 - (25\*d^3\*e^5\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/8

**Rubi in Sympy [A]** time = 90.7145, size = 245, normalized size = 1.13

$$\begin{aligned} & -\frac{d^7 \sqrt{d^2 - e^2 x^2}}{5x^5} - \frac{3d^6 e \sqrt{d^2 - e^2 x^2}}{4x^4} - \frac{4d^5 e^2 \sqrt{d^2 - e^2 x^2}}{15x^3} + \frac{23d^4 e^3 \sqrt{d^2 - e^2 x^2}}{8x^2} \\ & + \frac{13d^3 e^5 \operatorname{atan} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right)}{2} - \frac{25d^3 e^5 \operatorname{atanh} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right)}{8} + \frac{82d^3 e^4 \sqrt{d^2 - e^2 x^2}}{15x} \\ & + d^2 e^5 \sqrt{d^2 - e^2 x^2} + \frac{3de^6 x \sqrt{d^2 - e^2 x^2}}{2} - \frac{e^5 (d^2 - e^2 x^2)^{3/2}}{3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*6, x)

[Out] -d\*\*7\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(5\*x\*\*5) - 3\*d\*\*6\*e\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(4\*x\*\*4) - 4\*d\*\*5\*e\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*x\*\*3) +



$$23*d^{**4}*e^{**3}*sqrt(d^{**2} - e^{**2}*x^{**2})/(8*x^{**2}) + 13*d^{**3}*e^{**5}*atan(e*x/sqrt(d^{**2} - e^{**2}*x^{**2}))/2 - 25*d^{**3}*e^{**5}*atanh(sqrt(d^{**2} - e^{**2}*x^{**2})/d)/8 + 82*d^{**3}*e^{**4}*sqrt(d^{**2} - e^{**2}*x^{**2})/(15*x) + d^{**2}*e^{**5}*sqrt(d^{**2} - e^{**2}*x^{**2}) + 3*d*e^{**6}*x*sqrt(d^{**2} - e^{**2}*x^{**2})/2 - e^{**5}*(d^{**2} - e^{**2}*x^{**2})^{**3/2}/3$$

**Mathematica [A]** time = 0.486642, size = 167, normalized size = 0.77

$$\frac{1}{8} \left( 25d^3 e^5 \log(x) - 25d^3 e^5 \log\left(\sqrt{d^2 - e^2 x^2} + d\right) + 52d^3 e^5 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right) + \frac{\sqrt{d^2 - e^2 x^2} (-24d^7 - 90d^6 ex - 32d^5 e^2 x^2 + 345d^4 e^3 x^3 + 656d^3 e^4 x^4 + 80d^2 e^5 x^5 + 180de^6 x^6 + 40e^7 x^7)}{15x^5} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2))/x^6, x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(-24\*d^7 - 90\*d^6\*e\*x - 32\*d^5\*e^2\*x^2 + 345\*d^4\*e^3\*x^3 + 656\*d^3\*e^4\*x^4 + 80\*d^2\*e^5\*x^5 + 180\*d\*e^6\*x^6 + 40\*e^7\*x^7))/(15\*x^5) + 52\*d^3\*e^5\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]]) + 25\*d^3\*e^5\*Log[x] - 25\*d^3\*e^5\*Log[d + Sqrt[d^2 - e^2\*x^2]])/8

**Maple [A]** time = 0.035, size = 327, normalized size = 1.5

$$\begin{aligned} & -\frac{d}{5x^5}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{13e^2}{15dx^3}(-e^2x^2 + d^2)^{\frac{7}{2}} + \frac{52e^4}{15d^3x}(-e^2x^2 + d^2)^{\frac{7}{2}} + \frac{52e^6x}{15d^3}(-e^2x^2 + d^2)^{\frac{5}{2}} \\ & + \frac{13e^6x}{3d}(-e^2x^2 + d^2)^{\frac{3}{2}} + \frac{13de^6x}{2}\sqrt{-e^2x^2 + d^2} + \frac{13d^3e^6}{2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2 + d^2}}\right)\frac{1}{\sqrt{e^2}} \\ & + \frac{5e^3}{8d^2x^2}(-e^2x^2 + d^2)^{\frac{7}{2}} + \frac{5e^5}{8d^2}(-e^2x^2 + d^2)^{\frac{5}{2}} + \frac{25e^5}{24}(-e^2x^2 + d^2)^{\frac{3}{2}} + \frac{25e^5d^2}{8}\sqrt{-e^2x^2 + d^2} \\ & - \frac{25e^5d^4}{8}\ln\left(\frac{1}{x}\left(2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2}\right)\right)\frac{1}{\sqrt{d^2}} - \frac{3e}{4x^4}(-e^2x^2 + d^2)^{\frac{7}{2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(-e^2\*x^2+d^2)^(5/2)/x^6, x)

[Out] -1/5\*d\*(-e^2\*x^2+d^2)^(7/2)/x^5-13/15/d\*e^2/x^3\*(-e^2\*x^2+d^2)^(7/2)+52/15/d^3\*e^4/x\*(-e^2\*x^2+d^2)^(7/2)+52/15/d^3\*e^6\*x\*(-e^2\*x^2+d^2)^(5/2)+13/3/d\*e^6\*x\*(-e^2\*x^2+d^2)^(3/2)+13/2\*d\*e^6\*x\*(-e^2\*x^2+d^2)^(1/2)+13/2\*d^3\*e^6/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))+5/8\*e^3/d^2/x^2\*(-e^2\*x^2+d^2)^(7/2)+5/8\*e^5/d^2\*(-e^2\*x^2+d^2)^(5/2)+25/24\*e^5\*(-e^2\*x^2+d^2)^(3/2)+25/8\*e^5\*d^2\*(-e^2\*x^2+d^2)^(1/2)-25/8\*e^5\*d^4/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)-3/4\*e\*(-e^2\*x^2+d^2)^(7/2)/x^4

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3/x^6, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.303481, size = 994, normalized size = 4.6

$$320 d e^{15} x^{15} + 1440 d^2 e^{14} x^{14} - 2960 d^3 e^{13} x^{13} - 10592 d^4 e^{12} x^{12} + 9160 d^5 e^{11} x^{11} - 9024 d^6 e^{10} x^{10} - 34920 d^7 e^9 x^9 + 123456 d^8 e^8 x^8 - 101760 d^9 e^7 x^7 - 193472 d^{10} e^6 x^6 - 134880 d^{11} e^5 x^5 + 87680 d^{12} e^4 x^4 + 72960 d^{13} e^3 x^3 + 3584 d^{14} e^2 x^2 - 11520 d^{15} e x - 3072 d^{16}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3/x^6,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/120*(320*d*e^{15}*x^{15} + 1440*d^2*e^{14}*x^{14} - 2960*d^3*e^{13}*x^{13} \\ & - 10592*d^4*e^{12}*x^{12} + 9160*d^5*e^{11}*x^{11} - 9024*d^6*e^{10}*x^{10} \\ & - 34920*d^7*e^9*x^9 + 123456*d^8*e^8*x^8 + 101760*d^9*e^7*x^7 - 1 \\ & 93472*d^{10}*e^6*x^6 - 134880*d^{11}*e^5*x^5 + 87680*d^{12}*e^4*x^4 + 7 \\ & 2960*d^{13}*e^3*x^3 + 3584*d^{14}*e^2*x^2 - 11520*d^{15}*e*x - 3072*d^{16} \\ & + 1560*(d^3*e^{13}*x^{13} - 32*d^5*e^{11}*x^{11} + 160*d^7*e^9*x^9 - 25 \\ & 6*d^9*e^7*x^7 + 128*d^{11}*e^5*x^5 + 8*(d^4*e^{11}*x^{11} - 10*d^6*e^9* \\ & x^9 + 24*d^8*e^7*x^7 - 16*d^{10}*e^5*x^5)*\sqrt{-e^2*x^2 + d^2})*\arctan \\ & \left(\frac{-d - \sqrt{-e^2*x^2 + d^2}}{e*x}\right) - 375*(d^3*e^{13}*x^{13} - 32* \\ & d^5*e^{11}*x^{11} + 160*d^7*e^9*x^9 - 256*d^9*e^7*x^7 + 128*d^{11}*e^5* \\ & x^5 + 8*(d^4*e^{11}*x^{11} - 10*d^6*e^9*x^9 + 24*d^8*e^7*x^7 - 16*d^{10} \\ & e^5*x^5)*\sqrt{-e^2*x^2 + d^2})*\log\left(\frac{-d - \sqrt{-e^2*x^2 + d^2}}{x}\right) - \\ & (40*e^{15}*x^{15} + 180*d*e^{14}*x^{14} - 1200*d^2*e^{13}*x^{13} - 5104*d^3* \\ & e^{12}*x^{12} + 4825*d^4*e^{11}*x^{11} + 7776*d^5*e^{10}*x^{10} - 14970*d^6* \\ & e^9*x^9 + 59880*d^7*e^8*x^8 + 58080*d^8*e^7*x^7 - 149248*d^9*e^6* \\ & x^6 - 102720*d^{10}*e^5*x^5 + 88320*d^{11}*e^4*x^4 + 67200*d^{12}*e^3* \\ & x^3 + 2048*d^{13}*e^2*x^2 - 11520*d^{14}*e*x - 3072*d^{15})*\sqrt{-e^2* \\ & x^2 + d^2})/(e^8*x^{13} - 32*d^2*e^6*x^{11} + 160*d^4*e^4*x^9 - 256*d^6* \\ & e^2*x^7 + 128*d^8*x^5 + 8*(d*e^6*x^{11} - 10*d^3*e^4*x^9 + 24*d^5* \\ & e^2*x^7 - 16*d^7*x^5)*\sqrt{-e^2*x^2 + d^2}) \end{aligned}$$

**Sympy [A]** time = 43.5764, size = 1178, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*6,x)

[Out] 
$$\begin{aligned} & d^{**7} \text{Piecewise}((3*I*d^{**3}*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}})/(-15*d^{**2}*x^{**5} \\ & + 15*e^{**2}*x^{**7}) - 4*I*d*e^{**2}*x^{**2}*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}})/(-15 \\ & *d^{**2}*x^{**5} + 15*e^{**2}*x^{**7}) + 2*I*e^{**6}*x^{**6}*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}} \\ & )/(-15*d^{**5}*x^{**5} + 15*d^{**3}*e^{**2}*x^{**7}) - I*e^{**4}*x^{**4}*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}} \\ & )/(-15*d^{**3}*x^{**5} + 15*d*e^{**2}*x^{**7}), \text{Abs}(e^{**2}*x^{**2}/d^{**2}) > 1), (3*d^{**3}*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}})/(-15*d^{**2}*x^{**5} + 15*e^{**2}*x^{**7}) - 4*d*e^{**2}*x^{**2}*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}})/(-15*d^{**2}*x^{**5} + 15*e^{**2}*x^{**7}) + 2*e^{**6}*x^{**6}*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}})/(-15*d^{**5}*x^{**5} + 15*d^{**3}*e^{**2}*x^{**7}) - e^{**4}*x^{**4}*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}})/(-15*d^{**3}*x^{**5} + 15*d*e^{**2}*x^{**7}), \text{True})) + 3*d^{**6}*e*\text{Piecewise}((-d^{**2}/(4*e*x^{**5}*\sqrt{d^{**2}/(e^{**2}*x^{**2}) - 1}) + 3*e/(8*x^{**3}*\sqrt{d^{**2}/(e^{**2}*x^{**2}) - 1})) - e^{**3}/(8*d^{**2}*x*\sqrt{d^{**2}/(e^{**2}*x^{**2}) - 1})) + e^{**4}*\text{acosh}(d/(e*x))/(8*d^{**3}), \text{Abs}(d^{**2}/(e^{**2}*x^{**2})) > 1), (I*d^{**2}/(4*e*x^{**5}*\sqrt{-d^{**2}/(e^{**2}*x^{**2}) + 1}) - 3*I*e/(8*x^{**3}*\sqrt{-d^{**2}/(e^{**2}*x^{**2}) + 1})) + I*e^{**3}/(8*d^{**2}*x*\sqrt{-d^{**2}/(e^{**2}*x^{**2}) + 1})) - I*e^{**4}*\text{asin}(d/(e*x))/(8*d^{**3}), \text{True})) + d^{**5}*e^{**2}*\text{Piecewise}((-e*\sqrt{d^{**2}/(e^{**2}*x^{**2}) - 1})/(3*x^{**2}) + e^{**3}*\sqrt{d^{**2}/(e^{**2}*x^{**2}) - 1})/(3*d^{**2}), \text{Abs}(d^{**2}/(e^{**2}*x^{**2})) > 1), (-I*e*\sqrt{-d^{**2}/(e^{**2}*x^{**2}) + 1})/(3*x^{**2}) + I*e^{**3}*\sqrt{-d^{**2}/(e^{**2}*x^{**2}) + 1})/(3*d^{**2}), \text{True})) - 5*d^{**4}*e^{**3}*\text{Piecewise}((-d^{**2}/(2*e*x^{**3}*\sqrt{d^{**2}/(e^{**2}*x^{**2}) - 1})) + e/(2*x*\sqrt{d^{**2}/(e^{**2}*x^{**2}) - 1})) + e^{**2}*\text{acos} \end{aligned}$$

```

h(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e
**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d), True)) - 5*d**
3*e**4*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e
*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**
2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*
x/(d*sqrt(1 - e**2*x**2/d**2)), True)) + d**2*e**5*Piecewise((d**
2/(e*x*sqrt(d**2/(e**2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(
d**2/(e**2*x**2) - 1), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*
sqrt(-d**2/(e**2*x**2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d*
**2/(e**2*x**2) + 1), True)) + 3*d*e**6*Piecewise((-I*d**2*acosh(e
*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(
2*d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*a
sin(e*x/d)/(2*e) + d*x*sqrt(1 - e**2*x**2/d**2)/2, True)) + e**7*
Piecewise((x**2*sqrt(d**2)/2, Eq(e**2, 0)), -(d**2 - e**2*x**2)*
*(3/2)/(3*e**2), True)

```

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**GIAC/XCAS [A]** time = 0.288121, size = 1, normalized size = 0.

*Done*

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3/x^6,x, algorithm="giac")
```

```
[Out] Done
```

$$3.77 \quad \int \frac{(d+ex)^3 (d^2 - e^2 x^2)^{5/2}}{x^7} dx$$

**Optimal.** Leaf size=214

$$\frac{d(d^2 - e^2 x^2)^{7/2}}{6x^6} - \frac{3e(d^2 - e^2 x^2)^{7/2}}{5x^5} - \frac{e^2(85d + 12ex)(d^2 - e^2 x^2)^{5/2}}{120x^4} - \frac{1}{2}d^2 e^6 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right) - \frac{85}{16}d^2 e^6 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right) - \frac{de^5(8d - 85ex)\sqrt{d^2 - e^2 x^2}}{16x} + \frac{de^3(8d + 85ex)(d^2 - e^2 x^2)^{3/2}}{48x^3}$$

[Out]  $-(d^5 e^6 (8d - 85ex) \sqrt{d^2 - e^2 x^2}) / (16x) + (d^3 e^6 (8d + 85ex) (d^2 - e^2 x^2)^{3/2}) / (48x^3) - (e^6 (85d + 12ex) (d^2 - e^2 x^2)^{5/2}) / (120x^4) - (d^2 e^6 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)) / 2 - (85d^2 e^6 \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)) / 16 - (de^5 (8d - 85ex) \sqrt{d^2 - e^2 x^2}) / 16x$

**Rubi [A]** time = 0.619716, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{d(d^2 - e^2 x^2)^{7/2}}{6x^6} - \frac{3e(d^2 - e^2 x^2)^{7/2}}{5x^5} - \frac{e^2(85d + 12ex)(d^2 - e^2 x^2)^{5/2}}{120x^4} - \frac{1}{2}d^2 e^6 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right) - \frac{85}{16}d^2 e^6 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right) - \frac{de^5(8d - 85ex)\sqrt{d^2 - e^2 x^2}}{16x} + \frac{de^3(8d + 85ex)(d^2 - e^2 x^2)^{3/2}}{48x^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + ex)^3 (d^2 - e^2 x^2)^{5/2} / x^7, x]$

[Out]  $-(d^5 e^6 (8d - 85ex) \sqrt{d^2 - e^2 x^2}) / (16x) + (d^3 e^6 (8d + 85ex) (d^2 - e^2 x^2)^{3/2}) / (48x^3) - (e^6 (85d + 12ex) (d^2 - e^2 x^2)^{5/2}) / (120x^4) - (d^2 e^6 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)) / 2 - (85d^2 e^6 \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)) / 16 - (de^5 (8d - 85ex) \sqrt{d^2 - e^2 x^2}) / 16x$

**Rubi in Sympy [A]** time = 91.8144, size = 248, normalized size = 1.16

$$\frac{d^7 \sqrt{d^2 - e^2 x^2}}{6x^6} - \frac{3d^6 e \sqrt{d^2 - e^2 x^2}}{5x^5} - \frac{5d^5 e^2 \sqrt{d^2 - e^2 x^2}}{24x^4} + \frac{28d^4 e^3 \sqrt{d^2 - e^2 x^2}}{15x^3} + \frac{43d^3 e^4 \sqrt{d^2 - e^2 x^2}}{16x^2} - \frac{d^2 e^6 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{2} - \frac{85d^2 e^6 \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{16} - \frac{34d^2 e^5 \sqrt{d^2 - e^2 x^2}}{15x} + 3de^6 \sqrt{d^2 - e^2 x^2} + \frac{e^7 x \sqrt{d^2 - e^2 x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((ex+d)**3*(-e**2*x**2+d**2)**(5/2)/x**7,x)$

[Out]  $-d**7*\sqrt{d**2 - e**2*x**2}/(6*x**6) - 3*d**6*e*\sqrt{d**2 - e**2*x**2}/(5*x**5) - 5*d**5*e**2*\sqrt{d**2 - e**2*x**2}/(24*x**4) + 28*d**4*e**3*\sqrt{d**2 - e**2*x**2}/(15*x**3) + 43*d**3*e**4*\sqrt{d**2 - e**2*x**2}/(16*x**2) - d**2*e**6*\operatorname{atan}(ex/\sqrt{d**2 - e**2*x**2})/2 - 85*d**2*e**6*\operatorname{atanh}(\sqrt{d**2 - e**2*x**2}/d)/16 - 34*d**2*e**5*\sqrt{d**2 - e**2*x**2}/(15*x) + 3*d*e**6*\sqrt{d**2 - e**2*x**2} + e**7*x*\sqrt{d**2 - e**2*x**2}/2$

---

**Mathematica [A]** time = 0.365655, size = 167, normalized size = 0.78

$$\frac{1}{16} \left( 85d^2e^6 \log(x) - 85d^2e^6 \log(\sqrt{d^2 - e^2x^2} + d) - 8d^2e^6 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) + \frac{\sqrt{d^2 - e^2x^2} (-40d^7 - 144d^6ex - 50d^5e^2x^2 + 448d^4e^3x^3 + 645d^3e^4x^4 - 544d^2e^5x^5 + 720de^6x^6 + 120e^7x^7)}{15x^6} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2))/x^7, x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(-40\*d^7 - 144\*d^6\*e\*x - 50\*d^5\*e^2\*x^2 + 448\*d^4\*e^3\*x^3 + 645\*d^3\*e^4\*x^4 - 544\*d^2\*e^5\*x^5 + 720\*d\*e^6\*x^6 + 120\*e^7\*x^7))/(15\*x^6) - 8\*d^2\*e^6\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] + 85\*d^2\*e^6\*Log[x] - 85\*d^2\*e^6\*Log[d + Sqrt[d^2 - e^2\*x^2]])/16

---

**Maple [A]** time = 0.045, size = 352, normalized size = 1.6

$$\begin{aligned} & -\frac{d}{6x^6} (-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{17e^2}{24dx^4} (-e^2x^2 + d^2)^{\frac{7}{2}} + \frac{17e^4}{16d^3x^2} (-e^2x^2 + d^2)^{\frac{7}{2}} + \frac{17e^6}{16d^3} (-e^2x^2 + d^2)^{\frac{5}{2}} \\ & + \frac{85e^6}{48d} (-e^2x^2 + d^2)^{\frac{3}{2}} + \frac{85de^6}{16} \sqrt{-e^2x^2 + d^2} - \frac{85d^3e^6}{16} \ln\left(\frac{1}{x} \left(2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}} \\ & + \frac{e^3}{15d^2x^3} (-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{4e^5}{15d^4x} (-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{4e^7x}{15d^4} (-e^2x^2 + d^2)^{\frac{5}{2}} - \frac{e^7x}{3d^2} (-e^2x^2 + d^2)^{\frac{3}{2}} \\ & - \frac{e^7x}{2} \sqrt{-e^2x^2 + d^2} - \frac{e^7d^2}{2} \arctan\left(x\sqrt{e^2} \frac{1}{\sqrt{-e^2x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} - \frac{3e}{5x^5} (-e^2x^2 + d^2)^{\frac{7}{2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(-e^2\*x^2+d^2)^(5/2)/x^7, x)

[Out] -1/6\*d\*(-e^2\*x^2+d^2)^(7/2)/x^6-17/24/d\*e^2/x^4\*(-e^2\*x^2+d^2)^(7/2)+17/16/d^3\*e^4/x^2\*(-e^2\*x^2+d^2)^(7/2)+17/16/d^3\*e^6\*(-e^2\*x^2+d^2)^(5/2)+85/48/d\*e^6\*(-e^2\*x^2+d^2)^(3/2)+85/16\*d\*e^6\*(-e^2\*x^2+d^2)^(1/2)-85/16\*d^3\*e^6/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)+1/15\*e^3/d^2/x^3\*(-e^2\*x^2+d^2)^(7/2)-4/15\*e^5/d^4/x\*(-e^2\*x^2+d^2)^(7/2)-4/15\*e^7/d^4\*x\*(-e^2\*x^2+d^2)^(5/2)-1/3\*e^7/d^2\*x\*(-e^2\*x^2+d^2)^(3/2)-1/2\*e^7\*x\*(-e^2\*x^2+d^2)^(1/2)-1/2\*e^7\*d^2/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))-3/5\*e\*(-e^2\*x^2+d^2)^(7/2)/x^5

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3/x^7, x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.301436, size = 994, normalized size = 4.64

$$960 d e^{15} x^{15} + 5040 d^2 e^{14} x^{14} - 14912 d^3 e^{13} x^{13} - 35160 d^4 e^{12} x^{12} + 84096 d^5 e^{11} x^{11} + 23480 d^6 e^{10} x^{10} - 226944 d^7 e^9 x^9 + 133440 d^8 e^8 x^8 - 103424 d^9 e^7 x^7 + 6400 d^{10} e^6 x^6 - 252160 d^{11} e^5 x^5 + 87680 d^{12} e^4 x^4 + 103424 d^{13} e^3 x^3 + 6400 d^{14} e^2 x^2 - 18432 d^{15} e x - 5120 d^{16} - 240 (d^2 e^{14} x^{14} - 32 d^4 e^{12} x^{12} + 160 d^6 e^{10} x^{10} - 256 d^8 e^8 x^8 + 128 d^{10} e^6 x^6 + 8 (d^3 e^{12} x^{12} - 10 d^5 e^{10} x^{10} + 24 d^7 e^8 x^8 - 16 d^9 e^6 x^6)) \sqrt{-e^2 x^2 + d^2}) \arctan\left(\frac{d - \sqrt{-e^2 x^2 + d^2}}{e x}\right) - 1275 (d^2 e^{14} x^{14} - 32 d^4 e^{12} x^{12} + 160 d^6 e^{10} x^{10} - 256 d^8 e^8 x^8 + 128 d^{10} e^6 x^6 + 8 (d^3 e^{12} x^{12} - 10 d^5 e^{10} x^{10} + 24 d^7 e^8 x^8 - 16 d^9 e^6 x^6)) \sqrt{-e^2 x^2 + d^2}) \log\left(\frac{d - \sqrt{-e^2 x^2 + d^2}}{x}\right) - (120 e^{15} x^{15} + 720 d e^{14} x^{14} - 4384 d^2 e^{13} x^{13} - 16635 d^3 e^{12} x^{12} + 37056 d^4 e^{11} x^{11} + 36910 d^5 e^{10} x^{10} - 132240 d^6 e^9 x^9 + 58680 d^7 e^8 x^8 + 230912 d^8 e^7 x^7 - 171840 d^9 e^6 x^6 - 207360 d^{10} e^5 x^5 + 88960 d^{11} e^4 x^4 + 94208 d^{12} e^3 x^3 + 3840 d^{13} e^2 x^2 - 18432 d^{14} e x - 5120 d^{15}) \sqrt{-e^2 x^2 + d^2}) / (e^8 x^{14} - 32 d^2 e^6 x^{12} + 160 d^4 e^4 x^{10} - 256 d^6 e^2 x^8 + 128 d^8 x^6 + 8 (d e^6 x^{12} - 10 d^3 e^4 x^{10} + 24 d^5 e^2 x^8 - 16 d^7 x^6)) \sqrt{-e^2 x^2 + d^2})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3/x^7,x, algorithm="fricas")

[Out] -1/240\*(960\*d\*e^15\*x^15 + 5040\*d^2\*e^14\*x^14 - 14912\*d^3\*e^13\*x^13 - 35160\*d^4\*e^12\*x^12 + 84096\*d^5\*e^11\*x^11 + 23480\*d^6\*e^10\*x^10 - 226944\*d^7\*e^9\*x^9 + 133440\*d^8\*e^8\*x^8 + 323968\*d^9\*e^7\*x^7 - 216480\*d^10\*e^6\*x^6 - 252160\*d^11\*e^5\*x^5 + 87680\*d^12\*e^4\*x^4 + 103424\*d^13\*e^3\*x^3 + 6400\*d^14\*e^2\*x^2 - 18432\*d^15\*e\*x - 5120\*d^16 - 240\*(d^2\*e^14\*x^14 - 32\*d^4\*e^12\*x^12 + 160\*d^6\*e^10\*x^10 - 256\*d^8\*e^8\*x^8 + 128\*d^10\*e^6\*x^6 + 8\*(d^3\*e^12\*x^12 - 10\*d^5\*e^10\*x^10 + 24\*d^7\*e^8\*x^8 - 16\*d^9\*e^6\*x^6))\*sqrt(-e^2\*x^2 + d^2))\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) - 1275\*(d^2\*e^14\*x^14 - 32\*d^4\*e^12\*x^12 + 160\*d^6\*e^10\*x^10 - 256\*d^8\*e^8\*x^8 + 128\*d^10\*e^6\*x^6 + 8\*(d^3\*e^12\*x^12 - 10\*d^5\*e^10\*x^10 + 24\*d^7\*e^8\*x^8 - 16\*d^9\*e^6\*x^6))\*sqrt(-e^2\*x^2 + d^2))\*log(-(d - sqrt(-e^2\*x^2 + d^2))/x) - (120\*e^15\*x^15 + 720\*d\*e^14\*x^14 - 4384\*d^2\*e^13\*x^13 - 16635\*d^3\*e^12\*x^12 + 37056\*d^4\*e^11\*x^11 + 36910\*d^5\*e^10\*x^10 - 132240\*d^6\*e^9\*x^9 + 58680\*d^7\*e^8\*x^8 + 230912\*d^8\*e^7\*x^7 - 171840\*d^9\*e^6\*x^6 - 207360\*d^10\*e^5\*x^5 + 88960\*d^11\*e^4\*x^4 + 94208\*d^12\*e^3\*x^3 + 3840\*d^13\*e^2\*x^2 - 18432\*d^14\*e\*x - 5120\*d^15)\*sqrt(-e^2\*x^2 + d^2))/(e^8\*x^14 - 32\*d^2\*e^6\*x^12 + 160\*d^4\*e^4\*x^10 - 256\*d^6\*e^2\*x^8 + 128\*d^8\*x^6 + 8\*(d\*e^6\*x^12 - 10\*d^3\*e^4\*x^10 + 24\*d^5\*e^2\*x^8 - 16\*d^7\*x^6))\*sqrt(-e^2\*x^2 + d^2))

**Sympy [A]** time = 61.4377, size = 1397, normalized size = 6.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*7,x)

[Out] d\*\*7\*Piecewise((-d\*\*2/(6\*e\*x\*\*7\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) + 5\*e/(24\*x\*\*5\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) + e\*\*3/(48\*d\*\*2\*x\*\*3\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) - e\*\*5/(16\*d\*\*4\*x\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) + e\*\*6\*acosh(d/(e\*x))/(16\*d\*\*5), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (I\*d\*\*2/(6\*e\*x\*\*7\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) - 5\*I\*e/(24\*x\*\*5\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) - I\*e\*\*3/(48\*d\*\*2\*x\*\*3\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) + I\*e\*\*5/(16\*d\*\*4\*x\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) - I\*e\*\*6\*asin(d/(e\*x))/(16\*d\*\*5), True)) + 3\*d\*\*6\*e\*Piecewise((3\*I\*d\*\*3\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)/(-15\*d\*\*2\*x\*\*5 + 15\*e\*\*2\*x\*\*7) - 4\*I\*d\*e\*\*2\*x\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)/(-15\*d\*\*2\*x\*\*5 + 15\*e\*\*2\*x\*\*7) + 2\*I\*e\*\*6\*x\*\*6\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)/(-15\*d\*\*5\*x\*\*5 + 15\*d\*\*3\*e\*\*2\*x\*\*7) - I\*e\*\*4\*x\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)/(-15\*d\*\*3\*x\*\*5 + 15\*d\*e\*\*2\*x\*\*7), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (3\*d\*\*3\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)/(-15\*d\*\*2\*x\*\*5 + 15\*e\*\*2\*x\*\*7) - 4\*d\*e\*\*2\*x\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)/(-15\*d\*\*5\*x\*\*5 + 15\*d\*\*3\*e\*\*2\*x\*\*7) - e\*\*4\*x\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)/(-15\*d\*\*3\*x\*\*5 + 15\*d\*e\*\*2\*x\*\*7), True)) + d\*\*5\*e\*\*2\*Piecewise((-d\*\*2/(4\*e\*x\*\*5\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) + 3\*e/(8\*x\*\*3\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) - e\*\*3/(8\*d\*\*2\*x\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) + e\*\*4\*acosh(d/(e\*x))/(8\*d\*\*3), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (I\*d\*\*2/(4\*e\*x\*\*5\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) - 3\*I\*e/(8\*x\*\*3\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) + I\*e\*\*3/(8\*d\*\*2\*x\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) - I\*e\*\*4\*asin(d/(e\*x))/(8\*d\*\*3), True)) - 5\*d\*\*4\*e\*\*3\*Piecewise((-e\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)/(3\*x\*\*2) + e\*\*3\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)/(3\*d\*\*2), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (-I\*e\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)/(3

```

*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) - 5*
d**3*e**4*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1))
+ e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d),
Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2
*x) - I*e**2*asin(d/(e*x))/(2*d), True)) + d**2*e**5*Piecewise((I
*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d
*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqr
t(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*
x**2/d**2)), True)) + 3*d*e**6*Piecewise((d**2/(e*x*sqrt(d**2/(e
**2*x**2) - 1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1
), Abs(d**2/(e**2*x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**
2) + 1)) + I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1),
True)) + e**7*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*s
qrt(-1 + e**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/
d**2)), Abs(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*s
qrt(1 - e**2*x**2/d**2)/2, True))

```

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**GIAC/XCAS [A]** time = 0.296405, size = 1, normalized size = 0.

*Done*

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3/x^7,x, algorithm="giac")
```

```
[Out] Done
```

$$3.78 \quad \int \frac{(d+ex)^3 (d^2 - e^2 x^2)^{5/2}}{x^8} dx$$

**Optimal.** Leaf size=206

$$\frac{d(d^2 - e^2 x^2)^{7/2}}{7x^7} - \frac{e(d^2 - e^2 x^2)^{7/2}}{2x^6} - \frac{e^2(24d + 5ex)(d^2 - e^2 x^2)^{5/2}}{40x^5} - 3de^7 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right) - \frac{15}{16}de^7 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right) - \frac{3e^6(16d - 5ex)\sqrt{d^2 - e^2 x^2}}{16x} + \frac{e^4(16d + 5ex)(d^2 - e^2 x^2)^{3/2}}{16x^3}$$

[Out]  $(-3 * e^6 * (16 * d - 5 * e * x) * \text{Sqrt}[d^2 - e^2 * x^2]) / (16 * x) + (e^4 * (16 * d + 5 * e * x) * (d^2 - e^2 * x^2)^{(3/2)}) / (16 * x^3) - (e^2 * (24 * d + 5 * e * x) * (d^2 - e^2 * x^2)^{(5/2)}) / (40 * x^5) - (d * (d^2 - e^2 * x^2)^{(7/2)}) / (7 * x^7) - (e * (d^2 - e^2 * x^2)^{(7/2)}) / (2 * x^6) - 3 * d * e^7 * \text{ArcTan}[(e * x) / \text{Sqrt}[d^2 - e^2 * x^2]] - (15 * d * e^7 * \text{ArcTanh}[\text{Sqrt}[d^2 - e^2 * x^2] / d]) / 16$

**Rubi [A]** time = 0.621374, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{d(d^2 - e^2 x^2)^{7/2}}{7x^7} - \frac{e(d^2 - e^2 x^2)^{7/2}}{2x^6} - \frac{e^2(24d + 5ex)(d^2 - e^2 x^2)^{5/2}}{40x^5} - 3de^7 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right) - \frac{15}{16}de^7 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right) - \frac{3e^6(16d - 5ex)\sqrt{d^2 - e^2 x^2}}{16x} + \frac{e^4(16d + 5ex)(d^2 - e^2 x^2)^{3/2}}{16x^3}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2))/x^8, x]

[Out]  $(-3 * e^6 * (16 * d - 5 * e * x) * \text{Sqrt}[d^2 - e^2 * x^2]) / (16 * x) + (e^4 * (16 * d + 5 * e * x) * (d^2 - e^2 * x^2)^{(3/2)}) / (16 * x^3) - (e^2 * (24 * d + 5 * e * x) * (d^2 - e^2 * x^2)^{(5/2)}) / (40 * x^5) - (d * (d^2 - e^2 * x^2)^{(7/2)}) / (7 * x^7) - (e * (d^2 - e^2 * x^2)^{(7/2)}) / (2 * x^6) - 3 * d * e^7 * \text{ArcTan}[(e * x) / \text{Sqrt}[d^2 - e^2 * x^2]] - (15 * d * e^7 * \text{ArcTanh}[\text{Sqrt}[d^2 - e^2 * x^2] / d]) / 16$

**Rubi in Sympy [A]** time = 97.3131, size = 245, normalized size = 1.19

$$\frac{d^7 \sqrt{d^2 - e^2 x^2}}{7x^7} - \frac{d^6 e \sqrt{d^2 - e^2 x^2}}{2x^6} - \frac{6d^5 e^2 \sqrt{d^2 - e^2 x^2}}{35x^5} + \frac{11d^4 e^3 \sqrt{d^2 - e^2 x^2}}{8x^4} + \frac{62d^3 e^4 \sqrt{d^2 - e^2 x^2}}{35x^3} - \frac{15d^2 e^5 \sqrt{d^2 - e^2 x^2}}{16x^2} - 3de^7 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right) - \frac{15de^7 \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{16} - \frac{156de^6 \sqrt{d^2 - e^2 x^2}}{35x} + e^7 \sqrt{d^2 - e^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*8, x)

[Out]  $-d**7 * \text{sqrt}(d**2 - e**2 * x**2) / (7 * x**7) - d**6 * e * \text{sqrt}(d**2 - e**2 * x**2) / (2 * x**6) - 6 * d**5 * e**2 * \text{sqrt}(d**2 - e**2 * x**2) / (35 * x**5) + 11 * d**4 * e**3 * \text{sqrt}(d**2 - e**2 * x**2) / (8 * x**4) + 62 * d**3 * e**4 * \text{sqrt}(d**2 - e**2 * x**2) / (35 * x**3) - 15 * d**2 * e**5 * \text{sqrt}(d**2 - e**2 * x**2) / (16 * x**2) - 3 * d * e**7 * \operatorname{atan}(e * x / \text{sqrt}(d**2 - e**2 * x**2)) - 15 * d * e**7 * \operatorname{atanh}(\text{sqrt}(d**2 - e**2 * x**2) / d) / 16 - 156 * d * e**6 * \text{sqrt}(d**2 - e**2 * x**2) / (35 * x) + e**7 * \text{sqrt}(d**2 - e**2 * x**2)$



**Mathematica [A]** time = 0.273002, size = 161, normalized size = 0.78

$$-\frac{15}{16}de^7 \log\left(\sqrt{d^2 - e^2x^2} + d\right) - 3de^7 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) \\ \frac{\sqrt{d^2 - e^2x^2} (80d^7 + 280d^6ex + 96d^5e^2x^2 - 770d^4e^3x^3 - 992d^3e^4x^4 + 525d^2e^5x^5 + 2496de^6x^6 - 560e^7x^7)}{560x^7} \\ + \frac{15}{16}de^7 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2))/x^8, x]

[Out] -(Sqrt[d^2 - e^2\*x^2]\*(80\*d^7 + 280\*d^6\*e\*x + 96\*d^5\*e^2\*x^2 - 770\*d^4\*e^3\*x^3 - 992\*d^3\*e^4\*x^4 + 525\*d^2\*e^5\*x^5 + 2496\*d\*e^6\*x^6 - 560\*e^7\*x^7))/(560\*x^7) - 3\*d\*e^7\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] + (15\*d\*e^7\*Log[x])/16 - (15\*d\*e^7\*Log[d + Sqrt[d^2 - e^2\*x^2]])/16

**Maple [B]** time = 0.062, size = 377, normalized size = 1.8

$$-\frac{d}{7x^7}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{e^3}{8d^2x^4}(-e^2x^2 + d^2)^{\frac{7}{2}} + \frac{3e^5}{16d^4x^2}(-e^2x^2 + d^2)^{\frac{7}{2}} + \frac{3e^7}{16d^4}(-e^2x^2 + d^2)^{\frac{5}{2}} \\ + \frac{5e^7}{16d^2}(-e^2x^2 + d^2)^{\frac{3}{2}} + \frac{15e^7}{16}\sqrt{-e^2x^2 + d^2} - \frac{15e^7d^2}{16}\ln\left(\frac{1}{x}\left(2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}} \\ - \frac{3e^2}{5dx^5}(-e^2x^2 + d^2)^{\frac{7}{2}} + \frac{2e^4}{5d^3x^3}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{8e^6}{5d^5x}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{8e^8x}{5d^5}(-e^2x^2 + d^2)^{\frac{5}{2}} \\ - 2\frac{e^8x(-e^2x^2 + d^2)^{3/2}}{d^3} - 3\frac{e^8x\sqrt{-e^2x^2 + d^2}}{d} - 3\frac{de^8}{\sqrt{e^2}}\arctan\left(\frac{\sqrt{e^2}x}{\sqrt{-e^2x^2 + d^2}}\right) - \frac{e}{2x^6}(-e^2x^2 + d^2)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(-e^2\*x^2+d^2)^(5/2)/x^8, x)

[Out] -1/7\*d\*(-e^2\*x^2+d^2)^(7/2)/x^7-1/8\*e^3/d^2/x^4\*(-e^2\*x^2+d^2)^(7/2)+3/16\*e^5/d^4/x^2\*(-e^2\*x^2+d^2)^(7/2)+3/16\*e^7/d^4\*(-e^2\*x^2+d^2)^(5/2)+5/16\*e^7/d^2\*(-e^2\*x^2+d^2)^(3/2)+15/16\*e^7\*(-e^2\*x^2+d^2)^(1/2)-15/16\*e^7\*d^2/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)-3/5/d\*e^2/x^5\*(-e^2\*x^2+d^2)^(7/2)+2/5/d^3\*e^4/x^3\*(-e^2\*x^2+d^2)^(7/2)-8/5/d^5\*e^6/x\*(-e^2\*x^2+d^2)^(7/2)-8/5/d^5\*e^8\*x\*(-e^2\*x^2+d^2)^(5/2)-2/d^3\*e^8\*x\*(-e^2\*x^2+d^2)^(3/2)-3/d\*e^8\*x\*(-e^2\*x^2+d^2)^(1/2)-3\*d\*e^8/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))-1/2\*e\*(-e^2\*x^2+d^2)^(7/2)/x^6

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3/x^8, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.302656, size = 988, normalized size = 4.8

$$3920 d e^{15} x^{15} - 19968 d^2 e^{14} x^{14} - 35560 d^3 e^{13} x^{13} + 227584 d^4 e^{12} x^{12} + 115080 d^5 e^{11} x^{11} - 766976 d^6 e^{10} x^{10} - 248640 d^7 e^9 x^9$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3/x^8,x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/560*(3920*d*e^{15}*x^{15} - 19968*d^2*e^{14}*x^{14} - 35560*d^3*e^{13}*x^{13} + 227584*d^4*e^{12}*x^{12} + 115080*d^5*e^{11}*x^{11} - 766976*d^6*e^{10}*x^{10} - 248640*d^7*e^9*x^9 + 1076352*d^8*e^8*x^8 + 402080*d^9*e^7*x^7 - 656000*d^{10}*e^6*x^6 - 389760*d^{11}*e^5*x^5 + 135936*d^{12}*e^4*x^4 + 188160*d^{13}*e^3*x^3 + 13312*d^{14}*e^2*x^2 - 35840*d^{15}*e*x - 10240*d^{16} - 3360*(d*e^{15}*x^{15} - 32*d^3*e^{13}*x^{13} + 160*d^5*e^{11}*x^{11} - 256*d^7*e^9*x^9 + 128*d^9*e^7*x^7 + 8*(d^2*e^{13}*x^{13} - 10*d^4*e^{11}*x^{11} + 24*d^6*e^9*x^9 - 16*d^8*e^7*x^7)*\sqrt{-e^2*x^2 + d^2})*\arctan(-(d - \sqrt{-e^2*x^2 + d^2})/(e*x)) - 525*(d*e^{15}*x^{15} - 32*d^3*e^{13}*x^{13} + 160*d^5*e^{11}*x^{11} - 256*d^7*e^9*x^9 + 128*d^9*e^7*x^7 + 8*(d^2*e^{13}*x^{13} - 10*d^4*e^{11}*x^{11} + 24*d^6*e^9*x^9 - 16*d^8*e^7*x^7)*\sqrt{-e^2*x^2 + d^2})*\log(-(d - \sqrt{-e^2*x^2 + d^2})/x) - (560*e^{15}*x^{15} - 2496*d*e^{14}*x^{14} - 13965*d^2*e^{13}*x^{13} + 80864*d^3*e^{12}*x^{12} + 62370*d^4*e^{11}*x^{11} - 431200*d^5*e^{10}*x^{10} - 144760*d^6*e^9*x^9 + 800688*d^7*e^8*x^8 + 266560*d^8*e^7*x^7 - 586240*d^9*e^6*x^6 - 309120*d^{10}*e^5*x^5 + 138752*d^{11}*e^4*x^4 + 170240*d^{12}*e^3*x^3 + 8192*d^{13}*e^2*x^2 - 35840*d^{14}*e*x - 10240*d^{15})*\sqrt{-e^2*x^2 + d^2})/(e^8*x^{15} - 32*d^2*e^6*x^{13} + 160*d^4*e^4*x^{11} - 256*d^6*e^2*x^9 + 128*d^8*x^7 + 8*(d*e^6*x^{13} - 10*d^3*e^4*x^{11} + 24*d^5*e^2*x^9 - 16*d^7*x^7)*\sqrt{-e^2*x^2 + d^2})) \end{aligned}$$

**Sympy [A]** time = 60.842, size = 1513, normalized size = 7.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*8,x)

[Out] 
$$\begin{aligned} & d^{**7} \text{Piecewise}((-e*\sqrt{d^{**2}/(e^{**2}*x^{**2})} - 1)/(7*x^{**6}) + e^{**3}*\sqrt{d^{**2}/(e^{**2}*x^{**2})} - 1)/(35*d^{**2}*x^{**4}) + 4*e^{**5}*\sqrt{d^{**2}/(e^{**2}*x^{**2})} - 1)/(105*d^{**4}*x^{**2}) + 8*e^{**7}*\sqrt{d^{**2}/(e^{**2}*x^{**2})} - 1)/(105*d^{**6}), \text{Abs}(d^{**2}/(e^{**2}*x^{**2})) > 1), (-I*e*\sqrt{-d^{**2}/(e^{**2}*x^{**2})} + 1)/(7*x^{**6}) + I*e^{**3}*\sqrt{-d^{**2}/(e^{**2}*x^{**2})} + 1)/(35*d^{**2}*x^{**4}) + 4*I*e^{**5}*\sqrt{-d^{**2}/(e^{**2}*x^{**2})} + 1)/(105*d^{**4}*x^{**2}) + 8*I*e^{**7}*\sqrt{-d^{**2}/(e^{**2}*x^{**2})} + 1)/(105*d^{**6}), \text{True})) + 3*d^{**6}*e*\text{Piecewise}((-d^{**2}/(6*e*x^{**7}*\sqrt{d^{**2}/(e^{**2}*x^{**2})} - 1)) + 5*e/(24*x^{**5}*\sqrt{d^{**2}/(e^{**2}*x^{**2})} - 1)) + e^{**3}/(48*d^{**2}*x^{**3}*\sqrt{d^{**2}/(e^{**2}*x^{**2})} - 1)) - e^{**5}/(16*d^{**4}*x*\sqrt{d^{**2}/(e^{**2}*x^{**2})} - 1)) + e^{**6}*\text{acosh}(d/(e*x))/(16*d^{**5}), \text{Abs}(d^{**2}/(e^{**2}*x^{**2})) > 1), (I*d^{**2}/(6*e*x^{**7}*\sqrt{-d^{**2}/(e^{**2}*x^{**2})} + 1)) - 5*I*e/(24*x^{**5}*\sqrt{-d^{**2}/(e^{**2}*x^{**2})} + 1)) - I*e^{**3}/(48*d^{**2}*x^{**3}*\sqrt{-d^{**2}/(e^{**2}*x^{**2})} + 1)) + I*e^{**5}/(16*d^{**4}*x*\sqrt{-d^{**2}/(e^{**2}*x^{**2})} + 1)) - I*e^{**6}*\text{asin}(d/(e*x))/(16*d^{**5}), \text{True})) + d^{**5}*e^{**2}*\text{Piecewise}((3*I*d^{**3}*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}})/(-15*d^{**2}*x^{**5} + 15*e^{**2}*x^{**7}) - 4*I*d^{**2}*x^{**2}*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}})/(-15*d^{**2}*x^{**5} + 15*e^{**2}*x^{**7}) + 2*I*e^{**6}*x^{**6}*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}})/(-15*d^{**5}*x^{**5} + 15*d^{**3}*e^{**2}*x^{**7}) - I*e^{**4}*x^{**4}*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}})/(-15*d^{**3}*x^{**5} + 15*d^{**2}*e^{**2}*x^{**7}), \text{Abs}(e^{**2}*x^{**2}/d^{**2}) > 1), (3*d^{**3}*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}})/(-15*d^{**2}*x^{**5} + 15*e^{**2}*x^{**7}) - 4*d^{**2}*e^{**2}*x^{**2}*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}})/(-15*d^{**2}*x^{**5} + 15*e^{**2}*x^{**7}) + 2*e^{**6}*x^{**6}*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}})/(-15*d^{**5}*x^{**5} + 15*d^{**3}*e^{**2}*x^{**7}) - e^{**4}*x^{**4}*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}})/(-15*d^{**3}*x^{**5} + 15*d^{**2}*e^{**2}*x^{**7}), \text{True})) - 5*d^{**4}*e^{**3}*\text{Piecewise}((-d^{**2}/(4*e*x^{**5}*\sqrt{d^{**2}/(e^{**2}*x^{**2})} - 1)) + 5*e/(24*x^{**5}*\sqrt{d^{**2}/(e^{**2}*x^{**2})} - 1)) + e^{**3}/(48*d^{**2}*x^{**3}*\sqrt{d^{**2}/(e^{**2}*x^{**2})} - 1)) - e^{**5}/(16*d^{**4}*x*\sqrt{d^{**2}/(e^{**2}*x^{**2})} - 1)) + e^{**6}*\text{acosh}(d/(e*x))/(16*d^{**5}), \text{Abs}(d^{**2}/(e^{**2}*x^{**2})) > 1), (I*d^{**2}/(6*e*x^{**7}*\sqrt{-d^{**2}/(e^{**2}*x^{**2})} + 1)) - 5*I*e/(24*x^{**5}*\sqrt{-d^{**2}/(e^{**2}*x^{**2})} + 1)) - I*e^{**3}/(48*d^{**2}*x^{**3}*\sqrt{-d^{**2}/(e^{**2}*x^{**2})} + 1)) + I*e^{**5}/(16*d^{**4}*x*\sqrt{-d^{**2}/(e^{**2}*x^{**2})} + 1)) - I*e^{**6}*\text{asin}(d/(e*x))/(16*d^{**5}), \text{True})) \end{aligned}$$

$$\begin{aligned}
& *2*x^{**2}) - 1)) + 3*e/(8*x^{**3}*sqrt(d^{**2}/(e^{**2}*x^{**2}) - 1)) - e^{**3}/( \\
& 8*d^{**2}*x*sqrt(d^{**2}/(e^{**2}*x^{**2}) - 1)) + e^{**4}*acosh(d/(e*x))/(8*d^{** \\
& 3), Abs(d^{**2}/(e^{**2}*x^{**2})) > 1), (I*d^{**2}/(4*e*x^{**5}*sqrt(-d^{**2}/(e^{** \\
& 2*x^{**2}) + 1)) - 3*I*e/(8*x^{**3}*sqrt(-d^{**2}/(e^{**2}*x^{**2}) + 1)) + I*e \\
& ^{**3}/(8*d^{**2}*x*sqrt(-d^{**2}/(e^{**2}*x^{**2}) + 1)) - I*e^{**4}*asin(d/(e*x))/ \\
& (8*d^{**3}), True)) - 5*d^{**3}*e^{**4}*Piecewise((-e*sqrt(d^{**2}/(e^{**2}*x^{**2} \\
& ) - 1)/(3*x^{**2}) + e^{**3}*sqrt(d^{**2}/(e^{**2}*x^{**2}) - 1)/(3*d^{**2}), Abs(d \\
& ^{**2}/(e^{**2}*x^{**2})) > 1), (-I*e*sqrt(-d^{**2}/(e^{**2}*x^{**2}) + 1)/(3*x^{**2}) \\
& + I*e^{**3}*sqrt(-d^{**2}/(e^{**2}*x^{**2}) + 1)/(3*d^{**2}), True)) + d^{**2}*e^{** \\
& 5*Piecewise((-d^{**2}/(2*e*x^{**3}*sqrt(d^{**2}/(e^{**2}*x^{**2}) - 1)) + e/(2*x \\
& *sqrt(d^{**2}/(e^{**2}*x^{**2}) - 1)) + e^{**2}*acosh(d/(e*x))/(2*d), Abs(d^{** \\
& 2}/(e^{**2}*x^{**2})) > 1), (-I*e*sqrt(-d^{**2}/(e^{**2}*x^{**2}) + 1)/(2*x) - I* \\
& e^{**2}*asin(d/(e*x))/(2*d), True)) + 3*d*e^{**6}*Piecewise((I*d/(x*sqrt \\
& t(-1 + e^{**2}*x^{**2}/d^{**2})) + I*e*acosh(e*x/d) - I*e^{**2}*x/(d*sqrt(-1 \\
& + e^{**2}*x^{**2}/d^{**2})), Abs(e^{**2}*x^{**2}/d^{**2}) > 1), (-d/(x*sqrt(1 - e^{** \\
& 2}*x^{**2}/d^{**2})) - e*asin(e*x/d) + e^{**2}*x/(d*sqrt(1 - e^{**2}*x^{**2}/d^{**2} \\
& )), True)) + e^{**7}*Piecewise((d^{**2}/(e*x*sqrt(d^{**2}/(e^{**2}*x^{**2}) - 1) \\
& ) - d*acosh(d/(e*x)) - e*x/sqrt(d^{**2}/(e^{**2}*x^{**2}) - 1), Abs(d^{**2}/( \\
& e^{**2}*x^{**2})) > 1), (-I*d^{**2}/(e*x*sqrt(-d^{**2}/(e^{**2}*x^{**2}) + 1)) + I* \\
& d*asin(d/(e*x)) + I*e*x/sqrt(-d^{**2}/(e^{**2}*x^{**2}) + 1), True))
\end{aligned}$$


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**GIAC/XCAS [A]** time = 0.298732, size = 1, normalized size = 0.

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3/x^8,x, algorithm="giac")

[Out] Done

$$3.79 \quad \int \frac{(d+ex)^3 (d^2 - e^2 x^2)^{5/2}}{x^9} dx$$

**Optimal.** Leaf size=204

$$\frac{d(d^2 - e^2 x^2)^{7/2}}{8x^8} - \frac{3e(d^2 - e^2 x^2)^{7/2}}{7x^7} - \frac{e^2(125d + 48ex)(d^2 - e^2 x^2)^{5/2}}{240x^6} + e^8 \left( -\tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) \right) + \frac{125}{128} e^8 \tanh^{-1} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right) - \frac{e^6(125d + 128ex)\sqrt{d^2 - e^2 x^2}}{128x^2} + \frac{e^4(125d + 64ex)(d^2 - e^2 x^2)^{3/2}}{192x^4}$$

[Out]  $-(e^6*(125*d + 128*e*x)*\text{Sqrt}[d^2 - e^2*x^2])/(128*x^2) + (e^4*(125*d + 64*e*x)*(d^2 - e^2*x^2)^{(3/2)})/(192*x^4) - (e^2*(125*d + 48*e*x)*(d^2 - e^2*x^2)^{(5/2)})/(240*x^6) - (d*(d^2 - e^2*x^2)^{(7/2)})/(8*x^8) - (3*e*(d^2 - e^2*x^2)^{(7/2)})/(7*x^7) - e^8*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]] + (125*e^8*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/128$

**Rubi [A]** time = 0.605544, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$

$$\frac{d(d^2 - e^2 x^2)^{7/2}}{8x^8} - \frac{3e(d^2 - e^2 x^2)^{7/2}}{7x^7} - \frac{e^2(125d + 48ex)(d^2 - e^2 x^2)^{5/2}}{240x^6} + e^8 \left( -\tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) \right) + \frac{125}{128} e^8 \tanh^{-1} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right) - \frac{e^6(125d + 128ex)\sqrt{d^2 - e^2 x^2}}{128x^2} + \frac{e^4(125d + 64ex)(d^2 - e^2 x^2)^{3/2}}{192x^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^3*(d^2 - e^2*x^2)^{(5/2)}/x^9, x]$

[Out]  $-(e^6*(125*d + 128*e*x)*\text{Sqrt}[d^2 - e^2*x^2])/(128*x^2) + (e^4*(125*d + 64*e*x)*(d^2 - e^2*x^2)^{(3/2)})/(192*x^4) - (e^2*(125*d + 48*e*x)*(d^2 - e^2*x^2)^{(5/2)})/(240*x^6) - (d*(d^2 - e^2*x^2)^{(7/2)})/(8*x^8) - (3*e*(d^2 - e^2*x^2)^{(7/2)})/(7*x^7) - e^8*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]] + (125*e^8*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/128$

**Rubi in Sympy [A]** time = 109.416, size = 248, normalized size = 1.22

$$\begin{aligned} & -\frac{d^7\sqrt{d^2 - e^2x^2}}{8x^8} - \frac{3d^6e\sqrt{d^2 - e^2x^2}}{7x^7} - \frac{7d^5e^2\sqrt{d^2 - e^2x^2}}{48x^6} + \frac{38d^4e^3\sqrt{d^2 - e^2x^2}}{35x^5} \\ & + \frac{253d^3e^4\sqrt{d^2 - e^2x^2}}{192x^4} - \frac{58d^2e^5\sqrt{d^2 - e^2x^2}}{105x^3} - \frac{259de^6\sqrt{d^2 - e^2x^2}}{128x^2} \\ & - e^8 \operatorname{atan} \left( \frac{ex}{\sqrt{d^2 - e^2x^2}} \right) + \frac{125e^8 \operatorname{atanh} \left( \frac{\sqrt{d^2 - e^2x^2}}{d} \right)}{128} - \frac{116e^7\sqrt{d^2 - e^2x^2}}{105x} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**9, x)$

[Out]  $-d**7*\text{sqrt}(d**2 - e**2*x**2)/(8*x**8) - 3*d**6*e*\text{sqrt}(d**2 - e**2*x**2)/(7*x**7) - 7*d**5*e**2*\text{sqrt}(d**2 - e**2*x**2)/(48*x**6) + 38*d**4*e**3*\text{sqrt}(d**2 - e**2*x**2)/(35*x**5) + 253*d**3*e**4*\text{sqrt}(d**2 - e**2*x**2)/(192*x**4) - 58*d**2*e**5*\text{sqrt}(d**2 - e**2*x**2)/(105*x**3) - 259*d*e**6*\text{sqrt}(d**2 - e**2*x**2)/(128*x**2) - e**8*\operatorname{atan}(e*x/\text{sqrt}(d**2 - e**2*x**2)) + 125*e**8*\operatorname{atanh}(\text{sqrt}(d**2 - e**2*x**2)/d)/128 - 116*e**7*\text{sqrt}(d**2 - e**2*x**2)/(105*x)$

---

**Mathematica [A]** time = 0.288094, size = 158, normalized size = 0.77

$$\frac{125}{128}e^8 \log\left(\sqrt{d^2 - e^2x^2} + d\right) + e^8 \left(-\tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)\right)$$

$$\frac{\sqrt{d^2 - e^2x^2} (1680d^7 + 5760d^6ex + 1960d^5e^2x^2 - 14592d^4e^3x^3 - 17710d^3e^4x^4 + 7424d^2e^5x^5 + 27195de^6x^6 + 14848e^7x^7)}{13440x^8}$$

$$- \frac{125}{128}e^8 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2))/x^9, x]

[Out] -(Sqrt[d^2 - e^2\*x^2]\*(1680\*d^7 + 5760\*d^6\*e\*x + 1960\*d^5\*e^2\*x^2 - 14592\*d^4\*e^3\*x^3 - 17710\*d^3\*e^4\*x^4 + 7424\*d^2\*e^5\*x^5 + 27195\*d\*e^6\*x^6 + 14848\*e^7\*x^7))/(13440\*x^8) - e^8\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] - (125\*e^8\*Log[x])/128 + (125\*e^8\*Log[d + Sqrt[d^2 - e^2\*x^2]])/128

---

**Maple [B]** time = 0.094, size = 402, normalized size = 2.

$$-\frac{d}{8x^8}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{25e^2}{48dx^6}(-e^2x^2 + d^2)^{\frac{7}{2}} + \frac{25e^4}{192d^3x^4}(-e^2x^2 + d^2)^{\frac{7}{2}}$$

$$- \frac{25e^6}{128d^5x^2}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{25e^8}{128d^5}(-e^2x^2 + d^2)^{\frac{5}{2}} - \frac{125e^8}{384d^3}(-e^2x^2 + d^2)^{\frac{3}{2}} - \frac{125e^8}{128d}\sqrt{-e^2x^2 + d^2}$$

$$+ \frac{125de^8}{128} \ln\left(\frac{1}{x}\left(2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}} - \frac{e^3}{5d^2x^5}(-e^2x^2 + d^2)^{\frac{7}{2}}$$

$$+ \frac{2e^5}{15d^4x^3}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{8e^7}{15d^6x}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{8e^9x}{15d^6}(-e^2x^2 + d^2)^{\frac{5}{2}} - \frac{2e^9x}{3d^4}(-e^2x^2 + d^2)^{\frac{3}{2}}$$

$$- \frac{e^9x}{d^2}\sqrt{-e^2x^2 + d^2} - e^9 \arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} - \frac{3e}{7x^7}(-e^2x^2 + d^2)^{\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(-e^2\*x^2+d^2)^(5/2)/x^9, x)

[Out] -1/8\*d\*(-e^2\*x^2+d^2)^(7/2)/x^8-25/48/d\*e^2/x^6\*(-e^2\*x^2+d^2)^(7/2)+25/192/d^3\*e^4/x^4\*(-e^2\*x^2+d^2)^(7/2)-25/128/d^5\*e^6/x^2\*(-e^2\*x^2+d^2)^(7/2)-25/128/d^5\*e^8\*(-e^2\*x^2+d^2)^(5/2)-125/384/d^3\*e^8\*(-e^2\*x^2+d^2)^(3/2)-125/128/d\*e^8\*(-e^2\*x^2+d^2)^(1/2)+125/128\*d\*e^8/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)-1/5\*e^3/d^2/x^5\*(-e^2\*x^2+d^2)^(7/2)+2/15\*e^5/d^4/x^3\*(-e^2\*x^2+d^2)^(7/2)-8/15\*e^7/d^6/x\*(-e^2\*x^2+d^2)^(7/2)-8/15\*e^9/d^6\*x\*(-e^2\*x^2+d^2)^(5/2)-2/3\*e^9/d^4\*x\*(-e^2\*x^2+d^2)^(3/2)-e^9/d^2\*x\*(-e^2\*x^2+d^2)^(1/2)-e^9/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))-3/7\*e\*(-e^2\*x^2+d^2)^(7/2)/x^7

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3/x^9, x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.553355, size = 980, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3/x^9,x, algorithm="fricas")

[Out] 1/13440\*(118784\*d\*e^15\*x^15 + 217560\*d^2\*e^14\*x^14 - 1247232\*d^3\*e^13\*x^13 - 2534840\*d^4\*e^12\*x^12 + 3268608\*d^5\*e^11\*x^11 + 8971200\*d^6\*e^10\*x^10 - 1401856\*d^7\*e^9\*x^9 - 13678560\*d^8\*e^8\*x^8 - 4951040\*d^9\*e^7\*x^7 + 9533440\*d^10\*e^6\*x^6 + 7186432\*d^11\*e^5\*x^5 - 2437120\*d^12\*e^4\*x^4 - 3710976\*d^13\*e^3\*x^3 - 286720\*d^14\*e^2\*x^2 + 737280\*d^15\*e\*x + 215040\*d^16 + 26880\*(e^16\*x^16 - 32\*d^2\*e^14\*x^14 + 160\*d^4\*e^12\*x^12 - 256\*d^6\*e^10\*x^10 + 128\*d^8\*e^8\*x^8 + 8\*(d\*e^14\*x^14 - 10\*d^3\*e^12\*x^12 + 24\*d^5\*e^10\*x^10 - 16\*d^7\*e^8\*x^8)\*sqrt(-e^2\*x^2 + d^2))\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) - 13125\*(e^16\*x^16 - 32\*d^2\*e^14\*x^14 + 160\*d^4\*e^12\*x^12 - 256\*d^6\*e^10\*x^10 + 128\*d^8\*e^8\*x^8 + 8\*(d\*e^14\*x^14 - 10\*d^3\*e^12\*x^12 + 24\*d^5\*e^10\*x^10 - 16\*d^7\*e^8\*x^8)\*sqrt(-e^2\*x^2 + d^2))\*log(-(d - sqrt(-e^2\*x^2 + d^2))/x) - (14848\*e^15\*x^15 + 27195\*d\*e^14\*x^14 - 467712\*d^2\*e^13\*x^13 - 887950\*d^3\*e^12\*x^12 + 2123520\*d^4\*e^11\*x^11 + 4919880\*d^5\*e^10\*x^10 - 2140544\*d^6\*e^9\*x^9 - 9856560\*d^7\*e^8\*x^8 - 2519040\*d^8\*e^7\*x^7 + 8274560\*d^9\*e^6\*x^6 + 5607424\*d^10\*e^5\*x^5 - 2499840\*d^11\*e^4\*x^4 - 3342336\*d^12\*e^3\*x^3 - 179200\*d^13\*e^2\*x^2 + 737280\*d^14\*e\*x + 215040\*d^15)\*sqrt(-e^2\*x^2 + d^2)/(e^8\*x^16 - 32\*d^2\*e^6\*x^14 + 160\*d^4\*e^4\*x^12 - 256\*d^6\*e^2\*x^10 + 128\*d^8\*x^8 + 8\*(d\*e^6\*x^14 - 10\*d^3\*e^4\*x^12 + 24\*d^5\*e^2\*x^10 - 16\*d^7\*x^8)\*sqrt(-e^2\*x^2 + d^2))

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**Sympy [A]** time = 89.1017, size = 1719, normalized size = 8.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*9,x)

[Out] d\*\*7\*Piecewise((-d\*\*2/(8\*e\*x\*\*9\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) + 7\*e/(48\*x\*\*7\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) + e\*\*3/(192\*d\*\*2\*x\*\*5\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) + 5\*e\*\*5/(384\*d\*\*4\*x\*\*3\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) - 5\*e\*\*7/(128\*d\*\*6\*x\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) + 5\*e\*\*8\*acosh(d/(e\*x))/(128\*d\*\*7), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (I\*d\*\*2/(8\*e\*x\*\*9\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) - 7\*I\*e/(48\*x\*\*7\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) - I\*e\*\*3/(192\*d\*\*2\*x\*\*5\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) - 5\*I\*e\*\*5/(384\*d\*\*4\*x\*\*3\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) + 5\*I\*e\*\*7/(128\*d\*\*6\*x\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) - 5\*I\*e\*\*8\*asin(d/(e\*x))/(128\*d\*\*7), True)) + 3\*d\*\*6\*e\*Piecewise((-e\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)/(7\*x\*\*6) + e\*\*3\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)/(35\*d\*\*2\*x\*\*4) + 4\*e\*\*5\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)/(105\*d\*\*4\*x\*\*2) + 8\*e\*\*7\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)/(105\*d\*\*6), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (-I\*e\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)/(7\*x\*\*6) + I\*e\*\*3\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)/(35\*d\*\*2\*x\*\*4) + 4\*I\*e\*\*5\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)/(105\*d\*\*4\*x\*\*2) + 8\*I\*e\*\*7\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)/(105\*d\*\*6), True)) + d\*\*5\*e\*\*2\*Piecewise((-d\*\*2/(6\*e\*x\*\*7\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) + 5\*e/(24\*x\*\*5\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) + e\*\*3/(48\*d\*\*2\*x\*\*3\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) - e\*\*5/(16\*d\*\*4\*x\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) + e\*\*6\*acosh(d/(e\*x))/(16\*d\*\*5), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (I\*d\*\*2/(6\*e\*x\*\*7\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) - 5\*I\*e/(24\*x\*\*5\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) - I\*e\*\*3/(48\*d\*\*2\*x\*\*3\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) + I\*e\*\*5/(16\*d\*\*4\*x\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) - I\*e\*\*6\*asin(d/(e\*x))/(16\*d\*\*5), True)) - 5\*d\*\*4\*e\*\*3\*Piecewise((3\*I\*d\*\*3\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)/(-1

```

5*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2
/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e
**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*
sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e
**2*x**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x
**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15
*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)
/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x*
**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) - 5*d**3*e**4*P
iecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x*
**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x
**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2))
> 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x
**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e
**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True)) + d**2*e*
**5*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(
d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e
*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x
**2) + 1)/(3*d**2), True)) + 3*d*e**6*Piecewise((-d**2/(2*e*x**3*
sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1))
+ e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sq
rt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d), Tru
e)) + e**7*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*aco
sh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2
/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e
**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True))

```

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**GIAC/XCAS [A]** time = 0.305713, size = 1, normalized size = 0.

*Done*

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3/x^9,x, algorithm="giac")
```

```
[Out] Done
```

$$3.80 \quad \int \frac{(d+ex)^3 (d^2 - e^2 x^2)^{5/2}}{x^{10}} dx$$

**Optimal.** Leaf size=187

$$\begin{aligned} & -\frac{d(d^2 - e^2 x^2)^{7/2}}{9x^9} - \frac{3e(d^2 - e^2 x^2)^{7/2}}{8x^8} - \frac{29e^2(d^2 - e^2 x^2)^{7/2}}{63dx^7} + \frac{55e^9 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{128d} \\ & - \frac{55e^7 \sqrt{d^2 - e^2 x^2}}{128x^2} + \frac{55e^5 (d^2 - e^2 x^2)^{3/2}}{192x^4} - \frac{11e^3 (d^2 - e^2 x^2)^{5/2}}{48x^6} \end{aligned}$$

[Out]  $(-55 * e^7 * \text{Sqrt}[d^2 - e^2 * x^2]) / (128 * x^2) + (55 * e^5 * (d^2 - e^2 * x^2)^{3/2}) / (192 * x^4) - (11 * e^3 * (d^2 - e^2 * x^2)^{5/2}) / (48 * x^6) - (d * (d^2 - e^2 * x^2)^{7/2}) / (9 * x^9) - (3 * e * (d^2 - e^2 * x^2)^{7/2}) / (8 * x^8) - (29 * e^2 * (d^2 - e^2 * x^2)^{7/2}) / (63 * d * x^7) + (55 * e^9 * \text{ArcTanh}[\text{Sqrt}[d^2 - e^2 * x^2] / d]) / (128 * d)$

**Rubi [A]** time = 0.437093, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\begin{aligned} & -\frac{d(d^2 - e^2 x^2)^{7/2}}{9x^9} - \frac{3e(d^2 - e^2 x^2)^{7/2}}{8x^8} - \frac{29e^2(d^2 - e^2 x^2)^{7/2}}{63dx^7} + \frac{55e^9 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{128d} \\ & - \frac{55e^7 \sqrt{d^2 - e^2 x^2}}{128x^2} + \frac{55e^5 (d^2 - e^2 x^2)^{3/2}}{192x^4} - \frac{11e^3 (d^2 - e^2 x^2)^{5/2}}{48x^6} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2))/x^10, x]

[Out]  $(-55 * e^7 * \text{Sqrt}[d^2 - e^2 * x^2]) / (128 * x^2) + (55 * e^5 * (d^2 - e^2 * x^2)^{3/2}) / (192 * x^4) - (11 * e^3 * (d^2 - e^2 * x^2)^{5/2}) / (48 * x^6) - (d * (d^2 - e^2 * x^2)^{7/2}) / (9 * x^9) - (3 * e * (d^2 - e^2 * x^2)^{7/2}) / (8 * x^8) - (29 * e^2 * (d^2 - e^2 * x^2)^{7/2}) / (63 * d * x^7) + (55 * e^9 * \text{ArcTanh}[\text{Sqrt}[d^2 - e^2 * x^2] / d]) / (128 * d)$

**Rubi in Sympy [A]** time = 122.167, size = 253, normalized size = 1.35

$$\begin{aligned} & -\frac{d^7 \sqrt{d^2 - e^2 x^2}}{9x^9} - \frac{3d^6 e \sqrt{d^2 - e^2 x^2}}{8x^8} - \frac{8d^5 e^2 \sqrt{d^2 - e^2 x^2}}{63x^7} + \frac{43d^4 e^3 \sqrt{d^2 - e^2 x^2}}{48x^6} \\ & + \frac{22d^3 e^4 \sqrt{d^2 - e^2 x^2}}{21x^5} - \frac{73d^2 e^5 \sqrt{d^2 - e^2 x^2}}{192x^4} - \frac{80de^6 \sqrt{d^2 - e^2 x^2}}{63x^3} \\ & - \frac{73e^7 \sqrt{d^2 - e^2 x^2}}{128x^2} + \frac{55e^9 \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{128d} + \frac{29e^8 \sqrt{d^2 - e^2 x^2}}{63dx} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*10, x)

[Out]  $-d^{**7} * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (9 * x^{**9}) - 3 * d^{**6} * e * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (8 * x^{**8}) - 8 * d^{**5} * e^{**2} * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (63 * x^{**7}) + 43 * d^{**4} * e^{**3} * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (48 * x^{**6}) + 22 * d^{**3} * e^{**4} * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (21 * x^{**5}) - 73 * d^{**2} * e^{**5} * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (192 * x^{**4}) - 80 * d * e^{**6} * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (63 * x^{**3}) - 73 * e^{**7} * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (128 * x^{**2}) + 55 * e^{**9} * \operatorname{atanh}(\text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / d) / (128 * d) + 29 * e^{**8} * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (63 * d * x)$



**Mathematica [A]** time = 0.304955, size = 150, normalized size = 0.8

$$\frac{-3465e^9x^9 \log\left(\sqrt{d^2 - e^2x^2} + d\right) + \sqrt{d^2 - e^2x^2} (896d^8 + 3024d^7ex + 1024d^6e^2x^2 - 7224d^5e^3x^3 - 8448d^4e^4x^4 + 3066d^3e^5x^5 - 3465d^2e^6x^6 + 4599de^7x^7 - 3712e^8x^8) + 3465e^9x^9 \text{Log}[x]}{8064dx^9}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2))/x^10, x]

[Out] -(Sqrt[d^2 - e^2\*x^2]\*(896\*d^8 + 3024\*d^7\*e\*x + 1024\*d^6\*e^2\*x^2 - 7224\*d^5\*e^3\*x^3 - 8448\*d^4\*e^4\*x^4 + 3066\*d^3\*e^5\*x^5 + 10240\*d^2\*e^6\*x^6 + 4599\*d\*e^7\*x^7 - 3712\*e^8\*x^8) + 3465\*e^9\*x^9\*Log[x] - 3465\*e^9\*x^9\*Log[d + Sqrt[d^2 - e^2\*x^2]])/(8064\*d\*x^9)

**Maple [A]** time = 0.134, size = 250, normalized size = 1.3

$$\begin{aligned} & -\frac{d}{9x^9}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{29e^2}{63dx^7}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{11e^3}{48d^2x^6}(-e^2x^2 + d^2)^{\frac{7}{2}} + \frac{11e^5}{192d^4x^4}(-e^2x^2 + d^2)^{\frac{7}{2}} \\ & - \frac{11e^7}{128d^6x^2}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{11e^9}{128d^6}(-e^2x^2 + d^2)^{\frac{5}{2}} - \frac{55e^9}{384d^4}(-e^2x^2 + d^2)^{\frac{3}{2}} \\ & - \frac{55e^9}{128d^2}\sqrt{-e^2x^2 + d^2} + \frac{55e^9}{128}\ln\left(\frac{1}{x}\left(2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2}\right)\right)\frac{1}{\sqrt{d^2}} - \frac{3e}{8x^8}(-e^2x^2 + d^2)^{\frac{7}{2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(-e^2\*x^2+d^2)^(5/2)/x^10, x)

[Out] -1/9\*d\*(-e^2\*x^2+d^2)^(7/2)/x^9-29/63\*e^2\*(-e^2\*x^2+d^2)^(7/2)/d/x^7-11/48\*e^3/d^2/x^6\*(-e^2\*x^2+d^2)^(7/2)+11/192\*e^5/d^4/x^4\*(-e^2\*x^2+d^2)^(7/2)-11/128\*e^7/d^6/x^2\*(-e^2\*x^2+d^2)^(7/2)-11/128\*e^9/d^6\*(-e^2\*x^2+d^2)^(5/2)-55/384\*e^9/d^4\*(-e^2\*x^2+d^2)^(3/2)-55/128\*e^9/d^2\*(-e^2\*x^2+d^2)^(1/2)+55/128\*e^9/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)-3/8\*e\*(-e^2\*x^2+d^2)^(7/2)/x^8

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3/x^10, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.579292, size = 887, normalized size = 4.74

$$3712e^{18}x^{18} - 4599de^{17}x^{17} - 162432d^2e^{16}x^{16} + 185493d^3e^{15}x^{15} + 1467648d^4e^{14}x^{14} - 1154790d^5e^{13}x^{13} - 5768448d^6e^{12}x^{12} - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3/x^10, x, algorithm="fricas")

```
[Out] 1/8064*(3712*e^18*x^18 - 4599*d*e^17*x^17 - 162432*d^2*e^16*x^16
+ 185493*d^3*e^15*x^15 + 1467648*d^4*e^14*x^14 - 1154790*d^5*e^13
*x^13 - 5768448*d^6*e^12*x^12 + 2006424*d^7*e^11*x^11 + 12064896*
d^8*e^10*x^10 + 1018416*d^9*e^9*x^9 - 14221440*d^10*e^8*x^8 - 679
7952*d^11*e^7*x^7 + 9022464*d^12*e^6*x^6 + 7951104*d^13*e^5*x^5 -
2267136*d^14*e^4*x^4 - 3978240*d^15*e^3*x^3 - 368640*d^16*e^2*x^
2 + 774144*d^17*e*x + 229376*d^18 - 3465*(9*d*e^17*x^17 - 120*d^3
*e^15*x^15 + 432*d^5*e^13*x^13 - 576*d^7*e^11*x^11 + 256*d^9*e^9*
x^9 - (e^17*x^17 - 40*d^2*e^15*x^15 + 240*d^4*e^13*x^13 - 448*d^6
*e^11*x^11 + 256*d^8*e^9*x^9)*sqrt(-e^2*x^2 + d^2))*log(-(d - sqr
t(-e^2*x^2 + d^2))/x) + (33408*d*e^16*x^16 - 41391*d^2*e^15*x^15
- 537600*d^3*e^14*x^14 + 524286*d^4*e^13*x^13 + 2908416*d^5*e^12*
x^12 - 1553832*d^6*e^11*x^11 - 7584768*d^7*e^10*x^10 + 430416*d^8
*e^9*x^9 + 10612864*d^9*e^8*x^8 + 4072320*d^10*e^7*x^7 - 7822336*
d^11*e^6*x^6 - 6252288*d^12*e^5*x^5 + 2365440*d^13*e^4*x^4 + 3591
168*d^14*e^3*x^3 + 253952*d^15*e^2*x^2 - 774144*d^16*e*x - 229376
*d^17)*sqrt(-e^2*x^2 + d^2))/(9*d^2*e^8*x^17 - 120*d^4*e^6*x^15 +
432*d^6*e^4*x^13 - 576*d^8*e^2*x^11 + 256*d^10*x^9 - (d*e^8*x^17
- 40*d^3*e^6*x^15 + 240*d^5*e^4*x^13 - 448*d^7*e^2*x^11 + 256*d^
9*x^9)*sqrt(-e^2*x^2 + d^2))
```

---

**Sympy [A]** time = 90.8948, size = 1889, normalized size = 10.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*10,x)

```
[Out] d**7*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(9*x**8) + e**3*sqrt
(d**2/(e**2*x**2) - 1)/(63*d**2*x**6) + 2*e**5*sqrt(d**2/(e**2*x
**2) - 1)/(105*d**4*x**4) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(31
5*d**6*x**2) + 16*e**9*sqrt(d**2/(e**2*x**2) - 1)/(315*d**8), Abs
(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(9*x**
8) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(63*d**2*x**6) + 2*I*e**5
*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**4) + 8*I*e**7*sqrt(-d**
2/(e**2*x**2) + 1)/(315*d**6*x**2) + 16*I*e**9*sqrt(-d**2/(e**2*x
**2) + 1)/(315*d**8), True)) + 3*d**6*e*Piecewise((-d**2/(8*e*x**
9*sqrt(d**2/(e**2*x**2) - 1)) + 7*e/(48*x**7*sqrt(d**2/(e**2*x**2
) - 1)) + e**3/(192*d**2*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**
5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 5*e**7/(128*d**6*x
*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e*x))/(128*d**7),
Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(e**2*x*
**2) + 1)) - 7*I*e/(48*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/
(192*d**2*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**5/(384*d**4*
x**3*sqrt(-d**2/(e**2*x**2) + 1)) + 5*I*e**7/(128*d**6*x*sqrt(-d*
**2/(e**2*x**2) + 1)) - 5*I*e**8*asin(d/(e*x))/(128*d**7), True))
+ d**5*e**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e
**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/
(e**2*x**2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) -
1)/(105*d**6), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**
2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d*
**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**2) +
8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True)) - 5*d**4
*e**3*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*
e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(
d**2/(e**2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) -
1)) + e**6*acosh(d/(e*x))/(16*d**5), Abs(d**2/(e**2*x**2)) > 1),
(I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*s
qrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-d**2/(e*
**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) -
I*e**6*asin(d/(e*x))/(16*d**5), True)) - 5*d**3*e**4*Piecewise((
3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7)
- 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*
e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x*
**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-
15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**
```

$$\begin{aligned}
& 3\sqrt{1 - e^{2x^2/d^2}}/(-15d^2x^5 + 15e^{2x^7}) - 4d^2e^{2x^2}\sqrt{1 - e^{2x^2/d^2}}/(-15d^2x^5 + 15e^{2x^7}) \\
& + 2e^{6x^6}\sqrt{1 - e^{2x^2/d^2}}/(-15d^5x^5 + 15d^3e^{2x^7}) - e^{4x^4}\sqrt{1 - e^{2x^2/d^2}}/(-15d^3x^5 + 15d^2e^{2x^7}), \text{ True}) \\
& + d^2e^{5x^5}\text{Piecewise}((-d^2/(4e^{x^5}\sqrt{d^2/(e^{2x^2}) - 1})) + 3e/(8x^3\sqrt{d^2/(e^{2x^2}) - 1})) - e^3/(8d^2x\sqrt{d^2/(e^{2x^2}) - 1})) + e^4\text{acosh}(d/(e^x))/(8d^3), \text{ Abs}(d^2/(e^{2x^2})) > 1), (I^d^2/(4e^{x^5}\sqrt{-d^2/(e^{2x^2}) + 1})) - 3I^e/(8x^3\sqrt{-d^2/(e^{2x^2}) + 1})) + I^e^3/(8d^2x\sqrt{-d^2/(e^{2x^2}) + 1})) - I^e^4\text{asin}(d/(e^x))/(8d^3), \text{ True})) \\
& + 3d^2e^{6x^6}\text{Piecewise}((-e\sqrt{d^2/(e^{2x^2}) - 1})/(3x^2) + e^3\sqrt{d^2/(e^{2x^2}) - 1})/(3d^2), \text{ Abs}(d^2/(e^{2x^2})) > 1), (-I^e\sqrt{-d^2/(e^{2x^2}) + 1})/(3x^2) + I^e^3\sqrt{-d^2/(e^{2x^2}) + 1})/(3d^2), \text{ True})) \\
& + e^{7x^7}\text{Piecewise}((-d^2/(2e^{x^3}\sqrt{d^2/(e^{2x^2}) - 1})) + e/(2x\sqrt{d^2/(e^{2x^2}) - 1})) + e^2\text{acosh}(d/(e^x))/(2d), \text{ Abs}(d^2/(e^{2x^2})) > 1), (-I^e\sqrt{-d^2/(e^{2x^2}) + 1})/(2x) - I^e^2\text{asin}(d/(e^x))/(2d), \text{ True}))
\end{aligned}$$

**GIAC/XCAS [A]** time = 0.300282, size = 1, normalized size = 0.01

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3/x^10,x, algorithm="giac")

[Out] Done

$$3.81 \quad \int \frac{(d+ex)^3 (d^2 - e^2 x^2)^{5/2}}{x^{11}} dx$$

**Optimal.** Leaf size=225

$$\begin{aligned} & -\frac{d(d^2 - e^2 x^2)^{7/2}}{10x^{10}} - \frac{e(d^2 - e^2 x^2)^{7/2}}{3x^9} - \frac{33e^2(d^2 - e^2 x^2)^{7/2}}{80dx^8} + \frac{33e^{10} \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{256d^2} \\ & - \frac{33e^8 \sqrt{d^2 - e^2 x^2}}{256dx^2} + \frac{11e^6 (d^2 - e^2 x^2)^{3/2}}{128dx^4} - \frac{11e^4 (d^2 - e^2 x^2)^{5/2}}{160dx^6} - \frac{5e^3 (d^2 - e^2 x^2)^{7/2}}{21d^2 x^7} \end{aligned}$$

[Out]  $(-33 * e^8 * \text{Sqrt}[d^2 - e^2 * x^2]) / (256 * d * x^2) + (11 * e^6 * (d^2 - e^2 * x^2)^{3/2}) / (128 * d * x^4) - (11 * e^4 * (d^2 - e^2 * x^2)^{5/2}) / (160 * d * x^6) - (d * (d^2 - e^2 * x^2)^{7/2}) / (10 * x^{10}) - (e * (d^2 - e^2 * x^2)^{7/2}) / (3 * x^9) - (33 * e^2 * (d^2 - e^2 * x^2)^{7/2}) / (80 * d * x^8) - (5 * e^3 * (d^2 - e^2 * x^2)^{7/2}) / (21 * d^2 * x^7) + (33 * e^{10} * \text{ArcTanh}[\text{Sqrt}[d^2 - e^2 * x^2] / d]) / (256 * d^2)$

**Rubi [A]** time = 0.535826, antiderivative size = 225, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\begin{aligned} & -\frac{d(d^2 - e^2 x^2)^{7/2}}{10x^{10}} - \frac{e(d^2 - e^2 x^2)^{7/2}}{3x^9} - \frac{33e^2(d^2 - e^2 x^2)^{7/2}}{80dx^8} + \frac{33e^{10} \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{256d^2} \\ & - \frac{33e^8 \sqrt{d^2 - e^2 x^2}}{256dx^2} + \frac{11e^6 (d^2 - e^2 x^2)^{3/2}}{128dx^4} - \frac{11e^4 (d^2 - e^2 x^2)^{5/2}}{160dx^6} - \frac{5e^3 (d^2 - e^2 x^2)^{7/2}}{21d^2 x^7} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e * x)^3 * (d^2 - e^2 * x^2)^{5/2} / x^{11}, x]$

[Out]  $(-33 * e^8 * \text{Sqrt}[d^2 - e^2 * x^2]) / (256 * d * x^2) + (11 * e^6 * (d^2 - e^2 * x^2)^{3/2}) / (128 * d * x^4) - (11 * e^4 * (d^2 - e^2 * x^2)^{5/2}) / (160 * d * x^6) - (d * (d^2 - e^2 * x^2)^{7/2}) / (10 * x^{10}) - (e * (d^2 - e^2 * x^2)^{7/2}) / (3 * x^9) - (33 * e^2 * (d^2 - e^2 * x^2)^{7/2}) / (80 * d * x^8) - (5 * e^3 * (d^2 - e^2 * x^2)^{7/2}) / (21 * d^2 * x^7) + (33 * e^{10} * \text{ArcTanh}[\text{Sqrt}[d^2 - e^2 * x^2] / d]) / (256 * d^2)$

**Rubi in Sympy [A]** time = 138.234, size = 280, normalized size = 1.24

$$\begin{aligned} & -\frac{d^7 \sqrt{d^2 - e^2 x^2}}{10x^{10}} - \frac{d^6 e \sqrt{d^2 - e^2 x^2}}{3x^9} - \frac{9d^5 e^2 \sqrt{d^2 - e^2 x^2}}{80x^8} + \frac{16d^4 e^3 \sqrt{d^2 - e^2 x^2}}{21x^7} \\ & + \frac{139d^3 e^4 \sqrt{d^2 - e^2 x^2}}{160x^6} - \frac{2d^2 e^5 \sqrt{d^2 - e^2 x^2}}{7x^5} - \frac{117de^6 \sqrt{d^2 - e^2 x^2}}{128x^4} \\ & - \frac{8e^7 \sqrt{d^2 - e^2 x^2}}{21x^3} + \frac{33e^8 \sqrt{d^2 - e^2 x^2}}{256dx^2} + \frac{33e^{10} \text{atanh}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{256d^2} + \frac{5e^9 \sqrt{d^2 - e^2 x^2}}{21d^2 x} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e * x + d)^3 * (-e^2 * x^2 + d^2)^{5/2} / x^{11}, x)$

[Out]  $-d^{**7} * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (10 * x^{**10}) - d^{**6} * e * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (3 * x^{**9}) - 9 * d^{**5} * e^{**2} * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (80 * x^{**8}) + 16 * d^{**4} * e^{**3} * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (21 * x^{**7}) + 139 * d^{**3} * e^{**4} * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (160 * x^{**6}) - 2 * d^{**2} * e^{**5} * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (7 * x^{**5}) - 117 * d * e^{**6} * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (128 * x^{**4}) - 8 * e^{**7} * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (21 * x^{**3}) + 33 * e^{**8} * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (256 * d * x^{**2}) + 33 * e^{**10} * \text{atanh}(\text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / d) / (256 * d^2)$

$$d^{**2}) + 5*e^{**9}*sqrt(d^{**2} - e^{**2}*x^{**2})/(21*d^{**2}*x)$$

**Mathematica [A]** time = 0.266482, size = 161, normalized size = 0.72

$$\frac{-3465e^{10}x^{10} \log\left(\sqrt{d^2 - e^2x^2} + d\right) + \sqrt{d^2 - e^2x^2} (2688d^9 + 8960d^8ex + 3024d^7e^2x^2 - 20480d^6e^3x^3 - 23352d^5e^4x^4 + 7680d^4e^5x^5 + 24570d^3e^6x^6 + 10240d^2e^7x^7 - 3465d^2e^8x^8 - 6400d^2e^9x^9) + 3465e^{10}x^{10} \text{Log}[x] - 3465e^{10}x^{10} \text{Log}[d + \text{Sqrt}[d^2 - e^2x^2]]}{26880d^2x^{10}}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2))/x^11, x]

[Out] -(Sqrt[d^2 - e^2\*x^2]\*(2688\*d^9 + 8960\*d^8\*e\*x + 3024\*d^7\*e^2\*x^2 - 20480\*d^6\*e^3\*x^3 - 23352\*d^5\*e^4\*x^4 + 7680\*d^4\*e^5\*x^5 + 24570\*d^3\*e^6\*x^6 + 10240\*d^2\*e^7\*x^7 - 3465\*d^2\*e^8\*x^8 - 6400\*d^2\*e^9\*x^9) + 3465\*e^10\*x^10\*Log[x] - 3465\*e^10\*x^10\*Log[d + Sqrt[d^2 - e^2\*x^2]])/(26880\*d^2\*x^10)

**Maple [A]** time = 0.209, size = 278, normalized size = 1.2

$$\begin{aligned} & -\frac{d}{10x^{10}}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{33e^2}{80dx^8}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{11e^4}{160d^3x^6}(-e^2x^2 + d^2)^{\frac{7}{2}} + \frac{11e^6}{640d^5x^4}(-e^2x^2 + d^2)^{\frac{7}{2}} \\ & - \frac{33e^8}{1280d^7x^2}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{33e^{10}}{1280d^7}(-e^2x^2 + d^2)^{\frac{5}{2}} - \frac{11e^{10}}{256d^5}(-e^2x^2 + d^2)^{\frac{3}{2}} - \frac{33e^{10}}{256d^3}\sqrt{-e^2x^2 + d^2} \\ & + \frac{33e^{10}}{256d} \ln\left(\frac{1}{x}\left(2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}} - \frac{5e^3}{21d^2x^7}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{e}{3x^9}(-e^2x^2 + d^2)^{\frac{7}{2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(-e^2\*x^2+d^2)^(5/2)/x^11, x)

[Out] -1/10\*d\*(-e^2\*x^2+d^2)^(7/2)/x^10-33/80\*e^2\*(-e^2\*x^2+d^2)^(7/2)/d/x^8-11/160/d^3\*e^4/x^6\*(-e^2\*x^2+d^2)^(7/2)+11/640/d^5\*e^6/x^4\*(-e^2\*x^2+d^2)^(7/2)-33/1280/d^7\*e^8/x^2\*(-e^2\*x^2+d^2)^(7/2)-33/1280/d^7\*e^10\*(-e^2\*x^2+d^2)^(5/2)-11/256/d^5\*e^10\*(-e^2\*x^2+d^2)^(3/2)-33/256/d^3\*e^10\*(-e^2\*x^2+d^2)^(1/2)+33/256/d\*e^10/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)-5/21\*e^3\*(-e^2\*x^2+d^2)^(7/2)/d^2/x^7-1/3\*e\*(-e^2\*x^2+d^2)^(7/2)/x^9

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3/x^11, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.983389, size = 980, normalized size = 4.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3/x^11,x, algorithm="fricas")
```

```
[Out] -1/26880*(64000*d^e^19*x^19 + 34650*d^2*e^18*x^18 - 1190400*d^3*e^
^17*x^17 - 834750*d^4*e^16*x^16 + 6988800*d^5*e^15*x^15 + 7293300
*d^6*e^14*x^14 - 17863680*d^7*e^13*x^13 - 30318960*d^8*e^12*x^12
+ 17236480*d^9*e^11*x^11 + 66909024*d^10*e^10*x^10 + 12582400*d^1
1*e^9*x^9 - 81177600*d^12*e^8*x^8 - 48742400*d^13*e^7*x^7 + 51340
800*d^14*e^6*x^6 + 50585600*d^15*e^5*x^5 - 12042240*d^16*e^4*x^4
- 24248320*d^17*e^3*x^3 - 2580480*d^18*e^2*x^2 + 4587520*d^19*e*x
+ 1376256*d^20 + 3465*(e^20*x^20 - 50*d^2*e^18*x^18 + 400*d^4*e^
16*x^16 - 1120*d^6*e^14*x^14 + 1280*d^8*e^12*x^12 - 512*d^10*e^10
*x^10 + 2*(5*d^e^18*x^18 - 80*d^3*e^16*x^16 + 336*d^5*e^14*x^14 -
512*d^7*e^12*x^12 + 256*d^9*e^10*x^10)*sqrt(-e^2*x^2 + d^2))*log
(-(d - sqrt(-e^2*x^2 + d^2))/x) - (6400*e^19*x^19 + 3465*d^e^18*x
^18 - 330240*d^2*e^17*x^17 - 197820*d^3*e^16*x^16 + 3064320*d^4*e
^15*x^15 + 2637852*d^5*e^14*x^14 - 10859520*d^6*e^13*x^13 - 14879
424*d^7*e^12*x^12 + 15555840*d^8*e^11*x^11 + 41442912*d^9*e^10*x^
10 + 857600*d^10*e^9*x^9 - 60453120*d^11*e^8*x^8 - 31109120*d^12*
e^7*x^7 + 44782080*d^13*e^6*x^6 + 40181760*d^14*e^5*x^5 - 1281638
4*d^15*e^4*x^4 - 21954560*d^16*e^3*x^3 - 1892352*d^17*e^2*x^2 + 4
587520*d^18*e*x + 1376256*d^19)*sqrt(-e^2*x^2 + d^2))/(d^2*e^10*x
^20 - 50*d^4*e^8*x^18 + 400*d^6*e^6*x^16 - 1120*d^8*e^4*x^14 + 12
80*d^10*e^2*x^12 - 512*d^12*x^10 + 2*(5*d^3*e^8*x^18 - 80*d^5*e^6
*x^16 + 336*d^7*e^4*x^14 - 512*d^9*e^2*x^12 + 256*d^11*x^10)*sqrt
(-e^2*x^2 + d^2))
```

---

**Sympy [A]** time = 137.099, size = 2159, normalized size = 9.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**11,x)
```

```
[Out] d**7*Piecewise((-d**2/(10*e*x**11*sqrt(d**2/(e**2*x**2) - 1)) + 9
*e/(80*x**9*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(480*d**2*x**7*sq
rt(d**2/(e**2*x**2) - 1)) + 7*e**5/(1920*d**4*x**5*sqrt(d**2/(e**2
*x**2) - 1)) + 7*e**7/(768*d**6*x**3*sqrt(d**2/(e**2*x**2) - 1))
- 7*e**9/(256*d**8*x*sqrt(d**2/(e**2*x**2) - 1)) + 7*e**10*acosh(
d/(e*x))/(256*d**9), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(10*e*x*
**11*sqrt(-d**2/(e**2*x**2) + 1)) - 9*I*e/(80*x**9*sqrt(-d**2/(e**
2*x**2) + 1)) - I*e**3/(480*d**2*x**7*sqrt(-d**2/(e**2*x**2) + 1)
) - 7*I*e**5/(1920*d**4*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e
**7/(768*d**6*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + 7*I*e**9/(256*d
**8*x*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e**10*asin(d/(e*x))/(256
*d**9), True)) + 3*d**6*e*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1
)/(9*x**8) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(63*d**2*x**6) + 2*e
**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x**4) + 8*e**7*sqrt(d**2
/(e**2*x**2) - 1)/(315*d**6*x**2) + 16*e**9*sqrt(d**2/(e**2*x**2)
- 1)/(315*d**8), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e
**2*x**2) + 1)/(9*x**8) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(63*
d**2*x**6) + 2*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**4)
+ 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(315*d**6*x**2) + 16*I*e
**9*sqrt(-d**2/(e**2*x**2) + 1)/(315*d**8), True)) + d**5*e**2*Pie
cewise((-d**2/(8*e*x**9*sqrt(d**2/(e**2*x**2) - 1)) + 7*e/(48*x**
7*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(192*d**2*x**5*sqrt(d**2/(e*
**2*x**2) - 1)) + 5*e**5/(384*d**4*x**3*sqrt(d**2/(e**2*x**2) - 1)
) - 5*e**7/(128*d**6*x*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**8*acosh
(d/(e*x))/(128*d**7), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(8*e*x*
**9*sqrt(-d**2/(e**2*x**2) + 1)) - 7*I*e/(48*x**7*sqrt(-d**2/(e**2
*x**2) + 1)) - I*e**3/(192*d**2*x**5*sqrt(-d**2/(e**2*x**2) + 1))
- 5*I*e**5/(384*d**4*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + 5*I*e**
7/(128*d**6*x*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**8*asin(d/(e*x
))/(128*d**7), True)) - 5*d**4*e**3*Piecewise((-e*sqrt(d**2/(e**2
*x**2) - 1)/(7*x**6) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(35*d**2*x
**4) + 4*e**5*sqrt(d**2/(e**2*x**2) - 1)/(105*d**4*x**2) + 8*e**7
*sqrt(d**2/(e**2*x**2) - 1)/(105*d**6), Abs(d**2/(e**2*x**2)) > 1
```

```

), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(7*x**6) + I*e**3*sqrt(-d**2
/(e**2*x**2) + 1)/(35*d**2*x**4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2
) + 1)/(105*d**4*x**2) + 8*I*e**7*sqrt(-d**2/(e**2*x**2) + 1)/(10
5*d**6), True)) - 5*d**3*e**4*Piecewise((-d**2/(6*e*x**7*sqrt(d**
2/(e**2*x**2) - 1)) + 5*e/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) +
e**3/(48*d**2*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**5/(16*d**4*x*
sqrt(d**2/(e**2*x**2) - 1)) + e**6*acosh(d/(e*x))/(16*d**5), Abs(
d**2/(e**2*x**2)) > 1), (I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2)
+ 1)) - 5*I*e/(24*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*
d**2*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-
d**2/(e**2*x**2) + 1)) - I*e**6*asin(d/(e*x))/(16*d**5), True)) +
d**2*e**5*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**
2*x**5 + 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2
)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x
**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(
-1 + e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x
**2/d**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 +
15*e**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2
*x**5 + 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15
*d**5*x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d*
**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), True)) + 3*d*e**6*Piecewise
((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(
d**2/(e**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1
)) + e**4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I
*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt
(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2
) + 1)) - I*e**4*asin(d/(e*x))/(8*d**3), True)) + e**7*Piecewise(
(-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x*
**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(
e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*
d**2), True))

```

---

**GIAC/XCAS [A]** time = 0.31593, size = 1, normalized size = 0.

*Done*

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3/x^11,x, algorithm="giac")
```

```
[Out] Done
```

$$3.82 \quad \int \frac{(d+ex)^3 (d^2 - e^2 x^2)^{5/2}}{x^{12}} dx$$

**Optimal.** Leaf size=254

$$\frac{d(d^2 - e^2 x^2)^{7/2}}{11x^{11}} - \frac{3e(d^2 - e^2 x^2)^{7/2}}{10x^{10}} - \frac{37e^2(d^2 - e^2 x^2)^{7/2}}{99dx^9} - \frac{19e^9 \sqrt{d^2 - e^2 x^2}}{256d^2 x^2} + \frac{19e^7 (d^2 - e^2 x^2)^{3/2}}{384d^2 x^4} \\ - \frac{19e^5 (d^2 - e^2 x^2)^{5/2}}{480d^2 x^6} - \frac{19e^3 (d^2 - e^2 x^2)^{7/2}}{80d^2 x^8} + \frac{19e^{11} \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{256d^3} - \frac{74e^4 (d^2 - e^2 x^2)^{7/2}}{693d^3 x^7}$$

[Out]  $(-19e^9 \sqrt{d^2 - e^2 x^2}) / (256d^2 x^2) + (19e^7 (d^2 - e^2 x^2)^{3/2}) / (384d^2 x^4) - (19e^5 (d^2 - e^2 x^2)^{5/2}) / (480d^2 x^6) - (d^7 (d^2 - e^2 x^2)^{7/2}) / (11x^{11}) - (3e^2 (d^2 - e^2 x^2)^{7/2}) / (99d^2 x^9) - (19e^3 (d^2 - e^2 x^2)^{7/2}) / (80d^2 x^8) - (74e^4 (d^2 - e^2 x^2)^{7/2}) / (693d^3 x^7) + (19e^{11} \operatorname{ArcTanh}[\sqrt{d^2 - e^2 x^2} / d]) / (256d^3)$

**Rubi [A]** time = 0.637824, antiderivative size = 254, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\frac{d(d^2 - e^2 x^2)^{7/2}}{11x^{11}} - \frac{3e(d^2 - e^2 x^2)^{7/2}}{10x^{10}} - \frac{37e^2(d^2 - e^2 x^2)^{7/2}}{99dx^9} - \frac{19e^9 \sqrt{d^2 - e^2 x^2}}{256d^2 x^2} + \frac{19e^7 (d^2 - e^2 x^2)^{3/2}}{384d^2 x^4} \\ - \frac{19e^5 (d^2 - e^2 x^2)^{5/2}}{480d^2 x^6} - \frac{19e^3 (d^2 - e^2 x^2)^{7/2}}{80d^2 x^8} + \frac{19e^{11} \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{256d^3} - \frac{74e^4 (d^2 - e^2 x^2)^{7/2}}{693d^3 x^7}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(d + e x)^3 (d^2 - e^2 x^2)^{5/2} / x^{12}, x]$

[Out]  $(-19e^9 \sqrt{d^2 - e^2 x^2}) / (256d^2 x^2) + (19e^7 (d^2 - e^2 x^2)^{3/2}) / (384d^2 x^4) - (19e^5 (d^2 - e^2 x^2)^{5/2}) / (480d^2 x^6) - (d^7 (d^2 - e^2 x^2)^{7/2}) / (11x^{11}) - (3e^2 (d^2 - e^2 x^2)^{7/2}) / (99d^2 x^9) - (19e^3 (d^2 - e^2 x^2)^{7/2}) / (80d^2 x^8) - (74e^4 (d^2 - e^2 x^2)^{7/2}) / (693d^3 x^7) + (19e^{11} \operatorname{ArcTanh}[\sqrt{d^2 - e^2 x^2} / d]) / (256d^3)$

**Rubi in Sympy [A]** time = 164.825, size = 309, normalized size = 1.22

$$-\frac{d^7 \sqrt{d^2 - e^2 x^2}}{11x^{11}} - \frac{3d^6 e \sqrt{d^2 - e^2 x^2}}{10x^{10}} - \frac{10d^5 e^2 \sqrt{d^2 - e^2 x^2}}{99x^9} + \frac{53d^4 e^3 \sqrt{d^2 - e^2 x^2}}{80x^8} \\ + \frac{514d^3 e^4 \sqrt{d^2 - e^2 x^2}}{693x^7} - \frac{109d^2 e^5 \sqrt{d^2 - e^2 x^2}}{480x^6} - \frac{164d e^6 \sqrt{d^2 - e^2 x^2}}{231x^5} - \frac{109e^7 \sqrt{d^2 - e^2 x^2}}{384x^4} \\ + \frac{37e^8 \sqrt{d^2 - e^2 x^2}}{693dx^3} + \frac{19e^9 \sqrt{d^2 - e^2 x^2}}{256d^2 x^2} + \frac{19e^{11} \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{256d^3} + \frac{74e^{10} \sqrt{d^2 - e^2 x^2}}{693d^3 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{rubi\_integrate}((e x + d)^3 (-e^2 x^2 + d^2)^{5/2} / x^{12}, x)$

[Out]  $-d^7 \sqrt{d^2 - e^2 x^2} / (11x^{11}) - 3d^6 e \sqrt{d^2 - e^2 x^2} / (10x^{10}) - 10d^5 e^2 \sqrt{d^2 - e^2 x^2} / (99x^9) + 53d^4 e^3 \sqrt{d^2 - e^2 x^2} / (80x^8) + 514d^3 e^4 \sqrt{d^2 - e^2 x^2} / (693x^7) - 109d^2 e^5 \sqrt{d^2 - e^2 x^2} / (480x^6) - 164d e^6 \sqrt{d^2 - e^2 x^2} / (231x^5) - 109e^7 \sqrt{d^2 - e^2 x^2} / (384x^4) + 37e^8 \sqrt{d^2 - e^2 x^2} / (693dx^3) + 19e^9 \sqrt{d^2 - e^2 x^2} / (256d^2 x^2) + 19e^{11} \operatorname{atanh}(\sqrt{d^2 - e^2 x^2} / d) / (256d^3) + 74e^{10} \sqrt{d^2 - e^2 x^2} / (693d^3 x)$



$$\frac{d^2 - e^{*2}x^{*2}}{(693d^*x^{*3})} + 19e^{*9}\sqrt{d^{*2} - e^{*2}x^{*2}}/(256d^{*2}x^{*2}) + 19e^{*11}\operatorname{atanh}(\sqrt{d^{*2} - e^{*2}x^{*2}}/d)/(256d^{*3}) + 74e^{*10}\sqrt{d^{*2} - e^{*2}x^{*2}}/(693d^{*3}x)$$

**Mathematica [A]** time = 0.251686, size = 172, normalized size = 0.68

$$\frac{65835e^{11}x^{11}\log\left(\sqrt{d^2 - e^2x^2} + d\right) + \sqrt{d^2 - e^2x^2}\left(-80640d^{10} - 266112d^9ex - 89600d^8e^2x^2 + 587664d^7e^3x^3 + 657920d^6e^4x^4\right)}{887040d^3}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2))/x^12,x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(-80640\*d^10 - 266112\*d^9\*e\*x - 89600\*d^8\*e^2\*x^2 + 587664\*d^7\*e^3\*x^3 + 657920\*d^6\*e^4\*x^4 - 201432\*d^5\*e^5\*x^5 - 629760\*d^4\*e^6\*x^6 - 251790\*d^3\*e^7\*x^7 + 47360\*d^2\*e^8\*x^8 + 65835\*d\*e^9\*x^9 + 94720\*e^10\*x^10) - 65835\*e^11\*x^11\*Log[x] + 65835\*e^11\*x^11\*Log[d + Sqrt[d^2 - e^2\*x^2]])/(887040\*d^3\*x^11)

**Maple [A]** time = 0.327, size = 303, normalized size = 1.2

$$\begin{aligned} & -\frac{d}{11x^{11}}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{37e^2}{99dx^9}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{74e^4}{693d^3x^7}(-e^2x^2 + d^2)^{\frac{7}{2}} \\ & - \frac{19e^3}{80d^2x^8}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{19e^5}{480d^4x^6}(-e^2x^2 + d^2)^{\frac{7}{2}} + \frac{19e^7}{1920d^6x^4}(-e^2x^2 + d^2)^{\frac{7}{2}} \\ & - \frac{19e^9}{1280d^8x^2}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{19e^{11}}{1280d^8}(-e^2x^2 + d^2)^{\frac{5}{2}} - \frac{19e^{11}}{768d^6}(-e^2x^2 + d^2)^{\frac{3}{2}} \\ & - \frac{19e^{11}}{256d^4}\sqrt{-e^2x^2 + d^2} + \frac{19e^{11}}{256d^2}\ln\left(\frac{1}{x}\left(2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2}\right)\right)\frac{1}{\sqrt{d^2}} - \frac{3e}{10x^{10}}(-e^2x^2 + d^2)^{\frac{7}{2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(-e^2\*x^2+d^2)^(5/2)/x^12,x)

[Out] -1/11\*d\*(-e^2\*x^2+d^2)^(7/2)/x^11-37/99\*e^2\*(-e^2\*x^2+d^2)^(7/2)/d/x^9-74/693\*e^4\*(-e^2\*x^2+d^2)^(7/2)/d^3/x^7-19/80\*e^3\*(-e^2\*x^2+d^2)^(7/2)/d^2/x^8-19/480\*e^5/d^4/x^6\*(-e^2\*x^2+d^2)^(7/2)+19/1920\*e^7/d^6/x^4\*(-e^2\*x^2+d^2)^(7/2)-19/1280\*e^9/d^8/x^2\*(-e^2\*x^2+d^2)^(7/2)-19/1280\*e^11/d^8\*(-e^2\*x^2+d^2)^(5/2)-19/768\*e^11/d^6\*(-e^2\*x^2+d^2)^(3/2)-19/256\*e^11/d^4\*(-e^2\*x^2+d^2)^(1/2)+19/256\*e^11/d^2/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)-3/10\*e\*(-e^2\*x^2+d^2)^(7/2)/x^10

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3/x^12,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.13114, size = 1068, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3/x^12,x, algorithm="fricas")`

[Out] 
$$\frac{1}{887040} (94720 e^{22} x^{22} + 65835 d e^{21} x^{21} - 5730560 d^2 e^{20} x^{20} - 4267725 d^3 e^{19} x^{19} + 55207680 d^4 e^{18} x^{18} + 55975458 d^5 e^{17} x^{17} - 154344960 d^6 e^{16} x^{16} - 298078704 d^7 e^{15} x^{15} - 154091520 d^8 e^{14} x^{14} + 700868784 d^9 e^{13} x^{13} + 1773249280 d^{10} e^{12} x^{12} - 396078144 d^{11} e^{11} x^{11} - 4238178560 d^{12} e^{10} x^{10} - 1466868480 d^{13} e^9 x^9 + 4999920640 d^{14} e^8 x^8 + 3445499904 d^{15} e^7 x^7 - 3011768320 d^{16} e^6 x^6 - 3252006912 d^{17} e^5 x^5 + 641597440 d^{18} e^4 x^4 + 1487388672 d^{19} e^3 x^3 + 176619520 d^{20} e^2 x^2 - 272498688 d^{21} e x - 82575360 d^{22} - 65835 (11 d e^{21} x^{21} - 220 d^3 e^{19} x^{19} + 1232 d^5 e^{17} x^{17} - 2816 d^7 e^{15} x^{15} + 2816 d^9 e^{13} x^{13} - 1024 d^{11} e^{11} x^{11} - (e^{21} x^{21} - 60 d^2 e^{19} x^{19} + 560 d^4 e^{17} x^{17} - 1792 d^6 e^{15} x^{15} + 2304 d^8 e^{13} x^{13} - 1024 d^{10} e^{11} x^{11}) \sqrt{-e^2 x^2 + d^2}) \log\left(\frac{-(d - \sqrt{-e^2 x^2 + d^2})}{x}\right) + (1041920 d e^{20} x^{20} + 724185 d^2 e^{19} x^{19} - 20317440 d^3 e^{18} x^{18} - 17253390 d^4 e^{17} x^{17} + 99348480 d^5 e^{16} x^{16} + 134286768 d^6 e^{15} x^{15} - 62599680 d^7 e^{14} x^{14} - 444817296 d^8 e^{13} x^{13} - 788226560 d^9 e^{12} x^{12} + 514054464 d^{10} e^{11} x^{11} + 2639159040 d^{11} e^{10} x^{10} + 573323520 d^{12} e^9 x^9 - 3767249920 d^{13} e^8 x^8 - 2292111360 d^{14} e^7 x^7 + 2650542080 d^{15} e^6 x^6 + 2610499584 d^{16} e^5 x^5 - 698941440 d^{17} e^4 x^4 - 1351139328 d^{18} e^3 x^3 - 135331840 d^{19} e^2 x^2 + 272498688 d^{20} e x + 82575360 d^{21}) \sqrt{-e^2 x^2 + d^2}) / (11 d^4 e^{10} x^{21} - 220 d^6 e^8 x^{19} + 1232 d^8 e^6 x^{17} - 2816 d^{10} e^4 x^{15} + 2816 d^{12} e^2 x^{13} - 1024 d^{14} x^{11} - (d^3 e^{10} x^{21} - 60 d^5 e^8 x^{19} + 560 d^7 e^6 x^{17} - 1792 d^9 e^4 x^{15} + 2304 d^{11} e^2 x^{13} - 1024 d^{13} x^{11}) \sqrt{-e^2 x^2 + d^2})$$

**Sympy [A]** time = 146.641, size = 2397, normalized size = 9.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**3*(-e**2*x**2+d**2)**(5/2)/x**12,x)`

[Out] 
$$d^{**7} \text{Piecewise}\left(\left(\frac{-e \sqrt{d^{**2}/(e^{**2} x^{**2})} - 1}{(11 x^{**10})} + e^{**3} \sqrt{d^{**2}/(e^{**2} x^{**2})} - 1\right) / (99 d^{**2} x^{**8}) + 8 e^{**5} \sqrt{d^{**2}/(e^{**2} x^{**2})} - 1\right) / (693 d^{**4} x^{**6}) + 16 e^{**7} \sqrt{d^{**2}/(e^{**2} x^{**2})} - 1\right) / (1155 d^{**6} x^{**4}) + 64 e^{**9} \sqrt{d^{**2}/(e^{**2} x^{**2})} - 1\right) / (3465 d^{**8} x^{**2}) + 128 e^{**11} \sqrt{d^{**2}/(e^{**2} x^{**2})} - 1\right) / (3465 d^{**10}), \text{Abs}(d^{**2}/(e^{**2} x^{**2})) > 1\right), \left(-I e \sqrt{-d^{**2}/(e^{**2} x^{**2})} + 1\right) / (11 x^{**10}) + I e^{**3} \sqrt{-d^{**2}/(e^{**2} x^{**2})} + 1\right) / (99 d^{**2} x^{**8}) + 8 I e^{**5} \sqrt{-d^{**2}/(e^{**2} x^{**2})} + 1\right) / (693 d^{**4} x^{**6}) + 16 I e^{**7} \sqrt{-d^{**2}/(e^{**2} x^{**2})} + 1\right) / (1155 d^{**6} x^{**4}) + 64 I e^{**9} \sqrt{-d^{**2}/(e^{**2} x^{**2})} + 1\right) / (3465 d^{**8} x^{**2}) + 128 I e^{**11} \sqrt{-d^{**2}/(e^{**2} x^{**2})} + 1\right) / (3465 d^{**10}), \text{True}\right) + 3 d^{**6} e \text{Piecewise}\left(\left(\frac{-d^{**2}/(10 e x^{**11}) \sqrt{d^{**2}/(e^{**2} x^{**2})} - 1\right) + 9 e / (80 x^{**9} \sqrt{d^{**2}/(e^{**2} x^{**2})} - 1)\right) + e^{**3} / (480 d^{**2} x^{**7} \sqrt{d^{**2}/(e^{**2} x^{**2})} - 1)\right) + 7 e^{**5} / (1920 d^{**4} x^{**5} \sqrt{d^{**2}/(e^{**2} x^{**2})} - 1)\right) + 7 e^{**7} / (768 d^{**6} x^{**3} \sqrt{d^{**2}/(e^{**2} x^{**2})} - 1)\right) - 7 e^{**9} / (256 d^{**8} x \sqrt{d^{**2}/(e^{**2} x^{**2})} - 1)\right) + 7 e^{**10} \text{acosh}(d / (e x)) / (256 d^{**9}), \text{Abs}(d^{**2}/(e^{**2} x^{**2})) > 1\right), \left(I d^{**2}/(10 e x^{**11}) \sqrt{-d^{**2}/(e^{**2} x^{**2})} + 1\right) - 9 I e / (80 x^{**9} \sqrt{-d^{**2}/(e^{**2} x^{**2})} + 1)\right) - I e^{**3} / (480 d^{**2} x^{**7} \sqrt{-d^{**2}/(e^{**2} x^{**2})} + 1)\right) - 7 I e^{**5} / (1920 d^{**4} x^{**5} \sqrt{-d^{**2}/(e^{**2} x^{**2})} + 1)\right) - 7 I e^{**7} / (768 d^{**6} x^{**3} \sqrt{-d^{**2}/(e^{**2} x^{**2})} + 1)\right) + 7 I e^{**9} / (256 d^{**8} x \sqrt{-d^{**2}/(e^{**2} x^{**2})} + 1)\right) - 7 I e^{**10} \text{asin}(d / (e x)) / (256 d^{**9}), \text{True}\right) + d^{**5} e^{**2} \text{Piec$$

```

ise((-e*sqrt(d**2/(e**2*x**2) - 1)/(9*x**8) + e**3*sqrt(d**2/(e**
2*x**2) - 1)/(63*d**2*x**6) + 2*e**5*sqrt(d**2/(e**2*x**2) - 1)/(
105*d**4*x**4) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(315*d**6*x**2
) + 16*e**9*sqrt(d**2/(e**2*x**2) - 1)/(315*d**8), Abs(d**2/(e**2
*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(9*x**8) + I*e**3
*sqrt(-d**2/(e**2*x**2) + 1)/(63*d**2*x**6) + 2*I*e**5*sqrt(-d**2
/(e**2*x**2) + 1)/(105*d**4*x**4) + 8*I*e**7*sqrt(-d**2/(e**2*x**
2) + 1)/(315*d**6*x**2) + 16*I*e**9*sqrt(-d**2/(e**2*x**2) + 1)/(
315*d**8), True)) - 5*d**4*e**3*Piecewise((-d**2/(8*e*x**9*sqrt(d
**2/(e**2*x**2) - 1)) + 7*e/(48*x**7*sqrt(d**2/(e**2*x**2) - 1))
+ e**3/(192*d**2*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 5*e**5/(384*d
**4*x**3*sqrt(d**2/(e**2*x**2) - 1)) - 5*e**7/(128*d**6*x*sqrt(d*
**2/(e**2*x**2) - 1)) + 5*e**8*acosh(d/(e*x))/(128*d**7), Abs(d**2
/(e**2*x**2)) > 1), (I*d**2/(8*e*x**9*sqrt(-d**2/(e**2*x**2) + 1)
) - 7*I*e/(48*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**3/(192*d**
2*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e**5/(384*d**4*x**3*sqr
t(-d**2/(e**2*x**2) + 1)) + 5*I*e**7/(128*d**6*x*sqrt(-d**2/(e**2
*x**2) + 1)) - 5*I*e**8*asin(d/(e*x))/(128*d**7), True)) - 5*d**3
*e**4*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(7*x**6) + e**3*sq
rt(d**2/(e**2*x**2) - 1)/(35*d**2*x**4) + 4*e**5*sqrt(d**2/(e**2*
x**2) - 1)/(105*d**4*x**2) + 8*e**7*sqrt(d**2/(e**2*x**2) - 1)/(1
05*d**6), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2
) + 1)/(7*x**6) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(35*d**2*x**
4) + 4*I*e**5*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**4*x**2) + 8*I*e
**7*sqrt(-d**2/(e**2*x**2) + 1)/(105*d**6), True)) + d**2*e**5*Pi
ecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e/(24*x*
**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d**2/(e*
**2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1)) + e*
**6*acosh(d/(e*x))/(16*d**5), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/
(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sqrt(-d**
2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-d**2/(e**2*x**2)
+ 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**6*
asin(d/(e*x))/(16*d**5), True)) + 3*d*e**6*Piecewise((3*I*d**3*sq
rt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I*d*e*
**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7)
+ 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**
3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d**3*x*
**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt(1 -
e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x**2*sq
rt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) + 2*e**6*x*
**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) -
e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x*
**7), True)) + e**7*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e**2*x**
2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/(8*d**2
*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d**3), Ab
s(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e**2*x**2
) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e**3/(8*
d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))/(8*d**
3), True))

```

---

**GIAC/XCAS [A]** time = 0.31805, size = 1, normalized size = 0.

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^3/x^12,x, algorithm="giac")

[Out] Done

$$3.83 \quad \int \frac{x^5(d+ex)^3}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=174

$$\frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} - \frac{13d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6} \\ + \frac{x\sqrt{d^2-e^2x^2}}{2e^5} + \frac{d^4(d+ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6(d^2-e^2x^2)^{3/2}}$$

[Out]  $(d^4*(d+e*x)^3)/(5*e^6*(d^2-e^2*x^2)^{(5/2)}) - (23*d^3*(d+e*x)^2)/(15*e^6*(d^2-e^2*x^2)^{(3/2)}) + (127*d^2*(d+e*x))/((15*e^6*\text{Sqrt}[d^2-e^2*x^2])) + (3*d*\text{Sqrt}[d^2-e^2*x^2])/e^6 + (x*\text{Sqrt}[d^2-e^2*x^2])/(2*e^5) - (13*d^2*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2-e^2*x^2]])/(2*e^6)$

**Rubi [A]** time = 0.595229, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\frac{127d^2(d+ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} - \frac{13d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6} \\ + \frac{x\sqrt{d^2-e^2x^2}}{2e^5} + \frac{d^4(d+ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{23d^3(d+ex)^2}{15e^6(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^5*(d+e*x)^3)/(d^2-e^2*x^2)^{(7/2)}, x]$

[Out]  $(d^4*(d+e*x)^3)/(5*e^6*(d^2-e^2*x^2)^{(5/2)}) - (23*d^3*(d+e*x)^2)/(15*e^6*(d^2-e^2*x^2)^{(3/2)}) + (127*d^2*(d+e*x))/((15*e^6*\text{Sqrt}[d^2-e^2*x^2])) + (3*d*\text{Sqrt}[d^2-e^2*x^2])/e^6 + (x*\text{Sqrt}[d^2-e^2*x^2])/(2*e^5) - (13*d^2*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2-e^2*x^2]])/(2*e^6)$

**Rubi in Sympy [A]** time = 68.4353, size = 156, normalized size = 0.9

$$\frac{d^4\sqrt{d^2-e^2x^2}}{5e^6(d-ex)^3} - \frac{23d^3\sqrt{d^2-e^2x^2}}{15e^6(d-ex)^2} - \frac{13d^2 \text{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6} + \frac{127d^2\sqrt{d^2-e^2x^2}}{15e^6(d-ex)} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} + \frac{x\sqrt{d^2-e^2x^2}}{2e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**5}*(e*x+d)^{**3}/(-e^{**2}*x^{**2}+d^{**2})^{**}(7/2), x)$

[Out]  $d^{**4}*\text{sqrt}(d^{**2}-e^{**2}*x^{**2})/(5*e^{**6}*(d-e*x)^{**3}) - 23*d^{**3}*\text{sqrt}(d^{**2}-e^{**2}*x^{**2})/(15*e^{**6}*(d-e*x)^{**2}) - 13*d^{**2}*\text{atan}(e*x/\text{sqrt}(d^{**2}-e^{**2}*x^{**2}))/((2*e^{**6})) + 127*d^{**2}*\text{sqrt}(d^{**2}-e^{**2}*x^{**2})/(15*e^{**6}*(d-e*x)) + 3*d*\text{sqrt}(d^{**2}-e^{**2}*x^{**2})/e^{**6} + x*\text{sqrt}(d^{**2}-e^{**2}*x^{**2})/(2*e^{**5})$

**Mathematica [A]** time = 0.134595, size = 108, normalized size = 0.62

$$\frac{195d^2(d-ex)^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \sqrt{d^2-e^2x^2}(-304d^4 + 717d^3ex - 479d^2e^2x^2 + 45de^3x^3 + 15e^4x^4)}{30e^6(ex-d)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5\*(d + e\*x)^3)/(d^2 - e^2\*x^2)^(7/2), x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(-304\*d^4 + 717\*d^3\*e\*x - 479\*d^2\*e^2\*x^2 + 45\*d\*e^3\*x^3 + 15\*e^4\*x^4) + 195\*d^2\*(d - e\*x)^3\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(30\*e^6\*(-d + e\*x)^3)

**Maple [A]** time = 0.014, size = 222, normalized size = 1.3

$$19 \frac{d^3 x^4}{e^2 (-e^2 x^2 + d^2)^{5/2}} - \frac{76 d^5 x^2}{3 e^4} (-e^2 x^2 + d^2)^{-5/2} + \frac{152 d^7}{15 e^6} (-e^2 x^2 + d^2)^{-5/2} \\ - \frac{e x^7}{2} (-e^2 x^2 + d^2)^{-5/2} + \frac{13 d^2 x^5}{10 e} (-e^2 x^2 + d^2)^{-5/2} - \frac{13 d^2 x^3}{6 e^3} (-e^2 x^2 + d^2)^{-3/2} \\ + \frac{13 d^2 x}{2 e^5} \frac{1}{\sqrt{-e^2 x^2 + d^2}} - \frac{13 d^2}{2 e^5} \arctan\left(x \sqrt{e^2} \frac{1}{\sqrt{-e^2 x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} - 3 \frac{d x^6}{(-e^2 x^2 + d^2)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(e\*x+d)^3/(-e^2\*x^2+d^2)^(7/2), x)

[Out] 19\*d^3\*x^4/e^2/(-e^2\*x^2+d^2)^(5/2)-76/3\*d^5/e^4\*x^2/(-e^2\*x^2+d^2)^(5/2)+152/15\*d^7/e^6/(-e^2\*x^2+d^2)^(5/2)-1/2\*e\*x^7/(-e^2\*x^2+d^2)^(5/2)+13/10/e\*d^2\*x^5/(-e^2\*x^2+d^2)^(5/2)-13/6/e^3\*d^2\*x^3/(-e^2\*x^2+d^2)^(3/2)+13/2/e^5\*d^2\*x/(-e^2\*x^2+d^2)^(1/2)-13/2/e^5\*d^2/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))-3\*d\*x^6/(-e^2\*x^2+d^2)^(5/2)

**Maxima [A]** time = 0.804762, size = 429, normalized size = 2.47

$$-\frac{e x^7}{2(-e^2 x^2 + d^2)^{5/2}} + \frac{13}{30} d^2 e x \left( \frac{15 x^4}{(-e^2 x^2 + d^2)^{5/2} e^2} - \frac{20 d^2 x^2}{(-e^2 x^2 + d^2)^{5/2} e^4} + \frac{8 d^4}{(-e^2 x^2 + d^2)^{5/2} e^6} \right) \\ - \frac{3 d x^6}{(-e^2 x^2 + d^2)^{5/2}} - \frac{13 d^2 x \left( \frac{3 x^2}{(-e^2 x^2 + d^2)^{3/2} e^2} - \frac{2 d^2}{(-e^2 x^2 + d^2)^{3/2} e^4} \right)}{6 e} + \frac{19 d^3 x^4}{(-e^2 x^2 + d^2)^{5/2} e^2} - \frac{76 d^5 x^2}{3(-e^2 x^2 + d^2)^{5/2} e^4} \\ + \frac{152 d^7}{15(-e^2 x^2 + d^2)^{5/2} e^6} + \frac{26 d^4 x}{15(-e^2 x^2 + d^2)^{3/2} e^5} - \frac{91 d^2 x}{30 \sqrt{-e^2 x^2 + d^2} e^5} - \frac{13 d^2 \arcsin\left(\frac{e^2 x}{\sqrt{d^2 e^2}}\right)}{2 \sqrt{e^2} e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*x^5/(-e^2\*x^2 + d^2)^(7/2), x, algorithm="maxima")

[Out] -1/2\*e\*x^7/(-e^2\*x^2 + d^2)^(5/2) + 13/30\*d^2\*e\*x\*(15\*x^4/((-e^2\*x^2 + d^2)^(5/2)\*e^2) - 20\*d^2\*x^2/((-e^2\*x^2 + d^2)^(5/2)\*e^4) + 8\*d^4/((-e^2\*x^2 + d^2)^(5/2)\*e^6)) - 3\*d\*x^6/(-e^2\*x^2 + d^2)^(5/2) - 13/6\*d^2\*x\*(3\*x^2/((-e^2\*x^2 + d^2)^(3/2)\*e^2) - 2\*d^2/((-e^2\*x^2 + d^2)^(3/2)\*e^4))/e + 19\*d^3\*x^4/((-e^2\*x^2 + d^2)^(5/2)\*e^2) - 76/3\*d^5\*x^2/((-e^2\*x^2 + d^2)^(5/2)\*e^4) + 152/15\*d^7/((-e^2\*x^2 + d^2)^(5/2)\*e^6) + 26/15\*d^4\*x/((-e^2\*x^2 + d^2)^(3/2)\*e^5) - 91/30\*d^2\*x/(sqrt(-e^2\*x^2 + d^2)\*e^5) - 13/2\*d^2\*arcsin(e^2\*x/sqrt(d^2\*e^2))/sqrt(e^2)\*e^5

**Fricas [A]** time = 0.30001, size = 729, normalized size = 4.19

$$15 e^9 x^9 + 120 d e^8 x^8 - 678 d^2 e^7 x^7 - 210 d^3 e^6 x^6 + 5421 d^4 e^5 x^5 - 6500 d^5 e^4 x^4 - 2860 d^6 e^3 x^3 + 7800 d^7 e^2 x^2 - 3120 d^8 e x - 3$$


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Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*x^5/(-e^2\*x^2 + d^2)^(7/2),x, algorithm="fricas")

[Out] 
$$-1/30*(15*e^9*x^9 + 120*d*e^8*x^8 - 678*d^2*e^7*x^7 - 210*d^3*e^6*x^6 + 5421*d^4*e^5*x^5 - 6500*d^5*e^4*x^4 - 2860*d^6*e^3*x^3 + 7800*d^7*e^2*x^2 - 3120*d^8*e*x - 390*(d^2*e^7*x^7 - 7*d^3*e^6*x^6 + 3*d^4*e^5*x^5 + 31*d^5*e^4*x^4 - 40*d^6*e^3*x^3 - 12*d^7*e^2*x^2 + 40*d^8*e*x - 16*d^9 + (d^2*e^6*x^6 + 2*d^3*e^5*x^5 - 19*d^4*e^4*x^4 + 20*d^5*e^3*x^3 + 20*d^6*e^2*x^2 - 40*d^7*e*x + 16*d^8)*\sqrt{-e^2*x^2 + d^2})*\arctan(-(d - \sqrt{-e^2*x^2 + d^2})/(e*x)) - (15*e^8*x^8 - 15*d*e^7*x^7 - 535*d^2*e^6*x^6 + 2821*d^3*e^5*x^5 - 2600*d^4*e^4*x^4 - 4420*d^5*e^3*x^3 + 7800*d^6*e^2*x^2 - 3120*d^7*e*x)*\sqrt{-e^2*x^2 + d^2})/(e^{13}*x^7 - 7*d*e^{12}*x^6 + 3*d^2*e^{11}*x^5 + 31*d^3*e^{10}*x^4 - 40*d^4*e^9*x^3 - 12*d^5*e^8*x^2 + 40*d^6*e^7*x - 16*d^7*e^6 + (e^{12}*x^6 + 2*d*e^{11}*x^5 - 19*d^2*e^{10}*x^4 + 20*d^3*e^9*x^3 + 20*d^4*e^8*x^2 - 40*d^5*e^7*x + 16*d^6*e^6)*\sqrt{-e^2*x^2 + d^2})$$

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**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 (d + ex)^3}{(-(-d + ex)(d + ex))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(e\*x+d)\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2),x)

[Out] Integral(x\*\*5\*(d + e\*x)\*\*3/(-(-d + e\*x)\*(d + e\*x))\*\*(7/2), x)

---

**GIAC/XCAS [A]** time = 0.30419, size = 159, normalized size = 0.91

$$-\frac{13}{2} d^2 \arcsin\left(\frac{xe}{d}\right) e^{(-6)} \text{sign}(d) - \frac{(304 d^7 e^{(-6)} + (195 d^6 e^{(-5)} - (760 d^5 e^{(-4)} + (455 d^4 e^{(-3)} - (570 d^3 e^{(-2)} + (299 d^2 e^{(-1)} - 15(xe + 6d)x)x)x)x)\sqrt{-x^2 e^2 + d^2})}{30(x^2 e^2 - d^2)^3}$$


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Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*x^5/(-e^2\*x^2 + d^2)^(7/2),x, algorithm="giac")

[Out] 
$$-13/2*d^2*\arcsin(x*e/d)*e^{(-6)}*\text{sign}(d) - 1/30*(304*d^7*e^{(-6)} + (195*d^6*e^{(-5)} - (760*d^5*e^{(-4)} + (455*d^4*e^{(-3)} - (570*d^3*e^{(-2)} + (299*d^2*e^{(-1)} - 15*(x*e + 6*d)*x)*x)*x)*x)*\sqrt{-x^2*e^2 + d^2})/(x^2*e^2 - d^2)^3$$

$$3.84 \quad \int \frac{x^4(d+ex)^3}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=142

$$-\frac{6d^2(d+ex)^2}{5e^5(d^2-e^2x^2)^{3/2}} + \frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{3d \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5} + \frac{d^3(d+ex)^3}{5e^5(d^2-e^2x^2)^{5/2}}$$

[Out]  $(d^3*(d+e*x)^3)/(5*e^5*(d^2-e^2*x^2)^{(5/2)}) - (6*d^2*(d+e*x)^2)/(5*e^5*(d^2-e^2*x^2)^{(3/2)}) + (24*d*(d+e*x))/(5*e^5*\text{Sqrt}[d^2-e^2*x^2]) + \text{Sqrt}[d^2-e^2*x^2]/e^5 - (3*d*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2-e^2*x^2]])/e^5$

**Rubi [A]** time = 0.480729, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$-\frac{6d^2(d+ex)^2}{5e^5(d^2-e^2x^2)^{3/2}} + \frac{24d(d+ex)}{5e^5\sqrt{d^2-e^2x^2}} + \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{3d \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5} + \frac{d^3(d+ex)^3}{5e^5(d^2-e^2x^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*(d+e\*x)^3)/(d^2-e^2\*x^2)^(7/2),x]

[Out]  $(d^3*(d+e*x)^3)/(5*e^5*(d^2-e^2*x^2)^{(5/2)}) - (6*d^2*(d+e*x)^2)/(5*e^5*(d^2-e^2*x^2)^{(3/2)}) + (24*d*(d+e*x))/(5*e^5*\text{Sqrt}[d^2-e^2*x^2]) + \text{Sqrt}[d^2-e^2*x^2]/e^5 - (3*d*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2-e^2*x^2]])/e^5$

**Rubi in Sympy [A]** time = 56.469, size = 128, normalized size = 0.9

$$\frac{d^3\sqrt{d^2-e^2x^2}}{5e^5(d-ex)^3} - \frac{6d^2\sqrt{d^2-e^2x^2}}{5e^5(d-ex)^2} - \frac{3d \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5} + \frac{24d\sqrt{d^2-e^2x^2}}{5e^5(d-ex)} + \frac{\sqrt{d^2-e^2x^2}}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(e\*x+d)\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2),x)

[Out]  $d**3*\text{sqrt}(d**2-e**2*x**2)/(5*e**5*(d-e*x)**3) - 6*d**2*\text{sqrt}(d**2-e**2*x**2)/(5*e**5*(d-e*x)**2) - 3*d*\text{atan}(e*x/\text{sqrt}(d**2-e**2*x**2))/e**5 + 24*d*\text{sqrt}(d**2-e**2*x**2)/(5*e**5*(d-e*x)) + \text{sqrt}(d**2-e**2*x**2)/e**5$

**Mathematica [A]** time = 0.13864, size = 86, normalized size = 0.61

$$\frac{\sqrt{d^2-e^2x^2}(24d^3-57d^2ex+39de^2x^2-5e^3x^3)}{(d-ex)^3} - 15d \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{5e^5}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4\*(d+e\*x)^3)/(d^2-e^2\*x^2)^(7/2),x]

[Out]  $((\text{Sqrt}[d^2-e^2*x^2]*(24*d^3-57*d^2*e*x+39*d*e^2*x^2-5*e^3*x^3))/(d-e*x)^3-15*d*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2-e^2*x^2]])/(5*e$

^5)

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**Maple [B]** time = 0.013, size = 262, normalized size = 1.9

$$\begin{aligned} & \frac{d^3 x^3}{2 e^2} (-e^2 x^2 + d^2)^{-\frac{5}{2}} - \frac{3 d^5 x}{10 e^4} (-e^2 x^2 + d^2)^{-\frac{5}{2}} + \frac{d^3 x}{10 e^4} (-e^2 x^2 + d^2)^{-\frac{3}{2}} + \frac{16 dx}{5 e^4} \frac{1}{\sqrt{-e^2 x^2 + d^2}} \\ & - ex^6 (-e^2 x^2 + d^2)^{-\frac{5}{2}} + 9 \frac{d^2 x^4}{e (-e^2 x^2 + d^2)^{5/2}} - 12 \frac{d^4 x^2}{e^3 (-e^2 x^2 + d^2)^{5/2}} + \frac{24 d^6}{5 e^5} (-e^2 x^2 + d^2)^{-\frac{5}{2}} \\ & + \frac{3 dx^5}{5} (-e^2 x^2 + d^2)^{-\frac{5}{2}} - \frac{dx^3}{e^2} (-e^2 x^2 + d^2)^{-\frac{3}{2}} - 3 \frac{d}{e^4 \sqrt{e^2}} \arctan \left( \frac{\sqrt{e^2} x}{\sqrt{-e^2 x^2 + d^2}} \right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(e\*x+d)^3/(-e^2\*x^2+d^2)^(7/2),x)

```
[Out] 1/2*d^3*x^3/e^2/(-e^2*x^2+d^2)^(5/2)-3/10*d^5/e^4*x/(-e^2*x^2+d^2)^(5/2)+1/10*d^3/e^4*x/(-e^2*x^2+d^2)^(3/2)+16/5*d/e^4*x/(-e^2*x^2+d^2)^(1/2)-e*x^6/(-e^2*x^2+d^2)^(5/2)+9/e*d^2*x^4/(-e^2*x^2+d^2)^(5/2)-12/e^3*d^4*x^2/(-e^2*x^2+d^2)^(5/2)+24/5/e^5*d^6/(-e^2*x^2+d^2)^(5/2)+3/5*d*x^5/(-e^2*x^2+d^2)^(5/2)-d/e^2*x^3/(-e^2*x^2+d^2)^(3/2)-3*d/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
```

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**Maxima [A]** time = 0.795114, size = 455, normalized size = 3.2

$$\begin{aligned} & \frac{1}{5} de^2 x \left( \frac{15 x^4}{(-e^2 x^2 + d^2)^{\frac{5}{2}} e^2} - \frac{20 d^2 x^2}{(-e^2 x^2 + d^2)^{\frac{5}{2}} e^4} + \frac{8 d^4}{(-e^2 x^2 + d^2)^{\frac{5}{2}} e^6} \right) - \frac{ex^6}{(-e^2 x^2 + d^2)^{\frac{5}{2}}} \\ & - dx \left( \frac{3 x^2}{(-e^2 x^2 + d^2)^{\frac{3}{2}} e^2} - \frac{2 d^2}{(-e^2 x^2 + d^2)^{\frac{3}{2}} e^4} \right) + \frac{9 d^2 x^4}{(-e^2 x^2 + d^2)^{\frac{5}{2}} e} + \frac{d^3 x^3}{2 (-e^2 x^2 + d^2)^{\frac{5}{2}} e^2} \\ & - \frac{12 d^4 x^2}{(-e^2 x^2 + d^2)^{\frac{5}{2}} e^3} - \frac{3 d^5 x}{10 (-e^2 x^2 + d^2)^{\frac{5}{2}} e^4} + \frac{24 d^6}{5 (-e^2 x^2 + d^2)^{\frac{5}{2}} e^5} \\ & + \frac{9 d^3 x}{10 (-e^2 x^2 + d^2)^{\frac{3}{2}} e^4} - \frac{6 dx}{5 \sqrt{-e^2 x^2 + d^2} e^4} - \frac{3 d \arcsin \left( \frac{e^2 x}{\sqrt{d^2 e^2}} \right)}{\sqrt{e^2} e^4} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*x^4/(-e^2\*x^2 + d^2)^(7/2),x, algorithm="maxima")

```
[Out] 1/5*d*e^2*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) - e*x^6/(-e^2*x^2 + d^2)^(5/2) - d*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4)) + 9*d^2*x^4/((-e^2*x^2 + d^2)^(5/2)*e) + 1/2*d^3*x^3/((-e^2*x^2 + d^2)^(5/2)*e^2) - 12*d^4*x^2/((-e^2*x^2 + d^2)^(5/2)*e^3) - 3/10*d^5*x/((-e^2*x^2 + d^2)^(5/2)*e^4) + 24/5*d^6/((-e^2*x^2 + d^2)^(5/2)*e^5) + 9/10*d^3*x/((-e^2*x^2 + d^2)^(3/2)*e^4) - 6/5*d*x/(sqrt(-e^2*x^2 + d^2)*e^4) - 3*d*arcsin(e^2*x/sqrt(d^2*e^2))/sqrt(e^2)*e^4
```

---

**Fricas [A]** time = 0.297375, size = 603, normalized size = 4.25

$$5 e^7 x^7 - 30 d e^6 x^6 + 158 d^2 e^5 x^5 - 175 d^3 e^4 x^4 - 140 d^4 e^3 x^3 + 300 d^5 e^2 x^2 - 120 d^6 e x + 30 \left( d e^6 x^6 + d^2 e^5 x^5 - 13 d^3 e^4 x^4 + 15 d^4 e^3 x^3 - 10 d^5 e^2 x^2 - 6 d^6 e x + 3 d^7 \right) + 30 \left( e^{11} x^6 + d e^{10} x^5 - 13 d^2 e^9 x^4 + \dots \right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^3*x^4/(-e^2*x^2 + d^2)^(7/2),x, algorithm="fricas")`

[Out] 
$$\frac{1}{5} \cdot (5 \cdot e^7 \cdot x^7 - 30 \cdot d \cdot e^6 \cdot x^6 + 158 \cdot d^2 \cdot e^5 \cdot x^5 - 175 \cdot d^3 \cdot e^4 \cdot x^4 - 140 \cdot d^4 \cdot e^3 \cdot x^3 + 300 \cdot d^5 \cdot e^2 \cdot x^2 - 120 \cdot d^6 \cdot e \cdot x + 30 \cdot (d \cdot e^6 \cdot x^6 + d^2 \cdot e^5 \cdot x^5 - 13 \cdot d^3 \cdot e^4 \cdot x^4 + 15 \cdot d^4 \cdot e^3 \cdot x^3 + 8 \cdot d^5 \cdot e^2 \cdot x^2 - 20 \cdot d^6 \cdot e \cdot x + 8 \cdot d^7 - (d \cdot e^5 \cdot x^5 - 6 \cdot d^2 \cdot e^4 \cdot x^4 + 5 \cdot d^3 \cdot e^3 \cdot x^3 + 12 \cdot d^4 \cdot e^2 \cdot x^2 - 20 \cdot d^5 \cdot e \cdot x + 8 \cdot d^6)) \cdot \sqrt{-e^2 \cdot x^2 + d^2}) \cdot \arctan\left(\frac{-d - \sqrt{-e^2 \cdot x^2 + d^2}}{e \cdot x}\right) + (5 \cdot e^6 \cdot x^6 - 43 \cdot d \cdot e^5 \cdot x^5 + 25 \cdot d^2 \cdot e^4 \cdot x^4 + 200 \cdot d^3 \cdot e^3 \cdot x^3 - 300 \cdot d^4 \cdot e^2 \cdot x^2 + 120 \cdot d^5 \cdot e \cdot x) \cdot \sqrt{-e^2 \cdot x^2 + d^2}) / (e^{11} \cdot x^6 + d \cdot e^{10} \cdot x^5 - 13 \cdot d^2 \cdot e^9 \cdot x^4 + 15 \cdot d^3 \cdot e^8 \cdot x^3 + 8 \cdot d^4 \cdot e^7 \cdot x^2 - 20 \cdot d^5 \cdot e^6 \cdot x + 8 \cdot d^6 \cdot e^5 - (e^{10} \cdot x^5 - 6 \cdot d \cdot e^9 \cdot x^4 + 5 \cdot d^2 \cdot e^8 \cdot x^3 + 12 \cdot d^3 \cdot e^7 \cdot x^2 - 20 \cdot d^4 \cdot e^6 \cdot x + 8 \cdot d^5 \cdot e^5) \cdot \sqrt{-e^2 \cdot x^2 + d^2})$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 (d + ex)^3}{(-(-d + ex)(d + ex))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)`

[Out] `Integral(x**4*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)`

**GIAC/XCAS [A]** time = 0.292283, size = 144, normalized size = 1.01

$$\frac{-3 d \arcsin\left(\frac{x e}{d}\right) e^{(-5)} \operatorname{sign}(d) + \left(24 d^6 e^{(-5)} + \left(15 d^5 e^{(-4)} - \left(60 d^4 e^{(-3)} + \left(35 d^3 e^{(-2)} - \left(45 d^2 e^{(-1)} - (5 x e - 24 d) x\right) x\right) x\right) x\right) \sqrt{-x^2 e^2 + d^2}}{5 (x^2 e^2 - d^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^3*x^4/(-e^2*x^2 + d^2)^(7/2),x, algorithm="giac")`

[Out] 
$$-3 \cdot d \cdot \arcsin(x \cdot e / d) \cdot e^{(-5)} \cdot \operatorname{sign}(d) - \frac{1}{5} \cdot (24 \cdot d^6 \cdot e^{(-5)} + (15 \cdot d^5 \cdot e^{(-4)} - (60 \cdot d^4 \cdot e^{(-3)} + (35 \cdot d^3 \cdot e^{(-2)} - (45 \cdot d^2 \cdot e^{(-1)} - (5 \cdot x \cdot e - 24 \cdot d) \cdot x) \cdot x) \cdot x) \cdot \sqrt{-x^2 \cdot e^2 + d^2}) / (x^2 \cdot e^2 - d^2)^3$$

$$3.85 \quad \int \frac{x^3(d+ex)^3}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=118

$$\frac{d^2(d+ex)^3}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{13d(d+ex)^2}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

[Out]  $(d^2*(d+e*x)^3)/(5*e^4*(d^2-e^2*x^2)^{(5/2)}) - (13*d*(d+e*x)^2)/(15*e^4*(d^2-e^2*x^2)^{(3/2)}) + (32*(d+e*x))/(15*e^4*\text{Sqrt}[d^2-e^2*x^2]) - \text{ArcTan}[(e*x)/\text{Sqrt}[d^2-e^2*x^2]]/e^4$

**Rubi [A]** time = 0.36896, antiderivative size = 118, normalized size of antiderivative = 1., number of rules used = 5, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$\frac{d^2(d+ex)^3}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{13d(d+ex)^2}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{32(d+ex)}{15e^4\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^3*(d+e*x)^3)/(d^2-e^2*x^2)^{(7/2)}, x]$

[Out]  $(d^2*(d+e*x)^3)/(5*e^4*(d^2-e^2*x^2)^{(5/2)}) - (13*d*(d+e*x)^2)/(15*e^4*(d^2-e^2*x^2)^{(3/2)}) + (32*(d+e*x))/(15*e^4*\text{Sqrt}[d^2-e^2*x^2]) - \text{ArcTan}[(e*x)/\text{Sqrt}[d^2-e^2*x^2]]/e^4$

**Rubi in Sympy [A]** time = 53.7238, size = 104, normalized size = 0.88

$$\frac{d^2\sqrt{d^2-e^2x^2}}{5e^4(d-ex)^3} - \frac{13d\sqrt{d^2-e^2x^2}}{15e^4(d-ex)^2} - \frac{\text{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4} + \frac{32\sqrt{d^2-e^2x^2}}{15e^4(d-ex)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**3}*(e*x+d)^{**3}/(-e^{**2}*x^{**2}+d^{**2})^{**}(7/2), x)$

[Out]  $d^{**2}*\text{sqrt}(d^{**2}-e^{**2}*x^{**2})/(5*e^{**4}*(d-e*x)^{**3}) - 13*d*\text{sqrt}(d^{**2}-e^{**2}*x^{**2})/(15*e^{**4}*(d-e*x)^{**2}) - \text{atan}(e*x/\text{sqrt}(d^{**2}-e^{**2}*x^{**2}))/e^{**4} + 32*\text{sqrt}(d^{**2}-e^{**2}*x^{**2})/(15*e^{**4}*(d-e*x))$

**Mathematica [A]** time = 0.0975452, size = 83, normalized size = 0.7

$$\frac{(-22d^2 + 51dex - 32e^2x^2)\sqrt{d^2-e^2x^2} + 15(d-ex)^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{15e^4(ex-d)^3}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(x^3*(d+e*x)^3)/(d^2-e^2*x^2)^{(7/2)}, x]$

[Out]  $((-22*d^2 + 51*d*e*x - 32*e^2*x^2)*\text{Sqrt}[d^2-e^2*x^2] + 15*(d-e*x)^3*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2-e^2*x^2]])/(15*e^4*(-d+e*x)^3)$

**Maple [B]** time = 0.02, size = 234, normalized size = 2.

$$\begin{aligned}
 & -\frac{11d^3x^2}{3e^2}(-e^2x^2+d^2)^{-\frac{5}{2}} + \frac{22d^5}{15e^4}(-e^2x^2+d^2)^{-\frac{5}{2}} + \frac{ex^5}{5}(-e^2x^2+d^2)^{-\frac{5}{2}} \\
 & -\frac{x^3}{3e}(-e^2x^2+d^2)^{-\frac{3}{2}} + \frac{8x}{5e^3}\frac{1}{\sqrt{-e^2x^2+d^2}} - \frac{1}{e^3}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}} \\
 & + \frac{3d^2x^3}{2e}(-e^2x^2+d^2)^{-\frac{5}{2}} - \frac{9d^4x}{10e^3}(-e^2x^2+d^2)^{-\frac{5}{2}} + \frac{3d^2x}{10e^3}(-e^2x^2+d^2)^{-\frac{3}{2}} + 3\frac{dx^4}{(-e^2x^2+d^2)^{5/2}}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)`

[Out] `-11/3*d^3*x^2/e^2/(-e^2*x^2+d^2)^(5/2)+22/15*d^5/e^4/(-e^2*x^2+d^2)^(5/2)+1/5*e*x^5/(-e^2*x^2+d^2)^(5/2)-1/3/e*x^3/(-e^2*x^2+d^2)^(3/2)+8/5/e^3*x/(-e^2*x^2+d^2)^(1/2)-1/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+3/2/e*d^2*x^3/(-e^2*x^2+d^2)^(5/2)-9/10/e^3*d^4*x/(-e^2*x^2+d^2)^(5/2)+3/10/e^3*d^2*x/(-e^2*x^2+d^2)^(3/2)+3*d*x^4/(-e^2*x^2+d^2)^(5/2)`

**Maxima [A]** time = 0.809819, size = 417, normalized size = 3.53

$$\begin{aligned}
 & \frac{1}{15}e^3x\left(\frac{15x^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{20d^2x^2}{(-e^2x^2+d^2)^{\frac{5}{2}}e^4} + \frac{8d^4}{(-e^2x^2+d^2)^{\frac{5}{2}}e^6}\right) \\
 & - \frac{1}{3}ex\left(\frac{3x^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^2} - \frac{2d^2}{(-e^2x^2+d^2)^{\frac{3}{2}}e^4}\right) + \frac{3dx^4}{(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{3d^2x^3}{2(-e^2x^2+d^2)^{\frac{5}{2}}e} \\
 & - \frac{11d^3x^2}{3(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{9d^4x}{10(-e^2x^2+d^2)^{\frac{5}{2}}e^3} + \frac{22d^5}{15(-e^2x^2+d^2)^{\frac{5}{2}}e^4} \\
 & + \frac{17d^2x}{30(-e^2x^2+d^2)^{\frac{3}{2}}e^3} + \frac{2x}{15\sqrt{-e^2x^2+d^2}e^3} - \frac{\arcsin\left(\frac{e^2x}{\sqrt{d^2e^2}}\right)}{\sqrt{e^2}e^3}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^3*x^3/(-e^2*x^2+d^2)^(7/2),x,algorithm="maxima")`

[Out] `1/15*e^3*x*(15*x^4/((-e^2*x^2+d^2)^(5/2)*e^2)-20*d^2*x^2/((-e^2*x^2+d^2)^(5/2)*e^4)+8*d^4/((-e^2*x^2+d^2)^(5/2)*e^6)-1/3*e*x*(3*x^2/((-e^2*x^2+d^2)^(3/2)*e^2)-2*d^2/((-e^2*x^2+d^2)^(3/2)*e^4))+3*d*x^4/(-e^2*x^2+d^2)^(5/2)+3/2*d^2*x^3/((-e^2*x^2+d^2)^(5/2)*e)-11/3*d^3*x^2/((-e^2*x^2+d^2)^(5/2)*e^2)-9/10*d^4*x/((-e^2*x^2+d^2)^(5/2)*e^3)+22/15*d^5/((-e^2*x^2+d^2)^(5/2)*e^4)+17/30*d^2*x/((-e^2*x^2+d^2)^(3/2)*e^3)+2/15*x/(sqrt(-e^2*x^2+d^2)*e^3)-arcsin(e^2*x/sqrt(d^2*e^2))/sqrt(e^2)*e^3`

**Fricas [A]** time = 0.294195, size = 454, normalized size = 3.85

$$\frac{54e^5x^5 - 65de^4x^4 - 85d^2e^3x^3 + 150d^3e^2x^2 - 60d^4ex + 30\left(e^5x^5 - 5de^4x^4 + 5d^2e^3x^3 + 5d^3e^2x^2 - 10d^4ex + 4d^5 + (e^4x^4 - 15(e^9x^5 - 5de^8x^4 + 5d^2e^7x^3 + 5d^3e^6x^2 - 10d^4e^5x + 4d^5)\right)}{15\left(e^9x^5 - 5de^8x^4 + 5d^2e^7x^3 + 5d^3e^6x^2 - 10d^4e^5x + 4d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^3*x^3/(-e^2*x^2+d^2)^(7/2),x,algorithm="fricas")`

[Out]  $\frac{1}{15} (54e^5x^5 - 65d^4e^4x^4 - 85d^2e^3x^3 + 150d^3e^2x^2 - 60d^4e^2x + 30(e^5x^5 - 5d^4e^4x^4 + 5d^2e^3x^3 + 5d^3e^2x^2 - 10d^4e^2x + 4d^5 + (e^4x^4 - 7d^2e^2x^2 + 10d^3e^2x - 4d^4) \sqrt{-e^2x^2 + d^2}) \arctan(-\frac{d - \sqrt{-e^2x^2 + d^2}}{e^2x}) - 5(2e^4x^4 - 23d^3e^3x^3 + 30d^2e^2x^2 - 12d^3e^2x) \sqrt{-e^2x^2 + d^2}) / (e^9x^5 - 5d^8e^8x^4 + 5d^2e^7x^3 + 5d^3e^6x^2 - 10d^4e^5x + 4d^5e^4 + (e^8x^4 - 7d^2e^6x^2 + 10d^3e^5x - 4d^4e^4) \sqrt{-e^2x^2 + d^2})$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 (d + ex)^3}{(-(-d + ex)(d + ex))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)`

[Out] `Integral(x**3*(d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)`

**GIAC/XCAS [A]** time = 0.29334, size = 128, normalized size = 1.08

$$-\arcsin\left(\frac{xe}{d}\right) e^{(-4)} \operatorname{sign}(d) - \frac{\left(22d^5e^{(-4)} + \left(15d^4e^{(-3)} - \left(55d^3e^{(-2)} + \left(35d^2e^{(-1)} - (32xe + 45d)x\right)x\right)x\right)x\right) \sqrt{-x^2e^2 + d^2}}{15(x^2e^2 - d^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^3*x^3/(-e^2*x^2 + d^2)^(7/2),x, algorithm="giac")`

[Out]  $-\arcsin(xe/d) e^{(-4)} \operatorname{sign}(d) - \frac{1}{15} (22d^5e^{(-4)} + (15d^4e^{(-3)} - (55d^3e^{(-2)} + (35d^2e^{(-1)} - (32xe + 45d)x)x)x) \sqrt{-x^2e^2 + d^2}) / (x^2e^2 - d^2)^3$

$$3.86 \quad \int \frac{x^2(d+ex)^3}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=93

$$\frac{d(d+ex)^3}{5e^3(d^2-e^2x^2)^{5/2}} - \frac{8(d+ex)^2}{15e^3(d^2-e^2x^2)^{3/2}} + \frac{7(d+ex)}{15de^3\sqrt{d^2-e^2x^2}}$$

[Out] (d\*(d + e\*x)^3)/(5\*e^3\*(d^2 - e^2\*x^2)^(5/2)) - (8\*(d + e\*x)^2)/(15\*e^3\*(d^2 - e^2\*x^2)^(3/2)) + (7\*(d + e\*x))/(15\*d\*e^3\*Sqrt[d^2 - e^2\*x^2])

**Rubi [A]** time = 0.273609, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{d(d+ex)^3}{5e^3(d^2-e^2x^2)^{5/2}} - \frac{8(d+ex)^2}{15e^3(d^2-e^2x^2)^{3/2}} + \frac{7(d+ex)}{15de^3\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(d + e\*x)^3)/(d^2 - e^2\*x^2)^(7/2), x]

[Out] (d\*(d + e\*x)^3)/(5\*e^3\*(d^2 - e^2\*x^2)^(5/2)) - (8\*(d + e\*x)^2)/(15\*e^3\*(d^2 - e^2\*x^2)^(3/2)) + (7\*(d + e\*x))/(15\*d\*e^3\*Sqrt[d^2 - e^2\*x^2])

**Rubi in Sympy [A]** time = 46.3438, size = 80, normalized size = 0.86

$$\frac{d\sqrt{d^2-e^2x^2}}{5e^3(d-ex)^3} - \frac{8\sqrt{d^2-e^2x^2}}{15e^3(d-ex)^2} + \frac{7\sqrt{d^2-e^2x^2}}{15de^3(d-ex)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(e\*x+d)\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out] d\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(5\*e\*\*3\*(d - e\*x)\*\*3) - 8\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*e\*\*3\*(d - e\*x)\*\*2) + 7\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*d\*e\*\*3\*(d - e\*x))

**Mathematica [A]** time = 0.045563, size = 53, normalized size = 0.57

$$\frac{\sqrt{d^2-e^2x^2}(2d^2-6dex+7e^2x^2)}{15de^3(d-ex)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(d + e\*x)^3)/(d^2 - e^2\*x^2)^(7/2), x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(2\*d^2 - 6\*d\*e\*x + 7\*e^2\*x^2))/(15\*d\*e^3\*(d - e\*x)^3)

**Maple [A]** time = 0.01, size = 55, normalized size = 0.6

$$\frac{(-ex + d)(ex + d)^4 (7e^2x^2 - 6dex + 2d^2)}{15de^3} (-e^2x^2 + d^2)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)`

[Out] `1/15*(-e*x+d)*(e*x+d)^4*(7*e^2*x^2-6*d*e*x+2*d^2)/d/e^3/(-e^2*x^2+d^2)^(7/2)`

**Maxima [A]** time = 0.726464, size = 208, normalized size = 2.24

$$\begin{aligned} & \frac{ex^4}{(-e^2x^2 + d^2)^{\frac{5}{2}}} + \frac{3dx^3}{2(-e^2x^2 + d^2)^{\frac{5}{2}}} - \frac{d^2x^2}{3(-e^2x^2 + d^2)^{\frac{5}{2}}e} - \frac{7d^3x}{10(-e^2x^2 + d^2)^{\frac{5}{2}}e^2} \\ & + \frac{2d^4}{15(-e^2x^2 + d^2)^{\frac{5}{2}}e^3} + \frac{7dx}{30(-e^2x^2 + d^2)^{\frac{3}{2}}e^2} + \frac{7x}{15\sqrt{-e^2x^2 + d^2}de^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^3*x^2/(-e^2*x^2 + d^2)^(7/2),x, algorithm="maxima")`

[Out] `e*x^4/(-e^2*x^2 + d^2)^(5/2) + 3/2*d*x^3/(-e^2*x^2 + d^2)^(5/2) - 1/3*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e) - 7/10*d^3*x/((-e^2*x^2 + d^2)^(5/2)*e^2) + 2/15*d^4/((-e^2*x^2 + d^2)^(5/2)*e^3) + 7/30*d*x/((-e^2*x^2 + d^2)^(3/2)*e^2) + 7/15*x/(sqrt(-e^2*x^2 + d^2)*d*e^2)`

**Fricas [A]** time = 0.282042, size = 212, normalized size = 2.28

$$\frac{9e^2x^5 + 5dex^4 - 20d^2x^3 - 5(ex^4 - 4dx^3)\sqrt{-e^2x^2 + d^2}}{15\left(de^5x^5 - 5d^2e^4x^4 + 5d^3e^3x^3 + 5d^4e^2x^2 - 10d^5ex + 4d^6 + (d^4x^4 - 7d^3e^2x^2 + 10d^4ex - 4d^5)\sqrt{-e^2x^2 + d^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^3*x^2/(-e^2*x^2 + d^2)^(7/2),x, algorithm="fricas")`

[Out] `1/15*(9*e^2*x^5 + 5*d*e*x^4 - 20*d^2*x^3 - 5*(e*x^4 - 4*d*x^3)*sqrt(-e^2*x^2 + d^2))/(d*e^5*x^5 - 5*d^2*e^4*x^4 + 5*d^3*e^3*x^3 + 5*d^4*e^2*x^2 - 10*d^5*e*x + 4*d^6 + (d*e^4*x^4 - 7*d^3*e^2*x^2 + 10*d^4*e*x - 4*d^5)*sqrt(-e^2*x^2 + d^2))`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2(d + ex)^3}{(-(-d + ex)(d + ex))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)`

[Out] `Integral(x**2*(d + e*x)**3/((-d + e*x)*(d + e*x))**(7/2), x)`

---

**GIAC/XCAS [A]** time = 0.297943, size = 97, normalized size = 1.04

$$\frac{\left(2d^4e^{(-3)} - \left(5d^2e^{(-1)} - \left(x\left(\frac{7xe^2}{d} + 15e\right) + 5d\right)x\right)x^2\right)\sqrt{-x^2e^2 + d^2}}{15(x^2e^2 - d^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*x^2/(-e^2\*x^2 + d^2)^(7/2),x, algorithm="giac")

[Out] -1/15\*(2\*d^4\*e^(-3) - (5\*d^2\*e^(-1) - (x\*(7\*x\*e^2/d + 15\*e) + 5\*d)\*x)\*x^2)\*sqrt(-x^2\*e^2 + d^2)/(x^2\*e^2 - d^2)^3

$$3.87 \quad \int \frac{x(d+ex)^3}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=86

$$\frac{(d+ex)^3}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{2(d+ex)}{5e^2(d^2-e^2x^2)^{3/2}} - \frac{x}{5d^2e\sqrt{d^2-e^2x^2}}$$

[Out]  $(d + e*x)^3/(5*e^2*(d^2 - e^2*x^2)^{(5/2)}) - (2*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^{(3/2)}) - x/(5*d^2*e*Sqrt[d^2 - e^2*x^2])$

**Rubi [A]** time = 0.13398, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$\frac{(d+ex)^3}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{2(d+ex)}{5e^2(d^2-e^2x^2)^{3/2}} - \frac{x}{5d^2e\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(x\*(d + e\*x)^3)/(d^2 - e^2\*x^2)^(7/2), x]

[Out]  $(d + e*x)^3/(5*e^2*(d^2 - e^2*x^2)^{(5/2)}) - (2*(d + e*x))/(5*e^2*(d^2 - e^2*x^2)^{(3/2)}) - x/(5*d^2*e*Sqrt[d^2 - e^2*x^2])$

**Rubi in Sympy [A]** time = 13.1859, size = 73, normalized size = 0.85

$$\frac{(d+ex)^3}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{2(d+ex)}{5e^2(d^2-e^2x^2)^{3/2}} - \frac{x}{5d^2e\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(e\*x+d)\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out]  $(d + e*x)**3/(5*e**2*(d**2 - e**2*x**2)**(5/2)) - 2*(d + e*x)/(5*e**2*(d**2 - e**2*x**2)**(3/2)) - x/(5*d**2*e*sqrt(d**2 - e**2*x**2))$

**Mathematica [A]** time = 0.0423667, size = 50, normalized size = 0.58

$$-\frac{\sqrt{d^2-e^2x^2}(d^2-3dex+e^2x^2)}{5d^2e^2(d-ex)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(d + e\*x)^3)/(d^2 - e^2\*x^2)^(7/2), x]

[Out]  $-(Sqrt[d^2 - e^2*x^2]*(d^2 - 3*d*e*x + e^2*x^2))/(5*d^2*e^2*(d - e*x)^3)$

**Maple [A]** time = 0.011, size = 52, normalized size = 0.6

$$-\frac{(-ex+d)(ex+d)^4(e^2x^2-3dex+d^2)}{5d^2e^2}(-e^2x^2+d^2)^{-7/2}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(e*x+d)^3/(-e^2*x^2+d^2)^(7/2),x)`

[Out]  $-1/5*(-e*x+d)*(e*x+d)^4*(e^2*x^2-3*d*e*x+d^2)/d^2/e^2/(-e^2*x^2+d^2)^(7/2)$

**Maxima [A]** time = 0.727091, size = 173, normalized size = 2.01

$$\frac{ex^3}{2(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{dx^2}{(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{3d^2x}{10(-e^2x^2+d^2)^{\frac{5}{2}}e} - \frac{d^3}{5(-e^2x^2+d^2)^{\frac{5}{2}}e^2} - \frac{x}{10(-e^2x^2+d^2)^{\frac{3}{2}}e} - \frac{x}{5\sqrt{-e^2x^2+d^2}d^2e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^3*x/(-e^2*x^2+d^2)^(7/2),x,algorithm="maxima")`

[Out]  $1/2*e*x^3/(-e^2*x^2+d^2)^(5/2) + d*x^2/(-e^2*x^2+d^2)^(5/2) + 3/10*d^2*x/((-e^2*x^2+d^2)^(5/2)*e) - 1/5*d^3/((-e^2*x^2+d^2)^(5/2)*e^2) - 1/10*x/((-e^2*x^2+d^2)^(3/2)*e) - 1/5*x/(sqrt(-e^2*x^2+d^2)*d^2*e)$

**Fricas [A]** time = 0.277197, size = 236, normalized size = 2.74

$$\frac{2e^3x^5 - 5de^2x^4 - 5d^2ex^3 + 10d^3x^2 + 5(dex^3 - 2d^2x^2)\sqrt{-e^2x^2+d^2}}{5\left(d^2e^5x^5 - 5d^3e^4x^4 + 5d^4e^3x^3 + 5d^5e^2x^2 - 10d^6ex + 4d^7 + (d^2e^4x^4 - 7d^4e^2x^2 + 10d^5ex - 4d^6)\sqrt{-e^2x^2+d^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^3*x/(-e^2*x^2+d^2)^(7/2),x,algorithm="fricas")`

[Out]  $-1/5*(2*e^3*x^5 - 5*d*e^2*x^4 - 5*d^2*e*x^3 + 10*d^3*x^2 + 5*(d*e*x^3 - 2*d^2*x^2)*sqrt(-e^2*x^2+d^2))/(d^2*e^5*x^5 - 5*d^3*e^4*x^4 + 5*d^4*e^3*x^3 + 5*d^5*e^2*x^2 - 10*d^6*e*x + 4*d^7 + (d^2*e^4*x^4 - 7*d^4*e^2*x^2 + 10*d^5*e*x - 4*d^6)*sqrt(-e^2*x^2+d^2))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x(d+ex)^3}{(-(-d+ex)(d+ex))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)`

[Out] `Integral(x*(d+e*x)**3/(-(-d+e*x)*(d+e*x))**(7/2),x)`

**GIAC/XCAS [A]** time = 0.295909, size = 81, normalized size = 0.94

$$\frac{\left(d^3e^{(-2)} + \left(x\left(\frac{x^2e^3}{d^2} - 5e\right) - 5d\right)x^2\right)\sqrt{-x^2e^2+d^2}}{5(x^2e^2-d^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^3*x/(-e^2*x^2 + d^2)^(7/2),x, algorithm="giac")
```

```
[Out] 1/5*(d^3*e^(-2) + (x*(x^2*e^3/d^2 - 5*e) - 5*d)*x^2)*sqrt(-x^2*e^2 + d^2)/(x^2*e^2 - d^2)^3
```

$$3.88 \quad \int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=103

$$\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2} + \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}$$

[Out] Sqrt[d^2 - e^2\*x^2]/(5\*d\*e\*(d - e\*x)^3) + (2\*Sqrt[d^2 - e^2\*x^2])/(15\*d^2\*e\*(d - e\*x)^2) + (2\*Sqrt[d^2 - e^2\*x^2])/(15\*d^3\*e\*(d - e\*x))

**Rubi [A]** time = 0.136076, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2} + \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^3/(d^2 - e^2\*x^2)^(7/2), x]

[Out] Sqrt[d^2 - e^2\*x^2]/(5\*d\*e\*(d - e\*x)^3) + (2\*Sqrt[d^2 - e^2\*x^2])/(15\*d^2\*e\*(d - e\*x)^2) + (2\*Sqrt[d^2 - e^2\*x^2])/(15\*d^3\*e\*(d - e\*x))

**Rubi in Sympy [A]** time = 23.7912, size = 80, normalized size = 0.78

$$\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2} + \frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out] sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(5\*d\*e\*(d - e\*x)\*\*3) + 2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*d\*\*2\*e\*(d - e\*x)\*\*2) + 2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*d\*\*3\*e\*(d - e\*x))

**Mathematica [A]** time = 0.0360413, size = 53, normalized size = 0.51

$$\frac{\sqrt{d^2-e^2x^2} (7d^2 - 6dex + 2e^2x^2)}{15d^3e(d-ex)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^3/(d^2 - e^2\*x^2)^(7/2), x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(7\*d^2 - 6\*d\*e\*x + 2\*e^2\*x^2))/(15\*d^3\*e\*(d - e\*x)^3)

**Maple [A]** time = 0.01, size = 55, normalized size = 0.5

$$\frac{(-ex + d)(ex + d)^4 (2e^2x^2 - 6dex + 7d^2)}{15d^3e} (-e^2x^2 + d^2)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^3/(-e^2*x^2+d^2)^(7/2), x)`

[Out] `1/15*(-e*x+d)*(e*x+d)^4*(2*e^2*x^2-6*d*e*x+7*d^2)/d^3/e/(-e^2*x^2+d^2)^(7/2)`

**Maxima [A]** time = 0.725776, size = 136, normalized size = 1.32

$$\frac{ex^2}{3(-e^2x^2 + d^2)^{\frac{5}{2}}} + \frac{4dx}{5(-e^2x^2 + d^2)^{\frac{5}{2}}} + \frac{7d^2}{15(-e^2x^2 + d^2)^{\frac{5}{2}}e} + \frac{x}{15(-e^2x^2 + d^2)^{\frac{3}{2}}d} + \frac{2x}{15\sqrt{-e^2x^2 + d^2}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^3/(-e^2*x^2 + d^2)^(7/2), x, algorithm="maxima")`

[Out] `1/3*e*x^2/(-e^2*x^2 + d^2)^(5/2) + 4/5*d*x/(-e^2*x^2 + d^2)^(5/2) + 7/15*d^2/((-e^2*x^2 + d^2)^(5/2)*e) + 1/15*x/((-e^2*x^2 + d^2)^(3/2)*d) + 2/15*x/(sqrt(-e^2*x^2 + d^2)*d^3)`

**Fricas [A]** time = 0.278937, size = 271, normalized size = 2.63

$$\frac{9e^4x^5 - 35de^3x^4 + 20d^2e^2x^3 + 60d^3ex^2 - 60d^4x + 5(e^3x^4 + 2de^2x^3 - 12d^2ex^2 + 12d^3x)\sqrt{-e^2x^2 + d^2}}{15(d^3e^5x^5 - 5d^4e^4x^4 + 5d^5e^3x^3 + 5d^6e^2x^2 - 10d^7ex + 4d^8 + (d^3e^4x^4 - 7d^5e^2x^2 + 10d^6ex - 4d^7)\sqrt{-e^2x^2 + d^2})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^3/(-e^2*x^2 + d^2)^(7/2), x, algorithm="fricas")`

[Out] `1/15*(9*e^4*x^5 - 35*d*e^3*x^4 + 20*d^2*e^2*x^3 + 60*d^3*e*x^2 - 60*d^4*x + 5*(e^3*x^4 + 2*d*e^2*x^3 - 12*d^2*e*x^2 + 12*d^3*x)*sqrt(-e^2*x^2 + d^2))/(d^3*e^5*x^5 - 5*d^4*e^4*x^4 + 5*d^5*e^3*x^3 + 5*d^6*e^2*x^2 - 10*d^7*e*x + 4*d^8 + (d^3*e^4*x^4 - 7*d^5*e^2*x^2 + 10*d^6*e*x - 4*d^7)*sqrt(-e^2*x^2 + d^2))`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^3}{(-(-d + ex)(d + ex))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**3/(-e**2*x**2+d**2)**(7/2), x)`

[Out] `Integral((d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)`

**GIAC/XCAS [A]** time = 0.289945, size = 95, normalized size = 0.92

$$\frac{\sqrt{-x^2e^2 + d^2} \left( 7d^2e^{(-1)} + \left( \left( x \left( \frac{2x^2e^4}{d^3} - \frac{5e^2}{d} \right) + 5e \right) x + 15d \right) x \right)}{15(x^2e^2 - d^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3/(-e^2\*x^2 + d^2)^(7/2),x, algorithm="giac")

[Out] -1/15\*sqrt(-x^2\*e^2 + d^2)\*(7\*d^2\*e^(-1) + ((x\*(2\*x^2\*e^4/d^3 - 5\*e^2/d) + 5\*e)\*x + 15\*d)\*x)/(x^2\*e^2 - d^2)^3

$$3.89 \quad \int \frac{(d+ex)^3}{x(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=114

$$\frac{4(d+ex)}{5(d^2-e^2x^2)^{5/2}} + \frac{5d+11ex}{15d^2(d^2-e^2x^2)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

[Out] (4\*(d + e\*x))/(5\*(d^2 - e^2\*x^2)^(5/2)) + (5\*d + 11\*e\*x)/(15\*d^2\*(d^2 - e^2\*x^2)^(3/2)) + (15\*d + 22\*e\*x)/(15\*d^4\*sqrt[d^2 - e^2\*x^2]) - ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]/d^4

**Rubi [A]** time = 0.346936, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{4(d+ex)}{5(d^2-e^2x^2)^{5/2}} + \frac{5d+11ex}{15d^2(d^2-e^2x^2)^{3/2}} + \frac{15d+22ex}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^3/(x\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out] (4\*(d + e\*x))/(5\*(d^2 - e^2\*x^2)^(5/2)) + (5\*d + 11\*e\*x)/(15\*d^2\*(d^2 - e^2\*x^2)^(3/2)) + (15\*d + 22\*e\*x)/(15\*d^4\*sqrt[d^2 - e^2\*x^2]) - ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]/d^4

**Rubi in Sympy [A]** time = 46.6765, size = 97, normalized size = 0.85

$$\frac{\sqrt{d^2-e^2x^2}}{5d^2(d-ex)^3} + \frac{7\sqrt{d^2-e^2x^2}}{15d^3(d-ex)^2} - \frac{\operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4} + \frac{22\sqrt{d^2-e^2x^2}}{15d^4(d-ex)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3/x/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out] sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(5\*d\*\*2\*(d - e\*x)\*\*3) + 7\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*d\*\*3\*(d - e\*x)\*\*2) - atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d)/d\*\*4 + 22\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*d\*\*4\*(d - e\*x))

**Mathematica [A]** time = 0.145697, size = 77, normalized size = 0.68

$$\frac{\sqrt{d^2-e^2x^2}(32d^2-51dex+22e^2x^2)}{(d-ex)^3} - \frac{15 \log\left(\sqrt{d^2-e^2x^2} + d\right) + 15 \log(x)}{15d^4}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^3/(x\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(32\*d^2 - 51\*d\*e\*x + 22\*e^2\*x^2))/(d - e\*x)^3 + 15\*Log[x] - 15\*Log[d + Sqrt[d^2 - e^2\*x^2]])/(15\*d^4)

**Maple [A]** time = 0.017, size = 158, normalized size = 1.4

$$\begin{aligned} & \frac{4d}{5} (-e^2x^2 + d^2)^{-\frac{5}{2}} + \frac{1}{3d} (-e^2x^2 + d^2)^{-\frac{3}{2}} + \frac{1}{d^3} \frac{1}{\sqrt{-e^2x^2 + d^2}} \\ & - \frac{1}{d^3} \ln\left(\frac{1}{x} \left(2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}} \\ & + \frac{4ex}{5} (-e^2x^2 + d^2)^{-\frac{5}{2}} + \frac{11ex}{15d^2} (-e^2x^2 + d^2)^{-\frac{3}{2}} + \frac{22ex}{15d^4} \frac{1}{\sqrt{-e^2x^2 + d^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^3/x/(-e^2*x^2+d^2)^(7/2), x)`

[Out] `4/5*d/(-e^2*x^2+d^2)^(5/2)+1/3/d/(-e^2*x^2+d^2)^(3/2)+1/d^3/(-e^2*x^2+d^2)^(1/2)-1/d^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+4/5*e*x/(-e^2*x^2+d^2)^(5/2)+11/15*e/d^2*x/(-e^2*x^2+d^2)^(3/2)+22/15*e/d^4*x/(-e^2*x^2+d^2)^(1/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^3/((-e^2*x^2 + d^2)^(7/2)*x), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.289158, size = 447, normalized size = 3.92

$$\frac{54e^5x^5 - 145de^4x^4 - 5d^2e^3x^3 + 270d^3e^2x^2 - 180d^4ex + 15(e^5x^5 - 5de^4x^4 + 5d^2e^3x^3 + 5d^3e^2x^2 - 10d^4ex + 4d^5 + (e^4x^4 - 4d^4e^3x^3 + 6d^5e^2x^2 - 4d^6ex + d^7))\sqrt{-e^2x^2 + d^2}}{15(d^4e^5x^5 - 5d^5e^4x^4 + 5d^6e^3x^3 + 5d^7e^2x^2 - 10d^8ex + d^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^3/((-e^2*x^2 + d^2)^(7/2)*x), x, algorithm="fricas")`

[Out] `1/15*(54*e^5*x^5 - 145*d*e^4*x^4 - 5*d^2*e^3*x^3 + 270*d^3*e^2*x^2 - 180*d^4*e*x + 15*(e^5*x^5 - 5*d*e^4*x^4 + 5*d^2*e^3*x^3 + 5*d^3*e^2*x^2 - 10*d^4*e*x + 4*d^5 + (e^4*x^4 - 4*d^4*e^3*x^3 + 6*d^5*e^2*x^2 - 4*d^6*e*x + d^7))*sqrt(-e^2*x^2 + d^2))/((d^4*e^5*x^5 - 5*d^5*e^4*x^4 + 5*d^6*e^3*x^3 + 5*d^7*e^2*x^2 - 10*d^8*e*x + d^9)*sqrt(-e^2*x^2 + d^2))`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^3}{x(-(-d + ex)(d + ex))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**3/x/(-e**2*x**2+d**2)**(7/2), x)`

[Out] Integral((d + e\*x)\*\*3/(x\*(-(-d + e\*x)\*(d + e\*x))\*\*(7/2)), x)

---

**GIAC/XCAS [A]** time = 0.293, size = 158, normalized size = 1.39

$$\frac{\sqrt{-x^2e^2 + d^2} \left( \left( \left( x \left( \frac{22xe^5}{d^4} + \frac{15e^4}{d^3} \right) - \frac{55e^3}{d^2} \right) x - \frac{35e^2}{d} \right) x + 45e \right) x + 32d}{15(x^2e^2 - d^2)^3} - \frac{\ln \left( \frac{|-2de - 2\sqrt{-x^2e^2 + d^2}e|e^{(-2)}}{2|x|} \right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3/((-e^2\*x^2 + d^2)^(7/2)\*x),x, algorithm="giac")

[Out] -1/15\*sqrt(-x^2\*e^2 + d^2)\*(((x\*(22\*x\*e^5/d^4 + 15\*e^4/d^3) - 55\*e^3/d^2)\*x - 35\*e^2/d)\*x + 45\*e)\*x + 32\*d)/(x^2\*e^2 - d^2)^3 - 1/n(1/2\*abs(-2\*d\*e - 2\*sqrt(-x^2\*e^2 + d^2)\*e)\*e^(-2)/abs(x))/d^4



$$3.90 \quad \int \frac{(d+ex)^3}{x^2(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=145

$$\frac{4e(d+ex)}{5d(d^2-e^2x^2)^{5/2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} + \frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{3e \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5} + \frac{e(5d+7ex)}{5d^3(d^2-e^2x^2)^{3/2}}$$

[Out]  $(4*e*(d+e*x))/(5*d*(d^2-e^2*x^2)^(5/2)) + (e*(5*d+7*e*x))/(5*d^3*(d^2-e^2*x^2)^(3/2)) + (e*(15*d+19*e*x))/(5*d^5*\text{Sqrt}[d^2-e^2*x^2]) - \text{Sqrt}[d^2-e^2*x^2]/(d^5*x) - (3*e*\text{ArcTanh}[\text{Sqrt}[d^2-e^2*x^2]/d])/d^5$

**Rubi [A]** time = 0.448311, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\frac{4e(d+ex)}{5d(d^2-e^2x^2)^{5/2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} + \frac{e(15d+19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{3e \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5} + \frac{e(5d+7ex)}{5d^3(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^3/(x^2\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out]  $(4*e*(d+e*x))/(5*d*(d^2-e^2*x^2)^(5/2)) + (e*(5*d+7*e*x))/(5*d^3*(d^2-e^2*x^2)^(3/2)) + (e*(15*d+19*e*x))/(5*d^5*\text{Sqrt}[d^2-e^2*x^2]) - \text{Sqrt}[d^2-e^2*x^2]/(d^5*x) - (3*e*\text{ArcTanh}[\text{Sqrt}[d^2-e^2*x^2]/d])/d^5$

**Rubi in Sympy [A]** time = 55.9283, size = 124, normalized size = 0.86

$$\frac{e\sqrt{d^2-e^2x^2}}{5d^3(d-ex)^3} + \frac{4e\sqrt{d^2-e^2x^2}}{5d^4(d-ex)^2} - \frac{3e \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5} + \frac{19e\sqrt{d^2-e^2x^2}}{5d^5(d-ex)} - \frac{\sqrt{d^2-e^2x^2}}{d^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3/x\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out]  $e*\text{sqrt}(d**2-e**2*x**2)/(5*d**3*(d-e*x)**3) + 4*e*\text{sqrt}(d**2-e**2*x**2)/(5*d**4*(d-e*x)**2) - 3*e*\text{atanh}(\text{sqrt}(d**2-e**2*x**2)/d)/d**5 + 19*e*\text{sqrt}(d**2-e**2*x**2)/(5*d**5*(d-e*x)) - \text{sqrt}(d**2-e**2*x**2)/(d**5*x)$

**Mathematica [A]** time = 0.151774, size = 94, normalized size = 0.65

$$\frac{-15e \log\left(\sqrt{d^2-e^2x^2}+d\right) + \frac{\sqrt{d^2-e^2x^2}(5d^3-39d^2ex+57de^2x^2-24e^3x^3)}{x(ex-d)^3} + 15e \log(x)}{5d^5}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^3/(x^2\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out]  $((\text{Sqrt}[d^2-e^2*x^2]*(5*d^3-39*d^2*e*x+57*d*e^2*x^2-24*e^3*x^3))/(x*(-d+e*x)^3) + 15*e*\text{Log}[x] - 15*e*\text{Log}[d+\text{Sqrt}[d^2-e^2*x^2]])/d^5$

$$^2 * x^2 ]]) / (5 * d^5)$$

**Maple [A]** time = 0.015, size = 190, normalized size = 1.3

$$\begin{aligned} & -\frac{d}{x} (-e^2 x^2 + d^2)^{-\frac{5}{2}} + \frac{9 e^2 x}{5 d} (-e^2 x^2 + d^2)^{-\frac{5}{2}} + \frac{12 e^2 x}{5 d^3} (-e^2 x^2 + d^2)^{-\frac{3}{2}} \\ & + \frac{24 e^2 x}{5 d^5} \frac{1}{\sqrt{-e^2 x^2 + d^2}} + \frac{4 e}{5} (-e^2 x^2 + d^2)^{-\frac{5}{2}} + \frac{e}{d^2} (-e^2 x^2 + d^2)^{-\frac{3}{2}} \\ & + 3 \frac{e}{d^4 \sqrt{-e^2 x^2 + d^2}} - 3 \frac{e}{d^4 \sqrt{d^2}} \ln \left( \frac{2 d^2 + 2 \sqrt{d^2} \sqrt{-e^2 x^2 + d^2}}{x} \right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3/x^2/(-e^2\*x^2+d^2)^(7/2),x)

[Out] -d/x/(-e^2\*x^2+d^2)^(5/2)+9/5/d\*e^2\*x/(-e^2\*x^2+d^2)^(5/2)+12/5/d^3\*e^2\*x/(-e^2\*x^2+d^2)^(3/2)+24/5/d^5\*e^2\*x/(-e^2\*x^2+d^2)^(1/2)+4/5\*e/(-e^2\*x^2+d^2)^(5/2)+e/d^2/(-e^2\*x^2+d^2)^(3/2)+3\*e/d^4/(-e^2\*x^2+d^2)^(1/2)-3\*e/d^4/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3/((-e^2\*x^2 + d^2)^(7/2)\*x^2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.295272, size = 620, normalized size = 4.28

$$\frac{153 d e^6 x^6 - 330 d^2 e^5 x^5 - 70 d^3 e^4 x^4 + 525 d^4 e^3 x^3 - 220 d^5 e^2 x^2 - 100 d^6 e x + 40 d^7 + 15 (e^7 x^7 + d e^6 x^6 - 13 d^2 e^5 x^5 + 15 d^3 e^4 x^4 - 13 d^4 e^3 x^3 + 8 d^5 e^2 x^2 - 6 d^6 e x + 4 d^7)}{5 (d^5 e^6 x^7 + d^6 e^5 x^6 - 13 d^7)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3/((-e^2\*x^2 + d^2)^(7/2)\*x^2),x, algorithm="fricas")

[Out] 1/5\*(153\*d\*e^6\*x^6 - 330\*d^2\*e^5\*x^5 - 70\*d^3\*e^4\*x^4 + 525\*d^4\*e^3\*x^3 - 220\*d^5\*e^2\*x^2 - 100\*d^6\*e\*x + 40\*d^7 + 15\*(e^7\*x^7 + d\*e^6\*x^6 - 13\*d^2\*e^5\*x^5 + 15\*d^3\*e^4\*x^4 + 8\*d^4\*e^3\*x^3 - 20\*d^5\*e^2\*x^2 + 8\*d^6\*e\*x - (e^6\*x^6 - 6\*d\*e^5\*x^5 + 5\*d^2\*e^4\*x^4 + 12\*d^3\*e^3\*x^3 - 20\*d^4\*e^2\*x^2 + 8\*d^5\*e\*x)\*sqrt(-e^2\*x^2 + d^2))\*log(-(d - sqrt(-e^2\*x^2 + d^2))/x) - (48\*e^6\*x^6 - 105\*d\*e^5\*x^5 - 165\*d^2\*e^4\*x^4 + 475\*d^3\*e^3\*x^3 - 200\*d^4\*e^2\*x^2 - 100\*d^5\*e\*x + 40\*d^6)\*sqrt(-e^2\*x^2 + d^2))/(d^5\*e^6\*x^7 + d^6\*e^5\*x^6 - 13\*d^7 - 13\*d^7\*e^4\*x^4 + 8\*d^5\*e^3\*x^3 - 20\*d^6\*e^2\*x^2 + 8\*d^7)\*sqrt(-e^2\*x^2 + d^2))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^3}{x^2 (-(-d + ex)(d + ex))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3/x\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2),x)

[Out] Integral((d + e\*x)\*\*3/(x\*\*2\*(-(-d + e\*x)\*(d + e\*x))\*\*(7/2)), x)

**GIAC/XCAS [A]** time = 0.30003, size = 250, normalized size = 1.72

$$\begin{aligned} & -\frac{\sqrt{-x^2e^2 + d^2} \left( \left( \left( x \left( \frac{19xe^6}{d^5} + \frac{15e^5}{d^4} \right) - \frac{45e^4}{d^3} \right) x - \frac{35e^3}{d^2} \right) x + \frac{30e^2}{d} \right) x + 24e}{5(x^2e^2 - d^2)^3} \\ & - \frac{3e \ln \left( \frac{|-2de - 2\sqrt{-x^2e^2 + d^2}e|e^{(-2)}}{2|x|} \right)}{d^5} + \frac{xe^3}{2(de + \sqrt{-x^2e^2 + d^2}e)d^5} - \frac{(de + \sqrt{-x^2e^2 + d^2}e)e^{(-1)}}{2d^5x} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3/((-e^2\*x^2 + d^2)^(7/2)\*x^2),x, algorithm="giac")

[Out] -1/5\*sqrt(-x^2\*e^2 + d^2)\*(((x\*(19\*x\*e^6/d^5 + 15\*e^5/d^4) - 45\*e^4/d^3)\*x - 35\*e^3/d^2)\*x + 30\*e^2/d)\*x + 24\*e)/(x^2\*e^2 - d^2)^3 - 3\*e\*ln(1/2\*abs(-2\*d\*e - 2\*sqrt(-x^2\*e^2 + d^2)\*e)\*e^(-2)/abs(x))/d^5 + 1/2\*x\*e^3/((d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*d^5) - 1/2\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*e^(-1)/(d^5\*x)

$$3.91 \quad \int \frac{(d+ex)^3}{x^3(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=182

$$\frac{4e^2(d+ex)}{5d^2(d^2-e^2x^2)^{5/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{3e\sqrt{d^2-e^2x^2}}{d^6x} \\ - \frac{13e^2 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{e^2(25d+31ex)}{15d^4(d^2-e^2x^2)^{3/2}}$$

[Out]  $(4*e^2*(d+e*x))/(5*d^2*(d^2-e^2*x^2)^(5/2)) + (e^2*(25*d+31*e*x))/(15*d^4*(d^2-e^2*x^2)^(3/2)) + (e^2*(90*d+107*e*x))/(15*d^6*\text{Sqrt}[d^2-e^2*x^2]) - \text{Sqrt}[d^2-e^2*x^2]/(2*d^5*x^2) - (3*e*\text{Sqrt}[d^2-e^2*x^2])/(d^6*x) - (13*e^2*\text{ArcTanh}[\text{Sqrt}[d^2-e^2*x^2]/d])/(2*d^6)$

**Rubi [A]** time = 0.563133, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{4e^2(d+ex)}{5d^2(d^2-e^2x^2)^{5/2}} + \frac{e^2(90d+107ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{3e\sqrt{d^2-e^2x^2}}{d^6x} \\ - \frac{13e^2 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{e^2(25d+31ex)}{15d^4(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d+e*x)^3/(x^3*(d^2-e^2*x^2)^(7/2)),x]$

[Out]  $(4*e^2*(d+e*x))/(5*d^2*(d^2-e^2*x^2)^(5/2)) + (e^2*(25*d+31*e*x))/(15*d^4*(d^2-e^2*x^2)^(3/2)) + (e^2*(90*d+107*e*x))/(15*d^6*\text{Sqrt}[d^2-e^2*x^2]) - \text{Sqrt}[d^2-e^2*x^2]/(2*d^5*x^2) - (3*e*\text{Sqrt}[d^2-e^2*x^2])/(d^6*x) - (13*e^2*\text{ArcTanh}[\text{Sqrt}[d^2-e^2*x^2]/d])/(2*d^6)$

**Rubi in Sympy [A]** time = 65.8072, size = 158, normalized size = 0.87

$$\frac{e^2\sqrt{d^2-e^2x^2}}{5d^4(d-ex)^3} + \frac{17e^2\sqrt{d^2-e^2x^2}}{15d^5(d-ex)^2} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} - \frac{13e^2 \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6} + \frac{107e^2\sqrt{d^2-e^2x^2}}{15d^6(d-ex)} - \frac{3e\sqrt{d^2-e^2x^2}}{d^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)**3/x**3/(-e**2*x**2+d**2)**(7/2),x)$

[Out]  $e**2*\text{sqrt}(d**2-e**2*x**2)/(5*d**4*(d-e*x)**3) + 17*e**2*\text{sqrt}(d**2-e**2*x**2)/(15*d**5*(d-e*x)**2) - \text{sqrt}(d**2-e**2*x**2)/(2*d**5*x**2) - 13*e**2*\text{atanh}(\text{sqrt}(d**2-e**2*x**2)/d)/(2*d**6) + 107*e**2*\text{sqrt}(d**2-e**2*x**2)/(15*d**6*(d-e*x)) - 3*e*\text{sqrt}(d**2-e**2*x**2)/(d**6*x)$

**Mathematica [A]** time = 0.182114, size = 109, normalized size = 0.6

$$\frac{-195e^2 \log\left(\sqrt{d^2-e^2x^2}+d\right) + \frac{\sqrt{d^2-e^2x^2}(15d^4+45d^3ex-479d^2e^2x^2+717de^3x^3-304e^4x^4)}{x^2(ex-d)^3} + 195e^2 \log(x)}{30d^6}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^3/(x^3\*(d^2 - e^2\*x^2)^(7/2)),x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(15\*d^4 + 45\*d^3\*e\*x - 479\*d^2\*e^2\*x^2 + 717\*d\*e^3\*x^3 - 304\*e^4\*x^4))/(x^2\*(-d + e\*x)^3) + 195\*e^2\*Log[x] - 195\*e^2\*Log[d + Sqrt[d^2 - e^2\*x^2]])/(30\*d^6)

**Maple [A]** time = 0.018, size = 222, normalized size = 1.2

$$\begin{aligned} & \frac{19 e^3 x}{5 d^2} (-e^2 x^2 + d^2)^{-\frac{5}{2}} + \frac{76 e^3 x}{15 d^4} (-e^2 x^2 + d^2)^{-\frac{3}{2}} + \frac{152 e^3 x}{15 d^6} \frac{1}{\sqrt{-e^2 x^2 + d^2}} \\ & - \frac{d}{2 x^2} (-e^2 x^2 + d^2)^{-\frac{5}{2}} + \frac{13 e^2}{10 d} (-e^2 x^2 + d^2)^{-\frac{5}{2}} + \frac{13 e^2}{6 d^3} (-e^2 x^2 + d^2)^{-\frac{3}{2}} \\ & + \frac{13 e^2}{2 d^5} \frac{1}{\sqrt{-e^2 x^2 + d^2}} - \frac{13 e^2}{2 d^5} \ln \left( \frac{1}{x} \left( 2 d^2 + 2 \sqrt{d^2} \sqrt{-e^2 x^2 + d^2} \right) \right) \frac{1}{\sqrt{d^2}} - 3 \frac{e}{x (-e^2 x^2 + d^2)^{5/2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3/x^3/(-e^2\*x^2+d^2)^(7/2),x)

[Out] 19/5\*e^3\*x/d^2/(-e^2\*x^2+d^2)^(5/2)+76/15\*e^3/d^4\*x/(-e^2\*x^2+d^2)^(3/2)+152/15\*e^3/d^6\*x/(-e^2\*x^2+d^2)^(1/2)-1/2\*d/x^2/(-e^2\*x^2+d^2)^(5/2)+13/10/d\*e^2/(-e^2\*x^2+d^2)^(5/2)+13/6/d^3\*e^2/(-e^2\*x^2+d^2)^(3/2)+13/2/d^5\*e^2/(-e^2\*x^2+d^2)^(1/2)-13/2/d^5\*e^2/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)-3\*e/x/(-e^2\*x^2+d^2)^(5/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3/((-e^2\*x^2 + d^2)^(7/2)\*x^3),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.301746, size = 765, normalized size = 4.2

558 e^9 x^9 - 975 d e^8 x^8 - 4776 d^2 e^7 x^7 + 9880 d^3 e^6 x^6 + 2540 d^4 e^5 x^5 - 13215 d^5 e^4 x^4 + 3540 d^6 e^3 x^3 + 3540 d^7 e^2 x^2 - 840 d^8 e x -

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3/((-e^2\*x^2 + d^2)^(7/2)\*x^3),x, algorithm="fricas")

[Out] 1/30\*(558\*e^9\*x^9 - 975\*d\*e^8\*x^8 - 4776\*d^2\*e^7\*x^7 + 9880\*d^3\*e^6\*x^6 + 2540\*d^4\*e^5\*x^5 - 13215\*d^5\*e^4\*x^4 + 3540\*d^6\*e^3\*x^3 + 3540\*d^7\*e^2\*x^2 - 840\*d^8\*e\*x - 240\*d^9 + 195\*(e^9\*x^9 - 7\*d\*e^8\*x^8 + 3\*d^2\*e^7\*x^7 + 31\*d^3\*e^6\*x^6 - 40\*d^4\*e^5\*x^5 - 12\*d^5\*e^4\*x^4 + 40\*d^6\*e^3\*x^3 - 16\*d^7\*e^2\*x^2 + (e^8\*x^8 + 2\*d\*e^7\*x^7 - 19\*d^2\*e^6\*x^6 + 20\*d^3\*e^5\*x^5 + 20\*d^4\*e^4\*x^4 - 40\*d^5\*e^3\*x^3 + 16\*d^6\*e^2\*x^2)\*sqrt(-e^2\*x^2 + d^2))\*log(-(d - sqrt(-e^2

$(x^2 + d^2)/x) - (50e^8x^8 - 2441d^7e^7x^7 + 4525d^2e^6x^6 + 3995d^3e^5x^5 - 11535d^4e^4x^4 + 3120d^5e^3x^3 + 3420d^6e^2x^2 - 840d^7e^1x - 240d^8) \sqrt{-e^2x^2 + d^2} / (d^6e^7x^9 - 7d^7e^6x^8 + 3d^8e^5x^7 + 31d^9e^4x^6 - 40d^10e^3x^5 - 12d^11e^2x^4 + 40d^12e^1x^3 - 16d^13x^2 + (d^6e^6x^8 + 2d^7e^5x^7 - 19d^8e^4x^6 + 20d^9e^3x^5 + 20d^10e^2x^4 - 40d^11e^1x^3 + 16d^12x^2) \sqrt{-e^2x^2 + d^2})$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^3}{x^3 (-(-d + ex)(d + ex))^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3/x\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2),x)

[Out] Integral((d + e\*x)\*\*3/(x\*\*3\*(-(-d + e\*x)\*(d + e\*x))\*\*(7/2)), x)

**GIAC/XCAS [A]** time = 0.310203, size = 350, normalized size = 1.92

$$\frac{\sqrt{-x^2e^2 + d^2} \left( \left( \left( x \left( \frac{107xe^7}{d^6} + \frac{90e^6}{d^5} \right) - \frac{245e^5}{d^4} \right) x - \frac{205e^4}{d^3} \right) x + \frac{150e^3}{d^2} \right) x + \frac{127e^2}{d}}{15(x^2e^2 - d^2)^3} - \frac{13e^2 \ln \left( \frac{|-2de - 2\sqrt{-x^2e^2 + d^2}e|e^{(-2)}}{2|x|} \right)}{2d^6} + \frac{x^2 \left( \frac{12(de + \sqrt{-x^2e^2 + d^2}e)e^4}{x} + e^6 \right)}{8(de + \sqrt{-x^2e^2 + d^2}e)^2 d^6} - \frac{\left( \frac{12(de + \sqrt{-x^2e^2 + d^2}e)d^6e^8}{x} + \frac{(de + \sqrt{-x^2e^2 + d^2}e)^2 d^6e^6}{x^2} \right) e^{(-8)}}{8d^{12}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3/((-e^2\*x^2 + d^2)^(7/2)\*x^3),x, algorithm="giac")

[Out]  $-1/15 \sqrt{-x^2e^2 + d^2} \left( \left( \left( x \left( \frac{107xe^7}{d^6} + \frac{90e^6}{d^5} \right) - \frac{245e^5}{d^4} \right) x - \frac{205e^4}{d^3} \right) x + \frac{150e^3}{d^2} \right) x + \frac{127e^2}{d} / (x^2e^2 - d^2)^3 - 13/2 e^2 \ln(1/2 \operatorname{abs}(-2de - 2\sqrt{-x^2e^2 + d^2}e) \operatorname{abs}(x)) / d^6 + 1/8 x^2 (12(d^6e^8 + (de + \sqrt{-x^2e^2 + d^2}e)^2 d^6e^6) / x + e^6) / ((de + \sqrt{-x^2e^2 + d^2}e)^2 d^6) - 1/8 (12(d^6e^8 + (de + \sqrt{-x^2e^2 + d^2}e)^2 d^6e^6) / x + (de + \sqrt{-x^2e^2 + d^2}e)^2 d^6e^6) / d^{12}$

$$3.92 \quad \int \frac{x^4 \sqrt{d^2 - e^2 x^2}}{d + ex} dx$$

**Optimal.** Leaf size=147

$$\frac{x^4 \sqrt{d^2 - e^2 x^2}}{5e} - \frac{dx^3 \sqrt{d^2 - e^2 x^2}}{4e^2} + \frac{4d^2 x^2 \sqrt{d^2 - e^2 x^2}}{15e^3} + \frac{3d^5 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{8e^5} + \frac{d^3(64d - 45ex)\sqrt{d^2 - e^2 x^2}}{120e^5}$$

[Out] (4\*d^2\*x^2\*Sqrt[d^2 - e^2\*x^2])/(15\*e^3) - (d\*x^3\*Sqrt[d^2 - e^2\*x^2])/(4\*e^2) + (x^4\*Sqrt[d^2 - e^2\*x^2])/(5\*e) + (d^3\*(64\*d - 45\*e\*x)\*Sqrt[d^2 - e^2\*x^2])/(120\*e^5) + (3\*d^5\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(8\*e^5)

**Rubi [A]** time = 0.472769, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\frac{x^4 \sqrt{d^2 - e^2 x^2}}{5e} - \frac{dx^3 \sqrt{d^2 - e^2 x^2}}{4e^2} + \frac{4d^2 x^2 \sqrt{d^2 - e^2 x^2}}{15e^3} + \frac{3d^5 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{8e^5} + \frac{d^3(64d - 45ex)\sqrt{d^2 - e^2 x^2}}{120e^5}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*Sqrt[d^2 - e^2\*x^2])/(d + e\*x), x]

[Out] (4\*d^2\*x^2\*Sqrt[d^2 - e^2\*x^2])/(15\*e^3) - (d\*x^3\*Sqrt[d^2 - e^2\*x^2])/(4\*e^2) + (x^4\*Sqrt[d^2 - e^2\*x^2])/(5\*e) + (d^3\*(64\*d - 45\*e\*x)\*Sqrt[d^2 - e^2\*x^2])/(120\*e^5) + (3\*d^5\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(8\*e^5)

**Rubi in Sympy [A]** time = 52.0239, size = 129, normalized size = 0.88

$$\frac{3d^5 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{8e^5} + \frac{d^3(64d - 45ex)\sqrt{d^2 - e^2 x^2}}{120e^5} + \frac{4d^2 x^2 \sqrt{d^2 - e^2 x^2}}{15e^3} - \frac{dx^3 \sqrt{d^2 - e^2 x^2}}{4e^2} + \frac{x^4 \sqrt{d^2 - e^2 x^2}}{5e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2)/(e\*x+d), x)

[Out] 3\*d\*\*5\*atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2))/(8\*e\*\*5) + d\*\*3\*(64\*d - 45\*e\*x)\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(120\*e\*\*5) + 4\*d\*\*2\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*e\*\*3) - d\*x\*\*3\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(4\*e\*\*2) + x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(5\*e)

**Mathematica [A]** time = 0.100923, size = 91, normalized size = 0.62

$$\frac{45d^5 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right) + \sqrt{d^2 - e^2 x^2} (64d^4 - 45d^3 ex + 32d^2 e^2 x^2 - 30de^3 x^3 + 24e^4 x^4)}{120e^5}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4\*Sqrt[d^2 - e^2\*x^2])/(d + e\*x), x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(64\*d^4 - 45\*d^3\*e\*x + 32\*d^2\*e^2\*x^2 - 30\*d\*e^3\*x^3 + 24\*e^4\*x^4) + 45\*d^5\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])

)/(120\*e^5)

**Maple [A]** time = 0.028, size = 208, normalized size = 1.4

$$\begin{aligned}
 & -\frac{x^2}{5e^3}(-e^2x^2+d^2)^{\frac{3}{2}} - \frac{7d^2}{15e^5}(-e^2x^2+d^2)^{\frac{3}{2}} + \frac{d^4}{e^5}\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)} \\
 & + \frac{d^5}{e^4}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)\frac{1}{\sqrt{e^2}} - \frac{5d^3x}{8e^4}\sqrt{-e^2x^2+d^2} \\
 & - \frac{5d^5}{8e^4}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}} + \frac{dx}{4e^4}(-e^2x^2+d^2)^{\frac{3}{2}}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(-e^2\*x^2+d^2)^(1/2)/(e\*x+d),x)

[Out] -1/5/e^3\*x^2\*(-e^2\*x^2+d^2)^(3/2)-7/15\*d^2/e^5\*(-e^2\*x^2+d^2)^(3/2)+d^4/e^5\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)+d^5/e^4/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))-5/8\*d^3/e^4\*x\*(-e^2\*x^2+d^2)^(1/2)-5/8\*d^5/e^4/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(e^2\*x^2+d^2)^(1/2))+1/4\*d/e^4\*x\*(-e^2\*x^2+d^2)^(3/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)\*x^4/(e\*x + d),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.292162, size = 450, normalized size = 3.06

$$\frac{24e^{10}x^{10} - 30de^9x^9 - 280d^2e^8x^8 + 345d^3e^7x^7 + 320d^4e^6x^6 - 255d^5e^5x^5 - 780d^7e^3x^3 + 720d^9ex - 90(5d^6e^4x^4 - 20d^8e^2x^2)}{120(5de^9x^4 - 20d^8e^2x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)\*x^4/(e\*x + d),x, algorithm="fricas")

[Out] 1/120\*(24\*e^10\*x^10 - 30\*d\*e^9\*x^9 - 280\*d^2\*e^8\*x^8 + 345\*d^3\*e^7\*x^7 + 320\*d^4\*e^6\*x^6 - 255\*d^5\*e^5\*x^5 - 780\*d^7\*e^3\*x^3 + 720\*d^9\*e\*x - 90\*(5\*d^6\*e^4\*x^4 - 20\*d^8\*e^2\*x^2 + 16\*d^10 - (d^5\*e^4\*x^4 - 12\*d^7\*e^2\*x^2 + 16\*d^9)\*sqrt(-e^2\*x^2 + d^2)))\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) + 5\*(24\*d\*e^8\*x^8 - 30\*d^2\*e^7\*x^7 - 64\*d^3\*e^6\*x^6 + 75\*d^4\*e^5\*x^5 + 84\*d^6\*e^3\*x^3 - 144\*d^8\*e\*x)\*sqrt(-e^2\*x^2 + d^2)/(5\*d\*e^9\*x^4 - 20\*d^3\*e^7\*x^2 + 16\*d^5\*e^5 - (e^9\*x^4 - 12\*d^2\*e^7\*x^2 + 16\*d^4\*e^5)\*sqrt(-e^2\*x^2 + d^2))



**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \sqrt{-(-d + ex)(d + ex)}}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2)/(e\*x+d), x)

[Out] Integral(x\*\*4\*sqrt(-(-d + e\*x)\*(d + e\*x))/(d + e\*x), x)

**GIAC/XCAS [A]** time = 0.297028, size = 104, normalized size = 0.71

$$\frac{3}{8} d^5 \arcsin\left(\frac{xe}{d}\right) e^{(-5)} \text{sign}(d) + \frac{1}{120} \left( 64 d^4 e^{(-5)} - \left( 45 d^3 e^{(-4)} - 2 \left( 16 d^2 e^{(-3)} + 3 \left( 4 x e^{(-1)} - 5 d e^{(-2)} \right) x \right) x \right) \sqrt{-x^2 e^2 + d^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)\*x^4/(e\*x + d), x, algorithm="giac")

[Out] 3/8\*d^5\*arcsin(x\*e/d)\*e^(-5)\*sign(d) + 1/120\*(64\*d^4\*e^(-5) - (45\*d^3\*e^(-4) - 2\*(16\*d^2\*e^(-3) + 3\*(4\*x\*e^(-1) - 5\*d\*e^(-2))\*x)\*x)\*sqrt(-x^2\*e^2 + d^2)

$$3.93 \quad \int \frac{x^3 \sqrt{d^2 - e^2 x^2}}{d + ex} dx$$

**Optimal.** Leaf size=118

$$-\frac{dx^2 \sqrt{d^2 - e^2 x^2}}{3e^2} + \frac{x^3 \sqrt{d^2 - e^2 x^2}}{4e} - \frac{d^2(16d - 9ex) \sqrt{d^2 - e^2 x^2}}{24e^4} - \frac{3d^4 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{8e^4}$$

[Out]  $-(d*x^2*\text{Sqrt}[d^2 - e^2*x^2])/(3*e^2) + (x^3*\text{Sqrt}[d^2 - e^2*x^2])/(4*e) - (d^2*(16*d - 9*e*x)*\text{Sqrt}[d^2 - e^2*x^2])/(24*e^4) - (3*d^4*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(8*e^4)$

**Rubi [A]** time = 0.351921, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$-\frac{dx^2 \sqrt{d^2 - e^2 x^2}}{3e^2} + \frac{x^3 \sqrt{d^2 - e^2 x^2}}{4e} - \frac{d^2(16d - 9ex) \sqrt{d^2 - e^2 x^2}}{24e^4} - \frac{3d^4 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{8e^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^3*\text{Sqrt}[d^2 - e^2*x^2])/(d + e*x), x]$

[Out]  $-(d*x^2*\text{Sqrt}[d^2 - e^2*x^2])/(3*e^2) + (x^3*\text{Sqrt}[d^2 - e^2*x^2])/(4*e) - (d^2*(16*d - 9*e*x)*\text{Sqrt}[d^2 - e^2*x^2])/(24*e^4) - (3*d^4*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(8*e^4)$

**Rubi in Sympy [A]** time = 40.4693, size = 102, normalized size = 0.86

$$-\frac{3d^4 \text{atan}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{8e^4} - \frac{d^2(16d - 9ex) \sqrt{d^2 - e^2 x^2}}{24e^4} - \frac{dx^2 \sqrt{d^2 - e^2 x^2}}{3e^2} + \frac{x^3 \sqrt{d^2 - e^2 x^2}}{4e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**3}*(-e^{**2}*x^{**2}+d^{**2})^{**}(1/2)/(e*x+d), x)$

[Out]  $-3*d^{**4}*\text{atan}(e*x/\text{sqrt}(d^{**2} - e^{**2}*x^{**2}))/ (8*e^{**4}) - d^{**2}*(16*d - 9*e*x)*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(24*e^{**4}) - d*x^{**2}*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(3*e^{**2}) + x^{**3}*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(4*e)$

**Mathematica [A]** time = 0.0789366, size = 80, normalized size = 0.68

$$\frac{\sqrt{d^2 - e^2 x^2} (-16d^3 + 9d^2 ex - 8de^2 x^2 + 6e^3 x^3) - 9d^4 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{24e^4}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(x^3*\text{Sqrt}[d^2 - e^2*x^2])/(d + e*x), x]$

[Out]  $(\text{Sqrt}[d^2 - e^2*x^2]*(-16*d^3 + 9*d^2*e*x - 8*d*e^2*x^2 + 6*e^3*x^3) - 9*d^4*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(24*e^4)$

**Maple [A]** time = 0.015, size = 185, normalized size = 1.6

$$-\frac{x}{4e^3}(-e^2x^2+d^2)^{\frac{3}{2}}+\frac{5d^2x}{8e^3}\sqrt{-e^2x^2+d^2}+\frac{5d^4}{8e^3}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}}$$

$$+\frac{d}{3e^4}(-e^2x^2+d^2)^{\frac{3}{2}}-\frac{d^3}{e^4}\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}$$

$$-\frac{d^4}{e^3}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)\frac{1}{\sqrt{e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x)`

[Out] `-1/4/e^3*x*(-e^2*x^2+d^2)^(3/2)+5/8*d^2/e^3*x*(-e^2*x^2+d^2)^(1/2)+5/8/e^3*d^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+1/3*d/e^4*(-e^2*x^2+d^2)^(3/2)-d^3/e^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-d^4/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2+d^2)*x^3/(e*x+d),x,algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.28909, size = 387, normalized size = 3.28

$$\frac{24de^7x^7-32d^2e^6x^6-36d^3e^5x^5+48d^4e^4x^4-60d^5e^3x^3+72d^7ex-18(d^4e^4x^4-8d^6e^2x^2+8d^8+4(d^5e^2x^2-2d^7)\sqrt{-e^2x^2+d^2})}{24(e^8x^4-8d^2e^6x^2+8d^4e^4+4(d^5e^2x^2-2d^7)\sqrt{-e^2x^2+d^2})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2+d^2)*x^3/(e*x+d),x,algorithm="fricas")`

[Out] `-1/24*(24*d*e^7*x^7-32*d^2*e^6*x^6-36*d^3*e^5*x^5+48*d^4*e^4*x^4-60*d^5*e^3*x^3+72*d^7*e*x-18*(d^4*e^4*x^4-8*d^6*e^2*x^2+8*d^8+4*(d^5*e^2*x^2-2*d^7)*sqrt(-e^2*x^2+d^2))*arctan(-(d-sqrt(-e^2*x^2+d^2))/(e*x))-((6*e^7*x^7-8*d*e^6*x^6-39*d^2*e^5*x^5+48*d^3*e^4*x^4-24*d^4*e^3*x^3+72*d^6*e*x)*sqrt(-e^2*x^2+d^2))/(e^8*x^4-8*d^2*e^6*x^2+8*d^4*e^4+4*(d*e^6*x^2-2*d^3*e^4)*sqrt(-e^2*x^2+d^2))`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3\sqrt{-(-d+ex)(d+ex)}}{d+ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)`

[Out] `Integral(x**3*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)`

**GIAC/XCAS [A]** time = 0.294434, size = 89, normalized size = 0.75

$$-\frac{3}{8}d^4 \arcsin\left(\frac{xe}{d}\right) e^{(-4)} \operatorname{sign}(d) - \frac{1}{24} \left(16d^3 e^{(-4)} - \left(9d^2 e^{(-3)} + 2\left(3xe^{(-1)} - 4de^{(-2)}\right)x\right)x\right) \sqrt{-x^2 e^2 + d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2 + d^2)*x^3/(e*x + d),x, algorithm="giac")`

[Out] `-3/8*d^4*arcsin(x*e/d)*e^(-4)*sign(d) - 1/24*(16*d^3*e^(-4) - (9*d^2*e^(-3) + 2*(3*x*e^(-1) - 4*d*e^(-2))*x)*x)*sqrt(-x^2*e^2 + d^2)`

$$3.94 \quad \int \frac{x^2 \sqrt{d^2 - e^2 x^2}}{d + ex} dx$$

**Optimal.** Leaf size=86

$$\frac{d(2d - ex)\sqrt{d^2 - e^2 x^2}}{2e^3} - \frac{(d^2 - e^2 x^2)^{3/2}}{3e^3} + \frac{d^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{2e^3}$$

[Out] (d\*(2\*d - e\*x)\*Sqrt[d^2 - e^2\*x^2])/(2\*e^3) - (d^2 - e^2\*x^2)^(3/2)/(3\*e^3) + (d^3\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(2\*e^3)

**Rubi [A]** time = 0.253148, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{d(2d - ex)\sqrt{d^2 - e^2 x^2}}{2e^3} - \frac{(d^2 - e^2 x^2)^{3/2}}{3e^3} + \frac{d^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{2e^3}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*Sqrt[d^2 - e^2\*x^2])/(d + e\*x), x]

[Out] (d\*(2\*d - e\*x)\*Sqrt[d^2 - e^2\*x^2])/(2\*e^3) - (d^2 - e^2\*x^2)^(3/2)/(3\*e^3) + (d^3\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(2\*e^3)

**Rubi in Sympy [A]** time = 29.5293, size = 87, normalized size = 1.01

$$\frac{d^3 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{2e^3} + \frac{d^2 \sqrt{d^2 - e^2 x^2}}{e^3} - \frac{dx \sqrt{d^2 - e^2 x^2}}{2e^2} - \frac{(d^2 - e^2 x^2)^{\frac{3}{2}}}{3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2)/(e\*x+d), x)

[Out] d\*\*3\*atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2))/(2\*e\*\*3) + d\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/e\*\*3 - d\*x\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(2\*e\*\*2) - (d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(3\*e\*\*3)

**Mathematica [A]** time = 0.0665955, size = 69, normalized size = 0.8

$$\frac{\sqrt{d^2 - e^2 x^2} (4d^2 - 3dex + 2e^2 x^2) + 3d^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{6e^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*Sqrt[d^2 - e^2\*x^2])/(d + e\*x), x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(4\*d^2 - 3\*d\*e\*x + 2\*e^2\*x^2) + 3\*d^3\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(6\*e^3)

**Maple [B]** time = 0.016, size = 160, normalized size = 1.9

$$-\frac{1}{3e^3}(-e^2x^2+d^2)^{\frac{3}{2}}-\frac{dx}{2e^2}\sqrt{-e^2x^2+d^2}-\frac{d^3}{2e^2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}} \\ +\frac{d^2}{e^3}\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}+\frac{d^3}{e^2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)\frac{1}{\sqrt{e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x)`

[Out] `-1/3*(-e^2*x^2+d^2)^(3/2)/e^3-1/2*d*x*(-e^2*x^2+d^2)^(1/2)/e^2-1/2*d^3/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+d^2/e^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+d^3/e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2+d^2)*x^2/(e*x+d),x,algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.283985, size = 301, normalized size = 3.5

$$\frac{2e^6x^6-3de^5x^5-6d^2e^4x^4+15d^3e^3x^3-12d^5ex-6\left(3d^4e^2x^2-4d^6-(d^3e^2x^2-4d^5)\sqrt{-e^2x^2+d^2}\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)}{6\left(3de^5x^2-4d^3e^3-(e^5x^2-4d^2e^3)\sqrt{-e^2x^2+d^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2+d^2)*x^2/(e*x+d),x,algorithm="fricas")`

[Out] `1/6*(2*e^6*x^6-3*d*e^5*x^5-6*d^2*e^4*x^4+15*d^3*e^3*x^3-12*d^5*e*x-6*(3*d^4*e^2*x^2-4*d^6-(d^3*e^2*x^2-4*d^5)*sqrt(-e^2*x^2+d^2))*arctan(-(d-sqrt(-e^2*x^2+d^2))/(e*x))+3*(2*d*e^4*x^4-3*d^2*e^3*x^3+4*d^4*e*x)*sqrt(-e^2*x^2+d^2))/(3*d*e^5*x^2-4*d^3*e^3-(e^5*x^2-4*d^2*e^3)*sqrt(-e^2*x^2+d^2))`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2\sqrt{-(-d+ex)(d+ex)}}{d+ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)`

[Out] `Integral(x**2*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)`

---

**GIAC/XCAS [A]** time = 0.28904, size = 73, normalized size = 0.85

$$\frac{1}{2}d^3 \arcsin\left(\frac{xe}{d}\right) e^{(-3)} \operatorname{sign}(d) + \frac{1}{6} \sqrt{-x^2e^2 + d^2} \left(4d^2e^{(-3)} + (2xe^{(-1)} - 3de^{(-2)})x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2 + d^2)*x^2/(e*x + d),x, algorithm="giac")`

[Out] `1/2*d^3*arcsin(x*e/d)*e^(-3)*sign(d) + 1/6*sqrt(-x^2*e^2 + d^2)*(4*d^2*e^(-3) + (2*x*e^(-1) - 3*d*e^(-2))*x)`

$$3.95 \quad \int \frac{x\sqrt{d^2 - e^2x^2}}{d + ex} dx$$

**Optimal.** Leaf size=62

$$-\frac{(2d - ex)\sqrt{d^2 - e^2x^2}}{2e^2} - \frac{d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{2e^2}$$

[Out]  $-\frac{(2d - ex)\sqrt{d^2 - e^2x^2}}{2e^2} - \frac{d^2 \text{ArcTan}\left[\frac{ex}{\sqrt{d^2 - e^2x^2}}\right]}{2e^2}$

**Rubi [A]** time = 0.135751, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$-\frac{(2d - ex)\sqrt{d^2 - e^2x^2}}{2e^2} - \frac{d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{2e^2}$$

Antiderivative was successfully verified.

[In] Int[(x\*Sqrt[d^2 - e^2\*x^2])/(d + e\*x), x]

[Out]  $-\frac{(2d - ex)\sqrt{d^2 - e^2x^2}}{2e^2} - \frac{d^2 \text{ArcTan}\left[\frac{ex}{\sqrt{d^2 - e^2x^2}}\right]}{2e^2}$

**Rubi in Sympy [A]** time = 16.8796, size = 71, normalized size = 1.15

$$-\frac{d^2 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{2e^2} - \frac{d\sqrt{d^2 - e^2x^2}}{2e^2} - \frac{(d^2 - e^2x^2)^{\frac{3}{2}}}{2e^2(d + ex)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2)/(e\*x+d), x)

[Out]  $-\frac{d^2 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{2e^2} - \frac{d\sqrt{d^2 - e^2x^2}}{2e^2} - \frac{(d^2 - e^2x^2)^{\frac{3}{2}}}{2e^2(d + ex)}$

**Mathematica [A]** time = 0.0528263, size = 57, normalized size = 0.92

$$\frac{(ex - 2d)\sqrt{d^2 - e^2x^2} - d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{2e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*Sqrt[d^2 - e^2\*x^2])/(d + e\*x), x]

[Out]  $\frac{(-2d + ex)\sqrt{d^2 - e^2x^2} - d^2 \text{ArcTan}\left[\frac{ex}{\sqrt{d^2 - e^2x^2}}\right]}{2e^2}$



**Maple [B]** time = 0.011, size = 140, normalized size = 2.3

$$\frac{x}{2e} \sqrt{-e^2 x^2 + d^2} + \frac{d^2}{2e} \arctan\left(x \sqrt{e^2} \frac{1}{\sqrt{-e^2 x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} - \frac{d}{e^2} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right)} - \frac{d^2}{e} \arctan\left(x \sqrt{e^2} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right)}}\right) \frac{1}{\sqrt{e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d),x)`

[Out] `1/2/e*x*(-e^2*x^2+d^2)^(1/2)+1/2/e*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-d/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-d^2/e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2 + d^2)*x/(e*x + d),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.289408, size = 232, normalized size = 3.74

$$\frac{2de^3x^3 - 2d^2e^2x^2 - 2d^3ex - 2\left(d^2e^2x^2 - 2d^4 + 2\sqrt{-e^2x^2 + d^2}d^3\right) \arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - (e^3x^3 - 2de^2x^2 - 2d^2ex) \sqrt{-e^2x^2 + d^2}}{2\left(e^4x^2 - 2d^2e^2 + 2\sqrt{-e^2x^2 + d^2}de^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2 + d^2)*x/(e*x + d),x, algorithm="fricas")`

[Out] `-1/2*(2*d*e^3*x^3 - 2*d^2*e^2*x^2 - 2*d^3*e*x - 2*(d^2*e^2*x^2 - 2*d^4 + 2*sqrt(-e^2*x^2 + d^2)*d^3)*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (e^3*x^3 - 2*d^2*e^2*x^2 - 2*d^2*e*x)*sqrt(-e^2*x^2 + d^2))/(e^4*x^2 - 2*d^2*e^2 + 2*sqrt(-e^2*x^2 + d^2)*d*e^2)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sqrt{-(-d + ex)(d + ex)}}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(-e**2*x**2+d**2)**(1/2)/(e*x+d),x)`

[Out] `Integral(x*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)`

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**GIAC/XCAS [A]** time = 0.293226, size = 58, normalized size = 0.94

$$-\frac{1}{2} d^2 \arcsin\left(\frac{xe}{d}\right) e^{(-2)} \operatorname{sign}(d) + \frac{1}{2} \sqrt{-x^2 e^2 + d^2} (xe^{(-1)} - 2 de^{(-2)})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)\*x/(e\*x + d),x, algorithm="giac")

[Out] -1/2\*d^2\*arcsin(x\*e/d)\*e^(-2)\*sign(d) + 1/2\*sqrt(-x^2\*e^2 + d^2)\*(x\*e^(-1) - 2\*d\*e^(-2))

$$3.96 \quad \int \frac{\sqrt{d^2 - e^2 x^2}}{d + ex} dx$$

**Optimal.** Leaf size=46

$$\frac{\sqrt{d^2 - e^2 x^2}}{e} + \frac{d \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{e}$$

[Out] Sqrt[d^2 - e^2\*x^2]/e + (d\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/e

**Rubi [A]** time = 0.0446184, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{\sqrt{d^2 - e^2 x^2}}{e} + \frac{d \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{e}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d^2 - e^2\*x^2]/(d + e\*x), x]

[Out] Sqrt[d^2 - e^2\*x^2]/e + (d\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/e

**Rubi in Sympy [A]** time = 12.3079, size = 36, normalized size = 0.78

$$\frac{d \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{e} + \frac{\sqrt{d^2 - e^2 x^2}}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2)/(e\*x+d), x)

[Out] d\*atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2))/e + sqrt(d\*\*2 - e\*\*2\*x\*\*2)/e

**Mathematica [A]** time = 0.0272258, size = 43, normalized size = 0.93

$$\frac{\sqrt{d^2 - e^2 x^2} + d \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{e}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d^2 - e^2\*x^2]/(d + e\*x), x]

[Out] (Sqrt[d^2 - e^2\*x^2] + d\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/e

**Maple [A]** time = 0.005, size = 77, normalized size = 1.7

$$\frac{1}{e} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)} + d \arctan\left(x \sqrt{e^2} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)}}\right) \frac{1}{\sqrt{e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-e^2*x^2+d^2)^(1/2)/(e*x+d),x)`

[Out]  $1/e * (- (x+d/e)^2 * e^2 + 2 * d * e * (x+d/e))^{1/2} + d / (e^2)^{1/2} * \arctan((e^2)^{1/2} * x / (- (x+d/e)^2 * e^2 + 2 * d * e * (x+d/e))^{1/2})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2 + d^2)/(e*x + d),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.28337, size = 111, normalized size = 2.41

$$\frac{e^2 x^2 - 2 \left( d^2 - \sqrt{-e^2 x^2 + d^2} d \right) \arctan \left( -\frac{d - \sqrt{-e^2 x^2 + d^2}}{e x} \right)}{d e - \sqrt{-e^2 x^2 + d^2} e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2 + d^2)/(e*x + d),x, algorithm="fricas")`

[Out]  $(e^2 * x^2 - 2 * (d^2 - \sqrt{-e^2 * x^2 + d^2} * d) * \arctan(-(d - \sqrt{-e^2 * x^2 + d^2}) / (e * x))) / (d * e - \sqrt{-e^2 * x^2 + d^2} * e)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-d + ex)(d + ex)}}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e**2*x**2+d**2)**(1/2)/(e*x+d),x)`

[Out] `Integral(sqrt(-(-d + e*x)*(d + e*x))/(d + e*x), x)`

**GIAC/XCAS [A]** time = 0.2846, size = 42, normalized size = 0.91

$$d \arcsin \left( \frac{x e}{d} \right) e^{(-1)} \operatorname{sign}(d) + \sqrt{-x^2 e^2 + d^2} e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2 + d^2)/(e*x + d),x, algorithm="giac")`

[Out]  $d * \arcsin(x * e / d) * e^{(-1)} * \operatorname{sign}(d) + \sqrt{-x^2 * e^2 + d^2} * e^{(-1)}$

$$3.97 \quad \int \frac{\sqrt{d^2 - e^2 x^2}}{x(d + ex)} dx$$

**Optimal.** Leaf size=46

$$-\tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right) - \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)$$

[Out] -ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] - ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]

**Rubi [A]** time = 0.190585, antiderivative size = 46, normalized size of antiderivative = 1., number of rules used = 7, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$-\tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right) - \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d^2 - e^2\*x^2]/(x\*(d + e\*x)),x]

[Out] -ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] - ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]

**Rubi in Sympy [A]** time = 24.1138, size = 36, normalized size = 0.78

$$-\operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right) - \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2)/x/(e\*x+d),x)

[Out] -atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2)) - atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d)

**Mathematica [A]** time = 0.0242314, size = 46, normalized size = 1.

$$-\log\left(\sqrt{d^2 - e^2 x^2} + d\right) - \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right) + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d^2 - e^2\*x^2]/(x\*(d + e\*x)),x]

[Out] -ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] + Log[x] - Log[d + Sqrt[d^2 - e^2\*x^2]]

**Maple [B]** time = 0.016, size = 137, normalized size = 3.

$$\frac{1}{d}\sqrt{-e^2x^2+d^2}-d\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}-\frac{1}{d}\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}-e\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)\frac{1}{\sqrt{e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-e^2*x^2+d^2)^(1/2)/x/(e*x+d), x)`

[Out] `(-e^2*x^2+d^2)^(1/2)/d-d/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/d*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2+d^2)/((e*x+d)*x), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.289655, size = 73, normalized size = 1.59

$$2\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)+\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2+d^2)/((e*x+d)*x), x, algorithm="fricas")`

[Out] `2*arctan(-(d-sqrt(-e^2*x^2+d^2))/(e*x))+log(-(d-sqrt(-e^2*x^2+d^2))/x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-d+ex)(d+ex)}}{x(d+ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e**2*x**2+d**2)**(1/2)/x/(e*x+d), x)`

[Out] `Integral(sqrt(-(-d+e*x)*(d+e*x))/(x*(d+e*x)), x)`

**GIAC/XCAS [A]** time = 0.28776, size = 65, normalized size = 1.41

$$-\arcsin\left(\frac{xe}{d}\right) \operatorname{sign}(d) - \ln\left(\frac{\left|-2de - 2\sqrt{-x^2e^2 + d^2}e\right|e^{(-2)}}{2|x|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)/((e\*x + d)\*x),x, algorithm="giac")

[Out] -arcsin(x\*e/d)\*sign(d) - ln(1/2\*abs(-2\*d\*e - 2\*sqrt(-x^2\*e^2 + d^2)\*e)\*e^(-2)/abs(x))

$$3.98 \quad \int \frac{\sqrt{d^2 - e^2 x^2}}{x^2(d + ex)} dx$$

**Optimal.** Leaf size=51

$$\frac{e \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{d} - \frac{\sqrt{d^2 - e^2 x^2}}{dx}$$

[Out]  $-(\text{Sqrt}[d^2 - e^2 x^2]/(d * x)) + (e * \text{ArcTanh}[\text{Sqrt}[d^2 - e^2 x^2]/d])/d$

**Rubi [A]** time = 0.185852, antiderivative size = 51, normalized size of antiderivative = 1., number of rules used = 5, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\frac{e \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{d} - \frac{\sqrt{d^2 - e^2 x^2}}{dx}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[d^2 - e^2 x^2]/(x^2 * (d + e * x)), x]$

[Out]  $-(\text{Sqrt}[d^2 - e^2 x^2]/(d * x)) + (e * \text{ArcTanh}[\text{Sqrt}[d^2 - e^2 x^2]/d])/d$

**Rubi in Sympy [A]** time = 16.3632, size = 36, normalized size = 0.71

$$\frac{e \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{d} - \frac{\sqrt{d^2 - e^2 x^2}}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((-e^{**2} * x^{**2} + d^{**2})^{**}(1/2)/x^{**2}/(e * x + d), x)$

[Out]  $e * \operatorname{atanh}(\text{sqrt}(d^{**2} - e^{**2} * x^{**2})/d)/d - \text{sqrt}(d^{**2} - e^{**2} * x^{**2})/(d * x)$

**Mathematica [A]** time = 0.0440418, size = 53, normalized size = 1.04

$$\frac{\sqrt{d^2 - e^2 x^2} - ex \log\left(\sqrt{d^2 - e^2 x^2} + d\right) + ex \log(x)}{dx}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[\text{Sqrt}[d^2 - e^2 x^2]/(x^2 * (d + e * x)), x]$

[Out]  $-((\text{Sqrt}[d^2 - e^2 x^2] + e * x * \text{Log}[x] - e * x * \text{Log}[d + \text{Sqrt}[d^2 - e^2 x^2]]))/(d * x)$



**Maple [B]** time = 0.02, size = 222, normalized size = 4.4

$$\begin{aligned}
 & -\frac{1}{d^3 x} (-e^2 x^2 + d^2)^{\frac{3}{2}} - \frac{e^2 x}{d^3} \sqrt{-e^2 x^2 + d^2} - \frac{e^2}{d} \arctan\left(x \sqrt{e^2} \frac{1}{\sqrt{-e^2 x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} \\
 & + \frac{e}{d^2} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)} + \frac{e^2}{d} \arctan\left(x \sqrt{e^2} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)}}\right) \frac{1}{\sqrt{e^2}} \\
 & - \frac{e}{d^2} \sqrt{-e^2 x^2 + d^2} + e \ln\left(\frac{1}{x} \left(2 d^2 + 2 \sqrt{d^2} \sqrt{-e^2 x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d), x)`

[Out] `-1/d^3/x*(-e^2*x^2+d^2)^(3/2)-1/d^3*e^2*x*(-e^2*x^2+d^2)^(1/2)-1/d*e^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+e/d^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+e^2/d/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))-e/d^2*(-e^2*x^2+d^2)^(1/2)+e/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-e^2 x^2 + d^2}}{(e x + d) x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)*x^2), x, algorithm="maxima")`

[Out] `integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)*x^2), x)`

**Fricas [A]** time = 0.284498, size = 142, normalized size = 2.78

$$\frac{e^2 x^2 - d^2 + \left(d e x - \sqrt{-e^2 x^2 + d^2} e x\right) \log\left(-\frac{d - \sqrt{-e^2 x^2 + d^2}}{x}\right) + \sqrt{-e^2 x^2 + d^2} d}{d^2 x - \sqrt{-e^2 x^2 + d^2} d x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)*x^2), x, algorithm="fricas")`

[Out] `-(e^2*x^2 - d^2 + (d*e*x - sqrt(-e^2*x^2 + d^2)*e*x)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + sqrt(-e^2*x^2 + d^2)*d)/(d^2*x - sqrt(-e^2*x^2 + d^2)*d*x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-d + e x)(d + e x)}}{x^2 (d + e x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2)/x\*\*2/(e\*x+d), x)

[Out] Integral(sqrt(-(-d + e\*x)\*(d + e\*x))/(x\*\*2\*(d + e\*x)), x)

**GIAC/XCAS [A]** time = 0.299843, size = 138, normalized size = 2.71

$$\frac{e \ln \left( \frac{|-2de - 2\sqrt{-x^2e^2 + d^2}e|e^{(-2)}}{2|x|} \right)}{d} + \frac{xe^3}{2 \left( de + \sqrt{-x^2e^2 + d^2}e \right) d} - \frac{\left( de + \sqrt{-x^2e^2 + d^2}e \right) e^{(-1)}}{2 dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)/((e\*x + d)\*x^2), x, algorithm="giac")

[Out] e\*ln(1/2\*abs(-2\*d\*e - 2\*sqrt(-x^2\*e^2 + d^2)\*e)\*e^(-2)/abs(x))/d + 1/2\*x\*e^3/((d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*d) - 1/2\*(d\*e + sqrt(-x^2\*e^2 + d^2)\*e)\*e^(-1)/(d\*x)

$$3.99 \quad \int \frac{\sqrt{d^2 - e^2 x^2}}{x^3(d+ex)} dx$$

**Optimal.** Leaf size=82

$$\frac{e\sqrt{d^2 - e^2 x^2}}{d^2 x} - \frac{\sqrt{d^2 - e^2 x^2}}{2dx^2} - \frac{e^2 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{2d^2}$$

[Out]  $-\text{Sqrt}[d^2 - e^2 x^2]/(2*d*x^2) + (e*\text{Sqrt}[d^2 - e^2 x^2])/(d^2*x) - (e^2*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2 x^2]/d])/(2*d^2)$

**Rubi [A]** time = 0.264977, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{e\sqrt{d^2 - e^2 x^2}}{d^2 x} - \frac{\sqrt{d^2 - e^2 x^2}}{2dx^2} - \frac{e^2 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{2d^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[d^2 - e^2 x^2]/(x^3*(d + e*x)), x]$

[Out]  $-\text{Sqrt}[d^2 - e^2 x^2]/(2*d*x^2) + (e*\text{Sqrt}[d^2 - e^2 x^2])/(d^2*x) - (e^2*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2 x^2]/d])/(2*d^2)$

**Rubi in Sympy [A]** time = 25.9517, size = 65, normalized size = 0.79

$$-\frac{\sqrt{d^2 - e^2 x^2}}{2dx^2} - \frac{e^2 \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{2d^2} + \frac{e\sqrt{d^2 - e^2 x^2}}{d^2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((-e^{**2}*x^{**2}+d^{**2})^{**}(1/2)/x^{**3}/(e*x+d), x)$

[Out]  $-\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(2*d*x^{**2}) - e^{**2}*\operatorname{atanh}(\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/d)/(2*d^{**2}) + e*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(d^{**2}*x)$

**Mathematica [A]** time = 0.0811528, size = 70, normalized size = 0.85

$$\frac{(d - 2ex)\sqrt{d^2 - e^2 x^2} + e^2 x^2 \log\left(\sqrt{d^2 - e^2 x^2} + d\right) - e^2 x^2 \log(x)}{2d^2 x^2}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[\text{Sqrt}[d^2 - e^2 x^2]/(x^3*(d + e*x)), x]$

[Out]  $-((d - 2*e*x)*\text{Sqrt}[d^2 - e^2 x^2] - e^2*x^2*\text{Log}[x] + e^2*x^2*\text{Log}[d + \text{Sqrt}[d^2 - e^2 x^2]])/(2*d^2*x^2)$

**Maple [B]** time = 0.016, size = 254, normalized size = 3.1

$$\begin{aligned}
 & -\frac{1}{2d^3x^2}(-e^2x^2+d^2)^{\frac{3}{2}} + \frac{e^2}{2d^3}\sqrt{-e^2x^2+d^2} - \frac{e^2}{2d}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}} \\
 & - \frac{e^2}{d^3}\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)} - \frac{e^3}{d^2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)\frac{1}{\sqrt{e^2}} \\
 & + \frac{e}{d^4x}(-e^2x^2+d^2)^{\frac{3}{2}} + \frac{e^3x}{d^4}\sqrt{-e^2x^2+d^2} + \frac{e^3}{d^2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-e^2*x^2+d^2)^(1/2)/x^3/(e*x+d), x)`

[Out] 
$$\begin{aligned}
 & -1/2/d^3/x^2*(-e^2*x^2+d^2)^(3/2)+1/2/d^3*e^2*(-e^2*x^2+d^2)^(1/2) \\
 & -1/2/d*e^2/(d^2)^(1/2)*\ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/d^3*e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-1/d^2*e^3/ \\
 & (e^2)^(1/2)*\arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))+e/d^4/x*(-e^2*x^2+d^2)^(3/2)+e^3/d^4*x*(-e^2*x^2+d^2)^(1/2) \\
 & +e^3/d^2/(e^2)^(1/2)*\arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))
 \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-e^2x^2+d^2}}{(ex+d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2+d^2)/((e*x+d)*x^3), x, algorithm="maxima")`

[Out] `integrate(sqrt(-e^2*x^2+d^2)/((e*x+d)*x^3), x)`

**Fricas [A]** time = 0.279977, size = 259, normalized size = 3.16

$$\frac{4de^3x^3-2d^2e^2x^2-4d^3ex+2d^4-\left(e^4x^4-2d^2e^2x^2+2\sqrt{-e^2x^2+d^2}de^2x^2\right)\log\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{x}\right)-(2e^3x^3-de^2x^2-4d^2e^2x^2-2d^3ex+2d^4)}{2\left(d^2e^2x^4-2d^4x^2+2\sqrt{-e^2x^2+d^2}d^3x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2+d^2)/((e*x+d)*x^3), x, algorithm="fricas")`

[Out] 
$$\begin{aligned}
 & -1/2*(4*d*e^3*x^3-2*d^2*e^2*x^2-4*d^3*e*x+2*d^4-(e^4*x^4- \\
 & 2*d^2*e^2*x^2+2*\sqrt{-e^2*x^2+d^2}*d*e^2*x^2)*\log(-(d-\sqrt{-e^2*x^2+d^2})/x)- \\
 & (2*e^3*x^3-d*e^2*x^2-4*d^2*e*x+2*d^4)*\sqrt{-e^2*x^2+d^2})/(d^2*e^2*x^4-2*d^4*x^2+2*\sqrt{-e^2*x^2+d^2}*d^3*x^2)
 \end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-d+ex)(d+ex)}}{x^3(d+ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x**3/(e*x+d), x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x**3*(d + e*x)), x)
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)*x^3), x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.100 \quad \int \frac{\sqrt{d^2 - e^2 x^2}}{x^4(d+ex)} dx$$

**Optimal.** Leaf size=114

$$\frac{e\sqrt{d^2 - e^2 x^2}}{2d^2 x^2} - \frac{\sqrt{d^2 - e^2 x^2}}{3dx^3} - \frac{2e^2\sqrt{d^2 - e^2 x^2}}{3d^3 x} + \frac{e^3 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{2d^3}$$

[Out]  $-\text{Sqrt}[d^2 - e^2 x^2]/(3*d*x^3) + (e*\text{Sqrt}[d^2 - e^2 x^2])/(2*d^2*x^2) - (2*e^2*\text{Sqrt}[d^2 - e^2 x^2])/(3*d^3*x) + (e^3*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2 x^2]/d])/(2*d^3)$

**Rubi [A]** time = 0.364858, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{e\sqrt{d^2 - e^2 x^2}}{2d^2 x^2} - \frac{\sqrt{d^2 - e^2 x^2}}{3dx^3} - \frac{2e^2\sqrt{d^2 - e^2 x^2}}{3d^3 x} + \frac{e^3 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{2d^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[d^2 - e^2 x^2]/(x^4*(d + e*x)), x]$

[Out]  $-\text{Sqrt}[d^2 - e^2 x^2]/(3*d*x^3) + (e*\text{Sqrt}[d^2 - e^2 x^2])/(2*d^2*x^2) - (2*e^2*\text{Sqrt}[d^2 - e^2 x^2])/(3*d^3*x) + (e^3*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2 x^2]/d])/(2*d^3)$

**Rubi in Sympy [A]** time = 38.3383, size = 94, normalized size = 0.82

$$-\frac{\sqrt{d^2 - e^2 x^2}}{3dx^3} + \frac{e\sqrt{d^2 - e^2 x^2}}{2d^2 x^2} + \frac{e^3 \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{2d^3} - \frac{2e^2\sqrt{d^2 - e^2 x^2}}{3d^3 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((-e^{**2}*x^{**2}+d^{**2})^{**}(1/2)/x^{**4}/(e*x+d), x)$

[Out]  $-\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(3*d*x^{**3}) + e*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(2*d^{**2}*x^{**2}) + e^{**3}*\text{atanh}(\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/d)/(2*d^{**3}) - 2*e^{**2}*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(3*d^{**3}*x)$

**Mathematica [A]** time = 0.0764567, size = 84, normalized size = 0.74

$$\frac{(-2d^2 + 3dex - 4e^2x^2)\sqrt{d^2 - e^2x^2} + 3e^3x^3 \log(\sqrt{d^2 - e^2x^2} + d) - 3e^3x^3 \log(x)}{6d^3x^3}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[\text{Sqrt}[d^2 - e^2 x^2]/(x^4*(d + e*x)), x]$

[Out]  $((-2*d^2 + 3*d*e*x - 4*e^2*x^2)*\text{Sqrt}[d^2 - e^2*x^2] - 3*e^3*x^3*\text{Log}[x] + 3*e^3*x^3*\text{Log}[d + \text{Sqrt}[d^2 - e^2*x^2]])/(6*d^3*x^3)$

**Maple [B]** time = 0.017, size = 280, normalized size = 2.5

$$\begin{aligned}
 & -\frac{1}{3d^3x^3}(-e^2x^2+d^2)^{\frac{3}{2}} + \frac{e^3}{d^4}\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)} \\
 & + \frac{e^4}{d^3}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)\frac{1}{\sqrt{e^2}} - \frac{e^2}{d^5x}(-e^2x^2+d^2)^{\frac{3}{2}} \\
 & - \frac{e^4x}{d^5}\sqrt{-e^2x^2+d^2} - \frac{e^4}{d^3}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}} - \frac{e^3}{2d^4}\sqrt{-e^2x^2+d^2} \\
 & + \frac{e^3}{2d^2}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}} + \frac{e}{2d^4x^2}(-e^2x^2+d^2)^{\frac{3}{2}}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-e^2*x^2+d^2)^(1/2)/x^4/(e*x+d),x)`

[Out] `-1/3/d^3/x^3*(-e^2*x^2+d^2)^(3/2)+1/d^4*e^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/d^3*e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))-1/d^5*e^2/x*(-e^2*x^2+d^2)^(3/2)-1/d^5*e^4*x*(-e^2*x^2+d^2)^(1/2)-1/d^3*e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/2/d^4*e^3*(-e^2*x^2+d^2)^(1/2)+1/2/d^2*e^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/2*e/d^4/x^2*(-e^2*x^2+d^2)^(3/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-e^2x^2+d^2}}{(ex+d)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2+d^2)/((e*x+d)*x^4),x,algorithm="maxima")`

[Out] `integrate(sqrt(-e^2*x^2+d^2)/((e*x+d)*x^4),x)`

**Fricas [A]** time = 0.284932, size = 355, normalized size = 3.11

$$\frac{4e^6x^6 - 3de^5x^5 - 18d^2e^4x^4 + 15d^3e^3x^3 + 6d^4e^2x^2 - 12d^5ex + 8d^6 + 3(3de^5x^5 - 4d^3e^3x^3 - (e^5x^5 - 4d^2e^3x^3)\sqrt{-e^2x^2+d^2})}{6(3d^4e^2x^5 - 4d^6x^3 - (d^3e^2x^5 - 4d^5x^3)\sqrt{-e^2x^2+d^2})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2+d^2)/((e*x+d)*x^4),x,algorithm="fricas")`

[Out] `-1/6*(4*e^6*x^6 - 3*d*e^5*x^5 - 18*d^2*e^4*x^4 + 15*d^3*e^3*x^3 + 6*d^4*e^2*x^2 - 12*d^5*e*x + 8*d^6 + 3*(3*d*e^5*x^5 - 4*d^3*e^3*x^3 - (e^5*x^5 - 4*d^2*e^3*x^3)*sqrt(-e^2*x^2+d^2))*log(-(d - sqrt(-e^2*x^2+d^2))/x) + (12*d*e^4*x^4 - 9*d^2*e^3*x^3 - 10*d^3*e^2*x^2 + 12*d^4*e*x - 8*d^5)*sqrt(-e^2*x^2+d^2))/(3*d^4*e^2*x^5 - 4*d^6*x^3 - (d^3*e^2*x^5 - 4*d^5*x^3)*sqrt(-e^2*x^2+d^2))`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-d + ex)(d + ex)}}{x^4(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2)/x\*\*4/(e\*x+d), x)

[Out] Integral(sqrt(-(-d + e\*x)\*(d + e\*x))/(x\*\*4\*(d + e\*x)), x)

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)/((e\*x + d)\*x^4), x, algorithm="giac")

[Out] Exception raised: NotImplementedError



$$3.101 \quad \int \frac{\sqrt{d^2 - e^2 x^2}}{x^5(d+ex)} dx$$

**Optimal.** Leaf size=143

$$-\frac{\sqrt{d^2 - e^2 x^2}}{4dx^4} + \frac{e\sqrt{d^2 - e^2 x^2}}{3d^2 x^3} - \frac{3e^4 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{8d^4} + \frac{2e^3 \sqrt{d^2 - e^2 x^2}}{3d^4 x} - \frac{3e^2 \sqrt{d^2 - e^2 x^2}}{8d^3 x^2}$$

[Out]  $-\text{Sqrt}[d^2 - e^2 x^2]/(4*d*x^4) + (e*\text{Sqrt}[d^2 - e^2 x^2])/(3*d^2*x^3) - (3*e^4*\text{Sqrt}[d^2 - e^2 x^2])/(8*d^3*x^2) + (2*e^3*\text{Sqrt}[d^2 - e^2 x^2])/(3*d^4*x) - (3*e^4*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2 x^2]/d])/(8*d^4)$

**Rubi [A]** time = 0.455178, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-\frac{\sqrt{d^2 - e^2 x^2}}{4dx^4} + \frac{e\sqrt{d^2 - e^2 x^2}}{3d^2 x^3} - \frac{3e^4 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{8d^4} + \frac{2e^3 \sqrt{d^2 - e^2 x^2}}{3d^4 x} - \frac{3e^2 \sqrt{d^2 - e^2 x^2}}{8d^3 x^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[d^2 - e^2 x^2]/(x^5*(d + e*x)), x]$

[Out]  $-\text{Sqrt}[d^2 - e^2 x^2]/(4*d*x^4) + (e*\text{Sqrt}[d^2 - e^2 x^2])/(3*d^2*x^3) - (3*e^4*\text{Sqrt}[d^2 - e^2 x^2])/(8*d^3*x^2) + (2*e^3*\text{Sqrt}[d^2 - e^2 x^2])/(3*d^4*x) - (3*e^4*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2 x^2]/d])/(8*d^4)$

**Rubi in Sympy [A]** time = 50.6556, size = 122, normalized size = 0.85

$$-\frac{\sqrt{d^2 - e^2 x^2}}{4dx^4} + \frac{e\sqrt{d^2 - e^2 x^2}}{3d^2 x^3} - \frac{3e^2 \sqrt{d^2 - e^2 x^2}}{8d^3 x^2} - \frac{3e^4 \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{8d^4} + \frac{2e^3 \sqrt{d^2 - e^2 x^2}}{3d^4 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((-e**2*x**2+d**2)**(1/2)/x**5/(e*x+d), x)$

[Out]  $-\text{sqrt}(d**2 - e**2*x**2)/(4*d*x**4) + e*\text{sqrt}(d**2 - e**2*x**2)/(3*d**2*x**3) - 3*e**2*\text{sqrt}(d**2 - e**2*x**2)/(8*d**3*x**2) - 3*e**4*\text{atanh}(\text{sqrt}(d**2 - e**2*x**2)/d)/(8*d**4) + 2*e**3*\text{sqrt}(d**2 - e**2*x**2)/(3*d**4*x)$

**Mathematica [A]** time = 0.0947802, size = 95, normalized size = 0.66

$$\frac{-9e^4 x^4 \log\left(\sqrt{d^2 - e^2 x^2} + d\right) + \sqrt{d^2 - e^2 x^2} (-6d^3 + 8d^2 ex - 9de^2 x^2 + 16e^3 x^3) + 9e^4 x^4 \log(x)}{24d^4 x^4}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[\text{Sqrt}[d^2 - e^2 x^2]/(x^5*(d + e*x)), x]$

[Out]  $(\text{Sqrt}[d^2 - e^2 x^2]*(-6*d^3 + 8*d^2*e*x - 9*d*e^2*x^2 + 16*e^3*x^3) + 9*e^4*x^4*\text{Log}[x] - 9*e^4*x^4*\text{Log}[d + \text{Sqrt}[d^2 - e^2 x^2]])/$

$(24 \cdot d^4 \cdot x^4)$

**Maple [B]** time = 0.018, size = 304, normalized size = 2.1

$$\begin{aligned}
 & -\frac{1}{4d^3x^4}(-e^2x^2+d^2)^{\frac{3}{2}} - \frac{5e^2}{8d^5x^2}(-e^2x^2+d^2)^{\frac{3}{2}} + \frac{3e^4}{8d^5}\sqrt{-e^2x^2+d^2} \\
 & - \frac{3e^4}{8d^3} \ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}\right)\right) \frac{1}{\sqrt{d^2}} - \frac{e^4}{d^5}\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)} \\
 & - \frac{e^5}{d^4} \arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right) \frac{1}{\sqrt{e^2}} + \frac{e^3}{d^6x}(-e^2x^2+d^2)^{\frac{3}{2}} \\
 & + \frac{e^5x}{d^6}\sqrt{-e^2x^2+d^2} + \frac{e^5}{d^4} \arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right) \frac{1}{\sqrt{e^2}} + \frac{e}{3d^4x^3}(-e^2x^2+d^2)^{\frac{3}{2}}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-e^2*x^2+d^2)^(1/2)/x^5/(e*x+d), x)`

[Out]  $-1/4/d^3/x^4 \cdot (-e^2 \cdot x^2 + d^2)^{3/2} - 5/8/d^5 \cdot e^2/x^2 \cdot (-e^2 \cdot x^2 + d^2)^{3/2} + 3/8/d^5 \cdot e^4 \cdot (-e^2 \cdot x^2 + d^2)^{1/2} - 3/8/d^3 \cdot e^4/(d^2)^{1/2} \cdot \ln\left(\frac{2 \cdot d^2 + 2 \cdot (d^2)^{1/2} \cdot (-e^2 \cdot x^2 + d^2)^{1/2}}{x}\right) - 1/d^5 \cdot e^4 \cdot (-x + d/e)^{1/2} \cdot e^2 + 2 \cdot d \cdot e \cdot (x + d/e)^{1/2} - 1/d^4 \cdot e^5/(e^2)^{1/2} \cdot \arctan\left(\frac{e^2 \cdot x}{(-x + d/e)^{1/2} \cdot e^2 + 2 \cdot d \cdot e \cdot (x + d/e)^{1/2}}\right) + 1/d^6 \cdot e^3/x \cdot (-e^2 \cdot x^2 + d^2)^{3/2} + 1/d^6 \cdot e^5 \cdot x \cdot (-e^2 \cdot x^2 + d^2)^{3/2} + 1/d^4 \cdot e^5/(e^2)^{1/2} \cdot \arctan\left(\frac{e^2 \cdot x}{(-e^2 \cdot x^2 + d^2)^{1/2}}\right) + 1/3 \cdot e/d^4/x^3 \cdot (-e^2 \cdot x^2 + d^2)^{3/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-e^2x^2+d^2}}{(ex+d)x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2+d^2)/((e*x+d)*x^5), x, algorithm="maxima")`

[Out] `integrate(sqrt(-e^2*x^2+d^2)/((e*x+d)*x^5), x)`

**Fricas [A]** time = 0.289201, size = 428, normalized size = 2.99

$$\frac{64de^7x^7 - 36d^2e^6x^6 - 160d^3e^5x^5 + 84d^4e^4x^4 + 32d^5e^3x^3 + 64d^7ex - 48d^8 - 9(e^8x^8 - 8d^2e^6x^6 + 8d^4e^4x^4 + 4(de^6x^6 - 24(d^4e^4x^8 - 8d^6e^2x^6 + 12d^8e^2x^4 - 8d^10e^2x^2 + d^{12})))}{24(d^4e^4x^8 - 8d^6e^2x^6 + 12d^8e^2x^4 - 8d^{10}e^2x^2 + d^{12})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2+d^2)/((e*x+d)*x^5), x, algorithm="fricas")`

[Out]  $-1/24 \cdot (64 \cdot d \cdot e^7 \cdot x^7 - 36 \cdot d^2 \cdot e^6 \cdot x^6 - 160 \cdot d^3 \cdot e^5 \cdot x^5 + 84 \cdot d^4 \cdot e^4 \cdot x^4 + 32 \cdot d^5 \cdot e^3 \cdot x^3 + 64 \cdot d^7 \cdot e \cdot x - 48 \cdot d^8 - 9 \cdot (e^8 \cdot x^8 - 8 \cdot d^2 \cdot e^6 \cdot x^6 + 8 \cdot d^4 \cdot e^4 \cdot x^4 + 4 \cdot (de^6 \cdot x^6 - 24 \cdot (d^4 \cdot e^4 \cdot x^8 - 8 \cdot d^6 \cdot e^2 \cdot x^6 + 12 \cdot d^8 \cdot e^2 \cdot x^4 - 8 \cdot d^{10} \cdot e^2 \cdot x^2 + d^{12})))) \cdot \sqrt{-e^2 \cdot x^2 + d^2} \cdot \log\left(\frac{-d - \sqrt{-e^2 \cdot x^2 + d^2}}{x}\right) - (16 \cdot e^7 \cdot x^7 - 9 \cdot d \cdot e^6 \cdot x^6 - 120 \cdot d^2 \cdot e^5 \cdot x^5 + 66 \cdot d^3 \cdot e^4 \cdot x^4 + 64 \cdot d^4 \cdot e^3 \cdot x^3$

$$\frac{-24d^5e^2x^2 + 64d^6e^2x - 48d^7}{(d^4e^4x^8 - 8d^6e^2x^6 + 8d^8x^4 + 4(d^5e^2x^6 - 2d^7x^4))\sqrt{-e^2x^2 + d^2}}$$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-d+ex)(d+ex)}}{x^5(d+ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2)/x\*\*5/(e\*x+d), x)

[Out] Integral(sqrt(-(-d + e\*x)\*(d + e\*x))/(x\*\*5\*(d + e\*x)), x)

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)/((e\*x + d)\*x^5), x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.102 \quad \int \frac{x^2(d^2 - e^2x^2)^{3/2}}{d+ex} dx$$

**Optimal.** Leaf size=113

$$\frac{d(4d - 3ex)(d^2 - e^2x^2)^{3/2}}{12e^3} - \frac{(d^2 - e^2x^2)^{5/2}}{5e^3} + \frac{d^5 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{8e^3} + \frac{d^3x\sqrt{d^2 - e^2x^2}}{8e^2}$$

[Out]  $(d^3x\sqrt{d^2 - e^2x^2})/(8e^2) + (d(4d - 3ex)(d^2 - e^2x^2)^{3/2})/(12e^3) - (d^2 - e^2x^2)^{5/2}/(5e^3) + (d^5 \text{ArcTan}[(ex)/\sqrt{d^2 - e^2x^2}])/(8e^3)$

**Rubi [A]** time = 0.269821, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\frac{d(4d - 3ex)(d^2 - e^2x^2)^{3/2}}{12e^3} - \frac{(d^2 - e^2x^2)^{5/2}}{5e^3} + \frac{d^5 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{8e^3} + \frac{d^3x\sqrt{d^2 - e^2x^2}}{8e^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^2(d^2 - e^2x^2)^{3/2})/(d + ex), x]$

[Out]  $(d^3x\sqrt{d^2 - e^2x^2})/(8e^2) + (d(4d - 3ex)(d^2 - e^2x^2)^{3/2})/(12e^3) - (d^2 - e^2x^2)^{5/2}/(5e^3) + (d^5 \text{ArcTan}[(ex)/\sqrt{d^2 - e^2x^2}])/(8e^3)$

**Rubi in Sympy [A]** time = 33.4359, size = 112, normalized size = 0.99

$$\frac{d^5 \text{atan}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{8e^3} + \frac{d^3x\sqrt{d^2 - e^2x^2}}{8e^2} + \frac{d^2(d^2 - e^2x^2)^{3/2}}{3e^3} - \frac{dx(d^2 - e^2x^2)^{3/2}}{4e^2} - \frac{(d^2 - e^2x^2)^{5/2}}{5e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^2*(-e^2*x^2+d^2)^{3/2}/(e*x+d), x)$

[Out]  $d^5*\text{atan}(e*x/\text{sqrt}(d^2 - e^2*x^2))/(8*e^3) + d^3*x*\text{sqrt}(d^2 - e^2*x^2)/(8*e^2) + d^2*(d^2 - e^2*x^2)^{3/2}/(3*e^3) - d*x*(d^2 - e^2*x^2)^{3/2}/(4*e^2) - (d^2 - e^2*x^2)^{5/2}/(5*e^3)$

**Mathematica [A]** time = 0.0851792, size = 91, normalized size = 0.81

$$\frac{15d^5 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) + \sqrt{d^2 - e^2x^2}(16d^4 - 15d^3ex + 8d^2e^2x^2 + 30de^3x^3 - 24e^4x^4)}{120e^3}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(x^2(d^2 - e^2x^2)^{3/2})/(d + ex), x]$

[Out]  $(\text{sqrt}[d^2 - e^2x^2]*(16*d^4 - 15*d^3*e*x + 8*d^2*e^2*x^2 + 30*d*e^3*x^3 - 24*e^4*x^4) + 15*d^5*\text{ArcTan}[(e*x)/\text{sqrt}[d^2 - e^2*x^2]])/(120*e^3)$

---

**Maple [B]** time = 0.018, size = 222, normalized size = 2.

$$\begin{aligned}
 & -\frac{1}{5e^3}(-e^2x^2+d^2)^{\frac{5}{2}} - \frac{dx}{4e^2}(-e^2x^2+d^2)^{\frac{3}{2}} - \frac{3d^3x}{8e^2}\sqrt{-e^2x^2+d^2} \\
 & - \frac{3d^5}{8e^2} \arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right) \frac{1}{\sqrt{e^2}} + \frac{d^2}{3e^3} \left(-\left(x+\frac{d}{e}\right)^2 e^2 + 2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}} \\
 & + \frac{d^3x}{2e^2} \sqrt{-\left(x+\frac{d}{e}\right)^2 e^2 + 2de\left(x+\frac{d}{e}\right)} + \frac{d^5}{2e^2} \arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-\left(x+\frac{d}{e}\right)^2 e^2 + 2de\left(x+\frac{d}{e}\right)}}\right) \frac{1}{\sqrt{e^2}}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(-e^2\*x^2+d^2)^(3/2)/(e\*x+d),x)

[Out] -1/5\*(-e^2\*x^2+d^2)^(5/2)/e^3-1/4\*d\*x\*(-e^2\*x^2+d^2)^(3/2)/e^2-3/8\*d^3\*x\*(-e^2\*x^2+d^2)^(1/2)/e^2-3/8\*d^5/e^2/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))+1/3\*d^2/e^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)+1/2\*d^3/e^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x+1/2\*d^5/e^2/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*x^2/(e\*x + d),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.288049, size = 479, normalized size = 4.24

$$\frac{24e^{10}x^{10} - 30de^9x^9 - 320d^2e^8x^8 + 405d^3e^7x^7 + 760d^4e^6x^6 - 1035d^5e^5x^5 - 480d^6e^4x^4 + 900d^7e^3x^3 - 240d^9ex + 30}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*x^2/(e\*x + d),x, algorithm="fricas")

[Out] -1/120\*(24\*e^10\*x^10 - 30\*d\*e^9\*x^9 - 320\*d^2\*e^8\*x^8 + 405\*d^3\*e^7\*x^7 + 760\*d^4\*e^6\*x^6 - 1035\*d^5\*e^5\*x^5 - 480\*d^6\*e^4\*x^4 + 900\*d^7\*e^3\*x^3 - 240\*d^9\*e\*x + 30\*(5\*d^6\*e^4\*x^4 - 20\*d^8\*e^2\*x^2 + 16\*d^10 - (d^5\*e^4\*x^4 - 12\*d^7\*e^2\*x^2 + 16\*d^9)\*sqrt(-e^2\*x^2 + d^2)))\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) + 5\*(24\*d^8\*x^8 - 30\*d^2\*e^7\*x^7 - 104\*d^3\*e^6\*x^6 + 135\*d^4\*e^5\*x^5 + 96\*d^5\*e^4\*x^4 - 156\*d^6\*e^3\*x^3 + 48\*d^8\*e\*x)\*sqrt(-e^2\*x^2 + d^2))/(5\*d^7\*x^4 - 20\*d^3\*e^5\*x^2 + 16\*d^5\*e^3 - (e^7\*x^4 - 12\*d^2\*e^5\*x^2 + 16\*d^4\*e^3)\*sqrt(-e^2\*x^2 + d^2))

**Sympy [A]** time = 16.7882, size = 279, normalized size = 2.47

$$d \left( \begin{array}{l} \left( -\frac{id^4 \operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{id^3 x}{8e^2 \sqrt{-1 + \frac{e^2 x^2}{d^2}}} - \frac{3idx^3}{8\sqrt{-1 + \frac{e^2 x^2}{d^2}}} + \frac{ie^2 x^5}{4d\sqrt{-1 + \frac{e^2 x^2}{d^2}}} \right) \text{ for } \left| \frac{e^2 x^2}{d^2} \right| > 1 \\ \left( \frac{d^4 \operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3 x}{8e^2 \sqrt{1 - \frac{e^2 x^2}{d^2}}} + \frac{3dx^3}{8\sqrt{1 - \frac{e^2 x^2}{d^2}}} - \frac{e^2 x^5}{4d\sqrt{1 - \frac{e^2 x^2}{d^2}}} \right) \text{ otherwise} \end{array} \right) \\ - e \left( \begin{array}{l} \left( -\frac{2d^4 \sqrt{d^2 - e^2 x^2}}{15e^4} - \frac{d^2 x^2 \sqrt{d^2 - e^2 x^2}}{15e^2} + \frac{x^4 \sqrt{d^2 - e^2 x^2}}{5} \right) \text{ for } e \neq 0 \\ \left( \frac{x^4 \sqrt{d^2}}{4} \right) \text{ otherwise} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2)/(e\*x+d),x)

[Out] d\*Piecewise((-I\*d\*\*4\*acosh(e\*x/d)/(8\*e\*\*3) + I\*d\*\*3\*x/(8\*e\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 3\*I\*d\*x\*\*3/(8\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*5/(4\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*4\*asin(e\*x/d)/(8\*e\*\*3) - d\*\*3\*x/(8\*e\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 3\*d\*x\*\*3/(8\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*\*2\*x\*\*5/(4\*d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True)) - e\*Piecewise((-2\*d\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*e\*\*4) - d\*\*2\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*e\*\*2) + x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/5, Ne(e, 0)), (x\*\*4\*sqrt(d\*\*2)/4, True))

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(3/2)\*x^2/(e\*x + d),x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.103 \quad \int \frac{x^4 (d^2 - e^2 x^2)^{5/2}}{d + ex} dx$$

**Optimal.** Leaf size=201

$$\begin{aligned} & \frac{x^4 (d^2 - e^2 x^2)^{5/2}}{9e} - \frac{dx^3 (d^2 - e^2 x^2)^{5/2}}{8e^2} + \frac{4d^2 x^2 (d^2 - e^2 x^2)^{5/2}}{63e^3} + \frac{3d^9 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{128e^5} \\ & + \frac{3d^7 x \sqrt{d^2 - e^2 x^2}}{128e^4} + \frac{d^5 x (d^2 - e^2 x^2)^{3/2}}{64e^4} + \frac{d^3 (128d - 315ex) (d^2 - e^2 x^2)^{5/2}}{5040e^5} \end{aligned}$$

[Out]  $(3*d^7*x*\text{Sqrt}[d^2 - e^2*x^2])/(128*e^4) + (d^5*x*(d^2 - e^2*x^2)^(3/2))/(64*e^4) + (4*d^2*x^2*(d^2 - e^2*x^2)^(5/2))/(63*e^3) - (d*x^3*(d^2 - e^2*x^2)^(5/2))/(8*e^2) + (x^4*(d^2 - e^2*x^2)^(5/2))/(9*e) + (d^3*(128*d - 315*e*x)*(d^2 - e^2*x^2)^(5/2))/(5040*e^5) + (3*d^9*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(128*e^5)$

**Rubi [A]** time = 0.512918, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\begin{aligned} & \frac{x^4 (d^2 - e^2 x^2)^{5/2}}{9e} - \frac{dx^3 (d^2 - e^2 x^2)^{5/2}}{8e^2} + \frac{4d^2 x^2 (d^2 - e^2 x^2)^{5/2}}{63e^3} + \frac{3d^9 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{128e^5} \\ & + \frac{3d^7 x \sqrt{d^2 - e^2 x^2}}{128e^4} + \frac{d^5 x (d^2 - e^2 x^2)^{3/2}}{64e^4} + \frac{d^3 (128d - 315ex) (d^2 - e^2 x^2)^{5/2}}{5040e^5} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^4*(d^2 - e^2*x^2)^(5/2))/(d + e*x), x]$

[Out]  $(3*d^7*x*\text{Sqrt}[d^2 - e^2*x^2])/(128*e^4) + (d^5*x*(d^2 - e^2*x^2)^(3/2))/(64*e^4) + (4*d^2*x^2*(d^2 - e^2*x^2)^(5/2))/(63*e^3) - (d*x^3*(d^2 - e^2*x^2)^(5/2))/(8*e^2) + (x^4*(d^2 - e^2*x^2)^(5/2))/(9*e) + (d^3*(128*d - 315*e*x)*(d^2 - e^2*x^2)^(5/2))/(5040*e^5) + (3*d^9*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(128*e^5)$

**Rubi in Sympy [A]** time = 63.692, size = 178, normalized size = 0.89

$$\begin{aligned} & \frac{3d^9 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{128e^5} + \frac{3d^7 x \sqrt{d^2 - e^2 x^2}}{128e^4} + \frac{d^5 x (d^2 - e^2 x^2)^{3/2}}{64e^4} + \frac{d^3 (384d - 945ex) (d^2 - e^2 x^2)^{5/2}}{15120e^5} \\ & + \frac{4d^2 x^2 (d^2 - e^2 x^2)^{5/2}}{63e^3} - \frac{dx^3 (d^2 - e^2 x^2)^{5/2}}{8e^2} + \frac{x^4 (d^2 - e^2 x^2)^{5/2}}{9e} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**4*(-e**2*x**2+d**2)**(5/2)/(e*x+d), x)$

[Out]  $3*d**9*\operatorname{atan}(e*x/\text{sqrt}(d**2 - e**2*x**2))/(128*e**5) + 3*d**7*x*\text{sqrt}(d**2 - e**2*x**2)/(128*e**4) + d**5*x*(d**2 - e**2*x**2)**(3/2)/(64*e**4) + d**3*(384*d - 945*e*x)*(d**2 - e**2*x**2)**(5/2)/(15120*e**5) + 4*d**2*x**2*(d**2 - e**2*x**2)**(5/2)/(63*e**3) - d*x**3*(d**2 - e**2*x**2)**(5/2)/(8*e**2) + x**4*(d**2 - e**2*x**2)**(5/2)/(9*e)$

**Mathematica [A]** time = 0.139971, size = 135, normalized size = 0.67

$$945d^9 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right) + \sqrt{d^2 - e^2 x^2} (1024d^8 - 945d^7 ex + 512d^6 e^2 x^2 - 630d^5 e^3 x^3 + 384d^4 e^4 x^4 + 7560d^3 e^5 x^5 - 6400d^2 e^6 x^6$$

40320e^5

Antiderivative was successfully verified.

[In] Integrate[(x^4\*(d^2 - e^2\*x^2)^(5/2))/(d + e\*x), x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(1024\*d^8 - 945\*d^7\*e\*x + 512\*d^6\*e^2\*x^2 - 630\*d^5\*e^3\*x^3 + 384\*d^4\*e^4\*x^4 + 7560\*d^3\*e^5\*x^5 - 6400\*d^2\*e^6\*x^6 - 5040\*d\*e^7\*x^7 + 4480\*e^8\*x^8) + 945\*d^9\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(40320\*e^5)

**Maple [A]** time = 0.019, size = 330, normalized size = 1.6

$$\begin{aligned} & -\frac{x^2}{9e^3}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{11d^2}{63e^5}(-e^2x^2 + d^2)^{\frac{7}{2}} + \frac{d^4}{5e^5}\left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{5}{2}} \\ & + \frac{d^5x}{4e^4}\left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{3}{2}} + \frac{3d^7x}{8e^4}\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)} \\ & + \frac{3d^9}{8e^4}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)}}\right)\frac{1}{\sqrt{e^2}} \\ & - \frac{3d^3x}{16e^4}(-e^2x^2 + d^2)^{\frac{5}{2}} - \frac{15d^5x}{64e^4}(-e^2x^2 + d^2)^{\frac{3}{2}} - \frac{45d^7x}{128e^4}\sqrt{-e^2x^2 + d^2} \\ & - \frac{45d^9}{128e^4}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2 + d^2}}\right)\frac{1}{\sqrt{e^2}} + \frac{dx}{8e^4}(-e^2x^2 + d^2)^{\frac{7}{2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(-e^2\*x^2+d^2)^(5/2)/(e\*x+d), x)

[Out] -1/9/e^3\*x^2\*(-e^2\*x^2+d^2)^(7/2)-11/63\*d^2/e^5\*(-e^2\*x^2+d^2)^(7/2)+1/5\*d^4/e^5\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(5/2)+1/4\*d^5/e^4\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)\*x+3/8\*d^7/e^4\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x+3/8\*d^9/e^4/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))-3/16\*d^3/e^4\*x\*(-e^2\*x^2+d^2)^(5/2)-15/64\*d^5\*x\*(-e^2\*x^2+d^2)^(3/2)/e^4-45/128\*d^7\*x\*(-e^2\*x^2+d^2)^(1/2)/e^4-45/128\*d^9/e^4/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))+1/8\*d/e^4\*x\*(-e^2\*x^2+d^2)^(7/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x^4/(e\*x + d), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.293448, size = 806, normalized size = 4.01

4480 e^18\*x^18 - 5040 de^17\*x^17 - 190080 d^2e^16\*x^16 + 214200 d^3e^15\*x^15 + 1517184 d^4e^14\*x^14 - 1721790 d^5e^13\*x^13 - 4889472 d^6e^12\*x^12

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x^4/(e\*x + d),x, algorithm="fricas")

[Out] 1/40320\*(4480\*e^18\*x^18 - 5040\*d\*e^17\*x^17 - 190080\*d^2\*e^16\*x^16 + 214200\*d^3\*e^15\*x^15 + 1517184\*d^4\*e^14\*x^14 - 1721790\*d^5\*e^13\*x^13 - 4889472\*d^6\*e^12\*x^12 + 5609205\*d^7\*e^11\*x^11 + 7644672\*d^8\*e^10\*x^10 - 8887095\*d^9\*e^9\*x^9 - 5806080\*d^10\*e^8\*x^8 + 6781320\*d^11\*e^7\*x^7 + 1720320\*d^12\*e^6\*x^6 - 1728720\*d^13\*e^5\*x^5 - 504000\*d^15\*e^3\*x^3 + 241920\*d^17\*e\*x - 1890\*(9\*d^10\*e^8\*x^8 - 120\*d^12\*e^6\*x^6 + 432\*d^14\*e^4\*x^4 - 576\*d^16\*e^2\*x^2 + 256\*d^18 - (d^9\*e^8\*x^8 - 40\*d^11\*e^6\*x^6 + 240\*d^13\*e^4\*x^4 - 448\*d^15\*e^2\*x^2 + 256\*d^17)\*sqrt(-e^2\*x^2 + d^2))\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) + 3\*(13440\*d\*e^16\*x^16 - 15120\*d^2\*e^15\*x^15 - 198400\*d^3\*e^14\*x^14 + 224280\*d^4\*e^13\*x^13 + 902272\*d^5\*e^12\*x^12 - 1030050\*d^6\*e^11\*x^11 - 1795584\*d^7\*e^10\*x^10 + 2078685\*d^8\*e^9\*x^9 + 1648640\*d^9\*e^8\*x^8 - 1934520\*d^10\*e^7\*x^7 - 573440\*d^11\*e^6\*x^6 + 630000\*d^12\*e^5\*x^5 + 127680\*d^14\*e^3\*x^3 - 80640\*d^16\*e\*x)\*sqrt(-e^2\*x^2 + d^2))/(9\*d^5\*e^13\*x^8 - 120\*d^3\*e^11\*x^6 + 432\*d^5\*e^9\*x^4 - 576\*d^7\*e^7\*x^2 + 256\*d^9\*e^5 - (e^13\*x^8 - 40\*d^2\*e^11\*x^6 + 240\*d^4\*e^9\*x^4 - 448\*d^6\*e^7\*x^2 + 256\*d^8\*e^5)\*sqrt(-e^2\*x^2 + d^2))

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**Sympy [A]** time = 62.5952, size = 830, normalized size = 4.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/(e\*x+d),x)

[Out] d\*\*3\*Piecewise((-I\*d\*\*6\*acosh(e\*x/d)/(16\*e\*\*5) + I\*d\*\*5\*x/(16\*e\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - I\*d\*\*3\*x\*\*3/(48\*e\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 5\*I\*d\*x\*\*5/(24\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*7/(6\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*6\*asin(e\*x/d)/(16\*e\*\*5) - d\*\*5\*x/(16\*e\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + d\*\*3\*x\*\*3/(48\*e\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 5\*d\*x\*\*5/(24\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*\*2\*x\*\*7/(6\*d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True)) - d\*\*2\*e\*Piecewise((-8\*d\*\*6\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(105\*e\*\*6) - 4\*d\*\*4\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(105\*e\*\*4) - d\*\*2\*x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(35\*e\*\*2) + x\*\*6\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/7, Ne(e, 0)), (x\*\*6\*sqrt(d\*\*2)/6, True)) - d\*\*2\*Piecewise(e((-5\*I\*d\*\*8\*acosh(e\*x/d)/(128\*e\*\*7) + 5\*I\*d\*\*7\*x/(128\*e\*\*6\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 5\*I\*d\*\*5\*x\*\*3/(384\*e\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - I\*d\*\*3\*x\*\*5/(192\*e\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 7\*I\*d\*x\*\*7/(48\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*9/(8\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (5\*d\*\*8\*asin(e\*x/d)/(128\*e\*\*7) - 5\*d\*\*7\*x/(128\*e\*\*6\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 5\*d\*\*5\*x\*\*3/(384\*e\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + d\*\*3\*x\*\*5/(192\*e\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 7\*d\*x\*\*7/(48\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*\*2\*x\*\*9/(8\*d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True)) + e\*\*3\*Piecewise((-16\*d\*\*8\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(315\*e\*\*8) - 8\*d\*\*6\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(315\*e\*\*6) - 2\*d\*\*4\*x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(105\*e\*\*4) - d\*\*2\*x\*\*6\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(63\*e\*\*2) + x\*\*8\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/9, Ne(e, 0)), (x\*\*8\*sqrt(d\*\*2)/8, True))

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**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x^4/(e\*x + d),x, algorithm="giac")

```
[Out] Exception raised: NotImplementedError
```

$$3.104 \quad \int \frac{x^3 (d^2 - e^2 x^2)^{5/2}}{d + ex} dx$$

**Optimal.** Leaf size=172

$$\begin{aligned} & -\frac{dx^2 (d^2 - e^2 x^2)^{5/2}}{7e^2} + \frac{x^3 (d^2 - e^2 x^2)^{5/2}}{8e} - \frac{d^2(32d - 35ex) (d^2 - e^2 x^2)^{5/2}}{560e^4} \\ & - \frac{3d^8 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{128e^4} - \frac{3d^6 x \sqrt{d^2 - e^2 x^2}}{128e^3} - \frac{d^4 x (d^2 - e^2 x^2)^{3/2}}{64e^3} \end{aligned}$$

[Out]  $(-3*d^6*x*\text{Sqrt}[d^2 - e^2*x^2])/(128*e^3) - (d^4*x*(d^2 - e^2*x^2)^{(3/2)})/(64*e^3) - (d*x^2*(d^2 - e^2*x^2)^{(5/2)})/(7*e^2) + (x^3*(d^2 - e^2*x^2)^{(5/2)})/(8*e) - (d^2*(32*d - 35*e*x)*(d^2 - e^2*x^2)^{(5/2)})/(560*e^4) - (3*d^8*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(128*e^4)$

**Rubi [A]** time = 0.388477, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\begin{aligned} & -\frac{dx^2 (d^2 - e^2 x^2)^{5/2}}{7e^2} + \frac{x^3 (d^2 - e^2 x^2)^{5/2}}{8e} - \frac{d^2(32d - 35ex) (d^2 - e^2 x^2)^{5/2}}{560e^4} \\ & - \frac{3d^8 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{128e^4} - \frac{3d^6 x \sqrt{d^2 - e^2 x^2}}{128e^3} - \frac{d^4 x (d^2 - e^2 x^2)^{3/2}}{64e^3} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^3*(d^2 - e^2*x^2)^{(5/2)})/(d + e*x), x]$

[Out]  $(-3*d^6*x*\text{Sqrt}[d^2 - e^2*x^2])/(128*e^3) - (d^4*x*(d^2 - e^2*x^2)^{(3/2)})/(64*e^3) - (d*x^2*(d^2 - e^2*x^2)^{(5/2)})/(7*e^2) + (x^3*(d^2 - e^2*x^2)^{(5/2)})/(8*e) - (d^2*(32*d - 35*e*x)*(d^2 - e^2*x^2)^{(5/2)})/(560*e^4) - (3*d^8*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(128*e^4)$

**Rubi in Sympy [A]** time = 47.2577, size = 151, normalized size = 0.88

$$\begin{aligned} & -\frac{3d^8 \text{atan}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{128e^4} - \frac{3d^6 x \sqrt{d^2 - e^2 x^2}}{128e^3} - \frac{d^4 x (d^2 - e^2 x^2)^{\frac{3}{2}}}{64e^3} \\ & - \frac{d^2 (96d - 105ex) (d^2 - e^2 x^2)^{\frac{5}{2}}}{1680e^4} - \frac{dx^2 (d^2 - e^2 x^2)^{\frac{5}{2}}}{7e^2} + \frac{x^3 (d^2 - e^2 x^2)^{\frac{5}{2}}}{8e} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**3*(-e**2*x**2+d**2)**(5/2)/(e*x+d), x)$

[Out]  $-3*d**8*\text{atan}(e*x/\text{sqrt}(d**2 - e**2*x**2))/(128*e**4) - 3*d**6*x*\text{sqrt}(d**2 - e**2*x**2)/(128*e**3) - d**4*x*(d**2 - e**2*x**2)**(3/2)/(64*e**3) - d**2*(96*d - 105*e*x)*(d**2 - e**2*x**2)**(5/2)/(1680*e**4) - d*x**2*(d**2 - e**2*x**2)**(5/2)/(7*e**2) + x**3*(d**2 - e**2*x**2)**(5/2)/(8*e)$

**Mathematica [A]** time = 0.121789, size = 124, normalized size = 0.72

$$\frac{\sqrt{d^2 - e^2 x^2} (-256d^7 + 105d^6 ex - 128d^5 e^2 x^2 + 70d^4 e^3 x^3 + 1024d^3 e^4 x^4 - 840d^2 e^5 x^5 - 640de^6 x^6 + 560e^7 x^7) - 105d^8 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{4480e^4}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(d^2 - e^2\*x^2)^(5/2))/(d + e\*x), x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(-256\*d^7 + 105\*d^6\*e\*x - 128\*d^5\*e^2\*x^2 + 70\*d^4\*e^3\*x^3 + 1024\*d^3\*e^4\*x^4 - 840\*d^2\*e^5\*x^5 - 640\*d\*e^6\*x^6 + 560\*e^7\*x^7) - 105\*d^8\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(480\*e^4)

**Maple [B]** time = 0.016, size = 305, normalized size = 1.8

$$\begin{aligned}
 & -\frac{x}{8e^3}(-e^2x^2 + d^2)^{\frac{7}{2}} + \frac{3d^2x}{16e^3}(-e^2x^2 + d^2)^{\frac{5}{2}} + \frac{15d^4x}{64e^3}(-e^2x^2 + d^2)^{\frac{3}{2}} \\
 & + \frac{45d^6x}{128e^3}\sqrt{-e^2x^2 + d^2} + \frac{45d^8}{128e^3}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2 + d^2}}\right)\frac{1}{\sqrt{e^2}} \\
 & + \frac{d}{7e^4}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{d^3}{5e^4}\left(-\left(x + \frac{d}{e}\right)^2e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{5}{2}} \\
 & - \frac{d^4x}{4e^3}\left(-\left(x + \frac{d}{e}\right)^2e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{3}{2}} - \frac{3d^6x}{8e^3}\sqrt{-\left(x + \frac{d}{e}\right)^2e^2 + 2de\left(x + \frac{d}{e}\right)} \\
 & - \frac{3d^8}{8e^3}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2e^2 + 2de\left(x + \frac{d}{e}\right)}}\right)\frac{1}{\sqrt{e^2}}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(-e^2\*x^2+d^2)^(5/2)/(e\*x+d), x)

[Out] -1/8/e^3\*x\*(-e^2\*x^2+d^2)^(7/2)+3/16\*d^2/e^3\*x\*(-e^2\*x^2+d^2)^(5/2)+15/64\*d^4\*x\*(-e^2\*x^2+d^2)^(3/2)/e^3+45/128\*d^6\*x\*(-e^2\*x^2+d^2)^(1/2)/e^3+45/128\*d^8/e^3/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))+1/7\*d/e^4\*(-e^2\*x^2+d^2)^(7/2)-1/5\*d^3/e^4\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(5/2)-1/4\*d^4/e^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)\*x-3/8\*d^6/e^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x-3/8\*d^8/e^3/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x^3/(e\*x + d), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.295509, size = 744, normalized size = 4.33

$$4480de^{15}x^{15} - 5120d^2e^{14}x^{14} - 56000d^3e^{13}x^{13} + 64512d^4e^{12}x^{12} + 226800d^5e^{11}x^{11} - 265216d^6e^{10}x^{10} - 413000d^7e^9x^9 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x^3/(e\*x + d),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/4480*(4480*d^3*e^{15}*x^{15} - 5120*d^2*e^{14}*x^{14} - 56000*d^3*e^{13}*x^{13} \\ & + 64512*d^4*e^{12}*x^{12} + 226800*d^5*e^{11}*x^{11} - 265216*d^6*e^{10}*x^{10} - 413000*d^7*e^9*x^9 \\ & + 492800*d^8*e^8*x^8 + 350280*d^9*e^7*x^7 - 430080*d^{10}*e^6*x^6 - 101360*d^{11}*e^5*x^5 + 143360*d^{12}*e^4*x^4 \\ & - 24640*d^{13}*e^3*x^3 + 13440*d^{15}*e*x - 210*(d^8*e^8*x^8 - 32*d^{10}*e^6*x^6 + 160*d^{12}*e^4*x^4 \\ & - 256*d^{14}*e^2*x^2 + 128*d^{16} + 8*(d^9*e^6*x^6 - 10*d^{11}*e^4*x^4 + 24*d^{13}*e^2*x^2 - 16*d^{15})* \\ & \text{sqrt}(-e^2*x^2 + d^2))*\arctan(-(d - \text{sqrt}(-e^2*x^2 + d^2))/(e*x)) - \\ & (560*e^{15}*x^{15} - 640*d^3*e^{14}*x^{14} - 18760*d^2*e^{13}*x^{13} + 21504*d^3*e^{12}*x^{12} + 116550*d^4*e^{11}*x^{11} \\ & - 135296*d^5*e^{10}*x^{10} - 279895*d^6*e^9*x^9 + 331520*d^7*e^8*x^8 + 294560*d^8*e^7*x^7 - 358400*d^9*e^6*x^6 \\ & - 108640*d^{10}*e^5*x^5 + 143360*d^{11}*e^4*x^4 - 17920*d^{12}*e^3*x^3 + 13440*d^{14}*e*x)* \\ & \text{sqrt}(-e^2*x^2 + d^2))/(e^{12}*x^8 - 32*d^2*e^{10}*x^6 + 160*d^4*e^8*x^4 - 256*d^6*e^6*x^2 + 128*d^8*e^4 + 8*(d^5*e^{10}*x^6 \\ & - 10*d^3*e^8*x^4 + 24*d^5*e^6*x^2 - 16*d^7*e^4)*\text{sqrt}(-e^2*x^2 + d^2)) \end{aligned}$$

**Sympy [A]** time = 82.4995, size = 775, normalized size = 4.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/(e\*x+d),x)

[Out] 
$$\begin{aligned} & d^{**3}\text{Piecewise}((-2*d^{**4}\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(15*e^{**4}) - d^{**2}*x^{**2}\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(15*e^{**2}) + x^{**4}\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/5, \text{Ne}(e, 0)), (x^{**4}\text{sqrt}(d^{**2})/4, \text{True})) - d^{**2}*e*\text{Piecewise}((-I*d^{**6}\text{acosh}(e*x/d)/(16*e^{**5}) + I*d^{**5}*x/(16*e^{**4}\text{sqrt}(-1 + e^{**2}*x^{**2}/d^{**2})) - I*d^{**3}*x^{**3}/(48*e^{**2}\text{sqrt}(-1 + e^{**2}*x^{**2}/d^{**2})) - 5*I*d*x^{**5}/(24*\text{sqrt}(-1 + e^{**2}*x^{**2}/d^{**2})) + I*e^{**2}*x^{**7}/(6*d*\text{sqrt}(-1 + e^{**2}*x^{**2}/d^{**2})), \text{Abs}(e^{**2}*x^{**2}/d^{**2}) > 1), (d^{**6}\text{asin}(e*x/d)/(16*e^{**5}) - d^{**5}*x/(16*e^{**4}\text{sqrt}(1 - e^{**2}*x^{**2}/d^{**2})) + d^{**3}*x^{**3}/(48*e^{**2}\text{sqrt}(1 - e^{**2}*x^{**2}/d^{**2})) + 5*d*x^{**5}/(24*\text{sqrt}(1 - e^{**2}*x^{**2}/d^{**2})) - e^{**2}*x^{**7}/(6*d*\text{sqrt}(1 - e^{**2}*x^{**2}/d^{**2})), \text{True})) - d*e^{**2}*\text{Piecewise}((-8*d^{**6}\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(105*e^{**6}) - 4*d^{**4}*x^{**2}\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(105*e^{**4}) - d^{**2}*x^{**4}\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(35*e^{**2}) + x^{**6}\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/7, \text{Ne}(e, 0)), (x^{**6}\text{sqrt}(d^{**2})/6, \text{True})) + e^{**3}*\text{Piecewise}((-5*I*d^{**8}\text{acosh}(e*x/d)/(128*e^{**7}) + 5*I*d^{**7}*x/(128*e^{**6}\text{sqrt}(-1 + e^{**2}*x^{**2}/d^{**2})) - 5*I*d^{**5}*x^{**3}/(384*e^{**4}\text{sqrt}(-1 + e^{**2}*x^{**2}/d^{**2})) - I*d^{**3}*x^{**5}/(192*e^{**2}\text{sqrt}(-1 + e^{**2}*x^{**2}/d^{**2})) - 7*I*d*x^{**7}/(48*\text{sqrt}(-1 + e^{**2}*x^{**2}/d^{**2})) + I*e^{**2}*x^{**9}/(8*d*\text{sqrt}(-1 + e^{**2}*x^{**2}/d^{**2})), \text{Abs}(e^{**2}*x^{**2}/d^{**2}) > 1), (5*d^{**8}\text{asin}(e*x/d)/(128*e^{**7}) - 5*d^{**7}*x/(128*e^{**6}\text{sqrt}(1 - e^{**2}*x^{**2}/d^{**2})) + 5*d^{**5}*x^{**3}/(384*e^{**4}\text{sqrt}(1 - e^{**2}*x^{**2}/d^{**2})) + d^{**3}*x^{**5}/(192*e^{**2}\text{sqrt}(1 - e^{**2}*x^{**2}/d^{**2})) + 7*d*x^{**7}/(48*\text{sqrt}(1 - e^{**2}*x^{**2}/d^{**2})) - e^{**2}*x^{**9}/(8*d*\text{sqrt}(1 - e^{**2}*x^{**2}/d^{**2})), \text{True})) \end{aligned}$$

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x^3/(e\*x + d),x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.105 \quad \int \frac{x^2(d^2 - e^2x^2)^{5/2}}{d+ex} dx$$

**Optimal.** Leaf size=140

$$\frac{d(6d - 5ex)(d^2 - e^2x^2)^{5/2}}{30e^3} - \frac{(d^2 - e^2x^2)^{7/2}}{7e^3} + \frac{d^7 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{16e^3} + \frac{d^5x\sqrt{d^2 - e^2x^2}}{16e^2} + \frac{d^3x(d^2 - e^2x^2)^{3/2}}{24e^2}$$

[Out] (d^5\*x\*Sqrt[d^2 - e^2\*x^2])/(16\*e^2) + (d^3\*x\*(d^2 - e^2\*x^2)^(3/2))/(24\*e^2) + (d\*(6\*d - 5\*e\*x)\*(d^2 - e^2\*x^2)^(5/2))/(30\*e^3) - (d^2 - e^2\*x^2)^(7/2)/(7\*e^3) + (d^7\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(16\*e^3)

**Rubi [A]** time = 0.303145, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\frac{d(6d - 5ex)(d^2 - e^2x^2)^{5/2}}{30e^3} - \frac{(d^2 - e^2x^2)^{7/2}}{7e^3} + \frac{d^7 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{16e^3} + \frac{d^5x\sqrt{d^2 - e^2x^2}}{16e^2} + \frac{d^3x(d^2 - e^2x^2)^{3/2}}{24e^2}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(d^2 - e^2\*x^2)^(5/2))/(d + e\*x), x]

[Out] (d^5\*x\*Sqrt[d^2 - e^2\*x^2])/(16\*e^2) + (d^3\*x\*(d^2 - e^2\*x^2)^(3/2))/(24\*e^2) + (d\*(6\*d - 5\*e\*x)\*(d^2 - e^2\*x^2)^(5/2))/(30\*e^3) - (d^2 - e^2\*x^2)^(7/2)/(7\*e^3) + (d^7\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(16\*e^3)

**Rubi in Sympy [A]** time = 38.5871, size = 136, normalized size = 0.97

$$\frac{d^7 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{16e^3} + \frac{d^5x\sqrt{d^2 - e^2x^2}}{16e^2} + \frac{d^3x(d^2 - e^2x^2)^{\frac{3}{2}}}{24e^2} + \frac{d^2(d^2 - e^2x^2)^{\frac{5}{2}}}{5e^3} - \frac{dx(d^2 - e^2x^2)^{\frac{5}{2}}}{6e^2} - \frac{(d^2 - e^2x^2)^{\frac{7}{2}}}{7e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/(e\*x+d), x)

[Out] d\*\*7\*atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2))/(16\*e\*\*3) + d\*\*5\*x\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(16\*e\*\*2) + d\*\*3\*x\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(24\*e\*\*2) + d\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)/(5\*e\*\*3) - d\*x\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)/(6\*e\*\*2) - (d\*\*2 - e\*\*2\*x\*\*2)\*\*(7/2)/(7\*e\*\*3)

**Mathematica [A]** time = 0.108985, size = 113, normalized size = 0.81

$$\frac{105d^7 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) + \sqrt{d^2 - e^2x^2} (96d^6 - 105d^5ex + 48d^4e^2x^2 + 490d^3e^3x^3 - 384d^2e^4x^4 - 280de^5x^5 + 240e^6x^6)}{1680e^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(d^2 - e^2\*x^2)^(5/2))/(d + e\*x), x]

[Out]  $(\text{Sqrt}[d^2 - e^2 x^2]) \cdot (96 d^6 - 105 d^5 e x + 48 d^4 e^2 x^2 + 490 d^3 e^3 x^3 - 384 d^2 e^4 x^4 - 280 d e^5 x^5 + 240 e^6 x^6) + 105 d^7 \text{ArcTan}[(e x) / \text{Sqrt}[d^2 - e^2 x^2]] / (1680 e^3)$

**Maple [B]** time = 0.017, size = 282, normalized size = 2.

$$\begin{aligned} & -\frac{1}{7 e^3} (-e^2 x^2 + d^2)^{\frac{7}{2}} - \frac{d x}{6 e^2} (-e^2 x^2 + d^2)^{\frac{5}{2}} - \frac{5 d^3 x}{24 e^2} (-e^2 x^2 + d^2)^{\frac{3}{2}} - \frac{5 d^5 x}{16 e^2} \sqrt{-e^2 x^2 + d^2} \\ & - \frac{5 d^7}{16 e^2} \arctan\left(x \sqrt{e^2} \frac{1}{\sqrt{-e^2 x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} + \frac{d^2}{5 e^3} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)\right)^{\frac{5}{2}} \\ & + \frac{d^3 x}{4 e^2} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)\right)^{\frac{3}{2}} + \frac{3 d^5 x}{8 e^2} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)} \\ & + \frac{3 d^7}{8 e^2} \arctan\left(x \sqrt{e^2} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)}}\right) \frac{1}{\sqrt{e^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(-e^2*x^2+d^2)^(5/2)/(e*x+d),x)`

[Out]  $-1/7 * (-e^2 * x^2 + d^2)^{(7/2)} / e^3 - 1/6 * d * x * (-e^2 * x^2 + d^2)^{(5/2)} / e^2 - 5/24 * d^3 * x * (-e^2 * x^2 + d^2)^{(3/2)} / e^2 - 5/16 * d^5 * x * (-e^2 * x^2 + d^2)^{(1/2)} / e^2 - 5/16 * d^7 / e^2 / (e^2)^{(1/2)} * \arctan((e^2)^{(1/2)} * x / (-e^2 * x^2 + d^2)^{(1/2)}) + 1/5 * d^2 / e^3 * (- (x+d/e)^2 * e^2 + 2 * d * e * (x+d/e))^{(5/2)} + 1/4 * d^3 / e^2 * (- (x+d/e)^2 * e^2 + 2 * d * e * (x+d/e))^{(3/2)} * x + 3/8 * d^5 / e^2 * (- (x+d/e)^2 * e^2 + 2 * d * e * (x+d/e))^{(1/2)} * x + 3/8 * d^7 / e^2 / (e^2)^{(1/2)} * \arctan((e^2)^{(1/2)} * x / (- (x+d/e)^2 * e^2 + 2 * d * e * (x+d/e))^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)*x^2/(e*x + d),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.286915, size = 657, normalized size = 4.69

$240 e^{14} x^{14} - 280 d e^{13} x^{13} - 6384 d^2 e^{12} x^{12} + 7490 d^3 e^{11} x^{11} + 34608 d^4 e^{10} x^{10} - 41475 d^5 e^9 x^9 - 75600 d^6 e^8 x^8 + 93905 d^7 e^7 x^7$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)*x^2/(e*x + d),x, algorithm="fricas")`

[Out]  $1/1680 * (240 * e^{14} * x^{14} - 280 * d * e^{13} * x^{13} - 6384 * d^2 * e^{12} * x^{12} + 7490 * d^3 * e^{11} * x^{11} + 34608 * d^4 * e^{10} * x^{10} - 41475 * d^5 * e^9 * x^9 - 75600 * d^6 * e^8 * x^8 + 93905 * d^7 * e^7 * x^7 + 73920 * d^8 * e^6 * x^6 - 99400 * d^9 * e^5 * x^5 - 26880 * d^{10} * e^4 * x^4 + 46480 * d^{11} * e^3 * x^3 - 6720 * d^{13} * e * x - 210 * (7 * d^8 * e^6 * x^6 - 56 * d^{10} * e^4 * x^4 + 112 * d^{12} * e^2 * x^2 - 64 * d^{14})) / (1680 * e^3)$

$$d^{14} - (d^7 e^6 x^6 - 24 d^9 e^4 x^4 + 80 d^{11} e^2 x^2 - 64 d^{13}) \sqrt{-e^2 x^2 + d^2} \arctan\left(\frac{d - \sqrt{-e^2 x^2 + d^2}}{e x}\right) + 7(240 d e^{12} x^{12} - 280 d^2 e^{11} x^{11} - 2304 d^3 e^{10} x^{10} + 2730 d^4 e^9 x^9 + 6960 d^5 e^8 x^8 - 8505 d^6 e^7 x^7 - 8640 d^7 e^6 x^6 + 11240 d^8 e^5 x^5 + 3840 d^9 e^4 x^4 - 6160 d^{10} e^3 x^3 + 960 d^{12} e^2 x^2) \sqrt{-e^2 x^2 + d^2} / (7 d e^9 x^6 - 56 d^3 e^7 x^4 + 112 d^5 e^5 x^2 - 64 d^7 e^3 - (e^9 x^6 - 24 d^2 e^7 x^4 + 80 d^4 e^5 x^2 - 64 d^6 e^3) \sqrt{-e^2 x^2 + d^2})$$

**Sympy [A]** time = 40.5211, size = 653, normalized size = 4.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/(e\*x+d),x)

[Out]  $d^3 \text{Piecewise}\left(\left(-I d^4 \operatorname{acosh}\left(\frac{e x}{d}\right) / \left(8 e^3\right) + I d^3 x / \left(8 e^2 \sqrt{-1 + e^2 x^2 / d^2}\right) - 3 I d^2 x^3 / \left(8 \sqrt{-1 + e^2 x^2 / d^2}\right) + I e^2 x^5 / \left(4 d \sqrt{-1 + e^2 x^2 / d^2}\right), \operatorname{Abs}\left(e^2 x^2 / d^2\right) > 1\right), \left(d^4 \operatorname{asin}\left(\frac{e x}{d}\right) / \left(8 e^3\right) - d^3 x / \left(8 e^2 \sqrt{1 - e^2 x^2 / d^2}\right) + 3 d^2 x^3 / \left(8 \sqrt{1 - e^2 x^2 / d^2}\right) - e^2 x^5 / \left(4 d \sqrt{1 - e^2 x^2 / d^2}\right), \operatorname{True}\right) - d^2 e \text{Piecewise}\left(\left(-2 d^4 \sqrt{d^2 - e^2 x^2} / \left(15 e^4\right) - d^2 x^2 \sqrt{d^2 - e^2 x^2} / \left(15 e^2\right) + x^4 \sqrt{d^2 - e^2 x^2} / 5, \operatorname{Ne}\left(e, 0\right)\right), \left(x^4 \sqrt{d^2} / 4, \operatorname{True}\right) - d e^2 \text{Piecewise}\left(\left(-I d^6 \operatorname{acosh}\left(\frac{e x}{d}\right) / \left(16 e^5\right) + I d^5 x / \left(16 e^4 \sqrt{-1 + e^2 x^2 / d^2}\right) - I d^3 x^3 / \left(48 e^2 \sqrt{-1 + e^2 x^2 / d^2}\right) - 5 I d^2 x^5 / \left(24 \sqrt{-1 + e^2 x^2 / d^2}\right) + I e^2 x^7 / \left(6 d \sqrt{-1 + e^2 x^2 / d^2}\right), \operatorname{Abs}\left(e^2 x^2 / d^2\right) > 1\right), \left(d^6 \operatorname{asin}\left(\frac{e x}{d}\right) / \left(16 e^5\right) - d^5 x / \left(16 e^4 \sqrt{1 - e^2 x^2 / d^2}\right) + d^3 x^3 / \left(48 e^2 \sqrt{1 - e^2 x^2 / d^2}\right) + 5 d^2 x^5 / \left(24 \sqrt{1 - e^2 x^2 / d^2}\right) - e^2 x^7 / \left(6 d \sqrt{1 - e^2 x^2 / d^2}\right), \operatorname{True}\right) + e^3 \text{Piecewise}\left(\left(-8 d^6 \sqrt{d^2 - e^2 x^2} / \left(105 e^6\right) - 4 d^4 x^2 \sqrt{d^2 - e^2 x^2} / \left(105 e^4\right) - d^2 x^4 \sqrt{d^2 - e^2 x^2} / \left(35 e^2\right) + x^6 \sqrt{d^2 - e^2 x^2} / 7, \operatorname{Ne}\left(e, 0\right)\right), \left(x^6 \sqrt{d^2} / 6, \operatorname{True}\right)\right)$

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x^2/(e\*x + d),x, algorithm="giac")

[Out] Exception raised: NotImplementedError



$$3.106 \quad \int \frac{x(d^2 - e^2 x^2)^{5/2}}{d + ex} dx$$

**Optimal.** Leaf size=116

$$-\frac{d^2 x (d^2 - e^2 x^2)^{3/2}}{24e} - \frac{(6d - 5ex)(d^2 - e^2 x^2)^{5/2}}{30e^2} - \frac{d^6 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{16e^2} - \frac{d^4 x \sqrt{d^2 - e^2 x^2}}{16e}$$

[Out]  $-(d^4 * x * \text{Sqrt}[d^2 - e^2 * x^2]) / (16 * e) - (d^2 * x * (d^2 - e^2 * x^2)^{(3/2)}) / (24 * e) - ((6 * d - 5 * e * x) * (d^2 - e^2 * x^2)^{(5/2)}) / (30 * e^2) - (d^6 * \text{ArcTan}[(e * x) / \text{Sqrt}[d^2 - e^2 * x^2]]) / (16 * e^2)$

**Rubi [A]** time = 0.172586, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{d^2 x (d^2 - e^2 x^2)^{3/2}}{24e} - \frac{(6d - 5ex)(d^2 - e^2 x^2)^{5/2}}{30e^2} - \frac{d^6 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{16e^2} - \frac{d^4 x \sqrt{d^2 - e^2 x^2}}{16e}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x * (d^2 - e^2 * x^2)^{(5/2)}) / (d + e * x), x]$

[Out]  $-(d^4 * x * \text{Sqrt}[d^2 - e^2 * x^2]) / (16 * e) - (d^2 * x * (d^2 - e^2 * x^2)^{(3/2)}) / (24 * e) - ((6 * d - 5 * e * x) * (d^2 - e^2 * x^2)^{(5/2)}) / (30 * e^2) - (d^6 * \text{ArcTan}[(e * x) / \text{Sqrt}[d^2 - e^2 * x^2]]) / (16 * e^2)$

**Rubi in Sympy [A]** time = 22.0632, size = 116, normalized size = 1.

$$-\frac{d^6 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{16e^2} - \frac{d^4 x \sqrt{d^2 - e^2 x^2}}{16e} - \frac{d^2 x (d^2 - e^2 x^2)^{3/2}}{24e} - \frac{d (d^2 - e^2 x^2)^{5/2}}{30e^2} - \frac{(d^2 - e^2 x^2)^{7/2}}{6e^2 (d + ex)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x * (-e^{**2} * x^{**2} + d^{**2})^{** (5/2)} / (e * x + d), x)$

[Out]  $-d^{**6} * \operatorname{atan}(e * x / \operatorname{sqrt}(d^{**2} - e^{**2} * x^{**2})) / (16 * e^{**2}) - d^{**4} * x * \operatorname{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (16 * e) - d^{**2} * x * (d^{**2} - e^{**2} * x^{**2})^{** (3/2)} / (24 * e) - d * (d^{**2} - e^{**2} * x^{**2})^{** (5/2)} / (30 * e^{**2}) - (d^{**2} - e^{**2} * x^{**2})^{** (7/2)} / (6 * e^{**2} * (d + e * x))$

**Mathematica [A]** time = 0.094787, size = 102, normalized size = 0.88

$$\frac{\sqrt{d^2 - e^2 x^2} (-48d^5 + 15d^4 ex + 96d^3 e^2 x^2 - 70d^2 e^3 x^3 - 48de^4 x^4 + 40e^5 x^5) - 15d^6 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{240e^2}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(x * (d^2 - e^2 * x^2)^{(5/2)}) / (d + e * x), x]$

[Out]  $(\text{Sqrt}[d^2 - e^2 * x^2] * (-48 * d^5 + 15 * d^4 * e * x + 96 * d^3 * e^2 * x^2 - 70 * d^2 * e^3 * x^3 - 48 * d * e^4 * x^4 + 40 * e^5 * x^5) - 15 * d^6 * \text{ArcTan}[(e * x) / \text{Sqrt}[d^2 - e^2 * x^2]]) / (240 * e^2)$

---

**Maple [B]** time = 0.013, size = 260, normalized size = 2.2

$$\begin{aligned} & \frac{x}{6e} (-e^2x^2 + d^2)^{\frac{5}{2}} + \frac{5d^2x}{24e} (-e^2x^2 + d^2)^{\frac{3}{2}} + \frac{5d^4x}{16e} \sqrt{-e^2x^2 + d^2} \\ & + \frac{5d^6}{16e} \arctan\left(x\sqrt{e^2} \frac{1}{\sqrt{-e^2x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} - \frac{d}{5e^2} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{5}{2}} \\ & - \frac{d^2x}{4e} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{3}{2}} - \frac{3d^4x}{8e} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)} \\ & - \frac{3d^6}{8e} \arctan\left(x\sqrt{e^2} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)}}\right) \frac{1}{\sqrt{e^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(-e^2*x^2+d^2)^(5/2)/(e*x+d), x)`

[Out] `1/6*x*(-e^2*x^2+d^2)^(5/2)/e+5/24*d^2*x*(-e^2*x^2+d^2)^(3/2)/e+5/16*d^4*x*(-e^2*x^2+d^2)^(1/2)/e+5/16*d^6/e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/5*d/e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)-1/4*d^2/e*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)*x-3/8*d^4/e*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)*x-3/8*d^6/e/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)*x/(e*x + d), x, algorithm="maxima")`

[Out] Exception raised: ValueError

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**Fricas [A]** time = 0.2907, size = 598, normalized size = 5.16

$$240 de^{11}x^{11} - 288 d^2e^{10}x^{10} - 1940 d^3e^9x^9 + 2400 d^4e^8x^8 + 5310 d^5e^7x^7 - 6960 d^6e^6x^6 - 6330 d^7e^5x^5 + 8640 d^8e^4x^4 + 3200$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)*x/(e*x + d), x, algorithm="fricas")`

[Out] `-1/240*(240*d*e^11*x^11 - 288*d^2*e^10*x^10 - 1940*d^3*e^9*x^9 + 2400*d^4*e^8*x^8 + 5310*d^5*e^7*x^7 - 6960*d^6*e^6*x^6 - 6330*d^7*e^5*x^5 + 8640*d^8*e^4*x^4 + 3200*d^9*e^3*x^3 - 3840*d^10*e^2*x^2 - 480*d^11*e*x - 30*(d^6*e^6*x^6 - 18*d^8*e^4*x^4 + 48*d^10*e^2*x^2 - 32*d^12 + 2*(3*d^7*e^4*x^4 - 16*d^9*e^2*x^2 + 16*d^11)*sqrt(-e^2*x^2 + d^2))*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - (40*e^11*x^11 - 48*d*e^10*x^10 - 790*d^2*e^9*x^9 + 960*d^3*e^8*x^8 + 3195*d^4*e^7*x^7 - 4080*d^5*e^6*x^6 - 4910*d^6*e^5*x^5 + 6720*d^7*e^4*x^4 + 2960*d^8*e^3*x^3 - 3840*d^9*e^2*x^2 - 480*d^10*e*x)*sqrt(-e^2*x^2 + d^2))/(e^8*x^6 - 18*d^2*e^6*x^4 + 48*d^4*e^4*x^2`

$$- 32*d^6*e^2 + 2*(3*d*e^6*x^4 - 16*d^3*e^4*x^2 + 16*d^5*e^2)*\sqrt{(-e^2*x^2 + d^2)}$$

**Sympy [A]** time = 35.5144, size = 580, normalized size = 5.

$$d^3 \left( \begin{cases} \frac{x^2\sqrt{d^2}}{2} & \text{for } e^2 = 0 \\ -\frac{(d^2 - e^2x^2)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases} \right) - d^2 e \left( \begin{cases} -\frac{id^4 \operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{id^3 x}{8e^2\sqrt{-1 + \frac{e^2x^2}{d^2}}} - \frac{3idx^3}{8\sqrt{-1 + \frac{e^2x^2}{d^2}}} + \frac{ie^2x^5}{4d\sqrt{-1 + \frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^4 \operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3x}{8e^2\sqrt{1 - \frac{e^2x^2}{d^2}}} + \frac{3dx^3}{8\sqrt{1 - \frac{e^2x^2}{d^2}}} - \frac{e^2x^5}{4d\sqrt{1 - \frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases} \right) - de^2 \left( \begin{cases} -\frac{2d^4\sqrt{d^2 - e^2x^2}}{15e^4} - \frac{d^2x^2\sqrt{d^2 - e^2x^2}}{15e^2} + \frac{x^4\sqrt{d^2 - e^2x^2}}{5} & \text{for } e \neq 0 \\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise} \end{cases} \right) + e^3 \left( \begin{cases} -\frac{id^6 \operatorname{acosh}\left(\frac{ex}{d}\right)}{16e^5} + \frac{id^5x}{16e^4\sqrt{-1 + \frac{e^2x^2}{d^2}}} - \frac{id^3x^3}{48e^2\sqrt{-1 + \frac{e^2x^2}{d^2}}} - \frac{5idx^5}{24\sqrt{-1 + \frac{e^2x^2}{d^2}}} + \frac{ie^2x^7}{6d\sqrt{-1 + \frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^6 \operatorname{asin}\left(\frac{ex}{d}\right)}{16e^5} - \frac{d^5x}{16e^4\sqrt{1 - \frac{e^2x^2}{d^2}}} + \frac{d^3x^3}{48e^2\sqrt{1 - \frac{e^2x^2}{d^2}}} + \frac{5dx^5}{24\sqrt{1 - \frac{e^2x^2}{d^2}}} - \frac{e^2x^7}{6d\sqrt{1 - \frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/(e\*x+d),x)

[Out] d\*\*3\*Piecewise((x\*\*2\*sqrt(d\*\*2)/2, Eq(e\*\*2, 0)), (- (d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(3\*e\*\*2), True)) - d\*\*2\*e\*Piecewise((-I\*d\*\*4\*acosh(e\*x/d)/(8\*e\*\*3) + I\*d\*\*3\*x/(8\*e\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 3\*I\*d\*x\*\*3/(8\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*5/(4\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*4\*asin(e\*x/d)/(8\*e\*\*3) - d\*\*3\*x/(8\*e\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 3\*d\*x\*\*3/(8\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*\*2\*x\*\*5/(4\*d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True)) - d\*e\*\*2\*Piecewise((-2\*d\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*e\*\*4) - d\*\*2\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*e\*\*2) + x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/5, Ne(e, 0)), (x\*\*4\*sqrt(d\*\*2)/4, True)) + e\*\*3\*Piecewise((-I\*d\*\*6\*acosh(e\*x/d)/(16\*e\*\*5) + I\*d\*\*5\*x/(16\*e\*\*4\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - I\*d\*\*3\*x\*\*3/(48\*e\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 5\*I\*d\*x\*\*5/(24\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*7/(6\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*6\*asin(e\*x/d)/(16\*e\*\*5) - d\*\*5\*x/(16\*e\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + d\*\*3\*x\*\*3/(48\*e\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 5\*d\*x\*\*5/(24\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*\*2\*x\*\*7/(6\*d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True))

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x/(e\*x + d),x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.107 \quad \int \frac{(d^2 - e^2 x^2)^{5/2}}{d + ex} dx$$

**Optimal.** Leaf size=100

$$\frac{1}{4} dx (d^2 - e^2 x^2)^{3/2} + \frac{(d^2 - e^2 x^2)^{5/2}}{5e} + \frac{3d^5 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{8e} + \frac{3}{8} d^3 x \sqrt{d^2 - e^2 x^2}$$

[Out] (3\*d^3\*x\*Sqrt[d^2 - e^2\*x^2])/8 + (d\*x\*(d^2 - e^2\*x^2)^(3/2))/4 + (d^2 - e^2\*x^2)^(5/2)/(5\*e) + (3\*d^5\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(8\*e)

**Rubi [A]** time = 0.0742063, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{1}{4} dx (d^2 - e^2 x^2)^{3/2} + \frac{(d^2 - e^2 x^2)^{5/2}}{5e} + \frac{3d^5 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{8e} + \frac{3}{8} d^3 x \sqrt{d^2 - e^2 x^2}$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^(5/2)/(d + e\*x), x]

[Out] (3\*d^3\*x\*Sqrt[d^2 - e^2\*x^2])/8 + (d\*x\*(d^2 - e^2\*x^2)^(3/2))/4 + (d^2 - e^2\*x^2)^(5/2)/(5\*e) + (3\*d^5\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(8\*e)

**Rubi in Sympy [A]** time = 16.7284, size = 83, normalized size = 0.83

$$\frac{3d^5 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{8e} + \frac{3d^3 x \sqrt{d^2 - e^2 x^2}}{8} + \frac{dx (d^2 - e^2 x^2)^{\frac{3}{2}}}{4} + \frac{(d^2 - e^2 x^2)^{\frac{5}{2}}}{5e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/(e\*x+d), x)

[Out] 3\*d\*\*5\*atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2))/(8\*e) + 3\*d\*\*3\*x\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/8 + d\*x\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/4 + (d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)/(5\*e)

**Mathematica [A]** time = 0.0915401, size = 91, normalized size = 0.91

$$\frac{15d^5 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right) + \sqrt{d^2 - e^2 x^2} (8d^4 + 25d^3 ex - 16d^2 e^2 x^2 - 10de^3 x^3 + 8e^4 x^4)}{40e}$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^(5/2)/(d + e\*x), x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(8\*d^4 + 25\*d^3\*e\*x - 16\*d^2\*e^2\*x^2 - 10\*d\*e^3\*x^3 + 8\*e^4\*x^4) + 15\*d^5\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(40\*e)

**Maple [A]** time = 0.009, size = 147, normalized size = 1.5

$$\frac{1}{5e} \left( - \left( x + \frac{d}{e} \right)^2 e^2 + 2de \left( x + \frac{d}{e} \right) \right)^{\frac{5}{2}} + \frac{dx}{4} \left( - \left( x + \frac{d}{e} \right)^2 e^2 + 2de \left( x + \frac{d}{e} \right) \right)^{\frac{3}{2}} + \frac{3d^3x}{8} \sqrt{- \left( x + \frac{d}{e} \right)^2 e^2 + 2de \left( x + \frac{d}{e} \right)} + \frac{3d^5}{8} \arctan \left( x \sqrt{e^2} \frac{1}{\sqrt{- \left( x + \frac{d}{e} \right)^2 e^2 + 2de \left( x + \frac{d}{e} \right)}} \right) \frac{1}{\sqrt{e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^(5/2)/(e\*x+d), x)

[Out] 1/5/e\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(5/2)+1/4\*d\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)\*x+3/8\*d^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x+3/8\*d^5/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))

**Maxima [A]** time = 0.789761, size = 147, normalized size = 1.47

$$-\frac{3i d^5 \arcsin\left(\frac{ex}{d} + 2\right)}{8e} + \frac{3}{8} \sqrt{e^2 x^2 + 4dex + 3d^2} d^3 x + \frac{3 \sqrt{e^2 x^2 + 4dex + 3d^2} d^4}{4e} + \frac{1}{4} (-e^2 x^2 + d^2)^{\frac{3}{2}} dx + \frac{(-e^2 x^2 + d^2)^{\frac{5}{2}}}{5e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/(e\*x + d), x, algorithm="maxima")

[Out] -3/8\*I\*d^5\*arcsin(e\*x/d + 2)/e + 3/8\*sqrt(e^2\*x^2 + 4\*d\*e\*x + 3\*d^2)\*d^3\*x + 3/4\*sqrt(e^2\*x^2 + 4\*d\*e\*x + 3\*d^2)\*d^4/e + 1/4\*(-e^2\*x^2 + d^2)^(3/2)\*d\*x + 1/5\*(-e^2\*x^2 + d^2)^(5/2)/e

**Fricas [A]** time = 0.290756, size = 504, normalized size = 5.04

$$8e^{10}x^{10} - 10de^9x^9 - 120d^2e^8x^8 + 155d^3e^7x^7 + 440d^4e^6x^6 - 605d^5e^5x^5 - 640d^6e^4x^4 + 860d^7e^3x^3 + 320d^8e^2x^2 - 400d^9e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/(e\*x + d), x, algorithm="fricas")

[Out] 1/40\*(8\*e^10\*x^10 - 10\*d\*e^9\*x^9 - 120\*d^2\*e^8\*x^8 + 155\*d^3\*e^7\*x^7 + 440\*d^4\*e^6\*x^6 - 605\*d^5\*e^5\*x^5 - 640\*d^6\*e^4\*x^4 + 860\*d^7\*e^3\*x^3 + 320\*d^8\*e^2\*x^2 - 400\*d^9\*e) \* sqrt(-e^2\*x^2 + d^2) \* arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) + 5\*(8\*d^8\*e^8\*x^8 - 10\*d^2\*e^7\*x^7 - 48\*d^3\*e^6\*x^6 + 65\*d^4\*e^5\*x^5 + 96\*d^5\*e^4\*x^4 - 132\*d^6\*e^3\*x^3 - 64\*d^7\*e^2\*x^2 + 80\*d^8\*e\*x) \* sqrt(-e^2\*x^2 + d^2) / (5\*d^5\*x^4 - 20\*d^3\*e^3\*x^2 + 16\*d^5\*e - (e^5\*x^4 - 12\*d^2\*e^3\*x^2 + 16\*d^4\*e) \* sqrt(-e^2\*x^2 + d^2))

**Sympy [A]** time = 21.9166, size = 435, normalized size = 4.35

$$\begin{aligned}
 & d^3 \left( \begin{cases} -\frac{id^2 \operatorname{acosh}\left(\frac{ex}{d}\right)}{2e} - \frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^2 \operatorname{asin}\left(\frac{ex}{d}\right)}{2e} + \frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise} \end{cases} \right) \\
 & - d^2 e \left( \begin{cases} \frac{x^2\sqrt{d^2}}{2} & \text{for } e^2 = 0 \\ -\frac{(d^2-e^2x^2)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases} \right) \\
 & - de^2 \left( \begin{cases} -\frac{id^4 \operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^4 \operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases} \right) \\
 & + e^3 \left( \begin{cases} -\frac{2d^4\sqrt{d^2-e^2x^2}}{15e^4} - \frac{d^2x^2\sqrt{d^2-e^2x^2}}{15e^2} + \frac{x^4\sqrt{d^2-e^2x^2}}{5} & \text{for } e \neq 0 \\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise} \end{cases} \right)
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/(e\*x+d),x)

[Out] d\*\*3\*Piecewise((-I\*d\*\*2\*acosh(e\*x/d)/(2\*e) - I\*d\*x/(2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*3/(2\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*2\*asin(e\*x/d)/(2\*e) + d\*x\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)/2, True)) - d\*\*2\*e\*Piecewise((x\*\*2\*sqrt(d\*\*2)/2, Eq(e\*\*2, 0)), (- (d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(3\*e\*\*2), True)) - d\*e\*\*2\*Piecewise((-I\*d\*\*4\*acosh(e\*x/d)/(8\*e\*\*3) + I\*d\*\*3\*x/(8\*e\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 3\*I\*d\*x\*\*3/(8\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*5/(4\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*4\*asin(e\*x/d)/(8\*e\*\*3) - d\*\*3\*x/(8\*e\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 3\*d\*x\*\*3/(8\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*\*2\*x\*\*5/(4\*d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True)) + e\*\*3\*Piecewise((-2\*d\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*e\*\*4) - d\*\*2\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*e\*\*2) + x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/5, Ne(e, 0)), (x\*\*4\*sqrt(d\*\*2)/4, True))

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2+ d^2)^(5/2)/(e\*x + d),x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.108 \quad \int \frac{(d^2 - e^2 x^2)^{5/2}}{x(d + ex)} dx$$

**Optimal.** Leaf size=113

$$\frac{1}{8}d^2(8d - 3ex)\sqrt{d^2 - e^2x^2} + \frac{1}{12}(4d - 3ex)(d^2 - e^2x^2)^{3/2} - \frac{3}{8}d^4 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) - d^4 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)$$

[Out]  $(d^2*(8*d - 3*e*x)*\text{Sqrt}[d^2 - e^2*x^2])/8 + ((4*d - 3*e*x)*(d^2 - e^2*x^2)^{(3/2)})/12 - (3*d^4*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/8 - d^4*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d]$

**Rubi [A]** time = 0.357984, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$

$$\frac{1}{8}d^2(8d - 3ex)\sqrt{d^2 - e^2x^2} + \frac{1}{12}(4d - 3ex)(d^2 - e^2x^2)^{3/2} - \frac{3}{8}d^4 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) - d^4 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^(5/2)/(x\*(d + e\*x)), x]

[Out]  $(d^2*(8*d - 3*e*x)*\text{Sqrt}[d^2 - e^2*x^2])/8 + ((4*d - 3*e*x)*(d^2 - e^2*x^2)^{(3/2)})/12 - (3*d^4*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/8 - d^4*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d]$

**Rubi in Sympy [A]** time = 49.9324, size = 95, normalized size = 0.84

$$-\frac{3d^4 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{8} - d^4 \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right) + \frac{d^2(8d - 3ex)\sqrt{d^2 - e^2x^2}}{8} + \frac{(4d - 3ex)(d^2 - e^2x^2)^{3/2}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x/(e\*x+d), x)

[Out]  $-3*d**4*\operatorname{atan}(e*x/\operatorname{sqrt}(d**2 - e**2*x**2))/8 - d**4*\operatorname{atanh}(\operatorname{sqrt}(d**2 - e**2*x**2)/d) + d**2*(8*d - 3*e*x)*\operatorname{sqrt}(d**2 - e**2*x**2)/8 + (4*d - 3*e*x)*(d**2 - e**2*x**2)**(3/2)/12$

**Mathematica [A]** time = 0.101536, size = 108, normalized size = 0.96

$$d^4 \log(x) - d^4 \log\left(\sqrt{d^2 - e^2x^2} + d\right) - \frac{3}{8}d^4 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) + \frac{1}{24}\sqrt{d^2 - e^2x^2}(32d^3 - 15d^2ex - 8de^2x^2 + 6e^3x^3)$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^(5/2)/(x\*(d + e\*x)), x]

[Out]  $(\sqrt{d^2 - e^2 x^2}) \cdot (32d^3 - 15d^2 e x - 8d e^2 x^2 + 6e^3 x^3) / 24 - (3d^4 \operatorname{ArcTan}[(e x) / \sqrt{d^2 - e^2 x^2}]) / 8 + d^4 \operatorname{Log}[x] - d^4 \operatorname{Log}[d + \sqrt{d^2 - e^2 x^2}]$

**Maple [B]** time = 0.015, size = 245, normalized size = 2.2

$$\begin{aligned} & \frac{1}{5d} (-e^2 x^2 + d^2)^{\frac{5}{2}} + \frac{d}{3} (-e^2 x^2 + d^2)^{\frac{3}{2}} + d^3 \sqrt{-e^2 x^2 + d^2} \\ & - d^5 \ln \left( \frac{1}{x} \left( 2d^2 + 2\sqrt{d^2} \sqrt{-e^2 x^2 + d^2} \right) \right) \frac{1}{\sqrt{d^2}} - \frac{1}{5d} \left( - \left( x + \frac{d}{e} \right)^2 e^2 + 2de \left( x + \frac{d}{e} \right) \right)^{\frac{5}{2}} \\ & - \frac{ex}{4} \left( - \left( x + \frac{d}{e} \right)^2 e^2 + 2de \left( x + \frac{d}{e} \right) \right)^{\frac{3}{2}} - \frac{3d^2 ex}{8} \sqrt{- \left( x + \frac{d}{e} \right)^2 e^2 + 2de \left( x + \frac{d}{e} \right)} \\ & - \frac{3d^4 e}{8} \arctan \left( x \sqrt{e^2} \frac{1}{\sqrt{- \left( x + \frac{d}{e} \right)^2 e^2 + 2de \left( x + \frac{d}{e} \right)}} \right) \frac{1}{\sqrt{e^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-e^2*x^2+d^2)^(5/2)/x/(e*x+d),x)`

[Out]  $1/5/d * (-e^2 * x^2 + d^2)^{(5/2)} + 1/3 * d * (-e^2 * x^2 + d^2)^{(3/2)} + d^3 * (-e^2 * x^2 + d^2)^{(1/2)} - d^5 / (d^2)^{(1/2)} * \ln((2 * d^2 + 2 * (d^2)^{(1/2)} * (-e^2 * x^2 + d^2)^{(1/2)}) / x) - 1/5/d * (- (x+d/e)^2 * e^2 + 2 * d * e * (x+d/e))^{(5/2)} - 1/4 * e * (- (x+d/e)^2 * e^2 + 2 * d * e * (x+d/e))^{(3/2)} * x - 3/8 * d^2 * e * (- (x+d/e)^2 * e^2 + 2 * d * e * (x+d/e))^{(1/2)} * x - 3/8 * d^4 * e / (e^2)^{(1/2)} * \arctan((e^2)^{(1/2)} * x / (- (x+d/e)^2 * e^2 + 2 * d * e * (x+d/e))^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)*x),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.291915, size = 524, normalized size = 4.64

$$24 d e^7 x^7 - 32 d^2 e^6 x^6 - 132 d^3 e^5 x^5 + 192 d^4 e^4 x^4 + 228 d^5 e^3 x^3 - 192 d^6 e^2 x^2 - 120 d^7 e x - 18 \left( d^4 e^4 x^4 - 8 d^6 e^2 x^2 + 8 d^8 + 4 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)*x),x, algorithm="fricas")`

[Out]  $-1/24 * (24 * d * e^7 * x^7 - 32 * d^2 * e^6 * x^6 - 132 * d^3 * e^5 * x^5 + 192 * d^4 * e^4 * x^4 + 228 * d^5 * e^3 * x^3 - 192 * d^6 * e^2 * x^2 - 120 * d^7 * e * x - 18 * (d^4 * e^4 * x^4 - 8 * d^6 * e^2 * x^2 + 8 * d^8 + 4 * (d^5 * e^2 * x^2 - 2 * d^7) * \sqrt{-e^2 * x^2 + d^2})) * \arctan(-(d - \sqrt{-e^2 * x^2 + d^2}) / (e * x)) - 24 * (d^4 * e^4 * x^4 - 8 * d^6 * e^2 * x^2 + 8 * d^8 + 4 * (d^5 * e^2 * x^2 - 2 * d^7) * \sqrt{-e^2 * x^2 + d^2}) * \log(-(d - \sqrt{-e^2 * x^2 + d^2}) / x) - (6 * e^7 * x$



$$^7 - 8*d*e^6*x^6 - 63*d^2*e^5*x^5 + 96*d^3*e^4*x^4 + 168*d^4*e^3*x^3 - 192*d^5*e^2*x^2 - 120*d^6*e*x)*sqrt(-e^2*x^2 + d^2))/(e^4*x^4 - 8*d^2*e^2*x^2 + 8*d^4 + 4*(d*e^2*x^2 - 2*d^3)*sqrt(-e^2*x^2 + d^2))$$

**Sympy [A]** time = 26.7394, size = 469, normalized size = 4.15

$$d^3 \left( \begin{array}{l} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}} - d \operatorname{acosh}\left(\frac{d}{ex}\right) - \frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} \quad \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}} + id \operatorname{asin}\left(\frac{d}{ex}\right) + \frac{ix}{\sqrt{-\frac{d^2}{e^2x^2}+1}} \quad \text{otherwise} \end{array} \right) \\ - d^2 e \left( \begin{array}{l} \frac{id^2 \operatorname{acosh}\left(\frac{ex}{d}\right)}{2e} - \frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} \quad \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^2 \operatorname{asin}\left(\frac{ex}{d}\right)}{2e} + \frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} \quad \text{otherwise} \end{array} \right) \\ - de^2 \left( \begin{array}{l} \frac{x^2\sqrt{d^2}}{2} \quad \text{for } e^2 = 0 \\ -\frac{(d^2-e^2x^2)^{\frac{3}{2}}}{3e^2} \quad \text{otherwise} \end{array} \right) \\ + e^3 \left( \begin{array}{l} -\frac{id^4 \operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} \quad \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^4 \operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} \quad \text{otherwise} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x/(e\*x+d),x)

[Out] d\*\*3\*Piecewise((d\*\*2/(e\*x\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) - d\*acosh(d/(e\*x)) - e\*x/sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (-I\*d\*\*2/(e\*x\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) + I\*d\*asin(d/(e\*x)) + I\*e\*x/sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1), True)) - d\*\*2\*e\*Piecewise((-I\*d\*\*2\*acosh(e\*x/d)/(2\*e) - I\*d\*x/(2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*3/(2\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2))), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*2\*asin(e\*x/d)/(2\*e) + d\*x\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)/2, True)) - d\*e\*\*2\*Piecewise((x\*\*2\*sqrt(d\*\*2)/2, Eq(e\*\*2, 0)), (- (d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(3\*e\*\*2), True)) + e\*\*3\*Piecewise((-I\*d\*\*4\*acosh(e\*x/d)/(8\*e\*\*3) + I\*d\*\*3\*x/(8\*e\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 3\*I\*d\*x\*\*3/(8\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*5/(4\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2))), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*4\*asin(e\*x/d)/(8\*e\*\*3) - d\*\*3\*x/(8\*e\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 3\*d\*x\*\*3/(8\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*\*2\*x\*\*5/(4\*d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2))), True))

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)\*x),x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.109 \quad \int \frac{(d^2 - e^2 x^2)^{5/2}}{x^2(d + ex)} dx$$

**Optimal.** Leaf size=115

$$-\frac{1}{2}de(2d + 3ex)\sqrt{d^2 - e^2x^2} - \frac{(3d + ex)(d^2 - e^2x^2)^{3/2}}{3x} - \frac{3}{2}d^3e \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) + d^3e \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)$$

[Out]  $-(d*e*(2*d + 3*e*x)*\text{Sqrt}[d^2 - e^2*x^2])/2 - ((3*d + e*x)*(d^2 - e^2*x^2)^{(3/2)})/(3*x) - (3*d^3*e*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/2 + d^3*e*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d]$

**Rubi [A]** time = 0.359344, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{1}{2}de(2d + 3ex)\sqrt{d^2 - e^2x^2} - \frac{(3d + ex)(d^2 - e^2x^2)^{3/2}}{3x} - \frac{3}{2}d^3e \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) + d^3e \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d^2 - e^2*x^2)^{(5/2)}/(x^2*(d + e*x)), x]$

[Out]  $-(d*e*(2*d + 3*e*x)*\text{Sqrt}[d^2 - e^2*x^2])/2 - ((3*d + e*x)*(d^2 - e^2*x^2)^{(3/2)})/(3*x) - (3*d^3*e*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/2 + d^3*e*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d]$

**Rubi in Sympy [A]** time = 47.8671, size = 99, normalized size = 0.86

$$-\frac{3d^3e \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{2} + d^3e \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right) - \frac{de(4d + 6ex)\sqrt{d^2 - e^2x^2}}{4} - \frac{(3d + ex)(d^2 - e^2x^2)^{3/2}}{3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((-e^{**2}*x^{**2}+d^{**2})^{**}(5/2)/x^{**2}/(e*x+d), x)$

[Out]  $-3*d^{**3}*e*\operatorname{atan}(e*x/\operatorname{sqrt}(d^{**2} - e^{**2}*x^{**2}))/2 + d^{**3}*e*\operatorname{atanh}(\operatorname{sqrt}(d^{**2} - e^{**2}*x^{**2})/d) - d*e*(4*d + 6*e*x)*\operatorname{sqrt}(d^{**2} - e^{**2}*x^{**2})/4 - (3*d + e*x)*(d^{**2} - e^{**2}*x^{**2})^{**}(3/2)/(3*x)$

**Mathematica [A]** time = 0.151217, size = 114, normalized size = 0.99

$$-d^3e \log(x) + d^3e \log\left(\sqrt{d^2 - e^2x^2} + d\right) - \frac{3}{2}d^3e \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) + \sqrt{d^2 - e^2x^2} \left(-\frac{d^3}{x} - \frac{4d^2e}{3} - \frac{1}{2}de^2x + \frac{e^3x^2}{3}\right)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(d^2 - e^2*x^2)^{(5/2)}/(x^2*(d + e*x)), x]$

[Out]  $\text{Sqrt}[d^2 - e^2 x^2] * ((-4 d^2 e)/3 - d^3/x - (d e^2 x)/2 + (e^3 x^2)/3) - (3 d^3 e \text{ArcTan}[(e x)/\text{Sqrt}[d^2 - e^2 x^2]])/2 - d^3 e \text{Log}[x] + d^3 e \text{Log}[d + \text{Sqrt}[d^2 - e^2 x^2]]$

**Maple [B]** time = 0.016, size = 380, normalized size = 3.3

$$\begin{aligned} & -\frac{1}{d^3 x} (-e^2 x^2 + d^2)^{\frac{7}{2}} - \frac{e^2 x}{d^3} (-e^2 x^2 + d^2)^{\frac{5}{2}} - \frac{5 e^2 x}{4 d} (-e^2 x^2 + d^2)^{\frac{3}{2}} - \frac{15 d e^2 x}{8} \sqrt{-e^2 x^2 + d^2} \\ & - \frac{15 e^2 d^3}{8} \arctan\left(x \sqrt{e^2} \frac{1}{\sqrt{-e^2 x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} + \frac{e}{5 d^2} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)\right)^{\frac{5}{2}} \\ & + \frac{e^2 x}{4 d} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)\right)^{\frac{3}{2}} + \frac{3 d e^2 x}{8} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)} \\ & + \frac{3 e^2 d^3}{8} \arctan\left(x \sqrt{e^2} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)}}\right) \frac{1}{\sqrt{e^2}} - \frac{e}{5 d^2} (-e^2 x^2 + d^2)^{\frac{5}{2}} \\ & - \frac{e}{3} (-e^2 x^2 + d^2)^{\frac{3}{2}} - e d^2 \sqrt{-e^2 x^2 + d^2} + e d^4 \ln\left(\frac{1}{x} \left(2 d^2 + 2 \sqrt{d^2} \sqrt{-e^2 x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((-e^2 x^2 + d^2)^{(5/2)}/x^2/(e x + d), x)$

[Out]  $-1/d^3/x * (-e^2 x^2 + d^2)^{(7/2)} - 1/d^3 * e^2 * x * (-e^2 x^2 + d^2)^{(5/2)} - 5/4 * e^2/d * x * (-e^2 x^2 + d^2)^{(3/2)} - 15/8 * d * e^2 * x * (-e^2 x^2 + d^2)^{(1/2)} - 15/8 * e^2 * d^3/(e^2)^{(1/2)} * \arctan((e^2)^{(1/2)} * x/(-e^2 x^2 + d^2)^{(1/2)}) + 1/5 * e/d^2 * (-x + d/e)^2 * e^2 + 2 * d * e * (x + d/e)^{(5/2)} + 1/4 * e^2/d * (-x + d/e)^2 * e^2 + 2 * d * e * (x + d/e)^{(3/2)} * x + 3/8 * e^2 * d * (-x + d/e)^2 * e^2 + 2 * d * e * (x + d/e)^{(1/2)} * x + 3/8 * e^2 * d^3/(e^2)^{(1/2)} * \arctan((e^2)^{(1/2)} * x/(-x + d/e)^2 * e^2 + 2 * d * e * (x + d/e)^{(1/2)}) - 1/5 * e/d^2 * (-e^2 x^2 + d^2)^{(5/2)} - 1/3 * e * (-e^2 x^2 + d^2)^{(3/2)} - e * d^2 * (-e^2 x^2 + d^2)^{(1/2)} + e * d^4/(d^2)^{(1/2)} * \ln((2 * d^2 + 2 * (d^2)^{(1/2)} * (-e^2 x^2 + d^2)^{(1/2)})/x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((-e^2 x^2 + d^2)^{(5/2)}/((e x + d) * x^2), x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.298381, size = 532, normalized size = 4.63

$$8 d e^7 x^7 - 12 d^2 e^6 x^6 - 48 d^3 e^5 x^5 + 12 d^4 e^4 x^4 + 48 d^5 e^3 x^3 + 48 d^6 e^2 x^2 - 48 d^8 - 18 \left( d^3 e^5 x^5 - 8 d^5 e^3 x^3 + 8 d^7 e x + 4 (d^4 e^3 x^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((-e^2 x^2 + d^2)^{(5/2)}/((e x + d) * x^2), x, \text{algorithm}="fricas")$

[Out] 
$$-1/6*(8*d*e^7*x^7 - 12*d^2*e^6*x^6 - 48*d^3*e^5*x^5 + 12*d^4*e^4*x^4 + 48*d^5*e^3*x^3 + 48*d^6*e^2*x^2 - 48*d^8 - 18*(d^3*e^5*x^5 - 8*d^5*e^3*x^3 + 8*d^7*e*x + 4*(d^4*e^3*x^3 - 2*d^6*e*x)*\sqrt{-e^2*x^2 + d^2})*\arctan(-(d - \sqrt{-e^2*x^2 + d^2})/(e*x)) + 6*(d^3*e^5*x^5 - 8*d^5*e^3*x^3 + 8*d^7*e*x + 4*(d^4*e^3*x^3 - 2*d^6*e*x)*\sqrt{-e^2*x^2 + d^2})*\log(-(d - \sqrt{-e^2*x^2 + d^2})/x) - (2*e^7*x^7 - 3*d*e^6*x^6 - 24*d^2*e^5*x^5 + 18*d^3*e^4*x^4 + 48*d^4*e^3*x^3 + 24*d^5*e^2*x^2 - 48*d^7)*\sqrt{-e^2*x^2 + d^2})/(e^4*x^5 - 8*d^2*e^2*x^3 + 8*d^4*x + 4*(d*e^2*x^3 - 2*d^3*x)*\sqrt{-e^2*x^2 + d^2})$$

**Sympy [A]** time = 21.0631, size = 386, normalized size = 3.36

$$d^3 \left( \begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}} + ie \operatorname{acosh}\left(\frac{ex}{d}\right) - \frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}} - e \operatorname{asin}\left(\frac{ex}{d}\right) + \frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases} \right) - d^2e \left( \begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}} - d \operatorname{acosh}\left(\frac{d}{ex}\right) - \frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}} + id \operatorname{asin}\left(\frac{d}{ex}\right) + \frac{ie^2x}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases} \right) - de^2 \left( \begin{cases} \frac{id^2 \operatorname{acosh}\left(\frac{ex}{d}\right)}{2e} - \frac{id^2x}{2\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^2 \operatorname{asin}\left(\frac{ex}{d}\right)}{2e} + \frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise} \end{cases} \right) + e^3 \left( \begin{cases} \frac{x^2\sqrt{d^2}}{2} & \text{for } e^2 = 0 \\ -\frac{(d^2-e^2x^2)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e**2*x**2+d**2)**(5/2)/x**2/(e*x+d),x)`

[Out] 
$$d^{**3}\operatorname{Piecewise}((I*d/(x*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}})) + I*e*\operatorname{acosh}(e*x/d) - I*e^{**2}*x/(d*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}}), \operatorname{Abs}(e^{**2}*x^{**2}/d^{**2}) > 1), (-d/(x*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}})) - e*\operatorname{asin}(e*x/d) + e^{**2}*x/(d*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}}), \operatorname{True})) - d^{**2}*e*\operatorname{Piecewise}((d^{**2}/(e*x*\sqrt{d^{**2}/(e^{**2}*x^{**2}) - 1})) - d*\operatorname{acosh}(d/(e*x)) - e*x/\sqrt{d^{**2}/(e^{**2}*x^{**2}) - 1}, \operatorname{Abs}(d^{**2}/(e^{**2}*x^{**2})) > 1), (-I*d^{**2}/(e*x*\sqrt{-d^{**2}/(e^{**2}*x^{**2}) + 1})) + I*d*\operatorname{asin}(d/(e*x)) + I*e*x/\sqrt{-d^{**2}/(e^{**2}*x^{**2}) + 1}, \operatorname{True})) - d*e^{**2}*\operatorname{Piecewise}((-I*d^{**2}*\operatorname{acosh}(e*x/d)/(2*e) - I*d*x/(2*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}})) + I*e^{**2}*x^{**3}/(2*d*\sqrt{-1 + e^{**2}*x^{**2}/d^{**2}}), \operatorname{Abs}(e^{**2}*x^{**2}/d^{**2}) > 1), (d^{**2}*\operatorname{asin}(e*x/d)/(2*e) + d*x*\sqrt{1 - e^{**2}*x^{**2}/d^{**2}}/2, \operatorname{True})) + e^{**3}*\operatorname{Piecewise}((x^{**2}*\sqrt{d^{**2}}/2, \operatorname{Eq}(e^{**2}, 0)), (-d^{**2} - e^{**2}*x^{**2})^{**3/2}/(3*e^{**2}), \operatorname{True}))$$

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)*x^2),x, algorithm="giac")`

[Out] Exception raised: NotImplementedError

$$3.110 \quad \int \frac{(d^2 - e^2 x^2)^{5/2}}{x^3(d+ex)} dx$$

**Optimal.** Leaf size=121

$$\frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{2x^2} + \frac{3}{2}d^2e^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{3}{2}d^2e^2 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

[Out] (3\*d\*e\*(d - e\*x)\*Sqrt[d^2 - e^2\*x^2])/(2\*x) - ((d + e\*x)\*(d^2 - e^2\*x^2)^(3/2))/(2\*x^2) + (3\*d^2\*e^2\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/2 + (3\*d^2\*e^2\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/2

**Rubi [A]** time = 0.369046, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$

$$\frac{3de(d-ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d+ex)(d^2-e^2x^2)^{3/2}}{2x^2} + \frac{3}{2}d^2e^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{3}{2}d^2e^2 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^(5/2)/(x^3\*(d + e\*x)), x]

[Out] (3\*d\*e\*(d - e\*x)\*Sqrt[d^2 - e^2\*x^2])/(2\*x) - ((d + e\*x)\*(d^2 - e^2\*x^2)^(3/2))/(2\*x^2) + (3\*d^2\*e^2\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/2 + (3\*d^2\*e^2\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/2

**Rubi in Sympy [A]** time = 50.646, size = 112, normalized size = 0.93

$$\frac{3d^2e^2 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2} + \frac{3d^2e^2 \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2} + \frac{3de(4d-4ex)\sqrt{d^2-e^2x^2}}{8x} - \frac{(2d+2ex)(d^2-e^2x^2)^{3/2}}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*3/(e\*x+d), x)

[Out] 3\*d\*\*2\*e\*\*2\*atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2))/2 + 3\*d\*\*2\*e\*\*2\*atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d)/2 + 3\*d\*e\*(4\*d - 4\*e\*x)\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(8\*x) - (2\*d + 2\*e\*x)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(4\*x\*\*2)

**Mathematica [A]** time = 0.200284, size = 119, normalized size = 0.98

$$\frac{1}{2} \left( 3d^2e^2 \log\left(\sqrt{d^2 - e^2x^2} + d\right) + 3d^2e^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) - 3d^2e^2 \log(x) + \frac{\sqrt{d^2 - e^2x^2}(-d^3 + 2d^2ex - 2de^2x^2 + e^3x^3)}{x^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^(5/2)/(x^3\*(d + e\*x)), x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(-d^3 + 2\*d^2\*e\*x - 2\*d\*e^2\*x^2 + e^3\*x^3))/x^2 + 3\*d^2\*e^2\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]]) - 3\*d^2\*e^2\*Lo

$g[x] + 3*d^2*e^2*Log[d + Sqrt[d^2 - e^2*x^2]])/2$

**Maple [B]** time = 0.018, size = 411, normalized size = 3.4

$$\begin{aligned}
 & -\frac{1}{2d^3x^2}(-e^2x^2+d^2)^{\frac{7}{2}} - \frac{3e^2}{10d^3}(-e^2x^2+d^2)^{\frac{5}{2}} - \frac{e^2}{2d}(-e^2x^2+d^2)^{\frac{3}{2}} - \frac{3de^2}{2}\sqrt{-e^2x^2+d^2} \\
 & + \frac{3e^2d^3}{2}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}} - \frac{e^2}{5d^3}\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}} \\
 & - \frac{e^3x}{4d^2}\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}} - \frac{3e^3x}{8}\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)} \\
 & - \frac{3d^2e^3}{8}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)\frac{1}{\sqrt{e^2}} \\
 & + \frac{e}{d^4x}(-e^2x^2+d^2)^{\frac{7}{2}} + \frac{e^3x}{d^4}(-e^2x^2+d^2)^{\frac{5}{2}} + \frac{5e^3x}{4d^2}(-e^2x^2+d^2)^{\frac{3}{2}} \\
 & + \frac{15e^3x}{8}\sqrt{-e^2x^2+d^2} + \frac{15d^2e^3}{8}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d), x)`

[Out]  $-1/2/d^3/x^2*(-e^2*x^2+d^2)^(7/2)-3/10/d^3*e^2*(-e^2*x^2+d^2)^(5/2)-1/2*e^2/d*(-e^2*x^2+d^2)^(3/2)-3/2*d*e^2*(-e^2*x^2+d^2)^(1/2)+3/2*e^2*d^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-1/5/d^3*e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)-1/4/d^2*e^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)*x-3/8*e^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)*x-3/8*d^2*e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))+e/d^4/x*(-e^2*x^2+d^2)^(7/2)+e^3/d^4*x*(-e^2*x^2+d^2)^(5/2)+5/4*e^3/d^2*x*(-e^2*x^2+d^2)^(3/2)+15/8*e^3*x*(-e^2*x^2+d^2)^(1/2)+15/8*e^3*d^2/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)*x^3), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.298586, size = 576, normalized size = 4.76

$$4de^7x^7 - 6d^2e^6x^6 - 4d^3e^5x^5 + 4d^4e^4x^4 - 16d^5e^3x^3 + 12d^6e^2x^2 + 16d^7ex - 8d^8 + 6(d^2e^6x^6 - 8d^4e^4x^4 + 8d^6e^2x^2 + 4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)*x^3), x, algorithm="fricas")`

```
[Out] -1/2*(4*d*e^7*x^7 - 6*d^2*e^6*x^6 - 4*d^3*e^5*x^5 + 4*d^4*e^4*x^4
- 16*d^5*e^3*x^3 + 12*d^6*e^2*x^2 + 16*d^7*e*x - 8*d^8 + 6*(d^2*
e^6*x^6 - 8*d^4*e^4*x^4 + 8*d^6*e^2*x^2 + 4*(d^3*e^4*x^4 - 2*d^5*
e^2*x^2)*sqrt(-e^2*x^2 + d^2))*arctan(-(d - sqrt(-e^2*x^2 + d^2))
/(e*x)) + 3*(d^2*e^6*x^6 - 8*d^4*e^4*x^4 + 8*d^6*e^2*x^2 + 4*(d^3
*e^4*x^4 - 2*d^5*e^2*x^2)*sqrt(-e^2*x^2 + d^2))*log(-(d - sqrt(-e
^2*x^2 + d^2))/x) - (e^7*x^7 - 2*d*e^6*x^6 - 6*d^2*e^5*x^5 + 7*d^
3*e^4*x^4 - 8*d^4*e^3*x^3 + 8*d^5*e^2*x^2 + 16*d^6*e*x - 8*d^7)*s
qrt(-e^2*x^2 + d^2))/(e^4*x^6 - 8*d^2*e^2*x^4 + 8*d^4*x^2 + 4*(d*
e^2*x^4 - 2*d^3*x^2)*sqrt(-e^2*x^2 + d^2))
```

---

**Sympy [A]** time = 25.719, size = 461, normalized size = 3.81

$$d^3 \left( \begin{array}{l} \left( -\frac{d^2}{2ex^3\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e}{2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^2 \operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} \quad \text{for } \left| \frac{d^2}{e^2x^2} \right| > 1 \right) \\ \left( -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{2x} - \frac{ie^2 \operatorname{asin}\left(\frac{d}{ex}\right)}{2d} \quad \text{otherwise} \right) \end{array} \right) \\ - d^2 e \left( \begin{array}{l} \left( \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}} + ie \operatorname{acosh}\left(\frac{ex}{d}\right) - \frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} \quad \text{for } \left| \frac{e^2x^2}{d^2} \right| > 1 \right) \\ \left( -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}} - e \operatorname{asin}\left(\frac{ex}{d}\right) + \frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} \quad \text{otherwise} \right) \end{array} \right) \\ - de^2 \left( \begin{array}{l} \left( \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}} - d \operatorname{acosh}\left(\frac{d}{ex}\right) - \frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} \quad \text{for } \left| \frac{d^2}{e^2x^2} \right| > 1 \right) \\ \left( -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}} + id \operatorname{asin}\left(\frac{d}{ex}\right) + \frac{iox}{\sqrt{-\frac{d^2}{e^2x^2}+1}} \quad \text{otherwise} \right) \end{array} \right) \\ + e^3 \left( \begin{array}{l} \left( -\frac{id^2 \operatorname{acosh}\left(\frac{ex}{d}\right)}{2e} - \frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} \quad \text{for } \left| \frac{e^2x^2}{d^2} \right| > 1 \right) \\ \left( \frac{d^2 \operatorname{asin}\left(\frac{ex}{d}\right)}{2e} + \frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} \quad \text{otherwise} \right) \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**3/(e*x+d), x)
```

```
[Out] d**3*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(
2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(
d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) -
I*e**2*asin(d/(e*x))/(2*d), True)) - d**2*e*Piecewise((I*d/(x*sq
rt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1
+ e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1 - e
**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**
2)), True)) - d*e**2*Piecewise((d**2/(e*x*sqrt(d**2/(e**2*x**2) -
1)) - d*acosh(d/(e*x)) - e*x/sqrt(d**2/(e**2*x**2) - 1), Abs(d**
2/(e**2*x**2)) > 1), (-I*d**2/(e*x*sqrt(-d**2/(e**2*x**2) + 1)) +
I*d*asin(d/(e*x)) + I*e*x/sqrt(-d**2/(e**2*x**2) + 1), True)) +
e**3*Piecewise((-I*d**2*acosh(e*x/d)/(2*e) - I*d*x/(2*sqrt(-1 + e
**2*x**2/d**2)) + I*e**2*x**3/(2*d*sqrt(-1 + e**2*x**2/d**2)), Ab
s(e**2*x**2/d**2) > 1), (d**2*asin(e*x/d)/(2*e) + d*x*sqrt(1 - e
**2*x**2/d**2)/2, True))
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)*x^3), x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.111 \quad \int \frac{(d^2 - e^2 x^2)^{5/2}}{x^4(d+ex)} dx$$

**Optimal.** Leaf size=120

$$\frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d-3ex)(d^2-e^2x^2)^{3/2}}{6x^3} + de^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{3}{2}de^3 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

[Out]  $(e^2(2d+3ex)\sqrt{d^2-e^2x^2})/(2x) - ((2d-3ex)(d^2-e^2x^2)^{3/2})/(6x^3) + d^3e^3 \operatorname{ArcTan}[(ex)/\sqrt{d^2-e^2x^2}] - (3d^3e^3 \operatorname{ArcTanh}[\sqrt{d^2-e^2x^2}/d])/2$

**Rubi [A]** time = 0.370038, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{e^2(2d+3ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(2d-3ex)(d^2-e^2x^2)^{3/2}}{6x^3} + de^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{3}{2}de^3 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(d^2 - e^2 x^2)^{5/2}/(x^4(d + e x)), x]$

[Out]  $(e^2(2d+3ex)\sqrt{d^2-e^2x^2})/(2x) - ((2d-3ex)(d^2-e^2x^2)^{3/2})/(6x^3) + d^3e^3 \operatorname{ArcTan}[(ex)/\sqrt{d^2-e^2x^2}] - (3d^3e^3 \operatorname{ArcTanh}[\sqrt{d^2-e^2x^2}/d])/2$

**Rubi in Sympy [A]** time = 51.1633, size = 104, normalized size = 0.87

$$de^3 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{3de^3 \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2} + \frac{e^2(4d+6ex)\sqrt{d^2-e^2x^2}}{4x} - \frac{(2d-3ex)(d^2-e^2x^2)^{3/2}}{6x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{rubi\_integrate}((-e^{**2}x^{**2}+d^{**2})^{**}(5/2)/x^{**4}/(e*x+d), x)$

[Out]  $d^3e^3 \operatorname{atan}(ex/\sqrt{d^2-e^2x^2}) - 3d^3e^3 \operatorname{atanh}(\sqrt{d^2-e^2x^2}/d)/2 + e^2(4d+6ex)\sqrt{d^2-e^2x^2}/(4x) - (2d-3ex)(d^2-e^2x^2)^{3/2}/(6x^3)$

**Mathematica [A]** time = 0.162312, size = 116, normalized size = 0.97

$$-\frac{3}{2}de^3 \log\left(\sqrt{d^2-e^2x^2}+d\right) + de^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \left(-\frac{d^3}{3x^3} + \frac{d^2e}{2x^2} + \frac{4de^2}{3x} + e^3\right)\sqrt{d^2-e^2x^2} + \frac{3}{2}de^3 \log(x)$$

Antiderivative was successfully verified.

[In]  $\operatorname{Integrate}[(d^2 - e^2 x^2)^{5/2}/(x^4(d + e x)), x]$

[Out]  $(e^3 - d^3/(3x^3) + (d^2e)/(2x^2) + (4d^2e^2)/(3x))\sqrt{d^2 - e^2x^2} + d^3e^3 \operatorname{ArcTan}[(ex)/\sqrt{d^2 - e^2x^2}] + (3d^3e^3 \operatorname{Log}[\sqrt{d^2 - e^2x^2} + d])/2$



$\log[x])/2 - (3*d*e^3*\text{Log}[d + \text{Sqrt}[d^2 - e^2*x^2]])/2$

**Maple [B]** time = 0.018, size = 439, normalized size = 3.7

$$\begin{aligned}
 & -\frac{1}{3d^3x^3}(-e^2x^2+d^2)^{\frac{7}{2}} + \frac{e^2}{3d^5x}(-e^2x^2+d^2)^{\frac{7}{2}} \\
 & + \frac{e^4x}{3d^5}(-e^2x^2+d^2)^{\frac{5}{2}} + \frac{5e^4x}{12d^3}(-e^2x^2+d^2)^{\frac{3}{2}} + \frac{5e^4x}{8d}\sqrt{-e^2x^2+d^2} \\
 & + \frac{5de^4}{8}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}} + \frac{e^3}{5d^4}\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}} \\
 & + \frac{e^4x}{4d^3}\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}} + \frac{3e^4x}{8d}\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)} \\
 & + \frac{3de^4}{8}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)\frac{1}{\sqrt{e^2}} \\
 & + \frac{e}{2d^4x^2}(-e^2x^2+d^2)^{\frac{7}{2}} + \frac{3e^3}{10d^4}(-e^2x^2+d^2)^{\frac{5}{2}} + \frac{e^3}{2d^2}(-e^2x^2+d^2)^{\frac{3}{2}} \\
 & + \frac{3e^3}{2}\sqrt{-e^2x^2+d^2} - \frac{3e^3d^2}{2}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d),x)`

[Out]  $-1/3/d^3/x^3*(-e^2*x^2+d^2)^{(7/2)}+1/3/d^5*e^2/x*(-e^2*x^2+d^2)^{(7/2)}+1/3/d^5*e^4*x*(-e^2*x^2+d^2)^{(5/2)}+5/12*e^4/d^3*x*(-e^2*x^2+d^2)^{(3/2)}+5/8*e^4/d*x*(-e^2*x^2+d^2)^{(1/2)}+5/8*d*e^4/(e^2)^{(1/2)}*\arctan((e^2)^{(1/2)}*x/(-e^2*x^2+d^2)^{(1/2)})+1/5/d^4*e^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^{(5/2)}+1/4/d^3*e^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^{(3/2)}*x+3/8/d*e^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^{(1/2)}*x+3/8*d*e^4/(e^2)^{(1/2)}*\arctan((e^2)^{(1/2)}*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^{(1/2)})+1/2*e/d^4/x^2*(-e^2*x^2+d^2)^{(7/2)}+3/10*e^3/d^4*(-e^2*x^2+d^2)^{(5/2)}+1/2*e^3/d^2*(-e^2*x^2+d^2)^{(3/2)}+3/2*e^3*(-e^2*x^2+d^2)^{(1/2)}-3/2*e^3*d^2/(d^2)^{(1/2)}*\ln((2*d^2+2*(d^2)^{(1/2)}*(-e^2*x^2+d^2)^{(1/2)})/x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)*x^4),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.296722, size = 572, normalized size = 4.77

$$18de^7x^7 + 32d^2e^6x^6 - 12d^3e^5x^5 - 104d^4e^4x^4 - 36d^5e^3x^3 + 88d^6e^2x^2 + 24d^7ex - 16d^8 + 12\left(de^7x^7 - 8d^3e^5x^5 + 8d^5e^3x^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)\*x^4),x, algorithm="fricas")

[Out] 
$$-1/6*(18*d*e^7*x^7 + 32*d^2*e^6*x^6 - 12*d^3*e^5*x^5 - 104*d^4*e^4*x^4 - 36*d^5*e^3*x^3 + 88*d^6*e^2*x^2 + 24*d^7*e*x - 16*d^8 + 12*(d*e^7*x^7 - 8*d^3*e^5*x^5 + 8*d^5*e^3*x^3 + 4*(d^2*e^5*x^5 - 2*d^4*e^3*x^3)*\sqrt{-e^2*x^2 + d^2})*\arctan(-(d - \sqrt{-e^2*x^2 + d^2})/(e*x)) - 9*(d*e^7*x^7 - 8*d^3*e^5*x^5 + 8*d^5*e^3*x^3 + 4*(d^2*e^5*x^5 - 2*d^4*e^3*x^3)*\sqrt{-e^2*x^2 + d^2})*\log(-(d - \sqrt{-e^2*x^2 + d^2})/x) - (6*e^7*x^7 + 8*d*e^6*x^6 - 21*d^2*e^5*x^5 - 66*d^3*e^4*x^4 - 24*d^4*e^3*x^3 + 80*d^5*e^2*x^2 + 24*d^6*e*x - 16*d^7)*\sqrt{-e^2*x^2 + d^2})/(e^4*x^7 - 8*d^2*e^2*x^5 + 8*d^4*x^3 + 4*(d*e^2*x^5 - 2*d^3*x^3)*\sqrt{-e^2*x^2 + d^2})$$

**Sympy [A]** time = 22.5667, size = 457, normalized size = 3.81

$$d^3 \left( \begin{array}{l} \left( -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2} + \frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} \quad \text{for } \left| \frac{d^2}{e^2x^2} \right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2} + \frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} \quad \text{otherwise} \end{array} \right) \\ - d^2 e \left( \begin{array}{l} \left( -\frac{d^2}{2ex^3\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e}{2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^2 \operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} \quad \text{for } \left| \frac{d^2}{e^2x^2} \right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{2x} - \frac{ie^2 \operatorname{asin}\left(\frac{d}{ex}\right)}{2d} \quad \text{otherwise} \end{array} \right) \\ - de^2 \left( \begin{array}{l} \left( \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}} + ie \operatorname{acosh}\left(\frac{ex}{d}\right) - \frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} \quad \text{for } \left| \frac{e^2x^2}{d^2} \right| > 1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}} - e \operatorname{asin}\left(\frac{ex}{d}\right) + \frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} \quad \text{otherwise} \end{array} \right) \\ + e^3 \left( \begin{array}{l} \left( \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}} - d \operatorname{acosh}\left(\frac{d}{ex}\right) - \frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} \quad \text{for } \left| \frac{d^2}{e^2x^2} \right| > 1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}} + id \operatorname{asin}\left(\frac{d}{ex}\right) + \frac{ie^2x}{\sqrt{-\frac{d^2}{e^2x^2}+1}} \quad \text{otherwise} \end{array} \right) \end{array}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*4/(e\*x+d),x)

[Out] 
$$d^{**3} \operatorname{Piecewise}\left(\left(-e \sqrt{d^{**2}/(e^{**2}x^{**2}) - 1}/(3x^{**2}) + e^{**3} \sqrt{d^{**2}/(e^{**2}x^{**2}) - 1}/(3d^{**2})\right), \operatorname{Abs}(d^{**2}/(e^{**2}x^{**2})) > 1\right), \left(-I e \sqrt{-d^{**2}/(e^{**2}x^{**2}) + 1}/(3x^{**2}) + I e^{**3} \sqrt{-d^{**2}/(e^{**2}x^{**2}) + 1}/(3d^{**2})\right), \operatorname{True}) - d^{**2} e \operatorname{Piecewise}\left(\left(-d^{**2}/(2e x \sqrt{d^{**2}/(e^{**2}x^{**2}) - 1}) + e/(2x \sqrt{d^{**2}/(e^{**2}x^{**2}) - 1})\right), \operatorname{Abs}(d^{**2}/(e^{**2}x^{**2})) > 1\right), \left(-I e \sqrt{-d^{**2}/(e^{**2}x^{**2}) + 1}/(2x) - I e^{**2} \operatorname{asin}(d/(e x))/(2d)\right), \operatorname{True}) - d e^{**2} \operatorname{Piecewise}\left(\left(I d/(x \sqrt{-1 + e^{**2}x^{**2}/d^{**2}}) + I e \operatorname{acosh}(e x/d) - I e^{**2} x/(d \sqrt{-1 + e^{**2}x^{**2}/d^{**2}})\right), \operatorname{Abs}(e^{**2}x^{**2}/d^{**2}) > 1\right), \left(-d/(x \sqrt{1 - e^{**2}x^{**2}/d^{**2}}) - e \operatorname{asin}(e x/d) + e^{**2} x/(d \sqrt{1 - e^{**2}x^{**2}/d^{**2}})\right), \operatorname{True}) + e^{**3} \operatorname{Piecewise}\left(\left(d^{**2}/(e x \sqrt{d^{**2}/(e^{**2}x^{**2}) - 1}) - d \operatorname{acosh}(d/(e x)) - e x/\sqrt{d^{**2}/(e^{**2}x^{**2}) - 1}\right), \operatorname{Abs}(d^{**2}/(e^{**2}x^{**2})) > 1\right), \left(-I d^{**2}/(e x \sqrt{-d^{**2}/(e^{**2}x^{**2}) + 1}) + I d \operatorname{asin}(d/(e x)) + I e x/\sqrt{-d^{**2}/(e^{**2}x^{**2}) + 1}\right), \operatorname{True})$$

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)\*x^4),x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.112 \quad \int \frac{(d^2 - e^2 x^2)^{5/2}}{x^5(d+ex)} dx$$

**Optimal.** Leaf size=119

$$\frac{e^2(3d - 8ex)\sqrt{d^2 - e^2x^2}}{8x^2} - \frac{(3d - 4ex)(d^2 - e^2x^2)^{3/2}}{12x^4} + e^4 \left( -\tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2x^2}} \right) \right) - \frac{3}{8} e^4 \tanh^{-1} \left( \frac{\sqrt{d^2 - e^2x^2}}{d} \right)$$

[Out] (e^2\*(3\*d - 8\*e\*x)\*Sqrt[d^2 - e^2\*x^2])/(8\*x^2) - ((3\*d - 4\*e\*x)\*(d^2 - e^2\*x^2)^(3/2))/(12\*x^4) - e^4\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] - (3\*e^4\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/8

**Rubi [A]** time = 0.363376, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$

$$\frac{e^2(3d - 8ex)\sqrt{d^2 - e^2x^2}}{8x^2} - \frac{(3d - 4ex)(d^2 - e^2x^2)^{3/2}}{12x^4} + e^4 \left( -\tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2x^2}} \right) \right) - \frac{3}{8} e^4 \tanh^{-1} \left( \frac{\sqrt{d^2 - e^2x^2}}{d} \right)$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^(5/2)/(x^5\*(d + e\*x)), x]

[Out] (e^2\*(3\*d - 8\*e\*x)\*Sqrt[d^2 - e^2\*x^2])/(8\*x^2) - ((3\*d - 4\*e\*x)\*(d^2 - e^2\*x^2)^(3/2))/(12\*x^4) - e^4\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] - (3\*e^4\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/8

**Rubi in Sympy [A]** time = 51.7225, size = 102, normalized size = 0.86

$$-e^4 \operatorname{atan} \left( \frac{ex}{\sqrt{d^2 - e^2x^2}} \right) - \frac{3e^4 \operatorname{atanh} \left( \frac{\sqrt{d^2 - e^2x^2}}{d} \right)}{8} + \frac{e^2(6d - 16ex)\sqrt{d^2 - e^2x^2}}{16x^2} - \frac{(3d - 4ex)(d^2 - e^2x^2)^{3/2}}{12x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*5/(e\*x+d), x)

[Out] -e\*\*4\*atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2)) - 3\*e\*\*4\*atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d)/8 + e\*\*2\*(6\*d - 16\*e\*x)\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(16\*x\*\*2) - (3\*d - 4\*e\*x)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(12\*x\*\*4)

**Mathematica [A]** time = 0.222882, size = 111, normalized size = 0.93

$$\frac{1}{24} \left( -9e^4 \log \left( \sqrt{d^2 - e^2x^2} + d \right) - 24e^4 \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2x^2}} \right) + \frac{\sqrt{d^2 - e^2x^2} (-6d^3 + 8d^2ex + 15de^2x^2 - 32e^3x^3)}{x^4} + 9e^4 \log(x) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^(5/2)/(x^5\*(d + e\*x)), x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(-6\*d^3 + 8\*d^2\*e\*x + 15\*d\*e^2\*x^2 - 32\*e^3\*x^3))/x^4 - 24\*e^4\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] + 9\*e^4\*Log[x] - 9\*e^4\*Log[d + Sqrt[d^2 - e^2\*x^2]])/24

**Maple [B]** time = 0.019, size = 463, normalized size = 3.9

$$\begin{aligned}
& -\frac{1}{4d^3x^4}(-e^2x^2+d^2)^{\frac{7}{2}} - \frac{e^2}{8d^5x^2}(-e^2x^2+d^2)^{\frac{7}{2}} \\
& + \frac{3e^4}{40d^5}(-e^2x^2+d^2)^{\frac{5}{2}} + \frac{e^4}{8d^3}(-e^2x^2+d^2)^{\frac{3}{2}} + \frac{3e^4}{8d}\sqrt{-e^2x^2+d^2} \\
& - \frac{3de^4}{8}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}} - \frac{e^4}{5d^5}\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}} \\
& - \frac{e^5x}{4d^4}\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}} - \frac{3e^5x}{8d^2}\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)} \\
& - \frac{3e^5}{8}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)\frac{1}{\sqrt{e^2}} + \frac{e}{3d^4x^3}(-e^2x^2+d^2)^{\frac{7}{2}} \\
& - \frac{e^3}{3d^6x}(-e^2x^2+d^2)^{\frac{7}{2}} - \frac{e^5x}{3d^6}(-e^2x^2+d^2)^{\frac{5}{2}} - \frac{5e^5x}{12d^4}(-e^2x^2+d^2)^{\frac{3}{2}} \\
& - \frac{5e^5x}{8d^2}\sqrt{-e^2x^2+d^2} - \frac{5e^5}{8}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^(5/2)/x^5/(e\*x+d), x)

[Out] -1/4/d^3/x^4\*(-e^2\*x^2+d^2)^(7/2)-1/8/d^5\*e^2/x^2\*(-e^2\*x^2+d^2)^(7/2)+3/40/d^5\*e^4\*(-e^2\*x^2+d^2)^(5/2)+1/8\*e^4/d^3\*(-e^2\*x^2+d^2)^(3/2)+3/8\*e^4/d\*(-e^2\*x^2+d^2)^(1/2)-3/8\*d\*e^4/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)-1/5/d^5\*e^4\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(5/2)-1/4/d^4\*e^5\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)\*x-3/8/d^2\*e^5\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x-3/8\*e^5/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))+1/3\*e/d^4/x^3\*(-e^2\*x^2+d^2)^(7/2)-1/3\*e^3/d^6/x\*(-e^2\*x^2+d^2)^(7/2)-1/3\*e^5/d^6\*x\*(-e^2\*x^2+d^2)^(5/2)-5/12\*e^5/d^4\*x\*(-e^2\*x^2+d^2)^(3/2)-5/8\*e^5/d^2\*x\*(-e^2\*x^2+d^2)^(1/2)-5/8\*e^5/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)\*x^5), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.30952, size = 564, normalized size = 4.74

$$128 d e^7 x^7 - 60 d^2 e^6 x^6 - 416 d^3 e^5 x^5 + 204 d^4 e^4 x^4 + 352 d^5 e^3 x^3 - 192 d^6 e^2 x^2 - 64 d^7 e x + 48 d^8 + 48 \left( e^8 x^8 - 8 d^2 e^6 x^6 + 8 d^4 e^4 x^4 - 8 d^6 e^2 x^2 + 8 d^8 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)\*x^5), x, algorithm="fricas")

[Out]  $\frac{1}{24} \cdot (128 d^7 e^7 x^7 - 60 d^6 e^6 x^6 - 416 d^5 e^5 x^5 + 204 d^4 e^4 x^4 + 352 d^3 e^3 x^3 - 192 d^2 e^2 x^2 - 64 d e x + 48 d^8 + 48 (e^8 x^8 - 8 d^2 e^6 x^6 + 8 d^4 e^4 x^4 - 8 d^6 e^2 x^2 + 8 d^8)) \cdot \sqrt{-e^2 x^2 + d^2} \cdot \arctan\left(\frac{-d - \sqrt{-e^2 x^2 + d^2}}{e x}\right) + 9 \cdot (e^8 x^8 - 8 d^2 e^6 x^6 + 8 d^4 e^4 x^4 + 4 (d^2 e^6 x^6 - 2 d^3 e^4 x^4) \sqrt{-e^2 x^2 + d^2}) \cdot \log\left(\frac{-d - \sqrt{-e^2 x^2 + d^2}}{e x}\right) - (32 e^7 x^7 - 15 d e^6 x^6 - 264 d^2 e^5 x^5 + 126 d^3 e^4 x^4 + 320 d^4 e^3 x^3 - 168 d^5 e^2 x^2 - 64 d^6 e x + 48 d^7) \sqrt{-e^2 x^2 + d^2} / (e^4 x^8 - 8 d^2 e^2 x^6 + 8 d^4 x^4 + 4 (d^2 e^2 x^6 - 2 d^3 x^4) \sqrt{-e^2 x^2 + d^2})$

**Sympy [A]** time = 30.9284, size = 541, normalized size = 4.55

$$d^3 \left( \begin{array}{l} \left( -\frac{d^2}{4ex^5\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{3e}{8x^3\sqrt{\frac{d^2}{e^2x^2}-1}} - \frac{e^3}{8d^2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^4 \operatorname{acosh}\left(\frac{d}{ex}\right)}{8d^3} \right) \text{ for } \left| \frac{d^2}{e^2x^2} \right| > 1 \\ \left( \frac{id^2}{4ex^5\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{3ie}{8x^3\sqrt{-\frac{d^2}{e^2x^2}+1}} + \frac{ie^3}{8d^2x\sqrt{-\frac{d^2}{e^2x^2}+1}} - \frac{ie^4 \operatorname{asin}\left(\frac{d}{ex}\right)}{8d^3} \right) \text{ otherwise} \end{array} \right) \\ - d^2 e \left( \begin{array}{l} \left( -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2} + \frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} \right) \text{ for } \left| \frac{d^2}{e^2x^2} \right| > 1 \\ \left( -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2} + \frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} \right) \text{ otherwise} \end{array} \right) \\ - de^2 \left( \begin{array}{l} \left( -\frac{d^2}{2ex^3\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e}{2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^2 \operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} \right) \text{ for } \left| \frac{d^2}{e^2x^2} \right| > 1 \\ \left( -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{2x} - \frac{ie^2 \operatorname{asin}\left(\frac{d}{ex}\right)}{2d} \right) \text{ otherwise} \end{array} \right) \\ + e^3 \left( \begin{array}{l} \left( \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}} + ie \operatorname{acosh}\left(\frac{ex}{d}\right) - \frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} \right) \text{ for } \left| \frac{e^2x^2}{d^2} \right| > 1 \\ \left( -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}} - e \operatorname{asin}\left(\frac{ex}{d}\right) + \frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} \right) \text{ otherwise} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*5/(e\*x+d), x)

[Out]  $d^{**3} \operatorname{Piecewise}(( -d^{**2}/(4 * e * x^{**5} * \sqrt{d^{**2}/(e^{**2} * x^{**2}) - 1}) + 3 * e / (8 * x^{**3} * \sqrt{d^{**2}/(e^{**2} * x^{**2}) - 1}) - e^{**3}/(8 * d^{**2} * x * \sqrt{d^{**2}/(e^{**2} * x^{**2}) - 1}) + e^{**4} * \operatorname{acosh}(d/(e * x)) / (8 * d^{**3}), \operatorname{Abs}(d^{**2}/(e^{**2} * x^{**2})) > 1), (I * d^{**2}/(4 * e * x^{**5} * \sqrt{-d^{**2}/(e^{**2} * x^{**2}) + 1}) - 3 * I * e / (8 * x^{**3} * \sqrt{-d^{**2}/(e^{**2} * x^{**2}) + 1}) + I * e^{**3}/(8 * d^{**2} * x * \sqrt{-d^{**2}/(e^{**2} * x^{**2}) + 1}) - I * e^{**4} * \operatorname{asin}(d/(e * x)) / (8 * d^{**3}), \operatorname{True})) - d^{**2} * e * \operatorname{Piecewise}(( -e * \sqrt{d^{**2}/(e^{**2} * x^{**2}) - 1} / (3 * x^{**2}) + e^{**3} * \sqrt{d^{**2}/(e^{**2} * x^{**2}) - 1} / (3 * d^{**2}), \operatorname{Abs}(d^{**2}/(e^{**2} * x^{**2})) > 1), (-I * e * \sqrt{-d^{**2}/(e^{**2} * x^{**2}) + 1} / (3 * x^{**2}) + I * e^{**3} * \sqrt{-d^{**2}/(e^{**2} * x^{**2}) + 1} / (3 * d^{**2}), \operatorname{True})) - d * e^{**2} * \operatorname{Piecewise}(( -d^{**2}/(2 * e * x^{**3} * \sqrt{d^{**2}/(e^{**2} * x^{**2}) - 1}) + e / (2 * x * \sqrt{d^{**2}/(e^{**2} * x^{**2}) - 1}) + e^{**2} * \operatorname{acosh}(d/(e * x)) / (2 * d), \operatorname{Abs}(d^{**2}/(e^{**2} * x^{**2})) > 1), (-I * e * \sqrt{-d^{**2}/(e^{**2} * x^{**2}) + 1} / (2 * x) - I * e^{**2} * \operatorname{asin}(d/(e * x)) / (2 * d), \operatorname{True})) + e^{**3} * \operatorname{Piecewise}(( I * d / (x * \sqrt{-1 + e^{**2} * x^{**2} / d^{**2}}) + I * e * \operatorname{acosh}(e * x / d) - I * e^{**2} * x / (d * \sqrt{-1 + e^{**2} * x^{**2} / d^{**2}}), \operatorname{Abs}(e^{**2} * x^{**2} / d^{**2}) > 1), (-d / (x * \sqrt{1 - e^{**2} * x^{**2} / d^{**2}}) - e * \operatorname{asin}(e * x / d) + e^{**2} * x / (d * \sqrt{1 - e^{**2} * x^{**2} / d^{**2}}), \operatorname{True}))$

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**GIAC/XCAS** [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)*x^5),x, algorithm="giac")`

[Out] Exception raised: NotImplementedError

$$3.113 \quad \int \frac{(d^2 - e^2 x^2)^{5/2}}{x^6(d+ex)} dx$$

**Optimal.** Leaf size=108

$$-\frac{(d^2 - e^2 x^2)^{5/2}}{5dx^5} + \frac{e(d^2 - e^2 x^2)^{3/2}}{4x^4} + \frac{3e^5 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{8d} - \frac{3e^3 \sqrt{d^2 - e^2 x^2}}{8x^2}$$

[Out]  $(-3 * e^3 * \text{Sqrt}[d^2 - e^2 * x^2]) / (8 * x^2) + (e * (d^2 - e^2 * x^2)^{(3/2)}) / (4 * x^4) - (d^2 - e^2 * x^2)^{(5/2)} / (5 * d * x^5) + (3 * e^5 * \text{ArcTanh}[\text{Sqrt}[d^2 - e^2 * x^2] / d]) / (8 * d)$

**Rubi [A]** time = 0.26445, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-\frac{(d^2 - e^2 x^2)^{5/2}}{5dx^5} + \frac{e(d^2 - e^2 x^2)^{3/2}}{4x^4} + \frac{3e^5 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{8d} - \frac{3e^3 \sqrt{d^2 - e^2 x^2}}{8x^2}$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^(5/2)/(x^6\*(d + e\*x)), x]

[Out]  $(-3 * e^3 * \text{Sqrt}[d^2 - e^2 * x^2]) / (8 * x^2) + (e * (d^2 - e^2 * x^2)^{(3/2)}) / (4 * x^4) - (d^2 - e^2 * x^2)^{(5/2)} / (5 * d * x^5) + (3 * e^5 * \text{ArcTanh}[\text{Sqrt}[d^2 - e^2 * x^2] / d]) / (8 * d)$

**Rubi in Sympy [A]** time = 25.8976, size = 88, normalized size = 0.81

$$-\frac{3e^3 \sqrt{d^2 - e^2 x^2}}{8x^2} + \frac{e(d^2 - e^2 x^2)^{3/2}}{4x^4} + \frac{3e^5 \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{8d} - \frac{(d^2 - e^2 x^2)^{5/2}}{5dx^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*6/(e\*x+d), x)

[Out]  $-3 * e^3 * \text{sqrt}(d^2 - e^2 * x^2) / (8 * x^2) + e * (d^2 - e^2 * x^2)^{(3/2)} / (4 * x^4) + 3 * e^5 * \operatorname{atanh}(\text{sqrt}(d^2 - e^2 * x^2) / d) / (8 * d) - (d^2 - e^2 * x^2)^{(5/2)} / (5 * d * x^5)$

**Mathematica [A]** time = 0.108551, size = 106, normalized size = 0.98

$$\frac{15e^5 x^5 \log\left(\sqrt{d^2 - e^2 x^2} + d\right) + \sqrt{d^2 - e^2 x^2} (-8d^4 + 10d^3 ex + 16d^2 e^2 x^2 - 25de^3 x^3 - 8e^4 x^4) - 15e^5 x^5 \log(x)}{40dx^5}$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^(5/2)/(x^6\*(d + e\*x)), x]

[Out]  $(\text{Sqrt}[d^2 - e^2 * x^2] * (-8 * d^4 + 10 * d^3 * e * x + 16 * d^2 * e^2 * x^2 - 25 * d * e^3 * x^3 - 8 * e^4 * x^4) - 15 * e^5 * x^5 * \text{Log}[x] + 15 * e^5 * x^5 * \text{Log}[d + \text{Sqrt}[d^2 - e^2 * x^2]]) / (40 * d * x^5)$



**Maple [B]** time = 0.021, size = 493, normalized size = 4.6

$$\begin{aligned}
& -\frac{1}{5d^3x^5}(-e^2x^2+d^2)^{\frac{7}{2}} - \frac{e^2}{5d^5x^3}(-e^2x^2+d^2)^{\frac{7}{2}} - \frac{e^4}{5d^7x}(-e^2x^2+d^2)^{\frac{7}{2}} \\
& - \frac{e^6x}{5d^7}(-e^2x^2+d^2)^{\frac{5}{2}} - \frac{e^6x}{4d^5}(-e^2x^2+d^2)^{\frac{3}{2}} - \frac{3e^6x}{8d^3}\sqrt{-e^2x^2+d^2} \\
& - \frac{3e^6}{8d}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}} + \frac{e^5}{5d^6}\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}} \\
& + \frac{e^6x}{4d^5}\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}} + \frac{3e^6x}{8d^3}\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)} \\
& + \frac{3e^6}{8d}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)\frac{1}{\sqrt{e^2}} - \frac{3e^5}{40d^6}(-e^2x^2+d^2)^{\frac{5}{2}} \\
& - \frac{e^5}{8d^4}(-e^2x^2+d^2)^{\frac{3}{2}} - \frac{3e^5}{8d^2}\sqrt{-e^2x^2+d^2} + \frac{3e^5}{8}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}} \\
& + \frac{e^3}{8d^6x^2}(-e^2x^2+d^2)^{\frac{7}{2}} + \frac{e}{4d^4x^4}(-e^2x^2+d^2)^{\frac{7}{2}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-e^2*x^2+d^2)^(5/2)/x^6/(e*x+d),x)`

[Out] `-1/5/d^3/x^5*(-e^2*x^2+d^2)^(7/2)-1/5/d^5*e^2/x^3*(-e^2*x^2+d^2)^(7/2)-1/5/d^7*e^4/x*(-e^2*x^2+d^2)^(7/2)-1/5/d^7*e^6*x*(-e^2*x^2+d^2)^(5/2)-1/4/d^5*e^6*x*(-e^2*x^2+d^2)^(3/2)-3/8/d^3*e^6*x*(-e^2*x^2+d^2)^(1/2)-3/8/d*e^6/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+1/5/d^6*e^5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)+1/4/d^5*e^6*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)*x+3/8/d^3*e^6*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)*x+3/8/d*e^6/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))-3/40/d^6*e^5*(-e^2*x^2+d^2)^(5/2)-1/8*e^5/d^4*(-e^2*x^2+d^2)^(3/2)-3/8*e^5/d^2*(-e^2*x^2+d^2)^(1/2)+3/8*e^5/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/8/d^6*e^3/x^2*(-e^2*x^2+d^2)^(7/2)+1/4*e/d^4/x^4*(-e^2*x^2+d^2)^(7/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2+d^2)^(5/2)/((e*x+d)*x^6),x,algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.307, size = 531, normalized size = 4.92

$$8e^{10}x^{10} + 25de^9x^9 - 120d^2e^8x^8 - 335d^3e^7x^7 + 440d^4e^6x^6 + 830d^5e^5x^5 - 680d^6e^4x^4 - 680d^7e^3x^3 + 480d^8e^2x^2 + 160d^9e^1x^1 + 160d^{10}e^0x^0$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2+d^2)^(5/2)/((e*x+d)*x^6),x,algorithm="fricas")`

```
[Out] -1/40*(8*e^10*x^10 + 25*d*e^9*x^9 - 120*d^2*e^8*x^8 - 335*d^3*e^7
*x^7 + 440*d^4*e^6*x^6 + 830*d^5*e^5*x^5 - 680*d^6*e^4*x^4 - 680*
d^7*e^3*x^3 + 480*d^8*e^2*x^2 + 160*d^9*e*x - 128*d^10 + 15*(5*d*
e^9*x^9 - 20*d^3*e^7*x^7 + 16*d^5*e^5*x^5 - (e^9*x^9 - 12*d^2*e^7
*x^7 + 16*d^4*e^5*x^5)*sqrt(-e^2*x^2 + d^2))*log(-(d - sqrt(-e^2*
x^2 + d^2))/x) + (40*d*e^8*x^8 + 125*d^2*e^7*x^7 - 240*d^3*e^6*x^
6 - 550*d^4*e^5*x^5 + 488*d^5*e^4*x^4 + 600*d^6*e^3*x^3 - 416*d^7
*e^2*x^2 - 160*d^8*e*x + 128*d^9)*sqrt(-e^2*x^2 + d^2))/(5*d^2*e^
4*x^9 - 20*d^4*e^2*x^7 + 16*d^6*x^5 - (d*e^4*x^9 - 12*d^3*e^2*x^7
+ 16*d^5*x^5)*sqrt(-e^2*x^2 + d^2))
```

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**Sympy [A]** time = 30.5357, size = 774, normalized size = 7.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**6/(e*x+d),x)
```

```
[Out] d**3*Piecewise((3*I*d**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5
+ 15*e**2*x**7) - 4*I*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15
*d**2*x**5 + 15*e**2*x**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d*
**2)/(-15*d**5*x**5 + 15*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e
**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d*
**2) > 1), (3*d**3*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e
**2*x**7) - 4*d*e**2*x**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**2*x**5
+ 15*e**2*x**7) + 2*e**6*x**6*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*
x**5 + 15*d**3*e**2*x**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-
15*d**3*x**5 + 15*d*e**2*x**7), True)) - d**2*e*Piecewise((-d**2/
(4*e*x**5*sqrt(d**2/(e**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e
**2*x**2) - 1)) - e**3/(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**
4*acosh(d/(e*x))/(8*d**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4
*e*x**5*sqrt(-d**2/(e**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(
e**2*x**2) + 1)) + I*e**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1))
- I*e**4*asin(d/(e*x))/(8*d**3), True)) - d*e**2*Piecewise((-e*sq
rt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) -
1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x
**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2),
True)) + e**3*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) -
1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(
2*d), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) +
1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d), True))
```

---

**GIAC/XCAS [A]** time = 0.294825, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)*x^6),x, algorithm="giac")
```

```
[Out] Done
```

$$3.114 \quad \int \frac{(d^2 - e^2 x^2)^{5/2}}{x^7(d+ex)} dx$$

**Optimal.** Leaf size=143

$$-\frac{(d^2 - e^2 x^2)^{5/2}}{6dx^6} + \frac{e(d^2 - e^2 x^2)^{5/2}}{5d^2 x^5} - \frac{e^2(d^2 - e^2 x^2)^{3/2}}{24dx^4} - \frac{e^6 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{16d^2} + \frac{e^4 \sqrt{d^2 - e^2 x^2}}{16dx^2}$$

[Out] (e^4\*Sqrt[d^2 - e^2\*x^2])/((16\*d\*x^2) - (e^2\*(d^2 - e^2\*x^2)^(3/2))/(24\*d\*x^4) - (d^2 - e^2\*x^2)^(5/2)/(6\*d\*x^6) + (e\*(d^2 - e^2\*x^2)^(5/2))/(5\*d^2\*x^5) - (e^6\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]))/(16\*d^2)

**Rubi [A]** time = 0.348569, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$-\frac{(d^2 - e^2 x^2)^{5/2}}{6dx^6} + \frac{e(d^2 - e^2 x^2)^{5/2}}{5d^2 x^5} - \frac{e^2(d^2 - e^2 x^2)^{3/2}}{24dx^4} - \frac{e^6 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{16d^2} + \frac{e^4 \sqrt{d^2 - e^2 x^2}}{16dx^2}$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^(5/2)/(x^7\*(d + e\*x)), x]

[Out] (e^4\*Sqrt[d^2 - e^2\*x^2])/((16\*d\*x^2) - (e^2\*(d^2 - e^2\*x^2)^(3/2))/(24\*d\*x^4) - (d^2 - e^2\*x^2)^(5/2)/(6\*d\*x^6) + (e\*(d^2 - e^2\*x^2)^(5/2))/(5\*d^2\*x^5) - (e^6\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]))/(16\*d^2)

**Rubi in Sympy [A]** time = 37.8971, size = 116, normalized size = 0.81

$$\frac{e^4 \sqrt{d^2 - e^2 x^2}}{16dx^2} - \frac{e^2 (d^2 - e^2 x^2)^{3/2}}{24dx^4} - \frac{(d^2 - e^2 x^2)^{5/2}}{6dx^6} - \frac{e^6 \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{16d^2} + \frac{e (d^2 - e^2 x^2)^{5/2}}{5d^2 x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*7/(e\*x+d), x)

[Out] e\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(16\*d\*x\*\*2) - e\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(24\*d\*x\*\*4) - (d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)/(6\*d\*x\*\*6) - e\*\*6\*atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d)/(16\*d\*\*2) + e\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)/(5\*d\*\*2\*x\*\*5)

**Mathematica [A]** time = 0.118481, size = 117, normalized size = 0.82

$$\frac{-15e^6 x^6 \log\left(\sqrt{d^2 - e^2 x^2} + d\right) + \sqrt{d^2 - e^2 x^2} (-40d^5 + 48d^4 ex + 70d^3 e^2 x^2 - 96d^2 e^3 x^3 - 15de^4 x^4 + 48e^5 x^5) + 15e^6 x^6 \log(x)}{240d^2 x^6}$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^(5/2)/(x^7\*(d + e\*x)), x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(-40\*d^5 + 48\*d^4\*e\*x + 70\*d^3\*e^2\*x^2 - 96\*d^2\*e^3\*x^3 - 15\*d\*e^4\*x^4 + 48\*e^5\*x^5) + 15\*e^6\*x^6\*Log[x] - 15

\*e^6\*x^6\*Log[d + Sqrt[d^2 - e^2\*x^2]]/(240\*d^2\*x^6)

**Maple [B]** time = 0.021, size = 521, normalized size = 3.6

$$\begin{aligned}
 & -\frac{1}{6d^3x^6}(-e^2x^2+d^2)^{\frac{7}{2}} - \frac{5e^2}{24d^5x^4}(-e^2x^2+d^2)^{\frac{7}{2}} - \frac{3e^4}{16d^7x^2}(-e^2x^2+d^2)^{\frac{7}{2}} \\
 & + \frac{e^6}{80d^7}(-e^2x^2+d^2)^{\frac{5}{2}} + \frac{e^6}{48d^5}(-e^2x^2+d^2)^{\frac{3}{2}} + \frac{e^6}{16d^3}\sqrt{-e^2x^2+d^2} \\
 & - \frac{e^6}{16d}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}\right)\right) \frac{1}{\sqrt{d^2}} - \frac{e^6}{5d^7}\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}} \\
 & - \frac{e^7x}{4d^6}\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}} - \frac{3e^7x}{8d^4}\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)} \\
 & - \frac{3e^7}{8d^2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right) \frac{1}{\sqrt{e^2}} + \frac{e}{5d^4x^5}(-e^2x^2+d^2)^{\frac{7}{2}} \\
 & + \frac{e^3}{5d^6x^3}(-e^2x^2+d^2)^{\frac{7}{2}} + \frac{e^5}{5d^8x}(-e^2x^2+d^2)^{\frac{7}{2}} + \frac{e^7x}{5d^8}(-e^2x^2+d^2)^{\frac{5}{2}} \\
 & + \frac{e^7x}{4d^6}(-e^2x^2+d^2)^{\frac{3}{2}} + \frac{3e^7x}{8d^4}\sqrt{-e^2x^2+d^2} + \frac{3e^7}{8d^2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right) \frac{1}{\sqrt{e^2}}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^(5/2)/x^7/(e\*x+d), x)

[Out] -1/6/d^3/x^6\*(-e^2\*x^2+d^2)^(7/2)-5/24/d^5\*e^2/x^4\*(-e^2\*x^2+d^2)^(7/2)-3/16/d^7\*e^4/x^2\*(-e^2\*x^2+d^2)^(7/2)+1/80/d^7\*e^6\*(-e^2\*x^2+d^2)^(5/2)+1/48\*e^6/d^5\*(-e^2\*x^2+d^2)^(3/2)+1/16\*e^6/d^3\*(-e^2\*x^2+d^2)^(1/2)-1/16\*e^6/d/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)-1/5/d^7\*e^6\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(5/2)-1/4/d^6\*e^7\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)\*x-3/8/d^4\*e^7\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x-3/8/d^2\*e^7/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))+1/5\*e/d^4/x^5\*(-e^2\*x^2+d^2)^(7/2)+1/5\*e^3/d^6/x^3\*(-e^2\*x^2+d^2)^(7/2)+1/5\*e^5/d^8/x\*(-e^2\*x^2+d^2)^(7/2)+1/5\*e^7/d^8\*x\*(-e^2\*x^2+d^2)^(5/2)+1/4\*e^7/d^6\*x\*(-e^2\*x^2+d^2)^(3/2)+3/8\*e^7/d^4\*x\*(-e^2\*x^2+d^2)^(1/2)+3/8\*e^7/d^2/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)\*x^7), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.352663, size = 624, normalized size = 4.36

288 d e<sup>11</sup> x<sup>11</sup> - 90 d<sup>2</sup> e<sup>10</sup> x<sup>10</sup> - 2400 d<sup>3</sup> e<sup>9</sup> x<sup>9</sup> + 990 d<sup>4</sup> e<sup>8</sup> x<sup>8</sup> + 7008 d<sup>5</sup> e<sup>7</sup> x<sup>7</sup> - 3860 d<sup>6</sup> e<sup>6</sup> x<sup>6</sup> - 9504 d<sup>7</sup> e<sup>5</sup> x<sup>5</sup> + 6480 d<sup>8</sup> e<sup>4</sup> x<sup>4</sup> + 6144 d<sup>9</sup> e<sup>3</sup> x<sup>3</sup> + 3072 d<sup>10</sup> e<sup>2</sup> x<sup>2</sup> + 768 d<sup>11</sup> e x + 288 d<sup>12</sup>

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)*x^7),x, algorithm="fricas")
```

```
[Out] -1/240*(288*d*e^11*x^11 - 90*d^2*e^10*x^10 - 2400*d^3*e^9*x^9 + 9
90*d^4*e^8*x^8 + 7008*d^5*e^7*x^7 - 3860*d^6*e^6*x^6 - 9504*d^7*e
^5*x^5 + 6480*d^8*e^4*x^4 + 6144*d^9*e^3*x^3 - 4800*d^10*e^2*x^2
- 1536*d^11*e*x + 1280*d^12 - 15*(e^12*x^12 - 18*d^2*e^10*x^10 +
48*d^4*e^8*x^8 - 32*d^6*e^6*x^6 + 2*(3*d^10*x^10 - 16*d^3*e^8*x
^8 + 16*d^5*e^6*x^6)*sqrt(-e^2*x^2 + d^2))*log(-(d - sqrt(-e^2*x^
2 + d^2))/x) - (48*e^11*x^11 - 15*d^2*e^10*x^10 - 960*d^2*e^9*x^9 +
340*d^3*e^8*x^8 + 4080*d^4*e^7*x^7 - 2020*d^5*e^6*x^6 - 7008*d^6
*e^5*x^5 + 4560*d^7*e^4*x^4 + 5376*d^8*e^3*x^3 - 4160*d^9*e^2*x^2
- 1536*d^10*e*x + 1280*d^11)*sqrt(-e^2*x^2 + d^2))/(d^2*e^6*x^12
- 18*d^4*e^4*x^10 + 48*d^6*e^2*x^8 - 32*d^8*x^6 + 2*(3*d^3*e^4*x
^10 - 16*d^5*e^2*x^8 + 16*d^7*x^6)*sqrt(-e^2*x^2 + d^2))
```

---

**Sympy [A]** time = 79.6741, size = 918, normalized size = 6.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**7/(e*x+d),x)
```

```
[Out] d**3*Piecewise((-d**2/(6*e*x**7*sqrt(d**2/(e**2*x**2) - 1)) + 5*e
/(24*x**5*sqrt(d**2/(e**2*x**2) - 1)) + e**3/(48*d**2*x**3*sqrt(d
**2/(e**2*x**2) - 1)) - e**5/(16*d**4*x*sqrt(d**2/(e**2*x**2) - 1
)) + e**6*acosh(d/(e*x))/(16*d**5), Abs(d**2/(e**2*x**2)) > 1), (
I*d**2/(6*e*x**7*sqrt(-d**2/(e**2*x**2) + 1)) - 5*I*e/(24*x**5*sq
rt(-d**2/(e**2*x**2) + 1)) - I*e**3/(48*d**2*x**3*sqrt(-d**2/(e**
2*x**2) + 1)) + I*e**5/(16*d**4*x*sqrt(-d**2/(e**2*x**2) + 1)) -
I*e**6*asin(d/(e*x))/(16*d**5), True)) - d**2*e*Piecewise((3*I*d*
**3*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*I
*d*e**2*x**2*sqrt(-1 + e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x
**7) + 2*I*e**6*x**6*sqrt(-1 + e**2*x**2/d**2)/(-15*d**5*x**5 + 1
5*d**3*e**2*x**7) - I*e**4*x**4*sqrt(-1 + e**2*x**2/d**2)/(-15*d*
**3*x**5 + 15*d*e**2*x**7), Abs(e**2*x**2/d**2) > 1), (3*d**3*sqrt
(1 - e**2*x**2/d**2)/(-15*d**2*x**5 + 15*e**2*x**7) - 4*d*e**2*x*
**2*sqrt(1 - e**2*x**2/d**2)/(-15*d**5*x**5 + 15*d**3*e**2*x*
**7) - e**4*x**4*sqrt(1 - e**2*x**2/d**2)/(-15*d**3*x**5 + 15*d*e
**2*x**7), True)) - d*e**2*Piecewise((-d**2/(4*e*x**5*sqrt(d**2/(e
**2*x**2) - 1)) + 3*e/(8*x**3*sqrt(d**2/(e**2*x**2) - 1)) - e**3/
(8*d**2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**4*acosh(d/(e*x))/(8*d*
**3), Abs(d**2/(e**2*x**2)) > 1), (I*d**2/(4*e*x**5*sqrt(-d**2/(e*
**2*x**2) + 1)) - 3*I*e/(8*x**3*sqrt(-d**2/(e**2*x**2) + 1)) + I*e
**3/(8*d**2*x*sqrt(-d**2/(e**2*x**2) + 1)) - I*e**4*asin(d/(e*x))
/(8*d**3), True)) + e**3*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)
/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e
**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e
**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True))
```

---

**GIAC/XCAS [A]** time = 0.315372, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)*x^7),x, algorithm="giac")
```

```
[Out] Done
```

$$3.115 \quad \int \frac{(d^2 - e^2 x^2)^{5/2}}{x^8(d+ex)} dx$$

**Optimal.** Leaf size=172

$$\begin{aligned} & -\frac{(d^2 - e^2 x^2)^{5/2}}{7dx^7} + \frac{e(d^2 - e^2 x^2)^{5/2}}{6d^2 x^6} - \frac{e^5 \sqrt{d^2 - e^2 x^2}}{16d^2 x^2} \\ & + \frac{e^3 (d^2 - e^2 x^2)^{3/2}}{24d^2 x^4} - \frac{2e^2 (d^2 - e^2 x^2)^{5/2}}{35d^3 x^5} + \frac{e^7 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{16d^3} \end{aligned}$$

[Out]  $-(e^5 \sqrt{d^2 - e^2 x^2}) / (16 d^2 x^2) + (e^3 (d^2 - e^2 x^2)^{3/2}) / (24 d^2 x^4) - (d^2 - e^2 x^2)^{5/2} / (7 d x^7) + (e (d^2 - e^2 x^2)^{5/2}) / (6 d^2 x^6) - (2 e^2 (d^2 - e^2 x^2)^{5/2}) / (35 d^3 x^5) + (e^7 \operatorname{ArcTanh}[\sqrt{d^2 - e^2 x^2} / d]) / (16 d^3)$

**Rubi [A]** time = 0.439908, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\begin{aligned} & -\frac{(d^2 - e^2 x^2)^{5/2}}{7dx^7} + \frac{e(d^2 - e^2 x^2)^{5/2}}{6d^2 x^6} - \frac{e^5 \sqrt{d^2 - e^2 x^2}}{16d^2 x^2} \\ & + \frac{e^3 (d^2 - e^2 x^2)^{3/2}}{24d^2 x^4} - \frac{2e^2 (d^2 - e^2 x^2)^{5/2}}{35d^3 x^5} + \frac{e^7 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{16d^3} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^(5/2)/(x^8\*(d + e\*x)), x]

[Out]  $-(e^5 \sqrt{d^2 - e^2 x^2}) / (16 d^2 x^2) + (e^3 (d^2 - e^2 x^2)^{3/2}) / (24 d^2 x^4) - (d^2 - e^2 x^2)^{5/2} / (7 d x^7) + (e (d^2 - e^2 x^2)^{5/2}) / (6 d^2 x^6) - (2 e^2 (d^2 - e^2 x^2)^{5/2}) / (35 d^3 x^5) + (e^7 \operatorname{ArcTanh}[\sqrt{d^2 - e^2 x^2} / d]) / (16 d^3)$

**Rubi in Sympy [A]** time = 53.7654, size = 146, normalized size = 0.85

$$\begin{aligned} & -\frac{(d^2 - e^2 x^2)^{5/2}}{7dx^7} - \frac{e^5 \sqrt{d^2 - e^2 x^2}}{16d^2 x^2} + \frac{e^3 (d^2 - e^2 x^2)^{3/2}}{24d^2 x^4} \\ & + \frac{e (d^2 - e^2 x^2)^{5/2}}{6d^2 x^6} + \frac{e^7 \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{16d^3} - \frac{2e^2 (d^2 - e^2 x^2)^{5/2}}{35d^3 x^5} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*8/(e\*x+d), x)

[Out]  $-(d^2 - e^2 x^2)^{5/2} / (7 d x^7) - e^5 \sqrt{d^2 - e^2 x^2} / (16 d^2 x^2) + e^3 (d^2 - e^2 x^2)^{3/2} / (24 d^2 x^4) + e (d^2 - e^2 x^2)^{5/2} / (6 d^2 x^6) + e^7 \operatorname{atanh}(\sqrt{d^2 - e^2 x^2} / d) / (16 d^3) - 2 e^2 (d^2 - e^2 x^2)^{5/2} / (35 d^3 x^5)$

**Mathematica [A]** time = 0.128306, size = 128, normalized size = 0.74

$$105e^7 x^7 \log\left(\sqrt{d^2 - e^2 x^2} + d\right) + \sqrt{d^2 - e^2 x^2} (-240d^6 + 280d^5 ex + 384d^4 e^2 x^2 - 490d^3 e^3 x^3 - 48d^2 e^4 x^4 + 105de^5 x^5 - 96e^6 x^6) / 1680d^3 x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-240*d^6 + 280*d^5*e*x + 384*d^4*e^2*x^2 -
490*d^3*e^3*x^3 - 48*d^2*e^4*x^4 + 105*d*e^5*x^5 - 96*e^6*x^6) -
105*e^7*x^7*Log[x] + 105*e^7*x^7*Log[d + Sqrt[d^2 - e^2*x^2]])/(1
680*d^3*x^7)
```

**Maple [B]** time = 0.022, size = 546, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d),x)
```

```
[Out] 1/5/d^8*e^7*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)-1/7/d^3/x^7*(-e^
2*x^2+d^2)^(7/2)-1/48*e^7/d^6*(-e^2*x^2+d^2)^(3/2)-1/16*e^7/d^4*(
-e^2*x^2+d^2)^(1/2)+1/6*e/d^4/x^6*(-e^2*x^2+d^2)^(7/2)+5/24*e^3/d
^6/x^4*(-e^2*x^2+d^2)^(7/2)+3/16/d^8*e^5/x^2*(-e^2*x^2+d^2)^(7/2)
-1/5/d^5*e^2/x^5*(-e^2*x^2+d^2)^(7/2)-1/5/d^7*e^4/x^3*(-e^2*x^2+d
^2)^(7/2)-1/5/d^9*e^6/x*(-e^2*x^2+d^2)^(7/2)-1/5/d^9*e^8*x*(-e^2*
x^2+d^2)^(5/2)-1/4/d^7*e^8*x*(-e^2*x^2+d^2)^(3/2)-3/8/d^5*e^8*x*(
-e^2*x^2+d^2)^(1/2)-3/8/d^3*e^8/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/
(-e^2*x^2+d^2)^(1/2))+1/4/d^7*e^8*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^
(3/2)*x+3/8/d^5*e^8*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)*x+3/8/d^
3*e^8/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d
/e))^(1/2))+1/16*e^7/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^
2*x^2+d^2)^(1/2))/x)-1/80/d^8*e^7*(-e^2*x^2+d^2)^(5/2)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)*x^8),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 0.381634, size = 711, normalized size = 4.13

$$96 e^{14} x^{14} - 105 d e^{13} x^{13} - 2352 d^2 e^{12} x^{12} + 3115 d^3 e^{11} x^{11} + 8400 d^4 e^{10} x^{10} - 23450 d^5 e^9 x^9 + 1008 d^6 e^8 x^8 + 73080 d^7 e^7 x^7 - \dots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)*x^8),x, algorithm="fricas")
```

```
[Out] -1/1680*(96*e^14*x^14 - 105*d*e^13*x^13 - 2352*d^2*e^12*x^12 + 31
15*d^3*e^11*x^11 + 8400*d^4*e^10*x^10 - 23450*d^5*e^9*x^9 + 1008*
d^6*e^8*x^8 + 73080*d^7*e^7*x^7 - 46704*d^8*e^6*x^6 - 106400*d^9*
e^5*x^5 + 83328*d^10*e^4*x^4 + 71680*d^11*e^3*x^3 - 59136*d^12*e^
2*x^2 - 17920*d^13*e*x + 15360*d^14 + 105*(7*d*e^13*x^13 - 56*d^3
```

$$\begin{aligned} & e^{11}x^{11} + 112d^5e^9x^9 - 64d^7e^7x^7 - (e^{13}x^{13} - 24d^2e^{11}x^{11} + 80d^4e^9x^9 - 64d^6e^7x^7) \sqrt{-e^2x^2 + d^2} \\ & \log(-(d - \sqrt{-e^2x^2 + d^2})/x) + (672d^2e^{12}x^{12} - 735d^4e^{11}x^{11} - 5040d^3e^{10}x^{10} + 9310d^4e^9x^9 + 5376d^5e^8x^8 - 41160d^6e^7x^7 + 22416d^7e^6x^6 + 77280d^8e^5x^5 - 59520d^9e^4x^4 - 62720d^{10}e^3x^3 + 51456d^{11}e^2x^2 + 17920d^{12}ex - 15360d^{13}) \sqrt{-e^2x^2 + d^2} \\ & / (7d^4e^6x^{13} - 56d^6e^4x^{11} + 112d^8e^2x^9 - 64d^{10}x^7 - (d^3e^6x^{13} - 24d^5e^4x^{11} + 80d^7e^2x^9 - 64d^9x^7) \sqrt{-e^2x^2 + d^2}) \end{aligned}$$

**Sympy [A]** time = 49.1006, size = 1037, normalized size = 6.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*8/(e\*x+d),x)

[Out]  $d^{*3} \text{Piecewise}((-e \sqrt{d^{*2}/(e^{*2}x^{*2}) - 1}/(7x^{*6}) + e^{*3} \sqrt{d^{*2}/(e^{*2}x^{*2}) - 1}/(35d^{*2}x^{*4}) + 4e^{*5} \sqrt{d^{*2}/(e^{*2}x^{*2}) - 1}/(105d^{*4}x^{*2}) + 8e^{*7} \sqrt{d^{*2}/(e^{*2}x^{*2}) - 1}/(105d^{*6}), \text{Abs}(d^{*2}/(e^{*2}x^{*2})) > 1), (-Ie \sqrt{-d^{*2}/(e^{*2}x^{*2}) + 1}/(7x^{*6}) + Ie^{*3} \sqrt{-d^{*2}/(e^{*2}x^{*2}) + 1}/(35d^{*2}x^{*4}) + 4Ie^{*5} \sqrt{-d^{*2}/(e^{*2}x^{*2}) + 1}/(105d^{*4}x^{*2}) + 8Ie^{*7} \sqrt{-d^{*2}/(e^{*2}x^{*2}) + 1}/(105d^{*6}), \text{True})) - d^{*2}e \text{Piecewise}((-d^{*2}/(6e^x \sqrt{d^{*2}/(e^{*2}x^{*2}) - 1}) + 5e/(24x^5 \sqrt{d^{*2}/(e^{*2}x^{*2}) - 1})) + e^3/(48d^2x^3 \sqrt{d^{*2}/(e^{*2}x^{*2}) - 1})) - e^5/(16d^4x \sqrt{d^{*2}/(e^{*2}x^{*2}) - 1})) + e^6 \text{acosh}(d/(e^x))/(16d^5), \text{Abs}(d^{*2}/(e^{*2}x^{*2})) > 1), (Ie^{*2}/(6e^x \sqrt{-d^{*2}/(e^{*2}x^{*2}) + 1})) - 5Ie/(24x^5 \sqrt{-d^{*2}/(e^{*2}x^{*2}) + 1})) - Ie^{*3}/(48d^2x^3 \sqrt{-d^{*2}/(e^{*2}x^{*2}) + 1})) + Ie^{*5}/(16d^4x \sqrt{-d^{*2}/(e^{*2}x^{*2}) + 1})) - Ie^{*6} \text{asin}(d/(e^x))/(16d^5), \text{True})) - d^{*2}e^{*2} \text{Piecewise}((3Ie^{*3} \sqrt{-1 + e^{*2}x^{*2}/d^{*2}})/(-15d^{*2}x^{*5} + 15e^{*2}x^{*7}) - 4Ie^{*2}x^{*2} \sqrt{-1 + e^{*2}x^{*2}/d^{*2}})/(-15d^{*2}x^{*5} + 15e^{*2}x^{*7}) + 2Ie^{*6}x^{*6} \sqrt{-1 + e^{*2}x^{*2}/d^{*2}})/(-15d^{*5}x^{*5} + 15d^{*3}e^{*2}x^{*7}) - Ie^{*4}x^{*4} \sqrt{-1 + e^{*2}x^{*2}/d^{*2}})/(-15d^{*3}x^{*5} + 15d^{*2}e^{*2}x^{*7}), \text{Abs}(e^{*2}x^{*2}/d^{*2}) > 1), (3d^{*3} \sqrt{1 - e^{*2}x^{*2}/d^{*2}})/(-15d^{*2}x^{*5} + 15e^{*2}x^{*7}) - 4d^{*2}e^{*2}x^{*2} \sqrt{1 - e^{*2}x^{*2}/d^{*2}})/(-15d^{*2}x^{*5} + 15d^{*3}e^{*2}x^{*7}) - e^{*4}x^{*4} \sqrt{1 - e^{*2}x^{*2}/d^{*2}})/(-15d^{*3}x^{*5} + 15d^{*2}e^{*2}x^{*7}), \text{True})) + e^{*3} \text{Piecewise}((-d^{*2}/(4e^x \sqrt{d^{*2}/(e^{*2}x^{*2}) - 1})) + 3e/(8x^3 \sqrt{d^{*2}/(e^{*2}x^{*2}) - 1})) - e^3/(8d^2x \sqrt{d^{*2}/(e^{*2}x^{*2}) - 1})) + e^4 \text{acosh}(d/(e^x))/(8d^3), \text{Abs}(d^{*2}/(e^{*2}x^{*2})) > 1), (Ie^{*2}/(4e^x \sqrt{-d^{*2}/(e^{*2}x^{*2}) + 1})) - 3Ie/(8x^3 \sqrt{-d^{*2}/(e^{*2}x^{*2}) + 1})) + Ie^{*3}/(8d^2x \sqrt{-d^{*2}/(e^{*2}x^{*2}) + 1})) - Ie^{*4} \text{asin}(d/(e^x))/(8d^3), \text{True}))$

**GIAC/XCAS [A]** time = 0.313949, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)\*x^8),x, algorithm="giac")

[Out] Done



$$3.116 \quad \int \frac{(d^2 - e^2 x^2)^{5/2}}{x^9(d+ex)} dx$$

**Optimal.** Leaf size=201

$$\begin{aligned} & -\frac{(d^2 - e^2 x^2)^{5/2}}{8dx^8} + \frac{e(d^2 - e^2 x^2)^{5/2}}{7d^2 x^7} - \frac{3e^8 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{128d^4} + \frac{2e^3 (d^2 - e^2 x^2)^{5/2}}{35d^4 x^5} \\ & - \frac{e^2 (d^2 - e^2 x^2)^{5/2}}{16d^3 x^6} + \frac{3e^6 \sqrt{d^2 - e^2 x^2}}{128d^3 x^2} - \frac{e^4 (d^2 - e^2 x^2)^{3/2}}{64d^3 x^4} \end{aligned}$$

[Out]  $(3 * e^6 * \text{Sqrt}[d^2 - e^2 * x^2]) / (128 * d^3 * x^2) - (e^4 * (d^2 - e^2 * x^2)^{(3/2)}) / (64 * d^3 * x^4) - (d^2 - e^2 * x^2)^{(5/2)} / (8 * d * x^8) + (e * (d^2 - e^2 * x^2)^{(5/2)}) / (7 * d^2 * x^7) - (e^2 * (d^2 - e^2 * x^2)^{(5/2)}) / (16 * d^3 * x^6) + (2 * e^3 * (d^2 - e^2 * x^2)^{(5/2)}) / (35 * d^4 * x^5) - (3 * e^8 * \text{ArcTanh}[\text{Sqrt}[d^2 - e^2 * x^2] / d]) / (128 * d^4)$

**Rubi [A]** time = 0.534406, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\begin{aligned} & -\frac{(d^2 - e^2 x^2)^{5/2}}{8dx^8} + \frac{e(d^2 - e^2 x^2)^{5/2}}{7d^2 x^7} - \frac{3e^8 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{128d^4} + \frac{2e^3 (d^2 - e^2 x^2)^{5/2}}{35d^4 x^5} \\ & - \frac{e^2 (d^2 - e^2 x^2)^{5/2}}{16d^3 x^6} + \frac{3e^6 \sqrt{d^2 - e^2 x^2}}{128d^3 x^2} - \frac{e^4 (d^2 - e^2 x^2)^{3/2}}{64d^3 x^4} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d^2 - e^2 * x^2)^{(5/2)} / (x^9 * (d + e * x)), x]$

[Out]  $(3 * e^6 * \text{Sqrt}[d^2 - e^2 * x^2]) / (128 * d^3 * x^2) - (e^4 * (d^2 - e^2 * x^2)^{(3/2)}) / (64 * d^3 * x^4) - (d^2 - e^2 * x^2)^{(5/2)} / (8 * d * x^8) + (e * (d^2 - e^2 * x^2)^{(5/2)}) / (7 * d^2 * x^7) - (e^2 * (d^2 - e^2 * x^2)^{(5/2)}) / (16 * d^3 * x^6) + (2 * e^3 * (d^2 - e^2 * x^2)^{(5/2)}) / (35 * d^4 * x^5) - (3 * e^8 * \text{ArcTanh}[\text{Sqrt}[d^2 - e^2 * x^2] / d]) / (128 * d^4)$

**Rubi in Sympy [A]** time = 63.8596, size = 175, normalized size = 0.87

$$\begin{aligned} & -\frac{(d^2 - e^2 x^2)^{5/2}}{8dx^8} + \frac{e(d^2 - e^2 x^2)^{5/2}}{7d^2 x^7} + \frac{3e^6 \sqrt{d^2 - e^2 x^2}}{128d^3 x^2} - \frac{e^4 (d^2 - e^2 x^2)^{3/2}}{64d^3 x^4} \\ & - \frac{e^2 (d^2 - e^2 x^2)^{5/2}}{16d^3 x^6} - \frac{3e^8 \text{atanh}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{128d^4} + \frac{2e^3 (d^2 - e^2 x^2)^{5/2}}{35d^4 x^5} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((-e^{**2} * x^{**2} + d^{**2})^{**}(5/2) / x^{**9} / (e * x + d), x)$

[Out]  $-(d^{**2} - e^{**2} * x^{**2})^{**}(5/2) / (8 * d * x^{**8}) + e * (d^{**2} - e^{**2} * x^{**2})^{**}(5/2) / (7 * d^{**2} * x^{**7}) + 3 * e^{**6} * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (128 * d^{**3} * x^{**2}) - e^{**4} * (d^{**2} - e^{**2} * x^{**2})^{**}(3/2) / (64 * d^{**3} * x^{**4}) - e^{**2} * (d^{**2} - e^{**2} * x^{**2})^{**}(5/2) / (16 * d^{**3} * x^{**6}) - 3 * e^{**8} * \text{atanh}(\text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / d) / (128 * d^{**4}) + 2 * e^{**3} * (d^{**2} - e^{**2} * x^{**2})^{**}(5/2) / (35 * d^{**4} * x^{**5})$

**Mathematica [A]** time = 0.146368, size = 139, normalized size = 0.69

$$\frac{-105e^8 x^8 \log\left(\sqrt{d^2 - e^2 x^2} + d\right) + \sqrt{d^2 - e^2 x^2} (-560d^7 + 640d^6 ex + 840d^5 e^2 x^2 - 1024d^4 e^3 x^3 - 70d^3 e^4 x^4 + 128d^2 e^5 x^5 - 105e^6 x^6)}{4480d^4 x^8}$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^(5/2)/(x^9\*(d + e\*x)),x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(-560\*d^7 + 640\*d^6\*e\*x + 840\*d^5\*e^2\*x^2 - 1024\*d^4\*e^3\*x^3 - 70\*d^3\*e^4\*x^4 + 128\*d^2\*e^5\*x^5 - 105\*d\*e^6\*x^6 + 256\*e^7\*x^7) + 105\*e^8\*x^8\*Log[x] - 105\*e^8\*x^8\*Log[d + Sqrt[d^2 - e^2\*x^2]])/(4480\*d^4\*x^8)

**Maple [B]** time = 0.024, size = 571, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^(5/2)/x^9/(e\*x+d),x)

[Out] 1/128\*e^8/d^7\*(-e^2\*x^2+d^2)^(3/2)+3/128\*e^8/d^5\*(-e^2\*x^2+d^2)^(1/2)+1/5/d^6\*e^3/x^5\*(-e^2\*x^2+d^2)^(7/2)+1/5/d^8\*e^5/x^3\*(-e^2\*x^2+d^2)^(7/2)+1/5/d^10\*e^7/x\*(-e^2\*x^2+d^2)^(7/2)+1/5/d^10\*e^9\*x\*(-e^2\*x^2+d^2)^(5/2)+1/4/d^8\*e^9\*x\*(-e^2\*x^2+d^2)^(3/2)+3/8/d^6\*e^9\*x\*(-e^2\*x^2+d^2)^(1/2)+3/8/d^4\*e^9/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))-1/4/d^8\*e^9\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)\*x-3/8/d^6\*e^9\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x-3/8/d^4\*e^9/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))-3/16/d^5\*e^2/x^6\*(-e^2\*x^2+d^2)^(7/2)-13/64/d^7\*e^4/x^4\*(-e^2\*x^2+d^2)^(7/2)-25/128/d^9\*e^6/x^2\*(-e^2\*x^2+d^2)^(7/2)-3/128\*e^8/d^3/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)-1/5/d^9\*e^8\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(5/2)-1/8/d^3/x^8\*(-e^2\*x^2+d^2)^(7/2)+3/640/d^9\*e^8\*(-e^2\*x^2+d^2)^(5/2)+1/7\*e/d^4/x^7\*(-e^2\*x^2+d^2)^(7/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)\*x^9),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.541481, size = 799, normalized size = 3.98

2048 d e<sup>15</sup> x<sup>15</sup> - 840 d<sup>2</sup> e<sup>14</sup> x<sup>14</sup> - 21504 d<sup>3</sup> e<sup>13</sup> x<sup>13</sup> + 8680 d<sup>4</sup> e<sup>12</sup> x<sup>12</sup> + 50176 d<sup>5</sup> e<sup>11</sup> x<sup>11</sup> - 15680 d<sup>6</sup> e<sup>10</sup> x<sup>10</sup> + 48128 d<sup>7</sup> e<sup>9</sup> x<sup>9</sup> - 63840

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)\*x^9),x, algorithm="fricas")

[Out] -1/4480\*(2048\*d\*e^15\*x^15 - 840\*d^2\*e^14\*x^14 - 21504\*d^3\*e^13\*x^13 + 8680\*d^4\*e^12\*x^12 + 50176\*d^5\*e^11\*x^11 - 15680\*d^6\*e^10\*x^10 + 48128\*d^7\*e^9\*x^9 - 63840\*d^8\*e^8\*x^8 - 343040\*d^9\*e^7\*x^7 + 286720\*d^10\*e^6\*x^6 + 518144\*d^11\*e^5\*x^5 - 430080\*d^12\*e^4\*x^4

$$\begin{aligned}
& - 335872*d^{13}*e^3*x^3 + 286720*d^{14}*e^2*x^2 + 81920*d^{15}*e*x - 71680*d^{16} \\
& - 105*(e^{16}*x^{16} - 32*d^2*e^{14}*x^{14} + 160*d^4*e^{12}*x^{12} - 256*d^6*e^{10}*x^{10} \\
& + 128*d^8*e^8*x^8 + 8*(d*e^{14}*x^{14} - 10*d^3*e^{12}*x^{12} + 24*d^5*e^{10}*x^{10} - 16*d^7*e^8*x^8) \\
& *sqrt(-e^2*x^2 + d^2)) * log(-(d - sqrt(-e^2*x^2 + d^2))/x) - (256*e^{15}*x^{15} - 105*d*e^{14} \\
& *x^{14} - 8064*d^2*e^{13}*x^{13} + 3290*d^3*e^{12}*x^{12} + 35840*d^4*e^{11}*x^{11} - 13720*d^5 \\
& *e^{10}*x^{10} - 11648*d^6*e^9*x^9 - 11760*d^7*e^8*x^8 - 184320*d^8*e^7*x^7 + 156800*d^9 \\
& *e^6*x^6 + 380928*d^{10}*e^5*x^5 - 313600*d^{11}*e^4*x^4 - 294912*d^{12}*e^3*x^3 + 250880*d^{13} \\
& *e^2*x^2 + 81920*d^{14}*e*x - 71680*d^{15}) * sqrt(-e^2*x^2 + d^2)) / (d^4*e^8*x^{16} - 32*d^6 \\
& *e^6*x^{14} + 160*d^8*e^4*x^{12} - 256*d^{10}*e^2*x^{10} + 128*d^{12}*x^8 + 8*(d^5*e^6*x^{14} - 10*d^7 \\
& *e^4*x^{12} + 24*d^9*e^2*x^{10} - 16*d^{11}*x^8) * sqrt(-e^2*x^2 + d^2))
\end{aligned}$$

**Sympy [A]** time = 72.919, size = 1159, normalized size = 5.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*9/(e\*x+d), x)

[Out]  $d^{**3} \text{Piecewise}((-d^{**2}/(8*e*x^{**9}*sqrt(d^{**2}/(e^{**2}*x^{**2}) - 1)) + 7*e/(48*x^{**7}*sqrt(d^{**2}/(e^{**2}*x^{**2}) - 1)) + e^{**3}/(192*d^{**2}*x^{**5}*sqrt(d^{**2}/(e^{**2}*x^{**2}) - 1)) + 5*e^{**5}/(384*d^{**4}*x^{**3}*sqrt(d^{**2}/(e^{**2}*x^{**2}) - 1)) - 5*e^{**7}/(128*d^{**6}*x*sqrt(d^{**2}/(e^{**2}*x^{**2}) - 1)) + 5*e^{**8}*acosh(d/(e*x))/(128*d^{**7}), Abs(d^{**2}/(e^{**2}*x^{**2})) > 1), (I*d^{**2}/(8*e*x^{**9}*sqrt(-d^{**2}/(e^{**2}*x^{**2}) + 1)) - 7*I*e/(48*x^{**7}*sqrt(-d^{**2}/(e^{**2}*x^{**2}) + 1)) - I*e^{**3}/(192*d^{**2}*x^{**5}*sqrt(-d^{**2}/(e^{**2}*x^{**2}) + 1)) - 5*I*e^{**5}/(384*d^{**4}*x^{**3}*sqrt(-d^{**2}/(e^{**2}*x^{**2}) + 1)) + 5*I*e^{**7}/(128*d^{**6}*x*sqrt(-d^{**2}/(e^{**2}*x^{**2}) + 1)) - 5*I*e^{**8}*asin(d/(e*x))/(128*d^{**7}), True)) - d^{**2}*e*Piecewise((-e*sqrt(d^{**2}/(e^{**2}*x^{**2}) - 1)/(7*x^{**6}) + e^{**3}*sqrt(d^{**2}/(e^{**2}*x^{**2}) - 1)/(35*d^{**2}*x^{**4}) + 4*e^{**5}*sqrt(d^{**2}/(e^{**2}*x^{**2}) - 1)/(105*d^{**4}*x^{**2}) + 8*e^{**7}*sqrt(d^{**2}/(e^{**2}*x^{**2}) - 1)/(105*d^{**6}), Abs(d^{**2}/(e^{**2}*x^{**2})) > 1), (-I*e*sqrt(-d^{**2}/(e^{**2}*x^{**2}) + 1)/(7*x^{**6}) + I*e^{**3}*sqrt(-d^{**2}/(e^{**2}*x^{**2}) + 1)/(35*d^{**2}*x^{**4}) + 4*I*e^{**5}*sqrt(-d^{**2}/(e^{**2}*x^{**2}) + 1)/(105*d^{**4}*x^{**2}) + 8*I*e^{**7}*sqrt(-d^{**2}/(e^{**2}*x^{**2}) + 1)/(105*d^{**6}), True)) - d*e^{**2}*Piecewise((-d^{**2}/(6*e*x^{**7}*sqrt(d^{**2}/(e^{**2}*x^{**2}) - 1)) + 5*e/(24*x^{**5}*sqrt(d^{**2}/(e^{**2}*x^{**2}) - 1)) + e^{**3}/(48*d^{**2}*x^{**3}*sqrt(d^{**2}/(e^{**2}*x^{**2}) - 1)) - e^{**5}/(16*d^{**4}*x*sqrt(d^{**2}/(e^{**2}*x^{**2}) - 1)) + e^{**6}*acosh(d/(e*x))/(16*d^{**5}), Abs(d^{**2}/(e^{**2}*x^{**2})) > 1), (I*d^{**2}/(6*e*x^{**7}*sqrt(-d^{**2}/(e^{**2}*x^{**2}) + 1)) - 5*I*e/(24*x^{**5}*sqrt(-d^{**2}/(e^{**2}*x^{**2}) + 1)) - I*e^{**3}/(48*d^{**2}*x^{**3}*sqrt(-d^{**2}/(e^{**2}*x^{**2}) + 1)) + I*e^{**5}/(16*d^{**4}*x*sqrt(-d^{**2}/(e^{**2}*x^{**2}) + 1)) - I*e^{**6}*asin(d/(e*x))/(16*d^{**5}), True)) + e^{**3}*Piecewise((3*I*d^{**3}*sqrt(-1 + e^{**2}*x^{**2}/d^{**2})/(-15*d^{**2}*x^{**5} + 15*e^{**2}*x^{**7}) - 4*I*d*e^{**2}*x^{**2}*sqrt(-1 + e^{**2}*x^{**2}/d^{**2})/(-15*d^{**2}*x^{**5} + 15*e^{**2}*x^{**7}) + 2*I*e^{**6}*x^{**6}*sqrt(-1 + e^{**2}*x^{**2}/d^{**2})/(-15*d^{**5}*x^{**5} + 15*d^{**3}*e^{**2}*x^{**7}) - I*e^{**4}*x^{**4}*sqrt(-1 + e^{**2}*x^{**2}/d^{**2})/(-15*d^{**3}*x^{**5} + 15*d*e^{**2}*x^{**7}), Abs(e^{**2}*x^{**2}/d^{**2}) > 1), (3*d^{**3}*sqrt(1 - e^{**2}*x^{**2}/d^{**2})/(-15*d^{**2}*x^{**5} + 15*e^{**2}*x^{**7}) - 4*d*e^{**2}*x^{**2}*sqrt(1 - e^{**2}*x^{**2}/d^{**2})/(-15*d^{**2}*x^{**5} + 15*e^{**2}*x^{**7}) + 2*e^{**6}*x^{**6}*sqrt(1 - e^{**2}*x^{**2}/d^{**2})/(-15*d^{**5}*x^{**5} + 15*d^{**3}*e^{**2}*x^{**7}) - e^{**4}*x^{**4}*sqrt(1 - e^{**2}*x^{**2}/d^{**2})/(-15*d^{**3}*x^{**5} + 15*d*e^{**2}*x^{**7}), True))$

**GIAC/XCAS [A]** time = 0.325729, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)\*x^9), x, algorithm="giac")

[Out] Done

$$3.117 \quad \int \frac{x\sqrt{1-x^2}}{1+x} dx$$

**Optimal.** Leaf size=27

$$-\frac{1}{2}\sqrt{1-x^2}(2-x) - \frac{1}{2}\sin^{-1}(x)$$

[Out]  $-\left((2-x)\sqrt{1-x^2}\right)/2 - \text{ArcSin}[x]/2$

**Rubi [A]** time = 0.0681749, antiderivative size = 27, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{1}{2}\sqrt{1-x^2}(2-x) - \frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

[In]  $\text{Int}\left[\frac{x\sqrt{1-x^2}}{1+x}, x\right]$

[Out]  $-\left((2-x)\sqrt{1-x^2}\right)/2 - \text{ArcSin}[x]/2$

**Rubi in Sympy [A]** time = 5.2659, size = 29, normalized size = 1.07

$$-\frac{\sqrt{-x^2+1}}{2} - \frac{\text{asin}(x)}{2} - \frac{(-x^2+1)^{\frac{3}{2}}}{2(x+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x*(-x^2+1)^{(1/2)}/(1+x), x)$

[Out]  $-\text{sqrt}(-x^2+1)/2 - \text{asin}(x)/2 - (-x^2+1)^{(3/2)}/(2*(x+1))$

**Mathematica [A]** time = 0.0261381, size = 26, normalized size = 0.96

$$\left(\frac{x}{2}-1\right)\sqrt{1-x^2} - \frac{1}{2}\sin^{-1}(x)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}\left[\frac{x\sqrt{1-x^2}}{1+x}, x\right]$

[Out]  $(-1+x/2)\sqrt{1-x^2} - \text{ArcSin}[x]/2$

**Maple [A]** time = 0.009, size = 34, normalized size = 1.3

$$\frac{x}{2}\sqrt{-x^2+1} - \frac{\arcsin(x)}{2} - \sqrt{-(1+x)^2+2+2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x*(-x^2+1)^{(1/2)}/(1+x), x)$

[Out]  $\frac{1}{2}x(-x^2+1)^{1/2}-\frac{1}{2}\arcsin(x)-(-(1+x)^2+2+2x)^{1/2}$

**Maxima [A]** time = 0.785224, size = 38, normalized size = 1.41

$$\frac{1}{2}\sqrt{-x^2+1}x - \sqrt{-x^2+1} - \frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-x^2 + 1)*x/(x + 1),x, algorithm="maxima")`

[Out]  $\frac{1}{2}\sqrt{-x^2 + 1}x - \sqrt{-x^2 + 1} - \frac{1}{2}\arcsin(x)$

**Fricas [A]** time = 0.285531, size = 123, normalized size = 4.56

$$\frac{2x^3 - 2x^2 - 2(x^2 + 2\sqrt{-x^2 + 1} - 2)\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right) - (x^3 - 2x^2 - 2x)\sqrt{-x^2 + 1} - 2x}{2(x^2 + 2\sqrt{-x^2 + 1} - 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-x^2 + 1)*x/(x + 1),x, algorithm="fricas")`

[Out]  $-\frac{1}{2}(2x^3 - 2x^2 - 2(x^2 + 2\sqrt{-x^2 + 1} - 2)\arctan((\sqrt{-x^2 + 1} - 1)/x) - (x^3 - 2x^2 - 2x)\sqrt{-x^2 + 1} - 2x)/2$

**Sympy [A]** time = 5.61977, size = 29, normalized size = 1.07

$$\begin{cases} \frac{x\sqrt{-x^2+1}}{2} - \sqrt{-x^2+1} - \frac{\arcsin(x)}{2} & \text{for } x > -1 \wedge x < 1 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(-x**2+1)**(1/2)/(1+x),x)`

[Out] `Piecewise((x*sqrt(-x**2 + 1)/2 - sqrt(-x**2 + 1) - asin(x)/2, (x > -1) & (x < 1)))`

**GIAC/XCAS [A]** time = 0.283352, size = 26, normalized size = 0.96

$$\frac{1}{2}\sqrt{-x^2+1}(x-2) - \frac{1}{2}\arcsin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-x^2 + 1)*x/(x + 1),x, algorithm="giac")`

[Out]  $\frac{1}{2}\sqrt{-x^2 + 1}(x - 2) - \frac{1}{2}\arcsin(x)$

$$3.118 \quad \int \frac{(1-a^2x^2)^{3/2}}{x^2(1-ax)} dx$$

**Optimal.** Leaf size=51

$$-\frac{\sqrt{1-a^2x^2}(1-ax)}{x} - a \tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - a \sin^{-1}(ax)$$

[Out] -(((1 - a\*x)\*Sqrt[1 - a^2\*x^2])/x) - a\*ArcSin[a\*x] - a\*ArcTanh[Sqrt[1 - a^2\*x^2]]

**Rubi [A]** time = 0.241016, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$

$$-\frac{\sqrt{1-a^2x^2}(1-ax)}{x} - a \tanh^{-1}\left(\sqrt{1-a^2x^2}\right) - a \sin^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Int[(1 - a^2\*x^2)^(3/2)/(x^2\*(1 - a\*x)), x]

[Out] -(((1 - a\*x)\*Sqrt[1 - a^2\*x^2])/x) - a\*ArcSin[a\*x] - a\*ArcTanh[Sqrt[1 - a^2\*x^2]]

**Rubi in Sympy [A]** time = 21.4625, size = 41, normalized size = 0.8

$$-a \operatorname{asin}(ax) - a \operatorname{atanh}\left(\sqrt{-a^2x^2+1}\right) - \frac{(-ax+1)\sqrt{-a^2x^2+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-a\*\*2\*x\*\*2+1)\*\*(3/2)/x\*\*2/(-a\*x+1), x)

[Out] -a\*asin(a\*x) - a\*atanh(sqrt(-a\*\*2\*x\*\*2 + 1)) - (-a\*x + 1)\*sqrt(-a\*\*2\*x\*\*2 + 1)/x

**Mathematica [A]** time = 0.0640654, size = 54, normalized size = 1.06

$$\sqrt{1-a^2x^2}\left(a - \frac{1}{x}\right) - a \log\left(\sqrt{1-a^2x^2}+1\right) + a \log(x) - a \sin^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Integrate[(1 - a^2\*x^2)^(3/2)/(x^2\*(1 - a\*x)), x]

[Out] (a - x^(-1))\*Sqrt[1 - a^2\*x^2] - a\*ArcSin[a\*x] + a\*Log[x] - a\*Log[1 + Sqrt[1 - a^2\*x^2]]

**Maple [B]** time = 0.03, size = 238, normalized size = 4.7

$$\begin{aligned}
 & -\frac{a}{3} \left( -(x - a^{-1})^2 a^2 - 2(x - a^{-1})a \right)^{\frac{3}{2}} + \frac{a^2 x}{2} \sqrt{-(x - a^{-1})^2 a^2 - 2(x - a^{-1})a} \\
 & + \frac{a^2}{2} \arctan \left( x \sqrt{a^2} \frac{1}{\sqrt{-(x - a^{-1})^2 a^2 - 2(x - a^{-1})a}} \right) \frac{1}{\sqrt{a^2}} - \frac{1}{x} (-a^2 x^2 + 1)^{\frac{5}{2}} \\
 & - a^2 x (-a^2 x^2 + 1)^{\frac{3}{2}} - \frac{3 a^2 x}{2} \sqrt{-a^2 x^2 + 1} - \frac{3 a^2}{2} \arctan \left( x \sqrt{a^2} \frac{1}{\sqrt{-a^2 x^2 + 1}} \right) \frac{1}{\sqrt{a^2}} \\
 & + \frac{a}{3} (-a^2 x^2 + 1)^{\frac{3}{2}} + a \sqrt{-a^2 x^2 + 1} - a \operatorname{Artanh} \left( \frac{1}{\sqrt{-a^2 x^2 + 1}} \right)
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-a^2*x^2+1)^(3/2)/x^2/(-a*x+1),x)`

[Out] `-1/3*a*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(3/2)+1/2*a^2*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(1/2)*x+1/2*a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(1/2))-1/x*(-a^2*x^2+1)^(5/2)-a^2*x*(-a^2*x^2+1)^(3/2)-3/2*a^2*x*(-a^2*x^2+1)^(1/2)-3/2*a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-a^2*x^2+1)^(1/2))+1/3*a*(-a^2*x^2+1)^(3/2)+a*(-a^2*x^2+1)^(1/2)-a*artanh(1/(-a^2*x^2+1)^(1/2))`

**Maxima [A]** time = 0.792898, size = 111, normalized size = 2.18

$$-\frac{a^2 \arcsin\left(\frac{a^2 x}{\sqrt{a^2}}\right)}{\sqrt{a^2}} - a \log\left(\frac{2\sqrt{-a^2 x^2 + 1}}{|x|} + \frac{2}{|x|}\right) + \sqrt{-a^2 x^2 + 1} a - \frac{\sqrt{-a^2 x^2 + 1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(-a^2*x^2+1)^(3/2)/((a*x-1)*x^2),x,algorithm="maxima")`

[Out] `-a^2*arcsin(a^2*x/sqrt(a^2))/sqrt(a^2) - a*log(2*sqrt(-a^2*x^2+1)/abs(x) + 2/abs(x)) + sqrt(-a^2*x^2+1)*a - sqrt(-a^2*x^2+1)/x`

**Fricas [A]** time = 0.297304, size = 242, normalized size = 4.75

$$\frac{a^3 x^3 - 2 a^2 x^2 - 2 \left( a^3 x^3 + 2 \sqrt{-a^2 x^2 + 1} a x - 2 a x \right) \arctan\left(\frac{\sqrt{-a^2 x^2 + 1} - 1}{a x}\right) - \left( a^3 x^3 + 2 \sqrt{-a^2 x^2 + 1} a x - 2 a x \right) \log\left(\frac{\sqrt{-a^2 x^2 + 1}}{x}\right)}{a^2 x^3 + 2 \sqrt{-a^2 x^2 + 1} x - 2 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(-a^2*x^2+1)^(3/2)/((a*x-1)*x^2),x,algorithm="fricas")`

[Out] `-(a^3*x^3 - 2*a^2*x^2 - 2*(a^3*x^3 + 2*sqrt(-a^2*x^2+1)*a*x - 2*a*x)*arctan((sqrt(-a^2*x^2+1)-1)/(a*x)) - (a^3*x^3 + 2*sqrt(-a^2*x^2+1)*a*x - 2*a*x)*log((sqrt(-a^2*x^2+1)-1)/x) - (a^3*x^3 - a^2*x^2 + 2)*sqrt(-a^2*x^2+1) + 2)/(a^2*x^3 + 2*sqrt(-a^2*x^2+1)*x - 2*x)`



**Sympy [A]** time = 12.0071, size = 170, normalized size = 3.33

$$a \left( \begin{cases} i\sqrt{a^2x^2 - 1} - \log(ax) + \frac{\log(a^2x^2)}{2} + i \operatorname{asin}\left(\frac{1}{ax}\right) & \text{for } |a^2x^2| > 1 \\ \sqrt{-a^2x^2 + 1} + \frac{\log(a^2x^2)}{2} - \log\left(\sqrt{-a^2x^2 + 1} + 1\right) & \text{otherwise} \end{cases} \right) \\ + \begin{cases} -\frac{ia^2x}{\sqrt{a^2x^2-1}} + ia \operatorname{acosh}(ax) + \frac{i}{x\sqrt{a^2x^2-1}} & \text{for } |a^2x^2| > 1 \\ \frac{a^2x}{\sqrt{-a^2x^2+1}} - a \operatorname{asin}(ax) - \frac{1}{x\sqrt{-a^2x^2+1}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a\*\*2\*x\*\*2+1)\*\*(3/2)/x\*\*2/(-a\*x+1),x)

[Out] a\*Piecewise((I\*sqrt(a\*\*2\*x\*\*2 - 1) - log(a\*x) + log(a\*\*2\*x\*\*2)/2 + I\*asin(1/(a\*x)), Abs(a\*\*2\*x\*\*2) > 1), (sqrt(-a\*\*2\*x\*\*2 + 1) + log(a\*\*2\*x\*\*2)/2 - log(sqrt(-a\*\*2\*x\*\*2 + 1) + 1), True)) + Piecewise((-I\*a\*\*2\*x/sqrt(a\*\*2\*x\*\*2 - 1) + I\*a\*acosh(a\*x) + I/(x\*sqrt(a\*\*2\*x\*\*2 - 1))), Abs(a\*\*2\*x\*\*2) > 1), (a\*\*2\*x/sqrt(-a\*\*2\*x\*\*2 + 1) - a\*asin(a\*x) - 1/(x\*sqrt(-a\*\*2\*x\*\*2 + 1)), True))

**GIAC/XCAS [A]** time = 0.289853, size = 169, normalized size = 3.31

$$\frac{a^4x}{2\left(\sqrt{-a^2x^2+1}|a|+a\right)|a|} - \frac{a^2 \arcsin(ax) \operatorname{sign}(a)}{|a|} \\ - \frac{a^2 \ln\left(\frac{|-2\sqrt{-a^2x^2+1}|a|-2a|}{2a^2|x|}\right)}{|a|} + \sqrt{-a^2x^2+1}a - \frac{\sqrt{-a^2x^2+1}|a|+a}{2x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(-a^2\*x^2 + 1)^(3/2)/((a\*x - 1)\*x^2),x, algorithm="giac")

[Out] 1/2\*a^4\*x/((sqrt(-a^2\*x^2 + 1)\*abs(a) + a)\*abs(a)) - a^2\*arcsin(a\*x)\*sign(a)/abs(a) - a^2\*ln(1/2\*abs(-2\*sqrt(-a^2\*x^2 + 1)\*abs(a) - 2\*a)/(a^2\*abs(x)))/abs(a) + sqrt(-a^2\*x^2 + 1)\*a - 1/2\*(sqrt(-a^2\*x^2 + 1)\*abs(a) + a)/(x\*abs(a))

$$3.119 \quad \int \frac{x^4}{(d+ex)\sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=118

$$\frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5} - \frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3} - \frac{3d^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}$$

[Out]  $(x^3*(d - e*x))/(e^2*\text{Sqrt}[d^2 - e^2*x^2]) - (4*x^2*\text{Sqrt}[d^2 - e^2*x^2])/(3*e^3) - (d*(16*d - 9*e*x)*\text{Sqrt}[d^2 - e^2*x^2])/(6*e^5) - (3*d^3*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(2*e^5)$

**Rubi [A]** time = 0.37154, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{x^3(d-ex)}{e^2\sqrt{d^2-e^2x^2}} - \frac{d(16d-9ex)\sqrt{d^2-e^2x^2}}{6e^5} - \frac{4x^2\sqrt{d^2-e^2x^2}}{3e^3} - \frac{3d^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}$$

Antiderivative was successfully verified.

[In] Int[x^4/((d + e\*x)\*Sqrt[d^2 - e^2\*x^2]),x]

[Out]  $(x^3*(d - e*x))/(e^2*\text{Sqrt}[d^2 - e^2*x^2]) - (4*x^2*\text{Sqrt}[d^2 - e^2*x^2])/(3*e^3) - (d*(16*d - 9*e*x)*\text{Sqrt}[d^2 - e^2*x^2])/(6*e^5) - (3*d^3*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(2*e^5)$

**Rubi in Sympy [A]** time = 44.1517, size = 116, normalized size = 0.98

$$-\frac{3d^3 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5} - \frac{d^3\sqrt{d^2-e^2x^2}}{e^5(d+ex)} - \frac{2d^2\sqrt{d^2-e^2x^2}}{e^5} + \frac{dx\sqrt{d^2-e^2x^2}}{2e^4} + \frac{(d^2-e^2x^2)^{\frac{3}{2}}}{3e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2),x)

[Out]  $-3*d^3*\operatorname{atan}(e*x/\text{sqrt}(d^2 - e^2*x^2))/(2*e^5) - d^3*\text{sqrt}(d^2 - e^2*x^2)/(e^5*(d + e*x)) - 2*d^2*\text{sqrt}(d^2 - e^2*x^2)/e^5 + d*x*\text{sqrt}(d^2 - e^2*x^2)/(2*e^4) + (d^2 - e^2*x^2)^{3/2}/(3*e^5)$

**Mathematica [A]** time = 0.0985823, size = 93, normalized size = 0.79

$$\sqrt{d^2-e^2x^2} \left( -\frac{d^3}{e^5(d+ex)} - \frac{5d^2}{3e^5} + \frac{dx}{2e^4} - \frac{x^2}{3e^3} \right) - \frac{3d^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/((d + e\*x)\*Sqrt[d^2 - e^2\*x^2]),x]

[Out]  $\text{Sqrt}[d^2 - e^2*x^2]*((-5*d^2)/(3*e^5) + (d*x)/(2*e^4) - x^2/(3*e^3) - d^3/(e^5*(d + e*x))) - (3*d^3*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(2*e^5)$

---

**Maple [A]** time = 0.018, size = 147, normalized size = 1.3

$$-\frac{x^2}{3e^3}\sqrt{-e^2x^2+d^2}-\frac{5d^2}{3e^5}\sqrt{-e^2x^2+d^2}-\frac{d^3}{e^6}\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\left(x+\frac{d}{e}\right)^{-1}}$$

$$-\frac{3d^3}{2e^4}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}}+\frac{dx}{2e^4}\sqrt{-e^2x^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)`

[Out] `-1/3*x^2*(-e^2*x^2+d^2)^(1/2)/e^3-5/3/e^5*d^2*(-e^2*x^2+d^2)^(1/2)-d^3/e^6/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-3/2*d^3/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+1/2*d/e^4*x*(-e^2*x^2+d^2)^(1/2)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(sqrt(-e^2*x^2+d^2)*(e*x+d)),x,algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.287899, size = 491, normalized size = 4.16

$$\frac{2e^7x^7+5de^6x^6-12d^2e^5x^5+21d^3e^4x^4+60d^4e^3x^3-36d^5e^2x^2-72d^6ex+18(d^3e^4x^4-3d^4e^3x^3-8d^5e^2x^2+4d^6ex+8d^7)}{6(e^9x^4-3de^8x^3-8d^2e^7x^2+8d^3e^6x-8d^4e^5x+8d^5e^4x-8d^6e^3x+8d^7e^2x-8d^8ex+8d^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/(sqrt(-e^2*x^2+d^2)*(e*x+d)),x,algorithm="fricas")`

[Out] `1/6*(2*e^7*x^7+5*d*e^6*x^6-12*d^2*e^5*x^5+21*d^3*e^4*x^4+60*d^4*e^3*x^3-36*d^5*e^2*x^2-72*d^6*e*x+18*(d^3*e^4*x^4-3*d^4*e^3*x^3-8*d^5*e^2*x^2+4*d^6*e*x+8*d^7+(d^3*e^4*x^3+4*d^4*e^3*x^2-4*d^5*e^2*x-8*d^6)*sqrt(-e^2*x^2+d^2))*arctan(-(d-sqrt(-e^2*x^2+d^2))/(e*x))-((2*e^6*x^6-9*d*e^5*x^5+3*d^2*e^4*x^4+24*d^3*e^3*x^3-36*d^4*e^2*x^2-72*d^5*e*x)*sqrt(-e^2*x^2+d^2))/(e^9*x^4-3*d*e^8*x^3-8*d^2*e^7*x^2+4*d^3*e^6*x+8*d^4*e^5x+(e^8*x^3+4*d*e^7*x^2-4*d^2*e^6*x-8*d^3*e^5)*sqrt(-e^2*x^2+d^2))`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{-(-d+ex)(d+ex)}(d+ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)/(-e**2*x**2+d**2)**(1/2), x)
```

```
[Out] Integral(x**4/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(sqrt(-e^2*x^2 + d^2)*(e*x + d)), x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.120 \quad \int \frac{x^3}{(d+ex)\sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=91

$$\frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}} + \frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4} + \frac{3d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}$$

[Out] (x^2\*(d - e\*x))/(e^2\*Sqrt[d^2 - e^2\*x^2]) + ((4\*d - 3\*e\*x)\*Sqrt[d^2 - e^2\*x^2])/(2\*e^4) + (3\*d^2\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(2\*e^4)

**Rubi [A]** time = 0.247766, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\frac{x^2(d-ex)}{e^2\sqrt{d^2-e^2x^2}} + \frac{(4d-3ex)\sqrt{d^2-e^2x^2}}{2e^4} + \frac{3d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}$$

Antiderivative was successfully verified.

[In] Int[x^3/((d + e\*x)\*Sqrt[d^2 - e^2\*x^2]),x]

[Out] (x^2\*(d - e\*x))/(e^2\*Sqrt[d^2 - e^2\*x^2]) + ((4\*d - 3\*e\*x)\*Sqrt[d^2 - e^2\*x^2])/(2\*e^4) + (3\*d^2\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(2\*e^4)

**Rubi in Sympy [A]** time = 30.4651, size = 92, normalized size = 1.01

$$\frac{3d^2 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4} + \frac{d^2\sqrt{d^2-e^2x^2}}{e^4(d+ex)} + \frac{d\sqrt{d^2-e^2x^2}}{e^4} - \frac{x\sqrt{d^2-e^2x^2}}{2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2),x)

[Out] 3\*d\*\*2\*atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2))/(2\*e\*\*4) + d\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(e\*\*4\*(d + e\*x)) + d\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/e\*\*4 - x\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(2\*e\*\*3)

**Mathematica [A]** time = 0.115552, size = 70, normalized size = 0.77

$$\frac{\sqrt{d^2-e^2x^2}\left(\frac{2d^2}{d+ex} + 2d - ex\right) + 3d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((d + e\*x)\*Sqrt[d^2 - e^2\*x^2]),x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(2\*d - e\*x + (2\*d^2)/(d + e\*x)) + 3\*d^2\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(2\*e^4)

**Maple [A]** time = 0.015, size = 120, normalized size = 1.3

$$\frac{3d^2}{2e^3} \arctan\left(x\sqrt{e^2} \frac{1}{\sqrt{-e^2x^2+d^2}}\right) \frac{1}{\sqrt{e^2}} - \frac{x}{2e^3} \sqrt{-e^2x^2+d^2} + \frac{d}{e^4} \sqrt{-e^2x^2+d^2} + \frac{d^2}{e^5} \sqrt{-\left(x+\frac{d}{e}\right)^2 e^2 + 2de\left(x+\frac{d}{e}\right)\left(x+\frac{d}{e}\right)^{-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)`

[Out] `3/2*d^2/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/2/e^3*x*(-e^2*x^2+d^2)^(1/2)+d/e^4*(-e^2*x^2+d^2)^(1/2)+d^2/e^5/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(sqrt(-e^2*x^2+d^2)*(e*x+d)),x,algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.283686, size = 370, normalized size = 4.07

$$\frac{e^5x^5 - 4de^4x^4 - 7d^2e^3x^3 + 6d^3e^2x^2 + 12d^4ex + 6(d^2e^3x^3 + 3d^3e^2x^2 - 2d^4ex - 4d^5 - (d^2e^2x^2 - 2d^3ex - 4d^4)\sqrt{-e^2x^2+d^2})}{2(e^7x^3 + 3de^6x^2 - 2d^2e^5x - 4d^3e^4 - (e^6x^2 - 2de^5x - 4d^4)\sqrt{-e^2x^2+d^2})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(sqrt(-e^2*x^2+d^2)*(e*x+d)),x,algorithm="fricas")`

[Out] `-1/2*(e^5*x^5 - 4*d*e^4*x^4 - 7*d^2*e^3*x^3 + 6*d^3*e^2*x^2 + 12*d^4*e*x + 6*(d^2*e^3*x^3 + 3*d^3*e^2*x^2 - 2*d^4*e*x - 4*d^5 - (d^2*e^2*x^2 - 2*d^3*e*x - 4*d^4)*sqrt(-e^2*x^2+d^2))*arctan(-(d - sqrt(-e^2*x^2+d^2))/(e*x)) + (e^4*x^4 + d*e^3*x^3 - 6*d^2*e^2*x^2 - 12*d^3*e*x)*sqrt(-e^2*x^2+d^2))/(e^7*x^3 + 3*d*e^6*x^2 - 2*d^2*e^5*x - 4*d^3*e^4 - (e^6*x^2 - 2*d*e^5*x - 4*d^4)*sqrt(-e^2*x^2+d^2))`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{-(-d+ex)(d+ex)}(d+ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)`

[Out] `Integral(x**3/(sqrt(-(-d+e*x)*(d+e*x))*(d+e*x)),x)`

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(sqrt(-e^2*x^2 + d^2)*(e*x + d)),x, algorithm="giac")`

[Out] Exception raised: NotImplementedError

$$3.121 \quad \int \frac{x^2}{(d+ex)\sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=77

$$-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{\sqrt{d^2-e^2x^2}}{e^3} - \frac{d \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

[Out]  $-(\text{Sqrt}[d^2 - e^2x^2]/e^3) - (d*\text{Sqrt}[d^2 - e^2x^2])/(e^3*(d + e*x)) - (d*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2x^2]])/e^3$

**Rubi [A]** time = 0.198237, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$-\frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{\sqrt{d^2-e^2x^2}}{e^3} - \frac{d \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2/((d + e*x)*\text{Sqrt}[d^2 - e^2*x^2]), x]$

[Out]  $-(\text{Sqrt}[d^2 - e^2*x^2]/e^3) - (d*\text{Sqrt}[d^2 - e^2*x^2])/(e^3*(d + e*x)) - (d*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/e^3$

**Rubi in Sympy [A]** time = 22.1858, size = 65, normalized size = 0.84

$$-\frac{d \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3} - \frac{d\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{\sqrt{d^2-e^2x^2}}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**2}/(e*x+d)/(-e^{**2}*x^{**2}+d^{**2})^{**(1/2)}, x)$

[Out]  $-d*\operatorname{atan}(e*x/\operatorname{sqrt}(d^{**2} - e^{**2}*x^{**2}))/e^{**3} - d*\operatorname{sqrt}(d^{**2} - e^{**2}*x^{**2})/(e^{**3}*(d + e*x)) - \operatorname{sqrt}(d^{**2} - e^{**2}*x^{**2})/e^{**3}$

**Mathematica [A]** time = 0.116152, size = 59, normalized size = 0.77

$$-\frac{\frac{\sqrt{d^2-e^2x^2}(2d+ex)}{d+ex} + d \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^3}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^2/((d + e*x)*\text{Sqrt}[d^2 - e^2*x^2]), x]$

[Out]  $-(((2*d + e*x)*\text{Sqrt}[d^2 - e^2*x^2])/(d + e*x) + d*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/e^3$

**Maple [A]** time = 0.014, size = 97, normalized size = 1.3

$$-\frac{1}{e^3}\sqrt{-e^2x^2+d^2} - \frac{d}{e^2}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right) - \frac{1}{\sqrt{e^2}} - \frac{d}{e^4}\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}\left(x+\frac{d}{e}\right)^{-1}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)`

[Out]  $-(e^2x^2+d^2)^{1/2}/e^3-d/e^2/(e^2)^{1/2}*\arctan((e^2)^{1/2}*x/(-e^2*x^2+d^2)^{1/2})-d/e^4/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(sqrt(-e^2*x^2 + d^2)*(e*x + d)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.293096, size = 243, normalized size = 3.16

$$\frac{e^3x^3 + de^2x^2 + 2d^2ex + 2\left(de^2x^2 - d^2ex - 2d^3 + \sqrt{-e^2x^2 + d^2}(dex + 2d^2)\right)\arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right) - (e^2x^2 + 2dex)\sqrt{-e^2x^2 + d^2}}{e^5x^2 - de^4x - 2d^2e^3 + (e^4x + 2de^3)\sqrt{-e^2x^2 + d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(sqrt(-e^2*x^2 + d^2)*(e*x + d)),x, algorithm="fricas")`

[Out]  $(e^3x^3 + d^2e^2x^2 + 2d^2ex + 2(d^2e^2x^2 - d^2ex - 2d^3 + \sqrt{-e^2x^2 + d^2}(dex + 2d^2))\arctan(-\frac{d - \sqrt{-e^2x^2 + d^2}}{ex}) - (e^2x^2 + 2dex)\sqrt{-e^2x^2 + d^2})/(e^5x^2 - d^2e^4x - 2d^2e^3 + (e^4x + 2de^3)\sqrt{-e^2x^2 + d^2})$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-(-d + ex)(d + ex)}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)`

[Out] `Integral(x**2/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)`

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(sqrt(-e^2*x^2 + d^2)*(e*x + d)),x, algorithm="giac")`

[Out] Exception raised: NotImplementedError

$$3.122 \quad \int \frac{x}{(d+ex)\sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=52

$$\frac{\sqrt{d^2 - e^2x^2}}{e^2(d + ex)} + \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{e^2}$$

[Out] Sqrt[d^2 - e^2\*x^2]/(e^2\*(d + e\*x)) + ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]]/e^2

**Rubi [A]** time = 0.081524, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$\frac{\sqrt{d^2 - e^2x^2}}{e^2(d + ex)} + \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{e^2}$$

Antiderivative was successfully verified.

[In] Int[x/((d + e\*x)\*Sqrt[d^2 - e^2\*x^2]),x]

[Out] Sqrt[d^2 - e^2\*x^2]/(e^2\*(d + e\*x)) + ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]]/e^2

**Rubi in Sympy [A]** time = 13.1962, size = 42, normalized size = 0.81

$$\frac{\text{atan}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{e^2} + \frac{\sqrt{d^2 - e^2x^2}}{e^2(d + ex)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x/(e\*x+d)/((-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2)),x)

[Out] atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2))/e\*\*2 + sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(e\*\*2\*(d + e\*x))

**Mathematica [A]** time = 0.0535149, size = 49, normalized size = 0.94

$$\frac{\frac{\sqrt{d^2 - e^2x^2}}{d+ex} + \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{e^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/((d + e\*x)\*Sqrt[d^2 - e^2\*x^2]),x]

[Out] (Sqrt[d^2 - e^2\*x^2]/(d + e\*x) + ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/e^2

**Maple [A]** time = 0.013, size = 74, normalized size = 1.4

$$\frac{1}{e} \arctan\left(x\sqrt{e^2} \frac{1}{\sqrt{-e^2x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} + \frac{1}{e^3} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\left(x + \frac{d}{e}\right)^{-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)`

[Out]  $1/e/(e^2)^{(1/2)}*\arctan((e^2)^{(1/2)}*x/(-e^2*x^2+d^2)^{(1/2)})+1/e^3/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^{(1/2)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(sqrt(-e^2*x^2 + d^2)*(e*x + d)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.285467, size = 119, normalized size = 2.29

$$\frac{2\left(ex - \left(ex + d - \sqrt{-e^2x^2 + d^2}\right) \arctan\left(-\frac{d - \sqrt{-e^2x^2 + d^2}}{ex}\right)\right)}{e^3x + de^2 - \sqrt{-e^2x^2 + d^2}e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(sqrt(-e^2*x^2 + d^2)*(e*x + d)),x, algorithm="fricas")`

[Out]  $2*(e*x - (e*x + d - \sqrt{-e^2*x^2 + d^2}))*\arctan(-(d - \sqrt{-e^2*x^2 + d^2})/(e*x))/(e^3*x + d*e^2 - \sqrt{-e^2*x^2 + d^2}*e^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-(-d + ex)(d + ex)}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)`

[Out] `Integral(x/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)`

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(sqrt(-e^2*x^2 + d^2)*(e*x + d)),x, algorithm="giac")`

[Out] Exception raised: NotImplementedError

$$3.123 \quad \int \frac{1}{(d+ex)\sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=31

$$-\frac{\sqrt{d^2 - e^2x^2}}{de(d + ex)}$$

[Out] -(Sqrt[d^2 - e^2\*x^2]/(d\*e\*(d + e\*x)))

**Rubi [A]** time = 0.0338779, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$-\frac{\sqrt{d^2 - e^2x^2}}{de(d + ex)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)\*Sqrt[d^2 - e^2\*x^2]),x]

[Out] -(Sqrt[d^2 - e^2\*x^2]/(d\*e\*(d + e\*x)))

**Rubi in Sympy [A]** time = 5.3728, size = 22, normalized size = 0.71

$$-\frac{\sqrt{d^2 - e^2x^2}}{de(d + ex)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2),x)

[Out] -sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(d\*e\*(d + e\*x))

**Mathematica [A]** time = 0.0238119, size = 31, normalized size = 1.

$$-\frac{\sqrt{d^2 - e^2x^2}}{de(d + ex)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x)\*Sqrt[d^2 - e^2\*x^2]),x]

[Out] -(Sqrt[d^2 - e^2\*x^2]/(d\*e\*(d + e\*x)))

**Maple [A]** time = 0.009, size = 29, normalized size = 0.9

$$-\frac{-ex + d}{de} \frac{1}{\sqrt{-e^2x^2 + d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x+d)/(-e^2\*x^2+d^2)^(1/2),x)

[Out]  $-(-e^x+d)/d/e/(-e^2x^2+d^2)^{(1/2)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.276379, size = 41, normalized size = 1.32

$$-\frac{2x}{dex + d^2 - \sqrt{-e^2x^2 + d^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)),x, algorithm="fricas")`

[Out]  $-2*x/(d*e*x + d^2 - \sqrt{-e^2*x^2 + d^2}*d)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(-d + ex)(d + ex)}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)`

[Out] `Integral(1/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)`

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)),x, algorithm="giac")`

[Out] Exception raised: NotImplementedError

$$3.124 \quad \int \frac{1}{x(d+ex)\sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=54

$$\frac{\sqrt{d^2 - e^2x^2}}{d^2(d + ex)} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)}{d^2}$$

[Out] Sqrt[d^2 - e^2\*x^2]/(d^2\*(d + e\*x)) - ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]/d^2

**Rubi [A]** time = 0.143968, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\frac{\sqrt{d^2 - e^2x^2}}{d^2(d + ex)} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(d + e\*x)\*Sqrt[d^2 - e^2\*x^2]), x]

[Out] Sqrt[d^2 - e^2\*x^2]/(d^2\*(d + e\*x)) - ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]/d^2

**Rubi in Sympy [A]** time = 15.7826, size = 41, normalized size = 0.76

$$\frac{d - ex}{d^2\sqrt{d^2 - e^2x^2}} - \frac{\operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2), x)

[Out] (d - e\*x)/(d\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) - atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d)/d\*\*2

**Mathematica [A]** time = 0.0643559, size = 52, normalized size = 0.96

$$\frac{\frac{\sqrt{d^2 - e^2x^2}}{d + ex} - \log\left(\sqrt{d^2 - e^2x^2} + d\right) + \log(x)}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(d + e\*x)\*Sqrt[d^2 - e^2\*x^2]), x]

[Out] (Sqrt[d^2 - e^2\*x^2]/(d + e\*x) + Log[x] - Log[d + Sqrt[d^2 - e^2\*x^2]])/d^2

**Maple [A]** time = 0.016, size = 88, normalized size = 1.6

$$-\frac{1}{d} \ln\left(\frac{1}{x} \left(2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}} + \frac{1}{ed^2} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\left(x + \frac{d}{e}\right)^{-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(e*x+d)/(-e^2*x^2+d^2)^(1/2),x)`

[Out]  $-1/d/(d^2)^{(1/2)} * \ln((2*d^2+2*(d^2)^{(1/2)}*(-e^2*x^2+d^2)^{(1/2)})/x) + 1/d^2/e/(x+d/e) * (-(x+d/e)^2*e^2+2*d*e*(x+d/e))^{(1/2)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)*x),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.283199, size = 112, normalized size = 2.07

$$\frac{2ex + \left( ex + d - \sqrt{-e^2x^2 + d^2} \right) \log\left( -\frac{d - \sqrt{-e^2x^2 + d^2}}{x} \right)}{d^2ex + d^3 - \sqrt{-e^2x^2 + d^2}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)*x),x, algorithm="fricas")`

[Out]  $(2*e*x + (e*x + d - \sqrt{-e^2*x^2 + d^2})*\log(-(d - \sqrt{-e^2*x^2 + d^2}))/x))/(d^2*e*x + d^3 - \sqrt{-e^2*x^2 + d^2}*d^2)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-(-d + ex)(d + ex)}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(e*x+d)/(-e**2*x**2+d**2)**(1/2),x)`

[Out] `Integral(1/(x*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)), x)`

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)*x),x, algorithm="giac")`

[Out] Exception raised: NotImplementedError

$$3.125 \quad \int \frac{1}{x^2(d+ex)\sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=81

$$\frac{\sqrt{d^2 - e^2x^2}}{d^2x(d + ex)} - \frac{2\sqrt{d^2 - e^2x^2}}{d^3x} + \frac{e \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)}{d^3}$$

[Out]  $(-2*\text{Sqrt}[d^2 - e^2*x^2])/(d^3*x) + \text{Sqrt}[d^2 - e^2*x^2]/(d^2*x*(d + e*x)) + (e*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/d^3$

**Rubi [A]** time = 0.215206, antiderivative size = 81, normalized size of antiderivative = 1., number of rules used = 5, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\frac{\sqrt{d^2 - e^2x^2}}{d^2x(d + ex)} - \frac{2\sqrt{d^2 - e^2x^2}}{d^3x} + \frac{e \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)}{d^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^2*(d + e*x)*\text{Sqrt}[d^2 - e^2*x^2]), x]$

[Out]  $(-2*\text{Sqrt}[d^2 - e^2*x^2])/(d^3*x) + \text{Sqrt}[d^2 - e^2*x^2]/(d^2*x*(d + e*x)) + (e*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/d^3$

**Rubi in Sympy [A]** time = 21.3509, size = 65, normalized size = 0.8

$$\frac{d - ex}{d^2x\sqrt{d^2 - e^2x^2}} + \frac{e \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)}{d^3} - \frac{2\sqrt{d^2 - e^2x^2}}{d^3x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x^{**2}/(e*x+d)/(-e^{**2}*x^{**2}+d^{**2})^{**}(1/2), x)$

[Out]  $(d - e*x)/(d^{**2}*x*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})) + e*\operatorname{atanh}(\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/d)/d^{**3} - 2*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(d^{**3}*x)$

**Mathematica [A]** time = 0.104666, size = 65, normalized size = 0.8

$$\frac{-\frac{\sqrt{d^2 - e^2x^2}(d+2ex)}{x(d+ex)} + e \log\left(\sqrt{d^2 - e^2x^2} + d\right) - e \log(x)}{d^3}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[1/(x^2*(d + e*x)*\text{Sqrt}[d^2 - e^2*x^2]), x]$

[Out]  $(-(((d + 2*e*x)*\text{Sqrt}[d^2 - e^2*x^2])/(x*(d + e*x)))) - e*\text{Log}[x] + e*\text{Log}[d + \text{Sqrt}[d^2 - e^2*x^2]])/d^3$



**Maple [A]** time = 0.016, size = 108, normalized size = 1.3

$$-\frac{1}{d^3 x} \sqrt{-e^2 x^2 + d^2} - \frac{1}{d^3} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right) \left(x + \frac{d}{e}\right)^{-1}} + \frac{e}{d^2} \ln\left(\frac{1}{x} \left(2 d^2 + 2 \sqrt{d^2} \sqrt{-e^2 x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(e\*x+d)/(-e^2\*x^2+d^2)^(1/2), x)

[Out] -(-e^2\*x^2+d^2)^(1/2)/d^3/x-1/d^3/(x+d/e)\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)+e/d^2/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-e^2 x^2 + d^2} (e x + d) x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(-e^2\*x^2 + d^2)\*(e\*x + d)\*x^2), x, algorithm="maxima")

[Out] integrate(1/(sqrt(-e^2\*x^2 + d^2)\*(e\*x + d)\*x^2), x)

**Fricas [A]** time = 0.284395, size = 271, normalized size = 3.35

$$\frac{e^3 x^3 + 4 d e^2 x^2 - d^2 e x - 2 d^3 - \left(e^3 x^3 - d e^2 x^2 - 2 d^2 e x + (e^2 x^2 + 2 d e x) \sqrt{-e^2 x^2 + d^2}\right) \log\left(-\frac{d - \sqrt{-e^2 x^2 + d^2}}{x}\right) - (3 e^2 x^2 - d e x)}{d^3 e^2 x^3 - d^4 e x^2 - 2 d^5 x + (d^3 e x^2 + 2 d^4 x) \sqrt{-e^2 x^2 + d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(-e^2\*x^2 + d^2)\*(e\*x + d)\*x^2), x, algorithm="fricas")

[Out] (e^3\*x^3 + 4\*d\*e^2\*x^2 - d^2\*e\*x - 2\*d^3 - (e^3\*x^3 - d\*e^2\*x^2 - 2\*d^2\*e\*x + (e^2\*x^2 + 2\*d\*e\*x)\*sqrt(-e^2\*x^2 + d^2))\*log(-(d - sqrt(-e^2\*x^2 + d^2))/x) - (3\*e^2\*x^2 - d\*e\*x - 2\*d^2)\*sqrt(-e^2\*x^2 + d^2))/(d^3\*e^2\*x^3 - d^4\*e\*x^2 - 2\*d^5\*x + (d^3\*e\*x^2 + 2\*d^4\*x)\*sqrt(-e^2\*x^2 + d^2))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt{-(-d + e x)(d + e x)} (d + e x)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2), x)

[Out] Integral(1/(x\*\*2\*sqrt(-(-d + e\*x)\*(d + e\*x))\*(d + e\*x)), x)

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**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)*x^2),x, algorithm="giac")`

[Out] Exception raised: NotImplementedError

$$3.126 \quad \int \frac{1}{x^3(d+ex)\sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=113

$$\frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} + \frac{2e\sqrt{d^2-e^2x^2}}{d^4x} - \frac{3e^2 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4} - \frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2}$$

[Out]  $(-3*\text{Sqrt}[d^2 - e^2*x^2])/(2*d^3*x^2) + (2*e*\text{Sqrt}[d^2 - e^2*x^2])/(d^4*x) + \text{Sqrt}[d^2 - e^2*x^2]/(d^2*x^2*(d + e*x)) - (3*e^2*\text{ArcTan}[\text{Sqrt}[d^2 - e^2*x^2]/d])/(2*d^4)$

**Rubi [A]** time = 0.305292, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{\sqrt{d^2-e^2x^2}}{d^2x^2(d+ex)} + \frac{2e\sqrt{d^2-e^2x^2}}{d^4x} - \frac{3e^2 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4} - \frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^3*(d + e*x)*\text{Sqrt}[d^2 - e^2*x^2]), x]$

[Out]  $(-3*\text{Sqrt}[d^2 - e^2*x^2])/(2*d^3*x^2) + (2*e*\text{Sqrt}[d^2 - e^2*x^2])/(d^4*x) + \text{Sqrt}[d^2 - e^2*x^2]/(d^2*x^2*(d + e*x)) - (3*e^2*\text{ArcTan}[\text{Sqrt}[d^2 - e^2*x^2]/d])/(2*d^4)$

**Rubi in Sympy [A]** time = 33.8534, size = 97, normalized size = 0.86

$$\frac{d-ex}{d^2x^2\sqrt{d^2-e^2x^2}} - \frac{3\sqrt{d^2-e^2x^2}}{2d^3x^2} - \frac{3e^2 \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^4} + \frac{2e\sqrt{d^2-e^2x^2}}{d^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x^{**3}/(e*x+d)/(-e^{**2}*x^{**2}+d^{**2})^{**}(1/2), x)$

[Out]  $(d - e*x)/(d^{**2}*x^{**2}*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})) - 3*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(2*d^{**3}*x^{**2}) - 3*e^{**2}*\operatorname{atanh}(\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/d)/(2*d^{**4}) + 2*e*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(d^{**4}*x)$

**Mathematica [A]** time = 0.14159, size = 84, normalized size = 0.74

$$\frac{\frac{\sqrt{d^2-e^2x^2}(-d^2+dex+4e^2x^2)}{x^2(d+ex)}}{2d^4} - 3e^2 \log\left(\sqrt{d^2-e^2x^2} + d\right) + 3e^2 \log(x)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[1/(x^3*(d + e*x)*\text{Sqrt}[d^2 - e^2*x^2]), x]$

[Out]  $((\text{Sqrt}[d^2 - e^2*x^2]*(-d^2 + d*e*x + 4*e^2*x^2))/(x^2*(d + e*x)) + 3*e^2*\text{Log}[x] - 3*e^2*\text{Log}[d + \text{Sqrt}[d^2 - e^2*x^2]])/(2*d^4)$

**Maple [A]** time = 0.017, size = 133, normalized size = 1.2

$$-\frac{1}{2d^3x^2}\sqrt{-e^2x^2+d^2}-\frac{3e^2}{2d^3}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}+\frac{e}{d^4}\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\left(x+\frac{d}{e}\right)^{-1}}+\frac{e}{d^4x}\sqrt{-e^2x^2+d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(e\*x+d)/(-e^2\*x^2+d^2)^(1/2),x)

[Out] -1/2\*(-e^2\*x^2+d^2)^(1/2)/d^3/x^2-3/2/d^3\*e^2/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)+1/d^4\*e/(x+d/e)\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)+e\*(-e^2\*x^2+d^2)^(1/2)/d^4/x

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-e^2x^2+d^2}(ex+d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(-e^2\*x^2 + d^2)\*(e\*x + d)\*x^3),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-e^2\*x^2 + d^2)\*(e\*x + d)\*x^3), x)

**Fricas [A]** time = 0.286798, size = 410, normalized size = 3.63

$$\frac{6e^5x^5 - 5de^4x^4 - 16d^2e^3x^3 + 9d^3e^2x^2 + 6d^4ex - 4d^5 + 3(e^5x^5 + 3de^4x^4 - 2d^2e^3x^3 - 4d^3e^2x^2 - (e^4x^4 - 2de^3x^3 - 4d^2e^2x^2 - 3de^3x^3 + 3d^5e^2x^4 - 2d^6ex^3 - 4d^7x^2 - (d^4e^2x^4 - 2d^5e^3x^3 - 4d^6e^2x^2 - 3d^7ex - d^8))\sqrt{-e^2x^2+d^2})}{2(d^4e^3x^5 + 3d^5e^2x^4 - 2d^6ex^3 - 4d^7x^2 - (d^4e^2x^4 - 2d^5e^3x^3 - 4d^6e^2x^2 - 3d^7ex - d^8))\sqrt{-e^2x^2+d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(-e^2\*x^2 + d^2)\*(e\*x + d)\*x^3),x, algorithm="fricas")

[Out] 1/2\*(6\*e^5\*x^5 - 5\*d\*e^4\*x^4 - 16\*d^2\*e^3\*x^3 + 9\*d^3\*e^2\*x^2 + 6\*d^4\*e\*x - 4\*d^5 + 3\*(e^5\*x^5 + 3\*d\*e^4\*x^4 - 2\*d^2\*e^3\*x^3 - 4\*d^3\*e^2\*x^2 - (e^4\*x^4 - 2\*d\*e^3\*x^3 - 4\*d^2\*e^2\*x^2 - 3\*d\*e^3\*x^3 + 3\*d^5\*e^2\*x^4 - 2\*d^6\*e\*x^3 - 4\*d^7\*x^2 - (d^4\*e^2\*x^4 - 2\*d^5\*e^3\*x^3 - 4\*d^6\*e^2\*x^2 - 3\*d^7\*e\*x - d^8))\*sqrt(-e^2\*x^2 + d^2)))\*log(-(d - sqrt(-e^2\*x^2 + d^2))/x) + (2\*e^4\*x^4 + 13\*d\*e^3\*x^3 - 7\*d^2\*e^2\*x^2 - 6\*d^3\*e\*x + 4\*d^4)\*sqrt(-e^2\*x^2 + d^2)/(d^4\*e^3\*x^5 + 3\*d^5\*e^2\*x^4 - 2\*d^6\*e\*x^3 - 4\*d^7\*x^2 - (d^4\*e^2\*x^4 - 2\*d^5\*e^3\*x^3 - 4\*d^6\*e^2\*x^2 - 3\*d^7\*e\*x - d^8))\*sqrt(-e^2\*x^2 + d^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3\sqrt{-(-d+ex)(d+ex)(d+ex)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2),x)

[Out]  $\text{Integral}(1/(x^{*3}\sqrt{-(-d + e*x)*(d + e*x)}*(d + e*x)), x)$

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**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)*x^3),x, algorithm="giac")`

[Out] Exception raised: NotImplementedError

$$3.127 \quad \int \frac{x^5}{(d+ex)(d^2-e^2x^2)^{3/2}} dx$$

**Optimal.** Leaf size=128

$$\frac{x^4(d-ex)}{3e^2(d^2-e^2x^2)^{3/2}} - \frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6} - \frac{5d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6} - \frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}}$$

[Out]  $(x^4*(d - e*x))/(3*e^2*(d^2 - e^2*x^2)^(3/2)) - (x^2*(4*d - 5*e*x))/(3*e^4*\text{Sqrt}[d^2 - e^2*x^2]) - ((16*d - 15*e*x)*\text{Sqrt}[d^2 - e^2*x^2])/(6*e^6) - (5*d^2*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(2*e^6)$

**Rubi [A]** time = 0.367588, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\frac{x^4(d-ex)}{3e^2(d^2-e^2x^2)^{3/2}} - \frac{(16d-15ex)\sqrt{d^2-e^2x^2}}{6e^6} - \frac{5d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6} - \frac{x^2(4d-5ex)}{3e^4\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[x^5/((d + e\*x)\*(d^2 - e^2\*x^2)^(3/2)), x]

[Out]  $(x^4*(d - e*x))/(3*e^2*(d^2 - e^2*x^2)^(3/2)) - (x^2*(4*d - 5*e*x))/(3*e^4*\text{Sqrt}[d^2 - e^2*x^2]) - ((16*d - 15*e*x)*\text{Sqrt}[d^2 - e^2*x^2])/(6*e^6) - (5*d^2*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(2*e^6)$

**Rubi in Sympy [A]** time = 59.4011, size = 163, normalized size = 1.27

$$\frac{d^4}{3e^6(d+ex)\sqrt{d^2-e^2x^2}} - \frac{2d^3}{e^6\sqrt{d^2-e^2x^2}} + \frac{4d^2x}{3e^5\sqrt{d^2-e^2x^2}} - \frac{5d^2 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6} - \frac{d\sqrt{d^2-e^2x^2}}{e^6} + \frac{x^3}{e^3\sqrt{d^2-e^2x^2}} + \frac{3x\sqrt{d^2-e^2x^2}}{2e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2), x)

[Out]  $d**4/(3*e**6*(d + e*x)*\text{sqrt}(d**2 - e**2*x**2)) - 2*d**3/(e**6*\text{sqrt}(d**2 - e**2*x**2)) + 4*d**2*x/(3*e**5*\text{sqrt}(d**2 - e**2*x**2)) - 5*d**2*\text{atan}(e*x/\text{sqrt}(d**2 - e**2*x**2))/(2*e**6) - d*\text{sqrt}(d**2 - e**2*x**2)/e**6 + x**3/(e**3*\text{sqrt}(d**2 - e**2*x**2)) + 3*x*\text{sqrt}(d**2 - e**2*x**2)/(2*e**5)$

**Mathematica [A]** time = 0.11664, size = 106, normalized size = 0.83

$$\frac{\sqrt{d^2-e^2x^2}(16d^4+d^3ex-23d^2e^2x^2-3de^3x^3+3e^4x^4)}{(ex-d)(d+ex)^2} - 15d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{6e^6}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((d + e\*x)\*(d^2 - e^2\*x^2)^(3/2)), x]

[Out]  $((\sqrt{d^2 - e^2 x^2}) * (16 d^4 + d^3 e x - 23 d^2 e^2 x^2 - 3 d e^3 x^3 + 3 e^4 x^4)) / ((-d + e x) * (d + e x)^2) - 15 d^2 \operatorname{ArcTan}[(e x) / \sqrt{d^2 - e^2 x^2}] / (6 e^6)$

**Maple [A]** time = 0.026, size = 208, normalized size = 1.6

$$\begin{aligned} & \frac{7 d^2 x}{2 e^5} \frac{1}{\sqrt{-e^2 x^2 + d^2}} - \frac{x^3}{2 e^3} \frac{1}{\sqrt{-e^2 x^2 + d^2}} - \frac{5 d^2}{2 e^5} \operatorname{arctan}\left(x \sqrt{e^2} \frac{1}{\sqrt{-e^2 x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} \\ & + \frac{d x^2}{e^4} \frac{1}{\sqrt{-e^2 x^2 + d^2}} - 3 \frac{d^3}{e^6 \sqrt{-e^2 x^2 + d^2}} + \frac{d^4}{3 e^7} \left(x + \frac{d}{e}\right)^{-1} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)}} \\ & - \frac{2 d^2 x}{3 e^5} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)`

[Out]  $7/2/e^5*d^2*x/(-e^2*x^2+d^2)^(1/2)-1/2/e^3*x^3/(-e^2*x^2+d^2)^(1/2)-5/2/e^5*d^2/(e^2)^(1/2)*\operatorname{arctan}((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))+d/e^4*x^2/(-e^2*x^2+d^2)^(1/2)-3*d^3/e^6/(-e^2*x^2+d^2)^(1/2)+1/3*d^4/e^7/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-2/3*d^2/e^5/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)*x$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.282576, size = 605, normalized size = 4.73

$$\frac{3 e^8 x^8 - 3 d e^7 x^7 - 47 d^2 e^6 x^6 - 39 d^3 e^5 x^5 + 160 d^4 e^4 x^4 + 160 d^5 e^3 x^3 - 120 d^6 e^2 x^2 - 120 d^7 e x + 30 \left(4 d^3 e^5 x^5 + 4 d^4 e^4 x^4 - 12 d^5 e^3 x^3 - 12 d^6 e^2 x^2 - 12 d^7 e x + 30\right)}{6 \left(4 d e^{11} x^5 + 4 d^2 e^{10} x^4 - 12 d^3 e^9 x^3 - 12 d^4 e^8 x^2 + 8 d^5 e^7 x + 8 d^6 e^6 - (e^{11} x^5 + d^6 e^6) \sqrt{-e^2 x^2 + d^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)),x, algorithm="fricas")`

[Out]  $1/6*(3*e^8*x^8 - 3*d*e^7*x^7 - 47*d^2*e^6*x^6 - 39*d^3*e^5*x^5 + 160*d^4*e^4*x^4 + 160*d^5*e^3*x^3 - 120*d^6*e^2*x^2 - 120*d^7*e*x + 30*(4*d^3*e^5*x^5 + 4*d^4*e^4*x^4 - 12*d^5*e^3*x^3 - 12*d^6*e^2*x^2 + 8*d^7*e*x + 8*d^6*e^6 - (d^2*e^5*x^5 + d^3*e^4*x^4 - 8*d^4*e^3*x^3 - 8*d^5*e^2*x^2 + 8*d^6*e*x + 8*d^7)*\operatorname{sqrt}(-e^2*x^2 + d^2))*\operatorname{arctan}(-(d - \operatorname{sqrt}(-e^2*x^2 + d^2))/(e*x)) + 4*(3*d^6*e^6*x^6 + d^2*e^5*x^5 - 25*d^3*e^4*x^4 - 25*d^4*e^3*x^3 + 30*d^5*e^2*x^2 + 30*d^6*e*x)*\operatorname{sqrt}(-e^2*x^2 + d^2))/(4*d^6*e^{11}*x^5 + 4*d^7*e^{10}*x^4 - 12*d^8*e^9*x^3 - 12*d^9*e^8*x^2 + 8*d^10*e^7*x + 8*d^11*e^6 - (e^{11}*x^5 + d^6*e^6)*\operatorname{sqrt}(-e^2*x^2 + d^2))$

`*d^5*e^6)*sqrt(-e^2*x^2 + d^2))`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{(-(-d + ex)(d + ex))^{\frac{3}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**5/(e*x+d)/(-e**2*x**2+d**2)**(3/2), x)`

[Out] `Integral(x**5/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

`[undef, undef, undef, undef, undef, 1]`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)), x, algorithm="giac")`

[Out] `[undef, undef, undef, undef, undef, 1]`



$$3.128 \quad \int \frac{x^4}{(d+ex)(d^2-e^2x^2)^{3/2}} dx$$

**Optimal.** Leaf size=113

$$\frac{x^3(d-ex)}{3e^2(d^2-e^2x^2)^{3/2}} + \frac{8\sqrt{d^2-e^2x^2}}{3e^5} + \frac{d \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5} - \frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}}$$

[Out]  $(x^3*(d - e*x))/(3*e^2*(d^2 - e^2*x^2)^(3/2)) - (x*(3*d - 4*e*x))/(3*e^4*\text{Sqrt}[d^2 - e^2*x^2]) + (8*\text{Sqrt}[d^2 - e^2*x^2])/(3*e^5) + (d*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/e^5$

**Rubi [A]** time = 0.317433, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\frac{x^3(d-ex)}{3e^2(d^2-e^2x^2)^{3/2}} + \frac{8\sqrt{d^2-e^2x^2}}{3e^5} + \frac{d \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5} - \frac{x(3d-4ex)}{3e^4\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[x^4/((d + e\*x)\*(d^2 - e^2\*x^2)^(3/2)), x]

[Out]  $(x^3*(d - e*x))/(3*e^2*(d^2 - e^2*x^2)^(3/2)) - (x*(3*d - 4*e*x))/(3*e^4*\text{Sqrt}[d^2 - e^2*x^2]) + (8*\text{Sqrt}[d^2 - e^2*x^2])/(3*e^5) + (d*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/e^5$

**Rubi in Sympy [A]** time = 44.3673, size = 112, normalized size = 0.99

$$-\frac{d^3}{3e^5(d+ex)\sqrt{d^2-e^2x^2}} + \frac{2d^2}{e^5\sqrt{d^2-e^2x^2}} - \frac{4dx}{3e^4\sqrt{d^2-e^2x^2}} + \frac{d \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5} + \frac{\sqrt{d^2-e^2x^2}}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2), x)

[Out]  $-d**3/(3*e**5*(d + e*x)*\text{sqrt}(d**2 - e**2*x**2)) + 2*d**2/(e**5*\text{sqrt}(d**2 - e**2*x**2)) - 4*d*x/(3*e**4*\text{sqrt}(d**2 - e**2*x**2)) + d*\operatorname{atan}(e*x/\text{sqrt}(d**2 - e**2*x**2))/e**5 + \text{sqrt}(d**2 - e**2*x**2)/e**5$

**Mathematica [A]** time = 0.130206, size = 93, normalized size = 0.82

$$\frac{3d \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{\sqrt{d^2-e^2x^2}(8d^3+5d^2ex-7de^2x^2-3e^3x^3)}{(d-ex)(d+ex)^2}}{3e^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/((d + e\*x)\*(d^2 - e^2\*x^2)^(3/2)), x]

[Out]  $((\text{Sqrt}[d^2 - e^2*x^2])*(8*d^3 + 5*d^2*e*x - 7*d*e^2*x^2 - 3*e^3*x^3))/((d - e*x)*(d + e*x)^2) + 3*d*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]]/(3*e^5)$

---

**Maple [A]** time = 0.014, size = 179, normalized size = 1.6

$$\begin{aligned} &-\frac{x^2}{e^3} \frac{1}{\sqrt{-e^2x^2+d^2}} + 3 \frac{d^2}{e^5\sqrt{-e^2x^2+d^2}} - \frac{d^3}{3e^6} \left(x + \frac{d}{e}\right)^{-1} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)}} \\ &+ \frac{2dx}{3e^4} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)}} - 2 \frac{dx}{e^4\sqrt{-e^2x^2+d^2}} + \frac{d}{e^4} \arctan\left(x\sqrt{e^2} \frac{1}{\sqrt{-e^2x^2+d^2}}\right) \frac{1}{\sqrt{e^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(e\*x+d)/(-e^2\*x^2+d^2)^(3/2), x)

[Out] -1/e^3\*x^2/(-e^2\*x^2+d^2)^(1/2)+3\*d^2/e^5/(-e^2\*x^2+d^2)^(1/2)-1/3\*d^3/e^6/(x+d/e)/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)+2/3\*d/e^4/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x-2\*d/e^4\*x/(-e^2\*x^2+d^2)^(1/2)+d/e^4/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.292401, size = 481, normalized size = 4.26

$$\frac{de^5x^5 + 13d^2e^4x^4 + 13d^3e^3x^3 - 12d^4e^2x^2 - 12d^5ex + 6(d^5e^5x^5 + d^2e^4x^4 - 5d^3e^3x^3 - 5d^4e^2x^2 + 4d^5ex + 4d^6 + (3d^2e^3x^3 - 3d^3e^4x^4 + 3d^4e^5x^5))\sqrt{-e^2x^2+d^2}}{3(e^{10}x^5 + de^9x^4 - 5d^2e^8x^3 - 5d^3e^7x^2 + 4d^4e^6x + 3d^5e^5x^5 + 13d^2e^4x^4 + 13d^3e^3x^3 - 12d^4e^2x^2 - 12d^5ex + 6(d^5e^5x^5 + d^2e^4x^4 - 5d^3e^3x^3 - 5d^4e^2x^2 + 4d^5ex + 4d^6 + (3d^2e^3x^3 - 3d^3e^4x^4 + 3d^4e^5x^5))\sqrt{-e^2x^2+d^2})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)),x, algorithm="fricas")

[Out] -1/3\*(d\*e^5\*x^5 + 13\*d^2\*e^4\*x^4 + 13\*d^3\*e^3\*x^3 - 12\*d^4\*e^2\*x^2 - 12\*d^5\*e\*x + 6\*(d\*e^5\*x^5 + d^2\*e^4\*x^4 - 5\*d^3\*e^3\*x^3 - 5\*d^4\*e^2\*x^2 + 4\*d^5\*e\*x + 4\*d^6 + (3\*d^2\*e^3\*x^3 + 3\*d^3\*e^4\*x^4 - 4\*d^4\*e^5\*x - 4\*d^5)\*sqrt(-e^2\*x^2 + d^2))\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) - (3\*e^5\*x^5 + 7\*d\*e^4\*x^4 + 7\*d^2\*e^3\*x^3 - 12\*d^3\*e^2\*x^2 - 12\*d^4\*e\*x)\*sqrt(-e^2\*x^2 + d^2))/(e^10\*x^5 + d\*e^9\*x^4 - 5\*d^2\*e^8\*x^3 - 5\*d^3\*e^7\*x^2 + 4\*d^4\*e^6\*x + 4\*d^5\*e^5 + (3\*d^2\*e^3\*x^3 + 3\*d^3\*e^4\*x^4 - 4\*d^4\*e^5\*x - 4\*d^5)\*sqrt(-e^2\*x^2 + d^2))

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(-(-d + ex)(d + ex))^{\frac{3}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)`

[Out] `Integral(x**4/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

`[undef, undef, undef, undef, 1]`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)),x, algorithm="giac")`

[Out] `[undef, undef, undef, undef, 1]`

$$3.129 \quad \int \frac{x^3}{(d+ex)(d^2-e^2x^2)^{3/2}} dx$$

**Optimal.** Leaf size=89

$$\frac{x^2(d-ex)}{3e^2(d^2-e^2x^2)^{3/2}} - \frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

[Out]  $(x^2*(d - e*x))/(3*e^2*(d^2 - e^2*x^2)^(3/2)) - (2*d - 3*e*x)/(3*e^4*\text{Sqrt}[d^2 - e^2*x^2]) - \text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]]/e^4$

**Rubi [A]** time = 0.238978, antiderivative size = 89, normalized size of antiderivative = 1., number of rules used = 5, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\frac{x^2(d-ex)}{3e^2(d^2-e^2x^2)^{3/2}} - \frac{2d-3ex}{3e^4\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Antiderivative was successfully verified.

[In] `Int[x^3/((d + e*x)*(d^2 - e^2*x^2)^(3/2)), x]`

[Out]  $(x^2*(d - e*x))/(3*e^2*(d^2 - e^2*x^2)^(3/2)) - (2*d - 3*e*x)/(3*e^4*\text{Sqrt}[d^2 - e^2*x^2]) - \text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]]/e^4$

**Rubi in Sympy [A]** time = 30.5979, size = 88, normalized size = 0.99

$$\frac{d^2}{3e^4(d+ex)\sqrt{d^2-e^2x^2}} - \frac{d}{e^4\sqrt{d^2-e^2x^2}} + \frac{4x}{3e^3\sqrt{d^2-e^2x^2}} - \frac{\text{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**3/(e*x+d)/((-e**2*x**2+d**2)**(3/2)), x)`

[Out]  $d**2/(3*e**4*(d + e*x)*\text{sqrt}(d**2 - e**2*x**2)) - d/(e**4*\text{sqrt}(d**2 - e**2*x**2)) + 4*x/(3*e**3*\text{sqrt}(d**2 - e**2*x**2)) - \text{atan}(e*x/\text{sqrt}(d**2 - e**2*x**2))/e**4$

**Mathematica [A]** time = 0.133322, size = 80, normalized size = 0.9

$$\frac{\sqrt{d^2-e^2x^2}(-2d^2+dex+4e^2x^2)}{(d-ex)(d+ex)^2} - 3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)$$

$$3e^4$$

Antiderivative was successfully verified.

[In] `Integrate[x^3/((d + e*x)*(d^2 - e^2*x^2)^(3/2)), x]`

[Out]  $((\text{Sqrt}[d^2 - e^2*x^2]*(-2*d^2 + d*e*x + 4*e^2*x^2))/((d - e*x)*(d + e*x)^2) - 3*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(3*e^4)$

**Maple [A]** time = 0.014, size = 153, normalized size = 1.7

$$2 \frac{x}{e^3 \sqrt{-e^2 x^2 + d^2}} - \frac{1}{e^3} \arctan \left( x \sqrt{e^2} \frac{1}{\sqrt{-e^2 x^2 + d^2}} \right) \frac{1}{\sqrt{e^2}} - \frac{d}{e^4} \frac{1}{\sqrt{-e^2 x^2 + d^2}}$$

$$+ \frac{d^2}{3 e^5} \left( x + \frac{d}{e} \right)^{-1} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)}} - \frac{2 x}{3 e^3} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)`

[Out] `2/e^3*x/(-e^2*x^2+d^2)^(1/2)-1/e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-d/e^4/(-e^2*x^2+d^2)^(1/2)+1/3*d^2/e^5/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-2/3/e^3/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)*x`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.290756, size = 369, normalized size = 4.15

$$\frac{4 e^4 x^4 + 5 d e^3 x^3 - 6 d^2 e^2 x^2 - 6 d^3 e x - 6 \left( 2 d e^3 x^3 + 2 d^2 e^2 x^2 - 2 d^3 e x - 2 d^4 - (e^3 x^3 + d e^2 x^2 - 2 d^2 e x - 2 d^3) \sqrt{-e^2 x^2 + d^2} \right)}{3 \left( 2 d e^7 x^3 + 2 d^2 e^6 x^2 - 2 d^3 e^5 x - 2 d^4 e^4 - (e^7 x^3 + d e^6 x^2 - 2 d^2 e^5 x - 2 d^3) \sqrt{-e^2 x^2 + d^2} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)),x, algorithm="fricas")`

[Out] `-1/3*(4*e^4*x^4 + 5*d*e^3*x^3 - 6*d^2*e^2*x^2 - 6*d^3*e*x - 6*(2*d*e^3*x^3 + 2*d^2*e^2*x^2 - 2*d^3*e*x - 2*d^4 - (e^3*x^3 + d*e^2*x^2 - 2*d^2*e*x - 2*d^3)*sqrt(-e^2*x^2 + d^2))*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) - 2*(e^3*x^3 - 3*d*e^2*x^2 - 3*d^2*e*x)*sqrt(-e^2*x^2 + d^2))/(2*d*e^7*x^3 + 2*d^2*e^6*x^2 - 2*d^3*e^5*x - 2*d^4*e^4 - (e^7*x^3 + d*e^6*x^2 - 2*d^2*e^5*x - 2*d^3)*sqrt(-e^2*x^2 + d^2))`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(-(-d + ex)(d + ex))^{\frac{3}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)`

[Out]  $\text{Integral}(x^{**3}/((-(-d + e*x)*(d + e*x))^{** (3/2)}*(d + e*x)), x)$

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

*[undef, undef, undef, 1]*

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)),x, algorithm="giac")`

[Out] *[undef, undef, undef, 1]*

$$3.130 \quad \int \frac{x^2}{(d+ex)(d^2-e^2x^2)^{3/2}} dx$$

**Optimal.** Leaf size=60

$$\frac{2}{3e^3\sqrt{d^2-e^2x^2}} - \frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

[Out] 2/(3\*e^3\*Sqrt[d^2 - e^2\*x^2]) - x^2/(3\*d\*e\*(d + e\*x)\*Sqrt[d^2 - e^2\*x^2])

**Rubi [A]** time = 0.115035, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{2}{3e^3\sqrt{d^2-e^2x^2}} - \frac{x^2}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/((d + e\*x)\*(d^2 - e^2\*x^2)^(3/2)), x]

[Out] 2/(3\*e^3\*Sqrt[d^2 - e^2\*x^2]) - x^2/(3\*d\*e\*(d + e\*x)\*Sqrt[d^2 - e^2\*x^2])

**Rubi in Sympy [A]** time = 9.20806, size = 46, normalized size = 0.77

$$\frac{2}{3e^3\sqrt{d^2-e^2x^2}} - \frac{x^2(d-ex)}{3de(d^2-e^2x^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2), x)

[Out] 2/(3\*e\*\*3\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) - x\*\*2\*(d - e\*x)/(3\*d\*e\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2))

**Mathematica [A]** time = 0.0434962, size = 60, normalized size = 1.

$$\frac{\sqrt{d^2-e^2x^2}(2d^2+2dex-e^2x^2)}{3de^3(d-ex)(d+ex)^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((d + e\*x)\*(d^2 - e^2\*x^2)^(3/2)), x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(2\*d^2 + 2\*d\*e\*x - e^2\*x^2))/(3\*d\*e^3\*(d - e\*x)\*(d + e\*x)^2)

**Maple [A]** time = 0.011, size = 48, normalized size = 0.8

$$\frac{(-ex+d)(-e^2x^2+2dex+2d^2)}{3de^3}(-e^2x^2+d^2)^{-3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)`

[Out] `1/3*(-e*x+d)*(-e^2*x^2+2*d*e*x+2*d^2)/d/e^3/(-e^2*x^2+d^2)^(3/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.280372, size = 158, normalized size = 2.63

$$\frac{ex^4 + 2dx^3 - 2\sqrt{-e^2x^2 + d^2}x^3}{3\left(2d^2e^3x^3 + 2d^3e^2x^2 - 2d^4ex - 2d^5 - (de^3x^3 + d^2e^2x^2 - 2d^3ex - 2d^4)\sqrt{-e^2x^2 + d^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)),x, algorithm="fricas")`

[Out] `1/3*(e*x^4 + 2*d*x^3 - 2*sqrt(-e^2*x^2 + d^2)*x^3)/(2*d^2*e^3*x^3 + 2*d^3*e^2*x^2 - 2*d^4*e*x - 2*d^5 - (d*e^3*x^3 + d^2*e^2*x^2 - 2*d^3*e*x - 2*d^4)*sqrt(-e^2*x^2 + d^2))`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(-(-d + ex)(d + ex))^{\frac{3}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)`

[Out] `Integral(x**2/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)`

**GIAC/XCAS [A]** time = 0.288718, size = 1, normalized size = 0.02

+∞

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)),x, algorithm="giac")`

[Out] +Infinity



$$3.131 \quad \int \frac{x}{(d+ex)(d^2-e^2x^2)^{3/2}} dx$$

**Optimal.** Leaf size=58

$$\frac{x}{3d^2e\sqrt{d^2-e^2x^2}} + \frac{1}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}$$

[Out]  $x/(3*d^2*e*\text{Sqrt}[d^2 - e^2*x^2]) + 1/(3*e^2*(d + e*x)*\text{Sqrt}[d^2 - e^2*x^2])$

**Rubi [A]** time = 0.0822795, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$\frac{x}{3d^2e\sqrt{d^2-e^2x^2}} + \frac{1}{3e^2(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x/((d + e*x)*(d^2 - e^2*x^2)^{(3/2)}), x]$

[Out]  $x/(3*d^2*e*\text{Sqrt}[d^2 - e^2*x^2]) + 1/(3*e^2*(d + e*x)*\text{Sqrt}[d^2 - e^2*x^2])$

**Rubi in Sympy [A]** time = 6.72052, size = 46, normalized size = 0.79

$$\frac{1}{3e^2(d+ex)\sqrt{d^2-e^2x^2}} + \frac{x}{3d^2e\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x/(e*x+d)/(-e**2*x**2+d**2)**(3/2), x)$

[Out]  $1/(3*e**2*(d + e*x)*\text{sqrt}(d**2 - e**2*x**2)) + x/(3*d**2*e*\text{sqrt}(d**2 - e**2*x**2))$

**Mathematica [A]** time = 0.0379849, size = 56, normalized size = 0.97

$$\frac{\sqrt{d^2 - e^2x^2} (d^2 + dex + e^2x^2)}{3d^2e^2(d - ex)(d + ex)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x/((d + e*x)*(d^2 - e^2*x^2)^{(3/2)}), x]$

[Out]  $(\text{Sqrt}[d^2 - e^2*x^2]*(d^2 + d*e*x + e^2*x^2))/(3*d^2*e^2*(d - e*x)*(d + e*x)^2)$

**Maple [A]** time = 0.011, size = 44, normalized size = 0.8

$$\frac{(-ex + d)(e^2x^2 + dex + d^2)}{3d^2e^2} (-e^2x^2 + d^2)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)`

[Out]  $1/3*(-e*x+d)*(e^2*x^2+d*e*x+d^2)/d^2/e^2/(-e^2*x^2+d^2)^(3/2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.283917, size = 186, normalized size = 3.21

$$\frac{e^2x^4 - dex^3 - 3d^2x^2 + \sqrt{-e^2x^2 + d^2}(ex^3 + 3dx^2)}{3\left(2d^3e^3x^3 + 2d^4e^2x^2 - 2d^5ex - 2d^6 - (d^2e^3x^3 + d^3e^2x^2 - 2d^4ex - 2d^5)\sqrt{-e^2x^2 + d^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)),x, algorithm="fricas")`

[Out]  $-1/3*(e^2*x^4 - d*e*x^3 - 3*d^2*x^2 + \sqrt{-e^2*x^2 + d^2}*(e*x^3 + 3*d*x^2))/(2*d^3*e^3*x^3 + 2*d^4*e^2*x^2 - 2*d^5*e*x - 2*d^6 - (d^2*e^3*x^3 + d^3*e^2*x^2 - 2*d^4*e*x - 2*d^5)*\sqrt{-e^2*x^2 + d^2})$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(-(-d + ex)(d + ex))^{\frac{3}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)`

[Out] `Integral(x/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

*undef*

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)),x, algorithm="giac")`

[Out] undef

$$3.132 \quad \int \frac{1}{(d+ex)(d^2-e^2x^2)^{3/2}} dx$$

**Optimal.** Leaf size=58

$$\frac{2x}{3d^3\sqrt{d^2-e^2x^2}} - \frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

[Out] (2\*x)/(3\*d^3\*Sqrt[d^2 - e^2\*x^2]) - 1/(3\*d\*e\*(d + e\*x)\*Sqrt[d^2 - e^2\*x^2])

**Rubi [A]** time = 0.0473002, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{2x}{3d^3\sqrt{d^2-e^2x^2}} - \frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)\*(d^2 - e^2\*x^2)^(3/2)), x]

[Out] (2\*x)/(3\*d^3\*Sqrt[d^2 - e^2\*x^2]) - 1/(3\*d\*e\*(d + e\*x)\*Sqrt[d^2 - e^2\*x^2])

**Rubi in Sympy [A]** time = 6.19475, size = 46, normalized size = 0.79

$$-\frac{1}{3de(d+ex)\sqrt{d^2-e^2x^2}} + \frac{2x}{3d^3\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2), x)

[Out] -1/(3\*d\*e\*(d + e\*x)\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) + 2\*x/(3\*d\*\*3\*sqrt(d\*\*2 - e\*\*2\*x\*\*2))

**Mathematica [A]** time = 0.0396245, size = 58, normalized size = 1.

$$-\frac{(d^2 - 2dex - 2e^2x^2)\sqrt{d^2 - e^2x^2}}{3d^3e(d - ex)(d + ex)^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x)\*(d^2 - e^2\*x^2)^(3/2)), x]

[Out] -((d^2 - 2\*d\*e\*x - 2\*e^2\*x^2)\*Sqrt[d^2 - e^2\*x^2])/(3\*d^3\*e\*(d - e\*x)\*(d + e\*x)^2)

**Maple [A]** time = 0.009, size = 46, normalized size = 0.8

$$-\frac{(-ex + d)(-2e^2x^2 - 2dex + d^2)}{3d^3e} (-e^2x^2 + d^2)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x+d)/(-e^2*x^2+d^2)^(3/2),x)`

[Out] `-1/3*(-e*x+d)*(-2*e^2*x^2-2*d*e*x+d^2)/d^3/e/(-e^2*x^2+d^2)^(3/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.283863, size = 213, normalized size = 3.67

$$\frac{2e^3x^4 + 4de^2x^3 - 3d^2ex^2 - 6d^3x - (e^2x^3 - 3dex^2 - 6d^2x)\sqrt{-e^2x^2 + d^2}}{3\left(2d^4e^3x^3 + 2d^5e^2x^2 - 2d^6ex - 2d^7 - (d^3e^3x^3 + d^4e^2x^2 - 2d^5ex - 2d^6)\sqrt{-e^2x^2 + d^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)),x, algorithm="fricas")`

[Out] `-1/3*(2*e^3*x^4 + 4*d*e^2*x^3 - 3*d^2*e*x^2 - 6*d^3*x - (e^2*x^3 - 3*d*e*x^2 - 6*d^2*x)*sqrt(-e^2*x^2 + d^2))/(2*d^4*e^3*x^3 + 2*d^5*e^2*x^2 - 2*d^6*e*x - 2*d^7 - (d^3*e^3*x^3 + d^4*e^2*x^2 - 2*d^5*e*x - 2*d^6)*sqrt(-e^2*x^2 + d^2))`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-(-d + ex)(d + ex))^{\frac{3}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)`

[Out] `Integral(1/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

*undef*

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)),x, algorithm="giac")`

[Out] `undef`

$$3.133 \quad \int \frac{1}{x(d+ex)(d^2-e^2x^2)^{3/2}} dx$$

**Optimal.** Leaf size=88

$$\frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} + \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

[Out] (3\*d - 2\*e\*x)/(3\*d^4\*Sqrt[d^2 - e^2\*x^2]) + 1/(3\*d^2\*(d + e\*x)\*Sqrt[d^2 - e^2\*x^2]) - ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]/d^4

**Rubi [A]** time = 0.239111, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{1}{3d^2(d+ex)\sqrt{d^2-e^2x^2}} + \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(d + e\*x)\*(d^2 - e^2\*x^2)^(3/2)), x]

[Out] (3\*d - 2\*e\*x)/(3\*d^4\*Sqrt[d^2 - e^2\*x^2]) + 1/(3\*d^2\*(d + e\*x)\*Sqrt[d^2 - e^2\*x^2]) - ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]/d^4

**Rubi in Sympy [A]** time = 27.8675, size = 70, normalized size = 0.8

$$\frac{d-ex}{3d^2(d^2-e^2x^2)^{3/2}} + \frac{3d-2ex}{3d^4\sqrt{d^2-e^2x^2}} - \frac{\operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2), x)

[Out] (d - e\*x)/(3\*d\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)) + (3\*d - 2\*e\*x)/(3\*d\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) - atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d)/d\*\*4

**Mathematica [A]** time = 0.132128, size = 83, normalized size = 0.94

$$\frac{\sqrt{d^2-e^2x^2}(4d^2+dex-2e^2x^2)}{(d-ex)(d+ex)^2} - 3 \log\left(\sqrt{d^2-e^2x^2}+d\right) + 3 \log(x)$$

$$3d^4$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(d + e\*x)\*(d^2 - e^2\*x^2)^(3/2)), x]

[Out] (((4\*d^2 + d\*e\*x - 2\*e^2\*x^2)\*Sqrt[d^2 - e^2\*x^2])/((d - e\*x)\*(d + e\*x)^2) + 3\*Log[x] - 3\*Log[d + Sqrt[d^2 - e^2\*x^2]])/(3\*d^4)

**Maple [A]** time = 0.017, size = 142, normalized size = 1.6

$$\frac{1}{d^3} \frac{1}{\sqrt{-e^2 x^2 + d^2}} - \frac{1}{d^3} \ln \left( \frac{1}{x} \left( 2d^2 + 2\sqrt{d^2} \sqrt{-e^2 x^2 + d^2} \right) \right) \frac{1}{\sqrt{d^2}} + \frac{1}{3ed^2} \left( x + \frac{d}{e} \right)^{-1} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right)}} - \frac{2ex}{3d^4} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(e\*x+d)/(-e^2\*x^2+d^2)^(3/2), x)

[Out] 1/d^3/(-e^2\*x^2+d^2)^(1/2)-1/d^3/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)+1/3/d^2/e/(x+d/e)/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)-2/3/d^4\*e/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-e^2 x^2 + d^2)^{\frac{3}{2}} (ex + d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)\*x), x, algorithm="maxima")

[Out] integrate(1/((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)\*x), x)

**Fricas [A]** time = 0.288523, size = 335, normalized size = 3.81

$$\frac{2e^4 x^4 + 7de^3 x^3 - 6d^3 ex + 3 \left( 2de^3 x^3 + 2d^2 e^2 x^2 - 2d^3 ex - 2d^4 - (e^3 x^3 + de^2 x^2 - 2d^2 ex - 2d^3) \sqrt{-e^2 x^2 + d^2} \right) \log \left( -\frac{d - \sqrt{-e^2 x^2 + d^2}}{e^2 x^2 + d^2} \right)}{3 \left( 2d^5 e^3 x^3 + 2d^6 e^2 x^2 - 2d^7 ex - 2d^8 - (d^4 e^3 x^3 + d^5 e^2 x^2 - 2d^6 ex - 2d^7) \sqrt{-e^2 x^2 + d^2} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)\*x), x, algorithm="fricas")

[Out] 1/3\*(2\*e^4\*x^4 + 7\*d\*e^3\*x^3 - 6\*d^3\*e\*x + 3\*(2\*d\*e^3\*x^3 + 2\*d^2\*e^2\*x^2 - 2\*d^3\*ex - 2\*d^4 - (e^3\*x^3 + d\*e^2\*x^2 - 2\*d^2\*ex - 2\*d^3)\*sqrt(-e^2\*x^2 + d^2))\*log(-(d - sqrt(-e^2\*x^2 + d^2))/x) - 2\*(2\*e^3\*x^3 - 3\*d^2\*e\*x)\*sqrt(-e^2\*x^2 + d^2))/(2\*d^5\*e^3\*x^3 + 2\*d^6\*e^2\*x^2 - 2\*d^7\*ex - 2\*d^8 - (d^4\*e^3\*x^3 + d^5\*e^2\*x^2 - 2\*d^6\*ex - 2\*d^7)\*sqrt(-e^2\*x^2 + d^2))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x(-(-d + ex)(d + ex))^{\frac{3}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2), x)

[Out]  $\text{Integral}(1/(x*(-(-d + e*x)*(d + e*x))^{3/2}*(d + e*x)), x)$

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

*[undef, undef, undef, 1]*

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x),x, algorithm="giac")`

[Out] *[undef, undef, undef, 1]*

$$3.134 \quad \int \frac{1}{x^2(d+ex)(d^2-e^2x^2)^{3/2}} dx$$

**Optimal.** Leaf size=120

$$\frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{8\sqrt{d^2-e^2x^2}}{3d^5x} + \frac{e \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5} + \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}}$$

[Out]  $(4*d - 3*e*x)/(3*d^4*x*\text{Sqrt}[d^2 - e^2*x^2]) + 1/(3*d^2*x*(d + e*x)*\text{Sqrt}[d^2 - e^2*x^2]) - (8*\text{Sqrt}[d^2 - e^2*x^2])/(3*d^5*x) + (e*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/d^5$

**Rubi [A]** time = 0.326629, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{1}{3d^2x(d+ex)\sqrt{d^2-e^2x^2}} - \frac{8\sqrt{d^2-e^2x^2}}{3d^5x} + \frac{e \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5} + \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] `Int[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2)), x]`

[Out]  $(4*d - 3*e*x)/(3*d^4*x*\text{Sqrt}[d^2 - e^2*x^2]) + 1/(3*d^2*x*(d + e*x)*\text{Sqrt}[d^2 - e^2*x^2]) - (8*\text{Sqrt}[d^2 - e^2*x^2])/(3*d^5*x) + (e*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/d^5$

**Rubi in Sympy [A]** time = 35.5169, size = 97, normalized size = 0.81

$$\frac{d-ex}{3d^2x(d^2-e^2x^2)^{3/2}} + \frac{4d-3ex}{3d^4x\sqrt{d^2-e^2x^2}} + \frac{e \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5} - \frac{8\sqrt{d^2-e^2x^2}}{3d^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x**2/(e*x+d)/(-e**2*x**2+d**2)**(3/2), x)`

[Out]  $(d - e*x)/(3*d**2*x*(d**2 - e**2*x**2)**(3/2)) + (4*d - 3*e*x)/(3*d**4*x*\text{sqrt}(d**2 - e**2*x**2)) + e*\operatorname{atanh}(\text{sqrt}(d**2 - e**2*x**2)/d)/d**5 - 8*\text{sqrt}(d**2 - e**2*x**2)/(3*d**5*x)$

**Mathematica [A]** time = 0.143957, size = 101, normalized size = 0.84

$$\frac{3e \log\left(\sqrt{d^2 - e^2x^2} + d\right) + \frac{\sqrt{d^2 - e^2x^2}(3d^3 + 7d^2ex - 5de^2x^2 - 8e^3x^3)}{x(ex-d)(d+ex)^2} - 3e \log(x)}{3d^5}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x^2*(d + e*x)*(d^2 - e^2*x^2)^(3/2)), x]`

[Out]  $((\text{Sqrt}[d^2 - e^2*x^2]*(3*d^3 + 7*d^2*e*x - 5*d*e^2*x^2 - 8*e^3*x^3))/(x*(-d + e*x)*(d + e*x)^2) - 3*e*\text{Log}[x] + 3*e*\text{Log}[d + \text{Sqrt}[d^2 - e^2*x^2]])/(3*d^5)$



**Maple [A]** time = 0.021, size = 188, normalized size = 1.6

$$\begin{aligned} & -\frac{1}{d^3 x} \frac{1}{\sqrt{-e^2 x^2 + d^2}} + 2 \frac{e^2 x}{d^5 \sqrt{-e^2 x^2 + d^2}} - \frac{1}{3 d^3} \left(x + \frac{d}{e}\right)^{-1} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)}} \\ & + \frac{2 e^2 x}{3 d^5} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)}} - \frac{e}{d^4} \frac{1}{\sqrt{-e^2 x^2 + d^2}} \\ & + \frac{e}{d^4} \ln \left( \frac{1}{x} \left( 2 d^2 + 2 \sqrt{d^2} \sqrt{-e^2 x^2 + d^2} \right) \right) \frac{1}{\sqrt{d^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(e*x+d)/(-e^2*x^2+d^2)^(3/2), x)`

[Out] `-1/d^3/x/(-e^2*x^2+d^2)^(1/2)+2/d^5*e^2*x/(-e^2*x^2+d^2)^(1/2)-1/3/d^3/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+2/3*e^2/d^5/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)*x-e/d^4/(-e^2*x^2+d^2)^(1/2)+e/d^4/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-e^2 x^2 + d^2)^{\frac{3}{2}} (e x + d) x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x^2), x, algorithm="maxima")`

[Out] `integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x^2), x)`

**Fricas [A]** time = 0.288043, size = 514, normalized size = 4.28

$$\frac{4 e^6 x^6 - 20 d e^5 x^5 - 35 d^2 e^4 x^4 + 33 d^3 e^3 x^3 + 45 d^4 e^2 x^2 - 12 d^5 e x - 12 d^6 + 3 \left( e^6 x^6 + d e^5 x^5 - 5 d^2 e^4 x^4 - 5 d^3 e^3 x^3 + 4 d^4 e^2 x^2 - 12 d^5 e x - 12 d^6 + 3 \left( d^5 e^5 x^6 + d^6 e^4 x^5 - 5 d^7 e^3 x^4 - 5 d^8 e^2 x^3 - 4 d^9 e x^2 - 4 d^{10} \right) \right)}{3 \left( d^5 e^5 x^6 + d^6 e^4 x^5 - 5 d^7 e^3 x^4 - 5 d^8 e^2 x^3 - 4 d^9 e x^2 - 4 d^{10} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x^2), x, algorithm="fricas")`

[Out] `-1/3*(4*e^6*x^6 - 20*d*e^5*x^5 - 35*d^2*e^4*x^4 + 33*d^3*e^3*x^3 + 45*d^4*e^2*x^2 - 12*d^5*e*x - 12*d^6 + 3*(e^6*x^6 + d*e^5*x^5 - 5*d^2*e^4*x^4 - 5*d^3*e^3*x^3 + 4*d^4*e^2*x^2 - 12*d^5*e*x - 12*d^6 + 3*(d^5*e^5*x^6 + d^6*e^4*x^5 - 5*d^7*e^3*x^4 - 5*d^8*e^2*x^3 - 4*d^9*e*x^2 - 4*d^10)))*sqrt(-e^2*x^2 + d^2)*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (8*e^5*x^5 + 17*d*e^4*x^4 - 27*d^2*e^3*x^3 - 39*d^3*e^2*x^2 + 12*d^4*e*x + 12*d^5)*sqrt(-e^2*x^2 + d^2)/(d^5*e^5*x^6 + d^6*e^4*x^5 - 5*d^7*e^3*x^4 - 5*d^8*e^2*x^3 + 4*d^9*e*x^2 + 4*d^10*x + (3*d^6*e^3*x^4 + 3*d^7*e^2*x^3 - 4*d^8*e*x^2 - 4*d^9*x)*sqrt(-e^2*x^2 + d^2))`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (-(-d + ex)(d + ex))^{\frac{3}{2}} (d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)`

[Out] `Integral(1/(x**2*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)`

---

**GIAC/XCAS [A]** time = 0.293485, size = 1, normalized size = 0.01

+∞

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x^2),x, algorithm="giac")`

[Out] +Infinity

$$3.135 \quad \int \frac{1}{x^3(d+ex)(d^2-e^2x^2)^{3/2}} dx$$

**Optimal.** Leaf size=152

$$\frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} + \frac{8e\sqrt{d^2-e^2x^2}}{3d^6x} - \frac{5e^2 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6} - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}}$$

[Out]  $(5*d - 4*e*x)/(3*d^4*x^2*\text{Sqrt}[d^2 - e^2*x^2]) + 1/(3*d^2*x^2*(d + e*x)*\text{Sqrt}[d^2 - e^2*x^2]) - (5*\text{Sqrt}[d^2 - e^2*x^2])/(2*d^5*x^2) + (8*e*\text{Sqrt}[d^2 - e^2*x^2])/(3*d^6*x) - (5*e^2*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/(2*d^6)$

**Rubi [A]** time = 0.412433, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\frac{1}{3d^2x^2(d+ex)\sqrt{d^2-e^2x^2}} + \frac{8e\sqrt{d^2-e^2x^2}}{3d^6x} - \frac{5e^2 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6} - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] `Int[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2)), x]`

[Out]  $(5*d - 4*e*x)/(3*d^4*x^2*\text{Sqrt}[d^2 - e^2*x^2]) + 1/(3*d^2*x^2*(d + e*x)*\text{Sqrt}[d^2 - e^2*x^2]) - (5*\text{Sqrt}[d^2 - e^2*x^2])/(2*d^5*x^2) + (8*e*\text{Sqrt}[d^2 - e^2*x^2])/(3*d^6*x) - (5*e^2*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/(2*d^6)$

**Rubi in Sympy [A]** time = 47.3971, size = 131, normalized size = 0.86

$$\frac{d-ex}{3d^2x^2(d^2-e^2x^2)^{3/2}} + \frac{5d-4ex}{3d^4x^2\sqrt{d^2-e^2x^2}} - \frac{5\sqrt{d^2-e^2x^2}}{2d^5x^2} - \frac{5e^2 \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6} + \frac{8e\sqrt{d^2-e^2x^2}}{3d^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x**3/(e*x+d)/(-e**2*x**2+d**2)**(3/2), x)`

[Out]  $(d - e*x)/(3*d**2*x**2*(d**2 - e**2*x**2)**(3/2)) + (5*d - 4*e*x)/(3*d**4*x**2*\text{sqrt}(d**2 - e**2*x**2)) - 5*\text{sqrt}(d**2 - e**2*x**2)/(2*d**5*x**2) - 5*e**2*\operatorname{atanh}(\text{sqrt}(d**2 - e**2*x**2)/d)/(2*d**6) + 8*e*\text{sqrt}(d**2 - e**2*x**2)/(3*d**6*x)$

**Mathematica [A]** time = 0.106961, size = 115, normalized size = 0.76

$$\frac{-15e^2 \log\left(\sqrt{d^2-e^2x^2}+d\right) + \frac{\sqrt{d^2-e^2x^2}(3d^4-3d^3ex-23d^2e^2x^2+de^3x^3+16e^4x^4)}{x^2(ex-d)(d+ex)^2} + 15e^2 \log(x)}{6d^6}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x^3*(d + e*x)*(d^2 - e^2*x^2)^(3/2)), x]`

[Out]  $((\text{Sqrt}[d^2 - e^2*x^2])*(3*d^4 - 3*d^3*e*x - 23*d^2*e^2*x^2 + d*e^3*x^3 + 16*e^4*x^4))/(x^2*(-d + e*x)*(d + e*x)^2) + 15*e^2*\text{Log}[x]$

$$- 15 * e^2 * \text{Log}[d + \text{Sqrt}[d^2 - e^2 * x^2]] / (6 * d^6)$$

**Maple [A]** time = 0.021, size = 216, normalized size = 1.4

$$\begin{aligned} & -\frac{1}{2d^3x^2} \frac{1}{\sqrt{-e^2x^2+d^2}} + \frac{5e^2}{2d^5} \frac{1}{\sqrt{-e^2x^2+d^2}} - \frac{5e^2}{2d^5} \ln\left(\frac{1}{x} \left(2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2+d^2}\right)\right) \frac{1}{\sqrt{d^2}} \\ & + \frac{e}{3d^4} \left(x + \frac{d}{e}\right)^{-1} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)}} \\ & - \frac{2e^3x}{3d^6} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)}} + \frac{e}{d^4x} \frac{1}{\sqrt{-e^2x^2+d^2}} - 2 \frac{e^3x}{d^6\sqrt{-e^2x^2+d^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(e\*x+d)/(-e^2\*x^2+d^2)^(3/2), x)

[Out] -1/2/d^3/x^2/(-e^2\*x^2+d^2)^(1/2)+5/2/d^5\*e^2/(-e^2\*x^2+d^2)^(1/2)-5/2/d^5\*e^2/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)+1/3/d^4\*e/(x+d/e)/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)-2/3/d^6\*e^3/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x+e/d^4/x/(-e^2\*x^2+d^2)^(1/2)-2\*e^3/d^6\*x/(-e^2\*x^2+d^2)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-e^2x^2 + d^2)^{\frac{3}{2}}(ex + d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)\*x^3), x, algorithm="maxima")

[Out] integrate(1/((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)\*x^3), x)

**Fricas [A]** time = 0.289073, size = 653, normalized size = 4.3

$$\frac{16e^8x^8 + 57de^7x^7 - 95d^2e^6x^6 - 179d^3e^5x^5 + 147d^4e^4x^4 + 144d^5e^3x^3 - 96d^6e^2x^2 - 24d^7ex + 24d^8 + 15(4de^7x^7 + 4d^2e^5x^5 + 4d^7e^5x^5)}{6(4d^7e^5x^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)\*x^3), x, algorithm="fricas")

[Out] 1/6\*(16\*e^8\*x^8 + 57\*d\*e^7\*x^7 - 95\*d^2\*e^6\*x^6 - 179\*d^3\*e^5\*x^5 + 147\*d^4\*e^4\*x^4 + 144\*d^5\*e^3\*x^3 - 96\*d^6\*e^2\*x^2 - 24\*d^7\*e\*x + 24\*d^8 + 15\*(4\*d\*e^7\*x^7 + 4\*d^2\*e^5\*x^5 + 4\*d^7\*e^5\*x^5) \* sqrt(-e^2\*x^2 + d^2) \* log(-(d - sqrt(-e^2\*x^2 + d^2))/x) - 2\*(7\*e^7\*x^7 - 25\*d\*e^6\*x^6 - 58\*d^2\*e^5\*x^5 + 54\*d^3\*e^4\*x^4 + 66\*d^4\*e^3\*x^3 - 42\*d^5\*e^2\*x^2 - 12\*d^6\*e\*x + 12\*d^7)\*sqrt(-e^2\*x^2 + d^2))/(4\*d^7\*e^5\*x^7 + 4\*d^8\*e^4\*x^6 - 12\*d^9\*e^3\*x^5 - 12\*d^10\*e^2\*x^4 + 8\*d^11\*e\*x^3 + 8\*d^12\*x^2 - (d^6\*e^5\*x^7 + d^7\*e^4\*x^6 - 8\*d^8\*e^3\*x^5 - 8\*d^9\*e^2\*x^4 + 8\*d^10\*e\*x^3 + 8\*d^11\*x^2)\*sq

$\text{rt}(-e^2 x^2 + d^2)$

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**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (-(-d + ex)(d + ex))^{\frac{3}{2}} (d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(e*x+d)/(-e**2*x**2+d**2)**(3/2),x)`

[Out] `Integral(1/(x**3*(-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)), x)`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

`[undef, undef, undef, 1]`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)*x^3),x, algorithm="giac")`

[Out] `[undef, undef, undef, 1]`

$$3.136 \quad \int \frac{x^7}{(d+ex)(d^2-e^2x^2)^{5/2}} dx$$

**Optimal.** Leaf size=162

$$\frac{x^6(d-ex)}{5e^2(d^2-e^2x^2)^{5/2}} + \frac{(32d-35ex)\sqrt{d^2-e^2x^2}}{10e^8} + \frac{7d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8} + \frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{x^4(6d-7ex)}{15e^4(d^2-e^2x^2)^{3/2}}$$

[Out] (x^6\*(d - e\*x))/(5\*e^2\*(d^2 - e^2\*x^2)^(5/2)) - (x^4\*(6\*d - 7\*e\*x))/(15\*e^4\*(d^2 - e^2\*x^2)^(3/2)) + (x^2\*(24\*d - 35\*e\*x))/(15\*e^6\*Sqrt[d^2 - e^2\*x^2]) + ((32\*d - 35\*e\*x)\*Sqrt[d^2 - e^2\*x^2])/(10\*e^8) + (7\*d^2\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(2\*e^8)

**Rubi [A]** time = 0.510835, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\frac{x^6(d-ex)}{5e^2(d^2-e^2x^2)^{5/2}} + \frac{(32d-35ex)\sqrt{d^2-e^2x^2}}{10e^8} + \frac{7d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8} + \frac{x^2(24d-35ex)}{15e^6\sqrt{d^2-e^2x^2}} - \frac{x^4(6d-7ex)}{15e^4(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^7/((d + e\*x)\*(d^2 - e^2\*x^2)^(5/2)), x]

[Out] (x^6\*(d - e\*x))/(5\*e^2\*(d^2 - e^2\*x^2)^(5/2)) - (x^4\*(6\*d - 7\*e\*x))/(15\*e^4\*(d^2 - e^2\*x^2)^(3/2)) + (x^2\*(24\*d - 35\*e\*x))/(15\*e^6\*Sqrt[d^2 - e^2\*x^2]) + ((32\*d - 35\*e\*x)\*Sqrt[d^2 - e^2\*x^2])/(10\*e^8) + (7\*d^2\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(2\*e^8)

**Rubi in Sympy [A]** time = 98.3398, size = 260, normalized size = 1.6

$$\frac{d^6}{5e^8(d+ex)(d^2-e^2x^2)^{3/2}} - \frac{d^5}{e^8(d^2-e^2x^2)^{3/2}} + \frac{d^4x}{15e^7(d^2-e^2x^2)^{3/2}} + \frac{3d^3}{e^8\sqrt{d^2-e^2x^2}}$$

$$+ \frac{2d^2x^3}{3e^5(d^2-e^2x^2)^{3/2}} - \frac{13d^2x}{15e^7\sqrt{d^2-e^2x^2}} + \frac{7d^2 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^8}$$

$$+ \frac{d\sqrt{d^2-e^2x^2}}{e^8} + \frac{x^5}{3e^3(d^2-e^2x^2)^{3/2}} - \frac{5x^3}{3e^5\sqrt{d^2-e^2x^2}} - \frac{5x\sqrt{d^2-e^2x^2}}{2e^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*7/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2), x)

[Out] d\*\*6/(5\*e\*\*8\*(d + e\*x)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)) - d\*\*5/(e\*\*8\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)) + d\*\*4\*x/(15\*e\*\*7\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)) + 3\*d\*\*3/(e\*\*8\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) + 2\*d\*\*2\*x\*\*3/(3\*e\*\*5\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)) - 13\*d\*\*2\*x/(15\*e\*\*7\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) + 7\*d\*\*2\*atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2))/(2\*e\*\*8) + d\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/e\*\*8 + x\*\*5/(3\*e\*\*3\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)) - 5\*x\*\*3/(3\*e\*\*5\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) - 5\*x\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(2\*e\*\*7)

**Mathematica [A]** time = 0.15006, size = 128, normalized size = 0.79

$$\frac{105d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{\sqrt{d^2-e^2x^2}(96d^6-9d^5ex-249d^4e^2x^2-4d^3e^3x^3+176d^2e^4x^4+15de^5x^5-15e^6x^6)}{(d-ex)^2(d+ex)^3}}{30e^8}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((d + e*x)*(d^2 - e^2*x^2)^(5/2)),x]
```

```
[Out] ((Sqrt[d^2 - e^2*x^2]*(96*d^6 - 9*d^5*e*x - 249*d^4*e^2*x^2 - 4*d^3*e^3*x^3 + 176*d^2*e^4*x^4 + 15*d*e^5*x^5 - 15*e^6*x^6))/((d - e*x)^2*(d + e*x)^3) + 105*d^2*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/(30*e^8)
```

**Maple [F]** time = 180., size = 0, normalized size = 0.

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Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

```
[Out] int(x^7/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 0.326503, size = 963, normalized size = 5.94

$$15 e^{12} x^{12} - 15 d e^{11} x^{11} - 446 d^2 e^{10} x^{10} - 302 d^3 e^9 x^9 + 3561 d^4 e^8 x^8 + 3441 d^5 e^7 x^7 - 9282 d^6 e^6 x^6 - 9282 d^7 e^5 x^5 + 9520 d^8 e^4 x^4 - 3360 d^9 e^3 x^3 - 3360 d^{10} e^2 x^2 + 210 d^{11} e x - 15 d^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)),x, algorithm="fricas")
```

```
[Out] -1/30*(15*e^12*x^12 - 15*d*e^11*x^11 - 446*d^2*e^10*x^10 - 302*d^3*e^9*x^9 + 3561*d^4*e^8*x^8 + 3441*d^5*e^7*x^7 - 9282*d^6*e^6*x^6 - 9282*d^7*e^5*x^5 + 9520*d^8*e^4*x^4 + 9520*d^9*e^3*x^3 - 3360*d^10*e^2*x^2 - 3360*d^11*e*x + 210*(6*d^3*e^9*x^9 + 6*d^4*e^8*x^8 - 44*d^5*e^7*x^7 - 44*d^6*e^6*x^6 + 102*d^7*e^5*x^5 + 102*d^8*e^4*x^4 - 96*d^9*e^3*x^3 - 96*d^10*e^2*x^2 + 32*d^11*e*x + 32*d^12 - (d^2*e^9*x^9 + d^3*e^8*x^8 - 19*d^4*e^7*x^7 - 19*d^5*e^6*x^6 + 66*d^6*e^5*x^5 + 66*d^7*e^4*x^4 - 80*d^8*e^3*x^3 - 80*d^9*e^2*x^2 + 32*d^10*e*x + 32*d^11)*sqrt(-e^2*x^2 + d^2))*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 2*(45*d*e^10*x^10 + 3*d^2*e^9*x^9 - 720*d^3*e^8*x^8 - 660*d^4*e^7*x^7 + 2891*d^5*e^6*x^6 + 2891*d^6*e^5*x^5 - 3920*d^7*e^4*x^4 - 3920*d^8*e^3*x^3 + 1680*d^9*e^2*x^2 + 1680*d^10*e*x)*sqrt(-e^2*x^2 + d^2))/(6*d*e^17*x^9 + 6*d^2*e^16*x^8 - 44*d^3*e^15*x^7 - 44*d^4*e^14*x^6 + 102*d^5*e^13*x^5 + 102*d^6*e^12*x^4 - 96*d^7*e^11*x^3 - 96*d^8*e^10*x^2 + 32*d^9*e^9*x + 32*d^10*e^8 - (e^17*x^9 + d*e^16*x^8 - 19*d^2*e^15*x^7 - 19*d^3*e^14*x^6 + 66*d^4*e^13*x^5 + 66*d^5*e^12*x^4 - 80*d^6*e^11*x^3 - 80*d^7*e^10*x^2 + 32*d^8*e^9*x + 32*d^9)*sqrt(-e^2*x^2 + d^2))
```

$$0*d^7*e^{10*x^2} + 32*d^8*e^9*x + 32*d^9*e^8)*\text{sqrt}(-e^2*x^2 + d^2))$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^7}{(-(-d + ex)(d + ex))^{\frac{5}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*7/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2), x)

[Out] Integral(x\*\*7/((-(-d + e\*x)\*(d + e\*x))\*\*(5/2)\*(d + e\*x)), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7/((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)), x, algorithm="giac")

[Out] [undef, undef, undef, undef, undef, undef, undef, 1]



$$3.137 \quad \int \frac{x^6}{(d+ex)(d^2-e^2x^2)^{5/2}} dx$$

**Optimal.** Leaf size=148

$$\frac{x^5(d-ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7} + \frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{x^3(5d-6ex)}{15e^4(d^2-e^2x^2)^{3/2}}$$

[Out] (x^5\*(d - e\*x))/(5\*e^2\*(d^2 - e^2\*x^2)^(5/2)) - (x^3\*(5\*d - 6\*e\*x))/(15\*e^4\*(d^2 - e^2\*x^2)^(3/2)) + (x\*(5\*d - 8\*e\*x))/(5\*e^6\*sqrt[d^2 - e^2\*x^2]) - (16\*sqrt[d^2 - e^2\*x^2])/(5\*e^7) - (d\*ArcTan[(e\*x)/sqrt[d^2 - e^2\*x^2]])/e^7

**Rubi [A]** time = 0.454123, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\frac{x^5(d-ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{16\sqrt{d^2-e^2x^2}}{5e^7} - \frac{d \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7} + \frac{x(5d-8ex)}{5e^6\sqrt{d^2-e^2x^2}} - \frac{x^3(5d-6ex)}{15e^4(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^6/((d + e\*x)\*(d^2 - e^2\*x^2)^(5/2)), x]

[Out] (x^5\*(d - e\*x))/(5\*e^2\*(d^2 - e^2\*x^2)^(5/2)) - (x^3\*(5\*d - 6\*e\*x))/(15\*e^4\*(d^2 - e^2\*x^2)^(3/2)) + (x\*(5\*d - 8\*e\*x))/(5\*e^6\*sqrt[d^2 - e^2\*x^2]) - (16\*sqrt[d^2 - e^2\*x^2])/(5\*e^7) - (d\*ArcTan[(e\*x)/sqrt[d^2 - e^2\*x^2]])/e^7

**Rubi in Sympy [A]** time = 77.2055, size = 182, normalized size = 1.23

$$\begin{aligned} & -\frac{d^5}{5e^7(d+ex)(d^2-e^2x^2)^{\frac{3}{2}}} + \frac{d^4}{e^7(d^2-e^2x^2)^{\frac{3}{2}}} - \frac{d^3x}{15e^6(d^2-e^2x^2)^{\frac{3}{2}}} - \frac{3d^2}{e^7\sqrt{d^2-e^2x^2}} \\ & - \frac{2dx^3}{3e^4(d^2-e^2x^2)^{\frac{3}{2}}} + \frac{13dx}{15e^6\sqrt{d^2-e^2x^2}} - \frac{d \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^7} - \frac{\sqrt{d^2-e^2x^2}}{e^7} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*6/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2), x)

[Out] -d\*\*5/(5\*e\*\*7\*(d + e\*x)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)) + d\*\*4/(e\*\*7\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)) - d\*\*3\*x/(15\*e\*\*6\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)) - 3\*d\*\*2/(e\*\*7\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) - 2\*d\*x\*\*3/(3\*e\*\*4\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)) + 13\*d\*x/(15\*e\*\*6\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) - d\*atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2))/e\*\*7 - sqrt(d\*\*2 - e\*\*2\*x\*\*2)/e\*\*7

**Mathematica [A]** time = 0.161397, size = 115, normalized size = 0.78

$$\frac{15d \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{\sqrt{d^2-e^2x^2}(48d^5+33d^4ex-87d^3e^2x^2-52d^2e^3x^3+38de^4x^4+15e^5x^5)}{(d-ex)^2(d+ex)^3}}{15e^7}$$

Antiderivative was successfully verified.

[In] Integrate[x^6/((d + e\*x)\*(d^2 - e^2\*x^2)^(5/2)),x]

[Out] -((Sqrt[d^2 - e^2\*x^2]\*(48\*d^5 + 33\*d^4\*e\*x - 87\*d^3\*e^2\*x^2 - 52\*d^2\*e^3\*x^3 + 38\*d\*e^4\*x^4 + 15\*e^5\*x^5))/((d - e\*x)^2\*(d + e\*x)^3) + 15\*d\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(15\*e^7)

**Maple [B]** time = 0.045, size = 288, normalized size = 2.

$$\begin{aligned}
 & -\frac{x^4}{e^3}(-e^2x^2 + d^2)^{-\frac{3}{2}} + 5\frac{d^2x^2}{e^5(-e^2x^2 + d^2)^{\frac{3}{2}}} - 3\frac{d^4}{e^7(-e^2x^2 + d^2)^{\frac{3}{2}}} \\
 & -\frac{d^5}{5e^8}\left(x + \frac{d}{e}\right)^{-1}\left(-\left(x + \frac{d}{e}\right)^2e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{-\frac{3}{2}} + \frac{4d^3x}{15e^6}\left(-\left(x + \frac{d}{e}\right)^2e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{-\frac{3}{2}} \\
 & + \frac{8dx}{15e^6}\frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2e^2 + 2de\left(x + \frac{d}{e}\right)}} - \frac{2d^3x}{3e^6}(-e^2x^2 + d^2)^{-\frac{3}{2}} \\
 & + \frac{2dx}{3e^6}\frac{1}{\sqrt{-e^2x^2 + d^2}} - \frac{dx^3}{3e^4}(-e^2x^2 + d^2)^{-\frac{3}{2}} - \frac{d}{e^6}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2 + d^2}}\right)\frac{1}{\sqrt{e^2}}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^6/(e\*x+d)/(-e^2\*x^2+d^2)^(5/2),x)

[Out] -1/e^3\*x^4/(-e^2\*x^2+d^2)^(3/2)+5/e^5\*d^2\*x^2/(-e^2\*x^2+d^2)^(3/2)-3\*d^4/e^7/(-e^2\*x^2+d^2)^(3/2)-1/5\*d^5/e^8/(x+d/e)/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)+4/15\*d^3/e^6/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)\*x+8/15\*d/e^6/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x-2/3\*d^3/e^6\*x/(-e^2\*x^2+d^2)^(3/2)+2/3\*d/e^6\*x/(-e^2\*x^2+d^2)^(1/2)-1/3\*d/e^4\*x^3/(-e^2\*x^2+d^2)^(3/2)-d/e^6/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.306824, size = 838, normalized size = 5.66

$27de^9x^9 + 142d^2e^8x^8 + 112d^3e^7x^7 - 523d^4e^6x^6 - 523d^5e^5x^5 + 620d^6e^4x^4 + 620d^7e^3x^3 - 240d^8e^2x^2 - 240d^9ex + 30(de$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)),x, algorithm="fricas")

[Out] 1/15\*(27\*d\*e^9\*x^9 + 142\*d^2\*e^8\*x^8 + 112\*d^3\*e^7\*x^7 - 523\*d^4\*e^6\*x^6 - 523\*d^5\*e^5\*x^5 + 620\*d^6\*e^4\*x^4 + 620\*d^7\*e^3\*x^3 - 240\*d^8\*e^2\*x^2 - 240\*d^9\*e\*x + 30\*(d\*e^9\*x^9 + d^2\*e^8\*x^8 - 14\*d^3\*e^7\*x^7 - 14\*d^4\*e^6\*x^6 + 41\*d^5\*e^5\*x^5 + 41\*d^6\*e^4\*x^4 - 4

$$4*d^7*e^3*x^3 - 44*d^8*e^2*x^2 + 16*d^9*e*x + 16*d^{10} + (5*d^2*e^7*x^7 + 5*d^3*e^6*x^6 - 25*d^4*e^5*x^5 - 25*d^5*e^4*x^4 + 36*d^6*e^3*x^3 + 36*d^7*e^2*x^2 - 16*d^8*e*x - 16*d^9)*\sqrt{-e^2*x^2 + d^2})*\arctan(-(d - \sqrt{-e^2*x^2 + d^2})/(e*x)) - (15*e^9*x^9 + 38*d*e^8*x^8 + 8*d^2*e^7*x^7 - 303*d^3*e^6*x^6 - 303*d^4*e^5*x^5 + 500*d^5*e^4*x^4 + 500*d^6*e^3*x^3 - 240*d^7*e^2*x^2 - 240*d^8*e*x)*\sqrt{-e^2*x^2 + d^2})/(e^{16}*x^9 + d*e^{15}*x^8 - 14*d^2*e^{14}*x^7 - 14*d^3*e^{13}*x^6 + 41*d^4*e^{12}*x^5 + 41*d^5*e^{11}*x^4 - 44*d^6*e^{10}*x^3 - 44*d^7*e^9*x^2 + 16*d^8*e^8*x + 16*d^9*e^7 + (5*d*e^{14}*x^7 + 5*d^2*e^{13}*x^6 - 25*d^3*e^{12}*x^5 - 25*d^4*e^{11}*x^4 + 36*d^5*e^{10}*x^3 + 36*d^6*e^9*x^2 - 16*d^7*e^8*x - 16*d^8*e^7)*\sqrt{-e^2*x^2 + d^2}))$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^6}{(-(-d + ex)(d + ex))^{\frac{5}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*6/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2), x)

[Out] Integral(x\*\*6/((-(-d + e\*x)\*(d + e\*x))\*\*(5/2)\*(d + e\*x)), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)), x, algorithm="giac")

[Out] [undef, undef, undef, undef, undef, undef, 1]

$$3.138 \quad \int \frac{x^5}{(d+ex)(d^2-e^2x^2)^{5/2}} dx$$

**Optimal.** Leaf size=122

$$\frac{x^4(d-ex)}{5e^2(d^2-e^2x^2)^{5/2}} + \frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6} - \frac{x^2(4d-5ex)}{15e^4(d^2-e^2x^2)^{3/2}}$$

[Out]  $(x^4*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^2*(4*d - 5*e*x))/(15*e^4*(d^2 - e^2*x^2)^(3/2)) + (8*d - 15*e*x)/(15*e^6*\text{Sqrt}[d^2 - e^2*x^2]) + \text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]]/e^6$

**Rubi [A]** time = 0.350774, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\frac{x^4(d-ex)}{5e^2(d^2-e^2x^2)^{5/2}} + \frac{8d-15ex}{15e^6\sqrt{d^2-e^2x^2}} + \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6} - \frac{x^2(4d-5ex)}{15e^4(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^5/((d + e*x)*(d^2 - e^2*x^2)^(5/2)), x]$

[Out]  $(x^4*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (x^2*(4*d - 5*e*x))/(15*e^4*(d^2 - e^2*x^2)^(3/2)) + (8*d - 15*e*x)/(15*e^6*\text{Sqrt}[d^2 - e^2*x^2]) + \text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]]/e^6$

**Rubi in Sympy [A]** time = 58.691, size = 160, normalized size = 1.31

$$\frac{d^4}{5e^6(d+ex)(d^2-e^2x^2)^{3/2}} - \frac{2d^3}{3e^6(d^2-e^2x^2)^{3/2}} + \frac{d^2x}{15e^5(d^2-e^2x^2)^{3/2}} + \frac{d}{e^6\sqrt{d^2-e^2x^2}} + \frac{2x^3}{3e^3(d^2-e^2x^2)^{3/2}} - \frac{13x}{15e^5\sqrt{d^2-e^2x^2}} + \frac{\text{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**5/(e*x+d)/(-e**2*x**2+d**2)**(5/2), x)$

[Out]  $d**4/(5*e**6*(d + e*x)*(d**2 - e**2*x**2)**(3/2)) - 2*d**3/(3*e**6*(d**2 - e**2*x**2)**(3/2)) + d**2*x/(15*e**5*(d**2 - e**2*x**2)**(3/2)) + d/(e**6*\text{sqrt}(d**2 - e**2*x**2)) + 2*x**3/(3*e**3*(d**2 - e**2*x**2)**(3/2)) - 13*x/(15*e**5*\text{sqrt}(d**2 - e**2*x**2)) + a \tan(e*x/\text{sqrt}(d**2 - e**2*x**2))/e**6$

**Mathematica [A]** time = 0.117251, size = 103, normalized size = 0.84

$$\frac{15 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{\sqrt{d^2-e^2x^2}(8d^4-7d^3ex-27d^2e^2x^2+8de^3x^3+23e^4x^4)}{(d-ex)^2(d+ex)^3}}{15e^6}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^5/((d + e*x)*(d^2 - e^2*x^2)^(5/2)), x]$

[Out]  $((\sqrt{d^2 - e^2 x^2}) * (8d^4 - 7d^3 e x - 27d^2 e^2 x^2 + 8d e^3 x^3 + 23e^4 x^4)) / ((d - e x)^2 (d + e x)^3 + 15 \operatorname{ArcTan}[e x] / \sqrt{d^2 - e^2 x^2}) / (15 e^6)$

**Maple [B]** time = 0.015, size = 259, normalized size = 2.1

$$\begin{aligned} & \frac{2d^2 x}{3e^5} (-e^2 x^2 + d^2)^{-\frac{3}{2}} - \frac{2x}{3e^5} \frac{1}{\sqrt{-e^2 x^2 + d^2}} + \frac{x^3}{3e^3} (-e^2 x^2 + d^2)^{-\frac{3}{2}} \\ & + \frac{1}{e^5} \arctan\left(x \sqrt{e^2} \frac{1}{\sqrt{-e^2 x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} - \frac{dx^2}{e^4} (-e^2 x^2 + d^2)^{-\frac{3}{2}} \\ & + \frac{d^3}{3e^6} (-e^2 x^2 + d^2)^{-\frac{3}{2}} + \frac{d^4}{5e^7} \left(x + \frac{d}{e}\right)^{-1} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right)\right)^{-\frac{3}{2}} \\ & - \frac{4d^2 x}{15e^5} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right)\right)^{-\frac{3}{2}} - \frac{8x}{15e^5} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right)}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)`

[Out]  $2/3 * d^2 / e^5 * x / (-e^2 * x^2 + d^2)^{(3/2)} - 2/3 / e^5 * x / (-e^2 * x^2 + d^2)^{(1/2)} + 1/3 / e^3 * x^3 / (-e^2 * x^2 + d^2)^{(3/2)} + 1 / e^5 / (e^2)^{(1/2)} * \arctan((e^2)^{(1/2)} * x / (-e^2 * x^2 + d^2)^{(1/2)}) - d / e^4 * x^2 / (-e^2 * x^2 + d^2)^{(3/2)} + 1/3 * d^3 / e^6 / (-e^2 * x^2 + d^2)^{(3/2)} + 1/5 * d^4 / e^7 / (x + d/e) / (- (x + d/e)^2 * e^2 + 2 * d * e * (x + d/e))^{(3/2)} - 4/15 * d^2 / e^5 / (- (x + d/e)^2 * e^2 + 2 * d * e * (x + d/e))^{(3/2)} * x - 8/15 / e^5 / (- (x + d/e)^2 * e^2 + 2 * d * e * (x + d/e))^{(1/2)} * x$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.306031, size = 726, normalized size = 5.95

$$\frac{23e^8 x^8 + 40de^7 x^7 - 179d^2 e^6 x^6 - 199d^3 e^5 x^5 + 280d^4 e^4 x^4 + 280d^5 e^3 x^3 - 120d^6 e^2 x^2 - 120d^7 ex - 30(4de^7 x^7 + 4d^2 e^6 x^6)}{15(4de^{13} x^7 + 4d^2 e^6 x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^5/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)),x, algorithm="fricas")`

[Out]  $1/15 * (23 * e^8 * x^8 + 40 * d * e^7 * x^7 - 179 * d^2 * e^6 * x^6 - 199 * d^3 * e^5 * x^5 + 280 * d^4 * e^4 * x^4 + 280 * d^5 * e^3 * x^3 - 120 * d^6 * e^2 * x^2 - 120 * d^7 * e * x - 30 * (4 * d * e^7 * x^7 + 4 * d^2 * e^6 * x^6 - 16 * d^3 * e^5 * x^5 - 16 * d^4 * e^4 * x^4 + 20 * d^5 * e^3 * x^3 + 20 * d^6 * e^2 * x^2 - 8 * d^7 * e * x - 8 * d^8 - (e^7 * x^7 + d * e^6 * x^6 - 9 * d^2 * e^5 * x^5 - 9 * d^3 * e^4 * x^4 + 16 * d^4 * e^3 * x^3 + 16 * d^5 * e^2 * x^2 - 8 * d^6 * e * x - 8 * d^7) * \sqrt{-e^2 * x^2 + d^2})) * \arctan(-(d - \sqrt{-e^2 * x^2 + d^2}) / (e * x)) - 4 * (2 * e^7 * x^7 - 21 * d * e^6 * x^6)$

$$\begin{aligned} & ^6x^6 - 26d^2e^5x^5 + 55d^3e^4x^4 + 55d^4e^3x^3 - 30d^5e^2x^2 - 30d^6e^1x) \cdot \sqrt{-e^2x^2 + d^2}) / (4d^7e^{13}x^7 + 4d^8e^{12}x^6 - 16d^3e^{11}x^5 - 16d^4e^{10}x^4 + 20d^5e^9x^3 + 20d^6e^8x^2 - 8d^7e^7x - 8d^8e^6 - (e^{13}x^7 + d^8e^{12}x^6 - 9d^2e^{11}x^5 - 9d^3e^{10}x^4 + 16d^4e^9x^3 + 16d^5e^8x^2 - 8d^6e^7x - 8d^7e^6) \cdot \sqrt{-e^2x^2 + d^2})) \end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{(-(-d + ex)(d + ex))^{\frac{5}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2), x)

[Out] Integral(x\*\*5/((-(-d + e\*x)\*(d + e\*x))\*\*(5/2)\*(d + e\*x)), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)), x, algorithm="giac")

[Out] [undef, undef, undef, undef, undef, 1]

$$3.139 \quad \int \frac{x^4}{(d+ex)(d^2-e^2x^2)^{5/2}} dx$$

**Optimal.** Leaf size=85

$$-\frac{x^4(d-ex)}{5de(d^2-e^2x^2)^{5/2}} - \frac{4}{5e^5\sqrt{d^2-e^2x^2}} + \frac{4d^2}{15e^5(d^2-e^2x^2)^{3/2}}$$

[Out]  $-(x^4*(d - e*x))/(5*d*e*(d^2 - e^2*x^2)^(5/2)) + (4*d^2)/(15*e^5*(d^2 - e^2*x^2)^(3/2)) - 4/(5*e^5*sqrt[d^2 - e^2*x^2])$

**Rubi [A]** time = 0.2541, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$-\frac{x^4(d-ex)}{5de(d^2-e^2x^2)^{5/2}} - \frac{4}{5e^5\sqrt{d^2-e^2x^2}} + \frac{4d^2}{15e^5(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^4/((d + e\*x)\*(d^2 - e^2\*x^2)^(5/2)), x]

[Out]  $-(x^4*(d - e*x))/(5*d*e*(d^2 - e^2*x^2)^(5/2)) + (4*d^2)/(15*e^5*(d^2 - e^2*x^2)^(3/2)) - 4/(5*e^5*sqrt[d^2 - e^2*x^2])$

**Rubi in Sympy [A]** time = 21.0612, size = 70, normalized size = 0.82

$$\frac{4d^2}{15e^5(d^2-e^2x^2)^{3/2}} - \frac{4}{5e^5\sqrt{d^2-e^2x^2}} - \frac{x^4(d-ex)}{5de(d^2-e^2x^2)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2), x)

[Out]  $4*d**2/(15*e**5*(d**2 - e**2*x**2)**(3/2)) - 4/(5*e**5*sqrt(d**2 - e**2*x**2)) - x**4*(d - e*x)/(5*d*e*(d**2 - e**2*x**2)**(5/2))$

**Mathematica [A]** time = 0.0657341, size = 82, normalized size = 0.96

$$\frac{\sqrt{d^2 - e^2x^2} (8d^4 + 8d^3ex - 12d^2e^2x^2 - 12de^3x^3 + 3e^4x^4)}{15de^5(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/((d + e\*x)\*(d^2 - e^2\*x^2)^(5/2)), x]

[Out]  $-(sqrt[d^2 - e^2*x^2]*(8*d^4 + 8*d^3*e*x - 12*d^2*e^2*x^2 - 12*d*e^3*x^3 + 3*e^4*x^4))/(15*d*e^5*(d - e*x)^2*(d + e*x)^3)$

**Maple [A]** time = 0.011, size = 70, normalized size = 0.8

$$\frac{(-ex + d)(3x^4e^4 - 12x^3de^3 - 12d^2x^2e^2 + 8d^3xe + 8d^4)}{15de^5} (-e^2x^2 + d^2)^{-5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x+d)/(-e^2*x^2+d^2)^(5/2), x)
```

```
[Out] -1/15*(-e*x+d)*(3*e^4*x^4-12*d*e^3*x^3-12*d^2*e^2*x^2+8*d^3*e*x+8*d^4)/d/e^5/(-e^2*x^2+d^2)^(5/2)
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 0.292448, size = 336, normalized size = 3.95

$$\frac{3e^3x^8 + 20de^2x^7 - 4d^2ex^6 - 24d^3x^5 - 4(2e^2x^7 - dex^6 - 6d^2x^5)\sqrt{-e^2x^2 + d^2}}{15(4d^2e^7x^7 + 4d^3e^6x^6 - 16d^4e^5x^5 - 16d^5e^4x^4 + 20d^6e^3x^3 + 20d^7e^2x^2 - 8d^8ex - 8d^9 - (de^7x^7 + d^2e^6x^6 - 9d^3e^5x^5 - 9d^4e^4x^4 + 16d^5e^3x^3 + 16d^6e^2x^2 - 8d^7ex - 8d^8 - d^9)\sqrt{-e^2x^2 + d^2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)),x, algorithm="fricas")
```

```
[Out] -1/15*(3*e^3*x^8 + 20*d*e^2*x^7 - 4*d^2*e*x^6 - 24*d^3*x^5 - 4*(2*e^2*x^7 - d*e*x^6 - 6*d^2*x^5)*sqrt(-e^2*x^2 + d^2))/(4*d^2*e^7*x^7 + 4*d^3*e^6*x^6 - 16*d^4*e^5*x^5 - 16*d^5*e^4*x^4 + 20*d^6*e^3*x^3 + 20*d^7*e^2*x^2 - 8*d^8*e*x - 8*d^9 - (d*e^7*x^7 + d^2*e^6*x^6 - 9*d^3*e^5*x^5 - 9*d^4*e^4*x^4 + 16*d^5*e^3*x^3 + 16*d^6*e^2*x^2 - 8*d^7*e*x - 8*d^8)*sqrt(-e^2*x^2 + d^2))
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(-(-d + ex)(d + ex))^{\frac{5}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)/(-e**2*x**2+d**2)**(5/2), x)
```

```
[Out] Integral(x**4/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, 1]
```



$$3.140 \quad \int \frac{x^3}{(d+ex)(d^2-e^2x^2)^{5/2}} dx$$

**Optimal.** Leaf size=91

$$\frac{x^2(d-ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{2d-3ex}{15e^4(d^2-e^2x^2)^{3/2}} - \frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}$$

[Out]  $(x^2*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (2*d - 3*e*x)/(15*e^4*(d^2 - e^2*x^2)^(3/2)) - x/(5*d^2*e^3*Sqrt[d^2 - e^2*x^2])$

**Rubi [A]** time = 0.250177, antiderivative size = 91, normalized size of antiderivative = 1., number of rules used = 4, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$\frac{x^2(d-ex)}{5e^2(d^2-e^2x^2)^{5/2}} - \frac{2d-3ex}{15e^4(d^2-e^2x^2)^{3/2}} - \frac{x}{5d^2e^3\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[x^3/((d + e\*x)\*(d^2 - e^2\*x^2)^(5/2)), x]

[Out]  $(x^2*(d - e*x))/(5*e^2*(d^2 - e^2*x^2)^(5/2)) - (2*d - 3*e*x)/(15*e^4*(d^2 - e^2*x^2)^(3/2)) - x/(5*d^2*e^3*Sqrt[d^2 - e^2*x^2])$

**Rubi in Sympy [A]** time = 15.4255, size = 80, normalized size = 0.88

$$-\frac{x^3(d-ex)}{5de(d^2-e^2x^2)^{5/2}} - \frac{2}{15de^4\sqrt{d^2-e^2x^2}} + \frac{x^2(d+3ex)}{15d^2e^2(d^2-e^2x^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2), x)

[Out]  $-x**3*(d - e*x)/(5*d*e*(d**2 - e**2*x**2)**(5/2)) - 2/(15*d*e**4*\text{sqrt}(d**2 - e**2*x**2)) + x**2*(d + 3*e*x)/(15*d**2*e**2*(d**2 - e**2*x**2)**(3/2))$

**Mathematica [A]** time = 0.0579947, size = 82, normalized size = 0.9

$$\frac{\sqrt{d^2 - e^2x^2}(-2d^4 - 2d^3ex + 3d^2e^2x^2 + 3de^3x^3 + 3e^4x^4)}{15d^2e^4(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((d + e\*x)\*(d^2 - e^2\*x^2)^(5/2)), x]

[Out]  $(\text{Sqrt}[d^2 - e^2*x^2]*(-2*d^4 - 2*d^3*e*x + 3*d^2*e^2*x^2 + 3*d*e^3*x^3 + 3*e^4*x^4))/(15*d^2*e^4*(d - e*x)^2*(d + e*x)^3)$

**Maple [A]** time = 0.011, size = 70, normalized size = 0.8

$$-\frac{(-ex+d)(-3x^4e^4 - 3x^3de^3 - 3x^2d^2e^2 + 2d^3xe + 2d^4)}{15d^2e^4} (-e^2x^2 + d^2)^{-5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(e*x+d)/(-e^2*x^2+d^2)^(5/2),x)`

[Out]  $-1/15*(-e*x+d)*(-3*e^4*x^4-3*d*e^3*x^3-3*d^2*e^2*x^2+2*d^3*e*x+2*d^4)/d^2/e^4/(-e^2*x^2+d^2)^(5/2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.285406, size = 367, normalized size = 4.03

$$15 \frac{3e^4x^8 - 5de^3x^7 - 29d^2e^2x^6 + 6d^3ex^5 + 30d^4x^4 + 2(e^3x^7 + 7de^2x^6 - 3d^2ex^5 - 9d^3e^4x^4 + 4d^4e^5x^3 - 16d^5e^6x^2 + 20d^6e^7x - 8d^7e^8 - (d^2e^7x^7 + d^3e^6x^6 - 9d^4e^5x^5 - 9d^5e^4x^4 + 16d^6e^3x^3 + 20d^7e^2x^2 - 8d^8ex - 8d^{10} - (d^2e^7x^7 + d^3e^6x^6 - 9d^4e^5x^5 - 9d^5e^4x^4 + 16d^6e^3x^3 + 20d^7e^2x^2 - 8d^8ex - 8d^{10}))^2}{(4d^3e^7x^7 + 4d^4e^6x^6 - 16d^5e^5x^5 - 16d^6e^4x^4 + 20d^7e^3x^3 + 20d^8e^2x^2 - 8d^9ex - 8d^{10} - (d^2e^7x^7 + d^3e^6x^6 - 9d^4e^5x^5 - 9d^5e^4x^4 + 16d^6e^3x^3 + 20d^7e^2x^2 - 8d^8ex - 8d^{10}))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)),x, algorithm="fricas")`

[Out]  $1/15*(3*e^4*x^8 - 5*d*e^3*x^7 - 29*d^2*e^2*x^6 + 6*d^3*e*x^5 + 30*d^4*x^4 + 2*(e^3*x^7 + 7*d*e^2*x^6 - 3*d^2*e*x^5 - 15*d^3*x^4)*\sqrt[3]{(-e^2*x^2 + d^2)})/(4*d^3*e^7*x^7 + 4*d^4*e^6*x^6 - 16*d^5*e^5*x^5 - 16*d^6*e^4*x^4 + 20*d^7*e^3*x^3 + 20*d^8*e^2*x^2 - 8*d^9*e*x - 8*d^{10} - (d^2*e^7*x^7 + d^3*e^6*x^6 - 9*d^4*e^5*x^5 - 9*d^5*e^4*x^4 + 16*d^6*e^3*x^3 + 16*d^7*e^2*x^2 - 8*d^8*e*x - 8*d^9)*\sqrt{(-e^2*x^2 + d^2)})$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(-(-d + ex)(d + ex))^{\frac{5}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(e*x+d)/(-e**2*x**2+d**2)**(5/2),x)`

[Out] `Integral(x**3/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)),x, algorithm="giac")`

```
[Out] [undef, undef, undef, undef, undef, 1]
```

$$3.141 \quad \int \frac{x^2}{(d+ex)(d^2-e^2x^2)^{5/2}} dx$$

**Optimal.** Leaf size=95

$$-\frac{x^2}{5de(d+ex)(d^2-e^2x^2)^{3/2}} + \frac{2(d+ex)}{15de^3(d^2-e^2x^2)^{3/2}} - \frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}$$

[Out]  $-x^2/(5*d*e*(d+e*x)*(d^2-e^2*x^2)^{(3/2)}) + (2*(d+e*x))/(15*d*e^3*(d^2-e^2*x^2)^{(3/2)}) - (2*x)/(15*d^3*e^2*sqrt[d^2-e^2*x^2])$

**Rubi [A]** time = 0.17747, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$-\frac{x^2}{5de(d+ex)(d^2-e^2x^2)^{3/2}} + \frac{2(d+ex)}{15de^3(d^2-e^2x^2)^{3/2}} - \frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/((d + e\*x)\*(d^2 - e^2\*x^2)^(5/2)), x]

[Out]  $-x^2/(5*d*e*(d+e*x)*(d^2-e^2*x^2)^{(3/2)}) + (2*(d+e*x))/(15*d*e^3*(d^2-e^2*x^2)^{(3/2)}) - (2*x)/(15*d^3*e^2*sqrt[d^2-e^2*x^2])$

**Rubi in Sympy [A]** time = 13.8867, size = 80, normalized size = 0.84

$$-\frac{x^2(d-ex)}{5de(d^2-e^2x^2)^{5/2}} + \frac{2d+2ex}{15de^3(d^2-e^2x^2)^{3/2}} - \frac{2x}{15d^3e^2\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2), x)

[Out]  $-x**2*(d-e*x)/(5*d*e*(d**2-e**2*x**2)**(5/2)) + (2*d+2*e*x)/(15*d*e**3*(d**2-e**2*x**2)**(3/2)) - 2*x/(15*d**3*e**2*sqrt(d**2-e**2*x**2))$

**Mathematica [A]** time = 0.054273, size = 82, normalized size = 0.86

$$\frac{\sqrt{d^2-e^2x^2}(2d^4+2d^3ex-3d^2e^2x^2+2de^3x^3+2e^4x^4)}{15d^3e^3(d-ex)^2(d+ex)^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((d + e\*x)\*(d^2 - e^2\*x^2)^(5/2)), x]

[Out]  $(sqrt[d^2-e^2*x^2]*(2*d^4+2*d^3*e*x-3*d^2*e^2*x^2+2*d*e^3*x^3+2*e^4*x^4))/(15*d^3*e^3*(d-e*x)^2*(d+e*x)^3)$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, 1]
```

$$3.142 \quad \int \frac{x}{(d+ex)(d^2-e^2x^2)^{5/2}} dx$$

**Optimal.** Leaf size=85

$$\frac{x}{15d^2e(d^2-e^2x^2)^{3/2}} + \frac{1}{5e^2(d+ex)(d^2-e^2x^2)^{3/2}} + \frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}$$

[Out]  $x/(15*d^2*e*(d^2 - e^2*x^2)^{(3/2)}) + 1/(5*e^2*(d + e*x)*(d^2 - e^2*x^2)^{(3/2)}) + (2*x)/(15*d^4*e*\text{Sqrt}[d^2 - e^2*x^2])$

**Rubi [A]** time = 0.0973462, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$\frac{x}{15d^2e(d^2-e^2x^2)^{3/2}} + \frac{1}{5e^2(d+ex)(d^2-e^2x^2)^{3/2}} + \frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[x/((d + e\*x)\*(d^2 - e^2\*x^2)^(5/2)), x]

[Out]  $x/(15*d^2*e*(d^2 - e^2*x^2)^{(3/2)}) + 1/(5*e^2*(d + e*x)*(d^2 - e^2*x^2)^{(3/2)}) + (2*x)/(15*d^4*e*\text{Sqrt}[d^2 - e^2*x^2])$

**Rubi in Sympy [A]** time = 8.70396, size = 70, normalized size = 0.82

$$\frac{1}{5e^2(d+ex)(d^2-e^2x^2)^{3/2}} + \frac{x}{15d^2e(d^2-e^2x^2)^{3/2}} + \frac{2x}{15d^4e\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2), x)

[Out]  $1/(5*e**2*(d + e*x)*(d**2 - e**2*x**2)**(3/2)) + x/(15*d**2*e*(d**2 - e**2*x**2)**(3/2)) + 2*x/(15*d**4*e*\text{sqrt}(d**2 - e**2*x**2))$

**Mathematica [A]** time = 0.0543011, size = 82, normalized size = 0.96

$$\frac{\sqrt{d^2 - e^2x^2} (3d^4 + 3d^3ex + 3d^2e^2x^2 - 2de^3x^3 - 2e^4x^4)}{15d^4e^2(d - ex)^2(d + ex)^3}$$

Antiderivative was successfully verified.

[In] Integrate[x/((d + e\*x)\*(d^2 - e^2\*x^2)^(5/2)), x]

[Out]  $(\text{Sqrt}[d^2 - e^2*x^2]*(3*d^4 + 3*d^3*e*x + 3*d^2*e^2*x^2 - 2*d*e^3*x^3 - 2*e^4*x^4))/(15*d^4*e^2*(d - e*x)^2*(d + e*x)^3)$

**Maple [A]** time = 0.011, size = 70, normalized size = 0.8

$$\frac{(-ex + d)(-2e^4x^4 - 2e^3x^3d + 3x^2d^2e^2 + 3xd^3e + 3d^4)}{15d^4e^2} (-e^2x^2 + d^2)^{-\frac{5}{2}}$$





```
[In] integrate(x/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, 1]
```

$$3.143 \quad \int \frac{1}{(d+ex)(d^2-e^2x^2)^{5/2}} dx$$

**Optimal.** Leaf size=82

$$-\frac{1}{5de(d+ex)(d^2-e^2x^2)^{3/2}} + \frac{8x}{15d^5\sqrt{d^2-e^2x^2}} + \frac{4x}{15d^3(d^2-e^2x^2)^{3/2}}$$

[Out]  $(4*x)/(15*d^3*(d^2 - e^2*x^2)^{(3/2)}) - 1/(5*d*e*(d + e*x)*(d^2 - e^2*x^2)^{(3/2)}) + (8*x)/(15*d^5*\text{Sqrt}[d^2 - e^2*x^2])$

**Rubi [A]** time = 0.0593204, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$-\frac{1}{5de(d+ex)(d^2-e^2x^2)^{3/2}} + \frac{8x}{15d^5\sqrt{d^2-e^2x^2}} + \frac{4x}{15d^3(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)\*(d^2 - e^2\*x^2)^(5/2)), x]

[Out]  $(4*x)/(15*d^3*(d^2 - e^2*x^2)^{(3/2)}) - 1/(5*d*e*(d + e*x)*(d^2 - e^2*x^2)^{(3/2)}) + (8*x)/(15*d^5*\text{Sqrt}[d^2 - e^2*x^2])$

**Rubi in Sympy [A]** time = 8.01028, size = 68, normalized size = 0.83

$$-\frac{1}{5de(d+ex)(d^2-e^2x^2)^{3/2}} + \frac{4x}{15d^3(d^2-e^2x^2)^{3/2}} + \frac{8x}{15d^5\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2), x)

[Out]  $-1/(5*d*e*(d + e*x)*(d**2 - e**2*x**2)**(3/2)) + 4*x/(15*d**3*(d**2 - e**2*x**2)**(3/2)) + 8*x/(15*d**5*\text{sqrt}(d**2 - e**2*x**2))$

**Mathematica [A]** time = 0.0517924, size = 82, normalized size = 1.

$$-\frac{\sqrt{d^2 - e^2x^2} (3d^4 - 12d^3ex - 12d^2e^2x^2 + 8de^3x^3 + 8e^4x^4)}{15d^5e(d - ex)^2(d + ex)^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x)\*(d^2 - e^2\*x^2)^(5/2)), x]

[Out]  $-(\text{Sqrt}[d^2 - e^2*x^2]*(3*d^4 - 12*d^3*e*x - 12*d^2*e^2*x^2 + 8*d*e^3*x^3 + 8*e^4*x^4))/(15*d^5*e*(d - e*x)^2*(d + e*x)^3)$

**Maple [A]** time = 0.011, size = 70, normalized size = 0.9

$$-\frac{(-ex + d)(8e^4x^4 + 8e^3x^3d - 12e^2x^2d^2 - 12xd^3e + 3d^4)}{15d^5e} (-e^2x^2 + d^2)^{-5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x+d)/(-e^2*x^2+d^2)^(5/2), x)`

[Out]  $-1/15*(-e*x+d)*(8*e^4*x^4+8*d*e^3*x^3-12*d^2*e^2*x^2-12*d^3*e*x+3*d^4)/d^5/e/(-e^2*x^2+d^2)^(5/2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.295314, size = 452, normalized size = 5.51

$$\frac{8e^7x^8 + 20de^6x^7 - 64d^2e^5x^6 - 124d^3e^4x^5 + 115d^4e^3x^4 + 220d^5e^2x^3 - 60d^6ex^2 - 120d^7x - (3e^6x^7 - 29de^5x^6 - 59d^2e^4x^5 + 85d^3e^3x^4 + 160d^4e^2x^3 - 60d^5ex^2 - 120d^6x)}{15(4d^6e^7x^7 + 4d^7e^6x^6 - 16d^8e^5x^5 - 16d^9e^4x^4 + 20d^{10}e^3x^3 + 20d^{11}e^2x^2 - 8d^{12}ex - 8d^{13} - (d^5e^7x^7 + d^6e^6x^6 - 9d^7e^5x^5 + 85d^8e^4x^4 + 160d^9e^3x^3 - 60d^{10}ex^2 - 120d^{11}x))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)),x, algorithm="fricas")`

[Out]  $-1/15*(8*e^7*x^8 + 20*d*e^6*x^7 - 64*d^2*e^5*x^6 - 124*d^3*e^4*x^5 + 115*d^4*e^3*x^4 + 220*d^5*e^2*x^3 - 60*d^6*e*x^2 - 120*d^7*x - (3*e^6*x^7 - 29*d*e^5*x^6 - 59*d^2*e^4*x^5 + 85*d^3*e^3*x^4 + 160*d^4*e^2*x^3 - 60*d^5*e*x^2 - 120*d^6*x)*\sqrt{-e^2*x^2 + d^2})/(4*d^6*e^7*x^7 + 4*d^7*e^6*x^6 - 16*d^8*e^5*x^5 - 16*d^9*e^4*x^4 + 20*d^{10}*e^3*x^3 + 20*d^{11}*e^2*x^2 - 8*d^{12}*e*x - 8*d^{13} - (d^5*e^7*x^7 + d^6*e^6*x^6 - 9*d^7*e^5*x^5 + 85*d^8*e^4*x^4 + 16*d^9*e^3*x^3 + 16*d^{10}*e^2*x^2 - 8*d^{11}*e*x - 8*d^{12})*\sqrt{-e^2*x^2 + d^2})$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-(-d + ex)(d + ex))^{\frac{5}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x+d)/(-e**2*x**2+d**2)**(5/2), x)`

[Out] `Integral(1/((-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)),x, algorithm="giac")
```

```
[Out] [undef, undef, undef, undef, undef, 1]
```

$$3.144 \quad \int \frac{1}{x(d+ex)(d^2-e^2x^2)^{5/2}} dx$$

**Optimal.** Leaf size=119

$$\frac{1}{5d^2(d+ex)(d^2-e^2x^2)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6} + \frac{5d-4ex}{15d^4(d^2-e^2x^2)^{3/2}}$$

[Out] (5\*d - 4\*e\*x)/(15\*d^4\*(d^2 - e^2\*x^2)^(3/2)) + 1/(5\*d^2\*(d + e\*x)\*(d^2 - e^2\*x^2)^(3/2)) + (15\*d - 8\*e\*x)/(15\*d^6\*sqrt[d^2 - e^2\*x^2]) - ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]/d^6

**Rubi [A]** time = 0.33132, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{1}{5d^2(d+ex)(d^2-e^2x^2)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6} + \frac{5d-4ex}{15d^4(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(d + e\*x)\*(d^2 - e^2\*x^2)^(5/2)), x]

[Out] (5\*d - 4\*e\*x)/(15\*d^4\*(d^2 - e^2\*x^2)^(3/2)) + 1/(5\*d^2\*(d + e\*x)\*(d^2 - e^2\*x^2)^(3/2)) + (15\*d - 8\*e\*x)/(15\*d^6\*sqrt[d^2 - e^2\*x^2]) - ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]/d^6

**Rubi in Sympy [A]** time = 40.3862, size = 97, normalized size = 0.82

$$\frac{d-ex}{5d^2(d^2-e^2x^2)^{5/2}} + \frac{5d-4ex}{15d^4(d^2-e^2x^2)^{3/2}} + \frac{15d-8ex}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2), x)

[Out] (d - e\*x)/(5\*d\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)) + (5\*d - 4\*e\*x)/(15\*d\*\*4\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)) + (15\*d - 8\*e\*x)/(15\*d\*\*6\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) - atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d)/d\*\*6

**Mathematica [A]** time = 0.110204, size = 106, normalized size = 0.89

$$\frac{-15 \log\left(\sqrt{d^2 - e^2x^2} + d\right) + \frac{\sqrt{d^2 - e^2x^2}(23d^4 + 8d^3ex - 27d^2e^2x^2 - 7de^3x^3 + 8e^4x^4)}{(d-ex)^2(d+ex)^3} + 15 \log(x)}{15d^6}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(d + e\*x)\*(d^2 - e^2\*x^2)^(5/2)), x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(23\*d^4 + 8\*d^3\*e\*x - 27\*d^2\*e^2\*x^2 - 7\*d\*e^3\*x^3 + 8\*e^4\*x^4))/((d - e\*x)^2\*(d + e\*x)^3) + 15\*Log[x] - 15\*Log[d + Sqrt[d^2 - e^2\*x^2]])/(15\*d^6)

---

**Maple [A]** time = 0.019, size = 196, normalized size = 1.7

$$\frac{1}{3d^3}(-e^2x^2 + d^2)^{-\frac{3}{2}} + \frac{1}{d^5} \frac{1}{\sqrt{-e^2x^2 + d^2}} - \frac{1}{d^5} \ln\left(\frac{1}{x} \left(2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}}$$

$$+ \frac{1}{5ed^2} \left(x + \frac{d}{e}\right)^{-1} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{-\frac{3}{2}}$$

$$- \frac{4ex}{15d^4} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{-\frac{3}{2}} - \frac{8ex}{15d^6} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(e\*x+d)/(-e^2\*x^2+d^2)^(5/2),x)

[Out] 1/3/d^3/(-e^2\*x^2+d^2)^(3/2)+1/d^5/(-e^2\*x^2+d^2)^(1/2)-1/d^5/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)+1/5/d^2/e/(x+d/e)/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)-4/15/d^4\*e/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)\*x-8/15/d^6\*e/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-e^2x^2 + d^2)^{\frac{5}{2}}(ex + d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)\*x),x, algorithm="maxima")

[Out] integrate(1/((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)\*x), x)

---

**Fricas [A]** time = 0.307014, size = 717, normalized size = 6.03

$$\frac{8e^8x^8 + 85de^7x^7 + d^2e^6x^6 - 304d^3e^5x^5 - 65d^4e^4x^4 + 340d^5e^3x^3 + 60d^6e^2x^2 - 120d^7ex + 15(4de^7x^7 + 4d^2e^6x^6 - 16d^3e^5x^5 - 65d^4e^4x^4 + 340d^5e^3x^3 + 60d^6e^2x^2 - 120d^7ex + 15(4de^7x^7 + 4d^2e^6x^6 - 16d^3e^5x^5 - 65d^4e^4x^4 + 340d^5e^3x^3 + 60d^6e^2x^2 - 120d^7ex + 15(4de^7x^7 + 4d^2e^6x^6 - 16d^3e^5x^5 - 65d^4e^4x^4 + 340d^5e^3x^3 + 60d^6e^2x^2 - 120d^7ex + 15(4d^7e^7x^7 + 4d^8e^6x^6 - 16d^9e^5x^5 - 16d^10e^4x^4 + 20d^11e^3x^3 + 20d^12e^2x^2 - 8d^13e^1x - 8d^14 - (d^6e^7x^7 + d^7e^6x^6 - 9d^8e^5x^5 - 9d^9e^4x^4 + 16d^10e^3x^3 + 16d^11e^2x^2 - 8d^12e^1x - 8d^13)*sqrt(-e^2x^2 + d^2)))/x) - (23e^7x^7 - 9d^8e^6x^6 - 179d^9e^5x^5 - 35d^10e^4x^4 + 280d^11e^3x^3 + 60d^12e^2x^2 - 120d^13e^1x - 8d^14 - (d^6e^7x^7 + d^7e^6x^6 - 9d^8e^5x^5 - 9d^9e^4x^4 + 16d^10e^3x^3 + 16d^11e^2x^2 - 8d^12e^1x - 8d^13)*sqrt(-e^2x^2 + d^2))}{(4d^7e^7x^7 + 4d^8e^6x^6 - 16d^9e^5x^5 - 16d^10e^4x^4 + 20d^11e^3x^3 + 20d^12e^2x^2 - 8d^13e^1x - 8d^14 - (d^6e^7x^7 + d^7e^6x^6 - 9d^8e^5x^5 - 9d^9e^4x^4 + 16d^10e^3x^3 + 16d^11e^2x^2 - 8d^12e^1x - 8d^13)*sqrt(-e^2x^2 + d^2))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)\*x),x, algorithm="fricas")

[Out] 1/15\*(8\*e^8\*x^8 + 85\*d\*e^7\*x^7 + d^2\*e^6\*x^6 - 304\*d^3\*e^5\*x^5 - 65\*d^4\*e^4\*x^4 + 340\*d^5\*e^3\*x^3 + 60\*d^6\*e^2\*x^2 - 120\*d^7\*e\*x + 15\*(4\*d^7\*e^7\*x^7 + 4\*d^8\*e^6\*x^6 - 16\*d^9\*e^5\*x^5 - 16\*d^10\*e^4\*x^4 + 20\*d^11\*e^3\*x^3 + 20\*d^12\*e^2\*x^2 - 8\*d^13\*e\*x - 8\*d^14 - (d^6\*e^7\*x^7 + d^7\*e^6\*x^6 - 9\*d^8\*e^5\*x^5 - 9\*d^9\*e^4\*x^4 + 16\*d^10\*e^3\*x^3 + 16\*d^11\*e^2\*x^2 - 8\*d^12\*e\*x - 8\*d^13)\*sqrt(-e^2\*x^2 + d^2)))/x) - (23\*e^7\*x^7 - 9\*d^8\*e^6\*x^6 - 179\*d^9\*e^5\*x^5 - 35\*d^10\*e^4\*x^4 + 280\*d^11\*e^3\*x^3 + 60\*d^12\*e^2\*x^2 - 120\*d^13\*e\*x - 8\*d^14 - (d^6\*e^7\*x^7 + d^7\*e^6\*x^6 - 9\*d^8\*e^5\*x^5 - 9\*d^9\*e^4\*x^4 + 16\*d^10\*e^3\*x^3 + 16\*d^11\*e^2\*x^2 - 8\*d^12\*e\*x - 8\*d^13)\*sqrt(-e^2\*x^2 + d^2))

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x(-d+ex)(d+ex)^{\frac{5}{2}}(d+ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2),x)

[Out] Integral(1/(x\*(-(d + e\*x)\*(d + e\*x))\*\*(5/2)\*(d + e\*x)), x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)\*x),x, algorithm="giac")

[Out] [undef, undef, undef, undef, undef, 1]

$$3.145 \quad \int \frac{1}{x^2(d+ex)(d^2-e^2x^2)^{5/2}} dx$$

**Optimal.** Leaf size=154

$$\frac{1}{5d^2x(d+ex)(d^2-e^2x^2)^{3/2}} - \frac{16\sqrt{d^2-e^2x^2}}{5d^7x} + \frac{e \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7} + \frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}} + \frac{6d-5ex}{15d^4x(d^2-e^2x^2)^{3/2}}$$

[Out] (6\*d - 5\*e\*x)/(15\*d^4\*x\*(d^2 - e^2\*x^2)^(3/2)) + 1/(5\*d^2\*x\*(d + e\*x)\*(d^2 - e^2\*x^2)^(3/2)) + (8\*d - 5\*e\*x)/(5\*d^6\*x\*Sqrt[d^2 - e^2\*x^2]) - (16\*Sqrt[d^2 - e^2\*x^2])/(5\*d^7\*x) + (e\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/d^7

**Rubi [A]** time = 0.430702, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{1}{5d^2x(d+ex)(d^2-e^2x^2)^{3/2}} - \frac{16\sqrt{d^2-e^2x^2}}{5d^7x} + \frac{e \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7} + \frac{8d-5ex}{5d^6x\sqrt{d^2-e^2x^2}} + \frac{6d-5ex}{15d^4x(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(d + e\*x)\*(d^2 - e^2\*x^2)^(5/2)), x]

[Out] (6\*d - 5\*e\*x)/(15\*d^4\*x\*(d^2 - e^2\*x^2)^(3/2)) + 1/(5\*d^2\*x\*(d + e\*x)\*(d^2 - e^2\*x^2)^(3/2)) + (8\*d - 5\*e\*x)/(5\*d^6\*x\*Sqrt[d^2 - e^2\*x^2]) - (16\*Sqrt[d^2 - e^2\*x^2])/(5\*d^7\*x) + (e\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/d^7

**Rubi in Sympy [A]** time = 50.9282, size = 126, normalized size = 0.82

$$\frac{d-ex}{5d^2x(d^2-e^2x^2)^{5/2}} + \frac{6d-5ex}{15d^4x(d^2-e^2x^2)^{3/2}} + \frac{24d-15ex}{15d^6x\sqrt{d^2-e^2x^2}} + \frac{e \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^7} - \frac{16\sqrt{d^2-e^2x^2}}{5d^7x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2), x)

[Out] (d - e\*x)/(5\*d\*\*2\*x\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)) + (6\*d - 5\*e\*x)/(15\*d\*\*4\*x\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)) + (24\*d - 15\*e\*x)/(15\*d\*\*6\*x\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) + e\*atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d)/d\*\*7 - 16\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(5\*d\*\*7\*x)

**Mathematica [A]** time = 0.148569, size = 122, normalized size = 0.79

$$\frac{-15e \log\left(\sqrt{d^2 - e^2x^2} + d\right) + \frac{\sqrt{d^2 - e^2x^2}(15d^5 + 38d^4ex - 52d^3e^2x^2 - 87d^2e^3x^3 + 33de^4x^4 + 48e^5x^5)}{x(d-ex)^2(d+ex)^3} + 15e \log(x)}{15d^7}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(d + e\*x)\*(d^2 - e^2\*x^2)^(5/2)), x]

[Out] -((Sqrt[d^2 - e^2\*x^2]\*(15\*d^5 + 38\*d^4\*e\*x - 52\*d^3\*e^2\*x^2 - 87\*d^2\*e^3\*x^3 + 33\*d\*e^4\*x^4 + 48\*e^5\*x^5))/(x\*(d - e\*x)^2\*(d + e



$$x)^3) + 15 \cdot e \cdot \text{Log}[x] - 15 \cdot e \cdot \text{Log}[d + \text{Sqrt}[d^2 - e^2 \cdot x^2]] / (15 \cdot d^7)$$

**Maple [A]** time = 0.02, size = 268, normalized size = 1.7

$$\begin{aligned} & -\frac{1}{d^3 x} (-e^2 x^2 + d^2)^{-\frac{3}{2}} + \frac{4 e^2 x}{3 d^5} (-e^2 x^2 + d^2)^{-\frac{3}{2}} + \frac{8 e^2 x}{3 d^7} \frac{1}{\sqrt{-e^2 x^2 + d^2}} \\ & - \frac{1}{5 d^3} \left(x + \frac{d}{e}\right)^{-1} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)\right)^{-\frac{3}{2}} + \frac{4 e^2 x}{15 d^5} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)\right)^{-\frac{3}{2}} \\ & + \frac{8 e^2 x}{15 d^7} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)}} - \frac{e}{3 d^4} (-e^2 x^2 + d^2)^{-\frac{3}{2}} \\ & - \frac{e}{d^6} \frac{1}{\sqrt{-e^2 x^2 + d^2}} + \frac{e}{d^6} \ln\left(\frac{1}{x} \left(2 d^2 + 2 \sqrt{d^2} \sqrt{-e^2 x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(e\*x+d)/(-e^2\*x^2+d^2)^(5/2), x)

[Out]  $-1/d^3/x/(-e^2*x^2+d^2)^{3/2}+4/3*e^2/d^5*x/(-e^2*x^2+d^2)^{3/2}+8/3*e^2/d^7*x/(-e^2*x^2+d^2)^{1/2}-1/5/d^3/(x+d/e)/(-x+d/e)^2*e^2+2*d*e*(x+d/e)^{3/2}+4/15*e^2/d^5/(-x+d/e)^2*e^2+2*d*e*(x+d/e)^{3/2}*x+8/15*e^2/d^7/(-x+d/e)^2*e^2+2*d*e*(x+d/e)^{1/2}*x-1/3*e/d^4/(-e^2*x^2+d^2)^{3/2}-e/d^6/(-e^2*x^2+d^2)^{1/2}+e/d^6/(d^2)^{1/2}*ln((2*d^2+2*(d^2)^{1/2}*(-e^2*x^2+d^2)^{1/2})/x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-e^2 x^2 + d^2)^{\frac{5}{2}} (e x + d) x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)\*x^2), x, algorithm="maxima")

[Out] integrate(1/((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)\*x^2), x)

**Fricas [A]** time = 0.307297, size = 871, normalized size = 5.66

$$23 e^{10} x^{10} - 217 d e^9 x^9 - 487 d^2 e^8 x^8 + 1073 d^3 e^7 x^7 + 1863 d^4 e^6 x^6 - 1755 d^5 e^5 x^5 - 2655 d^6 e^4 x^4 + 1140 d^7 e^3 x^3 + 1500 d^8 e^2 x^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)\*x^2), x, algorithm="fricas")

[Out]  $-1/15*(23*e^{10}*x^{10} - 217*d*e^9*x^9 - 487*d^2*e^8*x^8 + 1073*d^3*e^7*x^7 + 1863*d^4*e^6*x^6 - 1755*d^5*e^5*x^5 - 2655*d^6*e^4*x^4 + 1140*d^7*e^3*x^3 + 1500*d^8*e^2*x^2 - 240*d^9*e*x - 240*d^{10} + 15*(e^{10}*x^{10} + d*e^9*x^9 - 14*d^2*e^8*x^8 - 14*d^3*e^7*x^7 + 41*d^4*e^6*x^6 + 41*d^5*e^5*x^5 - 44*d^6*e^4*x^4 - 44*d^7*e^3*x^3 + 16*d^8*e^2*x^2 + 16*d^9*e*x + (5*d*e^8*x^8 + 5*d^2*e^7*x^7 - 25*d^3*e^6*x^6 - 25*d^4*e^5*x^5 + 36*d^5*e^4*x^4 + 36*d^6*e^3*x^3 - 16*d^7*e^2*x^2 - 16*d^8*e*x)*sqrt(-e^2*x^2 + d^2))*log(-(d - sqrt($

$$\begin{aligned}
& -e^2x^2 + d^2)) / x) + (48e^9x^9 + 148d^8e^8x^8 - 548d^2e^7x^7 \\
& - 1023d^3e^6x^6 + 1275d^4e^5x^5 + 1995d^5e^4x^4 - 1020d^6e^3x^3 \\
& - 1380d^7e^2x^2 + 240d^8e^1x + 240d^9) \sqrt{-e^2x^2 + d^2}) / (d^7e^9x^{10} + d^8e^8x^9 - 14d^9e^7x^8 - 14d^{10}e^6x^7 \\
& + 41d^{11}e^5x^6 + 41d^{12}e^4x^5 - 44d^{13}e^3x^4 - 44d^{14}e^2x^3 + 16d^{15}e^1x^2 + 16d^{16}x + (5d^8e^7x^8 + 5d^9e^6x^7 \\
& - 25d^{10}e^5x^6 - 25d^{11}e^4x^5 + 36d^{12}e^3x^4 + 36d^{13}e^2x^3 - 16d^{14}e^1x^2 - 16d^{15}x) \sqrt{-e^2x^2 + d^2})
\end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (-(-d + ex)(d + ex))^{\frac{5}{2}} (d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2), x)

[Out] Integral(1/(x\*\*2\*(-(-d + e\*x)\*(d + e\*x))\*\*(5/2)\*(d + e\*x)), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)\*x^2), x, algorithm="giac")

[Out] [undef, undef, undef, undef, undef, 1]

$$3.146 \quad \int \frac{1}{x^3(d+ex)(d^2-e^2x^2)^{5/2}} dx$$

**Optimal.** Leaf size=186

$$\frac{1}{5d^2x^2(d+ex)(d^2-e^2x^2)^{3/2}} + \frac{16e\sqrt{d^2-e^2x^2}}{5d^8x} - \frac{7e^2 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^8}$$

$$- \frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} + \frac{7d-6ex}{15d^4x^2(d^2-e^2x^2)^{3/2}}$$

[Out] (7\*d - 6\*e\*x)/(15\*d^4\*x^2\*(d^2 - e^2\*x^2)^(3/2)) + 1/(5\*d^2\*x^2\*(d + e\*x)\*(d^2 - e^2\*x^2)^(3/2)) + (35\*d - 24\*e\*x)/(15\*d^6\*x^2\*Sqrt[d^2 - e^2\*x^2]) - (7\*Sqrt[d^2 - e^2\*x^2])/(2\*d^7\*x^2) + (16\*e\*Sqrt[d^2 - e^2\*x^2])/(5\*d^8\*x) - (7\*e^2\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/(2\*d^8)

**Rubi [A]** time = 0.524456, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\frac{1}{5d^2x^2(d+ex)(d^2-e^2x^2)^{3/2}} + \frac{16e\sqrt{d^2-e^2x^2}}{5d^8x} - \frac{7e^2 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^8}$$

$$- \frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}} + \frac{7d-6ex}{15d^4x^2(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*(d + e\*x)\*(d^2 - e^2\*x^2)^(5/2)), x]

[Out] (7\*d - 6\*e\*x)/(15\*d^4\*x^2\*(d^2 - e^2\*x^2)^(3/2)) + 1/(5\*d^2\*x^2\*(d + e\*x)\*(d^2 - e^2\*x^2)^(3/2)) + (35\*d - 24\*e\*x)/(15\*d^6\*x^2\*Sqrt[d^2 - e^2\*x^2]) - (7\*Sqrt[d^2 - e^2\*x^2])/(2\*d^7\*x^2) + (16\*e\*Sqrt[d^2 - e^2\*x^2])/(5\*d^8\*x) - (7\*e^2\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/(2\*d^8)

**Rubi in Sympy [A]** time = 65.1641, size = 162, normalized size = 0.87

$$\frac{d-ex}{5d^2x^2(d^2-e^2x^2)^{5/2}} + \frac{7d-6ex}{15d^4x^2(d^2-e^2x^2)^{3/2}} + \frac{35d-24ex}{15d^6x^2\sqrt{d^2-e^2x^2}}$$

$$- \frac{7\sqrt{d^2-e^2x^2}}{2d^7x^2} - \frac{7e^2 \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^8} + \frac{16e\sqrt{d^2-e^2x^2}}{5d^8x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*3/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2), x)

[Out] (d - e\*x)/(5\*d\*\*2\*x\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)) + (7\*d - 6\*e\*x)/(15\*d\*\*4\*x\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)) + (35\*d - 24\*e\*x)/(15\*d\*\*6\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) - 7\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(2\*d\*\*7\*x\*\*2) - 7\*e\*\*2\*atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d)/(2\*d\*\*8) + 16\*e\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(5\*d\*\*8\*x)

**Mathematica [A]** time = 0.151049, size = 137, normalized size = 0.74

$$-105e^2 \log\left(\sqrt{d^2 - e^2x^2} + d\right) + \frac{\sqrt{d^2 - e^2x^2}(-15d^6 + 15d^5ex + 176d^4e^2x^2 - 4d^3e^3x^3 - 249d^2e^4x^4 - 9de^5x^5 + 96e^6x^6)}{x^2(d-ex)^2(d+ex)^3} + 105e^2 \log(x)$$

$$30d^8$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(d + e\*x)\*(d^2 - e^2\*x^2)^(5/2)),x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(-15\*d^6 + 15\*d^5\*e\*x + 176\*d^4\*e^2\*x^2 - 4\*d^3\*e^3\*x^3 - 249\*d^2\*e^4\*x^4 - 9\*d\*e^5\*x^5 + 96\*e^6\*x^6))/(x^2\*(d - e\*x)^2\*(d + e\*x)^3) + 105\*e^2\*Log[x] - 105\*e^2\*Log[d + Sqrt[d^2 - e^2\*x^2]])/(30\*d^8)

**Maple [A]** time = 0.023, size = 298, normalized size = 1.6

$$\begin{aligned} & -\frac{1}{2d^3x^2}(-e^2x^2 + d^2)^{-\frac{3}{2}} + \frac{7e^2}{6d^5}(-e^2x^2 + d^2)^{-\frac{3}{2}} + \frac{7e^2}{2d^7} \frac{1}{\sqrt{-e^2x^2 + d^2}} \\ & - \frac{7e^2}{2d^7} \ln\left(\frac{1}{x}\left(2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}} \\ & + \frac{e}{5d^4}\left(x + \frac{d}{e}\right)^{-1} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{-\frac{3}{2}} \\ & - \frac{4e^3x}{15d^6} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{-\frac{3}{2}} - \frac{8e^3x}{15d^8} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)}} \\ & + \frac{e}{d^4x}(-e^2x^2 + d^2)^{-\frac{3}{2}} - \frac{4e^3x}{3d^6}(-e^2x^2 + d^2)^{-\frac{3}{2}} - \frac{8e^3x}{3d^8} \frac{1}{\sqrt{-e^2x^2 + d^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(e\*x+d)/(-e^2\*x^2+d^2)^(5/2),x)

[Out] -1/2/d^3/x^2/(-e^2\*x^2+d^2)^(3/2)+7/6\*e^2/d^5/(-e^2\*x^2+d^2)^(3/2)+7/2\*e^2/d^7/(-e^2\*x^2+d^2)^(1/2)-7/2\*e^2/d^7/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)+1/5/d^4\*e/(x+d/e)/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)-4/15/d^6\*e^3/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)\*x-8/15/d^8\*e^3/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x+d^4/x/(-e^2\*x^2+d^2)^(3/2)-4/3\*e^3/d^6\*x/(-e^2\*x^2+d^2)^(3/2)-8/3\*e^3/d^8\*x/(-e^2\*x^2+d^2)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-e^2x^2 + d^2)^{\frac{5}{2}}(ex + d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)\*x^3),x, algorithm="maxima")

[Out] integrate(1/((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)\*x^3), x)

**Fricas [A]** time = 0.330299, size = 1010, normalized size = 5.43

$$96e^{12}x^{12} + 687de^{11}x^{11} - 1281d^2e^{10}x^{10} - 4946d^3e^9x^9 + 4162d^4e^8x^8 + 11487d^5e^7x^7 - 6375d^6e^6x^6 - 11310d^7e^5x^5 + 5550d^8e^4x^4 - 11310d^9e^3x^3 + 5550d^{10}e^2x^2 - 11310d^{11}ex + 5550d^{12}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)\*x^3),x, algorithm="fricas")

[Out] 1/30\*(96\*e^12\*x^12 + 687\*d\*e^11\*x^11 - 1281\*d^2\*e^10\*x^10 - 4946\*d^3\*e^9\*x^9 + 4162\*d^4\*e^8\*x^8 + 11487\*d^5\*e^7\*x^7 - 6375\*d^6\*e^6\*x^6 - 11310\*d^7\*e^5\*x^5 + 5550\*d^8\*e^4\*x^4 + 4560\*d^9\*e^3\*x^3 - 2640\*d^10\*e^2\*x^2 - 480\*d^11\*e\*x + 480\*d^12 + 105\*(6\*d\*e^11\*x^11 + 6\*d^2\*e^10\*x^10 - 44\*d^3\*e^9\*x^9 - 44\*d^4\*e^8\*x^8 + 102\*d^5\*e^7\*x^7 + 102\*d^6\*e^6\*x^6 - 96\*d^7\*e^5\*x^5 - 96\*d^8\*e^4\*x^4 + 32\*d^9\*e^3\*x^3 + 32\*d^10\*e^2\*x^2 - (e^11\*x^11 + d\*e^10\*x^10 - 19\*d^2\*e^9\*x^9 - 19\*d^3\*e^8\*x^8 + 66\*d^4\*e^7\*x^7 + 66\*d^5\*e^6\*x^6 - 80\*d^6\*e^5\*x^5 - 80\*d^7\*e^4\*x^4 + 32\*d^8\*e^3\*x^3 + 32\*d^9\*e^2\*x^2)\*sqrt(-e^2\*x^2 + d^2))\*log(-(d - sqrt(-e^2\*x^2 + d^2))/x) - 2\*(58\*e^11\*x^11 - 230\*d\*e^10\*x^10 - 1075\*d^2\*e^9\*x^9 + 1181\*d^3\*e^8\*x^8 + 3696\*d^4\*e^7\*x^7 - 2220\*d^5\*e^6\*x^6 - 4605\*d^6\*e^5\*x^5 + 2205\*d^7\*e^4\*x^4 + 2160\*d^8\*e^3\*x^3 - 1200\*d^9\*e^2\*x^2 - 240\*d^10\*e\*x + 240\*d^11)\*sqrt(-e^2\*x^2 + d^2))/(6\*d^9\*e^9\*x^11 + 6\*d^10\*e^8\*x^10 - 44\*d^11\*e^7\*x^9 - 44\*d^12\*e^6\*x^8 + 102\*d^13\*e^5\*x^7 + 102\*d^14\*e^4\*x^6 - 96\*d^15\*e^3\*x^5 - 96\*d^16\*e^2\*x^4 + 32\*d^17\*e\*x^3 + 32\*d^18\*x^2 - (d^8\*e^9\*x^11 + d^9\*e^8\*x^10 - 19\*d^10\*e^7\*x^9 - 19\*d^11\*e^6\*x^8 + 66\*d^12\*e^5\*x^7 + 66\*d^13\*e^4\*x^6 - 80\*d^14\*e^3\*x^5 - 80\*d^15\*e^2\*x^4 + 32\*d^16\*e\*x^3 + 32\*d^17\*x^2)\*sqrt(-e^2\*x^2 + d^2))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (-(-d + ex)(d + ex))^{\frac{5}{2}} (d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2),x)

[Out] Integral(1/(x\*\*3\*(-(-d + e\*x)\*(d + e\*x))\*\*(5/2)\*(d + e\*x)), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)\*x^3),x, algorithm="giac")

[Out] [undef, undef, undef, undef, undef, 1]

$$3.147 \quad \int \frac{1}{x^4(d+ex)(d^2-e^2x^2)^{5/2}} dx$$

**Optimal.** Leaf size=215

$$\frac{1}{5d^2x^3(d+ex)(d^2-e^2x^2)^{3/2}} - \frac{128e^2\sqrt{d^2-e^2x^2}}{15d^9x} + \frac{7e^3 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^9}$$

$$+ \frac{7e\sqrt{d^2-e^2x^2}}{2d^8x^2} - \frac{64\sqrt{d^2-e^2x^2}}{15d^7x^3} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} + \frac{8d-7ex}{15d^4x^3(d^2-e^2x^2)^{3/2}}$$

[Out]  $(8*d - 7*e*x)/(15*d^4*x^3*(d^2 - e^2*x^2)^{(3/2)}) + 1/(5*d^2*x^3*(d + e*x)*(d^2 - e^2*x^2)^{(3/2)}) + (48*d - 35*e*x)/(15*d^6*x^3*\text{Sqrt}[d^2 - e^2*x^2]) - (64*\text{Sqrt}[d^2 - e^2*x^2])/(15*d^7*x^3) + (7*e*\text{Sqrt}[d^2 - e^2*x^2])/(2*d^8*x^2) - (128*e^2*\text{Sqrt}[d^2 - e^2*x^2])/(15*d^9*x) + (7*e^3*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/(2*d^9)$

**Rubi [A]** time = 0.640537, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\frac{1}{5d^2x^3(d+ex)(d^2-e^2x^2)^{3/2}} - \frac{128e^2\sqrt{d^2-e^2x^2}}{15d^9x} + \frac{7e^3 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^9}$$

$$+ \frac{7e\sqrt{d^2-e^2x^2}}{2d^8x^2} - \frac{64\sqrt{d^2-e^2x^2}}{15d^7x^3} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} + \frac{8d-7ex}{15d^4x^3(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^4*(d + e*x)*(d^2 - e^2*x^2)^{(5/2)}), x]$

[Out]  $(8*d - 7*e*x)/(15*d^4*x^3*(d^2 - e^2*x^2)^{(3/2)}) + 1/(5*d^2*x^3*(d + e*x)*(d^2 - e^2*x^2)^{(3/2)}) + (48*d - 35*e*x)/(15*d^6*x^3*\text{Sqrt}[d^2 - e^2*x^2]) - (64*\text{Sqrt}[d^2 - e^2*x^2])/(15*d^7*x^3) + (7*e*\text{Sqrt}[d^2 - e^2*x^2])/(2*d^8*x^2) - (128*e^2*\text{Sqrt}[d^2 - e^2*x^2])/(15*d^9*x) + (7*e^3*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/(2*d^9)$

**Rubi in Sympy [A]** time = 79.0883, size = 189, normalized size = 0.88

$$\frac{d-ex}{5d^2x^3(d^2-e^2x^2)^{5/2}} + \frac{8d-7ex}{15d^4x^3(d^2-e^2x^2)^{3/2}} + \frac{48d-35ex}{15d^6x^3\sqrt{d^2-e^2x^2}} - \frac{64\sqrt{d^2-e^2x^2}}{15d^7x^3}$$

$$+ \frac{7e\sqrt{d^2-e^2x^2}}{2d^8x^2} + \frac{7e^3 \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^9} - \frac{128e^2\sqrt{d^2-e^2x^2}}{15d^9x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x^{**4}/(e*x+d)/(-e^{**2}*x^{**2}+d^{**2})^{**}(5/2), x)$

[Out]  $(d - e*x)/(5*d^{**2}*x^{**3}*(d^{**2} - e^{**2}*x^{**2})^{**}(5/2)) + (8*d - 7*e*x)/(15*d^{**4}*x^{**3}*(d^{**2} - e^{**2}*x^{**2})^{**}(3/2)) + (48*d - 35*e*x)/(15*d^{**6}*x^{**3}*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})) - 64*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(15*d^{**7}*x^{**3}) + 7*e*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(2*d^{**8}*x^{**2}) + 7*e^{**3}*\text{atanh}(\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/d)/(2*d^{**9}) - 128*e^{**2}*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(15*d^{**9}*x)$

**Mathematica [A]** time = 0.181395, size = 148, normalized size = 0.69

$$\frac{-105e^3 \log\left(\sqrt{d^2 - e^2x^2} + d\right) + \frac{\sqrt{d^2 - e^2x^2}(10d^7 - 5d^6ex + 75d^5e^2x^2 + 236d^4e^3x^3 - 244d^3e^4x^4 - 489d^2e^5x^5 + 151de^6x^6 + 256e^7x^7)}{x^3(d-ex)^2(d+ex)^3}}{30d^9} + 105e^3 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^4\*(d + e\*x)\*(d^2 - e^2\*x^2)^(5/2)), x]

[Out] -((Sqrt[d^2 - e^2\*x^2]\*(10\*d^7 - 5\*d^6\*e\*x + 75\*d^5\*e^2\*x^2 + 236\*d^4\*e^3\*x^3 - 244\*d^3\*e^4\*x^4 - 489\*d^2\*e^5\*x^5 + 151\*d\*e^6\*x^6 + 256\*e^7\*x^7))/(x^3\*(d - e\*x)^2\*(d + e\*x)^3) + 105\*e^3\*Log[x] - 105\*e^3\*Log[d + Sqrt[d^2 - e^2\*x^2]])/(30\*d^9)

**Maple [A]** time = 0.025, size = 326, normalized size = 1.5

$$\begin{aligned} & -\frac{1}{3d^3x^3}(-e^2x^2 + d^2)^{-\frac{3}{2}} - 3\frac{e^2}{d^5x(-e^2x^2 + d^2)^{3/2}} + 4\frac{e^4x}{d^7(-e^2x^2 + d^2)^{3/2}} + 8\frac{e^4x}{d^9\sqrt{-e^2x^2 + d^2}} \\ & -\frac{e^2}{5d^5}\left(x + \frac{d}{e}\right)^{-1}\left(-\left(x + \frac{d}{e}\right)^2e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{-\frac{3}{2}} + \frac{4e^4x}{15d^7}\left(-\left(x + \frac{d}{e}\right)^2e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{-\frac{3}{2}} \\ & + \frac{8e^4x}{15d^9}\frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2e^2 + 2de\left(x + \frac{d}{e}\right)}} - \frac{7e^3}{6d^6}(-e^2x^2 + d^2)^{-\frac{3}{2}} - \frac{7e^3}{2d^8}\frac{1}{\sqrt{-e^2x^2 + d^2}} \\ & + \frac{7e^3}{2d^8}\ln\left(\frac{1}{x}\left(2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2}\right)\right)\frac{1}{\sqrt{d^2}} + \frac{e}{2d^4x^2}(-e^2x^2 + d^2)^{-\frac{3}{2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^4/(e\*x+d)/(-e^2\*x^2+d^2)^(5/2), x)

[Out] -1/3/d^3/x^3/(-e^2\*x^2+d^2)^(3/2)-3/d^5\*e^2/x/(-e^2\*x^2+d^2)^(3/2)+4/d^7\*e^4\*x/(-e^2\*x^2+d^2)^(3/2)+8/d^9\*e^4\*x/(-e^2\*x^2+d^2)^(1/2)-1/5/d^5\*e^2/(x+d/e)/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)+4/15/d^7\*e^4/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)\*x+8/15/d^9\*e^4/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x-7/6/d^6\*e^3/(-e^2\*x^2+d^2)^(3/2)-7/2/d^8\*e^3/(-e^2\*x^2+d^2)^(1/2)+7/2/d^8\*e^3/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)+1/2\*e/d^4/x^2/(-e^2\*x^2+d^2)^(3/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-e^2x^2 + d^2)^{\frac{5}{2}}(ex + d)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)\*x^4), x, algorithm="maxima")

[Out] integrate(1/((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)\*x^4), x)

**Fricas [A]** time = 0.329672, size = 1125, normalized size = 5.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x^4),x, algorithm="fricas")`

[Out] 
$$-1/30*(116*e^{14}*x^{14} - 1676*d*e^{13}*x^{13} - 4073*d^2*e^{12}*x^{12} + 14743*d^3*e^{11}*x^{11} + 25128*d^4*e^{10}*x^{10} - 42744*d^5*e^9*x^9 - 59869*d^6*e^8*x^8 + 55635*d^7*e^7*x^7 + 65250*d^8*e^6*x^6 - 33880*d^9*e^5*x^5 - 30880*d^{10}*e^4*x^4 + 8240*d^{11}*e^3*x^3 + 3680*d^{12}*e^2*x^2 - 320*d^{13}*e*x + 640*d^{14} + 105*(e^{14}*x^{14} + d*e^{13}*x^{13} - 26*d^2*e^{12}*x^{12} - 26*d^3*e^{11}*x^{11} + 129*d^4*e^{10}*x^{10} + 129*d^5*e^9*x^9 - 248*d^6*e^8*x^8 - 248*d^7*e^7*x^7 + 208*d^8*e^6*x^6 + 208*d^9*e^5*x^5 - 64*d^{10}*e^4*x^4 - 64*d^{11}*e^3*x^3 + (7*d*e^{12}*x^{12} + 7*d^2*e^{11}*x^{11} - 63*d^3*e^{10}*x^{10} - 63*d^4*e^9*x^9 + 168*d^5*e^8*x^8 + 168*d^6*e^7*x^7 - 176*d^7*e^6*x^6 - 176*d^8*e^5*x^5 + 64*d^9*e^4*x^4 + 64*d^{10}*e^3*x^3)*\sqrt{-e^2*x^2 + d^2})*\log(-(d - \sqrt{-e^2*x^2 + d^2})/x) + (256*e^{13}*x^{13} + 963*d*e^{12}*x^{12} - 5821*d^2*e^{11}*x^{11} - 11176*d^3*e^{10}*x^{10} + 25144*d^4*e^9*x^9 + 37499*d^5*e^8*x^8 - 41685*d^6*e^7*x^7 - 51390*d^7*e^6*x^6 + 29880*d^8*e^5*x^5 + 28800*d^9*e^4*x^4 - 8080*d^{10}*e^3*x^3 - 4000*d^{11}*e^2*x^2 + 320*d^{12}*e*x - 640*d^{13})*\sqrt{-e^2*x^2 + d^2})/(d^9*e^{11}*x^{14} + d^{10}*e^{10}*x^{13} - 26*d^{11}*e^9*x^{12} - 26*d^{12}*e^8*x^{11} + 129*d^{13}*e^7*x^{10} + 129*d^{14}*e^6*x^9 - 248*d^{15}*e^5*x^8 - 248*d^{16}*e^4*x^7 + 208*d^{17}*e^3*x^6 + 208*d^{18}*e^2*x^5 - 64*d^{19}*e*x^4 - 64*d^{20}*x^3 + (7*d^{10}*e^9*x^{12} + 7*d^{11}*e^8*x^{11} - 63*d^{12}*e^7*x^{10} - 63*d^{13}*e^6*x^9 + 168*d^{14}*e^5*x^8 + 168*d^{15}*e^4*x^7 - 176*d^{16}*e^3*x^6 - 176*d^{17}*e^2*x^5 + 64*d^{18}*e*x^4 + 64*d^{19}*x^3)*\sqrt{-e^2*x^2 + d^2})$$

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**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^4 (-(-d + ex)(d + ex))^{\frac{5}{2}} (d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(e*x+d)/((-e**2*x**2+d**2)**(5/2),x)`

[Out] `Integral(1/(x**4*(-(-d + e*x)*(d + e*x))**(5/2)*(d + e*x)), x)`

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

`[undef, undef, undef, undef, undef, 1]`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-e^2*x^2 + d^2)^(5/2)*(e*x + d)*x^4),x, algorithm="giac")`

[Out] `[undef, undef, undef, undef, undef, 1]`



$$3.148 \quad \int \frac{x^3}{(d+ex)(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=118

$$\frac{x^2(d-ex)}{7e^2(d^2-e^2x^2)^{7/2}} - \frac{2d-3ex}{35e^4(d^2-e^2x^2)^{5/2}} - \frac{x}{35d^2e^3(d^2-e^2x^2)^{3/2}} - \frac{2x}{35d^4e^3\sqrt{d^2-e^2x^2}}$$

[Out]  $(x^2*(d - e*x))/(7*e^2*(d^2 - e^2*x^2)^(7/2)) - (2*d - 3*e*x)/(35*e^4*(d^2 - e^2*x^2)^(5/2)) - x/(35*d^2*e^3*(d^2 - e^2*x^2)^(3/2)) - (2*x)/(35*d^4*e^3*sqrt[d^2 - e^2*x^2])$

**Rubi [A]** time = 0.268329, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\frac{x^2(d-ex)}{7e^2(d^2-e^2x^2)^{7/2}} - \frac{2d-3ex}{35e^4(d^2-e^2x^2)^{5/2}} - \frac{x}{35d^2e^3(d^2-e^2x^2)^{3/2}} - \frac{2x}{35d^4e^3\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[x^3/((d + e\*x)\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out]  $(x^2*(d - e*x))/(7*e^2*(d^2 - e^2*x^2)^(7/2)) - (2*d - 3*e*x)/(35*e^4*(d^2 - e^2*x^2)^(5/2)) - x/(35*d^2*e^3*(d^2 - e^2*x^2)^(3/2)) - (2*x)/(35*d^4*e^3*sqrt[d^2 - e^2*x^2])$

**Rubi in Sympy [A]** time = 26.1506, size = 116, normalized size = 0.98

$$-\frac{x^3(d-ex)}{7de(d^2-e^2x^2)^{7/2}} + \frac{x^2(3d+3ex)}{35d^2e^2(d^2-e^2x^2)^{5/2}} - \frac{6d-6ex}{105d^2e^4(d^2-e^2x^2)^{3/2}} - \frac{2x}{35d^4e^3\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out]  $-x**3*(d - e*x)/(7*d*e*(d**2 - e**2*x**2)**(7/2)) + x**2*(3*d + 3*e*x)/(35*d**2*e**2*(d**2 - e**2*x**2)**(5/2)) - (6*d - 6*e*x)/(105*d**2*e**4*(d**2 - e**2*x**2)**(3/2)) - 2*x/(35*d**4*e**3*sqrt(d**2 - e**2*x**2))$

**Mathematica [A]** time = 0.0834439, size = 104, normalized size = 0.88

$$\frac{\sqrt{d^2 - e^2x^2} (2d^6 + 2d^5ex - 5d^4e^2x^2 - 5d^3e^3x^3 - 5d^2e^4x^4 + 2de^5x^5 + 2e^6x^6)}{35d^4e^4(d-ex)^3(d+ex)^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((d + e\*x)\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out]  $-(sqrt[d^2 - e^2*x^2]*(2*d^6 + 2*d^5*e*x - 5*d^4*e^2*x^2 - 5*d^3*e^3*x^3 - 5*d^2*e^4*x^4 + 2*d*e^5*x^5 + 2*e^6*x^6))/(35*d^4*e^4*(d - e*x)^3*(d + e*x)^4)$

**Maple [A]** time = 0.012, size = 92, normalized size = 0.8

$$\frac{(-ex + d)(2e^6x^6 + 2e^5x^5d - 5x^4d^2e^4 - 5x^3d^3e^3 - 5x^2d^4e^2 + 2d^5xe + 2d^6)}{35d^4e^4} (-e^2x^2 + d^2)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(e\*x+d)/(-e^2\*x^2+d^2)^(7/2), x)

[Out] -1/35\*(-e\*x+d)\*(2\*e^6\*x^6+2\*d\*e^5\*x^5-5\*d^2\*e^4\*x^4-5\*d^3\*e^3\*x^3-5\*d^4\*e^2\*x^2+2\*d^5\*e\*x+2\*d^6)/d^4/e^4/(-e^2\*x^2+d^2)^(7/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/((-e^2\*x^2 + d^2)^(7/2)\*(e\*x + d)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.341498, size = 605, normalized size = 5.13

$$\frac{2e^8x^{12} - 10de^7x^{11} - 53d^2e^6x^{10} + 59d^3e^5x^9 + 281d^4e^4x^8 - 104d^5e^3x^7 + 35(6d^5e^{11}x^{11} + 6d^6e^{10}x^{10} - 50d^7e^9x^9 - 50d^8e^8x^8 + 146d^9e^7x^7 + 146d^{10}e^6x^6 - 198d^{11}e^5x^5 - 198d^{12}e^4x^4 + 128d^{13}e^3x^3 - 128d^{14}e^2x^2 + 32d^{15}e^1x^1 - 32d^{16})\sqrt{-e^2x^2 + d^2}}{(6d^5e^{11}x^{11} + 6d^6e^{10}x^{10} - 50d^7e^9x^9 - 50d^8e^8x^8 + 146d^9e^7x^7 + 146d^{10}e^6x^6 - 198d^{11}e^5x^5 - 198d^{12}e^4x^4 + 128d^{13}e^3x^3 - 128d^{14}e^2x^2 + 32d^{15}e^1x^1 - 32d^{16})\sqrt{-e^2x^2 + d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/((-e^2\*x^2 + d^2)^(7/2)\*(e\*x + d)), x, algorithm="fricas")

[Out] 1/35\*(2\*e^8\*x^12 - 10\*d\*e^7\*x^11 - 53\*d^2\*e^6\*x^10 + 59\*d^3\*e^5\*x^9 + 281\*d^4\*e^4\*x^8 - 104\*d^5\*e^3\*x^7 - 504\*d^6\*e^2\*x^6 + 56\*d^7\*e\*x^5 + 280\*d^8\*x^4 + 2\*(e^7\*x^11 + 7\*d\*e^6\*x^10 - 14\*d^2\*e^5\*x^9 - 67\*d^3\*e^4\*x^8 + 38\*d^4\*e^3\*x^7 + 182\*d^5\*e^2\*x^6 - 28\*d^6\*e\*x^5 - 140\*d^7\*x^4)\*sqrt(-e^2\*x^2 + d^2))/(6\*d^5\*e^11\*x^11 + 6\*d^6\*e^10\*x^10 - 50\*d^7\*e^9\*x^9 - 50\*d^8\*e^8\*x^8 + 146\*d^9\*e^7\*x^7 + 146\*d^10\*e^6\*x^6 - 198\*d^11\*e^5\*x^5 - 198\*d^12\*e^4\*x^4 + 128\*d^13\*e^3\*x^3 + 128\*d^14\*e^2\*x^2 - 32\*d^15\*e\*x - 32\*d^16 - (d^4\*e^11\*x^11 + d^5\*e^10\*x^10 - 20\*d^6\*e^9\*x^9 - 20\*d^7\*e^8\*x^8 + 85\*d^8\*e^7\*x^7 + 85\*d^9\*e^6\*x^6 - 146\*d^10\*e^5\*x^5 - 146\*d^11\*e^4\*x^4 + 112\*d^12\*e^3\*x^3 + 112\*d^13\*e^2\*x^2 - 32\*d^14\*e\*x - 32\*d^15)\*sqrt(-e^2\*x^2 + d^2))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(-(-d + ex)(d + ex))^{\frac{7}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out] Integral(x\*\*3/((-(-d + e\*x)\*(d + e\*x))\*\*(7/2)\*(d + e\*x)), x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/((-e^2\*x^2 + d^2)^(7/2)\*(e\*x + d)),x, algorithm="giac")

[Out] [undef, undef, undef, undef, undef, undef, undef, 1]

$$3.149 \quad \int \frac{x^2}{(d+ex)(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=123

$$-\frac{x^2}{7de(d+ex)(d^2-e^2x^2)^{5/2}} + \frac{2(d+2ex)}{35de^3(d^2-e^2x^2)^{5/2}} - \frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}} - \frac{4x}{105d^3e^2(d^2-e^2x^2)^{3/2}}$$

[Out]  $-x^2/(7*d*e*(d+e*x)*(d^2-e^2*x^2)^(5/2)) + (2*(d+2*e*x))/(35*d*e^3*(d^2-e^2*x^2)^(5/2)) - (4*x)/(105*d^3*e^2*(d^2-e^2*x^2)^(3/2)) - (8*x)/(105*d^5*e^2*sqrt[d^2-e^2*x^2])$

**Rubi [A]** time = 0.201322, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$-\frac{x^2}{7de(d+ex)(d^2-e^2x^2)^{5/2}} + \frac{2(d+2ex)}{35de^3(d^2-e^2x^2)^{5/2}} - \frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}} - \frac{4x}{105d^3e^2(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/((d + e\*x)\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out]  $-x^2/(7*d*e*(d+e*x)*(d^2-e^2*x^2)^(5/2)) + (2*(d+2*e*x))/(35*d*e^3*(d^2-e^2*x^2)^(5/2)) - (4*x)/(105*d^3*e^2*(d^2-e^2*x^2)^(3/2)) - (8*x)/(105*d^5*e^2*sqrt[d^2-e^2*x^2])$

**Rubi in Sympy [A]** time = 17.1857, size = 105, normalized size = 0.85

$$-\frac{x^2(d-ex)}{7de(d^2-e^2x^2)^{7/2}} + \frac{2d+4ex}{35de^3(d^2-e^2x^2)^{5/2}} - \frac{4x}{105d^3e^2(d^2-e^2x^2)^{3/2}} - \frac{8x}{105d^5e^2\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out]  $-x**2*(d-e*x)/(7*d*e*(d**2-e**2*x**2)**(7/2)) + (2*d+4*e*x)/(35*d*e**3*(d**2-e**2*x**2)**(5/2)) - 4*x/(105*d**3*e**2*(d**2-e**2*x**2)**(3/2)) - 8*x/(105*d**5*e**2*sqrt(d**2-e**2*x**2))$

**Mathematica [A]** time = 0.0699624, size = 104, normalized size = 0.85

$$\frac{\sqrt{d^2-e^2x^2}(6d^6+6d^5ex-15d^4e^2x^2+20d^3e^3x^3+20d^2e^4x^4-8de^5x^5-8e^6x^6)}{105d^5e^3(d-ex)^3(d+ex)^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((d + e\*x)\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out]  $(sqrt[d^2-e^2*x^2]*(6*d^6+6*d^5*e*x-15*d^4*e^2*x^2+20*d^3*e^3*x^3+20*d^2*e^4*x^4-8*d*e^5*x^5-8*e^6*x^6))/(105*d^5*e^3*(d-e*x)^3*(d+e*x)^4)$

**Maple [A]** time = 0.012, size = 92, normalized size = 0.8

$$\frac{(-ex + d)(-8e^6x^6 - 8e^5x^5d + 20e^4x^4d^2 + 20x^3d^3e^3 - 15x^2d^4e^2 + 6xd^5e + 6d^6)}{105d^5e^3} (-e^2x^2 + d^2)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(e*x+d)/(-e^2*x^2+d^2)^(7/2), x)`

[Out] `1/105*(-e*x+d)*(-8*e^6*x^6-8*d*e^5*x^5+20*d^2*e^4*x^4+20*d^3*e^3*x^3-15*d^4*e^2*x^2+6*d^5*e*x+6*d^6)/d^5/e^3/(-e^2*x^2+d^2)^(7/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.344249, size = 636, normalized size = 5.17

$$\frac{8e^9x^{12} + 44de^8x^{11} - 128d^2e^7x^{10} - 464d^3e^6x^9 + 459d^4e^5x^8 + 1614d^5e^4x^7 - 616d^6e^3x^6 - 105(6d^6e^{11}x^{11} + 6d^7e^{10}x^{10} - 50d^8e^9x^9 - 50d^9e^8x^8 + 146d^{10}e^7x^7 + 146d^{11}e^6x^6 - 198d^{12}e^5x^5 - 198d^{13}e^4x^4 + 128d^{14}e^3x^3 - 128d^{15}e^2x^2 - 32d^{16}e^1x - 32d^{17})\sqrt{-e^2x^2 + d^2}}{(6d^6e^{11}x^{11} + 6d^7e^{10}x^{10} - 50d^8e^9x^9 - 50d^9e^8x^8 + 146d^{10}e^7x^7 + 146d^{11}e^6x^6 - 198d^{12}e^5x^5 - 198d^{13}e^4x^4 + 128d^{14}e^3x^3 - 128d^{15}e^2x^2 - 32d^{16}e^1x - 32d^{17})\sqrt{-e^2x^2 + d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)), x, algorithm="fricas")`

[Out] `1/105*(8*e^9*x^12 + 44*d*e^8*x^11 - 128*d^2*e^7*x^10 - 464*d^3*e^6*x^9 + 459*d^4*e^5*x^8 + 1614*d^5*e^4*x^7 - 616*d^6*e^3*x^6 - 22*96*d^7*e^2*x^5 + 280*d^8*e*x^4 + 1120*d^9*x^3 - 2*(3*e^8*x^11 - 2*1*d*e^7*x^10 - 84*d^2*e^6*x^9 + 128*d^3*e^5*x^8 + 443*d^4*e^4*x^7 - 238*d^5*e^3*x^6 - 868*d^6*e^2*x^5 + 140*d^7*e*x^4 + 560*d^8*x^3)*sqrt(-e^2*x^2 + d^2))/(6*d^6*e^11*x^11 + 6*d^7*e^10*x^10 - 50*d^8*e^9*x^9 - 50*d^9*e^8*x^8 + 146*d^10*e^7*x^7 + 146*d^11*e^6*x^6 - 198*d^12*e^5*x^5 - 198*d^13*e^4*x^4 + 128*d^14*e^3*x^3 + 128*d^15*e^2*x^2 - 32*d^16*e*x - 32*d^17 - (d^5*e^11*x^11 + d^6*e^10*x^10 - 20*d^7*e^9*x^9 - 20*d^8*e^8*x^8 + 85*d^9*e^7*x^7 + 85*d^10*e^6*x^6 - 146*d^11*e^5*x^5 - 146*d^12*e^4*x^4 + 112*d^13*e^3*x^3 + 112*d^14*e^2*x^2 - 32*d^15*e*x - 32*d^16)*sqrt(-e^2*x^2 + d^2))`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(-(-d + ex)(d + ex))^{\frac{7}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(e*x+d)/(-e**2*x**2+d**2)**(7/2), x)`

[Out]  $\text{Integral}(x^{**2}/((-(-d + e*x)*(d + e*x))^{** (7/2)}*(d + e*x)), x)$

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

[*undef, undef, undef, undef, undef, undef, undef, 1*]

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)),x, algorithm="giac")`

[Out] [*undef, undef, undef, undef, undef, undef, undef, 1*]

$$3.150 \quad \int \frac{x^3}{(1+ax)\sqrt{1-a^2x^2}} dx$$

**Optimal.** Leaf size=66

$$\frac{3 \sin^{-1}(ax)}{2a^4} + \frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}} + \frac{(4-3ax)\sqrt{1-a^2x^2}}{2a^4}$$

[Out] (x^2\*(1 - a\*x))/(a^2\*Sqrt[1 - a^2\*x^2]) + ((4 - 3\*a\*x)\*Sqrt[1 - a^2\*x^2])/(2\*a^4) + (3\*ArcSin[a\*x])/(2\*a^4)

**Rubi [A]** time = 0.222484, antiderivative size = 66, normalized size of antiderivative = 1., number of rules used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{3 \sin^{-1}(ax)}{2a^4} + \frac{x^2(1-ax)}{a^2\sqrt{1-a^2x^2}} + \frac{(4-3ax)\sqrt{1-a^2x^2}}{2a^4}$$

Antiderivative was successfully verified.

[In] Int[x^3/((1 + a\*x)\*Sqrt[1 - a^2\*x^2]), x]

[Out] (x^2\*(1 - a\*x))/(a^2\*Sqrt[1 - a^2\*x^2]) + ((4 - 3\*a\*x)\*Sqrt[1 - a^2\*x^2])/(2\*a^4) + (3\*ArcSin[a\*x])/(2\*a^4)

**Rubi in Sympy [A]** time = 17.9313, size = 65, normalized size = 0.98

$$-\frac{x\sqrt{-a^2x^2+1}}{2a^3} + \frac{\sqrt{-a^2x^2+1}}{a^4} + \frac{3 \operatorname{asin}(ax)}{2a^4} + \frac{\sqrt{-a^2x^2+1}}{a^4(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3/(a\*x+1)/(-a\*\*2\*x\*\*2+1)\*\*(1/2), x)

[Out] -x\*sqrt(-a\*\*2\*x\*\*2 + 1)/(2\*a\*\*3) + sqrt(-a\*\*2\*x\*\*2 + 1)/a\*\*4 + 3\*asin(a\*x)/(2\*a\*\*4) + sqrt(-a\*\*2\*x\*\*2 + 1)/(a\*\*4\*(a\*x + 1))

**Mathematica [A]** time = 0.0909491, size = 44, normalized size = 0.67

$$\frac{\sqrt{1-a^2x^2}\left(-ax + \frac{2}{ax+1} + 2\right) + 3 \sin^{-1}(ax)}{2a^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((1 + a\*x)\*Sqrt[1 - a^2\*x^2]), x]

[Out] (Sqrt[1 - a^2\*x^2]\*(2 - a\*x + 2/(1 + a\*x)) + 3\*ArcSin[a\*x])/(2\*a^4)

**Maple [A]** time = 0.018, size = 100, normalized size = 1.5

$$\frac{3}{2a^3} \arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right) - \frac{1}{\sqrt{a^2}} - \frac{x}{2a^3}\sqrt{-a^2x^2+1} + \frac{1}{a^4}\sqrt{-a^2x^2+1} + \frac{1}{a^5(x+a^{-1})}\sqrt{-(x+a^{-1})^2a^2+2(x+a^{-1})a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(a*x+1)/(-a^2*x^2+1)^(1/2),x)`

[Out]  $\frac{3}{2} \frac{1}{a^3} \frac{1}{(a^2)^{1/2}} \arctan\left(\frac{(a^2)^{1/2} x}{(-a^2 x^2 + 1)^{1/2}}\right) - \frac{1}{2} \frac{1}{a^3 x} \frac{1}{(-a^2 x^2 + 1)^{1/2}} + \frac{1}{a^4} \frac{1}{(-a^2 x^2 + 1)^{1/2}} + \frac{1}{a^5} \frac{1}{(x + 1/a)} \frac{1}{(-a^2 x^2 + 1)^{1/2}} - \frac{1}{2} \frac{1}{a^2} \frac{1}{(x + 1/a)} \frac{1}{(-a^2 x^2 + 1)^{1/2}}$

**Maxima [A]** time = 0.790013, size = 92, normalized size = 1.39

$$\frac{\sqrt{-a^2x^2+1}}{a^5x+a^4} - \frac{\sqrt{-a^2x^2+1}x}{2a^3} + \frac{3 \arcsin(ax)}{2a^4} + \frac{\sqrt{-a^2x^2+1}}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(sqrt(-a^2*x^2+1)*(a*x+1)),x,algorithm="maxima")`

[Out]  $\frac{\sqrt{-a^2x^2+1}}{(a^5x+a^4)} - \frac{1}{2} \frac{\sqrt{-a^2x^2+1}x}{a^3} + \frac{3}{2} \frac{\arcsin(ax)}{a^4} + \frac{\sqrt{-a^2x^2+1}}{a^4}$

**Fricas [A]** time = 0.292551, size = 286, normalized size = 4.33

$$\frac{a^5x^5 - 4a^4x^4 - 7a^3x^3 + 6a^2x^2 + 12ax + 6 \left( a^3x^3 + 3a^2x^2 - 2ax - (a^2x^2 - 2ax - 4)\sqrt{-a^2x^2+1} - 4 \right) \arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right)}{2 \left( a^7x^3 + 3a^6x^2 - 2a^5x - 4a^4 - (a^6x^2 - 2a^5x - 4a^4)\sqrt{-a^2x^2+1} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(sqrt(-a^2*x^2+1)*(a*x+1)),x,algorithm="fricas")`

[Out]  $-\frac{1}{2} \frac{(a^5x^5 - 4a^4x^4 - 7a^3x^3 + 6a^2x^2 + 12ax + 6(a^3x^3 + 3a^2x^2 - 2ax - (a^2x^2 - 2ax - 4)\sqrt{-a^2x^2+1} - 4)) \arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) + (a^4x^4 + a^3x^3 - 6a^2x^2 - 12ax)\sqrt{-a^2x^2+1}}{(a^7x^3 + 3a^6x^2 - 2a^5x - 4a^4)\sqrt{-a^2x^2+1}}$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{-(ax-1)(ax+1)(ax+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(a*x+1)/(-a**2*x**2+1)**(1/2),x)`

[Out] `Integral(x**3/(sqrt(-(a*x-1)*(a*x+1))*(a*x+1)),x)`

**GIAC/XCAS [A]** time = 0.284264, size = 105, normalized size = 1.59

$$-\frac{1}{2} \sqrt{-a^2x^2+1} \left( \frac{x}{a^3} - \frac{2}{a^4} \right) + \frac{3 \arcsin(ax) \operatorname{sign}(a)}{2a^3|a|} - \frac{2}{a^3 \left( \frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} + 1 \right) |a|}$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(sqrt(-a^2*x^2 + 1)*(a*x + 1)),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-a^2*x^2 + 1)*(x/a^3 - 2/a^4) + 3/2*arcsin(a*x)*sign(a)
/(a^3*abs(a)) - 2/(a^3*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) +
1)*abs(a))
```

$$3.151 \quad \int \frac{x^2}{(1+ax)\sqrt{1-a^2x^2}} dx$$

**Optimal.** Leaf size=55

$$-\frac{\sin^{-1}(ax)}{a^3} - \frac{\sqrt{1-a^2x^2}}{a^3(ax+1)} - \frac{\sqrt{1-a^2x^2}}{a^3}$$

[Out] -(Sqrt[1 - a^2\*x^2]/a^3) - Sqrt[1 - a^2\*x^2]/(a^3\*(1 + a\*x)) - ArcSin[a\*x]/a^3

**Rubi [A]** time = 0.182163, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$-\frac{\sin^{-1}(ax)}{a^3} - \frac{\sqrt{1-a^2x^2}}{a^3(ax+1)} - \frac{\sqrt{1-a^2x^2}}{a^3}$$

Antiderivative was successfully verified.

[In] Int[x^2/((1 + a\*x)\*Sqrt[1 - a^2\*x^2]), x]

[Out] -(Sqrt[1 - a^2\*x^2]/a^3) - Sqrt[1 - a^2\*x^2]/(a^3\*(1 + a\*x)) - ArcSin[a\*x]/a^3

**Rubi in Sympy [A]** time = 13.0625, size = 44, normalized size = 0.8

$$-\frac{\sqrt{-a^2x^2+1}}{a^3} - \frac{\text{asin}(ax)}{a^3} - \frac{\sqrt{-a^2x^2+1}}{a^3(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(a\*x+1)/(-a\*\*2\*x\*\*2+1)\*\*(1/2), x)

[Out] -sqrt(-a\*\*2\*x\*\*2 + 1)/a\*\*3 - asin(a\*x)/a\*\*3 - sqrt(-a\*\*2\*x\*\*2 + 1)/(a\*\*3\*(a\*x + 1))

**Mathematica [A]** time = 0.0814731, size = 37, normalized size = 0.67

$$-\frac{\frac{\sqrt{1-a^2x^2}(ax+2)}{ax+1} + \sin^{-1}(ax)}{a^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((1 + a\*x)\*Sqrt[1 - a^2\*x^2]), x]

[Out] -(((2 + a\*x)\*Sqrt[1 - a^2\*x^2])/(1 + a\*x) + ArcSin[a\*x])/a^3)

**Maple [A]** time = 0.011, size = 84, normalized size = 1.5

$$-\frac{1}{a^4(x+a^{-1})}\sqrt{-(x+a^{-1})^2a^2+2(x+a^{-1})a} - \frac{1}{a^3}\sqrt{-a^2x^2+1} - \frac{1}{a^2}\arctan\left(x\sqrt{a^2}\frac{1}{\sqrt{-a^2x^2+1}}\right)\frac{1}{\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(a*x+1)/(-a^2*x^2+1)^(1/2),x)`

[Out]  $-1/a^4/(x+1/a) * (- (x+1/a)^2 * a^2 + 2 * (x+1/a) * a)^{1/2} - (-a^2 * x^2 + 1)^{1/2} / a^3 - 1/a^2 / (a^2)^{1/2} * \arctan((a^2)^{1/2} * x / (-a^2 * x^2 + 1)^{1/2})$

**Maxima [A]** time = 0.789106, size = 70, normalized size = 1.27

$$-\frac{\sqrt{-a^2x^2+1}}{a^4x+a^3} - \frac{\arcsin(ax)}{a^3} - \frac{\sqrt{-a^2x^2+1}}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(sqrt(-a^2*x^2+1)*(a*x+1)),x,algorithm="maxima")`

[Out]  $-\sqrt{-a^2x^2+1}/(a^4x+a^3) - \arcsin(ax)/a^3 - \sqrt{-a^2x^2+1}/a^3$

**Fricas [A]** time = 0.292976, size = 197, normalized size = 3.58

$$\frac{a^3x^3 + a^2x^2 + 2ax + 2 \left( a^2x^2 - ax + \sqrt{-a^2x^2+1}(ax+2) - 2 \right) \arctan\left(\frac{\sqrt{-a^2x^2+1}-1}{ax}\right) - (a^2x^2 + 2ax)\sqrt{-a^2x^2+1}}{a^5x^2 - a^4x - 2a^3 + (a^4x + 2a^3)\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(sqrt(-a^2*x^2+1)*(a*x+1)),x,algorithm="fricas")`

[Out]  $(a^3x^3 + a^2x^2 + 2ax + 2(a^2x^2 - ax + \sqrt{-a^2x^2+1} - 1)/(ax)) * \arctan((\sqrt{-a^2x^2+1} - 1)/(ax)) - (a^2x^2 + 2ax) * \sqrt{-a^2x^2+1} / (a^5x^2 - a^4x - 2a^3 + (a^4x + 2a^3) * \sqrt{-a^2x^2+1})$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-(ax-1)(ax+1)(ax+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(a*x+1)/(-a**2*x**2+1)**(1/2),x)`

[Out] `Integral(x**2/(sqrt(-(a*x-1)*(a*x+1))*(a*x+1)),x)`

**GIAC/XCAS [A]** time = 0.288345, size = 95, normalized size = 1.73

$$-\frac{\arcsin(ax) \operatorname{sign}(a)}{a^2|a|} - \frac{\sqrt{-a^2x^2+1}}{a^3} + \frac{2}{a^2 \left( \frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} + 1 \right) |a|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(sqrt(-a^2*x^2+1)*(a*x+1)),x,algorithm="giac")`

```
[Out] -arcsin(a*x)*sign(a)/(a^2*abs(a)) - sqrt(-a^2*x^2 + 1)/a^3 + 2/(a  
^2*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))
```

$$3.152 \quad \int \frac{x}{(1+ax)\sqrt{1-a^2x^2}} dx$$

**Optimal.** Leaf size=34

$$\frac{\sqrt{1-a^2x^2}}{a^2(ax+1)} + \frac{\sin^{-1}(ax)}{a^2}$$

[Out] Sqrt[1 - a^2\*x^2]/(a^2\*(1 + a\*x)) + ArcSin[a\*x]/a^2

**Rubi [A]** time = 0.0737862, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{\sqrt{1-a^2x^2}}{a^2(ax+1)} + \frac{\sin^{-1}(ax)}{a^2}$$

Antiderivative was successfully verified.

[In] Int[x/((1 + a\*x)\*Sqrt[1 - a^2\*x^2]), x]

[Out] Sqrt[1 - a^2\*x^2]/(a^2\*(1 + a\*x)) + ArcSin[a\*x]/a^2

**Rubi in Sympy [A]** time = 7.21762, size = 27, normalized size = 0.79

$$\frac{\text{asin}(ax)}{a^2} + \frac{\sqrt{-a^2x^2 + 1}}{a^2(ax + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x/(a\*x+1)/(-a\*\*2\*x\*\*2+1)\*\*(1/2), x)

[Out] asin(a\*x)/a\*\*2 + sqrt(-a\*\*2\*x\*\*2 + 1)/(a\*\*2\*(a\*x + 1))

**Mathematica [A]** time = 0.0370208, size = 31, normalized size = 0.91

$$\frac{\frac{\sqrt{1-a^2x^2}}{ax+1} + \sin^{-1}(ax)}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[x/((1 + a\*x)\*Sqrt[1 - a^2\*x^2]), x]

[Out] (Sqrt[1 - a^2\*x^2]/(1 + a\*x) + ArcSin[a\*x])/a^2

**Maple [A]** time = 0.011, size = 65, normalized size = 1.9

$$\frac{1}{a} \arctan\left(x\sqrt{a^2} \frac{1}{\sqrt{-a^2x^2 + 1}}\right) \frac{1}{\sqrt{a^2}} + \frac{1}{a^3(x + a^{-1})} \sqrt{-(x + a^{-1})^2 a^2 + 2(x + a^{-1})a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(a\*x+1)/(-a^2\*x^2+1)^(1/2), x)

[Out]  $1/a/(a^2)^{(1/2)} * \arctan((a^2)^{(1/2)} * x/(-a^2 * x^2 + 1)^{(1/2)}) + 1/a^3/(x + 1/a) * (-(x+1/a)^2 * a^2 + 2 * (x+1/a) * a)^{(1/2)}$

**Maxima [A]** time = 0.788177, size = 45, normalized size = 1.32

$$\frac{\sqrt{-a^2x^2 + 1}}{a^3x + a^2} + \frac{\arcsin(ax)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(sqrt(-a^2*x^2 + 1)*(a*x + 1)),x, algorithm="maxima")`

[Out] `sqrt(-a^2*x^2 + 1)/(a^3*x + a^2) + arcsin(a*x)/a^2`

**Fricas [A]** time = 0.282774, size = 104, normalized size = 3.06

$$\frac{2 \left( ax - \left( ax - \sqrt{-a^2x^2 + 1} + 1 \right) \arctan \left( \frac{\sqrt{-a^2x^2 + 1} - 1}{ax} \right) \right)}{a^3x - \sqrt{-a^2x^2 + 1}a^2 + a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(sqrt(-a^2*x^2 + 1)*(a*x + 1)),x, algorithm="fricas")`

[Out] `2*(a*x - (a*x - sqrt(-a^2*x^2 + 1) + 1)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)))/(a^3*x - sqrt(-a^2*x^2 + 1)*a^2 + a^2)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-(ax-1)(ax+1)(ax+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(a*x+1)/(-a**2*x**2+1)**(1/2),x)`

[Out] `Integral(x/(sqrt(-(a*x - 1)*(a*x + 1))*(a*x + 1)), x)`

**GIAC/XCAS [A]** time = 0.311539, size = 70, normalized size = 2.06

$$\frac{\arcsin(ax) \operatorname{sign}(a)}{a|a|} - \frac{2}{a \left( \frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} + 1 \right) |a|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(sqrt(-a^2*x^2 + 1)*(a*x + 1)),x, algorithm="giac")`

[Out] `arcsin(a*x)*sign(a)/(a*abs(a)) - 2/(a*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) + 1)*abs(a))`

$$3.153 \quad \int \frac{1}{(1+ax)\sqrt{1-a^2x^2}} dx$$

**Optimal.** Leaf size=26

$$-\frac{\sqrt{1-a^2x^2}}{a(ax+1)}$$

[Out] -(Sqrt[1 - a^2\*x^2]/(a\*(1 + a\*x)))

**Rubi [A]** time = 0.0317436, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$-\frac{\sqrt{1-a^2x^2}}{a(ax+1)}$$

Antiderivative was successfully verified.

[In] Int[1/((1 + a\*x)\*Sqrt[1 - a^2\*x^2]),x]

[Out] -(Sqrt[1 - a^2\*x^2]/(a\*(1 + a\*x)))

**Rubi in Sympy [A]** time = 4.62726, size = 19, normalized size = 0.73

$$-\frac{\sqrt{-a^2x^2+1}}{a(ax+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(a\*x+1)/(-a\*\*2\*x\*\*2+1)\*\*(1/2),x)

[Out] -sqrt(-a\*\*2\*x\*\*2 + 1)/(a\*(a\*x + 1))

**Mathematica [A]** time = 0.0168394, size = 23, normalized size = 0.88

$$\frac{ax-1}{a\sqrt{1-a^2x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 + a\*x)\*Sqrt[1 - a^2\*x^2]),x]

[Out] (-1 + a\*x)/(a\*Sqrt[1 - a^2\*x^2])

**Maple [A]** time = 0.007, size = 22, normalized size = 0.9

$$\frac{ax-1}{a\sqrt{-a^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(a\*x+1)/(-a^2\*x^2+1)^(1/2),x)

[Out]  $(a*x-1)/a/(-a^2*x^2+1)^{(1/2)}$

**Maxima [A]** time = 0.78227, size = 31, normalized size = 1.19

$$-\frac{\sqrt{-a^2x^2+1}}{a^2x+a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-a^2*x^2 + 1)*(a*x + 1)),x, algorithm="maxima")`

[Out]  $-\text{sqrt}(-a^2*x^2 + 1)/(a^2*x + a)$

**Fricas [A]** time = 0.26686, size = 32, normalized size = 1.23

$$-\frac{2x}{ax - \sqrt{-a^2x^2 + 1} + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-a^2*x^2 + 1)*(a*x + 1)),x, algorithm="fricas")`

[Out]  $-2*x/(a*x - \text{sqrt}(-a^2*x^2 + 1) + 1)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(ax-1)(ax+1)(ax+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(a*x+1)/(-a**2*x**2+1)**(1/2),x)`

[Out] `Integral(1/(sqrt(-(a*x - 1)*(a*x + 1))*(a*x + 1)), x)`

**GIAC/XCAS [A]** time = 0.299968, size = 46, normalized size = 1.77

$$\frac{2}{\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} + 1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-a^2*x^2 + 1)*(a*x + 1)),x, algorithm="giac")`

[Out]  $2/(((\text{sqrt}(-a^2*x^2 + 1)*\text{abs}(a) + a)/(a^2*x) + 1)*\text{abs}(a))$



$$3.154 \quad \int \frac{1}{x(1-ax)\sqrt{1-a^2x^2}} dx$$

**Optimal.** Leaf size=41

$$\frac{\sqrt{1-a^2x^2}}{1-ax} - \tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

[Out] Sqrt[1 - a^2\*x^2]/(1 - a\*x) - ArcTanh[Sqrt[1 - a^2\*x^2]]

**Rubi [A]** time = 0.131529, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$

$$\frac{\sqrt{1-a^2x^2}}{1-ax} - \tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(1 - a\*x)\*Sqrt[1 - a^2\*x^2]), x]

[Out] Sqrt[1 - a^2\*x^2]/(1 - a\*x) - ArcTanh[Sqrt[1 - a^2\*x^2]]

**Rubi in Sympy [A]** time = 11.0707, size = 29, normalized size = 0.71

$$\frac{ax+1}{\sqrt{-a^2x^2+1}} - \operatorname{atanh}\left(\sqrt{-a^2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(-a\*x+1)/(-a\*\*2\*x\*\*2+1)\*\*(1/2), x)

[Out] (a\*x + 1)/sqrt(-a\*\*2\*x\*\*2 + 1) - atanh(sqrt(-a\*\*2\*x\*\*2 + 1))

**Mathematica [A]** time = 0.0620213, size = 45, normalized size = 1.1

$$-\frac{\sqrt{1-a^2x^2}}{ax-1} - \log\left(\sqrt{1-a^2x^2}+1\right) + \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(1 - a\*x)\*Sqrt[1 - a^2\*x^2]), x]

[Out] -(Sqrt[1 - a^2\*x^2]/(-1 + a\*x)) + Log[x] - Log[1 + Sqrt[1 - a^2\*x^2]]

**Maple [A]** time = 0.016, size = 58, normalized size = 1.4

$$-\frac{1}{a}\sqrt{-(x-a^{-1})^2a^2-2(x-a^{-1})a(x-a^{-1})^{-1}} - \operatorname{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(-a\*x+1)/(-a^2\*x^2+1)^(1/2), x)

[Out]  $-1/a/(x-1/a) * (-(x-1/a)^2 * a^2 - 2 * (x-1/a) * a)^{1/2} - \operatorname{arctanh}(1/(-a^2 * x^2 + 1)^{1/2})$

**Maxima [A]** time = 0.89859, size = 78, normalized size = 1.9

$$-a \left( \frac{\log\left(\frac{2\sqrt{-a^2x^2+1}}{|x|} + \frac{2}{|x|}\right)}{a} + \frac{\sqrt{-a^2x^2+1}}{a^2x-a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/(sqrt(-a^2*x^2 + 1)*(a*x - 1)*x), x, algorithm="maxima")`

[Out]  $-a * (\log(2 * \sqrt{-a^2 * x^2 + 1} / \operatorname{abs}(x) + 2 / \operatorname{abs}(x)) / a + \sqrt{-a^2 * x^2 + 1} / (a^2 * x - a))$

**Fricas [A]** time = 0.285353, size = 84, normalized size = 2.05

$$\frac{2ax + (ax + \sqrt{-a^2x^2 + 1} - 1) \log\left(\frac{\sqrt{-a^2x^2 + 1} - 1}{x}\right)}{ax + \sqrt{-a^2x^2 + 1} - 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/(sqrt(-a^2*x^2 + 1)*(a*x - 1)*x), x, algorithm="fricas")`

[Out]  $(2 * a * x + (a * x + \sqrt{-a^2 * x^2 + 1} - 1) * \log((\sqrt{-a^2 * x^2 + 1} - 1) / x)) / (a * x + \sqrt{-a^2 * x^2 + 1} - 1)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{ax^2\sqrt{-a^2x^2+1} - x\sqrt{-a^2x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(-a*x+1)/(-a**2*x**2+1)**(1/2), x)`

[Out]  $-\operatorname{Integral}(1/(a * x^2 * \sqrt{-a^2 * x^2 + 1} - x * \sqrt{-a^2 * x^2 + 1}), x)$

**GIAC/XCAS [A]** time = 0.300939, size = 100, normalized size = 2.44

$$-\frac{a \ln\left(\frac{|-2\sqrt{-a^2x^2+1}|a|-2a|}{2a^2|x|}\right)}{|a|} + \frac{2a}{\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} - 1\right)|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/(sqrt(-a^2*x^2 + 1)*(a*x - 1)*x), x, algorithm="giac")`

```
[Out] -a*ln(1/2*abs(-2*sqrt(-a^2*x^2 + 1)*abs(a) - 2*a)/(a^2*abs(x)))/a  
bs(a) + 2*a/(((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)*abs(a)  
)
```

$$3.155 \quad \int \frac{1}{x^2(1-ax)\sqrt{1-a^2x^2}} dx$$

**Optimal.** Leaf size=64

$$-\frac{2\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x(1-ax)} - a \tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

[Out]  $(-2*\text{Sqrt}[1 - a^2*x^2])/x + \text{Sqrt}[1 - a^2*x^2]/(x*(1 - a*x)) - a*\text{ArcTanh}[\text{Sqrt}[1 - a^2*x^2]]$

**Rubi [A]** time = 0.199951, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$

$$-\frac{2\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x(1-ax)} - a \tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^2*(1 - a*x)*\text{Sqrt}[1 - a^2*x^2]), x]$

[Out]  $(-2*\text{Sqrt}[1 - a^2*x^2])/x + \text{Sqrt}[1 - a^2*x^2]/(x*(1 - a*x)) - a*\text{ArcTanh}[\text{Sqrt}[1 - a^2*x^2]]$

**Rubi in Sympy [A]** time = 15.8018, size = 48, normalized size = 0.75

$$-a \operatorname{atanh}\left(\sqrt{-a^2x^2 + 1}\right) + \frac{ax + 1}{x\sqrt{-a^2x^2 + 1}} - \frac{2\sqrt{-a^2x^2 + 1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x^{**2}/(-a*x+1)/(-a^{**2}*x^{**2}+1)^{(1/2}), x)$

[Out]  $-a*\operatorname{atanh}(\text{sqrt}(-a^{**2}*x^{**2} + 1)) + (a*x + 1)/(x*\text{sqrt}(-a^{**2}*x^{**2} + 1)) - 2*\text{sqrt}(-a^{**2}*x^{**2} + 1)/x$

**Mathematica [A]** time = 0.0770596, size = 54, normalized size = 0.84

$$-\sqrt{1-a^2x^2}\left(\frac{a}{ax-1} + \frac{1}{x}\right) - a \log\left(\sqrt{1-a^2x^2} + 1\right) + a \log(x)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[1/(x^2*(1 - a*x)*\text{Sqrt}[1 - a^2*x^2]), x]$

[Out]  $-(\text{Sqrt}[1 - a^2*x^2]*(x^{(-1)} + a/(-1 + a*x))) + a*\text{Log}[x] - a*\text{Log}[1 + \text{Sqrt}[1 - a^2*x^2]]$

**Maple [A]** time = 0.016, size = 73, normalized size = 1.1

$$-1\sqrt{-(x - a^{-1})^2 a^2 - 2(x - a^{-1})a(x - a^{-1})^{-1}} - \frac{1}{x}\sqrt{-a^2x^2 + 1} - a\operatorname{Artanh}\left(\frac{1}{\sqrt{-a^2x^2 + 1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(-a*x+1)/(-a^2*x^2+1)^(1/2),x)`

[Out]  $-1/(x-1/a) * (- (x-1/a)^2 * a^2 - 2 * (x-1/a) * a)^{1/2} - (-a^2 * x^2 + 1)^{1/2} / x - a * \operatorname{arctanh}(1/(-a^2 * x^2 + 1)^{1/2})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{-a^2x^2 + 1}(ax - 1)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/(sqrt(-a^2*x^2 + 1)*(a*x - 1)*x^2),x, algorithm="maxima")`

[Out] `-integrate(1/(sqrt(-a^2*x^2 + 1)*(a*x - 1)*x^2), x)`

**Fricas [A]** time = 0.284937, size = 211, normalized size = 3.3

$$\frac{a^3x^3 - 4a^2x^2 - ax - \left(a^3x^3 + a^2x^2 - 2ax - (a^2x^2 - 2ax)\sqrt{-a^2x^2 + 1}\right) \log\left(\frac{\sqrt{-a^2x^2 + 1} - 1}{x}\right) + (3a^2x^2 + ax - 2)\sqrt{-a^2x^2 + 1}}{a^2x^3 + ax^2 - \sqrt{-a^2x^2 + 1}(ax^2 - 2x) - 2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/(sqrt(-a^2*x^2 + 1)*(a*x - 1)*x^2),x, algorithm="fricas")`

[Out]  $-(a^3x^3 - 4a^2x^2 - ax - (a^3x^3 + a^2x^2 - 2ax - (a^2x^2 - 2ax)\sqrt{-a^2x^2 + 1})\sqrt{-a^2x^2 + 1}) \log((\sqrt{-a^2x^2 + 1} - 1)/x) + (3a^2x^2 + ax - 2)\sqrt{-a^2x^2 + 1} / (a^2x^3 + ax^2 - \sqrt{-a^2x^2 + 1}(ax^2 - 2x) - 2x)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{ax^3\sqrt{-a^2x^2 + 1} - x^2\sqrt{-a^2x^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(-a*x+1)/(-a**2*x**2+1)**(1/2),x)`

[Out] `-Integral(1/(a*x**3*sqrt(-a**2*x**2 + 1) - x**2*sqrt(-a**2*x**2 + 1)), x)`

**GIAC/XCAS [A]** time = 0.288667, size = 203, normalized size = 3.17

$$\frac{a^2 \ln\left(\frac{|-2\sqrt{-a^2x^2+1}|a|-2a|}{2a^2|x|}\right)}{|a|} - \frac{\left(a^2 - \frac{5(\sqrt{-a^2x^2+1}|a|+a)}{x}\right)a^2x}{2(\sqrt{-a^2x^2+1}|a|+a)\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} - 1\right)|a|} - \frac{\sqrt{-a^2x^2+1}|a|+a}{2x|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/(sqrt(-a^2\*x^2 + 1)\*(a\*x - 1)\*x^2),x, algorithm="giac")

[Out] 
$$-a^2 \ln\left(\frac{1}{2} \operatorname{abs}(-2 \sqrt{-a^2 x^2 + 1} \operatorname{abs}(a) - 2a) / (a^2 \operatorname{abs}(x))\right) / \operatorname{abs}(a) - \frac{1}{2} (a^2 - 5 \sqrt{-a^2 x^2 + 1} \operatorname{abs}(a) + a) / x * a^2 x / (\sqrt{-a^2 x^2 + 1} \operatorname{abs}(a) + a) * ((\sqrt{-a^2 x^2 + 1} \operatorname{abs}(a) + a) / (a^2 x - 1) \operatorname{abs}(a)) - \frac{1}{2} (\sqrt{-a^2 x^2 + 1} \operatorname{abs}(a) + a) / (x \operatorname{abs}(a))$$

$$3.156 \quad \int \frac{1}{x^3(1-ax)\sqrt{1-a^2x^2}} dx$$

**Optimal.** Leaf size=90

$$-\frac{2a\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} - \frac{3\sqrt{1-a^2x^2}}{2x^2} - \frac{3}{2}a^2 \tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

[Out]  $(-3*\text{Sqrt}[1 - a^2*x^2])/(2*x^2) - (2*a*\text{Sqrt}[1 - a^2*x^2])/x + \text{Sqrt}[1 - a^2*x^2]/(x^2*(1 - a*x)) - (3*a^2*\text{ArcTanh}[\text{Sqrt}[1 - a^2*x^2]])/2$

**Rubi [A]** time = 0.274768, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$-\frac{2a\sqrt{1-a^2x^2}}{x} + \frac{\sqrt{1-a^2x^2}}{x^2(1-ax)} - \frac{3\sqrt{1-a^2x^2}}{2x^2} - \frac{3}{2}a^2 \tanh^{-1}\left(\sqrt{1-a^2x^2}\right)$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*(1 - a\*x)\*Sqrt[1 - a^2\*x^2]), x]

[Out]  $(-3*\text{Sqrt}[1 - a^2*x^2])/(2*x^2) - (2*a*\text{Sqrt}[1 - a^2*x^2])/x + \text{Sqrt}[1 - a^2*x^2]/(x^2*(1 - a*x)) - (3*a^2*\text{ArcTanh}[\text{Sqrt}[1 - a^2*x^2]])/2$

**Rubi in Sympy [A]** time = 22.9582, size = 75, normalized size = 0.83

$$-\frac{3a^2 \operatorname{atanh}\left(\sqrt{-a^2x^2+1}\right)}{2} - \frac{2a\sqrt{-a^2x^2+1}}{x} + \frac{ax+1}{x^2\sqrt{-a^2x^2+1}} - \frac{3\sqrt{-a^2x^2+1}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*3/(-a\*x+1)/(-a\*\*2\*x\*\*2+1)\*\*(1/2), x)

[Out]  $-3*a**2*\operatorname{atanh}(\operatorname{sqrt}(-a**2*x**2+1))/2 - 2*a*\operatorname{sqrt}(-a**2*x**2+1)/x + (a*x+1)/(x**2*\operatorname{sqrt}(-a**2*x**2+1)) - 3*\operatorname{sqrt}(-a**2*x**2+1)/(2*x**2)$

**Mathematica [A]** time = 0.142581, size = 74, normalized size = 0.82

$$-\sqrt{1-a^2x^2} \left( \frac{a^2}{ax-1} + \frac{a}{x} + \frac{1}{2x^2} \right) - \frac{3}{2}a^2 \log\left(\sqrt{1-a^2x^2}+1\right) + \frac{3}{2}a^2 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(1 - a\*x)\*Sqrt[1 - a^2\*x^2]), x]

[Out]  $-(\text{Sqrt}[1 - a^2*x^2]*(1/(2*x^2) + a/x + a^2/(-1 + a*x))) + (3*a^2*\text{Log}[x])/2 - (3*a^2*\text{Log}[1 + \text{Sqrt}[1 - a^2*x^2]])/2$

**Maple [A]** time = 0.017, size = 94, normalized size = 1.

$$-a\sqrt{-(x-a^{-1})^2 a^2 - 2(x-a^{-1})a(x-a^{-1})^{-1}} - \frac{1}{2x^2}\sqrt{-a^2x^2+1} - \frac{3a^2}{2}\operatorname{Artanh}\left(\frac{1}{\sqrt{-a^2x^2+1}}\right) - \frac{a}{x}\sqrt{-a^2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(-a*x+1)/(-a^2*x^2+1)^(1/2), x)`

[Out] `-a/(x-1/a)*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(1/2)-1/2*(-a^2*x^2+1)^(1/2)/x^2-3/2*a^2*atanh(1/(-a^2*x^2+1)^(1/2))-a*(-a^2*x^2+1)^(1/2)/x`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{\sqrt{-a^2x^2+1}(ax-1)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/(sqrt(-a^2*x^2+1)*(a*x-1)*x^3), x, algorithm="maxima")`

[Out] `-integrate(1/(sqrt(-a^2*x^2+1)*(a*x-1)*x^3), x)`

**Fricas [A]** time = 0.287877, size = 317, normalized size = 3.52

$$\frac{6a^5x^5 + 5a^4x^4 - 16a^3x^3 - 9a^2x^2 + 6ax + 3(a^5x^5 - 3a^4x^4 - 2a^3x^3 + 4a^2x^2 + (a^4x^4 + 2a^3x^3 - 4a^2x^2)\sqrt{-a^2x^2+1}) \log\left(\frac{2(a^3x^5 - 3a^2x^4 - 2ax^3 + 4x^2 + (a^2x^4 + 2ax^3 - 4x^2)\sqrt{-a^2x^2+1}}{2(a^3x^5 - 3a^2x^4 - 2ax^3 + 4x^2 + (a^2x^4 + 2ax^3 - 4x^2)\sqrt{-a^2x^2+1}}\right)}{2(a^3x^5 - 3a^2x^4 - 2ax^3 + 4x^2 + (a^2x^4 + 2ax^3 - 4x^2)\sqrt{-a^2x^2+1})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-1/(sqrt(-a^2*x^2+1)*(a*x-1)*x^3), x, algorithm="fricas")`

[Out] `1/2*(6*a^5*x^5 + 5*a^4*x^4 - 16*a^3*x^3 - 9*a^2*x^2 + 6*a*x + 3*(a^5*x^5 - 3*a^4*x^4 - 2*a^3*x^3 + 4*a^2*x^2 + (a^4*x^4 + 2*a^3*x^3 - 4*a^2*x^2)*sqrt(-a^2*x^2+1))*log((sqrt(-a^2*x^2+1)-1)/x) - (2*a^4*x^4 - 13*a^3*x^3 - 7*a^2*x^2 + 6*a*x + 4)*sqrt(-a^2*x^2+1) + 4)/(a^3*x^5 - 3*a^2*x^4 - 2*a*x^3 + 4*x^2 + (a^2*x^4 + 2*a*x^3 - 4*x^2)*sqrt(-a^2*x^2+1))`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{ax^4\sqrt{-a^2x^2+1} - x^3\sqrt{-a^2x^2+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(-a*x+1)/(-a**2*x**2+1)**(1/2), x)`

[Out] `-Integral(1/(a*x**4*sqrt(-a**2*x**2+1) - x**3*sqrt(-a**2*x**2+1)), x)`



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**GIAC/XCAS [A]** time = 0.287118, size = 288, normalized size = 3.2

$$\frac{\left(a^3 + \frac{3(\sqrt{-a^2x^2+1}|a|+a)a}{x} - \frac{20(\sqrt{-a^2x^2+1}|a|+a)^2}{ax^2}\right)a^4x^2}{8\left(\sqrt{-a^2x^2+1}|a|+a\right)^2\left(\frac{\sqrt{-a^2x^2+1}|a|+a}{a^2x} - 1\right)|a|} - \frac{3a^3\ln\left(\frac{|-2\sqrt{-a^2x^2+1}|a|-2a|}{2a^2|x|}\right)}{2|a|} - \frac{4\left(\sqrt{-a^2x^2+1}|a|+a\right)a|a|}{x} + \frac{\left(\sqrt{-a^2x^2+1}|a|+a\right)^2|a|}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-1/(sqrt(-a^2\*x^2 + 1)\*(a\*x - 1)\*x^3),x, algorithm="giac")

[Out] -1/8\*(a^3 + 3\*(sqrt(-a^2\*x^2 + 1)\*abs(a) + a)\*a/x - 20\*(sqrt(-a^2\*x^2 + 1)\*abs(a) + a)^2/(a\*x^2))\*a^4\*x^2/((sqrt(-a^2\*x^2 + 1)\*abs(a) + a)^2\*((sqrt(-a^2\*x^2 + 1)\*abs(a) + a)/(a^2\*x) - 1)\*abs(a)) - 3/2\*a^3\*ln(1/2\*abs(-2\*sqrt(-a^2\*x^2 + 1)\*abs(a) - 2\*a)/(a^2\*abs(x)))/abs(a) - 1/8\*(4\*(sqrt(-a^2\*x^2 + 1)\*abs(a) + a)\*a\*abs(a)/x + (sqrt(-a^2\*x^2 + 1)\*abs(a) + a)^2\*abs(a)/(a\*x^2))/a^2

$$3.157 \quad \int \frac{x^5 (d^2 - e^2 x^2)^{5/2}}{(d + ex)^2} dx$$

**Optimal.** Leaf size=229

$$-\frac{1}{9}x^6 (d^2 - e^2 x^2)^{3/2} + \frac{dx^5 (d^2 - e^2 x^2)^{3/2}}{4e} - \frac{5d^2 x^4 (d^2 - e^2 x^2)^{3/2}}{21e^2} - \frac{5d^9 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{64e^6} \\ - \frac{5d^7 x \sqrt{d^2 - e^2 x^2}}{64e^5} - \frac{d^5 (256d - 315ex) (d^2 - e^2 x^2)^{3/2}}{2016e^6} - \frac{4d^4 x^2 (d^2 - e^2 x^2)^{3/2}}{21e^4} + \frac{5d^3 x^3 (d^2 - e^2 x^2)^{3/2}}{24e^3}$$

[Out]  $(-5*d^7*x*\text{Sqrt}[d^2 - e^2*x^2])/(64*e^5) - (4*d^4*x^2*(d^2 - e^2*x^2)^{(3/2)})/(21*e^4) + (5*d^3*x^3*(d^2 - e^2*x^2)^{(3/2)})/(24*e^3) - (5*d^2*x^4*(d^2 - e^2*x^2)^{(3/2)})/(21*e^2) + (d*x^5*(d^2 - e^2*x^2)^{(3/2)})/(4*e) - (x^6*(d^2 - e^2*x^2)^{(3/2)})/9 - (d^5*(256*d - 315*e*x)*(d^2 - e^2*x^2)^{(3/2)})/(2016*e^6) - (5*d^9*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(64*e^6)$

**Rubi [A]** time = 0.814862, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$-\frac{1}{9}x^6 (d^2 - e^2 x^2)^{3/2} + \frac{dx^5 (d^2 - e^2 x^2)^{3/2}}{4e} - \frac{5d^2 x^4 (d^2 - e^2 x^2)^{3/2}}{21e^2} - \frac{5d^9 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{64e^6} \\ - \frac{5d^7 x \sqrt{d^2 - e^2 x^2}}{64e^5} - \frac{d^5 (256d - 315ex) (d^2 - e^2 x^2)^{3/2}}{2016e^6} - \frac{4d^4 x^2 (d^2 - e^2 x^2)^{3/2}}{21e^4} + \frac{5d^3 x^3 (d^2 - e^2 x^2)^{3/2}}{24e^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^5*(d^2 - e^2*x^2)^{(5/2)})/(d + e*x)^2, x]$

[Out]  $(-5*d^7*x*\text{Sqrt}[d^2 - e^2*x^2])/(64*e^5) - (4*d^4*x^2*(d^2 - e^2*x^2)^{(3/2)})/(21*e^4) + (5*d^3*x^3*(d^2 - e^2*x^2)^{(3/2)})/(24*e^3) - (5*d^2*x^4*(d^2 - e^2*x^2)^{(3/2)})/(21*e^2) + (d*x^5*(d^2 - e^2*x^2)^{(3/2)})/(4*e) - (x^6*(d^2 - e^2*x^2)^{(3/2)})/9 - (d^5*(256*d - 315*e*x)*(d^2 - e^2*x^2)^{(3/2)})/(2016*e^6) - (5*d^9*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(64*e^6)$

**Rubi in Sympy [A]** time = 89.4965, size = 212, normalized size = 0.93

$$-\frac{5d^9 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{64e^6} + \frac{5d^7 x \sqrt{d^2 - e^2 x^2}}{64e^5} - \frac{2d^6 (d^2 - e^2 x^2)^{\frac{3}{2}}}{3e^6} + \frac{5d^5 x^3 \sqrt{d^2 - e^2 x^2}}{96e^3} \\ + \frac{d^4 (d^2 - e^2 x^2)^{\frac{5}{2}}}{e^6} + \frac{d^3 x^5 \sqrt{d^2 - e^2 x^2}}{24e} - \frac{4d^2 (d^2 - e^2 x^2)^{\frac{7}{2}}}{7e^6} - \frac{dex^7 \sqrt{d^2 - e^2 x^2}}{4} + \frac{(d^2 - e^2 x^2)^{\frac{9}{2}}}{9e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**5}*(-e^{**2}*x^{**2}+d^{**2})^{**}(5/2)/(e*x+d)^{**2}, x)$

[Out]  $-5*d^{**9}*\operatorname{atan}(e*x/\text{sqrt}(d^{**2} - e^{**2}*x^{**2}))/ (64*e^{**6}) + 5*d^{**7}*x*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/ (64*e^{**5}) - 2*d^{**6}*(d^{**2} - e^{**2}*x^{**2})^{**}(3/2)/ (3*e^{**6}) + 5*d^{**5}*x^{**3}*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/ (96*e^{**3}) + d^{**4}*(d^{**2} - e^{**2}*x^{**2})^{**}(5/2)/e^{**6} + d^{**3}*x^{**5}*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/ (24*e) - 4*d^{**2}*(d^{**2} - e^{**2}*x^{**2})^{**}(7/2)/ (7*e^{**6}) - d^{**1} * e^{**7} * \text{sqrt}(d^{**2} - e^{**2}*x^{**2})/4 + (d^{**2} - e^{**2}*x^{**2})^{**}(9/2)/ (9*e^{**6})$

**Mathematica [A]** time = 0.134909, size = 135, normalized size = 0.59

$$\frac{\sqrt{d^2 - e^2 x^2} (-512d^8 + 315d^7 ex - 256d^6 e^2 x^2 + 210d^5 e^3 x^3 - 192d^4 e^4 x^4 + 168d^3 e^5 x^5 + 512d^2 e^6 x^6 - 1008de^7 x^7 + 448e^8 x^8) - 4032e^6}{4032e^6}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5\*(d^2 - e^2\*x^2)^(5/2))/(d + e\*x)^2,x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(-512\*d^8 + 315\*d^7\*e\*x - 256\*d^6\*e^2\*x^2 + 210\*d^5\*e^3\*x^3 - 192\*d^4\*e^4\*x^4 + 168\*d^3\*e^5\*x^5 + 512\*d^2\*e^6\*x^6 - 1008\*d\*e^7\*x^7 + 448\*e^8\*x^8) - 315\*d^9\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(4032\*e^6)

**Maple [A]** time = 0.027, size = 375, normalized size = 1.6

$$\begin{aligned} & -\frac{x^2}{9e^4}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{29d^2}{63e^6}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{17d^3x}{24e^5}(-e^2x^2 + d^2)^{\frac{5}{2}} \\ & - \frac{85d^5x}{96e^5}(-e^2x^2 + d^2)^{\frac{3}{2}} - \frac{85d^7x}{64e^5}\sqrt{-e^2x^2 + d^2} - \frac{85d^9}{64e^5}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2 + d^2}}\right)\frac{1}{\sqrt{e^2}} \\ & + \frac{dx}{4e^5}(-e^2x^2 + d^2)^{\frac{7}{2}} + \frac{2d^4}{3e^6}\left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{5}{2}} \\ & + \frac{5d^5x}{6e^5}\left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{3}{2}} + \frac{5d^7x}{4e^5}\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)} \\ & + \frac{5d^9}{4e^5}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)}}\right)\frac{1}{\sqrt{e^2}} \\ & - \frac{d^4}{3e^8}\left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{7}{2}}\left(x + \frac{d}{e}\right)^{-2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(-e^2\*x^2+d^2)^(5/2)/(e\*x+d)^2,x)

[Out] -1/9/e^4\*x^2\*(-e^2\*x^2+d^2)^(7/2)-29/63\*d^2/e^6\*(-e^2\*x^2+d^2)^(7/2)-17/24\*d^3/e^5\*x\*(-e^2\*x^2+d^2)^(5/2)-85/96\*d^5/e^5\*x\*(-e^2\*x^2+d^2)^(3/2)-85/64\*d^7\*x\*(-e^2\*x^2+d^2)^(1/2)/e^5-85/64\*d^9/e^5/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))+1/4\*d/e^5\*x\*(-e^2\*x^2+d^2)^(7/2)+2/3/e^6\*d^4\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(5/2)+5/6/e^5\*d^5\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)\*x+5/4/e^5\*d^7\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x+5/4/e^5\*d^9/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))-1/3\*d^4/e^8/(x+d/e)^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x^5/(e\*x + d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

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**Fricas [A]** time = 0.294488, size = 806, normalized size = 3.52

$$448 e^{18} x^{18} - 1008 d e^{17} x^{17} - 17856 d^2 e^{16} x^{16} + 41496 d^3 e^{15} x^{15} + 104256 d^4 e^{14} x^{14} - 288918 d^5 e^{13} x^{13} - 157248 d^6 e^{12} x^{12} + 732249 d^7 e^{11} x^{11} - 80640 d^8 e^{10} x^{10} - 779331 d^9 e^9 x^9 + 322560 d^{10} e^8 x^8 + 320040 d^{11} e^7 x^7 - 172032 d^{12} e^6 x^6 - 111888 d^{13} e^5 x^5 + 168000 d^{15} e^3 x^3 - 80640 d^{17} e x + 630 (9 d^{10} e^8 x^8 - 120 d^{12} e^6 x^6 + 432 d^{14} e^4 x^4 - 576 d^{16} e^2 x^2 + 256 d^{18} - (d^9 e^8 x^8 - 40 d^{11} e^6 x^6 + 240 d^{13} e^4 x^4 - 448 d^{15} e^2 x^2 + 256 d^{17})) \sqrt{-e^2 x^2 + d^2} \arctan\left(\frac{-d - \sqrt{-e^2 x^2 + d^2}}{e x}\right) + 3 (1344 d e^{16} x^{16} - 3024 d^2 e^{15} x^{15} - 16384 d^3 e^{14} x^{14} + 40824 d^4 e^{13} x^{13} + 43456 d^5 e^{12} x^{12} - 151242 d^6 e^{11} x^{11} - 5376 d^7 e^{10} x^{10} + 210273 d^8 e^9 x^9 - 78848 d^9 e^8 x^8 - 100632 d^{10} e^7 x^7 + 57344 d^{11} e^6 x^6 + 19376 d^{12} e^5 x^5 - 42560 d^{14} e^3 x^3 + 26880 d^{16} e x) \sqrt{-e^2 x^2 + d^2} / (9 d e^{14} x^8 - 120 d^3 e^{12} x^6 + 432 d^5 e^{10} x^4 - 576 d^7 e^8 x^2 + 256 d^9 e^6 - (e^{14} x^8 - 40 d^2 e^{12} x^6 + 240 d^4 e^{10} x^4 - 448 d^6 e^8 x^2 + 256 d^8 e^6)) \sqrt{-e^2 x^2 + d^2}$$


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Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x^5/(e\*x + d)^2,x, algorithm="fricas")

[Out] 1/4032\*(448\*e^18\*x^18 - 1008\*d\*e^17\*x^17 - 17856\*d^2\*e^16\*x^16 + 41496\*d^3\*e^15\*x^15 + 104256\*d^4\*e^14\*x^14 - 288918\*d^5\*e^13\*x^13 - 157248\*d^6\*e^12\*x^12 + 732249\*d^7\*e^11\*x^11 - 80640\*d^8\*e^10\*x^10 - 779331\*d^9\*e^9\*x^9 + 322560\*d^10\*e^8\*x^8 + 320040\*d^11\*e^7\*x^7 - 172032\*d^12\*e^6\*x^6 - 111888\*d^13\*e^5\*x^5 + 168000\*d^15\*e^3\*x^3 - 80640\*d^17\*e\*x + 630\*(9\*d^10\*e^8\*x^8 - 120\*d^12\*e^6\*x^6 + 432\*d^14\*e^4\*x^4 - 576\*d^16\*e^2\*x^2 + 256\*d^18 - (d^9\*e^8\*x^8 - 40\*d^11\*e^6\*x^6 + 240\*d^13\*e^4\*x^4 - 448\*d^15\*e^2\*x^2 + 256\*d^17))\*sqrt(-e^2\*x^2 + d^2)\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) + 3\*(1344\*d\*e^16\*x^16 - 3024\*d^2\*e^15\*x^15 - 16384\*d^3\*e^14\*x^14 + 40824\*d^4\*e^13\*x^13 + 43456\*d^5\*e^12\*x^12 - 151242\*d^6\*e^11\*x^11 - 5376\*d^7\*e^10\*x^10 + 210273\*d^8\*e^9\*x^9 - 78848\*d^9\*e^8\*x^8 - 100632\*d^10\*e^7\*x^7 + 57344\*d^11\*e^6\*x^6 + 19376\*d^12\*e^5\*x^5 - 42560\*d^14\*e^3\*x^3 + 26880\*d^16\*e\*x)\*sqrt(-e^2\*x^2 + d^2)/(9\*d\*e^14\*x^8 - 120\*d^3\*e^12\*x^6 + 432\*d^5\*e^10\*x^4 - 576\*d^7\*e^8\*x^2 + 256\*d^9\*e^6 - (e^14\*x^8 - 40\*d^2\*e^12\*x^6 + 240\*d^4\*e^10\*x^4 - 448\*d^6\*e^8\*x^2 + 256\*d^8\*e^6))\*sqrt(-e^2\*x^2 + d^2)

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**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/(e\*x+d)\*\*2,x)

[Out] Timed out

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**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x^5/(e\*x + d)^2,x, algorithm="giac")

[Out] Timed out

$$3.158 \quad \int \frac{x^4(d^2 - e^2x^2)^{5/2}}{(d+ex)^2} dx$$

**Optimal.** Leaf size=200

$$-\frac{1}{8}x^5(d^2 - e^2x^2)^{3/2} + \frac{2dx^4(d^2 - e^2x^2)^{3/2}}{7e} - \frac{13d^2x^3(d^2 - e^2x^2)^{3/2}}{48e^2} + \frac{13d^8 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{128e^5} \\ + \frac{13d^6x\sqrt{d^2 - e^2x^2}}{128e^4} + \frac{d^4(1024d - 1365ex)(d^2 - e^2x^2)^{3/2}}{6720e^5} + \frac{8d^3x^2(d^2 - e^2x^2)^{3/2}}{35e^3}$$

[Out]  $(13*d^6*x*sqrt[d^2 - e^2*x^2])/(128*e^4) + (8*d^3*x^2*(d^2 - e^2*x^2)^{3/2})/(35*e^3) - (13*d^2*x^3*(d^2 - e^2*x^2)^{3/2})/(48*e^2) + (2*d*x^4*(d^2 - e^2*x^2)^{3/2})/(7*e) - (x^5*(d^2 - e^2*x^2)^{3/2})/8 + (d^4*(1024*d - 1365*e*x)*(d^2 - e^2*x^2)^{3/2})/(6720*e^5) + (13*d^8*ArcTan[(e*x)/sqrt[d^2 - e^2*x^2]])/(128*e^5)$

**Rubi [A]** time = 0.67569, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$-\frac{1}{8}x^5(d^2 - e^2x^2)^{3/2} + \frac{2dx^4(d^2 - e^2x^2)^{3/2}}{7e} - \frac{13d^2x^3(d^2 - e^2x^2)^{3/2}}{48e^2} + \frac{13d^8 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{128e^5} \\ + \frac{13d^6x\sqrt{d^2 - e^2x^2}}{128e^4} + \frac{d^4(1024d - 1365ex)(d^2 - e^2x^2)^{3/2}}{6720e^5} + \frac{8d^3x^2(d^2 - e^2x^2)^{3/2}}{35e^3}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*(d^2 - e^2\*x^2)^(5/2))/(d + e\*x)^2, x]

[Out]  $(13*d^6*x*sqrt[d^2 - e^2*x^2])/(128*e^4) + (8*d^3*x^2*(d^2 - e^2*x^2)^{3/2})/(35*e^3) - (13*d^2*x^3*(d^2 - e^2*x^2)^{3/2})/(48*e^2) + (2*d*x^4*(d^2 - e^2*x^2)^{3/2})/(7*e) - (x^5*(d^2 - e^2*x^2)^{3/2})/8 + (d^4*(1024*d - 1365*e*x)*(d^2 - e^2*x^2)^{3/2})/(6720*e^5) + (13*d^8*ArcTan[(e*x)/sqrt[d^2 - e^2*x^2]])/(128*e^5)$

**Rubi in Sympy [A]** time = 75.3018, size = 196, normalized size = 0.98

$$\frac{13d^8 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{128e^5} - \frac{13d^6x\sqrt{d^2 - e^2x^2}}{128e^4} + \frac{2d^5(d^2 - e^2x^2)^{3/2}}{3e^5} - \frac{13d^4x^3\sqrt{d^2 - e^2x^2}}{192e^2} \\ - \frac{4d^3(d^2 - e^2x^2)^{5/2}}{5e^5} + \frac{7d^2x^5\sqrt{d^2 - e^2x^2}}{48} + \frac{2d(d^2 - e^2x^2)^{7/2}}{7e^5} + \frac{e^2x^7\sqrt{d^2 - e^2x^2}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/(e\*x+d)\*\*2, x)

[Out]  $13*d**8*atan(e*x/sqrt(d**2 - e**2*x**2))/(128*e**5) - 13*d**6*x*sqrt(d**2 - e**2*x**2)/(128*e**4) + 2*d**5*(d**2 - e**2*x**2)**(3/2)/(3*e**5) - 13*d**4*x**3*sqrt(d**2 - e**2*x**2)/(192*e**2) - 4*d**3*(d**2 - e**2*x**2)**(5/2)/(5*e**5) + 7*d**2*x**5*sqrt(d**2 - e**2*x**2)/48 + 2*d*(d**2 - e**2*x**2)**(7/2)/(7*e**5) + e**2*x**7*sqrt(d**2 - e**2*x**2)/8$

**Mathematica [A]** time = 0.120537, size = 124, normalized size = 0.62

$$1365d^8 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) + \sqrt{d^2 - e^2x^2} (2048d^7 - 1365d^6ex + 1024d^5e^2x^2 - 910d^4e^3x^3 + 768d^3e^4x^4 + 1960d^2e^5x^5 - 3840de^6x^6 + 13440e^7x^7)$$

13440e<sup>5</sup>

Antiderivative was successfully verified.

[In] Integrate[(x^4\*(d^2 - e^2\*x^2)^(5/2))/(d + e\*x)^2,x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(2048\*d^7 - 1365\*d^6\*e\*x + 1024\*d^5\*e^2\*x^2 - 910\*d^4\*e^3\*x^3 + 768\*d^3\*e^4\*x^4 + 1960\*d^2\*e^5\*x^5 - 3840\*d\*e^6\*x^6 + 1680\*e^7\*x^7) + 1365\*d^8\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(13440\*e^5)

**Maple [B]** time = 0.02, size = 350, normalized size = 1.8

$$\begin{aligned}
 & -\frac{x}{8e^4}(-e^2x^2 + d^2)^{\frac{7}{2}} + \frac{25d^2x}{48e^4}(-e^2x^2 + d^2)^{\frac{5}{2}} + \frac{125d^4x}{192e^4}(-e^2x^2 + d^2)^{\frac{3}{2}} \\
 & + \frac{125d^6x}{128e^4}\sqrt{-e^2x^2 + d^2} + \frac{125d^8}{128e^4}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2 + d^2}}\right)\frac{1}{\sqrt{e^2}} \\
 & + \frac{d^3}{3e^7}\left(-\left(x + \frac{d}{e}\right)^2e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{7}{2}}\left(x + \frac{d}{e}\right)^{-2} - \frac{7d^3}{15e^5}\left(-\left(x + \frac{d}{e}\right)^2e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{5}{2}} \\
 & - \frac{7d^4x}{12e^4}\left(-\left(x + \frac{d}{e}\right)^2e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{3}{2}} - \frac{7d^6x}{8e^4}\sqrt{-\left(x + \frac{d}{e}\right)^2e^2 + 2de\left(x + \frac{d}{e}\right)} \\
 & - \frac{7d^8}{8e^4}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2e^2 + 2de\left(x + \frac{d}{e}\right)}}\right)\frac{1}{\sqrt{e^2}} + \frac{2d}{7e^5}(-e^2x^2 + d^2)^{\frac{7}{2}}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(-e^2\*x^2+d^2)^(5/2)/(e\*x+d)^2,x)

[Out] -1/8/e^4\*x\*(-e^2\*x^2+d^2)^(7/2)+25/48\*d^2/e^4\*x\*(-e^2\*x^2+d^2)^(5/2)+125/192/e^4\*d^4\*x\*(-e^2\*x^2+d^2)^(3/2)+125/128\*d^6\*x\*(-e^2\*x^2+d^2)^(1/2)/e^4+125/128/e^4\*d^8/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))+1/3\*d^3/e^7/(x+d/e)^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)-7/15\*d^3/e^5\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(5/2)-7/12\*d^4/e^4\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)\*x-7/8\*d^6/e^4\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x-7/8\*d^8/e^4/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))+2/7\*d/e^5\*(-e^2\*x^2+d^2)^(7/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x^4/(e\*x + d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.294021, size = 714, normalized size = 3.57

13440 d e<sup>15</sup> x<sup>15</sup> - 30720 d<sup>2</sup> e<sup>14</sup> x<sup>14</sup> - 132160 d<sup>3</sup> e<sup>13</sup> x<sup>13</sup> + 344064 d<sup>4</sup> e<sup>12</sup> x<sup>12</sup> + 277200 d<sup>5</sup> e<sup>11</sup> x<sup>11</sup> - 1103872 d<sup>6</sup> e<sup>10</sup> x<sup>10</sup> + 64680 d<sup>7</sup> e<sup>9</sup> x<sup>9</sup> - 30720 d<sup>8</sup> e<sup>8</sup> x<sup>8</sup> + 64680 d<sup>9</sup> e<sup>7</sup> x<sup>7</sup> - 132160 d<sup>10</sup> e<sup>6</sup> x<sup>6</sup> + 30720 d<sup>11</sup> e<sup>5</sup> x<sup>5</sup> - 64680 d<sup>12</sup> e<sup>4</sup> x<sup>4</sup> + 13440 d<sup>13</sup> e<sup>3</sup> x<sup>3</sup> - 30720 d<sup>14</sup> e<sup>2</sup> x<sup>2</sup> + 13440 d<sup>15</sup> e x - 13440 d<sup>16</sup>

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)*x^4/(e*x + d)^2,x, algorithm="fricas")`

[Out] 
$$-1/13440*(13440*d*e^{15}x^{15} - 30720*d^2*e^{14}x^{14} - 132160*d^3*e^{13}x^{13} + 344064*d^4*e^{12}x^{12} + 277200*d^5*e^{11}x^{11} - 1103872*d^6*e^{10}x^{10} + 64680*d^7*e^9x^9 + 1361920*d^8*e^8x^8 - 539560*d^9*e^7x^7 - 573440*d^{10}e^6x^6 + 170800*d^{11}e^5x^5 + 320320*d^{13}e^3x^3 - 174720*d^{15}e*x + 2730*(d^8e^8x^8 - 32*d^{10}e^6x^6 + 160*d^{12}e^4x^4 - 256*d^{14}e^2x^2 + 128*d^{16} + 8*(d^9e^6x^6 - 10*d^{11}e^4x^4 + 24*d^{13}e^2x^2 - 16*d^{15})*\sqrt{-e^2x^2 + d^2}))*\arctan(-(d - \sqrt{-e^2x^2 + d^2})/(e*x)) - (1680*e^{15}x^{15} - 3840*d*e^{14}x^{14} - 51800*d^2e^{13}x^{13} + 123648*d^3e^{12}x^{12} + 205170*d^4e^{11}x^{11} - 637952*d^5e^{10}x^{10} - 88725*d^6e^9x^9 + 1075200*d^7e^8x^8 - 388640*d^8e^7x^7 - 573440*d^9e^6x^6 + 265440*d^{10}e^5x^5 + 232960*d^{12}e^3x^3 - 174720*d^{14}e*x)*\sqrt{-e^2x^2 + d^2})/(e^{13}x^8 - 32*d^2e^{11}x^6 + 160*d^4e^9x^4 - 256*d^6e^7x^2 + 128*d^8e^5 + 8*(d*e^{11}x^6 - 10*d^3e^9x^4 + 24*d^5e^7x^2 - 16*d^7e^5)*\sqrt{-e^2x^2 + d^2})$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)`

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)*x^4/(e*x + d)^2,x, algorithm="giac")`

[Out] Timed out

$$3.159 \quad \int \frac{x^3 (d^2 - e^2 x^2)^{5/2}}{(d + ex)^2} dx$$

**Optimal.** Leaf size=171

$$\begin{aligned} & -\frac{11d^2 x^2 (d^2 - e^2 x^2)^{3/2}}{35e^2} - \frac{1}{7} x^4 (d^2 - e^2 x^2)^{3/2} + \frac{dx^3 (d^2 - e^2 x^2)^{3/2}}{3e} \\ & - \frac{d^7 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{8e^4} - \frac{d^5 x \sqrt{d^2 - e^2 x^2}}{8e^3} - \frac{d^3 (88d - 105ex) (d^2 - e^2 x^2)^{3/2}}{420e^4} \end{aligned}$$

[Out]  $-(d^5 x \sqrt{d^2 - e^2 x^2}) / (8 e^4) - (11 d^2 x^2 (d^2 - e^2 x^2)^{3/2}) / (35 e^2) + (d^3 x^3 (d^2 - e^2 x^2)^{3/2}) / (3 e) - (x^4 (d^2 - e^2 x^2)^{3/2}) / 7 - (d^3 (88 d - 105 e x) (d^2 - e^2 x^2)^{3/2}) / (420 e^4) - (d^7 \operatorname{ArcTan}[(e x) / \sqrt{d^2 - e^2 x^2}]) / (8 e^4)$

**Rubi [A]** time = 0.532324, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\begin{aligned} & -\frac{11d^2 x^2 (d^2 - e^2 x^2)^{3/2}}{35e^2} - \frac{1}{7} x^4 (d^2 - e^2 x^2)^{3/2} + \frac{dx^3 (d^2 - e^2 x^2)^{3/2}}{3e} \\ & - \frac{d^7 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{8e^4} - \frac{d^5 x \sqrt{d^2 - e^2 x^2}}{8e^3} - \frac{d^3 (88d - 105ex) (d^2 - e^2 x^2)^{3/2}}{420e^4} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(x^3 (d^2 - e^2 x^2)^{5/2}) / (d + e x)^2, x]$

[Out]  $-(d^5 x \sqrt{d^2 - e^2 x^2}) / (8 e^4) - (11 d^2 x^2 (d^2 - e^2 x^2)^{3/2}) / (35 e^2) + (d^3 x^3 (d^2 - e^2 x^2)^{3/2}) / (3 e) - (x^4 (d^2 - e^2 x^2)^{3/2}) / 7 - (d^3 (88 d - 105 e x) (d^2 - e^2 x^2)^{3/2}) / (420 e^4) - (d^7 \operatorname{ArcTan}[(e x) / \sqrt{d^2 - e^2 x^2}]) / (8 e^4)$

**Rubi in Sympy [A]** time = 69.4826, size = 162, normalized size = 0.95

$$\begin{aligned} & -\frac{d^7 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{8e^4} + \frac{d^5 x \sqrt{d^2 - e^2 x^2}}{8e^3} - \frac{2d^4 (d^2 - e^2 x^2)^{3/2}}{3e^4} \\ & + \frac{d^3 x^3 \sqrt{d^2 - e^2 x^2}}{12e} + \frac{3d^2 (d^2 - e^2 x^2)^{5/2}}{5e^4} - \frac{dex^5 \sqrt{d^2 - e^2 x^2}}{3} - \frac{(d^2 - e^2 x^2)^{7/2}}{7e^4} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{rubi\_integrate}(x^3 (-e^2 x^2 + d^2)^{5/2} / (e x + d)^2, x)$

[Out]  $-d^7 \operatorname{atan}(e x / \sqrt{d^2 - e^2 x^2}) / (8 e^4) + d^5 x \sqrt{d^2 - e^2 x^2} / (8 e^3) - 2 d^4 (d^2 - e^2 x^2)^{3/2} / (3 e^4) + d^3 x^3 \sqrt{d^2 - e^2 x^2} / (12 e) + 3 d^2 (d^2 - e^2 x^2)^{5/2} / (5 e^4) - d e x^5 \sqrt{d^2 - e^2 x^2} / 3 - (d^2 - e^2 x^2)^{7/2} / (7 e^4)$

**Mathematica [A]** time = 0.105648, size = 113, normalized size = 0.66

$$\frac{\sqrt{d^2 - e^2 x^2} (-176 d^6 + 105 d^5 e x - 88 d^4 e^2 x^2 + 70 d^3 e^3 x^3 + 144 d^2 e^4 x^4 - 280 d e^5 x^5 + 120 e^6 x^6) - 105 d^7 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{840 e^4}$$



Antiderivative was successfully verified.

[In] Integrate[(x^3\*(d^2 - e^2\*x^2)^(5/2))/(d + e\*x)^2,x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(-176\*d^6 + 105\*d^5\*e\*x - 88\*d^4\*e^2\*x^2 + 70\*d^3\*e^3\*x^3 + 144\*d^2\*e^4\*x^4 - 280\*d\*e^5\*x^5 + 120\*e^6\*x^6) - 105\*d^7\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(840\*e^4)

**Maple [B]** time = 0.02, size = 327, normalized size = 1.9

$$\begin{aligned}
 & -\frac{1}{7e^4}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{dx}{3e^3}(-e^2x^2 + d^2)^{\frac{5}{2}} - \frac{5d^3x}{12e^3}(-e^2x^2 + d^2)^{\frac{3}{2}} - \frac{5d^5x}{8e^3}\sqrt{-e^2x^2 + d^2} \\
 & - \frac{5d^7}{8e^3} \arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} + \frac{4d^2}{15e^4} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{5}{2}} \\
 & + \frac{d^3x}{3e^3} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{3}{2}} + \frac{d^5x}{2e^3} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)} \\
 & + \frac{d^7}{2e^3} \arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)}}\right) \frac{1}{\sqrt{e^2}} \\
 & - \frac{d^2}{3e^6} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{7}{2}} \left(x + \frac{d}{e}\right)^{-2}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(-e^2\*x^2+d^2)^(5/2)/(e\*x+d)^2,x)

[Out] -1/7/e^4\*(-e^2\*x^2+d^2)^(7/2)-1/3\*d/e^3\*x\*(-e^2\*x^2+d^2)^(5/2)-5/12/e^3\*d^3\*x\*(-e^2\*x^2+d^2)^(3/2)-5/8\*d^5\*x\*(-e^2\*x^2+d^2)^(1/2)/e^3-5/8/e^3\*d^7/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))+4/15/e^4\*d^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(5/2)+1/3/e^3\*d^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)\*x+1/2/e^3\*d^5\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x+1/2/e^3\*d^7/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))-1/3\*d^2/e^6/(x+d/e)^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x^3/(e\*x + d)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.289817, size = 657, normalized size = 3.84

120 e^14 x^14 - 280 d e^13 x^13 - 2856 d^2 e^12 x^12 + 7070 d^3 e^11 x^11 + 8792 d^4 e^10 x^10 - 30765 d^5 e^9 x^9 - 280 d^6 e^8 x^8 + 44975 d^7 e^7 x^7 - 19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^(5/2)*x^3/(e*x + d)^2,x, algorithm="fricas")
```

```
[Out] 1/840*(120*e^14*x^14 - 280*d*e^13*x^13 - 2856*d^2*e^12*x^12 + 7070*d^3*e^11*x^11 + 8792*d^4*e^10*x^10 - 30765*d^5*e^9*x^9 - 280*d^6*e^8*x^8 + 44975*d^7*e^7*x^7 - 19040*d^8*e^6*x^6 - 17080*d^9*e^5*x^5 + 13440*d^10*e^4*x^4 - 10640*d^11*e^3*x^3 + 6720*d^13*e*x + 210*(7*d^8*e^6*x^6 - 56*d^10*e^4*x^4 + 112*d^12*e^2*x^2 - 64*d^14 - (d^7*e^6*x^6 - 24*d^9*e^4*x^4 + 80*d^11*e^2*x^2 - 64*d^13)*sqrt(-e^2*x^2 + d^2))*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + 7*(120*d*e^12*x^12 - 280*d^2*e^11*x^11 - 816*d^3*e^10*x^10 + 2310*d^4*e^9*x^9 + 680*d^5*e^8*x^8 - 4935*d^6*e^7*x^7 + 1760*d^7*e^6*x^6 + 2840*d^8*e^5*x^5 - 1920*d^9*e^4*x^4 + 1040*d^10*e^3*x^3 - 960*d^12*e*x)*sqrt(-e^2*x^2 + d^2))/(7*d*e^10*x^6 - 56*d^3*e^8*x^4 + 112*d^5*e^6*x^2 - 64*d^7*e^4 - (e^10*x^6 - 24*d^2*e^8*x^4 + 80*d^4*e^6*x^2 - 64*d^6*e^4)*sqrt(-e^2*x^2 + d^2))
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)
```

```
[Out] Timed out
```

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^(5/2)*x^3/(e*x + d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.160 \quad \int \frac{x^2(d^2 - e^2x^2)^{5/2}}{(d+ex)^2} dx$$

**Optimal.** Leaf size=142

$$\frac{2dx^2(d^2 - e^2x^2)^{3/2}}{5e} - \frac{1}{6}x^3(d^2 - e^2x^2)^{3/2} + \frac{d^2(32d - 45ex)(d^2 - e^2x^2)^{3/2}}{120e^3} + \frac{3d^6 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{16e^3} + \frac{3d^4x\sqrt{d^2 - e^2x^2}}{16e^2}$$

[Out] (3\*d^4\*x\*Sqrt[d^2 - e^2\*x^2])/(16\*e^2) + (2\*d\*x^2\*(d^2 - e^2\*x^2)^(3/2))/(5\*e) - (x^3\*(d^2 - e^2\*x^2)^(3/2))/6 + (d^2\*(32\*d - 45\*e\*x)\*(d^2 - e^2\*x^2)^(3/2))/(120\*e^3) + (3\*d^6\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(16\*e^3)

**Rubi [A]** time = 0.411938, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\frac{2dx^2(d^2 - e^2x^2)^{3/2}}{5e} - \frac{1}{6}x^3(d^2 - e^2x^2)^{3/2} + \frac{d^2(32d - 45ex)(d^2 - e^2x^2)^{3/2}}{120e^3} + \frac{3d^6 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{16e^3} + \frac{3d^4x\sqrt{d^2 - e^2x^2}}{16e^2}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(d^2 - e^2\*x^2)^(5/2))/(d + e\*x)^2, x]

[Out] (3\*d^4\*x\*Sqrt[d^2 - e^2\*x^2])/(16\*e^2) + (2\*d\*x^2\*(d^2 - e^2\*x^2)^(3/2))/(5\*e) - (x^3\*(d^2 - e^2\*x^2)^(3/2))/6 + (d^2\*(32\*d - 45\*e\*x)\*(d^2 - e^2\*x^2)^(3/2))/(120\*e^3) + (3\*d^6\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(16\*e^3)

**Rubi in Sympy [A]** time = 57.5202, size = 144, normalized size = 1.01

$$\frac{3d^6 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{16e^3} - \frac{3d^4x\sqrt{d^2 - e^2x^2}}{16e^2} + \frac{2d^3(d^2 - e^2x^2)^{\frac{3}{2}}}{3e^3} + \frac{5d^2x^3\sqrt{d^2 - e^2x^2}}{24} - \frac{2d(d^2 - e^2x^2)^{\frac{5}{2}}}{5e^3} + \frac{e^2x^5\sqrt{d^2 - e^2x^2}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/(e\*x+d)\*\*2, x)

[Out] 3\*d\*\*6\*atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2))/(16\*e\*\*3) - 3\*d\*\*4\*x\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(16\*e\*\*2) + 2\*d\*\*3\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(3\*e\*\*3) + 5\*d\*\*2\*x\*\*3\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/24 - 2\*d\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)/(5\*e\*\*3) + e\*\*2\*x\*\*5\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/6

**Mathematica [A]** time = 0.0985375, size = 102, normalized size = 0.72

$$\frac{45d^6 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) + \sqrt{d^2 - e^2x^2} (64d^5 - 45d^4ex + 32d^3e^2x^2 + 50d^2e^3x^3 - 96de^4x^4 + 40e^5x^5)}{240e^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(d^2 - e^2\*x^2)^(5/2))/(d + e\*x)^2, x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(64\*d^5 - 45\*d^4\*e\*x + 32\*d^3\*e^2\*x^2 + 50\*d^2\*e^3\*x^3 - 96\*d\*e^4\*x^4 + 40\*e^5\*x^5) + 45\*d^6\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(240\*e^3)

**Maple [B]** time = 0.017, size = 303, normalized size = 2.1

$$\begin{aligned} & \frac{x}{6e^2}(-e^2x^2 + d^2)^{\frac{5}{2}} + \frac{5d^2x}{24e^2}(-e^2x^2 + d^2)^{\frac{3}{2}} + \frac{5d^4x}{16e^2}\sqrt{-e^2x^2 + d^2} \\ & + \frac{5d^6}{16e^2} \arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} + \frac{d}{3e^5}\left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{7}{2}} \left(x + \frac{d}{e}\right)^{-2} \\ & - \frac{d}{15e^3}\left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{5}{2}} - \frac{d^2x}{12e^2}\left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{3}{2}} \\ & - \frac{d^4x}{8e^2}\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)} - \frac{d^6}{8e^2} \arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)}}\right) \frac{1}{\sqrt{e^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(-e^2\*x^2+d^2)^(5/2)/(e\*x+d)^2, x)

[Out] 1/6\*x\*(-e^2\*x^2+d^2)^(5/2)/e^2+5/24/e^2\*d^2\*x\*(-e^2\*x^2+d^2)^(3/2)+5/16\*d^4\*x\*(-e^2\*x^2+d^2)^(1/2)/e^2+5/16/e^2\*d^6/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))+1/3\*d/e^5/(x+d/e)^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)-1/15\*d/e^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(5/2)-1/12\*d^2/e^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)\*x-1/8\*d^4/e^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x-1/8\*d^6/e^2/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x^2/(e\*x + d)^2, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.288379, size = 568, normalized size = 4.

$$240de^{11}x^{11} - 576d^2e^{10}x^{10} - 1220d^3e^9x^9 + 3840d^4e^8x^8 + 390d^5e^7x^7 - 7040d^6e^6x^6 + 3630d^7e^5x^5 + 3840d^8e^4x^4 - 4480d^9e^3x^3 - 320d^{10}e^2x^2 + 160d^{11}e^1x - 160d^{12}e^0$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x^2/(e\*x + d)^2, x, algorithm="fricas")

[Out] -1/240\*(240\*d\*e^11\*x^11 - 576\*d^2\*e^10\*x^10 - 1220\*d^3\*e^9\*x^9 + 3840\*d^4\*e^8\*x^8 + 390\*d^5\*e^7\*x^7 - 7040\*d^6\*e^6\*x^6 + 3630\*d^7\*e^5\*x^5 + 3840\*d^8\*e^4\*x^4 - 4480\*d^9\*e^3\*x^3 - 320\*d^{10}\*e^2\*x^2 + 160\*d^{11}\*e^1\*x - 160\*d^{12})

$$e^5 x^5 + 3840 d^8 e^4 x^4 - 4480 d^9 e^3 x^3 + 1440 d^{11} e x + 90 (d^6 e^6 x^6 - 18 d^8 e^4 x^4 + 48 d^{10} e^2 x^2 - 32 d^{12} + 2 (3 d^7 e^4 x^4 - 16 d^9 e^2 x^2 + 16 d^{11})) \sqrt{-e^2 x^2 + d^2}) \operatorname{arctan}(- (d - \sqrt{-e^2 x^2 + d^2}) / (e x)) - (40 e^{11} x^{11} - 96 d e^{10} x^{10} - 670 d^2 e^9 x^9 + 1760 d^3 e^8 x^8 + 975 d^4 e^7 x^7 - 5120 d^5 e^6 x^6 + 1930 d^6 e^5 x^5 + 3840 d^7 e^4 x^4 - 3760 d^8 e^3 x^3 + 1440 d^{10} e x) \sqrt{-e^2 x^2 + d^2}) / (e^9 x^6 - 18 d^2 e^7 x^4 + 48 d^4 e^5 x^2 - 32 d^6 e^3 + 2 (3 d e^7 x^4 - 16 d^3 e^5 x^2 + 16 d^5 e^3)) \sqrt{-e^2 x^2 + d^2})$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/(e\*x+d)\*\*2,x)

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x^2/(e\*x + d)^2,x, algorithm="giac")

[Out] Timed out

$$3.161 \quad \int \frac{x(d^2 - e^2 x^2)^{5/2}}{(d + ex)^2} dx$$

**Optimal.** Leaf size=136

$$-\frac{dx (d^2 - e^2 x^2)^{3/2}}{6e} - \frac{(d^2 - e^2 x^2)^{7/2}}{3e^2(d + ex)^2} - \frac{2(d^2 - e^2 x^2)^{5/2}}{15e^2} - \frac{d^5 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{4e^2} - \frac{d^3 x \sqrt{d^2 - e^2 x^2}}{4e}$$

[Out]  $-(d^3 x \sqrt{d^2 - e^2 x^2}) / (4 e) - (d^3 x (d^2 - e^2 x^2)^{3/2}) / (6 e) - (2 (d^2 - e^2 x^2)^{5/2}) / (15 e^2) - (d^2 - e^2 x^2)^{7/2} / (3 e^2 (d + e x)^2) - (d^5 \operatorname{ArcTan}[(e x) / \sqrt{d^2 - e^2 x^2}]) / (4 e^2)$

**Rubi [A]** time = 0.150591, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$-\frac{dx (d^2 - e^2 x^2)^{3/2}}{6e} - \frac{(d^2 - e^2 x^2)^{7/2}}{3e^2(d + ex)^2} - \frac{2(d^2 - e^2 x^2)^{5/2}}{15e^2} - \frac{d^5 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{4e^2} - \frac{d^3 x \sqrt{d^2 - e^2 x^2}}{4e}$$

Antiderivative was successfully verified.

[In] Int[(x\*(d^2 - e^2\*x^2)^(5/2))/(d + e\*x)^2, x]

[Out]  $-(d^3 x \sqrt{d^2 - e^2 x^2}) / (4 e) - (d^3 x (d^2 - e^2 x^2)^{3/2}) / (6 e) - (2 (d^2 - e^2 x^2)^{5/2}) / (15 e^2) - (d^2 - e^2 x^2)^{7/2} / (3 e^2 (d + e x)^2) - (d^5 \operatorname{ArcTan}[(e x) / \sqrt{d^2 - e^2 x^2}]) / (4 e^2)$

**Rubi in Sympy [A]** time = 22.1974, size = 116, normalized size = 0.85

$$-\frac{d^5 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{4e^2} - \frac{d^3 x \sqrt{d^2 - e^2 x^2}}{4e} - \frac{dx (d^2 - e^2 x^2)^{3/2}}{6e} - \frac{2 (d^2 - e^2 x^2)^{5/2}}{15e^2} - \frac{(d^2 - e^2 x^2)^{7/2}}{3e^2 (d + ex)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/(e\*x+d)\*\*2, x)

[Out]  $-d^5 \operatorname{atan}(e x / \sqrt{d^2 - e^2 x^2}) / (4 e^2) - d^3 x \sqrt{d^2 - e^2 x^2} / (4 e) - d^3 x (d^2 - e^2 x^2)^{3/2} / (6 e) - 2 (d^2 - e^2 x^2)^{5/2} / (15 e^2) - (d^2 - e^2 x^2)^{7/2} / (3 e^2 (d + e x)^2)$

**Mathematica [A]** time = 0.0859724, size = 91, normalized size = 0.67

$$\frac{\sqrt{d^2 - e^2 x^2} (-28 d^4 + 15 d^3 e x + 16 d^2 e^2 x^2 - 30 d e^3 x^3 + 12 e^4 x^4) - 15 d^5 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{60 e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(d^2 - e^2\*x^2)^(5/2))/(d + e\*x)^2, x]

[Out]  $(\sqrt{d^2 - e^2 x^2} (-28 d^4 + 15 d^3 e x + 16 d^2 e^2 x^2 - 30 d e^3 x^3 + 12 e^4 x^4) - 15 d^5 \operatorname{ArcTan}[(e x) / \sqrt{d^2 - e^2 x^2}]) / (60 e^2)$

)]/(60\*e^2)

**Maple [A]** time = 0.015, size = 198, normalized size = 1.5

$$\begin{aligned}
 & -\frac{2}{15e^2} \left( -\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right) \right)^{\frac{5}{2}} - \frac{dx}{6e} \left( -\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right) \right)^{\frac{3}{2}} \\
 & - \frac{d^3x}{4e} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right)} - \frac{d^5}{4e} \arctan \left( x\sqrt{e^2} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right)}} \right) \frac{1}{\sqrt{e^2}} \\
 & - \frac{1}{3e^4} \left( -\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right) \right)^{\frac{7}{2}} \left(x + \frac{d}{e}\right)^{-2}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(-e^2\*x^2+d^2)^(5/2)/(e\*x+d)^2,x)

[Out] -2/15/e^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(5/2)-1/6/e\*d\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)\*x-1/4/e\*d^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x-1/4/e\*d^5/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))-1/3/e^4/(x+d/e)^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)

**Maxima [A]** time = 0.792095, size = 225, normalized size = 1.65

$$\begin{aligned}
 & \frac{id^5 \arcsin\left(\frac{ex}{d} + 2\right)}{4e^2} - \frac{\sqrt{e^2x^2 + 4dex + 3d^2d^3x}}{4e} - \frac{(-e^2x^2 + d^2)^{\frac{5}{2}}d}{4(e^3x + de^2)} \\
 & - \frac{\sqrt{e^2x^2 + 4dex + 3d^2d^4}}{2e^2} + \frac{(-e^2x^2 + d^2)^{\frac{3}{2}}dx}{4e} - \frac{5(-e^2x^2 + d^2)^{\frac{3}{2}}d^2}{12e^2} + \frac{(-e^2x^2 + d^2)^{\frac{5}{2}}}{5e^2}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x/(e\*x + d)^2,x, algorithm="maxima")

[Out] 1/4\*I\*d^5\*arcsin(e\*x/d + 2)/e^2 - 1/4\*sqrt(e^2\*x^2 + 4\*d\*e\*x + 3\*d^2)\*d^3\*x/e - 1/4\*(-e^2\*x^2 + d^2)^(5/2)\*d/(e^3\*x + d\*e^2) - 1/2\*sqrt(e^2\*x^2 + 4\*d\*e\*x + 3\*d^2)\*d^4/e^2 + 1/4\*(-e^2\*x^2 + d^2)^(3/2)\*d\*x/e - 5/12\*(-e^2\*x^2 + d^2)^(3/2)\*d^2/e^2 + 1/5\*(-e^2\*x^2 + d^2)^(5/2)/e^2

**Fricas [A]** time = 0.295152, size = 509, normalized size = 3.74

$$12e^{10}x^{10} - 30de^9x^9 - 140d^2e^8x^8 + 405d^3e^7x^7 + 100d^4e^6x^6 - 1035d^5e^5x^5 + 480d^6e^4x^4 + 900d^7e^3x^3 - 480d^8e^2x^2 - 240d^9ex - 30d^{10}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x/(e\*x + d)^2,x, algorithm="fricas")

[Out] 1/60\*(12\*e^10\*x^10 - 30\*d\*e^9\*x^9 - 140\*d^2\*e^8\*x^8 + 405\*d^3\*e^7\*x^7 + 100\*d^4\*e^6\*x^6 - 1035\*d^5\*e^5\*x^5 + 480\*d^6\*e^4\*x^4 + 900\*d^7\*e^3\*x^3 - 480\*d^8\*e^2\*x^2 - 240\*d^9\*e\*x + 30\*(5\*d^6\*e^4\*x^4

$$- 20*d^8*e^2*x^2 + 16*d^10 - (d^5*e^4*x^4 - 12*d^7*e^2*x^2 + 16*d^9)*\sqrt{-e^2*x^2 + d^2}*\arctan(-(d - \sqrt{-e^2*x^2 + d^2})/(e*x)) + 5*(12*d^8*e^8*x^8 - 30*d^12*e^7*x^7 - 32*d^13*e^6*x^6 + 135*d^14*e^5*x^5 - 48*d^15*e^4*x^4 - 156*d^16*e^3*x^3 + 96*d^17*e^2*x^2 + 48*d^18*e*x)*\sqrt{-e^2*x^2 + d^2})/(5*d^6*e^6*x^4 - 20*d^13*e^4*x^2 + 16*d^15*e^2 - (e^6*x^4 - 12*d^12*e^4*x^2 + 16*d^14*e^2)*\sqrt{-e^2*x^2 + d^2}))$$

**Sympy [A]** time = 21.0022, size = 321, normalized size = 2.36

$$d^2 \left( \begin{cases} \frac{x^2\sqrt{d^2}}{2} & \text{for } e^2 = 0 \\ -\frac{(d^2 - e^2x^2)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases} \right) - 2de \left( \begin{cases} -\frac{id^4 \operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{id^3x}{8e^2\sqrt{-1 + \frac{e^2x^2}{d^2}}} - \frac{3idx^3}{8\sqrt{-1 + \frac{e^2x^2}{d^2}}} + \frac{ie^2x^5}{4d\sqrt{-1 + \frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^4 \operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3x}{8e^2\sqrt{1 - \frac{e^2x^2}{d^2}}} + \frac{3dx^3}{8\sqrt{1 - \frac{e^2x^2}{d^2}}} - \frac{e^2x^5}{4d\sqrt{1 - \frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases} \right) + e^2 \left( \begin{cases} -\frac{2d^4\sqrt{d^2 - e^2x^2}}{15e^4} - \frac{d^2x^2\sqrt{d^2 - e^2x^2}}{15e^2} + \frac{x^4\sqrt{d^2 - e^2x^2}}{5} & \text{for } e \neq 0 \\ \frac{x^4\sqrt{d^2}}{4} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/(e\*x+d)\*\*2,x)

[Out] d\*\*2\*Piecewise((x\*\*2\*sqrt(d\*\*2)/2, Eq(e\*\*2, 0)), (- (d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(3\*e\*\*2), True)) - 2\*d\*e\*Piecewise((-I\*d\*\*4\*acosh(e\*x/d)/(8\*e\*\*3) + I\*d\*\*3\*x/(8\*e\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 3\*I\*d\*x\*\*3/(8\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*5/(4\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*4\*asin(e\*x/d)/(8\*e\*\*3) - d\*\*3\*x/(8\*e\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 3\*d\*x\*\*3/(8\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*\*2\*x\*\*5/(4\*d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2))), True)) + e\*\*2\*Piecewise((-2\*d\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*e\*\*4) - d\*\*2\*x\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*e\*\*2) + x\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/5, Ne(e, 0)), (x\*\*4\*sqrt(d\*\*2)/4, True))

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x/(e\*x + d)^2,x, algorithm="giac")

[Out] Timed out



$$3.162 \quad \int \frac{(d^2 - e^2 x^2)^{5/2}}{(d + ex)^2} dx$$

**Optimal.** Leaf size=108

$$\frac{5}{8} d^2 x \sqrt{d^2 - e^2 x^2} + \frac{5d (d^2 - e^2 x^2)^{3/2}}{12e} + \frac{(d - ex) (d^2 - e^2 x^2)^{3/2}}{4e} + \frac{5d^4 \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right)}{8e}$$

[Out] (5\*d^2\*x\*Sqrt[d^2 - e^2\*x^2])/8 + (5\*d\*(d^2 - e^2\*x^2)^(3/2))/(12\*e) + ((d - e\*x)\*(d^2 - e^2\*x^2)^(3/2))/(4\*e) + (5\*d^4\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(8\*e)

**Rubi [A]** time = 0.112332, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{5}{8} d^2 x \sqrt{d^2 - e^2 x^2} + \frac{5d (d^2 - e^2 x^2)^{3/2}}{12e} + \frac{(d - ex) (d^2 - e^2 x^2)^{3/2}}{4e} + \frac{5d^4 \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right)}{8e}$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^(5/2)/(d + e\*x)^2, x]

[Out] (5\*d^2\*x\*Sqrt[d^2 - e^2\*x^2])/8 + (5\*d\*(d^2 - e^2\*x^2)^(3/2))/(12\*e) + ((d - e\*x)\*(d^2 - e^2\*x^2)^(3/2))/(4\*e) + (5\*d^4\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(8\*e)

**Rubi in Sympy [A]** time = 24.3507, size = 90, normalized size = 0.83

$$\frac{5d^4 \operatorname{atan} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right)}{8e} + \frac{5d^2 x \sqrt{d^2 - e^2 x^2}}{8} + \frac{5d (d^2 - e^2 x^2)^{3/2}}{12e} + \frac{(d - ex) (d^2 - e^2 x^2)^{3/2}}{4e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/(e\*x+d)\*\*2, x)

[Out] 5\*d\*\*4\*atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2))/(8\*e) + 5\*d\*\*2\*x\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/8 + 5\*d\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(12\*e) + (d - e\*x)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(4\*e)

**Mathematica [A]** time = 0.0685157, size = 80, normalized size = 0.74

$$\frac{15d^4 \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) + \sqrt{d^2 - e^2 x^2} (16d^3 + 9d^2 ex - 16de^2 x^2 + 6e^3 x^3)}{24e}$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^(5/2)/(d + e\*x)^2, x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(16\*d^3 + 9\*d^2\*e\*x - 16\*d\*e^2\*x^2 + 6\*e^3\*x^3) + 15\*d^4\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(24\*e)

**Maple [B]** time = 0.009, size = 194, normalized size = 1.8

$$\begin{aligned} & \frac{1}{3e^3d} \left( -\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right) \right)^{\frac{7}{2}} \left(x + \frac{d}{e}\right)^{-2} + \frac{1}{3de} \left( -\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right) \right)^{\frac{5}{2}} \\ & + \frac{5x}{12} \left( -\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right) \right)^{\frac{3}{2}} + \frac{5d^2x}{8} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right)} \\ & + \frac{5d^4}{8} \arctan \left( x\sqrt{e^2} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right)}} \right) \frac{1}{\sqrt{e^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^(5/2)/(e\*x+d)^2, x)

[Out] 1/3/e^3/d/(x+d/e)^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)+1/3/e/d\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(5/2)+5/12\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)\*x+5/8\*d^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x+5/8\*d^4/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))

**Maxima [A]** time = 0.792243, size = 161, normalized size = 1.49

$$\begin{aligned} & -\frac{5id^4 \arcsin\left(\frac{ex}{d} + 2\right)}{8e} + \frac{5}{8} \sqrt{e^2x^2 + 4dex + 3d^2d^2}x \\ & + \frac{5\sqrt{e^2x^2 + 4dex + 3d^2d^2}}{4e} + \frac{(-e^2x^2 + d^2)^{\frac{5}{2}}}{4(e^2x + de)} + \frac{5(-e^2x^2 + d^2)^{\frac{3}{2}}}{12e} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/(e\*x + d)^2, x, algorithm="maxima")

[Out] -5/8\*I\*d^4\*arcsin(e\*x/d + 2)/e + 5/8\*sqrt(e^2\*x^2 + 4\*d\*e\*x + 3\*d^2\*d^2)\*d^2\*x + 5/4\*sqrt(e^2\*x^2 + 4\*d\*e\*x + 3\*d^2\*d^2)\*d^3/e + 1/4\*(-e^2\*x^2 + d^2)^(5/2)/(e^2\*x + d\*e) + 5/12\*(-e^2\*x^2 + d^2)^(3/2)\*d/e

**Fricas [A]** time = 0.288018, size = 412, normalized size = 3.81

$$\frac{24de^7x^7 - 64d^2e^6x^6 - 36d^3e^5x^5 + 240d^4e^4x^4 - 60d^5e^3x^3 - 192d^6e^2x^2 + 72d^7ex + 30(d^4e^4x^4 - 8d^6e^2x^2 + 8d^8 + 4(d^5e^3x^3 - 12d^2e^5x^5 + 6d^4e^6x^6 - 3d^7e^7x^7))}{24(e^5x^4 - 8d^2e^3x^2 + d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/(e\*x + d)^2, x, algorithm="fricas")

[Out] -1/24\*(24\*d\*e^7\*x^7 - 64\*d^2\*e^6\*x^6 - 36\*d^3\*e^5\*x^5 + 240\*d^4\*e^4\*x^4 - 60\*d^5\*e^3\*x^3 - 192\*d^6\*e^2\*x^2 + 72\*d^7\*e\*x + 30\*(d^4\*e^4\*x^4 - 8\*d^6\*e^2\*x^2 + 8\*d^8 + 4\*(d^5\*e^3\*x^3 - 12\*d^2\*e^5\*x^5 + 6\*d^4\*e^6\*x^6 - 3\*d^7\*e^7\*x^7)))/24\*(e^5\*x^4 - 8\*d^2\*e^3\*x^2 + d^4)

**Sympy [A]** time = 23.4732, size = 350, normalized size = 3.24

$$d^2 \left( \begin{array}{l} \left( -\frac{id^2 \operatorname{acosh}\left(\frac{ex}{d}\right)}{2e} - \frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} \quad \text{for } \left| \frac{e^2x^2}{d^2} \right| > 1 \\ \frac{d^2 \operatorname{asin}\left(\frac{ex}{d}\right)}{2e} + \frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} \quad \text{otherwise} \end{array} \right) - 2de \left( \begin{array}{l} \frac{x^2\sqrt{d^2}}{2} \quad \text{for } e^2 = 0 \\ -\frac{(d^2 - e^2x^2)^{\frac{3}{2}}}{3e^2} \quad \text{otherwise} \end{array} \right) \\ + e^2 \left( \begin{array}{l} \left( -\frac{id^4 \operatorname{acosh}\left(\frac{ex}{d}\right)}{8e^3} + \frac{id^3x}{8e^2\sqrt{-1+\frac{e^2x^2}{d^2}}} - \frac{3idx^3}{8\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^5}{4d\sqrt{-1+\frac{e^2x^2}{d^2}}} \quad \text{for } \left| \frac{e^2x^2}{d^2} \right| > 1 \\ \frac{d^4 \operatorname{asin}\left(\frac{ex}{d}\right)}{8e^3} - \frac{d^3x}{8e^2\sqrt{1-\frac{e^2x^2}{d^2}}} + \frac{3dx^3}{8\sqrt{1-\frac{e^2x^2}{d^2}}} - \frac{e^2x^5}{4d\sqrt{1-\frac{e^2x^2}{d^2}}} \quad \text{otherwise} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/(e\*x+d)\*\*2,x)

[Out] d\*\*2\*Piecewise((-I\*d\*\*2\*acosh(e\*x/d)/(2\*e) - I\*d\*x/(2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*3/(2\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*2\*asin(e\*x/d)/(2\*e) + d\*x\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)/2, True)) - 2\*d\*e\*Piecewise((x\*\*2\*sqrt(d\*\*2)/2, Eq(e\*\*2, 0)), (- (d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(3\*e\*\*2), True)) + e\*\*2\*Piecewise((-I\*d\*\*4\*acosh(e\*x/d)/(8\*e\*\*3) + I\*d\*\*3\*x/(8\*e\*\*2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) - 3\*I\*d\*x\*\*3/(8\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*5/(4\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*4\*asin(e\*x/d)/(8\*e\*\*3) - d\*\*3\*x/(8\*e\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) + 3\*d\*x\*\*3/(8\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*\*2\*x\*\*5/(4\*d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True))

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/(e\*x + d)^2,x, algorithm="giac")

[Out] Timed out

$$3.163 \quad \int \frac{(d^2 - e^2 x^2)^{5/2}}{x(d + ex)^2} dx$$

**Optimal.** Leaf size=96

$$d(d - ex)\sqrt{d^2 - e^2 x^2} - \frac{1}{3} (d^2 - e^2 x^2)^{3/2} + d^3 \left( -\tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) \right) - d^3 \tanh^{-1} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right)$$

[Out] d\*(d - e\*x)\*Sqrt[d^2 - e^2\*x^2] - (d^2 - e^2\*x^2)^(3/2)/3 - d^3\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] - d^3\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]

**Rubi [A]** time = 0.390941, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$d(d - ex)\sqrt{d^2 - e^2 x^2} - \frac{1}{3} (d^2 - e^2 x^2)^{3/2} + d^3 \left( -\tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) \right) - d^3 \tanh^{-1} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right)$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^(5/2)/(x\*(d + e\*x)^2), x]

[Out] d\*(d - e\*x)\*Sqrt[d^2 - e^2\*x^2] - (d^2 - e^2\*x^2)^(3/2)/3 - d^3\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] - d^3\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]

**Rubi in Sympy [A]** time = 34.9221, size = 92, normalized size = 0.96

$$-d^3 \operatorname{atan} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) - d^3 \operatorname{atanh} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right) + d^2 \sqrt{d^2 - e^2 x^2} - dex \sqrt{d^2 - e^2 x^2} - \frac{(d^2 - e^2 x^2)^{3/2}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x/(e\*x+d)\*\*2, x)

[Out] -d\*\*3\*atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2)) - d\*\*3\*atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d) + d\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) - d\*e\*x\*sqrt(d\*\*2 - e\*\*2\*x\*\*2) - (d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/3

**Mathematica [A]** time = 0.0855334, size = 96, normalized size = 1.

$$d^3 \log(x) + \sqrt{d^2 - e^2 x^2} \left( \frac{2d^2}{3} - dex + \frac{e^2 x^2}{3} \right) - d^3 \log \left( \sqrt{d^2 - e^2 x^2} + d \right) + d^3 \left( -\tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^(5/2)/(x\*(d + e\*x)^2), x]

[Out] Sqrt[d^2 - e^2\*x^2]\*((2\*d^2)/3 - d\*e\*x + (e^2\*x^2)/3) - d^3\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] + d^3\*Log[x] - d^3\*Log[d + Sqrt[d^2 - e^2\*x^2]]

**Maple [B]** time = 0.017, size = 290, normalized size = 3.

$$\begin{aligned} & \frac{1}{5d^2} (-e^2x^2 + d^2)^{\frac{5}{2}} + \frac{1}{3} (-e^2x^2 + d^2)^{\frac{3}{2}} + d^2\sqrt{-e^2x^2 + d^2} \\ & - d^4 \ln\left(\frac{1}{x} \left(2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}} - \frac{8}{15d^2} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{5}{2}} \\ & - \frac{2ex}{3d} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{3}{2}} - de\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)x} \\ & - d^3e \arctan\left(x\sqrt{e^2} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)}}\right) \frac{1}{\sqrt{e^2}} \\ & - \frac{1}{3d^2e^2} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{7}{2}} \left(x + \frac{d}{e}\right)^{-2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^(5/2)/x/(e\*x+d)^2, x)

[Out] 1/5/d^2\*(-e^2\*x^2+d^2)^(5/2)+1/3\*(-e^2\*x^2+d^2)^(3/2)+d^2\*(-e^2\*x^2+d^2)^(1/2)-d^4/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)-8/15/d^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(5/2)-2/3/d\*e\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)\*x-d\*e\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x-d^3\*e/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))-1/3/d^2/e^2/(x+d/e)^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)^2\*x), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.300798, size = 392, normalized size = 4.08

$$\frac{e^6x^6 - 3de^5x^5 - 3d^2e^4x^4 + 15d^3e^3x^3 - 12d^5ex + 6\left(3d^4e^2x^2 - 4d^6 - (d^3e^2x^2 - 4d^5)\sqrt{-e^2x^2 + d^2}\right) \arctan\left(-\frac{d-\sqrt{-e^2x^2+d^2}}{ex}\right)}{3\left(3de^2x^2 - 4d^3 - (e^2x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)^2\*x), x, algorithm="fricas")

[Out] 1/3\*(e^6\*x^6 - 3\*d\*e^5\*x^5 - 3\*d^2\*e^4\*x^4 + 15\*d^3\*e^3\*x^3 - 12\*d^5\*e\*x + 6\*(3\*d^4\*e^2\*x^2 - 4\*d^6 - (d^3\*e^2\*x^2 - 4\*d^5)\*sqrt(-e^2\*x^2 + d^2))\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) + 3\*(3\*d^4\*e^2\*x^2 - 4\*d^6 - (d^3\*e^2\*x^2 - 4\*d^5)\*sqrt(-e^2\*x^2 + d^2))\*log(-(d - sqrt(-e^2\*x^2 + d^2))/x) + 3\*(d\*e^4\*x^4 - 3\*d^2\*e^3\*x^3 + 4\*d^4\*e\*x)\*sqrt(-e^2\*x^2 + d^2)/(3\*d\*e^2\*x^2 - 4\*d^3 - (e^2\*x^2 - 4\*d^2)\*sqrt(-e^2\*x^2 + d^2))

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**Sympy [A]** time = 19.0629, size = 267, normalized size = 2.78

$$d^2 \left( \begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}} - d \operatorname{acosh}\left(\frac{d}{ex}\right) - \frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}} + id \operatorname{asin}\left(\frac{d}{ex}\right) + \frac{iox}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases} \right) \\ -2de \left( \begin{cases} -\frac{id^2 \operatorname{acosh}\left(\frac{ex}{d}\right)}{2e} - \frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^2 \operatorname{asin}\left(\frac{ex}{d}\right)}{2e} + \frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise} \end{cases} \right) + e^2 \left( \begin{cases} \frac{x^2\sqrt{d^2}}{2} & \text{for } e^2 = 0 \\ -\frac{(d^2-e^2x^2)^{\frac{3}{2}}}{3e^2} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x/(e\*x+d)\*\*2,x)

[Out] d\*\*2\*Piecewise((d\*\*2/(e\*x\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) - d\*acosh(d/(e\*x)) - e\*x/sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (-I\*d\*\*2/(e\*x\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) + I\*d\*asin(d/(e\*x)) + I\*e\*x/sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1), True)) - 2\*d\*e\*Piecewise((-I\*d\*\*2\*acosh(e\*x/d)/(2\*e) - I\*d\*x/(2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*3/(2\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*2\*asin(e\*x/d)/(2\*e) + d\*x\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)/2, True)) + e\*\*2\*Piecewise((x\*\*2\*sqrt(d\*\*2)/2, Eq(e\*\*2, 0)), -(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(3\*e\*\*2), True))

---

**GIAC/XCAS [A]** time = 4.91639, size = 1, normalized size = 0.01

+∞

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)^2\*x),x, algorithm="giac")

[Out] +Infinity

$$3.164 \quad \int \frac{(d^2 - e^2 x^2)^{5/2}}{x^2(d+ex)^2} dx$$

**Optimal.** Leaf size=105

$$-\frac{(d^2 - e^2 x^2)^{3/2}}{x} - \frac{1}{2}e(4d + ex)\sqrt{d^2 - e^2 x^2} - \frac{1}{2}d^2 e \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right) + 2d^2 e \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)$$

[Out]  $-(e*(4*d + e*x)*\text{Sqrt}[d^2 - e^2*x^2])/2 - (d^2 - e^2*x^2)^{(3/2)}/x - (d^2*e*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/2 + 2*d^2*e*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d]$

**Rubi [A]** time = 0.382593, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{(d^2 - e^2 x^2)^{3/2}}{x} - \frac{1}{2}e(4d + ex)\sqrt{d^2 - e^2 x^2} - \frac{1}{2}d^2 e \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right) + 2d^2 e \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d^2 - e^2*x^2)^{(5/2)}/(x^2*(d + e*x)^2), x]$

[Out]  $-(e*(4*d + e*x)*\text{Sqrt}[d^2 - e^2*x^2])/2 - (d^2 - e^2*x^2)^{(3/2)}/x - (d^2*e*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/2 + 2*d^2*e*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d]$

**Rubi in Sympy [A]** time = 36.7024, size = 105, normalized size = 1.

$$-\frac{d^2 e \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{2} + 2d^2 e \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right) - \frac{d^2 \sqrt{d^2 - e^2 x^2}}{x} - 2de\sqrt{d^2 - e^2 x^2} + \frac{e^2 x \sqrt{d^2 - e^2 x^2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((-e**2*x**2+d**2)**(5/2)/x**2/(e*x+d)**2, x)$

[Out]  $-d**2*e*\operatorname{atan}(e*x/\text{sqrt}(d**2 - e**2*x**2))/2 + 2*d**2*e*\operatorname{atanh}(\text{sqrt}(d**2 - e**2*x**2)/d) - d**2*\text{sqrt}(d**2 - e**2*x**2)/x - 2*d*e*\text{sqrt}(d**2 - e**2*x**2) + e**2*x*\text{sqrt}(d**2 - e**2*x**2)/2$

**Mathematica [A]** time = 0.114991, size = 100, normalized size = 0.95

$$\left(-\frac{d^2}{x} - 2de + \frac{e^2 x}{2}\right)\sqrt{d^2 - e^2 x^2} + 2d^2 e \log\left(\sqrt{d^2 - e^2 x^2} + d\right) - \frac{1}{2}d^2 e \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right) - 2d^2 e \log(x)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(d^2 - e^2*x^2)^{(5/2)}/(x^2*(d + e*x)^2), x]$

[Out]  $(-2*d*e - d^2/x + (e^2*x)/2)*\text{Sqrt}[d^2 - e^2*x^2] - (d^2*e*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/2 - 2*d^2*e*\text{Log}[x] + 2*d^2*e*\text{Log}[d + \text{Sqrt}[d^2 - e^2*x^2]]$

**Maple [B]** time = 0.019, size = 425, normalized size = 4.1

$$\begin{aligned}
& -\frac{1}{d^4 x} (-e^2 x^2 + d^2)^{\frac{7}{2}} - \frac{e^2 x}{d^4} (-e^2 x^2 + d^2)^{\frac{5}{2}} - \frac{5 e^2 x}{4 d^2} (-e^2 x^2 + d^2)^{\frac{3}{2}} \\
& - \frac{15 e^2 x}{8} \sqrt{-e^2 x^2 + d^2} - \frac{15 d^2 e^2}{8} \arctan\left(x \sqrt{e^2} \frac{1}{\sqrt{-e^2 x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} \\
& + \frac{1}{3 d^3 e} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)\right)^{\frac{7}{2}} \left(x + \frac{d}{e}\right)^{-2} + \frac{11 e}{15 d^3} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)\right)^{\frac{5}{2}} \\
& + \frac{11 e^2 x}{12 d^2} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)\right)^{\frac{3}{2}} + \frac{11 e^2 x}{8} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)} \\
& + \frac{11 d^2 e^2}{8} \arctan\left(x \sqrt{e^2} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)}}\right) \frac{1}{\sqrt{e^2}} - \frac{2 e}{5 d^3} (-e^2 x^2 + d^2)^{\frac{5}{2}} \\
& - \frac{2 e}{3 d} (-e^2 x^2 + d^2)^{\frac{3}{2}} - 2 d e \sqrt{-e^2 x^2 + d^2} + 2 \frac{d^3 e}{\sqrt{d^2}} \ln\left(\frac{2 d^2 + 2 \sqrt{d^2} \sqrt{-e^2 x^2 + d^2}}{x}\right)
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^2,x)`

[Out] 
$$\begin{aligned}
& -1/d^4/x * (-e^2*x^2+d^2)^(7/2) - 1/d^4 * e^2 * x * (-e^2*x^2+d^2)^(5/2) - 5/ \\
& 4/d^4 * e^2 * x * (-e^2*x^2+d^2)^(3/2) - 15/8 * e^2 * x * (-e^2*x^2+d^2)^(1/2) - \\
& 15/8 * d^2 * e^2 / (e^2)^(1/2) * \arctan((e^2)^(1/2) * x / (-e^2*x^2+d^2)^(1/2) \\
& ) + 1/3/d^3/e / (x+d/e)^2 * (- (x+d/e)^2 * e^2 + 2 * d * e * (x+d/e))^(7/2) + 11/15 \\
& /d^3 * e * (- (x+d/e)^2 * e^2 + 2 * d * e * (x+d/e))^(5/2) + 11/12/d^2 * e^2 * (- (x+d/ \\
& e)^2 * e^2 + 2 * d * e * (x+d/e))^(3/2) * x + 11/8 * e^2 * (- (x+d/e)^2 * e^2 + 2 * d * e * (x \\
& +d/e))^(1/2) * x + 11/8 * d^2 * e^2 / (e^2)^(1/2) * \arctan((e^2)^(1/2) * x / (- (x \\
& +d/e)^2 * e^2 + 2 * d * e * (x+d/e))^(1/2)) - 2/5/d^3 * e * (-e^2*x^2+d^2)^(5/2) - \\
& 2/3/d * e * (-e^2*x^2+d^2)^(3/2) - 2 * d * e * (-e^2*x^2+d^2)^(1/2) + 2 * d^3 * e / ( \\
& d^2)^(1/2) * \ln((2 * d^2 + 2 * (d^2)^(1/2) * (-e^2*x^2+d^2)^(1/2)) / x)
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^2*x^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.295227, size = 429, normalized size = 4.09

$$\frac{e^6 x^6 - 4 d e^5 x^5 - 7 d^2 e^4 x^4 + 8 d^3 e^3 x^3 + 14 d^4 e^2 x^2 - 8 d^6 + 2 \left(3 d^3 e^3 x^3 - 4 d^5 e x - (d^2 e^3 x^3 - 4 d^4 e x) \sqrt{-e^2 x^2 + d^2}\right) \arctan\left(\frac{1}{\sqrt{e^2}}\right)}{2 \left(3 d e^2 x^3 - \dots\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^2*x^2),x, algorithm="fricas")`

[Out] 
$$\begin{aligned}
& 1/2 * (e^6 * x^6 - 4 * d * e^5 * x^5 - 7 * d^2 * e^4 * x^4 + 8 * d^3 * e^3 * x^3 + 14 * d \\
& ^4 * e^2 * x^2 - 8 * d^6 + 2 * (3 * d^3 * e^3 * x^3 - 4 * d^5 * e * x - (d^2 * e^3 * x^3 \\
& - 4 * d^4 * e * x) * \sqrt{-e^2 * x^2 + d^2})) * \arctan(-(d - \sqrt{-e^2 * x^2 + d^2}) / x)
\end{aligned}$$



$$\begin{aligned} &^2)) / (e^x)) - 4 * (3 * d^3 * e^3 * x^3 - 4 * d^5 * e^x - (d^2 * e^3 * x^3 - 4 * d^4 \\ &* e^x) * \sqrt{-e^2 * x^2 + d^2}) * \log(-(d - \sqrt{-e^2 * x^2 + d^2}) / x) + \\ &(3 * d * e^4 * x^4 - 8 * d^2 * e^3 * x^3 - 10 * d^3 * e^2 * x^2 + 8 * d^5) * \sqrt{-e^2 * \\ &x^2 + d^2}) / (3 * d * e^2 * x^3 - 4 * d^3 * x - (e^2 * x^3 - 4 * d^2 * x) * \sqrt{-e^2 * \\ &x^2 + d^2}) \end{aligned}$$


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**Sympy [A]** time = 22.7376, size = 347, normalized size = 3.3

$$\begin{aligned} &d^2 \left( \begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}} + ie \operatorname{acosh}\left(\frac{ex}{d}\right) - \frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}} - e \operatorname{asin}\left(\frac{ex}{d}\right) + \frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases} \right) \\ &- 2de \left( \begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}} - d \operatorname{acosh}\left(\frac{d}{ex}\right) - \frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}} + id \operatorname{asin}\left(\frac{d}{ex}\right) + \frac{iox}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases} \right) \\ &+ e^2 \left( \begin{cases} -\frac{id^2 \operatorname{acosh}\left(\frac{ex}{d}\right)}{2e} - \frac{idx}{2\sqrt{-1+\frac{e^2x^2}{d^2}}} + \frac{ie^2x^3}{2d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ \frac{d^2 \operatorname{asin}\left(\frac{ex}{d}\right)}{2e} + \frac{dx\sqrt{1-\frac{e^2x^2}{d^2}}}{2} & \text{otherwise} \end{cases} \right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*2/(e\*x+d)\*\*2,x)

[Out] d\*\*2\*Piecewise((I\*d/(x\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*acosh(e\*x/d) - I\*e\*\*2\*x/(d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (-d/(x\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*asin(e\*x/d) + e\*\*2\*x/(d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)), True)) - 2\*d\*e\*Piecewise((d\*\*2/(e\*x\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) - d\*acosh(d/(e\*x)) - e\*x/sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (-I\*d\*\*2/(e\*x\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) + I\*d\*asin(d/(e\*x)) + I\*e\*x/sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1), True)) + e\*\*2\*Piecewise((-I\*d\*\*2\*acosh(e\*x/d)/(2\*e) - I\*d\*x/(2\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*\*2\*x\*\*3/(2\*d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (d\*\*2\*asin(e\*x/d)/(2\*e) + d\*x\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)/2, True))

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**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)^2\*x^2),x, algorithm="giac")

[Out] Timed out

$$3.165 \quad \int \frac{(d^2 - e^2 x^2)^{5/2}}{x^3(d+ex)^2} dx$$

**Optimal.** Leaf size=110

$$\frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d^2-e^2x^2)^{3/2}}{2x^2} + 2de^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{2}de^2 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

[Out] (e\*(4\*d + e\*x)\*Sqrt[d^2 - e^2\*x^2])/(2\*x) - (d^2 - e^2\*x^2)^(3/2)/(2\*x^2) + 2\*d\*e^2\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] - (d\*e^2\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/2

**Rubi [A]** time = 0.397948, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{e(4d+ex)\sqrt{d^2-e^2x^2}}{2x} - \frac{(d^2-e^2x^2)^{3/2}}{2x^2} + 2de^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{1}{2}de^2 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^(5/2)/(x^3\*(d + e\*x)^2), x]

[Out] (e\*(4\*d + e\*x)\*Sqrt[d^2 - e^2\*x^2])/(2\*x) - (d^2 - e^2\*x^2)^(3/2)/(2\*x^2) + 2\*d\*e^2\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] - (d\*e^2\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/2

**Rubi in Sympy [A]** time = 39.8365, size = 107, normalized size = 0.97

$$-\frac{d^2\sqrt{d^2-e^2x^2}}{2x^2} + 2de^2 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) - \frac{de^2 \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2} + \frac{2de\sqrt{d^2-e^2x^2}}{x} + e^2\sqrt{d^2-e^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*3/(e\*x+d)\*\*2, x)

[Out] -d\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(2\*x\*\*2) + 2\*d\*e\*\*2\*atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2)) - d\*e\*\*2\*atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d)/2 + 2\*d\*e\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/x + e\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)

**Mathematica [A]** time = 0.127396, size = 102, normalized size = 0.93

$$\left(-\frac{d^2}{2x^2} + \frac{2de}{x} + e^2\right)\sqrt{d^2-e^2x^2} - \frac{1}{2}de^2 \log\left(\sqrt{d^2-e^2x^2} + d\right) + 2de^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{1}{2}de^2 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^(5/2)/(x^3\*(d + e\*x)^2), x]

[Out] (e^2 - d^2/(2\*x^2) + (2\*d\*e)/x)\*Sqrt[d^2 - e^2\*x^2] + 2\*d\*e^2\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] + (d\*e^2\*Log[x])/2 - (d\*e^2\*Log[d + Sqrt[d^2 - e^2\*x^2]])/2

**Maple [B]** time = 0.018, size = 456, normalized size = 4.2

$$\begin{aligned}
& -\frac{1}{2d^4x^2}(-e^2x^2+d^2)^{\frac{7}{2}} + \frac{e^2}{10d^4}(-e^2x^2+d^2)^{\frac{5}{2}} + \frac{e^2}{6d^2}(-e^2x^2+d^2)^{\frac{3}{2}} + \frac{e^2}{2}\sqrt{-e^2x^2+d^2} \\
& - \frac{d^2e^2}{2} \ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}\right)\right) \frac{1}{\sqrt{d^2}} - \frac{14e^2}{15d^4}\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}} \\
& - \frac{7e^3x}{6d^3}\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}} - \frac{7e^3x}{4d}\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)} \\
& - \frac{7de^3}{4} \arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right) \frac{1}{\sqrt{e^2}} + 2\frac{e(-e^2x^2+d^2)^{7/2}}{d^5x} \\
& + 2\frac{e^3x(-e^2x^2+d^2)^{5/2}}{d^5} + \frac{5e^3x}{2d^3}(-e^2x^2+d^2)^{\frac{3}{2}} + \frac{15e^3x}{4d}\sqrt{-e^2x^2+d^2} \\
& + \frac{15de^3}{4} \arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right) \frac{1}{\sqrt{e^2}} - \frac{1}{3d^4}\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}\left(x+\frac{d}{e}\right)^{-2}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-e^2*x^2+d^2)^(5/2)/x^3/(e*x+d)^2,x)`

[Out] `-1/2/d^4/x^2*(-e^2*x^2+d^2)^(7/2)+1/10/d^4*e^2*(-e^2*x^2+d^2)^(5/2)+1/6/d^2*e^2*(-e^2*x^2+d^2)^(3/2)+1/2*e^2*(-e^2*x^2+d^2)^(1/2)-1/2*d^2*e^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-14/15/d^4*e^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)-7/6/d^3*e^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)*x-7/4/d*e^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)*x-7/4*d*e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))+2/d^5*e/x*(-e^2*x^2+d^2)^(7/2)+2/d^5*e^3*x*(-e^2*x^2+d^2)^(5/2)+5/2/d^3*e^3*x*(-e^2*x^2+d^2)^(3/2)+15/4/d*e^3*x*(-e^2*x^2+d^2)^(1/2)+15/4*d*e^3/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/3/d^4/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2+d^2)^(5/2)/((e*x+d)^2*x^3),x,algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.293698, size = 470, normalized size = 4.27

$$\frac{2e^6x^6 + 4de^5x^5 - 5d^2e^4x^4 - 20d^3e^3x^3 + 5d^4e^2x^2 + 16d^5ex - 4d^6 - 8\left(3d^2e^4x^4 - 4d^4e^2x^2 - (de^4x^4 - 4d^3e^2x^2)\sqrt{-e^2x^2+d^2}\right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2+d^2)^(5/2)/((e*x+d)^2*x^3),x,algorithm="fricas")`

[Out] `1/2*(2*e^6*x^6 + 4*d*e^5*x^5 - 5*d^2*e^4*x^4 - 20*d^3*e^3*x^3 + 5*d^4*e^2*x^2 + 16*d^5*e*x - 4*d^6 - 8*(3*d^2*e^4*x^4 - 4*d^4*e^2*x^2 - (d*e^4*x^4 - 4*d^3*e^2*x^2)*sqrt(-e^2*x^2+d^2)))/((e*x+d)^2*x^3)`

$$x^2 - (d^2 e^4 x^4 - 4 d^3 e^2 x^2) \sqrt{-e^2 x^2 + d^2} \arctan\left(\frac{d - \sqrt{-e^2 x^2 + d^2}}{e x}\right) + (3 d^2 e^4 x^4 - 4 d^4 e^2 x^2 - (d^2 e^4 x^4 - 4 d^3 e^2 x^2) \sqrt{-e^2 x^2 + d^2}) \log\left(\frac{d - \sqrt{-e^2 x^2 + d^2}}{x}\right) + (4 d^2 e^4 x^4 + 12 d^2 e^3 x^3 - 3 d^3 e^2 x^2 - 16 d^4 e x + 4 d^5) \sqrt{-e^2 x^2 + d^2} / (3 d^2 e^2 x^4 - 4 d^3 x^2 - (e^2 x^4 - 4 d^2 x^2) \sqrt{-e^2 x^2 + d^2})$$

**Sympy [A]** time = 23.3053, size = 347, normalized size = 3.15

$$d^2 \left( \begin{cases} -\frac{d^2}{2ex^3\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e}{2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^2 \operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{2x} - \frac{ie^2 \operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases} \right) - 2de \left( \begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}} + ie \operatorname{acosh}\left(\frac{ex}{d}\right) - \frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left|\frac{e^2x^2}{d^2}\right| > 1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}} - e \operatorname{asin}\left(\frac{ex}{d}\right) + \frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases} \right) + e^2 \left( \begin{cases} \frac{d^2}{ex\sqrt{\frac{d^2}{e^2x^2}-1}} - d \operatorname{acosh}\left(\frac{d}{ex}\right) - \frac{ex}{\sqrt{\frac{d^2}{e^2x^2}-1}} & \text{for } \left|\frac{d^2}{e^2x^2}\right| > 1 \\ -\frac{id^2}{ex\sqrt{-\frac{d^2}{e^2x^2}+1}} + id \operatorname{asin}\left(\frac{d}{ex}\right) + \frac{iox}{\sqrt{-\frac{d^2}{e^2x^2}+1}} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*3/(e\*x+d)\*\*2,x)

[Out] d\*\*2\*Piecewise((-d\*\*2/(2\*e\*x\*\*3\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) + e/(2\*x\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) + e\*\*2\*acosh(d/(e\*x))/(2\*d), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (-I\*e\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)/(2\*x) - I\*e\*\*2\*asin(d/(e\*x))/(2\*d), True)) - 2\*d\*e\*Piecewise((I\*d/(x\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2)) + I\*e\*acosh(e\*x/d) - I\*e\*\*2\*x/(d\*sqrt(-1 + e\*\*2\*x\*\*2/d\*\*2))), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (-d/(x\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)) - e\*asin(e\*x/d) + e\*\*2\*x/(d\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2))), True)) + e\*\*2\*Piecewise((d\*\*2/(e\*x\*sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1)) - d\*acosh(d/(e\*x)) - e\*x/sqrt(d\*\*2/(e\*\*2\*x\*\*2) - 1), Abs(d\*\*2/(e\*\*2\*x\*\*2)) > 1), (-I\*d\*\*2/(e\*x\*sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1)) + I\*d\*asin(d/(e\*x)) + I\*e\*x/sqrt(-d\*\*2/(e\*\*2\*x\*\*2) + 1), True))

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)^2\*x^3),x, algorithm="giac")

[Out] Timed out

$$3.166 \quad \int \frac{(d^2 - e^2 x^2)^{5/2}}{x^4 (d + ex)^2} dx$$

**Optimal.** Leaf size=102

$$\frac{e(d - ex)\sqrt{d^2 - e^2 x^2}}{x^2} - \frac{(d^2 - e^2 x^2)^{3/2}}{3x^3} + e^3 \left( -\tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) \right) - e^3 \tanh^{-1} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right)$$

[Out] (e\*(d - e\*x)\*Sqrt[d^2 - e^2\*x^2])/x^2 - (d^2 - e^2\*x^2)^(3/2)/(3\*x^3) - e^3\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] - e^3\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]

**Rubi [A]** time = 0.38979, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{e(d - ex)\sqrt{d^2 - e^2 x^2}}{x^2} - \frac{(d^2 - e^2 x^2)^{3/2}}{3x^3} + e^3 \left( -\tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) \right) - e^3 \tanh^{-1} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right)$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^(5/2)/(x^4\*(d + e\*x)^2), x]

[Out] (e\*(d - e\*x)\*Sqrt[d^2 - e^2\*x^2])/x^2 - (d^2 - e^2\*x^2)^(3/2)/(3\*x^3) - e^3\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] - e^3\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]

**Rubi in Sympy [A]** time = 37.3722, size = 99, normalized size = 0.97

$$\frac{de\sqrt{d^2 - e^2 x^2}}{x^2} - e^3 \operatorname{atan} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) - e^3 \operatorname{atanh} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right) - \frac{e^2 \sqrt{d^2 - e^2 x^2}}{x} - \frac{(d^2 - e^2 x^2)^{3/2}}{3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*4/(e\*x+d)\*\*2, x)

[Out] d\*e\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/x\*\*2 - e\*\*3\*atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2)) - e\*\*3\*atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d) - e\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/x - (d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(3\*x\*\*3)

**Mathematica [A]** time = 0.164665, size = 96, normalized size = 0.94

$$-\frac{\sqrt{d^2 - e^2 x^2} (d^2 - 3dex + 2e^2 x^2)}{3x^3} - e^3 \log \left( \sqrt{d^2 - e^2 x^2} + d \right) + e^3 \left( -\tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) \right) + e^3 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^(5/2)/(x^4\*(d + e\*x)^2), x]

[Out] -(Sqrt[d^2 - e^2\*x^2]\*(d^2 - 3\*d\*e\*x + 2\*e^2\*x^2))/(3\*x^3) - e^3\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] + e^3\*Log[x] - e^3\*Log[d + Sqrt[d^2 - e^2\*x^2]]

**Maple [B]** time = 0.02, size = 479, normalized size = 4.7

$$\begin{aligned}
& -\frac{1}{3d^4x^3}(-e^2x^2+d^2)^{\frac{7}{2}} - \frac{5e^2}{3d^6x}(-e^2x^2+d^2)^{\frac{7}{2}} - \frac{5e^4x}{3d^6}(-e^2x^2+d^2)^{\frac{5}{2}} \\
& - \frac{25e^4x}{12d^4}(-e^2x^2+d^2)^{\frac{3}{2}} - \frac{25e^4x}{8d^2}\sqrt{-e^2x^2+d^2} - \frac{25e^4}{8}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}} \\
& + \frac{e}{3d^5}\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}\left(x+\frac{d}{e}\right)^{-2} + \frac{17e^3}{15d^5}\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}} \\
& + \frac{17e^4x}{12d^4}\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}} + \frac{17e^4x}{8d^2}\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)} \\
& + \frac{17e^4}{8}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)\frac{1}{\sqrt{e^2}} \\
& + \frac{e^3}{5d^5}(-e^2x^2+d^2)^{\frac{5}{2}} + \frac{e^3}{3d^3}(-e^2x^2+d^2)^{\frac{3}{2}} + \frac{e^3}{d}\sqrt{-e^2x^2+d^2} \\
& - de^3\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}} + \frac{e}{d^5x^2}(-e^2x^2+d^2)^{\frac{7}{2}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-e^2*x^2+d^2)^(5/2)/x^4/(e*x+d)^2,x)`

[Out] 
$$\begin{aligned}
& -1/3/d^4/x^3*(-e^2*x^2+d^2)^(7/2)-5/3/d^6*e^2/x*(-e^2*x^2+d^2)^(7/2) \\
& -5/3/d^6*e^4*x*(-e^2*x^2+d^2)^(5/2)-25/12/d^4*e^4*x*(-e^2*x^2+d^2)^(3/2) \\
& -25/8/d^2*e^4*x*(-e^2*x^2+d^2)^(1/2)-25/8*e^4/(e^2)^(1/2)*\arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)) \\
& +1/3/d^5*e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2) \\
& +17/15/d^5*e^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2) \\
& +17/12/d^4*e^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2) \\
& *x+17/8/d^2*e^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)*x+17/8*e^4/(e^2)^(1/2)*\arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2)) \\
& +1/5/d^5*e^3*(-e^2*x^2+d^2)^(5/2)+1/3/d^3*e^3*(-e^2*x^2+d^2)^(3/2) \\
& +1/d*e^3*(-e^2*x^2+d^2)^(1/2)-d*e^3/(d^2)^(1/2)*\ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x) \\
& +1/d^5*e/x^2*(-e^2*x^2+d^2)^(7/2)
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^2*x^4),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.293064, size = 463, normalized size = 4.54

$$2e^6x^6 - 3de^5x^5 - 9d^2e^4x^4 + 15d^3e^3x^3 + 3d^4e^2x^2 - 12d^5ex + 4d^6 - 6\left(3de^5x^5 - 4d^3e^3x^3 - (e^5x^5 - 4d^2e^3x^3)\sqrt{-e^2x^2 + d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^2*x^4),x, algorithm="fricas")`

```
[Out] -1/3*(2*e^6*x^6 - 3*d*e^5*x^5 - 9*d^2*e^4*x^4 + 15*d^3*e^3*x^3 +
3*d^4*e^2*x^2 - 12*d^5*e*x + 4*d^6 - 6*(3*d*e^5*x^5 - 4*d^3*e^3*x^
^3 - (e^5*x^5 - 4*d^2*e^3*x^3)*sqrt(-e^2*x^2 + d^2))*arctan(-(d -
sqrt(-e^2*x^2 + d^2))/(e*x)) - 3*(3*d*e^5*x^5 - 4*d^3*e^3*x^3 -
(e^5*x^5 - 4*d^2*e^3*x^3)*sqrt(-e^2*x^2 + d^2))*log(-(d - sqrt(-e
^2*x^2 + d^2))/x) + (6*d*e^4*x^4 - 9*d^2*e^3*x^3 - 5*d^3*e^2*x^2
+ 12*d^4*e*x - 4*d^5)*sqrt(-e^2*x^2 + d^2))/(3*d*e^2*x^5 - 4*d^3*
x^3 - (e^2*x^5 - 4*d^2*x^3)*sqrt(-e^2*x^2 + d^2))
```

---

**Sympy [A]** time = 26.1695, size = 338, normalized size = 3.31

$$d^2 \left( \begin{cases} -\frac{e\sqrt{\frac{d^2}{e^2x^2}-1}}{3x^2} + \frac{e^3\sqrt{\frac{d^2}{e^2x^2}-1}}{3d^2} & \text{for } \left| \frac{d^2}{e^2x^2} \right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{3x^2} + \frac{ie^3\sqrt{-\frac{d^2}{e^2x^2}+1}}{3d^2} & \text{otherwise} \end{cases} \right) - 2de \left( \begin{cases} -\frac{d^2}{2ex^3\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e}{2x\sqrt{\frac{d^2}{e^2x^2}-1}} + \frac{e^2\operatorname{acosh}\left(\frac{d}{ex}\right)}{2d} & \text{for } \left| \frac{d^2}{e^2x^2} \right| > 1 \\ -\frac{ie\sqrt{-\frac{d^2}{e^2x^2}+1}}{2x} - \frac{ie^2\operatorname{asin}\left(\frac{d}{ex}\right)}{2d} & \text{otherwise} \end{cases} \right) + e^2 \left( \begin{cases} \frac{id}{x\sqrt{-1+\frac{e^2x^2}{d^2}}} + ie\operatorname{acosh}\left(\frac{ex}{d}\right) - \frac{ie^2x}{d\sqrt{-1+\frac{e^2x^2}{d^2}}} & \text{for } \left| \frac{e^2x^2}{d^2} \right| > 1 \\ -\frac{d}{x\sqrt{1-\frac{e^2x^2}{d^2}}} - e\operatorname{asin}\left(\frac{ex}{d}\right) + \frac{e^2x}{d\sqrt{1-\frac{e^2x^2}{d^2}}} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**4/(e*x+d)**2,x)
```

```
[Out] d**2*Piecewise((-e*sqrt(d**2/(e**2*x**2) - 1)/(3*x**2) + e**3*sqrt(d**2/(e**2*x**2) - 1)/(3*d**2), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(3*x**2) + I*e**3*sqrt(-d**2/(e**2*x**2) + 1)/(3*d**2), True)) - 2*d*Piecewise((-d**2/(2*e*x**3*sqrt(d**2/(e**2*x**2) - 1)) + e/(2*x*sqrt(d**2/(e**2*x**2) - 1)) + e**2*acosh(d/(e*x))/(2*d), Abs(d**2/(e**2*x**2)) > 1), (-I*e*sqrt(-d**2/(e**2*x**2) + 1)/(2*x) - I*e**2*asin(d/(e*x))/(2*d), True)) + e**2*Piecewise((I*d/(x*sqrt(-1 + e**2*x**2/d**2)) + I*e*acosh(e*x/d) - I*e**2*x/(d*sqrt(-1 + e**2*x**2/d**2)), Abs(e**2*x**2/d**2) > 1), (-d/(x*sqrt(1 - e**2*x**2/d**2)) - e*asin(e*x/d) + e**2*x/(d*sqrt(1 - e**2*x**2/d**2)), True))
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^2*x^4),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.167 \quad \int \frac{(d^2 - e^2 x^2)^{5/2}}{x^5 (d + ex)^2} dx$$

**Optimal.** Leaf size=108

$$-\frac{5e^2 \sqrt{d^2 - e^2 x^2}}{8x^2} - \frac{(d^2 - e^2 x^2)^{3/2}}{4x^4} + \frac{2e (d^2 - e^2 x^2)^{3/2}}{3dx^3} + \frac{5e^4 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{8d}$$

[Out]  $(-5 * e^2 * \text{Sqrt}[d^2 - e^2 * x^2]) / (8 * x^2) - (d^2 - e^2 * x^2)^{(3/2)} / (4 * x^4) + (2 * e * (d^2 - e^2 * x^2)^{(3/2)}) / (3 * d * x^3) + (5 * e^4 * \text{ArcTanh}[\text{Sqrt}[d^2 - e^2 * x^2] / d]) / (8 * d)$

**Rubi [A]** time = 0.328318, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$-\frac{5e^2 \sqrt{d^2 - e^2 x^2}}{8x^2} - \frac{(d^2 - e^2 x^2)^{3/2}}{4x^4} + \frac{2e (d^2 - e^2 x^2)^{3/2}}{3dx^3} + \frac{5e^4 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{8d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d^2 - e^2 * x^2)^{(5/2)} / (x^5 * (d + e * x)^2), x]$

[Out]  $(-5 * e^2 * \text{Sqrt}[d^2 - e^2 * x^2]) / (8 * x^2) - (d^2 - e^2 * x^2)^{(3/2)} / (4 * x^4) + (2 * e * (d^2 - e^2 * x^2)^{(3/2)}) / (3 * d * x^3) + (5 * e^4 * \text{ArcTanh}[\text{Sqrt}[d^2 - e^2 * x^2] / d]) / (8 * d)$

**Rubi in Sympy [A]** time = 38.5662, size = 94, normalized size = 0.87

$$-\frac{d^2 \sqrt{d^2 - e^2 x^2}}{4x^4} - \frac{3e^2 \sqrt{d^2 - e^2 x^2}}{8x^2} + \frac{5e^4 \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{8d} + \frac{2e (d^2 - e^2 x^2)^{\frac{3}{2}}}{3dx^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((-e^{**2} * x^{**2} + d^{**2})^{** (5/2)} / x^{**5} / (e * x + d)^{**2}, x)$

[Out]  $-d^{**2} * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (4 * x^{**4}) - 3 * e^{**2} * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (8 * x^{**2}) + 5 * e^{**4} * \operatorname{atanh}(\text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / d) / (8 * d) + 2 * e * (d^{**2} - e^{**2} * x^{**2})^{** (3/2)} / (3 * d * x^{**3})$

**Mathematica [A]** time = 0.122108, size = 95, normalized size = 0.88

$$\frac{-15e^4 x^4 \log\left(\sqrt{d^2 - e^2 x^2} + d\right) + \sqrt{d^2 - e^2 x^2} (6d^3 - 16d^2 ex + 9de^2 x^2 + 16e^3 x^3) + 15e^4 x^4 \log(x)}{24dx^4}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(d^2 - e^2 * x^2)^{(5/2)} / (x^5 * (d + e * x)^2), x]$

[Out]  $-(\text{Sqrt}[d^2 - e^2 * x^2] * (6 * d^3 - 16 * d^2 * e * x + 9 * d * e^2 * x^2 + 16 * e^3 * x^3) + 15 * e^4 * x^4 * \text{Log}[x] - 15 * e^4 * x^4 * \text{Log}[d + \text{Sqrt}[d^2 - e^2 * x^2]]) / (24 * d * x^4)$



**Maple [B]** time = 0.02, size = 513, normalized size = 4.8

$$\begin{aligned}
& -\frac{1}{4d^4x^4}(-e^2x^2+d^2)^{\frac{7}{2}} - \frac{9e^2}{8d^6x^2}(-e^2x^2+d^2)^{\frac{7}{2}} \\
& - \frac{e^4}{8d^6}(-e^2x^2+d^2)^{\frac{5}{2}} - \frac{5e^4}{24d^4}(-e^2x^2+d^2)^{\frac{3}{2}} - \frac{5e^4}{8d^2}\sqrt{-e^2x^2+d^2} \\
& + \frac{5e^4}{8}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}} - \frac{4e^4}{3d^6}\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{5}{2}} \\
& - \frac{5e^5x}{3d^5}\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}} - \frac{5e^5x}{2d^3}\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)} \\
& - \frac{5e^5}{2d}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)\frac{1}{\sqrt{e^2}} \\
& + \frac{4e^3}{3d^7x}(-e^2x^2+d^2)^{\frac{7}{2}} + \frac{4e^5x}{3d^7}(-e^2x^2+d^2)^{\frac{5}{2}} + \frac{5e^5x}{3d^5}(-e^2x^2+d^2)^{\frac{3}{2}} \\
& + \frac{5e^5x}{2d^3}\sqrt{-e^2x^2+d^2} + \frac{5e^5}{2d}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}} \\
& - \frac{e^2}{3d^6}\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}\left(x+\frac{d}{e}\right)^{-2} + \frac{2e}{3d^5x^3}(-e^2x^2+d^2)^{\frac{7}{2}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-e^2*x^2+d^2)^(5/2)/x^5/(e*x+d)^2,x)`

[Out]  $-1/4/d^4/x^4*(-e^2*x^2+d^2)^(7/2)-9/8/d^6*e^2/x^2*(-e^2*x^2+d^2)^(7/2)-1/8/d^6*e^4*(-e^2*x^2+d^2)^(5/2)-5/24/d^4*e^4*(-e^2*x^2+d^2)^(3/2)-5/8/d^2*e^4*(-e^2*x^2+d^2)^(1/2)+5/8*e^4/(d^2)^(1/2)*\ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)-4/3/d^6*e^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)-5/3/d^5*e^5*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)*x-5/2/d*e^5/(e^2)^(1/2)*\arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2))+4/3/d^7*e^3/x*(-e^2*x^2+d^2)^(7/2)+4/3/d^7*e^5*x*(-e^2*x^2+d^2)^(5/2)+5/3/d^5*e^5*x*(-e^2*x^2+d^2)^(3/2)+5/2/d^3*e^5*x*(-e^2*x^2+d^2)^(1/2)+5/2/d*e^5/(e^2)^(1/2)*\arctan((e^2)^(1/2)*x/(-(e^2*x^2+d^2)^(1/2))-1/3/d^6*e^2/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(7/2)+2/3/d^5*e/x^3*(-e^2*x^2+d^2)^(7/2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2+d^2)^(5/2)/((e*x+d)^2*x^5),x,algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.291319, size = 425, normalized size = 3.94

$$\frac{64de^7x^7 + 36d^2e^6x^6 - 256d^3e^5x^5 - 84d^4e^4x^4 + 320d^5e^3x^3 - 128d^7ex + 48d^8 - 15\left(e^8x^8 - 8d^2e^6x^6 + 8d^4e^4x^4 + 4(de^6x^6 - 8d^2e^4x^4 + 4d^4e^2x^2 - 4d^6)\right)}{24\left(de^4x^8 - 8d^3e^2x^6 + 8d^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^2*x^5),x, algorithm="fricas")`

[Out] 
$$\frac{1}{24} (64 d e^7 x^7 + 36 d^2 e^6 x^6 - 256 d^3 e^5 x^5 - 84 d^4 e^4 x^4 + 320 d^5 e^3 x^3 - 128 d^7 e x + 48 d^8 - 15 (e^8 x^8 - 8 d^2 e^6 x^6 + 8 d^4 e^4 x^4 + 4 (d e^6 x^6 - 2 d^3 e^4 x^4) \sqrt{-e^2 x^2 + d^2})) \log\left(\frac{d - \sqrt{-e^2 x^2 + d^2}}{x}\right) - (16 e^7 x^7 + 9 d e^6 x^6 - 144 d^2 e^5 x^5 - 66 d^3 e^4 x^4 + 256 d^4 e^3 x^3 + 24 d^5 e^2 x^2 - 128 d^6 e x + 48 d^7) \sqrt{-e^2 x^2 + d^2}) / (d e^4 x^8 - 8 d^3 e^2 x^6 + 8 d^5 x^4 + 4 (d^2 e^2 x^6 - 2 d^4 x^4) \sqrt{-e^2 x^2 + d^2})$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e**2*x**2+d**2)**(5/2)/x**5/(e*x+d)**2,x)`

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^2*x^5),x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.168 \quad \int \frac{(d^2 - e^2 x^2)^{5/2}}{x^6 (d + ex)^2} dx$$

**Optimal.** Leaf size=140

$$-\frac{(d^2 - e^2 x^2)^{3/2}}{5x^5} + \frac{e(d^2 - e^2 x^2)^{3/2}}{2dx^4} - \frac{7e^2(d^2 - e^2 x^2)^{3/2}}{15d^2 x^3} - \frac{e^5 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{4d^2} + \frac{e^3 \sqrt{d^2 - e^2 x^2}}{4dx^2}$$

[Out] (e^3\*Sqrt[d^2 - e^2\*x^2])/(4\*d\*x^2) - (d^2 - e^2\*x^2)^(3/2)/(5\*x^5) + (e\*(d^2 - e^2\*x^2)^(3/2))/(2\*d\*x^4) - (7\*e^2\*(d^2 - e^2\*x^2)^(3/2))/(15\*d^2\*x^3) - (e^5\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/(4\*d^2)

**Rubi [A]** time = 0.415775, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$

$$-\frac{(d^2 - e^2 x^2)^{3/2}}{5x^5} + \frac{e(d^2 - e^2 x^2)^{3/2}}{2dx^4} - \frac{7e^2(d^2 - e^2 x^2)^{3/2}}{15d^2 x^3} - \frac{e^5 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{4d^2} + \frac{e^3 \sqrt{d^2 - e^2 x^2}}{4dx^2}$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^(5/2)/(x^6\*(d + e\*x)^2), x]

[Out] (e^3\*Sqrt[d^2 - e^2\*x^2])/(4\*d\*x^2) - (d^2 - e^2\*x^2)^(3/2)/(5\*x^5) + (e\*(d^2 - e^2\*x^2)^(3/2))/(2\*d\*x^4) - (7\*e^2\*(d^2 - e^2\*x^2)^(3/2))/(15\*d^2\*x^3) - (e^5\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/(4\*d^2)

**Rubi in Sympy [A]** time = 40.298, size = 116, normalized size = 0.83

$$\frac{de\sqrt{d^2 - e^2x^2}}{2x^4} - \frac{(d^2 - e^2x^2)^{3/2}}{5x^5} - \frac{e^3\sqrt{d^2 - e^2x^2}}{4dx^2} - \frac{e^5 \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)}{4d^2} - \frac{7e^2(d^2 - e^2x^2)^{3/2}}{15d^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*6/(e\*x+d)\*\*2, x)

[Out] d\*e\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(2\*x\*\*4) - (d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(5\*x\*\*5) - e\*\*3\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(4\*d\*x\*\*2) - e\*\*5\*atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d)/(4\*d\*\*2) - 7\*e\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(15\*d\*\*2\*x\*\*3)

**Mathematica [A]** time = 0.124901, size = 106, normalized size = 0.76

$$\frac{-15e^5x^5 \log\left(\sqrt{d^2 - e^2x^2} + d\right) + \sqrt{d^2 - e^2x^2}(-12d^4 + 30d^3ex - 16d^2e^2x^2 - 15de^3x^3 + 28e^4x^4) + 15e^5x^5 \log(x)}{60d^2x^5}$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^(5/2)/(x^6\*(d + e\*x)^2), x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(-12\*d^4 + 30\*d^3\*e\*x - 16\*d^2\*e^2\*x^2 - 15\*d\*e^3\*x^3 + 28\*e^4\*x^4) + 15\*e^5\*x^5\*Log[x] - 15\*e^5\*x^5\*Log[d +

$\text{Sqrt}[d^2 - e^2 x^2]]/(60 d^2 x^5)$

**Maple [B]** time = 0.022, size = 541, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((-e^2 x^2 + d^2)^{(5/2)}/x^6/(e x + d)^2, x)$

[Out] 
$$\begin{aligned} & -1/5/d^4/x^5 * (-e^2 x^2 + d^2)^{(7/2)} - 13/15/d^6 * e^2/x^3 * (-e^2 x^2 + d^2)^{(7/2)} - 23/15/d^8 * e^4/x * (-e^2 x^2 + d^2)^{(7/2)} - 23/15/d^8 * e^6 * x * (-e^2 x^2 + d^2)^{(5/2)} \\ & - 23/12/d^6 * e^6 * x * (-e^2 x^2 + d^2)^{(3/2)} - 23/8/d^4 * e^6 * x * (-e^2 x^2 + d^2)^{(1/2)} - 23/8/d^2 * e^6/(e^2)^{(1/2)} * \arctan((e^2)^{(1/2)} * x / (-e^2 x^2 + d^2)^{(1/2)}) \\ & + 1/3/d^7 * e^3/(x + d/e)^2 * (-x + d/e)^2 * e^2 + 2 * d * e * (x + d/e)^{(7/2)} + 23/15/d^7 * e^5 * (-x + d/e)^2 * e^2 + 2 * d * e * (x + d/e)^{(5/2)} \\ & + 23/12/d^6 * e^6 * (-x + d/e)^2 * e^2 + 2 * d * e * (x + d/e)^{(3/2)} * x + 23/8/d^4 * e^6 * (-x + d/e)^2 * e^2 + 2 * d * e * (x + d/e)^{(1/2)} * x + 23/8/d^2 * e^6/(e^2)^{(1/2)} * \arctan((e^2)^{(1/2)} * x / (-x + d/e)^2 * e^2 + 2 * d * e * (x + d/e)^{(1/2)}) \\ & + 1/20/d^7 * e^5 * (-e^2 x^2 + d^2)^{(5/2)} + 1/12/d^5 * e^5 * (-e^2 x^2 + d^2)^{(3/2)} + 1/4/d^3 * e^5 * (-e^2 x^2 + d^2)^{(1/2)} - 1/4/d * e^5/(d^2)^{(1/2)} * \ln((2 * d^2 + 2 * (d^2)^{(1/2)} * (-e^2 x^2 + d^2)^{(1/2)})/x) + 5/4/d^7 * e^3/x^2 * (-e^2 x^2 + d^2)^{(7/2)} \\ & + 1/2/d^5 * e/x^4 * (-e^2 x^2 + d^2)^{(7/2)} \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((-e^2 x^2 + d^2)^{(5/2)}/((e x + d)^2 x^6), x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.304752, size = 533, normalized size = 3.81

$28 e^{10} x^{10} - 15 d e^9 x^9 - 380 d^2 e^8 x^8 + 225 d^3 e^7 x^7 + 980 d^4 e^6 x^6 - 810 d^5 e^5 x^5 - 740 d^6 e^4 x^4 + 1080 d^7 e^3 x^3 - 80 d^8 e^2 x^2 - 480 d^9 e x - 16 d^{10}$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((-e^2 x^2 + d^2)^{(5/2)}/((e x + d)^2 x^6), x, \text{algorithm}="fricas")$

[Out] 
$$\begin{aligned} & 1/60 * (28 * e^{10} * x^{10} - 15 * d * e^9 * x^9 - 380 * d^2 * e^8 * x^8 + 225 * d^3 * e^7 * x^7 + 980 * d^4 * e^6 * x^6 - 810 * d^5 * e^5 * x^5 - 740 * d^6 * e^4 * x^4 + 1080 * d^7 * e^3 * x^3 - 80 * d^8 * e^2 * x^2 - 480 * d^9 * e * x - 16 * d^{10}) \\ & * \sqrt{-e^2 x^2 + d^2} * \log(-(d - \sqrt{-e^2 x^2 + d^2})/x) + (140 * d * e^8 * x^8 - 75 * d^2 * e^7 * x^7 - 640 * d^3 * e^6 * x^6 + 450 * d^4 * e^5 * x^5 + 708 * d^5 * e^4 * x^4 - 840 * d^6 * e^3 * x^3 - 16 * d^7 * e^2 * x^2 + 480 * d^8 * e * x - 192 * d^9) * \sqrt{-e^2 x^2 + d^2} / (5 * d^3 * e^4 * x^9 - 20 * d^5 * e^2 * x^7 + 16 * d^7 * x^5 - (d^2 * e^4 * x^9 - 12 * d^4 * e^2 * x^7 + 16 * d^6 * x^5) * \sqrt{-e^2 x^2 + d^2}) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(5/2)/x**6/(e*x+d)**2,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^2*x^6),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.169 \quad \int \frac{(d^2 - e^2 x^2)^{5/2}}{x^7 (d + ex)^2} dx$$

**Optimal.** Leaf size=169

$$\begin{aligned} & -\frac{(d^2 - e^2 x^2)^{3/2}}{6x^6} + \frac{2e(d^2 - e^2 x^2)^{3/2}}{5dx^5} - \frac{3e^2(d^2 - e^2 x^2)^{3/2}}{8d^2 x^4} \\ & - \frac{3e^4 \sqrt{d^2 - e^2 x^2}}{16d^2 x^2} + \frac{3e^6 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{16d^3} + \frac{4e^3(d^2 - e^2 x^2)^{3/2}}{15d^3 x^3} \end{aligned}$$

[Out]  $(-3 * e^4 * \text{Sqrt}[d^2 - e^2 * x^2]) / (16 * d^2 * x^2) - (d^2 - e^2 * x^2)^{(3/2)} / (6 * x^6) + (2 * e * (d^2 - e^2 * x^2)^{(3/2)}) / (5 * d * x^5) - (3 * e^2 * (d^2 - e^2 * x^2)^{(3/2)}) / (8 * d^2 * x^4) + (4 * e^3 * (d^2 - e^2 * x^2)^{(3/2)}) / (15 * d^3 * x^3) + (3 * e^6 * \text{ArcTanh}[\text{Sqrt}[d^2 - e^2 * x^2] / d]) / (16 * d^3)$

**Rubi [A]** time = 0.499429, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$

$$\begin{aligned} & -\frac{(d^2 - e^2 x^2)^{3/2}}{6x^6} + \frac{2e(d^2 - e^2 x^2)^{3/2}}{5dx^5} - \frac{3e^2(d^2 - e^2 x^2)^{3/2}}{8d^2 x^4} \\ & - \frac{3e^4 \sqrt{d^2 - e^2 x^2}}{16d^2 x^2} + \frac{3e^6 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{16d^3} + \frac{4e^3(d^2 - e^2 x^2)^{3/2}}{15d^3 x^3} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d^2 - e^2 * x^2)^{(5/2)} / (x^7 * (d + e * x)^2), x]$

[Out]  $(-3 * e^4 * \text{Sqrt}[d^2 - e^2 * x^2]) / (16 * d^2 * x^2) - (d^2 - e^2 * x^2)^{(3/2)} / (6 * x^6) + (2 * e * (d^2 - e^2 * x^2)^{(3/2)}) / (5 * d * x^5) - (3 * e^2 * (d^2 - e^2 * x^2)^{(3/2)}) / (8 * d^2 * x^4) + (4 * e^3 * (d^2 - e^2 * x^2)^{(3/2)}) / (15 * d^3 * x^3) + (3 * e^6 * \text{ArcTanh}[\text{Sqrt}[d^2 - e^2 * x^2] / d]) / (16 * d^3)$

**Rubi in Sympy [A]** time = 53.5247, size = 150, normalized size = 0.89

$$\begin{aligned} & -\frac{d^2 \sqrt{d^2 - e^2 x^2}}{6x^6} - \frac{5e^2 \sqrt{d^2 - e^2 x^2}}{24x^4} + \frac{2e(d^2 - e^2 x^2)^{3/2}}{5dx^5} \\ & + \frac{3e^4 \sqrt{d^2 - e^2 x^2}}{16d^2 x^2} + \frac{3e^6 \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{16d^3} + \frac{4e^3(d^2 - e^2 x^2)^{3/2}}{15d^3 x^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((-e^{**2} * x^{**2} + d^{**2})^{** (5/2)} / x^{**7} / (e * x + d)^{**2}, x)$

[Out]  $-d^{**2} * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (6 * x^{**6}) - 5 * e^{**2} * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (24 * x^{**4}) + 2 * e * (d^{**2} - e^{**2} * x^{**2})^{** (3/2)} / (5 * d * x^{**5}) + 3 * e^{**4} * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (16 * d^{**2} * x^{**2}) + 3 * e^{**6} * \operatorname{atanh}(\text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / d) / (16 * d^{**3}) + 4 * e^{**3} * (d^{**2} - e^{**2} * x^{**2})^{** (3/2)} / (15 * d^{**3} * x^{**3})$

**Mathematica [A]** time = 0.167923, size = 117, normalized size = 0.69

$$\frac{-45e^6 x^6 \log\left(\sqrt{d^2 - e^2 x^2} + d\right) + \sqrt{d^2 - e^2 x^2} (40d^5 - 96d^4 ex + 50d^3 e^2 x^2 + 32d^2 e^3 x^3 - 45de^4 x^4 + 64e^5 x^5) + 45e^6 x^6 \log(x)}{240d^3 x^6}$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^(5/2)/(x^7\*(d + e\*x)^2),x]

[Out]  $-(\text{Sqrt}[d^2 - e^2*x^2])*(40*d^5 - 96*d^4*e*x + 50*d^3*e^2*x^2 + 32*d^2*e^3*x^3 - 45*d*e^4*x^4 + 64*e^5*x^5) + 45*e^6*x^6*\text{Log}[x] - 45*e^6*x^6*\text{Log}[d + \text{Sqrt}[d^2 - e^2*x^2]]/(240*d^3*x^6)$

**Maple [B]** time = 0.025, size = 566, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^(5/2)/x^7/(e\*x+d)^2,x)

[Out]  $2/5/d^5*e/x^5*(-e^2*x^2+d^2)^{7/2}+16/15/d^7*e^3/x^3*(-e^2*x^2+d^2)^{7/2}-1/3/d^8*e^4/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^{7/2}+26/15/d^9*e^5/x*(-e^2*x^2+d^2)^{7/2}+26/15/d^9*e^7*x*(-e^2*x^2+d^2)^{5/2}+13/6/d^7*e^7*x*(-e^2*x^2+d^2)^{3/2}+13/4/d^5*e^7*x*(-e^2*x^2+d^2)^{1/2}+13/4/d^3*e^7/(e^2)^{1/2}*\text{arctan}((e^2)^{1/2}*x/(-e^2*x^2+d^2)^{1/2})-13/6/d^7*e^7*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^{3/2}*x-13/4/d^5*e^7*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^{1/2}*x-13/4/d^3*e^7/(e^2)^{1/2}*\text{arctan}((e^2)^{1/2}*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^{1/2})-17/24/d^6*e^2/x^4*(-e^2*x^2+d^2)^{7/2}-23/16/d^8*e^4/x^2*(-e^2*x^2+d^2)^{7/2}+3/16/d^2*e^6/(d^2)^{1/2}*\ln((2*d^2+2*(d^2)^{1/2}*(-e^2*x^2+d^2)^{1/2})/x)-26/15/d^8*e^6*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^{5/2}-1/6/d^4/x^6*(-e^2*x^2+d^2)^{7/2}-3/80/d^8*e^6*(-e^2*x^2+d^2)^{5/2}-1/16/d^6*e^6*(-e^2*x^2+d^2)^{3/2}-3/16/d^4*e^6*(-e^2*x^2+d^2)^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)^2\*x^7),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.360245, size = 624, normalized size = 3.69

$384 d e^{11} x^{11} - 270 d^2 e^{10} x^{10} - 2240 d^3 e^9 x^9 + 2010 d^4 e^8 x^8 + 2304 d^5 e^7 x^7 - 4540 d^6 e^6 x^6 + 3648 d^7 e^5 x^5 + 3120 d^8 e^4 x^4 - 7168 d^9 e^3 x^3 + 960 d^{10} e^2 x^2 + 3072 d^{11} e x - 1280 d^{12} - 45 (e^{12} x^{12} - 18 d^2 e^{10} x^{10} + 48 d^4 e^8 x^8 - 32 d^6 e^6 x^6 + 2 (3 d^8 e^4 x^4 - 16 d^{10} e^2 x^2 + 3 d^{12}))$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)^2\*x^7),x, algorithm="fricas")

[Out]  $1/240*(384*d*e^{11}*x^{11} - 270*d^2*e^{10}*x^{10} - 2240*d^3*e^9*x^9 + 2010*d^4*e^8*x^8 + 2304*d^5*e^7*x^7 - 4540*d^6*e^6*x^6 + 3648*d^7*e^5*x^5 + 3120*d^8*e^4*x^4 - 7168*d^9*e^3*x^3 + 960*d^{10}*e^2*x^2 + 3072*d^{11}*e*x - 1280*d^{12} - 45*(e^{12}*x^{12} - 18*d^2*e^{10}*x^{10} + 48*d^4*e^8*x^8 - 32*d^6*e^6*x^6 + 2*(3*d^8*e^4*x^4 - 16*d^{10}*e^2*x^2 + 3*d^{12})))$

$$\begin{aligned} &^8 + 16*d^5*e^6*x^6)*\sqrt{-e^2*x^2 + d^2})*\log(-(d - \sqrt{-e^2*x^2 \\ &2 + d^2}))/x) - (64*e^{11}*x^{11} - 45*d*e^{10}*x^{10} - 1120*d^2*e^9*x^9 \\ &+ 860*d^3*e^8*x^8 + 2400*d^4*e^7*x^7 - 3020*d^5*e^6*x^6 + 1216*d^6 \\ &6*e^5*x^5 + 3120*d^7*e^4*x^4 - 5632*d^8*e^3*x^3 + 320*d^9*e^2*x^2 \\ &+ 3072*d^{10}*e*x - 1280*d^{11})*\sqrt{-e^2*x^2 + d^2})/(d^3*e^6*x^{12} \\ &- 18*d^5*e^4*x^{10} + 48*d^7*e^2*x^8 - 32*d^9*x^6 + 2*(3*d^4*e^4*x \\ &^{10} - 16*d^6*e^2*x^8 + 16*d^8*x^6)*\sqrt{-e^2*x^2 + d^2}) \end{aligned}$$


---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*7/(e\*x+d)\*\*2,x)

[Out] Timed out

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)^2\*x^7),x, algorithm="giac")

[Out] Exception raised: TypeError



$$3.170 \quad \int \frac{(d^2 - e^2 x^2)^{5/2}}{x^8 (d + ex)^2} dx$$

**Optimal.** Leaf size=198

$$\begin{aligned} & -\frac{(d^2 - e^2 x^2)^{3/2}}{7x^7} + \frac{e(d^2 - e^2 x^2)^{3/2}}{3dx^6} - \frac{11e^2(d^2 - e^2 x^2)^{3/2}}{35d^2x^5} - \frac{e^7 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{8d^4} \\ & - \frac{22e^4(d^2 - e^2 x^2)^{3/2}}{105d^4x^3} + \frac{e^5\sqrt{d^2 - e^2 x^2}}{8d^3x^2} + \frac{e^3(d^2 - e^2 x^2)^{3/2}}{4d^3x^4} \end{aligned}$$

[Out] (e^5\*Sqrt[d^2 - e^2\*x^2])/(8\*d^3\*x^2) - (d^2 - e^2\*x^2)^(3/2)/(7\*x^7) + (e\*(d^2 - e^2\*x^2)^(3/2))/(3\*d\*x^6) - (11\*e^2\*(d^2 - e^2\*x^2)^(3/2))/(35\*d^2\*x^5) + (e^3\*(d^2 - e^2\*x^2)^(3/2))/(4\*d^3\*x^4) - (22\*e^4\*(d^2 - e^2\*x^2)^(3/2))/(105\*d^4\*x^3) - (e^7\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/(8\*d^4)

**Rubi [A]** time = 0.605738, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$

$$\begin{aligned} & -\frac{(d^2 - e^2 x^2)^{3/2}}{7x^7} + \frac{e(d^2 - e^2 x^2)^{3/2}}{3dx^6} - \frac{11e^2(d^2 - e^2 x^2)^{3/2}}{35d^2x^5} - \frac{e^7 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{8d^4} \\ & - \frac{22e^4(d^2 - e^2 x^2)^{3/2}}{105d^4x^3} + \frac{e^5\sqrt{d^2 - e^2 x^2}}{8d^3x^2} + \frac{e^3(d^2 - e^2 x^2)^{3/2}}{4d^3x^4} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^(5/2)/(x^8\*(d + e\*x)^2), x]

[Out] (e^5\*Sqrt[d^2 - e^2\*x^2])/(8\*d^3\*x^2) - (d^2 - e^2\*x^2)^(3/2)/(7\*x^7) + (e\*(d^2 - e^2\*x^2)^(3/2))/(3\*d\*x^6) - (11\*e^2\*(d^2 - e^2\*x^2)^(3/2))/(35\*d^2\*x^5) + (e^3\*(d^2 - e^2\*x^2)^(3/2))/(4\*d^3\*x^4) - (22\*e^4\*(d^2 - e^2\*x^2)^(3/2))/(105\*d^4\*x^3) - (e^7\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d])/(8\*d^4)

**Rubi in Sympy [A]** time = 54.1685, size = 168, normalized size = 0.85

$$\begin{aligned} & \frac{de\sqrt{d^2 - e^2x^2}}{3x^6} - \frac{(d^2 - e^2x^2)^{3/2}}{7x^7} - \frac{e^3\sqrt{d^2 - e^2x^2}}{12dx^4} - \frac{11e^2(d^2 - e^2x^2)^{3/2}}{35d^2x^5} \\ & - \frac{e^5\sqrt{d^2 - e^2x^2}}{8d^3x^2} - \frac{e^7 \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)}{8d^4} - \frac{22e^4(d^2 - e^2x^2)^{3/2}}{105d^4x^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*8/(e\*x+d)\*\*2, x)

[Out] d\*e\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(3\*x\*\*6) - (d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(7\*x\*\*7) - e\*\*3\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(12\*d\*x\*\*4) - 11\*e\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(35\*d\*\*2\*x\*\*5) - e\*\*5\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(8\*d\*\*3\*x\*\*2) - e\*\*7\*atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d)/(8\*d\*\*4) - 22\*e\*\*4\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(105\*d\*\*4\*x\*\*3)

**Mathematica [A]** time = 0.13369, size = 128, normalized size = 0.65

$$\frac{-105e^7x^7 \log\left(\sqrt{d^2 - e^2x^2} + d\right) + \sqrt{d^2 - e^2x^2}(-120d^6 + 280d^5ex - 144d^4e^2x^2 - 70d^3e^3x^3 + 88d^2e^4x^4 - 105de^5x^5 + 176e^6x^6)}{840d^4x^7}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d^2 - e^2*x^2)^(5/2)/(x^8*(d + e*x)^2),x]
```

```
[Out] (Sqrt[d^2 - e^2*x^2]*(-120*d^6 + 280*d^5*e*x - 144*d^4*e^2*x^2 -
70*d^3*e^3*x^3 + 88*d^2*e^4*x^4 - 105*d*e^5*x^5 + 176*e^6*x^6) +
105*e^7*x^7*Log[x] - 105*e^7*x^7*Log[d + Sqrt[d^2 - e^2*x^2]])/(8
40*d^4*x^7)
```

**Maple [B]** time = 0.025, size = 591, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-e^2*x^2+d^2)^(5/2)/x^8/(e*x+d)^2,x)
```

```
[Out] 1/3/d^5*e/x^6*(-e^2*x^2+d^2)^(7/2)-3/5/d^6*e^2/x^5*(-e^2*x^2+d^2)
^(7/2)+11/12/d^7*e^3/x^4*(-e^2*x^2+d^2)^(7/2)-19/15/d^8*e^4/x^3*(
-e^2*x^2+d^2)^(7/2)+13/8/d^9*e^5/x^2*(-e^2*x^2+d^2)^(7/2)-29/15/d
^10*e^6/x*(-e^2*x^2+d^2)^(7/2)-29/15/d^10*e^8*x*(-e^2*x^2+d^2)^(5
/2)-29/12/d^8*e^8*x*(-e^2*x^2+d^2)^(3/2)-29/8/d^6*e^8*x*(-e^2*x^2
+d^2)^(1/2)-29/8/d^4*e^8/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x
^2+d^2)^(1/2))+1/3/d^9*e^5/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e
))^7/2+29/12/d^8*e^8*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^3/2*x+29/
8/d^6*e^8*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)*x+29/8/d^4*e^8/(e^
2)^(1/2)*arctan((e^2)^(1/2)*x/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2
))-1/8/d^3*e^7/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)
^(1/2))/x)+1/40/d^9*e^7*(-e^2*x^2+d^2)^(5/2)+1/24/d^7*e^7*(-e^2*x
^2+d^2)^(3/2)+1/8/d^5*e^7*(-e^2*x^2+d^2)^(1/2)+29/15/d^9*e^7*(-(x
+d/e)^2*e^2+2*d*e*(x+d/e))^(5/2)-1/7/d^4/x^7*(-e^2*x^2+d^2)^(7/2)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^2*x^8),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 0.388651, size = 711, normalized size = 3.59

```
176 e^14 x^14 - 105 d e^13 x^13 - 4312 d^2 e^12 x^12 + 2555 d^3 e^11 x^11 + 15960 d^4 e^10 x^10 - 8890 d^5 e^9 x^9 - 12712 d^6 e^8 x^8 + 840 d^7 e^7 x^7 - 13388 d^8 e^6 x^6 - 44800 d^9 e^5 x^5 + 13888 d^10 e^4 x^4 - 44800 d^11 e^3 x^3 + 8064 d^12 e^2 x^2 - 105 d^13 e x + 176 d^14
```

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^2*x^8),x, algorithm="fricas")
```

```
[Out] 1/840*(176*e^14*x^14 - 105*d*e^13*x^13 - 4312*d^2*e^12*x^12 + 255
5*d^3*e^11*x^11 + 15960*d^4*e^10*x^10 - 8890*d^5*e^9*x^9 - 12712*
d^6*e^8*x^8 + 840*d^7*e^7*x^7 - 13384*d^8*e^6*x^6 + 32480*d^9*e^5
*x^5 + 13888*d^10*e^4*x^4 - 44800*d^11*e^3*x^3 + 8064*d^12*e^2*x^2
```

$$2 + 17920*d^{13}*e*x - 7680*d^{14} + 105*(7*d*e^{13}*x^{13} - 56*d^3*e^{11}*x^{11} + 112*d^5*e^9*x^9 - 64*d^7*e^7*x^7 - (e^{13}*x^{13} - 24*d^2*e^{11}*x^{11} + 80*d^4*e^9*x^9 - 64*d^6*e^7*x^7)*\sqrt{-e^2*x^2 + d^2})*\log(-(d - \sqrt{-e^2*x^2 + d^2})/x) + (1232*d*e^{12}*x^{12} - 735*d^2*e^{11}*x^{11} - 9240*d^3*e^{10}*x^{10} + 5390*d^4*e^9*x^9 + 13776*d^5*e^8*x^8 - 5880*d^6*e^7*x^7 + 5816*d^7*e^6*x^6 - 16800*d^8*e^5*x^5 - 15040*d^9*e^4*x^4 + 35840*d^{10}*e^3*x^3 - 4224*d^{11}*e^2*x^2 - 17920*d^{12}*e*x + 7680*d^{13})*\sqrt{-e^2*x^2 + d^2})/(7*d^5*e^6*x^{13} - 56*d^7*e^4*x^{11} + 112*d^9*e^2*x^9 - 64*d^{11}*x^7 - (d^4*e^6*x^{13} - 24*d^6*e^4*x^{11} + 80*d^8*e^2*x^9 - 64*d^{10}*x^7)*\sqrt{-e^2*x^2 + d^2}))$$


---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*8/(e\*x+d)\*\*2,x)

[Out] Timed out

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)^2\*x^8),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.171 \quad \int \frac{x^4}{(d+ex)^2(d^2-e^2x^2)^{3/2}} dx$$

**Optimal.** Leaf size=123

$$\frac{17d^2(d-ex)}{15e^5(d^2-e^2x^2)^{3/2}} - \frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5} - \frac{d^3(d-ex)^2}{5e^5(d^2-e^2x^2)^{5/2}}$$

[Out]  $-(d^3*(d - e*x)^2)/(5*e^5*(d^2 - e^2*x^2)^{(5/2)}) + (17*d^2*(d - e*x))/(15*e^5*(d^2 - e^2*x^2)^{(3/2)}) - (2*(15*d - 13*e*x))/(15*e^5*\text{Sqrt}[d^2 - e^2*x^2]) - \text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]]/e^5$

**Rubi [A]** time = 0.402498, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{17d^2(d-ex)}{15e^5(d^2-e^2x^2)^{3/2}} - \frac{2(15d-13ex)}{15e^5\sqrt{d^2-e^2x^2}} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5} - \frac{d^3(d-ex)^2}{5e^5(d^2-e^2x^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[x^4/((d + e\*x)^2\*(d^2 - e^2\*x^2)^(3/2)), x]

[Out]  $-(d^3*(d - e*x)^2)/(5*e^5*(d^2 - e^2*x^2)^{(5/2)}) + (17*d^2*(d - e*x))/(15*e^5*(d^2 - e^2*x^2)^{(3/2)}) - (2*(15*d - 13*e*x))/(15*e^5*\text{Sqrt}[d^2 - e^2*x^2]) - \text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]]/e^5$

**Rubi in Sympy [A]** time = 43.0961, size = 121, normalized size = 0.98

$$-\frac{d^3}{5e^5(d+ex)^2\sqrt{d^2-e^2x^2}} + \frac{17d^2}{15e^5(d+ex)\sqrt{d^2-e^2x^2}} - \frac{2d}{e^5\sqrt{d^2-e^2x^2}} + \frac{26x}{15e^4\sqrt{d^2-e^2x^2}} - \frac{\text{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(e\*x+d)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2), x)

[Out]  $-d**3/(5*e**5*(d + e*x)**2*\text{sqrt}(d**2 - e**2*x**2)) + 17*d**2/(15*e**5*(d + e*x)*\text{sqrt}(d**2 - e**2*x**2)) - 2*d/(e**5*\text{sqrt}(d**2 - e**2*x**2)) + 26*x/(15*e**4*\text{sqrt}(d**2 - e**2*x**2)) - \text{atan}(e*x/\text{sqrt}(d**2 - e**2*x**2))/e**5$

**Mathematica [A]** time = 0.115957, size = 106, normalized size = 0.86

$$\sqrt{d^2 - e^2x^2} \left( -\frac{d^2}{10e^5(d+ex)^3} + \frac{31d}{60e^5(d+ex)^2} - \frac{1}{8e^5(ex-d)} - \frac{193}{120e^5(d+ex)} \right) - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/((d + e\*x)^2\*(d^2 - e^2\*x^2)^(3/2)), x]

[Out]  $\text{Sqrt}[d^2 - e^2*x^2]*(-1/(8*e^5*(-d + e*x)) - d^2/(10*e^5*(d + e*x)^3) + (31*d)/(60*e^5*(d + e*x)^2) - 193/(120*e^5*(d + e*x))) - \text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]]/e^5$

**Maple [A]** time = 0.02, size = 198, normalized size = 1.6

$$\begin{aligned} & 4 \frac{x}{e^4 \sqrt{-e^2 x^2 + d^2}} - \frac{1}{e^4} \arctan \left( x \sqrt{e^2} \frac{1}{\sqrt{-e^2 x^2 + d^2}} \right) \frac{1}{\sqrt{e^2}} \\ & - \frac{d^3}{5 e^7} \left( x + \frac{d}{e} \right)^{-2} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)}} \\ & + \frac{17 d^2}{15 e^6} \left( x + \frac{d}{e} \right)^{-1} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)}} \\ & - \frac{34 x}{15 e^4} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)}} - 2 \frac{d}{e^5 \sqrt{-e^2 x^2 + d^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)`

[Out] `4/e^4*x/(-e^2*x^2+d^2)^(1/2)-1/e^4/(e^2)^(1/2)*arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/5*d^3/e^7/(x+d/e)^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+17/15*d^2/e^6/(x+d/e)/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-34/15/e^4/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)*x-2*d/e^5/(-e^2*x^2+d^2)^(1/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.292889, size = 547, normalized size = 4.45

$$\frac{16 e^6 x^6 - 46 d e^5 x^5 - 130 d^2 e^4 x^4 - 5 d^3 e^3 x^3 + 120 d^4 e^2 x^2 + 60 d^5 e x - 30 \left( e^6 x^6 + 2 d e^5 x^5 - 4 d^2 e^4 x^4 - 10 d^3 e^3 x^3 - d^4 e^2 x^2 \right)}{15 \left( e^{11} x^6 + 2 d e^{10} x^5 - 4 d^2 e^9 x^4 - 10 d^3 e^8 x^3 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2),x, algorithm="fricas")`

[Out] `-1/15*(16*e^6*x^6 - 46*d*e^5*x^5 - 130*d^2*e^4*x^4 - 5*d^3*e^3*x^3 + 120*d^4*e^2*x^2 + 60*d^5*e*x - 30*(e^6*x^6 + 2*d*e^5*x^5 - 4*d^2*e^4*x^4 - 10*d^3*e^3*x^3 - d^4*e^2*x^2 + 8*d^5*e*x + 4*d^6 + (3*d*e^4*x^4 + 6*d^2*e^3*x^3 - d^3*e^2*x^2 - 8*d^4*e*x - 4*d^5)*sqrt(-e^2*x^2 + d^2))*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (26*e^5*x^5 + 70*d*e^4*x^4 - 25*d^2*e^3*x^3 - 120*d^3*e^2*x^2 - 60*d^4*e*x)*sqrt(-e^2*x^2 + d^2))/(e^11*x^6 + 2*d*e^10*x^5 - 4*d^2*e^9*x^4 - 10*d^3*e^8*x^3 - d^4*e^7*x^2 + 8*d^5*e^6*x + 4*d^6*e^5 + (3*d*e^9*x^4 + 6*d^2*e^8*x^3 - d^3*e^7*x^2 - 8*d^4*e^6*x - 4*d^5*e^5)*sqrt(-e^2*x^2 + d^2))`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(-(-d + ex)(d + ex))^{\frac{3}{2}}(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)`

[Out] `Integral(x**4/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)`

---

**GIAC/XCAS [A]** time = 0.67318, size = 4, normalized size = 0.03

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2),x, algorithm="giac")`

[Out] `sage0*x`

$$3.172 \quad \int \frac{x^3}{(d+ex)^2(d^2-e^2x^2)^{3/2}} dx$$

**Optimal.** Leaf size=99

$$\frac{d^2(d-ex)^2}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{4d(d-ex)}{5e^4(d^2-e^2x^2)^{3/2}} + \frac{5d-2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

[Out]  $(d^2*(d - e*x)^2)/(5*e^4*(d^2 - e^2*x^2)^{(5/2)}) - (4*d*(d - e*x))/(5*e^4*(d^2 - e^2*x^2)^{(3/2)}) + (5*d - 2*e*x)/(5*d*e^4*sqrt[d^2 - e^2*x^2])$

**Rubi [A]** time = 0.37498, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{d^2(d-ex)^2}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{4d(d-ex)}{5e^4(d^2-e^2x^2)^{3/2}} + \frac{5d-2ex}{5de^4\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[x^3/((d + e\*x)^2\*(d^2 - e^2\*x^2)^(3/2)), x]

[Out]  $(d^2*(d - e*x)^2)/(5*e^4*(d^2 - e^2*x^2)^{(5/2)}) - (4*d*(d - e*x))/(5*e^4*(d^2 - e^2*x^2)^{(3/2)}) + (5*d - 2*e*x)/(5*d*e^4*sqrt[d^2 - e^2*x^2])$

**Rubi in Sympy [A]** time = 30.9277, size = 97, normalized size = 0.98

$$\frac{d^2}{5e^4(d+ex)^2\sqrt{d^2-e^2x^2}} - \frac{4d}{5e^4(d+ex)\sqrt{d^2-e^2x^2}} + \frac{1}{e^4\sqrt{d^2-e^2x^2}} - \frac{2x}{5de^3\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3/(e\*x+d)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2), x)

[Out]  $d**2/(5*e**4*(d + e*x)**2*sqrt(d**2 - e**2*x**2)) - 4*d/(5*e**4*(d + e*x)*sqrt(d**2 - e**2*x**2)) + 1/(e**4*sqrt(d**2 - e**2*x**2)) - 2*x/(5*d*e**3*sqrt(d**2 - e**2*x**2))$

**Mathematica [A]** time = 0.0521825, size = 70, normalized size = 0.71

$$\frac{\sqrt{d^2 - e^2x^2} (2d^3 + 4d^2ex + de^2x^2 - 2e^3x^3)}{5de^4(d-ex)(d+ex)^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((d + e\*x)^2\*(d^2 - e^2\*x^2)^(3/2)), x]

[Out]  $(sqrt[d^2 - e^2*x^2]*(2*d^3 + 4*d^2*e*x + d*e^2*x^2 - 2*e^3*x^3))/(5*d*e^4*(d - e*x)*(d + e*x)^3)$

**Maple [A]** time = 0.011, size = 65, normalized size = 0.7

$$\frac{(-ex + d)(-2e^3x^3 + de^2x^2 + 4d^2ex + 2d^3)}{(5ex + 5d)de^4} (-e^2x^2 + d^2)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)`

[Out] `1/5*(-e*x+d)*(-2*e^3*x^3+d*e^2*x^2+4*d^2*e*x+2*d^3)/(e*x+d)/d/e^4/(-e^2*x^2+d^2)^(3/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.281263, size = 246, normalized size = 2.48

$$\frac{2e^2x^6 - 2dex^5 - 5d^2x^4 + (2ex^5 + 5dx^4)\sqrt{-e^2x^2 + d^2}}{5(d^6e^6x^6 + 2d^2e^5x^5 - 4d^3e^4x^4 - 10d^4e^3x^3 - d^5e^2x^2 + 8d^6ex + 4d^7 + (3d^2e^4x^4 + 6d^3e^3x^3 - d^4e^2x^2 - 8d^5ex - 4d^6)\sqrt{-e^2x^2 + d^2})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2),x, algorithm="fricas")`

[Out] `1/5*(2*e^2*x^6 - 2*d*e*x^5 - 5*d^2*x^4 + (2*e*x^5 + 5*d*x^4)*sqrt(-e^2*x^2 + d^2))/(d*e^6*x^6 + 2*d^2*e^5*x^5 - 4*d^3*e^4*x^4 - 10*d^4*e^3*x^3 - d^5*e^2*x^2 + 8*d^6*e*x + 4*d^7 + (3*d^2*e^4*x^4 + 6*d^3*e^3*x^3 - d^4*e^2*x^2 - 8*d^5*e*x - 4*d^6)*sqrt(-e^2*x^2 + d^2))`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(-(-d + ex)(d + ex))^{\frac{3}{2}}(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)`

[Out] `Integral(x**3/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)`

**GIAC/XCAS [A]** time = 0.665414, size = 4, normalized size = 0.04

*sage<sub>0</sub>x*



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2),x, algorithm="giac")
```

```
[Out] sage0*x
```

$$3.173 \quad \int \frac{x^2}{(d+ex)^2(d^2-e^2x^2)^{3/2}} dx$$

**Optimal.** Leaf size=89

$$\frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}} - \frac{d}{5e^3(d+ex)^2\sqrt{d^2-e^2x^2}} + \frac{7}{15e^3(d+ex)\sqrt{d^2-e^2x^2}}$$

[Out]  $x/(15*d^2*e^2*\text{Sqrt}[d^2 - e^2*x^2]) - d/(5*e^3*(d + e*x)^2*\text{Sqrt}[d^2 - e^2*x^2]) + 7/(15*e^3*(d + e*x)*\text{Sqrt}[d^2 - e^2*x^2])$

**Rubi [A]** time = 0.311352, antiderivative size = 89, normalized size of antiderivative = 1., number of rules used = 4, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$\frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}} - \frac{d(d-ex)^2}{5e^3(d^2-e^2x^2)^{5/2}} + \frac{7(d-ex)}{15e^3(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2/((d + e*x)^2*(d^2 - e^2*x^2)^{(3/2)}), x]$

[Out]  $-(d*(d - e*x)^2)/(5*e^3*(d^2 - e^2*x^2)^{(5/2)}) + (7*(d - e*x))/(15*e^3*(d^2 - e^2*x^2)^{(3/2)}) + x/(15*d^2*e^2*\text{Sqrt}[d^2 - e^2*x^2])$

**Rubi in Sympy [A]** time = 25.727, size = 75, normalized size = 0.84

$$-\frac{d}{5e^3(d+ex)^2\sqrt{d^2-e^2x^2}} + \frac{7}{15e^3(d+ex)\sqrt{d^2-e^2x^2}} + \frac{x}{15d^2e^2\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**2/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2), x)$

[Out]  $-d/(5*e**3*(d + e*x)**2*\text{sqrt}(d**2 - e**2*x**2)) + 7/(15*e**3*(d + e*x)*\text{sqrt}(d**2 - e**2*x**2)) + x/(15*d**2*e**2*\text{sqrt}(d**2 - e**2*x**2))$

**Mathematica [A]** time = 0.0483472, size = 70, normalized size = 0.79

$$\frac{\sqrt{d^2 - e^2x^2} (4d^3 + 8d^2ex + 2de^2x^2 + e^3x^3)}{15d^2e^3(d - ex)(d + ex)^3}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^2/((d + e*x)^2*(d^2 - e^2*x^2)^{(3/2)}), x]$

[Out]  $(\text{Sqrt}[d^2 - e^2*x^2]*(4*d^3 + 8*d^2*e*x + 2*d*e^2*x^2 + e^3*x^3))/(15*d^2*e^3*(d - e*x)*(d + e*x)^3)$

**Maple [A]** time = 0.012, size = 65, normalized size = 0.7

$$\frac{(-ex + d)(e^3x^3 + 2de^2x^2 + 8xd^2e + 4d^3)}{(15ex + 15d)d^2e^3} (-e^2x^2 + d^2)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)`

[Out]  $1/15*(-e*x+d)*(e^3*x^3+2*d*e^2*x^2+8*d^2*e*x+4*d^3)/(e*x+d)/d^2/e^3/(-e^2*x^2+d^2)^(3/2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.282937, size = 278, normalized size = 3.12

$$\frac{4e^3x^6 + 11de^2x^5 - 10d^2ex^4 - 20d^3x^3 - (e^2x^5 - 10dex^4 - 20d^2x^3)\sqrt{-e^2x^2 + d^2}}{15(d^2e^6x^6 + 2d^3e^5x^5 - 4d^4e^4x^4 - 10d^5e^3x^3 - d^6e^2x^2 + 8d^7ex + 4d^8 + (3d^3e^4x^4 + 6d^4e^3x^3 - d^5e^2x^2 - 8d^6ex - 4d^7)\sqrt{-e^2x^2 + d^2})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2),x, algorithm="fricas")`

[Out]  $1/15*(4*e^3*x^6 + 11*d*e^2*x^5 - 10*d^2*e*x^4 - 20*d^3*x^3 - (e^2*x^5 - 10*d*e*x^4 - 20*d^2*x^3)*\sqrt{-e^2*x^2 + d^2})/(d^2*e^6*x^6 + 2*d^3*e^5*x^5 - 4*d^4*e^4*x^4 - 10*d^5*e^3*x^3 - d^6*e^2*x^2 + 8*d^7*e*x + 4*d^8 + (3*d^3*e^4*x^4 + 6*d^4*e^3*x^3 - d^5*e^2*x^2 - 8*d^6*e*x - 4*d^7)*\sqrt{-e^2*x^2 + d^2})$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(-(-d + ex)(d + ex))^{\frac{3}{2}}(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)`

[Out] `Integral(x**2/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)`

**GIAC/XCAS [A]** time = 0.632812, size = 4, normalized size = 0.04

*sage*<sub>0</sub>*x*

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2),x, algorithm="giac")`

[Out] `sage0*x`

$$3.174 \quad \int \frac{x}{(d+ex)^2(d^2-e^2x^2)^{3/2}} dx$$

**Optimal.** Leaf size=91

$$-\frac{2}{15de^2(d+ex)\sqrt{d^2-e^2x^2}} + \frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}} + \frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}$$

[Out]  $(4*x)/(15*d^3*e*\text{Sqrt}[d^2 - e^2*x^2]) + 1/(5*e^2*(d + e*x)^2*\text{Sqrt}[d^2 - e^2*x^2]) - 2/(15*d*e^2*(d + e*x)*\text{Sqrt}[d^2 - e^2*x^2])$

**Rubi [A]** time = 0.121642, antiderivative size = 91, normalized size of antiderivative = 1., number of rules used = 3, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$-\frac{2}{15de^2(d+ex)\sqrt{d^2-e^2x^2}} + \frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}} + \frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[x/((d + e\*x)^2\*(d^2 - e^2\*x^2)^(3/2)), x]

[Out]  $(4*x)/(15*d^3*e*\text{Sqrt}[d^2 - e^2*x^2]) + 1/(5*e^2*(d + e*x)^2*\text{Sqrt}[d^2 - e^2*x^2]) - 2/(15*d*e^2*(d + e*x)*\text{Sqrt}[d^2 - e^2*x^2])$

**Rubi in Sympy [A]** time = 11.0992, size = 76, normalized size = 0.84

$$\frac{1}{5e^2(d+ex)^2\sqrt{d^2-e^2x^2}} - \frac{2}{15de^2(d+ex)\sqrt{d^2-e^2x^2}} + \frac{4x}{15d^3e\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x/(e\*x+d)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2), x)

[Out]  $1/(5*e**2*(d + e*x)**2*\text{sqrt}(d**2 - e**2*x**2)) - 2/(15*d*e**2*(d + e*x)*\text{sqrt}(d**2 - e**2*x**2)) + 4*x/(15*d**3*e*\text{sqrt}(d**2 - e**2*x**2))$

**Mathematica [A]** time = 0.0458552, size = 69, normalized size = 0.76

$$\frac{\sqrt{d^2 - e^2x^2} (d^3 + 2d^2ex + 8de^2x^2 + 4e^3x^3)}{15d^3e^2(d - ex)(d + ex)^3}$$

Antiderivative was successfully verified.

[In] Integrate[x/((d + e\*x)^2\*(d^2 - e^2\*x^2)^(3/2)), x]

[Out]  $(\text{Sqrt}[d^2 - e^2*x^2]*(d^3 + 2*d^2*e*x + 8*d*e^2*x^2 + 4*e^3*x^3))/(15*d^3*e^2*(d - e*x)*(d + e*x)^3)$

**Maple [A]** time = 0.011, size = 64, normalized size = 0.7

$$\frac{(-ex + d)(4e^3x^3 + 8de^2x^2 + 2d^2ex + d^3)}{(15ex + 15d)d^3e^2} (-e^2x^2 + d^2)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2),x)`

[Out]  $1/15*(-e*x+d)*(4*e^3*x^3+8*d*e^2*x^2+2*d^2*e*x+d^3)/(e*x+d)/d^3/e^2/(-e^2*x^2+d^2)^(3/2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.283599, size = 308, normalized size = 3.38

$$\frac{e^4x^6 + 14de^3x^5 + 20d^2e^2x^4 - 20d^3ex^3 - 30d^4x^2 - (4e^3x^5 + 5de^2x^4 - 20d^2ex^3 - 30d^3x^2)\sqrt{-e^2x^2 + d^2}}{15(d^3e^6x^6 + 2d^4e^5x^5 - 4d^5e^4x^4 - 10d^6e^3x^3 - d^7e^2x^2 + 8d^8ex + 4d^9 + (3d^4e^4x^4 + 6d^5e^3x^3 - d^6e^2x^2 - 8d^7ex - 4d^8)\sqrt{-e^2x^2 + d^2})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2),x, algorithm="fricas")`

[Out]  $1/15*(e^4*x^6 + 14*d*e^3*x^5 + 20*d^2*e^2*x^4 - 20*d^3*e*x^3 - 30*d^4*x^2 - (4*e^3*x^5 + 5*d*e^2*x^4 - 20*d^2*e*x^3 - 30*d^3*x^2)*\sqrt{-e^2*x^2 + d^2})/(d^3*e^6*x^6 + 2*d^4*e^5*x^5 - 4*d^5*e^4*x^4 - 10*d^6*e^3*x^3 - d^7*e^2*x^2 + 8*d^8*e*x + 4*d^9 + (3*d^4*e^4*x^4 + 6*d^5*e^3*x^3 - d^6*e^2*x^2 - 8*d^7*e*x - 4*d^8)*\sqrt{-e^2*x^2 + d^2})$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(-(-d + ex)(d + ex))^{\frac{3}{2}}(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2),x)`

[Out] `Integral(x/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)`

**GIAC/XCAS [A]** time = 0.64673, size = 4, normalized size = 0.04

*sage0*x

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2),x, algorithm="giac")`

[Out] `sage0*x`

$$3.175 \quad \int \frac{1}{(d+ex)^2(d^2-e^2x^2)^{3/2}} dx$$

**Optimal.** Leaf size=91

$$-\frac{1}{5d^2e(d+ex)\sqrt{d^2-e^2x^2}} - \frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}} + \frac{2x}{5d^4\sqrt{d^2-e^2x^2}}$$

[Out] (2\*x)/(5\*d^4\*Sqrt[d^2 - e^2\*x^2]) - 1/(5\*d\*e\*(d + e\*x)^2\*Sqrt[d^2 - e^2\*x^2]) - 1/(5\*d^2\*e\*(d + e\*x)\*Sqrt[d^2 - e^2\*x^2])

**Rubi [A]** time = 0.0837866, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$-\frac{1}{5d^2e(d+ex)\sqrt{d^2-e^2x^2}} - \frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}} + \frac{2x}{5d^4\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)^2\*(d^2 - e^2\*x^2)^(3/2)), x]

[Out] (2\*x)/(5\*d^4\*Sqrt[d^2 - e^2\*x^2]) - 1/(5\*d\*e\*(d + e\*x)^2\*Sqrt[d^2 - e^2\*x^2]) - 1/(5\*d^2\*e\*(d + e\*x)\*Sqrt[d^2 - e^2\*x^2])

**Rubi in Sympy [A]** time = 10.3946, size = 75, normalized size = 0.82

$$-\frac{1}{5de(d+ex)^2\sqrt{d^2-e^2x^2}} - \frac{1}{5d^2e(d+ex)\sqrt{d^2-e^2x^2}} + \frac{2x}{5d^4\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(e\*x+d)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2), x)

[Out] -1/(5\*d\*e\*(d + e\*x)\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) - 1/(5\*d\*\*2\*e\*(d + e\*x)\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)) + 2\*x/(5\*d\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2))

**Mathematica [A]** time = 0.0533533, size = 70, normalized size = 0.77

$$\frac{\sqrt{d^2 - e^2x^2}(-2d^3 + d^2ex + 4de^2x^2 + 2e^3x^3)}{5d^4e(d - ex)(d + ex)^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x)^2\*(d^2 - e^2\*x^2)^(3/2)), x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(-2\*d^3 + d^2\*e\*x + 4\*d\*e^2\*x^2 + 2\*e^3\*x^3))/(5\*d^4\*e\*(d - e\*x)\*(d + e\*x)^3)

**Maple [A]** time = 0.01, size = 66, normalized size = 0.7

$$-\frac{(-ex + d)(-2e^3x^3 - 4e^2x^2d - xd^2e + 2d^3)}{(5ex + 5d)d^4e} (-e^2x^2 + d^2)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2), x)`

[Out]  $-1/5*(-e*x+d)*(-2*e^3*x^3-4*d*e^2*x^2-d^2*e*x+2*d^3)/(e*x+d)/d^4/e/(-e^2*x^2+d^2)^(3/2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.282319, size = 332, normalized size = 3.65

$$\frac{2e^5x^6 - 2de^4x^5 - 20d^2e^3x^4 - 15d^3e^2x^3 + 20d^4ex^2 + 20d^5x + (2e^4x^5 + 10de^3x^4 + 5d^2e^2x^3 - 20d^3ex^2 - 20d^4x)\sqrt{-e^2x^2 + d^2}}{5(d^4e^6x^6 + 2d^5e^5x^5 - 4d^6e^4x^4 - 10d^7e^3x^3 - d^8e^2x^2 + 8d^9ex + 4d^{10} + (3d^5e^4x^4 + 6d^6e^3x^3 - d^7e^2x^2 - 8d^8ex - 4d^9)\sqrt{-e^2x^2 + d^2})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2), x, algorithm="fricas")`

[Out]  $-1/5*(2*e^5*x^6 - 2*d*e^4*x^5 - 20*d^2*e^3*x^4 - 15*d^3*e^2*x^3 + 20*d^4*e*x^2 + 20*d^5*x + (2*e^4*x^5 + 10*d*e^3*x^4 + 5*d^2*e^2*x^3 - 20*d^3*e*x^2 - 20*d^4*x)*\sqrt{-e^2*x^2 + d^2})/(d^4*e^6*x^6 + 2*d^5*e^5*x^5 - 4*d^6*e^4*x^4 - 10*d^7*e^3*x^3 - d^8*e^2*x^2 + 8*d^9*e*x + 4*d^{10} + (3*d^5*e^4*x^4 + 6*d^6*e^3*x^3 - d^7*e^2*x^2 - 8*d^8*e*x - 4*d^9)*\sqrt{-e^2*x^2 + d^2})$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-(-d + ex)(d + ex))^{\frac{3}{2}}(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2), x)`

[Out] `Integral(1/((-(-d + e*x)*(d + e*x))**(3/2)*(d + e*x)**2), x)`

**GIAC/XCAS [A]** time = 0.639643, size = 4, normalized size = 0.04

`sage0*x`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2), x, algorithm="giac")`

[Out] `sage0*x`

$$3.176 \quad \int \frac{1}{x(d+ex)^2(d^2-e^2x^2)^{3/2}} dx$$

**Optimal.** Leaf size=118

$$\frac{2(d-ex)}{5d(d^2-e^2x^2)^{5/2}} + \frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5} + \frac{5d-8ex}{15d^3(d^2-e^2x^2)^{3/2}}$$

[Out]  $(2*(d - e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + (5*d - 8*e*x)/(15*d^3*(d^2 - e^2*x^2)^(3/2)) + (15*d - 16*e*x)/(15*d^5*\text{Sqrt}[d^2 - e^2*x^2]) - \text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d]/d^5$

**Rubi [A]** time = 0.42476, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\frac{2(d-ex)}{5d(d^2-e^2x^2)^{5/2}} + \frac{15d-16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5} + \frac{5d-8ex}{15d^3(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] `Int[1/(x*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)), x]`

[Out]  $(2*(d - e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) + (5*d - 8*e*x)/(15*d^3*(d^2 - e^2*x^2)^(3/2)) + (15*d - 16*e*x)/(15*d^5*\text{Sqrt}[d^2 - e^2*x^2]) - \text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d]/d^5$

**Rubi in Sympy [A]** time = 33.4342, size = 114, normalized size = 0.97

$$\frac{1}{5d^2(d+ex)^2\sqrt{d^2-e^2x^2}} + \frac{8}{15d^3(d+ex)\sqrt{d^2-e^2x^2}} + \frac{1}{d^4\sqrt{d^2-e^2x^2}} - \frac{16ex}{15d^5\sqrt{d^2-e^2x^2}} - \frac{\text{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x/(e*x+d)**2/(-e**2*x**2+d**2)**(3/2), x)`

[Out]  $1/(5*d**2*(d + e*x)**2*\text{sqrt}(d**2 - e**2*x**2)) + 8/(15*d**3*(d + e*x)*\text{sqrt}(d**2 - e**2*x**2)) + 1/(d**4*\text{sqrt}(d**2 - e**2*x**2)) - 16*e*x/(15*d**5*\text{sqrt}(d**2 - e**2*x**2)) - \text{atanh}(\text{sqrt}(d**2 - e**2*x**2)/d)/d**5$

**Mathematica [A]** time = 0.0898883, size = 95, normalized size = 0.81

$$\frac{-15 \log\left(\sqrt{d^2 - e^2x^2} + d\right) + \frac{\sqrt{d^2 - e^2x^2}(26d^3 + 22d^2ex - 17de^2x^2 - 16e^3x^3)}{(d-ex)(d+ex)^3} + 15 \log(x)}{15d^5}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x*(d + e*x)^2*(d^2 - e^2*x^2)^(3/2)), x]`

[Out]  $((\text{Sqrt}[d^2 - e^2*x^2]*(26*d^3 + 22*d^2*e*x - 17*d*e^2*x^2 - 16*e^3*x^3))/((d - e*x)*(d + e*x)^3) + 15*\text{Log}[x] - 15*\text{Log}[d + \text{Sqrt}[d^2 - e^2*x^2]])/(15*d^5)$



**Maple [A]** time = 0.017, size = 187, normalized size = 1.6

$$\frac{1}{d^4} \frac{1}{\sqrt{-e^2 x^2 + d^2}} - \frac{1}{d^4} \ln\left(\frac{1}{x} \left(2d^2 + 2\sqrt{d^2} \sqrt{-e^2 x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}}$$

$$+ \frac{8}{15d^3 e} \left(x + \frac{d}{e}\right)^{-1} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)}}$$

$$- \frac{16ex}{15d^5} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)}} + \frac{1}{5d^2 e^2} \left(x + \frac{d}{e}\right)^{-2} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(e\*x+d)^2/(-e^2\*x^2+d^2)^(3/2), x)

[Out] 1/d^4/(-e^2\*x^2+d^2)^(1/2)-1/d^4/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)+8/15/d^3/e/(x+d/e)/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)-16/15/d^5\*e/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x+1/5/d^2/e^2/(x+d/e)^2/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-e^2 x^2 + d^2)^{\frac{3}{2}} (ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)^2\*x), x, algorithm="maxima")

[Out] integrate(1/((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)^2\*x), x)

**Fricas [A]** time = 0.290636, size = 539, normalized size = 4.57

$$\frac{26e^6x^6 + 4de^5x^5 - 155d^2e^4x^4 - 130d^3e^3x^3 + 120d^4e^2x^2 + 120d^5ex + 15(e^6x^6 + 2de^5x^5 - 4d^2e^4x^4 - 10d^3e^3x^3 - d^4e^2x^2 + 120d^5ex + 15(d^5e^6x^6 + 2d^6e^5x^5 - 4d^7e^4x^4 - 10d^8e^3x^3 - 10d^9e^2x^2 + 8d^{10}ex + 4d^{11} + 3d^6e^4x^4 + 6d^7e^3x^3 - d^8e^2x^2 - 8d^9ex - 4d^{10})) \sqrt{-e^2x^2 + d^2}}{15(d^5e^6x^6 + 2d^6e^5x^5 - 4d^7e^4x^4 - 10d^8e^3x^3 - 10d^9e^2x^2 + 8d^{10}ex + 4d^{11} + 3d^6e^4x^4 + 6d^7e^3x^3 - d^8e^2x^2 - 8d^9ex - 4d^{10}) \sqrt{-e^2x^2 + d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)^2\*x), x, algorithm="fricas")

[Out] 1/15\*(26\*e^6\*x^6 + 4\*d\*e^5\*x^5 - 155\*d^2\*e^4\*x^4 - 130\*d^3\*e^3\*x^3 + 120\*d^4\*e^2\*x^2 + 120\*d^5\*e\*x + 15\*(e^6\*x^6 + 2\*d\*e^5\*x^5 - 4\*d^2\*e^4\*x^4 - 10\*d^3\*e^3\*x^3 - d^4\*e^2\*x^2 + 8\*d^5\*e\*x + 4\*d^6 + (3\*d\*e^4\*x^4 + 6\*d^2\*e^3\*x^3 - d^3\*e^2\*x^2 - 8\*d^4\*e\*x - 4\*d^5)\*sqrt(-e^2\*x^2 + d^2))\*log(-(d - sqrt(-e^2\*x^2 + d^2))/x) + (16\*e^5\*x^5 + 95\*d\*e^4\*x^4 + 70\*d^2\*e^3\*x^3 - 120\*d^3\*e^2\*x^2 - 120\*d^4\*e\*x)\*sqrt(-e^2\*x^2 + d^2))/(d^5\*e^6\*x^6 + 2\*d^6\*e^5\*x^5 - 4\*d^7\*e^4\*x^4 - 10\*d^8\*e^3\*x^3 - d^9\*e^2\*x^2 + 8\*d^10\*e\*x + 4\*d^11 + (3\*d^6\*e^4\*x^4 + 6\*d^7\*e^3\*x^3 - d^8\*e^2\*x^2 - 8\*d^9\*e\*x - 4\*d^10)\*sqrt(-e^2\*x^2 + d^2))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x(-(-d + ex)(d + ex))^{\frac{3}{2}}(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e\*x+d)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2), x)

[Out] Integral(1/(x\*(-(-d + e\*x)\*(d + e\*x))\*\*(3/2)\*(d + e\*x)\*\*2), x)

**GIAC/XCAS [A]** time = 0.678814, size = 4, normalized size = 0.03

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)^2\*x), x, algorithm="giac")

[Out] sage0\*x

$$3.177 \quad \int \frac{1}{x^2(d+ex)^2(d^2-e^2x^2)^{3/2}} dx$$

**Optimal.** Leaf size=146

$$-\frac{2e(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}} - \frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} + \frac{2e \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6} - \frac{e(10d-13ex)}{15d^4(d^2-e^2x^2)^{3/2}}$$

[Out]  $(-2*e*(d - e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) - (e*(10*d - 13*e*x))/(15*d^4*(d^2 - e^2*x^2)^(3/2)) - (e*(30*d - 41*e*x))/(15*d^6*\text{Sqrt}[d^2 - e^2*x^2]) - \text{Sqrt}[d^2 - e^2*x^2]/(d^6*x) + (2*e*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/d^6$

**Rubi [A]** time = 0.51836, antiderivative size = 146, normalized size of antiderivative = 1., number of rules used = 8, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-\frac{2e(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}} - \frac{e(30d-41ex)}{15d^6\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^6x} + \frac{2e \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6} - \frac{e(10d-13ex)}{15d^4(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(d + e\*x)^2\*(d^2 - e^2\*x^2)^(3/2)), x]

[Out]  $(-2*e*(d - e*x))/(5*d^2*(d^2 - e^2*x^2)^(5/2)) - (e*(10*d - 13*e*x))/(15*d^4*(d^2 - e^2*x^2)^(3/2)) - (e*(30*d - 41*e*x))/(15*d^6*\text{Sqrt}[d^2 - e^2*x^2]) - \text{Sqrt}[d^2 - e^2*x^2]/(d^6*x) + (2*e*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/d^6$

**Rubi in Sympy [A]** time = 44.4392, size = 143, normalized size = 0.98

$$-\frac{e}{5d^3(d+ex)^2\sqrt{d^2-e^2x^2}} - \frac{13e}{15d^4(d+ex)\sqrt{d^2-e^2x^2}} - \frac{1}{d^4x\sqrt{d^2-e^2x^2}} - \frac{2e}{d^5\sqrt{d^2-e^2x^2}} + \frac{56e^2x}{15d^6\sqrt{d^2-e^2x^2}} + \frac{2e \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(e\*x+d)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2), x)

[Out]  $-e/(5*d**3*(d + e*x)**2*\text{sqrt}(d**2 - e**2*x**2)) - 13*e/(15*d**4*(d + e*x)*\text{sqrt}(d**2 - e**2*x**2)) - 1/(d**4*x*\text{sqrt}(d**2 - e**2*x**2)) - 2*e/(d**5*\text{sqrt}(d**2 - e**2*x**2)) + 56*e**2*x/(15*d**6*\text{sqrt}(d**2 - e**2*x**2)) + 2*e*\operatorname{atanh}(\text{sqrt}(d**2 - e**2*x**2)/d)/d**6$

**Mathematica [A]** time = 0.108616, size = 112, normalized size = 0.77

$$\frac{30e \log\left(\sqrt{d^2 - e^2x^2} + d\right) + \frac{\sqrt{d^2 - e^2x^2}(15d^4 + 76d^3ex + 32d^2e^2x^2 - 82de^3x^3 - 56e^4x^4)}{x(ex-d)(d+ex)^3}}{15d^6} - 30e \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(d + e\*x)^2\*(d^2 - e^2\*x^2)^(3/2)), x]

[Out]  $((\text{Sqrt}[d^2 - e^2 x^2]) * (15 d^4 + 76 d^3 e x + 32 d^2 e^2 x^2 - 82 d e^3 x^3 - 56 e^4 x^4)) / (x (-d + e x) (d + e x)^3) - 30 e \text{Log}[x] + 30 e \text{Log}[d + \text{Sqrt}[d^2 - e^2 x^2]]) / (15 d^6)$

**Maple [A]** time = 0.018, size = 234, normalized size = 1.6

$$\begin{aligned} & -\frac{1}{d^4 x} \frac{1}{\sqrt{-e^2 x^2 + d^2}} + 2 \frac{e^2 x}{d^6 \sqrt{-e^2 x^2 + d^2}} - \frac{1}{5 d^3 e} \left(x + \frac{d}{e}\right)^{-2} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)}} \\ & - \frac{13}{15 d^4} \left(x + \frac{d}{e}\right)^{-1} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)}} + \frac{26 e^2 x}{15 d^6} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)}} \\ & - 2 \frac{e}{d^5 \sqrt{-e^2 x^2 + d^2}} + 2 \frac{e}{d^5 \sqrt{d^2}} \ln\left(\frac{2 d^2 + 2 \sqrt{d^2} \sqrt{-e^2 x^2 + d^2}}{x}\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(e*x+d)^2/(-e^2*x^2+d^2)^(3/2), x)`

[Out]  $-1/d^4/x/(-e^2 x^2+d^2)^{(1/2)}+2 e^2/d^6 x/(-e^2 x^2+d^2)^{(1/2)}-1/5/d^3/e/(x+d/e)^2/(-(x+d/e)^2 e^2+2 d e(x+d/e))^{(1/2)}-13/15/d^4/(x+d/e)/(-(x+d/e)^2 e^2+2 d e(x+d/e))^{(1/2)}+26/15/d^6 e^2/(-(x+d/e)^2 e^2+2 d e(x+d/e))^{(1/2)} x-2/d^5 e/(-e^2 x^2+d^2)^{(1/2)}+2/d^5 e/(d^2)^{(1/2)} \ln((2 d^2+2(d^2)^{(1/2)}(-e^2 x^2+d^2)^{(1/2)})/x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-e^2 x^2 + d^2)^{3/2} (e x + d)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2*x^2), x, algorithm="maxima")`

[Out] `integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2*x^2), x)`

**Fricas [A]** time = 0.287475, size = 670, normalized size = 4.59

$$56 e^8 x^8 + 266 d e^7 x^7 - 112 d^2 e^6 x^6 - 1100 d^3 e^5 x^5 - 415 d^4 e^4 x^4 + 1080 d^5 e^3 x^3 + 600 d^6 e^2 x^2 - 240 d^7 e x - 120 d^8 + 30 (4 d e^7$$

15

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-e^2*x^2 + d^2)^(3/2)*(e*x + d)^2*x^2), x, algorithm="fricas")`

[Out]  $-1/15*(56 e^8 x^8 + 266 d e^7 x^7 - 112 d^2 e^6 x^6 - 1100 d^3 e^5 x^5 + 415 d^4 e^4 x^4 + 1080 d^5 e^3 x^3 + 600 d^6 e^2 x^2 - 240 d^7 e x - 120 d^8 + 30*(4 d e^7 x^7 + 8 d^2 e^6 x^6 - 8 d^3 e^5 x^5 - 24 d^4 e^4 x^4 - 4 d^5 e^3 x^3 + 16 d^6 e^2 x^2 + 8 d^7 e x - (e^7 x^7 + 2 d e^6 x^6 - 7 d^2 e^5 x^5 - 16 d^3 e^4 x^4 + 16 d^4 e^3 x^3 + 8 d^5 e^2 x^2 + 8 d^6 e x) \sqrt{-e^2 x^2 + d^2}) \log(-(d - \sqrt{-e^2 x^2 + d^2})/x) - 2*(23 e^7 x^7 - 66 d e^6 x^6 - 325 d^2 e^5 x^5 - 80 d^3 e^4 x^4 + 480 d^4 e^3 x^3 + 270 d^5 e^2 x^2 - 120 d^6 e x - 120 d^7 + 30 d^8)/x$

$$\frac{x - 60d^7 \sqrt{-e^2 x^2 + d^2}}{(4d^7 e^6 x^7 + 8d^8 e^5 x^6 - 8d^9 e^4 x^5 - 24d^{10} e^3 x^4 - 4d^{11} e^2 x^3 + 16d^{12} e x^2 + 8d^{13} x - (d^6 e^6 x^7 + 2d^7 e^5 x^6 - 7d^8 e^4 x^5 - 16d^9 e^3 x^4 + 16d^{11} e x^2 + 8d^{12} x) \sqrt{-e^2 x^2 + d^2})}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 (-(-d + ex)(d + ex))^{\frac{3}{2}} (d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(e\*x+d)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2),x)

[Out] Integral(1/(x\*\*2\*(-(-d + e\*x)\*(d + e\*x))\*\*(3/2)\*(d + e\*x)\*\*2), x)

**GIAC/XCAS [A]** time = 0.665126, size = 4, normalized size = 0.03

*sage0*x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)^2\*x^2),x, algorithm="giac")

[Out] sage0\*x

$$3.178 \quad \int \frac{1}{x^3(d+ex)^2(d^2-e^2x^2)^{3/2}} dx$$

**Optimal.** Leaf size=183

$$\frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} + \frac{2e\sqrt{d^2-e^2x^2}}{d^7x} - \frac{9e^2 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7}$$

$$- \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} + \frac{e^2(5d-6ex)}{5d^5(d^2-e^2x^2)^{3/2}} + \frac{2e^2(d-ex)}{5d^3(d^2-e^2x^2)^{5/2}}$$

[Out]  $(2*e^2*(d - e*x))/(5*d^3*(d^2 - e^2*x^2)^{(5/2)}) + (e^2*(5*d - 6*e*x))/(5*d^5*(d^2 - e^2*x^2)^{(3/2)}) + (2*e^2*(10*d - 11*e*x))/(5*d^7*\text{Sqrt}[d^2 - e^2*x^2]) - \text{Sqrt}[d^2 - e^2*x^2]/(2*d^6*x^2) + (2*e^2*\text{Sqrt}[d^2 - e^2*x^2])/(d^7*x) - (9*e^2*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/(2*d^7)$

**Rubi [A]** time = 0.637886, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\frac{2e^2(10d-11ex)}{5d^7\sqrt{d^2-e^2x^2}} + \frac{2e\sqrt{d^2-e^2x^2}}{d^7x} - \frac{9e^2 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7}$$

$$- \frac{\sqrt{d^2-e^2x^2}}{2d^6x^2} + \frac{e^2(5d-6ex)}{5d^5(d^2-e^2x^2)^{3/2}} + \frac{2e^2(d-ex)}{5d^3(d^2-e^2x^2)^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^3*(d + e*x)^2*(d^2 - e^2*x^2)^{(3/2)}), x]$

[Out]  $(2*e^2*(d - e*x))/(5*d^3*(d^2 - e^2*x^2)^{(5/2)}) + (e^2*(5*d - 6*e*x))/(5*d^5*(d^2 - e^2*x^2)^{(3/2)}) + (2*e^2*(10*d - 11*e*x))/(5*d^7*\text{Sqrt}[d^2 - e^2*x^2]) - \text{Sqrt}[d^2 - e^2*x^2]/(2*d^6*x^2) + (2*e^2*\text{Sqrt}[d^2 - e^2*x^2])/(d^7*x) - (9*e^2*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/(2*d^7)$

**Rubi in Sympy [A]** time = 61.4604, size = 199, normalized size = 1.09

$$\frac{e^2}{5d^4(d+ex)^2\sqrt{d^2-e^2x^2}} + \frac{1}{d^4x^2\sqrt{d^2-e^2x^2}} + \frac{6e^2}{5d^5(d+ex)\sqrt{d^2-e^2x^2}} + \frac{2e}{d^5x\sqrt{d^2-e^2x^2}}$$

$$+ \frac{3e^2}{d^6\sqrt{d^2-e^2x^2}} - \frac{3\sqrt{d^2-e^2x^2}}{2d^6x^2} - \frac{32e^3x}{5d^7\sqrt{d^2-e^2x^2}} - \frac{9e^2 \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x^{**3}/(e*x+d)^{**2}/(-e^{**2}*x^{**2}+d^{**2})^{**3/2}, x)$

[Out]  $e^{**2}/(5*d^{**4}*(d + e*x)^{**2}*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})) + 1/(d^{**4}*x^{**2}*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})) + 6*e^{**2}/(5*d^{**5}*(d + e*x)*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})) + 2*e/(d^{**5}*x*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})) + 3*e^{**2}/(d^{**6}*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})) - 3*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(2*d^{**6}*x^{**2}) - 32*e^{**3}*x/(5*d^{**7}*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})) - 9*e^{**2}*\operatorname{atanh}(\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/d)/(2*d^{**7})$

**Mathematica [A]** time = 0.114964, size = 127, normalized size = 0.69

$$\frac{-45e^2 \log\left(\sqrt{d^2 - e^2x^2} + d\right) + \frac{\sqrt{d^2 - e^2x^2}(5d^5 - 10d^4ex - 94d^3e^2x^2 - 58d^2e^3x^3 + 83de^4x^4 + 64e^5x^5)}{x^2(ex-d)(d+ex)^3} + 45e^2 \log(x)}{10d^7}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(d + e\*x)^2\*(d^2 - e^2\*x^2)^(3/2)), x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(5\*d^5 - 10\*d^4\*e\*x - 94\*d^3\*e^2\*x^2 - 58\*d^2\*e^3\*x^3 + 83\*d\*e^4\*x^4 + 64\*e^5\*x^5))/(x^2\*(-d + e\*x)\*(d + e\*x)^3) + 45\*e^2\*Log[x] - 45\*e^2\*Log[d + Sqrt[d^2 - e^2\*x^2]])/(10\*d^7)

**Maple [A]** time = 0.021, size = 259, normalized size = 1.4

$$\begin{aligned} & -\frac{1}{2d^4x^2} \frac{1}{\sqrt{-e^2x^2+d^2}} + \frac{9e^2}{2d^6} \frac{1}{\sqrt{-e^2x^2+d^2}} - \frac{9e^2}{2d^6} \ln\left(\frac{1}{x} \left(2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2+d^2}\right)\right) \frac{1}{\sqrt{d^2}} \\ & + \frac{6e}{5d^5} \left(x + \frac{d}{e}\right)^{-1} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)}} - \frac{12e^3x}{5d^7} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)}} \\ & + 2 \frac{e}{d^5x\sqrt{-e^2x^2+d^2}} - 4 \frac{e^3x}{d^7\sqrt{-e^2x^2+d^2}} + \frac{1}{5d^4} \left(x + \frac{d}{e}\right)^{-2} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(e\*x+d)^2/(-e^2\*x^2+d^2)^(3/2), x)

[Out] -1/2/d^4/x^2/(-e^2\*x^2+d^2)^(1/2)+9/2/d^6\*e^2/(-e^2\*x^2+d^2)^(1/2)-9/2/d^6\*e^2/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)+6/5/d^5\*e/(x+d/e)/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)-12/5/d^7\*e^3/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x+2/d^5\*e/x/(-e^2\*x^2+d^2)^(1/2)-4/d^7\*e^3\*x/(-e^2\*x^2+d^2)^(1/2)+1/5/d^4/(x+d/e)^2/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-e^2x^2 + d^2)^{\frac{3}{2}}(ex + d)^2x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)^2\*x^3), x, algorithm="maxima")

[Out] integrate(1/((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)^2\*x^3), x)

**Fricas [A]** time = 0.292167, size = 830, normalized size = 4.54

$$54e^{10}x^{10} - 212de^9x^9 - 1063d^2e^8x^8 + 166d^3e^7x^7 + 2940d^4e^6x^6 + 890d^5e^5x^5 - 2585d^6e^4x^4 - 1000d^7e^3x^3 + 740d^8e^2x^2 + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)^2\*x^3),x, algorithm="fricas")

[Out] 1/10\*(54\*e^10\*x^10 - 212\*d\*e^9\*x^9 - 1063\*d^2\*e^8\*x^8 + 166\*d^3\*e^7\*x^7 + 2940\*d^4\*e^6\*x^6 + 890\*d^5\*e^5\*x^5 - 2585\*d^6\*e^4\*x^4 - 1000\*d^7\*e^3\*x^3 + 740\*d^8\*e^2\*x^2 + 160\*d^9\*e\*x - 80\*d^10 + 45\*(e^10\*x^10 + 2\*d\*e^9\*x^9 - 12\*d^2\*e^8\*x^8 - 26\*d^3\*e^7\*x^7 + 15\*d^4\*e^6\*x^6 + 56\*d^5\*e^5\*x^5 + 12\*d^6\*e^4\*x^4 - 32\*d^7\*e^3\*x^3 - 16\*d^8\*e^2\*x^2 + (5\*d\*e^8\*x^8 + 10\*d^2\*e^7\*x^7 - 15\*d^3\*e^6\*x^6 - 40\*d^4\*e^5\*x^5 - 4\*d^5\*e^4\*x^4 + 32\*d^6\*e^3\*x^3 + 16\*d^7\*e^2\*x^2)\*sqrt(-e^2\*x^2 + d^2))\*log(-(d - sqrt(-e^2\*x^2 + d^2))/x) + (64\*e^9\*x^9 + 353\*d\*e^8\*x^8 - 286\*d^2\*e^7\*x^7 - 1900\*d^3\*e^6\*x^6 - 450\*d^4\*e^5\*x^5 + 2245\*d^5\*e^4\*x^4 + 920\*d^6\*e^3\*x^3 - 700\*d^7\*e^2\*x^2 - 160\*d^8\*e\*x + 80\*d^9)\*sqrt(-e^2\*x^2 + d^2))/(d^7\*e^8\*x^10 + 2\*d^8\*e^7\*x^9 - 12\*d^9\*e^6\*x^8 - 26\*d^10\*e^5\*x^7 + 15\*d^11\*e^4\*x^6 + 56\*d^12\*e^3\*x^5 + 12\*d^13\*e^2\*x^4 - 32\*d^14\*e\*x^3 - 16\*d^15\*x^2 + (5\*d^8\*e^6\*x^8 + 10\*d^9\*e^5\*x^7 - 15\*d^10\*e^4\*x^6 - 40\*d^11\*e^3\*x^5 - 4\*d^12\*e^2\*x^4 + 32\*d^13\*e\*x^3 + 16\*d^14\*x^2)\*sqrt(-e^2\*x^2 + d^2))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (-(-d + ex)(d + ex))^{\frac{3}{2}} (d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/(e\*x+d)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(3/2),x)

[Out] Integral(1/(x\*\*3\*(-(-d + e\*x)\*(d + e\*x))\*\*(3/2)\*(d + e\*x)\*\*2), x)

**GIAC/XCAS [A]** time = 0.654497, size = 4, normalized size = 0.02

sage<sub>0</sub>x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-e^2\*x^2 + d^2)^(3/2)\*(e\*x + d)^2\*x^3),x, algorithm="giac")

[Out] sage<sub>0</sub>x



$$3.179 \quad \int \frac{x^5}{(d+ex)^3 \sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=177

$$\frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} + \frac{13d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6} - \frac{x\sqrt{d^2-e^2x^2}}{2e^5} + \frac{d^4(d-ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6(d^2-e^2x^2)^{3/2}}$$

[Out]  $(d^4*(d - e*x)^3)/(5*e^6*(d^2 - e^2*x^2)^{(5/2)}) - (23*d^3*(d - e*x)^2)/(15*e^6*(d^2 - e^2*x^2)^{(3/2)}) + (127*d^2*(d - e*x))/(15*e^6*\text{Sqrt}[d^2 - e^2*x^2]) + (3*d*\text{Sqrt}[d^2 - e^2*x^2])/e^6 - (x*\text{Sqrt}[d^2 - e^2*x^2])/(2*e^5) + (13*d^2*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(2*e^6)$

**Rubi [A]** time = 0.723625, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{127d^2(d-ex)}{15e^6\sqrt{d^2-e^2x^2}} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} + \frac{13d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6} - \frac{x\sqrt{d^2-e^2x^2}}{2e^5} + \frac{d^4(d-ex)^3}{5e^6(d^2-e^2x^2)^{5/2}} - \frac{23d^3(d-ex)^2}{15e^6(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^5/((d + e\*x)^3\*Sqrt[d^2 - e^2\*x^2]), x]

[Out]  $(d^4*(d - e*x)^3)/(5*e^6*(d^2 - e^2*x^2)^{(5/2)}) - (23*d^3*(d - e*x)^2)/(15*e^6*(d^2 - e^2*x^2)^{(3/2)}) + (127*d^2*(d - e*x))/(15*e^6*\text{Sqrt}[d^2 - e^2*x^2]) + (3*d*\text{Sqrt}[d^2 - e^2*x^2])/e^6 - (x*\text{Sqrt}[d^2 - e^2*x^2])/(2*e^5) + (13*d^2*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(2*e^6)$

**Rubi in Sympy [A]** time = 55.2287, size = 156, normalized size = 0.88

$$\frac{d^4\sqrt{d^2-e^2x^2}}{5e^6(d+ex)^3} - \frac{23d^3\sqrt{d^2-e^2x^2}}{15e^6(d+ex)^2} + \frac{13d^2 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6} + \frac{127d^2\sqrt{d^2-e^2x^2}}{15e^6(d+ex)} + \frac{3d\sqrt{d^2-e^2x^2}}{e^6} - \frac{x\sqrt{d^2-e^2x^2}}{2e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5/(e\*x+d)\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2), x)

[Out]  $d**4*\text{sqrt}(d**2 - e**2*x**2)/(5*e**6*(d + e*x)**3) - 23*d**3*\text{sqrt}(d**2 - e**2*x**2)/(15*e**6*(d + e*x)**2) + 13*d**2*\text{atan}(e*x/\text{sqrt}(d**2 - e**2*x**2))/(2*e**6) + 127*d**2*\text{sqrt}(d**2 - e**2*x**2)/(15*e**6*(d + e*x)) + 3*d*\text{sqrt}(d**2 - e**2*x**2)/e**6 - x*\text{sqrt}(d**2 - e**2*x**2)/(2*e**5)$

**Mathematica [A]** time = 0.149257, size = 98, normalized size = 0.55

$$\frac{195d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{\sqrt{d^2-e^2x^2}(304d^4+717d^3ex+479d^2e^2x^2+45de^3x^3-15e^4x^4)}{(d+ex)^3}}{30e^6}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((d + e\*x)^3\*Sqrt[d^2 - e^2\*x^2]),x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(304\*d^4 + 717\*d^3\*e\*x + 479\*d^2\*e^2\*x^2 + 45\*d\*e^3\*x^3 - 15\*e^4\*x^4))/(d + e\*x)^3 + 195\*d^2\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(30\*e^6)

**Maple [A]** time = 0.025, size = 212, normalized size = 1.2

$$\begin{aligned}
 & -\frac{x}{2e^5}\sqrt{-e^2x^2+d^2} + \frac{13d^2}{2e^5}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}} \\
 & + 3\frac{d\sqrt{-e^2x^2+d^2}}{e^6} + \frac{127d^2}{15e^7}\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}\left(x+\frac{d}{e}\right)^{-1} \\
 & - \frac{23d^3}{15e^8}\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}\left(x+\frac{d}{e}\right)^{-2} \\
 & + \frac{d^4}{5e^9}\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}\left(x+\frac{d}{e}\right)^{-3}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(e\*x+d)^3/(-e^2\*x^2+d^2)^(1/2),x)

[Out] -1/2\*x\*(-e^2\*x^2+d^2)^(1/2)/e^5+13/2/e^5\*d^2/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))+3\*d\*(-e^2\*x^2+d^2)^(1/2)/e^6+127/15/e^7\*d^2/(x+d/e)\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)-23/15/e^8\*d^3/(x+d/e)^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)+1/5\*d^4/e^9/(x+d/e)^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(sqrt(-e^2\*x^2 + d^2)\*(e\*x + d)^3),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.304596, size = 730, normalized size = 4.12

$$15e^9x^9 - 120de^8x^8 - 678d^2e^7x^7 + 210d^3e^6x^6 + 5421d^4e^5x^5 + 6500d^5e^4x^4 - 2860d^6e^3x^3 - 7800d^7e^2x^2 - 3120d^8ex + 3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(sqrt(-e^2\*x^2 + d^2)\*(e\*x + d)^3),x, algorithm="fricas")

[Out] -1/30\*(15\*e^9\*x^9 - 120\*d\*e^8\*x^8 - 678\*d^2\*e^7\*x^7 + 210\*d^3\*e^6\*x^6 + 5421\*d^4\*e^5\*x^5 + 6500\*d^5\*e^4\*x^4 - 2860\*d^6\*e^3\*x^3 - 7800\*d^7\*e^2\*x^2 - 3120\*d^8\*e\*x + 390\*(d^2\*e^7\*x^7 + 7\*d^3\*e^6\*x^6 + 3\*d^4\*e^5\*x^5 - 31\*d^5\*e^4\*x^4 - 40\*d^6\*e^3\*x^3 + 12\*d^7\*e^2\*x

$$\begin{aligned} &^2 + 40*d^8*e*x + 16*d^9 - (d^2*e^6*x^6 - 2*d^3*e^5*x^5 - 19*d^4* \\ &e^4*x^4 - 20*d^5*e^3*x^3 + 20*d^6*e^2*x^2 + 40*d^7*e*x + 16*d^8)* \\ &\text{sqrt}(-e^2*x^2 + d^2))*\text{arctan}(-(d - \text{sqrt}(-e^2*x^2 + d^2))/(e*x)) + \\ &(15*e^8*x^8 + 15*d*e^7*x^7 - 535*d^2*e^6*x^6 - 2821*d^3*e^5*x^5 \\ &- 2600*d^4*e^4*x^4 + 4420*d^5*e^3*x^3 + 7800*d^6*e^2*x^2 + 3120*d \\ &^7*e*x)*\text{sqrt}(-e^2*x^2 + d^2))/(e^{13}*x^7 + 7*d*e^{12}*x^6 + 3*d^2*e^{11}*x^5 \\ &- 31*d^3*e^{10}*x^4 - 40*d^4*e^9*x^3 + 12*d^5*e^8*x^2 + 40*d \\ &^6*e^7*x + 16*d^7*e^6 - (e^{12}*x^6 - 2*d*e^{11}*x^5 - 19*d^2*e^{10}*x^4 \\ &- 20*d^3*e^9*x^3 + 20*d^4*e^8*x^2 + 40*d^5*e^7*x + 16*d^6*e^6))* \\ &\text{sqrt}(-e^2*x^2 + d^2)) \end{aligned}$$


---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{-(-d+ex)(d+ex)}(d+ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5/(e\*x+d)\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2),x)

[Out] Integral(x\*\*5/(sqrt(-(-d + e\*x)\*(d + e\*x))\*(d + e\*x)\*\*3), x)

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(sqrt(-e^2\*x^2 + d^2)\*(e\*x + d)^3),x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.180 \quad \int \frac{x^4}{(d+ex)^3 \sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=146

$$\frac{6d^2(d-ex)^2}{5e^5(d^2-e^2x^2)^{3/2}} - \frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{3d \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5} - \frac{d^3(d-ex)^3}{5e^5(d^2-e^2x^2)^{5/2}}$$

[Out]  $-(d^3*(d - e*x)^3)/(5*e^5*(d^2 - e^2*x^2)^{(5/2)}) + (6*d^2*(d - e*x)^2)/(5*e^5*(d^2 - e^2*x^2)^{(3/2)}) - (24*d*(d - e*x))/(5*e^5*\text{Sqrt}[d^2 - e^2*x^2]) - \text{Sqrt}[d^2 - e^2*x^2]/e^5 - (3*d*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/e^5$

**Rubi [A]** time = 0.594183, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\frac{6d^2(d-ex)^2}{5e^5(d^2-e^2x^2)^{3/2}} - \frac{24d(d-ex)}{5e^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{e^5} - \frac{3d \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5} - \frac{d^3(d-ex)^3}{5e^5(d^2-e^2x^2)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[x^4/((d + e\*x)^3\*Sqrt[d^2 - e^2\*x^2]), x]

[Out]  $-(d^3*(d - e*x)^3)/(5*e^5*(d^2 - e^2*x^2)^{(5/2)}) + (6*d^2*(d - e*x)^2)/(5*e^5*(d^2 - e^2*x^2)^{(3/2)}) - (24*d*(d - e*x))/(5*e^5*\text{Sqrt}[d^2 - e^2*x^2]) - \text{Sqrt}[d^2 - e^2*x^2]/e^5 - (3*d*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/e^5$

**Rubi in Sympy [A]** time = 45.0295, size = 128, normalized size = 0.88

$$-\frac{d^3\sqrt{d^2-e^2x^2}}{5e^5(d+ex)^3} + \frac{6d^2\sqrt{d^2-e^2x^2}}{5e^5(d+ex)^2} - \frac{3d \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5} - \frac{24d\sqrt{d^2-e^2x^2}}{5e^5(d+ex)} - \frac{\sqrt{d^2-e^2x^2}}{e^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(e\*x+d)\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2), x)

[Out]  $-d**3*\text{sqrt}(d**2 - e**2*x**2)/(5*e**5*(d + e*x)**3) + 6*d**2*\text{sqrt}(d**2 - e**2*x**2)/(5*e**5*(d + e*x)**2) - 3*d*\text{atan}(e*x/\text{sqrt}(d**2 - e**2*x**2))/e**5 - 24*d*\text{sqrt}(d**2 - e**2*x**2)/(5*e**5*(d + e*x)) - \text{sqrt}(d**2 - e**2*x**2)/e**5$

**Mathematica [A]** time = 0.164934, size = 85, normalized size = 0.58

$$-\frac{15d \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{\sqrt{d^2-e^2x^2}(24d^3+57d^2ex+39de^2x^2+5e^3x^3)}{(d+ex)^3}}{5e^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/((d + e\*x)^3\*Sqrt[d^2 - e^2\*x^2]), x]

[Out]  $-((\text{Sqrt}[d^2 - e^2*x^2]*(24*d^3 + 57*d^2*e*x + 39*d*e^2*x^2 + 5*e^3*x^3))/(d + e*x)^3 + 15*d*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(5*$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.181 \quad \int \frac{x^3}{(d+ex)^3 \sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=120

$$\frac{d^2(d-ex)^3}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{13d(d-ex)^2}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}} + \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

[Out]  $(d^2*(d - e*x)^3)/(5*e^4*(d^2 - e^2*x^2)^{(5/2)}) - (13*d*(d - e*x)^2)/(15*e^4*(d^2 - e^2*x^2)^{(3/2)}) + (32*(d - e*x))/(15*e^4*\text{Sqrt}[d^2 - e^2*x^2]) + \text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]]/e^4$

**Rubi [A]** time = 0.482841, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\frac{d^2(d-ex)^3}{5e^4(d^2-e^2x^2)^{5/2}} - \frac{13d(d-ex)^2}{15e^4(d^2-e^2x^2)^{3/2}} + \frac{32(d-ex)}{15e^4\sqrt{d^2-e^2x^2}} + \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4}$$

Antiderivative was successfully verified.

[In] Int[x^3/((d + e\*x)^3\*Sqrt[d^2 - e^2\*x^2]), x]

[Out]  $(d^2*(d - e*x)^3)/(5*e^4*(d^2 - e^2*x^2)^{(5/2)}) - (13*d*(d - e*x)^2)/(15*e^4*(d^2 - e^2*x^2)^{(3/2)}) + (32*(d - e*x))/(15*e^4*\text{Sqrt}[d^2 - e^2*x^2]) + \text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]]/e^4$

**Rubi in Sympy [A]** time = 40.5074, size = 104, normalized size = 0.87

$$\frac{d^2\sqrt{d^2-e^2x^2}}{5e^4(d+ex)^3} - \frac{13d\sqrt{d^2-e^2x^2}}{15e^4(d+ex)^2} + \frac{\text{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4} + \frac{32\sqrt{d^2-e^2x^2}}{15e^4(d+ex)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3/(e\*x+d)\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2), x)

[Out]  $d^2*\text{sqrt}(d^2 - e^2*x^2)/(5*e^4*(d + e*x)^3) - 13*d*\text{sqrt}(d^2 - e^2*x^2)/(15*e^4*(d + e*x)^2) + \text{atan}(e*x/\text{sqrt}(d^2 - e^2*x^2))/e^4 + 32*\text{sqrt}(d^2 - e^2*x^2)/(15*e^4*(d + e*x))$

**Mathematica [A]** time = 0.110382, size = 73, normalized size = 0.61

$$\frac{\sqrt{d^2-e^2x^2}(22d^2+51dex+32e^2x^2)}{(d+ex)^3} + 15 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)$$

$15e^4$

Antiderivative was successfully verified.

[In] Integrate[x^3/((d + e\*x)^3\*Sqrt[d^2 - e^2\*x^2]), x]

[Out]  $((\text{Sqrt}[d^2 - e^2*x^2]*(22*d^2 + 51*d*e*x + 32*e^2*x^2))/(d + e*x)^3 + 15*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(15*e^4)$

**Maple [A]** time = 0.016, size = 163, normalized size = 1.4

$$\frac{1}{e^3} \arctan\left(x\sqrt{e^2} \frac{1}{\sqrt{-e^2x^2+d^2}}\right) \frac{1}{\sqrt{e^2}} + \frac{32}{15e^5} \sqrt{-\left(x+\frac{d}{e}\right)^2 e^2 + 2de\left(x+\frac{d}{e}\right)\left(x+\frac{d}{e}\right)^{-1}} \\ - \frac{13d}{15e^6} \sqrt{-\left(x+\frac{d}{e}\right)^2 e^2 + 2de\left(x+\frac{d}{e}\right)\left(x+\frac{d}{e}\right)^{-2}} + \frac{d^2}{5e^7} \sqrt{-\left(x+\frac{d}{e}\right)^2 e^2 + 2de\left(x+\frac{d}{e}\right)\left(x+\frac{d}{e}\right)^{-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(e\*x+d)^3/(-e^2\*x^2+d^2)^(1/2), x)

[Out] 1/e^3/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))+32/15/e^5/(x+d/e)\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)-13/15/e^6\*d/(x+d/e)^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)+1/5\*d^2/e^7/(x+d/e)^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(sqrt(-e^2\*x^2 + d^2)\*(e\*x + d)^3), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.291176, size = 456, normalized size = 3.8

$$\frac{54e^5x^5 + 65de^4x^4 - 85d^2e^3x^3 - 150d^3e^2x^2 - 60d^4ex - 30(e^5x^5 + 5de^4x^4 + 5d^2e^3x^3 - 5d^3e^2x^2 - 10d^4ex - 4d^5 - (e^4x^4 - 4d^4e^3x^3 + 6d^5e^2x^2 - 4d^6ex + d^7))}{15(e^9x^5 + 5de^8x^4 + 5d^2e^7x^3 - 5d^3e^6x^2 - 10d^4e^5x - 4d^5 - (e^4x^4 - 4d^4e^3x^3 + 6d^5e^2x^2 - 4d^6ex + d^7))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(sqrt(-e^2\*x^2 + d^2)\*(e\*x + d)^3), x, algorithm="fricas")

[Out] 1/15\*(54\*e^5\*x^5 + 65\*d\*e^4\*x^4 - 85\*d^2\*e^3\*x^3 - 150\*d^3\*e^2\*x^2 - 60\*d^4\*e\*x - 30\*(e^5\*x^5 + 5\*d\*e^4\*x^4 + 5\*d^2\*e^3\*x^3 - 5\*d^3\*e^2\*x^2 - 10\*d^4\*e\*x - 4\*d^5 - (e^4\*x^4 - 7\*d^2\*e^2\*x^2 - 10\*d^3\*e\*x - 4\*d^4)))\*sqrt(-e^2\*x^2 + d^2))\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) + 5\*(2\*e^4\*x^4 + 23\*d\*e^3\*x^3 + 30\*d^2\*e^2\*x^2 + 12\*d^3\*e\*x)\*sqrt(-e^2\*x^2 + d^2))/(e^9\*x^5 + 5\*d\*e^8\*x^4 + 5\*d^2\*e^7\*x^3 - 5\*d^3\*e^6\*x^2 - 10\*d^4\*e^5\*x - 4\*d^5 - (e^4\*x^4 - 7\*d^2\*e^2\*x^2 - 10\*d^3\*e\*x - 4\*d^4))\*sqrt(-e^2\*x^2 + d^2))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{-(-d+ex)(d+ex)}(d+ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(e\*x+d)\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2), x)



[Out]  $\text{Integral}(x^{**3}/(\text{sqrt}(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)$

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3),x, algorithm="giac")`

[Out] Exception raised: NotImplementedError

$$3.182 \quad \int \frac{x^2}{(d+ex)^3 \sqrt{d^2 - e^2 x^2}} dx$$

**Optimal.** Leaf size=95

$$-\frac{d\sqrt{d^2 - e^2 x^2}}{5e^3(d+ex)^3} + \frac{8\sqrt{d^2 - e^2 x^2}}{15e^3(d+ex)^2} - \frac{7\sqrt{d^2 - e^2 x^2}}{15de^3(d+ex)}$$

[Out]  $-(d*\text{Sqrt}[d^2 - e^2*x^2])/(5*e^3*(d + e*x)^3) + (8*\text{Sqrt}[d^2 - e^2*x^2])/(15*e^3*(d + e*x)^2) - (7*\text{Sqrt}[d^2 - e^2*x^2])/(15*d*e^3*(d + e*x))$

**Rubi [A]** time = 0.274871, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$-\frac{d\sqrt{d^2 - e^2 x^2}}{5e^3(d+ex)^3} + \frac{8\sqrt{d^2 - e^2 x^2}}{15e^3(d+ex)^2} - \frac{7\sqrt{d^2 - e^2 x^2}}{15de^3(d+ex)}$$

Antiderivative was successfully verified.

[In] Int[x^2/((d + e\*x)^3\*Sqrt[d^2 - e^2\*x^2]), x]

[Out]  $-(d*\text{Sqrt}[d^2 - e^2*x^2])/(5*e^3*(d + e*x)^3) + (8*\text{Sqrt}[d^2 - e^2*x^2])/(15*e^3*(d + e*x)^2) - (7*\text{Sqrt}[d^2 - e^2*x^2])/(15*d*e^3*(d + e*x))$

**Rubi in Sympy [A]** time = 30.1672, size = 80, normalized size = 0.84

$$-\frac{d\sqrt{d^2 - e^2 x^2}}{5e^3(d+ex)^3} + \frac{8\sqrt{d^2 - e^2 x^2}}{15e^3(d+ex)^2} - \frac{7\sqrt{d^2 - e^2 x^2}}{15de^3(d+ex)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(e\*x+d)\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2), x)

[Out]  $-d*\text{sqrt}(d**2 - e**2*x**2)/(5*e**3*(d + e*x)**3) + 8*\text{sqrt}(d**2 - e**2*x**2)/(15*e**3*(d + e*x)**2) - 7*\text{sqrt}(d**2 - e**2*x**2)/(15*d*e**3*(d + e*x))$

**Mathematica [A]** time = 0.0436854, size = 52, normalized size = 0.55

$$-\frac{\sqrt{d^2 - e^2 x^2} (2d^2 + 6dex + 7e^2 x^2)}{15de^3(d+ex)^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((d + e\*x)^3\*Sqrt[d^2 - e^2\*x^2]), x]

[Out]  $-(\text{Sqrt}[d^2 - e^2*x^2]*(2*d^2 + 6*d*e*x + 7*e^2*x^2))/(15*d*e^3*(d + e*x)^3)$

**Maple [A]** time = 0.011, size = 55, normalized size = 0.6

$$-\frac{(-ex + d)(7e^2x^2 + 6dex + 2d^2)}{15e^3d(ex + d)^2} \frac{1}{\sqrt{-e^2x^2 + d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)`

[Out] `-1/15*(-e*x+d)*(7*e^2*x^2+6*d*e*x+2*d^2)/(e*x+d)^2/d/e^3/(-e^2*x^2+d^2)^(1/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.283272, size = 213, normalized size = 2.24

$$\frac{9e^2x^5 - 5dex^4 - 20d^2x^3 + 5(ex^4 + 4dx^3)\sqrt{-e^2x^2 + d^2}}{15\left(de^5x^5 + 5d^2e^4x^4 + 5d^3e^3x^3 - 5d^4e^2x^2 - 10d^5ex - 4d^6 - (de^4x^4 - 7d^3e^2x^2 - 10d^4ex - 4d^5)\sqrt{-e^2x^2 + d^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3),x, algorithm="fricas")`

[Out] `-1/15*(9*e^2*x^5 - 5*d*e*x^4 - 20*d^2*x^3 + 5*(e*x^4 + 4*d*x^3)*sqrt(-e^2*x^2 + d^2))/(d*e^5*x^5 + 5*d^2*e^4*x^4 + 5*d^3*e^3*x^3 - 5*d^4*e^2*x^2 - 10*d^5*e*x - 4*d^6 - (d*e^4*x^4 - 7*d^3*e^2*x^2 - 10*d^4*e*x - 4*d^5)*sqrt(-e^2*x^2 + d^2))`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{-(-d + ex)(d + ex)}(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)`

[Out] `Integral(x**2/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)`

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3),x, algorithm="giac")`

[Out] Exception raised: NotImplementedError

$$3.183 \quad \int \frac{x}{(d+ex)^3 \sqrt{d^2 - e^2 x^2}} dx$$

**Optimal.** Leaf size=97

$$-\frac{\sqrt{d^2 - e^2 x^2}}{5d^2 e^2 (d + ex)} - \frac{\sqrt{d^2 - e^2 x^2}}{5de^2 (d + ex)^2} + \frac{\sqrt{d^2 - e^2 x^2}}{5e^2 (d + ex)^3}$$

[Out] Sqrt[d^2 - e^2\*x^2]/(5\*e^2\*(d + e\*x)^3) - Sqrt[d^2 - e^2\*x^2]/(5\*d\*e^2\*(d + e\*x)^2) - Sqrt[d^2 - e^2\*x^2]/(5\*d^2\*e^2\*(d + e\*x))

**Rubi [A]** time = 0.144726, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$-\frac{\sqrt{d^2 - e^2 x^2}}{5d^2 e^2 (d + ex)} - \frac{\sqrt{d^2 - e^2 x^2}}{5de^2 (d + ex)^2} + \frac{\sqrt{d^2 - e^2 x^2}}{5e^2 (d + ex)^3}$$

Antiderivative was successfully verified.

[In] Int[x/((d + e\*x)^3\*Sqrt[d^2 - e^2\*x^2]),x]

[Out] Sqrt[d^2 - e^2\*x^2]/(5\*e^2\*(d + e\*x)^3) - Sqrt[d^2 - e^2\*x^2]/(5\*d\*e^2\*(d + e\*x)^2) - Sqrt[d^2 - e^2\*x^2]/(5\*d^2\*e^2\*(d + e\*x))

**Rubi in Sympy [A]** time = 13.6547, size = 78, normalized size = 0.8

$$\frac{\sqrt{d^2 - e^2 x^2}}{5e^2 (d + ex)^3} - \frac{\sqrt{d^2 - e^2 x^2}}{5de^2 (d + ex)^2} - \frac{\sqrt{d^2 - e^2 x^2}}{5d^2 e^2 (d + ex)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x/(e\*x+d)\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2),x)

[Out] sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(5\*e\*\*2\*(d + e\*x)\*\*3) - sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(5\*d\*e\*\*2\*(d + e\*x)\*\*2) - sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(5\*d\*\*2\*e\*\*2\*(d + e\*x))

**Mathematica [A]** time = 0.0390686, size = 49, normalized size = 0.51

$$-\frac{\sqrt{d^2 - e^2 x^2} (d^2 + 3dex + e^2 x^2)}{5d^2 e^2 (d + ex)^3}$$

Antiderivative was successfully verified.

[In] Integrate[x/((d + e\*x)^3\*Sqrt[d^2 - e^2\*x^2]),x]

[Out] -(Sqrt[d^2 - e^2\*x^2]\*(d^2 + 3\*d\*e\*x + e^2\*x^2))/(5\*d^2\*e^2\*(d + e\*x)^3)

**Maple [A]** time = 0.011, size = 52, normalized size = 0.5

$$-\frac{(-ex + d)(e^2 x^2 + 3dex + d^2)}{5e^2 d^2 (ex + d)^2} \frac{1}{\sqrt{-e^2 x^2 + d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)`

[Out] `-1/5*(-e*x+d)*(e^2*x^2+3*d*e*x+d^2)/(e*x+d)^2/d^2/e^2/(-e^2*x^2+d^2)^(1/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.281395, size = 238, normalized size = 2.45

$$\frac{2e^3x^5 + 5de^2x^4 - 5d^2ex^3 - 10d^3x^2 + 5(dex^3 + 2d^2x^2)\sqrt{-e^2x^2 + d^2}}{5(d^2e^5x^5 + 5d^3e^4x^4 + 5d^4e^3x^3 - 5d^5e^2x^2 - 10d^6ex - 4d^7 - (d^2e^4x^4 - 7d^4e^2x^2 - 10d^5ex - 4d^6)\sqrt{-e^2x^2 + d^2})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3),x, algorithm="fricas")`

[Out] `-1/5*(2*e^3*x^5 + 5*d*e^2*x^4 - 5*d^2*e*x^3 - 10*d^3*x^2 + 5*(d*e*x^3 + 2*d^2*x^2)*sqrt(-e^2*x^2 + d^2))/(d^2*e^5*x^5 + 5*d^3*e^4*x^4 + 5*d^4*e^3*x^3 - 5*d^5*e^2*x^2 - 10*d^6*e*x - 4*d^7 - (d^2*e^4*x^4 - 7*d^4*e^2*x^2 - 10*d^5*e*x - 4*d^6)*sqrt(-e^2*x^2 + d^2))`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{-(-d + ex)(d + ex)}(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)`

[Out] `Integral(x/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)`

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3),x, algorithm="giac")`

[Out] Exception raised: NotImplementedError

$$3.184 \quad \int \frac{1}{(d+ex)^3 \sqrt{d^2 - e^2 x^2}} dx$$

**Optimal.** Leaf size=100

$$-\frac{2\sqrt{d^2 - e^2 x^2}}{15d^2 e(d+ex)^2} - \frac{\sqrt{d^2 - e^2 x^2}}{5de(d+ex)^3} - \frac{2\sqrt{d^2 - e^2 x^2}}{15d^3 e(d+ex)}$$

[Out]  $-\text{Sqrt}[d^2 - e^2 x^2]/(5*d*e*(d + e*x)^3) - (2*\text{Sqrt}[d^2 - e^2 x^2])/(15*d^2*e*(d + e*x)^2) - (2*\text{Sqrt}[d^2 - e^2 x^2])/(15*d^3*e*(d + e*x))$

**Rubi [A]** time = 0.110936, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$-\frac{2\sqrt{d^2 - e^2 x^2}}{15d^2 e(d+ex)^2} - \frac{\sqrt{d^2 - e^2 x^2}}{5de(d+ex)^3} - \frac{2\sqrt{d^2 - e^2 x^2}}{15d^3 e(d+ex)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/((d + e*x)^3*\text{Sqrt}[d^2 - e^2*x^2]),x]$

[Out]  $-\text{Sqrt}[d^2 - e^2*x^2]/(5*d*e*(d + e*x)^3) - (2*\text{Sqrt}[d^2 - e^2*x^2])/(15*d^2*e*(d + e*x)^2) - (2*\text{Sqrt}[d^2 - e^2*x^2])/(15*d^3*e*(d + e*x))$

**Rubi in Sympy [A]** time = 13.122, size = 82, normalized size = 0.82

$$-\frac{\sqrt{d^2 - e^2 x^2}}{5de(d+ex)^3} - \frac{2\sqrt{d^2 - e^2 x^2}}{15d^2 e(d+ex)^2} - \frac{2\sqrt{d^2 - e^2 x^2}}{15d^3 e(d+ex)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)$

[Out]  $-\text{sqrt}(d**2 - e**2*x**2)/(5*d*e*(d + e*x)**3) - 2*\text{sqrt}(d**2 - e**2*x**2)/(15*d**2*e*(d + e*x)**2) - 2*\text{sqrt}(d**2 - e**2*x**2)/(15*d**3*e*(d + e*x))$

**Mathematica [A]** time = 0.034506, size = 52, normalized size = 0.52

$$-\frac{\sqrt{d^2 - e^2 x^2} (7d^2 + 6dex + 2e^2 x^2)}{15d^3 e(d+ex)^3}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[1/((d + e*x)^3*\text{Sqrt}[d^2 - e^2*x^2]),x]$

[Out]  $-(\text{Sqrt}[d^2 - e^2*x^2]*(7*d^2 + 6*d*e*x + 2*e^2*x^2))/(15*d^3*e*(d + e*x)^3)$

**Maple [A]** time = 0.01, size = 55, normalized size = 0.6

$$-\frac{(-ex + d)(2e^2 x^2 + 6dex + 7d^2)}{15ed^3(ex + d)^2} \frac{1}{\sqrt{-e^2 x^2 + d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2),x)`

[Out]  $-1/15 * (-e*x+d) * (2*e^2*x^2+6*d*e*x+7*d^2)/(e*x+d)^2/d^3/e/(-e^2*x^2+d^2)^(1/2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.280696, size = 273, normalized size = 2.73

$$\frac{9e^4x^5 + 35de^3x^4 + 20d^2e^2x^3 - 60d^3ex^2 - 60d^4x - 5(e^3x^4 - 2de^2x^3 - 12d^2ex^2 - 12d^3x)\sqrt{-e^2x^2 + d^2}}{15(d^3e^5x^5 + 5d^4e^4x^4 + 5d^5e^3x^3 - 5d^6e^2x^2 - 10d^7ex - 4d^8 - (d^3e^4x^4 - 7d^5e^2x^2 - 10d^6ex - 4d^7)\sqrt{-e^2x^2 + d^2})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3),x, algorithm="fricas")`

[Out]  $-1/15 * (9 * e^4 * x^5 + 35 * d * e^3 * x^4 + 20 * d^2 * e^2 * x^3 - 60 * d^3 * e * x^2 - 60 * d^4 * x - 5 * (e^3 * x^4 - 2 * d * e^2 * x^3 - 12 * d^2 * e * x^2 - 12 * d^3 * x) * \text{sqrt}(-e^2 * x^2 + d^2)) / (d^3 * e^5 * x^5 + 5 * d^4 * e^4 * x^4 + 5 * d^5 * e^3 * x^3 - 5 * d^6 * e^2 * x^2 - 10 * d^7 * e * x - 4 * d^8 - (d^3 * e^4 * x^4 - 7 * d^5 * e^2 * x^2 - 10 * d^6 * e * x - 4 * d^7) * \text{sqrt}(-e^2 * x^2 + d^2))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-(-d + ex)(d + ex)}(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)`

[Out] `Integral(1/(sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)`

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3),x, algorithm="giac")`

[Out] Exception raised: NotImplementedError

$$3.185 \quad \int \frac{1}{x(d+ex)^3 \sqrt{d^2 - e^2 x^2}} dx$$

**Optimal.** Leaf size=115

$$\frac{5d - 11ex}{15d^2 (d^2 - e^2 x^2)^{3/2}} + \frac{4(d - ex)}{5(d^2 - e^2 x^2)^{5/2}} + \frac{15d - 22ex}{15d^4 \sqrt{d^2 - e^2 x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{d^4}$$

[Out] (4\*(d - e\*x))/(5\*(d^2 - e^2\*x^2)^(5/2)) + (5\*d - 11\*e\*x)/(15\*d^2\*(d^2 - e^2\*x^2)^(3/2)) + (15\*d - 22\*e\*x)/(15\*d^4\*Sqrt[d^2 - e^2\*x^2]) - ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]/d^4

**Rubi [A]** time = 0.426337, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\frac{5d - 11ex}{15d^2 (d^2 - e^2 x^2)^{3/2}} + \frac{4(d - ex)}{5(d^2 - e^2 x^2)^{5/2}} + \frac{15d - 22ex}{15d^4 \sqrt{d^2 - e^2 x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{d^4}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(d + e\*x)^3\*Sqrt[d^2 - e^2\*x^2]),x]

[Out] (4\*(d - e\*x))/(5\*(d^2 - e^2\*x^2)^(5/2)) + (5\*d - 11\*e\*x)/(15\*d^2\*(d^2 - e^2\*x^2)^(3/2)) + (15\*d - 22\*e\*x)/(15\*d^4\*Sqrt[d^2 - e^2\*x^2]) - ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]/d^4

**Rubi in Sympy [A]** time = 35.9242, size = 97, normalized size = 0.84

$$\frac{\sqrt{d^2 - e^2 x^2}}{5d^2 (d + ex)^3} + \frac{7\sqrt{d^2 - e^2 x^2}}{15d^3 (d + ex)^2} - \frac{\operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{d^4} + \frac{22\sqrt{d^2 - e^2 x^2}}{15d^4 (d + ex)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(e\*x+d)\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2),x)

[Out] sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(5\*d\*\*2\*(d + e\*x)\*\*3) + 7\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*d\*\*3\*(d + e\*x)\*\*2) - atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d)/d\*\*4 + 22\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*d\*\*4\*(d + e\*x))

**Mathematica [A]** time = 0.13618, size = 76, normalized size = 0.66

$$\frac{\sqrt{d^2 - e^2 x^2} (32d^2 + 51dex + 22e^2 x^2)}{(d+ex)^3} - \frac{15 \log\left(\sqrt{d^2 - e^2 x^2} + d\right) + 15 \log(x)}{15d^4}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(d + e\*x)^3\*Sqrt[d^2 - e^2\*x^2]),x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(32\*d^2 + 51\*d\*e\*x + 22\*e^2\*x^2))/(d + e\*x)^3 + 15\*Log[x] - 15\*Log[d + Sqrt[d^2 - e^2\*x^2]])/(15\*d^4)



**Maple [A]** time = 0.017, size = 179, normalized size = 1.6

$$\begin{aligned}
 & -\frac{1}{d^3} \ln\left(\frac{1}{x} \left(2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}} + \frac{22}{15d^4e} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\left(x + \frac{d}{e}\right)^{-1}} \\
 & + \frac{7}{15e^2d^3} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\left(x + \frac{d}{e}\right)^{-2}} \\
 & + \frac{1}{5d^2e^3} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\left(x + \frac{d}{e}\right)^{-3}}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(e*x+d)^3/(-e^2*x^2+d^2)^(1/2), x)`

[Out] `-1/d^3/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+22/15/d^4/e/(x+d/e)*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+7/15/e^2/d^3/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)+1/5/d^2/e^3/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-e^2x^2 + d^2}(ex + d)^3x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3*x), x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3*x), x)`

**Fricas [A]** time = 0.28651, size = 450, normalized size = 3.91

$$\frac{54e^5x^5 + 145de^4x^4 - 5d^2e^3x^3 - 270d^3e^2x^2 - 180d^4ex + 15(e^5x^5 + 5de^4x^4 + 5d^2e^3x^3 - 5d^3e^2x^2 - 10d^4ex - 4d^5 - (e^4x^4 - 4d^4e^3x^3 + 6d^5e^2x^2 - 4d^6ex - d^7))\sqrt{-e^2x^2 + d^2}}{15(d^4e^5x^5 + 5d^5e^4x^4 + 5d^6e^3x^3 - 5d^7e^2x^2 - 10d^8ex - d^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3*x), x, algorithm="fricas")`

[Out] `1/15*(54*e^5*x^5 + 145*d*e^4*x^4 - 5*d^2*e^3*x^3 - 270*d^3*e^2*x^2 - 180*d^4*e*x + 15*(e^5*x^5 + 5*d*e^4*x^4 + 5*d^2*e^3*x^3 - 5*d^3*e^2*x^2 - 10*d^4*e*x - 4*d^5 - (e^4*x^4 - 7*d^2*e^2*x^2 - 10*d^3*e*x - 4*d^4)*sqrt(-e^2*x^2 + d^2))*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - 5*(2*e^4*x^4 - 19*d*e^3*x^3 - 54*d^2*e^2*x^2 - 36*d^3*e*x)*sqrt(-e^2*x^2 + d^2))/(d^4*e^5*x^5 + 5*d^5*e^4*x^4 + 5*d^6*e^3*x^3 - 5*d^7*e^2*x^2 - 10*d^8*e*x - d^9)*sqrt(-e^2*x^2 + d^2)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{-(-d + ex)(d + ex)}(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)**3/(-e**2*x**2+d**2)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(-(-d + e*x)*(d + e*x))*(d + e*x)**3), x)
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(-e^2*x^2 + d^2)*(e*x + d)^3*x),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.186 \quad \int \frac{1}{x^2(d+ex)^3\sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=146

$$-\frac{4e(d-ex)}{5d(d^2-e^2x^2)^{5/2}} - \frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} + \frac{3e \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5} - \frac{e(5d-7ex)}{5d^3(d^2-e^2x^2)^{3/2}}$$

[Out]  $(-4*e*(d - e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) - (e*(5*d - 7*e*x))/(5*d^3*(d^2 - e^2*x^2)^(3/2)) - (e*(15*d - 19*e*x))/(5*d^5*\text{Sqrt}[d^2 - e^2*x^2]) - \text{Sqrt}[d^2 - e^2*x^2]/(d^5*x) + (3*e*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/d^5$

**Rubi [A]** time = 0.530599, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-\frac{4e(d-ex)}{5d(d^2-e^2x^2)^{5/2}} - \frac{e(15d-19ex)}{5d^5\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^5x} + \frac{3e \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5} - \frac{e(5d-7ex)}{5d^3(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(d + e\*x)^3\*Sqrt[d^2 - e^2\*x^2]), x]

[Out]  $(-4*e*(d - e*x))/(5*d*(d^2 - e^2*x^2)^(5/2)) - (e*(5*d - 7*e*x))/(5*d^3*(d^2 - e^2*x^2)^(3/2)) - (e*(15*d - 19*e*x))/(5*d^5*\text{Sqrt}[d^2 - e^2*x^2]) - \text{Sqrt}[d^2 - e^2*x^2]/(d^5*x) + (3*e*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/d^5$

**Rubi in Sympy [A]** time = 44.2998, size = 124, normalized size = 0.85

$$-\frac{e\sqrt{d^2-e^2x^2}}{5d^3(d+ex)^3} - \frac{4e\sqrt{d^2-e^2x^2}}{5d^4(d+ex)^2} + \frac{3e \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^5} - \frac{19e\sqrt{d^2-e^2x^2}}{5d^5(d+ex)} - \frac{\sqrt{d^2-e^2x^2}}{d^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(e\*x+d)\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2), x)

[Out]  $-e*\text{sqrt}(d**2 - e**2*x**2)/(5*d**3*(d + e*x)**3) - 4*e*\text{sqrt}(d**2 - e**2*x**2)/(5*d**4*(d + e*x)**2) + 3*e*\operatorname{atanh}(\text{sqrt}(d**2 - e**2*x**2)/d)/d**5 - 19*e*\text{sqrt}(d**2 - e**2*x**2)/(5*d**5*(d + e*x)) - \text{sqrt}(d**2 - e**2*x**2)/(d**5*x)$

**Mathematica [A]** time = 0.216613, size = 92, normalized size = 0.63

$$-\frac{15e \log\left(\sqrt{d^2-e^2x^2} + d\right) + \frac{\sqrt{d^2-e^2x^2}(5d^3+39d^2ex+57de^2x^2+24e^3x^3)}{x(d+ex)^3} + 15e \log(x)}{5d^5}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(d + e\*x)^3\*Sqrt[d^2 - e^2\*x^2]), x]

[Out]  $-((\text{Sqrt}[d^2 - e^2*x^2]*(5*d^3 + 39*d^2*e*x + 57*d*e^2*x^2 + 24*e^3*x^3))/(x*(d + e*x)^3) + 15*e*\text{Log}[x] - 15*e*\text{Log}[d + \text{Sqrt}[d^2 - e$

$$^2 * x^2 ] ] ) / ( 5 * d^5 )$$

**Maple [A]** time = 0.018, size = 199, normalized size = 1.4

$$\begin{aligned} & -\frac{1}{d^5 x} \sqrt{-e^2 x^2 + d^2} - \frac{1}{5 d^3 e^2} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right) \left(x + \frac{d}{e}\right)^{-3}} \\ & - \frac{4}{5 d^4 e} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right) \left(x + \frac{d}{e}\right)^{-2}} \\ & - \frac{19}{5 d^5} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right) \left(x + \frac{d}{e}\right)^{-1}} + 3 \frac{e}{d^4 \sqrt{d^2}} \ln \left( \frac{2 d^2 + 2 \sqrt{d^2} \sqrt{-e^2 x^2 + d^2}}{x} \right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(e\*x+d)^3/(-e^2\*x^2+d^2)^(1/2), x)

[Out]  $-( -e^2 x^2 + d^2 )^{1/2} / d^5 / x - 1/5 / d^3 / e^2 / (x+d/e)^3 * ( - (x+d/e)^2 * e^2 + 2 * d * e * (x+d/e) )^{1/2} - 4/5 / d^4 / e / (x+d/e)^2 * ( - (x+d/e)^2 * e^2 + 2 * d * e * (x+d/e) )^{1/2} - 19/5 / d^5 / (x+d/e) * ( - (x+d/e)^2 * e^2 + 2 * d * e * (x+d/e) )^{1/2} + 3 * e / d^4 / (d^2)^{1/2} * \ln( (2 * d^2 + 2 * (d^2)^{1/2} * ( -e^2 x^2 + d^2 )^{1/2} ) / x )$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-e^2 x^2 + d^2} (e x + d)^3 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(-e^2\*x^2 + d^2)\*(e\*x + d)^3\*x^2), x, algorithm="maxima")

[Out] integrate(1/(sqrt(-e^2\*x^2 + d^2)\*(e\*x + d)^3\*x^2), x)

**Fricas [A]** time = 0.292916, size = 620, normalized size = 4.25

$$\frac{153 d e^6 x^6 + 330 d^2 e^5 x^5 - 70 d^3 e^4 x^4 - 525 d^4 e^3 x^3 - 220 d^5 e^2 x^2 + 100 d^6 e x + 40 d^7 - 15 (e^7 x^7 - d e^6 x^6 - 13 d^2 e^5 x^5 - 15 d^3 e^4 x^4 - 12 d^4 e^3 x^3 - 20 d^5 e^2 x^2 - 8 d^6 e x + 4 d^7) \sqrt{-e^2 x^2 + d^2}}{5 (d^5 e^6 x^7 - d^6 e^5 x^6 - 13 d^2 e^5 x^5 - 15 d^3 e^4 x^4 - 12 d^4 e^3 x^3 - 20 d^5 e^2 x^2 - 8 d^6 e x + 4 d^7) \sqrt{-e^2 x^2 + d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(-e^2\*x^2 + d^2)\*(e\*x + d)^3\*x^2), x, algorithm="fricas")

[Out]  $1/5 * ( 153 * d * e^6 * x^6 + 330 * d^2 * e^5 * x^5 - 70 * d^3 * e^4 * x^4 - 525 * d^4 * e^3 * x^3 - 220 * d^5 * e^2 * x^2 + 100 * d^6 * e * x + 40 * d^7 - 15 * ( e^7 * x^7 - d * e^6 * x^6 - 13 * d^2 * e^5 * x^5 - 15 * d^3 * e^4 * x^4 + 8 * d^4 * e^3 * x^3 + 20 * d^5 * e^2 * x^2 + 8 * d^6 * e * x + ( e^6 * x^6 + 6 * d * e^5 * x^5 + 5 * d^2 * e^4 * x^4 - 12 * d^3 * e^3 * x^3 - 20 * d^4 * e^2 * x^2 - 8 * d^5 * e * x ) * \sqrt{-e^2 * x^2 + d^2} ) * \log( -(d - \sqrt{-e^2 * x^2 + d^2}) / x ) - ( 48 * e^6 * x^6 + 105 * d * e^5 * x^5 - 165 * d^2 * e^4 * x^4 - 475 * d^3 * e^3 * x^3 - 200 * d^4 * e^2 * x^2 + 100 * d^5 * e * x + 40 * d^6 ) * \sqrt{-e^2 * x^2 + d^2} ) / ( d^5 * e^6 * x^7 - d^6 * e^5 * x^6 - 13 * d^2 * e^5 * x^5 - 15 * d^3 * e^4 * x^4 + 8 * d^4 * e^3 * x^3 + 20 * d^5 * e^2 * x^2 + 8 * d^6 * e * x + ( d^5 * e^5 * x^6 + 6 * d^6 * e^4 * x^5 + 5 * d^7 * e^3 * x^4 - 12 * d^8 * e^2 * x^3 - 20 * d^9 * e * x^2 - 8 * d^{10} * x ) * \sqrt{-e^2 * x^2 + d^2} )$

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**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt{-(-d + ex)(d + ex)(d + ex)^3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(e\*x+d)\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2),x)

[Out] Integral(1/(x\*\*2\*sqrt(-(-d + e\*x)\*(d + e\*x))\*(d + e\*x)\*\*3), x)

---

**GIAC/XCAS [A]** time = 0.30971, size = 1, normalized size = 0.01

+∞

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(-e^2\*x^2 + d^2)\*(e\*x + d)^3\*x^2),x, algorithm="giac")

[Out] +Infinity

$$3.187 \quad \int \frac{1}{x^3(d+ex)^3\sqrt{d^2-e^2x^2}} dx$$

**Optimal.** Leaf size=183

$$\frac{4e^2(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} + \frac{3e\sqrt{d^2-e^2x^2}}{d^6x} \\ - \frac{13e^2 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{e^2(25d-31ex)}{15d^4(d^2-e^2x^2)^{3/2}}$$

[Out]  $(4e^2(d-ex))/(5d^2(d^2-e^2x^2)^{5/2}) + (e^2(25d-31e^2x))/(15d^4(d^2-e^2x^2)^{3/2}) + (e^2(90d-107e^2x))/(15d^6\sqrt{d^2-e^2x^2}) - \sqrt{d^2-e^2x^2}/(2d^5x^2) + (3e^2\sqrt{d^2-e^2x^2})/(d^6x) - (13e^2\text{ArcTanh}[\sqrt{d^2-e^2x^2}/d])/(2d^6)$

**Rubi [A]** time = 0.648789, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\frac{4e^2(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}} + \frac{e^2(90d-107ex)}{15d^6\sqrt{d^2-e^2x^2}} + \frac{3e\sqrt{d^2-e^2x^2}}{d^6x} \\ - \frac{13e^2 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} + \frac{e^2(25d-31ex)}{15d^4(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*(d+e\*x)^3\*sqrt[d^2-e^2\*x^2]),x]

[Out]  $(4e^2(d-ex))/(5d^2(d^2-e^2x^2)^{5/2}) + (e^2(25d-31e^2x))/(15d^4(d^2-e^2x^2)^{3/2}) + (e^2(90d-107e^2x))/(15d^6\sqrt{d^2-e^2x^2}) - \sqrt{d^2-e^2x^2}/(2d^5x^2) + (3e^2\sqrt{d^2-e^2x^2})/(d^6x) - (13e^2\text{ArcTanh}[\sqrt{d^2-e^2x^2}/d])/(2d^6)$

**Rubi in Sympy [A]** time = 57.7122, size = 158, normalized size = 0.86

$$\frac{e^2\sqrt{d^2-e^2x^2}}{5d^4(d+ex)^3} + \frac{17e^2\sqrt{d^2-e^2x^2}}{15d^5(d+ex)^2} - \frac{\sqrt{d^2-e^2x^2}}{2d^5x^2} - \frac{13e^2 \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^6} + \frac{107e^2\sqrt{d^2-e^2x^2}}{15d^6(d+ex)} + \frac{3e\sqrt{d^2-e^2x^2}}{d^6x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*3/(e\*x+d)\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2),x)

[Out]  $e^2\sqrt{d^2-e^2x^2}/(5d^4(d+e*x)^3) + 17e^2\sqrt{d^2-e^2x^2}/(15d^5(d+e*x)^2) - \sqrt{d^2-e^2x^2}/(2d^5x^2) - 13e^2\operatorname{atanh}(\sqrt{d^2-e^2x^2}/d)/(2d^6) + 107e^2\sqrt{d^2-e^2x^2}/(15d^6(d+e*x)) + 3e\sqrt{d^2-e^2x^2}/(d^6x)$

**Mathematica [A]** time = 0.186904, size = 107, normalized size = 0.58

$$\frac{-195e^2 \log\left(\sqrt{d^2-e^2x^2}+d\right) + \frac{\sqrt{d^2-e^2x^2}(-15d^4+45d^3ex+479d^2e^2x^2+717de^3x^3+304e^4x^4)}{x^2(d+ex)^3}}{30d^6} + 195e^2 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(d + e\*x)^3\*Sqrt[d^2 - e^2\*x^2]),x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(-15\*d^4 + 45\*d^3\*e\*x + 479\*d^2\*e^2\*x^2 + 7  
17\*d\*e^3\*x^3 + 304\*e^4\*x^4))/(x^2\*(d + e\*x)^3) + 195\*e^2\*Log[x] -  
195\*e^2\*Log[d + Sqrt[d^2 - e^2\*x^2]])/(30\*d^6)

**Maple [A]** time = 0.019, size = 222, normalized size = 1.2

$$-\frac{1}{2d^5x^2}\sqrt{-e^2x^2+d^2}-\frac{13e^2}{2d^5}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}$$

$$+\frac{107e}{15d^6}\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\left(x+\frac{d}{e}\right)^{-1}+3\frac{e\sqrt{-e^2x^2+d^2}}{d^6x}}$$

$$+\frac{17}{15d^5}\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\left(x+\frac{d}{e}\right)^{-2}}$$

$$+\frac{1}{5d^4e}\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\left(x+\frac{d}{e}\right)^{-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(e\*x+d)^3/(-e^2\*x^2+d^2)^(1/2),x)

[Out] -1/2\*(-e^2\*x^2+d^2)^(1/2)/d^5/x^2-13/2/d^5\*e^2/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)+107/15/d^6\*e/(x+d/e)\*(-  
(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)+3\*e\*(-e^2\*x^2+d^2)^(1/2)/d^6/  
x+17/15/d^5/(x+d/e)^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)+1/5/d^4/e/(x+d/e)^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-e^2x^2+d^2}(ex+d)^3x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(-e^2\*x^2 + d^2)\*(e\*x + d)^3\*x^3),x, algorithm="maxima")

[Out] integrate(1/(sqrt(-e^2\*x^2 + d^2)\*(e\*x + d)^3\*x^3), x)

**Fricas [A]** time = 0.300411, size = 767, normalized size = 4.19

558 e^9 x^9 + 975 d e^8 x^8 - 4776 d^2 e^7 x^7 - 9880 d^3 e^6 x^6 + 2540 d^4 e^5 x^5 + 13215 d^5 e^4 x^4 + 3540 d^6 e^3 x^3 - 3540 d^7 e^2 x^2 - 840 d^8 e x +

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(-e^2\*x^2 + d^2)\*(e\*x + d)^3\*x^3),x, algorithm="fricas")

[Out] 1/30\*(558\*e^9\*x^9 + 975\*d\*e^8\*x^8 - 4776\*d^2\*e^7\*x^7 - 9880\*d^3\*e^6\*x^6 + 2540\*d^4\*e^5\*x^5 + 13215\*d^5\*e^4\*x^4 + 3540\*d^6\*e^3\*x^3 - 3540\*d^7\*e^2\*x^2 - 840\*d^8\*e\*x + 240\*d^9 + 195\*(e^9\*x^9 + 7\*d\*e

$$\begin{aligned} & e^8 x^8 + 3 d^2 e^7 x^7 - 31 d^3 e^6 x^6 - 40 d^4 e^5 x^5 + 12 d^5 e^4 x^4 + 40 d^6 e^3 x^3 + 16 d^7 e^2 x^2 - (e^8 x^8 - 2 d e^7 x^7 - 19 d^2 e^6 x^6 - 20 d^3 e^5 x^5 + 20 d^4 e^4 x^4 + 40 d^5 e^3 x^3 + 16 d^6 e^2 x^2) \sqrt{-e^2 x^2 + d^2} \log(-(d - \sqrt{-e^2 x^2 + d^2})/x) \\ & + (50 e^8 x^8 + 2441 d e^7 x^7 + 4525 d^2 e^6 x^6 - 3995 d^3 e^5 x^5 - 11535 d^4 e^4 x^4 - 3120 d^5 e^3 x^3 + 3420 d^6 e^2 x^2 + 840 d^7 e x - 240 d^8) \sqrt{-e^2 x^2 + d^2} / (d^6 e^7 x^9 + 7 d^7 e^6 x^8 + 3 d^8 e^5 x^7 - 31 d^9 e^4 x^6 - 40 d^{10} e^3 x^5 + 12 d^{11} e^2 x^4 + 40 d^{12} e x^3 + 16 d^{13} x^2 - (d^6 e^6 x^8 - 2 d^7 e^5 x^7 - 19 d^8 e^4 x^6 - 20 d^9 e^3 x^5 + 20 d^{10} e^2 x^4 + 40 d^{11} e x^3 + 16 d^{12} x^2) \sqrt{-e^2 x^2 + d^2}) \end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \sqrt{-(-d + ex)(d + ex)} (d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/(e\*x+d)\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2),x)

[Out] Integral(1/(x\*\*3\*sqrt(-(-d + e\*x)\*(d + e\*x))\*(d + e\*x)\*\*3), x)

**GIAC/XCAS [A]** time = 0.319938, size = 1, normalized size = 0.01

+∞

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(-e^2\*x^2 + d^2)\*(e\*x + d)^3\*x^3),x, algorithm="giac")

[Out] +Infinity



$$3.188 \quad \int \frac{x^5 \sqrt{d^2 - e^2 x^2}}{(d + ex)^4} dx$$

**Optimal.** Leaf size=204

$$\frac{10d^2(d - ex)^2}{e^6 \sqrt{d^2 - e^2 x^2}} + \frac{59d^2 \sqrt{d^2 - e^2 x^2}}{3e^6} - \frac{2dx \sqrt{d^2 - e^2 x^2}}{e^5} + \frac{x^2 \sqrt{d^2 - e^2 x^2}}{3e^4} \\ + \frac{d^4(d - ex)^4}{5e^6 (d^2 - e^2 x^2)^{5/2}} - \frac{8d^3(d - ex)^3}{5e^6 (d^2 - e^2 x^2)^{3/2}} + \frac{18d^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{e^6}$$

[Out]  $(d^4 (d - e^* x)^4) / (5^* e^6 (d^2 - e^2 * x^2)^{(5/2)}) - (8^* d^3 (d - e^* x)^3) / (5^* e^6 (d^2 - e^2 * x^2)^{(3/2)}) + (10^* d^2 (d - e^* x)^2) / (e^6 * \text{Sqrt}[d^2 - e^2 * x^2]) + (59^* d^2 * \text{Sqrt}[d^2 - e^2 * x^2]) / (3^* e^6) - (2^* d^* x * \text{Sqrt}[d^2 - e^2 * x^2]) / e^5 + (x^2 * \text{Sqrt}[d^2 - e^2 * x^2]) / (3^* e^4) + (18^* d^3 * \text{ArcTan}[(e^* x) / \text{Sqrt}[d^2 - e^2 * x^2]]) / e^6$

**Rubi [A]** time = 0.930435, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{10d^2(d - ex)^2}{e^6 \sqrt{d^2 - e^2 x^2}} + \frac{59d^2 \sqrt{d^2 - e^2 x^2}}{3e^6} - \frac{2dx \sqrt{d^2 - e^2 x^2}}{e^5} + \frac{x^2 \sqrt{d^2 - e^2 x^2}}{3e^4} \\ + \frac{d^4(d - ex)^4}{5e^6 (d^2 - e^2 x^2)^{5/2}} - \frac{8d^3(d - ex)^3}{5e^6 (d^2 - e^2 x^2)^{3/2}} + \frac{18d^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{e^6}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^5 * \text{Sqrt}[d^2 - e^2 * x^2]) / (d + e * x)^4, x]$

[Out]  $(d^4 (d - e^* x)^4) / (5^* e^6 (d^2 - e^2 * x^2)^{(5/2)}) - (8^* d^3 (d - e^* x)^3) / (5^* e^6 (d^2 - e^2 * x^2)^{(3/2)}) + (10^* d^2 (d - e^* x)^2) / (e^6 * \text{Sqrt}[d^2 - e^2 * x^2]) + (59^* d^2 * \text{Sqrt}[d^2 - e^2 * x^2]) / (3^* e^6) - (2^* d^* x * \text{Sqrt}[d^2 - e^2 * x^2]) / e^5 + (x^2 * \text{Sqrt}[d^2 - e^2 * x^2]) / (3^* e^4) + (18^* d^3 * \text{ArcTan}[(e^* x) / \text{Sqrt}[d^2 - e^2 * x^2]]) / e^6$

**Rubi in Sympy [A]** time = 55.5822, size = 175, normalized size = 0.86

$$\frac{d^4 (d^2 - e^2 x^2)^{\frac{3}{2}}}{5e^6 (d + ex)^4} + \frac{18d^3 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{e^6} + \frac{20d^3 \sqrt{d^2 - e^2 x^2}}{e^6 (d + ex)} \\ - \frac{8d^3 (d^2 - e^2 x^2)^{\frac{3}{2}}}{5e^6 (d + ex)^3} + \frac{10d^2 \sqrt{d^2 - e^2 x^2}}{e^6} - \frac{2dx \sqrt{d^2 - e^2 x^2}}{e^5} - \frac{(d^2 - e^2 x^2)^{\frac{3}{2}}}{3e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**5} * (-e^{**2} * x^{**2} + d^{**2})^{** (1/2)} / (e * x + d)^{**4}, x)$

[Out]  $d^{**4} * (d^{**2} - e^{**2} * x^{**2})^{** (3/2)} / (5^* e^{**6} * (d + e * x)^{**4}) + 18^* d^{**3} * \operatorname{atan}(e * x / \text{sqrt}(d^{**2} - e^{**2} * x^{**2})) / e^{**6} + 20^* d^{**3} * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (e^{**6} * (d + e * x)) - 8^* d^{**3} * (d^{**2} - e^{**2} * x^{**2})^{** (3/2)} / (5^* e^{**6} * (d + e * x)^{**3}) + 10^* d^{**2} * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / e^{**6} - 2^* d^* x * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / e^{**5} - (d^{**2} - e^{**2} * x^{**2})^{** (3/2)} / (3^* e^{**6})$

**Mathematica [A]** time = 0.114647, size = 109, normalized size = 0.53

$$\frac{270d^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right) + \frac{\sqrt{d^2 - e^2 x^2} (424d^5 + 1002d^4 ex + 674d^3 e^2 x^2 + 70d^2 e^3 x^3 - 15de^4 x^4 + 5e^5 x^5)}{(d + ex)^3}}{15e^6}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5\*Sqrt[d^2 - e^2\*x^2])/(d + e\*x)^4,x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(424\*d^5 + 1002\*d^4\*e\*x + 674\*d^3\*e^2\*x^2 + 70\*d^2\*e^3\*x^3 - 15\*d\*e^4\*x^4 + 5\*e^5\*x^5))/(d + e\*x)^3 + 270\*d^3\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(15\*e^6)

**Maple [A]** time = 0.032, size = 297, normalized size = 1.5

$$\begin{aligned}
 & -\frac{1}{3e^6}(-e^2x^2 + d^2)^{\frac{3}{2}} - 2\frac{dx\sqrt{-e^2x^2 + d^2}}{e^5} - 2\frac{d^3}{e^5\sqrt{e^2}}\arctan\left(\frac{\sqrt{e^2}x}{\sqrt{-e^2x^2 + d^2}}\right) \\
 & + 20\frac{d^2}{e^6}\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)} + 20\frac{d^3}{e^5\sqrt{e^2}}\arctan\left(\frac{\sqrt{e^2}x}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)}}\right) \\
 & + 10\frac{d^2}{e^8}\left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{3}{2}}\left(x + \frac{d}{e}\right)^{-2} \\
 & - \frac{8d^3}{5e^9}\left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{3}{2}}\left(x + \frac{d}{e}\right)^{-3} \\
 & + \frac{d^4}{5e^{10}}\left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{3}{2}}\left(x + \frac{d}{e}\right)^{-4}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(-e^2\*x^2+d^2)^(1/2)/(e\*x+d)^4,x)

[Out] -1/3/e^6\*(-e^2\*x^2+d^2)^(3/2)-2\*d\*x\*(-e^2\*x^2+d^2)^(1/2)/e^5-2/e^5\*d^3/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))+20/e^6\*d^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)+20/e^5\*d^3/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))+10/e^8\*d^2/(x+d/e)^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)-8/5/e^9\*d^3/(x+d/e)^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)+1/5\*d^4/e^10/(x+d/e)^4\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)\*x^5/(e\*x + d)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.3122, size = 845, normalized size = 4.14

$$5e^{11}x^{11} + 10de^{10}x^{10} - 95d^2e^9x^9 + 770d^3e^8x^8 + 4924d^4e^7x^7 + 2710d^5e^6x^6 - 17532d^6e^5x^5 - 23400d^7e^4x^4 + 5760d^8e^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)\*x^5/(e\*x + d)^4,x, algorithm="fricas")

[Out] 
$$-1/15*(5*e^{11}*x^{11} + 10*d*e^{10}*x^{10} - 95*d^2*e^9*x^9 + 770*d^3*e^8*x^8 + 4924*d^4*e^7*x^7 + 2710*d^5*e^6*x^6 - 17532*d^6*e^5*x^5 - 23400*d^7*e^4*x^4 + 5760*d^8*e^3*x^3 + 21600*d^9*e^2*x^2 + 8640*d^{10}*e*x + 540*(d^3*e^8*x^8 - 3*d^4*e^7*x^7 - 27*d^5*e^6*x^6 - 21*d^6*e^5*x^5 + 70*d^7*e^4*x^4 + 100*d^8*e^3*x^3 - 16*d^9*e^2*x^2 - 80*d^{10}*e*x - 32*d^{11} + (d^3*e^7*x^7 + 8*d^4*e^6*x^6 + d^5*e^5*x^5 - 50*d^6*e^4*x^4 - 60*d^7*e^3*x^3 + 32*d^8*e^2*x^2 + 80*d^9*e*x + 32*d^{10})*\sqrt{-e^2*x^2 + d^2})*\arctan(-(d - \sqrt{-e^2*x^2 + d^2}))/e^x) - (5*e^{10}*x^{10} - 45*d*e^9*x^9 + 100*d^2*e^8*x^8 + 1018*d^3*e^7*x^7 - 890*d^4*e^6*x^6 - 11412*d^5*e^5*x^5 - 12600*d^6*e^4*x^4 + 10080*d^7*e^3*x^3 + 21600*d^8*e^2*x^2 + 8640*d^9*e*x)*\sqrt{-e^2*x^2 + d^2})/(e^{14}*x^8 - 3*d*e^{13}*x^7 - 27*d^2*e^{12}*x^6 - 21*d^3*e^{11}*x^5 + 70*d^4*e^{10}*x^4 + 100*d^5*e^9*x^3 - 16*d^6*e^8*x^2 - 80*d^7*e^7*x - 32*d^8*e^6 + (e^{13}*x^7 + 8*d*e^{12}*x^6 + d^2*e^{11}*x^5 - 50*d^3*e^{10}*x^4 - 60*d^4*e^9*x^3 + 32*d^5*e^8*x^2 + 80*d^6*e^7*x + 32*d^7*e^6)*\sqrt{-e^2*x^2 + d^2})$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 \sqrt{-(-d + ex)(d + ex)}}{(d + ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2)/(e\*x+d)\*\*4,x)

[Out] Integral(x\*\*5\*sqrt(-(-d + e\*x)\*(d + e\*x))/(d + e\*x)\*\*4, x)

**GIAC/XCAS [A]** time = 0.324126, size = 1, normalized size = 0.

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)\*x^5/(e\*x + d)^4,x, algorithm="giac")

[Out] Done

$$3.189 \quad \int \frac{x^4 \sqrt{d^2 - e^2 x^2}}{(d + ex)^4} dx$$

**Optimal.** Leaf size=160

$$\frac{19d^2(d - ex)^3}{15e^5(d^2 - e^2x^2)^{3/2}} - \frac{6d(d - ex)^2}{e^5\sqrt{d^2 - e^2x^2}} - \frac{(20d - ex)\sqrt{d^2 - e^2x^2}}{2e^5} - \frac{19d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{2e^5} - \frac{d^3(d - ex)^4}{5e^5(d^2 - e^2x^2)^{5/2}}$$

[Out]  $-(d^3*(d - e*x)^4)/(5*e^5*(d^2 - e^2*x^2)^{(5/2)}) + (19*d^2*(d - e*x)^3)/(15*e^5*(d^2 - e^2*x^2)^{(3/2)}) - (6*d*(d - e*x)^2)/(e^5*\text{Sqrt}[d^2 - e^2*x^2]) - ((20*d - e*x)*\text{Sqrt}[d^2 - e^2*x^2])/(2*e^5) - (19*d^2*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(2*e^5)$

**Rubi [A]** time = 0.714212, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\frac{19d^2(d - ex)^3}{15e^5(d^2 - e^2x^2)^{3/2}} - \frac{6d(d - ex)^2}{e^5\sqrt{d^2 - e^2x^2}} - \frac{(20d - ex)\sqrt{d^2 - e^2x^2}}{2e^5} - \frac{19d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{2e^5} - \frac{d^3(d - ex)^4}{5e^5(d^2 - e^2x^2)^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^4*\text{Sqrt}[d^2 - e^2*x^2])/(d + e*x)^4, x]$

[Out]  $-(d^3*(d - e*x)^4)/(5*e^5*(d^2 - e^2*x^2)^{(5/2)}) + (19*d^2*(d - e*x)^3)/(15*e^5*(d^2 - e^2*x^2)^{(3/2)}) - (6*d*(d - e*x)^2)/(e^5*\text{Sqrt}[d^2 - e^2*x^2]) - ((20*d - e*x)*\text{Sqrt}[d^2 - e^2*x^2])/(2*e^5) - (19*d^2*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(2*e^5)$

**Rubi in Sympy [A]** time = 48.6786, size = 155, normalized size = 0.97

$$\frac{d^3(d^2 - e^2x^2)^{\frac{3}{2}}}{5e^5(d + ex)^4} - \frac{19d^2 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{2e^5} - \frac{12d^2\sqrt{d^2 - e^2x^2}}{e^5(d + ex)} + \frac{19d^2(d^2 - e^2x^2)^{\frac{3}{2}}}{15e^5(d + ex)^3} - \frac{4d\sqrt{d^2 - e^2x^2}}{e^5} + \frac{x\sqrt{d^2 - e^2x^2}}{2e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**4}*(-e^{**2}*x^{**2}+d^{**2})^{**}(1/2)/(e*x+d)^{**4}, x)$

[Out]  $-d^{**3}*(d^{**2} - e^{**2}*x^{**2})^{**}(3/2)/(5*e^{**5}*(d + e*x)^{**4}) - 19*d^{**2}*a \tan(e*x/\text{sqrt}(d^{**2} - e^{**2}*x^{**2}))/ (2*e^{**5}) - 12*d^{**2}*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(e^{**5}*(d + e*x)) + 19*d^{**2}*(d^{**2} - e^{**2}*x^{**2})^{**}(3/2)/(15*e^{**5}*(d + e*x)^{**3}) - 4*d*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/e^{**5} + x*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(2*e^{**4})$

**Mathematica [A]** time = 0.183477, size = 98, normalized size = 0.61

$$\frac{285d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) + \frac{\sqrt{d^2 - e^2x^2}(448d^4 + 1059d^3ex + 713d^2e^2x^2 + 75de^3x^3 - 15e^4x^4)}{(d+ex)^3}}{30e^5}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(x^4*\text{Sqrt}[d^2 - e^2*x^2])/(d + e*x)^4, x]$

[Out]  $-\left(\sqrt{d^2 - e^2 x^2} \cdot (448 d^4 + 1059 d^3 e x + 713 d^2 e^2 x^2 + 75 d e^3 x^3 - 15 e^4 x^4)\right) / (d + e x)^3 + 285 d^2 \operatorname{ArcTan}\left(\frac{e x}{\sqrt{d^2 - e^2 x^2}}\right) / (30 e^5)$

**Maple [A]** time = 0.018, size = 273, normalized size = 1.7

$$\begin{aligned} & \frac{x}{2e^4} \sqrt{-e^2 x^2 + d^2} + \frac{d^2}{2e^4} \arctan\left(x \sqrt{e^2} \frac{1}{\sqrt{-e^2 x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} \\ & - \frac{d^3}{5e^9} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{3}{2}} \left(x + \frac{d}{e}\right)^{-4} \\ & + \frac{19d^2}{15e^8} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{3}{2}} \left(x + \frac{d}{e}\right)^{-3} - 10 \frac{d}{e^5} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)} \\ & - 10 \frac{d^2}{e^4 \sqrt{e^2}} \arctan\left(\frac{\sqrt{e^2} x}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)}}\right) \\ & - 6 \frac{d}{e^7} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{3/2} \left(x + \frac{d}{e}\right)^{-2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x)`

[Out]  $1/2/e^4*x*(-e^2*x^2+d^2)^(1/2)+1/2/e^4*d^2/(e^2)^(1/2)*\arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-1/5*d^3/e^9/(x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+19/15/e^8*d^2/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)-10/e^5*d*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(1/2)-10/e^4*d^2/(e^2)^(1/2)*\arctan((e^2)^(1/2)*x/(-e^2*x^2+d^2)^(1/2))-6/e^7*d/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2 + d^2)*x^4/(e*x + d)^4,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.301768, size = 730, normalized size = 4.56

$15 e^9 x^9 - 150 d e^8 x^8 - 906 d^2 e^7 x^7 + 270 d^3 e^6 x^6 + 7827 d^4 e^5 x^5 + 9500 d^5 e^4 x^4 - 4180 d^6 e^3 x^3 - 11400 d^7 e^2 x^2 - 4560 d^8 e x + 5$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2 + d^2)*x^4/(e*x + d)^4,x, algorithm="fricas")`

[Out]  $1/30*(15*e^9*x^9 - 150*d*e^8*x^8 - 906*d^2*e^7*x^7 + 270*d^3*e^6*x^6 + 7827*d^4*e^5*x^5 + 9500*d^5*e^4*x^4 - 4180*d^6*e^3*x^3 - 11400*d^7*e^2*x^2 - 4560*d^8*e*x + 5)$

$$400*d^7*e^2*x^2 - 4560*d^8*e*x + 570*(d^2*e^7*x^7 + 7*d^3*e^6*x^6 + 3*d^4*e^5*x^5 - 31*d^5*e^4*x^4 - 40*d^6*e^3*x^3 + 12*d^7*e^2*x^2 + 40*d^8*e*x + 16*d^9 - (d^2*e^6*x^6 - 2*d^3*e^5*x^5 - 19*d^4*e^4*x^4 - 20*d^5*e^3*x^3 + 20*d^6*e^2*x^2 + 40*d^7*e*x + 16*d^8)*\sqrt{-e^2*x^2 + d^2})*\arctan(-(d - \sqrt{-e^2*x^2 + d^2})/(e*x)) + (15*e^8*x^8 - 15*d*e^7*x^7 - 745*d^2*e^6*x^6 - 4027*d^3*e^5*x^5 - 3800*d^4*e^4*x^4 + 6460*d^5*e^3*x^3 + 11400*d^6*e^2*x^2 + 4560*d^7*e*x)*\sqrt{-e^2*x^2 + d^2})/(e^{12}*x^7 + 7*d*e^{11}*x^6 + 3*d^2*e^{10}*x^5 - 31*d^3*e^9*x^4 - 40*d^4*e^8*x^3 + 12*d^5*e^7*x^2 + 40*d^6*e^6*x + 16*d^7*e^5 - (e^{11}*x^6 - 2*d*e^{10}*x^5 - 19*d^2*e^9*x^4 - 20*d^3*e^8*x^3 + 20*d^4*e^7*x^2 + 40*d^5*e^6*x + 16*d^6*e^5)*\sqrt{-e^2*x^2 + d^2}))$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \sqrt{-(-d + ex)(d + ex)}}{(d + ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2)/(e\*x+d)\*\*4,x)

[Out] Integral(x\*\*4\*sqrt(-(-d + e\*x)\*(d + e\*x))/(d + e\*x)\*\*4, x)

**GIAC/XCAS [A]** time = 0.32269, size = 1, normalized size = 0.01

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)\*x^4/(e\*x + d)^4,x, algorithm="giac")

[Out] Done

$$3.190 \quad \int \frac{x^3 \sqrt{d^2 - e^2 x^2}}{(d + ex)^4} dx$$

**Optimal.** Leaf size=148

$$\frac{d^2 (d^2 - e^2 x^2)^{3/2}}{5e^4 (d + ex)^4} - \frac{14d (d^2 - e^2 x^2)^{3/2}}{15e^4 (d + ex)^3} + \frac{8d \sqrt{d^2 - e^2 x^2}}{e^4 (d + ex)} - \frac{(d^2 - e^2 x^2)^{3/2}}{e^4 (d + ex)^2} + \frac{4d \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right)}{e^4}$$

[Out]  $(8*d*\text{Sqrt}[d^2 - e^2*x^2])/(e^4*(d + e*x)) + (d^2*(d^2 - e^2*x^2)^(3/2))/(5*e^4*(d + e*x)^4) - (14*d*(d^2 - e^2*x^2)^(3/2))/(15*e^4*(d + e*x)^3) - (d^2 - e^2*x^2)^(3/2)/(e^4*(d + e*x)^2) + (4*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^4$

**Rubi [A]** time = 0.423754, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\frac{d^2 (d^2 - e^2 x^2)^{3/2}}{5e^4 (d + ex)^4} - \frac{14d (d^2 - e^2 x^2)^{3/2}}{15e^4 (d + ex)^3} + \frac{8d \sqrt{d^2 - e^2 x^2}}{e^4 (d + ex)} - \frac{(d^2 - e^2 x^2)^{3/2}}{e^4 (d + ex)^2} + \frac{4d \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right)}{e^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^3*\text{Sqrt}[d^2 - e^2*x^2])/(d + e*x)^4, x]$

[Out]  $(8*d*\text{Sqrt}[d^2 - e^2*x^2])/(e^4*(d + e*x)) + (d^2*(d^2 - e^2*x^2)^(3/2))/(5*e^4*(d + e*x)^4) - (14*d*(d^2 - e^2*x^2)^(3/2))/(15*e^4*(d + e*x)^3) - (d^2 - e^2*x^2)^(3/2)/(e^4*(d + e*x)^2) + (4*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]])/e^4$

**Rubi in Sympy [A]** time = 40.9128, size = 124, normalized size = 0.84

$$\frac{d^2 (d^2 - e^2 x^2)^{3/2}}{5e^4 (d + ex)^4} + \frac{4d \operatorname{atan} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right)}{e^4} + \frac{6d \sqrt{d^2 - e^2 x^2}}{e^4 (d + ex)} - \frac{14d (d^2 - e^2 x^2)^{3/2}}{15e^4 (d + ex)^3} + \frac{\sqrt{d^2 - e^2 x^2}}{e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**3}*(-e^{**2}*x^{**2}+d^{**2})^{**}(1/2)/(e*x+d)^{**4}, x)$

[Out]  $d^{**2}*(d^{**2} - e^{**2}*x^{**2})^{**}(3/2)/(5*e^{**4}*(d + e*x)^{**4}) + 4*d*\operatorname{atan}(e*x/\operatorname{sqrt}(d^{**2} - e^{**2}*x^{**2}))/e^{**4} + 6*d*\operatorname{sqrt}(d^{**2} - e^{**2}*x^{**2})/(e^{**4}*(d + e*x)) - 14*d*(d^{**2} - e^{**2}*x^{**2})^{**}(3/2)/(15*e^{**4}*(d + e*x)^{**3}) + \operatorname{sqrt}(d^{**2} - e^{**2}*x^{**2})/e^{**4}$

**Mathematica [A]** time = 0.121448, size = 85, normalized size = 0.57

$$\frac{60d \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) + \frac{\sqrt{d^2 - e^2 x^2} (94d^3 + 222d^2 ex + 149de^2 x^2 + 15e^3 x^3)}{(d + ex)^3}}{15e^4}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(x^3*\text{Sqrt}[d^2 - e^2*x^2])/(d + e*x)^4, x]$

[Out]  $((\text{Sqrt}[d^2 - e^2*x^2])*(94*d^3 + 222*d^2*e*x + 149*d*e^2*x^2 + 15*e^3*x^3))/(d + e*x)^3 + 60*d*ArcTan[(e*x)/Sqrt[d^2 - e^2*x^2]]/($

$15 * e^4$ )

**Maple [A]** time = 0.015, size = 212, normalized size = 1.4

$$4 \frac{1}{e^4} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right)} + 4 \frac{d}{e^3 \sqrt{e^2}} \arctan \left( \frac{\sqrt{e^2} x}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right)}} \right) + 3 \frac{1}{e^6} \left( -\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right) \right)^{3/2} \left(x + \frac{d}{e}\right)^{-2} - \frac{14d}{15e^7} \left( -\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right) \right)^{3/2} \left(x + \frac{d}{e}\right)^{-3} + \frac{d^2}{5e^8} \left( -\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right) \right)^{3/2} \left(x + \frac{d}{e}\right)^{-4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x)`

[Out]  $4/e^4 * (- (x+d/e)^2 * e^2 + 2*d*e * (x+d/e))^{1/2} + 4/e^3 * d / (e^2)^{1/2} * \arctan((e^2)^{1/2} * x / (- (x+d/e)^2 * e^2 + 2*d*e * (x+d/e))^{1/2}) + 3/e^6 / (x+d/e)^2 * (- (x+d/e)^2 * e^2 + 2*d*e * (x+d/e))^{3/2} - 14/15 * d / e^7 / (x+d/e)^3 * (- (x+d/e)^2 * e^2 + 2*d*e * (x+d/e))^{3/2} + 1/5 * d^2 / e^8 / (x+d/e)^4 * (- (x+d/e)^2 * e^2 + 2*d*e * (x+d/e))^{3/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2 + d^2)*x^3/(e*x + d)^4,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.297332, size = 605, normalized size = 4.09

$$\frac{15e^7x^7 + 100de^6x^6 + 643d^2e^5x^5 + 730d^3e^4x^4 - 560d^4e^3x^3 - 1200d^5e^2x^2 - 480d^6ex + 120(de^6x^6 - d^2e^5x^5 - 13d^3e^4x^4)}{15(e^{10}x^6 - de^9x^5 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2 + d^2)*x^3/(e*x + d)^4,x, algorithm="fricas")`

[Out]  $-1/15 * (15 * e^7 * x^7 + 100 * d * e^6 * x^6 + 643 * d^2 * e^5 * x^5 + 730 * d^3 * e^4 * x^4 - 560 * d^4 * e^3 * x^3 - 1200 * d^5 * e^2 * x^2 - 480 * d^6 * e * x + 120 * (d * e^6 * x^6 - d^2 * e^5 * x^5 - 13 * d^3 * e^4 * x^4 - 15 * d^4 * e^3 * x^3 + 8 * d^5 * e^2 * x^2 + 20 * d^6 * e * x + 8 * d^7 + (d * e^5 * x^5 + 6 * d^2 * e^4 * x^4 + 5 * d^3 * e^3 * x^3 - 12 * d^4 * e^2 * x^2 - 20 * d^5 * e * x - 8 * d^6) * \sqrt{-e^2 * x^2 + d^2}) * \arctan(-(d - \sqrt{-e^2 * x^2 + d^2}) / (e * x)) - (15 * e^6 * x^6 + 183 * d * e^5 * x^5 + 130 * d^2 * e^4 * x^4 - 800 * d^3 * e^3 * x^3 - 1200 * d^4 * e^2 * x^2 - 480 * d^5 * e * x) * \sqrt{-e^2 * x^2 + d^2}) / (e^{10} * x^6 - d * e^9 * x^5 - 13 * d^3 * e^4 * x^4 - 15 * d^4 * e^3 * x^3 + 8 * d^5 * e^2 * x^2 + 20 * d^6 * e * x + 8 * d^7)$



$$d^2 e^{8x^4} - 15 d^3 e^{7x^3} + 8 d^4 e^{6x^2} + 20 d^5 e^{5x} + 8 d^6 e^4 + (e^9 x^5 + 6 d e^{8x^4} + 5 d^2 e^{7x^3} - 12 d^3 e^{6x^2} - 20 d^4 e^{5x} - 8 d^5 e^4) \sqrt{-e^{2x^2} + d^2}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \sqrt{-(-d + ex)(d + ex)}}{(d + ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2)/(e\*x+d)\*\*4,x)

[Out] Integral(x\*\*3\*sqrt(-(-d + e\*x)\*(d + e\*x))/(d + e\*x)\*\*4, x)

**GIAC/XCAS [A]** time = 0.296806, size = 1, normalized size = 0.01

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)\*x^3/(e\*x + d)^4,x, algorithm="giac")

[Out] Done

$$3.191 \quad \int \frac{x^2 \sqrt{d^2 - e^2 x^2}}{(d + ex)^4} dx$$

**Optimal.** Leaf size=115

$$\frac{3(d^2 - e^2 x^2)^{3/2}}{5e^3(d + ex)^3} - \frac{d(d^2 - e^2 x^2)^{3/2}}{5e^3(d + ex)^4} - \frac{2\sqrt{d^2 - e^2 x^2}}{e^3(d + ex)} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{e^3}$$

[Out]  $(-2*\text{Sqrt}[d^2 - e^2*x^2])/(e^3*(d + e*x)) - (d*(d^2 - e^2*x^2)^(3/2))/(5*e^3*(d + e*x)^4) + (3*(d^2 - e^2*x^2)^(3/2))/(5*e^3*(d + e*x)^3) - \text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]]/e^3$

**Rubi [A]** time = 0.278848, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{3(d^2 - e^2 x^2)^{3/2}}{5e^3(d + ex)^3} - \frac{d(d^2 - e^2 x^2)^{3/2}}{5e^3(d + ex)^4} - \frac{2\sqrt{d^2 - e^2 x^2}}{e^3(d + ex)} - \frac{\tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{e^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^2*\text{Sqrt}[d^2 - e^2*x^2])/(d + e*x)^4, x]$

[Out]  $(-2*\text{Sqrt}[d^2 - e^2*x^2])/(e^3*(d + e*x)) - (d*(d^2 - e^2*x^2)^(3/2))/(5*e^3*(d + e*x)^4) + (3*(d^2 - e^2*x^2)^(3/2))/(5*e^3*(d + e*x)^3) - \text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]]/e^3$

**Rubi in Sympy [A]** time = 31.4054, size = 99, normalized size = 0.86

$$-\frac{d(d^2 - e^2 x^2)^{3/2}}{5e^3(d + ex)^4} - \frac{\text{atan}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{e^3} - \frac{2\sqrt{d^2 - e^2 x^2}}{e^3(d + ex)} + \frac{3(d^2 - e^2 x^2)^{3/2}}{5e^3(d + ex)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**2}*(-e^{**2}*x^{**2}+d^{**2})^{**}(1/2)/(e*x+d)^{**4}, x)$

[Out]  $-d*(d^{**2} - e^{**2}*x^{**2})^{**}(3/2)/(5*e^{**3}*(d + e*x)^{**4}) - \text{atan}(e*x/\text{sqrt}(d^{**2} - e^{**2}*x^{**2}))/e^{**3} - 2*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(e^{**3}*(d + e*x)) + 3*(d^{**2} - e^{**2}*x^{**2})^{**}(3/2)/(5*e^{**3}*(d + e*x)^{**3})$

**Mathematica [A]** time = 0.135356, size = 73, normalized size = 0.63

$$-\frac{\sqrt{d^2 - e^2 x^2}(8d^2 + 19dex + 13e^2 x^2)}{(d + ex)^3} + 5 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(x^2*\text{Sqrt}[d^2 - e^2*x^2])/(d + e*x)^4, x]$

[Out]  $-((\text{Sqrt}[d^2 - e^2*x^2]*(8*d^2 + 19*d*e*x + 13*e^2*x^2))/(d + e*x)^3 + 5*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(5*e^3)$

**Maple [B]** time = 0.016, size = 214, normalized size = 1.9

$$\begin{aligned}
 & -\frac{1}{e^5 d} \left( -\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right) \right)^{\frac{3}{2}} \left(x + \frac{d}{e}\right)^{-2} - \frac{1}{e^3 d} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)} \\
 & - \frac{1}{e^2} \arctan \left( x \sqrt{e^2} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)}} \right) \frac{1}{\sqrt{e^2}} \\
 & - \frac{d}{5 e^7} \left( -\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right) \right)^{\frac{3}{2}} \left(x + \frac{d}{e}\right)^{-4} \\
 & + \frac{3}{5 e^6} \left( -\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right) \right)^{\frac{3}{2}} \left(x + \frac{d}{e}\right)^{-3}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(-e^2\*x^2+d^2)^(1/2)/(e\*x+d)^4,x)

[Out] -1/e^5/d/(x+d/e)^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)-1/e^3/d\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)-1/e^2/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))-1/5\*d/e^7/(x+d/e)^4\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)+3/5/e^6/(x+d/e)^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)\*x^2/(e\*x + d)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.294436, size = 455, normalized size = 3.96

$$\frac{21 e^5 x^5 + 20 d e^4 x^4 - 35 d^2 e^3 x^3 - 50 d^3 e^2 x^2 - 20 d^4 e x - 10 \left( e^5 x^5 + 5 d e^4 x^4 + 5 d^2 e^3 x^3 - 5 d^3 e^2 x^2 - 10 d^4 e x - 4 d^5 - (e^4 x^4 + 4 d e^3 x^3 + 5 d^2 e^2 x^2 + 4 d^3 e x + d^4) \sqrt{-e^2 x^2 + d^2} \right)}{5 \left( e^8 x^5 + 5 d e^7 x^4 + 5 d^2 e^6 x^3 - 5 d^3 e^5 x^2 - 10 d^4 e^4 x - 4 d^5 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)\*x^2/(e\*x + d)^4,x, algorithm="fricas")

[Out] -1/5\*(21\*e^5\*x^5 + 20\*d\*e^4\*x^4 - 35\*d^2\*e^3\*x^3 - 50\*d^3\*e^2\*x^2 - 20\*d^4\*e\*x - 10\*(e^5\*x^5 + 5\*d\*e^4\*x^4 + 5\*d^2\*e^3\*x^3 - 5\*d^3\*e^2\*x^2 - 10\*d^4\*e\*x - 4\*d^5 - (e^4\*x^4 + 4\*d\*e^3\*x^3 + 5\*d^2\*e^2\*x^2 + 4\*d^3\*e\*x + d^4)\*sqrt(-e^2\*x^2 + d^2))\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) + 5\*(e^4\*x^4 + 9\*d\*e^3\*x^3 + 10\*d^2\*e^2\*x^2 + 4\*d^3\*e\*x)\*sqrt(-e^2\*x^2 + d^2)/(e^8\*x^5 + 5\*d\*e^7\*x^4 + 5\*d^2\*e^6\*x^3 - 5\*d^3\*e^5\*x^2 - 10\*d^4\*e^4\*x - 4\*d^5\*e^3 - (e^7\*x^4 + 7\*d^2\*e^6\*x^3 + 5\*d^3\*e^5\*x^2 - 10\*d^4\*e^4\*x - 4\*d^4\*e^3)\*sqrt(-e^2\*x^2 + d^2))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \sqrt{-(-d + ex)(d + ex)}}{(d + ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2)/(e\*x+d)\*\*4,x)

[Out] Integral(x\*\*2\*sqrt(-(-d + e\*x)\*(d + e\*x))/(d + e\*x)\*\*4, x)

**GIAC/XCAS [A]** time = 0.295544, size = 1, normalized size = 0.01

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)\*x^2/(e\*x + d)^4,x, algorithm="giac")

[Out] Done

$$3.192 \quad \int \frac{x\sqrt{d^2 - e^2x^2}}{(d+ex)^4} dx$$

**Optimal.** Leaf size=64

$$\frac{(d^2 - e^2x^2)^{3/2}}{5e^2(d+ex)^4} - \frac{4(d^2 - e^2x^2)^{3/2}}{15de^2(d+ex)^3}$$

[Out]  $(d^2 - e^2x^2)^{3/2}/(5e^2(d+ex)^4) - (4(d^2 - e^2x^2)^{3/2})/(15d^2e^2(d+ex)^3)$

**Rubi [A]** time = 0.100966, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$\frac{(d^2 - e^2x^2)^{3/2}}{5e^2(d+ex)^4} - \frac{4(d^2 - e^2x^2)^{3/2}}{15de^2(d+ex)^3}$$

Antiderivative was successfully verified.

[In] Int[(x\*sqrt[d^2 - e^2\*x^2])/(d + e\*x)^4, x]

[Out]  $(d^2 - e^2x^2)^{3/2}/(5e^2(d+ex)^4) - (4(d^2 - e^2x^2)^{3/2})/(15d^2e^2(d+ex)^3)$

**Rubi in Sympy [A]** time = 8.86658, size = 53, normalized size = 0.83

$$\frac{(d^2 - e^2x^2)^{3/2}}{5e^2(d+ex)^4} - \frac{4(d^2 - e^2x^2)^{3/2}}{15de^2(d+ex)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2)/(e\*x+d)\*\*4, x)

[Out]  $(d**2 - e**2*x**2)**(3/2)/(5*e**2*(d + e*x)**4) - 4*(d**2 - e**2*x**2)**(3/2)/(15*d*e**2*(d + e*x)**3)$

**Mathematica [A]** time = 0.0341371, size = 50, normalized size = 0.78

$$-\frac{(d^2 + 3dex - 4e^2x^2)\sqrt{d^2 - e^2x^2}}{15de^2(d+ex)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*sqrt[d^2 - e^2\*x^2])/(d + e\*x)^4, x]

[Out]  $-((d^2 + 3*d*e*x - 4*e^2*x^2)*sqrt[d^2 - e^2*x^2])/(15*d^2*e^2*(d + e*x)^3)$

**Maple [A]** time = 0.01, size = 42, normalized size = 0.7

$$-\frac{(4ex + d)(-ex + d)\sqrt{-e^2x^2 + d^2}}{15(ex + d)^3 de^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x)`

[Out] `-1/15*(-e*x+d)*(4*e*x+d)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3/d/e^2`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2 + d^2)*x/(e*x + d)^4,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.285373, size = 242, normalized size = 3.78

$$15 \frac{3e^3x^5 - 20de^2x^4 - 5d^2ex^3 + 30d^3x^2 + 5(e^2x^4 + dex^3 - 6d^2x^2)\sqrt{-e^2x^2 + d^2}}{(de^5x^5 + 5d^2e^4x^4 + 5d^3e^3x^3 - 5d^4e^2x^2 - 10d^5ex - 4d^6 - (de^4x^4 - 7d^3e^2x^2 - 10d^4ex - 4d^5)\sqrt{-e^2x^2 + d^2})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2 + d^2)*x/(e*x + d)^4,x, algorithm="fricas")`

[Out] `1/15*(3*e^3*x^5 - 20*d*e^2*x^4 - 5*d^2*e*x^3 + 30*d^3*x^2 + 5*(e^2*x^4 + d*e*x^3 - 6*d^2*x^2)*sqrt(-e^2*x^2 + d^2))/(d*e^5*x^5 + 5*d^2*e^4*x^4 + 5*d^3*e^3*x^3 - 5*d^4*e^2*x^2 - 10*d^5*e*x - 4*d^6 - (d*e^4*x^4 - 7*d^3*e^2*x^2 - 10*d^4*e*x - 4*d^5)*sqrt(-e^2*x^2 + d^2))`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{-(-d+ex)(d+ex)}}{(d+ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)`

[Out] `Integral(x*sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)`

**GIAC/XCAS [A]** time = 0.287887, size = 1, normalized size = 0.02

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2 + d^2)*x/(e*x + d)^4,x, algorithm="giac")`

[Out] Done

$$3.193 \quad \int \frac{\sqrt{d^2 - e^2 x^2}}{(d + ex)^4} dx$$

**Optimal.** Leaf size=67

$$-\frac{(d^2 - e^2 x^2)^{3/2}}{15d^2 e (d + ex)^3} - \frac{(d^2 - e^2 x^2)^{3/2}}{5de (d + ex)^4}$$

[Out]  $-(d^2 - e^2 x^2)^{(3/2)}/(5*d*e*(d + e*x)^4) - (d^2 - e^2 x^2)^{(3/2)}/(15*d^2*e*(d + e*x)^3)$

**Rubi [A]** time = 0.0674137, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$-\frac{(d^2 - e^2 x^2)^{3/2}}{15d^2 e (d + ex)^3} - \frac{(d^2 - e^2 x^2)^{3/2}}{5de (d + ex)^4}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d^2 - e^2\*x^2]/(d + e\*x)^4, x]

[Out]  $-(d^2 - e^2 x^2)^{(3/2)}/(5*d*e*(d + e*x)^4) - (d^2 - e^2 x^2)^{(3/2)}/(15*d^2*e*(d + e*x)^3)$

**Rubi in Sympy [A]** time = 8.53228, size = 53, normalized size = 0.79

$$-\frac{(d^2 - e^2 x^2)^{\frac{3}{2}}}{5de (d + ex)^4} - \frac{(d^2 - e^2 x^2)^{\frac{3}{2}}}{15d^2 e (d + ex)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2)/(e\*x+d)\*\*4, x)

[Out]  $-(d**2 - e**2*x**2)**(3/2)/(5*d*e*(d + e*x)**4) - (d**2 - e**2*x**2)**(3/2)/(15*d**2*e*(d + e*x)**3)$

**Mathematica [A]** time = 0.030512, size = 51, normalized size = 0.76

$$\frac{\sqrt{d^2 - e^2 x^2} (-4d^2 + 3dex + e^2 x^2)}{15d^2 e (d + ex)^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d^2 - e^2\*x^2]/(d + e\*x)^4, x]

[Out]  $(\text{Sqrt}[d^2 - e^2 x^2] * (-4*d^2 + 3*d*e*x + e^2*x^2))/(15*d^2*e*(d + e*x)^3)$

**Maple [A]** time = 0.007, size = 43, normalized size = 0.6

$$-\frac{(ex + 4d)(-ex + d)\sqrt{-e^2 x^2 + d^2}}{15(ex + d)^3 d^2 e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-e^2*x^2+d^2)^(1/2)/(e*x+d)^4,x)`

[Out] `-1/15*(-e*x+d)*(e*x+4*d)*(-e^2*x^2+d^2)^(1/2)/(e*x+d)^3/d^2/e`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2 + d^2)/(e*x + d)^4,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.273223, size = 271, normalized size = 4.04

$$\frac{3e^4x^5 + 20de^3x^4 + 35d^2e^2x^3 - 30d^3ex^2 - 60d^4x - 5(e^3x^4 + de^2x^3 - 6d^2ex^2 - 12d^3x)\sqrt{-e^2x^2 + d^2}}{15\left(d^2e^5x^5 + 5d^3e^4x^4 + 5d^4e^3x^3 - 5d^5e^2x^2 - 10d^6ex - 4d^7 - (d^2e^4x^4 - 7d^4e^2x^2 - 10d^5ex - 4d^6)\sqrt{-e^2x^2 + d^2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2 + d^2)/(e*x + d)^4,x, algorithm="fricas")`

[Out] `-1/15*(3*e^4*x^5 + 20*d*e^3*x^4 + 35*d^2*e^2*x^3 - 30*d^3*e*x^2 - 60*d^4*x - 5*(e^3*x^4 + d*e^2*x^3 - 6*d^2*e*x^2 - 12*d^3*x)*sqrt(-e^2*x^2 + d^2))/(d^2*e^5*x^5 + 5*d^3*e^4*x^4 + 5*d^4*e^3*x^3 - 5*d^5*e^2*x^2 - 10*d^6*e*x - 4*d^7 - (d^2*e^4*x^4 - 7*d^4*e^2*x^2 - 10*d^5*e*x - 4*d^6)*sqrt(-e^2*x^2 + d^2))`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-d + ex)(d + ex)}}{(d + ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e**2*x**2+d**2)**(1/2)/(e*x+d)**4,x)`

[Out] `Integral(sqrt(-(-d + e*x)*(d + e*x))/(d + e*x)**4, x)`

**GIAC/XCAS [A]** time = 0.290401, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2 + d^2)/(e*x + d)^4,x, algorithm="giac")`

[Out] Done



$$3.194 \quad \int \frac{\sqrt{d^2 - e^2 x^2}}{x(d+ex)^4} dx$$

**Optimal.** Leaf size=110

$$-\frac{4ex}{5d(d^2 - e^2x^2)^{3/2}} + \frac{8d(d - ex)}{5(d^2 - e^2x^2)^{5/2}} + \frac{5d - 8ex}{5d^3\sqrt{d^2 - e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)}{d^3}$$

[Out]  $(8*d*(d - e*x))/(5*(d^2 - e^2*x^2)^{(5/2)}) - (4*e*x)/(5*d*(d^2 - e^2*x^2)^{(3/2)}) + (5*d - 8*e*x)/(5*d^3*\text{Sqrt}[d^2 - e^2*x^2]) - \text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d]/d^3$

**Rubi [A]** time = 0.424799, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$-\frac{4ex}{5d(d^2 - e^2x^2)^{3/2}} + \frac{8d(d - ex)}{5(d^2 - e^2x^2)^{5/2}} + \frac{5d - 8ex}{5d^3\sqrt{d^2 - e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)}{d^3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d^2 - e^2\*x^2]/(x\*(d + e\*x)^4), x]

[Out]  $(8*d*(d - e*x))/(5*(d^2 - e^2*x^2)^{(5/2)}) - (4*e*x)/(5*d*(d^2 - e^2*x^2)^{(3/2)}) + (5*d - 8*e*x)/(5*d^3*\text{Sqrt}[d^2 - e^2*x^2]) - \text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d]/d^3$

**Rubi in Sympy [A]** time = 48.2365, size = 95, normalized size = 0.86

$$\frac{(d^2 - e^2x^2)^{\frac{3}{2}}}{5d^2(d+ex)^4} - \frac{\text{atanh}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)}{d^3} + \frac{2\sqrt{d^2 - e^2x^2}}{d^3(d+ex)} + \frac{2(d^2 - e^2x^2)^{\frac{3}{2}}}{5d^3(d+ex)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2)/x/(e\*x+d)\*\*4, x)

[Out]  $(d**2 - e**2*x**2)**(3/2)/(5*d**2*(d + e*x)**4) - \text{atanh}(\text{sqrt}(d**2 - e**2*x**2)/d)/d**3 + 2*\text{sqrt}(d**2 - e**2*x**2)/(d**3*(d + e*x)) + 2*(d**2 - e**2*x**2)**(3/2)/(5*d**3*(d + e*x)**3)$

**Mathematica [A]** time = 0.123273, size = 76, normalized size = 0.69

$$\frac{\sqrt{d^2 - e^2x^2}(13d^2 + 19dex + 8e^2x^2)}{(d+ex)^3} - 5 \log\left(\sqrt{d^2 - e^2x^2} + d\right) + 5 \log(x)$$

$$5d^3$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d^2 - e^2\*x^2]/(x\*(d + e\*x)^4), x]

[Out]  $((\text{Sqrt}[d^2 - e^2*x^2]*(13*d^2 + 19*d*e*x + 8*e^2*x^2))/(d + e*x)^3 + 5*\text{Log}[x] - 5*\text{Log}[d + \text{Sqrt}[d^2 - e^2*x^2]])/(5*d^3)$

**Maple [B]** time = 0.017, size = 196, normalized size = 1.8

$$\begin{aligned} & \frac{1}{d^4} \sqrt{-e^2 x^2 + d^2} - \frac{1}{d^2} \ln \left( \frac{1}{x} \left( 2d^2 + 2\sqrt{d^2} \sqrt{-e^2 x^2 + d^2} \right) \right) \frac{1}{\sqrt{d^2}} \\ & + \frac{1}{d^4 e^2} \left( - \left( x + \frac{d}{e} \right)^2 e^2 + 2de \left( x + \frac{d}{e} \right) \right)^{\frac{3}{2}} \left( x + \frac{d}{e} \right)^{-2} \\ & + \frac{2}{5e^3 d^3} \left( - \left( x + \frac{d}{e} \right)^2 e^2 + 2de \left( x + \frac{d}{e} \right) \right)^{\frac{3}{2}} \left( x + \frac{d}{e} \right)^{-3} \\ & + \frac{1}{5d^2 e^4} \left( - \left( x + \frac{d}{e} \right)^2 e^2 + 2de \left( x + \frac{d}{e} \right) \right)^{\frac{3}{2}} \left( x + \frac{d}{e} \right)^{-4} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-e^2*x^2+d^2)^(1/2)/x/(e*x+d)^4,x)`

[Out] `1/d^4*(-e^2*x^2+d^2)^(1/2)-1/d^2/(d^2)^(1/2)*ln((2*d^2+2*(d^2)^(1/2)*(-e^2*x^2+d^2)^(1/2))/x)+1/d^4/e^2/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+2/5/e^3/d^3/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)+1/5/d^2/e^4/(x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^(3/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-e^2 x^2 + d^2}}{(ex + d)^4 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)^4*x),x, algorithm="maxima")`

[Out] `integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)^4*x), x)`

**Fricas [A]** time = 0.287757, size = 448, normalized size = 4.07

$$\frac{21e^5x^5 + 60d^4e^4x^4 + 5d^2e^3x^3 - 110d^3e^2x^2 - 80d^4ex + 5(e^5x^5 + 5de^4x^4 + 5d^2e^3x^3 - 5d^3e^2x^2 - 10d^4ex - 4d^5 - (e^4x^4 - 5(d^3e^5x^5 + 5d^4e^4x^4 + 5d^5e^3x^3 - 5d^6e^2x^2 - 10d^7ex - 4d^8)))}{5(d^3e^5x^5 + 5d^4e^4x^4 + 5d^5e^3x^3 - 5d^6e^2x^2 - 10d^7ex - 4d^8)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)^4*x),x, algorithm="fricas")`

[Out] `1/5*(21*e^5*x^5 + 60*d^4*e^4*x^4 + 5*d^2*e^3*x^3 - 110*d^3*e^2*x^2 - 80*d^4*e*x + 5*(e^5*x^5 + 5*d*e^4*x^4 + 5*d^2*e^3*x^3 - 5*d^3*e^2*x^2 - 10*d^4*e*x - 4*d^5 - (e^4*x^4 - 7*d^2*e^2*x^2 - 10*d^3*e*x - 4*d^4))*sqrt(-e^2*x^2 + d^2))*log(-(d - sqrt(-e^2*x^2 + d^2))/x) - 5*(e^4*x^4 - 7*d*e^3*x^3 - 22*d^2*e^2*x^2 - 16*d^3*e*x)*sqrt(-e^2*x^2 + d^2)/(d^3*e^5*x^5 + 5*d^4*e^4*x^4 + 5*d^5*e^3*x^3 - 5*d^6*e^2*x^2 - 10*d^7*e*x - 4*d^8 - (d^3*e^4*x^4 - 7*d^5*e^2*x^2 - 10*d^6*e*x - 4*d^7))*sqrt(-e^2*x^2 + d^2)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-d + ex)(d + ex)}}{x(d + ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e**2*x**2+d**2)**(1/2)/x/(e*x+d)**4,x)
```

```
[Out] Integral(sqrt(-(-d + e*x)*(d + e*x))/(x*(d + e*x)**4), x)
```

---

**GIAC/XCAS [A]** time = 0.300639, size = 1, normalized size = 0.01

*Done*

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-e^2*x^2 + d^2)/((e*x + d)^4*x),x, algorithm="giac")
```

```
[Out] Done
```

$$3.195 \quad \int \frac{\sqrt{d^2 - e^2 x^2}}{x^2(d+ex)^4} dx$$

**Optimal.** Leaf size=143

$$-\frac{4e(5d-8ex)}{15d^2(d^2-e^2x^2)^{3/2}} - \frac{8e(d-ex)}{5(d^2-e^2x^2)^{5/2}} - \frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^4x} + \frac{4e \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

[Out]  $(-8*e*(d - e*x))/(5*(d^2 - e^2*x^2)^(5/2)) - (4*e*(5*d - 8*e*x))/(15*d^2*(d^2 - e^2*x^2)^(3/2)) - (e*(60*d - 79*e*x))/(15*d^4*\text{Sqrt}[d^2 - e^2*x^2]) - \text{Sqrt}[d^2 - e^2*x^2]/(d^4*x) + (4*e*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/d^4$

**Rubi [A]** time = 0.526751, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$-\frac{4e(5d-8ex)}{15d^2(d^2-e^2x^2)^{3/2}} - \frac{8e(d-ex)}{5(d^2-e^2x^2)^{5/2}} - \frac{e(60d-79ex)}{15d^4\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{d^4x} + \frac{4e \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^4}$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)^4), x]`

[Out]  $(-8*e*(d - e*x))/(5*(d^2 - e^2*x^2)^(5/2)) - (4*e*(5*d - 8*e*x))/(15*d^2*(d^2 - e^2*x^2)^(3/2)) - (e*(60*d - 79*e*x))/(15*d^4*\text{Sqrt}[d^2 - e^2*x^2]) - \text{Sqrt}[d^2 - e^2*x^2]/(d^4*x) + (4*e*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/d^4$

**Rubi in Sympy [A]** time = 60.2056, size = 122, normalized size = 0.85

$$-\frac{e(d^2 - e^2x^2)^{\frac{3}{2}}}{5d^3(d+ex)^4} + \frac{4e \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)}{d^4} - \frac{6e\sqrt{d^2 - e^2x^2}}{d^4(d+ex)} - \frac{11e(d^2 - e^2x^2)^{\frac{3}{2}}}{15d^4(d+ex)^3} - \frac{\sqrt{d^2 - e^2x^2}}{d^4x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((-e**2*x**2+d**2)**(1/2)/x**2/(e*x+d)**4, x)`

[Out]  $-e*(d**2 - e**2*x**2)**(3/2)/(5*d**3*(d + e*x)**4) + 4*e*\operatorname{atanh}(\text{sqrt}(d**2 - e**2*x**2)/d)/d**4 - 6*e*\text{sqrt}(d**2 - e**2*x**2)/(d**4*(d + e*x)) - 11*e*(d**2 - e**2*x**2)**(3/2)/(15*d**4*(d + e*x)**3) - \text{sqrt}(d**2 - e**2*x**2)/(d**4*x)$

**Mathematica [A]** time = 0.215231, size = 92, normalized size = 0.64

$$-\frac{60e \log\left(\sqrt{d^2 - e^2x^2} + d\right) + \frac{\sqrt{d^2 - e^2x^2}(15d^3 + 149d^2ex + 222de^2x^2 + 94e^3x^3)}{x(d+ex)^3} + 60e \log(x)}{15d^4}$$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[d^2 - e^2*x^2]/(x^2*(d + e*x)^4), x]`

[Out]  $-((\text{Sqrt}[d^2 - e^2*x^2]*(15*d^3 + 149*d^2*e*x + 222*d*e^2*x^2 + 94*e^3*x^3))/(x*(d + e*x)^3) + 60*e*\text{Log}[x] - 60*e*\text{Log}[d + \text{Sqrt}[d^2$

-  $e^{2x^2}]])/(15d^4)$

**Maple [B]** time = 0.018, size = 361, normalized size = 2.5

$$\begin{aligned}
 & -\frac{1}{d^6x}(-e^2x^2+d^2)^{\frac{3}{2}}-\frac{e^2x}{d^6}\sqrt{-e^2x^2+d^2}-\frac{e^2}{d^4}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}} \\
 & -\frac{1}{5d^3e^3}\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}\left(x+\frac{d}{e}\right)^{-4} \\
 & -\frac{11}{15d^4e^2}\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}\left(x+\frac{d}{e}\right)^{-3}-4\frac{e\sqrt{-e^2x^2+d^2}}{d^5} \\
 & +4\frac{e}{d^3\sqrt{d^2}}\ln\left(\frac{2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}}{x}\right)+\frac{e}{d^5}\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)} \\
 & +\frac{e^2}{d^4}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)\frac{1}{\sqrt{e^2}} \\
 & -3\frac{1}{d^5e}\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{3}{2}}\left(x+\frac{d}{e}\right)^{-2}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-e^2*x^2+d^2)^(1/2)/x^2/(e*x+d)^4,x)`

[Out]  $-1/d^6/x*(-e^2*x^2+d^2)^{(3/2)}-1/d^6*e^2*x*(-e^2*x^2+d^2)^{(1/2)}-1/d^4*e^2/(e^2)^{(1/2)}*\arctan((e^2)^{(1/2)}*x/(-e^2*x^2+d^2)^{(1/2)})-1/5/d^3/e^3/(x+d/e)^4*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^{(3/2)}-11/15/d^4/e^2/(x+d/e)^3*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^{(3/2)}-4/d^5*e*(-e^2*x^2+d^2)^{(1/2)}+4/d^3*e/(d^2)^{(1/2)}*\ln((2*d^2+2*(d^2)^{(1/2)}*(-e^2*x^2+d^2)^{(1/2)})/x)+1/d^5*e*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^{(1/2)}+1/d^4*e^2/(e^2)^{(1/2)}*\arctan((e^2)^{(1/2)}*x/(-e^2*x^2+d^2)^{(1/2)})-3/d^5/e/(x+d/e)^2*(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^{(3/2)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-e^2x^2+d^2}}{(ex+d)^4x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-e^2*x^2+d^2)/((e*x+d)^4*x^2),x,algorithm="maxima")`

[Out] `integrate(sqrt(-e^2*x^2+d^2)/((e*x+d)^4*x^2),x)`

**Fricas [A]** time = 0.291846, size = 629, normalized size = 4.4

$$10e^7x^7 - 608de^6x^6 - 1415d^2e^5x^5 + 130d^3e^4x^4 + 2115d^4e^3x^3 + 1020d^5e^2x^2 - 300d^6ex - 120d^7 + 60(e^7x^7 - de^6x^6 - 15(d^4e^6x^7 -$$

$$15(d^4e^6x^7 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)/((e\*x + d)^4\*x^2),x, algorithm="fricas")

[Out] 
$$-1/15*(10*e^7*x^7 - 608*d*e^6*x^6 - 1415*d^2*e^5*x^5 + 130*d^3*e^4*x^4 + 2115*d^4*e^3*x^3 + 1020*d^5*e^2*x^2 - 300*d^6*e*x - 120*d^7 + 60*(e^7*x^7 - d*e^6*x^6 - 13*d^2*e^5*x^5 - 15*d^3*e^4*x^4 + 8*d^4*e^3*x^3 + 20*d^5*e^2*x^2 + 8*d^6*e*x + (e^6*x^6 + 6*d*e^5*x^5 + 5*d^2*e^4*x^4 - 12*d^3*e^3*x^3 - 20*d^4*e^2*x^2 - 8*d^5*e*x) * \sqrt{-e^2*x^2 + d^2}) * \log(-(d - \sqrt{-e^2*x^2 + d^2})/x) + (198*e^6*x^6 + 470*d*e^5*x^5 - 595*d^2*e^4*x^4 - 1965*d^3*e^3*x^3 - 960*d^4*e^2*x^2 + 300*d^5*e*x + 120*d^6) * \sqrt{-e^2*x^2 + d^2}) / (d^4 * e^6*x^7 - d^5*e^5*x^6 - 13*d^6*e^4*x^5 - 15*d^7*e^3*x^4 + 8*d^8*e^2*x^3 + 20*d^9*e*x^2 + 8*d^{10}*x + (d^4*e^5*x^6 + 6*d^5*e^4*x^5 + 5*d^6*e^3*x^4 - 12*d^7*e^2*x^3 - 20*d^8*e*x^2 - 8*d^9*x) * \sqrt{-e^2*x^2 + d^2})$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-d + ex)(d + ex)}}{x^2 (d + ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2)/x\*\*2/(e\*x+d)\*\*4,x)

[Out] Integral(sqrt(-(-d + e\*x)\*(d + e\*x))/(x\*\*2\*(d + e\*x)\*\*4), x)

**GIAC/XCAS [A]** time = 0.293035, size = 1, normalized size = 0.01

+∞

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)/((e\*x + d)^4\*x^2),x, algorithm="giac")

[Out] +Infinity

$$3.196 \quad \int \frac{\sqrt{d^2 - e^2 x^2}}{x^3(d+ex)^4} dx$$

**Optimal.** Leaf size=183

$$\frac{8e^2(d-ex)}{5d(d^2-e^2x^2)^{5/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} + \frac{4e\sqrt{d^2-e^2x^2}}{d^5x} - \frac{19e^2 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^5} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{4e^2(10d-13ex)}{15d^3(d^2-e^2x^2)^{3/2}}$$

[Out]  $(8*e^2*(d - e*x))/(5*d*(d^2 - e^2*x^2)^{(5/2)}) + (4*e^2*(10*d - 13*e*x))/(15*d^3*(d^2 - e^2*x^2)^{(3/2)}) + (e^2*(135*d - 164*e*x))/(15*d^5*\text{Sqrt}[d^2 - e^2*x^2]) - \text{Sqrt}[d^2 - e^2*x^2]/(2*d^4*x^2) + (4*e*\text{Sqrt}[d^2 - e^2*x^2])/(d^5*x) - (19*e^2*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/(2*d^5)$

**Rubi [A]** time = 0.645175, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\frac{8e^2(d-ex)}{5d(d^2-e^2x^2)^{5/2}} + \frac{e^2(135d-164ex)}{15d^5\sqrt{d^2-e^2x^2}} + \frac{4e\sqrt{d^2-e^2x^2}}{d^5x} - \frac{19e^2 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d^5} - \frac{\sqrt{d^2-e^2x^2}}{2d^4x^2} + \frac{4e^2(10d-13ex)}{15d^3(d^2-e^2x^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[d^2 - e^2*x^2]/(x^3*(d + e*x)^4), x]$

[Out]  $(8*e^2*(d - e*x))/(5*d*(d^2 - e^2*x^2)^{(5/2)}) + (4*e^2*(10*d - 13*e*x))/(15*d^3*(d^2 - e^2*x^2)^{(3/2)}) + (e^2*(135*d - 164*e*x))/(15*d^5*\text{Sqrt}[d^2 - e^2*x^2]) - \text{Sqrt}[d^2 - e^2*x^2]/(2*d^4*x^2) + (4*e*\text{Sqrt}[d^2 - e^2*x^2])/(d^5*x) - (19*e^2*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/(2*d^5)$

**Rubi in Sympy [A]** time = 73.9085, size = 156, normalized size = 0.85

$$\frac{e^2(d^2 - e^2x^2)^{\frac{3}{2}}}{5d^4(d+ex)^4} - \frac{\sqrt{d^2 - e^2x^2}}{2d^4x^2} - \frac{19e^2 \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)}{2d^5} + \frac{12e^2\sqrt{d^2 - e^2x^2}}{d^5(d+ex)} + \frac{16e^2(d^2 - e^2x^2)^{\frac{3}{2}}}{15d^5(d+ex)^3} + \frac{4e\sqrt{d^2 - e^2x^2}}{d^5x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((-e^{**2}*x^{**2}+d^{**2})^{**}(1/2)/x^{**3}/(e*x+d)^{**4}, x)$

[Out]  $e^{**2}*(d^{**2} - e^{**2}*x^{**2})^{**}(3/2)/(5*d^{**4}*(d + e*x)^{**4}) - \text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(2*d^{**4}*x^{**2}) - 19*e^{**2}*\text{atanh}(\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/d)/(2*d^{**5}) + 12*e^{**2}*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(d^{**5}*(d + e*x)) + 16*e^{**2}*(d^{**2} - e^{**2}*x^{**2})^{**}(3/2)/(15*d^{**5}*(d + e*x)^{**3}) + 4*e*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(d^{**5}*x)$

**Mathematica [A]** time = 0.175925, size = 107, normalized size = 0.58

$$\frac{-285e^2 \log\left(\sqrt{d^2 - e^2x^2} + d\right) + \frac{\sqrt{d^2 - e^2x^2}(-15d^4 + 75d^3ex + 713d^2e^2x^2 + 1059de^3x^3 + 448e^4x^4)}{x^2(d+ex)^3} + 285e^2 \log(x)}{30d^5}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d^2 - e^2\*x^2]/(x^3\*(d + e\*x)^4), x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(-15\*d^4 + 75\*d^3\*e\*x + 713\*d^2\*e^2\*x^2 + 1059\*d\*e^3\*x^3 + 448\*e^4\*x^4))/(x^2\*(d + e\*x)^3) + 285\*e^2\*Log[x] - 285\*e^2\*Log[d + Sqrt[d^2 - e^2\*x^2]])/(30\*d^5)

**Maple [B]** time = 0.02, size = 389, normalized size = 2.1

$$\begin{aligned}
 & -\frac{1}{2d^6x^2}(-e^2x^2 + d^2)^{\frac{3}{2}} + \frac{19e^2}{2d^6}\sqrt{-e^2x^2 + d^2} \\
 & - \frac{19e^2}{2d^4} \ln\left(\frac{1}{x}\left(2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}} - 4\frac{e^2}{d^6}\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)} \\
 & - 4\frac{e^3}{d^5\sqrt{e^2}} \arctan\left(\frac{\sqrt{e^2}x}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)}}\right) \\
 & + 4\frac{e(-e^2x^2 + d^2)^{3/2}}{d^7x} + 4\frac{e^3x\sqrt{-e^2x^2 + d^2}}{d^7} + 4\frac{e^3}{d^5\sqrt{e^2}} \arctan\left(\frac{\sqrt{e^2}x}{\sqrt{-e^2x^2 + d^2}}\right) \\
 & + 6\frac{1}{d^6}\left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{3/2}\left(x + \frac{d}{e}\right)^{-2} \\
 & + \frac{16}{15d^5e}\left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{3/2}\left(x + \frac{d}{e}\right)^{-3} \\
 & + \frac{1}{5d^4e^2}\left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{3/2}\left(x + \frac{d}{e}\right)^{-4}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^(1/2)/x^3/(e\*x+d)^4, x)

[Out] -1/2/d^6/x^2\*(-e^2\*x^2+d^2)^(3/2)+19/2/d^6\*e^2\*(-e^2\*x^2+d^2)^(1/2)-19/2/d^4\*e^2/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)-4/d^6\*e^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)-4/d^5\*e^3/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))+4/d^7\*e/x\*(-e^2\*x^2+d^2)^(3/2)+4/d^7\*e^3\*x\*(-e^2\*x^2+d^2)^(1/2)+4/d^5\*e^3/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))+6/d^6/(x+d/e)^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)+16/15/d^5/e/(x+d/e)^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)+1/5/d^4/e^2/(x+d/e)^4\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-e^2x^2 + d^2}}{(ex + d)^4 x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)/((e\*x + d)^4\*x^3), x, algorithm="maxima")

[Out] integrate(sqrt(-e^2\*x^2 + d^2)/((e\*x + d)^4\*x^3), x)



**Fricas [A]** time = 0.301618, size = 767, normalized size = 4.19

$$846 e^9 x^9 + 1605 d e^8 x^8 - 6972 d^2 e^7 x^7 - 15340 d^3 e^6 x^6 + 2750 d^4 e^5 x^5 + 19815 d^5 e^4 x^4 + 6300 d^6 e^3 x^3 - 4740 d^7 e^2 x^2 - 1320 d^8 e x + 240 d^9 + 285 (e^9 x^9 + 7 d e^8 x^8 + 3 d^2 e^7 x^7 - 31 d^3 e^6 x^6 - 40 d^4 e^5 x^5 + 12 d^5 e^4 x^4 + 40 d^6 e^3 x^3 + 16 d^7 e^2 x^2 - (e^8 x^8 - 2 d e^7 x^7 - 19 d^2 e^6 x^6 - 20 d^3 e^5 x^5 + 20 d^4 e^4 x^4 + 40 d^5 e^3 x^3 + 16 d^6 e^2 x^2)) \sqrt{-e^2 x^2 + d^2} \log(-(d - \sqrt{-e^2 x^2 + d^2})/x) + (50 e^8 x^8 + 3647 d e^7 x^7 + 7135 d^2 e^6 x^6 - 5405 d^3 e^5 x^5 - 17535 d^4 e^4 x^4 - 5640 d^5 e^3 x^3 + 4620 d^6 e^2 x^2 + 1320 d^7 e x - 240 d^8) \sqrt{-e^2 x^2 + d^2} / (d^5 e^7 x^9 + 7 d^6 e^6 x^8 + 3 d^7 e^5 x^7 - 31 d^8 e^4 x^6 - 40 d^9 e^3 x^5 + 12 d^{10} e^2 x^4 + 40 d^{11} e x^3 + 16 d^{12} x^2 - (d^5 e^6 x^8 - 2 d^6 e^5 x^7 - 19 d^7 e^4 x^6 - 20 d^8 e^3 x^5 + 20 d^9 e^2 x^4 + 40 d^{10} e x^3 + 16 d^{11} x^2)) \sqrt{-e^2 x^2 + d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)/((e\*x + d)^4\*x^3),x, algorithm="fricas")

[Out] 1/30\*(846\*e^9\*x^9 + 1605\*d\*e^8\*x^8 - 6972\*d^2\*e^7\*x^7 - 15340\*d^3\*e^6\*x^6 + 2750\*d^4\*e^5\*x^5 + 19815\*d^5\*e^4\*x^4 + 6300\*d^6\*e^3\*x^3 - 4740\*d^7\*e^2\*x^2 - 1320\*d^8\*e\*x + 240\*d^9 + 285\*(e^9\*x^9 + 7\*d\*e^8\*x^8 + 3\*d^2\*e^7\*x^7 - 31\*d^3\*e^6\*x^6 - 40\*d^4\*e^5\*x^5 + 12\*d^5\*e^4\*x^4 + 40\*d^6\*e^3\*x^3 + 16\*d^7\*e^2\*x^2 - (e^8\*x^8 - 2\*d\*e^7\*x^7 - 19\*d^2\*e^6\*x^6 - 20\*d^3\*e^5\*x^5 + 20\*d^4\*e^4\*x^4 + 40\*d^5\*e^3\*x^3 + 16\*d^6\*e^2\*x^2))\*sqrt(-e^2\*x^2 + d^2))\*log(-(d - sqrt(-e^2\*x^2 + d^2))/x) + (50\*e^8\*x^8 + 3647\*d\*e^7\*x^7 + 7135\*d^2\*e^6\*x^6 - 5405\*d^3\*e^5\*x^5 - 17535\*d^4\*e^4\*x^4 - 5640\*d^5\*e^3\*x^3 + 4620\*d^6\*e^2\*x^2 + 1320\*d^7\*e\*x - 240\*d^8)\*sqrt(-e^2\*x^2 + d^2))/(d^5\*e^7\*x^9 + 7\*d^6\*e^6\*x^8 + 3\*d^7\*e^5\*x^7 - 31\*d^8\*e^4\*x^6 - 40\*d^9\*e^3\*x^5 + 12\*d^10\*e^2\*x^4 + 40\*d^11\*e\*x^3 + 16\*d^12\*x^2 - (d^5\*e^6\*x^8 - 2\*d^6\*e^5\*x^7 - 19\*d^7\*e^4\*x^6 - 20\*d^8\*e^3\*x^5 + 20\*d^9\*e^2\*x^4 + 40\*d^10\*e\*x^3 + 16\*d^11\*x^2))\*sqrt(-e^2\*x^2 + d^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-d + ex)(d + ex)}}{x^3(d + ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2)/x\*\*3/(e\*x+d)\*\*4,x)

[Out] Integral(sqrt(-(-d + e\*x)\*(d + e\*x))/(x\*\*3\*(d + e\*x)\*\*4), x)

**GIAC/XCAS [A]** time = 0.29573, size = 1, normalized size = 0.01

+∞

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)/((e\*x + d)^4\*x^3),x, algorithm="giac")

[Out] +Infinity

$$3.197 \quad \int \frac{\sqrt{d^2 - e^2 x^2}}{x^4(d+ex)^4} dx$$

**Optimal.** Leaf size=210

$$\begin{aligned} & -\frac{8e^3(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}} - \frac{29e^2\sqrt{d^2-e^2x^2}}{3d^6x} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} \\ & + \frac{18e^3 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6} + \frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2} - \frac{\sqrt{d^2-e^2x^2}}{3d^4x^3} - \frac{4e^3(5d-6ex)}{5d^4(d^2-e^2x^2)^{3/2}} \end{aligned}$$

[Out]  $(-8*e^3*(d - e*x))/(5*d^2*(d^2 - e^2*x^2)^{(5/2)}) - (4*e^3*(5*d - 6*e*x))/(5*d^4*(d^2 - e^2*x^2)^{(3/2)}) - (e^3*(80*d - 93*e*x))/(5*d^6*\text{Sqrt}[d^2 - e^2*x^2]) - \text{Sqrt}[d^2 - e^2*x^2]/(3*d^4*x^3) + (2*e*\text{Sqrt}[d^2 - e^2*x^2])/(d^5*x^2) - (29*e^2*\text{Sqrt}[d^2 - e^2*x^2])/(3*d^6*x) + (18*e^3*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/d^6$

**Rubi [A]** time = 0.795451, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\begin{aligned} & -\frac{8e^3(d-ex)}{5d^2(d^2-e^2x^2)^{5/2}} - \frac{29e^2\sqrt{d^2-e^2x^2}}{3d^6x} - \frac{e^3(80d-93ex)}{5d^6\sqrt{d^2-e^2x^2}} \\ & + \frac{18e^3 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^6} + \frac{2e\sqrt{d^2-e^2x^2}}{d^5x^2} - \frac{\sqrt{d^2-e^2x^2}}{3d^4x^3} - \frac{4e^3(5d-6ex)}{5d^4(d^2-e^2x^2)^{3/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[d^2 - e^2*x^2]/(x^4*(d + e*x)^4), x]$

[Out]  $(-8*e^3*(d - e*x))/(5*d^2*(d^2 - e^2*x^2)^{(5/2)}) - (4*e^3*(5*d - 6*e*x))/(5*d^4*(d^2 - e^2*x^2)^{(3/2)}) - (e^3*(80*d - 93*e*x))/(5*d^6*\text{Sqrt}[d^2 - e^2*x^2]) - \text{Sqrt}[d^2 - e^2*x^2]/(3*d^4*x^3) + (2*e*\text{Sqrt}[d^2 - e^2*x^2])/(d^5*x^2) - (29*e^2*\text{Sqrt}[d^2 - e^2*x^2])/(3*d^6*x) + (18*e^3*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/d^6$

**Rubi in Sympy [A]** time = 82.1194, size = 180, normalized size = 0.86

$$\begin{aligned} & -\frac{e^3(d^2 - e^2x^2)^{\frac{3}{2}}}{5d^5(d+ex)^4} + \frac{2e\sqrt{d^2 - e^2x^2}}{d^5x^2} + \frac{18e^3 \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)}{d^6} \\ & - \frac{20e^3\sqrt{d^2 - e^2x^2}}{d^6(d+ex)} - \frac{7e^3(d^2 - e^2x^2)^{\frac{3}{2}}}{5d^6(d+ex)^3} - \frac{10e^2\sqrt{d^2 - e^2x^2}}{d^6x} - \frac{(d^2 - e^2x^2)^{\frac{3}{2}}}{3d^6x^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((-e^{**2}*x^{**2}+d^{**2})^{**}(1/2)/x^{**4}/(e*x+d)^{**4}, x)$

[Out]  $-e^{**3}*(d^{**2} - e^{**2}*x^{**2})^{**}(3/2)/(5*d^{**5}*(d + e*x)^{**4}) + 2*e*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(d^{**5}*x^{**2}) + 18*e^{**3}*\text{atanh}(\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/d)/d^{**6} - 20*e^{**3}*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(d^{**6}*(d + e*x)) - 7*e^{**3}*(d^{**2} - e^{**2}*x^{**2})^{**}(3/2)/(5*d^{**6}*(d + e*x)^{**3}) - 10*e^{**2}*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(d^{**6}*x) - (d^{**2} - e^{**2}*x^{**2})^{**}(3/2)/(3*d^{**6}*x^{**3})$

**Mathematica [A]** time = 0.24788, size = 118, normalized size = 0.56

$$\frac{-270e^3 \log\left(\sqrt{d^2 - e^2x^2} + d\right) + \frac{\sqrt{d^2 - e^2x^2}(5d^5 - 15d^4ex + 70d^3e^2x^2 + 674d^2e^3x^3 + 1002de^4x^4 + 424e^5x^5)}{x^3(d+ex)^3} + 270e^3 \log(x)}{15d^6}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d^2 - e^2\*x^2]/(x^4\*(d + e\*x)^4), x]

[Out] -((Sqrt[d^2 - e^2\*x^2]\*(5\*d^5 - 15\*d^4\*e\*x + 70\*d^3\*e^2\*x^2 + 674\*d^2\*e^3\*x^3 + 1002\*d\*e^4\*x^4 + 424\*e^5\*x^5))/(x^3\*(d + e\*x)^3) + 270\*e^3\*Log[x] - 270\*e^3\*Log[d + Sqrt[d^2 - e^2\*x^2]])/(15\*d^6)

**Maple [B]** time = 0.02, size = 412, normalized size = 2.

$$\begin{aligned} & -\frac{1}{3d^6x^3}(-e^2x^2 + d^2)^{\frac{3}{2}} - \frac{1}{5d^5e} \left( -\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right) \right)^{\frac{3}{2}} \left(x + \frac{d}{e}\right)^{-4} \\ & - \frac{7}{5d^6} \left( -\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right) \right)^{\frac{3}{2}} \left(x + \frac{d}{e}\right)^{-3} - 18 \frac{e^3 \sqrt{-e^2x^2 + d^2}}{d^7} \\ & + 18 \frac{e^3}{d^5 \sqrt{d^2}} \ln \left( \frac{2d^2 + 2\sqrt{d^2} \sqrt{-e^2x^2 + d^2}}{x} \right) + 10 \frac{e^3}{d^7} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right)} \\ & + 10 \frac{e^4}{d^6 \sqrt{e^2}} \arctan \left( \frac{\sqrt{e^2}x \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right)}}}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right)}} \right) \\ & - 10 \frac{e^2(-e^2x^2 + d^2)^{3/2}}{d^8x} - 10 \frac{e^4x\sqrt{-e^2x^2 + d^2}}{d^8} - 10 \frac{e^4}{d^6\sqrt{e^2}} \arctan \left( \frac{\sqrt{e^2}x}{\sqrt{-e^2x^2 + d^2}} \right) \\ & - 10 \frac{e}{d^7} \left( -\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right) \right)^{3/2} \left(x + \frac{d}{e}\right)^{-2} + 2 \frac{e(-e^2x^2 + d^2)^{3/2}}{d^7x^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^(1/2)/x^4/(e\*x+d)^4, x)

[Out] -1/3/d^6/x^3\*(-e^2\*x^2+d^2)^(3/2)-1/5/d^5/e/(x+d/e)^4\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)-7/5/d^6/(x+d/e)^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)-18/d^7\*e^3\*(-e^2\*x^2+d^2)^(1/2)+18/d^5\*e^3/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)+10/d^7\*e^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)+10/d^6\*e^4/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-((x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)))-10/d^8\*e^2/x\*(-e^2\*x^2+d^2)^(3/2)-10/d^8\*e^4\*x\*(-e^2\*x^2+d^2)^(1/2)-10/d^6\*e^4/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))-10/d^7\*e/(x+d/e)^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)+2/d^7\*e/x^2\*(-e^2\*x^2+d^2)^(3/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-e^2x^2 + d^2}}{(ex + d)^4x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)/((e\*x + d)^4\*x^4), x, algorithm="maxima")

[Out] integrate(sqrt(-e^2\*x^2 + d^2)/((e\*x + d)^4\*x^4), x)

**Fricas [A]** time = 0.310684, size = 882, normalized size = 4.2

$100 e^{11} x^{11} + 4094 d e^{10} x^{10} + 6800 d^2 e^9 x^9 - 16272 d^3 e^8 x^8 - 34165 d^4 e^7 x^7 + 7670 d^5 e^6 x^6 + 38895 d^6 e^5 x^5 + 8210 d^7 e^4 x^4 - 10900 d^8 e^3 x^3 - 2240 d^9 e^2 x^2 + 560 d^{10} e x - 160 d^{11} - 270 (e^{11} x^{11} - 3 d e^{10} x^{10} - 27 d^2 e^9 x^9 - 21 d^3 e^8 x^8 + 70 d^4 e^7 x^7 + 100 d^5 e^6 x^6 - 16 d^6 e^5 x^5 - 80 d^7 e^4 x^4 - 32 d^8 e^3 x^3 + (e^{10} x^{10} + 8 d e^9 x^9 + d^2 e^8 x^8 - 50 d^3 e^7 x^7 - 60 d^4 e^6 x^6 + 32 d^5 e^5 x^5 + 80 d^6 e^4 x^4 + 32 d^7 e^3 x^3) \sqrt{-e^2 x^2 + d^2}) \log(- (d - \sqrt{-e^2 x^2 + d^2}) / x) - (748 e^{10} x^{10} + 1050 d e^9 x^9 - 10102 d^2 e^8 x^8 - 18630 d^3 e^7 x^7 + 10885 d^4 e^6 x^6 + 33655 d^5 e^5 x^5 + 7030 d^6 e^4 x^4 - 10620 d^7 e^3 x^3 - 2320 d^8 e^2 x^2 + 560 d^9 e x - 160 d^{10}) \sqrt{-e^2 x^2 + d^2}) / (d^6 e^8 x^{11} - 3 d^7 e^7 x^{10} - 27 d^8 e^6 x^9 - 21 d^9 e^5 x^8 + 70 d^{10} e^4 x^7 + 100 d^{11} e^3 x^6 - 16 d^{12} e^2 x^5 - 80 d^{13} e x^4 - 32 d^{14} x^3 + (d^6 e^7 x^{10} + 8 d^7 e^6 x^9 + d^8 e^5 x^8 - 50 d^9 e^4 x^7 - 60 d^{10} e^3 x^6 + 32 d^{11} e^2 x^5 + 80 d^{12} e x^4 + 32 d^{13} x^3) \sqrt{-e^2 x^2 + d^2})$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(-d + ex)(d + ex)}}{x^4 (d + ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(1/2)/x\*\*4/(e\*x+d)\*\*4,x)

[Out] Integral(sqrt(-(-d + e\*x)\*(d + e\*x))/(x\*\*4\*(d + e\*x)\*\*4), x)

**GIAC/XCAS [A]** time = 0.336527, size = 1, normalized size = 0.

+∞

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-e^2\*x^2 + d^2)/((e\*x + d)^4\*x^4),x, algorithm="giac")

[Out] +Infinity

$$3.198 \quad \int \frac{x^5 (d^2 - e^2 x^2)^{5/2}}{(d + ex)^4} dx$$

**Optimal.** Leaf size=252

$$\frac{1}{7}x^6\sqrt{d^2 - e^2x^2} - \frac{2dx^5\sqrt{d^2 - e^2x^2}}{3e} + \frac{11d^2x^4\sqrt{d^2 - e^2x^2}}{7e^2} + \frac{65d^7 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{4e^6}$$

$$+ \frac{515d^6\sqrt{d^2 - e^2x^2}}{21e^6} - \frac{49d^5x\sqrt{d^2 - e^2x^2}}{4e^5} + \frac{d^4(d - ex)^4}{e^6\sqrt{d^2 - e^2x^2}} + \frac{121d^4x^2\sqrt{d^2 - e^2x^2}}{21e^4} - \frac{17d^3x^3\sqrt{d^2 - e^2x^2}}{6e^3}$$

[Out]  $(d^4*(d - e*x)^4)/(e^6*\text{Sqrt}[d^2 - e^2*x^2]) + (515*d^6*\text{Sqrt}[d^2 - e^2*x^2])/(21*e^6) - (49*d^5*x*\text{Sqrt}[d^2 - e^2*x^2])/(4*e^5) + (121*d^4*x^2*\text{Sqrt}[d^2 - e^2*x^2])/(21*e^4) - (17*d^3*x^3*\text{Sqrt}[d^2 - e^2*x^2])/(6*e^3) + (11*d^2*x^4*\text{Sqrt}[d^2 - e^2*x^2])/(7*e^2) - (2*d*x^5*\text{Sqrt}[d^2 - e^2*x^2])/(3*e) + (x^6*\text{Sqrt}[d^2 - e^2*x^2])/7 + (65*d^7*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(4*e^6)$

**Rubi [A]** time = 0.974018, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{1}{7}x^6\sqrt{d^2 - e^2x^2} - \frac{2dx^5\sqrt{d^2 - e^2x^2}}{3e} + \frac{11d^2x^4\sqrt{d^2 - e^2x^2}}{7e^2} + \frac{65d^7 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{4e^6}$$

$$+ \frac{515d^6\sqrt{d^2 - e^2x^2}}{21e^6} - \frac{49d^5x\sqrt{d^2 - e^2x^2}}{4e^5} + \frac{d^4(d - ex)^4}{e^6\sqrt{d^2 - e^2x^2}} + \frac{121d^4x^2\sqrt{d^2 - e^2x^2}}{21e^4} - \frac{17d^3x^3\sqrt{d^2 - e^2x^2}}{6e^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^5*(d^2 - e^2*x^2)^(5/2))/(d + e*x)^4, x]$

[Out]  $(d^4*(d - e*x)^4)/(e^6*\text{Sqrt}[d^2 - e^2*x^2]) + (515*d^6*\text{Sqrt}[d^2 - e^2*x^2])/(21*e^6) - (49*d^5*x*\text{Sqrt}[d^2 - e^2*x^2])/(4*e^5) + (121*d^4*x^2*\text{Sqrt}[d^2 - e^2*x^2])/(21*e^4) - (17*d^3*x^3*\text{Sqrt}[d^2 - e^2*x^2])/(6*e^3) + (11*d^2*x^4*\text{Sqrt}[d^2 - e^2*x^2])/(7*e^2) - (2*d*x^5*\text{Sqrt}[d^2 - e^2*x^2])/(3*e) + (x^6*\text{Sqrt}[d^2 - e^2*x^2])/7 + (65*d^7*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(4*e^6)$

**Rubi in Sympy [A]** time = 120.445, size = 218, normalized size = 0.87

$$\frac{65d^7 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{4e^6} + \frac{8d^7\sqrt{d^2 - e^2x^2}}{e^6(d + ex)} + \frac{24d^6\sqrt{d^2 - e^2x^2}}{e^6} - \frac{33d^5x\sqrt{d^2 - e^2x^2}}{4e^5}$$

$$- \frac{25d^4(d^2 - e^2x^2)^{3/2}}{3e^6} - \frac{17d^3x^3\sqrt{d^2 - e^2x^2}}{6e^3} + \frac{2d^2(d^2 - e^2x^2)^{5/2}}{e^6} - \frac{2dx^5\sqrt{d^2 - e^2x^2}}{3e} - \frac{(d^2 - e^2x^2)^{7/2}}{7e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**5*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4, x)$

[Out]  $65*d**7*\operatorname{atan}(e*x/\text{sqrt}(d**2 - e**2*x**2))/(4*e**6) + 8*d**7*\text{sqrt}(d**2 - e**2*x**2)/(e**6*(d + e*x)) + 24*d**6*\text{sqrt}(d**2 - e**2*x**2)/e**6 - 33*d**5*x*\text{sqrt}(d**2 - e**2*x**2)/(4*e**5) - 25*d**4*(d**2 - e**2*x**2)**(3/2)/(3*e**6) - 17*d**3*x**3*\text{sqrt}(d**2 - e**2*x**2)/(6*e**3) + 2*d**2*(d**2 - e**2*x**2)**(5/2)/e**6 - 2*d*x**5*\text{sqrt}(d**2 - e**2*x**2)/(3*e) - (d**2 - e**2*x**2)**(7/2)/(7*e**6)$

**Mathematica [A]** time = 0.168722, size = 131, normalized size = 0.52

$$\frac{1365d^7 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + \frac{\sqrt{d^2-e^2x^2}(2144d^7+779d^6ex-293d^5e^2x^2+162d^4e^3x^3-106d^3e^4x^4+76d^2e^5x^5-44de^6x^6+12e^7x^7)}{d+ex}}{84e^6}$$

Antiderivative was successfully verified.

[In] Integrate[(x^5\*(d^2 - e^2\*x^2)^(5/2))/(d + e\*x)^4, x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(2144\*d^7 + 779\*d^6\*e\*x - 293\*d^5\*e^2\*x^2 + 162\*d^4\*e^3\*x^3 - 106\*d^3\*e^4\*x^4 + 76\*d^2\*e^5\*x^5 - 44\*d\*e^6\*x^6 + 12\*e^7\*x^7))/(d + e\*x) + 1365\*d^7\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(84\*e^6)

**Maple [A]** time = 0.031, size = 416, normalized size = 1.7

$$\begin{aligned} & -\frac{1}{7e^6}(-e^2x^2 + d^2)^{\frac{7}{2}} - \frac{2dx}{3e^5}(-e^2x^2 + d^2)^{\frac{5}{2}} - \frac{5d^3x}{6e^5}(-e^2x^2 + d^2)^{\frac{3}{2}} - \frac{5d^5x}{4e^5}\sqrt{-e^2x^2 + d^2} \\ & - \frac{5d^7}{4e^5} \arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} + \frac{28d^2}{3e^6} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{5}{2}} \\ & + \frac{35d^3x}{3e^5} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{3}{2}} + \frac{35d^5x}{2e^5} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)} \\ & + \frac{35d^7}{2e^5} \arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)}}\right) \frac{1}{\sqrt{e^2}} \\ & + \frac{22d^2}{3e^8} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{7}{2}} \left(x + \frac{d}{e}\right)^{-2} \\ & + 8\frac{d^3}{e^9} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{7/2} \left(x + \frac{d}{e}\right)^{-3} \\ & + \frac{d^4}{e^{10}} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{7}{2}} \left(x + \frac{d}{e}\right)^{-4} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(-e^2\*x^2+d^2)^(5/2)/(e\*x+d)^4, x)

[Out] -1/7/e^6\*(-e^2\*x^2+d^2)^(7/2)-2/3/e^5\*d\*x\*(-e^2\*x^2+d^2)^(5/2)-5/6/e^5\*d^3\*x\*(-e^2\*x^2+d^2)^(3/2)-5/4\*d^5\*x\*(-e^2\*x^2+d^2)^(1/2)/e^5-5/4/e^5\*d^7/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))+28/3/e^6\*d^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(5/2)+35/3/e^5\*d^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)\*x+35/2/e^5\*d^5\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x+35/2/e^5\*d^7/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))+22/3/e^8\*d^2/(x+d/e)^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)+8/e^9\*d^3/(x+d/e)^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)+d^4/e^10/(x+d/e)^4\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x^5/(e\*x + d)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.302983, size = 967, normalized size = 3.84

$$12e^{15}x^{15} + 40de^{14}x^{14} - 616d^2e^{13}x^{13} + 1162d^3e^{12}x^{12} + 1372d^4e^{11}x^{11} - 5719d^5e^{10}x^{10} + 3640d^6e^9x^9 + 7805d^7e^8x^8 + 20440d^8e^7x^7 - 55720d^9e^6x^6 - 173264d^{10}e^5x^5 + 138320d^{11}e^4x^4 + 320320d^{12}e^3x^3 - 87360d^{13}e^2x^2 - 174720d^{14}e^1x + 2730(d^7e^8x^8 - 7d^8e^7x^7 - 32d^9e^6x^6 + 56d^{10}e^5x^5 + 160d^{11}e^4x^4 - 112d^{12}e^3x^3 - 256d^{13}e^2x^2 + 64d^{14}e^1x + 128d^{15}) \sqrt{-e^2x^2 + d^2} \arctan\left(\frac{d - \sqrt{-e^2x^2 + d^2}}{e^1x}\right) - (12e^{14}x^{14} - 140d^1e^{13}x^{13} + 140d^2e^{12}x^{12} + 1302d^3e^{11}x^{11} - 3374d^4e^{10}x^{10} + 1211d^5e^9x^9 + 4515d^6e^8x^8 - 672d^7e^7x^7 - 19320d^8e^6x^6 - 78624d^9e^5x^5 + 94640d^{10}e^4x^4 + 232960d^{11}e^3x^3 - 87360d^{12}e^2x^2 - 174720d^{13}e^1x) \sqrt{-e^2x^2 + d^2} / (e^{14}x^8 - 7d^1e^{13}x^7 - 32d^2e^{12}x^6 + 56d^3e^{11}x^5 + 160d^4e^{10}x^4 - 112d^5e^9x^3 - 256d^6e^8x^2 + 64d^7e^7x + 128d^8e^6 + (e^{13}x^7 + 8d^1e^{12}x^6 - 24d^2e^{11}x^5 - 80d^3e^{10}x^4 + 80d^4e^9x^3 + 192d^5e^8x^2 - 64d^6e^7x - 128d^7e^6) \sqrt{-e^2x^2 + d^2})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x^5/(e\*x + d)^4,x, algorithm="fricas")

[Out] -1/84\*(12\*e^15\*x^15 + 40\*d\*e^14\*x^14 - 616\*d^2\*e^13\*x^13 + 1162\*d^3\*e^12\*x^12 + 1372\*d^4\*e^11\*x^11 - 5719\*d^5\*e^10\*x^10 + 3640\*d^6\*e^9\*x^9 + 7805\*d^7\*e^8\*x^8 + 20440\*d^8\*e^7\*x^7 - 55720\*d^9\*e^6\*x^6 - 173264\*d^10\*e^5\*x^5 + 138320\*d^11\*e^4\*x^4 + 320320\*d^12\*e^3\*x^3 - 87360\*d^13\*e^2\*x^2 - 174720\*d^14\*e^1\*x + 2730\*(d^7\*e^8\*x^8 - 7\*d^8\*e^7\*x^7 - 32\*d^9\*e^6\*x^6 + 56\*d^10\*e^5\*x^5 + 160\*d^11\*e^4\*x^4 - 112\*d^12\*e^3\*x^3 - 256\*d^13\*e^2\*x^2 + 64\*d^14\*e^1\*x + 128\*d^15 + (d^7\*e^8\*x^8 + 8\*d^8\*e^7\*x^7 - 24\*d^9\*e^6\*x^6 - 80\*d^10\*e^5\*x^5 - 80\*d^11\*e^4\*x^4 + 80\*d^12\*e^3\*x^3 + 192\*d^13\*e^2\*x^2 - 64\*d^14\*e^1\*x - 128\*d^15)\*sqrt(-e^2\*x^2 + d^2))\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) - (12\*e^14\*x^14 - 140\*d^1\*e^13\*x^13 + 140\*d^2\*e^12\*x^12 + 1302\*d^3\*e^11\*x^11 - 3374\*d^4\*e^10\*x^10 + 1211\*d^5\*e^9\*x^9 + 4515\*d^6\*e^8\*x^8 - 672\*d^7\*e^7\*x^7 - 19320\*d^8\*e^6\*x^6 - 78624\*d^9\*e^5\*x^5 + 94640\*d^10\*e^4\*x^4 + 232960\*d^11\*e^3\*x^3 - 87360\*d^12\*e^2\*x^2 - 174720\*d^13\*e^1\*x)\*sqrt(-e^2\*x^2 + d^2)/(e^14\*x^8 - 7\*d^1\*e^13\*x^7 - 32\*d^2\*e^12\*x^6 + 56\*d^3\*e^11\*x^5 + 160\*d^4\*e^10\*x^4 - 112\*d^5\*e^9\*x^3 - 256\*d^6\*e^8\*x^2 + 64\*d^7\*e^7\*x + 128\*d^8\*e^6 + (e^13\*x^7 + 8\*d^1\*e^12\*x^6 - 24\*d^2\*e^11\*x^5 - 80\*d^3\*e^10\*x^4 + 80\*d^4\*e^9\*x^3 + 192\*d^5\*e^8\*x^2 - 64\*d^6\*e^7\*x - 128\*d^7\*e^6)\*sqrt(-e^2\*x^2 + d^2))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/(e\*x+d)\*\*4,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.302464, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x^5/(e\*x + d)^4,x, algorithm="giac")

[Out] Done

$$3.199 \quad \int \frac{x^4(d^2 - e^2x^2)^{5/2}}{(d+ex)^4} dx$$

**Optimal.** Leaf size=224

$$\begin{aligned} & \frac{1}{6}x^5\sqrt{d^2 - e^2x^2} - \frac{4dx^4\sqrt{d^2 - e^2x^2}}{5e} + \frac{47d^2x^3\sqrt{d^2 - e^2x^2}}{24e^2} - \frac{239d^6 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{16e^5} \\ & - \frac{337d^5\sqrt{d^2 - e^2x^2}}{15e^5} + \frac{175d^4x\sqrt{d^2 - e^2x^2}}{16e^4} - \frac{d^3(d - ex)^4}{e^5\sqrt{d^2 - e^2x^2}} - \frac{71d^3x^2\sqrt{d^2 - e^2x^2}}{15e^3} \end{aligned}$$

[Out]  $-\left(\frac{d^3(d - ex)^4}{e^5\sqrt{d^2 - e^2x^2}}\right) - \left(\frac{337d^5\sqrt{d^2 - e^2x^2}}{15e^5} + \frac{175d^4x\sqrt{d^2 - e^2x^2}}{16e^4} - \frac{d^3(d - ex)^4}{e^5\sqrt{d^2 - e^2x^2}} - \frac{71d^3x^2\sqrt{d^2 - e^2x^2}}{15e^3} - \frac{47d^2x^3\sqrt{d^2 - e^2x^2}}{24e^2} + \frac{239d^6 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{16e^5} - \frac{1}{6}x^5\sqrt{d^2 - e^2x^2}\right)$

**Rubi [A]** time = 0.804343, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\begin{aligned} & \frac{1}{6}x^5\sqrt{d^2 - e^2x^2} - \frac{4dx^4\sqrt{d^2 - e^2x^2}}{5e} + \frac{47d^2x^3\sqrt{d^2 - e^2x^2}}{24e^2} - \frac{239d^6 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{16e^5} \\ & - \frac{337d^5\sqrt{d^2 - e^2x^2}}{15e^5} + \frac{175d^4x\sqrt{d^2 - e^2x^2}}{16e^4} - \frac{d^3(d - ex)^4}{e^5\sqrt{d^2 - e^2x^2}} - \frac{71d^3x^2\sqrt{d^2 - e^2x^2}}{15e^3} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}\left[\frac{x^4(d^2 - e^2x^2)^{5/2}}{(d + ex)^4}, x\right]$

[Out]  $-\left(\frac{d^3(d - ex)^4}{e^5\sqrt{d^2 - e^2x^2}}\right) - \left(\frac{337d^5\sqrt{d^2 - e^2x^2}}{15e^5} + \frac{175d^4x\sqrt{d^2 - e^2x^2}}{16e^4} - \frac{d^3(d - ex)^4}{e^5\sqrt{d^2 - e^2x^2}} - \frac{71d^3x^2\sqrt{d^2 - e^2x^2}}{15e^3} - \frac{47d^2x^3\sqrt{d^2 - e^2x^2}}{24e^2} + \frac{239d^6 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{16e^5} - \frac{1}{6}x^5\sqrt{d^2 - e^2x^2}\right)$

**Rubi in Sympy [A]** time = 93.2033, size = 194, normalized size = 0.87

$$\begin{aligned} & -\frac{239d^6 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{16e^5} - \frac{8d^6\sqrt{d^2 - e^2x^2}}{e^5(d + ex)} - \frac{20d^5\sqrt{d^2 - e^2x^2}}{e^5} + \frac{111d^4x\sqrt{d^2 - e^2x^2}}{16e^4} \\ & + \frac{16d^3(d^2 - e^2x^2)^{3/2}}{3e^5} + \frac{47d^2x^3\sqrt{d^2 - e^2x^2}}{24e^2} - \frac{4d(d^2 - e^2x^2)^{5/2}}{5e^5} + \frac{x^5\sqrt{d^2 - e^2x^2}}{6} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**4}(-e^{**2}x^{**2}+d^{**2})^{**}(5/2)/(e*x+d)^{**4}, x)$

[Out]  $-239*d^{**6}*atan(e*x/sqrt(d^{**2} - e^{**2}*x^{**2}))/16e^{**5} - 8*d^{**6}*sqrt(d^{**2} - e^{**2}*x^{**2})/(e^{**5}*(d + e*x)) - 20*d^{**5}*sqrt(d^{**2} - e^{**2}*x^{**2})/e^{**5} + 111*d^{**4}*x*sqrt(d^{**2} - e^{**2}*x^{**2})/16e^{**4} + 16*d^{**3}*(d^{**2} - e^{**2}*x^{**2})^{**}(3/2)/(3e^{**5}) + 47*d^{**2}*x^{**3}*sqrt(d^{**2} - e^{**2}*x^{**2})/(24e^{**2}) - 4*d*(d^{**2} - e^{**2}*x^{**2})^{**}(5/2)/(5e^{**5}) + x^{**5}*sqrt(d^{**2} - e^{**2}*x^{**2})/6$



**Mathematica [A]** time = 0.137922, size = 125, normalized size = 0.56

$$\frac{\sqrt{d^2 - e^2 x^2} (-5632d^6 - 2047d^5 ex + 769d^4 e^2 x^2 - 426d^3 e^3 x^3 + 278d^2 e^4 x^4 - 152de^5 x^5 + 40e^6 x^6) - 3585d^6(d + ex) \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{240e^5(d + ex)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4\*(d^2 - e^2\*x^2)^(5/2))/(d + e\*x)^4,x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(-5632\*d^6 - 2047\*d^5\*e\*x + 769\*d^4\*e^2\*x^2 - 426\*d^3\*e^3\*x^3 + 278\*d^2\*e^4\*x^4 - 152\*d\*e^5\*x^5 + 40\*e^6\*x^6) - 3585\*d^6\*(d + e\*x)\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(240\*e^5\*(d + e\*x))

**Maple [B]** time = 0.021, size = 393, normalized size = 1.8

$$\begin{aligned} & \frac{x}{6e^4} (-e^2x^2 + d^2)^{\frac{5}{2}} + \frac{5d^2x}{24e^4} (-e^2x^2 + d^2)^{\frac{3}{2}} + \frac{5d^4x}{16e^4} \sqrt{-e^2x^2 + d^2} \\ & + \frac{5d^6}{16e^4} \arctan\left(x\sqrt{e^2} \frac{1}{\sqrt{-e^2x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} - \frac{d^3}{e^9} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{7}{2}} \left(x + \frac{d}{e}\right)^{-4} \\ & - 7 \frac{d^2}{e^8} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{7}{2}} \left(x + \frac{d}{e}\right)^{-3} \\ & - \frac{22d}{3e^7} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{7}{2}} \left(x + \frac{d}{e}\right)^{-2} - \frac{122d}{15e^5} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{5}{2}} \\ & - \frac{61d^2x}{6e^4} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)\right)^{\frac{3}{2}} - \frac{61d^4x}{4e^4} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)} \\ & - \frac{61d^6}{4e^4} \arctan\left(x\sqrt{e^2} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de\left(x + \frac{d}{e}\right)}}\right) \frac{1}{\sqrt{e^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(-e^2\*x^2+d^2)^(5/2)/(e\*x+d)^4,x)

[Out] 1/6/e^4\*x\*(-e^2\*x^2+d^2)^(5/2)+5/24/e^4\*d^2\*x\*(-e^2\*x^2+d^2)^(3/2)+5/16\*d^4\*x\*(-e^2\*x^2+d^2)^(1/2)/e^4+5/16\*d^6/e^4/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))-d^3/e^9/(x+d/e)^4\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)-7\*d^2/e^8/(x+d/e)^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)-22/3\*d/e^7/(x+d/e)^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)-122/15\*d/e^5\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(5/2)-61/6\*d^2/e^4\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)\*x-61/4\*d^4/e^4\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x-61/4\*d^6/e^4/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x^4/(e\*x + d)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.301213, size = 849, normalized size = 3.79

$$40 e^{13} x^{13} - 432 d e^{12} x^{12} + 622 d^2 e^{11} x^{11} + 2604 d^3 e^{10} x^{10} - 7845 d^4 e^9 x^9 + 4030 d^5 e^8 x^8 - 5545 d^6 e^7 x^7 + 35750 d^7 e^6 x^6 + 154120 d^8 e^5 x^5 - 152960 d^9 e^4 x^4 - 363280 d^{10} e^3 x^3 + 114720 d^{11} e^2 x^2 + 229440 d^{12} e x + 7170 (d^6 e^7 x^7 + 7 d^7 e^6 x^6 - 18 d^8 e^5 x^5 - 56 d^9 e^4 x^4 + 48 d^{10} e^3 x^3 + 112 d^{11} e^2 x^2 - 32 d^{12} e x - 64 d^{13}) - (d^6 e^6 x^6 - 6 d^7 e^5 x^5 - 24 d^8 e^4 x^4 + 32 d^9 e^3 x^3 + 80 d^{10} e^2 x^2 - 32 d^{11} e x - 64 d^{12}) \sqrt{-e^2 x^2 + d^2} \arctan\left(\frac{-d - \sqrt{-e^2 x^2 + d^2}}{e x}\right) + (40 e^{12} x^{12} + 88 d e^{11} x^{11} - 1594 d^2 e^{10} x^{10} + 3610 d^3 e^9 x^9 - 395 d^4 e^8 x^8 - 6985 d^5 e^7 x^7 - 2290 d^6 e^6 x^6 - 58520 d^7 e^5 x^5 + 95600 d^8 e^4 x^4 + 248560 d^9 e^3 x^3 - 114720 d^{10} e^2 x^2 - 229440 d^{11} e x) \sqrt{-e^2 x^2 + d^2} / (e^{12} x^7 + 7 d e^{11} x^6 - 18 d^2 e^{10} x^5 - 56 d^3 e^9 x^4 + 48 d^4 e^8 x^3 + 112 d^5 e^7 x^2 - 32 d^6 e^6 x - 64 d^7 e^5 - (e^{11} x^6 - 6 d e^{10} x^5 - 24 d^2 e^9 x^4 + 32 d^3 e^8 x^3 + 80 d^4 e^7 x^2 - 32 d^5 e^6 x - 64 d^6 e^5) \sqrt{-e^2 x^2 + d^2})$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x^4/(e\*x + d)^4,x, algorithm="fricas")

[Out] 1/240\*(40\*e^13\*x^13 - 432\*d\*e^12\*x^12 + 622\*d^2\*e^11\*x^11 + 2604\*d^3\*e^10\*x^10 - 7845\*d^4\*e^9\*x^9 + 4030\*d^5\*e^8\*x^8 - 5545\*d^6\*e^7\*x^7 + 35750\*d^7\*e^6\*x^6 + 154120\*d^8\*e^5\*x^5 - 152960\*d^9\*e^4\*x^4 - 363280\*d^10\*e^3\*x^3 + 114720\*d^11\*e^2\*x^2 + 229440\*d^12\*e\*x + 7170\*(d^6\*e^7\*x^7 + 7\*d^7\*e^6\*x^6 - 18\*d^8\*e^5\*x^5 - 56\*d^9\*e^4\*x^4 + 48\*d^10\*e^3\*x^3 + 112\*d^11\*e^2\*x^2 - 32\*d^12\*e\*x - 64\*d^13) - (d^6\*e^6\*x^6 - 6\*d^7\*e^5\*x^5 - 24\*d^8\*e^4\*x^4 + 32\*d^9\*e^3\*x^3 + 80\*d^10\*e^2\*x^2 - 32\*d^11\*e\*x - 64\*d^12)\*sqrt(-e^2\*x^2 + d^2))\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) + (40\*e^12\*x^12 + 88\*d\*e^11\*x^11 - 1594\*d^2\*e^10\*x^10 + 3610\*d^3\*e^9\*x^9 - 395\*d^4\*e^8\*x^8 - 6985\*d^5\*e^7\*x^7 - 2290\*d^6\*e^6\*x^6 - 58520\*d^7\*e^5\*x^5 + 95600\*d^8\*e^4\*x^4 + 248560\*d^9\*e^3\*x^3 - 114720\*d^10\*e^2\*x^2 - 229440\*d^11\*e\*x)\*sqrt(-e^2\*x^2 + d^2)/(e^12\*x^7 + 7\*d\*e^11\*x^6 - 18\*d^2\*e^10\*x^5 - 56\*d^3\*e^9\*x^4 + 48\*d^4\*e^8\*x^3 + 112\*d^5\*e^7\*x^2 - 32\*d^6\*e^6\*x - 64\*d^7\*e^5 - (e^11\*x^6 - 6\*d\*e^10\*x^5 - 24\*d^2\*e^9\*x^4 + 32\*d^3\*e^8\*x^3 + 80\*d^4\*e^7\*x^2 - 32\*d^5\*e^6\*x - 64\*d^6\*e^5)\*sqrt(-e^2\*x^2 + d^2))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/(e\*x+d)\*\*4,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.306932, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x^4/(e\*x + d)^4,x, algorithm="giac")

[Out] Done

$$3.200 \quad \int \frac{x^3(d^2 - e^2x^2)^{5/2}}{(d+ex)^4} dx$$

**Optimal.** Leaf size=192

$$\frac{18d^2x^2\sqrt{d^2 - e^2x^2}}{5e^2} + \frac{1}{5}x^4\sqrt{d^2 - e^2x^2} - \frac{dx^3\sqrt{d^2 - e^2x^2}}{e} + \frac{d^2(d - ex)^4}{e^4\sqrt{d^2 - e^2x^2}}$$

$$+ \frac{27d^5 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{2e^4} + \frac{101d^4\sqrt{d^2 - e^2x^2}}{5e^4} - \frac{19d^3x\sqrt{d^2 - e^2x^2}}{2e^3}$$

[Out] (d^2\*(d - e\*x)^4)/(e^4\*Sqrt[d^2 - e^2\*x^2]) + (101\*d^4\*Sqrt[d^2 - e^2\*x^2])/(5\*e^4) - (19\*d^3\*x\*Sqrt[d^2 - e^2\*x^2])/(2\*e^3) + (18\*d^2\*x^2\*Sqrt[d^2 - e^2\*x^2])/(5\*e^2) - (d\*x^3\*Sqrt[d^2 - e^2\*x^2])/e + (x^4\*Sqrt[d^2 - e^2\*x^2])/5 + (27\*d^5\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(2\*e^4)

**Rubi [A]** time = 0.665694, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\frac{18d^2x^2\sqrt{d^2 - e^2x^2}}{5e^2} + \frac{1}{5}x^4\sqrt{d^2 - e^2x^2} - \frac{dx^3\sqrt{d^2 - e^2x^2}}{e} + \frac{d^2(d - ex)^4}{e^4\sqrt{d^2 - e^2x^2}}$$

$$+ \frac{27d^5 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{2e^4} + \frac{101d^4\sqrt{d^2 - e^2x^2}}{5e^4} - \frac{19d^3x\sqrt{d^2 - e^2x^2}}{2e^3}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(d^2 - e^2\*x^2)^(5/2))/(d + e\*x)^4, x]

[Out] (d^2\*(d - e\*x)^4)/(e^4\*Sqrt[d^2 - e^2\*x^2]) + (101\*d^4\*Sqrt[d^2 - e^2\*x^2])/(5\*e^4) - (19\*d^3\*x\*Sqrt[d^2 - e^2\*x^2])/(2\*e^3) + (18\*d^2\*x^2\*Sqrt[d^2 - e^2\*x^2])/(5\*e^2) - (d\*x^3\*Sqrt[d^2 - e^2\*x^2])/e + (x^4\*Sqrt[d^2 - e^2\*x^2])/5 + (27\*d^5\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(2\*e^4)

**Rubi in Sympy [A]** time = 88.9786, size = 163, normalized size = 0.85

$$\frac{27d^5 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right)}{2e^4} + \frac{8d^5\sqrt{d^2 - e^2x^2}}{e^4(d + ex)} + \frac{16d^4\sqrt{d^2 - e^2x^2}}{e^4}$$

$$- \frac{11d^3x\sqrt{d^2 - e^2x^2}}{2e^3} - \frac{3d^2(d^2 - e^2x^2)^{3/2}}{e^4} - \frac{dx^3\sqrt{d^2 - e^2x^2}}{e} + \frac{(d^2 - e^2x^2)^{5/2}}{5e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/(e\*x+d)\*\*4, x)

[Out] 27\*d\*\*5\*atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2))/(2\*e\*\*4) + 8\*d\*\*5\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(e\*\*4\*(d + e\*x)) + 16\*d\*\*4\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/e\*\*4 - 11\*d\*\*3\*x\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(2\*e\*\*3) - 3\*d\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/e\*\*4 - d\*x\*\*3\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/e + (d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)/(5\*e\*\*4)

**Mathematica [A]** time = 0.133912, size = 109, normalized size = 0.57

$$\frac{135d^5 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) + \frac{\sqrt{d^2 - e^2x^2}(212d^5 + 77d^4ex - 29d^3e^2x^2 + 16d^2e^3x^3 - 8de^4x^4 + 2e^5x^5)}{d+ex}}{10e^4}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(d^2 - e^2\*x^2)^(5/2))/(d + e\*x)^4,x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(212\*d^5 + 77\*d^4\*e\*x - 29\*d^3\*e^2\*x^2 + 16\*d^2\*e^3\*x^3 - 8\*d\*e^4\*x^4 + 2\*e^5\*x^5))/(d + e\*x) + 135\*d^5\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(10\*e^4)

**Maple [A]** time = 0.019, size = 285, normalized size = 1.5

$$\begin{aligned} & \frac{36}{5e^4} \left( - \left( x + \frac{d}{e} \right)^2 e^2 + 2de \left( x + \frac{d}{e} \right) \right)^{\frac{5}{2}} + 9 \frac{dx}{e^3} \left( - \left( x + \frac{d}{e} \right)^2 e^2 + 2de \left( x + \frac{d}{e} \right) \right)^{3/2} \\ & + \frac{27d^3x}{2e^3} \sqrt{- \left( x + \frac{d}{e} \right)^2 e^2 + 2de \left( x + \frac{d}{e} \right)} \\ & + \frac{27d^5}{2e^3} \arctan \left( x \sqrt{e^2} \frac{1}{\sqrt{- \left( x + \frac{d}{e} \right)^2 e^2 + 2de \left( x + \frac{d}{e} \right)}} \right) \frac{1}{\sqrt{e^2}} \\ & + 7 \frac{1}{e^6} \left( - \left( x + \frac{d}{e} \right)^2 e^2 + 2de \left( x + \frac{d}{e} \right) \right)^{7/2} \left( x + \frac{d}{e} \right)^{-2} \\ & + 6 \frac{d}{e^7} \left( - \left( x + \frac{d}{e} \right)^2 e^2 + 2de \left( x + \frac{d}{e} \right) \right)^{7/2} \left( x + \frac{d}{e} \right)^{-3} \\ & + \frac{d^2}{e^8} \left( - \left( x + \frac{d}{e} \right)^2 e^2 + 2de \left( x + \frac{d}{e} \right) \right)^{\frac{7}{2}} \left( x + \frac{d}{e} \right)^{-4} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(-e^2\*x^2+d^2)^(5/2)/(e\*x+d)^4,x)

[Out] 36/5/e^4\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(5/2)+9/e^3\*d\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)\*x+27/2/e^3\*d^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x+27/2/e^3\*d^5/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))+7/e^6/(x+d/e)^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)+6\*d/e^7/(x+d/e)^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)+d^2/e^8/(x+d/e)^4\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x^3/(e\*x + d)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.296126, size = 729, normalized size = 3.8

$$2e^{11}x^{11} + 2de^{10}x^{10} - 60d^2e^9x^9 + 155d^3e^8x^8 - 100d^4e^7x^7 + 235d^5e^6x^6 + 1890d^6e^5x^5 - 2420d^7e^4x^4 - 5760d^8e^3x^3 + 2160d^9e^2x^2 - 4320d^{10}e^1x - 4320d^{11}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)*x^3/(e*x + d)^4,x, algorithm="fricas")`

[Out] 
$$-1/10*(2*e^{11}*x^{11} + 2*d*e^{10}*x^{10} - 60*d^2*e^9*x^9 + 155*d^3*e^8*x^8 - 100*d^4*e^7*x^7 + 235*d^5*e^6*x^6 + 1890*d^6*e^5*x^5 - 2420*d^7*e^4*x^4 - 5760*d^8*e^3*x^3 + 2160*d^9*e^2*x^2 + 4320*d^{10}*e*x + 270*(d^5*e^6*x^6 - 5*d^6*e^5*x^5 - 18*d^7*e^4*x^4 + 20*d^8*e^3*x^3 + 48*d^9*e^2*x^2 - 16*d^{10}*e*x - 32*d^{11} + (d^5*e^5*x^5 + 6*d^6*e^4*x^4 - 12*d^7*e^3*x^3 - 32*d^8*e^2*x^2 + 16*d^9*e*x + 32*d^{10})*\sqrt{-e^2*x^2 + d^2})*\arctan(-(d - \sqrt{-e^2*x^2 + d^2}))/e*x) - (2*e^{10}*x^{10} - 20*d*e^9*x^9 + 40*d^2*e^8*x^8 + 35*d^3*e^7*x^7 - 165*d^4*e^6*x^6 + 630*d^5*e^5*x^5 - 1340*d^6*e^4*x^4 - 3600*d^7*e^3*x^3 + 2160*d^8*e^2*x^2 + 4320*d^9*e*x)*\sqrt{-e^2*x^2 + d^2})/(e^{10}*x^6 - 5*d*e^9*x^5 - 18*d^2*e^8*x^4 + 20*d^3*e^7*x^3 + 48*d^4*e^6*x^2 - 16*d^5*e^5*x - 32*d^6*e^4 + (e^9*x^5 + 6*d*e^8*x^4 - 12*d^2*e^7*x^3 - 32*d^3*e^6*x^2 + 16*d^4*e^5*x + 32*d^5*e^4)*\sqrt{-e^2*x^2 + d^2})$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**4,x)`

[Out] Timed out

**GIAC/XCAS [A]** time = 0.335365, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)*x^3/(e*x + d)^4,x, algorithm="giac")`

[Out] Done

$$3.201 \quad \int \frac{x^2(d^2 - e^2x^2)^{5/2}}{(d+ex)^4} dx$$

**Optimal.** Leaf size=182

$$\begin{aligned} & -\frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3} - \frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3} - \frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} \\ & - \frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3} - \frac{95d^4 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3} - \frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3} \end{aligned}$$

[Out]  $-\left(\frac{d^2(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}}\right) - \frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3} - \frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3} - \frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3} - \frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3} - \frac{95d^4 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3} - \frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3}$

**Rubi [A]** time = 0.482291, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\begin{aligned} & -\frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3} - \frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3} - \frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} \\ & - \frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3} - \frac{95d^4 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3} - \frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^2(d^2 - e^2x^2)^{(5/2)})/(d + ex)^4, x]$

[Out]  $-\left(\frac{d^2(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}}\right) - \frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3} - \frac{95d^2(d-ex)\sqrt{d^2-e^2x^2}}{24e^3} - \frac{19d(d-ex)^2\sqrt{d^2-e^2x^2}}{12e^3} - \frac{d(d-ex)^4}{e^3\sqrt{d^2-e^2x^2}} - \frac{(d-ex)^3\sqrt{d^2-e^2x^2}}{4e^3} - \frac{95d^4 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3} - \frac{95d^3\sqrt{d^2-e^2x^2}}{8e^3}$

**Rubi in Sympy [A]** time = 62.5597, size = 143, normalized size = 0.79

$$\begin{aligned} & -\frac{95d^4 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3} - \frac{8d^4\sqrt{d^2-e^2x^2}}{e^3(d+ex)} - \frac{12d^3\sqrt{d^2-e^2x^2}}{e^3} \\ & + \frac{31d^2x\sqrt{d^2-e^2x^2}}{8e^2} + \frac{4d(d^2-e^2x^2)^{3/2}}{3e^3} + \frac{x^3\sqrt{d^2-e^2x^2}}{4} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**2}*(-e^{**2}*x^{**2}+d^{**2})^{**}(5/2)/(e*x+d)^{**4}, x)$

[Out]  $-95d^{**4} \operatorname{atan}(ex/\sqrt{d^{**2}-e^{**2}x^{**2}})/(8e^{**3}) - 8d^{**4} \sqrt{d^{**2}-e^{**2}x^{**2}}/(e^{**3}(d+ex)) - 12d^{**3} \sqrt{d^{**2}-e^{**2}x^{**2}}/e^{**3} + 31d^{**2}x \sqrt{d^{**2}-e^{**2}x^{**2}}/(8e^{**2}) + 4d^{**4}(d^{**2}-e^{**2}x^{**2})^{**}(3/2)/(3e^{**3}) + x^{**3} \sqrt{d^{**2}-e^{**2}x^{**2}}/4$

**Mathematica [A]** time = 0.143537, size = 103, normalized size = 0.57

$$\sqrt{d^2 - e^2x^2} \left( -\frac{8d^4}{e^3(d+ex)} - \frac{32d^3}{3e^3} + \frac{31d^2x}{8e^2} - \frac{4dx^2}{3e} + \frac{x^3}{4} \right) - \frac{95d^4 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{8e^3}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(d^2 - e^2\*x^2)^(5/2))/(d + e\*x)^4, x]

[Out] Sqrt[d^2 - e^2\*x^2]\*((-32\*d^3)/(3\*e^3) + (31\*d^2\*x)/(8\*e^2) - (4\*d\*x^2)/(3\*e) + x^3/4 - (8\*d^4)/(e^3\*(d + e\*x))) - (95\*d^4\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(8\*e^3)

**Maple [A]** time = 0.017, size = 288, normalized size = 1.6

$$\begin{aligned}
 & -\frac{19}{3e^5d} \left( -\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right) \right)^{\frac{7}{2}} \left(x + \frac{d}{e}\right)^{-2} - \frac{19}{3e^3d} \left( -\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right) \right)^{\frac{5}{2}} \\
 & - \frac{95x}{12e^2} \left( -\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right) \right)^{\frac{3}{2}} - \frac{95d^2x}{8e^2} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right)} \\
 & - \frac{95d^4}{8e^2} \arctan \left( x\sqrt{e^2} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right)}} \right) \frac{1}{\sqrt{e^2}} \\
 & - \frac{d}{e^7} \left( -\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right) \right)^{\frac{7}{2}} \left(x + \frac{d}{e}\right)^{-4} \\
 & - 5 \frac{1}{e^6} \left( -\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right) \right)^{7/2} \left(x + \frac{d}{e}\right)^{-3}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(-e^2\*x^2+d^2)^(5/2)/(e\*x+d)^4, x)

[Out] -19/3/e^5/d/(x+d/e)^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)-19/3/e^3/d\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(5/2)-95/12/e^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)\*x-95/8/e^2\*d^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x-95/8/e^2\*d^4/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))-d/e^7/(x+d/e)^4\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)-5/e^6/(x+d/e)^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x^2/(e\*x + d)^4, x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.292427, size = 612, normalized size = 3.36

$$6e^9x^9 - 56de^8x^8 + 143d^2e^7x^7 - 140d^3e^6x^6 - 1041d^4e^5x^5 + 2220d^5e^4x^4 + 4812d^6e^3x^3 - 2280d^7e^2x^2 - 4560d^8ex + 570$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x^2/(e\*x + d)^4,x, algorithm="fricas")

[Out]  $\frac{1}{24} (6e^9x^9 - 56d^2e^8x^8 + 143d^2e^7x^7 - 140d^3e^6x^6 - 1041d^4e^5x^5 + 2220d^5e^4x^4 + 4812d^6e^3x^3 - 2280d^7e^2x^2 - 4560d^8e^1x + 570(d^4e^5x^5 + 5d^5e^4x^4 - 8d^6e^3x^3 - 20d^7e^2x^2 + 8d^8e^1x + 16d^9 - (d^4e^4x^4 - 4d^5e^3x^3 - 12d^6e^2x^2 + 8d^7e^1x + 16d^8) \sqrt{-e^2x^2 + d^2}) \arctan\left(\frac{-(d - \sqrt{-e^2x^2 + d^2})}{e^1x}\right) + (6e^8x^8 - 2d^2e^7x^7 - 115d^2e^6x^6 + 345d^3e^5x^5 - 1080d^4e^4x^4 - 2532d^5e^3x^3 + 2280d^6e^2x^2 + 4560d^7e^1x) \sqrt{-e^2x^2 + d^2}) / (e^8x^5 + 5d^2e^7x^4 - 8d^2e^6x^3 - 20d^3e^5x^2 + 8d^4e^4x + 16d^5e^3 - (e^7x^4 - 4d^2e^6x^3 - 12d^2e^5x^2 + 8d^3e^4x + 16d^4e^3) \sqrt{-e^2x^2 + d^2})$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/(e\*x+d)\*\*4,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.331462, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x^2/(e\*x + d)^4,x, algorithm="giac")

[Out] Done



$$3.202 \quad \int \frac{x(d^2 - e^2 x^2)^{5/2}}{(d + ex)^4} dx$$

**Optimal.** Leaf size=130

$$\frac{(d^2 - e^2 x^2)^{7/2}}{e^2 (d + ex)^4} + \frac{8 (d^2 - e^2 x^2)^{5/2}}{e^2 (d + ex)^2} + \frac{20 (d^2 - e^2 x^2)^{3/2}}{3e^2} + \frac{10 dx \sqrt{d^2 - e^2 x^2}}{e} + \frac{10 d^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{e^2}$$

[Out] (10\*d\*x\*Sqrt[d^2 - e^2\*x^2])/e + (20\*(d^2 - e^2\*x^2)^(3/2))/(3\*e^2) + (8\*(d^2 - e^2\*x^2)^(5/2))/(e^2\*(d + e\*x)^2) + (d^2 - e^2\*x^2)^(7/2)/(e^2\*(d + e\*x)^4) + (10\*d^3\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/e^2

**Rubi [A]** time = 0.16634, antiderivative size = 130, normalized size of antiderivative = 1., number of rules used = 6, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{(d^2 - e^2 x^2)^{7/2}}{e^2 (d + ex)^4} + \frac{8 (d^2 - e^2 x^2)^{5/2}}{e^2 (d + ex)^2} + \frac{20 (d^2 - e^2 x^2)^{3/2}}{3e^2} + \frac{10 dx \sqrt{d^2 - e^2 x^2}}{e} + \frac{10 d^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{e^2}$$

Antiderivative was successfully verified.

[In] Int[(x\*(d^2 - e^2\*x^2)^(5/2))/(d + e\*x)^4, x]

[Out] (10\*d\*x\*Sqrt[d^2 - e^2\*x^2])/e + (20\*(d^2 - e^2\*x^2)^(3/2))/(3\*e^2) + (8\*(d^2 - e^2\*x^2)^(5/2))/(e^2\*(d + e\*x)^2) + (d^2 - e^2\*x^2)^(7/2)/(e^2\*(d + e\*x)^4) + (10\*d^3\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/e^2

**Rubi in Sympy [A]** time = 25.2626, size = 116, normalized size = 0.89

$$\frac{10d^3 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{e^2} + \frac{10dx \sqrt{d^2 - e^2 x^2}}{e} + \frac{20 (d^2 - e^2 x^2)^{3/2}}{3e^2} + \frac{8 (d^2 - e^2 x^2)^{5/2}}{e^2 (d + ex)^2} + \frac{(d^2 - e^2 x^2)^{7/2}}{e^2 (d + ex)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/(e\*x+d)\*\*4, x)

[Out] 10\*d\*\*3\*atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2))/e\*\*2 + 10\*d\*x\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/e + 20\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)/(3\*e\*\*2) + 8\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)/(e\*\*2\*(d + e\*x)\*\*2) + (d\*\*2 - e\*\*2\*x\*\*2)\*\*(7/2)/(e\*\*2\*(d + e\*x)\*\*4)

**Mathematica [A]** time = 0.131566, size = 83, normalized size = 0.64

$$\frac{1}{3} \sqrt{d^2 - e^2 x^2} \left( \frac{24d^3}{e^2 (d + ex)} + \frac{23d^2}{e^2} - \frac{6dx}{e} + x^2 \right) + \frac{10d^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(d^2 - e^2\*x^2)^(5/2))/(d + e\*x)^4, x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*((23\*d^2)/e^2 - (6\*d\*x)/e + x^2 + (24\*d^3)/(e^2\*(d + e\*x))))/3 + (10\*d^3\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/e

^2

---

**Maple [B]** time = 0.016, size = 290, normalized size = 2.2

$$\begin{aligned}
& 4 \frac{1}{e^5 d} \left( - \left( x + \frac{d}{e} \right)^2 e^2 + 2 d e \left( x + \frac{d}{e} \right) \right)^{7/2} \left( x + \frac{d}{e} \right)^{-3} \\
& + \frac{16}{3 e^4 d^2} \left( - \left( x + \frac{d}{e} \right)^2 e^2 + 2 d e \left( x + \frac{d}{e} \right) \right)^{7/2} \left( x + \frac{d}{e} \right)^{-2} + \frac{16}{3 e^2 d^2} \left( - \left( x + \frac{d}{e} \right)^2 e^2 + 2 d e \left( x + \frac{d}{e} \right) \right)^{5/2} \\
& + \frac{20 x}{3 d e} \left( - \left( x + \frac{d}{e} \right)^2 e^2 + 2 d e \left( x + \frac{d}{e} \right) \right)^{3/2} + 10 \frac{d x}{e} \sqrt{- \left( x + \frac{d}{e} \right)^2 e^2 + 2 d e \left( x + \frac{d}{e} \right)} \\
& + 10 \frac{d^3}{e \sqrt{e^2}} \arctan \left( \frac{\sqrt{e^2} x}{\sqrt{- \left( x + \frac{d}{e} \right)^2 e^2 + 2 d e \left( x + \frac{d}{e} \right)}} \right) \\
& + \frac{1}{e^6} \left( - \left( x + \frac{d}{e} \right)^2 e^2 + 2 d e \left( x + \frac{d}{e} \right) \right)^{7/2} \left( x + \frac{d}{e} \right)^{-4}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(-e^2\*x^2+d^2)^(5/2)/(e\*x+d)^4,x)

[Out] 4/e^5/d/(x+d/e)^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)+16/3/e^4/d^2/(x+d/e)^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)+16/3/e^2/d^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(5/2)+20/3/e/d\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)\*x+10/e\*d\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x+10/e\*d^3/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))+1/e^6/(x+d/e)^4\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)

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**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x/(e\*x + d)^4,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.292027, size = 489, normalized size = 3.76

$$\frac{e^7 x^7 - 2 d e^6 x^6 - 6 d^2 e^5 x^5 + 87 d^3 e^4 x^4 + 174 d^4 e^3 x^3 - 108 d^5 e^2 x^2 - 240 d^6 e x + 60 (d^3 e^4 x^4 - 3 d^4 e^3 x^3 - 8 d^5 e^2 x^2 + 4 d^6 e x)}{3 (e^6 x^4 - 3 d e^5 x^3 - 8 d^2 e^4 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x/(e\*x + d)^4,x, algorithm="fricas")

[Out] -1/3\*(e^7\*x^7 - 2\*d\*e^6\*x^6 - 6\*d^2\*e^5\*x^5 + 87\*d^3\*e^4\*x^4 + 174\*d^4\*e^3\*x^3 - 108\*d^5\*e^2\*x^2 - 240\*d^6\*e\*x + 60\*(d^3\*e^4\*x^4 -

$$3*d^4*e^3*x^3 - 8*d^5*e^2*x^2 + 4*d^6*e*x + 8*d^7 + (d^3*e^3*x^3 + 4*d^4*e^2*x^2 - 4*d^5*e*x - 8*d^6)*\sqrt{-e^2*x^2 + d^2}*\arctan\left(\frac{-d - \sqrt{-e^2*x^2 + d^2}}{e*x}\right) - (e^6*x^6 - 9*d*e^5*x^5 + 33*d^2*e^4*x^4 + 54*d^3*e^3*x^3 - 108*d^4*e^2*x^2 - 240*d^5*e*x)*\sqrt{-e^2*x^2 + d^2}/(e^6*x^4 - 3*d*e^5*x^3 - 8*d^2*e^4*x^2 + 4*d^3*e^3*x + 8*d^4*e^2 + (e^5*x^3 + 4*d*e^4*x^2 - 4*d^2*e^3*x - 8*d^3*e^2)*\sqrt{-e^2*x^2 + d^2})$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x(-(-d+ex)(d+ex))^{\frac{5}{2}}}{(d+ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/(e\*x+d)\*\*4,x)

[Out] Integral(x\*(-(-d + e\*x)\*(d + e\*x))\*\*(5/2)/(d + e\*x)\*\*4, x)

**GIAC/XCAS [A]** time = 0.30959, size = 1, normalized size = 0.01

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*x/(e\*x + d)^4,x, algorithm="giac")

[Out] Done

$$3.203 \quad \int \frac{(d^2 - e^2 x^2)^{5/2}}{(d + ex)^4} dx$$

**Optimal.** Leaf size=113

$$-\frac{2(d^2 - e^2 x^2)^{5/2}}{e(d + ex)^3} - \frac{5(d^2 - e^2 x^2)^{3/2}}{2e(d + ex)} - \frac{15d\sqrt{d^2 - e^2 x^2}}{2e} - \frac{15d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{2e}$$

[Out]  $(-15*d*\text{Sqrt}[d^2 - e^2*x^2])/(2*e) - (5*(d^2 - e^2*x^2)^(3/2))/(2*e*(d + e*x)) - (2*(d^2 - e^2*x^2)^(5/2))/(e*(d + e*x)^3) - (15*d^2*2*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(2*e)$

**Rubi [A]** time = 0.115866, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{2(d^2 - e^2 x^2)^{5/2}}{e(d + ex)^3} - \frac{5(d^2 - e^2 x^2)^{3/2}}{2e(d + ex)} - \frac{15d\sqrt{d^2 - e^2 x^2}}{2e} - \frac{15d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{2e}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^4, x]$

[Out]  $(-15*d*\text{Sqrt}[d^2 - e^2*x^2])/(2*e) - (5*(d^2 - e^2*x^2)^(3/2))/(2*e*(d + e*x)) - (2*(d^2 - e^2*x^2)^(5/2))/(e*(d + e*x)^3) - (15*d^2*2*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(2*e)$

**Rubi in Sympy [A]** time = 23.0227, size = 95, normalized size = 0.84

$$-\frac{15d^2 \text{atan}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{2e} - \frac{15d\sqrt{d^2 - e^2 x^2}}{2e} - \frac{5(d^2 - e^2 x^2)^{3/2}}{2e(d + ex)} - \frac{2(d^2 - e^2 x^2)^{5/2}}{e(d + ex)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((-e^{**2}*x^{**2}+d^{**2})^{**}(5/2)/(e*x+d)^{**4}, x)$

[Out]  $-15*d^{**2}*\text{atan}(e*x/\text{sqrt}(d^{**2} - e^{**2}*x^{**2}))/ (2*e) - 15*d*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/ (2*e) - 5*(d^{**2} - e^{**2}*x^{**2})^{**}(3/2)/ (2*e*(d + e*x)) - 2*(d^{**2} - e^{**2}*x^{**2})^{**}(5/2)/ (e*(d + e*x)^{**3})$

**Mathematica [A]** time = 0.0997608, size = 75, normalized size = 0.66

$$\sqrt{d^2 - e^2 x^2} \left( -\frac{8d^2}{e(d + ex)} - \frac{4d}{e} + \frac{x}{2} \right) - \frac{15d^2 \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2 x^2}}\right)}{2e}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(d^2 - e^2*x^2)^(5/2)/(d + e*x)^4, x]$

[Out]  $\text{Sqrt}[d^2 - e^2*x^2]*((-4*d)/e + x/2 - (8*d^2)/(e*(d + e*x))) - (15*d^2*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(2*e)$

**Maple [B]** time = 0.008, size = 284, normalized size = 2.5

$$\begin{aligned}
 & -\frac{1}{e^5 d} \left( -\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right) \right)^{\frac{7}{2}} \left(x + \frac{d}{e}\right)^{-4} \\
 & -3 \frac{1}{e^4 d^2} \left( -\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right) \right)^{7/2} \left(x + \frac{d}{e}\right)^{-3} \\
 & -4 \frac{1}{e^3 d^3} \left( -\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right) \right)^{7/2} \left(x + \frac{d}{e}\right)^{-2} \\
 & -4 \frac{1}{e d^3} \left( -\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right) \right)^{5/2} \\
 & -5 \frac{x}{d^2} \left( -\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right) \right)^{3/2} - \frac{15 x}{2} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)} \\
 & - \frac{15 d^2}{2} \arctan \left( x \sqrt{e^2} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)}} \right) \frac{1}{\sqrt{e^2}}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-e^2*x^2+d^2)^(5/2)/(e*x+d)^4,x)`

[Out]  $-1/e^5/d/(x+d/e)^4 * (- (x+d/e)^2 * e^2 + 2*d*e*(x+d/e))^(7/2) - 3/e^4/d^2 / (x+d/e)^3 * (- (x+d/e)^2 * e^2 + 2*d*e*(x+d/e))^(7/2) - 4/e^3/d^3 / (x+d/e)^2 * (- (x+d/e)^2 * e^2 + 2*d*e*(x+d/e))^(7/2) - 4/e/d^3 * (- (x+d/e)^2 * e^2 + 2*d*e*(x+d/e))^(5/2) - 5/d^2 * (- (x+d/e)^2 * e^2 + 2*d*e*(x+d/e))^(3/2) * x - 15/2 * (- (x+d/e)^2 * e^2 + 2*d*e*(x+d/e))^(1/2) * x - 15/2 * d^2 / (e^2)^(1/2) * \arctan((e^2)^(1/2) * x / (- (x+d/e)^2 * e^2 + 2*d*e*(x+d/e))^(1/2))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)/(e*x + d)^4,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.289165, size = 366, normalized size = 3.24

$$\frac{e^5 x^5 - 10 d e^4 x^4 - 29 d^2 e^3 x^3 + 18 d^3 e^2 x^2 + 68 d^4 e x + 30 \left( d^2 e^3 x^3 + 3 d^3 e^2 x^2 - 2 d^4 e x - 4 d^5 - (d^2 e^2 x^2 - 2 d^3 e x - 4 d^4) \sqrt{-e^2 x^2 + d^2} \right)}{2 \left( e^4 x^3 + 3 d e^3 x^2 - 2 d^2 e^2 x - 4 d^3 e - (e^3 x^2 - 2 d e^2 x - \dots) \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)/(e*x + d)^4,x, algorithm="fricas")`

[Out]  $1/2 * (e^5 * x^5 - 10 * d * e^4 * x^4 - 29 * d^2 * e^3 * x^3 + 18 * d^3 * e^2 * x^2 + 68 * d^4 * e * x + 30 * (d^2 * e^3 * x^3 + 3 * d^3 * e^2 * x^2 - 2 * d^4 * e * x - 4 * d^5 - (d^2 * e^2 * x^2 - 2 * d^3 * e * x - 4 * d^4) * \sqrt{-e^2 * x^2 + d^2})) * \arctan(- (d - \sqrt{-e^2 * x^2 + d^2}) / (e * x)) + (e^4 * x^3 + 3 * d * e^3 * x^2 - 2 * d^2 * e^2 * x - 4 * d^3 * e - (e^3 * x^2 - 2 * d * e^2 * x - \dots)) * \sqrt{-e^2 * x^2 + d^2} / (e^4 * x^3 + 3 * d * e^3 * x^2 - 2 * d^2 * e^2 * x - 4 * d^3 * e - (e^3 * x^2 - 2 * d * e^2 * x - \dots))$

$x^2 - 2d^2e^2x - 4d^3e - (e^3x^2 - 2de^2x - 4d^2e) \sqrt{-e^2x^2 + d^2}$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(-d + ex)(d + ex))^{\frac{5}{2}}}{(d + ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/(e\*x+d)\*\*4,x)

[Out] Integral((-(-d + e\*x)\*(d + e\*x))\*\*(5/2)/(d + e\*x)\*\*4, x)

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**GIAC/XCAS [A]** time = 0.301862, size = 1, normalized size = 0.01

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/(e\*x + d)^4,x, algorithm="giac")

[Out] Done

$$3.204 \quad \int \frac{(d^2 - e^2 x^2)^{5/2}}{x(d+ex)^4} dx$$

**Optimal.** Leaf size=89

$$\frac{8d(d-ex)}{\sqrt{d^2 - e^2 x^2}} + \sqrt{d^2 - e^2 x^2} + 4d \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) - d \tanh^{-1} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right)$$

[Out] (8\*d\*(d - e\*x))/Sqrt[d^2 - e^2\*x^2] + Sqrt[d^2 - e^2\*x^2] + 4\*d\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] - d\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]

**Rubi [A]** time = 0.392879, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{8d(d-ex)}{\sqrt{d^2 - e^2 x^2}} + \sqrt{d^2 - e^2 x^2} + 4d \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) - d \tanh^{-1} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right)$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^(5/2)/(x\*(d + e\*x)^4), x]

[Out] (8\*d\*(d - e\*x))/Sqrt[d^2 - e^2\*x^2] + Sqrt[d^2 - e^2\*x^2] + 4\*d\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] - d\*ArcTanh[Sqrt[d^2 - e^2\*x^2]/d]

**Rubi in Sympy [A]** time = 31.794, size = 75, normalized size = 0.84

$$4d \operatorname{atan} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) - d \operatorname{atanh} \left( \frac{\sqrt{d^2 - e^2 x^2}}{d} \right) + \frac{8d\sqrt{d^2 - e^2 x^2}}{d + ex} + \sqrt{d^2 - e^2 x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x/(e\*x+d)\*\*4, x)

[Out] 4\*d\*atan(e\*x/sqrt(d\*\*2 - e\*\*2\*x\*\*2)) - d\*atanh(sqrt(d\*\*2 - e\*\*2\*x\*\*2)/d) + 8\*d\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(d + e\*x) + sqrt(d\*\*2 - e\*\*2\*x\*\*2)

**Mathematica [A]** time = 0.118311, size = 79, normalized size = 0.89

$$\sqrt{d^2 - e^2 x^2} \left( \frac{8d}{d + ex} + 1 \right) - d \log \left( \sqrt{d^2 - e^2 x^2} + d \right) + 4d \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right) + d \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^(5/2)/(x\*(d + e\*x)^4), x]

[Out] Sqrt[d^2 - e^2\*x^2]\*(1 + (8\*d)/(d + e\*x)) + 4\*d\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]] + d\*Log[x] - d\*Log[d + Sqrt[d^2 - e^2\*x^2]]

**Maple [B]** time = 0.017, size = 378, normalized size = 4.3

$$\begin{aligned} & \frac{1}{5d^4} (-e^2x^2 + d^2)^{\frac{5}{2}} + \frac{1}{3d^2} (-e^2x^2 + d^2)^{\frac{3}{2}} + \sqrt{-e^2x^2 + d^2} \\ & - d^2 \ln \left( \frac{1}{x} \left( 2d^2 + 2\sqrt{d^2} \sqrt{-e^2x^2 + d^2} \right) \right) \frac{1}{\sqrt{d^2}} + \frac{32}{15d^4} \left( - \left( x + \frac{d}{e} \right)^2 e^2 + 2de \left( x + \frac{d}{e} \right) \right)^{\frac{5}{2}} \\ & + \frac{8ex}{3d^3} \left( - \left( x + \frac{d}{e} \right)^2 e^2 + 2de \left( x + \frac{d}{e} \right) \right)^{\frac{3}{2}} + 4 \frac{ex}{d} \sqrt{- \left( x + \frac{d}{e} \right)^2 e^2 + 2de \left( x + \frac{d}{e} \right)} \\ & + 4 \frac{de}{\sqrt{e^2}} \arctan \left( \frac{\sqrt{e^2}x}{\sqrt{- \left( x + \frac{d}{e} \right)^2 e^2 + 2de \left( x + \frac{d}{e} \right)}} \right) \\ & + \frac{7}{3d^4e^2} \left( - \left( x + \frac{d}{e} \right)^2 e^2 + 2de \left( x + \frac{d}{e} \right) \right)^{\frac{7}{2}} \left( x + \frac{d}{e} \right)^{-2} \\ & + 2 \frac{1}{e^3d^3} \left( - \left( x + \frac{d}{e} \right)^2 e^2 + 2de \left( x + \frac{d}{e} \right) \right)^{7/2} \left( x + \frac{d}{e} \right)^{-3} \\ & + \frac{1}{d^2e^4} \left( - \left( x + \frac{d}{e} \right)^2 e^2 + 2de \left( x + \frac{d}{e} \right) \right)^{\frac{7}{2}} \left( x + \frac{d}{e} \right)^{-4} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^(5/2)/x/(e\*x+d)^4,x)

[Out] 1/5/d^4\*(-e^2\*x^2+d^2)^(5/2)+1/3/d^2\*(-e^2\*x^2+d^2)^(3/2)+(-e^2\*x^2+d^2)^(1/2)-d^2/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)+32/15/d^4\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(5/2)+8/3/d^3\*e\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)\*x+4/d\*e\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x+4\*d\*e/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))+7/3/d^4/e^2/(x+d/e)^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)+2/e^3/d^3/(x+d/e)^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)+1/d^2/e^4/(x+d/e)^4\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^{\frac{5}{2}}}{(ex + d)^4x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)^4\*x),x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)^4\*x), x)

**Fricas [A]** time = 0.292718, size = 328, normalized size = 3.69

$$\frac{e^3x^3 + de^2x^2 + 16d^2ex + 8 \left( de^2x^2 - d^2ex - 2d^3 + \sqrt{-e^2x^2 + d^2}(dex + 2d^2) \right) \arctan \left( -\frac{d-\sqrt{-e^2x^2+d^2}}{ex} \right) - \left( de^2x^2 - d^2ex - e^2x^2 - dex - 2d^2 + \sqrt{-e^2x^2 + d^2}(ex + 2d^2) \right)}{e^2x^2 - dex - 2d^2 + \sqrt{-e^2x^2 + d^2}(ex + 2d^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)^4\*x),x, algorithm="fricas")



[Out]  $-(e^3 x^3 + d e^2 x^2 + 16 d^2 e x + 8(d e^2 x^2 - d^2 e x - 2 d^3 + \sqrt{-e^2 x^2 + d^2})(d e x + 2 d^2)) \arctan\left(\frac{-(d - \sqrt{-e^2 x^2 + d^2})}{e x}\right) - (d e^2 x^2 - d^2 e x - 2 d^3 + \sqrt{-e^2 x^2 + d^2})(d e x + 2 d^2) \log\left(\frac{-(d - \sqrt{-e^2 x^2 + d^2})}{x}\right) - (e^2 x^2 + 16 d e x) \sqrt{-e^2 x^2 + d^2} / (e^2 x^2 - d e x - 2 d^2 + \sqrt{-e^2 x^2 + d^2})(e x + 2 d)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(-d + ex)(d + ex))^{\frac{5}{2}}}{x(d + ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e**2*x**2+d**2)**(5/2)/x/(e*x+d)**4,x)`

[Out] `Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x*(d + e*x)**4), x)`

**GIAC/XCAS [A]** time = 0.305844, size = 1, normalized size = 0.01

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x),x, algorithm="giac")`

[Out] Done

$$3.205 \quad \int \frac{(d^2 - e^2 x^2)^{5/2}}{x^2(d+ex)^4} dx$$

**Optimal.** Leaf size=94

$$-\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{x} - e \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + 4e \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

[Out]  $(-8*e*(d - e*x))/\text{Sqrt}[d^2 - e^2*x^2] - \text{Sqrt}[d^2 - e^2*x^2]/x - e*$   
 $\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]] + 4*e*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d]$

**Rubi [A]** time = 0.394382, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$-\frac{8e(d-ex)}{\sqrt{d^2-e^2x^2}} - \frac{\sqrt{d^2-e^2x^2}}{x} - e \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right) + 4e \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d^2 - e^2*x^2)^{(5/2)}/(x^2*(d + e*x)^4), x]$

[Out]  $(-8*e*(d - e*x))/\text{Sqrt}[d^2 - e^2*x^2] - \text{Sqrt}[d^2 - e^2*x^2]/x - e*$   
 $\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]] + 4*e*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d]$

**Rubi in Sympy [A]** time = 32.5711, size = 76, normalized size = 0.81

$$-e \operatorname{atan}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) + 4e \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right) - \frac{8e\sqrt{d^2 - e^2x^2}}{d + ex} - \frac{\sqrt{d^2 - e^2x^2}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((-e^{**2}*x^{**2}+d^{**2})^{**}(5/2)/x^{**2}/(e*x+d)^{**4}, x)$

[Out]  $-e*\operatorname{atan}(e*x/\text{sqrt}(d^{**2} - e^{**2}*x^{**2})) + 4*e*\operatorname{atanh}(\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/d) - 8*e*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(d + e*x) - \text{sqrt}(d^{**2} - e^{**2}*x^{**2})/x$

**Mathematica [A]** time = 0.164373, size = 84, normalized size = 0.89

$$\sqrt{d^2 - e^2x^2} \left( -\frac{8e}{d+ex} - \frac{1}{x} \right) + 4e \log(\sqrt{d^2 - e^2x^2} + d) - e \tan^{-1}\left(\frac{ex}{\sqrt{d^2 - e^2x^2}}\right) - 4e \log(x)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(d^2 - e^2*x^2)^{(5/2)}/(x^2*(d + e*x)^4), x]$

[Out]  $\text{Sqrt}[d^2 - e^2*x^2]*(-x^{(-1)} - (8*e)/(d + e*x)) - e*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]] - 4*e*\text{Log}[x] + 4*e*\text{Log}[d + \text{Sqrt}[d^2 - e^2*x^2]]$

**Maple [B]** time = 0.019, size = 515, normalized size = 5.5

$$\begin{aligned}
& -\frac{1}{d^6 x} (-e^2 x^2 + d^2)^{\frac{7}{2}} - \frac{e^2 x}{d^6} (-e^2 x^2 + d^2)^{\frac{5}{2}} - \frac{5 e^2 x}{4 d^4} (-e^2 x^2 + d^2)^{\frac{3}{2}} - \frac{15 e^2 x}{8 d^2} \sqrt{-e^2 x^2 + d^2} \\
& - \frac{15 e^2}{8} \arctan\left(x \sqrt{e^2} \frac{1}{\sqrt{-e^2 x^2 + d^2}}\right) \frac{1}{\sqrt{e^2}} - \frac{1}{d^3 e^3} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)\right)^{\frac{7}{2}} \left(x + \frac{d}{e}\right)^{-4} \\
& - \frac{1}{d^4 e^2} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)\right)^{\frac{7}{2}} \left(x + \frac{d}{e}\right)^{-3} \\
& - \frac{1}{3 d^5 e} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)\right)^{\frac{7}{2}} \left(x + \frac{d}{e}\right)^{-2} \\
& + \frac{7 e}{15 d^5} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)\right)^{\frac{5}{2}} \\
& + \frac{7 e^2 x}{12 d^4} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)\right)^{\frac{3}{2}} + \frac{7 e^2 x}{8 d^2} \sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)} \\
& + \frac{7 e^2}{8} \arctan\left(x \sqrt{e^2} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)}}\right) \frac{1}{\sqrt{e^2}} - \frac{4 e}{5 d^5} (-e^2 x^2 + d^2)^{\frac{5}{2}} \\
& - \frac{4 e}{3 d^3} (-e^2 x^2 + d^2)^{\frac{3}{2}} - 4 \frac{e \sqrt{-e^2 x^2 + d^2}}{d} + 4 \frac{d e}{\sqrt{d^2}} \ln\left(\frac{2 d^2 + 2 \sqrt{d^2} \sqrt{-e^2 x^2 + d^2}}{x}\right)
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-e^2*x^2+d^2)^(5/2)/x^2/(e*x+d)^4,x)`

[Out] 
$$\begin{aligned}
& -1/d^6/x^2 * (-e^2*x^2+d^2)^(7/2) - 1/d^6 * e^2 * x * (-e^2*x^2+d^2)^(5/2) - 5/4/d^4 * e^2 * x * (-e^2*x^2+d^2)^(3/2) - 15/8/d^2 * e^2 * x * (-e^2*x^2+d^2)^(1/2) \\
& - 15/8 * e^2/(e^2)^(1/2) * \arctan((e^2)^(1/2) * x/(-e^2*x^2+d^2)^(1/2)) - 1/d^3/e^3/(x+d/e)^4 * (- (x+d/e)^2 * e^2 + 2 * d * e * (x+d/e))^(7/2) - 1/d^4/e^2/(x+d/e)^3 * (- (x+d/e)^2 * e^2 + 2 * d * e * (x+d/e))^(7/2) - 1/3/d^5/e/(x+d/e)^2 * (- (x+d/e)^2 * e^2 + 2 * d * e * (x+d/e))^(7/2) + 7/15/d^5 * e * (- (x+d/e)^2 * e^2 + 2 * d * e * (x+d/e))^(5/2) + 7/12/d^4 * e^2 * (- (x+d/e)^2 * e^2 + 2 * d * e * (x+d/e))^(3/2) * x + 7/8/d^2 * e^2 * (- (x+d/e)^2 * e^2 + 2 * d * e * (x+d/e))^(1/2) * x + 7/8 * e^2/(e^2)^(1/2) * \arctan((e^2)^(1/2) * x/(- (x+d/e)^2 * e^2 + 2 * d * e * (x+d/e))^(1/2)) - 4/5/d^5 * e * (-e^2*x^2+d^2)^(5/2) - 4/3/d^3 * e * (-e^2*x^2+d^2)^(3/2) - 4/d * e * (-e^2*x^2+d^2)^(1/2) + 4 * d * e/(d^2)^(1/2) * \ln((2 * d^2 + 2 * (d^2)^(1/2) * (-e^2*x^2+d^2)^(1/2))/x)
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2 x^2 + d^2)^{\frac{5}{2}}}{(e x + d)^4 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^2),x, algorithm="maxima")`

[Out] `integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^2), x)`

**Fricas [A]** time = 0.293123, size = 367, normalized size = 3.9

$$\frac{e^3 x^3 + 18 d e^2 x^2 - d^2 e x - 2 d^3 + 2 \left( e^3 x^3 - d e^2 x^2 - 2 d^2 e x + (e^2 x^2 + 2 d e x) \sqrt{-e^2 x^2 + d^2} \right) \arctan\left( -\frac{d - \sqrt{-e^2 x^2 + d^2}}{e x} \right) - 4 \left( e^3 x^3 - d e^2 x^2 - 2 d^2 e x + \sqrt{-e^2 x^2 + d^2} \right)}{e^2 x^3 - d e x^2 - 2 d^2 x + \sqrt{-e^2 x^2 + d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^2),x, algorithm="fricas")`

[Out]  $(e^3x^3 + 18d^2e^2x^2 - d^2e^2x - 2d^3 + 2(e^3x^3 - d^2e^2x^2 - 2d^2e^2x + (e^2x^2 + 2d^2e^2x)\sqrt{-e^2x^2 + d^2})\arctan(-\frac{d - \sqrt{-e^2x^2 + d^2}}{e^2x}) - 4(e^3x^3 - d^2e^2x^2 - 2d^2e^2x + (e^2x^2 + 2d^2e^2x)\sqrt{-e^2x^2 + d^2})\log(-\frac{d - \sqrt{-e^2x^2 + d^2}}{e^2x}) - (17e^2x^2 - d^2e^2x - 2d^2)\sqrt{-e^2x^2 + d^2})/(e^2x^3 - d^2e^2x^2 - 2d^2e^2x + \sqrt{-e^2x^2 + d^2})(e^2x^2 + 2d^2e^2x)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(-d + ex)(d + ex))^{\frac{5}{2}}}{x^2(d + ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e**2*x**2+d**2)**(5/2)/x**2/(e*x+d)**4,x)`

[Out] `Integral((-(-d + e*x)*(d + e*x))**(5/2)/(x**2*(d + e*x)**4), x)`

**GIAC/XCAS [A]** time = 0.341448, size = 1, normalized size = 0.01

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^2),x, algorithm="giac")`

[Out] Done

$$3.206 \quad \int \frac{(d^2 - e^2 x^2)^{5/2}}{x^3(d+ex)^4} dx$$

**Optimal.** Leaf size=110

$$\frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} + \frac{4e\sqrt{d^2-e^2x^2}}{dx} - \frac{\sqrt{d^2-e^2x^2}}{2x^2} - \frac{15e^2 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}$$

[Out]  $(8*e^2*(d - e*x))/(d*\text{Sqrt}[d^2 - e^2*x^2]) - \text{Sqrt}[d^2 - e^2*x^2]/(2*x^2) + (4*e*\text{Sqrt}[d^2 - e^2*x^2])/(d*x) - (15*e^2*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/(2*d)$

**Rubi [A]** time = 0.412657, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\frac{8e^2(d-ex)}{d\sqrt{d^2-e^2x^2}} + \frac{4e\sqrt{d^2-e^2x^2}}{dx} - \frac{\sqrt{d^2-e^2x^2}}{2x^2} - \frac{15e^2 \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{2d}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d^2 - e^2*x^2)^{(5/2)}/(x^3*(d + e*x)^4), x]$

[Out]  $(8*e^2*(d - e*x))/(d*\text{Sqrt}[d^2 - e^2*x^2]) - \text{Sqrt}[d^2 - e^2*x^2]/(2*x^2) + (4*e*\text{Sqrt}[d^2 - e^2*x^2])/(d*x) - (15*e^2*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/(2*d)$

**Rubi in Sympy [A]** time = 33.3633, size = 88, normalized size = 0.8

$$-\frac{\sqrt{d^2 - e^2x^2}}{2x^2} - \frac{15e^2 \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)}{2d} + \frac{8e^2\sqrt{d^2 - e^2x^2}}{d(d+ex)} + \frac{4e\sqrt{d^2 - e^2x^2}}{dx}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((-e^{**2}*x^{**2}+d^{**2})^{**}(5/2)/x^{**3}/(e*x+d)^{**4}, x)$

[Out]  $-\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(2*x^{**2}) - 15*e^{**2}*\text{atanh}(\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/d)/(2*d) + 8*e^{**2}*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(d*(d + e*x)) + 4*e*\text{sqrt}(d^{**2} - e^{**2}*x^{**2})/(d*x)$

**Mathematica [A]** time = 0.200272, size = 85, normalized size = 0.77

$$\frac{\frac{\sqrt{d^2 - e^2x^2}(-d^2 + 7dex + 24e^2x^2)}{x^2(d+ex)} - 15e^2 \log\left(\sqrt{d^2 - e^2x^2} + d\right) + 15e^2 \log(x)}{2d}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(d^2 - e^2*x^2)^{(5/2)}/(x^3*(d + e*x)^4), x]$

[Out]  $((\text{Sqrt}[d^2 - e^2*x^2]*(-d^2 + 7*d*e*x + 24*e^2*x^2))/(x^2*(d + e*x)) + 15*e^2*\text{Log}[x] - 15*e^2*\text{Log}[d + \text{Sqrt}[d^2 - e^2*x^2]])/(2*d)$

**Maple [B]** time = 0.021, size = 504, normalized size = 4.6

$$\begin{aligned}
 & -\frac{1}{2d^6x^2}(-e^2x^2+d^2)^{\frac{7}{2}} + \frac{3e^2}{2d^6}(-e^2x^2+d^2)^{\frac{5}{2}} + \frac{5e^2}{2d^4}(-e^2x^2+d^2)^{\frac{3}{2}} + \frac{15e^2}{2d^2}\sqrt{-e^2x^2+d^2} \\
 & -\frac{15e^2}{2}\ln\left(\frac{1}{x}\left(2d^2+2\sqrt{d^2}\sqrt{-e^2x^2+d^2}\right)\right)\frac{1}{\sqrt{d^2}}-4\frac{e^2}{d^6}\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{5/2} \\
 & -5\frac{e^3x}{d^5}\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{3/2}-\frac{15e^3x}{2d^3}\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)} \\
 & -\frac{15e^3}{2d}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)}}\right)\frac{1}{\sqrt{e^2}}+4\frac{e(-e^2x^2+d^2)^{7/2}}{d^7x} \\
 & +4\frac{e^3x(-e^2x^2+d^2)^{5/2}}{d^7}+5\frac{e^3x(-e^2x^2+d^2)^{3/2}}{d^5}+\frac{15e^3x}{2d^3}\sqrt{-e^2x^2+d^2} \\
 & +\frac{15e^3}{2d}\arctan\left(x\sqrt{e^2}\frac{1}{\sqrt{-e^2x^2+d^2}}\right)\frac{1}{\sqrt{e^2}}-2\frac{1}{d^6}\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{7/2}\left(x+\frac{d}{e}\right)^{-2} \\
 & +\frac{1}{d^4e^2}\left(-\left(x+\frac{d}{e}\right)^2e^2+2de\left(x+\frac{d}{e}\right)\right)^{\frac{7}{2}}\left(x+\frac{d}{e}\right)^{-4}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^(5/2)/x^3/(e\*x+d)^4,x)

[Out] -1/2/d^6/x^2\*(-e^2\*x^2+d^2)^(7/2)+3/2/d^6\*e^2\*(-e^2\*x^2+d^2)^(5/2)+5/2/d^4\*e^2\*(-e^2\*x^2+d^2)^(3/2)+15/2/d^2\*e^2\*(-e^2\*x^2+d^2)^(1/2)-15/2\*e^2/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)-4/d^6\*e^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(5/2)-5/d^5\*e^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)\*x-15/2/d^3\*e^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x-15/2/d\*e^3/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))+4/d^7\*e/x\*(-e^2\*x^2+d^2)^(7/2)+4/d^7\*e^3\*x\*(-e^2\*x^2+d^2)^(5/2)+5/d^5\*e^3\*x\*(-e^2\*x^2+d^2)^(3/2)+15/2/d^3\*e^3\*x\*(-e^2\*x^2+d^2)^(1/2)+15/2/d\*e^3/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))-2/d^6/(x+d/e)^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)+1/d^4/e^2/(x+d/e)^4\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2+d^2)^{\frac{5}{2}}}{(ex+d)^4x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2+d^2)^(5/2)/((e\*x+d)^4\*x^3),x,algorithm="maxima")

[Out] integrate((-e^2\*x^2+d^2)^(5/2)/((e\*x+d)^4\*x^3),x)

**Fricas [A]** time = 0.281926, size = 405, normalized size = 3.68

$$\frac{40e^5x^5-17de^4x^4-102d^2e^3x^3+21d^3e^2x^2+30d^4ex-4d^5+15\left(e^5x^5+3de^4x^4-2d^2e^3x^3-4d^3e^2x^2-(e^4x^4-2de^3x^3-2d^2e^2x^2-d^3ex^2-d^4x^2-d^5)\right)}{2\left(de^3x^5+3d^2e^2x^4-2d^3ex^3-4d^4x^2-(de^2x^2+d^3ex-d^4)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)^4\*x^3),x, algorithm="fricas")

[Out]  $\frac{1}{2} \cdot (40 \cdot e^5 \cdot x^5 - 17 \cdot d \cdot e^4 \cdot x^4 - 102 \cdot d^2 \cdot e^3 \cdot x^3 + 21 \cdot d^3 \cdot e^2 \cdot x^2 + 30 \cdot d^4 \cdot e \cdot x - 4 \cdot d^5 + 15 \cdot (e^5 \cdot x^5 + 3 \cdot d \cdot e^4 \cdot x^4 - 2 \cdot d^2 \cdot e^3 \cdot x^3 - 4 \cdot d^3 \cdot e^2 \cdot x^2 - (e^4 \cdot x^4 - 2 \cdot d \cdot e^3 \cdot x^3 - 4 \cdot d^2 \cdot e^2 \cdot x^2) \cdot \sqrt{-e^2 \cdot x^2 + d^2})) \cdot \log(-\frac{d - \sqrt{-e^2 \cdot x^2 + d^2}}{x}) + (8 \cdot e^4 \cdot x^4 + 87 \cdot d \cdot e^3 \cdot x^3 - 19 \cdot d^2 \cdot e^2 \cdot x^2 - 30 \cdot d^3 \cdot e \cdot x + 4 \cdot d^4) \cdot \sqrt{-e^2 \cdot x^2 + d^2}) / (d \cdot e^3 \cdot x^5 + 3 \cdot d^2 \cdot e^2 \cdot x^4 - 2 \cdot d^3 \cdot e \cdot x^3 - 4 \cdot d^4 \cdot x^2 - (d \cdot e^2 \cdot x^4 - 2 \cdot d^2 \cdot e \cdot x^3 - 4 \cdot d^3 \cdot x^2) \cdot \sqrt{-e^2 \cdot x^2 + d^2})$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*3/(e\*x+d)\*\*4,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.338371, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)^4\*x^3),x, algorithm="giac")

[Out] Done

$$3.207 \quad \int \frac{(d^2 - e^2 x^2)^{5/2}}{x^4 (d + ex)^4} dx$$

**Optimal.** Leaf size=137

$$-\frac{23e^2\sqrt{d^2 - e^2x^2}}{3d^2x} + \frac{2e\sqrt{d^2 - e^2x^2}}{dx^2} - \frac{\sqrt{d^2 - e^2x^2}}{3x^3} - \frac{8e^3(d - ex)}{d^2\sqrt{d^2 - e^2x^2}} + \frac{10e^3 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)}{d^2}$$

[Out]  $(-8*e^3*(d - e*x))/(d^2*\text{Sqrt}[d^2 - e^2*x^2]) - \text{Sqrt}[d^2 - e^2*x^2]/(3*x^3) + (2*e*\text{Sqrt}[d^2 - e^2*x^2])/(d*x^2) - (23*e^2*\text{Sqrt}[d^2 - e^2*x^2])/(3*d^2*x) + (10*e^3*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/d^2$

**Rubi [A]** time = 0.526274, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$-\frac{23e^2\sqrt{d^2 - e^2x^2}}{3d^2x} + \frac{2e\sqrt{d^2 - e^2x^2}}{dx^2} - \frac{\sqrt{d^2 - e^2x^2}}{3x^3} - \frac{8e^3(d - ex)}{d^2\sqrt{d^2 - e^2x^2}} + \frac{10e^3 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)}{d^2}$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^(5/2)/(x^4\*(d + e\*x)^4), x]

[Out]  $(-8*e^3*(d - e*x))/(d^2*\text{Sqrt}[d^2 - e^2*x^2]) - \text{Sqrt}[d^2 - e^2*x^2]/(3*x^3) + (2*e*\text{Sqrt}[d^2 - e^2*x^2])/(d*x^2) - (23*e^2*\text{Sqrt}[d^2 - e^2*x^2])/(3*d^2*x) + (10*e^3*\text{ArcTanh}[\text{Sqrt}[d^2 - e^2*x^2]/d])/d^2$

**Rubi in Sympy [A]** time = 41.996, size = 117, normalized size = 0.85

$$-\frac{\sqrt{d^2 - e^2x^2}}{3x^3} + \frac{2e\sqrt{d^2 - e^2x^2}}{dx^2} + \frac{10e^3 \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)}{d^2} - \frac{8e^3\sqrt{d^2 - e^2x^2}}{d^2(d + ex)} - \frac{23e^2\sqrt{d^2 - e^2x^2}}{3d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*4/(e\*x+d)\*\*4, x)

[Out]  $-\text{sqrt}(d^2 - e^2*x^2)/(3*x^3) + 2*e*\text{sqrt}(d^2 - e^2*x^2)/(d*x^2) + 10*e^3*\operatorname{atanh}(\text{sqrt}(d^2 - e^2*x^2)/d)/d^2 - 8*e^3*\text{sqrt}(d^2 - e^2*x^2)/(d^2*(d + e*x)) - 23*e^2*\text{sqrt}(d^2 - e^2*x^2)/(3*d^2*x)$

**Mathematica [A]** time = 0.217721, size = 94, normalized size = 0.69

$$-\frac{30e^3 \log\left(\sqrt{d^2 - e^2x^2} + d\right) + \frac{\sqrt{d^2 - e^2x^2}(d^3 - 5d^2ex + 17de^2x^2 + 47e^3x^3)}{x^3(d + ex)}}{3d^2} + 30e^3 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^(5/2)/(x^4\*(d + e\*x)^4), x]

[Out]  $-((\text{Sqrt}[d^2 - e^2*x^2]*(d^3 - 5*d^2*e*x + 17*d*e^2*x^2 + 47*e^3*x^3))/(x^3*(d + e*x)) + 30*e^3*\text{Log}[x] - 30*e^3*\text{Log}[d + \text{Sqrt}[d^2 -$



$$e^{2x^2}]/(3d^2)$$

**Maple [B]** time = 0.024, size = 575, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((-e^{2x^2+d^2})^{5/2}/x^4/(e^x+d)^4, x)$

[Out] 
$$\begin{aligned} & -1/3/d^6/x^3*(-e^{2x^2+d^2})^{7/2}-26/3/d^8*e^2/x*(-e^{2x^2+d^2})^{7/2}-26/3/d^8*e^4*x*(-e^{2x^2+d^2})^{5/2}-65/6/d^6*e^4*x*(-e^{2x^2+d^2})^{3/2}-65/4/d^4*e^4*x*(-e^{2x^2+d^2})^{1/2}-65/4/d^2*e^4/(e^2)^{1/2}*\arctan((e^2)^{1/2}*x/(-e^{2x^2+d^2})^{1/2})-1/d^5/e/(x+d/e)^4*(-(x+d/e)^2*e^{2+2*d*e*(x+d/e)}^{7/2}+1/d^6/(x+d/e)^3*(-(x+d/e)^2*e^{2+2*d*e*(x+d/e)}^{7/2}+14/3/d^7*e/(x+d/e)^2*(-(x+d/e)^2*e^{2+2*d*e*(x+d/e)}^{7/2}-2/d^7*e^3*(-e^{2x^2+d^2})^{5/2}-10/3/d^5*e^3*(-e^{2x^2+d^2})^{3/2}-10/d^3*e^3*(-e^{2x^2+d^2})^{1/2}+10/d*e^3/(d^2)^{1/2}*\ln((2*d^2+2*(d^2)^{1/2}*(-e^{2x^2+d^2})^{1/2})/x)+26/3/d^7*e^3*(-(x+d/e)^2*e^{2+2*d*e*(x+d/e)}^{5/2}+65/6/d^6*e^4*(-(x+d/e)^2*e^{2+2*d*e*(x+d/e)}^{3/2})*x+65/4/d^4*e^4*(-(x+d/e)^2*e^{2+2*d*e*(x+d/e)}^{1/2})*x+65/4/d^2*e^4/(e^2)^{1/2}*\arctan((e^2)^{1/2}*x/(-(x+d/e)^2*e^{2+2*d*e*(x+d/e)}^{1/2}))+2/d^7*e/x^2*(-e^{2x^2+d^2})^{7/2} \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^{\frac{5}{2}}}{(ex + d)^4x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((-e^{2x^2+d^2})^{5/2}/((e^x+d)^4*x^4), x, \text{algorithm}="maxima")$

[Out]  $\text{integrate}((-e^{2x^2+d^2})^{5/2}/((e^x+d)^4*x^4), x)$

**Fricas [A]** time = 0.276153, size = 528, normalized size = 3.85

$$\frac{23e^7x^7 + 230de^6x^6 - 138d^2e^5x^5 - 434d^3e^4x^4 + 159d^4e^3x^3 + 148d^5e^2x^2 - 44d^6ex + 8d^7 - 30(e^7x^7 - 3de^6x^6 - 8d^2e^5x^5 - 3(d^2e^4x^7 - 3d^3e^3x^6))}{3(d^2e^4x^7 - 3d^3e^3x^6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((-e^{2x^2+d^2})^{5/2}/((e^x+d)^4*x^4), x, \text{algorithm}="fricas")$

[Out] 
$$\begin{aligned} & 1/3*(23*e^7*x^7 + 230*d*e^6*x^6 - 138*d^2*e^5*x^5 - 434*d^3*e^4*x^4 + 159*d^4*e^3*x^3 + 148*d^5*e^2*x^2 - 44*d^6*e*x + 8*d^7 - 30*(e^7*x^7 - 3*d*e^6*x^6 - 8*d^2*e^5*x^5 + 4*d^3*e^4*x^4 + 8*d^4*e^3*x^3 + (e^6*x^6 + 4*d*e^5*x^5 - 4*d^2*e^4*x^4 - 8*d^3*e^3*x^3)*\sqrt{-e^{2x^2+d^2}}*\log(-(d - \sqrt{-e^{2x^2+d^2}})/x) - (71*e^6*x^6 - 75*d*e^5*x^5 - 357*d^2*e^4*x^4 + 137*d^3*e^3*x^3 + 152*d^4*e^2*x^2 - 44*d^5*e*x + 8*d^6)*\sqrt{-e^{2x^2+d^2}})/(d^2*e^4*x^7 - 3*d^3*e^3*x^6 - 8*d^4*e^2*x^5 + 4*d^5*e*x^4 + 8*d^6*x^3 + (d^2*e^3*x^6 + 4*d^3*e^2*x^5 - 4*d^4*e*x^4 - 8*d^5*x^3)*\sqrt{-e^{2x^2+d^2}}) \end{aligned}$$

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**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e**2*x**2+d**2)**(5/2)/x**4/(e*x+d)**4,x)`

[Out] Timed out

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**GIAC/XCAS [A]** time = 0.392081, size = 1, normalized size = 0.01

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)/((e*x + d)^4*x^4),x, algorithm="giac")`

[Out] Done

$$3.208 \quad \int \frac{(d^2 - e^2 x^2)^{5/2}}{x^5 (d + ex)^4} dx$$

**Optimal.** Leaf size=170

$$\begin{aligned} & -\frac{31e^2\sqrt{d^2 - e^2x^2}}{8d^2x^2} - \frac{\sqrt{d^2 - e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2 - e^2x^2}}{3dx^3} + \frac{8e^4(d - ex)}{d^3\sqrt{d^2 - e^2x^2}} \\ & - \frac{95e^4 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)}{8d^3} + \frac{32e^3\sqrt{d^2 - e^2x^2}}{3d^3x} \end{aligned}$$

[Out]  $(8 * e^4 * (d - e * x)) / (d^3 * \text{Sqrt}[d^2 - e^2 * x^2]) - \text{Sqrt}[d^2 - e^2 * x^2] / (4 * x^4) + (4 * e * \text{Sqrt}[d^2 - e^2 * x^2]) / (3 * d * x^3) - (31 * e^2 * \text{Sqrt}[d^2 - e^2 * x^2]) / (8 * d^2 * x^2) + (32 * e^3 * \text{Sqrt}[d^2 - e^2 * x^2]) / (3 * d^3 * x) - (95 * e^4 * \text{ArcTanh}[\text{Sqrt}[d^2 - e^2 * x^2] / d]) / (8 * d^3)$

**Rubi [A]** time = 0.648433, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\begin{aligned} & -\frac{31e^2\sqrt{d^2 - e^2x^2}}{8d^2x^2} - \frac{\sqrt{d^2 - e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2 - e^2x^2}}{3dx^3} + \frac{8e^4(d - ex)}{d^3\sqrt{d^2 - e^2x^2}} \\ & - \frac{95e^4 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)}{8d^3} + \frac{32e^3\sqrt{d^2 - e^2x^2}}{3d^3x} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d^2 - e^2 * x^2)^{(5/2)} / (x^5 * (d + e * x)^4), x]$

[Out]  $(8 * e^4 * (d - e * x)) / (d^3 * \text{Sqrt}[d^2 - e^2 * x^2]) - \text{Sqrt}[d^2 - e^2 * x^2] / (4 * x^4) + (4 * e * \text{Sqrt}[d^2 - e^2 * x^2]) / (3 * d * x^3) - (31 * e^2 * \text{Sqrt}[d^2 - e^2 * x^2]) / (8 * d^2 * x^2) + (32 * e^3 * \text{Sqrt}[d^2 - e^2 * x^2]) / (3 * d^3 * x) - (95 * e^4 * \text{ArcTanh}[\text{Sqrt}[d^2 - e^2 * x^2] / d]) / (8 * d^3)$

**Rubi in Sympy [A]** time = 54.6238, size = 148, normalized size = 0.87

$$\begin{aligned} & -\frac{\sqrt{d^2 - e^2x^2}}{4x^4} + \frac{4e\sqrt{d^2 - e^2x^2}}{3dx^3} - \frac{31e^2\sqrt{d^2 - e^2x^2}}{8d^2x^2} \\ & - \frac{95e^4 \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2x^2}}{d}\right)}{8d^3} + \frac{8e^4\sqrt{d^2 - e^2x^2}}{d^3(d + ex)} + \frac{32e^3\sqrt{d^2 - e^2x^2}}{3d^3x} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((-e^{**2} * x^{**2} + d^{**2})^{** (5/2)} / x^{**5} / (e * x + d)^{**4}, x)$

[Out]  $-\text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (4 * x^{**4}) + 4 * e * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (3 * d * x^{**3}) - 31 * e^{**2} * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (8 * d^{**2} * x^{**2}) - 95 * e^{**4} * \operatorname{atanh}(\text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / d) / (8 * d^{**3}) + 8 * e^{**4} * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (d^{**3} * (d + e * x)) + 32 * e^{**3} * \text{sqrt}(d^{**2} - e^{**2} * x^{**2}) / (3 * d^{**3} * x)$

**Mathematica [A]** time = 0.193004, size = 107, normalized size = 0.63

$$\frac{-285e^4 \log\left(\sqrt{d^2 - e^2x^2} + d\right) + \frac{\sqrt{d^2 - e^2x^2}(-6d^4 + 26d^3ex - 61d^2e^2x^2 + 163de^3x^3 + 448e^4x^4)}{x^4(d+ex)}}{24d^3} + 285e^4 \log(x)$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^(5/2)/(x^5\*(d + e\*x)^4), x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(-6\*d^4 + 26\*d^3\*e\*x - 61\*d^2\*e^2\*x^2 + 163\*d\*e^3\*x^3 + 448\*e^4\*x^4))/(x^4\*(d + e\*x)) + 285\*e^4\*Log[x] - 285\*e^4\*Log[d + Sqrt[d^2 - e^2\*x^2]])/(24\*d^3)

**Maple [B]** time = 0.023, size = 600, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^(5/2)/x^5/(e\*x+d)^4, x)

[Out] 4/3/d^7\*e/x^3\*(-e^2\*x^2+d^2)^(7/2)-2/d^7\*e/(x+d/e)^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)-23/3/d^8\*e^2/(x+d/e)^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)+44/3/d^9\*e^3/x\*(-e^2\*x^2+d^2)^(7/2)+44/3/d^9\*e^5\*x\*(-e^2\*x^2+d^2)^(5/2)+55/3/d^7\*e^5\*x\*(-e^2\*x^2+d^2)^(3/2)+55/2/d^5\*e^5\*x\*(-e^2\*x^2+d^2)^(1/2)+55/2/d^3\*e^5/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))-55/3/d^7\*e^5\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)\*x-55/2/d^5\*e^5\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x-55/2/d^3\*e^5/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))-37/8/d^8\*e^2/x^2\*(-e^2\*x^2+d^2)^(7/2)-95/8/d^2\*e^4/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)+1/d^6/(x+d/e)^4\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)-44/3/d^8\*e^4\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(5/2)-1/4/d^6/x^4\*(-e^2\*x^2+d^2)^(7/2)+19/8/d^8\*e^4\*(-e^2\*x^2+d^2)^(5/2)+95/24/d^6\*e^4\*(-e^2\*x^2+d^2)^(3/2)+95/8/d^4\*e^4\*(-e^2\*x^2+d^2)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^{\frac{5}{2}}}{(ex + d)^4x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)^4\*x^5), x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)^4\*x^5), x)

**Fricas [A]** time = 0.285901, size = 648, normalized size = 3.81

640 e^9x^9 - 1117 de^8x^8 - 5996 d^2e^7x^7 + 4147 d^3e^6x^6 + 8732 d^4e^5x^5 - 4190 d^5e^4x^4 - 2528 d^6e^3x^3 + 1064 d^7e^2x^2 - 464 d^8ex +

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)^4\*x^5), x, algorithm="fricas")

[Out] 1/24\*(640\*e^9\*x^9 - 1117\*d\*e^8\*x^8 - 5996\*d^2\*e^7\*x^7 + 4147\*d^3\*e^6\*x^6 + 8732\*d^4\*e^5\*x^5 - 4190\*d^5\*e^4\*x^4 - 2528\*d^6\*e^3\*x^3 + 1064\*d^7\*e^2\*x^2 - 464\*d^8\*e\*x + 96\*d^9 + 285\*(e^9\*x^9 + 5\*d\*e^8\*x^8 - 8\*d^2\*e^7\*x^7 - 20\*d^3\*e^6\*x^6 + 8\*d^4\*e^5\*x^5 + 16\*d^5\*e

$$\begin{aligned} & ^4x^4 - (e^8x^8 - 4d^7e^7x^7 - 12d^2e^6x^6 + 8d^3e^5x^5 \\ & + 16d^4e^4x^4) \sqrt{-e^2x^2 + d^2}) \log(-(d - \sqrt{-e^2x^2 + \\ & d^2}))/x + (256e^8x^8 + 2723d^7e^7x^7 - 2481d^2e^6x^6 - 72 \\ & 94d^3e^5x^5 + 3622d^4e^4x^4 + 2760d^5e^3x^3 - 1112d^6e \\ & ^2x^2 + 464d^7e^1x - 96d^8) \sqrt{-e^2x^2 + d^2}) / (d^3e^5x^9 \\ & + 5d^4e^4x^8 - 8d^5e^3x^7 - 20d^6e^2x^6 + 8d^7e^1x^5 + \\ & 16d^8x^4 - (d^3e^4x^8 - 4d^4e^3x^7 - 12d^5e^2x^6 + 8d \\ & ^6e^1x^5 + 16d^7x^4) \sqrt{-e^2x^2 + d^2}) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*5/(e\*x+d)\*\*4,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.353409, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)^4\*x^5),x, algorithm="giac")

[Out] Done

$$3.209 \quad \int \frac{(d^2 - e^2 x^2)^{5/2}}{x^6 (d + ex)^4} dx$$

**Optimal.** Leaf size=196

$$\begin{aligned} & -\frac{\sqrt{d^2 - e^2 x^2}}{5x^5} + \frac{e\sqrt{d^2 - e^2 x^2}}{dx^4} - \frac{13e^2\sqrt{d^2 - e^2 x^2}}{5d^2x^3} - \frac{8e^5(d - ex)}{d^4\sqrt{d^2 - e^2 x^2}} \\ & + \frac{27e^5 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{2d^4} - \frac{66e^4\sqrt{d^2 - e^2 x^2}}{5d^4x} + \frac{11e^3\sqrt{d^2 - e^2 x^2}}{2d^3x^2} \end{aligned}$$

[Out]  $(-8e^5(d - ex))/(d^4\sqrt{d^2 - e^2x^2}) - \sqrt{d^2 - e^2x^2}/(5x^5) + (e\sqrt{d^2 - e^2x^2})/(d^4x) - (13e^2\sqrt{d^2 - e^2x^2})/(5d^2x^3) + (11e^3\sqrt{d^2 - e^2x^2})/(2d^3x^2) - (66e^4\sqrt{d^2 - e^2x^2})/(5d^4x) + (27e^5 \operatorname{ArcTanh}[\sqrt{d^2 - e^2x^2}/d])/(2d^4)$

**Rubi [A]** time = 0.796705, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\begin{aligned} & -\frac{\sqrt{d^2 - e^2 x^2}}{5x^5} + \frac{e\sqrt{d^2 - e^2 x^2}}{dx^4} - \frac{13e^2\sqrt{d^2 - e^2 x^2}}{5d^2x^3} - \frac{8e^5(d - ex)}{d^4\sqrt{d^2 - e^2 x^2}} \\ & + \frac{27e^5 \tanh^{-1}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{2d^4} - \frac{66e^4\sqrt{d^2 - e^2 x^2}}{5d^4x} + \frac{11e^3\sqrt{d^2 - e^2 x^2}}{2d^3x^2} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(d^2 - e^2x^2)^{5/2}/(x^6(d + ex)^4), x]$

[Out]  $(-8e^5(d - ex))/(d^4\sqrt{d^2 - e^2x^2}) - \sqrt{d^2 - e^2x^2}/(5x^5) + (e\sqrt{d^2 - e^2x^2})/(d^4x) - (13e^2\sqrt{d^2 - e^2x^2})/(5d^2x^3) + (11e^3\sqrt{d^2 - e^2x^2})/(2d^3x^2) - (66e^4\sqrt{d^2 - e^2x^2})/(5d^4x) + (27e^5 \operatorname{ArcTanh}[\sqrt{d^2 - e^2x^2}/d])/(2d^4)$

**Rubi in Sympy [A]** time = 67.506, size = 172, normalized size = 0.88

$$\begin{aligned} & -\frac{\sqrt{d^2 - e^2 x^2}}{5x^5} + \frac{e\sqrt{d^2 - e^2 x^2}}{dx^4} - \frac{13e^2\sqrt{d^2 - e^2 x^2}}{5d^2x^3} + \frac{11e^3\sqrt{d^2 - e^2 x^2}}{2d^3x^2} \\ & + \frac{27e^5 \operatorname{atanh}\left(\frac{\sqrt{d^2 - e^2 x^2}}{d}\right)}{2d^4} - \frac{8e^5\sqrt{d^2 - e^2 x^2}}{d^4(d + ex)} - \frac{66e^4\sqrt{d^2 - e^2 x^2}}{5d^4x} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{rubi\_integrate}((-e^{**2}x^{**2}+d^{**2})^{**}(5/2)/x^{**6}/(e^*x+d)^{**4}, x)$

[Out]  $-\sqrt{d^{**2} - e^{**2}x^{**2}}/(5x^{**5}) + e*\sqrt{d^{**2} - e^{**2}x^{**2}}/(d^*x^{**4}) - 13e^{**2}*\sqrt{d^{**2} - e^{**2}x^{**2}}/(5d^{**2}x^{**3}) + 11e^{**3}*\sqrt{d^{**2} - e^{**2}x^{**2}}/(2d^{**3}x^{**2}) + 27e^{**5}*\operatorname{atanh}(\sqrt{d^{**2} - e^{**2}x^{**2}}/d)/(2d^{**4}) - 8e^{**5}*\sqrt{d^{**2} - e^{**2}x^{**2}}/(d^{**4}(d + e^*x)) - 66e^{**4}*\sqrt{d^{**2} - e^{**2}x^{**2}}/(5d^{**4}x)$

**Mathematica [A]** time = 0.298614, size = 118, normalized size = 0.6

$$\frac{-135e^5 \log\left(\sqrt{d^2 - e^2 x^2} + d\right) + \frac{\sqrt{d^2 - e^2 x^2}(2d^5 - 8d^4 ex + 16d^3 e^2 x^2 - 29d^2 e^3 x^3 + 77de^4 x^4 + 212e^5 x^5)}{x^5(d + ex)} + 135e^5 \log(x)}{10d^4}$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^(5/2)/(x^6\*(d + e\*x)^4), x]

[Out] -((Sqrt[d^2 - e^2\*x^2]\*(2\*d^5 - 8\*d^4\*e\*x + 16\*d^3\*e^2\*x^2 - 29\*d^2\*e^3\*x^3 + 77\*d\*e^4\*x^4 + 212\*e^5\*x^5))/(x^5\*(d + e\*x)) + 135\*e^5\*Log[x] - 135\*e^5\*Log[d + Sqrt[d^2 - e^2\*x^2]])/(10\*d^4)

**Maple [B]** time = 0.024, size = 628, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^(5/2)/x^6/(e\*x+d)^4, x)

[Out] 1/d^7\*e/x^4\*(-e^2\*x^2+d^2)^(7/2)+17/2/d^9\*e^3/x^2\*(-e^2\*x^2+d^2)^(7/2)-1/d^7\*e/(x+d/e)^4\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)+3/d^8\*e^2/(x+d/e)^3\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)+11/d^9\*e^3/(x+d/e)^2\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(7/2)+111/4/d^8\*e^6\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)\*x+333/8/d^6\*e^6\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)\*x+333/8/d^4\*e^6/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))-16/5/d^8\*e^2/x^3\*(-e^2\*x^2+d^2)^(7/2)-111/5/d^10\*e^4/x\*(-e^2\*x^2+d^2)^(7/2)-111/5/d^10\*e^6\*x\*(-e^2\*x^2+d^2)^(5/2)-111/4/d^8\*e^6\*x\*(-e^2\*x^2+d^2)^(3/2)-333/8/d^6\*e^6\*x\*(-e^2\*x^2+d^2)^(1/2)-333/8/d^4\*e^6/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2))+27/2/d^3\*e^5/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)^(1/2)\*(-e^2\*x^2+d^2)^(1/2))/x)-27/10/d^9\*e^5\*(-e^2\*x^2+d^2)^(5/2)-9/2/d^7\*e^5\*(-e^2\*x^2+d^2)^(3/2)-27/2/d^5\*e^5\*(-e^2\*x^2+d^2)^(1/2)+111/5/d^9\*e^5\*(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(5/2)-1/5/d^6/x^5\*(-e^2\*x^2+d^2)^(7/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^{\frac{5}{2}}}{(ex + d)^4x^6} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)^4\*x^6), x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)^4\*x^6), x)

**Fricas [A]** time = 0.285192, size = 765, normalized size = 3.9

132 e^11x^11 + 1537 de^10x^10 - 2020 d^2e^9x^9 - 7355 d^3e^8x^8 + 5390 d^4e^7x^7 + 8622 d^5e^6x^6 - 4550 d^6e^5x^5 - 2036 d^7e^4x^4 + 760 d^8e^3x^3 - 544 d^9e^2x^2 + 288 d^10e^1x - 288 d^10

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)^4\*x^6), x, algorithm="fricas")

[Out] 1/10\*(132\*e^11\*x^11 + 1537\*d\*e^10\*x^10 - 2020\*d^2\*e^9\*x^9 - 7355\*d^3\*e^8\*x^8 + 5390\*d^4\*e^7\*x^7 + 8622\*d^5\*e^6\*x^6 - 4550\*d^6\*e^5\*x^5 - 2036\*d^7\*e^4\*x^4 + 760\*d^8\*e^3\*x^3 - 544\*d^9\*e^2\*x^2 + 288\*d^10\*e^1\*x - 288\*d^10)

$$d^{10}e^x - 64d^{11} - 135(e^{11}x^{11} - 5d^*e^{10}x^{10} - 18d^2e^9x^9 + 20d^3e^8x^8 + 48d^4e^7x^7 - 16d^5e^6x^6 - 32d^6e^5x^5 + (e^{10}x^{10} + 6d^*e^9x^9 - 12d^2e^8x^8 - 32d^3e^7x^7 + 16d^4e^6x^6 + 32d^5e^5x^5)*\sqrt{-e^2x^2 + d^2})*\log(- (d - \sqrt{-e^2x^2 + d^2})/x) - (292e^{10}x^{10} - 715d^*e^9x^9 - 3995d^2e^8x^8 + 3490d^3e^7x^7 + 7380d^4e^6x^6 - 4062d^5e^5x^5 - 2332d^6e^4x^4 + 904d^7e^3x^3 - 576d^8e^2x^2 + 288d^9e^x - 64d^{10})*\sqrt{-e^2x^2 + d^2})/(d^4e^6x^{11} - 5d^5e^5x^{10} - 18d^6e^4x^9 + 20d^7e^3x^8 + 48d^8e^2x^7 - 16d^9e^x^6 - 32d^{10}x^5 + (d^4e^5x^{10} + 6d^5e^4x^9 - 12d^6e^3x^8 - 32d^7e^2x^7 + 16d^8e^x^6 + 32d^9x^5)*\sqrt{-e^2x^2 + d^2}))$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/x\*\*6/(e\*x+d)\*\*4,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.34795, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)/((e\*x + d)^4\*x^6),x, algorithm="giac")

[Out] Done



$$3.210 \quad \int \frac{x^2 \sqrt{1-a^2x^2}}{(1-ax)^4} dx$$

**Optimal.** Leaf size=95

$$-\frac{\sin^{-1}(ax)}{a^3} - \frac{3(1-a^2x^2)^{3/2}}{5a^3(1-ax)^3} + \frac{(1-a^2x^2)^{3/2}}{5a^3(1-ax)^4} + \frac{2\sqrt{1-a^2x^2}}{a^3(1-ax)}$$

[Out] (2\*Sqrt[1 - a^2\*x^2])/(a^3\*(1 - a\*x)) + (1 - a^2\*x^2)^(3/2)/(5\*a^3\*(1 - a\*x)^4) - (3\*(1 - a^2\*x^2)^(3/2))/(5\*a^3\*(1 - a\*x)^3) - ArcSin[a\*x]/a^3

**Rubi [A]** time = 0.250216, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$

$$-\frac{\sin^{-1}(ax)}{a^3} - \frac{3(1-a^2x^2)^{3/2}}{5a^3(1-ax)^3} + \frac{(1-a^2x^2)^{3/2}}{5a^3(1-ax)^4} + \frac{2\sqrt{1-a^2x^2}}{a^3(1-ax)}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*Sqrt[1 - a^2\*x^2])/(1 - a\*x)^4, x]

[Out] (2\*Sqrt[1 - a^2\*x^2])/(a^3\*(1 - a\*x)) + (1 - a^2\*x^2)^(3/2)/(5\*a^3\*(1 - a\*x)^4) - (3\*(1 - a^2\*x^2)^(3/2))/(5\*a^3\*(1 - a\*x)^3) - ArcSin[a\*x]/a^3

**Rubi in Sympy [A]** time = 24.4743, size = 78, normalized size = 0.82

$$-\frac{\text{asin}(ax)}{a^3} + \frac{2\sqrt{-a^2x^2+1}}{a^3(-ax+1)} - \frac{3(-a^2x^2+1)^{3/2}}{5a^3(-ax+1)^3} + \frac{(-a^2x^2+1)^{3/2}}{5a^3(-ax+1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(-a\*\*2\*x\*\*2+1)\*\*(1/2)/(-a\*x+1)\*\*4, x)

[Out] -asin(a\*x)/a\*\*3 + 2\*sqrt(-a\*\*2\*x\*\*2 + 1)/(a\*\*3\*(-a\*x + 1)) - 3\*(-a\*\*2\*x\*\*2 + 1)\*\*(3/2)/(5\*a\*\*3\*(-a\*x + 1)\*\*3) + (-a\*\*2\*x\*\*2 + 1)\*\*(3/2)/(5\*a\*\*3\*(-a\*x + 1)\*\*4)

**Mathematica [A]** time = 0.084815, size = 50, normalized size = 0.53

$$\frac{(-13a^2x^2+19ax-8)\sqrt{1-a^2x^2}}{(ax-1)^3} - 5\sin^{-1}(ax)$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*Sqrt[1 - a^2\*x^2])/(1 - a\*x)^4, x]

[Out] (((-8 + 19\*a\*x - 13\*a^2\*x^2)\*Sqrt[1 - a^2\*x^2])/(-1 + a\*x)^3 - 5\*ArcSin[a\*x])/(5\*a^3)

**Maple [B]** time = 0.025, size = 200, normalized size = 2.1

$$\begin{aligned} & \frac{1}{5a^7} \left( -(x-a^{-1})^2 a^2 - 2(x-a^{-1})a \right)^{\frac{3}{2}} (x-a^{-1})^{-4} \\ & + \frac{3}{5a^6} \left( -(x-a^{-1})^2 a^2 - 2(x-a^{-1})a \right)^{\frac{3}{2}} (x-a^{-1})^{-3} \\ & + \frac{1}{a^5} \left( -(x-a^{-1})^2 a^2 - 2(x-a^{-1})a \right)^{\frac{3}{2}} (x-a^{-1})^{-2} + \frac{1}{a^3} \sqrt{-(x-a^{-1})^2 a^2 - 2(x-a^{-1})a} \\ & - \frac{1}{a^2} \arctan \left( x\sqrt{a^2} \frac{1}{\sqrt{-(x-a^{-1})^2 a^2 - 2(x-a^{-1})a}} \right) \frac{1}{\sqrt{a^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^4,x)`

[Out] `1/5/a^7/(x-1/a)^4*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(3/2)+3/5/a^6/(x-1/a)^3*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(3/2)+1/a^5/(x-1/a)^2*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(3/2)+1/a^3*(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(1/2)-1/a^2/(a^2)^(1/2)*arctan((a^2)^(1/2)*x/(-(x-1/a)^2*a^2-2*(x-1/a)*a)^(1/2))`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-a^2x^2 + 1x^2}}{(ax - 1)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-a^2*x^2 + 1)*x^2/(a*x - 1)^4,x, algorithm="maxima")`

[Out] `integrate(sqrt(-a^2*x^2 + 1)*x^2/(a*x - 1)^4, x)`

**Fricas [A]** time = 0.289239, size = 352, normalized size = 3.71

$$\frac{21a^5x^5 - 20a^4x^4 - 35a^3x^3 + 50a^2x^2 - 20ax + 10(a^5x^5 - 5a^4x^4 + 5a^3x^3 + 5a^2x^2 - 10ax + (a^4x^4 - 7a^2x^2 + 10ax - 4)\sqrt{-a^2x^2 + 1})}{5(a^8x^5 - 5a^7x^4 + 5a^6x^3 + 5a^5x^2 - 10a^4x + 4a^3 + (a^7x^4 - 7a^5x^2 - 4a^3)\sqrt{-a^2x^2 + 1})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-a^2*x^2 + 1)*x^2/(a*x - 1)^4,x, algorithm="fricas")`

[Out] `1/5*(21*a^5*x^5 - 20*a^4*x^4 - 35*a^3*x^3 + 50*a^2*x^2 - 20*a*x + 10*(a^5*x^5 - 5*a^4*x^4 + 5*a^3*x^3 + 5*a^2*x^2 - 10*a*x + (a^4*x^4 - 7*a^2*x^2 + 10*a*x - 4)*sqrt(-a^2*x^2 + 1) + 4)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x)) - 5*(a^4*x^4 - 9*a^3*x^3 + 10*a^2*x^2 - 4*a*x)*sqrt(-a^2*x^2 + 1))/(a^8*x^5 - 5*a^7*x^4 + 5*a^6*x^3 + 5*a^5*x^2 - 10*a^4*x + 4*a^3 + (a^7*x^4 - 7*a^5*x^2 + 10*a^4*x - 4*a^3)*sqrt(-a^2*x^2 + 1))`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \sqrt{-(ax-1)(ax+1)}}{(ax-1)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(-a**2*x**2+1)**(1/2)/(-a*x+1)**4,x)`

[Out] `Integral(x**2*sqrt(-(a*x - 1)*(a*x + 1))/(a*x - 1)**4, x)`

**GIAC/XCAS [A]** time = 0.275597, size = 217, normalized size = 2.28

$$\frac{\frac{\arcsin(ax) \operatorname{sign}(a)}{a^2 |a|}}{2 \left( \frac{35 (\sqrt{-a^2 x^2 + 1} |a| + a)}{a^2 x} - \frac{55 (\sqrt{-a^2 x^2 + 1} |a| + a)^2}{a^4 x^2} + \frac{25 (\sqrt{-a^2 x^2 + 1} |a| + a)^3}{a^6 x^3} - \frac{5 (\sqrt{-a^2 x^2 + 1} |a| + a)^4}{a^8 x^4} - 8 \right)}{5 a^2 \left( \frac{\sqrt{-a^2 x^2 + 1} |a| + a}{a^2 x} - 1 \right)^5 |a|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-a^2*x^2 + 1)*x^2/(a*x - 1)^4,x, algorithm="giac")`

[Out] `-arcsin(a*x)*sign(a)/(a^2*abs(a)) - 2/5*(35*(sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 55*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^2/(a^4*x^2) + 25*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^3/(a^6*x^3) - 5*(sqrt(-a^2*x^2 + 1)*abs(a) + a)^4/(a^8*x^4) - 8)/(a^2*((sqrt(-a^2*x^2 + 1)*abs(a) + a)/(a^2*x) - 1)^5*abs(a))`

$$3.211 \quad \int \frac{x^2 \sqrt{1-a^2x^2}}{(1-ax)^5} dx$$

**Optimal.** Leaf size=88

$$\frac{23(1-a^2x^2)^{3/2}}{105a^3(1-ax)^3} - \frac{12(1-a^2x^2)^{3/2}}{35a^3(1-ax)^4} + \frac{(1-a^2x^2)^{3/2}}{7a^3(1-ax)^5}$$

[Out]  $(1 - a^2x^2)^{3/2}/(7a^3(1 - ax)^5) - (12(1 - a^2x^2)^{3/2})/(35a^3(1 - ax)^4) + (23(1 - a^2x^2)^{3/2})/(105a^3(1 - ax)^3)$

**Rubi [A]** time = 0.252023, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{23(1-a^2x^2)^{3/2}}{105a^3(1-ax)^3} - \frac{12(1-a^2x^2)^{3/2}}{35a^3(1-ax)^4} + \frac{(1-a^2x^2)^{3/2}}{7a^3(1-ax)^5}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*sqrt[1 - a^2\*x^2])/(1 - a\*x)^5, x]

[Out]  $(1 - a^2x^2)^{3/2}/(7a^3(1 - ax)^5) - (12(1 - a^2x^2)^{3/2})/(35a^3(1 - ax)^4) + (23(1 - a^2x^2)^{3/2})/(105a^3(1 - ax)^3)$

**Rubi in Sympy [A]** time = 25.7614, size = 73, normalized size = 0.83

$$\frac{23(-a^2x^2 + 1)^{3/2}}{105a^3(-ax + 1)^3} - \frac{12(-a^2x^2 + 1)^{3/2}}{35a^3(-ax + 1)^4} + \frac{(-a^2x^2 + 1)^{3/2}}{7a^3(-ax + 1)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(-a\*\*2\*x\*\*2+1)\*\*(1/2)/(-a\*x+1)\*\*5, x)

[Out]  $23(-a^2x^2 + 1)^{3/2}/(105a^3(-ax + 1)^3) - 12(-a^2x^2 + 1)^{3/2}/(35a^3(-ax + 1)^4) + (-a^2x^2 + 1)^{3/2}/(7a^3(-ax + 1)^5)$

**Mathematica [A]** time = 0.0377398, size = 50, normalized size = 0.57

$$\frac{\sqrt{1-a^2x^2}(23a^3x^3 + 13a^2x^2 - 8ax + 2)}{105a^3(ax - 1)^4}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*sqrt[1 - a^2\*x^2])/(1 - a\*x)^5, x]

[Out]  $(\text{sqrt}[1 - a^2x^2]*(2 - 8ax + 13a^2x^2 + 23a^3x^3))/(105a^3(-1 + ax)^4)$

**Maple [A]** time = 0.009, size = 44, normalized size = 0.5

$$\frac{(23a^2x^2 - 10ax + 2)(ax + 1)\sqrt{-a^2x^2 + 1}}{105(ax - 1)^4a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(-a^2*x^2+1)^(1/2)/(-a*x+1)^5,x)`

[Out] `1/105*(-a^2*x^2+1)^(1/2)*(23*a^2*x^2-10*a*x+2)*(a*x+1)/(a*x-1)^4/a^3`

**Maxima [A]** time = 0.721859, size = 207, normalized size = 2.35

$$\frac{2\sqrt{-a^2x^2+1}}{7(a^7x^4-4a^6x^3+6a^5x^2-4a^4x+a^3)} + \frac{29\sqrt{-a^2x^2+1}}{35(a^6x^3-3a^5x^2+3a^4x-a^3)} + \frac{82\sqrt{-a^2x^2+1}}{105(a^5x^2-2a^4x+a^3)} + \frac{23\sqrt{-a^2x^2+1}}{105(a^4x-a^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-sqrt(-a^2*x^2+1)*x^2/(a*x-1)^5,x, algorithm="maxima")`

[Out] `2/7*sqrt(-a^2*x^2+1)/(a^7*x^4-4*a^6*x^3+6*a^5*x^2-4*a^4*x+a^3)+29/35*sqrt(-a^2*x^2+1)/(a^6*x^3-3*a^5*x^2+3*a^4*x-a^3)+82/105*sqrt(-a^2*x^2+1)/(a^5*x^2-2*a^4*x+a^3)+2/105*sqrt(-a^2*x^2+1)/(a^4*x-a^3)`

**Fricas [A]** time = 0.281416, size = 251, normalized size = 2.85

$$\frac{25a^4x^7 - 56a^3x^6 - 259a^2x^5 + 70ax^4 + 280x^3 + 7(3a^3x^6 + 17a^2x^5 - 10ax^4 - 40x^3)\sqrt{-a^2x^2+1}}{105(a^7x^7 - 14a^5x^5 + 28a^4x^4 - 7a^3x^3 - 28a^2x^2 + 28ax - (a^6x^6 - 7a^5x^5 + 11a^4x^4 + 7a^3x^3 - 32a^2x^2 + 28ax - 8)\sqrt{-a^2x^2+1})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-sqrt(-a^2*x^2+1)*x^2/(a*x-1)^5,x, algorithm="fricas")`

[Out] `1/105*(25*a^4*x^7-56*a^3*x^6-259*a^2*x^5+70*a*x^4+280*x^3+7*(3*a^3*x^6+17*a^2*x^5-10*a*x^4-40*x^3)*sqrt(-a^2*x^2+1))/(a^7*x^7-14*a^5*x^5+28*a^4*x^4-7*a^3*x^3-28*a^2*x^2+28*a*x-(a^6*x^6-7*a^5*x^5+11*a^4*x^4+7*a^3*x^3-32*a^2*x^2+28*a*x-8)*sqrt(-a^2*x^2+1)-8)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{x^2\sqrt{-a^2x^2+1}}{a^5x^5-5a^4x^4+10a^3x^3-10a^2x^2+5ax-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(-a**2*x**2+1)**(1/2)/(-a*x+1)**5,x)`

[Out] `-Integral(x**2*sqrt(-a**2*x**2+1)/(a**5*x**5-5*a**4*x**4+10*a**3*x**3-10*a**2*x**2+5*a*x-1),x)`

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**GIAC/XCAS [A]** time = 0.282837, size = 274, normalized size = 3.11

$$-\frac{1}{420} \left( \frac{92 i \operatorname{sign}\left(\frac{1}{ax-1}\right) \operatorname{sign}(a)}{a^4} + \frac{140 \left(-\frac{2}{ax-1} - 1\right)^{\frac{3}{2}} \operatorname{sign}\left(\frac{1}{ax-1}\right) \operatorname{sign}(a) - 28 \left(3 \left(\frac{2}{ax-1} + 1\right)^2 \sqrt{-\frac{2}{ax-1} - 1} + 5 \left(-\frac{2}{ax-1} - 1\right)^{\frac{3}{2}}\right)}{a^4} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-sqrt(-a^2\*x^2 + 1)\*x^2/(a\*x - 1)^5,x, algorithm="giac")

[Out] -1/420\*(92\*i\*sign(1/(a\*x - 1))\*sign(a)/a^4 + (140\*(-2/(a\*x - 1) - 1)^(3/2)\*sign(1/(a\*x - 1))\*sign(a) - 28\*(3\*(2/(a\*x - 1) + 1)^2\*sqrt(-2/(a\*x - 1) - 1) + 5\*(-2/(a\*x - 1) - 1)^(3/2))\*sign(1/(a\*x - 1))\*sign(a) - (15\*a^12\*(2/(a\*x - 1) + 1)^3\*sqrt(-2/(a\*x - 1) - 1) - 42\*a^12\*(2/(a\*x - 1) + 1)^2\*sqrt(-2/(a\*x - 1) - 1) - 35\*a^12\*(-2/(a\*x - 1) - 1)^(3/2))\*sign(1/(a\*x - 1))\*sign(a)/a^12)/a^4)\*abs(a)

$$3.212 \quad \int \frac{x^3}{(d+ex)^4(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=209

$$\begin{aligned} & \frac{d^2}{13e^4(d+ex)^4(d^2-e^2x^2)^{5/2}} - \frac{30d}{143e^4(d+ex)^3(d^2-e^2x^2)^{5/2}} \\ & + \frac{21}{143e^4(d+ex)^2(d^2-e^2x^2)^{5/2}} + \frac{4}{1001de^4(d+ex)(d^2-e^2x^2)^{5/2}} \\ & - \frac{64x}{5005d^7e^3\sqrt{d^2-e^2x^2}} - \frac{32x}{5005d^5e^3(d^2-e^2x^2)^{3/2}} - \frac{24x}{5005d^3e^3(d^2-e^2x^2)^{5/2}} \end{aligned}$$

[Out]  $(-24*x)/(5005*d^3*e^3*(d^2 - e^2*x^2)^{(5/2)}) + d^2/(13*e^4*(d + e*x)^4*(d^2 - e^2*x^2)^{(5/2)}) - (30*d)/(143*e^4*(d + e*x)^3*(d^2 - e^2*x^2)^{(5/2)}) + 21/(143*e^4*(d + e*x)^2*(d^2 - e^2*x^2)^{(5/2)}) + 4/(1001*d*e^4*(d + e*x)*(d^2 - e^2*x^2)^{(5/2)}) - (32*x)/(5005*d^5*e^3*(d^2 - e^2*x^2)^{(3/2)}) - (64*x)/(5005*d^7*e^3*\text{Sqrt}[d^2 - e^2*x^2])$

**Rubi [A]** time = 0.571417, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\begin{aligned} & \frac{d^2}{13e^4(d+ex)^4(d^2-e^2x^2)^{5/2}} - \frac{30d}{143e^4(d+ex)^3(d^2-e^2x^2)^{5/2}} \\ & + \frac{21}{143e^4(d+ex)^2(d^2-e^2x^2)^{5/2}} + \frac{4}{1001de^4(d+ex)(d^2-e^2x^2)^{5/2}} \\ & - \frac{64x}{5005d^7e^3\sqrt{d^2-e^2x^2}} - \frac{32x}{5005d^5e^3(d^2-e^2x^2)^{3/2}} - \frac{24x}{5005d^3e^3(d^2-e^2x^2)^{5/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3/((d + e*x)^4*(d^2 - e^2*x^2)^{(7/2)}), x]$

[Out]  $(-24*x)/(5005*d^3*e^3*(d^2 - e^2*x^2)^{(5/2)}) + d^2/(13*e^4*(d + e*x)^4*(d^2 - e^2*x^2)^{(5/2)}) - (30*d)/(143*e^4*(d + e*x)^3*(d^2 - e^2*x^2)^{(5/2)}) + 21/(143*e^4*(d + e*x)^2*(d^2 - e^2*x^2)^{(5/2)}) + 4/(1001*d*e^4*(d + e*x)*(d^2 - e^2*x^2)^{(5/2)}) - (32*x)/(5005*d^5*e^3*(d^2 - e^2*x^2)^{(3/2)}) - (64*x)/(5005*d^7*e^3*\text{Sqrt}[d^2 - e^2*x^2])$

**Rubi in Sympy [A]** time = 73.4009, size = 187, normalized size = 0.89

$$\begin{aligned} & \frac{d^2}{13e^4(d+ex)^4(d^2-e^2x^2)^{5/2}} - \frac{30d}{143e^4(d+ex)^3(d^2-e^2x^2)^{5/2}} \\ & + \frac{21}{143e^4(d+ex)^2(d^2-e^2x^2)^{5/2}} + \frac{4}{1001de^4(d+ex)(d^2-e^2x^2)^{5/2}} \\ & - \frac{64x}{5005d^3e^3(d^2-e^2x^2)^{5/2}} - \frac{32x}{5005d^5e^3(d^2-e^2x^2)^{3/2}} - \frac{24x}{5005d^7e^3\sqrt{d^2-e^2x^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**3/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2), x)$

[Out]  $d**2/(13*e**4*(d + e*x)**4*(d**2 - e**2*x**2)**(5/2)) - 30*d/(143*e**4*(d + e*x)**3*(d**2 - e**2*x**2)**(5/2)) + 21/(143*e**4*(d + e*x)**2*(d**2 - e**2*x**2)**(5/2)) + 4/(1001*d*e**4*(d + e*x)*(d**2 - e**2*x**2)**(5/2)) - 24*x/(5005*d**3*e**3*(d**2 - e**2*x**2)$

$$\left. \right)^{5/2} - 32x / (5005d^5e^3(d^2 - e^2x^2)^{3/2}) - 64x / (5005d^7e^3\sqrt{d^2 - e^2x^2})$$

**Mathematica [A]** time = 0.110981, size = 137, normalized size = 0.66

$$\frac{\sqrt{d^2 - e^2x^2} (90d^9 + 360d^8ex + 315d^7e^2x^2 - 540d^6e^3x^3 + 160d^5e^4x^4 + 776d^4e^5x^5 + 384d^3e^6x^6 - 224d^2e^7x^7 - 256de^8x^8 - 64e^9x^9)}{5005d^7e^4(d - ex)^3(d + ex)^7}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((d + e\*x)^4\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(90\*d^9 + 360\*d^8\*e\*x + 315\*d^7\*e^2\*x^2 - 540\*d^6\*e^3\*x^3 + 160\*d^5\*e^4\*x^4 + 776\*d^4\*e^5\*x^5 + 384\*d^3\*e^6\*x^6 - 224\*d^2\*e^7\*x^7 - 256\*d\*e^8\*x^8 - 64\*e^9\*x^9))/(5005\*d^7\*e^4\*(d - e\*x)^3\*(d + e\*x)^7)

**Maple [A]** time = 0.015, size = 132, normalized size = 0.6

$$\frac{(-ex + d)(-64e^9x^9 - 256e^8x^8d - 224e^7x^7d^2 + 384e^6x^6d^3 + 776e^5x^5d^4 + 160x^4d^5e^4 - 540x^3d^6e^3 + 315x^2d^7e^2 + 360d^8ex - 64e^9x^9)}{5005e^4d^7(ex + d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(e\*x+d)^4/(-e^2\*x^2+d^2)^(7/2), x)

[Out] 1/5005\*(-e\*x+d)\*(-64\*e^9\*x^9-256\*d\*e^8\*x^8-224\*d^2\*e^7\*x^7+384\*d^3\*e^6\*x^6+776\*d^4\*e^5\*x^5+160\*d^5\*e^4\*x^4-540\*d^6\*e^3\*x^3+315\*d^7\*e^2\*x^2+360\*d^8\*e\*x-64\*e^9\*x^9)/(e\*x+d)^3/d^7/e^4/(-e^2\*x^2+d^2)^(7/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/((-e^2\*x^2 + d^2)^(7/2)\*(e\*x + d)^4), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.52139, size = 961, normalized size = 4.6

$$\frac{90e^{14}x^{18} - 216de^{13}x^{17} - 5634d^2e^{12}x^{16} - 9456d^3e^{11}x^{15} + 43716d^4e^{10}x^{14} + 121416d^5e^9x^{13} - 7906d^6e^8x^{12} - 3704d^7e^7x^{11} + 106d^8e^6x^{10} - 168d^9e^5x^9 + 1280d^{10}e^4x^8 + 846d^{11}e^3x^7 + 1280d^{12}e^2x^6 + 43716d^{13}ex^5 + 90d^{14}x^4}{5005(d^7e^{18}x^{18} + 4d^8e^{17}x^{17} - 37d^9e^{16}x^{16} - 168d^{10}e^{15}x^{15} + 106d^{11}e^{14}x^{14} + 1280d^{12}e^{13}x^{13} + 846d^{13}e^{12}x^{12} - 3704d^{14}e^{11}x^{11} + 106d^{15}e^{10}x^{10} - 168d^{16}e^9x^9 + 1280d^{17}e^8x^8 + 43716d^{18}e^7x^7 - 7906d^{19}e^6x^6 - 3704d^{20}e^5x^5 + 106d^{21}e^4x^4 - 168d^{22}e^3x^3 + 1280d^{23}e^2x^2 - 3704d^{24}ex + 90d^{25})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/((-e^2\*x^2 + d^2)^(7/2)\*(e\*x + d)^4), x, algorithm="fricas")

[Out] 1/5005\*(90\*e^14\*x^18 - 216\*d\*e^13\*x^17 - 5634\*d^2\*e^12\*x^16 - 9456\*d^3\*e^11\*x^15 + 43716\*d^4\*e^10\*x^14 + 121416\*d^5\*e^9\*x^13 - 7906\*d^6\*e^8\*x^12 - 3704\*d^7\*e^7\*x^11 + 106\*d^8\*e^6\*x^10 - 168\*d^9\*e^5\*x^9 + 1280\*d^10\*e^4\*x^8 + 846\*d^11\*e^3\*x^7 + 1280\*d^12\*e^2\*x^6 + 43716\*d^13\*e\*x^5 + 90\*d^14\*x^4 - 168\*d^15\*e^3\*x^3 + 1280\*d^16\*e^2\*x^2 - 3704\*d^17\*e\*x + 90\*d^18)



$$\begin{aligned}
& 92*d^6*e^8*x^{12} - 491244*d^7*e^7*x^{11} - 154011*d^8*e^6*x^{10} + 901 \\
& 472*d^9*e^5*x^9 + 692120*d^{10}*e^4*x^8 - 777920*d^{11}*e^3*x^7 - 816 \\
& 816*d^{12}*e^2*x^6 + 256256*d^{13}*e*x^5 + 320320*d^{14}*x^4 + (64*e^{13} \\
& *x^{17} + 1066*d*e^{12}*x^{16} + 904*d^2*e^{11}*x^{15} - 18184*d^3*e^{10}*x^{14} \\
& 4 - 40816*d^4*e^9*x^{13} + 64220*d^5*e^8*x^{12} + 252148*d^6*e^7*x^{11} \\
& + 14157*d^7*e^6*x^{10} - 608608*d^8*e^5*x^9 - 403832*d^9*e^4*x^8 + \\
& 649792*d^{10}*e^3*x^7 + 656656*d^{11}*e^2*x^6 - 256256*d^{12}*e*x^5 - \\
& 320320*d^{13}*x^4)*\sqrt{-e^2*x^2 + d^2})/(d^7*e^{18}*x^{18} + 4*d^8*e^{17} \\
& *x^{17} - 37*d^9*e^{16}*x^{16} - 168*d^{10}*e^{15}*x^{15} + 106*d^{11}*e^{14}*x^{14} \\
& + 1280*d^{12}*e^{13}*x^{13} + 846*d^{13}*e^{12}*x^{12} - 3704*d^{14}*e^{11}*x^{11} \\
& - 5011*d^{15}*e^{10}*x^{10} + 4284*d^{16}*e^9*x^9 + 10519*d^{17}*e^8*x^8 \\
& + 32*d^{18}*e^7*x^7 - 10536*d^{19}*e^6*x^6 - 4544*d^{20}*e^5*x^5 + 468 \\
& 8*d^{21}*e^4*x^4 + 3840*d^{22}*e^3*x^3 - 320*d^{23}*e^2*x^2 - 1024*d^{24} \\
& *e*x - 256*d^{25} + (9*d^8*e^{16}*x^{16} + 36*d^9*e^{15}*x^{15} - 84*d^{10}*e^{14} \\
& *x^{14} - 516*d^{11}*e^{13}*x^{13} - 138*d^{12}*e^{12}*x^{12} + 2172*d^{13}*e^{11} \\
& *x^{11} + 2388*d^{14}*e^{10}*x^{10} - 3516*d^{15}*e^9*x^9 - 6839*d^{16}*e^8 \\
& *x^8 + 1120*d^{17}*e^7*x^7 + 8392*d^{18}*e^6*x^6 + 3008*d^{19}*e^5*x^5 \\
& - 4432*d^{20}*e^4*x^4 - 3328*d^{21}*e^3*x^3 + 448*d^{22}*e^2*x^2 + 1024 \\
& *d^{23}*e*x + 256*d^{24})*\sqrt{-e^2*x^2 + d^2})
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(e\*x+d)\*\*4/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2),x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/((-e^2\*x^2 + d^2)^(7/2)\*(e\*x + d)^4),x, algorithm="giac")

[Out] [undef, undef, undef, undef, undef, undef, undef, 1]

$$3.213 \quad \int \frac{x^2}{(d+ex)^4(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=209

$$\begin{aligned} & -\frac{d}{13e^3(d+ex)^4(d^2-e^2x^2)^{5/2}} + \frac{17}{143e^3(d+ex)^3(d^2-e^2x^2)^{5/2}} \\ & -\frac{1287de^3(d+ex)^2(d^2-e^2x^2)^{5/2}}{112x} - \frac{1287d^2e^3(d+ex)(d^2-e^2x^2)^{5/2}}{56x} \\ & + \frac{112x}{6435d^8e^2\sqrt{d^2-e^2x^2}} + \frac{56x}{6435d^6e^2(d^2-e^2x^2)^{3/2}} + \frac{14x}{2145d^4e^2(d^2-e^2x^2)^{5/2}} \end{aligned}$$

[Out] (14\*x)/(2145\*d^4\*e^2\*(d^2 - e^2\*x^2)^(5/2)) - d/(13\*e^3\*(d + e\*x)^4\*(d^2 - e^2\*x^2)^(5/2)) + 17/(143\*e^3\*(d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2)) - 7/(1287\*d\*e^3\*(d + e\*x)^2\*(d^2 - e^2\*x^2)^(5/2)) - 7/(1287\*d^2\*e^3\*(d + e\*x)\*(d^2 - e^2\*x^2)^(5/2)) + (56\*x)/(6435\*d^6\*e^2\*(d^2 - e^2\*x^2)^(3/2)) + (112\*x)/(6435\*d^8\*e^2\*sqrt[d^2 - e^2\*x^2])

**Rubi [A]** time = 0.415857, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\begin{aligned} & -\frac{d}{13e^3(d+ex)^4(d^2-e^2x^2)^{5/2}} + \frac{17}{143e^3(d+ex)^3(d^2-e^2x^2)^{5/2}} \\ & -\frac{1287de^3(d+ex)^2(d^2-e^2x^2)^{5/2}}{112x} - \frac{1287d^2e^3(d+ex)(d^2-e^2x^2)^{5/2}}{56x} \\ & + \frac{112x}{6435d^8e^2\sqrt{d^2-e^2x^2}} + \frac{56x}{6435d^6e^2(d^2-e^2x^2)^{3/2}} + \frac{14x}{2145d^4e^2(d^2-e^2x^2)^{5/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^2/((d + e\*x)^4\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out] (14\*x)/(2145\*d^4\*e^2\*(d^2 - e^2\*x^2)^(5/2)) - d/(13\*e^3\*(d + e\*x)^4\*(d^2 - e^2\*x^2)^(5/2)) + 17/(143\*e^3\*(d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2)) - 7/(1287\*d\*e^3\*(d + e\*x)^2\*(d^2 - e^2\*x^2)^(5/2)) - 7/(1287\*d^2\*e^3\*(d + e\*x)\*(d^2 - e^2\*x^2)^(5/2)) + (56\*x)/(6435\*d^6\*e^2\*(d^2 - e^2\*x^2)^(3/2)) + (112\*x)/(6435\*d^8\*e^2\*sqrt[d^2 - e^2\*x^2])

**Rubi in Sympy [A]** time = 62.1787, size = 187, normalized size = 0.89

$$\begin{aligned} & -\frac{d}{13e^3(d+ex)^4(d^2-e^2x^2)^{5/2}} + \frac{17}{143e^3(d+ex)^3(d^2-e^2x^2)^{5/2}} \\ & -\frac{1287de^3(d+ex)^2(d^2-e^2x^2)^{5/2}}{14x} - \frac{1287d^2e^3(d+ex)(d^2-e^2x^2)^{5/2}}{56x} \\ & + \frac{112x}{2145d^4e^2(d^2-e^2x^2)^{5/2}} + \frac{56x}{6435d^6e^2(d^2-e^2x^2)^{3/2}} + \frac{14x}{6435d^8e^2\sqrt{d^2-e^2x^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(e\*x+d)\*\*4/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out] -d/(13\*e\*\*3\*(d + e\*x)\*\*4\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)) + 17/(143\*e\*\*3\*(d + e\*x)\*\*3\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)) - 7/(1287\*d\*e\*\*3\*(d + e\*x)\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)) - 7/(1287\*d\*\*2\*e\*\*3\*(d + e\*x)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)) + 14\*x/(2145\*d\*\*4\*e\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)) + 56\*x/(6435\*d\*\*6\*e\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)) + 14\*x/(6435\*d\*\*8\*e\*\*2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2))

$$2)^{(5/2)} + 56x / (6435d^6e^{2(d^2 - e^2x^2)}^{3/2}) + 112x / (6435d^8e^{2\sqrt{d^2 - e^2x^2}})$$

**Mathematica [A]** time = 0.0974611, size = 137, normalized size = 0.66

$$\frac{\sqrt{d^2 - e^2x^2} (200d^9 + 800d^8ex + 700d^7e^2x^2 + 945d^6e^3x^3 - 280d^5e^4x^4 - 1358d^4e^5x^5 - 672d^3e^6x^6 + 392d^2e^7x^7 + 448de^8x^8 + 112e^9x^9)}{6435d^8e^3(d - ex)^3(d + ex)^7}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((d + e\*x)^4\*(d^2 - e^2\*x^2)^(7/2)),x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(200\*d^9 + 800\*d^8\*e\*x + 700\*d^7\*e^2\*x^2 + 945\*d^6\*e^3\*x^3 - 280\*d^5\*e^4\*x^4 - 1358\*d^4\*e^5\*x^5 - 672\*d^3\*e^6\*x^6 + 392\*d^2\*e^7\*x^7 + 448\*d\*e^8\*x^8 + 112\*e^9\*x^9))/(6435\*d^8\*e^3\*(d - e\*x)^3\*(d + e\*x)^7)

**Maple [A]** time = 0.015, size = 132, normalized size = 0.6

$$\frac{(-ex + d) (112 e^9 x^9 + 448 e^8 x^8 d + 392 e^7 x^7 d^2 - 672 e^6 x^6 d^3 - 1358 e^5 x^5 d^4 - 280 e^4 x^4 d^5 + 945 x^3 d^6 e^3 + 700 x^2 d^7 e^2 + 800 x d^8 e + 112 d^9 e)}{6435 e^3 d^8 (ex + d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(e\*x+d)^4/(-e^2\*x^2+d^2)^(7/2),x)

[Out] 1/6435\*(-e\*x+d)\*(112\*e^9\*x^9+448\*d\*e^8\*x^8+392\*d^2\*e^7\*x^7-672\*d^3\*e^6\*x^6-1358\*d^4\*e^5\*x^5-280\*d^5\*e^4\*x^4+945\*d^6\*e^3\*x^3+700\*d^7\*e^2\*x^2+800\*d^8\*e\*x+200\*d^9)/(e\*x+d)^3/d^8/e^3/(-e^2\*x^2+d^2)^(7/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((-e^2\*x^2 + d^2)^(7/2)\*(e\*x + d)^4),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.51384, size = 992, normalized size = 4.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((-e^2\*x^2 + d^2)^(7/2)\*(e\*x + d)^4),x, algorithm="fricas")

[Out] 1/6435\*(200\*e^15\*x^18 + 1808\*d\*e^14\*x^17 - 3368\*d^2\*e^13\*x^16 - 43512\*d^3\*e^12\*x^15 - 38608\*d^4\*e^11\*x^14 + 245122\*d^5\*e^10\*x^13 + 440856\*d^6\*e^9\*x^12 - 464503\*d^7\*e^8\*x^11 - 1510652\*d^8\*e^7\*x^10 + 392\*d^9\*e^6\*x^9 + 280\*d^8\*e^5\*x^8 - 1358\*d^7\*e^4\*x^7 + 672\*d^6\*e^3\*x^6 - 945\*d^5\*e^2\*x^5 + 800\*d^4\*e\*x^4 + 112\*d^3\*x^3 + 112\*d^2\*x^2 + 112\*d\*x + 112)

$$\begin{aligned}
& - 33176*d^9*e^6*x^9 + 2402400*d^10*e^5*x^8 + 1201200*d^11*e^4*x^7 \\
& - 1839552*d^12*e^3*x^6 - 1455168*d^13*e^2*x^5 + 549120*d^14*e*x^4 \\
& + 549120*d^15*x^3 - (112*e^14*x^17 - 1352*d*e^13*x^16 - 11288*d^2*e^12*x^15 \\
& - 1792*d^3*e^11*x^14 + 113042*d^4*e^10*x^13 + 161720*d^5*e^9*x^12 \\
& - 335231*d^6*e^8*x^11 - 827684*d^7*e^7*x^10 + 193336*d^8*e^6*x^9 \\
& + 1688544*d^9*e^5*x^8 + 679536*d^10*e^4*x^7 - 1564992*d^11*e^3*x^6 \\
& - 1180608*d^12*e^2*x^5 + 549120*d^13*e*x^4 + 549120*d^14*x^3) * \sqrt{-e^2*x^2 + d^2}) / (d^8*e^18*x^18 + 4*d^9*e^17*x^17 \\
& - 37*d^10*e^16*x^16 - 168*d^11*e^15*x^15 + 106*d^12*e^14*x^14 + 1280*d^13*e^13*x^13 \\
& + 846*d^14*e^12*x^12 - 3704*d^15*e^11*x^11 - 5011*d^16*e^10*x^10 + 4284*d^17*e^9*x^9 \\
& + 10519*d^18*e^8*x^8 + 32*d^19*e^7*x^7 - 10536*d^20*e^6*x^6 - 4544*d^21*e^5*x^5 + 4688*d^22*e^4*x^4 \\
& + 3840*d^23*e^3*x^3 - 320*d^24*e^2*x^2 - 1024*d^25*e*x - 256*d^26 + (9*d^9*e^16*x^16 + 36*d^10*e^15*x^15 \\
& - 84*d^11*e^14*x^14 - 516*d^12*e^13*x^13 - 138*d^13*e^12*x^12 + 2172*d^14*e^11*x^11 \\
& + 2388*d^15*e^10*x^10 - 3516*d^16*e^9*x^9 - 6839*d^17*e^8*x^8 + 1120*d^18*e^7*x^7 \\
& + 8392*d^19*e^6*x^6 + 3008*d^20*e^5*x^5 - 4432*d^21*e^4*x^4 - 3328*d^22*e^3*x^3 \\
& + 448*d^23*e^2*x^2 + 1024*d^24*e*x + 256*d^25) * \sqrt{-e^2*x^2 + d^2})
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(e\*x+d)\*\*4/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2),x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((-e^2\*x^2 + d^2)^(7/2)\*(e\*x + d)^4),x, algorithm="giac")

[Out] [undef, undef, undef, undef, undef, undef, undef, 1]

$$3.214 \quad \int \frac{x}{(d+ex)^4(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=211

$$\begin{aligned} & -\frac{32}{1287d^2e^2(d+ex)^2(d^2-e^2x^2)^{5/2}} - \frac{4}{143de^2(d+ex)^3(d^2-e^2x^2)^{5/2}} \\ & + \frac{1}{13e^2(d+ex)^4(d^2-e^2x^2)^{5/2}} + \frac{512x}{6435d^9e\sqrt{d^2-e^2x^2}} + \frac{256x}{6435d^7e(d^2-e^2x^2)^{3/2}} \\ & + \frac{64x}{2145d^5e(d^2-e^2x^2)^{5/2}} - \frac{32}{1287d^3e^2(d+ex)(d^2-e^2x^2)^{5/2}} \end{aligned}$$

[Out] (64\*x)/(2145\*d^5\*e\*(d^2 - e^2\*x^2)^(5/2)) + 1/(13\*e^2\*(d + e\*x)^4\*(d^2 - e^2\*x^2)^(5/2)) - 4/(143\*d\*e^2\*(d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2)) - 32/(1287\*d^2\*e^2\*(d + e\*x)^2\*(d^2 - e^2\*x^2)^(5/2)) - 32/(1287\*d^3\*e^2\*(d + e\*x)\*(d^2 - e^2\*x^2)^(5/2)) + (256\*x)/(6435\*d^7\*e\*(d^2 - e^2\*x^2)^(3/2)) + (512\*x)/(6435\*d^9\*e\*sqrt[d^2 - e^2\*x^2])

**Rubi [A]** time = 0.260977, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\begin{aligned} & -\frac{32}{1287d^2e^2(d+ex)^2(d^2-e^2x^2)^{5/2}} - \frac{4}{143de^2(d+ex)^3(d^2-e^2x^2)^{5/2}} \\ & + \frac{1}{13e^2(d+ex)^4(d^2-e^2x^2)^{5/2}} + \frac{512x}{6435d^9e\sqrt{d^2-e^2x^2}} + \frac{256x}{6435d^7e(d^2-e^2x^2)^{3/2}} \\ & + \frac{64x}{2145d^5e(d^2-e^2x^2)^{5/2}} - \frac{32}{1287d^3e^2(d+ex)(d^2-e^2x^2)^{5/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x/((d + e\*x)^4\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out] (64\*x)/(2145\*d^5\*e\*(d^2 - e^2\*x^2)^(5/2)) + 1/(13\*e^2\*(d + e\*x)^4\*(d^2 - e^2\*x^2)^(5/2)) - 4/(143\*d\*e^2\*(d + e\*x)^3\*(d^2 - e^2\*x^2)^(5/2)) - 32/(1287\*d^2\*e^2\*(d + e\*x)^2\*(d^2 - e^2\*x^2)^(5/2)) - 32/(1287\*d^3\*e^2\*(d + e\*x)\*(d^2 - e^2\*x^2)^(5/2)) + (256\*x)/(6435\*d^7\*e\*(d^2 - e^2\*x^2)^(3/2)) + (512\*x)/(6435\*d^9\*e\*sqrt[d^2 - e^2\*x^2])

**Rubi in Sympy [A]** time = 29.7649, size = 185, normalized size = 0.88

$$\begin{aligned} & \frac{1}{13e^2(d+ex)^4(d^2-e^2x^2)^{5/2}} - \frac{4}{143de^2(d+ex)^3(d^2-e^2x^2)^{5/2}} \\ & - \frac{32}{1287d^2e^2(d+ex)^2(d^2-e^2x^2)^{5/2}} - \frac{32}{1287d^3e^2(d+ex)(d^2-e^2x^2)^{5/2}} \\ & + \frac{64x}{2145d^5e(d^2-e^2x^2)^{5/2}} + \frac{256x}{6435d^7e(d^2-e^2x^2)^{3/2}} + \frac{512x}{6435d^9e\sqrt{d^2-e^2x^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x/(e\*x+d)\*\*4/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out] 1/(13\*e\*\*2\*(d + e\*x)\*\*4\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)) - 4/(143\*d\*e\*\*2\*(d + e\*x)\*\*3\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)) - 32/(1287\*d\*\*2\*e\*\*2\*(d + e\*x)\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)) - 32/(1287\*d\*\*3\*e\*\*2\*(d + e\*x)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)) + 64\*x/(2145\*d\*\*5\*e\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(5/2)) + 256\*x/(6435\*d\*\*7\*e\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(3/2)) + 512\*x/(6435\*d\*\*9\*e\*sqrt(d\*\*2 - e\*\*2\*x\*\*2))

$2^{**2}^{**}(5/2)) + 256*x/(6435*d^{**7}*e*(d^{**2} - e^{**2}*x^{**2})^{**}(3/2)) + 51$   
 $2*x/(6435*d^{**9}*e*\text{sqrt}(d^{**2} - e^{**2}*x^{**2}))$

**Mathematica [A]** time = 0.101131, size = 137, normalized size = 0.65

$$\frac{\sqrt{d^2 - e^2 x^2} (-5d^9 - 20d^8 ex + 3200d^7 e^2 x^2 + 4320d^6 e^3 x^3 - 1280d^5 e^4 x^4 - 6208d^4 e^5 x^5 - 3072d^3 e^6 x^6 + 1792d^2 e^7 x^7 + 2048de^8 x^8)}{6435d^9 e^2 (d - ex)^3 (d + ex)^7}$$

Antiderivative was successfully verified.

[In] Integrate[x/((d + e\*x)^4\*(d^2 - e^2\*x^2)^(7/2)),x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(-5\*d^9 - 20\*d^8\*e\*x + 3200\*d^7\*e^2\*x^2 + 4320\*d^6\*e^3\*x^3 - 1280\*d^5\*e^4\*x^4 - 6208\*d^4\*e^5\*x^5 - 3072\*d^3\*e^6\*x^6 + 1792\*d^2\*e^7\*x^7 + 2048\*d\*e^8\*x^8 + 512\*e^9\*x^9))/(6435\*d^9\*e^2\*(d - e\*x)^3\*(d + e\*x)^7)

**Maple [A]** time = 0.014, size = 132, normalized size = 0.6

$$\frac{(-ex + d)(-512e^9x^9 - 2048e^8x^8d - 1792e^7x^7d^2 + 3072e^6x^6d^3 + 6208e^5x^5d^4 + 1280e^4x^4d^5 - 4320e^3x^3d^6 - 3200x^2d^7e^8)}{6435e^2d^9(ex + d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(e\*x+d)^4/(-e^2\*x^2+d^2)^(7/2),x)

[Out] -1/6435\*(-e\*x+d)\*(-512\*e^9\*x^9-2048\*d\*e^8\*x^8-1792\*d^2\*e^7\*x^7+3072\*d^3\*e^6\*x^6+6208\*d^4\*e^5\*x^5+1280\*d^5\*e^4\*x^4-4320\*d^6\*e^3\*x^3-3200\*d^7\*e^2\*x^2+20\*d^8\*e\*x+5\*d^9)/(e\*x+d)^3/d^9/e^2/(-e^2\*x^2+d^2)^(7/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((-e^2\*x^2 + d^2)^(7/2)\*(e\*x + d)^4),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.517365, size = 1021, normalized size = 4.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((-e^2\*x^2 + d^2)^(7/2)\*(e\*x + d)^4),x, algorithm="fricas")

[Out] -1/6435\*(5\*e^16\*x^18 - 4588\*d\*e^15\*x^17 - 18617\*d^2\*e^14\*x^16 + 44472\*d^3\*e^13\*x^15 + 273938\*d^4\*e^12\*x^14 + 56128\*d^5\*e^11\*x^13 - 1237626\*d^6\*e^10\*x^12 - 1281592\*d^7\*e^9\*x^11 + 2299297\*d^8\*e^8\*x^10 - 2299297\*d^9\*e^7\*x^9 + 1281592\*d^10\*e^6\*x^8 - 44472\*d^11\*e^5\*x^7 + 18617\*d^12\*e^4\*x^6 - 4588\*d^13\*e^3\*x^5 + 5\*d^14\*e^2\*x^4 - d^15\*e\*x^3 + d^16)

$$\begin{aligned} & ^{10} + 4122976*d^9*e^7*x^9 - 1304160*d^{10}*e^6*x^8 - 5903040*d^{11}*e \\ & ^5*x^7 - 1386528*d^{12}*e^4*x^6 + 4063488*d^{13}*e^3*x^5 + 2196480*d^ \\ & ^{14}*e^2*x^4 - 1098240*d^{15}*e*x^3 - 823680*d^{16}*x^2 + (512*e^{15}*x^1 \\ & ^7 + 2093*d*e^{14}*x^{16} - 18508*d^2*e^{13}*x^{15} - 85412*d^3*e^{12}*x^{14} \\ & + 42412*d^4*e^{11}*x^{13} + 612430*d^5*e^{10}*x^{12} + 464204*d^6*e^9*x^{11} \\ & - 1588444*d^7*e^8*x^{10} - 2352064*d^8*e^7*x^9 + 1431144*d^9*e^6* \\ & x^8 + 4283136*d^{10}*e^5*x^7 + 597168*d^{11}*e^4*x^6 - 3514368*d^{12}*e \\ & ^3*x^5 - 1784640*d^{13}*e^2*x^4 + 1098240*d^{14}*e*x^3 + 823680*d^{15}* \\ & x^2)*\text{sqrt}(-e^2*x^2 + d^2))/(d^9*e^{18}*x^{18} + 4*d^{10}*e^{17}*x^{17} - 37 \\ & *d^{11}*e^{16}*x^{16} - 168*d^{12}*e^{15}*x^{15} + 106*d^{13}*e^{14}*x^{14} + 1280* \\ & d^{14}*e^{13}*x^{13} + 846*d^{15}*e^{12}*x^{12} - 3704*d^{16}*e^{11}*x^{11} - 5011* \\ & d^{17}*e^{10}*x^{10} + 4284*d^{18}*e^9*x^9 + 10519*d^{19}*e^8*x^8 + 32*d^{20} \\ & *e^7*x^7 - 10536*d^{21}*e^6*x^6 - 4544*d^{22}*e^5*x^5 + 4688*d^{23}*e^4 \\ & *x^4 + 3840*d^{24}*e^3*x^3 - 320*d^{25}*e^2*x^2 - 1024*d^{26}*e*x - 256 \\ & *d^{27} + (9*d^{10}*e^{16}*x^{16} + 36*d^{11}*e^{15}*x^{15} - 84*d^{12}*e^{14}*x^{14} \\ & - 516*d^{13}*e^{13}*x^{13} - 138*d^{14}*e^{12}*x^{12} + 2172*d^{15}*e^{11}*x^{11} \\ & + 2388*d^{16}*e^{10}*x^{10} - 3516*d^{17}*e^9*x^9 - 6839*d^{18}*e^8*x^8 + 1 \\ & 120*d^{19}*e^7*x^7 + 8392*d^{20}*e^6*x^6 + 3008*d^{21}*e^5*x^5 - 4432*d \\ & ^{22}*e^4*x^4 - 3328*d^{23}*e^3*x^3 + 448*d^{24}*e^2*x^2 + 1024*d^{25}*e \\ & x + 256*d^{26})*\text{sqrt}(-e^2*x^2 + d^2)) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e\*x+d)\*\*4/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((-e^2\*x^2 + d^2)^(7/2)\*(e\*x + d)^4), x, algorithm="giac")

[Out] [undef, undef, undef, undef, undef, undef, undef, 1]

$$3.215 \quad \int \frac{1}{(d+ex)^4(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=205

$$\begin{aligned} & -\frac{9}{143d^2e(d+ex)^3(d^2-e^2x^2)^{5/2}} - \frac{1}{13de(d+ex)^4(d^2-e^2x^2)^{5/2}} \\ & + \frac{128x}{715d^{10}\sqrt{d^2-e^2x^2}} + \frac{64x}{715d^8(d^2-e^2x^2)^{3/2}} + \frac{48x}{715d^6(d^2-e^2x^2)^{5/2}} \\ & - \frac{143d^4e(d+ex)(d^2-e^2x^2)^{5/2}}{143d^3e(d+ex)^2(d^2-e^2x^2)^{5/2}} \end{aligned}$$

[Out]  $(48*x)/(715*d^6*(d^2 - e^2*x^2)^{(5/2)}) - 1/(13*d*e*(d + e*x)^4*(d^2 - e^2*x^2)^{(5/2)}) - 9/(143*d^2*e*(d + e*x)^3*(d^2 - e^2*x^2)^{(5/2)}) - 8/(143*d^3*e*(d + e*x)^2*(d^2 - e^2*x^2)^{(5/2)}) - 8/(143*d^4*e*(d + e*x)*(d^2 - e^2*x^2)^{(5/2)}) + (64*x)/(715*d^8*(d^2 - e^2*x^2)^{(3/2)}) + (128*x)/(715*d^{10}*Sqrt[d^2 - e^2*x^2])$

**Rubi [A]** time = 0.217793, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\begin{aligned} & -\frac{9}{143d^2e(d+ex)^3(d^2-e^2x^2)^{5/2}} - \frac{1}{13de(d+ex)^4(d^2-e^2x^2)^{5/2}} \\ & + \frac{128x}{715d^{10}\sqrt{d^2-e^2x^2}} + \frac{64x}{715d^8(d^2-e^2x^2)^{3/2}} + \frac{48x}{715d^6(d^2-e^2x^2)^{5/2}} \\ & - \frac{143d^4e(d+ex)(d^2-e^2x^2)^{5/2}}{143d^3e(d+ex)^2(d^2-e^2x^2)^{5/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)^4\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out]  $(48*x)/(715*d^6*(d^2 - e^2*x^2)^{(5/2)}) - 1/(13*d*e*(d + e*x)^4*(d^2 - e^2*x^2)^{(5/2)}) - 9/(143*d^2*e*(d + e*x)^3*(d^2 - e^2*x^2)^{(5/2)}) - 8/(143*d^3*e*(d + e*x)^2*(d^2 - e^2*x^2)^{(5/2)}) - 8/(143*d^4*e*(d + e*x)*(d^2 - e^2*x^2)^{(5/2)}) + (64*x)/(715*d^8*(d^2 - e^2*x^2)^{(3/2)}) + (128*x)/(715*d^{10}*Sqrt[d^2 - e^2*x^2])$

**Rubi in Sympy [A]** time = 29.2195, size = 177, normalized size = 0.86

$$\begin{aligned} & -\frac{1}{13de(d+ex)^4(d^2-e^2x^2)^{5/2}} - \frac{9}{143d^2e(d+ex)^3(d^2-e^2x^2)^{5/2}} - \frac{8}{143d^3e(d+ex)^2(d^2-e^2x^2)^{5/2}} \\ & - \frac{143d^4e(d+ex)(d^2-e^2x^2)^{5/2}}{143d^3e(d+ex)^2(d^2-e^2x^2)^{5/2}} + \frac{48x}{715d^6(d^2-e^2x^2)^{5/2}} + \frac{64x}{715d^8(d^2-e^2x^2)^{3/2}} + \frac{128x}{715d^{10}\sqrt{d^2-e^2x^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(e\*x+d)\*\*4/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out]  $-1/(13*d*e*(d + e*x)**4*(d**2 - e**2*x**2)**(5/2)) - 9/(143*d**2*e*(d + e*x)**3*(d**2 - e**2*x**2)**(5/2)) - 8/(143*d**3*e*(d + e*x)**2*(d**2 - e**2*x**2)**(5/2)) - 8/(143*d**4*e*(d + e*x)*(d**2 - e**2*x**2)**(5/2)) + 48*x/(715*d**6*(d**2 - e**2*x**2)**(5/2)) + 64*x/(715*d**8*(d**2 - e**2*x**2)**(3/2)) + 128*x/(715*d**10*sqrt(d**2 - e**2*x**2))$



**Mathematica [A]** time = 0.0992181, size = 137, normalized size = 0.67

$$\frac{\sqrt{d^2 - e^2 x^2} (-180d^9 - 5d^8 ex + 800d^7 e^2 x^2 + 1080d^6 e^3 x^3 - 320d^5 e^4 x^4 - 1552d^4 e^5 x^5 - 768d^3 e^6 x^6 + 448d^2 e^7 x^7 + 512de^8 x^8 + 128e^9 x^9)}{715d^{10}e(d - ex)^3(d + ex)^7}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x)^4\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(-180\*d^9 - 5\*d^8\*e\*x + 800\*d^7\*e^2\*x^2 + 1080\*d^6\*e^3\*x^3 - 320\*d^5\*e^4\*x^4 - 1552\*d^4\*e^5\*x^5 - 768\*d^3\*e^6\*x^6 + 448\*d^2\*e^7\*x^7 + 512\*d\*e^8\*x^8 + 128\*e^9\*x^9))/(715\*d^10\*e\*(d - e\*x)^3\*(d + e\*x)^7)

**Maple [A]** time = 0.014, size = 132, normalized size = 0.6

$$\frac{(-ex + d)(-128e^9x^9 - 512e^8x^8d - 448e^7x^7d^2 + 768e^6x^6d^3 + 1552e^5x^5d^4 + 320e^4x^4d^5 - 1080e^3x^3d^6 - 800e^2x^2d^7 + 512exd^8 - 128d^9)}{715ed^{10}(ex + d)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x+d)^4/(-e^2\*x^2+d^2)^(7/2), x)

[Out] -1/715\*(-e\*x+d)\*(-128\*e^9\*x^9-512\*d\*e^8\*x^8-448\*d^2\*e^7\*x^7+768\*d^3\*e^6\*x^6+1552\*d^4\*e^5\*x^5+320\*d^5\*e^4\*x^4-1080\*d^6\*e^3\*x^3-800\*d^7\*e^2\*x^2+5\*d^8\*e\*x+180\*d^9)/(e\*x+d)^3/d^10/e/(-e^2\*x^2+d^2)^(7/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-e^2\*x^2 + d^2)^(7/2)\*(e\*x + d)^4), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.518105, size = 1045, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-e^2\*x^2 + d^2)^(7/2)\*(e\*x + d)^4), x, algorithm="fricas")

[Out] -1/715\*(180\*e^17\*x^18 - 432\*d\*e^16\*x^17 - 11268\*d^2\*e^15\*x^16 - 18912\*d^3\*e^14\*x^15 + 87432\*d^4\*e^13\*x^14 + 242832\*d^5\*e^12\*x^13 - 158184\*d^6\*e^11\*x^12 - 982488\*d^7\*e^10\*x^11 - 320892\*d^8\*e^9\*x^10 + 1796509\*d^9\*e^8\*x^9 + 1555840\*d^10\*e^7\*x^8 - 1470040\*d^11\*e^6\*x^7 - 2251392\*d^12\*e^5\*x^6 + 203632\*d^13\*e^4\*x^5 + 1464320\*d^14\*e^3\*x^4 + 411840\*d^15\*e^2\*x^3 - 366080\*d^16\*e\*x^2 - 183040\*d^17\*x + (128\*e^16\*x^17 + 2132\*d\*e^15\*x^16 + 1808\*d^2\*e^14\*x^15 - 36368\*d^3\*e^13\*x^14 - 81632\*d^4\*e^12\*x^13 + 128440\*d^5\*e^11\*x^12 + 504296\*d^6\*e^10\*x^11 + 29744\*d^7\*e^9\*x^10 - 1216501\*d^8\*e^8\*x^9 - 86

$$4864*d^9*e^7*x^8 + 1270984*d^{10}*e^6*x^7 + 1656512*d^{11}*e^5*x^6 - 340912*d^{12}*e^4*x^5 - 1281280*d^{13}*e^3*x^4 - 320320*d^{14}*e^2*x^3 + 366080*d^{15}*e*x^2 + 183040*d^{16}*x)*\sqrt{-e^2*x^2 + d^2})/(d^{10}*e^{18}*x^{18} + 4*d^{11}*e^{17}*x^{17} - 37*d^{12}*e^{16}*x^{16} - 168*d^{13}*e^{15}*x^{15} + 106*d^{14}*e^{14}*x^{14} + 1280*d^{15}*e^{13}*x^{13} + 846*d^{16}*e^{12}*x^{12} - 3704*d^{17}*e^{11}*x^{11} - 5011*d^{18}*e^{10}*x^{10} + 4284*d^{19}*e^9*x^9 + 10519*d^{20}*e^8*x^8 + 32*d^{21}*e^7*x^7 - 10536*d^{22}*e^6*x^6 - 4544*d^{23}*e^5*x^5 + 4688*d^{24}*e^4*x^4 + 3840*d^{25}*e^3*x^3 - 320*d^{26}*e^2*x^2 - 1024*d^{27}*e*x - 256*d^{28} + (9*d^{11}*e^{16}*x^{16} + 36*d^{12}*e^{15}*x^{15} - 84*d^{13}*e^{14}*x^{14} - 516*d^{14}*e^{13}*x^{13} - 138*d^{15}*e^{12}*x^{12} + 2172*d^{16}*e^{11}*x^{11} + 2388*d^{17}*e^{10}*x^{10} - 3516*d^{18}*e^9*x^9 - 6839*d^{19}*e^8*x^8 + 1120*d^{20}*e^7*x^7 + 8392*d^{21}*e^6*x^6 + 3008*d^{22}*e^5*x^5 - 4432*d^{23}*e^4*x^4 - 3328*d^{24}*e^3*x^3 + 448*d^{25}*e^2*x^2 + 1024*d^{26}*e*x + 256*d^{27})*\sqrt{-e^2*x^2 + d^2}))$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x+d)\*\*4/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-e^2\*x^2 + d^2)^(7/2)\*(e\*x + d)^4), x, algorithm="giac")

[Out] [undef, undef, undef, undef, undef, undef, undef, 1]

$$3.216 \quad \int \frac{1}{x(d+ex)^4(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=234

$$\begin{aligned} & -\frac{4ex}{13d(d^2-e^2x^2)^{11/2}} + \frac{8d(d-ex)}{13(d^2-e^2x^2)^{13/2}} + \frac{819d-1024ex}{819d^{11}\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^{11}} \\ & + \frac{273d-512ex}{819d^9(d^2-e^2x^2)^{3/2}} + \frac{273d-640ex}{1365d^7(d^2-e^2x^2)^{5/2}} + \frac{117d-320ex}{819d^5(d^2-e^2x^2)^{7/2}} + \frac{13d-40ex}{117d^3(d^2-e^2x^2)^{9/2}} \end{aligned}$$

[Out]  $(8*d*(d - e*x))/(13*(d^2 - e^2*x^2)^(13/2)) - (4*e*x)/(13*d*(d^2 - e^2*x^2)^(11/2)) + (13*d - 40*e*x)/(117*d^3*(d^2 - e^2*x^2)^(9/2)) + (117*d - 320*e*x)/(819*d^5*(d^2 - e^2*x^2)^(7/2)) + (273*d - 640*e*x)/(1365*d^7*(d^2 - e^2*x^2)^(5/2)) + (273*d - 512*e*x)/(819*d^9*(d^2 - e^2*x^2)^(3/2)) + (819*d - 1024*e*x)/(819*d^11*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]/d^11$

**Rubi [A]** time = 0.855179, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$

$$\begin{aligned} & -\frac{4ex}{13d(d^2-e^2x^2)^{11/2}} + \frac{8d(d-ex)}{13(d^2-e^2x^2)^{13/2}} + \frac{819d-1024ex}{819d^{11}\sqrt{d^2-e^2x^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^{11}} \\ & + \frac{273d-512ex}{819d^9(d^2-e^2x^2)^{3/2}} + \frac{273d-640ex}{1365d^7(d^2-e^2x^2)^{5/2}} + \frac{117d-320ex}{819d^5(d^2-e^2x^2)^{7/2}} + \frac{13d-40ex}{117d^3(d^2-e^2x^2)^{9/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(d + e\*x)^4\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out]  $(8*d*(d - e*x))/(13*(d^2 - e^2*x^2)^(13/2)) - (4*e*x)/(13*d*(d^2 - e^2*x^2)^(11/2)) + (13*d - 40*e*x)/(117*d^3*(d^2 - e^2*x^2)^(9/2)) + (117*d - 320*e*x)/(819*d^5*(d^2 - e^2*x^2)^(7/2)) + (273*d - 640*e*x)/(1365*d^7*(d^2 - e^2*x^2)^(5/2)) + (273*d - 512*e*x)/(819*d^9*(d^2 - e^2*x^2)^(3/2)) + (819*d - 1024*e*x)/(819*d^11*Sqrt[d^2 - e^2*x^2]) - ArcTanh[Sqrt[d^2 - e^2*x^2]/d]/d^11$

**Rubi in Sympy [A]** time = 87.1559, size = 257, normalized size = 1.1

$$\begin{aligned} & \frac{1}{13d^2(d+ex)^4(d^2-e^2x^2)^{5/2}} + \frac{2}{13d^3(d+ex)^3(d^2-e^2x^2)^{5/2}} + \frac{29}{117d^4(d+ex)^2(d^2-e^2x^2)^{5/2}} \\ & + \frac{320}{819d^5(d+ex)(d^2-e^2x^2)^{5/2}} + \frac{1}{5d^6(d^2-e^2x^2)^{5/2}} - \frac{128ex}{273d^7(d^2-e^2x^2)^{5/2}} + \frac{1}{3d^8(d^2-e^2x^2)^{3/2}} \\ & - \frac{512ex}{819d^9(d^2-e^2x^2)^{3/2}} + \frac{1}{d^{10}\sqrt{d^2-e^2x^2}} - \frac{1024ex}{819d^{11}\sqrt{d^2-e^2x^2}} - \frac{\operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^{11}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(e\*x+d)\*\*4/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out]  $1/(13*d**2*(d + e*x)**4*(d**2 - e**2*x**2)**(5/2)) + 2/(13*d**3*(d + e*x)**3*(d**2 - e**2*x**2)**(5/2)) + 29/(117*d**4*(d + e*x)**2*(d**2 - e**2*x**2)**(5/2)) + 320/(819*d**5*(d + e*x)*(d**2 - e**2*x**2)**(5/2)) + 1/(5*d**6*(d**2 - e**2*x**2)**(5/2)) - 128*e*x/(273*d**7*(d**2 - e**2*x**2)**(5/2)) + 1/(3*d**8*(d**2 - e**2*x**2)**(3/2)) - 512*e*x/(819*d**9*(d**2 - e**2*x**2)**(3/2)) + 1/(d**10*sqrt(d**2 - e**2*x**2)) - 1024*e*x/(819*d**11*sqrt(d**2 - e**2*x**2))$

$*2*x**2)) - \operatorname{atanh}(\sqrt{d**2 - e**2*x**2}/d)/d**11$

**Mathematica [A]** time = 0.177508, size = 161, normalized size = 0.69

$$-4095 \log\left(\sqrt{d^2 - e^2 x^2} + d\right) + \frac{\sqrt{d^2 - e^2 x^2}(9839d^9 + 22976d^8 ex - 4466d^7 e^2 x^2 - 56304d^6 e^3 x^3 - 34156d^5 e^4 x^4 + 40240d^4 e^5 x^5 + 45735d^3 e^6 x^6 - 1540d^2 e^7 x^7 - 16385d e^8 x^8 + 5120e^9 x^9)}{(d - ex)^3(d + ex)^7}$$

$$4095d^{11}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(d + e\*x)^4\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out] ((Sqrt[d^2 - e^2\*x^2]\*(9839\*d^9 + 22976\*d^8\*e\*x - 4466\*d^7\*e^2\*x^2 - 56304\*d^6\*e^3\*x^3 - 34156\*d^5\*e^4\*x^4 + 40240\*d^4\*e^5\*x^5 + 45735\*d^3\*e^6\*x^6 - 1540\*d^2\*e^7\*x^7 - 16385\*d\*e^8\*x^8 - 5120\*e^9\*x^9))/((d - e\*x)^3\*(d + e\*x)^7) + 4095\*Log[x] - 4095\*Log[d + Sqrt[d^2 - e^2\*x^2]])/(4095\*d^11)

**Maple [A]** time = 0.034, size = 385, normalized size = 1.7

$$\frac{1}{13d^2e^4} \left(x + \frac{d}{e}\right)^{-4} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right)\right)^{-\frac{5}{2}}$$

$$+ \frac{2}{13d^3e^3} \left(x + \frac{d}{e}\right)^{-3} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right)\right)^{-\frac{5}{2}}$$

$$+ \frac{29}{117d^4e^2} \left(x + \frac{d}{e}\right)^{-2} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right)\right)^{-\frac{5}{2}}$$

$$+ \frac{320}{819d^5e} \left(x + \frac{d}{e}\right)^{-1} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right)\right)^{-\frac{5}{2}}$$

$$- \frac{128ex}{273d^7} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right)\right)^{-\frac{5}{2}} - \frac{512ex}{819d^9} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right)\right)^{-\frac{3}{2}}$$

$$- \frac{1024ex}{819d^{11}} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2de \left(x + \frac{d}{e}\right)}} + \frac{1}{5d^6} (-e^2x^2 + d^2)^{-\frac{5}{2}} + \frac{1}{3d^8} (-e^2x^2 + d^2)^{-\frac{3}{2}}$$

$$+ \frac{1}{d^{10}} \frac{1}{\sqrt{-e^2x^2 + d^2}} - \frac{1}{d^{10}} \ln\left(\frac{1}{x} \left(2d^2 + 2\sqrt{d^2}\sqrt{-e^2x^2 + d^2}\right)\right) \frac{1}{\sqrt{d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(e\*x+d)^4/(-e^2\*x^2+d^2)^(7/2), x)

[Out] 1/13/d^2/e^4/(x+d/e)^4/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(5/2)+2/13/d^3/e^3/(x+d/e)^3/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(5/2)+29/117/d^4/e^2/(x+d/e)^2/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(5/2)+320/819/d^5/e/(x+d/e)/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(5/2)-128/273/d^7\*e/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(5/2)-512/819/d^9\*e/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(3/2)-1024/819/d^11\*e/(-(x+d/e)^2\*e^2+2\*d\*e\*(x+d/e))^(1/2)+1/5/d^6/(-e^2\*x^2+d^2)^(5/2)+1/3/d^8/(-e^2\*x^2+d^2)^(3/2)+1/d^10/(-e^2\*x^2+d^2)^(1/2)-1/d^10/(d^2)^(1/2)\*ln((2\*d^2+2\*(d^2)\*sqrt(-e^2\*x^2+d^2))/x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-e^2x^2 + d^2)^{\frac{7}{2}}(ex + d)^4x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4*x),x, algorithm="maxima")`

[Out] `integrate(1/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4*x), x)`

**Fricas [A]** time = 0.655524, size = 1608, normalized size = 6.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4*x),x, algorithm="fricas")`

[Out] 
$$\begin{aligned} & 1/4095*(9839*e^{18}*x^{18} - 6724*d*e^{17}*x^{17} - 511508*d^2*e^{16}*x^{16} \\ & - 1052412*d^3*e^{15}*x^{15} + 3420749*d^4*e^{14}*x^{14} + 10929040*d^5*e^{13}*x^{13} - 4550130*d^6*e^{12}*x^{12} - 39495352*d^7*e^{11}*x^{11} - 160494 \\ & 23*d^8*e^{10}*x^{10} + 66073540*d^9*e^9*x^9 + 58827600*d^{10}*e^8*x^8 - 50338080*d^{11}*e^7*x^7 - 75391680*d^{12}*e^6*x^6 + 7949760*d^{13}*e^5 \\ & *x^5 + 44204160*d^{14}*e^4*x^4 + 10133760*d^{15}*e^3*x^3 - 9959040*d^{16}*e^2*x^2 - 4193280*d^{17}*e*x + 4095*(e^{18}*x^{18} + 4*d*e^{17}*x^{17} - \\ & 37*d^2*e^{16}*x^{16} - 168*d^3*e^{15}*x^{15} + 106*d^4*e^{14}*x^{14} + 1280*d^5*e^{13}*x^{13} + 846*d^6*e^{12}*x^{12} - 3704*d^7*e^{11}*x^{11} - 5011*d^8 \\ & *e^{10}*x^{10} + 4284*d^9*e^9*x^9 + 10519*d^{10}*e^8*x^8 + 32*d^{11}*e^7*x^7 - 10536*d^{12}*e^6*x^6 - 4544*d^{13}*e^5*x^5 + 4688*d^{14}*e^4*x^4 \\ & + 3840*d^{15}*e^3*x^3 - 320*d^{16}*e^2*x^2 - 1024*d^{17}*e*x - 256*d^{18} \\ & + (9*d*e^{16}*x^{16} + 36*d^2*e^{15}*x^{15} - 84*d^3*e^{14}*x^{14} - 516*d^4 \\ & *e^{13}*x^{13} - 138*d^5*e^{12}*x^{12} + 2172*d^6*e^{11}*x^{11} + 2388*d^7*e^{10}*x^{10} - 3516*d^8*e^9*x^9 - 6839*d^9*e^8*x^8 + 1120*d^{10}*e^7*x^7 \\ & + 8392*d^{11}*e^6*x^6 + 3008*d^{12}*e^5*x^5 - 4432*d^{13}*e^4*x^4 - 33 \\ & 28*d^{14}*e^3*x^3 + 448*d^{15}*e^2*x^2 + 1024*d^{16}*e*x + 256*d^{17})*sqrt(-e^2 \\ & *x^2 + d^2))*log(-(d - sqrt(-e^2*x^2 + d^2))/x) + (5120*e^{17}*x^{17} + 104936*d*e^{16}*x^{16} + 150944*d^2*e^{15}*x^{15} - 1527611*d^3 \\ & *e^{14}*x^{14} - 3949964*d^4*e^{13}*x^{13} + 4438174*d^5*e^{12}*x^{12} + 2111 \\ & 2052*d^6*e^{11}*x^{11} + 3816878*d^7*e^{10}*x^{10} - 45905860*d^8*e^9*x^9 \\ & - 34596120*d^9*e^8*x^8 + 43873440*d^{10}*e^7*x^7 + 57024240*d^{11}*e^6*x^6 - 11444160*d^{12}*e^5*x^5 - 39224640*d^{13}*e^4*x^4 - 8037120* \\ & d^{14}*e^3*x^3 + 9959040*d^{15}*e^2*x^2 + 4193280*d^{16}*e*x)*sqrt(-e^2 \\ & *x^2 + d^2))/(d^{11}*e^{18}*x^{18} + 4*d^{12}*e^{17}*x^{17} - 37*d^{13}*e^{16}*x^{16} - 168*d^{14}*e^{15}*x^{15} + 106*d^{15}*e^{14}*x^{14} + 1280*d^{16}*e^{13}*x^{13} \\ & + 846*d^{17}*e^{12}*x^{12} - 3704*d^{18}*e^{11}*x^{11} - 5011*d^{19}*e^{10}*x^{10} + 4284*d^{20}*e^9*x^9 + 10519*d^{21}*e^8*x^8 + 32*d^{22}*e^7*x^7 - 10 \\ & 536*d^{23}*e^6*x^6 - 4544*d^{24}*e^5*x^5 + 4688*d^{25}*e^4*x^4 + 3840*d^{26}*e^3*x^3 - 320*d^{27}*e^2*x^2 - 1024*d^{28}*e*x - 256*d^{29} + (9*d^{12}*e^{16}*x^{16} + 36*d^{13}*e^{15}*x^{15} - 84*d^{14}*e^{14}*x^{14} - 516*d^{15}*e^{13}*x^{13} - 138*d^{16}*e^{12}*x^{12} + 2172*d^{17}*e^{11}*x^{11} + 2388*d^{18}*e^{10}*x^{10} - 3516*d^{19}*e^9*x^9 - 6839*d^{20}*e^8*x^8 + 1120*d^{21}*e^7*x^7 + 8392*d^{22}*e^6*x^6 + 3008*d^{23}*e^5*x^5 - 4432*d^{24}*e^4*x^4 - 3328*d^{25}*e^3*x^3 + 448*d^{26}*e^2*x^2 + 1024*d^{27}*e*x + 256*d^{28})*sqrt(-e^2*x^2 + d^2)) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)`

[Out] Timed out

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4*x),x, algorithm="giac")`

[Out] [undef, undef, undef, undef, undef, undef, undef, 1]

$$3.217 \quad \int \frac{1}{x^2(d+ex)^4(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=271

$$\begin{aligned} & -\frac{4e(13d-24ex)}{143d^2(d^2-e^2x^2)^{11/2}} - \frac{8e(d-ex)}{13(d^2-e^2x^2)^{13/2}} - \frac{e(36036d-52175ex)}{9009d^{12}\sqrt{d^2-e^2x^2}} \\ & - \frac{\sqrt{d^2-e^2x^2}}{d^{12}x} + \frac{4e \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^{12}} - \frac{e(12012d-21583ex)}{9009d^{10}(d^2-e^2x^2)^{3/2}} \\ & - \frac{e(12012d-23225ex)}{15015d^8(d^2-e^2x^2)^{5/2}} - \frac{e(5148d-10111ex)}{9009d^6(d^2-e^2x^2)^{7/2}} - \frac{e(572d-1103ex)}{1287d^4(d^2-e^2x^2)^{9/2}} \end{aligned}$$

[Out]  $(-8*e*(d - e*x))/(13*(d^2 - e^2*x^2)^{(13/2)}) - (4*e*(13*d - 24*e*x))/(143*d^2*(d^2 - e^2*x^2)^{(11/2)}) - (e*(572*d - 1103*e*x))/(1287*d^4*(d^2 - e^2*x^2)^{(9/2)}) - (e*(5148*d - 10111*e*x))/(9009*d^6*(d^2 - e^2*x^2)^{(7/2)}) - (e*(12012*d - 23225*e*x))/(15015*d^8*(d^2 - e^2*x^2)^{(5/2)}) - (e*(12012*d - 21583*e*x))/(9009*d^{10}*(d^2 - e^2*x^2)^{(3/2)}) - (e*(36036*d - 52175*e*x))/(9009*d^{12}*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(d^{12}*x) + (4*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^{12}$

**Rubi [A]** time = 1.03711, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\begin{aligned} & -\frac{4e(13d-24ex)}{143d^2(d^2-e^2x^2)^{11/2}} - \frac{8e(d-ex)}{13(d^2-e^2x^2)^{13/2}} - \frac{e(36036d-52175ex)}{9009d^{12}\sqrt{d^2-e^2x^2}} \\ & - \frac{\sqrt{d^2-e^2x^2}}{d^{12}x} + \frac{4e \tanh^{-1}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^{12}} - \frac{e(12012d-21583ex)}{9009d^{10}(d^2-e^2x^2)^{3/2}} \\ & - \frac{e(12012d-23225ex)}{15015d^8(d^2-e^2x^2)^{5/2}} - \frac{e(5148d-10111ex)}{9009d^6(d^2-e^2x^2)^{7/2}} - \frac{e(572d-1103ex)}{1287d^4(d^2-e^2x^2)^{9/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(d + e\*x)^4\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out]  $(-8*e*(d - e*x))/(13*(d^2 - e^2*x^2)^{(13/2)}) - (4*e*(13*d - 24*e*x))/(143*d^2*(d^2 - e^2*x^2)^{(11/2)}) - (e*(572*d - 1103*e*x))/(1287*d^4*(d^2 - e^2*x^2)^{(9/2)}) - (e*(5148*d - 10111*e*x))/(9009*d^6*(d^2 - e^2*x^2)^{(7/2)}) - (e*(12012*d - 23225*e*x))/(15015*d^8*(d^2 - e^2*x^2)^{(5/2)}) - (e*(12012*d - 21583*e*x))/(9009*d^{10}*(d^2 - e^2*x^2)^{(3/2)}) - (e*(36036*d - 52175*e*x))/(9009*d^{12}*Sqrt[d^2 - e^2*x^2]) - Sqrt[d^2 - e^2*x^2]/(d^{12}*x) + (4*e*ArcTanh[Sqrt[d^2 - e^2*x^2]/d])/d^{12}$

**Rubi in Sympy [A]** time = 117.476, size = 296, normalized size = 1.09

$$\begin{aligned} & -\frac{e}{13d^3(d+ex)^4(d^2-e^2x^2)^{\frac{5}{2}}} - \frac{35e}{143d^4(d+ex)^3(d^2-e^2x^2)^{\frac{5}{2}}} - \frac{709e}{1287d^5(d+ex)^2(d^2-e^2x^2)^{\frac{5}{2}}} \\ & - \frac{10111e}{9009d^6(d+ex)(d^2-e^2x^2)^{\frac{5}{2}}} - \frac{1}{d^6x(d^2-e^2x^2)^{\frac{5}{2}}} - \frac{5d^7(d^2-e^2x^2)^{\frac{5}{2}}}{4e} \\ & + \frac{7648e^2x}{3003d^8(d^2-e^2x^2)^{\frac{5}{2}}} - \frac{4e}{3d^9(d^2-e^2x^2)^{\frac{3}{2}}} + \frac{30592e^2x}{9009d^{10}(d^2-e^2x^2)^{\frac{3}{2}}} \\ & - \frac{4e}{d^{11}\sqrt{d^2-e^2x^2}} + \frac{61184e^2x}{9009d^{12}\sqrt{d^2-e^2x^2}} + \frac{4e \operatorname{atanh}\left(\frac{\sqrt{d^2-e^2x^2}}{d}\right)}{d^{12}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x**2/(e*x+d)**4/(-e**2*x**2+d**2)**(7/2),x)`

[Out] 
$$-e/(13*d**3*(d+e*x)**4*(d**2-e**2*x**2)**(5/2)) - 35*e/(143*d**4*(d+e*x)**3*(d**2-e**2*x**2)**(5/2)) - 709*e/(1287*d**5*(d+e*x)**2*(d**2-e**2*x**2)**(5/2)) - 10111*e/(9009*d**6*(d+e*x)*(d**2-e**2*x**2)**(5/2)) - 1/(d**6*x*(d**2-e**2*x**2)**(5/2)) - 4*e/(5*d**7*(d**2-e**2*x**2)**(5/2)) + 7648*e**2*x/(3003*d**8*(d**2-e**2*x**2)**(5/2)) - 4*e/(3*d**9*(d**2-e**2*x**2)**(3/2)) + 30592*e**2*x/(9009*d**10*(d**2-e**2*x**2)**(3/2)) - 4*e/(d**11*sqrt(d**2-e**2*x**2)) + 61184*e**2*x/(9009*d**12*sqrt(d**2-e**2*x**2)) + 4*e*atanh(sqrt(d**2-e**2*x**2)/d)/d**12$$

**Mathematica [A]** time = 0.180735, size = 183, normalized size = 0.68

$$-\frac{4e \log(x)}{d^{12}} + \frac{4e \log(\sqrt{d^2 - e^2 x^2} + d)}{d^{12}} + \frac{\sqrt{d^2 - e^2 x^2} (45045 d^{10} + 546316 d^9 e x + 1014094 d^8 e^2 x^2 - 700504 d^7 e^3 x^3 - 3157776 d^6 e^4 x^4 - 1301264 d^5 e^5 x^5 + 2748320 d^4 e^6 x^6 - 1043500 d^3 e^7 x^7 + 2496180 d^2 e^8 x^8 - 1043500 d e^9 x^9 - 305920 e^{10} x^{10})}{45045 d^{12} x (e x - d)^3 (d + e x)^7}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x^2*(d+e*x)^4*(d^2-e^2*x^2)^(7/2)),x]`

[Out] 
$$(\text{Sqrt}[d^2 - e^2 x^2] * (45045 d^{10} + 546316 d^9 e x + 1014094 d^8 e^2 x^2 - 700504 d^7 e^3 x^3 - 3157776 d^6 e^4 x^4 - 1301264 d^5 e^5 x^5 + 2748320 d^4 e^6 x^6 + 2496180 d^3 e^7 x^7 - 350000 d^2 e^8 x^8 - 1043500 d e^9 x^9 - 305920 e^{10} x^{10})) / (45045 d^{12} x * (-d + e x)^3 * (d + e x)^7) - (4 e * \text{Log}[x]) / d^{12} + (4 e * \text{Log}[d + \text{Sqrt}[d^2 - e^2 x^2]]) / d^{12}$$

**Maple [B]** time = 0.021, size = 484, normalized size = 1.8

$$\begin{aligned} & -\frac{1}{13 d^3 e^3} \left(x + \frac{d}{e}\right)^{-4} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)\right)^{-\frac{5}{2}} \\ & -\frac{35}{143 d^4 e^2} \left(x + \frac{d}{e}\right)^{-3} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)\right)^{-\frac{5}{2}} \\ & -\frac{709}{1287 d^5 e} \left(x + \frac{d}{e}\right)^{-2} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)\right)^{-\frac{5}{2}} \\ & +\frac{20222 e^2 x}{15015 d^8} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)\right)^{-\frac{5}{2}} \\ & +\frac{80888 e^2 x}{45045 d^{10}} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)\right)^{-\frac{3}{2}} \\ & +\frac{161776 e^2 x}{45045 d^{12}} \frac{1}{\sqrt{-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)}} + \frac{6 e^2 x}{5 d^8} (-e^2 x^2 + d^2)^{-\frac{5}{2}} \\ & +\frac{8 e^2 x}{5 d^{10}} (-e^2 x^2 + d^2)^{-\frac{3}{2}} + \frac{16 e^2 x}{5 d^{12}} \frac{1}{\sqrt{-e^2 x^2 + d^2}} + 4 \frac{e}{d^{11} \sqrt{d^2}} \ln\left(\frac{2 d^2 + 2 \sqrt{d^2} \sqrt{-e^2 x^2 + d^2}}{x}\right) \\ & -\frac{10111}{9009 d^6} \left(x + \frac{d}{e}\right)^{-1} \left(-\left(x + \frac{d}{e}\right)^2 e^2 + 2 d e \left(x + \frac{d}{e}\right)\right)^{-\frac{5}{2}} - \frac{1}{d^6 x} (-e^2 x^2 + d^2)^{-\frac{5}{2}} \\ & -\frac{4 e}{5 d^7} (-e^2 x^2 + d^2)^{-\frac{5}{2}} - \frac{4 e}{3 d^9} (-e^2 x^2 + d^2)^{-\frac{3}{2}} - 4 \frac{e}{d^{11} \sqrt{-e^2 x^2 + d^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int(1/x^2/(e*x+d)^4/(-e^2*x^2+d^2)^(7/2),x)`

[Out] 
$$\begin{aligned} & -1/13/d^3/e^3/(x+d/e)^4/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^{5/2} - 35/143/d^4/e^2/(x+d/e)^3/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^{5/2} - 709/1287/d^5/e/(x+d/e)^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^{5/2} + 20222/15015/d^8*e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^{5/2} * x + 80888/45045/d^{10} * e^2/(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^{3/2} * x + 161776/45045/d^{12} * e^2 /(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^{1/2} * x + 6/5/d^8 * e^2 * x /(-e^2*x^2+d^2)^{5/2} + 8/5/d^{10} * e^2 * x /(-e^2*x^2+d^2)^{3/2} + 16/5/d^{12} * e^2 * x /(-e^2*x^2+d^2)^{1/2} + 4/d^{11} * e / (d^2)^{1/2} * \ln((2*d^2+2*(d^2)^{1/2}) * (-e^2*x^2+d^2)^{1/2}) / x - 10111/9009/d^6/(x+d/e) /(-(x+d/e)^2*e^2+2*d*e*(x+d/e))^{5/2} - 1/d^6/x /(-e^2*x^2+d^2)^{5/2} - 4/5/d^7 * e /(-e^2*x^2+d^2)^{5/2} - 4/3/d^9 * e /(-e^2*x^2+d^2)^{3/2} - 4/d^{11} * e /(-e^2*x^2+d^2)^{1/2} \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(-e^2x^2 + d^2)^{\frac{7}{2}}(ex + d)^4x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4*x^2),x, algorithm="maxima")`

[Out] `integrate(1/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4*x^2), x)`

**Fricas [A]** time = 0.947354, size = 1769, normalized size = 6.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^4*x^2),x, algorithm="fricas")`

[Out] 
$$\begin{aligned} & -1/45045 * (305920 * e^{20} * x^{20} + 4704860 * d * e^{19} * x^{19} - 300560 * d^2 * e^{18} * x^{18} - 102268860 * d^3 * e^{17} * x^{17} - 161498240 * d^4 * e^{16} * x^{16} + 562549336 * d^5 * e^{15} * x^{15} + 1390771024 * d^6 * e^{14} * x^{14} - 996944104 * d^7 * e^{13} * x^{13} - 4697402606 * d^8 * e^{12} * x^{12} - 596708684 * d^9 * e^{11} * x^{11} + 7949190535 * d^{10} * e^{10} * x^{10} + 4548721320 * d^{11} * e^9 * x^9 - 6934162950 * d^{12} * e^8 * x^8 - 6612365760 * d^{13} * e^7 * x^7 + 2583300720 * d^{14} * e^6 * x^6 + 4345461120 * d^{15} * e^5 * x^5 + 135014880 * d^{16} * e^4 * x^4 - 1245404160 * d^{17} * e^3 * x^3 - 288288000 * d^{18} * e^2 * x^2 + 92252160 * d^{19} * e * x + 23063040 * d^{20} + 180180 * (10 * d * e^{19} * x^{19} + 40 * d^2 * e^{18} * x^{18} - 130 * d^3 * e^{17} * x^{17} - 720 * d^4 * e^{16} * x^{16} + 52 * d^5 * e^{15} * x^{15} + 3968 * d^6 * e^{14} * x^{14} + 3372 * d^7 * e^{13} * x^{13} - 9392 * d^8 * e^{12} * x^{12} - 14238 * d^9 * e^{11} * x^{11} + 8920 * d^{10} * e^{10} * x^{10} + 25750 * d^{11} * e^9 * x^9 + 1920 * d^{12} * e^8 * x^8 - 23360 * d^{13} * e^7 * x^7 - 10880 * d^{14} * e^6 * x^6 + 9568 * d^{15} * e^5 * x^5 + 8192 * d^{16} * e^4 * x^4 - 512 * d^{17} * e^3 * x^3 - 2048 * d^{18} * e^2 * x^2 - 512 * d^{19} * e * x - (e^{19} * x^{19} + 4 * d * e^{18} * x^{18} - 46 * d^2 * e^{17} * x^{17} - 204 * d^3 * e^{16} * x^{16} + 190 * d^4 * e^{15} * x^{15} + 1796 * d^5 * e^{14} * x^{14} + 984 * d^6 * e^{13} * x^{13} - 5876 * d^7 * e^{12} * x^{12} - 7399 * d^8 * e^{11} * x^{11} + 7800 * d^9 * e^{10} * x^{10} + 17358 * d^{10} * e^9 * x^9 - 1088 * d^{11} * e^8 * x^8 - 18928 * d^{12} * e^7 * x^7 - 7552 * d^{13} * e^6 * x^6 + 9120 * d^{14} * e^5 * x^5 + 7168 * d^{15} * e^4 * x^4 - 768 * d^{16} * e^3 * x^3 - 2048 * d^{17} * e^2 * x^2 - 512 * d^{18} * e * x) * \sqrt{-e^2 * x^2 + d^2} * \log(-(d - \sqrt{-e^2 * x^2 + d^2}) / x) - 2 * (183068 * e^{19} * x^{19} - 797328 * d * e^{18} * x^{18} - 13638628 * d^2 * e^{17} * x^{17} - 14622272 * d^3 * e^{16} * x^{16} + 130743820 * d^4 * e^{15} * x^{15} + 267742608 * d^5 * e^{14} * x^{14} - 376677808 * d^6 * e^{13} * x^{13} - 1272331008 * d^7 * e^{12} * x^{12} + 119066948 * d^8 * e^{11} * x^{11} + 2709942950 * d^9 * e^{10} * x^{10} + 1254061380 * d^{10} * e^9 * x^9 - 2837832855 * d^{11} * e^8 * x^8 - 2438916480 * d^{12} * e^7 * x^7 + 1274953680 * d^{13} * e^6 * x^6 + 1878676800 * d^{14} * e^5 * x^5 - 240240 * d^{15} * e^4 * x^4 - 599639040 * d^{16} * e^3 * x^3 - 138378240 * d^{17} * e^2 * x^2 + 46126080 * d^{18} * e * x + 115315 \end{aligned}$$

$$20*d^{19}*sqrt(-e^2*x^2 + d^2))/(10*d^{13}*e^{18}*x^{19} + 40*d^{14}*e^{17}*x^{18} - 130*d^{15}*e^{16}*x^{17} - 720*d^{16}*e^{15}*x^{16} + 52*d^{17}*e^{14}*x^{15} + 3968*d^{18}*e^{13}*x^{14} + 3372*d^{19}*e^{12}*x^{13} - 9392*d^{20}*e^{11}*x^{12} - 14238*d^{21}*e^{10}*x^{11} + 8920*d^{22}*e^9*x^{10} + 25750*d^{23}*e^8*x^9 + 1920*d^{24}*e^7*x^8 - 23360*d^{25}*e^6*x^7 - 10880*d^{26}*e^5*x^6 + 9568*d^{27}*e^4*x^5 + 8192*d^{28}*e^3*x^4 - 512*d^{29}*e^2*x^3 - 2048*d^{30}*e*x^2 - 512*d^{31}*x - (d^{12}*e^{18}*x^{19} + 4*d^{13}*e^{17}*x^{18} - 46*d^{14}*e^{16}*x^{17} - 204*d^{15}*e^{15}*x^{16} + 190*d^{16}*e^{14}*x^{15} + 1796*d^{17}*e^{13}*x^{14} + 984*d^{18}*e^{12}*x^{13} - 5876*d^{19}*e^{11}*x^{12} - 7399*d^{20}*e^{10}*x^{11} + 7800*d^{21}*e^9*x^{10} + 17358*d^{22}*e^8*x^9 - 1088*d^{23}*e^7*x^8 - 18928*d^{24}*e^6*x^7 - 7552*d^{25}*e^5*x^6 + 9120*d^{26}*e^4*x^5 + 7168*d^{27}*e^3*x^4 - 768*d^{28}*e^2*x^3 - 2048*d^{29}*e*x^2 - 512*d^{30}*x)*sqrt(-e^2*x^2 + d^2))$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(e\*x+d)\*\*4/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2),x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((-e^2\*x^2 + d^2)^(7/2)\*(e\*x + d)^4\*x^2),x, algorithm="giac")

[Out] [undef, undef, undef, undef, undef, undef, undef, 1]

$$3.218 \quad \int \frac{\sqrt{c-acx}\sqrt{1-a^2x^2}}{x^2} dx$$

**Optimal.** Leaf size=102

$$-\frac{c^2(1-a^2x^2)^{3/2}}{x(c-acx)^{3/2}} - \frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}} + a\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

[Out] -((a\*c\*Sqrt[1 - a^2\*x^2])/Sqrt[c - a\*c\*x]) - (c^2\*(1 - a^2\*x^2)^(3/2))/(x\*(c - a\*c\*x)^(3/2)) + a\*Sqrt[c]\*ArcTanh[(Sqrt[c]\*Sqrt[1 - a^2\*x^2])/Sqrt[c - a\*c\*x]]

**Rubi [A]** time = 0.374969, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$

$$-\frac{c^2(1-a^2x^2)^{3/2}}{x(c-acx)^{3/2}} - \frac{ac\sqrt{1-a^2x^2}}{\sqrt{c-acx}} + a\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-acx}}\right)$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[c - a\*c\*x]\*Sqrt[1 - a^2\*x^2])/x^2, x]

[Out] -((a\*c\*Sqrt[1 - a^2\*x^2])/Sqrt[c - a\*c\*x]) - (c^2\*(1 - a^2\*x^2)^(3/2))/(x\*(c - a\*c\*x)^(3/2)) + a\*Sqrt[c]\*ArcTanh[(Sqrt[c]\*Sqrt[1 - a^2\*x^2])/Sqrt[c - a\*c\*x]]

**Rubi in Sympy [A]** time = 22.9082, size = 87, normalized size = 0.85

$$a\sqrt{c} \operatorname{atanh}\left(\frac{\sqrt{c}\sqrt{-a^2x^2+1}}{\sqrt{-acx+c}}\right) - \frac{ac\sqrt{-a^2x^2+1}}{\sqrt{-acx+c}} - \frac{c^2(-a^2x^2+1)^{3/2}}{x(-acx+c)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-a\*c\*x+c)\*\*(1/2)\*(-a\*\*2\*x\*\*2+1)\*\*(1/2)/x\*\*2, x)

[Out] a\*sqrt(c)\*atanh(sqrt(c)\*sqrt(-a\*\*2\*x\*\*2+1)/sqrt(-a\*c\*x+c)) - a\*c\*sqrt(-a\*\*2\*x\*\*2+1)/sqrt(-a\*c\*x+c) - c\*\*2\*(-a\*\*2\*x\*\*2+1)\*\*(3/2)/(x\*(-a\*c\*x+c)\*\*(3/2))

**Mathematica [A]** time = 0.173624, size = 92, normalized size = 0.9

$$\sqrt{1-a^2x^2} \left( \frac{3a}{ax-1} - \frac{1}{x} \right) \sqrt{-c(ax-1)} - a\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{1-a^2x^2}\sqrt{-c(ax-1)}}{\sqrt{c(ax-1)}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[c - a\*c\*x]\*Sqrt[1 - a^2\*x^2])/x^2, x]

[Out] Sqrt[-(c\*(-1 + a\*x))]\*Sqrt[1 - a^2\*x^2]\*(-x^(-1) + (3\*a)/(-1 + a\*x)) - a\*Sqrt[c]\*ArcTanh[(Sqrt[-(c\*(-1 + a\*x))]\*Sqrt[1 - a^2\*x^2])/Sqrt[c]\*(-1 + a\*x)]

**Maple [A]** time = 0.04, size = 95, normalized size = 0.9

$$\frac{1}{(ax-1)x} \left( -\operatorname{Artanh} \left( 1\sqrt{c(ax+1)} \frac{1}{\sqrt{c}} \right) xac + 2xa\sqrt{c(ax+1)}\sqrt{c} + \sqrt{c(ax+1)}\sqrt{c} \right) \sqrt{-c(ax-1)}\sqrt{-a^2x^2+1} \frac{1}{\sqrt{c(ax+1)}} \frac{1}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a\*c\*x+c)^(1/2)\*(-a^2\*x^2+1)^(1/2)/x^2,x)

[Out] (-arctanh((c\*(a\*x+1))^(1/2)/c^(1/2))\*x\*a\*c+2\*x\*a\*(c\*(a\*x+1))^(1/2)\*c^(1/2)+(c\*(a\*x+1))^(1/2)\*c^(1/2))\*(-c\*(a\*x-1))^(1/2)\*(-a^2\*x^2+1)^(1/2)/(a\*x-1)/(c\*(a\*x+1))^(1/2)/x/c^(1/2)

**Maxima [A]** time = 0.796273, size = 72, normalized size = 0.71

$$-\frac{1}{2}a\sqrt{c} \left( 4\sqrt{ax+1} + \frac{2\sqrt{ax+1}}{ax} - \log(\sqrt{ax+1}+1) + \log(\sqrt{ax+1}-1) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-a^2\*x^2+1)\*sqrt(-a\*c\*x+c)/x^2,x, algorithm="maxima")

[Out] -1/2\*a\*sqrt(c)\*(4\*sqrt(a\*x+1)+2\*sqrt(a\*x+1)/(a\*x)-log(sqrt(a\*x+1)+1)+log(sqrt(a\*x+1)-1))

**Fricas [A]** time = 0.294326, size = 1, normalized size = 0.01

$$\left[ \frac{4a^3cx^3 + 2a^2cx^2 + \sqrt{-a^2x^2+1}\sqrt{-acx+ca}\sqrt{cx} \log\left(-\frac{a^2cx^2+acx-2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c-2c}}{ax^2-x}\right) - 4acx - 2c}{2\sqrt{-a^2x^2+1}\sqrt{-acx+cx}}, \frac{2a^3cx^3 + a^2cx^2 - 4acx - 2c}{2\sqrt{-a^2x^2+1}\sqrt{-acx+cx}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-a^2\*x^2+1)\*sqrt(-a\*c\*x+c)/x^2,x, algorithm="fricas")

[Out] [1/2\*(4\*a^3\*c\*x^3 + 2\*a^2\*c\*x^2 + sqrt(-a^2\*x^2+1)\*sqrt(-a\*c\*x+c)\*a\*sqrt(c)\*x\*log(-(a^2\*c\*x^2+a\*c\*x-2\*sqrt(-a^2\*x^2+1)\*sqrt(-a\*c\*x+c)\*sqrt(c)-2\*c)/(a\*x^2-x))-4\*a\*c\*x-2\*c)/(sqrt(-a^2\*x^2+1)\*sqrt(-a\*c\*x+c)\*x), (2\*a^3\*c\*x^3+a^2\*c\*x^2-sqrt(-a^2\*x^2+1)\*sqrt(-a\*c\*x+c)\*a\*sqrt(-c)\*x\*arctan(sqrt(-a^2\*x^2+1)\*sqrt(-a\*c\*x+c)/((a^2\*x^2-1)\*sqrt(-c)))-2\*a\*c\*x-c)/(sqrt(-a^2\*x^2+1)\*sqrt(-a\*c\*x+c)\*x)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c(ax-1)}\sqrt{-(ax-1)(ax+1)}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a\*c\*x+c)\*\*(1/2)\*(-a\*\*2\*x\*\*2+1)\*\*(1/2)/x\*\*2,x)

[Out] Integral(sqrt(-c\*(a\*x-1))\*sqrt(-(a\*x-1)\*(a\*x+1))/x\*\*2,x)

---

**GIAC/XCAS [A]** time = 0.303064, size = 154, normalized size = 1.51

$$\frac{\left(2 \left(\frac{c \arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} + 2 \sqrt{acx+c} + \frac{\sqrt{acx+c}}{ax}\right) a^2 c^2 - \frac{\sqrt{2} \left(\sqrt{2} a^2 c^3 \arctan\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{-c}}\right) + 6 a^2 \sqrt{-c} c^{\frac{5}{2}}\right)}{\sqrt{-c}}\right) |c|}{2 a c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(-a^2\*x^2 + 1)\*sqrt(-a\*c\*x + c)/x^2,x, algorithm="giac")

[Out] -1/2\*(2\*(c\*arctan(sqrt(a\*c\*x + c)/sqrt(-c))/sqrt(-c) + 2\*sqrt(a\*c\*x + c) + sqrt(a\*c\*x + c)/(a\*x))\*a^2\*c^2 - sqrt(2)\*(sqrt(2)\*a^2\*c^3\*arctan(sqrt(2)\*sqrt(c)/sqrt(-c)) + 6\*a^2\*sqrt(-c)\*c^(5/2))/sqrt(-c))\*abs(c)/(a\*c^3)

$$3.219 \quad \int \frac{\sqrt{c-ax}}{x\sqrt{1-a^2x^2}} dx$$

**Optimal.** Leaf size=39

$$-2\sqrt{c} \tanh^{-1} \left( \frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-ax}} \right)$$

[Out]  $-2*\text{Sqrt}[c]*\text{ArcTanh}[(\text{Sqrt}[c]*\text{Sqrt}[1 - a^2*x^2])/\text{Sqrt}[c - a*c*x]]$

**Rubi [A]** time = 0.134639, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$-2\sqrt{c} \tanh^{-1} \left( \frac{\sqrt{c}\sqrt{1-a^2x^2}}{\sqrt{c-ax}} \right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[c - a*c*x]/(x*\text{Sqrt}[1 - a^2*x^2]), x]$

[Out]  $-2*\text{Sqrt}[c]*\text{ArcTanh}[(\text{Sqrt}[c]*\text{Sqrt}[1 - a^2*x^2])/\text{Sqrt}[c - a*c*x]]$

**Rubi in Sympy [A]** time = 10.1385, size = 36, normalized size = 0.92

$$-2\sqrt{c} \operatorname{atanh} \left( \frac{\sqrt{c}\sqrt{-a^2x^2+1}}{\sqrt{-ax+c}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((-a*c*x+c)**(1/2)/x/(-a**2*x**2+1)**(1/2), x)$

[Out]  $-2*\text{sqrt}(c)*\text{atanh}(\text{sqrt}(c)*\text{sqrt}(-a**2*x**2 + 1)/\text{sqrt}(-a*c*x + c))$

**Mathematica [A]** time = 0.0324818, size = 47, normalized size = 1.21

$$2\sqrt{c} \tanh^{-1} \left( \frac{\sqrt{1-a^2x^2}\sqrt{-c(ax-1)}}{\sqrt{c}(ax-1)} \right)$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[\text{Sqrt}[c - a*c*x]/(x*\text{Sqrt}[1 - a^2*x^2]), x]$

[Out]  $2*\text{Sqrt}[c]*\text{ArcTanh}[(\text{Sqrt}[-(c*(-1 + a*x))]*\text{Sqrt}[1 - a^2*x^2])/( \text{Sqrt}[c]*(-1 + a*x))]$

**Maple [A]** time = 0.017, size = 58, normalized size = 1.5

$$2 \frac{\sqrt{-c(ax-1)}\sqrt{-a^2x^2+1}\sqrt{c}}{(ax-1)\sqrt{c}(ax+1)} \operatorname{Artanh} \left( \frac{\sqrt{c(ax+1)}}{\sqrt{c}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-a*c*x+c)^(1/2)/x/(-a^2*x^2+1)^(1/2),x)`

[Out]  $2*(-c*(a*x-1))^{1/2}*(-a^2*x^2+1)^{1/2}/(a*x-1)/(c*(a*x+1))^{1/2}$   
 $*c^{1/2}*\operatorname{arctanh}((c*(a*x+1))^{1/2}/c^{1/2})$

**Maxima [A]** time = 0.769428, size = 38, normalized size = 0.97

$$-\sqrt{c}\left(\log\left(\sqrt{ax+1}+1\right)-\log\left(\sqrt{ax+1}-1\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-a*c*x+c)/(sqrt(-a^2*x^2+1)*x),x,algorithm="maxima")`

[Out]  $-\operatorname{sqrt}(c)*(\log(\operatorname{sqrt}(a*x+1)+1)-\log(\operatorname{sqrt}(a*x+1)-1))$

**Fricas [A]** time = 0.290903, size = 1, normalized size = 0.03

$$\left[\sqrt{c}\log\left(-\frac{a^2cx^2+acx+2\sqrt{-a^2x^2+1}\sqrt{-acx+c}\sqrt{c}-2c}{ax^2-x}\right),2\sqrt{-c}\arctan\left(\frac{\sqrt{-a^2x^2+1}\sqrt{-acx+c}}{(a^2x^2-1)\sqrt{-c}}\right)\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-a*c*x+c)/(sqrt(-a^2*x^2+1)*x),x,algorithm="fricas")`

[Out]  $[\operatorname{sqrt}(c)*\log(-\frac{a^2*c*x^2+a*c*x+2*\operatorname{sqrt}(-a^2*x^2+1)*\operatorname{sqrt}(-a*c*x+c)*\operatorname{sqrt}(c)-2*c}{a*x^2-x}),2*\operatorname{sqrt}(-c)*\operatorname{arctan}(\operatorname{sqrt}(-a^2*x^2+1)*\operatorname{sqrt}(-a*c*x+c)/((a^2*x^2-1)*\operatorname{sqrt}(-c)))]$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-c}(ax-1)}{x\sqrt{-(ax-1)(ax+1)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-a*c*x+c)**(1/2)/x/(-a**2*x**2+1)**(1/2),x)`

[Out] `Integral(sqrt(-c*(a*x-1))/(x*sqrt(-(a*x-1)*(a*x+1))),x)`

**GIAC/XCAS [A]** time = 0.29579, size = 69, normalized size = 1.77

$$\frac{2c^2\left(\frac{\arctan\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{-c}}\right)}{\sqrt{-c}}-\frac{\arctan\left(\frac{\sqrt{acx+c}}{\sqrt{-c}}\right)}{\sqrt{-c}}\right)}{|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-a*c*x+c)/(sqrt(-a^2*x^2+1)*x),x,algorithm="giac")`

[Out]  $-2*c^2*(\operatorname{arctan}(\operatorname{sqrt}(2)*\operatorname{sqrt}(c)/\operatorname{sqrt}(-c))/\operatorname{sqrt}(-c)-\operatorname{arctan}(\operatorname{sqrt}(a*c*x+c)/\operatorname{sqrt}(-c))/\operatorname{sqrt}(-c))/\operatorname{abs}(c)$

$$3.220 \quad \int \frac{\sqrt{1-ax}}{\sqrt{x}} dx$$

**Optimal.** Leaf size=35

$$\sqrt{x}\sqrt{1-ax} + \frac{\sin^{-1}(\sqrt{a}\sqrt{x})}{\sqrt{a}}$$

[Out] Sqrt[x]\*Sqrt[1 - a\*x] + ArcSin[Sqrt[a]\*Sqrt[x]]/Sqrt[a]

**Rubi [A]** time = 0.032716, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\sqrt{x}\sqrt{1-ax} + \frac{\sin^{-1}(\sqrt{a}\sqrt{x})}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - a\*x]/Sqrt[x], x]

[Out] Sqrt[x]\*Sqrt[1 - a\*x] + ArcSin[Sqrt[a]\*Sqrt[x]]/Sqrt[a]

**Rubi in Sympy [A]** time = 5.92644, size = 29, normalized size = 0.83

$$\sqrt{x}\sqrt{-ax+1} + \frac{\text{asin}(\sqrt{a}\sqrt{x})}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-a\*x+1)\*\*(1/2)/x\*\*(1/2), x)

[Out] sqrt(x)\*sqrt(-a\*x + 1) + asin(sqrt(a)\*sqrt(x))/sqrt(a)

**Mathematica [A]** time = 0.0265896, size = 35, normalized size = 1.

$$\sqrt{x}\sqrt{1-ax} + \frac{\sin^{-1}(\sqrt{a}\sqrt{x})}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - a\*x]/Sqrt[x], x]

[Out] Sqrt[x]\*Sqrt[1 - a\*x] + ArcSin[Sqrt[a]\*Sqrt[x]]/Sqrt[a]

**Maple [B]** time = 0.026, size = 62, normalized size = 1.8

$$\sqrt{x}\sqrt{-ax+1} + \frac{1}{2}\sqrt{(-ax+1)x} \arctan\left(1\sqrt{a}\left(x - \frac{1}{2a}\right) \frac{1}{\sqrt{-ax^2+x}}\right) \frac{1}{\sqrt{x}} \frac{1}{\sqrt{-ax+1}} \frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-a\*x+1)^(1/2)/x^(1/2), x)



[Out]  $x^{(1/2)} * (-a*x+1)^{(1/2)} + 1/2 * ((-a*x+1)*x)^{(1/2)} / (-a*x+1)^{(1/2)} / x^{(1/2)} / a^{(1/2)} * \arctan(a^{(1/2)} * (x-1/2/a) / (-a*x^2+x)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-a*x + 1)/sqrt(x),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.287702, size = 1, normalized size = 0.03

$$\left[ \frac{2\sqrt{-ax+1}\sqrt{-a}\sqrt{x} + \log\left(-2\sqrt{-ax+1}a\sqrt{x} - (2ax-1)\sqrt{-a}\right)}{2\sqrt{-a}}, \frac{\sqrt{-ax+1}\sqrt{a}\sqrt{x} - \arctan\left(\frac{\sqrt{-ax+1}}{\sqrt{a}\sqrt{x}}\right)}{\sqrt{a}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-a*x + 1)/sqrt(x),x, algorithm="fricas")`

[Out]  $[1/2 * (2 * \sqrt{-a*x + 1} * \sqrt{-a} * \sqrt{x} + \log(-2 * \sqrt{-a*x + 1} * a * \sqrt{x} - (2 * a * x - 1) * \sqrt{-a})) / \sqrt{-a}, (\sqrt{-a*x + 1} * \sqrt{a} * \sqrt{x} - \arctan(\sqrt{-a*x + 1} / (\sqrt{a} * \sqrt{x}))) / \sqrt{a}]$

**Sympy [A]** time = 5.87043, size = 83, normalized size = 2.37

$$\begin{cases} \frac{iax^{\frac{3}{2}}}{\sqrt{ax-1}} - \frac{i\sqrt{x}}{\sqrt{ax-1}} - \frac{i \operatorname{acosh}(\sqrt{a}\sqrt{x})}{\sqrt{a}} & \text{for } |ax| > 1 \\ \sqrt{x}\sqrt{-ax+1} + \frac{\operatorname{asin}(\sqrt{a}\sqrt{x})}{\sqrt{a}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-a*x+1)**(1/2)/x**(1/2),x)`

[Out] `Piecewise((I*a*x**(3/2)/sqrt(a*x - 1) - I*sqrt(x)/sqrt(a*x - 1) - I*acosh(sqrt(a)*sqrt(x))/sqrt(a), Abs(a*x) > 1), (sqrt(x)*sqrt(-a*x + 1) + asin(sqrt(a)*sqrt(x))/sqrt(a), True))`

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-a*x + 1)/sqrt(x),x, algorithm="giac")`

[Out] Exception raised: NotImplementedError

$$3.221 \quad \int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1+ax}} dx$$

**Optimal.** Leaf size=35

$$\sqrt{x}\sqrt{1-ax} + \frac{\sin^{-1}(\sqrt{a}\sqrt{x})}{\sqrt{a}}$$

[Out] Sqrt[x]\*Sqrt[1 - a\*x] + ArcSin[Sqrt[a]\*Sqrt[x]]/Sqrt[a]

**Rubi [A]** time = 0.108697, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$

$$\sqrt{x}\sqrt{1-ax} + \frac{\sin^{-1}(\sqrt{a}\sqrt{x})}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - a^2\*x^2]/(Sqrt[x]\*Sqrt[1 + a\*x]), x]

[Out] Sqrt[x]\*Sqrt[1 - a\*x] + ArcSin[Sqrt[a]\*Sqrt[x]]/Sqrt[a]

**Rubi in Sympy [A]** time = 9.75256, size = 29, normalized size = 0.83

$$\sqrt{x}\sqrt{-ax+1} + \frac{\text{asin}(\sqrt{a}\sqrt{x})}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-a\*\*2\*x\*\*2+1)\*\*(1/2)/x\*\*(1/2)/(a\*x+1)\*\*(1/2), x)

[Out] sqrt(x)\*sqrt(-a\*x + 1) + asin(sqrt(a)\*sqrt(x))/sqrt(a)

**Mathematica [C]** time = 0.107416, size = 80, normalized size = 2.29

$$\frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{ax+1}} + \frac{i \log\left(\frac{2\sqrt{1-a^2x^2}}{\sqrt{ax+1}} - 2i\sqrt{a}\sqrt{x}\right)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - a^2\*x^2]/(Sqrt[x]\*Sqrt[1 + a\*x]), x]

[Out] (Sqrt[x]\*Sqrt[1 - a^2\*x^2])/Sqrt[1 + a\*x] + (I\*Log[(-2\*I)\*Sqrt[a]\*Sqrt[x] + (2\*Sqrt[1 - a^2\*x^2])/Sqrt[1 + a\*x]])/Sqrt[a]

**Maple [B]** time = 0.019, size = 76, normalized size = 2.2

$$\frac{1}{2}\sqrt{-a^2x^2+1}\sqrt{x} \left( 2\sqrt{a}\sqrt{-x(ax-1)} + \arctan\left(\frac{2ax-1}{2}\frac{1}{\sqrt{a}\sqrt{-x(ax-1)}}\right) \right) \frac{1}{\sqrt{ax+1}} \frac{1}{\sqrt{-x(ax-1)}} \frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-a^2*x^2+1)^(1/2)/x^(1/2)/(a*x+1)^(1/2), x)`

[Out]  $\frac{1}{2} \cdot (-a^2 x^2 + 1)^{1/2} \cdot x^{1/2} / (a x + 1)^{1/2} \cdot (2 \cdot a^{1/2}) \cdot (-x \cdot (a x - 1))^{1/2} + \arctan\left(\frac{1/2 \cdot a^{1/2} \cdot (2 \cdot a x - 1)}{(-x \cdot (a x - 1))^{1/2}}\right) / (-x \cdot (a x - 1))^{1/2} / a^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-a^2*x^2 + 1)/(sqrt(a*x + 1)*sqrt(x)), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.311637, size = 1, normalized size = 0.03

$$\left[ \frac{4 \sqrt{-a^2 x^2 + 1} \sqrt{a x + 1} \sqrt{-a} \sqrt{x} + (a x + 1) \log\left(-\frac{4 \sqrt{-a^2 x^2 + 1} (2 a^2 x - a) \sqrt{a x + 1} \sqrt{x} + (8 a^3 x^3 - 7 a x + 1) \sqrt{-a}}{a x + 1}\right)}{4 (a x + 1) \sqrt{-a}}, \frac{2 \sqrt{-a^2 x^2 + 1} \sqrt{a x + 1} \sqrt{a} \sqrt{x}}{4 (a x + 1) \sqrt{-a}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-a^2*x^2 + 1)/(sqrt(a*x + 1)*sqrt(x)), x, algorithm="fricas")`

[Out]  $\left[ \frac{1}{4} \cdot (4 \cdot \sqrt{-a^2 x^2 + 1}) \cdot \sqrt{a x + 1} \cdot \sqrt{-a} \cdot \sqrt{x} + (a x + 1) \cdot \log\left(-\frac{4 \cdot \sqrt{-a^2 x^2 + 1} \cdot (2 \cdot a^2 x - a) \cdot \sqrt{a x + 1} \cdot \sqrt{x} + (8 \cdot a^3 x^3 - 7 \cdot a x + 1) \cdot \sqrt{-a}}{a x + 1}\right)}{4 (a x + 1) \sqrt{-a}}, \frac{1}{2} \cdot (2 \cdot \sqrt{-a^2 x^2 + 1}) \cdot \sqrt{a x + 1} \cdot \sqrt{a} \cdot \sqrt{x} - (a x + 1) \cdot \arctan\left(\frac{2 \cdot \sqrt{-a^2 x^2 + 1} \cdot \sqrt{a x + 1} \cdot \sqrt{a} \cdot \sqrt{x}}{2 \cdot a^2 x^2 + a x - 1}\right)}{4 (a x + 1) \sqrt{-a}} \right]$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax-1)(ax+1)}}{\sqrt{x}\sqrt{ax+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-a**2*x**2+1)**(1/2)/x**(1/2)/(a*x+1)**(1/2), x)`

[Out] `Integral(sqrt(-(a*x - 1)*(a*x + 1))/(sqrt(x)*sqrt(a*x + 1)), x)`

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-a^2*x^2 + 1)/(sqrt(a*x + 1)*sqrt(x)), x, algorithm="giac")`

[Out] Exception raised: NotImplementedError

$$3.222 \quad \int \frac{\sqrt{1+ax}}{\sqrt{x}} dx$$

**Optimal.** Leaf size=34

$$\sqrt{x}\sqrt{ax+1} + \frac{\sinh^{-1}(\sqrt{a}\sqrt{x})}{\sqrt{a}}$$

[Out] Sqrt[x]\*Sqrt[1 + a\*x] + ArcSinh[Sqrt[a]\*Sqrt[x]]/Sqrt[a]

**Rubi [A]** time = 0.0278372, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\sqrt{x}\sqrt{ax+1} + \frac{\sinh^{-1}(\sqrt{a}\sqrt{x})}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + a\*x]/Sqrt[x], x]

[Out] Sqrt[x]\*Sqrt[1 + a\*x] + ArcSinh[Sqrt[a]\*Sqrt[x]]/Sqrt[a]

**Rubi in Sympy [A]** time = 5.08801, size = 29, normalized size = 0.85

$$\sqrt{x}\sqrt{ax+1} + \frac{\operatorname{asinh}(\sqrt{a}\sqrt{x})}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a\*x+1)\*\*(1/2)/x\*\*(1/2), x)

[Out] sqrt(x)\*sqrt(a\*x + 1) + asinh(sqrt(a)\*sqrt(x))/sqrt(a)

**Mathematica [A]** time = 0.0222779, size = 34, normalized size = 1.

$$\sqrt{x}\sqrt{ax+1} + \frac{\sinh^{-1}(\sqrt{a}\sqrt{x})}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 + a\*x]/Sqrt[x], x]

[Out] Sqrt[x]\*Sqrt[1 + a\*x] + ArcSinh[Sqrt[a]\*Sqrt[x]]/Sqrt[a]

**Maple [B]** time = 0.008, size = 57, normalized size = 1.7

$$\sqrt{x}\sqrt{ax+1} + \frac{1}{2}\sqrt{(ax+1)x} \ln\left(1 + \left(\frac{1}{2} + ax\right) \frac{1}{\sqrt{a}} + \sqrt{ax^2+x}\right) \frac{1}{\sqrt{x}} \frac{1}{\sqrt{ax+1}} \frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*x+1)^(1/2)/x^(1/2),x)`

[Out]  $x^{1/2} * (a*x+1)^{1/2} + 1/2 * ((a*x+1)*x)^{1/2} / (a*x+1)^{1/2} / x^{1/2} * \ln((1/2+a*x)/a^{1/2} + (a*x^2+x)^{1/2}) / a^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(a*x + 1)/sqrt(x),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.287526, size = 1, normalized size = 0.03

$$\left[ \frac{2\sqrt{ax+1}\sqrt{a}\sqrt{x} + \log\left(2\sqrt{ax+1}a\sqrt{x} + (2ax+1)\sqrt{a}\right)}{2\sqrt{a}}, \frac{\sqrt{ax+1}\sqrt{-a}\sqrt{x} + \arctan\left(\frac{\sqrt{ax+1}\sqrt{-a}}{a\sqrt{x}}\right)}{\sqrt{-a}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(a*x + 1)/sqrt(x),x, algorithm="fricas")`

[Out]  $[1/2*(2*\sqrt{a*x + 1}*\sqrt{a}*\sqrt{x} + \log(2*\sqrt{a*x + 1}*a*\sqrt{t(x) + (2*a*x + 1)*\sqrt{a}}))/\sqrt{a}, (\sqrt{a*x + 1}*\sqrt{-a}*\sqrt{t(x) + \arctan(\sqrt{a*x + 1}*\sqrt{-a}/(a*\sqrt{x}))})/\sqrt{-a}]$

**Sympy [A]** time = 5.78381, size = 29, normalized size = 0.85

$$\sqrt{x}\sqrt{ax+1} + \frac{\operatorname{asinh}(\sqrt{a}\sqrt{x})}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*x+1)**(1/2)/x**(1/2),x)`

[Out]  $\sqrt{x} * \sqrt{a*x + 1} + \operatorname{asinh}(\sqrt{a} * \sqrt{x}) / \sqrt{a}$

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(a*x + 1)/sqrt(x),x, algorithm="giac")`

[Out] Exception raised: NotImplementedError

$$3.223 \quad \int \frac{\sqrt{1-a^2x^2}}{\sqrt{x}\sqrt{1-ax}} dx$$

**Optimal.** Leaf size=34

$$\sqrt{x}\sqrt{ax+1} + \frac{\sinh^{-1}(\sqrt{a}\sqrt{x})}{\sqrt{a}}$$

[Out] Sqrt[x]\*Sqrt[1 + a\*x] + ArcSinh[Sqrt[a]\*Sqrt[x]]/Sqrt[a]

**Rubi [A]** time = 0.111975, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\sqrt{x}\sqrt{ax+1} + \frac{\sinh^{-1}(\sqrt{a}\sqrt{x})}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 - a^2\*x^2]/(Sqrt[x]\*Sqrt[1 - a\*x]), x]

[Out] Sqrt[x]\*Sqrt[1 + a\*x] + ArcSinh[Sqrt[a]\*Sqrt[x]]/Sqrt[a]

**Rubi in Sympy [A]** time = 9.62407, size = 29, normalized size = 0.85

$$\sqrt{x}\sqrt{ax+1} + \frac{\operatorname{asinh}(\sqrt{a}\sqrt{x})}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-a\*\*2\*x\*\*2+1)\*\*(1/2)/x\*\*(1/2)/(-a\*x+1)\*\*(1/2), x)

[Out] sqrt(x)\*sqrt(a\*x + 1) + asinh(sqrt(a)\*sqrt(x))/sqrt(a)

**Mathematica [B]** time = 0.14217, size = 100, normalized size = 2.94

$$\frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1-ax}} - \frac{\log\left(a^2x^{3/2} + \sqrt{a}\sqrt{1-ax}\sqrt{1-a^2x^2} - a\sqrt{x}\right)}{\sqrt{a}} + \frac{\log(1-ax)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[1 - a^2\*x^2]/(Sqrt[x]\*Sqrt[1 - a\*x]), x]

[Out] (Sqrt[x]\*Sqrt[1 - a^2\*x^2])/Sqrt[1 - a\*x] + Log[1 - a\*x]/Sqrt[a] - Log[-(a\*Sqrt[x]) + a^2\*x^(3/2) + Sqrt[a]\*Sqrt[1 - a\*x]\*Sqrt[1 - a^2\*x^2]]/Sqrt[a]

**Maple [B]** time = 0.017, size = 86, normalized size = 2.5

$$-\frac{1}{2ax-2}\sqrt{-a^2x^2+1}\sqrt{x}\sqrt{-ax+1}\left(2\sqrt{(ax+1)x}\sqrt{a}+\ln\left(\frac{1}{2}\left(2\sqrt{(ax+1)x}\sqrt{a}+2ax+1\right)\frac{1}{\sqrt{a}}\right)\right)\frac{1}{\sqrt{(ax+1)x}}\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-a^2*x^2+1)^(1/2)/x^(1/2)/(-a*x+1)^(1/2), x)`

[Out] 
$$-1/2 * (-a^2 * x^2 + 1)^{(1/2)} * x^{(1/2)} * (-a * x + 1)^{(1/2)} * (2 * ((a * x + 1) * x)^{(1/2)} * a^{(1/2)} + \ln(1/2 * (2 * ((a * x + 1) * x)^{(1/2)} * a^{(1/2)} + 2 * a * x + 1) / a^{(1/2)})) / (a * x - 1) / ((a * x + 1) * x)^{(1/2)} / a^{(1/2)}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-a^2*x^2 + 1)/(sqrt(-a*x + 1)*sqrt(x)), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.31755, size = 1, normalized size = 0.03

$$\left[ \frac{4 \sqrt{-a^2 x^2 + 1} \sqrt{-ax + 1} \sqrt{a} \sqrt{x} - (ax - 1) \log \left( \frac{4 \sqrt{-a^2 x^2 + 1} (2 a^2 x + a) \sqrt{-ax + 1} \sqrt{x} - (8 a^3 x^3 - 7 a x - 1) \sqrt{a}}{ax - 1} \right)}{4 (ax - 1) \sqrt{a}}, \right. \\ \left. - \frac{2 \sqrt{-a^2 x^2 + 1} \sqrt{-ax + 1} \sqrt{-a} \sqrt{x} + (ax - 1) \arctan \left( \frac{2 \sqrt{-a^2 x^2 + 1} \sqrt{-ax + 1} \sqrt{-a} \sqrt{x}}{2 a^2 x^2 - ax - 1} \right)}{2 (ax - 1) \sqrt{-a}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-a^2*x^2 + 1)/(sqrt(-a*x + 1)*sqrt(x)), x, algorithm="fricas")`

[Out] 
$$[-1/4 * (4 * \sqrt{-a^2 * x^2 + 1} * \sqrt{-a * x + 1} * \sqrt{a} * \sqrt{x} - (a * x - 1) * \log((4 * \sqrt{-a^2 * x^2 + 1} * (2 * a^2 * x + a) * \sqrt{-a * x + 1} * \sqrt{x} - (8 * a^3 * x^3 - 7 * a * x - 1) * \sqrt{a}) / (a * x - 1))) / ((a * x - 1) * \sqrt{a}), -1/2 * (2 * \sqrt{-a^2 * x^2 + 1} * \sqrt{-a * x + 1} * \sqrt{-a} * \sqrt{x} + (a * x - 1) * \arctan(2 * \sqrt{-a^2 * x^2 + 1} * \sqrt{-a * x + 1} * \sqrt{-a} * \sqrt{x} / (2 * a^2 * x^2 - a * x - 1))) / ((a * x - 1) * \sqrt{-a})]$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(ax - 1)(ax + 1)}}{\sqrt{x} \sqrt{-ax + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-a**2*x**2+1)**(1/2)/x**(1/2)/(-a*x+1)**(1/2), x)`

[Out] `Integral(sqrt(-(a*x - 1)*(a*x + 1))/(sqrt(x)*sqrt(-a*x + 1)), x)`

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-a^2*x^2 + 1)/(sqrt(-a*x + 1)*sqrt(x)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```



### 3.224 $\int \sqrt{x}\sqrt{1-ax} dx$

**Optimal.** Leaf size=63

$$\frac{\sin^{-1}(\sqrt{a}\sqrt{x})}{4a^{3/2}} + \frac{1}{2}x^{3/2}\sqrt{1-ax} - \frac{\sqrt{x}\sqrt{1-ax}}{4a}$$

[Out]  $-(\text{Sqrt}[x] * \text{Sqrt}[1 - a*x]) / (4*a) + (x^{(3/2)} * \text{Sqrt}[1 - a*x]) / 2 + \text{ArcSin}[\text{Sqrt}[a] * \text{Sqrt}[x]] / (4*a^{(3/2)})$

**Rubi [A]** time = 0.0490371, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{\sin^{-1}(\sqrt{a}\sqrt{x})}{4a^{3/2}} + \frac{1}{2}x^{3/2}\sqrt{1-ax} - \frac{\sqrt{x}\sqrt{1-ax}}{4a}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[x] * \text{Sqrt}[1 - a*x], x]$

[Out]  $-(\text{Sqrt}[x] * \text{Sqrt}[1 - a*x]) / (4*a) + (x^{(3/2)} * \text{Sqrt}[1 - a*x]) / 2 + \text{ArcSin}[\text{Sqrt}[a] * \text{Sqrt}[x]] / (4*a^{(3/2)})$

**Rubi in Sympy [A]** time = 8.20873, size = 51, normalized size = 0.81

$$-\frac{\sqrt{x}(-ax+1)^{3/2}}{2a} + \frac{\sqrt{x}\sqrt{-ax+1}}{4a} + \frac{\text{asin}(\sqrt{a}\sqrt{x})}{4a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{(1/2)} * (-a*x+1)^{(1/2)}, x)$

[Out]  $-\text{sqrt}(x) * (-a*x + 1)^{(3/2)} / (2*a) + \text{sqrt}(x) * \text{sqrt}(-a*x + 1) / (4*a) + \text{asin}(\text{sqrt}(a) * \text{sqrt}(x)) / (4*a^{(3/2)})$

**Mathematica [A]** time = 0.0406145, size = 49, normalized size = 0.78

$$\frac{\sqrt{a}\sqrt{x}\sqrt{1-ax}(2ax-1) + \sin^{-1}(\sqrt{a}\sqrt{x})}{4a^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[\text{Sqrt}[x] * \text{Sqrt}[1 - a*x], x]$

[Out]  $(\text{Sqrt}[a] * \text{Sqrt}[x] * \text{Sqrt}[1 - a*x] * (-1 + 2*a*x) + \text{ArcSin}[\text{Sqrt}[a] * \text{Sqrt}[x]]) / (4*a^{(3/2)})$

**Maple [A]** time = 0.007, size = 79, normalized size = 1.3

$$\frac{1}{2}x^{3/2}\sqrt{-ax+1} - \frac{1}{4a}\sqrt{x}\sqrt{-ax+1} + \frac{1}{8}\sqrt{(-ax+1)x} \arctan\left(1\sqrt{a}\left(x - \frac{1}{2a}\right) \frac{1}{\sqrt{-ax^2+x}}\right) a^{-3/2} \frac{1}{\sqrt{-ax+1}} \frac{1}{\sqrt{x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^(1/2)*(-a*x+1)^(1/2),x)`

[Out]  $\frac{1}{2}x^{3/2}(-ax+1)^{1/2} - \frac{1}{4}x^{1/2}(-ax+1)^{1/2}/a + \frac{1}{8}a^{3/2}((-ax+1)x)^{1/2}/(-ax+1)^{1/2}/x^{1/2} \arctan(a^{1/2}(x-1/2/a)/(-ax^2+x)^{1/2})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-a*x + 1)*sqrt(x),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.287385, size = 1, normalized size = 0.02

$$\left[ \frac{2(2ax-1)\sqrt{-ax+1}\sqrt{-a}\sqrt{x} + \log\left(-2\sqrt{-ax+1}a\sqrt{x} - (2ax-1)\sqrt{-a}\right)}{8\sqrt{-aa}}, \frac{(2ax-1)\sqrt{-ax+1}\sqrt{a}\sqrt{x} - \arctan\left(\frac{\sqrt{-ax+1}}{\sqrt{a}\sqrt{x}}\right)}{4a^{3/2}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(-a*x + 1)*sqrt(x),x, algorithm="fricas")`

[Out]  $\left[ \frac{1}{8} \left( 2 \cdot (2 \cdot a \cdot x - 1) \cdot \sqrt{-a \cdot x + 1} \cdot \sqrt{-a} \cdot \sqrt{x} + \log(-2 \cdot \sqrt{-a \cdot x + 1} \cdot a \cdot \sqrt{x} - (2 \cdot a \cdot x - 1) \cdot \sqrt{-a}) \right) / (\sqrt{-a} \cdot a), \frac{1}{4} \left( (2 \cdot a \cdot x - 1) \cdot \sqrt{-a \cdot x + 1} \cdot \sqrt{a} \cdot \sqrt{x} - \arctan(\sqrt{-a \cdot x + 1} / (\sqrt{a} \cdot \sqrt{x})) \right) / (a^{3/2}) \right]$

**Sympy [A]** time = 10.2354, size = 148, normalized size = 2.35

$$\begin{cases} \frac{iax^{5/2}}{2\sqrt{ax-1}} - \frac{3ix^{3/2}}{4\sqrt{ax-1}} + \frac{i\sqrt{x}}{4a\sqrt{ax-1}} - \frac{i \operatorname{acosh}(\sqrt{a}\sqrt{x})}{4a^{3/2}} & \text{for } |ax| > 1 \\ -\frac{ax^{5/2}}{2\sqrt{-ax+1}} + \frac{3x^{3/2}}{4\sqrt{-ax+1}} - \frac{\sqrt{x}}{4a\sqrt{-ax+1}} + \frac{\operatorname{asin}(\sqrt{a}\sqrt{x})}{4a^{3/2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**(1/2)*(-a*x+1)**(1/2),x)`

[Out] `Piecewise((I*a*x**(5/2)/(2*sqrt(a*x - 1)) - 3*I*x**(3/2)/(4*sqrt(a*x - 1)) + I*sqrt(x)/(4*a*sqrt(a*x - 1)) - I*acosh(sqrt(a)*sqrt(x))/(4*a**(3/2)), Abs(a*x) > 1), (-a*x**(5/2)/(2*sqrt(-a*x + 1)) + 3*x**(3/2)/(4*sqrt(-a*x + 1)) - sqrt(x)/(4*a*sqrt(-a*x + 1)) + asin(sqrt(a)*sqrt(x))/(4*a**(3/2)), True))`

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-a*x + 1)*sqrt(x),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.225 \quad \int \frac{\sqrt{x}\sqrt{1-a^2x^2}}{\sqrt{1+ax}} dx$$

**Optimal.** Leaf size=63

$$\frac{\sin^{-1}(\sqrt{a}\sqrt{x})}{4a^{3/2}} + \frac{1}{2}x^{3/2}\sqrt{1-ax} - \frac{\sqrt{x}\sqrt{1-ax}}{4a}$$

[Out]  $-(\text{Sqrt}[x] * \text{Sqrt}[1 - a * x]) / (4 * a) + (x^{(3/2)} * \text{Sqrt}[1 - a * x]) / 2 + \text{ArcS}$   
 $\text{in}[\text{Sqrt}[a] * \text{Sqrt}[x]] / (4 * a^{(3/2)})$

**Rubi [A]** time = 0.120076, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$

$$\frac{\sin^{-1}(\sqrt{a}\sqrt{x})}{4a^{3/2}} + \frac{1}{2}x^{3/2}\sqrt{1-ax} - \frac{\sqrt{x}\sqrt{1-ax}}{4a}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Sqrt}[x] * \text{Sqrt}[1 - a^2 * x^2]) / \text{Sqrt}[1 + a * x], x]$

[Out]  $-(\text{Sqrt}[x] * \text{Sqrt}[1 - a * x]) / (4 * a) + (x^{(3/2)} * \text{Sqrt}[1 - a * x]) / 2 + \text{ArcS}$   
 $\text{in}[\text{Sqrt}[a] * \text{Sqrt}[x]] / (4 * a^{(3/2)})$

**Rubi in Sympy [A]** time = 11.9661, size = 49, normalized size = 0.78

$$\frac{x^{3/2}\sqrt{-ax+1}}{2} - \frac{\sqrt{x}\sqrt{-ax+1}}{4a} + \frac{\text{asin}(\sqrt{a}\sqrt{x})}{4a^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{(1/2)} * (-a^{**2} * x^{**2} + 1)^{(1/2)} / (a * x + 1)^{(1/2)}, x)$

[Out]  $x^{(3/2)} * \text{sqrt}(-a * x + 1) / 2 - \text{sqrt}(x) * \text{sqrt}(-a * x + 1) / (4 * a) + \text{asin}(\text{sqrt}(a) * \text{sqrt}(x)) / (4 * a^{(3/2)})$

**Mathematica [C]** time = 0.141263, size = 94, normalized size = 1.49

$$\frac{\sqrt{x}\sqrt{1-a^2x^2}(2ax-1)}{4a\sqrt{ax+1}} + \frac{i \log\left(\frac{2\sqrt{1-a^2x^2}}{\sqrt{ax+1}} - 2i\sqrt{a}\sqrt{x}\right)}{4a^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(\text{Sqrt}[x] * \text{Sqrt}[1 - a^2 * x^2]) / \text{Sqrt}[1 + a * x], x]$

[Out]  $(\text{Sqrt}[x] * (-1 + 2 * a * x) * \text{Sqrt}[1 - a^2 * x^2]) / (4 * a * \text{Sqrt}[1 + a * x]) + (($   
 $\text{I} / 4) * \text{Log}[(-2 * \text{I}) * \text{Sqrt}[a] * \text{Sqrt}[x] + (2 * \text{Sqrt}[1 - a^2 * x^2]) / \text{Sqrt}[1 +$   
 $a * x]]) / a^{(3/2)}$

**Maple [B]** time = 0.016, size = 92, normalized size = 1.5

$$\frac{1}{8}\sqrt{x}\sqrt{-a^2x^2+1}\left(4xa^{3/2}\sqrt{-x(ax-1)}-2\sqrt{a}\sqrt{-x(ax-1)}+\arctan\left(\frac{2ax-1}{2}\frac{1}{\sqrt{a}\sqrt{-x(ax-1)}}\right)\right)a^{-3/2}\frac{1}{\sqrt{ax+1}}\frac{1}{\sqrt{-x(ax-1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^{(1/2)} * (-a^2 * x^2 + 1)^{(1/2)} / (a * x + 1)^{(1/2)}, x)$

[Out]  $1/8 * x^{(1/2)} * (-a^2 * x^2 + 1)^{(1/2)} / a^{(3/2)} * (4 * x * a^{(3/2)} * (-x * (a * x - 1))^{(1/2)} - 2 * a^{(1/2)} * (-x * (a * x - 1))^{(1/2)} + \arctan(1/2 / a^{(1/2)} * (2 * a * x - 1) / (-x * (a * x - 1))^{(1/2)})) / (a * x + 1)^{(1/2)} / (-x * (a * x - 1))^{(1/2)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\text{sqrt}(-a^2 * x^2 + 1) * \text{sqrt}(x) / \text{sqrt}(a * x + 1), x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.318782, size = 1, normalized size = 0.02

$$\left[ \frac{4 \sqrt{-a^2 x^2 + 1} (2 a x - 1) \sqrt{a x + 1} \sqrt{-a} \sqrt{x} + (a x + 1) \log\left(-\frac{4 \sqrt{-a^2 x^2 + 1} (2 a^2 x - a) \sqrt{a x + 1} \sqrt{x} + (8 a^3 x^3 - 7 a x + 1) \sqrt{-a}}{a x + 1}\right)}{16 (a^2 x + a) \sqrt{-a}}, \frac{2 \sqrt{-a^2 x^2 + 1} (2 a x - 1) \sqrt{a x + 1} \sqrt{-a} \sqrt{x} + (a x + 1) \log\left(-\frac{4 \sqrt{-a^2 x^2 + 1} (2 a^2 x - a) \sqrt{a x + 1} \sqrt{x} + (8 a^3 x^3 - 7 a x + 1) \sqrt{-a}}{a x + 1}\right)}{16 (a^2 x + a) \sqrt{-a}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\text{sqrt}(-a^2 * x^2 + 1) * \text{sqrt}(x) / \text{sqrt}(a * x + 1), x, \text{algorithm}="fricas")$

[Out]  $[1/16 * (4 * \text{sqrt}(-a^2 * x^2 + 1) * (2 * a * x - 1) * \text{sqrt}(a * x + 1) * \text{sqrt}(-a) * \text{sqrt}(x) + (a * x + 1) * \log(-4 * \text{sqrt}(-a^2 * x^2 + 1) * (2 * a^2 * x - a) * \text{sqrt}(a * x + 1) * \text{sqrt}(x) + (8 * a^3 * x^3 - 7 * a * x + 1) * \text{sqrt}(-a)) / (a * x + 1))) / ((a^2 * x + a) * \text{sqrt}(-a)), 1/8 * (2 * \text{sqrt}(-a^2 * x^2 + 1) * (2 * a * x - 1) * \text{sqrt}(a * x + 1) * \text{sqrt}(a) * \text{sqrt}(x) - (a * x + 1) * \arctan(2 * \text{sqrt}(-a^2 * x^2 + 1) * \text{sqrt}(a * x + 1) * \text{sqrt}(a) * \text{sqrt}(x) / (2 * a^2 * x^2 + a * x - 1))) / ((a^2 * x + a) * \text{sqrt}(a))]$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x} \sqrt{-(ax - 1)(ax + 1)}}{\sqrt{ax + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(x^{(1/2)} * (-a^2 * x^2 + 1)^{(1/2)} / (a * x + 1)^{(1/2)}, x)$

[Out]  $\text{Integral}(\text{sqrt}(x) * \text{sqrt}(-(a * x - 1) * (a * x + 1)) / \text{sqrt}(a * x + 1), x)$

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(-a^2*x^2 + 1)*sqrt(x)/sqrt(a*x + 1),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.226 \quad \int (gx)^m (d + ex)^3 (d^2 - e^2 x^2)^{5/2} dx$$

**Optimal.** Leaf size=250

$$\begin{aligned} & \frac{e (d^2 - e^2 x^2)^{7/2} (gx)^{m+2}}{g^2 (m+9)} - \frac{3d (d^2 - e^2 x^2)^{7/2} (gx)^{m+1}}{g(m+8)} \\ & + \frac{d^7 (4m+11) \sqrt{d^2 - e^2 x^2} (gx)^{m+1} {}_2F_1\left(-\frac{5}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \frac{e^2 x^2}{d^2}\right)}{g(m+1)(m+8) \sqrt{1 - \frac{e^2 x^2}{d^2}}} \\ & + \frac{d^6 e (4m+29) \sqrt{d^2 - e^2 x^2} (gx)^{m+2} {}_2F_1\left(-\frac{5}{2}, \frac{m+2}{2}, \frac{m+4}{2}, \frac{e^2 x^2}{d^2}\right)}{g^2 (m+2)(m+9) \sqrt{1 - \frac{e^2 x^2}{d^2}}} \end{aligned}$$

[Out]  $(-3*d*(g*x)^{(1+m)}*(d^2 - e^2*x^2)^{(7/2)})/(g*(8+m)) - (e*(g*x)^{(2+m)}*(d^2 - e^2*x^2)^{(7/2)})/(g^2*(9+m)) + (d^7*(11+4*m)*(g*x)^{(1+m)}*\text{Sqrt}[d^2 - e^2*x^2]*\text{Hypergeometric2F1}[-5/2, (1+m)/2, (3+m)/2, (e^2*x^2)/d^2])/(g*(1+m)*(8+m)*\text{Sqrt}[1 - (e^2*x^2)/d^2]) + (d^6*e*(29+4*m)*(g*x)^{(2+m)}*\text{Sqrt}[d^2 - e^2*x^2]*\text{Hypergeometric2F1}[-5/2, (2+m)/2, (4+m)/2, (e^2*x^2)/d^2])/(g^2*(2+m)*(9+m)*\text{Sqrt}[1 - (e^2*x^2)/d^2])$

**Rubi [A]** time = 0.675867, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$

$$\begin{aligned} & \frac{e (d^2 - e^2 x^2)^{7/2} (gx)^{m+2}}{g^2 (m+9)} - \frac{3d (d^2 - e^2 x^2)^{7/2} (gx)^{m+1}}{g(m+8)} \\ & + \frac{d^7 (4m+11) \sqrt{d^2 - e^2 x^2} (gx)^{m+1} {}_2F_1\left(-\frac{5}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \frac{e^2 x^2}{d^2}\right)}{g(m+1)(m+8) \sqrt{1 - \frac{e^2 x^2}{d^2}}} \\ & + \frac{d^6 e (4m+29) \sqrt{d^2 - e^2 x^2} (gx)^{m+2} {}_2F_1\left(-\frac{5}{2}, \frac{m+2}{2}, \frac{m+4}{2}, \frac{e^2 x^2}{d^2}\right)}{g^2 (m+2)(m+9) \sqrt{1 - \frac{e^2 x^2}{d^2}}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(g*x)^m*(d + e*x)^3*(d^2 - e^2*x^2)^{(5/2)}, x]$

[Out]  $(-3*d*(g*x)^{(1+m)}*(d^2 - e^2*x^2)^{(7/2)})/(g*(8+m)) - (e*(g*x)^{(2+m)}*(d^2 - e^2*x^2)^{(7/2)})/(g^2*(9+m)) + (d^7*(11+4*m)*(g*x)^{(1+m)}*\text{Sqrt}[d^2 - e^2*x^2]*\text{Hypergeometric2F1}[-5/2, (1+m)/2, (3+m)/2, (e^2*x^2)/d^2])/(g*(1+m)*(8+m)*\text{Sqrt}[1 - (e^2*x^2)/d^2]) + (d^6*e*(29+4*m)*(g*x)^{(2+m)}*\text{Sqrt}[d^2 - e^2*x^2]*\text{Hypergeometric2F1}[-5/2, (2+m)/2, (4+m)/2, (e^2*x^2)/d^2])/(g^2*(2+m)*(9+m)*\text{Sqrt}[1 - (e^2*x^2)/d^2])$

**Rubi in Sympy [A]** time = 72.8265, size = 287, normalized size = 1.15

$$\begin{aligned} & \frac{d^7 (gx)^{m+1} \sqrt{d^2 - e^2 x^2} {}_2F_1\left(-\frac{5}{2}, \frac{m}{2} + \frac{1}{2}, \frac{m}{2} + \frac{3}{2}, \frac{e^2 x^2}{d^2}\right)}{g \sqrt{1 - \frac{e^2 x^2}{d^2}} (m+1)} + \frac{3d^6 e (gx)^{m+2} \sqrt{d^2 - e^2 x^2} {}_2F_1\left(-\frac{5}{2}, \frac{m}{2} + 1, \frac{m}{2} + 2, \frac{e^2 x^2}{d^2}\right)}{g^2 \sqrt{1 - \frac{e^2 x^2}{d^2}} (m+2)} \\ & + \frac{3d^5 e^2 (gx)^{m+3} \sqrt{d^2 - e^2 x^2} {}_2F_1\left(-\frac{5}{2}, \frac{m}{2} + \frac{3}{2}, \frac{m}{2} + \frac{5}{2}, \frac{e^2 x^2}{d^2}\right)}{g^3 \sqrt{1 - \frac{e^2 x^2}{d^2}} (m+3)} + \frac{d^4 e^3 (gx)^{m+4} \sqrt{d^2 - e^2 x^2} {}_2F_1\left(-\frac{5}{2}, \frac{m}{2} + 2, \frac{m}{2} + 3, \frac{e^2 x^2}{d^2}\right)}{g^4 \sqrt{1 - \frac{e^2 x^2}{d^2}} (m+4)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x)**m*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2),x)`

[Out]  $d^{7/2}(g*x)^{m+1}\sqrt{d^2 - e^2*x^2}\operatorname{hyper}((-5/2, m/2 + 1/2), (m/2 + 3/2, ), e^2*x^2/d^2)/(g*\sqrt{1 - e^2*x^2/d^2})^{m+1} + 3*d^{6/2}*e*(g*x)^{m+2}\sqrt{d^2 - e^2*x^2}\operatorname{hyper}((-5/2, m/2 + 1), (m/2 + 2, ), e^2*x^2/d^2)/(g^2*\sqrt{1 - e^2*x^2/d^2})^{m+2} + 3*d^{5/2}*e^2*(g*x)^{m+3}\sqrt{d^2 - e^2*x^2}\operatorname{hyper}((-5/2, m/2 + 3/2), (m/2 + 5/2, ), e^2*x^2/d^2)/(g^3*\sqrt{1 - e^2*x^2/d^2})^{m+3} + d^{4/2}*e^3*(g*x)^{m+4}\sqrt{d^2 - e^2*x^2}\operatorname{hyper}((-5/2, m/2 + 2), (m/2 + 3, ), e^2*x^2/d^2)/(g^4*\sqrt{1 - e^2*x^2/d^2})^{m+4}$

**Mathematica [A]** time = 0.715923, size = 377, normalized size = 1.51

$$d^2 x \sqrt{1 - \frac{e^2 x^2}{d^2}} (g x)^m \left( \frac{e^7 x^7 {}_2F_1\left(-\frac{1}{2}, \frac{m}{2} + 4; \frac{m}{2} + 5; \frac{e^2 x^2}{d^2}\right)}{m+8} + \frac{3 d e^6 x^6 {}_2F_1\left(-\frac{1}{2}, \frac{m+7}{2}; \frac{m+9}{2}; \frac{e^2 x^2}{d^2}\right)}{m+7} + \frac{d^2 e^5 x^5 {}_2F_1\left(-\frac{1}{2}, \frac{m}{2} + 3; \frac{m}{2} + 4; \frac{e^2 x^2}{d^2}\right)}{m+6} + \frac{d^7 {}_2F_1\left(-\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}\right)}{m+1} \right) \sqrt{d^2}$$

Antiderivative was successfully verified.

[In] `Integrate[(g*x)^m*(d + e*x)^3*(d^2 - e^2*x^2)^(5/2),x]`

[Out]  $(d^2*x*(g*x)^m*\sqrt{1 - (e^2*x^2)/d^2})*((3*d^6*e*x*\operatorname{Hypergeometric2F1}[-1/2, 1 + m/2, 2 + m/2, (e^2*x^2)/d^2])/(2 + m) - (5*d^4*e^3*x^3*\operatorname{Hypergeometric2F1}[-1/2, 2 + m/2, 3 + m/2, (e^2*x^2)/d^2])/(4 + m) + (d^2*e^5*x^5*\operatorname{Hypergeometric2F1}[-1/2, 3 + m/2, 4 + m/2, (e^2*x^2)/d^2])/(6 + m) + (e^7*x^7*\operatorname{Hypergeometric2F1}[-1/2, 4 + m/2, 5 + m/2, (e^2*x^2)/d^2])/(8 + m) + (d^7*\operatorname{Hypergeometric2F1}[-1/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2])/(1 + m) + (d^5*e^2*x^2*\operatorname{Hypergeometric2F1}[-1/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2])/(3 + m) - (5*d^3*e^4*x^4*\operatorname{Hypergeometric2F1}[-1/2, (5 + m)/2, (7 + m)/2, (e^2*x^2)/d^2])/(5 + m) + (3*d*e^6*x^6*\operatorname{Hypergeometric2F1}[-1/2, (7 + m)/2, (9 + m)/2, (e^2*x^2)/d^2])/(7 + m))/\sqrt{d^2 - e^2*x^2}$

**Maple [F]** time = 0.099, size = 0, normalized size = 0.

$$\int (g x)^m (e x + d)^3 (-e^2 x^2 + d^2)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)`

[Out] `int((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^(5/2),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (-e^2 x^2 + d^2)^{\frac{5}{2}} (e x + d)^3 (g x)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3*(g*x)^m,x, algorithm="maxima")`



[Out] `integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3*(g*x)^m, x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

`integral((e^7*x^7 + 3*d*e^6*x^6 + d^2*e^5*x^5 - 5*d^3*e^4*x^4 - 5*d^4*e^3*x^3 + d^5*e^2*x^2 + 3*d^6*e*x + d^7)*sqrt(-e^2*x^2 + d^2)*(g*x)^m, x)`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3*(g*x)^m, x, algorithm="fricas")`

[Out] `integral((e^7*x^7 + 3*d*e^6*x^6 + d^2*e^5*x^5 - 5*d^3*e^4*x^4 - 5*d^4*e^3*x^3 + d^5*e^2*x^2 + 3*d^6*e*x + d^7)*sqrt(-e^2*x^2 + d^2)*(g*x)^m, x)`

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x)**m*(e*x+d)**3*(-e**2*x**2+d**2)**(5/2), x)`

[Out] Timed out

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int (-e^2x^2 + d^2)^{\frac{5}{2}}(ex + d)^3 (gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3*(g*x)^m, x, algorithm="giac")`

[Out] `integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^3*(g*x)^m, x)`

$$3.227 \quad \int (gx)^m (d + ex)^2 (d^2 - e^2 x^2)^{5/2} dx$$

**Optimal.** Leaf size=206

$$\frac{(d^2 - e^2 x^2)^{7/2} (gx)^{m+1}}{g(m+8)} + \frac{d^6 (2m+9) \sqrt{d^2 - e^2 x^2} (gx)^{m+1} {}_2F_1\left(-\frac{5}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \frac{e^2 x^2}{d^2}\right)}{g(m+1)(m+8) \sqrt{1 - \frac{e^2 x^2}{d^2}}} \\ + \frac{2d^5 e \sqrt{d^2 - e^2 x^2} (gx)^{m+2} {}_2F_1\left(-\frac{5}{2}, \frac{m+2}{2}; \frac{m+4}{2}; \frac{e^2 x^2}{d^2}\right)}{g^2 (m+2) \sqrt{1 - \frac{e^2 x^2}{d^2}}}$$

[Out] -(((g\*x)^(1+m)\*(d^2 - e^2\*x^2)^(7/2))/(g\*(8+m))) + (d^6\*(9+2\*m)\*(g\*x)^(1+m)\*Sqrt[d^2 - e^2\*x^2]\*Hypergeometric2F1[-5/2, (1+m)/2, (3+m)/2, (e^2\*x^2)/d^2])/(g\*(1+m)\*(8+m)\*Sqrt[1 - (e^2\*x^2)/d^2]) + (2\*d^5\*e\*(g\*x)^(2+m)\*Sqrt[d^2 - e^2\*x^2]\*Hypergeometric2F1[-5/2, (2+m)/2, (4+m)/2, (e^2\*x^2)/d^2])/(g^2\*(2+m)\*Sqrt[1 - (e^2\*x^2)/d^2])

**Rubi [A]** time = 0.423761, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$

$$\frac{(d^2 - e^2 x^2)^{7/2} (gx)^{m+1}}{g(m+8)} + \frac{d^6 (2m+9) \sqrt{d^2 - e^2 x^2} (gx)^{m+1} {}_2F_1\left(-\frac{5}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \frac{e^2 x^2}{d^2}\right)}{g(m+1)(m+8) \sqrt{1 - \frac{e^2 x^2}{d^2}}} \\ + \frac{2d^5 e \sqrt{d^2 - e^2 x^2} (gx)^{m+2} {}_2F_1\left(-\frac{5}{2}, \frac{m+2}{2}; \frac{m+4}{2}; \frac{e^2 x^2}{d^2}\right)}{g^2 (m+2) \sqrt{1 - \frac{e^2 x^2}{d^2}}}$$

Antiderivative was successfully verified.

[In] Int[(g\*x)^m\*(d + e\*x)^2\*(d^2 - e^2\*x^2)^(5/2), x]

[Out] -(((g\*x)^(1+m)\*(d^2 - e^2\*x^2)^(7/2))/(g\*(8+m))) + (d^6\*(9+2\*m)\*(g\*x)^(1+m)\*Sqrt[d^2 - e^2\*x^2]\*Hypergeometric2F1[-5/2, (1+m)/2, (3+m)/2, (e^2\*x^2)/d^2])/(g\*(1+m)\*(8+m)\*Sqrt[1 - (e^2\*x^2)/d^2]) + (2\*d^5\*e\*(g\*x)^(2+m)\*Sqrt[d^2 - e^2\*x^2]\*Hypergeometric2F1[-5/2, (2+m)/2, (4+m)/2, (e^2\*x^2)/d^2])/(g^2\*(2+m)\*Sqrt[1 - (e^2\*x^2)/d^2])

**Rubi in Sympy [A]** time = 52.6554, size = 214, normalized size = 1.04

$$\frac{d^6 (gx)^{m+1} \sqrt{d^2 - e^2 x^2} {}_2F_1\left(-\frac{5}{2}, \frac{m}{2} + \frac{1}{2}; \frac{m}{2} + \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{g \sqrt{1 - \frac{e^2 x^2}{d^2}} (m+1)} + \frac{2d^5 e (gx)^{m+2} \sqrt{d^2 - e^2 x^2} {}_2F_1\left(-\frac{5}{2}, \frac{m}{2} + 1; \frac{m}{2} + 2; \frac{e^2 x^2}{d^2}\right)}{g^2 \sqrt{1 - \frac{e^2 x^2}{d^2}} (m+2)} \\ + \frac{d^4 e^2 (gx)^{m+3} \sqrt{d^2 - e^2 x^2} {}_2F_1\left(-\frac{5}{2}, \frac{m}{2} + \frac{3}{2}; \frac{m}{2} + \frac{5}{2}; \frac{e^2 x^2}{d^2}\right)}{g^3 \sqrt{1 - \frac{e^2 x^2}{d^2}} (m+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x)\*\*m\*(e\*x+d)\*\*2\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2), x)

[Out] d\*\*6\*(g\*x)\*\*(m+1)\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)\*hyper((-5/2, m/2 + 1/2), (m/2 + 3/2, ), e\*\*2\*x\*\*2/d\*\*2)/(g\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)\*(m +

1)) + 2\*d\*\*5\*e\*(g\*x)\*\*(m+2)\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)\*hyper((-5/2, m/2 + 1), (m/2 + 2, ), e\*\*2\*x\*\*2/d\*\*2)/(g\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)\*(m+2)) + d\*\*4\*e\*\*2\*(g\*x)\*\*(m+3)\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)\*hyper((-5/2, m/2 + 3/2), (m/2 + 5/2, ), e\*\*2\*x\*\*2/d\*\*2)/(g\*\*3\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)\*(m+3))

**Mathematica [A]** time = 0.555276, size = 335, normalized size = 1.63

$$d^2 x \sqrt{1 - \frac{e^2 x^2}{d^2}} (g x)^m \left( \frac{e^6 x^6 {}_2F_1\left(-\frac{1}{2}, \frac{m+7}{2}; \frac{m+9}{2}; \frac{e^2 x^2}{d^2}\right)}{m+7} + \frac{2 d e^5 x^5 {}_2F_1\left(-\frac{1}{2}, \frac{m}{2}+3; \frac{m}{2}+4; \frac{e^2 x^2}{d^2}\right)}{m+6} - \frac{d^2 e^4 x^4 {}_2F_1\left(-\frac{1}{2}, \frac{m+5}{2}; \frac{m+7}{2}; \frac{e^2 x^2}{d^2}\right)}{m+5} + \frac{d^6 {}_2F_1\left(-\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \frac{e^2 x^2}{d^2}\right)}{m+1} \right) \sqrt{d^2 - e^2 x^2}$$

Antiderivative was successfully verified.

[In] Integrate[(g\*x)^m\*(d + e\*x)^2\*(d^2 - e^2\*x^2)^(5/2), x]

[Out] (d^2\*x\*(g\*x)^m\*Sqrt[1 - (e^2\*x^2)/d^2]\*((2\*d^5\*e\*x\*Hypergeometric2F1[-1/2, 1 + m/2, 2 + m/2, (e^2\*x^2)/d^2])/(2 + m) - (4\*d^3\*e^3\*x^3\*Hypergeometric2F1[-1/2, 2 + m/2, 3 + m/2, (e^2\*x^2)/d^2])/(4 + m) + (2\*d\*e^5\*x^5\*Hypergeometric2F1[-1/2, 3 + m/2, 4 + m/2, (e^2\*x^2)/d^2])/(6 + m) + (d^6\*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, (e^2\*x^2)/d^2])/(1 + m) - (d^4\*e^2\*x^2\*Hypergeometric2F1[-1/2, (3 + m)/2, (5 + m)/2, (e^2\*x^2)/d^2])/(3 + m) - (d^2\*e^4\*x^4\*Hypergeometric2F1[-1/2, (5 + m)/2, (7 + m)/2, (e^2\*x^2)/d^2])/(5 + m) + (e^6\*x^6\*Hypergeometric2F1[-1/2, (7 + m)/2, (9 + m)/2, (e^2\*x^2)/d^2])/(7 + m))/Sqrt[d^2 - e^2\*x^2]

**Maple [F]** time = 0.038, size = 0, normalized size = 0.

$$\int (g x)^m (e x + d)^2 (-e^2 x^2 + d^2)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x)^m\*(e\*x+d)^2\*(-e^2\*x^2+d^2)^(5/2), x)

[Out] int((g\*x)^m\*(e\*x+d)^2\*(-e^2\*x^2+d^2)^(5/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (-e^2 x^2 + d^2)^{\frac{5}{2}} (e x + d)^2 (g x)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^2\*(g\*x)^m, x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)^2\*(g\*x)^m, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^6 x^6 + 2 d e^5 x^5 - d^2 e^4 x^4 - 4 d^3 e^3 x^3 - d^4 e^2 x^2 + 2 d^5 e x + d^6\right) \sqrt{-e^2 x^2 + d^2} (g x)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^2*(g*x)^m,x, algorithm="fricas")`

[Out] `integral((e^6*x^6 + 2*d*e^5*x^5 - d^2*e^4*x^4 - 4*d^3*e^3*x^3 - d^4*e^2*x^2 + 2*d^5*e*x + d^6)*sqrt(-e^2*x^2 + d^2)*(g*x)^m, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x)**m*(e*x+d)**2*(-e**2*x**2+d**2)**(5/2),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (-e^2x^2 + d^2)^{\frac{5}{2}}(ex + d)^2(gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^2*(g*x)^m,x, algorithm="giac")`

[Out] `integrate((-e^2*x^2 + d^2)^(5/2)*(e*x + d)^2*(g*x)^m, x)`

$$3.228 \quad \int (gx)^m (d + ex) (d^2 - e^2 x^2)^{5/2} dx$$

**Optimal.** Leaf size=162

$$\frac{d^5 \sqrt{d^2 - e^2 x^2} (gx)^{m+1} {}_2F_1\left(-\frac{5}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \frac{e^2 x^2}{d^2}\right)}{g(m+1) \sqrt{1 - \frac{e^2 x^2}{d^2}}} + \frac{d^4 e \sqrt{d^2 - e^2 x^2} (gx)^{m+2} {}_2F_1\left(-\frac{5}{2}, \frac{m+2}{2}, \frac{m+4}{2}, \frac{e^2 x^2}{d^2}\right)}{g^2(m+2) \sqrt{1 - \frac{e^2 x^2}{d^2}}}$$

[Out]  $(d^5 (g*x)^{(1+m)} \sqrt{d^2 - e^2*x^2} \text{Hypergeometric2F1}[-5/2, (1+m)/2, (3+m)/2, (e^2*x^2)/d^2]) / (g*(1+m) \sqrt{1 - (e^2*x^2)/d^2}) + (d^4*e*(g*x)^{(2+m)} \sqrt{d^2 - e^2*x^2} \text{Hypergeometric2F1}[-5/2, (2+m)/2, (4+m)/2, (e^2*x^2)/d^2]) / (g^2*(2+m) \sqrt{1 - (e^2*x^2)/d^2})$

**Rubi [A]** time = 0.224537, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{d^5 \sqrt{d^2 - e^2 x^2} (gx)^{m+1} {}_2F_1\left(-\frac{5}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \frac{e^2 x^2}{d^2}\right)}{g(m+1) \sqrt{1 - \frac{e^2 x^2}{d^2}}} + \frac{d^4 e \sqrt{d^2 - e^2 x^2} (gx)^{m+2} {}_2F_1\left(-\frac{5}{2}, \frac{m+2}{2}, \frac{m+4}{2}, \frac{e^2 x^2}{d^2}\right)}{g^2(m+2) \sqrt{1 - \frac{e^2 x^2}{d^2}}}$$

Antiderivative was successfully verified.

[In] Int[(g\*x)^m\*(d + e\*x)\*(d^2 - e^2\*x^2)^(5/2), x]

[Out]  $(d^5 (g*x)^{(1+m)} \sqrt{d^2 - e^2*x^2} \text{Hypergeometric2F1}[-5/2, (1+m)/2, (3+m)/2, (e^2*x^2)/d^2]) / (g*(1+m) \sqrt{1 - (e^2*x^2)/d^2}) + (d^4*e*(g*x)^{(2+m)} \sqrt{d^2 - e^2*x^2} \text{Hypergeometric2F1}[-5/2, (2+m)/2, (4+m)/2, (e^2*x^2)/d^2]) / (g^2*(2+m) \sqrt{1 - (e^2*x^2)/d^2})$

**Rubi in Sympy [A]** time = 26.5831, size = 138, normalized size = 0.85

$$\frac{d^5 (gx)^{m+1} \sqrt{d^2 - e^2 x^2} {}_2F_1\left(-\frac{5}{2}, \frac{m}{2} + \frac{1}{2}, \frac{m}{2} + \frac{3}{2}, \frac{e^2 x^2}{d^2}\right)}{g \sqrt{1 - \frac{e^2 x^2}{d^2}} (m+1)} + \frac{d^4 e (gx)^{m+2} \sqrt{d^2 - e^2 x^2} {}_2F_1\left(-\frac{5}{2}, \frac{m}{2} + 1, \frac{m}{2} + 2, \frac{e^2 x^2}{d^2}\right)}{g^2 \sqrt{1 - \frac{e^2 x^2}{d^2}} (m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x)\*\*m\*(e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2), x)

[Out]  $d**5*(g*x)**(m+1)*\text{sqrt}(d**2 - e**2*x**2)*\text{hyper}((-5/2, m/2 + 1/2), (m/2 + 3/2, ), e**2*x**2/d**2)/(g*\text{sqrt}(1 - e**2*x**2/d**2))*(m+1) + d**4*e*(g*x)**(m+2)*\text{sqrt}(d**2 - e**2*x**2)*\text{hyper}((-5/2, m/2 + 1), (m/2 + 2, ), e**2*x**2/d**2)/(g**2*\text{sqrt}(1 - e**2*x**2/d**2))*(m+2)$

**Mathematica [A]** time = 0.396327, size = 289, normalized size = 1.78

$$\frac{d^2 x \sqrt{1 - \frac{e^2 x^2}{d^2}} (gx)^m \left( \frac{e^5 x^5 {}_2F_1\left(-\frac{1}{2}, \frac{m}{2}+3, \frac{m}{2}+4, \frac{e^2 x^2}{d^2}\right)}{m+6} + \frac{d e^4 x^4 {}_2F_1\left(-\frac{1}{2}, \frac{m+5}{2}, \frac{m+7}{2}, \frac{e^2 x^2}{d^2}\right)}{m+5} - \frac{2 d^2 e^3 x^3 {}_2F_1\left(-\frac{1}{2}, \frac{m}{2}+2, \frac{m}{2}+3, \frac{e^2 x^2}{d^2}\right)}{m+4} + \frac{d^5 {}_2F_1\left(-\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \frac{e^2 x^2}{d^2}\right)}{m+1} \right)}{\sqrt{d^2 - e^2 x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(g\*x)^m\*(d + e\*x)\*(d^2 - e^2\*x^2)^(5/2),x]

[Out] (d^2\*x\*(g\*x)^m\*sqrt[1 - (e^2\*x^2)/d^2]\*((d^4\*e\*x\*Hypergeometric2F1[-1/2, 1 + m/2, 2 + m/2, (e^2\*x^2)/d^2])/(2 + m) - (2\*d^2\*e^3\*x^3\*Hypergeometric2F1[-1/2, 2 + m/2, 3 + m/2, (e^2\*x^2)/d^2])/(4 + m) + (e^5\*x^5\*Hypergeometric2F1[-1/2, 3 + m/2, 4 + m/2, (e^2\*x^2)/d^2])/(6 + m) + (d^5\*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, (e^2\*x^2)/d^2])/(1 + m) - (2\*d^3\*e^2\*x^2\*Hypergeometric2F1[-1/2, (3 + m)/2, (5 + m)/2, (e^2\*x^2)/d^2])/(3 + m) + (d\*e^4\*x^4\*Hypergeometric2F1[-1/2, (5 + m)/2, (7 + m)/2, (e^2\*x^2)/d^2])/(5 + m)))/sqrt[d^2 - e^2\*x^2]

**Maple [F]** time = 0.03, size = 0, normalized size = 0.

$$\int (gx)^m (ex + d) (-e^2x^2 + d^2)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x)^m\*(e\*x+d)\*(-e^2\*x^2+d^2)^(5/2),x)

[Out] int((g\*x)^m\*(e\*x+d)\*(-e^2\*x^2+d^2)^(5/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (-e^2x^2 + d^2)^{\frac{5}{2}}(ex + d)(gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)\*(g\*x)^m,x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)\*(g\*x)^m, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^5x^5 + de^4x^4 - 2d^2e^3x^3 - 2d^3e^2x^2 + d^4ex + d^5\right)\sqrt{-e^2x^2 + d^2}(gx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)\*(g\*x)^m,x, algorithm="fricas")

[Out] integral((e^5\*x^5 + d\*e^4\*x^4 - 2\*d^2\*e^3\*x^3 - 2\*d^3\*e^2\*x^2 + d^4\*e\*x + d^5)\*sqrt(-e^2\*x^2 + d^2)\*(g\*x)^m, x)

**Sympy [A]** time = 176.499, size = 374, normalized size = 2.31

$$\frac{d^6 g^m x x^m \left(\frac{m}{2} + \frac{1}{2}\right) {}_2F_1\left(-\frac{1}{2}, \frac{m}{2} + \frac{1}{2} \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{2\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{d^5 e g^m x^2 x^m \left(\frac{m}{2} + 1\right) {}_2F_1\left(-\frac{1}{2}, \frac{m}{2} + 1 \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{2\left(\frac{m}{2} + 2\right)}$$

$$- \frac{d^4 e^2 g^m x^3 x^m \left(\frac{m}{2} + \frac{3}{2}\right) {}_2F_1\left(-\frac{1}{2}, \frac{m}{2} + \frac{3}{2} \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{\left(\frac{m}{2} + \frac{5}{2}\right)} - \frac{d^3 e^3 g^m x^4 x^m \left(\frac{m}{2} + 2\right) {}_2F_1\left(-\frac{1}{2}, \frac{m}{2} + 2 \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{\left(\frac{m}{2} + 3\right)}$$

$$+ \frac{d^2 e^4 g^m x^5 x^m \left(\frac{m}{2} + \frac{5}{2}\right) {}_2F_1\left(-\frac{1}{2}, \frac{m}{2} + \frac{5}{2} \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{2\left(\frac{m}{2} + \frac{7}{2}\right)} + \frac{d e^5 g^m x^6 x^m \left(\frac{m}{2} + 3\right) {}_2F_1\left(-\frac{1}{2}, \frac{m}{2} + 3 \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{2\left(\frac{m}{2} + 4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x)\*\*m\*(e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2),x)

[Out] d\*\*6\*g\*\*m\*x\*x\*\*m\*gamma(m/2 + 1/2)\*hyper((-1/2, m/2 + 1/2), (m/2 + 3/2, ), e\*\*2\*x\*\*2\*exp\_polar(2\*I\*pi)/d\*\*2)/(2\*gamma(m/2 + 3/2)) + d\*\*5\*e\*g\*\*m\*x\*\*2\*x\*\*m\*gamma(m/2 + 1)\*hyper((-1/2, m/2 + 1), (m/2 + 2, ), e\*\*2\*x\*\*2\*exp\_polar(2\*I\*pi)/d\*\*2)/(2\*gamma(m/2 + 2)) - d\*\*4\*e\*\*2\*g\*\*m\*x\*\*3\*x\*\*m\*gamma(m/2 + 3/2)\*hyper((-1/2, m/2 + 3/2), (m/2 + 5/2, ), e\*\*2\*x\*\*2\*exp\_polar(2\*I\*pi)/d\*\*2)/gamma(m/2 + 5/2) - d\*\*3\*e\*\*3\*g\*\*m\*x\*\*4\*x\*\*m\*gamma(m/2 + 2)\*hyper((-1/2, m/2 + 2), (m/2 + 3, ), e\*\*2\*x\*\*2\*exp\_polar(2\*I\*pi)/d\*\*2)/gamma(m/2 + 3) + d\*\*2\*e\*\*4\*g\*\*m\*x\*\*5\*x\*\*m\*gamma(m/2 + 5/2)\*hyper((-1/2, m/2 + 5/2), (m/2 + 7/2, ), e\*\*2\*x\*\*2\*exp\_polar(2\*I\*pi)/d\*\*2)/(2\*gamma(m/2 + 7/2)) + d\*e\*\*5\*g\*\*m\*x\*\*6\*x\*\*m\*gamma(m/2 + 3)\*hyper((-1/2, m/2 + 3), (m/2 + 4, ), e\*\*2\*x\*\*2\*exp\_polar(2\*I\*pi)/d\*\*2)/(2\*gamma(m/2 + 4))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (-e^2 x^2 + d^2)^{\frac{5}{2}} (ex + d)(gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)\*(g\*x)^m,x, algorithm="giac")

[Out] integrate((-e^2\*x^2 + d^2)^(5/2)\*(e\*x + d)\*(g\*x)^m, x)

$$3.229 \quad \int (gx)^m (d^2 - e^2 x^2)^{5/2} dx$$

**Optimal.** Leaf size=80

$$\frac{d^4 \sqrt{d^2 - e^2 x^2} (gx)^{m+1} {}_2F_1\left(-\frac{5}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \frac{e^2 x^2}{d^2}\right)}{g(m+1) \sqrt{1 - \frac{e^2 x^2}{d^2}}}$$

[Out] (d^4\*(g\*x)^(1+m)\*Sqrt[d^2 - e^2\*x^2]\*Hypergeometric2F1[-5/2, (1+m)/2, (3+m)/2, (e^2\*x^2)/d^2])/(g\*(1+m)\*Sqrt[1 - (e^2\*x^2)/d^2])

**Rubi [A]** time = 0.0723568, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{d^4 \sqrt{d^2 - e^2 x^2} (gx)^{m+1} {}_2F_1\left(-\frac{5}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \frac{e^2 x^2}{d^2}\right)}{g(m+1) \sqrt{1 - \frac{e^2 x^2}{d^2}}}$$

Antiderivative was successfully verified.

[In] Int[(g\*x)^m\*(d^2 - e^2\*x^2)^(5/2), x]

[Out] (d^4\*(g\*x)^(1+m)\*Sqrt[d^2 - e^2\*x^2]\*Hypergeometric2F1[-5/2, (1+m)/2, (3+m)/2, (e^2\*x^2)/d^2])/(g\*(1+m)\*Sqrt[1 - (e^2\*x^2)/d^2])

**Rubi in Sympy [A]** time = 11.674, size = 68, normalized size = 0.85

$$\frac{d^4 (gx)^{m+1} \sqrt{d^2 - e^2 x^2} {}_2F_1\left(-\frac{5}{2}, \frac{m}{2} + \frac{1}{2}; \frac{m}{2} + \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{g \sqrt{1 - \frac{e^2 x^2}{d^2}} (m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x)\*\*m\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2), x)

[Out] d\*\*4\*(g\*x)\*\*(m+1)\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)\*hyper((-5/2, m/2 + 1/2), (m/2 + 3/2, ), e\*\*2\*x\*\*2/d\*\*2)/(g\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)\*(m+1))

**Mathematica [B]** time = 0.165613, size = 183, normalized size = 2.29

$$\frac{x \sqrt{d^2 - e^2 x^2} (gx)^m \left( d^4 (m^2 + 8m + 15) {}_2F_1\left(-\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \frac{e^2 x^2}{d^2}\right) - e^2 (m+1) x^2 \left( 2d^2 (m+5) {}_2F_1\left(-\frac{1}{2}, \frac{m+3}{2}; \frac{m+5}{2}; \frac{e^2 x^2}{d^2}\right) - e^2 (m+1) x^2 \right) \right)}{(m+1)(m+3)(m+5) \sqrt{1 - \frac{e^2 x^2}{d^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(g\*x)^m\*(d^2 - e^2\*x^2)^(5/2), x]

[Out] (x\*(g\*x)^m\*Sqrt[d^2 - e^2\*x^2]\*(d^4\*(15 + 8\*m + m^2)\*Hypergeometric2F1[-1/2, (1+m)/2, (3+m)/2, (e^2\*x^2)/d^2] - e^2\*(1+m)\*x^2)



$$2 \cdot (2 \cdot d^2 \cdot (5 + m) \cdot \text{Hypergeometric2F1}[-1/2, (3 + m)/2, (5 + m)/2, (e^{2 \cdot x^2})/d^2] - e^{2 \cdot (3 + m) \cdot x^2} \cdot \text{Hypergeometric2F1}[-1/2, (5 + m)/2, (7 + m)/2, (e^{2 \cdot x^2})/d^2])) / ((1 + m) \cdot (3 + m) \cdot (5 + m) \cdot \text{Sqrt}[1 - (e^{2 \cdot x^2})/d^2])$$

**Maple [F]** time = 0.028, size = 0, normalized size = 0.

$$\int (gx)^m (-e^2x^2 + d^2)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x)^m\*(-e^2\*x^2+d^2)^(5/2),x)

[Out] int((g\*x)^m\*(-e^2\*x^2+d^2)^(5/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (-e^2x^2 + d^2)^{\frac{5}{2}} (gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(g\*x)^m,x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^(5/2)\*(g\*x)^m, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^4x^4 - 2d^2e^2x^2 + d^4\right)\sqrt{-e^2x^2 + d^2}(gx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(g\*x)^m,x, algorithm="fricas")

[Out] integral((e^4\*x^4 - 2\*d^2\*e^2\*x^2 + d^4)\*sqrt(-e^2\*x^2 + d^2)\*(g\*x)^m, x)

**Sympy [A]** time = 134.14, size = 61, normalized size = 0.76

$$\frac{d^5 g^m x x^m \left(\frac{m}{2} + \frac{1}{2}\right) {}_2F_1\left(-\frac{5}{2}, \frac{m}{2} + \frac{1}{2} \mid \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{2 \left(\frac{m}{2} + \frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x)\*\*m\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2),x)

[Out] d\*\*5\*g\*\*m\*x\*x\*\*m\*gamma(m/2 + 1/2)\*hyper((-5/2, m/2 + 1/2), (m/2 + 3/2, ), e\*\*2\*x\*\*2\*exp\_polar(2\*I\*pi)/d\*\*2)/(2\*gamma(m/2 + 3/2))

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (-e^2 x^2 + d^2)^{\frac{5}{2}} (gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m,x, algorithm="giac")`

[Out] `integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m, x)`

$$3.230 \quad \int \frac{(gx)^m (d^2 - e^2 x^2)^{5/2}}{d + ex} dx$$

**Optimal.** Leaf size=163

$$\frac{d^3 \sqrt{d^2 - e^2 x^2} (gx)^{m+1} {}_2F_1\left(-\frac{3}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \frac{e^2 x^2}{d^2}\right)}{g(m+1) \sqrt{1 - \frac{e^2 x^2}{d^2}}} - \frac{d^2 e \sqrt{d^2 - e^2 x^2} (gx)^{m+2} {}_2F_1\left(-\frac{3}{2}, \frac{m+2}{2}; \frac{m+4}{2}; \frac{e^2 x^2}{d^2}\right)}{g^2 (m+2) \sqrt{1 - \frac{e^2 x^2}{d^2}}}$$

[Out]  $(d^3 (g^* x)^{(1+m)} \sqrt{d^2 - e^2 x^2} \text{Hypergeometric2F1}[-3/2, (1+m)/2, (3+m)/2, (e^2 x^2)/d^2]) / (g^* (1+m) \sqrt{1 - (e^2 x^2)/d^2}) - (d^2 e^* (g^* x)^{(2+m)} \sqrt{d^2 - e^2 x^2} \text{Hypergeometric2F1}[-3/2, (2+m)/2, (4+m)/2, (e^2 x^2)/d^2]) / (g^2 (2+m) \sqrt{1 - (e^2 x^2)/d^2})$

**Rubi [A]** time = 0.433023, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$\frac{d^3 \sqrt{d^2 - e^2 x^2} (gx)^{m+1} {}_2F_1\left(-\frac{3}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \frac{e^2 x^2}{d^2}\right)}{g(m+1) \sqrt{1 - \frac{e^2 x^2}{d^2}}} - \frac{d^2 e \sqrt{d^2 - e^2 x^2} (gx)^{m+2} {}_2F_1\left(-\frac{3}{2}, \frac{m+2}{2}; \frac{m+4}{2}; \frac{e^2 x^2}{d^2}\right)}{g^2 (m+2) \sqrt{1 - \frac{e^2 x^2}{d^2}}}$$

Antiderivative was successfully verified.

[In] Int[((g\*x)^m\*(d^2 - e^2\*x^2)^(5/2))/(d + e\*x), x]

[Out]  $(d^3 (g^* x)^{(1+m)} \sqrt{d^2 - e^2 x^2} \text{Hypergeometric2F1}[-3/2, (1+m)/2, (3+m)/2, (e^2 x^2)/d^2]) / (g^* (1+m) \sqrt{1 - (e^2 x^2)/d^2}) - (d^2 e^* (g^* x)^{(2+m)} \sqrt{d^2 - e^2 x^2} \text{Hypergeometric2F1}[-3/2, (2+m)/2, (4+m)/2, (e^2 x^2)/d^2]) / (g^2 (2+m) \sqrt{1 - (e^2 x^2)/d^2})$

**Rubi in Sympy [A]** time = 50.8948, size = 138, normalized size = 0.85

$$\frac{d^3 (gx)^{m+1} \sqrt{d^2 - e^2 x^2} {}_2F_1\left(-\frac{3}{2}, \frac{m}{2} + \frac{1}{2}; \frac{m}{2} + \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{g \sqrt{1 - \frac{e^2 x^2}{d^2}} (m+1)} - \frac{d^2 e (gx)^{m+2} \sqrt{d^2 - e^2 x^2} {}_2F_1\left(-\frac{3}{2}, \frac{m}{2} + 1; \frac{m}{2} + 2; \frac{e^2 x^2}{d^2}\right)}{g^2 \sqrt{1 - \frac{e^2 x^2}{d^2}} (m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x)\*\*m\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/(e\*x+d), x)

[Out]  $d^{*3} (g^* x)^{(m+1)} \sqrt{d^{*2} - e^{*2} x^{*2}} \text{hyper}((-3/2, m/2 + 1/2), (m/2 + 3/2, ), e^{*2} x^{*2}/d^{*2}) / (g^* \sqrt{1 - e^{*2} x^{*2}/d^{*2}})^{(m+1)} - d^{*2} e^* (g^* x)^{(m+2)} \sqrt{d^{*2} - e^{*2} x^{*2}} \text{hyper}((-3/2, m/2 + 1), (m/2 + 2, ), e^{*2} x^{*2}/d^{*2}) / (g^{*2} \sqrt{1 - e^{*2} x^{*2}/d^{*2}})^{(m+2)}$

**Mathematica [A]** time = 0.226541, size = 203, normalized size = 1.25

$$\frac{d^3 x \sqrt{1 - \frac{e^2 x^2}{d^2}} (gx)^m \left( -\frac{d^2 e x {}_2F_1\left(-\frac{1}{2}, \frac{m}{2} + 1; \frac{m}{2} + 2; \frac{e^2 x^2}{d^2}\right)}{m+2} - \frac{d e^2 x^2 {}_2F_1\left(-\frac{1}{2}, \frac{m+3}{2}; \frac{m+5}{2}; \frac{e^2 x^2}{d^2}\right)}{m+3} + \frac{e^3 x^3 {}_2F_1\left(-\frac{1}{2}, \frac{m}{2} + 2; \frac{m}{2} + 3; \frac{e^2 x^2}{d^2}\right)}{m+4} + \frac{d^3 {}_2F_1\left(-\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}\right)}{m+1} \right)}{\sqrt{d^2 - e^2 x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[((g\*x)^m\*(d^2 - e^2\*x^2)^(5/2))/(d + e\*x),x]

[Out] (d^2\*x\*(g\*x)^m\*sqrt[1 - (e^2\*x^2)/d^2]\*(-(d^2\*e\*x\*Hypergeometric2F1[-1/2, 1 + m/2, 2 + m/2, (e^2\*x^2)/d^2])/(2 + m)) + (e^3\*x^3\*Hypergeometric2F1[-1/2, 2 + m/2, 3 + m/2, (e^2\*x^2)/d^2])/(4 + m) + (d^3\*Hypergeometric2F1[-1/2, (1 + m)/2, (3 + m)/2, (e^2\*x^2)/d^2])/(1 + m) - (d\*e^2\*x^2\*Hypergeometric2F1[-1/2, (3 + m)/2, (5 + m)/2, (e^2\*x^2)/d^2])/(3 + m))/sqrt[d^2 - e^2\*x^2]

**Maple [F]** time = 0.049, size = 0, normalized size = 0.

$$\int \frac{(gx)^m}{ex+d} (-e^2x^2 + d^2)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x)^m\*(-e^2\*x^2+d^2)^(5/2)/(e\*x+d),x)

[Out] int((g\*x)^m\*(-e^2\*x^2+d^2)^(5/2)/(e\*x+d),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^{\frac{5}{2}} (gx)^m}{ex+d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(g\*x)^m/(e\*x + d),x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^(5/2)\*(g\*x)^m/(e\*x + d), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^3 - de^2x^2 - d^2ex + d^3\right)\sqrt{-e^2x^2 + d^2}(gx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(g\*x)^m/(e\*x + d),x, algorithm="fricas")

[Out] integral((e^3\*x^3 - d\*e^2\*x^2 - d^2\*e\*x + d^3)\*sqrt(-e^2\*x^2 + d^2)\*(g\*x)^m, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x)\*\*m\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/(e\*x+d),x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^{\frac{5}{2}} (gx)^m}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d), x, algorithm="giac")`

[Out] `integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d), x)`

$$3.231 \quad \int \frac{(gx)^m (d^2 - e^2 x^2)^{5/2}}{(d+ex)^2} dx$$

**Optimal.** Leaf size=204

$$\frac{2de\sqrt{d^2 - e^2 x^2} (gx)^{m+2} {}_2F_1\left(-\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, \frac{e^2 x^2}{d^2}\right)}{g^2(m+2)\sqrt{1 - \frac{e^2 x^2}{d^2}}} + \frac{d^2(2m+5)\sqrt{d^2 - e^2 x^2} (gx)^{m+1} {}_2F_1\left(-\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \frac{e^2 x^2}{d^2}\right)}{g(m+1)(m+4)\sqrt{1 - \frac{e^2 x^2}{d^2}}} - \frac{(d^2 - e^2 x^2)^{3/2} (gx)^{m+1}}{g(m+4)}$$

[Out] -(((g\*x)^(1+m)\*(d^2 - e^2\*x^2)^(3/2))/(g\*(4+m))) + (d^2\*(5+2\*m)\*(g\*x)^(1+m)\*Sqrt[d^2 - e^2\*x^2]\*Hypergeometric2F1[-1/2, (1+m)/2, (3+m)/2, (e^2\*x^2)/d^2])/(g\*(1+m)\*(4+m)\*Sqrt[1 - (e^2\*x^2)/d^2]) - (2\*d\*e\*(g\*x)^(2+m)\*Sqrt[d^2 - e^2\*x^2]\*Hypergeometric2F1[-1/2, (2+m)/2, (4+m)/2, (e^2\*x^2)/d^2])/(g^2\*(2+m)\*Sqrt[1 - (e^2\*x^2)/d^2])

**Rubi [A]** time = 0.485507, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$\frac{2de\sqrt{d^2 - e^2 x^2} (gx)^{m+2} {}_2F_1\left(-\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, \frac{e^2 x^2}{d^2}\right)}{g^2(m+2)\sqrt{1 - \frac{e^2 x^2}{d^2}}} + \frac{d^2(2m+5)\sqrt{d^2 - e^2 x^2} (gx)^{m+1} {}_2F_1\left(-\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \frac{e^2 x^2}{d^2}\right)}{g(m+1)(m+4)\sqrt{1 - \frac{e^2 x^2}{d^2}}} - \frac{(d^2 - e^2 x^2)^{3/2} (gx)^{m+1}}{g(m+4)}$$

Antiderivative was successfully verified.

[In] Int[((g\*x)^m\*(d^2 - e^2\*x^2)^(5/2))/(d + e\*x)^2, x]

[Out] -(((g\*x)^(1+m)\*(d^2 - e^2\*x^2)^(3/2))/(g\*(4+m))) + (d^2\*(5+2\*m)\*(g\*x)^(1+m)\*Sqrt[d^2 - e^2\*x^2]\*Hypergeometric2F1[-1/2, (1+m)/2, (3+m)/2, (e^2\*x^2)/d^2])/(g\*(1+m)\*(4+m)\*Sqrt[1 - (e^2\*x^2)/d^2]) - (2\*d\*e\*(g\*x)^(2+m)\*Sqrt[d^2 - e^2\*x^2]\*Hypergeometric2F1[-1/2, (2+m)/2, (4+m)/2, (e^2\*x^2)/d^2])/(g^2\*(2+m)\*Sqrt[1 - (e^2\*x^2)/d^2])

**Rubi in Sympy [A]** time = 77.7446, size = 209, normalized size = 1.02

$$\frac{d^2 (gx)^{m+1} \sqrt{d^2 - e^2 x^2} {}_2F_1\left(-\frac{1}{2}, \frac{m}{2} + \frac{1}{2}, \frac{m}{2} + \frac{3}{2}, \frac{e^2 x^2}{d^2}\right)}{g\sqrt{1 - \frac{e^2 x^2}{d^2}} (m+1)} - \frac{2de (gx)^{m+2} \sqrt{d^2 - e^2 x^2} {}_2F_1\left(-\frac{1}{2}, \frac{m}{2} + 1, \frac{m}{2} + 2, \frac{e^2 x^2}{d^2}\right)}{g^2\sqrt{1 - \frac{e^2 x^2}{d^2}} (m+2)} + \frac{e^2 (gx)^{m+3} \sqrt{d^2 - e^2 x^2} {}_2F_1\left(-\frac{1}{2}, \frac{m}{2} + \frac{3}{2}, \frac{m}{2} + \frac{5}{2}, \frac{e^2 x^2}{d^2}\right)}{g^3\sqrt{1 - \frac{e^2 x^2}{d^2}} (m+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x)\*\*m\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*(5/2)/(e\*x+d)\*\*2, x)

[Out]  $d^{2*(g*x)^{(m+1)}\sqrt{d^2 - e^{2*x^2}}\text{hyper}((-1/2, m/2 + 1/2), (m/2 + 3/2, ), e^{2*x^2}/d^2)/(g*\sqrt{1 - e^{2*x^2}/d^2})^{(m+1)} - 2*d*e*(g*x)^{(m+2)}\sqrt{d^2 - e^{2*x^2}}\text{hyper}((-1/2, m/2 + 1), (m/2 + 2, ), e^{2*x^2}/d^2)/(g^2*\sqrt{1 - e^{2*x^2}/d^2})^{(m+2)} + e^{2*x^2}(g*x)^{(m+3)}\sqrt{d^2 - e^{2*x^2}}\text{hyper}((-1/2, m/2 + 3/2), (m/2 + 5/2, ), e^{2*x^2}/d^2)/(g^3*\sqrt{1 - e^{2*x^2}/d^2})^{(m+3)}$

---

**Mathematica [A]** time = 0.164661, size = 175, normalized size = 0.86

$$\frac{x\sqrt{d^2 - e^2x^2}(gx)^m \left( (m+2) \left( d^2(m+3) {}_2F_1 \left( -\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}; \frac{e^2x^2}{d^2} \right) + e^2(m+1)x^2 {}_2F_1 \left( -\frac{1}{2}, \frac{m+3}{2}, \frac{m+5}{2}; \frac{e^2x^2}{d^2} \right) \right) - 2de(m^2 + 4m + 3)}{(m+1)(m+2)(m+3)\sqrt{1 - \frac{e^2x^2}{d^2}}}$$

Antiderivative was successfully verified.

[In] Integrate[((g\*x)^m\*(d^2 - e^2\*x^2)^(5/2))/(d + e\*x)^2,x]

[Out]  $(x*(g*x)^m*\text{Sqrt}[d^2 - e^2*x^2]*(-2*d*e*(3 + 4*m + m^2)*x*\text{Hypergeometric2F1}[-1/2, 1 + m/2, 2 + m/2, (e^2*x^2)/d^2] + (2 + m)*(d^2*(3 + m)*\text{Hypergeometric2F1}[-1/2, (1 + m)/2, (3 + m)/2, (e^2*x^2)/d^2] + e^2*(1 + m)*x^2*\text{Hypergeometric2F1}[-1/2, (3 + m)/2, (5 + m)/2, (e^2*x^2)/d^2]))/((1 + m)*(2 + m)*(3 + m)*\text{Sqrt}[1 - (e^2*x^2)/d^2])$

---

**Maple [F]** time = 0.042, size = 0, normalized size = 0.

$$\int \frac{(gx)^m}{(ex + d)^2} (-e^2x^2 + d^2)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x)^m\*(-e^2\*x^2+d^2)^(5/2)/(e\*x+d)^2,x)

[Out] int((g\*x)^m\*(-e^2\*x^2+d^2)^(5/2)/(e\*x+d)^2,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^{\frac{5}{2}}(gx)^m}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(g\*x)^m/(e\*x + d)^2,x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^(5/2)\*(g\*x)^m/(e\*x + d)^2, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((e^2x^2 - 2dex + d^2)\sqrt{-e^2x^2 + d^2}(gx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d)^2,x, algorithm="fricas")`

[Out] `integral((e^2*x^2 - 2*d*e*x + d^2)*sqrt(-e^2*x^2 + d^2)*(g*x)^m, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x)**m*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**2,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^{\frac{5}{2}}(gx)^m}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d)^2,x, algorithm="giac")`

[Out] `integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d)^2, x)`



$$3.232 \quad \int \frac{(gx)^m (d^2 - e^2 x^2)^{5/2}}{(d+ex)^3} dx$$

**Optimal.** Leaf size=250

$$\frac{d^2 e (4m+11) \sqrt{1 - \frac{e^2 x^2}{d^2}} (gx)^{m+2} {}_2F_1\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, \frac{e^2 x^2}{d^2}\right)}{g^2 (m+2)(m+3) \sqrt{d^2 - e^2 x^2}} + \frac{e \sqrt{d^2 - e^2 x^2} (gx)^{m+2}}{g^2 (m+3)}$$

$$- \frac{3d \sqrt{d^2 - e^2 x^2} (gx)^{m+1}}{g(m+2)} + \frac{d^3 (4m+5) \sqrt{1 - \frac{e^2 x^2}{d^2}} (gx)^{m+1} {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \frac{e^2 x^2}{d^2}\right)}{g(m+1)(m+2) \sqrt{d^2 - e^2 x^2}}$$

[Out]  $(-3*d*(g*x)^{(1+m)*\text{Sqrt}[d^2 - e^2*x^2]}/(g*(2+m)) + (e*(g*x)^{(2+m)*\text{Sqrt}[d^2 - e^2*x^2]}/(g^2*(3+m)) + (d^3*(5+4*m)*(g*x)^{(1+m)*\text{Sqrt}[1 - (e^2*x^2)/d^2]}*\text{Hypergeometric2F1}[1/2, (1+m)/2, (3+m)/2, (e^2*x^2)/d^2]}/(g*(1+m)*(2+m)*\text{Sqrt}[d^2 - e^2*x^2]) - (d^2*e*(11+4*m)*(g*x)^{(2+m)*\text{Sqrt}[1 - (e^2*x^2)/d^2]}*\text{Hypergeometric2F1}[1/2, (2+m)/2, (4+m)/2, (e^2*x^2)/d^2]}/(g^2*(2+m)*(3+m)*\text{Sqrt}[d^2 - e^2*x^2])$

**Rubi [A]** time = 0.736592, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$\frac{d^2 e (4m+11) \sqrt{1 - \frac{e^2 x^2}{d^2}} (gx)^{m+2} {}_2F_1\left(\frac{1}{2}, \frac{m+2}{2}, \frac{m+4}{2}, \frac{e^2 x^2}{d^2}\right)}{g^2 (m+2)(m+3) \sqrt{d^2 - e^2 x^2}} + \frac{e \sqrt{d^2 - e^2 x^2} (gx)^{m+2}}{g^2 (m+3)}$$

$$- \frac{3d \sqrt{d^2 - e^2 x^2} (gx)^{m+1}}{g(m+2)} + \frac{d^3 (4m+5) \sqrt{1 - \frac{e^2 x^2}{d^2}} (gx)^{m+1} {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \frac{e^2 x^2}{d^2}\right)}{g(m+1)(m+2) \sqrt{d^2 - e^2 x^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}(((g*x)^m*(d^2 - e^2*x^2)^{(5/2)})/(d + e*x)^3, x)$

[Out]  $(-3*d*(g*x)^{(1+m)*\text{Sqrt}[d^2 - e^2*x^2]}/(g*(2+m)) + (e*(g*x)^{(2+m)*\text{Sqrt}[d^2 - e^2*x^2]}/(g^2*(3+m)) + (d^3*(5+4*m)*(g*x)^{(1+m)*\text{Sqrt}[1 - (e^2*x^2)/d^2]}*\text{Hypergeometric2F1}[1/2, (1+m)/2, (3+m)/2, (e^2*x^2)/d^2]}/(g*(1+m)*(2+m)*\text{Sqrt}[d^2 - e^2*x^2]) - (d^2*e*(11+4*m)*(g*x)^{(2+m)*\text{Sqrt}[1 - (e^2*x^2)/d^2]}*\text{Hypergeometric2F1}[1/2, (2+m)/2, (4+m)/2, (e^2*x^2)/d^2]}/(g^2*(2+m)*(3+m)*\text{Sqrt}[d^2 - e^2*x^2])$

**Rubi in Sympy [A]** time = 113.819, size = 274, normalized size = 1.1

$$\frac{d (gx)^{m+1} \sqrt{d^2 - e^2 x^2} {}_2F_1\left(\frac{1}{2}, \frac{m}{2} + \frac{1}{2}, \frac{m}{2} + \frac{3}{2}, \frac{e^2 x^2}{d^2}\right)}{g \sqrt{1 - \frac{e^2 x^2}{d^2}} (m+1)} - \frac{3e (gx)^{m+2} \sqrt{d^2 - e^2 x^2} {}_2F_1\left(\frac{1}{2}, \frac{m}{2} + 1, \frac{m}{2} + 2, \frac{e^2 x^2}{d^2}\right)}{g^2 \sqrt{1 - \frac{e^2 x^2}{d^2}} (m+2)}$$

$$+ \frac{3e^2 (gx)^{m+3} \sqrt{d^2 - e^2 x^2} {}_2F_1\left(\frac{1}{2}, \frac{m}{2} + \frac{3}{2}, \frac{m}{2} + \frac{5}{2}, \frac{e^2 x^2}{d^2}\right)}{d g^3 \sqrt{1 - \frac{e^2 x^2}{d^2}} (m+3)} - \frac{e^3 (gx)^{m+4} \sqrt{d^2 - e^2 x^2} {}_2F_1\left(\frac{1}{2}, \frac{m}{2} + 2, \frac{m}{2} + 3, \frac{e^2 x^2}{d^2}\right)}{d^2 g^4 \sqrt{1 - \frac{e^2 x^2}{d^2}} (m+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((g*x)**m*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**3, x)$

[Out]  $d*(g*x)^{(m+1)}*\sqrt{d^2 - e^2*x^2}*\text{hyper}((1/2, m/2 + 1/2), (m/2 + 3/2, ), e^2*x^2/d^2)/(g*\sqrt{1 - e^2*x^2/d^2})^{(m+1)}$   
 $- 3*e*(g*x)^{(m+2)}*\sqrt{d^2 - e^2*x^2}*\text{hyper}((1/2, m/2 + 1), (m/2 + 2, ), e^2*x^2/d^2)/(g^2*\sqrt{1 - e^2*x^2/d^2})^{(m+2)}$   
 $+ 3*e^2*(g*x)^{(m+3)}*\sqrt{d^2 - e^2*x^2}*\text{hyper}((1/2, m/2 + 3/2), (m/2 + 5/2, ), e^2*x^2/d^2)/(d*g^3*\sqrt{1 - e^2*x^2/d^2})^{(m+3)}$   
 $- e^3*(g*x)^{(m+4)}*\sqrt{d^2 - e^2*x^2}*\text{hyper}((1/2, m/2 + 2), (m/2 + 3, ), e^2*x^2/d^2)/(d^2*g^4*\sqrt{1 - e^2*x^2/d^2})^{(m+4)}$

**Mathematica [C]** time = 1.27142, size = 272, normalized size = 1.09

$$x(gx)^m \left( \frac{8d^3(m+2)^2\sqrt{d-ex}F_1\left(m+1;-\frac{1}{2},\frac{1}{2};m+2;\frac{ex}{d},-\frac{ex}{d}\right)}{\sqrt{d+ex}\left(2d(m+2)F_1\left(m+1;-\frac{1}{2},\frac{1}{2};m+2;\frac{ex}{d},-\frac{ex}{d}\right)-ex\left({}_2F_1\left(\frac{1}{2},\frac{m}{2}+1;\frac{m}{2}+2;\frac{e^2x^2}{d^2}\right)+F_1\left(m+2;-\frac{1}{2},\frac{3}{2};m+3;\frac{ex}{d},-\frac{ex}{d}\right)\right)}\right) + \frac{\sqrt{d^2-e^2x^2}\left(e(m+1)x {}_2F_1\left(-\frac{1}{2},\frac{m}{2}\right)\right)}{(m+1)(m+2)} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[((g\*x)^m\*(d^2 - e^2\*x^2)^(5/2))/(d + e\*x)^3,x]

[Out]  $(x*(g*x)^m*((\text{Sqrt}[d^2 - e^2*x^2])*(e*(1+m)*x*\text{Hypergeometric2F1}[-1/2, 1+m/2, 2+m/2, (e^2*x^2)/d^2] - 3*d*(2+m)*\text{Hypergeometric2F1}[-1/2, (1+m)/2, (3+m)/2, (e^2*x^2)/d^2]))/\text{Sqrt}[1 - (e^2*x^2)/d^2] + (8*d^3*(2+m)^2*\text{Sqrt}[d - e*x]*\text{AppellF1}[1+m, -1/2, 1/2, 2+m, (e*x)/d, -((e*x)/d)])/(\text{Sqrt}[d + e*x]*(2*d*(2+m)*\text{AppellF1}[1+m, -1/2, 1/2, 2+m, (e*x)/d, -((e*x)/d)] - e*x*(\text{AppellF1}[2+m, -1/2, 3/2, 3+m, (e*x)/d, -((e*x)/d)] + \text{HypergeometricPFQ}[\{1/2, 1+m/2\}, \{2+m/2\}, (e^2*x^2)/d^2])))))/((1+m)*(2+m))$

**Maple [F]** time = 0.041, size = 0, normalized size = 0.

$$\int \frac{(gx)^m}{(ex+d)^3} (-e^2x^2 + d^2)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x)^m\*(-e^2\*x^2+d^2)^(5/2)/(e\*x+d)^3,x)

[Out] int((g\*x)^m\*(-e^2\*x^2+d^2)^(5/2)/(e\*x+d)^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^{\frac{5}{2}}(gx)^m}{(ex+d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^(5/2)\*(g\*x)^m/(e\*x + d)^3,x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^(5/2)\*(g\*x)^m/(e\*x + d)^3, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e^2x^2 - 2dex + d^2)\sqrt{-e^2x^2 + d^2}(gx)^m}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d)^3,x, algorithm="fricas")`

[Out] `integral((e^2*x^2 - 2*d*e*x + d^2)*sqrt(-e^2*x^2 + d^2)*(g*x)^m/(e*x + d), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x)**m*(-e**2*x**2+d**2)**(5/2)/(e*x+d)**3,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^{\frac{5}{2}}(gx)^m}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d)^3,x, algorithm="giac")`

[Out] `integrate((-e^2*x^2 + d^2)^(5/2)*(g*x)^m/(e*x + d)^3, x)`

$$3.233 \quad \int \frac{(gx)^m(d+ex)^3}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=213

$$\frac{4(d+ex)(gx)^{m+1}}{5g(d^2-e^2x^2)^{5/2}} + \frac{e(7-4m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2} {}_2F_1\left(\frac{5}{2}, \frac{m+2}{2}, \frac{m+4}{2}, \frac{e^2x^2}{d^2}\right)}{5d^4g^2(m+2)\sqrt{d^2-e^2x^2}}$$

$$+ \frac{(1-4m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1} {}_2F_1\left(\frac{5}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \frac{e^2x^2}{d^2}\right)}{5d^3g(m+1)\sqrt{d^2-e^2x^2}}$$

[Out]  $(4*(g*x)^{(1+m)*(d+e*x)})/(5*g*(d^2-e^2*x^2)^{(5/2)}) + ((1-4*m)*(g*x)^{(1+m)*\text{Sqrt}[1-(e^2*x^2)/d^2]}*\text{Hypergeometric2F1}[5/2, (1+m)/2, (3+m)/2, (e^2*x^2)/d^2])/(5*d^4*g^2*(1+m)*\text{Sqrt}[d^2-e^2*x^2]) + (e*(7-4*m)*(g*x)^{(2+m)*\text{Sqrt}[1-(e^2*x^2)/d^2]}*\text{Hypergeometric2F1}[5/2, (2+m)/2, (4+m)/2, (e^2*x^2)/d^2])/(5*d^4*g^2*(2+m)*\text{Sqrt}[d^2-e^2*x^2])$

**Rubi [A]** time = 0.430359, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$

$$\frac{4(d+ex)(gx)^{m+1}}{5g(d^2-e^2x^2)^{5/2}} + \frac{e(7-4m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2} {}_2F_1\left(\frac{5}{2}, \frac{m+2}{2}, \frac{m+4}{2}, \frac{e^2x^2}{d^2}\right)}{5d^4g^2(m+2)\sqrt{d^2-e^2x^2}}$$

$$+ \frac{(1-4m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1} {}_2F_1\left(\frac{5}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \frac{e^2x^2}{d^2}\right)}{5d^3g(m+1)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int $\left[\frac{(g*x)^m*(d+e*x)^3}{(d^2-e^2*x^2)^{(7/2)}, x\right]$

[Out]  $(4*(g*x)^{(1+m)*(d+e*x)})/(5*g*(d^2-e^2*x^2)^{(5/2)}) + ((1-4*m)*(g*x)^{(1+m)*\text{Sqrt}[1-(e^2*x^2)/d^2]}*\text{Hypergeometric2F1}[5/2, (1+m)/2, (3+m)/2, (e^2*x^2)/d^2])/(5*d^4*g^2*(1+m)*\text{Sqrt}[d^2-e^2*x^2]) + (e*(7-4*m)*(g*x)^{(2+m)*\text{Sqrt}[1-(e^2*x^2)/d^2]}*\text{Hypergeometric2F1}[5/2, (2+m)/2, (4+m)/2, (e^2*x^2)/d^2])/(5*d^4*g^2*(2+m)*\text{Sqrt}[d^2-e^2*x^2])$

**Rubi in Sympy [A]** time = 71.1337, size = 280, normalized size = 1.31

$$\frac{(gx)^{m+1} \sqrt{d^2-e^2x^2} {}_2F_1\left(\frac{7}{2}, \frac{m}{2} + \frac{1}{2} \middle| \frac{e^2x^2}{d^2}\right)}{d^5g\sqrt{1-\frac{e^2x^2}{d^2}}(m+1)} + \frac{3e(gx)^{m+2} \sqrt{d^2-e^2x^2} {}_2F_1\left(\frac{7}{2}, \frac{m}{2} + 1 \middle| \frac{e^2x^2}{d^2}\right)}{d^6g^2\sqrt{1-\frac{e^2x^2}{d^2}}(m+2)}$$

$$+ \frac{3e^2(gx)^{m+3} \sqrt{d^2-e^2x^2} {}_2F_1\left(\frac{7}{2}, \frac{m}{2} + \frac{3}{2} \middle| \frac{e^2x^2}{d^2}\right)}{d^7g^3\sqrt{1-\frac{e^2x^2}{d^2}}(m+3)} + \frac{e^3(gx)^{m+4} \sqrt{d^2-e^2x^2} {}_2F_1\left(\frac{7}{2}, \frac{m}{2} + 2 \middle| \frac{e^2x^2}{d^2}\right)}{d^8g^4\sqrt{1-\frac{e^2x^2}{d^2}}(m+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x)**m*(e*x+d)**3/(-e**2*x**2+d**2)**(7/2), x)`

[Out]  $(g*x)**(m+1)*\text{sqrt}(d**2-e**2*x**2)*\text{hyper}((7/2, m/2+1/2), (m/2+3/2, ), e**2*x**2/d**2)/(d**5*g*\text{sqrt}(1-e**2*x**2/d**2))*(m+$

$1)) + 3e(gx)^{m+2} \sqrt{d^2 - e^2 x^2} \operatorname{hyper}\left(\frac{7}{2}, \frac{m}{2} + 1, \left(\frac{m}{2} + 2, \right), \frac{e^2 x^2 / d^2}{(d^2 - e^2 x^2)^{3/2}}\right) + 3e^2 (gx)^{m+3} \sqrt{d^2 - e^2 x^2} \operatorname{hyper}\left(\frac{7}{2}, \frac{m}{2} + \frac{3}{2}, \left(\frac{m}{2} + \frac{5}{2}, \right), \frac{e^2 x^2 / d^2}{(d^2 - e^2 x^2)^{5/2}}\right) + e^3 (gx)^{m+4} \sqrt{d^2 - e^2 x^2} \operatorname{hyper}\left(\frac{7}{2}, \frac{m}{2} + 2, \left(\frac{m}{2} + 3, \right), \frac{e^2 x^2 / d^2}{(d^2 - e^2 x^2)^{7/2}}\right)$

**Mathematica [A]** time = 0.35739, size = 240, normalized size = 1.13

$$x \sqrt{1 - \frac{e^2 x^2}{d^2}} (gx)^m \left( 3d^2 e (m^3 + 8m^2 + 19m + 12) x {}_2F_1\left(\frac{7}{2}, \frac{m}{2} + 1; \frac{m}{2} + 2; \frac{e^2 x^2}{d^2}\right) + (m+2) \left( d(m+4) \left( d^2(m+3) {}_2F_1\left(\frac{7}{2}, \frac{m+1}{2}; \frac{m+2}{2}; \frac{e^2 x^2}{d^2}\right) \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((gx)^m\*(d + e\*x)^3)/(d^2 - e^2\*x^2)^(7/2), x]

[Out] (x\*(gx)^m\*Sqrt[1 - (e^2\*x^2)/d^2]\*(3\*d^2\*e\*(12 + 19\*m + 8\*m^2 + m^3)\*x\*Hypergeometric2F1[7/2, 1 + m/2, 2 + m/2, (e^2\*x^2)/d^2] + (2 + m)\*(e^3\*(3 + 4\*m + m^2)\*x^3\*Hypergeometric2F1[7/2, 2 + m/2, 3 + m/2, (e^2\*x^2)/d^2] + d\*(4 + m)\*(d^2\*(3 + m)\*Hypergeometric2F1[7/2, (1 + m)/2, (3 + m)/2, (e^2\*x^2)/d^2] + 3\*e^2\*(1 + m)\*x^2\*Hypergeometric2F1[7/2, (3 + m)/2, (5 + m)/2, (e^2\*x^2)/d^2])))/(d^6\*(1 + m)\*(2 + m)\*(3 + m)\*(4 + m)\*Sqrt[d^2 - e^2\*x^2])

**Maple [F]** time = 0.035, size = 0, normalized size = 0.

$$\int (gx)^m (ex + d)^3 (-e^2 x^2 + d^2)^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((gx)^m\*(e\*x+d)^3/(-e^2\*x^2+d^2)^(7/2), x)

[Out] int((gx)^m\*(e\*x+d)^3/(-e^2\*x^2+d^2)^(7/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^3 (gx)^m}{(-e^2 x^2 + d^2)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(gx)^m/(-e^2\*x^2 + d^2)^(7/2), x, algorithm="maxima")

[Out] integrate((e\*x + d)^3\*(gx)^m/(-e^2\*x^2 + d^2)^(7/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(-\frac{(gx)^m}{(e^3 x^3 - 3 d e^2 x^2 + 3 d^2 e x - d^3) \sqrt{-e^2 x^2 + d^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(g\*x)^m/(-e^2\*x^2 + d^2)^(7/2), x, algorithm="fricas")

[Out] integral(-(g\*x)^m/((e^3\*x^3 - 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x - d^3)\*sqrt(-e^2\*x^2 + d^2)), x)

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx)^m (d + ex)^3}{(-(-d + ex)(d + ex))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x)\*\*m\*(e\*x+d)\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out] Integral((g\*x)\*\*m\*(d + e\*x)\*\*3/((-(-d + e\*x)\*(d + e\*x))\*\*(7/2)), x)

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^3 (gx)^m}{(-e^2x^2 + d^2)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(g\*x)^m/(-e^2\*x^2 + d^2)^(7/2), x, algorithm="giac")

[Out] integrate((e\*x + d)^3\*(g\*x)^m/(-e^2\*x^2 + d^2)^(7/2), x)

$$3.234 \quad \int \frac{(gx)^m(d+ex)^2}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=216

$$\frac{2(d+ex)(gx)^{m+1}}{5dg(d^2-e^2x^2)^{5/2}} + \frac{2e(3-m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2} {}_2F_1\left(\frac{5}{2}, \frac{m+2}{2}; \frac{m+4}{2}; \frac{e^2x^2}{d^2}\right)}{5d^5g^2(m+2)\sqrt{d^2-e^2x^2}} \\ + \frac{(3-2m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1} {}_2F_1\left(\frac{5}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \frac{e^2x^2}{d^2}\right)}{5d^4g(m+1)\sqrt{d^2-e^2x^2}}$$

[Out]  $(2*(g*x)^{(1+m)*(d+e*x)})/(5*d*g*(d^2-e^2*x^2)^{(5/2)}) + ((3-2*m)*(g*x)^{(1+m)*\text{Sqrt}[1-(e^2*x^2)/d^2]}*\text{Hypergeometric2F1}[5/2, (1+m)/2, (3+m)/2, (e^2*x^2)/d^2])/(5*d^4*g*(1+m)*\text{Sqrt}[d^2-e^2*x^2]) + (2*e*(3-m)*(g*x)^{(2+m)*\text{Sqrt}[1-(e^2*x^2)/d^2]}*\text{Hypergeometric2F1}[5/2, (2+m)/2, (4+m)/2, (e^2*x^2)/d^2])/(5*d^5*g^2*(2+m)*\text{Sqrt}[d^2-e^2*x^2])$

**Rubi [A]** time = 0.426606, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$

$$\frac{2(d+ex)(gx)^{m+1}}{5dg(d^2-e^2x^2)^{5/2}} + \frac{2e(3-m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2} {}_2F_1\left(\frac{5}{2}, \frac{m+2}{2}; \frac{m+4}{2}; \frac{e^2x^2}{d^2}\right)}{5d^5g^2(m+2)\sqrt{d^2-e^2x^2}} \\ + \frac{(3-2m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1} {}_2F_1\left(\frac{5}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \frac{e^2x^2}{d^2}\right)}{5d^4g(m+1)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}(((g*x)^m*(d+e*x)^2)/(d^2-e^2*x^2)^{(7/2}), x)$

[Out]  $(2*(g*x)^{(1+m)*(d+e*x)})/(5*d*g*(d^2-e^2*x^2)^{(5/2)}) + ((3-2*m)*(g*x)^{(1+m)*\text{Sqrt}[1-(e^2*x^2)/d^2]}*\text{Hypergeometric2F1}[5/2, (1+m)/2, (3+m)/2, (e^2*x^2)/d^2])/(5*d^4*g*(1+m)*\text{Sqrt}[d^2-e^2*x^2]) + (2*e*(3-m)*(g*x)^{(2+m)*\text{Sqrt}[1-(e^2*x^2)/d^2]}*\text{Hypergeometric2F1}[5/2, (2+m)/2, (4+m)/2, (e^2*x^2)/d^2])/(5*d^5*g^2*(2+m)*\text{Sqrt}[d^2-e^2*x^2])$

**Rubi in Sympy [A]** time = 52.2686, size = 209, normalized size = 0.97

$$\frac{(gx)^{m+1} \sqrt{d^2-e^2x^2} {}_2F_1\left(\frac{7}{2}, \frac{m}{2} + \frac{1}{2}; \frac{m}{2} + \frac{3}{2}; \frac{e^2x^2}{d^2}\right)}{d^6g\sqrt{1-\frac{e^2x^2}{d^2}}(m+1)} + \frac{2e(gx)^{m+2} \sqrt{d^2-e^2x^2} {}_2F_1\left(\frac{7}{2}, \frac{m}{2} + 1; \frac{m}{2} + 2; \frac{e^2x^2}{d^2}\right)}{d^7g^2\sqrt{1-\frac{e^2x^2}{d^2}}(m+2)} \\ + \frac{e^2(gx)^{m+3} \sqrt{d^2-e^2x^2} {}_2F_1\left(\frac{7}{2}, \frac{m}{2} + \frac{3}{2}; \frac{m}{2} + \frac{5}{2}; \frac{e^2x^2}{d^2}\right)}{d^8g^3\sqrt{1-\frac{e^2x^2}{d^2}}(m+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((g*x)**m*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2), x)$

[Out]  $(g*x)**(m+1)*\text{sqrt}(d**2-e**2*x**2)*\text{hyper}((7/2, m/2+1/2), (m/2+3/2, ), e**2*x**2/d**2)/(d**6*g*\text{sqrt}(1-e**2*x**2/d**2))*(m+$

1)) + 2\*e\*(g\*x)\*\*(m+2)\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)\*hyper((7/2, m/2 + 1), (m/2 + 2, ), e\*\*2\*x\*\*2/d\*\*2)/(d\*\*7\*g\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)\*\*(m+2)) + e\*\*2\*(g\*x)\*\*(m+3)\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)\*hyper((7/2, m/2 + 3/2), (m/2 + 5/2, ), e\*\*2\*x\*\*2/d\*\*2)/(d\*\*8\*g\*\*3\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)\*\*(m+3))

**Mathematica [A]** time = 0.216172, size = 178, normalized size = 0.82

$$\frac{x\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^m \left( 2de(m^2+4m+3) {}_2F_1\left(\frac{7}{2}, \frac{m}{2}+1; \frac{m}{2}+2; \frac{e^2x^2}{d^2}\right) + (m+2) \left( d^2(m+3) {}_2F_1\left(\frac{7}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \frac{e^2x^2}{d^2}\right) + e^2(m+1) \right) \right)}{d^6(m+1)(m+2)(m+3)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[((g\*x)^m\*(d + e\*x)^2)/(d^2 - e^2\*x^2)^(7/2), x]

[Out] (x\*(g\*x)^m\*Sqrt[1 - (e^2\*x^2)/d^2]\*(2\*d\*e\*(3 + 4\*m + m^2)\*x\*Hypergeometric2F1[7/2, 1 + m/2, 2 + m/2, (e^2\*x^2)/d^2] + (2 + m)\*(d^2\*(3 + m)\*Hypergeometric2F1[7/2, (1 + m)/2, (3 + m)/2, (e^2\*x^2)/d^2] + e^2\*(1 + m)\*x^2\*Hypergeometric2F1[7/2, (3 + m)/2, (5 + m)/2, (e^2\*x^2)/d^2]))/(d^6\*(1 + m)\*(2 + m)\*(3 + m)\*Sqrt[d^2 - e^2\*x^2])

**Maple [F]** time = 0.035, size = 0, normalized size = 0.

$$\int (gx)^m (ex + d)^2 (-e^2x^2 + d^2)^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x)^m\*(e\*x+d)^2/(-e^2\*x^2+d^2)^(7/2), x)

[Out] int((g\*x)^m\*(e\*x+d)^2/(-e^2\*x^2+d^2)^(7/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^2 (gx)^m}{(-e^2x^2 + d^2)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(g\*x)^m/(-e^2\*x^2 + d^2)^(7/2), x, algorithm="maxima")

[Out] integrate((e\*x + d)^2\*(g\*x)^m/(-e^2\*x^2 + d^2)^(7/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(gx)^m}{(e^4x^4 - 2de^3x^3 + 2d^3ex - d^4)\sqrt{-e^2x^2 + d^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(g\*x)^m/(-e^2\*x^2 + d^2)^(7/2), x, algorithm="fricas")



[Out] `integral(-(g*x)^m/((e^4*x^4 - 2*d*e^3*x^3 + 2*d^3*e*x - d^4)*sqrt(-e^2*x^2 + d^2)), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx)^m (d + ex)^2}{(-(-d + ex)(d + ex))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x)**m*(e*x+d)**2/(-e**2*x**2+d**2)**(7/2), x)`

[Out] `Integral((g*x)**m*(d + e*x)**2/(-(-d + e*x)*(d + e*x))** (7/2), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^2 (gx)^m}{(-e^2x^2 + d^2)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^2*(g*x)^m/(-e^2*x^2 + d^2)^(7/2), x, algorithm="giac")`

[Out] `integrate((e*x + d)^2*(g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)`

$$3.235 \quad \int \frac{(gx)^m(d+ex)}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=124

$$\frac{e(gx)^{m+2} {}_2F_1\left(1, \frac{m-3}{2}; \frac{m+4}{2}; \frac{e^2x^2}{d^2}\right)}{d^2g^2(m+2)(d^2-e^2x^2)^{5/2}} + \frac{(gx)^{m+1} {}_2F_1\left(1, \frac{m-4}{2}; \frac{m+3}{2}; \frac{e^2x^2}{d^2}\right)}{dg(m+1)(d^2-e^2x^2)^{5/2}}$$

[Out]  $((g*x)^{(1+m)} \text{Hypergeometric2F1}[1, (-4+m)/2, (3+m)/2, (e^2*x^2)/d^2]) / (d*g*(1+m)*(d^2-e^2*x^2)^{(5/2)}) + (e*(g*x)^{(2+m)} \text{Hypergeometric2F1}[1, (-3+m)/2, (4+m)/2, (e^2*x^2)/d^2]) / (d^2*g^{2*(2+m)}*(d^2-e^2*x^2)^{(5/2)})$

**Rubi [A]** time = 0.237815, antiderivative size = 162, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{e\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2} {}_2F_1\left(\frac{7}{2}, \frac{m+2}{2}, \frac{m+4}{2}, \frac{e^2x^2}{d^2}\right)}{d^6g^2(m+2)\sqrt{d^2-e^2x^2}} + \frac{\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1} {}_2F_1\left(\frac{7}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \frac{e^2x^2}{d^2}\right)}{d^5g(m+1)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[((g\*x)^m\*(d+e\*x))/(d^2-e^2\*x^2)^(7/2), x]

[Out]  $((g*x)^{(1+m)} \text{Sqrt}[1 - (e^2*x^2)/d^2] \text{Hypergeometric2F1}[7/2, (1+m)/2, (3+m)/2, (e^2*x^2)/d^2]) / (d^5*g*(1+m)*\text{Sqrt}[d^2 - e^2*x^2]) + (e*(g*x)^{(2+m)} \text{Sqrt}[1 - (e^2*x^2)/d^2] \text{Hypergeometric2F1}[7/2, (2+m)/2, (4+m)/2, (e^2*x^2)/d^2]) / (d^6*g^{2*(2+m)}*\text{Sqrt}[d^2 - e^2*x^2])$

**Rubi in Sympy [A]** time = 26.5333, size = 134, normalized size = 1.08

$$\frac{(gx)^{m+1} \sqrt{d^2-e^2x^2} {}_2F_1\left(\frac{7}{2}, \frac{m}{2} + \frac{1}{2} \middle| \frac{e^2x^2}{d^2}\right)}{d^7g\sqrt{1-\frac{e^2x^2}{d^2}}(m+1)} + \frac{e(gx)^{m+2} \sqrt{d^2-e^2x^2} {}_2F_1\left(\frac{7}{2}, \frac{m}{2} + 1 \middle| \frac{e^2x^2}{d^2}\right)}{d^8g^2\sqrt{1-\frac{e^2x^2}{d^2}}(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x)\*\*m\*(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out]  $(g*x)^{(m+1)} \text{sqrt}(d^2 - e^2*x^2) \text{hyper}((7/2, m/2 + 1/2), (m/2 + 3/2, ), e^2*x^2/d^2) / (d^7*g*\text{sqrt}(1 - e^2*x^2/d^2))^{(m+1)} + e*(g*x)^{(m+2)} \text{sqrt}(d^2 - e^2*x^2) \text{hyper}((7/2, m/2 + 1), (m/2 + 2, ), e^2*x^2/d^2) / (d^8*g^{2*(m+2)}*\text{sqrt}(1 - e^2*x^2/d^2))^{(m+2)}$

**Mathematica [A]** time = 0.145272, size = 121, normalized size = 0.98

$$\frac{x\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^m \left( e(m+1)x {}_2F_1\left(\frac{7}{2}, \frac{m}{2} + 1; \frac{m}{2} + 2; \frac{e^2x^2}{d^2}\right) + d(m+2) {}_2F_1\left(\frac{7}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \frac{e^2x^2}{d^2}\right) \right)}{d^6(m+1)(m+2)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[((g\*x)^m\*(d + e\*x))/(d^2 - e^2\*x^2)^(7/2), x]

[Out] (x\*(g\*x)^m\*Sqrt[1 - (e^2\*x^2)/d^2]\*(e\*(1 + m)\*x\*Hypergeometric2F1[7/2, 1 + m/2, 2 + m/2, (e^2\*x^2)/d^2] + d\*(2 + m)\*Hypergeometric2F1[7/2, (1 + m)/2, (3 + m)/2, (e^2\*x^2)/d^2]))/(d^6\*(1 + m)^(2 + m)\*Sqrt[d^2 - e^2\*x^2])

**Maple [F]** time = 0.031, size = 0, normalized size = 0.

$$\int (gx)^m (ex + d) (-e^2x^2 + d^2)^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x)^m\*(e\*x+d)/(-e^2\*x^2+d^2)^(7/2), x)

[Out] int((g\*x)^m\*(e\*x+d)/(-e^2\*x^2+d^2)^(7/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)(gx)^m}{(-e^2x^2 + d^2)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(g\*x)^m/(-e^2\*x^2 + d^2)^(7/2), x, algorithm="maxima")

[Out] integrate((e\*x + d)\*(g\*x)^m/(-e^2\*x^2 + d^2)^(7/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(gx)^m}{(e^5x^5 - de^4x^4 - 2d^2e^3x^3 + 2d^3e^2x^2 + d^4ex - d^5)\sqrt{-e^2x^2 + d^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(g\*x)^m/(-e^2\*x^2 + d^2)^(7/2), x, algorithm="fricas")

[Out] integral(-(g\*x)^m/((e^5\*x^5 - d\*e^4\*x^4 - 2\*d^2\*e^3\*x^3 + 2\*d^3\*e^2\*x^2 + d^4\*e\*x - d^5)\*sqrt(-e^2\*x^2 + d^2)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x)\*\*m\*(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)(gx)^m}{(-e^2x^2 + d^2)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(g\*x)^m/(-e^2\*x^2 + d^2)^(7/2), x, algorithm="giac")

[Out] integrate((e\*x + d)\*(g\*x)^m/(-e^2\*x^2 + d^2)^(7/2), x)

$$3.236 \quad \int \frac{(gx)^m}{(d^2 - e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=80

$$\frac{\sqrt{1 - \frac{e^2x^2}{d^2}}(gx)^{m+1} {}_2F_1\left(\frac{7}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \frac{e^2x^2}{d^2}\right)}{d^6g(m+1)\sqrt{d^2 - e^2x^2}}$$

[Out] ((g\*x)^(1 + m)\*Sqrt[1 - (e^2\*x^2)/d^2]\*Hypergeometric2F1[7/2, (1 + m)/2, (3 + m)/2, (e^2\*x^2)/d^2])/(d^6\*g\*(1 + m)\*Sqrt[d^2 - e^2\*x^2])

**Rubi [A]** time = 0.0725677, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{\sqrt{1 - \frac{e^2x^2}{d^2}}(gx)^{m+1} {}_2F_1\left(\frac{7}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \frac{e^2x^2}{d^2}\right)}{d^6g(m+1)\sqrt{d^2 - e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(g\*x)^m/(d^2 - e^2\*x^2)^(7/2), x]

[Out] ((g\*x)^(1 + m)\*Sqrt[1 - (e^2\*x^2)/d^2]\*Hypergeometric2F1[7/2, (1 + m)/2, (3 + m)/2, (e^2\*x^2)/d^2])/(d^6\*g\*(1 + m)\*Sqrt[d^2 - e^2\*x^2])

**Rubi in Sympy [A]** time = 11.5775, size = 66, normalized size = 0.82

$$\frac{(gx)^{m+1} \sqrt{d^2 - e^2x^2} {}_2F_1\left(\frac{7}{2}, \frac{m}{2} + \frac{1}{2}, \frac{m}{2} + \frac{3}{2}, \frac{e^2x^2}{d^2}\right)}{d^8g\sqrt{1 - \frac{e^2x^2}{d^2}}(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x)\*\*m/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out] (g\*x)\*\*(m + 1)\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)\*hyper((7/2, m/2 + 1/2), (m/2 + 3/2, ), e\*\*2\*x\*\*2/d\*\*2)/(d\*\*8\*g\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)\*(m + 1))

**Mathematica [A]** time = 0.0417155, size = 76, normalized size = 0.95

$$\frac{x\sqrt{1 - \frac{e^2x^2}{d^2}}(gx)^m {}_2F_1\left(\frac{7}{2}, \frac{m+1}{2}, \frac{m+3}{2}, \frac{e^2x^2}{d^2}\right)}{d^6(m+1)\sqrt{d^2 - e^2x^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(g\*x)^m/(d^2 - e^2\*x^2)^(7/2), x]

[Out] (x\*(g\*x)^m\*Sqrt[1 - (e^2\*x^2)/d^2]\*Hypergeometric2F1[7/2, (1 + m)/2, (3 + m)/2, (e^2\*x^2)/d^2])/(d^6\*(1 + m)\*Sqrt[d^2 - e^2\*x^2])

---

**Maple [F]** time = 0.029, size = 0, normalized size = 0.

$$\int (gx)^m (-e^2x^2 + d^2)^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x)^m/(-e^2*x^2+d^2)^(7/2), x)`

[Out] `int((g*x)^m/(-e^2*x^2+d^2)^(7/2), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx)^m}{(-e^2x^2 + d^2)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x)^m/(-e^2*x^2 + d^2)^(7/2), x, algorithm="maxima")`

[Out] `integrate((g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(gx)^m}{(e^6x^6 - 3d^2e^4x^4 + 3d^4e^2x^2 - d^6)\sqrt{-e^2x^2 + d^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x)^m/(-e^2*x^2 + d^2)^(7/2), x, algorithm="fricas")`

[Out] `integral(-(g*x)^m/((e^6*x^6 - 3*d^2*e^4*x^4 + 3*d^4*e^2*x^2 - d^6)*sqrt(-e^2*x^2 + d^2)), x)`

---

**Sympy [A]** time = 155.627, size = 60, normalized size = 0.75

$$\frac{g^m x x^m \left(\frac{m}{2} + \frac{1}{2}\right) {}_2F_1\left(\frac{7}{2}, \frac{m}{2} + \frac{1}{2} \mid \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{2d^7 \left(\frac{m}{2} + \frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x)**m/(-e**2*x**2+d**2)**(7/2), x)`

[Out] `g**m*x*x**m*gamma(m/2 + 1/2)*hyper((7/2, m/2 + 1/2), (m/2 + 3/2, ), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*d**7*gamma(m/2 + 3/2))`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx)^m}{(-e^2x^2 + d^2)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/(-e^2*x^2 + d^2)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((g*x)^m/(-e^2*x^2 + d^2)^(7/2), x)
```

$$3.237 \quad \int \frac{(gx)^m}{(d+ex)(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=163

$$\frac{\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1} {}_2F_1\left(\frac{9}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \frac{e^2x^2}{d^2}\right)}{d^7 g(m+1)\sqrt{d^2-e^2x^2}} - \frac{e\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2} {}_2F_1\left(\frac{9}{2}, \frac{m+2}{2}; \frac{m+4}{2}; \frac{e^2x^2}{d^2}\right)}{d^8 g^2(m+2)\sqrt{d^2-e^2x^2}}$$

[Out] ((g\*x)^(1+m)\*Sqrt[1-(e^2\*x^2)/d^2]\*Hypergeometric2F1[9/2, (1+m)/2, (3+m)/2, (e^2\*x^2)/d^2])/(d^7\*g\*(1+m)\*Sqrt[d^2-e^2\*x^2]) - (e\*(g\*x)^(2+m)\*Sqrt[1-(e^2\*x^2)/d^2]\*Hypergeometric2F1[9/2, (2+m)/2, (4+m)/2, (e^2\*x^2)/d^2])/(d^8\*g^2\*(2+m)\*Sqrt[d^2-e^2\*x^2])

**Rubi [A]** time = 0.443327, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$\frac{\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1} {}_2F_1\left(\frac{9}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \frac{e^2x^2}{d^2}\right)}{d^7 g(m+1)\sqrt{d^2-e^2x^2}} - \frac{e\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2} {}_2F_1\left(\frac{9}{2}, \frac{m+2}{2}; \frac{m+4}{2}; \frac{e^2x^2}{d^2}\right)}{d^8 g^2(m+2)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(g\*x)^m/((d+e\*x)\*(d^2-e^2\*x^2)^(7/2)),x]

[Out] ((g\*x)^(1+m)\*Sqrt[1-(e^2\*x^2)/d^2]\*Hypergeometric2F1[9/2, (1+m)/2, (3+m)/2, (e^2\*x^2)/d^2])/(d^7\*g\*(1+m)\*Sqrt[d^2-e^2\*x^2]) - (e\*(g\*x)^(2+m)\*Sqrt[1-(e^2\*x^2)/d^2]\*Hypergeometric2F1[9/2, (2+m)/2, (4+m)/2, (e^2\*x^2)/d^2])/(d^8\*g^2\*(2+m)\*Sqrt[d^2-e^2\*x^2])

**Rubi in Sympy [A]** time = 51.3974, size = 134, normalized size = 0.82

$$\frac{(gx)^{m+1} \sqrt{d^2-e^2x^2} {}_2F_1\left(\frac{9}{2}, \frac{m}{2} + \frac{1}{2}; \frac{m}{2} + \frac{3}{2}; \frac{e^2x^2}{d^2}\right)}{d^9 g \sqrt{1-\frac{e^2x^2}{d^2}} (m+1)} - \frac{e (gx)^{m+2} \sqrt{d^2-e^2x^2} {}_2F_1\left(\frac{9}{2}, \frac{m}{2} + 1; \frac{m}{2} + 2; \frac{e^2x^2}{d^2}\right)}{d^{10} g^2 \sqrt{1-\frac{e^2x^2}{d^2}} (m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x)\*\*m/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2),x)

[Out] (g\*x)\*\*(m+1)\*sqrt(d\*\*2-e\*\*2\*x\*\*2)\*hyper((9/2, m/2+1/2), (m/2+3/2, ), e\*\*2\*x\*\*2/d\*\*2)/(d\*\*9\*g\*sqrt(1-e\*\*2\*x\*\*2/d\*\*2)\*(m+1)) - e\*(g\*x)\*\*(m+2)\*sqrt(d\*\*2-e\*\*2\*x\*\*2)\*hyper((9/2, m/2+1), (m/2+2, ), e\*\*2\*x\*\*2/d\*\*2)/(d\*\*10\*g\*\*2\*sqrt(1-e\*\*2\*x\*\*2/d\*\*2)\*(m+2))

**Mathematica [C]** time = 1.00901, size = 161, normalized size = 0.99

$$\frac{2d(m+2)x(gx)^m F_1\left(m+1; \frac{7}{2}, \frac{9}{2}; m+2; \frac{ex}{d}, -\frac{ex}{d}\right)}{(m+1)(d-ex)^{7/2}(d+ex)^{9/2} \left( ex \left( {}_2F_1\left(\frac{9}{2}, \frac{m}{2} + 1; \frac{m}{2} + 2; \frac{e^2x^2}{d^2}\right) - 9F_1\left(m+2; \frac{7}{2}, \frac{11}{2}; m+3; \frac{ex}{d}, -\frac{ex}{d}\right) \right) + 2d(m+2)F_1\left(m+1; \frac{7}{2}, \frac{9}{2}; m+2; \frac{ex}{d}, -\frac{ex}{d}\right) \right)}$$

Warning: Unable to verify antiderivative.



[In] Integrate[(g\*x)^m/((d + e\*x)\*(d^2 - e^2\*x^2)^(7/2)),x]

[Out] (2\*d\*(2 + m)\*x\*(g\*x)^m\*AppellF1[1 + m, 7/2, 9/2, 2 + m, (e\*x)/d, -((e\*x)/d)]/((1 + m)\*(d - e\*x)^(7/2)\*(d + e\*x)^(9/2)\*(2\*d\*(2 + m)\*AppellF1[1 + m, 7/2, 9/2, 2 + m, (e\*x)/d, -((e\*x)/d)] + e\*x\*(-9\*AppellF1[2 + m, 7/2, 11/2, 3 + m, (e\*x)/d, -((e\*x)/d)] + 7\*HypergeometricPFQ[{9/2, 1 + m/2}, {2 + m/2}, (e^2\*x^2)/d^2]))

**Maple [F]** time = 0.041, size = 0, normalized size = 0.

$$\int \frac{(gx)^m}{ex + d} (-e^2x^2 + d^2)^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x)^m/(e\*x+d)/(-e^2\*x^2+d^2)^(7/2),x)

[Out] int((g\*x)^m/(e\*x+d)/(-e^2\*x^2+d^2)^(7/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx)^m}{(-e^2x^2 + d^2)^{\frac{7}{2}}(ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x)^m/((-e^2\*x^2 + d^2)^(7/2)\*(e\*x + d)),x, algorithm="maxima")

[Out] integrate((g\*x)^m/((-e^2\*x^2 + d^2)^(7/2)\*(e\*x + d)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(gx)^m}{(e^7x^7 + de^6x^6 - 3d^2e^5x^5 - 3d^3e^4x^4 + 3d^4e^3x^3 + 3d^5e^2x^2 - d^6ex - d^7)\sqrt{-e^2x^2 + d^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x)^m/((-e^2\*x^2 + d^2)^(7/2)\*(e\*x + d)),x, algorithm="fricas")

[Out] integral(-(g\*x)^m/((e^7\*x^7 + d\*e^6\*x^6 - 3\*d^2\*e^5\*x^5 - 3\*d^3\*e^4\*x^4 + 3\*d^4\*e^3\*x^3 + 3\*d^5\*e^2\*x^2 - d^6\*e\*x - d^7)\*sqrt(-e^2\*x^2 + d^2)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x)\*\*m/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2),x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx)^m}{(-e^2x^2 + d^2)^{\frac{7}{2}}(ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)),x, algorithm="giac")`

[Out] `integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)), x)`

$$3.238 \quad \int \frac{(gx)^m}{(d+ex)^2(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=217

$$\frac{2(d-ex)(gx)^{m+1}}{9dg(d^2-e^2x^2)^{9/2}} - \frac{2e(7-m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2} {}_2F_1\left(\frac{9}{2}, \frac{m+2}{2}, \frac{m+4}{2}; \frac{e^2x^2}{d^2}\right)}{9d^9g^2(m+2)\sqrt{d^2-e^2x^2}} \\ + \frac{(7-2m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1} {}_2F_1\left(\frac{9}{2}, \frac{m+1}{2}, \frac{m+3}{2}; \frac{e^2x^2}{d^2}\right)}{9d^8g(m+1)\sqrt{d^2-e^2x^2}}$$

[Out] (2\*(g\*x)^(1+m)\*(d-e\*x))/(9\*d\*g\*(d^2-e^2\*x^2)^(9/2)) + ((7-2\*m)\*(g\*x)^(1+m)\*Sqrt[1-(e^2\*x^2)/d^2]\*Hypergeometric2F1[9/2, (1+m)/2, (3+m)/2, (e^2\*x^2)/d^2])/(9\*d^8\*g\*(1+m)\*Sqrt[d^2-e^2\*x^2]) - (2\*e\*(7-m)\*(g\*x)^(2+m)\*Sqrt[1-(e^2\*x^2)/d^2]\*Hypergeometric2F1[9/2, (2+m)/2, (4+m)/2, (e^2\*x^2)/d^2])/(9\*d^9\*g^2\*(2+m)\*Sqrt[d^2-e^2\*x^2])

**Rubi [A]** time = 0.505332, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$\frac{2(d-ex)(gx)^{m+1}}{9dg(d^2-e^2x^2)^{9/2}} - \frac{2e(7-m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2} {}_2F_1\left(\frac{9}{2}, \frac{m+2}{2}, \frac{m+4}{2}; \frac{e^2x^2}{d^2}\right)}{9d^9g^2(m+2)\sqrt{d^2-e^2x^2}} \\ + \frac{(7-2m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1} {}_2F_1\left(\frac{9}{2}, \frac{m+1}{2}, \frac{m+3}{2}; \frac{e^2x^2}{d^2}\right)}{9d^8g(m+1)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(g\*x)^m/((d+e\*x)^2\*(d^2-e^2\*x^2)^(7/2)), x]

[Out] (2\*(g\*x)^(1+m)\*(d-e\*x))/(9\*d\*g\*(d^2-e^2\*x^2)^(9/2)) + ((7-2\*m)\*(g\*x)^(1+m)\*Sqrt[1-(e^2\*x^2)/d^2]\*Hypergeometric2F1[9/2, (1+m)/2, (3+m)/2, (e^2\*x^2)/d^2])/(9\*d^8\*g\*(1+m)\*Sqrt[d^2-e^2\*x^2]) - (2\*e\*(7-m)\*(g\*x)^(2+m)\*Sqrt[1-(e^2\*x^2)/d^2]\*Hypergeometric2F1[9/2, (2+m)/2, (4+m)/2, (e^2\*x^2)/d^2])/(9\*d^9\*g^2\*(2+m)\*Sqrt[d^2-e^2\*x^2])

**Rubi in Sympy [A]** time = 86.4397, size = 209, normalized size = 0.96

$$\frac{(gx)^{m+1} \sqrt{d^2-e^2x^2} {}_2F_1\left(\frac{11}{2}, \frac{m}{2} + \frac{1}{2} \middle| \frac{e^2x^2}{d^2}\right)}{d^{10}g\sqrt{1-\frac{e^2x^2}{d^2}}(m+1)} - \frac{2e(gx)^{m+2} \sqrt{d^2-e^2x^2} {}_2F_1\left(\frac{11}{2}, \frac{m}{2} + 1 \middle| \frac{e^2x^2}{d^2}\right)}{d^{11}g^2\sqrt{1-\frac{e^2x^2}{d^2}}(m+2)} \\ + \frac{e^2(gx)^{m+3} \sqrt{d^2-e^2x^2} {}_2F_1\left(\frac{11}{2}, \frac{m}{2} + \frac{3}{2} \middle| \frac{e^2x^2}{d^2}\right)}{d^{12}g^3\sqrt{1-\frac{e^2x^2}{d^2}}(m+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x)\*\*m/(e\*x+d)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out] (g\*x)\*\*(m+1)\*sqrt(d\*\*2-e\*\*2\*x\*\*2)\*hyper((11/2, m/2+1/2), (m/2+3/2, ), e\*\*2\*x\*\*2/d\*\*2)/(d\*\*10\*g\*sqrt(1-e\*\*2\*x\*\*2/d\*\*2))\*(m

+ 1)) - 2\*e\*(g\*x)\*\*(m + 2)\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)\*hyper((11/2, m/2 + 1), (m/2 + 2, ), e\*\*2\*x\*\*2/d\*\*2)/(d\*\*11\*g\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)\*(m + 2)) + e\*\*2\*(g\*x)\*\*(m + 3)\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)\*hyper((11/2, m/2 + 3/2), (m/2 + 5/2, ), e\*\*2\*x\*\*2/d\*\*2)/(d\*\*12\*g\*\*3\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)\*(m + 3))

**Mathematica [C]** time = 1.05167, size = 157, normalized size = 0.72

$$\frac{2d(m+2)x(gx)^m F_1\left(m+1; \frac{7}{2}, \frac{11}{2}; m+2; \frac{ex}{d}, -\frac{ex}{d}\right)}{(m+1)(d-ex)^{7/2}(d+ex)^{11/2} \left(2d(m+2)F_1\left(m+1; \frac{7}{2}, \frac{11}{2}; m+2; \frac{ex}{d}, -\frac{ex}{d}\right) + ex \left(7F_1\left(m+2; \frac{9}{2}, \frac{11}{2}; m+3; \frac{ex}{d}, -\frac{ex}{d}\right) - 11F_1\left(m+2; \frac{7}{2}, \frac{11}{2}; m+3; \frac{ex}{d}, -\frac{ex}{d}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(g\*x)^m/((d + e\*x)^2\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out] (2\*d\*(2 + m)\*x\*(g\*x)^m\*AppellF1[1 + m, 7/2, 11/2, 2 + m, (e\*x)/d, -((e\*x)/d)]/((1 + m)\*(d - e\*x)^(7/2)\*(d + e\*x)^(11/2)\*(2\*d\*(2 + m)\*AppellF1[1 + m, 7/2, 11/2, 2 + m, (e\*x)/d, -((e\*x)/d)] + e\*x\*(-11\*AppellF1[2 + m, 7/2, 13/2, 3 + m, (e\*x)/d, -((e\*x)/d)] + 7\*AppellF1[2 + m, 9/2, 11/2, 3 + m, (e\*x)/d, -((e\*x)/d)]))

**Maple [F]** time = 0.041, size = 0, normalized size = 0.

$$\int \frac{(gx)^m}{(ex + d)^2} (-e^2x^2 + d^2)^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x)^m/(e\*x+d)^2/(-e^2\*x^2+d^2)^(7/2), x)

[Out] int((g\*x)^m/(e\*x+d)^2/(-e^2\*x^2+d^2)^(7/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx)^m}{(-e^2x^2 + d^2)^{\frac{7}{2}}(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x)^m/((-e^2\*x^2 + d^2)^(7/2)\*(e\*x + d)^2), x, algorithm="maxima")

[Out] integrate((g\*x)^m/((-e^2\*x^2 + d^2)^(7/2)\*(e\*x + d)^2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(gx)^m}{(e^8x^8 + 2de^7x^7 - 2d^2e^6x^6 - 6d^3e^5x^5 + 6d^5e^3x^3 + 2d^6e^2x^2 - 2d^7ex - d^8)\sqrt{-e^2x^2 + d^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x)^m/((-e^2\*x^2 + d^2)^(7/2)\*(e\*x + d)^2), x, algorithm="fricas")

```
[Out] integral(-(g*x)^m/((e^8*x^8 + 2*d*e^7*x^7 - 2*d^2*e^6*x^6 - 6*d^3
*e^5*x^5 + 6*d^5*e^3*x^3 + 2*d^6*e^2*x^2 - 2*d^7*e*x - d^8)*sqrt(
-e^2*x^2 + d^2)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m/(e*x+d)**2/(-e**2*x**2+d**2)**(7/2),x)
```

[Out] Timed out

**GIAC/XCAS [A]** time = 0.653896, size = 4, normalized size = 0.02

$sage_0x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^2),x, algorithm="giac")
```

[Out] sage0\*x

$$3.239 \quad \int \frac{(gx)^m}{(d+ex)^3(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=214

$$\frac{4(d-ex)(gx)^{m+1}}{11g(d^2-e^2x^2)^{11/2}} - \frac{e(25-4m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2} {}_2F_1\left(\frac{11}{2}, \frac{m+2}{2}, \frac{m+4}{2}; \frac{e^2x^2}{d^2}\right)}{11d^{10}g^2(m+2)\sqrt{d^2-e^2x^2}}$$

$$+ \frac{(7-4m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1} {}_2F_1\left(\frac{11}{2}, \frac{m+1}{2}, \frac{m+3}{2}; \frac{e^2x^2}{d^2}\right)}{11d^9g(m+1)\sqrt{d^2-e^2x^2}}$$

[Out]  $(4*(g*x)^{(1+m)}*(d-e*x))/(11*g*(d^2-e^2*x^2)^{(11/2)}) + ((7-4*m)*(g*x)^{(1+m)}*sqrt[1-(e^2*x^2)/d^2]*Hypergeometric2F1[11/2, (1+m)/2, (3+m)/2, (e^2*x^2)/d^2])/(11*d^9*g*(1+m)*sqrt[d^2-e^2*x^2]) - (e*(25-4*m)*(g*x)^{(2+m)}*sqrt[1-(e^2*x^2)/d^2]*Hypergeometric2F1[11/2, (2+m)/2, (4+m)/2, (e^2*x^2)/d^2])/(11*d^{10}*g^2*(2+m)*sqrt[d^2-e^2*x^2])$

**Rubi [A]** time = 0.506485, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$\frac{4(d-ex)(gx)^{m+1}}{11g(d^2-e^2x^2)^{11/2}} - \frac{e(25-4m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+2} {}_2F_1\left(\frac{11}{2}, \frac{m+2}{2}, \frac{m+4}{2}; \frac{e^2x^2}{d^2}\right)}{11d^{10}g^2(m+2)\sqrt{d^2-e^2x^2}}$$

$$+ \frac{(7-4m)\sqrt{1-\frac{e^2x^2}{d^2}}(gx)^{m+1} {}_2F_1\left(\frac{11}{2}, \frac{m+1}{2}, \frac{m+3}{2}; \frac{e^2x^2}{d^2}\right)}{11d^9g(m+1)\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[(g\*x)^m/((d+e\*x)^3\*(d^2-e^2\*x^2)^(7/2)), x]

[Out]  $(4*(g*x)^{(1+m)}*(d-e*x))/(11*g*(d^2-e^2*x^2)^{(11/2)}) + ((7-4*m)*(g*x)^{(1+m)}*sqrt[1-(e^2*x^2)/d^2]*Hypergeometric2F1[11/2, (1+m)/2, (3+m)/2, (e^2*x^2)/d^2])/(11*d^9*g*(1+m)*sqrt[d^2-e^2*x^2]) - (e*(25-4*m)*(g*x)^{(2+m)}*sqrt[1-(e^2*x^2)/d^2]*Hypergeometric2F1[11/2, (2+m)/2, (4+m)/2, (e^2*x^2)/d^2])/(11*d^{10}*g^2*(2+m)*sqrt[d^2-e^2*x^2])$

**Rubi in Sympy [A]** time = 115.693, size = 280, normalized size = 1.31

$$\frac{(gx)^{m+1} \sqrt{d^2-e^2x^2} {}_2F_1\left(\frac{13}{2}, \frac{m}{2} + \frac{1}{2} \middle| \frac{e^2x^2}{d^2}\right)}{d^{11}g\sqrt{1-\frac{e^2x^2}{d^2}}(m+1)} - \frac{3e(gx)^{m+2} \sqrt{d^2-e^2x^2} {}_2F_1\left(\frac{13}{2}, \frac{m}{2} + 1 \middle| \frac{e^2x^2}{d^2}\right)}{d^{12}g^2\sqrt{1-\frac{e^2x^2}{d^2}}(m+2)}$$

$$+ \frac{3e^2(gx)^{m+3} \sqrt{d^2-e^2x^2} {}_2F_1\left(\frac{13}{2}, \frac{m}{2} + \frac{3}{2} \middle| \frac{e^2x^2}{d^2}\right)}{d^{13}g^3\sqrt{1-\frac{e^2x^2}{d^2}}(m+3)} - \frac{e^3(gx)^{m+4} \sqrt{d^2-e^2x^2} {}_2F_1\left(\frac{13}{2}, \frac{m}{2} + 2 \middle| \frac{e^2x^2}{d^2}\right)}{d^{14}g^4\sqrt{1-\frac{e^2x^2}{d^2}}(m+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x)\*\*m/(e\*x+d)\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out]  $(g*x)**(m+1)*sqrt(d**2-e**2*x**2)*hyper((13/2, m/2+1/2), (m/2+3/2, ), e**2*x**2/d**2)/(d**11*g*sqrt(1-e**2*x**2/d**2))*(m$

+ 1)) - 3\*e\*(g\*x)\*\*(m + 2)\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)\*hyper((13/2, m/2 + 1), (m/2 + 2, ), e\*\*2\*x\*\*2/d\*\*2)/(d\*\*12\*g\*\*2\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)\*(m + 2)) + 3\*e\*\*2\*(g\*x)\*\*(m + 3)\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)\*hyper((13/2, m/2 + 3/2), (m/2 + 5/2, ), e\*\*2\*x\*\*2/d\*\*2)/(d\*\*13\*g\*\*3\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)\*(m + 3)) - e\*\*3\*(g\*x)\*\*(m + 4)\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)\*hyper((13/2, m/2 + 2), (m/2 + 3, ), e\*\*2\*x\*\*2/d\*\*2)/(d\*\*14\*g\*\*4\*sqrt(1 - e\*\*2\*x\*\*2/d\*\*2)\*(m + 4))

**Mathematica [C]** time = 1.04234, size = 157, normalized size = 0.73

$$\frac{2d(m+2)x(gx)^m F_1\left(m+1; \frac{7}{2}, \frac{13}{2}; m+2; \frac{ex}{d}, -\frac{ex}{d}\right)}{(m+1)(d-ex)^{7/2}(d+ex)^{13/2} \left(2d(m+2)F_1\left(m+1; \frac{7}{2}, \frac{13}{2}; m+2; \frac{ex}{d}, -\frac{ex}{d}\right) + ex \left(7F_1\left(m+2; \frac{9}{2}, \frac{13}{2}; m+3; \frac{ex}{d}, -\frac{ex}{d}\right) - 13F_1\left(m+3; \frac{11}{2}, \frac{13}{2}; m+4; \frac{ex}{d}, -\frac{ex}{d}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(g\*x)^m/((d + e\*x)^3\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out] (2\*d\*(2 + m)\*x\*(g\*x)^m\*AppellF1[1 + m, 7/2, 13/2, 2 + m, (e\*x)/d, -((e\*x)/d)]/((1 + m)\*(d - e\*x)^(7/2)\*(d + e\*x)^(13/2)\*(2\*d\*(2 + m)\*AppellF1[1 + m, 7/2, 13/2, 2 + m, (e\*x)/d, -((e\*x)/d)] + e\*x\*(-13\*AppellF1[2 + m, 7/2, 15/2, 3 + m, (e\*x)/d, -((e\*x)/d)] + 7\*AppellF1[2 + m, 9/2, 13/2, 3 + m, (e\*x)/d, -((e\*x)/d)]))

**Maple [F]** time = 0.043, size = 0, normalized size = 0.

$$\int \frac{(gx)^m}{(ex+d)^3} (-e^2x^2 + d^2)^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x)^m/(e\*x+d)^3/(-e^2\*x^2+d^2)^(7/2), x)

[Out] int((g\*x)^m/(e\*x+d)^3/(-e^2\*x^2+d^2)^(7/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx)^m}{(-e^2x^2 + d^2)^{\frac{7}{2}}(ex+d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x)^m/((-e^2\*x^2 + d^2)^(7/2)\*(e\*x + d)^3), x, algorithm="maxima")

[Out] integrate((g\*x)^m/((-e^2\*x^2 + d^2)^(7/2)\*(e\*x + d)^3), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(gx)^m}{(e^9x^9 + 3de^8x^8 - 8d^3e^6x^6 - 6d^4e^5x^5 + 6d^5e^4x^4 + 8d^6e^3x^3 - 3d^8ex - d^9)\sqrt{-e^2x^2 + d^2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x)^m/((-e^2\*x^2 + d^2)^(7/2)\*(e\*x + d)^3), x, algorithm="fricas")

```
[Out] integral(-(g*x)^m/((e^9*x^9 + 3*d*e^8*x^8 - 8*d^3*e^6*x^6 - 6*d^4
*e^5*x^5 + 6*d^5*e^4*x^4 + 8*d^6*e^3*x^3 - 3*d^8*e*x - d^9)*sqrt(
-e^2*x^2 + d^2)), x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m/(e*x+d)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Timed out
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx)^m}{(-e^2x^2 + d^2)^{\frac{7}{2}}(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^3),x, algorithm="giac")
```

```
[Out] integrate((g*x)^m/((-e^2*x^2 + d^2)^(7/2)*(e*x + d)^3), x)
```



### 3.240 $\int x^5(d + ex)(d^2 - e^2x^2)^p dx$

**Optimal.** Leaf size=148

$$\frac{1}{7}ex^7(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{7}{2}, -p; \frac{9}{2}; \frac{e^2x^2}{d^2}\right) - \frac{d(d^2 - e^2x^2)^{p+3}}{2e^6(p+3)} - \frac{d^5(d^2 - e^2x^2)^{p+1}}{2e^6(p+1)} + \frac{d^3(d^2 - e^2x^2)^{p+2}}{e^6(p+2)}$$

[Out]  $-(d^5*(d^2 - e^2*x^2)^(1 + p))/(2*e^6*(1 + p)) + (d^3*(d^2 - e^2*x^2)^(2 + p))/(e^6*(2 + p)) - (d*(d^2 - e^2*x^2)^(3 + p))/(2*e^6*(3 + p)) + (e*x^7*(d^2 - e^2*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(7*(1 - (e^2*x^2)/d^2)^p)$

**Rubi [A]** time = 0.226445, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$

$$\frac{1}{7}ex^7(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{7}{2}, -p; \frac{9}{2}; \frac{e^2x^2}{d^2}\right) - \frac{d(d^2 - e^2x^2)^{p+3}}{2e^6(p+3)} - \frac{d^5(d^2 - e^2x^2)^{p+1}}{2e^6(p+1)} + \frac{d^3(d^2 - e^2x^2)^{p+2}}{e^6(p+2)}$$

Antiderivative was successfully verified.

[In] Int[x^5\*(d + e\*x)\*(d^2 - e^2\*x^2)^p, x]

[Out]  $-(d^5*(d^2 - e^2*x^2)^(1 + p))/(2*e^6*(1 + p)) + (d^3*(d^2 - e^2*x^2)^(2 + p))/(e^6*(2 + p)) - (d*(d^2 - e^2*x^2)^(3 + p))/(2*e^6*(3 + p)) + (e*x^7*(d^2 - e^2*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(7*(1 - (e^2*x^2)/d^2)^p)$

**Rubi in Sympy [A]** time = 38.1723, size = 121, normalized size = 0.82

$$-\frac{d^5(d^2 - e^2x^2)^{p+1}}{2e^6(p+1)} + \frac{d^3(d^2 - e^2x^2)^{p+2}}{e^6(p+2)} - \frac{d(d^2 - e^2x^2)^{p+3}}{2e^6(p+3)} + \frac{ex^7 \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(-p, \frac{7}{2}; \frac{9}{2}; \frac{e^2x^2}{d^2}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5\*(e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p, x)

[Out]  $-d**5*(d**2 - e**2*x**2)**(p + 1)/(2*e**6*(p + 1)) + d**3*(d**2 - e**2*x**2)**(p + 2)/(e**6*(p + 2)) - d*(d**2 - e**2*x**2)**(p + 3)/(2*e**6*(p + 3)) + e*x**7*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper((-p, 7/2), (9/2, ), e**2*x**2/d**2)/7$

**Mathematica [A]** time = 0.244916, size = 208, normalized size = 1.41

$$\frac{(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} \left(2e^7(p^3 + 6p^2 + 11p + 6)x^7 {}_2F_1\left(\frac{7}{2}, -p; \frac{9}{2}; \frac{e^2x^2}{d^2}\right) - 7d \left(-e^6(p^2 + 3p + 2)x^6 \left(1 - \frac{e^2x^2}{d^2}\right)^p + d^2e^4p\right)}{14e^6(p+1)(p+2)(p+3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(d + e\*x)\*(d^2 - e^2\*x^2)^p,x]

[Out]  $((d^2 - e^2 x^2)^p (-7 d (2 d^4 e^2 p x^2 (1 - (e^2 x^2)/d^2)^p + d^2 e^4 p (1 + p) x^4 (1 - (e^2 x^2)/d^2)^p - e^6 (2 + 3 p + p^2) x^6 (1 - (e^2 x^2)/d^2)^p + 2 d^6 (-1 + (1 - (e^2 x^2)/d^2)^p) + 2 e^7 (6 + 11 p + 6 p^2 + p^3) x^7 \text{Hypergeometric2F1}[7/2, -p, 9/2, (e^2 x^2)/d^2])) / (14 e^6 (1 + p) (2 + p) (3 + p) (1 - (e^2 x^2)/d^2)^p)$

**Maple [F]** time = 0.127, size = 0, normalized size = 0.

$$\int x^5 (ex + d) (-e^2 x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(e\*x+d)\*(-e^2\*x^2+d^2)^p,x)

[Out] int(x^5\*(e\*x+d)\*(-e^2\*x^2+d^2)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$e \int x^6 e^{(p \log(ex+d) + p \log(-ex+d))} dx + \frac{((p^2 + 3p + 2)e^6 x^6 - (p^2 + p)d^2 e^4 x^4 - 2d^4 e^2 p x^2 - 2d^6)(-e^2 x^2 + d^2)^p d}{2(p^3 + 6p^2 + 11p + 6)e^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(-e^2\*x^2 + d^2)^p\*x^5,x, algorithm="maxima")

[Out]  $e \cdot \text{integrate}(x^6 \cdot e^{(p \cdot \log(e \cdot x + d) + p \cdot \log(-e \cdot x + d))}, x) + 1/2 \cdot ((p^2 + 3 \cdot p + 2) \cdot e^6 \cdot x^6 - (p^2 + p) \cdot d^2 \cdot e^4 \cdot x^4 - 2 \cdot d^4 \cdot e^2 \cdot p \cdot x^2 - 2 \cdot d^6) \cdot (-e^2 \cdot x^2 + d^2)^p \cdot d / ((p^3 + 6 \cdot p^2 + 11 \cdot p + 6) \cdot e^6)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( (ex^6 + dx^5) (-e^2 x^2 + d^2)^p, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(-e^2\*x^2 + d^2)^p\*x^5,x, algorithm="fricas")

[Out] integral((e\*x^6 + d\*x^5)\*(-e^2\*x^2 + d^2)^p, x)

**Sympy [A]** time = 19.0671, size = 972, normalized size = 6.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p,x)

```
[Out] d*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x)/
(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e
+ x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*
d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log
(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*
d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e
**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4
*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e
**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 -
8*d**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e
+ x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*
e**6 + 2*e**8*x**2) - 2*d**4/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**
2*e**2*x**2*log(-d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e
**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) + e**4*x**4/(-
2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d/e + x)/(2*e
**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(4*
e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 +
12*e**6*p**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 -
e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6)
- d**2*e**4*p**2*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**
6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p*x**4*(d**2 - e**2*x**
2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) + e**6*p
**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e
**6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p
**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e
**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), T
rue)) + d**(2*p)*e*x**7*hyper((7/2, -p), (9/2, ), e**2*x**2*exp_po
lar(2*I*pi)/d**2)/7
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(-e^2x^2 + d^2)^p x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^5,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^5, x)
```

### 3.241 $\int x^4(d + ex) (d^2 - e^2x^2)^p dx$

**Optimal.** Leaf size=147

$$\frac{1}{5}dx^5 (d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; \frac{e^2x^2}{d^2}\right) + \frac{d^2 (d^2 - e^2x^2)^{p+2}}{e^5(p+2)} - \frac{(d^2 - e^2x^2)^{p+3}}{2e^5(p+3)} - \frac{d^4 (d^2 - e^2x^2)^{p+1}}{2e^5(p+1)}$$

[Out]  $-(d^4*(d^2 - e^2*x^2)^(1 + p))/(2*e^5*(1 + p)) + (d^2*(d^2 - e^2*x^2)^(2 + p))/(e^5*(2 + p)) - (d^2 - e^2*x^2)^(3 + p)/(2*e^5*(3 + p)) + (d*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(1 - (e^2*x^2)/d^2)^p)$

**Rubi [A]** time = 0.214114, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$

$$\frac{1}{5}dx^5 (d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; \frac{e^2x^2}{d^2}\right) + \frac{d^2 (d^2 - e^2x^2)^{p+2}}{e^5(p+2)} - \frac{(d^2 - e^2x^2)^{p+3}}{2e^5(p+3)} - \frac{d^4 (d^2 - e^2x^2)^{p+1}}{2e^5(p+1)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^4*(d + e*x)*(d^2 - e^2*x^2)^p, x]$

[Out]  $-(d^4*(d^2 - e^2*x^2)^(1 + p))/(2*e^5*(1 + p)) + (d^2*(d^2 - e^2*x^2)^(2 + p))/(e^5*(2 + p)) - (d^2 - e^2*x^2)^(3 + p)/(2*e^5*(3 + p)) + (d*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(1 - (e^2*x^2)/d^2)^p)$

**Rubi in Sympy [A]** time = 38.6495, size = 119, normalized size = 0.81

$$-\frac{d^4 (d^2 - e^2x^2)^{p+1}}{2e^5 (p+1)} + \frac{d^2 (d^2 - e^2x^2)^{p+2}}{e^5 (p+2)} + \frac{dx^5 \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(\frac{-p}{2}, \frac{5}{2}; \frac{e^2x^2}{d^2}\right)}{5} - \frac{(d^2 - e^2x^2)^{p+3}}{2e^5 (p+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**4}*(e*x+d)*(-e^{**2}*x^{**2}+d^{**2})^{**p}, x)$

[Out]  $-d^{**4}*(d^{**2} - e^{**2}*x^{**2})^{**p}*(p + 1)/(2*e^{**5}*(p + 1)) + d^{**2}*(d^{**2} - e^{**2}*x^{**2})^{**p}*(p + 2)/(e^{**5}*(p + 2)) + d*x^{**5}*(1 - e^{**2}*x^{**2}/d^{**2})^{**p}*(-p)*(d^{**2} - e^{**2}*x^{**2})^{**p}*hyper((-p, 5/2), (7/2, ), e^{**2}*x^{**2}/d^{**2})/5 - (d^{**2} - e^{**2}*x^{**2})^{**p}*(p + 3)/(2*e^{**5}*(p + 3))$

**Mathematica [A]** time = 0.210056, size = 208, normalized size = 1.41

$$\frac{(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} \left(2de^5(p^3 + 6p^2 + 11p + 6)x^5 {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; \frac{e^2x^2}{d^2}\right) - 5\left(-e^6(p^2 + 3p + 2)x^6 \left(1 - \frac{e^2x^2}{d^2}\right)^p + d^2e^4p\right)}{10e^5(p+1)(p+2)(p+3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(d + e\*x)\*(d^2 - e^2\*x^2)^p,x]

[Out]  $((d^2 - e^2x^2)^p * (-5 * (2 * d^4 * e^2 * p * x^2 * (1 - (e^2x^2)/d^2)^p + d^2 * e^4 * p * (1 + p) * x^4 * (1 - (e^2x^2)/d^2)^p - e^6 * (2 + 3 * p + p^2) * x^6 * (1 - (e^2x^2)/d^2)^p + 2 * d^6 * (-1 + (1 - (e^2x^2)/d^2)^p)) + 2 * d * e^5 * (6 + 11 * p + 6 * p^2 + p^3) * x^5 * \text{Hypergeometric2F1}[5/2, -p, 7/2, (e^2x^2)/d^2]) / (10 * e^5 * (1 + p) * (2 + p) * (3 + p) * (1 - (e^2x^2)/d^2)^p)$

**Maple [F]** time = 0.073, size = 0, normalized size = 0.

$$\int x^4 (ex + d) (-e^2x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(e\*x+d)\*(-e^2\*x^2+d^2)^p,x)

[Out] int(x^4\*(e\*x+d)\*(-e^2\*x^2+d^2)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(-e^2x^2 + d^2)^p x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(-e^2\*x^2 + d^2)^p\*x^4,x, algorithm="maxima")

[Out] integrate((e\*x + d)\*(-e^2\*x^2 + d^2)^p\*x^4, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex^5 + dx^4)(-e^2x^2 + d^2)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(-e^2\*x^2 + d^2)^p\*x^4,x, algorithm="fricas")

[Out] integral((e\*x^5 + d\*x^4)\*(-e^2\*x^2 + d^2)^p, x)

**Sympy [A]** time = 16.9238, size = 972, normalized size = 6.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p,x)

[Out]  $d * d^{2p} * x^5 * \text{hyper}((5/2, -p), (7/2,)), e^{2x^2} * \exp_{\text{polar}}(2 * I * \pi) / d^{2p} / 5 + e * \text{Piecewise}((x^{6p} * (d^{2p})^{p/6}, \text{Eq}(e, 0)), (-2 * d^{4p} * \log(-d/e + x) / (4 * d^{4p} * e^{6p} - 8 * d^{2p} * e^{8p} * x^{2p} + 4 * e^{10p} * x^{4p}) - 2$

```

*d**4*log(d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4)
) - 3*d**4/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**
*2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**
*10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**6 - 8*d**2*e**
**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2/(4*d**4*e**6 - 8*d**2
*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**
*6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/
(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*
d**4*log(-d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4*log(d/e
+ x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4/(-2*d**2*e**6 + 2*e**8
*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**2*e**6 + 2*e**8*x**
*2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2)
+ e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(
-d/e + x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**
*4) - x**4/(4*e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(
2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p
*x**2*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6
*p + 12*e**6) - d**2*e**4*p**2*x**4*(d**2 - e**2*x**2)**p/(2*e**6
*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p*x**4*(d
**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*
e**6) + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**
*6*p**2 + 22*e**6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)
**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x
**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p
+ 12*e**6), True))

```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(-e^2x^2 + d^2)^p x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(-e^2\*x^2 + d^2)^p\*x^4,x, algorithm="giac")

[Out] integrate((e\*x + d)\*(-e^2\*x^2 + d^2)^p\*x^4, x)

### 3.242 $\int x^3(d + ex)(d^2 - e^2x^2)^p dx$

**Optimal.** Leaf size=120

$$\frac{1}{5}ex^5(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; \frac{e^2x^2}{d^2}\right) + \frac{d(d^2 - e^2x^2)^{p+2}}{2e^4(p+2)} - \frac{d^3(d^2 - e^2x^2)^{p+1}}{2e^4(p+1)}$$

[Out]  $-(d^3*(d^2 - e^2*x^2)^(1 + p))/(2*e^4*(1 + p)) + (d*(d^2 - e^2*x^2)^(2 + p))/(2*e^4*(2 + p)) + (e*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(1 - (e^2*x^2)/d^2)^p)$

**Rubi [A]** time = 0.171427, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$

$$\frac{1}{5}ex^5(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; \frac{e^2x^2}{d^2}\right) + \frac{d(d^2 - e^2x^2)^{p+2}}{2e^4(p+2)} - \frac{d^3(d^2 - e^2x^2)^{p+1}}{2e^4(p+1)}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(d + e\*x)\*(d^2 - e^2\*x^2)^p, x]

[Out]  $-(d^3*(d^2 - e^2*x^2)^(1 + p))/(2*e^4*(1 + p)) + (d*(d^2 - e^2*x^2)^(2 + p))/(2*e^4*(2 + p)) + (e*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(1 - (e^2*x^2)/d^2)^p)$

**Rubi in Sympy [A]** time = 33.398, size = 97, normalized size = 0.81

$$\frac{d^3(d^2 - e^2x^2)^{p+1}}{2e^4(p+1)} + \frac{d(d^2 - e^2x^2)^{p+2}}{2e^4(p+2)} + \frac{ex^5 \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(\frac{-p}{2}, \frac{5}{2}; \frac{e^2x^2}{d^2}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p, x)

[Out]  $-d**3*(d**2 - e**2*x**2)**(p + 1)/(2*e**4*(p + 1)) + d*(d**2 - e**2*x**2)**(p + 2)/(2*e**4*(p + 2)) + e*x**5*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper((-p, 5/2), (7/2, ), e**2*x**2/d**2)/5$

**Mathematica [A]** time = 0.150373, size = 161, normalized size = 1.34

$$\frac{(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} \left(2e^5(p^2 + 3p + 2)x^5 {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; \frac{e^2x^2}{d^2}\right) + 5de^4(p+1)x^4 \left(1 - \frac{e^2x^2}{d^2}\right)^p - 5d^5 \left(\left(1 - \frac{e^2x^2}{d^2}\right)^p - 1\right)\right)}{10e^4(p+1)(p+2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(d + e\*x)\*(d^2 - e^2\*x^2)^p, x]

[Out]  $((d^2 - e^2*x^2)^p*(-5*d^3*e^2*p*x^2*(1 - (e^2*x^2)/d^2)^p + 5*d*e^4*(1 + p)*x^4*(1 - (e^2*x^2)/d^2)^p - 5*d^5*(-1 + (1 - (e^2*x^2)/d^2)^p) + 2*e^5*(2 + 3*p + p^2)*x^5*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(10*e^4*(1 + p)*(2 + p)*(1 - (e^2*x^2)/d^2))$

$^p)$

**Maple [F]** time = 0.066, size = 0, normalized size = 0.

$$\int x^3 (ex + d) (-e^2x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(e*x+d)*(-e^2*x^2+d^2)^p,x)`

[Out] `int(x^3*(e*x+d)*(-e^2*x^2+d^2)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$e \int x^4 e^{(p \log(ex+d) + p \log(-ex+d))} dx + \frac{(e^4(p+1)x^4 - d^2 e^2 p x^2 - d^4) (-e^2 x^2 + d^2)^p d}{2(p^2 + 3p + 2)e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^3,x, algorithm="maxima")`

[Out] `e*integrate(x^4*e^(p*log(e*x + d) + p*log(-e*x + d)), x) + 1/2*(e^4*(p + 1)*x^4 - d^2*e^2*p*x^2 - d^4)*(-e^2*x^2 + d^2)^p*d/((p^2 + 3*p + 2)*e^4)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex^4 + dx^3) (-e^2x^2 + d^2)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^3,x, algorithm="fricas")`

[Out] `integral((e*x^4 + d*x^3)*(-e^2*x^2 + d^2)^p, x)`

**Sympy [A]** time = 12.0498, size = 382, normalized size = 3.18

$$d \left( \begin{array}{l} \left( \begin{array}{l} \frac{x^4 (d^2)^p}{4} \\ \frac{d^2 \log\left(-\frac{d}{e} + x\right)}{-2d^2 e^4 + 2e^6 x^2} - \frac{d^2 \log\left(\frac{d}{e} + x\right)}{-2d^2 e^4 + 2e^6 x^2} - \frac{d^2}{-2d^2 e^4 + 2e^6 x^2} + \frac{e^2 x^2 \log\left(-\frac{d}{e} + x\right)}{-2d^2 e^4 + 2e^6 x^2} + \frac{e^2 x^2 \log\left(\frac{d}{e} + x\right)}{-2d^2 e^4 + 2e^6 x^2} \end{array} \right) \text{ for } e = 0 \\ \left( \begin{array}{l} \frac{d^2 \log\left(-\frac{d}{e} + x\right)}{2e^4} - \frac{d^2 \log\left(\frac{d}{e} + x\right)}{2e^4} - \frac{x^2}{2e^2} \\ \frac{d^4 (d^2 - e^2 x^2)^p}{2e^4 p^2 + 6e^4 p + 4e^4} - \frac{d^2 e^2 p x^2 (d^2 - e^2 x^2)^p}{2e^4 p^2 + 6e^4 p + 4e^4} + \frac{e^4 p x^4 (d^2 - e^2 x^2)^p}{2e^4 p^2 + 6e^4 p + 4e^4} + \frac{e^4 x^4 (d^2 - e^2 x^2)^p}{2e^4 p^2 + 6e^4 p + 4e^4} \end{array} \right) \text{ for } p = -2 \\ \left( \begin{array}{l} \frac{d^2 \log\left(-\frac{d}{e} + x\right)}{2e^4} - \frac{d^2 \log\left(\frac{d}{e} + x\right)}{2e^4} - \frac{x^2}{2e^2} \\ \frac{d^4 (d^2 - e^2 x^2)^p}{2e^4 p^2 + 6e^4 p + 4e^4} - \frac{d^2 e^2 p x^2 (d^2 - e^2 x^2)^p}{2e^4 p^2 + 6e^4 p + 4e^4} + \frac{e^4 p x^4 (d^2 - e^2 x^2)^p}{2e^4 p^2 + 6e^4 p + 4e^4} + \frac{e^4 x^4 (d^2 - e^2 x^2)^p}{2e^4 p^2 + 6e^4 p + 4e^4} \end{array} \right) \text{ for } p = -1 \\ \text{otherwise} \end{array} \right) + \frac{d^{2p} e x^5 {}_2F_1\left(\frac{5}{2}, -p \mid \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(e*x+d)*(-e**2*x**2+d**2)**p,x)`



```
[Out] d*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(-
2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*
e**6*x**2) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d
/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2
*d**2*e**4 + 2*e**6*x**2), Eq(p, -2)), (-d**2*log(-d/e + x)/(2*e*
**4) - d**2*log(d/e + x)/(2*e**4) - x**2/(2*e**2), Eq(p, -1)), (-d
**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) - d**
2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e
**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p
+ 4*e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4
*p + 4*e**4), True)) + d**(2*p)*e*x**5*hyper((5/2, -p), (7/2,), e
**2*x**2*exp_polar(2*I*pi)/d**2)/5
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(-e^2x^2 + d^2)^p x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^3, x)
```

### 3.243 $\int x^2(d + ex) (d^2 - e^2x^2)^p dx$

**Optimal.** Leaf size=119

$$\frac{1}{3}dx^3 (d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; \frac{e^2x^2}{d^2}\right) - \frac{d^2 (d^2 - e^2x^2)^{p+1}}{2e^3(p+1)} + \frac{(d^2 - e^2x^2)^{p+2}}{2e^3(p+2)}$$

[Out]  $-(d^2*(d^2 - e^2*x^2)^(1 + p))/(2*e^3*(1 + p)) + (d^2 - e^2*x^2)^(2 + p)/(2*e^3*(2 + p)) + (d*x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(1 - (e^2*x^2)/d^2)^p)$

**Rubi [A]** time = 0.170065, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$

$$\frac{1}{3}dx^3 (d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; \frac{e^2x^2}{d^2}\right) - \frac{d^2 (d^2 - e^2x^2)^{p+1}}{2e^3(p+1)} + \frac{(d^2 - e^2x^2)^{p+2}}{2e^3(p+2)}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(d + e\*x)\*(d^2 - e^2\*x^2)^p,x]

[Out]  $-(d^2*(d^2 - e^2*x^2)^(1 + p))/(2*e^3*(1 + p)) + (d^2 - e^2*x^2)^(2 + p)/(2*e^3*(2 + p)) + (d*x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(1 - (e^2*x^2)/d^2)^p)$

**Rubi in Sympy [A]** time = 34.8804, size = 95, normalized size = 0.8

$$-\frac{d^2 (d^2 - e^2x^2)^{p+1}}{2e^3(p+1)} + \frac{dx^3 \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(-p, \frac{3}{2}; \frac{5}{2}; \frac{e^2x^2}{d^2}\right)}{3} + \frac{(d^2 - e^2x^2)^{p+2}}{2e^3(p+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p,x)

[Out]  $-d**2*(d**2 - e**2*x**2)**(p + 1)/(2*e**3*(p + 1)) + d*x**3*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper((-p, 3/2), (5/2, ), e**2*x**2/d**2)/3 + (d**2 - e**2*x**2)**(p + 2)/(2*e**3*(p + 2))$

**Mathematica [A]** time = 0.119976, size = 161, normalized size = 1.35

$$\frac{(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} \left(-3d^2e^2px^2 \left(1 - \frac{e^2x^2}{d^2}\right)^p + 3e^4(p+1)x^4 \left(1 - \frac{e^2x^2}{d^2}\right)^p + 2de^3(p^2 + 3p + 2)x^3 {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; \frac{e^2x^2}{d^2}\right)\right)}{6e^3(p+1)(p+2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(d + e\*x)\*(d^2 - e^2\*x^2)^p,x]

[Out]  $((d^2 - e^2*x^2)^p*(-3*d^2*e^2*p*x^2*(1 - (e^2*x^2)/d^2)^p + 3*e^4*(1 + p)*x^4*(1 - (e^2*x^2)/d^2)^p - 3*d^4*(-1 + (1 - (e^2*x^2)/d^2)^p) + 2*d*e^3*(2 + 3*p + p^2)*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(6*e^3*(1 + p)*(2 + p)*(1 - (e^2*x^2)/d^2)^p)$

p)

**Maple [F]** time = 0.057, size = 0, normalized size = 0.

$$\int x^2 (ex + d) (-e^2x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(e\*x+d)\*(-e^2\*x^2+d^2)^p,x)

[Out] int(x^2\*(e\*x+d)\*(-e^2\*x^2+d^2)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(-e^2x^2 + d^2)^p x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(-e^2\*x^2 + d^2)^p\*x^2,x, algorithm="maxima")

[Out] integrate((e\*x + d)\*(-e^2\*x^2 + d^2)^p\*x^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex^3 + dx^2)(-e^2x^2 + d^2)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(-e^2\*x^2 + d^2)^p\*x^2,x, algorithm="fricas")

[Out] integral((e\*x^3 + d\*x^2)\*(-e^2\*x^2 + d^2)^p, x)

**Sympy [A]** time = 10.8007, size = 382, normalized size = 3.21

$$\frac{dd^{2p}x^3 {}_2F_1\left(\frac{3}{2}, -p \middle| \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3} + e \left( \begin{array}{l} \left( \frac{x^4(d^2)^p}{4} - \frac{d^2 \log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} - \frac{d^2 \log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} - \frac{d^2}{-2d^2e^4+2e^6x^2} + \frac{e^2x^2 \log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} + \frac{e^2x^2 \log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} \right) \text{ for } e = 0 \\ \left( \frac{d^2 \log\left(-\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} - \frac{d^2 \log\left(\frac{d}{e}+x\right)}{-2d^2e^4+2e^6x^2} - \frac{x^2}{2e^2} \right) \text{ for } p = -2 \\ \left( -\frac{d^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} - \frac{d^2e^2px^2(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} + \frac{e^4px^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} + \frac{e^4x^4(d^2-e^2x^2)^p}{2e^4p^2+6e^4p+4e^4} \right) \text{ otherwise} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p,x)

```
[Out] d*d**(2*p)*x**3*hyper((3/2, -p), (5/2, ), e**2*x**2*exp_polar(2*I*pi)/d**2)/3 + e*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4), True))
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(-e^2x^2 + d^2)^p x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x^2, x)
```

### 3.244 $\int x(d + ex) (d^2 - e^2x^2)^p dx$

**Optimal.** Leaf size=89

$$\frac{1}{3}ex^3 (d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; \frac{e^2x^2}{d^2}\right) - \frac{d(d^2 - e^2x^2)^{p+1}}{2e^2(p+1)}$$

[Out]  $-(d*(d^2 - e^2*x^2)^(1 + p))/(2*e^2*(1 + p)) + (e*x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(1 - (e^2*x^2)/d^2)^p)$

**Rubi [A]** time = 0.0939422, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$\frac{1}{3}ex^3 (d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; \frac{e^2x^2}{d^2}\right) - \frac{d(d^2 - e^2x^2)^{p+1}}{2e^2(p+1)}$$

Antiderivative was successfully verified.

[In] Int[x\*(d + e\*x)\*(d^2 - e^2\*x^2)^p, x]

[Out]  $-(d*(d^2 - e^2*x^2)^(1 + p))/(2*e^2*(1 + p)) + (e*x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(1 - (e^2*x^2)/d^2)^p)$

**Rubi in Sympy [A]** time = 17.8137, size = 71, normalized size = 0.8

$$-\frac{d(d^2 - e^2x^2)^{p+1}}{2e^2(p+1)} + \frac{ex^3 \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(-p, \frac{3}{2} \middle| \frac{e^2x^2}{d^2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p, x)

[Out]  $-d*(d**2 - e**2*x**2)**(p + 1)/(2*e**2*(p + 1)) + e*x**3*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper((-p, 3/2), (5/2, ), e**2*x**2/d**2)/3$

**Mathematica [A]** time = 0.108477, size = 121, normalized size = 1.36

$$\frac{(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} \left(3de^2x^2 \left(1 - \frac{e^2x^2}{d^2}\right)^p + 2e^3(p+1)x^3 {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; \frac{e^2x^2}{d^2}\right) - 3d^3 \left(\left(1 - \frac{e^2x^2}{d^2}\right)^p - 1\right)\right)}{6e^2(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(d + e\*x)\*(d^2 - e^2\*x^2)^p, x]

[Out]  $((d^2 - e^2*x^2)^p*(3*d*e^2*x^2*(1 - (e^2*x^2)/d^2)^p - 3*d^3*(-1 + (1 - (e^2*x^2)/d^2)^p) + 2*e^3*(1 + p)*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(6*e^2*(1 + p)*(1 - (e^2*x^2)/d^2)^p)$

---

**Maple [F]** time = 0.052, size = 0, normalized size = 0.

$$\int x(ex + d)(-e^2x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(e\*x+d)\*(-e^2\*x^2+d^2)^p,x)

[Out] int(x\*(e\*x+d)\*(-e^2\*x^2+d^2)^p,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(-e^2\*x^2 + d^2)^p\*x,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex^2 + dx)(-e^2x^2 + d^2)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(-e^2\*x^2 + d^2)^p\*x,x, algorithm="fricas")

[Out] integral((e\*x^2 + d\*x)\*(-e^2\*x^2 + d^2)^p, x)

---

**Sympy [A]** time = 6.98514, size = 85, normalized size = 0.96

$$d \left( \begin{cases} \frac{x^2(d^2)^p}{2} & \text{for } e^2 = 0 \\ \frac{(d^2 - e^2x^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ -\frac{\log(d^2 - e^2x^2)}{2e^2} & \text{otherwise} \end{cases} \right) + \frac{d^{2p}ex^3 {}_2F_1\left(\frac{3}{2}, -p \mid \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p,x)

[Out] d\*Piecewise((x\*\*2\*(d\*\*2)\*\*p/2, Eq(e\*\*2, 0)), (-Piecewise(((d\*\*2 - e\*\*2\*x\*\*2)\*\*(p + 1)/(p + 1), Ne(p, -1)), (log(d\*\*2 - e\*\*2\*x\*\*2), True))/(2\*e\*\*2), True)) + d\*\*(2\*p)\*e\*x\*\*3\*hyper((3/2, -p), (5/2, ), e\*\*2\*x\*\*2\*exp\_polar(2\*I\*pi)/d\*\*2)/3

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(-e^2x^2 + d^2)^p x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*x, x)
```

### 3.245 $\int (d + ex) (d^2 - e^2 x^2)^p dx$

**Optimal.** Leaf size=83

$$dx (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right) - \frac{(d^2 - e^2 x^2)^{p+1}}{2e(p+1)}$$

[Out]  $-(d^2 - e^2 x^2)^{(1+p)}/(2^*e*(1+p)) + (d*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p$

**Rubi [A]** time = 0.0567295, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$dx (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right) - \frac{(d^2 - e^2 x^2)^{p+1}}{2e(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)\*(d^2 - e^2\*x^2)^p, x]

[Out]  $-(d^2 - e^2*x^2)^{(1+p)}/(2^*e*(1+p)) + (d*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p$

**Rubi in Sympy [A]** time = 25.1072, size = 66, normalized size = 0.8

$$\frac{2d \left(\frac{d+ex}{d}\right)^{-p} (d-ex)^{-p} (d-ex)^{p+1} (d^2 - e^2 x^2)^p {}_2F_1\left(\begin{matrix} -p-1, p+1 \\ p+2 \end{matrix} \middle| \frac{d-ex}{d}\right)}{e(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p, x)

[Out]  $-2*d*((d/2 + e*x/2)/d)**(-p)*(d - e*x)**(-p)*(d - e*x)**(p+1)*(d**2 - e**2*x**2)**p*hyper((-p-1, p+1), (p+2, ), (d/2 - e*x/2)/d)/(e*(p+1))$

**Mathematica [A]** time = 0.0719559, size = 116, normalized size = 1.4

$$\frac{(d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} \left(2de(p+1)x {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right) + e^2 x^2 \left(1 - \frac{e^2 x^2}{d^2}\right)^p - d^2 \left(\left(1 - \frac{e^2 x^2}{d^2}\right)^p - 1\right)\right)}{2e(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)\*(d^2 - e^2\*x^2)^p, x]

[Out]  $((d^2 - e^2*x^2)^p*(e^2*x^2*(1 - (e^2*x^2)/d^2)^p - d^2*(-1 + (1 - (e^2*x^2)/d^2)^p) + 2*d*e*(1+p)*x*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(2^*e*(1+p)*(1 - (e^2*x^2)/d^2)^p)$



**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int (ex + d)(-e^2x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)\*(-e^2\*x^2+d^2)^p, x)

[Out] int((e\*x+d)\*(-e^2\*x^2+d^2)^p, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(-e^2x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(-e^2\*x^2 + d^2)^p, x, algorithm="maxima")

[Out] integrate((e\*x + d)\*(-e^2\*x^2 + d^2)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex + d)(-e^2x^2 + d^2)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(-e^2\*x^2 + d^2)^p, x, algorithm="fricas")

[Out] integral((e\*x + d)\*(-e^2\*x^2 + d^2)^p, x)

**Sympy [A]** time = 6.31458, size = 82, normalized size = 0.99

$$d d^{2p} x {}_2F_1\left(\frac{1}{2}, -p \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right) + e \left( \begin{array}{ll} \frac{x^2 (d^2)^p}{2} & \text{for } e^2 = 0 \\ \frac{(d^2 - e^2 x^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \frac{\log(d^2 - e^2 x^2)}{2e^2} & \text{otherwise} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p, x)

[Out] d\*d\*\*(2\*p)\*x\*hyper((1/2, -p), (3/2, ), e\*\*2\*x\*\*2\*exp\_polar(2\*I\*pi)/d\*\*2) + e\*Piecewise((x\*\*2\*(d\*\*2)\*\*p/2, Eq(e\*\*2, 0)), (-Piecewise(((d\*\*2 - e\*\*2\*x\*\*2)\*\*(p + 1)/(p + 1), Ne(p, -1)), (log(d\*\*2 - e\*\*2\*x\*\*2), True))/(2\*e\*\*2), True))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(-e^2x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)*(-e^2*x^2 + d^2)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p, x)
```

$$3.246 \quad \int \frac{(d+ex)(d^2-e^2x^2)^p}{x} dx$$

**Optimal.** Leaf size=104

$$ex (d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2x^2}{d^2}\right) - \frac{(d^2 - e^2x^2)^{p+1} {}_2F_1\left(1, p+1; p+2; 1 - \frac{e^2x^2}{d^2}\right)}{2d(p+1)}$$

[Out] (e\*x\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[1/2, -p, 3/2, (e^2\*x^2)/d^2])/(1 - (e^2\*x^2)/d^2)^p - ((d^2 - e^2\*x^2)^(1 + p)\*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2\*x^2)/d^2])/(2\*d\*(1 + p))

**Rubi [A]** time = 0.126423, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$

$$ex (d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2x^2}{d^2}\right) - \frac{(d^2 - e^2x^2)^{p+1} {}_2F_1\left(1, p+1; p+2; 1 - \frac{e^2x^2}{d^2}\right)}{2d(p+1)}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*(d^2 - e^2\*x^2)^p)/x, x]

[Out] (e\*x\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[1/2, -p, 3/2, (e^2\*x^2)/d^2])/(1 - (e^2\*x^2)/d^2)^p - ((d^2 - e^2\*x^2)^(1 + p)\*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2\*x^2)/d^2])/(2\*d\*(1 + p))

**Rubi in Sympy [A]** time = 30.4703, size = 82, normalized size = 0.79

$$ex \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(-p, \frac{1}{2}; \frac{3}{2}; \frac{e^2x^2}{d^2}\right) - \frac{(d^2 - e^2x^2)^{p+1} {}_2F_1\left(1, p+1; p+2; 1 - \frac{e^2x^2}{d^2}\right)}{2d(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x, x)

[Out] e\*x\*(1 - e\*\*2\*x\*\*2/d\*\*2)\*\*(-p)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p\*hyper((-p, 1/2), (3/2, ), e\*\*2\*x\*\*2/d\*\*2) - (d\*\*2 - e\*\*2\*x\*\*2)\*\*(p + 1)\*hyper((1, p + 1), (p + 2, ), 1 - e\*\*2\*x\*\*2/d\*\*2)/(2\*d\*(p + 1))

**Mathematica [A]** time = 0.0755646, size = 104, normalized size = 1.

$$\frac{1}{2} (d^2 - e^2x^2)^p \left( \frac{d \left(1 - \frac{d^2}{e^2x^2}\right)^{-p} {}_2F_1\left(-p, -p; 1 - p; \frac{d^2}{e^2x^2}\right)}{p} + 2ex \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2x^2}{d^2}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(d^2 - e^2\*x^2)^p)/x, x]

[Out] ((d^2 - e^2\*x^2)^p\*((2\*e\*x\*Hypergeometric2F1[1/2, -p, 3/2, (e^2\*x^2)/d^2])/(1 - (e^2\*x^2)/d^2)^p + (d\*Hypergeometric2F1[-p, -p, 1

- p,  $d^2/(e^2 x^2)] / (p(1 - d^2/(e^2 x^2))^p) / 2$

**Maple [F]** time = 0.038, size = 0, normalized size = 0.

$$\int \frac{(ex + d)(-e^2 x^2 + d^2)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)*(-e^2*x^2+d^2)^p/x, x)`

[Out] `int((e*x+d)*(-e^2*x^2+d^2)^p/x, x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)(-e^2 x^2 + d^2)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x, x, algorithm="maxima")`

[Out] `integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex + d)(-e^2 x^2 + d^2)^p}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x, x, algorithm="fricas")`

[Out] `integral((e*x + d)*(-e^2*x^2 + d^2)^p/x, x)`

**Sympy [A]** time = 9.95561, size = 78, normalized size = 0.75

$$-\frac{de^{2p}x^{2p}e^{i\pi p}(-p)_2F_1\left(-p, -p \middle| \frac{d^2}{e^2 x^2}\right)}{2(-p+1)} + d^{2p}ex_2F_1\left(\frac{1}{2}, -p \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(-e**2*x**2+d**2)**p/x, x)`

[Out] `-d*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (-p + 1, ), d**2/(e**2*x**2))/(2*gamma(-p + 1)) + d**(2*p)*e*x*hyper((1/2, -p), (3/2, ), e**2*x**2*exp_polar(2*I*pi)/d**2)`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)(-e^2x^2 + d^2)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x, x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x, x)
```

$$3.247 \quad \int \frac{(d+ex)(d^2-e^2x^2)^p}{x^2} dx$$

**Optimal.** Leaf size=108

$$\frac{d(d^2-e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; \frac{e^2x^2}{d^2}\right)}{x} - \frac{e(d^2-e^2x^2)^{p+1} {}_2F_1\left(1, p+1; p+2; 1 - \frac{e^2x^2}{d^2}\right)}{2d^2(p+1)}$$

[Out] -((d\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[-1/2, -p, 1/2, (e^2\*x^2)/d^2])/(x\*(1 - (e^2\*x^2)/d^2)^p)) - (e\*(d^2 - e^2\*x^2)^(1+p)\*Hypergeometric2F1[1, 1+p, 2+p, 1 - (e^2\*x^2)/d^2])/(2\*d^2\*(1+p))

**Rubi [A]** time = 0.139177, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$

$$\frac{d(d^2-e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; \frac{e^2x^2}{d^2}\right)}{x} - \frac{e(d^2-e^2x^2)^{p+1} {}_2F_1\left(1, p+1; p+2; 1 - \frac{e^2x^2}{d^2}\right)}{2d^2(p+1)}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*(d^2 - e^2\*x^2)^p)/x^2, x]

[Out] -((d\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[-1/2, -p, 1/2, (e^2\*x^2)/d^2])/(x\*(1 - (e^2\*x^2)/d^2)^p)) - (e\*(d^2 - e^2\*x^2)^(1+p)\*Hypergeometric2F1[1, 1+p, 2+p, 1 - (e^2\*x^2)/d^2])/(2\*d^2\*(1+p))

**Rubi in Sympy [A]** time = 22.1809, size = 88, normalized size = 0.81

$$\frac{d \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(-p, -\frac{1}{2} \middle| \frac{e^2x^2}{d^2}\right)}{x} - \frac{e(d^2 - e^2x^2)^{p+1} {}_2F_1\left(1, p+1 \middle| 1 - \frac{e^2x^2}{d^2}\right)}{2d^2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x\*\*2, x)

[Out] -d\*(1 - e\*\*2\*x\*\*2/d\*\*2)\*\*(-p)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p\*hyper((-p, -1/2), (1/2, ), e\*\*2\*x\*\*2/d\*\*2)/x - e\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(p+1)\*hyper((1, p+1), (p+2, ), 1 - e\*\*2\*x\*\*2/d\*\*2)/(2\*d\*\*2\*(p+1))

**Mathematica [A]** time = 0.0699195, size = 106, normalized size = 0.98

$$\frac{1}{2} (d^2 - e^2x^2)^p \left( \frac{e \left(1 - \frac{d^2}{e^2x^2}\right)^{-p} {}_2F_1\left(-p, -p; 1-p; \frac{d^2}{e^2x^2}\right)}{p} - \frac{2d \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; \frac{e^2x^2}{d^2}\right)}{x} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(d^2 - e^2\*x^2)^p)/x^2, x]

[Out] ((d^2 - e^2\*x^2)^p\*((-2\*d\*Hypergeometric2F1[-1/2, -p, 1/2, (e^2\*x^2)/d^2])/(x\*(1 - (e^2\*x^2)/d^2)^p) + (e\*Hypergeometric2F1[-p, -p

,  $1 - p$ ,  $d^2/(e^2 x^2)] / (p(1 - d^2/(e^2 x^2))^p) / 2$

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int \frac{(ex + d)(-e^2 x^2 + d^2)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)*(-e^2*x^2+d^2)^p/x^2,x)`

[Out] `int((e*x+d)*(-e^2*x^2+d^2)^p/x^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)(-e^2 x^2 + d^2)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x^2,x, algorithm="maxima")`

[Out] `integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x^2, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex + d)(-e^2 x^2 + d^2)^p}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x^2,x, algorithm="fricas")`

[Out] `integral((e*x + d)*(-e^2*x^2 + d^2)^p/x^2, x)`

**Sympy [A]** time = 10.0617, size = 82, normalized size = 0.76

$$\frac{d d^{2p} {}_2F_1\left(-\frac{1}{2}, -p \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{x} - \frac{e e^{2p} x^{2p} e^{i\pi p} (-p) {}_2F_1\left(-p, -p \middle| \frac{d^2}{e^2 x^2}\right)}{2(-p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(-e**2*x**2+d**2)**p/x**2,x)`

[Out] `-d*d**(2*p)*hyper((-1/2, -p), (1/2, ), e**2*x**2*exp_polar(2*I*pi)/d**2)/x - e*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (-p + 1, ), d**2/(e**2*x**2))/(2*gamma(-p + 1))`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)(-e^2x^2 + d^2)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x^2, x)
```



$$3.248 \quad \int \frac{(d+ex)(d^2-e^2x^2)^p}{x^3} dx$$

**Optimal.** Leaf size=110

$$\frac{e(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; \frac{e^2x^2}{d^2}\right)}{x} - \frac{e^2(d^2 - e^2x^2)^{p+1} {}_2F_1\left(2, p+1; p+2; 1 - \frac{e^2x^2}{d^2}\right)}{2d^3(p+1)}$$

[Out] -((e\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[-1/2, -p, 1/2, (e^2\*x^2)/d^2])/(x\*(1 - (e^2\*x^2)/d^2)^p)) - (e^2\*(d^2 - e^2\*x^2)^(1+p)\*Hypergeometric2F1[2, 1+p, 2+p, 1 - (e^2\*x^2)/d^2])/(2\*d^3\*(1+p))

**Rubi [A]** time = 0.146365, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$

$$\frac{e(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; \frac{e^2x^2}{d^2}\right)}{x} - \frac{e^2(d^2 - e^2x^2)^{p+1} {}_2F_1\left(2, p+1; p+2; 1 - \frac{e^2x^2}{d^2}\right)}{2d^3(p+1)}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*(d^2 - e^2\*x^2)^p)/x^3, x]

[Out] -((e\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[-1/2, -p, 1/2, (e^2\*x^2)/d^2])/(x\*(1 - (e^2\*x^2)/d^2)^p)) - (e^2\*(d^2 - e^2\*x^2)^(1+p)\*Hypergeometric2F1[2, 1+p, 2+p, 1 - (e^2\*x^2)/d^2])/(2\*d^3\*(1+p))

**Rubi in Sympy [A]** time = 23.6107, size = 90, normalized size = 0.82

$$\frac{e\left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(-p, -\frac{1}{2} \middle| \frac{e^2x^2}{d^2} \right)}{x} - \frac{e^2(d^2 - e^2x^2)^{p+1} {}_2F_1\left(2, p+1 \middle| 1 - \frac{e^2x^2}{d^2} \right)}{2d^3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x\*\*3, x)

[Out] -e\*(1 - e\*\*2\*x\*\*2/d\*\*2)\*\*(-p)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p\*hyper((-p, -1/2), (1/2, ), e\*\*2\*x\*\*2/d\*\*2)/x - e\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(p+1)\*hyper((2, p+1), (p+2, ), 1 - e\*\*2\*x\*\*2/d\*\*2)/(2\*d\*\*3\*(p+1))

**Mathematica [A]** time = 0.103446, size = 111, normalized size = 1.01

$$\frac{(d^2 - e^2x^2)^p \left( \frac{d\left(1 - \frac{d^2}{e^2x^2}\right)^{-p} {}_2F_1\left(1-p, -p; 2-p; \frac{d^2}{e^2x^2}\right)}{p-1} - 2ex \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; \frac{e^2x^2}{d^2}\right) \right)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(d^2 - e^2\*x^2)^p)/x^3, x]

[Out] ((d^2 - e^2\*x^2)^p\*((-2\*e\*x\*Hypergeometric2F1[-1/2, -p, 1/2, (e^2\*x^2)/d^2])/(1 - (e^2\*x^2)/d^2)^p + (d\*Hypergeometric2F1[1 - p, -

$p, 2 - p, d^2/(e^2 x^2)]/((-1 + p) * (1 - d^2/(e^2 x^2))^p)) / (2 * x^2)$

**Maple [F]** time = 0.054, size = 0, normalized size = 0.

$$\int \frac{(ex + d)(-e^2 x^2 + d^2)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)*(-e^2*x^2+d^2)^p/x^3,x)`

[Out] `int((e*x+d)*(-e^2*x^2+d^2)^p/x^3,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)(-e^2 x^2 + d^2)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x^3,x, algorithm="maxima")`

[Out] `integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x^3, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex + d)(-e^2 x^2 + d^2)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)*(-e^2*x^2 + d^2)^p/x^3,x, algorithm="fricas")`

[Out] `integral((e*x + d)*(-e^2*x^2 + d^2)^p/x^3, x)`

**Sympy [A]** time = 12.1377, size = 85, normalized size = 0.77

$$-\frac{de^{2p}x^{2p}e^{i\pi p}(-p+1)_2F_1\left(-p,-p+1\left|\frac{d^2}{e^2x^2}\right.\right)}{2x^2(-p+2)} - \frac{d^{2p}e_2F_1\left(-\frac{1}{2},-p\left|\frac{e^2x^2e^{2i\pi}}{d^2}\right.\right)}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(-e**2*x**2+d**2)**p/x**3,x)`

[Out] `-d*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p + 1)*hyper((-p, -p + 1), (-p + 2, ), d**2/(e**2*x**2))/(2*x**2*gamma(-p + 2)) - d**(2*p)*e*hyper((-1/2, -p), (1/2, ), e**2*x**2*exp_polar(2*I*pi)/d**2)/x`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)(-e^2x^2 + d^2)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(-e^2\*x^2 + d^2)^p/x^3,x, algorithm="giac")

[Out] integrate((e\*x + d)\*(-e^2\*x^2 + d^2)^p/x^3, x)

### 3.249 $\int x^5(d+ex)^2(d^2-e^2x^2)^p dx$

**Optimal.** Leaf size=178

$$\frac{2}{7}dex^7(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)-\frac{2d^2(d^2-e^2x^2)^{p+3}}{e^6(p+3)} \\ +\frac{(d^2-e^2x^2)^{p+4}}{2e^6(p+4)}-\frac{d^6(d^2-e^2x^2)^{p+1}}{e^6(p+1)}+\frac{5d^4(d^2-e^2x^2)^{p+2}}{2e^6(p+2)}$$

[Out]  $-\left(\frac{d^6(d^2-e^2x^2)^{(1+p)}}{e^6(1+p)}\right)+\left(\frac{5d^4(d^2-e^2x^2)^{(2+p)}}{2e^6(2+p)}\right)-\left(\frac{2d^2(d^2-e^2x^2)^{(3+p)}}{e^6(3+p)}\right)+\left(\frac{d^2(d^2-e^2x^2)^{(4+p)}}{2e^6(4+p)}\right)+\left(\frac{2d^2e^2x^7(d^2-e^2x^2)^p \operatorname{Hypergeometric2F1}\left[\frac{7}{2},-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right]}{7(1-(e^2x^2)/d^2)^p}\right)$

**Rubi [A]** time = 0.316122, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{2}{7}dex^7(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)-\frac{2d^2(d^2-e^2x^2)^{p+3}}{e^6(p+3)} \\ +\frac{(d^2-e^2x^2)^{p+4}}{2e^6(p+4)}-\frac{d^6(d^2-e^2x^2)^{p+1}}{e^6(p+1)}+\frac{5d^4(d^2-e^2x^2)^{p+2}}{2e^6(p+2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^5*(d+e*x)^2*(d^2-e^2*x^2)^p,x]$

[Out]  $-\left(\frac{d^6(d^2-e^2x^2)^{(1+p)}}{e^6(1+p)}\right)+\left(\frac{5d^4(d^2-e^2x^2)^{(2+p)}}{2e^6(2+p)}\right)-\left(\frac{2d^2(d^2-e^2x^2)^{(3+p)}}{e^6(3+p)}\right)+\left(\frac{d^2(d^2-e^2x^2)^{(4+p)}}{2e^6(4+p)}\right)+\left(\frac{2d^2e^2x^7(d^2-e^2x^2)^p \operatorname{Hypergeometric2F1}\left[\frac{7}{2},-p,\frac{9}{2},\frac{e^2x^2}{d^2}\right]}{7(1-(e^2x^2)/d^2)^p}\right)$

**Rubi in Sympy [A]** time = 82.6202, size = 150, normalized size = 0.84

$$-\frac{d^6(d^2-e^2x^2)^{p+1}}{e^6(p+1)}+\frac{5d^4(d^2-e^2x^2)^{p+2}}{2e^6(p+2)}-\frac{2d^2(d^2-e^2x^2)^{p+3}}{e^6(p+3)} \\ +\frac{2dex^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p{}_2F_1\left(-p,\frac{7}{2}\left|\frac{e^2x^2}{d^2}\right.\right)}{7}+\frac{(d^2-e^2x^2)^{p+4}}{2e^6(p+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**5*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)$

[Out]  $-d**6*(d**2-e**2*x**2)**(p+1)/(e**6*(p+1))+5*d**4*(d**2-e**2*x**2)**(p+2)/(2*e**6*(p+2))-2*d**2*(d**2-e**2*x**2)**(p+3)/(e**6*(p+3))+2*d**e*x**7*(1-e**2*x**2/d**2)**(-p)*(d**2-e**2*x**2)**p*hyper((-p,7/2),(9/2,),(e**2*x**2/d**2))/7+(d**2-e**2*x**2)**(p+4)/(2*e**6*(p+4))$

**Mathematica [A]** time = 0.296819, size = 268, normalized size = 1.51

$$(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(4de^7(p^4+10p^3+35p^2+50p+24)x^7{}_2F_1\left(\frac{7}{2},-p;\frac{9}{2};\frac{e^2x^2}{d^2}\right)-7\left(-e^8(p^3+6p^2+11p+6)x^8\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(d + e\*x)^2\*(d^2 - e^2\*x^2)^p,x]

[Out]  $((d^2 - e^2 x^2)^p (-7 (2 d^6 e^2 p (7 + p) x^2 (1 - (e^2 x^2)/d^2)^p + d^4 e^4 p (7 + 8 p + p^2) x^4 (1 - (e^2 x^2)/d^2)^p - 4 d^2 e^6 (2 + 3 p + p^2) x^6 (1 - (e^2 x^2)/d^2)^p - e^8 (6 + 11 p + 6 p^2 + p^3) x^8 (1 - (e^2 x^2)/d^2)^p + 2 d^8 (7 + p) (-1 + (1 - (e^2 x^2)/d^2)^p)) + 4 d e^7 (24 + 50 p + 35 p^2 + 10 p^3 + p^4) x^7 \text{Hypergeometric2F1}[7/2, -p, 9/2, (e^2 x^2)/d^2]) / (14 e^6 (1 + p) (2 + p) (3 + p) (4 + p) (1 - (e^2 x^2)/d^2)^p)$

**Maple [F]** time = 0.093, size = 0, normalized size = 0.

$$\int x^5 (ex + d)^2 (-e^2 x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(e\*x+d)^2\*(-e^2\*x^2+d^2)^p,x)

[Out] int(x^5\*(e\*x+d)^2\*(-e^2\*x^2+d^2)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{((p^2 + 3p + 2)e^6 x^6 - (p^2 + p)d^2 e^4 x^4 - 2d^4 e^2 p x^2 - 2d^6)(-e^2 x^2 + d^2)^p d^2}{2(p^3 + 6p^2 + 11p + 6)e^6} + \int (e^2 x^7 + 2dex^6) e^{(p \log(ex+d) + p \log(-ex+d))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p\*x^5,x, algorithm="maxima")

[Out]  $1/2 * ((p^2 + 3*p + 2) * e^6 * x^6 - (p^2 + p) * d^2 * e^4 * x^4 - 2 * d^4 * e^2 * p * x^2 - 2 * d^6) * (-e^2 * x^2 + d^2)^p * d^2 / ((p^3 + 6 * p^2 + 11 * p + 6) * e^6) + \text{integrate}((e^2 * x^7 + 2 * d * e * x^6) * e^{(p * \log(e * x + d) + p * \log(-e * x + d))}, x)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( (e^2 x^7 + 2 dex^6 + d^2 x^5) (-e^2 x^2 + d^2)^p, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p\*x^5,x, algorithm="fricas")

[Out] integral((e^2\*x^7 + 2\*d\*e\*x^6 + d^2\*x^5)\*(-e^2\*x^2 + d^2)^p, x)

**Sympy [A]** time = 36.1297, size = 2924, normalized size = 16.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(e\*x+d)\*\*2\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p,x)

[Out]  $d^{**2} \text{Piecewise}((x^{**6}(d^{**2})^{**p}/6, \text{Eq}(e, 0)), (-2*d^{**4} \log(-d/e + x)/(4*d^{**4}e^{**6} - 8*d^{**2}e^{**8}x^{**2} + 4*e^{**10}x^{**4}) - 2*d^{**4} \log(d/e + x)/(4*d^{**4}e^{**6} - 8*d^{**2}e^{**8}x^{**2} + 4*e^{**10}x^{**4}) - 3*d^{**4}/(4*d^{**4}e^{**6} - 8*d^{**2}e^{**8}x^{**2} + 4*e^{**10}x^{**4}) + 4*d^{**2}e^{**2}x^{**2} \log(-d/e + x)/(4*d^{**4}e^{**6} - 8*d^{**2}e^{**8}x^{**2} + 4*e^{**10}x^{**4}) + 4*d^{**2}e^{**2}x^{**2} \log(d/e + x)/(4*d^{**4}e^{**6} - 8*d^{**2}e^{**8}x^{**2} + 4*e^{**10}x^{**4}) + 4*d^{**2}e^{**2}x^{**2}/(4*d^{**4}e^{**6} - 8*d^{**2}e^{**8}x^{**2} + 4*e^{**10}x^{**4}) - 2*e^{**4}x^{**4} \log(-d/e + x)/(4*d^{**4}e^{**6} - 8*d^{**2}e^{**8}x^{**2} + 4*e^{**10}x^{**4}) - 2*e^{**4}x^{**4} \log(d/e + x)/(4*d^{**4}e^{**6} - 8*d^{**2}e^{**8}x^{**2} + 4*e^{**10}x^{**4}), \text{Eq}(p, -3)), (-2*d^{**4} \log(-d/e + x)/(-2*d^{**2}e^{**6} + 2*e^{**8}x^{**2}) - 2*d^{**4} \log(d/e + x)/(-2*d^{**2}e^{**6} + 2*e^{**8}x^{**2}) - 2*d^{**4}/(-2*d^{**2}e^{**6} + 2*e^{**8}x^{**2}) + 2*d^{**2}e^{**2}x^{**2} \log(-d/e + x)/(-2*d^{**2}e^{**6} + 2*e^{**8}x^{**2}) + 2*d^{**2}e^{**2}x^{**2} \log(d/e + x)/(-2*d^{**2}e^{**6} + 2*e^{**8}x^{**2}) + e^{**4}x^{**4}/(-2*d^{**2}e^{**6} + 2*e^{**8}x^{**2}), \text{Eq}(p, -2)), (-d^{**4} \log(-d/e + x)/(2*e^{**6}) - d^{**4} \log(d/e + x)/(2*e^{**6}) - d^{**2}x^{**2}/(2*e^{**4}) - x^{**4}/(4*e^{**2}), \text{Eq}(p, -1)), (-2*d^{**6}(d^{**2} - e^{**2}x^{**2})^{**p}/(2*e^{**6}p^{**3} + 12*e^{**6}p^{**2} + 22*e^{**6}p + 12*e^{**6}) - 2*d^{**4}e^{**2}p^{**2}x^{**2}(d^{**2} - e^{**2}x^{**2})^{**p}/(2*e^{**6}p^{**3} + 12*e^{**6}p^{**2} + 22*e^{**6}p + 12*e^{**6}) - d^{**2}e^{**4}p^{**2}x^{**4}(d^{**2} - e^{**2}x^{**2})^{**p}/(2*e^{**6}p^{**3} + 12*e^{**6}p^{**2} + 22*e^{**6}p + 12*e^{**6}) - d^{**2}e^{**4}p^{**2}x^{**4}(d^{**2} - e^{**2}x^{**2})^{**p}/(2*e^{**6}p^{**3} + 12*e^{**6}p^{**2} + 22*e^{**6}p + 12*e^{**6}) + e^{**6}p^{**2}x^{**6}(d^{**2} - e^{**2}x^{**2})^{**p}/(2*e^{**6}p^{**3} + 12*e^{**6}p^{**2} + 22*e^{**6}p + 12*e^{**6}) + 3*e^{**6}p^{**2}x^{**6}(d^{**2} - e^{**2}x^{**2})^{**p}/(2*e^{**6}p^{**3} + 12*e^{**6}p^{**2} + 22*e^{**6}p + 12*e^{**6}) + 2*e^{**6}x^{**6}(d^{**2} - e^{**2}x^{**2})^{**p}/(2*e^{**6}p^{**3} + 12*e^{**6}p^{**2} + 22*e^{**6}p + 12*e^{**6}), \text{True})) + 2*d*d^{**2}p^{**2}e^{**7} \text{hyper}((7/2, -p), (9/2, ), e^{**2}x^{**2} \text{exp\_polar}(2*I*pi)/d^{**2})/7 + e^{**2} \text{Piecewise}((x^{**8}(d^{**2})^{**p}/8, \text{Eq}(e, 0)), (-6*d^{**6} \log(-d/e + x)/(-12*d^{**6}e^{**8} + 36*d^{**4}e^{**10}x^{**2} - 36*d^{**2}e^{**12}x^{**4} + 12*e^{**14}x^{**6}) - 6*d^{**6} \log(d/e + x)/(-12*d^{**6}e^{**8} + 36*d^{**4}e^{**10}x^{**2} - 36*d^{**2}e^{**12}x^{**4} + 12*e^{**14}x^{**6}) - 11*d^{**6}/(-12*d^{**6}e^{**8} + 36*d^{**4}e^{**10}x^{**2} - 36*d^{**2}e^{**12}x^{**4} + 12*e^{**14}x^{**6}) + 18*d^{**4}e^{**2}x^{**2} \log(-d/e + x)/(-12*d^{**6}e^{**8} + 36*d^{**4}e^{**10}x^{**2} - 36*d^{**2}e^{**12}x^{**4} + 12*e^{**14}x^{**6}) + 18*d^{**4}e^{**2}x^{**2} \log(d/e + x)/(-12*d^{**6}e^{**8} + 36*d^{**4}e^{**10}x^{**2} - 36*d^{**2}e^{**12}x^{**4} + 12*e^{**14}x^{**6}) + 27*d^{**4}e^{**2}x^{**2}/(-12*d^{**6}e^{**8} + 36*d^{**4}e^{**10}x^{**2} - 36*d^{**2}e^{**12}x^{**4} + 12*e^{**14}x^{**6}) - 18*d^{**2}e^{**4}x^{**4} \log(-d/e + x)/(-12*d^{**6}e^{**8} + 36*d^{**4}e^{**10}x^{**2} - 36*d^{**2}e^{**12}x^{**4} + 12*e^{**14}x^{**6}) - 18*d^{**2}e^{**4}x^{**4} \log(d/e + x)/(-12*d^{**6}e^{**8} + 36*d^{**4}e^{**10}x^{**2} - 36*d^{**2}e^{**12}x^{**4} + 12*e^{**14}x^{**6}) + 6*e^{**6}x^{**6} \log(-d/e + x)/(-12*d^{**6}e^{**8} + 36*d^{**4}e^{**10}x^{**2} - 36*d^{**2}e^{**12}x^{**4} + 12*e^{**14}x^{**6}) + 6*e^{**6}x^{**6} \log(d/e + x)/(-12*d^{**6}e^{**8} + 36*d^{**4}e^{**10}x^{**2} - 36*d^{**2}e^{**12}x^{**4} + 12*e^{**14}x^{**6}), \text{Eq}(p, -4)), (-6*d^{**6} \log(-d/e + x)/(4*d^{**4}e^{**8} - 8*d^{**2}e^{**10}x^{**2} + 4*e^{**12}x^{**4}) - 6*d^{**6} \log(d/e + x)/(4*d^{**4}e^{**8} - 8*d^{**2}e^{**10}x^{**2} + 4*e^{**12}x^{**4}) - 9*d^{**6}/(4*d^{**4}e^{**8} - 8*d^{**2}e^{**10}x^{**2} + 4*e^{**12}x^{**4}) + 12*d^{**4}e^{**2}x^{**2} \log(-d/e + x)/(4*d^{**4}e^{**8} - 8*d^{**2}e^{**10}x^{**2} + 4*e^{**12}x^{**4}) + 12*d^{**4}e^{**2}x^{**2} \log(d/e + x)/(4*d^{**4}e^{**8} - 8*d^{**2}e^{**10}x^{**2} + 4*e^{**12}x^{**4}) + 12*d^{**4}e^{**2}x^{**2}/(4*d^{**4}e^{**8} - 8*d^{**2}e^{**10}x^{**2} + 4*e^{**12}x^{**4}) - 6*d^{**2}e^{**4}x^{**4} \log(-d/e + x)/(4*d^{**4}e^{**8} - 8*d^{**2}e^{**10}x^{**2} + 4*e^{**12}x^{**4}) - 6*d^{**2}e^{**4}x^{**4} \log(d/e + x)/(4*d^{**4}e^{**8} - 8*d^{**2}e^{**10}x^{**2} + 4*e^{**12}x^{**4}) - 2*e^{**6}x^{**6}/(4*d^{**4}e^{**8} - 8*d^{**2}e^{**10}x^{**2} + 4*e^{**12}x^{**4}), \text{Eq}(p, -3)), (-6*d^{**6} \log(-d/e + x)/(-4*d^{**2}e^{**8} + 4*e^{**10}x^{**2}) - 6*d^{**6} \log(d/e + x)/(-4*d^{**2}e^{**8} + 4*e^{**10}x^{**2}) - 6*d^{**6}/(-4*d^{**2}e^{**8} + 4*e^{**10}x^{**2}) + 6*d^{**4}e^{**2}x^{**2} \log(-d/e + x)/(-4*d^{**2}e^{**8} + 4*e^{**10}x^{**2}) + 6*d^{**4}e^{**2}x^{**2} \log(d/e + x)/(-4*d^{**2}e^{**8} + 4*e^{**10}x^{**2}) + 3*d^{**2}e^{**4}x^{**4}/(-4*d^{**2}e^{**8} + 4*e^{**10}x^{**2}) + e^{**6}x^{**6}/(-4*d^{**2}e^{**8} + 4*e^{**10}x^{**2}), \text{Eq}(p, -2)), (-d^{**6} \log(-d/e + x)/(2*e^{**8}) - d^{**6} \log(d/e + x)/(2*e^{**8}) - d^{**4}x^{**2}/(2*e^{**6}) - d^{**2}x^{**4}/(4*e^{**4}) - x^{**6}/(6*e^{**2}), \text{Eq}(p, -1)), (-6*d^{**8}(d^{**2} - e^{**2}x^{**2})^{**p}/(2*e^{**8}p^{**4} + 20*e^{**8}p^{**3} + 70*e^{**8}p^{**2} + 100*e^{**8}p + 48*e^{**8}) - 6*d^{**6}e^{**2}p^{**2}x^{**2}(d^{**2} - e^{**2}x^{**2})^{**p}/(2*e^{**8}p^{**4} + 20*e^{**8}p^{**3} + 70*e^{**8}p^{**2} + 100*e^{**8}p + 48*e^{**8}) - 3*d^{**4}e^{**4}p^{**2}x^{**4}(d^{**2}$

```

- e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*
e**8*p + 48*e**8) - 3*d**4*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e
**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) -
d**2*e**6*p**3*x**6*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*
p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 3*d**2*e**6*p**2*x*
**6*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p*
**2 + 100*e**8*p + 48*e**8) - 2*d**2*e**6*p*x**6*(d**2 - e**2*x**2
)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48
*e**8) + e**8*p**3*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e
**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) + 6*e**8*p**2*x**
8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**
2 + 100*e**8*p + 48*e**8) + 11*e**8*p*x**8*(d**2 - e**2*x**2)**p/
(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8
) + 6*e**8*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3
+ 70*e**8*p**2 + 100*e**8*p + 48*e**8), True))

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2(-e^2x^2 + d^2)^p x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p\*x^5,x, algorithm="giac")

[Out] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p\*x^5, x)

$$3.250 \quad \int x^4(d+ex)^2(d^2-e^2x^2)^p dx$$

**Optimal.** Leaf size=185

$$\frac{2d^2(p+6)x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; \frac{e^2x^2}{d^2}\right)}{5(2p+7)} - \frac{x^5(d^2-e^2x^2)^{p+1}}{2p+7} - \frac{d(d^2-e^2x^2)^{p+3}}{e^5(p+3)} - \frac{d^5(d^2-e^2x^2)^{p+1}}{e^5(p+1)} + \frac{2d^3(d^2-e^2x^2)^{p+2}}{e^5(p+2)}$$

[Out]  $-\left(\frac{d^5(d^2-e^2x^2)^{(1+p)}}{e^{5(1+p)}}\right) - \left(\frac{x^5(d^2-e^2x^2)^{(1+p)}}{7+2p}\right) + \left(\frac{2d^3(d^2-e^2x^2)^{(2+p)}}{e^{5(2+p)}}\right) - \left(\frac{d(d^2-e^2x^2)^{(3+p)}}{e^{5(3+p)}}\right) + \left(\frac{2d^2(6+p)x^5(d^2-e^2x^2)^p \operatorname{Hypergeometric2F1}\left[\frac{5}{2}, -p, \frac{7}{2}, \frac{e^2x^2}{d^2}\right]}{5(7+2p)(1-(e^2x^2/d^2)^p)}\right)$

**Rubi [A]** time = 0.328758, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$\frac{2d^2(p+6)x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; \frac{e^2x^2}{d^2}\right)}{5(2p+7)} - \frac{x^5(d^2-e^2x^2)^{p+1}}{2p+7} - \frac{d(d^2-e^2x^2)^{p+3}}{e^5(p+3)} - \frac{d^5(d^2-e^2x^2)^{p+1}}{e^5(p+1)} + \frac{2d^3(d^2-e^2x^2)^{p+2}}{e^5(p+2)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^4(d+e*x)^2(d^2-e^2*x^2)^p, x]$

[Out]  $-\left(\frac{d^5(d^2-e^2x^2)^{(1+p)}}{e^{5(1+p)}}\right) - \left(\frac{x^5(d^2-e^2x^2)^{(1+p)}}{7+2p}\right) + \left(\frac{2d^3(d^2-e^2x^2)^{(2+p)}}{e^{5(2+p)}}\right) - \left(\frac{d(d^2-e^2x^2)^{(3+p)}}{e^{5(3+p)}}\right) + \left(\frac{2d^2(6+p)x^5(d^2-e^2x^2)^p \operatorname{Hypergeometric2F1}\left[\frac{5}{2}, -p, \frac{7}{2}, \frac{e^2x^2}{d^2}\right]}{5(7+2p)(1-(e^2x^2/d^2)^p)}\right)$

**Rubi in Sympy [A]** time = 64.092, size = 172, normalized size = 0.93

$$-\frac{d^5(d^2-e^2x^2)^{p+1}}{e^5(p+1)} + \frac{2d^3(d^2-e^2x^2)^{p+2}}{e^5(p+2)} + \frac{d^2x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p {}_2F_1\left(\frac{-p}{2}, \frac{5}{2}; \frac{e^2x^2}{d^2}\right)}{5} - \frac{d(d^2-e^2x^2)^{p+3}}{e^5(p+3)} + \frac{e^2x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p {}_2F_1\left(\frac{-p}{2}, \frac{9}{2}; \frac{e^2x^2}{d^2}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{rubi\_integrate}(x^{**4}(e*x+d)^{**2}(-e^{**2}*x^{**2}+d^{**2})^{**p}, x)$

[Out]  $-d^{**5}(d^{**2}-e^{**2}*x^{**2})^{**p}(p+1)/(e^{**5}(p+1)) + 2*d^{**3}(d^{**2}-e^{**2}*x^{**2})^{**p}(p+2)/(e^{**5}(p+2)) + d^{**2}*x^{**5}(1-e^{**2}*x^{**2}/d^{**2})^{**p}(-p)(d^{**2}-e^{**2}*x^{**2})^{**p} \operatorname{hyper}((-p, 5/2), (7/2, ), e^{**2}*x^{**2}/d^{**2})/5 - d(d^{**2}-e^{**2}*x^{**2})^{**p}(p+3)/(e^{**5}(p+3)) + e^{**2}*x^{**7}(1-e^{**2}*x^{**2}/d^{**2})^{**p}(-p)(d^{**2}-e^{**2}*x^{**2})^{**p} \operatorname{hyper}((-p, 7/2), (9/2, ), e^{**2}*x^{**2}/d^{**2})/7$



**Mathematica [A]** time = 0.394187, size = 252, normalized size = 1.36

$$\frac{(d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} \left(5e^7 (p^3 + 6p^2 + 11p + 6) x^7 {}_2F_1\left(\frac{7}{2}, -p; \frac{9}{2}; \frac{e^2 x^2}{d^2}\right) + 7d^2 e^5 (p^3 + 6p^2 + 11p + 6) x^5 {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; \frac{e^2 x^2}{d^2}\right)\right)}{35e^5(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(d + e\*x)^2\*(d^2 - e^2\*x^2)^p,x]

[Out] ((d^2 - e^2\*x^2)^p\*(-35\*d\*(2\*d^4\*e^2\*p\*x^2\*(1 - (e^2\*x^2)/d^2)^p + d^2\*e^4\*p\*(1 + p)\*x^4\*(1 - (e^2\*x^2)/d^2)^p - e^6\*(2 + 3\*p + p^2)\*x^6\*(1 - (e^2\*x^2)/d^2)^p + 2\*d^6\*(-1 + (1 - (e^2\*x^2)/d^2)^p) + 7\*d^2\*e^5\*(6 + 11\*p + 6\*p^2 + p^3)\*x^5\*Hypergeometric2F1[5/2, -p, 7/2, (e^2\*x^2)/d^2] + 5\*e^7\*(6 + 11\*p + 6\*p^2 + p^3)\*x^7\*Hypergeometric2F1[7/2, -p, 9/2, (e^2\*x^2)/d^2]))/(35\*e^5\*(1 + p)\*(2 + p)\*(3 + p)\*(1 - (e^2\*x^2)/d^2)^p)

**Maple [F]** time = 0.086, size = 0, normalized size = 0.

$$\int x^4 (ex + d)^2 (-e^2 x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(e\*x+d)^2\*(-e^2\*x^2+d^2)^p,x)

[Out] int(x^4\*(e\*x+d)^2\*(-e^2\*x^2+d^2)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2 (-e^2 x^2 + d^2)^p x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p\*x^4,x, algorithm="maxima")

[Out] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p\*x^4, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((e^2 x^6 + 2 d e x^5 + d^2 x^4) (-e^2 x^2 + d^2)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p\*x^4,x, algorithm="fricas")

[Out] integral((e^2\*x^6 + 2\*d\*e\*x^5 + d^2\*x^4)\*(-e^2\*x^2 + d^2)^p, x)

**Sympy [A]** time = 22.5418, size = 1015, normalized size = 5.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(e\*x+d)\*\*2\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p,x)

[Out] d\*\*2\*d\*\*(2\*p)\*x\*\*5\*hyper((5/2, -p), (7/2, ), e\*\*2\*x\*\*2\*exp\_polar(2\*I\*pi)/d\*\*2)/5 + 2\*d\*e\*Piecewise((x\*\*6\*(d\*\*2)\*\*p/6, Eq(e, 0)), (-2\*d\*\*4\*log(-d/e + x)/(4\*d\*\*4\*e\*\*6 - 8\*d\*\*2\*e\*\*8\*x\*\*2 + 4\*e\*\*10\*x\*\*4) - 2\*d\*\*4\*log(d/e + x)/(4\*d\*\*4\*e\*\*6 - 8\*d\*\*2\*e\*\*8\*x\*\*2 + 4\*e\*\*10\*x\*\*4) - 3\*d\*\*4/(4\*d\*\*4\*e\*\*6 - 8\*d\*\*2\*e\*\*8\*x\*\*2 + 4\*e\*\*10\*x\*\*4) + 4\*d\*\*2\*e\*\*2\*x\*\*2\*log(-d/e + x)/(4\*d\*\*4\*e\*\*6 - 8\*d\*\*2\*e\*\*8\*x\*\*2 + 4\*e\*\*10\*x\*\*4) + 4\*d\*\*2\*e\*\*2\*x\*\*2\*log(d/e + x)/(4\*d\*\*4\*e\*\*6 - 8\*d\*\*2\*e\*\*8\*x\*\*2 + 4\*e\*\*10\*x\*\*4) + 4\*d\*\*2\*e\*\*2\*x\*\*2/(4\*d\*\*4\*e\*\*6 - 8\*d\*\*2\*e\*\*8\*x\*\*2 + 4\*e\*\*10\*x\*\*4) - 2\*e\*\*4\*x\*\*4\*log(-d/e + x)/(4\*d\*\*4\*e\*\*6 - 8\*d\*\*2\*e\*\*8\*x\*\*2 + 4\*e\*\*10\*x\*\*4) - 2\*e\*\*4\*x\*\*4\*log(d/e + x)/(4\*d\*\*4\*e\*\*6 - 8\*d\*\*2\*e\*\*8\*x\*\*2 + 4\*e\*\*10\*x\*\*4), Eq(p, -3)), (-2\*d\*\*4\*log(-d/e + x)/(-2\*d\*\*2\*e\*\*6 + 2\*e\*\*8\*x\*\*2) - 2\*d\*\*4\*log(d/e + x)/(-2\*d\*\*2\*e\*\*6 + 2\*e\*\*8\*x\*\*2) - 2\*d\*\*4/(-2\*d\*\*2\*e\*\*6 + 2\*e\*\*8\*x\*\*2) + 2\*d\*\*2\*e\*\*2\*x\*\*2\*log(-d/e + x)/(-2\*d\*\*2\*e\*\*6 + 2\*e\*\*8\*x\*\*2) + 2\*d\*\*2\*e\*\*2\*x\*\*2\*log(d/e + x)/(-2\*d\*\*2\*e\*\*6 + 2\*e\*\*8\*x\*\*2) + e\*\*4\*x\*\*4/(-2\*d\*\*2\*e\*\*6 + 2\*e\*\*8\*x\*\*2), Eq(p, -2)), (-d\*\*4\*log(-d/e + x)/(2\*e\*\*6) - d\*\*4\*log(d/e + x)/(2\*e\*\*6) - d\*\*2\*x\*\*2/(2\*e\*\*4) - x\*\*4/(4\*e\*\*2), Eq(p, -1)), (-2\*d\*\*6\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p/(2\*e\*\*6\*p\*\*3 + 12\*e\*\*6\*p\*\*2 + 22\*e\*\*6\*p + 12\*e\*\*6) - 2\*d\*\*4\*e\*\*2\*p\*x\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p/(2\*e\*\*6\*p\*\*3 + 12\*e\*\*6\*p\*\*2 + 22\*e\*\*6\*p + 12\*e\*\*6) - d\*\*2\*e\*\*4\*p\*\*2\*x\*\*4\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p/(2\*e\*\*6\*p\*\*3 + 12\*e\*\*6\*p\*\*2 + 22\*e\*\*6\*p + 12\*e\*\*6) - d\*\*2\*e\*\*4\*p\*x\*\*4\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p/(2\*e\*\*6\*p\*\*3 + 12\*e\*\*6\*p\*\*2 + 22\*e\*\*6\*p + 12\*e\*\*6) + e\*\*6\*p\*\*2\*x\*\*6\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p/(2\*e\*\*6\*p\*\*3 + 12\*e\*\*6\*p\*\*2 + 22\*e\*\*6\*p + 12\*e\*\*6) + 3\*e\*\*6\*p\*x\*\*6\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p/(2\*e\*\*6\*p\*\*3 + 12\*e\*\*6\*p\*\*2 + 22\*e\*\*6\*p + 12\*e\*\*6) + 2\*e\*\*6\*x\*\*6\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p/(2\*e\*\*6\*p\*\*3 + 12\*e\*\*6\*p\*\*2 + 22\*e\*\*6\*p + 12\*e\*\*6), True)) + d\*\*(2\*p)\*e\*\*2\*x\*\*7\*hyper((7/2, -p), (9/2, ), e\*\*2\*x\*\*2\*exp\_polar(2\*I\*pi)/d\*\*2)/7

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2(-e^2x^2 + d^2)^p x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p\*x^4,x, algorithm="giac")

[Out] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p\*x^4, x)

### 3.251 $\int x^3(d+ex)^2(d^2-e^2x^2)^p dx$

**Optimal.** Leaf size=149

$$\frac{2}{5}dex^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right) + \frac{3d^2(d^2-e^2x^2)^{p+2}}{2e^4(p+2)} - \frac{(d^2-e^2x^2)^{p+3}}{2e^4(p+3)} - \frac{d^4(d^2-e^2x^2)^{p+1}}{e^4(p+1)}$$

[Out]  $-\left(\frac{d^4(d^2-e^2x^2)^{(1+p)}}{e^4(1+p)}\right) + \left(\frac{3d^2(d^2-e^2x^2)^{(2+p)}}{2e^4(2+p)}\right) - \frac{(d^2-e^2x^2)^{(3+p)}}{2e^4(3+p)} + \left(\frac{2d^5e^2x^5(d^2-e^2x^2)^p \operatorname{Hypergeometric2F1}\left[\frac{5}{2}, -p, \frac{7}{2}, \frac{e^2x^2}{d^2}\right]}{5\left(1-\frac{e^2x^2}{d^2}\right)^p}\right)$

**Rubi [A]** time = 0.266099, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{2}{5}dex^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{5}{2},-p;\frac{7}{2};\frac{e^2x^2}{d^2}\right) + \frac{3d^2(d^2-e^2x^2)^{p+2}}{2e^4(p+2)} - \frac{(d^2-e^2x^2)^{p+3}}{2e^4(p+3)} - \frac{d^4(d^2-e^2x^2)^{p+1}}{e^4(p+1)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[x^3(d+e*x)^2(d^2-e^2*x^2)^p, x]$

[Out]  $-\left(\frac{d^4(d^2-e^2x^2)^{(1+p)}}{e^4(1+p)}\right) + \left(\frac{3d^2(d^2-e^2x^2)^{(2+p)}}{2e^4(2+p)}\right) - \frac{(d^2-e^2x^2)^{(3+p)}}{2e^4(3+p)} + \left(\frac{2d^5e^2x^5(d^2-e^2x^2)^p \operatorname{Hypergeometric2F1}\left[\frac{5}{2}, -p, \frac{7}{2}, \frac{e^2x^2}{d^2}\right]}{5\left(1-\frac{e^2x^2}{d^2}\right)^p}\right)$

**Rubi in Sympy [A]** time = 64.773, size = 124, normalized size = 0.83

$$-\frac{d^4(d^2-e^2x^2)^{p+1}}{e^4(p+1)} + \frac{3d^2(d^2-e^2x^2)^{p+2}}{2e^4(p+2)} + \frac{2dex^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p{}_2F_1\left(-p, \frac{5}{2}; \frac{7}{2}; \frac{e^2x^2}{d^2}\right)}{5} - \frac{(d^2-e^2x^2)^{p+3}}{2e^4(p+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{rubi\_integrate}(x^{**3}*(e*x+d)^{**2}*(-e^{**2}*x^{**2}+d^{**2})^{**p}, x)$

[Out]  $-d^{**4}*(d^{**2}-e^{**2}*x^{**2})^{**p}*(p+1)/(e^{**4}*(p+1)) + 3*d^{**2}*(d^{**2}-e^{**2}*x^{**2})^{**p}*(p+2)/(2*e^{**4}*(p+2)) + 2*d^5*e^2*x^5*(1-e^{**2}*x^{**2}/d^{**2})^{**(-p)}*(d^{**2}-e^{**2}*x^{**2})^{**p}*\operatorname{hyper}((-p, 5/2), (7/2, ), e^{**2}*x^{**2}/d^{**2})/5 - (d^{**2}-e^{**2}*x^{**2})^{**p}*(p+3)/(2*e^{**4}*(p+3))$

**Mathematica [A]** time = 0.23915, size = 211, normalized size = 1.42

$$\frac{(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}\left(5e^6(p^2+3p+2)x^6\left(1-\frac{e^2x^2}{d^2}\right)^p + 4de^5(p^3+6p^2+11p+6)x^5{}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; \frac{e^2x^2}{d^2}\right) + 15d^2e^4(p+1)(p+2)(p+3)\right)}{10e^4(p+1)(p+2)(p+3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(d + e\*x)^2\*(d^2 - e^2\*x^2)^p,x]

[Out]  $((d^2 - e^2 x^2)^p (-5 d^4 e^2 p (5 + p) x^2 (1 - (e^2 x^2)/d^2)^p + 15 d^2 e^4 (1 + p) x^4 (1 - (e^2 x^2)/d^2)^p + 5 e^6 (2 + 3 p + p^2) x^6 (1 - (e^2 x^2)/d^2)^p - 5 d^6 (5 + p) (-1 + (1 - (e^2 x^2)/d^2)^p) + 4 d e^5 (6 + 11 p + 6 p^2 + p^3) x^5 \text{Hypergeometric2F1}[5/2, -p, 7/2, (e^2 x^2)/d^2]) / (10 e^4 (1 + p) (2 + p) (3 + p) (1 - (e^2 x^2)/d^2)^p)$

**Maple [F]** time = 0.073, size = 0, normalized size = 0.

$$\int x^3 (ex + d)^2 (-e^2 x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(e\*x+d)^2\*(-e^2\*x^2+d^2)^p,x)

[Out] int(x^3\*(e\*x+d)^2\*(-e^2\*x^2+d^2)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(e^4(p+1)x^4 - d^2 e^2 p x^2 - d^4)(-e^2 x^2 + d^2)^p d^2}{2(p^2 + 3p + 2)e^4} + \int (e^2 x^5 + 2 dex^4) e^{(p \log(ex+d) + p \log(-ex+d))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p\*x^3,x, algorithm="maxima")

[Out]  $1/2*(e^4*(p+1)*x^4 - d^2*e^2*p*x^2 - d^4)*(-e^2*x^2 + d^2)^p*d^2/((p^2 + 3*p + 2)*e^4) + \text{integrate}((e^2*x^5 + 2*d*e*x^4)*e^{(p*\log(e*x + d) + p*\log(-e*x + d))}, x)$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((e^2 x^5 + 2 dex^4 + d^2 x^3)(-e^2 x^2 + d^2)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p\*x^3,x, algorithm="fricas")

[Out] integral((e^2\*x^5 + 2\*d\*e\*x^4 + d^2\*x^3)\*(-e^2\*x^2 + d^2)^p, x)

**Sympy [A]** time = 21.3446, size = 1328, normalized size = 8.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(e\*x+d)\*\*2\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p,x)

```
[Out] d**2*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)
/(-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 +
2*e**6*x**2) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log
(-d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/
(-2*d**2*e**4 + 2*e**6*x**2), Eq(p, -2)), (-d**2*log(-d/e + x)/(2
*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2/(2*e**2), Eq(p, -1)),
(-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) -
d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p +
4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4
*p + 4*e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e
**4*p + 4*e**4), True)) + 2*d*d**(2*p)*e*x**5*hyper((5/2, -p), (7
/2, ), e**2*x**2*exp_polar(2*I*pi)/d**2)/5 + e**2*Piecewise((x**6*
(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x)/(4*d**4*e**6 - 8*d
**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*d**4*e**6
- 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d**2
*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d
**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log
(d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**
2*e**2*x**2/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e
**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*
x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2
+ 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2*e**6
+ 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2
) - 2*d**4/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d
/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e +
x)/(-2*d**2*e**6 + 2*e**8*x**2) + e**4*x**4/(-2*d**2*e**6 + 2*e
**8*x**2), Eq(p, -2)), (-d**4*log(-d/e + x)/(2*e**6) - d**4*log(d/
e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(4*e**2), Eq(p, -1)),
(-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*
e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e
**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p**2*x
**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p
+ 12*e**6) - d**2*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3
+ 12*e**6*p**2 + 22*e**6*p + 12*e**6) + e**6*p**2*x**6*(d**2 - e
**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) +
3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2
+ 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(2*e**
6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True))
```

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2(-e^2x^2 + d^2)^p x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^3, x)
```

### 3.252 $\int x^2(d + ex)^2 (d^2 - e^2x^2)^p dx$

**Optimal.** Leaf size=155

$$\frac{2d^2(p+4)x^3(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; \frac{e^2x^2}{d^2}\right)}{3(2p+5)} - \frac{x^3(d^2 - e^2x^2)^{p+1}}{2p+5} + \frac{d(d^2 - e^2x^2)^{p+2}}{e^3(p+2)} - \frac{d^3(d^2 - e^2x^2)^{p+1}}{e^3(p+1)}$$

[Out]  $-\left(\frac{d^3(d^2 - e^2x^2)^{p+1}}{e^3(p+1)} - \frac{d(d^2 - e^2x^2)^{p+2}}{e^3(p+2)} + \frac{x^3(d^2 - e^2x^2)^{p+1}}{2p+5}\right) / (e^3(1+p)) - (x^3(d^2 - e^2x^2)^{p+1} / (5+2p) + (d(d^2 - e^2x^2)^{p+2} / (e^3(2+p))) + (2d^2(4+p)x^3(d^2 - e^2x^2)^p \text{Hypergeometric2F1}[3/2, -p, 5/2, (e^2x^2)/d^2]) / (3(5+2p)(1 - (e^2x^2)/d^2)^p)$

**Rubi [A]** time = 0.275449, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$\frac{2d^2(p+4)x^3(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; \frac{e^2x^2}{d^2}\right)}{3(2p+5)} - \frac{x^3(d^2 - e^2x^2)^{p+1}}{2p+5} + \frac{d(d^2 - e^2x^2)^{p+2}}{e^3(p+2)} - \frac{d^3(d^2 - e^2x^2)^{p+1}}{e^3(p+1)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*(d + e*x)^2*(d^2 - e^2*x^2)^p, x]$

[Out]  $-\left(\frac{d^3(d^2 - e^2x^2)^{p+1}}{e^3(p+1)} - \frac{d(d^2 - e^2x^2)^{p+2}}{e^3(p+2)} + \frac{x^3(d^2 - e^2x^2)^{p+1}}{2p+5}\right) / (e^3(1+p)) - (x^3(d^2 - e^2x^2)^{p+1} / (5+2p) + (d(d^2 - e^2x^2)^{p+2} / (e^3(2+p))) + (2d^2(4+p)x^3(d^2 - e^2x^2)^p \text{Hypergeometric2F1}[3/2, -p, 5/2, (e^2x^2)/d^2]) / (3(5+2p)(1 - (e^2x^2)/d^2)^p)$

**Rubi in Sympy [A]** time = 50.1032, size = 146, normalized size = 0.94

$$-\frac{d^3(d^2 - e^2x^2)^{p+1}}{e^3(p+1)} + \frac{d^2x^3 \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(-p, \frac{3}{2}; \frac{5}{2}; \frac{e^2x^2}{d^2}\right)}{3} + \frac{d(d^2 - e^2x^2)^{p+2}}{e^3(p+2)} + \frac{e^2x^5 \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(-p, \frac{5}{2}; \frac{7}{2}; \frac{e^2x^2}{d^2}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**2*(e*x+d)**2*(-e**2*x**2+d**2)**p, x)$

[Out]  $-\frac{d^3(d^2 - e^2x^2)^{p+1}}{e^3(p+1)} + \frac{d^2x^3 \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(-p, \frac{3}{2}; \frac{5}{2}; \frac{e^2x^2}{d^2}\right)}{3} + \frac{d(d^2 - e^2x^2)^{p+2}}{e^3(p+2)} + \frac{e^2x^5 \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(-p, \frac{5}{2}; \frac{7}{2}; \frac{e^2x^2}{d^2}\right)}{5}$

**Mathematica [A]** time = 0.245525, size = 202, normalized size = 1.3

$$\frac{(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} \left(5d^2e^3(p^2 + 3p + 2)x^3 {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; \frac{e^2x^2}{d^2}\right) + 3\left(e^5(p^2 + 3p + 2)x^5 {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; \frac{e^2x^2}{d^2}\right) + 5de^4(p^2 + 3p + 2)\right)}{15e^3(p+1)(p+2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(d + e\*x)^2\*(d^2 - e^2\*x^2)^p,x]

[Out]  $((d^2 - e^2 x^2)^p (5 d^2 e^3 (2 + 3 p + p^2) x^3 \text{Hypergeometric2F1}[3/2, -p, 5/2, (e^2 x^2)/d^2] + 3 (-5 d^3 e^2 p x^2 (1 - (e^2 x^2)/d^2)^p + 5 d e^4 (1 + p) x^4 (1 - (e^2 x^2)/d^2)^p - 5 d^5 (-1 + (1 - (e^2 x^2)/d^2)^p) + e^5 (2 + 3 p + p^2) x^5 \text{Hypergeometric2F1}[5/2, -p, 7/2, (e^2 x^2)/d^2])) / (15 e^3 (1 + p)^2 (1 - (e^2 x^2)/d^2)^p)$

**Maple [F]** time = 0.066, size = 0, normalized size = 0.

$$\int x^2 (ex + d)^2 (-e^2 x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(e\*x+d)^2\*(-e^2\*x^2+d^2)^p,x)

[Out] int(x^2\*(e\*x+d)^2\*(-e^2\*x^2+d^2)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2 (-e^2 x^2 + d^2)^p x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p\*x^2,x, algorithm="maxima")

[Out] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p\*x^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2 x^4 + 2 d e x^3 + d^2 x^2\right) \left(-e^2 x^2 + d^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p\*x^2,x, algorithm="fricas")

[Out] integral((e^2\*x^4 + 2\*d\*e\*x^3 + d^2\*x^2)\*(-e^2\*x^2 + d^2)^p, x)

**Sympy [A]** time = 14.3772, size = 425, normalized size = 2.74

$$\frac{d^2 d^{2p} x^3 {}_2F_1\left(\frac{3}{2}, -p \mid \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{3} + 2de \left( \begin{array}{l} \left( \frac{x^4 (d^2)^p}{4} \right. \\ \left. \frac{d^2 \log\left(-\frac{d}{e} + x\right)}{-2d^2 e^4 + 2e^6 x^2} - \frac{d^2 \log\left(\frac{d}{e} + x\right)}{-2d^2 e^4 + 2e^6 x^2} - \frac{d^2}{-2d^2 e^4 + 2e^6 x^2} + \frac{e^2 x^2 \log\left(-\frac{d}{e} + x\right)}{-2d^2 e^4 + 2e^6 x^2} + \frac{e^2 x^2 \log\left(\frac{d}{e} + x\right)}{-2d^2 e^4 + 2e^6 x^2} \right) \\ \left( \frac{d^2 \log\left(-\frac{d}{e} + x\right)}{2e^4} - \frac{d^2 \log\left(\frac{d}{e} + x\right)}{2e^4} - \frac{x^2}{2e^2} \right) \\ \left( -\frac{d^4 (d^2 - e^2 x^2)^p}{2e^4 p^2 + 6e^4 p + 4e^4} - \frac{d^2 e^2 p x^2 (d^2 - e^2 x^2)^p}{2e^4 p^2 + 6e^4 p + 4e^4} + \frac{e^4 p x^4 (d^2 - e^2 x^2)^p}{2e^4 p^2 + 6e^4 p + 4e^4} + \frac{e^4 x^4 (d^2 - e^2 x^2)^p}{2e^4 p^2 + 6e^4 p + 4e^4} \right) \end{array} \right. \begin{array}{l} \text{for } e = 0 \\ \text{for } p = -2 \\ \text{for } p = -1 \\ \text{otherwise} \end{array} \\ + \frac{d^{2p} e^2 x^5 {}_2F_1\left(\frac{5}{2}, -p \mid \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**2*d**(2*p)*x**3*hyper((3/2, -p), (5/2, ), e**2*x**2*exp_polar(2*I*pi)/d**2)/3 + 2*d*e*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4), True)) + d**(2*p)*e**2*x**5*hyper((5/2, -p), (7/2, ), e**2*x**2*exp_polar(2*I*pi)/d**2)/5
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2 (-e^2 x^2 + d^2)^p x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x^2, x)
```



### 3.253 $\int x(d + ex)^2 (d^2 - e^2x^2)^p dx$

**Optimal.** Leaf size=118

$$\frac{2}{3}dex^3 (d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; \frac{e^2x^2}{d^2}\right) - \frac{d^2 (d^2 - e^2x^2)^{p+1}}{e^2(p+1)} + \frac{(d^2 - e^2x^2)^{p+2}}{2e^2(p+2)}$$

[Out]  $-\left(\frac{d^2 (d^2 - e^2x^2)^{p+1}}{e^2(p+1)} + \frac{(d^2 - e^2x^2)^{p+2}}{2e^2(p+2)}\right) + \frac{2d^2 e^2 x^3 (d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; \frac{e^2x^2}{d^2}\right)}{3}$

**Rubi [A]** time = 0.182293, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$

$$\frac{2}{3}dex^3 (d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; \frac{e^2x^2}{d^2}\right) - \frac{d^2 (d^2 - e^2x^2)^{p+1}}{e^2(p+1)} + \frac{(d^2 - e^2x^2)^{p+2}}{2e^2(p+2)}$$

Antiderivative was successfully verified.

[In] Int[x\*(d + e\*x)^2\*(d^2 - e^2\*x^2)^p, x]

[Out]  $-\left(\frac{d^2 (d^2 - e^2x^2)^{p+1}}{e^2(p+1)} + \frac{(d^2 - e^2x^2)^{p+2}}{2e^2(p+2)}\right) + \frac{2d^2 e^2 x^3 (d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; \frac{e^2x^2}{d^2}\right)}{3}$

**Rubi in Sympy [A]** time = 34.0773, size = 102, normalized size = 0.86

$$\frac{4d^3 \left(\frac{d+ex}{d}\right)^{-p} (d-ex)^{-p} (d-ex)^{p+1} (d^2 - e^2x^2)^p {}_2F_1\left(-p-2, p+1; p+2; \frac{d-ex}{d}\right)}{e^2(p+1)(p+2)} - \frac{(d+ex)^2 (d^2 - e^2x^2)^{p+1}}{2e^2(p+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(e\*x+d)\*\*2\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p, x)

[Out]  $-4d^3 \left(\frac{d+ex}{d}\right)^{-p} (d-ex)^{-p} (d-ex)^{p+1} (d^2 - e^2x^2)^p {}_2F_1\left(-p-2, p+1; p+2; \frac{d-ex}{d}\right) - \frac{(d+ex)^2 (d^2 - e^2x^2)^{p+1}}{2e^2(p+2)}$

**Mathematica [A]** time = 0.156604, size = 163, normalized size = 1.38

$$\frac{(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} \left(6d^2 e^2 x^2 \left(1 - \frac{e^2x^2}{d^2}\right)^p + 3e^4(p+1)x^4 \left(1 - \frac{e^2x^2}{d^2}\right)^p + 4de^3(p^2 + 3p + 2)x^3 {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; \frac{e^2x^2}{d^2}\right) - 3\right)}{6e^2(p+1)(p+2)}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(d + e\*x)^2\*(d^2 - e^2\*x^2)^p, x]

[Out]  $\frac{(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} \left(6d^2 e^2 x^2 \left(1 - \frac{e^2x^2}{d^2}\right)^p + 3e^4(p+1)x^4 \left(1 - \frac{e^2x^2}{d^2}\right)^p + 4de^3(p^2 + 3p + 2)x^3 {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; \frac{e^2x^2}{d^2}\right) - 3\right)}{6e^2(p+1)(p+2)}$

$d^2)^p$

**Maple [F]** time = 0.058, size = 0, normalized size = 0.

$$\int x (ex + d)^2 (-e^2x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(e\*x+d)^2\*(-e^2\*x^2+d^2)^p,x)

[Out] int(x\*(e\*x+d)^2\*(-e^2\*x^2+d^2)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p\*x,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^3 + 2dex^2 + d^2x\right)\left(-e^2x^2 + d^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p\*x,x, algorithm="fricas")

[Out] integral((e^2\*x^3 + 2\*d\*e\*x^2 + d^2\*x)\*(-e^2\*x^2 + d^2)^p, x)

**Sympy [A]** time = 11.7053, size = 440, normalized size = 3.73

$$d^2 \left( \begin{cases} \frac{x^2(d^2)^p}{2} & \text{for } e^2 = 0 \\ \frac{(d^2 - e^2x^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \frac{\log(d^2 - e^2x^2)}{2e^2} & \text{otherwise} \end{cases} \right) + \frac{2dd^2pex^3{}_2F_1\left(\frac{3}{2}, -p \mid \frac{e^2x^2e^{2i\pi}}{d^2}\right)}{3}$$

$$+ e^2 \left( \begin{cases} \frac{x^4(d^2)^p}{4} & \text{for } e = 0 \\ \frac{d^2 \log\left(-\frac{d}{e} + x\right)}{-2d^2e^4 + 2e^6x^2} - \frac{d^2 \log\left(\frac{d}{e} + x\right)}{-2d^2e^4 + 2e^6x^2} - \frac{d^2}{-2d^2e^4 + 2e^6x^2} + \frac{e^2x^2 \log\left(-\frac{d}{e} + x\right)}{-2d^2e^4 + 2e^6x^2} + \frac{e^2x^2 \log\left(\frac{d}{e} + x\right)}{-2d^2e^4 + 2e^6x^2} & \text{for } p = -2 \\ \frac{d^2 \log\left(-\frac{d}{e} + x\right)}{2e^4} - \frac{d^2 \log\left(\frac{d}{e} + x\right)}{2e^4} - \frac{x^2}{2e^2} & \text{for } p = -1 \\ -\frac{d^4(d^2 - e^2x^2)^p}{2e^4p^2 + 6e^4p + 4e^4} - \frac{d^2e^2px^2(d^2 - e^2x^2)^p}{2e^4p^2 + 6e^4p + 4e^4} + \frac{e^4px^4(d^2 - e^2x^2)^p}{2e^4p^2 + 6e^4p + 4e^4} + \frac{e^4x^4(d^2 - e^2x^2)^p}{2e^4p^2 + 6e^4p + 4e^4} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x+d)\*\*2\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p,x)

```
[Out] d**2*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**
2 - e**2*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**
2), True))/(2*e**2), True)) + 2*d*d**(2*p)*e*x**3*hyper((3/2, -p)
, (5/2, ), e**2*x**2*exp_polar(2*I*pi)/d**2)/3 + e**2*Piecewise((x
**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(-2*d**2*e**4 +
2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d
**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d*
**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2
*e**6*x**2), Eq(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log
(d/e + x)/(2*e**4) - x**2/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e*
**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) - d**2*e**2*p*x**2*
(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) + e**4*p*
x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) + e*
**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4),
True))
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2 (-e^2x^2 + d^2)^p x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*x, x)
```

$$3.254 \quad \int (d + ex)^2 (d^2 - e^2 x^2)^p dx$$

**Optimal.** Leaf size=71

$$\frac{d^{2p+2} \left(\frac{ex}{d} + 1\right)^{-p-1} (d^2 - e^2 x^2)^{p+1} {}_2F_1\left(-p-2, p+1; p+2; \frac{d-ex}{2d}\right)}{e(p+1)}$$

[Out] -((2^(2 + p)\*d\*(1 + (e\*x)/d)^(-1 - p)\*(d^2 - e^2\*x^2)^(1 + p)\*Hypergeometric2F1[-2 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/(e\*(1 + p)))

**Rubi [A]** time = 0.0775095, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{d^{2p+2} \left(\frac{ex}{d} + 1\right)^{-p-1} (d^2 - e^2 x^2)^{p+1} {}_2F_1\left(-p-2, p+1; p+2; \frac{d-ex}{2d}\right)}{e(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^2\*(d^2 - e^2\*x^2)^p,x]

[Out] -((2^(2 + p)\*d\*(1 + (e\*x)/d)^(-1 - p)\*(d^2 - e^2\*x^2)^(1 + p)\*Hypergeometric2F1[-2 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/(e\*(1 + p)))

**Rubi in Sympy [A]** time = 26.7078, size = 68, normalized size = 0.96

$$\frac{4d^2 \left(\frac{d+ex}{2}\right)^{-p} (d-ex)^{-p} (d-ex)^{p+1} (d^2 - e^2 x^2)^p {}_2F_1\left(-p-2, p+1; p+2; \frac{d-ex}{d}\right)}{e(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*2\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p,x)

[Out] -4\*d\*\*2\*((d/2 + e\*x/2)/d)\*\*(-p)\*(d - e\*x)\*\*(-p)\*(d - e\*x)\*\*(p + 1)\*((d\*\*2 - e\*\*2\*x\*\*2)\*\*p\*hyper((-p - 2, p + 1), (p + 2, ), (d/2 - e\*x/2)/d))/(e\*(p + 1))

**Mathematica [B]** time = 0.182647, size = 150, normalized size = 2.11

$$\frac{(d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} \left(3d^2 e(p+1)x {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right) + 3de^2 x^2 \left(1 - \frac{e^2 x^2}{d^2}\right)^p + e^3(p+1)x^3 {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; \frac{e^2 x^2}{d^2}\right) - 3d^3\right)}{3e(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^2\*(d^2 - e^2\*x^2)^p,x]

[Out] ((d^2 - e^2\*x^2)^p\*(3\*d\*e^2\*x^2\*(1 - (e^2\*x^2)/d^2)^p - 3\*d^3\*(-1 + (1 - (e^2\*x^2)/d^2)^p) + 3\*d^2\*e\*(1 + p)\*x\*Hypergeometric2F1[1/2, -p, 3/2, (e^2\*x^2)/d^2] + e^3\*(1 + p)\*x^3\*Hypergeometric2F1[3/2, -p, 5/2, (e^2\*x^2)/d^2]))/(3\*e\*(1 + p)\*(1 - (e^2\*x^2)/d^2)^p)

---

**Maple [F]** time = 0.053, size = 0, normalized size = 0.

$$\int (ex + d)^2 (-e^2x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^2\*(-e^2\*x^2+d^2)^p,x)

[Out] int((e\*x+d)^2\*(-e^2\*x^2+d^2)^p,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2 (-e^2x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p,x, algorithm="maxima")

[Out] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^2 + 2dex + d^2\right)\left(-e^2x^2 + d^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p,x, algorithm="fricas")

[Out] integral((e^2\*x^2 + 2\*d\*e\*x + d^2)\*(-e^2\*x^2 + d^2)^p, x)

---

**Sympy [A]** time = 8.68044, size = 124, normalized size = 1.75

$$d^2 d^{2p} x {}_2F_1\left(\frac{1}{2}, -p \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right) + 2de \left( \begin{array}{ll} \frac{x^2 (d^2)^p}{2} & \text{for } e^2 = 0 \\ \frac{(d^2 - e^2 x^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ -\frac{\log(d^2 - e^2 x^2)}{2e^2} & \text{otherwise} \end{array} \right)$$

$$+ \frac{d^{2p} e^2 x^3 {}_2F_1\left(\frac{3}{2}, -p \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p,x)

[Out] d\*\*2\*d\*\*(2\*p)\*x\*hyper((1/2, -p), (3/2, ), e\*\*2\*x\*\*2\*exp\_polar(2\*I\*pi)/d\*\*2) + 2\*d\*e\*Piecewise((x\*\*2\*(d\*\*2)\*\*p/2, Eq(e\*\*2, 0)), (-Piecewise(((d\*\*2 - e\*\*2\*x\*\*2)\*\*(p + 1)/(p + 1), Ne(p, -1)), (log(d\*\*2 - e\*\*2\*x\*\*2), True))/(2\*e\*\*2), True)) + d\*\*(2\*p)\*e\*\*2\*x\*\*3\*hyp

$er((3/2, -p), (5/2, ), e^{2x^2} \exp_{\text{polar}}(2i\pi)/d^2)/3$

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2 (-e^2x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p,x, algorithm="giac")

[Out] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p, x)

$$3.255 \quad \int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x} dx$$

**Optimal.** Leaf size=128

$$\frac{2dex(d^2-e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2x^2}{d^2}\right)}{(d^2-e^2x^2)^{p+1} {}_2F_1\left(1, p+1; p+2; 1 - \frac{e^2x^2}{d^2}\right)} - \frac{(d^2-e^2x^2)^{p+1}}{2(p+1)}$$

[Out]  $-(d^2 - e^2x^2)^{(1+p)}/(2*(1+p)) + (2*d*e*x*(d^2 - e^2x^2)^p * \text{Hypergeometric2F1}[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p - ((d^2 - e^2*x^2)^{(1+p)} * \text{Hypergeometric2F1}[1, 1+p, 2+p, 1 - (e^2*x^2)/d^2])/(2*(1+p))$

**Rubi [A]** time = 0.185525, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$\frac{2dex(d^2-e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2x^2}{d^2}\right)}{(d^2-e^2x^2)^{p+1} {}_2F_1\left(1, p+1; p+2; 1 - \frac{e^2x^2}{d^2}\right)} - \frac{(d^2-e^2x^2)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^2\*(d^2 - e^2\*x^2)^p)/x, x]

[Out]  $-(d^2 - e^2x^2)^{(1+p)}/(2*(1+p)) + (2*d*e*x*(d^2 - e^2x^2)^p * \text{Hypergeometric2F1}[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p - ((d^2 - e^2*x^2)^{(1+p)} * \text{Hypergeometric2F1}[1, 1+p, 2+p, 1 - (e^2*x^2)/d^2])/(2*(1+p))$

**Rubi in Sympy [A]** time = 37.8625, size = 102, normalized size = 0.8

$$2dex \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(-p, \frac{1}{2}; \frac{3}{2}; \frac{e^2x^2}{d^2}\right) - \frac{(d^2 - e^2x^2)^{p+1} {}_2F_1\left(1, p+1; p+2; 1 - \frac{e^2x^2}{d^2}\right)}{2(p+1)} - \frac{(d^2 - e^2x^2)^{p+1}}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*2\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x, x)

[Out]  $2*d*e*x*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*\text{hyper}((-p, 1/2), (3/2, ), e**2*x**2/d**2) - (d**2 - e**2*x**2)**(p+1)*\text{hyper}((1, p+1), (p+2, ), 1 - e**2*x**2/d**2)/(2*(p+1)) - (d**2 - e**2*x**2)**(p+1)/(2*(p+1))$

**Mathematica [A]** time = 0.419431, size = 147, normalized size = 1.15

$$\frac{1}{2} (d^2 - e^2x^2)^p \left( \frac{d^2 \left(1 - \frac{d^2}{e^2x^2}\right)^{-p} {}_2F_1\left(-p, -p; 1-p; \frac{d^2}{e^2x^2}\right)}{p} + 4dex \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2x^2}{d^2}\right) + \frac{d^2 \left(\left(1 - \frac{e^2x^2}{d^2}\right)^{-p} - 1\right)}{p+1} + \frac{e^2x^2}{p+1} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^2\*(d^2 - e^2\*x^2)^p)/x, x]

[Out] ((d^2 - e^2\*x^2)^p\*((e^2\*x^2)/(1 + p) + (d^2\*(-1 + (1 - (e^2\*x^2)/d^2)^(-p)))/(1 + p) + (4\*d\*e\*x\*Hypergeometric2F1[1/2, -p, 3/2, (e^2\*x^2)/d^2])/(1 - (e^2\*x^2)/d^2)^p + (d^2\*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2\*x^2)])/(p\*(1 - d^2/(e^2\*x^2))^p))/2

**Maple [F]** time = 0.041, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^2 (-e^2x^2 + d^2)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^2\*(-e^2\*x^2+d^2)^p/x, x)

[Out] int((e\*x+d)^2\*(-e^2\*x^2+d^2)^p/x, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^2 (-e^2x^2 + d^2)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p/x, x, algorithm="maxima")

[Out] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p/x, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e^2x^2 + 2dex + d^2)(-e^2x^2 + d^2)^p}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p/x, x, algorithm="fricas")

[Out] integral((e^2\*x^2 + 2\*d\*e\*x + d^2)\*(-e^2\*x^2 + d^2)^p/x, x)

**Sympy [A]** time = 10.2874, size = 136, normalized size = 1.06

$$\frac{d^2 e^{2p} x^{2p} e^{i\pi p} (-p) {}_2F_1\left(\begin{matrix} -p, -p \\ -p+1 \end{matrix} \middle| \frac{d^2}{e^2 x^2}\right)}{2(-p+1)} + 2dd^{2p} ex {}_2F_1\left(\begin{matrix} \frac{1}{2}, -p \\ \frac{3}{2} \end{matrix} \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right) + e^2 \left( \begin{array}{l} \frac{x^2 (d^2)^p}{2} \quad \text{for } e^2 = 0 \\ \frac{(d^2 - e^2 x^2)^{p+1}}{p+1} \quad \text{for } p \neq -1 \\ \frac{\log(d^2 - e^2 x^2)}{2e^2} \quad \text{otherwise} \end{array} \right)$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(-e**2*x**2+d**2)**p/x,x)
```

```
[Out] -d**2*e**(2*p)*x**(2*p)*exp(I*pi*p)*gamma(-p)*hyper((-p, -p), (-p
+ 1, ), d**2/(e**2*x**2))/(2*gamma(-p + 1)) + 2*d*d**(2*p)*e*x*hy
per((1/2, -p), (3/2, ), e**2*x**2*exp_polar(2*I*pi)/d**2) + e**2*P
iecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e*
*2*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), Tr
ue))/(2*e**2), True))
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^2(-e^2x^2 + d^2)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p/x,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p/x, x)
```

$$3.256 \quad \int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x^2} dx$$

**Optimal.** Leaf size=128

$$\frac{-2e^2px(d^2-e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2x^2}{d^2}\right) + e(d^2-e^2x^2)^{p+1} {}_2F_1\left(1, p+1; p+2; 1 - \frac{e^2x^2}{d^2}\right)}{d(p+1)} - \frac{(d^2-e^2x^2)^{p+1}}{x}$$

[Out]  $-\left(\frac{d^2 - e^2x^2}{x}\right)^{(1+p)} - (2e^2px^2(d^2 - e^2x^2)^p \text{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, \frac{e^2x^2}{d^2}\right]) / (1 - (e^2x^2)/d^2)^p - (e^2(d^2 - e^2x^2)^{p+1} \text{Hypergeometric2F1}\left[1, 1+p, 2+p, 1 - \frac{e^2x^2}{d^2}\right]) / (d(1+p))$

**Rubi [A]** time = 0.217803, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{-2e^2px(d^2-e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2x^2}{d^2}\right) + e(d^2-e^2x^2)^{p+1} {}_2F_1\left(1, p+1; p+2; 1 - \frac{e^2x^2}{d^2}\right)}{d(p+1)} - \frac{(d^2-e^2x^2)^{p+1}}{x}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^2\*(d^2 - e^2\*x^2)^p)/x^2, x]

[Out]  $-\left(\frac{d^2 - e^2x^2}{x}\right)^{(1+p)} - (2e^2px^2(d^2 - e^2x^2)^p \text{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, \frac{e^2x^2}{d^2}\right]) / (1 - (e^2x^2)/d^2)^p - (e^2(d^2 - e^2x^2)^{p+1} \text{Hypergeometric2F1}\left[1, 1+p, 2+p, 1 - \frac{e^2x^2}{d^2}\right]) / (d(1+p))$

**Rubi in Sympy [A]** time = 44.8743, size = 133, normalized size = 1.04

$$\frac{d^2 \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(-p, -\frac{1}{2}; \frac{1}{2}; \frac{e^2x^2}{d^2}\right)}{x} + e^2x \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(-p, \frac{1}{2}; \frac{3}{2}; \frac{e^2x^2}{d^2}\right) - \frac{e(d^2 - e^2x^2)^{p+1} {}_2F_1\left(1, p+1; p+2; 1 - \frac{e^2x^2}{d^2}\right)}{d(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*2\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x\*\*2, x)

[Out]  $-d^2(1 - e^2x^2/d^2)^{-p}(d^2 - e^2x^2)^p \text{hyper}((-p, -1/2), (1/2, ), e^2x^2/d^2)/x + e^2x(1 - e^2x^2/d^2)^{-p}(d^2 - e^2x^2)^p \text{hyper}((-p, 1/2), (3/2, ), e^2x^2/d^2) - e^2(d^2 - e^2x^2)^{p+1} \text{hyper}((1, p+1), (p+2, ), 1 - e^2x^2/d^2) / (d(p+1))$

**Mathematica [A]** time = 0.205905, size = 148, normalized size = 1.16

$$(d^2 - e^2 x^2)^p \left( e \left( \frac{d \left(1 - \frac{d^2}{e^2 x^2}\right)^{-p} {}_2F_1\left(-p, -p; 1 - p; \frac{d^2}{e^2 x^2}\right)}{p} + ex \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right) \right) - \frac{d^2 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{x} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^2\*(d^2 - e^2\*x^2)^p)/x^2, x]

[Out] (d^2 - e^2\*x^2)^p\*(-((d^2\*Hypergeometric2F1[-1/2, -p, 1/2, (e^2\*x^2)/d^2])/(x\*(1 - (e^2\*x^2)/d^2)^p)) + e\*((e\*x\*Hypergeometric2F1[1/2, -p, 3/2, (e^2\*x^2)/d^2])/(1 - (e^2\*x^2)/d^2)^p + (d\*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2\*x^2)])/(p\*(1 - d^2/(e^2\*x^2))^p)))

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^2 (-e^2 x^2 + d^2)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^2\*(-e^2\*x^2+d^2)^p/x^2, x)

[Out] int((e\*x+d)^2\*(-e^2\*x^2+d^2)^p/x^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^2 (-e^2 x^2 + d^2)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p/x^2, x, algorithm="maxima")

[Out] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p/x^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e^2 x^2 + 2 d e x + d^2) (-e^2 x^2 + d^2)^p}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p/x^2, x, algorithm="fricas")

[Out] integral((e^2\*x^2 + 2\*d\*e\*x + d^2)\*(-e^2\*x^2 + d^2)^p/x^2, x)

---

**Sympy [A]** time = 12.699, size = 116, normalized size = 0.91

$$\frac{d^2 d^{2p} {}_2F_1\left(-\frac{1}{2}, -p \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{x} - \frac{d e^{2p} x^{2p} e^{i\pi p} (-p) {}_2F_1\left(-p, -p \middle| \frac{d^2}{e^2 x^2}\right)}{(-p+1)} + d^{2p} e^2 x {}_2F_1\left(\frac{1}{2}, -p \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x\*\*2,x)

[Out] -d\*\*2\*d\*\*(2\*p)\*hyper((-1/2, -p), (1/2,), e\*\*2\*x\*\*2\*exp\_polar(2\*I\*pi)/d\*\*2)/x - d\*e\*e\*\*(2\*p)\*x\*\*(2\*p)\*exp(I\*pi\*p)\*gamma(-p)\*hyper((-p, -p), (-p+1,), d\*\*2/(e\*\*2\*x\*\*2))/gamma(-p+1) + d\*\*(2\*p)\*e\*\*2\*x\*hyper((1/2, -p), (3/2,), e\*\*2\*x\*\*2\*exp\_polar(2\*I\*pi)/d\*\*2)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2(-e^2x^2+d^2)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p/x^2,x, algorithm="giac")

[Out] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p/x^2, x)

$$3.257 \quad \int \frac{(d+ex)^2(d^2-e^2x^2)^p}{x^3} dx$$

**Optimal.** Leaf size=139

$$\frac{2de(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; \frac{e^2x^2}{d^2}\right)}{x} - \frac{e^2(1-p)(d^2 - e^2x^2)^{p+1} {}_2F_1\left(1, p+1; p+2; 1 - \frac{e^2x^2}{d^2}\right)}{2d^2(p+1)} - \frac{(d^2 - e^2x^2)^{p+1}}{2x^2}$$

[Out]  $-(d^2 - e^2x^2)^{(1+p)}/(2x^2) - (2d^2e^2(d^2 - e^2x^2)^p \text{Hypergeometric2F1}[-1/2, -p, 1/2, (e^2x^2)/d^2])/(x^2(1 - (e^2x^2)/d^2)^p) - (e^2(1-p)(d^2 - e^2x^2)^{(p+1)} \text{Hypergeometric2F1}[1, 1+p, 2+p, 1 - (e^2x^2)/d^2])/(2d^2(1+p))$

**Rubi [A]** time = 0.248563, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{2de(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; \frac{e^2x^2}{d^2}\right)}{x} - \frac{e^2(1-p)(d^2 - e^2x^2)^{p+1} {}_2F_1\left(1, p+1; p+2; 1 - \frac{e^2x^2}{d^2}\right)}{2d^2(p+1)} - \frac{(d^2 - e^2x^2)^{p+1}}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^2\*(d^2 - e^2\*x^2)^p)/x^3, x]

[Out]  $-(d^2 - e^2x^2)^{(1+p)}/(2x^2) - (2d^2e^2(d^2 - e^2x^2)^p \text{Hypergeometric2F1}[-1/2, -p, 1/2, (e^2x^2)/d^2])/(x^2(1 - (e^2x^2)/d^2)^p) - (e^2(1-p)(d^2 - e^2x^2)^{(p+1)} \text{Hypergeometric2F1}[1, 1+p, 2+p, 1 - (e^2x^2)/d^2])/(2d^2(1+p))$

**Rubi in Sympy [A]** time = 35.0233, size = 136, normalized size = 0.98

$$\frac{2de \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(-p, -\frac{1}{2} \middle| \frac{e^2x^2}{d^2}\right)}{x} - \frac{e^2 (d^2 - e^2x^2)^{p+1} {}_2F_1\left(1, p+1 \middle| 1 - \frac{e^2x^2}{d^2}\right)}{2d^2(p+1)} - \frac{e^2 (d^2 - e^2x^2)^{p+1} {}_2F_1\left(2, p+1 \middle| 1 - \frac{e^2x^2}{d^2}\right)}{2d^2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*2\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x\*\*3, x)

[Out]  $-2*d^2*e^2(1 - e^2*x^2/d^2)^{-p}(d^2 - e^2*x^2)^p \text{hyper}((-p, -1/2), (1/2, ), e^2*x^2/d^2)/x - e^2*(d^2 - e^2*x^2)^{(p+1)} \text{hyper}((1, p+1), (p+2, ), 1 - e^2*x^2/d^2)/(2*d^2*(p+1)) - e^2*(d^2 - e^2*x^2)^{(p+1)} \text{hyper}((2, p+1), (p+2, ), 1 - e^2*x^2/d^2)/(2*d^2*(p+1))$

**Mathematica [A]** time = 0.233056, size = 152, normalized size = 1.09

$$\frac{(d^2 - e^2 x^2)^p \left( \frac{\left(1 - \frac{d^2}{e^2 x^2}\right)^{-p} \left(d^2 {}_2F_1\left(1-p, -p; 2-p; \frac{d^2}{e^2 x^2}\right) + e^2 (p-1) x^2 {}_2F_1\left(-p, -p; 1-p; \frac{d^2}{e^2 x^2}\right)\right)}{(p-1)p} - 4dex \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right) \right)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^2\*(d^2 - e^2\*x^2)^p)/x^3, x]

[Out] ((d^2 - e^2\*x^2)^p\*((-4\*d\*e\*x\*Hypergeometric2F1[-1/2, -p, 1/2, (e^2\*x^2)/d^2])/(1 - (e^2\*x^2)/d^2)^p + (d^2\*p\*Hypergeometric2F1[1 - p, -p, 2 - p, d^2/(e^2\*x^2)] + e^2\*(-1 + p)\*x^2\*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2\*x^2)])/((-1 + p)\*p\*(1 - d^2/(e^2\*x^2))^p))/(2\*x^2)

**Maple [F]** time = 0.057, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^2 (-e^2 x^2 + d^2)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^2\*(-e^2\*x^2+d^2)^p/x^3, x)

[Out] int((e\*x+d)^2\*(-e^2\*x^2+d^2)^p/x^3, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^2 (-e^2 x^2 + d^2)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p/x^3, x, algorithm="maxima")

[Out] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p/x^3, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e^2 x^2 + 2 dex + d^2)(-e^2 x^2 + d^2)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p/x^3, x, algorithm="fricas")

[Out] integral((e^2\*x^2 + 2\*d\*e\*x + d^2)\*(-e^2\*x^2 + d^2)^p/x^3, x)

**Sympy [A]** time = 15.2524, size = 139, normalized size = 1.

$$\frac{d^2 e^{2p} x^{2p} e^{i\pi p} (-p+1) {}_2F_1\left(-p, -p+1 \middle| \frac{d^2}{e^2 x^2}\right)}{2x^2(-p+2)} - \frac{2dd^{2p} e {}_2F_1\left(-\frac{1}{2}, -p \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{x} - \frac{e^2 e^{2p} x^{2p} e^{i\pi p} (-p) {}_2F_1\left(-p, -p \middle| \frac{d^2}{e^2 x^2}\right)}{2(-p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x\*\*3,x)

[Out]  $-d^{2p} e^{2p} x^{2p} \exp(i\pi p) \gamma(-p+1) \text{hyper}((-p, -p+1), (-p+2, ), d^2/(e^{2p} x^2)) / (2x^{2p} \gamma(-p+2)) - 2d^{2p} e^{2p} x^{2p} \exp(i\pi p) \text{hyper}((-1/2, -p), (1/2, ), e^{2p} x^2 \exp_{\text{polar}}(2i\pi)/d^2) / x - e^{2p} e^{2p} x^{2p} \exp(i\pi p) \gamma(-p) \text{hyper}((-p, -p), (-p+1, ), d^2/(e^{2p} x^2)) / (2\gamma(-p+1))$

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2 (-e^2 x^2 + d^2)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p/x^3,x, algorithm="giac")

[Out] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p/x^3, x)

### 3.258 $\int x^5(d+ex)^3(d^2-e^2x^2)^p dx$

**Optimal.** Leaf size=222

$$\frac{2d^2e(3p+17)x^7(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2}, -p; \frac{9}{2}; \frac{e^2x^2}{d^2}\right) - ex^7(d^2-e^2x^2)^{p+1}}{7(2p+9)} + \frac{3d(d^2-e^2x^2)^{p+4}}{2e^6(p+4)} - \frac{2d^7(d^2-e^2x^2)^{p+1}}{e^6(p+1)} + \frac{11d^5(d^2-e^2x^2)^{p+2}}{2e^6(p+2)} - \frac{5d^3(d^2-e^2x^2)^{p+3}}{e^6(p+3)}$$

[Out]  $(-2*d^7*(d^2 - e^2*x^2)^(1 + p))/(e^6*(1 + p)) - (e*x^7*(d^2 - e^2*x^2)^(1 + p))/(9 + 2*p) + (11*d^5*(d^2 - e^2*x^2)^(2 + p))/(2*e^6*(2 + p)) - (5*d^3*(d^2 - e^2*x^2)^(3 + p))/(e^6*(3 + p)) + (3*d*(d^2 - e^2*x^2)^(4 + p))/(2*e^6*(4 + p)) + (2*d^2*e*(17 + 3*p)*x^7*(d^2 - e^2*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(7*(9 + 2*p)*(1 - (e^2*x^2)/d^2)^p)$

**Rubi [A]** time = 0.411273, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{2d^2e(3p+17)x^7(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p}{}_2F_1\left(\frac{7}{2}, -p; \frac{9}{2}; \frac{e^2x^2}{d^2}\right) - ex^7(d^2-e^2x^2)^{p+1}}{7(2p+9)} + \frac{3d(d^2-e^2x^2)^{p+4}}{2e^6(p+4)} - \frac{2d^7(d^2-e^2x^2)^{p+1}}{e^6(p+1)} + \frac{11d^5(d^2-e^2x^2)^{p+2}}{2e^6(p+2)} - \frac{5d^3(d^2-e^2x^2)^{p+3}}{e^6(p+3)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^5*(d+e*x)^3*(d^2-e^2*x^2)^p, x]$

[Out]  $(-2*d^7*(d^2 - e^2*x^2)^(1 + p))/(e^6*(1 + p)) - (e*x^7*(d^2 - e^2*x^2)^(1 + p))/(9 + 2*p) + (11*d^5*(d^2 - e^2*x^2)^(2 + p))/(2*e^6*(2 + p)) - (5*d^3*(d^2 - e^2*x^2)^(3 + p))/(e^6*(3 + p)) + (3*d*(d^2 - e^2*x^2)^(4 + p))/(2*e^6*(4 + p)) + (2*d^2*e*(17 + 3*p)*x^7*(d^2 - e^2*x^2)^p*Hypergeometric2F1[7/2, -p, 9/2, (e^2*x^2)/d^2])/(7*(9 + 2*p)*(1 - (e^2*x^2)/d^2)^p)$

**Rubi in Sympy [A]** time = 101.881, size = 207, normalized size = 0.93

$$-\frac{2d^7(d^2-e^2x^2)^{p+1}}{e^6(p+1)} + \frac{11d^5(d^2-e^2x^2)^{p+2}}{2e^6(p+2)} - \frac{5d^3(d^2-e^2x^2)^{p+3}}{e^6(p+3)} + \frac{3d^2ex^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p{}_2F_1\left(\frac{7}{2}, -p; \frac{9}{2}; \frac{e^2x^2}{d^2}\right)}{7} + \frac{3d(d^2-e^2x^2)^{p+4}}{2e^6(p+4)} + \frac{e^3x^9\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p{}_2F_1\left(\frac{9}{2}, -p; \frac{11}{2}; \frac{e^2x^2}{d^2}\right)}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**5*(e*x+d)**3*(-e**2*x**2+d**2)**p, x)$

[Out]  $-2*d**7*(d**2 - e**2*x**2)**(p + 1)/(e**6*(p + 1)) + 11*d**5*(d**2 - e**2*x**2)**(p + 2)/(2*e**6*(p + 2)) - 5*d**3*(d**2 - e**2*x**2)**(p + 3)/(e**6*(p + 3)) + 3*d**2*e*x**7*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper((-p, 7/2), (9/2, ), e**2*x**2/d**2)$



$\frac{2}{7} + 3d(d^2 - e^2x^2)^p(p+4)/(2e^{6p}(p+4)) + e^{3x} \frac{9(1 - e^{2x^2}/d^2)^p(-p)(d^2 - e^{2x^2})^p \operatorname{hyper}((-p, 9/2), (11/2), e^{2x^2}/d^2)}{9}$

**Mathematica [A]** time = 0.472572, size = 320, normalized size = 1.44

$(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} \left(14e^9(p^4 + 10p^3 + 35p^2 + 50p + 24)x^9 {}_2F_1\left(\frac{9}{2}, -p; \frac{11}{2}; \frac{e^2x^2}{d^2}\right) + 54d^2e^7(p^4 + 10p^3 + 35p^2 + 50p + 24)\right)$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(d + e\*x)^3\*(d^2 - e^2\*x^2)^p,x]

[Out]  $((d^2 - e^2x^2)^p(-63d(2d^6e^{2p}(13+p)x^2(1 - (e^2x^2)/d^2)^p + d^4e^{4p}(13 + 14p + p^2)x^4(1 - (e^2x^2)/d^2)^p + 2d^2e^{6p}(-4 - 4p + p^2 + p^3)x^6(1 - (e^2x^2)/d^2)^p - 3e^{8p}(6 + 11p + 6p^2 + p^3)x^8(1 - (e^2x^2)/d^2)^p + 2d^8(13 + p)(-1 + (1 - (e^2x^2)/d^2)^p)) + 54d^2e^{7p}(24 + 50p + 35p^2 + 10p^3 + p^4)x^7 \operatorname{Hypergeometric2F1}[7/2, -p, 9/2, (e^2x^2)/d^2] + 14e^{9p}(24 + 50p + 35p^2 + 10p^3 + p^4)x^9 \operatorname{Hypergeometric2F1}[9/2, -p, 11/2, (e^2x^2)/d^2])/(126e^{6p}(1+p)^2(2+p)^3(3+p)^4(4+p)(1 - (e^2x^2)/d^2)^p)$

**Maple [F]** time = 0.083, size = 0, normalized size = 0.

$$\int x^5 (ex + d)^3 (-e^2x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(e\*x+d)^3\*(-e^2\*x^2+d^2)^p,x)

[Out] int(x^5\*(e\*x+d)^3\*(-e^2\*x^2+d^2)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{((p^2 + 3p + 2)e^6x^6 - (p^2 + p)d^2e^4x^4 - 2d^4e^2px^2 - 2d^6)(-e^2x^2 + d^2)^p d^3}{2(p^3 + 6p^2 + 11p + 6)e^6} + \int (e^3x^8 + 3de^2x^7 + 3d^2ex^6) e^{(p \log(ex+d) + p \log(-ex+d))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p\*x^5,x, algorithm="maxima")

[Out]  $\frac{1}{2}((p^2 + 3p + 2)e^{6p}x^6 - (p^2 + p)d^2e^{4p}x^4 - 2d^4e^{2p}px^2 - 2d^6)e^{3p \log(-e^2x^2 + d^2)} d^3 / ((p^3 + 6p^2 + 11p + 6)e^{6p}) + \int (e^{3p \log(ex+d)} x^8 + 3d e^{2p \log(ex+d)} x^7 + 3d^2 e^{p \log(ex+d)} x^6) e^{p \log(-ex+d)} dx$

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left((e^3x^8 + 3de^2x^7 + 3d^2ex^6 + d^3x^5)(-e^2x^2 + d^2)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x^5,x, algorithm="fricas")
```

```
[Out] integral((e^3*x^8 + 3*d*e^2*x^7 + 3*d^2*e*x^6 + d^3*x^5)*(-e^2*x^2 + d^2)^p, x)
```

**Sympy [A]** time = 44.3248, size = 2966, normalized size = 13.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**3*Piecewise((x**6*(d**2)**p/6, Eq(e, 0)), (-2*d**4*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*d**4*log(d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 3*d**4/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) + 4*d**2*e**2*x**2*log(d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(-d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4) - 2*e**4*x**4*log(d/e + x)/(4*d**4*e**6 - 8*d**2*e**8*x**2 + 4*e**10*x**4), Eq(p, -3)), (-2*d**4*log(-d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) - 2*d**4/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(-d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) + 2*d**2*e**2*x**2*log(d/e + x)/(-2*d**2*e**6 + 2*e**8*x**2) + e**4*x**4/(-2*d**2*e**6 + 2*e**8*x**2), Eq(p, -2)), (-d**4*log(-d/e + x)/(2*e**6) - d**4*log(d/e + x)/(2*e**6) - d**2*x**2/(2*e**4) - x**4/(4*e**2), Eq(p, -1)), (-2*d**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - 2*d**4*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) - d**2*e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) + e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) + 3*e**6*p*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6) + 2*e**6*x**6*(d**2 - e**2*x**2)**p/(2*e**6*p**3 + 12*e**6*p**2 + 22*e**6*p + 12*e**6), True)) + 3*d**2*d*(2*p)*e*x**7*hyper((7/2, -p), (9/2, ), e**2*x**2*exp_polar(2*I*pi)/d**2)/7 + 3*d*e**2*Piecewise((x**8*(d**2)**p/8, Eq(e, 0)), (-6*d**6*log(-d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 6*d**6*log(d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 11*d**6/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 18*d**4*e**2*x**2*log(-d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 18*d**4*e**2*x**2*log(d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 27*d**4*e**2*x**2/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 18*d**2*e**4*x**4*log(-d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 18*d**2*e**4*x**4*log(d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) - 18*d**2*e**4*x**4/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 6*e**6*x**6*log(-d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6) + 6*e**6*x**6*log(d/e + x)/(-12*d**6*e**8 + 36*d**4*e**10*x**2 - 36*d**2*e**12*x**4 + 12*e**14*x**6), Eq(p, -4)), (-6*d**6*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) - 6*d**6*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) - 9*d**6/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) + 12*d**4*e**2*x**2*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) + 12*d**4*e**2*x**2*log(d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4) +
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12*d**4*e**2*x**2/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4
) - 6*d**2*e**4*x**4*log(-d/e + x)/(4*d**4*e**8 - 8*d**2*e**10*x**
*2 + 4*e**12*x**4) - 6*d**2*e**4*x**4*log(d/e + x)/(4*d**4*e**8 -
8*d**2*e**10*x**2 + 4*e**12*x**4) - 2*e**6*x**6/(4*d**4*e**8 - 8
*d**2*e**10*x**2 + 4*e**12*x**4), Eq(p, -3)), (-6*d**6*log(-d/e +
x)/(-4*d**2*e**8 + 4*e**10*x**2) - 6*d**6*log(d/e + x)/(-4*d**2*
e**8 + 4*e**10*x**2) - 6*d**6/(-4*d**2*e**8 + 4*e**10*x**2) + 6*d
**4*e**2*x**2*log(-d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) + 6*d**
4*e**2*x**2*log(d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) + 3*d**2*e
**4*x**4/(-4*d**2*e**8 + 4*e**10*x**2) + e**6*x**6/(-4*d**2*e**8
+ 4*e**10*x**2), Eq(p, -2)), (-d**6*log(-d/e + x)/(2*e**8) - d**6
*log(d/e + x)/(2*e**8) - d**4*x**2/(2*e**6) - d**2*x**4/(4*e**4)
- x**6/(6*e**2), Eq(p, -1)), (-6*d**8*(d**2 - e**2*x**2)**p/(2*e
**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 6
*d**6*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p
**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 3*d**4*e**4*p**2*x**4
*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2
+ 100*e**8*p + 48*e**8) - 3*d**4*e**4*p*x**4*(d**2 - e**2*x**2)*
*p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*
e**8) - d**2*e**6*p**3*x**6*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 2
0*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 3*d**2*e**6*
p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*
e**8*p**2 + 100*e**8*p + 48*e**8) - 2*d**2*e**6*p*x**6*(d**2 - e
**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8
*p + 48*e**8) + e**8*p**3*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4
+ 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) + 6*e**8*p
**2*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*
e**8*p**2 + 100*e**8*p + 48*e**8) + 11*e**8*p*x**8*(d**2 - e**2*x
**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p +
48*e**8) + 6*e**8*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e
**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8), True)) + d**(2*p)
*e**3*x**9*hyper((9/2, -p), (11/2, ), e**2*x**2*exp_polar(2*I*pi)/
d**2)/9

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3 (-e^2x^2 + d^2)^p x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p\*x^5,x, algorithm="giac")

[Out] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p\*x^5, x)

$$3.259 \quad \int x^4(d+ex)^3(d^2-e^2x^2)^p dx$$

**Optimal.** Leaf size=218

$$\frac{3dx^5(d^2-e^2x^2)^{p+1}}{2p+7} - \frac{3d^2(d^2-e^2x^2)^{p+3}}{e^5(p+3)} + \frac{(d^2-e^2x^2)^{p+4}}{2e^5(p+4)} - \frac{2d^6(d^2-e^2x^2)^{p+1}}{e^5(p+1)} \\ + \frac{9d^4(d^2-e^2x^2)^{p+2}}{2e^5(p+2)} + \frac{2d^3(p+11)x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; \frac{e^2x^2}{d^2}\right)}{5(2p+7)}$$

[Out]  $(-2*d^6*(d^2 - e^2*x^2)^(1 + p))/(e^5*(1 + p)) - (3*d*x^5*(d^2 - e^2*x^2)^(1 + p))/(7 + 2*p) + (9*d^4*(d^2 - e^2*x^2)^(2 + p))/(2*e^5*(2 + p)) - (3*d^2*(d^2 - e^2*x^2)^(3 + p))/(e^5*(3 + p)) + (d^2 - e^2*x^2)^(4 + p)/(2*e^5*(4 + p)) + (2*d^3*(11 + p)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(7 + 2*p)*(1 - (e^2*x^2)/d^2)^p)$

**Rubi [A]** time = 0.399901, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{3dx^5(d^2-e^2x^2)^{p+1}}{2p+7} - \frac{3d^2(d^2-e^2x^2)^{p+3}}{e^5(p+3)} + \frac{(d^2-e^2x^2)^{p+4}}{2e^5(p+4)} - \frac{2d^6(d^2-e^2x^2)^{p+1}}{e^5(p+1)} \\ + \frac{9d^4(d^2-e^2x^2)^{p+2}}{2e^5(p+2)} + \frac{2d^3(p+11)x^5(d^2-e^2x^2)^p\left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; \frac{e^2x^2}{d^2}\right)}{5(2p+7)}$$

Antiderivative was successfully verified.

[In] Int[x^4\*(d+e\*x)^3\*(d^2-e^2\*x^2)^p,x]

[Out]  $(-2*d^6*(d^2 - e^2*x^2)^(1 + p))/(e^5*(1 + p)) - (3*d*x^5*(d^2 - e^2*x^2)^(1 + p))/(7 + 2*p) + (9*d^4*(d^2 - e^2*x^2)^(2 + p))/(2*e^5*(2 + p)) - (3*d^2*(d^2 - e^2*x^2)^(3 + p))/(e^5*(3 + p)) + (d^2 - e^2*x^2)^(4 + p)/(2*e^5*(4 + p)) + (2*d^3*(11 + p)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(7 + 2*p)*(1 - (e^2*x^2)/d^2)^p)$

**Rubi in Sympy [A]** time = 94.6582, size = 204, normalized size = 0.94

$$\frac{2d^6(d^2-e^2x^2)^{p+1}}{e^5(p+1)} + \frac{9d^4(d^2-e^2x^2)^{p+2}}{2e^5(p+2)} + \frac{d^3x^5\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p {}_2F_1\left(-p, \frac{5}{2}; \frac{7}{2}; \frac{e^2x^2}{d^2}\right)}{5} \\ - \frac{3d^2(d^2-e^2x^2)^{p+3}}{e^5(p+3)} + \frac{3de^2x^7\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p {}_2F_1\left(-p, \frac{7}{2}; \frac{9}{2}; \frac{e^2x^2}{d^2}\right)}{7} + \frac{(d^2-e^2x^2)^{p+4}}{2e^5(p+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p,x)

[Out]  $-2*d**6*(d**2 - e**2*x**2)**(p + 1)/(e**5*(p + 1)) + 9*d**4*(d**2 - e**2*x**2)**(p + 2)/(2*e**5*(p + 2)) + d**3*x**5*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper((-p, 5/2), (7/2, ), e**2*x**2/d**2)/5 - 3*d**2*(d**2 - e**2*x**2)**(p + 3)/(e**5*(p + 3)) + 3*d*e**2*x**7*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper((-p, 7/2), (9/2, ), e**2*x**2/d**2)/7 + (d**2 - e**2*x**2)**(p + 4)/(2*e**5*(p + 4))$

$$*(p + 4)/(2*e**5*(p + 4))$$

**Mathematica [A]** time = 0.447252, size = 323, normalized size = 1.48

$$(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} \left(30de^7 (p^4 + 10p^3 + 35p^2 + 50p + 24) x^7 {}_2F_1\left(\frac{7}{2}, -p; \frac{9}{2}; \frac{e^2x^2}{d^2}\right) + 14d^3e^5 (p^4 + 10p^3 + 35p^2 + 50p + \right.$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(d + e\*x)^3\*(d^2 - e^2\*x^2)^p,x]

[Out] ((d^2 - e^2\*x^2)^p\*(-35\*(6\*d^6\*e^2\*p\*(5 + p)\*x^2\*(1 - (e^2\*x^2)/d^2)^p + 3\*d^4\*e^4\*p\*(5 + 6\*p + p^2)\*x^4\*(1 - (e^2\*x^2)/d^2)^p - 2\*d^2\*e^6\*(12 + 20\*p + 9\*p^2 + p^3)\*x^6\*(1 - (e^2\*x^2)/d^2)^p - e^8\*(6 + 11\*p + 6\*p^2 + p^3)\*x^8\*(1 - (e^2\*x^2)/d^2)^p + 6\*d^8\*(5 + p)\*(-1 + (1 - (e^2\*x^2)/d^2)^p)) + 14\*d^3\*e^5\*(24 + 50\*p + 35\*p^2 + 10\*p^3 + p^4)\*x^5\*Hypergeometric2F1[5/2, -p, 7/2, (e^2\*x^2)/d^2] + 30\*d\*e^7\*(24 + 50\*p + 35\*p^2 + 10\*p^3 + p^4)\*x^7\*Hypergeometric2F1[7/2, -p, 9/2, (e^2\*x^2)/d^2])/(70\*e^5\*(1 + p)\*(2 + p)\*(3 + p)\*(4 + p)\*(1 - (e^2\*x^2)/d^2)^p)

**Maple [F]** time = 0.092, size = 0, normalized size = 0.

$$\int x^4 (ex + d)^3 (-e^2x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(e\*x+d)^3\*(-e^2\*x^2+d^2)^p,x)

[Out] int(x^4\*(e\*x+d)^3\*(-e^2\*x^2+d^2)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3 (-e^2x^2 + d^2)^p x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p\*x^4,x, algorithm="maxima")

[Out] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p\*x^4, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^7 + 3de^2x^6 + 3d^2ex^5 + d^3x^4\right)\left(-e^2x^2 + d^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p\*x^4,x, algorithm="fricas")

[Out] integral((e^3\*x^7 + 3\*d\*e^2\*x^6 + 3\*d^2\*e\*x^5 + d^3\*x^4)\*(-e^2\*x^2 + d^2)^p, x)

**Sympy [A]** time = 39.7092, size = 2966, normalized size = 13.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p,x)

[Out]  $d^{3p}d^{2p}x^{5p}\text{hyper}\left(\frac{5}{2}, -p, \frac{7}{2}, e^{2x^2}\text{exp\_polar}(2I\pi)/d^{2p}\right)/5 + 3d^{2p}e^p\text{Piecewise}\left(\left(x^{6p}(d^{2p})^p/6, \text{Eq}(e, 0)\right), \left(-2d^{4p}\log(-d/e+x)/(4d^{4p}e^{6p}-8d^{2p}e^{8p}x^{2p}+4e^{10p}x^{4p})-2d^{4p}\log(d/e+x)/(4d^{4p}e^{6p}-8d^{2p}e^{8p}x^{2p}+4e^{10p}x^{4p})-3d^{4p}/(4d^{4p}e^{6p}-8d^{2p}e^{8p}x^{2p}+4e^{10p}x^{4p})+4d^{2p}e^{2p}x^{2p}\log(-d/e+x)/(4d^{4p}e^{6p}-8d^{2p}e^{8p}x^{2p}+4e^{10p}x^{4p})+4d^{2p}e^{2p}x^{2p}\log(d/e+x)/(4d^{4p}e^{6p}-8d^{2p}e^{8p}x^{2p}+4e^{10p}x^{4p})+4d^{2p}e^{2p}x^{2p}/(4d^{4p}e^{6p}-8d^{2p}e^{8p}x^{2p}+4e^{10p}x^{4p})-2e^{4p}x^{4p}\log(-d/e+x)/(4d^{4p}e^{6p}-8d^{2p}e^{8p}x^{2p}+4e^{10p}x^{4p})-2e^{4p}x^{4p}\log(d/e+x)/(4d^{4p}e^{6p}-8d^{2p}e^{8p}x^{2p}+4e^{10p}x^{4p}), \text{Eq}(p, -3)\right), \left(-2d^{4p}\log(-d/e+x)/(-2d^{2p}e^{6p}+2e^{8p}x^{2p})-2d^{4p}\log(d/e+x)/(-2d^{2p}e^{6p}+2e^{8p}x^{2p})-2d^{4p}/(-2d^{2p}e^{6p}+2e^{8p}x^{2p})+2d^{2p}e^{2p}x^{2p}\log(-d/e+x)/(-2d^{2p}e^{6p}+2e^{8p}x^{2p})+2d^{2p}e^{2p}x^{2p}\log(d/e+x)/(-2d^{2p}e^{6p}+2e^{8p}x^{2p})+e^{4p}x^{4p}/(-2d^{2p}e^{6p}+2e^{8p}x^{2p}), \text{Eq}(p, -2)\right), \left(-d^{4p}\log(-d/e+x)/(2e^{6p})-d^{4p}\log(d/e+x)/(2e^{6p})-d^{2p}x^{2p}/(2e^{4p})-x^{4p}/(4e^{2p}), \text{Eq}(p, -1)\right), \left(-2d^{6p}(d^{2p}-e^{2p}x^{2p})^p/(2e^{6p}p^3+12e^{6p}p^2+22e^{6p}p+12e^{6p})-2d^{4p}e^{2p}x^{2p}(d^{2p}-e^{2p}x^{2p})^p/(2e^{6p}p^3+12e^{6p}p^2+22e^{6p}p+12e^{6p})-d^{2p}e^{4p}x^{4p}(d^{2p}-e^{2p}x^{2p})^p/(2e^{6p}p^3+12e^{6p}p^2+22e^{6p}p+12e^{6p})+e^{6p}x^{6p}(d^{2p}-e^{2p}x^{2p})^p/(2e^{6p}p^3+12e^{6p}p^2+22e^{6p}p+12e^{6p})+3e^{6p}x^{6p}(d^{2p}-e^{2p}x^{2p})^p/(2e^{6p}p^3+12e^{6p}p^2+22e^{6p}p+12e^{6p})+2e^{6p}x^{6p}(d^{2p}-e^{2p}x^{2p})^p/(2e^{6p}p^3+12e^{6p}p^2+22e^{6p}p+12e^{6p}), \text{True}\right) + 3d^{2p}d^{2p}e^{2p}x^{7p}\text{hyper}\left(\frac{7}{2}, -p, \frac{9}{2}, e^{2x^2}\text{exp\_polar}(2I\pi)/d^{2p}\right)/7 + e^{3p}\text{Piecewise}\left(\left(x^{8p}(d^{2p})^p/8, \text{Eq}(e, 0)\right), \left(-6d^{6p}\log(-d/e+x)/(-12d^{6p}e^{8p}+36d^{4p}e^{10p}x^{2p}-36d^{2p}e^{12p}x^{4p}+12e^{14p}x^{6p})-6d^{6p}\log(d/e+x)/(-12d^{6p}e^{8p}+36d^{4p}e^{10p}x^{2p}-36d^{2p}e^{12p}x^{4p}+12e^{14p}x^{6p})-11d^{6p}/(-12d^{6p}e^{8p}+36d^{4p}e^{10p}x^{2p}-36d^{2p}e^{12p}x^{4p}+12e^{14p}x^{6p})+18d^{4p}e^{2p}x^{2p}\log(-d/e+x)/(-12d^{6p}e^{8p}+36d^{4p}e^{10p}x^{2p}-36d^{2p}e^{12p}x^{4p}+12e^{14p}x^{6p})+18d^{4p}e^{2p}x^{2p}\log(d/e+x)/(-12d^{6p}e^{8p}+36d^{4p}e^{10p}x^{2p}-36d^{2p}e^{12p}x^{4p}+12e^{14p}x^{6p})+27d^{4p}e^{2p}x^{2p}/(-12d^{6p}e^{8p}+36d^{4p}e^{10p}x^{2p}-36d^{2p}e^{12p}x^{4p}+12e^{14p}x^{6p})-18d^{2p}e^{4p}x^{4p}\log(-d/e+x)/(-12d^{6p}e^{8p}+36d^{4p}e^{10p}x^{2p}-36d^{2p}e^{12p}x^{4p}+12e^{14p}x^{6p})-18d^{2p}e^{4p}x^{4p}/(-12d^{6p}e^{8p}+36d^{4p}e^{10p}x^{2p}-36d^{2p}e^{12p}x^{4p}+12e^{14p}x^{6p})+6e^{6p}x^{6p}\log(-d/e+x)/(-12d^{6p}e^{8p}+36d^{4p}e^{10p}x^{2p}-36d^{2p}e^{12p}x^{4p}+12e^{14p}x^{6p})+6e^{6p}x^{6p}\log(d/e+x)/(-12d^{6p}e^{8p}+36d^{4p}e^{10p}x^{2p}-36d^{2p}e^{12p}x^{4p}+12e^{14p}x^{6p}), \text{Eq}(p, -4)\right), \left(-6d^{6p}\log(-d/e+x)/(4d^{4p}e^{8p}-8d^{2p}e^{10p}x^{2p}+4e^{12p}x^{4p})-6d^{6p}\log(d/e+x)/(4d^{4p}e^{8p}-8d^{2p}e^{10p}x^{2p}+4e^{12p}x^{4p})-9d^{6p}/(4d^{4p}e^{8p}-8d^{2p}e^{10p}x^{2p}+4e^{12p}x^{4p})+12d^{4p}e^{2p}x^{2p}\log(-d/e+x)/(4d^{4p}e^{8p}-8d^{2p}e^{10p}x^{2p}+4e^{12p}x^{4p})+12d^{4p}e^{2p}x^{2p}\log(d/e+x)/(4d^{4p}e^{8p}-8d^{2p}e^{10p}x^{2p}+4e^{12p}x^{4p})+12d^{4p}e^{2p}x^{2p}/(4d^{4p}e^{8p}-8d^{2p}e^{10p}x^{2p}+4e^{12p}x^{4p})-6d^{2p}e^{4p}x^{4p}\log(-d/e+x)/(4d^{4p}e^{8p}-8d^{2p}e^{10p}x^{2p}+4e^{12p}x^{4p})-6d^{2p}e^{4p}x^{4p}\log(d/e+x)/(4d^{4p}e^{8p}-8d^{2p}e^{10p}x^{2p}+4e^{12p}x^{4p})-2e^{6p}x^{6p}\right)$

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/(4*d**4*e**8 - 8*d**2*e**10*x**2 + 4*e**12*x**4), Eq(p, -3)), (-
6*d**6*log(-d/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) - 6*d**6*log(d
/e + x)/(-4*d**2*e**8 + 4*e**10*x**2) - 6*d**6/(-4*d**2*e**8 + 4*
e**10*x**2) + 6*d**4*e**2*x**2*log(-d/e + x)/(-4*d**2*e**8 + 4*e*
**10*x**2) + 6*d**4*e**2*x**2*log(d/e + x)/(-4*d**2*e**8 + 4*e**10
*x**2) + 3*d**2*e**4*x**4/(-4*d**2*e**8 + 4*e**10*x**2) + e**6*x*
**6/(-4*d**2*e**8 + 4*e**10*x**2), Eq(p, -2)), (-d**6*log(-d/e + x
)/(2*e**8) - d**6*log(d/e + x)/(2*e**8) - d**4*x**2/(2*e**6) - d*
**2*x**4/(4*e**4) - x**6/(6*e**2), Eq(p, -1)), (-6*d**8*(d**2 - e*
**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8
*p + 48*e**8) - 6*d**6*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**8*
p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 3*d*
**4*e**4*p**2*x**4*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p*
**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 3*d**4*e**4*p*x**4*(d
**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 +
100*e**8*p + 48*e**8) - d**2*e**6*p**3*x**6*(d**2 - e**2*x**2)**p
/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**
8) - 3*d**2*e**6*p**2*x**6*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 2
0*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) - 2*d**2*e**6*
p*x**6*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**
8*p**2 + 100*e**8*p + 48*e**8) + e**8*p**3*x**8*(d**2 - e**2*x**2
)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48
*e**8) + 6*e**8*p**2*x**8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20
*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8) + 11*e**8*p*x**
8*(d**2 - e**2*x**2)**p/(2*e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**
2 + 100*e**8*p + 48*e**8) + 6*e**8*x**8*(d**2 - e**2*x**2)**p/(2*
e**8*p**4 + 20*e**8*p**3 + 70*e**8*p**2 + 100*e**8*p + 48*e**8),
True))

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3 (-e^2x^2 + d^2)^p x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p\*x^4,x, algorithm="giac")

[Out] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p\*x^4, x)

### 3.260 $\int x^3(d + ex)^3 (d^2 - e^2x^2)^p dx$

**Optimal.** Leaf size=193

$$\frac{2d^2e(3p+13)x^5(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; \frac{e^2x^2}{d^2}\right)}{5(2p+7)} - \frac{ex^5(d^2 - e^2x^2)^{p+1}}{2p+7} - \frac{3d(d^2 - e^2x^2)^{p+3}}{2e^4(p+3)} - \frac{2d^5(d^2 - e^2x^2)^{p+1}}{e^4(p+1)} + \frac{7d^3(d^2 - e^2x^2)^{p+2}}{2e^4(p+2)}$$

[Out]  $(-2*d^5*(d^2 - e^2*x^2)^(1 + p))/(e^4*(1 + p)) - (e*x^5*(d^2 - e^2*x^2)^(1 + p))/(7 + 2*p) + (7*d^3*(d^2 - e^2*x^2)^(2 + p))/(2*e^4*(2 + p)) - (3*d*(d^2 - e^2*x^2)^(3 + p))/(2*e^4*(3 + p)) + (2*d^2*e*(13 + 3*p)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(7 + 2*p)*(1 - (e^2*x^2)/d^2)^p)$

**Rubi [A]** time = 0.36716, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{2d^2e(3p+13)x^5(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; \frac{e^2x^2}{d^2}\right)}{5(2p+7)} - \frac{ex^5(d^2 - e^2x^2)^{p+1}}{2p+7} - \frac{3d(d^2 - e^2x^2)^{p+3}}{2e^4(p+3)} - \frac{2d^5(d^2 - e^2x^2)^{p+1}}{e^4(p+1)} + \frac{7d^3(d^2 - e^2x^2)^{p+2}}{2e^4(p+2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3*(d + e*x)^3*(d^2 - e^2*x^2)^p, x]$

[Out]  $(-2*d^5*(d^2 - e^2*x^2)^(1 + p))/(e^4*(1 + p)) - (e*x^5*(d^2 - e^2*x^2)^(1 + p))/(7 + 2*p) + (7*d^3*(d^2 - e^2*x^2)^(2 + p))/(2*e^4*(2 + p)) - (3*d*(d^2 - e^2*x^2)^(3 + p))/(2*e^4*(3 + p)) + (2*d^2*e*(13 + 3*p)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/(5*(7 + 2*p)*(1 - (e^2*x^2)/d^2)^p)$

**Rubi in Sympy [A]** time = 82.2782, size = 182, normalized size = 0.94

$$-\frac{2d^5(d^2 - e^2x^2)^{p+1}}{e^4(p+1)} + \frac{7d^3(d^2 - e^2x^2)^{p+2}}{2e^4(p+2)} + \frac{3d^2ex^5\left(1 - \frac{e^2x^2}{d^2}\right)^{-p}(d^2 - e^2x^2)^p {}_2F_1\left(-p, \frac{5}{2}; \frac{7}{2}; \frac{e^2x^2}{d^2}\right)}{5} - \frac{3d(d^2 - e^2x^2)^{p+3}}{2e^4(p+3)} + \frac{e^3x^7\left(1 - \frac{e^2x^2}{d^2}\right)^{-p}(d^2 - e^2x^2)^p {}_2F_1\left(-p, \frac{7}{2}; \frac{9}{2}; \frac{e^2x^2}{d^2}\right)}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**3*(e*x+d)**3*(-e**2*x**2+d**2)**p, x)$

[Out]  $-2*d**5*(d**2 - e**2*x**2)**(p + 1)/(e**4*(p + 1)) + 7*d**3*(d**2 - e**2*x**2)**(p + 2)/(2*e**4*(p + 2)) + 3*d**2*e*x**5*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper((-p, 5/2), (7/2, ), e**2*x**2/d**2)/5 - 3*d*(d**2 - e**2*x**2)**(p + 3)/(2*e**4*(p + 3)) + e**3*x**7*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper((-p, 7/2), (9/2, ), e**2*x**2/d**2)/7$



**Mathematica [A]** time = 0.360297, size = 262, normalized size = 1.36

$$(d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} \left(10e^7 (p^3 + 6p^2 + 11p + 6) x^7 {}_2F_1\left(\frac{7}{2}, -p; \frac{9}{2}; \frac{e^2 x^2}{d^2}\right) + 42d^2 e^5 (p^3 + 6p^2 + 11p + 6) x^5 {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; \frac{e^2 x^2}{d^2}\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(d + e\*x)^3\*(d^2 - e^2\*x^2)^p,x]

[Out] ((d^2 - e^2\*x^2)^p\*(-35\*d\*(d^4\*e^2\*p\*(9 + p)\*x^2\*(1 - (e^2\*x^2)/d^2)^p + d^2\*e^4\*(-3 - p + 2\*p^2)\*x^4\*(1 - (e^2\*x^2)/d^2)^p - 3\*e^6\*(2 + 3\*p + p^2)\*x^6\*(1 - (e^2\*x^2)/d^2)^p + d^6\*(9 + p)\*(-1 + (1 - (e^2\*x^2)/d^2)^p)) + 42\*d^2\*e^5\*(6 + 11\*p + 6\*p^2 + p^3)\*x^5\*Hypergeometric2F1[5/2, -p, 7/2, (e^2\*x^2)/d^2] + 10\*e^7\*(6 + 11\*p + 6\*p^2 + p^3)\*x^7\*Hypergeometric2F1[7/2, -p, 9/2, (e^2\*x^2)/d^2])/((70\*e^4\*(1 + p)\*(2 + p)\*(3 + p)\*(1 - (e^2\*x^2)/d^2)^p))

**Maple [F]** time = 0.084, size = 0, normalized size = 0.

$$\int x^3 (ex + d)^3 (-e^2 x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(e\*x+d)^3\*(-e^2\*x^2+d^2)^p,x)

[Out] int(x^3\*(e\*x+d)^3\*(-e^2\*x^2+d^2)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(e^4(p+1)x^4 - d^2 e^2 p x^2 - d^4)(-e^2 x^2 + d^2)^p d^3}{2(p^2 + 3p + 2)e^4} + \int (e^3 x^6 + 3 d e^2 x^5 + 3 d^2 e x^4) e^{(p \log(ex+d) + p \log(-ex+d))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p\*x^3,x, algorithm="maxima")

[Out] 1/2\*(e^4\*(p + 1)\*x^4 - d^2\*e^2\*p\*x^2 - d^4)\*(-e^2\*x^2 + d^2)^p\*d^3/((p^2 + 3\*p + 2)\*e^4) + integrate((e^3\*x^6 + 3\*d\*e^2\*x^5 + 3\*d^2\*e\*x^4)\*e^(p\*log(e\*x + d) + p\*log(-e\*x + d)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3 x^6 + 3 d e^2 x^5 + 3 d^2 e x^4 + d^3 x^3\right)\left(-e^2 x^2 + d^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p\*x^3,x, algorithm="fricas")

[Out] integral((e^3\*x^6 + 3\*d\*e^2\*x^5 + 3\*d^2\*e\*x^4 + d^3\*x^3)\*(-e^2\*x^2 + d^2)^p, x)

**Sympy [A]** time = 26.9542, size = 1370, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p,x)

[Out] d\*\*3\*Piecewise((x\*\*4\*(d\*\*2)\*\*p/4, Eq(e, 0)), (-d\*\*2\*log(-d/e + x)/(-2\*d\*\*2\*e\*\*4 + 2\*e\*\*6\*x\*\*2) - d\*\*2\*log(d/e + x)/(-2\*d\*\*2\*e\*\*4 + 2\*e\*\*6\*x\*\*2) - d\*\*2/(-2\*d\*\*2\*e\*\*4 + 2\*e\*\*6\*x\*\*2) + e\*\*2\*x\*\*2\*log(-d/e + x)/(-2\*d\*\*2\*e\*\*4 + 2\*e\*\*6\*x\*\*2) + e\*\*2\*x\*\*2\*log(d/e + x)/(-2\*d\*\*2\*e\*\*4 + 2\*e\*\*6\*x\*\*2), Eq(p, -2)), (-d\*\*2\*log(-d/e + x)/(2\*e\*\*4) - d\*\*2\*log(d/e + x)/(2\*e\*\*4) - x\*\*2/(2\*e\*\*2), Eq(p, -1)), (-d\*\*4\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p/(2\*e\*\*4\*p\*\*2 + 6\*e\*\*4\*p + 4\*e\*\*4) - d\*\*2\*e\*\*2\*p\*x\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p/(2\*e\*\*4\*p\*\*2 + 6\*e\*\*4\*p + 4\*e\*\*4) + e\*\*4\*p\*x\*\*4\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p/(2\*e\*\*4\*p\*\*2 + 6\*e\*\*4\*p + 4\*e\*\*4) + e\*\*4\*x\*\*4\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p/(2\*e\*\*4\*p\*\*2 + 6\*e\*\*4\*p + 4\*e\*\*4), True)) + 3\*d\*\*2\*d\*\*(2\*p)\*e\*x\*\*5\*hyper((5/2, -p), (7/2, ), e\*\*2\*x\*\*2\*exp\_polar(2\*I\*pi)/d\*\*2)/5 + 3\*d\*e\*\*2\*Piecewise((x\*\*6\*(d\*\*2)\*\*p/6, Eq(e, 0)), (-2\*d\*\*4\*log(-d/e + x)/(4\*d\*\*4\*e\*\*6 - 8\*d\*\*2\*e\*\*8\*x\*\*2 + 4\*e\*\*10\*x\*\*4) - 2\*d\*\*4\*log(d/e + x)/(4\*d\*\*4\*e\*\*6 - 8\*d\*\*2\*e\*\*8\*x\*\*2 + 4\*e\*\*10\*x\*\*4) - 3\*d\*\*4/(4\*d\*\*4\*e\*\*6 - 8\*d\*\*2\*e\*\*8\*x\*\*2 + 4\*e\*\*10\*x\*\*4) + 4\*d\*\*2\*e\*\*2\*x\*\*2\*log(-d/e + x)/(4\*d\*\*4\*e\*\*6 - 8\*d\*\*2\*e\*\*8\*x\*\*2 + 4\*e\*\*10\*x\*\*4) + 4\*d\*\*2\*e\*\*2\*x\*\*2\*log(d/e + x)/(4\*d\*\*4\*e\*\*6 - 8\*d\*\*2\*e\*\*8\*x\*\*2 + 4\*e\*\*10\*x\*\*4) + 4\*d\*\*2\*e\*\*2\*x\*\*2/(4\*d\*\*4\*e\*\*6 - 8\*d\*\*2\*e\*\*8\*x\*\*2 + 4\*e\*\*10\*x\*\*4) - 2\*e\*\*4\*x\*\*4\*log(-d/e + x)/(4\*d\*\*4\*e\*\*6 - 8\*d\*\*2\*e\*\*8\*x\*\*2 + 4\*e\*\*10\*x\*\*4) - 2\*e\*\*4\*x\*\*4\*log(d/e + x)/(4\*d\*\*4\*e\*\*6 - 8\*d\*\*2\*e\*\*8\*x\*\*2 + 4\*e\*\*10\*x\*\*4), Eq(p, -3)), (-2\*d\*\*4\*log(-d/e + x)/(-2\*d\*\*2\*e\*\*6 + 2\*e\*\*8\*x\*\*2) - 2\*d\*\*4\*log(d/e + x)/(-2\*d\*\*2\*e\*\*6 + 2\*e\*\*8\*x\*\*2) - 2\*d\*\*4/(-2\*d\*\*2\*e\*\*6 + 2\*e\*\*8\*x\*\*2) + 2\*d\*\*2\*e\*\*2\*x\*\*2\*log(-d/e + x)/(-2\*d\*\*2\*e\*\*6 + 2\*e\*\*8\*x\*\*2) + 2\*d\*\*2\*e\*\*2\*x\*\*2\*log(d/e + x)/(-2\*d\*\*2\*e\*\*6 + 2\*e\*\*8\*x\*\*2) + e\*\*4\*x\*\*4/(-2\*d\*\*2\*e\*\*6 + 2\*e\*\*8\*x\*\*2), Eq(p, -2)), (-d\*\*4\*log(-d/e + x)/(2\*e\*\*6) - d\*\*4\*log(d/e + x)/(2\*e\*\*6) - d\*\*2\*x\*\*2/(2\*e\*\*4) - x\*\*4/(4\*e\*\*2), Eq(p, -1)), (-2\*d\*\*6\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p/(2\*e\*\*6\*p\*\*3 + 12\*e\*\*6\*p\*\*2 + 22\*e\*\*6\*p + 12\*e\*\*6) - 2\*d\*\*4\*e\*\*2\*p\*x\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p/(2\*e\*\*6\*p\*\*3 + 12\*e\*\*6\*p\*\*2 + 22\*e\*\*6\*p + 12\*e\*\*6) - d\*\*2\*e\*\*4\*p\*x\*\*4\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p/(2\*e\*\*6\*p\*\*3 + 12\*e\*\*6\*p\*\*2 + 22\*e\*\*6\*p + 12\*e\*\*6) + e\*\*6\*p\*\*2\*x\*\*6\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p/(2\*e\*\*6\*p\*\*3 + 12\*e\*\*6\*p\*\*2 + 22\*e\*\*6\*p + 12\*e\*\*6) + 3\*e\*\*6\*p\*x\*\*6\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p/(2\*e\*\*6\*p\*\*3 + 12\*e\*\*6\*p\*\*2 + 22\*e\*\*6\*p + 12\*e\*\*6), True)) + d\*\*(2\*p)\*e\*\*3\*x\*\*7\*hyper((7/2, -p), (9/2, ), e\*\*2\*x\*\*2\*exp\_polar(2\*I\*pi)/d\*\*2)/7

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3(-e^2x^2 + d^2)^p x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p\*x^3,x, algorithm="giac")

[Out] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p\*x^3, x)

### 3.261 $\int x^2(d + ex)^3 (d^2 - e^2x^2)^p dx$

**Optimal.** Leaf size=189

$$\begin{aligned} & -\frac{3dx^3(d^2 - e^2x^2)^{p+1}}{2p+5} + \frac{5d^2(d^2 - e^2x^2)^{p+2}}{2e^3(p+2)} - \frac{(d^2 - e^2x^2)^{p+3}}{2e^3(p+3)} - \frac{2d^4(d^2 - e^2x^2)^{p+1}}{e^3(p+1)} \\ & + \frac{2d^3(p+7)x^3(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; \frac{e^2x^2}{d^2}\right)}{3(2p+5)} \end{aligned}$$

[Out]  $(-2*d^4*(d^2 - e^2*x^2)^(1 + p))/(e^3*(1 + p)) - (3*d*x^3*(d^2 - e^2*x^2)^(1 + p))/(5 + 2*p) + (5*d^2*(d^2 - e^2*x^2)^(2 + p))/(2*e^3*(2 + p)) - (d^2 - e^2*x^2)^(3 + p)/(2*e^3*(3 + p)) + (2*d^3*(7 + p)*x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(5 + 2*p)*(1 - (e^2*x^2)/d^2)^p)$

**Rubi [A]** time = 0.361741, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\begin{aligned} & -\frac{3dx^3(d^2 - e^2x^2)^{p+1}}{2p+5} + \frac{5d^2(d^2 - e^2x^2)^{p+2}}{2e^3(p+2)} - \frac{(d^2 - e^2x^2)^{p+3}}{2e^3(p+3)} - \frac{2d^4(d^2 - e^2x^2)^{p+1}}{e^3(p+1)} \\ & + \frac{2d^3(p+7)x^3(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; \frac{e^2x^2}{d^2}\right)}{3(2p+5)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(d + e\*x)^3\*(d^2 - e^2\*x^2)^p, x]

[Out]  $(-2*d^4*(d^2 - e^2*x^2)^(1 + p))/(e^3*(1 + p)) - (3*d*x^3*(d^2 - e^2*x^2)^(1 + p))/(5 + 2*p) + (5*d^2*(d^2 - e^2*x^2)^(2 + p))/(2*e^3*(2 + p)) - (d^2 - e^2*x^2)^(3 + p)/(2*e^3*(3 + p)) + (2*d^3*(7 + p)*x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2])/(3*(5 + 2*p)*(1 - (e^2*x^2)/d^2)^p)$

**Rubi in Sympy [A]** time = 77.5815, size = 178, normalized size = 0.94

$$\begin{aligned} & -\frac{2d^4(d^2 - e^2x^2)^{p+1}}{e^3(p+1)} + \frac{d^3x^3\left(1 - \frac{e^2x^2}{d^2}\right)^{-p}(d^2 - e^2x^2)^p {}_2F_1\left(-p, \frac{3}{2}; \frac{5}{2}; \frac{e^2x^2}{d^2}\right)}{3} + \frac{5d^2(d^2 - e^2x^2)^{p+2}}{2e^3(p+2)} \\ & + \frac{3de^2x^5\left(1 - \frac{e^2x^2}{d^2}\right)^{-p}(d^2 - e^2x^2)^p {}_2F_1\left(-p, \frac{5}{2}; \frac{7}{2}; \frac{e^2x^2}{d^2}\right)}{5} - \frac{(d^2 - e^2x^2)^{p+3}}{2e^3(p+3)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p, x)

[Out]  $-2*d^4*(d^2 - e^2*x^2)**(p + 1)/(e^3*(p + 1)) + d^3*x^3*(1 - e^2*x^2/d^2)**(-p)*(d^2 - e^2*x^2)**p*hyper((-p, 3/2), (5/2, ), e^2*x^2/d^2)/3 + 5*d^2*(d^2 - e^2*x^2)**(p + 2)/(2*e^3*(p + 2)) + 3*d*e^2*x^5*(1 - e^2*x^2/d^2)**(-p)*(d^2 - e^2*x^2)**p*hyper((-p, 5/2), (7/2, ), e^2*x^2/d^2)/5 - (d^2 - e^2*x^2)**(p + 3)/(2*e^3*(p + 3))$

**Mathematica [A]** time = 0.344381, size = 269, normalized size = 1.42

$$(d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} \left(10 d^3 e^3 (p^3 + 6 p^2 + 11 p + 6) x^3 {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; \frac{e^2 x^2}{d^2}\right) - 3 \left(-5 e^6 (p^2 + 3 p + 2) x^6 \left(1 - \frac{e^2 x^2}{d^2}\right)^p - 6 d\right.\right.$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(d + e\*x)^3\*(d^2 - e^2\*x^2)^p,x]

[Out] ((d^2 - e^2\*x^2)^p\*(10\*d^3\*e^3\*(6 + 11\*p + 6\*p^2 + p^3)\*x^3\*Hypergeometric2F1[3/2, -p, 5/2, (e^2\*x^2)/d^2] - 3\*(5\*d^4\*e^2\*p\*(11 + 3\*p)\*x^2\*(1 - (e^2\*x^2)/d^2)^p - 5\*d^2\*e^4\*(9 + 11\*p + 2\*p^2)\*x^4\*(1 - (e^2\*x^2)/d^2)^p - 5\*e^6\*(2 + 3\*p + p^2)\*x^6\*(1 - (e^2\*x^2)/d^2)^p + 5\*d^6\*(11 + 3\*p)\*(-1 + (1 - (e^2\*x^2)/d^2)^p) - 6\*d\*e^5\*(6 + 11\*p + 6\*p^2 + p^3)\*x^5\*Hypergeometric2F1[5/2, -p, 7/2, (e^2\*x^2)/d^2]))/(30\*e^3\*(1 + p)\*(2 + p)\*(3 + p)\*(1 - (e^2\*x^2)/d^2)^p)

**Maple [F]** time = 0.076, size = 0, normalized size = 0.

$$\int x^2 (ex + d)^3 (-e^2 x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(e\*x+d)^3\*(-e^2\*x^2+d^2)^p,x)

[Out] int(x^2\*(e\*x+d)^3\*(-e^2\*x^2+d^2)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3 (-e^2 x^2 + d^2)^p x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p\*x^2,x, algorithm="maxima")

[Out] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p\*x^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3 x^5 + 3 d e^2 x^4 + 3 d^2 e x^3 + d^3 x^2\right)\left(-e^2 x^2 + d^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p\*x^2,x, algorithm="fricas")

[Out] integral((e^3\*x^5 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^3 + d^3\*x^2)\*(-e^2\*x^2 + d^2)^p, x)

**Sympy [A]** time = 23.9146, size = 1370, normalized size = 7.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p,x)

[Out]  $d^{3}d^{(2p)}x^{3}\text{hyper}\left(\frac{3}{2}, -p, \frac{5}{2}, e^{2x^2}\text{exp\_polar}(2I\pi)/d^2\right)/3 + 3d^{2}e\text{Piecewise}\left(\left(x^4(d^2)^p/4, \text{Eq}(e, 0)\right), \left(-d^2\log(-d/e+x)/(-2d^2e^4+2e^6x^2) - d^2\log(d/e+x)/(-2d^2e^4+2e^6x^2) - d^2/(-2d^2e^4+2e^6x^2) + e^{2x^2}\log(-d/e+x)/(-2d^2e^4+2e^6x^2) + e^{2x^2}\log(d/e+x)/(-2d^2e^4+2e^6x^2), \text{Eq}(p, -2)\right), \left(-d^2\log(-d/e+x)/(2e^4) - d^2\log(d/e+x)/(2e^4) - x^2/(2e^2), \text{Eq}(p, -1)\right), \left(-d^4(d^2 - e^{2x^2})^p/(2e^4p^2 + 6e^4p + 4e^4) - d^2e^{2x^2}p^2(d^2 - e^{2x^2})^p/(2e^4p^2 + 6e^4p + 4e^4) + e^{4x^4}(d^2 - e^{2x^2})^p/(2e^4p^2 + 6e^4p + 4e^4) + e^{4x^4}(d^2 - e^{2x^2})^p/(2e^4p^2 + 6e^4p + 4e^4), \text{True}\right) + 3d^2d^{(2p)}e^{2x^5}\text{hyper}\left(\frac{5}{2}, -p, \frac{7}{2}, e^{2x^2}\text{exp\_polar}(2I\pi)/d^2\right)/5 + e^3\text{Piecewise}\left(\left(x^6(d^2)^p/6, \text{Eq}(e, 0)\right), \left(-2d^4\log(-d/e+x)/(4d^4e^6 - 8d^2e^8x^2 + 4e^{10}x^4) - 2d^4\log(d/e+x)/(4d^4e^6 - 8d^2e^8x^2 + 4e^{10}x^4) - 3d^4/(4d^4e^6 - 8d^2e^8x^2 + 4e^{10}x^4) + 4d^2e^{2x^2}\log(-d/e+x)/(4d^4e^6 - 8d^2e^8x^2 + 4e^{10}x^4) + 4d^2e^{2x^2}\log(d/e+x)/(4d^4e^6 - 8d^2e^8x^2 + 4e^{10}x^4) + 4d^2e^{2x^2}/(4d^4e^6 - 8d^2e^8x^2 + 4e^{10}x^4) - 2e^4x^4\log(-d/e+x)/(4d^4e^6 - 8d^2e^8x^2 + 4e^{10}x^4) - 2e^4x^4\log(d/e+x)/(4d^4e^6 - 8d^2e^8x^2 + 4e^{10}x^4), \text{Eq}(p, -3)\right), \left(-2d^4\log(-d/e+x)/(-2d^2e^6 + 2e^8x^2) - 2d^4\log(d/e+x)/(-2d^2e^6 + 2e^8x^2) - 2d^4/(-2d^2e^6 + 2e^8x^2) + 2d^2e^{2x^2}\log(-d/e+x)/(-2d^2e^6 + 2e^8x^2) + 2d^2e^{2x^2}\log(d/e+x)/(-2d^2e^6 + 2e^8x^2) + e^4x^4/(-2d^2e^6 + 2e^8x^2), \text{Eq}(p, -2)\right), \left(-d^4\log(-d/e+x)/(2e^6) - d^4\log(d/e+x)/(2e^6) - d^2x^2/(2e^4) - x^4/(4e^2), \text{Eq}(p, -1)\right), \left(-2d^6(d^2 - e^{2x^2})^p/(2e^6p^3 + 12e^6p^2 + 22e^6p + 12e^6) - 2d^4e^{2x^2}p^2(d^2 - e^{2x^2})^p/(2e^6p^3 + 12e^6p^2 + 22e^6p + 12e^6) - d^2e^{4x^4}(d^2 - e^{2x^2})^p/(2e^6p^3 + 12e^6p^2 + 22e^6p + 12e^6) - d^2e^{4x^4}p^2(d^2 - e^{2x^2})^p/(2e^6p^3 + 12e^6p^2 + 22e^6p + 12e^6) + e^6p^2x^6(d^2 - e^{2x^2})^p/(2e^6p^3 + 12e^6p^2 + 22e^6p + 12e^6) + 22e^6p^2 + 22e^6p + 12e^6) + 2e^6x^6(d^2 - e^{2x^2})^p/(2e^6p^3 + 12e^6p^2 + 22e^6p + 12e^6) + 2e^6x^6(d^2 - e^{2x^2})^p/(2e^6p^3 + 12e^6p^2 + 22e^6p + 12e^6), \text{True}\right)$

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3(-e^2x^2 + d^2)^p x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p\*x^2,x, algorithm="giac")

[Out] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p\*x^2, x)

### 3.262 $\int x(d + ex)^3 (d^2 - e^2x^2)^p dx$

**Optimal.** Leaf size=116

$$\frac{(d + ex)^3 (d^2 - e^2x^2)^{p+1}}{e^2(2p + 5)} - \frac{3d^3 2^{p+3} (d^2 - e^2x^2)^{p+1} \left(\frac{ex}{d} + 1\right)^{-p-1} {}_2F_1\left(-p - 3, p + 1; p + 2; \frac{d - ex}{2d}\right)}{e^2(p + 1)(2p + 5)}$$

[Out] -(((d + e\*x)^3\*(d^2 - e^2\*x^2)^(1 + p))/(e^2\*(5 + 2\*p))) - (3\*2^(3 + p)\*d^3\*(1 + (e\*x)/d)^(-1 - p)\*(d^2 - e^2\*x^2)^(1 + p)\*Hypergeometric2F1[-3 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/(e^2\*(1 + p)\*(5 + 2\*p))

**Rubi [A]** time = 0.170124, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$

$$\frac{(d + ex)^3 (d^2 - e^2x^2)^{p+1}}{e^2(2p + 5)} - \frac{3d^3 2^{p+3} (d^2 - e^2x^2)^{p+1} \left(\frac{ex}{d} + 1\right)^{-p-1} {}_2F_1\left(-p - 3, p + 1; p + 2; \frac{d - ex}{2d}\right)}{e^2(p + 1)(2p + 5)}$$

Antiderivative was successfully verified.

[In] Int[x\*(d + e\*x)^3\*(d^2 - e^2\*x^2)^p,x]

[Out] -(((d + e\*x)^3\*(d^2 - e^2\*x^2)^(1 + p))/(e^2\*(5 + 2\*p))) - (3\*2^(3 + p)\*d^3\*(1 + (e\*x)/d)^(-1 - p)\*(d^2 - e^2\*x^2)^(1 + p)\*Hypergeometric2F1[-3 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/(e^2\*(1 + p)\*(5 + 2\*p))

**Rubi in Sympy [A]** time = 33.3364, size = 104, normalized size = 0.9

$$\frac{24d^4 \left(\frac{d + ex}{2d}\right)^{-p} (d - ex)^{-p} (d - ex)^{p+1} (d^2 - e^2x^2)^p {}_2F_1\left(-p - 3, p + 1; p + 2; \frac{d - ex}{2d}\right)}{e^2(p + 1)(2p + 5)} - \frac{(d + ex)^3 (d^2 - e^2x^2)^{p+1}}{e^2(2p + 5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p,x)

[Out] -24\*d\*\*4\*((d/2 + e\*x/2)/d)\*\*(-p)\*(d - e\*x)\*\*(-p)\*(d - e\*x)\*\*(p + 1)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p\*hyper((-p - 3, p + 1), (p + 2, ), (d/2 - e\*x/2)/d)/(e\*\*2\*(p + 1)\*(2\*p + 5)) - (d + e\*x)\*\*3\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(p + 1)/(e\*\*2\*(2\*p + 5))

**Mathematica [A]** time = 0.253076, size = 207, normalized size = 1.78

$$\frac{(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} \left(2e^5(p^2 + 3p + 2)x^5 {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; \frac{e^2x^2}{d^2}\right) + 10d^2e^3(p^2 + 3p + 2)x^3 {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; \frac{e^2x^2}{d^2}\right) - 5d(2d^2 - e^2x^2) {}_2F_1\left(-p - 3, p + 1; p + 2; \frac{d - ex}{2d}\right)\right)}{10e^2(p + 1)(p + 2)}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(d + e\*x)^3\*(d^2 - e^2\*x^2)^p,x]

[Out] ((d^2 - e^2\*x^2)^p\*(-5\*d\*(2\*d^2\*e^2\*(-1 + p)\*x^2\*(1 - (e^2\*x^2)/d^2)^p - 3\*e^4\*(1 + p)\*x^4\*(1 - (e^2\*x^2)/d^2)^p + d^4\*(5 + p)\*(-1

$$+ (1 - (e^2*x^2)/d^2)^p) + 10*d^2*e^3*(2 + 3*p + p^2)*x^3*Hypergeometric2F1[3/2, -p, 5/2, (e^2*x^2)/d^2] + 2*e^5*(2 + 3*p + p^2)*x^5*Hypergeometric2F1[5/2, -p, 7/2, (e^2*x^2)/d^2])/((10*e^2*(1 + p)*(2 + p)*(1 - (e^2*x^2)/d^2)^p)$$

**Maple [F]** time = 0.079, size = 0, normalized size = 0.

$$\int x (ex + d)^3 (-e^2x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(e\*x+d)^3\*(-e^2\*x^2+d^2)^p,x)

[Out] int(x\*(e\*x+d)^3\*(-e^2\*x^2+d^2)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p\*x,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^4 + 3de^2x^3 + 3d^2ex^2 + d^3x\right)\left(-e^2x^2 + d^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p\*x,x, algorithm="fricas")

[Out] integral((e^3\*x^4 + 3\*d\*e^2\*x^3 + 3\*d^2\*e\*x^2 + d^3\*x)\*(-e^2\*x^2 + d^2)^p, x)

**Sympy [A]** time = 15.4426, size = 479, normalized size = 4.13

$$d^3 \left( \begin{array}{ll} \left( \frac{x^2(d^2)^p}{2} & \text{for } e^2 = 0 \right. \\ \left( \frac{(d^2 - e^2x^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \log(d^2 - e^2x^2) & \text{otherwise} \end{array} \right) + d^2 d^{2p} e x^3 {}_2F_1 \left( \frac{3}{2}, -p \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2} \right) \\ + 3de^2 \left( \begin{array}{ll} \left( \frac{x^4(d^2)^p}{4} & \text{for } e = 0 \right. \\ \frac{d^2 \log\left(-\frac{d}{e} + x\right)}{-2d^2 e^4 + 2e^6 x^2} - \frac{d^2 \log\left(\frac{d}{e} + x\right)}{-2d^2 e^4 + 2e^6 x^2} - \frac{d^2}{-2d^2 e^4 + 2e^6 x^2} + \frac{e^2 x^2 \log\left(-\frac{d}{e} + x\right)}{-2d^2 e^4 + 2e^6 x^2} + \frac{e^2 x^2 \log\left(\frac{d}{e} + x\right)}{-2d^2 e^4 + 2e^6 x^2} & \text{for } p = -2 \\ \frac{d^2 \log\left(-\frac{d}{e} + x\right)}{2e^4} - \frac{d^2 \log\left(\frac{d}{e} + x\right)}{2e^4} - \frac{x^2}{2e^2} & \text{for } p = -1 \\ \frac{d^4 (d^2 - e^2 x^2)^p}{2e^4 p^2 + 6e^4 p + 4e^4} - \frac{d^2 e^2 p x^2 (d^2 - e^2 x^2)^p}{2e^4 p^2 + 6e^4 p + 4e^4} + \frac{e^4 p x^4 (d^2 - e^2 x^2)^p}{2e^4 p^2 + 6e^4 p + 4e^4} + \frac{e^4 x^4 (d^2 - e^2 x^2)^p}{2e^4 p^2 + 6e^4 p + 4e^4} & \text{otherwise} \end{array} \right) \\ + \frac{d^{2p} e^3 x^5 {}_2F_1 \left( \frac{5}{2}, -p \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2} \right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d**3*Piecewise((x**2*(d**2)**p/2, Eq(e**2, 0)), (-Piecewise(((d**2 - e**2*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(d**2 - e**2*x**2), True))/(2*e**2), True)) + d**2*d**(2*p)*e*x**3*hyper((3/2, -p), (5/2,), e**2*x**2*exp_polar(2*I*pi)/d**2) + 3*d*e**2*Piecewise((x**4*(d**2)**p/4, Eq(e, 0)), (-d**2*log(-d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) - d**2/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(-d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2) + e**2*x**2*log(d/e + x)/(-2*d**2*e**4 + 2*e**6*x**2), Eq(p, -2)), (-d**2*log(-d/e + x)/(2*e**4) - d**2*log(d/e + x)/(2*e**4) - x**2/(2*e**2), Eq(p, -1)), (-d**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) - d**2*e**2*p*x**2*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) + e**4*p*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4) + e**4*x**4*(d**2 - e**2*x**2)**p/(2*e**4*p**2 + 6*e**4*p + 4*e**4), True)) + d**(2*p)*e**3*x**5*hyper((5/2, -p), (7/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/5
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3 (-e^2x^2 + d^2)^p x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*x, x)
```



### 3.263 $\int (d + ex)^3 (d^2 - e^2x^2)^p dx$

**Optimal.** Leaf size=73

$$\frac{d^2 2^{p+3} \left(\frac{ex}{d} + 1\right)^{-p-1} (d^2 - e^2x^2)^{p+1} {}_2F_1\left(-p-3, p+1; p+2; \frac{d-ex}{2d}\right)}{e(p+1)}$$

[Out] -((2^(3 + p)\*d^2\*(1 + (e\*x)/d)^(-1 - p)\*(d^2 - e^2\*x^2)^(1 + p)\*Hypergeometric2F1[-3 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/(e\*(1 + p)))

**Rubi [A]** time = 0.0725257, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{d^2 2^{p+3} \left(\frac{ex}{d} + 1\right)^{-p-1} (d^2 - e^2x^2)^{p+1} {}_2F_1\left(-p-3, p+1; p+2; \frac{d-ex}{2d}\right)}{e(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^3\*(d^2 - e^2\*x^2)^p, x]

[Out] -((2^(3 + p)\*d^2\*(1 + (e\*x)/d)^(-1 - p)\*(d^2 - e^2\*x^2)^(1 + p)\*Hypergeometric2F1[-3 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/(e\*(1 + p)))

**Rubi in Sympy [A]** time = 26.1018, size = 68, normalized size = 0.93

$$\frac{8d^3 \left(\frac{d+ex}{2}\right)^{-p} (d-ex)^{-p} (d-ex)^{p+1} (d^2 - e^2x^2)^p {}_2F_1\left(-p-3, p+1; p+2; \frac{d-ex}{d}\right)}{e(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p, x)

[Out] -8\*d\*\*3\*((d/2 + e\*x/2)/d)\*\*(-p)\*(d - e\*x)\*\*(-p)\*(d - e\*x)\*\*(p + 1)\*\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p\*hyper((-p - 3, p + 1), (p + 2, ), (d/2 - e\*x/2)/d)/(e\*(p + 1))

**Mathematica [B]** time = 0.229539, size = 271, normalized size = 3.71

$$\frac{(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} \left(3d^4p + 7d^4 + 6d^2e^2x^2 \left(1 - \frac{e^2x^2}{d^2}\right)^p + 2d^2e^2px^2 \left(1 - \frac{e^2x^2}{d^2}\right)^p + e^4x^4 \left(1 - \frac{e^2x^2}{d^2}\right)^p + e^4px^4 \left(1 - \frac{e^2x^2}{d^2}\right)^p\right)}{2e(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^3\*(d^2 - e^2\*x^2)^p, x]

[Out] ((d^2 - e^2\*x^2)^p\*(7\*d^4 + 3\*d^4\*p - 7\*d^4\*(1 - (e^2\*x^2)/d^2)^p - 3\*d^4\*p\*(1 - (e^2\*x^2)/d^2)^p + 6\*d^2\*e^2\*x^2\*(1 - (e^2\*x^2)/d^2)^p + 2\*d^2\*e^2\*p\*x^2\*(1 - (e^2\*x^2)/d^2)^p + e^4\*x^4\*(1 - (e^2\*x^2)/d^2)^p + e^4\*p\*x^4\*(1 - (e^2\*x^2)/d^2)^p + 2\*d^3\*e\*(2 + 3\*p

$$+ p^2) * x * \text{Hypergeometric2F1}[1/2, -p, 3/2, (e^2 * x^2)/d^2] + 2 * d * e^{\frac{3 * (2 + 3 * p + p^2) * x^3 * \text{Hypergeometric2F1}[3/2, -p, 5/2, (e^2 * x^2)/d^2]}{(2 * e * (1 + p) * (2 + p) * (1 - (e^2 * x^2)/d^2)^p)}$$

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int (ex + d)^3 (-e^2 x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(-e^2\*x^2+d^2)^p,x)

[Out] int((e\*x+d)^3\*(-e^2\*x^2+d^2)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3 (-e^2 x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p,x, algorithm="maxima")

[Out] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3 x^3 + 3 d e^2 x^2 + 3 d^2 e x + d^3\right) \left(-e^2 x^2 + d^2\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p,x, algorithm="fricas")

[Out] integral((e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3)\*(-e^2\*x^2 + d^2)^p, x)

**Sympy [A]** time = 13.5108, size = 476, normalized size = 6.52

$$d^3 d^{2p} x {}_2F_1\left(\frac{1}{2}, -p \left| \frac{e^2 x^2 e^{2i\pi}}{d^2} \right. \right) + 3d^2 e \left( \begin{array}{ll} \frac{x^2 (d^2)^p}{2} & \text{for } e^2 = 0 \\ \frac{(d^2 - e^2 x^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \frac{\log(d^2 - e^2 x^2)}{2e^2} & \text{otherwise} \end{array} \right)$$

$$+ dd^{2p} e^2 x^3 {}_2F_1\left(\frac{3}{2}, -p \left| \frac{e^2 x^2 e^{2i\pi}}{d^2} \right. \right)$$

$$+ e^3 \left( \begin{array}{ll} \frac{x^4 (d^2)^p}{4} & \text{for } e = 0 \\ \frac{d^2 \log\left(-\frac{d}{e} + x\right)}{-2d^2 e^4 + 2e^6 x^2} - \frac{d^2 \log\left(\frac{d}{e} + x\right)}{-2d^2 e^4 + 2e^6 x^2} - \frac{d^2}{-2d^2 e^4 + 2e^6 x^2} + \frac{e^2 x^2 \log\left(-\frac{d}{e} + x\right)}{-2d^2 e^4 + 2e^6 x^2} + \frac{e^2 x^2 \log\left(\frac{d}{e} + x\right)}{-2d^2 e^4 + 2e^6 x^2} & \text{for } p = -2 \\ \frac{d^2 \log\left(-\frac{d}{e} + x\right)}{-2e^4} - \frac{d^2 \log\left(\frac{d}{e} + x\right)}{2e^4} - \frac{x^2}{2e^2} & \text{for } p = -1 \\ -\frac{d^4 (d^2 - e^2 x^2)^p}{2e^4 p^2 + 6e^4 p + 4e^4} - \frac{d^2 e^2 p x^2 (d^2 - e^2 x^2)^p}{2e^4 p^2 + 6e^4 p + 4e^4} + \frac{e^4 p x^4 (d^2 - e^2 x^2)^p}{2e^4 p^2 + 6e^4 p + 4e^4} + \frac{e^4 x^4 (d^2 - e^2 x^2)^p}{2e^4 p^2 + 6e^4 p + 4e^4} & \text{otherwise} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p,x)

[Out] d\*\*3\*d\*\*(2\*p)\*x\*hyper((1/2, -p), (3/2, ), e\*\*2\*x\*\*2\*exp\_polar(2\*I\*pi)/d\*\*2) + 3\*d\*\*2\*e\*Piecewise((x\*\*2\*(d\*\*2)\*\*p/2, Eq(e\*\*2, 0)), (-Piecewise(((d\*\*2 - e\*\*2\*x\*\*2)\*\*(p + 1)/(p + 1), Ne(p, -1)), (log(d\*\*2 - e\*\*2\*x\*\*2), True))/(2\*e\*\*2), True)) + d\*d\*\*(2\*p)\*e\*\*2\*x\*\*3\*hyper((3/2, -p), (5/2, ), e\*\*2\*x\*\*2\*exp\_polar(2\*I\*pi)/d\*\*2) + e\*\*3\*Piecewise((x\*\*4\*(d\*\*2)\*\*p/4, Eq(e, 0)), (-d\*\*2\*log(-d/e + x)/(-2\*d\*\*2\*e\*\*4 + 2\*e\*\*6\*x\*\*2) - d\*\*2\*log(d/e + x)/(-2\*d\*\*2\*e\*\*4 + 2\*e\*\*6\*x\*\*2) - d\*\*2/(-2\*d\*\*2\*e\*\*4 + 2\*e\*\*6\*x\*\*2) + e\*\*2\*x\*\*2\*log(-d/e + x)/(-2\*d\*\*2\*e\*\*4 + 2\*e\*\*6\*x\*\*2) + e\*\*2\*x\*\*2\*log(d/e + x)/(-2\*d\*\*2\*e\*\*4 + 2\*e\*\*6\*x\*\*2), Eq(p, -2)), (-d\*\*2\*log(-d/e + x)/(2\*e\*\*4) - d\*\*2\*log(d/e + x)/(2\*e\*\*4) - x\*\*2/(2\*e\*\*2), Eq(p, -1)), (-d\*\*4\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p/(2\*e\*\*4\*p\*\*2 + 6\*e\*\*4\*p + 4\*e\*\*4) - d\*\*2\*e\*\*2\*p\*x\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p/(2\*e\*\*4\*p\*\*2 + 6\*e\*\*4\*p + 4\*e\*\*4) + e\*\*4\*p\*x\*\*4\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p/(2\*e\*\*4\*p\*\*2 + 6\*e\*\*4\*p + 4\*e\*\*4) + e\*\*4\*x\*\*4\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p/(2\*e\*\*4\*p\*\*2 + 6\*e\*\*4\*p + 4\*e\*\*4), True))

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3(-e^2x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p,x, algorithm="giac")

[Out] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p, x)

$$3.264 \quad \int \frac{(d+ex)^3 (d^2 - e^2 x^2)^p}{x} dx$$

**Optimal.** Leaf size=171

$$\frac{2d^2 e(3p+5)x (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{2p+3} - \frac{d (d^2 - e^2 x^2)^{p+1} {}_2F_1\left(1, p+1; p+2; 1 - \frac{e^2 x^2}{d^2}\right)}{2(p+1)} - \frac{ex (d^2 - e^2 x^2)^{p+1}}{2p+3} - \frac{3d (d^2 - e^2 x^2)^{p+1}}{2(p+1)}$$

[Out]  $(-3*d*(d^2 - e^2*x^2)^(1+p))/(2*(1+p)) - (e*x*(d^2 - e^2*x^2)^(1+p))/(3+2*p) + (2*d^2*e*(5+3*p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/((3+2*p)*(1 - (e^2*x^2)/d^2)^p) - (d*(d^2 - e^2*x^2)^(1+p)*Hypergeometric2F1[1, 1+p, 2+p, 1 - (e^2*x^2)/d^2])/(2*(1+p))$

**Rubi [A]** time = 0.257204, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$\frac{2d^2 e(3p+5)x (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{2p+3} - \frac{d (d^2 - e^2 x^2)^{p+1} {}_2F_1\left(1, p+1; p+2; 1 - \frac{e^2 x^2}{d^2}\right)}{2(p+1)} - \frac{ex (d^2 - e^2 x^2)^{p+1}}{2p+3} - \frac{3d (d^2 - e^2 x^2)^{p+1}}{2(p+1)}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(d^2 - e^2\*x^2)^p)/x,x]

[Out]  $(-3*d*(d^2 - e^2*x^2)^(1+p))/(2*(1+p)) - (e*x*(d^2 - e^2*x^2)^(1+p))/(3+2*p) + (2*d^2*e*(5+3*p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/((3+2*p)*(1 - (e^2*x^2)/d^2)^p) - (d*(d^2 - e^2*x^2)^(1+p)*Hypergeometric2F1[1, 1+p, 2+p, 1 - (e^2*x^2)/d^2])/(2*(1+p))$

**Rubi in Sympy [A]** time = 51.8461, size = 160, normalized size = 0.94

$$3d^2 ex \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p, \frac{1}{2} \middle| \frac{e^2 x^2}{d^2}\right) - \frac{d (d^2 - e^2 x^2)^{p+1} {}_2F_1\left(1, p+1 \middle| 1 - \frac{e^2 x^2}{d^2}\right)}{2(p+1)} - \frac{3d (d^2 - e^2 x^2)^{p+1}}{2(p+1)} + \frac{e^3 x^3 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p, \frac{3}{2} \middle| \frac{e^2 x^2}{d^2}\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x,x)

[Out]  $3*d**2*e*x*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper((-p, 1/2), (3/2, ), e**2*x**2/d**2) - d*(d**2 - e**2*x**2)**(p+1)*hyper((1, p+1), (p+2, ), 1 - e**2*x**2/d**2)/(2*(p+1)) - 3*d*(d**2 - e**2*x**2)**(p+1)/(2*(p+1)) + e**3*x**3*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper((-p, 3/2), (5/2, ), e**2*x**2/d**2)/3$

**Mathematica [A]** time = 0.732501, size = 194, normalized size = 1.13

$$\frac{1}{6} (d^2 - e^2 x^2)^p \left( 18 d^2 e x \left( 1 - \frac{e^2 x^2}{d^2} \right)^{-p} {}_2F_1 \left( \frac{1}{2}, -p; \frac{3}{2}; \frac{e^2 x^2}{d^2} \right) + \frac{9 d \left( d^2 \left( \left( 1 - \frac{e^2 x^2}{d^2} \right)^{-p} - 1 \right) + e^2 x^2 \right)}{p + 1} \right. \\ \left. + 2 e^3 x^3 \left( 1 - \frac{e^2 x^2}{d^2} \right)^{-p} {}_2F_1 \left( \frac{3}{2}, -p; \frac{5}{2}; \frac{e^2 x^2}{d^2} \right) + \frac{3 d^3 \left( 1 - \frac{d^2}{e^2 x^2} \right)^{-p} {}_2F_1 \left( -p, -p; 1 - p; \frac{d^2}{e^2 x^2} \right)}{p} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(d^2 - e^2\*x^2)^p)/x,x]

[Out] ((d^2 - e^2\*x^2)^p\*((9\*d\*(e^2\*x^2 + d^2\*(-1 + (1 - (e^2\*x^2)/d^2)^(-p))))/(1 + p) + (18\*d^2\*e\*x\*Hypergeometric2F1[1/2, -p, 3/2, (e^2\*x^2)/d^2])/(1 - (e^2\*x^2)/d^2)^p + (2\*e^3\*x^3\*Hypergeometric2F1[3/2, -p, 5/2, (e^2\*x^2)/d^2])/(1 - (e^2\*x^2)/d^2)^p + (3\*d^3\*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2\*x^2)])/(p\*(1 - d^2/(e^2\*x^2))^p))/6

**Maple [F]** time = 0.041, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^3 (-e^2x^2 + d^2)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(-e^2\*x^2+d^2)^p/x,x)

[Out] int((e\*x+d)^3\*(-e^2\*x^2+d^2)^p/x,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^3 (-e^2x^2 + d^2)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p/x,x, algorithm="maxima")

[Out] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p/x, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)(-e^2x^2 + d^2)^p}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p/x,x, algorithm="fricas")

[Out] integral((e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3)\*(-e^2\*x^2 + d^2)^p/x, x)

**Sympy [A]** time = 13.9263, size = 178, normalized size = 1.04

$$\begin{aligned} & -\frac{d^3 e^{2p} x^{2p} e^{i\pi p} (-p) {}_2F_1\left(-p, -p \middle| \frac{d^2}{e^2 x^2}\right)}{2(-p+1)} + 3d^2 d^{2p} e x {}_2F_1\left(\frac{1}{2}, -p \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right) \\ & + 3de^2 \left( \begin{array}{ll} \left\{ \begin{array}{l} \frac{x^2 (d^2)^p}{2} \\ \frac{(d^2 - e^2 x^2)^{p+1}}{p+1} \\ \log(d^2 - e^2 x^2) \end{array} \right. & \begin{array}{l} \text{for } e^2 = 0 \\ \text{for } p \neq -1 \\ \text{otherwise} \end{array} \end{array} \right) + \frac{d^{2p} e^3 x^3 {}_2F_1\left(\frac{3}{2}, -p \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x,x)

[Out] -d\*\*3\*e\*\*(2\*p)\*x\*\*(2\*p)\*exp(I\*pi\*p)\*gamma(-p)\*hyper((-p, -p), (-p + 1, ), d\*\*2/(e\*\*2\*x\*\*2))/(2\*gamma(-p + 1)) + 3\*d\*\*2\*d\*\*(2\*p)\*e\*x\*hyper((1/2, -p), (3/2, ), e\*\*2\*x\*\*2\*exp\_polar(2\*I\*pi)/d\*\*2) + 3\*d\*e\*\*2\*Piecewise((x\*\*2\*(d\*\*2)\*\*p/2, Eq(e\*\*2, 0)), (-Piecewise(((d\*\*2 - e\*\*2\*x\*\*2)\*\*(p + 1)/(p + 1), Ne(p, -1)), (log(d\*\*2 - e\*\*2\*x\*\*2), True)))/(2\*e\*\*2), True)) + d\*\*(2\*p)\*e\*\*3\*x\*\*3\*hyper((3/2, -p), (5/2, ), e\*\*2\*x\*\*2\*exp\_polar(2\*I\*pi)/d\*\*2)/3

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^3 (-e^2 x^2 + d^2)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p/x,x, algorithm="giac")

[Out] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p/x, x)

$$3.265 \quad \int \frac{(d+ex)^3 (d^2 - e^2 x^2)^p}{x^2} dx$$

**Optimal.** Leaf size=159

$$2de^2(1-p)x(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2x^2}{d^2}\right) \\ - \frac{3e(d^2 - e^2x^2)^{p+1} {}_2F_1\left(1, p+1; p+2; 1 - \frac{e^2x^2}{d^2}\right)}{2(p+1)} - \frac{e(d^2 - e^2x^2)^{p+1}}{2(p+1)} - \frac{d(d^2 - e^2x^2)^{p+1}}{x}$$

[Out]  $-(e*(d^2 - e^2*x^2)^(1 + p))/(2*(1 + p)) - (d*(d^2 - e^2*x^2)^(1 + p))/x + (2*d*e^2*(1 - p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p - (3*e*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*(1 + p))$

**Rubi [A]** time = 0.323462, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.32$

$$2de^2(1-p)x(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2x^2}{d^2}\right) \\ - \frac{3e(d^2 - e^2x^2)^{p+1} {}_2F_1\left(1, p+1; p+2; 1 - \frac{e^2x^2}{d^2}\right)}{2(p+1)} - \frac{e(d^2 - e^2x^2)^{p+1}}{2(p+1)} - \frac{d(d^2 - e^2x^2)^{p+1}}{x}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(d^2 - e^2\*x^2)^p)/x^2, x]

[Out]  $-(e*(d^2 - e^2*x^2)^(1 + p))/(2*(1 + p)) - (d*(d^2 - e^2*x^2)^(1 + p))/x + (2*d*e^2*(1 - p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p - (3*e*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*(1 + p))$

**Rubi in Sympy [A]** time = 51.8829, size = 158, normalized size = 0.99

$$\frac{d^3 \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(-p, -\frac{1}{2}; \frac{1}{2}; \frac{e^2x^2}{d^2}\right)}{x} \\ + 3de^2x \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(-p, \frac{1}{2}; \frac{3}{2}; \frac{e^2x^2}{d^2}\right) \\ - \frac{3e(d^2 - e^2x^2)^{p+1} {}_2F_1\left(1, p+1; p+2; 1 - \frac{e^2x^2}{d^2}\right)}{2(p+1)} - \frac{e(d^2 - e^2x^2)^{p+1}}{2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x\*\*2, x)

[Out]  $-d**3*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper((-p, -1/2), (1/2, ), e**2*x**2/d**2)/x + 3*d*e**2*x*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper((-p, 1/2), (3/2, ), e**2*x**2/d**2) - 3*e*(d**2 - e**2*x**2)**(p + 1)*hyper((1, p + 1), (p + 2, ), 1 - e**2*x**2/d**2)/(2*(p + 1)) - e*(d**2 - e**2*x**2)**(p + 1)/(2*(p + 1))$

---

**Mathematica [A]** time = 0.362842, size = 264, normalized size = 1.66

$$\left(1 - \frac{d^2}{e^2 x^2}\right)^{-p} (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} \left( ex \left( 6dep(p+1)x \left(1 - \frac{d^2}{e^2 x^2}\right)^p {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right) + 3d^2(p+1) \left(1 - \frac{e^2 x^2}{d^2}\right)^p {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(d^2 - e^2\*x^2)^p)/x^2, x]

[Out] ((d^2 - e^2\*x^2)^p\*(-2\*d^3\*p\*(1+p)\*(1 - d^2/(e^2\*x^2))^p\*Hypergeometric2F1[-1/2, -p, 1/2, (e^2\*x^2)/d^2] + e\*x\*(-(p\*(1 - d^2/(e^2\*x^2))^p\*(-(e^2\*x^2\*(1 - (e^2\*x^2)/d^2)^p) + d^2\*(-1 + (1 - (e^2\*x^2)/d^2)^p))) + 6\*d\*e\*p\*(1+p)\*(1 - d^2/(e^2\*x^2))^p\*x\*Hypergeometric2F1[1/2, -p, 3/2, (e^2\*x^2)/d^2] + 3\*d^2\*(1+p)\*(1 - (e^2\*x^2)/d^2)^p\*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2\*x^2)])))/(2\*p\*(1+p)\*(1 - d^2/(e^2\*x^2))^p\*x\*(1 - (e^2\*x^2)/d^2)^p)

---

**Maple [F]** time = 0.047, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^3 (-e^2 x^2 + d^2)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(-e^2\*x^2+d^2)^p/x^2, x)

[Out] int((e\*x+d)^3\*(-e^2\*x^2+d^2)^p/x^2, x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^3 (-e^2 x^2 + d^2)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p/x^2, x, algorithm="maxima")

[Out] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p/x^2, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e^3 x^3 + 3 d e^2 x^2 + 3 d^2 e x + d^3) (-e^2 x^2 + d^2)^p}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p/x^2, x, algorithm="fricas")

[Out] integral((e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3)\*(-e^2\*x^2 + d^2)^p/x^2, x)



---

**Sympy [A]** time = 14.0684, size = 177, normalized size = 1.11

$$\frac{d^3 d^{2p} {}_2F_1\left(-\frac{1}{2}, -p \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{x} - \frac{3d^2 e e^{2p} x^{2p} e^{i\pi p} (-p) {}_2F_1\left(-p, -p \middle| \frac{d^2}{e^2 x^2}\right)}{2(-p+1)} + 3d d^{2p} e^2 x {}_2F_1\left(\frac{1}{2}, -p \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right) + e^3 \begin{cases} \frac{x^2 (d^2)^p}{2} & \text{for } e^2 = 0 \\ \frac{(d^2 - e^2 x^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ -\frac{\log(d^2 - e^2 x^2)}{2e^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x\*\*2,x)

[Out]  $-d^{3}d^{2p}\text{hyper}((-1/2, -p), (1/2, ), e^{2}x^{2}\text{exp\_polar}(2*I\pi)/d^{2})/x - 3d^{2}e^{2}e^{2p}x^{2p}\text{exp}(I\pi p)\text{gamma}(-p)\text{hyper}((-p, -p), (-p + 1, ), d^{2}/(e^{2}x^{2}))/ (2\text{gamma}(-p + 1)) + 3d^{2}d^{2p}e^{2}x\text{hyper}(1/2, -p), (3/2, ), e^{2}x^{2}\text{exp\_polar}(2I\pi)/d^{2} + e^{3}\text{Piecewise}((x^{2}(d^{2})^{p}/2, \text{Eq}(e^{2}, 0)), (-\text{Piecewise}((d^{2} - e^{2}x^{2})^{p+1}/(p + 1), \text{Ne}(p, -1)), (\log(d^{2} - e^{2}x^{2}), \text{True}))/ (2e^{2}), \text{True}))$

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^3 (-e^2 x^2 + d^2)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p/x^2,x, algorithm="giac")

[Out] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p/x^2, x)

$$3.266 \quad \int \frac{(d+ex)^3(d^2-e^2x^2)^p}{x^3} dx$$

**Optimal.** Leaf size=166

$$\begin{aligned} & -\frac{e^2(3-p)(d^2-e^2x^2)^{p+1} {}_2F_1\left(1, p+1; p+2; 1-\frac{e^2x^2}{d^2}\right)}{2d(p+1)} - \frac{3e(d^2-e^2x^2)^{p+1}}{x} \\ & - \frac{d(d^2-e^2x^2)^{p+1}}{2x^2} - 2e^3(3p+1)x(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2x^2}{d^2}\right) \end{aligned}$$

[Out]  $-(d*(d^2 - e^2*x^2)^(1 + p))/(2*x^2) - (3*e*(d^2 - e^2*x^2)^(1 + p))/x - (2*e^3*(1 + 3*p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p - (e^2*(3 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*d*(1 + p))$

**Rubi [A]** time = 0.367427, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\begin{aligned} & -\frac{e^2(3-p)(d^2-e^2x^2)^{p+1} {}_2F_1\left(1, p+1; p+2; 1-\frac{e^2x^2}{d^2}\right)}{2d(p+1)} - \frac{3e(d^2-e^2x^2)^{p+1}}{x} \\ & - \frac{d(d^2-e^2x^2)^{p+1}}{2x^2} - 2e^3(3p+1)x(d^2-e^2x^2)^p \left(1-\frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2x^2}{d^2}\right) \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(d^2 - e^2\*x^2)^p)/x^3, x]

[Out]  $-(d*(d^2 - e^2*x^2)^(1 + p))/(2*x^2) - (3*e*(d^2 - e^2*x^2)^(1 + p))/x - (2*e^3*(1 + 3*p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2])/(1 - (e^2*x^2)/d^2)^p - (e^2*(3 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 - (e^2*x^2)/d^2])/(2*d*(1 + p))$

**Rubi in Sympy [A]** time = 58.7051, size = 182, normalized size = 1.1

$$\begin{aligned} & \frac{3d^2e\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p {}_2F_1\left(-p, -\frac{1}{2}; \frac{1}{2}; \frac{e^2x^2}{d^2}\right)}{x} \\ & + e^3x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p {}_2F_1\left(-p, \frac{1}{2}; \frac{3}{2}; \frac{e^2x^2}{d^2}\right) \\ & - \frac{3e^2(d^2-e^2x^2)^{p+1} {}_2F_1\left(1, p+1; p+2; 1-\frac{e^2x^2}{d^2}\right)}{2d(p+1)} - \frac{e^2(d^2-e^2x^2)^{p+1} {}_2F_1\left(2, p+1; p+2; 1-\frac{e^2x^2}{d^2}\right)}{2d(p+1)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x\*\*3, x)

[Out]  $-3*d**2*e*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper((-p, -1/2), (1/2, ), e**2*x**2/d**2)/x + e**3*x*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper((-p, 1/2), (3/2, ), e**2*x**2/d**2) - 3*e**2*(d**2 - e**2*x**2)**(p + 1)*hyper((1, p + 1), (p + 2, ), 1 - e**2*x**2/d**2)/(2*d*(p + 1)) - e**2*(d**2 - e**2*x**2)**(p + 1)*hyper((2, p + 1), (p + 2, ), 1 - e**2*x**2/d**2)/(2*d*($

p + 1))

---

**Mathematica [A]** time = 0.290167, size = 201, normalized size = 1.21

$$\frac{(d^2 - e^2 x^2)^p \left( \frac{d \left(1 - \frac{d^2}{e^2 x^2}\right)^{-p} \left( d^2 p {}_2F_1\left(1-p, -p; 2-p; \frac{d^2}{e^2 x^2}\right) + 3e^2 (p-1)x^2 {}_2F_1\left(-p, -p; 1-p; \frac{d^2}{e^2 x^2}\right) \right)}{(p-1)^p} - 6d^2 e x \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right) \right)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(d^2 - e^2\*x^2)^p)/x^3, x]

[Out] ((d^2 - e^2\*x^2)^p\*((-6\*d^2\*e\*x\*Hypergeometric2F1[-1/2, -p, 1/2, (e^2\*x^2)/d^2])/(1 - (e^2\*x^2)/d^2)^p + (2\*e^3\*x^3\*Hypergeometric2F1[1/2, -p, 3/2, (e^2\*x^2)/d^2])/(1 - (e^2\*x^2)/d^2)^p + (d\*(d^2\*p\*Hypergeometric2F1[1 - p, -p, 2 - p, d^2/(e^2\*x^2)] + 3\*e^2\*(-1 + p)\*x^2\*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2\*x^2)])))/((-1 + p)\*p\*(1 - d^2/(e^2\*x^2))^p))/(2\*x^2)

---

**Maple [F]** time = 0.058, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^3 (-e^2x^2 + d^2)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(-e^2\*x^2+d^2)^p/x^3, x)

[Out] int((e\*x+d)^3\*(-e^2\*x^2+d^2)^p/x^3, x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^3 (-e^2x^2 + d^2)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p/x^3, x, algorithm="maxima")

[Out] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p/x^3, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)(-e^2x^2 + d^2)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(-e^2\*x^2 + d^2)^p/x^3, x, algorithm="fricas")

[Out] integral((e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3)\*(-e^2\*x^2 + d^2)^p/x^3, x)

---

**Sympy [A]** time = 18.313, size = 177, normalized size = 1.07

$$\frac{d^3 e^{2p} x^{2p} e^{i\pi p} (-p+1) {}_2F_1\left(-p, -p+1 \middle| \frac{d^2}{e^2 x^2}\right)}{2x^2 (-p+2)} - \frac{3d^2 d^{2p} e_2 F_1\left(-\frac{1}{2}, -p \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{x}$$

$$- \frac{3de^2 e^{2p} x^{2p} e^{i\pi p} (-p) {}_2F_1\left(-p, -p \middle| \frac{d^2}{e^2 x^2}\right)}{2(-p+1)} + d^{2p} e^3 x {}_2F_1\left(\frac{1}{2}, -p \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x\*\*3,x)

[Out] -d\*\*3\*e\*\*(2\*p)\*x\*\*(2\*p)\*exp(I\*pi\*p)\*gamma(-p+1)\*hyper((-p, -p+1), (-p+2, ), d\*\*2/(e\*\*2\*x\*\*2))/(2\*x\*\*2\*gamma(-p+2)) - 3\*d\*\*2\*d\*\*(2\*p)\*e\*hyper((-1/2, -p), (1/2, ), e\*\*2\*x\*\*2\*exp\_polar(2\*I\*pi)/d\*\*2)/x - 3\*d\*e\*\*2\*e\*\*(2\*p)\*x\*\*(2\*p)\*exp(I\*pi\*p)\*gamma(-p)\*hyper((-p, -p), (-p+1, ), d\*\*2/(e\*\*2\*x\*\*2))/(2\*gamma(-p+1)) + d\*\*(2\*p)\*e\*\*3\*x\*hyper((1/2, -p), (3/2, ), e\*\*2\*x\*\*2\*exp\_polar(2\*I\*pi)/d\*\*2)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3(-e^2x^2+d^2)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(-e^2\*x^2+d^2)^p/x^3,x, algorithm="giac")

[Out] integrate((e\*x+d)^3\*(-e^2\*x^2+d^2)^p/x^3, x)

$$3.267 \quad \int \frac{x^4 (d^2 - e^2 x^2)^p}{d + ex} dx$$

**Optimal.** Leaf size=148

$$\frac{x^5 (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{5}{2}, 1 - p; \frac{7}{2}; \frac{e^2 x^2}{d^2}\right)}{5d} - \frac{d^2 (d^2 - e^2 x^2)^{p+1}}{e^5(p+1)} + \frac{(d^2 - e^2 x^2)^{p+2}}{2e^5(p+2)} + \frac{d^4 (d^2 - e^2 x^2)^p}{2e^5 p}$$

[Out] (d^4\*(d^2 - e^2\*x^2)^p)/(2\*e^5\*p) - (d^2\*(d^2 - e^2\*x^2)^(1 + p))/(e^5\*(1 + p)) + (d^2 - e^2\*x^2)^(2 + p)/(2\*e^5\*(2 + p)) + (x^5\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[5/2, 1 - p, 7/2, (e^2\*x^2)/d^2])/((5\*d\*(1 - (e^2\*x^2)/d^2)^p))

**Rubi [A]** time = 0.296313, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{x^5 (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{5}{2}, 1 - p; \frac{7}{2}; \frac{e^2 x^2}{d^2}\right)}{5d} - \frac{d^2 (d^2 - e^2 x^2)^{p+1}}{e^5(p+1)} + \frac{(d^2 - e^2 x^2)^{p+2}}{2e^5(p+2)} + \frac{d^4 (d^2 - e^2 x^2)^p}{2e^5 p}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*(d^2 - e^2\*x^2)^p)/(d + e\*x), x]

[Out] (d^4\*(d^2 - e^2\*x^2)^p)/(2\*e^5\*p) - (d^2\*(d^2 - e^2\*x^2)^(1 + p))/(e^5\*(1 + p)) + (d^2 - e^2\*x^2)^(2 + p)/(2\*e^5\*(2 + p)) + (x^5\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[5/2, 1 - p, 7/2, (e^2\*x^2)/d^2])/((5\*d\*(1 - (e^2\*x^2)/d^2)^p))

**Rubi in Sympy [A]** time = 56.8357, size = 116, normalized size = 0.78

$$\frac{d^4 (d^2 - e^2 x^2)^p}{2e^5 p} - \frac{d^2 (d^2 - e^2 x^2)^{p+1}}{e^5 (p+1)} + \frac{(d^2 - e^2 x^2)^{p+2}}{2e^5 (p+2)} + \frac{x^5 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p + 1, \frac{5}{2}; \frac{7}{2}; \frac{e^2 x^2}{d^2}\right)}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d), x)

[Out] d\*\*4\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p/(2\*e\*\*5\*p) - d\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(p + 1)/(e\*\*5\*(p + 1)) + (d\*\*2 - e\*\*2\*x\*\*2)\*\*(p + 2)/(2\*e\*\*5\*(p + 2)) + x\*\*5\*(1 - e\*\*2\*x\*\*2/d\*\*2)\*\*(-p)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p\*hyper((-p + 1, 5/2), (7/2, ), e\*\*2\*x\*\*2/d\*\*2)/(5\*d)

**Mathematica [C]** time = 0.738148, size = 225, normalized size = 1.52

$$\frac{6de^5(p+1)(p+2)x^{10}(d-ex)^p(d+ex)^{p-1}F_1\left(5; -p, 1-p; 6; \frac{ex}{d}, -\frac{ex}{d}\right) + 5\left(e^6(p^3+2p^2-p-2)x^6F_1\left(6; -p, 2-p; 7; \frac{ex}{d}, -\frac{ex}{d}\right) + 6de^5(p^2+3p+2)x^5F_1\left(5; -p, 1-p; 6; \frac{ex}{d}, -\frac{ex}{d}\right) + 3d^2e^4p(p+1)x^4\right)}{5d}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^4\*(d^2 - e^2\*x^2)^p)/(d + e\*x), x]

[Out]  $(6*d*e^5*(1+p)*(2+p)*x^{10}(d-e*x)^p(d+e*x)^{-1+p} \text{AppellF1}[5, -p, 1-p, 6, (e*x)/d, -((e*x)/d)] / (5*(6*d^4*e^2*p*x^2*(1-(e^2*x^2)/d^2)^p + 3*d^2*e^4*p*(1+p)*x^4*(1-(e^2*x^2)/d^2)^p + 6*d^6*(-1+(1-(e^2*x^2)/d^2)^p) + 6*d*e^5*(2+3*p+p^2)*x^5 \text{AppellF1}[5, -p, 1-p, 6, (e*x)/d, -((e*x)/d)] + e^6*(-2-p+2*p^2+p^3)*x^6 \text{AppellF1}[6, -p, 2-p, 7, (e*x)/d, -((e*x)/d)])$

**Maple [F]** time = 0.085, size = 0, normalized size = 0.

$$\int \frac{x^4 (-e^2 x^2 + d^2)^p}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(-e^2*x^2+d^2)^p/(e*x+d),x)`

[Out] `int(x^4*(-e^2*x^2+d^2)^p/(e*x+d),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2 x^2 + d^2)^p x^4}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d),x, algorithm="maxima")`

[Out] `integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2 x^2 + d^2)^p x^4}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d),x, algorithm="fricas")`

[Out] `integral((-e^2*x^2 + d^2)^p*x^4/(e*x + d), x)`

**Sympy [A]** time = 57.2155, size = 4442, normalized size = 30.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(-e**2*x**2+d**2)**p/(e*x+d),x)`

[Out] `Piecewise((-6*0**p*d**4*d**(2*p)*p*log(d**2/(e**2*x**2))*gamma(-p)*gamma(-p-1/2)*gamma(p+1)*gamma(p+3)/(12*e**5*p*gamma(-p)*gamma(-p-1/2)*gamma(p+1)*gamma(p+3) + 12*e**5*gamma(-p)*gam`



```

3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)
+ 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) +
12*0**p*d**4*d**(2*p)*p*atanh(d/(e*x))*gamma(-p)*gamma(-p - 1/2)
*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*g
amma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamm
a(p + 1)*gamma(p + 3)) - 6*0**p*d**4*d**(2*p)*log(d**2/(e**2*x**2
))*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p
*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*ga
mma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 6*0**p*d**4*
d**(2*p)*log(-d**2/(e**2*x**2) + 1)*gamma(-p)*gamma(-p - 1/2)*gam
ma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma
(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p
+ 1)*gamma(p + 3)) + 12*0**p*d**4*d**(2*p)*atanh(d/(e*x))*gamma(-
p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)
*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*ga
mma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) - 12*0**p*d**3*d**(2*p)*
e*p*x*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**
5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5
*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) - 12*0**p*d
**3*d**(2*p)*e*x*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p +
3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3
) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3))
+ 6*0**p*d**2*d**(2*p)*e**2*p*x**2*gamma(-p)*gamma(-p - 1/2)*gamm
a(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(
p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p +
1)*gamma(p + 3)) + 6*0**p*d**2*d**(2*p)*e**2*x**2*gamma(-p)*gamm
a(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(
-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p
- 1/2)*gamma(p + 1)*gamma(p + 3)) - 4*0**p*d*d**(2*p)*e**3*p*x**3
*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)/(12*e**5*p*g
amma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamm
a(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) - 4*0**p*d*d**(2
*p)*e**3*x**3*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)
/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) +
12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 3
*0**p*d**(2*p)*e**4*p*x**4*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)
*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*g
amma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamm
a(p + 3)) + 3*0**p*d**(2*p)*e**4*x**4*gamma(-p)*gamma(-p - 1/2)*g
amma(p + 1)*gamma(p + 3)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gam
ma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(
p + 1)*gamma(p + 3)) + 12*d**4*e**(2*p)*x**(2*p)*(-d**2/(e**2*x**
2) + 1)**p*exp(I*pi*p)*gamma(-p)*gamma(p)*gamma(-p - 1/2)*gamma(p
+ 2)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p +
3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)
) + 12*d**2*e**2*e**(2*p)*p*x**2*x**(2*p)*(-d**2/(e**2*x**2) + 1)
**p*exp(I*pi*p)*gamma(-p)*gamma(p)*gamma(-p - 1/2)*gamma(p + 2)/(
12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 1
2*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 6*d
**3*e**(2*p)*p**2*x**3*x**(2*p)*exp(I*pi*p)*gamma(-p)*gamma(p)*
gamma(-p - 3/2)*gamma(p + 3)*hyper((-p + 1, -p - 3/2), (-p - 1/2,
), d**2/(e**2*x**2))/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p
+ 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p +
1)*gamma(p + 3)) + 6*d*e**3*e**(2*p)*p*x**3*x**(2*p)*exp(I*pi*p)*
gamma(-p)*gamma(p)*gamma(-p - 3/2)*gamma(p + 3)*hyper((-p + 1, -p
- 3/2), (-p - 1/2, ), d**2/(e**2*x**2))/(12*e**5*p*gamma(-p)*gamm
a(-p - 1/2)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-
p - 1/2)*gamma(p + 1)*gamma(p + 3)) + 6*e**4*e**(2*p)*p**2*x**4*x
**(2*p)*(-d**2/(e**2*x**2) + 1)**p*exp(I*pi*p)*gamma(-p)*gamma(p)
*gamma(-p - 1/2)*gamma(p + 2)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2
)*gamma(p + 1)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*g
amma(p + 1)*gamma(p + 3)) + 6*e**4*e**(2*p)*p*x**4*x**(2*p)*(-d**
2/(e**2*x**2) + 1)**p*exp(I*pi*p)*gamma(-p)*gamma(p)*gamma(-p - 1
/2)*gamma(p + 2)/(12*e**5*p*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1
)*gamma(p + 3) + 12*e**5*gamma(-p)*gamma(-p - 1/2)*gamma(p + 1)*g
amma(p + 3)), True))

```



GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2 x^2 + d^2)^p x^4}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d), x)
```

$$3.268 \quad \int \frac{x^3(d^2 - e^2x^2)^p}{d + ex} dx$$

**Optimal.** Leaf size=121

$$-\frac{ex^5(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{5}{2}, 1 - p; \frac{7}{2}; \frac{e^2x^2}{d^2}\right)}{5d^2} + \frac{d(d^2 - e^2x^2)^{p+1}}{2e^4(p+1)} - \frac{d^3(d^2 - e^2x^2)^p}{2e^4p}$$

[Out]  $-(d^3*(d^2 - e^2*x^2)^p)/(2*e^4*p) + (d*(d^2 - e^2*x^2)^{(1+p)})/(2*e^4*(1+p)) - (e*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, 1 - p, 7/2, (e^2*x^2)/d^2])/(5*d^2*(1 - (e^2*x^2)/d^2)^p)$

**Rubi [A]** time = 0.241402, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$-\frac{ex^5(d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{5}{2}, 1 - p; \frac{7}{2}; \frac{e^2x^2}{d^2}\right)}{5d^2} + \frac{d(d^2 - e^2x^2)^{p+1}}{2e^4(p+1)} - \frac{d^3(d^2 - e^2x^2)^p}{2e^4p}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(d^2 - e^2\*x^2)^p)/(d + e\*x), x]

[Out]  $-(d^3*(d^2 - e^2*x^2)^p)/(2*e^4*p) + (d*(d^2 - e^2*x^2)^{(1+p)})/(2*e^4*(1+p)) - (e*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, 1 - p, 7/2, (e^2*x^2)/d^2])/(5*d^2*(1 - (e^2*x^2)/d^2)^p)$

**Rubi in Sympy [A]** time = 44.9319, size = 97, normalized size = 0.8

$$-\frac{d^3(d^2 - e^2x^2)^p}{2e^4p} + \frac{d(d^2 - e^2x^2)^{p+1}}{2e^4(p+1)} - \frac{ex^5 \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(-p + 1, \frac{5}{2}; \frac{7}{2}; \frac{e^2x^2}{d^2}\right)}{5d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d), x)

[Out]  $-d**3*(d**2 - e**2*x**2)**p/(2*e**4*p) + d*(d**2 - e**2*x**2)**(p + 1)/(2*e**4*(p + 1)) - e*x**5*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper((-p + 1, 5/2), (7/2, ), e**2*x**2/d**2)/(5*d**2)$

**Mathematica [B]** time = 0.463687, size = 245, normalized size = 2.02

$$\frac{\left(\frac{ex}{d} + 1\right)^{-p} (d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} \left(6d^2e(p+1)x \left(\frac{ex}{d} + 1\right)^p {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2x^2}{d^2}\right) + 3d \left(d(d - ex) \left(2 - \frac{2e^2x^2}{d^2}\right)^p {}_2F_1\left(1 - p, \dots\right)\right)}{6e^4(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(d^2 - e^2\*x^2)^p)/(d + e\*x), x]

[Out]  $((d^2 - e^2*x^2)^p*(6*d^2*e*(1+p)*x*(1 + (e*x)/d)^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + 2*e^3*(1+p)*x^3*(1 + (e*x$

$\int \frac{x^3 (-e^2 x^2 + d^2)^p}{ex + d} dx$

**Maple [F]** time = 0.074, size = 0, normalized size = 0.

$$\int \frac{x^3 (-e^2 x^2 + d^2)^p}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(-e^2*x^2+d^2)^p/(e*x+d),x)`

[Out] `int(x^3*(-e^2*x^2+d^2)^p/(e*x+d),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2 x^2 + d^2)^p x^3}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d),x, algorithm="maxima")`

[Out] `integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2 x^2 + d^2)^p x^3}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d),x, algorithm="fricas")`

[Out] `integral((-e^2*x^2 + d^2)^p*x^3/(e*x + d), x)`

**Sympy [A]** time = 36.6861, size = 5090, normalized size = 42.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(-e**2*x**2+d**2)**p/(e*x+d),x)`

[Out] `Piecewise((3**0**p*d**3*d**(2*p)*p*log(d**2/(e**2*x**2))*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3*0**p*d**3*d**(2*p)*p*log(d**2/(e**2*x**2) - 1)*gamma(-p - 1/2)*gamma(p + 1)/(6*e**4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 6*`

$$\begin{aligned}
& 0^*p^*d^{**3}d^{**}(2^*p)^*p^*acoth(d/(e^*x))^*gamma(-p - 1/2)^*gamma(p + 1)/ \\
& (6^*e^{**4}p^*gamma(-p - 1/2)^*gamma(p + 1) + 6^*e^{**4}gamma(-p - 1/2)^*g \\
& amma(p + 1)) + 3^*0^*p^*d^{**3}d^{**}(2^*p)^*log(d^{**2}/(e^{**2}x^{**2}))^*gamma(- \\
& p - 1/2)^*gamma(p + 1)/(6^*e^{**4}p^*gamma(-p - 1/2)^*gamma(p + 1) + 6^* \\
& e^{**4}gamma(-p - 1/2)^*gamma(p + 1)) - 3^*0^*p^*d^{**3}d^{**}(2^*p)^*log(d^{**} \\
& 2/(e^{**2}x^{**2}) - 1)^*gamma(-p - 1/2)^*gamma(p + 1)/(6^*e^{**4}p^*gamma(- \\
& p - 1/2)^*gamma(p + 1) + 6^*e^{**4}gamma(-p - 1/2)^*gamma(p + 1)) - 6^* \\
& 0^*p^*d^{**3}d^{**}(2^*p)^*acoth(d/(e^*x))^*gamma(-p - 1/2)^*gamma(p + 1)/(6 \\
& ^*e^{**4}p^*gamma(-p - 1/2)^*gamma(p + 1) + 6^*e^{**4}gamma(-p - 1/2)^*gam \\
& ma(p + 1)) + 6^*0^*p^*d^{**2}d^{**}(2^*p)^*e^*p^*x^*gamma(-p - 1/2)^*gamma \\
& (p + 1)/(6^*e^{**4}p^*gamma(-p - 1/2)^*gamma(p + 1) + 6^*e^{**4}gamma(-p - 1/ \\
& 2)^*gamma(p + 1)) + 6^*0^*p^*d^{**2}d^{**}(2^*p)^*e^*x^*gamma(-p - 1/2)^*gamma \\
& (p + 1)/(6^*e^{**4}p^*gamma(-p - 1/2)^*gamma(p + 1) + 6^*e^{**4}gamma(-p - \\
& 1/2)^*gamma(p + 1)) - 3^*0^*p^*d^*d^{**}(2^*p)^*e^{**2}p^*x^{**2}^*gamma(-p - 1 \\
& /2)^*gamma(p + 1)/(6^*e^{**4}p^*gamma(-p - 1/2)^*gamma(p + 1) + 6^*e^{**4} \\
& gamma(-p - 1/2)^*gamma(p + 1)) - 3^*0^*p^*d^*d^{**}(2^*p)^*e^{**2}x^{**2}^*gamma \\
& (-p - 1/2)^*gamma(p + 1)/(6^*e^{**4}p^*gamma(-p - 1/2)^*gamma(p + 1) + \\
& 6^*e^{**4}gamma(-p - 1/2)^*gamma(p + 1)) + 2^*0^*p^*d^{**}(2^*p)^*e^{**3}p^*x^{**} \\
& 3^*gamma(-p - 1/2)^*gamma(p + 1)/(6^*e^{**4}p^*gamma(-p - 1/2)^*gamma(p \\
& + 1) + 6^*e^{**4}gamma(-p - 1/2)^*gamma(p + 1)) + 2^*0^*p^*d^{**}(2^*p)^*e^{**} \\
& 3^*x^{**3}^*gamma(-p - 1/2)^*gamma(p + 1)/(6^*e^{**4}p^*gamma(-p - 1/2)^*gam \\
& ma(p + 1) + 6^*e^{**4}gamma(-p - 1/2)^*gamma(p + 1)) - 3^*d^{**3}d^{**}(2^*p \\
& )^*(-1 + e^{**2}x^{**2}/d^{**2})^*p^*exp(I*pi*p)^*gamma(p)^*gamma(-p - 1/2)/( \\
& 6^*e^{**4}p^*gamma(-p - 1/2)^*gamma(p + 1) + 6^*e^{**4}gamma(-p - 1/2)^*ga \\
& mma(p + 1)) - 3^*d^*d^{**}(2^*p)^*e^{**2}p^*x^{**2}^*(-1 + e^{**2}x^{**2}/d^{**2})^*p^*e \\
& xp(I*pi*p)^*gamma(p)^*gamma(-p - 1/2)/(6^*e^{**4}p^*gamma(-p - 1/2)^*gam \\
& ma(p + 1) + 6^*e^{**4}gamma(-p - 1/2)^*gamma(p + 1)) - 3^*e^{**3}e^{**}(2^*p \\
& )^*p^{**2}x^{**3}x^{**}(2^*p)^*exp(I*pi*p)^*gamma(p)^*gamma(-p - 3/2)^*hyper(( \\
& -p + 1, -p - 3/2), (-p - 1/2, ), d^{**2}/(e^{**2}x^{**2}))/ (6^*e^{**4}p^*gamma \\
& (-p - 1/2)^*gamma(p + 1) + 6^*e^{**4}gamma(-p - 1/2)^*gamma(p + 1)) - \\
& 3^*e^{**3}e^{**}(2^*p)^*p^*x^{**3}x^{**}(2^*p)^*exp(I*pi*p)^*gamma(p)^*gamma(-p - 3 \\
& /2)^*hyper((-p + 1, -p - 3/2), (-p - 1/2, ), d^{**2}/(e^{**2}x^{**2}))/ (6^*e \\
& ^**4p^*gamma(-p - 1/2)^*gamma(p + 1) + 6^*e^{**4}gamma(-p - 1/2)^*gamma \\
& (p + 1)), (Abs(e^{**2}x^{**2}/d^{**2}) > 1) & (Abs(d^{**2}/(e^{**2}x^{**2})) > 1) \\
& ), (3^*0^*p^*d^{**3}d^{**}(2^*p)^*p^*log(d^{**2}/(e^{**2}x^{**2}))^*gamma(-p - 1/2)^* \\
& gamma(p + 1)/(6^*e^{**4}p^*gamma(-p - 1/2)^*gamma(p + 1) + 6^*e^{**4}gamm \\
& a(-p - 1/2)^*gamma(p + 1)) - 3^*0^*p^*d^{**3}d^{**}(2^*p)^*p^*log(-d^{**2}/(e^{**} \\
& 2^*x^{**2}) + 1)^*gamma(-p - 1/2)^*gamma(p + 1)/(6^*e^{**4}p^*gamma(-p - 1/ \\
& 2)^*gamma(p + 1) + 6^*e^{**4}gamma(-p - 1/2)^*gamma(p + 1)) - 6^*0^*p^*d \\
& ^**3d^{**}(2^*p)^*p^*atanh(d/(e^*x))^*gamma(-p - 1/2)^*gamma(p + 1)/(6^*e^{**} \\
& 4^*p^*gamma(-p - 1/2)^*gamma(p + 1) + 6^*e^{**4}gamma(-p - 1/2)^*gamma(p \\
& + 1)) + 3^*0^*p^*d^{**3}d^{**}(2^*p)^*log(d^{**2}/(e^{**2}x^{**2}))^*gamma(-p - 1/ \\
& 2)^*gamma(p + 1)/(6^*e^{**4}p^*gamma(-p - 1/2)^*gamma(p + 1) + 6^*e^{**4}g \\
& amma(-p - 1/2)^*gamma(p + 1)) - 3^*0^*p^*d^{**3}d^{**}(2^*p)^*log(-d^{**2}/(e^ \\
& ^*2^*x^{**2}) + 1)^*gamma(-p - 1/2)^*gamma(p + 1)/(6^*e^{**4}p^*gamma(-p - 1 \\
& /2)^*gamma(p + 1) + 6^*e^{**4}gamma(-p - 1/2)^*gamma(p + 1)) - 6^*0^*p^*p \\
& ^*d^{**3}d^{**}(2^*p)^*atanh(d/(e^*x))^*gamma(-p - 1/2)^*gamma(p + 1)/(6^*e^{**4} \\
& ^*p^*gamma(-p - 1/2)^*gamma(p + 1) + 6^*e^{**4}gamma(-p - 1/2)^*gamma(p \\
& + 1)) + 6^*0^*p^*d^{**2}d^{**}(2^*p)^*e^*p^*x^*gamma(-p - 1/2)^*gamma(p + 1)/( \\
& 6^*e^{**4}p^*gamma(-p - 1/2)^*gamma(p + 1) + 6^*e^{**4}gamma(-p - 1/2)^*ga \\
& mma(p + 1)) + 6^*0^*p^*d^{**2}d^{**}(2^*p)^*e^*x^*gamma(-p - 1/2)^*gamma(p + \\
& 1)/(6^*e^{**4}p^*gamma(-p - 1/2)^*gamma(p + 1) + 6^*e^{**4}gamma(-p - 1/2) \\
& )^*gamma(p + 1)) - 3^*0^*p^*d^*d^{**}(2^*p)^*e^{**2}p^*x^{**2}^*gamma(-p - 1/2)^*g \\
& amma(p + 1)/(6^*e^{**4}p^*gamma(-p - 1/2)^*gamma(p + 1) + 6^*e^{**4}gamma \\
& (-p - 1/2)^*gamma(p + 1)) - 3^*0^*p^*d^*d^{**}(2^*p)^*e^{**2}x^{**2}^*gamma(-p - \\
& 1/2)^*gamma(p + 1)/(6^*e^{**4}p^*gamma(-p - 1/2)^*gamma(p + 1) + 6^*e^{**} \\
& 4^*gamma(-p - 1/2)^*gamma(p + 1)) + 2^*0^*p^*d^{**}(2^*p)^*e^{**3}p^*x^{**3}^*gam \\
& ma(-p - 1/2)^*gamma(p + 1)/(6^*e^{**4}p^*gamma(-p - 1/2)^*gamma(p + 1) \\
& + 6^*e^{**4}gamma(-p - 1/2)^*gamma(p + 1)) + 2^*0^*p^*d^{**}(2^*p)^*e^{**3}x^{**} \\
& 3^*gamma(-p - 1/2)^*gamma(p + 1)/(6^*e^{**4}p^*gamma(-p - 1/2)^*gamma(p \\
& + 1) + 6^*e^{**4}gamma(-p - 1/2)^*gamma(p + 1)) - 3^*d^{**3}d^{**}(2^*p)^*(-1 \\
& + e^{**2}x^{**2}/d^{**2})^*p^*exp(I*pi*p)^*gamma(p)^*gamma(-p - 1/2)/(6^*e^{**} \\
& 4^*p^*gamma(-p - 1/2)^*gamma(p + 1) + 6^*e^{**4}gamma(-p - 1/2)^*gamma(p \\
& + 1)) - 3^*d^*d^{**}(2^*p)^*e^{**2}p^*x^{**2}^*(-1 + e^{**2}x^{**2}/d^{**2})^*p^*exp(I* \\
& pi*p)^*gamma(p)^*gamma(-p - 1/2)/(6^*e^{**4}p^*gamma(-p - 1/2)^*gamma(p \\
& + 1) + 6^*e^{**4}gamma(-p - 1/2)^*gamma(p + 1)) - 3^*e^{**3}e^{**}(2^*p)^*p^{**} \\
& 2^*x^{**3}x^{**}(2^*p)^*exp(I*pi*p)^*gamma(p)^*gamma(-p - 3/2)^*hyper((-p + \\
& 1, -p - 3/2), (-p - 1/2, ), d^{**2}/(e^{**2}x^{**2}))/ (6^*e^{**4}p^*gamma(-p - \\
& 1/2)^*gamma(p + 1) + 6^*e^{**4}gamma(-p - 1/2)^*gamma(p + 1)) - 3^*e^{**} \\
& 3^*e^{**}(2^*p)^*p^*x^{**3}x^{**}(2^*p)^*exp(I*pi*p)^*gamma(p)^*gamma(-p - 3/2)^*h \\
& yper((-p + 1, -p - 3/2), (-p - 1/2, ), d^{**2}/(e^{**2}x^{**2}))/ (6^*e^{**4}p \\
& ^*gamma(-p - 1/2)^*gamma(p + 1) + 6^*e^{**4}gamma(-p - 1/2)^*gamma(p +
\end{aligned}$$

$$\begin{aligned}
& 1)), \text{Abs}(e^{**2*x**2/d**2} > 1), (3^{*0**p*d**3*d**(2*p)} * p * \log(d^{**2}/( \\
& e^{**2*x**2})) * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) / (6^{*e**4*p} * \text{gamma}(-p - 1/2) \\
& ) * \text{gamma}(p + 1) + 6^{*e**4} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1)) - 3^{*0**p*d* \\
& *3*d**(2*p)} * p * \log(d^{**2}/(e^{**2*x**2}) - 1) * \text{gamma}(-p - 1/2) * \text{gamma}(p + \\
& 1) / (6^{*e**4*p} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) + 6^{*e**4} * \text{gamma}(-p - 1/ \\
& 2) * \text{gamma}(p + 1)) - 6^{*0**p*d**3*d**(2*p)} * p * \text{acoth}(d/(e*x)) * \text{gamma}(-p \\
& - 1/2) * \text{gamma}(p + 1) / (6^{*e**4*p} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) + 6^{*e \\
& **4} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1)) + 3^{*0**p*d**3*d**(2*p)} * \log(d^{**2} \\
& /(e^{**2*x**2})) * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) / (6^{*e**4*p} * \text{gamma}(-p - 1 \\
& /2) * \text{gamma}(p + 1) + 6^{*e**4} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1)) - 3^{*0**p* \\
& d**3*d**(2*p)} * \log(d^{**2}/(e^{**2*x**2}) - 1) * \text{gamma}(-p - 1/2) * \text{gamma}(p + \\
& 1) / (6^{*e**4*p} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) + 6^{*e**4} * \text{gamma}(-p - 1/ \\
& 2) * \text{gamma}(p + 1)) - 6^{*0**p*d**3*d**(2*p)} * \text{acoth}(d/(e*x)) * \text{gamma}(-p - \\
& 1/2) * \text{gamma}(p + 1) / (6^{*e**4*p} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) + 6^{*e** \\
& 4} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1)) + 6^{*0**p*d**2*d**(2*p)} * e * p * x * \text{gamma} \\
& a(-p - 1/2) * \text{gamma}(p + 1) / (6^{*e**4*p} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) + \\
& 6^{*e**4} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1)) + 6^{*0**p*d**2*d**(2*p)} * e * x * \\
& \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) / (6^{*e**4*p} * \text{gamma}(-p - 1/2) * \text{gamma}(p + \\
& 1) + 6^{*e**4} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1)) - 3^{*0**p*d*d**(2*p)} * e** \\
& 2 * p * x**2 * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) / (6^{*e**4*p} * \text{gamma}(-p - 1/2) * \text{g} \\
& \text{amma}(p + 1) + 6^{*e**4} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1)) - 3^{*0**p*d*d** \\
& (2*p)} * e**2 * x**2 * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) / (6^{*e**4*p} * \text{gamma}(-p - \\
& 1/2) * \text{gamma}(p + 1) + 6^{*e**4} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1)) + 2^{*0** \\
& p*d**(2*p)} * e**3 * p * x**3 * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) / (6^{*e**4*p} * \text{gamma} \\
& a(-p - 1/2) * \text{gamma}(p + 1) + 6^{*e**4} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1)) \\
& + 2^{*0**p*d**(2*p)} * e**3 * x**3 * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) / (6^{*e**4* \\
& p} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) + 6^{*e**4} * \text{gamma}(-p - 1/2) * \text{gamma}(p + \\
& 1)) - 3^{*d**3*d**(2*p)} * (1 - e^{**2*x**2/d**2}) ** p * \text{gamma}(p) * \text{gamma}(-p \\
& - 1/2) / (6^{*e**4*p} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) + 6^{*e**4} * \text{gamma}(-p - \\
& 1/2) * \text{gamma}(p + 1)) - 3^{*d*d**(2*p)} * e**2 * p * x**2 * (1 - e^{**2*x**2/d** \\
& 2}) ** p * \text{gamma}(p) * \text{gamma}(-p - 1/2) / (6^{*e**4*p} * \text{gamma}(-p - 1/2) * \text{gamma}(p \\
& + 1) + 6^{*e**4} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1)) - 3^{*e**3*e**(2*p)} * p** \\
& 2 * x**3 * x** (2*p) * \exp(I*pi*p) * \text{gamma}(p) * \text{gamma}(-p - 3/2) * \text{hyper}((-p + \\
& 1, -p - 3/2), (-p - 1/2, ), d**2/(e**2*x**2)) / (6^{*e**4*p} * \text{gamma}(-p - \\
& 1/2) * \text{gamma}(p + 1) + 6^{*e**4} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1)) - 3^{*e** \\
& 3 * e** (2*p) * p * x**3 * x** (2*p) * \exp(I*pi*p) * \text{gamma}(p) * \text{gamma}(-p - 3/2) * h \\
& yper((-p + 1, -p - 3/2), (-p - 1/2, ), d**2/(e**2*x**2)) / (6^{*e**4*p} \\
& * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) + 6^{*e**4} * \text{gamma}(-p - 1/2) * \text{gamma}(p + \\
& 1)), \text{Abs}(d^{**2}/(e^{**2*x**2}) > 1), (3^{*0**p*d**3*d**(2*p)} * p * \log(d^{**2} \\
& /(e^{**2*x**2})) * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) / (6^{*e**4*p} * \text{gamma}(-p - 1 \\
& /2) * \text{gamma}(p + 1) + 6^{*e**4} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1)) - 3^{*0**p* \\
& d**3*d**(2*p)} * p * \log(-d^{**2}/(e^{**2*x**2}) + 1) * \text{gamma}(-p - 1/2) * \text{gamma}( \\
& p + 1) / (6^{*e**4*p} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) + 6^{*e**4} * \text{gamma}(-p - \\
& 1/2) * \text{gamma}(p + 1)) - 6^{*0**p*d**3*d**(2*p)} * p * \text{atanh}(d/(e*x)) * \text{gamma} \\
& (-p - 1/2) * \text{gamma}(p + 1) / (6^{*e**4*p} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) + \\
& 6^{*e**4} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1)) + 3^{*0**p*d**3*d**(2*p)} * \log(d \\
& **2/(e^{**2*x**2})) * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) / (6^{*e**4*p} * \text{gamma}(-p \\
& - 1/2) * \text{gamma}(p + 1) + 6^{*e**4} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1)) - 3^{*0* \\
& *p*d**3*d**(2*p)} * \log(-d^{**2}/(e^{**2*x**2}) + 1) * \text{gamma}(-p - 1/2) * \text{gamma} \\
& (p + 1) / (6^{*e**4*p} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) + 6^{*e**4} * \text{gamma}(-p \\
& - 1/2) * \text{gamma}(p + 1)) - 6^{*0**p*d**3*d**(2*p)} * \text{atanh}(d/(e*x)) * \text{gamma}( \\
& -p - 1/2) * \text{gamma}(p + 1) / (6^{*e**4*p} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) + 6 \\
& *e**4} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1)) + 6^{*0**p*d**2*d**(2*p)} * e * p * x * \\
& \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) / (6^{*e**4*p} * \text{gamma}(-p - 1/2) * \text{gamma}(p + \\
& 1) + 6^{*e**4} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1)) + 6^{*0**p*d**2*d**(2*p)} * \\
& e * x * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) / (6^{*e**4*p} * \text{gamma}(-p - 1/2) * \text{gamma}( \\
& p + 1) + 6^{*e**4} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1)) - 3^{*0**p*d*d**(2*p)} \\
& * e**2 * p * x**2 * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) / (6^{*e**4*p} * \text{gamma}(-p - 1/ \\
& 2) * \text{gamma}(p + 1) + 6^{*e**4} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1)) - 3^{*0**p*d \\
& *d**(2*p)} * e**2 * x**2 * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) / (6^{*e**4*p} * \text{gamma}( \\
& -p - 1/2) * \text{gamma}(p + 1) + 6^{*e**4} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1)) + 2 \\
& *0**p*d**(2*p)} * e**3 * p * x**3 * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) / (6^{*e**4*p} \\
& * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) + 6^{*e**4} * \text{gamma}(-p - 1/2) * \text{gamma}(p + \\
& 1)) + 2^{*0**p*d**(2*p)} * e**3 * x**3 * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) / (6^{*e \\
& **4*p} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) + 6^{*e**4} * \text{gamma}(-p - 1/2) * \text{gamma} \\
& (p + 1)) - 3^{*d**3*d**(2*p)} * (1 - e^{**2*x**2/d**2}) ** p * \text{gamma}(p) * \text{gamma} \\
& (-p - 1/2) / (6^{*e**4*p} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1) + 6^{*e**4} * \text{gamma}( \\
& -p - 1/2) * \text{gamma}(p + 1)) - 3^{*d*d**(2*p)} * e**2 * p * x**2 * (1 - e^{**2*x**2 \\
& /d**2}) ** p * \text{gamma}(p) * \text{gamma}(-p - 1/2) / (6^{*e**4*p} * \text{gamma}(-p - 1/2) * \text{gamma} \\
& a(p + 1) + 6^{*e**4} * \text{gamma}(-p - 1/2) * \text{gamma}(p + 1)) - 3^{*e**3*e**(2*p)} \\
& * p**2 * x**3 * x** (2*p) * \exp(I*pi*p) * \text{gamma}(p) * \text{gamma}(-p - 3/2) * \text{hyper}((- \\
& p + 1, -p - 3/2), (-p - 1/2, ), d**2/(e**2*x**2)) / (6^{*e**4*p} * \text{gamma}(
\end{aligned}$$

```
-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(p + 1)) - 3
*e**3*e**(2*p)*p*x**3*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p - 3/
2)*hyper((-p + 1, -p - 3/2), (-p - 1/2, ), d**2/(e**2*x**2))/(6*e*
*4*p*gamma(-p - 1/2)*gamma(p + 1) + 6*e**4*gamma(-p - 1/2)*gamma(
p + 1)), True))
```

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x^3}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x^3/(e\*x + d),x, algorithm="giac")

[Out] integrate((-e^2\*x^2 + d^2)^p\*x^3/(e\*x + d), x)

$$3.269 \quad \int \frac{x^2 (d^2 - e^2 x^2)^p}{d + ex} dx$$

**Optimal.** Leaf size=119

$$\frac{x^3 (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{3}{2}, 1 - p; \frac{5}{2}; \frac{e^2 x^2}{d^2}\right)}{3d} + \frac{d^2 (d^2 - e^2 x^2)^p}{2e^3 p} - \frac{(d^2 - e^2 x^2)^{p+1}}{2e^3 (p+1)}$$

[Out]  $(d^2 (d^2 - e^2 x^2)^p) / (2 e^3 p) - (d^2 - e^2 x^2)^{(1+p)} / (2 e^3 (1+p)) + (x^3 (d^2 - e^2 x^2)^p \text{Hypergeometric2F1}[3/2, 1-p, 5/2, (e^2 x^2)/d^2]) / (3 d (1 - (e^2 x^2)/d^2)^p)$

**Rubi [A]** time = 0.235812, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{x^3 (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{3}{2}, 1 - p; \frac{5}{2}; \frac{e^2 x^2}{d^2}\right)}{3d} + \frac{d^2 (d^2 - e^2 x^2)^p}{2e^3 p} - \frac{(d^2 - e^2 x^2)^{p+1}}{2e^3 (p+1)}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(d^2 - e^2\*x^2)^p)/(d + e\*x), x]

[Out]  $(d^2 (d^2 - e^2 x^2)^p) / (2 e^3 p) - (d^2 - e^2 x^2)^{(1+p)} / (2 e^3 (1+p)) + (x^3 (d^2 - e^2 x^2)^p \text{Hypergeometric2F1}[3/2, 1-p, 5/2, (e^2 x^2)/d^2]) / (3 d (1 - (e^2 x^2)/d^2)^p)$

**Rubi in Sympy [A]** time = 44.7078, size = 92, normalized size = 0.77

$$\frac{d^2 (d^2 - e^2 x^2)^p}{2e^3 p} - \frac{(d^2 - e^2 x^2)^{p+1}}{2e^3 (p+1)} + \frac{x^3 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+1, \frac{3}{2}; \frac{5}{2}; \frac{e^2 x^2}{d^2}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d), x)

[Out]  $d^2 (d^2 - e^2 x^2)^p / (2 e^3 p) - (d^2 - e^2 x^2)^{(p+1)} / (2 e^3 (p+1)) + x^3 (1 - e^2 x^2/d^2)^{-p} (d^2 - e^2 x^2)^p \text{hyper}((-p+1, 3/2), (5/2, ), e^2 x^2/d^2) / (3 d)$

**Mathematica [A]** time = 0.427147, size = 198, normalized size = 1.66

$$\frac{\left(\frac{ex}{d} + 1\right)^{-p} (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} \left(2de(p+1)x \left(\frac{ex}{d} + 1\right)^p {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right) + d(d-ex) \left(2 - \frac{2e^2 x^2}{d^2}\right)^p {}_2F_1\left(1-p, p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)\right)}{2e^3 (p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(d^2 - e^2\*x^2)^p)/(d + e\*x), x]

[Out]  $-((d^2 - e^2 x^2)^p ((1 + (e*x)/d)^p (- (e^2 x^2 (1 - (e^2 x^2)/d^2)) / d^2)^p) + d^2 (-1 + (1 - (e^2 x^2)/d^2)^p) + 2*d*e*(1+p)*x*(1 + (e*x)/d)^p \text{Hypergeometric2F1}[1/2, -p, 3/2, (e^2 x^2)/d^2] + d*(d$

$-e^x)^*(2 - (2*e^2*x^2)/d^2)^p*$ Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/(2\*e^3\*(1 + p)\*(1 + (e\*x)/d)^p\*(1 - (e^2\*x^2)/d^2)^p)

**Maple [F]** time = 0.065, size = 0, normalized size = 0.

$$\int \frac{x^2 (-e^2 x^2 + d^2)^p}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(-e^2\*x^2+d^2)^p/(e\*x+d), x)

[Out] int(x^2\*(-e^2\*x^2+d^2)^p/(e\*x+d), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2 x^2 + d^2)^p x^2}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x^2/(e\*x + d), x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p\*x^2/(e\*x + d), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2 x^2 + d^2)^p x^2}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x^2/(e\*x + d), x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p\*x^2/(e\*x + d), x)

**Sympy [A]** time = 23.2794, size = 4124, normalized size = 34.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d), x)

[Out] Piecewise((-0\*\*p\*d\*\*2\*d\*\*(2\*p)\*p\*log(d\*\*2/(e\*\*2\*x\*\*2))\*gamma(-p + 1/2)\*gamma(p + 1)/(2\*e\*\*3\*p\*gamma(-p + 1/2)\*gamma(p + 1) + 2\*e\*\*3\*gamma(-p + 1/2)\*gamma(p + 1)) + 0\*\*p\*d\*\*2\*d\*\*(2\*p)\*p\*log(d\*\*2/(e\*\*2\*x\*\*2) - 1)\*gamma(-p + 1/2)\*gamma(p + 1)/(2\*e\*\*3\*p\*gamma(-p + 1/2)\*gamma(p + 1) + 2\*e\*\*3\*gamma(-p + 1/2)\*gamma(p + 1)) + 2\*0\*\*p\*d\*\*2\*d\*\*(2\*p)\*p\*acoth(d/(e\*x))\*gamma(-p + 1/2)\*gamma(p + 1)/(2\*e\*\*3\*p\*gamma(-p + 1/2)\*gamma(p + 1) + 2\*e\*\*3\*gamma(-p + 1/2)\*gamma(p + 1))





```

gamma(p + 1)) + 2*0**p*d**2*d**(2*p)*atanh(d/(e*x))*gamma(-p + 1/
2)*gamma(p + 1)/(2*e**3*p*gamma(-p + 1/2)*gamma(p + 1) + 2*e**3*g
amma(-p + 1/2)*gamma(p + 1)) - 2*0**p*d*d**(2*p)*e*p*x*gamma(-p +
1/2)*gamma(p + 1)/(2*e**3*p*gamma(-p + 1/2)*gamma(p + 1) + 2*e**
3*gamma(-p + 1/2)*gamma(p + 1)) - 2*0**p*d*d**(2*p)*e*x*gamma(-p
+ 1/2)*gamma(p + 1)/(2*e**3*p*gamma(-p + 1/2)*gamma(p + 1) + 2*e*
**3*gamma(-p + 1/2)*gamma(p + 1)) + 0**p*d**(2*p)*e**2*p*x**2*gamm
a(-p + 1/2)*gamma(p + 1)/(2*e**3*p*gamma(-p + 1/2)*gamma(p + 1) +
2*e**3*gamma(-p + 1/2)*gamma(p + 1)) + 0**p*d**(2*p)*e**2*x**2*g
amma(-p + 1/2)*gamma(p + 1)/(2*e**3*p*gamma(-p + 1/2)*gamma(p + 1
) + 2*e**3*gamma(-p + 1/2)*gamma(p + 1)) + d**2*d**(2*p)*(-1 + e*
**2*x**2/d**2)**p*exp(I*pi*p)*gamma(p)*gamma(-p + 1/2)/(2*e**3*p*g
amma(-p + 1/2)*gamma(p + 1) + 2*e**3*gamma(-p + 1/2)*gamma(p + 1
) + d*e*e**(2*p)*p**2*x*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p -
1/2)*hyper((-p + 1, -p - 1/2), (-p + 1/2, ), d**2/(e**2*x**2))/(2*
e**3*p*gamma(-p + 1/2)*gamma(p + 1) + 2*e**3*gamma(-p + 1/2)*gamm
a(p + 1)) + d*e*e**(2*p)*p*x*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(
-p - 1/2)*hyper((-p + 1, -p - 1/2), (-p + 1/2, ), d**2/(e**2*x**2
))/(2*e**3*p*gamma(-p + 1/2)*gamma(p + 1) + 2*e**3*gamma(-p + 1/2)
*gamma(p + 1)) + d**(2*p)*e**2*p*x**2*(-1 + e**2*x**2/d**2)**p*ex
p(I*pi*p)*gamma(p)*gamma(-p + 1/2)/(2*e**3*p*gamma(-p + 1/2)*gamm
a(p + 1) + 2*e**3*gamma(-p + 1/2)*gamma(p + 1)), Abs(e**2*x**2/d*
**2) > 1), (-0**p*d**2*d**(2*p)*p*log(d**2/(e**2*x**2))*gamma(-p +
1/2)*gamma(p + 1)/(2*e**3*p*gamma(-p + 1/2)*gamma(p + 1) + 2*e**
3*gamma(-p + 1/2)*gamma(p + 1)) + 0**p*d**2*d**(2*p)*p*log(-d**2/
(e**2*x**2) + 1)*gamma(-p + 1/2)*gamma(p + 1)/(2*e**3*p*gamma(-p
+ 1/2)*gamma(p + 1) + 2*e**3*gamma(-p + 1/2)*gamma(p + 1)) + 2*0*
*p*d**2*d**(2*p)*p*atanh(d/(e*x))*gamma(-p + 1/2)*gamma(p + 1)/(2
*e**3*p*gamma(-p + 1/2)*gamma(p + 1) + 2*e**3*gamma(-p + 1/2)*gam
ma(p + 1)) - 0**p*d**2*d**(2*p)*log(d**2/(e**2*x**2))*gamma(-p +
1/2)*gamma(p + 1)/(2*e**3*p*gamma(-p + 1/2)*gamma(p + 1) + 2*e**3
*gamma(-p + 1/2)*gamma(p + 1)) + 0**p*d**2*d**(2*p)*log(-d**2/(e*
**2*x**2) + 1)*gamma(-p + 1/2)*gamma(p + 1)/(2*e**3*p*gamma(-p + 1
/2)*gamma(p + 1) + 2*e**3*gamma(-p + 1/2)*gamma(p + 1)) + 2*0**p*
d**2*d**(2*p)*atanh(d/(e*x))*gamma(-p + 1/2)*gamma(p + 1)/(2*e**3
*p*gamma(-p + 1/2)*gamma(p + 1) + 2*e**3*gamma(-p + 1/2)*gamma(p
+ 1)) - 2*0**p*d*d**(2*p)*e*p*x*gamma(-p + 1/2)*gamma(p + 1)/(2*e
**3*p*gamma(-p + 1/2)*gamma(p + 1) + 2*e**3*gamma(-p + 1/2)*gamma
(p + 1)) - 2*0**p*d*d**(2*p)*e*x*gamma(-p + 1/2)*gamma(p + 1)/(2*
e**3*p*gamma(-p + 1/2)*gamma(p + 1) + 2*e**3*gamma(-p + 1/2)*gamm
a(p + 1)) + 0**p*d**(2*p)*e**2*p*x**2*gamma(-p + 1/2)*gamma(p + 1
)/(2*e**3*p*gamma(-p + 1/2)*gamma(p + 1) + 2*e**3*gamma(-p + 1/2)
*gamma(p + 1)) + 0**p*d**(2*p)*e**2*x**2*gamma(-p + 1/2)*gamma(p
+ 1)/(2*e**3*p*gamma(-p + 1/2)*gamma(p + 1) + 2*e**3*gamma(-p + 1
/2)*gamma(p + 1)) + d**2*d**(2*p)*(1 - e**2*x**2/d**2)**p*gamma(p
)*gamma(-p + 1/2)/(2*e**3*p*gamma(-p + 1/2)*gamma(p + 1) + 2*e**3
*gamma(-p + 1/2)*gamma(p + 1)) + d*e*e**(2*p)*p**2*x*x**(2*p)*exp
(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((-p + 1, -p - 1/2), (-p +
1/2, ), d**2/(e**2*x**2))/(2*e**3*p*gamma(-p + 1/2)*gamma(p + 1)
+ 2*e**3*gamma(-p + 1/2)*gamma(p + 1)) + d*e*e**(2*p)*p*x*x**(2*p
)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((-p + 1, -p - 1/2),
(-p + 1/2, ), d**2/(e**2*x**2))/(2*e**3*p*gamma(-p + 1/2)*gamma(p
+ 1) + 2*e**3*gamma(-p + 1/2)*gamma(p + 1)) + d**(2*p)*e**2*p*x**
2*(1 - e**2*x**2/d**2)**p*gamma(p)*gamma(-p + 1/2)/(2*e**3*p*gamm
a(-p + 1/2)*gamma(p + 1) + 2*e**3*gamma(-p + 1/2)*gamma(p + 1)),
True))

```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x^2}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x^2/(e\*x + d),x, algorithm="giac")

[Out] integrate((-e^2\*x^2 + d^2)^p\*x^2/(e\*x + d), x)

$$3.270 \quad \int \frac{x(d^2 - e^2x^2)^p}{d + ex} dx$$

**Optimal.** Leaf size=90

$$\frac{ex^3 (d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{3}{2}, 1 - p; \frac{5}{2}; \frac{e^2x^2}{d^2}\right)}{3d^2} - \frac{d (d^2 - e^2x^2)^p}{2e^2p}$$

[Out]  $-(d*(d^2 - e^2*x^2)^p)/(2*e^2*p) - (e*x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, 1 - p, 5/2, (e^2*x^2)/d^2])/(3*d^2*(1 - (e^2*x^2)/d^2)^p)$

**Rubi [A]** time = 0.166434, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$

$$\frac{ex^3 (d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{3}{2}, 1 - p; \frac{5}{2}; \frac{e^2x^2}{d^2}\right)}{3d^2} - \frac{d (d^2 - e^2x^2)^p}{2e^2p}$$

Antiderivative was successfully verified.

[In] Int[(x\*(d^2 - e^2\*x^2)^p)/(d + e\*x), x]

[Out]  $-(d*(d^2 - e^2*x^2)^p)/(2*e^2*p) - (e*x^3*(d^2 - e^2*x^2)^p*Hypergeometric2F1[3/2, 1 - p, 5/2, (e^2*x^2)/d^2])/(3*d^2*(1 - (e^2*x^2)/d^2)^p)$

**Rubi in Sympy [A]** time = 29.389, size = 73, normalized size = 0.81

$$\frac{d (d^2 - e^2x^2)^p}{2e^2p} - \frac{ex^3 \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(-p + 1, \frac{3}{2}; \frac{5}{2}; \frac{e^2x^2}{d^2}\right)}{3d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d), x)

[Out]  $-d*(d**2 - e**2*x**2)**p/(2*e**2*p) - e*x**3*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper((-p + 1, 3/2), (5/2, ), e**2*x**2/d**2)/(3*d**2)$

**Mathematica [A]** time = 0.144862, size = 147, normalized size = 1.63

$$\frac{2^{p-1} \left(\frac{ex}{d} + 1\right)^{-p} (d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} \left(2e(p+1)x \left(\frac{ex}{2d} + \frac{1}{2}\right)^p {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2x^2}{d^2}\right) + (d - ex) \left(1 - \frac{e^2x^2}{d^2}\right)^p {}_2F_1\left(1 - p, p; 2, 1 - p; \frac{e^2x^2}{d^2}\right)\right)}{e^2(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(d^2 - e^2\*x^2)^p)/(d + e\*x), x]

[Out]  $(2^{(-1 + p)}*(d^2 - e^2*x^2)^p*(2*e*(1 + p)*x*(1/2 + (e*x)/(2*d)))^p*Hypergeometric2F1[1/2, -p, 3/2, (e^2*x^2)/d^2] + (d - e*x)*(1 - (e^2*x^2)/d^2)^p*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e*x)$

)/(2\*d])))/(e^2\*(1 + p)\*(1 + (e\*x)/d)^p\*(1 - (e^2\*x^2)/d^2)^p)

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int \frac{x(-e^2x^2 + d^2)^p}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(-e^2\*x^2+d^2)^p/(e\*x+d), x)

[Out] int(x\*(-e^2\*x^2+d^2)^p/(e\*x+d), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x/(e\*x + d), x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p\*x/(e\*x + d), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p x}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x/(e\*x + d), x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p\*x/(e\*x + d), x)

**Sympy [A]** time = 15.0963, size = 427, normalized size = 4.74

$$\left\{ \begin{array}{l} \frac{0^p d d^{2p} \log\left(\frac{d^2}{e^2 x^2}\right)}{2e^2} - \frac{0^p d d^{2p} \log\left(\frac{d^2}{e^2 x^2} - 1\right)}{2e^2} - \frac{0^p d d^{2p} \operatorname{acoth}\left(\frac{d}{ex}\right)}{e^2} + \frac{0^p d^{2p} x}{e} - \frac{e^{2p} p x x^{2p} e^{i\pi p} (p) (-p - \frac{1}{2}) {}_2F_1\left(-p + 1, -p - \frac{1}{2} \middle| \frac{d^2}{e^2 x^2}\right)}{2e(-p + \frac{1}{2})(p+1)} - \frac{d^{2p} x^2 (p)}{e^2} \\ \frac{0^p d d^{2p} \log\left(\frac{d^2}{e^2 x^2}\right)}{2e^2} - \frac{0^p d d^{2p} \log\left(-\frac{d^2}{e^2 x^2} + 1\right)}{2e^2} - \frac{0^p d d^{2p} \operatorname{atanh}\left(\frac{d}{ex}\right)}{e^2} + \frac{0^p d^{2p} x}{e} - \frac{e^{2p} p x x^{2p} e^{i\pi p} (p) (-p - \frac{1}{2}) {}_2F_1\left(-p + 1, -p - \frac{1}{2} \middle| \frac{d^2}{e^2 x^2}\right)}{2e(-p + \frac{1}{2})(p+1)} - \frac{d^{2p} x^2 (p)}{e^2} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d), x)

[Out] Piecewise((0\*\*p\*d\*d\*\*(2\*p)\*log(d\*\*2/(e\*\*2\*x\*\*2))/(2\*e\*\*2) - 0\*\*p\*d\*d\*\*(2\*p)\*log(d\*\*2/(e\*\*2\*x\*\*2) - 1)/(2\*e\*\*2) - 0\*\*p\*d\*d\*\*(2\*p)\*a

```

coth(d/(e*x))/e**2 + 0**p*d**(2*p)*x/e - e**(2*p)*p*x*x**(2*p)*ex
p(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((-p + 1, -p - 1/2), (-p
+ 1/2, ), d**2/(e**2*x**2))/(2*e*gamma(-p + 1/2)*gamma(p + 1)) - d
**(2*p)*x**2*gamma(p)*gamma(-p + 1)*hyper((2, 1, -p + 1), (2, 2),
e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*d*gamma(-p)*gamma(p + 1)),
Abs(d**2/(e**2*x**2)) > 1), (0**p*d*d**(2*p)*log(d**2/(e**2*x**2)
)/(2*e**2) - 0**p*d*d**(2*p)*log(-d**2/(e**2*x**2) + 1)/(2*e**2)
- 0**p*d*d**(2*p)*atanh(d/(e*x))/e**2 + 0**p*d**(2*p)*x/e - e**(2
*p)*p*x*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p - 1/2)*hyper((-p +
1, -p - 1/2), (-p + 1/2, ), d**2/(e**2*x**2))/(2*e*gamma(-p + 1/2
)*gamma(p + 1)) - d**(2*p)*x**2*gamma(p)*gamma(-p + 1)*hyper((2,
1, -p + 1), (2, 2), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*d*gamma(
-p)*gamma(p + 1)), True))

```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x/(e\*x + d),x, algorithm="giac")

[Out] integrate((-e^2\*x^2 + d^2)^p\*x/(e\*x + d), x)

$$3.271 \quad \int \frac{(d^2 - e^2 x^2)^p}{d + ex} dx$$

**Optimal.** Leaf size=73

$$\frac{2^{p-1} \left(\frac{ex}{d} + 1\right)^{-p-1} (d^2 - e^2 x^2)^{p+1} {}_2F_1\left(1 - p, p + 1; p + 2; \frac{d - ex}{2d}\right)}{d^2 e(p + 1)}$$

[Out] -((2^(-1 + p) \* (1 + (e\*x)/d)^(-1 - p) \* (d^2 - e^2\*x^2)^(1 + p) \* Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)]) / (d^2 \* e \* (1 + p)))

**Rubi [A]** time = 0.0808427, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{2^{p-1} \left(\frac{ex}{d} + 1\right)^{-p-1} (d^2 - e^2 x^2)^{p+1} {}_2F_1\left(1 - p, p + 1; p + 2; \frac{d - ex}{2d}\right)}{d^2 e(p + 1)}$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^p/(d + e\*x), x]

[Out] -((2^(-1 + p) \* (1 + (e\*x)/d)^(-1 - p) \* (d^2 - e^2\*x^2)^(1 + p) \* Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)]) / (d^2 \* e \* (1 + p)))

**Rubi in Sympy [A]** time = 24.4089, size = 41, normalized size = 0.56

$$\frac{\left(\frac{\frac{d}{2} - \frac{ex}{2}}{d}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p, p \left| \frac{\frac{d}{2} + \frac{ex}{2}}{d} \right. \right)}{ep}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d), x)

[Out] ((d/2 - e\*x/2)/d)\*\*(-p) \* (d\*\*2 - e\*\*2\*x\*\*2)\*\*p \* hyper((-p, p), (p + 1, ), (d/2 + e\*x/2)/d)/(e\*p)

**Mathematica [A]** time = 0.0460123, size = 75, normalized size = 1.03

$$\frac{2^{p-1} (d - ex) \left(\frac{ex}{d} + 1\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(1 - p, p + 1; p + 2; \frac{d - ex}{2d}\right)}{de(p + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^p/(d + e\*x), x]

[Out] -((2^(-1 + p) \* (d - e\*x) \* (d^2 - e^2\*x^2)^p \* Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)]) / (d \* e \* (1 + p) \* (1 + (e\*x)/d)^p))

**Maple [F]** time = 0.063, size = 0, normalized size = 0.

$$\int \frac{(-e^2 x^2 + d^2)^p}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^p/(e\*x+d), x)

[Out] int((-e^2\*x^2+d^2)^p/(e\*x+d), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2 x^2 + d^2)^p}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/(e\*x + d), x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p/(e\*x + d), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2 x^2 + d^2)^p}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/(e\*x + d), x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p/(e\*x + d), x)

**Sympy [A]** time = 13.389, size = 321, normalized size = 4.4

$$\left\{ \begin{array}{l} \frac{0^p \log\left(-1 + \frac{e^2 x^2}{d^2}\right)}{2e} + \frac{0^p \operatorname{acoth}\left(\frac{ex}{d}\right)}{e} + \frac{d e^{2p} p x^{2p} e^{i\pi p} (p) (-p + \frac{1}{2}) {}_2F_1\left(-p + 1, -p + \frac{1}{2} \middle| \frac{d^2}{e^2 x^2}\right)}{2e^2 x (-p + \frac{3}{2}) (p+1)} + \frac{d^{2p} e x^2 (p) (-p+1) {}_3F_2\left(2, 1, -p + 1 \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{2d^2 (-p) (p+1)} \\ \frac{0^p \log\left(1 - \frac{e^2 x^2}{d^2}\right)}{2e} + \frac{0^p \operatorname{atanh}\left(\frac{ex}{d}\right)}{e} + \frac{d e^{2p} p x^{2p} e^{i\pi p} (p) (-p + \frac{1}{2}) {}_2F_1\left(-p + 1, -p + \frac{1}{2} \middle| \frac{d^2}{e^2 x^2}\right)}{2e^2 x (-p + \frac{3}{2}) (p+1)} + \frac{d^{2p} e x^2 (p) (-p+1) {}_3F_2\left(2, 1, -p + 1 \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{2d^2 (-p) (p+1)} \end{array} \right. \quad \begin{array}{l} \text{for} \\ \text{oth}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d), x)

[Out] Piecewise((0\*\*p\*log(-1 + e\*\*2\*x\*\*2/d\*\*2)/(2\*e) + 0\*\*p\*acoth(e\*x/d)/e + d\*e\*\*(2\*p)\*p\*x\*\*(2\*p)\*exp(I\*pi\*p)\*gamma(p)\*gamma(-p + 1/2)\*hyper((-p + 1, -p + 1/2), (-p + 3/2, ), d\*\*2/(e\*\*2\*x\*\*2))/(2\*e\*\*2\*x\*gamma(-p + 3/2)\*gamma(p + 1)) + d\*\*(2\*p)\*e\*x\*\*2\*gamma(p)\*gamma(-p + 1)\*hyper((2, 1, -p + 1), (2, 2), e\*\*2\*x\*\*2\*exp\_polar(2\*I\*pi)/d\*\*2)/(2\*d\*\*2\*gamma(-p)\*gamma(p + 1)), Abs(e\*\*2\*x\*\*2/d\*\*2) > 1), (0\*\*p\*log(1 - e\*\*2\*x\*\*2/d\*\*2)/(2\*e) + 0\*\*p\*atanh(e\*x/d)/e + d\*e

```

*(2*p)*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p + 1/2)*hyper((-p
+ 1, -p + 1/2), (-p + 3/2, ), d**2/(e**2*x**2))/(2*e**2*x*gamma(-p
+ 3/2)*gamma(p + 1)) + d**(2*p)*e*x**2*gamma(p)*gamma(-p + 1)*hy
per((2, 1, -p + 1), (2, 2), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*
d**2*gamma(-p)*gamma(p + 1)), True))

```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/(e\*x + d),x, algorithm="giac")

[Out] integrate((-e^2\*x^2 + d^2)^p/(e\*x + d), x)



$$3.272 \quad \int \frac{(d^2 - e^2 x^2)^p}{x(d + ex)} dx$$

**Optimal.** Leaf size=104

$$\frac{ex (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2}, 1 - p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^2} - \frac{(d^2 - e^2 x^2)^p {}_2F_1\left(1, p; p + 1; 1 - \frac{e^2 x^2}{d^2}\right)}{2dp}$$

[Out] -((e\*x\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[1/2, 1 - p, 3/2, (e^2\*x^2)/d^2])/(d^2\*(1 - (e^2\*x^2)/d^2)^p)) - ((d^2 - e^2\*x^2)^p\*Hypergeometric2F1[1, p, 1 + p, 1 - (e^2\*x^2)/d^2])/(2\*d\*p)

**Rubi [A]** time = 0.1861, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{ex (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{1}{2}, 1 - p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^2} - \frac{(d^2 - e^2 x^2)^p {}_2F_1\left(1, p; p + 1; 1 - \frac{e^2 x^2}{d^2}\right)}{2dp}$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^p/(x\*(d + e\*x)),x]

[Out] -((e\*x\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[1/2, 1 - p, 3/2, (e^2\*x^2)/d^2])/(d^2\*(1 - (e^2\*x^2)/d^2)^p)) - ((d^2 - e^2\*x^2)^p\*Hypergeometric2F1[1, p, 1 + p, 1 - (e^2\*x^2)/d^2])/(2\*d\*p)

**Rubi in Sympy [A]** time = 41.5426, size = 82, normalized size = 0.79

$$\frac{(d^2 - e^2 x^2)^p {}_2F_1\left(1, p; p + 1; 1 - \frac{e^2 x^2}{d^2}\right)}{2dp} - \frac{ex \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p + 1, \frac{1}{2}; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x/(e\*x+d),x)

[Out] -(d\*\*2 - e\*\*2\*x\*\*2)\*\*p\*hyper((1, p), (p + 1, ), 1 - e\*\*2\*x\*\*2/d\*\*2)/(2\*d\*p) - e\*x\*(1 - e\*\*2\*x\*\*2/d\*\*2)\*\*(-p)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p\*hyper((-p + 1, 1/2), (3/2, ), e\*\*2\*x\*\*2/d\*\*2)/d\*\*2

**Mathematica [A]** time = 0.145778, size = 151, normalized size = 1.45

$$\frac{2^{p-1} \left(1 - \frac{d^2}{e^2 x^2}\right)^{-p} \left(\frac{ex}{d} + 1\right)^{-p} (d^2 - e^2 x^2)^p \left(p(d - ex) \left(1 - \frac{d^2}{e^2 x^2}\right)^p {}_2F_1\left(1 - p, p + 1; p + 2; \frac{d - ex}{2d}\right) + d(p + 1) \left(\frac{ex}{2d} + \frac{1}{2}\right)^p {}_2F_1\left(1 - p, p + 1; p + 2; \frac{d - ex}{2d}\right)\right)}{d^2 p (p + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^p/(x\*(d + e\*x)),x]

[Out] (2^(-1 + p)\*(d^2 - e^2\*x^2)^p\*(p\*(1 - d^2/(e^2\*x^2))^p\*(d - e\*x)\*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)] + d\*(1 + p)\*(1/2 + (e\*x)/(2\*d))^p\*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2\*x^2)]))

$$2^*x^2]))/(d^2*p*(1+p)*(1-d^2/(e^2*x^2))^p*(1+(e*x)/d)^p)$$

**Maple [F]** time = 0.065, size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{x(ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^p/x/(e\*x+d), x)

[Out] int((-e^2\*x^2+d^2)^p/x/(e\*x+d), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)\*x), x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)\*x), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p}{ex^2 + dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)\*x), x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p/(e\*x^2 + d\*x), x)

**Sympy [A]** time = 14.8412, size = 355, normalized size = 3.41

$$\left\{ \begin{array}{l} -\frac{0^p d^{2p} \log\left(\frac{d^2}{e^2 x^2} - 1\right)}{2d} - \frac{0^p d^{2p} \operatorname{acoth}\left(\frac{d}{ex}\right)}{d} + \frac{d e^{2p} p x^{2p} e^{i\pi p} (p) (-p+1) {}_2F_1\left(\begin{matrix} -p+1, -p+1 \\ -p+2 \end{matrix} \middle| \frac{d^2}{e^2 x^2}\right)}{2e^2 x^2 (-p+2)(p+1)} - \frac{e^{2p} p x^{2p} e^{i\pi p} (p) (-p+\frac{1}{2}) {}_2F_1\left(\begin{matrix} -p+1, -p+\frac{1}{2} \\ -p+\frac{3}{2} \end{matrix} \middle| \frac{d^2}{e^2 x^2}\right)}{2ex(-p+\frac{3}{2})(p+1)} \\ -\frac{0^p d^{2p} \log\left(-\frac{d^2}{e^2 x^2} + 1\right)}{2d} - \frac{0^p d^{2p} \operatorname{atanh}\left(\frac{d}{ex}\right)}{d} + \frac{d e^{2p} p x^{2p} e^{i\pi p} (p) (-p+1) {}_2F_1\left(\begin{matrix} -p+1, -p+1 \\ -p+2 \end{matrix} \middle| \frac{d^2}{e^2 x^2}\right)}{2e^2 x^2 (-p+2)(p+1)} - \frac{e^{2p} p x^{2p} e^{i\pi p} (p) (-p+\frac{1}{2}) {}_2F_1\left(\begin{matrix} -p+1, -p+\frac{1}{2} \\ -p+\frac{3}{2} \end{matrix} \middle| \frac{d^2}{e^2 x^2}\right)}{2ex(-p+\frac{3}{2})(p+1)} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x/(e\*x+d), x)

[Out] Piecewise((-0\*\*p\*d\*\*(2\*p)\*log(d\*\*2/(e\*\*2\*x\*\*2) - 1)/(2\*d) - 0\*\*p\*d\*\*(2\*p)\*acoth(d/(e\*x))/d + d\*e\*\*(2\*p)\*p\*x\*\*(2\*p)\*exp(I\*pi\*p)\*gam

```

ma(p)*gamma(-p + 1)*hyper((-p + 1, -p + 1), (-p + 2, ), d**2/(e**2
*x**2))/(2*e**2*x**2*gamma(-p + 2)*gamma(p + 1)) - e**(2*p)*p*x**
(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p + 1/2)*hyper((-p + 1, -p + 1/
2), (-p + 3/2, ), d**2/(e**2*x**2))/(2*e*x*gamma(-p + 3/2)*gamma(p
+ 1)), Abs(d**2/(e**2*x**2)) > 1), (-0**p*d**(2*p)*log(-d**2/(e*
*2*x**2) + 1)/(2*d) - 0**p*d**(2*p)*atanh(d/(e*x))/d + d*e**(2*p)
*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p + 1)*hyper((-p + 1, -p
+ 1), (-p + 2, ), d**2/(e**2*x**2))/(2*e**2*x**2*gamma(-p + 2)*gam
ma(p + 1)) - e**(2*p)*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p +
1/2)*hyper((-p + 1, -p + 1/2), (-p + 3/2, ), d**2/(e**2*x**2))/(2*
e*x*gamma(-p + 3/2)*gamma(p + 1)), True))

```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)\*x), x, algorithm="giac")

[Out] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)\*x), x)

$$3.273 \quad \int \frac{(d^2 - e^2 x^2)^p}{x^2(d+ex)} dx$$

**Optimal.** Leaf size=106

$$\frac{e (d^2 - e^2 x^2)^p {}_2F_1\left(1, p; p+1; 1 - \frac{e^2 x^2}{d^2}\right)}{2d^2 p} - \frac{(d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2}, 1-p; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{dx}$$

[Out] -(((d^2 - e^2\*x^2)^p\*Hypergeometric2F1[-1/2, 1 - p, 1/2, (e^2\*x^2)/d^2])/(d\*x\*(1 - (e^2\*x^2)/d^2)^p)) + (e\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[1, p, 1 + p, 1 - (e^2\*x^2)/d^2])/(2\*d^2\*p)

**Rubi [A]** time = 0.212387, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{e (d^2 - e^2 x^2)^p {}_2F_1\left(1, p; p+1; 1 - \frac{e^2 x^2}{d^2}\right)}{2d^2 p} - \frac{(d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2}, 1-p; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{dx}$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^p/(x^2\*(d + e\*x)), x]

[Out] -(((d^2 - e^2\*x^2)^p\*Hypergeometric2F1[-1/2, 1 - p, 1/2, (e^2\*x^2)/d^2])/(d\*x\*(1 - (e^2\*x^2)/d^2)^p)) + (e\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[1, p, 1 + p, 1 - (e^2\*x^2)/d^2])/(2\*d^2\*p)

**Rubi in Sympy [A]** time = 33.4714, size = 82, normalized size = 0.77

$$-\frac{\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+1, -\frac{1}{2} \middle| \frac{e^2 x^2}{d^2}\right)}{dx} + \frac{e (d^2 - e^2 x^2)^p {}_2F_1\left(1, p \middle| 1 - \frac{e^2 x^2}{d^2}\right)}{2d^2 p}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x\*\*2/(e\*x+d), x)

[Out] -(1 - e\*\*2\*x\*\*2/d\*\*2)\*\*(-p)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p\*hyper((-p + 1, -1/2), (1/2, ), e\*\*2\*x\*\*2/d\*\*2)/(d\*x) + e\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p\*hyper((1, p), (p + 1, ), 1 - e\*\*2\*x\*\*2/d\*\*2)/(2\*d\*\*2\*p)

**Mathematica [A]** time = 0.2559, size = 167, normalized size = 1.58

$$\frac{(d^2 - e^2 x^2)^p \left( -\frac{de \left(1 - \frac{d^2}{e^2 x^2}\right)^{-p} {}_2F_1\left(-p, -p; 1-p; \frac{d^2}{e^2 x^2}\right)}{p} - \frac{2d^2 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{x} + \frac{e^{2p} (ex-d) \left(\frac{ex}{d} + 1\right)^{-p} {}_2F_1\left(1-p, p+1; p+2; \frac{d-ex}{2d}\right)}{p+1} \right)}{2d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^p/(x^2\*(d + e\*x)), x]

[Out] ((d^2 - e^2\*x^2)^p\*((-2\*d^2\*Hypergeometric2F1[-1/2, -p, 1/2, (e^2\*x^2)/d^2])/(x\*(1 - (e^2\*x^2)/d^2)^p) + (2^p\*e\*(-d + e\*x)\*Hypergeometric2F1[1, p, 1 + p, 1 - (e^2\*x^2)/d^2])/(2\*d^2\*p))

ometric2F1[1 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)]/((1 + p)\*(1 + (e\*x)/d)^p) - (d\*e\*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2\*x^2)])/(p\*(1 - d^2/(e^2\*x^2))^p))/(2\*d^3)

**Maple [F]** time = 0.077, size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{x^2(ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^p/x^2/(e\*x+d), x)

[Out] int((-e^2\*x^2+d^2)^p/x^2/(e\*x+d), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)\*x^2), x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)\*x^2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p}{ex^3 + dx^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)\*x^2), x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p/(e\*x^3 + d\*x^2), x)

**Sympy [A]** time = 18.2182, size = 450, normalized size = 4.25

$$\left\{ \begin{array}{l} -\frac{0^p d^{2p}}{dx} - \frac{0^p d^{2p} e \log\left(\frac{e^2 x^2}{d^2}\right)}{2d^2} + \frac{0^p d^{2p} e \log\left(-1 + \frac{e^2 x^2}{d^2}\right)}{2d^2} + \frac{0^p d^{2p} e \operatorname{acoth}\left(\frac{ex}{d}\right)}{d^2} + \frac{de^{2p} px^{2p} e^{i\pi p} (p) (-p + \frac{3}{2}) {}_2F_1\left(-p + 1, -p + \frac{3}{2} \middle| -\frac{d^2}{e^2 x^2}\right)}{2e^2 x^3 (-p + \frac{5}{2}) (p+1)} - \frac{e^{2p} px^{2p} e^{i\pi p}}{2e^2 x^3 (-p + \frac{5}{2}) (p+1)} \\ -\frac{0^p d^{2p}}{dx} - \frac{0^p d^{2p} e \log\left(\frac{e^2 x^2}{d^2}\right)}{2d^2} + \frac{0^p d^{2p} e \log\left(1 - \frac{e^2 x^2}{d^2}\right)}{2d^2} + \frac{0^p d^{2p} e \operatorname{atanh}\left(\frac{ex}{d}\right)}{d^2} + \frac{de^{2p} px^{2p} e^{i\pi p} (p) (-p + \frac{3}{2}) {}_2F_1\left(-p + 1, -p + \frac{3}{2} \middle| \frac{d^2}{e^2 x^2}\right)}{2e^2 x^3 (-p + \frac{5}{2}) (p+1)} - \frac{e^{2p} px^{2p} e^{i\pi p}}{2e^2 x^3 (-p + \frac{5}{2}) (p+1)} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x\*\*2/(e\*x+d), x)

```
[Out] Piecewise((-0**p*d**(2*p)/(d*x) - 0**p*d**(2*p)*e*log(e**2*x**2/d
**2)/(2*d**2) + 0**p*d**(2*p)*e*log(-1 + e**2*x**2/d**2)/(2*d**2)
+ 0**p*d**(2*p)*e*acoth(e*x/d)/d**2 + d*e**(2*p)*p*x**(2*p)*exp(
I*pi*p)*gamma(p)*gamma(-p + 3/2)*hyper((-p + 1, -p + 3/2), (-p +
5/2,), d**2/(e**2*x**2))/(2*e**2*x**3*gamma(-p + 5/2)*gamma(p + 1
)) - e**(2*p)*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p + 1)*hyper
((-p + 1, -p + 1), (-p + 2,), d**2/(e**2*x**2))/(2*e*x**2*gamma(-
p + 2)*gamma(p + 1)), Abs(e**2*x**2/d**2) > 1), (-0**p*d**(2*p)/(
d*x) - 0**p*d**(2*p)*e*log(e**2*x**2/d**2)/(2*d**2) + 0**p*d**(2*
p)*e*log(1 - e**2*x**2/d**2)/(2*d**2) + 0**p*d**(2*p)*e*atanh(e*x
/d)/d**2 + d*e**(2*p)*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p +
3/2)*hyper((-p + 1, -p + 3/2), (-p + 5/2,), d**2/(e**2*x**2))/(2*
e**2*x**3*gamma(-p + 5/2)*gamma(p + 1)) - e**(2*p)*p*x**(2*p)*exp
(I*pi*p)*gamma(p)*gamma(-p + 1)*hyper((-p + 1, -p + 1), (-p + 2,
), d**2/(e**2*x**2))/(2*e*x**2*gamma(-p + 2)*gamma(p + 1)), True))
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x^2),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x^2), x)
```

$$3.274 \quad \int \frac{(d^2 - e^2 x^2)^p}{x^3(d+ex)} dx$$

**Optimal.** Leaf size=108

$$\frac{e(d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2}, 1-p; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{d^2 x} - \frac{e^2 (d^2 - e^2 x^2)^p {}_2F_1\left(2, p; p+1; 1 - \frac{e^2 x^2}{d^2}\right)}{2d^3 p}$$

[Out] (e\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[-1/2, 1 - p, 1/2, (e^2\*x^2)/d^2])/(d^2\*x\*(1 - (e^2\*x^2)/d^2)^p) - (e^2\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[2, p, 1 + p, 1 - (e^2\*x^2)/d^2])/(2\*d^3\*p)

**Rubi [A]** time = 0.217953, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{e(d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2}, 1-p; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{d^2 x} - \frac{e^2 (d^2 - e^2 x^2)^p {}_2F_1\left(2, p; p+1; 1 - \frac{e^2 x^2}{d^2}\right)}{2d^3 p}$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^p/(x^3\*(d + e\*x)), x]

[Out] (e\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[-1/2, 1 - p, 1/2, (e^2\*x^2)/d^2])/(d^2\*x\*(1 - (e^2\*x^2)/d^2)^p) - (e^2\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[2, p, 1 + p, 1 - (e^2\*x^2)/d^2])/(2\*d^3\*p)

**Rubi in Sympy [A]** time = 35.4839, size = 87, normalized size = 0.81

$$\frac{e\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p + 1, -\frac{1}{2} \middle| \frac{e^2 x^2}{d^2}\right)}{d^2 x} - \frac{e^2 (d^2 - e^2 x^2)^p {}_2F_1\left(2, p \middle| 1 - \frac{e^2 x^2}{d^2}\right)}{2d^3 p}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x\*\*3/(e\*x+d), x)

[Out] e\*(1 - e\*\*2\*x\*\*2/d\*\*2)\*\*(-p)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p\*hyper((-p + 1, -1/2), (1/2, ), e\*\*2\*x\*\*2/d\*\*2)/(d\*\*2\*x) - e\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p\*hyper((2, p), (p + 1, ), 1 - e\*\*2\*x\*\*2/d\*\*2)/(2\*d\*\*3\*p)

**Mathematica [B]** time = 1.23429, size = 219, normalized size = 2.03

$$\frac{(d^2 - e^2 x^2)^p \left( \frac{2d^2 e \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{x} + \left(1 - \frac{d^2}{e^2 x^2}\right)^{-p} \left( e^2 \left( \frac{(d-ex) \left(2 - \frac{2d^2}{e^2 x^2}\right)^p \left(\frac{ex}{d} + 1\right)^{-p} {}_2F_1\left(1-p, p+1; p+2; \frac{d-ex}{2d}\right)}{p+1} + \frac{d {}_2F_1\left(-p, -p; 1, 1; \frac{d-ex}{2d}\right)}{p} \right) \right)}{2d^4}$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^p/(x^3\*(d + e\*x)), x]

[Out] ((d^2 - e^2\*x^2)^p\*((2\*d^2\*e\*Hypergeometric2F1[-1/2, -p, 1/2, (e^2\*x^2)/d^2])/(x\*(1 - (e^2\*x^2)/d^2)^p) + ((d^3\*Hypergeometric2F1[

$$1 - p, -p, 2 - p, d^2/(e^2 x^2)]/((-1 + p) x^2) + e^2 * ((2 - (2 * d^2)/(e^2 x^2))^p * (d - e x) * \text{Hypergeometric2F1}[1 - p, 1 + p, 2 + p, (d - e x)/(2 * d)]/((1 + p) * (1 + (e x)/d)^p) + (d * \text{Hypergeometric2F1}[-p, -p, 1 - p, d^2/(e^2 x^2)]/p))/(1 - d^2/(e^2 x^2))^p)/(2 * d^4)$$

**Maple [F]** time = 0.091, size = 0, normalized size = 0.

$$\int \frac{(-e^2 x^2 + d^2)^p}{x^3 (ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^p/x^3/(e\*x+d), x)

[Out] int((-e^2\*x^2+d^2)^p/x^3/(e\*x+d), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2 x^2 + d^2)^p}{(ex + d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)\*x^3), x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)\*x^3), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2 x^2 + d^2)^p}{ex^4 + dx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)\*x^3), x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p/(e\*x^4 + d\*x^3), x)

**Sympy [A]** time = 22.4695, size = 498, normalized size = 4.61

$$\left\{ \begin{array}{l} -\frac{0^p d^{2p}}{2dx^2} + \frac{0^p d^{2p} e}{d^2 x} + \frac{0^p d^{2p} e^2 \log\left(\frac{e^2 x^2}{d^2}\right)}{2d^3} - \frac{0^p d^{2p} e^2 \log\left(-1 + \frac{e^2 x^2}{d^2}\right)}{2d^3} - \frac{0^p d^{2p} e^2 \operatorname{acoth}\left(\frac{ex}{d}\right)}{d^3} + \frac{de^{2p} px^{2p} e^{i\pi p} (p)(-p+2)_2F_1\left(-p+1, -p+2 \middle| \frac{d^2}{e^2 x^2} \right)}{2e^2 x^4 (-p+3)(p+1)} \\ -\frac{0^p d^{2p}}{2dx^2} + \frac{0^p d^{2p} e}{d^2 x} + \frac{0^p d^{2p} e^2 \log\left(\frac{e^2 x^2}{d^2}\right)}{2d^3} - \frac{0^p d^{2p} e^2 \log\left(1 - \frac{e^2 x^2}{d^2}\right)}{2d^3} - \frac{0^p d^{2p} e^2 \operatorname{atanh}\left(\frac{ex}{d}\right)}{d^3} + \frac{de^{2p} px^{2p} e^{i\pi p} (p)(-p+2)_2F_1\left(-p+1, -p+2 \middle| \frac{d^2}{e^2 x^2} \right)}{2e^2 x^4 (-p+3)(p+1)} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x\*\*3/(e\*x+d), x)



```
[Out] Piecewise((-0**p*d**(2*p)/(2*d*x**2) + 0**p*d**(2*p)*e/(d**2*x) +
  0**p*d**(2*p)*e**2*log(e**2*x**2/d**2)/(2*d**3) - 0**p*d**(2*p)*
e**2*log(-1 + e**2*x**2/d**2)/(2*d**3) - 0**p*d**(2*p)*e**2*acoth
(e*x/d)/d**3 + d*e**(2*p)*p*x**(2*p)*exp(I*pi*p)*gamma(p)*gamma(-
p + 2)*hyper((-p + 1, -p + 2), (-p + 3, ), d**2/(e**2*x**2))/(2*e*
**2*x**4*gamma(-p + 3)*gamma(p + 1)) - e**(2*p)*p*x**(2*p)*exp(I*pi
i*p)*gamma(p)*gamma(-p + 3/2)*hyper((-p + 1, -p + 3/2), (-p + 5/2
, ), d**2/(e**2*x**2))/(2*e*x**3*gamma(-p + 5/2)*gamma(p + 1)), Ab
s(e**2*x**2/d**2) > 1), (-0**p*d**(2*p)/(2*d*x**2) + 0**p*d**(2*p)
)*e/(d**2*x) + 0**p*d**(2*p)*e**2*log(e**2*x**2/d**2)/(2*d**3) -
0**p*d**(2*p)*e**2*log(1 - e**2*x**2/d**2)/(2*d**3) - 0**p*d**(2*
p)*e**2*atanh(e*x/d)/d**3 + d*e**(2*p)*p*x**(2*p)*exp(I*pi*p)*gam
ma(p)*gamma(-p + 2)*hyper((-p + 1, -p + 2), (-p + 3, ), d**2/(e**2
*x**2))/(2*e**2*x**4*gamma(-p + 3)*gamma(p + 1)) - e**(2*p)*p*x**
(2*p)*exp(I*pi*p)*gamma(p)*gamma(-p + 3/2)*hyper((-p + 1, -p + 3/
2), (-p + 5/2, ), d**2/(e**2*x**2))/(2*e*x**3*gamma(-p + 5/2)*gamm
a(p + 1)), True))
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x^3),x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/((e*x + d)*x^3), x)
```

$$3.275 \quad \int \frac{x^5 (d^2 - e^2 x^2)^p}{(d + ex)^2} dx$$

**Optimal.** Leaf size=179

$$\frac{2d^2 (d^2 - e^2 x^2)^{p+1}}{e^6 (p+1)} + \frac{(d^2 - e^2 x^2)^{p+2}}{2e^6 (p+2)} + \frac{d^6 (d^2 - e^2 x^2)^{p-1}}{e^6 (1-p)} + \frac{5d^4 (d^2 - e^2 x^2)^p}{2e^6 p} - \frac{2ex^7 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(\frac{7}{2}, 2-p; \frac{9}{2}; \frac{e^2 x^2}{d^2}\right)}{7d^3}$$

[Out]  $(d^6 (d^2 - e^2 x^2)^{-1+p}) / (e^6 (1-p)) + (5d^4 (d^2 - e^2 x^2)^p) / (2e^6 p) - (2d^2 (d^2 - e^2 x^2)^{1+p}) / (e^6 (1+p)) + (d^2 - e^2 x^2)^{2+p} / (2e^6 (2+p)) - (2e^7 x^7 (d^2 - e^2 x^2)^p) / (2e^6 (1-p)) - (2e^7 x^7 (d^2 - e^2 x^2)^p \text{Hypergeometric2F1}[7/2, 2-p, 9/2, (e^2 x^2)/d^2]) / (7d^3 (1 - (e^2 x^2)/d^2)^p)$

**Rubi [A]** time = 0.408152, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$\frac{2d^2 (d^2 - e^2 x^2)^{p+1}}{e^6 (p+1)} + \frac{(d^2 - e^2 x^2)^{p+2}}{2e^6 (p+2)} + \frac{d^6 (d^2 - e^2 x^2)^{p-1}}{e^6 (1-p)} + \frac{5d^4 (d^2 - e^2 x^2)^p}{2e^6 p} - \frac{2ex^7 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(\frac{7}{2}, 2-p; \frac{9}{2}; \frac{e^2 x^2}{d^2}\right)}{7d^3}$$

Antiderivative was successfully verified.

[In] Int[(x^5\*(d^2 - e^2\*x^2)^p)/(d + e\*x)^2,x]

[Out]  $(d^6 (d^2 - e^2 x^2)^{-1+p}) / (e^6 (1-p)) + (5d^4 (d^2 - e^2 x^2)^p) / (2e^6 p) - (2d^2 (d^2 - e^2 x^2)^{1+p}) / (e^6 (1+p)) + (d^2 - e^2 x^2)^{2+p} / (2e^6 (2+p)) - (2e^7 x^7 (d^2 - e^2 x^2)^p) / (2e^6 (1-p)) - (2e^7 x^7 (d^2 - e^2 x^2)^p \text{Hypergeometric2F1}[7/2, 2-p, 9/2, (e^2 x^2)/d^2]) / (7d^3 (1 - (e^2 x^2)/d^2)^p)$

**Rubi in Sympy [A]** time = 111.53, size = 148, normalized size = 0.83

$$\frac{d^6 (d^2 - e^2 x^2)^{p-1}}{e^6 (-p+1)} + \frac{5d^4 (d^2 - e^2 x^2)^p}{2e^6 p} - \frac{2d^2 (d^2 - e^2 x^2)^{p+1}}{e^6 (p+1)} + \frac{(d^2 - e^2 x^2)^{p+2}}{2e^6 (p+2)} - \frac{2ex^7 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+2, \frac{7}{2}; \frac{9}{2}; \frac{e^2 x^2}{d^2}\right)}{7d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d)\*\*2,x)

[Out]  $d^6 (d^2 - e^2 x^2)^{p-1} / (e^6 (-p+1)) + 5d^4 (d^2 - e^2 x^2)^p / (2e^6 p) - 2d^2 (d^2 - e^2 x^2)^{p+1} / (e^6 (p+1)) + (d^2 - e^2 x^2)^{p+2} / (2e^6 (p+2)) - 2e^7 x^7 (1 - e^2 x^2/d^2)^{-p} (d^2 - e^2 x^2)^p \text{hyper}((-p+2, 7/2), (9/2, ), e^2 x^2/d^2) / (7d^3)$

**Mathematica [C]** time = 0.455235, size = 140, normalized size = 0.78

$$\frac{7dx^6(d-ex)^p(d+ex)^{p-2} {}_2F_1\left(6, -p, 2-p; 7; \frac{ex}{d}, -\frac{ex}{d}\right)}{6\left(ex \left({}_2F_1\left(7, 1-p, 2-p; 8; \frac{ex}{d}, -\frac{ex}{d}\right) - (p-2) {}_2F_1\left(7, -p, 3-p; 8; \frac{ex}{d}, -\frac{ex}{d}\right)\right) - 7d {}_2F_1\left(6, -p, 2-p; 7; \frac{ex}{d}, -\frac{ex}{d}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^5\*(d^2 - e^2\*x^2)^p)/(d + e\*x)^2,x]

[Out]  $(-7*d*x^6*(d - e*x)^p*(d + e*x)^{-2 + p}*AppellF1[6, -p, 2 - p, 7, (e*x)/d, -((e*x)/d)]/(6*(-7*d*AppellF1[6, -p, 2 - p, 7, (e*x)/d, -((e*x)/d)] + e*x*(p*AppellF1[7, 1 - p, 2 - p, 8, (e*x)/d, -((e*x)/d)] - (-2 + p)*AppellF1[7, -p, 3 - p, 8, (e*x)/d, -((e*x)/d)]))$

**Maple** [F] time = 0.107, size = 0, normalized size = 0.

$$\int \frac{x^5 (-e^2x^2 + d^2)^p}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(-e^2\*x^2+d^2)^p/(e\*x+d)^2,x)

[Out] int(x^5\*(-e^2\*x^2+d^2)^p/(e\*x+d)^2,x)

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x^5}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x^5/(e\*x + d)^2,x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p\*x^5/(e\*x + d)^2, x)

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p x^5}{e^2x^2 + 2dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x^5/(e\*x + d)^2,x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p\*x^5/(e^2\*x^2 + 2\*d\*e\*x + d^2), x)

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5 (-(-d + ex)(d + ex))^p}{(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d)\*\*2,x)

[Out] Integral( $x^{**5}*(-(-d + e*x)*(d + e*x))^{**p}/(d + e*x)^{**2}, x$ )

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x^5}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x^5/(e\*x + d)^2,x, algorithm="giac")

[Out] integrate((-e^2\*x^2 + d^2)^p\*x^5/(e\*x + d)^2, x)

$$3.276 \quad \int \frac{x^4(d^2 - e^2x^2)^p}{(d+ex)^2} dx$$

**Optimal.** Leaf size=184

$$\frac{2(p+4)x^5 \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(\frac{5}{2}, 2-p; \frac{7}{2}; \frac{e^2x^2}{d^2}\right)}{5d^2(2p+3)} - \frac{x^5 (d^2 - e^2x^2)^{p-1}}{2p+3} \\ + \frac{d (d^2 - e^2x^2)^{p+1}}{e^5(p+1)} - \frac{d^5 (d^2 - e^2x^2)^{p-1}}{e^5(1-p)} - \frac{2d^3 (d^2 - e^2x^2)^p}{e^5p}$$

[Out]  $-\left(\frac{d^5 (d^2 - e^2x^2)^{-1+p}}{e^5(1-p)}\right) - \frac{x^5 (d^2 - e^2x^2)^{-1+p}}{(3+2p)} - \frac{2d^3 (d^2 - e^2x^2)^p}{e^5p} + \frac{d (d^2 - e^2x^2)^{1+p}}{e^5(1+p)} + \frac{2(4+p)x^5 (d^2 - e^2x^2)^p \text{Hypergeometric2F1}\left[\frac{5}{2}, 2-p, \frac{7}{2}, \frac{e^2x^2}{d^2}\right]}{5d^2(3+2p)(1 - (e^2x^2)/d^2)^p}$

**Rubi [A]** time = 0.430833, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.32$

$$\frac{2(p+4)x^5 \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(\frac{5}{2}, 2-p; \frac{7}{2}; \frac{e^2x^2}{d^2}\right)}{5d^2(2p+3)} - \frac{x^5 (d^2 - e^2x^2)^{p-1}}{2p+3} \\ + \frac{d (d^2 - e^2x^2)^{p+1}}{e^5(p+1)} - \frac{d^5 (d^2 - e^2x^2)^{p-1}}{e^5(1-p)} - \frac{2d^3 (d^2 - e^2x^2)^p}{e^5p}$$

Antiderivative was successfully verified.

[In]  $\text{Int}\left[\frac{x^4 (d^2 - e^2x^2)^p}{(d+ex)^2}, x\right]$

[Out]  $-\left(\frac{d^5 (d^2 - e^2x^2)^{-1+p}}{e^5(1-p)}\right) - \frac{x^5 (d^2 - e^2x^2)^{-1+p}}{(3+2p)} - \frac{2d^3 (d^2 - e^2x^2)^p}{e^5p} + \frac{d (d^2 - e^2x^2)^{1+p}}{e^5(1+p)} + \frac{2(4+p)x^5 (d^2 - e^2x^2)^p \text{Hypergeometric2F1}\left[\frac{5}{2}, 2-p, \frac{7}{2}, \frac{e^2x^2}{d^2}\right]}{5d^2(3+2p)(1 - (e^2x^2)/d^2)^p}$

**Rubi in Sympy [A]** time = 87.2523, size = 172, normalized size = 0.93

$$-\frac{d^5 (d^2 - e^2x^2)^{p-1}}{e^5(-p+1)} - \frac{2d^3 (d^2 - e^2x^2)^p}{e^5p} + \frac{d (d^2 - e^2x^2)^{p+1}}{e^5(p+1)} \\ + \frac{x^5 \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(-p+2, \frac{5}{2}; \frac{7}{2}; \frac{e^2x^2}{d^2}\right)}{5d^2} \\ + \frac{e^2x^7 \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(-p+2, \frac{7}{2}; \frac{9}{2}; \frac{e^2x^2}{d^2}\right)}{7d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**4} * (-e^{**2} * x^{**2} + d^{**2})^{**p} / (e * x + d)^{**2}, x)$

[Out]  $-d^{**5} * (d^{**2} - e^{**2} * x^{**2})^{**p} / (e^{**5} * (-p + 1)) - 2 * d^{**3} * (d^{**2} - e^{**2} * x^{**2})^{**p} / (e^{**5} * p) + d * (d^{**2} - e^{**2} * x^{**2})^{**p} / (e^{**5} * (p + 1)) + x^{**5} * (1 - e^{**2} * x^{**2} / d^{**2})^{**p} * (d^{**2} - e^{**2} * x^{**2})^{**p} * \text{hyper}((-p + 2, 5/2), (7/2, ), e^{**2} * x^{**2} / d^{**2}) / (5 * d^{**2}) + e^{**2} * x^{**7} * (1 - e^{**2} * x^{**2} / d^{**2})^{**p} * (d^{**2} - e^{**2} * x^{**2})^{**p} * \text{hyper}((-p + 2, 7/2, 9/2, e^{**2} * x^{**2} / d^{**2})) / (7 * d^{**4})$

2), (9/2, ),  $e^{2x^2/d^2}/(7d^4)$

**Mathematica [C]** time = 0.42535, size = 140, normalized size = 0.76

$$\frac{6dx^5(d-ex)^p(d+ex)^{p-2}F_1\left(5; -p, 2-p; 6; \frac{ex}{d}, -\frac{ex}{d}\right)}{5\left(ex\left(pF_1\left(6; 1-p, 2-p; 7; \frac{ex}{d}, -\frac{ex}{d}\right) - (p-2)F_1\left(6; -p, 3-p; 7; \frac{ex}{d}, -\frac{ex}{d}\right)\right) - 6dF_1\left(5; -p, 2-p; 6; \frac{ex}{d}, -\frac{ex}{d}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^4\*(d^2 - e^2\*x^2)^p)/(d + e\*x)^2, x]

[Out] (-6\*d\*x^5\*(d - e\*x)^p\*(d + e\*x)^(-2 + p)\*AppellF1[5, -p, 2 - p, 6, (e\*x)/d, -((e\*x)/d)]/(5\*(-6\*d\*AppellF1[5, -p, 2 - p, 6, (e\*x)/d, -((e\*x)/d)] + e\*x\*(p\*AppellF1[6, 1 - p, 2 - p, 7, (e\*x)/d, -((e\*x)/d)] - (-2 + p)\*AppellF1[6, -p, 3 - p, 7, (e\*x)/d, -((e\*x)/d)]))

**Maple [F]** time = 0.116, size = 0, normalized size = 0.

$$\int \frac{x^4 (-e^2x^2 + d^2)^p}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(-e^2\*x^2+d^2)^p/(e\*x+d)^2, x)

[Out] int(x^4\*(-e^2\*x^2+d^2)^p/(e\*x+d)^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x^4}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x^4/(e\*x + d)^2, x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p\*x^4/(e\*x + d)^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p x^4}{e^2x^2 + 2dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x^4/(e\*x + d)^2, x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p\*x^4/(e^2\*x^2 + 2\*d\*e\*x + d^2), x)

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 (-(-d + ex)(d + ex))^p}{(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d)\*\*2,x)

[Out] Integral(x\*\*4\*(-(-d + e\*x)\*(d + e\*x))\*\*p/(d + e\*x)\*\*2, x)

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x^4}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x^4/(e\*x + d)^2,x, algorithm="giac")

[Out] integrate((-e^2\*x^2 + d^2)^p\*x^4/(e\*x + d)^2, x)

$$3.277 \quad \int \frac{x^3 (d^2 - e^2 x^2)^p}{(d + ex)^2} dx$$

**Optimal.** Leaf size=150

$$\frac{3d^2 (d^2 - e^2 x^2)^p}{2e^4 p} - \frac{(d^2 - e^2 x^2)^{p+1}}{2e^4 (p+1)} + \frac{d^4 (d^2 - e^2 x^2)^{p-1}}{e^4 (1-p)} - \frac{2ex^5 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(\frac{5}{2}, 2-p; \frac{7}{2}; \frac{e^2 x^2}{d^2}\right)}{5d^3}$$

[Out]  $(d^4 (d^2 - e^2 x^2)^{-1+p}) / (e^4 (1-p)) + (3d^2 (d^2 - e^2 x^2)^p) / (2e^4 p) - (d^2 - e^2 x^2)^{1+p} / (2e^4 (1+p)) - (2e^5 x^5 (d^2 - e^2 x^2)^p \text{Hypergeometric2F1}[5/2, 2-p, 7/2, (e^2 x^2)/d^2]) / (5d^3 (1 - (e^2 x^2)/d^2)^p)$

**Rubi [A]** time = 0.36949, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$\frac{3d^2 (d^2 - e^2 x^2)^p}{2e^4 p} - \frac{(d^2 - e^2 x^2)^{p+1}}{2e^4 (p+1)} + \frac{d^4 (d^2 - e^2 x^2)^{p-1}}{e^4 (1-p)} - \frac{2ex^5 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(\frac{5}{2}, 2-p; \frac{7}{2}; \frac{e^2 x^2}{d^2}\right)}{5d^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^3 (d^2 - e^2 x^2)^p) / (d + e x)^2, x]$

[Out]  $(d^4 (d^2 - e^2 x^2)^{-1+p}) / (e^4 (1-p)) + (3d^2 (d^2 - e^2 x^2)^p) / (2e^4 p) - (d^2 - e^2 x^2)^{1+p} / (2e^4 (1+p)) - (2e^5 x^5 (d^2 - e^2 x^2)^p \text{Hypergeometric2F1}[5/2, 2-p, 7/2, (e^2 x^2)/d^2]) / (5d^3 (1 - (e^2 x^2)/d^2)^p)$

**Rubi in Sympy [A]** time = 88.3953, size = 122, normalized size = 0.81

$$\frac{d^4 (d^2 - e^2 x^2)^{p-1}}{e^4 (-p+1)} + \frac{3d^2 (d^2 - e^2 x^2)^p}{2e^4 p} - \frac{(d^2 - e^2 x^2)^{p+1}}{2e^4 (p+1)} - \frac{2ex^5 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+2, \frac{5}{2}; \frac{7}{2}; \frac{e^2 x^2}{d^2}\right)}{5d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^3 * (-e^2 * x^2 + d^2)^p / (e * x + d)^2, x)$

[Out]  $d^4 (d^2 - e^2 x^2)^{p-1} / (e^4 (-p+1)) + 3d^2 (d^2 - e^2 x^2)^p / (2e^4 p) - (d^2 - e^2 x^2)^{p+1} / (2e^4 (p+1)) - 2e^5 x^5 (1 - e^2 x^2 / d^2)^{-p} (d^2 - e^2 x^2)^p \text{hyper}((-p+2, 5/2), (7/2, ), e^2 x^2 / d^2) / (5d^3)$

**Mathematica [B]** time = 0.435732, size = 332, normalized size = 2.21

$$2^{p-2} \left(\frac{ex}{d} + 1\right)^{-p} (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} \left(-8de(p+1)x \left(\frac{ex}{2d} + \frac{1}{2}\right)^p {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right) - 6d(d-ex) \left(1 - \frac{e^2 x^2}{d^2}\right)^p {}_2F_1\left(1 - \frac{p}{2}, -p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)\right)$$



Antiderivative was successfully verified.

[In] Integrate[(x^3\*(d^2 - e^2\*x^2)^p)/(d + e\*x)^2,x]

[Out]  $(2^{(-2 + p)}(d^2 - e^2x^2)^p(2d^{1/2 + (e^*x)/(2*d)})^p - 2d^{2(1/2 + (e^*x)/(2*d))}p(1 - (e^2x^2)/d^2)^p + 2e^2x^{2(1/2 + (e^*x)/(2*d))}p(1 - (e^2x^2)/d^2)^p - 8d^*e^*(1 + p)x^{1/2 + (e^*x)/(2*d)}p\text{Hypergeometric2F1}[1/2, -p, 3/2, (e^2x^2)/d^2] - 6d^*(d - e^*x)(1 - (e^2x^2)/d^2)^p\text{Hypergeometric2F1}[1 - p, 1 + p, 2 + p, (d - e^*x)/(2*d)] + d^2(1 - (e^2x^2)/d^2)^p\text{Hypergeometric2F1}[2 - p, 1 + p, 2 + p, (d - e^*x)/(2*d)] - d^*e^*x(1 - (e^2x^2)/d^2)^p\text{Hypergeometric2F1}[2 - p, 1 + p, 2 + p, (d - e^*x)/(2*d)])/(e^4(1 + p)(1 + (e^*x)/d)^p(1 - (e^2x^2)/d^2)^p)$

**Maple [F]** time = 0.107, size = 0, normalized size = 0.

$$\int \frac{x^3 (-e^2x^2 + d^2)^p}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(-e^2\*x^2+d^2)^p/(e\*x+d)^2,x)

[Out] int(x^3\*(-e^2\*x^2+d^2)^p/(e\*x+d)^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x^3}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x^3/(e\*x + d)^2,x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p\*x^3/(e\*x + d)^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p x^3}{e^2x^2 + 2dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x^3/(e\*x + d)^2,x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p\*x^3/(e^2\*x^2 + 2\*d\*e\*x + d^2), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 (-(-d + ex)(d + ex))^p}{(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)`

[Out] `Integral(x**3*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x^3}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d)^2,x, algorithm="giac")`

[Out] `integrate((-e^2*x^2 + d^2)^p*x^3/(e*x + d)^2, x)`

$$3.278 \quad \int \frac{x^2(d^2 - e^2x^2)^p}{(d+ex)^2} dx$$

**Optimal.** Leaf size=156

$$\frac{2(p+2)x^3 \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(\frac{3}{2}, 2-p; \frac{5}{2}; \frac{e^2x^2}{d^2}\right)}{3d^2(2p+1)} - \frac{x^3 (d^2 - e^2x^2)^{p-1}}{2p+1} - \frac{d (d^2 - e^2x^2)^p}{e^3p} - \frac{d^3 (d^2 - e^2x^2)^{p-1}}{e^3(1-p)}$$

[Out]  $-\left(\frac{d^3(d^2 - e^2x^2)^{-1+p}}{e^3(1-p)}\right) - \frac{x^3(d^2 - e^2x^2)^{-1+p}}{(1+2p)} - \frac{d(d^2 - e^2x^2)^p}{e^3p} + (2+p)x^3 \frac{(d^2 - e^2x^2)^p \text{Hypergeometric2F1}\left[\frac{3}{2}, 2-p, \frac{5}{2}, \frac{e^2x^2}{d^2}\right]}{(3d^2(1+2p)(1 - \frac{e^2x^2}{d^2})^p)}$

**Rubi [A]** time = 0.383072, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.32$

$$\frac{2(p+2)x^3 \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(\frac{3}{2}, 2-p; \frac{5}{2}; \frac{e^2x^2}{d^2}\right)}{3d^2(2p+1)} - \frac{x^3 (d^2 - e^2x^2)^{p-1}}{2p+1} - \frac{d (d^2 - e^2x^2)^p}{e^3p} - \frac{d^3 (d^2 - e^2x^2)^{p-1}}{e^3(1-p)}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(d^2 - e^2\*x^2)^p)/(d + e\*x)^2, x]

[Out]  $-\left(\frac{d^3(d^2 - e^2x^2)^{-1+p}}{e^3(1-p)}\right) - \frac{x^3(d^2 - e^2x^2)^{-1+p}}{(1+2p)} - \frac{d(d^2 - e^2x^2)^p}{e^3p} + (2+p)x^3 \frac{(d^2 - e^2x^2)^p \text{Hypergeometric2F1}\left[\frac{3}{2}, 2-p, \frac{5}{2}, \frac{e^2x^2}{d^2}\right]}{(3d^2(1+2p)(1 - \frac{e^2x^2}{d^2})^p)}$

**Rubi in Sympy [A]** time = 73.4424, size = 146, normalized size = 0.94

$$-\frac{d^3 (d^2 - e^2x^2)^{p-1}}{e^3(-p+1)} - \frac{d (d^2 - e^2x^2)^p}{e^3p} + \frac{x^3 \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(-p+2, \frac{3}{2}; \frac{5}{2}; \frac{e^2x^2}{d^2}\right)}{3d^2} + \frac{e^2x^5 \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(-p+2, \frac{5}{2}; \frac{7}{2}; \frac{e^2x^2}{d^2}\right)}{5d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d)\*\*2, x)

[Out]  $-d**3*(d**2 - e**2*x**2)**(p-1)/(e**3*(-p+1)) - d*(d**2 - e**2*x**2)**p/(e**3*p) + x**3*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper((-p+2, 3/2), (5/2, ), e**2*x**2/d**2)/(3*d**2) + e**2*x**5*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper((-p+2, 5/2), (7/2, ), e**2*x**2/d**2)/(5*d**4)$

**Mathematica [A]** time = 0.195556, size = 177, normalized size = 1.13

$$\frac{2^{p-2} \left(\frac{ex}{d} + 1\right)^{-p} (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} \left(4e(p+1)x \left(\frac{ex}{2d} + \frac{1}{2}\right)^p {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right) + (d - ex) \left(1 - \frac{e^2 x^2}{d^2}\right)^p \left({}_4F_1\left(1 - p, \dots\right)\right)}{e^3(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(d^2 - e^2\*x^2)^p)/(d + e\*x)^2, x]

[Out] (2^(-2 + p)\*(d^2 - e^2\*x^2)^p\*(4\*e\*(1 + p)\*x\*(1/2 + (e\*x)/(2\*d))^p\*Hypergeometric2F1[1/2, -p, 3/2, (e^2\*x^2)/d^2] + (d - e\*x)\*(1 - (e^2\*x^2)/d^2)^p\*(4\*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)] - Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])))/(e^3\*(1 + p)\*(1 + (e\*x)/d)^p\*(1 - (e^2\*x^2)/d^2)^p)

**Maple [F]** time = 0.079, size = 0, normalized size = 0.

$$\int \frac{x^2 (-e^2 x^2 + d^2)^p}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(-e^2\*x^2+d^2)^p/(e\*x+d)^2, x)

[Out] int(x^2\*(-e^2\*x^2+d^2)^p/(e\*x+d)^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2 x^2 + d^2)^p x^2}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x^2/(e\*x + d)^2, x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p\*x^2/(e\*x + d)^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2 x^2 + d^2)^p x^2}{e^2 x^2 + 2 dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x^2/(e^2\*x^2 + 2\*d\*e\*x + d^2), x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p\*x^2/(e^2\*x^2 + 2\*d\*e\*x + d^2), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 (-(-d + ex)(d + ex))^p}{(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)`

[Out] `Integral(x**2*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x^2}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^2,x, algorithm="giac")`

[Out] `integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^2, x)`

$$3.279 \quad \int \frac{x(d^2 - e^2 x^2)^p}{(d + ex)^2} dx$$

**Optimal.** Leaf size=115

$$\frac{(d^2 - e^2 x^2)^{p+1}}{2e^2(1-p)(d+ex)^2} - \frac{2^{p-1} \left(\frac{ex}{d} + 1\right)^{-p-1} (d^2 - e^2 x^2)^{p+1} {}_2F_1\left(1-p, p+1; p+2; \frac{d-ex}{2d}\right)}{d^2 e^2 (1-p^2)}$$

[Out]  $(d^2 - e^2 x^2)^{p+1} / (2 e^2 (1-p) (d+ex)^2) - (2^{p-1} (ex/d + 1)^{-p-1} (d^2 - e^2 x^2)^{p+1} {}_2F_1(1-p, p+1; p+2; (d-ex)/(2d))) / (d^2 e^2 (1-p^2))$

**Rubi [A]** time = 0.154419, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$

$$\frac{(d^2 - e^2 x^2)^{p+1}}{2e^2(1-p)(d+ex)^2} - \frac{2^{p-1} \left(\frac{ex}{d} + 1\right)^{-p-1} (d^2 - e^2 x^2)^{p+1} {}_2F_1\left(1-p, p+1; p+2; \frac{d-ex}{2d}\right)}{d^2 e^2 (1-p^2)}$$

Antiderivative was successfully verified.

[In] Int[(x\*(d^2 - e^2\*x^2)^p)/(d + e\*x)^2, x]

[Out]  $(d^2 - e^2 x^2)^{p+1} / (2 e^2 (1-p) (d+ex)^2) - (2^{p-1} (ex/d + 1)^{-p-1} (d^2 - e^2 x^2)^{p+1} {}_2F_1(1-p, p+1; p+2; (d-ex)/(2d))) / (d^2 e^2 (1-p^2))$

**Rubi in Sympy [A]** time = 48.4315, size = 75, normalized size = 0.65

$$\frac{(d-ex)^2 (d^2 - e^2 x^2)^{p-1}}{2e^2(-p+1)} + \frac{\left(\frac{d-ex}{2d}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(\frac{-p, p}{p+1} \middle| \frac{d+ex}{d}\right)}{e^2 p (-p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d)\*\*2, x)

[Out]  $(d - e*x)^2 * (d^2 - e^2*x^2)^{p-1} / (2*e^2*(-p+1)) + ((d/2 - e*x/2)/d)^{-p} * (d^2 - e^2*x^2)^p * \text{hyper}((-p, p), (p+1), (d/2 + e*x/2)/d) / (e^2*p*(-p+1))$

**Mathematica [A]** time = 0.0878113, size = 102, normalized size = 0.89

$$\frac{2^{p-2} (d-ex) \left(\frac{ex}{d} + 1\right)^{-p} (d^2 - e^2 x^2)^p \left( {}_2F_1\left(2-p, p+1; p+2; \frac{d-ex}{2d}\right) - 2 {}_2F_1\left(1-p, p+1; p+2; \frac{d-ex}{2d}\right) \right)}{d e^2 (p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(d^2 - e^2\*x^2)^p)/(d + e\*x)^2, x]

[Out]  $(2^{p-2} (d-ex) (ex/d + 1)^{-p} (d^2 - e^2 x^2)^p ({}_2F_1(2-p, p+1; p+2; (d-ex)/(2d)) - 2 {}_2F_1(1-p, p+1; p+2; (d-ex)/(2d)))) / (d e^2 (p+1))$

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**Maple [F]** time = 0.088, size = 0, normalized size = 0.

$$\int \frac{x(-e^2x^2 + d^2)^p}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(-e^2\*x^2+d^2)^p/(e\*x+d)^2, x)

[Out] int(x\*(-e^2\*x^2+d^2)^p/(e\*x+d)^2, x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x/(e\*x + d)^2, x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p\*x/(e\*x + d)^2, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p x}{e^2x^2 + 2dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x/(e\*x + d)^2, x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p\*x/(e^2\*x^2 + 2\*d\*e\*x + d^2), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x(-(-d + ex)(d + ex))^p}{(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d)\*\*2, x)

[Out] Integral(x\*(-(-d + e\*x)\*(d + e\*x))\*\*p/(d + e\*x)\*\*2, x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^2,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^2, x)
```



$$3.280 \quad \int \frac{(d^2 - e^2 x^2)^p}{(d + ex)^2} dx$$

**Optimal.** Leaf size=73

$$\frac{2^{p-2} \left(\frac{ex}{d} + 1\right)^{-p-1} (d^2 - e^2 x^2)^{p+1} {}_2F_1\left(2 - p, p + 1; p + 2; \frac{d - ex}{2d}\right)}{d^3 e(p + 1)}$$

[Out] -((2^(-2 + p)\*(1 + (e\*x)/d)^(-1 - p)\*(d^2 - e^2\*x^2)^(1 + p)\*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/(d^3\*e\*(1 + p)))

**Rubi [A]** time = 0.0823163, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{2^{p-2} \left(\frac{ex}{d} + 1\right)^{-p-1} (d^2 - e^2 x^2)^{p+1} {}_2F_1\left(2 - p, p + 1; p + 2; \frac{d - ex}{2d}\right)}{d^3 e(p + 1)}$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^p/(d + e\*x)^2,x]

[Out] -((2^(-2 + p)\*(1 + (e\*x)/d)^(-1 - p)\*(d^2 - e^2\*x^2)^(1 + p)\*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/(d^3\*e\*(1 + p)))

**Rubi in Sympy [A]** time = 26.7232, size = 66, normalized size = 0.9

$$\frac{\left(\frac{\frac{d}{2} + \frac{ex}{2}}{d}\right)^{-p} (d - ex)^{-p} (d - ex)^{p+1} (d^2 - e^2 x^2)^p {}_2F_1\left(\begin{matrix} -p + 2, p + 1 \\ p + 2 \end{matrix} \middle| \frac{\frac{d}{2} - \frac{ex}{2}}{d}\right)}{4d^2 e(p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d)\*\*2,x)

[Out] -((d/2 + e\*x/2)/d)\*\*(-p)\*(d - e\*x)\*\*(-p)\*(d - e\*x)\*\*(p + 1)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p\*hyper((-p + 2, p + 1), (p + 2, ), (d/2 - e\*x/2)/d)/(4\*d\*\*2\*e\*(p + 1))

**Mathematica [A]** time = 0.0529265, size = 75, normalized size = 1.03

$$\frac{2^{p-2}(d - ex) \left(\frac{ex}{d} + 1\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(2 - p, p + 1; p + 2; \frac{d - ex}{2d}\right)}{d^2 e(p + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^p/(d + e\*x)^2,x]

[Out] -((2^(-2 + p)\*(d - e\*x)\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/(d^2\*e\*(1 + p)\*(1 + (e\*x)/d)^p))

---

**Maple [F]** time = 0.075, size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^P}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^p/(e\*x+d)^2, x)

[Out] int((-e^2\*x^2+d^2)^p/(e\*x+d)^2, x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^P}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/(e\*x + d)^2, x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p/(e\*x + d)^2, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^P}{e^2x^2 + 2dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/(e\*x + d)^2, x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p/(e^2\*x^2 + 2\*d\*e\*x + d^2), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(-d + ex)(d + ex))^P}{(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d)\*\*2, x)

[Out] Integral((-(-d + e\*x)\*(d + e\*x))\*\*p/(d + e\*x)\*\*2, x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^P}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^p/(e*x + d)^2,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/(e*x + d)^2, x)
```

$$3.281 \quad \int \frac{(d^2 - e^2 x^2)^p}{x(d+ex)^2} dx$$

**Optimal.** Leaf size=128

$$\frac{(d^2 - e^2 x^2)^p {}_2F_1\left(1, p; p+1; 1 - \frac{e^2 x^2}{d^2}\right)}{2d^2 p} + \frac{(d^2 - e^2 x^2)^{p-1}}{1-p} - \frac{2ex \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(\frac{1}{2}, 2-p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^3}$$

[Out] (d^2 - e^2\*x^2)^(-1 + p)/(1 - p) - (2\*e\*x\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[1/2, 2 - p, 3/2, (e^2\*x^2)/d^2])/(d^3\*(1 - (e^2\*x^2)/d^2)^p) - ((d^2 - e^2\*x^2)^p\*Hypergeometric2F1[1, p, 1 + p, 1 - (e^2\*x^2)/d^2])/(2\*d^2\*p)

**Rubi [A]** time = 0.274992, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.32$

$$\frac{(d^2 - e^2 x^2)^p {}_2F_1\left(1, p; p+1; 1 - \frac{e^2 x^2}{d^2}\right)}{2d^2 p} + \frac{(d^2 - e^2 x^2)^{p-1}}{1-p} - \frac{2ex \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(\frac{1}{2}, 2-p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^3}$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^p/(x\*(d + e\*x)^2), x]

[Out] (d^2 - e^2\*x^2)^(-1 + p)/(1 - p) - (2\*e\*x\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[1/2, 2 - p, 3/2, (e^2\*x^2)/d^2])/(d^3\*(1 - (e^2\*x^2)/d^2)^p) - ((d^2 - e^2\*x^2)^p\*Hypergeometric2F1[1, p, 1 + p, 1 - (e^2\*x^2)/d^2])/(2\*d^2\*p)

**Rubi in Sympy [A]** time = 60.8306, size = 102, normalized size = 0.8

$$\frac{(d^2 - e^2 x^2)^{p-1} {}_2F_1\left(1, p-1 \middle| 1 - \frac{e^2 x^2}{d^2}\right)}{2(-p+1)} + \frac{(d^2 - e^2 x^2)^{p-1}}{2(-p+1)} - \frac{2ex \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+2, \frac{1}{2} \middle| \frac{e^2 x^2}{d^2}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x/(e\*x+d)\*\*2, x)

[Out] (d\*\*2 - e\*\*2\*x\*\*2)\*\*(p - 1)\*hyper((1, p - 1), (p, ), 1 - e\*\*2\*x\*\*2/d\*\*2)/(2\*(-p + 1)) + (d\*\*2 - e\*\*2\*x\*\*2)\*\*(p - 1)/(2\*(-p + 1)) - 2\*e\*x\*(1 - e\*\*2\*x\*\*2/d\*\*2)\*\*(-p)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p\*hyper((-p + 2, 1/2), (3/2, ), e\*\*2\*x\*\*2/d\*\*2)/d\*\*3

**Mathematica [A]** time = 0.223476, size = 201, normalized size = 1.57

$$\frac{2^{p-2} \left(1 - \frac{d^2}{e^2 x^2}\right)^{-p} \left(\frac{ex}{d} + 1\right)^{-p} (d^2 - e^2 x^2)^p \left(2p(d - ex) \left(1 - \frac{d^2}{e^2 x^2}\right)^p {}_2F_1\left(1 - p, p + 1; p + 2; \frac{d - ex}{2d}\right) + p(d - ex) \left(1 - \frac{d^2}{e^2 x^2}\right)^p\right)}{d^3 p(p + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^p/(x\*(d + e\*x)^2), x]

[Out] (2^(-2 + p)\*(d^2 - e^2\*x^2)^p\*(2\*p\*(1 - d^2/(e^2\*x^2))^p\*(d - e\*x)\*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)] + p\*(1 - d^2/(e^2\*x^2))^p\*(d - e\*x)\*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)] + 2\*d\*(1 + p)\*(1/2 + (e\*x)/(2\*d))^p\*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2\*x^2)])/(d^3\*p\*(1 + p)\*(1 - d^2/(e^2\*x^2))^p\*(1 + (e\*x)/d)^p)

**Maple [F]** time = 0.095, size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{x(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^p/x/(e\*x+d)^2, x)

[Out] int((-e^2\*x^2+d^2)^p/x/(e\*x+d)^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^2x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^2\*x), x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^2\*x), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p}{e^2x^3 + 2dex^2 + d^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^2\*x), x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p/(e^2\*x^3 + 2\*d\*e\*x^2 + d^2\*x), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(-d + ex)(d + ex))^p}{x(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x/(e\*x+d)\*\*2,x)

[Out] Integral((-(-d + e\*x)\*(d + e\*x))\*\*p/(x\*(d + e\*x)\*\*2), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^2x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^2\*x),x, algorithm="giac")

[Out] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^2\*x), x)

$$3.282 \quad \int \frac{(d^2 - e^2 x^2)^p}{x^2 (d + ex)^2} dx$$

**Optimal.** Leaf size=137

$$\frac{e (d^2 - e^2 x^2)^{p-1} {}_2F_1\left(1, p-1; p; 1 - \frac{e^2 x^2}{d^2}\right)}{d(1-p)} - \frac{(d^2 - e^2 x^2)^{p-1}}{x} + \frac{2e^2(2-p)x \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(\frac{1}{2}, 2-p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^4}$$

[Out]  $-\left(\frac{d^2 - e^2 x^2}{d^2}\right)^{-1+p}/x + (2e^2(2-p)x \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left[\frac{1}{2}, 2-p, \frac{3}{2}, \frac{e^2 x^2}{d^2}\right]) / (d^4 (1 - (e^2 x^2)/d^2)^p) - (e^2 (d^2 - e^2 x^2)^{-1+p} {}_2F_1[1, -1+p, p, 1 - (e^2 x^2)/d^2]) / (d(1-p))$

**Rubi [A]** time = 0.340382, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$\frac{e (d^2 - e^2 x^2)^{p-1} {}_2F_1\left(1, p-1; p; 1 - \frac{e^2 x^2}{d^2}\right)}{d(1-p)} - \frac{(d^2 - e^2 x^2)^{p-1}}{x} + \frac{2e^2(2-p)x \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(\frac{1}{2}, 2-p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^4}$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^p/(x^2\*(d + e\*x)^2), x]

[Out]  $-\left(\frac{d^2 - e^2 x^2}{d^2}\right)^{-1+p}/x + (2e^2(2-p)x \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left[\frac{1}{2}, 2-p, \frac{3}{2}, \frac{e^2 x^2}{d^2}\right]) / (d^4 (1 - (e^2 x^2)/d^2)^p) - (e^2 (d^2 - e^2 x^2)^{-1+p} {}_2F_1[1, -1+p, p, 1 - (e^2 x^2)/d^2]) / (d(1-p))$

**Rubi in Sympy [A]** time = 68.7371, size = 134, normalized size = 0.98

$$\frac{e (d^2 - e^2 x^2)^{p-1} {}_2F_1\left(1, p-1; p; 1 - \frac{e^2 x^2}{d^2}\right)}{d(-p+1)} - \frac{\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+2, -\frac{1}{2}; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{d^2 x} + \frac{e^2 x \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+2, \frac{1}{2}; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x\*\*2/(e\*x+d)\*\*2, x)

[Out]  $-e^2(d^2 - e^2 x^2)^p (p-1) \text{hyper}((1, p-1), (p,), 1 - e^2 x^2/d^2) / (d^2 (-p+1)) - (1 - e^2 x^2/d^2)^{-p} (d^2 - e^2 x^2)^p \text{hyper}((-p+2, -1/2), (1/2,), e^2 x^2/d^2) / (d^2 x) + e^2 x (1 - e^2 x^2/d^2)^{-p} (d^2 - e^2 x^2)^p \text{hyper}((-p+2, 1/2), (3/2,), e^2 x^2/d^2) / d^4$

**Mathematica [C]** time = 0.475811, size = 195, normalized size = 1.42

$$\frac{2e(p-2)(d-ex)^p(d+ex)^{p-2}F_1\left(3-2p; -p, 2-p; 4-2p; \frac{d}{ex}, -\frac{d}{ex}\right)}{(2p-3)\left(2e(p-2)x F_1\left(3-2p; -p, 2-p; 4-2p; \frac{d}{ex}, -\frac{d}{ex}\right) + dp F_1\left(4-2p; 1-p, 2-p; 5-2p; \frac{d}{ex}, -\frac{d}{ex}\right) - d(p-2)F_1\left(4-2p; 1-p, 2-p; 5-2p; \frac{d}{ex}, -\frac{d}{ex}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(d^2 - e^2\*x^2)^p/(x^2\*(d + e\*x)^2), x]

[Out] (2\*e\*(-2 + p)\*(d - e\*x)^p\*(d + e\*x)^(-2 + p)\*AppellF1[3 - 2\*p, -p, 2 - p, 4 - 2\*p, d/(e\*x), -(d/(e\*x))])/((-3 + 2\*p)\*(2\*e\*(-2 + p)\*x\*AppellF1[3 - 2\*p, -p, 2 - p, 4 - 2\*p, d/(e\*x), -(d/(e\*x))] + d\*p\*AppellF1[4 - 2\*p, 1 - p, 2 - p, 5 - 2\*p, d/(e\*x), -(d/(e\*x))] - d\*(-2 + p)\*AppellF1[4 - 2\*p, -p, 3 - p, 5 - 2\*p, d/(e\*x), -(d/(e\*x))]))

**Maple [F]** time = 0.08, size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{x^2(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^p/x^2/(e\*x+d)^2, x)

[Out] int((-e^2\*x^2+d^2)^p/x^2/(e\*x+d)^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^2x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^2\*x^2), x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^2\*x^2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p}{e^2x^4 + 2dex^3 + d^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^2\*x^2), x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p/(e^2\*x^4 + 2\*d\*e\*x^3 + d^2\*x^2), x)



**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(-d + ex)(d + ex))^p}{x^2 (d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x\*\*2/(e\*x+d)\*\*2, x)

[Out] Integral((-(-d + e\*x)\*(d + e\*x))\*\*p/(x\*\*2\*(d + e\*x)\*\*2), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^2x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^2\*x^2), x, algorithm="giac")

[Out] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^2\*x^2), x)

$$3.283 \quad \int \frac{(d^2 - e^2 x^2)^p}{x^3 (d + ex)^2} dx$$

**Optimal.** Leaf size=143

$$\frac{e^2(3-p)(d^2 - e^2 x^2)^{p-1} {}_2F_1\left(1, p-1; p; 1 - \frac{e^2 x^2}{d^2}\right)}{2d^2(1-p)} - \frac{(d^2 - e^2 x^2)^{p-1}}{2x^2} + \frac{2e\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-\frac{1}{2}, 2-p; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{d^3 x}$$

[Out]  $-(d^2 - e^2 x^2)^{-1+p}/(2x^2) + (2e^*(d^2 - e^2 x^2)^p \text{Hypergeometric2F1}[-1/2, 2-p, 1/2, (e^2 x^2)/d^2])/(d^3 x^*(1 - (e^2 x^2)/d^2)^p) + (e^2*(3-p)*(d^2 - e^2 x^2)^{-1+p} \text{Hypergeometric2F1}[1, -1+p, p, 1 - (e^2 x^2)/d^2])/(2*d^2*(1-p))$

**Rubi [A]** time = 0.356275, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$\frac{e^2(3-p)(d^2 - e^2 x^2)^{p-1} {}_2F_1\left(1, p-1; p; 1 - \frac{e^2 x^2}{d^2}\right)}{2d^2(1-p)} - \frac{(d^2 - e^2 x^2)^{p-1}}{2x^2} + \frac{2e\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-\frac{1}{2}, 2-p; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{d^3 x}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d^2 - e^2 x^2)^p / (x^3 (d + e x)^2), x]$

[Out]  $-(d^2 - e^2 x^2)^{-1+p}/(2x^2) + (2e^*(d^2 - e^2 x^2)^p \text{Hypergeometric2F1}[-1/2, 2-p, 1/2, (e^2 x^2)/d^2])/(d^3 x^*(1 - (e^2 x^2)/d^2)^p) + (e^2*(3-p)*(d^2 - e^2 x^2)^{-1+p} \text{Hypergeometric2F1}[1, -1+p, p, 1 - (e^2 x^2)/d^2])/(2*d^2*(1-p))$

**Rubi in Sympy [A]** time = 58.8943, size = 133, normalized size = 0.93

$$\frac{e^2 (d^2 - e^2 x^2)^{p-1} {}_2F_1\left(1, p-1 \middle| 1 - \frac{e^2 x^2}{d^2}\right)}{2d^2(-p+1)} + \frac{e^2 (d^2 - e^2 x^2)^{p-1} {}_2F_1\left(2, p-1 \middle| 1 - \frac{e^2 x^2}{d^2}\right)}{2d^2(-p+1)} + \frac{2e\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+2, -\frac{1}{2} \middle| \frac{e^2 x^2}{d^2}\right)}{d^3 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((-e^{**2}x^{**2}+d^{**2})^{**p}/x^{**3}/(e*x+d)^{**2}, x)$

[Out]  $e^{**2}*(d^{**2} - e^{**2}x^{**2})^{**p} \text{hyper}((1, p-1), (p, ), 1 - e^{**2}x^{**2}/d^{**2})/(2*d^{**2}*(-p+1)) + e^{**2}*(d^{**2} - e^{**2}x^{**2})^{**p} \text{hyper}((2, p-1), (p, ), 1 - e^{**2}x^{**2}/d^{**2})/(2*d^{**2}*(-p+1)) + 2*e*(1 - e^{**2}x^{**2}/d^{**2})^{**p}*(d^{**2} - e^{**2}x^{**2})^{**p} \text{hyper}((-p+2, -1/2), (1/2, ), e^{**2}x^{**2}/d^{**2})/(d^{**3}x)$

**Mathematica [A]** time = 1.18532, size = 283, normalized size = 1.98

$$\frac{(d^2 - e^2 x^2)^p \left( \frac{6de^2 \left(1 - \frac{d^2}{e^2 x^2}\right)^{-p} {}_2F_1\left(-p, -p; 1-p; \frac{d^2}{e^2 x^2}\right)}{p} + \frac{8d^2 e \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{x} + \frac{2d^3 \left(1 - \frac{d^2}{e^2 x^2}\right)^{-p} {}_2F_1\left(1-p, -p; 2-p; \frac{d^2}{e^2 x^2}\right)}{(p-1)x^2} + \frac{3e^2 2^p}{4d^5} \right)}{4d^5}$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^p/(x^3\*(d + e\*x)^2), x]

[Out] ((d^2 - e^2\*x^2)^p\*((8\*d^2\*e\*Hypergeometric2F1[-1/2, -p, 1/2, (e^2\*x^2)/d^2])/(x\*(1 - (e^2\*x^2)/d^2)^p) + (2\*d^3\*Hypergeometric2F1[1 - p, -p, 2 - p, d^2/(e^2\*x^2)])/((-1 + p)\*(1 - d^2/(e^2\*x^2))^p\*x^2) + (3\*2^(1 + p)\*e^2\*(d - e\*x)\*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/((1 + p)\*(1 + (e\*x)/d)^p) + (2^p\*e^2\*(d - e\*x)\*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/((1 + p)\*(1 + (e\*x)/d)^p) + (6\*d\*e^2\*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2\*x^2)])/(p\*(1 - d^2/(e^2\*x^2))^p))/(4\*d^5)

**Maple [F]** time = 0.089, size = 0, normalized size = 0.

$$\int \frac{(-e^2 x^2 + d^2)^p}{x^3 (ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^p/x^3/(e\*x+d)^2, x)

[Out] int((-e^2\*x^2+d^2)^p/x^3/(e\*x+d)^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2 x^2 + d^2)^p}{(ex + d)^2 x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^2\*x^3), x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^2\*x^3), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2 x^2 + d^2)^p}{e^2 x^5 + 2 d e x^4 + d^2 x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^2\*x^3), x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p/(e^2\*x^5 + 2\*d\*e\*x^4 + d^2\*x^3), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(-d + ex)(d + ex))^p}{x^3 (d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x\*\*3/(e\*x+d)\*\*2, x)

[Out] Integral((-(-d + e\*x)\*(d + e\*x))\*\*p/(x\*\*3\*(d + e\*x)\*\*2), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^2x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^2\*x^3), x, algorithm="giac")

[Out] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^2\*x^3), x)

$$3.284 \quad \int \frac{(d^2 - e^2 x^2)^p}{x^4 (d + ex)^2} dx$$

**Optimal.** Leaf size=145

$$\frac{(d^2 - e^2 x^2)^{p-1}}{3x^3} - \frac{2e^2(4-p) \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-\frac{1}{2}, 2-p; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{3d^4 x} - \frac{e^3 (d^2 - e^2 x^2)^{p-1} {}_2F_1\left(2, p-1; p; 1 - \frac{e^2 x^2}{d^2}\right)}{d^3(1-p)}$$

[Out]  $-(d^2 - e^2 x^2)^{-1+p}/(3x^3) - (2e^2(4-p)(d^2 - e^2 x^2)^p \text{Hypergeometric2F1}[-1/2, 2-p, 1/2, (e^2 x^2)/d^2])/(3d^4 x^3 (1 - (e^2 x^2)/d^2)^p) - (e^3 (d^2 - e^2 x^2)^{p-1} \text{Hypergeometric2F1}[2, -1+p, p, 1 - (e^2 x^2)/d^2])/(d^3(1-p))$

**Rubi [A]** time = 0.380129, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$\frac{(d^2 - e^2 x^2)^{p-1}}{3x^3} - \frac{2e^2(4-p) \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-\frac{1}{2}, 2-p; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{3d^4 x} - \frac{e^3 (d^2 - e^2 x^2)^{p-1} {}_2F_1\left(2, p-1; p; 1 - \frac{e^2 x^2}{d^2}\right)}{d^3(1-p)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d^2 - e^2 x^2)^p / (x^4 (d + e x)^2), x]$

[Out]  $-(d^2 - e^2 x^2)^{-1+p}/(3x^3) - (2e^2(4-p)(d^2 - e^2 x^2)^p \text{Hypergeometric2F1}[-1/2, 2-p, 1/2, (e^2 x^2)/d^2])/(3d^4 x^3 (1 - (e^2 x^2)/d^2)^p) - (e^3 (d^2 - e^2 x^2)^{p-1} \text{Hypergeometric2F1}[2, -1+p, p, 1 - (e^2 x^2)/d^2])/(d^3(1-p))$

**Rubi in Sympy [A]** time = 62.504, size = 146, normalized size = 1.01

$$\frac{\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+2, -\frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{3d^2 x^3} - \frac{e^3 (d^2 - e^2 x^2)^{p-1} {}_2F_1\left(2, p-1; p; 1 - \frac{e^2 x^2}{d^2}\right)}{d^3(-p+1)} - \frac{e^2 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+2, -\frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{d^4 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((-e^{**2}x^{**2}+d^{**2})^{**p}/x^{**4}/(e*x+d)^{**2}, x)$

[Out]  $-(1 - e^{**2}x^{**2}/d^{**2})^{**(-p)}(d^{**2} - e^{**2}x^{**2})^{**p} \text{hyper}((-p + 2, -3/2), (-1/2, ), e^{**2}x^{**2}/d^{**2})/(3d^{**2}x^{**3}) - e^{**3}(d^{**2} - e^{**2}x^{**2})^{**p} \text{hyper}((2, p - 1), (p, ), 1 - e^{**2}x^{**2}/d^{**2})/(d^{**3}(-p + 1)) - e^{**2}(1 - e^{**2}x^{**2}/d^{**2})^{**(-p)}(d^{**2} - e^{**2}x^{**2})^{**p} \text{hyper}((-p + 2, -1/2), (1/2, ), e^{**2}x^{**2}/d^{**2})/(d^{**4}x)$

**Mathematica [B]** time = 0.602752, size = 334, normalized size = 2.3

$$(d^2 - e^2 x^2)^p \left( -\frac{36d^2 e^2 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{x} - \frac{24de^3 \left(1 - \frac{d^2}{e^2 x^2}\right)^{-p} {}_2F_1\left(-p, -p; 1-p; \frac{d^2}{e^2 x^2}\right)}{p} - \frac{4d^4 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{3}{2}, -p; -\frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{x^3} - \dots \right)$$

---

12d<sup>6</sup>

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^p/(x^4\*(d + e\*x)^2), x]

[Out] ((d^2 - e^2\*x^2)^p\*((-4\*d^4\*Hypergeometric2F1[-3/2, -p, -1/2, (e^2\*x^2)/d^2])/(x^3\*(1 - (e^2\*x^2)/d^2)^p) - (36\*d^2\*e^2\*Hypergeometric2F1[-1/2, -p, 1/2, (e^2\*x^2)/d^2])/(x\*(1 - (e^2\*x^2)/d^2)^p) - (12\*d^3\*e\*Hypergeometric2F1[1 - p, -p, 2 - p, d^2/(e^2\*x^2)])/((-1 + p)\*(1 - d^2/(e^2\*x^2))^p\*x^2) + (3\*2^(3 + p)\*e^3\*(-d + e\*x)\*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/((1 + p)\*(1 + (e\*x)/d)^p) + (3\*2^p\*e^3\*(-d + e\*x)\*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/((1 + p)\*(1 + (e\*x)/d)^p) - (2\*4\*d\*e^3\*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2\*x^2)])/(p\*(1 - d^2/(e^2\*x^2))^p))/(12\*d^6)

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**Maple [F]** time = 0.109, size = 0, normalized size = 0.

$$\int \frac{(-e^2 x^2 + d^2)^p}{x^4 (ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^p/x^4/(e\*x+d)^2, x)

[Out] int((-e^2\*x^2+d^2)^p/x^4/(e\*x+d)^2, x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2 x^2 + d^2)^p}{(ex + d)^2 x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^2\*x^4), x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^2\*x^4), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2 x^2 + d^2)^p}{e^2 x^6 + 2 dex^5 + d^2 x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^2\*x^4), x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p/(e^2\*x^6 + 2\*d\*e\*x^5 + d^2\*x^4), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(-d + ex)(d + ex))^p}{x^4 (d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x\*\*4/(e\*x+d)\*\*2,x)

[Out] Integral((-(-d + e\*x)\*(d + e\*x))\*\*p/(x\*\*4\*(d + e\*x)\*\*2), x)

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^2x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^2\*x^4),x, algorithm="giac")

[Out] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^2\*x^4), x)

$$3.285 \quad \int \frac{(d^2 - e^2 x^2)^p}{x^5 (d + ex)^2} dx$$

**Optimal.** Leaf size=145

$$\begin{aligned} & -\frac{(d^2 - e^2 x^2)^{p-1}}{4x^4} + \frac{e^4(5-p)(d^2 - e^2 x^2)^{p-1} {}_2F_1\left(2, p-1; p; 1 - \frac{e^2 x^2}{d^2}\right)}{4d^4(1-p)} \\ & + \frac{2e\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-\frac{3}{2}, 2-p; -\frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{3d^3 x^3} \end{aligned}$$

[Out]  $-(d^2 - e^2 x^2)^{-1+p}/(4x^4) + (2e^4(d^2 - e^2 x^2)^{p-1} \text{Hypergeometric2F1}[-3/2, 2-p, -1/2, (e^2 x^2)/d^2])/(3d^4 x^3 (1 - (e^2 x^2)/d^2)^p) + (2e(d^2 - e^2 x^2)^p \text{Hypergeometric2F1}[2, -1+p, p, 1 - (e^2 x^2)/d^2])/(4d^4 (1-p))$

**Rubi [A]** time = 0.366456, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$\begin{aligned} & -\frac{(d^2 - e^2 x^2)^{p-1}}{4x^4} + \frac{e^4(5-p)(d^2 - e^2 x^2)^{p-1} {}_2F_1\left(2, p-1; p; 1 - \frac{e^2 x^2}{d^2}\right)}{4d^4(1-p)} \\ & + \frac{2e\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-\frac{3}{2}, 2-p; -\frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{3d^3 x^3} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d^2 - e^2 x^2)^p / (x^5 (d + e x)^2), x]$

[Out]  $-(d^2 - e^2 x^2)^{-1+p}/(4x^4) + (2e^4(d^2 - e^2 x^2)^{p-1} \text{Hypergeometric2F1}[-3/2, 2-p, -1/2, (e^2 x^2)/d^2])/(3d^4 x^3 (1 - (e^2 x^2)/d^2)^p) + (2e(d^2 - e^2 x^2)^p \text{Hypergeometric2F1}[2, -1+p, p, 1 - (e^2 x^2)/d^2])/(4d^4 (1-p))$

**Rubi in Sympy [A]** time = 64.3299, size = 138, normalized size = 0.95

$$\begin{aligned} & \frac{2e\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+2, -\frac{3}{2}; -\frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{3d^3 x^3} \\ & + \frac{e^4 (d^2 - e^2 x^2)^{p-1} {}_2F_1\left(2, p-1; p; 1 - \frac{e^2 x^2}{d^2}\right)}{2d^4 (-p+1)} + \frac{e^4 (d^2 - e^2 x^2)^{p-1} {}_2F_1\left(3, p-1; p; 1 - \frac{e^2 x^2}{d^2}\right)}{2d^4 (-p+1)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((-e^{**2}x^{**2}+d^{**2})^{**p}/x^{**5}/(e*x+d)^{**2}, x)$

[Out]  $2e^4(1 - e^{**2}x^{**2}/d^{**2})^{**(-p)}(d^{**2} - e^{**2}x^{**2})^{**p} \text{hyper}((-p+2, -3/2), (-1/2, ), e^{**2}x^{**2}/d^{**2})/(3d^{**3}x^{**3}) + e^{**4}(d^{**2} - e^{**2}x^{**2})^{**p} \text{hyper}((2, p-1), (p, ), 1 - e^{**2}x^{**2}/d^{**2})/(2d^{**4}(-p+1)) + e^{**4}(d^{**2} - e^{**2}x^{**2})^{**p} \text{hyper}((3, p-1), (p, ), 1 - e^{**2}x^{**2}/d^{**2})/(2d^{**4}(-p+1))$



**Mathematica [B]** time = 1.00442, size = 389, normalized size = 2.68

$$(d^2 - e^2 x^2)^p \left( \frac{30 d e^4 \left(1 - \frac{d^2}{e^2 x^2}\right)^{-p} {}_2F_1\left(-p, -p; 1-p; \frac{d^2}{e^2 x^2}\right)}{p} + \frac{48 d^2 e^3 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{x} + \frac{6 d^5 \left(1 - \frac{d^2}{e^2 x^2}\right)^{-p} {}_2F_1\left(2-p, -p; 3-p; \frac{d^2}{e^2 x^2}\right)}{(p-2)x^4} + \frac{8 d^4}{(p-2)x^4} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^p/(x^5\*(d + e\*x)^2),x]

[Out] ((d^2 - e^2\*x^2)^p\*((8\*d^4\*e\*Hypergeometric2F1[-3/2, -p, -1/2, (e^2\*x^2)/d^2])/(x^3\*(1 - (e^2\*x^2)/d^2)^p) + (48\*d^2\*e^3\*Hypergeometric2F1[-1/2, -p, 1/2, (e^2\*x^2)/d^2])/(x\*(1 - (e^2\*x^2)/d^2)^p) + (18\*d^3\*e^2\*Hypergeometric2F1[1 - p, -p, 2 - p, d^2/(e^2\*x^2)])/((-1 + p)\*(1 - d^2/(e^2\*x^2))^p\*x^2) + (15\*2^(1 + p)\*e^4\*(d - e\*x)\*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/((1 + p)\*(1 + (e\*x)/d)^p) + (6\*d^5\*Hypergeometric2F1[2 - p, -p, 3 - p, d^2/(e^2\*x^2)])/((-2 + p)\*(1 - d^2/(e^2\*x^2))^p\*x^4) + (3\*2^p\*e^4\*(d - e\*x)\*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/((1 + p)\*(1 + (e\*x)/d)^p) + (30\*d\*e^4\*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2\*x^2)])/(p\*(1 - d^2/(e^2\*x^2))^p))/((12\*d^7)

**Maple [F]** time = 0.125, size = 0, normalized size = 0.

$$\int \frac{(-e^2 x^2 + d^2)^p}{x^5 (e x + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^p/x^5/(e\*x+d)^2,x)

[Out] int((-e^2\*x^2+d^2)^p/x^5/(e\*x+d)^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2 x^2 + d^2)^p}{(e x + d)^2 x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^2\*x^5),x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^2\*x^5), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2 x^2 + d^2)^p}{e^2 x^7 + 2 d e x^6 + d^2 x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^2\*x^5),x, algorithm="fricas")

[Out] `integral((-e^2*x^2 + d^2)^p/(e^2*x^7 + 2*d*e*x^6 + d^2*x^5), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(-d + ex)(d + ex))^p}{x^5 (d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e**2*x**2+d**2)**p/x**5/(e*x+d)**2,x)`

[Out] `Integral((-(-d + e*x)*(d + e*x))**p/(x**5*(d + e*x)**2), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^2x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^5),x, algorithm="giac")`

[Out] `integrate((-e^2*x^2 + d^2)^p/((e*x + d)^2*x^5), x)`

$$3.286 \quad \int \frac{x^4 (d^2 - e^2 x^2)^p}{(d + ex)^3} dx$$

**Optimal.** Leaf size=220

$$\begin{aligned} & -\frac{3dx^5 (d^2 - e^2 x^2)^{p-2}}{2p+1} + \frac{3d^2 (d^2 - e^2 x^2)^p}{e^5 p} - \frac{(d^2 - e^2 x^2)^{p+1}}{2e^5 (p+1)} - \frac{2d^6 (d^2 - e^2 x^2)^{p-2}}{e^5 (2-p)} \\ & + \frac{9d^4 (d^2 - e^2 x^2)^{p-1}}{2e^5 (1-p)} + \frac{2(p+8)x^5 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(\frac{5}{2}, 3-p; \frac{7}{2}; \frac{e^2 x^2}{d^2}\right)}{5d^3 (2p+1)} \end{aligned}$$

[Out]  $(-2*d^6*(d^2 - e^2*x^2)^{(-2 + p)})/(e^5*(2 - p)) - (3*d*x^5*(d^2 - e^2*x^2)^{(-2 + p)})/(1 + 2*p) + (9*d^4*(d^2 - e^2*x^2)^{(-1 + p)})/(2*e^5*(1 - p)) + (3*d^2*(d^2 - e^2*x^2)^p)/(e^5*p) - (d^2 - e^2*x^2)^{(1 + p)}/(2*e^5*(1 + p)) + (2*(8 + p)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, 3 - p, 7/2, (e^2*x^2)/d^2])/(5*d^3*(1 + 2*p))*(1 - (e^2*x^2)/d^2)^p$

**Rubi [A]** time = 0.531737, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$\begin{aligned} & -\frac{3dx^5 (d^2 - e^2 x^2)^{p-2}}{2p+1} + \frac{3d^2 (d^2 - e^2 x^2)^p}{e^5 p} - \frac{(d^2 - e^2 x^2)^{p+1}}{2e^5 (p+1)} - \frac{2d^6 (d^2 - e^2 x^2)^{p-2}}{e^5 (2-p)} \\ & + \frac{9d^4 (d^2 - e^2 x^2)^{p-1}}{2e^5 (1-p)} + \frac{2(p+8)x^5 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(\frac{5}{2}, 3-p; \frac{7}{2}; \frac{e^2 x^2}{d^2}\right)}{5d^3 (2p+1)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*(d^2 - e^2\*x^2)^p)/(d + e\*x)^3, x]

[Out]  $(-2*d^6*(d^2 - e^2*x^2)^{(-2 + p)})/(e^5*(2 - p)) - (3*d*x^5*(d^2 - e^2*x^2)^{(-2 + p)})/(1 + 2*p) + (9*d^4*(d^2 - e^2*x^2)^{(-1 + p)})/(2*e^5*(1 - p)) + (3*d^2*(d^2 - e^2*x^2)^p)/(e^5*p) - (d^2 - e^2*x^2)^{(1 + p)}/(2*e^5*(1 + p)) + (2*(8 + p)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, 3 - p, 7/2, (e^2*x^2)/d^2])/(5*d^3*(1 + 2*p))*(1 - (e^2*x^2)/d^2)^p$

**Rubi in Sympy [A]** time = 132.337, size = 202, normalized size = 0.92

$$\begin{aligned} & -\frac{2d^6 (d^2 - e^2 x^2)^{p-2}}{e^5 (-p+2)} + \frac{9d^4 (d^2 - e^2 x^2)^{p-1}}{2e^5 (-p+1)} + \frac{3d^2 (d^2 - e^2 x^2)^p}{e^5 p} \\ & - \frac{(d^2 - e^2 x^2)^{p+1}}{2e^5 (p+1)} + \frac{x^5 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+3, \frac{5}{2}; \frac{7}{2}; \frac{e^2 x^2}{d^2}\right)}{5d^3} \\ & + \frac{3e^2 x^7 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+3, \frac{7}{2}; \frac{9}{2}; \frac{e^2 x^2}{d^2}\right)}{7d^5} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d)\*\*3, x)

[Out]  $-2*d**6*(d**2 - e**2*x**2)**(p - 2)/(e**5*(-p + 2)) + 9*d**4*(d**2 - e**2*x**2)**(p - 1)/(2*e**5*(-p + 1)) + 3*d**2*(d**2 - e**2*x**2)**p/(e**5*p) - (d**2 - e**2*x**2)**(p + 1)/(2*e**5*(p + 1)) + x**5*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper((-p$

+ 3, 5/2), (7/2, ), e\*\*2\*x\*\*2/d\*\*2)/(5\*d\*\*3) + 3\*e\*\*2\*x\*\*7\*(1 - e\*\*2\*x\*\*2/d\*\*2)\*\*(-p)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p\*hyper((-p + 3, 7/2), (9/2, ), e\*\*2\*x\*\*2/d\*\*2)/(7\*d\*\*5)

**Mathematica [C]** time = 0.436041, size = 140, normalized size = 0.64

$$\frac{6dx^5(d-ex)^p(d+ex)^{p-3}F_1\left(5; -p, 3-p; 6; \frac{ex}{d}, -\frac{ex}{d}\right)}{5\left(ex\left(pF_1\left(6; 1-p, 3-p; 7; \frac{ex}{d}, -\frac{ex}{d}\right) - (p-3)F_1\left(6; -p, 4-p; 7; \frac{ex}{d}, -\frac{ex}{d}\right)\right) - 6dF_1\left(5; -p, 3-p; 6; \frac{ex}{d}, -\frac{ex}{d}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^4\*(d^2 - e^2\*x^2)^p)/(d + e\*x)^3, x]

[Out] (-6\*d\*x^5\*(d - e\*x)^p\*(d + e\*x)^(-3 + p)\*AppellF1[5, -p, 3 - p, 6, (e\*x)/d, -((e\*x)/d)]/(5\*(-6\*d\*AppellF1[5, -p, 3 - p, 6, (e\*x)/d, -((e\*x)/d)] + e\*x\*(p\*AppellF1[6, 1 - p, 3 - p, 7, (e\*x)/d, -((e\*x)/d)] - (-3 + p)\*AppellF1[6, -p, 4 - p, 7, (e\*x)/d, -((e\*x)/d)]))

**Maple [F]** time = 0.153, size = 0, normalized size = 0.

$$\int \frac{x^4 (-e^2x^2 + d^2)^p}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(-e^2\*x^2+d^2)^p/(e\*x+d)^3, x)

[Out] int(x^4\*(-e^2\*x^2+d^2)^p/(e\*x+d)^3, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x^4}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x^4/(e\*x + d)^3, x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p\*x^4/(e\*x + d)^3, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p x^4}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x^4/(e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3), x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p\*x^4/(e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 (-(-d + ex)(d + ex))^p}{(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d)\*\*3,x)

[Out] Integral(x\*\*4\*(-(-d + e\*x)\*(d + e\*x))\*\*p/(d + e\*x)\*\*3, x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x^4}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x^4/(e\*x + d)^3,x, algorithm="giac")

[Out] integrate((-e^2\*x^2 + d^2)^p\*x^4/(e\*x + d)^3, x)

$$3.287 \quad \int \frac{x^3(d^2 - e^2x^2)^p}{(d+ex)^3} dx$$

**Optimal.** Leaf size=194

$$\frac{ex^5(d^2 - e^2x^2)^{p-2}}{2p+1} - \frac{3d(d^2 - e^2x^2)^p}{2e^4p} + \frac{2d^5(d^2 - e^2x^2)^{p-2}}{e^4(2-p)} - \frac{2e(3p+4)x^5\left(1 - \frac{e^2x^2}{d^2}\right)^{-p}(d^2 - e^2x^2)^p {}_2F_1\left(\frac{5}{2}, 3-p; \frac{7}{2}; \frac{e^2x^2}{d^2}\right)}{5d^4(2p+1)} - \frac{7d^3(d^2 - e^2x^2)^{p-1}}{2e^4(1-p)}$$

[Out]  $(2*d^5*(d^2 - e^2*x^2)^{(-2 + p)})/(e^4*(2 - p)) + (e*x^5*(d^2 - e^2*x^2)^{(-2 + p)})/(1 + 2*p) - (7*d^3*(d^2 - e^2*x^2)^{(-1 + p)})/(2*e^4*(1 - p)) - (3*d*(d^2 - e^2*x^2)^p)/(2*e^4*p) - (2*e*(4 + 3*p)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, 3 - p, 7/2, (e^2*x^2)/d^2])/(5*d^4*(1 + 2*p)*(1 - (e^2*x^2)/d^2)^p)$

**Rubi [A]** time = 0.492174, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$\frac{ex^5(d^2 - e^2x^2)^{p-2}}{2p+1} - \frac{3d(d^2 - e^2x^2)^p}{2e^4p} + \frac{2d^5(d^2 - e^2x^2)^{p-2}}{e^4(2-p)} - \frac{2e(3p+4)x^5\left(1 - \frac{e^2x^2}{d^2}\right)^{-p}(d^2 - e^2x^2)^p {}_2F_1\left(\frac{5}{2}, 3-p; \frac{7}{2}; \frac{e^2x^2}{d^2}\right)}{5d^4(2p+1)} - \frac{7d^3(d^2 - e^2x^2)^{p-1}}{2e^4(1-p)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^3*(d^2 - e^2*x^2)^p)/(d + e*x)^3, x]$

[Out]  $(2*d^5*(d^2 - e^2*x^2)^{(-2 + p)})/(e^4*(2 - p)) + (e*x^5*(d^2 - e^2*x^2)^{(-2 + p)})/(1 + 2*p) - (7*d^3*(d^2 - e^2*x^2)^{(-1 + p)})/(2*e^4*(1 - p)) - (3*d*(d^2 - e^2*x^2)^p)/(2*e^4*p) - (2*e*(4 + 3*p)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, 3 - p, 7/2, (e^2*x^2)/d^2])/(5*d^4*(1 + 2*p)*(1 - (e^2*x^2)/d^2)^p)$

**Rubi in Sympy [A]** time = 109.922, size = 182, normalized size = 0.94

$$\frac{2d^5(d^2 - e^2x^2)^{p-2}}{e^4(-p+2)} - \frac{7d^3(d^2 - e^2x^2)^{p-1}}{2e^4(-p+1)} - \frac{3d(d^2 - e^2x^2)^p}{2e^4p} - \frac{3ex^5\left(1 - \frac{e^2x^2}{d^2}\right)^{-p}(d^2 - e^2x^2)^p {}_2F_1\left(-p+3, \frac{5}{2}; \frac{7}{2}; \frac{e^2x^2}{d^2}\right)}{5d^4} - \frac{e^3x^7\left(1 - \frac{e^2x^2}{d^2}\right)^{-p}(d^2 - e^2x^2)^p {}_2F_1\left(-p+3, \frac{7}{2}; \frac{9}{2}; \frac{e^2x^2}{d^2}\right)}{7d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**3*(-e**2*x**2+d**2)**p/(e*x+d)**3, x)$

[Out]  $2*d**5*(d**2 - e**2*x**2)**(p - 2)/(e**4*(-p + 2)) - 7*d**3*(d**2 - e**2*x**2)**(p - 1)/(2*e**4*(-p + 1)) - 3*d*(d**2 - e**2*x**2)**p/(2*e**4*p) - 3*e*x**5*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper((-p + 3, 5/2), (7/2, ), e**2*x**2/d**2)/(5*d**4) - e**3*x**7*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper$

$((-p + 3, 7/2), (9/2, ), e^{*2*x^{*2}/d^{*2}})/(7*d^{*6})$

**Mathematica [C]** time = 0.444837, size = 140, normalized size = 0.72

$$\frac{5dx^4(d - ex)^p(d + ex)^{p-3}F_1\left(4; -p, 3 - p; 5; \frac{ex}{d}, -\frac{ex}{d}\right)}{4\left(ex\left(pF_1\left(5; 1 - p, 3 - p; 6; \frac{ex}{d}, -\frac{ex}{d}\right) - (p - 3)F_1\left(5; -p, 4 - p; 6; \frac{ex}{d}, -\frac{ex}{d}\right)\right) - 5dF_1\left(4; -p, 3 - p; 5; \frac{ex}{d}, -\frac{ex}{d}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^3\*(d^2 - e^2\*x^2)^p)/(d + e\*x)^3, x]

[Out]  $(-5*d*x^4*(d - e*x)^p*(d + e*x)^{-3 + p}*AppellF1[4, -p, 3 - p, 5, (e*x)/d, -((e*x)/d)]/(4*(-5*d*AppellF1[4, -p, 3 - p, 5, (e*x)/d, -((e*x)/d)] + e*x*(p*AppellF1[5, 1 - p, 3 - p, 6, (e*x)/d, -((e*x)/d)] - (-3 + p)*AppellF1[5, -p, 4 - p, 6, (e*x)/d, -((e*x)/d)]))$

**Maple [F]** time = 0.118, size = 0, normalized size = 0.

$$\int \frac{x^3 (-e^2x^2 + d^2)^p}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(-e^2\*x^2+d^2)^p/(e\*x+d)^3, x)

[Out] int(x^3\*(-e^2\*x^2+d^2)^p/(e\*x+d)^3, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x^3}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x^3/(e\*x + d)^3, x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p\*x^3/(e\*x + d)^3, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p x^3}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x^3/(e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3), x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p\*x^3/(e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 (-(-d + ex)(d + ex))^p}{(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d)\*\*3,x)

[Out] Integral(x\*\*3\*(-(-d + e\*x)\*(d + e\*x))\*\*p/(d + e\*x)\*\*3, x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x^3}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x^3/(e\*x + d)^3,x, algorithm="giac")

[Out] integrate((-e^2\*x^2 + d^2)^p\*x^3/(e\*x + d)^3, x)



$$3.288 \quad \int \frac{x^2(d^2 - e^2x^2)^p}{(d+ex)^3} dx$$

**Optimal.** Leaf size=157

$$\frac{2^{p-3}(p+4)(d^2 - e^2x^2)^{p+1} \left(\frac{ex}{d} + 1\right)^{-p-1} {}_2F_1\left(2-p, p+1; p+2; \frac{d-ex}{2d}\right)}{d^2e^3(2-p)p(p+1)} - \frac{(d^2 - e^2x^2)^{p+1}}{2e^3p(d+ex)^2} - \frac{d(d^2 - e^2x^2)^{p+1}}{2e^3(2-p)(d+ex)^3}$$

[Out]  $-(d*(d^2 - e^2*x^2)^(1+p))/(2*e^3*(2-p)*(d+e*x)^3) - (d^2 - e^2*x^2)^(1+p)/(2*e^3*p*(d+e*x)^2) + (2^(-3+p)*(4+p)*(1 + (e*x)/d)^(-1-p)*(d^2 - e^2*x^2)^(1+p)*Hypergeometric2F1[2 - p, 1+p, 2+p, (d - e*x)/(2*d)])/(d^2*e^3*(2-p)*p*(1+p))$

**Rubi [A]** time = 0.361206, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{2^{p-3}(p+4)(d^2 - e^2x^2)^{p+1} \left(\frac{ex}{d} + 1\right)^{-p-1} {}_2F_1\left(2-p, p+1; p+2; \frac{d-ex}{2d}\right)}{d^2e^3(2-p)p(p+1)} - \frac{(d^2 - e^2x^2)^{p+1}}{2e^3p(d+ex)^2} - \frac{d(d^2 - e^2x^2)^{p+1}}{2e^3(2-p)(d+ex)^3}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(d^2 - e^2\*x^2)^p)/(d + e\*x)^3, x]

[Out]  $-(d*(d^2 - e^2*x^2)^(1+p))/(2*e^3*(2-p)*(d+e*x)^3) - (d^2 - e^2*x^2)^(1+p)/(2*e^3*p*(d+e*x)^2) + (2^(-3+p)*(4+p)*(1 + (e*x)/d)^(-1-p)*(d^2 - e^2*x^2)^(1+p)*Hypergeometric2F1[2 - p, 1+p, 2+p, (d - e*x)/(2*d)])/(d^2*e^3*(2-p)*p*(1+p))$

**Rubi in Sympy [A]** time = 125.777, size = 202, normalized size = 1.29

$$\frac{4d^2 \left(\frac{d-ex}{2d}\right)^{-p} (d+ex)^{-p} (d+ex)^{p-2} (d^2 - e^2x^2)^p {}_2F_1\left(-p-2, p-2; p-1; \frac{d-ex}{d}\right)}{e^3(-p+2)} - \frac{\left(\frac{d+ex}{d}\right)^{-p} (d-ex)^{-p} (d-ex)^{p+1} (d^2 - e^2x^2)^p {}_2F_1\left(-p+3, p+1; p+2; \frac{d-ex}{d}\right)}{8de^3(p+1)} + \frac{\left(\frac{d+ex}{d}\right)^{-p} (d-ex)^{-p} (d-ex)^{p+2} (d^2 - e^2x^2)^p {}_2F_1\left(-p+3, p+2; p+3; \frac{d-ex}{d}\right)}{4d^2e^3(p+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d)\*\*3, x)

[Out]  $-4*d**2*((d/2 - e*x/2)/d)**(-p)*(d+e*x)**(-p)*(d+e*x)**(p-2)*(d**2 - e**2*x**2)**p*hyper((-p-2, p-2), (p-1), (d/2 + e*x/2)/d)/(e**3*(-p+2)) - ((d/2 + e*x/2)/d)**(-p)*(d - e*x)**(-p)*(d - e*x)**(p+1)*(d**2 - e**2*x**2)**p*hyper((-p+3, p+1), (p+2), (d/2 - e*x/2)/d)/(8*d*e**3*(p+1)) + ((d/2 + e*x/2)/d)**(-p)*(d - e*x)**(-p)*(d - e*x)**(p+2)*(d**2 - e**2*x**2)**p*hyper((-p+3, p+2), (p+3), (d/2 - e*x/2)/d)/(4*d**2*e**3*(p+2))$

**Mathematica [A]** time = 0.127838, size = 130, normalized size = 0.83

$$\frac{2^{p-3}(d-ex)\left(\frac{ex}{d}+1\right)^{-p}(d^2-e^2x^2)^p\left(4{}_2F_1\left(1-p,p+1;p+2;\frac{d-ex}{2d}\right)-4{}_2F_1\left(2-p,p+1;p+2;\frac{d-ex}{2d}\right)+{}_2F_1\left(3-p,p+1;p+2;\frac{d-ex}{2d}\right)\right)}{de^3(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(d^2 - e^2\*x^2)^p)/(d + e\*x)^3,x]

[Out] -((2^(-3 + p)\*(d - e\*x)\*(d^2 - e^2\*x^2)^p\*(4\*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)] - 4\*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)] + Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)]))/(d\*e^3\*(1 + p)\*(1 + (e\*x)/d)^p))

**Maple [F]** time = 0.117, size = 0, normalized size = 0.

$$\int \frac{x^2 (-e^2x^2 + d^2)^p}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(-e^2\*x^2+d^2)^p/(e\*x+d)^3,x)

[Out] int(x^2\*(-e^2\*x^2+d^2)^p/(e\*x+d)^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x^2}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x^2/(e\*x + d)^3,x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p\*x^2/(e\*x + d)^3, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p x^2}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x^2/(e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3), x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p\*x^2/(e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 (-(-d + ex)(d + ex))^p}{(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)`

[Out] `Integral(x**2*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x^2}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^3,x, algorithm="giac")`

[Out] `integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^3, x)`

$$3.289 \quad \int \frac{x(d^2 - e^2 x^2)^p}{(d + ex)^3} dx$$

**Optimal.** Leaf size=118

$$\frac{(d^2 - e^2 x^2)^{p+1}}{2e^2(2-p)(d+ex)^3} - \frac{3 \cdot 2^{p-3} \left(\frac{ex}{d} + 1\right)^{-p-1} (d^2 - e^2 x^2)^{p+1} {}_2F_1\left(2-p, p+1; p+2; \frac{d-ex}{2d}\right)}{d^3 e^2 (2-p)(p+1)}$$

[Out] (d^2 - e^2\*x^2)^(1+p)/(2\*e^2\*(2-p)\*(d+e\*x)^3) - (3\*2^(-3+p)\*(1+(e\*x)/d)^(-1-p)\*(d^2 - e^2\*x^2)^(1+p)\*Hypergeometric2F1[2-p, 1+p, 2+p, (d-e\*x)/(2\*d)])/(d^3\*e^2\*(2-p)\*(1+p))

**Rubi [A]** time = 0.150818, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$

$$\frac{(d^2 - e^2 x^2)^{p+1}}{2e^2(2-p)(d+ex)^3} - \frac{3 \cdot 2^{p-3} \left(\frac{ex}{d} + 1\right)^{-p-1} (d^2 - e^2 x^2)^{p+1} {}_2F_1\left(2-p, p+1; p+2; \frac{d-ex}{2d}\right)}{d^3 e^2 (2-p)(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(x\*(d^2 - e^2\*x^2)^p)/(d + e\*x)^3, x]

[Out] (d^2 - e^2\*x^2)^(1+p)/(2\*e^2\*(2-p)\*(d+e\*x)^3) - (3\*2^(-3+p)\*(1+(e\*x)/d)^(-1-p)\*(d^2 - e^2\*x^2)^(1+p)\*Hypergeometric2F1[2-p, 1+p, 2+p, (d-e\*x)/(2\*d)])/(d^3\*e^2\*(2-p)\*(1+p))

**Rubi in Sympy [A]** time = 60.6845, size = 114, normalized size = 0.97

$$\frac{(d-ex)^3 (d^2 - e^2 x^2)^{p-2}}{2e^2(-p+2)} - \frac{3 \left(\frac{d+ex}{2d}\right)^{-p} (d^2 - e^2 x^2)^p (d^2 e - de^2 x)^{-p} (d^2 e - de^2 x)^{p+1} {}_2F_1\left(-p+2, p+1; p+2; \frac{d-ex}{d}\right)}{8d^3 e^3 (-p+2)(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d)\*\*3, x)

[Out] (d - e\*x)\*\*3\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(p - 2)/(2\*e\*\*2\*(-p + 2)) - 3\*((d/2 + e\*x/2)/d)\*\*(-p)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p\*(d\*\*2\*e - d\*e\*\*2\*x)\*\*(-p)\*(d\*\*2\*e - d\*e\*\*2\*x)\*\*(p + 1)\*hyper((-p + 2, p + 1), (p + 2), (d/2 - e\*x/2)/d)/(8\*d\*\*3\*e\*\*3\*(-p + 2)\*(p + 1))

**Mathematica [A]** time = 0.0922655, size = 102, normalized size = 0.86

$$\frac{2^{p-3}(d-ex)\left(\frac{ex}{d} + 1\right)^{-p} (d^2 - e^2 x^2)^p \left({}_2F_1\left(3-p, p+1; p+2; \frac{d-ex}{2d}\right) - 2 {}_2F_1\left(2-p, p+1; p+2; \frac{d-ex}{2d}\right)\right)}{d^2 e^2 (p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(d^2 - e^2\*x^2)^p)/(d + e\*x)^3, x]

[Out]  $(2^{(-3 + p)}(d - e^*x)(d^2 - e^2x^2)^p(-2\text{Hypergeometric2F1}[2 - p, 1 + p, 2 + p, (d - e^*x)/(2*d)] + \text{Hypergeometric2F1}[3 - p, 1 + p, 2 + p, (d - e^*x)/(2*d)])) / (d^2e^2(1 + p)(1 + (e^*x)/d)^p)$

**Maple [F]** time = 0.111, size = 0, normalized size = 0.

$$\int \frac{x(-e^2x^2 + d^2)^p}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)`

[Out] `int(x*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^3,x, algorithm="maxima")`

[Out] `integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^3, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p x}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^3,x, algorithm="fricas")`

[Out] `integral((-e^2*x^2 + d^2)^p*x/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x(-(-d + ex)(d + ex))^p}{(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)`

[Out] `Integral(x*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**3, x)`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2 x^2 + d^2)^p x}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^3,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^3, x)
```

$$3.290 \quad \int \frac{(d^2 - e^2 x^2)^p}{(d + ex)^3} dx$$

**Optimal.** Leaf size=73

$$\frac{2^{p-3} \left(\frac{ex}{d} + 1\right)^{-p-1} (d^2 - e^2 x^2)^{p+1} {}_2F_1\left(3 - p, p + 1; p + 2; \frac{d - ex}{2d}\right)}{d^4 e (p + 1)}$$

[Out] -((2^(-3 + p)\*(1 + (e\*x)/d)^(-1 - p)\*(d^2 - e^2\*x^2)^(1 + p)\*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/(d^4\*e\*(1 + p)))

**Rubi [A]** time = 0.0751352, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{2^{p-3} \left(\frac{ex}{d} + 1\right)^{-p-1} (d^2 - e^2 x^2)^{p+1} {}_2F_1\left(3 - p, p + 1; p + 2; \frac{d - ex}{2d}\right)}{d^4 e (p + 1)}$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^p/(d + e\*x)^3,x]

[Out] -((2^(-3 + p)\*(1 + (e\*x)/d)^(-1 - p)\*(d^2 - e^2\*x^2)^(1 + p)\*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/(d^4\*e\*(1 + p)))

**Rubi in Sympy [A]** time = 24.3032, size = 66, normalized size = 0.9

$$\frac{\left(\frac{\frac{d}{2} + \frac{ex}{2}}{d}\right)^{-p} (d - ex)^{-p} (d - ex)^{p+1} (d^2 - e^2 x^2)^p {}_2F_1\left(\begin{matrix} -p + 3, p + 1 \\ p + 2 \end{matrix} \middle| \frac{\frac{d}{2} - \frac{ex}{2}}{d}\right)}{8d^3 e (p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d)\*\*3,x)

[Out] -((d/2 + e\*x/2)/d)\*\*(-p)\*(d - e\*x)\*\*(-p)\*(d - e\*x)\*\*(p + 1)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p\*hyper((-p + 3, p + 1), (p + 2, ), (d/2 - e\*x/2)/d)/(8\*d\*\*3\*e\*(p + 1))

**Mathematica [A]** time = 0.0526782, size = 75, normalized size = 1.03

$$\frac{2^{p-3} (d - ex) \left(\frac{ex}{d} + 1\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(3 - p, p + 1; p + 2; \frac{d - ex}{2d}\right)}{d^3 e (p + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^p/(d + e\*x)^3,x]

[Out] -((2^(-3 + p)\*(d - e\*x)\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/(d^3\*e\*(1 + p)\*(1 + (e\*x)/d)^p))

---

**Maple [F]** time = 0.108, size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^p/(e\*x+d)^3, x)

[Out] int((-e^2\*x^2+d^2)^p/(e\*x+d)^3, x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/(e\*x + d)^3, x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p/(e\*x + d)^3, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/(e\*x + d)^3, x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p/(e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(-d + ex)(d + ex))^p}{(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d)\*\*3, x)

[Out] Integral((-(-d + e\*x)\*(d + e\*x))\*\*p/(d + e\*x)\*\*3, x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^3} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^p/(e*x + d)^3,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/(e*x + d)^3, x)
```

$$3.291 \quad \int \frac{(d^2 - e^2 x^2)^p}{x(d+ex)^3} dx$$

**Optimal.** Leaf size=175

$$\frac{(d^2 - e^2 x^2)^{p-1} {}_2F_1\left(1, p-1; p; 1 - \frac{e^2 x^2}{d^2}\right)}{2d(1-p)} - \frac{ex (d^2 - e^2 x^2)^{p-2}}{3-2p} + \frac{2d (d^2 - e^2 x^2)^{p-2}}{2-p} - \frac{2e(4-3p)x \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(\frac{1}{2}, 3-p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^4(3-2p)}$$

[Out] (2\*d\*(d^2 - e^2\*x^2)^(-2 + p))/(2 - p) - (e\*x\*(d^2 - e^2\*x^2)^(-2 + p))/(3 - 2\*p) - (2\*e\*(4 - 3\*p)\*x\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[1/2, 3 - p, 3/2, (e^2\*x^2)/d^2])/(d^4\*(3 - 2\*p)\*(1 - (e^2\*x^2)/d^2)^p) + ((d^2 - e^2\*x^2)^(-1 + p)\*Hypergeometric2F1[1, -1 + p, p, 1 - (e^2\*x^2)/d^2])/(2\*d\*(1 - p))

**Rubi [A]** time = 0.359227, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.32$

$$\frac{(d^2 - e^2 x^2)^{p-1} {}_2F_1\left(1, p-1; p; 1 - \frac{e^2 x^2}{d^2}\right)}{2d(1-p)} - \frac{ex (d^2 - e^2 x^2)^{p-2}}{3-2p} + \frac{2d (d^2 - e^2 x^2)^{p-2}}{2-p} - \frac{2e(4-3p)x \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(\frac{1}{2}, 3-p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^4(3-2p)}$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^p/(x\*(d + e\*x)^3), x]

[Out] (2\*d\*(d^2 - e^2\*x^2)^(-2 + p))/(2 - p) - (e\*x\*(d^2 - e^2\*x^2)^(-2 + p))/(3 - 2\*p) - (2\*e\*(4 - 3\*p)\*x\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[1/2, 3 - p, 3/2, (e^2\*x^2)/d^2])/(d^4\*(3 - 2\*p)\*(1 - (e^2\*x^2)/d^2)^p) + ((d^2 - e^2\*x^2)^(-1 + p)\*Hypergeometric2F1[1, -1 + p, p, 1 - (e^2\*x^2)/d^2])/(2\*d\*(1 - p))

**Rubi in Sympy [A]** time = 75.9342, size = 163, normalized size = 0.93

$$\frac{d (d^2 - e^2 x^2)^{p-2} {}_2F_1\left(1, p-2 \middle| 1 - \frac{e^2 x^2}{d^2}\right)}{2(-p+2)} + \frac{3d (d^2 - e^2 x^2)^{p-2}}{2(-p+2)} - \frac{3ex \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+3, \frac{1}{2} \middle| \frac{e^2 x^2}{d^2}\right)}{d^4} - \frac{e^3 x^3 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+3, \frac{3}{2} \middle| \frac{e^2 x^2}{d^2}\right)}{3d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x/(e\*x+d)\*\*3, x)

[Out] d\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(p - 2)\*hyper((1, p - 2), (p - 1, ), 1 - e\*\*2\*x\*\*2/d\*\*2)/(2\*(-p + 2)) + 3\*d\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(p - 2)/(2\*(-p + 2)) - 3\*e\*x\*(1 - e\*\*2\*x\*\*2/d\*\*2)\*\*(-p)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p

hyper((-p + 3, 1/2), (3/2, ), e\*\*2\*x\*\*2/d\*\*2)/d\*\*4 - e\*\*3\*x\*\*3\*(1 - e\*\*2\*x\*\*2/d\*\*2)\*\*(-p)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p\*hyper((-p + 3, 3/2), (5/2, ), e\*\*2\*x\*\*2/d\*\*2)/(3\*d\*\*6)

**Mathematica [C]** time = 0.474111, size = 196, normalized size = 1.12

$$\frac{2e(p-2)x(d-ex)^p(d+ex)^{p-3}F_1\left(3-2p; -p, 3-p; 4-2p; \frac{d}{ex}, -\frac{d}{ex}\right)}{(2p-3)\left(2e(p-2)x F_1\left(3-2p; -p, 3-p; 4-2p; \frac{d}{ex}, -\frac{d}{ex}\right) + d p F_1\left(4-2p; 1-p, 3-p; 5-2p; \frac{d}{ex}, -\frac{d}{ex}\right) - d(p-3) F_1\left(4-2p; 1-p, 3-p; 5-2p; \frac{d}{ex}, -\frac{d}{ex}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(d^2 - e^2\*x^2)^p/(x\*(d + e\*x)^3), x]

[Out] (2\*e\*(-2 + p)\*x\*(d - e\*x)^p\*(d + e\*x)^(-3 + p)\*AppellF1[3 - 2\*p, -p, 3 - p, 4 - 2\*p, d/(e\*x), -(d/(e\*x))]/((-3 + 2\*p)\*(2\*e\*(-2 + p)\*x\*AppellF1[3 - 2\*p, -p, 3 - p, 4 - 2\*p, d/(e\*x), -(d/(e\*x))] + d\*p\*AppellF1[4 - 2\*p, 1 - p, 3 - p, 5 - 2\*p, d/(e\*x), -(d/(e\*x))] - d\*(-3 + p)\*AppellF1[4 - 2\*p, -p, 4 - p, 5 - 2\*p, d/(e\*x), -(d/(e\*x))])))

**Maple [F]** time = 0.072, size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{x(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^p/x/(e\*x+d)^3, x)

[Out] int((-e^2\*x^2+d^2)^p/x/(e\*x+d)^3, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^3x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^3\*x), x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^3\*x), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p}{e^3x^4 + 3de^2x^3 + 3d^2ex^2 + d^3x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^3\*x), x, algorithm="fricas")

[Out] `integral((-e^2*x^2 + d^2)^p/(e^3*x^4 + 3*d*e^2*x^3 + 3*d^2*e*x^2 + d^3*x), x)`

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(-d + ex)(d + ex))^p}{x(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e**2*x**2+d**2)**p/x/(e*x+d)**3,x)`

[Out] `Integral((-(-d + e*x)*(d + e*x))**p/(x*(d + e*x)**3), x)`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^3x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x),x, algorithm="giac")`

[Out] `integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x), x)`

$$3.292 \quad \int \frac{(d^2 - e^2 x^2)^p}{x^2(d+ex)^3} dx$$

**Optimal.** Leaf size=166

$$\frac{3e(d^2 - e^2 x^2)^{p-1} {}_2F_1\left(1, p-1; p; 1 - \frac{e^2 x^2}{d^2}\right)}{2d^2(1-p)} - \frac{2e(d^2 - e^2 x^2)^{p-2}}{2-p} - \frac{d(d^2 - e^2 x^2)^{p-2}}{x} + \frac{2e^2(4-p)x\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(\frac{1}{2}, 3-p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^5}$$

[Out]  $(-2 * e * (d^2 - e^2 * x^2)^{(-2 + p)}) / (2 - p) - (d * (d^2 - e^2 * x^2)^{(-2 + p)}) / x + (2 * e^2 * (4 - p) * x * (d^2 - e^2 * x^2)^p * \text{Hypergeometric2F1}[1/2, 3 - p, 3/2, (e^2 * x^2) / d^2]) / (d^5 * (1 - (e^2 * x^2) / d^2)^p) - (3 * e * (d^2 - e^2 * x^2)^{(-1 + p)} * \text{Hypergeometric2F1}[1, -1 + p, p, 1 - (e^2 * x^2) / d^2]) / (2 * d^2 * (1 - p))$

**Rubi [A]** time = 0.430773, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.36$

$$\frac{3e(d^2 - e^2 x^2)^{p-1} {}_2F_1\left(1, p-1; p; 1 - \frac{e^2 x^2}{d^2}\right)}{2d^2(1-p)} - \frac{2e(d^2 - e^2 x^2)^{p-2}}{2-p} - \frac{d(d^2 - e^2 x^2)^{p-2}}{x} + \frac{2e^2(4-p)x\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(\frac{1}{2}, 3-p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^5}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d^2 - e^2 * x^2)^p / (x^2 * (d + e * x)^3), x]$

[Out]  $(-2 * e * (d^2 - e^2 * x^2)^{(-2 + p)}) / (2 - p) - (d * (d^2 - e^2 * x^2)^{(-2 + p)}) / x + (2 * e^2 * (4 - p) * x * (d^2 - e^2 * x^2)^p * \text{Hypergeometric2F1}[1/2, 3 - p, 3/2, (e^2 * x^2) / d^2]) / (d^5 * (1 - (e^2 * x^2) / d^2)^p) - (3 * e * (d^2 - e^2 * x^2)^{(-1 + p)} * \text{Hypergeometric2F1}[1, -1 + p, p, 1 - (e^2 * x^2) / d^2]) / (2 * d^2 * (1 - p))$

**Rubi in Sympy [A]** time = 74.2349, size = 160, normalized size = 0.96

$$\frac{3e(d^2 - e^2 x^2)^{p-2} {}_2F_1\left(1, p-2; p-1; 1 - \frac{e^2 x^2}{d^2}\right)}{2(-p+2)} - \frac{e(d^2 - e^2 x^2)^{p-2}}{2(-p+2)} - \frac{\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+3, -\frac{1}{2}; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{d^3 x} + \frac{3e^2 x \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+3, \frac{1}{2}; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((-e^{**2} * x^{**2} + d^{**2})^{**p} / x^{**2} / (e * x + d)^{**3}, x)$

[Out]  $-3 * e * (d^{**2} - e^{**2} * x^{**2})^{**p} * \text{hyper}((1, p - 2), (p - 1, ), 1 - e^{**2} * x^{**2} / d^{**2}) / (2 * (-p + 2)) - e * (d^{**2} - e^{**2} * x^{**2})^{**p} * \text{hyper}((-p + 2), (-p + 2), (1 - e^{**2} * x^{**2} / d^{**2})^{**(-p)} * (d^{**2} - e^{**2} * x^{**2})^{**p} * \text{hyper}((-p + 3, -1/2), (1/2, ), e^{**2} * x^{**2} / d^{**2}) / (d^{**3} * x) + 3 * e^{**2} * x * (1 -$

$$e^{2x} x^2 / d^2)^{-p} (d^2 - e^{2x} x^2)^p \operatorname{hyper}((-p + 3, 1/2), (3/2, ), e^{2x} x^2 / d^2) / d^5$$

**Mathematica [C]** time = 0.481672, size = 198, normalized size = 1.19

$$\frac{e(2p-5)(d-ex)^p(d+ex)^{p-3}F_1\left(4-2p; -p, 3-p; 5-2p; \frac{d}{ex}, -\frac{d}{ex}\right)}{2(p-2)\left(e(2p-5)x F_1\left(4-2p; -p, 3-p; 5-2p; \frac{d}{ex}, -\frac{d}{ex}\right) + d p F_1\left(5-2p; 1-p, 3-p; 6-2p; \frac{d}{ex}, -\frac{d}{ex}\right) - d(p-3)F_1\left(5-2p; 1-p, 3-p; 6-2p; \frac{d}{ex}, -\frac{d}{ex}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(d^2 - e^2\*x^2)^p/(x^2\*(d + e\*x)^3), x]

[Out] (e\*(-5 + 2\*p)\*(d - e\*x)^p\*(d + e\*x)^(-3 + p)\*AppellF1[4 - 2\*p, -p, 3 - p, 5 - 2\*p, d/(e\*x), -(d/(e\*x))])/(2\*(-2 + p)\*(e\*(-5 + 2\*p)\*x\*AppellF1[4 - 2\*p, -p, 3 - p, 5 - 2\*p, d/(e\*x), -(d/(e\*x))] + d\*p\*AppellF1[5 - 2\*p, 1 - p, 3 - p, 6 - 2\*p, d/(e\*x), -(d/(e\*x))]) - d\*(-3 + p)\*AppellF1[5 - 2\*p, -p, 4 - p, 6 - 2\*p, d/(e\*x), -(d/(e\*x))]))

**Maple [F]** time = 0.105, size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{x^2(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^p/x^2/(e\*x+d)^3, x)

[Out] int((-e^2\*x^2+d^2)^p/x^2/(e\*x+d)^3, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^3x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^3\*x^2), x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^3\*x^2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(-e^2x^2 + d^2)^p}{e^3x^5 + 3de^2x^4 + 3d^2ex^3 + d^3x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^3\*x^2), x, algorithm="fricas")

[Out] `integral((-e^2*x^2 + d^2)^p/(e^3*x^5 + 3*d*e^2*x^4 + 3*d^2*e*x^3 + d^3*x^2), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(-d + ex)(d + ex))^p}{x^2(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e**2*x**2+d**2)**p/x**2/(e*x+d)**3,x)`

[Out] `Integral((-(-d + e*x)*(d + e*x))**p/(x**2*(d + e*x)**3), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^3x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^2),x, algorithm="giac")`

[Out] `integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^2), x)`

$$3.293 \quad \int \frac{(d^2 - e^2 x^2)^p}{x^3 (d + ex)^3} dx$$

**Optimal.** Leaf size=173

$$\frac{e^2(6-p)(d^2 - e^2 x^2)^{p-2} {}_2F_1\left(1, p-2; p-1; 1 - \frac{e^2 x^2}{d^2}\right)}{2d(2-p)} + \frac{3e(d^2 - e^2 x^2)^{p-2}}{x} - \frac{d(d^2 - e^2 x^2)^{p-2}}{2x^2} - \frac{2e^3(8-3p)x\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(\frac{1}{2}, 3-p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^6}$$

[Out]  $-(d*(d^2 - e^2*x^2)^{(-2 + p)})/(2*x^2) + (3*e*(d^2 - e^2*x^2)^{(-2 + p)})/x - (2*e^3*(8 - 3*p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 3 - p, 3/2, (e^2*x^2)/d^2])/(d^6*(1 - (e^2*x^2)/d^2)^p) + (e^2*(6 - p)*(d^2 - e^2*x^2)^{(-2 + p)}*Hypergeometric2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(2*d*(2 - p))$

**Rubi [A]** time = 0.48634, antiderivative size = 173, normalized size of antiderivative = 1., number of rules used = 8, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$\frac{e^2(6-p)(d^2 - e^2 x^2)^{p-2} {}_2F_1\left(1, p-2; p-1; 1 - \frac{e^2 x^2}{d^2}\right)}{2d(2-p)} + \frac{3e(d^2 - e^2 x^2)^{p-2}}{x} - \frac{d(d^2 - e^2 x^2)^{p-2}}{2x^2} - \frac{2e^3(8-3p)x\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(\frac{1}{2}, 3-p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^6}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^3), x]$

[Out]  $-(d*(d^2 - e^2*x^2)^{(-2 + p)})/(2*x^2) + (3*e*(d^2 - e^2*x^2)^{(-2 + p)})/x - (2*e^3*(8 - 3*p)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 3 - p, 3/2, (e^2*x^2)/d^2])/(d^6*(1 - (e^2*x^2)/d^2)^p) + (e^2*(6 - p)*(d^2 - e^2*x^2)^{(-2 + p)}*Hypergeometric2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(2*d*(2 - p))$

**Rubi in Sympy [A]** time = 82.9973, size = 185, normalized size = 1.07

$$\frac{3e^2(d^2 - e^2 x^2)^{p-2} {}_2F_1\left(1, p-2; p-1; 1 - \frac{e^2 x^2}{d^2}\right)}{2d(-p+2)} + \frac{e^2(d^2 - e^2 x^2)^{p-2} {}_2F_1\left(2, p-2; p-1; 1 - \frac{e^2 x^2}{d^2}\right)}{2d(-p+2)} + \frac{3e\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+3, -\frac{1}{2}; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{d^4 x} - \frac{e^3 x \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+3, \frac{1}{2}; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((-e^{**2}*x^{**2}+d^{**2})^{**p}/x^{**3}/(e*x+d)^{**3}, x)$

[Out]  $3*e^{**2}*(d^{**2} - e^{**2}*x^{**2})^{**p}*(p - 2)*\text{hyper}((1, p - 2), (p - 1, ), 1 - e^{**2}*x^{**2}/d^{**2})/(2*d*(-p + 2)) + e^{**2}*(d^{**2} - e^{**2}*x^{**2})^{**p}*(p - 2)*\text{hyper}((2, p - 2), (p - 1, ), 1 - e^{**2}*x^{**2}/d^{**2})/(2*d*(-p + 2)) + 3*e*(1 - e^{**2}*x^{**2}/d^{**2})^{**(-p)}*(d^{**2} - e^{**2}*x^{**2})^{**p}*\text{hyper}((-p$



+ 3, -1/2), (1/2, ), e\*\*2\*x\*\*2/d\*\*2)/(d\*\*4\*x) - e\*\*3\*x\*(1 - e\*\*2\*x\*\*2/d\*\*2)\*\*(-p)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p\*hyper((-p + 3, 1/2), (3/2, ), e\*\*2\*x\*\*2/d\*\*2)/d\*\*6

**Mathematica [A]** time = 1.5881, size = 341, normalized size = 1.97

$$(d^2 - e^2 x^2)^p \left( \frac{24 d e^2 \left(1 - \frac{d^2}{e^2 x^2}\right)^{-p} {}_2F_1\left(-p, -p; 1-p; \frac{d^2}{e^2 x^2}\right)}{p} + \frac{24 d^2 e \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{x} + \frac{4 d^3 \left(1 - \frac{d^2}{e^2 x^2}\right)^{-p} {}_2F_1\left(1-p, -p; 2-p; \frac{d^2}{e^2 x^2}\right)}{(p-1)x^2} + \frac{3 e^2 d^4 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(-p, -p; -p; \frac{e^2 x^2}{d^2}\right)}{x^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^p/(x^3\*(d + e\*x)^3), x]

[Out] ((d^2 - e^2\*x^2)^p\*((24\*d^2\*e\*Hypergeometric2F1[-1/2, -p, 1/2, (e^2\*x^2)/d^2])/(x\*(1 - (e^2\*x^2)/d^2)^p) + (4\*d^3\*Hypergeometric2F1[1 - p, -p, 2 - p, d^2/(e^2\*x^2)])/((1 + p)\*(1 - d^2/(e^2\*x^2)))^p\*x^2) + (3\*2^(3 + p)\*e^2\*(d - e\*x)\*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/((1 + p)\*(1 + (e\*x)/d)^p) + (3\*2^(1 + p)\*e^2\*(d - e\*x)\*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/((1 + p)\*(1 + (e\*x)/d)^p) + (2^p\*e^2\*(d - e\*x)\*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/((1 + p)\*(1 + (e\*x)/d)^p) + (24\*d\*e^2\*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2\*x^2)])/(p\*(1 - d^2/(e^2\*x^2))^p))/(8\*d^6)

**Maple [F]** time = 0.128, size = 0, normalized size = 0.

$$\int \frac{(-e^2 x^2 + d^2)^p}{x^3 (e x + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^p/x^3/(e\*x+d)^3, x)

[Out] int((-e^2\*x^2+d^2)^p/x^3/(e\*x+d)^3, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2 x^2 + d^2)^p}{(e x + d)^3 x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^3\*x^3), x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^3\*x^3), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2 x^2 + d^2)^p}{e^3 x^6 + 3 d e^2 x^5 + 3 d^2 e x^4 + d^3 x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^3\*x^3),x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p/(e^3\*x^6 + 3\*d\*e^2\*x^5 + 3\*d^2\*e\*x^4 + d^3\*x^3), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(-d + ex)(d + ex))^p}{x^3(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x\*\*3/(e\*x+d)\*\*3,x)

[Out] Integral((-(-d + e\*x)\*(d + e\*x))\*\*p/(x\*\*3\*(d + e\*x)\*\*3), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^3x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^3\*x^3),x, algorithm="giac")

[Out] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^3\*x^3), x)

$$3.294 \quad \int \frac{(d^2 - e^2 x^2)^p}{x^4 (d + ex)^3} dx$$

**Optimal.** Leaf size=179

$$\frac{3e(d^2 - e^2 x^2)^{p-2}}{2x^2} - \frac{d(d^2 - e^2 x^2)^{p-2}}{3x^3} - \frac{e^3(10 - 3p)(d^2 - e^2 x^2)^{p-2} {}_2F_1\left(1, p-2; p-1; 1 - \frac{e^2 x^2}{d^2}\right)}{2d^2(2-p)}$$

$$- \frac{2e^2(8-p)\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-\frac{1}{2}, 3-p; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{3d^5 x}$$

[Out]  $-(d*(d^2 - e^2*x^2)^{(-2 + p)})/(3*x^3) + (3*e*(d^2 - e^2*x^2)^{(-2 + p)})/(2*x^2) - (2*e^2*(8 - p)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1[-1/2, 3 - p, 1/2, (e^2*x^2)/d^2]])/(3*d^5*x*(1 - (e^2*x^2)/d^2)^p) - (e^3*(10 - 3*p)*(d^2 - e^2*x^2)^{(-2 + p)}*Hypergeometric2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*(2 - p))$

**Rubi [A]** time = 0.516534, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$\frac{3e(d^2 - e^2 x^2)^{p-2}}{2x^2} - \frac{d(d^2 - e^2 x^2)^{p-2}}{3x^3} - \frac{e^3(10 - 3p)(d^2 - e^2 x^2)^{p-2} {}_2F_1\left(1, p-2; p-1; 1 - \frac{e^2 x^2}{d^2}\right)}{2d^2(2-p)}$$

$$- \frac{2e^2(8-p)\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-\frac{1}{2}, 3-p; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{3d^5 x}$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^p/(x^4\*(d + e\*x)^3), x]

[Out]  $-(d*(d^2 - e^2*x^2)^{(-2 + p)})/(3*x^3) + (3*e*(d^2 - e^2*x^2)^{(-2 + p)})/(2*x^2) - (2*e^2*(8 - p)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1[-1/2, 3 - p, 1/2, (e^2*x^2)/d^2]])/(3*d^5*x*(1 - (e^2*x^2)/d^2)^p) - (e^3*(10 - 3*p)*(d^2 - e^2*x^2)^{(-2 + p)}*Hypergeometric2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(2*d^2*(2 - p))$

**Rubi in Sympy [A]** time = 72.2599, size = 196, normalized size = 1.09

$$\frac{e^3(d^2 - e^2 x^2)^{p-2} {}_2F_1\left(1, p-2; p-1; 1 - \frac{e^2 x^2}{d^2}\right)}{2d^2(-p+2)} - \frac{3e^3(d^2 - e^2 x^2)^{p-2} {}_2F_1\left(2, p-2; p-1; 1 - \frac{e^2 x^2}{d^2}\right)}{2d^2(-p+2)}$$

$$- \frac{\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+3, -\frac{3}{2}; -\frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{3d^3 x^3}$$

$$- \frac{3e^2\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+3, -\frac{1}{2}; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{d^5 x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x\*\*4/(e\*x+d)\*\*3, x)

[Out]  $-e**3*(d**2 - e**2*x**2)**(p - 2)*hyper((1, p - 2), (p - 1, ), 1 - e**2*x**2/d**2)/(2*d**2*(-p + 2)) - 3*e**3*(d**2 - e**2*x**2)**(p - 2)*hyper((2, p - 2), (p - 1, ), 1 - e**2*x**2/d**2)/(2*d**2*(-p + 2)) - (1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper($

$(-p + 3, -3/2), (-1/2, ), e^{2x^2}/d^2)/(3d^3x^3) - 3e^{2x^2}(1 - e^{2x^2}/d^2)^{-p}(d^2 - e^{2x^2})^p \text{hyper}((-p + 3, -1/2), (1/2, ), e^{2x^2}/d^2)/(d^5x)$

**Mathematica [B]** time = 0.74982, size = 393, normalized size = 2.2

$$(d^2 - e^{2x^2})^p \left( -\frac{144d^2e^2\left(1 - \frac{e^{2x^2}}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; \frac{e^{2x^2}}{d^2}\right)}{x} - \frac{120de^3\left(1 - \frac{d^2}{e^{2x^2}}\right)^{-p} {}_2F_1\left(-p, -p; 1-p; \frac{d^2}{e^{2x^2}}\right)}{p} - \frac{8d^4\left(1 - \frac{e^{2x^2}}{d^2}\right)^{-p} {}_2F_1\left(-\frac{3}{2}, -p; -\frac{1}{2}; \frac{e^{2x^2}}{d^2}\right)}{x^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^p/(x^4\*(d + e\*x)^3), x]

[Out]  $((d^2 - e^{2x^2})^p((-8d^4 \text{Hypergeometric2F1}[-3/2, -p, -1/2, (e^{2x^2}/d^2)]/(x^3(1 - (e^{2x^2}/d^2))^p) - (144d^2e^2 \text{Hypergeometric2F1}[-1/2, -p, 1/2, (e^{2x^2}/d^2)]/(x(1 - (e^{2x^2}/d^2))^p) - (36d^3e \text{Hypergeometric2F1}[1 - p, -p, 2 - p, d^2/(e^{2x^2})]])/((-1 + p)(1 - d^2/(e^{2x^2}))^p x^2) + (15 \cdot 2^{3+p})e^3(-d + e^x) \text{Hypergeometric2F1}[1 - p, 1 + p, 2 + p, (d - e^x)/(2d)]/((1 + p)(1 + (e^x)/d)^p) + (3 \cdot 2^{3+p})e^3(-d + e^x) \text{Hypergeometric2F1}[2 - p, 1 + p, 2 + p, (d - e^x)/(2d)]/((1 + p)(1 + (e^x)/d)^p) + (3 \cdot 2^p)e^3(-d + e^x) \text{Hypergeometric2F1}[3 - p, 1 + p, 2 + p, (d - e^x)/(2d)]/((1 + p)(1 + (e^x)/d)^p) - (120d^2e^3 \text{Hypergeometric2F1}[-p, -p, 1 - p, d^2/(e^{2x^2})])/(p(1 - d^2/(e^{2x^2}))^p)))/(24d^7)$

**Maple [F]** time = 0.137, size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{x^4(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^p/x^4/(e\*x+d)^3, x)

[Out] int((-e^2\*x^2+d^2)^p/x^4/(e\*x+d)^3, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^3x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^3\*x^4), x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^3\*x^4), x)

**Ericsas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p}{e^3x^7 + 3de^2x^6 + 3d^2ex^5 + d^3x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^4),x, algorithm="fricas")`

[Out] `integral((-e^2*x^2 + d^2)^p/(e^3*x^7 + 3*d*e^2*x^6 + 3*d^2*e*x^5 + d^3*x^4), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(-d + ex)(d + ex))^p}{x^4(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e**2*x**2+d**2)**p/x**4/(e*x+d)**3,x)`

[Out] `Integral((-(-d + e*x)*(d + e*x))**p/(x**4*(d + e*x)**3), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^3x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^4),x, algorithm="giac")`

[Out] `integrate((-e^2*x^2 + d^2)^p/((e*x + d)^3*x^4), x)`

$$3.295 \quad \int \frac{(d^2 - e^2 x^2)^p}{x^5 (d + ex)^3} dx$$

**Optimal.** Leaf size=174

$$\begin{aligned} & -\frac{d(d^2 - e^2 x^2)^{p-2}}{4x^4} + \frac{e(d^2 - e^2 x^2)^{p-2}}{x^3} \\ & + \frac{2e^3(4-p)\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-\frac{1}{2}, 3-p; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{d^6 x} \\ & + \frac{e^4(10-p)(d^2 - e^2 x^2)^{p-2} {}_2F_1\left(2, p-2; p-1; 1 - \frac{e^2 x^2}{d^2}\right)}{4d^3(2-p)} \end{aligned}$$

[Out]  $-(d*(d^2 - e^2*x^2)^{(-2 + p)})/(4*x^4) + (e*(d^2 - e^2*x^2)^{(-2 + p)})/x^3 + (2*e^3*(4 - p)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 3 - p, 1/2, (e^2*x^2)/d^2])/(d^6*x*(1 - (e^2*x^2)/d^2)^p) + (e^4*(10 - p)*(d^2 - e^2*x^2)^{(-2 + p)}*Hypergeometric2F1[2, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(4*d^3*(2 - p))$

**Rubi [A]** time = 0.519324, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$\begin{aligned} & -\frac{d(d^2 - e^2 x^2)^{p-2}}{4x^4} + \frac{e(d^2 - e^2 x^2)^{p-2}}{x^3} \\ & + \frac{2e^3(4-p)\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-\frac{1}{2}, 3-p; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{d^6 x} \\ & + \frac{e^4(10-p)(d^2 - e^2 x^2)^{p-2} {}_2F_1\left(2, p-2; p-1; 1 - \frac{e^2 x^2}{d^2}\right)}{4d^3(2-p)} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d^2 - e^2*x^2)^p/(x^5*(d + e*x)^3), x]$

[Out]  $-(d*(d^2 - e^2*x^2)^{(-2 + p)})/(4*x^4) + (e*(d^2 - e^2*x^2)^{(-2 + p)})/x^3 + (2*e^3*(4 - p)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 3 - p, 1/2, (e^2*x^2)/d^2])/(d^6*x*(1 - (e^2*x^2)/d^2)^p) + (e^4*(10 - p)*(d^2 - e^2*x^2)^{(-2 + p)}*Hypergeometric2F1[2, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(4*d^3*(2 - p))$

**Rubi in Sympy [A]** time = 74.2036, size = 192, normalized size = 1.1

$$\begin{aligned} & \frac{3e^4(d^2 - e^2 x^2)^{p-2} {}_2F_1\left(2, p-2; p-1; 1 - \frac{e^2 x^2}{d^2}\right)}{2d^3(-p+2)} + \frac{e^4(d^2 - e^2 x^2)^{p-2} {}_2F_1\left(3, p-2; p-1; 1 - \frac{e^2 x^2}{d^2}\right)}{2d^3(-p+2)} \\ & + \frac{e\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+3, -\frac{3}{2}; -\frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{d^4 x^3} \\ & + \frac{e^3\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+3, -\frac{1}{2}; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{d^6 x} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((-e**2*x**2+d**2)**p/x**5/(e*x+d)**3, x)$

[Out]  $3e^{4x^2}(d^2 - e^{2x^2})^{p-2} \text{hyper}((2, p-2), (p-1, ), 1 - e^{2x^2}/d^2)/(2d^3(-p+2)) + e^{4x^2}(d^2 - e^{2x^2})^{p-2} \text{hyper}((3, p-2), (p-1, ), 1 - e^{2x^2}/d^2)/(2d^3(-p+2)) + e^{(1 - e^{2x^2}/d^2)^{-p}}(d^2 - e^{2x^2})^p \text{hyper}((-p+3, -3/2), (-1/2, ), e^{2x^2}/d^2)/(d^4x^3) + e^{3(1 - e^{2x^2}/d^2)^{-p}}(d^2 - e^{2x^2})^p \text{hyper}((-p+3, -1/2), (1/2, ), e^{2x^2}/d^2)/(d^6x)$

**Mathematica [B]** time = 1.01305, size = 446, normalized size = 2.56

$$(d^2 - e^2x^2)^p \left( \frac{60de^4 \left(1 - \frac{d^2}{e^2x^2}\right)^{-p} {}_2F_1\left(-p, -p; 1-p; \frac{d^2}{e^2x^2}\right)}{p} + \frac{80d^2e^3 \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; \frac{e^2x^2}{d^2}\right)}{x} + \frac{4d^5 \left(1 - \frac{d^2}{e^2x^2}\right)^{-p} {}_2F_1\left(2-p, -p; 3-p; \frac{d^2}{e^2x^2}\right)}{(p-2)x^4} + \frac{8d^4}{(p-2)x^4} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^p/(x^5\*(d + e\*x)^3),x]

[Out]  $((d^2 - e^2x^2)^p ((8d^4e \text{Hypergeometric2F1}[-3/2, -p, -1/2, (e^2x^2)/d^2])/(x^3(1 - (e^2x^2)/d^2)^p) + (80d^2e^3 \text{Hypergeometric2F1}[-1/2, -p, 1/2, (e^2x^2)/d^2])/(x(1 - (e^2x^2)/d^2)^p) + (24d^3e^2 \text{Hypergeometric2F1}[1-p, -p, 2-p, d^2/(e^2x^2)])/((-1+p)(1 - d^2/(e^2x^2))^p x^2) + (15^2(2+p)e^4(d - ex) \text{Hypergeometric2F1}[1-p, 1+p, 2+p, (d - ex)/(2d)])/((1+p)(1 + (ex)/d)^p) + (4d^5 \text{Hypergeometric2F1}[2-p, -p, 3-p, d^2/(e^2x^2)])/((-2+p)(1 - d^2/(e^2x^2))^p x^4) + (5^2(1+p)e^4(d - ex) \text{Hypergeometric2F1}[2-p, 1+p, 2+p, (d - ex)/(2d)])/((1+p)(1 + (ex)/d)^p) + (2^p e^4(d - ex) \text{Hypergeometric2F1}[3-p, 1+p, 2+p, (d - ex)/(2d)])/((1+p)(1 + (ex)/d)^p) + (60d^4e \text{Hypergeometric2F1}[-p, -p, 1-p, d^2/(e^2x^2)])/(p(1 - d^2/(e^2x^2))^p)))/(8d^8)$

**Maple [F]** time = 0.109, size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{x^5(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^p/x^5/(e\*x+d)^3,x)

[Out] int((-e^2\*x^2+d^2)^p/x^5/(e\*x+d)^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^3x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^3\*x^5),x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^3\*x^5), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^P}{e^3x^8 + 3de^2x^7 + 3d^2ex^6 + d^3x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^3\*x^5),x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p/(e^3\*x^8 + 3\*d\*e^2\*x^7 + 3\*d^2\*e\*x^6 + d^3\*x^5), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(-d + ex)(d + ex))^P}{x^5(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x\*\*5/(e\*x+d)\*\*3,x)

[Out] Integral((-(-d + e\*x)\*(d + e\*x))\*\*p/(x\*\*5\*(d + e\*x)\*\*3), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^P}{(ex + d)^3x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^3\*x^5),x, algorithm="giac")

[Out] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^3\*x^5), x)



$$3.296 \quad \int \frac{x^4 (d^2 - e^2 x^2)^p}{(d + ex)^4} dx$$

**Optimal.** Leaf size=265

$$\frac{d^2(12p+13)x^5 (d^2 - e^2 x^2)^{p-3}}{1 - 4p^2} - \frac{e^2 x^7 (d^2 - e^2 x^2)^{p-3}}{2p+1} - \frac{2d (d^2 - e^2 x^2)^p}{e^5 p} - \frac{4d^7 (d^2 - e^2 x^2)^{p-3}}{e^5(3-p)} + \frac{10d^5 (d^2 - e^2 x^2)^{p-2}}{e^5(2-p)} - \frac{4(p^2 + 15p + 16) x^5 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(\frac{5}{2}, 4-p; \frac{7}{2}; \frac{e^2 x^2}{d^2}\right)}{5d^4(1-4p^2)} - \frac{8d^3 (d^2 - e^2 x^2)^{p-1}}{e^5(1-p)}$$

[Out]  $(-4*d^7*(d^2 - e^2*x^2)^{(-3 + p)})/(e^5*(3 - p)) + (d^2*(13 + 12*p)*x^5*(d^2 - e^2*x^2)^{(-3 + p)})/(1 - 4*p^2) - (e^2*x^7*(d^2 - e^2*x^2)^{(-3 + p)})/(2*p + 1) + (10*d^5*(d^2 - e^2*x^2)^{(-2 + p)})/(e^5*(2 - p)) - (8*d^7*(d^2 - e^2*x^2)^{(-1 + p)})/(e^5*(3 - p)) - (2*d*(d^2 - e^2*x^2)^p)/(e^5*p) - (4*(16 + 15*p + p^2)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, 4 - p, 7/2, (e^2*x^2)/d^2])/(5*d^4*(1 - 4*p^2)*(1 - (e^2*x^2)/d^2)^p)$

**Rubi [A]** time = 0.74319, antiderivative size = 265, normalized size of antiderivative = 1., number of rules used = 10, number of rules used = 9, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.36$

$$\frac{d^2(12p+13)x^5 (d^2 - e^2 x^2)^{p-3}}{1 - 4p^2} - \frac{e^2 x^7 (d^2 - e^2 x^2)^{p-3}}{2p+1} - \frac{2d (d^2 - e^2 x^2)^p}{e^5 p} - \frac{4d^7 (d^2 - e^2 x^2)^{p-3}}{e^5(3-p)} + \frac{10d^5 (d^2 - e^2 x^2)^{p-2}}{e^5(2-p)} - \frac{4(p^2 + 15p + 16) x^5 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(\frac{5}{2}, 4-p; \frac{7}{2}; \frac{e^2 x^2}{d^2}\right)}{5d^4(1-4p^2)} - \frac{8d^3 (d^2 - e^2 x^2)^{p-1}}{e^5(1-p)}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*(d^2 - e^2\*x^2)^p)/(d + e\*x)^4, x]

[Out]  $(-4*d^7*(d^2 - e^2*x^2)^{(-3 + p)})/(e^5*(3 - p)) + (d^2*(13 + 12*p)*x^5*(d^2 - e^2*x^2)^{(-3 + p)})/(1 - 4*p^2) - (e^2*x^7*(d^2 - e^2*x^2)^{(-3 + p)})/(2*p + 1) + (10*d^5*(d^2 - e^2*x^2)^{(-2 + p)})/(e^5*(2 - p)) - (8*d^7*(d^2 - e^2*x^2)^{(-1 + p)})/(e^5*(3 - p)) - (2*d*(d^2 - e^2*x^2)^p)/(e^5*p) - (4*(16 + 15*p + p^2)*x^5*(d^2 - e^2*x^2)^p*Hypergeometric2F1[5/2, 4 - p, 7/2, (e^2*x^2)/d^2])/(5*d^4*(1 - 4*p^2)*(1 - (e^2*x^2)/d^2)^p)$

**Rubi in Sympy [A]** time = 143.648, size = 257, normalized size = 0.97

$$-\frac{4d^7 (d^2 - e^2 x^2)^{p-3}}{e^5(-p+3)} + \frac{10d^5 (d^2 - e^2 x^2)^{p-2}}{e^5(-p+2)} - \frac{8d^3 (d^2 - e^2 x^2)^{p-1}}{e^5(-p+1)} - \frac{2d (d^2 - e^2 x^2)^p}{e^5 p} + \frac{x^5 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+4, \frac{5}{2}; \frac{7}{2}; \frac{e^2 x^2}{d^2}\right)}{5d^4} + \frac{6e^2 x^7 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+4, \frac{7}{2}; \frac{9}{2}; \frac{e^2 x^2}{d^2}\right)}{7d^6} + \frac{e^4 x^9 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+4, \frac{9}{2}; \frac{11}{2}; \frac{e^2 x^2}{d^2}\right)}{9d^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**4*(-e**2*x**2+d**2)**p/(e*x+d)**4,x)`

[Out] 
$$-4*d^{7*(d^2 - e^2*x^2)^p}*(p - 3)/(e^{5*(-p + 3)}) + 10*d^{5*(d^2 - e^2*x^2)^p}*(p - 2)/(e^{5*(-p + 2)}) - 8*d^{3*(d^2 - e^2*x^2)^p}*(p - 1)/(e^{5*(-p + 1)}) - 2*d*(d^2 - e^2*x^2)^p/(e^{5*p}) + x^{5*(1 - e^2*x^2/d^2)^p}*(-p)*(d^2 - e^2*x^2)^p*\text{hyper}((-p + 4, 5/2), (7/2, ), e^{2*x^2/d^2})/(5*d^4) + 6*e^{2*x^7*(1 - e^2*x^2/d^2)^p}*(-p)*(d^2 - e^2*x^2)^p*\text{hyper}((-p + 4, 7/2), (9/2, ), e^{2*x^2/d^2})/(7*d^6) + e^{4*x^9*(1 - e^2*x^2/d^2)^p}*(-p)*(d^2 - e^2*x^2)^p*\text{hyper}((-p + 4, 9/2), (11/2, ), e^{2*x^2/d^2})/(9*d^8)$$

**Mathematica [C]** time = 0.469677, size = 140, normalized size = 0.53

$$\frac{6dx^5(d - ex)^p(d + ex)^{p-4}F_1\left(5; -p, 4 - p; 6; \frac{ex}{d}, -\frac{ex}{d}\right)}{5\left(ex\left(pF_1\left(6; 1 - p, 4 - p; 7; \frac{ex}{d}, -\frac{ex}{d}\right) - (p - 4)F_1\left(6; -p, 5 - p; 7; \frac{ex}{d}, -\frac{ex}{d}\right)\right) - 6dF_1\left(5; -p, 4 - p; 6; \frac{ex}{d}, -\frac{ex}{d}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(x^4*(d^2 - e^2*x^2)^p)/(d + e*x)^4,x]`

[Out] 
$$(-6*d*x^5*(d - e*x)^p*(d + e*x)^{-4 + p}*\text{AppellF1}[5, -p, 4 - p, 6, (e*x)/d, -((e*x)/d)]/(5*(-6*d*\text{AppellF1}[5, -p, 4 - p, 6, (e*x)/d, -((e*x)/d)] + e*x*(p*\text{AppellF1}[6, 1 - p, 4 - p, 7, (e*x)/d, -((e*x)/d)] - (-4 + p)*\text{AppellF1}[6, -p, 5 - p, 7, (e*x)/d, -((e*x)/d)]))$$

**Maple [F]** time = 0.184, size = 0, normalized size = 0.

$$\int \frac{x^4 (-e^2x^2 + d^2)^p}{(ex + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)`

[Out] `int(x^4*(-e^2*x^2+d^2)^p/(e*x+d)^4,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x^4}{(ex + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^4,x, algorithm="maxima")`

[Out] `integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^4, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p x^4}{e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^4,x, algorithm="fricas")`

[Out] `integral((-e^2*x^2 + d^2)^p*x^4/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 (-(-d + ex)(d + ex))^p}{(d + ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(-e**2*x**2+d**2)**p/(e*x+d)**4,x)`

[Out] `Integral(x**4*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**4, x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x^4}{(ex + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^4,x, algorithm="giac")`

[Out] `integrate((-e^2*x^2 + d^2)^p*x^4/(e*x + d)^4, x)`

$$3.297 \quad \int \frac{x^3(d^2 - e^2x^2)^p}{(d+ex)^4} dx$$

**Optimal.** Leaf size=211

$$\frac{3 \cdot 2^{p-2}(p+2)(d^2 - e^2x^2)^{p+1} \left(\frac{ex}{d} + 1\right)^{-p-1} {}_2F_1\left(3-p, p+1; p+2; \frac{d-ex}{2d}\right)}{\frac{d^2 e^4(1-2p)(3-p)p(p+1)}{2e^4 p(d+ex)^2} - \frac{d(2p+1)(d^2 - e^2x^2)^{p+1}}{e^4(1-2p)p(d+ex)^3} + \frac{d^2(d^2 - e^2x^2)^{p+1}}{2e^4(3-p)(d+ex)^4}}$$

[Out]  $(d^2*(d^2 - e^2*x^2)^(1 + p))/(2*e^4*(3 - p)*(d + e*x)^4) - (d*(1 + 2*p)*(d^2 - e^2*x^2)^(1 + p))/(e^4*(1 - 2*p)*p*(d + e*x)^3) - (d^2 - e^2*x^2)^(1 + p)/(2*e^4*p*(d + e*x)^2) + (3*2^(-2 + p)*(2 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^2*e^4*(1 - 2*p)*(3 - p)*p*(1 + p))$

**Rubi [A]** time = 0.678008, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{3 \cdot 2^{p-2}(p+2)(d^2 - e^2x^2)^{p+1} \left(\frac{ex}{d} + 1\right)^{-p-1} {}_2F_1\left(3-p, p+1; p+2; \frac{d-ex}{2d}\right)}{\frac{d^2 e^4(1-2p)(3-p)p(p+1)}{2e^4 p(d+ex)^2} - \frac{d(2p+1)(d^2 - e^2x^2)^{p+1}}{e^4(1-2p)p(d+ex)^3} + \frac{d^2(d^2 - e^2x^2)^{p+1}}{2e^4(3-p)(d+ex)^4}}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(d^2 - e^2\*x^2)^p)/(d + e\*x)^4, x]

[Out]  $(d^2*(d^2 - e^2*x^2)^(1 + p))/(2*e^4*(3 - p)*(d + e*x)^4) - (d*(1 + 2*p)*(d^2 - e^2*x^2)^(1 + p))/(e^4*(1 - 2*p)*p*(d + e*x)^3) - (d^2 - e^2*x^2)^(1 + p)/(2*e^4*p*(d + e*x)^2) + (3*2^(-2 + p)*(2 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^2*e^4*(1 - 2*p)*(3 - p)*p*(1 + p))$

**Rubi in Sympy [A]** time = 141.427, size = 206, normalized size = 0.98

$$\frac{4d^6(d^2 - e^2x^2)^{p-3}}{e^4(-p+3)} - \frac{8d^4(d^2 - e^2x^2)^{p-2}}{e^4(-p+2)} + \frac{9d^2(d^2 - e^2x^2)^{p-1}}{2e^4(-p+1)} + \frac{(d^2 - e^2x^2)^p}{2e^4p} - \frac{4ex^5 \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(-p+4, \frac{5}{2} \middle| \frac{e^2x^2}{d^2}\right)}{5d^5} - \frac{4e^3x^7 \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} (d^2 - e^2x^2)^p {}_2F_1\left(-p+4, \frac{7}{2} \middle| \frac{e^2x^2}{d^2}\right)}{7d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d)\*\*4, x)

[Out]  $4*d**6*(d**2 - e**2*x**2)**(p - 3)/(e**4*(-p + 3)) - 8*d**4*(d**2 - e**2*x**2)**(p - 2)/(e**4*(-p + 2)) + 9*d**2*(d**2 - e**2*x**2)**(p - 1)/(2*e**4*(-p + 1)) + (d**2 - e**2*x**2)**p/(2*e**4*p) - 4*e*x**5*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper($

$(-p + 4, 5/2), (7/2, ), e^{2x^2/d^2}/(5d^5) - 4e^{3x^2/d^2}(1 - e^{2x^2/d^2})^{p-1}d^{-2} - e^{2x^2/d^2}/(7d^7)$

**Mathematica [C]** time = 0.457616, size = 140, normalized size = 0.66

$$\frac{5dx^4(d - ex)^p(d + ex)^{p-4}F_1\left(4; -p, 4 - p; 5; \frac{ex}{d}, -\frac{ex}{d}\right)}{4\left(ex\left(pF_1\left(5; 1 - p, 4 - p; 6; \frac{ex}{d}, -\frac{ex}{d}\right) - (p - 4)F_1\left(5; -p, 5 - p; 6; \frac{ex}{d}, -\frac{ex}{d}\right)\right) - 5dF_1\left(4; -p, 4 - p; 5; \frac{ex}{d}, -\frac{ex}{d}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^3\*(d^2 - e^2\*x^2)^p)/(d + e\*x)^4, x]

[Out]  $(-5d^4x^4(d - ex)^p(d + ex)^{-4+p} \text{AppellF1}[4, -p, 4 - p, 5, (ex)/d, -((ex)/d)] / (4(-5d \text{AppellF1}[4, -p, 4 - p, 5, (ex)/d, -((ex)/d)] + ex(p \text{AppellF1}[5, 1 - p, 4 - p, 6, (ex)/d, -((ex)/d)] - (-4 + p) \text{AppellF1}[5, -p, 5 - p, 6, (ex)/d, -((ex)/d)])))$

**Maple [F]** time = 0.163, size = 0, normalized size = 0.

$$\int \frac{x^3 (-e^2x^2 + d^2)^p}{(ex + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(-e^2\*x^2+d^2)^p/(e\*x+d)^4, x)

[Out] int(x^3\*(-e^2\*x^2+d^2)^p/(e\*x+d)^4, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x^3}{(ex + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x^3/(e\*x + d)^4, x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p\*x^3/(e\*x + d)^4, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p x^3}{e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x^3/(e\*x + d)^4, x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p\*x^3/(e^4\*x^4 + 4\*d\*e^3\*x^3 + 6\*d^2\*e^2\*x^2 + 4\*d^3\*e\*x + d^4), x)

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**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 (-(-d + ex)(d + ex))^p}{(d + ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d)\*\*4,x)

[Out] Integral(x\*\*3\*(-(-d + e\*x)\*(d + e\*x))\*\*p/(d + e\*x)\*\*4, x)

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x^3}{(ex + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x^3/(e\*x + d)^4,x, algorithm="giac")

[Out] integrate((-e^2\*x^2 + d^2)^p\*x^3/(e\*x + d)^4, x)

$$3.298 \quad \int \frac{x^2(d^2 - e^2x^2)^p}{(d+ex)^4} dx$$

**Optimal.** Leaf size=163

$$\frac{(d^2 - e^2x^2)^{p+1}}{e^3(1-2p)(d+ex)^3} - \frac{d(d^2 - e^2x^2)^{p+1}}{2e^3(3-p)(d+ex)^4} - \frac{2^{p-3}(p+7)(d^2 - e^2x^2)^{p+1} \left(\frac{ex}{d} + 1\right)^{-p-1} {}_2F_1\left(3-p, p+1; p+2; \frac{d-ex}{2d}\right)}{d^3e^3(1-2p)(3-p)(p+1)}$$

[Out]  $-(d*(d^2 - e^2*x^2)^(1 + p))/(2*e^3*(3 - p)*(d + e*x)^4) + (d^2 - e^2*x^2)^(1 + p)/(e^3*(1 - 2*p)*(d + e*x)^3) - (2^(-3 + p)*(7 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^3*e^3*(1 - 2*p)*(3 - p)*(1 + p))$

**Rubi [A]** time = 0.396839, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{(d^2 - e^2x^2)^{p+1}}{e^3(1-2p)(d+ex)^3} - \frac{d(d^2 - e^2x^2)^{p+1}}{2e^3(3-p)(d+ex)^4} - \frac{2^{p-3}(p+7)(d^2 - e^2x^2)^{p+1} \left(\frac{ex}{d} + 1\right)^{-p-1} {}_2F_1\left(3-p, p+1; p+2; \frac{d-ex}{2d}\right)}{d^3e^3(1-2p)(3-p)(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(d^2 - e^2\*x^2)^p)/(d + e\*x)^4, x]

[Out]  $-(d*(d^2 - e^2*x^2)^(1 + p))/(2*e^3*(3 - p)*(d + e*x)^4) + (d^2 - e^2*x^2)^(1 + p)/(e^3*(1 - 2*p)*(d + e*x)^3) - (2^(-3 + p)*(7 + p)*(1 + (e*x)/d)^(-1 - p)*(d^2 - e^2*x^2)^(1 + p)*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e*x)/(2*d)])/(d^3*e^3*(1 - 2*p)*(3 - p)*(1 + p))$

**Rubi in Sympy [A]** time = 102.308, size = 202, normalized size = 1.24

$$\frac{\left(\frac{d+ex}{d}\right)^{-p} (d-ex)^{-p} (d-ex)^{p+1} (d^2 - e^2x^2)^p {}_2F_1\left(-p+4, p+1; p+2; \frac{d-ex}{d}\right)}{16d^2e^3(p+1)} + \frac{\left(\frac{d+ex}{d}\right)^{-p} (d-ex)^{-p} (d-ex)^{p+2} (d^2 - e^2x^2)^p {}_2F_1\left(-p+4, p+2; p+3; \frac{d-ex}{d}\right)}{8d^3e^3(p+2)} - \frac{\left(\frac{d+ex}{d}\right)^{-p} (d-ex)^{-p} (d-ex)^{p+3} (d^2 - e^2x^2)^p {}_2F_1\left(-p+4, p+3; p+4; \frac{d-ex}{d}\right)}{16d^4e^3(p+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d)\*\*4, x)

[Out]  $-\left(\frac{d}{2} + \frac{e*x}{2}\right)/d)^{-p}*(d - e*x)^{-p}*(d - e*x)^{p+1}*(d^{**2} - e^{**2}*x^{**2})^{**p}*hyper((-p + 4, p + 1), (p + 2, ), (d/2 - e*x/2)/d)/(16*d^{**2}*e^{**3}*(p + 1)) + \left(\frac{d}{2} + \frac{e*x}{2}\right)/d)^{-p}*(d - e*x)^{-p}*(d - e*x)^{p+2}*(d^{**2} - e^{**2}*x^{**2})^{**p}*hyper((-p + 4, p + 2),$

$$(p + 3, ), (d/2 - e^*x/2)/d)/(8*d^{**3}*e^{**3}*(p + 2)) - ((d/2 + e^*x/2)/d)^{**}(-p)*(d - e^*x)^{**}(-p)*(d - e^*x)^{**}(p + 3)*(d^{**2} - e^{**2}*x^{**2})^{**}p*hyper((-p + 4, p + 3), (p + 4, ), (d/2 - e^*x/2)/d)/(16*d^{**4}*e^{**3}*(p + 3))$$

**Mathematica [A]** time = 0.129357, size = 130, normalized size = 0.8

$$\frac{2^{p-4}(d - ex) \left(\frac{ex}{d} + 1\right)^{-p} (d^2 - e^2x^2)^p \left(4 {}_2F_1\left(2 - p, p + 1; p + 2; \frac{d-ex}{2d}\right) - 4 {}_2F_1\left(3 - p, p + 1; p + 2; \frac{d-ex}{2d}\right) + {}_2F_1\left(4 - p, p + 1; p + 2; \frac{d-ex}{2d}\right)\right)}{d^2 e^3 (p + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(d^2 - e^2\*x^2)^p)/(d + e\*x)^4, x]

[Out] -((2^(-4 + p)\*(d - e\*x)\*(d^2 - e^2\*x^2)^p\*(4\*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)] - 4\*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)] + Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)]))/(d^2\*e^3\*(1 + p)\*(1 + (e\*x)/d)^p))

**Maple [F]** time = 0.151, size = 0, normalized size = 0.

$$\int \frac{x^2 (-e^2x^2 + d^2)^p}{(ex + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(-e^2\*x^2+d^2)^p/(e\*x+d)^4, x)

[Out] int(x^2\*(-e^2\*x^2+d^2)^p/(e\*x+d)^4, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x^2}{(ex + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x^2/(e\*x + d)^4, x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p\*x^2/(e\*x + d)^4, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p x^2}{e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x^2/(e\*x + d)^4, x, algorithm="fricas")



[Out] `integral((-e^2*x^2 + d^2)^p*x^2/(e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x + d^4), x)`

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 (-(-d + ex)(d + ex))^p}{(d + ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(-e**2*x**2+d**2)**p/(e*x+d)**4, x)`

[Out] `Integral(x**2*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**4, x)`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x^2}{(ex + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^4, x, algorithm="giac")`

[Out] `integrate((-e^2*x^2 + d^2)^p*x^2/(e*x + d)^4, x)`

$$3.299 \quad \int \frac{x(d^2 - e^2 x^2)^p}{(d + ex)^4} dx$$

**Optimal.** Leaf size=118

$$\frac{(d^2 - e^2 x^2)^{p+1}}{2e^2(3-p)(d+ex)^4} - \frac{2^{p-2} \left(\frac{ex}{d} + 1\right)^{-p-1} (d^2 - e^2 x^2)^{p+1} {}_2F_1\left(3-p, p+1; p+2; \frac{d-ex}{2d}\right)}{d^4 e^2 (3-p)(p+1)}$$

[Out]  $(d^2 - e^2 x^2)^{(1+p)} / (2 e^2 (3-p) (d+e x)^4) - (2^{(-2+p)} (1 + (e x)/d)^{(-1-p)} (d^2 - e^2 x^2)^{(1+p)} \text{Hypergeometric2F1}[3-p, 1+p, 2+p, (d - e x)/(2 d)]) / (d^4 e^2 (3-p) (1+p))$

**Rubi [A]** time = 0.157093, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$

$$\frac{(d^2 - e^2 x^2)^{p+1}}{2e^2(3-p)(d+ex)^4} - \frac{2^{p-2} \left(\frac{ex}{d} + 1\right)^{-p-1} (d^2 - e^2 x^2)^{p+1} {}_2F_1\left(3-p, p+1; p+2; \frac{d-ex}{2d}\right)}{d^4 e^2 (3-p)(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(x\*(d^2 - e^2\*x^2)^p)/(d + e\*x)^4, x]

[Out]  $(d^2 - e^2 x^2)^{(1+p)} / (2 e^2 (3-p) (d+e x)^4) - (2^{(-2+p)} (1 + (e x)/d)^{(-1-p)} (d^2 - e^2 x^2)^{(1+p)} \text{Hypergeometric2F1}[3-p, 1+p, 2+p, (d - e x)/(2 d)]) / (d^4 e^2 (3-p) (1+p))$

**Rubi in Sympy [A]** time = 27.9677, size = 112, normalized size = 0.95

$$\frac{(d - ex)^4 (d^2 - e^2 x^2)^{p-3}}{2e^2(-p+3)} - \frac{\left(\frac{d+ex}{d}\right)^{-p} (d^2 - e^2 x^2)^p (d^2 e - de^2 x)^{-p} (d^2 e - de^2 x)^{p+1} {}_2F_1\left(-p+3, p+1; p+2; \frac{d-ex}{d}\right)}{4d^4 e^3 (-p+3)(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d)\*\*4, x)

[Out]  $(d - e x)^4 (d^2 - e^2 x^2)^{p-3} / (2 e^2 (-p+3)) - ((d/2 + e x/2)/d)^{-p} (d^2 - e^2 x^2)^p (d^2 e - de^2 x)^{-p} (d^2 e - de^2 x)^{p+1} \text{hyper}((-p+3, p+1), (p+2), (d/2 - e x/2)/d) / (4 d^4 e^3 (-p+3) (p+1))$

**Mathematica [A]** time = 0.0985804, size = 102, normalized size = 0.86

$$\frac{2^{p-4} (d - ex) \left(\frac{ex}{d} + 1\right)^{-p} (d^2 - e^2 x^2)^p \left({}_2F_1\left(4-p, p+1; p+2; \frac{d-ex}{2d}\right) - 2 {}_2F_1\left(3-p, p+1; p+2; \frac{d-ex}{2d}\right)\right)}{d^3 e^2 (p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(d^2 - e^2\*x^2)^p)/(d + e\*x)^4, x]

[Out]  $(2^{(-4+p)} (d - e x) (d^2 - e^2 x^2)^p (-2 \text{Hypergeometric2F1}[3-p, 1+p, 2+p, (d - e x)/(2 d)] + \text{Hypergeometric2F1}[4-p, 1 +$

$p, 2 + p, (d - e^*x)/(2*d)])))/(d^3*e^2*(1 + p)*(1 + (e^*x)/d)^p$

**Maple [F]** time = 0.146, size = 0, normalized size = 0.

$$\int \frac{x(-e^2x^2 + d^2)^p}{(ex + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(-e^2\*x^2+d^2)^p/(e\*x+d)^4, x)

[Out] int(x\*(-e^2\*x^2+d^2)^p/(e\*x+d)^4, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x}{(ex + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x/(e\*x + d)^4, x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p\*x/(e\*x + d)^4, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p x}{e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*x/(e\*x + d)^4, x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p\*x/(e^4\*x^4 + 4\*d\*e^3\*x^3 + 6\*d^2\*e^2\*x^2 + 4\*d^3\*e\*x + d^4), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x(-(-d + ex)(d + ex))^p}{(d + ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d)\*\*4, x)

[Out] Integral(x\*(-(-d + e\*x)\*(d + e\*x))\*\*p/(d + e\*x)\*\*4, x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p x}{(ex + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^4,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p*x/(e*x + d)^4, x)
```

$$3.300 \quad \int \frac{(d^2 - e^2 x^2)^p}{(d + ex)^4} dx$$

**Optimal.** Leaf size=73

$$\frac{2^{p-4} \left(\frac{ex}{d} + 1\right)^{-p-1} (d^2 - e^2 x^2)^{p+1} {}_2F_1\left(4 - p, p + 1; p + 2; \frac{d - ex}{2d}\right)}{d^5 e(p + 1)}$$

[Out] -((2^(-4 + p)\*(1 + (e\*x)/d)^(-1 - p)\*(d^2 - e^2\*x^2)^(1 + p)\*Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/(d^5\*e\*(1 + p)))

**Rubi [A]** time = 0.0763143, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{2^{p-4} \left(\frac{ex}{d} + 1\right)^{-p-1} (d^2 - e^2 x^2)^{p+1} {}_2F_1\left(4 - p, p + 1; p + 2; \frac{d - ex}{2d}\right)}{d^5 e(p + 1)}$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^p/(d + e\*x)^4, x]

[Out] -((2^(-4 + p)\*(1 + (e\*x)/d)^(-1 - p)\*(d^2 - e^2\*x^2)^(1 + p)\*Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/(d^5\*e\*(1 + p)))

**Rubi in Sympy [A]** time = 11.8307, size = 66, normalized size = 0.9

$$\frac{\left(\frac{\frac{d}{2} + \frac{ex}{2}}{d}\right)^{-p} (d - ex)^{-p} (d - ex)^{p+1} (d^2 - e^2 x^2)^p {}_2F_1\left(\begin{matrix} -p + 4, p + 1 \\ p + 2 \end{matrix} \middle| \frac{\frac{d}{2} - \frac{ex}{2}}{d}\right)}{16d^4 e(p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d)\*\*4, x)

[Out] -((d/2 + e\*x/2)/d)\*\*(-p)\*(d - e\*x)\*\*(-p)\*(d - e\*x)\*\*(p + 1)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p\*hyper((-p + 4, p + 1), (p + 2, ), (d/2 - e\*x/2)/d)/(16\*d\*\*4\*e\*(p + 1))

**Mathematica [A]** time = 0.0516472, size = 75, normalized size = 1.03

$$\frac{2^{p-4} (d - ex) \left(\frac{ex}{d} + 1\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(4 - p, p + 1; p + 2; \frac{d - ex}{2d}\right)}{d^4 e(p + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^p/(d + e\*x)^4, x]

[Out] -((2^(-4 + p)\*(d - e\*x)\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/(d^4\*e\*(1 + p)\*(1 + (e\*x)/d)^p))

---

**Maple [F]** time = 0.14, size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^p/(e\*x+d)^4, x)

[Out] int((-e^2\*x^2+d^2)^p/(e\*x+d)^4, x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/(e\*x + d)^4, x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p/(e\*x + d)^4, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p}{e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/(e\*x + d)^4, x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p/(e^4\*x^4 + 4\*d\*e^3\*x^3 + 6\*d^2\*e^2\*x^2 + 4\*d^3\*e\*x + d^4), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(-d + ex)(d + ex))^p}{(d + ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d)\*\*4, x)

[Out] Integral((-(-d + e\*x)\*(d + e\*x))\*\*p/(d + e\*x)\*\*4, x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-e^2*x^2 + d^2)^p/(e*x + d)^4,x, algorithm="giac")
```

```
[Out] integrate((-e^2*x^2 + d^2)^p/(e*x + d)^4, x)
```

$$3.301 \quad \int \frac{(d^2 - e^2 x^2)^p}{x(d+ex)^4} dx$$

**Optimal.** Leaf size=204

$$\frac{(d^2 - e^2 x^2)^{p-2} {}_2F_1\left(1, p-2; p-1; 1 - \frac{e^2 x^2}{d^2}\right)}{2(2-p)} - \frac{4dex (d^2 - e^2 x^2)^{p-3}}{5-2p} + \frac{4d^2 (d^2 - e^2 x^2)^{p-3}}{3-p} - \frac{(d^2 - e^2 x^2)^{p-2}}{2(2-p)} - \frac{8e(2-p)x \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(\frac{1}{2}, 4-p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^5(5-2p)}$$

[Out] (4\*d^2\*(d^2 - e^2\*x^2)^(-3 + p))/(3 - p) - (4\*d\*e\*x\*(d^2 - e^2\*x^2)^(-3 + p))/(5 - 2\*p) - (d^2 - e^2\*x^2)^(-2 + p)/(2\*(2 - p)) - (8\*e\*(2 - p)\*x\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[1/2, 4 - p, 3/2, (e^2\*x^2)/d^2])/(d^5\*(5 - 2\*p)\*(1 - (e^2\*x^2)/d^2)^p) + ((d^2 - e^2\*x^2)^(-2 + p)\*Hypergeometric2F1[1, -2 + p, -1 + p, 1 - (e^2\*x^2)/d^2])/(2\*(2 - p))

**Rubi [A]** time = 0.463041, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.36$

$$\frac{(d^2 - e^2 x^2)^{p-2} {}_2F_1\left(1, p-2; p-1; 1 - \frac{e^2 x^2}{d^2}\right)}{2(2-p)} - \frac{4dex (d^2 - e^2 x^2)^{p-3}}{5-2p} + \frac{4d^2 (d^2 - e^2 x^2)^{p-3}}{3-p} - \frac{(d^2 - e^2 x^2)^{p-2}}{2(2-p)} - \frac{8e(2-p)x \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(\frac{1}{2}, 4-p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^5(5-2p)}$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^p/(x\*(d + e\*x)^4), x]

[Out] (4\*d^2\*(d^2 - e^2\*x^2)^(-3 + p))/(3 - p) - (4\*d\*e\*x\*(d^2 - e^2\*x^2)^(-3 + p))/(5 - 2\*p) - (d^2 - e^2\*x^2)^(-2 + p)/(2\*(2 - p)) - (8\*e\*(2 - p)\*x\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[1/2, 4 - p, 3/2, (e^2\*x^2)/d^2])/(d^5\*(5 - 2\*p)\*(1 - (e^2\*x^2)/d^2)^p) + ((d^2 - e^2\*x^2)^(-2 + p)\*Hypergeometric2F1[1, -2 + p, -1 + p, 1 - (e^2\*x^2)/d^2])/(2\*(2 - p))

**Rubi in Sympy [A]** time = 48.2519, size = 187, normalized size = 0.92

$$\frac{d^2 (d^2 - e^2 x^2)^{p-3} {}_2F_1\left(1, p-3; p-2; 1 - \frac{e^2 x^2}{d^2}\right)}{2(-p+3)} + \frac{7d^2 (d^2 - e^2 x^2)^{p-3}}{2(-p+3)} - \frac{(d^2 - e^2 x^2)^{p-2}}{2(-p+2)} - \frac{4ex \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+4, \frac{1}{2}; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^5} - \frac{4e^3 x^3 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+4, \frac{3}{2}; \frac{5}{2}; \frac{e^2 x^2}{d^2}\right)}{3d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x/(e\*x+d)\*\*4, x)

[Out] d\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(p - 3)\*hyper((1, p - 3), (p - 2, ), 1 - e\*\*2\*x\*\*2/d\*\*2)/(2\*(-p + 3)) + 7\*d\*\*2\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(p - 3)



$$\frac{(2(-p+3)) - (d^2 - e^2 x^2)^{p-2} / (2(-p+2)) - 4e^2 x^2 (1 - e^2 x^2 / d^2)^{-p} (d^2 - e^2 x^2)^p \operatorname{hyper}((-p+4, 1/2), (3/2, ), e^2 x^2 / d^2) / d^5 - 4e^3 x^3 (1 - e^2 x^2 / d^2)^{-p} (d^2 - e^2 x^2)^p \operatorname{hyper}((-p+4, 3/2), (5/2, ), e^2 x^2 / d^2) / (3d^7)}{e(2p-5)x(d-ex)^p(d+ex)^{p-4} F_1\left(4-2p; -p, 4-p; 5-2p; \frac{d}{ex}, -\frac{d}{ex}\right)}$$

**Mathematica [C]** time = 0.498571, size = 199, normalized size = 0.98

$$\frac{e(2p-5)x(d-ex)^p(d+ex)^{p-4} F_1\left(4-2p; -p, 4-p; 5-2p; \frac{d}{ex}, -\frac{d}{ex}\right)}{2(p-2)\left(e(2p-5)x F_1\left(4-2p; -p, 4-p; 5-2p; \frac{d}{ex}, -\frac{d}{ex}\right) + d p F_1\left(5-2p; 1-p, 4-p; 6-2p; \frac{d}{ex}, -\frac{d}{ex}\right) - d(p-4) F_1\left(5-2p; 1-p, 4-p; 6-2p; \frac{d}{ex}, -\frac{d}{ex}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(d^2 - e^2 x^2)^p / (x (d + e x)^4), x]

[Out] (e^(-5 + 2\*p) \* x \* (d - e\*x)^p \* (d + e\*x)^(-4 + p) \* AppellF1[4 - 2\*p, -p, 4 - p, 5 - 2\*p, d/(e\*x), -(d/(e\*x))]) / (2 \* (-2 + p) \* (e^(-5 + 2\*p) \* x \* AppellF1[4 - 2\*p, -p, 4 - p, 5 - 2\*p, d/(e\*x), -(d/(e\*x))] + d \* p \* AppellF1[5 - 2\*p, 1 - p, 4 - p, 6 - 2\*p, d/(e\*x), -(d/(e\*x))] - d \* (-4 + p) \* AppellF1[5 - 2\*p, -p, 5 - p, 6 - 2\*p, d/(e\*x), -(d/(e\*x))]))

**Maple [F]** time = 0.071, size = 0, normalized size = 0.

$$\int \frac{(-e^2 x^2 + d^2)^p}{x (ex + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^p/x/(e\*x+d)^4,x)

[Out] int((-e^2\*x^2+d^2)^p/x/(e\*x+d)^4,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2 x^2 + d^2)^p}{(ex + d)^4 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p / ((e\*x + d)^4 \* x), x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p / ((e\*x + d)^4 \* x), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(-e^2 x^2 + d^2)^p}{e^4 x^5 + 4 d e^3 x^4 + 6 d^2 e^2 x^3 + 4 d^3 e x^2 + d^4 x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x),x, algorithm="fricas")`

[Out] `integral((-e^2*x^2 + d^2)^p/(e^4*x^5 + 4*d*e^3*x^4 + 6*d^2*e^2*x^3 + 4*d^3*e*x^2 + d^4*x), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(-d + ex)(d + ex))^p}{x(d + ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e**2*x**2+d**2)**p/x/(e*x+d)**4,x)`

[Out] `Integral((-(-d + e*x)*(d + e*x))**p/(x*(d + e*x)**4), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^4x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x),x, algorithm="giac")`

[Out] `integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x), x)`

$$3.302 \quad \int \frac{(d^2 - e^2 x^2)^p}{x^2 (d + ex)^4} dx$$

**Optimal.** Leaf size=207

$$\begin{aligned} & -\frac{2e(d^2 - e^2 x^2)^{p-2} {}_2F_1\left(1, p-2; p-1; 1 - \frac{e^2 x^2}{d^2}\right)}{d(2-p)} + \frac{e^2 x (d^2 - e^2 x^2)^{p-3}}{5-2p} - \frac{4de (d^2 - e^2 x^2)^{p-3}}{3-p} \\ & - \frac{d^2 (d^2 - e^2 x^2)^{p-3}}{x} + \frac{4e^2 (p^2 - 9p + 16) x \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(\frac{1}{2}, 4-p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^6(5-2p)} \end{aligned}$$

[Out]  $(-4*d*e*(d^2 - e^2*x^2)^{(-3 + p)})/(3 - p) - (d^2*(d^2 - e^2*x^2)^{(-3 + p)})/x + (e^2*x*(d^2 - e^2*x^2)^{(-3 + p)})/(5 - 2*p) + (4*e^2*(16 - 9*p + p^2)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 4 - p, 3/2, (e^2*x^2)/d^2])/(d^6*(5 - 2*p)*(1 - (e^2*x^2)/d^2)^p) - (2*e*(d^2 - e^2*x^2)^{(-2 + p)}*Hypergeometric2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(d*(2 - p))$

**Rubi [A]** time = 0.527922, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.36$

$$\begin{aligned} & -\frac{2e(d^2 - e^2 x^2)^{p-2} {}_2F_1\left(1, p-2; p-1; 1 - \frac{e^2 x^2}{d^2}\right)}{d(2-p)} + \frac{e^2 x (d^2 - e^2 x^2)^{p-3}}{5-2p} - \frac{4de (d^2 - e^2 x^2)^{p-3}}{3-p} \\ & - \frac{d^2 (d^2 - e^2 x^2)^{p-3}}{x} + \frac{4e^2 (p^2 - 9p + 16) x \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(\frac{1}{2}, 4-p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^6(5-2p)} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^4), x]$

[Out]  $(-4*d*e*(d^2 - e^2*x^2)^{(-3 + p)})/(3 - p) - (d^2*(d^2 - e^2*x^2)^{(-3 + p)})/x + (e^2*x*(d^2 - e^2*x^2)^{(-3 + p)})/(5 - 2*p) + (4*e^2*(16 - 9*p + p^2)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 4 - p, 3/2, (e^2*x^2)/d^2])/(d^6*(5 - 2*p)*(1 - (e^2*x^2)/d^2)^p) - (2*e*(d^2 - e^2*x^2)^{(-2 + p)}*Hypergeometric2F1[1, -2 + p, -1 + p, 1 - (e^2*x^2)/d^2])/(d*(2 - p))$

**Rubi in Sympy [A]** time = 73.6857, size = 216, normalized size = 1.04

$$\begin{aligned} & -\frac{2de (d^2 - e^2 x^2)^{p-3} {}_2F_1\left(1, p-3; p-2; 1 - \frac{e^2 x^2}{d^2}\right)}{-p+3} - \frac{2de (d^2 - e^2 x^2)^{p-3}}{-p+3} \\ & - \frac{\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+4, -\frac{1}{2}; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{d^4 x} \\ & + \frac{6e^2 x \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+4, \frac{1}{2}; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^6} \\ & + \frac{e^4 x^3 \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+4, \frac{3}{2}; \frac{5}{2}; \frac{e^2 x^2}{d^2}\right)}{3d^8} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((-e**2*x**2+d**2)**p/x**2/(e*x+d)**4,x)`

[Out]  $-2*d*e*(d**2 - e**2*x**2)**(p - 3)*\text{hyper}((1, p - 3), (p - 2,), 1 - e**2*x**2/d**2)/(-p + 3) - 2*d*e*(d**2 - e**2*x**2)**(p - 3)/(-p + 3) - (1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*\text{hyper}((-p + 4, -1/2), (1/2,), e**2*x**2/d**2)/(d**4*x) + 6*e**2*x*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*\text{hyper}((-p + 4, 1/2), (3/2,), e**2*x**2/d**2)/d**6 + e**4*x**3*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*\text{hyper}((-p + 4, 3/2), (5/2,), e**2*x**2/d**2)/(3*d**8)$

**Mathematica [C]** time = 0.53431, size = 195, normalized size = 0.94

$$\frac{2e(p-3)(d-ex)^p(d+ex)^{p-4}F_1\left(5-2p; -p, 4-p; 6-2p; \frac{d}{ex}, -\frac{d}{ex}\right)}{(2p-5)\left(2e(p-3)xF_1\left(5-2p; -p, 4-p; 6-2p; \frac{d}{ex}, -\frac{d}{ex}\right) + dpF_1\left(6-2p; 1-p, 4-p; 7-2p; \frac{d}{ex}, -\frac{d}{ex}\right) - d(p-4)F_1\left(6-2p; 1-p, 4-p; 7-2p; \frac{d}{ex}, -\frac{d}{ex}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(d^2 - e^2*x^2)^p/(x^2*(d + e*x)^4),x]`

[Out]  $(2*e*(-3 + p)*(d - e*x)^p*(d + e*x)^{(-4 + p)}*\text{AppellF1}[5 - 2*p, -p, 4 - p, 6 - 2*p, d/(e*x), -(d/(e*x))])/((-5 + 2*p)*(2*e*(-3 + p)*x*\text{AppellF1}[5 - 2*p, -p, 4 - p, 6 - 2*p, d/(e*x), -(d/(e*x))] + d*p*\text{AppellF1}[6 - 2*p, 1 - p, 4 - p, 7 - 2*p, d/(e*x), -(d/(e*x))]) - d*(-4 + p)*\text{AppellF1}[6 - 2*p, -p, 5 - p, 7 - 2*p, d/(e*x), -(d/(e*x))])$

**Maple [F]** time = 0.108, size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{x^2(ex + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^4,x)`

[Out] `int((-e^2*x^2+d^2)^p/x^2/(e*x+d)^4,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^4x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^2),x, algorithm="maxima")`

[Out] `integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p}{e^4x^6 + 4de^3x^5 + 6d^2e^2x^4 + 4d^3ex^3 + d^4x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^2),x, algorithm="fricas")`

[Out] `integral((-e^2*x^2 + d^2)^p/(e^4*x^6 + 4*d*e^3*x^5 + 6*d^2*e^2*x^4 + 4*d^3*e*x^3 + d^4*x^2), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(-d + ex)(d + ex))^p}{x^2(d + ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e**2*x**2+d**2)**p/x**2/(e*x+d)**4,x)`

[Out] `Integral((-(-d + e*x)*(d + e*x))**p/(x**2*(d + e*x)**4), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^4x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^2),x, algorithm="giac")`

[Out] `integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^2), x)`

$$3.303 \quad \int \frac{(d^2 - e^2 x^2)^p}{x^3 (d + ex)^4} dx$$

**Optimal.** Leaf size=211

$$\frac{e^2(10-p)(d^2 - e^2 x^2)^{p-2} {}_2F_1\left(1, p-2; p-1; 1 - \frac{e^2 x^2}{d^2}\right)}{2d^2(2-p)} + \frac{e^2(11-p)(d^2 - e^2 x^2)^{p-3}}{2(3-p)} + \frac{4de(d^2 - e^2 x^2)^{p-3}}{x} - \frac{d^2(d^2 - e^2 x^2)^{p-3}}{2x^2} - \frac{8e^3(4-p)x\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p}(d^2 - e^2 x^2)^p {}_2F_1\left(\frac{1}{2}, 4-p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^7}$$

[Out]  $(e^2(11-p)(d^2 - e^2 x^2)^{-3+p})/(2(3-p)) - (d^2(d^2 - e^2 x^2)^{-3+p})/(2x^2) + (4de(d^2 - e^2 x^2)^{-3+p})/x - (8e^3(4-p)x(d^2 - e^2 x^2)^p \text{Hypergeometric2F1}[1/2, 4-p, 3/2, (e^2 x^2)/d^2])/(d^7(1 - (e^2 x^2)/d^2)^p) + (e^2(10-p)(d^2 - e^2 x^2)^{-2+p} \text{Hypergeometric2F1}[1, -2+p, -1+p, 1 - (e^2 x^2)/d^2])/(2d^2(2-p))$

**Rubi [A]** time = 0.634138, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.36$

$$\frac{e^2(10-p)(d^2 - e^2 x^2)^{p-2} {}_2F_1\left(1, p-2; p-1; 1 - \frac{e^2 x^2}{d^2}\right)}{2d^2(2-p)} + \frac{e^2(11-p)(d^2 - e^2 x^2)^{p-3}}{2(3-p)} + \frac{4de(d^2 - e^2 x^2)^{p-3}}{x} - \frac{d^2(d^2 - e^2 x^2)^{p-3}}{2x^2} - \frac{8e^3(4-p)x\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p}(d^2 - e^2 x^2)^p {}_2F_1\left(\frac{1}{2}, 4-p; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^7}$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^p/(x^3\*(d + e\*x)^4), x]

[Out]  $(e^2(11-p)(d^2 - e^2 x^2)^{-3+p})/(2(3-p)) - (d^2(d^2 - e^2 x^2)^{-3+p})/(2x^2) + (4de(d^2 - e^2 x^2)^{-3+p})/x - (8e^3(4-p)x(d^2 - e^2 x^2)^p \text{Hypergeometric2F1}[1/2, 4-p, 3/2, (e^2 x^2)/d^2])/(d^7(1 - (e^2 x^2)/d^2)^p) + (e^2(10-p)(d^2 - e^2 x^2)^{-2+p} \text{Hypergeometric2F1}[1, -2+p, -1+p, 1 - (e^2 x^2)/d^2])/(2d^2(2-p))$

**Rubi in Sympy [A]** time = 88.1931, size = 204, normalized size = 0.97

$$\frac{3e^2(d^2 - e^2 x^2)^{p-3} {}_2F_1\left(1, p-3; p-2; 1 - \frac{e^2 x^2}{d^2}\right)}{-p+3} + \frac{e^2(d^2 - e^2 x^2)^{p-3} {}_2F_1\left(2, p-3; p-2; 1 - \frac{e^2 x^2}{d^2}\right)}{2(-p+3)} + \frac{e^2(d^2 - e^2 x^2)^{p-3}}{2(-p+3)} + \frac{4e\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p}(d^2 - e^2 x^2)^p {}_2F_1\left(-p+4, -\frac{1}{2}; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{d^5 x} - \frac{4e^3 x\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p}(d^2 - e^2 x^2)^p {}_2F_1\left(-p+4, \frac{1}{2}; \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((-e**2*x**2+d**2)**p/x**3/(e*x+d)**4,x)`

[Out]  $3e^{2x^2}(d^2 - e^{2x^2})^{p-3} \operatorname{hyper}((1, p-3), (p-2, ), 1 - e^{2x^2}/d^2)/(-p+3) + e^{2x^2}(d^2 - e^{2x^2})^{p-3} \operatorname{hyper}((2, p-3), (p-2, ), 1 - e^{2x^2}/d^2)/(2(-p+3)) + e^{2x^2}(d^2 - e^{2x^2})^{p-3}/(2(-p+3)) + 4e^{2x^2}(1 - e^{2x^2}/d^2)^{p-1} \operatorname{hyper}((-p+4, -1/2), (1/2, ), e^{2x^2}/d^2)/(d^5x) - 4e^{2x^2}(1 - e^{2x^2}/d^2)^{p-1} \operatorname{hyper}((-p+4, 1/2), (3/2, ), e^{2x^2}/d^2)/d^7$

**Mathematica [A]** time = 2.03137, size = 399, normalized size = 1.89

$$(d^2 - e^{2x^2})^p \left( \frac{80de^2 \left(1 - \frac{d^2}{e^{2x^2}}\right)^{-p} {}_2F_1\left(-p, -p; 1-p; \frac{d^2}{e^{2x^2}}\right)}{p} + \frac{64d^2 e \left(1 - \frac{e^{2x^2}}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; \frac{e^{2x^2}}{d^2}\right)}{x} + \frac{8d^3 \left(1 - \frac{d^2}{e^{2x^2}}\right)^{-p} {}_2F_1\left(1-p, -p; 2-p; \frac{d^2}{e^{2x^2}}\right)}{(p-1)x^2} + \frac{5e^{2x^2}}{d^7} \right)$$

Antiderivative was successfully verified.

[In] `Integrate[(d^2 - e^2*x^2)^p/(x^3*(d + e*x)^4),x]`

[Out]  $((d^2 - e^{2x^2})^p * ((64*d^2*e*Hypergeometric2F1[-1/2, -p, 1/2, (e^{2x^2}/d^2)]/(x*(1 - (e^{2x^2}/d^2))^p) + (8*d^3*Hypergeometric2F1[1-p, -p, 2-p, d^2/(e^{2x^2})]]/((-1+p)*(1 - d^2/(e^{2x^2})))^p * x^2) + (5*2^(4+p)*e^{2x^2}*(d - e*x)*Hypergeometric2F1[1-p, 1+p, 2+p, (d - e*x)/(2*d)])/((1+p)*(1 + (e*x)/d)^p) + (3*2^(3+p)*e^{2x^2}*(d - e*x)*Hypergeometric2F1[2-p, 1+p, 2+p, (d - e*x)/(2*d)])/((1+p)*(1 + (e*x)/d)^p) + (3*2^(1+p)*e^{2x^2}*(d - e*x)*Hypergeometric2F1[3-p, 1+p, 2+p, (d - e*x)/(2*d)])/((1+p)*(1 + (e*x)/d)^p) + (2^p*e^{2x^2}*(d - e*x)*Hypergeometric2F1[4-p, 1+p, 2+p, (d - e*x)/(2*d)])/((1+p)*(1 + (e*x)/d)^p) + (80*d*e^{2x^2}*Hypergeometric2F1[-p, -p, 1-p, d^2/(e^{2x^2})]]/(p*(1 - d^2/(e^{2x^2}))^p)))/(16*d^7)$

**Maple [F]** time = 0.127, size = 0, normalized size = 0.

$$\int \frac{(-e^{2x^2} + d^2)^p}{x^3 (ex + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^4,x)`

[Out] `int((-e^2*x^2+d^2)^p/x^3/(e*x+d)^4,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^{2x^2} + d^2)^p}{(ex + d)^4 x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^3),x, algorithm="maxima")`

[Out] `integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^3), x)`

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**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p}{e^4x^7 + 4de^3x^6 + 6d^2e^2x^5 + 4d^3ex^4 + d^4x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^3), x, algorithm="fricas")`

[Out] `integral((-e^2*x^2 + d^2)^p/(e^4*x^7 + 4*d*e^3*x^6 + 6*d^2*e^2*x^5 + 4*d^3*e*x^4 + d^4*x^3), x)`

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**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(-d + ex)(d + ex))^p}{x^3(d + ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e**2*x**2+d**2)**p/x**3/(e*x+d)**4, x)`

[Out] `Integral((-(-d + e*x)*(d + e*x))**p/(x**3*(d + e*x)**4), x)`

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**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^4x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^3), x, algorithm="giac")`

[Out] `integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^3), x)`



$$3.304 \quad \int \frac{(d^2 - e^2 x^2)^p}{x^4 (d + ex)^4} dx$$

**Optimal.** Leaf size=210

$$\begin{aligned} & -\frac{e^2(27-2p)(d^2-e^2x^2)^{p-3}}{3x} + \frac{2de(d^2-e^2x^2)^{p-3}}{x^2} - \frac{d^2(d^2-e^2x^2)^{p-3}}{3x^3} \\ & - \frac{2e^3(5-p)(d^2-e^2x^2)^{p-3} {}_2F_1\left(1, p-3; p-2; 1-\frac{e^2x^2}{d^2}\right)}{d(3-p)} \\ & + \frac{4e^4(p^2-17p+48)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p {}_2F_1\left(\frac{1}{2}, 4-p; \frac{3}{2}; \frac{e^2x^2}{d^2}\right)}{3d^8} \end{aligned}$$

[Out]  $-(d^2*(d^2 - e^2*x^2)^{(-3 + p)})/(3*x^3) + (2*d*e*(d^2 - e^2*x^2)^{(-3 + p)})/x^2 - (e^2*(27 - 2*p)*(d^2 - e^2*x^2)^{(-3 + p)})/(3*x) + (4*e^4*(48 - 17*p + p^2)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 4 - p, 3/2, (e^2*x^2)/d^2])/(3*d^8*(1 - (e^2*x^2)/d^2)^p) - (2*e^3*(5 - p)*(d^2 - e^2*x^2)^{(-3 + p)}*Hypergeometric2F1[1, -3 + p, -2 + p, 1 - (e^2*x^2)/d^2])/(d*(3 - p))$

**Rubi [A]** time = 0.665328, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$\begin{aligned} & -\frac{e^2(27-2p)(d^2-e^2x^2)^{p-3}}{3x} + \frac{2de(d^2-e^2x^2)^{p-3}}{x^2} - \frac{d^2(d^2-e^2x^2)^{p-3}}{3x^3} \\ & - \frac{2e^3(5-p)(d^2-e^2x^2)^{p-3} {}_2F_1\left(1, p-3; p-2; 1-\frac{e^2x^2}{d^2}\right)}{d(3-p)} \\ & + \frac{4e^4(p^2-17p+48)x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p {}_2F_1\left(\frac{1}{2}, 4-p; \frac{3}{2}; \frac{e^2x^2}{d^2}\right)}{3d^8} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^p/(x^4\*(d + e\*x)^4), x]

[Out]  $-(d^2*(d^2 - e^2*x^2)^{(-3 + p)})/(3*x^3) + (2*d*e*(d^2 - e^2*x^2)^{(-3 + p)})/x^2 - (e^2*(27 - 2*p)*(d^2 - e^2*x^2)^{(-3 + p)})/(3*x) + (4*e^4*(48 - 17*p + p^2)*x*(d^2 - e^2*x^2)^p*Hypergeometric2F1[1/2, 4 - p, 3/2, (e^2*x^2)/d^2])/(3*d^8*(1 - (e^2*x^2)/d^2)^p) - (2*e^3*(5 - p)*(d^2 - e^2*x^2)^{(-3 + p)}*Hypergeometric2F1[1, -3 + p, -2 + p, 1 - (e^2*x^2)/d^2])/(d*(3 - p))$

**Rubi in Sympy [A]** time = 96.312, size = 240, normalized size = 1.14

$$\begin{aligned} & -\frac{2e^3(d^2-e^2x^2)^{p-3} {}_2F_1\left(1, p-3; p-2; 1-\frac{e^2x^2}{d^2}\right)}{d(-p+3)} - \frac{2e^3(d^2-e^2x^2)^{p-3} {}_2F_1\left(2, p-3; p-2; 1-\frac{e^2x^2}{d^2}\right)}{d(-p+3)} \\ & - \frac{\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p {}_2F_1\left(-p+4, -\frac{3}{2}; -\frac{1}{2}; \frac{e^2x^2}{d^2}\right)}{3d^4x^3} \\ & - \frac{6e^2\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p {}_2F_1\left(-p+4, -\frac{1}{2}; \frac{1}{2}; \frac{e^2x^2}{d^2}\right)}{d^6x} \\ & + \frac{e^4x\left(1-\frac{e^2x^2}{d^2}\right)^{-p}(d^2-e^2x^2)^p {}_2F_1\left(-p+4, \frac{1}{2}; \frac{3}{2}; \frac{e^2x^2}{d^2}\right)}{d^8} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((-e**2*x**2+d**2)**p/x**4/(e*x+d)**4,x)`

[Out]  $-2e^{2x^2/d^2}(d^2 - e^{2x^2/d^2})^{p-3} \operatorname{hyper}((1, p-3), (p-2,), 1 - e^{2x^2/d^2}/d^2)/(d^{p-3}) - 2e^{2x^2/d^2}(d^2 - e^{2x^2/d^2})^{p-3} \operatorname{hyper}((2, p-3), (p-2,), 1 - e^{2x^2/d^2}/d^2)/(d^{p-3}) - (1 - e^{2x^2/d^2})^{-p}(d^2 - e^{2x^2/d^2})^p \operatorname{hyper}((-p+4, -3/2), (-1/2,), e^{2x^2/d^2}/(3d^4x^3) - 6e^{2x^2/d^2}(1 - e^{2x^2/d^2})^{-p}(d^2 - e^{2x^2/d^2})^p \operatorname{hyper}((-p+4, -1/2), (1/2,), e^{2x^2/d^2}/(d^6x) + e^{4x^2/d^2}(1 - e^{2x^2/d^2})^{-p}(d^2 - e^{2x^2/d^2})^p \operatorname{hyper}((-p+4, 1/2), (3/2,), e^{2x^2/d^2}/d^8)$

**Mathematica [B]** time = 1.0128, size = 452, normalized size = 2.15

$$(d^2 - e^2x^2)^p \left( -\frac{480d^2e^2\left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; \frac{e^2x^2}{d^2}\right)}{x} - \frac{480de^3\left(1 - \frac{d^2}{e^2x^2}\right)^{-p} {}_2F_1\left(-p, -p; 1-p; \frac{d^2}{e^2x^2}\right)}{p} - \frac{16d^4\left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{3}{2}, -p; -\frac{1}{2}; \frac{e^2x^2}{d^2}\right)}{x^3} \right)$$

Antiderivative was successfully verified.

[In] `Integrate[(d^2 - e^2*x^2)^p/(x^4*(d + e*x)^4),x]`

[Out]  $((d^2 - e^2x^2)^p \operatorname{Hypergeometric2F1}[-3/2, -p, -1/2, (e^2x^2/d^2)]/(x^3(1 - (e^2x^2/d^2))^p) - (480d^2e^2 \operatorname{Hypergeometric2F1}[-1/2, -p, 1/2, (e^2x^2/d^2)]/(x(1 - (e^2x^2/d^2))^p) - (96d^3e \operatorname{Hypergeometric2F1}[1-p, -p, 2-p, d^2/(e^2x^2)]/((1+p)(1 - d^2/(e^2x^2))^p x^2) + (152^{5+p}e^3(-d + ex) \operatorname{Hypergeometric2F1}[1-p, 1+p, 2+p, (d - ex)/(2d)]/((1+p)(1 + (ex)/d)^p) + (152^{3+p}e^3(-d + ex) \operatorname{Hypergeometric2F1}[2-p, 1+p, 2+p, (d - ex)/(2d)]/((1+p)(1 + (ex)/d)^p) + (32^{3+p}e^3(-d + ex) \operatorname{Hypergeometric2F1}[3-p, 1+p, 2+p, (d - ex)/(2d)]/((1+p)(1 + (ex)/d)^p) + (32^p e^3(-d + ex) \operatorname{Hypergeometric2F1}[4-p, 1+p, 2+p, (d - ex)/(2d)]/((1+p)(1 + (ex)/d)^p) - (480d^2e^3 \operatorname{Hypergeometric2F1}[-p, -p, 1-p, d^2/(e^2x^2)]/(p(1 - d^2/(e^2x^2))^p)))/(48d^8)$

**Maple [F]** time = 0.092, size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{x^4(ex + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^4,x)`

[Out] `int((-e^2*x^2+d^2)^p/x^4/(e*x+d)^4,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^4x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^4),x, algorithm="maxima")`

[Out] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^4\*x^4), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^P}{e^4x^8 + 4de^3x^7 + 6d^2e^2x^6 + 4d^3ex^5 + d^4x^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^4\*x^4),x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p/(e^4\*x^8 + 4\*d\*e^3\*x^7 + 6\*d^2\*e^2\*x^6 + 4\*d^3\*e\*x^5 + d^4\*x^4), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(-d + ex)(d + ex))^P}{x^4(d + ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x\*\*4/(e\*x+d)\*\*4,x)

[Out] Integral((-(-d + e\*x)\*(d + e\*x))\*\*p/(x\*\*4\*(d + e\*x)\*\*4), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^P}{(ex + d)^4x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^4\*x^4),x, algorithm="giac")

[Out] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^4\*x^4), x)

$$3.305 \quad \int \frac{(d^2 - e^2 x^2)^p}{x^5 (d + ex)^4} dx$$

**Optimal.** Leaf size=216

$$\begin{aligned} & -\frac{e^2(17-p)(d^2 - e^2 x^2)^{p-3}}{4x^2} - \frac{d^2(d^2 - e^2 x^2)^{p-3}}{4x^4} + \frac{4de(d^2 - e^2 x^2)^{p-3}}{3x^3} \\ & + \frac{e^4(p^2 - 21p + 70)(d^2 - e^2 x^2)^{p-3} {}_2F_1\left(1, p-3; p-2; 1 - \frac{e^2 x^2}{d^2}\right)}{4d^2(3-p)} \\ & + \frac{8e^3(6-p)\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-\frac{1}{2}, 4-p; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{3d^7 x} \end{aligned}$$

[Out]  $-(d^2*(d^2 - e^2*x^2)^{(-3 + p)})/(4*x^4) + (4*d*e*(d^2 - e^2*x^2)^{(-3 + p)})/(3*x^3) - (e^2*(17 - p)*(d^2 - e^2*x^2)^{(-3 + p)})/(4*x^2) + (8*e^3*(6 - p)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 4 - p, 1/2, (e^2*x^2)/d^2])/(3*d^7*x*(1 - (e^2*x^2)/d^2)^p) + (e^4*(70 - 21*p + p^2)*(d^2 - e^2*x^2)^{(-3 + p)}*Hypergeometric2F1[1, -3 + p, -2 + p, 1 - (e^2*x^2)/d^2])/(4*d^2*(3 - p))$

**Rubi [A]** time = 0.695903, antiderivative size = 216, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$\begin{aligned} & -\frac{e^2(17-p)(d^2 - e^2 x^2)^{p-3}}{4x^2} - \frac{d^2(d^2 - e^2 x^2)^{p-3}}{4x^4} + \frac{4de(d^2 - e^2 x^2)^{p-3}}{3x^3} \\ & + \frac{e^4(p^2 - 21p + 70)(d^2 - e^2 x^2)^{p-3} {}_2F_1\left(1, p-3; p-2; 1 - \frac{e^2 x^2}{d^2}\right)}{4d^2(3-p)} \\ & + \frac{8e^3(6-p)\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-\frac{1}{2}, 4-p; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{3d^7 x} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(d^2 - e^2\*x^2)^p/(x^5\*(d + e\*x)^4), x]

[Out]  $-(d^2*(d^2 - e^2*x^2)^{(-3 + p)})/(4*x^4) + (4*d*e*(d^2 - e^2*x^2)^{(-3 + p)})/(3*x^3) - (e^2*(17 - p)*(d^2 - e^2*x^2)^{(-3 + p)})/(4*x^2) + (8*e^3*(6 - p)*(d^2 - e^2*x^2)^p*Hypergeometric2F1[-1/2, 4 - p, 1/2, (e^2*x^2)/d^2])/(3*d^7*x*(1 - (e^2*x^2)/d^2)^p) + (e^4*(70 - 21*p + p^2)*(d^2 - e^2*x^2)^{(-3 + p)}*Hypergeometric2F1[1, -3 + p, -2 + p, 1 - (e^2*x^2)/d^2])/(4*d^2*(3 - p))$

**Rubi in Sympy [A]** time = 87.492, size = 238, normalized size = 1.1

$$\begin{aligned} & \frac{e^4(d^2 - e^2 x^2)^{p-3} {}_2F_1\left(1, p-3; p-2; 1 - \frac{e^2 x^2}{d^2}\right)}{2d^2(-p+3)} + \frac{3e^4(d^2 - e^2 x^2)^{p-3} {}_2F_1\left(2, p-3; p-2; 1 - \frac{e^2 x^2}{d^2}\right)}{d^2(-p+3)} \\ & + \frac{e^4(d^2 - e^2 x^2)^{p-3} {}_2F_1\left(3, p-3; p-2; 1 - \frac{e^2 x^2}{d^2}\right)}{2d^2(-p+3)} + \frac{4e\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+4, -\frac{3}{2}; -\frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{3d^5 x^3} \\ & + \frac{4e^3\left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+4, -\frac{1}{2}; \frac{1}{2}; \frac{e^2 x^2}{d^2}\right)}{d^7 x} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*\*2\*x\*\*2+d\*\*2)\*\*p/x\*\*5/(e\*x+d)\*\*4,x)

[Out] e\*\*4\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(p - 3)\*hyper((1, p - 3), (p - 2, ), 1 - e\*\*2\*x\*\*2/d\*\*2)/(2\*d\*\*2\*(-p + 3)) + 3\*e\*\*4\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(p - 3)\*hyper((2, p - 3), (p - 2, ), 1 - e\*\*2\*x\*\*2/d\*\*2)/(d\*\*2\*(-p + 3)) + e\*\*4\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*(p - 3)\*hyper((3, p - 3), (p - 2, ), 1 - e\*\*2\*x\*\*2/d\*\*2)/(2\*d\*\*2\*(-p + 3)) + 4\*e\*(1 - e\*\*2\*x\*\*2/d\*\*2)\*\*(-p)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p\*hyper((-p + 4, -3/2), (-1/2, ), e\*\*2\*x\*\*2/d\*\*2)/(3\*d\*\*5\*x\*\*3) + 4\*e\*\*3\*(1 - e\*\*2\*x\*\*2/d\*\*2)\*\*(-p)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p\*hyper((-p + 4, -1/2), (1/2, ), e\*\*2\*x\*\*2/d\*\*2)/(d\*\*7\*x)

**Mathematica [B]** time = 1.42239, size = 505, normalized size = 2.34

$$(d^2 - e^2x^2)^p \left( \frac{840de^4 \left(1 - \frac{d^2}{e^2x^2}\right)^{-p} {}_2F_1\left(-p, -p; 1-p; -\frac{d^2}{e^2x^2}\right)}{p} + \frac{960d^2e^3 \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; \frac{e^2x^2}{d^2}\right)}{x} + \frac{24d^5 \left(1 - \frac{d^2}{e^2x^2}\right)^{-p} {}_2F_1\left(2-p, -p; 3-p; \frac{d^2}{e^2x^2}\right)}{(p-2)x^4} + \dots \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d^2 - e^2\*x^2)^p/(x^5\*(d + e\*x)^4),x]

[Out] ((d^2 - e^2\*x^2)^p\*((64\*d^4\*e\*Hypergeometric2F1[-3/2, -p, -1/2, (e^2\*x^2)/d^2])/(x^3\*(1 - (e^2\*x^2)/d^2)^p) + (960\*d^2\*e^3\*Hypergeometric2F1[-1/2, -p, 1/2, (e^2\*x^2)/d^2])/(x\*(1 - (e^2\*x^2)/d^2)^p) + (240\*d^3\*e^2\*Hypergeometric2F1[1 - p, -p, 2 - p, d^2/(e^2\*x^2)])/((-1 + p)\*(1 - d^2/(e^2\*x^2))^p\*x^2) + (105\*2^(3 + p)\*e^4\*(d - e\*x)\*Hypergeometric2F1[1 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/((1 + p)\*(1 + (e\*x)/d)^p) + (24\*d^5\*Hypergeometric2F1[2 - p, -p, 3 - p, d^2/(e^2\*x^2)])/((-2 + p)\*(1 - d^2/(e^2\*x^2))^p\*x^4) + (45\*2^(2 + p)\*e^4\*(d - e\*x)\*Hypergeometric2F1[2 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/((1 + p)\*(1 + (e\*x)/d)^p) + (15\*2^(1 + p)\*e^4\*(d - e\*x)\*Hypergeometric2F1[3 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/((1 + p)\*(1 + (e\*x)/d)^p) + (3\*2^p\*e^4\*(d - e\*x)\*Hypergeometric2F1[4 - p, 1 + p, 2 + p, (d - e\*x)/(2\*d)])/((1 + p)\*(1 + (e\*x)/d)^p) + (840\*d\*e^4\*Hypergeometric2F1[-p, -p, 1 - p, d^2/(e^2\*x^2)])/(p\*(1 - d^2/(e^2\*x^2))^p))/(48\*d^9)

**Maple [F]** time = 0.104, size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{x^5 (ex + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e^2\*x^2+d^2)^p/x^5/(e\*x+d)^4,x)

[Out] int((-e^2\*x^2+d^2)^p/x^5/(e\*x+d)^4,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p}{(ex + d)^4 x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p/((e\*x + d)^4\*x^5),x, algorithm="maxima")

[Out] `integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^5), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^P}{e^4x^9 + 4de^3x^8 + 6d^2e^2x^7 + 4d^3ex^6 + d^4x^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^5),x, algorithm="fricas")`

[Out] `integral((-e^2*x^2 + d^2)^p/(e^4*x^9 + 4*d*e^3*x^8 + 6*d^2*e^2*x^7 + 4*d^3*e*x^6 + d^4*x^5), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-(-d + ex)(d + ex))^p}{x^5(d + ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e**2*x**2+d**2)**p/x**5/(e*x+d)**4,x)`

[Out] `Integral((-(-d + e*x)*(d + e*x))**p/(x**5*(d + e*x)**4), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^P}{(ex + d)^4x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^5),x, algorithm="giac")`

[Out] `integrate((-e^2*x^2 + d^2)^p/((e*x + d)^4*x^5), x)`

### 3.306 $\int (gx)^m (d + ex)^3 (d^2 - e^2 x^2)^p dx$

**Optimal.** Leaf size=264

$$\frac{2d^2 e(2m + 3p + 7)(gx)^{m+2} (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{m+2}{2}, -p; \frac{m+4}{2}; \frac{e^2 x^2}{d^2}\right)}{g^2(m+2)(m+2p+4)} - \frac{e(gx)^{m+2} (d^2 - e^2 x^2)^{p+1}}{g^2(m+2p+4)} - \frac{3d(gx)^{m+1} (d^2 - e^2 x^2)^{p+1}}{g(m+2p+3)} + \frac{2d^3(2m+p+3)(gx)^{m+1} (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+3}{2}; \frac{e^2 x^2}{d^2}\right)}{g(m+1)(m+2p+3)}$$

[Out]  $(-3*d*(g*x)^{(1+m)}*(d^2 - e^2*x^2)^{(1+p)})/(g*(3+m+2*p)) - (e*(g*x)^{(2+m)}*(d^2 - e^2*x^2)^{(1+p)})/(g^2*(4+m+2*p)) + (2*d^3*(3+2*m+p)*(g*x)^{(1+m)}*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1+m)/2, -p, (3+m)/2, (e^2*x^2)/d^2])/(g*(1+m)*(3+m+2*p)*(1 - (e^2*x^2)/d^2)^p) + (2*d^2*e*(7+2*m+3*p)*(g*x)^{(2+m)}*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2+m)/2, -p, (4+m)/2, (e^2*x^2)/d^2])/(g^2*(2+m)*(4+m+2*p)*(1 - (e^2*x^2)/d^2)^p)$

**Rubi [A]** time = 0.654608, antiderivative size = 264, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$\frac{2d^2 e(2m + 3p + 7)(gx)^{m+2} (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{m+2}{2}, -p; \frac{m+4}{2}; \frac{e^2 x^2}{d^2}\right)}{g^2(m+2)(m+2p+4)} - \frac{e(gx)^{m+2} (d^2 - e^2 x^2)^{p+1}}{g^2(m+2p+4)} - \frac{3d(gx)^{m+1} (d^2 - e^2 x^2)^{p+1}}{g(m+2p+3)} + \frac{2d^3(2m+p+3)(gx)^{m+1} (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+3}{2}; \frac{e^2 x^2}{d^2}\right)}{g(m+1)(m+2p+3)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(g*x)^m*(d + e*x)^3*(d^2 - e^2*x^2)^p, x]$

[Out]  $(-3*d*(g*x)^{(1+m)}*(d^2 - e^2*x^2)^{(1+p)})/(g*(3+m+2*p)) - (e*(g*x)^{(2+m)}*(d^2 - e^2*x^2)^{(1+p)})/(g^2*(4+m+2*p)) + (2*d^3*(3+2*m+p)*(g*x)^{(1+m)}*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1+m)/2, -p, (3+m)/2, (e^2*x^2)/d^2])/(g*(1+m)*(3+m+2*p)*(1 - (e^2*x^2)/d^2)^p) + (2*d^2*e*(7+2*m+3*p)*(g*x)^{(2+m)}*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(2+m)/2, -p, (4+m)/2, (e^2*x^2)/d^2])/(g^2*(2+m)*(4+m+2*p)*(1 - (e^2*x^2)/d^2)^p)$

**Rubi in Sympy [A]** time = 68.3166, size = 262, normalized size = 0.99

$$\frac{d^3 (gx)^{m+1} \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p, \frac{m}{2} + \frac{1}{2} \middle| \frac{e^2 x^2}{d^2}\right)}{g(m+1)} + \frac{3d^2 e (gx)^{m+2} \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p, \frac{m}{2} + 1 \middle| \frac{e^2 x^2}{d^2}\right)}{g^2(m+2)} + \frac{3de^2 (gx)^{m+3} \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p, \frac{m}{2} + \frac{3}{2} \middle| \frac{e^2 x^2}{d^2}\right)}{g^3(m+3)} + \frac{e^3 (gx)^{m+4} \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p, \frac{m}{2} + 2 \middle| \frac{e^2 x^2}{d^2}\right)}{g^4(m+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x)**m*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)`

[Out] `d**3*(g*x)**(m+1)*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper((-p, m/2 + 1/2), (m/2 + 3/2, ), e**2*x**2/d**2)/(g*(m+1)) + 3*d**2*e*(g*x)**(m+2)*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper((-p, m/2 + 1), (m/2 + 2, ), e**2*x**2/d**2)/(g**2*(m+2)) + 3*d*e**2*(g*x)**(m+3)*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper((-p, m/2 + 3/2), (m/2 + 5/2, ), e**2*x**2/d**2)/(g**3*(m+3)) + e**3*(g*x)**(m+4)*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper((-p, m/2 + 2), (m/2 + 3, ), e**2*x**2/d**2)/(g**4*(m+4))`

**Mathematica [A]** time = 0.240428, size = 198, normalized size = 0.75

$$x(gx)^m (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} \left( \frac{3d^2 e x {}_2F_1\left(\frac{m}{2} + 1, -p; \frac{m}{2} + 2; \frac{e^2 x^2}{d^2}\right)}{m+2} + \frac{3de^2 x^2 {}_2F_1\left(\frac{m+3}{2}, -p; \frac{m+5}{2}; \frac{e^2 x^2}{d^2}\right)}{m+3} + \frac{e^3 x^3 {}_2F_1\left(\frac{m}{2} + 2, -p; \frac{m}{2} + 3; \frac{e^2 x^2}{d^2}\right)}{m+4} + \frac{d^3 {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+3}{2}; \frac{e^2 x^2}{d^2}\right)}{m+1} \right)$$

Antiderivative was successfully verified.

[In] `Integrate[(g*x)^m*(d + e*x)^3*(d^2 - e^2*x^2)^p,x]`

[Out] `(x*(g*x)^m*(d^2 - e^2*x^2)^p*((3*d^2*e*x*Hypergeometric2F1[1 + m/2, -p, 2 + m/2, (e^2*x^2)/d^2])/(2 + m) + (e^3*x^3*Hypergeometric2F1[2 + m/2, -p, 3 + m/2, (e^2*x^2)/d^2])/(4 + m) + (d^3*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, (e^2*x^2)/d^2])/(1 + m) + (3*d*e^2*x^2*Hypergeometric2F1[(3 + m)/2, -p, (5 + m)/2, (e^2*x^2)/d^2])/(3 + m))/(1 - (e^2*x^2)/d^2)^p`

**Maple [F]** time = 0.19, size = 0, normalized size = 0.

$$\int (gx)^m (ex + d)^3 (-e^2 x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `int((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)`

[Out] `int((g*x)^m*(e*x+d)^3*(-e^2*x^2+d^2)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3 (-e^2x^2 + d^2)^p (gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*(g*x)^m,x, algorithm="maxima")`

[Out] `integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*(g*x)^m, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3\right)\left(-e^2x^2 + d^2\right)^p (gx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*(g*x)^m,x, algorithm="fricas")`

[Out] `integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*(-e^2*x^2 + d^2)^p*(g*x)^m, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x)**m*(e*x+d)**3*(-e**2*x**2+d**2)**p,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3 (-e^2x^2 + d^2)^p (gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*(g*x)^m,x, algorithm="giac")`

[Out] `integrate((e*x + d)^3*(-e^2*x^2 + d^2)^p*(g*x)^m, x)`

### 3.307 $\int (gx)^m (d + ex)^2 (d^2 - e^2 x^2)^p dx$

**Optimal.** Leaf size=206

$$\frac{2de(gx)^{m+2} (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{m+2}{2}, -p; \frac{m+4}{2}; \frac{e^2 x^2}{d^2}\right)}{g^2(m+2)} + \frac{2d^2(m+p+2)(gx)^{m+1} (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+3}{2}; \frac{e^2 x^2}{d^2}\right)}{g(m+1)(m+2p+3)} - \frac{(gx)^{m+1} (d^2 - e^2 x^2)^{p+1}}{g(m+2p+3)}$$

[Out] -(((g\*x)^(1+m)\*(d^2 - e^2\*x^2)^(1+p))/(g\*(3+m+2\*p))) + (2\*d^2\*(2+m+p)\*(g\*x)^(1+m)\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[(1+m)/2, -p, (3+m)/2, (e^2\*x^2)/d^2])/(g\*(1+m)\*(3+m+2\*p)\*(1 - (e^2\*x^2)/d^2)^p) + (2\*d\*e\*(g\*x)^(2+m)\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[(2+m)/2, -p, (4+m)/2, (e^2\*x^2)/d^2])/(g^2\*(2+m)\*(1 - (e^2\*x^2)/d^2)^p)

**Rubi [A]** time = 0.350055, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$\frac{2de(gx)^{m+2} (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{m+2}{2}, -p; \frac{m+4}{2}; \frac{e^2 x^2}{d^2}\right)}{g^2(m+2)} + \frac{2d^2(m+p+2)(gx)^{m+1} (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+3}{2}; \frac{e^2 x^2}{d^2}\right)}{g(m+1)(m+2p+3)} - \frac{(gx)^{m+1} (d^2 - e^2 x^2)^{p+1}}{g(m+2p+3)}$$

Antiderivative was successfully verified.

[In] Int[(g\*x)^m\*(d + e\*x)^2\*(d^2 - e^2\*x^2)^p,x]

[Out] -(((g\*x)^(1+m)\*(d^2 - e^2\*x^2)^(1+p))/(g\*(3+m+2\*p))) + (2\*d^2\*(2+m+p)\*(g\*x)^(1+m)\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[(1+m)/2, -p, (3+m)/2, (e^2\*x^2)/d^2])/(g\*(1+m)\*(3+m+2\*p)\*(1 - (e^2\*x^2)/d^2)^p) + (2\*d\*e\*(g\*x)^(2+m)\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[(2+m)/2, -p, (4+m)/2, (e^2\*x^2)/d^2])/(g^2\*(2+m)\*(1 - (e^2\*x^2)/d^2)^p)

**Rubi in Sympy [A]** time = 50.2772, size = 194, normalized size = 0.94

$$\frac{d^2 (gx)^{m+1} \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p, \frac{m}{2} + \frac{1}{2} \middle| \frac{e^2 x^2}{d^2}\right)}{g(m+1)} + \frac{2de (gx)^{m+2} \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p, \frac{m}{2} + 1 \middle| \frac{e^2 x^2}{d^2}\right)}{g^2(m+2)} + \frac{e^2 (gx)^{m+3} \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p, \frac{m}{2} + \frac{3}{2} \middle| \frac{e^2 x^2}{d^2}\right)}{g^3(m+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x)**m*(e*x+d)**2*(-e**2*x**2+d**2)**p,x)`

[Out]  $d^{2m}(g^m x^{m+1}) (1 - e^{2x^2}/d)^{-p} (d^2 - e^{2x^2})^{p-m} \text{hyper}((-p, m/2 + 1/2), (m/2 + 3/2, ), e^{2x^2}/d) / (g^{m+1}) + 2d^m e^{2x^2} (g^m x^{m+2}) (1 - e^{2x^2}/d)^{-p} (d^2 - e^{2x^2})^{p-m} \text{hyper}((-p, m/2 + 1), (m/2 + 2, ), e^{2x^2}/d) / (g^{m+2}) + e^{2x^2} (g^m x^{m+3}) (1 - e^{2x^2}/d)^{-p} (d^2 - e^{2x^2})^{p-m} \text{hyper}((-p, m/2 + 3/2), (m/2 + 5/2, ), e^{2x^2}/d) / (g^{m+3})$

**Mathematica [A]** time = 0.168299, size = 173, normalized size = 0.84

$$\frac{x(gx)^m (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} \left(2de (m^2 + 4m + 3) x {}_2F_1\left(\frac{m}{2} + 1, -p; \frac{m}{2} + 2; \frac{e^2 x^2}{d^2}\right) + (m+2) \left(d^2(m+3) {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+1}{2}\right)\right)}{(m+1)(m+2)(m+3)}$$

Antiderivative was successfully verified.

[In] `Integrate[(g*x)^m*(d + e*x)^2*(d^2 - e^2*x^2)^p,x]`

[Out]  $(x^m (g^m x^{m+2}) (d^2 - e^2 x^2)^p (2d^2 e^{2x^2} (3 + 4m + m^2) x \text{Hypergeometric2F1}[1 + m/2, -p, 2 + m/2, (e^{2x^2})/d^2] + (2 + m) (d^2 (3 + m) \text{Hypergeometric2F1}[(1 + m)/2, -p, (3 + m)/2, (e^{2x^2})/d^2] + e^{2x^2} (1 + m) x^2 \text{Hypergeometric2F1}[(3 + m)/2, -p, (5 + m)/2, (e^{2x^2})/d^2])) / ((1 + m) (2 + m) (3 + m) (1 - (e^{2x^2})/d^2)^p)$

**Maple [F]** time = 0.082, size = 0, normalized size = 0.

$$\int (gx)^m (ex + d)^2 (-e^2 x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)`

[Out] `int((g*x)^m*(e*x+d)^2*(-e^2*x^2+d^2)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2 (-e^2 x^2 + d^2)^p (gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*(g*x)^m,x, algorithm="maxima")`

[Out] `integrate((e*x + d)^2*(-e^2*x^2 + d^2)^p*(g*x)^m, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2 x^2 + 2 dex + d^2\right) \left(-e^2 x^2 + d^2\right)^p (gx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p\*(g\*x)^m,x, algorithm="fricas")

[Out] integral((e^2\*x^2 + 2\*d\*e\*x + d^2)\*(-e^2\*x^2 + d^2)^p\*(g\*x)^m, x)

**Sympy [A]** time = 100.798, size = 192, normalized size = 0.93

$$\frac{d^2 d^{2p} g^m x x^m \left(\frac{m}{2} + \frac{1}{2}\right) {}_2F_1\left(-p, \frac{m}{2} + \frac{1}{2} \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{2\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{d d^{2p} e g^m x^2 x^m \left(\frac{m}{2} + 1\right) {}_2F_1\left(-p, \frac{m}{2} + 1 \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{\left(\frac{m}{2} + 2\right)} + \frac{d^{2p} e^2 g^m x^3 x^m \left(\frac{m}{2} + \frac{3}{2}\right) {}_2F_1\left(-p, \frac{m}{2} + \frac{3}{2} \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{2\left(\frac{m}{2} + \frac{5}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x)\*\*m\*(e\*x+d)\*\*2\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p,x)

[Out] d\*\*2\*d\*\*(2\*p)\*g\*\*m\*x\*x\*\*m\*gamma(m/2 + 1/2)\*hyper((-p, m/2 + 1/2), (m/2 + 3/2, ), e\*\*2\*x\*\*2\*exp\_polar(2\*I\*pi)/d\*\*2)/(2\*gamma(m/2 + 3/2)) + d\*d\*\*(2\*p)\*e\*g\*\*m\*x\*\*2\*x\*\*m\*gamma(m/2 + 1)\*hyper((-p, m/2 + 1), (m/2 + 2, ), e\*\*2\*x\*\*2\*exp\_polar(2\*I\*pi)/d\*\*2)/gamma(m/2 + 2) + d\*\*(2\*p)\*e\*\*2\*g\*\*m\*x\*\*3\*x\*\*m\*gamma(m/2 + 3/2)\*hyper((-p, m/2 + 3/2), (m/2 + 5/2, ), e\*\*2\*x\*\*2\*exp\_polar(2\*I\*pi)/d\*\*2)/(2\*gamma(m/2 + 5/2))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2 (-e^2x^2 + d^2)^p (gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p\*(g\*x)^m,x, algorithm="giac")

[Out] integrate((e\*x + d)^2\*(-e^2\*x^2 + d^2)^p\*(g\*x)^m, x)

### 3.308 $\int (gx)^m (d + ex) (d^2 - e^2 x^2)^P dx$

**Optimal.** Leaf size=153

$$\frac{e(gx)^{m+2} (d^2 - e^2 x^2)^P \left(1 - \frac{e^2 x^2}{d^2}\right)^{-P} {}_2F_1\left(\frac{m+2}{2}, -p; \frac{m+4}{2}; \frac{e^2 x^2}{d^2}\right)}{g^2(m+2)} + \frac{d(gx)^{m+1} (d^2 - e^2 x^2)^P \left(1 - \frac{e^2 x^2}{d^2}\right)^{-P} {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+3}{2}; \frac{e^2 x^2}{d^2}\right)}{g(m+1)}$$

[Out]  $(d*(g*x)^{(1+m)}*(d^2 - e^2*x^2)^p*$ Hypergeometric2F1 $[(1+m)/2, -p, (3+m)/2, (e^2*x^2)/d^2])/(g*(1+m)*(1 - (e^2*x^2)/d^2)^p) + (e*(g*x)^{(2+m)}*(d^2 - e^2*x^2)^p*$ Hypergeometric2F1 $[(2+m)/2, -p, (4+m)/2, (e^2*x^2)/d^2])/(g^2*(2+m)*(1 - (e^2*x^2)/d^2)^p)$

**Rubi [A]** time = 0.166817, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$\frac{e(gx)^{m+2} (d^2 - e^2 x^2)^P \left(1 - \frac{e^2 x^2}{d^2}\right)^{-P} {}_2F_1\left(\frac{m+2}{2}, -p; \frac{m+4}{2}; \frac{e^2 x^2}{d^2}\right)}{g^2(m+2)} + \frac{d(gx)^{m+1} (d^2 - e^2 x^2)^P \left(1 - \frac{e^2 x^2}{d^2}\right)^{-P} {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+3}{2}; \frac{e^2 x^2}{d^2}\right)}{g(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(g\*x)^m\*(d + e\*x)\*(d^2 - e^2\*x^2)^p,x]

[Out]  $(d*(g*x)^{(1+m)}*(d^2 - e^2*x^2)^p*$ Hypergeometric2F1 $[(1+m)/2, -p, (3+m)/2, (e^2*x^2)/d^2])/(g*(1+m)*(1 - (e^2*x^2)/d^2)^p) + (e*(g*x)^{(2+m)}*(d^2 - e^2*x^2)^p*$ Hypergeometric2F1 $[(2+m)/2, -p, (4+m)/2, (e^2*x^2)/d^2])/(g^2*(2+m)*(1 - (e^2*x^2)/d^2)^p)$

**Rubi in Sympy [A]** time = 25.8605, size = 122, normalized size = 0.8

$$\frac{d(gx)^{m+1} \left(1 - \frac{e^2 x^2}{d^2}\right)^{-P} (d^2 - e^2 x^2)^P {}_2F_1\left(-p, \frac{m}{2} + \frac{1}{2} \middle| \frac{e^2 x^2}{d^2}\right)}{g(m+1)} + \frac{e(gx)^{m+2} \left(1 - \frac{e^2 x^2}{d^2}\right)^{-P} (d^2 - e^2 x^2)^P {}_2F_1\left(-p, \frac{m}{2} + 1 \middle| \frac{e^2 x^2}{d^2}\right)}{g^2(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x)\*\*m\*(e\*x+d)\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p,x)

[Out]  $d*(g*x)**(m+1)*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*$ hyper $((-p, m/2 + 1/2), (m/2 + 3/2), e**2*x**2/d**2)/(g*(m+1)) + e*(g*x)**(m+2)*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*$ hyper $((-p, m/2 + 1), (m/2 + 2), e**2*x**2/d**2)/(g**2*(m+2))$

**Mathematica [A]** time = 0.108571, size = 116, normalized size = 0.76

$$\frac{x(gx)^m (d^2 - e^2x^2)^p \left(1 - \frac{e^2x^2}{d^2}\right)^{-p} \left(e(m+1)x {}_2F_1\left(\frac{m}{2} + 1, -p; \frac{m}{2} + 2; \frac{e^2x^2}{d^2}\right) + d(m+2) {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+3}{2}; \frac{e^2x^2}{d^2}\right)\right)}{(m+1)(m+2)}$$

Antiderivative was successfully verified.

[In] Integrate[(g\*x)^m\*(d + e\*x)\*(d^2 - e^2\*x^2)^p,x]

[Out] (x\*(g\*x)^m\*(d^2 - e^2\*x^2)^p\*(e\*(1+m)\*x\*Hypergeometric2F1[1+m/2, -p, 2+m/2, (e^2\*x^2)/d^2] + d\*(2+m)\*Hypergeometric2F1[(1+m)/2, -p, (3+m)/2, (e^2\*x^2)/d^2]))/((1+m)\*(2+m)\*(1-(e^2\*x^2)/d^2)^p)

**Maple [F]** time = 0.083, size = 0, normalized size = 0.

$$\int (gx)^m (ex + d) (-e^2x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x)^m\*(e\*x+d)\*(-e^2\*x^2+d^2)^p,x)

[Out] int((g\*x)^m\*(e\*x+d)\*(-e^2\*x^2+d^2)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(-e^2x^2 + d^2)^p (gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(-e^2\*x^2 + d^2)^p\*(g\*x)^m,x, algorithm="maxima")

[Out] integrate((e\*x + d)\*(-e^2\*x^2 + d^2)^p\*(g\*x)^m, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex + d)(-e^2x^2 + d^2)^p (gx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(-e^2\*x^2 + d^2)^p\*(g\*x)^m,x, algorithm="fricas")

[Out] integral((e\*x + d)\*(-e^2\*x^2 + d^2)^p\*(g\*x)^m, x)

**Sympy [A]** time = 52.7187, size = 122, normalized size = 0.8

$$\frac{d^{2p} g^m x x^m \left(\frac{m}{2} + \frac{1}{2}\right) {}_2F_1\left(-p, \frac{m}{2} + \frac{1}{2} \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{2\left(\frac{m}{2} + \frac{3}{2}\right)} + \frac{d^{2p} e g^m x^2 x^m \left(\frac{m}{2} + 1\right) {}_2F_1\left(-p, \frac{m}{2} + 1 \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{2\left(\frac{m}{2} + 2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x)**m*(e*x+d)*(-e**2*x**2+d**2)**p,x)
```

```
[Out] d*d**(2*p)*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2)) + d**(2*p)*e*g**m*x**2*x**m*gamma(m/2 + 1)*hyper((-p, m/2 + 1), (m/2 + 2,), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 2))
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(-e^2x^2 + d^2)^p (gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*(g*x)^m,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(-e^2*x^2 + d^2)^p*(g*x)^m, x)
```

### 3.309 $\int (gx)^m (d^2 - e^2 x^2)^p dx$

**Optimal.** Leaf size=75

$$\frac{(gx)^{m+1} (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+3}{2}; \frac{e^2 x^2}{d^2}\right)}{g(m+1)}$$

[Out]  $((g*x)^{(1+m)}*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1+m)/2, -p, (3+m)/2, (e^2*x^2)/d^2])/(g*(1+m)*(1 - (e^2*x^2)/d^2)^p)$

**Rubi [A]** time = 0.0534599, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{(gx)^{m+1} (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+3}{2}; \frac{e^2 x^2}{d^2}\right)}{g(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(g\*x)^m\*(d^2 - e^2\*x^2)^p,x]

[Out]  $((g*x)^{(1+m)}*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1+m)/2, -p, (3+m)/2, (e^2*x^2)/d^2])/(g*(1+m)*(1 - (e^2*x^2)/d^2)^p)$

**Rubi in Sympy [A]** time = 11.4165, size = 60, normalized size = 0.8

$$\frac{(gx)^{m+1} \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p, \frac{m}{2} + \frac{1}{2} \middle| \frac{e^2 x^2}{d^2}\right)}{g(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x)\*\*m\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p,x)

[Out]  $(g*x)**(m+1)*(1 - e**2*x**2/d**2)**(-p)*(d**2 - e**2*x**2)**p*hyper((-p, m/2 + 1/2), (m/2 + 3/2, ), e**2*x**2/d**2)/(g*(m+1))$

**Mathematica [A]** time = 0.0405495, size = 73, normalized size = 0.97

$$\frac{x(gx)^m (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+1}{2} + 1; \frac{e^2 x^2}{d^2}\right)}{m+1}$$

Antiderivative was successfully verified.

[In] Integrate[(g\*x)^m\*(d^2 - e^2\*x^2)^p,x]

[Out]  $(x*(g*x)^m*(d^2 - e^2*x^2)^p*Hypergeometric2F1[(1+m)/2, -p, 1+(1+m)/2, (e^2*x^2)/d^2])/((1+m)*(1 - (e^2*x^2)/d^2)^p)$

**Maple [F]** time = 0.099, size = 0, normalized size = 0.

$$\int (gx)^m (-e^2 x^2 + d^2)^p dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x)^m*(-e^2*x^2+d^2)^p,x)`

[Out] `int((g*x)^m*(-e^2*x^2+d^2)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (-e^2x^2 + d^2)^P (gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p*(g*x)^m,x, algorithm="maxima")`

[Out] `integrate((-e^2*x^2 + d^2)^p*(g*x)^m, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-e^2x^2 + d^2\right)^P (gx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p*(g*x)^m,x, algorithm="fricas")`

[Out] `integral((-e^2*x^2 + d^2)^p*(g*x)^m, x)`

**Sympy [A]** time = 15.2955, size = 61, normalized size = 0.81

$$\frac{d^{2p} g^m x x^m \left(\frac{m}{2} + \frac{1}{2}\right) {}_2F_1\left(-p, \frac{m}{2} + \frac{1}{2} \middle| \frac{e^2 x^2 e^{2i\pi}}{d^2}\right)}{2 \left(\frac{m}{2} + \frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x)**m*(-e**2*x**2+d**2)**p,x)`

[Out] `d**(2*p)*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2, ), e**2*x**2*exp_polar(2*I*pi)/d**2)/(2*gamma(m/2 + 3/2))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (-e^2x^2 + d^2)^P (gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p*(g*x)^m,x, algorithm="giac")`

[Out] `integrate((-e^2*x^2 + d^2)^p*(g*x)^m, x)`

$$3.310 \quad \int \frac{(gx)^m (d^2 - e^2 x^2)^p}{d + ex} dx$$

**Optimal.** Leaf size=163

$$\frac{(gx)^{m+1} (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{m+1}{2}, 1-p; \frac{m+3}{2}; \frac{e^2 x^2}{d^2}\right)}{dg(m+1)} - \frac{e(gx)^{m+2} (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{m+2}{2}, 1-p; \frac{m+4}{2}; \frac{e^2 x^2}{d^2}\right)}{d^2 g^2(m+2)}$$

[Out] ((g\*x)^(1+m)\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[(1+m)/2, 1-p, (3+m)/2, (e^2\*x^2)/d^2])/(d\*g\*(1+m)\*(1 - (e^2\*x^2)/d^2)^p) - (e\*(g\*x)^(2+m)\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[(2+m)/2, 1-p, (4+m)/2, (e^2\*x^2)/d^2])/(d^2\*g^2\*(2+m)\*(1 - (e^2\*x^2)/d^2)^p)

**Rubi [A]** time = 0.374606, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\frac{(gx)^{m+1} (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{m+1}{2}, 1-p; \frac{m+3}{2}; \frac{e^2 x^2}{d^2}\right)}{dg(m+1)} - \frac{e(gx)^{m+2} (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{m+2}{2}, 1-p; \frac{m+4}{2}; \frac{e^2 x^2}{d^2}\right)}{d^2 g^2(m+2)}$$

Antiderivative was successfully verified.

[In] Int[((g\*x)^m\*(d^2 - e^2\*x^2)^p)/(d + e\*x), x]

[Out] ((g\*x)^(1+m)\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[(1+m)/2, 1-p, (3+m)/2, (e^2\*x^2)/d^2])/(d\*g\*(1+m)\*(1 - (e^2\*x^2)/d^2)^p) - (e\*(g\*x)^(2+m)\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[(2+m)/2, 1-p, (4+m)/2, (e^2\*x^2)/d^2])/(d^2\*g^2\*(2+m)\*(1 - (e^2\*x^2)/d^2)^p)

**Rubi in Sympy [A]** time = 43.4996, size = 126, normalized size = 0.77

$$\frac{(gx)^{m+1} \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+1, \frac{m}{2} + \frac{1}{2} \middle| \frac{e^2 x^2}{d^2}\right)}{dg(m+1)} - \frac{e(gx)^{m+2} \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+1, \frac{m}{2} + 1 \middle| \frac{e^2 x^2}{d^2}\right)}{d^2 g^2(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x)\*\*m\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d), x)

[Out] (g\*x)\*\*(m+1)\*(1 - e\*\*2\*x\*\*2/d\*\*2)\*\*(-p)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p\*hyper((-p+1, m/2+1/2), (m/2+3/2, ), e\*\*2\*x\*\*2/d\*\*2)/(d\*g\*(m+1)) - e\*(g\*x)\*\*(m+2)\*(1 - e\*\*2\*x\*\*2/d\*\*2)\*\*(-p)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p\*hyper((-p+1, m/2+1), (m/2+2, ), e\*\*2\*x\*\*2/d\*\*2)/(d\*\*2\*g\*\*2\*(m+2))

**Mathematica [C]** time = 0.474102, size = 168, normalized size = 1.03

$$\frac{d(m+2)x(gx)^m(d-ex)^p(d+ex)^{p-1}F_1\left(m+1; -p, 1-p; m+2; \frac{ex}{d}, -\frac{ex}{d}\right)}{(m+1)\left(ex\left((p-1)F_1\left(m+2; -p, 2-p; m+3; \frac{ex}{d}, -\frac{ex}{d}\right) - p {}_2F_1\left(\frac{m}{2}+1, 1-p; \frac{m}{2}+2; \frac{e^2x^2}{d^2}\right)\right) + d(m+2)F_1\left(m+1; -p, 1-p\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((g\*x)^m\*(d^2 - e^2\*x^2)^p)/(d + e\*x), x]

[Out] (d\*(2 + m)\*x\*(g\*x)^m\*(d - e\*x)^p\*(d + e\*x)^(-1 + p)\*AppellF1[1 + m, -p, 1 - p, 2 + m, (e\*x)/d, -((e\*x)/d)]/((1 + m)\*(d\*(2 + m)\*AppellF1[1 + m, -p, 1 - p, 2 + m, (e\*x)/d, -((e\*x)/d)] + e\*x\*((-1 + p)\*AppellF1[2 + m, -p, 2 - p, 3 + m, (e\*x)/d, -((e\*x)/d)] - p\*HypergeometricPFQ[{1 + m/2, 1 - p}, {2 + m/2}, (e^2\*x^2)/d^2]))

**Maple [F]** time = 0.099, size = 0, normalized size = 0.

$$\int \frac{(gx)^m (-e^2x^2 + d^2)^p}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x)^m\*(-e^2\*x^2+d^2)^p/(e\*x+d), x)

[Out] int((g\*x)^m\*(-e^2\*x^2+d^2)^p/(e\*x+d), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p (gx)^m}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*(g\*x)^m/(e\*x + d), x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p\*(g\*x)^m/(e\*x + d), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p (gx)^m}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*(g\*x)^m/(e\*x + d), x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p\*(g\*x)^m/(e\*x + d), x)

**Sympy [A]** time = 44.2349, size = 337, normalized size = 2.07

$$\frac{0^p d d^{2p} g^m m x^m \left( \frac{d^2}{e^2 x^2}, 1, -\frac{m}{2} + \frac{1}{2} \right) \left( -\frac{m}{2} + \frac{1}{2} \right)}{4e^2 x \left( -\frac{m}{2} + \frac{3}{2} \right)} + \frac{0^p d d^{2p} g^m m x^m \left( \frac{d^2}{e^2 x^2}, 1, -\frac{m}{2} + \frac{1}{2} \right) \left( -\frac{m}{2} + \frac{1}{2} \right)}{4e^2 x \left( -\frac{m}{2} + \frac{3}{2} \right)} + \frac{0^p d^{2p} g^m m x^m \left( \frac{d^2}{e^2 x^2}, 1, \frac{m e^{i\pi}}{2} \right) \left( -\frac{m}{2} \right)}{4e \left( -\frac{m}{2} + 1 \right)}$$

$$+ \frac{d e^{2p} g^m p x^m x^{2p} e^{i\pi p} (p) \left( -\frac{m}{2} - p + \frac{1}{2} \right) {}_2F_1 \left( \begin{matrix} -p + 1, -\frac{m}{2} - p + \frac{1}{2} \\ -\frac{m}{2} - p + \frac{3}{2} \end{matrix} \middle| \frac{d^2}{e^2 x^2} \right)}{2e^2 x (p + 1) \left( -\frac{m}{2} - p + \frac{3}{2} \right)}$$

$$+ \frac{e^{2p} g^m p x^m x^{2p} e^{i\pi p} (p) \left( -\frac{m}{2} - p \right) {}_2F_1 \left( \begin{matrix} -p + 1, -\frac{m}{2} - p \\ -\frac{m}{2} - p + 1 \end{matrix} \middle| \frac{d^2}{e^2 x^2} \right)}{2e (p + 1) \left( -\frac{m}{2} - p + 1 \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x)\*\*m\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d),x)

[Out] -0\*\*p\*d\*d\*\*(2\*p)\*g\*\*m\*m\*x\*\*m\*lerchphi(d\*\*2/(e\*\*2\*x\*\*2), 1, -m/2 + 1/2)\*gamma(-m/2 + 1/2)/(4\*e\*\*2\*x\*gamma(-m/2 + 3/2)) + 0\*\*p\*d\*d\*\*(2\*p)\*g\*\*m\*x\*\*m\*lerchphi(d\*\*2/(e\*\*2\*x\*\*2), 1, -m/2 + 1/2)\*gamma(-m/2 + 1/2)/(4\*e\*\*2\*x\*gamma(-m/2 + 3/2)) + 0\*\*p\*d\*\*(2\*p)\*g\*\*m\*m\*x\*\*m\*lerchphi(d\*\*2/(e\*\*2\*x\*\*2), 1, m\*exp\_polar(I\*pi)/2)\*gamma(-m/2)/(4\*e\*gamma(-m/2 + 1)) + d\*e\*\*(2\*p)\*g\*\*m\*p\*x\*\*m\*x\*\*(2\*p)\*exp(I\*pi\*p)\*gamma(p)\*gamma(-m/2 - p + 1/2)\*hyper((-p + 1, -m/2 - p + 1/2), (-m/2 - p + 3/2, ), d\*\*2/(e\*\*2\*x\*\*2))/(2\*e\*\*2\*x\*gamma(p + 1)\*gamma(-m/2 - p + 3/2)) - e\*\*(2\*p)\*g\*\*m\*p\*x\*\*m\*x\*\*(2\*p)\*exp(I\*pi\*p)\*gamma(p)\*gamma(-m/2 - p)\*hyper((-p + 1, -m/2 - p), (-m/2 - p + 1, ), d\*\*2/(e\*\*2\*x\*\*2))/(2\*e\*gamma(p + 1)\*gamma(-m/2 - p + 1))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2 x^2 + d^2)^p (g x)^m}{e x + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*(g\*x)^m/(e\*x + d),x, algorithm="giac")

[Out] integrate((-e^2\*x^2 + d^2)^p\*(g\*x)^m/(e\*x + d), x)

$$3.311 \quad \int \frac{(gx)^m (d^2 - e^2 x^2)^p}{(d+ex)^2} dx$$

**Optimal.** Leaf size=214

$$\frac{2(m+p)(gx)^{m+1} (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{m+1}{2}, 2-p; \frac{m+3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^2 g(m+1)(-m-2p+1)} + \frac{(gx)^{m+1} (d^2 - e^2 x^2)^{p-1}}{g(-m-2p+1)} - \frac{2e(gx)^{m+2} (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{m+2}{2}, 2-p; \frac{m+4}{2}; \frac{e^2 x^2}{d^2}\right)}{d^3 g^2(m+2)}$$

[Out] ((g\*x)^(1+m)\*(d^2 - e^2\*x^2)^(-1+p))/(g\*(1-m-2\*p)) - (2\*(m+p)\*(g\*x)^(1+m)\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[(1+m)/2, 2-p, (3+m)/2, (e^2\*x^2)/d^2])/(d^2\*g\*(1+m)\*(1-m-2\*p)\*(1 - (e^2\*x^2)/d^2)^p) - (2\*e\*(g\*x)^(2+m)\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[(2+m)/2, 2-p, (4+m)/2, (e^2\*x^2)/d^2])/(d^3\*g^2\*(2+m)\*(1 - (e^2\*x^2)/d^2)^p)

**Rubi [A]** time = 0.482796, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\frac{2(m+p)(gx)^{m+1} (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{m+1}{2}, 2-p; \frac{m+3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^2 g(m+1)(-m-2p+1)} + \frac{(gx)^{m+1} (d^2 - e^2 x^2)^{p-1}}{g(-m-2p+1)} - \frac{2e(gx)^{m+2} (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{m+2}{2}, 2-p; \frac{m+4}{2}; \frac{e^2 x^2}{d^2}\right)}{d^3 g^2(m+2)}$$

Antiderivative was successfully verified.

[In] Int[((g\*x)^m\*(d^2 - e^2\*x^2)^p)/(d + e\*x)^2, x]

[Out] ((g\*x)^(1+m)\*(d^2 - e^2\*x^2)^(-1+p))/(g\*(1-m-2\*p)) - (2\*(m+p)\*(g\*x)^(1+m)\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[(1+m)/2, 2-p, (3+m)/2, (e^2\*x^2)/d^2])/(d^2\*g\*(1+m)\*(1-m-2\*p)\*(1 - (e^2\*x^2)/d^2)^p) - (2\*e\*(g\*x)^(2+m)\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[(2+m)/2, 2-p, (4+m)/2, (e^2\*x^2)/d^2])/(d^3\*g^2\*(2+m)\*(1 - (e^2\*x^2)/d^2)^p)

**Rubi in Sympy [A]** time = 83.4721, size = 199, normalized size = 0.93

$$\frac{(gx)^{m+1} \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+2, \frac{m}{2} + \frac{1}{2}; \frac{m}{2} + \frac{3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^2 g(m+1)} - \frac{2e(gx)^{m+2} \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+2, \frac{m}{2} + 1; \frac{m}{2} + 2; \frac{e^2 x^2}{d^2}\right)}{d^3 g^2(m+2)} + \frac{e^2 (gx)^{m+3} \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p+2, \frac{m}{2} + \frac{3}{2}; \frac{m}{2} + \frac{5}{2}; \frac{e^2 x^2}{d^2}\right)}{d^4 g^3(m+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x)\*\*m\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d)\*\*2, x)

[Out] (g\*x)\*\*(m+1)\*(1 - e\*\*2\*x\*\*2/d\*\*2)\*\*(-p)\*(d\*\*2 - e\*\*2\*x\*\*2)\*\*p\*hyper((-p+2, m/2 + 1/2), (m/2 + 3/2, ), e\*\*2\*x\*\*2/d\*\*2)/(d\*\*2\*g\*(

$$(m + 1)) - 2 * e * (g * x)^{(m + 2)} * (1 - e^{2 * x^2 / d^2})^{(-p)} * (d^2 - e^{2 * x^2})^{p * \text{hyper}((-p + 2, m/2 + 1), (m/2 + 2), e^{2 * x^2 / d^2})} / (d^3 * g^{2 * (m + 2)}) + e^{2 * x^2} * (g * x)^{(m + 3)} * (1 - e^{2 * x^2 / d^2})^{(-p)} * (d^2 - e^{2 * x^2})^{p * \text{hyper}((-p + 2, m/2 + 3/2), (m/2 + 5/2), e^{2 * x^2 / d^2})} / (d^4 * g^{3 * (m + 3)})$$

**Mathematica [C]** time = 0.480381, size = 166, normalized size = 0.78

$$\frac{d(m+2)x(gx)^m(d-ex)^p(d+ex)^{p-2}F_1\left(m+1; -p, 2-p; m+2; \frac{ex}{d}, -\frac{ex}{d}\right)}{(m+1)\left(d(m+2)F_1\left(m+1; -p, 2-p; m+2; \frac{ex}{d}, -\frac{ex}{d}\right) + ex\left((p-2)F_1\left(m+2; -p, 3-p; m+3; \frac{ex}{d}, -\frac{ex}{d}\right) - pF_1\left(m+2; 1-p, 2-p; m+3; \frac{ex}{d}, -\frac{ex}{d}\right)\right)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((g\*x)^m\*(d^2 - e^2\*x^2)^p)/(d + e\*x)^2, x]

[Out] (d\*(2 + m)\*x\*(g\*x)^m\*(d - e\*x)^p\*(d + e\*x)^(-2 + p)\*AppellF1[1 + m, -p, 2 - p, 2 + m, (e\*x)/d, -((e\*x)/d)]/((1 + m)\*(d\*(2 + m)\*AppellF1[1 + m, -p, 2 - p, 2 + m, (e\*x)/d, -((e\*x)/d)] + e\*x\*(-p\*AppellF1[2 + m, 1 - p, 2 - p, 3 + m, (e\*x)/d, -((e\*x)/d)])) + (-2 + p)\*AppellF1[2 + m, -p, 3 - p, 3 + m, (e\*x)/d, -((e\*x)/d)])

**Maple [F]** time = 0.089, size = 0, normalized size = 0.

$$\int \frac{(gx)^m (-e^2x^2 + d^2)^p}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x)^m\*(-e^2\*x^2+d^2)^p/(e\*x+d)^2, x)

[Out] int((g\*x)^m\*(-e^2\*x^2+d^2)^p/(e\*x+d)^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p (gx)^m}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*(g\*x)^m/(e\*x + d)^2, x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p\*(g\*x)^m/(e\*x + d)^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p (gx)^m}{e^2x^2 + 2dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*(g\*x)^m/(e\*x + d)^2, x, algorithm="fricas")

[Out] `integral((-e^2*x^2 + d^2)^p*(g*x)^m/(e^2*x^2 + 2*d*e*x + d^2), x)`

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx)^m (-(-d + ex)(d + ex))^p}{(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x)**m*(-e**2*x**2+d**2)**p/(e*x+d)**2,x)`

[Out] `Integral((g*x)**m*(-(-d + e*x)*(d + e*x))**p/(d + e*x)**2, x)`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p (gx)^m}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d)^2,x, algorithm="giac")`

[Out] `integrate((-e^2*x^2 + d^2)^p*(g*x)^m/(e*x + d)^2, x)`

$$3.312 \quad \int \frac{(gx)^m (d^2 - e^2 x^2)^p}{(d+ex)^3} dx$$

**Optimal.** Leaf size=275

$$\begin{aligned} & -\frac{e(gx)^{m+2} (d^2 - e^2 x^2)^{p-2}}{g^2(-m-2p+2)} + \frac{3d(gx)^{m+1} (d^2 - e^2 x^2)^{p-2}}{g(-m-2p+3)} \\ & - \frac{2e(-2m-3p+2)(gx)^{m+2} (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{m+2}{2}, 3-p; \frac{m+4}{2}; \frac{e^2 x^2}{d^2}\right)}{d^4 g^2(m+2)(-m-2p+2)} \\ & - \frac{2(2m+p)(gx)^{m+1} (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{m+1}{2}, 3-p; \frac{m+3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^3 g(m+1)(-m-2p+3)} \end{aligned}$$

[Out] (3\*d\*(g\*x)^(1+m)\*(d^2 - e^2\*x^2)^(-2+p))/(g\*(3-m-2\*p)) - (e\*(g\*x)^(2+m)\*(d^2 - e^2\*x^2)^(-2+p))/(g^2\*(2-m-2\*p)) - (2\*(2\*m+p)\*(g\*x)^(1+m)\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[(1+m)/2, 3-p, (3+m)/2, (e^2\*x^2)/d^2])/(d^3\*g\*(1+m)\*(3-m-2\*p)\*(1 - (e^2\*x^2)/d^2)^p) - (2\*e\*(2-2\*m-3\*p)\*(g\*x)^(2+m)\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[(2+m)/2, 3-p, (4+m)/2, (e^2\*x^2)/d^2])/(d^4\*g^2\*(2+m)\*(2-m-2\*p)\*(1 - (e^2\*x^2)/d^2)^p)

**Rubi [A]** time = 0.809638, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\begin{aligned} & -\frac{e(gx)^{m+2} (d^2 - e^2 x^2)^{p-2}}{g^2(-m-2p+2)} + \frac{3d(gx)^{m+1} (d^2 - e^2 x^2)^{p-2}}{g(-m-2p+3)} \\ & - \frac{2e(-2m-3p+2)(gx)^{m+2} (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{m+2}{2}, 3-p; \frac{m+4}{2}; \frac{e^2 x^2}{d^2}\right)}{d^4 g^2(m+2)(-m-2p+2)} \\ & - \frac{2(2m+p)(gx)^{m+1} (d^2 - e^2 x^2)^p \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} {}_2F_1\left(\frac{m+1}{2}, 3-p; \frac{m+3}{2}; \frac{e^2 x^2}{d^2}\right)}{d^3 g(m+1)(-m-2p+3)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[((g\*x)^m\*(d^2 - e^2\*x^2)^p)/(d + e\*x)^3,x]

[Out] (3\*d\*(g\*x)^(1+m)\*(d^2 - e^2\*x^2)^(-2+p))/(g\*(3-m-2\*p)) - (e\*(g\*x)^(2+m)\*(d^2 - e^2\*x^2)^(-2+p))/(g^2\*(2-m-2\*p)) - (2\*(2\*m+p)\*(g\*x)^(1+m)\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[(1+m)/2, 3-p, (3+m)/2, (e^2\*x^2)/d^2])/(d^3\*g\*(1+m)\*(3-m-2\*p)\*(1 - (e^2\*x^2)/d^2)^p) - (2\*e\*(2-2\*m-3\*p)\*(g\*x)^(2+m)\*(d^2 - e^2\*x^2)^p\*Hypergeometric2F1[(2+m)/2, 3-p, (4+m)/2, (e^2\*x^2)/d^2])/(d^4\*g^2\*(2+m)\*(2-m-2\*p)\*(1 - (e^2\*x^2)/d^2)^p)



**Rubi in Sympy [A]** time = 108.302, size = 267, normalized size = 0.97

$$\frac{(gx)^{m+1} \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p + 3, \frac{m}{2} + \frac{1}{2} \middle| \frac{e^2 x^2}{d^2}\right)}{d^3 g (m+1)} - \frac{3e (gx)^{m+2} \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p + 3, \frac{m}{2} + 1 \middle| \frac{e^2 x^2}{d^2}\right)}{d^4 g^2 (m+2)} + \frac{3e^2 (gx)^{m+3} \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p + 3, \frac{m}{2} + \frac{3}{2} \middle| \frac{e^2 x^2}{d^2}\right)}{d^5 g^3 (m+3)} - \frac{e^3 (gx)^{m+4} \left(1 - \frac{e^2 x^2}{d^2}\right)^{-p} (d^2 - e^2 x^2)^p {}_2F_1\left(-p + 3, \frac{m}{2} + 2 \middle| \frac{e^2 x^2}{d^2}\right)}{d^6 g^4 (m+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x)**m*(-e**2*x**2+d**2)**p/(e*x+d)**3,x)`

[Out] `(g*x)**(m+1)*(1-e**2*x**2/d**2)**(-p)*(d**2-e**2*x**2)**p*hyper((-p+3, m/2+1/2), (m/2+3/2, ), e**2*x**2/d**2)/(d**3*g*(m+1))-3*e*(g*x)**(m+2)*(1-e**2*x**2/d**2)**(-p)*(d**2-e**2*x**2)**p*hyper((-p+3, m/2+1), (m/2+2, ), e**2*x**2/d**2)/(d**4*g**2*(m+2))+3*e**2*(g*x)**(m+3)*(1-e**2*x**2/d**2)**(-p)*(d**2-e**2*x**2)**p*hyper((-p+3, m/2+3/2), (m/2+5/2, ), e**2*x**2/d**2)/(d**5*g**3*(m+3))-e**3*(g*x)**(m+4)*(1-e**2*x**2/d**2)**(-p)*(d**2-e**2*x**2)**p*hyper((-p+3, m/2+2), (m/2+3, ), e**2*x**2/d**2)/(d**6*g**4*(m+4))`

**Mathematica [C]** time = 0.529032, size = 166, normalized size = 0.6

$$\frac{d(m+2)x(gx)^m(d-ex)^p(d+ex)^{p-3}F_1\left(m+1; -p, 3-p; m+2; \frac{ex}{d}, -\frac{ex}{d}\right)}{(m+1)(d(m+2)F_1\left(m+1; -p, 3-p; m+2; \frac{ex}{d}, -\frac{ex}{d}\right) + ex((p-3)F_1\left(m+2; -p, 4-p; m+3; \frac{ex}{d}, -\frac{ex}{d}\right) - pF_1\left(m+2; 1-p, 3-p; m+3; \frac{ex}{d}, -\frac{ex}{d}\right))}$$

Warning: Unable to verify antiderivative.

[In] `Integrate[((g*x)^m*(d^2 - e^2*x^2)^p)/(d + e*x)^3,x]`

[Out] `(d*(2+m)*x*(g*x)^m*(d-e*x)^p*(d+e*x)^(-3+p)*AppellF1[1+m, -p, 3-p, 2+m, (e*x)/d, -((e*x)/d)]/((1+m)*(d*(2+m)*AppellF1[1+m, -p, 3-p, 2+m, (e*x)/d, -((e*x)/d)] + e*x*(-p*AppellF1[2+m, 1-p, 3-p, 3+m, (e*x)/d, -((e*x)/d)] + (-3+p)*AppellF1[2+m, -p, 4-p, 3+m, (e*x)/d, -((e*x)/d)]))`

**Maple [F]** time = 0.088, size = 0, normalized size = 0.

$$\int \frac{(gx)^m (-e^2 x^2 + d^2)^p}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)`

[Out] `int((g*x)^m*(-e^2*x^2+d^2)^p/(e*x+d)^3,x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p (gx)^m}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*(g\*x)^m/(e\*x + d)^3,x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p\*(g\*x)^m/(e\*x + d)^3, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-e^2x^2 + d^2)^p (gx)^m}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*(g\*x)^m/(e\*x + d)^3,x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p\*(g\*x)^m/(e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x)\*\*m\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p/(e\*x+d)\*\*3,x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-e^2x^2 + d^2)^p (gx)^m}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*(g\*x)^m/(e\*x + d)^3,x, algorithm="giac")

[Out] integrate((-e^2\*x^2 + d^2)^p\*(g\*x)^m/(e\*x + d)^3, x)

$$3.313 \quad \int \frac{(gx)^m (1-a^2x^2)^p}{1+ax} dx$$

**Optimal.** Leaf size=89

$$\frac{(gx)^{m+1} {}_2F_1\left(\frac{m+1}{2}, 1-p; \frac{m+3}{2}; a^2x^2\right)}{g(m+1)} - \frac{a(gx)^{m+2} {}_2F_1\left(\frac{m+2}{2}, 1-p; \frac{m+4}{2}; a^2x^2\right)}{g^2(m+2)}$$

[Out] ((g\*x)^(1+m)\*Hypergeometric2F1[(1+m)/2, 1-p, (3+m)/2, a^2\*x^2])/(g\*(1+m)) - (a\*(g\*x)^(2+m)\*Hypergeometric2F1[(2+m)/2, 1-p, (4+m)/2, a^2\*x^2])/(g^2\*(2+m))

**Rubi [A]** time = 0.211705, antiderivative size = 89, normalized size of antiderivative = 1., number of rules used = 6, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{(gx)^{m+1} {}_2F_1\left(\frac{m+1}{2}, 1-p; \frac{m+3}{2}; a^2x^2\right)}{g(m+1)} - \frac{a(gx)^{m+2} {}_2F_1\left(\frac{m+2}{2}, 1-p; \frac{m+4}{2}; a^2x^2\right)}{g^2(m+2)}$$

Antiderivative was successfully verified.

[In] Int[((g\*x)^m\*(1-a^2\*x^2)^p)/(1+a\*x), x]

[Out] ((g\*x)^(1+m)\*Hypergeometric2F1[(1+m)/2, 1-p, (3+m)/2, a^2\*x^2])/(g\*(1+m)) - (a\*(g\*x)^(2+m)\*Hypergeometric2F1[(2+m)/2, 1-p, (4+m)/2, a^2\*x^2])/(g^2\*(2+m))

**Rubi in Sympy [A]** time = 21.6605, size = 63, normalized size = 0.71

$$-\frac{a(gx)^{m+2} {}_2F_1\left(-p+1, \frac{m}{2}+1; \frac{m}{2}+2; a^2x^2\right)}{g^2(m+2)} + \frac{(gx)^{m+1} {}_2F_1\left(-p+1, \frac{m}{2}+\frac{1}{2}; \frac{m}{2}+\frac{3}{2}; a^2x^2\right)}{g(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x)\*\*m\*(-a\*\*2\*x\*\*2+1)\*\*p/(a\*x+1), x)

[Out] -a\*(g\*x)\*\*(m+2)\*hyper((-p+1, m/2+1), (m/2+2, ), a\*\*2\*x\*\*2)/(g\*\*2\*(m+2)) + (g\*x)\*\*(m+1)\*hyper((-p+1, m/2+1/2), (m/2+3/2, ), a\*\*2\*x\*\*2)/(g\*(m+1))

**Mathematica [C]** time = 0.276596, size = 145, normalized size = 1.63

$$\frac{(m+2)x(1-ax)^p(ax+1)^{p-1}(gx)^m F_1(m+1; -p, 1-p; m+2; ax, -ax)}{(m+1)\left(ax\left((p-1)F_1(m+2; -p, 2-p; m+3; ax, -ax) - p {}_2F_1\left(\frac{m}{2}+1, 1-p; \frac{m}{2}+2; a^2x^2\right)\right) + (m+2)F_1(m+1; -p, 1-p; m+2; ax, -ax)\right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((g\*x)^m\*(1-a^2\*x^2)^p)/(1+a\*x), x]

[Out] ((2+m)\*x\*(g\*x)^m\*(1-a\*x)^p\*(1+a\*x)^(-1+p)\*AppellF1[1+m, -p, 1-p, 2+m, a\*x, -(a\*x)]/((1+m)\*((2+m)\*AppellF1[1+m, -p, 1-p, 2+m, a\*x, -(a\*x)] + a\*x\*(-1+p)\*AppellF1[2+m, -p, 2-p, 3+m, a\*x, -(a\*x)] - p\*HypergeometricPFQ[{1+m/2, 1-p}, {2+m/2}, a^2\*x^2]))

---

**Maple [F]** time = 0.405, size = 0, normalized size = 0.

$$\int \frac{(gx)^m (-a^2x^2 + 1)^p}{ax + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x)^m\*(-a^2\*x^2+1)^p/(a\*x+1),x)

[Out] int((g\*x)^m\*(-a^2\*x^2+1)^p/(a\*x+1),x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-a^2x^2 + 1)^p (gx)^m}{ax + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a^2\*x^2 + 1)^p\*(g\*x)^m/(a\*x + 1),x, algorithm="maxima")

[Out] integrate((-a^2\*x^2 + 1)^p\*(g\*x)^m/(a\*x + 1), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(-a^2x^2 + 1)^p (gx)^m}{ax + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a^2\*x^2 + 1)^p\*(g\*x)^m/(a\*x + 1),x, algorithm="fricas")

[Out] integral((-a^2\*x^2 + 1)^p\*(g\*x)^m/(a\*x + 1), x)

---

**Sympy [A]** time = 42.96, size = 308, normalized size = 3.46

$$\begin{aligned} & \frac{0^p g^m m x^m \left(\frac{1}{a^2 x^2}, 1, \frac{m e^{i\pi}}{2}\right) \left(-\frac{m}{2}\right)}{4a \left(-\frac{m}{2} + 1\right)} - \frac{0^p g^m m x^m \left(\frac{1}{a^2 x^2}, 1, -\frac{m}{2} + \frac{1}{2}\right) \left(-\frac{m}{2} + \frac{1}{2}\right)}{4a^2 x \left(-\frac{m}{2} + \frac{3}{2}\right)} \\ & + \frac{0^p g^m x^m \left(\frac{1}{a^2 x^2}, 1, -\frac{m}{2} + \frac{1}{2}\right) \left(-\frac{m}{2} + \frac{1}{2}\right)}{4a^2 x \left(-\frac{m}{2} + \frac{3}{2}\right)} \\ & - \frac{a^{2p} g^m p x^m x^{2p} e^{i\pi p} (p) \left(-\frac{m}{2} - p\right) {}_2F_1\left(-p + 1, -\frac{m}{2} - p \middle| \frac{1}{a^2 x^2}\right)}{2a (p + 1) \left(-\frac{m}{2} - p + 1\right)} \\ & + \frac{a^{2p} g^m p x^m x^{2p} e^{i\pi p} (p) \left(-\frac{m}{2} - p + \frac{1}{2}\right) {}_2F_1\left(-p + 1, -\frac{m}{2} - p + \frac{1}{2} \middle| \frac{1}{a^2 x^2}\right)}{2a^2 x (p + 1) \left(-\frac{m}{2} - p + \frac{3}{2}\right)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x)\*\*m\*(-a\*\*2\*x\*\*2+1)\*\*p/(a\*x+1),x)

[Out] 0\*\*p\*g\*\*m\*m\*x\*\*m\*lerchphi(1/(a\*\*2\*x\*\*2), 1, m\*exp\_polar(I\*pi)/2)\*  
gamma(-m/2)/(4\*a\*gamma(-m/2 + 1)) - 0\*\*p\*g\*\*m\*m\*x\*\*m\*lerchphi(1/(  
a\*\*2\*x\*\*2), 1, -m/2 + 1/2)\*gamma(-m/2 + 1/2)/(4\*a\*\*2\*x\*gamma(-m/2  
+ 3/2)) + 0\*\*p\*g\*\*m\*x\*\*m\*lerchphi(1/(a\*\*2\*x\*\*2), 1, -m/2 + 1/2)\*  
gamma(-m/2 + 1/2)/(4\*a\*\*2\*x\*gamma(-m/2 + 3/2)) - a\*\*(2\*p)\*g\*\*m\*p\*  
x\*\*m\*x\*\*(2\*p)\*exp(I\*pi\*p)\*gamma(p)\*gamma(-m/2 - p)\*hyper((-p + 1,  
-m/2 - p), (-m/2 - p + 1,), 1/(a\*\*2\*x\*\*2))/(2\*a\*gamma(p + 1)\*gam  
ma(-m/2 - p + 1)) + a\*\*(2\*p)\*g\*\*m\*p\*x\*\*m\*x\*\*(2\*p)\*exp(I\*pi\*p)\*gam  
ma(p)\*gamma(-m/2 - p + 1/2)\*hyper((-p + 1, -m/2 - p + 1/2), (-m/2  
- p + 3/2,), 1/(a\*\*2\*x\*\*2))/(2\*a\*\*2\*x\*gamma(p + 1)\*gamma(-m/2 -  
p + 3/2))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-a^2x^2 + 1)^p (gx)^m}{ax + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-a^2\*x^2 + 1)^p\*(g\*x)^m/(a\*x + 1),x, algorithm="giac")

[Out] integrate((-a^2\*x^2 + 1)^p\*(g\*x)^m/(a\*x + 1), x)

### 3.314 $\int (gx)^m (d + ex)^n (d^2 - e^2 x^2)^p dx$

**Optimal.** Leaf size=96

$$\frac{(gx)^{m+1} (d + ex)^n \left(1 - \frac{ex}{d}\right)^{-p} (d^2 - e^2 x^2)^p \left(\frac{ex}{d} + 1\right)^{-n-p} F_1\left(m + 1; -p, -n - p; m + 2; \frac{ex}{d}, -\frac{ex}{d}\right)}{g(m + 1)}$$

[Out]  $((g*x)^{(1+m)}*(d+e*x)^n*(1+(e*x)/d)^{(-n-p)}*(d^2-e^2*x^2)^p*AppellF1[1+m, -p, -n-p, 2+m, (e*x)/d, -((e*x)/d)]/(g*(1+m)*(1-(e*x)/d)^p)$

**Rubi [A]** time = 0.217819, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{(gx)^{m+1} (d + ex)^n \left(1 - \frac{ex}{d}\right)^{-p} (d^2 - e^2 x^2)^p \left(\frac{ex}{d} + 1\right)^{-n-p} F_1\left(m + 1; -p, -n - p; m + 2; \frac{ex}{d}, -\frac{ex}{d}\right)}{g(m + 1)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(g*x)^m*(d+e*x)^n*(d^2-e^2*x^2)^p, x]$

[Out]  $((g*x)^{(1+m)}*(d+e*x)^n*(1+(e*x)/d)^{(-n-p)}*(d^2-e^2*x^2)^p*AppellF1[1+m, -p, -n-p, 2+m, (e*x)/d, -((e*x)/d)]/(g*(1+m)*(1-(e*x)/d)^p)$

**Rubi in Sympy [A]** time = 36.0669, size = 78, normalized size = 0.81

$$\frac{(gx)^{m+1} \left(1 - \frac{ex}{d}\right)^{-p} \left(1 + \frac{ex}{d}\right)^{-n-p} (d + ex)^{-p} (d + ex)^{n+p} (d^2 - e^2 x^2)^p \text{appellf}_1\left(m + 1, -p, -n - p, m + 2, \frac{ex}{d}, -\frac{ex}{d}\right)}{g(m + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((g*x)**m*(e*x+d)**n*(-e**2*x**2+d**2)**p, x)$

[Out]  $(g*x)**(m+1)*(1-e*x/d)**(-p)*(1+e*x/d)**(-n-p)*(d+e*x)**(-p)*(d+e*x)**(n+p)*(d**2-e**2*x**2)**p*\text{appellf}_1(m+1, -p, -n-p, m+2, e*x/d, -e*x/d)/(g*(m+1))$

**Mathematica [A]** time = 0.576574, size = 175, normalized size = 1.82

$$\frac{d(m+2)x(gx)^m(d-ex)^p(d+ex)^{n+p}F_1\left(m+1; -p, -n-p; m+2; \frac{ex}{d}, -\frac{ex}{d}\right)}{(m+1)\left(d(m+2)F_1\left(m+1; -p, -n-p; m+2; \frac{ex}{d}, -\frac{ex}{d}\right) + ex\left((n+p)F_1\left(m+2; -p, -n-p+1; m+3; \frac{ex}{d}, -\frac{ex}{d}\right) - pF_1\left(m+1; -p, -n-p; m+2; \frac{ex}{d}, -\frac{ex}{d}\right)\right)}$$

Warning: Unable to verify antiderivative.

[In]  $\text{Integrate}[(g*x)^m*(d+e*x)^n*(d^2-e^2*x^2)^p, x]$

[Out]  $(d*(2+m)*x*(g*x)^m*(d-e*x)^p*(d+e*x)^{(n+p)}*AppellF1[1+m, -p, -n-p, 2+m, (e*x)/d, -((e*x)/d)]/((1+m)*(d*(2+m)*AppellF1[1+m, -p, -n-p, 2+m, (e*x)/d, -((e*x)/d)] + e*x*(-(p*AppellF1[2+m, 1-p, -n-p, 3+m, (e*x)/d, -((e*x)/d)]) + (n+p)*AppellF1[2+m, -p, 1-n-p, 3+m, (e*x)/d, -((e*x)/d)]))$

---

**Maple [F]** time = 0.224, size = 0, normalized size = 0.

$$\int (gx)^m (ex + d)^n (-e^2x^2 + d^2)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x)^m\*(e\*x+d)^n\*(-e^2\*x^2+d^2)^p,x)

[Out] int((g\*x)^m\*(e\*x+d)^n\*(-e^2\*x^2+d^2)^p,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (-e^2x^2 + d^2)^p (ex + d)^n (gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*(e\*x + d)^n\*(g\*x)^m,x, algorithm="maxima")

[Out] integrate((-e^2\*x^2 + d^2)^p\*(e\*x + d)^n\*(g\*x)^m, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-e^2x^2 + d^2\right)^p (ex + d)^n (gx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*(e\*x + d)^n\*(g\*x)^m,x, algorithm="fricas")

[Out] integral((-e^2\*x^2 + d^2)^p\*(e\*x + d)^n\*(g\*x)^m, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x)\*\*m\*(e\*x+d)\*\*n\*(-e\*\*2\*x\*\*2+d\*\*2)\*\*p,x)

[Out] Timed out

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (-e^2x^2 + d^2)^p (ex + d)^n (gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e^2\*x^2 + d^2)^p\*(e\*x + d)^n\*(g\*x)^m,x, algorithm="giac")

[Out] integrate((-e^2\*x^2 + d^2)^p\*(e\*x + d)^n\*(g\*x)^m, x)

$$3.315 \quad \int \frac{x\sqrt{1+x}}{1+x^2} dx$$

Optimal. Leaf size=214

$$\begin{aligned} & 2\sqrt{x+1} + \frac{1}{2}\sqrt{\frac{1}{2}(1+\sqrt{2})} \log\left(x - \sqrt{2(1+\sqrt{2})}\sqrt{x+1} + \sqrt{2} + 1\right) \\ & - \frac{1}{2}\sqrt{\frac{1}{2}(1+\sqrt{2})} \log\left(x + \sqrt{2(1+\sqrt{2})}\sqrt{x+1} + \sqrt{2} + 1\right) \\ & + \frac{\tan^{-1}\left(\frac{\sqrt{2(1+\sqrt{2})-2\sqrt{x+1}}}{\sqrt{2(\sqrt{2}-1)}}\right)}{\sqrt{2(1+\sqrt{2})}} - \frac{\tan^{-1}\left(\frac{2\sqrt{x+1} + \sqrt{2(1+\sqrt{2})}}{\sqrt{2(\sqrt{2}-1)}}\right)}{\sqrt{2(1+\sqrt{2})}} \end{aligned}$$

[Out] 2\*Sqrt[1 + x] + ArcTan[(Sqrt[2\*(1 + Sqrt[2])] - 2\*Sqrt[1 + x])/Sqrt[2\*(-1 + Sqrt[2])]]/Sqrt[2\*(1 + Sqrt[2])] - ArcTan[(Sqrt[2\*(1 + Sqrt[2])] + 2\*Sqrt[1 + x])/Sqrt[2\*(-1 + Sqrt[2])]]/Sqrt[2\*(1 + Sqrt[2])] + (Sqrt[(1 + Sqrt[2])/2]\*Log[1 + Sqrt[2] + x - Sqrt[2\*(1 + Sqrt[2])]\*Sqrt[1 + x]])/2 - (Sqrt[(1 + Sqrt[2])/2]\*Log[1 + Sqrt[2] + x + Sqrt[2\*(1 + Sqrt[2])]\*Sqrt[1 + x]])/2

Rubi [A] time = 0.526942, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$

$$\begin{aligned} & 2\sqrt{x+1} + \frac{1}{2}\sqrt{\frac{1}{2}(1+\sqrt{2})} \log\left(x - \sqrt{2(1+\sqrt{2})}\sqrt{x+1} + \sqrt{2} + 1\right) \\ & - \frac{1}{2}\sqrt{\frac{1}{2}(1+\sqrt{2})} \log\left(x + \sqrt{2(1+\sqrt{2})}\sqrt{x+1} + \sqrt{2} + 1\right) \\ & + \frac{\tan^{-1}\left(\frac{\sqrt{2(1+\sqrt{2})-2\sqrt{x+1}}}{\sqrt{2(\sqrt{2}-1)}}\right)}{\sqrt{2(1+\sqrt{2})}} - \frac{\tan^{-1}\left(\frac{2\sqrt{x+1} + \sqrt{2(1+\sqrt{2})}}{\sqrt{2(\sqrt{2}-1)}}\right)}{\sqrt{2(1+\sqrt{2})}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(x\*Sqrt[1 + x])/(1 + x^2), x]

[Out] 2\*Sqrt[1 + x] + ArcTan[(Sqrt[2\*(1 + Sqrt[2])] - 2\*Sqrt[1 + x])/Sqrt[2\*(-1 + Sqrt[2])]]/Sqrt[2\*(1 + Sqrt[2])] - ArcTan[(Sqrt[2\*(1 + Sqrt[2])] + 2\*Sqrt[1 + x])/Sqrt[2\*(-1 + Sqrt[2])]]/Sqrt[2\*(1 + Sqrt[2])] + (Sqrt[(1 + Sqrt[2])/2]\*Log[1 + Sqrt[2] + x - Sqrt[2\*(1 + Sqrt[2])]\*Sqrt[1 + x]])/2 - (Sqrt[(1 + Sqrt[2])/2]\*Log[1 + Sqrt[2] + x + Sqrt[2\*(1 + Sqrt[2])]\*Sqrt[1 + x]])/2



**Rubi in Sympy [A]** time = 51.1913, size = 323, normalized size = 1.51

$$\frac{2\sqrt{x+1} + \frac{\left(\frac{\sqrt{2}}{2} + 1\right) \log\left(x - \sqrt{2}\sqrt{1+\sqrt{2}}\sqrt{x+1} + 1 + \sqrt{2}\right)}{2\sqrt{1+\sqrt{2}}}}{\frac{\left(\frac{\sqrt{2}}{2} + 1\right) \log\left(x + \sqrt{2}\sqrt{1+\sqrt{2}}\sqrt{x+1} + 1 + \sqrt{2}\right)}{2\sqrt{1+\sqrt{2}}}} - \frac{\sqrt{2} \left( -\frac{\sqrt{2}\sqrt{1+\sqrt{2}}(\sqrt{2}+2)}{2} + 2\sqrt{2}\sqrt{1+\sqrt{2}} \right) \operatorname{atan}\left( \frac{\sqrt{2}\left(\sqrt{x+1} - \frac{\sqrt{2}+2\sqrt{2}}{2}\right)}{\sqrt{-1+\sqrt{2}}} \right)}{2\sqrt{-1+\sqrt{2}}\sqrt{1+\sqrt{2}}} - \frac{\sqrt{2} \left( -\frac{\sqrt{2}\sqrt{1+\sqrt{2}}(\sqrt{2}+2)}{2} + 2\sqrt{2}\sqrt{1+\sqrt{2}} \right) \operatorname{atan}\left( \frac{\sqrt{2}\left(\sqrt{x+1} + \frac{\sqrt{2}+2\sqrt{2}}{2}\right)}{\sqrt{-1+\sqrt{2}}} \right)}{2\sqrt{-1+\sqrt{2}}\sqrt{1+\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x*(1+x)**(1/2)/(x**2+1), x)`

[Out] `2*sqrt(x + 1) + (sqrt(2)/2 + 1)*log(x - sqrt(2)*sqrt(1 + sqrt(2)) *sqrt(x + 1) + 1 + sqrt(2))/(2*sqrt(1 + sqrt(2))) - (sqrt(2)/2 + 1)*log(x + sqrt(2)*sqrt(1 + sqrt(2))*sqrt(x + 1) + 1 + sqrt(2))/(2*sqrt(1 + sqrt(2))) - sqrt(2)*(-sqrt(2)*sqrt(1 + sqrt(2)))*(sqrt(2) + 2)/2 + 2*sqrt(2)*sqrt(1 + sqrt(2))*atan(sqrt(2)*(sqrt(x + 1) - sqrt(2 + 2*sqrt(2)))/2)/sqrt(-1 + sqrt(2)))/(2*sqrt(-1 + sqrt(2))*sqrt(1 + sqrt(2))) - sqrt(2)*(-sqrt(2)*sqrt(1 + sqrt(2)))*(sqrt(2) + 2)/2 + 2*sqrt(2)*sqrt(1 + sqrt(2))*atan(sqrt(2)*(sqrt(x + 1) + sqrt(2 + 2*sqrt(2)))/2)/sqrt(-1 + sqrt(2)))/(2*sqrt(-1 + sqrt(2))*sqrt(1 + sqrt(2)))`

**Mathematica [C]** time = 0.0304022, size = 60, normalized size = 0.28

$$2\sqrt{x+1} - \sqrt{1-i} \tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1-i}}\right) - \sqrt{1+i} \tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{1+i}}\right)$$

Antiderivative was successfully verified.

[In] `Integrate[(x*Sqrt[1 + x])/(1 + x^2), x]`

[Out] `2*Sqrt[1 + x] - Sqrt[1 - I]*ArcTanh[Sqrt[1 + x]/Sqrt[1 - I]] - Sqrt[1 + I]*ArcTanh[Sqrt[1 + x]/Sqrt[1 + I]]`

**Maple [A]** time = 0.121, size = 240, normalized size = 1.1

$$\begin{aligned}
& 2\sqrt{1+x} + \frac{\sqrt{2+2\sqrt{2}}}{4} \ln\left(1+x+\sqrt{2}-\sqrt{1+x}\sqrt{2+2\sqrt{2}}\right) \\
& - \frac{\sqrt{2}}{\sqrt{-2+2\sqrt{2}}} \arctan\left(\frac{1}{\sqrt{-2+2\sqrt{2}}}\left(2\sqrt{1+x}-\sqrt{2+2\sqrt{2}}\right)\right) \\
& + \frac{1}{\sqrt{-2+2\sqrt{2}}} \arctan\left(\frac{1}{\sqrt{-2+2\sqrt{2}}}\left(2\sqrt{1+x}-\sqrt{2+2\sqrt{2}}\right)\right) \\
& - \frac{\sqrt{2+2\sqrt{2}}}{4} \ln\left(1+x+\sqrt{2}+\sqrt{1+x}\sqrt{2+2\sqrt{2}}\right) \\
& - \frac{\sqrt{2}}{\sqrt{-2+2\sqrt{2}}} \arctan\left(\frac{1}{\sqrt{-2+2\sqrt{2}}}\left(2\sqrt{1+x}+\sqrt{2+2\sqrt{2}}\right)\right) \\
& + \frac{1}{\sqrt{-2+2\sqrt{2}}} \arctan\left(\frac{1}{\sqrt{-2+2\sqrt{2}}}\left(2\sqrt{1+x}+\sqrt{2+2\sqrt{2}}\right)\right)
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(1+x)^(1/2)/(x^2+1), x)

[Out]  $2*(1+x)^{(1/2)}+1/4*\ln(1+x+2^{(1/2)}-(1+x)^{(1/2)}*(2+2*2^{(1/2)})^{(1/2)})$   
 $* (2+2*2^{(1/2)})^{(1/2)}-1/(-2+2*2^{(1/2)})^{(1/2)}*\arctan((2*(1+x)^{(1/2)}$   
 $-(2+2*2^{(1/2)})^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})*2^{(1/2)}+1/(-2+2*2^{(1/2)}$   
 $)^{(1/2)}*\arctan((2*(1+x)^{(1/2)}-(2+2*2^{(1/2)})^{(1/2)})/(-2+2*2^{(1/2)}$   
 $)^{(1/2)})-1/4*\ln(1+x+2^{(1/2)}+(1+x)^{(1/2)}*(2+2*2^{(1/2)})^{(1/2)})*(2+$   
 $2*2^{(1/2)})^{(1/2)}-1/(-2+2*2^{(1/2)})^{(1/2)}*\arctan((2*(1+x)^{(1/2)}+(2+$   
 $2*2^{(1/2)})^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})*2^{(1/2)}+1/(-2+2*2^{(1/2)})^{(1/2)}$   
 $*\arctan((2*(1+x)^{(1/2)}+(2+2*2^{(1/2)})^{(1/2)})/(-2+2*2^{(1/2)})^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+1}x}{x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x + 1)\*x/(x^2 + 1), x, algorithm="maxima")

[Out] integrate(sqrt(x + 1)\*x/(x^2 + 1), x)

**Fricas [A]** time = 0.304006, size = 711, normalized size = 3.32

$$\sqrt{2} \left( 4\sqrt{2}\sqrt{x+1}(\sqrt{2}+1)\sqrt{\frac{\sqrt{2}+2}{2\sqrt{2}+3}} - 2^{\frac{1}{4}}(\sqrt{2}+1) \log\left(\frac{2^{\frac{3}{4}}\sqrt{x+1}(41\sqrt{2}+58)\sqrt{\frac{\sqrt{2}+2}{2\sqrt{2}+3}}+24\sqrt{2}(x+1)+2\sqrt{2}(12\sqrt{2}+17)+34x+34}{2(12\sqrt{2}+17)}}\right) + 2^{\frac{1}{4}}(\sqrt{2}+1) \log\left(\frac{2^{\frac{3}{4}}\sqrt{x+1}(41\sqrt{2}+58)\sqrt{\frac{\sqrt{2}+2}{2\sqrt{2}+3}}+24\sqrt{2}(x+1)+2\sqrt{2}(12\sqrt{2}+17)+34x+34}{2(12\sqrt{2}+17)}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x + 1)\*x/(x^2 + 1), x, algorithm="fricas")

[Out]  $1/4*\sqrt{2}*(4*\sqrt{2})*\sqrt{x+1}*(\sqrt{2}+1)*\sqrt{((\sqrt{2}+2)/(2*\sqrt{2}+3))}$   
 $- 2^{(1/4)}*(\sqrt{2}+1)*\log(1/2*(2^{(3/4)})*\sqrt{2}$

$$(x + 1) \cdot (41\sqrt{2} + 58) \cdot \sqrt{(\sqrt{2} + 2)/(2\sqrt{2} + 3)} + 2 \cdot 4\sqrt{2} \cdot (x + 1) + 2\sqrt{2} \cdot (12\sqrt{2} + 17) + 34x + 34 / (12\sqrt{2} + 17) + 2^{1/4} \cdot (\sqrt{2} + 1) \cdot \log(-1/2 \cdot (2^{3/4}) \cdot \sqrt{x + 1}) \cdot (41\sqrt{2} + 58) \cdot \sqrt{(\sqrt{2} + 2)/(2\sqrt{2} + 3)} - 24\sqrt{2} \cdot (x + 1) - 2\sqrt{2} \cdot (12\sqrt{2} + 17) - 34x - 34 / (12\sqrt{2} + 17) + 4 \cdot 2^{1/4} \cdot \arctan(2^{1/4} / (\sqrt{2} \cdot \sqrt{1/2} \cdot (\sqrt{2} + 1) \cdot \sqrt{((2^{3/4}) \cdot \sqrt{x + 1}) \cdot (41\sqrt{2} + 58) \cdot \sqrt{(\sqrt{2} + 2)/(2\sqrt{2} + 3)} + 24\sqrt{2} \cdot (x + 1) + 2\sqrt{2} \cdot (12\sqrt{2} + 17) + 34x + 34) / (12\sqrt{2} + 17)) \cdot \sqrt{(\sqrt{2} + 2)/(2\sqrt{2} + 3)} + \sqrt{2} \cdot \sqrt{x + 1} \cdot (\sqrt{2} + 1) \cdot \sqrt{(\sqrt{2} + 2)/(2\sqrt{2} + 3)} + 2^{1/4} \cdot (\sqrt{2} + 1))) + 4 \cdot 2^{1/4} \cdot \arctan(2^{1/4} / (\sqrt{2} \cdot \sqrt{1/2} \cdot (\sqrt{2} + 1) \cdot \sqrt{-(2^{3/4}) \cdot \sqrt{x + 1}} \cdot (41\sqrt{2} + 58) \cdot \sqrt{(\sqrt{2} + 2)/(2\sqrt{2} + 3)} - 24\sqrt{2} \cdot (x + 1) - 2\sqrt{2} \cdot (12\sqrt{2} + 17) - 34x - 34) / (12\sqrt{2} + 17)) \cdot \sqrt{(\sqrt{2} + 2)/(2\sqrt{2} + 3)} + \sqrt{2} \cdot \sqrt{x + 1} \cdot (\sqrt{2} + 1) \cdot \sqrt{(\sqrt{2} + 2)/(2\sqrt{2} + 3)} - 2^{1/4} \cdot (\sqrt{2} + 1))) / ((\sqrt{2} + 1) \cdot \sqrt{(\sqrt{2} + 2)/(2\sqrt{2} + 3)})$$

**Sympy [A]** time = 11.3025, size = 68, normalized size = 0.32

$$2\sqrt{x+1} - 4\text{RootSum}\left(512t^4 + 32t^2 + 1, \left(t \mapsto t \log\left(-128t^3 + \sqrt{x+1}\right)\right)\right) + 2\text{RootSum}\left(128t^4 + 16t^2 + 1, \left(t \mapsto t \log\left(64t^3 + 4t + \sqrt{x+1}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(1+x)\*\*(1/2)/(x\*\*2+1), x)

[Out] 2\*sqrt(x + 1) - 4\*RootSum(512\*\_t\*\*4 + 32\*\_t\*\*2 + 1, Lambda(\_t, \_t\*log(-128\*\_t\*\*3 + sqrt(x + 1)))) + 2\*RootSum(128\*\_t\*\*4 + 16\*\_t\*\*2 + 1, Lambda(\_t, \_t\*log(64\*\_t\*\*3 + 4\*\_t + sqrt(x + 1))))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+1}x}{x^2+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x + 1)\*x/(x^2 + 1), x, algorithm="giac")

[Out] integrate(sqrt(x + 1)\*x/(x^2 + 1), x)

$$3.316 \quad \int \frac{x^4 \sqrt{a+cx^2}}{d+ex} dx$$

**Optimal.** Leaf size=255

$$\begin{aligned} & - \frac{d(-a^2e^4 + 4acd^2e^2 + 8c^2d^4) \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{8c^{3/2}e^6} + \frac{(a+cx^2)^{3/2}(47cd^2 - 8ae^2)}{60c^2e^3} \\ & - \frac{d^4\sqrt{ae^2 + cd^2} \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^6} + \frac{d\sqrt{a+cx^2}(8cd^3 - ex(4cd^2 - ae^2))}{8ce^5} \\ & - \frac{13d(a+cx^2)^{3/2}(d+ex)}{20ce^3} + \frac{(a+cx^2)^{3/2}(d+ex)^2}{5ce^3} \end{aligned}$$

[Out] (d\*(8\*c\*d^3 - e\*(4\*c\*d^2 - a\*e^2)\*x)\*Sqrt[a + c\*x^2])/(8\*c\*e^5) + ((47\*c\*d^2 - 8\*a\*e^2)\*(a + c\*x^2)^(3/2))/(60\*c^2\*e^3) - (13\*d\*(d + e\*x)\*(a + c\*x^2)^(3/2))/(20\*c\*e^3) + ((d + e\*x)^2\*(a + c\*x^2)^(3/2))/(5\*c\*e^3) - (d\*(8\*c^2\*d^4 + 4\*a\*c\*d^2\*e^2 - a^2\*e^4)\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(8\*c^(3/2)\*e^6) - (d^4\*Sqrt[c\*d^2 + a\*e^2]\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]))/e^6

**Rubi [A]** time = 1.10185, antiderivative size = 255, normalized size of antiderivative = 1., number of rules used = 9, number of rules used = 6, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\begin{aligned} & - \frac{d(-a^2e^4 + 4acd^2e^2 + 8c^2d^4) \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{8c^{3/2}e^6} + \frac{(a+cx^2)^{3/2}(47cd^2 - 8ae^2)}{60c^2e^3} \\ & - \frac{d^4\sqrt{ae^2 + cd^2} \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^6} + \frac{d\sqrt{a+cx^2}(8cd^3 - ex(4cd^2 - ae^2))}{8ce^5} \\ & - \frac{13d(a+cx^2)^{3/2}(d+ex)}{20ce^3} + \frac{(a+cx^2)^{3/2}(d+ex)^2}{5ce^3} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*Sqrt[a + c\*x^2])/(d + e\*x), x]

[Out] (d\*(8\*c\*d^3 - e\*(4\*c\*d^2 - a\*e^2)\*x)\*Sqrt[a + c\*x^2])/(8\*c\*e^5) + ((47\*c\*d^2 - 8\*a\*e^2)\*(a + c\*x^2)^(3/2))/(60\*c^2\*e^3) - (13\*d\*(d + e\*x)\*(a + c\*x^2)^(3/2))/(20\*c\*e^3) + ((d + e\*x)^2\*(a + c\*x^2)^(3/2))/(5\*c\*e^3) - (d\*(8\*c^2\*d^4 + 4\*a\*c\*d^2\*e^2 - a^2\*e^4)\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(8\*c^(3/2)\*e^6) - (d^4\*Sqrt[c\*d^2 + a\*e^2]\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]))/e^6

**Rubi in Sympy [A]** time = 65.1515, size = 287, normalized size = 1.13

$$\begin{aligned} & \frac{a^2d \operatorname{atanh}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{8c^{\frac{3}{2}}e^2} - \frac{adx\sqrt{a+cx^2}}{8ce^2} - \frac{a(a+cx^2)^{\frac{3}{2}}}{3c^2e} - \frac{ad^3 \operatorname{atanh}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{2\sqrt{ce^4}} \\ & - \frac{\sqrt{cd^5} \operatorname{atanh}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{e^6} + \frac{d^4\sqrt{a+cx^2}}{e^5} - \frac{d^4\sqrt{ae^2 + cd^2} \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^6} \\ & - \frac{d^3x\sqrt{a+cx^2}}{2e^4} - \frac{dx^3\sqrt{a+cx^2}}{4e^2} + \frac{d^2(a+cx^2)^{\frac{3}{2}}}{3ce^3} + \frac{(a+cx^2)^{\frac{5}{2}}}{5c^2e} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(c\*x\*\*2+a)\*\*(1/2)/(e\*x+d), x)

[Out]  $a^{**2}d*atanh(sqrt(c)*x/sqrt(a + c*x**2))/(8*c**(3/2)*e**2) - a*d*x*sqrt(a + c*x**2)/(8*c*e**2) - a*(a + c*x**2)**(3/2)/(3*c**2*e) - a*d**3*atanh(sqrt(c)*x/sqrt(a + c*x**2))/(2*sqrt(c)*e**4) - sqrt(c)*d**5*atanh(sqrt(c)*x/sqrt(a + c*x**2))/e**6 + d**4*sqrt(a + c*x**2)/e**5 - d**4*sqrt(a*e**2 + c*d**2)*atanh((a*e - c*d*x)/(sqrt(a + c*x**2)*sqrt(a*e**2 + c*d**2)))/e**6 - d**3*x*sqrt(a + c*x**2)/(2*e**4) - d*x**3*sqrt(a + c*x**2)/(4*e**2) + d**2*(a + c*x**2)**(3/2)/(3*c*e**3) + (a + c*x**2)**(5/2)/(5*c**2*e)$

**Mathematica [A]** time = 0.413962, size = 250, normalized size = 0.98

$$-15\sqrt{cd}(-a^2e^4 + 4acd^2e^2 + 8c^2d^4) \log\left(\sqrt{c}\sqrt{a+cx^2} + cx\right) + e\sqrt{a+cx^2}(-16a^2e^4 + ace^2(40d^2 - 15dex + 8e^2x^2) + 2c^2(60$$

Antiderivative was successfully verified.

[In] Integrate[(x^4\*Sqrt[a + c\*x^2])/(d + e\*x), x]

[Out]  $(e*\text{Sqrt}[a + c*x^2]*(-16*a^2*e^4 + a*c*e^2*(40*d^2 - 15*d*e*x + 8*e^2*x^2) + 2*c^2*(60*d^4 - 30*d^3*e*x + 20*d^2*e^2*x^2 - 15*d*e^3*x^3 + 12*e^4*x^4)) + 120*c^2*d^4*\text{Sqrt}[c*d^2 + a*e^2]*\text{Log}[d + e*x] - 15*\text{Sqrt}[c]*d*(8*c^2*d^4 + 4*a*c*d^2*e^2 - a^2*e^4)*\text{Log}[c*x + \text{Sqrt}[c]*\text{Sqrt}[a + c*x^2]] - 120*c^2*d^4*\text{Sqrt}[c*d^2 + a*e^2]*\text{Log}[a*e - c*d*x + \text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2]])/(120*c^2*e^6)$

**Maple [B]** time = 0.034, size = 560, normalized size = 2.2

$$\begin{aligned} & \frac{x^2}{5ce} (cx^2 + a)^{\frac{3}{2}} - \frac{2a}{15ec^2} (cx^2 + a)^{\frac{3}{2}} + \frac{d^2}{3e^3c} (cx^2 + a)^{\frac{3}{2}} + \frac{d^4}{e^5} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2\frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}} \\ & - \frac{d^5}{e^6} \sqrt{c} \ln\left(1 - \frac{cd}{e} + c\left(x + \frac{d}{e}\right)\right) \frac{1}{\sqrt{c}} + \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2\frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}} \\ & - \frac{d^4 a}{e^5} \ln\left(1 - \left(2\frac{ae^2 + cd^2}{e^2} - 2\frac{cd}{e} \left(x + \frac{d}{e}\right) + 2\sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2\frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}}\right) \left(x + \frac{d}{e}\right)^{-1}\right) \frac{1}{\sqrt{\frac{ae^2 + cd^2}{e^2}}} \\ & - \frac{d^6 c}{e^7} \ln\left(1 - \left(2\frac{ae^2 + cd^2}{e^2} - 2\frac{cd}{e} \left(x + \frac{d}{e}\right) + 2\sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2\frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}}\right) \left(x + \frac{d}{e}\right)^{-1}\right) \frac{1}{\sqrt{\frac{ae^2 + cd^2}{e^2}}} \\ & - \frac{d^3 x}{2e^4} \sqrt{cx^2 + a} - \frac{d^3 a}{2e^4} \ln\left(x\sqrt{c} + \sqrt{cx^2 + a}\right) \frac{1}{\sqrt{c}} - \frac{dx}{4ce^2} (cx^2 + a)^{\frac{3}{2}} \\ & + \frac{adx}{8ce^2} \sqrt{cx^2 + a} + \frac{da^2}{8e^2} \ln\left(x\sqrt{c} + \sqrt{cx^2 + a}\right) c^{-\frac{3}{2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(c\*x^2+a)^(1/2)/(e\*x+d), x)

[Out]  $1/5/e*x^2*(c*x^2+a)^(3/2)/c-2/15/e*a/c^2*(c*x^2+a)^(3/2)+1/3*d^2/e^3*(c*x^2+a)^(3/2)/c+d^4/e^5*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)-d^5/e^6*c^(1/2)*ln((-c*d/e+c*(x+d/e))/c^(1/2)+((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))-d^4/e^5/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2))*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)*a-d^6/e^7/((a*e^2+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2))*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)*c-1/2*d^3/e^4*x*(c*x^2+a)^(1/2)-1/2*d^3/e^4*a/c^(1/2)*ln(x*c^(1/2)+(c*x^2+a)^(1/2))-1/4*d/e^2*x*(c*x^2+a)^(3/2)/c+1/8*d/e^2*a/c*x*($

$$c^2 x^2 + a)^{1/2} + 1/8 * d / e^2 * a^2 / c^{3/2} * \ln(x * c^{1/2} + (c^2 x^2 + a)^{1/2})$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)\*x^4/(e\*x + d),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 8.80388, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)\*x^4/(e\*x + d),x, algorithm="fricas")

[Out] [1/240\*(120\*sqrt(c\*d^2 + a\*e^2)\*c^(5/2)\*d^4\*log((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2 - 2\*sqrt(c\*d^2 + a\*e^2)\*(c\*d\*x - a\*e)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)) + 2\*(24\*c^2\*e^5\*x^4 - 30\*c^2\*d\*e^4\*x^3 + 120\*c^2\*d^4\*e + 40\*a\*c\*d^2\*e^3 - 16\*a^2\*e^5 + 8\*(5\*c^2\*d^2\*e^3 + a\*c\*e^5)\*x^2 - 15\*(4\*c^2\*d^3\*e^2 + a\*c\*d\*e^4)\*x)\*sqrt(c\*x^2 + a)\*sqrt(c) - 15\*(8\*c^3\*d^5 + 4\*a\*c^2\*d^3\*e^2 - a^2\*c\*d\*e^4)\*log(-2\*sqrt(c\*x^2 + a)\*c\*x - (2\*c\*x^2 + a)\*sqrt(c))/(c^(5/2)\*e^6), 1/240\*(240\*sqrt(-c\*d^2 - a\*e^2)\*c^(5/2)\*d^4\*arctan((c\*d\*x - a\*e)/(sqrt(-c\*d^2 - a\*e^2)\*sqrt(c\*x^2 + a))) + 2\*(24\*c^2\*e^5\*x^4 - 30\*c^2\*d\*e^4\*x^3 + 120\*c^2\*d^4\*e + 40\*a\*c\*d^2\*e^3 - 16\*a^2\*e^5 + 8\*(5\*c^2\*d^2\*e^3 + a\*c\*e^5)\*x^2 - 15\*(4\*c^2\*d^3\*e^2 + a\*c\*d\*e^4)\*x)\*sqrt(c\*x^2 + a)\*sqrt(c) - 15\*(8\*c^3\*d^5 + 4\*a\*c^2\*d^3\*e^2 - a^2\*c\*d\*e^4)\*log(-2\*sqrt(c\*x^2 + a)\*c\*x - (2\*c\*x^2 + a)\*sqrt(c))/(c^(5/2)\*e^6), 1/120\*(60\*sqrt(c\*d^2 + a\*e^2)\*sqrt(-c)\*c^2\*d^4\*log((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2 - 2\*sqrt(c\*d^2 + a\*e^2)\*(c\*d\*x - a\*e)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)) + (24\*c^2\*e^5\*x^4 - 30\*c^2\*d\*e^4\*x^3 + 120\*c^2\*d^4\*e + 40\*a\*c\*d^2\*e^3 - 16\*a^2\*e^5 + 8\*(5\*c^2\*d^2\*e^3 + a\*c\*e^5)\*x^2 - 15\*(4\*c^2\*d^3\*e^2 + a\*c\*d\*e^4)\*x)\*sqrt(c\*x^2 + a)\*sqrt(-c) - 15\*(8\*c^3\*d^5 + 4\*a\*c^2\*d^3\*e^2 - a^2\*c\*d\*e^4)\*arctan(sqrt(-c)\*x/sqrt(c\*x^2 + a))/(sqrt(-c)\*c^2\*e^6), 1/120\*(120\*sqrt(-c\*d^2 - a\*e^2)\*sqrt(-c)\*c^2\*d^4\*arctan((c\*d\*x - a\*e)/(sqrt(-c\*d^2 - a\*e^2)\*sqrt(c\*x^2 + a))) + (24\*c^2\*e^5\*x^4 - 30\*c^2\*d\*e^4\*x^3 + 120\*c^2\*d^4\*e + 40\*a\*c\*d^2\*e^3 - 16\*a^2\*e^5 + 8\*(5\*c^2\*d^2\*e^3 + a\*c\*e^5)\*x^2 - 15\*(4\*c^2\*d^3\*e^2 + a\*c\*d\*e^4)\*x)\*sqrt(c\*x^2 + a)\*sqrt(-c) - 15\*(8\*c^3\*d^5 + 4\*a\*c^2\*d^3\*e^2 - a^2\*c\*d\*e^4)\*arctan(sqrt(-c)\*x/sqrt(c\*x^2 + a))/(sqrt(-c)\*c^2\*e^6)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4 \sqrt{a + cx^2}}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(c\*x\*\*2+a)\*\*(1/2)/(e\*x+d),x)

[Out] Integral( $x^4 \sqrt{a + cx^2} / (d + ex)$ , x)

---

**GIAC/XCAS [A]** time = 0.298813, size = 340, normalized size = 1.33

$$\frac{2(cd^6 + ad^4e^2) \arctan\left(-\frac{(\sqrt{cx} - \sqrt{cx^2+a})e + \sqrt{cd}}{\sqrt{-cd^2 - ae^2}}\right) e^{(-6)}}{\sqrt{-cd^2 - ae^2}} + \frac{1}{120} \sqrt{cx^2 + a} \left( \left( 2 \left( 3 \left( 4xe^{(-1)} - 5de^{(-2)} \right) x + \frac{4(5c^3d^2e^{18} + ac^2e^{20})e^{(-21)}}{c^3} \right) x - \frac{15(4c^3d^3e^{17} + ac^2de^{19})e^{(-21)}}{c^3} \right) x + \frac{8(15c^3d^4e^{16} + 5a^2c^2d^2e^{18} - 2a^2c^2e^{20})e^{(-21)}}{c^3} \right) + \frac{(8c^{\frac{5}{2}}d^5 + 4ac^{\frac{3}{2}}d^3e^2 - a^2\sqrt{cde^4})e^{(-6)} \ln\left(\left|-\sqrt{cx} + \sqrt{cx^2 + a}\right|\right)}{8c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)\*x^4/(e\*x + d),x, algorithm="giac")

[Out]  $2*(c*d^6 + a*d^4*e^2)*\arctan(-((\sqrt{c}*x - \sqrt{c*x^2 + a})*e + \sqrt{c}*d)/\sqrt{-c*d^2 - a*e^2})*e^{(-6)}/\sqrt{-c*d^2 - a*e^2} + 1/120*\sqrt{c*x^2 + a}*((2*(3*(4*x*e^{(-1)} - 5*d*e^{(-2)})*x + 4*(5*c^3*d^2*e^{18} + a*c^2*e^{20})*e^{(-21)}/c^3)*x - 15*(4*c^3*d^3*e^{17} + a*c^2*d*e^{19})*e^{(-21)}/c^3)*x + 8*(15*c^3*d^4*e^{16} + 5*a^2*c^2*d^2*e^{18} - 2*a^2*c^2*e^{20})*e^{(-21)}/c^3) + 1/8*(8*c^{(5/2)}*d^5 + 4*a*c^{(3/2)}*d^3*e^2 - a^2*\sqrt{c}*d*e^4)*e^{(-6)}*\ln(\text{abs}(-\sqrt{c}*x + \sqrt{c*x^2 + a}))/c^2$

$$3.317 \quad \int \frac{x^3 \sqrt{a+cx^2}}{d+ex} dx$$

**Optimal.** Leaf size=211

$$\frac{(-a^2e^4 + 4acd^2e^2 + 8c^2d^4) \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{8c^{3/2}e^5} + \frac{d^3\sqrt{ae^2 + cd^2} \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^5} \\ - \frac{\sqrt{a+cx^2}(8cd^3 - ex(4cd^2 - ae^2))}{8ce^4} - \frac{7d(a+cx^2)^{3/2}}{12ce^2} + \frac{(a+cx^2)^{3/2}(d+ex)}{4ce^2}$$

[Out]  $-\left(\frac{8c^3d^3 - e(4c^2d^2 - a^2e^2)x}{8c^3e^4} \sqrt{a+cx^2}\right) - \frac{7d^3(a+cx^2)^{3/2}}{12ce^2} + \frac{(d+ex)(a+cx^2)^{3/2}}{4ce^2} + \frac{(8c^2d^4 + 4a^2c^2d^2e^2 - a^2e^4) \operatorname{ArcTanh}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{8c^{3/2}e^5} + \frac{d^3\sqrt{ae^2 + cd^2} \operatorname{ArcTanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^5}$

**Rubi [A]** time = 0.739254, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{(-a^2e^4 + 4acd^2e^2 + 8c^2d^4) \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{8c^{3/2}e^5} + \frac{d^3\sqrt{ae^2 + cd^2} \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^5} \\ - \frac{\sqrt{a+cx^2}(8cd^3 - ex(4cd^2 - ae^2))}{8ce^4} - \frac{7d(a+cx^2)^{3/2}}{12ce^2} + \frac{(a+cx^2)^{3/2}(d+ex)}{4ce^2}$$

Antiderivative was successfully verified.

[In] `Int[(x^3*Sqrt[a + c*x^2])/(d + e*x), x]`

[Out]  $-\left(\frac{8c^3d^3 - e(4c^2d^2 - a^2e^2)x}{8c^3e^4} \sqrt{a+cx^2}\right) - \frac{7d^3(a+cx^2)^{3/2}}{12ce^2} + \frac{(d+ex)(a+cx^2)^{3/2}}{4ce^2} + \frac{(8c^2d^4 + 4a^2c^2d^2e^2 - a^2e^4) \operatorname{ArcTanh}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{8c^{3/2}e^5} + \frac{d^3\sqrt{ae^2 + cd^2} \operatorname{ArcTanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^5}$

**Rubi in Sympy [A]** time = 54.5576, size = 240, normalized size = 1.14

$$-\frac{a^2 \operatorname{atanh}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{8c^{3/2}e} + \frac{ax\sqrt{a+cx^2}}{8ce} + \frac{ad^2 \operatorname{atanh}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{2\sqrt{ce^3}} + \frac{\sqrt{cd^4} \operatorname{atanh}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{e^5} - \frac{d^3\sqrt{a+cx^2}}{e^4} \\ + \frac{d^3\sqrt{ae^2 + cd^2} \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^5} + \frac{d^2x\sqrt{a+cx^2}}{2e^3} + \frac{x^3\sqrt{a+cx^2}}{4e} - \frac{d(a+cx^2)^{3/2}}{3ce^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**3*(c*x**2+a)**(1/2)/(e*x+d), x)`

[Out]  $-a^{2/2} \operatorname{atanh}\left(\frac{\sqrt{c}x}{\sqrt{a+c*x^2}}\right) / (8c^{3/2}e) + a*x*\sqrt{a+c*x^2} / (8c^2e) + a*d^{2/2} \operatorname{atanh}\left(\frac{\sqrt{c}x}{\sqrt{a+c*x^2}}\right) / (2*\sqrt{c}*e^3) + \sqrt{c}*d^{4/2} \operatorname{atanh}\left(\frac{\sqrt{c}x}{\sqrt{a+c*x^2}}\right) / e^5 - d^{3/2}*\sqrt{a+c*x^2} / e^4 + d^{3/2}*\sqrt{a*e^2+c*d^2} \operatorname{atanh}\left(\frac{a*e-c*d*x}{\sqrt{a+c*x^2}*\sqrt{a*e^2+c*d^2}}\right) / e^5 + d^{2/2}*x*\sqrt{a+c*x^2} / (2*e^3) + x^3*\sqrt{a+c*x^2} / (4*e) - d*(a+c*x^2)^{3/2} / (3*c*e^2)$



**Mathematica [A]** time = 0.482812, size = 219, normalized size = 1.04

$$\frac{3(-a^2e^4 + 4acd^2e^2 + 8c^2d^4) \log(\sqrt{c}\sqrt{a+cx^2} + cx) - 24c^{3/2}d^3\sqrt{ae^2+cd^2} \log(d+ex) + \sqrt{c} \left( 24cd^3\sqrt{ae^2+cd^2} \log(\sqrt{a+cx^2} + cx) \right)}{24c^{3/2}e^5}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*Sqrt[a + c\*x^2])/(d + e\*x), x]

[Out] (-24\*c^(3/2)\*d^3\*Sqrt[c\*d^2 + a\*e^2]\*Log[d + e\*x] + 3\*(8\*c^2\*d^4 + 4\*a\*c\*d^2\*e^2 - a^2\*e^4)\*Log[c\*x + Sqrt[c]\*Sqrt[a + c\*x^2]] + Sqrt[c]\*(e\*Sqrt[a + c\*x^2]\*(a\*e^2\*(-8\*d + 3\*e\*x) + c\*(-24\*d^3 + 12\*d^2\*e\*x - 8\*d\*e^2\*x^2 + 6\*e^3\*x^3)) + 24\*c\*d^3\*Sqrt[c\*d^2 + a\*e^2]\*Log[a\*e - c\*d\*x + Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]])/(24\*c^(3/2)\*e^5)

**Maple [B]** time = 0.014, size = 515, normalized size = 2.4

$$\begin{aligned} & \frac{d^2x}{2e^3}\sqrt{cx^2+a} + \frac{d^2a}{2e^3}\ln\left(x\sqrt{c} + \sqrt{cx^2+a}\right) \frac{1}{\sqrt{c}} + \frac{x}{4ce}(cx^2+a)^{\frac{3}{2}} - \frac{ax}{8ce}\sqrt{cx^2+a} \\ & - \frac{a^2}{8e}\ln\left(x\sqrt{c} + \sqrt{cx^2+a}\right) c^{-\frac{3}{2}} - \frac{d}{3ce^2}(cx^2+a)^{\frac{3}{2}} - \frac{d^3}{e^4}\sqrt{\left(x+\frac{d}{e}\right)^2 c - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + \frac{ae^2+cd^2}{e^2}} \\ & + \frac{d^4}{e^5}\sqrt{c}\ln\left(1\left(-\frac{cd}{e} + c\left(x+\frac{d}{e}\right)\right) \frac{1}{\sqrt{c}} + \sqrt{\left(x+\frac{d}{e}\right)^2 c - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + \frac{ae^2+cd^2}{e^2}}\right) \\ & + \frac{d^3a}{e^4}\ln\left(1\left(2\frac{ae^2+cd^2}{e^2} - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + 2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2 c - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + \frac{ae^2+cd^2}{e^2}}\right)\left(x+\frac{d}{e}\right)^{-1}\right) \frac{1}{\sqrt{\frac{ae^2+cd^2}{e^2}}} \\ & + \frac{d^5c}{e^6}\ln\left(1\left(2\frac{ae^2+cd^2}{e^2} - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + 2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2 c - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + \frac{ae^2+cd^2}{e^2}}\right)\left(x+\frac{d}{e}\right)^{-1}\right) \frac{1}{\sqrt{\frac{ae^2+cd^2}{e^2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(c\*x^2+a)^(1/2)/(e\*x+d), x)

[Out] 1/2\*d^2/e^3\*x\*(c\*x^2+a)^(1/2)+1/2\*d^2/e^3\*a/c^(1/2)\*ln(x\*c^(1/2)+(c\*x^2+a)^(1/2))+1/4/e\*x\*(c\*x^2+a)^(3/2)/c-1/8/e\*a/c\*x\*(c\*x^2+a)^(1/2)-1/8/e\*a^2/c^(3/2)\*ln(x\*c^(1/2)+(c\*x^2+a)^(1/2))-1/3\*d\*(c\*x^2+a)^(3/2)/c/e^2-d^3/e^4\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2)+d^4/e^5\*c^(1/2)\*ln((-c\*d/e+c\*(x+d/e))/c^(1/2)+((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2))+d^3/e^4/((a\*e^2+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2+c\*d^2)/e^2-2\*c\*d/e\*(x+d/e)+2\*((a\*e^2+c\*d^2)/e^2)^(1/2))\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2))/(x+d/e))\*a+d^5/e^6/(((a\*e^2+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2+c\*d^2)/e^2-2\*c\*d/e\*(x+d/e)+2\*((a\*e^2+c\*d^2)/e^2)^(1/2))\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2))/(x+d/e))\*c

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)\*x^3/(e\*x + d), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 6.77092, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)\*x^3/(e\*x + d), x, algorithm="fricas")

[Out] [1/48\*(24\*sqrt(c\*d^2 + a\*e^2)\*c^(3/2)\*d^3\*log((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2 + 2\*sqrt(c\*d^2 + a\*e^2)\*(c\*d\*x - a\*e)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)) + 2\*(6\*c\*e^4\*x^3 - 8\*c\*d\*e^3\*x^2 - 24\*c\*d^3\*e - 8\*a\*d\*e^3 + 3\*(4\*c\*d^2\*e^2 + a\*e^4)\*x)\*sqrt(c\*x^2 + a)\*sqrt(c) - 3\*(8\*c^2\*d^4 + 4\*a\*c\*d^2\*e^2 - a^2\*e^4)\*log(2\*sqrt(c\*x^2 + a)\*c\*x - (2\*c\*x^2 + a)\*sqrt(c)))/(c^(3/2)\*e^5), -1/48\*(48\*sqrt(-c\*d^2 - a\*e^2)\*c^(3/2)\*d^3\*arctan((c\*d\*x - a\*e)/(sqrt(-c\*d^2 - a\*e^2)\*sqrt(c\*x^2 + a))) - 2\*(6\*c\*e^4\*x^3 - 8\*c\*d\*e^3\*x^2 - 24\*c\*d^3\*e - 8\*a\*d\*e^3 + 3\*(4\*c\*d^2\*e^2 + a\*e^4)\*x)\*sqrt(c\*x^2 + a)\*sqrt(c) + 3\*(8\*c^2\*d^4 + 4\*a\*c\*d^2\*e^2 - a^2\*e^4)\*log(2\*sqrt(c\*x^2 + a)\*c\*x - (2\*c\*x^2 + a)\*sqrt(c)))/(c^(3/2)\*e^5), 1/24\*(12\*sqrt(c\*d^2 + a\*e^2)\*sqrt(-c)\*c\*d^3\*log((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2 + 2\*sqrt(c\*d^2 + a\*e^2)\*(c\*d\*x - a\*e)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)) + (6\*c\*e^4\*x^3 - 8\*c\*d\*e^3\*x^2 - 24\*c\*d^3\*e - 8\*a\*d\*e^3 + 3\*(4\*c\*d^2\*e^2 + a\*e^4)\*x)\*sqrt(c\*x^2 + a)\*sqrt(-c) + 3\*(8\*c^2\*d^4 + 4\*a\*c\*d^2\*e^2 - a^2\*e^4)\*arctan(sqrt(-c)\*x/sqrt(c\*x^2 + a)))/(sqrt(-c)\*c\*e^5), -1/24\*(24\*sqrt(-c\*d^2 - a\*e^2)\*sqrt(-c)\*c\*d^3\*arctan((c\*d\*x - a\*e)/(sqrt(-c\*d^2 - a\*e^2)\*sqrt(c\*x^2 + a))) - (6\*c\*e^4\*x^3 - 8\*c\*d\*e^3\*x^2 - 24\*c\*d^3\*e - 8\*a\*d\*e^3 + 3\*(4\*c\*d^2\*e^2 + a\*e^4)\*x)\*sqrt(c\*x^2 + a)\*sqrt(-c) - 3\*(8\*c^2\*d^4 + 4\*a\*c\*d^2\*e^2 - a^2\*e^4)\*arctan(sqrt(-c)\*x/sqrt(c\*x^2 + a)))/(sqrt(-c)\*c\*e^5)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3 \sqrt{a + cx^2}}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(c\*x\*\*2+a)\*\*(1/2)/(e\*x+d), x)

[Out] Integral(x\*\*3\*sqrt(a + c\*x\*\*2)/(d + e\*x), x)

**GIAC/XCAS [A]** time = 0.30515, size = 271, normalized size = 1.28

$$\frac{2(cd^5 + ad^3e^2) \arctan\left(-\frac{(\sqrt{cx} - \sqrt{cx^2+a})e + \sqrt{cd}}{\sqrt{-cd^2 - ae^2}}\right) e^{(-5)}}{\sqrt{-cd^2 - ae^2}} + \frac{1}{24} \sqrt{cx^2 + a} \left( \left( 2(3xe^{(-1)} - 4de^{(-2)})x + \frac{3(4c^2d^2e^{12} + ace^{14})e^{(-15)}}{c^2} \right) x - \frac{8(3c^2d^3e^{11} + acde^{13})e^{(-15)}}{c^2} \right) - \frac{(8c^2d^4 + 4acd^2e^2 - a^2e^4)e^{(-5)} \ln\left(\left|-\sqrt{cx} + \sqrt{cx^2 + a}\right|\right)}{8c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + a)*x^3/(e*x + d),x, algorithm="giac")
```

```
[Out] -2*(c*d^5 + a*d^3*e^2)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e +
sqrt(c)*d)/sqrt(-c*d^2 - a*e^2))*e^(-5)/sqrt(-c*d^2 - a*e^2) + 1
/24*sqrt(c*x^2 + a)*((2*(3*x*e^(-1) - 4*d*e^(-2))*x + 3*(4*c^2*d^
2*e^12 + a*c*e^14)*e^(-15)/c^2)*x - 8*(3*c^2*d^3*e^11 + a*c*d*e^1
3)*e^(-15)/c^2) - 1/8*(8*c^2*d^4 + 4*a*c*d^2*e^2 - a^2*e^4)*e^(-5
)*ln(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^(3/2)
```

$$3.318 \quad \int \frac{x^2 \sqrt{a+cx^2}}{d+ex} dx$$

**Optimal.** Leaf size=153

$$-\frac{d^2 \sqrt{ae^2 + cd^2} \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4} - \frac{d(ae^2 + 2cd^2) \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{2\sqrt{ce^4}} + \frac{d\sqrt{a+cx^2}(2d-ex)}{2e^3} + \frac{(a+cx^2)^{3/2}}{3ce}$$

[Out] (d\*(2\*d - e\*x)\*Sqrt[a + c\*x^2])/(2\*e^3) + (a + c\*x^2)^(3/2)/(3\*c\*e) - (d\*(2\*c\*d^2 + a\*e^2)\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(2\*Sqrt[c]\*e^4) - (d^2\*Sqrt[c\*d^2 + a\*e^2]\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]]))/e^4

**Rubi [A]** time = 0.443296, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$

$$-\frac{d^2 \sqrt{ae^2 + cd^2} \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4} - \frac{d(ae^2 + 2cd^2) \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{2\sqrt{ce^4}} + \frac{d\sqrt{a+cx^2}(2d-ex)}{2e^3} + \frac{(a+cx^2)^{3/2}}{3ce}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*Sqrt[a + c\*x^2])/(d + e\*x), x]

[Out] (d\*(2\*d - e\*x)\*Sqrt[a + c\*x^2])/(2\*e^3) + (a + c\*x^2)^(3/2)/(3\*c\*e) - (d\*(2\*c\*d^2 + a\*e^2)\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(2\*Sqrt[c]\*e^4) - (d^2\*Sqrt[c\*d^2 + a\*e^2]\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]]))/e^4

**Rubi in Sympy [A]** time = 44.1001, size = 167, normalized size = 1.09

$$-\frac{ad \operatorname{atanh}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{2\sqrt{ce^2}} - \frac{\sqrt{cd^3} \operatorname{atanh}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{e^4} + \frac{d^2 \sqrt{a+cx^2}}{e^3} - \frac{d^2 \sqrt{ae^2 + cd^2} \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4} - \frac{dx \sqrt{a+cx^2}}{2e^2} + \frac{(a+cx^2)^{3/2}}{3ce}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(c\*x\*\*2+a)\*\*(1/2)/(e\*x+d), x)

[Out] -a\*d\*atanh(sqrt(c)\*x/sqrt(a + c\*x\*\*2))/(2\*sqrt(c)\*e\*\*2) - sqrt(c)\*d\*\*3\*atanh(sqrt(c)\*x/sqrt(a + c\*x\*\*2))/e\*\*4 + d\*\*2\*sqrt(a + c\*x\*\*2)/e\*\*3 - d\*\*2\*sqrt(a\*e\*\*2 + c\*d\*\*2)\*atanh((a\*e - c\*d\*x)/(sqrt(a + c\*x\*\*2)\*sqrt(a\*e\*\*2 + c\*d\*\*2)))/e\*\*4 - d\*x\*sqrt(a + c\*x\*\*2)/(2\*e\*\*2) + (a + c\*x\*\*2)\*\*(3/2)/(3\*c\*e)

**Mathematica [A]** time = 0.185174, size = 179, normalized size = 1.17

$$\frac{e\sqrt{a+cx^2}(2ae^2+c(6d^2-3dex+2e^2x^2))-6cd^2\sqrt{ae^2+cd^2}\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)-3\sqrt{cd}(ae^2+2cd^2)}{6ce^4}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*Sqrt[a + c\*x^2])/(d + e\*x), x]

[Out] (e\*Sqrt[a + c\*x^2]\*(2\*a\*e^2 + c\*(6\*d^2 - 3\*d\*e\*x + 2\*e^2\*x^2)) + 6\*c\*d^2\*Sqrt[c\*d^2 + a\*e^2]\*Log[d + e\*x] - 3\*Sqrt[c]\*d\*(2\*c\*d^2 + a\*e^2)\*Log[c\*x + Sqrt[c]\*Sqrt[a + c\*x^2]] - 6\*c\*d^2\*Sqrt[c\*d^2 + a\*e^2]\*Log[a\*e - c\*d\*x + Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]])/(6\*c\*e^4)

**Maple [B]** time = 0.014, size = 448, normalized size = 2.9

$$\begin{aligned} & \frac{1}{3ce} (cx^2 + a)^{\frac{3}{2}} - \frac{dx}{2e^2} \sqrt{cx^2 + a} - \frac{ad}{2e^2} \ln(x\sqrt{c} + \sqrt{cx^2 + a}) \frac{1}{\sqrt{c}} \\ & + \frac{d^2}{e^3} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2\frac{cd}{e}\left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}} \\ & - \frac{d^3}{e^4} \sqrt{c} \ln\left(1 - \frac{cd}{e} + c\left(x + \frac{d}{e}\right)\right) \frac{1}{\sqrt{c}} + \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2\frac{cd}{e}\left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}} \\ & - \frac{d^2 a}{e^3} \ln\left(1 - \left(2\frac{ae^2 + cd^2}{e^2} - 2\frac{cd}{e}\left(x + \frac{d}{e}\right) + 2\sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2\frac{cd}{e}\left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}}\right)\left(x + \frac{d}{e}\right)^{-1}\right) \frac{1}{\sqrt{\frac{ae^2 + cd^2}{e^2}}} \\ & - \frac{d^4 c}{e^5} \ln\left(1 - \left(2\frac{ae^2 + cd^2}{e^2} - 2\frac{cd}{e}\left(x + \frac{d}{e}\right) + 2\sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2\frac{cd}{e}\left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}}\right)\left(x + \frac{d}{e}\right)^{-1}\right) \frac{1}{\sqrt{\frac{ae^2 + cd^2}{e^2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(c\*x^2+a)^(1/2)/(e\*x+d), x)

[Out] 1/3\*(c\*x^2+a)^(3/2)/c/e-1/2\*d/e^2\*x\*(c\*x^2+a)^(1/2)-1/2/e^2\*d\*a/c^(1/2)\*ln(x\*c^(1/2)+(c\*x^2+a)^(1/2))+d^2/e^3\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2)-d^3/e^4\*c^(1/2)\*ln((-c\*d/e+c\*(x+d/e))/c^(1/2)+((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2))-d^2/e^3/((a\*e^2+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2+c\*d^2)/e^2-2\*c\*d/e\*(x+d/e)+2\*((a\*e^2+c\*d^2)/e^2)^(1/2))\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2))/(x+d/e)\*a-d^4/e^5/((a\*e^2+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2+c\*d^2)/e^2-2\*c\*d/e\*(x+d/e)+2\*((a\*e^2+c\*d^2)/e^2)^(1/2))\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2))/(x+d/e))\*c

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)\*x^2/(e\*x + d), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.571219, size = 1, normalized size = 0.01

$$\left[ \frac{6\sqrt{cd^2 + ae^2}c^{\frac{3}{2}}d^2 \log\left(\frac{2acdex - acd^2 - 2a^2e^2 - (2c^2d^2 + ace^2)x^2 - 2\sqrt{cd^2 + ae^2}(cdx - ae)\sqrt{cx^2 + a}}{e^2x^2 + 2dex + d^2}\right) + 2(2ce^3x^2 - 3cde^2x + 6cd^2e + 2ae^3)}{12c^{\frac{3}{2}}e^4} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)\*x^2/(e\*x + d),x, algorithm="fricas")

[Out] [1/12\*(6\*sqrt(c\*d^2 + a\*e^2)\*c^(3/2)\*d^2\*log((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2 - 2\*sqrt(c\*d^2 + a\*e^2)\*(c\*d\*x - a\*e)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)) + 2\*(2\*c\*e^3\*x^2 - 3\*c\*d\*e^2\*x + 6\*c\*d^2\*e + 2\*a\*e^3)\*sqrt(c\*x^2 + a)\*sqrt(c) + 3\*(2\*c^2\*d^3 + a\*c\*d\*e^2)\*log(2\*sqrt(c\*x^2 + a)\*c\*x - (2\*c\*x^2 + a)\*sqrt(c))/(c^(3/2)\*e^4), 1/12\*(12\*sqrt(-c\*d^2 - a\*e^2)\*c^(3/2)\*d^2\*arctan((c\*d\*x - a\*e)/(sqrt(-c\*d^2 - a\*e^2)\*sqrt(c\*x^2 + a))) + 2\*(2\*c\*e^3\*x^2 - 3\*c\*d\*e^2\*x + 6\*c\*d^2\*e + 2\*a\*e^3)\*sqrt(c\*x^2 + a)\*sqrt(c) + 3\*(2\*c^2\*d^3 + a\*c\*d\*e^2)\*log(2\*sqrt(c\*x^2 + a)\*c\*x - (2\*c\*x^2 + a)\*sqrt(c))/(c^(3/2)\*e^4), 1/6\*(3\*sqrt(c\*d^2 + a\*e^2)\*sqrt(-c)\*c\*d^2\*log((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2 - 2\*sqrt(c\*d^2 + a\*e^2)\*(c\*d\*x - a\*e)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)) + (2\*c\*e^3\*x^2 - 3\*c\*d\*e^2\*x + 6\*c\*d^2\*e + 2\*a\*e^3)\*sqrt(c\*x^2 + a)\*sqrt(-c) - 3\*(2\*c^2\*d^3 + a\*c\*d\*e^2)\*arctan(sqrt(-c)\*x/sqrt(c\*x^2 + a)))/(sqrt(-c)\*c\*e^4), 1/6\*(6\*sqrt(-c\*d^2 - a\*e^2)\*sqrt(-c)\*c\*d^2\*arctan((c\*d\*x - a\*e)/(sqrt(-c\*d^2 - a\*e^2)\*sqrt(c\*x^2 + a))) + (2\*c\*e^3\*x^2 - 3\*c\*d\*e^2\*x + 6\*c\*d^2\*e + 2\*a\*e^3)\*sqrt(c\*x^2 + a)\*sqrt(-c) - 3\*(2\*c^2\*d^3 + a\*c\*d\*e^2)\*arctan(sqrt(-c)\*x/sqrt(c\*x^2 + a)))/(sqrt(-c)\*c\*e^4)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 \sqrt{a + cx^2}}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(c\*x\*\*2+a)\*\*(1/2)/(e\*x+d),x)

[Out] Integral(x\*\*2\*sqrt(a + c\*x\*\*2)/(d + e\*x), x)

**GIAC/XCAS [A]** time = 0.278268, size = 212, normalized size = 1.39

$$\frac{2(cd^4 + ad^2e^2) \arctan\left(-\frac{(\sqrt{cx} - \sqrt{cx^2+a})e + \sqrt{cd}}{\sqrt{-cd^2 - ae^2}}\right) e^{(-4)}}{\sqrt{-cd^2 - ae^2}} + \frac{(2cd^3 + ade^2) e^{(-4)} \ln\left(\left|-\sqrt{cx} + \sqrt{cx^2+a}\right|\right)}{2\sqrt{c}} + \frac{1}{6} \sqrt{cx^2+a} \left( (2xe^{(-1)} - 3de^{(-2)})x + \frac{2(3cd^2e^7 + ae^9)e^{(-10)}}{c} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)\*x^2/(e\*x + d),x, algorithm="giac")

[Out] 2\*(c\*d^4 + a\*d^2\*e^2)\*arctan(-((sqrt(c)\*x - sqrt(c\*x^2 + a))\*e + sqrt(c)\*d)/sqrt(-c\*d^2 - a\*e^2))\*e^(-4)/sqrt(-c\*d^2 - a\*e^2) + 1/2\*(2\*c\*d^3 + a\*d\*e^2)\*e^(-4)\*ln(abs(-sqrt(c)\*x + sqrt(c\*x^2 + a)))/sqrt(c) + 1/6\*sqrt(c\*x^2 + a)\*((2\*x\*e^(-1) - 3\*d\*e^(-2))\*x + 2\*(3\*c\*d^2\*e^7 + a\*e^9)\*e^(-10)/c)

$$3.319 \quad \int \frac{x\sqrt{a+cx^2}}{d+ex} dx$$

**Optimal.** Leaf size=127

$$\frac{(ae^2 + 2cd^2) \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{2\sqrt{ce^3}} + \frac{d\sqrt{ae^2 + cd^2} \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^3} - \frac{\sqrt{a+cx^2}(2d-ex)}{2e^2}$$

[Out]  $-\left((2*d - e*x)*\text{Sqrt}[a + c*x^2]\right)/(2*e^2) + \left(\left(2*c*d^2 + a*e^2\right)*\text{ArcTanh}\left[\left(\text{Sqrt}[c]*x\right)/\text{Sqrt}[a + c*x^2]\right]\right)/(2*\text{Sqrt}[c]*e^3) + \left(d*\text{Sqrt}[c*d^2 + a*e^2]*\text{ArcTanh}\left[\left(a*e - c*d*x\right)/\left(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2]\right)\right]\right)/e^3$

**Rubi [A]** time = 0.284924, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{(ae^2 + 2cd^2) \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{2\sqrt{ce^3}} + \frac{d\sqrt{ae^2 + cd^2} \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^3} - \frac{\sqrt{a+cx^2}(2d-ex)}{2e^2}$$

Antiderivative was successfully verified.

[In] Int[(x\*Sqrt[a + c\*x^2])/(d + e\*x), x]

[Out]  $-\left((2*d - e*x)*\text{Sqrt}[a + c*x^2]\right)/(2*e^2) + \left(\left(2*c*d^2 + a*e^2\right)*\text{ArcTanh}\left[\left(\text{Sqrt}[c]*x\right)/\text{Sqrt}[a + c*x^2]\right]\right)/(2*\text{Sqrt}[c]*e^3) + \left(d*\text{Sqrt}[c*d^2 + a*e^2]*\text{ArcTanh}\left[\left(a*e - c*d*x\right)/\left(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2]\right)\right]\right)/e^3$

**Rubi in Sympy [A]** time = 34.4748, size = 114, normalized size = 0.9

$$\frac{d\sqrt{ae^2 + cd^2} \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^3} - \frac{\sqrt{a+cx^2}(2d-ex)}{2e^2} + \frac{(ae^2 + 2cd^2) \operatorname{atanh}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{2\sqrt{ce^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(c\*x\*\*2+a)\*\*(1/2)/(e\*x+d), x)

[Out]  $d*\text{sqrt}(a*e**2 + c*d**2)*\text{atanh}\left(\left(a*e - c*d*x\right)/\left(\text{sqrt}(a + c*x**2)*\text{sqrt}(a*e**2 + c*d**2)\right)\right)/e**3 - \text{sqrt}(a + c*x**2)*(2*d - e*x)/(2*e**2) + \left(a*e**2 + 2*c*d**2\right)*\text{atanh}\left(\text{sqrt}(c)*x/\text{sqrt}(a + c*x**2)\right)/(2*\text{sqrt}(c)*e**3)$

**Mathematica [A]** time = 0.173257, size = 158, normalized size = 1.24

$$\frac{(ae^2 + 2cd^2) \log\left(\sqrt{c}\sqrt{a+cx^2} + cx\right)}{2\sqrt{ce^3}} + \frac{d\sqrt{ae^2 + cd^2} \log\left(\sqrt{a+cx^2}\sqrt{ae^2 + cd^2} + ae - cdx\right)}{e^3} - \frac{d\sqrt{ae^2 + cd^2} \log(d + ex)}{e^3} + \sqrt{a+cx^2} \left(\frac{x}{2e} - \frac{d}{e^2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(x\*Sqrt[a + c\*x^2])/(d + e\*x), x]

[Out]  $(-(d/e^2) + x/(2*e))*\text{Sqrt}[a + c*x^2] - (d*\text{Sqrt}[c*d^2 + a*e^2]*\text{Log}[d + e*x])/e^3 + ((2*c*d^2 + a*e^2)*\text{Log}[c*x + \text{Sqrt}[c]*\text{Sqrt}[a + c*x^2]])/(2*\text{Sqrt}[c]*e^3) + (d*\text{Sqrt}[c*d^2 + a*e^2]*\text{Log}[a*e - c*d*x + \text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2]])/e^3$

**Maple [B]** time = 0.009, size = 423, normalized size = 3.3

$$\begin{aligned} & \frac{x}{2e} \sqrt{cx^2 + a} + \frac{a}{2e} \ln \left( x\sqrt{c} + \sqrt{cx^2 + a} \right) \frac{1}{\sqrt{c}} - \frac{d}{e^2} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}} \\ & + \frac{d^2}{e^3} \sqrt{c} \ln \left( 1 \left( -\frac{cd}{e} + c \left(x + \frac{d}{e}\right) \right) \frac{1}{\sqrt{c}} + \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}} \right) \\ & + \frac{ad}{e^2} \ln \left( 1 \left( 2 \frac{ae^2 + cd^2}{e^2} - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + 2 \sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}} \right) \left(x + \frac{d}{e}\right)^{-1} \right) \frac{1}{\sqrt{\frac{ae^2 + cd^2}{e^2}}} \\ & + \frac{cd^3}{e^4} \ln \left( 1 \left( 2 \frac{ae^2 + cd^2}{e^2} - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + 2 \sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}} \right) \left(x + \frac{d}{e}\right)^{-1} \right) \frac{1}{\sqrt{\frac{ae^2 + cd^2}{e^2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(c*x^2+a)^(1/2)/(e*x+d), x)`

[Out]  $1/2/e*x*(c*x^2+a)^(1/2)+1/2/e*a/c^(1/2)*\ln(x*c^(1/2)+(c*x^2+a)^(1/2))-d/e^2*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2)+d^2/e^3*c^(1/2)*\ln((-c*d/e+c*(x+d/e))/c^(1/2)+((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))+d/e^2/((a*e^2+c*d^2)/e^2)^(1/2)*\ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)*a+d^3/e^4/((a*e^2+c*d^2)/e^2)^(1/2)*\ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e))*c$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + a)*x/(e*x + d), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.535017, size = 1, normalized size = 0.01

$$\left[ \frac{2 \sqrt{cd^2 + ae^2} \sqrt{cd} \log \left( \frac{2 acdex - acd^2 - 2 a^2 e^2 - (2 c^2 d^2 + ace^2) x^2 + 2 \sqrt{cd^2 + ae^2} (cdx - ae) \sqrt{cx^2 + a}}{e^2 x^2 + 2 dex + d^2} \right) + 2 (e^2 x - 2 de) \sqrt{cx^2 + a} \sqrt{c} + (2 cd^2 + ae^2) \sqrt{c}}{4 \sqrt{ce^3}} \right. \\ \left. \frac{4 \sqrt{-cd^2 - ae^2} \sqrt{cd} \arctan \left( \frac{cdx - ae}{\sqrt{-cd^2 - ae^2} \sqrt{cx^2 + a}} \right) - 2 (e^2 x - 2 de) \sqrt{cx^2 + a} \sqrt{c} - (2 cd^2 + ae^2) \log \left( -2 \sqrt{cx^2 + acx} - (2 cx^2 + a) \sqrt{c} \right)}{4 \sqrt{ce^3}} \right. \\ \left. \frac{2 \sqrt{-cd^2 - ae^2} \sqrt{-cd} \arctan \left( \frac{cdx - ae}{\sqrt{-cd^2 - ae^2} \sqrt{cx^2 + a}} \right) - (e^2 x - 2 de) \sqrt{cx^2 + a} \sqrt{-c} - (2 cd^2 + ae^2) \arctan \left( \frac{\sqrt{-cx}}{\sqrt{cx^2 + a}} \right)}{2 \sqrt{-ce^3}} \right]$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)\*x/(e\*x + d),x, algorithm="fricas")

[Out] [1/4\*(2\*sqrt(c\*d^2 + a\*e^2)\*sqrt(c)\*d\*log((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2 + 2\*sqrt(c\*d^2 + a\*e^2)\*(c\*d\*x - a\*e)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)) + 2\*(e^2\*x - 2\*d\*e)\*sqrt(c\*x^2 + a)\*sqrt(c) + (2\*c\*d^2 + a\*e^2)\*log(-2\*sqrt(c\*x^2 + a)\*c\*x - (2\*c\*x^2 + a)\*sqrt(c))/(sqrt(c)\*e^3), -1/4\*(4\*sqrt(-c\*d^2 - a\*e^2)\*sqrt(c)\*d\*arctan((c\*d\*x - a\*e)/(sqrt(-c\*d^2 - a\*e^2)\*sqrt(c\*x^2 + a))) - 2\*(e^2\*x - 2\*d\*e)\*sqrt(c\*x^2 + a)\*sqrt(c) - (2\*c\*d^2 + a\*e^2)\*log(-2\*sqrt(c\*x^2 + a)\*c\*x - (2\*c\*x^2 + a)\*sqrt(c))/(sqrt(c)\*e^3), 1/2\*(sqrt(c\*d^2 + a\*e^2)\*sqrt(-c)\*d\*log((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2 + 2\*sqrt(c\*d^2 + a\*e^2)\*(c\*d\*x - a\*e)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)) + (e^2\*x - 2\*d\*e)\*sqrt(c\*x^2 + a)\*sqrt(-c) + (2\*c\*d^2 + a\*e^2)\*arctan(sqrt(-c)\*x/sqrt(c\*x^2 + a))/(sqrt(-c)\*e^3), -1/2\*(2\*sqrt(-c\*d^2 - a\*e^2)\*sqrt(-c)\*d\*arctan((c\*d\*x - a\*e)/(sqrt(-c\*d^2 - a\*e^2)\*sqrt(c\*x^2 + a))) - (e^2\*x - 2\*d\*e)\*sqrt(c\*x^2 + a)\*sqrt(-c) - (2\*c\*d^2 + a\*e^2)\*arctan(sqrt(-c)\*x/sqrt(c\*x^2 + a)))/(sqrt(-c)\*e^3)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x\sqrt{a+cx^2}}{d+ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(c\*x\*\*2+a)\*\*(1/2)/(e\*x+d),x)

[Out] Integral(x\*sqrt(a + c\*x\*\*2)/(d + e\*x), x)

**GIAC/XCAS [A]** time = 0.276095, size = 182, normalized size = 1.43

$$\frac{2(cd^3 + ade^2) \arctan\left(-\frac{(\sqrt{cx}-\sqrt{cx^2+a})e+\sqrt{cd}}{\sqrt{-cd^2-ae^2}}\right) e^{(-3)}}{\sqrt{-cd^2-ae^2}} - \frac{(2c^{\frac{3}{2}}d^2 + a\sqrt{ce^2})e^{(-3)}\ln\left(\left|-\sqrt{cx} + \sqrt{cx^2+a}\right|\right)}{2c} + \frac{1}{2}\sqrt{cx^2+a}\left(xe^{(-1)} - 2de^{(-2)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)\*x/(e\*x + d),x, algorithm="giac")

[Out] -2\*(c\*d^3 + a\*d\*e^2)\*arctan(-((sqrt(c)\*x - sqrt(c\*x^2 + a))\*e + sqrt(c)\*d)/sqrt(-c\*d^2 - a\*e^2))\*e^(-3)/sqrt(-c\*d^2 - a\*e^2) - 1/2\*(2\*c^(3/2)\*d^2 + a\*sqrt(c)\*e^2)\*e^(-3)\*ln(abs(-sqrt(c)\*x + sqrt(c\*x^2 + a)))/c + 1/2\*sqrt(c\*x^2 + a)\*(x\*e^(-1) - 2\*d\*e^(-2))

$$3.320 \quad \int \frac{\sqrt{a+cx^2}}{d+ex} dx$$

**Optimal.** Leaf size=103

$$-\frac{\sqrt{ae^2+cd^2} \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2} - \frac{\sqrt{cd} \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{e^2} + \frac{\sqrt{a+cx^2}}{e}$$

[Out] Sqrt[a + c\*x^2]/e - (Sqrt[c]\*d\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/e^2 - (Sqrt[c\*d^2 + a\*e^2]\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]))/e^2

**Rubi [A]** time = 0.187291, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$

$$-\frac{\sqrt{ae^2+cd^2} \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2} - \frac{\sqrt{cd} \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{e^2} + \frac{\sqrt{a+cx^2}}{e}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c\*x^2]/(d + e\*x), x]

[Out] Sqrt[a + c\*x^2]/e - (Sqrt[c]\*d\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/e^2 - (Sqrt[c\*d^2 + a\*e^2]\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]))/e^2

**Rubi in Sympy [A]** time = 26.0591, size = 90, normalized size = 0.87

$$-\frac{\sqrt{cd} \operatorname{atanh}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{e^2} + \frac{\sqrt{a+cx^2}}{e} - \frac{\sqrt{ae^2+cd^2} \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+a)\*\*(1/2)/(e\*x+d), x)

[Out] -sqrt(c)\*d\*atanh(sqrt(c)\*x/sqrt(a + c\*x\*\*2))/e\*\*2 + sqrt(a + c\*x\*\*2)/e - sqrt(a\*e\*\*2 + c\*d\*\*2)\*atanh((a\*e - c\*d\*x)/(sqrt(a + c\*x\*\*2)\*sqrt(a\*e\*\*2 + c\*d\*\*2)))/e\*\*2

**Mathematica [A]** time = 0.064514, size = 124, normalized size = 1.2

$$\frac{-\sqrt{ae^2+cd^2} \log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2} + ae - cdx\right) + \sqrt{ae^2+cd^2} \log(d+ex) - \sqrt{cd} \log\left(\sqrt{c}\sqrt{a+cx^2} + cx\right) + e\sqrt{a+cx^2}}{e^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + c\*x^2]/(d + e\*x), x]

[Out] (e\*Sqrt[a + c\*x^2] + Sqrt[c\*d^2 + a\*e^2]\*Log[d + e\*x] - Sqrt[c]\*d\*Log[c\*x + Sqrt[c]\*Sqrt[a + c\*x^2]] - Sqrt[c\*d^2 + a\*e^2]\*Log[a\*e - c\*d\*x + Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]])/e^2

**Maple [B]** time = 0.006, size = 381, normalized size = 3.7

$$\begin{aligned} & \frac{1}{e} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}} \\ & - \frac{d}{e^2} \sqrt{c} \ln \left( 1 \left( -\frac{cd}{e} + c \left(x + \frac{d}{e}\right) \right) \frac{1}{\sqrt{c}} + \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}} \right) \\ & - \frac{a}{e} \ln \left( 1 \left( 2 \frac{ae^2 + cd^2}{e^2} - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + 2 \sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}} \right) \left(x + \frac{d}{e}\right)^{-1} \right) \frac{1}{\sqrt{\frac{ae^2 + cd^2}{e^2}}} \\ & - \frac{cd^2}{e^3} \ln \left( 1 \left( 2 \frac{ae^2 + cd^2}{e^2} - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + 2 \sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}} \right) \left(x + \frac{d}{e}\right)^{-1} \right) \frac{1}{\sqrt{\frac{ae^2 + cd^2}{e^2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+a)^(1/2)/(e\*x+d), x)

[Out]  $\frac{1}{e} \left( \left( x + \frac{d}{e} \right)^2 c - 2 \frac{cd}{e} \left( x + \frac{d}{e} \right) + \frac{ae^2 + cd^2}{e^2} \right)^{1/2} - \frac{1}{e^2} c^{1/2} d \ln \left( \frac{-c \frac{d}{e} + c \left( x + \frac{d}{e} \right)}{c^{1/2}} + \frac{\left( x + \frac{d}{e} \right)^2 c - 2 \frac{cd}{e} \left( x + \frac{d}{e} \right) + \frac{ae^2 + cd^2}{e^2}}{\left( x + \frac{d}{e} \right)^2 c - 2 \frac{cd}{e} \left( x + \frac{d}{e} \right) + \frac{ae^2 + cd^2}{e^2}} \right) - \frac{1}{e} \frac{\ln \left( 2 \frac{ae^2 + cd^2}{e^2} - 2 \frac{cd}{e} \left( x + \frac{d}{e} \right) + 2 \sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left( x + \frac{d}{e} \right)^2 c - 2 \frac{cd}{e} \left( x + \frac{d}{e} \right) + \frac{ae^2 + cd^2}{e^2}} \right)}{\sqrt{\frac{ae^2 + cd^2}{e^2}}} \left( x + \frac{d}{e} \right)^{-1} - \frac{cd^2}{e^3} \frac{\ln \left( 2 \frac{ae^2 + cd^2}{e^2} - 2 \frac{cd}{e} \left( x + \frac{d}{e} \right) + 2 \sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left( x + \frac{d}{e} \right)^2 c - 2 \frac{cd}{e} \left( x + \frac{d}{e} \right) + \frac{ae^2 + cd^2}{e^2}} \right)}{\sqrt{\frac{ae^2 + cd^2}{e^2}}} \left( x + \frac{d}{e} \right)^{-1}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)/(e\*x + d), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.365704, size = 1, normalized size = 0.01

$$\left[ \frac{\sqrt{cd} \log \left( -2 cx^2 + 2 \sqrt{cx^2 + a} \sqrt{cx} - a \right) + 2 \sqrt{cx^2 + a} e + \sqrt{cd^2 + ae^2} \log \left( \frac{2 acdex - acd^2 - 2 a^2 e^2 - (2 c^2 d^2 + ace^2) x^2 - 2 \sqrt{cd^2 + ae^2} (cdx - ae)}{e^2 x^2 + 2 dex + d^2} \right)}{2 e^2}, \right. \\ \left. \frac{2 \sqrt{-cd} \arctan \left( \frac{cx}{\sqrt{cx^2 + a} \sqrt{-c}} \right) - 2 \sqrt{cx^2 + a} e - \sqrt{cd^2 + ae^2} \log \left( \frac{2 acdex - acd^2 - 2 a^2 e^2 - (2 c^2 d^2 + ace^2) x^2 - 2 \sqrt{cd^2 + ae^2} (cdx - ae) \sqrt{cx^2 + a}}{e^2 x^2 + 2 dex + d^2} \right)}{2 e^2}, \right. \\ \left. \frac{\sqrt{-cd} \arctan \left( \frac{cx}{\sqrt{cx^2 + a} \sqrt{-c}} \right) - \sqrt{cx^2 + a} e - \sqrt{-cd^2 - ae^2} \arctan \left( \frac{cdx - ae}{\sqrt{-cd^2 - ae^2} \sqrt{cx^2 + a}} \right)}{e^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)/(e\*x + d), x, algorithm="fricas")

[Out]  $\frac{1}{2} \sqrt{c} d \log \left( -2 c x^2 + 2 \sqrt{c x^2 + a} \sqrt{c} x - a \right) + 2 \sqrt{c x^2 + a} e + \sqrt{c d^2 + a e^2} \log \left( \frac{2 a c d e x - a c d^2 - 2 a^2 e^2 - (2 c^2 d^2 + a c e^2) x^2 - 2 \sqrt{c d^2 + a e^2} (c d x - a e)}{e^2 x^2 + 2 d e x + d^2} \right)$

```
*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e
^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2))/e^
2, -1/2*(2*sqrt(-c)*d*arctan(c*x/(sqrt(c*x^2 + a)*sqrt(-c))) - 2*
sqrt(c*x^2 + a)*e - sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^
2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)
*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2))/e^2,
1/2*(sqrt(c)*d*log(-2*c*x^2 + 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) +
2*sqrt(c*x^2 + a)*e + 2*sqrt(-c*d^2 - a*e^2)*arctan((c*d*x - a*e)
/(sqrt(-c*d^2 - a*e^2)*sqrt(c*x^2 + a)))/e^2, -(sqrt(-c)*d*arcta
n(c*x/(sqrt(c*x^2 + a)*sqrt(-c))) - sqrt(c*x^2 + a)*e - sqrt(-c*d
^2 - a*e^2)*arctan((c*d*x - a*e)/(sqrt(-c*d^2 - a*e^2)*sqrt(c*x^2
+ a)))/e^2]
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+cx^2}}{d+ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+a)\*\*(1/2)/(e\*x+d),x)

[Out] Integral(sqrt(a + c\*x\*\*2)/(d + e\*x), x)

**GIAC/XCAS [A]** time = 0.27515, size = 147, normalized size = 1.43

$$\sqrt{cde}e^{(-2)}\ln\left(\left|-\sqrt{cx} + \sqrt{cx^2 + a}\right|\right) + \frac{2(cd^2 + ae^2)\arctan\left(-\frac{(\sqrt{cx}-\sqrt{cx^2+a})e+\sqrt{cd}}{\sqrt{-cd^2-ae^2}}\right)e^{(-2)}}{\sqrt{-cd^2-ae^2}} + \sqrt{cx^2 + ae}e^{(-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)/(e\*x + d),x, algorithm="giac")

[Out] sqrt(c)\*d\*e^(-2)\*ln(abs(-sqrt(c)\*x + sqrt(c\*x^2 + a))) + 2\*(c\*d^2 + a\*e^2)\*arctan(-((sqrt(c)\*x - sqrt(c\*x^2 + a))\*e + sqrt(c)\*d)/sqrt(-c\*d^2 - a\*e^2))\*e^(-2)/sqrt(-c\*d^2 - a\*e^2) + sqrt(c\*x^2 + a)\*e^(-1)

$$3.321 \quad \int \frac{\sqrt{a+cx^2}}{x(d+ex)} dx$$

**Optimal.** Leaf size=116

$$\frac{\sqrt{ae^2 + cd^2} \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{de} - \frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d} + \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{e}$$

[Out] (Sqrt[c]\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/e + (Sqrt[c\*d^2 + a\*e^2]\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])])/(d\*e) - (Sqrt[a]\*ArcTanh[Sqrt[a + c\*x^2]/Sqrt[a]])/d

**Rubi [A]** time = 0.284895, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$\frac{\sqrt{ae^2 + cd^2} \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{de} - \frac{\sqrt{a} \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d} + \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{e}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c\*x^2]/(x\*(d + e\*x)), x]

[Out] (Sqrt[c]\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/e + (Sqrt[c\*d^2 + a\*e^2]\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])])/(d\*e) - (Sqrt[a]\*ArcTanh[Sqrt[a + c\*x^2]/Sqrt[a]])/d

**Rubi in Sympy [A]** time = 28.4214, size = 99, normalized size = 0.85

$$-\frac{\sqrt{a} \operatorname{atanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d} + \frac{\sqrt{c} \operatorname{atanh}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{e} + \frac{\sqrt{ae^2 + cd^2} \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+a)\*\*(1/2)/x/(e\*x+d), x)

[Out] -sqrt(a)\*atanh(sqrt(a + c\*x\*\*2)/sqrt(a))/d + sqrt(c)\*atanh(sqrt(c)\*x/sqrt(a + c\*x\*\*2))/e + sqrt(a\*e\*\*2 + c\*d\*\*2)\*atanh((a\*e - c\*d\*x)/(sqrt(a + c\*x\*\*2)\*sqrt(a\*e\*\*2 + c\*d\*\*2)))/(d\*e)

**Mathematica [A]** time = 0.0848857, size = 150, normalized size = 1.29

$$\frac{\sqrt{ae^2 + cd^2} \log\left(\sqrt{a + cx^2}\sqrt{ae^2 + cd^2} + ae - cdx\right) - \sqrt{ae^2 + cd^2} \log(d + ex) + \sqrt{cd} \log\left(\sqrt{c}\sqrt{a + cx^2} + cx\right) - \sqrt{ae} \log\left(\sqrt{a}\sqrt{a + cx^2}\right)}{de}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + c\*x^2]/(x\*(d + e\*x)), x]

[Out] (Sqrt[a]\*e\*Log[x] - Sqrt[c\*d^2 + a\*e^2]\*Log[d + e\*x] - Sqrt[a]\*e\*Log[a + Sqrt[a]\*Sqrt[a + c\*x^2]]) + Sqrt[c]\*d\*Log[c\*x + Sqrt[c]\*Sqrt[a + c\*x^2]] + Sqrt[c\*d^2 + a\*e^2]\*Log[a\*e - c\*d\*x + Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]]/(d\*e)

**Maple [B]** time = 0.015, size = 420, normalized size = 3.6

$$\begin{aligned}
& -\frac{1}{d}\sqrt{a}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{cx^2+a}\right)\right)+\frac{1}{d}\sqrt{cx^2+a}-\frac{1}{d}\sqrt{\left(x+\frac{d}{e}\right)^2c-2\frac{cd}{e}\left(x+\frac{d}{e}\right)+\frac{ae^2+cd^2}{e^2}} \\
& +\frac{1}{e}\sqrt{c}\ln\left(1-\frac{cd}{e}+c\left(x+\frac{d}{e}\right)\right)\frac{1}{\sqrt{c}}+\sqrt{\left(x+\frac{d}{e}\right)^2c-2\frac{cd}{e}\left(x+\frac{d}{e}\right)+\frac{ae^2+cd^2}{e^2}} \\
& +\frac{a}{d}\ln\left(1\left(2\frac{ae^2+cd^2}{e^2}-2\frac{cd}{e}\left(x+\frac{d}{e}\right)+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c-2\frac{cd}{e}\left(x+\frac{d}{e}\right)+\frac{ae^2+cd^2}{e^2}}\right)\left(x+\frac{d}{e}\right)^{-1}\right)\frac{1}{\sqrt{\frac{ae^2+cd^2}{e^2}}} \\
& +\frac{cd}{e^2}\ln\left(1\left(2\frac{ae^2+cd^2}{e^2}-2\frac{cd}{e}\left(x+\frac{d}{e}\right)+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c-2\frac{cd}{e}\left(x+\frac{d}{e}\right)+\frac{ae^2+cd^2}{e^2}}\right)\left(x+\frac{d}{e}\right)^{-1}\right)\frac{1}{\sqrt{\frac{ae^2+cd^2}{e^2}}}
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+a)^(1/2)/x/(e\*x+d), x)

[Out] 
$$\begin{aligned}
& -1/d*a^{(1/2)}*\ln((2*a+2*a^{(1/2)}*(c*x^2+a)^{(1/2)})/x)+1/d*(c*x^2+a)^{(1/2)}-1/d*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^{(1/2)}+c \\
& ^{(1/2)}/e*\ln((-c*d/e+c*(x+d/e))/c^{(1/2)}+((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^{(1/2)})+1/d/((a*e^2+c*d^2)/e^2)^{(1/2)}*\ln((2* \\
& (a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^{(1/2)})/(x+d/e)) \\
& *a+d/e^2/((a*e^2+c*d^2)/e^2)^{(1/2)}*\ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a* \\
& e^2+c*d^2)/e^2)^{(1/2)})/(x+d/e))*c
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)/((e\*x + d)\*x), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.03029, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)/((e\*x + d)\*x), x, algorithm="fricas")

[Out] 
$$\begin{aligned}
& [1/2*(\sqrt{c}*d*\log(-2*c*x^2-2*\sqrt{c*x^2+a}*\sqrt{c}*x-a)+\sqrt{a}*e*\log(-(c*x^2-2*\sqrt{c*x^2+a}*\sqrt{a}+2*a)/x^2)+ \\
& \sqrt{c*d^2+a*e^2}*\log((2*a*c*d*e*x-a*c*d^2-2*a^2*e^2-(2*c^2*d^2+a*c*e^2)*x^2+2*\sqrt{c*d^2+a*e^2}*(c*d*x-a*e)*\sqrt{c*x^2+a}))/ \\
& (e^2*x^2+2*d*e*x+d^2)))/(d*e), 1/2*(2*\sqrt{-c}*d*\arctan(c*x/(\sqrt{c*x^2+a}*\sqrt{-c}))+\sqrt{a}*e*\log(-(c*x^2- \\
& 2*\sqrt{c*x^2+a}*\sqrt{a}+2*a)/x^2)+\sqrt{c*d^2+a*e^2}*\log((2*a*c*d*e*x-a*c*d^2-2*a^2*e^2-(2*c^2*d^2+a*c*e^2)*x^2+2* \\
& \sqrt{c*d^2+a*e^2}*(c*d*x-a*e)*\sqrt{c*x^2+a}))/e^2*x^2+2*d*e*x+d^2)))/(d*e), 1/2*(\sqrt{c}*d*\log(-2*c*x^2-2*\sqrt{c*x^2+a}
\end{aligned}$$

```
+ a)*sqrt(c)*x - a) + sqrt(a)*e*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*sqrt(-c*d^2 - a*e^2)*arctan((c*d*x - a*e)/(sqrt(-c*d^2 - a*e^2)*sqrt(c*x^2 + a)))/(d*e), 1/2*(2*sqrt(-c)*d*arctan(c*x/(sqrt(c*x^2 + a)*sqrt(-c))) + sqrt(a)*e*log(-(c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(a) + 2*a)/x^2) - 2*sqrt(-c*d^2 - a*e^2)*arctan((c*d*x - a*e)/(sqrt(-c*d^2 - a*e^2)*sqrt(c*x^2 + a)))/(d*e), -1/2*(2*sqrt(-a)*e*arctan(a/(sqrt(c*x^2 + a)*sqrt(-a))) - sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) - sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), 1/2*(2*sqrt(-c)*d*arctan(c*x/(sqrt(c*x^2 + a)*sqrt(-c))) - 2*sqrt(-a)*e*arctan(a/(sqrt(c*x^2 + a)*sqrt(-a))) + sqrt(c*d^2 + a*e^2)*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 + 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/(d*e), -1/2*(2*sqrt(-a)*e*arctan(a/(sqrt(c*x^2 + a)*sqrt(-a))) - sqrt(c)*d*log(-2*c*x^2 - 2*sqrt(c*x^2 + a)*sqrt(c)*x - a) + 2*sqrt(-c*d^2 - a*e^2)*arctan((c*d*x - a*e)/(sqrt(-c*d^2 - a*e^2)*sqrt(c*x^2 + a)))/(d*e), (sqrt(-c)*d*arctan(c*x/(sqrt(c*x^2 + a)*sqrt(-c))) - sqrt(-a)*e*arctan(a/(sqrt(c*x^2 + a)*sqrt(-a))) - sqrt(-c*d^2 - a*e^2)*arctan((c*d*x - a*e)/(sqrt(-c*d^2 - a*e^2)*sqrt(c*x^2 + a)))/(d*e))]
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2}}{x(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+a)\*\*(1/2)/x/(e\*x+d),x)

[Out] Integral(sqrt(a + c\*x\*\*2)/(x\*(d + e\*x)), x)

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)/((e\*x + d)\*x),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.322 \quad \int \frac{\sqrt{a+cx^2}}{x^2(d+ex)} dx$$

**Optimal.** Leaf size=105

$$-\frac{\sqrt{ae^2+cd^2} \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2} + \frac{\sqrt{ae} \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2} - \frac{\sqrt{a+cx^2}}{dx}$$

[Out]  $-(\text{Sqrt}[a + c*x^2]/(d*x)) - (\text{Sqrt}[c*d^2 + a*e^2]*\text{ArcTanh}[(a*e - c*d*x)/(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2])])/d^2 + (\text{Sqrt}[a]*e*\text{ArcTanh}[\text{Sqrt}[a + c*x^2]/\text{Sqrt}[a]])/d^2$

**Rubi [A]** time = 0.410984, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 11, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$-\frac{\sqrt{ae^2+cd^2} \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2} + \frac{\sqrt{ae} \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2} - \frac{\sqrt{a+cx^2}}{dx}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c\*x^2]/(x^2\*(d + e\*x)), x]

[Out]  $-(\text{Sqrt}[a + c*x^2]/(d*x)) - (\text{Sqrt}[c*d^2 + a*e^2]*\text{ArcTanh}[(a*e - c*d*x)/(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2])])/d^2 + (\text{Sqrt}[a]*e*\text{ArcTanh}[\text{Sqrt}[a + c*x^2]/\text{Sqrt}[a]])/d^2$

**Rubi in Sympy [A]** time = 49.4292, size = 90, normalized size = 0.86

$$\frac{\sqrt{ae} \operatorname{atanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^2} - \frac{\sqrt{a+cx^2}}{dx} - \frac{\sqrt{ae^2+cd^2} \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+a)\*\*(1/2)/x\*\*2/(e\*x+d), x)

[Out]  $\text{sqrt}(a)*e*\operatorname{atanh}(\text{sqrt}(a + c*x^2)/\text{sqrt}(a))/d^2 - \text{sqrt}(a + c*x^2)/(d*x) - \text{sqrt}(a*e^2 + c*d^2)*\operatorname{atanh}((a*e - c*d*x)/(\text{sqrt}(a + c*x^2)*\text{sqrt}(a*e^2 + c*d^2)))/d^2$

**Mathematica [A]** time = 0.0787273, size = 139, normalized size = 1.32

$$\frac{-x\sqrt{ae^2+cd^2} \log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2} + ae - cdx\right) + x\sqrt{ae^2+cd^2} \log(d+ex) + d\left(-\sqrt{a+cx^2}\right) + \sqrt{a}ex \log\left(\sqrt{a}\sqrt{a+cx^2} + d\right)}{d^2x}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + c\*x^2]/(x^2\*(d + e\*x)), x]

[Out]  $(-(d*\text{Sqrt}[a + c*x^2]) - \text{Sqrt}[a]*e*x*\text{Log}[x] + \text{Sqrt}[c*d^2 + a*e^2]*x*\text{Log}[d + e*x] + \text{Sqrt}[a]*e*x*\text{Log}[a + \text{Sqrt}[a]*\text{Sqrt}[a + c*x^2]]) - \text{Sqrt}[c*d^2 + a*e^2]*x*\text{Log}[a*e - c*d*x + \text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2]])/(d^2*x)$



**Maple [B]** time = 0.017, size = 486, normalized size = 4.6

$$\begin{aligned}
 & -\frac{1}{adx} (cx^2 + a)^{\frac{3}{2}} + \frac{cx}{ad} \sqrt{cx^2 + a} + \frac{1}{d} \sqrt{c} \ln \left( x\sqrt{c} + \sqrt{cx^2 + a} \right) + \frac{e}{d^2} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}} \\
 & -\frac{1}{d} \sqrt{c} \ln \left( 1 \left( -\frac{cd}{e} + c \left(x + \frac{d}{e}\right) \right) \frac{1}{\sqrt{c}} + \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}} \right) \\
 & -\frac{ae}{d^2} \ln \left( 1 \left( 2 \frac{ae^2 + cd^2}{e^2} - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + 2 \sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}} \right) \left(x + \frac{d}{e}\right)^{-1} \right) \frac{1}{\sqrt{\frac{ae^2 + cd^2}{e^2}}} \\
 & -\frac{c}{e} \ln \left( 1 \left( 2 \frac{ae^2 + cd^2}{e^2} - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + 2 \sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}} \right) \left(x + \frac{d}{e}\right)^{-1} \right) \frac{1}{\sqrt{\frac{ae^2 + cd^2}{e^2}}} \\
 & + \frac{e}{d^2} \sqrt{a} \ln \left( \frac{1}{x} \left( 2a + 2\sqrt{a}\sqrt{cx^2 + a} \right) \right) - \frac{e}{d^2} \sqrt{cx^2 + a}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+a)^(1/2)/x^2/(e\*x+d), x)

[Out] 
$$\begin{aligned}
 & -1/d/a/x*(c*x^2+a)^(3/2)+1/d*c/a*x*(c*x^2+a)^(1/2)+1/d*c^(1/2)*\ln \\
 & (x*c^(1/2)+(c*x^2+a)^(1/2))+e/d^2*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a \\
 & *e^2+c*d^2)/e^2)^(1/2)-1/d*c^(1/2)*\ln((-c*d/e+c*(x+d/e))/c^(1/2)+ \\
 & ((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))-e/d^2/((a* \\
 & e^2+c*d^2)/e^2)^(1/2)*\ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*( \\
 & (a*e^2+c*d^2)/e^2)^(1/2)*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2) \\
 & /e^2)^(1/2))/(x+d/e)*a-1/e/((a*e^2+c*d^2)/e^2)^(1/2)*\ln((2*(a* \\
 & e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^(1/2)*((x+d/ \\
 & e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^(1/2))/(x+d/e)*c+e/d^2 \\
 & *a^(1/2)*\ln((2*a+2*a^(1/2)*(c*x^2+a)^(1/2))/x)-e/d^2*(c*x^2+a)^(1 \\
 & /2)
 \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}}{(ex + d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)/((e\*x + d)\*x^2), x, algorithm="maxima")

[Out] integrate(sqrt(c\*x^2 + a)/((e\*x + d)\*x^2), x)

**Fricas [A]** time = 0.327652, size = 1, normalized size = 0.01

$$\left[ \frac{\sqrt{a}ex \log \left( -\frac{cx^2 + 2\sqrt{cx^2 + a}\sqrt{a} + 2a}{x^2} \right) + \sqrt{cd^2 + ae^2}x \log \left( \frac{2acdex - acd^2 - 2a^2e^2 - (2c^2d^2 + ace^2)x^2 - 2\sqrt{cd^2 + ae^2}(cdx - ae)\sqrt{cx^2 + a}}{e^2x^2 + 2dex + d^2} \right)}{2d^2x} \right] - 2\sqrt{cx^2 + a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)/((e\*x + d)\*x^2), x, algorithm="fricas")

[Out] 
$$\begin{aligned}
 & [1/2*(\sqrt{a}*e*x*\log(-(c*x^2 + 2*\sqrt{c*x^2 + a})*\sqrt{a} + 2*a)/ \\
 & x^2) + \sqrt{c*d^2 + a*e^2}*x*\log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e
 \end{aligned}$$

$$\begin{aligned} &^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*\sqrt{c*d^2 + a*e^2}*(c*d*x - a \\ &*e)*\sqrt{c*x^2 + a})/(e^2*x^2 + 2*d*e*x + d^2)) - 2*\sqrt{c*x^2 + \\ &a)*d)/(d^2*x), 1/2*(\sqrt{a}*e*x*\log(-(c*x^2 + 2*\sqrt{c*x^2 + a})*\sqrt{a} \\ &+ 2*a)/x^2) + 2*\sqrt{-c*d^2 - a*e^2}*x*\arctan((c*d*x - a*e \\ &)/(\sqrt{-c*d^2 - a*e^2}*\sqrt{c*x^2 + a})) - 2*\sqrt{c*x^2 + a)*d)/ \\ &(d^2*x), 1/2*(2*\sqrt{-a}*e*x*\arctan(a/(\sqrt{c*x^2 + a}*\sqrt{-a}))) \\ &+ \sqrt{c*d^2 + a*e^2}*x*\log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - \\ &(2*c^2*d^2 + a*c*e^2)*x^2 - 2*\sqrt{c*d^2 + a*e^2}*(c*d*x - a*e)* \\ &\sqrt{c*x^2 + a}))/(\sqrt{c*x^2 + a}*\sqrt{-a}))) - 2*\sqrt{c*x^2 + a)*d \\ &)/(d^2*x), (\sqrt{-a}*e*x*\arctan(a/(\sqrt{c*x^2 + a}*\sqrt{-a}))) + \sqrt{ \\ &-c*d^2 - a*e^2}*x*\arctan((c*d*x - a*e)/(\sqrt{-c*d^2 - a*e^2}*\sqrt{ \\ &c*x^2 + a})) - \sqrt{c*x^2 + a)*d)/(d^2*x)] \end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+cx^2}}{x^2(d+ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+a)\*\*(1/2)/x\*\*2/(e\*x+d), x)

[Out] Integral(sqrt(a + c\*x\*\*2)/(x\*\*2\*(d + e\*x)), x)

**GIAC/XCAS [A]** time = 0.275415, size = 196, normalized size = 1.87

$$-\frac{2a \arctan\left(-\frac{\sqrt{cx}-\sqrt{cx^2+a}}{\sqrt{-a}}\right) e}{\sqrt{-ad^2}} + \frac{2a\sqrt{c}}{\left(\left(\sqrt{cx}-\sqrt{cx^2+a}\right)^2 - a\right)d} + \frac{2(cd^2 + ae^2) \arctan\left(-\frac{(\sqrt{cx}-\sqrt{cx^2+a})e+\sqrt{cd}}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)/((e\*x + d)\*x^2), x, algorithm="giac")

[Out] -2\*a\*arctan(-(sqrt(c)\*x - sqrt(c\*x^2 + a))/sqrt(-a))\*e/(sqrt(-a)\*d^2) + 2\*a\*sqrt(c)/(((sqrt(c)\*x - sqrt(c\*x^2 + a))^2 - a)\*d) + 2\*(c\*d^2 + a\*e^2)\*arctan(-((sqrt(c)\*x - sqrt(c\*x^2 + a))\*e + sqrt(c)\*d)/sqrt(-c\*d^2 - a\*e^2))/sqrt(-c\*d^2 - a\*e^2)\*d^2)

$$3.323 \quad \int \frac{\sqrt{a+cx^2}}{x^3(d+ex)} dx$$

**Optimal.** Leaf size=160

$$\begin{aligned} & -\frac{\sqrt{ae^2} \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{e\sqrt{ae^2+cd^2} \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3} \\ & -\frac{\sqrt{a+cx^2}}{2dx^2} - \frac{c \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{ad}} \end{aligned}$$

[Out] -Sqrt[a + c\*x^2]/(2\*d\*x^2) + (e\*Sqrt[a + c\*x^2])/(d^2\*x) + (e\*Sqrt[c\*d^2 + a\*e^2]\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]))/d^3 - (c\*ArcTanh[Sqrt[a + c\*x^2]/Sqrt[a]])/(2\*Sqrt[a]\*d) - (Sqrt[a]\*e^2\*ArcTanh[Sqrt[a + c\*x^2]/Sqrt[a]])/d^3

**Rubi [A]** time = 0.504806, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 12, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$

$$\begin{aligned} & -\frac{\sqrt{ae^2} \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{e\sqrt{ae^2+cd^2} \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3} \\ & -\frac{\sqrt{a+cx^2}}{2dx^2} - \frac{c \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{ad}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c\*x^2]/(x^3\*(d + e\*x)), x]

[Out] -Sqrt[a + c\*x^2]/(2\*d\*x^2) + (e\*Sqrt[a + c\*x^2])/(d^2\*x) + (e\*Sqrt[c\*d^2 + a\*e^2]\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]))/d^3 - (c\*ArcTanh[Sqrt[a + c\*x^2]/Sqrt[a]])/(2\*Sqrt[a]\*d) - (Sqrt[a]\*e^2\*ArcTanh[Sqrt[a + c\*x^2]/Sqrt[a]])/d^3

**Rubi in Sympy [A]** time = 57.7648, size = 141, normalized size = 0.88

$$\begin{aligned} & -\frac{\sqrt{ae^2} \operatorname{atanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^3} - \frac{\sqrt{a+cx^2}}{2dx^2} + \frac{e\sqrt{a+cx^2}}{d^2x} + \frac{e\sqrt{ae^2+cd^2} \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3} - \frac{c \operatorname{atanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{ad}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+a)\*\*(1/2)/x\*\*3/(e\*x+d), x)

[Out] -sqrt(a)\*e\*\*2\*atanh(sqrt(a + c\*x\*\*2)/sqrt(a))/d\*\*3 - sqrt(a + c\*x\*\*2)/(2\*d\*x\*\*2) + e\*sqrt(a + c\*x\*\*2)/(d\*\*2\*x) + e\*sqrt(a\*e\*\*2 + c\*d\*\*2)\*atanh((a\*e - c\*d\*x)/(sqrt(a + c\*x\*\*2)\*sqrt(a\*e\*\*2 + c\*d\*\*2)))/d\*\*3 - c\*atanh(sqrt(a + c\*x\*\*2)/sqrt(a))/(2\*sqrt(a)\*d)

**Mathematica [A]** time = 0.263801, size = 170, normalized size = 1.06

$$\begin{aligned} & -\frac{(2ae^2+cd^2) \log\left(\frac{\sqrt{a}\sqrt{a+cx^2}+a}{\sqrt{a}}\right)}{2d^3} + 2e\sqrt{ae^2+cd^2} \log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right) + \frac{\log(x)(2ae^2+cd^2)}{\sqrt{a}} - 2e\sqrt{ae^2+cd^2} \log(d+ex) \end{aligned}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + c\*x^2]/(x^3\*(d + e\*x)),x]

[Out] ((d\*(-d + 2\*e\*x)\*Sqrt[a + c\*x^2])/x^2 + ((c\*d^2 + 2\*a\*e^2)\*Log[x])/Sqrt[a] - 2\*e\*Sqrt[c\*d^2 + a\*e^2]\*Log[d + e\*x] - ((c\*d^2 + 2\*a\*e^2)\*Log[a + Sqrt[a]\*Sqrt[a + c\*x^2]])/Sqrt[a] + 2\*e\*Sqrt[c\*d^2 + a\*e^2]\*Log[a\*e - c\*d\*x + Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]])/(2\*d^3)

**Maple [B]** time = 0.017, size = 567, normalized size = 3.5

$$\begin{aligned}
 & -\frac{1}{2adx^2} (cx^2 + a)^{\frac{3}{2}} - \frac{c}{2d} \ln\left(\frac{1}{x} (2a + 2\sqrt{a}\sqrt{cx^2 + a})\right) \frac{1}{\sqrt{a}} + \frac{c}{2ad} \sqrt{cx^2 + a} \\
 & - \frac{e^2}{d^3} \sqrt{a} \ln\left(\frac{1}{x} (2a + 2\sqrt{a}\sqrt{cx^2 + a})\right) + \frac{e^2}{d^3} \sqrt{cx^2 + a} - \frac{e^2}{d^3} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2\frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}} \\
 & + \frac{e}{d^2} \sqrt{c} \ln\left(1 - \left(\frac{cd}{e} + c\left(x + \frac{d}{e}\right)\right) \frac{1}{\sqrt{c}} + \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2\frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}}\right) \\
 & + \frac{ae^2}{d^3} \ln\left(1 - \left(2\frac{ae^2 + cd^2}{e^2} - 2\frac{cd}{e} \left(x + \frac{d}{e}\right) + 2\sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2\frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}}\right) \left(x + \frac{d}{e}\right)^{-1}\right) \frac{1}{\sqrt{\frac{ae^2 + cd^2}{e^2}}} \\
 & + \frac{c}{d} \ln\left(1 - \left(2\frac{ae^2 + cd^2}{e^2} - 2\frac{cd}{e} \left(x + \frac{d}{e}\right) + 2\sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2\frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}}\right) \left(x + \frac{d}{e}\right)^{-1}\right) \frac{1}{\sqrt{\frac{ae^2 + cd^2}{e^2}}} \\
 & + \frac{e}{d^2ax} (cx^2 + a)^{\frac{3}{2}} - \frac{cex}{d^2a} \sqrt{cx^2 + a} - \frac{e}{d^2} \sqrt{c} \ln\left(x\sqrt{c} + \sqrt{cx^2 + a}\right)
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+a)^(1/2)/x^3/(e\*x+d),x)

[Out] -1/2/d/a/x^2\*(c\*x^2+a)^(3/2)-1/2/d\*c/a^(1/2)\*ln((2\*a+2\*a^(1/2)\*(c\*x^2+a)^(1/2))/x)+1/2/d\*c/a\*(c\*x^2+a)^(1/2)-1/d^3\*e^2\*a^(1/2)\*ln((2\*a+2\*a^(1/2)\*(c\*x^2+a)^(1/2))/x)+1/d^3\*e^2\*(c\*x^2+a)^(1/2)-1/d^3\*e^2\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2)+1/d^2\*e\*c^(1/2)\*ln((-c\*d/e+c\*(x+d/e))/c^(1/2)+((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2))+1/d^3\*e^2/((a\*e^2+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2+c\*d^2)/e^2-2\*c\*d/e\*(x+d/e)+2\*((a\*e^2+c\*d^2)/e^2)^(1/2))\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2))/(x+d/e)\*a+1/d/((a\*e^2+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2+c\*d^2)/e^2-2\*c\*d/e\*(x+d/e)+2\*((a\*e^2+c\*d^2)/e^2)^(1/2))\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2))/(x+d/e)\*c+e/d^2/a/x\*(c\*x^2+a)^(3/2)-e/d^2\*c/a\*x\*(c\*x^2+a)^(1/2)-e/d^2\*c^(1/2)\*ln(x\*c^(1/2)+(c\*x^2+a)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}}{(ex + d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)/((e\*x + d)\*x^3),x, algorithm="maxima")

[Out] integrate(sqrt(c\*x^2 + a)/((e\*x + d)\*x^3), x)

**Fricas [A]** time = 0.342078, size = 1, normalized size = 0.01

$$\left[ \frac{2\sqrt{cd^2 + ae^2}\sqrt{aex^2} \log\left(\frac{2acdex - acd^2 - 2a^2e^2 - (2c^2d^2 + ace^2)x^2 + 2\sqrt{cd^2 + ae^2}(cdx - ae)\sqrt{cx^2 + a}}{e^2x^2 + 2dex + d^2}\right) + (cd^2 + 2ae^2)x^2 \log\left(-\frac{(cx^2 + 2a)\sqrt{a-2a}}{x^2}\right)}{4\sqrt{ad^3}x^2} \right. \\ \left. \frac{4\sqrt{-cd^2 - ae^2}\sqrt{aex^2} \arctan\left(\frac{cdx - ae}{\sqrt{-cd^2 - ae^2}\sqrt{cx^2 + a}}\right) - (cd^2 + 2ae^2)x^2 \log\left(-\frac{(cx^2 + 2a)\sqrt{a-2}\sqrt{cx^2 + aa}}{x^2}\right) - 2(2dex - d^2)\sqrt{cx^2 + a}}{4\sqrt{ad^3}x^2} \right. \\ \left. \frac{2\sqrt{-cd^2 - ae^2}\sqrt{aex^2} \arctan\left(\frac{cdx - ae}{\sqrt{-cd^2 - ae^2}\sqrt{cx^2 + a}}\right) + (cd^2 + 2ae^2)x^2 \arctan\left(\frac{\sqrt{-a}}{\sqrt{cx^2 + a}}\right) - (2dex - d^2)\sqrt{cx^2 + a}\sqrt{-a}}{2\sqrt{-ad^3}x^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)/((e\*x + d)\*x^3), x, algorithm="fricas")

[Out] [1/4\*(2\*sqrt(c\*d^2 + a\*e^2)\*sqrt(a)\*e\*x^2\*log((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2 + 2\*sqrt(c\*d^2 + a\*e^2)\*(c\*d\*x - a\*e)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)) + (c\*d^2 + 2\*a\*e^2)\*x^2\*log(-((c\*x^2 + 2\*a)\*sqrt(a) - 2\*sqrt(c\*x^2 + a)\*a)/x^2) + 2\*(2\*d\*e\*x - d^2)\*sqrt(c\*x^2 + a)\*sqrt(a)/(sqrt(a)\*d^3\*x^2), -1/4\*(4\*sqrt(-c\*d^2 - a\*e^2)\*sqrt(a)\*e\*x^2\*arctan((c\*d\*x - a\*e)/(sqrt(-c\*d^2 - a\*e^2)\*sqrt(c\*x^2 + a))) - (c\*d^2 + 2\*a\*e^2)\*x^2\*log(-((c\*x^2 + 2\*a)\*sqrt(a) - 2\*sqrt(c\*x^2 + a)\*a)/x^2) - 2\*(2\*d\*e\*x - d^2)\*sqrt(c\*x^2 + a)\*sqrt(a)/(sqrt(a)\*d^3\*x^2), 1/2\*(sqrt(c\*d^2 + a\*e^2)\*sqrt(-a)\*e\*x^2\*log((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2 + 2\*sqrt(c\*d^2 + a\*e^2)\*(c\*d\*x - a\*e)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)) - (c\*d^2 + 2\*a\*e^2)\*x^2\*arctan(sqrt(-a)/sqrt(c\*x^2 + a)) + (2\*d\*e\*x - d^2)\*sqrt(c\*x^2 + a)\*sqrt(-a)/(sqrt(-a)\*d^3\*x^2), -1/2\*(2\*sqrt(-c\*d^2 - a\*e^2)\*sqrt(-a)\*e\*x^2\*arctan((c\*d\*x - a\*e)/(sqrt(-c\*d^2 - a\*e^2)\*sqrt(c\*x^2 + a))) + (c\*d^2 + 2\*a\*e^2)\*x^2\*arctan(sqrt(-a)/sqrt(c\*x^2 + a)) - (2\*d\*e\*x - d^2)\*sqrt(c\*x^2 + a)\*sqrt(-a)/(sqrt(-a)\*d^3\*x^2)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2}}{x^3(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+a)\*\*(1/2)/x\*\*3/(e\*x+d), x)

[Out] Integral(sqrt(a + c\*x\*\*2)/(x\*\*3\*(d + e\*x)), x)

**GIAC/XCAS [A]** time = 0.280075, size = 311, normalized size = 1.94

$$\frac{2(cd^2e + ae^3) \arctan\left(-\frac{(\sqrt{cx} - \sqrt{cx^2 + a})e + \sqrt{cd}}{\sqrt{-cd^2 - ae^2}}\right)}{\sqrt{-cd^2 - ae^2}d^3} + \frac{(cd^2 + 2ae^2) \arctan\left(-\frac{\sqrt{cx} - \sqrt{cx^2 + a}}{\sqrt{-a}}\right)}{\sqrt{-ad^3}} \\ + \frac{(\sqrt{cx} - \sqrt{cx^2 + a})^3 cd - 2(\sqrt{cx} - \sqrt{cx^2 + a})^2 a\sqrt{ce} + (\sqrt{cx} - \sqrt{cx^2 + a})acd + 2a^2\sqrt{ce}}{\left((\sqrt{cx} - \sqrt{cx^2 + a})^2 - a\right)^2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)/((e\*x + d)\*x^3),x, algorithm="giac")

[Out] 
$$\begin{aligned} & -2*(c*d^2*e + a*e^3)*\arctan(-((\sqrt{c})x - \sqrt{c*x^2 + a})*e + \sqrt{c}*d)/\sqrt{-c*d^2 - a*e^2})/(\sqrt{-c*d^2 - a*e^2}*d^3) + (c*d \\ & ^2 + 2*a*e^2)*\arctan(-(\sqrt{c})x - \sqrt{c*x^2 + a})/\sqrt{-a})/(\sqrt{-a}*d^3) + ((\sqrt{c})x - \sqrt{c*x^2 + a})^3*c*d - 2*(\sqrt{c})x \\ & - \sqrt{c*x^2 + a})^2*a*\sqrt{c}*e + (\sqrt{c})x - \sqrt{c*x^2 + a}) \\ & *a*c*d + 2*a^2*\sqrt{c}*e)/(((\sqrt{c})x - \sqrt{c*x^2 + a})^2 - a)^2*d^2) \end{aligned}$$

$$3.324 \quad \int \frac{\sqrt{a+cx^2}}{x^4(d+ex)} dx$$

**Optimal.** Leaf size=191

$$\frac{\sqrt{ae^3} \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^4} - \frac{e^2\sqrt{a+cx^2}}{d^3x} + \frac{e\sqrt{a+cx^2}}{2d^2x^2} + \frac{ce \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{ad^2}} - \frac{e^2\sqrt{ae^2+cd^2} \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^4} - \frac{(a+cx^2)^{3/2}}{3adx^3}$$

[Out] (e\*Sqrt[a + c\*x^2])/(2\*d^2\*x^2) - (e^2\*Sqrt[a + c\*x^2])/(d^3\*x) - (a + c\*x^2)^(3/2)/(3\*a\*d\*x^3) - (e^2\*Sqrt[c\*d^2 + a\*e^2]\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])])/d^4 + (c\*e\*ArcTanh[Sqrt[a + c\*x^2]/Sqrt[a]])/(2\*Sqrt[a]\*d^2) + (Sqrt[a]\*e^3\*ArcTanh[Sqrt[a + c\*x^2]/Sqrt[a]])/d^4

**Rubi [A]** time = 0.549099, antiderivative size = 191, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 13, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.591$

$$\frac{\sqrt{ae^3} \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^4} - \frac{e^2\sqrt{a+cx^2}}{d^3x} + \frac{e\sqrt{a+cx^2}}{2d^2x^2} + \frac{ce \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{ad^2}} - \frac{e^2\sqrt{ae^2+cd^2} \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^4} - \frac{(a+cx^2)^{3/2}}{3adx^3}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c\*x^2]/(x^4\*(d + e\*x)), x]

[Out] (e\*Sqrt[a + c\*x^2])/(2\*d^2\*x^2) - (e^2\*Sqrt[a + c\*x^2])/(d^3\*x) - (a + c\*x^2)^(3/2)/(3\*a\*d\*x^3) - (e^2\*Sqrt[c\*d^2 + a\*e^2]\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])])/d^4 + (c\*e\*ArcTanh[Sqrt[a + c\*x^2]/Sqrt[a]])/(2\*Sqrt[a]\*d^2) + (Sqrt[a]\*e^3\*ArcTanh[Sqrt[a + c\*x^2]/Sqrt[a]])/d^4

**Rubi in Sympy [A]** time = 62.9514, size = 170, normalized size = 0.89

$$\frac{\sqrt{ae^3} \operatorname{atanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^4} + \frac{e\sqrt{a+cx^2}}{2d^2x^2} - \frac{e^2\sqrt{a+cx^2}}{d^3x} - \frac{e^2\sqrt{ae^2+cd^2} \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^4} - \frac{(a+cx^2)^{3/2}}{3adx^3} + \frac{ce \operatorname{atanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{ad^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+a)\*\*(1/2)/x\*\*4/(e\*x+d), x)

[Out] sqrt(a)\*e\*\*3\*atanh(sqrt(a + c\*x\*\*2)/sqrt(a))/d\*\*4 + e\*sqrt(a + c\*x\*\*2)/(2\*d\*\*2\*x\*\*2) - e\*\*2\*sqrt(a + c\*x\*\*2)/(d\*\*3\*x) - e\*\*2\*sqrt(a\*e\*\*2 + c\*d\*\*2)\*atanh((a\*e - c\*d\*x)/(sqrt(a + c\*x\*\*2)\*sqrt(a\*e\*\*2 + c\*d\*\*2)))/d\*\*4 - (a + c\*x\*\*2)\*\*(3/2)/(3\*a\*d\*x\*\*3) + c\*e\*atanh(sqrt(a + c\*x\*\*2)/sqrt(a))/(2\*sqrt(a)\*d\*\*2)

**Mathematica [A]** time = 0.282952, size = 216, normalized size = 1.13

$$\frac{3\sqrt{a}ex^3 \log(x)(2ae^2 + cd^2) - 6ae^2x^3\sqrt{ae^2 + cd^2} \log(d + ex) + d\sqrt{a + cx^2}(2x^2(3ae^2 + cd^2) + 2ad^2 - 3adex) - 3\sqrt{a}ex^3(2 - 3ad^2x^3)}{6ad^4x^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + c\*x^2]/(x^4\*(d + e\*x)), x]

[Out]  $-(d*\text{Sqrt}[a + c*x^2]*(2*a*d^2 - 3*a*d*e*x + 2*(c*d^2 + 3*a*e^2)*x^2) + 3*\text{Sqrt}[a]*e*(c*d^2 + 2*a*e^2)*x^3*\text{Log}[x] - 6*a*e^2*\text{Sqrt}[c*d^2 + a*e^2]*x^3*\text{Log}[d + e*x] - 3*\text{Sqrt}[a]*e*(c*d^2 + 2*a*e^2)*x^3*\text{Log}[a + \text{Sqrt}[a]*\text{Sqrt}[a + c*x^2]] + 6*a*e^2*\text{Sqrt}[c*d^2 + a*e^2]*x^3*\text{Log}[a*e - c*d*x + \text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2]])/(6*a*d^4*x^3)$

**Maple [B]** time = 0.017, size = 600, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+a)^(1/2)/x^4/(e\*x+d), x)

[Out]  $-1/3*(c*x^2+a)^{3/2}/a/d/x^3+1/d^4*e^3*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^{1/2}-1/d^3*e^2*c^{1/2}*\ln((-c*d/e+c*(x+d/e))/c^{1/2}+(x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^{1/2})-1/d^4*e^3/((a*e^2+c*d^2)/e^2)^{1/2}*\ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^{1/2}*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^{1/2})/(x+d/e))*a-1/d^2*e/((a*e^2+c*d^2)/e^2)^{1/2}*\ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^{1/2}*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^{1/2})/(x+d/e))*c-1/d^3*e^2/a/x*(c*x^2+a)^{3/2}+1/d^3*e^2*c/a*x*(c*x^2+a)^{1/2}+1/d^3*e^2*c^{1/2}*\ln(x*c^{1/2}+(c*x^2+a)^{1/2})+1/d^4*e^3*a^{1/2}*\ln((2*a+2*a^{1/2}*(c*x^2+a)^{1/2}))/x)-1/d^4*e^3*(c*x^2+a)^{1/2}+1/2*e/d^2/a/x^2*(c*x^2+a)^{3/2}+1/2*e/d^2*c/a^{1/2}*\ln((2*a+2*a^{1/2}*(c*x^2+a)^{1/2}))/x)-1/2*e/d^2*c/a*(c*x^2+a)^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}}{(ex + d)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)/((e\*x + d)\*x^4), x, algorithm="maxima")

[Out] integrate(sqrt(c\*x^2 + a)/((e\*x + d)\*x^4), x)

**Fricas [A]** time = 0.341783, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)/((e\*x + d)\*x^4), x, algorithm="fricas")



```
[Out] [1/12*(6*sqrt(c*d^2 + a*e^2)*a^(3/2)*e^2*x^3*log((2*a*c*d*e*x - a
*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 - 2*sqrt(c*d^2 + a
*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) +
3*(a*c*d^2*e + 2*a^2*e^3)*x^3*log(-((c*x^2 + 2*a)*sqrt(a) + 2*sq
rt(c*x^2 + a)*a)/x^2) + 2*(3*a*d^2*e*x - 2*a*d^3 - 2*(c*d^3 + 3*a
*d*e^2)*x^2)*sqrt(c*x^2 + a)*sqrt(a))/(a^(3/2)*d^4*x^3), 1/12*(12
*sqrt(-c*d^2 - a*e^2)*a^(3/2)*e^2*x^3*arctan((c*d*x - a*e)/(sqrt(
-c*d^2 - a*e^2)*sqrt(c*x^2 + a))) + 3*(a*c*d^2*e + 2*a^2*e^3)*x^3
*log(-((c*x^2 + 2*a)*sqrt(a) + 2*sqrt(c*x^2 + a)*a)/x^2) + 2*(3*a
*d^2*e*x - 2*a*d^3 - 2*(c*d^3 + 3*a*d*e^2)*x^2)*sqrt(c*x^2 + a)*s
qrt(a))/(a^(3/2)*d^4*x^3), 1/6*(3*sqrt(c*d^2 + a*e^2)*sqrt(-a)*a
e^2*x^3*log((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c
*e^2)*x^2 - 2*sqrt(c*d^2 + a*e^2)*(c*d*x - a*e)*sqrt(c*x^2 + a))/
(e^2*x^2 + 2*d*e*x + d^2)) + 3*(a*c*d^2*e + 2*a^2*e^3)*x^3*arctan
(sqrt(-a)/sqrt(c*x^2 + a)) + (3*a*d^2*e*x - 2*a*d^3 - 2*(c*d^3 +
3*a*d*e^2)*x^2)*sqrt(c*x^2 + a)*sqrt(-a))/(sqrt(-a)*a*d^4*x^3), 1
/6*(6*sqrt(-c*d^2 - a*e^2)*sqrt(-a)*a*e^2*x^3*arctan((c*d*x - a*e
)/(sqrt(-c*d^2 - a*e^2)*sqrt(c*x^2 + a))) + 3*(a*c*d^2*e + 2*a^2
e^3)*x^3*arctan(sqrt(-a)/sqrt(c*x^2 + a)) + (3*a*d^2*e*x - 2*a*d
^3 - 2*(c*d^3 + 3*a*d*e^2)*x^2)*sqrt(c*x^2 + a)*sqrt(-a))/(sqrt(-a
)*a*d^4*x^3)]
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+cx^2}}{x^4(dx+e)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/x**4/(e*x+d), x)
```

```
[Out] Integral(sqrt(a + c*x**2)/(x**4*(d + e*x)), x)
```

---

**GIAC/XCAS [A]** time = 0.297902, size = 417, normalized size = 2.18

$$\frac{2(cd^2e^2 + ae^4) \arctan\left(-\frac{(\sqrt{cx}-\sqrt{cx^2+a})e+\sqrt{cd}}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}d^4} - \frac{(cd^2e + 2ae^3) \arctan\left(-\frac{\sqrt{cx}-\sqrt{cx^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}d^4} - \frac{3\left(\sqrt{cx}-\sqrt{cx^2+a}\right)^5cde - 6\left(\sqrt{cx}-\sqrt{cx^2+a}\right)^4c^{\frac{3}{2}}d^2 - 6\left(\sqrt{cx}-\sqrt{cx^2+a}\right)^4a\sqrt{ce^2} - 3\left(\sqrt{cx}-\sqrt{cx^2+a}\right)a^2cde - 2a^2c^{\frac{3}{2}}}{3\left(\left(\sqrt{cx}-\sqrt{cx^2+a}\right)^2 - a\right)^3}d^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + a)/((e*x + d)*x^4), x, algorithm="giac")
```

```
[Out] 2*(c*d^2*e^2 + a*e^4)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e +
sqrt(c)*d)/sqrt(-c*d^2 - a*e^2))/sqrt(-c*d^2 - a*e^2)*d^4) - (c*
d^2*e + 2*a*e^3)*arctan(-(sqrt(c)*x - sqrt(c*x^2 + a))/sqrt(-a))/
(sqrt(-a)*d^4) - 1/3*(3*(sqrt(c)*x - sqrt(c*x^2 + a))^5*c*d*e - 6
*(sqrt(c)*x - sqrt(c*x^2 + a))^4*c^(3/2)*d^2 - 6*(sqrt(c)*x - sq
rt(c*x^2 + a))^4*a*sqrt(c)*e^2 - 3*(sqrt(c)*x - sqrt(c*x^2 + a))*a
^2*c*d*e - 2*a^2*c^(3/2)*d^2 + 12*(sqrt(c)*x - sqrt(c*x^2 + a))^2
*a^2*sqrt(c)*e^2 - 6*a^3*sqrt(c)*e^2)/(((sqrt(c)*x - sqrt(c*x^2 +
a))^2 - a)^3*d^3)
```

### 3.325 $\int \frac{\sqrt{a+cx^2}}{x^5(d+ex)} dx$

**Optimal.** Leaf size=274

$$\frac{c^2 \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{8a^{3/2}d} - \frac{\sqrt{ae^4} \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^5} + \frac{e^3\sqrt{a+cx^2}}{d^4x} - \frac{e^2\sqrt{a+cx^2}}{2d^3x^2} - \frac{ce^2 \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{ad^3}}$$

$$+ \frac{e(a+cx^2)^{3/2}}{3ad^2x^3} + \frac{e^3\sqrt{ae^2+cd^2} \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^5} - \frac{c\sqrt{a+cx^2}}{8adx^2} - \frac{\sqrt{a+cx^2}}{4dx^4}$$

[Out] -Sqrt[a + c\*x^2]/(4\*d\*x^4) - (c\*Sqrt[a + c\*x^2])/(8\*a\*d\*x^2) - (e^2\*Sqrt[a + c\*x^2])/(2\*d^3\*x^2) + (e^3\*Sqrt[a + c\*x^2])/(d^4\*x) + (e\*(a + c\*x^2)^(3/2))/(3\*a\*d^2\*x^3) + (e^3\*Sqrt[c\*d^2 + a\*e^2]\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])])/d^5 + (c^2\*ArcTanh[Sqrt[a + c\*x^2]/Sqrt[a]])/(8\*a^(3/2)\*d) - (c\*e^2\*ArcTanh[Sqrt[a + c\*x^2]/Sqrt[a]])/(2\*Sqrt[a]\*d^3) - (Sqrt[a]\*e^4\*ArcTanh[Sqrt[a + c\*x^2]/Sqrt[a]])/d^5

**Rubi [A]** time = 0.701576, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 14, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$

$$\frac{c^2 \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{8a^{3/2}d} - \frac{\sqrt{ae^4} \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^5} + \frac{e^3\sqrt{a+cx^2}}{d^4x} - \frac{e^2\sqrt{a+cx^2}}{2d^3x^2} - \frac{ce^2 \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{ad^3}}$$

$$+ \frac{e(a+cx^2)^{3/2}}{3ad^2x^3} + \frac{e^3\sqrt{ae^2+cd^2} \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^5} - \frac{c\sqrt{a+cx^2}}{8adx^2} - \frac{\sqrt{a+cx^2}}{4dx^4}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c\*x^2]/(x^5\*(d + e\*x)), x]

[Out] -Sqrt[a + c\*x^2]/(4\*d\*x^4) - (c\*Sqrt[a + c\*x^2])/(8\*a\*d\*x^2) - (e^2\*Sqrt[a + c\*x^2])/(2\*d^3\*x^2) + (e^3\*Sqrt[a + c\*x^2])/(d^4\*x) + (e\*(a + c\*x^2)^(3/2))/(3\*a\*d^2\*x^3) + (e^3\*Sqrt[c\*d^2 + a\*e^2]\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])])/d^5 + (c^2\*ArcTanh[Sqrt[a + c\*x^2]/Sqrt[a]])/(8\*a^(3/2)\*d) - (c\*e^2\*ArcTanh[Sqrt[a + c\*x^2]/Sqrt[a]])/(2\*Sqrt[a]\*d^3) - (Sqrt[a]\*e^4\*ArcTanh[Sqrt[a + c\*x^2]/Sqrt[a]])/d^5

**Rubi in Sympy [A]** time = 73.7002, size = 243, normalized size = 0.89

$$-\frac{\sqrt{ae^4} \operatorname{atanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{d^5} - \frac{\sqrt{a+cx^2}}{4dx^4} - \frac{e^2\sqrt{a+cx^2}}{2d^3x^2} + \frac{e^3\sqrt{a+cx^2}}{d^4x}$$

$$+ \frac{e^3\sqrt{ae^2+cd^2} \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^5} - \frac{c\sqrt{a+cx^2}}{8adx^2}$$

$$+ \frac{e(a+cx^2)^{\frac{3}{2}}}{3ad^2x^3} - \frac{ce^2 \operatorname{atanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2\sqrt{ad^3}} + \frac{c^2 \operatorname{atanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{8a^{\frac{3}{2}}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+a)\*\*(1/2)/x\*\*5/(e\*x+d), x)

[Out] -sqrt(a)\*e\*\*4\*atanh(sqrt(a + c\*x\*\*2)/sqrt(a))/d\*\*5 - sqrt(a + c\*x\*\*2)/(4\*d\*x\*\*4) - e\*\*2\*sqrt(a + c\*x\*\*2)/(2\*d\*\*3\*x\*\*2) + e\*\*3\*sqrt(a + c\*x\*\*2)/(d\*\*4\*x) + e\*\*3\*sqrt(a\*e\*\*2 + c\*d\*\*2)\*atanh((a\*e - c

$$\frac{d^5 x}{\sqrt{a + cx^2} \sqrt{ae^2 + cd^2}} - \frac{c \sqrt{a + cx^2}}{(8a^2 d^2 x^2) + e^2 (a + cx^2)^{3/2}} - \frac{c^2 \sqrt{a + cx^2} \operatorname{atanh}\left(\frac{\sqrt{a + cx^2}}{\sqrt{a}}\right)}{(2 \sqrt{a})^3 d^3} + \frac{c^2 \operatorname{atanh}\left(\frac{\sqrt{a + cx^2}}{\sqrt{a}}\right)}{(8a^2)^{3/2} d}$$

**Mathematica [A]** time = 0.644674, size = 248, normalized size = 0.91

$$\frac{3(-8a^2e^4 - 4acd^2e^2 + c^2d^4) \log\left(\sqrt{a}\sqrt{a+cx^2}+a\right)}{a^{3/2}} + \frac{3\log(x)(8a^2e^4 + 4acd^2e^2 - c^2d^4)}{a^{3/2}} + 24e^3\sqrt{ae^2 + cd^2} \log\left(\sqrt{a + cx^2}\sqrt{ae^2 + cd^2} + ae - cdx\right) - 24d^5$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + c\*x^2]/(x^5\*(d + e\*x)), x]

[Out]  $\left(\frac{d \sqrt{a + cx^2} (c^2 d^2 x^2 (-3d + 8ex) + a(-6d^3 + 8d^2 ex - 12d^2 e^2 x^2 + 24e^3 x^3))}{a^2 x^4} + \frac{3(-c^2 d^4 + 4a^2 c^2 d^2 e^2 + 8a^2 e^4) \operatorname{Log}[x]}{a^{3/2}} - \frac{24e^3 \sqrt{c^2 d^2 + a^2 e^2} \operatorname{Log}[d + ex]}{a^{3/2}} + \frac{3(c^2 d^4 - 4a^2 c^2 d^2 e^2 - 8a^2 e^4) \operatorname{Log}[a + \sqrt{a} \sqrt{a + cx^2}]}{a^{3/2}} + \frac{24e^3 \sqrt{c^2 d^2 + a^2 e^2} \operatorname{Log}[a^2 e - c^2 d^2 x + \sqrt{c^2 d^2 + a^2 e^2} \sqrt{a + cx^2}]}{24d^5}\right)$

**Maple [B]** time = 0.02, size = 703, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+a)^(1/2)/x^5/(e\*x+d), x)

[Out]  $-1/4/d/a/x^4*(c*x^2+a)^{3/2} + 1/8/d*c/a^2/x^2*(c*x^2+a)^{3/2} + 1/8/d*c^2/a^{3/2}*\ln\left(\frac{(2*a+2*a^{1/2}*(c*x^2+a)^{1/2})}{x}\right) - 1/8/d*c^2/a^2*(c*x^2+a)^{1/2} - 1/d^5*e^4*a^{1/2}*\ln\left(\frac{(2*a+2*a^{1/2}*(c*x^2+a)^{1/2})}{x}\right) + 1/d^5*e^4*(c*x^2+a)^{1/2} - 1/2/d^3*e^2/a/x^2*(c*x^2+a)^{3/2} - 1/2/d^3*e^2*c/a^{1/2}*\ln\left(\frac{(2*a+2*a^{1/2}*(c*x^2+a)^{1/2})}{x}\right) + 1/2/d^3*e^2*c/a*(c*x^2+a)^{1/2} - 1/d^5*e^4*((x+d/e)^2*c - 2*c*d/e*(x+d/e) + (a^2*e^2 + c*d^2)/e^2)^{1/2} + 1/d^4*e^3*c^{1/2}*\ln\left(\frac{(-c*d/e + c*(x+d/e))}{c^{1/2} + ((x+d/e)^2*c - 2*c*d/e*(x+d/e) + (a^2*e^2 + c*d^2)/e^2)^{1/2}}\right) + 1/d^5*e^4/((a^2*e^2 + c*d^2)/e^2)^{1/2}*\ln\left(\frac{(2*(a^2*e^2 + c*d^2)/e^2 - 2*c*d/e*(x+d/e) + 2*((a^2*e^2 + c*d^2)/e^2)^{1/2}*((x+d/e)^2*c - 2*c*d/e*(x+d/e) + (a^2*e^2 + c*d^2)/e^2)^{1/2}}{(x+d/e)}\right) + 1/d^3*e^2/((a^2*e^2 + c*d^2)/e^2)^{1/2}*\ln\left(\frac{(2*(a^2*e^2 + c*d^2)/e^2 - 2*c*d/e*(x+d/e) + 2*((a^2*e^2 + c*d^2)/e^2)^{1/2}*((x+d/e)^2*c - 2*c*d/e*(x+d/e) + (a^2*e^2 + c*d^2)/e^2)^{1/2}}{(x+d/e)}\right) + c + 1/d^4*e^3/a/x*(c*x^2+a)^{3/2} - 1/d^4*e^3*c/a*x*(c*x^2+a)^{1/2} - 1/d^4*e^3*c^{1/2}*\ln(x*c^{1/2} + (c*x^2+a)^{1/2}) + 1/3*e*(c*x^2+a)^{3/2}/a/d^2/x^3$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}}{(ex + d)x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)/((e\*x + d)\*x^5), x, algorithm="maxima")

[Out] integrate(sqrt(c\*x^2 + a)/((e\*x + d)\*x^5), x)

---

**Fricas [A]** time = 0.419208, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)/((e\*x + d)\*x^5), x, algorithm="fricas")

[Out] [1/48\*(24\*sqrt(c\*d^2 + a\*e^2)\*a^(3/2)\*e^3\*x^4\*log((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2 + 2\*sqrt(c\*d^2 + a\*e^2)\*(c\*d\*x - a\*e)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)) - 3\*(c^2\*d^4 - 4\*a\*c\*d^2\*e^2 - 8\*a^2\*e^4)\*x^4\*log(-((c\*x^2 + 2\*a)\*sqrt(a) - 2\*sqrt(c\*x^2 + a)\*a)/x^2) + 2\*(8\*a\*d^3\*e\*x - 6\*a\*d^4 + 8\*(c\*d^3\*e + 3\*a\*d^2\*e^3)\*x^3 - 3\*(c\*d^4 + 4\*a\*d^2\*e^2)\*x^2)\*sqrt(c\*x^2 + a)\*sqrt(a)/(a^(3/2)\*d^5\*x^4), -1/48\*(48\*sqrt(-c\*d^2 - a\*e^2)\*a^(3/2)\*e^3\*x^4\*arctan((c\*d\*x - a\*e)/(sqrt(-c\*d^2 - a\*e^2)\*sqrt(c\*x^2 + a))) + 3\*(c^2\*d^4 - 4\*a\*c\*d^2\*e^2 - 8\*a^2\*e^4)\*x^4\*log(-((c\*x^2 + 2\*a)\*sqrt(a) - 2\*sqrt(c\*x^2 + a)\*a)/x^2) - 2\*(8\*a\*d^3\*e\*x - 6\*a\*d^4 + 8\*(c\*d^3\*e + 3\*a\*d^2\*e^3)\*x^3 - 3\*(c\*d^4 + 4\*a\*d^2\*e^2)\*x^2)\*sqrt(c\*x^2 + a)\*sqrt(a)/(a^(3/2)\*d^5\*x^4), 1/24\*(12\*sqrt(c\*d^2 + a\*e^2)\*sqrt(-a)\*a\*e^3\*x^4\*log((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2 + 2\*sqrt(c\*d^2 + a\*e^2)\*(c\*d\*x - a\*e)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)) + 3\*(c^2\*d^4 - 4\*a\*c\*d^2\*e^2 - 8\*a^2\*e^4)\*x^4\*arctan(sqrt(-a)/sqrt(c\*x^2 + a)) + (8\*a\*d^3\*e\*x - 6\*a\*d^4 + 8\*(c\*d^3\*e + 3\*a\*d^2\*e^3)\*x^3 - 3\*(c\*d^4 + 4\*a\*d^2\*e^2)\*x^2)\*sqrt(c\*x^2 + a)\*sqrt(-a)/(sqrt(-a)\*a\*d^5\*x^4), -1/24\*(24\*sqrt(-c\*d^2 - a\*e^2)\*sqrt(-a)\*a\*e^3\*x^4\*arctan((c\*d\*x - a\*e)/(sqrt(-c\*d^2 - a\*e^2)\*sqrt(c\*x^2 + a))) - 3\*(c^2\*d^4 - 4\*a\*c\*d^2\*e^2 - 8\*a^2\*e^4)\*x^4\*arctan(sqrt(-a)/sqrt(c\*x^2 + a)) - (8\*a\*d^3\*e\*x - 6\*a\*d^4 + 8\*(c\*d^3\*e + 3\*a\*d^2\*e^3)\*x^3 - 3\*(c\*d^4 + 4\*a\*d^2\*e^2)\*x^2)\*sqrt(c\*x^2 + a)\*sqrt(-a)/(sqrt(-a)\*a\*d^5\*x^4)]

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2}}{x^5(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+a)\*\*(1/2)/x\*\*5/(e\*x+d), x)

[Out] Integral(sqrt(a + c\*x\*\*2)/(x\*\*5\*(d + e\*x)), x)

---

**GIAC/XCAS [A]** time = 0.288165, size = 805, normalized size = 2.94

$$\frac{2(c^2e^3 + ae^5) \arctan\left(-\frac{(\sqrt{cx - \sqrt{cx^2 + a}})e + \sqrt{cd}}{\sqrt{-cd^2 - ae^2}}\right)}{\sqrt{-cd^2 - ae^2}d^5} - \frac{(c^2d^4 - 4acd^2e^2 - 8a^2e^4) \arctan\left(-\frac{\sqrt{cx - \sqrt{cx^2 + a}}}{\sqrt{-a}}\right)}{4\sqrt{-aad^5}} + \frac{3(\sqrt{cx - \sqrt{cx^2 + a}})^7 c^2d^3 - 24(\sqrt{cx - \sqrt{cx^2 + a}})^6 ac^{\frac{3}{2}}d^2e + 21(\sqrt{cx - \sqrt{cx^2 + a}})^5 ac^2d^3 + 12(\sqrt{cx - \sqrt{cx^2 + a}})^7 acde^2 + \dots}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)/((e\*x + d)\*x^5), x, algorithm="giac")

```
[Out] -2*(c*d^2*e^3 + a*e^5)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e +
sqrt(c)*d)/sqrt(-c*d^2 - a*e^2))/sqrt(-c*d^2 - a*e^2)*d^5) - 1/
4*(c^2*d^4 - 4*a*c*d^2*e^2 - 8*a^2*e^4)*arctan(-(sqrt(c)*x - sqrt
(c*x^2 + a))/sqrt(-a))/sqrt(-a)*a*d^5) + 1/12*(3*(sqrt(c)*x - sq
rt(c*x^2 + a))^7*c^2*d^3 - 24*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a*c
^(3/2)*d^2*e + 21*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a*c^2*d^3 + 12*
(sqrt(c)*x - sqrt(c*x^2 + a))^7*a*c*d*e^2 + 24*(sqrt(c)*x - sqrt(
c*x^2 + a))^4*a^2*c^(3/2)*d^2*e + 21*(sqrt(c)*x - sqrt(c*x^2 + a)
)^3*a^2*c^2*d^3 - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^5*a^2*c*d*e^2
- 24*(sqrt(c)*x - sqrt(c*x^2 + a))^6*a^2*sqrt(c)*e^3 - 8*(sqrt(c)
*x - sqrt(c*x^2 + a))^2*a^3*c^(3/2)*d^2*e + 3*(sqrt(c)*x - sqrt(c
*x^2 + a))*a^3*c^2*d^3 - 12*(sqrt(c)*x - sqrt(c*x^2 + a))^3*a^3*c
*d*e^2 + 72*(sqrt(c)*x - sqrt(c*x^2 + a))^4*a^3*sqrt(c)*e^3 + 8*a
^4*c^(3/2)*d^2*e + 12*(sqrt(c)*x - sqrt(c*x^2 + a))*a^4*c*d*e^2 -
72*(sqrt(c)*x - sqrt(c*x^2 + a))^2*a^4*sqrt(c)*e^3 + 24*a^5*sqrt
(c)*e^3)/(((sqrt(c)*x - sqrt(c*x^2 + a))^2 - a)^4*a*d^4)
```

$$3.326 \quad \int \frac{x^4}{(d+ex)\sqrt{a+cx^2}} dx$$

Optimal. Leaf size=195

$$\frac{d(2cd^2 - ae^2) \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^4} + \frac{\sqrt{a+cx^2}(11cd^2 - 4ae^2)}{6c^2e^3} - \frac{d^4 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4\sqrt{ae^2+cd^2}} - \frac{7d\sqrt{a+cx^2}(d+ex)}{6ce^3} + \frac{\sqrt{a+cx^2}(d+ex)^2}{3ce^3}$$

[Out]  $((11*c*d^2 - 4*a*e^2)*\text{Sqrt}[a + c*x^2])/(6*c^2*e^3) - (7*d*(d + e*x)*\text{Sqrt}[a + c*x^2])/(6*c*e^3) + ((d + e*x)^2*\text{Sqrt}[a + c*x^2])/(3*c*e^3) - (d*(2*c*d^2 - a*e^2)*\text{ArcTanh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a + c*x^2]])/(2*c^{(3/2)}*e^4) - (d^4*\text{ArcTanh}[(a*e - c*d*x)/(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2])])/(e^4*\text{Sqrt}[c*d^2 + a*e^2])$

Rubi [A] time = 0.833369, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$\frac{d(2cd^2 - ae^2) \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^4} + \frac{\sqrt{a+cx^2}(11cd^2 - 4ae^2)}{6c^2e^3} - \frac{d^4 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4\sqrt{ae^2+cd^2}} - \frac{7d\sqrt{a+cx^2}(d+ex)}{6ce^3} + \frac{\sqrt{a+cx^2}(d+ex)^2}{3ce^3}$$

Antiderivative was successfully verified.

[In] Int[x^4/((d + e\*x)\*Sqrt[a + c\*x^2]), x]

[Out]  $((11*c*d^2 - 4*a*e^2)*\text{Sqrt}[a + c*x^2])/(6*c^2*e^3) - (7*d*(d + e*x)*\text{Sqrt}[a + c*x^2])/(6*c*e^3) + ((d + e*x)^2*\text{Sqrt}[a + c*x^2])/(3*c*e^3) - (d*(2*c*d^2 - a*e^2)*\text{ArcTanh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a + c*x^2]])/(2*c^{(3/2)}*e^4) - (d^4*\text{ArcTanh}[(a*e - c*d*x)/(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2])])/(e^4*\text{Sqrt}[c*d^2 + a*e^2])$

Rubi in Sympy [A] time = 38.4035, size = 189, normalized size = 0.97

$$\frac{a\sqrt{a+cx^2}}{c^2e} + \frac{ad \operatorname{atanh}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^2} - \frac{d^4 \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4\sqrt{ae^2+cd^2}} + \frac{d^2\sqrt{a+cx^2}}{ce^3} - \frac{dx\sqrt{a+cx^2}}{2ce^2} + \frac{(a+cx^2)^{3/2}}{3c^2e} - \frac{d^3 \operatorname{atanh}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{\sqrt{ce^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(e\*x+d)/(c\*x\*\*2+a)\*\*(1/2), x)

[Out]  $-a*\text{sqrt}(a + c*x**2)/(c**2*e) + a*d*\text{atanh}(\text{sqrt}(c)*x/\text{sqrt}(a + c*x**2))/(2*c**{(3/2)}*e**2) - d**4*\text{atanh}((a*e - c*d*x)/(\text{sqrt}(a + c*x**2)*\text{sqrt}(a*e**2 + c*d**2)))/(e**4*\text{sqrt}(a*e**2 + c*d**2)) + d**2*\text{sqrt}(a + c*x**2)/(c*e**3) - d*x*\text{sqrt}(a + c*x**2)/(2*c*e**2) + (a + c*x**2)**{(3/2)}/(3*c**2*e) - d**3*\text{atanh}(\text{sqrt}(c)*x/\text{sqrt}(a + c*x**2))/(\text{sqrt}(c)*e**4)$

**Mathematica [A]** time = 0.798338, size = 178, normalized size = 0.91

$$\frac{-\frac{3d(2cd^2-ae^2)\log(\sqrt{c}\sqrt{a+cx^2+cx})}{c^{3/2}} + \frac{e\sqrt{a+cx^2}(c(6d^2-3dex+2e^2x^2)-4ae^2)}{c^2} - \frac{6d^4\log(\sqrt{a+cx^2}\sqrt{ae^2+cd^2+ae-cdx})}{\sqrt{ae^2+cd^2}} + \frac{6d^4\log(d+ex)}{\sqrt{ae^2+cd^2}}}{6e^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/((d + e\*x)\*Sqrt[a + c\*x^2]),x]

[Out] ((e\*Sqrt[a + c\*x^2]\*(-4\*a\*e^2 + c\*(6\*d^2 - 3\*d\*e\*x + 2\*e^2\*x^2)))/c^2 + (6\*d^4\*Log[d + e\*x])/Sqrt[c\*d^2 + a\*e^2] - (3\*d\*(2\*c\*d^2 - a\*e^2)\*Log[c\*x + Sqrt[c]\*Sqrt[a + c\*x^2]])/c^(3/2) - (6\*d^4\*Log[a\*e - c\*d\*x + Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])/Sqrt[c\*d^2 + a\*e^2])/ (6\*e^4)

**Maple [A]** time = 0.021, size = 260, normalized size = 1.3

$$\begin{aligned} & \frac{x^2}{3ce}\sqrt{cx^2+a} - \frac{2a}{3ec^2}\sqrt{cx^2+a} + \frac{d^2}{e^3c}\sqrt{cx^2+a} \\ & - \frac{d^4}{e^5}\ln\left(1\left(2\frac{ae^2+cd^2}{e^2} - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + 2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + \frac{ae^2+cd^2}{e^2}}\right)\left(x+\frac{d}{e}\right)^{-1}\right)\frac{1}{\sqrt{\frac{ae^2+cd^2}{e^2}}} \\ & - \frac{d^3}{e^4}\ln\left(x\sqrt{c} + \sqrt{cx^2+a}\right)\frac{1}{\sqrt{c}} - \frac{dx}{2e^2c}\sqrt{cx^2+a} + \frac{ad}{2e^2}\ln\left(x\sqrt{c} + \sqrt{cx^2+a}\right)c^{-\frac{3}{2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(e\*x+d)/(c\*x^2+a)^(1/2),x)

[Out] 1/3/e\*x^2/c\*(c\*x^2+a)^(1/2)-2/3/e\*a/c^2\*(c\*x^2+a)^(1/2)+d^2/e^3/c\*(c\*x^2+a)^(1/2)-d^4/e^5/((a\*e^2+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2+c\*d^2)/e^2-2\*c\*d/e\*(x+d/e)+2\*((a\*e^2+c\*d^2)/e^2)^(1/2)\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2))/(x+d/e))-d^3/e^4\*ln(x\*c^(1/2)+(c\*x^2+a)^(1/2))/c^(1/2)-1/2\*d/e^2\*x/c\*(c\*x^2+a)^(1/2)+1/2\*d/e^2\*a/c^(3/2)\*ln(x\*c^(1/2)+(c\*x^2+a)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(sqrt(c\*x^2 + a)\*(e\*x + d)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.92123, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(sqrt(c\*x^2 + a)\*(e\*x + d)),x, algorithm="fricas")

```
[Out] [1/12*(6*c^(5/2)*d^4*log(((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2)*sqrt(c*d^2 + a*e^2) + 2*(a*c*d^2*e + a^2*e^3 - (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*(2*c*e^3*x^2 - 3*c*d*e^2*x + 6*c*d^2*e - 4*a*e^3)*sqrt(c*d^2 + a*e^2)*sqrt(c*x^2 + a)*sqrt(c) - 3*(2*c^2*d^3 - a*c*d*e^2)*sqrt(c*d^2 + a*e^2)*log(-2*sqrt(c*x^2 + a)*c*x - (2*c*x^2 + a)*sqrt(c)))/(sqrt(c*d^2 + a*e^2)*c^(5/2)*e^4), 1/12*(12*c^(5/2)*d^4*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)/((c*d^2 + a*e^2)*sqrt(c*x^2 + a))) + 2*(2*c*e^3*x^2 - 3*c*d*e^2*x + 6*c*d^2*e - 4*a*e^3)*sqrt(-c*d^2 - a*e^2)*sqrt(c*x^2 + a)*sqrt(c) - 3*(2*c^2*d^3 - a*c*d*e^2)*sqrt(-c*d^2 - a*e^2)*log(-2*sqrt(c*x^2 + a)*c*x - (2*c*x^2 + a)*sqrt(c)))/(sqrt(-c*d^2 - a*e^2)*c^(5/2)*e^4), 1/6*(3*sqrt(-c)*c^2*d^4*log(((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2)*sqrt(c*d^2 + a*e^2) + 2*(a*c*d^2*e + a^2*e^3 - (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) + (2*c*e^3*x^2 - 3*c*d*e^2*x + 6*c*d^2*e - 4*a*e^3)*sqrt(c*d^2 + a*e^2)*sqrt(c*x^2 + a)*sqrt(-c) - 3*(2*c^2*d^3 - a*c*d*e^2)*sqrt(c*d^2 + a*e^2)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)))/(sqrt(c*d^2 + a*e^2)*sqrt(-c)*c^2*e^4), 1/6*(6*sqrt(-c)*c^2*d^4*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)/((c*d^2 + a*e^2)*sqrt(c*x^2 + a))) + (2*c*e^3*x^2 - 3*c*d*e^2*x + 6*c*d^2*e - 4*a*e^3)*sqrt(-c*d^2 - a*e^2)*sqrt(c*x^2 + a)*sqrt(-c) - 3*(2*c^2*d^3 - a*c*d*e^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)))/(sqrt(-c*d^2 - a*e^2)*sqrt(-c)*c^2*e^4)]
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{a+cx^2}(d+ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**4/(sqrt(a + c*x**2)*(d + e*x)), x)
```

**GIAC/XCAS [A]** time = 0.278003, size = 220, normalized size = 1.13

$$\frac{2d^4 \arctan\left(-\frac{(\sqrt{cx}-\sqrt{cx^2+a})e+\sqrt{cd}}{\sqrt{-cd^2-ae^2}}\right)e^{(-4)}}{\sqrt{-cd^2-ae^2}} + \frac{1}{6}\sqrt{cx^2+a}\left(x\left(\frac{2xe^{(-1)}}{c}-\frac{3de^{(-2)}}{c}\right)+\frac{2(3c^2d^2e^7-2ace^9)e^{(-10)}}{c^3}\right) + \frac{(2c^{\frac{3}{2}}d^3-a\sqrt{cde^2})e^{(-4)}\ln\left(\left|-\sqrt{cx}+\sqrt{cx^2+a}\right|\right)}{2c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(sqrt(c*x^2 + a)*(e*x + d)),x, algorithm="giac")
```

```
[Out] 2*d^4*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 - a*e^2))*e^(-4)/sqrt(-c*d^2 - a*e^2) + 1/6*sqrt(c*x^2 + a)*(x*(2*x*e^(-1)/c - 3*d*e^(-2)/c) + 2*(3*c^2*d^2*e^7 - 2*a*c*e^9)*e^(-10)/c^3) + 1/2*(2*c^(3/2)*d^3 - a*sqrt(c)*d*e^2)*e^(-4)*ln(abs(-sqrt(c)*x + sqrt(c*x^2 + a)))/c^2
```



$$3.327 \quad \int \frac{x^3}{(d+ex)\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=152

$$\frac{(2cd^2 - ae^2) \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^3} + \frac{d^3 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^3\sqrt{ae^2+cd^2}} - \frac{3d\sqrt{a+cx^2}}{2ce^2} + \frac{\sqrt{a+cx^2}(d+ex)}{2ce^2}$$

[Out]  $(-3*d*\text{Sqrt}[a + c*x^2])/(2*c*e^2) + ((d + e*x)*\text{Sqrt}[a + c*x^2])/(2*c*e^2) + ((2*c*d^2 - a*e^2)*\text{ArcTanh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a + c*x^2]])/(2*c^{3/2}*e^3) + (d^3*\text{ArcTanh}[(a*e - c*d*x)/(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2])])/(e^3*\text{Sqrt}[c*d^2 + a*e^2])$

**Rubi [A]** time = 0.503231, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$\frac{(2cd^2 - ae^2) \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^3} + \frac{d^3 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^3\sqrt{ae^2+cd^2}} - \frac{3d\sqrt{a+cx^2}}{2ce^2} + \frac{\sqrt{a+cx^2}(d+ex)}{2ce^2}$$

Antiderivative was successfully verified.

[In] Int[x^3/((d + e\*x)\*Sqrt[a + c\*x^2]),x]

[Out]  $(-3*d*\text{Sqrt}[a + c*x^2])/(2*c*e^2) + ((d + e*x)*\text{Sqrt}[a + c*x^2])/(2*c*e^2) + ((2*c*d^2 - a*e^2)*\text{ArcTanh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a + c*x^2]])/(2*c^{3/2}*e^3) + (d^3*\text{ArcTanh}[(a*e - c*d*x)/(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2])])/(e^3*\text{Sqrt}[c*d^2 + a*e^2])$

**Rubi in Sympy [A]** time = 28.2701, size = 146, normalized size = 0.96

$$-\frac{a \operatorname{atanh}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{2c^{\frac{3}{2}}e} + \frac{d^3 \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^3\sqrt{ae^2+cd^2}} - \frac{d\sqrt{a+cx^2}}{ce^2} + \frac{x\sqrt{a+cx^2}}{2ce} + \frac{d^2 \operatorname{atanh}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{\sqrt{ce^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3/(e\*x+d)/(c\*x\*\*2+a)\*\*(1/2),x)

[Out]  $-a*\operatorname{atanh}(\text{sqrt}(c)*x/\text{sqrt}(a + c*x**2))/(2*c**(3/2)*e) + d**3*\operatorname{atanh}((a*e - c*d*x)/(\text{sqrt}(a + c*x**2)*\text{sqrt}(a*e**2 + c*d**2)))/(e**3*\text{sqrt}(a*e**2 + c*d**2)) - d*\text{sqrt}(a + c*x**2)/(c*e**2) + x*\text{sqrt}(a + c*x**2)/(2*c*e) + d**2*\operatorname{atanh}(\text{sqrt}(c)*x/\text{sqrt}(a + c*x**2))/(\text{sqrt}(c)*e**3)$

**Mathematica [A]** time = 0.402967, size = 155, normalized size = 1.02

$$\frac{(2cd^2 - ae^2) \log(\sqrt{c}\sqrt{a+cx^2}+cx)}{c^{3/2}} + \frac{2d^3 \log(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx)}{\sqrt{ae^2+cd^2}} - \frac{2d^3 \log(d+ex)}{\sqrt{ae^2+cd^2}} + \frac{e\sqrt{a+cx^2}(ex-2d)}{c}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((d + e\*x)\*Sqrt[a + c\*x^2]),x]

[Out]  $((e^{(-2*d + e*x)}*\text{Sqrt}[a + c*x^2])/c - (2*d^3*\text{Log}[d + e*x])/(\text{Sqrt}[c*d^2 + a*e^2] + ((2*c*d^2 - a*e^2)*\text{Log}[c*x + \text{Sqrt}[c]*\text{Sqrt}[a + c*x^2]))/c^{(3/2)} + (2*d^3*\text{Log}[a*e - c*d*x + \text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2]))/(\text{Sqrt}[c*d^2 + a*e^2]))/(2*e^3)$

**Maple [A]** time = 0.013, size = 217, normalized size = 1.4

$$\frac{d^2}{e^3} \ln\left(x\sqrt{c} + \sqrt{cx^2 + a}\right) \frac{1}{\sqrt{c}} + \frac{x}{2ce} \sqrt{cx^2 + a} - \frac{a}{2e} \ln\left(x\sqrt{c} + \sqrt{cx^2 + a}\right) c^{-\frac{3}{2}} - \frac{d}{ce^2} \sqrt{cx^2 + a} + \frac{d^3}{e^4} \ln\left(1 \left(2 \frac{ae^2 + cd^2}{e^2} - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + 2 \sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}}\right) \left(x + \frac{d}{e}\right)^{-1}\right) \frac{1}{\sqrt{\frac{ae^2 + cd^2}{e^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(e*x+d)/(c*x^2+a)^(1/2), x)`

[Out]  $d^2/e^3 * \ln(x*c^{(1/2)} + (c*x^2+a)^{(1/2)})/c^{(1/2)} + 1/2/e*x/c*(c*x^2+a)^{(1/2)} - 1/2/e*a/c^{(3/2)} * \ln(x*c^{(1/2)} + (c*x^2+a)^{(1/2)}) - d*(c*x^2+a)^{(1/2)}/c/e^2 + d^3/e^4 / ((a*e^2 + c*d^2)/e^2)^{(1/2)} * \ln((2*(a*e^2 + c*d^2)/e^2 - 2*c*d/e*(x+d/e) + 2*((a*e^2 + c*d^2)/e^2)^{(1/2)} * ((x+d/e)^2*c - 2*c*d/e*(x+d/e) + (a*e^2 + c*d^2)/e^2)^{(1/2)}) / (x+d/e)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(sqrt(c*x^2 + a)*(e*x + d)), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.9921, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(sqrt(c*x^2 + a)*(e*x + d)), x, algorithm="fricas")`

[Out]  $[1/4*(2*c^{(3/2)}*d^3*\log(((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2)*\text{sqrt}(c*d^2 + a*e^2) - 2*(a*c*d^2*e + a^2*e^3 - (c^2*d^3 + a*c*d*e^2)*x)*\text{sqrt}(c*x^2 + a)))/(e^2*x^2 + 2*d*e*x + d^2)) + 2*\text{sqrt}(c*d^2 + a*e^2)*(e^2*x - 2*d*e)*\text{sqrt}(c*x^2 + a)*\text{sqrt}(c) - (2*c*d^2 - a*e^2)*\text{sqrt}(c*d^2 + a*e^2)*\log(2*\text{sqrt}(c*x^2 + a)*c*x - (2*c*x^2 + a)*\text{sqrt}(c)))/(\text{sqrt}(c*d^2 + a*e^2)*c^{(3/2)}*e^3), -1/4*(4*c^{(3/2)}*d^3*\arctan(\text{sqrt}(-c*d^2 - a*e^2)*(c*d*x - a*e)/((c*d^2 + a*e^2)*\text{sqrt}(c*x^2 + a)))) - 2*\text{sqrt}(-c*d^2 - a*e^2)*(e^2*x - 2*d*e)*\text{sqrt}(c*x^2 + a)*\text{sqrt}(c) + (2*c*d^2 - a*e^2)*\text{sqrt}(-c*d^2 - a*e^2)*\log(2*\text{sqrt}(c*x^2 + a)*c*x - (2*c*x^2 + a)*\text{sqrt}(c)))/(\text{sqrt}(-c*d^2 - a*e^2)*c^{(3/2)}*e^3), 1/2*(\text{sqrt}(-c)*c*d^3*\log(((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2)*\text{sqrt}(c*d^2 + a*e^2) - 2*(a*c*d^2*e + a^2*e^3 - (c^2*d^3 + a*c*d*e^2)*x)*\text{sqrt}(c*x^2 + a)))/(e^2*x^2 + 2*d*e*x + d^2)) + \text{sqrt}(c*d^2 + a*e^2)*(e^2*x - 2*d*e)*\text{sqrt}(c*x^2 + a)*\text{sqrt}(-c) + (2*c*d^2 - a*e^2)*\text{sqrt}(c*d^2 + a*e^2)*\arctan(\text{sqrt}(-c)*x/\text{sqrt}(c*x^2 + a)))/(\text{sqrt}(c*d$

$$\begin{aligned} &^2 + a^*e^2)*\text{sqrt}(-c)*c^*e^3), -1/2*(2*\text{sqrt}(-c)*c^*d^3*\text{arctan}(\text{sqrt}(- \\ &c^*d^2 - a^*e^2)*(c^*d*x - a^*e)/((c^*d^2 + a^*e^2)*\text{sqrt}(c^*x^2 + a))) - \\ &\text{sqrt}(-c^*d^2 - a^*e^2)*(e^2*x - 2*d^*e)*\text{sqrt}(c^*x^2 + a)*\text{sqrt}(-c) - \\ &(2*c^*d^2 - a^*e^2)*\text{sqrt}(-c^*d^2 - a^*e^2)*\text{arctan}(\text{sqrt}(-c)*x/\text{sqrt}(c^*x \\ &^2 + a)))/(\text{sqrt}(-c^*d^2 - a^*e^2)*\text{sqrt}(-c)*c^*e^3)] \end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a + cx^2}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(e\*x+d)/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] Integral(x\*\*3/(sqrt(a + c\*x\*\*2)\*(d + e\*x)), x)

**GIAC/XCAS [A]** time = 0.281944, size = 174, normalized size = 1.14

$$\begin{aligned} &2 d^3 \arctan\left(-\frac{(\sqrt{cx}-\sqrt{cx^2+a})e+\sqrt{cd}}{\sqrt{-cd^2-ae^2}}\right) e^{(-3)} \\ &-\frac{\sqrt{-cd^2-ae^2}}{\sqrt{-cd^2-ae^2}} + \frac{1}{2} \sqrt{cx^2+a} \left(\frac{xe^{(-1)}}{c} - \frac{2de^{(-2)}}{c}\right) \\ &-\frac{(2cd^2-ae^2)e^{(-3)}\ln\left(\left|-\sqrt{cx}+\sqrt{cx^2+a}\right|\right)}{2c^{\frac{3}{2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(sqrt(c\*x^2 + a)\*(e\*x + d)),x, algorithm="giac")

[Out] -2\*d^3\*arctan(-((sqrt(c)\*x - sqrt(c\*x^2 + a))\*e + sqrt(c)\*d)/sqrt(-c\*d^2 - a\*e^2))\*e^(-3)/sqrt(-c\*d^2 - a\*e^2) + 1/2\*sqrt(c\*x^2 + a)\*(x\*e^(-1)/c - 2\*d\*e^(-2)/c) - 1/2\*(2\*c\*d^2 - a\*e^2)\*e^(-3)\*ln(abs(-sqrt(c)\*x + sqrt(c\*x^2 + a)))/c^(3/2)

$$3.328 \quad \int \frac{x^2}{(d+ex)\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=109

$$-\frac{d^2 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2\sqrt{ae^2+cd^2}} - \frac{d \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{\sqrt{ce^2}} + \frac{\sqrt{a+cx^2}}{ce}$$

[Out] Sqrt[a + c\*x^2]/(c\*e) - (d\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(Sqrt[c]\*e^2) - (d^2\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]))/(e^2\*Sqrt[c\*d^2 + a\*e^2])

**Rubi [A]** time = 0.269357, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$-\frac{d^2 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2\sqrt{ae^2+cd^2}} - \frac{d \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{\sqrt{ce^2}} + \frac{\sqrt{a+cx^2}}{ce}$$

Antiderivative was successfully verified.

[In] Int[x^2/((d + e\*x)\*Sqrt[a + c\*x^2]),x]

[Out] Sqrt[a + c\*x^2]/(c\*e) - (d\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(Sqrt[c]\*e^2) - (d^2\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]))/(e^2\*Sqrt[c\*d^2 + a\*e^2])

**Rubi in Sympy [A]** time = 21.915, size = 95, normalized size = 0.87

$$-\frac{d^2 \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2\sqrt{ae^2+cd^2}} + \frac{\sqrt{a+cx^2}}{ce} - \frac{d \operatorname{atanh}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{\sqrt{ce^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(e\*x+d)/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] -d\*\*2\*atanh((a\*e - c\*d\*x)/(sqrt(a + c\*x\*\*2)\*sqrt(a\*e\*\*2 + c\*d\*\*2)))/(e\*\*2\*sqrt(a\*e\*\*2 + c\*d\*\*2)) + sqrt(a + c\*x\*\*2)/(c\*e) - d\*atanh(sqrt(c)\*x/sqrt(a + c\*x\*\*2))/(sqrt(c)\*e\*\*2)

**Mathematica [A]** time = 0.158137, size = 133, normalized size = 1.22

$$-\frac{d^2 \log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{\sqrt{ae^2+cd^2}} + \frac{d^2 \log(d+ex)}{\sqrt{ae^2+cd^2}} - \frac{d \log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{\sqrt{c}} + \frac{e\sqrt{a+cx^2}}{c}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((d + e\*x)\*Sqrt[a + c\*x^2]),x]

[Out] ((e\*Sqrt[a + c\*x^2])/c + (d^2\*Log[d + e\*x])/Sqrt[c\*d^2 + a\*e^2] - (d\*Log[c\*x + Sqrt[c]\*Sqrt[a + c\*x^2]])/Sqrt[c] - (d^2\*Log[a\*e - c\*d\*x + Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]])/Sqrt[c\*d^2 + a\*e^2])/e^2

**Maple [A]** time = 0.013, size = 172, normalized size = 1.6

$$\frac{1}{ce} \sqrt{cx^2 + a} - \frac{d}{e^2} \ln \left( x\sqrt{c} + \sqrt{cx^2 + a} \right) \frac{1}{\sqrt{c}}$$

$$- \frac{d^2}{e^3} \ln \left( 1 \left( 2 \frac{ae^2 + cd^2}{e^2} - 2 \frac{cd}{e} \left( x + \frac{d}{e} \right) + 2 \sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left( x + \frac{d}{e} \right)^2 c - 2 \frac{cd}{e} \left( x + \frac{d}{e} \right) + \frac{ae^2 + cd^2}{e^2}} \right) \left( x + \frac{d}{e} \right)^{-1} \right) \frac{1}{\sqrt{\frac{ae^2 + cd^2}{e^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(e\*x+d)/(c\*x^2+a)^(1/2), x)

[Out] (c\*x^2+a)^(1/2)/c/e-1/e^2\*d\*ln(x\*c^(1/2)+(c\*x^2+a)^(1/2))/c^(1/2)  
-d^2/e^3/((a\*e^2+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2+c\*d^2)/e^2-2\*c\*d/  
e\*(x+d/e)+2\*((a\*e^2+c\*d^2)/e^2)^(1/2)\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)  
)+(a\*e^2+c\*d^2)/e^2)^(1/2))/(x+d/e))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(sqrt(c\*x^2 + a)\*(e\*x + d)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.402016, size = 1, normalized size = 0.01

$$\frac{c^{\frac{3}{2}} d^2 \log \left( \frac{(2acdex - acd^2 - 2a^2e^2 - (2c^2d^2 + ace^2)x^2) \sqrt{cd^2 + ae^2} + 2(acd^2e + a^2e^3 - (c^2d^3 + acde^2)x) \sqrt{cx^2 + a}}{e^2x^2 + 2dex + d^2} \right) + \sqrt{cd^2 + ae^2} cd \log \left( 2 \sqrt{cx^2 + acx} \right)}{2 \sqrt{cd^2 + ae^2} c^{\frac{3}{2}} e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(sqrt(c\*x^2 + a)\*(e\*x + d)), x, algorithm="fricas")

[Out] [1/2\*(c^(3/2)\*d^2\*log(((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2)\*sqrt(c\*d^2 + a\*e^2) + 2\*(a\*c\*d^2\*e + a^2\*e^3 - (c^2\*d^3 + a\*c\*d^2\*e^2)\*x)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)) + sqrt(c\*d^2 + a\*e^2)\*c\*d\*log(2\*sqrt(c\*x^2 + a)\*c\*x - (2\*c\*x^2 + a)\*sqrt(c)) + 2\*sqrt(c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a)\*sqrt(c)\*e/(sqrt(c\*d^2 + a\*e^2)\*c^(3/2)\*e^2), 1/2\*(2\*c^(3/2)\*d^2\*arctan(sqrt(-c\*d^2 - a\*e^2)\*(c\*d\*x - a\*e)/((c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a))) + sqrt(-c\*d^2 - a\*e^2)\*c\*d\*log(2\*sqrt(c\*x^2 + a)\*c\*x - (2\*c\*x^2 + a)\*sqrt(c)) + 2\*sqrt(-c\*d^2 - a\*e^2)\*sqrt(c\*x^2 + a)\*sqrt(c)\*e/(sqrt(-c\*d^2 - a\*e^2)\*c^(3/2)\*e^2), 1/2\*(sqrt(-c)\*c\*d^2\*log(((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2)\*sqrt(c\*d^2 + a\*e^2) + 2\*(a\*c\*d^2\*e + a^2\*e^3 - (c^2\*d^3 + a\*c\*d^2\*e^2)\*x)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)) - 2\*sqrt(c\*d^2 + a\*e^2)\*c\*d\*arctan(sqrt(-c)\*x/sqrt(c\*x^2 + a)) + 2\*sqrt(c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a)\*sqrt(-c)\*e/(sqrt(c\*d^2 + a\*e^2)\*sqrt(-c)\*c\*e^2), (sqrt(-c)\*c\*d^2\*arctan(sqrt(-c\*d^2 - a\*e^2)\*(c\*d\*x - a\*e)/((c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a))) - sqrt(-c\*d^2 - a\*e^2)\*c\*d\*arctan(sqrt(-c)\*x/sqrt(c\*x^2 + a)) + sqrt(-c\*d^2 - a\*e^2)\*sqrt(c\*x^2 + a)\*sqrt(-c)\*e/(sqrt(-c\*d^2 - a\*e^2)\*sqrt(-c)\*c\*e^2)]

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**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a + cx^2}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(e\*x+d)/(c\*x\*\*2+a)\*\*(1/2), x)

[Out] Integral(x\*\*2/(sqrt(a + c\*x\*\*2)\*(d + e\*x)), x)

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**GIAC/XCAS [A]** time = 0.275943, size = 142, normalized size = 1.3

$$\frac{2d^2 \arctan\left(-\frac{(\sqrt{cx}-\sqrt{cx^2+a})e+\sqrt{cd}}{\sqrt{-cd^2-ae^2}}\right)e^{(-2)}}{\sqrt{-cd^2-ae^2}} + \frac{de^{(-2)}\ln\left(\left|-\sqrt{cx}+\sqrt{cx^2+a}\right|\right)}{\sqrt{c}} + \frac{\sqrt{cx^2+ae}^{(-1)}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(sqrt(c\*x^2 + a)\*(e\*x + d)), x, algorithm="giac")

[Out] 2\*d^2\*arctan(-((sqrt(c)\*x - sqrt(c\*x^2 + a))\*e + sqrt(c)\*d)/sqrt(-c\*d^2 - a\*e^2))\*e^(-2)/sqrt(-c\*d^2 - a\*e^2) + d\*e^(-2)\*ln(abs(-sqrt(c)\*x + sqrt(c\*x^2 + a)))/sqrt(c) + sqrt(c\*x^2 + a)\*e^(-1)/c

$$3.329 \quad \int \frac{x}{(d+ex)\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=86

$$\frac{d \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e\sqrt{ae^2+cd^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{ce}}$$

[Out] ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]]/(Sqrt[c]\*e) + (d\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])])/(e\*Sqrt[c\*d^2 + a\*e^2])

**Rubi [A]** time = 0.126241, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{d \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e\sqrt{ae^2+cd^2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{ce}}$$

Antiderivative was successfully verified.

[In] Int[x/((d + e\*x)\*Sqrt[a + c\*x^2]),x]

[Out] ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]]/(Sqrt[c]\*e) + (d\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])])/(e\*Sqrt[c\*d^2 + a\*e^2])

**Rubi in Sympy [A]** time = 15.9739, size = 75, normalized size = 0.87

$$\frac{d \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e\sqrt{ae^2+cd^2}} + \frac{\operatorname{atanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{ce}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x/(e\*x+d)/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] d\*atanh((a\*e - c\*d\*x)/(sqrt(a + c\*x\*\*2)\*sqrt(a\*e\*\*2 + c\*d\*\*2)))/(e\*sqrt(a\*e\*\*2 + c\*d\*\*2)) + atanh(sqrt(c)\*x/sqrt(a + c\*x\*\*2))/(sqrt(c)\*e)

**Mathematica [A]** time = 0.100612, size = 111, normalized size = 1.29

$$\frac{\frac{d \log(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx)}{\sqrt{ae^2+cd^2}} - \frac{d \log(d+ex)}{\sqrt{ae^2+cd^2}} + \frac{\log(\sqrt{c}\sqrt{a+cx^2}+cx)}{\sqrt{c}}}{e}$$

Antiderivative was successfully verified.

[In] Integrate[x/((d + e\*x)\*Sqrt[a + c\*x^2]),x]

[Out] (-((d\*Log[d + e\*x])/Sqrt[c\*d^2 + a\*e^2])) + Log[c\*x + Sqrt[c]\*Sqrt[a + c\*x^2]]/Sqrt[c] + (d\*Log[a\*e - c\*d\*x + Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]])/Sqrt[c\*d^2 + a\*e^2])/e

**Maple [B]** time = 0.009, size = 151, normalized size = 1.8

$$\frac{1}{e} \ln \left( x\sqrt{c} + \sqrt{cx^2 + a} \right) \frac{1}{\sqrt{c}} + \frac{d}{e^2} \ln \left( 1 \left( 2 \frac{ae^2 + cd^2}{e^2} - 2 \frac{cd}{e} \left( x + \frac{d}{e} \right) + 2 \sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left( x + \frac{d}{e} \right)^2 c - 2 \frac{cd}{e} \left( x + \frac{d}{e} \right) + \frac{ae^2 + cd^2}{e^2}} \right) \left( x + \frac{d}{e} \right)^{-1} \right) \frac{1}{\sqrt{\frac{ae^2 + cd^2}{e^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(e\*x+d)/(c\*x^2+a)^(1/2), x)

[Out] 1/e\*ln(x\*c^(1/2)+(c\*x^2+a)^(1/2))/c^(1/2)+d/e^2/((a\*e^2+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2+c\*d^2)/e^2-2\*c\*d/e\*(x+d/e)+2\*((a\*e^2+c\*d^2)/e^2)^(1/2)\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2))/(x+d/e))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(sqrt(c\*x^2 + a)\*(e\*x + d)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.349945, size = 1, normalized size = 0.01

$$\left[ \frac{\sqrt{cd} \log \left( \frac{(2acdex - acd^2 - 2a^2e^2 - (2c^2d^2 + ace^2)x^2)\sqrt{cd^2 + ae^2} - 2(acd^2e + a^2e^3 - (c^2d^3 + acde^2)x)\sqrt{cx^2 + a}}{e^2x^2 + 2dex + d^2} \right) + \sqrt{cd^2 + ae^2} \log \left( -2\sqrt{cx^2 + acx} \right)}{2\sqrt{cd^2 + ae^2}\sqrt{ce}} \right. \\ \left. \frac{2\sqrt{cd} \arctan \left( \frac{\sqrt{-cd^2 - ae^2}(cdx - ae)}{(cd^2 + ae^2)\sqrt{cx^2 + a}} \right) - \sqrt{-cd^2 - ae^2} \log \left( -2\sqrt{cx^2 + acx} - (2cx^2 + a)\sqrt{c} \right)}{2\sqrt{-cd^2 - ae^2}\sqrt{ce}}, \frac{\sqrt{-cd} \log \left( \frac{(2acdex - acd^2 - 2a^2e^2 - (2c^2d^2 + ace^2)x^2)\sqrt{-cd^2 - ae^2} - 2(acd^2e + a^2e^3 - (c^2d^3 + acde^2)x)\sqrt{-cx^2 + a}}{e^2x^2 + 2dex + d^2} \right)}{\sqrt{-cd^2 - ae^2}\sqrt{-ce}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(sqrt(c\*x^2 + a)\*(e\*x + d)),x, algorithm="fricas")

[Out] [1/2\*(sqrt(c)\*d\*log(((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2)\*sqrt(c\*d^2 + a\*e^2) - 2\*(a\*c\*d^2\*e + a^2\*e^3 - (c^2\*d^3 + a\*c\*d\*e^2)\*x)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)) + sqrt(c\*d^2 + a\*e^2)\*log(-2\*sqrt(c\*x^2 + a)\*c\*x - (2\*c\*x^2 + a)\*sqrt(c)))/(sqrt(c\*d^2 + a\*e^2)\*sqrt(c)\*e), -1/2\*(2\*sqrt(c)\*d\*arctan(sqrt(-c\*d^2 - a\*e^2)\*(c\*d\*x - a\*e)/((c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a))) - sqrt(-c\*d^2 - a\*e^2)\*log(-2\*sqrt(c\*x^2 + a)\*c\*x - (2\*c\*x^2 + a)\*sqrt(c)))/(sqrt(-c\*d^2 - a\*e^2)\*sqrt(c)\*e), 1/2\*(sqrt(-c)\*d\*log(((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2)\*sqrt(c\*d^2 + a\*e^2) - 2\*(a\*c\*d^2\*e + a^2\*e^3 - (c^2\*d^3 + a\*c\*d\*e^2)\*x)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)) + 2\*sqrt(c\*d^2 + a\*e^2)\*arctan(sqrt(-c)\*x/sqrt(c\*x^2 + a)))/(sqrt(c\*d^2 + a\*e^2)\*sqrt(-c)\*e), -(sqrt(-c)\*d\*arctan(sqrt(-c\*d^2 - a



$e^2) * (c * d * x - a * e) / ((c * d^2 + a * e^2) * \sqrt{c * x^2 + a}) - \sqrt{-c * d^2 - a * e^2} * \arctan(\sqrt{-c} * x / \sqrt{c * x^2 + a}) / (\sqrt{-c * d^2 - a * e^2} * \sqrt{-c} * e]$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a + cx^2} (d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e\*x+d)/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] Integral(x/(sqrt(a + c\*x\*\*2)\*(d + e\*x)), x)

**GIAC/XCAS [A]** time = 0.27681, size = 119, normalized size = 1.38

$$-\frac{2d \arctan\left(-\frac{(\sqrt{cx}-\sqrt{cx^2+a})e+\sqrt{cd}}{\sqrt{-cd^2-ae^2}}\right) e^{(-1)}}{\sqrt{-cd^2-ae^2}} - \frac{e^{(-1)} \ln\left(\left|-\sqrt{cx} + \sqrt{cx^2+a}\right|\right)}{\sqrt{c}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(sqrt(c\*x^2 + a)\*(e\*x + d)),x, algorithm="giac")

[Out] -2\*d\*arctan(-((sqrt(c)\*x - sqrt(c\*x^2 + a))\*e + sqrt(c)\*d)/sqrt(-c\*d^2 - a\*e^2))\*e^(-1)/sqrt(-c\*d^2 - a\*e^2) - e^(-1)\*ln(abs(-sqrt(c)\*x + sqrt(c\*x^2 + a)))/sqrt(c)

$$3.330 \quad \int \frac{1}{(d+ex)\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=54

$$-\frac{\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}}$$

[Out] -(ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])]/Sqrt[c\*d^2 + a\*e^2])

**Rubi [A]** time = 0.0453227, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$-\frac{\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)\*Sqrt[a + c\*x^2]),x]

[Out] -(ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])]/Sqrt[c\*d^2 + a\*e^2])

**Rubi in Sympy [A]** time = 8.77869, size = 48, normalized size = 0.89

$$-\frac{\operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{\sqrt{ae^2+cd^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(e\*x+d)/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] -atanh((a\*e - c\*d\*x)/(sqrt(a + c\*x\*\*2)\*sqrt(a\*e\*\*2 + c\*d\*\*2)))/sqrt(a\*e\*\*2 + c\*d\*\*2)

**Mathematica [A]** time = 0.0376777, size = 62, normalized size = 1.15

$$\frac{\log(d+ex) - \log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2} + ae - cdx\right)}{\sqrt{ae^2+cd^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x)\*Sqrt[a + c\*x^2]),x]

[Out] (Log[d + e\*x] - Log[a\*e - c\*d\*x + Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])]/Sqrt[c\*d^2 + a\*e^2]

**Maple [B]** time = 0.007, size = 127, normalized size = 2.4

$$-\frac{1}{e} \ln \left( 1 \left( 2 \frac{ae^2 + cd^2}{e^2} - 2 \frac{cd}{e} \left( x + \frac{d}{e} \right) + 2 \sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left( x + \frac{d}{e} \right)^2 - c - 2 \frac{cd}{e} \left( x + \frac{d}{e} \right) + \frac{ae^2 + cd^2}{e^2}} \right) \left( x + \frac{d}{e} \right)^{-1} \right) \frac{1}{\sqrt{\frac{ae^2 + cd^2}{e^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x+d)/(c*x^2+a)^(1/2),x)`

[Out]  $-1/e/((a*e^2+c*d^2)/e^2)^{(1/2)}*\ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^{(1/2)})/(x+d/e)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*x^2 + a)*(e*x + d)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.29886, size = 1, normalized size = 0.02

$$\left[ \frac{\log\left(\frac{(2acdex-acd^2-2a^2e^2-(2c^2d^2+ace^2)x^2)\sqrt{cd^2+ae^2}+2(acd^2e+a^2e^3-(c^2d^3+acde^2)x)\sqrt{cx^2+a}}{e^2x^2+2dex+d^2}\right)}{2\sqrt{cd^2+ae^2}}, \frac{\arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)}{(cd^2+ae^2)\sqrt{cx^2+a}}\right)}{\sqrt{-cd^2-ae^2}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*x^2 + a)*(e*x + d)),x, algorithm="fricas")`

[Out]  $[1/2*\log(((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2)*sqrt(c*d^2 + a*e^2) + 2*(a*c*d^2*e + a^2*e^3 - (c^2*d^3 + a*c*d^2*e^2)*x)*sqrt(c*x^2 + a)))/(e^2*x^2 + 2*d*e*x + d^2))/sqrt(c*d^2 + a*e^2), \arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)/((c*d^2 + a*e^2)*sqrt(c*x^2 + a)))/sqrt(-c*d^2 - a*e^2)]$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+cx^2}(d+ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x+d)/(c*x**2+a)**(1/2),x)`

[Out] `Integral(1/(sqrt(a + c*x**2)*(d + e*x)), x)`

**GIAC/XCAS [A]** time = 0.271451, size = 80, normalized size = 1.48

$$\frac{2 \arctan\left(-\frac{(\sqrt{cx}-\sqrt{cx^2+a})e+\sqrt{cd}}{\sqrt{-cd^2-ae^2}}\right)}{\sqrt{-cd^2-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)),x, algorithm="giac")
```

```
[Out] 2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d  
^2 - a*e^2))/sqrt(-c*d^2 - a*e^2)
```

$$3.331 \quad \int \frac{1}{x(d+ex)\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=86

$$\frac{e \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d\sqrt{ae^2+cd^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{ad}}$$

[Out] (e\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]))/(d\*Sqrt[c\*d^2 + a\*e^2]) - ArcTanh[Sqrt[a + c\*x^2]/Sqrt[a]]/(Sqrt[a]\*d)

**Rubi [A]** time = 0.216069, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{e \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d\sqrt{ae^2+cd^2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{ad}}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(d + e\*x)\*Sqrt[a + c\*x^2]), x]

[Out] (e\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]))/(d\*Sqrt[c\*d^2 + a\*e^2]) - ArcTanh[Sqrt[a + c\*x^2]/Sqrt[a]]/(Sqrt[a]\*d)

**Rubi in Sympy [A]** time = 21.133, size = 73, normalized size = 0.85

$$\frac{e \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d\sqrt{ae^2+cd^2}} - \frac{\operatorname{atanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{ad}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(e\*x+d)/(c\*x\*\*2+a)\*\*(1/2), x)

[Out] e\*atanh((a\*e - c\*d\*x)/(sqrt(a + c\*x\*\*2)\*sqrt(a\*e\*\*2 + c\*d\*\*2)))/(d\*sqrt(a\*e\*\*2 + c\*d\*\*2)) - atanh(sqrt(a + c\*x\*\*2)/sqrt(a))/(sqrt(a)\*d)

**Mathematica [A]** time = 0.12827, size = 118, normalized size = 1.37

$$\frac{e \log(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx)}{\sqrt{ae^2+cd^2}} - \frac{e \log(d+ex)}{\sqrt{ae^2+cd^2}} - \frac{\log(\sqrt{a}\sqrt{a+cx^2}+a)}{\sqrt{a}} + \frac{\log(x)}{\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(d + e\*x)\*Sqrt[a + c\*x^2]), x]

[Out] (Log[x]/Sqrt[a] - (e\*Log[d + e\*x])/Sqrt[c\*d^2 + a\*e^2] - Log[a + Sqrt[a]\*Sqrt[a + c\*x^2]]/Sqrt[a] + (e\*Log[a\*e - c\*d\*x + Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]]/Sqrt[c\*d^2 + a\*e^2])/d

**Maple [B]** time = 0.014, size = 158, normalized size = 1.8

$$-\frac{1}{d} \ln \left( \frac{1}{x} \left( 2a + 2\sqrt{a}\sqrt{cx^2+a} \right) \right) \frac{1}{\sqrt{a}}$$

$$+ \frac{1}{d} \ln \left( 1 \left( 2 \frac{ae^2 + cd^2}{e^2} - 2 \frac{cd}{e} \left( x + \frac{d}{e} \right) + 2 \sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left( x + \frac{d}{e} \right)^2 c - 2 \frac{cd}{e} \left( x + \frac{d}{e} \right) + \frac{ae^2 + cd^2}{e^2}} \right) \left( x + \frac{d}{e} \right)^{-1} \right) \frac{1}{\sqrt{\frac{ae^2 + cd^2}{e^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(e\*x+d)/(c\*x^2+a)^(1/2), x)

[Out] -1/d/a^(1/2)\*ln((2\*a+2\*a^(1/2)\*(c\*x^2+a)^(1/2))/x)+1/d/((a\*e^2+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2+c\*d^2)/e^2-2\*c\*d/e\*(x+d/e)+2\*((a\*e^2+c\*d^2)/e^2)^(1/2)\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2))/(x+d/e))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)\*x), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.313643, size = 1, normalized size = 0.01

$$\left[ \frac{\sqrt{ae} \log \left( \frac{(2acdex - acd^2 - 2a^2e^2 - (2c^2d^2 + ace^2)x^2)\sqrt{cd^2 + ae^2} - 2(acd^2e + a^2e^3 - (c^2d^3 + acde^2)x)\sqrt{cx^2 + a}}{e^2x^2 + 2dex + d^2} \right) + \sqrt{cd^2 + ae^2} \log \left( -\frac{(cx^2 + 2a)\sqrt{a} - 2\sqrt{cx^2 + a}}{x^2} \right)}{2\sqrt{cd^2 + ae^2}\sqrt{ad}} \right.$$

$$\left. \frac{2\sqrt{ae} \arctan \left( \frac{\sqrt{-cd^2 - ae^2}(cdx - ae)}{(cd^2 + ae^2)\sqrt{cx^2 + a}} \right) - \sqrt{-cd^2 - ae^2} \log \left( -\frac{(cx^2 + 2a)\sqrt{a} - 2\sqrt{cx^2 + a}}{x^2} \right)}{2\sqrt{-cd^2 - ae^2}\sqrt{ad}}, \frac{\sqrt{-ae} \arctan \left( \frac{\sqrt{-cd^2 - ae^2}(cdx - ae)}{(cd^2 + ae^2)\sqrt{cx^2 + a}} \right) + \sqrt{-cd^2 - ae^2} \arctan \left( \frac{\sqrt{-a}}{\sqrt{cx^2 + a}} \right)}{\sqrt{-cd^2 - ae^2}\sqrt{-ad}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)\*x), x, algorithm="fricas")

[Out] [1/2\*(sqrt(a)\*e\*log(((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2)\*sqrt(c\*d^2 + a\*e^2) - 2\*(a\*c\*d^2\*e + a^2\*e^3 - (c^2\*d^3 + a\*c\*d\*e^2)\*x)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)) + sqrt(c\*d^2 + a\*e^2)\*log(-((c\*x^2 + 2\*a)\*sqrt(a) - 2\*sqrt(c\*x^2 + a)\*a)/x^2))/(sqrt(c\*d^2 + a\*e^2)\*sqrt(a)\*d), -1/2\*(2\*sqrt(a)\*e\*arctan(sqrt(-c\*d^2 - a\*e^2)\*(c\*d\*x - a\*e)/((c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a))) - sqrt(-c\*d^2 - a\*e^2)\*log(-((c\*x^2 + 2\*a)\*sqrt(a) - 2\*sqrt(c\*x^2 + a)\*a)/x^2))/(sqrt(-c\*d^2 - a\*e^2)\*sqrt(a)\*d), 1/2\*(sqrt(-a)\*e\*log(((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2)\*sqrt(c\*d^2 + a\*e^2) - 2\*(a\*c\*d^2\*e + a^2\*e^3 - (c^2\*d^3 + a\*c\*d\*e^2)\*x)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)) - 2\*sqrt(c\*d^2 + a\*e^2)\*arctan(sqrt(-a)/sqrt(c\*x^2 + a)))]

```
/(sqrt(c*d^2 + a*e^2)*sqrt(-a)*d), -(sqrt(-a)*e*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)/((c*d^2 + a*e^2)*sqrt(c*x^2 + a)))) + sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-a)/sqrt(c*x^2 + a))/(sqrt(-c*d^2 - a*e^2)*sqrt(-a)*d]
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+cx^2}(d+ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x*sqrt(a + c*x**2)*(d + e*x)), x)
```

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*x),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.332 \quad \int \frac{1}{x^2(d+ex)\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=111

$$-\frac{e^2 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\sqrt{ae^2+cd^2}} + \frac{e \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{ad^2}} - \frac{\sqrt{a+cx^2}}{adx}$$

[Out]  $-(\text{Sqrt}[a + c*x^2]/(a*d*x)) - (e^2*\text{ArcTanh}[(a*e - c*d*x)/(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2])])/(d^2*\text{Sqrt}[c*d^2 + a*e^2]) + (e*\text{ArcTanh}[\text{Sqrt}[a + c*x^2]/\text{Sqrt}[a]])/(\text{Sqrt}[a]*d^2)$

**Rubi [A]** time = 0.260002, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$

$$-\frac{e^2 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\sqrt{ae^2+cd^2}} + \frac{e \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{ad^2}} - \frac{\sqrt{a+cx^2}}{adx}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^2*(d + e*x)*\text{Sqrt}[a + c*x^2]), x]$

[Out]  $-(\text{Sqrt}[a + c*x^2]/(a*d*x)) - (e^2*\text{ArcTanh}[(a*e - c*d*x)/(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2])])/(d^2*\text{Sqrt}[c*d^2 + a*e^2]) + (e*\text{ArcTanh}[\text{Sqrt}[a + c*x^2]/\text{Sqrt}[a]])/(\text{Sqrt}[a]*d^2)$

**Rubi in Sympy [A]** time = 25.3921, size = 95, normalized size = 0.86

$$-\frac{e^2 \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2\sqrt{ae^2+cd^2}} - \frac{\sqrt{a+cx^2}}{adx} + \frac{e \operatorname{atanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{ad^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x^{**2}/(e*x+d)/(c*x^{**2}+a)^{(1/2}), x)$

[Out]  $-e^{**2}*\operatorname{atanh}((a*e - c*d*x)/(\text{sqrt}(a + c*x^{**2})*\text{sqrt}(a*e^{**2} + c*d^{**2}))) / (d^{**2}*\text{sqrt}(a*e^{**2} + c*d^{**2})) - \text{sqrt}(a + c*x^{**2}) / (a*d*x) + e*\operatorname{atanh}(\text{sqrt}(a + c*x^{**2})/\text{sqrt}(a)) / (\text{sqrt}(a)*d^{**2})$

**Mathematica [A]** time = 0.170995, size = 144, normalized size = 1.3

$$\frac{-\frac{e^2 \log(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx)}{\sqrt{ae^2+cd^2}} + \frac{e^2 \log(d+ex)}{\sqrt{ae^2+cd^2}} - \frac{d\sqrt{a+cx^2}}{ax} + \frac{e \log(\sqrt{a}\sqrt{a+cx^2}+a)}{\sqrt{a}} - \frac{e \log(x)}{\sqrt{a}}}{d^2}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[1/(x^2*(d + e*x)*\text{Sqrt}[a + c*x^2]), x]$

[Out]  $(-((d*\text{Sqrt}[a + c*x^2])/(a*x)) - (e*\text{Log}[x])/(\text{Sqrt}[a] + (e^2*\text{Log}[d + e*x])/(\text{Sqrt}[c*d^2 + a*e^2] + (e*\text{Log}[a + \text{Sqrt}[a]*\text{Sqrt}[a + c*x^2])])/\text{Sqrt}[a] - (e^2*\text{Log}[a*e - c*d*x + \text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2])])/\text{Sqrt}[c*d^2 + a*e^2])/d^2$



**Maple [A]** time = 0.017, size = 180, normalized size = 1.6

$$-\frac{1}{adx} \sqrt{cx^2 + a}$$

$$-\frac{e}{d^2} \ln \left( 1 \left( 2 \frac{ae^2 + cd^2}{e^2} - 2 \frac{cd}{e} \left( x + \frac{d}{e} \right) + 2 \sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left( x + \frac{d}{e} \right)^2 c - 2 \frac{cd}{e} \left( x + \frac{d}{e} \right) + \frac{ae^2 + cd^2}{e^2}} \right) \left( x + \frac{d}{e} \right)^{-1} \right) \frac{1}{\sqrt{\frac{ae^2 + cd^2}{e^2}}}$$

$$+\frac{e}{d^2} \ln \left( \frac{1}{x} \left( 2a + 2\sqrt{a}\sqrt{cx^2 + a} \right) \right) \frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(e\*x+d)/(c\*x^2+a)^(1/2), x)

[Out]  $-(c*x^2+a)^{(1/2)}/a/d/x-1/d^2*e/((a*e^2+c*d^2)/e^2)^{(1/2)}*\ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^{(1/2)})/(x+d/e))+e/d^2/a^{(1/2)}*\ln((2*a+2*a^{(1/2)}*(c*x^2+a)^{(1/2)})/x)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + a}(ex + d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)\*x^2), x, algorithm="maxima")

[Out] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)\*x^2), x)

**Fricas [A]** time = 0.336558, size = 1, normalized size = 0.01

$$\frac{a^{\frac{3}{2}}e^2x \log \left( \frac{(2acdex - acd^2 - 2a^2e^2 - (2c^2d^2 + ace^2)x^2)\sqrt{cd^2 + ae^2} + 2(acd^2e + a^2e^3 - (c^2d^3 + acde^2)x)\sqrt{cx^2 + a}}{e^2x^2 + 2dex + d^2} \right) + \sqrt{cd^2 + ae^2}aex \log \left( -\frac{(cx^2 + 2a)\sqrt{cx^2 + a}}{2\sqrt{cd^2 + ae^2}a^{\frac{3}{2}}d^2x} \right)}{2\sqrt{cd^2 + ae^2}a^{\frac{3}{2}}d^2x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)\*x^2), x, algorithm="fricas")

[Out]  $[1/2*(a^{(3/2)}*e^2*x*\log(((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2)*\sqrt{c*d^2 + a*e^2} + 2*(a*c*d^2*e + a^2*e^3 - (c^2*d^3 + a*c*d*e^2)*x)*\sqrt{c*x^2 + a}))/((e^2*x^2 + 2*d*e*x + d^2)) + \sqrt{c*d^2 + a*e^2}*a*e*x*\log(-((c*x^2 + 2*a)*\sqrt{a} + 2*\sqrt{c*x^2 + a}*a)/x^2) - 2*\sqrt{c*d^2 + a*e^2}*\sqrt{c*x^2 + a}*\sqrt{a}*d)/(\sqrt{c*d^2 + a*e^2}*a^{(3/2)}*d^2*x), 1/2*(2*a^{(3/2)}*e^2*x*\arctan(\sqrt{-c*d^2 - a*e^2}*(c*d*x - a*e)/((c*d^2 + a*e^2)*\sqrt{c*x^2 + a})) + \sqrt{-c*d^2 - a*e^2}*a*e*x*\log(-((c*x^2 + 2*a)*\sqrt{a} + 2*\sqrt{c*x^2 + a}*a)/x^2) - 2*\sqrt{-c*d^2 - a*e^2}*\sqrt{c*x^2 + a}*\sqrt{a}*d)/(\sqrt{-c*d^2 - a*e^2}*a^{(3/2)}*d^2*x), 1/2*(\sqrt{-a}*a*e^2*x*\log(((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2)*\sqrt{c*d^2 + a*e^2} + 2*(a*c*d^2*e + a^2*e^3 - (c^2*d^3 + a*c*d*e^2)*x)*\sqrt{c*x^2 + a}))/((e^2*x^2 + 2*d*e*x + d^2)) + 2*\sqrt{c*d^2 + a*e^2}*a*e*x*\arctan(\sqrt{-a}/\sqrt{c*x^2 + a}) - 2*\sqrt{c*d^2 + a*e^2}*\sqrt{c*x^2 + a}*\sqrt{-a}*d)/(\sqrt{c*d^2 + a*e^2}*\sqrt{-a}*a*d^2*x), (\sqrt{-a}*a*e^2*x*\arctan(\sqrt{-a}/\sqrt{c*x^2 + a})) + \sqrt{-a}*a*e^2*x*\log(-((c*x^2 + 2*a)*\sqrt{a} + 2*\sqrt{c*x^2 + a}*a)/x^2) - 2*\sqrt{-a}*a*e^2*x*\sqrt{c*x^2 + a}*\sqrt{-a}*d)/(\sqrt{-a}*a^{(3/2)}*d^2*x)$

```
t(-c*d^2 - a*e^2)*(c*d*x - a*e)/((c*d^2 + a*e^2)*sqrt(c*x^2 + a))
) + sqrt(-c*d^2 - a*e^2)*a*e*x*arctan(sqrt(-a)/sqrt(c*x^2 + a)) -
sqrt(-c*d^2 - a*e^2)*sqrt(c*x^2 + a)*sqrt(-a)*d/(sqrt(-c*d^2 -
a*e^2)*sqrt(-a)*a*d^2*x]
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt{a + cx^2} (d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x+d)/(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a + c*x**2)*(d + e*x)), x)
```

**GIAC/XCAS [A]** time = 0.279711, size = 192, normalized size = 1.73

$$2c \left( \frac{\arctan\left(-\frac{(\sqrt{cx}-\sqrt{cx^2+a})e+\sqrt{cd}}{\sqrt{-cd^2-ae^2}}\right)e^2}{\sqrt{-cd^2-ae^2}cd^2} - \frac{\arctan\left(-\frac{\sqrt{cx}-\sqrt{cx^2+a}}{\sqrt{-a}}\right)e}{\sqrt{-acd^2}} + \frac{1}{\left(\left(\sqrt{cx}-\sqrt{cx^2+a}\right)^2-a\right)\sqrt{cd}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*x^2),x, algorithm="giac")
```

```
[Out] 2*c*(arctan(-((sqrt(c)*x - sqrt(c*x^2 + a))*e + sqrt(c)*d)/sqrt(-
c*d^2 - a*e^2))*e^2/(sqrt(-c*d^2 - a*e^2)*c*d^2) - arctan(-(sqrt(
c)*x - sqrt(c*x^2 + a))/sqrt(-a))*e/(sqrt(-a)*c*d^2) + 1/(((sqrt(
c)*x - sqrt(c*x^2 + a))^2 - a)*sqrt(c)*d)
```

$$3.333 \quad \int \frac{1}{x^3(d+ex)\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=168

$$\frac{c \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d} - \frac{e^2 \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3} + \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{e^3 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3\sqrt{ae^2+cd^2}} - \frac{\sqrt{a+cx^2}}{2adx^2}$$

[Out] -Sqrt[a + c\*x^2]/(2\*a\*d\*x^2) + (e\*Sqrt[a + c\*x^2])/(a\*d^2\*x) + (e^3\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])])/(d^3\*Sqrt[c\*d^2 + a\*e^2]) + (c\*ArcTanh[Sqrt[a + c\*x^2]/Sqrt[a]])/(2\*a^(3/2)\*d) - (e^2\*ArcTanh[Sqrt[a + c\*x^2]/Sqrt[a]])/(Sqrt[a]\*d^3)

**Rubi [A]** time = 0.356062, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$\frac{c \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d} - \frac{e^2 \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3} + \frac{e\sqrt{a+cx^2}}{ad^2x} + \frac{e^3 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3\sqrt{ae^2+cd^2}} - \frac{\sqrt{a+cx^2}}{2adx^2}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*(d + e\*x)\*Sqrt[a + c\*x^2]),x]

[Out] -Sqrt[a + c\*x^2]/(2\*a\*d\*x^2) + (e\*Sqrt[a + c\*x^2])/(a\*d^2\*x) + (e^3\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])])/(d^3\*Sqrt[c\*d^2 + a\*e^2]) + (c\*ArcTanh[Sqrt[a + c\*x^2]/Sqrt[a]])/(2\*a^(3/2)\*d) - (e^2\*ArcTanh[Sqrt[a + c\*x^2]/Sqrt[a]])/(Sqrt[a]\*d^3)

**Rubi in Sympy [A]** time = 32.5291, size = 146, normalized size = 0.87

$$\frac{e^3 \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3\sqrt{ae^2+cd^2}} - \frac{\sqrt{a+cx^2}}{2adx^2} + \frac{e\sqrt{a+cx^2}}{ad^2x} - \frac{e^2 \operatorname{atanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3} + \frac{c \operatorname{atanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*3/(e\*x+d)/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] e\*\*3\*atanh((a\*e - c\*d\*x)/(sqrt(a + c\*x\*\*2)\*sqrt(a\*e\*\*2 + c\*d\*\*2)))/(d\*\*3\*sqrt(a\*e\*\*2 + c\*d\*\*2)) - sqrt(a + c\*x\*\*2)/(2\*a\*d\*x\*\*2) + e\*sqrt(a + c\*x\*\*2)/(a\*d\*\*2\*x) - e\*\*2\*atanh(sqrt(a + c\*x\*\*2)/sqrt(a))/(sqrt(a)\*d\*\*3) + c\*atanh(sqrt(a + c\*x\*\*2)/sqrt(a))/(2\*a\*\*(3/2)\*d)

**Mathematica [A]** time = 0.404555, size = 177, normalized size = 1.05

$$\frac{(cd^2-2ae^2) \log(\sqrt{a}\sqrt{a+cx^2}+a)}{a^{3/2}} + \frac{\log(x)(2ae^2-cd^2)}{a^{3/2}} + \frac{2e^3 \log(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx)}{\sqrt{ae^2+cd^2}} - \frac{2e^3 \log(d+ex)}{\sqrt{ae^2+cd^2}} + \frac{d\sqrt{a+cx^2}(2ex-d)}{ax^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(d + e\*x)\*Sqrt[a + c\*x^2]),x]

[Out]  $((d*(-d + 2*e*x)*\text{Sqrt}[a + c*x^2])/(a*x^2) + ((-(c*d^2) + 2*a*e^2) * \text{Log}[x])/a^{(3/2)} - (2*e^3*\text{Log}[d + e*x])/ \text{Sqrt}[c*d^2 + a*e^2] + ((c*d^2 - 2*a*e^2)* \text{Log}[a + \text{Sqrt}[a]*\text{Sqrt}[a + c*x^2]])/a^{(3/2)} + (2*e^3*\text{Log}[a*e - c*d*x + \text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2]])/ \text{Sqrt}[c*d^2 + a*e^2])/(2*d^3)$

**Maple [A]** time = 0.021, size = 236, normalized size = 1.4

$$-\frac{1}{2ad^2}\sqrt{cx^2+a} + \frac{c}{2d}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{cx^2+a}\right)\right)a^{-\frac{3}{2}} - \frac{e^2}{d^3}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{cx^2+a}\right)\right)\frac{1}{\sqrt{a}}$$

$$+ \frac{e^2}{d^3}\ln\left(1\left(2\frac{ae^2+cd^2}{e^2} - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + 2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2 - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + \frac{ae^2+cd^2}{e^2}}\right)\left(x+\frac{d}{e}\right)^{-1}\right)\frac{1}{\sqrt{\frac{ae^2+cd^2}{e^2}}}$$

$$+ \frac{e}{ad^2x}\sqrt{cx^2+a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(e*x+d)/(c*x^2+a)^(1/2), x)`

[Out]  $-1/2*(c*x^2+a)^{(1/2)}/a/d/x^2+1/2/d*c/a^{(3/2)}*\ln((2*a+2*a^{(1/2)}*(c*x^2+a)^{(1/2)})/x)-1/d^3*e^2/a^{(1/2)}*\ln((2*a+2*a^{(1/2)}*(c*x^2+a)^{(1/2)})/x)+1/d^3*e^2/((a*e^2+c*d^2)/e^2)^{(1/2)}*\ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^{(1/2)})/(x+d/e))+e*(c*x^2+a)^{(1/2)}/a/d^2/x$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}(ex+d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*x^3), x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*x^3), x)`

**Fricas [A]** time = 0.357967, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*x^3), x, algorithm="fricas")`

[Out]  $[1/4*(2*a^{(3/2)}*e^3*x^2*\log(((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2)*\text{sqrt}(c*d^2 + a*e^2) - 2*(a*c*d^2*e + a^2*e^3 - (c^2*d^3 + a*c*d*e^2)*x)*\text{sqrt}(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)) - \text{sqrt}(c*d^2 + a*e^2)*(c*d^2 - 2*a*e^2)*x^2*\log(-((c*x^2 + 2*a)*\text{sqrt}(a) - 2*\text{sqrt}(c*x^2 + a)*a)/x^2) + 2*\text{sqrt}(c*d^2 + a*e^2)*(2*d*e*x - d^2)*\text{sqrt}(c*x^2 + a)*\text{sqrt}(a))/(\text{sqrt}(c*d^2 + a*e^2)*a^{(3/2)}*d^3*x^2), -1/4*(4*a^{(3/2)}*e^3*x^2*\arctan(\text{sqrt}(-c*d^2 - a*e^2)*(c*d*x - a*e)/((c*d^2 + a*e^2)*\text{sqrt}(c*x^2 + a)) + (c*d^2 - 2*a*e^2)*\text{sqrt}(-c*d^2 - a*e^2)*x^2*\log(-((c*x^2 + 2*a)*\text{sqrt}(a) - 2*\text{sqrt}(c*x^2 + a)*a)/x^2) - 2*\text{sqrt}(-c*d^2 - a*e^2)*(2*d*e*x - d^2)*\text{sqrt}(c*x^2 + a)*\text{sqrt}(a))/(\text{sqrt}(-c*d^2 - a*e^2)*a^{(3/2)}*d^3*$

$x^2)$ ,  $1/2*(\sqrt{-a}*a*e^3*x^2*\log(((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2)*\sqrt{c*d^2 + a*e^2} - 2*(a*c*d^2*e + a^2*e^3 - (c^2*d^3 + a*c*d*e^2)*x)*\sqrt{c*x^2 + a}))/((e^2*x^2 + 2*d*e*x + d^2)) + \sqrt{c*d^2 + a*e^2}*(c*d^2 - 2*a*e^2)*x^2*a$   
 $\arctan(\sqrt{-a}/\sqrt{c*x^2 + a}) + \sqrt{c*d^2 + a*e^2}*(2*d*e*x - d^2)*\sqrt{c*x^2 + a}*\sqrt{-a}))/(\sqrt{c*d^2 + a*e^2}*\sqrt{-a})*a*d^3*x^2)$ ,  $-1/2*(2*\sqrt{-a}*a*e^3*x^2*\arctan(\sqrt{-c*d^2 - a*e^2}*(c*d*x - a*e)/((c*d^2 + a*e^2)*\sqrt{c*x^2 + a}))) - (c*d^2 - 2*a*e^2)*\sqrt{-c*d^2 - a*e^2}*x^2*\arctan(\sqrt{-a}/\sqrt{c*x^2 + a}) - \sqrt{-c*d^2 - a*e^2}*(2*d*e*x - d^2)*\sqrt{c*x^2 + a}*\sqrt{-a}))/(\sqrt{-c*d^2 - a*e^2}*\sqrt{-a})*a*d^3*x^2)]$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \sqrt{a + cx^2} (d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/(e\*x+d)/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] Integral(1/(x\*\*3\*sqrt(a + c\*x\*\*2)\*(d + e\*x)), x)

**GIAC/XCAS [A]** time = 0.286229, size = 323, normalized size = 1.92

$$-c^{\frac{3}{2}} \left( \frac{2 \arctan \left( -\frac{(\sqrt{cx} - \sqrt{cx^2+a})e + \sqrt{cd}}{\sqrt{-cd^2 - ae^2}} \right) e^3}{\sqrt{-cd^2 - ae^2} c^{\frac{3}{2}} d^3} + \frac{(cd^2 - 2ae^2) \arctan \left( -\frac{\sqrt{cx} - \sqrt{cx^2+a}}{\sqrt{-a}} \right)}{\sqrt{-aac^{\frac{3}{2}} d^3}} - \frac{(\sqrt{cx} - \sqrt{cx^2+a})^3 \sqrt{cd} - 2(\sqrt{cx} - \sqrt{cx^2+a})}{\left( (\sqrt{cx} - \sqrt{cx^2+a})^3 \sqrt{cd} - 2(\sqrt{cx} - \sqrt{cx^2+a}) \right)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)\*x^3),x, algorithm="giac")

[Out]  $-c^{3/2}*(2*\arctan(-((\sqrt{c}*x - \sqrt{c*x^2 + a})*e + \sqrt{c})*d)/\sqrt{-c*d^2 - a*e^2})*e^3/(\sqrt{-c*d^2 - a*e^2})*c^{3/2}*d^3) + (c*d^2 - 2*a*e^2)*\arctan(-(\sqrt{c}*x - \sqrt{c*x^2 + a})/\sqrt{-a}))/(\sqrt{-a})*a*c^{3/2}*d^3 - ((\sqrt{c}*x - \sqrt{c*x^2 + a})^3*\sqrt{c}*d - 2*(\sqrt{c}*x - \sqrt{c*x^2 + a})^2*a*e + (\sqrt{c}*x - \sqrt{c*x^2 + a})*a*\sqrt{c}*d + 2*a^2*e)/(((\sqrt{c}*x - \sqrt{c*x^2 + a})^2 - a)^2*a*c*d^2)$

$$3.334 \quad \int \frac{x^4}{(d+ex)(a+cx^2)^{3/2}} dx$$

**Optimal.** Leaf size=146

$$-\frac{d \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^2} + \frac{a(ae+cdx)}{c^2\sqrt{a+cx^2}(ae^2+cd^2)} + \frac{\sqrt{a+cx^2}}{c^2e} - \frac{d^4 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2(ae^2+cd^2)^{3/2}}$$

[Out] (a\*(a\*e + c\*d\*x))/(c^2\*(c\*d^2 + a\*e^2)\*Sqrt[a + c\*x^2]) + Sqrt[a + c\*x^2]/(c^2\*e) - (d\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(c^(3/2)\*e^2) - (d^4\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]))/(e^2\*(c\*d^2 + a\*e^2)^(3/2))

**Rubi [A]** time = 0.569879, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$-\frac{d \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^2} + \frac{a(ae+cdx)}{c^2\sqrt{a+cx^2}(ae^2+cd^2)} + \frac{\sqrt{a+cx^2}}{c^2e} - \frac{d^4 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2(ae^2+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^4/((d + e\*x)\*(a + c\*x^2)^(3/2)), x]

[Out] (a\*(a\*e + c\*d\*x))/(c^2\*(c\*d^2 + a\*e^2)\*Sqrt[a + c\*x^2]) + Sqrt[a + c\*x^2]/(c^2\*e) - (d\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(c^(3/2)\*e^2) - (d^4\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]))/(e^2\*(c\*d^2 + a\*e^2)^(3/2))

**Rubi in Sympy [A]** time = 54.9717, size = 209, normalized size = 1.43

$$\frac{a}{c^2e\sqrt{a+cx^2}} - \frac{d^4 \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2(ae^2+cd^2)^{3/2}} - \frac{d^2}{ce^3\sqrt{a+cx^2}} + \frac{dx}{ce^2\sqrt{a+cx^2}} + \frac{\sqrt{a+cx^2}}{c^2e} - \frac{d \operatorname{atanh}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^2} + \frac{d^4(ae+cdx)}{ae^4\sqrt{a+cx^2}(ae^2+cd^2)} - \frac{d^3x}{ae^4\sqrt{a+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(e\*x+d)/(c\*x\*\*2+a)\*\*(3/2), x)

[Out] a/(c\*\*2\*e\*sqr(a + c\*x\*\*2)) - d\*\*4\*atanh((a\*e - c\*d\*x)/(sqr(a + c\*x\*\*2)\*sqr(a\*e\*\*2 + c\*d\*\*2)))/(e\*\*2\*(a\*e\*\*2 + c\*d\*\*2)\*\*(3/2)) - d\*\*2/(c\*e\*\*3\*sqr(a + c\*x\*\*2)) + d\*x/(c\*e\*\*2\*sqr(a + c\*x\*\*2)) + sqr(a + c\*x\*\*2)/(c\*\*2\*e) - d\*atanh(sqr(c)\*x/sqr(a + c\*x\*\*2))/(c\*\*(3/2)\*e\*\*2) + d\*\*4\*(a\*e + c\*d\*x)/(a\*e\*\*4\*sqr(a + c\*x\*\*2)\*(a\*e\*\*2 + c\*d\*\*2)) - d\*\*3\*x/(a\*e\*\*4\*sqr(a + c\*x\*\*2))

**Mathematica [A]** time = 0.503546, size = 173, normalized size = 1.18

$$-\frac{d \log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{c^{3/2}e^2} + \frac{\sqrt{a+cx^2}\left(\frac{a(ae+cdx)}{(a+cx^2)(ae^2+cd^2)} + \frac{1}{e}\right)}{c^2} - \frac{d^4 \log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{e^2(ae^2+cd^2)^{3/2}} + \frac{d^4 \log(d+ex)}{e^2(ae^2+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/((d + e\*x)\*(a + c\*x^2)^(3/2)),x]

[Out] (Sqrt[a + c\*x^2]\*(e^(-1) + (a\*(a\*e + c\*d\*x))/((c\*d^2 + a\*e^2)\*(a + c\*x^2))))/c^2 + (d^4\*Log[d + e\*x])/(e^2\*(c\*d^2 + a\*e^2)^(3/2)) - (d\*Log[c\*x + Sqrt[c]\*Sqrt[a + c\*x^2]])/(c^(3/2)\*e^2) - (d^4\*Log[a\*e - c\*d\*x + Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]])/(e^2\*(c\*d^2 + a\*e^2)^(3/2))

**Maple [B]** time = 0.024, size = 396, normalized size = 2.7

$$\begin{aligned} & \frac{x^2}{ce} \frac{1}{\sqrt{cx^2+a}} + 2 \frac{a}{ec^2} \frac{1}{\sqrt{cx^2+a}} - \frac{d^2}{e^3c} \frac{1}{\sqrt{cx^2+a}} + \frac{d^4}{e^3(ae^2+cd^2)} \frac{1}{\sqrt{\left(x+\frac{d}{e}\right)^2 c - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + \frac{ae^2+cd^2}{e^2}}} \\ & + \frac{d^5cx}{e^4(ae^2+cd^2)a} \frac{1}{\sqrt{\left(x+\frac{d}{e}\right)^2 c - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + \frac{ae^2+cd^2}{e^2}}} \\ & - \frac{d^4}{e^3(ae^2+cd^2)} \ln\left(1\left(2\frac{ae^2+cd^2}{e^2} - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + 2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2 c - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + \frac{ae^2+cd^2}{e^2}}\right)\left(x+\frac{d}{e}\right)^{-1}\right) \\ & - \frac{d^3x}{e^4a} \frac{1}{\sqrt{cx^2+a}} + \frac{dx}{e^2c} \frac{1}{\sqrt{cx^2+a}} - \frac{d}{e^2} \ln\left(x\sqrt{c} + \sqrt{cx^2+a}\right) c^{-\frac{3}{2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(e\*x+d)/(c\*x^2+a)^(3/2),x)

[Out] 1/e\*x^2/c/(c\*x^2+a)^(1/2)+2/e\*a/c^2/(c\*x^2+a)^(1/2)-d^2/e^3/c/(c\*x^2+a)^(1/2)+d^4/e^3/(a\*e^2+c\*d^2)/((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2)+d^5/e^4/(a\*e^2+c\*d^2)/a/((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2)\*c\*x-d^4/e^3/(a\*e^2+c\*d^2)/((a\*e^2+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2+c\*d^2)/e^2-2\*c\*d/e\*(x+d/e)+2\*((a\*e^2+c\*d^2)/e^2)^(1/2)\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2))/(x+d/e))-d^3/e^4\*x/a/(c\*x^2+a)^(1/2)+d/e^2\*x/c/(c\*x^2+a)^(1/2)-d/e^2/c^(3/2)\*ln(x\*c^(1/2)+(c\*x^2+a)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/((c\*x^2 + a)^(3/2)\*(e\*x + d)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 5.06737, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/((c\*x^2 + a)^(3/2)\*(e\*x + d)),x, algorithm="fricas")

```
[Out] [1/2*(2*(a*c*d*e^2*x + a*c*d^2*e + 2*a^2*e^3 + (c^2*d^2*e + a*c*e^3)*x^2)*sqrt(c*d^2 + a*e^2)*sqrt(c*x^2 + a)*sqrt(c) + (a*c^2*d^3 + a^2*c*d*e^2 + (c^3*d^3 + a*c^2*d*e^2)*x^2)*sqrt(c*d^2 + a*e^2)*log(2*sqrt(c*x^2 + a)*c*x - (2*c*x^2 + a)*sqrt(c)) + (c^3*d^4*x^2 + a*c^2*d^4)*sqrt(c)*log(((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2)*sqrt(c*d^2 + a*e^2) + 2*(a*c*d^2*e + a^2*e^3 - (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/((a*c^3*d^2*e^2 + a^2*c^2*e^4 + (c^4*d^2*e^2 + a*c^3*e^4)*x^2)*sqrt(c*d^2 + a*e^2)*sqrt(c)), 1/2*(2*(a*c*d*e^2*x + a*c*d^2*e + 2*a^2*e^3 + (c^2*d^2*e + a*c*e^3)*x^2)*sqrt(-c*d^2 - a*e^2)*sqrt(c*x^2 + a)*sqrt(c) + 2*(c^3*d^4*x^2 + a*c^2*d^4)*sqrt(c)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)/((c*d^2 + a*e^2)*sqrt(c*x^2 + a))) + (a*c^2*d^3 + a^2*c*d*e^2 + (c^3*d^3 + a*c^2*d*e^2)*x^2)*sqrt(-c*d^2 - a*e^2)*log(2*sqrt(c*x^2 + a)*c*x - (2*c*x^2 + a)*sqrt(c)))/((a*c^3*d^2*e^2 + a^2*c^2*e^4 + (c^4*d^2*e^2 + a*c^3*e^4)*x^2)*sqrt(-c*d^2 - a*e^2)*sqrt(c)), 1/2*(2*(a*c*d*e^2*x + a*c*d^2*e + 2*a^2*e^3 + (c^2*d^2*e + a*c*e^3)*x^2)*sqrt(c*d^2 + a*e^2)*sqrt(c*x^2 + a)*sqrt(-c) - 2*(a*c^2*d^3 + a^2*c*d*e^2 + (c^3*d^3 + a*c^2*d*e^2)*x^2)*sqrt(c*d^2 + a*e^2)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (c^3*d^4*x^2 + a*c^2*d^4)*sqrt(-c)*log(((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2)*sqrt(c*d^2 + a*e^2) + 2*(a*c*d^2*e + a^2*e^3 - (c^2*d^3 + a*c*d*e^2)*x)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/((a*c^3*d^2*e^2 + a^2*c^2*e^4 + (c^4*d^2*e^2 + a*c^3*e^4)*x^2)*sqrt(c*d^2 + a*e^2)*sqrt(-c)), ((a*c*d*e^2*x + a*c*d^2*e + 2*a^2*e^3 + (c^2*d^2*e + a*c*e^3)*x^2)*sqrt(-c*d^2 - a*e^2)*sqrt(c*x^2 + a)*sqrt(-c) - (a*c^2*d^3 + a^2*c*d*e^2 + (c^3*d^3 + a*c^2*d*e^2)*x^2)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (c^3*d^4*x^2 + a*c^2*d^4)*sqrt(-c)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)/((c*d^2 + a*e^2)*sqrt(c*x^2 + a)))))/((a*c^3*d^2*e^2 + a^2*c^2*e^4 + (c^4*d^2*e^2 + a*c^3*e^4)*x^2)*sqrt(-c*d^2 - a*e^2)*sqrt(-c))]
```

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**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{(a + cx^2)^{\frac{3}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4/(e\*x+d)/(c\*x\*\*2+a)\*\*(3/2),x)

[Out] Integral(x\*\*4/((a + c\*x\*\*2)\*\*(3/2)\*(d + e\*x)), x)

---

**GIAC/XCAS [A]** time = 0.281573, size = 404, normalized size = 2.77

$$\frac{2d^4 \arctan\left(-\frac{(\sqrt{cx-\sqrt{cx^2+a}})e+\sqrt{cd}}{\sqrt{-cd^2-ae^2}}\right)}{(cd^2e^2 + ae^4)\sqrt{-cd^2 - ae^2}} + \frac{de^{(-2)}\ln\left(\left|-\sqrt{cx} + \sqrt{cx^2 + a}\right|\right)}{c^{\frac{3}{2}}} + \frac{\left(\frac{(c^4d^4e^5+2ac^3d^2e^7+a^2c^2e^9)x}{c^5d^4e^6+2ac^4d^2e^8+a^2c^3e^{10}} + \frac{ac^3d^3e^6+a^2c^2de^8}{c^5d^4e^6+2ac^4d^2e^8+a^2c^3e^{10}}\right)x + \frac{ac^3d^4e^5+3a^2c^2d^2e^7+2a^3ce^9}{c^5d^4e^6+2ac^4d^2e^8+a^2c^3e^{10}}}{\sqrt{cx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/((c\*x^2 + a)^(3/2)\*(e\*x + d)),x, algorithm="giac")

[Out] 2\*d^4\*arctan(-((sqrt(c)\*x - sqrt(c\*x^2 + a))\*e + sqrt(c)\*d)/sqrt(-c\*d^2 - a\*e^2))/((c\*d^2\*e^2 + a\*e^4)\*sqrt(-c\*d^2 - a\*e^2)) + d\*e^(-2)\*ln(abs(-sqrt(c)\*x + sqrt(c\*x^2 + a)))/c^(3/2) + (((c^4\*d^4\*e^5 + 2\*a\*c^3\*d^2\*e^7 + a^2\*c^2\*d^2\*e^9)\*x/(c^5\*d^4\*e^6 + 2\*a\*c^4\*d^2\*e^8 + a^2\*c^3\*d^2\*e^10) + (a\*c^3\*d^3\*e^6 + a^2\*c^2\*d\*e^8)/(c^5\*d^4\*e^6 + 2\*a\*c^4\*d^2\*e^8 + a^2\*c^3\*d^2\*e^10)))/sqrt(c\*x^2 + a)



$$\frac{^6 + 2*a*c^4*d^2*e^8 + a^2*c^3*e^{10}) * x + (a*c^3*d^4*e^5 + 3*a^2*c^2*d^2*e^7 + 2*a^3*c*e^9)}{(c^5*d^4*e^6 + 2*a*c^4*d^2*e^8 + a^2*c^3*e^{10})} / \text{sqrt}(c*x^2 + a)$$

$$3.335 \quad \int \frac{x^3}{(d+ex)(a+cx^2)^{3/2}} dx$$

**Optimal.** Leaf size=123

$$\frac{\tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{c^{3/2}e} + \frac{a(d-ex)}{c\sqrt{a+cx^2}(ae^2+cd^2)} + \frac{d^3 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e(ae^2+cd^2)^{3/2}}$$

[Out]  $(a*(d - e*x))/(c*(c*d^2 + a*e^2)*\text{Sqrt}[a + c*x^2]) + \text{ArcTanh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a + c*x^2]]/(c^{(3/2)*e}) + (d^3*\text{ArcTanh}[(a*e - c*d*x)/(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2])])/(e*(c*d^2 + a*e^2)^{(3/2)})$

**Rubi [A]** time = 0.329412, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{c^{3/2}e} + \frac{a(d-ex)}{c\sqrt{a+cx^2}(ae^2+cd^2)} + \frac{d^3 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e(ae^2+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^3/((d + e\*x)\*(a + c\*x^2)^(3/2)), x]

[Out]  $(a*(d - e*x))/(c*(c*d^2 + a*e^2)*\text{Sqrt}[a + c*x^2]) + \text{ArcTanh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a + c*x^2]]/(c^{(3/2)*e}) + (d^3*\text{ArcTanh}[(a*e - c*d*x)/(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2])])/(e*(c*d^2 + a*e^2)^{(3/2)})$

**Rubi in Sympy [A]** time = 44.6447, size = 167, normalized size = 1.36

$$\frac{d^3 \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e(ae^2+cd^2)^{3/2}} + \frac{d}{ce^2\sqrt{a+cx^2}} - \frac{x}{ce\sqrt{a+cx^2}} + \frac{\operatorname{atanh}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{c^{3/2}e} - \frac{d^3(ae+cdx)}{ae^3\sqrt{a+cx^2}(ae^2+cd^2)} + \frac{d^2x}{ae^3\sqrt{a+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3/(e\*x+d)/(c\*x\*\*2+a)\*\*(3/2), x)

[Out]  $d**3*\operatorname{atanh}((a*e - c*d*x)/(\text{sqrt}(a + c*x**2)*\text{sqrt}(a*e**2 + c*d**2)))/(e*(a*e**2 + c*d**2)**(3/2)) + d/(c*e**2*\text{sqrt}(a + c*x**2)) - x/(c*e*\text{sqrt}(a + c*x**2)) + \operatorname{atanh}(\text{sqrt}(c)*x/\text{sqrt}(a + c*x**2))/(c** (3/2)*e) - d**3*(a*e + c*d*x)/(a*e**3*\text{sqrt}(a + c*x**2)*(a*e**2 + c*d**2)) + d**2*x/(a*e**3*\text{sqrt}(a + c*x**2))$

**Mathematica [A]** time = 0.441252, size = 157, normalized size = 1.28

$$\frac{\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{c^{3/2}e} + \frac{ad-ae^2x}{c\sqrt{a+cx^2}(ae^2+cd^2)} + \frac{d^3 \log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{e(ae^2+cd^2)^{3/2}} - \frac{d^3 \log(d+ex)}{e(ae^2+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((d + e\*x)\*(a + c\*x^2)^(3/2)), x]

[Out]  $(a*d - a*e*x)/(c*(c*d^2 + a*e^2)*\text{Sqrt}[a + c*x^2]) - (d^3*\text{Log}[d + e*x])/(e*(c*d^2 + a*e^2)^{(3/2)}) + \text{Log}[c*x + \text{Sqrt}[c]*\text{Sqrt}[a + c*x^2]]/(c^{(3/2)}*e) + (d^3*\text{Log}[a*e - c*d*x + \text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2]])/(e*(c*d^2 + a*e^2)^{(3/2)})$

**Maple [B]** time = 0.012, size = 354, normalized size = 2.9

$$\begin{aligned} & \frac{d^2 x}{e^3 a \sqrt{c x^2 + a}} - \frac{x}{c e \sqrt{c x^2 + a}} + \frac{1}{e} \ln \left( x \sqrt{c} + \sqrt{c x^2 + a} \right) c^{-\frac{3}{2}} \\ & + \frac{d}{e^2 c \sqrt{c x^2 + a}} - \frac{d^3}{e^2 (a e^2 + c d^2)} \frac{1}{\sqrt{\left(x + \frac{d}{e}\right)^2 c - 2 \frac{c d}{e} \left(x + \frac{d}{e}\right) + \frac{a e^2 + c d^2}{e^2}}} \\ & - \frac{d^4 c x}{e^3 (a e^2 + c d^2) a} \frac{1}{\sqrt{\left(x + \frac{d}{e}\right)^2 c - 2 \frac{c d}{e} \left(x + \frac{d}{e}\right) + \frac{a e^2 + c d^2}{e^2}}} \\ & + \frac{d^3}{e^2 (a e^2 + c d^2)} \ln \left( 1 \left( 2 \frac{a e^2 + c d^2}{e^2} - 2 \frac{c d}{e} \left(x + \frac{d}{e}\right) + 2 \sqrt{\frac{a e^2 + c d^2}{e^2}} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2 \frac{c d}{e} \left(x + \frac{d}{e}\right) + \frac{a e^2 + c d^2}{e^2}} \right) \left(x + \frac{d}{e}\right)^{-1} \right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(e*x+d)/(c*x^2+a)^(3/2),x)`

[Out]  $d^2/e^3*x/a/(c*x^2+a)^{(1/2)} - 1/e*x/c/(c*x^2+a)^{(1/2)} + 1/e/c^{(3/2)}*1n(x*c^{(1/2)}+(c*x^2+a)^{(1/2)})+d/e^2/c/(c*x^2+a)^{(1/2)} - d^3/e^2/(a*e^2+c*d^2)/((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^{(1/2)} - d^4/e^3/(a*e^2+c*d^2)/a/((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^{(1/2)}*c*x+d^3/e^2/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^{(1/2)}*1n((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^{(1/2)})*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^{(1/2)}/(x+d/e)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/((c*x^2 + a)^(3/2)*(e*x + d)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 4.93785, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/((c*x^2 + a)^(3/2)*(e*x + d)),x, algorithm="fricas")`

[Out]  $[-1/2*(2*(a*e^2*x - a*d*e)*\text{sqrt}(c*d^2 + a*e^2)*\text{sqrt}(c*x^2 + a)*\text{sqrt}(c) - (a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)*\text{sqrt}(c*d^2 + a*e^2)*\text{log}(-2*\text{sqrt}(c*x^2 + a)*c*x - (2*c*x^2 + a)*\text{sqrt}(c)) - (c^2*d^3*x^2 + a*c*d^3)*\text{sqrt}(c)*\text{log}(((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2)*\text{sqrt}(c*d^2 + a*e^2) - 2*(a*c*d^2*e + a^2*e^3 - (c^2*d^3 + a*c*d*e^2)*x)*\text{sqrt}(c*x^2 + a)))/(e^2*x^2$

$$\begin{aligned} & 2 + 2*d*e*x + d^2)))/((a*c^2*d^2*e + a^2*c*e^3 + (c^3*d^2*e + a*c \\ & ^2*e^3)*x^2)*sqrt(c*d^2 + a*e^2)*sqrt(c)), -1/2*(2*(a*e^2*x - a*d \\ & *e)*sqrt(-c*d^2 - a*e^2)*sqrt(c*x^2 + a)*sqrt(c) + 2*(c^2*d^3*x^2 \\ & + a*c*d^3)*sqrt(c)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)/((c \\ & *d^2 + a*e^2)*sqrt(c*x^2 + a))) - (a*c*d^2 + a^2*e^2 + (c^2*d^2 + \\ & a*c*e^2)*x^2)*sqrt(-c*d^2 - a*e^2)*log(-2*sqrt(c*x^2 + a)*c*x - \\ & (2*c*x^2 + a)*sqrt(c))/((a*c^2*d^2*e + a^2*c*e^3 + (c^3*d^2*e + \\ & a*c^2*e^3)*x^2)*sqrt(-c*d^2 - a*e^2)*sqrt(c)), -1/2*(2*(a*e^2*x - \\ & a*d*e)*sqrt(c*d^2 + a*e^2)*sqrt(c*x^2 + a)*sqrt(-c) - 2*(a*c*d^2 \\ & + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)*sqrt(c*d^2 + a*e^2)*arctan( \\ & sqrt(-c)*x/sqrt(c*x^2 + a)) - (c^2*d^3*x^2 + a*c*d^3)*sqrt(-c)*lo \\ & g(((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2 \\ & )*sqrt(c*d^2 + a*e^2) - 2*(a*c*d^2*e + a^2*e^3 - (c^2*d^3 + a*c*d \\ & *e^2)*x)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2)))/((a*c^2*d^2 \\ & *e + a^2*c*e^3 + (c^3*d^2*e + a*c^2*e^3)*x^2)*sqrt(c*d^2 + a*e^2) \\ & *sqrt(-c)), -((a*e^2*x - a*d*e)*sqrt(-c*d^2 - a*e^2)*sqrt(c*x^2 + \\ & a)*sqrt(-c) - (a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)*sqrt \\ & (-c*d^2 - a*e^2)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a)) + (c^2*d^3*x^2 \\ & + a*c*d^3)*sqrt(-c)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)/ \\ & ((c*d^2 + a*e^2)*sqrt(c*x^2 + a))))/((a*c^2*d^2*e + a^2*c*e^3 + (c \\ & ^3*d^2*e + a*c^2*e^3)*x^2)*sqrt(-c*d^2 - a*e^2)*sqrt(-c))] \end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(a + cx^2)^{\frac{3}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(e\*x+d)/(c\*x\*\*2+a)\*\*(3/2),x)

[Out] Integral(x\*\*3/((a + c\*x\*\*2)\*\*(3/2)\*(d + e\*x)), x)

**GIAC/XCAS [A]** time = 0.28136, size = 296, normalized size = 2.41

$$\begin{aligned} & \frac{2d^3 \arctan\left(-\frac{(\sqrt{cx}-\sqrt{cx^2+a})e+\sqrt{cd}}{\sqrt{-cd^2-ae^2}}\right)}{(cd^2e + ae^3)\sqrt{-cd^2 - ae^2}} \\ & - \frac{\frac{ac^2d^2e^3+a^2ce^5}{c^4d^4e^2+2ac^3d^2e^4+a^2c^2e^6}x - \frac{ac^2d^3e^2+a^2cde^4}{c^4d^4e^2+2ac^3d^2e^4+a^2c^2e^6}}{\sqrt{cx^2 + a}} - \frac{e^{(-1)}\ln\left(\left|-\sqrt{cx} + \sqrt{cx^2 + a}\right|\right)}{c^{\frac{3}{2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/((c\*x^2 + a)^(3/2)\*(e\*x + d)),x, algorithm="giac")

[Out] -2\*d^3\*arctan(-((sqrt(c)\*x - sqrt(c\*x^2 + a))\*e + sqrt(c)\*d)/sqrt(-c\*d^2 - a\*e^2))/((c\*d^2\*e + a\*e^3)\*sqrt(-c\*d^2 - a\*e^2)) - ((a\*c^2\*d^2\*e^3 + a^2\*c\*e^5)\*x/(c^4\*d^4\*e^2 + 2\*a\*c^3\*d^2\*e^4 + a^2\*c^2\*e^6) - (a\*c^2\*d^3\*e^2 + a^2\*c\*d\*e^4)/(c^4\*d^4\*e^2 + 2\*a\*c^3\*d^2\*e^4 + a^2\*c^2\*e^6))/sqrt(c\*x^2 + a) - e^(-1)\*ln(abs(-sqrt(c)\*x + sqrt(c\*x^2 + a)))/c^(3/2)

$$3.336 \quad \int \frac{x^2}{(d+ex)(a+cx^2)^{3/2}} dx$$

**Optimal.** Leaf size=95

$$-\frac{ae+cdx}{c\sqrt{a+cx^2}(ae^2+cd^2)} - \frac{d^2 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}}$$

[Out]  $-\left(\frac{a^*e + c^*d^*x}{c^*(c^*d^{\wedge}2 + a^*e^{\wedge}2)^*\text{Sqrt}[a + c^*x^{\wedge}2]}\right) - \left(\frac{d^{\wedge}2*\text{ArcTanh}\left[\frac{a^*e - c^*d^*x}{\left(\text{Sqrt}[c^*d^{\wedge}2 + a^*e^{\wedge}2]\right)^*\text{Sqrt}[a + c^*x^{\wedge}2]}\right]}{c^*d^{\wedge}2 + a^*e^{\wedge}2}\right)^{\wedge}(3/2)$

**Rubi [A]** time = 0.180078, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$-\frac{ae+cdx}{c\sqrt{a+cx^2}(ae^2+cd^2)} - \frac{d^2 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^2/((d + e\*x)\*(a + c\*x^2)^(3/2)), x]

[Out]  $-\left(\frac{a^*e + c^*d^*x}{c^*(c^*d^{\wedge}2 + a^*e^{\wedge}2)^*\text{Sqrt}[a + c^*x^{\wedge}2]}\right) - \left(\frac{d^{\wedge}2*\text{ArcTanh}\left[\frac{a^*e - c^*d^*x}{\left(\text{Sqrt}[c^*d^{\wedge}2 + a^*e^{\wedge}2]\right)^*\text{Sqrt}[a + c^*x^{\wedge}2]}\right]}{c^*d^{\wedge}2 + a^*e^{\wedge}2}\right)^{\wedge}(3/2)$

**Rubi in Sympy [A]** time = 36.3881, size = 121, normalized size = 1.27

$$-\frac{d^2 \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{\frac{3}{2}}} - \frac{1}{ce\sqrt{a+cx^2}} + \frac{d^2(ae+cdx)}{ae^2\sqrt{a+cx^2}(ae^2+cd^2)} - \frac{dx}{ae^2\sqrt{a+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(e\*x+d)/(c\*x\*\*2+a)\*\*(3/2), x)

[Out]  $-d^{\wedge}2*\operatorname{atanh}\left(\frac{a^*e - c^*d^*x}{\left(\text{sqrt}(a + c^*x^{\wedge}2)\right)^*\text{sqrt}(a^*e^{\wedge}2 + c^*d^{\wedge}2)}\right) / \left(a^*e^{\wedge}2 + c^*d^{\wedge}2\right)^{\wedge}(3/2) - 1 / \left(c^*e*\text{sqrt}(a + c^*x^{\wedge}2)\right) + d^{\wedge}2*\left(a^*e + c^*d^*x\right) / \left(a^*e^{\wedge}2*\text{sqrt}(a + c^*x^{\wedge}2)\right)^*(a^*e^{\wedge}2 + c^*d^{\wedge}2) - d^*x / \left(a^*e^{\wedge}2*\text{sqrt}(a + c^*x^{\wedge}2)\right)$

**Mathematica [A]** time = 0.248969, size = 121, normalized size = 1.27

$$\frac{-ae-cdx}{c\sqrt{a+cx^2}(ae^2+cd^2)} - \frac{d^2 \log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2} + ae - cdx\right)}{(ae^2+cd^2)^{3/2}} + \frac{d^2 \log(d+ex)}{(ae^2+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((d + e\*x)\*(a + c\*x^2)^(3/2)), x]

[Out]  $\left(-\frac{a^*e - c^*d^*x}{c^*(c^*d^{\wedge}2 + a^*e^{\wedge}2)^*\text{Sqrt}[a + c^*x^{\wedge}2]}\right) + \left(\frac{d^{\wedge}2*\text{Log}[d + e^*x]}{c^*d^{\wedge}2 + a^*e^{\wedge}2}\right)^{\wedge}(3/2) - \left(\frac{d^{\wedge}2*\text{Log}[a^*e - c^*d^*x + \text{Sqrt}[c^*d^{\wedge}2 + a^*e^{\wedge}2]]}{c^*d^{\wedge}2 + a^*e^{\wedge}2}\right)^{\wedge}(3/2)$

$$2 + a \cdot e^2] \cdot \text{Sqrt}[a + c \cdot x^2]] / (c \cdot d^2 + a \cdot e^2)^{(3/2)}$$

**Maple [B]** time = 0.014, size = 311, normalized size = 3.3

$$\begin{aligned} & -\frac{1}{ce} \frac{1}{\sqrt{cx^2+a}} - \frac{dx}{ae^2} \frac{1}{\sqrt{cx^2+a}} + \frac{d^2}{e(ae^2+cd^2)} \frac{1}{\sqrt{\left(x+\frac{d}{e}\right)^2 c - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + \frac{ae^2+cd^2}{e^2}}} \\ & + \frac{cd^3x}{e^2(ae^2+cd^2)a} \frac{1}{\sqrt{\left(x+\frac{d}{e}\right)^2 c - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + \frac{ae^2+cd^2}{e^2}}} \\ & - \frac{d^2}{e(ae^2+cd^2)} \ln\left(1\left(2\frac{ae^2+cd^2}{e^2} - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + 2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2 c - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + \frac{ae^2+cd^2}{e^2}}\right)\left(x+\frac{d}{e}\right)^{-1}\right) \frac{1}{\sqrt{\left(x+\frac{d}{e}\right)^2 c - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + \frac{ae^2+cd^2}{e^2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(e\*x+d)/(c\*x^2+a)^(3/2), x)

[Out] 
$$-1/e/c/(c \cdot x^2+a)^{(1/2)} - 1/e^2 \cdot d \cdot x/a/(c \cdot x^2+a)^{(1/2)} + d^2/e/(a \cdot e^2+c \cdot d^2)/((x+d/e)^2 \cdot c - 2 \cdot c \cdot d/e \cdot (x+d/e) + (a \cdot e^2+c \cdot d^2)/e^2)^{(1/2)} + d^3/e^2/(a \cdot e^2+c \cdot d^2)/a/((x+d/e)^2 \cdot c - 2 \cdot c \cdot d/e \cdot (x+d/e) + (a \cdot e^2+c \cdot d^2)/e^2)^{(1/2)} \cdot c \cdot x - d^2/e/(a \cdot e^2+c \cdot d^2)/((a \cdot e^2+c \cdot d^2)/e^2)^{(1/2)} \cdot \ln\left(\frac{2 \cdot (a \cdot e^2+c \cdot d^2)/e^2 - 2 \cdot c \cdot d/e \cdot (x+d/e) + 2 \cdot \sqrt{(a \cdot e^2+c \cdot d^2)/e^2} \cdot \sqrt{(x+d/e)^2 \cdot c - 2 \cdot c \cdot d/e \cdot (x+d/e) + (a \cdot e^2+c \cdot d^2)/e^2}}{(x+d/e)^{-1}}\right)$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((c\*x^2 + a)^(3/2)\*(e\*x + d)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.336172, size = 1, normalized size = 0.01

$$\left[ \frac{2\sqrt{cd^2+ae^2}(cdx+ae)\sqrt{cx^2+a} - (c^2d^2x^2+acd^2) \log\left(\frac{(2acdex-acd^2-2a^2e^2-(2c^2d^2+ace^2)x^2)\sqrt{cd^2+ae^2}+2(acd^2e+a^2e^3-(c^2d^3+acd^2e)x^2)\sqrt{cd^2+ae^2}}{e^2x^2+2dex+d^2}\right)}{2(ac^2d^2+a^2ce^2+(c^3d^2+ac^2e^2)x^2)\sqrt{cd^2+ae^2}} \right. \\ \left. - \frac{\sqrt{-cd^2-ae^2}(cdx+ae)\sqrt{cx^2+a} - (c^2d^2x^2+acd^2) \arctan\left(\frac{\sqrt{-cd^2-ae^2}(cdx-ae)}{(cd^2+ae^2)\sqrt{cx^2+a}}\right)}{(ac^2d^2+a^2ce^2+(c^3d^2+ac^2e^2)x^2)\sqrt{-cd^2-ae^2}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((c\*x^2 + a)^(3/2)\*(e\*x + d)),x, algorithm="fricas")

[Out] 
$$[-1/2 \cdot (2 \cdot \text{sqrt}(c \cdot d^2 + a \cdot e^2)) \cdot (c \cdot d \cdot x + a \cdot e) \cdot \text{sqrt}(c \cdot x^2 + a) - (c^2 \cdot d^2 \cdot x^2 + a \cdot c \cdot d^2) \cdot \log(((2 \cdot a \cdot c \cdot d \cdot e \cdot x - a \cdot c \cdot d^2 - 2 \cdot a^2 \cdot e^2 - (2 \cdot c^2 \cdot d^2 + a \cdot c \cdot e^2) \cdot x^2) \cdot \text{sqrt}(c \cdot d^2 + a \cdot e^2) + 2 \cdot (a \cdot c \cdot d^2 \cdot e + a^2 \cdot e^3 - (c^2 \cdot d^3 + a \cdot c \cdot d^2 \cdot e) \cdot x) \cdot \text{sqrt}(c \cdot x^2 + a)) / (e^2 \cdot x^2 + 2 \cdot d \cdot e \cdot x + d^2)) / ((a \cdot c^2 \cdot d^2 + a^2 \cdot c \cdot e^2 + (c^3 \cdot d^2 + a \cdot c^2 \cdot e^2) \cdot x^2) \cdot \text{sqrt}(c \cdot d^2 + a \cdot e^2)) - (c^2 \cdot d^2 \cdot x^2 + a \cdot c \cdot d^2) \cdot \arctan\left(\frac{\sqrt{-c \cdot d^2 - a \cdot e^2} \cdot (c \cdot d \cdot x - a \cdot e)}{(c \cdot d^2 + a \cdot e^2) \cdot \text{sqrt}(c \cdot x^2 + a)}\right) / ((a \cdot c^2 \cdot d^2 + a^2 \cdot c \cdot e^2 + (c^3 \cdot d^2 + a \cdot c^2 \cdot e^2) \cdot x^2) \cdot \text{sqrt}(-c \cdot d^2 - a \cdot e^2))]$$

$\text{qrt}(c*d^2 + a*e^2)), -(\text{sqrt}(-c*d^2 - a*e^2)*(c*d*x + a*e)*\text{sqrt}(c*x^2 + a) - (c^2*d^2*x^2 + a*c*d^2)*\text{arctan}(\text{sqrt}(-c*d^2 - a*e^2)*(c*d*x - a*e)/((c*d^2 + a*e^2)*\text{sqrt}(c*x^2 + a))))/((a*c^2*d^2 + a^2*c*e^2 + (c^3*d^2 + a*c^2*e^2)*x^2)*\text{sqrt}(-c*d^2 - a*e^2))]$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(a + cx^2)^{\frac{3}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(e\*x+d)/(c\*x\*\*2+a)\*\*(3/2), x)

[Out] Integral(x\*\*2/((a + c\*x\*\*2)\*\*(3/2)\*(d + e\*x)), x)

**GIAC/XCAS [A]** time = 0.272045, size = 235, normalized size = 2.47

$$-\frac{2d^2 \arctan\left(\frac{(\sqrt{cx}-\sqrt{cx^2+a})e+\sqrt{cd}}{\sqrt{-cd^2-ae^2}}\right)}{(cd^2 + ae^2)\sqrt{-cd^2 - ae^2}} - \frac{\frac{(c^2d^3+acde^2)x}{c^3d^4+2ac^2d^2e^2+a^2ce^4} + \frac{acd^2e+a^2e^3}{c^3d^4+2ac^2d^2e^2+a^2ce^4}}{\sqrt{cx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((c\*x^2 + a)^(3/2)\*(e\*x + d)),x, algorithm="giac")

[Out] -2\*d^2\*arctan(((sqrt(c)\*x - sqrt(c\*x^2 + a))\*e + sqrt(c)\*d)/sqrt(-c\*d^2 - a\*e^2))/((c\*d^2 + a\*e^2)\*sqrt(-c\*d^2 - a\*e^2)) - ((c^2\*d^3 + a\*c\*d\*e^2)\*x/(c^3\*d^4 + 2\*a\*c^2\*d^2\*e^2 + a^2\*c\*e^4) + (a\*c\*d^2\*e + a^2\*e^3)/(c^3\*d^4 + 2\*a\*c^2\*d^2\*e^2 + a^2\*c\*e^4))/sqrt(c\*x^2 + a)

$$3.337 \quad \int \frac{x}{(d+ex)(a+cx^2)^{3/2}} dx$$

**Optimal.** Leaf size=88

$$\frac{de \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}} - \frac{d-ex}{\sqrt{a+cx^2}(ae^2+cd^2)}$$

[Out]  $-\left(\frac{d - e*x}{(c*d^2 + a*e^2)*\text{Sqrt}[a + c*x^2]}\right) + \left(\frac{d*e*\text{ArcTanh}\left[\frac{a*e - c*d*x}{\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2]}\right]}{(c*d^2 + a*e^2)^{3/2}}\right)$

**Rubi [A]** time = 0.142079, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{de \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}} - \frac{d-ex}{\sqrt{a+cx^2}(ae^2+cd^2)}$$

Antiderivative was successfully verified.

[In] Int[x/((d + e\*x)\*(a + c\*x^2)^(3/2)), x]

[Out]  $-\left(\frac{d - e*x}{(c*d^2 + a*e^2)*\text{Sqrt}[a + c*x^2]}\right) + \left(\frac{d*e*\text{ArcTanh}\left[\frac{a*e - c*d*x}{\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2]}\right]}{(c*d^2 + a*e^2)^{3/2}}\right)$

**Rubi in Sympy [A]** time = 23.8604, size = 75, normalized size = 0.85

$$\frac{de \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}} - \frac{d-ex}{\sqrt{a+cx^2}(ae^2+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x/(e\*x+d)/(c\*x\*\*2+a)\*\*(3/2), x)

[Out]  $d*e*\operatorname{atanh}\left(\frac{a*e - c*d*x}{\text{sqrt}(a + c*x**2)*\text{sqrt}(a*e**2 + c*d**2)}\right) / (a*e**2 + c*d**2)**(3/2) - (d - e*x) / (\text{sqrt}(a + c*x**2)*(a*e**2 + c*d**2))$

**Mathematica [A]** time = 0.208528, size = 113, normalized size = 1.28

$$\frac{ex-d}{\sqrt{a+cx^2}(ae^2+cd^2)} + \frac{de \log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2} + ae - cdx\right)}{(ae^2+cd^2)^{3/2}} - \frac{de \log(d+ex)}{(ae^2+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x/((d + e\*x)\*(a + c\*x^2)^(3/2)), x]

[Out]  $\left(-\frac{d + e*x}{(c*d^2 + a*e^2)*\text{Sqrt}[a + c*x^2]} - \frac{d*e*\text{Log}[d + e*x]}{(c*d^2 + a*e^2)^{3/2}} + \frac{d*e*\text{Log}\left[\frac{a*e - c*d*x + \text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2]}{(c*d^2 + a*e^2)^{3/2}}\right]}{(c*d^2 + a*e^2)^{3/2}}\right)$



**Maple [B]** time = 0.018, size = 283, normalized size = 3.2

$$\frac{x}{ae\sqrt{cx^2+a}} - \frac{d}{ae^2+cd^2} \frac{1}{\sqrt{\left(x+\frac{d}{e}\right)^2 c - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + \frac{ae^2+cd^2}{e^2}}}$$

$$- \frac{cd^2x}{e(ae^2+cd^2)a} \frac{1}{\sqrt{\left(x+\frac{d}{e}\right)^2 c - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + \frac{ae^2+cd^2}{e^2}}}$$

$$+ \frac{d}{ae^2+cd^2} \ln\left(1\left(2\frac{ae^2+cd^2}{e^2} - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + 2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2 c - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + \frac{ae^2+cd^2}{e^2}}\right)\left(x+\frac{d}{e}\right)^{-1}\right) \frac{1}{\sqrt{\frac{ae^2+cd^2}{e^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(e\*x+d)/(c\*x^2+a)^(3/2),x)

[Out] 1/e\*x/a/(c\*x^2+a)^(1/2)-d/(a\*e^2+c\*d^2)/((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2)-d^2/e/(a\*e^2+c\*d^2)/a/((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2)\*c\*x+d/(a\*e^2+c\*d^2)/((a\*e^2+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2+c\*d^2)/e^2-2\*c\*d/e\*(x+d/e)+2\*((a\*e^2+c\*d^2)/e^2)^(1/2)\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2))/(x+d/e))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((c\*x^2 + a)^(3/2)\*(e\*x + d)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.340594, size = 1, normalized size = 0.01

$$\frac{2\sqrt{cd^2+ae^2}\sqrt{cx^2+a}(ex-d) + (cdex^2+ade)\log\left(\frac{(2acdex-acd^2-2a^2e^2-(2c^2d^2+ace^2)x^2)\sqrt{cd^2+ae^2}-2(acd^2e+a^2e^3-(c^2d^3+acde^2)x)}{e^2x^2+2dex+d^2}\right)}{2(acd^2+a^2e^2+(c^2d^2+ace^2)x^2)\sqrt{cd^2+ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((c\*x^2 + a)^(3/2)\*(e\*x + d)),x, algorithm="fricas")

[Out] [1/2\*(2\*sqrt(c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a)\*(e\*x - d) + (c\*d\*e\*x^2 + a\*d\*e)\*log(((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2)\*sqrt(c\*d^2 + a\*e^2) - 2\*(a\*c\*d^2\*e + a^2\*e^3 - (c^2\*d^3 + a\*c\*d\*e^2)\*x)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)))/((a\*c\*d^2 + a^2\*e^2 + (c^2\*d^2 + a\*c\*e^2)\*x^2)\*sqrt(c\*d^2 + a\*e^2)), (sqrt(-c\*d^2 - a\*e^2)\*sqrt(c\*x^2 + a)\*(e\*x - d) - (c\*d\*e\*x^2 + a\*d\*e)\*arctan(sqrt(-c\*d^2 - a\*e^2)\*(c\*d\*x - a\*e)/((c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a))))/((a\*c\*d^2 + a^2\*e^2 + (c^2\*d^2 + a\*c\*e^2)\*x^2)\*sqrt(-c\*d^2 - a\*e^2))]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(a + cx^2)^{\frac{3}{2}} (d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e\*x+d)/(c\*x\*\*2+a)\*\*(3/2), x)

[Out] Integral(x/((a + c\*x\*\*2)\*\*(3/2)\*(d + e\*x)), x)

**GIAC/XCAS [A]** time = 0.274818, size = 219, normalized size = 2.49

$$\frac{2d \arctan\left(\frac{(\sqrt{cx}-\sqrt{cx^2+a})e+\sqrt{cd}}{\sqrt{-cd^2-ae^2}}\right)e}{(cd^2 + ae^2)\sqrt{-cd^2 - ae^2}} + \frac{\frac{(cd^2e+ae^3)x}{c^2d^4+2acd^2e^2+a^2e^4} - \frac{cd^3+ade^2}{c^2d^4+2acd^2e^2+a^2e^4}}{\sqrt{cx^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((c\*x^2 + a)^(3/2)\*(e\*x + d)),x, algorithm="giac")

[Out] 2\*d\*arctan(((sqrt(c)\*x - sqrt(c\*x^2 + a))\*e + sqrt(c)\*d)/sqrt(-c\*d^2 - a\*e^2))\*e/((c\*d^2 + a\*e^2)\*sqrt(-c\*d^2 - a\*e^2)) + ((c\*d^2\*e + a\*e^3)\*x/(c^2\*d^4 + 2\*a\*c\*d^2\*e^2 + a^2\*e^4) - (c\*d^3 + a\*d\*e^2)/(c^2\*d^4 + 2\*a\*c\*d^2\*e^2 + a^2\*e^4))/sqrt(c\*x^2 + a)

$$3.338 \quad \int \frac{1}{(d+ex)(a+cx^2)^{3/2}} dx$$

**Optimal.** Leaf size=94

$$\frac{ae + cdx}{a\sqrt{a + cx^2}(ae^2 + cd^2)} - \frac{e^2 \tanh^{-1}\left(\frac{ae - cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2 + cd^2)^{3/2}}$$

[Out] (a\*e + c\*d\*x)/(a\*(c\*d^2 + a\*e^2)\*Sqrt[a + c\*x^2]) - (e^2\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])])/(c\*d^2 + a\*e^2)^(3/2)

**Rubi [A]** time = 0.105669, antiderivative size = 94, normalized size of antiderivative = 1., number of rules used = 4, number of rules used = 4, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.21$

$$\frac{ae + cdx}{a\sqrt{a + cx^2}(ae^2 + cd^2)} - \frac{e^2 \tanh^{-1}\left(\frac{ae - cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2 + cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)\*(a + c\*x^2)^(3/2)), x]

[Out] (a\*e + c\*d\*x)/(a\*(c\*d^2 + a\*e^2)\*Sqrt[a + c\*x^2]) - (e^2\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])])/(c\*d^2 + a\*e^2)^(3/2)

**Rubi in Sympy [A]** time = 19.6743, size = 80, normalized size = 0.85

$$-\frac{e^2 \operatorname{atanh}\left(\frac{ae - cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2 + cd^2)^{3/2}} + \frac{ae + cdx}{a\sqrt{a + cx^2}(ae^2 + cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(e\*x+d)/(c\*x\*\*2+a)\*\*(3/2), x)

[Out] -e\*\*2\*atanh((a\*e - c\*d\*x)/(sqrt(a + c\*x\*\*2)\*sqrt(a\*e\*\*2 + c\*d\*\*2)))/(a\*e\*\*2 + c\*d\*\*2)\*\*(3/2) + (a\*e + c\*d\*x)/(a\*sqrt(a + c\*x\*\*2)\*(a\*e\*\*2 + c\*d\*\*2))

**Mathematica [A]** time = 0.188844, size = 119, normalized size = 1.27

$$\frac{ae + cdx}{a\sqrt{a + cx^2}(ae^2 + cd^2)} - \frac{e^2 \log\left(\sqrt{a + cx^2}\sqrt{ae^2 + cd^2} + ae - cdx\right)}{(ae^2 + cd^2)^{3/2}} + \frac{e^2 \log(d + ex)}{(ae^2 + cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x)\*(a + c\*x^2)^(3/2)), x]

[Out] (a\*e + c\*d\*x)/(a\*(c\*d^2 + a\*e^2)\*Sqrt[a + c\*x^2]) + (e^2\*Log[d + e\*x])/(c\*d^2 + a\*e^2)^(3/2) - (e^2\*Log[a\*e - c\*d\*x + Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]])/(c\*d^2 + a\*e^2)^(3/2)

**Maple [B]** time = 0.01, size = 260, normalized size = 2.8

$$\frac{e}{ae^2 + cd^2} \frac{1}{\sqrt{\left(x + \frac{d}{e}\right)^2 c - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}}} + \frac{cdx}{(ae^2 + cd^2)a} \frac{1}{\sqrt{\left(x + \frac{d}{e}\right)^2 c - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}}} - \frac{e}{ae^2 + cd^2} \ln \left( 1 \left( 2 \frac{ae^2 + cd^2}{e^2} - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + 2 \sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}} \right) \left(x + \frac{d}{e}\right)^{-1} \right) \frac{1}{\sqrt{\frac{ae^2 + cd^2}{e^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x+d)/(c\*x^2+a)^(3/2), x)

[Out] e/(a\*e^2+c\*d^2)/((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2)+d/(a\*e^2+c\*d^2)/a/((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2)\*c\*x-e/(a\*e^2+c\*d^2)/((a\*e^2+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2+c\*d^2)/e^2-2\*c\*d/e\*(x+d/e)+2\*((a\*e^2+c\*d^2)/e^2)^(1/2)\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2))/(x+d/e))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + a)^(3/2)\*(e\*x + d)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.335282, size = 1, normalized size = 0.01

$$\frac{2\sqrt{cd^2 + ae^2}(cdx + ae)\sqrt{cx^2 + a} + (ace^2x^2 + a^2e^2) \log\left(\frac{(2acdex - acd^2 - 2a^2e^2 - (2c^2d^2 + ace^2)x^2)\sqrt{cd^2 + ae^2} + 2(acd^2e + a^2e^3 - (c^2d^3 + acde^2)e^2x^2 + 2dex + d^2)}{e^2x^2 + 2dex + d^2}\right)}{2(a^2cd^2 + a^3e^2 + (ac^2d^2 + a^2ce^2)x^2)\sqrt{cd^2 + ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + a)^(3/2)\*(e\*x + d)), x, algorithm="fricas")

[Out] [1/2\*(2\*sqrt(c\*d^2 + a\*e^2)\*(c\*d\*x + a\*e)\*sqrt(c\*x^2 + a) + (a\*c\*e^2\*x^2 + a^2\*e^2)\*log(((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2)\*sqrt(c\*d^2 + a\*e^2) + 2\*(a\*c\*d^2\*e + a^2\*e^3 - (c^2\*d^3 + a\*c\*d\*e^2)\*x)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)))/((a^2\*c\*d^2 + a^3\*e^2 + (a\*c^2\*d^2 + a^2\*c\*e^2)\*x^2)\*sqrt(c\*d^2 + a\*e^2)), (sqrt(-c\*d^2 - a\*e^2)\*(c\*d\*x + a\*e)\*sqrt(c\*x^2 + a) + (a\*c\*e^2\*x^2 + a^2\*e^2)\*arctan(sqrt(-c\*d^2 - a\*e^2)\*(c\*d\*x - a\*e)/((c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a))))/((a^2\*c\*d^2 + a^3\*e^2 + (a\*c^2\*d^2 + a^2\*c\*e^2)\*x^2)\*sqrt(-c\*d^2 - a\*e^2))]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + cx^2)^{\frac{3}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x+d)/(c\*x\*\*2+a)\*\*(3/2),x)

[Out] Integral(1/((a + c\*x\*\*2)\*\*(3/2)\*(d + e\*x)), x)

**GIAC/XCAS [A]** time = 0.300377, size = 232, normalized size = 2.47

$$\frac{\frac{(c^2d^3+acde^2)x}{ac^2d^4+2a^2cd^2e^2+a^3e^4} + \frac{acd^2e+a^2e^3}{ac^2d^4+2a^2cd^2e^2+a^3e^4}}{\sqrt{cx^2+a}} - \frac{2 \arctan\left(\frac{(\sqrt{c}x-\sqrt{cx^2+a})e+\sqrt{cd}}{\sqrt{-cd^2-ae^2}}\right) e^2}{(cd^2+ae^2)\sqrt{-cd^2-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + a)^(3/2)\*(e\*x + d)),x, algorithm="giac")

[Out] ((c^2\*d^3 + a\*c\*d\*e^2)\*x/(a\*c^2\*d^4 + 2\*a^2\*c\*d^2\*e^2 + a^3\*e^4) + (a\*c\*d^2\*e + a^2\*e^3)/(a\*c^2\*d^4 + 2\*a^2\*c\*d^2\*e^2 + a^3\*e^4))/sqrt(c\*x^2 + a) - 2\*arctan(((sqrt(c)\*x - sqrt(c\*x^2 + a))\*e + sqrt(c)\*d)/sqrt(-c\*d^2 - a\*e^2))\*e^2/((c\*d^2 + a\*e^2)\*sqrt(-c\*d^2 - a\*e^2))

$$3.339 \quad \int \frac{1}{x(d+ex)(a+cx^2)^{3/2}} dx$$

**Optimal.** Leaf size=147

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{e(ae+cdx)}{ad\sqrt{a+cx^2}(ae^2+cd^2)} + \frac{e^3 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d(ae^2+cd^2)^{3/2}} + \frac{1}{ad\sqrt{a+cx^2}}$$

[Out] 1/(a\*d\*Sqrt[a + c\*x^2]) - (e\*(a\*e + c\*d\*x))/(a\*d\*(c\*d^2 + a\*e^2)\*Sqrt[a + c\*x^2]) + (e^3\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]))/(d\*(c\*d^2 + a\*e^2)^(3/2)) - ArcTanh[Sqrt[a + c\*x^2]/Sqrt[a]]/(a^(3/2)\*d)

**Rubi [A]** time = 0.325103, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{e(ae+cdx)}{ad\sqrt{a+cx^2}(ae^2+cd^2)} + \frac{e^3 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d(ae^2+cd^2)^{3/2}} + \frac{1}{ad\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(d + e\*x)\*(a + c\*x^2)^(3/2)), x]

[Out] 1/(a\*d\*Sqrt[a + c\*x^2]) - (e\*(a\*e + c\*d\*x))/(a\*d\*(c\*d^2 + a\*e^2)\*Sqrt[a + c\*x^2]) + (e^3\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]))/(d\*(c\*d^2 + a\*e^2)^(3/2)) - ArcTanh[Sqrt[a + c\*x^2]/Sqrt[a]]/(a^(3/2)\*d)

**Rubi in Sympy [A]** time = 39.3311, size = 124, normalized size = 0.84

$$\frac{e^3 \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d(ae^2+cd^2)^{3/2}} - \frac{e(ae+cdx)}{ad\sqrt{a+cx^2}(ae^2+cd^2)} + \frac{1}{ad\sqrt{a+cx^2}} - \frac{\operatorname{atanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(e\*x+d)/(c\*x\*\*2+a)\*\*(3/2), x)

[Out] e\*\*3\*atanh((a\*e - c\*d\*x)/(sqrt(a + c\*x\*\*2)\*sqrt(a\*e\*\*2 + c\*d\*\*2)))/(d\*(a\*e\*\*2 + c\*d\*\*2)\*\*(3/2)) - e\*(a\*e + c\*d\*x)/(a\*d\*sqrt(a + c\*x\*\*2)\*(a\*e\*\*2 + c\*d\*\*2)) + 1/(a\*d\*sqrt(a + c\*x\*\*2)) - atanh(sqrt(a + c\*x\*\*2)/sqrt(a))/(a\*\*(3/2)\*d)

**Mathematica [A]** time = 0.461422, size = 167, normalized size = 1.14

$$-\frac{\log\left(\sqrt{a}\sqrt{a+cx^2}+a\right)}{a^{3/2}d} + \frac{\log(x)}{a^{3/2}d} + \frac{cd-cex}{a\sqrt{a+cx^2}(ae^2+cd^2)} + \frac{e^3 \log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{d(ae^2+cd^2)^{3/2}} - \frac{e^3 \log(d+ex)}{d(ae^2+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(d + e\*x)\*(a + c\*x^2)^(3/2)), x]

[Out]  $(c*d - c*e*x)/(a*(c*d^2 + a*e^2)*\text{Sqrt}[a + c*x^2]) + \text{Log}[x]/(a^{(3/2)*d}) - (e^3*\text{Log}[d + e*x])/(d*(c*d^2 + a*e^2)^{(3/2)}) - \text{Log}[a + \text{Sqrt}[a]*\text{Sqrt}[a + c*x^2]]/(a^{(3/2)*d}) + (e^3*\text{Log}[a*e - c*d*x + \text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2]])/(d*(c*d^2 + a*e^2)^{(3/2)})$

**Maple [B]** time = 0.016, size = 318, normalized size = 2.2

$$\frac{1}{ad} \frac{1}{\sqrt{cx^2 + a}} - \frac{1}{d} \ln \left( \frac{1}{x} \left( 2a + 2\sqrt{a}\sqrt{cx^2 + a} \right) \right) a^{-\frac{3}{2}} - \frac{e^2}{d(ae^2 + cd^2)} \frac{1}{\sqrt{\left(x + \frac{d}{e}\right)^2 c - 2\frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}}}$$

$$- \frac{cex}{(ae^2 + cd^2)a} \frac{1}{\sqrt{\left(x + \frac{d}{e}\right)^2 c - 2\frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}}}$$

$$+ \frac{e^2}{d(ae^2 + cd^2)} \ln \left( 1 \left( 2\frac{ae^2 + cd^2}{e^2} - 2\frac{cd}{e} \left(x + \frac{d}{e}\right) + 2\sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2\frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}} \right) \left(x + \frac{d}{e}\right)^{-1} \right) \sqrt{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(e*x+d)/(c*x^2+a)^(3/2),x)`

[Out]  $1/a/d/(c*x^2+a)^{(1/2)} - 1/d/a^{(3/2)} * \ln((2*a+2*a^{(1/2)}*(c*x^2+a)^{(1/2)})/x) - 1/d/(a*e^2+c*d^2)*e^2/((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^{(1/2)} - e/(a*e^2+c*d^2)/a/((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^{(1/2)} * c*x + 1/d/(a*e^2+c*d^2)*e^2/((a*e^2+c*d^2)/e^2)^{(1/2)} * \ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^{(1/2)})/(x+d/e)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + a)^{\frac{3}{2}}(ex + d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((c*x^2 + a)^(3/2)*(e*x + d)*x),x, algorithm="maxima")`

[Out] `integrate(1/((c*x^2 + a)^(3/2)*(e*x + d)*x), x)`

**Fricas [A]** time = 0.543231, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((c*x^2 + a)^(3/2)*(e*x + d)*x),x, algorithm="fricas")`

[Out]  $[-1/2*(2*(c*d*e*x - c*d^2)*\text{sqrt}(c*d^2 + a*e^2)*\text{sqrt}(c*x^2 + a)*\text{sqrt}(a) - (a*c*e^3*x^2 + a^2*e^3)*\text{sqrt}(a)*\text{log}(((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2)*\text{sqrt}(c*d^2 + a*e^2) - 2*(a*c*d^2*e + a^2*e^3 - (c^2*d^3 + a*c*d*e^2)*x)*\text{sqrt}(c*x^2 + a)))/(e^2*x^2 + 2*d*e*x + d^2)) - (a*c*d^2 + a^2*e^2 + (c^2*d^2 + a*c*e^2)*x^2)*\text{sqrt}(c*d^2 + a*e^2)*\text{log}(-((c*x^2 + 2*a)*\text{sqrt}(a) - 2*\text{sqrt}(c*x^2 + a)*a)/x^2))/((a^2*c*d^3 + a^3*d*e^2 + (a*c^2*d^3 + a^2*c*d*e^2)*x^2)*\text{sqrt}(c*d^2 + a*e^2)*\text{sqrt}(a)), -1/2*(2*(c*d*e*x -$

$$\begin{aligned}
& c^2 d^2 \sqrt{-c^2 d^2 - a^2 e^2} \sqrt{c^2 x^2 + a} \sqrt{a} + 2 (a^2 c^2 e^3 x^2 + a^2 e^3) \sqrt{a} \arctan(\sqrt{-c^2 d^2 - a^2 e^2} (c^2 d x - a^2 e) / ((c^2 d^2 + a^2 e^2) \sqrt{c^2 x^2 + a})) - (a^2 c^2 d^2 + a^2 e^2 + (c^2 d^2 + a^2 e^2) x^2) \sqrt{-c^2 d^2 - a^2 e^2} \log(-((c^2 x^2 + 2 a) \sqrt{a} - 2 \sqrt{c^2 x^2 + a} a) / x^2) / ((a^2 c^2 d^3 + a^3 d^2 e^2 + (a^2 c^2 d^3 + a^2 c^2 d^2 e^2) x^2) \sqrt{-c^2 d^2 - a^2 e^2} \sqrt{a}), -1/2 (2 (c^2 d^2 e x - c^2 d^2) \sqrt{c^2 d^2 + a^2 e^2} \sqrt{c^2 x^2 + a} \sqrt{-a} + 2 (a^2 c^2 d^2 + a^2 e^2 + (c^2 d^2 + a^2 e^2) x^2) \sqrt{c^2 d^2 + a^2 e^2} \arctan(\sqrt{-a} / \sqrt{c^2 x^2 + a})) - (a^2 c^2 e^3 x^2 + a^2 e^3) \sqrt{-a} \log(((2 a^2 c^2 d^2 e x - a^2 c^2 d^2 - 2 a^2 e^2 - (2 c^2 d^2 + a^2 e^2) x^2) \sqrt{c^2 d^2 + a^2 e^2} - 2 (a^2 c^2 d^2 e + a^2 e^3 - (c^2 d^3 + a^2 c^2 d^2 e^2) x) \sqrt{c^2 x^2 + a}) / (e^2 x^2 + 2 d^2 e x + d^2)) / ((a^2 c^2 d^3 + a^3 d^2 e^2 + (a^2 c^2 d^3 + a^2 c^2 d^2 e^2) x^2) \sqrt{c^2 d^2 + a^2 e^2} \sqrt{-a}), -((c^2 d^2 e x - c^2 d^2) \sqrt{-c^2 d^2 - a^2 e^2} \sqrt{c^2 x^2 + a} \sqrt{-a} + (a^2 c^2 e^3 x^2 + a^2 e^3) \sqrt{-a} \arctan(\sqrt{-c^2 d^2 - a^2 e^2} (c^2 d x - a^2 e) / ((c^2 d^2 + a^2 e^2) \sqrt{c^2 x^2 + a})) + (a^2 c^2 d^2 + a^2 e^2 + (c^2 d^2 + a^2 e^2) x^2) \sqrt{-c^2 d^2 - a^2 e^2} \arctan(\sqrt{-a} / \sqrt{c^2 x^2 + a})) / ((a^2 c^2 d^3 + a^3 d^2 e^2 + (a^2 c^2 d^3 + a^2 c^2 d^2 e^2) x^2) \sqrt{-c^2 d^2 - a^2 e^2} \sqrt{-a})]
\end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x(a + cx^2)^{\frac{3}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e\*x+d)/(c\*x\*\*2+a)\*\*(3/2),x)

[Out] Integral(1/(x\*(a + c\*x\*\*2)\*\*(3/2)\*(d + e\*x)), x)

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + a)^(3/2)\*(e\*x + d)\*x),x, algorithm="giac")

[Out] Exception raised: TypeError



$$3.340 \quad \int \frac{1}{x^2(d+ex)(a+cx^2)^{3/2}} dx$$

**Optimal.** Leaf size=194

$$\frac{e \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2} - \frac{2cx}{a^2d\sqrt{a+cx^2}} + \frac{e^2(ae+cdx)}{ad^2\sqrt{a+cx^2}(ae^2+cd^2)}$$

$$- \frac{e^4 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2(ae^2+cd^2)^{3/2}} - \frac{e}{ad^2\sqrt{a+cx^2}} - \frac{1}{adx\sqrt{a+cx^2}}$$

[Out]  $-(e/(a*d^2*\text{Sqrt}[a + c*x^2])) - 1/(a*d*x*\text{Sqrt}[a + c*x^2]) - (2*c*x)/(a^2*d*\text{Sqrt}[a + c*x^2]) + (e^2*(a*e + c*d*x))/(a*d^2*(c*d^2 + a*e^2)*\text{Sqrt}[a + c*x^2]) - (e^4*\text{ArcTanh}[(a*e - c*d*x)/(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2])])/(d^2*(c*d^2 + a*e^2)^(3/2)) + (e*\text{ArcTanh}[\text{Sqrt}[a + c*x^2]/\text{Sqrt}[a]])/(a^(3/2)*d^2)$

**Rubi [A]** time = 0.401405, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{e \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2} - \frac{2cx}{a^2d\sqrt{a+cx^2}} + \frac{e^2(ae+cdx)}{ad^2\sqrt{a+cx^2}(ae^2+cd^2)}$$

$$- \frac{e^4 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2(ae^2+cd^2)^{3/2}} - \frac{e}{ad^2\sqrt{a+cx^2}} - \frac{1}{adx\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^2*(d + e*x)*(a + c*x^2)^(3/2)), x]$

[Out]  $-(e/(a*d^2*\text{Sqrt}[a + c*x^2])) - 1/(a*d*x*\text{Sqrt}[a + c*x^2]) - (2*c*x)/(a^2*d*\text{Sqrt}[a + c*x^2]) + (e^2*(a*e + c*d*x))/(a*d^2*(c*d^2 + a*e^2)*\text{Sqrt}[a + c*x^2]) - (e^4*\text{ArcTanh}[(a*e - c*d*x)/(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2])])/(d^2*(c*d^2 + a*e^2)^(3/2)) + (e*\text{ArcTanh}[\text{Sqrt}[a + c*x^2]/\text{Sqrt}[a]])/(a^(3/2)*d^2)$

**Rubi in Sympy [A]** time = 46.8053, size = 172, normalized size = 0.89

$$-\frac{e^4 \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2(ae^2+cd^2)^{3/2}} - \frac{1}{adx\sqrt{a+cx^2}} + \frac{e^2(ae+cdx)}{ad^2\sqrt{a+cx^2}(ae^2+cd^2)}$$

$$- \frac{e}{ad^2\sqrt{a+cx^2}} - \frac{2cx}{a^2d\sqrt{a+cx^2}} + \frac{e \operatorname{atanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x^{**2}/(e*x+d)/(c*x^{**2}+a)^{(3/2)}, x)$

[Out]  $-e^{*4}*\operatorname{atanh}((a*e - c*d*x)/(\text{sqrt}(a + c*x^{**2})*\text{sqrt}(a*e^{**2} + c*d^{**2}))) / (d^{**2}*(a*e^{**2} + c*d^{**2})^{**}(3/2)) - 1/(a*d*x*\text{sqrt}(a + c*x^{**2})) + e^{*2}*(a*e + c*d*x)/(a*d^{**2}*\text{sqrt}(a + c*x^{**2})*(a*e^{**2} + c*d^{**2})) - e/(a*d^{**2}*\text{sqrt}(a + c*x^{**2})) - 2*c*x/(a^{**2}*d*\text{sqrt}(a + c*x^{**2})) + e*\operatorname{atanh}(\text{sqrt}(a + c*x^{**2})/\text{sqrt}(a))/(a^{**}(3/2)*d^{**2})$

**Mathematica [A]** time = 0.589887, size = 188, normalized size = 0.97

$$\frac{e \log\left(\sqrt{a}\sqrt{a+cx^2}+a\right)}{a^{3/2}d^2} - \frac{e \log(x)}{a^{3/2}d^2} - \frac{\sqrt{a+cx^2}\left(\frac{c(ae+cdx)}{(a+cx^2)(ae^2+cd^2)} + \frac{1}{dx}\right)}{a^2}$$

$$- \frac{e^4 \log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{d^2(ae^2+cd^2)^{3/2}} + \frac{e^4 \log(d+ex)}{d^2(ae^2+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(d + e\*x)\*(a + c\*x^2)^(3/2)), x]

[Out] -((Sqrt[a + c\*x^2]\*(1/(d\*x) + (c\*(a\*e + c\*d\*x))/((c\*d^2 + a\*e^2)\*(a + c\*x^2))))/a^2) - (e\*Log[x])/(a^(3/2)\*d^2) + (e^4\*Log[d + e\*x])/(d^2\*(c\*d^2 + a\*e^2)^(3/2)) + (e\*Log[a + Sqrt[a]\*Sqrt[a + c\*x^2]])/(a^(3/2)\*d^2) - (e^4\*Log[a\*e - c\*d\*x + Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]])/(d^2\*(c\*d^2 + a\*e^2)^(3/2))

**Maple [B]** time = 0.017, size = 363, normalized size = 1.9

$$-\frac{1}{adx} \frac{1}{\sqrt{cx^2+a}} - 2 \frac{cx}{a^2 d \sqrt{cx^2+a}} + \frac{e^3}{d^2 (ae^2+cd^2)} \frac{1}{\sqrt{\left(x+\frac{d}{e}\right)^2 c - 2 \frac{cd}{e} \left(x+\frac{d}{e}\right) + \frac{ae^2+cd^2}{e^2}}}$$

$$+ \frac{e^2 cx}{d (ae^2+cd^2) a} \frac{1}{\sqrt{\left(x+\frac{d}{e}\right)^2 c - 2 \frac{cd}{e} \left(x+\frac{d}{e}\right) + \frac{ae^2+cd^2}{e^2}}}$$

$$- \frac{e^3}{d^2 (ae^2+cd^2)} \ln\left(1\left(2 \frac{ae^2+cd^2}{e^2} - 2 \frac{cd}{e} \left(x+\frac{d}{e}\right) + 2 \sqrt{\frac{ae^2+cd^2}{e^2}} \sqrt{\left(x+\frac{d}{e}\right)^2 c - 2 \frac{cd}{e} \left(x+\frac{d}{e}\right) + \frac{ae^2+cd^2}{e^2}}\right)\left(x+\frac{d}{e}\right)^{-1}\right)$$

$$- \frac{e}{ad^2} \frac{1}{\sqrt{cx^2+a}} + \frac{e}{d^2} \ln\left(\frac{1}{x} \left(2a + 2\sqrt{a}\sqrt{cx^2+a}\right)\right) a^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(e\*x+d)/(c\*x^2+a)^(3/2), x)

[Out] -1/a/d/x/(c\*x^2+a)^(1/2) - 2\*c\*x/a^2/d/(c\*x^2+a)^(1/2) + e^3/d^2/(a\*e^2+c\*d^2)/((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2) + e^2/d/(a\*e^2+c\*d^2)/a/((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2)\*c\*x - e^3/d^2/(a\*e^2+c\*d^2)/((a\*e^2+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2+c\*d^2)/e^2-2\*c\*d/e\*(x+d/e)+2\*((a\*e^2+c\*d^2)/e^2)^(1/2)\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2))/(x+d/e)) - e/a/d^2/(c\*x^2+a)^(1/2) + e/d^2/a^(3/2)\*ln((2\*a+2\*a^(1/2)\*(c\*x^2+a)^(1/2))/x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2+a)^{\frac{3}{2}}(ex+d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + a)^(3/2)\*(e\*x + d)\*x^2), x, algorithm="maxima")

[Out] integrate(1/((c\*x^2 + a)^(3/2)\*(e\*x + d)\*x^2), x)

---

**Fricas [A]** time = 0.534645, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + a)^(3/2)\*(e\*x + d)\*x^2),x, algorithm="fricas")

[Out] [-1/2\*(2\*(a\*c\*d^2\*e\*x + a\*c\*d^3 + a^2\*d\*e^2 + (2\*c^2\*d^3 + a\*c\*d\*e^2)\*x^2)\*sqrt(c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a)\*sqrt(a) - (a^2\*c\*e^4\*x^3 + a^3\*e^4\*x)\*sqrt(a)\*log(((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2)\*sqrt(c\*d^2 + a\*e^2) + 2\*(a\*c\*d^2\*e + a^2\*e^3 - (c^2\*d^3 + a\*c\*d\*e^2)\*x)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)) - ((a\*c^2\*d^2\*e + a^2\*c\*e^3)\*x^3 + (a^2\*c\*d^2\*e + a^3\*e^3)\*x)\*sqrt(c\*d^2 + a\*e^2)\*log(-((c\*x^2 + 2\*a)\*sqrt(a) + 2\*sqrt(c\*x^2 + a)\*a)/x^2))/(((a^2\*c^2\*d^4 + a^3\*c\*d^2\*e^2)\*x^3 + (a^3\*c\*d^4 + a^4\*d^2\*e^2)\*x)\*sqrt(c\*d^2 + a\*e^2)\*sqrt(a)), -1/2\*(2\*(a\*c\*d^2\*e\*x + a\*c\*d^3 + a^2\*d\*e^2 + (2\*c^2\*d^3 + a\*c\*d\*e^2)\*x^2)\*sqrt(-c\*d^2 - a\*e^2)\*sqrt(c\*x^2 + a)\*sqrt(a) - 2\*(a^2\*c\*e^4\*x^3 + a^3\*e^4\*x)\*sqrt(a)\*arctan(sqrt(-c\*d^2 - a\*e^2)\*(c\*d\*x - a\*e)/((c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a))) - ((a\*c^2\*d^2\*e + a^2\*c\*e^3)\*x^3 + (a^2\*c\*d^2\*e + a^3\*e^3)\*x)\*sqrt(-c\*d^2 - a\*e^2)\*log(-((c\*x^2 + 2\*a)\*sqrt(a) + 2\*sqrt(c\*x^2 + a)\*a)/x^2))/(((a^2\*c^2\*d^4 + a^3\*c\*d^2\*e^2)\*x^3 + (a^3\*c\*d^4 + a^4\*d^2\*e^2)\*x)\*sqrt(-c\*d^2 - a\*e^2)\*sqrt(a)), -1/2\*(2\*(a\*c\*d^2\*e\*x + a\*c\*d^3 + a^2\*d\*e^2 + (2\*c^2\*d^3 + a\*c\*d\*e^2)\*x^2)\*sqrt(c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a)\*sqrt(-a) - 2\*((a\*c^2\*d^2\*e + a^2\*c\*e^3)\*x^3 + (a^2\*c\*d^2\*e + a^3\*e^3)\*x)\*sqrt(c\*d^2 + a\*e^2)\*arctan(sqrt(-a)/sqrt(c\*x^2 + a)) - (a^2\*c\*e^4\*x^3 + a^3\*e^4\*x)\*sqrt(-a)\*log(((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2)\*sqrt(c\*d^2 + a\*e^2) + 2\*(a\*c\*d^2\*e + a^2\*e^3 - (c^2\*d^3 + a\*c\*d\*e^2)\*x)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2))/(((a^2\*c^2\*d^4 + a^3\*c\*d^2\*e^2)\*x^3 + (a^3\*c\*d^4 + a^4\*d^2\*e^2)\*x)\*sqrt(c\*d^2 + a\*e^2)\*sqrt(-a)), -((a\*c\*d^2\*e\*x + a\*c\*d^3 + a^2\*d\*e^2 + (2\*c^2\*d^3 + a\*c\*d\*e^2)\*x^2)\*sqrt(-c\*d^2 - a\*e^2)\*sqrt(c\*x^2 + a)\*sqrt(-a) - (a^2\*c\*e^4\*x^3 + a^3\*e^4\*x)\*sqrt(-a)\*arctan(sqrt(-c\*d^2 - a\*e^2)\*(c\*d\*x - a\*e)/((c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a))) - ((a\*c^2\*d^2\*e + a^2\*c\*e^3)\*x^3 + (a^2\*c\*d^2\*e + a^3\*e^3)\*x)\*sqrt(-c\*d^2 - a\*e^2)\*arctan(sqrt(-a)/sqrt(c\*x^2 + a)))/(((a^2\*c^2\*d^4 + a^3\*c\*d^2\*e^2)\*x^3 + (a^3\*c\*d^4 + a^4\*d^2\*e^2)\*x)\*sqrt(-c\*d^2 - a\*e^2)\*sqrt(-a))]

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2(a + cx^2)^{\frac{3}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(e\*x+d)/(c\*x\*\*2+a)\*\*(3/2),x)

[Out] Integral(1/(x\*\*2\*(a + c\*x\*\*2)\*\*(3/2)\*(d + e\*x)), x)

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + a)^(3/2)\*(e\*x + d)\*x^2),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.341 \quad \int \frac{1}{x^3(d+ex)(a+cx^2)^{3/2}} dx$$

**Optimal.** Leaf size=275

$$\begin{aligned} & -\frac{e^2 \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^3} + \frac{3c \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{5/2}d} + \frac{2cex}{a^2d^2\sqrt{a+cx^2}} - \frac{3\sqrt{a+cx^2}}{2a^2dx^2} + \frac{e^2}{ad^3\sqrt{a+cx^2}} \\ & + \frac{e}{ad^2x\sqrt{a+cx^2}} + \frac{e^5 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3(ae^2+cd^2)^{3/2}} - \frac{e^3(ae+cdx)}{ad^3\sqrt{a+cx^2}(ae^2+cd^2)} + \frac{1}{adx^2\sqrt{a+cx^2}} \end{aligned}$$

[Out]  $e^2/(a*d^3*\text{Sqrt}[a + c*x^2]) + 1/(a*d*x^2*\text{Sqrt}[a + c*x^2]) + e/(a*d^2*x*\text{Sqrt}[a + c*x^2]) + (2*c*e*x)/(a^2*d^2*\text{Sqrt}[a + c*x^2]) - (e^3*(a*e + c*d*x))/(a*d^3*(c*d^2 + a*e^2)*\text{Sqrt}[a + c*x^2]) - (3*\text{Sqrt}[a + c*x^2])/(2*a^2*d*x^2) + (e^5*\text{ArcTanh}[(a*e - c*d*x)/(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2])])/(d^3*(c*d^2 + a*e^2)^(3/2)) + (3*c*\text{ArcTanh}[\text{Sqrt}[a + c*x^2]/\text{Sqrt}[a]])/(2*a^(5/2)*d) - (e^2*\text{ArcTanh}[\text{Sqrt}[a + c*x^2]/\text{Sqrt}[a]])/(a^(3/2)*d^3)$

**Rubi [A]** time = 0.539899, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 12, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.546$

$$\begin{aligned} & -\frac{e^2 \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^3} + \frac{3c \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{5/2}d} + \frac{2cex}{a^2d^2\sqrt{a+cx^2}} - \frac{3\sqrt{a+cx^2}}{2a^2dx^2} + \frac{e^2}{ad^3\sqrt{a+cx^2}} \\ & + \frac{e}{ad^2x\sqrt{a+cx^2}} + \frac{e^5 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3(ae^2+cd^2)^{3/2}} - \frac{e^3(ae+cdx)}{ad^3\sqrt{a+cx^2}(ae^2+cd^2)} + \frac{1}{adx^2\sqrt{a+cx^2}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*(d + e\*x)\*(a + c\*x^2)^(3/2)), x]

[Out]  $e^2/(a*d^3*\text{Sqrt}[a + c*x^2]) + 1/(a*d*x^2*\text{Sqrt}[a + c*x^2]) + e/(a*d^2*x*\text{Sqrt}[a + c*x^2]) + (2*c*e*x)/(a^2*d^2*\text{Sqrt}[a + c*x^2]) - (e^3*(a*e + c*d*x))/(a*d^3*(c*d^2 + a*e^2)*\text{Sqrt}[a + c*x^2]) - (3*\text{Sqrt}[a + c*x^2])/(2*a^2*d*x^2) + (e^5*\text{ArcTanh}[(a*e - c*d*x)/(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2])])/(d^3*(c*d^2 + a*e^2)^(3/2)) + (3*c*\text{ArcTanh}[\text{Sqrt}[a + c*x^2]/\text{Sqrt}[a]])/(2*a^(5/2)*d) - (e^2*\text{ArcTanh}[\text{Sqrt}[a + c*x^2]/\text{Sqrt}[a]])/(a^(3/2)*d^3)$

**Rubi in Sympy [A]** time = 57.6849, size = 250, normalized size = 0.91

$$\begin{aligned} & \frac{e^5 \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3(ae^2+cd^2)^{3/2}} + \frac{1}{adx^2\sqrt{a+cx^2}} + \frac{e}{ad^2x\sqrt{a+cx^2}} - \frac{e^3(ae+cdx)}{ad^3\sqrt{a+cx^2}(ae^2+cd^2)} \\ & + \frac{e^2}{ad^3\sqrt{a+cx^2}} + \frac{2cex}{a^2d^2\sqrt{a+cx^2}} - \frac{3\sqrt{a+cx^2}}{2a^2dx^2} - \frac{e^2 \operatorname{atanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{a^{3/2}d^3} + \frac{3c \operatorname{atanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{5/2}d} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*3/(e\*x+d)/(c\*x\*\*2+a)\*\*(3/2), x)

[Out]  $e**5*\operatorname{atanh}((a*e - c*d*x)/(\text{sqrt}(a + c*x**2)*\text{sqrt}(a*e**2 + c*d**2)))/(d**3*(a*e**2 + c*d**2)**(3/2)) + 1/(a*d*x**2*\text{sqrt}(a + c*x**2)) + e/(a*d**2*x*\text{sqrt}(a + c*x**2)) - e**3*(a*e + c*d*x)/(a*d**3*\text{sqrt}(a + c*x**2)*(a*e**2 + c*d**2)) + e**2/(a*d**3*\text{sqrt}(a + c*x**2)) + 2*c*e*x/(a**2*d**2*\text{sqrt}(a + c*x**2)) - 3*\text{sqrt}(a + c*x**2)/(2*a$

$$**2*d*x**2) - e**2*atanh(sqrt(a + c*x**2)/sqrt(a))/(a**(3/2)*d**3) + 3*c*atanh(sqrt(a + c*x**2)/sqrt(a))/(2*a**(5/2)*d)$$

**Mathematica [A]** time = 0.87437, size = 226, normalized size = 0.82

$$\frac{1}{2} \left( \frac{(3cd^2 - 2ae^2) \log(\sqrt{a} \sqrt{a + cx^2} + a)}{a^{5/2} d^3} + \frac{\log(x) (2ae^2 - 3cd^2)}{a^{5/2} d^3} - \frac{\sqrt{a + cx^2} \left( \frac{2c^2(d-ex)}{(a+cx^2)(ae^2+cd^2)} - \frac{2e}{d^2x} + \frac{1}{dx^2} \right)}{a^2} + \frac{2e^5 \log(\sqrt{a + cx^2} \sqrt{ae^2 + cd^2} + ae - cdx)}{d^3 (ae^2 + cd^2)^{3/2}} - \frac{2e^5 \log(d + ex)}{d^3 (ae^2 + cd^2)^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(d + e\*x)\*(a + c\*x^2)^(3/2)), x]

[Out]  $-\left(\frac{\sqrt{a + cx^2}}{(c^2d^2 + a^2e^2)(a + cx^2)} - \frac{2e}{d^2x} + \frac{2c^2(d - ex)}{(a + cx^2)(ae^2 + cd^2)}\right) / a^2 + \frac{(-3c^2d^2 + 2a^2e^2) \operatorname{Log}[x]}{a^{5/2} d^3} - \frac{2e^5 \operatorname{Log}[d + ex]}{d^3 (ae^2 + cd^2)^{3/2}} + \frac{(3c^2d^2 - 2a^2e^2) \operatorname{Log}[a + \sqrt{a} \sqrt{a + cx^2}]}{a^{5/2} d^3} + \frac{2e^5 \operatorname{Log}[a^2e - c^2d^2x + \sqrt{c^2d^2 + a^2e^2} \sqrt{a + cx^2}]}{d^3 (c^2d^2 + a^2e^2)^{3/2}} / 2$

**Maple [A]** time = 0.019, size = 439, normalized size = 1.6

$$\begin{aligned} & -\frac{1}{2ad^2x^2} \frac{1}{\sqrt{cx^2 + a}} - \frac{3c}{2da^2} \frac{1}{\sqrt{cx^2 + a}} + \frac{3c}{2d} \ln\left(\frac{1}{x} (2a + 2\sqrt{a}\sqrt{cx^2 + a})\right) a^{-\frac{5}{2}} + \frac{e^2}{ad^3} \frac{1}{\sqrt{cx^2 + a}} \\ & - \frac{e^2}{d^3} \ln\left(\frac{1}{x} (2a + 2\sqrt{a}\sqrt{cx^2 + a})\right) a^{-\frac{3}{2}} - \frac{e^4}{d^3 (ae^2 + cd^2)} \frac{1}{\sqrt{\left(x + \frac{d}{e}\right)^2 c - 2\frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}}} \\ & - \frac{e^3 cx}{d^2 (ae^2 + cd^2) a} \frac{1}{\sqrt{\left(x + \frac{d}{e}\right)^2 c - 2\frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}}} \\ & + \frac{e^4}{d^3 (ae^2 + cd^2)} \ln\left(1 \left(2 \frac{ae^2 + cd^2}{e^2} - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + 2 \sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}}\right) \left(x + \frac{d}{e}\right)^{-1}\right) \\ & + \frac{e}{ad^2x} \frac{1}{\sqrt{cx^2 + a}} + 2 \frac{cex}{a^2 d^2 \sqrt{cx^2 + a}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(e\*x+d)/(c\*x^2+a)^(3/2), x)

[Out]  $-1/2/a/d/x^2/(c*x^2+a)^{(1/2)} - 3/2/d*c/a^2/(c*x^2+a)^{(1/2)} + 3/2/d*c/a^{5/2}*\ln((2*a+2*a^{(1/2)}*(c*x^2+a)^{(1/2)})/x) + e^2/a/d^3/(c*x^2+a)^{(1/2)} - 1/d^3*e^2/a^{(3/2)}*\ln((2*a+2*a^{(1/2)}*(c*x^2+a)^{(1/2)})/x) - 1/d^3*e^4/(a^2e^2+c^2d^2)/((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a^2e^2+c^2d^2)/e^2)^{(1/2)} - 1/d^3*e^3/(a^2e^2+c^2d^2)/a/((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a^2e^2+c^2d^2)/e^2)^{(1/2)}*c*x+1/d^3*e^4/(a^2e^2+c^2d^2)/((a^2e^2+c^2d^2)/e^2)^{(1/2)}*\ln(2*(a^2e^2+c^2d^2)/e^2-2*c*d/e*(x+d/e)+2*((a^2e^2+c^2d^2)/e^2)^{(1/2)}*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a^2e^2+c^2d^2)/e^2)^{(1/2)})/(x+d/e)+e/a/d^2/x/(c*x^2+a)^{(1/2)}+2*c*e*x/a^2/d^2/(c*x^2+a)^{(1/2)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + a)^{\frac{3}{2}}(ex + d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + a)^(3/2)\*(e\*x + d)\*x^3),x, algorithm="maxima")

[Out] integrate(1/((c\*x^2 + a)^(3/2)\*(e\*x + d)\*x^3), x)

**Fricas [A]** time = 0.887489, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + a)^(3/2)\*(e\*x + d)\*x^3),x, algorithm="fricas")

[Out] [-1/4\*(2\*(a\*c\*d^4 + a^2\*d^2\*e^2 - 2\*(2\*c^2\*d^3\*e + a\*c\*d\*e^3)\*x^3 + (3\*c^2\*d^4 + a\*c\*d^2\*e^2)\*x^2 - 2\*(a\*c\*d^3\*e + a^2\*d\*e^3)\*x)\*sqrt(c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a)\*sqrt(a) - 2\*(a^2\*c\*e^5\*x^4 + a^3\*e^5\*x^2)\*sqrt(a)\*log(((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2)\*sqrt(c\*d^2 + a\*e^2) - 2\*(a\*c\*d^2\*e + a^2\*e^3 - (c^2\*d^3 + a\*c\*d\*e^2)\*x)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)) + ((3\*c^3\*d^4 + a\*c^2\*d^2\*e^2 - 2\*a^2\*c\*e^4)\*x^4 + (3\*a\*c^2\*d^4 + a^2\*c\*d^2\*e^2 - 2\*a^3\*e^4)\*x^2)\*sqrt(c\*d^2 + a\*e^2)\*log(-(c\*x^2 + 2\*a)\*sqrt(a) - 2\*sqrt(c\*x^2 + a)\*a/x^2)/(((a^2\*c^2\*d^5 + a^3\*c\*d^3\*e^2)\*x^4 + (a^3\*c\*d^5 + a^4\*d^3\*e^2)\*x^2)\*sqrt(c\*d^2 + a\*e^2)\*sqrt(a)), -1/4\*(2\*(a\*c\*d^4 + a^2\*d^2\*e^2 - 2\*(2\*c^2\*d^3\*e + a\*c\*d\*e^3)\*x^3 + (3\*c^2\*d^4 + a\*c\*d^2\*e^2)\*x^2 - 2\*(a\*c\*d^3\*e + a^2\*d\*e^3)\*x)\*sqrt(-c\*d^2 - a\*e^2)\*sqrt(c\*x^2 + a)\*sqrt(a) + 4\*(a^2\*c\*e^5\*x^4 + a^3\*e^5\*x^2)\*sqrt(a)\*arctan(sqrt(-c\*d^2 - a\*e^2)\*(c\*d\*x - a\*e)/((c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a))) + ((3\*c^3\*d^4 + a\*c^2\*d^2\*e^2 - 2\*a^2\*c\*e^4)\*x^4 + (3\*a\*c^2\*d^4 + a^2\*c\*d^2\*e^2 - 2\*a^3\*e^4)\*x^2)\*sqrt(-c\*d^2 - a\*e^2)\*log(-(c\*x^2 + 2\*a)\*sqrt(a) - 2\*sqrt(c\*x^2 + a)\*a/x^2)/(((a^2\*c^2\*d^5 + a^3\*c\*d^3\*e^2)\*x^4 + (a^3\*c\*d^5 + a^4\*d^3\*e^2)\*x^2)\*sqrt(-c\*d^2 - a\*e^2)\*sqrt(a)), -1/2\*((a\*c\*d^4 + a^2\*d^2\*e^2 - 2\*(2\*c^2\*d^3\*e + a\*c\*d\*e^3)\*x^3 + (3\*c^2\*d^4 + a\*c\*d^2\*e^2)\*x^2 - 2\*(a\*c\*d^3\*e + a^2\*d\*e^3)\*x)\*sqrt(c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a)\*sqrt(-a) - ((3\*c^3\*d^4 + a\*c^2\*d^2\*e^2 - 2\*a^2\*c\*e^4)\*x^4 + (3\*a\*c^2\*d^4 + a^2\*c\*d^2\*e^2 - 2\*a^3\*e^4)\*x^2)\*sqrt(c\*d^2 + a\*e^2)\*arctan(sqrt(-a)/sqrt(c\*x^2 + a)) - (a^2\*c\*e^5\*x^4 + a^3\*e^5\*x^2)\*sqrt(-a)\*log(((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2)\*sqrt(c\*d^2 + a\*e^2) - 2\*(a\*c\*d^2\*e + a^2\*e^3 - (c^2\*d^3 + a\*c\*d\*e^2)\*x)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2))/(((a^2\*c^2\*d^5 + a^3\*c\*d^3\*e^2)\*x^4 + (a^3\*c\*d^5 + a^4\*d^3\*e^2)\*x^2)\*sqrt(c\*d^2 + a\*e^2)\*sqrt(-a)), -1/2\*((a\*c\*d^4 + a^2\*d^2\*e^2 - 2\*(2\*c^2\*d^3\*e + a\*c\*d\*e^3)\*x^3 + (3\*c^2\*d^4 + a\*c\*d^2\*e^2)\*x^2 - 2\*(a\*c\*d^3\*e + a^2\*d\*e^3)\*x)\*sqrt(-c\*d^2 - a\*e^2)\*sqrt(c\*x^2 + a)\*sqrt(-a) + 2\*(a^2\*c\*e^5\*x^4 + a^3\*e^5\*x^2)\*sqrt(-a)\*arctan(sqrt(-c\*d^2 - a\*e^2)\*(c\*d\*x - a\*e)/((c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a))) - ((3\*c^3\*d^4 + a\*c^2\*d^2\*e^2 - 2\*a^2\*c\*e^4)\*x^4 + (3\*a\*c^2\*d^4 + a^2\*c\*d^2\*e^2 - 2\*a^3\*e^4)\*x^2)\*sqrt(-c\*d^2 - a\*e^2)\*arctan(sqrt(-a)/sqrt(c\*x^2 + a))/(((a^2\*c^2\*d^5 + a^3\*c\*d^3\*e^2)\*x^4 + (a^3\*c\*d^5 + a^4\*d^3\*e^2)\*x^2)\*sqrt(-c\*d^2 - a\*e^2)\*sqrt(-a))]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3(a + cx^2)^{\frac{3}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x+d)/(c*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(a + c*x**2)**(3/2)*(d + e*x)), x)
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^2 + a)^(3/2)*(e*x + d)*x^3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```



$$3.342 \quad \int \frac{x^5}{(d+ex)^2 \sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=244

$$\begin{aligned} & \frac{d(4cd^2 - ae^2) \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^5} + \frac{\sqrt{a+cx^2}(13cd^2 - 2ae^2)}{3c^2e^4} + \frac{d^5\sqrt{a+cx^2}}{e^4(d+ex)(ae^2+cd^2)} \\ & - \frac{d^4(5ae^2 + 4cd^2) \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^5(ae^2+cd^2)^{3/2}} - \frac{5d\sqrt{a+cx^2}(d+ex)}{3ce^4} + \frac{\sqrt{a+cx^2}(d+ex)^2}{3ce^4} \end{aligned}$$

[Out]  $((13*c*d^2 - 2*a*e^2)*\text{Sqrt}[a + c*x^2])/(3*c^2*e^4) + (d^5*\text{Sqrt}[a + c*x^2])/(e^4*(c*d^2 + a*e^2)*(d + e*x)) - (5*d*(d + e*x)*\text{Sqrt}[a + c*x^2])/(3*c*e^4) + ((d + e*x)^2*\text{Sqrt}[a + c*x^2])/(3*c*e^4) - (d*(4*c*d^2 - a*e^2)*\text{ArcTanh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a + c*x^2]])/(c^(3/2)*e^5) - (d^4*(4*c*d^2 + 5*a*e^2)*\text{ArcTanh}[(a*e - c*d*x)/(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2])])/(e^5*(c*d^2 + a*e^2)^(3/2))$

**Rubi [A]** time = 1.48614, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\begin{aligned} & \frac{d(4cd^2 - ae^2) \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{c^{3/2}e^5} + \frac{\sqrt{a+cx^2}(13cd^2 - 2ae^2)}{3c^2e^4} + \frac{d^5\sqrt{a+cx^2}}{e^4(d+ex)(ae^2+cd^2)} \\ & - \frac{d^4(5ae^2 + 4cd^2) \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^5(ae^2+cd^2)^{3/2}} - \frac{5d\sqrt{a+cx^2}(d+ex)}{3ce^4} + \frac{\sqrt{a+cx^2}(d+ex)^2}{3ce^4} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^5/((d + e\*x)^2\*Sqrt[a + c\*x^2]),x]

[Out]  $((13*c*d^2 - 2*a*e^2)*\text{Sqrt}[a + c*x^2])/(3*c^2*e^4) + (d^5*\text{Sqrt}[a + c*x^2])/(e^4*(c*d^2 + a*e^2)*(d + e*x)) - (5*d*(d + e*x)*\text{Sqrt}[a + c*x^2])/(3*c*e^4) + ((d + e*x)^2*\text{Sqrt}[a + c*x^2])/(3*c*e^4) - (d*(4*c*d^2 - a*e^2)*\text{ArcTanh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a + c*x^2]])/(c^(3/2)*e^5) - (d^4*(4*c*d^2 + 5*a*e^2)*\text{ArcTanh}[(a*e - c*d*x)/(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2])])/(e^5*(c*d^2 + a*e^2)^(3/2))$

**Rubi in Sympy [A]** time = 56.7376, size = 282, normalized size = 1.16

$$\begin{aligned} & -\frac{a\sqrt{a+cx^2}}{c^2e^2} + \frac{ad \operatorname{atanh}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{c^{\frac{3}{2}}e^3} + \frac{cd^6 \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^5(ae^2+cd^2)^{\frac{3}{2}}} + \frac{d^5\sqrt{a+cx^2}}{e^4(d+ex)(ae^2+cd^2)} \\ & - \frac{5d^4 \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^5\sqrt{ae^2+cd^2}} + \frac{3d^2\sqrt{a+cx^2}}{ce^4} - \frac{dx\sqrt{a+cx^2}}{ce^3} + \frac{(a+cx^2)^{\frac{3}{2}}}{3c^2e^2} - \frac{4d^3 \operatorname{atanh}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{\sqrt{ce^5}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5/(e\*x+d)\*\*2/(c\*x\*\*2+a)\*\*(1/2),x)

[Out]  $-a*\text{sqrt}(a + c*x^2)/(c^2*e^2) + a*d*\text{atanh}(\text{sqrt}(c)*x/\text{sqrt}(a + c*x^2))/(c^(3/2)*e^3) + c*d^6*\text{atanh}((a*e - c*d*x)/(\text{sqrt}(a + c*x^2)*\text{sqrt}(a*e^2 + c*d^2)))/(e^5*(a*e^2 + c*d^2)^(3/2)) + d^5*\text{sqrt}(a + c*x^2)/(e^4*(d + e*x)*(a*e^2 + c*d^2)) - 5*d^4*a*\text{tanh}((a*e - c*d*x)/(\text{sqrt}(a + c*x^2)*\text{sqrt}(a*e^2 + c*d^2)))/(e^5*\text{sqrt}(a*e^2 + c*d^2)) + 3*d^2*\text{sqrt}(a + c*x^2)/(c*e^4) - d*x*\text{sqrt}(a + c*x^2)/(c*e^3) + (a + c*x^2)^(3/2)/(3*c^2*e^2) - \frac{4d^3 \operatorname{atanh}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{\sqrt{ce^5}}$

$$4*d**3*atanh(sqrt(c)*x/sqrt(a + c*x**2))/(sqrt(c)*e**5)$$

**Mathematica [A]** time = 0.773979, size = 230, normalized size = 0.94

$$\frac{-\frac{3d(4cd^2-ae^2)\log(\sqrt{c}\sqrt{a+cx^2+cx})}{c^{3/2}} + e\sqrt{a+cx^2}\left(-\frac{2ae^2}{c^2} + \frac{3d^5}{(d+ex)(ae^2+cd^2)} + \frac{9d^2-3dex+e^2x^2}{c}\right) - \frac{3d^4(5ae^2+4cd^2)\log(\sqrt{a+cx^2}\sqrt{ae^2+cd^2+ae-cd^2})}{(ae^2+cd^2)^{3/2}}}{3e^5}$$

Antiderivative was successfully verified.

[In] Integrate[x^5/((d + e\*x)^2\*Sqrt[a + c\*x^2]),x]

[Out] (e\*Sqrt[a + c\*x^2]\*((-2\*a\*e^2)/c^2 + (3\*d^5)/((c\*d^2 + a\*e^2)\*(d + e\*x)) + (9\*d^2 - 3\*d\*e\*x + e^2\*x^2)/c) + (3\*d^4\*(4\*c\*d^2 + 5\*a\*e^2)\*Log[d + e\*x])/(c\*d^2 + a\*e^2)^(3/2) - (3\*d\*(4\*c\*d^2 - a\*e^2)\*Log[c\*x + Sqrt[c]\*Sqrt[a + c\*x^2]])/c^(3/2) - (3\*d^4\*(4\*c\*d^2 + 5\*a\*e^2)\*Log[a\*e - c\*d\*x + Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]])/(c\*d^2 + a\*e^2)^(3/2))/(3\*e^5)

**Maple [B]** time = 0.025, size = 474, normalized size = 1.9

$$\begin{aligned} & \frac{x^2}{3ce^2}\sqrt{cx^2+a} - \frac{2a}{3c^2e^2}\sqrt{cx^2+a} - 4\frac{d^3\ln(x\sqrt{c}+\sqrt{cx^2+a})}{e^5\sqrt{c}} \\ & - \frac{dx}{e^3c}\sqrt{cx^2+a} + \frac{ad}{e^3}\ln(x\sqrt{c}+\sqrt{cx^2+a})c^{-\frac{3}{2}} + 3\frac{d^2\sqrt{cx^2+a}}{e^4c} \\ & - 5\frac{d^4}{e^6}\ln\left(1\left(2\frac{ae^2+cd^2}{e^2} - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + 2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + \frac{ae^2+cd^2}{e^2}}\right)\left(x+\frac{d}{e}\right)^{-1}\right)\frac{1}{\sqrt{\frac{ae^2+cd^2}{e^2}}} \\ & + \frac{d^5}{e^5(ae^2+cd^2)}\sqrt{\left(x+\frac{d}{e}\right)^2c - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + \frac{ae^2+cd^2}{e^2}}\left(x+\frac{d}{e}\right)^{-1} \\ & + \frac{d^6c}{e^6(ae^2+cd^2)}\ln\left(1\left(2\frac{ae^2+cd^2}{e^2} - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + 2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + \frac{ae^2+cd^2}{e^2}}\right)\left(x+\frac{d}{e}\right)^{-1}\right)\frac{1}{\sqrt{\frac{ae^2+cd^2}{e^2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(e\*x+d)^2/(c\*x^2+a)^(1/2),x)

[Out] 1/3/e^2\*x^2/c\*(c\*x^2+a)^(1/2)-2/3/e^2\*a/c^2\*(c\*x^2+a)^(1/2)-4\*d^3/e^5\*ln(x\*c^(1/2)+(c\*x^2+a)^(1/2))/c^(1/2)-d/e^3\*x/c\*(c\*x^2+a)^(1/2)+d/e^3\*a/c^(3/2)\*ln(x\*c^(1/2)+(c\*x^2+a)^(1/2))+3\*d^2/e^4/c\*(c\*x^2+a)^(1/2)-5/e^6\*d^4/((a\*e^2+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2+c\*d^2)/e^2-2\*c\*d/e\*(x+d/e)+2\*((a\*e^2+c\*d^2)/e^2)^(1/2)\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2))/(x+d/e)+d^5/e^5/(a\*e^2+c\*d^2)/(x+d/e)\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2)+d^6/e^6\*c/(a\*e^2+c\*d^2)/((a\*e^2+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2+c\*d^2)/e^2-2\*c\*d/e\*(x+d/e)+2\*((a\*e^2+c\*d^2)/e^2)^(1/2)\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2))/(x+d/e)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(sqrt(c\*x^2 + a)\*(e\*x + d)^2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas** [A] time = 48.9693, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(sqrt(c\*x^2 + a)\*(e\*x + d)^2),x, algorithm="fricas")

[Out] [1/6\*(2\*(12\*c^2\*d^5\*e + 7\*a\*c\*d^3\*e^3 - 2\*a^2\*d\*e^5 + (c^2\*d^2\*e^4 + a\*c\*e^6)\*x^3 - 2\*(c^2\*d^3\*e^3 + a\*c\*d\*e^5)\*x^2 + 2\*(3\*c^2\*d^4\*e^2 + 2\*a\*c\*d^2\*e^4 - a^2\*e^6)\*x)\*sqrt(c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a)\*sqrt(c) + 3\*(4\*c^3\*d^6 + 3\*a\*c^2\*d^4\*e^2 - a^2\*c\*d^2\*e^4 + (4\*c^3\*d^5\*e + 3\*a\*c^2\*d^3\*e^3 - a^2\*c\*d\*e^5)\*x)\*sqrt(c\*d^2 + a\*e^2)\*log(2\*sqrt(c\*x^2 + a)\*c\*x - (2\*c\*x^2 + a)\*sqrt(c)) + 3\*(4\*c^3\*d^7 + 5\*a\*c^2\*d^5\*e^2 + (4\*c^3\*d^6\*e + 5\*a\*c^2\*d^4\*e^3)\*x)\*sqrt(c)\*log(((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2)\*sqrt(c\*d^2 + a\*e^2) + 2\*(a\*c\*d^2\*e + a^2\*e^3 - (c^2\*d^3 + a\*c\*d\*e^2)\*x)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)))/((c^3\*d^3\*e^5 + a\*c^2\*d\*e^7 + (c^3\*d^2\*e^6 + a\*c^2\*e^8)\*x)\*sqrt(c\*d^2 + a\*e^2)\*sqrt(c)), 1/6\*(2\*(12\*c^2\*d^5\*e + 7\*a\*c\*d^3\*e^3 - 2\*a^2\*d\*e^5 + (c^2\*d^2\*e^4 + a\*c\*e^6)\*x^3 - 2\*(c^2\*d^3\*e^3 + a\*c\*d\*e^5)\*x^2 + 2\*(3\*c^2\*d^4\*e^2 + 2\*a\*c\*d^2\*e^4 - a^2\*e^6)\*x)\*sqrt(-c\*d^2 - a\*e^2)\*sqrt(c\*x^2 + a)\*sqrt(c) + 6\*(4\*c^3\*d^7 + 5\*a\*c^2\*d^5\*e^2 + (4\*c^3\*d^6\*e + 5\*a\*c^2\*d^4\*e^3)\*x)\*sqrt(c)\*arctan(sqrt(-c\*d^2 - a\*e^2)\*(c\*d\*x - a\*e)/((c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a))) + 3\*(4\*c^3\*d^6 + 3\*a\*c^2\*d^4\*e^2 - a^2\*c\*d^2\*e^4 + (4\*c^3\*d^5\*e + 3\*a\*c^2\*d^3\*e^3 - a^2\*c\*d\*e^5)\*x)\*sqrt(-c\*d^2 - a\*e^2)\*log(2\*sqrt(c\*x^2 + a)\*c\*x - (2\*c\*x^2 + a)\*sqrt(c)))/((c^3\*d^3\*e^5 + a\*c^2\*d\*e^7 + (c^3\*d^2\*e^6 + a\*c^2\*e^8)\*x)\*sqrt(-c\*d^2 - a\*e^2)\*sqrt(c)), 1/6\*(2\*(12\*c^2\*d^5\*e + 7\*a\*c\*d^3\*e^3 - 2\*a^2\*d\*e^5 + (c^2\*d^2\*e^4 + a\*c\*e^6)\*x^3 - 2\*(c^2\*d^3\*e^3 + a\*c\*d\*e^5)\*x^2 + 2\*(3\*c^2\*d^4\*e^2 + 2\*a\*c\*d^2\*e^4 - a^2\*e^6)\*x)\*sqrt(c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a)\*sqrt(-c) - 6\*(4\*c^3\*d^6 + 3\*a\*c^2\*d^4\*e^2 - a^2\*c\*d^2\*e^4 + (4\*c^3\*d^5\*e + 3\*a\*c^2\*d^3\*e^3 - a^2\*c\*d\*e^5)\*x)\*sqrt(c\*d^2 + a\*e^2)\*arctan(sqrt(-c)\*x/sqrt(c\*x^2 + a)) + 3\*(4\*c^3\*d^7 + 5\*a\*c^2\*d^5\*e^2 + (4\*c^3\*d^6\*e + 5\*a\*c^2\*d^4\*e^3)\*x)\*sqrt(-c)\*log(((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2)\*sqrt(c\*d^2 + a\*e^2) + 2\*(a\*c\*d^2\*e + a^2\*e^3 - (c^2\*d^3 + a\*c\*d\*e^2)\*x)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)))/((c^3\*d^3\*e^5 + a\*c^2\*d\*e^7 + (c^3\*d^2\*e^6 + a\*c^2\*e^8)\*x)\*sqrt(c\*d^2 + a\*e^2)\*sqrt(-c)), 1/3\*((12\*c^2\*d^5\*e + 7\*a\*c\*d^3\*e^3 - 2\*a^2\*d\*e^5 + (c^2\*d^2\*e^4 + a\*c\*e^6)\*x^3 - 2\*(c^2\*d^3\*e^3 + a\*c\*d\*e^5)\*x^2 + 2\*(3\*c^2\*d^4\*e^2 + 2\*a\*c\*d^2\*e^4 - a^2\*e^6)\*x)\*sqrt(-c\*d^2 - a\*e^2)\*sqrt(c\*x^2 + a)\*sqrt(-c) - 3\*(4\*c^3\*d^6 + 3\*a\*c^2\*d^4\*e^2 - a^2\*c\*d^2\*e^4 + (4\*c^3\*d^5\*e + 3\*a\*c^2\*d^3\*e^3 - a^2\*c\*d\*e^5)\*x)\*sqrt(-c\*d^2 - a\*e^2)\*arctan(sqrt(-c)\*x/sqrt(c\*x^2 + a)) + 3\*(4\*c^3\*d^7 + 5\*a\*c^2\*d^5\*e^2 + (4\*c^3\*d^6\*e + 5\*a\*c^2\*d^4\*e^3)\*x)\*sqrt(-c)\*arctan(sqrt(-c\*d^2 - a\*e^2)\*(c\*d\*x - a\*e)/((c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a)))/((c^3\*d^3\*e^5 + a\*c^2\*d\*e^7 + (c^3\*d^2\*e^6 + a\*c^2\*e^8)\*x)\*sqrt(-c\*d^2 - a\*e^2)\*sqrt(-c))]

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{a + cx^2}(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5/(e\*x+d)\*\*2/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] Integral( $x^{**5}/(\text{sqrt}(a + c*x^{**2})*(d + e*x)^{**2})$ , x)

---

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{\sqrt{cx^2 + a}(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate( $x^5/(\text{sqrt}(c*x^2 + a)*(e*x + d)^2)$ , x, algorithm="giac")

[Out] integrate( $x^5/(\text{sqrt}(c*x^2 + a)*(e*x + d)^2)$ , x)

$$3.343 \quad \int \frac{x^4}{(d+ex)^2 \sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=204

$$\frac{(6cd^2 - ae^2) \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^4} - \frac{d^4 \sqrt{a+cx^2}}{e^3(d+ex)(ae^2+cd^2)} + \frac{d^3(4ae^2+3cd^2) \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4(ae^2+cd^2)^{3/2}} - \frac{5d\sqrt{a+cx^2}}{2ce^3} + \frac{\sqrt{a+cx^2}(d+ex)}{2ce^3}$$

[Out]  $(-5*d*\text{Sqrt}[a + c*x^2])/(2*c*e^3) - (d^4*\text{Sqrt}[a + c*x^2])/(e^3*(c*d^2 + a*e^2)*(d + e*x)) + ((d + e*x)*\text{Sqrt}[a + c*x^2])/(2*c*e^3) + ((6*c*d^2 - a*e^2)*\text{ArcTanh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a + c*x^2]])/(2*c^(3/2)*e^4) + (d^3*(3*c*d^2 + 4*a*e^2)*\text{ArcTanh}[(a*e - c*d*x)/(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2])])/(e^4*(c*d^2 + a*e^2)^(3/2))$

**Rubi [A]** time = 0.906049, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{(6cd^2 - ae^2) \tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^4} - \frac{d^4 \sqrt{a+cx^2}}{e^3(d+ex)(ae^2+cd^2)} + \frac{d^3(4ae^2+3cd^2) \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4(ae^2+cd^2)^{3/2}} - \frac{5d\sqrt{a+cx^2}}{2ce^3} + \frac{\sqrt{a+cx^2}(d+ex)}{2ce^3}$$

Antiderivative was successfully verified.

[In] Int[x^4/((d + e\*x)^2\*Sqrt[a + c\*x^2]),x]

[Out]  $(-5*d*\text{Sqrt}[a + c*x^2])/(2*c*e^3) - (d^4*\text{Sqrt}[a + c*x^2])/(e^3*(c*d^2 + a*e^2)*(d + e*x)) + ((d + e*x)*\text{Sqrt}[a + c*x^2])/(2*c*e^3) + ((6*c*d^2 - a*e^2)*\text{ArcTanh}[(\text{Sqrt}[c]*x)/\text{Sqrt}[a + c*x^2]])/(2*c^(3/2)*e^4) + (d^3*(3*c*d^2 + 4*a*e^2)*\text{ArcTanh}[(a*e - c*d*x)/(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2])])/(e^4*(c*d^2 + a*e^2)^(3/2))$

**Rubi in Sympy [A]** time = 44.7607, size = 243, normalized size = 1.19

$$-\frac{a \operatorname{atanh}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{2c^{3/2}e^2} - \frac{cd^5 \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4(ae^2+cd^2)^{3/2}} - \frac{d^4 \sqrt{a+cx^2}}{e^3(d+ex)(ae^2+cd^2)} + \frac{4d^3 \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^4 \sqrt{ae^2+cd^2}} - \frac{2d\sqrt{a+cx^2}}{ce^3} + \frac{x\sqrt{a+cx^2}}{2ce^2} + \frac{3d^2 \operatorname{atanh}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{\sqrt{ce^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4/(e\*x+d)\*\*2/(c\*x\*\*2+a)\*\*(1/2),x)

[Out]  $-a*\operatorname{atanh}(\text{sqrt}(c)*x/\text{sqrt}(a + c*x**2))/(2*c**(3/2)*e**2) - c*d**5*a*\operatorname{atanh}((a*e - c*d*x)/(\text{sqrt}(a + c*x**2)*\text{sqrt}(a*e**2 + c*d**2)))/(e**4*(a*e**2 + c*d**2)**(3/2)) - d**4*\text{sqrt}(a + c*x**2)/(e**3*(d + e*x)*(a*e**2 + c*d**2)) + 4*d**3*\operatorname{atanh}((a*e - c*d*x)/(\text{sqrt}(a + c*x**2)*\text{sqrt}(a*e**2 + c*d**2)))/(e**4*\text{sqrt}(a*e**2 + c*d**2)) - 2*d*\text{sqrt}(a + c*x**2)/(c*e**3) + x*\text{sqrt}(a + c*x**2)/(2*c*e**2) + 3*d**2*\operatorname{atanh}(\text{sqrt}(c)*x/\text{sqrt}(a + c*x**2))/(\text{sqrt}(c)*e**4)$

**Mathematica [A]** time = 0.636178, size = 208, normalized size = 1.02

$$\frac{(6cd^2 - ae^2) \log(\sqrt{c}\sqrt{a+cx^2+cx})}{c^{3/2}} + e\sqrt{a+cx^2} \left( \frac{ex-4d}{c} - \frac{2d^4}{(d+ex)(ae^2+cd^2)} \right) + \frac{2d^3(4ae^2+3cd^2) \log(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx)}{(ae^2+cd^2)^{3/2}} - \frac{2d^3(4ae^2+3cd^2)}{(ae^2+cd^2)} \frac{1}{2e^4}$$

Antiderivative was successfully verified.

[In] Integrate[x^4/((d + e\*x)^2\*Sqrt[a + c\*x^2]),x]

[Out] (e\*Sqrt[a + c\*x^2]\*((-4\*d + e\*x)/c - (2\*d^4)/((c\*d^2 + a\*e^2)\*(d + e\*x))) - (2\*d^3\*(3\*c\*d^2 + 4\*a\*e^2)\*Log[d + e\*x])/(c\*d^2 + a\*e^2)^(3/2) + ((6\*c\*d^2 - a\*e^2)\*Log[c\*x + Sqrt[c]\*Sqrt[a + c\*x^2]])/c^(3/2) + (2\*d^3\*(3\*c\*d^2 + 4\*a\*e^2)\*Log[a\*e - c\*d\*x + Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]])/(c\*d^2 + a\*e^2)^(3/2))/(2\*e^4)

**Maple [B]** time = 0.018, size = 435, normalized size = 2.1

$$\begin{aligned} & \frac{x}{2ce^2} \sqrt{cx^2 + a} - \frac{a}{2e^2} \ln(x\sqrt{c} + \sqrt{cx^2 + a}) c^{-\frac{3}{2}} \\ & - \frac{d^4}{e^4(ae^2 + cd^2)} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2\frac{cd}{e}\left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}\left(x + \frac{d}{e}\right)^{-1}} \\ & - \frac{d^5c}{e^5(ae^2 + cd^2)} \ln\left(1\left(2\frac{ae^2 + cd^2}{e^2} - 2\frac{cd}{e}\left(x + \frac{d}{e}\right) + 2\sqrt{\frac{ae^2 + cd^2}{e^2}}\sqrt{\left(x + \frac{d}{e}\right)^2 c - 2\frac{cd}{e}\left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}}\right)\left(x + \frac{d}{e}\right)^{-1}\right) \\ & + 3\frac{d^2 \ln(x\sqrt{c} + \sqrt{cx^2 + a})}{e^4\sqrt{c}} - 2\frac{d\sqrt{cx^2 + a}}{ce^3} \\ & + 4\frac{d^3}{e^5} \ln\left(1\left(2\frac{ae^2 + cd^2}{e^2} - 2\frac{cd}{e}\left(x + \frac{d}{e}\right) + 2\sqrt{\frac{ae^2 + cd^2}{e^2}}\sqrt{\left(x + \frac{d}{e}\right)^2 c - 2\frac{cd}{e}\left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}}\right)\left(x + \frac{d}{e}\right)^{-1}\right) \frac{1}{\sqrt{\frac{ae^2 + cd^2}{e^2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(e\*x+d)^2/(c\*x^2+a)^(1/2),x)

[Out] 1/2/e^2\*x/c\*(c\*x^2+a)^(1/2)-1/2/e^2\*a/c^(3/2)\*ln(x\*c^(1/2)+(c\*x^2+a)^(1/2))-d^4/e^4/(a\*e^2+c\*d^2)/(x+d/e)\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2)-d^5/e^5\*c/(a\*e^2+c\*d^2)/((a\*e^2+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2+c\*d^2)/e^2-2\*c\*d/e\*(x+d/e)+2\*((a\*e^2+c\*d^2)/e^2)^(1/2)\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2))/(x+d/e))+3\*d^2/e^4\*ln(x\*c^(1/2)+(c\*x^2+a)^(1/2))/c^(1/2)-2\*d\*(c\*x^2+a)^(1/2)/c/e^3+4/e^5\*d^3/((a\*e^2+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2+c\*d^2)/e^2-2\*c\*d/e\*(x+d/e)+2\*((a\*e^2+c\*d^2)/e^2)^(1/2)\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2))/(x+d/e))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(sqrt(c\*x^2 + a)\*(e\*x + d)^2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 66.849, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(sqrt(c\*x^2 + a)\*(e\*x + d)^2),x, algorithm="fricas")

[Out] [-1/4\*(2\*(6\*c\*d^4\*e + 4\*a\*d^2\*e^3 - (c\*d^2\*e^3 + a\*e^5)\*x^2 + 3\*(c\*d^3\*e^2 + a\*d\*e^4)\*x)\*sqrt(c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a)\*sqrt(c) + (6\*c^2\*d^5 + 5\*a\*c\*d^3\*e^2 - a^2\*d\*e^4 + (6\*c^2\*d^4\*e + 5\*a\*c\*d^2\*e^3 - a^2\*e^5)\*x)\*sqrt(c\*d^2 + a\*e^2)\*log(2\*sqrt(c\*x^2 + a)\*c\*x - (2\*c\*x^2 + a)\*sqrt(c)) - 2\*(3\*c^2\*d^6 + 4\*a\*c\*d^4\*e^2 + (3\*c^2\*d^5\*e + 4\*a\*c\*d^3\*e^3)\*x)\*sqrt(c)\*log(((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2)\*sqrt(c\*d^2 + a\*e^2) - 2\*(a\*c\*d^2\*e + a^2\*e^3 - (c^2\*d^3 + a\*c\*d\*e^2)\*x)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)))/((c^2\*d^3\*e^4 + a\*c\*d\*e^6 + (c^2\*d^2\*e^5 + a\*c\*e^7)\*x)\*sqrt(c\*d^2 + a\*e^2)\*sqrt(c)), -1/4\*(2\*(6\*c\*d^4\*e + 4\*a\*d^2\*e^3 - (c\*d^2\*e^3 + a\*e^5)\*x^2 + 3\*(c\*d^3\*e^2 + a\*d\*e^4)\*x)\*sqrt(-c\*d^2 - a\*e^2)\*sqrt(c\*x^2 + a)\*sqrt(c) + 4\*(3\*c^2\*d^6 + 4\*a\*c\*d^4\*e^2 + (3\*c^2\*d^5\*e + 4\*a\*c\*d^3\*e^3)\*x)\*sqrt(c)\*arctan(sqrt(-c\*d^2 - a\*e^2)\*(c\*d\*x - a\*e)/((c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a))) + (6\*c^2\*d^5 + 5\*a\*c\*d^3\*e^2 - a^2\*d\*e^4 + (6\*c^2\*d^4\*e + 5\*a\*c\*d^2\*e^3 - a^2\*e^5)\*x)\*sqrt(-c\*d^2 - a\*e^2)\*log(2\*sqrt(c\*x^2 + a)\*c\*x - (2\*c\*x^2 + a)\*sqrt(c)))/((c^2\*d^3\*e^4 + a\*c\*d\*e^6 + (c^2\*d^2\*e^5 + a\*c\*e^7)\*x)\*sqrt(-c\*d^2 - a\*e^2)\*sqrt(c)), -1/2\*(6\*c\*d^4\*e + 4\*a\*d^2\*e^3 - (c\*d^2\*e^3 + a\*e^5)\*x^2 + 3\*(c\*d^3\*e^2 + a\*d\*e^4)\*x)\*sqrt(c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a)\*sqrt(-c) - (6\*c^2\*d^5 + 5\*a\*c\*d^3\*e^2 - a^2\*d\*e^4 + (6\*c^2\*d^4\*e + 5\*a\*c\*d^2\*e^3 - a^2\*e^5)\*x)\*sqrt(c\*d^2 + a\*e^2)\*arctan(sqrt(-c)\*x/sqrt(c\*x^2 + a)) - (3\*c^2\*d^6 + 4\*a\*c\*d^4\*e^2 + (3\*c^2\*d^5\*e + 4\*a\*c\*d^3\*e^3)\*x)\*sqrt(-c)\*log(((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2)\*sqrt(c\*d^2 + a\*e^2) - 2\*(a\*c\*d^2\*e + a^2\*e^3 - (c^2\*d^3 + a\*c\*d\*e^2)\*x)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)))/((c^2\*d^3\*e^4 + a\*c\*d\*e^6 + (c^2\*d^2\*e^5 + a\*c\*e^7)\*x)\*sqrt(c\*d^2 + a\*e^2)\*sqrt(-c)), -1/2\*((6\*c\*d^4\*e + 4\*a\*d^2\*e^3 - (c\*d^2\*e^3 + a\*e^5)\*x^2 + 3\*(c\*d^3\*e^2 + a\*d\*e^4)\*x)\*sqrt(-c\*d^2 - a\*e^2)\*sqrt(c\*x^2 + a)\*sqrt(-c) - (6\*c^2\*d^5 + 5\*a\*c\*d^3\*e^2 - a^2\*d\*e^4 + (6\*c^2\*d^4\*e + 5\*a\*c\*d^2\*e^3 - a^2\*e^5)\*x)\*sqrt(-c\*d^2 - a\*e^2)\*arctan(sqrt(-c)\*x/sqrt(c\*x^2 + a)) + 2\*(3\*c^2\*d^6 + 4\*a\*c\*d^4\*e^2 + (3\*c^2\*d^5\*e + 4\*a\*c\*d^3\*e^3)\*x)\*sqrt(-c)\*arctan(sqrt(-c\*d^2 - a\*e^2)\*(c\*d\*x - a\*e)/((c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a))))/((c^2\*d^3\*e^4 + a\*c\*d\*e^6 + (c^2\*d^2\*e^5 + a\*c\*e^7)\*x)\*sqrt(-c\*d^2 - a\*e^2)\*sqrt(-c))]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{a + cx^2}(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4/(e\*x+d)\*\*2/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] Integral(x\*\*4/(sqrt(a + c\*x\*\*2)\*(d + e\*x)\*\*2), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{\sqrt{cx^2 + a}(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(sqrt(c*x^2 + a)*(e*x + d)^2),x, algorithm="giac")
```

```
[Out] integrate(x^4/(sqrt(c*x^2 + a)*(e*x + d)^2), x)
```



$$3.344 \quad \int \frac{x^3}{(d+ex)^2 \sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=160

$$-\frac{d^2(3ae^2+2cd^2)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^3(ae^2+cd^2)^{3/2}} + \frac{d^3\sqrt{a+cx^2}}{e^2(d+ex)(ae^2+cd^2)} - \frac{2d\tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{\sqrt{ce^3}} + \frac{\sqrt{a+cx^2}}{ce^2}$$

[Out] Sqrt[a + c\*x^2]/(c\*e^2) + (d^3\*Sqrt[a + c\*x^2])/(e^2\*(c\*d^2 + a\*e^2)\*(d + e\*x)) - (2\*d\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(Sqrt[c]\*e^3) - (d^2\*(2\*c\*d^2 + 3\*a\*e^2)\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])])/(e^3\*(c\*d^2 + a\*e^2)^(3/2))

**Rubi [A]** time = 0.595137, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$-\frac{d^2(3ae^2+2cd^2)\tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^3(ae^2+cd^2)^{3/2}} + \frac{d^3\sqrt{a+cx^2}}{e^2(d+ex)(ae^2+cd^2)} - \frac{2d\tanh^{-1}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{\sqrt{ce^3}} + \frac{\sqrt{a+cx^2}}{ce^2}$$

Antiderivative was successfully verified.

[In] Int[x^3/((d + e\*x)^2\*Sqrt[a + c\*x^2]),x]

[Out] Sqrt[a + c\*x^2]/(c\*e^2) + (d^3\*Sqrt[a + c\*x^2])/(e^2\*(c\*d^2 + a\*e^2)\*(d + e\*x)) - (2\*d\*ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]])/(Sqrt[c]\*e^3) - (d^2\*(2\*c\*d^2 + 3\*a\*e^2)\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])])/(e^3\*(c\*d^2 + a\*e^2)^(3/2))

**Rubi in Sympy [A]** time = 38.5456, size = 189, normalized size = 1.18

$$\frac{cd^4 \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^3(ae^2+cd^2)^{3/2}} + \frac{d^3\sqrt{a+cx^2}}{e^2(d+ex)(ae^2+cd^2)} - \frac{3d^2 \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^3\sqrt{ae^2+cd^2}} + \frac{\sqrt{a+cx^2}}{ce^2} - \frac{2d \operatorname{atanh}\left(\frac{\sqrt{cx}}{\sqrt{a+cx^2}}\right)}{\sqrt{ce^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3/(e\*x+d)\*\*2/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] c\*d\*\*4\*atanh((a\*e - c\*d\*x)/(sqrt(a + c\*x\*\*2)\*sqrt(a\*e\*\*2 + c\*d\*\*2)))/(e\*\*3\*(a\*e\*\*2 + c\*d\*\*2)\*\*(3/2)) + d\*\*3\*sqrt(a + c\*x\*\*2)/(e\*\*2\*(d + e\*x)\*(a\*e\*\*2 + c\*d\*\*2)) - 3\*d\*\*2\*atanh((a\*e - c\*d\*x)/(sqrt(a + c\*x\*\*2)\*sqrt(a\*e\*\*2 + c\*d\*\*2)))/(e\*\*3\*sqrt(a\*e\*\*2 + c\*d\*\*2)) + sqrt(a + c\*x\*\*2)/(c\*e\*\*2) - 2\*d\*atanh(sqrt(c)\*x/sqrt(a + c\*x\*\*2))/(sqrt(c)\*e\*\*3)

**Mathematica [A]** time = 0.470515, size = 184, normalized size = 1.15

$$-\frac{d^2(3ae^2+2cd^2)\log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx\right)}{(ae^2+cd^2)^{3/2}} + \frac{d^2(3ae^2+2cd^2)\log(d+ex)}{(ae^2+cd^2)^{3/2}} + e\sqrt{a+cx^2}\left(\frac{d^3}{(d+ex)(ae^2+cd^2)} + \frac{1}{c}\right) - \frac{2d\log\left(\sqrt{c}\sqrt{a+cx^2}+cx\right)}{\sqrt{c}}$$

$e^3$

Antiderivative was successfully verified.

[In] Integrate[x^3/((d + e\*x)^2\*Sqrt[a + c\*x^2]),x]

[Out] (e\*Sqrt[a + c\*x^2]\*(c^(-1) + d^3/((c\*d^2 + a\*e^2)\*(d + e\*x))) + (d^2\*(2\*c\*d^2 + 3\*a\*e^2)\*Log[d + e\*x])/(c\*d^2 + a\*e^2)^(3/2) - (2\*d\*Log[c\*x + Sqrt[c]\*Sqrt[a + c\*x^2]])/Sqrt[c] - (d^2\*(2\*c\*d^2 + 3\*a\*e^2)\*Log[a\*e - c\*d\*x + Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]])/(c\*d^2 + a\*e^2)^(3/2))/e^3

**Maple [B]** time = 0.016, size = 386, normalized size = 2.4

$$\frac{1}{ce^2}\sqrt{cx^2+a}-2\frac{d\ln\left(x\sqrt{c}+\sqrt{cx^2+a}\right)}{e^3\sqrt{c}}$$

$$-3\frac{d^2}{e^4}\ln\left(1\left(2\frac{ae^2+cd^2}{e^2}-2\frac{cd}{e}\left(x+\frac{d}{e}\right)+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c-2\frac{cd}{e}\left(x+\frac{d}{e}\right)+\frac{ae^2+cd^2}{e^2}}\right)\left(x+\frac{d}{e}\right)^{-1}\right)\frac{1}{\sqrt{\frac{ae^2+cd^2}{e^2}}}$$

$$+\frac{d^3}{e^3(ae^2+cd^2)}\sqrt{\left(x+\frac{d}{e}\right)^2c-2\frac{cd}{e}\left(x+\frac{d}{e}\right)+\frac{ae^2+cd^2}{e^2}}\left(x+\frac{d}{e}\right)^{-1}$$

$$+\frac{d^4c}{e^4(ae^2+cd^2)}\ln\left(1\left(2\frac{ae^2+cd^2}{e^2}-2\frac{cd}{e}\left(x+\frac{d}{e}\right)+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c-2\frac{cd}{e}\left(x+\frac{d}{e}\right)+\frac{ae^2+cd^2}{e^2}}\right)\left(x+\frac{d}{e}\right)^{-1}\right)\frac{1}{\sqrt{\frac{ae^2+cd^2}{e^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(e\*x+d)^2/(c\*x^2+a)^(1/2),x)

[Out] (c\*x^2+a)^(1/2)/c/e^2-2/e^3\*d\*ln(x\*c^(1/2)+(c\*x^2+a)^(1/2))/c^(1/2)-3/e^4\*d^2/((a\*e^2+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2+c\*d^2)/e^2-2\*c\*d/e\*(x+d/e)+2\*((a\*e^2+c\*d^2)/e^2)^(1/2))\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2)/(x+d/e)+d^3/e^3/(a\*e^2+c\*d^2)/(x+d/e)\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2)+d^4/e^4\*c/(a\*e^2+c\*d^2)/((a\*e^2+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2+c\*d^2)/e^2-2\*c\*d/e\*(x+d/e)+2\*((a\*e^2+c\*d^2)/e^2)^(1/2))\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2)/(x+d/e))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(sqrt(c\*x^2 + a)\*(e\*x + d)^2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 6.6053, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(sqrt(c\*x^2 + a)\*(e\*x + d)^2),x, algorithm="fricas")

[Out] [1/2\*(2\*(2\*c\*d^3\*e + a\*d\*e^3 + (c\*d^2\*e^2 + a\*e^4)\*x)\*sqrt(c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a)\*sqrt(c) + 2\*(c^2\*d^4 + a\*c\*d^2\*e^2 + (c^2

$$\begin{aligned}
& 2*d^3*e + a*c*d*e^3)*x)*\sqrt{c*d^2 + a*e^2}*\log(2*\sqrt{c*x^2 + a} \\
& *c*x - (2*c*x^2 + a)*\sqrt{c})) + (2*c^2*d^5 + 3*a*c*d^3*e^2 + (2*c \\
& ^2*d^4*e + 3*a*c*d^2*e^3)*x)*\sqrt{c}*\log(((2*a*c*d*e*x - a*c*d^2 \\
& - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2)*\sqrt{c*d^2 + a*e^2} + 2* \\
& (a*c*d^2*e + a^2*e^3 - (c^2*d^3 + a*c*d*e^2)*x)*\sqrt{c*x^2 + a}))/ \\
& (e^2*x^2 + 2*d*e*x + d^2)))/((c^2*d^3*e^3 + a*c*d*e^5 + (c^2*d^2* \\
& e^4 + a*c*e^6)*x)*\sqrt{c*d^2 + a*e^2}*\sqrt{c}), ((2*c*d^3*e + a*d \\
& *e^3 + (c*d^2*e^2 + a*e^4)*x)*\sqrt{-c*d^2 - a*e^2}*\sqrt{c*x^2 + a} \\
& )*\sqrt{c} + (2*c^2*d^5 + 3*a*c*d^3*e^2 + (2*c^2*d^4*e + 3*a*c*d^2 \\
& *e^3)*x)*\sqrt{c}*\arctan(\sqrt{-c*d^2 - a*e^2}*(c*d*x - a*e)/((c*d^2 \\
& + a*e^2)*\sqrt{c*x^2 + a})) + (c^2*d^4 + a*c*d^2*e^2 + (c^2*d^3* \\
& e + a*c*d*e^3)*x)*\sqrt{-c*d^2 - a*e^2}*\log(2*\sqrt{c*x^2 + a}*c*x \\
& - (2*c*x^2 + a)*\sqrt{c}))/((c^2*d^3*e^3 + a*c*d*e^5 + (c^2*d^2*e^4 \\
& + a*c*e^6)*x)*\sqrt{-c*d^2 - a*e^2}*\sqrt{c}), 1/2*(2*(2*c*d^3*e \\
& + a*d*e^3 + (c*d^2*e^2 + a*e^4)*x)*\sqrt{c*d^2 + a*e^2}*\sqrt{c*x^2 \\
& + a}*\sqrt{-c} - 4*(c^2*d^4 + a*c*d^2*e^2 + (c^2*d^3*e + a*c*d*e^3 \\
& )*x)*\sqrt{c*d^2 + a*e^2}*\arctan(\sqrt{-c}*x/\sqrt{c*x^2 + a}) + (2 \\
& *c^2*d^5 + 3*a*c*d^3*e^2 + (2*c^2*d^4*e + 3*a*c*d^2*e^3)*x)*\sqrt{ \\
& -c}*\log(((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2 \\
& )*x^2)*\sqrt{c*d^2 + a*e^2} + 2*(a*c*d^2*e + a^2*e^3 - (c^2*d^3 + \\
& a*c*d*e^2)*x)*\sqrt{c*x^2 + a}))/((e^2*x^2 + 2*d*e*x + d^2)))/((c^2 \\
& *d^3*e^3 + a*c*d*e^5 + (c^2*d^2*e^4 + a*c*e^6)*x)*\sqrt{c*d^2 + a* \\
& e^2}*\sqrt{-c}), ((2*c*d^3*e + a*d*e^3 + (c*d^2*e^2 + a*e^4)*x)*\sqrt{ \\
& -c*d^2 - a*e^2}*\sqrt{c*x^2 + a}*\sqrt{-c} - 2*(c^2*d^4 + a*c*d^2 \\
& *e^2 + (c^2*d^3*e + a*c*d*e^3)*x)*\sqrt{-c*d^2 - a*e^2}*\arctan(\sqrt{ \\
& -c}*x/\sqrt{c*x^2 + a}) + (2*c^2*d^5 + 3*a*c*d^3*e^2 + (2*c^2*d \\
& ^4*e + 3*a*c*d^2*e^3)*x)*\sqrt{-c}*\arctan(\sqrt{-c*d^2 - a*e^2}*(c* \\
& d*x - a*e)/((c*d^2 + a*e^2)*\sqrt{c*x^2 + a}))/((c^2*d^3*e^3 + a* \\
& c*d*e^5 + (c^2*d^2*e^4 + a*c*e^6)*x)*\sqrt{-c*d^2 - a*e^2}*\sqrt{-c} \\
& ))]
\end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{a + cx^2}(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(e\*x+d)\*\*2/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] Integral(x\*\*3/(sqrt(a + c\*x\*\*2)\*(d + e\*x)\*\*2), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{cx^2 + a}(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(sqrt(c\*x^2 + a)\*(e\*x + d)^2),x, algorithm="giac")

[Out] integrate(x^3/(sqrt(c\*x^2 + a)\*(e\*x + d)^2), x)

$$3.345 \quad \int \frac{x^2}{(d+ex)^2 \sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=137

$$-\frac{d^2 \sqrt{a+cx^2}}{e(d+ex)(ae^2+cd^2)} + \frac{d(2ae^2+cd^2) \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2(ae^2+cd^2)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e^2}$$

[Out] -((d^2\*Sqrt[a + c\*x^2])/(e\*(c\*d^2 + a\*e^2)\*(d + e\*x))) + ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]]/(Sqrt[c]\*e^2) + (d\*(c\*d^2 + 2\*a\*e^2)\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])])/(e^2\*(c\*d^2 + a\*e^2)^(3/2))

**Rubi [A]** time = 0.349212, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$-\frac{d^2 \sqrt{a+cx^2}}{e(d+ex)(ae^2+cd^2)} + \frac{d(2ae^2+cd^2) \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2(ae^2+cd^2)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e^2}$$

Antiderivative was successfully verified.

[In] Int[x^2/((d + e\*x)^2\*Sqrt[a + c\*x^2]),x]

[Out] -((d^2\*Sqrt[a + c\*x^2])/(e\*(c\*d^2 + a\*e^2)\*(d + e\*x))) + ArcTanh[(Sqrt[c]\*x)/Sqrt[a + c\*x^2]]/(Sqrt[c]\*e^2) + (d\*(c\*d^2 + 2\*a\*e^2)\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])])/(e^2\*(c\*d^2 + a\*e^2)^(3/2))

**Rubi in Sympy [A]** time = 33.8045, size = 167, normalized size = 1.22

$$-\frac{cd^3 \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2(ae^2+cd^2)^{3/2}} - \frac{d^2 \sqrt{a+cx^2}}{e(d+ex)(ae^2+cd^2)} + \frac{2d \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{e^2 \sqrt{ae^2+cd^2}} + \frac{\operatorname{atanh}\left(\frac{\sqrt{c}x}{\sqrt{a+cx^2}}\right)}{\sqrt{c}e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(e\*x+d)\*\*2/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] -c\*d\*\*3\*atanh((a\*e - c\*d\*x)/(sqrt(a + c\*x\*\*2)\*sqrt(a\*e\*\*2 + c\*d\*\*2)))/(e\*\*2\*(a\*e\*\*2 + c\*d\*\*2)\*\*(3/2)) - d\*\*2\*sqrt(a + c\*x\*\*2)/(e\*(d + e\*x)\*(a\*e\*\*2 + c\*d\*\*2)) + 2\*d\*atanh((a\*e - c\*d\*x)/(sqrt(a + c\*x\*\*2)\*sqrt(a\*e\*\*2 + c\*d\*\*2)))/(e\*\*2\*sqrt(a\*e\*\*2 + c\*d\*\*2)) + atanh(sqrt(c)\*x/sqrt(a + c\*x\*\*2))/(sqrt(c)\*e\*\*2)

**Mathematica [A]** time = 0.608454, size = 172, normalized size = 1.26

$$\frac{d \left( \frac{(2ae^2+cd^2) \log(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx)}{(ae^2+cd^2)^{3/2}} - \frac{de\sqrt{a+cx^2}}{(d+ex)(ae^2+cd^2)} \right) - \frac{(2ade^2+cd^3) \log(d+ex)}{(ae^2+cd^2)^{3/2}} + \frac{\log(\sqrt{c}\sqrt{a+cx^2}+cx)}{\sqrt{c}}}{e^2}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((d + e\*x)^2\*Sqrt[a + c\*x^2]),x]

[Out]  $(-\left(\left(\left(c*d^3 + 2*a*d*e^2\right)*\text{Log}[d + e*x]\right)/\left(c*d^2 + a*e^2\right)^{3/2}\right) + \text{Log}[c*x + \text{Sqrt}[c]*\text{Sqrt}[a + c*x^2]]/\text{Sqrt}[c] + d*(-\left(\left(d*e*\text{Sqrt}[a + c*x^2]\right)/\left(\left(c*d^2 + a*e^2\right)*(d + e*x)\right)\right) + \left(\left(c*d^2 + 2*a*e^2\right)*\text{Log}[a*e - c*d*x + \text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2]]\right)/\left(c*d^2 + a*e^2\right)^{3/2})/e^2$

**Maple [B]** time = 0.015, size = 368, normalized size = 2.7

$$\begin{aligned} & \frac{1}{e^2} \ln\left(x\sqrt{c} + \sqrt{cx^2 + a}\right) \frac{1}{\sqrt{c}} - \frac{d^2}{e^2(ae^2 + cd^2)} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2\frac{cd}{e}\left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}} \left(x + \frac{d}{e}\right)^{-1} \\ & - \frac{cd^3}{e^3(ae^2 + cd^2)} \ln\left(1\left(2\frac{ae^2 + cd^2}{e^2} - 2\frac{cd}{e}\left(x + \frac{d}{e}\right) + 2\sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2\frac{cd}{e}\left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}}\right)\left(x + \frac{d}{e}\right)^{-1}\right) \\ & + 2\frac{d}{e^3} \ln\left(1\left(2\frac{ae^2 + cd^2}{e^2} - 2\frac{cd}{e}\left(x + \frac{d}{e}\right) + 2\sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2\frac{cd}{e}\left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}}\right)\left(x + \frac{d}{e}\right)^{-1}\right) \frac{1}{\sqrt{\frac{ae^2 + cd^2}{e^2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(e*x+d)^2/(c*x^2+a)^(1/2),x)`

[Out]  $\frac{1}{e^2} \ln\left(x^2 c^{1/2} + (c x^2 + a)^{1/2}\right) / c^{1/2} - d^2 / e^2 / (a^* e^2 + c^* d^2) / (x + d/e) * ((x + d/e)^2 c - 2^* c^* d / e^* (x + d/e) + (a^* e^2 + c^* d^2) / e^2)^{1/2} - d^3 / e^3 * c / (a^* e^2 + c^* d^2) / ((a^* e^2 + c^* d^2) / e^2)^{1/2} * \ln\left(\left(2^* (a^* e^2 + c^* d^2) / e^2 - 2^* c^* d / e^* (x + d/e) + 2^* ((a^* e^2 + c^* d^2) / e^2)^{1/2} * ((x + d/e)^2 c - 2^* c^* d / e^* (x + d/e) + (a^* e^2 + c^* d^2) / e^2)^{1/2}\right) / (x + d/e) + 2^* d / e^3 / ((a^* e^2 + c^* d^2) / e^2)^{1/2} * \ln\left(\left(2^* (a^* e^2 + c^* d^2) / e^2 - 2^* c^* d / e^* (x + d/e) + 2^* ((a^* e^2 + c^* d^2) / e^2)^{1/2} * ((x + d/e)^2 c - 2^* c^* d / e^* (x + d/e) + (a^* e^2 + c^* d^2) / e^2)^{1/2}\right) / (x + d/e)\right)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(sqrt(c*x^2 + a)*(e*x + d)^2),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 7.64733, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(sqrt(c*x^2 + a)*(e*x + d)^2),x, algorithm="fricas")`

[Out]  $[-1/2*(2*\text{sqrt}(c*d^2 + a*e^2)*\text{sqrt}(c*x^2 + a)*\text{sqrt}(c)*d^2*e - (c*d^3 + a*d*e^2 + (c*d^2*e + a*e^3)*x)*\text{sqrt}(c*d^2 + a*e^2)*\text{log}(-2*\text{sqrt}(c*x^2 + a)*c*x - (2*c*x^2 + a)*\text{sqrt}(c)) - (c*d^4 + 2*a*d^2*e^2 + (c*d^3*e + 2*a*d*e^3)*x)*\text{sqrt}(c)*\text{log}(((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2)*\text{sqrt}(c*d^2 + a*e^2) - 2*(a*c*d^2*e + a^2*e^3 - (c^2*d^3 + a*c*d*e^2)*x)*\text{sqrt}(c*x^2 + a)))/(e^2*x^2 + 2*d*e*x + d^2))/((c*d^3*e^2 + a*d*e^4 + (c*d^2*e^3 + a*e^5)*x)*\text{sqrt}(c*d^2 + a*e^2)*\text{sqrt}(c)), -1/2*(2*\text{sqrt}(-c*d^2 - a*e^2)$

```

2)*sqrt(c*x^2 + a)*sqrt(c)*d^2*e + 2*(c*d^4 + 2*a*d^2*e^2 + (c*d^
3*e + 2*a*d*e^3)*x)*sqrt(c)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x -
a*e)/((c*d^2 + a*e^2)*sqrt(c*x^2 + a))) - (c*d^3 + a*d*e^2 + (c*d
^2*e + a*e^3)*x)*sqrt(-c*d^2 - a*e^2)*log(-2*sqrt(c*x^2 + a)*c*x
- (2*c*x^2 + a)*sqrt(c))/((c*d^3*e^2 + a*d*e^4 + (c*d^2*e^3 + a*
e^5)*x)*sqrt(-c*d^2 - a*e^2)*sqrt(c)), -1/2*(2*sqrt(c*d^2 + a*e^2
)*sqrt(c*x^2 + a)*sqrt(-c)*d^2*e - 2*(c*d^3 + a*d*e^2 + (c*d^2*e
+ a*e^3)*x)*sqrt(c*d^2 + a*e^2)*arctan(sqrt(-c)*x/sqrt(c*x^2 + a
)) - (c*d^4 + 2*a*d^2*e^2 + (c*d^3*e + 2*a*d*e^3)*x)*sqrt(-c)*log(
((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2)*
sqrt(c*d^2 + a*e^2) - 2*(a*c*d^2*e + a^2*e^3 - (c^2*d^3 + a*c*d*e
^2)*x)*sqrt(c*x^2 + a))/(e^2*x^2 + 2*d*e*x + d^2))/((c*d^3*e^2 +
a*d*e^4 + (c*d^2*e^3 + a*e^5)*x)*sqrt(c*d^2 + a*e^2)*sqrt(-c)),
-(sqrt(-c*d^2 - a*e^2)*sqrt(c*x^2 + a)*sqrt(-c)*d^2*e - (c*d^3 +
a*d*e^2 + (c*d^2*e + a*e^3)*x)*sqrt(-c*d^2 - a*e^2)*arctan(sqrt(-
c)*x/sqrt(c*x^2 + a)) + (c*d^4 + 2*a*d^2*e^2 + (c*d^3*e + 2*a*d*e
^3)*x)*sqrt(-c)*arctan(sqrt(-c*d^2 - a*e^2)*(c*d*x - a*e)/((c*d^2
+ a*e^2)*sqrt(c*x^2 + a))))/((c*d^3*e^2 + a*d*e^4 + (c*d^2*e^3 +
a*e^5)*x)*sqrt(-c*d^2 - a*e^2)*sqrt(-c))]

```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{a + cx^2}(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(e\*x+d)\*\*2/(c\*x\*\*2+a)\*\*(1/2), x)

[Out] Integral(x\*\*2/(sqrt(a + c\*x\*\*2)\*(d + e\*x)\*\*2), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{cx^2 + a}(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(sqrt(c\*x^2 + a)\*(e\*x + d)^2), x, algorithm="giac")

[Out] integrate(x^2/(sqrt(c\*x^2 + a)\*(e\*x + d)^2), x)

$$3.346 \quad \int \frac{x}{(d+ex)^2 \sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=90

$$\frac{d\sqrt{a+cx^2}}{(d+ex)(ae^2+cd^2)} - \frac{ae \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}}$$

[Out] (d\*Sqrt[a + c\*x^2])/((c\*d^2 + a\*e^2)\*(d + e\*x)) - (a\*e\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])])/(c\*d^2 + a\*e^2)^2)^(3/2)

**Rubi [A]** time = 0.118103, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{d\sqrt{a+cx^2}}{(d+ex)(ae^2+cd^2)} - \frac{ae \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x/((d + e\*x)^2\*Sqrt[a + c\*x^2]),x]

[Out] (d\*Sqrt[a + c\*x^2])/((c\*d^2 + a\*e^2)\*(d + e\*x)) - (a\*e\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])])/(c\*d^2 + a\*e^2)^2)^(3/2)

**Rubi in Sympy [A]** time = 14.3386, size = 76, normalized size = 0.84

$$-\frac{ae \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}} + \frac{d\sqrt{a+cx^2}}{(d+ex)(ae^2+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x/(e\*x+d)\*\*2/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] -a\*e\*atanh((a\*e - c\*d\*x)/(sqrt(a + c\*x\*\*2)\*sqrt(a\*e\*\*2 + c\*d\*\*2)))/(a\*e\*\*2 + c\*d\*\*2)\*\*(3/2) + d\*sqrt(a + c\*x\*\*2)/((d + e\*x)\*(a\*e\*\*2 + c\*d\*\*2))

**Mathematica [A]** time = 0.122557, size = 114, normalized size = 1.27

$$\frac{d\sqrt{a+cx^2}}{(d+ex)(ae^2+cd^2)} - \frac{ae \log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2} + ae - cdx\right)}{(ae^2+cd^2)^{3/2}} + \frac{ae \log(d+ex)}{(ae^2+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x/((d + e\*x)^2\*Sqrt[a + c\*x^2]),x]

[Out] (d\*Sqrt[a + c\*x^2])/((c\*d^2 + a\*e^2)\*(d + e\*x)) + (a\*e\*Log[d + e\*x])/((c\*d^2 + a\*e^2)^(3/2)) - (a\*e\*Log[a\*e - c\*d\*x + Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]])/(c\*d^2 + a\*e^2)^(3/2)

**Maple [B]** time = 0.012, size = 340, normalized size = 3.8

$$\begin{aligned}
 & -\frac{1}{e^2} \ln \left( 1 \left( 2 \frac{ae^2 + cd^2}{e^2} - 2 \frac{cd}{e} \left( x + \frac{d}{e} \right) + 2 \sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left( x + \frac{d}{e} \right)^2 c - 2 \frac{cd}{e} \left( x + \frac{d}{e} \right) + \frac{ae^2 + cd^2}{e^2}} \right) \left( x + \frac{d}{e} \right)^{-1} \right) \frac{1}{\sqrt{\frac{ae^2 + cd^2}{e^2}}} \\
 & + \frac{d}{e(ae^2 + cd^2)} \sqrt{\left( x + \frac{d}{e} \right)^2 c - 2 \frac{cd}{e} \left( x + \frac{d}{e} \right) + \frac{ae^2 + cd^2}{e^2}} \left( x + \frac{d}{e} \right)^{-1} \\
 & + \frac{cd^2}{e^2(ae^2 + cd^2)} \ln \left( 1 \left( 2 \frac{ae^2 + cd^2}{e^2} - 2 \frac{cd}{e} \left( x + \frac{d}{e} \right) + 2 \sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left( x + \frac{d}{e} \right)^2 c - 2 \frac{cd}{e} \left( x + \frac{d}{e} \right) + \frac{ae^2 + cd^2}{e^2}} \right) \left( x + \frac{d}{e} \right)^{-1} \right) \frac{1}{\sqrt{\frac{ae^2 + cd^2}{e^2}}}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(e\*x+d)^2/(c\*x^2+a)^(1/2),x)

[Out]  $-1/e^2/((a^*e^2+c^*d^2)/e^2)^{(1/2)}*\ln((2*(a^*e^2+c^*d^2)/e^2-2*c^*d/e*(x+d/e)+2*((a^*e^2+c^*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c-2*c^*d/e*(x+d/e)+(a^*e^2+c^*d^2)/e^2)^{(1/2)})/(x+d/e))+d/e/(a^*e^2+c^*d^2)/(x+d/e)*((x+d/e)^2*c-2*c^*d/e*(x+d/e)+(a^*e^2+c^*d^2)/e^2)^{(1/2)}+d^2/e^2*c/(a^*e^2+c^*d^2)/((a^*e^2+c^*d^2)/e^2)^{(1/2)}*\ln((2*(a^*e^2+c^*d^2)/e^2-2*c^*d/e*(x+d/e)+2*((a^*e^2+c^*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c-2*c^*d/e*(x+d/e)+(a^*e^2+c^*d^2)/e^2)^{(1/2)})/(x+d/e))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(sqrt(c\*x^2 + a)\*(e\*x + d)^2),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.322112, size = 1, normalized size = 0.01

$$\frac{2\sqrt{cd^2 + ae^2}\sqrt{cx^2 + ad} + (ae^2x + ade) \log\left(\frac{(2acdex - acd^2 - 2a^2e^2 - (2c^2d^2 + ace^2)x^2)\sqrt{cd^2 + ae^2} + 2(acd^2e + a^2e^3 - (c^2d^3 + acde^2)x)\sqrt{cx^2 + a}}{e^2x^2 + 2dex + d^2}\right)}{2(cd^3 + ade^2 + (cd^2e + ae^3)x)\sqrt{cd^2 + ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(sqrt(c\*x^2 + a)\*(e\*x + d)^2),x, algorithm="fricas")

[Out]  $[1/2*(2*\sqrt{c^*d^2 + a^*e^2})*\sqrt{c^*x^2 + a}*d + (a^*e^2*x + a^*d^*e)*\log(((2*a^*c^*d^*e*x - a^*c^*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c^*e^2)*x^2)*\sqrt{c^*d^2 + a^*e^2} + 2*(a^*c^*d^2*e + a^2*e^3 - (c^2*d^3 + a^*c^*d^*e^2)*x)*\sqrt{c^*x^2 + a}))/((e^2*x^2 + 2*d^*e*x + d^2)))/((c^*d^3 + a^*d^*e^2 + (c^*d^2*e + a^*e^3)*x)*\sqrt{c^*d^2 + a^*e^2}), (\sqrt{-c^*d^2 - a^*e^2})*\sqrt{c^*x^2 + a}*d + (a^*e^2*x + a^*d^*e)*\arctan(\sqrt{-c^*d^2 - a^*e^2}*(c^*d*x - a^*e)/((c^*d^2 + a^*e^2)*\sqrt{c^*x^2 + a})))/((c^*d^3 + a^*d^*e^2 + (c^*d^2*e + a^*e^3)*x)*\sqrt{-c^*d^2 - a^*e^2})]$



**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{a + cx^2}(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e\*x+d)\*\*2/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] Integral(x/(sqrt(a + c\*x\*\*2)\*(d + e\*x)\*\*2), x)

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{cx^2 + a}(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(sqrt(c\*x^2 + a)\*(e\*x + d)^2),x, algorithm="giac")

[Out] integrate(x/(sqrt(c\*x^2 + a)\*(e\*x + d)^2), x)

$$3.347 \quad \int \frac{1}{(d+ex)^2 \sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=91

$$-\frac{e\sqrt{a+cx^2}}{(d+ex)(ae^2+cd^2)} - \frac{cd \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}}$$

[Out] -((e\*Sqrt[a + c\*x^2])/((c\*d^2 + a\*e^2)\*(d + e\*x))) - (c\*d\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])])/(c\*d^2 + a\*e^2)^(3/2)

**Rubi [A]** time = 0.0891377, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$

$$-\frac{e\sqrt{a+cx^2}}{(d+ex)(ae^2+cd^2)} - \frac{cd \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)^2\*Sqrt[a + c\*x^2]),x]

[Out] -((e\*Sqrt[a + c\*x^2])/((c\*d^2 + a\*e^2)\*(d + e\*x))) - (c\*d\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])])/(c\*d^2 + a\*e^2)^(3/2)

**Rubi in Sympy [A]** time = 13.8041, size = 78, normalized size = 0.86

$$-\frac{cd \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}} - \frac{e\sqrt{a+cx^2}}{(d+ex)(ae^2+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(e\*x+d)\*\*2/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] -c\*d\*atanh((a\*e - c\*d\*x)/(sqrt(a + c\*x\*\*2)\*sqrt(a\*e\*\*2 + c\*d\*\*2)))/(a\*e\*\*2 + c\*d\*\*2)\*\*(3/2) - e\*sqrt(a + c\*x\*\*2)/((d + e\*x)\*(a\*e\*\*2 + c\*d\*\*2))

**Mathematica [A]** time = 0.112902, size = 115, normalized size = 1.26

$$-\frac{e\sqrt{a+cx^2}}{(d+ex)(ae^2+cd^2)} - \frac{cd \log\left(\sqrt{a+cx^2}\sqrt{ae^2+cd^2} + ae - cdx\right)}{(ae^2+cd^2)^{3/2}} + \frac{cd \log(d+ex)}{(ae^2+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x)^2\*Sqrt[a + c\*x^2]),x]

[Out] -((e\*Sqrt[a + c\*x^2])/((c\*d^2 + a\*e^2)\*(d + e\*x))) + (c\*d\*Log[d + e\*x])/((c\*d^2 + a\*e^2)^(3/2)) - (c\*d\*Log[a\*e - c\*d\*x + Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])/(c\*d^2 + a\*e^2)^(3/2)

**Maple [B]** time = 0.007, size = 210, normalized size = 2.3

$$-\frac{1}{ae^2 + cd^2} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2} \left(x + \frac{d}{e}\right)^{-1}}$$

$$-\frac{cd}{e(ae^2 + cd^2)} \ln\left(1 \left(2 \frac{ae^2 + cd^2}{e^2} - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + 2 \sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}}\right) \left(x + \frac{d}{e}\right)^{-1}\right) \sqrt{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x+d)^2/(c\*x^2+a)^(1/2), x)

[Out] -1/(a\*e^2+c\*d^2)/(x+d/e)\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2)-1/e\*c\*d/(a\*e^2+c\*d^2)/((a\*e^2+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2+c\*d^2)/e^2-2\*c\*d/e\*(x+d/e)+2\*((a\*e^2+c\*d^2)/e^2)^(1/2))\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2))/(x+d/e)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)^2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.318173, size = 1, normalized size = 0.01

$$\left[ \frac{2 \sqrt{cd^2 + ae^2} \sqrt{cx^2 + ae} - (cdex + cd^2) \log\left(\frac{(2acdex - acd^2 - 2a^2e^2 - (2c^2d^2 + ace^2)x^2) \sqrt{cd^2 + ae^2} + 2(acd^2e + a^2e^3 - (c^2d^3 + acde^2)x) \sqrt{cx^2 + ae}}{e^2x^2 + 2dex + d^2}\right)}{2(cd^3 + ade^2 + (cd^2e + ae^3)x) \sqrt{cd^2 + ae^2}} \right]$$

$$-\frac{\sqrt{-cd^2 - ae^2} \sqrt{cx^2 + ae} - (cdex + cd^2) \arctan\left(\frac{\sqrt{-cd^2 - ae^2}(cdx - ae)}{(cd^2 + ae^2) \sqrt{cx^2 + ae}}\right)}{(cd^3 + ade^2 + (cd^2e + ae^3)x) \sqrt{-cd^2 - ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)^2), x, algorithm="fricas")

[Out] [-1/2\*(2\*sqrt(c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a)\*e - (c\*d\*e\*x + c\*d^2)\*log(((2\*a\*c\*d\*e\*x - a\*c\*d^2 - 2\*a^2\*e^2 - (2\*c^2\*d^2 + a\*c\*e^2)\*x^2)\*sqrt(c\*d^2 + a\*e^2) + 2\*(a\*c\*d^2\*e + a^2\*e^3 - (c^2\*d^3 + a\*c\*d\*e^2)\*x)\*sqrt(c\*x^2 + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)))/((c\*d^3 + a\*d\*e^2 + (c\*d^2\*e + a\*e^3)\*x)\*sqrt(c\*d^2 + a\*e^2)), -(sqrt(-c\*d^2 - a\*e^2)\*sqrt(c\*x^2 + a)\*e - (c\*d\*e\*x + c\*d^2)\*arctan(sqrt(-c\*d^2 - a\*e^2)\*(c\*d\*x - a\*e)/((c\*d^2 + a\*e^2)\*sqrt(c\*x^2 + a))))/((c\*d^3 + a\*d\*e^2 + (c\*d^2\*e + a\*e^3)\*x)\*sqrt(-c\*d^2 - a\*e^2))]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + cx^2} (d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x+d)\*\*2/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] Integral(1/(sqrt(a + c\*x\*\*2)\*(d + e\*x)\*\*2), x)

**GIAC/XCAS [A]** time = 1.43956, size = 439, normalized size = 4.82

$$\frac{\sqrt{cd^2 + ae^2}cd \ln \left( \left| -\sqrt{cd^2 + ae^2}cd + (cd^2 + ae^2) \left( \sqrt{c - \frac{2cd}{xe+d} + \frac{cd^2}{(xe+d)^2} + \frac{ae^2}{(xe+d)^2}} + \frac{\sqrt{cd^2e^2 + ae^4}e^{(-1)}}{xe+d} \right) \right| \right)}{(c^2d^4e + 2acd^2e^3 + a^2e^5) \operatorname{sign} \left( \frac{1}{xe+d} \right)}$$

$$+ \frac{\left( c^{\frac{3}{2}}d^2 + \sqrt{cd^2 + ae^2}cd \ln \left( \left| c^{\frac{3}{2}}d^2 - \sqrt{cd^2 + ae^2}cd + a\sqrt{ce^2} \right| \right) + a\sqrt{ce^2} \right) \operatorname{sign} \left( \frac{1}{xe+d} \right)}{c^2d^4 + 2acd^2e^2 + a^2e^4}$$

$$- \frac{\sqrt{c - \frac{2cd}{xe+d} + \frac{cd^2}{(xe+d)^2} + \frac{ae^2}{(xe+d)^2}}}{cd^2 \operatorname{sign} \left( \frac{1}{xe+d} \right) + ae^2 \operatorname{sign} \left( \frac{1}{xe+d} \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)^2),x, algorithm="giac")

[Out]  $-\sqrt{c*d^2 + a*e^2} * c*d*e * \ln(\operatorname{abs}(-\sqrt{c*d^2 + a*e^2}) * c*d + (c*d^2 + a*e^2) * (\sqrt{c - 2*c*d/(x*e + d)} + c*d^2/(x*e + d)^2 + a*e^2/(x*e + d)^2) + \sqrt{c*d^2*e^2 + a*e^4} * e^{(-1)}/(x*e + d))) / ((c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) * \operatorname{sign}(1/(x*e + d))) + (c^{(3/2)} * d^2 + \sqrt{c*d^2 + a*e^2} * c*d * \ln(\operatorname{abs}(c^{(3/2)} * d^2 - \sqrt{c*d^2 + a*e^2} * c*d + a * \sqrt{c} * e^2)) + a * \sqrt{c} * e^2) * \operatorname{sign}(1/(x*e + d))) / (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - \sqrt{c - 2*c*d/(x*e + d)} + c*d^2/(x*e + d)^2 + a*e^2/(x*e + d)^2) / (c*d^2 * \operatorname{sign}(1/(x*e + d))) + a * e^2 * \operatorname{sign}(1/(x*e + d)))$

$$3.348 \quad \int \frac{1}{x(d+ex)^2 \sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=179

$$\frac{e^2 \sqrt{a+cx^2}}{d(d+ex)(ae^2+cd^2)} + \frac{e \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2 \sqrt{ae^2+cd^2}} + \frac{ce \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{ad^2}}$$

[Out] (e^2\*Sqrt[a + c\*x^2])/(d\*(c\*d^2 + a\*e^2)\*(d + e\*x)) + (c\*e\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])])/(c\*d^2 + a\*e^2)^(3/2) + (e\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])])/(d^2\*Sqrt[c\*d^2 + a\*e^2]) - ArcTanh[Sqrt[a + c\*x^2]/Sqrt[a]]/(Sqrt[a]\*d^2)

**Rubi [A]** time = 0.345129, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$

$$\frac{e^2 \sqrt{a+cx^2}}{d(d+ex)(ae^2+cd^2)} + \frac{e \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2 \sqrt{ae^2+cd^2}} + \frac{ce \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{ad^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(d + e\*x)^2\*Sqrt[a + c\*x^2]),x]

[Out] (e^2\*Sqrt[a + c\*x^2])/(d\*(c\*d^2 + a\*e^2)\*(d + e\*x)) + (c\*e\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])])/(c\*d^2 + a\*e^2)^(3/2) + (e\*ArcTanh[(a\*e - c\*d\*x)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2])])/(d^2\*Sqrt[c\*d^2 + a\*e^2]) - ArcTanh[Sqrt[a + c\*x^2]/Sqrt[a]]/(Sqrt[a]\*d^2)

**Rubi in Sympy [A]** time = 33.4604, size = 158, normalized size = 0.88

$$\frac{ce \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{(ae^2+cd^2)^{3/2}} + \frac{e^2 \sqrt{a+cx^2}}{d(d+ex)(ae^2+cd^2)} + \frac{e \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2 \sqrt{ae^2+cd^2}} - \frac{\operatorname{atanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{ad^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(e\*x+d)\*\*2/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] c\*e\*atanh((a\*e - c\*d\*x)/(sqrt(a + c\*x\*\*2)\*sqrt(a\*e\*\*2 + c\*d\*\*2)))/(a\*e\*\*2 + c\*d\*\*2)\*\*(3/2) + e\*\*2\*sqrt(a + c\*x\*\*2)/(d\*(d + e\*x)\*(a\*e\*\*2 + c\*d\*\*2)) + e\*atanh((a\*e - c\*d\*x)/(sqrt(a + c\*x\*\*2)\*sqrt(a\*e\*\*2 + c\*d\*\*2)))/(d\*\*2\*sqrt(a\*e\*\*2 + c\*d\*\*2)) - atanh(sqrt(a + c\*x\*\*2)/sqrt(a))/(sqrt(a)\*d\*\*2)

**Mathematica [A]** time = 0.399877, size = 178, normalized size = 0.99

$$\frac{\frac{de^2 \sqrt{a+cx^2}}{(d+ex)(ae^2+cd^2)} + \frac{e(ae^2+2cd^2) \log(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx)}{(ae^2+cd^2)^{3/2}} - \frac{e(ae^2+2cd^2) \log(d+ex)}{(ae^2+cd^2)^{3/2}} - \frac{\log(\sqrt{a}\sqrt{a+cx^2}+a)}{\sqrt{a}} + \frac{\log(x)}{\sqrt{a}}}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(d + e\*x)^2\*Sqrt[a + c\*x^2]),x]

[Out]  $((d^2 e^2 \sqrt{a + c x^2}) / ((c d^2 + a e^2) (d + e x)) + \text{Log}[x] / \text{Sqrt}[a] - (e (2 c d^2 + a e^2) \text{Log}[d + e x]) / (c d^2 + a e^2)^{3/2} - \text{Log}[a + \text{Sqrt}[a] \text{Sqrt}[a + c x^2]] / \text{Sqrt}[a] + (e (2 c d^2 + a e^2) \text{Log}[a e - c d x + \text{Sqrt}[c d^2 + a e^2] \text{Sqrt}[a + c x^2]]) / (c d^2 + a e^2)^{3/2}) / d^2$

**Maple [B]** time = 0.016, size = 364, normalized size = 2.

$$-\frac{1}{d^2} \ln\left(\frac{1}{x} \left(2a + 2\sqrt{a}\sqrt{cx^2 + a}\right)\right) \frac{1}{\sqrt{a}}$$

$$+ \frac{1}{d^2} \ln\left(1 \left(2 \frac{ae^2 + cd^2}{e^2} - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + 2 \sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}}\right) \left(x + \frac{d}{e}\right)^{-1}\right) \frac{1}{\sqrt{\frac{ae^2 + cd^2}{e^2}}}$$

$$+ \frac{e}{d(ae^2 + cd^2)} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}} \left(x + \frac{d}{e}\right)^{-1}$$

$$+ \frac{c}{ae^2 + cd^2} \ln\left(1 \left(2 \frac{ae^2 + cd^2}{e^2} - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + 2 \sqrt{\frac{ae^2 + cd^2}{e^2}} \sqrt{\left(x + \frac{d}{e}\right)^2 c - 2 \frac{cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 + cd^2}{e^2}}\right) \left(x + \frac{d}{e}\right)^{-1}\right) \frac{1}{\sqrt{\frac{ae^2 + cd^2}{e^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(e*x+d)^2/(c*x^2+a)^(1/2),x)`

[Out]  $-1/d^2/a^{1/2} * \ln((2*a+2*a^{1/2}*(c*x^2+a)^{1/2})/x) + 1/d^2/((a*e^2+2*c*d^2)/e^2)^{1/2} * \ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^{1/2}*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^{1/2})/(x+d/e) + 1/d*e/(a*e^2+c*d^2)/(x+d/e)*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^{1/2} + c/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^{1/2} * \ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^{1/2}*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^{1/2})/(x+d/e)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + a}(ex + d)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*x),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*x), x)`

**Fricas [A]** time = 0.62225, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*x),x, algorithm="fricas")`

[Out]  $[1/2*(2*\text{sqrt}(c*d^2 + a*e^2)*\text{sqrt}(c*x^2 + a)*\text{sqrt}(a)*d*e^2 + (2*c*d^3*e + a*d^2*e^3 + (2*c*d^2*e^2 + a*e^4)*x)*\text{sqrt}(a)*\text{log}(((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2)*\text{sqrt}(c*d^2 + a*e^2) - 2*(a*c*d^2*e + a^2*e^3 - (c^2*d^3 + a*c*d^2*e^2)*x)*\text{sqrt}(a)))]$

$$\begin{aligned} & t(c*x^2 + a)/(e^2*x^2 + 2*d*e*x + d^2) + (c*d^3 + a*d*e^2 + (c*d^2*e + a*e^3)*x)*\sqrt{c*d^2 + a*e^2}*\log(-((c*x^2 + 2*a)*\sqrt{a} \\ & - 2*\sqrt{c*x^2 + a}*a)/x^2))/((c*d^5 + a*d^3*e^2 + (c*d^4*e + a*d^2*e^3)*x)*\sqrt{c*d^2 + a*e^2}*\sqrt{a}), 1/2*(2*\sqrt{-c*d^2 - a* \\ & e^2}*\sqrt{c*x^2 + a}*\sqrt{a}*d*e^2 - 2*(2*c*d^3*e + a*d*e^3 + (2*c*d^2*e^2 + a*e^4)*x)*\sqrt{a})*\arctan(\sqrt{-c*d^2 - a*e^2}*(c*d*x \\ & - a*e)/((c*d^2 + a*e^2)*\sqrt{c*x^2 + a})) + (c*d^3 + a*d*e^2 + (c*d^2*e + a*e^3)*x)*\sqrt{-c*d^2 - a*e^2}*\log(-((c*x^2 + 2*a)*\sqrt{a} \\ & - 2*\sqrt{c*x^2 + a}*a)/x^2))/((c*d^5 + a*d^3*e^2 + (c*d^4*e + a*d^2*e^3)*x)*\sqrt{-c*d^2 - a*e^2}*\sqrt{a}), 1/2*(2*\sqrt{c*d^2 + \\ & a*e^2}*\sqrt{c*x^2 + a}*\sqrt{-a}*d*e^2 - 2*(c*d^3 + a*d*e^2 + (c*d^2*e + a*e^3)*x)*\sqrt{c*d^2 + a*e^2})*\arctan(\sqrt{-a}/\sqrt{c*x^2 + \\ & a}) + (2*c*d^3*e + a*d*e^3 + (2*c*d^2*e^2 + a*e^4)*x)*\sqrt{-a}*\log(((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2) \\ & )*\sqrt{c*d^2 + a*e^2} - 2*(a*c*d^2*e + a^2*e^3 - (c^2*d^3 + a*c*d*e^2)*x)*\sqrt{c*x^2 + a}))/((e^2*x^2 + 2*d*e*x + d^2)))/((c*d^5 + \\ & a*d^3*e^2 + (c*d^4*e + a*d^2*e^3)*x)*\sqrt{c*d^2 + a*e^2}*\sqrt{-a}), (\sqrt{-c*d^2 - a*e^2}*\sqrt{c*x^2 + a}*\sqrt{-a}*d*e^2 - (2*c*d^3 \\ & e + a*d*e^3 + (2*c*d^2*e^2 + a*e^4)*x)*\sqrt{-a})*\arctan(\sqrt{-c*d^2 - a*e^2}*(c*d*x - a*e)/((c*d^2 + a*e^2)*\sqrt{c*x^2 + a})) - ( \\ & c*d^3 + a*d*e^2 + (c*d^2*e + a*e^3)*x)*\sqrt{-c*d^2 - a*e^2})*\arctan(\sqrt{-a}/\sqrt{c*x^2 + a}))/((c*d^5 + a*d^3*e^2 + (c*d^4*e + a*d^2 \\ & e^3)*x)*\sqrt{-c*d^2 - a*e^2}*\sqrt{-a})] \end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{a+cx^2}(d+ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e\*x+d)\*\*2/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] Integral(1/(x\*sqrt(a + c\*x\*\*2)\*(d + e\*x)\*\*2), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}(ex+d)^2x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)^2\*x),x, algorithm="giac")

[Out] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)^2\*x), x)

$$3.349 \quad \int \frac{1}{x^2(d+ex)^2\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=212

$$\frac{2e \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3} - \frac{ce^2 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d(ae^2+cd^2)^{3/2}} \\ - \frac{e^3\sqrt{a+cx^2}}{d^2(d+ex)(ae^2+cd^2)} - \frac{\sqrt{a+cx^2}}{ad^2x} - \frac{2e^2 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3\sqrt{ae^2+cd^2}}$$

[Out]  $-(\text{Sqrt}[a + c*x^2]/(a*d^2*x)) - (e^3*\text{Sqrt}[a + c*x^2])/(d^2*(c*d^2 + a*e^2)*(d + e*x)) - (c*e^2*\text{ArcTanh}[(a*e - c*d*x)/(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2])])/(d*(c*d^2 + a*e^2)^{(3/2)}) - (2*e^2*\text{ArcTanh}[(a*e - c*d*x)/(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2])])/(d^3*\text{Sqrt}[c*d^2 + a*e^2]) + (2*e*\text{ArcTanh}[\text{Sqrt}[a + c*x^2]/\text{Sqrt}[a]])/(\text{Sqrt}[a]*d^3)$

**Rubi [A]** time = 0.412162, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$

$$\frac{2e \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3} - \frac{ce^2 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d(ae^2+cd^2)^{3/2}} \\ - \frac{e^3\sqrt{a+cx^2}}{d^2(d+ex)(ae^2+cd^2)} - \frac{\sqrt{a+cx^2}}{ad^2x} - \frac{2e^2 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3\sqrt{ae^2+cd^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^2*(d + e*x)^2*\text{Sqrt}[a + c*x^2]), x]$

[Out]  $-(\text{Sqrt}[a + c*x^2]/(a*d^2*x)) - (e^3*\text{Sqrt}[a + c*x^2])/(d^2*(c*d^2 + a*e^2)*(d + e*x)) - (c*e^2*\text{ArcTanh}[(a*e - c*d*x)/(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2])])/(d*(c*d^2 + a*e^2)^{(3/2)}) - (2*e^2*\text{ArcTanh}[(a*e - c*d*x)/(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2])])/(d^3*\text{Sqrt}[c*d^2 + a*e^2]) + (2*e*\text{ArcTanh}[\text{Sqrt}[a + c*x^2]/\text{Sqrt}[a]])/(\text{Sqrt}[a]*d^3)$

**Rubi in Sympy [A]** time = 41.1157, size = 187, normalized size = 0.88

$$-\frac{ce^2 \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d(ae^2+cd^2)^{3/2}} - \frac{e^3\sqrt{a+cx^2}}{d^2(d+ex)(ae^2+cd^2)} \\ - \frac{2e^2 \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^3\sqrt{ae^2+cd^2}} - \frac{\sqrt{a+cx^2}}{ad^2x} + \frac{2e \operatorname{atanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x^{**2}/(e*x+d)^{**2}/(c*x^{**2}+a)^{**}(1/2), x)$

[Out]  $-c*e^{**2}*\operatorname{atanh}((a*e - c*d*x)/(\text{sqrt}(a + c*x^{**2})*\text{sqrt}(a*e^{**2} + c*d^{**2}))) / (d*(a*e^{**2} + c*d^{**2})^{**}(3/2)) - e^{**3}*\text{sqrt}(a + c*x^{**2}) / (d^{**2}*(d + e*x)*(a*e^{**2} + c*d^{**2})) - 2*e^{**2}*\operatorname{atanh}((a*e - c*d*x)/(\text{sqrt}(a + c*x^{**2})*\text{sqrt}(a*e^{**2} + c*d^{**2}))) / (d^{**3}*\text{sqrt}(a*e^{**2} + c*d^{**2})) - \text{sqrt}(a + c*x^{**2}) / (a*d^{**2}*x) + 2*e*\operatorname{atanh}(\text{sqrt}(a + c*x^{**2})/\text{sqrt}(a)) / (\text{sqrt}(a)*d^{**3})$



**Mathematica [A]** time = 0.563207, size = 197, normalized size = 0.93

$$\frac{-\frac{e^2(2ae^2+3cd^2)\log(\sqrt{a+cx^2}\sqrt{ae^2+cd^2}+ae-cdx)}{(ae^2+cd^2)^{3/2}} + \frac{e^2(2ae^2+3cd^2)\log(d+ex)}{(ae^2+cd^2)^{3/2}} - d\sqrt{a+cx^2}\left(\frac{e^3}{(d+ex)(ae^2+cd^2)} + \frac{1}{ax}\right) + \frac{2e\log(\sqrt{a}\sqrt{a+cx^2}+a)}{\sqrt{a}}}{d^3}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(d + e\*x)^2\*Sqrt[a + c\*x^2]), x]

[Out]  $(-(d*\text{Sqrt}[a + c*x^2]*(1/(a*x) + e^3/((c*d^2 + a*e^2)*(d + e*x)))) - (2*e*\text{Log}[x])/ \text{Sqrt}[a] + (e^2*(3*c*d^2 + 2*a*e^2)*\text{Log}[d + e*x])/ (c*d^2 + a*e^2)^{(3/2)} + (2*e*\text{Log}[a + \text{Sqrt}[a]*\text{Sqrt}[a + c*x^2]])/ \text{Sqrt}[a] - (e^2*(3*c*d^2 + 2*a*e^2)*\text{Log}[a*e - c*d*x + \text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2]])/ (c*d^2 + a*e^2)^{(3/2)})/d^3$

**Maple [B]** time = 0.016, size = 395, normalized size = 1.9

$$\begin{aligned} & -\frac{1}{ad^2x}\sqrt{cx^2+a} - \frac{e^2}{d^2(ae^2+cd^2)}\sqrt{\left(x+\frac{d}{e}\right)^2c-2\frac{cd}{e}\left(x+\frac{d}{e}\right)+\frac{ae^2+cd^2}{e^2}\left(x+\frac{d}{e}\right)^{-1}} \\ & -\frac{ce}{d(ae^2+cd^2)}\ln\left(1\left(2\frac{ae^2+cd^2}{e^2}-2\frac{cd}{e}\left(x+\frac{d}{e}\right)+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c-2\frac{cd}{e}\left(x+\frac{d}{e}\right)+\frac{ae^2+cd^2}{e^2}}\right)\left(x+\frac{d}{e}\right)^{-1}\right) \\ & +2\frac{e}{d^3\sqrt{a}}\ln\left(\frac{2a+2\sqrt{a}\sqrt{cx^2+a}}{x}\right) \\ & -2\frac{e}{d^3}\ln\left(1\left(2\frac{ae^2+cd^2}{e^2}-2\frac{cd}{e}\left(x+\frac{d}{e}\right)+2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c-2\frac{cd}{e}\left(x+\frac{d}{e}\right)+\frac{ae^2+cd^2}{e^2}}\right)\left(x+\frac{d}{e}\right)^{-1}\right)\frac{1}{\sqrt{\frac{ae^2+cd^2}{e^2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(e\*x+d)^2/(c\*x^2+a)^(1/2), x)

[Out]  $-(c*x^2+a)^{(1/2)}/a/d^2/x-1/d^2/(a*e^2+c*d^2)*e^2/(x+d/e)*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^{(1/2)}-1/d*c*e/(a*e^2+c*d^2)/((a*e^2+c*d^2)/e^2)^{(1/2)}*\ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^{(1/2)})/(x+d/e))+2/d^3*e/a^{(1/2)}*\ln((2*a+2*a^{(1/2)}*(c*x^2+a)^{(1/2)})/x)-2/d^3*e/((a*e^2+c*d^2)/e^2)^{(1/2)}*\ln((2*(a*e^2+c*d^2)/e^2-2*c*d/e*(x+d/e)+2*((a*e^2+c*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c-2*c*d/e*(x+d/e)+(a*e^2+c*d^2)/e^2)^{(1/2)})/(x+d/e))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}(ex+d)^2x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)^2\*x^2), x, algorithm="maxima")

[Out] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)^2\*x^2), x)

**Fricas [A]** time = 0.590383, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)^2\*x^2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/2*(2*(c*d^4 + a*d^2*e^2 + (c*d^3*e + 2*a*d*e^3)*x)*\sqrt{c*d^2 + a*e^2})*\sqrt{c*x^2 + a}*\sqrt{a} - ((3*a*c*d^2*e^3 + 2*a^2*e^5)*x^2 + (3*a*c*d^3*e^2 + 2*a^2*d*e^4)*x)*\sqrt{a}*\log(((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2)*\sqrt{c*d^2 + a*e^2}) + 2*(a*c*d^2*e + a^2*e^3 - (c^2*d^3 + a*c*d*e^2)*x)*\sqrt{c*x^2 + a})/(e^2*x^2 + 2*d*e*x + d^2)) - 2*\sqrt{c*d^2 + a*e^2}*((a*c*d^2*e^2 + a^2*e^4)*x^2 + (a*c*d^3*e + a^2*d*e^3)*x)*\log(-((c*x^2 + 2*a)*\sqrt{a} + 2*\sqrt{c*x^2 + a}*a)/x^2))/(\sqrt{c*d^2 + a*e^2}) * ((a*c*d^5*e + a^2*d^3*e^3)*x^2 + (a*c*d^6 + a^2*d^4*e^2)*x)*\sqrt{a}), -((c*d^4 + a*d^2*e^2 + (c*d^3*e + 2*a*d*e^3)*x)*\sqrt{-c*d^2 - a*e^2})*\sqrt{c*x^2 + a}*\sqrt{a} - ((3*a*c*d^2*e^3 + 2*a^2*e^5)*x^2 + (3*a*c*d^3*e^2 + 2*a^2*d*e^4)*x)*\sqrt{a}*\arctan(\sqrt{-c*d^2 - a*e^2}*(c*d*x - a*e)/((c*d^2 + a*e^2)*\sqrt{c*x^2 + a})) - \sqrt{-c*d^2 - a*e^2}*((a*c*d^2*e^2 + a^2*e^4)*x^2 + (a*c*d^3*e + a^2*d*e^3)*x)*\log(-((c*x^2 + 2*a)*\sqrt{a} + 2*\sqrt{c*x^2 + a}*a)/x^2))/(\sqrt{-c*d^2 - a*e^2}*((a*c*d^5*e + a^2*d^3*e^3)*x^2 + (a*c*d^6 + a^2*d^4*e^2)*x)*\sqrt{a}), -1/2*(2*(c*d^4 + a*d^2*e^2 + (c*d^3*e + 2*a*d*e^3)*x)*\sqrt{c*d^2 + a*e^2})*\sqrt{c*x^2 + a}*\sqrt{-a} - 4*\sqrt{c*d^2 + a*e^2}*((a*c*d^2*e^2 + a^2*e^4)*x^2 + (a*c*d^3*e + a^2*d*e^3)*x)*\arctan(\sqrt{-a}/\sqrt{c*x^2 + a}) - ((3*a*c*d^2*e^3 + 2*a^2*e^5)*x^2 + (3*a*c*d^3*e^2 + 2*a^2*d*e^4)*x)*\sqrt{-a}*\log(((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2)*\sqrt{c*d^2 + a*e^2}) + 2*(a*c*d^2*e + a^2*e^3 - (c^2*d^3 + a*c*d*e^2)*x)*\sqrt{c*x^2 + a})/(e^2*x^2 + 2*d*e*x + d^2)))/(\sqrt{c*d^2 + a*e^2}*((a*c*d^5*e + a^2*d^3*e^3)*x^2 + (a*c*d^6 + a^2*d^4*e^2)*x)*\sqrt{-a}), -((c*d^4 + a*d^2*e^2 + (c*d^3*e + 2*a*d*e^3)*x)*\sqrt{-c*d^2 - a*e^2})*\sqrt{c*x^2 + a}*\sqrt{-a} - ((3*a*c*d^2*e^3 + 2*a^2*e^5)*x^2 + (3*a*c*d^3*e^2 + 2*a^2*d*e^4)*x)*\sqrt{-a}*\arctan(\sqrt{-c*d^2 - a*e^2}*(c*d*x - a*e)/((c*d^2 + a*e^2)*\sqrt{c*x^2 + a})) - 2*\sqrt{-c*d^2 - a*e^2}*((a*c*d^2*e^2 + a^2*e^4)*x^2 + (a*c*d^3*e + a^2*d*e^3)*x)*\arctan(\sqrt{-a}/\sqrt{c*x^2 + a})/(\sqrt{-c*d^2 - a*e^2}*((a*c*d^5*e + a^2*d^3*e^3)*x^2 + (a*c*d^6 + a^2*d^4*e^2)*x)*\sqrt{-a}))] \end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt{a + cx^2} (d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(e\*x+d)\*\*2/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] Integral(1/(x\*\*2\*sqrt(a + c\*x\*\*2)\*(d + e\*x)\*\*2), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + a}(ex + d)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)^2\*x^2),x, algorithm="giac")

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*x^2), x)
```

$$3.350 \quad \int \frac{1}{x^3(d+ex)^2\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=268

$$\frac{c \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d^2} - \frac{3e^2 \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^4} + \frac{2e\sqrt{a+cx^2}}{ad^3x} + \frac{ce^3 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2(ae^2+cd^2)^{3/2}}$$

$$- \frac{\sqrt{a+cx^2}}{2ad^2x^2} + \frac{3e^3 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^4\sqrt{ae^2+cd^2}} + \frac{e^4\sqrt{a+cx^2}}{d^3(d+ex)(ae^2+cd^2)}$$

[Out]  $-\text{Sqrt}[a + c*x^2]/(2*a*d^2*x^2) + (2*e*\text{Sqrt}[a + c*x^2])/(a*d^3*x) + (e^4*\text{Sqrt}[a + c*x^2])/(d^3*(c*d^2 + a*e^2)*(d + e*x)) + (c*e^3*\text{ArcTanh}[(a*e - c*d*x)/(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2])])/(d^2*(c*d^2 + a*e^2)^{(3/2)}) + (3*e^3*\text{ArcTanh}[(a*e - c*d*x)/(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2])])/(d^4*\text{Sqrt}[c*d^2 + a*e^2]) + (c*\text{ArcTanh}[\text{Sqrt}[a + c*x^2]/\text{Sqrt}[a]])/(2*a^{(3/2)}*d^2) - (3*e^2*\text{ArcTanh}[\text{Sqrt}[a + c*x^2]/\text{Sqrt}[a]])/(\text{Sqrt}[a]*d^4)$

**Rubi [A]** time = 0.521565, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$

$$\frac{c \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d^2} - \frac{3e^2 \tanh^{-1}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^4} + \frac{2e\sqrt{a+cx^2}}{ad^3x} + \frac{ce^3 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2(ae^2+cd^2)^{3/2}}$$

$$- \frac{\sqrt{a+cx^2}}{2ad^2x^2} + \frac{3e^3 \tanh^{-1}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^4\sqrt{ae^2+cd^2}} + \frac{e^4\sqrt{a+cx^2}}{d^3(d+ex)(ae^2+cd^2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^3*(d + e*x)^2*\text{Sqrt}[a + c*x^2]), x]$

[Out]  $-\text{Sqrt}[a + c*x^2]/(2*a*d^2*x^2) + (2*e*\text{Sqrt}[a + c*x^2])/(a*d^3*x) + (e^4*\text{Sqrt}[a + c*x^2])/(d^3*(c*d^2 + a*e^2)*(d + e*x)) + (c*e^3*\text{ArcTanh}[(a*e - c*d*x)/(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2])])/(d^2*(c*d^2 + a*e^2)^{(3/2)}) + (3*e^3*\text{ArcTanh}[(a*e - c*d*x)/(\text{Sqrt}[c*d^2 + a*e^2]*\text{Sqrt}[a + c*x^2])])/(d^4*\text{Sqrt}[c*d^2 + a*e^2]) + (c*\text{ArcTanh}[\text{Sqrt}[a + c*x^2]/\text{Sqrt}[a]])/(2*a^{(3/2)}*d^2) - (3*e^2*\text{ArcTanh}[\text{Sqrt}[a + c*x^2]/\text{Sqrt}[a]])/(\text{Sqrt}[a]*d^4)$

**Rubi in Sympy [A]** time = 49.4382, size = 243, normalized size = 0.91

$$\frac{ce^3 \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^2(ae^2+cd^2)^{3/2}} + \frac{e^4\sqrt{a+cx^2}}{d^3(d+ex)(ae^2+cd^2)} + \frac{3e^3 \operatorname{atanh}\left(\frac{ae-cdx}{\sqrt{a+cx^2}\sqrt{ae^2+cd^2}}\right)}{d^4\sqrt{ae^2+cd^2}}$$

$$- \frac{\sqrt{a+cx^2}}{2ad^2x^2} + \frac{2e\sqrt{a+cx^2}}{ad^3x} - \frac{3e^2 \operatorname{atanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{\sqrt{a}d^4} + \frac{c \operatorname{atanh}\left(\frac{\sqrt{a+cx^2}}{\sqrt{a}}\right)}{2a^{3/2}d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x^{**3}/(e*x+d)^{**2}/(c*x^{**2}+a)^{**}(1/2), x)$

[Out]  $c*e^{**3}*\operatorname{atanh}((a*e - c*d*x)/(\text{sqrt}(a + c*x^{**2})*\text{sqrt}(a*e^{**2} + c*d^{**2}))) / (d^{**2}*(a*e^{**2} + c*d^{**2})^{**}(3/2)) + e^{**4}*\text{sqrt}(a + c*x^{**2}) / (d^{**3}*(d + e*x)*(a*e^{**2} + c*d^{**2})) + 3*e^{**3}*\operatorname{atanh}((a*e - c*d*x)/(\text{sqrt}(a + c*x^{**2})*\text{sqrt}(a*e^{**2} + c*d^{**2}))) / (d^{**4}*\text{sqrt}(a*e^{**2} + c*d^{**2})) - \text{sqrt}(a + c*x^{**2}) / (2*a*d^{**2}*x^{**2}) + 2*e*\text{sqrt}(a + c*x^{**2}) / (a*d^{**3})$

\*x) - 3\*e\*\*2\*atanh(sqrt(a + c\*x\*\*2)/sqrt(a))/(sqrt(a)\*d\*\*4) + c\*a  
tanh(sqrt(a + c\*x\*\*2)/sqrt(a))/(2\*a\*\*(3/2)\*d\*\*2)

**Mathematica [A]** time = 0.779016, size = 229, normalized size = 0.85

$$\frac{(cd^2-6ae^2) \log(\sqrt{a}\sqrt{a+cx^2+a})}{a^{3/2}} + \frac{\log(x)(6ae^2-cd^2)}{a^{3/2}} + d\sqrt{a+cx^2} \left( \frac{2e^4}{(d+ex)(ae^2+cd^2)} - \frac{d-4ex}{ax^2} \right) + \frac{2e^3(3ae^2+4cd^2) \log(\sqrt{a+cx^2}\sqrt{ae^2+cd^2+ae-cdx})}{(ae^2+cd^2)^{3/2}}$$


---


$$2d^4$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(d + e\*x)^2\*Sqrt[a + c\*x^2]),x]

[Out] (d\*Sqrt[a + c\*x^2]\*(-(d - 4\*e\*x)/(a\*x^2)) + (2\*e^4)/((c\*d^2 + a\*e^2)\*(d + e\*x))) + ((-(c\*d^2) + 6\*a\*e^2)\*Log[x])/a^(3/2) - (2\*e^3\*(4\*c\*d^2 + 3\*a\*e^2)\*Log[d + e\*x])/(c\*d^2 + a\*e^2)^(3/2) + ((c\*d^2 - 6\*a\*e^2)\*Log[a + Sqrt[a]\*Sqrt[a + c\*x^2]])/a^(3/2) + (2\*e^3\*(4\*c\*d^2 + 3\*a\*e^2)\*Log[a\*e - c\*d\*x + Sqrt[c\*d^2 + a\*e^2]\*Sqrt[a + c\*x^2]])/(c\*d^2 + a\*e^2)^(3/2)/(2\*d^4)

**Maple [A]** time = 0.019, size = 452, normalized size = 1.7

$$-\frac{1}{2ad^2x^2}\sqrt{cx^2+a} + \frac{c}{2d^2}\ln\left(\frac{1}{x}\left(2a+2\sqrt{a}\sqrt{cx^2+a}\right)\right)a^{-\frac{3}{2}} - 3\frac{e^2}{d^4\sqrt{a}}\ln\left(\frac{2a+2\sqrt{a}\sqrt{cx^2+a}}{x}\right)$$

$$+ 3\frac{e^2}{d^4}\ln\left(1\left(2\frac{ae^2+cd^2}{e^2} - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + 2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + \frac{ae^2+cd^2}{e^2}}\right)\left(x+\frac{d}{e}\right)^{-1}\right)\frac{1}{\sqrt{\frac{ae^2+cd^2}{e^2}}}$$

$$+ 2\frac{e\sqrt{cx^2+a}}{ad^3x} + \frac{e^3}{d^3(ae^2+cd^2)}\sqrt{\left(x+\frac{d}{e}\right)^2c - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + \frac{ae^2+cd^2}{e^2}}\left(x+\frac{d}{e}\right)^{-1}$$

$$+ \frac{e^2c}{d^2(ae^2+cd^2)}\ln\left(1\left(2\frac{ae^2+cd^2}{e^2} - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + 2\sqrt{\frac{ae^2+cd^2}{e^2}}\sqrt{\left(x+\frac{d}{e}\right)^2c - 2\frac{cd}{e}\left(x+\frac{d}{e}\right) + \frac{ae^2+cd^2}{e^2}}\right)\left(x+\frac{d}{e}\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(e\*x+d)^2/(c\*x^2+a)^(1/2),x)

[Out] -1/2\*(c\*x^2+a)^(1/2)/a/d^2/x^2+1/2/d^2\*c/a^(3/2)\*ln((2\*a+2\*a^(1/2)\*(c\*x^2+a)^(1/2))/x)-3/d^4\*e^2/a^(1/2)\*ln((2\*a+2\*a^(1/2)\*(c\*x^2+a)^(1/2))/x)+3/d^4\*e^2/((a\*e^2+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2+c\*d^2)/e^2-2\*c\*d/e\*(x+d/e)+2\*((a\*e^2+c\*d^2)/e^2)^(1/2)\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2))/(x+d/e))+2\*e\*(c\*x^2+a)^(1/2)/a/d^3/x+1/d^3\*e^3/(a\*e^2+c\*d^2)/(x+d/e)\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2)+1/d^2\*e^2\*c/(a\*e^2+c\*d^2)/((a\*e^2+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2+c\*d^2)/e^2-2\*c\*d/e\*(x+d/e)+2\*((a\*e^2+c\*d^2)/e^2)^(1/2)\*((x+d/e)^2\*c-2\*c\*d/e\*(x+d/e)+(a\*e^2+c\*d^2)/e^2)^(1/2))/(x+d/e))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}(ex+d)^2x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)^2\*x^3),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)^2\*x^3), x)

**Fricas [A]** time = 1.10475, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)^2\*x^3),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/4*(2*(c*d^5 + a*d^3*e^2 - 2*(2*c*d^3*e^2 + 3*a*d*e^4)*x^2 - 3 \\ & *(c*d^4*e + a*d^2*e^3)*x)*\sqrt{c*d^2 + a*e^2}*\sqrt{c*x^2 + a}*\sqrt{a} \\ & - 2*((4*a*c*d^2*e^4 + 3*a^2*e^6)*x^3 + (4*a*c*d^3*e^3 + 3*a^2*d*e^5)*x^2)*\sqrt{a}*\log(((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - ( \\ & 2*c^2*d^2 + a*c*e^2)*x^2)*\sqrt{c*d^2 + a*e^2} - 2*(a*c*d^2*e + a^2*e^3 - (c^2*d^3 + a*c*d*e^2)*x)*\sqrt{c*x^2 + a}))/ (e^2*x^2 + 2*d* \\ & e*x + d^2)) + ((c^2*d^4*e - 5*a*c*d^2*e^3 - 6*a^2*e^5)*x^3 + (c^2*d^5 - 5*a*c*d^3*e^2 - 6*a^2*d*e^4)*x^2)*\sqrt{c*d^2 + a*e^2}*\log( \\ & -((c*x^2 + 2*a)*\sqrt{a} - 2*\sqrt{c*x^2 + a}*a)/x^2))/(((a*c*d^6*e + a^2*d^4*e^3)*x^3 + (a*c*d^7 + a^2*d^5*e^2)*x^2)*\sqrt{c*d^2 + a \\ & *e^2}*\sqrt{a}), -1/4*(2*(c*d^5 + a*d^3*e^2 - 2*(2*c*d^3*e^2 + 3*a*d*e^4)*x^2 - 3*(c*d^4*e + a*d^2*e^3)*x)*\sqrt{-c*d^2 - a*e^2}*\sqrt{ \\ & c*x^2 + a}*\sqrt{a} + 4*((4*a*c*d^2*e^4 + 3*a^2*e^6)*x^3 + (4*a*c*d^3*e^3 + 3*a^2*d*e^5)*x^2)*\sqrt{a}*\arctan(\sqrt{-c*d^2 - a*e^2} \\ & *(c*d*x - a*e)/((c*d^2 + a*e^2)*\sqrt{c*x^2 + a})) + ((c^2*d^4*e - 5*a*c*d^2*e^3 - 6*a^2*e^5)*x^3 + (c^2*d^5 - 5*a*c*d^3*e^2 - 6*a^2*d*e^4)*x^2)*\sqrt{-c*d^2 - a*e^2}*\log(-((c*x^2 + 2*a)*\sqrt{a} - 2*\sqrt{c*x^2 + a}*a)/x^2))/(((a*c*d^6*e + a^2*d^4*e^3)*x^3 + (a*c*d^7 + a^2*d^5*e^2)*x^2)*\sqrt{-c*d^2 - a*e^2}*\sqrt{a}), -1/2*((c*d^5 + a*d^3*e^2 - 2*(2*c*d^3*e^2 + 3*a*d*e^4)*x^2 - 3*(c*d^4*e + a*d^2*e^3)*x)*\sqrt{c*d^2 + a*e^2}*\sqrt{c*x^2 + a}*\sqrt{-a} - ((c^2*d^4*e - 5*a*c*d^2*e^3 - 6*a^2*e^5)*x^3 + (c^2*d^5 - 5*a*c*d^3*e^2 - 6*a^2*d*e^4)*x^2)*\sqrt{c*d^2 + a*e^2}*\arctan(\sqrt{-a}/\sqrt{c*x^2 + a}) - ((4*a*c*d^2*e^4 + 3*a^2*e^6)*x^3 + (4*a*c*d^3*e^3 + 3*a^2*d*e^5)*x^2)*\sqrt{-a}*\log(((2*a*c*d*e*x - a*c*d^2 - 2*a^2*e^2 - (2*c^2*d^2 + a*c*e^2)*x^2)*\sqrt{c*d^2 + a*e^2} - 2*(a*c*d^2*e + a^2*e^3 - (c^2*d^3 + a*c*d*e^2)*x)*\sqrt{c*x^2 + a}))/ (e^2*x^2 + 2*d*e*x + d^2)))/(((a*c*d^6*e + a^2*d^4*e^3)*x^3 + (a*c*d^7 + a^2*d^5*e^2)*x^2)*\sqrt{c*d^2 + a*e^2}*\sqrt{-a}), -1/2*((c*d^5 + a*d^3*e^2 - 2*(2*c*d^3*e^2 + 3*a*d*e^4)*x^2 - 3*(c*d^4*e + a*d^2*e^3)*x)*\sqrt{-c*d^2 - a*e^2}*\sqrt{c*x^2 + a}*\sqrt{-a} + 2*((4*a*c*d^2*e^4 + 3*a^2*e^6)*x^3 + (4*a*c*d^3*e^3 + 3*a^2*d*e^5)*x^2)*\sqrt{-a}*\arctan(\sqrt{-c*d^2 - a*e^2}*(c*d*x - a*e)/((c*d^2 + a*e^2)*\sqrt{c*x^2 + a})) - ((c^2*d^4*e - 5*a*c*d^2*e^3 - 6*a^2*e^5)*x^3 + (c^2*d^5 - 5*a*c*d^3*e^2 - 6*a^2*d*e^4)*x^2)*\sqrt{-c*d^2 - a*e^2}*\arctan(\sqrt{-a}/\sqrt{c*x^2 + a}))/(((a*c*d^6*e + a^2*d^4*e^3)*x^3 + (a*c*d^7 + a^2*d^5*e^2)*x^2)*\sqrt{-c*d^2 - a*e^2}*\sqrt{-a})] \end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \sqrt{a + cx^2} (d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/(e\*x+d)\*\*2/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] Integral(1/(x\*\*3\*sqrt(a + c\*x\*\*2)\*(d + e\*x)\*\*2), x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + a}(ex + d)^2 x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*x^3),x, algorithm="giac")`

[Out] `integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^2*x^3), x)`

### 3.351 $\int x^2(a + bx)^n (c + dx^2) dx$

**Optimal.** Leaf size=135

$$\frac{a^2 (a^2d + b^2c) (a + bx)^{n+1}}{b^5(n+1)} - \frac{2a (2a^2d + b^2c) (a + bx)^{n+2}}{b^5(n+2)} + \frac{(6a^2d + b^2c) (a + bx)^{n+3}}{b^5(n+3)} - \frac{4ad(a + bx)^{n+4}}{b^5(n+4)} + \frac{d(a + bx)^{n+5}}{b^5(n+5)}$$

[Out]  $(a^2(b^2c + a^2d)(a + bx)^{(1+n)})/(b^5(1+n)) - (2a(2a^2d + b^2c)(a + bx)^{(2+n)})/(b^5(2+n)) + ((b^2c + 6a^2d)(a + bx)^{(3+n)})/(b^5(3+n)) - (4ad(a + bx)^{(4+n)})/(b^5(4+n)) + (d(a + bx)^{(5+n)})/(b^5(5+n))$

**Rubi [A]** time = 0.17588, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$\frac{a^2 (a^2d + b^2c) (a + bx)^{n+1}}{b^5(n+1)} - \frac{2a (2a^2d + b^2c) (a + bx)^{n+2}}{b^5(n+2)} + \frac{(6a^2d + b^2c) (a + bx)^{n+3}}{b^5(n+3)} - \frac{4ad(a + bx)^{n+4}}{b^5(n+4)} + \frac{d(a + bx)^{n+5}}{b^5(n+5)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2(a + bx)^n(c + dx^2), x]$

[Out]  $(a^2(b^2c + a^2d)(a + bx)^{(1+n)})/(b^5(1+n)) - (2a(2a^2d + b^2c)(a + bx)^{(2+n)})/(b^5(2+n)) + ((b^2c + 6a^2d)(a + bx)^{(3+n)})/(b^5(3+n)) - (4ad(a + bx)^{(4+n)})/(b^5(4+n)) + (d(a + bx)^{(5+n)})/(b^5(5+n))$

**Rubi in Sympy [A]** time = 31.6645, size = 122, normalized size = 0.9

$$\frac{a^2 (a + bx)^{n+1} (a^2d + b^2c)}{b^5(n+1)} - \frac{4ad(a + bx)^{n+4}}{b^5(n+4)} - \frac{2a(a + bx)^{n+2} (2a^2d + b^2c)}{b^5(n+2)} + \frac{d(a + bx)^{n+5}}{b^5(n+5)} + \frac{(a + bx)^{n+3} (6a^2d + b^2c)}{b^5(n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^2(b*x+a)^n(d*x^2+c), x)$

[Out]  $a^2(a + bx)^{(n+1)}(a^2d + b^2c)/(b^5(n+1)) - 4ad(a + bx)^{(n+4)}/(b^5(n+4)) - 2a(a + bx)^{(n+2)}(2a^2d + b^2c)/(b^5(n+2)) + d(a + bx)^{(n+5)}/(b^5(n+5)) + (a + bx)^{(n+3)}(6a^2d + b^2c)/(b^5(n+3))$

**Mathematica [A]** time = 0.135431, size = 158, normalized size = 1.17

$$\frac{(a + bx)^{n+1} (24a^4d - 24a^3bd(n+1)x + 2a^2b^2(c(n^2 + 9n + 20) + 6d(n^2 + 3n + 2)x^2) - 2ab^3(n+1)x(c(n^2 + 9n + 20) + 2d))}{b^5(n+1)(n+2)(n+3)(n+4)(n+5)}$$

Antiderivative was successfully verified.



[In] Integrate[x^2\*(a + b\*x)^n\*(c + d\*x^2), x]

[Out]  $((a + b*x)^{(1+n)} * (24*a^4*d - 24*a^3*b*d*(1+n)*x + b^4*(8 + 14*n + 7*n^2 + n^3)*x^2*(c*(5+n) + d*(3+n)*x^2) + 2*a^2*b^2*(c*(20 + 9*n + n^2) + 6*d*(2 + 3*n + n^2)*x^2) - 2*a*b^3*(1+n)*x*(c*(20 + 9*n + n^2) + 2*d*(6 + 5*n + n^2)*x^2)) / (b^5*(1+n)*(2+n)*(3+n)*(4+n)*(5+n))$

**Maple [B]** time = 0.01, size = 328, normalized size = 2.4

$(bx + a)^{1+n} (b^4 dn^4 x^4 + 10 b^4 dn^3 x^4 - 4 ab^3 dn^3 x^3 + b^4 cn^4 x^2 + 35 b^4 dn^2 x^4 - 24 ab^3 dn^2 x^3 + 12 b^4 cn^3 x^2 + 50 b^4 dn x^4 + 12 a^2 b^2$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(b\*x+a)^n\*(d\*x^2+c), x)

[Out]  $(b*x+a)^{(1+n)} * (b^4*d*n^4*x^4+10*b^4*d*n^3*x^4-4*a*b^3*d*n^3*x^3+b^4*c*n^4*x^2+35*b^4*d*n^2*x^4-24*a*b^3*d*n^2*x^3+12*b^4*c*n^3*x^2+50*b^4*d*n*x^4+12*a^2*b^2*d*n^2*x^2-2*a*b^3*c*n^3*x-44*a*b^3*d*n*x^3+49*b^4*c*n^2*x^2+24*b^4*d*x^4+36*a^2*b^2*d*n*x^2-20*a*b^3*c*n^2*x-24*a*b^3*d*x^3+78*b^4*c*n*x^2-24*a^3*b*d*n*x+2*a^2*b^2*c*n^2+24*a^2*b^2*d*x^2-58*a*b^3*c*n*x+40*b^4*c*x^2-24*a^3*b*d*x+18*a^2*b^2*c*n-40*a*b^3*c*x+24*a^4*d+40*a^2*b^2*c) / b^5 / (n^5+15*n^4+85*n^3+225*n^2+274*n+120)$

**Maxima [A]** time = 0.710032, size = 284, normalized size = 2.1

$$\frac{((n^2 + 3n + 2)b^3x^3 + (n^2 + n)ab^2x^2 - 2a^2bnx + 2a^3)(bx + a)^n c}{(n^3 + 6n^2 + 11n + 6)b^3}$$

$$+ \frac{((n^4 + 10n^3 + 35n^2 + 50n + 24)b^5x^5 + (n^4 + 6n^3 + 11n^2 + 6n)ab^4x^4 - 4(n^3 + 3n^2 + 2n)a^2b^3x^3 + 12(n^2 + n)a^3b^2x^2 - 2a^4b^2x + 2a^5)(bx + a)^n d}{(n^5 + 15n^4 + 85n^3 + 225n^2 + 274n + 120)b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)\*(b\*x + a)^n\*x^2, x, algorithm="maxima")

[Out]  $((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*a^3)*(b*x + a)^n*c / ((n^3 + 6*n^2 + 11*n + 6)*b^3) + ((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*d / ((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5)$

**Fricas [A]** time = 0.281342, size = 497, normalized size = 3.68

$(2 a^3 b^2 c n^2 + 18 a^3 b^2 c n + 40 a^3 b^2 c + 24 a^5 d + (b^5 d n^4 + 10 b^5 d n^3 + 35 b^5 d n^2 + 50 b^5 d n + 24 b^5 d) x^5 + (a b^4 d n^4 + 6 a b^4 d n^3 + 12 a^2 b^3 d n^2 + 12 a^3 b^2 d n + 2 a^4 b d) x^4 + (a^2 b^3 d n^3 + 6 a^3 b^2 d n^2 + 6 a^4 b d n + 2 a^5) x^3 + (a^2 b^3 d n^2 + 2 a^3 b^2 d n + 2 a^4 b d) x^2 + (a^2 b^3 d n + 2 a^3 b^2 d) x + a^2 b^3 d) (b x + a)^n$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)\*(b\*x + a)^n\*x^2, x, algorithm="fricas")

[Out]  $(2*a^3*b^2*c*n^2 + 18*a^3*b^2*c*n + 40*a^3*b^2*c + 24*a^5*d + (b^5*d*n^4 + 10*b^5*d*n^3 + 35*b^5*d*n^2 + 50*b^5*d*n + 24*b^5*d)*x^4 + (a*b^4*d*n^4 + 6*a*b^4*d*n^3 + 11*a*b^4*d*n^2 + 6*a*b^4*d*n)*x^3 + (a^2*b^3*d*n^3 + 6*a^3*b^2*d*n^2 + 6*a^4*b*d*n + 2*a^5)*x^2 + (a^2*b^3*d*n^2 + 2*a^3*b^2*d*n + 2*a^4*b*d)*x + a^2*b^3*d)$

$$x^4 + (b^5*c*n^4 + 40*b^5*c + 4*(3*b^5*c - a^2*b^3*d)*n^3 + (49*b^5*c - 12*a^2*b^3*d)*n^2 + 2*(39*b^5*c - 4*a^2*b^3*d)*n)*x^3 + (a*b^4*c*n^4 + 10*a*b^4*c*n^3 + (29*a*b^4*c + 12*a^3*b^2*d)*n^2 + 4*(5*a*b^4*c + 3*a^3*b^2*d)*n)*x^2 - 2*(a^2*b^3*c*n^3 + 9*a^2*b^3*c*n^2 + 4*(5*a^2*b^3*c + 3*a^4*b*d)*n)*x*(b*x + a)^n/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)$$

**Sympy [A]** time = 12.8097, size = 4078, normalized size = 30.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(b\*x+a)\*\*n\*(d\*x\*\*2+c),x)

[Out] Piecewise((a\*\*n\*(c\*x\*\*3/3 + d\*x\*\*5/5), Eq(b, 0)), (12\*a\*\*4\*d\*log(a/b + x)/(12\*a\*\*4\*b\*\*5 + 48\*a\*\*3\*b\*\*6\*x + 72\*a\*\*2\*b\*\*7\*x\*\*2 + 48\*a\*b\*\*8\*x\*\*3 + 12\*b\*\*9\*x\*\*4) + 25\*a\*\*4\*d/(12\*a\*\*4\*b\*\*5 + 48\*a\*\*3\*b\*\*6\*x + 72\*a\*\*2\*b\*\*7\*x\*\*2 + 48\*a\*b\*\*8\*x\*\*3 + 12\*b\*\*9\*x\*\*4) + 48\*a\*\*3\*b\*d\*x\*log(a/b + x)/(12\*a\*\*4\*b\*\*5 + 48\*a\*\*3\*b\*\*6\*x + 72\*a\*\*2\*b\*\*7\*x\*\*2 + 48\*a\*b\*\*8\*x\*\*3 + 12\*b\*\*9\*x\*\*4) + 88\*a\*\*3\*b\*d\*x/(12\*a\*\*4\*b\*\*5 + 48\*a\*\*3\*b\*\*6\*x + 72\*a\*\*2\*b\*\*7\*x\*\*2 + 48\*a\*b\*\*8\*x\*\*3 + 12\*b\*\*9\*x\*\*4) - a\*\*2\*b\*\*2\*c/(12\*a\*\*4\*b\*\*5 + 48\*a\*\*3\*b\*\*6\*x + 72\*a\*\*2\*b\*\*7\*x\*\*2 + 48\*a\*b\*\*8\*x\*\*3 + 12\*b\*\*9\*x\*\*4) + 72\*a\*\*2\*b\*\*2\*d\*x\*\*2\*log(a/b + x)/(12\*a\*\*4\*b\*\*5 + 48\*a\*\*3\*b\*\*6\*x + 72\*a\*\*2\*b\*\*7\*x\*\*2 + 48\*a\*b\*\*8\*x\*\*3 + 12\*b\*\*9\*x\*\*4) + 108\*a\*\*2\*b\*\*2\*d\*x\*\*2/(12\*a\*\*4\*b\*\*5 + 48\*a\*\*3\*b\*\*6\*x + 72\*a\*\*2\*b\*\*7\*x\*\*2 + 48\*a\*b\*\*8\*x\*\*3 + 12\*b\*\*9\*x\*\*4) - 4\*a\*b\*\*3\*c\*x/(12\*a\*\*4\*b\*\*5 + 48\*a\*\*3\*b\*\*6\*x + 72\*a\*\*2\*b\*\*7\*x\*\*2 + 48\*a\*b\*\*8\*x\*\*3 + 12\*b\*\*9\*x\*\*4) + 48\*a\*b\*\*3\*d\*x\*\*3\*log(a/b + x)/(12\*a\*\*4\*b\*\*5 + 48\*a\*\*3\*b\*\*6\*x + 72\*a\*\*2\*b\*\*7\*x\*\*2 + 48\*a\*b\*\*8\*x\*\*3 + 12\*b\*\*9\*x\*\*4) + 48\*a\*b\*\*3\*d\*x\*\*3/(12\*a\*\*4\*b\*\*5 + 48\*a\*\*3\*b\*\*6\*x + 72\*a\*\*2\*b\*\*7\*x\*\*2 + 48\*a\*b\*\*8\*x\*\*3 + 12\*b\*\*9\*x\*\*4) - 6\*b\*\*4\*c\*x\*\*2/(12\*a\*\*4\*b\*\*5 + 48\*a\*\*3\*b\*\*6\*x + 72\*a\*\*2\*b\*\*7\*x\*\*2 + 48\*a\*b\*\*8\*x\*\*3 + 12\*b\*\*9\*x\*\*4) + 12\*b\*\*4\*d\*x\*\*4\*log(a/b + x)/(12\*a\*\*4\*b\*\*5 + 48\*a\*\*3\*b\*\*6\*x + 72\*a\*\*2\*b\*\*7\*x\*\*2 + 48\*a\*b\*\*8\*x\*\*3 + 12\*b\*\*9\*x\*\*4), Eq(n, -5)), (-12\*a\*\*5\*d\*log(a/b + x)/(3\*a\*\*4\*b\*\*5 + 9\*a\*\*3\*b\*\*6\*x + 9\*a\*\*2\*b\*\*7\*x\*\*2 + 3\*a\*b\*\*8\*x\*\*3) - 4\*a\*\*5\*d/(3\*a\*\*4\*b\*\*5 + 9\*a\*\*3\*b\*\*6\*x + 9\*a\*\*2\*b\*\*7\*x\*\*2 + 3\*a\*b\*\*8\*x\*\*3) - 36\*a\*\*4\*b\*d\*x\*log(a/b + x)/(3\*a\*\*4\*b\*\*5 + 9\*a\*\*3\*b\*\*6\*x + 9\*a\*\*2\*b\*\*7\*x\*\*2 + 3\*a\*b\*\*8\*x\*\*3) - 36\*a\*\*3\*b\*\*2\*d\*x\*\*2\*log(a/b + x)/(3\*a\*\*4\*b\*\*5 + 9\*a\*\*3\*b\*\*6\*x + 9\*a\*\*2\*b\*\*7\*x\*\*2 + 3\*a\*b\*\*8\*x\*\*3) + 18\*a\*\*3\*b\*\*2\*d\*x\*\*2/(3\*a\*\*4\*b\*\*5 + 9\*a\*\*3\*b\*\*6\*x + 9\*a\*\*2\*b\*\*7\*x\*\*2 + 3\*a\*b\*\*8\*x\*\*3) - 12\*a\*\*2\*b\*\*3\*d\*x\*\*3\*log(a/b + x)/(3\*a\*\*4\*b\*\*5 + 9\*a\*\*3\*b\*\*6\*x + 9\*a\*\*2\*b\*\*7\*x\*\*2 + 3\*a\*b\*\*8\*x\*\*3) + 18\*a\*\*2\*b\*\*3\*d\*x\*\*3/(3\*a\*\*4\*b\*\*5 + 9\*a\*\*3\*b\*\*6\*x + 9\*a\*\*2\*b\*\*7\*x\*\*2 + 3\*a\*b\*\*8\*x\*\*3) + 3\*a\*b\*\*4\*d\*x\*\*4/(3\*a\*\*4\*b\*\*5 + 9\*a\*\*3\*b\*\*6\*x + 9\*a\*\*2\*b\*\*7\*x\*\*2 + 3\*a\*b\*\*8\*x\*\*3) + b\*\*5\*c\*x\*\*3/(3\*a\*\*4\*b\*\*5 + 9\*a\*\*3\*b\*\*6\*x + 9\*a\*\*2\*b\*\*7\*x\*\*2 + 3\*a\*b\*\*8\*x\*\*3), Eq(n, -4)), (12\*a\*\*4\*d\*log(a/b + x)/(2\*a\*\*2\*b\*\*5 + 4\*a\*b\*\*6\*x + 2\*b\*\*7\*x\*\*2) + 6\*a\*\*4\*d/(2\*a\*\*2\*b\*\*5 + 4\*a\*b\*\*6\*x + 2\*b\*\*7\*x\*\*2) + 24\*a\*\*3\*b\*d\*x\*log(a/b + x)/(2\*a\*\*2\*b\*\*5 + 4\*a\*b\*\*6\*x + 2\*b\*\*7\*x\*\*2) + 2\*a\*\*2\*b\*\*2\*c\*log(a/b + x)/(2\*a\*\*2\*b\*\*5 + 4\*a\*b\*\*6\*x + 2\*b\*\*7\*x\*\*2) + a\*\*2\*b\*\*2\*c/(2\*a\*\*2\*b\*\*5 + 4\*a\*b\*\*6\*x + 2\*b\*\*7\*x\*\*2) + 12\*a\*\*2\*b\*\*2\*d\*x\*\*2\*log(a/b + x)/(2\*a\*\*2\*b\*\*5 + 4\*a\*b\*\*6\*x + 2\*b\*\*7\*x\*\*2) - 12\*a\*\*2\*b\*\*2\*d\*x\*\*2/(2\*a\*\*2\*b\*\*5 + 4\*a\*b\*\*6\*x + 2\*b\*\*7\*x\*\*2) + 4\*a\*b\*\*3\*c\*x\*log(a/b + x)/(2\*a\*\*2\*b\*\*5 + 4\*a\*b\*\*6\*x + 2\*b\*\*7\*x\*\*2) - 4\*a\*b\*\*3\*d\*x\*\*3/(2\*a\*\*2\*b\*\*5 + 4\*a\*b\*\*6\*x + 2\*b\*\*7\*x\*\*2) + 2\*b\*\*4\*c\*x\*\*2\*log(a/b + x)/(2\*a\*\*2\*b\*\*5 + 4\*a\*b\*\*6\*x + 2\*b\*\*7\*x\*\*2) - 2\*b\*\*4\*c\*x\*\*2/(2\*a\*\*2\*b\*\*5 + 4\*a\*b\*\*6\*x + 2\*b\*\*7\*x\*\*2) + b\*\*4\*d\*x\*\*4/(2\*a\*\*2\*b\*\*5 + 4\*a\*b\*\*6\*x + 2\*b\*\*7\*x\*\*2), Eq(n, -3)), (-12\*a\*\*4\*d\*log(a/b + x)/(3\*a\*b\*\*5 + 3\*b\*\*6\*x) - 12\*a\*\*4\*d/(3\*a\*b\*\*5 + 3\*b\*\*6\*x) - 12\*a\*\*3\*b\*d\*x\*log(a/b + x)/(3\*a\*b\*\*5 + 3\*b\*\*6\*x) - 6\*a\*\*2\*b\*\*2\*c\*log(a/b + x)/(3\*a\*b\*\*5 + 3\*b\*\*6\*x) - 6\*a\*\*2\*b\*\*2\*c/(3\*a\*b\*\*5 + 3\*b\*\*6\*x) + 6\*a\*\*2\*b\*\*2\*d\*x\*\*2/(3\*a\*b\*\*5 + 3\*b\*\*6\*x) - 6\*a\*b\*\*3\*c\*x\*log(a/b + x)/(3\*a\*b\*\*5 + 3\*b\*\*6\*x) - 2\*a\*b\*\*3\*d\*x\*\*3/(3\*a\*b\*\*5 + 3\*b\*\*6\*x) + 3\*b\*\*4\*c\*x\*\*2/(3\*a\*b\*\*5 + 3\*b\*\*6\*x) + b\*\*4\*d\*x\*\*4/(3\*a\*b\*\*5 + 3\*b\*\*6\*x), Eq(n, -2)), (a\*\*4\*d\*log(a/b + x)

```

/b**5 - a**3*d*x/b**4 + a**2*c*log(a/b + x)/b**3 + a**2*d*x**2/(2
*b**3) - a*c*x/b**2 - a*d*x**3/(3*b**2) + c*x**2/(2*b) + d*x**4/(
4*b), Eq(n, -1)), (24*a**5*d*(a + b*x)**n/(b**5*n**5 + 15*b**5*n*
**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 24*a
**4*b*d*n*x*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3
+ 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 2*a**3*b**2*c*n**2*(a
+ b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n*
**2 + 274*b**5*n + 120*b**5) + 18*a**3*b**2*c*n*(a + b*x)**n/(b**5
*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n
+ 120*b**5) + 40*a**3*b**2*c*(a + b*x)**n/(b**5*n**5 + 15*b**5*n*
**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a
**3*b**2*d*n**2*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*
b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*a**3*b**2
*d*n*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 +
225*b**5*n**2 + 274*b**5*n + 120*b**5) - 2*a**2*b**3*c*n**3*x*(a
+ b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n*
**2 + 274*b**5*n + 120*b**5) - 18*a**2*b**3*c*n**2*x*(a + b*x)**n/
(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b*
**5*n + 120*b**5) - 40*a**2*b**3*c*n*x*(a + b*x)**n/(b**5*n**5 + 1
5*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**
5) - 4*a**2*b**3*d*n**3*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n*
**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 12*a
**2*b**3*d*n**2*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*
b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) - 8*a**2*b**3*
d*n*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 +
225*b**5*n**2 + 274*b**5*n + 120*b**5) + a*b**4*c*n**4*x**2*(a +
b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2
+ 274*b**5*n + 120*b**5) + 10*a*b**4*c*n**3*x**2*(a + b*x)**n/(b*
**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*
n + 120*b**5) + 29*a*b**4*c*n**2*x**2*(a + b*x)**n/(b**5*n**5 + 1
5*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**
5) + 20*a*b**4*c*n*x**2*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 +
85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + a*b**4*d*
n**4*x**4*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 +
225*b**5*n**2 + 274*b**5*n + 120*b**5) + 6*a*b**4*d*n**3*x**4*(a
+ b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n*
**2 + 274*b**5*n + 120*b**5) + 11*a*b**4*d*n**2*x**4*(a + b*x)**n/
(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b*
**5*n + 120*b**5) + 6*a*b**4*d*n*x**4*(a + b*x)**n/(b**5*n**5 + 15
*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5
) + b**5*c*n**4*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*
b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 12*b**5*c*n*
**3*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 2
25*b**5*n**2 + 274*b**5*n + 120*b**5) + 49*b**5*c*n**2*x**3*(a +
b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2
+ 274*b**5*n + 120*b**5) + 78*b**5*c*n*x**3*(a + b*x)**n/(b**5*n*
**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 1
20*b**5) + 40*b**5*c*x**3*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4
+ 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + b**5*d*
n**4*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 +
225*b**5*n**2 + 274*b**5*n + 120*b**5) + 10*b**5*d*n**3*x**5*(a
+ b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n*
**2 + 274*b**5*n + 120*b**5) + 35*b**5*d*n**2*x**5*(a + b*x)**n/(b*
**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*
n + 120*b**5) + 50*b**5*d*n*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**
5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) +
24*b**5*d*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n
**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5), True))

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**GIAC/XCAS [A]** time = 0.296885, size = 926, normalized size = 6.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)\*(b\*x + a)^n\*x^2,x, algorithm="giac")

[Out] (b^5\*d\*n^4\*x^5\*e^(n\*ln(b\*x + a)) + a\*b^4\*d\*n^4\*x^4\*e^(n\*ln(b\*x +

$$\begin{aligned}
& a)) + 10*b^5*d*n^3*x^5*e^{(n*\ln(b*x + a))} + b^5*c*n^4*x^3*e^{(n*\ln(b*x + a))} + 6*a*b^4*d*n^3*x^4*e^{(n*\ln(b*x + a))} + 35*b^5*d*n^2*x^5 \\
& 5*e^{(n*\ln(b*x + a))} + a*b^4*c*n^4*x^2*e^{(n*\ln(b*x + a))} + 12*b^5*c*n^3*x^3*e^{(n*\ln(b*x + a))} - 4*a^2*b^3*d*n^3*x^3*e^{(n*\ln(b*x + a))} \\
& + 11*a*b^4*d*n^2*x^4*e^{(n*\ln(b*x + a))} + 50*b^5*d*n*x^5*e^{(n*\ln(b*x + a))} + 10*a*b^4*c*n^3*x^2*e^{(n*\ln(b*x + a))} + 49*b^5*c*n^2 \\
& *x^3*e^{(n*\ln(b*x + a))} - 12*a^2*b^3*d*n^2*x^3*e^{(n*\ln(b*x + a))} + 6*a*b^4*d*n*x^4*e^{(n*\ln(b*x + a))} + 24*b^5*d*x^5*e^{(n*\ln(b*x + a))} \\
& - 2*a^2*b^3*c*n^3*x*e^{(n*\ln(b*x + a))} + 29*a*b^4*c*n^2*x^2*e^{(n*\ln(b*x + a))} + 12*a^3*b^2*d*n^2*x^2*e^{(n*\ln(b*x + a))} + 78*b^5*c \\
& *n*x^3*e^{(n*\ln(b*x + a))} - 8*a^2*b^3*d*n*x^3*e^{(n*\ln(b*x + a))} - 18*a^2*b^3*c*n^2*x*e^{(n*\ln(b*x + a))} + 20*a*b^4*c*n*x^2*e^{(n*\ln(b*x + a))} \\
& + 12*a^3*b^2*d*n*x^2*e^{(n*\ln(b*x + a))} + 40*b^5*c*x^3*e^{(n*\ln(b*x + a))} + 2*a^3*b^2*c*n^2*e^{(n*\ln(b*x + a))} - 40*a^2*b^3 \\
& *c*n*x*e^{(n*\ln(b*x + a))} - 24*a^4*b*d*n*x*e^{(n*\ln(b*x + a))} + 18*a^3*b^2*c*n*e^{(n*\ln(b*x + a))} + 40*a^3*b^2*c*e^{(n*\ln(b*x + a))} + \\
& 24*a^5*d*e^{(n*\ln(b*x + a))}/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)
\end{aligned}$$

### 3.352 $\int x(a + bx)^n (c + dx^2) dx$

**Optimal.** Leaf size=102

$$-\frac{a(a^2d + b^2c)(a + bx)^{n+1}}{b^4(n+1)} + \frac{(3a^2d + b^2c)(a + bx)^{n+2}}{b^4(n+2)} - \frac{3ad(a + bx)^{n+3}}{b^4(n+3)} + \frac{d(a + bx)^{n+4}}{b^4(n+4)}$$

[Out]  $-\left(\frac{a^2(b^2c + a^2d)(a + bx)^{1+n}}{b^4(1+n)}\right) + \left(\frac{(b^2c + 3a^2d)(a + bx)^{2+n}}{b^4(2+n)} - \frac{3ad(a + bx)^{3+n}}{b^4(3+n)} + \frac{d(a + bx)^{4+n}}{b^4(4+n)}\right)$

**Rubi [A]** time = 0.117719, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$-\frac{a(a^2d + b^2c)(a + bx)^{n+1}}{b^4(n+1)} + \frac{(3a^2d + b^2c)(a + bx)^{n+2}}{b^4(n+2)} - \frac{3ad(a + bx)^{n+3}}{b^4(n+3)} + \frac{d(a + bx)^{n+4}}{b^4(n+4)}$$

Antiderivative was successfully verified.

[In] Int[x\*(a + b\*x)^n\*(c + d\*x^2), x]

[Out]  $-\left(\frac{a^2(b^2c + a^2d)(a + bx)^{1+n}}{b^4(1+n)}\right) + \left(\frac{(b^2c + 3a^2d)(a + bx)^{2+n}}{b^4(2+n)} - \frac{3ad(a + bx)^{3+n}}{b^4(3+n)} + \frac{d(a + bx)^{4+n}}{b^4(4+n)}\right)$

**Rubi in Sympy [A]** time = 23.1907, size = 90, normalized size = 0.88

$$-\frac{3ad(a + bx)^{n+3}}{b^4(n+3)} - \frac{a(a + bx)^{n+1}(a^2d + b^2c)}{b^4(n+1)} + \frac{d(a + bx)^{n+4}}{b^4(n+4)} + \frac{(a + bx)^{n+2}(3a^2d + b^2c)}{b^4(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(b\*x+a)\*\*n\*(d\*x\*\*2+c), x)

[Out]  $-3ad(a + bx)^{n+3}/(b^4(n+3)) - a(a + bx)^{n+1}(a^2d + b^2c)/(b^4(n+1)) + d(a + bx)^{n+4}/(b^4(n+4)) + (a + bx)^{n+2}(3a^2d + b^2c)/(b^4(n+2))$

**Mathematica [A]** time = 0.113742, size = 109, normalized size = 1.07

$$\frac{(a + bx)^{n+1}(-6a^3d + 6a^2bd(n+1)x - ab^2(c(n^2 + 7n + 12) + 3d(n^2 + 3n + 2)x^2) + b^3(n^2 + 4n + 3)x(c(n+4) + d(n+2)x))}{b^4(n+1)(n+2)(n+3)(n+4)}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*x)^n\*(c + d\*x^2), x]

[Out]  $\left(\frac{(a + bx)^{1+n}(-6a^3d + 6a^2b^2d(1+n)x + b^3(3 + 4n + n^2)x^2(c(4+n) + d(2+n)x^2) - a^2b^2(c(12 + 7n + n^2) + 3d(2 + 3n + n^2)x^2))}{b^4(1+n)(2+n)(3+n)(4+n)}\right)$

**Maple [A]** time = 0.008, size = 195, normalized size = 1.9

$$\frac{(bx+a)^{1+n}(-b^3dn^3x^3-6b^3dn^2x^3+3ab^2dn^2x^2-b^3cn^3x-11b^3dnx^3+9ab^2dnx^2-8b^3cn^2x-6dx^3b^3-6a^2bdnx+a^2b^3)}{b^4(n^4+10n^3+35n^2+50n+24)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b*x+a)^n*(d*x^2+c), x)`

[Out]  $-(b*x+a)^{(1+n)}*(-b^3*d*n^3*x^3-6*b^3*d*n^2*x^3+3*a*b^2*d*n^2*x^2-b^3*c*n^3*x-11*b^3*d*n*x^3+9*a*b^2*d*n*x^2-8*b^3*c*n^2*x-6*b^3*d*x^3-6*a^2*b*d*n*x+a*b^2*c*n^2+6*a*b^2*d*x^2-19*b^3*c*n*x-6*a^2*b*d*x+7*a*b^2*c*n-12*b^3*c*x+6*a^3*d+12*a*b^2*c)/b^4/(n^4+10*n^3+35*n^2+50*n+24)$

**Maxima [A]** time = 0.712687, size = 197, normalized size = 1.93

$$\frac{(b^2(n+1)x^2+abnx-a^2)(bx+a)^nc}{(n^2+3n+2)b^2} + \frac{((n^3+6n^2+11n+6)b^4x^4+(n^3+3n^2+2n)ab^3x^3-3(n^2+n)a^2b^2x^2+6a^3bnx-6a^4)(bx+a)^nd}{(n^4+10n^3+35n^2+50n+24)b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^2+c)*(b*x+a)^n*x,x, algorithm="maxima")`

[Out]  $(b^2*(n+1)*x^2+a*b*n*x-a^2)*(b*x+a)^n*c/((n^2+3*n+2)*b^2)+((n^3+6*n^2+11*n+6)*b^4*x^4+(n^3+3*n^2+2*n)*a*b^3*x^3-3*(n^2+n)*a^2*b^2*x^2+6*a^3*b*n*x-6*a^4)*(b*x+a)^n*d/((n^4+10*n^3+35*n^2+50*n+24)*b^4)$

**Fricas [A]** time = 0.278833, size = 338, normalized size = 3.31

$$\frac{(a^2b^2cn^2+7a^2b^2cn+12a^2b^2c+6a^4d-(b^4dn^3+6b^4dn^2+11b^4dn+6b^4d)x^4-(ab^3dn^3+3ab^3dn^2+2ab^3dn)x^3-(b^4n^4+10b^4n^3+35b^4n^2+50b^4n+24b^4d)x^2-(a^2b^2cn^2+7a^2b^2cn+12a^2b^2c+6a^4d)(bx+a)^n)}{b^4n^4+10b^4n^3+35b^4n^2+50b^4n+24b^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^2+c)*(b*x+a)^n*x,x, algorithm="fricas")`

[Out]  $-(a^2*b^2*c*n^2+7*a^2*b^2*c*n+12*a^2*b^2*c+6*a^4*d-(b^4*d*n^3+6*b^4*d*n^2+11*b^4*d*n+6*b^4*d)*x^4-(a*b^3*d*n^3+3*a*b^3*d*n^2+2*a*b^3*d*n)*x^3-(b^4*c*n^3+12*b^4*c+(8*b^4*c-3*a^2*b^2*d)*n^2+(19*b^4*c-3*a^2*b^2*d)*n)*x^2-(a*b^3*c*n^3+7*a*b^3*c*n^2+6*(2*a*b^3*c+a^3*b*d)*n)*x*(b*x+a)^n/(b^4*n^4+10*b^4*n^3+35*b^4*n^2+50*b^4*n+24*b^4d)$

**Sympy [A]** time = 7.68216, size = 2241, normalized size = 21.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x+a)**n*(d*x**2+c), x)`

```
[Out] Piecewise((a**n*(c*x**2/2 + d*x**4/4), Eq(b, 0)), (6*a**5*d*log(a
/b + x)/(6*a**5*b**4 + 18*a**4*b**5*x + 18*a**3*b**6*x**2 + 6*a**
2*b**7*x**3) + 2*a**5*d/(6*a**5*b**4 + 18*a**4*b**5*x + 18*a**3*b
**6*x**2 + 6*a**2*b**7*x**3) + 18*a**4*b*d*x*log(a/b + x)/(6*a**5
*b**4 + 18*a**4*b**5*x + 18*a**3*b**6*x**2 + 6*a**2*b**7*x**3) +
18*a**3*b**2*d*x**2*log(a/b + x)/(6*a**5*b**4 + 18*a**4*b**5*x +
18*a**3*b**6*x**2 + 6*a**2*b**7*x**3) - 9*a**3*b**2*d*x**2/(6*a**
5*b**4 + 18*a**4*b**5*x + 18*a**3*b**6*x**2 + 6*a**2*b**7*x**3) +
6*a**2*b**3*d*x**3*log(a/b + x)/(6*a**5*b**4 + 18*a**4*b**5*x +
18*a**3*b**6*x**2 + 6*a**2*b**7*x**3) - 9*a**2*b**3*d*x**3/(6*a**
5*b**4 + 18*a**4*b**5*x + 18*a**3*b**6*x**2 + 6*a**2*b**7*x**3) +
3*a*b**4*c*x**2/(6*a**5*b**4 + 18*a**4*b**5*x + 18*a**3*b**6*x**
2 + 6*a**2*b**7*x**3) + b**5*c*x**3/(6*a**5*b**4 + 18*a**4*b**5*x
+ 18*a**3*b**6*x**2 + 6*a**2*b**7*x**3), Eq(n, -4)), (-6*a**4*d*
log(a/b + x)/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2) - 3*a*
**4*d/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2) - 12*a**3*b*d*
x*log(a/b + x)/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6*x**2) - 6*
a**2*b**2*d*x**2*log(a/b + x)/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*
b**6*x**2) + 6*a**2*b**2*d*x**2/(2*a**3*b**4 + 4*a**2*b**5*x + 2*
a*b**6*x**2) + 2*a*b**3*d*x**3/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a
*b**6*x**2) + b**4*c*x**2/(2*a**3*b**4 + 4*a**2*b**5*x + 2*a*b**6
*x**2), Eq(n, -3)), (6*a**3*d*log(a/b + x)/(2*a*b**4 + 2*b**5*x)
+ 6*a**3*d/(2*a*b**4 + 2*b**5*x) + 6*a**2*b*d*x*log(a/b + x)/(2*a
*b**4 + 2*b**5*x) + 2*a*b**2*c*log(a/b + x)/(2*a*b**4 + 2*b**5*x)
+ 2*a*b**2*c/(2*a*b**4 + 2*b**5*x) - 3*a*b**2*d*x**2/(2*a*b**4 +
2*b**5*x) + 2*b**3*c*x*log(a/b + x)/(2*a*b**4 + 2*b**5*x) + b**3
*d*x**3/(2*a*b**4 + 2*b**5*x), Eq(n, -2)), (-a**3*d*log(a/b + x)/
b**4 + a**2*d*x/b**3 - a*c*log(a/b + x)/b**2 - a*d*x**2/(2*b**2)
+ c*x/b + d*x**3/(3*b), Eq(n, -1)), (-6*a**4*d*(a + b*x)**n/(b**4
*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 6*a*
**3*b*d*n*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2
+ 50*b**4*n + 24*b**4) - a**2*b**2*c*n**2*(a + b*x)**n/(b**4*n**4
+ 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 7*a**2*b*
**2*c*n*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50
*b**4*n + 24*b**4) - 12*a**2*b**2*c*(a + b*x)**n/(b**4*n**4 + 10*
b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) - 3*a**2*b**2*d*n
**2*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 +
50*b**4*n + 24*b**4) - 3*a**2*b**2*d*n*x**2*(a + b*x)**n/(b**4*n*
**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + a*b**3*
c*n**3*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 +
50*b**4*n + 24*b**4) + 7*a*b**3*c*n**2*x*(a + b*x)**n/(b**4*n**4
+ 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 12*a*b**3*
c*n*x*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*
b**4*n + 24*b**4) + a*b**3*d*n**3*x**3*(a + b*x)**n/(b**4*n**4 +
10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 3*a*b**3*d*n
**2*x**3*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 +
50*b**4*n + 24*b**4) + 2*a*b**3*d*n*x**3*(a + b*x)**n/(b**4*n**4
+ 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + b**4*c*n**
3*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50
*b**4*n + 24*b**4) + 8*b**4*c*n**2*x**2*(a + b*x)**n/(b**4*n**4 +
10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 19*b**4*c*n
*x**2*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*
b**4*n + 24*b**4) + 12*b**4*c*x**2*(a + b*x)**n/(b**4*n**4 + 10*b
**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + b**4*d*n**3*x**4
*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*
n + 24*b**4) + 6*b**4*d*n**2*x**4*(a + b*x)**n/(b**4*n**4 + 10*b*
**4*n**3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4) + 11*b**4*d*n*x**4*
(a + b*x)**n/(b**4*n**4 + 10*b**4*n**3 + 35*b**4*n**2 + 50*b**4*n
+ 24*b**4) + 6*b**4*d*x**4*(a + b*x)**n/(b**4*n**4 + 10*b**4*n**
3 + 35*b**4*n**2 + 50*b**4*n + 24*b**4), True))
```

**GIAC/XCAS [A]** time = 0.279534, size = 610, normalized size = 5.98

$$b^4 d n^3 x^4 e^{(n \ln(bx+a))} + a b^3 d n^3 x^3 e^{(n \ln(bx+a))} + 6 b^4 d n^2 x^4 e^{(n \ln(bx+a))} + b^4 c n^3 x^2 e^{(n \ln(bx+a))} + 3 a b^3 d n^2 x^3 e^{(n \ln(bx+a))} + 11 b^4 d n x^4 e^{(n \ln(bx+a))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)\*(b\*x + a)^n\*x,x, algorithm="giac")

[Out]  $(b^4 d n^3 x^4 e^{n \ln(bx+a)} + a b^3 d n^3 x^3 e^{n \ln(bx+a)} + 6 b^4 d n^2 x^4 e^{n \ln(bx+a)} + b^4 c n^3 x^2 e^{n \ln(bx+a)} + 3 a b^3 d n^2 x^3 e^{n \ln(bx+a)} + 11 b^4 d n x^4 e^{n \ln(bx+a)} + a b^3 c n^3 x e^{n \ln(bx+a)} + 8 b^4 c n^2 x^2 e^{n \ln(bx+a)} - 3 a^2 b^2 d n^2 x^2 e^{n \ln(bx+a)} + 2 a b^3 d n x^3 e^{n \ln(bx+a)} + 6 b^4 d x^4 e^{n \ln(bx+a)} + 7 a b^3 c n^2 x e^{n \ln(bx+a)} + 19 b^4 c n x^2 e^{n \ln(bx+a)} - 3 a^2 b^2 d n x^2 e^{n \ln(bx+a)} - a^2 b^2 c n^2 e^{n \ln(bx+a)} + 12 a b^3 c n x e^{n \ln(bx+a)} + 6 a^3 b d n x e^{n \ln(bx+a)} + 12 b^4 c x^2 e^{n \ln(bx+a)} - 7 a^2 b^2 c n e^{n \ln(bx+a)} - 12 a^2 b^2 c e^{n \ln(bx+a)} - 6 a^4 d e^{n \ln(bx+a)}) / (b^4 n^4 + 10 b^4 n^3 + 35 b^4 n^2 + 50 b^4 n + 24 b^4)$



### 3.353 $\int (a + bx)^n (c + dx^2) dx$

**Optimal.** Leaf size=70

$$\frac{(a^2d + b^2c)(a + bx)^{n+1}}{b^3(n+1)} - \frac{2ad(a + bx)^{n+2}}{b^3(n+2)} + \frac{d(a + bx)^{n+3}}{b^3(n+3)}$$

[Out]  $((b^2c + a^2d)(a + bx)^{(1+n)})/(b^3(1+n)) - (2ad(a + bx)^{(2+n)})/(b^3(2+n)) + (d(a + bx)^{(3+n)})/(b^3(3+n))$

**Rubi [A]** time = 0.0678415, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{(a^2d + b^2c)(a + bx)^{n+1}}{b^3(n+1)} - \frac{2ad(a + bx)^{n+2}}{b^3(n+2)} + \frac{d(a + bx)^{n+3}}{b^3(n+3)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x)^n\*(c + d\*x^2), x]

[Out]  $((b^2c + a^2d)(a + bx)^{(1+n)})/(b^3(1+n)) - (2ad(a + bx)^{(2+n)})/(b^3(2+n)) + (d(a + bx)^{(3+n)})/(b^3(3+n))$

**Rubi in Sympy [A]** time = 16.1489, size = 61, normalized size = 0.87

$$-\frac{2ad(a + bx)^{n+2}}{b^3(n+2)} + \frac{d(a + bx)^{n+3}}{b^3(n+3)} + \frac{(a + bx)^{n+1}(a^2d + b^2c)}{b^3(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x+a)\*\*n\*(d\*x\*\*2+c), x)

[Out]  $-2ad(a + bx)^{(n+2)}/(b^3(n+2)) + d(a + bx)^{(n+3)}/(b^3(n+3)) + (a + bx)^{(n+1)}(a^2d + b^2c)/(b^3(n+1))$

**Mathematica [A]** time = 0.0619951, size = 65, normalized size = 0.93

$$\frac{(a + bx)^{n+1}(2a^2d - 2abd(n+1)x + b^2(n+2)(c(n+3) + d(n+1)x^2))}{b^3(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x)^n\*(c + d\*x^2), x]

[Out]  $((a + bx)^{(1+n)}(2a^2d - 2abdnx + b^2(n+2)(c + dx^2)))/(b^3(1+n)(2+n)(3+n))$

**Maple [A]** time = 0.006, size = 100, normalized size = 1.4

$$\frac{(bx + a)^{1+n}(b^2dn^2x^2 + 3b^2dnx^2 - 2abdnx + b^2cn^2 + 2dx^2b^2 - 2adxb + 5b^2cn + 2a^2d + 6b^2c)}{b^3(n^3 + 6n^2 + 11n + 6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x+a)^n*(d*x^2+c), x)`

[Out]  $(b*x+a)^{(1+n)} * (b^2*d*n^2*x^2+3*b^2*d*n*x^2-2*a*b*d*n*x+b^2*c*n^2+2*b^2*d*x^2-2*a*b*d*x+5*b^2*c*n+2*a^2*d+6*b^2*c)/b^3/(n^3+6*n^2+11*n+6)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^2 + c)*(b*x + a)^n,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.278955, size = 200, normalized size = 2.86

$$\frac{(ab^2cn^2 + 5ab^2cn + 6ab^2c + 2a^3d + (b^3dn^2 + 3b^3dn + 2b^3d)x^3 + (ab^2dn^2 + ab^2dn)x^2 + (b^3cn^2 + 6b^3c + (5b^3c - 2a^2bd)n)x + 2a^3d + 6ab^2c + 2a^2d + 6b^2c)}{b^3n^3 + 6b^3n^2 + 11b^3n + 6b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^2 + c)*(b*x + a)^n,x, algorithm="fricas")`

[Out]  $(a*b^2*c*n^2 + 5*a*b^2*c*n + 6*a*b^2*c + 2*a^3*d + (b^3*d*n^2 + 3*b^3*d*n + 2*b^3*d)*x^3 + (a*b^2*d*n^2 + a*b^2*d*n)*x^2 + (b^3*c*n^2 + 6*b^3*c + (5*b^3*c - 2*a^2*b*d)*n)*x*(b*x + a)^n/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6*b^3)$

**Sympy [A]** time = 4.31287, size = 978, normalized size = 13.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x+a)**n*(d*x**2+c), x)`

[Out]  $\text{Piecewise}((a**n*(c*x + d*x**3/3), \text{Eq}(b, 0)), (2*a**2*d*\log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + a**2*d/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 4*a*b*d*x*\log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) - b**2*c/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) + 2*b**2*d*x**2*\log(a/b + x)/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2) - 2*b**2*d*x**2/(2*a**2*b**3 + 4*a*b**4*x + 2*b**5*x**2), \text{Eq}(n, -3)), (-2*a**3*d*\log(a/b + x)/(a**2*b**3 + a*b**4*x) - 2*a**2*b*d*x*\log(a/b + x)/(a**2*b**3 + a*b**4*x) + 2*a**2*b*d*x/(a**2*b**3 + a*b**4*x) + a*b**2*d*x**2/(a**2*b**3 + a*b**4*x) + b**3*c*x/(a**2*b**3 + a*b**4*x), \text{Eq}(n, -2)), (a**2*d*\log(a/b + x)/b**3 - a*d*x/b**2 + c*\log(a/b + x)/b + d*x**2/(2*b), \text{Eq}(n, -1)), (2*a**3*d*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) - 2*a**2*b*d*n*x*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*c*n**2*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 5*a*b**2*c*n*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 6*a*b**2*c*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + a*b**2*d$

```

n**2*x**2*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b
**3) + a*b**2*d*n*x**2*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11
*b**3*n + 6*b**3) + b**3*c*n**2*x*(a + b*x)**n/(b**3*n**3 + 6*b**
3*n**2 + 11*b**3*n + 6*b**3) + 5*b**3*c*n*x*(a + b*x)**n/(b**3*n*
*3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 6*b**3*c*x*(a + b*x)**n/
(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + b**3*d*n**2*x**3
*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n + 6*b**3) + 3*
b**3*d*n*x**3*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 + 11*b**3*n +
6*b**3) + 2*b**3*d*x**3*(a + b*x)**n/(b**3*n**3 + 6*b**3*n**2 +
11*b**3*n + 6*b**3), True))

```

**GIAC/XCAS [A]** time = 0.265267, size = 355, normalized size = 5.07

$$\frac{b^3 d n^2 x^3 e^{(n \ln(bx+a))} + a b^2 d n^2 x^2 e^{(n \ln(bx+a))} + 3 b^3 d n x^3 e^{(n \ln(bx+a))} + b^3 c n^2 x e^{(n \ln(bx+a))} + a b^2 d n x^2 e^{(n \ln(bx+a))} + 2 b^3 d x^3 e^{(n \ln(bx+a))}}{b^3 n^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*(b*x + a)^n,x, algorithm="giac")
```

```
[Out] (b^3*d*n^2*x^3*e^(n*ln(b*x + a)) + a*b^2*d*n^2*x^2*e^(n*ln(b*x +
a)) + 3*b^3*d*n*x^3*e^(n*ln(b*x + a)) + b^3*c*n^2*x*e^(n*ln(b*x +
a)) + a*b^2*d*n*x^2*e^(n*ln(b*x + a)) + 2*b^3*d*x^3*e^(n*ln(b*x
+ a)) + a*b^2*c*n^2*e^(n*ln(b*x + a)) + 5*b^3*c*n*x*e^(n*ln(b*x +
a)) - 2*a^2*b*d*n*x*e^(n*ln(b*x + a)) + 5*a*b^2*c*n*e^(n*ln(b*x
+ a)) + 6*b^3*c*x*e^(n*ln(b*x + a)) + 6*a*b^2*c*e^(n*ln(b*x + a))
+ 2*a^3*d*e^(n*ln(b*x + a)))/(b^3*n^3 + 6*b^3*n^2 + 11*b^3*n + 6
*b^3)
```

$$3.354 \quad \int \frac{(a+bx)^n (c+dx^2)}{x} dx$$

**Optimal.** Leaf size=77

$$-\frac{ad(a+bx)^{n+1}}{b^2(n+1)} + \frac{d(a+bx)^{n+2}}{b^2(n+2)} - \frac{c(a+bx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{bx}{a} + 1\right)}{a(n+1)}$$

[Out]  $-\left(\frac{a \cdot d \cdot (a + b \cdot x)^{(1 + n)}}{(b^2 \cdot (1 + n))}\right) + \left(\frac{d \cdot (a + b \cdot x)^{(2 + n)}}{(b^2 \cdot (2 + n))} - \left(\frac{c \cdot (a + b \cdot x)^{(1 + n)} \cdot \text{Hypergeometric2F1}[1, 1 + n, 2 + n, 1 + (b \cdot x)/a]}{(a \cdot (1 + n))}\right)\right)$

**Rubi [A]** time = 0.120712, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{ad(a+bx)^{n+1}}{b^2(n+1)} + \frac{d(a+bx)^{n+2}}{b^2(n+2)} - \frac{c(a+bx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{bx}{a} + 1\right)}{a(n+1)}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x)^n\*(c + d\*x^2))/x, x]

[Out]  $-\left(\frac{a \cdot d \cdot (a + b \cdot x)^{(1 + n)}}{(b^2 \cdot (1 + n))}\right) + \left(\frac{d \cdot (a + b \cdot x)^{(2 + n)}}{(b^2 \cdot (2 + n))} - \left(\frac{c \cdot (a + b \cdot x)^{(1 + n)} \cdot \text{Hypergeometric2F1}[1, 1 + n, 2 + n, 1 + (b \cdot x)/a]}{(a \cdot (1 + n))}\right)\right)$

**Rubi in Sympy [A]** time = 14.2698, size = 61, normalized size = 0.79

$$-\frac{ad(a+bx)^{n+1}}{b^2(n+1)} + \frac{d(a+bx)^{n+2}}{b^2(n+2)} - \frac{c(a+bx)^{n+1} {}_2F_1\left(1, n+1; n+2; 1 + \frac{bx}{a}\right)}{a(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x+a)\*\*n\*(d\*x\*\*2+c)/x, x)

[Out]  $-a \cdot d \cdot (a + b \cdot x)^{(n + 1)} / (b^2 \cdot (n + 1)) + d \cdot (a + b \cdot x)^{(n + 2)} / (b^2 \cdot (n + 2)) - c \cdot (a + b \cdot x)^{(n + 1)} \cdot \text{hyper}((1, n + 1), (n + 2), 1 + b \cdot x/a) / (a \cdot (n + 1))$

**Mathematica [A]** time = 0.29242, size = 98, normalized size = 1.27

$$(a+bx)^n \left( \frac{d \left( a^2 \left( \left( \frac{bx}{a} + 1 \right)^{-n} - 1 \right) + abnx + b^2(n+1)x^2 \right)}{b^2(n+1)(n+2)} + \frac{c \left( \frac{a}{bx} + 1 \right)^{-n} {}_2F_1\left(-n, -n; 1-n; -\frac{a}{bx}\right)}{n} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x)^n\*(c + d\*x^2))/x, x]

[Out]  $(a + b \cdot x)^n \cdot \left( \frac{d \cdot (a \cdot b \cdot n \cdot x + b^2 \cdot (1 + n) \cdot x^2 + a^2 \cdot (-1 + (1 + (b \cdot x)/a)^{-n}))}{(b^2 \cdot (1 + n) \cdot (2 + n))} + \left( \frac{c \cdot \text{Hypergeometric2F1}[-n, -n, 1 - n, -(a/(b \cdot x))]}{(n \cdot (1 + a/(b \cdot x))^n)} \right) \right)$

---

**Maple [F]** time = 0.058, size = 0, normalized size = 0.

$$\int \frac{(bx + a)^n (dx^2 + c)}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)^n\*(d\*x^2+c)/x,x)

[Out] int((b\*x+a)^n\*(d\*x^2+c)/x,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2 + c)(bx + a)^n}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)\*(b\*x + a)^n/x,x, algorithm="maxima")

[Out] integrate((d\*x^2 + c)\*(b\*x + a)^n/x, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(dx^2 + c)(bx + a)^n}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)\*(b\*x + a)^n/x,x, algorithm="fricas")

[Out] integral((d\*x^2 + c)\*(b\*x + a)^n/x, x)

---

**Sympy [A]** time = 9.02149, size = 347, normalized size = 4.51

$$\frac{b^n cn \left(\frac{a}{b} + x\right)^n \left(1 + \frac{bx}{a}, 1, n + 1\right) (n + 1)}{(n + 2)} - \frac{b^n c \left(\frac{a}{b} + x\right)^n \left(1 + \frac{bx}{a}, 1, n + 1\right) (n + 1)}{(n + 2)} + d \left( \begin{array}{l} \frac{a^n x^2}{2} \quad \text{for } b = 0 \\ \frac{a \log\left(\frac{a}{b} + x\right)}{ab^2 + b^3 x} + \frac{bx \log\left(\frac{a}{b} + x\right)}{ab^2 + b^3 x} - \frac{bx}{ab^2 + b^3 x} \quad \text{for } n = -2 \\ -\frac{a \log\left(\frac{a}{b} + x\right)}{b^2} + \frac{x}{b} \quad \text{for } n = -1 \\ -\frac{a^2(a+bx)^n}{b^2 n^2 + 3b^2 n + 2b^2} + \frac{abnx(a+bx)^n}{b^2 n^2 + 3b^2 n + 2b^2} + \frac{b^2 nx^2(a+bx)^n}{b^2 n^2 + 3b^2 n + 2b^2} + \frac{b^2 x^2(a+bx)^n}{b^2 n^2 + 3b^2 n + 2b^2} \quad \text{otherwise} \end{array} \right) - \frac{bb^n cnx \left(\frac{a}{b} + x\right)^n \left(1 + \frac{bx}{a}, 1, n + 1\right) (n + 1)}{a(n + 2)} - \frac{bb^n cx \left(\frac{a}{b} + x\right)^n \left(1 + \frac{bx}{a}, 1, n + 1\right) (n + 1)}{a(n + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*\*n\*(d\*x\*\*2+c)/x,x)

```
[Out] -b**n*c*n*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)
/gamma(n + 2) - b**n*c*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)
*gamma(n + 1)/gamma(n + 2) + d*Piecewise((a**n*x**2/2, Eq(b, 0)),
(a*log(a/b + x)/(a*b**2 + b**3*x) + b*x*log(a/b + x)/(a*b**2 + b
**3*x) - b*x/(a*b**2 + b**3*x), Eq(n, -2)), (-a*log(a/b + x)/b**2
+ x/b, Eq(n, -1)), (-a**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2
*b**2) + a*b*n*x*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b
**2*n*x**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2) + b**2*x*
**2*(a + b*x)**n/(b**2*n**2 + 3*b**2*n + 2*b**2), True)) - b*b**n*
c*n*x*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*
gamma(n + 2)) - b*b**n*c*x*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n
+ 1)*gamma(n + 1)/(a*gamma(n + 2))
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2 + c)(bx + a)^n}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)*(b*x + a)^n/x,x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)*(b*x + a)^n/x, x)
```

### 3.355 $\int x^2(a + bx)^n (c + dx^2)^2 dx$

**Optimal.** Leaf size=232

$$\begin{aligned} & \frac{a^2 (a^2 d + b^2 c)^2 (a + bx)^{n+1}}{b^7 (n+1)} - \frac{2a (a^2 d + b^2 c) (3a^2 d + b^2 c) (a + bx)^{n+2}}{b^7 (n+2)} \\ & - \frac{4ad (5a^2 d + 2b^2 c) (a + bx)^{n+4}}{b^7 (n+4)} + \frac{d (15a^2 d + 2b^2 c) (a + bx)^{n+5}}{b^7 (n+5)} \\ & + \frac{(15a^4 d^2 + 12a^2 b^2 cd + b^4 c^2) (a + bx)^{n+3}}{b^7 (n+3)} - \frac{6ad^2 (a + bx)^{n+6}}{b^7 (n+6)} + \frac{d^2 (a + bx)^{n+7}}{b^7 (n+7)} \end{aligned}$$

[Out]  $(a^2 (b^2 c + a^2 d)^2 (a + bx)^{(1+n)}) / (b^7 (1+n)) - (2 a (b^2 c + a^2 d) (b^2 c + 3 a^2 d) (a + bx)^{(2+n)}) / (b^7 (2+n)) + ((b^4 c^2 + 12 a^2 b^2 c d + 15 a^4 d^2) (a + bx)^{(3+n)}) / (b^7 (3+n)) - (4 a d (5 a^2 d + 2 b^2 c) (a + bx)^{(4+n)}) / (b^7 (4+n)) + (d (15 a^2 d + 2 b^2 c) (a + bx)^{(5+n)}) / (b^7 (5+n)) - (6 a d^2 (a + bx)^{(6+n)}) / (b^7 (6+n)) + (d^2 (a + bx)^{(7+n)}) / (b^7 (7+n))$

**Rubi [A]** time = 0.30146, antiderivative size = 232, normalized size of antiderivative = 1., number of rules used = 2, number of rules used = 1, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$\begin{aligned} & \frac{a^2 (a^2 d + b^2 c)^2 (a + bx)^{n+1}}{b^7 (n+1)} - \frac{2a (a^2 d + b^2 c) (3a^2 d + b^2 c) (a + bx)^{n+2}}{b^7 (n+2)} \\ & - \frac{4ad (5a^2 d + 2b^2 c) (a + bx)^{n+4}}{b^7 (n+4)} + \frac{d (15a^2 d + 2b^2 c) (a + bx)^{n+5}}{b^7 (n+5)} \\ & + \frac{(15a^4 d^2 + 12a^2 b^2 cd + b^4 c^2) (a + bx)^{n+3}}{b^7 (n+3)} - \frac{6ad^2 (a + bx)^{n+6}}{b^7 (n+6)} + \frac{d^2 (a + bx)^{n+7}}{b^7 (n+7)} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2 (a + bx)^n (c + d x^2)^2, x]$

[Out]  $(a^2 (b^2 c + a^2 d)^2 (a + bx)^{(1+n)}) / (b^7 (1+n)) - (2 a (b^2 c + a^2 d) (b^2 c + 3 a^2 d) (a + bx)^{(2+n)}) / (b^7 (2+n)) + ((b^4 c^2 + 12 a^2 b^2 c d + 15 a^4 d^2) (a + bx)^{(3+n)}) / (b^7 (3+n)) - (4 a d (5 a^2 d + 2 b^2 c) (a + bx)^{(4+n)}) / (b^7 (4+n)) + (d (15 a^2 d + 2 b^2 c) (a + bx)^{(5+n)}) / (b^7 (5+n)) - (6 a d^2 (a + bx)^{(6+n)}) / (b^7 (6+n)) + (d^2 (a + bx)^{(7+n)}) / (b^7 (7+n))$

**Rubi in Sympy [A]** time = 61.9367, size = 218, normalized size = 0.94

$$\begin{aligned} & \frac{a^2 (a + bx)^{n+1} (a^2 d + b^2 c)^2}{b^7 (n+1)} - \frac{6ad^2 (a + bx)^{n+6}}{b^7 (n+6)} - \frac{4ad (a + bx)^{n+4} (5a^2 d + 2b^2 c)}{b^7 (n+4)} \\ & - \frac{2a (a + bx)^{n+2} (a^2 d + b^2 c) (3a^2 d + b^2 c)}{b^7 (n+2)} + \frac{d^2 (a + bx)^{n+7}}{b^7 (n+7)} \\ & + \frac{d (a + bx)^{n+5} (15a^2 d + 2b^2 c)}{b^7 (n+5)} + \frac{(a + bx)^{n+3} (15a^4 d^2 + 12a^2 b^2 cd + b^4 c^2)}{b^7 (n+3)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**2*(b*x+a)**n*(d*x**2+c)**2,x)$

[Out]  $a**2*(a + b*x)**(n + 1)*(a**2*d + b**2*c)**2/(b**7*(n + 1)) - 6*a*d**2*(a + b*x)**(n + 6)/(b**7*(n + 6)) - 4*a*d*(a + b*x)**(n + 4)$

$$) * (5*a**2*d + 2*b**2*c)/(b**7*(n + 4)) - 2*a*(a + b*x)**(n + 2) * (a**2*d + b**2*c) * (3*a**2*d + b**2*c)/(b**7*(n + 2)) + d**2*(a + b*x)**(n + 7)/(b**7*(n + 7)) + d*(a + b*x)**(n + 5) * (15*a**2*d + 2*b**2*c)/(b**7*(n + 5)) + (a + b*x)**(n + 3) * (15*a**4*d**2 + 12*a**2*b**2*c*d + b**4*c**2)/(b**7*(n + 3))$$

**Mathematica [A]** time = 0.383753, size = 378, normalized size = 1.63

$$(a + bx)^{n+1} (720a^6d^2 - 720a^5bd^2(n + 1)x + 24a^4b^2d(2c(n^2 + 13n + 42) + 15d(n^2 + 3n + 2)x^2) - 24a^3b^3d(n + 1)x(2c(n^2 + 13n + 42) + 15d(n^2 + 3n + 2)x^2) - 24a^2b^4d^2(n + 1)x^2(2c(n^2 + 13n + 42) + 15d(n^2 + 3n + 2)x^2) - 24ab^5d^3(n + 1)x^3(2c(n^2 + 13n + 42) + 15d(n^2 + 3n + 2)x^2) - 24b^6d^4(n + 1)x^4(2c(n^2 + 13n + 42) + 15d(n^2 + 3n + 2)x^2))$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(a + b\*x)^n\*(c + d\*x^2)^2,x]

$$[Out] ((a + b*x)^(1 + n) * (720*a^6*d^2 - 720*a^5*b*d^2*(1 + n)*x + 24*a^4*b^2*d*(2*c*(42 + 13*n + n^2) + 15*d*(2 + 3*n + n^2)*x^2) - 24*a^3*b^3*d*(1 + n)*x*(2*c*(42 + 13*n + n^2) + 5*d*(6 + 5*n + n^2)*x^2) + b^6*(48 + 92*n + 56*n^2 + 13*n^3 + n^4)*x^2*(c^2*(35 + 12*n + n^2) + 2*c*d*(21 + 10*n + n^2)*x^2 + d^2*(15 + 8*n + n^2)*x^4) + 2*a^2*b^4*(c^2*(840 + 638*n + 179*n^2 + 22*n^3 + n^4) + 12*c*d*(84 + 152*n + 83*n^2 + 16*n^3 + n^4)*x^2 + 15*d^2*(24 + 50*n + 35*n^2 + 10*n^3 + n^4)*x^4) - 2*a*b^5*(1 + n)*x*(c^2*(840 + 638*n + 179*n^2 + 22*n^3 + n^4) + 4*c*d*(252 + 288*n + 113*n^2 + 18*n^3 + n^4)*x^2 + 3*d^2*(120 + 154*n + 71*n^2 + 14*n^3 + n^4)*x^4)))/(b^7*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n)*(7 + n))$$

**Maple [B]** time = 0.017, size = 1000, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(b\*x+a)^n\*(d\*x^2+c)^2,x)

$$[Out] (b*x+a)^(1+n) * (b^6*d^2*n^6*x^6+21*b^6*d^2*n^5*x^6-6*a*b^5*d^2*n^5*x^5+2*b^6*c*d*n^6*x^4+175*b^6*d^2*n^4*x^6-90*a*b^5*d^2*n^4*x^5+46*b^6*c*d*n^5*x^4+735*b^6*d^2*n^3*x^6+30*a^2*b^4*d^2*n^4*x^4-8*a*b^5*c*d*n^5*x^3-510*a*b^5*d^2*n^3*x^5+b^6*c^2*n^6*x^2+414*b^6*c*d*n^4*x^4+1624*b^6*d^2*n^2*x^6+300*a^2*b^4*d^2*n^3*x^4-152*a*b^5*c*d*n^4*x^3-1350*a*b^5*d^2*n^2*x^5+25*b^6*c^2*n^5*x^2+1850*b^6*c*d*n^3*x^4+1764*b^6*d^2*n*x^6-120*a^3*b^3*d^2*n^3*x^3+24*a^2*b^4*c*d*n^4*x^2+1050*a^2*b^4*d^2*n^2*x^4-2*a*b^5*c^2*n^5*x-1048*a*b^5*c*d*n^3*x^3-1644*a*b^5*d^2*n*x^5+247*b^6*c^2*n^4*x^2+4288*b^6*c*d*n^2*x^4+720*b^6*d^2*x^6-720*a^3*b^3*d^2*n^2*x^3+384*a^2*b^4*c*d*n^3*x^2+1500*a^2*b^4*d^2*n*x^4-46*a*b^5*c^2*n^4*x-3208*a*b^5*c*d*n^2*x^3-720*a*b^5*d^2*x^5+1219*b^6*c^2*n^3*x^2+4824*b^6*c*d*n*x^4+360*a^4*b^2*d^2*n^2*x^2-48*a^3*b^3*c*d*n^3*x-1320*a^3*b^3*d^2*n*x^3+2*a^2*b^4*c^2*n^4+1992*a^2*b^4*c*d*n^2*x^2+720*a^2*b^4*d^2*x^4-402*a*b^5*c^2*n^3*x-4320*a*b^5*c*d*n*x^3+3112*b^6*c^2*n^2*x^2+2016*b^6*c*d*x^4+1080*a^4*b^2*d^2*n*x^2-672*a^3*b^3*c*d*n^2*x-720*a^3*b^3*d^2*x^3+44*a^2*b^4*c^2*n^3+3648*a^2*b^4*c*d*n*x^2-1634*a*b^5*c^2*n^2*x-2016*a*b^5*c*d*x^3+3796*b^6*c^2*n*x^2-720*a^5*b*d^2*n*x+48*a^4*b^2*c*d*n^2+720*a^4*b^2*d^2*x^2-2640*a^3*b^3*c*d*n*x+358*a^2*b^4*c^2*n^2+2016*a^2*b^4*c*d*x^2-2956*a*b^5*c^2*n*x+1680*b^6*c^2*x^2-720*a^5*b*d^2*x+624*a^4*b^2*c*d*n-2016*a^3*b^3*c*d*x+1276*a^2*b^4*c^2*n-1680*a*b^5*c^2*x+720*a^6*d^2+2016*a^4*b^2*c*d+1680*a^2*b^4*c^2)/b^7/(n^7+28*n^6+322*n^5+1960*n^4+6769*n^3+13132*n^2+13068*n+5040)$$



**Maxima [A]** time = 0.723801, size = 603, normalized size = 2.6

$$\frac{((n^2 + 3n + 2)b^3x^3 + (n^2 + n)ab^2x^2 - 2a^2bnx + 2a^3)(bx + a)^n c^2}{(n^3 + 6n^2 + 11n + 6)b^3} + \frac{2((n^4 + 10n^3 + 35n^2 + 50n + 24)b^5x^5 + (n^4 + 6n^3 + 11n^2 + 6n)ab^4x^4 - 4(n^3 + 3n^2 + 2n)a^2b^3x^3 + 12(n^2 + n)a^3b^2x^2 - (n^5 + 15n^4 + 85n^3 + 225n^2 + 274n + 120)b^5)}{(n^5 + 15n^4 + 85n^3 + 225n^2 + 274n + 120)b^5} + \frac{((n^6 + 21n^5 + 175n^4 + 735n^3 + 1624n^2 + 1764n + 720)b^7x^7 + (n^6 + 15n^5 + 85n^4 + 225n^3 + 274n^2 + 120n)ab^6x^6 - 6(n^5 + 10n^4 + 35n^3 + 50n^2 + 24n + 24)a^2b^5x^5 + 30(n^4 + 6n^3 + 11n^2 + 6n)a^3b^4x^4 - 120(n^3 + 3n^2 + 2n)a^4b^3x^3 + 360(n^2 + n)a^5b^2x^2 - 720a^6b^2x + 720a^7)(bx + a)^n d^2}{(n^7 + 28n^6 + 322n^5 + 1960n^4 + 6769n^3 + 13132n^2 + 13068n + 5040)b^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)^2\*(b\*x + a)^n\*x^2,x, algorithm="maxima")

[Out] ((n^2 + 3\*n + 2)\*b^3\*x^3 + (n^2 + n)\*a\*b^2\*x^2 - 2\*a^2\*b\*n\*x + 2\*a^3)\*(b\*x + a)^n\*c^2/((n^3 + 6\*n^2 + 11\*n + 6)\*b^3) + 2\*((n^4 + 10\*n^3 + 35\*n^2 + 50\*n + 24)\*b^5\*x^5 + (n^4 + 6\*n^3 + 11\*n^2 + 6\*n)\*a\*b^4\*x^4 - 4\*(n^3 + 3\*n^2 + 2\*n)\*a^2\*b^3\*x^3 + 12\*(n^2 + n)\*a^3\*b^2\*x^2 - 24\*a^4\*b\*n\*x + 24\*a^5)\*(b\*x + a)^n\*c^2/((n^5 + 15\*n^4 + 85\*n^3 + 225\*n^2 + 274\*n + 120)\*b^5) + ((n^6 + 21\*n^5 + 175\*n^4 + 735\*n^3 + 1624\*n^2 + 1764\*n + 720)\*b^7\*x^7 + (n^6 + 15\*n^5 + 85\*n^4 + 225\*n^3 + 274\*n^2 + 120\*n)\*a\*b^6\*x^6 - 6\*(n^5 + 10\*n^4 + 35\*n^3 + 50\*n^2 + 24\*n + 24)\*a^2\*b^5\*x^5 + 30\*(n^4 + 6\*n^3 + 11\*n^2 + 6\*n)\*a^3\*b^4\*x^4 - 120\*(n^3 + 3\*n^2 + 2\*n)\*a^4\*b^3\*x^3 + 360\*(n^2 + n)\*a^5\*b^2\*x^2 - 720\*a^6\*b^2\*x + 720\*a^7)\*(b\*x + a)^n\*d^2/((n^7 + 28\*n^6 + 322\*n^5 + 1960\*n^4 + 6769\*n^3 + 13132\*n^2 + 13068\*n + 5040)\*b^7)

**Fricas [A]** time = 0.285176, size = 1386, normalized size = 5.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)^2\*(b\*x + a)^n\*x^2,x, algorithm="fricas")

[Out] (2\*a^3\*b^4\*c^2\*n^4 + 44\*a^3\*b^4\*c^2\*n^3 + 1680\*a^3\*b^4\*c^2 + 2016\*a^5\*b^2\*c\*d + 720\*a^7\*d^2 + (b^7\*d^2\*n^6 + 21\*b^7\*d^2\*n^5 + 175\*b^7\*d^2\*n^4 + 735\*b^7\*d^2\*n^3 + 1624\*b^7\*d^2\*n^2 + 1764\*b^7\*d^2\*n + 720\*b^7\*d^2)\*x^7 + (a\*b^6\*d^2\*n^6 + 15\*a\*b^6\*d^2\*n^5 + 85\*a\*b^6\*d^2\*n^4 + 225\*a\*b^6\*d^2\*n^3 + 274\*a\*b^6\*d^2\*n^2 + 120\*a\*b^6\*d^2\*n)\*x^6 + 2\*(b^7\*c\*d\*n^6 + 1008\*b^7\*c\*d + (23\*b^7\*c\*d - 3\*a^2\*b^5\*d^2)\*n^5 + 3\*(69\*b^7\*c\*d - 10\*a^2\*b^5\*d^2)\*n^4 + 5\*(185\*b^7\*c\*d - 21\*a^2\*b^5\*d^2)\*n^3 + 2\*(1072\*b^7\*c\*d - 75\*a^2\*b^5\*d^2)\*n^2 + 36\*(67\*b^7\*c\*d - 2\*a^2\*b^5\*d^2)\*n)\*x^5 + 2\*(a\*b^6\*c\*d\*n^6 + 19\*a\*b^6\*c\*d\*n^5 + (131\*a\*b^6\*c\*d + 15\*a^3\*b^4\*d^2)\*n^4 + (401\*a\*b^6\*c\*d + 90\*a^3\*b^4\*d^2)\*n^3 + 15\*(36\*a\*b^6\*c\*d + 11\*a^3\*b^4\*d^2)\*n^2 + 18\*(14\*a\*b^6\*c\*d + 5\*a^3\*b^4\*d^2)\*n)\*x^4 + (b^7\*c^2\*n^6 + 1680\*b^7\*c^2 + (25\*b^7\*c^2 - 8\*a^2\*b^5\*c\*d)\*n^5 + (247\*b^7\*c^2 - 128\*a^2\*b^5\*c\*d)\*n^4 + (1219\*b^7\*c^2 - 664\*a^2\*b^5\*c\*d - 120\*a^4\*b^3\*d^2)\*n^3 + 8\*(389\*b^7\*c^2 - 152\*a^2\*b^5\*c\*d - 45\*a^4\*b^3\*d^2)\*n^2 + 4\*(949\*b^7\*c^2 - 168\*a^2\*b^5\*c\*d - 60\*a^4\*b^3\*d^2)\*n)\*x^3 + 2\*(179\*a^3\*b^4\*c^2 + 24\*a^5\*b^2\*c\*d)\*n^2 + (a\*b^6\*c^2\*n^6 + 23\*a\*b^6\*c^2\*n^5 + 3\*(67\*a\*b^6\*c^2 + 8\*a^3\*b^4\*c\*d)\*n^4 + (817\*a\*b^6\*c^2 + 336\*a^3\*b^4\*c\*d)\*n^3 + 2\*(739\*a\*b^6\*c^2 + 660\*a^3\*b^4\*c\*d + 180\*a^5\*b^2\*d^2)\*n^2 + 24\*(35\*a\*b^6\*c^2 + 42\*a^3\*b^4\*c\*d + 15\*a^5\*b^2\*d^2)\*n)\*x^2 + 4\*(319\*a^3\*b^4\*c^2 + 156\*a^5\*b^2\*c\*d)\*n - 2\*(a^2\*b^5\*c^2\*n^5 + 22\*a^2\*b^5\*c^2\*n^4 + (179\*a^2\*b^5\*c^2 + 24\*a^4\*b^3\*c\*d)\*n^3 + 2\*(319\*a^2\*b^5\*c^2 + 156\*a^4\*b^3\*c\*d)\*n^2 + 24\*(35\*a^2\*b^5\*c^2 + 42\*a^4\*b^3\*c\*d + 15\*a^6\*b\*d^2)\*n)\*x\*(b\*x + a)^n/(b^7\*n^7 + 28\*b^7\*n^6 + 322\*b^7\*n^5 + 1960\*b^7\*n^4 + 6769\*b^7\*n^3 + 13132\*b^7\*n^2 + 13068\*b^7\*n + 5040\*b^7)

**Sympy [A]** time = 39.9254, size = 14300, normalized size = 61.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(b\*x+a)\*\*n\*(d\*x\*\*2+c)\*\*2,x)

[Out] Piecewise((a\*\*n\*(c\*\*2\*x\*\*3/3 + 2\*c\*d\*x\*\*5/5 + d\*\*2\*x\*\*7/7), Eq(b, 0)), (60\*a\*\*6\*d\*\*2\*log(a/b + x)/(60\*a\*\*6\*b\*\*7 + 360\*a\*\*5\*b\*\*8\*x + 900\*a\*\*4\*b\*\*9\*x\*\*2 + 1200\*a\*\*3\*b\*\*10\*x\*\*3 + 900\*a\*\*2\*b\*\*11\*x\*\*4 + 360\*a\*b\*\*12\*x\*\*5 + 60\*b\*\*13\*x\*\*6) + 147\*a\*\*6\*d\*\*2/(60\*a\*\*6\*b\*\*7 + 360\*a\*\*5\*b\*\*8\*x + 900\*a\*\*4\*b\*\*9\*x\*\*2 + 1200\*a\*\*3\*b\*\*10\*x\*\*3 + 900\*a\*\*2\*b\*\*11\*x\*\*4 + 360\*a\*b\*\*12\*x\*\*5 + 60\*b\*\*13\*x\*\*6) + 360\*a\*\*5\*b\*d\*\*2\*x\*log(a/b + x)/(60\*a\*\*6\*b\*\*7 + 360\*a\*\*5\*b\*\*8\*x + 900\*a\*\*4\*b\*\*9\*x\*\*2 + 1200\*a\*\*3\*b\*\*10\*x\*\*3 + 900\*a\*\*2\*b\*\*11\*x\*\*4 + 360\*a\*b\*\*12\*x\*\*5 + 60\*b\*\*13\*x\*\*6) + 822\*a\*\*5\*b\*d\*\*2\*x/(60\*a\*\*6\*b\*\*7 + 360\*a\*\*5\*b\*\*8\*x + 900\*a\*\*4\*b\*\*9\*x\*\*2 + 1200\*a\*\*3\*b\*\*10\*x\*\*3 + 900\*a\*\*2\*b\*\*11\*x\*\*4 + 360\*a\*b\*\*12\*x\*\*5 + 60\*b\*\*13\*x\*\*6) - 4\*a\*\*4\*b\*\*2\*c\*d/(60\*a\*\*6\*b\*\*7 + 360\*a\*\*5\*b\*\*8\*x + 900\*a\*\*4\*b\*\*9\*x\*\*2 + 1200\*a\*\*3\*b\*\*10\*x\*\*3 + 900\*a\*\*2\*b\*\*11\*x\*\*4 + 360\*a\*b\*\*12\*x\*\*5 + 60\*b\*\*13\*x\*\*6) + 900\*a\*\*4\*b\*\*2\*d\*\*2\*x\*\*2\*log(a/b + x)/(60\*a\*\*6\*b\*\*7 + 360\*a\*\*5\*b\*\*8\*x + 900\*a\*\*4\*b\*\*9\*x\*\*2 + 1200\*a\*\*3\*b\*\*10\*x\*\*3 + 900\*a\*\*2\*b\*\*11\*x\*\*4 + 360\*a\*b\*\*12\*x\*\*5 + 60\*b\*\*13\*x\*\*6) + 1875\*a\*\*4\*b\*\*2\*d\*\*2\*x\*\*2/(60\*a\*\*6\*b\*\*7 + 360\*a\*\*5\*b\*\*8\*x + 900\*a\*\*4\*b\*\*9\*x\*\*2 + 1200\*a\*\*3\*b\*\*10\*x\*\*3 + 900\*a\*\*2\*b\*\*11\*x\*\*4 + 360\*a\*b\*\*12\*x\*\*5 + 60\*b\*\*13\*x\*\*6) - 24\*a\*\*3\*b\*\*3\*c\*d\*x/(60\*a\*\*6\*b\*\*7 + 360\*a\*\*5\*b\*\*8\*x + 900\*a\*\*4\*b\*\*9\*x\*\*2 + 1200\*a\*\*3\*b\*\*10\*x\*\*3 + 900\*a\*\*2\*b\*\*11\*x\*\*4 + 360\*a\*b\*\*12\*x\*\*5 + 60\*b\*\*13\*x\*\*6) + 1200\*a\*\*3\*b\*\*3\*d\*\*2\*x\*\*3\*log(a/b + x)/(60\*a\*\*6\*b\*\*7 + 360\*a\*\*5\*b\*\*8\*x + 900\*a\*\*4\*b\*\*9\*x\*\*2 + 1200\*a\*\*3\*b\*\*10\*x\*\*3 + 900\*a\*\*2\*b\*\*11\*x\*\*4 + 360\*a\*b\*\*12\*x\*\*5 + 60\*b\*\*13\*x\*\*6) + 2200\*a\*\*3\*b\*\*3\*d\*\*2\*x\*\*3/(60\*a\*\*6\*b\*\*7 + 360\*a\*\*5\*b\*\*8\*x + 900\*a\*\*4\*b\*\*9\*x\*\*2 + 1200\*a\*\*3\*b\*\*10\*x\*\*3 + 900\*a\*\*2\*b\*\*11\*x\*\*4 + 360\*a\*b\*\*12\*x\*\*5 + 60\*b\*\*13\*x\*\*6) - a\*\*2\*b\*\*4\*c\*\*2/(60\*a\*\*6\*b\*\*7 + 360\*a\*\*5\*b\*\*8\*x + 900\*a\*\*4\*b\*\*9\*x\*\*2 + 1200\*a\*\*3\*b\*\*10\*x\*\*3 + 900\*a\*\*2\*b\*\*11\*x\*\*4 + 360\*a\*b\*\*12\*x\*\*5 + 60\*b\*\*13\*x\*\*6) - 60\*a\*\*2\*b\*\*4\*c\*d\*x\*\*2/(60\*a\*\*6\*b\*\*7 + 360\*a\*\*5\*b\*\*8\*x + 900\*a\*\*4\*b\*\*9\*x\*\*2 + 1200\*a\*\*3\*b\*\*10\*x\*\*3 + 900\*a\*\*2\*b\*\*11\*x\*\*4 + 360\*a\*b\*\*12\*x\*\*5 + 60\*b\*\*13\*x\*\*6) + 900\*a\*\*2\*b\*\*4\*d\*\*2\*x\*\*4\*log(a/b + x)/(60\*a\*\*6\*b\*\*7 + 360\*a\*\*5\*b\*\*8\*x + 900\*a\*\*4\*b\*\*9\*x\*\*2 + 1200\*a\*\*3\*b\*\*10\*x\*\*3 + 900\*a\*\*2\*b\*\*11\*x\*\*4 + 360\*a\*b\*\*12\*x\*\*5 + 60\*b\*\*13\*x\*\*6) + 1350\*a\*\*2\*b\*\*4\*d\*\*2\*x\*\*4/(60\*a\*\*6\*b\*\*7 + 360\*a\*\*5\*b\*\*8\*x + 900\*a\*\*4\*b\*\*9\*x\*\*2 + 1200\*a\*\*3\*b\*\*10\*x\*\*3 + 900\*a\*\*2\*b\*\*11\*x\*\*4 + 360\*a\*b\*\*12\*x\*\*5 + 60\*b\*\*13\*x\*\*6) - 6\*a\*b\*\*5\*c\*\*2\*x/(60\*a\*\*6\*b\*\*7 + 360\*a\*\*5\*b\*\*8\*x + 900\*a\*\*4\*b\*\*9\*x\*\*2 + 1200\*a\*\*3\*b\*\*10\*x\*\*3 + 900\*a\*\*2\*b\*\*11\*x\*\*4 + 360\*a\*b\*\*12\*x\*\*5 + 60\*b\*\*13\*x\*\*6) - 80\*a\*b\*\*5\*c\*d\*x\*\*3/(60\*a\*\*6\*b\*\*7 + 360\*a\*\*5\*b\*\*8\*x + 900\*a\*\*4\*b\*\*9\*x\*\*2 + 1200\*a\*\*3\*b\*\*10\*x\*\*3 + 900\*a\*\*2\*b\*\*11\*x\*\*4 + 360\*a\*b\*\*12\*x\*\*5 + 60\*b\*\*13\*x\*\*6) + 360\*a\*b\*\*5\*d\*\*2\*x\*\*5\*log(a/b + x)/(60\*a\*\*6\*b\*\*7 + 360\*a\*\*5\*b\*\*8\*x + 900\*a\*\*4\*b\*\*9\*x\*\*2 + 1200\*a\*\*3\*b\*\*10\*x\*\*3 + 900\*a\*\*2\*b\*\*11\*x\*\*4 + 360\*a\*b\*\*12\*x\*\*5 + 60\*b\*\*13\*x\*\*6) + 360\*a\*b\*\*5\*d\*\*2\*x\*\*5/(60\*a\*\*6\*b\*\*7 + 360\*a\*\*5\*b\*\*8\*x + 900\*a\*\*4\*b\*\*9\*x\*\*2 + 1200\*a\*\*3\*b\*\*10\*x\*\*3 + 900\*a\*\*2\*b\*\*11\*x\*\*4 + 360\*a\*b\*\*12\*x\*\*5 + 60\*b\*\*13\*x\*\*6) - 15\*b\*\*6\*c\*\*2\*x\*\*2/(60\*a\*\*6\*b\*\*7 + 360\*a\*\*5\*b\*\*8\*x + 900\*a\*\*4\*b\*\*9\*x\*\*2 + 1200\*a\*\*3\*b\*\*10\*x\*\*3 + 900\*a\*\*2\*b\*\*11\*x\*\*4 + 360\*a\*b\*\*12\*x\*\*5 + 60\*b\*\*13\*x\*\*6) - 60\*b\*\*6\*c\*d\*x\*\*4/(60\*a\*\*6\*b\*\*7 + 360\*a\*\*5\*b\*\*8\*x + 900\*a\*\*4\*b\*\*9\*x\*\*2 + 1200\*a\*\*3\*b\*\*10\*x\*\*3 + 900\*a\*\*2\*b\*\*11\*x\*\*4 + 360\*a\*b\*\*12\*x\*\*5 + 60\*b\*\*13\*x\*\*6) + 60\*b\*\*6\*d\*\*2\*x\*\*6\*log(a/b + x)/(60\*a\*\*6\*b\*\*7 + 360\*a\*\*5\*b\*\*8\*x + 900\*a\*\*4\*b\*\*9\*x\*\*2 + 1200\*a\*\*3\*b\*\*10\*x\*\*3 + 900\*a\*\*2\*b\*\*11\*x\*\*4 + 360\*a\*b\*\*12\*x\*\*5 + 60\*b\*\*13\*x\*\*6), Eq(n, -7)), (-180\*a\*\*6\*d\*\*2\*log(a/b + x)/(30\*a\*\*5\*b\*\*7 + 150\*a\*\*4\*b\*\*8\*x + 300\*a\*\*3\*b\*\*9\*x\*\*2 + 300\*a\*\*2\*b\*\*10\*x\*\*3 + 150\*a\*b\*\*11\*x\*\*4 + 30\*b\*\*12\*x\*\*5) - 411\*a\*\*6\*d\*\*2/(30\*a\*\*5\*b\*\*7 + 150\*a\*\*4\*b\*\*8\*x + 300\*a\*\*3\*b\*\*9\*x\*\*2 + 300\*a\*\*2\*b\*\*10\*x\*\*3 + 150\*a\*b\*\*11\*x\*\*4 + 30\*b\*\*12\*x\*\*5) - 900\*a\*\*5\*b\*d\*\*2\*x\*log(a/b + x)/(30\*a\*\*5\*b\*\*7 + 150\*a\*\*4\*b\*\*8\*x + 300\*a\*\*3\*b\*\*9\*x\*\*2 + 300\*a\*\*2\*b\*\*10\*x\*\*3 + 150\*a\*b\*\*11\*x\*\*4 + 30\*b\*\*12\*x\*\*5) - 1875\*a\*\*5\*b\*d\*\*2\*x/(30\*a\*\*5\*b\*\*7 + 150\*a\*\*4\*b\*\*8\*x + 300\*a\*\*3\*b\*\*9\*x\*\*2 + 300\*a\*\*2\*b\*\*10\*x\*\*3 + 150\*a\*b\*\*11\*x\*\*4 + 30\*b\*\*12\*x\*\*5) - 12\*a\*\*4\*b\*\*2\*c\*d/(30\*a\*\*5\*b\*\*7 + 150\*a\*\*4\*b\*\*8\*x + 300\*a\*\*3\*b\*\*9\*x\*\*2 + 300\*a\*\*2\*b\*\*10\*x\*\*3 + 150\*a\*b\*\*11\*x\*\*4 + 30\*b\*\*12\*x\*\*5))



$$\begin{aligned}
& 4*b^{**7} + 48*a^{**3}*b^{**8}*x + 72*a^{**2}*b^{**9}*x^{**2} + 48*a*b^{**10}*x^{**3} + \\
& 12*b^{**11}*x^{**4}), \text{Eq}(n, -5)), (-60*a^{**7}*d^{**2}*\log(a/b + x)/(3*a^{**4}*b^{**7} \\
& + 9*a^{**3}*b^{**8}*x + 9*a^{**2}*b^{**9}*x^{**2} + 3*a*b^{**10}*x^{**3}) - 20*a^{**7} \\
& *d^{**2}/(3*a^{**4}*b^{**7} + 9*a^{**3}*b^{**8}*x + 9*a^{**2}*b^{**9}*x^{**2} + 3*a*b^{**10} \\
& *x^{**3}) - 180*a^{**6}*b*d^{**2}*x*\log(a/b + x)/(3*a^{**4}*b^{**7} + 9*a^{**3}*b^{**8} \\
& *x + 9*a^{**2}*b^{**9}*x^{**2} + 3*a*b^{**10}*x^{**3}) - 24*a^{**5}*b^{**2}*c*d*\log( \\
& a/b + x)/(3*a^{**4}*b^{**7} + 9*a^{**3}*b^{**8}*x + 9*a^{**2}*b^{**9}*x^{**2} + 3*a*b^{**10} \\
& *x^{**3}) - 8*a^{**5}*b^{**2}*c*d/(3*a^{**4}*b^{**7} + 9*a^{**3}*b^{**8}*x + 9*a^{**2} \\
& *b^{**9}*x^{**2} + 3*a*b^{**10}*x^{**3}) - 180*a^{**5}*b^{**2}*d^{**2}*x^{**2}*\log(a/b + \\
& x)/(3*a^{**4}*b^{**7} + 9*a^{**3}*b^{**8}*x + 9*a^{**2}*b^{**9}*x^{**2} + 3*a*b^{**10}*x^{**3}) \\
& + 90*a^{**5}*b^{**2}*d^{**2}*x^{**2}/(3*a^{**4}*b^{**7} + 9*a^{**3}*b^{**8}*x + 9*a^{**2} \\
& *b^{**9}*x^{**2} + 3*a*b^{**10}*x^{**3}) - 72*a^{**4}*b^{**3}*c*d*x*\log(a/b + x)/( \\
& 3*a^{**4}*b^{**7} + 9*a^{**3}*b^{**8}*x + 9*a^{**2}*b^{**9}*x^{**2} + 3*a*b^{**10}*x^{**3}) \\
& - 60*a^{**4}*b^{**3}*d^{**2}*x^{**3}*\log(a/b + x)/(3*a^{**4}*b^{**7} + 9*a^{**3}*b^{**8} \\
& *x + 9*a^{**2}*b^{**9}*x^{**2} + 3*a*b^{**10}*x^{**3}) + 90*a^{**4}*b^{**3}*d^{**2}*x^{**3}/( \\
& 3*a^{**4}*b^{**7} + 9*a^{**3}*b^{**8}*x + 9*a^{**2}*b^{**9}*x^{**2} + 3*a*b^{**10}*x^{**3}) \\
& - 72*a^{**3}*b^{**4}*c*d*x^{**2}*\log(a/b + x)/(3*a^{**4}*b^{**7} + 9*a^{**3}*b^{**8}*x \\
& + 9*a^{**2}*b^{**9}*x^{**2} + 3*a*b^{**10}*x^{**3}) + 36*a^{**3}*b^{**4}*c*d*x^{**2}/(3* \\
& a^{**4}*b^{**7} + 9*a^{**3}*b^{**8}*x + 9*a^{**2}*b^{**9}*x^{**2} + 3*a*b^{**10}*x^{**3}) + \\
& 15*a^{**3}*b^{**4}*d^{**2}*x^{**4}/(3*a^{**4}*b^{**7} + 9*a^{**3}*b^{**8}*x + 9*a^{**2}*b^{**9} \\
& *x^{**2} + 3*a*b^{**10}*x^{**3}) - 24*a^{**2}*b^{**5}*c*d*x^{**3}*\log(a/b + x)/(3*a \\
& **4*b^{**7} + 9*a^{**3}*b^{**8}*x + 9*a^{**2}*b^{**9}*x^{**2} + 3*a*b^{**10}*x^{**3}) + 3 \\
& 6*a^{**2}*b^{**5}*c*d*x^{**3}/(3*a^{**4}*b^{**7} + 9*a^{**3}*b^{**8}*x + 9*a^{**2}*b^{**9}*x \\
& **2 + 3*a*b^{**10}*x^{**3}) - 3*a^{**2}*b^{**5}*d^{**2}*x^{**5}/(3*a^{**4}*b^{**7} + 9*a \\
& **3*b^{**8}*x + 9*a^{**2}*b^{**9}*x^{**2} + 3*a*b^{**10}*x^{**3}) + 6*a*b^{**6}*c*d*x^{**4} \\
& / (3*a^{**4}*b^{**7} + 9*a^{**3}*b^{**8}*x + 9*a^{**2}*b^{**9}*x^{**2} + 3*a*b^{**10}*x^{**3}) \\
& + a*b^{**6}*d^{**2}*x^{**6}/(3*a^{**4}*b^{**7} + 9*a^{**3}*b^{**8}*x + 9*a^{**2}*b^{**9} \\
& *x^{**2} + 3*a*b^{**10}*x^{**3}) + b^{**7}*c^{**2}*x^{**3}/(3*a^{**4}*b^{**7} + 9*a^{**3}*b^{**8} \\
& *x + 9*a^{**2}*b^{**9}*x^{**2} + 3*a*b^{**10}*x^{**3}), \text{Eq}(n, -4)), (60*a^{**6}*d^{**2} \\
& *x*\log(a/b + x)/(4*a^{**2}*b^{**7} + 8*a*b^{**8}*x + 4*b^{**9}*x^{**2}) + 30*a^{**6} \\
& *d^{**2}/(4*a^{**2}*b^{**7} + 8*a*b^{**8}*x + 4*b^{**9}*x^{**2}) + 120*a^{**5}*b*d^{**2} \\
& *x*\log(a/b + x)/(4*a^{**2}*b^{**7} + 8*a*b^{**8}*x + 4*b^{**9}*x^{**2}) + 48*a^{**4} \\
& *b^{**2}*c*d*\log(a/b + x)/(4*a^{**2}*b^{**7} + 8*a*b^{**8}*x + 4*b^{**9}*x^{**2}) \\
& + 24*a^{**4}*b^{**2}*c*d/(4*a^{**2}*b^{**7} + 8*a*b^{**8}*x + 4*b^{**9}*x^{**2}) + 60* \\
& a^{**4}*b^{**2}*d^{**2}*x^{**2}*\log(a/b + x)/(4*a^{**2}*b^{**7} + 8*a*b^{**8}*x + 4*b^{**9} \\
& *x^{**2}) - 60*a^{**4}*b^{**2}*d^{**2}*x^{**2}/(4*a^{**2}*b^{**7} + 8*a*b^{**8}*x + 4*b \\
& **9*x^{**2}) + 96*a^{**3}*b^{**3}*c*d*x*\log(a/b + x)/(4*a^{**2}*b^{**7} + 8*a*b^{**8} \\
& *x + 4*b^{**9}*x^{**2}) - 20*a^{**3}*b^{**3}*d^{**2}*x^{**3}/(4*a^{**2}*b^{**7} + 8*a*b \\
& **8*x + 4*b^{**9}*x^{**2}) + 4*a^{**2}*b^{**4}*c^{**2}*\log(a/b + x)/(4*a^{**2}*b^{**7} \\
& + 8*a*b^{**8}*x + 4*b^{**9}*x^{**2}) + 2*a^{**2}*b^{**4}*c^{**2}/(4*a^{**2}*b^{**7} + 8* \\
& a*b^{**8}*x + 4*b^{**9}*x^{**2}) + 48*a^{**2}*b^{**4}*c*d*x^{**2}*\log(a/b + x)/(4*a \\
& **2*b^{**7} + 8*a*b^{**8}*x + 4*b^{**9}*x^{**2}) - 48*a^{**2}*b^{**4}*c*d*x^{**2}/(4*a \\
& **2*b^{**7} + 8*a*b^{**8}*x + 4*b^{**9}*x^{**2}) + 5*a^{**2}*b^{**4}*d^{**2}*x^{**4}/(4*a \\
& **2*b^{**7} + 8*a*b^{**8}*x + 4*b^{**9}*x^{**2}) + 8*a*b^{**5}*c^{**2}*x*\log(a/b + \\
& x)/(4*a^{**2}*b^{**7} + 8*a*b^{**8}*x + 4*b^{**9}*x^{**2}) - 16*a*b^{**5}*c*d*x^{**3}/ \\
& (4*a^{**2}*b^{**7} + 8*a*b^{**8}*x + 4*b^{**9}*x^{**2}) - 2*a*b^{**5}*d^{**2}*x^{**5}/(4* \\
& a^{**2}*b^{**7} + 8*a*b^{**8}*x + 4*b^{**9}*x^{**2}) + 4*b^{**6}*c^{**2}*x^{**2}*\log(a/b \\
& + x)/(4*a^{**2}*b^{**7} + 8*a*b^{**8}*x + 4*b^{**9}*x^{**2}) - 4*b^{**6}*c^{**2}*x^{**2}/ \\
& (4*a^{**2}*b^{**7} + 8*a*b^{**8}*x + 4*b^{**9}*x^{**2}) + 4*b^{**6}*c*d*x^{**4}/(4*a^{**2} \\
& *b^{**7} + 8*a*b^{**8}*x + 4*b^{**9}*x^{**2}) + b^{**6}*d^{**2}*x^{**6}/(4*a^{**2}*b^{**7} \\
& + 8*a*b^{**8}*x + 4*b^{**9}*x^{**2}), \text{Eq}(n, -3)), (-180*a^{**6}*d^{**2}*\log(a/b \\
& + x)/(30*a*b^{**7} + 30*b^{**8}*x) - 180*a^{**6}*d^{**2}/(30*a*b^{**7} + 30*b^{**8} \\
& *x) - 180*a^{**5}*b*d^{**2}*x*\log(a/b + x)/(30*a*b^{**7} + 30*b^{**8}*x) - 24 \\
& 0*a^{**4}*b^{**2}*c*d*\log(a/b + x)/(30*a*b^{**7} + 30*b^{**8}*x) - 240*a^{**4}*b \\
& **2*c*d/(30*a*b^{**7} + 30*b^{**8}*x) + 90*a^{**4}*b^{**2}*d^{**2}*x^{**2}/(30*a*b^{**7} \\
& + 30*b^{**8}*x) - 240*a^{**3}*b^{**3}*c*d*x*\log(a/b + x)/(30*a*b^{**7} + 3 \\
& 0*b^{**8}*x) - 30*a^{**3}*b^{**3}*d^{**2}*x^{**3}/(30*a*b^{**7} + 30*b^{**8}*x) - 60*a \\
& **2*b^{**4}*c^{**2}*\log(a/b + x)/(30*a*b^{**7} + 30*b^{**8}*x) - 60*a^{**2}*b^{**4} \\
& *c^{**2}/(30*a*b^{**7} + 30*b^{**8}*x) + 120*a^{**2}*b^{**4}*c*d*x^{**2}/(30*a*b^{**7} \\
& + 30*b^{**8}*x) + 15*a^{**2}*b^{**4}*d^{**2}*x^{**4}/(30*a*b^{**7} + 30*b^{**8}*x) - \\
& 60*a*b^{**5}*c^{**2}*x*\log(a/b + x)/(30*a*b^{**7} + 30*b^{**8}*x) - 40*a*b^{**5} \\
& *c*d*x^{**3}/(30*a*b^{**7} + 30*b^{**8}*x) - 9*a*b^{**5}*d^{**2}*x^{**5}/(30*a*b^{**7} \\
& + 30*b^{**8}*x) + 30*b^{**6}*c^{**2}*x^{**2}/(30*a*b^{**7} + 30*b^{**8}*x) + 20*b^{**6} \\
& *c*d*x^{**4}/(30*a*b^{**7} + 30*b^{**8}*x) + 6*b^{**6}*d^{**2}*x^{**6}/(30*a*b^{**7} \\
& + 30*b^{**8}*x), \text{Eq}(n, -2)), (a^{**6}*d^{**2}*\log(a/b + x)/b^{**7} - a^{**5}*d^{**2} \\
& *x/b^{**6} + 2*a^{**4}*c*d*\log(a/b + x)/b^{**5} + a^{**4}*d^{**2}*x^{**2}/(2*b^{**5} \\
& ) - 2*a^{**3}*c*d*x/b^{**4} - a^{**3}*d^{**2}*x^{**3}/(3*b^{**4}) + a^{**2}*c^{**2}*\log(a \\
& /b + x)/b^{**3} + a^{**2}*c*d*x^{**2}/b^{**3} + a^{**2}*d^{**2}*x^{**4}/(4*b^{**3}) - a*c \\
& **2*x/b^{**2} - 2*a*c*d*x^{**3}/(3*b^{**2}) - a*d^{**2}*x^{**5}/(5*b^{**2}) + c^{**2} \\
& *x^{**2}/(2*b) + c*d*x^{**4}/(2*b) + d^{**2}*x^{**6}/(6*b), \text{Eq}(n, -1)), (720*a \\
& **7*d^{**2}*(a + b*x)**n/(b^{**7}*n^{**7} + 28*b^{**7}*n^{**6} + 322*b^{**7}*n^{**5} + \\
& 1960*b^{**7}*n^{**4} + 6769*b^{**7}*n^{**3} + 13132*b^{**7}*n^{**2} + 13068*b^{**7}*n
\end{aligned}$$





```

**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7
) + b**7*c**2*n**6*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 +
322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2
+ 13068*b**7*n + 5040*b**7) + 25*b**7*c**2*n**5*x**3*(a + b*x)**
n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 67
69*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 247*
b**7*c**2*n**4*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*
b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 1
3068*b**7*n + 5040*b**7) + 1219*b**7*c**2*n**3*x**3*(a + b*x)**n/
(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769
*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 3112*b
**7*c**2*n**2*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b
**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13
068*b**7*n + 5040*b**7) + 3796*b**7*c**2*n*x**3*(a + b*x)**n/(b**
7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**
7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1680*b**7*
c**2*x**3*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5
+ 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*
n + 5040*b**7) + 2*b**7*c*d*n**6*x**5*(a + b*x)**n/(b**7*n**7 + 2
8*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 1
3132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 46*b**7*c*d*n**5*x**
5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b
**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040
*b**7) + 414*b**7*c*d*n**4*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7
*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b
**7*n**2 + 13068*b**7*n + 5040*b**7) + 1850*b**7*c*d*n**3*x**5*(a
+ b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*
n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**
7) + 4288*b**7*c*d*n**2*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n*
**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7
*n**2 + 13068*b**7*n + 5040*b**7) + 4824*b**7*c*d*n*x**5*(a + b*x
)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 +
6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 2
016*b**7*c*d*x**5*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b*
**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 130
68*b**7*n + 5040*b**7) + b**7*d**2*n**6*x**7*(a + b*x)**n/(b**7*n
**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n
**3 + 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 21*b**7*d**2*
n**5*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5
+ 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*
n + 5040*b**7) + 175*b**7*d**2*n**4*x**7*(a + b*x)**n/(b**7*n**7
+ 28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3
+ 13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 735*b**7*d**2*n**
3*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1
960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n +
5040*b**7) + 1624*b**7*d**2*n**2*x**7*(a + b*x)**n/(b**7*n**7 +
28*b**7*n**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 +
13132*b**7*n**2 + 13068*b**7*n + 5040*b**7) + 1764*b**7*d**2*n*x*
**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n**6 + 322*b**7*n**5 + 1960*
b**7*n**4 + 6769*b**7*n**3 + 13132*b**7*n**2 + 13068*b**7*n + 504
0*b**7) + 720*b**7*d**2*x**7*(a + b*x)**n/(b**7*n**7 + 28*b**7*n*
**6 + 322*b**7*n**5 + 1960*b**7*n**4 + 6769*b**7*n**3 + 13132*b**7
*n**2 + 13068*b**7*n + 5040*b**7), True))

```

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**GIAC/XCAS [A]** time = 0.275405, size = 1, normalized size = 0.

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)^2\*(b\*x + a)^n\*x^2,x, algorithm="giac")

[Out] Done

### 3.356 $\int x(a + bx)^n (c + dx^2)^2 dx$

**Optimal.** Leaf size=185

$$\frac{a(a^2d + b^2c)^2(a + bx)^{n+1}}{b^6(n+1)} + \frac{(a^2d + b^2c)(5a^2d + b^2c)(a + bx)^{n+2}}{b^6(n+2)} - \frac{2ad(5a^2d + 3b^2c)(a + bx)^{n+3}}{b^6(n+3)} + \frac{2d(5a^2d + b^2c)(a + bx)^{n+4}}{b^6(n+4)} - \frac{5ad^2(a + bx)^{n+5}}{b^6(n+5)} + \frac{d^2(a + bx)^{n+6}}{b^6(n+6)}$$

[Out]  $-\left(\frac{(a^2d + b^2c)^2(a + bx)^{n+1}}{b^6(n+1)} + \frac{(a^2d + b^2c)(5a^2d + b^2c)(a + bx)^{n+2}}{b^6(n+2)} - \frac{2ad(5a^2d + 3b^2c)(a + bx)^{n+3}}{b^6(n+3)} + \frac{2d(5a^2d + b^2c)(a + bx)^{n+4}}{b^6(n+4)} - \frac{5ad^2(a + bx)^{n+5}}{b^6(n+5)} + \frac{d^2(a + bx)^{n+6}}{b^6(n+6)}\right)$

**Rubi [A]** time = 0.230561, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$\frac{a(a^2d + b^2c)^2(a + bx)^{n+1}}{b^6(n+1)} + \frac{(a^2d + b^2c)(5a^2d + b^2c)(a + bx)^{n+2}}{b^6(n+2)} - \frac{2ad(5a^2d + 3b^2c)(a + bx)^{n+3}}{b^6(n+3)} + \frac{2d(5a^2d + b^2c)(a + bx)^{n+4}}{b^6(n+4)} - \frac{5ad^2(a + bx)^{n+5}}{b^6(n+5)} + \frac{d^2(a + bx)^{n+6}}{b^6(n+6)}$$

Antiderivative was successfully verified.

[In] `Int[x*(a + b*x)^n*(c + d*x^2)^2, x]`

[Out]  $-\left(\frac{(a^2d + b^2c)^2(a + bx)^{n+1}}{b^6(n+1)} + \frac{(a^2d + b^2c)(5a^2d + b^2c)(a + bx)^{n+2}}{b^6(n+2)} - \frac{2ad(5a^2d + 3b^2c)(a + bx)^{n+3}}{b^6(n+3)} + \frac{2d(5a^2d + b^2c)(a + bx)^{n+4}}{b^6(n+4)} - \frac{5ad^2(a + bx)^{n+5}}{b^6(n+5)} + \frac{d^2(a + bx)^{n+6}}{b^6(n+6)}\right)$

**Rubi in Sympy [A]** time = 47.599, size = 170, normalized size = 0.92

$$\frac{5ad^2(a + bx)^{n+5}}{b^6(n+5)} - \frac{2ad(a + bx)^{n+3}(5a^2d + 3b^2c)}{b^6(n+3)} - \frac{a(a + bx)^{n+1}(a^2d + b^2c)^2}{b^6(n+1)} + \frac{d^2(a + bx)^{n+6}}{b^6(n+6)} + \frac{2d(a + bx)^{n+4}(5a^2d + b^2c)}{b^6(n+4)} + \frac{(a + bx)^{n+2}(a^2d + b^2c)(5a^2d + b^2c)}{b^6(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x*(b*x+a)**n*(d*x**2+c)**2, x)`

[Out]  $-5ad^2(a + bx)^{n+5}/b^6(n+5) - 2ad(a + bx)^{n+3}(5a^2d + 3b^2c)/b^6(n+3) - a(a + bx)^{n+1}(a^2d + b^2c)^2/b^6(n+1) + d^2(a + bx)^{n+6}/b^6(n+6) + 2d(a + bx)^{n+4}(5a^2d + b^2c)/b^6(n+4) + (a + bx)^{n+2}(a^2d + b^2c)(5a^2d + b^2c)/b^6(n+2)$

**Mathematica [A]** time = 0.288424, size = 279, normalized size = 1.51

$$(a + bx)^{n+1} (-120a^5d^2 + 120a^4bd^2(n+1)x - 12a^3b^2d(c(n^2 + 11n + 30) + 5d(n^2 + 3n + 2)x^2) + 4a^2b^3d(n+1)x(3c(n^2 + 11n + 30) + 5d(n^2 + 3n + 2)x^2) + 4ab^4d^2(n+1)x^2 + b^5d^3(n+1)x^3 + d^4(n+1)x^4)$$



Antiderivative was successfully verified.

[In] Integrate[x\*(a + b\*x)^n\*(c + d\*x^2)^2,x]

[Out] 
$$\frac{((a + b*x)^{(1+n}) * (-120*a^5*d^2 + 120*a^4*b*d^2*(1+n)*x - 12*a^3*b^2*d*(c*(30 + 11*n + n^2) + 5*d*(2 + 3*n + n^2)*x^2) + 4*a^2*b^3*d*(1+n)*x*(3*c*(30 + 11*n + n^2) + 5*d*(6 + 5*n + n^2)*x^2) + b^5*(15 + 23*n + 9*n^2 + n^3)*x*(c^2*(24 + 10*n + n^2) + 2*c*d*(12 + 8*n + n^2)*x^2 + d^2*(8 + 6*n + n^2)*x^4) - a*b^4*(c^2*(360 + 342*n + 119*n^2 + 18*n^3 + n^4) + 6*c*d*(60 + 112*n + 65*n^2 + 14*n^3 + n^4)*x^2 + 5*d^2*(24 + 50*n + 35*n^2 + 10*n^3 + n^4)*x^4)) / (b^6*(1+n)*(2+n)*(3+n)*(4+n)*(5+n)*(6+n))$$

**Maple [B]** time = 0.015, size = 677, normalized size = 3.7

$$\frac{(bx + a)^{1+n} (-b^5 d^2 n^5 x^5 - 15 b^5 d^2 n^4 x^5 + 5 ab^4 d^2 n^4 x^4 - 2 b^5 cd n^5 x^3 - 85 b^5 d^2 n^3 x^5 + 50 ab^4 d^2 n^3 x^4 - 34 b^5 cd n^4 x^3 - 225 b^5 d^2 n^4 x^5)}{b^6 (n+1)(n+2)(n+3)(n+4)(n+5)(n+6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(b\*x+a)^n\*(d\*x^2+c)^2,x)

[Out] 
$$-(b*x+a)^{(1+n)} * (-b^5*d^2*n^5*x^5 - 15*b^5*d^2*n^4*x^5 + 5*a*b^4*d^2*n^4*x^4 - 2*b^5*c*d*n^5*x^3 - 85*b^5*d^2*n^3*x^5 + 50*a*b^4*d^2*n^3*x^4 - 34*b^5*c*d*n^4*x^3 - 225*b^5*d^2*n^2*x^5 - 20*a^2*b^3*d^2*n^3*x^3 + 6*a*b^4*c*d*n^4*x^2 + 175*a*b^4*d^2*n^2*x^4 - b^5*c^2*n^5*x - 214*b^5*c*d*n^3*x^3 - 274*b^5*d^2*n*x^5 - 120*a^2*b^3*d^2*n^2*x^3 + 84*a*b^4*c*d*n^3*x^2 + 250*a*b^4*d^2*n*x^4 - 19*b^5*c^2*n^4*x - 614*b^5*c*d*n^2*x^3 - 120*b^5*d^2*x^5 + 60*a^3*b^2*d^2*n^2*x^2 - 12*a^2*b^3*c*d*n^3*x - 220*a^2*b^3*d^2*n*x^3 + a*b^4*c^2*n^4 + 390*a*b^4*c*d*n^2*x^2 + 120*a*b^4*d^2*x^4 - 137*b^5*c^2*n^3*x - 792*b^5*c*d*n*x^3 + 180*a^3*b^2*d^2*n*x^2 - 144*a^2*b^3*c*d*n^2*x - 120*a^2*b^3*d^2*x^3 + 18*a*b^4*c^2*n^3 + 672*a*b^4*c*d*n*x^2 - 461*b^5*c^2*n^2*x - 360*b^5*c*d*x^3 - 120*a^4*b*d^2*n*x + 12*a^3*b^2*c*d*n^2 + 120*a^3*b^2*d^2*x^2 - 492*a^2*b^3*c*d*n*x + 119*a*b^4*c^2*n^2 + 360*a*b^4*c*d*x^2 - 702*b^5*c^2*n*x - 120*a^4*b*d^2*x + 132*a^3*b^2*c*d*n - 360*a^2*b^3*c*d*x + 342*a*b^4*c^2*n - 360*b^5*c^2*x + 120*a^5*d^2 + 360*a^3*b^2*c*d + 360*a*b^4*c^2) / b^6 / (n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)$$

**Maxima [A]** time = 0.722806, size = 452, normalized size = 2.44

$$\frac{(b^2(n+1)x^2 + abnx - a^2)(bx + a)^n c^2}{(n^2 + 3n + 2)b^2} + \frac{2((n^3 + 6n^2 + 11n + 6)b^4x^4 + (n^3 + 3n^2 + 2n)ab^3x^3 - 3(n^2 + n)a^2b^2x^2 + 6a^3bnx - 6a^4)(bx + a)^n cd}{(n^4 + 10n^3 + 35n^2 + 50n + 24)b^4} + \frac{((n^5 + 15n^4 + 85n^3 + 225n^2 + 274n + 120)b^6x^6 + (n^5 + 10n^4 + 35n^3 + 50n^2 + 24n)ab^5x^5 - 5(n^4 + 6n^3 + 11n^2 + 6n)a^2b^4x^4 + (n^4 + 10n^3 + 35n^2 + 50n + 24)b^4)(bx + a)^n cd}{(n^6 + 21n^5 + 175n^4 + 735n^3 + 1624n^2 + 1764n + 720)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)^2\*(b\*x + a)^n\*x,x, algorithm="maxima")

[Out] 
$$(b^2*(n+1)*x^2 + a*b*n*x - a^2)*(b*x + a)^n*c^2/((n^2 + 3*n + 2)*b^2) + 2*((n^3 + 6*n^2 + 11*n + 6)*b^4*x^4 + (n^3 + 3*n^2 + 2*n)*a*b^3*x^3 - 3*(n^2 + n)*a^2*b^2*x^2 + 6*a^3*b*n*x - 6*a^4)*(b*x + a)^n*c*d/((n^4 + 10*n^3 + 35*n^2 + 50*n + 24)*b^4) + ((n^5 + 15*n^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^6*x^6 + (n^5 + 10*n^4 + 35*n^3 + 50*n^2 + 24*n)*a*b^5*x^5 - 5*(n^4 + 6*n^3 + 11*n^2 + 6*n)*a^2*b^4*x^4 + 20*(n^3 + 3*n^2 + 2*n)*a^3*b^3*x^3 - 60*(n^2 + n)*a^4*b^2*x^2 + 120*a^5*b*n*x - 120*a^6)*(b*x + a)^n*d^2/((n^6 + 21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720))$$

$$21*n^5 + 175*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^6)$$

**Fricas [A]** time = 0.282535, size = 1022, normalized size = 5.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)^2\*(b\*x + a)^n\*x,x, algorithm="fricas")

[Out] 
$$-(a^2*b^4*c^2*n^4 + 18*a^2*b^4*c^2*n^3 + 360*a^2*b^4*c^2 + 360*a^4*b^2*c*d + 120*a^6*d^2 - (b^6*d^2*n^5 + 15*b^6*d^2*n^4 + 85*b^6*d^2*n^3 + 225*b^6*d^2*n^2 + 274*b^6*d^2*n + 120*b^6*d^2)*x^6 - (a*b^5*d^2*n^5 + 10*a*b^5*d^2*n^4 + 35*a*b^5*d^2*n^3 + 50*a*b^5*d^2*n^2 + 24*a*b^5*d^2*n)*x^5 - (2*b^6*c*d*n^5 + 360*b^6*c*d + (34*b^6*c*d - 5*a^2*b^4*d^2)*n^4 + 2*(107*b^6*c*d - 15*a^2*b^4*d^2)*n^3 + (614*b^6*c*d - 55*a^2*b^4*d^2)*n^2 + 6*(132*b^6*c*d - 5*a^2*b^4*d^2)*n)*x^4 - 2*(a*b^5*c*d*n^5 + 14*a*b^5*c*d*n^4 + 5*(13*a*b^5*c*d + 2*a^3*b^3*d^2)*n^3 + 2*(56*a*b^5*c*d + 15*a^3*b^3*d^2)*n^2 + 20*(3*a*b^5*c*d + a^3*b^3*d^2)*n)*x^3 + (119*a^2*b^4*c^2 + 12*a^4*b^2*c*d)*n^2 - (b^6*c^2*n^5 + 360*b^6*c^2 + (19*b^6*c^2 - 6*a^2*b^4*c*d)*n^4 + (137*b^6*c^2 - 72*a^2*b^4*c*d)*n^3 + (461*b^6*c^2 - 246*a^2*b^4*c*d - 60*a^4*b^2*d^2)*n^2 + 6*(117*b^6*c^2 - 30*a^2*b^4*c*d - 10*a^4*b^2*d^2)*n)*x^2 + 6*(57*a^2*b^4*c^2 + 22*a^4*b^2*c*d)*n - (a*b^5*c^2*n^5 + 18*a*b^5*c^2*n^4 + (119*a*b^5*c^2 + 12*a^3*b^3*c*d)*n^3 + 6*(57*a*b^5*c^2 + 22*a^3*b^3*c*d)*n^2 + 120*(3*a*b^5*c^2 + 3*a^3*b^3*c*d + a^5*b*d^2)*n)*x)*(b*x + a)^n/(b^6*n^6 + 21*b^6*n^5 + 175*b^6*n^4 + 735*b^6*n^3 + 1624*b^6*n^2 + 1764*b^6*n + 720*b^6)$$

**Sympy [A]** time = 25.6743, size = 9083, normalized size = 49.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x+a)\*\*n\*(d\*x\*\*2+c)\*\*2,x)

[Out] 
$$\text{Piecewise}((a**n*(c**2*x**2/2 + c*d*x**4/2 + d**2*x**6/6), \text{Eq}(b, 0)), (60*a**5*d**2*\log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 137*a**5*d**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 300*a**4*b*d**2*x*\log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 625*a**4*b*d**2*x/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) - 6*a**3*b**2*c*d/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 600*a**3*b**2*d**2*x**2*\log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 1100*a**3*b**2*d**2*x**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) - 30*a**2*b**3*c*d*x/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 600*a**2*b**3*d**2*x**3*\log(a/b + x)/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) + 900*a**2*b**3*d**2*x**3/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) - 3*a*b**4*c**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5) - 60*a*b**4*c*d*x**2/(60*a**5*b**6 + 300*a**4*b**7*x + 600*a**3*b**8*x**2 + 600*a**2*b**9*x**3 + 300*a*b**10*x**4 + 60*b**11*x**5)$$

$$\begin{aligned}
& b^{*8}x^{*2} + 600a^{*2}b^{*9}x^{*3} + 300a^{*b}^{*10}x^{*4} + 60b^{*11}x^{*5} \\
& ) + 300a^{*b}^{*4}d^{*2}x^{*4}\log(a/b + x)/(60a^{*5}b^{*6} + 300a^{*4}b^{*7}x \\
& + 600a^{*3}b^{*8}x^{*2} + 600a^{*2}b^{*9}x^{*3} + 300a^{*b}^{*10}x^{*4} \\
& + 60b^{*11}x^{*5}) + 300a^{*b}^{*4}d^{*2}x^{*4}/(60a^{*5}b^{*6} + 300a^{*4}b^{*7}x \\
& + 600a^{*3}b^{*8}x^{*2} + 600a^{*2}b^{*9}x^{*3} + 300a^{*b}^{*10}x^{*4} \\
& + 60b^{*11}x^{*5}) - 15b^{*5}c^{*2}x/(60a^{*5}b^{*6} + 300a^{*4}b^{*7}x \\
& + 600a^{*3}b^{*8}x^{*2} + 600a^{*2}b^{*9}x^{*3} + 300a^{*b}^{*10}x^{*4} \\
& + 60b^{*11}x^{*5}) - 60b^{*5}c^{*d}x^3/(60a^{*5}b^{*6} + 300a^{*4}b^{*7}x \\
& + 600a^{*3}b^{*8}x^{*2} + 600a^{*2}b^{*9}x^{*3} + 300a^{*b}^{*10}x^{*4} \\
& + 60b^{*11}x^{*5}) + 60b^{*5}d^{*2}x^{*5}\log(a/b + x)/(60a^{*5}b^{*6} + \\
& 300a^{*4}b^{*7}x + 600a^{*3}b^{*8}x^{*2} + 600a^{*2}b^{*9}x^{*3} + 300a^{*b}^{*10}x^{*4} \\
& + 60b^{*11}x^{*5}), \text{Eq}(n, -6)), (-60a^{*5}d^{*2}\log(a/b \\
& + x)/(12a^{*4}b^{*6} + 48a^{*3}b^{*7}x + 72a^{*2}b^{*8}x^{*2} + 48a^{*b}^{*9}x^{*3} \\
& + 12b^{*10}x^{*4}) - 125a^{*5}d^{*2}/(12a^{*4}b^{*6} + 48a^{*3}b^{*7}x + 72a^{*2}b^{*8}x^{*2} \\
& + 48a^{*b}^{*9}x^{*3} + 12b^{*10}x^{*4}) - 240a^{*4}b^{*d}^{*2}x\log(a/b + x)/(12a^{*4}b^{*6} + 48a^{*3}b^{*7}x + 72 \\
& a^{*2}b^{*8}x^{*2} + 48a^{*b}^{*9}x^{*3} + 12b^{*10}x^{*4}) - 440a^{*4}b^{*d}^{*2}x/(12a^{*4}b^{*6} + 48a^{*3}b^{*7}x + 72a^{*2}b^{*8}x^{*2} + 48a^{*b}^{*9}x^{*3} \\
& + 12b^{*10}x^{*4}) - 6a^{*3}b^{*2}c^d/(12a^{*4}b^{*6} + 48a^{*3}b^{*7}x + 72a^{*2}b^{*8}x^{*2} + 48a^{*b}^{*9}x^{*3} + 12b^{*10}x^{*4}) - \\
& 360a^{*3}b^{*2}d^{*2}x^{*2}\log(a/b + x)/(12a^{*4}b^{*6} + 48a^{*3}b^{*7}x + 72a^{*2}b^{*8}x^{*2} + 48a^{*b}^{*9}x^{*3} + 12b^{*10}x^{*4}) - 540a^{*3}b^{*2}d^{*2}x^{*2}/(12a^{*4}b^{*6} + 48a^{*3}b^{*7}x + 72a^{*2}b^{*8}x^{*2} \\
& + 48a^{*b}^{*9}x^{*3} + 12b^{*10}x^{*4}) - 24a^{*2}b^{*3}c^d x/(12a^{*4}b^{*6} + 48a^{*3}b^{*7}x + 72a^{*2}b^{*8}x^{*2} + 48a^{*b}^{*9}x^{*3} + 12b^{*10}x^{*4}) - 240a^{*2}b^{*3}d^{*2}x^{*3}\log(a/b + x)/(12a^{*4}b^{*6} + 48a^{*3}b^{*7}x + 72a^{*2}b^{*8}x^{*2} \\
& + 48a^{*b}^{*9}x^{*3} + 12b^{*10}x^{*4}) - a^{*b}^{*4}c^{*2}/(12a^{*4}b^{*6} + 48a^{*3}b^{*7}x + 72a^{*2}b^{*8}x^{*2} + 48a^{*b}^{*9}x^{*3} + 12b^{*10}x^{*4}) - 36a^{*b}^{*4}c^d x^2/(12a^{*4}b^{*6} + 48a^{*3}b^{*7}x + 72a^{*2}b^{*8}x^{*2} + 48a^{*b}^{*9}x^{*3} + 12b^{*10}x^{*4}) - \\
& 60a^{*b}^{*4}d^{*2}x^{*4}\log(a/b + x)/(12a^{*4}b^{*6} + 48a^{*3}b^{*7}x + 72a^{*2}b^{*8}x^{*2} + 48a^{*b}^{*9}x^{*3} + 12b^{*10}x^{*4}) - 4b^{*5}c^{*2}x/(12a^{*4}b^{*6} + 48a^{*3}b^{*7}x + 72a^{*2}b^{*8}x^{*2} + 48a^{*b}^{*9}x^{*3} + 12b^{*10}x^{*4}) - 24b^{*5}c^d x^3/(12a^{*4}b^{*6} + 48a^{*3}b^{*7}x + 72a^{*2}b^{*8}x^{*2} + 48a^{*b}^{*9}x^{*3} + 12b^{*10}x^{*4}) + \\
& 12b^{*5}d^{*2}x^{*5}/(12a^{*4}b^{*6} + 48a^{*3}b^{*7}x + 72a^{*2}b^{*8}x^{*2} + 48a^{*b}^{*9}x^{*3} + 12b^{*10}x^{*4}), \text{Eq}(n, -5)), (60a^{*7}d^{*2} \\
& \log(a/b + x)/(6a^{*5}b^{*6} + 18a^{*4}b^{*7}x + 18a^{*3}b^{*8}x^{*2} + 6a^{*2}b^{*9}x^{*3}) + 20a^{*7}d^{*2}/(6a^{*5}b^{*6} + 18a^{*4}b^{*7}x + \\
& 18a^{*3}b^{*8}x^{*2} + 6a^{*2}b^{*9}x^{*3}) + 180a^{*6}b^d x^2\log(a/b + x)/(6a^{*5}b^{*6} + 18a^{*4}b^{*7}x + 18a^{*3}b^{*8}x^{*2} + 6a^{*2}b^{*9}x^{*3}) + 12a^{*5}b^{*2}c^d \log(a/b + x)/(6a^{*5}b^{*6} + 18a^{*4}b^{*7}x + 18a^{*3}b^{*8}x^{*2} + 6a^{*2}b^{*9}x^{*3}) + 4a^{*5}b^{*2}c^d \\
& d/(6a^{*5}b^{*6} + 18a^{*4}b^{*7}x + 18a^{*3}b^{*8}x^{*2} + 6a^{*2}b^{*9}x^{*3}) + 180a^{*5}b^{*2}d^{*2}x^{*2}\log(a/b + x)/(6a^{*5}b^{*6} + 18a^{*4}b^{*7}x + 18a^{*3}b^{*8}x^{*2} + 6a^{*2}b^{*9}x^{*3}) - 90a^{*5}b^{*2} \\
& d^{*2}x^{*2}/(6a^{*5}b^{*6} + 18a^{*4}b^{*7}x + 18a^{*3}b^{*8}x^{*2} + 6a^{*2}b^{*9}x^{*3}) + 36a^{*4}b^{*3}c^d x \log(a/b + x)/(6a^{*5}b^{*6} + 18a^{*4}b^{*7}x + 18a^{*3}b^{*8}x^{*2} + 6a^{*2}b^{*9}x^{*3}) + 60a^{*4}b^{*3}d^{*2}x^{*3}\log(a/b + x)/(6a^{*5}b^{*6} + 18a^{*4}b^{*7}x + 18a^{*3}b^{*8}x^{*2} + 6a^{*2}b^{*9}x^{*3}) - 90a^{*4}b^{*3}d^{*2}x^{*3}/(6a^{*5} \\
& b^{*6} + 18a^{*4}b^{*7}x + 18a^{*3}b^{*8}x^{*2} + 6a^{*2}b^{*9}x^{*3}) + 36a^{*3}b^{*4}c^d x^2 \log(a/b + x)/(6a^{*5}b^{*6} + 18a^{*4}b^{*7}x + 18a^{*3}b^{*8}x^{*2} + 6a^{*2}b^{*9}x^{*3}) - 18a^{*3}b^{*4}c^d x^2/(6 \\
& a^{*5}b^{*6} + 18a^{*4}b^{*7}x + 18a^{*3}b^{*8}x^{*2} + 6a^{*2}b^{*9}x^{*3}) - 15a^{*3}b^{*4}d^{*2}x^{*4}/(6a^{*5}b^{*6} + 18a^{*4}b^{*7}x + 18a^{*3}b^{*8}x^{*2} + 6a^{*2}b^{*9}x^{*3}) + 12a^{*2}b^{*5}c^d x^3 \log(a/b \\
& + x)/(6a^{*5}b^{*6} + 18a^{*4}b^{*7}x + 18a^{*3}b^{*8}x^{*2} + 6a^{*2}b^{*9}x^{*3}) - 18a^{*2}b^{*5}c^d x^3/(6a^{*5}b^{*6} + 18a^{*4}b^{*7}x + 18a^{*3}b^{*8}x^{*2} + 6a^{*2}b^{*9}x^{*3}) + 3a^{*2}b^{*5}d^{*2}x^{*5}/(6 \\
& a^{*5}b^{*6} + 18a^{*4}b^{*7}x + 18a^{*3}b^{*8}x^{*2} + 6a^{*2}b^{*9}x^{*3}) + 3a^{*b}^{*6}c^{*2}x^{*2}/(6a^{*5}b^{*6} + 18a^{*4}b^{*7}x + 18a^{*3}b^{*8}x^{*2} + 6a^{*2}b^{*9}x^{*3}) + b^{*7}c^{*2}x^{*3}/(6a^{*5}b^{*6} + 18a^{*4}b^{*7}x + 18a^{*3}b^{*8}x^{*2} + 6a^{*2}b^{*9}x^{*3}), \text{Eq}(n, -4)), \\
& (-60a^{*6}d^{*2}\log(a/b + x)/(6a^{*3}b^{*6} + 12a^{*2}b^{*7}x + 6a^{*b}^{*8}x^{*2}) - 30a^{*6}d^{*2}/(6a^{*3}b^{*6} + 12a^{*2}b^{*7}x + 6a^{*b}^{*8} \\
& x^{*2}) - 120a^{*5}b^d x^2 \log(a/b + x)/(6a^{*3}b^{*6} + 12a^{*2}b^{*7}x + 6a^{*b}^{*8}x^{*2}) - 36a^{*4}b^{*2}c^d \log(a/b + x)/(6a^{*3}b^{*6} + 12a^{*2}b^{*7}x + 6a^{*b}^{*8}x^{*2}) - 18a^{*4}b^{*2}c^d/(6a^{*3}b^{*6} \\
& + 12a^{*2}b^{*7}x + 6a^{*b}^{*8}x^{*2}) - 60a^{*4}b^{*2}d^{*2}x^{*2}\log
\end{aligned}$$

$$\begin{aligned}
& (a/b + x)/(6*a^{**3}*b^{**6} + 12*a^{**2}*b^{**7}*x + 6*a*b^{**8}*x^{**2}) + 60*a^{**4}*b^{**2}*d^{**2}*x^{**2}/(6*a^{**3}*b^{**6} + 12*a^{**2}*b^{**7}*x + 6*a*b^{**8}*x^{**2}) - \\
& 72*a^{**3}*b^{**3}*c*d*x*log(a/b + x)/(6*a^{**3}*b^{**6} + 12*a^{**2}*b^{**7}*x + 6*a*b^{**8}*x^{**2}) + 20*a^{**3}*b^{**3}*d^{**2}*x^{**3}/(6*a^{**3}*b^{**6} + 12*a^{**2}*b^{**7}*x + 6*a*b^{**8}*x^{**2}) - \\
& 36*a^{**2}*b^{**4}*c*d*x^{**2}*log(a/b + x)/(6*a^{**3}*b^{**6} + 12*a^{**2}*b^{**7}*x + 6*a*b^{**8}*x^{**2}) + 36*a^{**2}*b^{**4}*c*d*x^{**2}/ \\
& (6*a^{**3}*b^{**6} + 12*a^{**2}*b^{**7}*x + 6*a*b^{**8}*x^{**2}) - 5*a^{**2}*b^{**4}*d^{**2}*x^{**4}/(6*a^{**3}*b^{**6} + 12*a^{**2}*b^{**7}*x + 6*a*b^{**8}*x^{**2}) + 12*a*b^{**5}* \\
& c*d*x^{**3}/(6*a^{**3}*b^{**6} + 12*a^{**2}*b^{**7}*x + 6*a*b^{**8}*x^{**2}) + 2*a*b^{**5}*d^{**2}*x^{**5}/(6*a^{**3}*b^{**6} + 12*a^{**2}*b^{**7}*x + 6*a*b^{**8}*x^{**2}) + 3*b^{**6}*c^{**2}*x^{**2}/(6*a^{**3}*b^{**6} + 12*a^{**2}*b^{**7}*x + 6*a*b^{**8}*x^{**2}), \text{Eq}(n \\
& , -3)), (60*a^{**5}*d^{**2}*log(a/b + x)/(12*a*b^{**6} + 12*b^{**7}*x) + 60*a^{**5}*d^{**2}/(12*a*b^{**6} + 12*b^{**7}*x) + 60*a^{**4}*b*d^{**2}*x*log(a/b + x)/ \\
& (12*a*b^{**6} + 12*b^{**7}*x) + 72*a^{**3}*b^{**2}*c*d*log(a/b + x)/(12*a*b^{**6} + 12*b^{**7}*x) + 72*a^{**3}*b^{**2}*c*d/(12*a*b^{**6} + 12*b^{**7}*x) - 30*a^{**3}*b^{**2}*d^{**2}*x^{**2}/(12*a*b^{**6} + 12*b^{**7}*x) + 72*a^{**2}*b^{**3}*c*d*x*log \\
& (a/b + x)/(12*a*b^{**6} + 12*b^{**7}*x) + 10*a^{**2}*b^{**3}*d^{**2}*x^{**3}/(12*a*b^{**6} + 12*b^{**7}*x) + 12*a*b^{**4}*c^{**2}*log(a/b + x)/(12*a*b^{**6} + 12* \\
& b^{**7}*x) + 12*a*b^{**4}*c^{**2}/(12*a*b^{**6} + 12*b^{**7}*x) - 36*a*b^{**4}*c*d*x^{**2}/(12*a*b^{**6} + 12*b^{**7}*x) - 5*a*b^{**4}*d^{**2}*x^{**4}/(12*a*b^{**6} + 12* \\
& b^{**7}*x) + 12*b^{**5}*c^{**2}*x*log(a/b + x)/(12*a*b^{**6} + 12*b^{**7}*x) + \\
& 12*b^{**5}*c*d*x^{**3}/(12*a*b^{**6} + 12*b^{**7}*x) + 3*b^{**5}*d^{**2}*x^{**5}/(12*a* \\
& b^{**6} + 12*b^{**7}*x), \text{Eq}(n, -2)), (-a^{**5}*d^{**2}*log(a/b + x)/b^{**6} + a^{**4}*d^{**2}*x/b^{**5} - 2*a^{**3}*c*d*log(a/b + x)/b^{**4} - a^{**3}*d^{**2}*x^{**2}/( \\
& 2*b^{**4}) + 2*a^{**2}*c*d*x/b^{**3} + a^{**2}*d^{**2}*x^{**3}/(3*b^{**3}) - a*c^{**2}*log \\
& (a/b + x)/b^{**2} - a*c*d*x^{**2}/b^{**2} - a*d^{**2}*x^{**4}/(4*b^{**2}) + c^{**2}*x \\
& /b + 2*c*d*x^{**3}/(3*b) + d^{**2}*x^{**5}/(5*b), \text{Eq}(n, -1)), (-120*a^{**6}*d^{**2}*(a + b*x)^**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 735* \\
& b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6}) + 120*a^{**5}*b \\
& *d^{**2}*n*x*(a + b*x)^**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} \\
& + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6}) - 12*a \\
& **4*b^{**2}*c*d*n^{**2}*(a + b*x)^**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b* \\
& **6*n^{**4} + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6} \\
& ) - 132*a^{**4}*b^{**2}*c*d*n*(a + b*x)^**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + \\
& 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 72 \\
& 0*b^{**6}) - 360*a^{**4}*b^{**2}*c*d*(a + b*x)^**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} \\
& + 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n \\
& + 720*b^{**6}) - 60*a^{**4}*b^{**2}*d^{**2}*n^{**2}*x^{**2}*(a + b*x)^**n/(b^{**6}*n^{**6} \\
& + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} \\
& + 1764*b^{**6}*n + 720*b^{**6}) - 60*a^{**4}*b^{**2}*d^{**2}*n*x^{**2}*(a + b*x)^**n \\
& /(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 1624 \\
& *b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6}) + 12*a^{**3}*b^{**3}*c*d*n^{**3}*x*(a \\
& + b*x)^**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 735*b^{**6}*n \\
& **3 + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6}) + 132*a^{**3}*b^{**3}*c* \\
& d*n^{**2}*x*(a + b*x)^**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + \\
& 735*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6}) + 360*a \\
& **3*b^{**3}*c*d*n*x*(a + b*x)^**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b* \\
& **6*n^{**4} + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6}) \\
& + 20*a^{**3}*b^{**3}*d^{**2}*n^{**3}*x^{**3}*(a + b*x)^**n/(b^{**6}*n^{**6} + 21*b^{**6}* \\
& n^{**5} + 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b^{**6} \\
& *n + 720*b^{**6}) + 60*a^{**3}*b^{**3}*d^{**2}*n^{**2}*x^{**3}*(a + b*x)^**n/(b^{**6}*n \\
& **6 + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n* \\
& **2 + 1764*b^{**6}*n + 720*b^{**6}) + 40*a^{**3}*b^{**3}*d^{**2}*n*x^{**3}*(a + b*x) \\
& **n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 1 \\
& 624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6}) - a^{**2}*b^{**4}*c^{**2}*n^{**4}*(a \\
& + b*x)^**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 735*b^{**6}*n* \\
& **3 + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6}) - 18*a^{**2}*b^{**4}*c^{**2} \\
& *n^{**3}*(a + b*x)^**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 73 \\
& 5*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6}) - 119*a^{**2} \\
& *b^{**4}*c^{**2}*n^{**2}*(a + b*x)^**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b* \\
& **6*n^{**4} + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6}) \\
& - 342*a^{**2}*b^{**4}*c^{**2}*n*(a + b*x)^**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 1 \\
& 75*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720 \\
& *b^{**6}) - 360*a^{**2}*b^{**4}*c^{**2}*(a + b*x)^**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} \\
& + 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n \\
& + 720*b^{**6}) - 6*a^{**2}*b^{**4}*c*d*n^{**4}*x^{**2}*(a + b*x)^**n/(b^{**6}*n^{**6} + \\
& 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 1624*b^{**6}*n^{**2} + \\
& 1764*b^{**6}*n + 720*b^{**6}) - 72*a^{**2}*b^{**4}*c*d*n^{**3}*x^{**2}*(a + b*x)^**n \\
& /(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 735*b^{**6}*n^{**3} + 1624 \\
& *b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6}) - 246*a^{**2}*b^{**4}*c*d*n^{**2}*x^{** \\
& 2*(a + b*x)^**n/(b^{**6}*n^{**6} + 21*b^{**6}*n^{**5} + 175*b^{**6}*n^{**4} + 735*b* \\
& **6*n^{**3} + 1624*b^{**6}*n^{**2} + 1764*b^{**6}*n + 720*b^{**6}) - 180*a^{**2}*b^{**
\end{aligned}$$



```

*2 + 1764*b**6*n + 720*b**6) + 85*b**6*d**2*n**3*x**6*(a + b*x)**
n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 162
4*b**6*n**2 + 1764*b**6*n + 720*b**6) + 225*b**6*d**2*n**2*x**6*(
a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*
n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 274*b**6*d**2*n
*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 73
5*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 120*b**6
*d**2*x**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4
+ 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6), True
))

```

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**GIAC/XCAS [A]** time = 0.294322, size = 1, normalized size = 0.01

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)^2\*(b\*x + a)^n\*x,x, algorithm="giac")

[Out] Done

$$3.357 \quad \int (a + bx)^n (c + dx^2)^2 dx$$

**Optimal.** Leaf size=140

$$\frac{(a^2d + b^2c)^2 (a + bx)^{n+1}}{b^5(n+1)} - \frac{4ad(a^2d + b^2c)(a + bx)^{n+2}}{b^5(n+2)} + \frac{2d(3a^2d + b^2c)(a + bx)^{n+3}}{b^5(n+3)} - \frac{4ad^2(a + bx)^{n+4}}{b^5(n+4)} + \frac{d^2(a + bx)^{n+5}}{b^5(n+5)}$$

[Out]  $((b^2c + a^2d)^2(a + bx)^{(1+n)})/(b^5(1+n)) - (4ad(b^2c + a^2d)(a + bx)^{(2+n)})/(b^5(2+n)) + (2d(b^2c + 3a^2d)(a + bx)^{(3+n)})/(b^5(3+n)) - (4ad^2(a + bx)^{(4+n)})/(b^5(4+n)) + (d^2(a + bx)^{(5+n)})/(b^5(5+n))$

**Rubi [A]** time = 0.150135, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{(a^2d + b^2c)^2 (a + bx)^{n+1}}{b^5(n+1)} - \frac{4ad(a^2d + b^2c)(a + bx)^{n+2}}{b^5(n+2)} + \frac{2d(3a^2d + b^2c)(a + bx)^{n+3}}{b^5(n+3)} - \frac{4ad^2(a + bx)^{n+4}}{b^5(n+4)} + \frac{d^2(a + bx)^{n+5}}{b^5(n+5)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x)^n\*(c + d\*x^2)^2, x]

[Out]  $((b^2c + a^2d)^2(a + bx)^{(1+n)})/(b^5(1+n)) - (4ad(b^2c + a^2d)(a + bx)^{(2+n)})/(b^5(2+n)) + (2d(b^2c + 3a^2d)(a + bx)^{(3+n)})/(b^5(3+n)) - (4ad^2(a + bx)^{(4+n)})/(b^5(4+n)) + (d^2(a + bx)^{(5+n)})/(b^5(5+n))$

**Rubi in Sympy [A]** time = 35.8763, size = 128, normalized size = 0.91

$$-\frac{4ad^2(a + bx)^{n+4}}{b^5(n+4)} - \frac{4ad(a + bx)^{n+2}(a^2d + b^2c)}{b^5(n+2)} + \frac{d^2(a + bx)^{n+5}}{b^5(n+5)} + \frac{2d(a + bx)^{n+3}(3a^2d + b^2c)}{b^5(n+3)} + \frac{(a + bx)^{n+1}(a^2d + b^2c)^2}{b^5(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x+a)\*\*n\*(d\*x\*\*2+c)\*\*2, x)

[Out]  $-4ad^2(a + bx)^{(n+4)}/(b^5(n+4)) - 4ad(a + bx)^{(n+2)}(a^2d + b^2c)/(b^5(n+2)) + d^2(a + bx)^{(n+5)}/(b^5(n+5)) + 2d(a + bx)^{(n+3)}(3a^2d + b^2c)/(b^5(n+3)) + (a + bx)^{(n+1)}(a^2d + b^2c)^2/(b^5(n+1))$

**Mathematica [A]** time = 0.160051, size = 184, normalized size = 1.31

$$\frac{(a + bx)^{n+1} (24a^4d^2 - 24a^3bd^2(n+1)x + 4a^2b^2d(c(n^2 + 9n + 20) + 3d(n^2 + 3n + 2)x^2) - 4ab^3d(n+1)x(c(n^2 + 9n + 20) + 3d(n^2 + 3n + 2)x^2) + b^5(n+1)(n+2)(n+3)(n+4))}{b^5(n+1)(n+2)(n+3)(n+4)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x)^n\*(c + d\*x^2)^2,x]

[Out] ((a + b\*x)^(1 + n)\*(24\*a^4\*d^2 - 24\*a^3\*b\*d^2\*(1 + n)\*x + 4\*a^2\*b^2\*d\*(c\*(20 + 9\*n + n^2) + 3\*d\*(2 + 3\*n + n^2)\*x^2) - 4\*a\*b^3\*d\*(1 + n)\*x\*(c\*(20 + 9\*n + n^2) + d\*(6 + 5\*n + n^2)\*x^2) + b^4\*(8 + 6\*n + n^2)\*(c^2\*(15 + 8\*n + n^2) + 2\*c\*d\*(5 + 6\*n + n^2)\*x^2 + d^2\*(3 + 4\*n + n^2)\*x^4)))/(b^5\*(1 + n)\*(2 + n)\*(3 + n)\*(4 + n)\*(5 + n))

**Maple [B]** time = 0.013, size = 420, normalized size = 3.

$$(bx + a)^{1+n} (b^4 d^2 n^4 x^4 + 10 b^4 d^2 n^3 x^4 - 4 ab^3 d^2 n^3 x^3 + 2 b^4 c d n^4 x^2 + 35 b^4 d^2 n^2 x^4 - 24 ab^3 d^2 n^2 x^3 + 24 b^4 c d n^3 x^2 + 50 b^4 d^2 n x^4)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)^n\*(d\*x^2+c)^2,x)

[Out] (b\*x+a)^(1+n)\*(b^4\*d^2\*n^4\*x^4+10\*b^4\*d^2\*n^3\*x^4-4\*a\*b^3\*d^2\*n^3\*x^3+2\*b^4\*c\*d\*n^4\*x^2+35\*b^4\*d^2\*n^2\*x^4-24\*a\*b^3\*d^2\*n^2\*x^3+24\*b^4\*c\*d\*n^3\*x^2+50\*b^4\*d^2\*n\*x^4+12\*a^2\*b^2\*d^2\*n^2\*x^2-4\*a\*b^3\*c\*d\*n^3\*x-44\*a\*b^3\*d^2\*n\*x^3+b^4\*c^2\*n^4+98\*b^4\*c\*d\*n^2\*x^2+24\*b^4\*d^2\*x^4+36\*a^2\*b^2\*d^2\*n\*x^2-40\*a\*b^3\*c\*d\*n^2\*x-24\*a\*b^3\*d^2\*x^3+14\*b^4\*c^2\*n^3+156\*b^4\*c\*d\*n\*x^2-24\*a^3\*b\*d^2\*n\*x+4\*a^2\*b^2\*c\*d\*n^2+24\*a^2\*b^2\*d^2\*x^2-116\*a\*b^3\*c\*d\*n\*x+71\*b^4\*c^2\*n^2+80\*b^4\*c\*d\*x^2-24\*a^3\*b\*d^2\*x+36\*a^2\*b^2\*c\*d\*n-80\*a\*b^3\*c\*d\*x+154\*b^4\*c^2\*n+24\*a^4\*d^2+80\*a^2\*b^2\*c\*d+120\*b^4\*c^2)/b^5/(n^5+15\*n^4+85\*n^3+225\*n^2+274\*n+120)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)^2\*(b\*x + a)^n,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.282003, size = 701, normalized size = 5.01

$$(ab^4c^2n^4 + 14ab^4c^2n^3 + 120ab^4c^2 + 80a^3b^2cd + 24a^5d^2 + (b^5d^2n^4 + 10b^5d^2n^3 + 35b^5d^2n^2 + 50b^5d^2n + 24b^5d^2)x^5 + (ab^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)^2\*(b\*x + a)^n,x, algorithm="fricas")

[Out] (a\*b^4\*c^2\*n^4 + 14\*a\*b^4\*c^2\*n^3 + 120\*a\*b^4\*c^2 + 80\*a^3\*b^2\*c\*d + 24\*a^5\*d^2 + (b^5\*d^2\*n^4 + 10\*b^5\*d^2\*n^3 + 35\*b^5\*d^2\*n^2 + 50\*b^5\*d^2\*n + 24\*b^5\*d^2)\*x^5 + (a\*b^4\*d^2\*n^4 + 6\*a\*b^4\*d^2\*n^3 + 11\*a\*b^4\*d^2\*n^2 + 6\*a\*b^4\*d^2\*n)\*x^4 + 2\*(b^5\*c\*d\*n^4 + 40\*b^5\*c\*d + 2\*(6\*b^5\*c\*d - a^2\*b^3\*d^2)\*n^3 + (49\*b^5\*c\*d - 6\*a^2\*b^3\*d^2)\*n^2 + 2\*(39\*b^5\*c\*d - 2\*a^2\*b^3\*d^2)\*n)\*x^3 + (71\*a\*b^4\*c^2 + 4\*a^3\*b^2\*c\*d)\*n^2 + 2\*(a\*b^4\*c\*d\*n^4 + 10\*a\*b^4\*c\*d\*n^3 + (29\*a\*b^4\*c\*d + 6\*a^3\*b^2\*d^2)\*n^2 + 2\*(10\*a\*b^4\*c\*d + 3\*a^3\*b^2\*d^2)\*n)\*x^2 + 2\*(77\*a\*b^4\*c^2 + 18\*a^3\*b^2\*c\*d)\*n + (b^5\*c^2\*n^4 +



$$120*b^5*c^2 + 2*(7*b^5*c^2 - 2*a^2*b^3*c*d)*n^3 + (71*b^5*c^2 - 36*a^2*b^3*c*d)*n^2 + 2*(77*b^5*c^2 - 40*a^2*b^3*c*d - 12*a^4*b*d^2)*n*x*(b*x + a)^n/(b^5*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^5)$$

**Sympy [A]** time = 15.7622, size = 5044, normalized size = 36.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*\*n\*(d\*x\*\*2+c)\*\*2,x)

[Out] Piecewise((a\*\*n\*(c\*\*2\*x + 2\*c\*d\*x\*\*3/3 + d\*\*2\*x\*\*5/5), Eq(b, 0)), (12\*a\*\*4\*d\*\*2\*log(a/b + x)/(12\*a\*\*4\*b\*\*5 + 48\*a\*\*3\*b\*\*6\*x + 72\*a\*\*2\*b\*\*7\*x\*\*2 + 48\*a\*b\*\*8\*x\*\*3 + 12\*b\*\*9\*x\*\*4) + 25\*a\*\*4\*d\*\*2/(12\*a\*\*4\*b\*\*5 + 48\*a\*\*3\*b\*\*6\*x + 72\*a\*\*2\*b\*\*7\*x\*\*2 + 48\*a\*b\*\*8\*x\*\*3 + 12\*b\*\*9\*x\*\*4) + 48\*a\*\*3\*b\*d\*\*2\*x\*log(a/b + x)/(12\*a\*\*4\*b\*\*5 + 48\*a\*\*3\*b\*\*6\*x + 72\*a\*\*2\*b\*\*7\*x\*\*2 + 48\*a\*b\*\*8\*x\*\*3 + 12\*b\*\*9\*x\*\*4) + 88\*a\*\*3\*b\*d\*\*2\*x/(12\*a\*\*4\*b\*\*5 + 48\*a\*\*3\*b\*\*6\*x + 72\*a\*\*2\*b\*\*7\*x\*\*2 + 48\*a\*b\*\*8\*x\*\*3 + 12\*b\*\*9\*x\*\*4) - 2\*a\*\*2\*b\*\*2\*c\*d/(12\*a\*\*4\*b\*\*5 + 48\*a\*\*3\*b\*\*6\*x + 72\*a\*\*2\*b\*\*7\*x\*\*2 + 48\*a\*b\*\*8\*x\*\*3 + 12\*b\*\*9\*x\*\*4) + 72\*a\*\*2\*b\*\*2\*d\*\*2\*x\*\*2\*log(a/b + x)/(12\*a\*\*4\*b\*\*5 + 48\*a\*\*3\*b\*\*6\*x + 72\*a\*\*2\*b\*\*7\*x\*\*2 + 48\*a\*b\*\*8\*x\*\*3 + 12\*b\*\*9\*x\*\*4) + 108\*a\*\*2\*b\*\*2\*d\*\*2\*x\*\*2/(12\*a\*\*4\*b\*\*5 + 48\*a\*\*3\*b\*\*6\*x + 72\*a\*\*2\*b\*\*7\*x\*\*2 + 48\*a\*b\*\*8\*x\*\*3 + 12\*b\*\*9\*x\*\*4) - 8\*a\*b\*\*3\*c\*d\*x/(12\*a\*\*4\*b\*\*5 + 48\*a\*\*3\*b\*\*6\*x + 72\*a\*\*2\*b\*\*7\*x\*\*2 + 48\*a\*b\*\*8\*x\*\*3 + 12\*b\*\*9\*x\*\*4) + 48\*a\*b\*\*3\*d\*\*2\*x\*\*3\*log(a/b + x)/(12\*a\*\*4\*b\*\*5 + 48\*a\*\*3\*b\*\*6\*x + 72\*a\*\*2\*b\*\*7\*x\*\*2 + 48\*a\*b\*\*8\*x\*\*3 + 12\*b\*\*9\*x\*\*4) + 48\*a\*b\*\*3\*d\*\*2\*x\*\*3/(12\*a\*\*4\*b\*\*5 + 48\*a\*\*3\*b\*\*6\*x + 72\*a\*\*2\*b\*\*7\*x\*\*2 + 48\*a\*b\*\*8\*x\*\*3 + 12\*b\*\*9\*x\*\*4) - 3\*b\*\*4\*c\*\*2/(12\*a\*\*4\*b\*\*5 + 48\*a\*\*3\*b\*\*6\*x + 72\*a\*\*2\*b\*\*7\*x\*\*2 + 48\*a\*b\*\*8\*x\*\*3 + 12\*b\*\*9\*x\*\*4) - 12\*b\*\*4\*c\*d\*x\*\*2/(12\*a\*\*4\*b\*\*5 + 48\*a\*\*3\*b\*\*6\*x + 72\*a\*\*2\*b\*\*7\*x\*\*2 + 48\*a\*b\*\*8\*x\*\*3 + 12\*b\*\*9\*x\*\*4) + 12\*b\*\*4\*d\*\*2\*x\*\*4\*log(a/b + x)/(12\*a\*\*4\*b\*\*5 + 48\*a\*\*3\*b\*\*6\*x + 72\*a\*\*2\*b\*\*7\*x\*\*2 + 48\*a\*b\*\*8\*x\*\*3 + 12\*b\*\*9\*x\*\*4), Eq(n, -5)), (-12\*a\*\*5\*d\*\*2\*log(a/b + x)/(3\*a\*\*4\*b\*\*5 + 9\*a\*\*3\*b\*\*6\*x + 9\*a\*\*2\*b\*\*7\*x\*\*2 + 3\*a\*b\*\*8\*x\*\*3) - 4\*a\*\*5\*d\*\*2/(3\*a\*\*4\*b\*\*5 + 9\*a\*\*3\*b\*\*6\*x + 9\*a\*\*2\*b\*\*7\*x\*\*2 + 3\*a\*b\*\*8\*x\*\*3) - 36\*a\*\*4\*b\*d\*\*2\*x\*log(a/b + x)/(3\*a\*\*4\*b\*\*5 + 9\*a\*\*3\*b\*\*6\*x + 9\*a\*\*2\*b\*\*7\*x\*\*2 + 3\*a\*b\*\*8\*x\*\*3) - 36\*a\*\*3\*b\*\*2\*d\*\*2\*x\*\*2\*log(a/b + x)/(3\*a\*\*4\*b\*\*5 + 9\*a\*\*3\*b\*\*6\*x + 9\*a\*\*2\*b\*\*7\*x\*\*2 + 3\*a\*b\*\*8\*x\*\*3) + 18\*a\*\*3\*b\*\*2\*d\*\*2\*x\*\*2/(3\*a\*\*4\*b\*\*5 + 9\*a\*\*3\*b\*\*6\*x + 9\*a\*\*2\*b\*\*7\*x\*\*2 + 3\*a\*b\*\*8\*x\*\*3) - 12\*a\*\*2\*b\*\*3\*d\*\*2\*x\*\*3\*log(a/b + x)/(3\*a\*\*4\*b\*\*5 + 9\*a\*\*3\*b\*\*6\*x + 9\*a\*\*2\*b\*\*7\*x\*\*2 + 3\*a\*b\*\*8\*x\*\*3) + 18\*a\*\*2\*b\*\*3\*d\*\*2\*x\*\*3/(3\*a\*\*4\*b\*\*5 + 9\*a\*\*3\*b\*\*6\*x + 9\*a\*\*2\*b\*\*7\*x\*\*2 + 3\*a\*b\*\*8\*x\*\*3) - a\*b\*\*4\*c\*\*2/(3\*a\*\*4\*b\*\*5 + 9\*a\*\*3\*b\*\*6\*x + 9\*a\*\*2\*b\*\*7\*x\*\*2 + 3\*a\*b\*\*8\*x\*\*3) + 3\*a\*b\*\*4\*d\*\*2\*x\*\*4/(3\*a\*\*4\*b\*\*5 + 9\*a\*\*3\*b\*\*6\*x + 9\*a\*\*2\*b\*\*7\*x\*\*2 + 3\*a\*b\*\*8\*x\*\*3) + 2\*b\*\*5\*c\*d\*x\*\*3/(3\*a\*\*4\*b\*\*5 + 9\*a\*\*3\*b\*\*6\*x + 9\*a\*\*2\*b\*\*7\*x\*\*2 + 3\*a\*b\*\*8\*x\*\*3), Eq(n, -4)), (12\*a\*\*4\*d\*\*2\*log(a/b + x)/(2\*a\*\*2\*b\*\*5 + 4\*a\*b\*\*6\*x + 2\*b\*\*7\*x\*\*2) + 6\*a\*\*4\*d\*\*2/(2\*a\*\*2\*b\*\*5 + 4\*a\*b\*\*6\*x + 2\*b\*\*7\*x\*\*2) + 24\*a\*\*3\*b\*d\*\*2\*x\*log(a/b + x)/(2\*a\*\*2\*b\*\*5 + 4\*a\*b\*\*6\*x + 2\*b\*\*7\*x\*\*2) + 4\*a\*\*2\*b\*\*2\*c\*d\*log(a/b + x)/(2\*a\*\*2\*b\*\*5 + 4\*a\*b\*\*6\*x + 2\*b\*\*7\*x\*\*2) + 2\*a\*\*2\*b\*\*2\*c\*d/(2\*a\*\*2\*b\*\*5 + 4\*a\*b\*\*6\*x + 2\*b\*\*7\*x\*\*2) + 12\*a\*\*2\*b\*\*2\*d\*\*2\*x\*\*2\*log(a/b + x)/(2\*a\*\*2\*b\*\*5 + 4\*a\*b\*\*6\*x + 2\*b\*\*7\*x\*\*2) - 12\*a\*\*2\*b\*\*2\*d\*\*2\*x\*\*2/(2\*a\*\*2\*b\*\*5 + 4\*a\*b\*\*6\*x + 2\*b\*\*7\*x\*\*2) + 8\*a\*b\*\*3\*c\*d\*x\*log(a/b + x)/(2\*a\*\*2\*b\*\*5 + 4\*a\*b\*\*6\*x + 2\*b\*\*7\*x\*\*2) - 4\*a\*b\*\*3\*d\*\*2\*x\*\*3/(2\*a\*\*2\*b\*\*5 + 4\*a\*b\*\*6\*x + 2\*b\*\*7\*x\*\*2) - b\*\*4\*c\*\*2/(2\*a\*\*2\*b\*\*5 + 4\*a\*b\*\*6\*x + 2\*b\*\*7\*x\*\*2) + 4\*b\*\*4\*c\*d\*x\*\*2\*log(a/b + x)/(2\*a\*\*2\*b\*\*5 + 4\*a\*b\*\*6\*x + 2\*b\*\*7\*x\*\*2) - 4\*b\*\*4\*c\*d\*x\*\*2/(2\*a\*\*2\*b\*\*5 + 4\*a\*b\*\*6\*x + 2\*b\*\*7\*x\*\*2) + b\*\*4\*d\*\*2\*x\*\*4/(2\*a\*\*2\*b\*\*5 + 4\*a\*b\*\*6\*x + 2\*b\*\*7\*x\*\*2), Eq(n, -3)), (-12\*a\*\*4\*d\*\*2\*log(a/b + x)/(3\*a\*b\*\*5 + 3\*b\*\*6\*x) - 12\*a\*\*4\*d\*\*2/(3\*a\*b\*\*5 + 3\*b\*\*6\*x) - 12\*a\*\*3\*b\*d\*\*2\*x\*log(a/b + x)/(3\*a\*b\*\*5 + 3\*b\*\*6\*x) - 12\*a\*\*2\*b\*\*2\*c\*d\*log(a/b + x)/(3\*a\*b\*\*5 + 3\*b\*\*6\*x) - 12\*a\*\*2\*b\*\*2\*c\*d/(3\*a\*b\*\*5 + 3\*b\*\*6\*x) + 6\*a\*\*2\*b\*\*

$$\begin{aligned}
& 2*d^{**2}*x^{**2}/(3*a*b^{**5} + 3*b^{**6}*x) - 12*a*b^{**3}*c*d*x*\log(a/b + x)/ \\
& (3*a*b^{**5} + 3*b^{**6}*x) - 2*a*b^{**3}*d^{**2}*x^{**3}/(3*a*b^{**5} + 3*b^{**6}*x) \\
& - 3*b^{**4}*c^{**2}/(3*a*b^{**5} + 3*b^{**6}*x) + 6*b^{**4}*c*d*x^{**2}/(3*a*b^{**5} + \\
& 3*b^{**6}*x) + b^{**4}*d^{**2}*x^{**4}/(3*a*b^{**5} + 3*b^{**6}*x), \text{Eq}(n, -2)), (a \\
& **4*d^{**2}*\log(a/b + x)/b^{**5} - a^{**3}*d^{**2}*x/b^{**4} + 2*a^{**2}*c*d*\log(a/ \\
& b + x)/b^{**3} + a^{**2}*d^{**2}*x^{**2}/(2*b^{**3}) - 2*a*c*d*x/b^{**2} - a*d^{**2}*x \\
& **3/(3*b^{**2}) + c^{**2}*\log(a/b + x)/b + c*d*x^{**2}/b + d^{**2}*x^{**4}/(4*b) \\
& , \text{Eq}(n, -1)), (24*a^{**5}*d^{**2}*(a + b*x)^{**n}/(b^{**5}*n^{**5} + 15*b^{**5}*n^{**4} \\
& + 85*b^{**5}*n^{**3} + 225*b^{**5}*n^{**2} + 274*b^{**5}*n + 120*b^{**5}) - 24*a^{**4} \\
& *b*d^{**2}*n*x*(a + b*x)^{**n}/(b^{**5}*n^{**5} + 15*b^{**5}*n^{**4} + 85*b^{**5}*n^{**3} \\
& + 225*b^{**5}*n^{**2} + 274*b^{**5}*n + 120*b^{**5}) + 4*a^{**3}*b^{**2}*c*d*n^{**2} \\
& *(a + b*x)^{**n}/(b^{**5}*n^{**5} + 15*b^{**5}*n^{**4} + 85*b^{**5}*n^{**3} + 225*b^{**5} \\
& *n^{**2} + 274*b^{**5}*n + 120*b^{**5}) + 36*a^{**3}*b^{**2}*c*d*n*(a + b*x)^{**n} \\
& /(b^{**5}*n^{**5} + 15*b^{**5}*n^{**4} + 85*b^{**5}*n^{**3} + 225*b^{**5}*n^{**2} + 274*b \\
& **5*n + 120*b^{**5}) + 80*a^{**3}*b^{**2}*c*d*(a + b*x)^{**n}/(b^{**5}*n^{**5} + 15 \\
& *b^{**5}*n^{**4} + 85*b^{**5}*n^{**3} + 225*b^{**5}*n^{**2} + 274*b^{**5}*n + 120*b^{**5} \\
& ) + 12*a^{**3}*b^{**2}*d^{**2}*n^{**2}*x^{**2}*(a + b*x)^{**n}/(b^{**5}*n^{**5} + 15*b^{**5} \\
& *n^{**4} + 85*b^{**5}*n^{**3} + 225*b^{**5}*n^{**2} + 274*b^{**5}*n + 120*b^{**5}) + 1 \\
& 2*a^{**3}*b^{**2}*d^{**2}*n*x^{**2}*(a + b*x)^{**n}/(b^{**5}*n^{**5} + 15*b^{**5}*n^{**4} + \\
& 85*b^{**5}*n^{**3} + 225*b^{**5}*n^{**2} + 274*b^{**5}*n + 120*b^{**5}) - 4*a^{**2}*b^{**3} \\
& *c*d*n^{**3}*x*(a + b*x)^{**n}/(b^{**5}*n^{**5} + 15*b^{**5}*n^{**4} + 85*b^{**5}*n^{**3} \\
& + 225*b^{**5}*n^{**2} + 274*b^{**5}*n + 120*b^{**5}) - 36*a^{**2}*b^{**3}*c*d*n^{**2} \\
& *x*(a + b*x)^{**n}/(b^{**5}*n^{**5} + 15*b^{**5}*n^{**4} + 85*b^{**5}*n^{**3} + 225* \\
& b^{**5}*n^{**2} + 274*b^{**5}*n + 120*b^{**5}) - 80*a^{**2}*b^{**3}*c*d*n*x*(a + b \\
& x)^{**n}/(b^{**5}*n^{**5} + 15*b^{**5}*n^{**4} + 85*b^{**5}*n^{**3} + 225*b^{**5}*n^{**2} + \\
& 274*b^{**5}*n + 120*b^{**5}) - 4*a^{**2}*b^{**3}*d^{**2}*n^{**3}*x^{**3}*(a + b*x)^{**n}/ \\
& (b^{**5}*n^{**5} + 15*b^{**5}*n^{**4} + 85*b^{**5}*n^{**3} + 225*b^{**5}*n^{**2} + 274*b^{**5} \\
& *n + 120*b^{**5}) - 12*a^{**2}*b^{**3}*d^{**2}*n^{**2}*x^{**3}*(a + b*x)^{**n}/(b^{**5} \\
& *n^{**5} + 15*b^{**5}*n^{**4} + 85*b^{**5}*n^{**3} + 225*b^{**5}*n^{**2} + 274*b^{**5}*n \\
& + 120*b^{**5}) - 8*a^{**2}*b^{**3}*d^{**2}*n*x^{**3}*(a + b*x)^{**n}/(b^{**5}*n^{**5} + 1 \\
& 5*b^{**5}*n^{**4} + 85*b^{**5}*n^{**3} + 225*b^{**5}*n^{**2} + 274*b^{**5}*n + 120*b^{**5} \\
& ) + a*b^{**4}*c^{**2}*n^{**4}*(a + b*x)^{**n}/(b^{**5}*n^{**5} + 15*b^{**5}*n^{**4} + 85 \\
& *b^{**5}*n^{**3} + 225*b^{**5}*n^{**2} + 274*b^{**5}*n + 120*b^{**5}) + 14*a*b^{**4}*c \\
& **2*n^{**3}*(a + b*x)^{**n}/(b^{**5}*n^{**5} + 15*b^{**5}*n^{**4} + 85*b^{**5}*n^{**3} + \\
& 225*b^{**5}*n^{**2} + 274*b^{**5}*n + 120*b^{**5}) + 71*a*b^{**4}*c^{**2}*n^{**2}*(a + \\
& b*x)^{**n}/(b^{**5}*n^{**5} + 15*b^{**5}*n^{**4} + 85*b^{**5}*n^{**3} + 225*b^{**5}*n^{**2} \\
& + 274*b^{**5}*n + 120*b^{**5}) + 154*a*b^{**4}*c^{**2}*n*(a + b*x)^{**n}/(b^{**5} \\
& *n^{**5} + 15*b^{**5}*n^{**4} + 85*b^{**5}*n^{**3} + 225*b^{**5}*n^{**2} + 274*b^{**5}*n + \\
& 120*b^{**5}) + 120*a*b^{**4}*c^{**2}*(a + b*x)^{**n}/(b^{**5}*n^{**5} + 15*b^{**5}*n^{**4} \\
& + 85*b^{**5}*n^{**3} + 225*b^{**5}*n^{**2} + 274*b^{**5}*n + 120*b^{**5}) + 2*a \\
& b^{**4}*c*d*n^{**4}*x^{**2}*(a + b*x)^{**n}/(b^{**5}*n^{**5} + 15*b^{**5}*n^{**4} + 85*b^{**5} \\
& *n^{**3} + 225*b^{**5}*n^{**2} + 274*b^{**5}*n + 120*b^{**5}) + 20*a*b^{**4}*c*d \\
& n^{**3}*x^{**2}*(a + b*x)^{**n}/(b^{**5}*n^{**5} + 15*b^{**5}*n^{**4} + 85*b^{**5}*n^{**3} + \\
& 225*b^{**5}*n^{**2} + 274*b^{**5}*n + 120*b^{**5}) + 58*a*b^{**4}*c*d*n^{**2}*x^{**2} \\
& *(a + b*x)^{**n}/(b^{**5}*n^{**5} + 15*b^{**5}*n^{**4} + 85*b^{**5}*n^{**3} + 225*b^{**5} \\
& *n^{**2} + 274*b^{**5}*n + 120*b^{**5}) + 40*a*b^{**4}*c*d*n*x^{**2}*(a + b*x)^{**n} \\
& n/(b^{**5}*n^{**5} + 15*b^{**5}*n^{**4} + 85*b^{**5}*n^{**3} + 225*b^{**5}*n^{**2} + 274* \\
& b^{**5}*n + 120*b^{**5}) + a*b^{**4}*d^{**2}*n^{**4}*x^{**4}*(a + b*x)^{**n}/(b^{**5}*n^{**5} \\
& + 15*b^{**5}*n^{**4} + 85*b^{**5}*n^{**3} + 225*b^{**5}*n^{**2} + 274*b^{**5}*n + 12 \\
& 0*b^{**5}) + 6*a*b^{**4}*d^{**2}*n^{**3}*x^{**4}*(a + b*x)^{**n}/(b^{**5}*n^{**5} + 15*b^{**5} \\
& *n^{**4} + 85*b^{**5}*n^{**3} + 225*b^{**5}*n^{**2} + 274*b^{**5}*n + 120*b^{**5}) + \\
& 11*a*b^{**4}*d^{**2}*n^{**2}*x^{**4}*(a + b*x)^{**n}/(b^{**5}*n^{**5} + 15*b^{**5}*n^{**4} \\
& + 85*b^{**5}*n^{**3} + 225*b^{**5}*n^{**2} + 274*b^{**5}*n + 120*b^{**5}) + 6*a*b^{**4} \\
& *d^{**2}*n*x^{**4}*(a + b*x)^{**n}/(b^{**5}*n^{**5} + 15*b^{**5}*n^{**4} + 85*b^{**5}*n^{**3} \\
& + 225*b^{**5}*n^{**2} + 274*b^{**5}*n + 120*b^{**5}) + b^{**5}*c^{**2}*n^{**4}*x*(a \\
& + b*x)^{**n}/(b^{**5}*n^{**5} + 15*b^{**5}*n^{**4} + 85*b^{**5}*n^{**3} + 225*b^{**5}*n^{**2} \\
& + 274*b^{**5}*n + 120*b^{**5}) + 14*b^{**5}*c^{**2}*n^{**3}*x*(a + b*x)^{**n}/(b \\
& **5*n^{**5} + 15*b^{**5}*n^{**4} + 85*b^{**5}*n^{**3} + 225*b^{**5}*n^{**2} + 274*b^{**5} \\
& *n + 120*b^{**5}) + 71*b^{**5}*c^{**2}*n^{**2}*x*(a + b*x)^{**n}/(b^{**5}*n^{**5} + 15 \\
& *b^{**5}*n^{**4} + 85*b^{**5}*n^{**3} + 225*b^{**5}*n^{**2} + 274*b^{**5}*n + 120*b^{**5} \\
& ) + 154*b^{**5}*c^{**2}*n*x*(a + b*x)^{**n}/(b^{**5}*n^{**5} + 15*b^{**5}*n^{**4} + 85 \\
& *b^{**5}*n^{**3} + 225*b^{**5}*n^{**2} + 274*b^{**5}*n + 120*b^{**5}) + 120*b^{**5}*c \\
& **2*x*(a + b*x)^{**n}/(b^{**5}*n^{**5} + 15*b^{**5}*n^{**4} + 85*b^{**5}*n^{**3} + 225* \\
& b^{**5}*n^{**2} + 274*b^{**5}*n + 120*b^{**5}) + 2*b^{**5}*c*d*n^{**4}*x^{**3}*(a + b \\
& x)^{**n}/(b^{**5}*n^{**5} + 15*b^{**5}*n^{**4} + 85*b^{**5}*n^{**3} + 225*b^{**5}*n^{**2} + \\
& 274*b^{**5}*n + 120*b^{**5}) + 24*b^{**5}*c*d*n^{**3}*x^{**3}*(a + b*x)^{**n}/(b^{**5} \\
& *n^{**5} + 15*b^{**5}*n^{**4} + 85*b^{**5}*n^{**3} + 225*b^{**5}*n^{**2} + 274*b^{**5}*n \\
& + 120*b^{**5}) + 98*b^{**5}*c*d*n^{**2}*x^{**3}*(a + b*x)^{**n}/(b^{**5}*n^{**5} + 15* \\
& b^{**5}*n^{**4} + 85*b^{**5}*n^{**3} + 225*b^{**5}*n^{**2} + 274*b^{**5}*n + 120*b^{**5}) \\
& + 156*b^{**5}*c*d*n*x^{**3}*(a + b*x)^{**n}/(b^{**5}*n^{**5} + 15*b^{**5}*n^{**4} + 8 \\
& 5*b^{**5}*n^{**3} + 225*b^{**5}*n^{**2} + 274*b^{**5}*n + 120*b^{**5}) + 80*b^{**5}*c \\
& d*x^{**3}*(a + b*x)^{**n}/(b^{**5}*n^{**5} + 15*b^{**5}*n^{**4} + 85*b^{**5}*n^{**3} + 22
\end{aligned}$$

```

5*b**5*n**2 + 274*b**5*n + 120*b**5) + b**5*d**2*n**4*x**5*(a + b
*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 +
274*b**5*n + 120*b**5) + 10*b**5*d**2*n**3*x**5*(a + b*x)**n/(b*
**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*
n + 120*b**5) + 35*b**5*d**2*n**2*x**5*(a + b*x)**n/(b**5*n**5 +
15*b**5*n**4 + 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b*
**5) + 50*b**5*d**2*n*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4
+ 85*b**5*n**3 + 225*b**5*n**2 + 274*b**5*n + 120*b**5) + 24*b**5
*d**2*x**5*(a + b*x)**n/(b**5*n**5 + 15*b**5*n**4 + 85*b**5*n**3
+ 225*b**5*n**2 + 274*b**5*n + 120*b**5), True))

```

**GIAC/XCAS [A]** time = 0.301347, size = 1260, normalized size = 9.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^2*(b*x + a)^n,x, algorithm="giac")
```

```
[Out] (b^5*d^2*n^4*x^5*e^(n*ln(b*x + a)) + a*b^4*d^2*n^4*x^4*e^(n*ln(b*
x + a)) + 10*b^5*d^2*n^3*x^5*e^(n*ln(b*x + a)) + 2*b^5*c*d*n^4*x^4
^3*e^(n*ln(b*x + a)) + 6*a*b^4*d^2*n^3*x^4*e^(n*ln(b*x + a)) + 35*
b^5*d^2*n^2*x^5*e^(n*ln(b*x + a)) + 2*a*b^4*c*d*n^4*x^2*e^(n*ln(b
*x + a)) + 24*b^5*c*d*n^3*x^3*e^(n*ln(b*x + a)) - 4*a^2*b^3*d^2*n
^3*x^3*e^(n*ln(b*x + a)) + 11*a*b^4*d^2*n^2*x^4*e^(n*ln(b*x + a))
+ 50*b^5*d^2*n*x^5*e^(n*ln(b*x + a)) + b^5*c^2*n^4*x*e^(n*ln(b*x
+ a)) + 20*a*b^4*c*d*n^3*x^2*e^(n*ln(b*x + a)) + 98*b^5*c*d*n^2*
x^3*e^(n*ln(b*x + a)) - 12*a^2*b^3*d^2*n^2*x^3*e^(n*ln(b*x + a))
+ 6*a*b^4*d^2*n*x^4*e^(n*ln(b*x + a)) + 24*b^5*d^2*x^5*e^(n*ln(b*
x + a)) + a*b^4*c^2*n^4*e^(n*ln(b*x + a)) + 14*b^5*c^2*n^3*x*e^(n
*ln(b*x + a)) - 4*a^2*b^3*c*d*n^3*x*e^(n*ln(b*x + a)) + 58*a*b^4*
c*d*n^2*x^2*e^(n*ln(b*x + a)) + 12*a^3*b^2*d^2*n^2*x^2*e^(n*ln(b*
x + a)) + 156*b^5*c*d*n*x^3*e^(n*ln(b*x + a)) - 8*a^2*b^3*d^2*n*x
^3*e^(n*ln(b*x + a)) + 14*a*b^4*c^2*n^3*e^(n*ln(b*x + a)) + 71*b^
5*c^2*n^2*x*e^(n*ln(b*x + a)) - 36*a^2*b^3*c*d*n^2*x*e^(n*ln(b*x
+ a)) + 40*a*b^4*c*d*n*x^2*e^(n*ln(b*x + a)) + 12*a^3*b^2*d^2*n*x
^2*e^(n*ln(b*x + a)) + 80*b^5*c*d*x^3*e^(n*ln(b*x + a)) + 71*a*b^
4*c^2*n^2*e^(n*ln(b*x + a)) + 4*a^3*b^2*c*d*n^2*e^(n*ln(b*x + a))
+ 154*b^5*c^2*n*x*e^(n*ln(b*x + a)) - 80*a^2*b^3*c*d*n*x*e^(n*ln
(b*x + a)) - 24*a^4*b*d^2*n*x*e^(n*ln(b*x + a)) + 154*a*b^4*c^2*n
*e^(n*ln(b*x + a)) + 36*a^3*b^2*c*d*n*e^(n*ln(b*x + a)) + 120*b^5
*c^2*x*e^(n*ln(b*x + a)) + 120*a*b^4*c^2*e^(n*ln(b*x + a)) + 80*a
^3*b^2*c*d*e^(n*ln(b*x + a)) + 24*a^5*d^2*e^(n*ln(b*x + a)))/(b^5
*n^5 + 15*b^5*n^4 + 85*b^5*n^3 + 225*b^5*n^2 + 274*b^5*n + 120*b^
5)

```

$$3.358 \quad \int \frac{(a+bx)^n (c+dx^2)^2}{x} dx$$

**Optimal.** Leaf size=148

$$\begin{aligned} & -\frac{ad(a^2d+2b^2c)(a+bx)^{n+1}}{b^4(n+1)} + \frac{d(3a^2d+2b^2c)(a+bx)^{n+2}}{b^4(n+2)} - \frac{3ad^2(a+bx)^{n+3}}{b^4(n+3)} \\ & + \frac{d^2(a+bx)^{n+4}}{b^4(n+4)} - \frac{c^2(a+bx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{bx}{a} + 1\right)}{a(n+1)} \end{aligned}$$

[Out]  $-\left(\frac{a^2 d^2 (2 b^2 c + a^2 d) (a + b x)^{1+n}}{b^4 (1+n)}\right) + \left(\frac{d (2 b^2 c + 3 a^2 d) (a + b x)^{2+n}}{b^4 (2+n)} - \frac{3 a^2 d^2 (a + b x)^{3+n}}{b^4 (3+n)} + \frac{d^2 (a + b x)^{4+n}}{b^4 (4+n)} - \frac{c^2 (a + b x)^{1+n} \operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, 1 + \frac{b x}{a}\right]}{a (1+n)}\right)$

**Rubi [A]** time = 0.179472, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\begin{aligned} & -\frac{ad(a^2d+2b^2c)(a+bx)^{n+1}}{b^4(n+1)} + \frac{d(3a^2d+2b^2c)(a+bx)^{n+2}}{b^4(n+2)} - \frac{3ad^2(a+bx)^{n+3}}{b^4(n+3)} \\ & + \frac{d^2(a+bx)^{n+4}}{b^4(n+4)} - \frac{c^2(a+bx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{bx}{a} + 1\right)}{a(n+1)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x)^n\*(c + d\*x^2)^2)/x, x]

[Out]  $-\left(\frac{a^2 d^2 (2 b^2 c + a^2 d) (a + b x)^{1+n}}{b^4 (1+n)}\right) + \left(\frac{d (2 b^2 c + 3 a^2 d) (a + b x)^{2+n}}{b^4 (2+n)} - \frac{3 a^2 d^2 (a + b x)^{3+n}}{b^4 (3+n)} + \frac{d^2 (a + b x)^{4+n}}{b^4 (4+n)} - \frac{c^2 (a + b x)^{1+n} \operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, 1 + \frac{b x}{a}\right]}{a (1+n)}\right)$

**Rubi in Sympy [A]** time = 32.3735, size = 129, normalized size = 0.87

$$\begin{aligned} & -\frac{3ad^2(a+bx)^{n+3}}{b^4(n+3)} - \frac{ad(a+bx)^{n+1}(a^2d+2b^2c)}{b^4(n+1)} + \frac{d^2(a+bx)^{n+4}}{b^4(n+4)} \\ & + \frac{d(a+bx)^{n+2}(3a^2d+2b^2c)}{b^4(n+2)} - \frac{c^2(a+bx)^{n+1} {}_2F_1\left(1, n+1; n+2; 1 + \frac{bx}{a}\right)}{a(n+1)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x+a)\*\*n\*(d\*x\*\*2+c)\*\*2/x, x)

[Out]  $-3 a^2 d^2 (a + b x)^{n+3} / (b^4 (n+3)) - a^2 d (a + b x)^{n+1} (a^2 d + 2 b^2 c) / (b^4 (n+1)) + d^2 (a + b x)^{n+4} / (b^4 (n+4)) + d (a + b x)^{n+2} (3 a^2 d + 2 b^2 c) / (b^4 (n+2)) - c^2 (a + b x)^{n+1} \operatorname{hyper}((1, n+1), (n+2, ), 1 + b x / a) / (a (n+1))$

**Mathematica [A]** time = 0.539422, size = 259, normalized size = 1.75

$$(a + bx)^n \left( \frac{2cd \left( a^2 \left( \left( \frac{bx}{a} + 1 \right)^{-n} - 1 \right) + abnx + b^2(n+1)x^2 \right)}{b^2(n+1)(n+2)} \right. \\ \left. + \frac{d^2 \left( \frac{bx}{a} + 1 \right)^{-n} \left( -6a^4 \left( \left( \frac{bx}{a} + 1 \right)^n - 1 \right) + 6a^3bnx \left( \frac{bx}{a} + 1 \right)^n - 3a^2b^2n(n+1)x^2 \left( \frac{bx}{a} + 1 \right)^n + b^4(n^3 + 6n^2 + 11n + 6) x^4 \left( \frac{bx}{a} + 1 \right)^n \right)}{b^4(n+1)(n+2)(n+3)(n+4)} \right. \\ \left. + \frac{c^2 \left( \frac{a}{bx} + 1 \right)^{-n} {}_2F_1 \left( -n, -n; 1 - n; -\frac{a}{bx} \right)}{n} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x)^n\*(c + d\*x^2)^2)/x,x]

[Out] (a + b\*x)^n\*((2\*c\*d\*(a\*b\*n\*x + b^2\*(1 + n)\*x^2 + a^2\*(-1 + (1 + (b\*x)/a)^(-n))))/(b^2\*(1 + n)\*(2 + n)) + (d^2\*(6\*a^3\*b\*n\*x\*(1 + (b\*x)/a)^n - 3\*a^2\*b^2\*n\*(1 + n)\*x^2\*(1 + (b\*x)/a)^n + a\*b^3\*n\*(2 + 3\*n + n^2)\*x^3\*(1 + (b\*x)/a)^n + b^4\*(6 + 11\*n + 6\*n^2 + n^3)\*x^4\*(1 + (b\*x)/a)^n - 6\*a^4\*(-1 + (1 + (b\*x)/a)^n))/(b^4\*(1 + n)\*(2 + n)\*(3 + n)\*(4 + n)\*(1 + (b\*x)/a)^n) + (c^2\*Hypergeometric2F1[-n, -n, 1 - n, -(a/(b\*x))])/(n\*(1 + a/(b\*x))^n)

**Maple [F]** time = 0.051, size = 0, normalized size = 0.

$$\int \frac{(bx + a)^n (dx^2 + c)^2}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)^n\*(d\*x^2+c)^2/x,x)

[Out] int((b\*x+a)^n\*(d\*x^2+c)^2/x,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2 + c)^2 (bx + a)^n}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)^2\*(b\*x + a)^n/x,x, algorithm="maxima")

[Out] integrate((d\*x^2 + c)^2\*(b\*x + a)^n/x, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{(d^2x^4 + 2cdx^2 + c^2)(bx + a)^n}{x}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)^2\*(b\*x + a)^n/x,x, algorithm="fricas")

[Out] integral((d^2\*x^4 + 2\*c\*d\*x^2 + c^2)\*(b\*x + a)^n/x, x)

**Sympy [A]** time = 15.5575, size = 1681, normalized size = 11.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*\*n\*(d\*x\*\*2+c)\*\*2/x,x)

[Out] 
$$-b^{n+1}c^{2n}(a/b+x)^n \operatorname{lerchphi}(1+b*x/a, 1, n+1) \frac{\Gamma(n+1)}{\Gamma(n+2)} - b^{n+1}c^{2n}(a/b+x)^n \operatorname{lerchphi}(1+b*x/a, 1, n+1) \frac{\Gamma(n+1)}{\Gamma(n+2)} + 2cd \operatorname{Piecewise}((a^n x^{2/2}, \operatorname{Eq}(b, 0)), (a \log(a/b+x)/(a^2 b^2 + b^3 x) + b x \log(a/b+x)/(a^2 b^2 + b^3 x) - b x/(a^2 b^2 + b^3 x), \operatorname{Eq}(n, -2)), (-a \log(a/b+x)/b^2 + x/b, \operatorname{Eq}(n, -1)), (-a^{2n}(a+b*x)^n/(b^{2n+2} + 3b^{2n+2} + 2b^{2n+2}) + a^2 b^n x^n (a+b*x)^n/(b^{2n+2} + 3b^{2n+2} + 2b^{2n+2}) + b^{2n+2} x^{2n} (a+b*x)^n/(b^{2n+2} + 3b^{2n+2} + 2b^{2n+2}), \operatorname{True})) + d^2 \operatorname{Piecewise}((a^n x^4/4, \operatorname{Eq}(b, 0)), (6a^3 \log(a/b+x)/(6a^3 b^4 + 18a^2 b^5 x + 18a b^6 x^2 + 6b^7 x^3) + 2a^3/(6a^3 b^4 + 18a^2 b^5 x + 18a b^6 x^2 + 6b^7 x^3) + 18a^2 b x \log(a/b+x)/(6a^3 b^4 + 18a^2 b^5 x + 18a b^6 x^2 + 6b^7 x^3) + 18a b^6 x^2 + 6b^7 x^3) + 18a b^6 x^2 + 6b^7 x^3) + 18a b^6 x^2 + 6b^7 x^3) - 9a^2 b^2 x^2/(6a^3 b^4 + 18a^2 b^5 x + 18a b^6 x^2 + 6b^7 x^3) + 6b^3 x^3 \log(a/b+x)/(6a^3 b^4 + 18a^2 b^5 x + 18a b^6 x^2 + 6b^7 x^3) - 9b^3 x^3/(6a^3 b^4 + 18a^2 b^5 x + 18a b^6 x^2 + 6b^7 x^3), \operatorname{Eq}(n, -4)), (-6a^3 \log(a/b+x)/(2a^2 b^4 + 4a b^5 x + 2b^6 x^2) - 3a^3/(2a^2 b^4 + 4a b^5 x + 2b^6 x^2) - 12a^2 b x \log(a/b+x)/(2a^2 b^4 + 4a b^5 x + 2b^6 x^2) - 6a b^2 x^2 \log(a/b+x)/(2a^2 b^4 + 4a b^5 x + 2b^6 x^2) + 6a b^2 x^2/(2a^2 b^4 + 4a b^5 x + 2b^6 x^2) + 2b^3 x^3/(2a^2 b^4 + 4a b^5 x + 2b^6 x^2), \operatorname{Eq}(n, -3)), (6a^3 \log(a/b+x)/(2a b^4 + 2b^5 x) + 6a^3/(2a b^4 + 2b^5 x) + 6a^2 b x \log(a/b+x)/(2a b^4 + 2b^5 x) - 3a^2 x^2/(2a b^4 + 2b^5 x) + b^3 x^3/(2a b^4 + 2b^5 x), \operatorname{Eq}(n, -2)), (-a^3 \log(a/b+x)/b^4 + a^2 x/b^3 - a x^2/(2b^2) + x^3/(3b), \operatorname{Eq}(n, -1)), (-6a^4 (a+b*x)^n/(b^4 n^4 + 10b^4 n^3 + 35b^4 n^2 + 50b^4 n + 24b^4) + 6a^3 b^n x^n (a+b*x)^n/(b^4 n^4 + 10b^4 n^3 + 35b^4 n^2 + 50b^4 n + 24b^4) - 3a^2 b^{2n} x^{2n} (a+b*x)^n/(b^4 n^4 + 10b^4 n^3 + 35b^4 n^2 + 50b^4 n + 24b^4) - 3a^2 b^{2n} x^{2n} (a+b*x)^n/(b^4 n^4 + 10b^4 n^3 + 35b^4 n^2 + 50b^4 n + 24b^4) + a b^3 n^3 x^{3n} (a+b*x)^n/(b^4 n^4 + 10b^4 n^3 + 35b^4 n^2 + 50b^4 n + 24b^4) + 3a b^3 n^2 x^{3n} (a+b*x)^n/(b^4 n^4 + 10b^4 n^3 + 35b^4 n^2 + 50b^4 n + 24b^4) + 2a b^3 n x^{3n} (a+b*x)^n/(b^4 n^4 + 10b^4 n^3 + 35b^4 n^2 + 50b^4 n + 24b^4) + b^4 n^3 x^{4n} (a+b*x)^n/(b^4 n^4 + 10b^4 n^3 + 35b^4 n^2 + 50b^4 n + 24b^4) + 6b^4 n^2 x^{4n} (a+b*x)^n/(b^4 n^4 + 10b^4 n^3 + 35b^4 n^2 + 50b^4 n + 24b^4) + 11b^4 n x^{4n} (a+b*x)^n/(b^4 n^4 + 10b^4 n^3 + 35b^4 n^2 + 50b^4 n + 24b^4) + 6b^4 x^{4n} (a+b*x)^n/(b^4 n^4 + 10b^4 n^3 + 35b^4 n^2 + 50b^4 n + 24b^4), \operatorname{True})) - b^{n+1}c^{2n} x^n (a/b+x)^n \operatorname{lerchphi}(1+b*x/a, 1, n+1) \frac{\Gamma(n+1)}{\Gamma(n+2)} - b^{n+1}c^{2n} x^n (a/b+x)^n \operatorname{lerchphi}(1+b*x/a, 1, n+1) \frac{\Gamma(n+1)}{\Gamma(n+2)}$$

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2 + c)^2 (bx + a)^n}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^2*(b*x + a)^n/x,x, algorithm="giac")
```

```
[Out] integrate((d*x^2 + c)^2*(b*x + a)^n/x, x)
```

$$3.359 \quad \int x^2(a + bx)^n (c + dx^2)^3 dx$$

**Optimal.** Leaf size=343

$$\begin{aligned} & -\frac{2ad^2(28a^2d + 9b^2c)(a + bx)^{n+6}}{b^9(n+6)} + \frac{d^2(28a^2d + 3b^2c)(a + bx)^{n+7}}{b^9(n+7)} + \frac{a^2(a^2d + b^2c)^3(a + bx)^{n+1}}{b^9(n+1)} \\ & -\frac{2a(a^2d + b^2c)^2(4a^2d + b^2c)(a + bx)^{n+2}}{b^9(n+2)} + \frac{(a^2d + b^2c)(28a^4d^2 + 17a^2b^2cd + b^4c^2)(a + bx)^{n+3}}{b^9(n+3)} \\ & -\frac{4ad(14a^4d^2 + 15a^2b^2cd + 3b^4c^2)(a + bx)^{n+4}}{b^9(n+4)} \\ & + \frac{d(70a^4d^2 + 45a^2b^2cd + 3b^4c^2)(a + bx)^{n+5}}{b^9(n+5)} - \frac{8ad^3(a + bx)^{n+8}}{b^9(n+8)} + \frac{d^3(a + bx)^{n+9}}{b^9(n+9)} \end{aligned}$$

[Out]  $(a^2(b^2c + a^2d)^3(a + bx)^{(1+n)})/(b^9(1+n)) - (2a(b^2c + a^2d)^2(b^2c + 4a^2d)(a + bx)^{(2+n)})/(b^9(2+n)) + ((b^2c + a^2d)(b^4c^2 + 17a^2b^2cd + 28a^4d^2)(a + bx)^{(3+n)})/(b^9(3+n)) - (4ad(3b^4c^2 + 15a^2b^2cd + 14a^4d^2)(a + bx)^{(4+n)})/(b^9(4+n)) + (d(3b^4c^2 + 45a^2b^2cd + 70a^4d^2)(a + bx)^{(5+n)})/(b^9(5+n)) - (2ad^2(9b^2c + 28a^2d)(a + bx)^{(6+n)})/(b^9(6+n)) + (d^2(3b^2c + 28a^2d)(a + bx)^{(7+n)})/(b^9(7+n)) - (8ad^3(a + bx)^{(8+n)})/(b^9(8+n)) + (d^3(a + bx)^{(9+n)})/(b^9(9+n))$

**Rubi [A]** time = 0.431498, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$\begin{aligned} & -\frac{2ad^2(28a^2d + 9b^2c)(a + bx)^{n+6}}{b^9(n+6)} + \frac{d^2(28a^2d + 3b^2c)(a + bx)^{n+7}}{b^9(n+7)} + \frac{a^2(a^2d + b^2c)^3(a + bx)^{n+1}}{b^9(n+1)} \\ & -\frac{2a(a^2d + b^2c)^2(4a^2d + b^2c)(a + bx)^{n+2}}{b^9(n+2)} + \frac{(a^2d + b^2c)(28a^4d^2 + 17a^2b^2cd + b^4c^2)(a + bx)^{n+3}}{b^9(n+3)} \\ & -\frac{4ad(14a^4d^2 + 15a^2b^2cd + 3b^4c^2)(a + bx)^{n+4}}{b^9(n+4)} \\ & + \frac{d(70a^4d^2 + 45a^2b^2cd + 3b^4c^2)(a + bx)^{n+5}}{b^9(n+5)} - \frac{8ad^3(a + bx)^{n+8}}{b^9(n+8)} + \frac{d^3(a + bx)^{n+9}}{b^9(n+9)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(a + b\*x)^n\*(c + d\*x^2)^3,x]

[Out]  $(a^2(b^2c + a^2d)^3(a + bx)^{(1+n)})/(b^9(1+n)) - (2a(b^2c + a^2d)^2(b^2c + 4a^2d)(a + bx)^{(2+n)})/(b^9(2+n)) + ((b^2c + a^2d)(b^4c^2 + 17a^2b^2cd + 28a^4d^2)(a + bx)^{(3+n)})/(b^9(3+n)) - (4ad(3b^4c^2 + 15a^2b^2cd + 14a^4d^2)(a + bx)^{(4+n)})/(b^9(4+n)) + (d(3b^4c^2 + 45a^2b^2cd + 70a^4d^2)(a + bx)^{(5+n)})/(b^9(5+n)) - (2ad^2(9b^2c + 28a^2d)(a + bx)^{(6+n)})/(b^9(6+n)) + (d^2(3b^2c + 28a^2d)(a + bx)^{(7+n)})/(b^9(7+n)) - (8ad^3(a + bx)^{(8+n)})/(b^9(8+n)) + (d^3(a + bx)^{(9+n)})/(b^9(9+n))$



**Rubi in Sympy [A]** time = 97.5123, size = 328, normalized size = 0.96

$$\frac{a^2(a+bx)^{n+1}(a^2d+b^2c)^3}{b^9(n+1)} - \frac{8ad^3(a+bx)^{n+8}}{b^9(n+8)} - \frac{2ad^2(a+bx)^{n+6}(28a^2d+9b^2c)}{b^9(n+6)}$$

$$- \frac{4ad(a+bx)^{n+4}(14a^4d^2+15a^2b^2cd+3b^4c^2)}{b^9(n+4)} - \frac{2a(a+bx)^{n+2}(a^2d+b^2c)^2(4a^2d+b^2c)}{b^9(n+2)}$$

$$+ \frac{d^3(a+bx)^{n+9}}{b^9(n+9)} + \frac{d^2(a+bx)^{n+7}(28a^2d+3b^2c)}{b^9(n+7)} + \frac{d(a+bx)^{n+5}(70a^4d^2+45a^2b^2cd+3b^4c^2)}{b^9(n+5)}$$

$$+ \frac{(a+bx)^{n+3}(a^2d+b^2c)(28a^4d^2+17a^2b^2cd+b^4c^2)}{b^9(n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**2*(b*x+a)**n*(d*x**2+c)**3,x)`

[Out]  $a^{**2}*(a + b*x)^{(n + 1)}*(a^{**2}*d + b^{**2}*c)^{**3}/(b^{**9}*(n + 1)) - 8*a^{**3}*(a + b*x)^{(n + 8)}/(b^{**9}*(n + 8)) - 2*a*d^{**2}*(a + b*x)^{(n + 6)}*(28*a^{**2}*d + 9*b^{**2}*c)/(b^{**9}*(n + 6)) - 4*a*d*(a + b*x)^{(n + 4)}*(14*a^{**4}*d^{**2} + 15*a^{**2}*b^{**2}*c*d + 3*b^{**4}*c^{**2})/(b^{**9}*(n + 4)) - 2*a*(a + b*x)^{(n + 2)}*(a^{**2}*d + b^{**2}*c)^{**2}*(4*a^{**2}*d + b^{**2}*c)/(b^{**9}*(n + 2)) + d^{**3}*(a + b*x)^{(n + 9)}/(b^{**9}*(n + 9)) + d^{**2}*(a + b*x)^{(n + 7)}*(28*a^{**2}*d + 3*b^{**2}*c)/(b^{**9}*(n + 7)) + d*(a + b*x)^{(n + 5)}*(70*a^{**4}*d^{**2} + 45*a^{**2}*b^{**2}*c*d + 3*b^{**4}*c^{**2})/(b^{**9}*(n + 5)) + (a + b*x)^{(n + 3)}*(a^{**2}*d + b^{**2}*c)*(28*a^{**4}*d^{**2} + 17*a^{**2}*b^{**2}*c*d + b^{**4}*c^{**2})/(b^{**9}*(n + 3))$

**Mathematica [B]** time = 1.1988, size = 746, normalized size = 2.17

$$\frac{(a+bx)^{n+1}(40320a^8d^3 - 40320a^7bd^3(n+1)x + 720a^6b^2d^2(3c(n^2+17n+72) + 28d(n^2+3n+2)x^2) - 240a^5b^3d^2(n+1)x^2)}{(b^9(n+1)(n+2)(n+3)(n+4)(n+5)(n+6)(n+7)(n+8)(n+9))}$$

Antiderivative was successfully verified.

[In] `Integrate[x^2*(a + b*x)^n*(c + d*x^2)^3,x]`

[Out]  $((a + b*x)^{(1 + n)}*(40320*a^8*d^3 - 40320*a^7*b*d^3*(1 + n)*x + 720*a^6*b^2*d^2*(3*c*(72 + 17*n + n^2) + 28*d*(2 + 3*n + n^2)*x^2) - 240*a^5*b^3*d^2*(1 + n)*x*(9*c*(72 + 17*n + n^2) + 28*d*(6 + 5*n + n^2)*x^2) + 24*a^4*b^4*d*(3*c^2*(3024 + 1650*n + 335*n^2 + 30*n^3 + n^4) + 45*c*d*(144 + 250*n + 125*n^2 + 20*n^3 + n^4)*x^2 + 70*d^2*(24 + 50*n + 35*n^2 + 10*n^3 + n^4)*x^4) - 24*a^3*b^5*d*(1 + n)*x*(3*c^2*(3024 + 1650*n + 335*n^2 + 30*n^3 + n^4) + 15*c*d*(432 + 462*n + 163*n^2 + 22*n^3 + n^4)*x^2 + 14*d^2*(120 + 154*n + 71*n^2 + 14*n^3 + n^4)*x^4) + b^8*(384 + 784*n + 540*n^2 + 160*n^3 + 21*n^4 + n^5)*x^2*(c^3*(315 + 143*n + 21*n^2 + n^3) + 3*c^2*d*(189 + 111*n + 19*n^2 + n^3)*x^2 + 3*c*d^2*(135 + 87*n + 17*n^2 + n^3)*x^4 + d^3*(105 + 71*n + 15*n^2 + n^3)*x^6) + 2*a^2*b^6*(c^3*(60480 + 60216*n + 24574*n^2 + 5265*n^3 + 625*n^4 + 39*n^5 + n^6) + 18*c^2*d*(6048 + 12372*n + 8644*n^2 + 2715*n^3 + 427*n^4 + 33*n^5 + n^6)*x^2 + 45*c*d^2*(1728 + 4008*n + 3394*n^2 + 1365*n^3 + 277*n^4 + 27*n^5 + n^6)*x^4 + 28*d^3*(720 + 1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5 + n^6)*x^6) - 2*a*b^7*(1 + n)*x*(c^3*(60480 + 60216*n + 24574*n^2 + 5265*n^3 + 625*n^4 + 39*n^5 + n^6) + 6*c^2*d*(18144 + 25020*n + 13284*n^2 + 3505*n^3 + 491*n^4 + 35*n^5 + n^6)*x^2 + 9*c*d^2*(8640 + 13128*n + 7850*n^2 + 2369*n^3 + 381*n^4 + 31*n^5 + n^6)*x^4 + 4*d^3*(5040 + 8028*n + 5104*n^2 + 1665*n^3 + 295*n^4 + 27*n^5 + n^6)*x^6))/(b^9*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n)*(7 + n)*(8 + n)*(9 + n))$

**Maple [B]** time = 0.024, size = 2232, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^2 \cdot (b \cdot x + a)^n \cdot (d \cdot x^2 + c)^3, x)$

[Out]  $(b \cdot x + a)^{(1+n)} \cdot (b^8 \cdot d^3 \cdot n^8 \cdot x^8 + 36 \cdot b^8 \cdot d^3 \cdot n^7 \cdot x^8 - 8 \cdot a \cdot b^7 \cdot d^3 \cdot n^7 \cdot x^7 + 3 \cdot b^8 \cdot c \cdot d^2 \cdot n^8 \cdot x^6 + 546 \cdot b^8 \cdot d^3 \cdot n^6 \cdot x^8 - 224 \cdot a \cdot b^7 \cdot d^3 \cdot n^6 \cdot x^7 + 114 \cdot b^8 \cdot c \cdot d^2 \cdot n^7 \cdot x^6 + 4536 \cdot b^8 \cdot d^3 \cdot n^5 \cdot x^8 + 56 \cdot a^2 \cdot b^6 \cdot d^3 \cdot n^6 \cdot x^6 - 18 \cdot a \cdot b^7 \cdot c \cdot d^2 \cdot n^7 \cdot x^5 - 2576 \cdot a \cdot b^7 \cdot d^3 \cdot n^5 \cdot x^7 + 3 \cdot b^8 \cdot c^2 \cdot d \cdot n^8 \cdot x^4 + 1812 \cdot b^8 \cdot c \cdot d^2 \cdot n^6 \cdot x^6 + 22449 \cdot b^8 \cdot d^3 \cdot n^4 \cdot x^8 + 1176 \cdot a^2 \cdot b^6 \cdot d^3 \cdot n^5 \cdot x^6 - 576 \cdot a \cdot b^7 \cdot c \cdot d^2 \cdot n^6 \cdot x^5 - 15680 \cdot a \cdot b^7 \cdot d^3 \cdot n^4 \cdot x^7 + 120 \cdot b^8 \cdot c^2 \cdot d \cdot n^7 \cdot x^4 + 15666 \cdot b^8 \cdot c \cdot d^2 \cdot n^5 \cdot x^6 + 67284 \cdot b^8 \cdot d^3 \cdot n^3 \cdot x^8 - 336 \cdot a^3 \cdot b^5 \cdot d^3 \cdot n^5 \cdot x^5 + 90 \cdot a^2 \cdot b^6 \cdot c \cdot d^2 \cdot n^6 \cdot x^4 + 9800 \cdot a^2 \cdot b^6 \cdot d^3 \cdot n^4 \cdot x^6 - 12 \cdot a \cdot b^7 \cdot c^2 \cdot d \cdot n^7 \cdot x^3 - 7416 \cdot a \cdot b^7 \cdot c \cdot d^2 \cdot n^5 \cdot x^5 - 54152 \cdot a \cdot b^7 \cdot d^3 \cdot n^3 \cdot x^7 + b^8 \cdot c^3 \cdot n^8 \cdot x^2 + 2010 \cdot b^8 \cdot c^2 \cdot d \cdot n^6 \cdot x^4 + 80157 \cdot b^8 \cdot c \cdot d^2 \cdot n^4 \cdot x^6 + 118124 \cdot b^8 \cdot d^3 \cdot n^2 \cdot x^8 - 5040 \cdot a^3 \cdot b^5 \cdot d^3 \cdot n^4 \cdot x^5 + 2430 \cdot a^2 \cdot b^6 \cdot c \cdot d^2 \cdot n^5 \cdot x^4 + 41160 \cdot a^2 \cdot b^6 \cdot d^3 \cdot n^3 \cdot x^6 - 432 \cdot a \cdot b^7 \cdot c^2 \cdot d \cdot n^6 \cdot x^3 - 49500 \cdot a \cdot b^7 \cdot c \cdot d^2 \cdot n^4 \cdot x^5 - 105056 \cdot a \cdot b^7 \cdot d^3 \cdot n^2 \cdot x^7 + 42 \cdot b^8 \cdot c^3 \cdot n^7 \cdot x^2 + 18300 \cdot b^8 \cdot c^2 \cdot d \cdot n^5 \cdot x^4 + 246876 \cdot b^8 \cdot c \cdot d^2 \cdot n^3 \cdot x^6 + 109584 \cdot b^8 \cdot d^3 \cdot n \cdot x^8 + 1680 \cdot a^4 \cdot b^4 \cdot d^3 \cdot n^4 \cdot x^4 - 360 \cdot a^3 \cdot b^5 \cdot c \cdot d^2 \cdot n^5 \cdot x^3 - 28560 \cdot a^3 \cdot b^5 \cdot d^3 \cdot n^3 \cdot x^5 + 36 \cdot a^2 \cdot b^6 \cdot c^2 \cdot d \cdot n^6 \cdot x^2 + 24930 \cdot a^2 \cdot b^6 \cdot c \cdot d^2 \cdot n^4 \cdot x^4 + 90944 \cdot a^2 \cdot b^6 \cdot d^3 \cdot n^2 \cdot x^6 - 2 \cdot a \cdot b^7 \cdot c^3 \cdot n^7 \cdot x - 6312 \cdot a \cdot b^7 \cdot c^2 \cdot d \cdot n^5 \cdot x^3 - 183942 \cdot a \cdot b^7 \cdot c \cdot d^2 \cdot n^3 \cdot x^5 - 104544 \cdot a \cdot b^7 \cdot d^3 \cdot n \cdot x^7 + 744 \cdot b^8 \cdot c^3 \cdot n^6 \cdot x^2 + 98319 \cdot b^8 \cdot c^2 \cdot d \cdot n^4 \cdot x^4 + 442908 \cdot b^8 \cdot c \cdot d^2 \cdot n^2 \cdot x^6 + 40320 \cdot b^8 \cdot d^3 \cdot x^8 + 16800 \cdot a^4 \cdot b^4 \cdot d^3 \cdot n^3 \cdot x^4 - 8280 \cdot a^3 \cdot b^5 \cdot c \cdot d^2 \cdot n^4 \cdot x^3 - 75600 \cdot a^3 \cdot b^5 \cdot d^3 \cdot n^2 \cdot x^5 + 1188 \cdot a^2 \cdot b^6 \cdot c^2 \cdot d \cdot n^5 \cdot x^2 + 122850 \cdot a^2 \cdot b^6 \cdot c \cdot d^2 \cdot n^3 \cdot x^4 + 98784 \cdot a^2 \cdot b^6 \cdot d^3 \cdot n \cdot x^6 - 80 \cdot a \cdot b^7 \cdot c^3 \cdot n^6 \cdot x - 47952 \cdot a \cdot b^7 \cdot c^2 \cdot d \cdot n^4 \cdot x^3 - 377604 \cdot a \cdot b^7 \cdot c \cdot d^2 \cdot n^2 \cdot x^5 - 40320 \cdot a \cdot b^7 \cdot d^3 \cdot x^7 + 7218 \cdot b^8 \cdot c^3 \cdot n^5 \cdot x^2 + 316380 \cdot b^8 \cdot c^2 \cdot d \cdot n^3 \cdot x^4 + 417744 \cdot b^8 \cdot c \cdot d^2 \cdot n \cdot x^6 - 6720 \cdot a^5 \cdot b^3 \cdot d^3 \cdot n^3 \cdot x^3 + 1080 \cdot a^4 \cdot b^4 \cdot c \cdot d^2 \cdot n^4 \cdot x^2 + 58800 \cdot a^4 \cdot b^4 \cdot d^3 \cdot n^2 \cdot x^4 - 72 \cdot a^3 \cdot b^5 \cdot c^2 \cdot d \cdot n^5 \cdot x - 66600 \cdot a^3 \cdot b^5 \cdot c \cdot d^2 \cdot n^3 \cdot x^3 - 92064 \cdot a^3 \cdot b^5 \cdot d^3 \cdot n \cdot x^5 + 2 \cdot a^2 \cdot b^6 \cdot c^3 \cdot n^6 + 15372 \cdot a^2 \cdot b^6 \cdot c^2 \cdot d \cdot n^4 \cdot x^2 + 305460 \cdot a^2 \cdot b^6 \cdot c \cdot d^2 \cdot n^2 \cdot x^4 + 40320 \cdot a^2 \cdot b^6 \cdot d^3 \cdot x^6 - 1328 \cdot a \cdot b^7 \cdot c^3 \cdot n^5 \cdot x - 201468 \cdot a \cdot b^7 \cdot c^2 \cdot d \cdot n^3 \cdot x^3 - 391824 \cdot a \cdot b^7 \cdot c \cdot d^2 \cdot n \cdot x^5 + 41619 \cdot b^8 \cdot c^3 \cdot n^4 \cdot x^2 + 589140 \cdot b^8 \cdot c^2 \cdot d \cdot n^2 \cdot x^4 + 155520 \cdot b^8 \cdot c \cdot d^2 \cdot x^6 - 40320 \cdot a^5 \cdot b^3 \cdot d^3 \cdot n^2 \cdot x^3 + 21600 \cdot a^4 \cdot b^4 \cdot c \cdot d^2 \cdot n^3 \cdot x^2 + 84000 \cdot a^4 \cdot b^4 \cdot d^3 \cdot n \cdot x^4 - 2232 \cdot a^3 \cdot b^5 \cdot c^2 \cdot d \cdot n^4 \cdot x - 225000 \cdot a^3 \cdot b^5 \cdot c \cdot d^2 \cdot n^2 \cdot x^3 - 40320 \cdot a^3 \cdot b^5 \cdot d^3 \cdot x^5 + 78 \cdot a^2 \cdot b^6 \cdot c^3 \cdot n^5 + 97740 \cdot a^2 \cdot b^6 \cdot c^2 \cdot d \cdot n^3 \cdot x^2 + 360720 \cdot a^2 \cdot b^6 \cdot c \cdot d^2 \cdot n \cdot x^4 - 11780 \cdot a \cdot b^7 \cdot c^3 \cdot n^4 \cdot x - 459648 \cdot a \cdot b^7 \cdot c^2 \cdot d \cdot n^2 \cdot x^3 - 155520 \cdot a \cdot b^7 \cdot c \cdot d^2 \cdot x^5 + 144468 \cdot b^8 \cdot c^3 \cdot n^3 \cdot x^2 + 572400 \cdot b^8 \cdot c^2 \cdot d \cdot n \cdot x^4 + 20160 \cdot a^6 \cdot b^2 \cdot d^3 \cdot n^2 \cdot x^2 - 2160 \cdot a^5 \cdot b^3 \cdot c \cdot d^2 \cdot n^3 \cdot x - 73920 \cdot a^5 \cdot b^3 \cdot d^3 \cdot n \cdot x^3 + 72 \cdot a^4 \cdot b^4 \cdot c^2 \cdot d \cdot n^4 + 135000 \cdot a^4 \cdot b^4 \cdot c \cdot d^2 \cdot n^2 \cdot x^2 + 40320 \cdot a^4 \cdot b^4 \cdot d^3 \cdot x^4 - 26280 \cdot a^3 \cdot b^5 \cdot c^2 \cdot d \cdot n^3 \cdot x - 321840 \cdot a^3 \cdot b^5 \cdot c \cdot d^2 \cdot n \cdot x^3 + 1250 \cdot a^2 \cdot b^6 \cdot c^3 \cdot n^4 + 311184 \cdot a^2 \cdot b^6 \cdot c^2 \cdot d \cdot n^2 \cdot x^2 + 155520 \cdot a^2 \cdot b^6 \cdot c \cdot d^2 \cdot x^4 - 59678 \cdot a \cdot b^7 \cdot c^3 \cdot n^3 \cdot x - 517968 \cdot a \cdot b^7 \cdot c^2 \cdot d \cdot n \cdot x^3 + 290276 \cdot b^8 \cdot c^3 \cdot n^2 \cdot x^2 + 217728 \cdot b^8 \cdot c^2 \cdot d \cdot x^4 + 60480 \cdot a^6 \cdot b^2 \cdot d^3 \cdot n \cdot x^2 - 38880 \cdot a^5 \cdot b^3 \cdot c \cdot d^2 \cdot n^2 \cdot x - 40320 \cdot a^5 \cdot b^3 \cdot d^3 \cdot x^3 + 2160 \cdot a^4 \cdot b^4 \cdot c^2 \cdot d \cdot n^3 + 270000 \cdot a^4 \cdot b^4 \cdot c \cdot d^2 \cdot n \cdot x^2 - 142920 \cdot a^3 \cdot b^5 \cdot c^2 \cdot d \cdot n^2 \cdot x - 155520 \cdot a^3 \cdot b^5 \cdot c \cdot d^2 \cdot x^3 + 10530 \cdot a^2 \cdot b^6 \cdot c^3 \cdot n^3 + 445392 \cdot a^2 \cdot b^6 \cdot c^2 \cdot d \cdot n \cdot x^2 - 169580 \cdot a \cdot b^7 \cdot c^3 \cdot n^2 \cdot x - 217728 \cdot a \cdot b^7 \cdot c^2 \cdot d \cdot x^3 + 301872 \cdot b^8 \cdot c^3 \cdot n \cdot x^2 - 40320 \cdot a^7 \cdot b \cdot d^3 \cdot n \cdot x + 2160 \cdot a^6 \cdot b^2 \cdot c \cdot d^2 \cdot n^2 + 40320 \cdot a^6 \cdot b^2 \cdot d^3 \cdot x^2 - 192240 \cdot a^5 \cdot b^3 \cdot c \cdot d^2 \cdot n \cdot x + 24120 \cdot a^4 \cdot b^4 \cdot c^2 \cdot d \cdot n^2 + 155520 \cdot a^4 \cdot b^4 \cdot c \cdot d^2 \cdot x^2 - 336528 \cdot a^3 \cdot b^5 \cdot c^2 \cdot d \cdot n \cdot x + 49148 \cdot a^2 \cdot b^6 \cdot c^3 \cdot n^2 + 217728 \cdot a^2 \cdot b^6 \cdot c^2 \cdot d \cdot x^2 - 241392 \cdot a \cdot b^7 \cdot c^3 \cdot n \cdot x + 120960 \cdot b^8 \cdot c^3 \cdot x^2 - 40320 \cdot a^7 \cdot b \cdot d^3 \cdot x + 36720 \cdot a^6 \cdot b^2 \cdot c \cdot d^2 \cdot n - 155520 \cdot a^5 \cdot b^3 \cdot c \cdot d^2 \cdot x + 118800 \cdot a^4 \cdot b^4 \cdot c^2 \cdot d \cdot n - 217728 \cdot a^3 \cdot b^5 \cdot c^2 \cdot d \cdot x + 120432 \cdot a^2 \cdot b^6 \cdot c^3 \cdot n - 120960 \cdot a \cdot b^7 \cdot c^3 \cdot x + 40320 \cdot a^8 \cdot d^3 + 155520 \cdot a^6 \cdot b^2 \cdot c \cdot d^2 + 217728 \cdot a^4 \cdot b^4 \cdot c^2 \cdot d + 120960 \cdot a^2 \cdot b^6 \cdot c^3) / b^9 / (n^9 + 45 \cdot n^8 + 870 \cdot n^7 + 9450 \cdot n^6 + 63273 \cdot n^5 + 269325 \cdot n^4 + 723680 \cdot n^3 + 1172700 \cdot n^2 + 1026576 \cdot n + 362880)$

**Maxima [A]** time = 0.725939, size = 1073, normalized size = 3.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((d \cdot x^2 + c)^3 \cdot (b \cdot x + a)^n \cdot x^2, x, \text{algorithm}="maxima")$

```
[Out] ((n^2 + 3*n + 2)*b^3*x^3 + (n^2 + n)*a*b^2*x^2 - 2*a^2*b*n*x + 2*
a^3)*(b*x + a)^n*c^3/((n^3 + 6*n^2 + 11*n + 6)*b^3) + 3*((n^4 + 1
0*n^3 + 35*n^2 + 50*n + 24)*b^5*x^5 + (n^4 + 6*n^3 + 11*n^2 + 6*n
)*a*b^4*x^4 - 4*(n^3 + 3*n^2 + 2*n)*a^2*b^3*x^3 + 12*(n^2 + n)*a^
3*b^2*x^2 - 24*a^4*b*n*x + 24*a^5)*(b*x + a)^n*c^2*d/((n^5 + 15*n
^4 + 85*n^3 + 225*n^2 + 274*n + 120)*b^5) + 3*((n^6 + 21*n^5 + 17
5*n^4 + 735*n^3 + 1624*n^2 + 1764*n + 720)*b^7*x^7 + (n^6 + 15*n^
5 + 85*n^4 + 225*n^3 + 274*n^2 + 120*n)*a*b^6*x^6 - 6*(n^5 + 10*n
^4 + 35*n^3 + 50*n^2 + 24*n)*a^2*b^5*x^5 + 30*(n^4 + 6*n^3 + 11*n
^2 + 6*n)*a^3*b^4*x^4 - 120*(n^3 + 3*n^2 + 2*n)*a^4*b^3*x^3 + 360
*(n^2 + n)*a^5*b^2*x^2 - 720*a^6*b*n*x + 720*a^7)*(b*x + a)^n*c^d
^2/((n^7 + 28*n^6 + 322*n^5 + 1960*n^4 + 6769*n^3 + 13132*n^2 + 1
3068*n + 5040)*b^7) + ((n^8 + 36*n^7 + 546*n^6 + 4536*n^5 + 22449
*n^4 + 67284*n^3 + 118124*n^2 + 109584*n + 40320)*b^9*x^9 + (n^8
+ 28*n^7 + 322*n^6 + 1960*n^5 + 6769*n^4 + 13132*n^3 + 13068*n^2
+ 5040*n)*a*b^8*x^8 - 8*(n^7 + 21*n^6 + 175*n^5 + 735*n^4 + 1624*
n^3 + 1764*n^2 + 720*n)*a^2*b^7*x^7 + 56*(n^6 + 15*n^5 + 85*n^4 +
225*n^3 + 274*n^2 + 120*n)*a^3*b^6*x^6 - 336*(n^5 + 10*n^4 + 35*
n^3 + 50*n^2 + 24*n)*a^4*b^5*x^5 + 1680*(n^4 + 6*n^3 + 11*n^2 + 6
*n)*a^5*b^4*x^4 - 6720*(n^3 + 3*n^2 + 2*n)*a^6*b^3*x^3 + 20160*(n
^2 + n)*a^7*b^2*x^2 - 40320*a^8*b*n*x + 40320*a^9)*(b*x + a)^n*d^
3/((n^9 + 45*n^8 + 870*n^7 + 9450*n^6 + 63273*n^5 + 269325*n^4 +
723680*n^3 + 1172700*n^2 + 1026576*n + 362880)*b^9)
```

---

**Fricas [A]** time = 0.294397, size = 2923, normalized size = 8.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2 + c)^3*(b*x + a)^n*x^2,x, algorithm="fricas")
```

```
[Out] (2*a^3*b^6*c^3*n^6 + 78*a^3*b^6*c^3*n^5 + 120960*a^3*b^6*c^3 + 21
7728*a^5*b^4*c^2*d + 155520*a^7*b^2*c*d^2 + 40320*a^9*d^3 + (b^9*
d^3*n^8 + 36*b^9*d^3*n^7 + 546*b^9*d^3*n^6 + 4536*b^9*d^3*n^5 + 2
2449*b^9*d^3*n^4 + 67284*b^9*d^3*n^3 + 118124*b^9*d^3*n^2 + 10958
4*b^9*d^3*n + 40320*b^9*d^3)*x^9 + (a*b^8*d^3*n^8 + 28*a*b^8*d^3*
n^7 + 322*a*b^8*d^3*n^6 + 1960*a*b^8*d^3*n^5 + 6769*a*b^8*d^3*n^4
+ 13132*a*b^8*d^3*n^3 + 13068*a*b^8*d^3*n^2 + 5040*a*b^8*d^3*n)*
x^8 + (3*b^9*c*d^2*n^8 + 155520*b^9*c*d^2 + 2*(57*b^9*c*d^2 - 4*a
^2*b^7*d^3)*n^7 + 12*(151*b^9*c*d^2 - 14*a^2*b^7*d^3)*n^6 + 14*(1
119*b^9*c*d^2 - 100*a^2*b^7*d^3)*n^5 + 21*(3817*b^9*c*d^2 - 280*a
^2*b^7*d^3)*n^4 + 28*(8817*b^9*c*d^2 - 464*a^2*b^7*d^3)*n^3 + 36*
(12303*b^9*c*d^2 - 392*a^2*b^7*d^3)*n^2 + 144*(2901*b^9*c*d^2 - 4
0*a^2*b^7*d^3)*n)*x^7 + (3*a*b^8*c*d^2*n^8 + 96*a*b^8*c*d^2*n^7 +
4*(309*a*b^8*c*d^2 + 14*a^3*b^6*d^3)*n^6 + 30*(275*a*b^8*c*d^2 +
28*a^3*b^6*d^3)*n^5 + (30657*a*b^8*c*d^2 + 4760*a^3*b^6*d^3)*n^4
+ 6*(10489*a*b^8*c*d^2 + 2100*a^3*b^6*d^3)*n^3 + 8*(8163*a*b^8*c
*d^2 + 1918*a^3*b^6*d^3)*n^2 + 960*(27*a*b^8*c*d^2 + 7*a^3*b^6*d^
3)*n)*x^6 + 3*(b^9*c^2*d*n^8 + 72576*b^9*c^2*d + 2*(20*b^9*c^2*d
- 3*a^2*b^7*c*d^2)*n^7 + 2*(335*b^9*c^2*d - 81*a^2*b^7*c*d^2)*n^6
+ 2*(3050*b^9*c^2*d - 831*a^2*b^7*c*d^2 - 56*a^4*b^5*d^3)*n^5 +
(32773*b^9*c^2*d - 8190*a^2*b^7*c*d^2 - 1120*a^4*b^5*d^3)*n^4 + 4
*(26365*b^9*c^2*d - 5091*a^2*b^7*c*d^2 - 980*a^4*b^5*d^3)*n^3 + 4
*(49095*b^9*c^2*d - 6012*a^2*b^7*c*d^2 - 1400*a^4*b^5*d^3)*n^2 +
48*(3975*b^9*c^2*d - 216*a^2*b^7*c*d^2 - 56*a^4*b^5*d^3)*n)*x^5 +
2*(625*a^3*b^6*c^3 + 36*a^5*b^4*c^2*d)*n^4 + 3*(a*b^8*c^2*d*n^8
+ 36*a*b^8*c^2*d*n^7 + 2*(263*a*b^8*c^2*d + 15*a^3*b^6*c*d^2)*n^6
+ 6*(666*a*b^8*c^2*d + 115*a^3*b^6*c*d^2)*n^5 + (16789*a*b^8*c^2
*d + 5550*a^3*b^6*c*d^2 + 560*a^5*b^4*d^3)*n^4 + 6*(6384*a*b^8*c^
2*d + 3125*a^3*b^6*c*d^2 + 560*a^5*b^4*d^3)*n^3 + 4*(10791*a*b^8*
c^2*d + 6705*a^3*b^6*c*d^2 + 1540*a^5*b^4*d^3)*n^2 + 96*(189*a*b^
8*c^2*d + 135*a^3*b^6*c*d^2 + 35*a^5*b^4*d^3)*n)*x^4 + 270*(39*a^
3*b^6*c^3 + 8*a^5*b^4*c^2*d)*n^3 + (b^9*c^3*n^8 + 120960*b^9*c^3
+ 6*(7*b^9*c^3 - 2*a^2*b^7*c^2*d)*n^7 + 12*(62*b^9*c^3 - 33*a^2*b
^7*c^2*d)*n^6 + 6*(1203*b^9*c^3 - 854*a^2*b^7*c^2*d - 60*a^4*b^5*
c^2*d^2)*n^5 + 3*(13873*b^9*c^3 - 10860*a^2*b^7*c^2*d - 2400*a^4*b^
5*c^2*d^2)*n^4 + 12*(12039*b^9*c^3 - 8644*a^2*b^7*c^2*d - 3750*a^4*
```

$$\begin{aligned}
& b^5 c^2 d^2 - 560 a^6 b^3 d^3) n^3 + 4 (72569 b^9 c^3 - 37116 a^2 b^7 c^2 d - 22500 a^4 b^5 c^2 d^2 - 5040 a^6 b^3 d^3) n^2 + 48 (6289 b^9 c^3 - 1512 a^2 b^7 c^2 d - 1080 a^4 b^5 c^2 d^2 - 280 a^6 b^3 d^3) n) x^3 + 4 (12287 a^3 b^6 c^3 + 6030 a^5 b^4 c^2 d + 540 a^7 b^2 c^2 d^2) n^2 + (a^8 b^8 c^3 n^8 + 40 a^6 b^8 c^3 n^7 + 4 (166 a^8 b^8 c^3 + 9 a^3 b^6 c^2 d) n^6 + 62 (95 a^8 b^8 c^3 + 18 a^3 b^6 c^2 d) n^5 + (29839 a^8 b^8 c^3 + 13140 a^3 b^6 c^2 d + 1080 a^5 b^4 c^2 d^2) n^4 + 10 (8479 a^8 b^8 c^3 + 7146 a^3 b^6 c^2 d + 1944 a^5 b^4 c^2 d^2) n^3 + 24 (5029 a^8 b^8 c^3 + 7011 a^3 b^6 c^2 d + 4005 a^5 b^4 c^2 d^2 + 840 a^7 b^2 d^3) n^2 + 576 (105 a^8 b^8 c^3 + 189 a^3 b^6 c^2 d + 135 a^5 b^4 c^2 d^2 + 35 a^7 b^2 d^3) n) x^2 + 48 (2509 a^3 b^6 c^3 + 2475 a^5 b^4 c^2 d + 765 a^7 b^2 c^2 d^2) n - 2 (a^2 b^7 c^3 n^7 + 39 a^2 b^7 c^3 n^6 + (625 a^2 b^7 c^3 + 36 a^4 b^5 c^2 d) n^5 + 135 (39 a^2 b^7 c^3 + 8 a^4 b^5 c^2 d) n^4 + 2 (12287 a^2 b^7 c^3 + 6030 a^4 b^5 c^2 d + 540 a^6 b^3 c^2 d^2) n^3 + 24 (2509 a^2 b^7 c^3 + 2475 a^4 b^5 c^2 d + 765 a^6 b^3 c^2 d^2) n^2 + 576 (105 a^2 b^7 c^3 + 189 a^4 b^5 c^2 d + 135 a^6 b^3 c^2 d^2 + 35 a^8 b^2 d^3) n) x) (b x + a)^n / (b^9 n^9 + 45 b^9 n^8 + 870 b^9 n^7 + 9450 b^9 n^6 + 63273 b^9 n^5 + 269325 b^9 n^4 + 723680 b^9 n^3 + 1172700 b^9 n^2 + 1026576 b^9 n + 362880 b^9)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(b\*x+a)\*\*n\*(d\*x\*\*2+c)\*\*3,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.308134, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)^3\*(b\*x + a)^n\*x^2,x, algorithm="giac")

[Out] Done

### 3.360 $\int x(a + bx)^n (c + dx^2)^3 dx$

**Optimal.** Leaf size=282

$$\begin{aligned} & -\frac{5ad^2(7a^2d + 3b^2c)(a + bx)^{n+5}}{b^8(n+5)} + \frac{3d^2(7a^2d + b^2c)(a + bx)^{n+6}}{b^8(n+6)} - \frac{a(a^2d + b^2c)^3(a + bx)^{n+1}}{b^8(n+1)} \\ & + \frac{(a^2d + b^2c)^2(7a^2d + b^2c)(a + bx)^{n+2}}{b^8(n+2)} - \frac{3ad(a^2d + b^2c)(7a^2d + 3b^2c)(a + bx)^{n+3}}{b^8(n+3)} \\ & + \frac{d(35a^4d^2 + 30a^2b^2cd + 3b^4c^2)(a + bx)^{n+4}}{b^8(n+4)} - \frac{7ad^3(a + bx)^{n+7}}{b^8(n+7)} + \frac{d^3(a + bx)^{n+8}}{b^8(n+8)} \end{aligned}$$

[Out]  $-((a*(b^2*c + a^2*d)^3*(a + b*x)^(1 + n))/(b^8*(1 + n))) + ((b^2*c + a^2*d)^2*(b^2*c + 7*a^2*d)*(a + b*x)^(2 + n))/(b^8*(2 + n)) - (3*a*d*(b^2*c + a^2*d)*(3*b^2*c + 7*a^2*d)*(a + b*x)^(3 + n))/(b^8*(3 + n)) + (d*(3*b^4*c^2 + 30*a^2*b^2*c*d + 35*a^4*d^2)*(a + b*x)^(4 + n))/(b^8*(4 + n)) - (5*a*d^2*(3*b^2*c + 7*a^2*d)*(a + b*x)^(5 + n))/(b^8*(5 + n)) + (3*d^2*(b^2*c + 7*a^2*d)*(a + b*x)^(6 + n))/(b^8*(6 + n)) - (7*a*d^3*(a + b*x)^(7 + n))/(b^8*(7 + n)) + (d^3*(a + b*x)^(8 + n))/(b^8*(8 + n))$

**Rubi [A]** time = 0.368504, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$

$$\begin{aligned} & -\frac{5ad^2(7a^2d + 3b^2c)(a + bx)^{n+5}}{b^8(n+5)} + \frac{3d^2(7a^2d + b^2c)(a + bx)^{n+6}}{b^8(n+6)} - \frac{a(a^2d + b^2c)^3(a + bx)^{n+1}}{b^8(n+1)} \\ & + \frac{(a^2d + b^2c)^2(7a^2d + b^2c)(a + bx)^{n+2}}{b^8(n+2)} - \frac{3ad(a^2d + b^2c)(7a^2d + 3b^2c)(a + bx)^{n+3}}{b^8(n+3)} \\ & + \frac{d(35a^4d^2 + 30a^2b^2cd + 3b^4c^2)(a + bx)^{n+4}}{b^8(n+4)} - \frac{7ad^3(a + bx)^{n+7}}{b^8(n+7)} + \frac{d^3(a + bx)^{n+8}}{b^8(n+8)} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x*(a + b*x)^n*(c + d*x^2)^3, x]$

[Out]  $-((a*(b^2*c + a^2*d)^3*(a + b*x)^(1 + n))/(b^8*(1 + n))) + ((b^2*c + a^2*d)^2*(b^2*c + 7*a^2*d)*(a + b*x)^(2 + n))/(b^8*(2 + n)) - (3*a*d*(b^2*c + a^2*d)*(3*b^2*c + 7*a^2*d)*(a + b*x)^(3 + n))/(b^8*(3 + n)) + (d*(3*b^4*c^2 + 30*a^2*b^2*c*d + 35*a^4*d^2)*(a + b*x)^(4 + n))/(b^8*(4 + n)) - (5*a*d^2*(3*b^2*c + 7*a^2*d)*(a + b*x)^(5 + n))/(b^8*(5 + n)) + (3*d^2*(b^2*c + 7*a^2*d)*(a + b*x)^(6 + n))/(b^8*(6 + n)) - (7*a*d^3*(a + b*x)^(7 + n))/(b^8*(7 + n)) + (d^3*(a + b*x)^(8 + n))/(b^8*(8 + n))$

**Rubi in Sympy [A]** time = 77.0045, size = 265, normalized size = 0.94

$$\begin{aligned} & -\frac{7ad^3(a + bx)^{n+7}}{b^8(n+7)} - \frac{5ad^2(a + bx)^{n+5}(7a^2d + 3b^2c)}{b^8(n+5)} - \frac{3ad(a + bx)^{n+3}(a^2d + b^2c)(7a^2d + 3b^2c)}{b^8(n+3)} \\ & - \frac{a(a + bx)^{n+1}(a^2d + b^2c)^3}{b^8(n+1)} + \frac{d^3(a + bx)^{n+8}}{b^8(n+8)} + \frac{3d^2(a + bx)^{n+6}(7a^2d + b^2c)}{b^8(n+6)} \\ & + \frac{d(a + bx)^{n+4}(35a^4d^2 + 30a^2b^2cd + 3b^4c^2)}{b^8(n+4)} + \frac{(a + bx)^{n+2}(a^2d + b^2c)^2(7a^2d + b^2c)}{b^8(n+2)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x*(b*x+a)**n*(d*x**2+c)**3, x)$

```
[Out] -7*a*d**3*(a + b*x)**(n + 7)/(b**8*(n + 7)) - 5*a*d**2*(a + b*x)**
*(n + 5)*(7*a**2*d + 3*b**2*c)/(b**8*(n + 5)) - 3*a*d*(a + b*x)**
(n + 3)*(a**2*d + b**2*c)*(7*a**2*d + 3*b**2*c)/(b**8*(n + 3)) -
a*(a + b*x)**(n + 1)*(a**2*d + b**2*c)**3/(b**8*(n + 1)) + d**3*(
a + b*x)**(n + 8)/(b**8*(n + 8)) + 3*d**2*(a + b*x)**(n + 6)*(7*a
**2*d + b**2*c)/(b**8*(n + 6)) + d*(a + b*x)**(n + 4)*(35*a**4*d*
*2 + 30*a**2*b**2*c*d + 3*b**4*c**2)/(b**8*(n + 4)) + (a + b*x)**
(n + 2)*(a**2*d + b**2*c)**2*(7*a**2*d + b**2*c)/(b**8*(n + 2))
```

**Mathematica [B]** time = 0.828552, size = 578, normalized size = 2.05

$$(a + bx)^{n+1} (-5040a^7d^3 + 5040a^6bd^3(n+1)x - 360a^5b^2d^2(c(n^2 + 15n + 56) + 7d(n^2 + 3n + 2)x^2) + 120a^4b^3d^2(n+1)x(3$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x)^n*(c + d*x^2)^3,x]
```

```
[Out] ((a + b*x)^(1 + n)*(-5040*a^7*d^3 + 5040*a^6*b*d^3*(1 + n)*x - 36
0*a^5*b^2*d^2*(c*(56 + 15*n + n^2) + 7*d*(2 + 3*n + n^2)*x^2) + 1
20*a^4*b^3*d^2*(1 + n)*x*(3*c*(56 + 15*n + n^2) + 7*d*(6 + 5*n +
n^2)*x^2) - 6*a^3*b^4*d*(3*c^2*(1680 + 1066*n + 251*n^2 + 26*n^3
+ n^4) + 30*c*d*(112 + 198*n + 103*n^2 + 18*n^3 + n^4)*x^2 + 35*d
^2*(24 + 50*n + 35*n^2 + 10*n^3 + n^4)*x^4) + 6*a^2*b^5*d*(1 + n)
*x*(3*c^2*(1680 + 1066*n + 251*n^2 + 26*n^3 + n^4) + 10*c*d*(336
+ 370*n + 137*n^2 + 20*n^3 + n^4)*x^2 + 7*d^2*(120 + 154*n + 71*n
^2 + 14*n^3 + n^4)*x^4) + b^7*(105 + 176*n + 86*n^2 + 16*n^3 + n^
4)*x*(c^3*(192 + 104*n + 18*n^2 + n^3) + 3*c^2*d*(96 + 76*n + 16*
n^2 + n^3)*x^2 + 3*c*d^2*(64 + 56*n + 14*n^2 + n^3)*x^4 + d^3*(48
+ 44*n + 12*n^2 + n^3)*x^6) - a*b^6*(c^3*(20160 + 24552*n + 1215
4*n^2 + 3135*n^3 + 445*n^4 + 33*n^5 + n^6) + 9*c^2*d*(3360 + 7172
*n + 5380*n^2 + 1871*n^3 + 331*n^4 + 29*n^5 + n^6)*x^2 + 15*c*d^2
*(1344 + 3160*n + 2734*n^2 + 1135*n^3 + 241*n^4 + 25*n^5 + n^6)*x
^4 + 7*d^3*(720 + 1764*n + 1624*n^2 + 735*n^3 + 175*n^4 + 21*n^5
+ n^6)*x^6)))/(b^8*(1 + n)*(2 + n)*(3 + n)*(4 + n)*(5 + n)*(6 + n)
*(7 + n)*(8 + n))
```

**Maple [B]** time = 0.019, size = 1639, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x+a)^n*(d*x^2+c)^3,x)
```

```
[Out] -(b*x+a)^(1+n)*(-b^7*d^3*n^7*x^7-28*b^7*d^3*n^6*x^7+7*a*b^6*d^3*n
^6*x^6-3*b^7*c*d^2*n^7*x^5-322*b^7*d^3*n^5*x^7+147*a*b^6*d^3*n^5*
x^6-90*b^7*c*d^2*n^6*x^5-1960*b^7*d^3*n^4*x^7-42*a^2*b^5*d^3*n^5*
x^5+15*a*b^6*c*d^2*n^6*x^4+1225*a*b^6*d^3*n^4*x^6-3*b^7*c^2*d*n^7
*x^3-1098*b^7*c*d^2*n^5*x^5-6769*b^7*d^3*n^3*x^7-630*a^2*b^5*d^3*
n^4*x^5+375*a*b^6*c*d^2*n^5*x^4+5145*a*b^6*d^3*n^3*x^6-96*b^7*c^2
*d*n^6*x^3-7020*b^7*c*d^2*n^4*x^5-13132*b^7*d^3*n^2*x^7+210*a^3*b
^4*d^3*n^4*x^4-60*a^2*b^5*c*d^2*n^5*x^3-3570*a^2*b^5*d^3*n^3*x^5+
9*a*b^6*c^2*d*n^6*x^2+3615*a*b^6*c*d^2*n^4*x^4+11368*a*b^6*d^3*n^
2*x^6-b^7*c^3*n^7*x-1254*b^7*c^2*d*n^5*x^3-25227*b^7*c*d^2*n^3*x^
5-13068*b^7*d^3*n*x^7+2100*a^3*b^4*d^3*n^3*x^4-1260*a^2*b^5*c*d^2
*n^4*x^3-9450*a^2*b^5*d^3*n^2*x^5+261*a*b^6*c^2*d*n^5*x^2+17025*a
*b^6*c*d^2*n^3*x^4+12348*a*b^6*d^3*n*x^6-34*b^7*c^3*n^6*x-8592*b^
7*c^2*d*n^4*x^3-50490*b^7*c*d^2*n^2*x^5-5040*b^7*d^3*x^7-840*a^4*
b^3*d^3*n^3*x^3+180*a^3*b^4*c*d^2*n^4*x^2+7350*a^3*b^4*d^3*n^2*x^
4-18*a^2*b^5*c^2*d*n^5*x-9420*a^2*b^5*c*d^2*n^3*x^3-11508*a^2*b^5
*d^3*n*x^5+a*b^6*c^3*n^6+2979*a*b^6*c^2*d*n^4*x^2+41010*a*b^6*c*d
```

$$\begin{aligned} &^2n^2x^4+5040a^6b^6d^3x^6-478b^7c^3n^5x-32979b^7c^2d^2n^3x^3-51432b^7c^2d^2n^3x^5-5040a^4b^3d^3n^2x^3+3240a^3b^4c^2d^2n^3x^2+10500a^3b^4d^3n^2x^4-486a^2b^5c^2d^2n^4x-30420a^2b^5c^2d^2n^2x^3-5040a^2b^5d^3x^5+33a^6b^6c^3n^5+16839a^6b^6c^2d^2n^3x^2+47400a^6b^6c^2d^2n^3x^4-3580b^7c^3n^4x-69936b^7c^2d^2n^2x^3-20160b^7c^2d^2x^5+2520a^5b^2d^3n^2x^2-360a^4b^3c^2d^2n^3x-9240a^4b^3d^3n^2x^3+18a^3b^4c^2d^2n^4+18540a^3b^4c^2d^2n^2x^2+5040a^3b^4d^3x^4-4986a^2b^5c^2d^2n^3x-42360a^2b^5c^2d^2n^2x^3+445a^6b^6c^3n^4+48420a^6b^6c^2d^2n^2x^2+20160a^6b^6c^2d^2x^4-15289b^7c^3n^3x-74628b^7c^2d^2n^2x^3+7560a^5b^2d^3n^2x^2-5760a^4b^3c^2d^2n^2x-5040a^4b^3d^3x^3+468a^3b^4c^2d^2n^3+35640a^3b^4c^2d^2n^2x^2-23706a^2b^5c^2d^2n^2x-20160a^2b^5c^2d^2x^3+3135a^6b^6c^3n^3+64548a^6b^6c^2d^2n^2x^2-36706b^7c^3n^2x-30240b^7c^2d^2x^3-5040a^6b^6d^3n^2x+360a^5b^2c^2d^2n^2+5040a^5b^2d^3x^2-25560a^4b^3c^2d^2n^2x+4518a^3b^4c^2d^2n^2+20160a^3b^4c^2d^2x^2-49428a^2b^5c^2d^2n^2x+12154a^6b^6c^3n^2+30240a^6b^6c^2d^2x^2-44712b^7c^3n^2x-5040a^6b^6d^3x+5400a^5b^2c^2d^2n-20160a^4b^3c^2d^2x+19188a^3b^4c^2d^2n-30240a^2b^5c^2d^2x+24552a^6b^6c^3n-20160b^7c^3x+5040a^7d^3+20160a^5b^2c^2d^2+30240a^3b^4c^2d+20160a^6b^6c^3)/b^8/(n^8+36n^7+546n^6+4536n^5+22449n^4+67284n^3+118124n^2+109584n+40320) \end{aligned}$$

**Maxima [A]** time = 0.714953, size = 844, normalized size = 2.99

$$\begin{aligned} &\frac{(b^2(n+1)x^2 + abnx - a^2)(bx + a)^n c^3}{(n^2 + 3n + 2)b^2} \\ &+ \frac{3((n^3 + 6n^2 + 11n + 6)b^4x^4 + (n^3 + 3n^2 + 2n)ab^3x^3 - 3(n^2 + n)a^2b^2x^2 + 6a^3bnx - 6a^4)(bx + a)^n c^2 d}{(n^4 + 10n^3 + 35n^2 + 50n + 24)b^4} \\ &+ \frac{3((n^5 + 15n^4 + 85n^3 + 225n^2 + 274n + 120)b^6x^6 + (n^5 + 10n^4 + 35n^3 + 50n^2 + 24n)ab^5x^5 - 5(n^4 + 6n^3 + 11n^2 + 6n)a^2b^4x^4 + (n^4 + 6n^3 + 11n^2 + 6n)ab^3x^3 - 3(n^3 + 3n^2 + 2n)ab^3x^3 - 3(n^2 + n)a^2b^2x^2 + 6a^3bnx - 6a^4)(bx + a)^n c^2 d}{(n^6 + 21n^5 + 175n^4 + 735n^3 + 1624n^2 + 1764n + 720)b^6} \\ &+ \frac{((n^7 + 28n^6 + 322n^5 + 1960n^4 + 6769n^3 + 13132n^2 + 13068n + 5040)b^8x^8 + (n^7 + 21n^6 + 175n^5 + 735n^4 + 1624n^3 + 1764n^2 + 720n + 5040)b^8x^8 + (n^7 + 21n^6 + 175n^5 + 735n^4 + 1624n^3 + 1764n^2 + 720n + 5040)a^2b^4x^4 + (n^7 + 21n^6 + 175n^5 + 735n^4 + 1624n^3 + 1764n^2 + 720n + 5040)ab^3x^3 - 3(n^6 + 21n^5 + 175n^4 + 735n^3 + 1624n^2 + 1764n + 720)a^2b^4x^4 + (n^6 + 21n^5 + 175n^4 + 735n^3 + 1624n^2 + 1764n + 720)ab^3x^3 - 3(n^5 + 15n^4 + 85n^3 + 225n^2 + 274n + 120)ab^3x^3 - 3(n^4 + 6n^3 + 11n^2 + 6n)a^2b^4x^4 + (n^4 + 6n^3 + 11n^2 + 6n)ab^3x^3 - 3(n^3 + 3n^2 + 2n)ab^3x^3 - 3(n^2 + n)a^2b^2x^2 + 6a^3bnx - 6a^4)(bx + a)^n c^2 d}{(n^8 + 36n^7 + 546n^6 + 4536n^5 + 22449n^4 + 67284n^3 + 118124n^2 + 109584n + 40320)b^8} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)^3\*(b\*x + a)^n\*x,x, algorithm="maxima")

[Out]  $(b^2(n+1)x^2 + a^6b^6d^3n^2x^2 - a^2)(bx + a)^n c^3 / ((n^2 + 3n + 2)b^2) + 3((n^3 + 6n^2 + 11n + 6)b^4x^4 + (n^3 + 3n^2 + 2n)ab^3x^3 - 3(n^2 + n)a^2b^2x^2 + 6a^3bnx - 6a^4)(bx + a)^n c^2 d / ((n^4 + 10n^3 + 35n^2 + 50n + 24)b^4) + 3((n^5 + 15n^4 + 85n^3 + 225n^2 + 274n + 120)b^6x^6 + (n^5 + 10n^4 + 35n^3 + 50n^2 + 24n)ab^5x^5 - 5(n^4 + 6n^3 + 11n^2 + 6n)a^2b^4x^4 + (n^4 + 6n^3 + 11n^2 + 6n)ab^3x^3 - 3(n^3 + 3n^2 + 2n)ab^3x^3 - 3(n^2 + n)a^2b^2x^2 + 6a^3bnx - 6a^4)(bx + a)^n c^2 d / ((n^6 + 21n^5 + 175n^4 + 735n^3 + 1624n^2 + 1764n + 720)b^6) + ((n^7 + 28n^6 + 322n^5 + 1960n^4 + 6769n^3 + 13132n^2 + 13068n + 5040)b^8x^8 + (n^7 + 21n^6 + 175n^5 + 735n^4 + 1624n^3 + 1764n^2 + 720n + 5040)b^8x^8 + (n^7 + 21n^6 + 175n^5 + 735n^4 + 1624n^3 + 1764n^2 + 720n + 5040)a^2b^4x^4 + (n^7 + 21n^6 + 175n^5 + 735n^4 + 1624n^3 + 1764n^2 + 720n + 5040)ab^3x^3 - 3(n^6 + 21n^5 + 175n^4 + 735n^3 + 1624n^2 + 1764n + 720)a^2b^4x^4 + (n^6 + 21n^5 + 175n^4 + 735n^3 + 1624n^2 + 1764n + 720)ab^3x^3 - 3(n^5 + 15n^4 + 85n^3 + 225n^2 + 274n + 120)ab^3x^3 - 3(n^4 + 6n^3 + 11n^2 + 6n)a^2b^4x^4 + (n^4 + 6n^3 + 11n^2 + 6n)ab^3x^3 - 3(n^3 + 3n^2 + 2n)ab^3x^3 - 3(n^2 + n)a^2b^2x^2 + 6a^3bnx - 6a^4)(bx + a)^n d^3 / ((n^8 + 36n^7 + 546n^6 + 4536n^5 + 22449n^4 + 67284n^3 + 118124n^2 + 109584n + 40320)b^8)$

**Fricas [A]** time = 0.294262, size = 2261, normalized size = 8.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)^3\*(b\*x + a)^n\*x,x, algorithm="fricas")

[Out]  $-(a^2 b^6 c^3 n^6 + 33 a^2 b^6 c^3 n^5 + 20160 a^2 b^6 c^3 + 30240 a^4 b^4 c^2 d + 20160 a^6 b^2 c^2 d^2 + 5040 a^8 d^3 - (b^8 d^3 n^7 + 28 b^8 d^3 n^6 + 322 b^8 d^3 n^5 + 1960 b^8 d^3 n^4 + 6769 b^8 d^3 n^3 + 13132 b^8 d^3 n^2 + 13068 b^8 d^3 n + 5040 b^8 d^3)) x^8 - (a b^7 d^3 n^7 + 21 a b^7 d^3 n^6 + 175 a b^7 d^3 n^5 + 735 a b^7 d^3 n^4 + 1624 a b^7 d^3 n^3 + 1764 a b^7 d^3 n^2 + 720 a b^7 d^3 n) x^7 - (3 b^8 c^2 d^2 n^7 + 20160 b^8 c^2 d^2 + (90 b^8 c^2 d^2 - 7 a^2 b^6 d^3) n^6 + 3(366 b^8 c^2 d^2 - 35 a^2 b^6 d^3) n^5 + 5(1404 b^8 c^2 d^2 - 119 a^2 b^6 d^3) n^4 + 9(2803 b^8 c^2 d^2 - 175 a^2 b^6 d^3) n^3 + 2(25245 b^8 c^2 d^2 - 959 a^2 b^6 d^3) n^2 + 24(2143 b^8 c^2 d^2 - 35 a^2 b^6 d^3) n) x^6 - 3(a b^7 c^2 d^2 n^7 + 25 a b^7 c^2 d^2 n^6 + (241 a b^7 c^2 d^2 + 14 a^3 b^5 d^3) n^5 + 5(227 a b^7 c^2 d^2 + 28 a^3 b^5 d^3) n^4 + 2(1367 a b^7 c^2 d^2 + 245 a^3 b^5 d^3) n^3 + 20(158 a b^7 c^2 d^2 + 35 a^3 b^5 d^3) n^2 + 336(4 a b^7 c^2 d^2 + a^3 b^5 d^3) n) x^5 + (445 a^2 b^6 c^3 + 18 a^4 b^4 c^2 d) n^4 - 3(b^8 c^2 d^2 n^7 + 10080 b^8 c^2 d^2 + (32 b^8 c^2 d^2 - 5 a^2 b^6 c^2 d^2) n^6 + (418 b^8 c^2 d^2 - 105 a^2 b^6 c^2 d^2) n^5 + (2864 b^8 c^2 d^2 - 785 a^2 b^6 c^2 d^2 - 70 a^4 b^4 d^3) n^4 + (10993 b^8 c^2 d^2 - 2535 a^2 b^6 c^2 d^2 - 420 a^4 b^4 d^3) n^3 + 2(11656 b^8 c^2 d^2 - 1765 a^2 b^6 c^2 d^2 - 385 a^4 b^4 d^3) n^2 + 12(2073 b^8 c^2 d^2 - 140 a^2 b^6 c^2 d^2 - 35 a^4 b^4 d^3) n) x^4 + 3(1045 a^2 b^6 c^3 + 156 a^4 b^4 c^2 d) n^3 - 3(a b^7 c^2 d^2 n^7 + 29 a b^7 c^2 d^2 n^6 + (331 a b^7 c^2 d^2 + 20 a^3 b^5 c^2 d^2) n^5 + (1871 a b^7 c^2 d^2 + 360 a^3 b^5 c^2 d^2) n^4 + 20(269 a b^7 c^2 d^2 + 103 a^3 b^5 c^2 d^2 + 14 a^5 b^3 d^3) n^3 + 4(1793 a b^7 c^2 d^2 + 990 a^3 b^5 c^2 d^2 + 210 a^5 b^3 d^3) n^2 + 560(6 a b^7 c^2 d^2 + 4 a^3 b^5 c^2 d^2 + a^5 b^3 d^3) n) x^3 + 2(6077 a^2 b^6 c^3 + 2259 a^4 b^4 c^2 d + 180 a^6 b^2 c^2 d^2) n^2 - (b^8 c^3 n^7 + 20160 b^8 c^3 + (34 b^8 c^3 - 9 a^2 b^6 c^2 d) n^6 + (478 b^8 c^3 - 243 a^2 b^6 c^2 d) n^5 + (3580 b^8 c^3 - 2493 a^2 b^6 c^2 d - 180 a^4 b^4 c^2 d^2) n^4 + (15289 b^8 c^3 - 11853 a^2 b^6 c^2 d - 2880 a^4 b^4 c^2 d^2) n^3 + 2(18353 b^8 c^3 - 12357 a^2 b^6 c^2 d - 6390 a^4 b^4 c^2 d^2 - 1260 a^6 b^2 d^3) n^2 + 72(621 b^8 c^3 - 210 a^2 b^6 c^2 d - 140 a^4 b^4 c^2 d^2 - 35 a^6 b^2 d^3) n) x^2 + 36(682 a^2 b^6 c^3 + 533 a^4 b^4 c^2 d + 150 a^6 b^2 c^2 d^2) n - (a b^7 c^3 n^7 + 33 a b^7 c^3 n^6 + (445 a b^7 c^3 + 18 a^3 b^5 c^2 d) n^5 + 3(1045 a b^7 c^3 + 156 a^3 b^5 c^2 d) n^4 + 2(6077 a b^7 c^3 + 2259 a^3 b^5 c^2 d + 180 a^5 b^3 c^2 d^2) n^3 + 36(682 a b^7 c^3 + 533 a^3 b^5 c^2 d + 150 a^5 b^3 c^2 d^2) n^2 + 5040(4 a b^7 c^3 + 6 a^3 b^5 c^2 d + 4 a^5 b^3 c^2 d^2 + a^7 b^1 d^3) n) x) (b*x + a)^n / (b^8 n^8 + 36 b^8 n^7 + 546 b^8 n^6 + 4536 b^8 n^5 + 22449 b^8 n^4 + 67284 b^8 n^3 + 118124 b^8 n^2 + 109584 b^8 n + 40320 b^8)$

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**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x+a)\*\*n\*(d\*x\*\*2+c)\*\*3,x)

[Out] Timed out

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**GIAC/XCAS [A]** time = 0.279571, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)^3\*(b\*x + a)^n\*x,x, algorithm="giac")



[Out] Done

### 3.361 $\int (a + bx)^n (c + dx^2)^3 dx$

**Optimal.** Leaf size=223

$$\begin{aligned} & -\frac{4ad^2(5a^2d+3b^2c)(a+bx)^{n+4}}{b^{7(n+4)}} + \frac{3d^2(5a^2d+b^2c)(a+bx)^{n+5}}{b^{7(n+5)}} \\ & + \frac{(a^2d+b^2c)^3(a+bx)^{n+1}}{b^{7(n+1)}} - \frac{6ad(a^2d+b^2c)^2(a+bx)^{n+2}}{b^{7(n+2)}} \\ & + \frac{3d(a^2d+b^2c)(5a^2d+b^2c)(a+bx)^{n+3}}{b^{7(n+3)}} - \frac{6ad^3(a+bx)^{n+6}}{b^{7(n+6)}} + \frac{d^3(a+bx)^{n+7}}{b^{7(n+7)}} \end{aligned}$$

[Out]  $((b^2c + a^2d)^3(a + bx)^{(1+n)})/(b^{7(1+n)}) - (6ad(b^2c + a^2d)^2(a + bx)^{(2+n)})/(b^{7(2+n)}) + (3d(b^2c + a^2d)(b^2c + 5a^2d)(a + bx)^{(3+n)})/(b^{7(3+n)}) - (4a^2d^2(3b^2c + 5a^2d)(a + bx)^{(4+n)})/(b^{7(4+n)}) + (3d^2(b^2c + 5a^2d)(a + bx)^{(5+n)})/(b^{7(5+n)}) - (6ad^3(a + bx)^{(6+n)})/(b^{7(6+n)}) + (d^3(a + bx)^{(7+n)})/(b^{7(7+n)})$

**Rubi [A]** time = 0.26396, antiderivative size = 223, normalized size of antiderivative = 1., number of rules used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\begin{aligned} & -\frac{4ad^2(5a^2d+3b^2c)(a+bx)^{n+4}}{b^{7(n+4)}} + \frac{3d^2(5a^2d+b^2c)(a+bx)^{n+5}}{b^{7(n+5)}} \\ & + \frac{(a^2d+b^2c)^3(a+bx)^{n+1}}{b^{7(n+1)}} - \frac{6ad(a^2d+b^2c)^2(a+bx)^{n+2}}{b^{7(n+2)}} \\ & + \frac{3d(a^2d+b^2c)(5a^2d+b^2c)(a+bx)^{n+3}}{b^{7(n+3)}} - \frac{6ad^3(a+bx)^{n+6}}{b^{7(n+6)}} + \frac{d^3(a+bx)^{n+7}}{b^{7(n+7)}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + bx)^n (c + dx^2)^3, x]$

[Out]  $((b^2c + a^2d)^3(a + bx)^{(1+n)})/(b^{7(1+n)}) - (6ad(b^2c + a^2d)^2(a + bx)^{(2+n)})/(b^{7(2+n)}) + (3d(b^2c + a^2d)(b^2c + 5a^2d)(a + bx)^{(3+n)})/(b^{7(3+n)}) - (4a^2d^2(3b^2c + 5a^2d)(a + bx)^{(4+n)})/(b^{7(4+n)}) + (3d^2(b^2c + 5a^2d)(a + bx)^{(5+n)})/(b^{7(5+n)}) - (6ad^3(a + bx)^{(6+n)})/(b^{7(6+n)}) + (d^3(a + bx)^{(7+n)})/(b^{7(7+n)})$

**Rubi in Sympy [A]** time = 60.525, size = 207, normalized size = 0.93

$$\begin{aligned} & -\frac{6ad^3(a+bx)^{n+6}}{b^{7(n+6)}} - \frac{4ad^2(a+bx)^{n+4}(5a^2d+3b^2c)}{b^{7(n+4)}} - \frac{6ad(a+bx)^{n+2}(a^2d+b^2c)^2}{b^{7(n+2)}} + \frac{d^3(a+bx)^{n+7}}{b^{7(n+7)}} \\ & + \frac{3d^2(a+bx)^{n+5}(5a^2d+b^2c)}{b^{7(n+5)}} + \frac{3d(a+bx)^{n+3}(a^2d+b^2c)(5a^2d+b^2c)}{b^{7(n+3)}} + \frac{(a+bx)^{n+1}(a^2d+b^2c)^3}{b^{7(n+1)}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((b*x+a)**n*(d*x**2+c)**3, x)$

[Out]  $-6ad^3(a + bx)^{(n+6)}/(b^{7(n+6)}) - 4a^2d^2(a + bx)^{(n+4)}/(b^{7(n+4)}) - 6ad^2(a + bx)^{(n+2)}/(b^{7(n+2)}) + d^3(a + bx)^{(n+7)}/(b^{7(n+7)}) + 3d^2(a + bx)^{(n+5)}/(b^{7(n+5)}) + 3d(a + bx)^{(n+3)}/(b^{7(n+3)}) + (a + bx)^{(n+1)}/(b^{7(n+1)})$

$$c)/(b^{**7*(n + 5)}) + 3*d*(a + b*x)^{(n + 3)}*(a^{**2*d + b^{**2*c}}*(5*a^{**2*d + b^{**2*c}})/(b^{**7*(n + 3)}) + (a + b*x)^{(n + 1)}*(a^{**2*d + b^{**2*c}})^{**3}/(b^{**7*(n + 1)})$$

**Mathematica [A]** time = 0.408894, size = 413, normalized size = 1.85

$$(a + bx)^{n+1} (720a^6d^3 - 720a^5bd^3(n+1)x + 72a^4b^2d^2(c(n^2 + 13n + 42) + 5d(n^2 + 3n + 2)x^2) - 24a^3b^3d^2(n+1)x(3c(n^2 +$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x)^n\*(c + d\*x^2)^3,x]

[Out] ((a + b\*x)^(1 + n)\*(720\*a^6\*d^3 - 720\*a^5\*b\*d^3\*(1 + n)\*x + 72\*a^4\*b^2\*d^2\*(c\*(42 + 13\*n + n^2) + 5\*d\*(2 + 3\*n + n^2)\*x^2) - 24\*a^3\*b^3\*d^2\*(1 + n)\*x\*(3\*c\*(42 + 13\*n + n^2) + 5\*d\*(6 + 5\*n + n^2)\*x^2) + 6\*a^2\*b^4\*d\*(c^2\*(840 + 638\*n + 179\*n^2 + 22\*n^3 + n^4) + 6\*c\*d\*(84 + 152\*n + 83\*n^2 + 16\*n^3 + n^4)\*x^2 + 5\*d^2\*(24 + 50\*n + 35\*n^2 + 10\*n^3 + n^4)\*x^4) - 6\*a\*b^5\*d\*(1 + n)\*x\*(c^2\*(840 + 638\*n + 179\*n^2 + 22\*n^3 + n^4) + 2\*c\*d\*(252 + 288\*n + 113\*n^2 + 18\*n^3 + n^4)\*x^2 + d^2\*(120 + 154\*n + 71\*n^2 + 14\*n^3 + n^4)\*x^4) + b^6\*(48 + 44\*n + 12\*n^2 + n^3)\*(c^3\*(105 + 71\*n + 15\*n^2 + n^3) + 3\*c^2\*d\*(35 + 47\*n + 13\*n^2 + n^3)\*x^2 + 3\*c\*d^2\*(21 + 31\*n + 11\*n^2 + n^3)\*x^4 + d^3\*(15 + 23\*n + 9\*n^2 + n^3)\*x^6)))/(b^7\*(1 + n)\*(2 + n)\*(3 + n)\*(4 + n)\*(5 + n)\*(6 + n)\*(7 + n))

**Maple [B]** time = 0.016, size = 1140, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)^n\*(d\*x^2+c)^3,x)

[Out] (b\*x+a)^(1+n)\*(b^6\*d^3\*n^6\*x^6+21\*b^6\*d^3\*n^5\*x^6-6\*a\*b^5\*d^3\*n^5\*x^5+3\*b^6\*c\*d^2\*n^6\*x^4+175\*b^6\*d^3\*n^4\*x^6-90\*a\*b^5\*d^3\*n^4\*x^5+69\*b^6\*c\*d^2\*n^5\*x^4+735\*b^6\*d^3\*n^3\*x^6+30\*a^2\*b^4\*d^3\*n^4\*x^4-12\*a\*b^5\*c\*d^2\*n^5\*x^3-510\*a\*b^5\*d^3\*n^3\*x^5+3\*b^6\*c^2\*d\*n^6\*x^2+621\*b^6\*c\*d^2\*n^4\*x^4+1624\*b^6\*d^3\*n^2\*x^6+300\*a^2\*b^4\*d^3\*n^3\*x^4-228\*a\*b^5\*c\*d^2\*n^4\*x^3-1350\*a\*b^5\*d^3\*n^2\*x^5+75\*b^6\*c^2\*d\*n^5\*x^2+2775\*b^6\*c\*d^2\*n^3\*x^4+1764\*b^6\*d^3\*n\*x^6-120\*a^3\*b^3\*d^3\*n^3\*x^3+36\*a^2\*b^4\*c\*d^2\*n^4\*x^2+1050\*a^2\*b^4\*d^3\*n^2\*x^4-6\*a\*b^5\*c^2\*d\*n^5\*x-1572\*a\*b^5\*c\*d^2\*n^3\*x^3-1644\*a\*b^5\*d^3\*n\*x^5+b^6\*c^3\*n^6+741\*b^6\*c^2\*d\*n^4\*x^2+6432\*b^6\*c\*d^2\*n^2\*x^4+720\*b^6\*d^3\*x^6-720\*a^3\*b^3\*d^3\*n^2\*x^3+576\*a^2\*b^4\*c\*d^2\*n^3\*x^2+1500\*a^2\*b^4\*d^3\*n\*x^4-138\*a\*b^5\*c^2\*d\*n^4\*x-4812\*a\*b^5\*c\*d^2\*n^2\*x^3-720\*a\*b^5\*d^3\*x^5+27\*b^6\*c^3\*n^5+3657\*b^6\*c^2\*d\*n^3\*x^2+7236\*b^6\*c\*d^2\*n\*x^4+360\*a^4\*b^2\*d^3\*n^2\*x^2-72\*a^3\*b^3\*c\*d^2\*n^3\*x-1320\*a^3\*b^3\*d^3\*n\*x^3+6\*a^2\*b^4\*c^2\*d\*n^4+2988\*a^2\*b^4\*c\*d^2\*n^2\*x^2+720\*a^2\*b^4\*d^3\*x^4-1206\*a\*b^5\*c^2\*d\*n^3\*x-6480\*a\*b^5\*c\*d^2\*n\*x^3+295\*b^6\*c^3\*n^4+9336\*b^6\*c^2\*d\*n^2\*x^2+3024\*b^6\*c\*d^2\*x^4+1080\*a^4\*b^2\*d^3\*n\*x^2-1008\*a^3\*b^3\*c\*d^2\*n^2\*x-720\*a^3\*b^3\*d^3\*x^3+132\*a^2\*b^4\*c^2\*d\*n^3+5472\*a^2\*b^4\*c\*d^2\*n\*x^2-4902\*a\*b^5\*c^2\*d\*n^2\*x-3024\*a\*b^5\*c\*d^2\*x^3+1665\*b^6\*c^3\*n^3+11388\*b^6\*c^2\*d\*n\*x^2-720\*a^5\*b\*d^3\*n\*x+72\*a^4\*b^2\*c\*d^2\*n^2+720\*a^4\*b^2\*d^3\*x^2-3960\*a^3\*b^3\*c\*d^2\*n\*x+1074\*a^2\*b^4\*c^2\*d\*n^2+3024\*a^2\*b^4\*c\*d^2\*x^2-8868\*a\*b^5\*c^2\*d\*n\*x+5104\*b^6\*c^3\*n^2+5040\*b^6\*c^2\*d\*x^2-720\*a^5\*b\*d^3\*x+936\*a^4\*b^2\*c\*d^2\*n-3024\*a^3\*b^3\*c\*d^2\*x+3828\*a^2\*b^4\*c^2\*d\*n-5040\*a\*b^5\*c^2\*d\*x+8028\*b^6\*c^3\*n+720\*a^6\*d^3+3024\*a^4\*b^2\*c\*d^2+5040\*a^2\*b^4\*c^2\*d+5040\*b^6\*c^3)/b^7/(n^7+28\*n^6+322\*n^5+1960\*n^4+6769\*n^3+13132\*n^2+13068\*n+5040)

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**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)^3\*(b\*x + a)^n,x, algorithm="maxima")

[Out] Exception raised: ValueError

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**Fricas [A]** time = 0.286411, size = 1679, normalized size = 7.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)^3\*(b\*x + a)^n,x, algorithm="fricas")

[Out] 
$$(a^6 b^6 c^3 n^6 + 27 a^6 b^6 c^3 n^5 + 5040 a^6 b^6 c^3 + 5040 a^3 b^4 c^2 d + 3024 a^5 b^2 c^2 d^2 + 720 a^7 d^3 + (b^7 d^3 n^6 + 21 b^7 d^3 n^5 + 175 b^7 d^3 n^4 + 735 b^7 d^3 n^3 + 1624 b^7 d^3 n^2 + 1764 b^7 d^3 n + 720 b^7 d^3) x^7 + (a^6 b^6 d^3 n^6 + 15 a^6 b^6 d^3 n^5 + 85 a^6 b^6 d^3 n^4 + 225 a^6 b^6 d^3 n^3 + 274 a^6 b^6 d^3 n^2 + 120 a^6 b^6 d^3 n) x^6 + 3 (b^7 c^2 d^2 n^6 + 1008 b^7 c^2 d^2 + (23 b^7 c^2 d^2 - 2 a^2 b^5 d^3) n^5 + (207 b^7 c^2 d^2 - 20 a^2 b^5 d^3) n^4 + 5 (185 b^7 c^2 d^2 - 14 a^2 b^5 d^3) n^3 + 4 (536 b^7 c^2 d^2 - 25 a^2 b^5 d^3) n^2 + 12 (201 b^7 c^2 d^2 - 4 a^2 b^5 d^3) n) x^5 + (295 a^6 b^6 c^3 + 6 a^3 b^4 c^2 d) n^4 + 3 (a^6 b^6 c^2 d^2 n^6 + 19 a^6 b^6 c^2 d^2 n^5 + (131 a^6 b^6 c^2 d^2 + 10 a^3 b^4 d^3) n^4 + (401 a^6 b^6 c^2 d^2 + 60 a^3 b^4 d^3) n^3 + 10 (54 a^6 b^6 c^2 d^2 + 11 a^3 b^4 d^3) n^2 + 12 (21 a^6 b^6 c^2 d^2 + 5 a^3 b^4 d^3) n) x^4 + 3 (55 a^6 b^6 c^3 + 44 a^3 b^4 c^2 d) n^3 + 3 (b^7 c^2 d^2 n^6 + 1680 b^7 c^2 d^2 + (25 b^7 c^2 d^2 - 4 a^2 b^5 c^2 d^2) n^5 + (247 b^7 c^2 d^2 - 64 a^2 b^5 c^2 d^2) n^4 + (1219 b^7 c^2 d^2 - 332 a^2 b^5 c^2 d^2 - 40 a^4 b^3 d^3) n^3 + 8 (389 b^7 c^2 d^2 - 76 a^2 b^5 c^2 d^2 - 15 a^4 b^3 d^3) n^2 + 4 (949 b^7 c^2 d^2 - 84 a^2 b^5 c^2 d^2 - 20 a^4 b^3 d^3) n) x^3 + 2 (2552 a^6 b^6 c^3 + 537 a^3 b^4 c^2 d + 36 a^5 b^2 c^2 d^2) n^2 + 3 (a^6 b^6 c^2 d^2 n^6 + 23 a^6 b^6 c^2 d^2 n^5 + 3 (67 a^6 b^6 c^2 d^2 + 4 a^3 b^4 c^2 d^2) n^4 + (817 a^6 b^6 c^2 d^2 + 168 a^3 b^4 c^2 d^2) n^3 + 2 (739 a^6 b^6 c^2 d^2 + 330 a^3 b^4 c^2 d^2 + 60 a^5 b^2 d^3) n^2 + 24 (35 a^6 b^6 c^2 d^2 + 21 a^3 b^4 c^2 d^2 + 5 a^5 b^2 d^3) n) x^2 + 12 (669 a^6 b^6 c^3 + 319 a^3 b^4 c^2 d + 78 a^5 b^2 c^2 d^2) n + (b^7 c^3 n^6 + 5040 b^7 c^3 + 3 (9 b^7 c^3 - 2 a^2 b^5 c^2 d) n^5 + (295 b^7 c^3 - 132 a^2 b^5 c^2 d) n^4 + 3 (555 b^7 c^3 - 358 a^2 b^5 c^2 d - 24 a^4 b^3 c^2 d^2) n^3 + 4 (1276 b^7 c^3 - 957 a^2 b^5 c^2 d - 234 a^4 b^3 c^2 d^2) n^2 + 36 (223 b^7 c^3 - 140 a^2 b^5 c^2 d - 84 a^4 b^3 c^2 d^2 - 20 a^6 b^3 d^3) n) x) (b*x + a)^n / (b^7 n^7 + 28 b^7 n^6 + 322 b^7 n^5 + 1960 b^7 n^4 + 6769 b^7 n^3 + 13132 b^7 n^2 + 13068 b^7 n + 5040 b^7)$$

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**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*\*n\*(d\*x\*\*2+c)\*\*3,x)

[Out] Timed out

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**GIAC/XCAS [A]** time = 0.26887, size = 1, normalized size = 0.

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((d*x^2 + c)^3*(b*x + a)^n,x, algorithm="giac")`

[Out] Done

$$3.362 \quad \int \frac{(a+bx)^n (c+dx^2)^3}{x} dx$$

**Optimal.** Leaf size=246

$$\begin{aligned} & -\frac{ad^2(10a^2d+9b^2c)(a+bx)^{n+3}}{b^6(n+3)} + \frac{d^2(10a^2d+3b^2c)(a+bx)^{n+4}}{b^6(n+4)} \\ & -\frac{ad(a^4d^2+3a^2b^2cd+3b^4c^2)(a+bx)^{n+1}}{b^6(n+1)} + \frac{d(5a^4d^2+9a^2b^2cd+3b^4c^2)(a+bx)^{n+2}}{b^6(n+2)} \\ & -\frac{5ad^3(a+bx)^{n+5}}{b^6(n+5)} + \frac{d^3(a+bx)^{n+6}}{b^6(n+6)} - \frac{c^3(a+bx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{bx}{a} + 1\right)}{a(n+1)} \end{aligned}$$

[Out]  $-\left((a^*d*(3*b^4*c^2 + 3*a^2*b^2*c*d + a^4*d^2)*(a + b*x)^{(1 + n)})/(b^6*(1 + n)) + (d*(3*b^4*c^2 + 9*a^2*b^2*c*d + 5*a^4*d^2)*(a + b*x)^{(2 + n)})/(b^6*(2 + n)) - (a^*d^2*(9*b^2*c + 10*a^2*d)*(a + b*x)^{(3 + n)})/(b^6*(3 + n)) + (d^2*(3*b^2*c + 10*a^2*d)*(a + b*x)^{(4 + n)})/(b^6*(4 + n)) - (5*a^*d^3*(a + b*x)^{(5 + n)})/(b^6*(5 + n)) + (d^3*(a + b*x)^{(6 + n)})/(b^6*(6 + n)) - (c^3*(a + b*x)^{(1 + n)})^*$   
Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b\*x)/a]/(a\*(1 + n))

**Rubi [A]** time = 0.286879, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\begin{aligned} & -\frac{ad^2(10a^2d+9b^2c)(a+bx)^{n+3}}{b^6(n+3)} + \frac{d^2(10a^2d+3b^2c)(a+bx)^{n+4}}{b^6(n+4)} \\ & -\frac{ad(a^4d^2+3a^2b^2cd+3b^4c^2)(a+bx)^{n+1}}{b^6(n+1)} + \frac{d(5a^4d^2+9a^2b^2cd+3b^4c^2)(a+bx)^{n+2}}{b^6(n+2)} \\ & -\frac{5ad^3(a+bx)^{n+5}}{b^6(n+5)} + \frac{d^3(a+bx)^{n+6}}{b^6(n+6)} - \frac{c^3(a+bx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{bx}{a} + 1\right)}{a(n+1)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[((a + b\*x)^n\*(c + d\*x^2)^3)/x, x]

[Out]  $-\left((a^*d*(3*b^4*c^2 + 3*a^2*b^2*c*d + a^4*d^2)*(a + b*x)^{(1 + n)})/(b^6*(1 + n)) + (d*(3*b^4*c^2 + 9*a^2*b^2*c*d + 5*a^4*d^2)*(a + b*x)^{(2 + n)})/(b^6*(2 + n)) - (a^*d^2*(9*b^2*c + 10*a^2*d)*(a + b*x)^{(3 + n)})/(b^6*(3 + n)) + (d^2*(3*b^2*c + 10*a^2*d)*(a + b*x)^{(4 + n)})/(b^6*(4 + n)) - (5*a^*d^3*(a + b*x)^{(5 + n)})/(b^6*(5 + n)) + (d^3*(a + b*x)^{(6 + n)})/(b^6*(6 + n)) - (c^3*(a + b*x)^{(1 + n)})^*$   
Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (b\*x)/a]/(a\*(1 + n))

**Rubi in Sympy [A]** time = 61.3429, size = 226, normalized size = 0.92

$$\begin{aligned} & -\frac{5ad^3(a+bx)^{n+5}}{b^6(n+5)} - \frac{ad^2(a+bx)^{n+3}(10a^2d+9b^2c)}{b^6(n+3)} \\ & -\frac{ad(a+bx)^{n+1}(a^4d^2+3a^2b^2cd+3b^4c^2)}{b^6(n+1)} + \frac{d^3(a+bx)^{n+6}}{b^6(n+6)} + \frac{d^2(a+bx)^{n+4}(10a^2d+3b^2c)}{b^6(n+4)} \\ & + \frac{d(a+bx)^{n+2}(5a^4d^2+9a^2b^2cd+3b^4c^2)}{b^6(n+2)} - \frac{c^3(a+bx)^{n+1} {}_2F_1\left(1, n+1; n+2; 1 + \frac{bx}{a}\right)}{a(n+1)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x+a)\*\*n\*(d\*x\*\*2+c)\*\*3/x, x)

[Out]  $-5*a*d^{3*(a+b*x)**(n+5)/(b^{6*(n+5)})} - a*d^{2*(a+b*x)**(n+3)*(10*a^{2*d}+9*b^{2*c})/(b^{6*(n+3)})} - a*d*(a+b*x)**(n+1)*(a^{4*d^2}+3*a^{2*b^{2*c}}d+3*b^{4*c^2})/(b^{6*(n+1)}) + d^{3*(a+b*x)**(n+6)/(b^{6*(n+6)})} + d^{2*(a+b*x)**(n+4)*(10*a^{2*d}+3*b^{2*c})/(b^{6*(n+4)})} + d*(a+b*x)**(n+2)*(5*a^{4*d^2}+9*a^{2*b^{2*c}}d+3*b^{4*c^2})/(b^{6*(n+2)}) - c^{3*(a+b*x)**(n+1)*\text{hyper}((1, n+1), (n+2, ), 1+b*x/a)/(a*(n+1))}$

**Mathematica [B]** time = 0.906103, size = 546, normalized size = 2.22

$$(a+bx)^n \left( \frac{3c^2d \left(\frac{bx}{a}+1\right)^{-n} \left(-a^2 \left(\left(\frac{bx}{a}+1\right)^n - 1\right) + b^2(n+1)x^2 \left(\frac{bx}{a}+1\right)^n + abnx \left(\frac{bx}{a}+1\right)^n\right)}{b^2(n+1)(n+2)} \right. \\ + \frac{3cd^2 \left(\frac{bx}{a}+1\right)^{-n} \left(-6a^4 \left(\left(\frac{bx}{a}+1\right)^n - 1\right) + 6a^3bnx \left(\frac{bx}{a}+1\right)^n - 3a^2b^2n(n+1)x^2 \left(\frac{bx}{a}+1\right)^n + b^4(n^3+6n^2+11n+6)x^4 \left(\frac{bx}{a}+1\right)^n\right)}{b^4(n+1)(n+2)(n+3)(n+4)} \\ + \frac{d^3 \left(\frac{bx}{a}+1\right)^{-n} \left(-120a^6 \left(\left(\frac{bx}{a}+1\right)^n - 1\right) + 120a^5bnx \left(\frac{bx}{a}+1\right)^n - 60a^4b^2n(n+1)x^2 \left(\frac{bx}{a}+1\right)^n + 20a^3b^3n(n^2+3n+2)x^3 \left(\frac{bx}{a}+1\right)^n\right)}{b^6(n+1)(n+2)(n+3)(n+4)(n+5)} \\ \left. + \frac{c^3 \left(\frac{a}{bx}+1\right)^{-n} {}_2F_1\left(-n, -n; 1-n; -\frac{a}{bx}\right)}{n} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((a + b\*x)^n\*(c + d\*x^2)^3)/x, x]

[Out]  $(a+b*x)^n * ((3*c^2*d*(a*b^n*x*(1+(b*x)/a)^n + b^2*(1+n)*x^2*(1+(b*x)/a)^n - a^2*(-1+(1+(b*x)/a)^n)))/(b^2*(1+n)*(2+n)*(1+(b*x)/a)^n) + (3*c*d^2*(6*a^3*b^n*x*(1+(b*x)/a)^n - 3*a^2*b^2*n*(1+n)*x^2*(1+(b*x)/a)^n + a*b^3*n*(2+3*n+n^2)*x^3*(1+(b*x)/a)^n + b^4*(6+11*n+6*n^2+n^3)*x^4*(1+(b*x)/a)^n - 6*a^4*(-1+(1+(b*x)/a)^n)))/(b^4*(1+n)*(2+n)*(3+n)*(4+n)*(1+(b*x)/a)^n) + (d^3*(120*a^5*b^n*x*(1+(b*x)/a)^n - 60*a^4*b^2*n*(1+n)*x^2*(1+(b*x)/a)^n + 20*a^3*b^3*n*(2+3*n+n^2)*x^3*(1+(b*x)/a)^n - 5*a^2*b^4*n*(6+11*n+6*n^2+n^3)*x^4*(1+(b*x)/a)^n + a*b^5*n*(24+50*n+35*n^2+10*n^3+n^4)*x^5*(1+(b*x)/a)^n + b^6*(120+274*n+225*n^2+85*n^3+15*n^4+n^5)*x^6*(1+(b*x)/a)^n - 120*a^6*(-1+(1+(b*x)/a)^n)))/(b^6*(1+n)*(2+n)*(3+n)*(4+n)*(5+n)*(6+n)*(1+(b*x)/a)^n) + (c^3*Hypergeometric2F1[-n, -n, 1-n, -(a/(b*x))])/(n*(1+a/(b*x))^n)$

**Maple [F]** time = 0.044, size = 0, normalized size = 0.

$$\int \frac{(bx+a)^n (dx^2+c)^3}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x+a)^n\*(d\*x^2+c)^3/x, x)

[Out] int((b\*x+a)^n\*(d\*x^2+c)^3/x, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2+c)^3(bx+a)^n}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)^3\*(b\*x + a)^n/x,x, algorithm="maxima")

[Out] integrate((d\*x^2 + c)^3\*(b\*x + a)^n/x, x)

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(d^3x^6 + 3cd^2x^4 + 3c^2dx^2 + c^3)(bx + a)^n}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)^3\*(b\*x + a)^n/x,x, algorithm="fricas")

[Out] integral((d^3\*x^6 + 3\*c\*d^2\*x^4 + 3\*c^2\*d\*x^2 + c^3)\*(b\*x + a)^n/x, x)

**Sympy** [A] time = 33.9662, size = 5702, normalized size = 23.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x+a)\*\*n\*(d\*x\*\*2+c)\*\*3/x,x)

[Out]  $-b^{**n}c^{**3}n*(a/b + x)^{**n}\text{lerchphi}(1 + b*x/a, 1, n + 1)*\text{gamma}(n + 1)/\text{gamma}(n + 2) - b^{**n}c^{**3}(a/b + x)^{**n}\text{lerchphi}(1 + b*x/a, 1, n + 1)*\text{gamma}(n + 1)/\text{gamma}(n + 2) + 3*c^{**2}d*\text{Piecewise}((a^{**n}x^{**2}/2, \text{Eq}(b, 0)), (a*\log(a/b + x)/(a*b^{**2} + b^{**3}x) + b*x*\log(a/b + x)/(a*b^{**2} + b^{**3}x) - b*x/(a*b^{**2} + b^{**3}x), \text{Eq}(n, -2)), (-a*\log(a/b + x)/b^{**2} + x/b, \text{Eq}(n, -1)), (-a^{**2}*(a + b*x)^{**n}/(b^{**2}n^{**2} + 3*b^{**2}n + 2*b^{**2}) + a*b*n*x*(a + b*x)^{**n}/(b^{**2}n^{**2} + 3*b^{**2}n + 2*b^{**2}) + b^{**2}n*x^{**2}*(a + b*x)^{**n}/(b^{**2}n^{**2} + 3*b^{**2}n + 2*b^{**2}), \text{True})) + 3*c*d^{**2}*\text{Piecewise}((a^{**n}x^{**4}/4, \text{Eq}(b, 0)), (6*a^{**3}\log(a/b + x)/(6*a^{**3}b^{**4} + 18*a^{**2}b^{**5}x + 18*a*b^{**6}x^{**2} + 6*b^{**7}x^{**3}) + 2*a^{**3}/(6*a^{**3}b^{**4} + 18*a^{**2}b^{**5}x + 18*a*b^{**6}x^{**2} + 6*b^{**7}x^{**3}) + 18*a^{**2}b*x*\log(a/b + x)/(6*a^{**3}b^{**4} + 18*a^{**2}b^{**5}x + 18*a*b^{**6}x^{**2} + 6*b^{**7}x^{**3}) + 18*a*b^{**2}x^{**2}*\log(a/b + x)/(6*a^{**3}b^{**4} + 18*a^{**2}b^{**5}x + 18*a*b^{**6}x^{**2} + 6*b^{**7}x^{**3}) - 9*a*b^{**2}x^{**2}/(6*a^{**3}b^{**4} + 18*a^{**2}b^{**5}x + 18*a*b^{**6}x^{**2} + 6*b^{**7}x^{**3}) + 6*b^{**3}x^{**3}*\log(a/b + x)/(6*a^{**3}b^{**4} + 18*a^{**2}b^{**5}x + 18*a*b^{**6}x^{**2} + 6*b^{**7}x^{**3}) - 9*b^{**3}x^{**3}/(6*a^{**3}b^{**4} + 18*a^{**2}b^{**5}x + 18*a*b^{**6}x^{**2} + 6*b^{**7}x^{**3}), \text{Eq}(n, -4)), (-6*a^{**3}*\log(a/b + x)/(2*a^{**2}b^{**4} + 4*a*b^{**5}x + 2*b^{**6}x^{**2}) - 3*a^{**3}/(2*a^{**2}b^{**4} + 4*a*b^{**5}x + 2*b^{**6}x^{**2}) - 12*a^{**2}b*x*\log(a/b + x)/(2*a^{**2}b^{**4} + 4*a*b^{**5}x + 2*b^{**6}x^{**2}) - 6*a*b^{**2}x^{**2}*\log(a/b + x)/(2*a^{**2}b^{**4} + 4*a*b^{**5}x + 2*b^{**6}x^{**2}) + 6*a*b^{**2}x^{**2}/(2*a^{**2}b^{**4} + 4*a*b^{**5}x + 2*b^{**6}x^{**2}), \text{Eq}(n, -3)), (6*a^{**3}*\log(a/b + x)/(2*a*b^{**4} + 2*b^{**5}x) + 6*a^{**3}/(2*a*b^{**4} + 2*b^{**5}x) + 6*a^{**2}b*x*\log(a/b + x)/(2*a*b^{**4} + 2*b^{**5}x) - 3*a*b^{**2}x^{**2}/(2*a*b^{**4} + 2*b^{**5}x) + b^{**3}x^{**3}/(2*a*b^{**4} + 2*b^{**5}x), \text{Eq}(n, -2)), (-a^{**3}*\log(a/b + x)/b^{**4} + a^{**2}x/b^{**3} - a*x^{**2}/(2*b^{**2}) + x^{**3}/(3*b), \text{Eq}(n, -1)), (-6*a^{**4}*(a + b*x)^{**n}/(b^{**4}n^{**4} + 10*b^{**4}n^{**3} + 35*b^{**4}n^{**2} + 50*b^{**4}n + 24*b^{**4}) + 6*a^{**3}b*n*x*(a + b*x)^{**n}/(b^{**4}n^{**4} + 10*b^{**4}n^{**3} + 35*b^{**4}n^{**2} + 50*b^{**4}n + 24*b^{**4}) - 3*a^{**2}b^{**2}n^{**2}x^{**2}*(a + b*x)^{**n}/(b^{**4}n^{**4} + 10*b^{**4}n^{**3} + 35*b^{**4}n^{**2} + 50*b^{**4}n + 24*b^{**4}) - 3*a^{**2}b^{**2}n*x^{**2}*(a + b*x)^{**n}/(b^{**4}n^{**4} + 10*b^{**4}n^{**3} + 35*b^{**4}n^{**2} + 50*b^{**4}n + 24*b^{**4}) + a*b^{**$



$$\begin{aligned}
& 3^n x^3 (a + b^4 x)^n / (b^4 n^4 + 10 b^4 n^3 + 35 b^4 n^2 + 50 b^4 n + 24 b^4) + 3 a b^3 n^2 x^3 (a + b^4 x)^n / (b^4 n^4 + 10 b^4 n^3 + 35 b^4 n^2 + 50 b^4 n + 24 b^4) + 2 a b^3 n x^3 (a + b^4 x)^n / (b^4 n^4 + 10 b^4 n^3 + 35 b^4 n^2 + 50 b^4 n + 24 b^4) + b^4 n^3 x^4 (a + b^4 x)^n / (b^4 n^4 + 10 b^4 n^3 + 35 b^4 n^2 + 50 b^4 n + 24 b^4) + 6 b^4 n^2 x^4 (a + b^4 x)^n / (b^4 n^4 + 10 b^4 n^3 + 35 b^4 n^2 + 50 b^4 n + 24 b^4) + 11 b^4 n x^4 (a + b^4 x)^n / (b^4 n^4 + 10 b^4 n^3 + 35 b^4 n^2 + 50 b^4 n + 24 b^4) + 6 b^4 x^4 (a + b^4 x)^n / (b^4 n^4 + 10 b^4 n^3 + 35 b^4 n^2 + 50 b^4 n + 24 b^4), \text{True})) + d^3 \text{Piecewise}((a^n x^6 / 6, \text{Eq}(b, 0)), (60 a^5 \log(a/b + x) / (60 a^5 b^6 + 300 a^4 b^7 x + 600 a^3 b^8 x^2 + 600 a^2 b^9 x^3 + 300 a b^{10} x^4 + 60 b^{11} x^5) + 137 a^5 / (60 a^5 b^6 + 300 a^4 b^7 x + 600 a^3 b^8 x^2 + 600 a^2 b^9 x^3 + 300 a b^{10} x^4 + 60 b^{11} x^5) + 300 a^4 b^7 x \log(a/b + x) / (60 a^5 b^6 + 300 a^4 b^7 x + 600 a^3 b^8 x^2 + 600 a^2 b^9 x^3 + 300 a b^{10} x^4 + 60 b^{11} x^5) + 625 a^4 b^7 x / (60 a^5 b^6 + 300 a^4 b^7 x + 600 a^3 b^8 x^2 + 600 a^2 b^9 x^3 + 300 a b^{10} x^4 + 60 b^{11} x^5) + 600 a^3 b^8 x^2 \log(a/b + x) / (60 a^5 b^6 + 300 a^4 b^7 x + 600 a^3 b^8 x^2 + 600 a^2 b^9 x^3 + 300 a b^{10} x^4 + 60 b^{11} x^5) + 1100 a^3 b^8 x^2 / (60 a^5 b^6 + 300 a^4 b^7 x + 600 a^3 b^8 x^2 + 600 a^2 b^9 x^3 + 300 a b^{10} x^4 + 60 b^{11} x^5) + 600 a^2 b^9 x^3 \log(a/b + x) / (60 a^5 b^6 + 300 a^4 b^7 x + 600 a^3 b^8 x^2 + 600 a^2 b^9 x^3 + 300 a b^{10} x^4 + 60 b^{11} x^5) + 900 a^2 b^9 x^3 / (60 a^5 b^6 + 300 a^4 b^7 x + 600 a^3 b^8 x^2 + 600 a^2 b^9 x^3 + 300 a b^{10} x^4 + 60 b^{11} x^5) + 300 a b^{10} x^4 \log(a/b + x) / (60 a^5 b^6 + 300 a^4 b^7 x + 600 a^3 b^8 x^2 + 600 a^2 b^9 x^3 + 300 a b^{10} x^4 + 60 b^{11} x^5) + 300 a^4 b^7 x + 600 a^3 b^8 x^2 + 600 a^2 b^9 x^3 + 300 a b^{10} x^4 + 60 b^{11} x^5) + 60 b^5 x^5 \log(a/b + x) / (60 a^5 b^6 + 300 a^4 b^7 x + 600 a^3 b^8 x^2 + 600 a^2 b^9 x^3 + 300 a b^{10} x^4 + 60 b^{11} x^5), \text{Eq}(n, -6)), (-60 a^5 \log(a/b + x) / (12 a^4 b^6 + 48 a^3 b^7 x + 72 a^2 b^8 x^2 + 48 a b^9 x^3 + 12 b^{10} x^4) - 125 a^5 / (12 a^4 b^6 + 48 a^3 b^7 x + 72 a^2 b^8 x^2 + 48 a b^9 x^3 + 12 b^{10} x^4) - 240 a^4 b^7 x \log(a/b + x) / (12 a^4 b^6 + 48 a^3 b^7 x + 72 a^2 b^8 x^2 + 48 a b^9 x^3 + 12 b^{10} x^4) - 440 a^4 b^7 x / (12 a^4 b^6 + 48 a^3 b^7 x + 72 a^2 b^8 x^2 + 48 a b^9 x^3 + 12 b^{10} x^4) - 360 a^3 b^8 x^2 \log(a/b + x) / (12 a^4 b^6 + 48 a^3 b^7 x + 72 a^2 b^8 x^2 + 48 a b^9 x^3 + 12 b^{10} x^4) - 540 a^3 b^8 x^2 / (12 a^4 b^6 + 48 a^3 b^7 x + 72 a^2 b^8 x^2 + 48 a b^9 x^3 + 12 b^{10} x^4) - 240 a^2 b^9 x^3 \log(a/b + x) / (12 a^4 b^6 + 48 a^3 b^7 x + 72 a^2 b^8 x^2 + 48 a b^9 x^3 + 12 b^{10} x^4) - 240 a^2 b^9 x^3 / (12 a^4 b^6 + 48 a^3 b^7 x + 72 a^2 b^8 x^2 + 48 a b^9 x^3 + 12 b^{10} x^4) - 60 a b^{10} x^4 \log(a/b + x) / (12 a^4 b^6 + 48 a^3 b^7 x + 72 a^2 b^8 x^2 + 48 a b^9 x^3 + 12 b^{10} x^4) + 12 b^{10} x^4) - 60 a b^{10} x^4 / (6 a^3 b^6 + 18 a^2 b^7 x + 18 a b^8 x^2 + 6 b^9 x^3) + 20 a^5 / (6 a^3 b^6 + 18 a^2 b^7 x + 18 a b^8 x^2 + 6 b^9 x^3) + 180 a^4 b^7 x \log(a/b + x) / (6 a^3 b^6 + 18 a^2 b^7 x + 18 a b^8 x^2 + 6 b^9 x^3) + 180 a^3 b^8 x^2 \log(a/b + x) / (6 a^3 b^6 + 18 a^2 b^7 x + 18 a b^8 x^2 + 6 b^9 x^3) - 90 a^3 b^8 x^2 / (6 a^3 b^6 + 18 a^2 b^7 x + 18 a b^8 x^2 + 6 b^9 x^3) + 60 a^2 b^9 x^3 \log(a/b + x) / (6 a^3 b^6 + 18 a^2 b^7 x + 18 a b^8 x^2 + 6 b^9 x^3) - 90 a^2 b^9 x^3 / (6 a^3 b^6 + 18 a^2 b^7 x + 18 a b^8 x^2 + 6 b^9 x^3) - 15 a b^9 x^3 / (6 a^3 b^6 + 18 a^2 b^7 x + 18 a b^8 x^2 + 6 b^9 x^3) + 3 b^9 x^3 / (6 a^3 b^6 + 18 a^2 b^7 x + 18 a b^8 x^2 + 6 b^9 x^3), \text{Eq}(n, -4)), (-60 a^5 \log(a/b + x) / (6 a^2 b^6 + 12 a b^7 x + 6 b^8 x^2) - 30 a^5 / (6 a^2 b^6 + 12 a b^7 x + 6 b^8 x^2) - 120 a^4 b^7 x \log(a/b + x) / (6 a^2 b^6 + 12 a b^7 x + 6 b^8 x^2) - 60 a^3 b^8 x^2 \log(a/b + x) / (6 a^2 b^6 + 12 a b^7 x + 6 b^8 x^2) + 60 a^3 b^8 x^2 / (6 a^2 b^6 + 12 a b^7 x + 6 b^8 x^2) + 20 a^2 b^9 x^3 / (6 a^2 b^6 + 12 a b^7 x + 6 b^8 x^2) - 5 a b^9 x^3 / (6 a^2 b^6 + 12 a b^7 x + 6 b^8 x^2) + 2 b^9 x^3 / (6 a^2 b^6 + 12 a b^7 x + 6 b^8 x^2), \text{Eq}(n, -3)), (60 a^5 \log(a/b + x) / (12 a b^6 + 12 b^7 x) + 60 a^4 b^7 x \log(a/b + x) / (
\end{aligned}$$

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12*a*b**6 + 12*b**7*x) - 30*a**3*b**2*x**2/(12*a*b**6 + 12*b**7*x
) + 10*a**2*b**3*x**3/(12*a*b**6 + 12*b**7*x) - 5*a*b**4*x**4/(12
*a*b**6 + 12*b**7*x) + 3*b**5*x**5/(12*a*b**6 + 12*b**7*x), Eq(n,
-2)), (-a**5*log(a/b + x)/b**6 + a**4*x/b**5 - a**3*x**2/(2*b**4
) + a**2*x**3/(3*b**3) - a*x**4/(4*b**2) + x**5/(5*b), Eq(n, -1))
, (-120*a**6*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n*
**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 1
20*a**5*b*n*x*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n
**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) -
60*a**4*b**2*n**2*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 1
75*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720
*b**6) - 60*a**4*b**2*n*x**2*(a + b*x)**n/(b**6*n**6 + 21*b**6*n*
**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n
+ 720*b**6) + 20*a**3*b**3*n**3*x**3*(a + b*x)**n/(b**6*n**6 + 2
1*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 17
64*b**6*n + 720*b**6) + 60*a**3*b**3*n**2*x**3*(a + b*x)**n/(b**6
*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*
n**2 + 1764*b**6*n + 720*b**6) + 40*a**3*b**3*n*x**3*(a + b*x)**n
/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624
*b**6*n**2 + 1764*b**6*n + 720*b**6) - 5*a**2*b**4*n**4*x**4*(a +
b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**
3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 30*a**2*b**4*n**3*
x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735
*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) - 55*a**2*b
**4*n**2*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n
**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) -
30*a**2*b**4*n*x**4*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*
b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720*b*
**6) + a*b**5*n**5*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 1
75*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n + 720
*b**6) + 10*a*b**5*n**4*x**5*(a + b*x)**n/(b**6*n**6 + 21*b**6*n*
**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*b**6*n
+ 720*b**6) + 35*a*b**5*n**3*x**5*(a + b*x)**n/(b**6*n**6 + 21*b
**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 + 1764*
b**6*n + 720*b**6) + 50*a*b**5*n**2*x**5*(a + b*x)**n/(b**6*n**6
+ 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**2 +
1764*b**6*n + 720*b**6) + 24*a*b**5*n*x**5*(a + b*x)**n/(b**6*n*
**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n**
2 + 1764*b**6*n + 720*b**6) + b**6*n**5*x**6*(a + b*x)**n/(b**6*n
**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**6*n*
**2 + 1764*b**6*n + 720*b**6) + 15*b**6*n**4*x**6*(a + b*x)**n/(b*
**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624*b**
6*n**2 + 1764*b**6*n + 720*b**6) + 85*b**6*n**3*x**6*(a + b*x)**n
/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 + 1624
*b**6*n**2 + 1764*b**6*n + 720*b**6) + 225*b**6*n**2*x**6*(a + b*
x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3 +
1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 274*b**6*n*x**6*(a +
b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3
+ 1624*b**6*n**2 + 1764*b**6*n + 720*b**6) + 120*b**6*x**6*(a +
b*x)**n/(b**6*n**6 + 21*b**6*n**5 + 175*b**6*n**4 + 735*b**6*n**3
+ 1624*b**6*n**2 + 1764*b**6*n + 720*b**6), True)) - b*b**n*c**3
*n*x*(a/b + x)**n*lerchphi(1 + b*x/a, 1, n + 1)*gamma(n + 1)/(a*g
amma(n + 2)) - b*b**n*c**3*x*(a/b + x)**n*lerchphi(1 + b*x/a, 1,
n + 1)*gamma(n + 1)/(a*gamma(n + 2))

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(dx^2 + c)^3 (bx + a)^n}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d\*x^2 + c)^3\*(b\*x + a)^n/x,x, algorithm="giac")

[Out] integrate((d\*x^2 + c)^3\*(b\*x + a)^n/x, x)

$$3.363 \quad \int \frac{x^4(d+ex)^n}{a+cx^2} dx$$

**Optimal.** Leaf size=250

$$\frac{(cd^2 - ae^2)(d+ex)^{n+1}}{c^2e^3(n+1)} + \frac{(-a)^{3/2}(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}-\sqrt{-ae}}\right)}{2c^2(n+1)(\sqrt{cd}-\sqrt{-ae})}$$

$$- \frac{(-a)^{3/2}(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}+\sqrt{-ae}}\right)}{2c^2(n+1)(\sqrt{-ae}+\sqrt{cd})} - \frac{2d(d+ex)^{n+2}}{ce^3(n+2)} + \frac{(d+ex)^{n+3}}{ce^3(n+3)}$$

[Out] ((c\*d^2 - a\*e^2)\*(d + e\*x)^(1 + n))/(c^2\*e^3\*(1 + n)) - (2\*d\*(d + e\*x)^(2 + n))/(c\*e^3\*(2 + n)) + (d + e\*x)^(3 + n)/(c\*e^3\*(3 + n)) + ((-a)^(3/2)\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d - Sqrt[-a]\*e)]/(2\*c^2\*(Sqrt[c]\*d - Sqrt[-a]\*e)^(1 + n)) - ((-a)^(3/2)\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d + Sqrt[-a]\*e)]/(2\*c^2\*(Sqrt[c]\*d + Sqrt[-a]\*e)^(1 + n)))

**Rubi [A]** time = 0.696263, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{(cd^2 - ae^2)(d+ex)^{n+1}}{c^2e^3(n+1)} + \frac{(-a)^{3/2}(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}-\sqrt{-ae}}\right)}{2c^2(n+1)(\sqrt{cd}-\sqrt{-ae})}$$

$$- \frac{(-a)^{3/2}(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}+\sqrt{-ae}}\right)}{2c^2(n+1)(\sqrt{-ae}+\sqrt{cd})} - \frac{2d(d+ex)^{n+2}}{ce^3(n+2)} + \frac{(d+ex)^{n+3}}{ce^3(n+3)}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*(d + e\*x)^n)/(a + c\*x^2), x]

[Out] ((c\*d^2 - a\*e^2)\*(d + e\*x)^(1 + n))/(c^2\*e^3\*(1 + n)) - (2\*d\*(d + e\*x)^(2 + n))/(c\*e^3\*(2 + n)) + (d + e\*x)^(3 + n)/(c\*e^3\*(3 + n)) + ((-a)^(3/2)\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d - Sqrt[-a]\*e)]/(2\*c^2\*(Sqrt[c]\*d - Sqrt[-a]\*e)^(1 + n)) - ((-a)^(3/2)\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d + Sqrt[-a]\*e)]/(2\*c^2\*(Sqrt[c]\*d + Sqrt[-a]\*e)^(1 + n)))

**Rubi in Sympy [A]** time = 89.9628, size = 204, normalized size = 0.82

$$- \frac{2d(d+ex)^{n+2}}{ce^3(n+2)} + \frac{(d+ex)^{n+3}}{ce^3(n+3)} - \frac{(-a)^{3/2}(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd+e\sqrt{-a}}}\right)}{2c^2(n+1)(\sqrt{cd+e\sqrt{-a}})}$$

$$+ \frac{(-a)^{3/2}(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd-e\sqrt{-a}}}\right)}{2c^2(n+1)(\sqrt{cd-e\sqrt{-a}})} - \frac{(d+ex)^{n+1}(ae^2 - cd^2)}{c^2e^3(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(e\*x+d)\*\*n/(c\*x\*\*2+a), x)

[Out] -2\*d\*(d + e\*x)\*\*(n + 2)/(c\*e\*\*3\*(n + 2)) + (d + e\*x)\*\*(n + 3)/(c\*e\*\*3\*(n + 3)) - (-a)\*\*(3/2)\*(d + e\*x)\*\*(n + 1)\*hyper((1, n + 1), (n + 2, ), sqrt(c)\*(d + e\*x)/(sqrt(c)\*d + e\*sqrt(-a)))/(2\*c\*\*2\*(n

$+ 1) \cdot (\sqrt{c} \cdot d + e \cdot \sqrt{-a})) + (-a)^{(3/2)} \cdot (d + e \cdot x)^{(n+1)} \cdot \operatorname{hyper}((1, n+1), (n+2, ), \sqrt{c} \cdot (d + e \cdot x) / (\sqrt{c} \cdot d - e \cdot \sqrt{-a})) / (2 \cdot c^{n+1} \cdot (\sqrt{c} \cdot d - e \cdot \sqrt{-a})) - (d + e \cdot x)^{(n+1)} \cdot (a \cdot e^{2n} - c \cdot d^{2n}) / (c^{n+1} \cdot e^{3n} \cdot (n+1))$

**Mathematica [C]** time = 0.915572, size = 354, normalized size = 1.42

$$(d + ex)^n \left( \frac{ia^{3/2} \sqrt{c} e^3 \left( \frac{\sqrt{c}(d+ex)}{e(\sqrt{cx+i\sqrt{a}})} \right)^{-n} {}_2F_1(-n, -n; 1-n; -\frac{\sqrt{c}d-i\sqrt{ae}}{\sqrt{cx+e+i\sqrt{a}}}) - \left( \frac{\sqrt{c}(d+ex)}{e(\sqrt{cx-i\sqrt{a}})} \right)^{-n} {}_2F_1(-n, -n; 1-n; \frac{\sqrt{c}d+i\sqrt{ae}}{i\sqrt{ae}-\sqrt{cex}})}{n} - \frac{2ace^2(d+ex)}{n+1} + \frac{2c^2 \left( 2d^3 \left( \frac{ex}{d} \right) \right)}{2c^3 e^3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^4\*(d + e\*x)^n)/(a + c\*x^2), x]

[Out]  $((d + e \cdot x)^n \cdot ((-2 \cdot a \cdot c \cdot e^{2n} \cdot (d + e \cdot x)) / (1 + n) + (2 \cdot c^2 \cdot (-2 \cdot d^2 \cdot e^n \cdot x \cdot (1 + (e \cdot x) / d)^n + d \cdot e^{2n} \cdot (1 + n) \cdot x^2 \cdot (1 + (e \cdot x) / d)^n + e^{3n} \cdot (2 + 3 \cdot n + n^2) \cdot x^3 \cdot (1 + (e \cdot x) / d)^n + 2 \cdot d^3 \cdot (-1 + (1 + (e \cdot x) / d)^n)) / ((1 + n) \cdot (2 + n) \cdot (3 + n) \cdot (1 + (e \cdot x) / d)^n + (I \cdot a^{3/2} \cdot \operatorname{Sqrt}[c] \cdot e^{3n} \cdot (-\operatorname{Hypergeometric2F1}[-n, -n, 1 - n, (\operatorname{Sqrt}[c] \cdot d + I \cdot \operatorname{Sqrt}[a] \cdot e) / (I \cdot \operatorname{Sqrt}[a] \cdot e - \operatorname{Sqrt}[c] \cdot e \cdot x)] / ((\operatorname{Sqrt}[c] \cdot (d + e \cdot x)) / (e \cdot ((-I) \cdot \operatorname{Sqrt}[a] + \operatorname{Sqrt}[c] \cdot x)))^n + \operatorname{Hypergeometric2F1}[-n, -n, 1 - n, -((\operatorname{Sqrt}[c] \cdot d - I \cdot \operatorname{Sqrt}[a] \cdot e) / (I \cdot \operatorname{Sqrt}[a] \cdot e + \operatorname{Sqrt}[c] \cdot e \cdot x))] / ((\operatorname{Sqrt}[c] \cdot (d + e \cdot x)) / (e \cdot (I \cdot \operatorname{Sqrt}[a] + \operatorname{Sqrt}[c] \cdot x)))^n)) / n)) / (2 \cdot c^3 \cdot e^3)$

**Maple [F]** time = 0.091, size = 0, normalized size = 0.

$$\int \frac{x^4 (ex + d)^n}{cx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(e\*x+d)^n/(c\*x^2+a), x)

[Out] int(x^4\*(e\*x+d)^n/(c\*x^2+a), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n x^4}{cx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^n\*x^4/(c\*x^2 + a), x, algorithm="maxima")

[Out] integrate((e\*x + d)^n\*x^4/(c\*x^2 + a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(ex + d)^n x^4}{cx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^n*x^4/(c*x^2 + a),x, algorithm="fricas")`

[Out] `integral((e*x + d)^n*x^4/(c*x^2 + a), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(e*x+d)**n/(c*x**2+a),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n x^4}{cx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^n*x^4/(c*x^2 + a),x, algorithm="giac")`

[Out] `integrate((e*x + d)^n*x^4/(c*x^2 + a), x)`

$$3.364 \quad \int \frac{x^3(d+ex)^n}{a+cx^2} dx$$

**Optimal.** Leaf size=209

$$\frac{a(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c(d+ex)}}{\sqrt{cd-\sqrt{-ae}}}\right)}{2c^{3/2}(n+1)(\sqrt{cd}-\sqrt{-ae})} + \frac{a(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c(d+ex)}}{\sqrt{cd+\sqrt{-ae}}}\right)}{2c^{3/2}(n+1)(\sqrt{-ae}+\sqrt{cd})} - \frac{d(d+ex)^{n+1}}{ce^2(n+1)} + \frac{(d+ex)^{n+2}}{ce^2(n+2)}$$

[Out]  $-\left(\frac{d(d+ex)^{n+1}}{c^2e^{2(n+1)}}\right) + \frac{(d+ex)^{n+2}}{c^2e^{2(n+2)}} + \frac{a(d+ex)^{n+1} \operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{\sqrt{c(d+ex)}}{\sqrt{cd-\sqrt{-ae}}}\right]}{2c^{3/2}(n+1)(\sqrt{cd}-\sqrt{-ae})} + \frac{a(d+ex)^{n+1} \operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{\sqrt{c(d+ex)}}{\sqrt{cd+\sqrt{-ae}}}\right]}{2c^{3/2}(n+1)(\sqrt{-ae}+\sqrt{cd})} - \frac{d(d+ex)^{n+1}}{ce^2(n+1)} + \frac{(d+ex)^{n+2}}{ce^2(n+2)}$

**Rubi [A]** time = 0.40901, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{a(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c(d+ex)}}{\sqrt{cd-\sqrt{-ae}}}\right)}{2c^{3/2}(n+1)(\sqrt{cd}-\sqrt{-ae})} + \frac{a(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c(d+ex)}}{\sqrt{cd+\sqrt{-ae}}}\right)}{2c^{3/2}(n+1)(\sqrt{-ae}+\sqrt{cd})} - \frac{d(d+ex)^{n+1}}{ce^2(n+1)} + \frac{(d+ex)^{n+2}}{ce^2(n+2)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\frac{x^3(d+ex)^n}{a+cx^2}, x\right]$

[Out]  $-\left(\frac{d(d+ex)^{n+1}}{c^2e^{2(n+1)}}\right) + \frac{(d+ex)^{n+2}}{c^2e^{2(n+2)}} + \frac{a(d+ex)^{n+1} \operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{\sqrt{c(d+ex)}}{\sqrt{cd-\sqrt{-ae}}}\right]}{2c^{3/2}(n+1)(\sqrt{cd}-\sqrt{-ae})} + \frac{a(d+ex)^{n+1} \operatorname{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{\sqrt{c(d+ex)}}{\sqrt{cd+\sqrt{-ae}}}\right]}{2c^{3/2}(n+1)(\sqrt{-ae}+\sqrt{cd})} - \frac{d(d+ex)^{n+1}}{ce^2(n+1)} + \frac{(d+ex)^{n+2}}{ce^2(n+2)}$

**Rubi in Sympy [A]** time = 59.8406, size = 167, normalized size = 0.8

$$\frac{a(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c(d+ex)}}{\sqrt{cd+e\sqrt{-a}}}\right)}{2c^{\frac{3}{2}}(n+1)(\sqrt{cd+e\sqrt{-a}})} + \frac{a(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c(d+ex)}}{\sqrt{cd-e\sqrt{-a}}}\right)}{2c^{\frac{3}{2}}(n+1)(\sqrt{cd-e\sqrt{-a}})} - \frac{d(d+ex)^{n+1}}{ce^2(n+1)} + \frac{(d+ex)^{n+2}}{ce^2(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{rubi\_integrate}(x^{**3}*(e*x+d)^{**n}/(c*x^{**2}+a), x)$

[Out]  $a*(d+e*x)^{**n+1}*\operatorname{hyper}((1, n+1), (n+2, ), \operatorname{sqrt}(c)*(d+e*x)/(\operatorname{sqrt}(c)*d+e*\operatorname{sqrt}(-a)))/(2*c^{**3/2}*(n+1)*(\operatorname{sqrt}(c)*d+e*\operatorname{sqrt}(-a))) + a*(d+e*x)^{**n+1}*\operatorname{hyper}((1, n+1), (n+2, ), \operatorname{sqrt}(c)*(d+e*x)/(\operatorname{sqrt}(c)*d-e*\operatorname{sqrt}(-a)))/(2*c^{**3/2}*(n+1)*(\operatorname{sqrt}(c)*d-e*\operatorname{sqrt}(-a))) - d*(d+e*x)^{**n+1}/(c*e^{**2}*(n+1)) + (d+e*x)^{**n+2}/(c*e^{**2}*(n+2))$

**Mathematica [C]** time = 1.1137, size = 275, normalized size = 1.32

$$\frac{(d + ex)^n \left( -a (n^2 + 3n + 2) \left( \frac{\sqrt{c}(d+ex)}{e(\sqrt{cx}-i\sqrt{a})} \right)^{-n} {}_2F_1 \left( -n, -n; 1 - n; \frac{\sqrt{c}d+i\sqrt{ae}}{i\sqrt{ae}-\sqrt{c}ex} \right) - a (n^2 + 3n + 2) \left( \frac{\sqrt{c}(d+ex)}{e(\sqrt{cx}+i\sqrt{a})} \right)^{-n} {}_2F_1 \left( -n, -n; \right. \right.}{2c^2n(n+1)(n+2)}$$

Antiderivative was successfully verified.

[In] Integrate[(x^3\*(d + e\*x)^n)/(a + c\*x^2), x]

[Out] ((d + e\*x)^n\*((2\*c\*d\*n^2\*x)/e + 2\*c\*n\*(1 + n)\*x^2 + (2\*c\*d^2\*n\*(-1 + (1 + (e\*x)/d)^(-n)))/e^2 - (a\*(2 + 3\*n + n^2)\*Hypergeometric2F1[-n, -n, 1 - n, (Sqrt[c]\*d + I\*Sqrt[a]\*e)/(I\*Sqrt[a]\*e - Sqrt[c]\*e\*x)])/((Sqrt[c]\*(d + e\*x))/(e\*((-I)\*Sqrt[a] + Sqrt[c]\*x)))^n - (a\*(2 + 3\*n + n^2)\*Hypergeometric2F1[-n, -n, 1 - n, -(Sqrt[c]\*d - I\*Sqrt[a]\*e)/(I\*Sqrt[a]\*e + Sqrt[c]\*e\*x)])/((Sqrt[c]\*(d + e\*x))/(e\*(I\*Sqrt[a] + Sqrt[c]\*x)))^n)/(2\*c^2\*n\*(1 + n)\*(2 + n))

**Maple [F]** time = 0.079, size = 0, normalized size = 0.

$$\int \frac{x^3 (ex + d)^n}{cx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(e\*x+d)^n/(c\*x^2+a), x)

[Out] int(x^3\*(e\*x+d)^n/(c\*x^2+a), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n x^3}{cx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^n\*x^3/(c\*x^2 + a), x, algorithm="maxima")

[Out] integrate((e\*x + d)^n\*x^3/(c\*x^2 + a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{(ex + d)^n x^3}{cx^2 + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^n\*x^3/(c\*x^2 + a), x, algorithm="fricas")

[Out] integral((e\*x + d)^n\*x^3/(c\*x^2 + a), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(e*x+d)**n/(c*x**2+a),x)`

[Out] Timed out

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n x^3}{cx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^n*x^3/(c*x^2 + a),x, algorithm="giac")`

[Out] `integrate((e*x + d)^n*x^3/(c*x^2 + a), x)`



$$3.365 \quad \int \frac{x^2(d+ex)^n}{a+cx^2} dx$$

**Optimal.** Leaf size=194

$$\frac{\sqrt{-a}(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}-\sqrt{-ae}}\right)}{2c(n+1)(\sqrt{cd}-\sqrt{-ae})} - \frac{\sqrt{-a}(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}+\sqrt{-ae}}\right)}{2c(n+1)(\sqrt{-ae}+\sqrt{cd})} + \frac{(d+ex)^{n+1}}{ce(n+1)}$$

[Out] (d + e\*x)^(1 + n)/(c\*e\*(1 + n)) + (Sqrt[-a]\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d - Sqrt[-a]\*e)]/(2\*c\*(Sqrt[c]\*d - Sqrt[-a]\*e)\*(1 + n)) - (Sqrt[-a]\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d + Sqrt[-a]\*e)]/(2\*c\*(Sqrt[c]\*d + Sqrt[-a]\*e)\*(1 + n))

**Rubi [A]** time = 0.470028, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{\sqrt{-a}(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}-\sqrt{-ae}}\right)}{2c(n+1)(\sqrt{cd}-\sqrt{-ae})} - \frac{\sqrt{-a}(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}+\sqrt{-ae}}\right)}{2c(n+1)(\sqrt{-ae}+\sqrt{cd})} + \frac{(d+ex)^{n+1}}{ce(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(d + e\*x)^n)/(a + c\*x^2), x]

[Out] (d + e\*x)^(1 + n)/(c\*e\*(1 + n)) + (Sqrt[-a]\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d - Sqrt[-a]\*e)]/(2\*c\*(Sqrt[c]\*d - Sqrt[-a]\*e)\*(1 + n)) - (Sqrt[-a]\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d + Sqrt[-a]\*e)]/(2\*c\*(Sqrt[c]\*d + Sqrt[-a]\*e)\*(1 + n))

**Rubi in Sympy [A]** time = 77.3805, size = 150, normalized size = 0.77

$$-\frac{\sqrt{-a}(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}+e\sqrt{-a}}\right)}{2c(n+1)(\sqrt{cd}+e\sqrt{-a})} + \frac{\sqrt{-a}(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}-e\sqrt{-a}}\right)}{2c(n+1)(\sqrt{cd}-e\sqrt{-a})} + \frac{(d+ex)^{n+1}}{ce(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(e\*x+d)\*\*n/(c\*x\*\*2+a), x)

[Out] -sqrt(-a)\*(d + e\*x)\*\*(n + 1)\*hyper((1, n + 1), (n + 2, ), sqrt(c)\*(d + e\*x)/(sqrt(c)\*d + e\*sqrt(-a)))/(2\*c\*(n + 1)\*(sqrt(c)\*d + e\*sqrt(-a))) + sqrt(-a)\*(d + e\*x)\*\*(n + 1)\*hyper((1, n + 1), (n + 2, ), sqrt(c)\*(d + e\*x)/(sqrt(c)\*d - e\*sqrt(-a)))/(2\*c\*(n + 1)\*(sqrt(c)\*d - e\*sqrt(-a))) + (d + e\*x)\*\*(n + 1)/(c\*e\*(n + 1))

**Mathematica [C]** time = 0.442627, size = 233, normalized size = 1.2

$$\frac{(d+ex)^n \left( \frac{2c(d+ex)}{n+1} - \frac{i\sqrt{a}\sqrt{ce} \left( \left( \frac{\sqrt{c}(d+ex)}{e(\sqrt{cx}+i\sqrt{a})} \right)^{-n} {}_2F_1\left(-n, -n; 1-n; -\frac{\sqrt{cd}-i\sqrt{ae}}{\sqrt{cx}+i\sqrt{ae}}\right) - \left( \frac{\sqrt{c}(d+ex)}{e(\sqrt{cx}-i\sqrt{a})} \right)^{-n} {}_2F_1\left(-n, -n; 1-n; \frac{\sqrt{cd}+i\sqrt{ae}}{i\sqrt{ae}-\sqrt{cex}}\right) \right)}{n} \right)}{2c^2e}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*(d + e\*x)^n)/(a + c\*x^2), x]

[Out]  $((d + e*x)^n * ((2*c*(d + e*x))/(1 + n) - (I*\text{Sqrt}[a]*\text{Sqrt}[c]*e*(-\text{Hypergeometric2F1}[-n, -n, 1 - n, (\text{Sqrt}[c]*d + I*\text{Sqrt}[a]*e)/(I*\text{Sqrt}[a]*e - \text{Sqrt}[c]*e*x)]/((\text{Sqrt}[c]*(d + e*x))/(e*((-I)*\text{Sqrt}[a] + \text{Sqrt}[c]*x)))^n) + \text{Hypergeometric2F1}[-n, -n, 1 - n, -((\text{Sqrt}[c]*d - I*\text{Sqrt}[a]*e)/(I*\text{Sqrt}[a]*e + \text{Sqrt}[c]*e*x))]/((\text{Sqrt}[c]*(d + e*x))/(e*(I*\text{Sqrt}[a] + \text{Sqrt}[c]*x)))^n))/n)/(2*c^2*e)$

**Maple** [F] time = 0.073, size = 0, normalized size = 0.

$$\int \frac{x^2 (ex + d)^n}{cx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(e\*x+d)^n/(c\*x^2+a), x)

[Out] int(x^2\*(e\*x+d)^n/(c\*x^2+a), x)

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n x^2}{cx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^n\*x^2/(c\*x^2 + a), x, algorithm="maxima")

[Out] integrate((e\*x + d)^n\*x^2/(c\*x^2 + a), x)

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex + d)^n x^2}{cx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^n\*x^2/(c\*x^2 + a), x, algorithm="fricas")

[Out] integral((e\*x + d)^n\*x^2/(c\*x^2 + a), x)

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2 (d + ex)^n}{a + cx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(e\*x+d)\*\*n/(c\*x\*\*2+a), x)

[Out] Integral(x\*\*2\*(d + e\*x)\*\*n/(a + c\*x\*\*2), x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n x^2}{cx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^n\*x^2/(c\*x^2 + a),x, algorithm="giac")

[Out] integrate((e\*x + d)^n\*x^2/(c\*x^2 + a), x)

$$3.366 \quad \int \frac{x(d+ex)^n}{a+cx^2} dx$$

**Optimal.** Leaf size=163

$$\frac{(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}-\sqrt{-ae}}\right)}{2\sqrt{c}(n+1)(\sqrt{cd}-\sqrt{-ae})} - \frac{(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}+\sqrt{-ae}}\right)}{2\sqrt{c}(n+1)(\sqrt{-ae}+\sqrt{cd})}$$

[Out]  $-\left(\frac{(d+e*x)^{(1+n)} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \left(\frac{\sqrt{c}}{\sqrt{cd}-\sqrt{-ae}}\right)\left(\frac{d+e*x}{\sqrt{c}}\right)\right]}{2\sqrt{c}\left(\sqrt{cd}-\sqrt{-ae}\right)} - \frac{(d+e*x)^{(1+n)} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \left(\frac{\sqrt{c}}{\sqrt{cd}+\sqrt{-ae}}\right)\left(\frac{d+e*x}{\sqrt{c}}\right)\right]}{2\sqrt{c}\left(\sqrt{-ae}+\sqrt{cd}\right)}\right)$

**Rubi [A]** time = 0.197806, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}-\sqrt{-ae}}\right)}{2\sqrt{c}(n+1)(\sqrt{cd}-\sqrt{-ae})} - \frac{(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}+\sqrt{-ae}}\right)}{2\sqrt{c}(n+1)(\sqrt{-ae}+\sqrt{cd})}$$

Antiderivative was successfully verified.

[In] Int[(x\*(d + e\*x)^n)/(a + c\*x^2), x]

[Out]  $-\left(\frac{(d+e*x)^{(1+n)} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \left(\frac{\sqrt{c}}{\sqrt{cd}-\sqrt{-ae}}\right)\left(\frac{d+e*x}{\sqrt{c}}\right)\right]}{2\sqrt{c}\left(\sqrt{cd}-\sqrt{-ae}\right)} - \frac{(d+e*x)^{(1+n)} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \left(\frac{\sqrt{c}}{\sqrt{cd}+\sqrt{-ae}}\right)\left(\frac{d+e*x}{\sqrt{c}}\right)\right]}{2\sqrt{c}\left(\sqrt{-ae}+\sqrt{cd}\right)}\right)$

**Rubi in Sympy [A]** time = 38.3642, size = 129, normalized size = 0.79

$$\frac{(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}+e\sqrt{-a}}\right)}{2\sqrt{c}(n+1)(\sqrt{cd}+e\sqrt{-a})} - \frac{(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}-e\sqrt{-a}}\right)}{2\sqrt{c}(n+1)(\sqrt{cd}-e\sqrt{-a})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(e\*x+d)\*\*n/(c\*x\*\*2+a), x)

[Out]  $-\left(\frac{(d+e*x)^{(n+1)} \text{hyper}\left((1, n+1), (n+2,), \sqrt{c}\left(\frac{d+e*x}{\sqrt{c}}\right)\right)}{2\sqrt{c}\left(\sqrt{cd}+e\sqrt{-a}\right)} - \frac{(d+e*x)^{(n+1)} \text{hyper}\left((1, n+1), (n+2,), \sqrt{c}\left(\frac{d+e*x}{\sqrt{c}}\right)\right)}{2\sqrt{c}\left(\sqrt{cd}-e\sqrt{-a}\right)}\right)$

**Mathematica [C]** time = 0.148661, size = 200, normalized size = 1.23

$$\frac{(d+ex)^n \left( \left( \frac{\sqrt{c}(d+ex)}{e(\sqrt{cx}-i\sqrt{a})} \right)^{-n} {}_2F_1\left(-n, -n; 1-n; \frac{\sqrt{cd}+i\sqrt{ae}}{i\sqrt{ae}-\sqrt{cex}}\right) + \left( \frac{\sqrt{c}(d+ex)}{e(\sqrt{cx}+i\sqrt{a})} \right)^{-n} {}_2F_1\left(-n, -n; 1-n; -\frac{\sqrt{cd}-i\sqrt{ae}}{\sqrt{cex}+i\sqrt{ae}}\right) \right)}{2cn}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(d + e\*x)^n)/(a + c\*x^2), x]

[Out]  $((d + e^x)^n \text{Hypergeometric2F1}[-n, -n, 1 - n, (\text{Sqrt}[c]^d + I \text{Sqrt}[a]^e) / (I \text{Sqrt}[a]^e - \text{Sqrt}[c]^e x)] / ((\text{Sqrt}[c]^d + e^x) / (e^{(-I \text{Sqrt}[a] + \text{Sqrt}[c]^x))^n} + \text{Hypergeometric2F1}[-n, -n, 1 - n, -((\text{Sqrt}[c]^d - I \text{Sqrt}[a]^e) / (I \text{Sqrt}[a]^e + \text{Sqrt}[c]^e x))] / ((\text{Sqrt}[c]^d + e^x) / (e^{(I \text{Sqrt}[a] + \text{Sqrt}[c]^x))^n}))) / (2^c n)$

**Maple** [F] time = 0.059, size = 0, normalized size = 0.

$$\int \frac{x(ex+d)^n}{cx^2+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(e*x+d)^n/(c*x^2+a), x)`

[Out] `int(x*(e*x+d)^n/(c*x^2+a), x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n x}{cx^2+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^n*x/(c*x^2 + a), x, algorithm="maxima")`

[Out] `integrate((e*x + d)^n*x/(c*x^2 + a), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^n x}{cx^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^n*x/(c*x^2 + a), x, algorithm="fricas")`

[Out] `integral((e*x + d)^n*x/(c*x^2 + a), x)`

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x(d+ex)^n}{a+cx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x+d)**n/(c*x**2+a), x)`

[Out] `Integral(x*(d + e*x)**n/(a + c*x**2), x)`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n x}{cx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^n*x/(c*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n*x/(c*x^2 + a), x)
```

$$3.367 \quad \int \frac{(d+ex)^n}{a+cx^2} dx$$

**Optimal.** Leaf size=167

$$\frac{(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}-\sqrt{-ae}}\right)}{2\sqrt{-a}(n+1)(\sqrt{cd}-\sqrt{-ae})} - \frac{(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}+\sqrt{-ae}}\right)}{2\sqrt{-a}(n+1)(\sqrt{-ae}+\sqrt{cd})}$$

[Out] ((d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d - Sqrt[-a]\*e)]/(2\*Sqrt[-a]\*(Sqrt[c]\*d - Sqrt[-a]\*e)^(1 + n)) - ((d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d + Sqrt[-a]\*e)]/(2\*Sqrt[-a]\*(Sqrt[c]\*d + Sqrt[-a]\*e)^(1 + n)))

**Rubi [A]** time = 0.237423, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$

$$\frac{(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}-\sqrt{-ae}}\right)}{2\sqrt{-a}(n+1)(\sqrt{cd}-\sqrt{-ae})} - \frac{(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}+\sqrt{-ae}}\right)}{2\sqrt{-a}(n+1)(\sqrt{-ae}+\sqrt{cd})}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^n/(a + c\*x^2), x]

[Out] ((d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d - Sqrt[-a]\*e)]/(2\*Sqrt[-a]\*(Sqrt[c]\*d - Sqrt[-a]\*e)^(1 + n)) - ((d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d + Sqrt[-a]\*e)]/(2\*Sqrt[-a]\*(Sqrt[c]\*d + Sqrt[-a]\*e)^(1 + n)))

**Rubi in Sympy [A]** time = 42.5933, size = 131, normalized size = 0.78

$$-\frac{(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}+e\sqrt{-a}}\right)}{2\sqrt{-a}(n+1)(\sqrt{cd}+e\sqrt{-a})} + \frac{(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}-e\sqrt{-a}}\right)}{2\sqrt{-a}(n+1)(\sqrt{cd}-e\sqrt{-a})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*n/(c\*x\*\*2+a), x)

[Out] -(d + e\*x)\*\*(n + 1)\*hyper((1, n + 1), (n + 2, ), sqrt(c)\*(d + e\*x)/(sqrt(c)\*d + e\*sqrt(-a)))/(2\*sqrt(-a)\*(n + 1)\*(sqrt(c)\*d + e\*sqrt(-a))) + (d + e\*x)\*\*(n + 1)\*hyper((1, n + 1), (n + 2, ), sqrt(c)\*(d + e\*x)/(sqrt(c)\*d - e\*sqrt(-a)))/(2\*sqrt(-a)\*(n + 1)\*(sqrt(c)\*d - e\*sqrt(-a)))

**Mathematica [C]** time = 0.122853, size = 210, normalized size = 1.26

$$\frac{i(d+ex)^n \left( \left( \frac{\sqrt{c}(d+ex)}{e(\sqrt{cx}-i\sqrt{a})} \right)^{-n} {}_2F_1\left(-n, -n; 1-n; \frac{\sqrt{cd}+i\sqrt{ae}}{i\sqrt{ae}-\sqrt{cex}}\right) - \left( \frac{\sqrt{c}(d+ex)}{e(\sqrt{cx}+i\sqrt{a})} \right)^{-n} {}_2F_1\left(-n, -n; 1-n; -\frac{\sqrt{cd}-i\sqrt{ae}}{\sqrt{cxe}+i\sqrt{ae}}\right) \right)}{2\sqrt{a}\sqrt{cn}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^n/(a + c\*x^2), x]

[Out]  $((-I/2)^*(d + e*x)^n*(\text{Hypergeometric2F1}[-n, -n, 1 - n, (\text{Sqrt}[c]*d + I*\text{Sqrt}[a]*e)/(\text{I}*\text{Sqrt}[a]*e - \text{Sqrt}[c]*e*x)]/((\text{Sqrt}[c]*(d + e*x))/(e*((-I)*\text{Sqrt}[a] + \text{Sqrt}[c]*x)))^n - \text{Hypergeometric2F1}[-n, -n, 1 - n, -((\text{Sqrt}[c]*d - I*\text{Sqrt}[a]*e)/(\text{I}*\text{Sqrt}[a]*e + \text{Sqrt}[c]*e*x))]/((\text{Sqrt}[c]*(d + e*x))/(e*(I*\text{Sqrt}[a] + \text{Sqrt}[c]*x)))^n))/(\text{Sqrt}[a]*\text{Sqrt}[c]^n)$

**Maple [F]** time = 0.06, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n}{cx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^n/(c*x^2+a), x)`

[Out] `int((e*x+d)^n/(c*x^2+a), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n}{cx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^n/(c*x^2 + a), x, algorithm="maxima")`

[Out] `integrate((e*x + d)^n/(c*x^2 + a), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex + d)^n}{cx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^n/(c*x^2 + a), x, algorithm="fricas")`

[Out] `integral((e*x + d)^n/(c*x^2 + a), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^n}{a + cx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**n/(c*x**2+a), x)`

[Out] `Integral((d + e*x)**n/(a + c*x**2), x)`



GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n}{cx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^n/(c*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n/(c*x^2 + a), x)
```

$$3.368 \quad \int \frac{(d+ex)^n}{x(a+cx^2)} dx$$

**Optimal.** Leaf size=207

$$\frac{\sqrt{c}(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}-\sqrt{-ae}}\right)}{2a(n+1)(\sqrt{cd}-\sqrt{-ae})} + \frac{\sqrt{c}(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}+\sqrt{-ae}}\right)}{2a(n+1)(\sqrt{-ae}+\sqrt{cd})} - \frac{(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{ex}{d}+1\right)}{ad(n+1)}$$

[Out] (Sqrt[c]\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d - Sqrt[-a]\*e)]/(2\*a\*(Sqrt[c]\*d - Sqrt[-a]\*e)^(1 + n)) + (Sqrt[c]\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d + Sqrt[-a]\*e)]/(2\*a\*(Sqrt[c]\*d + Sqrt[-a]\*e)^(1 + n)) - ((d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (e\*x)/d])/(a\*d\*(1 + n))

**Rubi [A]** time = 0.378911, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{\sqrt{c}(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}-\sqrt{-ae}}\right)}{2a(n+1)(\sqrt{cd}-\sqrt{-ae})} + \frac{\sqrt{c}(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}+\sqrt{-ae}}\right)}{2a(n+1)(\sqrt{-ae}+\sqrt{cd})} - \frac{(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{ex}{d}+1\right)}{ad(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^n/(x\*(a + c\*x^2)), x]

[Out] (Sqrt[c]\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d - Sqrt[-a]\*e)]/(2\*a\*(Sqrt[c]\*d - Sqrt[-a]\*e)^(1 + n)) + (Sqrt[c]\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d + Sqrt[-a]\*e)]/(2\*a\*(Sqrt[c]\*d + Sqrt[-a]\*e)^(1 + n)) - ((d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (e\*x)/d])/(a\*d\*(1 + n))

**Rubi in Sympy [A]** time = 55.5659, size = 158, normalized size = 0.76

$$\frac{\sqrt{c}(d+ex)^{n+1} {}_2F_1\left(1, n+1 \middle| \frac{\sqrt{c}(d+ex)}{\sqrt{cd+e\sqrt{-a}}}\right)}{2a(n+1)(\sqrt{cd+e\sqrt{-a}})} + \frac{\sqrt{c}(d+ex)^{n+1} {}_2F_1\left(1, n+1 \middle| \frac{\sqrt{c}(d+ex)}{\sqrt{cd-e\sqrt{-a}}}\right)}{2a(n+1)(\sqrt{cd-e\sqrt{-a}})} - \frac{(d+ex)^{n+1} {}_2F_1\left(1, n+1 \middle| 1 + \frac{ex}{d}\right)}{ad(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*n/x/(c\*x\*\*2+a), x)

[Out] sqrt(c)\*(d + e\*x)\*\*(n + 1)\*hyper((1, n + 1), (n + 2, ), sqrt(c)\*(d + e\*x)/(sqrt(c)\*d + e\*sqrt(-a)))/(2\*a\*(n + 1)\*(sqrt(c)\*d + e\*sqrt(-a))) + sqrt(c)\*(d + e\*x)\*\*(n + 1)\*hyper((1, n + 1), (n + 2, ), sqrt(c)\*(d + e\*x)/(sqrt(c)\*d - e\*sqrt(-a)))/(2\*a\*(n + 1)\*(sqrt(c)\*d - e\*sqrt(-a))) - (d + e\*x)\*\*(n + 1)\*hyper((1, n + 1), (n + 2, ), 1 + e\*x/d)/(a\*d\*(n + 1))

---

**Mathematica [C]** time = 0.481632, size = 239, normalized size = 1.15

$$\frac{(d + ex)^n \left( - \left( \frac{\sqrt{c}(d+ex)}{e(\sqrt{cx}-i\sqrt{a})} \right)^{-n} {}_2F_1 \left( -n, -n; 1-n; \frac{\sqrt{cd+i\sqrt{ae}}}{i\sqrt{ae}-\sqrt{cex}} \right) - \left( \frac{\sqrt{c}(d+ex)}{e(\sqrt{cx}+i\sqrt{a})} \right)^{-n} {}_2F_1 \left( -n, -n; 1-n; -\frac{\sqrt{cd-i\sqrt{ae}}}{\sqrt{cxe+i\sqrt{ae}}} \right) + 2 \left( \frac{d}{ex} + \dots \right) \right)}{2an}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^n/(x\*(a + c\*x^2)),x]

[Out] ((d + e\*x)^n\*((2\*Hypergeometric2F1[-n, -n, 1 - n, -(d/(e\*x))])/(1 + d/(e\*x))^n - Hypergeometric2F1[-n, -n, 1 - n, (Sqrt[c]\*d + I\*Sqrt[a]\*e)/(I\*Sqrt[a]\*e - Sqrt[c]\*e\*x)]/((Sqrt[c]\*(d + e\*x))/(e\*((-I)\*Sqrt[a] + Sqrt[c]\*x)))^n - Hypergeometric2F1[-n, -n, 1 - n, -((Sqrt[c]\*d - I\*Sqrt[a]\*e)/(I\*Sqrt[a]\*e + Sqrt[c]\*e\*x)]/((Sqrt[c]\*(d + e\*x))/(e\*(I\*Sqrt[a] + Sqrt[c]\*x)))^n))/(2\*a^n)

---

**Maple [F]** time = 0.053, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n}{x(cx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^n/x/(c\*x^2+a),x)

[Out] int((e\*x+d)^n/x/(c\*x^2+a),x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n}{(cx^2 + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^n/((c\*x^2 + a)\*x),x, algorithm="maxima")

[Out] integrate((e\*x + d)^n/((c\*x^2 + a)\*x), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex + d)^n}{cx^3 + ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^n/((c\*x^2 + a)\*x),x, algorithm="fricas")

[Out] integral((e\*x + d)^n/(c\*x^3 + a\*x), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^n}{x(a + cx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**n/x/(c*x**2+a),x)`

[Out] `Integral((d + e*x)**n/(x*(a + c*x**2)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n}{(cx^2 + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^n/((c*x^2 + a)*x),x, algorithm="giac")`

[Out] `integrate((e*x + d)^n/((c*x^2 + a)*x), x)`

$$3.369 \quad \int \frac{(d+ex)^n}{x^2(a+cx^2)} dx$$

**Optimal.** Leaf size=207

$$\frac{c(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}-\sqrt{-ae}}\right)}{2(-a)^{3/2}(n+1)(\sqrt{cd}-\sqrt{-ae})} - \frac{c(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}+\sqrt{-ae}}\right)}{2(-a)^{3/2}(n+1)(\sqrt{-ae}+\sqrt{cd})} + \frac{e(d+ex)^{n+1} {}_2F_1\left(2, n+1; n+2; \frac{ex}{d}+1\right)}{ad^2(n+1)}$$

[Out] (c\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*  
(d + e\*x))/(Sqrt[c]\*d - Sqrt[-a]\*e)]/(2\*(-a)^(3/2)\*(Sqrt[c]\*d -  
Sqrt[-a]\*e)^(1 + n)) - (c\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1,  
1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d + Sqrt[-a]\*e)]/(2\*(  
-a)^(3/2)\*(Sqrt[c]\*d + Sqrt[-a]\*e)^(1 + n)) + (e\*(d + e\*x)^(1 + n  
) \*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (e\*x)/d])/(a\*d^2\*(1 + n  
)

**Rubi [A]** time = 0.468437, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{c(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}-\sqrt{-ae}}\right)}{2(-a)^{3/2}(n+1)(\sqrt{cd}-\sqrt{-ae})} - \frac{c(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}+\sqrt{-ae}}\right)}{2(-a)^{3/2}(n+1)(\sqrt{-ae}+\sqrt{cd})} + \frac{e(d+ex)^{n+1} {}_2F_1\left(2, n+1; n+2; \frac{ex}{d}+1\right)}{ad^2(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^n/(x^2\*(a + c\*x^2)), x]

[Out] (c\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*  
(d + e\*x))/(Sqrt[c]\*d - Sqrt[-a]\*e)]/(2\*(-a)^(3/2)\*(Sqrt[c]\*d -  
Sqrt[-a]\*e)^(1 + n)) - (c\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1,  
1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d + Sqrt[-a]\*e)]/(2\*(  
-a)^(3/2)\*(Sqrt[c]\*d + Sqrt[-a]\*e)^(1 + n)) + (e\*(d + e\*x)^(1 + n  
) \*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (e\*x)/d])/(a\*d^2\*(1 + n  
)

**Rubi in Sympy [A]** time = 72.9256, size = 165, normalized size = 0.8

$$-\frac{c(d+ex)^{n+1} {}_2F_1\left(1, n+1 \middle| \frac{\sqrt{c}(d+ex)}{\sqrt{cd}+e\sqrt{-a}} \right)}{2(-a)^{\frac{3}{2}}(n+1)(\sqrt{cd}+e\sqrt{-a})} + \frac{c(d+ex)^{n+1} {}_2F_1\left(1, n+1 \middle| \frac{\sqrt{c}(d+ex)}{\sqrt{cd}-e\sqrt{-a}} \right)}{2(-a)^{\frac{3}{2}}(n+1)(\sqrt{cd}-e\sqrt{-a})} + \frac{e(d+ex)^{n+1} {}_2F_1\left(2, n+1 \middle| 1 + \frac{ex}{d} \right)}{ad^2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*n/x\*\*2/(c\*x\*\*2+a), x)

[Out] -c\*(d + e\*x)\*\*(n + 1)\*hyper((1, n + 1), (n + 2, ), sqrt(c)\*(d + e\*  
x)/(sqrt(c)\*d + e\*sqrt(-a)))/(2\*(-a)\*\*(3/2)\*(n + 1)\*(sqrt(c)\*d +  
e\*sqrt(-a))) + c\*(d + e\*x)\*\*(n + 1)\*hyper((1, n + 1), (n + 2, ), s  
qrt(c)\*(d + e\*x)/(sqrt(c)\*d - e\*sqrt(-a)))/(2\*(-a)\*\*(3/2)\*(n + 1)

$(\sqrt{c}d - e\sqrt{-a}) + e(d + ex)^{n+1} \operatorname{hyper}((2, n+1), (n+2, ), 1 + ex/d)/(a^2 d^{2(n+1)})$

**Mathematica [C]** time = 0.584545, size = 263, normalized size = 1.27

$$(d + ex)^n \left( \frac{2 \left( \frac{d}{ex} + 1 \right)^{-n} {}_2F_1\left(1-n, -n; 2-n; -\frac{d}{ex}\right)}{(n-1)x} + \frac{i\sqrt{c} \left( \frac{\sqrt{c}(d+ex)}{e(\sqrt{cx-i\sqrt{a}})} \right)^{-n} {}_2F_1\left(-n, -n; 1-n; \frac{\sqrt{cd+i\sqrt{ae}}}{i\sqrt{ae-\sqrt{c}ex}}\right) - \left( \frac{\sqrt{c}(d+ex)}{e(\sqrt{cx+i\sqrt{a}})} \right)^{-n} {}_2F_1\left(-n, -n; 1-n; -\frac{\sqrt{cd-i\sqrt{ae}}}{\sqrt{cx+i\sqrt{ae}}}\right)}{\sqrt{an}} \right)$$

2a

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^n/(x^2\*(a + c\*x^2)), x]

[Out] ((d + e\*x)^n\*((2\*Hypergeometric2F1[1 - n, -n, 2 - n, -(d/(e\*x))]) /((-1 + n)\*(1 + d/(e\*x))^n\*x) + (I\*Sqrt[c]\*(Hypergeometric2F1[-n, -n, 1 - n, (Sqrt[c]\*d + I\*Sqrt[a]\*e)/(I\*Sqrt[a]\*e - Sqrt[c]\*e\*x)] /((Sqrt[c]\*(d + e\*x))/(e\*((-I)\*Sqrt[a] + Sqrt[c]\*x)))^n - Hypergeometric2F1[-n, -n, 1 - n, -(Sqrt[c]\*d - I\*Sqrt[a]\*e)/(I\*Sqrt[a]\*e + Sqrt[c]\*e\*x)] /((Sqrt[c]\*(d + e\*x))/(e\*(I\*Sqrt[a] + Sqrt[c]\*x)))^n)) / (Sqrt[a]^n)) / (2\*a)

**Maple [F]** time = 0.081, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n}{x^2(cx^2 + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^n/x^2/(c\*x^2+a), x)

[Out] int((e\*x+d)^n/x^2/(c\*x^2+a), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n}{(cx^2 + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^n/((c\*x^2 + a)\*x^2), x, algorithm="maxima")

[Out] integrate((e\*x + d)^n/((c\*x^2 + a)\*x^2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(ex + d)^n}{cx^4 + ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^n/((c\*x^2 + a)\*x^2), x, algorithm="fricas")

[Out] `integral((e*x + d)^n/(c*x^4 + a*x^2), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**n/x**2/(c*x**2+a), x)`

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n}{(cx^2 + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^n/((c*x^2 + a)*x^2), x, algorithm="giac")`

[Out] `integrate((e*x + d)^n/((c*x^2 + a)*x^2), x)`

$$3.370 \quad \int \frac{x^4(d+ex)^n}{(a+cx^2)^2} dx$$

**Optimal.** Leaf size=332

$$\frac{(d+ex)^{n+1} (3\sqrt{-acd^2} + a\sqrt{cde} + \sqrt{-aae^2}(n+3)) {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd-\sqrt{-ae}}}\right)}{4c^2(n+1)(\sqrt{cd}-\sqrt{-ae})(ae^2+cd^2)} \\ - \frac{(d+ex)^{n+1} (3\sqrt{-acd^2} - a\sqrt{cde} + \sqrt{-aae^2}(n+3)) {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd+\sqrt{-ae}}}\right)}{4c^2(n+1)(\sqrt{-ae}+\sqrt{cd})(ae^2+cd^2)} \\ + \frac{a(d+ex)^{n+1}(ae+cdx)}{2c^2(a+cx^2)(ae^2+cd^2)} + \frac{(d+ex)^{n+1}}{c^2e(n+1)}$$

[Out] (d + e\*x)^(1 + n)/(c^2\*e\*(1 + n)) + (a\*(a\*e + c\*d\*x)\*(d + e\*x)^(1 + n))/(2\*c^2\*(c\*d^2 + a\*e^2)\*(a + c\*x^2)) + ((3\*Sqrt[-a]\*c\*d^2 + a\*Sqrt[c]\*d\*e\*n + Sqrt[-a]\*a\*e^2\*(3 + n))\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d - Sqrt[-a]\*e)]/(4\*c^2\*(Sqrt[c]\*d - Sqrt[-a]\*e)\*(c\*d^2 + a\*e^2)\*(1 + n)) - ((3\*Sqrt[-a]\*c\*d^2 - a\*Sqrt[c]\*d\*e\*n + Sqrt[-a]\*a\*e^2\*(3 + n))\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d + Sqrt[-a]\*e)]/(4\*c^2\*(Sqrt[c]\*d + Sqrt[-a]\*e)\*(c\*d^2 + a\*e^2)\*(1 + n))

**Rubi [A]** time = 0.797197, antiderivative size = 332, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{(d+ex)^{n+1} (3\sqrt{-acd^2} + a\sqrt{cde} + \sqrt{-aae^2}(n+3)) {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd-\sqrt{-ae}}}\right)}{4c^2(n+1)(\sqrt{cd}-\sqrt{-ae})(ae^2+cd^2)} \\ - \frac{(d+ex)^{n+1} (3\sqrt{-acd^2} - a\sqrt{cde} + \sqrt{-aae^2}(n+3)) {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd+\sqrt{-ae}}}\right)}{4c^2(n+1)(\sqrt{-ae}+\sqrt{cd})(ae^2+cd^2)} \\ + \frac{a(d+ex)^{n+1}(ae+cdx)}{2c^2(a+cx^2)(ae^2+cd^2)} + \frac{(d+ex)^{n+1}}{c^2e(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*(d + e\*x)^n)/(a + c\*x^2)^2, x]

[Out] (d + e\*x)^(1 + n)/(c^2\*e\*(1 + n)) + (a\*(a\*e + c\*d\*x)\*(d + e\*x)^(1 + n))/(2\*c^2\*(c\*d^2 + a\*e^2)\*(a + c\*x^2)) + ((3\*Sqrt[-a]\*c\*d^2 + a\*Sqrt[c]\*d\*e\*n + Sqrt[-a]\*a\*e^2\*(3 + n))\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d - Sqrt[-a]\*e)]/(4\*c^2\*(Sqrt[c]\*d - Sqrt[-a]\*e)\*(c\*d^2 + a\*e^2)\*(1 + n)) - ((3\*Sqrt[-a]\*c\*d^2 - a\*Sqrt[c]\*d\*e\*n + Sqrt[-a]\*a\*e^2\*(3 + n))\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d + Sqrt[-a]\*e)]/(4\*c^2\*(Sqrt[c]\*d + Sqrt[-a]\*e)\*(c\*d^2 + a\*e^2)\*(1 + n))

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(e\*x+d)\*\*n/(c\*x\*\*2+a)\*\*2, x)

[Out] Timed out



---

**Mathematica [A]** time = 0.166727, size = 0, normalized size = 0.

$$\int \frac{x^4(d+ex)^n}{(a+cx^2)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^4\*(d + e\*x)^n)/(a + c\*x^2)^2, x]

[Out] Integrate[(x^4\*(d + e\*x)^n)/(a + c\*x^2)^2, x]

---

**Maple [F]** time = 0.091, size = 0, normalized size = 0.

$$\int \frac{x^4(ex+d)^n}{(cx^2+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(e\*x+d)^n/(c\*x^2+a)^2, x)

[Out] int(x^4\*(e\*x+d)^n/(c\*x^2+a)^2, x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n x^4}{(cx^2+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^n\*x^4/(c\*x^2 + a)^2, x, algorithm="maxima")

[Out] integrate((e\*x + d)^n\*x^4/(c\*x^2 + a)^2, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)^n x^4}{c^2 x^4 + 2acx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^n\*x^4/(c\*x^2 + a)^2, x, algorithm="fricas")

[Out] integral((e\*x + d)^n\*x^4/(c^2\*x^4 + 2\*a\*c\*x^2 + a^2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x+d)**n/(c*x**2+a)**2,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n x^4}{(cx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^n*x^4/(c*x^2 + a)^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n*x^4/(c*x^2 + a)^2, x)
```

$$3.371 \quad \int \frac{x^3(d+ex)^n}{(a+cx^2)^2} dx$$

**Optimal.** Leaf size=297

$$\frac{(d+ex)^{n+1} (\sqrt{-a}\sqrt{cd}en + ae^2(n+2) + 2cd^2) {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd+\sqrt{-ae}}}\right)}{4c^{3/2}(n+1) (\sqrt{-ae} + \sqrt{cd}) (ae^2 + cd^2)} + \frac{(d+ex)^{n+1} \left(\sqrt{-a}den - \frac{ae^2(n+2)+2cd^2}{\sqrt{c}}\right) {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd-\sqrt{-ae}}}\right)}{4c(n+1) (\sqrt{cd} - \sqrt{-ae}) (ae^2 + cd^2)} + \frac{a(d-ex)(d+ex)^{n+1}}{2c(a+cx^2)(ae^2 + cd^2)}$$

[Out] (a\*(d - e\*x)\*(d + e\*x)^(1 + n))/(2\*c\*(c\*d^2 + a\*e^2)\*(a + c\*x^2)) + ((Sqrt[-a]\*d\*e^n - (2\*c\*d^2 + a\*e^2\*(2 + n))/Sqrt[c])\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d - Sqrt[-a]\*e)]/(4\*c\*(Sqrt[c]\*d - Sqrt[-a]\*e)\*(c\*d^2 + a\*e^2)\*(1 + n)) - ((2\*c\*d^2 + Sqrt[-a]\*Sqrt[c]\*d\*e^n + a\*e^2\*(2 + n))\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d + Sqrt[-a]\*e)]/(4\*c^(3/2)\*(Sqrt[c]\*d + Sqrt[-a]\*e)\*(c\*d^2 + a\*e^2)\*(1 + n)))

**Rubi [A]** time = 0.769574, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{(d+ex)^{n+1} (\sqrt{-a}\sqrt{cd}en + ae^2(n+2) + 2cd^2) {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd+\sqrt{-ae}}}\right)}{4c^{3/2}(n+1) (\sqrt{-ae} + \sqrt{cd}) (ae^2 + cd^2)} + \frac{(d+ex)^{n+1} \left(\sqrt{-a}den - \frac{ae^2(n+2)+2cd^2}{\sqrt{c}}\right) {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd-\sqrt{-ae}}}\right)}{4c(n+1) (\sqrt{cd} - \sqrt{-ae}) (ae^2 + cd^2)} + \frac{a(d-ex)(d+ex)^{n+1}}{2c(a+cx^2)(ae^2 + cd^2)}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(d + e\*x)^n)/(a + c\*x^2)^2, x]

[Out] (a\*(d - e\*x)\*(d + e\*x)^(1 + n))/(2\*c\*(c\*d^2 + a\*e^2)\*(a + c\*x^2)) + ((Sqrt[-a]\*d\*e^n - (2\*c\*d^2 + a\*e^2\*(2 + n))/Sqrt[c])\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d - Sqrt[-a]\*e)]/(4\*c\*(Sqrt[c]\*d - Sqrt[-a]\*e)\*(c\*d^2 + a\*e^2)\*(1 + n)) - ((2\*c\*d^2 + Sqrt[-a]\*Sqrt[c]\*d\*e^n + a\*e^2\*(2 + n))\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d + Sqrt[-a]\*e)]/(4\*c^(3/2)\*(Sqrt[c]\*d + Sqrt[-a]\*e)\*(c\*d^2 + a\*e^2)\*(1 + n)))

**Rubi in Sympy [A]** time = 165.105, size = 354, normalized size = 1.19

$$\frac{a(d-ex)(d+ex)^{n+1}}{2c(a+cx^2)(ae^2 + cd^2)} - \frac{en(d+ex)^{n+1} (ae - \sqrt{cd}\sqrt{-a}) {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd-e\sqrt{-a}}}\right)}{4c^{3/2}(n+1)(ae^2 + cd^2)(\sqrt{cd} - e\sqrt{-a})} - \frac{en(d+ex)^{n+1} (ae + \sqrt{cd}\sqrt{-a}) {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd+e\sqrt{-a}}}\right)}{4c^{3/2}(n+1)(ae^2 + cd^2)(\sqrt{cd} + e\sqrt{-a})} - \frac{(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd+e\sqrt{-a}}}\right)}{2c^{3/2}(n+1)(\sqrt{cd} + e\sqrt{-a})} - \frac{(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd-e\sqrt{-a}}}\right)}{2c^{3/2}(n+1)(\sqrt{cd} - e\sqrt{-a})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**3*(e*x+d)**n/(c*x**2+a)**2,x)`

[Out]  $a(d - ex)(d + ex)^{n+1}/(2c(a + cx^2)(ae^2 + cd^2)) - e^n(d + ex)^{n+1}(ae - \sqrt{c}d\sqrt{-a})\operatorname{hyper}((1, n+1), (n+2,), \sqrt{c}(d + ex)/(\sqrt{c}d - e\sqrt{-a}))/4c^{3/2}(n+1)(ae^2 + cd^2)(\sqrt{c}d - e\sqrt{-a}) - e^n(d + ex)^{n+1}(ae + \sqrt{c}d\sqrt{-a})\operatorname{hyper}((1, n+1), (n+2,), \sqrt{c}(d + ex)/(\sqrt{c}d + e\sqrt{-a}))/4c^{3/2}(n+1)(ae^2 + cd^2)(\sqrt{c}d + e\sqrt{-a}) - (d + ex)^{n+1}\operatorname{hyper}((1, n+1), (n+2,), \sqrt{c}(d + ex)/(\sqrt{c}d + e\sqrt{-a}))/2c^{3/2}(n+1)(\sqrt{c}d + e\sqrt{-a}) - (d + ex)^{n+1}\operatorname{hyper}((1, n+1), (n+2,), \sqrt{c}(d + ex)/(\sqrt{c}d - e\sqrt{-a}))/2c^{3/2}(n+1)(\sqrt{c}d - e\sqrt{-a})$

**Mathematica [A]** time = 0.140881, size = 0, normalized size = 0.

$$\int \frac{x^3(d + ex)^n}{(a + cx^2)^2} dx$$

Verification is Not applicable to the result.

[In] `Integrate[(x^3*(d + e*x)^n)/(a + c*x^2)^2,x]`

[Out] `Integrate[(x^3*(d + e*x)^n)/(a + c*x^2)^2, x]`

**Maple [F]** time = 0.087, size = 0, normalized size = 0.

$$\int \frac{x^3(ex + d)^n}{(cx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(e*x+d)^n/(c*x^2+a)^2,x)`

[Out] `int(x^3*(e*x+d)^n/(c*x^2+a)^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n x^3}{(cx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^n*x^3/(c*x^2 + a)^2,x, algorithm="maxima")`

[Out] `integrate((e*x + d)^n*x^3/(c*x^2 + a)^2, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(ex + d)^n x^3}{c^2 x^4 + 2acx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^n\*x^3/(c\*x^2 + a)^2,x, algorithm="fricas")

[Out] integral((e\*x + d)^n\*x^3/(c^2\*x^4 + 2\*a\*c\*x^2 + a^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(e\*x+d)\*\*n/(c\*x\*\*2+a)\*\*2,x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n x^3}{(cx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^n\*x^3/(c\*x^2 + a)^2,x, algorithm="giac")

[Out] integrate((e\*x + d)^n\*x^3/(c\*x^2 + a)^2, x)

$$3.372 \quad \int \frac{x^2(d+ex)^n}{(a+cx^2)^2} dx$$

**Optimal.** Leaf size=306

$$\frac{(d+ex)^{n+1}(-\sqrt{-a}\sqrt{c}den+ae^2(n+1)+cd^2) {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}-\sqrt{-ae}}\right)}{4\sqrt{-ac}(n+1)(\sqrt{cd}-\sqrt{-ae})(ae^2+cd^2)} \\ - \frac{(d+ex)^{n+1}(\sqrt{-a}\sqrt{c}den+ae^2(n+1)+cd^2) {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}+\sqrt{-ae}}\right)}{4\sqrt{-ac}(n+1)(\sqrt{-ae}+\sqrt{cd})(ae^2+cd^2)} \\ - \frac{(d+ex)^{n+1}(ae+cdx)}{2c(a+cx^2)(ae^2+cd^2)}$$

[Out]  $-\left((a^*e + c^*d^*x)^*(d + e^*x)^{(1 + n)}\right)/\left(2^*c^*(c^*d^{\wedge}2 + a^*e^{\wedge}2)^*(a + c^*x^{\wedge}2)\right) + \left((c^*d^{\wedge}2 - \text{Sqrt}[-a]^*\text{Sqrt}[c]^*d^*e^*n + a^*e^{\wedge}2^*(1 + n))^*(d + e^*x)^{(1 + n)}\right)^*\text{Hypergeometric2F1}[1, 1 + n, 2 + n, (\text{Sqrt}[c]^*(d + e^*x))/(\text{Sqrt}[c]^*d - \text{Sqrt}[-a]^*e)]/\left(4^*\text{Sqrt}[-a]^*c^*(\text{Sqrt}[c]^*d - \text{Sqrt}[-a]^*e)^*(c^*d^{\wedge}2 + a^*e^{\wedge}2)^*(1 + n)\right) - \left((c^*d^{\wedge}2 + \text{Sqrt}[-a]^*\text{Sqrt}[c]^*d^*e^*n + a^*e^{\wedge}2^*(1 + n))^*(d + e^*x)^{(1 + n)}\right)^*\text{Hypergeometric2F1}[1, 1 + n, 2 + n, (\text{Sqrt}[c]^*(d + e^*x))/(\text{Sqrt}[c]^*d + \text{Sqrt}[-a]^*e)]/\left(4^*\text{Sqrt}[-a]^*c^*(\text{Sqrt}[c]^*d + \text{Sqrt}[-a]^*e)^*(c^*d^{\wedge}2 + a^*e^{\wedge}2)^*(1 + n)\right)$

**Rubi [A]** time = 1.05898, antiderivative size = 306, normalized size of antiderivative = 1., number of rules used = 5, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{(d+ex)^{n+1}(-\sqrt{-a}\sqrt{c}den+ae^2(n+1)+cd^2) {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}-\sqrt{-ae}}\right)}{4\sqrt{-ac}(n+1)(\sqrt{cd}-\sqrt{-ae})(ae^2+cd^2)} \\ - \frac{(d+ex)^{n+1}(\sqrt{-a}\sqrt{c}den+ae^2(n+1)+cd^2) {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}+\sqrt{-ae}}\right)}{4\sqrt{-ac}(n+1)(\sqrt{-ae}+\sqrt{cd})(ae^2+cd^2)} \\ - \frac{(d+ex)^{n+1}(ae+cdx)}{2c(a+cx^2)(ae^2+cd^2)}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(d + e\*x)^n)/(a + c\*x^2)^2, x]

[Out]  $-\left((a^*e + c^*d^*x)^*(d + e^*x)^{(1 + n)}\right)/\left(2^*c^*(c^*d^{\wedge}2 + a^*e^{\wedge}2)^*(a + c^*x^{\wedge}2)\right) + \left((c^*d^{\wedge}2 - \text{Sqrt}[-a]^*\text{Sqrt}[c]^*d^*e^*n + a^*e^{\wedge}2^*(1 + n))^*(d + e^*x)^{(1 + n)}\right)^*\text{Hypergeometric2F1}[1, 1 + n, 2 + n, (\text{Sqrt}[c]^*(d + e^*x))/(\text{Sqrt}[c]^*d - \text{Sqrt}[-a]^*e)]/\left(4^*\text{Sqrt}[-a]^*c^*(\text{Sqrt}[c]^*d - \text{Sqrt}[-a]^*e)^*(c^*d^{\wedge}2 + a^*e^{\wedge}2)^*(1 + n)\right) - \left((c^*d^{\wedge}2 + \text{Sqrt}[-a]^*\text{Sqrt}[c]^*d^*e^*n + a^*e^{\wedge}2^*(1 + n))^*(d + e^*x)^{(1 + n)}\right)^*\text{Hypergeometric2F1}[1, 1 + n, 2 + n, (\text{Sqrt}[c]^*(d + e^*x))/(\text{Sqrt}[c]^*d + \text{Sqrt}[-a]^*e)]/\left(4^*\text{Sqrt}[-a]^*c^*(\text{Sqrt}[c]^*d + \text{Sqrt}[-a]^*e)^*(c^*d^{\wedge}2 + a^*e^{\wedge}2)^*(1 + n)\right)$

**Rubi in Sympy [A]** time = 161.51, size = 382, normalized size = 1.25

$$\begin{aligned} & -\frac{(d+ex)^{n+1}(ae+cdx)}{2c(a+cx^2)(ae^2+cd^2)} - \frac{(d+ex)^{n+1} {}_2F_1\left(1, n+1 \left| \frac{\sqrt{c}(d+ex)}{\sqrt{cd+e\sqrt{-a}}} \right. \right)}{2c\sqrt{-a}(n+1)(\sqrt{cd+e\sqrt{-a}})} \\ & + \frac{(d+ex)^{n+1} {}_2F_1\left(1, n+1 \left| \frac{\sqrt{c}(d+ex)}{\sqrt{cd-e\sqrt{-a}}} \right. \right)}{2c\sqrt{-a}(n+1)(\sqrt{cd-e\sqrt{-a}})} \\ & - \frac{(d+ex)^{n+1}(a\sqrt{c}den + \sqrt{-a}(ae^2(-n+1) + cd^2)) {}_2F_1\left(1, n+1 \left| \frac{\sqrt{c}(d+ex)}{\sqrt{cd+e\sqrt{-a}}} \right. \right)}{4ac(n+1)(ae^2+cd^2)(\sqrt{cd+e\sqrt{-a}})} \\ & + \frac{(d+ex)^{n+1}(-a\sqrt{c}den + \sqrt{-a}(ae^2(-n+1) + cd^2)) {}_2F_1\left(1, n+1 \left| \frac{\sqrt{c}(d+ex)}{\sqrt{cd-e\sqrt{-a}}} \right. \right)}{4ac(n+1)(ae^2+cd^2)(\sqrt{cd-e\sqrt{-a}})} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**2*(e*x+d)**n/(c*x**2+a)**2,x)`

[Out] `-(d + e*x)**(n + 1)*(a*e + c*d*x)/(2*c*(a + c*x**2)*(a*e**2 + c*d**2)) - (d + e*x)**(n + 1)*hyper((1, n + 1), (n + 2, ), sqrt(c)*(d + e*x)/(sqrt(c)*d + e*sqrt(-a)))/(2*c*sqrt(-a)*(n + 1)*(sqrt(c)*d + e*sqrt(-a))) + (d + e*x)**(n + 1)*hyper((1, n + 1), (n + 2, ), sqrt(c)*(d + e*x)/(sqrt(c)*d - e*sqrt(-a)))/(2*c*sqrt(-a)*(n + 1)*(sqrt(c)*d - e*sqrt(-a))) - (d + e*x)**(n + 1)*(a*sqrt(c)*d*e*n + sqrt(-a)*(a*e**2*(-n + 1) + c*d**2))*hyper((1, n + 1), (n + 2, ), sqrt(c)*(d + e*x)/(sqrt(c)*d + e*sqrt(-a)))/(4*a*c*(n + 1)*(a*e**2 + c*d**2)*(sqrt(c)*d + e*sqrt(-a))) + (d + e*x)**(n + 1)*(-a*sqrt(c)*d*e*n + sqrt(-a)*(a*e**2*(-n + 1) + c*d**2))*hyper((1, n + 1), (n + 2, ), sqrt(c)*(d + e*x)/(sqrt(c)*d - e*sqrt(-a)))/(4*a*c*(n + 1)*(a*e**2 + c*d**2)*(sqrt(c)*d - e*sqrt(-a)))`

**Mathematica [A]** time = 0.128038, size = 0, normalized size = 0.

$$\int \frac{x^2(d+ex)^n}{(a+cx^2)^2} dx$$

Verification is Not applicable to the result.

[In] `Integrate[(x^2*(d + e*x)^n)/(a + c*x^2)^2,x]`

[Out] `Integrate[(x^2*(d + e*x)^n)/(a + c*x^2)^2, x]`

**Maple [F]** time = 0.082, size = 0, normalized size = 0.

$$\int \frac{x^2(ex+d)^n}{(cx^2+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(e*x+d)^n/(c*x^2+a)^2,x)`

[Out] `int(x^2*(e*x+d)^n/(c*x^2+a)^2,x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n x^2}{(cx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^n\*x^2/(c\*x^2 + a)^2,x, algorithm="maxima")

[Out] integrate((e\*x + d)^n\*x^2/(c\*x^2 + a)^2, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex + d)^n x^2}{c^2 x^4 + 2 acx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^n\*x^2/(c\*x^2 + a)^2,x, algorithm="fricas")

[Out] integral((e\*x + d)^n\*x^2/(c^2\*x^4 + 2\*a\*c\*x^2 + a^2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(e\*x+d)\*\*n/(c\*x\*\*2+a)\*\*2,x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n x^2}{(cx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^n\*x^2/(c\*x^2 + a)^2,x, algorithm="giac")

[Out] integrate((e\*x + d)^n\*x^2/(c\*x^2 + a)^2, x)



$$3.373 \quad \int \frac{x(d+ex)^n}{(a+cx^2)^2} dx$$

**Optimal.** Leaf size=279

$$\frac{en(\sqrt{-ae} + \sqrt{cd})(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}-\sqrt{-ae}}\right)}{4\sqrt{-a}\sqrt{c}(n+1)(\sqrt{cd}-\sqrt{-ae})(ae^2+cd^2)} + \frac{en(\sqrt{-a}\sqrt{cd}+ae)(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}+\sqrt{-ae}}\right)}{4a\sqrt{c}(n+1)(\sqrt{-ae}+\sqrt{cd})(ae^2+cd^2)} - \frac{(d-ex)(d+ex)^{n+1}}{2(a+cx^2)(ae^2+cd^2)}$$

[Out]  $-\left((d - e*x)^*(d + e*x)^{(1 + n)} / (2*(c*d^2 + a*e^2)*(a + c*x^2)) + (e*(\text{Sqrt}[c]*d + \text{Sqrt}[-a]*e)^n*(d + e*x)^{(1 + n)} \text{Hypergeometric2F1}[1, 1 + n, 2 + n, (\text{Sqrt}[c]*(d + e*x))/(\text{Sqrt}[c]*d - \text{Sqrt}[-a]*e)]) / (4*\text{Sqrt}[-a]*\text{Sqrt}[c]*(\text{Sqrt}[c]*d - \text{Sqrt}[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + (e*(\text{Sqrt}[-a]*\text{Sqrt}[c]*d + a*e)^n*(d + e*x)^{(1 + n)} \text{Hypergeometric2F1}[1, 1 + n, 2 + n, (\text{Sqrt}[c]*(d + e*x))/(\text{Sqrt}[c]*d + \text{Sqrt}[-a]*e)]) / (4*a*\text{Sqrt}[c]*(\text{Sqrt}[c]*d + \text{Sqrt}[-a]*e)*(c*d^2 + a*e^2)*(1 + n))\right)$

**Rubi [A]** time = 0.663582, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{en(\sqrt{-ae} + \sqrt{cd})(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}-\sqrt{-ae}}\right)}{4\sqrt{-a}\sqrt{c}(n+1)(\sqrt{cd}-\sqrt{-ae})(ae^2+cd^2)} + \frac{en(\sqrt{-a}\sqrt{cd}+ae)(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}+\sqrt{-ae}}\right)}{4a\sqrt{c}(n+1)(\sqrt{-ae}+\sqrt{cd})(ae^2+cd^2)} - \frac{(d-ex)(d+ex)^{n+1}}{2(a+cx^2)(ae^2+cd^2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x*(d + e*x)^n)/(a + c*x^2)^2, x]$

[Out]  $-\left((d - e*x)^*(d + e*x)^{(1 + n)} / (2*(c*d^2 + a*e^2)*(a + c*x^2)) + (e*(\text{Sqrt}[c]*d + \text{Sqrt}[-a]*e)^n*(d + e*x)^{(1 + n)} \text{Hypergeometric2F1}[1, 1 + n, 2 + n, (\text{Sqrt}[c]*(d + e*x))/(\text{Sqrt}[c]*d - \text{Sqrt}[-a]*e)]) / (4*\text{Sqrt}[-a]*\text{Sqrt}[c]*(\text{Sqrt}[c]*d - \text{Sqrt}[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + (e*(\text{Sqrt}[-a]*\text{Sqrt}[c]*d + a*e)^n*(d + e*x)^{(1 + n)} \text{Hypergeometric2F1}[1, 1 + n, 2 + n, (\text{Sqrt}[c]*(d + e*x))/(\text{Sqrt}[c]*d + \text{Sqrt}[-a]*e)]) / (4*a*\text{Sqrt}[c]*(\text{Sqrt}[c]*d + \text{Sqrt}[-a]*e)*(c*d^2 + a*e^2)*(1 + n))\right)$

**Rubi in Sympy [A]** time = 104.559, size = 224, normalized size = 0.8

$$-\frac{(d-ex)(d+ex)^{n+1}}{2(a+cx^2)(ae^2+cd^2)} + \frac{en(d+ex)^{n+1}(ae-\sqrt{cd}\sqrt{-a}) {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}-e\sqrt{-a}}\right)}{4a\sqrt{c}(n+1)(ae^2+cd^2)(\sqrt{cd}-e\sqrt{-a})} + \frac{en(d+ex)^{n+1}(ae+\sqrt{cd}\sqrt{-a}) {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}+e\sqrt{-a}}\right)}{4a\sqrt{c}(n+1)(ae^2+cd^2)(\sqrt{cd}+e\sqrt{-a})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x*(e*x+d)**n/(c*x**2+a)**2, x)$

[Out]  $-(d - e*x)^*(d + e*x)**(n + 1) / (2*(a + c*x**2)*(a*e**2 + c*d**2)) + e*n*(d + e*x)**(n + 1)*(a*e - \text{sqrt}(c)*d*\text{sqrt}(-a))*\text{hyper}((1, n +$

$$1), (n + 2,), \sqrt{c} \cdot (d + e \cdot x) / (\sqrt{c} \cdot d - e \cdot \sqrt{-a}) / (4 \cdot a \cdot \sqrt{c} \cdot (n + 1) \cdot (a \cdot e^2 + c \cdot d^2) \cdot (\sqrt{c} \cdot d - e \cdot \sqrt{-a})) + e \cdot n \cdot (d + e \cdot x)^{n+1} \cdot (a \cdot e + \sqrt{c} \cdot d \cdot \sqrt{-a}) \cdot \text{hyper}((1, n + 1), (n + 2,), \sqrt{c} \cdot (d + e \cdot x) / (\sqrt{c} \cdot d + e \cdot \sqrt{-a})) / (4 \cdot a \cdot \sqrt{c} \cdot (n + 1) \cdot (a \cdot e^2 + c \cdot d^2) \cdot (\sqrt{c} \cdot d + e \cdot \sqrt{-a}))$$

**Mathematica [A]** time = 0.097271, size = 0, normalized size = 0.

$$\int \frac{x(d + ex)^n}{(a + cx^2)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x\*(d + e\*x)^n)/(a + c\*x^2)^2, x]

[Out] Integrate[(x\*(d + e\*x)^n)/(a + c\*x^2)^2, x]

**Maple [F]** time = 0.08, size = 0, normalized size = 0.

$$\int \frac{x(ex + d)^n}{(cx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(e\*x+d)^n/(c\*x^2+a)^2, x)

[Out] int(x\*(e\*x+d)^n/(c\*x^2+a)^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n x}{(cx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^n\*x/(c\*x^2 + a)^2, x, algorithm="maxima")

[Out] integrate((e\*x + d)^n\*x/(c\*x^2 + a)^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex + d)^n x}{c^2 x^4 + 2 a c x^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^n\*x/(c\*x^2 + a)^2, x, algorithm="fricas")

[Out] integral((e\*x + d)^n\*x/(c^2\*x^4 + 2\*a\*c\*x^2 + a^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x+d)**n/(c*x**2+a)**2,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n x}{(cx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^n*x/(c*x^2 + a)^2,x, algorithm="giac")`

[Out] `integrate((e*x + d)^n*x/(c*x^2 + a)^2, x)`

$$3.374 \quad \int \frac{(d+ex)^n}{(a+cx^2)^2} dx$$

**Optimal.** Leaf size=304

$$\begin{aligned} & \frac{(d+ex)^{n+1} (\sqrt{-a}\sqrt{c}den + ae^2(1-n) + cd^2) {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}-\sqrt{-ae}}\right)}{4(-a)^{3/2}(n+1) (\sqrt{cd}-\sqrt{-ae}) (ae^2+cd^2)} \\ & + \frac{(d+ex)^{n+1} (-\sqrt{-a}\sqrt{c}den + ae^2(1-n) + cd^2) {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}+\sqrt{-ae}}\right)}{4(-a)^{3/2}(n+1) (\sqrt{-ae}+\sqrt{cd}) (ae^2+cd^2)} \\ & + \frac{(d+ex)^{n+1}(ae+cdx)}{2a(a+cx^2)(ae^2+cd^2)} \end{aligned}$$

[Out] ((a\*e + c\*d\*x)\*(d + e\*x)^(1 + n))/(2\*a\*(c\*d^2 + a\*e^2)\*(a + c\*x^2)) - ((c\*d^2 + a\*e^2\*(1 - n) + Sqrt[-a]\*Sqrt[c]\*d\*e\*n)\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d - Sqrt[-a]\*e)])/(4\*(-a)^(3/2)\*(Sqrt[c]\*d - Sqrt[-a]\*e)\*(c\*d^2 + a\*e^2)\*(1 + n)) + ((c\*d^2 + a\*e^2\*(1 - n) - Sqrt[-a]\*Sqrt[c]\*d\*e\*n)\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d + Sqrt[-a]\*e)])/(4\*(-a)^(3/2)\*(Sqrt[c]\*d + Sqrt[-a]\*e)\*(c\*d^2 + a\*e^2)\*(1 + n))

**Rubi [A]** time = 0.837822, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$

$$\begin{aligned} & \frac{(d+ex)^{n+1} (\sqrt{-a}\sqrt{c}den + ae^2(1-n) + cd^2) {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}-\sqrt{-ae}}\right)}{4(-a)^{3/2}(n+1) (\sqrt{cd}-\sqrt{-ae}) (ae^2+cd^2)} \\ & + \frac{(d+ex)^{n+1} (-\sqrt{-a}\sqrt{c}den + ae^2(1-n) + cd^2) {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}+\sqrt{-ae}}\right)}{4(-a)^{3/2}(n+1) (\sqrt{-ae}+\sqrt{cd}) (ae^2+cd^2)} \\ & + \frac{(d+ex)^{n+1}(ae+cdx)}{2a(a+cx^2)(ae^2+cd^2)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^n/(a + c\*x^2)^2, x]

[Out] ((a\*e + c\*d\*x)\*(d + e\*x)^(1 + n))/(2\*a\*(c\*d^2 + a\*e^2)\*(a + c\*x^2)) - ((c\*d^2 + a\*e^2\*(1 - n) + Sqrt[-a]\*Sqrt[c]\*d\*e\*n)\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d - Sqrt[-a]\*e)])/(4\*(-a)^(3/2)\*(Sqrt[c]\*d - Sqrt[-a]\*e)\*(c\*d^2 + a\*e^2)\*(1 + n)) + ((c\*d^2 + a\*e^2\*(1 - n) - Sqrt[-a]\*Sqrt[c]\*d\*e\*n)\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d + Sqrt[-a]\*e)])/(4\*(-a)^(3/2)\*(Sqrt[c]\*d + Sqrt[-a]\*e)\*(c\*d^2 + a\*e^2)\*(1 + n))

**Rubi in Sympy [A]** time = 91.36, size = 246, normalized size = 0.81

$$\begin{aligned} & \frac{(d+ex)^{n+1}(ae+cdx)}{2a(a+cx^2)(ae^2+cd^2)} + \frac{(d+ex)^{n+1}(a\sqrt{c}den + \sqrt{-a}(ae^2(-n+1) + cd^2)) {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd+e\sqrt{-a}}}\right)}{4a^2(n+1)(ae^2+cd^2)(\sqrt{cd+e\sqrt{-a}})} \\ & - \frac{(d+ex)^{n+1}(-a\sqrt{c}den + \sqrt{-a}(ae^2(-n+1) + cd^2)) {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd-e\sqrt{-a}}}\right)}{4a^2(n+1)(ae^2+cd^2)(\sqrt{cd-e\sqrt{-a}})} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**n/(c*x**2+a)**2,x)`

[Out]  $(d + ex)^{n+1}(ae + cd^2x)/(2a(a + cx^2)(ae^2 + cd^2)) + (d + ex)^{n+1}(a\sqrt{c}d^ne^n + \sqrt{-a}(ae^{2(-n+1)} + cd^2))\operatorname{hyper}((1, n+1), (n+2, ), \sqrt{c}(d + ex)/(\sqrt{c}d + e\sqrt{-a}))/4a^2(n+1)(ae^2 + cd^2)(\sqrt{c}d + e\sqrt{-a}) - (d + ex)^{n+1}(-a\sqrt{c}d^ne^n + \sqrt{-a}(ae^{2(-n+1)} + cd^2))\operatorname{hyper}((1, n+1), (n+2, ), \sqrt{c}(d + ex)/(\sqrt{c}d - e\sqrt{-a}))/4a^2(n+1)(ae^2 + cd^2)(\sqrt{c}d - e\sqrt{-a})$

**Mathematica [A]** time = 0.0485241, size = 0, normalized size = 0.

$$\int \frac{(d + ex)^n}{(a + cx^2)^2} dx$$

Verification is Not applicable to the result.

[In] `Integrate[(d + e*x)^n/(a + c*x^2)^2,x]`

[Out] `Integrate[(d + e*x)^n/(a + c*x^2)^2, x]`

**Maple [F]** time = 0.078, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n}{(cx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^n/(c*x^2+a)^2,x)`

[Out] `int((e*x+d)^n/(c*x^2+a)^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n}{(cx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^n/(c*x^2 + a)^2,x, algorithm="maxima")`

[Out] `integrate((e*x + d)^n/(c*x^2 + a)^2, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(ex + d)^n}{c^2x^4 + 2acx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^n/(c*x^2 + a)^2,x, algorithm="fricas")`

[Out] `integral((e*x + d)^n/(c^2*x^4 + 2*a*c*x^2 + a^2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**n/(c*x**2+a)**2,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n}{(cx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^n/(c*x^2 + a)^2,x, algorithm="giac")`

[Out] `integrate((e*x + d)^n/(c*x^2 + a)^2, x)`

$$3.375 \quad \int \frac{(d+ex)^n}{x(a+cx^2)^2} dx$$

**Optimal.** Leaf size=489

$$\begin{aligned} & \frac{\sqrt{c}en(\sqrt{-a}\sqrt{cd}+ae)(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd+\sqrt{-ae}}}\right)}{4a^2(n+1)(\sqrt{-ae}+\sqrt{cd})(ae^2+cd^2)} \\ & + \frac{\sqrt{c}(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd-\sqrt{-ae}}}\right)}{2a^2(n+1)(\sqrt{cd}-\sqrt{-ae})} \\ & + \frac{\sqrt{c}(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd+\sqrt{-ae}}}\right)}{2a^2(n+1)(\sqrt{-ae}+\sqrt{cd})} - \frac{(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{ex}{d}+1\right)}{a^2d(n+1)} \\ & + \frac{\sqrt{c}en(\sqrt{-ae}+\sqrt{cd})(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd-\sqrt{-ae}}}\right)}{4(-a)^{3/2}(n+1)(\sqrt{cd}-\sqrt{-ae})(ae^2+cd^2)} + \frac{c(d-ex)(d+ex)^{n+1}}{2a(a+cx^2)(ae^2+cd^2)} \end{aligned}$$

[Out] (c\*(d - e\*x)\*(d + e\*x)^(1 + n))/(2\*a\*(c\*d^2 + a\*e^2)\*(a + c\*x^2)) + (Sqrt[c]\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d - Sqrt[-a]\*e)]/(2\*a^2\*(Sqrt[c]\*d - Sqrt[-a]\*e)\*(1 + n)) + (Sqrt[c]\*e\*(Sqrt[c]\*d + Sqrt[-a]\*e)\*n\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d - Sqrt[-a]\*e)]/(4\*(-a)^(3/2)\*(Sqrt[c]\*d - Sqrt[-a]\*e)\*(c\*d^2 + a\*e^2)\*(1 + n)) + (Sqrt[c]\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d + Sqrt[-a]\*e)]/(2\*a^2\*(Sqrt[c]\*d + Sqrt[-a]\*e)\*(1 + n)) - (Sqrt[c]\*e\*(Sqrt[-a]\*Sqrt[c]\*d + a\*e)\*n\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d + Sqrt[-a]\*e)]/(4\*a^2\*(Sqrt[c]\*d + Sqrt[-a]\*e)\*(c\*d^2 + a\*e^2)\*(1 + n)) - ((d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (e\*x)/d])/(a^2\*d\*(1 + n))

**Rubi [A]** time = 1.24952, antiderivative size = 489, normalized size of antiderivative = 1., number of rules used = 12, number of rules used = 5, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\begin{aligned} & \frac{\sqrt{c}en(\sqrt{-a}\sqrt{cd}+ae)(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd+\sqrt{-ae}}}\right)}{4a^2(n+1)(\sqrt{-ae}+\sqrt{cd})(ae^2+cd^2)} \\ & + \frac{\sqrt{c}(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd-\sqrt{-ae}}}\right)}{2a^2(n+1)(\sqrt{cd}-\sqrt{-ae})} \\ & + \frac{\sqrt{c}(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd+\sqrt{-ae}}}\right)}{2a^2(n+1)(\sqrt{-ae}+\sqrt{cd})} - \frac{(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{ex}{d}+1\right)}{a^2d(n+1)} \\ & + \frac{\sqrt{c}en(\sqrt{-ae}+\sqrt{cd})(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd-\sqrt{-ae}}}\right)}{4(-a)^{3/2}(n+1)(\sqrt{cd}-\sqrt{-ae})(ae^2+cd^2)} + \frac{c(d-ex)(d+ex)^{n+1}}{2a(a+cx^2)(ae^2+cd^2)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^n/(x\*(a + c\*x^2)^2), x]

[Out] (c\*(d - e\*x)\*(d + e\*x)^(1 + n))/(2\*a\*(c\*d^2 + a\*e^2)\*(a + c\*x^2)) + (Sqrt[c]\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d - Sqrt[-a]\*e)]/(2\*a^2\*(Sqrt[c]\*d - Sqrt[-a]\*e)\*(1 + n)) + (Sqrt[c]\*e\*(Sqrt[c]\*d + Sqrt[-a]\*e)\*n\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d - Sqrt[-a]\*e)]/(4\*(-a)^(3/2)\*(Sqrt[c]\*d - Sqrt[-a]\*e)\*(c\*d^2 + a\*e^2)\*(1 + n)) + (Sqrt[c]\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d + Sqrt[-a]\*e)]/(2\*a^2\*(Sqrt[c]\*d + Sqrt[-a]\*e)\*(1 + n)) - (Sqrt[c]\*e\*(Sqrt[-a]\*Sqrt[c]\*d + a\*e)\*n\*(d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]\*(d + e\*x))/(Sqrt[c]\*d + Sqrt[-a]\*e)]/(4\*a^2\*(Sqrt[c]\*d + Sqrt[-a]\*e)\*(c\*d^2 + a\*e^2)\*(1 + n)) - ((d + e\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (e\*x)/d])/(a^2\*d\*(1 + n))

$$\frac{/(4*a^2*(\text{Sqrt}[c]*d + \text{Sqrt}[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) - ((d + e*x)^{(1 + n)}*\text{Hypergeometric2F1}[1, 1 + n, 2 + n, 1 + (e*x)/d])/(a^2*d*(1 + n))}{}$$

**Rubi in Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**n/x/(c*x**2+a)**2,x)`

[Out] Timed out

**Mathematica** [A] time = 0.0977494, size = 0, normalized size = 0.

$$\int \frac{(d + ex)^n}{x(a + cx^2)^2} dx$$

Verification is Not applicable to the result.

[In] `Integrate[(d + e*x)^n/(x*(a + c*x^2)^2),x]`

[Out] `Integrate[(d + e*x)^n/(x*(a + c*x^2)^2), x]`

**Maple** [F] time = 0.068, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n}{x(cx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^n/x/(c*x^2+a)^2,x)`

[Out] `int((e*x+d)^n/x/(c*x^2+a)^2,x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n}{(cx^2 + a)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^n/((c*x^2 + a)^2*x),x, algorithm="maxima")`

[Out] `integrate((e*x + d)^n/((c*x^2 + a)^2*x), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex + d)^n}{c^2x^5 + 2acx^3 + a^2x}, x\right)$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^n/((c*x^2 + a)^2*x),x, algorithm="fricas")`

[Out] `integral((e*x + d)^n/(c^2*x^5 + 2*a*c*x^3 + a^2*x), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**n/x/(c*x**2+a)**2,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n}{(cx^2 + a)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^n/((c*x^2 + a)^2*x),x, algorithm="giac")`

[Out] `integrate((e*x + d)^n/((c*x^2 + a)^2*x), x)`

$$3.376 \quad \int \frac{(d+ex)^n}{x^2(a+cx^2)^2} dx$$

**Optimal.** Leaf size=513

$$\begin{aligned} & \frac{c(d+ex)^{n+1}(ae+cdx)}{2a^2(a+cx^2)(ae^2+cd^2)} + \frac{e(d+ex)^{n+1} {}_2F_1\left(2, n+1; n+2; \frac{ex}{d} + 1\right)}{a^2 d^2 (n+1)} \\ & - \frac{c(d+ex)^{n+1} \left(\sqrt{-a}\sqrt{cd}en + ae^2(1-n) + cd^2\right) {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}-\sqrt{-ae}}\right)}{4(-a)^{5/2}(n+1) (\sqrt{cd} - \sqrt{-ae}) (ae^2 + cd^2)} \\ & + \frac{c(d+ex)^{n+1} \left(-\sqrt{-a}\sqrt{cd}en + ae^2(1-n) + cd^2\right) {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}+\sqrt{-ae}}\right)}{4(-a)^{5/2}(n+1) (\sqrt{-ae} + \sqrt{cd}) (ae^2 + cd^2)} \\ & - \frac{c(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}-\sqrt{-ae}}\right)}{2(-a)^{5/2}(n+1) (\sqrt{cd} - \sqrt{-ae})} + \frac{c(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}+\sqrt{-ae}}\right)}{2(-a)^{5/2}(n+1) (\sqrt{-ae} + \sqrt{cd})} \end{aligned}$$

[Out]  $-(c*(a*e + c*d*x)*(d + e*x)^(1 + n))/(2*a^2*(c*d^2 + a*e^2)*(a + c*x^2)) - (c*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)]/(2*(-a)^(5/2)*(Sqrt[c]*d - Sqrt[-a]*e)^(1 + n)) - (c*(c*d^2 + a*e^2*(1 - n) + Sqrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)]/(4*(-a)^(5/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + (c*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]/(2*(-a)^(5/2)*(Sqrt[c]*d + Sqrt[-a]*e)^(1 + n)) + (c*(c*d^2 + a*e^2*(1 - n) - Sqrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]/(4*(-a)^(5/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + (e*(d + e*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (e*x)/d])/(a^2*d^2*(1 + n))$

**Rubi [A]** time = 1.5029, antiderivative size = 513, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\begin{aligned} & \frac{c(d+ex)^{n+1}(ae+cdx)}{2a^2(a+cx^2)(ae^2+cd^2)} + \frac{e(d+ex)^{n+1} {}_2F_1\left(2, n+1; n+2; \frac{ex}{d} + 1\right)}{a^2 d^2 (n+1)} \\ & - \frac{c(d+ex)^{n+1} \left(\sqrt{-a}\sqrt{cd}en + ae^2(1-n) + cd^2\right) {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}-\sqrt{-ae}}\right)}{4(-a)^{5/2}(n+1) (\sqrt{cd} - \sqrt{-ae}) (ae^2 + cd^2)} \\ & + \frac{c(d+ex)^{n+1} \left(-\sqrt{-a}\sqrt{cd}en + ae^2(1-n) + cd^2\right) {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}+\sqrt{-ae}}\right)}{4(-a)^{5/2}(n+1) (\sqrt{-ae} + \sqrt{cd}) (ae^2 + cd^2)} \\ & - \frac{c(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}-\sqrt{-ae}}\right)}{2(-a)^{5/2}(n+1) (\sqrt{cd} - \sqrt{-ae})} + \frac{c(d+ex)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{\sqrt{c}(d+ex)}{\sqrt{cd}+\sqrt{-ae}}\right)}{2(-a)^{5/2}(n+1) (\sqrt{-ae} + \sqrt{cd})} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^n/(x^2\*(a + c\*x^2)^2), x]

[Out]  $-(c*(a*e + c*d*x)*(d + e*x)^(1 + n))/(2*a^2*(c*d^2 + a*e^2)*(a + c*x^2)) - (c*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)]/(2*(-a)^(5/2)*(Sqrt[c]*d - Sqrt[-a]*e)^(1 + n)) - (c*(c*d^2 + a*e^2*(1 - n) + Sqrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d - Sqrt[-a]*e)]/(4*(-a)^(5/2)*(Sqrt[c]*d - Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + (c*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]/(2*(-a)^(5/2)*(Sqrt[c]*d + Sqrt[-a]*e)^(1 + n)) + (c*(c*d^2 + a*e^2*(1 - n) - Sqrt[-a]*Sqrt[c]*d*e*n)*(d + e*x)^(1 + n)*Hypergeometric2F1[1, 1 + n, 2 + n, (Sqrt[c]*(d + e*x))/(Sqrt[c]*d + Sqrt[-a]*e)]/(4*(-a)^(5/2)*(Sqrt[c]*d + Sqrt[-a]*e)*(c*d^2 + a*e^2)*(1 + n)) + (e*(d + e*x)^(1 + n)*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (e*x)/d])/(a^2*d^2*(1 + n))$

$\text{rt}[-a]^e \cdot (c \cdot d^2 + a \cdot e^2)^{n+1} + (e \cdot (d + e \cdot x)^{n+1}) \cdot \text{Hypergeometric2F1}[2, 1 + n, 2 + n, 1 + (e \cdot x)/d] / (a^2 \cdot d^2)^{n+1}$

**Rubi in Sympy [A]** time = 175.627, size = 422, normalized size = 0.82

$$\begin{aligned} & \frac{c(d+ex)^{n+1} {}_2F_1\left(1, n+1 \left| \frac{\sqrt{c(d+ex)}}{\sqrt{cd+e\sqrt{-a}}} \right. \right)}{2(-a)^{\frac{5}{2}}(n+1)(\sqrt{cd+e\sqrt{-a}})} - \frac{c(d+ex)^{n+1} {}_2F_1\left(1, n+1 \left| \frac{\sqrt{c(d+ex)}}{\sqrt{cd-e\sqrt{-a}}} \right. \right)}{2(-a)^{\frac{5}{2}}(n+1)(\sqrt{cd-e\sqrt{-a}})} \\ & - \frac{c(d+ex)^{n+1}(ae+cdx)}{2a^2(a+cx^2)(ae^2+cd^2)} + \frac{e(d+ex)^{n+1} {}_2F_1\left(2, n+1 \left| 1 + \frac{ex}{d} \right. \right)}{a^2 d^2 (n+1)} \\ & - \frac{c(d+ex)^{n+1}(a\sqrt{cd}en + \sqrt{-a}(ae^2(-n+1) + cd^2)) {}_2F_1\left(1, n+1 \left| \frac{\sqrt{c(d+ex)}}{\sqrt{cd+e\sqrt{-a}}} \right. \right)}{4a^3(n+1)(ae^2+cd^2)(\sqrt{cd+e\sqrt{-a}})} \\ & + \frac{c(d+ex)^{n+1}(-a\sqrt{cd}en + \sqrt{-a}(ae^2(-n+1) + cd^2)) {}_2F_1\left(1, n+1 \left| \frac{\sqrt{c(d+ex)}}{\sqrt{cd-e\sqrt{-a}}} \right. \right)}{4a^3(n+1)(ae^2+cd^2)(\sqrt{cd-e\sqrt{-a}})} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**n/x**2/(c*x**2+a)**2,x)`

[Out]  $c \cdot (d + e \cdot x)^{n+1} \cdot \text{hyper}((1, n+1), (n+2, ), \sqrt{c} \cdot (d + e \cdot x) / (\sqrt{c} \cdot d + e \cdot \sqrt{-a})) / (2 \cdot (-a)^{5/2} \cdot (n+1) \cdot (\sqrt{c} \cdot d + e \cdot \sqrt{-a})) - c \cdot (d + e \cdot x)^{n+1} \cdot \text{hyper}((1, n+1), (n+2, ), \sqrt{c} \cdot (d + e \cdot x) / (\sqrt{c} \cdot d - e \cdot \sqrt{-a})) / (2 \cdot (-a)^{5/2} \cdot (n+1) \cdot (\sqrt{c} \cdot d - e \cdot \sqrt{-a})) - c \cdot (d + e \cdot x)^{n+1} \cdot (a \cdot e + c \cdot d \cdot x) / (2 \cdot a^2 \cdot (a + c \cdot x^2) \cdot (a \cdot e^2 + c \cdot d^2)) + e \cdot (d + e \cdot x)^{n+1} \cdot \text{hyper}((2, n+1), (n+2, ), 1 + e \cdot x / d) / (a^2 \cdot d^2 \cdot (n+1)) - c \cdot (d + e \cdot x)^{n+1} \cdot (a \cdot \sqrt{c} \cdot d \cdot e \cdot n + \sqrt{-a} \cdot (a \cdot e^2 \cdot (-n+1) + c \cdot d^2)) \cdot \text{hyper}((1, n+1), (n+2, ), \sqrt{c} \cdot (d + e \cdot x) / (\sqrt{c} \cdot d + e \cdot \sqrt{-a})) / (4 \cdot a^3 \cdot (n+1) \cdot (a \cdot e^2 + c \cdot d^2) \cdot (\sqrt{c} \cdot d + e \cdot \sqrt{-a})) + c \cdot (d + e \cdot x)^{n+1} \cdot (-a \cdot \sqrt{c} \cdot d \cdot e \cdot n + \sqrt{-a} \cdot (a \cdot e^2 \cdot (-n+1) + c \cdot d^2)) \cdot \text{hyper}((1, n+1), (n+2, ), \sqrt{c} \cdot (d + e \cdot x) / (\sqrt{c} \cdot d - e \cdot \sqrt{-a})) / (4 \cdot a^3 \cdot (n+1) \cdot (a \cdot e^2 + c \cdot d^2) \cdot (\sqrt{c} \cdot d - e \cdot \sqrt{-a}))$

**Mathematica [A]** time = 0.121062, size = 0, normalized size = 0.

$$\int \frac{(d+ex)^n}{x^2(a+cx^2)^2} dx$$

Verification is Not applicable to the result.

[In] `Integrate[(d + e*x)^n/(x^2*(a + c*x^2)^2), x]`

[Out] `Integrate[(d + e*x)^n/(x^2*(a + c*x^2)^2), x]`

**Maple [F]** time = 0.106, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^n}{x^2(cx^2+a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^n/x^2/(c*x^2+a)^2,x)`

[Out] `int((e*x+d)^n/x^2/(c*x^2+a)^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n}{(cx^2 + a)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^n/((c*x^2 + a)^2*x^2),x, algorithm="maxima")`

[Out] `integrate((e*x + d)^n/((c*x^2 + a)^2*x^2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex + d)^n}{c^2x^6 + 2acx^4 + a^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^n/((c*x^2 + a)^2*x^2),x, algorithm="fricas")`

[Out] `integral((e*x + d)^n/(c^2*x^6 + 2*a*c*x^4 + a^2*x^2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**n/x**2/(c*x**2+a)**2,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n}{(cx^2 + a)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^n/((c*x^2 + a)^2*x^2),x, algorithm="giac")`

[Out] `integrate((e*x + d)^n/((c*x^2 + a)^2*x^2), x)`

$$3.377 \quad \int (gx)^m (d + ex)^n (a + cx^2)^2 dx$$

**Optimal.** Leaf size=399

$$\frac{(gx)^{m+1}(d+ex)^n \left(\frac{ex}{d} + 1\right)^{-n} \left(a^2 e^4 (m+n+2)(m+n+3)(m+n+4)(m+n+5) + cd^2 (m+1)(m+2) (2ae^2 (m^2 + m(2n+9) + n^2 + 9n + 20) + cd^2 (m^2 + 7m + 12))\right)}{e^4 g (m+1)(m+n+2)(m+n+3)(m+n+4)(m+n+5)} \\ - \frac{cd(m+2)(gx)^{m+1}(d+ex)^{n+1} (2ae^2 (m^2 + m(2n+9) + n^2 + 9n + 20) + cd^2 (m^2 + 7m + 12))}{e^4 g (m+n+2)(m+n+3)(m+n+4)(m+n+5)} \\ + \frac{c(gx)^{m+2}(d+ex)^{n+1} (2ae^2 (m^2 + m(2n+9) + n^2 + 9n + 20) + cd^2 (m^2 + 7m + 12))}{e^3 g^2 (m+n+3)(m+n+4)(m+n+5)} \\ - \frac{c^2 d(m+4)(gx)^{m+3}(d+ex)^{n+1}}{e^2 g^3 (m+n+4)(m+n+5)} + \frac{c^2 (gx)^{m+4}(d+ex)^{n+1}}{eg^4 (m+n+5)}$$

[Out]  $-\left(\left(c^*d^*(2+m)^*(c^*d^2*(12+7*m+m^2)+2*a^*e^2*(20+m^2+9*n+n^2+m*(9+2*n)))\right)^*(g^*x)^{(1+m)}*(d+e^*x)^{(1+n)}\right)/(e^4*g^*(2+m+n)^*(3+m+n)^*(4+m+n)^*(5+m+n)) + \left(c^*(c^*d^2*(12+7*m+m^2)+2*a^*e^2*(20+m^2+9*n+n^2+m*(9+2*n)))\right)^*(g^*x)^{(2+m)}*(d+e^*x)^{(1+n)}\right)/(e^3*g^2*(3+m+n)^*(4+m+n)^*(5+m+n)) - \left(c^2*d^*(4+m)^*(g^*x)^{(3+m)}*(d+e^*x)^{(1+n)}\right)/(e^2*g^3*(4+m+n)^*(5+m+n)) + \left(c^2*(g^*x)^{(4+m)}*(d+e^*x)^{(1+n)}\right)/(e*g^4*(5+m+n)) + \left(\left(a^2/e^4*(2+m+n)^*(3+m+n)^*(4+m+n)^*(5+m+n) + c^*d^2*(1+m)^*(2+m)^*(c^*d^2*(12+7*m+m^2)+2*a^*e^2*(20+m^2+9*n+n^2+m*(9+2*n)))\right)^*(g^*x)^{(1+m)}*(d+e^*x)^n * \text{Hypergeometric2F1}\left[1+m, -n, 2+m, -((e^*x)/d)\right]\right)/(e^4*g^*(1+m)^*(2+m+n)^*(3+m+n)^*(4+m+n)^*(5+m+n)^*(1+(e^*x)/d)^n)$

**Rubi [A]** time = 1.45151, antiderivative size = 377, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$\frac{(gx)^{m+1}(d+ex)^n \left(\frac{ex}{d} + 1\right)^{-n} \left(\frac{a^2}{m+1} + \frac{cd^2(m+2)(2ae^2(m^2+m(2n+9)+n^2+9n+20)+cd^2(m^2+7m+12))}{e^4(m+n+2)(m+n+3)(m+n+4)(m+n+5)}\right)}{e^4 g (m+1)(m+n+2)(m+n+3)(m+n+4)(m+n+5)} {}_2F_1\left(m+1, -n; m+2; -\frac{ex}{d}\right) \\ - \frac{cd(m+2)(gx)^{m+1}(d+ex)^{n+1} (2ae^2 (m^2 + m(2n+9) + n^2 + 9n + 20) + cd^2 (m^2 + 7m + 12))}{e^4 g (m+n+2)(m+n+3)(m+n+4)(m+n+5)} \\ + \frac{c(gx)^{m+2}(d+ex)^{n+1} (2ae^2 (m^2 + m(2n+9) + n^2 + 9n + 20) + cd^2 (m^2 + 7m + 12))}{e^3 g^2 (m+n+3)(m+n+4)(m+n+5)} \\ - \frac{c^2 d(m+4)(gx)^{m+3}(d+ex)^{n+1}}{e^2 g^3 (m+n+4)(m+n+5)} + \frac{c^2 (gx)^{m+4}(d+ex)^{n+1}}{eg^4 (m+n+5)}$$

Antiderivative was successfully verified.

[In] Int[(g\*x)^m\*(d+e\*x)^n\*(a+c\*x^2)^2,x]

[Out]  $-\left(\left(c^*d^*(2+m)^*(c^*d^2*(12+7*m+m^2)+2*a^*e^2*(20+m^2+9*n+n^2+m*(9+2*n)))\right)^*(g^*x)^{(1+m)}*(d+e^*x)^{(1+n)}\right)/(e^4*g^*(2+m+n)^*(3+m+n)^*(4+m+n)^*(5+m+n)) + \left(c^*(c^*d^2*(12+7*m+m^2)+2*a^*e^2*(20+m^2+9*n+n^2+m*(9+2*n)))\right)^*(g^*x)^{(2+m)}*(d+e^*x)^{(1+n)}\right)/(e^3*g^2*(3+m+n)^*(4+m+n)^*(5+m+n)) - \left(c^2*d^*(4+m)^*(g^*x)^{(3+m)}*(d+e^*x)^{(1+n)}\right)/(e^2*g^3*(4+m+n)^*(5+m+n)) + \left(c^2*(g^*x)^{(4+m)}*(d+e^*x)^{(1+n)}\right)/(e*g^4*(5+m+n)) + \left(\left(a^2/(1+m) + (c^*d^2*(2+m)^*(c^*d^2*(12+7*m+m^2)+2*a^*e^2*(20+m^2+9*n+n^2+m*(9+2*n)))\right)/(e^4*(2+m+n)^*(3+m+n)^*(4+m+n)^*(5+m+n))\right)^*(g^*x)^{(1+m)}*(d+e^*x)^n * \text{Hypergeometric2F1}\left[1+m, -n, 2+m, -((e^*x)/d)\right]\right)/(g^*(1+(e^*x)/d)^n)$

**Rubi in Sympy [A]** time = 88.0008, size = 289, normalized size = 0.72

$$\frac{c^2 d^4 (gx)^{m+1} \left(1 + \frac{ex}{d}\right)^{-n} (d + ex)^n {}_2F_1\left(\begin{matrix} -n - 4, m + 1 \\ m + 2 \end{matrix} \middle| -\frac{ex}{d}\right)}{e^4 g (m + 1)}$$

$$- \frac{4c^2 d^4 (gx)^{m+1} \left(1 + \frac{ex}{d}\right)^{-n} (d + ex)^n {}_2F_1\left(\begin{matrix} -n - 3, m + 1 \\ m + 2 \end{matrix} \middle| -\frac{ex}{d}\right)}{e^4 g (m + 1)}$$

$$- \frac{4cd^2 (gx)^{m+1} \left(1 + \frac{ex}{d}\right)^{-n} (d + ex)^n (ae^2 + cd^2) {}_2F_1\left(\begin{matrix} -n - 1, m + 1 \\ m + 2 \end{matrix} \middle| -\frac{ex}{d}\right)}{e^4 g (m + 1)}$$

$$+ \frac{2cd^2 (gx)^{m+1} \left(1 + \frac{ex}{d}\right)^{-n} (d + ex)^n (ae^2 + 3cd^2) {}_2F_1\left(\begin{matrix} -n - 2, m + 1 \\ m + 2 \end{matrix} \middle| -\frac{ex}{d}\right)}{e^4 g (m + 1)}$$

$$+ \frac{(gx)^{m+1} \left(1 + \frac{ex}{d}\right)^{-n} (d + ex)^n (ae^2 + cd^2)^2 {}_2F_1\left(\begin{matrix} -n, m + 1 \\ m + 2 \end{matrix} \middle| -\frac{ex}{d}\right)}{e^4 g (m + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x)**m*(e*x+d)**n*(c*x**2+a)**2,x)`

[Out] `c**2*d**4*(g*x)**(m+1)*(1+e*x/d)**(-n)*(d+e*x)**n*hyper((-n-4,m+1),(m+2,)-e*x/d)/(e**4*g*(m+1))-4*c**2*d**4*(g*x)**(m+1)*(1+e*x/d)**(-n)*(d+e*x)**n*hyper((-n-3,m+1),(m+2,)-e*x/d)/(e**4*g*(m+1))-4*c*d**2*(g*x)**(m+1)*(1+e*x/d)**(-n)*(d+e*x)**n*(a*e**2+c*d**2)*hyper((-n-1,m+1),(m+2,)-e*x/d)/(e**4*g*(m+1))+2*c*d**2*(g*x)**(m+1)*(1+e*x/d)**(-n)*(d+e*x)**n*(a*e**2+3*c*d**2)*hyper((-n-2,m+1),(m+2,)-e*x/d)/(e**4*g*(m+1))+(g*x)**(m+1)*(1+e*x/d)**(-n)*(d+e*x)**n*(a*e**2+c*d**2)**2*hyper((-n,m+1),(m+2,)-e*x/d)/(e**4*g*(m+1))`

**Mathematica [A]** time = 0.231068, size = 128, normalized size = 0.32

$$\frac{x(gx)^m(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}\left(a^2(m^2+8m+15) {}_2F_1\left(m+1,-n;m+2;-\frac{ex}{d}\right)+c(m+1)x^2(2a(m+5) {}_2F_1(m+3,-n;m+4;(m+1)(m+3)(m+5))\right)}{(m+1)(m+3)(m+5)}$$

Antiderivative was successfully verified.

[In] `Integrate[(g*x)^m*(d+e*x)^n*(a+c*x^2)^2,x]`

[Out] `(x*(g*x)^m*(d+e*x)^n*(a^2*(15+8*m+m^2)*Hypergeometric2F1[1+m,-n,2+m,-((e*x)/d)]+c*(1+m)*x^2*(2*a*(5+m)*Hypergeometric2F1[3+m,-n,4+m,-((e*x)/d)]+c*(3+m)*x^2*Hypergeometric2F1[5+m,-n,6+m,-((e*x)/d)]))/((1+m)*(3+m)*(5+m))*(1+(e*x)/d)^n`

**Maple [F]** time = 0.111, size = 0, normalized size = 0.

$$\int (gx)^m (ex+d)^n (cx^2+a)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x)^m*(e*x+d)^n*(c*x^2+a)^2,x)`

[Out] `int((g*x)^m*(e*x+d)^n*(c*x^2+a)^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (cx^2 + a)^2 (ex + d)^n (gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)^2*(e*x + d)^n*(g*x)^m,x, algorithm="maxima")`

[Out] `integrate((c*x^2 + a)^2*(e*x + d)^n*(g*x)^m, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((c^2x^4 + 2acx^2 + a^2)(ex + d)^n (gx)^m, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)^2*(e*x + d)^n*(g*x)^m,x, algorithm="fricas")`

[Out] `integral((c^2*x^4 + 2*a*c*x^2 + a^2)*(e*x + d)^n*(g*x)^m, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x)**m*(e*x+d)**n*(c*x**2+a)**2,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (cx^2 + a)^2 (ex + d)^n (gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)^2*(e*x + d)^n*(g*x)^m,x, algorithm="giac")`

[Out] `integrate((c*x^2 + a)^2*(e*x + d)^n*(g*x)^m, x)`

### 3.378 $\int (gx)^m (d + ex)^n (a + cx^2) dx$

**Optimal.** Leaf size=164

$$\frac{(gx)^{m+1}(d+ex)^n \left(\frac{ex}{d} + 1\right)^{-n} (ae^2(m+n+2)(m+n+3) + cd^2(m+1)(m+2)) {}_2F_1\left(m+1, -n; m+2; -\frac{ex}{d}\right)}{e^2g(m+1)(m+n+2)(m+n+3)} - \frac{cd(m+2)(gx)^{m+1}(d+ex)^{n+1}}{e^2g(m+n+2)(m+n+3)} + \frac{c(gx)^{m+2}(d+ex)^{n+1}}{eg^2(m+n+3)}$$

[Out]  $-\left(\frac{c^2d^2(2+m)(gx)^{1+m}(d+ex)^{1+n}}{e^{2+2m+n}} + \frac{c^2(gx)^{2+m}(d+ex)^{1+n}}{e^{2+2m+n}} + \frac{(c^2d^2(1+m)(2+m) + a^2e^{2+2m+n})(3+m+n)(gx)^{1+m}(d+ex)^n \operatorname{Hypergeometric2F1}\left[1+m, -n, 2+m, -\frac{ex}{d}\right]}{e^{2+2m+n}(1+m)(2+m+n)(3+m+n)(1+\frac{ex}{d})^n}\right)$

**Rubi [A]** time = 0.302571, antiderivative size = 150, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 4, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{(gx)^{m+1}(d+ex)^n \left(\frac{ex}{d} + 1\right)^{-n} \left(\frac{a}{m+1} + \frac{cd^2(m+2)}{e^2(m+n+2)(m+n+3)}\right) {}_2F_1\left(m+1, -n; m+2; -\frac{ex}{d}\right)}{e^2g(m+n+2)(m+n+3)} + \frac{c(gx)^{m+2}(d+ex)^{n+1}}{eg^2(m+n+3)}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[(gx)^m (d + ex)^n (a + cx^2), x\right]$

[Out]  $-\left(\frac{c^2d^2(2+m)(gx)^{1+m}(d+ex)^{1+n}}{e^{2+2m+n}} + \frac{c^2(gx)^{2+m}(d+ex)^{1+n}}{e^{2+2m+n}} + \frac{(a/(1+m) + c^2d^2(2+m))/(e^{2+2m+n}(3+m+n))(gx)^{1+m}(d+ex)^n \operatorname{Hypergeometric2F1}\left[1+m, -n, 2+m, -\frac{ex}{d}\right]}{g(1+\frac{ex}{d})^n}\right)$

**Rubi in Sympy [A]** time = 41.3975, size = 156, normalized size = 0.95

$$\frac{cd^2(gx)^{m+1} \left(1 + \frac{ex}{d}\right)^{-n} (d+ex)^n {}_2F_1\left(-n-2, m+1; m+2; -\frac{ex}{d}\right)}{e^2g(m+1)} - \frac{2cd^2(gx)^{m+1} \left(1 + \frac{ex}{d}\right)^{-n} (d+ex)^n {}_2F_1\left(-n-1, m+1; m+2; -\frac{ex}{d}\right)}{e^2g(m+1)} + \frac{(gx)^{m+1} \left(1 + \frac{ex}{d}\right)^{-n} (d+ex)^n (ae^2 + cd^2) {}_2F_1\left(-n, m+1; m+2; -\frac{ex}{d}\right)}{e^2g(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{rubi\_integrate}\left((gx)^m (e^x+d)^n (c^2x^2+a), x\right)$

[Out]  $c^2d^2(gx)^{m+1}(1+\frac{ex}{d})^{-n}(d+ex)^n \operatorname{hyper}\left((-n-2, m+1), (m+2), -\frac{ex}{d}\right) - 2c^2d^2(gx)^{m+1}(1+\frac{ex}{d})^{-n}(d+ex)^n \operatorname{hyper}\left((-n-1, m+1), (m+2), -\frac{ex}{d}\right) + (gx)^{m+1}(1+\frac{ex}{d})^{-n}(d+ex)^n (ae^2 + c^2d^2) \operatorname{hyper}\left((-n, m+1), (m+2), -\frac{ex}{d}\right)$



/d)/(e\*\*2\*g\*(m + 1))

**Mathematica [A]** time = 0.0993278, size = 84, normalized size = 0.51

$$\frac{x(gx)^m(d+ex)^n\left(\frac{ex}{d}+1\right)^{-n}\left(a(m+3) {}_2F_1\left(m+1, -n; m+2; -\frac{ex}{d}\right)+c(m+1)x^2 {}_2F_1\left(m+3, -n; m+4; -\frac{ex}{d}\right)\right)}{(m+1)(m+3)}$$

Antiderivative was successfully verified.

[In] Integrate[(g\*x)^m\*(d + e\*x)^n\*(a + c\*x^2), x]

[Out] (x\*(g\*x)^m\*(d + e\*x)^n\*(a\*(3 + m)\*Hypergeometric2F1[1 + m, -n, 2 + m, -(e\*x)/d] + c\*(1 + m)\*x^2\*Hypergeometric2F1[3 + m, -n, 4 + m, -(e\*x)/d]))/((1 + m)\*(3 + m)\*(1 + (e\*x)/d)^n)

**Maple [F]** time = 0.072, size = 0, normalized size = 0.

$$\int (gx)^m (ex + d)^n (cx^2 + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x)^m\*(e\*x+d)^n\*(c\*x^2+a), x)

[Out] int((g\*x)^m\*(e\*x+d)^n\*(c\*x^2+a), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (cx^2 + a)(ex + d)^n (gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)\*(e\*x + d)^n\*(g\*x)^m, x, algorithm="maxima")

[Out] integrate((c\*x^2 + a)\*(e\*x + d)^n\*(g\*x)^m, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}((cx^2 + a)(ex + d)^n (gx)^m, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)\*(e\*x + d)^n\*(g\*x)^m, x, algorithm="fricas")

[Out] integral((c\*x^2 + a)\*(e\*x + d)^n\*(g\*x)^m, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x)**m*(e*x+d)**n*(c*x**2+a),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (cx^2 + a)(ex + d)^n (gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)*(e*x + d)^n*(g*x)^m,x, algorithm="giac")`

[Out] `integrate((c*x^2 + a)*(e*x + d)^n*(g*x)^m, x)`

$$3.379 \quad \int \frac{(gx)^m (d+ex)^n}{a+cx^2} dx$$

**Optimal.** Leaf size=148

$$\frac{(gx)^{m+1}(d+ex)^n \left(\frac{ex}{d} + 1\right)^{-n} F_1\left(m+1; -n, 1; m+2; -\frac{ex}{d}, -\frac{\sqrt{cx}}{\sqrt{-a}}\right)}{2ag(m+1)} + \frac{(gx)^{m+1}(d+ex)^n \left(\frac{ex}{d} + 1\right)^{-n} F_1\left(m+1; -n, 1; m+2; -\frac{ex}{d}, \frac{\sqrt{cx}}{\sqrt{-a}}\right)}{2ag(m+1)}$$

[Out] ((g\*x)^(1+m)\*(d+e\*x)^n\*AppellF1[1+m, -n, 1, 2+m, -((e\*x)/d), -((Sqrt[c]\*x)/Sqrt[-a])])/(2\*a\*g\*(1+m)\*(1+(e\*x)/d)^n) + ((g\*x)^(1+m)\*(d+e\*x)^n\*AppellF1[1+m, -n, 1, 2+m, -((e\*x)/d), (Sqrt[c]\*x)/Sqrt[-a]])/(2\*a\*g\*(1+m)\*(1+(e\*x)/d)^n)

**Rubi [A]** time = 0.411911, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$\frac{(gx)^{m+1}(d+ex)^n \left(\frac{ex}{d} + 1\right)^{-n} F_1\left(m+1; -n, 1; m+2; -\frac{ex}{d}, -\frac{\sqrt{cx}}{\sqrt{-a}}\right)}{2ag(m+1)} + \frac{(gx)^{m+1}(d+ex)^n \left(\frac{ex}{d} + 1\right)^{-n} F_1\left(m+1; -n, 1; m+2; -\frac{ex}{d}, \frac{\sqrt{cx}}{\sqrt{-a}}\right)}{2ag(m+1)}$$

Antiderivative was successfully verified.

[In] Int[((g\*x)^m\*(d+e\*x)^n)/(a+c\*x^2),x]

[Out] ((g\*x)^(1+m)\*(d+e\*x)^n\*AppellF1[1+m, -n, 1, 2+m, -((e\*x)/d), -((Sqrt[c]\*x)/Sqrt[-a])])/(2\*a\*g\*(1+m)\*(1+(e\*x)/d)^n) + ((g\*x)^(1+m)\*(d+e\*x)^n\*AppellF1[1+m, -n, 1, 2+m, -((e\*x)/d), (Sqrt[c]\*x)/Sqrt[-a]])/(2\*a\*g\*(1+m)\*(1+(e\*x)/d)^n)

**Rubi in Sympy [A]** time = 61.0681, size = 112, normalized size = 0.76

$$\frac{(gx)^{m+1} \left(1 + \frac{ex}{d}\right)^{-n} (d+ex)^n \operatorname{appellf}_1\left(m+1, 1, -n, m+2, -\frac{\sqrt{cx}}{\sqrt{-a}}, -\frac{ex}{d}\right)}{2ag(m+1)} + \frac{(gx)^{m+1} \left(1 + \frac{ex}{d}\right)^{-n} (d+ex)^n \operatorname{appellf}_1\left(m+1, 1, -n, m+2, \frac{\sqrt{cx}}{\sqrt{-a}}, -\frac{ex}{d}\right)}{2ag(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x)\*\*m\*(e\*x+d)\*\*n/(c\*x\*\*2+a),x)

[Out] (g\*x)\*\*(m+1)\*(1+e\*x/d)\*\*(-n)\*(d+e\*x)\*\*n\*appellf1(m+1, 1, -n, m+2, -sqrt(c)\*x/sqrt(-a), -e\*x/d)/(2\*a\*g\*(m+1)) + (g\*x)\*\*(m+1)\*(1+e\*x/d)\*\*(-n)\*(d+e\*x)\*\*n\*appellf1(m+1, 1, -n, m+2, sqrt(c)\*x/sqrt(-a), -e\*x/d)/(2\*a\*g\*(m+1))

**Mathematica [A]** time = 0.0845539, size = 0, normalized size = 0.

$$\int \frac{(gx)^m (d+ex)^n}{a+cx^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[((g\*x)^m\*(d + e\*x)^n)/(a + c\*x^2), x]

[Out] Integrate[((g\*x)^m\*(d + e\*x)^n)/(a + c\*x^2), x]

**Maple** [F] time = 0.078, size = 0, normalized size = 0.

$$\int \frac{(gx)^m (ex + d)^n}{cx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x)^m\*(e\*x+d)^n/(c\*x^2+a), x)

[Out] int((g\*x)^m\*(e\*x+d)^n/(c\*x^2+a), x)

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n (gx)^m}{cx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^n\*(g\*x)^m/(c\*x^2 + a), x, algorithm="maxima")

[Out] integrate((e\*x + d)^n\*(g\*x)^m/(c\*x^2 + a), x)

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex + d)^n (gx)^m}{cx^2 + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^n\*(g\*x)^m/(c\*x^2 + a), x, algorithm="fricas")

[Out] integral((e\*x + d)^n\*(g\*x)^m/(c\*x^2 + a), x)

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x)\*\*m\*(e\*x+d)\*\*n/(c\*x\*\*2+a), x)

[Out] Timed out

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n (gx)^m}{cx^2 + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^n*(g*x)^m/(c*x^2 + a),x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^n*(g*x)^m/(c*x^2 + a), x)
```

$$3.380 \quad \int \frac{(gx)^m (d+ex)^n}{(a+cx^2)^2} dx$$

**Optimal.** Leaf size=295

$$\begin{aligned} & \frac{(gx)^{m+1}(d+ex)^n \left(\frac{ex}{d} + 1\right)^{-n} F_1\left(m+1; -n, 1; m+2; -\frac{ex}{d}, -\frac{\sqrt{cx}}{\sqrt{-a}}\right)}{4a^2g(m+1)} \\ & + \frac{(gx)^{m+1}(d+ex)^n \left(\frac{ex}{d} + 1\right)^{-n} F_1\left(m+1; -n, 1; m+2; -\frac{ex}{d}, \frac{\sqrt{cx}}{\sqrt{-a}}\right)}{4a^2g(m+1)} \\ & + \frac{(gx)^{m+1}(d+ex)^n \left(\frac{ex}{d} + 1\right)^{-n} F_1\left(m+1; -n, 2; m+2; -\frac{ex}{d}, -\frac{\sqrt{cx}}{\sqrt{-a}}\right)}{4a^2g(m+1)} \\ & + \frac{(gx)^{m+1}(d+ex)^n \left(\frac{ex}{d} + 1\right)^{-n} F_1\left(m+1; -n, 2; m+2; -\frac{ex}{d}, \frac{\sqrt{cx}}{\sqrt{-a}}\right)}{4a^2g(m+1)} \end{aligned}$$

[Out]  $((g*x)^{(1+m)}*(d+e*x)^n*AppellF1[1+m, -n, 1, 2+m, -((e*x)/d), -((Sqrt[c]*x)/Sqrt[-a])])/(4*a^2*g*(1+m)*(1+(e*x)/d)^n) + ((g*x)^{(1+m)}*(d+e*x)^n*AppellF1[1+m, -n, 1, 2+m, -((e*x)/d), (Sqrt[c]*x)/Sqrt[-a]])/(4*a^2*g*(1+m)*(1+(e*x)/d)^n) + ((g*x)^{(1+m)}*(d+e*x)^n*AppellF1[1+m, -n, 2, 2+m, -((e*x)/d), -((Sqrt[c]*x)/Sqrt[-a])])/(4*a^2*g*(1+m)*(1+(e*x)/d)^n) + ((g*x)^{(1+m)}*(d+e*x)^n*AppellF1[1+m, -n, 2, 2+m, -((e*x)/d), (Sqrt[c]*x)/Sqrt[-a]])/(4*a^2*g*(1+m)*(1+(e*x)/d)^n)$

**Rubi [A]** time = 0.962102, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\begin{aligned} & \frac{(gx)^{m+1}(d+ex)^n \left(\frac{ex}{d} + 1\right)^{-n} F_1\left(m+1; -n, 1; m+2; -\frac{ex}{d}, -\frac{\sqrt{cx}}{\sqrt{-a}}\right)}{4a^2g(m+1)} \\ & + \frac{(gx)^{m+1}(d+ex)^n \left(\frac{ex}{d} + 1\right)^{-n} F_1\left(m+1; -n, 1; m+2; -\frac{ex}{d}, \frac{\sqrt{cx}}{\sqrt{-a}}\right)}{4a^2g(m+1)} \\ & + \frac{(gx)^{m+1}(d+ex)^n \left(\frac{ex}{d} + 1\right)^{-n} F_1\left(m+1; -n, 2; m+2; -\frac{ex}{d}, -\frac{\sqrt{cx}}{\sqrt{-a}}\right)}{4a^2g(m+1)} \\ & + \frac{(gx)^{m+1}(d+ex)^n \left(\frac{ex}{d} + 1\right)^{-n} F_1\left(m+1; -n, 2; m+2; -\frac{ex}{d}, \frac{\sqrt{cx}}{\sqrt{-a}}\right)}{4a^2g(m+1)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[((g\*x)^m\*(d+e\*x)^n)/(a+c\*x^2)^2,x]

[Out]  $((g*x)^{(1+m)}*(d+e*x)^n*AppellF1[1+m, -n, 1, 2+m, -((e*x)/d), -((Sqrt[c]*x)/Sqrt[-a])])/(4*a^2*g*(1+m)*(1+(e*x)/d)^n) + ((g*x)^{(1+m)}*(d+e*x)^n*AppellF1[1+m, -n, 1, 2+m, -((e*x)/d), (Sqrt[c]*x)/Sqrt[-a]])/(4*a^2*g*(1+m)*(1+(e*x)/d)^n) + ((g*x)^{(1+m)}*(d+e*x)^n*AppellF1[1+m, -n, 2, 2+m, -((e*x)/d), -((Sqrt[c]*x)/Sqrt[-a])])/(4*a^2*g*(1+m)*(1+(e*x)/d)^n) + ((g*x)^{(1+m)}*(d+e*x)^n*AppellF1[1+m, -n, 2, 2+m, -((e*x)/d), (Sqrt[c]*x)/Sqrt[-a]])/(4*a^2*g*(1+m)*(1+(e*x)/d)^n)$

**Rubi in Sympy [A]** time = 152.239, size = 233, normalized size = 0.79

$$\begin{aligned} & \frac{(gx)^{m+1} \left(1 + \frac{ex}{d}\right)^{-n} (d + ex)^n \operatorname{appellf}_1\left(m + 1, 1, -n, m + 2, -\frac{\sqrt{cx}}{\sqrt{-a}}, -\frac{ex}{d}\right)}{4a^2g(m + 1)} \\ & + \frac{(gx)^{m+1} \left(1 + \frac{ex}{d}\right)^{-n} (d + ex)^n \operatorname{appellf}_1\left(m + 1, 1, -n, m + 2, \frac{\sqrt{cx}}{\sqrt{-a}}, -\frac{ex}{d}\right)}{4a^2g(m + 1)} \\ & + \frac{(gx)^{m+1} \left(1 + \frac{ex}{d}\right)^{-n} (d + ex)^n \operatorname{appellf}_1\left(m + 1, 2, -n, m + 2, -\frac{\sqrt{cx}}{\sqrt{-a}}, -\frac{ex}{d}\right)}{4a^2g(m + 1)} \\ & + \frac{(gx)^{m+1} \left(1 + \frac{ex}{d}\right)^{-n} (d + ex)^n \operatorname{appellf}_1\left(m + 1, 2, -n, m + 2, \frac{\sqrt{cx}}{\sqrt{-a}}, -\frac{ex}{d}\right)}{4a^2g(m + 1)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x)**m*(e*x+d)**n/(c*x**2+a)**2,x)`

[Out]  $(g*x)^{(m + 1)} * (1 + e*x/d)^{(-n)} * (d + e*x)^n * \operatorname{appellf}_1(m + 1, 1, -n, m + 2, -\sqrt{c}*x/\sqrt{-a}, -e*x/d)/(4*a**2*g*(m + 1)) + (g*x)^{(m + 1)} * (1 + e*x/d)^{(-n)} * (d + e*x)^n * \operatorname{appellf}_1(m + 1, 1, -n, m + 2, \sqrt{c}*x/\sqrt{-a}, -e*x/d)/(4*a**2*g*(m + 1)) + (g*x)^{(m + 1)} * (1 + e*x/d)^{(-n)} * (d + e*x)^n * \operatorname{appellf}_1(m + 1, 2, -n, m + 2, -\sqrt{c}*x/\sqrt{-a}, -e*x/d)/(4*a**2*g*(m + 1)) + (g*x)^{(m + 1)} * (1 + e*x/d)^{(-n)} * (d + e*x)^n * \operatorname{appellf}_1(m + 1, 2, -n, m + 2, \sqrt{c}*x/\sqrt{-a}, -e*x/d)/(4*a**2*g*(m + 1))$

**Mathematica [A]** time = 0.109072, size = 0, normalized size = 0.

$$\int \frac{(gx)^m (d + ex)^n}{(a + cx^2)^2} dx$$

Verification is Not applicable to the result.

[In] `Integrate[((g*x)^m*(d + e*x)^n)/(a + c*x^2)^2,x]`

[Out] `Integrate[((g*x)^m*(d + e*x)^n)/(a + c*x^2)^2, x]`

**Maple [F]** time = 0.084, size = 0, normalized size = 0.

$$\int \frac{(gx)^m (ex + d)^n}{(cx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x)^m*(e*x+d)^n/(c*x^2+a)^2,x)`

[Out] `int((g*x)^m*(e*x+d)^n/(c*x^2+a)^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n (gx)^m}{(cx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^n*(g*x)^m/(c*x^2 + a)^2,x, algorithm="maxima")`

[Out] `integrate((e*x + d)^n*(g*x)^m/(c*x^2 + a)^2, x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex + d)^n (gx)^m}{c^2x^4 + 2acx^2 + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^n*(g*x)^m/(c*x^2 + a)^2,x, algorithm="fricas")`

[Out] `integral((e*x + d)^n*(g*x)^m/(c^2*x^4 + 2*a*c*x^2 + a^2), x)`

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x)**m*(e*x+d)**n/(c*x**2+a)**2,x)`

[Out] Timed out

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^n (gx)^m}{(cx^2 + a)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^n*(g*x)^m/(c*x^2 + a)^2,x, algorithm="giac")`

[Out] `integrate((e*x + d)^n*(g*x)^m/(c*x^2 + a)^2, x)`



### 3.381 $\int x^5(d + ex)(a + bx^2)^p dx$

**Optimal.** Leaf size=125

$$\frac{a^2 d (a + bx^2)^{p+1}}{2b^3(p+1)} - \frac{ad (a + bx^2)^{p+2}}{b^3(p+2)} + \frac{d (a + bx^2)^{p+3}}{2b^3(p+3)} + \frac{1}{7} ex^7 (a + bx^2)^p \left( \frac{bx^2}{a} + 1 \right)^{-p} {}_2F_1 \left( \frac{7}{2}, -p; \frac{9}{2}; -\frac{bx^2}{a} \right)$$

[Out]  $(a^2 d (a + b x^2)^{(1+p)}) / (2 b^3 (1+p)) - (a d (a + b x^2)^{(2+p)}) / (b^3 (2+p)) + (d (a + b x^2)^{(3+p)}) / (2 b^3 (3+p)) + (e x^7 (a + b x^2)^p \text{Hypergeometric2F1}[7/2, -p, 9/2, -(b x^2)/a]) / (7 (1 + (b x^2)/a)^p)$

**Rubi [A]** time = 0.20005, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$

$$\frac{a^2 d (a + bx^2)^{p+1}}{2b^3(p+1)} - \frac{ad (a + bx^2)^{p+2}}{b^3(p+2)} + \frac{d (a + bx^2)^{p+3}}{2b^3(p+3)} + \frac{1}{7} ex^7 (a + bx^2)^p \left( \frac{bx^2}{a} + 1 \right)^{-p} {}_2F_1 \left( \frac{7}{2}, -p; \frac{9}{2}; -\frac{bx^2}{a} \right)$$

Antiderivative was successfully verified.

[In] Int[x^5\*(d + e\*x)\*(a + b\*x^2)^p, x]

[Out]  $(a^2 d (a + b x^2)^{(1+p)}) / (2 b^3 (1+p)) - (a d (a + b x^2)^{(2+p)}) / (b^3 (2+p)) + (d (a + b x^2)^{(3+p)}) / (2 b^3 (3+p)) + (e x^7 (a + b x^2)^p \text{Hypergeometric2F1}[7/2, -p, 9/2, -(b x^2)/a]) / (7 (1 + (b x^2)/a)^p)$

**Rubi in Sympy [A]** time = 28.9166, size = 104, normalized size = 0.83

$$\frac{a^2 d (a + bx^2)^{p+1}}{2b^3(p+1)} - \frac{ad (a + bx^2)^{p+2}}{b^3(p+2)} + \frac{ex^7 \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1 \left( \frac{-p}{2}, \frac{7}{2} \middle| -\frac{bx^2}{a} \right)}{7} + \frac{d (a + bx^2)^{p+3}}{2b^3(p+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5\*(e\*x+d)\*(b\*x\*\*2+a)\*\*p, x)

[Out]  $a^2 d (a + b x^2)^{(p+1)} / (2 b^3 (p+1)) - a d (a + b x^2)^{(p+2)} / (b^3 (p+2)) + e x^7 (1 + b x^2/a)^{-p} (a + b x^2)^p \text{hyper}((-p, 7/2), (9/2, ), -b x^2/a) / 7 + d (a + b x^2)^{(p+3)} / (2 b^3 (p+3))$

**Mathematica [A]** time = 0.240914, size = 183, normalized size = 1.46

$$\frac{(a + bx^2)^p \left( \frac{bx^2}{a} + 1 \right)^{-p} \left( 7d \left( 2a^3 \left( \left( \frac{bx^2}{a} + 1 \right)^p - 1 \right) - 2a^2 b p x^2 \left( \frac{bx^2}{a} + 1 \right)^p + b^3 (p^2 + 3p + 2) x^6 \left( \frac{bx^2}{a} + 1 \right)^p + ab^2 p(p+1)x^4 \right)}{14b^3(p+1)(p+2)(p+3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(d + e\*x)\*(a + b\*x^2)^p, x]

[Out]  $((a + b x^2)^p (7 d (-2 a^2 b p x^2 (1 + (b x^2)/a)^p + a b^2 p (1 + p) x^4 (1 + (b x^2)/a)^p + b^3 (2 + 3 p + p^2) x^6 (1 + (b x^2)/a)^p)) / (14 b^3 (p + 1) (p + 2) (p + 3))$

$2)/a)^p + 2*a^3*(-1 + (1 + (b*x^2)/a)^p) + 2*b^3*e*(6 + 11*p + 6*p^2 + p^3)*x^7*Hypergeometric2F1[7/2, -p, 9/2, -((b*x^2)/a)])/(14*b^3*(1 + p)*(2 + p)*(3 + p)*(1 + (b*x^2)/a)^p)$

**Maple [F]** time = 0.077, size = 0, normalized size = 0.

$$\int x^5 (ex + d) (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(e\*x+d)\*(b\*x^2+a)^p,x)

[Out] int(x^5\*(e\*x+d)\*(b\*x^2+a)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$e \int (bx^2 + a)^p x^6 dx + \frac{((p^2 + 3p + 2)b^3x^6 + (p^2 + p)ab^2x^4 - 2a^2bpx^2 + 2a^3)(bx^2 + a)^p d}{2(p^3 + 6p^2 + 11p + 6)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(b\*x^2 + a)^p\*x^5,x, algorithm="maxima")

[Out] e\*integrate((b\*x^2 + a)^p\*x^6, x) + 1/2\*((p^2 + 3\*p + 2)\*b^3\*x^6 + (p^2 + p)\*a\*b^2\*x^4 - 2\*a^2\*b\*p\*x^2 + 2\*a^3)\*(b\*x^2 + a)^p\*d/((p^3 + 6\*p^2 + 11\*p + 6)\*b^3)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex^6 + dx^5)(bx^2 + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(b\*x^2 + a)^p\*x^5,x, algorithm="fricas")

[Out] integral((e\*x^6 + d\*x^5)\*(b\*x^2 + a)^p, x)

**Sympy [A]** time = 117.074, size = 1012, normalized size = 8.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(e\*x+d)\*(b\*x\*\*2+a)\*\*p,x)

[Out] a\*\*p\*e\*x\*\*7\*hyper((7/2, -p), (9/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/7 + d\*Piecewise((a\*\*p\*x\*\*6/6, Eq(b, 0)), (2\*a\*\*2\*log(-I\*sqrt(a)\*sqrt(1/b) + x)/(4\*a\*\*2\*b\*\*3 + 8\*a\*b\*\*4\*x\*\*2 + 4\*b\*\*5\*x\*\*4) + 2\*a\*\*2\*log(I\*sqrt(a)\*sqrt(1/b) + x)/(4\*a\*\*2\*b\*\*3 + 8\*a\*b\*\*4\*x\*\*2 + 4\*b\*\*5\*x\*\*4) + 3\*a\*\*2/(4\*a\*\*2\*b\*\*3 + 8\*a\*b\*\*4\*x\*\*2 + 4\*b\*\*5\*x\*\*4) + 4\*a\*b\*x\*\*2\*log(-I\*sqrt(a)\*sqrt(1/b) + x)/(4\*a\*\*2\*b\*\*3 + 8\*a\*b\*\*4\*x\*\*2 + 4\*b\*\*5\*x\*\*4))

```

*2 + 4*b**5*x**4) + 4*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a
*2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3
+ 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(-I*sqrt(a)*sqrt(
1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x
*4*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*
b**5*x**4), Eq(p, -3)), (-2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2
*a*b**3 + 2*b**4*x**2) - 2*a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a
*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x
*2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a*b
*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) + b**
2*x**4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(-I*sqrt(a)
*sqrt(1/b) + x)/(2*b**3) + a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b
**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x
**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) - 2*a*
*2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3
*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 1
2*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**
p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*
x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12
*b**3) + 3*b**3*p*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**
2 + 22*b**3*p + 12*b**3) + 2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p
*3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3), True))

```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(bx^2 + a)^p x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(b\*x^2 + a)^p\*x^5,x, algorithm="giac")

[Out] integrate((e\*x + d)\*(b\*x^2 + a)^p\*x^5, x)

### 3.382 $\int x^4(d + ex) (a + bx^2)^p dx$

**Optimal.** Leaf size=125

$$\frac{a^2 e (a + bx^2)^{p+1}}{2b^3(p+1)} - \frac{ae (a + bx^2)^{p+2}}{b^3(p+2)} + \frac{e (a + bx^2)^{p+3}}{2b^3(p+3)} + \frac{1}{5} dx^5 (a + bx^2)^p \left( \frac{bx^2}{a} + 1 \right)^{-p} {}_2F_1 \left( \frac{5}{2}, -p; \frac{7}{2}; -\frac{bx^2}{a} \right)$$

[Out]  $(a^2 e^*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) - (a*e*(a + b*x^2)^(2 + p))/(b^3*(2 + p)) + (e*(a + b*x^2)^(3 + p))/(2*b^3*(3 + p)) + (d*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -(b*x^2)/a])/ (5*(1 + (b*x^2)/a)^p)$

**Rubi [A]** time = 0.1958, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$

$$\frac{a^2 e (a + bx^2)^{p+1}}{2b^3(p+1)} - \frac{ae (a + bx^2)^{p+2}}{b^3(p+2)} + \frac{e (a + bx^2)^{p+3}}{2b^3(p+3)} + \frac{1}{5} dx^5 (a + bx^2)^p \left( \frac{bx^2}{a} + 1 \right)^{-p} {}_2F_1 \left( \frac{5}{2}, -p; \frac{7}{2}; -\frac{bx^2}{a} \right)$$

Antiderivative was successfully verified.

[In] Int[x^4\*(d + e\*x)\*(a + b\*x^2)^p, x]

[Out]  $(a^2 e^*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) - (a*e*(a + b*x^2)^(2 + p))/(b^3*(2 + p)) + (e*(a + b*x^2)^(3 + p))/(2*b^3*(3 + p)) + (d*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -(b*x^2)/a])/ (5*(1 + (b*x^2)/a)^p)$

**Rubi in Sympy [A]** time = 28.7686, size = 104, normalized size = 0.83

$$\frac{a^2 e (a + bx^2)^{p+1}}{2b^3(p+1)} - \frac{ae (a + bx^2)^{p+2}}{b^3(p+2)} + \frac{dx^5 \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1 \left( \frac{-p, \frac{5}{2}}{\frac{7}{2}} \middle| -\frac{bx^2}{a} \right)}{5} + \frac{e (a + bx^2)^{p+3}}{2b^3(p+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(e\*x+d)\*(b\*x\*\*2+a)\*\*p, x)

[Out]  $a**2*e*(a + b*x**2)**(p + 1)/(2*b**3*(p + 1)) - a*e*(a + b*x**2)**(p + 2)/(b**3*(p + 2)) + d*x**5*(1 + b*x**2/a)**(-p)*(a + b*x**2)**p*hyper((-p, 5/2), (7/2, ), -b*x**2/a)/5 + e*(a + b*x**2)**(p + 3)/(2*b**3*(p + 3))$

**Mathematica [A]** time = 0.203704, size = 183, normalized size = 1.46

$$\frac{(a + bx^2)^p \left( \frac{bx^2}{a} + 1 \right)^{-p} \left( 5e \left( 2a^3 \left( \left( \frac{bx^2}{a} + 1 \right)^p - 1 \right) - 2a^2 b p x^2 \left( \frac{bx^2}{a} + 1 \right)^p + b^3 (p^2 + 3p + 2) x^6 \left( \frac{bx^2}{a} + 1 \right)^p + ab^2 p (p + 1) x^4 \right)}{10b^3(p+1)(p+2)(p+3)}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(d + e\*x)\*(a + b\*x^2)^p, x]

[Out]  $((a + b*x^2)^p*(5*e*(-2*a^2*b*p*x^2*(1 + (b*x^2)/a)^p + a*b^2*p*(1 + p)*x^4*(1 + (b*x^2)/a)^p + b^3*(2 + 3*p + p^2)*x^6*(1 + (b*x^2)/a)^p + e*(a + b*x^2)^{p+3}))/ (5*(1 + (b*x^2)/a)^p)$

$2)/a)^p + 2*a^3*(-1 + (1 + (b*x^2)/a)^p) + 2*b^3*d*(6 + 11*p + 6*p^2 + p^3)*x^5*Hypergeometric2F1[5/2, -p, 7/2, -((b*x^2)/a)])/(10*b^3*(1 + p)*(2 + p)*(3 + p)*(1 + (b*x^2)/a)^p)$

**Maple [F]** time = 0.079, size = 0, normalized size = 0.

$$\int x^4 (ex + d) (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(e\*x+d)\*(b\*x^2+a)^p,x)

[Out] int(x^4\*(e\*x+d)\*(b\*x^2+a)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(bx^2 + a)^p x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(b\*x^2 + a)^p\*x^4,x, algorithm="maxima")

[Out] integrate((e\*x + d)\*(b\*x^2 + a)^p\*x^4, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex^5 + dx^4)(bx^2 + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(b\*x^2 + a)^p\*x^4,x, algorithm="fricas")

[Out] integral((e\*x^5 + d\*x^4)\*(b\*x^2 + a)^p, x)

**Sympy [A]** time = 83.1233, size = 1012, normalized size = 8.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(e\*x+d)\*(b\*x\*\*2+a)\*\*p,x)

[Out]  $a**p*d*x**5*hyper((5/2, -p), (7/2, ), b*x**2*exp\_polar(I*pi)/a)/5 + e*Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3$

```

+ 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(-I*sqrt(a)*sqrt(
1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x*
**4*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*
b**5*x**4), Eq(p, -3)), (-2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2
*a*b**3 + 2*b**4*x**2) - 2*a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a
*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x*
**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a*b
*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) + b**
2*x**4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(-I*sqrt(a)
*sqrt(1/b) + x)/(2*b**3) + a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b
**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x
**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) - 2*a*
**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3
*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 1
2*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**
p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*
x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12
*b**3) + 3*b**3*p*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**
2 + 22*b**3*p + 12*b**3) + 2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p
**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3), True))

```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(bx^2 + a)^p x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(b\*x^2 + a)^p\*x^4,x, algorithm="giac")

[Out] integrate((e\*x + d)\*(b\*x^2 + a)^p\*x^4, x)

### 3.383 $\int x^3(d + ex)(a + bx^2)^p dx$

**Optimal.** Leaf size=100

$$-\frac{ad(a+bx^2)^{p+1}}{2b^2(p+1)} + \frac{d(a+bx^2)^{p+2}}{2b^2(p+2)} + \frac{1}{5}ex^5(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; -\frac{bx^2}{a}\right)$$

[Out]  $-(a*d*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (d*(a + b*x^2)^(2 + p))/(2*b^2*(2 + p)) + (e*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -(b*x^2)/a])/(5*(1 + (b*x^2)/a)^p)$

**Rubi [A]** time = 0.152345, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$

$$-\frac{ad(a+bx^2)^{p+1}}{2b^2(p+1)} + \frac{d(a+bx^2)^{p+2}}{2b^2(p+2)} + \frac{1}{5}ex^5(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; -\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[x^3\*(d + e\*x)\*(a + b\*x^2)^p, x]

[Out]  $-(a*d*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (d*(a + b*x^2)^(2 + p))/(2*b^2*(2 + p)) + (e*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -(b*x^2)/a])/(5*(1 + (b*x^2)/a)^p)$

**Rubi in Sympy [A]** time = 20.8856, size = 82, normalized size = 0.82

$$-\frac{ad(a+bx^2)^{p+1}}{2b^2(p+1)} + \frac{ex^5\left(1 + \frac{bx^2}{a}\right)^{-p}(a+bx^2)^p {}_2F_1\left(-p, \frac{5}{2} \middle| \frac{7}{2}; -\frac{bx^2}{a}\right)}{5} + \frac{d(a+bx^2)^{p+2}}{2b^2(p+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(e\*x+d)\*(b\*x\*\*2+a)\*\*p, x)

[Out]  $-a*d*(a + b*x**2)**(p + 1)/(2*b**2*(p + 1)) + e*x**5*(1 + b*x**2/a)**(-p)*(a + b*x**2)**p*hyper((-p, 5/2), (7/2, ), -b*x**2/a)/5 + d*(a + b*x**2)**(p + 2)/(2*b**2*(p + 2))$

**Mathematica [A]** time = 0.149363, size = 141, normalized size = 1.41

$$\frac{(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} \left(5d \left(-a^2 \left(\left(\frac{bx^2}{a} + 1\right)^p - 1\right) + b^2(p+1)x^4 \left(\frac{bx^2}{a} + 1\right)^p + abpx^2 \left(\frac{bx^2}{a} + 1\right)^p\right) + 2b^2e(p^2 + 3p + 2)x^5}{10b^2(p+1)(p+2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(d + e\*x)\*(a + b\*x^2)^p, x]

[Out]  $((a + b*x^2)^p*(5*d*(a*b*p*x^2*(1 + (b*x^2)/a)^p + b^2*(1 + p)*x^4*(1 + (b*x^2)/a)^p - a^2*(-1 + (1 + (b*x^2)/a)^p)) + 2*b^2*e*(2 + 3*p + p^2)*x^5*Hypergeometric2F1[5/2, -p, 7/2, -(b*x^2)/a])/(10*b^2*(1 + p)*(2 + p)*(1 + (b*x^2)/a)^p)$

**Maple [F]** time = 0.06, size = 0, normalized size = 0.

$$\int x^3 (ex + d) (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(e\*x+d)\*(b\*x^2+a)^p,x)

[Out] int(x^3\*(e\*x+d)\*(b\*x^2+a)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$e \int (bx^2 + a)^p x^4 dx + \frac{(b^2(p+1)x^4 + abpx^2 - a^2)(bx^2 + a)^p d}{2(p^2 + 3p + 2)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(b\*x^2 + a)^p\*x^3,x, algorithm="maxima")

[Out] e\*integrate((b\*x^2 + a)^p\*x^4, x) + 1/2\*(b^2\*(p + 1)\*x^4 + a\*b\*p\*x^2 - a^2)\*(b\*x^2 + a)^p\*d/((p^2 + 3\*p + 2)\*b^2)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex^4 + dx^3)(bx^2 + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(b\*x^2 + a)^p\*x^3,x, algorithm="fricas")

[Out] integral((e\*x^4 + d\*x^3)\*(b\*x^2 + a)^p, x)

**Sympy [A]** time = 62.1887, size = 394, normalized size = 3.94

$$\frac{a^p e x^5 {}_2F_1\left(\frac{5}{2}, -p \mid \frac{bx^2 e^{i\pi}}{a}\right)}{5} + d \begin{cases} \left( \frac{a^p x^4}{4} \right. & \text{for } b = 0 \\ \frac{a \log\left(-i\sqrt{a}\sqrt{\frac{1}{b}+x}\right)}{2ab^2+2b^3x^2} + \frac{a \log\left(i\sqrt{a}\sqrt{\frac{1}{b}+x}\right)}{2ab^2+2b^3x^2} + \frac{a}{2ab^2+2b^3x^2} + \frac{bx^2 \log\left(-i\sqrt{a}\sqrt{\frac{1}{b}+x}\right)}{2ab^2+2b^3x^2} + \frac{bx^2 \log\left(i\sqrt{a}\sqrt{\frac{1}{b}+x}\right)}{2ab^2+2b^3x^2} & \text{for } p = -2 \\ \left. -\frac{a \log\left(-i\sqrt{a}\sqrt{\frac{1}{b}+x}\right)}{2b^2} - \frac{a \log\left(i\sqrt{a}\sqrt{\frac{1}{b}+x}\right)}{2b^2} + \frac{x^2}{2b} \right) & \text{for } p = -1 \\ \left( -\frac{a^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{abpx^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2px^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2x^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} \right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(e\*x+d)\*(b\*x\*\*2+a)\*\*p,x)

[Out] a\*\*p\*e\*x\*\*5\*hyper((5/2, -p), (7/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/5 + d\*Piecewise((a\*\*p\*x\*\*4/4, Eq(b, 0)), (a\*log(-I\*sqrt(a)\*sqrt(1/b



```
) + x)/(2*a*b**2 + 2*b**3*x**2) + a*log(I*sqrt(a)*sqrt(1/b) + x)/
(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log
(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log
(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (
-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I*sqrt(a)*sqrt(
1/b) + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2)**
*p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p
/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/
(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*
b**2*p**2 + 6*b**2*p + 4*b**2), True))
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(bx^2 + a)^p x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)*(b*x^2 + a)^p*x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p*x^3, x)
```

### 3.384 $\int x^2(d + ex) (a + bx^2)^p dx$

**Optimal.** Leaf size=100

$$-\frac{ae(a+bx^2)^{p+1}}{2b^2(p+1)} + \frac{e(a+bx^2)^{p+2}}{2b^2(p+2)} + \frac{1}{3}dx^3(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; -\frac{bx^2}{a}\right)$$

[Out]  $-(a * e * (a + b * x^2)^{(1 + p)}) / (2 * b^2 * (1 + p)) + (e * (a + b * x^2)^{(2 + p)}) / (2 * b^2 * (2 + p)) + (d * x^3 * (a + b * x^2)^p * \text{Hypergeometric2F1}[3/2, -p, 5/2, -(b * x^2)/a]) / (3 * (1 + (b * x^2)/a)^p)$

**Rubi [A]** time = 0.147994, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$

$$-\frac{ae(a+bx^2)^{p+1}}{2b^2(p+1)} + \frac{e(a+bx^2)^{p+2}}{2b^2(p+2)} + \frac{1}{3}dx^3(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; -\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[x^2\*(d + e\*x)\*(a + b\*x^2)^p, x]

[Out]  $-(a * e * (a + b * x^2)^{(1 + p)}) / (2 * b^2 * (1 + p)) + (e * (a + b * x^2)^{(2 + p)}) / (2 * b^2 * (2 + p)) + (d * x^3 * (a + b * x^2)^p * \text{Hypergeometric2F1}[3/2, -p, 5/2, -(b * x^2)/a]) / (3 * (1 + (b * x^2)/a)^p)$

**Rubi in Sympy [A]** time = 21.4733, size = 82, normalized size = 0.82

$$-\frac{ae(a+bx^2)^{p+1}}{2b^2(p+1)} + \frac{dx^3 \left(1 + \frac{bx^2}{a}\right)^{-p} (a+bx^2)^p {}_2F_1\left(-p, \frac{3}{2} \middle| \frac{5}{2}; -\frac{bx^2}{a}\right)}{3} + \frac{e(a+bx^2)^{p+2}}{2b^2(p+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(e\*x+d)\*(b\*x\*\*2+a)\*\*p, x)

[Out]  $-a * e * (a + b * x^2)^{(p + 1)} / (2 * b^2 * (p + 1)) + d * x^3 * (1 + b * x^2 / a)^{-p} * (a + b * x^2)^p * \text{hyper}((-p, 3/2), (5/2, ), -b * x^2 / a) / 3 + e * (a + b * x^2)^{(p + 2)} / (2 * b^2 * (p + 2))$

**Mathematica [A]** time = 0.133301, size = 141, normalized size = 1.41

$$\frac{(a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} \left(3e \left(-a^2 \left(\left(\frac{bx^2}{a} + 1\right)^p - 1\right) + b^2(p+1)x^4 \left(\frac{bx^2}{a} + 1\right)^p + abpx^2 \left(\frac{bx^2}{a} + 1\right)^p\right) + 2b^2d(p^2 + 3p + 2)x^3}{6b^2(p+1)(p+2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(d + e\*x)\*(a + b\*x^2)^p, x]

[Out]  $((a + b * x^2)^p * (3 * e * (a * b * p * x^2 * (1 + (b * x^2)/a)^p + b^2 * (1 + p) * x^4 * (1 + (b * x^2)/a)^p - a^2 * (-1 + (1 + (b * x^2)/a)^p)) + 2 * b^2 * d * (2 + 3 * p + p^2) * x^3 * \text{Hypergeometric2F1}[3/2, -p, 5/2, -(b * x^2)/a]) / (6 * b^2 * (1 + p) * (2 + p) * (1 + (b * x^2)/a)^p)$

---

**Maple [F]** time = 0.053, size = 0, normalized size = 0.

$$\int x^2 (ex + d) (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(e\*x+d)\*(b\*x^2+a)^p,x)

[Out] int(x^2\*(e\*x+d)\*(b\*x^2+a)^p,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(bx^2 + a)^p x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(b\*x^2 + a)^p\*x^2,x, algorithm="maxima")

[Out] integrate((e\*x + d)\*(b\*x^2 + a)^p\*x^2, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex^3 + dx^2)(bx^2 + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(b\*x^2 + a)^p\*x^2,x, algorithm="fricas")

[Out] integral((e\*x^3 + d\*x^2)\*(b\*x^2 + a)^p, x)

---

**Sympy [A]** time = 42.36, size = 394, normalized size = 3.94

$$\frac{a^p dx^3 {}_2F_1\left(\frac{3}{2}, -p \mid \frac{bx^2 e^{i\pi}}{a}\right)}{3} + e \left( \begin{array}{ll} \left( \frac{a^p x^4}{4} \right. & \text{for } b = 0 \\ \frac{a \log(-i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2ab^2+2b^3x^2} + \frac{a \log(i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2ab^2+2b^3x^2} + \frac{a}{2ab^2+2b^3x^2} + \frac{bx^2 \log(-i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2ab^2+2b^3x^2} + \frac{bx^2 \log(i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2ab^2+2b^3x^2} & \text{for } p = -2 \\ \left. - \frac{a \log(-i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2b^2} - \frac{a \log(i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2b^2} + \frac{x^2}{2b} \right) & \text{for } p = -1 \\ \left( -\frac{a^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{abpx^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2px^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2x^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} \right) & \text{otherwise} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(e\*x+d)\*(b\*x\*\*2+a)\*\*p,x)

[Out] a\*\*p\*d\*x\*\*3\*hyper((3/2, -p), (5/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/3 + e\*Piecewise((a\*\*p\*x\*\*4/4, Eq(b, 0)), (a\*log(-I\*sqrt(a)\*sqrt(1/b

```
) + x)/(2*a*b**2 + 2*b**3*x**2) + a*log(I*sqrt(a)*sqrt(1/b) + x)/
(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log
(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log
(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (
-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I*sqrt(a)*sqrt(
1/b) + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2)**
*p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p
/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/
(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*
b**2*p**2 + 6*b**2*p + 4*b**2), True))
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(bx^2 + a)^p x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)*(b*x^2 + a)^p*x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p*x^2, x)
```

### 3.385 $\int x(d + ex) (a + bx^2)^p dx$

**Optimal.** Leaf size=75

$$\frac{d(a + bx^2)^{p+1}}{2b(p+1)} + \frac{1}{3}ex^3(a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; -\frac{bx^2}{a}\right)$$

[Out] (d\*(a + b\*x^2)^(1 + p))/(2\*b\*(1 + p)) + (e\*x^3\*(a + b\*x^2)^p\*Hypergeometric2F1[3/2, -p, 5/2, -(b\*x^2)/a])/(3\*(1 + (b\*x^2)/a)^p)

**Rubi [A]** time = 0.0858863, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{d(a + bx^2)^{p+1}}{2b(p+1)} + \frac{1}{3}ex^3(a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; -\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[x\*(d + e\*x)\*(a + b\*x^2)^p, x]

[Out] (d\*(a + b\*x^2)^(1 + p))/(2\*b\*(1 + p)) + (e\*x^3\*(a + b\*x^2)^p\*Hypergeometric2F1[3/2, -p, 5/2, -(b\*x^2)/a])/(3\*(1 + (b\*x^2)/a)^p)

**Rubi in Sympy [A]** time = 12.2335, size = 58, normalized size = 0.77

$$\frac{ex^3 \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, \frac{3}{2} \middle| -\frac{bx^2}{a}\right)}{3} + \frac{d(a + bx^2)^{p+1}}{2b(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(e\*x+d)\*(b\*x\*\*2+a)\*\*p, x)

[Out] e\*x\*\*3\*(1 + b\*x\*\*2/a)\*\*(-p)\*(a + b\*x\*\*2)\*\*p\*hyper((-p, 3/2), (5/2, ), -b\*x\*\*2/a)/3 + d\*(a + b\*x\*\*2)\*\*(p + 1)/(2\*b\*(p + 1))

**Mathematica [A]** time = 0.0892388, size = 102, normalized size = 1.36

$$\frac{(a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} \left(3d \left(bx^2 \left(\frac{bx^2}{a} + 1\right)^p + a \left(\left(\frac{bx^2}{a} + 1\right)^p - 1\right)\right) + 2be(p+1)x^3 {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; -\frac{bx^2}{a}\right)}{6b(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(d + e\*x)\*(a + b\*x^2)^p, x]

[Out] ((a + b\*x^2)^p\*(3\*d\*(b\*x^2\*(1 + (b\*x^2)/a)^p + a\*(-1 + (1 + (b\*x^2)/a)^p)) + 2\*b\*e\*(1 + p)\*x^3\*Hypergeometric2F1[3/2, -p, 5/2, -(b\*x^2)/a])/(6\*b\*(1 + p)\*(1 + (b\*x^2)/a)^p)

**Maple [F]** time = 0.047, size = 0, normalized size = 0.

$$\int x(ex + d)(bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(e*x+d)*(b*x^2+a)^p,x)`

[Out] `int(x*(e*x+d)*(b*x^2+a)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)*(b*x^2 + a)^p*x,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex^2 + dx)(bx^2 + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)*(b*x^2 + a)^p*x,x, algorithm="fricas")`

[Out] `integral((e*x^2 + d*x)*(b*x^2 + a)^p, x)`

**Sympy [A]** time = 32.2824, size = 65, normalized size = 0.87

$$\frac{a^p e x^3 {}_2F_1\left(\frac{3}{2}, -p \mid \frac{b x^2 e^{i\pi}}{a}\right)}{3} + d \left( \begin{cases} \frac{a^p x^2}{2} & \text{for } b = 0 \\ \frac{(a + b x^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \frac{\log(a + b x^2)}{2b} & \text{otherwise} \end{cases} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x+d)*(b*x**2+a)**p,x)`

[Out] `a**p*e*x**3*hyper((3/2, -p), (5/2, ), b*x**2*exp_polar(I*pi)/a)/3 + d*Piecewise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**2), True))/(2*b), True))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(bx^2 + a)^p x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)*(b*x^2 + a)^p*x,x, algorithm="giac")`

[Out] `integrate((e*x + d)*(b*x^2 + a)^p*x, x)`

### 3.386 $\int (d + ex) (a + bx^2)^p dx$

**Optimal.** Leaf size=70

$$dx (a + bx^2)^p \left( \frac{bx^2}{a} + 1 \right)^{-p} {}_2F_1 \left( \frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a} \right) + \frac{e (a + bx^2)^{p+1}}{2b(p+1)}$$

[Out] (e\*(a + b\*x^2)^(1 + p))/(2\*b\*(1 + p)) + (d\*x\*(a + b\*x^2)^p\*Hypergeometric2F1[1/2, -p, 3/2, -(b\*x^2)/a])/(1 + (b\*x^2)/a)^p

**Rubi [A]** time = 0.0491049, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$dx (a + bx^2)^p \left( \frac{bx^2}{a} + 1 \right)^{-p} {}_2F_1 \left( \frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a} \right) + \frac{e (a + bx^2)^{p+1}}{2b(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)\*(a + b\*x^2)^p, x]

[Out] (e\*(a + b\*x^2)^(1 + p))/(2\*b\*(1 + p)) + (d\*x\*(a + b\*x^2)^p\*Hypergeometric2F1[1/2, -p, 3/2, -(b\*x^2)/a])/(1 + (b\*x^2)/a)^p

**Rubi in Sympy [A]** time = 8.76371, size = 54, normalized size = 0.77

$$dx \left( 1 + \frac{bx^2}{a} \right)^{-p} (a + bx^2)^p {}_2F_1 \left( -p, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a} \right) + \frac{e (a + bx^2)^{p+1}}{2b(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*(b\*x\*\*2+a)\*\*p, x)

[Out] d\*x\*(1 + b\*x\*\*2/a)\*\*(-p)\*(a + b\*x\*\*2)\*\*p\*hyper((-p, 1/2), (3/2, ), -b\*x\*\*2/a) + e\*(a + b\*x\*\*2)\*\*(p + 1)/(2\*b\*(p + 1))

**Mathematica [A]** time = 0.0751096, size = 98, normalized size = 1.4

$$\frac{(a + bx^2)^p \left( \frac{bx^2}{a} + 1 \right)^{-p} \left( 2bd(p+1)x {}_2F_1 \left( \frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a} \right) + bex^2 \left( \frac{bx^2}{a} + 1 \right)^p + ae \left( \left( \frac{bx^2}{a} + 1 \right)^p - 1 \right) \right)}{2b(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)\*(a + b\*x^2)^p, x]

[Out] ((a + b\*x^2)^p\*(b\*e\*x^2\*(1 + (b\*x^2)/a)^p + a\*e\*(-1 + (1 + (b\*x^2)/a)^p) + 2\*b\*d\*(1 + p)\*x\*Hypergeometric2F1[1/2, -p, 3/2, -(b\*x^2)/a])/(2\*b\*(1 + p)\*(1 + (b\*x^2)/a)^p)

**Maple [F]** time = 0.046, size = 0, normalized size = 0.

$$\int (ex + d) (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)*(b*x^2+a)^p,x)`

[Out] `int((e*x+d)*(b*x^2+a)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)*(b*x^2 + a)^p,x, algorithm="maxima")`

[Out] `integrate((e*x + d)*(b*x^2 + a)^p, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex + d)(bx^2 + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)*(b*x^2 + a)^p,x, algorithm="fricas")`

[Out] `integral((e*x + d)*(b*x^2 + a)^p, x)`

**Sympy [A]** time = 20.7486, size = 61, normalized size = 0.87

$$a^p dx {}_2F_1\left(\frac{1}{2}, -p \mid \frac{bx^2 e^{i\pi}}{a}\right) + e \left( \begin{array}{ll} \left(\frac{a^p x^2}{2}\right) & \text{for } b = 0 \\ \left(\frac{(a+bx^2)^{p+1}}{p+1}\right) & \text{for } p \neq -1 \\ \left(\frac{\log(a+bx^2)}{2b}\right) & \text{otherwise} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(b*x**2+a)**p,x)`

[Out] `a**p*d*x*hyper((1/2, -p), (3/2, ), b*x**2*exp_polar(I*pi)/a) + e*P  
iecewise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*x**2)**(p +  
1)/(p + 1), Ne(p, -1)), (log(a + b*x**2), True)))/(2*b), True))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)*(b*x^2 + a)^p,x, algorithm="giac")`

[Out] `integrate((e*x + d)*(b*x^2 + a)^p, x)`



$$3.387 \quad \int \frac{(d+ex)(a+bx^2)^p}{x} dx$$

**Optimal.** Leaf size=88

$$ex(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right) - \frac{d(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a} + 1\right)}{2a(p+1)}$$

[Out] (e\*x\*(a + b\*x^2)^p\*Hypergeometric2F1[1/2, -p, 3/2, -(b\*x^2)/a]) / ((1 + (b\*x^2)/a)^p - (d\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b\*x^2)/a]) / (2\*a\*(1 + p))

**Rubi [A]** time = 0.10613, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$

$$ex(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right) - \frac{d(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a} + 1\right)}{2a(p+1)}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*(a + b\*x^2)^p)/x, x]

[Out] (e\*x\*(a + b\*x^2)^p\*Hypergeometric2F1[1/2, -p, 3/2, -(b\*x^2)/a]) / ((1 + (b\*x^2)/a)^p - (d\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b\*x^2)/a]) / (2\*a\*(1 + p))

**Rubi in Sympy [A]** time = 13.2189, size = 68, normalized size = 0.77

$$ex\left(1 + \frac{bx^2}{a}\right)^{-p} (a+bx^2)^p {}_2F_1\left(-p, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) - \frac{d(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; 1 + \frac{bx^2}{a}\right)}{2a(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*(b\*x\*\*2+a)\*\*p/x, x)

[Out] e\*x\*(1 + b\*x\*\*2/a)\*\*(-p)\*(a + b\*x\*\*2)\*\*p\*hyper((-p, 1/2), (3/2, ), -b\*x\*\*2/a) - d\*(a + b\*x\*\*2)\*\*(p + 1)\*hyper((1, p + 1), (p + 2, ), 1 + b\*x\*\*2/a)/(2\*a\*(p + 1))

**Mathematica [A]** time = 0.0744296, size = 91, normalized size = 1.03

$$\frac{1}{2} (a+bx^2)^p \left( \frac{d\left(\frac{a}{bx^2} + 1\right)^{-p} {}_2F_1\left(-p, -p; 1-p; -\frac{a}{bx^2}\right)}{p} + 2ex\left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(a + b\*x^2)^p)/x, x]

[Out] ((a + b\*x^2)^p\*((2\*e\*x\*Hypergeometric2F1[1/2, -p, 3/2, -(b\*x^2)/a]) / (1 + (b\*x^2)/a)^p + (d\*Hypergeometric2F1[-p, -p, 1 - p, -(a/

$(b^*x^2)))]/(p*(1 + a/(b^*x^2))^p))/2$

**Maple [F]** time = 0.038, size = 0, normalized size = 0.

$$\int \frac{(ex + d)(bx^2 + a)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)\*(b\*x^2+a)^p/x,x)

[Out] int((e\*x+d)\*(b\*x^2+a)^p/x,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)(bx^2 + a)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(b\*x^2 + a)^p/x,x, algorithm="maxima")

[Out] integrate((e\*x + d)\*(b\*x^2 + a)^p/x, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex + d)(bx^2 + a)^p}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(b\*x^2 + a)^p/x,x, algorithm="fricas")

[Out] integral((e\*x + d)\*(b\*x^2 + a)^p/x, x)

**Sympy [A]** time = 31.2369, size = 65, normalized size = 0.74

$$a^p e x {}_2F_1\left(\frac{1}{2}, -p \middle| \frac{bx^2 e^{i\pi}}{a}\right) - \frac{b^p dx^{2p} (-p) {}_2F_1\left(\frac{-p, -p}{-p+1} \middle| \frac{ae^{i\pi}}{bx^2}\right)}{2(-p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(b\*x\*\*2+a)\*\*p/x,x)

[Out] a\*\*p\*e\*x\*hyper((1/2, -p), (3/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a) - b\*\*p\*d\*x\*\*(2\*p)\*gamma(-p)\*hyper((-p, -p), (-p + 1, ), a\*exp\_polar(I\*pi)/(b\*x\*\*2))/(2\*gamma(-p + 1))

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)(bx^2 + a)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)*(b*x^2 + a)^p/x,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p/x, x)
```

$$3.388 \quad \int \frac{(d+ex)(a+bx^2)^p}{x^2} dx$$

**Optimal.** Leaf size=91

$$\frac{d(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; -\frac{bx^2}{a}\right)}{x} - \frac{e(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a} + 1\right)}{2a(p+1)}$$

[Out]  $-\left(\frac{d(a+bx^2)^p \text{Hypergeometric2F1}\left[-\frac{1}{2}, -p, \frac{1}{2}, -\left(\frac{bx^2}{a}\right)\right]}{x} - \frac{e(a+bx^2)^{p+1} \text{Hypergeometric2F1}\left[1, p+1, p+2, \frac{bx^2}{a} + 1\right]}{2a(p+1)}\right)$

**Rubi [A]** time = 0.120604, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$

$$\frac{d(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; -\frac{bx^2}{a}\right)}{x} - \frac{e(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a} + 1\right)}{2a(p+1)}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*(a + b\*x^2)^p)/x^2, x]

[Out]  $-\left(\frac{d(a+bx^2)^p \text{Hypergeometric2F1}\left[-\frac{1}{2}, -p, \frac{1}{2}, -\left(\frac{bx^2}{a}\right)\right]}{x} - \frac{e(a+bx^2)^{p+1} \text{Hypergeometric2F1}\left[1, p+1, p+2, \frac{bx^2}{a} + 1\right]}{2a(p+1)}\right)$

**Rubi in Sympy [A]** time = 15.142, size = 71, normalized size = 0.78

$$\frac{d\left(1 + \frac{bx^2}{a}\right)^{-p} (a+bx^2)^p {}_2F_1\left(-p, -\frac{1}{2} \middle| -\frac{bx^2}{a}\right)}{x} - \frac{e(a+bx^2)^{p+1} {}_2F_1\left(1, p+1 \middle| p+2 \middle| 1 + \frac{bx^2}{a}\right)}{2a(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*(b\*x\*\*2+a)\*\*p/x\*\*2, x)

[Out]  $-d(1 + b*x**2/a)**(-p)*(a + b*x**2)**p*hyper((-p, -1/2), (1/2, ), -b*x**2/a)/x - e*(a + b*x**2)**(p + 1)*hyper((1, p + 1), (p + 2, ), 1 + b*x**2/a)/(2*a*(p + 1))$

**Mathematica [A]** time = 0.0682181, size = 93, normalized size = 1.02

$$\frac{1}{2}(a+bx^2)^p \left( \frac{e\left(\frac{a}{bx^2} + 1\right)^{-p} {}_2F_1\left(-p, -p; 1-p; -\frac{a}{bx^2}\right)}{p} - \frac{2d\left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; -\frac{bx^2}{a}\right)}{x} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(a + b\*x^2)^p)/x^2, x]

[Out]  $\left(\frac{(a+bx^2)^p \left(-2d \text{Hypergeometric2F1}\left[-\frac{1}{2}, -p, \frac{1}{2}, -\left(\frac{bx^2}{a}\right)\right]}{p} + e \text{Hypergeometric2F1}\left[-p, -p, 1-p, \frac{bx^2}{a} + 1\right]\right)}{2}\right)$

$$-(a/(b*x^2))]/(p*(1 + a/(b*x^2))^p))/2$$

**Maple [F]** time = 0.044, size = 0, normalized size = 0.

$$\int \frac{(ex + d)(bx^2 + a)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)\*(b\*x^2+a)^p/x^2,x)

[Out] int((e\*x+d)\*(b\*x^2+a)^p/x^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)(bx^2 + a)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(b\*x^2 + a)^p/x^2,x, algorithm="maxima")

[Out] integrate((e\*x + d)\*(b\*x^2 + a)^p/x^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex + d)(bx^2 + a)^p}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(b\*x^2 + a)^p/x^2,x, algorithm="fricas")

[Out] integral((e\*x + d)\*(b\*x^2 + a)^p/x^2, x)

**Sympy [A]** time = 42.5218, size = 68, normalized size = 0.75

$$-\frac{a^p d {}_2F_1\left(-\frac{1}{2}, -p \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{x} - \frac{b^p e x^{2p} (-p) {}_2F_1\left(-p, -p \middle| \frac{a e^{i\pi}}{bx^2}\right)}{2(-p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(b\*x\*\*2+a)\*\*p/x\*\*2,x)

[Out] -a\*\*p\*d\*hyper((-1/2, -p), (1/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/x - b\*\*p\*e\*x\*\*(2\*p)\*gamma(-p)\*hyper((-p, -p), (-p + 1), a\*exp\_polar(I\*pi)/(b\*x\*\*2))/(2\*gamma(-p + 1))

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)(bx^2 + a)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(b\*x^2 + a)^p/x^2,x, algorithm="giac")

[Out] integrate((e\*x + d)\*(b\*x^2 + a)^p/x^2, x)

$$3.389 \quad \int \frac{(d+ex)(a+bx^2)^p}{x^3} dx$$

**Optimal.** Leaf size=92

$$\frac{bd(a+bx^2)^{p+1} {}_2F_1\left(2, p+1; p+2; \frac{bx^2}{a}+1\right)}{2a^2(p+1)} - \frac{e(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; -\frac{bx^2}{a}\right)}{x}$$

[Out]  $-\left(\frac{e(a+bx^2)^p \text{Hypergeometric2F1}\left[-\frac{1}{2}, -p, \frac{1}{2}, -\left(\frac{bx^2}{a}\right)\right]}{x(1+\frac{bx^2}{a})^p}\right) + \frac{bd(a+bx^2)^{p+1} \text{Hypergeometric2F1}\left[2, 1+p, 2+p, 1+\left(\frac{bx^2}{a}\right)\right]}{2a^2(1+p)}$

**Rubi [A]** time = 0.124937, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$

$$\frac{bd(a+bx^2)^{p+1} {}_2F_1\left(2, p+1; p+2; \frac{bx^2}{a}+1\right)}{2a^2(p+1)} - \frac{e(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; -\frac{bx^2}{a}\right)}{x}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*(a + b\*x^2)^p)/x^3, x]

[Out]  $-\left(\frac{e(a+bx^2)^p \text{Hypergeometric2F1}\left[-\frac{1}{2}, -p, \frac{1}{2}, -\left(\frac{bx^2}{a}\right)\right]}{x(1+\frac{bx^2}{a})^p}\right) + \frac{bd(a+bx^2)^{p+1} \text{Hypergeometric2F1}\left[2, 1+p, 2+p, 1+\left(\frac{bx^2}{a}\right)\right]}{2a^2(1+p)}$

**Rubi in Sympy [A]** time = 15.5928, size = 73, normalized size = 0.79

$$-\frac{e\left(1+\frac{bx^2}{a}\right)^{-p}(a+bx^2)^p {}_2F_1\left(-p, -\frac{1}{2}; \frac{1}{2}; -\frac{bx^2}{a}\right)}{x} + \frac{bd(a+bx^2)^{p+1} {}_2F_1\left(2, p+1; p+2; 1+\frac{bx^2}{a}\right)}{2a^2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*(b\*x\*\*2+a)\*\*p/x\*\*3, x)

[Out]  $-e(1+\frac{bx^2}{a})^{-p}(a+bx^2)^p \text{hyper}\left(-p, -\frac{1}{2}, \frac{1}{2}, -\frac{bx^2}{a}\right)/x + bd(a+bx^2)^{p+1} \text{hyper}\left(2, p+1, p+2, 1+\frac{bx^2}{a}\right)/(2a^2(p+1))$

**Mathematica [A]** time = 0.0960147, size = 98, normalized size = 1.07

$$\frac{(a+bx^2)^p \left( \frac{d\left(\frac{a}{bx^2}+1\right)^{-p} {}_2F_1\left(1-p, -p; 2-p; -\frac{a}{bx^2}\right)}{p-1} - 2ex \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; -\frac{bx^2}{a}\right) \right)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(a + b\*x^2)^p)/x^3, x]

[Out]  $\frac{(a+bx^2)^p \left( (-2e*x \text{Hypergeometric2F1}\left[-\frac{1}{2}, -p, \frac{1}{2}, -\left(\frac{bx^2}{a}\right)\right]) \right)}{(1+\frac{bx^2}{a})^p} + d \text{Hypergeometric2F1}\left[1-p, -p, 2-p\right]$

$$-(a/(b*x^2))]/((-1 + p)*(1 + a/(b*x^2))^p)]/(2*x^2)$$

**Maple [F]** time = 0.056, size = 0, normalized size = 0.

$$\int \frac{(ex + d)(bx^2 + a)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)\*(b\*x^2+a)^p/x^3,x)

[Out] int((e\*x+d)\*(b\*x^2+a)^p/x^3,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)(bx^2 + a)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(b\*x^2 + a)^p/x^3,x, algorithm="maxima")

[Out] integrate((e\*x + d)\*(b\*x^2 + a)^p/x^3, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex + d)(bx^2 + a)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(b\*x^2 + a)^p/x^3,x, algorithm="fricas")

[Out] integral((e\*x + d)\*(b\*x^2 + a)^p/x^3, x)

**Sympy [A]** time = 62.2845, size = 71, normalized size = 0.77

$$\frac{a^p e_2 F_1\left(-\frac{1}{2}, -p \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{x} - \frac{b^p dx^{2p} (-p + 1) {}_2F_1\left(-p, -p + 1 \middle| \frac{ae^{i\pi}}{bx^2}\right)}{2x^2 (-p + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(b\*x\*\*2+a)\*\*p/x\*\*3,x)

[Out] -a\*\*p\*e\*hyper((-1/2, -p), (1/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/x - b\*\*p\*d\*x\*\*(2\*p)\*gamma(-p + 1)\*hyper((-p, -p + 1), (-p + 2, ), a\*exp\_polar(I\*pi)/(b\*x\*\*2))/(2\*x\*\*2\*gamma(-p + 2))



GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)(bx^2 + a)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)*(b*x^2 + a)^p/x^3,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)*(b*x^2 + a)^p/x^3, x)
```

### 3.390 $\int x^5(d + ex)^2 (a + bx^2)^p dx$

**Optimal.** Leaf size=188

$$\frac{a^2 (bd^2 - ae^2) (a + bx^2)^{p+1}}{2b^4(p+1)} - \frac{a(2bd^2 - 3ae^2) (a + bx^2)^{p+2}}{2b^4(p+2)} + \frac{(bd^2 - 3ae^2) (a + bx^2)^{p+3}}{2b^4(p+3)} + \frac{e^2 (a + bx^2)^{p+4}}{2b^4(p+4)} + \frac{2}{7} dex^7 (a + bx^2)^p \left( \frac{bx^2}{a} + 1 \right)^{-p} {}_2F_1 \left( \frac{7}{2}, -p; \frac{9}{2}; -\frac{bx^2}{a} \right)$$

[Out]  $(a^2(bd^2 - ae^2)(a + bx^2)^{p+1}) / (2b^4(1 + p)) - (a^2(bd^2 - 3ae^2)(a + bx^2)^{p+2}) / (2b^4(2 + p)) + ((bd^2 - 3ae^2)(a + bx^2)^{p+3}) / (2b^4(3 + p)) + (e^2(a + bx^2)^{p+4}) / (2b^4(4 + p)) + (2d^2e^2x^7(a + bx^2)^p \text{Hypergeometric2F1}[7/2, -p, 9/2, -(bx^2/a)]) / (7(1 + (bx^2/a)^p))$

**Rubi [A]** time = 0.353377, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{a^2 (bd^2 - ae^2) (a + bx^2)^{p+1}}{2b^4(p+1)} - \frac{a(2bd^2 - 3ae^2) (a + bx^2)^{p+2}}{2b^4(p+2)} + \frac{(bd^2 - 3ae^2) (a + bx^2)^{p+3}}{2b^4(p+3)} + \frac{e^2 (a + bx^2)^{p+4}}{2b^4(p+4)} + \frac{2}{7} dex^7 (a + bx^2)^p \left( \frac{bx^2}{a} + 1 \right)^{-p} {}_2F_1 \left( \frac{7}{2}, -p; \frac{9}{2}; -\frac{bx^2}{a} \right)$$

Antiderivative was successfully verified.

[In] Int[x^5\*(d + e\*x)^2\*(a + b\*x^2)^p,x]

[Out]  $(a^2(bd^2 - ae^2)(a + bx^2)^{p+1}) / (2b^4(1 + p)) - (a^2(bd^2 - 3ae^2)(a + bx^2)^{p+2}) / (2b^4(2 + p)) + ((bd^2 - 3ae^2)(a + bx^2)^{p+3}) / (2b^4(3 + p)) + (e^2(a + bx^2)^{p+4}) / (2b^4(4 + p)) + (2d^2e^2x^7(a + bx^2)^p \text{Hypergeometric2F1}[7/2, -p, 9/2, -(bx^2/a)]) / (7(1 + (bx^2/a)^p))$

**Rubi in Sympy [A]** time = 62.9598, size = 212, normalized size = 1.13

$$\frac{a^3 e^2 (a + bx^2)^{p+1}}{2b^4(p+1)} + \frac{a^2 d^2 (a + bx^2)^{p+1}}{2b^3(p+1)} + \frac{3a^2 e^2 (a + bx^2)^{p+2}}{2b^4(p+2)} - \frac{ad^2 (a + bx^2)^{p+2}}{b^3(p+2)} - \frac{3ae^2 (a + bx^2)^{p+3}}{2b^4(p+3)} + \frac{2dex^7 \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1 \left( \frac{-p, \frac{7}{2}}{\frac{9}{2}}; -\frac{bx^2}{a} \right)}{7} + \frac{d^2 (a + bx^2)^{p+3}}{2b^3(p+3)} + \frac{e^2 (a + bx^2)^{p+4}}{2b^4(p+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*5\*(e\*x+d)\*\*2\*(b\*x\*\*2+a)\*\*p,x)

[Out]  $-a**3*e**2*(a + b*x**2)**(p + 1)/(2*b**4*(p + 1)) + a**2*d**2*(a + b*x**2)**(p + 1)/(2*b**3*(p + 1)) + 3*a**2*e**2*(a + b*x**2)**(p + 2)/(2*b**4*(p + 2)) - a*d**2*(a + b*x**2)**(p + 2)/(b**3*(p + 2)) - 3*a*e**2*(a + b*x**2)**(p + 3)/(2*b**4*(p + 3)) + 2*d*e*x**7*(1 + b*x**2/a)**(-p)*(a + b*x**2)**p*hyper((-p, 7/2), (9/2, ), -b*x**2/a)/7 + d**2*(a + b*x**2)**(p + 3)/(2*b**3*(p + 3)) + e**2*(a + b*x**2)**(p + 4)/(2*b**4*(p + 4))$

**Mathematica [A]** time = 0.478337, size = 303, normalized size = 1.61

$$(a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} \left(7 \left(-6a^4e^2 \left(\left(\frac{bx^2}{a} + 1\right)^p - 1\right) + 2a^3b \left(d^2(p+4) \left(\left(\frac{bx^2}{a} + 1\right)^p - 1\right) + 3e^2px^2 \left(\frac{bx^2}{a} + 1\right)^p\right) - a^2b^2px^2\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^5\*(d + e\*x)^2\*(a + b\*x^2)^p,x]

[Out] ((a + b\*x^2)^p\*(7\*(-(a^2\*b^2\*p\*x^2\*(1 + (b\*x^2)/a)^p\*(2\*d^2\*(4 + p) + 3\*e^2\*(1 + p)\*x^2)) + a\*b^3\*p\*(1 + p)\*x^4\*(1 + (b\*x^2)/a)^p\*(d^2\*(4 + p) + e^2\*(2 + p)\*x^2) + b^4\*(2 + 3\*p + p^2)\*x^6\*(1 + (b\*x^2)/a)^p\*(d^2\*(4 + p) + e^2\*(3 + p)\*x^2) - 6\*a^4\*e^2\*(-1 + (1 + (b\*x^2)/a)^p) + 2\*a^3\*b\*(3\*e^2\*p\*x^2\*(1 + (b\*x^2)/a)^p + d^2\*(4 + p)\*(-1 + (1 + (b\*x^2)/a)^p))) + 4\*b^4\*d\*e\*(24 + 50\*p + 35\*p^2 + 10\*p^3 + p^4)\*x^7\*Hypergeometric2F1[7/2, -p, 9/2, -(b\*x^2)/a])/(14\*b^4\*(1 + p)\*(2 + p)\*(3 + p)\*(4 + p)\*(1 + (b\*x^2)/a)^p)

**Maple [F]** time = 0.096, size = 0, normalized size = 0.

$$\int x^5 (ex + d)^2 (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5\*(e\*x+d)^2\*(b\*x^2+a)^p,x)

[Out] int(x^5\*(e\*x+d)^2\*(b\*x^2+a)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{((p^2 + 3p + 2)b^3x^6 + (p^2 + p)ab^2x^4 - 2a^2bpx^2 + 2a^3)(bx^2 + a)^p d^2}{2(p^3 + 6p^2 + 11p + 6)b^3} + \int (e^2x^7 + 2dex^6)(bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(b\*x^2 + a)^p\*x^5,x, algorithm="maxima")

[Out] 1/2\*((p^2 + 3\*p + 2)\*b^3\*x^6 + (p^2 + p)\*a\*b^2\*x^4 - 2\*a^2\*b\*p\*x^2 + 2\*a^3)\*(b\*x^2 + a)^p\*d^2/((p^3 + 6\*p^2 + 11\*p + 6)\*b^3) + integrate((e^2\*x^7 + 2\*d\*e\*x^6)\*(b\*x^2 + a)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^7 + 2dex^6 + d^2x^5\right)(bx^2 + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(b\*x^2 + a)^p\*x^5,x, algorithm="fricas")

[Out] integral((e^2\*x^7 + 2\*d\*e\*x^6 + d^2\*x^5)\*(b\*x^2 + a)^p, x)

Sympy [A] time = 170.363, size = 3046, normalized size = 16.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(e\*x+d)\*\*2\*(b\*x\*\*2+a)\*\*p,x)

[Out]  $2*a**p*d*e*x**7*hyper((7/2, -p), (9/2, ), b*x**2*exp\_polar(I*pi)/a)/7 + d**2*Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -3)), (-2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 3*b**3*p*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3), True)) + e**2*Piecewise((a**p*x**8/8, Eq(b, 0)), (6*a**3*log(-I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 11*a**3/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a**2*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a**2*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 27*a**2*b*x**2/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4*log(-I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 18*a*b**2*x**4*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*b**3*x**6*log(-I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6) + 6*b**3*x**6*log(I*sqrt(a)*sqrt(1/b) + x)/(12*a**3*b**4 + 36*a**2*b**5*x**2 + 36*a*b**6*x**4 + 12*b**7*x**6), Eq(p, -4)), (-6*a**3*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 6*a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 9*a**3/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 12*a**2*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 12*a**2*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 12*a**2*b*x**2/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 6*a*b**2*x**4*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) - 6*a*b**2*x**4*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4) + 2*b**3*x**6/(4*a**2*b**4 + 8*a*b**5*x**2 + 4*b**6*x**4), Eq(p, -3)), (6*a**3*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a*b**4 + 4*b**5*x**2) + 6*a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a*b**4 + 4*b**5*x**2) + 6*a**3/(4*a*b**4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a*b**4 + 4*b**5*x**2) + 6*a**2*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a*b**4 + 4*b**5*x**2) - 3*a*b**2*x**4/(4*a*b$

```

*4 + 4*b**5*x**2) + b**3*x**6/(4*a*b**4 + 4*b**5*x**2), Eq(p, -2)
), (-a**3*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**4) - a**3*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**4) + a**2*x**2/(2*b**3) - a*x**4/(4*b**2) + x**6/(6*b), Eq(p, -1)), (-6*a**4*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*a**3*b*p*x**2*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) - 3*a**2*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) - 3*a**2*b**2*p*x**4*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + a*b**3*p**3*x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 3*a*b**3*p**2*x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 2*a*b**3*p*x**6*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + b**4*p**3*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*b**4*p**2*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 11*b**4*p*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4) + 6*b**4*x**8*(a + b*x**2)**p/(2*b**4*p**4 + 20*b**4*p**3 + 70*b**4*p**2 + 100*b**4*p + 48*b**4), True))

```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2 (bx^2 + a)^p x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^2*(b*x^2 + a)^p*x^5,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p*x^5, x)
```

### 3.391 $\int x^4(d + ex)^2 (a + bx^2)^p dx$

**Optimal.** Leaf size=177

$$\frac{a^2 de (a + bx^2)^{p+1}}{b^3(p+1)} - \frac{2ade (a + bx^2)^{p+2}}{b^3(p+2)} + \frac{de (a + bx^2)^{p+3}}{b^3(p+3)} - \frac{x^5 (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (5ae^2 - bd^2(2p+7)) {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; -\frac{bx^2}{a}\right)}{5b(2p+7)} + \frac{e^2 x^5 (a + bx^2)^{p+1}}{b(2p+7)}$$

[Out]  $(a^2 d e (a + b x^2)^{(1+p)}) / (b^3 (1+p)) + (e^2 x^5 (a + b x^2)^{(1+p)}) / (b (7 + 2p)) - (2 a d e (a + b x^2)^{(2+p)}) / (b^3 (2+p)) + (d e (a + b x^2)^{(3+p)}) / (b^3 (3+p)) - ((5 a e^2 - b d^2 (7 + 2p)) x^5 (a + b x^2)^p \text{Hypergeometric2F1}[5/2, -p, 7/2, -(b x^2)/a]) / (5 b (7 + 2p) (1 + (b x^2)/a)^p)$

**Rubi [A]** time = 0.324414, antiderivative size = 169, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$\frac{a^2 de (a + bx^2)^{p+1}}{b^3(p+1)} - \frac{2ade (a + bx^2)^{p+2}}{b^3(p+2)} + \frac{de (a + bx^2)^{p+3}}{b^3(p+3)} + \frac{1}{5} x^5 (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} \left(d^2 - \frac{5ae^2}{2bp+7b}\right) {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; -\frac{bx^2}{a}\right) + \frac{e^2 x^5 (a + bx^2)^{p+1}}{b(2p+7)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^4*(d + e*x)^2*(a + b*x^2)^p, x]$

[Out]  $(a^2 d e (a + b x^2)^{(1+p)}) / (b^3 (1+p)) + (e^2 x^5 (a + b x^2)^{(1+p)}) / (b (7 + 2p)) - (2 a d e (a + b x^2)^{(2+p)}) / (b^3 (2+p)) + (d e (a + b x^2)^{(3+p)}) / (b^3 (3+p)) + ((d^2 - (5 a e^2) / (7 b + 2 b p)) x^5 (a + b x^2)^p \text{Hypergeometric2F1}[5/2, -p, 7/2, -(b x^2)/a]) / (5 (1 + (b x^2)/a)^p)$

**Rubi in Sympy [A]** time = 45.1335, size = 151, normalized size = 0.85

$$\frac{a^2 de (a + bx^2)^{p+1}}{b^3 (p+1)} - \frac{2ade (a + bx^2)^{p+2}}{b^3 (p+2)} + \frac{d^2 x^5 \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, \frac{5}{2}; \frac{7}{2}; -\frac{bx^2}{a}\right)}{5} + \frac{e^2 x^7 \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, \frac{7}{2}; \frac{9}{2}; -\frac{bx^2}{a}\right)}{7} + \frac{de (a + bx^2)^{p+3}}{b^3 (p+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**4}*(e*x+d)^{**2}*(b*x^{**2}+a)^{**p}, x)$

[Out]  $a^{**2}d*e*(a + b*x^{**2})^{**p} / (b^{**3}*(p + 1)) - 2*a*d*e*(a + b*x^{**2})^{**p} / (b^{**3}*(p + 2)) + d^{**2}*x^{**5}*(1 + b*x^{**2}/a)^{**(-p)}*(a + b*x^{**2})^{**p}*\text{hyper}((-p, 5/2), (7/2, ), -b*x^{**2}/a)/5 + e^{**2}*x^{**7}*(1 + b*x^{**2}/a)^{**(-p)}*(a + b*x^{**2})^{**p}*\text{hyper}((-p, 7/2), (9/2, ), -b*x^{**2}/a)/7 + d*e*(a + b*x^{**2})^{**p} / (b^{**3}*(p + 3))$

**Mathematica [A]** time = 0.291189, size = 229, normalized size = 1.29

$$\frac{(a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} \left(5e \left(7d \left(2a^3 \left(\left(\frac{bx^2}{a} + 1\right)^p - 1\right) - 2a^2 b p x^2 \left(\frac{bx^2}{a} + 1\right)^p + b^3 (p^2 + 3p + 2) x^6 \left(\frac{bx^2}{a} + 1\right)^p + ab^2 p(p + 1)\right)\right)}{35b^3(p + 1)(p + 2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^4\*(d + e\*x)^2\*(a + b\*x^2)^p,x]

[Out] ((a + b\*x^2)^p\*(7\*b^3\*d^2\*(6 + 11\*p + 6\*p^2 + p^3)\*x^5\*Hypergeometric2F1[5/2, -p, 7/2, -(b\*x^2)/a] + 5\*e\*(7\*d\*(-2\*a^2\*b\*p\*x^2\*(1 + (b\*x^2)/a)^p + a\*b^2\*p\*(1 + p)\*x^4\*(1 + (b\*x^2)/a)^p + b^3\*(2 + 3\*p + p^2)\*x^6\*(1 + (b\*x^2)/a)^p + 2\*a^3\*(-1 + (1 + (b\*x^2)/a)^p)) + b^3\*e\*(6 + 11\*p + 6\*p^2 + p^3)\*x^7\*Hypergeometric2F1[7/2, -p, 9/2, -(b\*x^2)/a]))/(35\*b^3\*(1 + p)\*(2 + p)\*(3 + p)\*(1 + (b\*x^2)/a)^p)

**Maple [F]** time = 0.085, size = 0, normalized size = 0.

$$\int x^4 (ex + d)^2 (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(e\*x+d)^2\*(b\*x^2+a)^p,x)

[Out] int(x^4\*(e\*x+d)^2\*(b\*x^2+a)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2 (bx^2 + a)^p x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(b\*x^2 + a)^p\*x^4,x, algorithm="maxima")

[Out] integrate((e\*x + d)^2\*(b\*x^2 + a)^p\*x^4, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^6 + 2dex^5 + d^2x^4\right)(bx^2 + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(b\*x^2 + a)^p\*x^4,x, algorithm="fricas")

[Out] integral((e^2\*x^6 + 2\*d\*e\*x^5 + d^2\*x^4)\*(b\*x^2 + a)^p, x)

**Sympy [A]** time = 151.911, size = 1047, normalized size = 5.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(e\*x+d)\*\*2\*(b\*x\*\*2+a)\*\*p,x)

[Out] a\*\*p\*d\*\*2\*x\*\*5\*hyper((5/2, -p), (7/2,)), b\*x\*\*2\*exp\_polar(I\*pi)/a/5 + a\*\*p\*e\*\*2\*x\*\*7\*hyper((7/2, -p), (9/2,)), b\*x\*\*2\*exp\_polar(I\*pi)/a/7 + 2\*d\*e\*Piecewise((a\*\*p\*x\*\*6/6, Eq(b, 0)), (2\*a\*\*2\*log(-I\*sqrt(a)\*sqrt(1/b) + x)/(4\*a\*\*2\*b\*\*3 + 8\*a\*b\*\*4\*x\*\*2 + 4\*b\*\*5\*x\*\*4) + 2\*a\*\*2\*log(I\*sqrt(a)\*sqrt(1/b) + x)/(4\*a\*\*2\*b\*\*3 + 8\*a\*b\*\*4\*x\*\*2 + 4\*b\*\*5\*x\*\*4) + 3\*a\*\*2/(4\*a\*\*2\*b\*\*3 + 8\*a\*b\*\*4\*x\*\*2 + 4\*b\*\*5\*x\*\*4) + 4\*a\*b\*x\*\*2\*log(-I\*sqrt(a)\*sqrt(1/b) + x)/(4\*a\*\*2\*b\*\*3 + 8\*a\*b\*\*4\*x\*\*2 + 4\*b\*\*5\*x\*\*4) + 4\*a\*b\*x\*\*2\*log(I\*sqrt(a)\*sqrt(1/b) + x)/(4\*a\*\*2\*b\*\*3 + 8\*a\*b\*\*4\*x\*\*2 + 4\*b\*\*5\*x\*\*4) + 4\*a\*b\*x\*\*2/(4\*a\*\*2\*b\*\*3 + 8\*a\*b\*\*4\*x\*\*2 + 4\*b\*\*5\*x\*\*4) + 2\*b\*\*2\*x\*\*4\*log(-I\*sqrt(a)\*sqrt(1/b) + x)/(4\*a\*\*2\*b\*\*3 + 8\*a\*b\*\*4\*x\*\*2 + 4\*b\*\*5\*x\*\*4) + 2\*b\*\*2\*x\*\*4\*log(I\*sqrt(a)\*sqrt(1/b) + x)/(4\*a\*\*2\*b\*\*3 + 8\*a\*b\*\*4\*x\*\*2 + 4\*b\*\*5\*x\*\*4), Eq(p, -3)), (-2\*a\*\*2\*log(-I\*sqrt(a)\*sqrt(1/b) + x)/(2\*a\*b\*\*3 + 2\*b\*\*4\*x\*\*2) - 2\*a\*\*2\*log(I\*sqrt(a)\*sqrt(1/b) + x)/(2\*a\*b\*\*3 + 2\*b\*\*4\*x\*\*2) - 2\*a\*b\*x\*\*2\*log(-I\*sqrt(a)\*sqrt(1/b) + x)/(2\*a\*b\*\*3 + 2\*b\*\*4\*x\*\*2) - 2\*a\*b\*x\*\*2\*log(I\*sqrt(a)\*sqrt(1/b) + x)/(2\*a\*b\*\*3 + 2\*b\*\*4\*x\*\*2) + b\*\*2\*x\*\*4/(2\*a\*b\*\*3 + 2\*b\*\*4\*x\*\*2), Eq(p, -2)), (a\*\*2\*log(-I\*sqrt(a)\*sqrt(1/b) + x)/(2\*b\*\*3) + a\*\*2\*log(I\*sqrt(a)\*sqrt(1/b) + x)/(2\*b\*\*3) - a\*x\*\*2/(2\*b\*\*2) + x\*\*4/(4\*b), Eq(p, -1)), (2\*a\*\*3\*(a + b\*x\*\*2)\*\*p/(2\*b\*\*3\*p\*\*3 + 12\*b\*\*3\*p\*\*2 + 22\*b\*\*3\*p + 12\*b\*\*3) - 2\*a\*\*2\*b\*p\*x\*\*2\*(a + b\*x\*\*2)\*\*p/(2\*b\*\*3\*p\*\*3 + 12\*b\*\*3\*p\*\*2 + 22\*b\*\*3\*p + 12\*b\*\*3) + a\*b\*\*2\*p\*\*2\*x\*\*4\*(a + b\*x\*\*2)\*\*p/(2\*b\*\*3\*p\*\*3 + 12\*b\*\*3\*p\*\*2 + 22\*b\*\*3\*p + 12\*b\*\*3) + a\*b\*\*2\*p\*x\*\*4\*(a + b\*x\*\*2)\*\*p/(2\*b\*\*3\*p\*\*3 + 12\*b\*\*3\*p\*\*2 + 22\*b\*\*3\*p + 12\*b\*\*3) + b\*\*3\*p\*\*2\*x\*\*6\*(a + b\*x\*\*2)\*\*p/(2\*b\*\*3\*p\*\*3 + 12\*b\*\*3\*p\*\*2 + 22\*b\*\*3\*p + 12\*b\*\*3) + 3\*b\*\*3\*p\*x\*\*6\*(a + b\*x\*\*2)\*\*p/(2\*b\*\*3\*p\*\*3 + 12\*b\*\*3\*p\*\*2 + 22\*b\*\*3\*p + 12\*b\*\*3) + 2\*b\*\*3\*x\*\*6\*(a + b\*x\*\*2)\*\*p/(2\*b\*\*3\*p\*\*3 + 12\*b\*\*3\*p\*\*2 + 22\*b\*\*3\*p + 12\*b\*\*3), True))

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2 (bx^2 + a)^p x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(b\*x^2 + a)^p\*x^4,x, algorithm="giac")

[Out] integrate((e\*x + d)^2\*(b\*x^2 + a)^p\*x^4, x)



### 3.392 $\int x^3(d + ex)^2 (a + bx^2)^p dx$

**Optimal.** Leaf size=149

$$\frac{a(bd^2 - ae^2)(a + bx^2)^{p+1}}{2b^3(p+1)} + \frac{(bd^2 - 2ae^2)(a + bx^2)^{p+2}}{2b^3(p+2)} + \frac{e^2(a + bx^2)^{p+3}}{2b^3(p+3)} + \frac{2}{5}dex^5(a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; -\frac{bx^2}{a}\right)$$

[Out]  $-(a*(b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) + ((b*d^2 - 2*a*e^2)*(a + b*x^2)^(2 + p))/(2*b^3*(2 + p)) + (e^2*(a + b*x^2)^(3 + p))/(2*b^3*(3 + p)) + (2*d*e*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -(b*x^2)/a])/(5*(1 + (b*x^2)/a)^p)$

**Rubi [A]** time = 0.288477, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{a(bd^2 - ae^2)(a + bx^2)^{p+1}}{2b^3(p+1)} + \frac{(bd^2 - 2ae^2)(a + bx^2)^{p+2}}{2b^3(p+2)} + \frac{e^2(a + bx^2)^{p+3}}{2b^3(p+3)} + \frac{2}{5}dex^5(a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; -\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3*(d + e*x)^2*(a + b*x^2)^p, x]$

[Out]  $-(a*(b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) + ((b*d^2 - 2*a*e^2)*(a + b*x^2)^(2 + p))/(2*b^3*(2 + p)) + (e^2*(a + b*x^2)^(3 + p))/(2*b^3*(3 + p)) + (2*d*e*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -(b*x^2)/a])/(5*(1 + (b*x^2)/a)^p)$

**Rubi in Sympy [A]** time = 45.4117, size = 158, normalized size = 1.06

$$\frac{a^2e^2(a + bx^2)^{p+1}}{2b^3(p+1)} - \frac{ad^2(a + bx^2)^{p+1}}{2b^2(p+1)} - \frac{ae^2(a + bx^2)^{p+2}}{b^3(p+2)} + \frac{2dex^5\left(1 + \frac{bx^2}{a}\right)^{-p}(a + bx^2)^p {}_2F_1\left(-p, \frac{5}{2}; \frac{7}{2}; -\frac{bx^2}{a}\right)}{5} + \frac{d^2(a + bx^2)^{p+2}}{2b^2(p+2)} + \frac{e^2(a + bx^2)^{p+3}}{2b^3(p+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**3*(e*x+d)**2*(b*x**2+a)**p, x)$

[Out]  $a**2*e**2*(a + b*x**2)**(p + 1)/(2*b**3*(p + 1)) - a*d**2*(a + b*x**2)**(p + 1)/(2*b**2*(p + 1)) - a*e**2*(a + b*x**2)**(p + 2)/(b**3*(p + 2)) + 2*d*e*x**5*(1 + b*x**2/a)**(-p)*(a + b*x**2)**p*hyper((-p, 5/2), (7/2, ), -b*x**2/a)/5 + d**2*(a + b*x**2)**(p + 2)/(2*b**2*(p + 2)) + e**2*(a + b*x**2)**(p + 3)/(2*b**3*(p + 3))$

**Mathematica [A]** time = 0.301101, size = 241, normalized size = 1.62

$$\frac{(a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} \left(5 \left(2a^3e^2 \left(\left(\frac{bx^2}{a} + 1\right)^p - 1\right) - a^2b \left(d^2(p+3) \left(\left(\frac{bx^2}{a} + 1\right)^p - 1\right) + 2e^2px^2 \left(\frac{bx^2}{a} + 1\right)^p\right) + b^3(p+1)x^4\right)}{10b^3}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(d + e\*x)^2\*(a + b\*x^2)^p,x]

[Out] ((a + b\*x^2)^p\*(5\*(a\*b^2\*p\*x^2\*(1 + (b\*x^2)/a)^p\*(d^2\*(3 + p) + e^2\*(1 + p)\*x^2) + b^3\*(1 + p)\*x^4\*(1 + (b\*x^2)/a)^p\*(d^2\*(3 + p) + e^2\*(2 + p)\*x^2) + 2\*a^3\*e^2\*(-1 + (1 + (b\*x^2)/a)^p) - a^2\*b\*(2\*e^2\*p\*x^2\*(1 + (b\*x^2)/a)^p + d^2\*(3 + p)\*(-1 + (1 + (b\*x^2)/a)^p))) + 4\*b^3\*d\*e\*(6 + 11\*p + 6\*p^2 + p^3)\*x^5\*Hypergeometric2F1[5/2, -p, 7/2, -((b\*x^2)/a)])/(10\*b^3\*(1 + p)\*(2 + p)\*(3 + p)\*(1 + (b\*x^2)/a)^p)

**Maple [F]** time = 0.075, size = 0, normalized size = 0.

$$\int x^3 (ex + d)^2 (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(e\*x+d)^2\*(b\*x^2+a)^p,x)

[Out] int(x^3\*(e\*x+d)^2\*(b\*x^2+a)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(b^2(p+1)x^4 + abpx^2 - a^2)(bx^2 + a)^p d^2}{2(p^2 + 3p + 2)b^2} + \int (e^2x^5 + 2dex^4)(bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(b\*x^2 + a)^p\*x^3,x, algorithm="maxima")

[Out] 1/2\*(b^2\*(p + 1)\*x^4 + a\*b\*p\*x^2 - a^2)\*(b\*x^2 + a)^p\*d^2/((p^2 + 3\*p + 2)\*b^2) + integrate((e^2\*x^5 + 2\*d\*e\*x^4)\*(b\*x^2 + a)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^5 + 2dex^4 + d^2x^3\right)(bx^2 + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(b\*x^2 + a)^p\*x^3,x, algorithm="fricas")

[Out] integral((e^2\*x^5 + 2\*d\*e\*x^4 + d^2\*x^3)\*(b\*x^2 + a)^p, x)

**Sympy [A]** time = 93.3054, size = 1386, normalized size = 9.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(e\*x+d)\*\*2\*(b\*x\*\*2+a)\*\*p,x)

[Out]  $2*a**p*d*e*x**5*hyper((5/2, -p), (7/2, ), b*x**2*exp\_polar(I*pi)/a)/5 + d**2*Piecewise((a**p*x**4/4, Eq(b, 0)), (a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2), True)) + e**2*Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -3)), (-2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**3) + a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 3*b**3*p*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3), True))$

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2 (bx^2 + a)^p x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(b\*x^2 + a)^p\*x^3,x, algorithm="giac")

[Out] integrate((e\*x + d)^2\*(b\*x^2 + a)^p\*x^3, x)

### 3.393 $\int x^2(d + ex)^2 (a + bx^2)^p dx$

**Optimal.** Leaf size=152

$$-\frac{ade(a+bx^2)^{p+1}}{b^2(p+1)} + \frac{de(a+bx^2)^{p+2}}{b^2(p+2)} - \frac{x^3(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (3ae^2 - bd^2(2p+5)) {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; -\frac{bx^2}{a}\right)}{3b(2p+5)} + \frac{e^2x^3(a+bx^2)^{p+1}}{b(2p+5)}$$

[Out]  $-\left(\frac{a^*d^*e^*(a+b*x^2)^{(1+p)}}{(b^2*(1+p))}\right) + \left(\frac{e^2*x^3*(a+b*x^2)^{(1+p)}}{(b*(5+2*p))}\right) + \left(\frac{d^*e^*(a+b*x^2)^{(2+p)}}{(b^2*(2+p))}\right) - \left(\frac{(3*a^*e^2 - b*d^2*(5+2*p))*x^3*(a+b*x^2)^p*Hypergeometric2F1\left[\frac{3}{2}, -p, \frac{5}{2}, -\left(\frac{b*x^2}{a}\right)\right]}{(3*b*(5+2*p))*(1+(b*x^2)/a)^p}\right)$

**Rubi [A]** time = 0.278796, antiderivative size = 144, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$-\frac{ade(a+bx^2)^{p+1}}{b^2(p+1)} + \frac{de(a+bx^2)^{p+2}}{b^2(p+2)} + \frac{1}{3}x^3(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} \left(d^2 - \frac{3ae^2}{2bp+5b}\right) {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; -\frac{bx^2}{a}\right) + \frac{e^2x^3(a+bx^2)^{p+1}}{b(2p+5)}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(d + e\*x)^2\*(a + b\*x^2)^p, x]

[Out]  $-\left(\frac{a^*d^*e^*(a+b*x^2)^{(1+p)}}{(b^2*(1+p))}\right) + \left(\frac{e^2*x^3*(a+b*x^2)^{(1+p)}}{(b*(5+2*p))}\right) + \left(\frac{d^*e^*(a+b*x^2)^{(2+p)}}{(b^2*(2+p))}\right) + \left(\frac{(d^2 - (3*a^*e^2)/(5*b + 2*b*p))*x^3*(a+b*x^2)^p*Hypergeometric2F1\left[\frac{3}{2}, -p, \frac{5}{2}, -\left(\frac{b*x^2}{a}\right)\right]}{(3*(1+(b*x^2)/a)^p)}\right)$

**Rubi in Sympy [A]** time = 34.3618, size = 126, normalized size = 0.83

$$-\frac{ade(a+bx^2)^{p+1}}{b^2(p+1)} + \frac{d^2x^3\left(1+\frac{bx^2}{a}\right)^{-p}(a+bx^2)^p {}_2F_1\left(-p, \frac{3}{2}; \frac{5}{2}; -\frac{bx^2}{a}\right)}{3} + \frac{e^2x^5\left(1+\frac{bx^2}{a}\right)^{-p}(a+bx^2)^p {}_2F_1\left(-p, \frac{5}{2}; \frac{7}{2}; -\frac{bx^2}{a}\right)}{5} + \frac{de(a+bx^2)^{p+2}}{b^2(p+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(e\*x+d)\*\*2\*(b\*x\*\*2+a)\*\*p, x)

[Out]  $-a^*d^*e^*(a+b*x**2)**(p+1)/(b**2*(p+1)) + d**2*x**3*(1+b*x**2/a)**(-p)*(a+b*x**2)**p*hyper((-p, 3/2), (5/2, ), -b*x**2/a)/3 + e**2*x**5*(1+b*x**2/a)**(-p)*(a+b*x**2)**p*hyper((-p, 5/2), (7/2, ), -b*x**2/a)/5 + d^*e^*(a+b*x**2)**(p+2)/(b**2*(p+2))$

**Mathematica [A]** time = 0.26068, size = 182, normalized size = 1.2

$$\frac{(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} \left(3e \left(5d \left(-a^2 \left(\left(\frac{bx^2}{a} + 1\right)^p - 1\right) + b^2(p+1)x^4 \left(\frac{bx^2}{a} + 1\right)^p + abpx^2 \left(\frac{bx^2}{a} + 1\right)^p\right) + b^2e(p^2 + 3p + 2)x^3\right)}{15b^2(p+1)(p+2)}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(d + e\*x)^2\*(a + b\*x^2)^p,x]

[Out]  $((a + b*x^2)^p*(5*b^2*d^2*(2 + 3*p + p^2)*x^3*Hypergeometric2F1[3/2, -p, 5/2, -(b*x^2)/a]) + 3*e*(5*d*(a*b*p*x^2*(1 + (b*x^2)/a)^p + b^2*(1 + p)*x^4*(1 + (b*x^2)/a)^p - a^2*(-1 + (1 + (b*x^2)/a)^p)) + b^2*e*(2 + 3*p + p^2)*x^5*Hypergeometric2F1[5/2, -p, 7/2, -(b*x^2)/a]))/(15*b^2*(1 + p)*(2 + p)*(1 + (b*x^2)/a)^p)$

**Maple [F]** time = 0.067, size = 0, normalized size = 0.

$$\int x^2 (ex + d)^2 (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(e\*x+d)^2\*(b\*x^2+a)^p,x)

[Out] int(x^2\*(e\*x+d)^2\*(b\*x^2+a)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2 (bx^2 + a)^p x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(b\*x^2 + a)^p\*x^2,x, algorithm="maxima")

[Out] integrate((e\*x + d)^2\*(b\*x^2 + a)^p\*x^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((e^2x^4 + 2dex^3 + d^2x^2)(bx^2 + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(b\*x^2 + a)^p\*x^2,x, algorithm="fricas")

[Out] integral((e^2\*x^4 + 2\*d\*e\*x^3 + d^2\*x^2)\*(b\*x^2 + a)^p, x)

**Sympy [A]** time = 81.6281, size = 430, normalized size = 2.83

$$\frac{a^p d^2 x^3 {}_2F_1\left(\frac{3}{2}, -p \mid \frac{bx^2 e^{i\pi}}{a}\right)}{3} + \frac{a^p e^2 x^5 {}_2F_1\left(\frac{5}{2}, -p \mid \frac{bx^2 e^{i\pi}}{a}\right)}{5}$$

$$+ 2de \left\{ \begin{array}{ll} \left( \frac{a^p x^4}{4} \right. & \text{for } b = 0 \\ \frac{a \log(-i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2ab^2+2b^3x^2} + \frac{a \log(i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2ab^2+2b^3x^2} + \frac{a}{2ab^2+2b^3x^2} + \frac{bx^2 \log(-i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2ab^2+2b^3x^2} + \frac{bx^2 \log(i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2ab^2+2b^3x^2} & \text{for } p = -2 \\ \frac{a \log(-i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2b^2} - \frac{a \log(i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2b^2} + \frac{x^2}{2b} & \text{for } p = -1 \\ \left. -\frac{a^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{abpx^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2px^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2x^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} \right) & \text{otherwise} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x+d)**2*(b*x**2+a)**p,x)
```

```
[Out] a**p*d**2*x**3*hyper((3/2, -p), (5/2, ), b*x**2*exp_polar(I*pi)/a)
/3 + a**p*e**2*x**5*hyper((5/2, -p), (7/2, ), b*x**2*exp_polar(I*pi)
/a)/5 + 2*d*e*Piecewise((a**p*x**4/4, Eq(b, 0)), (a*log(-I*sqrt
(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + a*log(I*sqrt(a)*sqr
t(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**3*x**2)
+ b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2)
+ b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2), E
q(p, -2)), (-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I*s
qrt(a)*sqrt(1/b) + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(
a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a
+ b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a +
b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*
x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2), True))
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2 (bx^2 + a)^p x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^2*(b*x^2 + a)^p*x^2,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p*x^2, x)
```

### 3.394 $\int x(d + ex)^2 (a + bx^2)^p dx$

**Optimal.** Leaf size=113

$$\frac{(bd^2 - ae^2)(a + bx^2)^{p+1}}{2b^2(p+1)} + \frac{e^2(a + bx^2)^{p+2}}{2b^2(p+2)} + \frac{2}{3}dex^3(a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; -\frac{bx^2}{a}\right)$$

[Out]  $((b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (e^2*(a + b*x^2)^(2 + p))/(2*b^2*(2 + p)) + (2*d*e*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -(b*x^2)/a])/(3*(1 + (b*x^2)/a)^p)$

**Rubi [A]** time = 0.190356, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{(bd^2 - ae^2)(a + bx^2)^{p+1}}{2b^2(p+1)} + \frac{e^2(a + bx^2)^{p+2}}{2b^2(p+2)} + \frac{2}{3}dex^3(a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; -\frac{bx^2}{a}\right)$$

Antiderivative was successfully verified.

[In] Int[x\*(d + e\*x)^2\*(a + b\*x^2)^p, x]

[Out]  $((b*d^2 - a*e^2)*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (e^2*(a + b*x^2)^(2 + p))/(2*b^2*(2 + p)) + (2*d*e*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -(b*x^2)/a])/(3*(1 + (b*x^2)/a)^p)$

**Rubi in Sympy [A]** time = 27.3233, size = 126, normalized size = 1.12

$$\frac{2adex \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, \frac{1}{2} \middle| -\frac{bx^2}{a}\right)}{b(2p+3)} + \frac{(a + bx^2)^{p+1} (d + ex)^2}{2b(p+2)} - \frac{(a + bx^2)^{p+1} (-4bdex(p+1) + (4p+6)(ae^2 - bd^2))}{4b^2(p+1)(p+2)(2p+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(e\*x+d)\*\*2\*(b\*x\*\*2+a)\*\*p, x)

[Out]  $-2*a*d*e*x*(1 + b*x**2/a)**(-p)*(a + b*x**2)**p*hyper((-p, 1/2), (3/2, ), -b*x**2/a)/(b*(2*p + 3)) + (a + b*x**2)**(p + 1)*(d + e*x)**2/(2*b*(p + 2)) - (a + b*x**2)**(p + 1)*(-4*b*d*e*x*(p + 1) + (4*p + 6)*(a*e**2 - b*d**2))/(4*b**2*(p + 1)*(p + 2)*(2*p + 3))$

**Mathematica [A]** time = 0.274021, size = 184, normalized size = 1.63

$$\frac{(a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} \left(-3a^2e^2 \left(\left(\frac{bx^2}{a} + 1\right)^p - 1\right) + 3b^2x^2 \left(\frac{bx^2}{a} + 1\right)^p (d^2(p+2) + e^2(p+1)x^2) + 4b^2de(p^2 + 3p + 2)x^3 {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; -\frac{bx^2}{a}\right)\right)}{6b^2(p+1)(p+2)}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(d + e\*x)^2\*(a + b\*x^2)^p, x]

[Out]  $((a + b*x^2)^p*(3*b^2*x^2*(1 + (b*x^2)/a)^p*(d^2*(2 + p) + e^2*(1 + p)*x^2) - 3*a^2*e^2*(-1 + (1 + (b*x^2)/a)^p) + 3*a*b*(e^2*p*x^2$

$$2 \cdot (1 + (b \cdot x^2)/a)^p + d^2 \cdot (2 + p) \cdot (-1 + (1 + (b \cdot x^2)/a)^p) + 4 \cdot b^2 \cdot d \cdot e \cdot (2 + 3 \cdot p + p^2) \cdot x^3 \cdot \text{Hypergeometric2F1}\left[\frac{3}{2}, -p, \frac{5}{2}, -\left(\frac{b \cdot x^2}{a}\right)\right] \Big/ (6 \cdot b^2 \cdot (1 + p) \cdot (2 + p) \cdot (1 + (b \cdot x^2)/a)^p)$$

**Maple [F]** time = 0.06, size = 0, normalized size = 0.

$$\int x (ex + d)^2 (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(e\*x+d)^2\*(b\*x^2+a)^p,x)

[Out] int(x\*(e\*x+d)^2\*(b\*x^2+a)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(b\*x^2 + a)^p\*x,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2 x^3 + 2 d e x^2 + d^2 x\right) (b x^2 + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(b\*x^2 + a)^p\*x,x, algorithm="fricas")

[Out] integral((e^2\*x^3 + 2\*d\*e\*x^2 + d^2\*x)\*(b\*x^2 + a)^p, x)

**Sympy [A]** time = 45.5209, size = 439, normalized size = 3.88

$$\frac{2a^p d e x^3 {}_2F_1\left(\frac{3}{2}, -p \mid \frac{b x^2 e^{i\pi}}{a}\right)}{3} + d^2 \left( \begin{array}{ll} \frac{a^p x^2}{2} & \text{for } b = 0 \\ \frac{(a + b x^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \frac{\log(a + b x^2)}{2b} & \text{otherwise} \end{array} \right)$$

$$+ e^2 \left( \begin{array}{ll} \frac{a^p x^4}{4} & \text{for } b = 0 \\ \frac{a \log(-i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2ab^2+2b^3x^2} + \frac{a \log(i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2ab^2+2b^3x^2} + \frac{a}{2ab^2+2b^3x^2} + \frac{bx^2 \log(-i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2ab^2+2b^3x^2} + \frac{bx^2 \log(i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2ab^2+2b^3x^2} & \text{for } p = -2 \\ -\frac{a \log(-i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2b^2} - \frac{a \log(i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2b^2} + \frac{x^2}{2b} & \text{for } p = -1 \\ -\frac{a^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{abpx^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2px^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2x^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} & \text{otherwise} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate(x*(e*x+d)**2*(b*x**2+a)**p,x)
```

```
[Out] 2*a**p*d*e*x**3*hyper((3/2, -p), (5/2,)), b*x**2*exp_polar(I*pi)/a
)/3 + d**2*Piecewise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*
x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**2), True))/(2*b
), True)) + e**2*Piecewise((a**p*x**4/4, Eq(b, 0)), (a*log(-I*sq
rt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + a*log(I*sqrt(a)*sq
rt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**3*x**2
) + b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2)
+ b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2),
Eq(p, -2)), (-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I*
sqrt(a)*sqrt(1/b) + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*
(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a
+ b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a
+ b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b
*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2), True))
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2 (bx^2 + a)^p x dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^2*(b*x^2 + a)^p*x,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p*x, x)
```

### 3.395 $\int (d + ex)^2 (a + bx^2)^p dx$

**Optimal.** Leaf size=133

$$\frac{x (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (ae^2 - bd^2(2p + 3)) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{b(2p + 3)} + \frac{e(d + ex)(a + bx^2)^{p+1}}{b(2p + 3)} + \frac{de(p + 2)(a + bx^2)^{p+1}}{b(p + 1)(2p + 3)}$$

[Out] (d\*e\*(2 + p)\*(a + b\*x^2)^(1 + p))/(b\*(1 + p)\*(3 + 2\*p)) + (e\*(d + e\*x)\*(a + b\*x^2)^(1 + p))/(b\*(3 + 2\*p)) - ((a\*e^2 - b\*d^2\*(3 + 2\*p))\*x\*(a + b\*x^2)^p\*Hypergeometric2F1[1/2, -p, 3/2, -(b\*x^2)/a])/b\*(3 + 2\*p)\*(1 + (b\*x^2)/a)^p

**Rubi [A]** time = 0.172484, antiderivative size = 125, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$

$$\frac{x (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} \left(d^2 - \frac{ae^2}{2bp + 3b}\right) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{b(2p + 3)} + \frac{e(d + ex)(a + bx^2)^{p+1}}{b(2p + 3)} + \frac{de(p + 2)(a + bx^2)^{p+1}}{b(p + 1)(2p + 3)}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^2\*(a + b\*x^2)^p, x]

[Out] (d\*e\*(2 + p)\*(a + b\*x^2)^(1 + p))/(b\*(1 + p)\*(3 + 2\*p)) + (e\*(d + e\*x)\*(a + b\*x^2)^(1 + p))/(b\*(3 + 2\*p)) + ((d^2 - (a\*e^2)/(3\*b + 2\*b\*p))\*x\*(a + b\*x^2)^p\*Hypergeometric2F1[1/2, -p, 3/2, -(b\*x^2)/a])/b\*(1 + (b\*x^2)/a)^p

**Rubi in Sympy [A]** time = 28.0547, size = 107, normalized size = 0.8

$$\frac{de(a + bx^2)^{p+1}(p + 2)}{b(p + 1)(2p + 3)} + \frac{e(a + bx^2)^{p+1}(d + ex)}{b(2p + 3)} - \frac{x \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p (ae^2 - bd^2(2p + 3)) {}_2F_1\left(-p, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right)}{b(2p + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*2\*(b\*x\*\*2+a)\*\*p, x)

[Out] d\*e\*(a + b\*x\*\*2)\*\*(p + 1)\*(p + 2)/(b\*(p + 1)\*(2\*p + 3)) + e\*(a + b\*x\*\*2)\*\*(p + 1)\*(d + e\*x)/(b\*(2\*p + 3)) - x\*(1 + b\*x\*\*2/a)\*\*(-p)\*(a + b\*x\*\*2)\*\*p\*(a\*e\*\*2 - b\*d\*\*2\*(2\*p + 3))\*hyper((-p, 1/2), (3/2, ), -b\*x\*\*2/a)/(b\*(2\*p + 3))

**Mathematica [A]** time = 0.154338, size = 133, normalized size = 1.

$$\frac{(a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} \left(3bd^2(p + 1)x {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right) + e \left(3d \left(bx^2 \left(\frac{bx^2}{a} + 1\right)^p + a \left(\left(\frac{bx^2}{a} + 1\right)^p - 1\right)\right) + be(p + 1)x^3 {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)\right)}{3b(p + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^2\*(a + b\*x^2)^p,x]

[Out]  $((a + b*x^2)^p*(3*b*d^2*(1 + p)*x*Hypergeometric2F1[1/2, -p, 3/2, -(b*x^2)/a]) + e*(3*d*(b*x^2*(1 + (b*x^2)/a)^p + a*(-1 + (1 + (b*x^2)/a)^p)) + b*e*(1 + p)*x^3*Hypergeometric2F1[3/2, -p, 5/2, -(b*x^2)/a]))/(3*b*(1 + p)*(1 + (b*x^2)/a)^p)$

**Maple [F]** time = 0.055, size = 0, normalized size = 0.

$$\int (ex + d)^2 (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^2\*(b\*x^2+a)^p,x)

[Out] int((e\*x+d)^2\*(b\*x^2+a)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2 (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(b\*x^2 + a)^p,x, algorithm="maxima")

[Out] integrate((e\*x + d)^2\*(b\*x^2 + a)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^2 + 2dex + d^2\right)(bx^2 + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(b\*x^2 + a)^p,x, algorithm="fricas")

[Out] integral((e^2\*x^2 + 2\*d\*e\*x + d^2)\*(b\*x^2 + a)^p, x)

**Sympy [A]** time = 40.8541, size = 97, normalized size = 0.73

$$a^p d^2 x {}_2F_1\left(\frac{1}{2}, -p \middle| \frac{bx^2 e^{i\pi}}{a}\right) + \frac{a^p e^2 x^3 {}_2F_1\left(\frac{3}{2}, -p \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3} + 2de \left( \begin{array}{l} \left( \frac{a^p x^2}{2} \right. \\ \left. \frac{(a+bx^2)^{p+1}}{p+1} \right. \\ \left. \frac{\log(a+bx^2)}{2b} \right) \end{array} \begin{array}{l} \text{for } b = 0 \\ \text{for } p \neq -1 \\ \text{otherwise} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2\*(b\*x\*\*2+a)\*\*p,x)

```
[Out] a**p*d**2*x*hyper((1/2, -p), (3/2, ), b*x**2*exp_polar(I*pi)/a) +
a**p*e**2*x**3*hyper((3/2, -p), (5/2, ), b*x**2*exp_polar(I*pi)/a)
/3 + 2*d*e*Piecewise((a**p*x**2/2, Eq(b, 0)), (Piecewise(((a + b*
x**2)**(p + 1)/(p + 1), Ne(p, -1)), (log(a + b*x**2), True))/(2*b
), True))
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2 (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^2*(b*x^2 + a)^p,x, algorithm="giac")
```

```
[Out] integrate((e*x + d)^2*(b*x^2 + a)^p, x)
```

$$3.396 \quad \int \frac{(d+ex)^2(a+bx^2)^p}{x} dx$$

**Optimal.** Leaf size=118

$$\frac{d^2(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a} + 1\right)}{2a(p+1)} + 2dex(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right) + \frac{e^2(a+bx^2)^{p+1}}{2b(p+1)}$$

[Out] (e^2\*(a + b\*x^2)^(1 + p))/(2\*b\*(1 + p)) + (2\*d\*e\*x\*(a + b\*x^2)^p\*Hypergeometric2F1[1/2, -p, 3/2, -(b\*x^2)/a])/(1 + (b\*x^2)/a)^p - (d^2\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b\*x^2)/a])/(2\*a\*(1 + p))

**Rubi [A]** time = 0.180749, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$\frac{d^2(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a} + 1\right)}{2a(p+1)} + 2dex(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right) + \frac{e^2(a+bx^2)^{p+1}}{2b(p+1)}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^2\*(a + b\*x^2)^p)/x, x]

[Out] (e^2\*(a + b\*x^2)^(1 + p))/(2\*b\*(1 + p)) + (2\*d\*e\*x\*(a + b\*x^2)^p\*Hypergeometric2F1[1/2, -p, 3/2, -(b\*x^2)/a])/(1 + (b\*x^2)/a)^p - (d^2\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b\*x^2)/a])/(2\*a\*(1 + p))

**Rubi in Sympy [A]** time = 19.9873, size = 94, normalized size = 0.8

$$2dex \left(1 + \frac{bx^2}{a}\right)^{-p} (a+bx^2)^p {}_2F_1\left(-p, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) + \frac{e^2(a+bx^2)^{p+1}}{2b(p+1)} - \frac{d^2(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; 1 + \frac{bx^2}{a}\right)}{2a(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*2\*(b\*x\*\*2+a)\*\*p/x, x)

[Out] 2\*d\*e\*x\*(1 + b\*x\*\*2/a)\*\*(-p)\*(a + b\*x\*\*2)\*\*p\*hyper((-p, 1/2), (3/2, ), -b\*x\*\*2/a) + e\*\*2\*(a + b\*x\*\*2)\*\*(p + 1)/(2\*b\*(p + 1)) - d\*\*2\*(a + b\*x\*\*2)\*\*(p + 1)\*hyper((1, p + 1), (p + 2, ), 1 + b\*x\*\*2/a)/(2\*a\*(p + 1))

**Mathematica [A]** time = 0.563401, size = 130, normalized size = 1.1

$$\frac{1}{2}(a+bx^2)^p \left( \frac{d^2 \left(\frac{a}{bx^2} + 1\right)^{-p} {}_2F_1\left(-p, -p; 1-p; -\frac{a}{bx^2}\right)}{p} + 4dex \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right) + \frac{e^2 \left(-a \left(\frac{bx^2}{a} + 1\right)^{-p} + a + bx^2\right)}{b(p+1)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^2\*(a + b\*x^2)^p)/x,x]

[Out] ((a + b\*x^2)^p\*((e^2\*(a + b\*x^2 - a/(1 + (b\*x^2)/a))^p)/(b\*(1 + p)) + (4\*d\*e\*x\*Hypergeometric2F1[1/2, -p, 3/2, -((b\*x^2)/a)])/(1 + (b\*x^2)/a)^p + (d^2\*Hypergeometric2F1[-p, -p, 1 - p, -(a/(b\*x^2))])/(p\*(1 + a/(b\*x^2))^p))/2

**Maple [F]** time = 0.059, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^2 (bx^2 + a)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^2\*(b\*x^2+a)^p/x,x)

[Out] int((e\*x+d)^2\*(b\*x^2+a)^p/x,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^2 (bx^2 + a)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(b\*x^2 + a)^p/x,x, algorithm="maxima")

[Out] integrate((e\*x + d)^2\*(b\*x^2 + a)^p/x, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e^2x^2 + 2dex + d^2)(bx^2 + a)^p}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(b\*x^2 + a)^p/x,x, algorithm="fricas")

[Out] integral((e^2\*x^2 + 2\*d\*e\*x + d^2)\*(b\*x^2 + a)^p/x, x)

**Sympy [A]** time = 37.7494, size = 109, normalized size = 0.92

$$2a^p dex {}_2F_1\left(\frac{1}{2}, -p \middle| \frac{bx^2 e^{i\pi}}{a}\right) - \frac{b^p d^2 x^{2p} (-p) {}_2F_1\left(-p, -p \middle| \frac{ae^{i\pi}}{bx^2}\right)}{2(-p+1)}$$

$$+ e^2 \left( \begin{array}{ll} \frac{a^p x^2}{2} & \text{for } b = 0 \\ \frac{(a+bx^2)^{p+1}}{p+1} & \text{for } p \neq -1 \\ \frac{\log(a+bx^2)}{2b} & \text{otherwise} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2\*(b\*x\*\*2+a)\*\*p/x,x)

[Out] 2\*a\*\*p\*d\*e\*x\*hyper((1/2, -p), (3/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a) -  
 b\*\*p\*d\*\*2\*x\*\*(2\*p)\*gamma(-p)\*hyper((-p, -p), (-p + 1, ), a\*exp\_polar(I\*pi)/(b\*x\*\*2))/(2\*gamma(-p + 1)) + e\*\*2\*Piecewise((a\*\*p\*x\*\*2/2, Eq(b, 0)), (Piecewise(((a + b\*x\*\*2)\*\*(p + 1)/(p + 1), Ne(p, -1)), (log(a + b\*x\*\*2), True)))/(2\*b), True))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^2 (bx^2 + a)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(b\*x^2 + a)^p/x,x, algorithm="giac")

[Out] integrate((e\*x + d)^2\*(b\*x^2 + a)^p/x, x)

$$3.397 \quad \int \frac{(d+ex)^2(a+bx^2)^p}{x^2} dx$$

**Optimal.** Leaf size=127

$$\frac{x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (ae^2 + bd^2(2p+1)) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{a} - \frac{d^2(a+bx^2)^{p+1}}{ax} - \frac{de(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a} + 1\right)}{a(p+1)}$$

[Out]  $-\left(\frac{d^2(a+bx^2)^{p+1}}{ax}\right) + \left(\frac{x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (ae^2 + bd^2(2p+1)) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{a}\right) - \left(\frac{de(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a} + 1\right)}{a(p+1)}\right)$

**Rubi [A]** time = 0.229028, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (ae^2 + bd^2(2p+1)) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{a} - \frac{d^2(a+bx^2)^{p+1}}{ax} - \frac{de(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a} + 1\right)}{a(p+1)}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^2\*(a + b\*x^2)^p)/x^2, x]

[Out]  $-\left(\frac{d^2(a+bx^2)^{p+1}}{ax}\right) + \left(\frac{x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (ae^2 + bd^2(2p+1)) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{a}\right) - \left(\frac{de(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a} + 1\right)}{a(p+1)}\right)$

**Rubi in Sympy [A]** time = 24.9215, size = 110, normalized size = 0.87

$$\frac{d^2 \left(1 + \frac{bx^2}{a}\right)^{-p} (a+bx^2)^p {}_2F_1\left(-p, -\frac{1}{2} \middle| -\frac{bx^2}{a}\right)}{x} + e^2 x \left(1 + \frac{bx^2}{a}\right)^{-p} (a+bx^2)^p {}_2F_1\left(-p, \frac{1}{2} \middle| -\frac{bx^2}{a}\right) - \frac{de(a+bx^2)^{p+1} {}_2F_1\left(1, p+1 \middle| 1 + \frac{bx^2}{a}\right)}{a(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*2\*(b\*x\*\*2+a)\*\*p/x\*\*2, x)

[Out]  $-d^2 \frac{(a+bx^2)^{p+1}}{ax} + e^2 x (a+bx^2)^p \left(1 + \frac{bx^2}{a}\right)^{-p} {}_2F_1\left(-p, \frac{1}{2} \middle| -\frac{bx^2}{a}\right) - \frac{de(a+bx^2)^{p+1} {}_2F_1\left(1, p+1 \middle| 1 + \frac{bx^2}{a}\right)}{a(p+1)}$



**Mathematica [A]** time = 0.18693, size = 131, normalized size = 1.03

$$(a + bx^2)^p \left( e \left( \frac{d \left( \frac{a}{bx^2} + 1 \right)^{-p} {}_2F_1 \left( -p, -p; 1 - p; -\frac{a}{bx^2} \right)}{p} + ex \left( \frac{bx^2}{a} + 1 \right)^{-p} {}_2F_1 \left( \frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a} \right) \right) - \frac{d^2 \left( \frac{bx^2}{a} + 1 \right)^{-p} {}_2F_1 \left( -\frac{1}{2}, -p; \frac{1}{2}; -\frac{bx^2}{a} \right)}{x} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^2\*(a + b\*x^2)^p)/x^2, x]

[Out] (a + b\*x^2)^p\*(-((d^2\*Hypergeometric2F1[-1/2, -p, 1/2, -(b\*x^2)/a])/x\*(1 + (b\*x^2)/a)^p)) + e\*((e\*x\*Hypergeometric2F1[1/2, -p, 3/2, -(b\*x^2)/a])/((1 + (b\*x^2)/a)^p + (d\*Hypergeometric2F1[-p, -p, 1 - p, -(a/(b\*x^2))])/(p\*(1 + a/(b\*x^2))^p)))

**Maple [F]** time = 0.058, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^2 (bx^2 + a)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^2\*(b\*x^2+a)^p/x^2, x)

[Out] int((e\*x+d)^2\*(b\*x^2+a)^p/x^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^2 (bx^2 + a)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(b\*x^2 + a)^p/x^2, x, algorithm="maxima")

[Out] integrate((e\*x + d)^2\*(b\*x^2 + a)^p/x^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{(e^2x^2 + 2dex + d^2)(bx^2 + a)^p}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(b\*x^2 + a)^p/x^2, x, algorithm="fricas")

[Out] integral((e^2\*x^2 + 2\*d\*e\*x + d^2)\*(b\*x^2 + a)^p/x^2, x)

**Sympy [A]** time = 57.6326, size = 95, normalized size = 0.75

$$-\frac{a^p d^2 {}_2F_1\left(-\frac{1}{2}, -p \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{x} + a^p e^2 x {}_2F_1\left(\frac{1}{2}, -p \middle| \frac{bx^2 e^{i\pi}}{a}\right) - \frac{b^p d e x^{2p} (-p) {}_2F_1\left(-p, -p \middle| \frac{a e^{i\pi}}{bx^2}\right)}{(-p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2\*(b\*x\*\*2+a)\*\*p/x\*\*2,x)

[Out] -a\*\*p\*d\*\*2\*hyper((-1/2, -p), (1/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/x + a\*\*p\*e\*\*2\*x\*hyper((1/2, -p), (3/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a) - b\*\*p\*d\*e\*x\*\*(2\*p)\*gamma(-p)\*hyper((-p, -p), (-p+1, ), a\*exp\_polar(I\*pi)/(b\*x\*\*2))/gamma(-p+1)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^2 (bx^2+a)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(b\*x^2 + a)^p/x^2,x, algorithm="giac")

[Out] integrate((e\*x + d)^2\*(b\*x^2 + a)^p/x^2, x)

$$3.398 \quad \int \frac{(d+ex)^2(a+bx^2)^p}{x^3} dx$$

**Optimal.** Leaf size=127

$$\frac{(a+bx^2)^{p+1}(ae^2+bd^2p) {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a}+1\right)}{2a^2(p+1)} - \frac{d^2(a+bx^2)^{p+1}}{2ax^2} - \frac{2de(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; -\frac{bx^2}{a}\right)}{x}$$

[Out]  $-(d^2(a+bx^2)^{(1+p)})/(2ax^2) - (2de(a+bx^2)^p \text{Hypergeometric2F1}[-1/2, -p, 1/2, -(bx^2/a)])/(x(1+(bx^2/a))^p) - ((a^2e^2+bd^2p)(a+bx^2)^{(1+p)} \text{Hypergeometric2F1}[1, 1+p, 2+p, 1+(bx^2/a)])/(2a^2(1+p))$

**Rubi [A]** time = 0.238728, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{(a+bx^2)^{p+1}(ae^2+bd^2p) {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a}+1\right)}{2a^2(p+1)} - \frac{d^2(a+bx^2)^{p+1}}{2ax^2} - \frac{2de(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; -\frac{bx^2}{a}\right)}{x}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^2\*(a + b\*x^2)^p)/x^3, x]

[Out]  $-(d^2(a+bx^2)^{(1+p)})/(2ax^2) - (2de(a+bx^2)^p \text{Hypergeometric2F1}[-1/2, -p, 1/2, -(bx^2/a)])/(x(1+(bx^2/a))^p) - ((a^2e^2+bd^2p)(a+bx^2)^{(1+p)} \text{Hypergeometric2F1}[1, 1+p, 2+p, 1+(bx^2/a)])/(2a^2(1+p))$

**Rubi in Sympy [A]** time = 25.6477, size = 112, normalized size = 0.88

$$\frac{2de \left(1 + \frac{bx^2}{a}\right)^{-p} (a+bx^2)^p {}_2F_1\left(-p, -\frac{1}{2} \middle| -\frac{bx^2}{a}\right)}{x} - \frac{e^2(a+bx^2)^{p+1} {}_2F_1\left(1, p+1 \middle| 1 + \frac{bx^2}{a}\right)}{2a(p+1)} + \frac{bd^2(a+bx^2)^{p+1} {}_2F_1\left(2, p+1 \middle| 1 + \frac{bx^2}{a}\right)}{2a^2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*2\*(b\*x\*\*2+a)\*\*p/x\*\*3, x)

[Out]  $-2*d*e*(1+b*x**2/a)**(-p)*(a+b*x**2)**p*\text{hyper}((-p, -1/2), (1/2, ), -b*x**2/a)/x - e**2*(a+b*x**2)**(p+1)*\text{hyper}((1, p+1), (p+2, ), 1+b*x**2/a)/(2*a*(p+1)) + b*d**2*(a+b*x**2)**(p+1)*\text{hyper}((2, p+1), (p+2, ), 1+b*x**2/a)/(2*a**2*(p+1))$

**Mathematica [A]** time = 0.275336, size = 138, normalized size = 1.09

$$\frac{(a + bx^2)^p \left( \frac{\left(\frac{-a}{bx^2} + 1\right)^{-p} \left( d^2 {}_2F_1\left(1-p, -p; 2-p; -\frac{a}{bx^2}\right) + e^{2(p-1)x^2} {}_2F_1\left(-p, -p; 1-p; -\frac{a}{bx^2}\right) \right)}{(p-1)p} - 4dex \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; -\frac{bx^2}{a}\right) \right)}{2x^2}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^2\*(a + b\*x^2)^p)/x^3, x]

[Out] ((a + b\*x^2)^p\*((-4\*d\*e\*x\*Hypergeometric2F1[-1/2, -p, 1/2, -(b\*x^2)/a])/(1 + (b\*x^2)/a)^p + (d^2\*p\*Hypergeometric2F1[1 - p, -p, 2 - p, -(a/(b\*x^2))]) + e^2\*(-1 + p)\*x^2\*Hypergeometric2F1[-p, -p, 1 - p, -(a/(b\*x^2))])/((-1 + p)\*p\*(1 + a/(b\*x^2))^p))/(2\*x^2)

**Maple [F]** time = 0.142, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^2 (bx^2 + a)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^2\*(b\*x^2+a)^p/x^3, x)

[Out] int((e\*x+d)^2\*(b\*x^2+a)^p/x^3, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^2 (bx^2 + a)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(b\*x^2 + a)^p/x^3, x, algorithm="maxima")

[Out] integrate((e\*x + d)^2\*(b\*x^2 + a)^p/x^3, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e^2x^2 + 2dex + d^2)(bx^2 + a)^p}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(b\*x^2 + a)^p/x^3, x, algorithm="fricas")

[Out] integral((e^2\*x^2 + 2\*d\*e\*x + d^2)\*(b\*x^2 + a)^p/x^3, x)

**Sympy [A]** time = 74.213, size = 119, normalized size = 0.94

$$\frac{2a^p de {}_2F_1\left(-\frac{1}{2}, -p \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{x} - \frac{b^p d^2 x^{2p} (-p + 1) {}_2F_1\left(-p, -p + 1 \middle| \frac{ae^{i\pi}}{bx^2}\right)}{2x^2(-p + 2)} - \frac{b^p e^2 x^{2p} (-p) {}_2F_1\left(-p, -p \middle| \frac{ae^{i\pi}}{bx^2}\right)}{2(-p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2\*(b\*x\*\*2+a)\*\*p/x\*\*3,x)

[Out]  $-2*a**p*d*e*hyper((-1/2, -p), (1/2, ), b*x**2*exp\_polar(I*pi)/a)/x$   
 $- b**p*d**2*x**(2*p)*gamma(-p + 1)*hyper((-p, -p + 1), (-p + 2, )$   
 $, a*exp\_polar(I*pi)/(b*x**2))/(2*x**2*gamma(-p + 2)) - b**p*e**2*$   
 $x**(2*p)*gamma(-p)*hyper((-p, -p), (-p + 1, ), a*exp\_polar(I*pi)/($   
 $b*x**2))/(2*gamma(-p + 1))$

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^2 (bx^2 + a)^P}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(b\*x^2 + a)^p/x^3,x, algorithm="giac")

[Out] integrate((e\*x + d)^2\*(b\*x^2 + a)^p/x^3, x)

### 3.399 $\int x^5(d + ex)^3 (a + bx^2)^p dx$

**Optimal.** Leaf size=247

$$\frac{a^2 d (bd^2 - 3ae^2) (a + bx^2)^{p+1}}{2b^4(p+1)} - \frac{ad (2bd^2 - 9ae^2) (a + bx^2)^{p+2}}{2b^4(p+2)} + \frac{d (bd^2 - 9ae^2) (a + bx^2)^{p+3}}{2b^4(p+3)} + \frac{3de^2 (a + bx^2)^{p+4}}{2b^4(p+4)} - \frac{ex^7 (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (7ae^2 - 3bd^2(2p+9)) {}_2F_1\left(\frac{7}{2}, -p; \frac{9}{2}; -\frac{bx^2}{a}\right)}{7b(2p+9)} + \frac{e^3 x^7 (a + bx^2)^{p+1}}{b(2p+9)}$$

[Out]  $(a^2 d (b^2 d^2 - 3 a^2 e^2) (a + b^2 x^2)^{(1+p)} / (2 b^4 (1+p)) + (e^3 x^7 (a + b^2 x^2)^{(1+p)} / (b^2 (9 + 2p))) - (a d (2 b^2 d^2 - 9 a^2 e^2) (a + b^2 x^2)^{(2+p)} / (2 b^4 (2+p)) + (d (b^2 d^2 - 9 a^2 e^2) (a + b^2 x^2)^{(3+p)} / (2 b^4 (3+p)) + (3 d^2 e^2 (a + b^2 x^2)^{(4+p)} / (2 b^4 (4+p)) - (e (7 a^2 e^2 - 3 b d^2 (9 + 2p))) x^7 (a + b^2 x^2)^p \text{Hypergeometric2F1}[7/2, -p, 9/2, -(b^2 x^2/a)] / (7 b^2 (9 + 2p)) (1 + (b^2 x^2/a)^p)$

**Rubi [A]** time = 0.502993, antiderivative size = 241, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{a^2 d (bd^2 - 3ae^2) (a + bx^2)^{p+1}}{2b^4(p+1)} - \frac{ad (2bd^2 - 9ae^2) (a + bx^2)^{p+2}}{2b^4(p+2)} + \frac{d (bd^2 - 9ae^2) (a + bx^2)^{p+3}}{2b^4(p+3)} + \frac{3de^2 (a + bx^2)^{p+4}}{2b^4(p+4)} + \frac{1}{7} ex^7 (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} \left(3d^2 - \frac{7ae^2}{2bp+9b}\right) {}_2F_1\left(\frac{7}{2}, -p; \frac{9}{2}; -\frac{bx^2}{a}\right) + \frac{e^3 x^7 (a + bx^2)^{p+1}}{b(2p+9)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^5 (d + e x)^3 (a + b x^2)^p, x]$

[Out]  $(a^2 d (b^2 d^2 - 3 a^2 e^2) (a + b^2 x^2)^{(1+p)} / (2 b^4 (1+p)) + (e^3 x^7 (a + b^2 x^2)^{(1+p)} / (b^2 (9 + 2p))) - (a d (2 b^2 d^2 - 9 a^2 e^2) (a + b^2 x^2)^{(2+p)} / (2 b^4 (2+p)) + (d (b^2 d^2 - 9 a^2 e^2) (a + b^2 x^2)^{(3+p)} / (2 b^4 (3+p)) + (3 d^2 e^2 (a + b^2 x^2)^{(4+p)} / (2 b^4 (4+p)) + (e (3 d^2 - (7 a^2 e^2) / (9 b + 2 b^2 p))) x^7 (a + b^2 x^2)^p \text{Hypergeometric2F1}[7/2, -p, 9/2, -(b^2 x^2/a)] / (7 (1 + (b^2 x^2/a)^p)$

**Rubi in SymPy [A]** time = 78.5849, size = 267, normalized size = 1.08

$$-\frac{3a^3 de^2 (a + bx^2)^{p+1}}{2b^4 (p+1)} + \frac{a^2 d^3 (a + bx^2)^{p+1}}{2b^3 (p+1)} + \frac{9a^2 de^2 (a + bx^2)^{p+2}}{2b^4 (p+2)} - \frac{ad^3 (a + bx^2)^{p+2}}{b^3 (p+2)} - \frac{9ade^2 (a + bx^2)^{p+3}}{2b^4 (p+3)} + \frac{3d^2 ex^7 \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, \frac{7}{2}; \frac{9}{2}; -\frac{bx^2}{a}\right)}{7} + \frac{e^3 x^9 \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, \frac{9}{2}; \frac{11}{2}; -\frac{bx^2}{a}\right)}{9} + \frac{d^3 (a + bx^2)^{p+3}}{2b^3 (p+3)} + \frac{3de^2 (a + bx^2)^{p+4}}{2b^4 (p+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**5*(e*x+d)**3*(b*x**2+a)**p,x)`

[Out]  $-3a^{*3}d^*e^{*2}(a + b^*x^{*2})^{*p}(p + 1)/(2^*b^{*4}(p + 1)) + a^{*2}d^{*3}(a + b^*x^{*2})^{*p}(p + 1)/(2^*b^{*3}(p + 1)) + 9a^{*2}d^*e^{*2}(a + b^*x^{*2})^{*p}(p + 2)/(2^*b^{*4}(p + 2)) - a^*d^{*3}(a + b^*x^{*2})^{*p}(p + 2)/(b^{*3}(p + 2)) - 9a^*d^*e^{*2}(a + b^*x^{*2})^{*p}(p + 3)/(2^*b^{*4}(p + 3)) + 3^*d^{*2}e^{*x^{*7}}(1 + b^*x^{*2}/a)^{*(-p)}(a + b^*x^{*2})^{*p}\text{hyper}((-p, 7/2), (9/2, ), -b^*x^{*2}/a)/7 + e^{*3}x^{*9}(1 + b^*x^{*2}/a)^{*(-p)}(a + b^*x^{*2})^{*p}\text{hyper}((-p, 9/2), (11/2, ), -b^*x^{*2}/a)/9 + d^{*3}(a + b^*x^{*2})^{*p}(p + 3)/(2^*b^{*3}(p + 3)) + 3^*d^*e^{*2}(a + b^*x^{*2})^{*p}(p + 4)/(2^*b^{*4}(p + 4))$

**Mathematica [A]** time = 0.701644, size = 356, normalized size = 1.44

$$(a + bx^2)^p \left( \frac{bx^2}{a} + 1 \right)^{-p} \left( 63d \left( -18a^4e^2 \left( \left( \frac{bx^2}{a} + 1 \right)^p - 1 \right) + 2a^3b \left( d^2(p+4) \left( \left( \frac{bx^2}{a} + 1 \right)^p - 1 \right) + 9e^2px^2 \left( \frac{bx^2}{a} + 1 \right)^p \right) - a^2b^2p \right)$$

Antiderivative was successfully verified.

[In] `Integrate[x^5*(d + e*x)^3*(a + b*x^2)^p,x]`

[Out]  $((a + b^*x^{*2})^{*p}(63^*d^*(-(a^{*2}b^{*2}p^*x^{*2}(1 + (b^*x^{*2})/a)^{*p}(2^*d^{*2}(4 + p) + 9^*e^{*2}(1 + p)^*x^{*2})) + a^*b^{*3}p^*(1 + p)^*x^{*4}(1 + (b^*x^{*2})/a)^{*p}(d^{*2}(4 + p) + 3^*e^{*2}(2 + p)^*x^{*2}) + b^{*4}(2 + 3^*p + p^{*2})^*x^{*6}(1 + (b^*x^{*2})/a)^{*p}(d^{*2}(4 + p) + 3^*e^{*2}(3 + p)^*x^{*2}) - 18^*a^{*4}e^{*2}(-1 + (1 + (b^*x^{*2})/a)^{*p}) + 2^*a^{*3}b^*(9^*e^{*2}p^*x^{*2}(1 + (b^*x^{*2})/a)^{*p} + d^{*2}(4 + p)^*(-1 + (1 + (b^*x^{*2})/a)^{*p}))) + 54^*b^{*4}d^{*2}e^*(24 + 50^*p + 35^*p^{*2} + 10^*p^{*3} + p^{*4})^*x^{*7}\text{Hypergeometric2F1}[7/2, -p, 9/2, -((b^*x^{*2})/a)] + 14^*b^{*4}e^{*3}(24 + 50^*p + 35^*p^{*2} + 10^*p^{*3} + p^{*4})^*x^{*9}\text{Hypergeometric2F1}[9/2, -p, 11/2, -((b^*x^{*2})/a)])))/(126^*b^{*4}(1 + p)^*(2 + p)^*(3 + p)^*(4 + p)^*(1 + (b^*x^{*2})/a)^{*p}$

**Maple [F]** time = 0.096, size = 0, normalized size = 0.

$$\int x^5 (ex + d)^3 (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5*(e*x+d)^3*(b*x^2+a)^p,x)`

[Out] `int(x^5*(e*x+d)^3*(b*x^2+a)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{((p^2 + 3p + 2)b^3x^6 + (p^2 + p)ab^2x^4 - 2a^2bpx^2 + 2a^3)(bx^2 + a)^p d^3}{2(p^3 + 6p^2 + 11p + 6)b^3} + \int (e^3x^8 + 3de^2x^7 + 3d^2ex^6)(bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^3*(b*x^2 + a)^p*x^5,x, algorithm="maxima")`

[Out]  $1/2^*(p^{*2} + 3^*p + 2)^*b^{*3}x^{*6} + (p^{*2} + p)^*a^*b^{*2}x^{*4} - 2^*a^{*2}b^*p^*x^{*2} + 2^*a^{*3})^*(b^*x^{*2} + a)^{*p}d^{*3}/((p^{*3} + 6^*p^{*2} + 11^*p + 6)^*b^{*3}) + \text{int}$

egrate((e^3\*x^8 + 3\*d\*e^2\*x^7 + 3\*d^2\*e\*x^6)\*(b\*x^2 + a)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^8 + 3de^2x^7 + 3d^2ex^6 + d^3x^5\right)(bx^2 + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(b\*x^2 + a)^p\*x^5,x, algorithm="fricas")

[Out] integral((e^3\*x^8 + 3\*d\*e^2\*x^7 + 3\*d^2\*e\*x^6 + d^3\*x^5)\*(b\*x^2 + a)^p, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5\*(e\*x+d)\*\*3\*(b\*x\*\*2+a)\*\*p,x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3 (bx^2 + a)^p x^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(b\*x^2 + a)^p\*x^5,x, algorithm="giac")

[Out] integrate((e\*x + d)^3\*(b\*x^2 + a)^p\*x^5, x)



### 3.400 $\int x^4(d + ex)^3 (a + bx^2)^p dx$

**Optimal.** Leaf size=249

$$\frac{a^2 e (3bd^2 - ae^2) (a + bx^2)^{p+1}}{2b^4(p+1)} - \frac{3ae (2bd^2 - ae^2) (a + bx^2)^{p+2}}{2b^4(p+2)} + \frac{3e (bd^2 - ae^2) (a + bx^2)^{p+3}}{2b^4(p+3)} + \frac{e^3 (a + bx^2)^{p+4}}{2b^4(p+4)} - \frac{dx^5 (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (15ae^2 - bd^2(2p+7)) {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; -\frac{bx^2}{a}\right)}{5b(2p+7)} + \frac{3de^2 x^5 (a + bx^2)^{p+1}}{b(2p+7)}$$

[Out]  $(a^2 e^* (3*b*d^2 - a*e^2) * (a + b*x^2)^(1 + p)) / (2*b^4*(1 + p)) + (3*d*e^2*x^5*(a + b*x^2)^(1 + p)) / (b*(7 + 2*p)) - (3*a*e*(2*b*d^2 - a*e^2) * (a + b*x^2)^(2 + p)) / (2*b^4*(2 + p)) + (3*e*(b*d^2 - a*e^2) * (a + b*x^2)^(3 + p)) / (2*b^4*(3 + p)) + (e^3*(a + b*x^2)^(4 + p)) / (2*b^4*(4 + p)) - (d*(15*a*e^2 - b*d^2*(7 + 2*p)) * x^5*(a + b*x^2)^p * Hypergeometric2F1[5/2, -p, 7/2, -(b*x^2)/a]) / (5*b*(7 + 2*p)*(1 + (b*x^2)/a)^p)$

**Rubi [A]** time = 0.50207, antiderivative size = 241, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{a^2 e (3bd^2 - ae^2) (a + bx^2)^{p+1}}{2b^4(p+1)} - \frac{3ae (2bd^2 - ae^2) (a + bx^2)^{p+2}}{2b^4(p+2)} + \frac{3e (bd^2 - ae^2) (a + bx^2)^{p+3}}{2b^4(p+3)} + \frac{e^3 (a + bx^2)^{p+4}}{2b^4(p+4)} + \frac{1}{5} dx^5 (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} \left(d^2 - \frac{15ae^2}{2bp+7b}\right) {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; -\frac{bx^2}{a}\right) + \frac{3de^2 x^5 (a + bx^2)^{p+1}}{b(2p+7)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^4*(d + e*x)^3*(a + b*x^2)^p, x]$

[Out]  $(a^2 e^* (3*b*d^2 - a*e^2) * (a + b*x^2)^(1 + p)) / (2*b^4*(1 + p)) + (3*d*e^2*x^5*(a + b*x^2)^(1 + p)) / (b*(7 + 2*p)) - (3*a*e*(2*b*d^2 - a*e^2) * (a + b*x^2)^(2 + p)) / (2*b^4*(2 + p)) + (3*e*(b*d^2 - a*e^2) * (a + b*x^2)^(3 + p)) / (2*b^4*(3 + p)) + (e^3*(a + b*x^2)^(4 + p)) / (2*b^4*(4 + p)) + (d*(d^2 - (15*a*e^2)/(7*b + 2*b*p)) * x^5*(a + b*x^2)^p * Hypergeometric2F1[5/2, -p, 7/2, -(b*x^2)/a]) / (5*(1 + (b*x^2)/a)^p)$

**Rubi in Sympy [A]** time = 75.3312, size = 267, normalized size = 1.07

$$-\frac{a^3 e^3 (a + bx^2)^{p+1}}{2b^4(p+1)} + \frac{3a^2 d^2 e (a + bx^2)^{p+1}}{2b^3(p+1)} + \frac{3a^2 e^3 (a + bx^2)^{p+2}}{2b^4(p+2)} - \frac{3ad^2 e (a + bx^2)^{p+2}}{b^3(p+2)} - \frac{3ae^3 (a + bx^2)^{p+3}}{2b^4(p+3)} + \frac{d^3 x^5 \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, \frac{5}{2}; \frac{7}{2}; -\frac{bx^2}{a}\right)}{5} + \frac{3de^2 x^7 \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, \frac{7}{2}; \frac{9}{2}; -\frac{bx^2}{a}\right)}{7} + \frac{3d^2 e (a + bx^2)^{p+3}}{2b^3(p+3)} + \frac{e^3 (a + bx^2)^{p+4}}{2b^4(p+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**4*(e*x+d)**3*(b*x**2+a)**p,x)`

[Out] 
$$-a^{**3}e^{**3}(a + b*x^{**2})^{**}(p + 1)/(2*b^{**4}(p + 1)) + 3*a^{**2}d^{**2}e^{**}(a + b*x^{**2})^{**}(p + 1)/(2*b^{**3}(p + 1)) + 3*a^{**2}e^{**3}(a + b*x^{**2})^{**}(p + 2)/(2*b^{**4}(p + 2)) - 3*a*d^{**2}e^{**}(a + b*x^{**2})^{**}(p + 2)/(b^{**3}(p + 2)) - 3*a*e^{**3}(a + b*x^{**2})^{**}(p + 3)/(2*b^{**4}(p + 3)) + d^{**3}x^{**5}(1 + b*x^{**2}/a)^{**}(-p)(a + b*x^{**2})^{**}p*hyper((-p, 5/2), (7/2, ), -b*x^{**2}/a)/5 + 3*d*e^{**2}x^{**7}(1 + b*x^{**2}/a)^{**}(-p)(a + b*x^{**2})^{**}p*hyper((-p, 7/2), (9/2, ), -b*x^{**2}/a)/7 + 3*d^{**2}e^{**}(a + b*x^{**2})^{**}(p + 3)/(2*b^{**3}(p + 3)) + e^{**3}(a + b*x^{**2})^{**}(p + 4)/(2*b^{**4}(p + 4))$$

**Mathematica [A]** time = 0.69797, size = 357, normalized size = 1.43

$$(a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} \left(14b^4d^3(p^4 + 10p^3 + 35p^2 + 50p + 24)x^5 {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; -\frac{bx^2}{a}\right) - 5e\left(7\left(6a^4e^2\left(\left(\frac{bx^2}{a} + 1\right)^p - 1\right) - 6\right.\right.\right.$$

Antiderivative was successfully verified.

[In] `Integrate[x^4*(d + e*x)^3*(a + b*x^2)^p,x]`

[Out] 
$$\left((a + b*x^2)^p(14*b^4*d^3(24 + 50*p + 35*p^2 + 10*p^3 + p^4)*x^5*Hypergeometric2F1[5/2, -p, 7/2, -(b*x^2)/a] - 5*e*(7*(3*a^2*b^2*p*x^2*(1 + (b*x^2)/a)^p*(2*d^2*(4 + p) + e^2*(1 + p)*x^2) - a*b^3*p*(1 + p)*x^4*(1 + (b*x^2)/a)^p*(3*d^2*(4 + p) + e^2*(2 + p)*x^2) - b^4*(2 + 3*p + p^2)*x^6*(1 + (b*x^2)/a)^p*(3*d^2*(4 + p) + e^2*(3 + p)*x^2) + 6*a^4*e^2*(-1 + (1 + (b*x^2)/a)^p) - 6*a^3*b*(e^2*p*x^2*(1 + (b*x^2)/a)^p + d^2*(4 + p)*(-1 + (1 + (b*x^2)/a)^p)) - 6*b^4*d*e*(24 + 50*p + 35*p^2 + 10*p^3 + p^4)*x^7*Hypergeometric2F1[7/2, -p, 9/2, -(b*x^2)/a])\right)/(70*b^4*(1 + p)^*(2 + p)^*(3 + p)^*(4 + p)^*(1 + (b*x^2)/a)^p$$

**Maple [F]** time = 0.101, size = 0, normalized size = 0.

$$\int x^4 (ex + d)^3 (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(e*x+d)^3*(b*x^2+a)^p,x)`

[Out] `int(x^4*(e*x+d)^3*(b*x^2+a)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3 (bx^2 + a)^p x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^3*(b*x^2 + a)^p*x^4,x, algorithm="maxima")`

[Out] `integrate((e*x + d)^3*(b*x^2 + a)^p*x^4, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^7 + 3de^2x^6 + 3d^2ex^5 + d^3x^4\right)(bx^2 + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^3*(b*x^2 + a)^p*x^4,x, algorithm="fricas")`

[Out] `integral((e^3*x^7 + 3*d*e^2*x^6 + 3*d^2*e*x^5 + d^3*x^4)*(b*x^2 + a)^p, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(e*x+d)**3*(b*x**2+a)**p,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3 (bx^2 + a)^p x^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^3*(b*x^2 + a)^p*x^4,x, algorithm="giac")`

[Out] `integrate((e*x + d)^3*(b*x^2 + a)^p*x^4, x)`

### 3.401 $\int x^3(d + ex)^3 (a + bx^2)^p dx$

**Optimal.** Leaf size=207

$$\begin{aligned} & -\frac{ad(bd^2 - 3ae^2)(a + bx^2)^{p+1}}{2b^3(p+1)} + \frac{d(bd^2 - 6ae^2)(a + bx^2)^{p+2}}{2b^3(p+2)} + \frac{3de^2(a + bx^2)^{p+3}}{2b^3(p+3)} \\ & - \frac{ex^5(a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (5ae^2 - 3bd^2(2p+7)) {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; -\frac{bx^2}{a}\right)}{5b(2p+7)} + \frac{e^3x^5(a + bx^2)^{p+1}}{b(2p+7)} \end{aligned}$$

[Out]  $-(a*d*(b*d^2 - 3*a*e^2)*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) + (e^3*x^5*(a + b*x^2)^(1 + p))/(b*(7 + 2*p)) + (d*(b*d^2 - 6*a*e^2)*(a + b*x^2)^(2 + p))/(2*b^3*(2 + p)) + (3*d*e^2*(a + b*x^2)^(3 + p))/(2*b^3*(3 + p)) - (e*(5*a*e^2 - 3*b*d^2*(7 + 2*p))*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -(b*x^2)/a])/(5*b*(7 + 2*p)*(1 + (b*x^2)/a)^p)$

**Rubi [A]** time = 0.418984, antiderivative size = 201, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\begin{aligned} & -\frac{ad(bd^2 - 3ae^2)(a + bx^2)^{p+1}}{2b^3(p+1)} + \frac{d(bd^2 - 6ae^2)(a + bx^2)^{p+2}}{2b^3(p+2)} + \frac{3de^2(a + bx^2)^{p+3}}{2b^3(p+3)} \\ & + \frac{1}{5}ex^5(a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} \left(3d^2 - \frac{5ae^2}{2bp+7b}\right) {}_2F_1\left(\frac{5}{2}, -p; \frac{7}{2}; -\frac{bx^2}{a}\right) + \frac{e^3x^5(a + bx^2)^{p+1}}{b(2p+7)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^3\*(d + e\*x)^3\*(a + b\*x^2)^p,x]

[Out]  $-(a*d*(b*d^2 - 3*a*e^2)*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) + (e^3*x^5*(a + b*x^2)^(1 + p))/(b*(7 + 2*p)) + (d*(b*d^2 - 6*a*e^2)*(a + b*x^2)^(2 + p))/(2*b^3*(2 + p)) + (3*d*e^2*(a + b*x^2)^(3 + p))/(2*b^3*(3 + p)) + (e*(3*d^2 - (5*a*e^2)/(7*b + 2*b*p))*x^5*(a + b*x^2)^p*Hypergeometric2F1[5/2, -p, 7/2, -(b*x^2)/a])/(5*(1 + (b*x^2)/a)^p)$

**Rubi in Sympy [A]** time = 58.3354, size = 212, normalized size = 1.02

$$\begin{aligned} & \frac{3a^2de^2(a + bx^2)^{p+1}}{2b^3(p+1)} - \frac{ad^3(a + bx^2)^{p+1}}{2b^2(p+1)} - \frac{3ade^2(a + bx^2)^{p+2}}{b^3(p+2)} \\ & + \frac{3d^2ex^5\left(1 + \frac{bx^2}{a}\right)^{-p}(a + bx^2)^p {}_2F_1\left(\frac{-p, \frac{5}{2}}{\frac{7}{2}} \middle| -\frac{bx^2}{a}\right)}{5} \\ & + \frac{e^3x^7\left(1 + \frac{bx^2}{a}\right)^{-p}(a + bx^2)^p {}_2F_1\left(\frac{-p, \frac{7}{2}}{\frac{9}{2}} \middle| -\frac{bx^2}{a}\right)}{7} + \frac{d^3(a + bx^2)^{p+2}}{2b^2(p+2)} + \frac{3de^2(a + bx^2)^{p+3}}{2b^3(p+3)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(e\*x+d)\*\*3\*(b\*x\*\*2+a)\*\*p,x)

[Out]  $3*a**2*d*e**2*(a + b*x**2)**(p + 1)/(2*b**3*(p + 1)) - a*d**3*(a + b*x**2)**(p + 1)/(2*b**2*(p + 1)) - 3*a*d*e**2*(a + b*x**2)**(p + 2)/(b**3*(p + 2)) + 3*d**2*e*x**5*(1 + b*x**2/a)**(-p)*(a + b*x**2)**p*hyper((-p, 5/2), (7/2, ), -b*x**2/a)/5 + e**3*x**7*(1 + b*x**2/a)**(-p)*(a + b*x**2)**p*hyper((-p, 7/2), (9/2, ), -b*x**2/a)$

$$\frac{d^3(a + bx^2)^p}{7} + \frac{d^3(a + bx^2)^{p+2}}{(2b^2(p+2))} + \frac{3d^2e^2(a + bx^2)^{p+3}}{(2b^3(p+3))}$$

**Mathematica [A]** time = 0.487161, size = 289, normalized size = 1.4

$$\frac{(a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} \left(35d \left(6a^3 e^2 \left(\left(\frac{bx^2}{a} + 1\right)^p - 1\right) - a^2 b \left(d^2(p+3) \left(\left(\frac{bx^2}{a} + 1\right)^p - 1\right) + 6e^2 p x^2 \left(\frac{bx^2}{a} + 1\right)^p\right) + b^3(p+1)\right)}{1}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(d + e\*x)^3\*(a + b\*x^2)^p,x]

[Out] ((a + b\*x^2)^p\*(35\*d\*(a\*b^2\*p\*x^2\*(1 + (b\*x^2)/a)^p\*(d^2\*(3 + p) + 3\*e^2\*(1 + p)\*x^2) + b^3\*(1 + p)\*x^4\*(1 + (b\*x^2)/a)^p\*(d^2\*(3 + p) + 3\*e^2\*(2 + p)\*x^2) + 6\*a^3\*e^2\*(-1 + (1 + (b\*x^2)/a)^p) - a^2\*b\*(6\*e^2\*p\*x^2\*(1 + (b\*x^2)/a)^p + d^2\*(3 + p)\*(-1 + (1 + (b\*x^2)/a)^p))) + 42\*b^3\*d^2\*e\*(6 + 11\*p + 6\*p^2 + p^3)\*x^5\*Hypergeometric2F1[5/2, -p, 7/2, -(b\*x^2)/a] + 10\*b^3\*e^3\*(6 + 11\*p + 6\*p^2 + p^3)\*x^7\*Hypergeometric2F1[7/2, -p, 9/2, -(b\*x^2)/a]))/(70\*b^3\*(1 + p)\*(2 + p)\*(3 + p)\*(1 + (b\*x^2)/a)^p)

**Maple [F]** time = 0.095, size = 0, normalized size = 0.

$$\int x^3 (ex + d)^3 (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(e\*x+d)^3\*(b\*x^2+a)^p,x)

[Out] int(x^3\*(e\*x+d)^3\*(b\*x^2+a)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(b^2(p+1)x^4 + abpx^2 - a^2)(bx^2 + a)^p d^3}{2(p^2 + 3p + 2)b^2} + \int (e^3 x^6 + 3de^2 x^5 + 3d^2 ex^4)(bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(b\*x^2 + a)^p\*x^3,x, algorithm="maxima")

[Out] 1/2\*(b^2\*(p + 1)\*x^4 + a\*b\*p\*x^2 - a^2)\*(b\*x^2 + a)^p\*d^3/((p^2 + 3\*p + 2)\*b^2) + integrate((e^3\*x^6 + 3\*d^2\*e^2\*x^5 + 3\*d^2\*e\*x^4)\*(b\*x^2 + a)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3 x^6 + 3de^2 x^5 + 3d^2 ex^4 + d^3 x^3\right)(bx^2 + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(b\*x^2 + a)^p\*x^3,x, algorithm="fricas")

[Out] integral((e^3\*x^6 + 3\*d\*e^2\*x^5 + 3\*d^2\*e\*x^4 + d^3\*x^3)\*(b\*x^2 + a)^p, x)

---

**Sympy [A]** time = 162.661, size = 1421, normalized size = 6.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(e\*x+d)\*\*3\*(b\*x\*\*2+a)\*\*p,x)

[Out]  $3*a**p*d**2*e*x**5*hyper((5/2, -p), (7/2, ), b*x**2*exp\_polar(I*pi)/a)/5 + a**p*e**3*x**7*hyper((7/2, -p), (9/2, ), b*x**2*exp\_polar(I*pi)/a)/7 + d**3*Piecewise((a**p*x**4/4, Eq(b, 0)), (a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + a/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2) + b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**2 + 2*b**3*x**2), Eq(p, -2)), (-a*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) - a*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + a*b*p*x**2*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*p*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2) + b**2*x**4*(a + b*x**2)**p/(2*b**2*p**2 + 6*b**2*p + 4*b**2), True)) + 3*d*e**2*Piecewise((a**p*x**6/6, Eq(b, 0)), (2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(-I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*b**2*x**4*log(I*sqrt(a)*sqrt(1/b) + x)/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4), Eq(p, -3)), (-2*a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a**2/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) - 2*a*b*x**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*a*b**3 + 2*b**4*x**2) + b**2*x**4/(2*a*b**3 + 2*b**4*x**2), Eq(p, -2)), (a**2*log(-I*sqrt(a)*sqrt(1/b) + x)/(2*b**3) + a**2*log(I*sqrt(a)*sqrt(1/b) + x)/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (2*a**3*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) - 2*a**2*b*p*x**2*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p**2*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + a*b**2*p*x**4*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + b**3*p**2*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 3*b**3*p*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3) + 2*b**3*x**6*(a + b*x**2)**p/(2*b**3*p**3 + 12*b**3*p**2 + 22*b**3*p + 12*b**3), True))$

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3 (bx^2 + a)^p x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(b\*x^2 + a)^p\*x^3,x, algorithm="giac")

[Out] integrate((e\*x + d)^3\*(b\*x^2 + a)^p\*x^3, x)

### 3.402 $\int x^2(d + ex)^3 (a + bx^2)^p dx$

**Optimal.** Leaf size=210

$$\frac{ae(3bd^2 - ae^2)(a + bx^2)^{p+1}}{2b^3(p+1)} + \frac{e(3bd^2 - 2ae^2)(a + bx^2)^{p+2}}{2b^3(p+2)} + \frac{e^3(a + bx^2)^{p+3}}{2b^3(p+3)} - \frac{dx^3(a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (9ae^2 - bd^2(2p+5)) {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; -\frac{bx^2}{a}\right)}{3b(2p+5)} + \frac{3de^2x^3(a + bx^2)^{p+1}}{b(2p+5)}$$

[Out]  $-(a^*e*(3*b*d^2 - a^*e^2)*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) + (3*d^3*x^3*(a + b*x^2)^p*(b*x^2/a + 1)^(-p)*(9*a*e^2 - b*d^2*(2*p + 5)){}_2F_1(3/2, -p, 5/2, -(b*x^2/a)))/(3*b*(2*p + 5)) + (e^3*(a + b*x^2)^(p + 3))/(2*b^3*(p + 3)) + (e*(3*b*d^2 - 2*a*e^2)*(a + b*x^2)^(p + 2))/(2*b^3*(p + 2))$

**Rubi [A]** time = 0.420691, antiderivative size = 202, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{ae(3bd^2 - ae^2)(a + bx^2)^{p+1}}{2b^3(p+1)} + \frac{e(3bd^2 - 2ae^2)(a + bx^2)^{p+2}}{2b^3(p+2)} + \frac{e^3(a + bx^2)^{p+3}}{2b^3(p+3)} + \frac{1}{3}dx^3(a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} \left(d^2 - \frac{9ae^2}{2bp + 5b}\right) {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; -\frac{bx^2}{a}\right) + \frac{3de^2x^3(a + bx^2)^{p+1}}{b(2p+5)}$$

Antiderivative was successfully verified.

[In] Int[x^2\*(d + e\*x)^3\*(a + b\*x^2)^p, x]

[Out]  $-(a^*e*(3*b*d^2 - a^*e^2)*(a + b*x^2)^(1 + p))/(2*b^3*(1 + p)) + (3*d^3*x^3*(a + b*x^2)^p*(b*x^2/a + 1)^(-p)*(d^2 - (9*a*e^2)/(5*b + 2*b*p)){}_2F_1(3/2, -p, 5/2, -(b*x^2/a)))/(3*(1 + (b*x^2/a)^p)) + (e^3*(a + b*x^2)^(p + 3))/(2*b^3*(p + 3)) + (e*(3*b*d^2 - 2*a*e^2)*(a + b*x^2)^(p + 2))/(2*b^3*(p + 2))$

**Rubi in Sympy [A]** time = 55.4724, size = 209, normalized size = 1.

$$\frac{a^2e^3(a + bx^2)^{p+1}}{2b^3(p+1)} - \frac{3ad^2e(a + bx^2)^{p+1}}{2b^2(p+1)} - \frac{ae^3(a + bx^2)^{p+2}}{b^3(p+2)} + \frac{d^3x^3\left(1 + \frac{bx^2}{a}\right)^{-p}(a + bx^2)^p {}_2F_1\left(-p, \frac{3}{2}; \frac{5}{2}; -\frac{bx^2}{a}\right)}{3} + \frac{3de^2x^5\left(1 + \frac{bx^2}{a}\right)^{-p}(a + bx^2)^p {}_2F_1\left(-p, \frac{5}{2}; \frac{7}{2}; -\frac{bx^2}{a}\right)}{5} + \frac{3d^2e(a + bx^2)^{p+2}}{2b^2(p+2)} + \frac{e^3(a + bx^2)^{p+3}}{2b^3(p+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(e\*x+d)\*\*3\*(b\*x\*\*2+a)\*\*p, x)

[Out]  $a**2*e**3*(a + b*x**2)**(p + 1)/(2*b**3*(p + 1)) - 3*a*d**2*e*(a + b*x**2)**(p + 1)/(2*b**2*(p + 1)) - a*e**3*(a + b*x**2)**(p + 2)/(b**3*(p + 2)) + d**3*x**3*(1 + b*x**2/a)**(-p)*(a + b*x**2)**p*hyper((-p, 3/2), (5/2, ), -b*x**2/a)/3 + 3*d*e**2*x**5*(1 + b*x**2/a)**(-p)*(a + b*x**2)**p*hyper((-p, 5/2), (7/2, ), -b*x**2/a)/5$

$$+ 3d^{*2}e^{*}(a + b^{*}x^{*2})^{*}(p + 2)/(2^{*}b^{*2}(p + 2)) + e^{*3}(a + b^{*}x^{*2})^{*}(p + 3)/(2^{*}b^{*3}(p + 3))$$

**Mathematica [A]** time = 0.474125, size = 291, normalized size = 1.39

$$(a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} \left(3e \left(5 \left(2a^3 e^2 \left(\left(\frac{bx^2}{a} + 1\right)^p - 1\right) - a^2 b \left(3d^2(p+3) \left(\left(\frac{bx^2}{a} + 1\right)^p - 1\right) + 2e^2 p x^2 \left(\frac{bx^2}{a} + 1\right)^p\right) + b^3(p + 3)\right)\right)$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*(d + e\*x)^3\*(a + b\*x^2)^p,x]

[Out] ((a + b\*x^2)^p\*(10\*b^3\*d^3\*(6 + 11\*p + 6\*p^2 + p^3)\*x^3\*Hypergeometric2F1[3/2, -p, 5/2, -(b\*x^2)/a]) + 3\*e\*(5\*(a\*b^2\*p\*x^2\*(1 + (b\*x^2)/a)^p\*(3\*d^2\*(3 + p) + e^2\*(1 + p)\*x^2) + b^3\*(1 + p)\*x^4\*(1 + (b\*x^2)/a)^p\*(3\*d^2\*(3 + p) + e^2\*(2 + p)\*x^2) + 2\*a^3\*e^2\*(-1 + (1 + (b\*x^2)/a)^p) - a^2\*b\*(2\*e^2\*p\*x^2\*(1 + (b\*x^2)/a)^p + 3\*d^2\*(3 + p)\*(-1 + (1 + (b\*x^2)/a)^p))) + 6\*b^3\*d\*e\*(6 + 11\*p + 6\*p^2 + p^3)\*x^5\*Hypergeometric2F1[5/2, -p, 7/2, -(b\*x^2)/a]))/(30\*b^3\*(1 + p)\*(2 + p)\*(3 + p)\*(1 + (b\*x^2)/a)^p)

**Maple [F]** time = 0.082, size = 0, normalized size = 0.

$$\int x^2 (ex + d)^3 (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(e\*x+d)^3\*(b\*x^2+a)^p,x)

[Out] int(x^2\*(e\*x+d)^3\*(b\*x^2+a)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3 (bx^2 + a)^p x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(b\*x^2 + a)^p\*x^2,x, algorithm="maxima")

[Out] integrate((e\*x + d)^3\*(b\*x^2 + a)^p\*x^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^5 + 3de^2x^4 + 3d^2ex^3 + d^3x^2\right)(bx^2 + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(b\*x^2 + a)^p\*x^2,x, algorithm="fricas")



[Out] integral((e^3\*x^5 + 3\*d\*e^2\*x^4 + 3\*d^2\*e\*x^3 + d^3\*x^2)\*(b\*x^2 + a)^p, x)

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**Sympy [A]** time = 114.196, size = 1421, normalized size = 6.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(e\*x+d)\*\*3\*(b\*x\*\*2+a)\*\*p,x)

[Out] a\*\*p\*d\*\*3\*x\*\*3\*hyper((3/2, -p), (5/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/3 + 3\*a\*\*p\*d\*\*2\*e\*\*2\*x\*\*5\*hyper((5/2, -p), (7/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/5 + 3\*d\*\*2\*e\*Piecewise((a\*\*p\*x\*\*4/4, Eq(b, 0)), (a\*log(-I\*sqrt(a)\*sqrt(1/b) + x)/(2\*a\*b\*\*2 + 2\*b\*\*3\*x\*\*2) + a\*log(I\*sqrt(a)\*sqrt(1/b) + x)/(2\*a\*b\*\*2 + 2\*b\*\*3\*x\*\*2) + a/(2\*a\*b\*\*2 + 2\*b\*\*3\*x\*\*2) + b\*x\*\*2\*log(-I\*sqrt(a)\*sqrt(1/b) + x)/(2\*a\*b\*\*2 + 2\*b\*\*3\*x\*\*2) + b\*x\*\*2\*log(I\*sqrt(a)\*sqrt(1/b) + x)/(2\*a\*b\*\*2 + 2\*b\*\*3\*x\*\*2), Eq(p, -2)), (-a\*log(-I\*sqrt(a)\*sqrt(1/b) + x)/(2\*b\*\*2) - a\*log(I\*sqrt(a)\*sqrt(1/b) + x)/(2\*b\*\*2) + x\*\*2/(2\*b), Eq(p, -1)), (-a\*\*2\*(a + b\*x\*\*2)\*\*p/(2\*b\*\*2\*p\*\*2 + 6\*b\*\*2\*p + 4\*b\*\*2) + a\*b\*p\*x\*\*2\*(a + b\*x\*\*2)\*\*p/(2\*b\*\*2\*p\*\*2 + 6\*b\*\*2\*p + 4\*b\*\*2) + b\*\*2\*p\*x\*\*4\*(a + b\*x\*\*2)\*\*p/(2\*b\*\*2\*p\*\*2 + 6\*b\*\*2\*p + 4\*b\*\*2) + b\*\*2\*x\*\*4\*(a + b\*x\*\*2)\*\*p/(2\*b\*\*2\*p\*\*2 + 6\*b\*\*2\*p + 4\*b\*\*2), True)) + e\*\*3\*Piecewise((a\*\*p\*x\*\*6/6, Eq(b, 0)), (2\*a\*\*2\*log(-I\*sqrt(a)\*sqrt(1/b) + x)/(4\*a\*\*2\*b\*\*3 + 8\*a\*b\*\*4\*x\*\*2 + 4\*b\*\*5\*x\*\*4) + 2\*a\*\*2\*log(I\*sqrt(a)\*sqrt(1/b) + x)/(4\*a\*\*2\*b\*\*3 + 8\*a\*b\*\*4\*x\*\*2 + 4\*b\*\*5\*x\*\*4) + 3\*a\*\*2/(4\*a\*\*2\*b\*\*3 + 8\*a\*b\*\*4\*x\*\*2 + 4\*b\*\*5\*x\*\*4) + 4\*a\*b\*x\*\*2\*log(-I\*sqrt(a)\*sqrt(1/b) + x)/(4\*a\*\*2\*b\*\*3 + 8\*a\*b\*\*4\*x\*\*2 + 4\*b\*\*5\*x\*\*4) + 4\*a\*b\*x\*\*2\*log(I\*sqrt(a)\*sqrt(1/b) + x)/(4\*a\*\*2\*b\*\*3 + 8\*a\*b\*\*4\*x\*\*2 + 4\*b\*\*5\*x\*\*4) + 4\*a\*b\*x\*\*2/(4\*a\*\*2\*b\*\*3 + 8\*a\*b\*\*4\*x\*\*2 + 4\*b\*\*5\*x\*\*4) + 2\*b\*\*2\*x\*\*4\*log(-I\*sqrt(a)\*sqrt(1/b) + x)/(4\*a\*\*2\*b\*\*3 + 8\*a\*b\*\*4\*x\*\*2 + 4\*b\*\*5\*x\*\*4) + 2\*b\*\*2\*x\*\*4\*log(I\*sqrt(a)\*sqrt(1/b) + x)/(4\*a\*\*2\*b\*\*3 + 8\*a\*b\*\*4\*x\*\*2 + 4\*b\*\*5\*x\*\*4), Eq(p, -3)), (-2\*a\*\*2\*log(-I\*sqrt(a)\*sqrt(1/b) + x)/(2\*a\*b\*\*3 + 2\*b\*\*4\*x\*\*2) - 2\*a\*\*2\*log(I\*sqrt(a)\*sqrt(1/b) + x)/(2\*a\*b\*\*3 + 2\*b\*\*4\*x\*\*2) - 2\*a\*\*2/(2\*a\*b\*\*3 + 2\*b\*\*4\*x\*\*2) - 2\*a\*b\*x\*\*2\*log(-I\*sqrt(a)\*sqrt(1/b) + x)/(2\*a\*b\*\*3 + 2\*b\*\*4\*x\*\*2) - 2\*a\*b\*x\*\*2\*log(I\*sqrt(a)\*sqrt(1/b) + x)/(2\*a\*b\*\*3 + 2\*b\*\*4\*x\*\*2) + b\*\*2\*x\*\*4/(2\*a\*b\*\*3 + 2\*b\*\*4\*x\*\*2), Eq(p, -2)), (a\*\*2\*log(-I\*sqrt(a)\*sqrt(1/b) + x)/(2\*b\*\*3) + a\*\*2\*log(I\*sqrt(a)\*sqrt(1/b) + x)/(2\*b\*\*3) - a\*x\*\*2/(2\*b\*\*2) + x\*\*4/(4\*b), Eq(p, -1)), (2\*a\*\*3\*(a + b\*x\*\*2)\*\*p/(2\*b\*\*3\*p\*\*3 + 12\*b\*\*3\*p\*\*2 + 22\*b\*\*3\*p + 12\*b\*\*3) - 2\*a\*\*2\*b\*p\*x\*\*2\*(a + b\*x\*\*2)\*\*p/(2\*b\*\*3\*p\*\*3 + 12\*b\*\*3\*p\*\*2 + 22\*b\*\*3\*p + 12\*b\*\*3) + a\*b\*\*2\*p\*\*2\*x\*\*4\*(a + b\*x\*\*2)\*\*p/(2\*b\*\*3\*p\*\*3 + 12\*b\*\*3\*p\*\*2 + 22\*b\*\*3\*p + 12\*b\*\*3) + a\*b\*\*2\*p\*x\*\*4\*(a + b\*x\*\*2)\*\*p/(2\*b\*\*3\*p\*\*3 + 12\*b\*\*3\*p\*\*2 + 22\*b\*\*3\*p + 12\*b\*\*3) + b\*\*3\*p\*\*2\*x\*\*6\*(a + b\*x\*\*2)\*\*p/(2\*b\*\*3\*p\*\*3 + 12\*b\*\*3\*p\*\*2 + 22\*b\*\*3\*p + 12\*b\*\*3) + 3\*b\*\*3\*p\*x\*\*6\*(a + b\*x\*\*2)\*\*p/(2\*b\*\*3\*p\*\*3 + 12\*b\*\*3\*p\*\*2 + 22\*b\*\*3\*p + 12\*b\*\*3) + 2\*b\*\*3\*x\*\*6\*(a + b\*x\*\*2)\*\*p/(2\*b\*\*3\*p\*\*3 + 12\*b\*\*3\*p\*\*2 + 22\*b\*\*3\*p + 12\*b\*\*3), True))

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3 (bx^2 + a)^p x^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(b\*x^2 + a)^p\*x^2,x, algorithm="giac")

[Out] integrate((e\*x + d)^3\*(b\*x^2 + a)^p\*x^2, x)

### 3.403 $\int x(d + ex)^3 (a + bx^2)^p dx$

**Optimal.** Leaf size=167

$$\frac{d(bd^2 - 3ae^2)(a + bx^2)^{p+1}}{2b^2(p+1)} + \frac{3de^2(a + bx^2)^{p+2}}{2b^2(p+2)} - \frac{ex^3(a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (ae^2 - bd^2(2p+5)) {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; -\frac{bx^2}{a}\right)}{b(2p+5)} + \frac{e^3x^3(a + bx^2)^{p+1}}{b(2p+5)}$$

[Out]  $(d*(b*d^2 - 3*a*e^2)*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (e^3*x^3*(a + b*x^2)^(1 + p))/(b*(5 + 2*p)) + (3*d*e^2*(a + b*x^2)^(2 + p))/(2*b^2*(2 + p)) - (e*(a*e^2 - b*d^2*(5 + 2*p))*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -(b*x^2)/a])/(b*(5 + 2*p)*(1 + (b*x^2)/a)^p)$

**Rubi [A]** time = 0.325148, antiderivative size = 159, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$

$$\frac{d(bd^2 - 3ae^2)(a + bx^2)^{p+1}}{2b^2(p+1)} + \frac{3de^2(a + bx^2)^{p+2}}{2b^2(p+2)} + ex^3(a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} \left(d^2 - \frac{ae^2}{2bp + 5b}\right) {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; -\frac{bx^2}{a}\right) + \frac{e^3x^3(a + bx^2)^{p+1}}{b(2p+5)}$$

Antiderivative was successfully verified.

[In] Int[x\*(d + e\*x)^3\*(a + b\*x^2)^p, x]

[Out]  $(d*(b*d^2 - 3*a*e^2)*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)) + (e^3*x^3*(a + b*x^2)^(1 + p))/(b*(5 + 2*p)) + (3*d*e^2*(a + b*x^2)^(2 + p))/(2*b^2*(2 + p)) + (e*(d^2 - (a*e^2)/(5*b + 2*b*p))*x^3*(a + b*x^2)^p*Hypergeometric2F1[3/2, -p, 5/2, -(b*x^2)/a])/(1 + (b*x^2)/a)^p$

**Rubi in Sympy [A]** time = 64.9472, size = 212, normalized size = 1.27

$$\frac{3aex \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p (ae^2 - 2bd^2p - 5bd^2) {}_2F_1\left(\frac{-p}{\frac{3}{2}}, \frac{1}{2}; -\frac{bx^2}{a}\right)}{b^2(2p+3)(2p+5)} + \frac{3d(a + bx^2)^{p+1}(d + ex)^2}{2b(p+2)(2p+5)} + \frac{(a + bx^2)^{p+1}(d + ex)^3}{b(2p+5)} - \frac{(a + bx^2)^{p+1}(6d(2p+3)(2ae^2p + 5ae^2 - bd^2) - 12ex(p+1)(-ae^2(p+2) + bd^2))}{4b^2(p+1)(p+2)(2p+3)(2p+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(e\*x+d)\*\*3\*(b\*x\*\*2+a)\*\*p, x)

[Out]  $3*a*e*x*(1 + b*x**2/a)**(-p)*(a + b*x**2)**p*(a*e**2 - 2*b*d**2*p - 5*b*d**2)*hyper((-p, 1/2), (3/2, ), -b*x**2/a)/(b**2*(2*p + 3)*(2*p + 5)) + 3*d*(a + b*x**2)**(p + 1)*(d + e*x)**2/(2*b*(p + 2)*(2*p + 5)) + (a + b*x**2)**(p + 1)*(d + e*x)**3/(b*(2*p + 5)) - (a + b*x**2)**(p + 1)*(6*d*(2*p + 3)*(2*a*e**2*p + 5*a*e**2 - b*d**2) - 12*e*x*(p + 1)*(-a*e**2*(p + 2) + b*d**2))/(4*b**2*(p + 1)*(p + 2)*(2*p + 3)*(2*p + 5))$

---

**Mathematica [A]** time = 0.367588, size = 228, normalized size = 1.37

$$\frac{(a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} \left(5d \left(-3a^2 e^2 \left(\left(\frac{bx^2}{a} + 1\right)^p - 1\right) + b^2 x^2 \left(\frac{bx^2}{a} + 1\right)^p (d^2(p+2) + 3e^2(p+1)x^2) + ab \left(d^2(p+2) \left(\left(\frac{bx^2}{a} + 1\right)^p - 1\right) + 3e^2(p+1)x^2\right)\right)}{10b^2(p+1)}$$

Antiderivative was successfully verified.

[In] Integrate[x\*(d + e\*x)^3\*(a + b\*x^2)^p,x]

[Out] ((a + b\*x^2)^p\*(5\*d\*(b^2\*x^2\*(1 + (b\*x^2)/a)^p\*(d^2\*(2 + p) + 3\*e^2\*(1 + p)\*x^2) - 3\*a^2\*e^2\*(-1 + (1 + (b\*x^2)/a)^p) + a\*b\*(3\*e^2\*p\*x^2\*(1 + (b\*x^2)/a)^p + d^2\*(2 + p)\*(-1 + (1 + (b\*x^2)/a)^p))) + 10\*b^2\*d^2\*e\*(2 + 3\*p + p^2)\*x^3\*Hypergeometric2F1[3/2, -p, 5/2, -(b\*x^2)/a] + 2\*b^2\*e^3\*(2 + 3\*p + p^2)\*x^5\*Hypergeometric2F1[5/2, -p, 7/2, -(b\*x^2)/a]))/(10\*b^2\*(1 + p)\*(2 + p)\*(1 + (b\*x^2)/a)^p)

---

**Maple [F]** time = 0.074, size = 0, normalized size = 0.

$$\int x(ex + d)^3 (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(e\*x+d)^3\*(b\*x^2+a)^p,x)

[Out] int(x\*(e\*x+d)^3\*(b\*x^2+a)^p,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(b\*x^2 + a)^p\*x,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3 x^4 + 3 d e^2 x^3 + 3 d^2 e x^2 + d^3 x\right) (b x^2 + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(b\*x^2 + a)^p\*x,x, algorithm="fricas")

[Out] integral((e^3\*x^4 + 3\*d\*e^2\*x^3 + 3\*d^2\*e\*x^2 + d^3\*x)\*(b\*x^2 + a)^p, x)

---

**Sympy [A]** time = 85.6439, size = 471, normalized size = 2.82

$$\begin{aligned}
 & a^p d^2 e x^3 {}_2F_1\left(\frac{3}{2}, -p \middle| \frac{b x^2 e^{i\pi}}{a}\right) + \frac{a^p e^3 x^5 {}_2F_1\left(\frac{5}{2}, -p \middle| \frac{b x^2 e^{i\pi}}{a}\right)}{5} + d^3 \left( \begin{array}{l} \left( \frac{a^p x^2}{2} \right. \\ \left. \frac{(a+bx^2)^{p+1}}{p+1} \right. \\ \left. \frac{\log(a+bx^2)}{2b} \right) \\ \text{for } b = 0 \\ \text{for } p \neq -1 \\ \text{otherwise} \end{array} \right) \\
 & + 3de^2 \left( \begin{array}{l} \left( \frac{a^p x^4}{4} \right. \\ \frac{a \log(-i\sqrt{a}\sqrt{1/b}+x)}{2ab^2+2b^3x^2} + \frac{a \log(i\sqrt{a}\sqrt{1/b}+x)}{2ab^2+2b^3x^2} + \frac{a}{2ab^2+2b^3x^2} + \frac{bx^2 \log(-i\sqrt{a}\sqrt{1/b}+x)}{2ab^2+2b^3x^2} + \frac{bx^2 \log(i\sqrt{a}\sqrt{1/b}+x)}{2ab^2+2b^3x^2} \\ \left. \frac{a \log(-i\sqrt{a}\sqrt{1/b}+x)}{2b^2} - \frac{a \log(i\sqrt{a}\sqrt{1/b}+x)}{2b^2} + \frac{x^2}{2b} \right) \\ \left. - \frac{a^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{abpx^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2px^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2x^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} \right) \\ \text{for } b = 0 \\ \text{for } p = -2 \\ \text{for } p = -1 \\ \text{otherwise} \end{array} \right)
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x+d)\*\*3\*(b\*x\*\*2+a)\*\*p,x)

[Out] a\*\*p\*d\*\*2\*e\*x\*\*3\*hyper((3/2, -p), (5/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a) + a\*\*p\*e\*\*3\*x\*\*5\*hyper((5/2, -p), (7/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/5 + d\*\*3\*Piecewise((a\*\*p\*x\*\*2/2, Eq(b, 0)), (Piecewise(((a + b\*x\*\*2)\*\*(p + 1)/(p + 1), Ne(p, -1)), (log(a + b\*x\*\*2), True))/(2\*b), True)) + 3\*d\*e\*\*2\*Piecewise((a\*\*p\*x\*\*4/4, Eq(b, 0)), (a\*log(-I\*sqrt(a)\*sqrt(1/b) + x)/(2\*a\*b\*\*2 + 2\*b\*\*3\*x\*\*2) + a\*log(I\*sqrt(a)\*sqrt(1/b) + x)/(2\*a\*b\*\*2 + 2\*b\*\*3\*x\*\*2) + a/(2\*a\*b\*\*2 + 2\*b\*\*3\*x\*\*2) + b\*x\*\*2\*log(-I\*sqrt(a)\*sqrt(1/b) + x)/(2\*a\*b\*\*2 + 2\*b\*\*3\*x\*\*2) + b\*x\*\*2\*log(I\*sqrt(a)\*sqrt(1/b) + x)/(2\*a\*b\*\*2 + 2\*b\*\*3\*x\*\*2), Eq(p, -2)), (-a\*log(-I\*sqrt(a)\*sqrt(1/b) + x)/(2\*b\*\*2) - a\*log(I\*sqrt(a)\*sqrt(1/b) + x)/(2\*b\*\*2) + x\*\*2/(2\*b), Eq(p, -1)), (-a\*\*2\*(a + b\*x\*\*2)\*\*p/(2\*b\*\*2\*p\*\*2 + 6\*b\*\*2\*p + 4\*b\*\*2) + a\*b\*p\*x\*\*2\*(a + b\*x\*\*2)\*\*p/(2\*b\*\*2\*p\*\*2 + 6\*b\*\*2\*p + 4\*b\*\*2) + b\*\*2\*p\*x\*\*4\*(a + b\*x\*\*2)\*\*p/(2\*b\*\*2\*p\*\*2 + 6\*b\*\*2\*p + 4\*b\*\*2) + b\*\*2\*x\*\*4\*(a + b\*x\*\*2)\*\*p/(2\*b\*\*2\*p\*\*2 + 6\*b\*\*2\*p + 4\*b\*\*2), True))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3 (bx^2 + a)^p x dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(b\*x^2 + a)^p\*x,x, algorithm="giac")

[Out] integrate((e\*x + d)^3\*(b\*x^2 + a)^p\*x, x)

### 3.404 $\int (d + ex)^3 (a + bx^2)^p dx$

**Optimal.** Leaf size=176

$$\frac{e (a + bx^2)^{p+1} (ae^2 - 3bd^2(p + 2))}{2b^2(p + 1)(p + 2)} dx (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (3ae^2 - bd^2(2p + 3)) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right) \\ + \frac{3de^2x (a + bx^2)^{p+1}}{b(2p + 3)} + \frac{e^3x^2 (a + bx^2)^{p+1}}{2b(p + 2)}$$

[Out]  $-(e^*(a^*e^2 - 3*b*d^2*(2 + p))*(a + b*x^2)^(1 + p))/(2*b^2*(1 + p)*(2 + p)) + (3*d*e^2*x*(a + b*x^2)^(1 + p))/(b*(3 + 2*p)) + (e^3*x^2*(a + b*x^2)^(1 + p))/(2*b*(2 + p)) - (d*(3*a*e^2 - b*d^2*(3 + 2*p))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -(b*x^2)/a])/ (b*(3 + 2*p)*(1 + (b*x^2)/a)^p)$

**Rubi [A]** time = 0.331236, antiderivative size = 169, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$

$$\frac{e (a + bx^2)^{p+1} ((2p + 3)(ae^2 - bd^2(2p + 5)) - 2bde(p + 1)(p + 3)x)}{2b^2(p + 2)(2p^2 + 5p + 3)} + dx (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} \left(d^2 - \frac{3ae^2}{2bp + 3b}\right) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right) + \frac{e(d + ex)^2 (a + bx^2)^{p+1}}{2b(p + 2)}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^3\*(a + b\*x^2)^p,x]

[Out]  $(e*(d + e*x)^2*(a + b*x^2)^(1 + p))/(2*b*(2 + p)) - (e*((3 + 2*p)*(a^*e^2 - b*d^2*(5 + 2*p)) - 2*b*d*e*(1 + p)*(3 + p)*x*(a + b*x^2)^(1 + p))/(2*b^2*(2 + p)*(3 + 5*p + 2*p^2)) + (d*(d^2 - (3*a*e^2)/(3*b + 2*b*p))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -(b*x^2)/a])/ (1 + (b*x^2)/a)^p)$

**Rubi in Sympy [A]** time = 45.1893, size = 160, normalized size = 0.91

$$\frac{dx \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p (3ae^2 - 2bd^2p - 3bd^2) {}_2F_1\left(\frac{-p, \frac{1}{2}}{\frac{3}{2}} \middle| -\frac{bx^2}{a}\right)}{b(2p + 3)} + \frac{e (a + bx^2)^{p+1} (d + ex)^2}{2b(p + 2)} \\ - \frac{e (a + bx^2)^{p+1} (-4bdex(p + 1)(p + 3) + (4p + 6)(ae^2 - 2bd^2p - 5bd^2))}{4b^2(p + 1)(p + 2)(2p + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3\*(b\*x\*\*2+a)\*\*p,x)

[Out]  $-d*x*(1 + b*x**2/a)**(-p)*(a + b*x**2)**p*(3*a*e**2 - 2*b*d**2*p - 3*b*d**2)*hyper((-p, 1/2), (3/2, ), -b*x**2/a)/(b*(2*p + 3)) + e*(a + b*x**2)**(p + 1)*(d + e*x)**2/(2*b*(p + 2)) - e*(a + b*x**2)**(p + 1)*(-4*b*d*e*x*(p + 1)*(p + 3) + (4*p + 6)*(a*e**2 - 2*b*d**2*p - 5*b*d**2))/(4*b**2*(p + 1)*(p + 2)*(2*p + 3))$

**Mathematica [A]** time = 0.353457, size = 223, normalized size = 1.27

$$\frac{(a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} \left(e \left(-a^2 e^2 \left(\left(\frac{bx^2}{a} + 1\right)^p - 1\right) + b^2 x^2 \left(\frac{bx^2}{a} + 1\right)^p (3d^2(p+2) + e^2(p+1)x^2) + 2b^2 d e (p^2 + 3p + 2) x^3 + 2b^2(p+1)x^2\right)}{2b^2(p+1)x^2}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^3\*(a + b\*x^2)^p,x]

[Out] ((a + b\*x^2)^p\*(2\*b^2\*d^3\*(2 + 3\*p + p^2)\*x\*Hypergeometric2F1[1/2, -p, 3/2, -(b\*x^2)/a]) + e\*(b^2\*x^2\*(1 + (b\*x^2)/a)^p\*(3\*d^2\*(2 + p) + e^2\*(1 + p)\*x^2) - a^2\*e^2\*(-1 + (1 + (b\*x^2)/a)^p) + a\*b\*(e^2\*p\*x^2\*(1 + (b\*x^2)/a)^p + 3\*d^2\*(2 + p)\*(-1 + (1 + (b\*x^2)/a)^p)) + 2\*b^2\*d\*e\*(2 + 3\*p + p^2)\*x^3\*Hypergeometric2F1[3/2, -p, 5/2, -(b\*x^2)/a]))/(2\*b^2\*(1 + p)\*(2 + p)\*(1 + (b\*x^2)/a)^p)

**Maple [F]** time = 0.064, size = 0, normalized size = 0.

$$\int (ex + d)^3 (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(b\*x^2+a)^p,x)

[Out] int((e\*x+d)^3\*(b\*x^2+a)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3 (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(b\*x^2 + a)^p,x, algorithm="maxima")

[Out] integrate((e\*x + d)^3\*(b\*x^2 + a)^p, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3 x^3 + 3 d e^2 x^2 + 3 d^2 e x + d^3\right) (b x^2 + a)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(b\*x^2 + a)^p,x, algorithm="fricas")

[Out] integral((e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3)\*(b\*x^2 + a)^p, x)

**Sympy [A]** time = 55.6537, size = 468, normalized size = 2.66

$$\begin{aligned}
 & a^p d^3 x {}_2F_1\left(\frac{1}{2}, -p \middle| \frac{bx^2 e^{i\pi}}{a}\right) + a^p d e^2 x^3 {}_2F_1\left(\frac{3}{2}, -p \middle| \frac{bx^2 e^{i\pi}}{a}\right) \\
 & + 3d^2 e \left( \begin{array}{l} \frac{\frac{a^p x^2}{2}}{\quad} \quad \text{for } b = 0 \\ \frac{(a+bx^2)^{p+1}}{p+1} \quad \text{for } p \neq -1 \\ \frac{\log(a+bx^2)}{2b} \quad \text{otherwise} \end{array} \right) \\
 & + e^3 \left( \begin{array}{l} \frac{\frac{a^p x^4}{4}}{\quad} \quad \text{for } b = 0 \\ \frac{a \log(-i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2ab^2+2b^3x^2} + \frac{a \log(i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2ab^2+2b^3x^2} + \frac{a}{2ab^2+2b^3x^2} + \frac{bx^2 \log(-i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2ab^2+2b^3x^2} + \frac{bx^2 \log(i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2ab^2+2b^3x^2} \quad \text{for } p = -2 \\ -\frac{a \log(-i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2b^2} - \frac{a \log(i\sqrt{a}\sqrt{\frac{1}{b}+x})}{2b^2} + \frac{x^2}{2b} \quad \text{for } p = -1 \\ -\frac{a^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{abpx^2(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2px^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} + \frac{b^2x^4(a+bx^2)^p}{2b^2p^2+6b^2p+4b^2} \quad \text{otherwise} \end{array} \right)
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(b\*x\*\*2+a)\*\*p,x)

[Out] a\*\*p\*d\*\*3\*x\*hyper((1/2, -p), (3/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a) + a\*\*p\*d\*e\*\*2\*x\*\*3\*hyper((3/2, -p), (5/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a) + 3\*d\*\*2\*e\*Piecewise((a\*\*p\*x\*\*2/2, Eq(b, 0)), (Piecewise(((a + b\*x\*\*2)\*\*(p + 1)/(p + 1), Ne(p, -1)), (log(a + b\*x\*\*2), True))/(2\*b), True)) + e\*\*3\*Piecewise((a\*\*p\*x\*\*4/4, Eq(b, 0)), (a\*log(-I\*sqrt(a)\*sqrt(1/b) + x)/(2\*a\*b\*\*2 + 2\*b\*\*3\*x\*\*2) + a\*log(I\*sqrt(a)\*sqrt(1/b) + x)/(2\*a\*b\*\*2 + 2\*b\*\*3\*x\*\*2) + a/(2\*a\*b\*\*2 + 2\*b\*\*3\*x\*\*2) + b\*x\*\*2\*log(-I\*sqrt(a)\*sqrt(1/b) + x)/(2\*a\*b\*\*2 + 2\*b\*\*3\*x\*\*2) + b\*x\*\*2\*log(I\*sqrt(a)\*sqrt(1/b) + x)/(2\*a\*b\*\*2 + 2\*b\*\*3\*x\*\*2), Eq(p, -2)), (-a\*log(-I\*sqrt(a)\*sqrt(1/b) + x)/(2\*b\*\*2) - a\*log(I\*sqrt(a)\*sqrt(1/b) + x)/(2\*b\*\*2) + x\*\*2/(2\*b), Eq(p, -1)), (-a\*\*2\*(a + b\*x\*\*2)\*\*p/(2\*b\*\*2\*p\*\*2 + 6\*b\*\*2\*p + 4\*b\*\*2) + a\*b\*p\*x\*\*2\*(a + b\*x\*\*2)\*\*p/(2\*b\*\*2\*p\*\*2 + 6\*b\*\*2\*p + 4\*b\*\*2) + b\*\*2\*p\*x\*\*4\*(a + b\*x\*\*2)\*\*p/(2\*b\*\*2\*p\*\*2 + 6\*b\*\*2\*p + 4\*b\*\*2) + b\*\*2\*x\*\*4\*(a + b\*x\*\*2)\*\*p/(2\*b\*\*2\*p\*\*2 + 6\*b\*\*2\*p + 4\*b\*\*2), True))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3 (bx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(b\*x^2 + a)^p,x, algorithm="giac")

[Out] integrate((e\*x + d)^3\*(b\*x^2 + a)^p, x)

$$3.405 \quad \int \frac{(d+ex)^3(a+bx^2)^p}{x} dx$$

**Optimal.** Leaf size=171

$$\frac{d^3 (a + bx^2)^{p+1} {}_2F_1\left(1, p + 1; p + 2; \frac{bx^2}{a} + 1\right)}{2a(p + 1)} - \frac{ex (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (ae^2 - 3bd^2(2p + 3)) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{b(2p + 3)} + \frac{3de^2 (a + bx^2)^{p+1}}{2b(p + 1)} + \frac{e^3 x (a + bx^2)^{p+1}}{b(2p + 3)}$$

[Out]  $(3*d*e^2*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) + (e^3*x*(a + b*x^2)^(1 + p))/(b*(3 + 2*p)) - (e*(a*e^2 - 3*b*d^2*(3 + 2*p))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -(b*x^2)/a])/ (b*(3 + 2*p)) * (1 + (b*x^2)/a)^p - (d^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/ (2*a*(1 + p))$

**Rubi [A]** time = 0.271101, antiderivative size = 165, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$\frac{d^3 (a + bx^2)^{p+1} {}_2F_1\left(1, p + 1; p + 2; \frac{bx^2}{a} + 1\right)}{2a(p + 1)} + ex (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} \left(3d^2 - \frac{ae^2}{2bp + 3b}\right) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right) + \frac{3de^2 (a + bx^2)^{p+1}}{2b(p + 1)} + \frac{e^3 x (a + bx^2)^{p+1}}{b(2p + 3)}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(a + b\*x^2)^p)/x, x]

[Out]  $(3*d*e^2*(a + b*x^2)^(1 + p))/(2*b*(1 + p)) + (e^3*x*(a + b*x^2)^(1 + p))/(b*(3 + 2*p)) + (e*(3*d^2 - (a*e^2)/(3*b + 2*b*p))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -(b*x^2)/a])/ (1 + (b*x^2)/a)^p - (d^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/ (2*a*(1 + p))$

**Rubi in Sympy [A]** time = 29.7222, size = 141, normalized size = 0.82

$$3d^2 ex \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(\frac{-p}{\frac{3}{2}}, \frac{1}{2} \middle| -\frac{bx^2}{a}\right) + \frac{e^3 x^3 \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(\frac{-p}{\frac{5}{2}}, \frac{3}{2} \middle| -\frac{bx^2}{a}\right)}{3} + \frac{3de^2 (a + bx^2)^{p+1}}{2b(p + 1)} - \frac{d^3 (a + bx^2)^{p+1} {}_2F_1\left(1, p + 1 \middle| 1 + \frac{bx^2}{a}\right)}{2a(p + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3\*(b\*x\*\*2+a)\*\*p/x, x)

[Out]  $3*d**2*e*x*(1 + b*x**2/a)**(-p)*(a + b*x**2)**p*hyper((-p, 1/2), (3/2, ), -b*x**2/a) + e**3*x**3*(1 + b*x**2/a)**(-p)*(a + b*x**2)**$



\*p\*hyper((-p, 3/2), (5/2, ), -b\*x\*\*2/a)/3 + 3\*d\*e\*\*2\*(a + b\*x\*\*2)\*  
 \*(p + 1)/(2\*b\*(p + 1)) - d\*\*3\*(a + b\*x\*\*2)\*\*(p + 1)\*hyper((1, p +  
 1), (p + 2, ), 1 + b\*x\*\*2/a)/(2\*a\*(p + 1))

**Mathematica [A]** time = 0.838119, size = 175, normalized size = 1.02

$$\frac{1}{6}(a+bx^2)^p \left( \frac{3d^3 \left(\frac{a}{bx^2} + 1\right)^{-p} {}_2F_1\left(-p, -p; 1-p; -\frac{a}{bx^2}\right)}{p} + 18d^2 ex \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right) \right. \\ \left. + \frac{9de^2 \left(-a \left(\frac{bx^2}{a} + 1\right)^{-p} + a + bx^2\right)}{bp + b} + 2e^3 x^3 \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; -\frac{bx^2}{a}\right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(a + b\*x^2)^p)/x,x]

[Out] ((a + b\*x^2)^p\*((9\*d\*e^2\*(a + b\*x^2 - a/(1 + (b\*x^2)/a)^p))/(b + b\*p) + (18\*d^2\*e\*x\*Hypergeometric2F1[1/2, -p, 3/2, -(b\*x^2)/a])/(1 + (b\*x^2)/a)^p + (2\*e^3\*x^3\*Hypergeometric2F1[3/2, -p, 5/2, -(b\*x^2)/a])/(1 + (b\*x^2)/a)^p + (3\*d^3\*Hypergeometric2F1[-p, -p, 1 - p, -(a/(b\*x^2))])/(p\*(1 + a/(b\*x^2))^p))/6

**Maple [F]** time = 0.055, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^3 (bx^2 + a)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(b\*x^2+a)^p/x,x)

[Out] int((e\*x+d)^3\*(b\*x^2+a)^p/x,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^3 (bx^2 + a)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(b\*x^2 + a)^p/x,x, algorithm="maxima")

[Out] integrate((e\*x + d)^3\*(b\*x^2 + a)^p/x, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)(bx^2 + a)^p}{x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(b\*x^2 + a)^p/x,x, algorithm="fricas")

[Out] integral((e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3)\*(b\*x^2 + a)^p/x, x)

**Sympy [A]** time = 63.7909, size = 144, normalized size = 0.84

$$3a^p d^2 e x {}_2F_1\left(\frac{1}{2}, -p \middle| \frac{bx^2 e^{i\pi}}{a}\right) + \frac{a^p e^3 x^3 {}_2F_1\left(\frac{3}{2}, -p \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{3}$$

$$- \frac{b^p d^3 x^{2p} (-p) {}_2F_1\left(-p, -p \middle| \frac{ae^{i\pi}}{bx^2}\right)}{2(-p+1)} + 3de^2 \left( \begin{array}{ll} \left( \frac{a^p x^2}{2} \right. & \text{for } b = 0 \\ \left. \frac{(a+bx^2)^{p+1}}{p+1} \right. & \text{for } p \neq -1 \\ \left. \frac{\log(a+bx^2)}{2b} \right. & \text{otherwise} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(b\*x\*\*2+a)\*\*p/x,x)

[Out] 3\*a\*\*p\*d\*\*2\*e\*x\*hyper((1/2, -p), (3/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a) + a\*\*p\*e\*\*3\*x\*\*3\*hyper((3/2, -p), (5/2, ), b\*x\*\*2\*exp\_polar(I\*pi)/a)/3 - b\*\*p\*d\*\*3\*x\*\*(2\*p)\*gamma(-p)\*hyper((-p, -p), (-p + 1, ), a\*exp\_polar(I\*pi)/(b\*x\*\*2))/(2\*gamma(-p + 1)) + 3\*d\*e\*\*2\*Piecewise(e((a\*\*p\*x\*\*2/2, Eq(b, 0)), (Piecewise(((a + b\*x\*\*2)\*\*(p + 1)/(p + 1), Ne(p, -1)), (log(a + b\*x\*\*2), True)))/(2\*b), True))

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^3 (bx^2 + a)^p}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(b\*x^2 + a)^p/x,x, algorithm="giac")

[Out] integrate((e\*x + d)^3\*(b\*x^2 + a)^p/x, x)

$$3.406 \quad \int \frac{(d+ex)^3(a+bx^2)^p}{x^2} dx$$

**Optimal.** Leaf size=159

$$\frac{d^3(a+bx^2)^{p+1}}{ax} + \frac{dx(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (3ae^2 + bd^2(2p+1)) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{a} \\ - \frac{3d^2e(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a} + 1\right)}{2a(p+1)} + \frac{e^3(a+bx^2)^{p+1}}{2b(p+1)}$$

[Out] (e^3\*(a + b\*x^2)^(1 + p))/(2\*b\*(1 + p)) - (d^3\*(a + b\*x^2)^(1 + p))/(a\*x) + (d\*(3\*a\*e^2 + b\*d^2\*(1 + 2\*p))\*x\*(a + b\*x^2)^p\*Hypergeometric2F1[1/2, -p, 3/2, -(b\*x^2)/a])/(a\*(1 + (b\*x^2)/a)^p) - (3\*d^2\*e\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b\*x^2)/a])/(2\*a\*(1 + p))

**Rubi [A]** time = 0.325915, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\frac{d^3(a+bx^2)^{p+1}}{ax} + \frac{dx(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (3ae^2 + bd^2(2p+1)) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{a} \\ - \frac{3d^2e(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a} + 1\right)}{2a(p+1)} + \frac{e^3(a+bx^2)^{p+1}}{2b(p+1)}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(a + b\*x^2)^p)/x^2, x]

[Out] (e^3\*(a + b\*x^2)^(1 + p))/(2\*b\*(1 + p)) - (d^3\*(a + b\*x^2)^(1 + p))/(a\*x) + (d\*(3\*a\*e^2 + b\*d^2\*(1 + 2\*p))\*x\*(a + b\*x^2)^p\*Hypergeometric2F1[1/2, -p, 3/2, -(b\*x^2)/a])/(a\*(1 + (b\*x^2)/a)^p) - (3\*d^2\*e\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b\*x^2)/a])/(2\*a\*(1 + p))

**Rubi in Sympy [A]** time = 29.5236, size = 139, normalized size = 0.87

$$-\frac{d^3 \left(1 + \frac{bx^2}{a}\right)^{-p} (a+bx^2)^p {}_2F_1\left(-p, -\frac{1}{2} \middle| -\frac{bx^2}{a}\right)}{x} + 3de^2x \left(1 + \frac{bx^2}{a}\right)^{-p} (a+bx^2)^p {}_2F_1\left(-p, \frac{1}{2} \middle| -\frac{bx^2}{a}\right) \\ + \frac{e^3(a+bx^2)^{p+1}}{2b(p+1)} - \frac{3d^2e(a+bx^2)^{p+1} {}_2F_1\left(1, p+1 \middle| 1 + \frac{bx^2}{a}\right)}{2a(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3\*(b\*x\*\*2+a)\*\*p/x\*\*2, x)

[Out] -d\*\*3\*(1 + b\*x\*\*2/a)\*\*(-p)\*(a + b\*x\*\*2)\*\*p\*hyper((-p, -1/2), (1/2, ), -b\*x\*\*2/a)/x + 3\*d\*e\*\*2\*x\*(1 + b\*x\*\*2/a)\*\*(-p)\*(a + b\*x\*\*2)\*\*p\*hyper((-p, 1/2), (3/2, ), -b\*x\*\*2/a) + e\*\*3\*(a + b\*x\*\*2)\*\*(p + 1)/(2\*b\*(p + 1)) - 3\*d\*\*2\*e\*(a + b\*x\*\*2)\*\*(p + 1)\*hyper((1, p + 1), (p + 2, ), 1 + b\*x\*\*2/a)/(2\*a\*(p + 1))

**Mathematica [A]** time = 0.955778, size = 174, normalized size = 1.09

$$\frac{1}{2} (a + bx^2)^p \left( e \left( \frac{3d^2 \left( \frac{a}{bx^2} + 1 \right)^{-p} {}_2F_1 \left( -p, -p; 1 - p; -\frac{a}{bx^2} \right)}{p} \right. \right. \\ \left. \left. + 6dex \left( \frac{bx^2}{a} + 1 \right)^{-p} {}_2F_1 \left( \frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a} \right) + \frac{e^2 \left( -a \left( \frac{bx^2}{a} + 1 \right)^{-p} + a + bx^2 \right)}{bp + b} \right) \right. \\ \left. - \frac{2d^3 \left( \frac{bx^2}{a} + 1 \right)^{-p} {}_2F_1 \left( -\frac{1}{2}, -p; \frac{1}{2}; -\frac{bx^2}{a} \right)}{x} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(a + b\*x^2)^p)/x^2,x]

[Out] ((a + b\*x^2)^p\*((-2\*d^3\*Hypergeometric2F1[-1/2, -p, 1/2, -(b\*x^2)/a])/x\*(1 + (b\*x^2)/a)^p + e\*((e^2\*(a + b\*x^2 - a/(1 + (b\*x^2)/a))^p)/(b + b\*p) + (6\*d\*e\*x\*Hypergeometric2F1[1/2, -p, 3/2, -(b\*x^2)/a])/((1 + (b\*x^2)/a)^p + (3\*d^2\*Hypergeometric2F1[-p, -p, 1 - p, -(a/(b\*x^2))])/((p\*(1 + a/(b\*x^2))^p)))))/2

**Maple [F]** time = 0.087, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^3 (bx^2 + a)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(b\*x^2+a)^p/x^2,x)

[Out] int((e\*x+d)^3\*(b\*x^2+a)^p/x^2,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^3 (bx^2 + a)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(b\*x^2 + a)^p/x^2,x, algorithm="maxima")

[Out] integrate((e\*x + d)^3\*(b\*x^2 + a)^p/x^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)(bx^2 + a)^p}{x^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(b\*x^2 + a)^p/x^2,x, algorithm="fricas")

[Out]  $\text{integral}((e^{3x^3} + 3d^2e^{2x^2} + 3d^2e^x + d^3)(bx^2 + a)^p/x^2, x)$

**Sympy [A]** time = 69.6426, size = 143, normalized size = 0.9

$$-\frac{a^p d^3 {}_2F_1\left(-\frac{1}{2}, -p \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{x} + 3a^p d e^2 x {}_2F_1\left(\frac{1}{2}, -p \middle| \frac{bx^2 e^{i\pi}}{a}\right) - \frac{3b^p d^2 e x^{2p} (-p) {}_2F_1\left(-p, -p \middle| \frac{a e^{i\pi}}{bx^2}\right)}{2(-p+1)} + e^3 \left( \begin{array}{ll} \left( \frac{a^p x^2}{2} \right) & \text{for } b = 0 \\ \left( \frac{(a+bx^2)^{p+1}}{p+1} \right) & \text{for } p \neq -1 \\ \left( \frac{\log(a+bx^2)}{2b} \right) & \text{otherwise} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**3*(b*x**2+a)**p/x**2,x)`

[Out]  $-a^p d^3 \text{hyper}((-1/2, -p), (1/2, ), b^p x^{2p} \exp(\pi i)/a)/x + 3a^p d^2 e^2 x \text{hyper}((1/2, -p), (3/2, ), b^p x^{2p} \exp(\pi i)/a) - 3b^p d^2 e x^{2p} \gamma(-p) \text{hyper}((-p, -p), (-p+1, ), a \exp(\pi i)/(b^p x^{2p}))/\gamma(-p+1) + e^3 \text{Piecewise}((a^p x^{2p/2}, \text{Eq}(b, 0)), (\text{Piecewise}(((a+b^p x^{2p})^{p+1}/(p+1)), \text{Ne}(p, -1)), (\log(a+b^p x^{2p}), \text{True}))/2b), \text{True}))$

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^3 (bx^2+a)^p}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^3*(b*x^2 + a)^p/x^2,x, algorithm="giac")`

[Out] `integrate((e*x + d)^3*(b*x^2 + a)^p/x^2, x)`

$$3.407 \quad \int \frac{(d+ex)^3 (a+bx^2)^p}{x^3} dx$$

**Optimal.** Leaf size=168

$$\begin{aligned} & -\frac{d(a+bx^2)^{p+1}(3ae^2+bd^2p) {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a}+1\right)}{2a^2(p+1)} - \frac{d^3(a+bx^2)^{p+1}}{2ax^2} \\ & + \frac{ex(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (ae^2+3bd^2(2p+1)) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{a} - \frac{3d^2e(a+bx^2)^{p+1}}{ax} \end{aligned}$$

[Out]  $-(d^3(a+bx^2)^{p+1}(3ae^2+bd^2p) {}_2F_1(1, p+1; p+2; \frac{bx^2}{a}+1))/2a^2(p+1) - (d^3(a+bx^2)^{p+1})/2ax^2 + (ex(a+bx^2)^p (\frac{bx^2}{a}+1)^{-p} (ae^2+3bd^2(2p+1)) {}_2F_1(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}))/a - (3d^2e(a+bx^2)^{p+1})/ax$

**Rubi [A]** time = 0.381711, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\begin{aligned} & -\frac{d(a+bx^2)^{p+1}(3ae^2+bd^2p) {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a}+1\right)}{2a^2(p+1)} - \frac{d^3(a+bx^2)^{p+1}}{2ax^2} \\ & + \frac{ex(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} (ae^2+3bd^2(2p+1)) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{a} - \frac{3d^2e(a+bx^2)^{p+1}}{ax} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(a + b\*x^2)^p)/x^3, x]

[Out]  $-(d^3(a+bx^2)^{p+1}(3ae^2+bd^2p) {}_2F_1(1, p+1; p+2; \frac{bx^2}{a}+1))/2a^2(p+1) - (d^3(a+bx^2)^{p+1})/2ax^2 + (ex(a+bx^2)^p (\frac{bx^2}{a}+1)^{-p} (ae^2+3bd^2(2p+1)) {}_2F_1(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}))/a - (3d^2e(a+bx^2)^{p+1})/ax$

**Rubi in Sympy [A]** time = 33.1041, size = 156, normalized size = 0.93

$$\begin{aligned} & -\frac{3d^2e \left(1 + \frac{bx^2}{a}\right)^{-p} (a+bx^2)^p {}_2F_1\left(-p, -\frac{1}{2}; \frac{1}{2}; -\frac{bx^2}{a}\right)}{x} + e^3x \left(1 + \frac{bx^2}{a}\right)^{-p} (a+bx^2)^p {}_2F_1\left(-p, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right) \\ & - \frac{3de^2(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; 1 + \frac{bx^2}{a}\right)}{2a(p+1)} + \frac{bd^3(a+bx^2)^{p+1} {}_2F_1\left(2, p+1; p+2; 1 + \frac{bx^2}{a}\right)}{2a^2(p+1)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3\*(b\*x\*\*2+a)\*\*p/x\*\*3, x)

[Out]  $-3*d**2*e*(1+b*x**2/a)**(-p)*(a+b*x**2)**p*hyper((-p, -1/2), (1/2, ), -b*x**2/a)/x + e**3*x*(1+b*x**2/a)**(-p)*(a+b*x**2)**p*hyper((-p, 1/2), (3/2, ), -b*x**2/a) - 3*d*e**2*(a+b*x**2)**(p+1)*hyper((1, p+1), (p+2, ), 1+b*x**2/a)/(2*a*(p+1)) + b*d**3*(a+b*x**2)**(p+1)*hyper((2, p+1), (p+2, ), 1+b*x**2/a)/(2*a**2*(p+1))$

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**Mathematica [A]** time = 0.662875, size = 176, normalized size = 1.05

$$\frac{1}{2} (a + bx^2)^p \left( d \left( \frac{a}{bx^2} + 1 \right)^{-p} \left( \frac{d^2 {}_2F_1 \left( 1 - p, -p; 2 - p; -\frac{a}{bx^2} \right)}{(p-1)x^2} + \frac{3e^2 {}_2F_1 \left( -p, -p; 1 - p; -\frac{a}{bx^2} \right)}{p} \right) \right. \\ \left. - \frac{6d^2 e \left( \frac{bx^2}{a} + 1 \right)^{-p} {}_2F_1 \left( -\frac{1}{2}, -p; \frac{1}{2}; -\frac{bx^2}{a} \right)}{x} + 2e^3 x \left( \frac{bx^2}{a} + 1 \right)^{-p} {}_2F_1 \left( \frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a} \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(a + b\*x^2)^p)/x^3, x]

[Out] ((a + b\*x^2)^p\*((-6\*d^2\*e\*Hypergeometric2F1[-1/2, -p, 1/2, -(b\*x^2)/a])/(x\*(1 + (b\*x^2)/a)^p) + (2\*e^3\*x\*Hypergeometric2F1[1/2, -p, 3/2, -(b\*x^2)/a])/(1 + (b\*x^2)/a)^p + (d\*((d^2\*Hypergeometric2F1[1 - p, -p, 2 - p, -(a/(b\*x^2))])/((-1 + p)\*x^2) + (3\*e^2\*Hypergeometric2F1[-p, -p, 1 - p, -(a/(b\*x^2))])/p))/(1 + a/(b\*x^2))^p)/2

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**Maple [F]** time = 0.063, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^3 (bx^2 + a)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(b\*x^2+a)^p/x^3, x)

[Out] int((e\*x+d)^3\*(b\*x^2+a)^p/x^3, x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^3 (bx^2 + a)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(b\*x^2 + a)^p/x^3, x, algorithm="maxima")

[Out] integrate((e\*x + d)^3\*(b\*x^2 + a)^p/x^3, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{(e^3 x^3 + 3 d e^2 x^2 + 3 d^2 e x + d^3) (b x^2 + a)^p}{x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(b\*x^2 + a)^p/x^3, x, algorithm="fricas")

[Out]  $\text{integral}((e^{3x} + 3d e^{2x} + 3d^2 e^x + d^3) \cdot (bx^2 + a)^p / x^3, x)$

**Sympy [A]** time = 94.4463, size = 150, normalized size = 0.89

$$-\frac{3a^p d^2 e {}_2F_1\left(-\frac{1}{2}, -p \middle| \frac{bx^2 e^{i\pi}}{a}\right)}{x} + a^p e^3 x {}_2F_1\left(\frac{1}{2}, -p \middle| \frac{bx^2 e^{i\pi}}{a}\right) - \frac{b^p d^3 x^{2p} (-p+1) {}_2F_1\left(-p, -p+1 \middle| \frac{ae^{i\pi}}{bx^2}\right)}{2x^2 (-p+2)} - \frac{3b^p d e^2 x^{2p} (-p) {}_2F_1\left(-p, -p \middle| \frac{ae^{i\pi}}{bx^2}\right)}{2(-p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**3*(b*x**2+a)**p/x**3,x)`

[Out]  $-3*a**p*d**2*e*\text{hyper}((-1/2, -p), (1/2, ), b*x**2*\text{exp\_polar}(I*\text{pi})/a)/x + a**p*e**3*x*\text{hyper}(1/2, -p), (3/2, ), b*x**2*\text{exp\_polar}(I*\text{pi})/a) - b**p*d**3*x**(2*p)*\text{gamma}(-p+1)*\text{hyper}((-p, -p+1), (-p+2, ), a*\text{exp\_polar}(I*\text{pi})/(b*x**2))/((2*x**2*\text{gamma}(-p+2)) - 3*b**p*d*e**2*x**(2*p)*\text{gamma}(-p)*\text{hyper}((-p, -p), (-p+1, ), a*\text{exp\_polar}(I*\text{pi})/(b*x**2))/((2*\text{gamma}(-p+1))$

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^3 (bx^2 + a)^p}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^3*(b*x^2 + a)^p/x^3,x, algorithm="giac")`

[Out] `integrate((e*x + d)^3*(b*x^2 + a)^p/x^3, x)`



$$3.408 \quad \int \frac{x^4(a+bx^2)^p}{d+ex} dx$$

**Optimal.** Leaf size=199

$$\frac{x^5(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{5}{2}; -p, 1; \frac{7}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{5d} + \frac{(bd^2 - ae^2)(a+bx^2)^{p+1}}{2b^2e^3(p+1)}$$

$$+ \frac{(a+bx^2)^{p+2}}{2b^2e(p+2)} - \frac{d^4(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e^3(p+1)(ae^2+bd^2)}$$

[Out] ((b\*d^2 - a\*e^2)\*(a + b\*x^2)^(1 + p))/(2\*b^2\*e^3\*(1 + p)) + (a + b\*x^2)^(2 + p)/(2\*b^2\*e\*(2 + p)) + (x^5\*(a + b\*x^2)^p\*AppellF1[5/2, -p, 1, 7/2, -(b\*x^2)/a, (e^2\*x^2)/d^2])/(5\*d\*(1 + (b\*x^2)/a)^p) - (d^4\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2\*(a + b\*x^2))/(b\*d^2 + a\*e^2)])/(2\*e^3\*(b\*d^2 + a\*e^2)\*(1 + p))

**Rubi [A]** time = 0.495727, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{x^5(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{5}{2}; -p, 1; \frac{7}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{5d} + \frac{(bd^2 - ae^2)(a+bx^2)^{p+1}}{2b^2e^3(p+1)}$$

$$+ \frac{(a+bx^2)^{p+2}}{2b^2e(p+2)} - \frac{d^4(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e^3(p+1)(ae^2+bd^2)}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*(a + b\*x^2)^p)/(d + e\*x), x]

[Out] ((b\*d^2 - a\*e^2)\*(a + b\*x^2)^(1 + p))/(2\*b^2\*e^3\*(1 + p)) + (a + b\*x^2)^(2 + p)/(2\*b^2\*e\*(2 + p)) + (x^5\*(a + b\*x^2)^p\*AppellF1[5/2, -p, 1, 7/2, -(b\*x^2)/a, (e^2\*x^2)/d^2])/(5\*d\*(1 + (b\*x^2)/a)^p) - (d^4\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2\*(a + b\*x^2))/(b\*d^2 + a\*e^2)])/(2\*e^3\*(b\*d^2 + a\*e^2)\*(1 + p))

**Rubi in Sympy [A]** time = 62.3532, size = 277, normalized size = 1.39

$$\frac{a(a+bx^2)^{p+1}}{2b^2e(p+1)}$$

$$+ \frac{d^4 \left(\frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} \left(-\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} (a+bx^2)^p \operatorname{appellf}_1\left(-2p, -p, -p, -2p+1, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}\right)}{2e^5p}$$

$$- \frac{d^3x \left(1 + \frac{bx^2}{a}\right)^{-p} (a+bx^2)^p {}_2F_1\left(-p, \frac{1}{2} \middle| -\frac{bx^2}{a}\right)}{e^4}$$

$$- \frac{dx^3 \left(1 + \frac{bx^2}{a}\right)^{-p} (a+bx^2)^p {}_2F_1\left(-p, \frac{3}{2} \middle| -\frac{bx^2}{a}\right)}{3e^2} + \frac{d^2(a+bx^2)^{p+1}}{2be^3(p+1)} + \frac{(a+bx^2)^{p+2}}{2b^2e(p+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(b\*x\*\*2+a)\*\*p/(e\*x+d), x)

[Out]  $-a*(a + b*x**2)**(p + 1)/(2*b**2*e*(p + 1)) + d**4*(e*(sqrt(b)*x + sqrt(-a))/(sqrt(b)*(d + e*x)))**(-p)*(-e*(-sqrt(b)*x + sqrt(-a))/(sqrt(b)*(d + e*x)))**(-p)*(a + b*x**2)**p*appellf1(-2*p, -p, -p, -2*p + 1, (d - e*sqrt(-a)/sqrt(b))/(d + e*x), (d + e*sqrt(-a)/sqrt(b))/(d + e*x))/(2*e**5*p) - d**3*x*(1 + b*x**2/a)**(-p)*(a + b*x**2)**p*hyper((-p, 1/2), (3/2, ), -b*x**2/a)/e**4 - d*x**3*(1 + b*x**2/a)**(-p)*(a + b*x**2)**p*hyper((-p, 3/2), (5/2, ), -b*x**2/a)/(3*e**2) + d**2*(a + b*x**2)**(p + 1)/(2*b*e**3*(p + 1)) + (a + b*x**2)**(p + 2)/(2*b**2*e*(p + 2))$

**Mathematica [A]** time = 0.970068, size = 0, normalized size = 0.

$$\int \frac{x^4 (a + bx^2)^p}{d + ex} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^4\*(a + b\*x^2)^p)/(d + e\*x), x]

[Out] Integrate[(x^4\*(a + b\*x^2)^p)/(d + e\*x), x]

**Maple [F]** time = 0.082, size = 0, normalized size = 0.

$$\int \frac{x^4 (bx^2 + a)^p}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(b\*x^2+a)^p/(e\*x+d), x)

[Out] int(x^4\*(b\*x^2+a)^p/(e\*x+d), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p x^4}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^p\*x^4/(e\*x + d), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^p\*x^4/(e\*x + d), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^p x^4}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^p\*x^4/(e\*x + d), x, algorithm="fricas")

[Out] `integral((b*x^2 + a)^p*x^4/(e*x + d), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(b*x**2+a)**p/(e*x+d), x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p x^4}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^4/(e*x + d), x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*x^4/(e*x + d), x)`

$$3.409 \quad \int \frac{x^3(a+bx^2)^p}{d+ex} dx$$

**Optimal.** Leaf size=163

$$\frac{ex^5(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{5}{2}; -p, 1; \frac{7}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{5d^2} + \frac{d^3(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e^2(p+1)(ae^2+bd^2)} - \frac{d(a+bx^2)^{p+1}}{2be^2(p+1)}$$

[Out]  $-(d*(a+b*x^2)^(1+p))/(2*b*e^2*(1+p)) - (e*x^5*(a+b*x^2)^p * AppellF1[5/2, -p, 1, 7/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(5*d^2*(1+(b*x^2)/a)^p) + (d^3*(a+b*x^2)^(1+p)*Hypergeometric2F1[1, 1+p, 2+p, (e^2*(a+b*x^2))/(b*d^2+a*e^2)])/(2*e^2*(b*d^2+a*e^2)*(1+p))$

**Rubi [A]** time = 0.370748, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{ex^5(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{5}{2}; -p, 1; \frac{7}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{5d^2} + \frac{d^3(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e^2(p+1)(ae^2+bd^2)} - \frac{d(a+bx^2)^{p+1}}{2be^2(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(a+b\*x^2)^p)/(d+e\*x), x]

[Out]  $-(d*(a+b*x^2)^(1+p))/(2*b*e^2*(1+p)) - (e*x^5*(a+b*x^2)^p * AppellF1[5/2, -p, 1, 7/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(5*d^2*(1+(b*x^2)/a)^p) + (d^3*(a+b*x^2)^(1+p)*Hypergeometric2F1[1, 1+p, 2+p, (e^2*(a+b*x^2))/(b*d^2+a*e^2)])/(2*e^2*(b*d^2+a*e^2)*(1+p))$

**Rubi in Sympy [A]** time = 46.6381, size = 230, normalized size = 1.41

$$\frac{d^3 \left(\frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} \left(-\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} (a+bx^2)^p \operatorname{appellf}_1\left(-2p, -p, -p, -2p+1, \frac{d-e\sqrt{-a}}{\sqrt{b}}, \frac{d+e\sqrt{-a}}{\sqrt{b}}\right)}{2e^4p} + \frac{d^2x \left(1+\frac{bx^2}{a}\right)^{-p} (a+bx^2)^p {}_2F_1\left(-p, \frac{1}{2} \middle| -\frac{bx^2}{a}\right)}{e^3} + \frac{x^3 \left(1+\frac{bx^2}{a}\right)^{-p} (a+bx^2)^p {}_2F_1\left(-p, \frac{3}{2} \middle| -\frac{bx^2}{a}\right)}{3e} - \frac{d(a+bx^2)^{p+1}}{2be^2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(b\*x\*\*2+a)\*\*p/(e\*x+d), x)

[Out]  $-d**3*(e*(\sqrt{b}*x + \sqrt{-a})/(\sqrt{b}*(d + e*x)))**(-p)*(-e*(-\sqrt{b}*x + \sqrt{-a})/(\sqrt{b}*(d + e*x)))**(-p)*(a + b*x**2)**p * \operatorname{appellf}_1(-2*p, -p, -p, -2*p + 1, (d - e*\sqrt{-a})/\sqrt{b})/(d + e*x), (d + e*\sqrt{-a})/\sqrt{b})/(d + e*x))/(2*e**4*p) + d**2*x*(1 +$

$b^2 x^2/a)^{-p} (a + b^2 x^2)^p \operatorname{hyper}((-p, 1/2), (3/2, ), -b^2 x^2/a)/e^{3x} + x^3 (1 + b^2 x^2/a)^{-p} (a + b^2 x^2)^p \operatorname{hyper}((-p, 3/2), (5/2, ), -b^2 x^2/a)/(3e) - d^p (a + b^2 x^2)^p (p + 1)/(2b^2 e^{2x} (p + 1))$

**Mathematica [A]** time = 0.531567, size = 260, normalized size = 1.6

$$(a + bx^2)^p \left( \frac{e^{\left(\frac{bx^2}{a} + 1\right)^{-p} \left( 6bd^2(p+1)x {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right) + e\left(2be(p+1)x^3 {}_2F_1\left(\frac{3}{2}, -p; \frac{5}{2}; -\frac{bx^2}{a}\right) - 3d\left(bx^2\left(\frac{bx^2}{a} + 1\right)^p + a\left(\left(\frac{bx^2}{a} + 1\right)^p - 1\right)\right)\right)}}{b(p+1)} - \frac{3d^3 \left(\frac{e^{x - \sqrt{-\frac{a}{b}}}}{d + ex}\right)}{6e^4} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^3\*(a + b\*x^2)^p)/(d + e\*x), x]

[Out] ((a + b\*x^2)^p\*((-3\*d^3\*AppellF1[-2\*p, -p, -p, 1 - 2\*p, (d - Sqrt[-(a/b)]\*e)/(d + e\*x), (d + Sqrt[-(a/b)]\*e)/(d + e\*x)]/(p\*((e\*(Sqrt[-(a/b)] + x))/(d + e\*x))^p) + (e\*(6\*b\*d^2\*(1 + p)\*x\*Hypergeometric2F1[1/2, -p, 3/2, -(b\*x^2)/a]) + e\*(-3\*d\*(b\*x^2\*(1 + (b\*x^2)/a)^p + a\*(-1 + (1 + (b\*x^2)/a)^p)) + 2\*b\*e\*(1 + p)\*x^3\*Hypergeometric2F1[3/2, -p, 5/2, -(b\*x^2)/a])))/(b\*(1 + p)\*(1 + (b\*x^2)/a)^p))/(6\*e^4)

**Maple [F]** time = 0.095, size = 0, normalized size = 0.

$$\int \frac{x^3 (bx^2 + a)^p}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(b\*x^2+a)^p/(e\*x+d), x)

[Out] int(x^3\*(b\*x^2+a)^p/(e\*x+d), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p x^3}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^p\*x^3/(e\*x + d), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^p\*x^3/(e\*x + d), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(bx^2 + a)^p x^3}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^3/(e*x + d),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p*x^3/(e*x + d), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(b*x**2+a)**p/(e*x+d),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p x^3}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^3/(e*x + d),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*x^3/(e*x + d), x)`

$$3.410 \quad \int \frac{x^2(a+bx^2)^p}{d+ex} dx$$

**Optimal.** Leaf size=161

$$\frac{x^3 (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{3}{2}; -p, 1; \frac{5}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{3d} - \frac{d^2 (a + bx^2)^{p+1} {}_2F_1\left(1, p + 1; p + 2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e(p+1)(ae^2 + bd^2)} + \frac{(a + bx^2)^{p+1}}{2be(p+1)}$$

[Out] (a + b\*x^2)^(1 + p)/(2\*b\*e\*(1 + p)) + (x^3\*(a + b\*x^2)^p\*AppellF1[3/2, -p, 1, 5/2, -(b\*x^2)/a, (e^2\*x^2)/d^2])/(3\*d\*(1 + (b\*x^2)/a)^p) - (d^2\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2\*(a + b\*x^2))/(b\*d^2 + a\*e^2)])/(2\*e\*(b\*d^2 + a\*e^2)\*(1 + p))

**Rubi [A]** time = 0.366441, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.3$

$$\frac{x^3 (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{3}{2}; -p, 1; \frac{5}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{3d} - \frac{d^2 (a + bx^2)^{p+1} {}_2F_1\left(1, p + 1; p + 2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e(p+1)(ae^2 + bd^2)} + \frac{(a + bx^2)^{p+1}}{2be(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(a + b\*x^2)^p)/(d + e\*x), x]

[Out] (a + b\*x^2)^(1 + p)/(2\*b\*e\*(1 + p)) + (x^3\*(a + b\*x^2)^p\*AppellF1[3/2, -p, 1, 5/2, -(b\*x^2)/a, (e^2\*x^2)/d^2])/(3\*d\*(1 + (b\*x^2)/a)^p) - (d^2\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2\*(a + b\*x^2))/(b\*d^2 + a\*e^2)])/(2\*e\*(b\*d^2 + a\*e^2)\*(1 + p))

**Rubi in Sympy [A]** time = 37.0512, size = 184, normalized size = 1.14

$$\frac{d^2 \left(\frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b}(d+ex)}\right)^{-p} \left(-\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b}(d+ex)}\right)^{-p} (a + bx^2)^p \text{appellf1}\left(-2p, -p, -p, -2p + 1, \frac{d - e\sqrt{-a}}{\sqrt{b}}, \frac{d + e\sqrt{-a}}{\sqrt{b}}\right)}{2e^3p} - \frac{dx \left(1 + \frac{bx^2}{a}\right)^{-p} (a + bx^2)^p {}_2F_1\left(-p, \frac{1}{2} \middle| -\frac{bx^2}{a}\right)}{e^2} + \frac{(a + bx^2)^{p+1}}{2be(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(b\*x\*\*2+a)\*\*p/(e\*x+d), x)

[Out] d\*\*2\*(e\*(sqrt(b)\*x + sqrt(-a))/(sqrt(b)\*(d + e\*x)))\*\*(-p)\*(-e\*(-sqrt(b)\*x + sqrt(-a))/(sqrt(b)\*(d + e\*x)))\*\*(-p)\*(a + b\*x\*\*2)\*\*p\*appellf1(-2\*p, -p, -p, -2\*p + 1, (d - e\*sqrt(-a)/sqrt(b))/(d + e\*x), (d + e\*sqrt(-a)/sqrt(b))/(d + e\*x))/(2\*e\*\*3\*p) - d\*x\*(1 + b\*x\*\*2/a)\*\*(-p)\*(a + b\*x\*\*2)\*\*p\*hyper((-p, 1/2), (3/2,), -b\*x\*\*2/a)/e\*\*2 + (a + b\*x\*\*2)\*\*(p + 1)/(2\*b\*e\*(p + 1))

---

**Mathematica [A]** time = 0.396893, size = 227, normalized size = 1.41

$$\frac{(a + bx^2)^p \left( bd^2(p+1) \left( \frac{e(x - \sqrt{-\frac{a}{b}})}{d+ex} \right)^{-p} \left( \frac{e(\sqrt{-\frac{a}{b}} + x)}{d+ex} \right)^{-p} F_1 \left( -2p; -p, -p; 1 - 2p; \frac{d - \sqrt{-\frac{a}{b}}e}{d+ex}, \frac{d + \sqrt{-\frac{a}{b}}e}{d+ex} \right) - 2bdep(p+1)x \left( \frac{bx^2}{a} + 1 \right) \right)}{2be^3p(p+1)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x^2\*(a + b\*x^2)^p)/(d + e\*x), x]

[Out] ((a + b\*x^2)^p\*(a\*e^2\*p + b\*e^2\*p\*x^2 - (a\*e^2\*p)/(1 + (b\*x^2)/a)^p + (b\*d^2\*(1 + p)\*AppellF1[-2\*p, -p, -p, 1 - 2\*p, (d - Sqrt[-(a/b)]\*e)/(d + e\*x), (d + Sqrt[-(a/b)]\*e)/(d + e\*x]])/((e\*(-Sqrt[-(a/b)] + x))/(d + e\*x))^p\*((e\*(Sqrt[-(a/b)] + x))/(d + e\*x))^p - (2\*b\*d\*e\*p\*(1 + p)\*x\*Hypergeometric2F1[1/2, -p, 3/2, -(b\*x^2)/a]))/(1 + (b\*x^2)/a)^p)/(2\*b\*e^3\*p\*(1 + p))

---

**Maple [F]** time = 0.076, size = 0, normalized size = 0.

$$\int \frac{x^2 (bx^2 + a)^p}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(b\*x^2+a)^p/(e\*x+d), x)

[Out] int(x^2\*(b\*x^2+a)^p/(e\*x+d), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p x^2}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^p\*x^2/(e\*x + d), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^p\*x^2/(e\*x + d), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{(bx^2 + a)^p x^2}{ex + d}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^p\*x^2/(e\*x + d), x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^p\*x^2/(e\*x + d), x)

---



**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b*x**2+a)**p/(e*x+d),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p x^2}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^2/(e*x + d),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*x^2/(e*x + d), x)`

$$3.411 \quad \int \frac{x(a+bx^2)^p}{d+ex} dx$$

**Optimal.** Leaf size=173

$$\frac{x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{e} + \frac{d(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2+bd^2)} + \frac{x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{e}$$

[Out]  $-\left(\frac{x(a+bx^2)^p \text{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{(bx^2)}{a}\right], (e^2x^2/d^2)}{e} + \frac{x(a+bx^2)^p \text{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{(bx^2)}{a}\right]}{e} + \frac{d(a+bx^2)^{p+1} \text{Hypergeometric2F1}\left[1, 1+p, 2+p, \frac{e^2(a+bx^2)}{bd^2+ae^2}\right]}{2(p+1)(ae^2+bd^2)}\right) / (2(bd^2+ae^2)(1+p))$

**Rubi [A]** time = 0.318123, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$

$$\frac{x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{e} + \frac{d(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2+bd^2)} + \frac{x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{e}$$

Antiderivative was successfully verified.

[In] Int[(x\*(a+b\*x^2)^p)/(d+e\*x), x]

[Out]  $-\left(\frac{x(a+bx^2)^p \text{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{(bx^2)}{a}\right], (e^2x^2/d^2)}{e} + \frac{x(a+bx^2)^p \text{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\frac{(bx^2)}{a}\right]}{e} + \frac{d(a+bx^2)^{p+1} \text{Hypergeometric2F1}\left[1, 1+p, 2+p, \frac{e^2(a+bx^2)}{bd^2+ae^2}\right]}{2(p+1)(ae^2+bd^2)}\right) / (2(bd^2+ae^2)(1+p))$

**Rubi in Sympy [A]** time = 28.5294, size = 160, normalized size = 0.92

$$\frac{d \left(\frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} \left(-\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} (a+bx^2)^p \text{appellf1}\left(-2p, -p, -p, -2p+1, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}\right)}{2e^2p} + \frac{x \left(1 + \frac{bx^2}{a}\right)^{-p} (a+bx^2)^p {}_2F_1\left(-p, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right)}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(b\*x\*\*2+a)\*\*p/(e\*x+d), x)

[Out]  $-d*(e*(\sqrt{b}*x + \sqrt{-a})/(\sqrt{b}*(d + e*x)))^{(-p)}*(-e*(-\sqrt{b}*x + \sqrt{-a})/(\sqrt{b}*(d + e*x)))^{(-p)}*(a + b*x**2)**p \text{appellf1}(-2*p, -p, -p, -2*p + 1, (d - e*\sqrt{-a}/\sqrt{b})/(d + e*x), (d + e*\sqrt{-a}/\sqrt{b})/(d + e*x))/(2*e**2*p) + x*(1 + b*x**2/a)**(-p)*(a + b*x**2)**p \text{hyper}((-p, 1/2), (3/2, ), -b*x**2/a)/e$

**Mathematica [A]** time = 0.350095, size = 172, normalized size = 0.99

$$\frac{(a + bx^2)^p \left( 2ex \left( \frac{bx^2}{a} + 1 \right)^{-p} {}_2F_1 \left( \frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a} \right) - \frac{d \left( \frac{e^{(x - \sqrt{-\frac{a}{b}})}}{d+ex} \right)^{-p} \left( \frac{e^{\left( \sqrt{-\frac{a}{b}} + x \right)}}{d+ex} \right)^{-p} F_1 \left( -2p; -p, -p; 1-2p; \frac{d - \sqrt{-\frac{a}{b}} e}{d+ex}, \frac{d + \sqrt{-\frac{a}{b}} e}{d+ex} \right)}{p} \right)}{2e^2}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(x\*(a + b\*x^2)^p)/(d + e\*x), x]

[Out] ((a + b\*x^2)^p\*(-((d\*AppellF1[-2\*p, -p, -p, 1 - 2\*p, (d - Sqrt[-(a/b)]\*e)/(d + e\*x), (d + Sqrt[-(a/b)]\*e)/(d + e\*x))]/(p\*((e\*(-Sqrt[-(a/b)] + x))/(d + e\*x))^p\*((e\*(Sqrt[-(a/b)] + x))/(d + e\*x))^p)) + (2\*e\*x\*Hypergeometric2F1[1/2, -p, 3/2, -(b\*x^2)/a]))/(1 + (b\*x^2)/a)^p)/(2\*e^2)

**Maple [F]** time = 0.131, size = 0, normalized size = 0.

$$\int \frac{x (bx^2 + a)^p}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(b\*x^2+a)^p/(e\*x+d), x)

[Out] int(x\*(b\*x^2+a)^p/(e\*x+d), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p x}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^p\*x/(e\*x + d), x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^p\*x/(e\*x + d), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{(bx^2 + a)^p x}{ex + d}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^p\*x/(e\*x + d), x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^p\*x/(e\*x + d), x)

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x (a + bx^2)^p}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x\*\*2+a)\*\*p/(e\*x+d), x)

[Out] Integral(x\*(a + b\*x\*\*2)\*\*p/(d + e\*x), x)

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p x}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^p\*x/(e\*x + d), x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^p\*x/(e\*x + d), x)

$$3.412 \quad \int \frac{(a+bx^2)^p}{d+ex} dx$$

**Optimal.** Leaf size=125

$$\frac{x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{d} - \frac{e(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2+bd^2)}$$

[Out] (x\*(a + b\*x^2)^p\*AppellF1[1/2, -p, 1, 3/2, -(b\*x^2)/a], (e^2\*x^2)/d^2)]/(d\*(1 + (b\*x^2)/a)^p) - (e\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2\*(a + b\*x^2))/(b\*d^2 + a\*e^2)])/(2\*(b\*d^2 + a\*e^2)\*(1 + p))

**Rubi [A]** time = 0.225172, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$

$$\frac{x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{d} - \frac{e(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2+bd^2)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^p/(d + e\*x), x]

[Out] (x\*(a + b\*x^2)^p\*AppellF1[1/2, -p, 1, 3/2, -(b\*x^2)/a], (e^2\*x^2)/d^2)]/(d\*(1 + (b\*x^2)/a)^p) - (e\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2\*(a + b\*x^2))/(b\*d^2 + a\*e^2)])/(2\*(b\*d^2 + a\*e^2)\*(1 + p))

**Rubi in Sympy [A]** time = 17.5953, size = 119, normalized size = 0.95

$$\frac{\left(\frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} \left(-\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} (a+bx^2)^p \operatorname{appellf}_1\left(-2p, -p, -p, -2p+1, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}\right)}{2ep}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*p/(e\*x+d), x)

[Out] (e\*(sqrt(b)\*x + sqrt(-a))/(sqrt(b)\*(d + e\*x)))\*\*(-p)\*(-e\*(-sqrt(b)\*x + sqrt(-a))/(sqrt(b)\*(d + e\*x)))\*\*(-p)\*(a + b\*x\*\*2)\*\*p\*appellf1(-2\*p, -p, -p, -2\*p + 1, (d - e\*sqrt(-a)/sqrt(b))/(d + e\*x), (d + e\*sqrt(-a)/sqrt(b))/(d + e\*x))/(2\*e\*p)

**Mathematica [A]** time = 0.102687, size = 131, normalized size = 1.05

$$\frac{(a+bx^2)^p \left(\frac{e\left(x-\sqrt{\frac{-a}{b}}\right)}{d+ex}\right)^{-p} \left(\frac{e\left(\sqrt{\frac{-a}{b}}+x\right)}{d+ex}\right)^{-p} F_1\left(-2p; -p, -p; 1-2p; \frac{d-\sqrt{\frac{-a}{b}}e}{d+ex}, \frac{d+\sqrt{\frac{-a}{b}}e}{d+ex}\right)}{2ep}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^2)^p/(d + e\*x), x]

[Out]  $((a + b*x^2)^p * \text{AppellF1}[-2*p, -p, -p, 1 - 2*p, (d - \text{Sqrt}[-(a/b)] * e)/(d + e*x), (d + \text{Sqrt}[-(a/b)] * e)/(d + e*x)]) / (2 * e * p * ((e * (-\text{Sqrt}[-(a/b)] + x)) / (d + e*x))^p)$

**Maple** [F] time = 0.057, size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^p/(e*x+d), x)`

[Out] `int((b*x^2+a)^p/(e*x+d), x)`

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/(e*x + d), x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p/(e*x + d), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^p}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/(e*x + d), x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p/(e*x + d), x)`

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2)^p}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**p/(e*x+d), x)`

[Out] `Integral((a + b*x**2)**p/(d + e*x), x)`

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*x^2 + a)^p/(e*x + d), x, algorithm="giac")
```

```
[Out] integrate((b*x^2 + a)^p/(e*x + d), x)
```

$$3.413 \quad \int \frac{(a+bx^2)^p}{x(d+ex)} dx$$

**Optimal.** Leaf size=176

$$\frac{ex(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{d^2} + \frac{e^2(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d(p+1)(ae^2+bd^2)} - \frac{(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a} + 1\right)}{2ad(p+1)}$$

[Out]  $-\left(\frac{e^2x^2(a+bx^2)^p \text{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right]}{d^2}\right) + \frac{e^2(a+bx^2)^{p+1} {}_2F_1\left[1, 1+p, 2+p, \frac{e^2(a+bx^2)}{bd^2+ae^2}\right]}{2d(p+1)(ae^2+bd^2)} - \frac{(a+bx^2)^{p+1} {}_2F_1\left[1, 1+p, 2+p, 1+\frac{bx^2}{a}\right]}{2ad(p+1)}$

**Rubi [A]** time = 0.346438, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$\frac{ex(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{d^2} + \frac{e^2(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d(p+1)(ae^2+bd^2)} - \frac{(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a} + 1\right)}{2ad(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^p/(x\*(d + e\*x)), x]

[Out]  $-\left(\frac{e^2x^2(a+bx^2)^p \text{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right]}{d^2}\right) + \frac{e^2(a+bx^2)^{p+1} {}_2F_1\left[1, 1+p, 2+p, \frac{e^2(a+bx^2)}{bd^2+ae^2}\right]}{2d(p+1)(ae^2+bd^2)} - \frac{(a+bx^2)^{p+1} {}_2F_1\left[1, 1+p, 2+p, 1+\frac{bx^2}{a}\right]}{2ad(p+1)}$

**Rubi in Sympy [A]** time = 32.9321, size = 153, normalized size = 0.87

$$\frac{\left(\frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} \left(-\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} (a+bx^2)^p \text{appellf1}\left(-2p, -p, -p, -2p+1, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}\right)}{2dp} - \frac{(a+bx^2)^{p+1} {}_2F_1\left(1, p+1 \middle| p+2 \middle| 1+\frac{bx^2}{a}\right)}{2ad(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*p/x/(e\*x+d), x)

[Out]  $-\left(\frac{e(\sqrt{b}x + \sqrt{-a})}{(\sqrt{b}(d + e*x))}\right)^{-p} \left(-\frac{e(-\sqrt{b}x + \sqrt{-a})}{(\sqrt{b}(d + e*x))}\right)^{-p} (a + b*x^2)^p \text{appellf1}\left(-2p, -p, -p, -2p + 1, \frac{d - e\sqrt{-a}/\sqrt{b}}{d + e*x}, \frac{d + e\sqrt{-a}/\sqrt{b}}{d + e*x}\right) - \frac{(a + b*x^2)^{p+1} {}_2F_1\left[1, p + 1, p + 2, 1 + \frac{bx^2}{a}\right]}{2ad(p + 1)}$



**Mathematica [A]** time = 0.377508, size = 170, normalized size = 0.97

$$\frac{(a + bx^2)^p \left( \left( \frac{a}{bx^2} + 1 \right)^{-p} {}_2F_1 \left( -p, -p; 1 - p; -\frac{a}{bx^2} \right) - \left( \frac{e^{(x - \sqrt{-\frac{a}{b}})}}{d + ex} \right)^{-p} \left( \frac{e^{(\sqrt{-\frac{a}{b}} + x)}}{d + ex} \right)^{-p} {}_F_1 \left( -2p; -p, -p; 1 - 2p; \frac{d - \sqrt{-\frac{a}{b}}e}{d + ex}, \frac{d + \sqrt{-\frac{a}{b}}e}{d + ex} \right) \right)}{2dp}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^2)^p/(x\*(d + e\*x)),x]

[Out] ((a + b\*x^2)^p\*(-(AppellF1[-2\*p, -p, -p, 1 - 2\*p, (d - Sqrt[-(a/b)])\*e]/(d + e\*x), (d + Sqrt[-(a/b)]\*e)/(d + e\*x)]/(((e\*(-Sqrt[-(a/b)] + x))/(d + e\*x))^p\*((e\*(Sqrt[-(a/b)] + x))/(d + e\*x))^p)) + Hypergeometric2F1[-p, -p, 1 - p, -(a/(b\*x^2))]/(1 + a/(b\*x^2))^p)/(2\*d\*p)

**Maple [F]** time = 0.06, size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x(ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^p/x/(e\*x+d),x)

[Out] int((b\*x^2+a)^p/x/(e\*x+d),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{(ex + d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^p/((e\*x + d)\*x),x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^p/((e\*x + d)\*x), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^p}{ex^2 + dx}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^p/((e\*x + d)\*x),x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^p/(e\*x^2 + d\*x), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx^2)^p}{x(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**p/x/(e*x+d),x)`

[Out] `Integral((a + b*x**2)**p/(x*(d + e*x)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{(ex + d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/((e*x + d)*x),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p/((e*x + d)*x), x)`

$$3.414 \quad \int \frac{(a+bx^2)^p}{x^2(d+ex)} dx$$

**Optimal.** Leaf size=178

$$\frac{(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(-\frac{1}{2}; -p, 1; \frac{1}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{dx} - \frac{e^3 (a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d^2(p+1)(ae^2+bd^2)} + \frac{e (a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a} + 1\right)}{2ad^2(p+1)}$$

[Out] -(((a + b\*x^2)^p\*AppellF1[-1/2, -p, 1, 1/2, -(b\*x^2)/a], (e^2\*x^2)/d^2))/(d\*x\*(1 + (b\*x^2)/a)^p) - (e^3\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2\*(a + b\*x^2))/(b\*d^2 + a\*e^2)])/(2\*d^2\*(b\*d^2 + a\*e^2)\*(1 + p)) + (e\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b\*x^2)/a])/(2\*a\*d^2\*(1 + p))

**Rubi [A]** time = 0.412854, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.35$

$$\frac{(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(-\frac{1}{2}; -p, 1; \frac{1}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{dx} - \frac{e^3 (a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d^2(p+1)(ae^2+bd^2)} + \frac{e (a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a} + 1\right)}{2ad^2(p+1)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^p/(x^2\*(d + e\*x)), x]

[Out] -(((a + b\*x^2)^p\*AppellF1[-1/2, -p, 1, 1/2, -(b\*x^2)/a], (e^2\*x^2)/d^2))/(d\*x\*(1 + (b\*x^2)/a)^p) - (e^3\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2\*(a + b\*x^2))/(b\*d^2 + a\*e^2)])/(2\*d^2\*(b\*d^2 + a\*e^2)\*(1 + p)) + (e\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b\*x^2)/a])/(2\*a\*d^2\*(1 + p))

**Rubi in Sympy [A]** time = 42.8834, size = 197, normalized size = 1.11

$$\frac{\left(1 + \frac{bx^2}{a}\right)^{-p} (a+bx^2)^p {}_2F_1\left(-p, -\frac{1}{2} \middle| -\frac{bx^2}{a}\right)}{dx} + \frac{e \left(\frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} \left(-\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} (a+bx^2)^p \operatorname{appellf}_1\left(-2p, -p, -p, -2p+1, \frac{d-\sqrt{-a}}{\sqrt{b}}, \frac{d+\sqrt{-a}}{\sqrt{b}}\right)}{2d^2p} + \frac{e (a+bx^2)^{p+1} {}_2F_1\left(1, p+1 \middle| 1 + \frac{bx^2}{a}\right)}{2ad^2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*p/x\*\*2/(e\*x+d), x)

[Out] -(1 + b\*x\*\*2/a)\*\*(-p)\*(a + b\*x\*\*2)\*\*p\*hyper((-p, -1/2), (1/2, ), -b\*x\*\*2/a)/(d\*x) + e\*(e\*(sqrt(b)\*x + sqrt(-a))/(sqrt(b)\*(d + e\*x))\*\*(-p)\*(-e\*(-sqrt(b)\*x + sqrt(-a))/(sqrt(b)\*(d + e\*x))\*\*(-p)\*(a

$+ b^2 x^2)^p \operatorname{appellf1}(-2p, -p, -p, -2p + 1, (d - e \sqrt{-a})/\sqrt{b})/\sqrt{b})/(d + e x), (d + e \sqrt{-a})/\sqrt{b})/(d + e x)/(2 d^2 p) + e (a + b^2 x^2)^p (p + 1) \operatorname{hyper}((1, p + 1), (p + 2, ), 1 + b^2 x^2/a)/(2 a d^2 (p + 1))$

**Mathematica [A]** time = 1.01607, size = 214, normalized size = 1.2

$$(a + bx^2)^p \left( \frac{e \left( \frac{e(x - \sqrt{-a/b})}{d + ex} \right)^{-p} \left( \frac{e(\sqrt{-a/b} + x)}{d + ex} \right)^{-p} F_1 \left( -2p; -p, -p; 1 - 2p; \frac{d - \sqrt{-a/b} e}{d + ex}, \frac{d + \sqrt{-a/b} e}{d + ex} \right)}{p} - \frac{2d \left( \frac{bx^2}{a} + 1 \right)^{-p} {}_2F_1 \left( -\frac{1}{2}, -p; \frac{1}{2}; -\frac{bx^2}{a} \right)}{x} - \frac{e \left( \frac{a}{bx^2} + 1 \right)^{-p} {}_2F_1 \left( -p, -1; -p, -1; -\frac{a}{bx^2} \right)}{p} \right)$$


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$$2d^2$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^2)^p/(x^2\*(d + e\*x)),x]

[Out] ((a + b\*x^2)^p\*((e\*AppellF1[-2\*p, -p, -p, 1 - 2\*p, (d - Sqrt[-(a/b)]\*e)/(d + e\*x), (d + Sqrt[-(a/b)]\*e)/(d + e\*x)])/(p\*((e\*(-Sqrt[-(a/b)] + x))/(d + e\*x))^p\*((e\*(Sqrt[-(a/b)] + x))/(d + e\*x))^p) - (2\*d\*Hypergeometric2F1[-1/2, -p, 1/2, -(b\*x^2)/a]))/(x\*(1 + (b\*x^2)/a)^p) - (e\*Hypergeometric2F1[-p, -p, 1 - p, -(a/(b\*x^2))]))/(p\*(1 + a/(b\*x^2))^p))/(2\*d^2)

**Maple [F]** time = 0.152, size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x^2(ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^p/x^2/(e\*x+d),x)

[Out] int((b\*x^2+a)^p/x^2/(e\*x+d),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{(ex + d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^p/((e\*x + d)\*x^2),x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^p/((e\*x + d)\*x^2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{(bx^2 + a)^p}{ex^3 + dx^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/((e*x + d)*x^2),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p/(e*x^3 + d*x^2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**p/x**2/(e*x+d),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{(ex + d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/((e*x + d)*x^2),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p/((e*x + d)*x^2), x)`

$$3.415 \quad \int \frac{(a+bx^2)^p}{x^3(d+ex)} dx$$

**Optimal.** Leaf size=213

$$\begin{aligned} & -\frac{(a+bx^2)^{p+1} (ae^2+bd^2p) {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a}+1\right)}{2a^2d^3(p+1)} \\ & + \frac{e(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} F_1\left(-\frac{1}{2}; -p, 1; \frac{1}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{d^2x} \\ & + \frac{e^4(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d^3(p+1)(ae^2+bd^2)} - \frac{(a+bx^2)^{p+1}}{2adx^2} \end{aligned}$$

[Out]  $-(a+b*x^2)^{(1+p)}/(2*a*d*x^2) + (e*(a+b*x^2)^p*AppellF1[-1/2, -p, 1, 1/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^2*x*(1+(b*x^2)/a)^p) + (e^4*(a+b*x^2)^{(1+p)}*Hypergeometric2F1[1, 1+p, 2+p, (e^2*(a+b*x^2))/(b*d^2+a*e^2)])/(2*d^3*(b*d^2+a*e^2)*(1+p)) - ((a*e^2+b*d^2*p)*(a+b*x^2)^{(1+p)}*Hypergeometric2F1[1, 1+p, 2+p, 1+(b*x^2)/a])/(2*a^2*d^3*(1+p))$

**Rubi [A]** time = 0.582735, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.4$

$$\begin{aligned} & -\frac{(a+bx^2)^{p+1} (ae^2+bd^2p) {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a}+1\right)}{2a^2d^3(p+1)} \\ & + \frac{e(a+bx^2)^p \left(\frac{bx^2}{a}+1\right)^{-p} F_1\left(-\frac{1}{2}; -p, 1; \frac{1}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{d^2x} \\ & + \frac{e^4(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d^3(p+1)(ae^2+bd^2)} - \frac{(a+bx^2)^{p+1}}{2adx^2} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^p/(x^3\*(d + e\*x)), x]

[Out]  $-(a+b*x^2)^{(1+p)}/(2*a*d*x^2) + (e*(a+b*x^2)^p*AppellF1[-1/2, -p, 1, 1/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^2*x*(1+(b*x^2)/a)^p) + (e^4*(a+b*x^2)^{(1+p)}*Hypergeometric2F1[1, 1+p, 2+p, (e^2*(a+b*x^2))/(b*d^2+a*e^2)])/(2*d^3*(b*d^2+a*e^2)*(1+p)) - ((a*e^2+b*d^2*p)*(a+b*x^2)^{(1+p)}*Hypergeometric2F1[1, 1+p, 2+p, 1+(b*x^2)/a])/(2*a^2*d^3*(1+p))$

**Rubi in Sympy [A]** time = 51.6582, size = 240, normalized size = 1.13

$$\begin{aligned} & \frac{e\left(1+\frac{bx^2}{a}\right)^{-p} (a+bx^2)^p {}_2F_1\left(-p, -\frac{1}{2} \middle| -\frac{bx^2}{a} \right)}{d^2x} \\ & \frac{e^2\left(\frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b}(d+ex)}\right)^{-p} \left(-\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b}(d+ex)}\right)^{-p} (a+bx^2)^p \operatorname{appellf}_1\left(-2p, -p, -p, -2p+1, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}\right)}{2d^3p} \\ & - \frac{e^2(a+bx^2)^{p+1} {}_2F_1\left(1, p+1 \middle| 1+\frac{bx^2}{a} \right)}{2ad^3(p+1)} + \frac{b(a+bx^2)^{p+1} {}_2F_1\left(2, p+1 \middle| 1+\frac{bx^2}{a} \right)}{2a^2d(p+1)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**p/x**3/(e*x+d),x)`

[Out]  $e^{(1 + b x^2/a)^{-p}} (a + b x^2)^p \operatorname{hyper}((-p, -1/2), (1/2, ), -b x^2/a)/(d^2 x) - e^{2 \sqrt{b} x + \sqrt{-a}} (\sqrt{b} (d + e x))^{-p} (-e^{-\sqrt{b} x + \sqrt{-a}} / (\sqrt{b} (d + e x)))^p (-p) (a + b x^2)^p \operatorname{appellf1}(-2p, -p, -p, -2p + 1, (d - e \sqrt{b} x) / \sqrt{b} (d + e x), (d + e \sqrt{-a}) / \sqrt{b} (d + e x)) / (2 d^3 p) - e^{2 \sqrt{a} x} (a + b x^2)^{p+1} \operatorname{hyper}((1, p+1), (p+2, ), 1 + b x^2/a) / (2 a d^3 (p+1)) + b (a + b x^2)^{p+1} \operatorname{hyper}((2, p+1), (p+2, ), 1 + b x^2/a) / (2 a^2 d (p+1))$

**Mathematica [A]** time = 0.519981, size = 256, normalized size = 1.2

$$(a + bx^2)^p \left( -\frac{e^2 \left( \frac{e^{(x - \sqrt{-a/b})}}{d + ex} \right)^{-p} \left( \frac{e^{(\sqrt{-a/b} + x)}}{d + ex} \right)^{-p} F_1 \left( -2p; -p, -p; 1 - 2p; \frac{d - \sqrt{-a/b} e}{d + ex}, \frac{d + \sqrt{-a/b} e}{d + ex} \right)}{p} + \left( \frac{a}{bx^2} + 1 \right)^{-p} \left( \frac{d^2 {}_2F_1(1-p, -p; 2-p; -\frac{a}{bx^2})}{(p-1)x^2} + \frac{e^2 {}_2F_1(-p, -p; 1-p; -\frac{a}{bx^2})}{(p-1)x^2} \right) \right) \frac{1}{2d^3}$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(a + b*x^2)^p/(x^3*(d + e*x)),x]`

[Out]  $((a + b x^2)^p (-((e^2 \operatorname{AppellF1}[-2p, -p, -p, 1 - 2p, (d - \operatorname{Sqrt}[-(a/b)]) e] / (d + e x), (d + \operatorname{Sqrt}[-(a/b)]) e] / (d + e x)) / (p ((e^{-\operatorname{Sqrt}[-(a/b)] + x}) / (d + e x))^p ((e^{\operatorname{Sqrt}[-(a/b)] + x}) / (d + e x))^p) + (2 d^2 e \operatorname{Hypergeometric2F1}[-1/2, -p, 1/2, -(b x^2/a)]) / (x^2 (1 + (b x^2/a)^p) + ((d^2 \operatorname{Hypergeometric2F1}[1 - p, -p, 2 - p, -(a/(b x^2))]) / ((-1 + p) x^2) + (e^2 \operatorname{Hypergeometric2F1}[-p, -p, 1 - p, -(a/(b x^2))]) / p) / (1 + a/(b x^2))^p) / (2 d^3)$

**Maple [F]** time = 0.092, size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x^3 (ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^p/x^3/(e*x+d),x)`

[Out] `int((b*x^2+a)^p/x^3/(e*x+d),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{(ex + d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/((e*x + d)*x^3),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p/((e*x + d)*x^3), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^p}{ex^4 + dx^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^p/((e\*x + d)\*x^3),x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^p/(e\*x^4 + d\*x^3), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*p/x\*\*3/(e\*x+d),x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{(ex + d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^p/((e\*x + d)\*x^3),x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^p/((e\*x + d)\*x^3), x)



$$3.416 \quad \int \frac{x^4(a+bx^2)^p}{(d+ex)^2} dx$$

**Optimal.** Leaf size=392

$$\frac{x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (a^2e^4 - 2abd^2e^2(3p+4) - 2b^2d^4(2p^2+7p+6)) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{be^4(2p+3)(ae^2+bd^2)} - \frac{2d^2x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (2ae^2+bd^2(p+2)) F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{e^4(ae^2+bd^2)} - \frac{d^4(a+bx^2)^{p+1}}{e^3(d+ex)(ae^2+bd^2)} + \frac{d^3(a+bx^2)^{p+1} (2ae^2+bd^2(p+2)) {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{e^3(p+1)(ae^2+bd^2)^2} - \frac{d(3p+4)(a+bx^2)^{p+1}}{be^3(p+1)(2p+3)} + \frac{(d+ex)(a+bx^2)^{p+1}}{be^3(2p+3)}$$

[Out]  $-\left(\frac{(d(4+3p)(a+bx^2)^{1+p})/(b^3e^3(1+p)(3+2p))}{(d^4(a+bx^2)^{1+p})/(e^3(bd^2+ae^2)(d+ex))} + \frac{(d+ex)(a+bx^2)^{1+p}/(b^3e^3(3+2p))}{(2d^2(2+a^2e^2+bd^2(2+p))x^2(a+bx^2)^p \text{AppellF1}[1/2, -p, 1, 3/2, -(bx^2/a), (e^2x^2)/d^2])/(e^4(bd^2+ae^2)(1+(bx^2/a)^p))} - \frac{(a^2e^4 - 2a^2bd^2e^2(4+3p) - 2b^2d^4(6+7p+2p^2))x^2(a+bx^2)^p \text{Hypergeometric2F1}[1/2, -p, 3/2, -(bx^2/a)]}{(b^3e^4(bd^2+ae^2)(3+2p)(1+(bx^2/a)^p) + (d^3(2a^2e^2+bd^2(2+p))(a+bx^2)^{1+p} \text{Hypergeometric2F1}[1, 1+p, 2+p, (e^2(a+bx^2))/(bd^2+ae^2)])/(e^3(bd^2+ae^2))^2(1+p))}\right)$

**Rubi [A]** time = 1.54403, antiderivative size = 392, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (a^2e^4 - 2abd^2e^2(3p+4) - 2b^2d^4(2p^2+7p+6)) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{be^4(2p+3)(ae^2+bd^2)} - \frac{2d^2x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (2ae^2+bd^2(p+2)) F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{e^4(ae^2+bd^2)} - \frac{d^4(a+bx^2)^{p+1}}{e^3(d+ex)(ae^2+bd^2)} + \frac{d^3(a+bx^2)^{p+1} (2ae^2+bd^2(p+2)) {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{e^3(p+1)(ae^2+bd^2)^2} - \frac{d(3p+4)(a+bx^2)^{p+1}}{be^3(p+1)(2p+3)} + \frac{(d+ex)(a+bx^2)^{p+1}}{be^3(2p+3)}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*(a+bx^2)^p)/(d+ex)^2, x]

[Out]  $-\left(\frac{(d(4+3p)(a+bx^2)^{1+p})/(b^3e^3(1+p)(3+2p))}{(d^4(a+bx^2)^{1+p})/(e^3(bd^2+ae^2)(d+ex))} + \frac{(d+ex)(a+bx^2)^{1+p}/(b^3e^3(3+2p))}{(2d^2(2+a^2e^2+bd^2(2+p))x^2(a+bx^2)^p \text{AppellF1}[1/2, -p, 1, 3/2, -(bx^2/a), (e^2x^2)/d^2])/(e^4(bd^2+ae^2)(1+(bx^2/a)^p))} - \frac{(a^2e^4 - 2a^2bd^2e^2(4+3p) - 2b^2d^4(6+7p+2p^2))x^2(a+bx^2)^p \text{Hypergeometric2F1}[1/2, -p, 3/2, -(bx^2/a)]}{(b^3e^4(bd^2+ae^2)(3+2p)(1+(bx^2/a)^p) + (d^3(2a^2e^2+bd^2(2+p))(a+bx^2)^{1+p} \text{Hypergeometric2F1}[1, 1+p, 2+p, (e^2(a+bx^2))/(bd^2+ae^2)])/(e^3(bd^2+ae^2))^2(1+p))}\right)$

**Rubi in Sympy [A]** time = 75.0462, size = 381, normalized size = 0.97

$$\frac{d^4 \left( \frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}} \right)^{-p} \left( -\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}} \right)^{-p} (a+bx^2)^p \left( \frac{1}{d+ex} \right)^{2p} \left( \frac{1}{d+ex} \right)^{-2p+1} \operatorname{appellf}_1 \left( -2p+1, -p, -p, -2p+2, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex} \right)}{e^5(-2p+1)}$$

$$\frac{2d^3 \left( \frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}} \right)^{-p} \left( -\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}} \right)^{-p} (a+bx^2)^p \operatorname{appellf}_1 \left( -2p, -p, -p, -2p+1, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex} \right)}{e^{5p}}$$

$$+ \frac{3d^2 x \left( 1 + \frac{bx^2}{a} \right)^{-p} (a+bx^2)^p {}_2F_1 \left( \begin{matrix} -p, \frac{1}{2} \\ \frac{3}{2} \end{matrix} \middle| -\frac{bx^2}{a} \right)}{e^4}$$

$$+ \frac{x^3 \left( 1 + \frac{bx^2}{a} \right)^{-p} (a+bx^2)^p {}_2F_1 \left( \begin{matrix} -p, \frac{3}{2} \\ \frac{5}{2} \end{matrix} \middle| -\frac{bx^2}{a} \right)}{3e^2} - \frac{d(a+bx^2)^{p+1}}{be^3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**4*(b*x**2+a)**p/(e*x+d)**2,x)`

[Out]  $-d^{4p} \left( \frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}} \right)^{-p} \left( -\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}} \right)^{-p} (a+bx^2)^p \left( \frac{1}{d+ex} \right)^{2p} \left( \frac{1}{d+ex} \right)^{-2p+1} \operatorname{appellf}_1(-2p+1, -p, -p, -2p+2, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}) - 2d^{3p} \left( \frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}} \right)^{-p} \left( -\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}} \right)^{-p} (a+bx^2)^p \operatorname{appellf}_1(-2p, -p, -p, -2p+1, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}) + 3d^{2p} x \left( 1 + \frac{bx^2}{a} \right)^{-p} (a+bx^2)^p \operatorname{hyper}((-p, 1/2), (3/2, ), -\frac{bx^2}{a}) / e^{4p} + x^3 \left( 1 + \frac{bx^2}{a} \right)^{-p} (a+bx^2)^p \operatorname{hyper}((-p, 3/2), (5/2, ), -\frac{bx^2}{a}) / (3e^{2p}) - d^{p+1} (a+bx^2)^p / (be^{3p}(p+1))$

**Mathematica [A]** time = 1.05896, size = 0, normalized size = 0.

$$\int \frac{x^4 (a+bx^2)^p}{(d+ex)^2} dx$$

Verification is Not applicable to the result.

[In] `Integrate[(x^4*(a+b*x^2)^p)/(d+e*x)^2,x]`

[Out] `Integrate[(x^4*(a+b*x^2)^p)/(d+e*x)^2,x]`

**Maple [F]** time = 0.1, size = 0, normalized size = 0.

$$\int \frac{x^4 (bx^2+a)^p}{(ex+d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(b*x^2+a)^p/(e*x+d)^2,x)`

[Out] `int(x^4*(b*x^2+a)^p/(e*x+d)^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p x^4}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^p\*x^4/(e\*x + d)^2,x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^p\*x^4/(e\*x + d)^2, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^p x^4}{e^2 x^2 + 2 dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^p\*x^4/(e\*x + d)^2,x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^p\*x^4/(e^2\*x^2 + 2\*d\*e\*x + d^2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(b\*x\*\*2+a)\*\*p/(e\*x+d)\*\*2,x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p x^4}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^p\*x^4/(e\*x + d)^2,x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^p\*x^4/(e\*x + d)^2, x)

$$3.417 \quad \int \frac{x^3(a+bx^2)^p}{(d+ex)^2} dx$$

**Optimal.** Leaf size=321

$$\frac{dx (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (3ae^2 + bd^2(2p + 3)) F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{e^3 (ae^2 + bd^2)}$$

$$- \frac{d^2 (a + bx^2)^{p+1} (3ae^2 + bd^2(2p + 3)) {}_2F_1\left(1, p + 1; p + 2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e^2(p + 1)(ae^2 + bd^2)^2}$$

$$- \frac{dx (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (2ae^2 + bd^2(2p + 3)) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{e^3 (ae^2 + bd^2)}$$

$$+ \frac{d^3 (a + bx^2)^{p+1}}{e^2(d + ex)(ae^2 + bd^2)} + \frac{(a + bx^2)^{p+1}}{2be^2(p + 1)}$$

[Out] (a + b\*x^2)^(1 + p)/(2\*b\*e^2\*(1 + p)) + (d^3\*(a + b\*x^2)^(1 + p))/(e^2\*(b\*d^2 + a\*e^2)\*(d + e\*x)) + (d\*(3\*a\*e^2 + b\*d^2\*(3 + 2\*p))\*x\*(a + b\*x^2)^p\*AppellF1[1/2, -p, 1, 3/2, -((b\*x^2)/a), (e^2\*x^2)/d^2])/(e^3\*(b\*d^2 + a\*e^2)\*(1 + (b\*x^2)/a)^p) - (d\*(2\*a\*e^2 + b\*d^2\*(3 + 2\*p))\*x\*(a + b\*x^2)^p\*Hypergeometric2F1[1/2, -p, 3/2, -((b\*x^2)/a)])/(e^3\*(b\*d^2 + a\*e^2)\*(1 + (b\*x^2)/a)^p) - (d^2\*(3\*a\*e^2 + b\*d^2\*(3 + 2\*p))\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2\*(a + b\*x^2))/(b\*d^2 + a\*e^2)])/(2\*e^2\*(b\*d^2 + a\*e^2)^2\*(1 + p))

**Rubi [A]** time = 0.999818, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{dx (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (3ae^2 + bd^2(2p + 3)) F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{e^3 (ae^2 + bd^2)}$$

$$- \frac{d^2 (a + bx^2)^{p+1} (3ae^2 + bd^2(2p + 3)) {}_2F_1\left(1, p + 1; p + 2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e^2(p + 1)(ae^2 + bd^2)^2}$$

$$- \frac{dx (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (2ae^2 + bd^2(2p + 3)) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{e^3 (ae^2 + bd^2)}$$

$$+ \frac{d^3 (a + bx^2)^{p+1}}{e^2(d + ex)(ae^2 + bd^2)} + \frac{(a + bx^2)^{p+1}}{2be^2(p + 1)}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(a + b\*x^2)^p)/(d + e\*x)^2, x]

[Out] (a + b\*x^2)^(1 + p)/(2\*b\*e^2\*(1 + p)) + (d^3\*(a + b\*x^2)^(1 + p))/(e^2\*(b\*d^2 + a\*e^2)\*(d + e\*x)) + (d\*(3\*a\*e^2 + b\*d^2\*(3 + 2\*p))\*x\*(a + b\*x^2)^p\*AppellF1[1/2, -p, 1, 3/2, -((b\*x^2)/a), (e^2\*x^2)/d^2])/(e^3\*(b\*d^2 + a\*e^2)\*(1 + (b\*x^2)/a)^p) - (d\*(2\*a\*e^2 + b\*d^2\*(3 + 2\*p))\*x\*(a + b\*x^2)^p\*Hypergeometric2F1[1/2, -p, 3/2, -((b\*x^2)/a)])/(e^3\*(b\*d^2 + a\*e^2)\*(1 + (b\*x^2)/a)^p) - (d^2\*(3\*a\*e^2 + b\*d^2\*(3 + 2\*p))\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2\*(a + b\*x^2))/(b\*d^2 + a\*e^2)])/(2\*e^2\*(b\*d^2 + a\*e^2)^2\*(1 + p))

**Rubi in Sympy [A]** time = 64.7212, size = 338, normalized size = 1.05

$$\frac{d^3 \left( \frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b}(d+ex)} \right)^{-p} \left( -\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b}(d+ex)} \right)^{-p} (a+bx^2)^p \left( \frac{1}{d+ex} \right)^{2p} \left( \frac{1}{d+ex} \right)^{-2p+1} \operatorname{appellf}_1 \left( -2p+1, -p, -p, -2p+2, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex} \right)}{e^4(-2p+1)} + \frac{3d^2 \left( \frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b}(d+ex)} \right)^{-p} \left( -\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b}(d+ex)} \right)^{-p} (a+bx^2)^p \operatorname{appellf}_1 \left( -2p, -p, -p, -2p+1, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex} \right)}{2e^4p} - \frac{2dx \left( 1 + \frac{bx^2}{a} \right)^{-p} (a+bx^2)^p {}_2F_1 \left( \begin{matrix} -p, \frac{1}{2} \\ \frac{3}{2} \end{matrix} \middle| -\frac{bx^2}{a} \right)}{e^3} + \frac{(a+bx^2)^{p+1}}{2be^2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**3*(b*x**2+a)**p/(e*x+d)**2,x)`

[Out]  $d^{*3} \left( e \left( \sqrt{b}x + \sqrt{-a} \right) / \left( \sqrt{b} \left( d + e^*x \right) \right) \right)^{*(-p)} \left( -e \left( -\sqrt{b}x + \sqrt{-a} \right) / \left( \sqrt{b} \left( d + e^*x \right) \right) \right)^{*(-p)} \left( a + b^*x^{*2} \right)^{*p} \left( \frac{1}{d + e^*x} \right)^{*(2^*p)} \left( \frac{1}{d + e^*x} \right)^{*(-2^*p + 1)} \operatorname{appellf}_1 \left( -2^*p + 1, -p, -p, -2^*p + 2, \frac{d - e^*\sqrt{-a} / \sqrt{b}}{d + e^*x}, \frac{d + e^*\sqrt{-a} / \sqrt{b}}{d + e^*x} \right) + 3^*d^{*2} \left( e \left( \sqrt{b}x + \sqrt{-a} \right) / \left( \sqrt{b} \left( d + e^*x \right) \right) \right)^{*(-p)} \left( -e \left( -\sqrt{b}x + \sqrt{-a} \right) / \left( \sqrt{b} \left( d + e^*x \right) \right) \right)^{*(-p)} \left( a + b^*x^{*2} \right)^{*p} \operatorname{appellf}_1 \left( -2^*p, -p, -p, -2^*p + 1, \frac{d - e^*\sqrt{-a} / \sqrt{b}}{d + e^*x}, \frac{d + e^*\sqrt{-a} / \sqrt{b}}{d + e^*x} \right) / \left( 2^*e^{*4} \right)^p - 2^*d^*x \left( 1 + \frac{b^*x^{*2}}{a} \right)^{*(-p)} \left( a + b^*x^{*2} \right)^{*p} \operatorname{hyper} \left( \left( -p, \frac{1}{2} \right), \left( \frac{3}{2}, \right), -\frac{b^*x^{*2}}{a} / e^{*3} + \left( a + b^*x^{*2} \right)^{*(p+1)} / \left( 2^*b^*e^{*2} \left( p + 1 \right) \right) \right)$

**Mathematica [A]** time = 1.47693, size = 343, normalized size = 1.07

$$(a+bx^2)^p \left( -\frac{2d^3 \left( \frac{e(x-\sqrt{-\frac{a}{b}})}{d+ex} \right)^{-p} \left( \frac{e(\sqrt{-\frac{a}{b}+x})}{d+ex} \right)^{-p} F_1 \left( 1-2p; -p, -p; 2-2p; \frac{d-\sqrt{-\frac{a}{b}}e}{d+ex}, \frac{d+\sqrt{-\frac{a}{b}}e}{d+ex} \right)}{(2p-1)(d+ex)} + \frac{3d^2 \left( \frac{e(x-\sqrt{-\frac{a}{b}})}{d+ex} \right)^{-p} \left( \frac{e(\sqrt{-\frac{a}{b}+x})}{d+ex} \right)^{-p} F_1 \left( -2p; -p, -p; 1-2p; \frac{d-\sqrt{-\frac{a}{b}}e}{d+ex}, \frac{d+\sqrt{-\frac{a}{b}}e}{d+ex} \right)}{p} \right) / 2e^4$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(x^3*(a+b*x^2)^p)/(d+e*x)^2,x]`

[Out]  $\left( \left( a + b^*x^{*2} \right)^{*p} \left( \left( -2^*d^{*3} \operatorname{AppellF}_1 \left[ 1 - 2^*p, -p, -p, 2 - 2^*p, \frac{d - \sqrt{-\frac{a}{b}}e}{d + e^*x}, \frac{d + \sqrt{-\frac{a}{b}}e}{d + e^*x} \right] \right) / \left( (-1 + 2^*p) \left( \left( e \left( -\sqrt{-\frac{a}{b}} \right) + x \right) / \left( d + e^*x \right) \right)^{*p} \left( \left( e \left( \sqrt{-\frac{a}{b}+x} \right) + x \right) / \left( d + e^*x \right) \right)^{*p} \left( d + e^*x \right) \right) + \left( 3^*d^{*2} \operatorname{AppellF}_1 \left[ -2^*p, -p, -p, 1 - 2^*p, \frac{d - \sqrt{-\frac{a}{b}}e}{d + e^*x}, \frac{d + \sqrt{-\frac{a}{b}}e}{d + e^*x} \right] \right) / \left( p \left( \left( e \left( -\sqrt{-\frac{a}{b}} \right) + x \right) / \left( d + e^*x \right) \right)^{*p} \left( \left( e \left( \sqrt{-\frac{a}{b}+x} \right) + x \right) / \left( d + e^*x \right) \right)^{*p} \right) + e \left( \left( a + b^*x^{*2} - \frac{a}{1 + \left( b^*x^{*2} \right) / a} \right) / \left( b + b^*p \right) - \left( 4^*d^*x \operatorname{Hypergeometric}_2F_1 \left[ \frac{1}{2}, -p, \frac{3}{2}, -\frac{\left( b^*x^{*2} \right)}{a} \right] \right) / \left( 1 + \frac{\left( b^*x^{*2} \right)}{a} \right) \right) \right) / \left( 2^*e^{*4} \right)$

**Maple [F]** time = 0.088, size = 0, normalized size = 0.

$$\int \frac{x^3 (bx^2 + a)^p}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(b*x^2+a)^p/(e*x+d)^2,x)`

[Out] `int(x^3*(b*x^2+a)^p/(e*x+d)^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p x^3}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^3/(e*x + d)^2,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p*x^3/(e*x + d)^2, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^p x^3}{e^2 x^2 + 2 dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^3/(e*x + d)^2,x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p*x^3/(e^2*x^2 + 2*d*e*x + d^2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(b*x**2+a)**p/(e*x+d)**2,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p x^3}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^3/(e*x + d)^2,x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*x^3/(e*x + d)^2, x)`

$$3.418 \quad \int \frac{x^2(a+bx^2)^p}{(d+ex)^2} dx$$

**Optimal.** Leaf size=281

$$\begin{aligned} & - \frac{2x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (ae^2 + bd^2(p+1)) F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{e^2(ae^2 + bd^2)} \\ & + \frac{x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (ae^2 + 2bd^2(p+1)) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{e^2(ae^2 + bd^2)} \\ & + \frac{d(a+bx^2)^{p+1} (ae^2 + bd^2(p+1)) {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{e(p+1)(ae^2 + bd^2)^2} - \frac{d^2(a+bx^2)^{p+1}}{e(d+ex)(ae^2 + bd^2)} \end{aligned}$$

[Out]  $-\left(\frac{d^2(a+bx^2)^{p+1}}{e^2(ae^2 + bd^2)}\right) / \left(\frac{e^2(bd^2 + ae^2)(d+ex)}{e^2(ae^2 + bd^2)}\right) - \left(2 \left(\frac{a^2e^2 + b^2d^2(1+p)}{e^2(ae^2 + bd^2)}\right) x \left(\frac{bx^2}{a} + 1\right)^{-p} \text{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\left(\frac{bx^2}{a}\right), \left(\frac{e^2x^2}{d^2}\right)\right] / \left(\frac{e^2(bd^2 + ae^2)(1 + (bx^2)/a)^p}{e^2(ae^2 + bd^2)}\right) + \left(\frac{a^2e^2 + 2b^2d^2(1+p)}{e^2(ae^2 + bd^2)}\right) x \left(\frac{bx^2}{a} + 1\right)^{-p} \text{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\left(\frac{bx^2}{a}\right)\right] / \left(\frac{e^2(bd^2 + ae^2)(1 + (bx^2)/a)^p}{e^2(ae^2 + bd^2)}\right) + \left(\frac{d^2(a+bx^2)^{p+1}}{e^2(ae^2 + bd^2)^2}\right) / \left(\frac{e^2(bd^2 + ae^2)(1 + (bx^2)/a)^p}{e^2(ae^2 + bd^2)}\right) - \frac{d^2(a+bx^2)^{p+1}}{e(d+ex)(ae^2 + bd^2)}\right)$

**Rubi [A]** time = 0.655827, antiderivative size = 277, normalized size of antiderivative = 0.99, number of steps used = 10, number of rules used = 9, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$

$$\begin{aligned} & - \frac{2x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (ae^2 + bd^2(p+1)) F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{e^2(ae^2 + bd^2)} \\ & + \frac{x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} \left(a + \frac{2bd^2(p+1)}{e^2}\right) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{ae^2 + bd^2} \\ & + \frac{d(a+bx^2)^{p+1} (ae^2 + bd^2(p+1)) {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{e(p+1)(ae^2 + bd^2)^2} - \frac{d^2(a+bx^2)^{p+1}}{e(d+ex)(ae^2 + bd^2)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(a + b\*x^2)^p)/(d + e\*x)^2, x]

[Out]  $-\left(\frac{d^2(a+bx^2)^{p+1}}{e^2(ae^2 + bd^2)}\right) / \left(\frac{e^2(bd^2 + ae^2)(d+ex)}{e^2(ae^2 + bd^2)}\right) - \left(2 \left(\frac{a^2e^2 + b^2d^2(1+p)}{e^2(ae^2 + bd^2)}\right) x \left(\frac{bx^2}{a} + 1\right)^{-p} \text{AppellF1}\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\left(\frac{bx^2}{a}\right), \left(\frac{e^2x^2}{d^2}\right)\right] / \left(\frac{e^2(bd^2 + ae^2)(1 + (bx^2)/a)^p}{e^2(ae^2 + bd^2)}\right) + \left(\frac{a + (2b^2d^2(1+p))/e^2}{ae^2 + bd^2}\right) x \left(\frac{bx^2}{a} + 1\right)^{-p} \text{Hypergeometric2F1}\left[\frac{1}{2}, -p, \frac{3}{2}, -\left(\frac{bx^2}{a}\right)\right] / \left(\frac{e^2(bd^2 + ae^2)(1 + (bx^2)/a)^p}{e^2(ae^2 + bd^2)}\right) + \left(\frac{d^2(a+bx^2)^{p+1}}{e^2(ae^2 + bd^2)^2}\right) / \left(\frac{e^2(bd^2 + ae^2)(1 + (bx^2)/a)^p}{e^2(ae^2 + bd^2)}\right) - \frac{d^2(a+bx^2)^{p+1}}{e(d+ex)(ae^2 + bd^2)}\right)$

**Rubi in Sympy [A]** time = 59.0981, size = 309, normalized size = 1.1

$$\frac{d^2 \left( \frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}} \right)^{-p} \left( -\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}} \right)^{-p} (a+bx^2)^p \left( \frac{1}{d+ex} \right)^{2p} \left( \frac{1}{d+ex} \right)^{-2p+1} \operatorname{appellf}_1 \left( -2p+1, -p, -p, -2p+2, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex} \right)}{e^3(-2p+1)} \\ + \frac{d \left( \frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}} \right)^{-p} \left( -\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}} \right)^{-p} (a+bx^2)^p \operatorname{appellf}_1 \left( -2p, -p, -p, -2p+1, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex} \right)}{e^3 p} \\ + \frac{x \left( 1 + \frac{bx^2}{a} \right)^{-p} (a+bx^2)^p {}_2F_1 \left( -p, \frac{1}{2} \middle| -\frac{bx^2}{a} \right)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**2*(b*x**2+a)**p/(e*x+d)**2,x)`

[Out] `-d**2*(e*(sqrt(b)*x + sqrt(-a))/(sqrt(b)*(d + e*x)))**(-p)*(-e*(-sqrt(b)*x + sqrt(-a))/(sqrt(b)*(d + e*x)))**(-p)*(a + b*x**2)**p*(1/(d + e*x))**(2*p)*(1/(d + e*x))**(-2*p + 1)*appellf1(-2*p + 1, -p, -p, -2*p + 2, (d - e*sqrt(-a)/sqrt(b))/(d + e*x), (d + e*sqrt(-a)/sqrt(b))/(d + e*x))/(e**3*(-2*p + 1)) - d*(e*(sqrt(b)*x + sqrt(-a))/(sqrt(b)*(d + e*x)))**(-p)*(-e*(-sqrt(b)*x + sqrt(-a))/(sqrt(b)*(d + e*x)))**(-p)*(a + b*x**2)**p*appellf1(-2*p, -p, -p, -2*p + 1, (d - e*sqrt(-a)/sqrt(b))/(d + e*x), (d + e*sqrt(-a)/sqrt(b))/(d + e*x))/(e**3*p) + x*(1 + b*x**2/a)**(-p)*(a + b*x**2)**p*hyper((-p, 1/2), (3/2, ), -b*x**2/a)/e**2`

**Mathematica [A]** time = 0.541552, size = 300, normalized size = 1.07

$$\frac{(a+bx^2)^p \left( \frac{d^2 \left( \frac{e(x-\sqrt{\frac{-a}{b}})}{d+ex} \right)^{-p} \left( \frac{e(\sqrt{\frac{-a}{b}+x})}{d+ex} \right)^{-p} F_1 \left( 1-2p; -p, -p; 2-2p; \frac{d-\sqrt{\frac{-a}{b}}e}{d+ex}, \frac{d+\sqrt{\frac{-a}{b}}e}{d+ex} \right)}{(2p-1)(d+ex)} - \frac{d \left( \frac{e(x-\sqrt{\frac{-a}{b}})}{d+ex} \right)^{-p} \left( \frac{e(\sqrt{\frac{-a}{b}+x})}{d+ex} \right)^{-p} F_1 \left( -2p; -p, -p; 1-2p; \frac{d-\sqrt{\frac{-a}{b}}e}{d+ex}, \frac{d+\sqrt{\frac{-a}{b}}e}{d+ex} \right)}{p} \right)}{e^3}$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(x^2*(a + b*x^2)^p)/(d + e*x)^2,x]`

[Out] `((a + b*x^2)^p*((d^2*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/((-1 + 2*p)*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p*(d + e*x) - (d*AppellF1[-2*p, -p, -p, 1 - 2*p, (d - Sqrt[-(a/b)]*e)/(d + e*x), (d + Sqrt[-(a/b)]*e)/(d + e*x)])/(p*((e*(-Sqrt[-(a/b)] + x))/(d + e*x))^p*((e*(Sqrt[-(a/b)] + x))/(d + e*x))^p) + (e*x*Hypergeometric2F1[1/2, -p, 3/2, -(b*x^2/a)])/(1 + (b*x^2/a)^p))/e^3`

**Maple [F]** time = 0.086, size = 0, normalized size = 0.

$$\int \frac{x^2 (bx^2 + a)^p}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*x^2+a)^p/(e*x+d)^2,x)`



[Out] `int(x^2*(b*x^2+a)^p/(e*x+d)^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p x^2}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^2/(e*x + d)^2,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p*x^2/(e*x + d)^2, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^p x^2}{e^2 x^2 + 2 dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^2/(e*x + d)^2,x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p*x^2/(e^2*x^2 + 2*d*e*x + d^2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b*x**2+a)**p/(e*x+d)**2,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p x^2}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^2/(e*x + d)^2,x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*x^2/(e*x + d)^2, x)`

$$3.419 \quad \int \frac{x(a+bx^2)^p}{(d+ex)^2} dx$$

**Optimal.** Leaf size=273

$$\frac{x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (ae^2 + bd^2(2p+1)) F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{de(ae^2 + bd^2)} - \frac{bd(2p+1)x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{e(ae^2 + bd^2)} - \frac{(a+bx^2)^{p+1} (ae^2 + bd^2(2p+1)) {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2 + bd^2)^2} + \frac{d(a+bx^2)^{p+1}}{(d+ex)(ae^2 + bd^2)}$$

[Out] (d\*(a + b\*x^2)^(1 + p))/((b\*d^2 + a\*e^2)\*(d + e\*x)) + ((a\*e^2 + b\*d^2\*(1 + 2\*p))\*x\*(a + b\*x^2)^p\*AppellF1[1/2, -p, 1, 3/2, -(b\*x^2)/a, (e^2\*x^2)/d^2])/((d\*e\*(b\*d^2 + a\*e^2)\*(1 + (b\*x^2)/a)^p) - (b\*d\*(1 + 2\*p)\*x\*(a + b\*x^2)^p\*Hypergeometric2F1[1/2, -p, 3/2, -(b\*x^2)/a]))/(e\*(b\*d^2 + a\*e^2)\*(1 + (b\*x^2)/a)^p) - ((a\*e^2 + b\*d^2\*(1 + 2\*p))\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2\*(a + b\*x^2))/(b\*d^2 + a\*e^2)]/(b\*d^2 + a\*e^2)]/(2\*(b\*d^2 + a\*e^2)^2\*(1 + p))

**Rubi [A]** time = 0.50916, antiderivative size = 273, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (ae^2 + bd^2(2p+1)) F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{de(ae^2 + bd^2)} - \frac{bd(2p+1)x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{e(ae^2 + bd^2)} - \frac{(a+bx^2)^{p+1} (ae^2 + bd^2(2p+1)) {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2 + bd^2)^2} + \frac{d(a+bx^2)^{p+1}}{(d+ex)(ae^2 + bd^2)}$$

Antiderivative was successfully verified.

[In] Int[(x\*(a + b\*x^2)^p)/(d + e\*x)^2, x]

[Out] (d\*(a + b\*x^2)^(1 + p))/((b\*d^2 + a\*e^2)\*(d + e\*x)) + ((a\*e^2 + b\*d^2\*(1 + 2\*p))\*x\*(a + b\*x^2)^p\*AppellF1[1/2, -p, 1, 3/2, -(b\*x^2)/a, (e^2\*x^2)/d^2])/((d\*e\*(b\*d^2 + a\*e^2)\*(1 + (b\*x^2)/a)^p) - (b\*d\*(1 + 2\*p)\*x\*(a + b\*x^2)^p\*Hypergeometric2F1[1/2, -p, 3/2, -(b\*x^2)/a]))/(e\*(b\*d^2 + a\*e^2)\*(1 + (b\*x^2)/a)^p) - ((a\*e^2 + b\*d^2\*(1 + 2\*p))\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2\*(a + b\*x^2))/(b\*d^2 + a\*e^2)]/(b\*d^2 + a\*e^2)]/(2\*(b\*d^2 + a\*e^2)^2\*(1 + p))

**Rubi in Sympy [A]** time = 48.7394, size = 230, normalized size = 0.84

$$\frac{bdx \left(1 + \frac{bx^2}{a}\right)^{-p} (a+bx^2)^p (2p+1) {}_2F_1\left(-p, \frac{1}{2}; \frac{3}{2}; -\frac{bx^2}{a}\right)}{e(ae^2 + bd^2)} + \frac{d(a+bx^2)^{p+1}}{(d+ex)(ae^2 + bd^2)} + \frac{\left(\frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} \left(-\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} (a+bx^2)^p (ae^2 + bd^2(2p+1)) \text{appellf}_1\left(-2p, -p, -p, -2p+1, \frac{d-e\sqrt{-a}}{d+ex}, \frac{d+e\sqrt{-a}}{d+ex}\right)}{2e^2p(ae^2 + bd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x*(b*x**2+a)**p/(e*x+d)**2,x)`

[Out]  $-b*d*x*(1 + b*x**2/a)**(-p)*(a + b*x**2)**p*(2*p + 1)*\text{hyper}((-p, 1/2), (3/2, ), -b*x**2/a)/(e*(a*e**2 + b*d**2)) + d*(a + b*x**2)**(p + 1)/((d + e*x)*(a*e**2 + b*d**2)) + (e*(\text{sqrt}(b)*x + \text{sqrt}(-a))/(\text{sqrt}(b)*(d + e*x)))**(-p)*(-e*(-\text{sqrt}(b)*x + \text{sqrt}(-a))/(\text{sqrt}(b)*(d + e*x)))**(-p)*(a + b*x**2)**p*(a*e**2 + b*d**2*(2*p + 1))*\text{appellf1}(-2*p, -p, -p, -2*p + 1, (d - e*\text{sqrt}(-a)/\text{sqrt}(b))/(\text{sqrt}(b)), (d + e*\text{sqrt}(-a)/\text{sqrt}(b))/(\text{sqrt}(b)))/(2*e**2*p*(a*e**2 + b*d**2))$

**Mathematica [A]** time = 0.23003, size = 223, normalized size = 0.82

$$\frac{(a + bx^2)^p \left( \frac{e^{(x - \sqrt{-a/b})}}{d + ex} \right)^{-p} \left( \frac{e^{(\sqrt{-a/b} + x)}}{d + ex} \right)^{-p} \left( (2p - 1)(d + ex)F_1 \left( -2p; -p, -p; 1 - 2p; \frac{d - \sqrt{-a/b}e}{d + ex}, \frac{d + \sqrt{-a/b}e}{d + ex} \right) - 2dpF_1 \left( 1 - 2p; -p, -p; 1 - 2p; \frac{d - \sqrt{-a/b}e}{d + ex}, \frac{d + \sqrt{-a/b}e}{d + ex} \right) \right)}{2e^{2p}(2p - 1)(d + ex)}$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(x*(a + b*x^2)^p)/(d + e*x)^2,x]`

[Out]  $((a + b*x^2)^p*(-2*d*p*\text{AppellF1}[1 - 2*p, -p, -p, 2 - 2*p, (d - \text{Sqrt}[-(a/b)]*e)/(d + e*x), (d + \text{Sqrt}[-(a/b)]*e)/(d + e*x)] + (-1 + 2*p)*(d + e*x)*\text{AppellF1}[-2*p, -p, -p, 1 - 2*p, (d - \text{Sqrt}[-(a/b)]*e)/(d + e*x), (d + \text{Sqrt}[-(a/b)]*e)/(d + e*x)]))/(2*e^2*p*(-1 + 2*p)*((e*(-\text{Sqrt}[-(a/b)] + x))/(d + e*x))^p*((e*(\text{Sqrt}[-(a/b)] + x))/(d + e*x))^p*(d + e*x))$

**Maple [F]** time = 0.077, size = 0, normalized size = 0.

$$\int \frac{x (bx^2 + a)^P}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b*x^2+a)^p/(e*x+d)^2,x)`

[Out] `int(x*(b*x^2+a)^p/(e*x+d)^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^P x}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x/(e*x + d)^2,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p*x/(e*x + d)^2, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{(bx^2 + a)^P x}{e^2 x^2 + 2dex + d^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x/(e*x + d)^2,x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p*x/(e^2*x^2 + 2*d*e*x + d^2), x)`

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(b*x**2+a)**p/(e*x+d)**2,x)`

[Out] Timed out

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p x}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x/(e*x + d)^2,x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*x/(e*x + d)^2, x)`

$$3.420 \quad \int \frac{(a+bx^2)^p}{(d+ex)^2} dx$$

**Optimal.** Leaf size=244

$$\begin{aligned} & - \frac{2bpx (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{ae^2 + bd^2} \\ & + \frac{b(2p+1)x (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{ae^2 + bd^2} \\ & - \frac{bde (a+bx^2)^{p+1} {}_2F_1\left(2, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{(p+1)(ae^2 + bd^2)^2} + \frac{e^2x (a+bx^2)^{p+1}}{(d^2 - e^2x^2)(ae^2 + bd^2)} \end{aligned}$$

[Out]  $(e^2x^2(a+bx^2)^{1+p})/((b^2d^2+a^2e^2)(d^2-e^2x^2)) - (2^2b^2p^2x^2(a+bx^2)^p \text{AppellF1}[1/2, -p, 1, 3/2, -(b^2x^2)/a], (e^2x^2)/d^2])/((b^2d^2+a^2e^2)(1+(b^2x^2)/a)^p) + (b^2(1+2p)^2x^2(a+bx^2)^p \text{Hypergeometric2F1}[1/2, -p, 3/2, -(b^2x^2)/a])/((b^2d^2+a^2e^2)(1+(b^2x^2)/a)^p) - (b^2d^2e^2(a+bx^2)^{1+p} \text{Hypergeometric2F1}[2, 1+p, 2+p, (e^2(a+bx^2))/(b^2d^2+a^2e^2)])/((b^2d^2+a^2e^2)^2(1+p))$

**Rubi [A]** time = 0.412003, antiderivative size = 191, normalized size of antiderivative = 0.78, number of steps used = 8, number of rules used = 7, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$

$$\begin{aligned} & \frac{x (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; -p, 2; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{d^2} \\ & + \frac{e^2x^3 (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{3}{2}; -p, 2; \frac{5}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{3d^4} \\ & - \frac{bde (a+bx^2)^{p+1} {}_2F_1\left(2, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{(p+1)(ae^2 + bd^2)^2} \end{aligned}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + b\*x^2)^p/(d + e\*x)^2, x]

[Out]  $(x^2(a+bx^2)^p \text{AppellF1}[1/2, -p, 2, 3/2, -(b^2x^2)/a], (e^2x^2)/d^2])/((d^2(1+(b^2x^2)/a)^p) + (e^2x^2)^3(a+bx^2)^p \text{AppellF1}[3/2, -p, 2, 5/2, -(b^2x^2)/a], (e^2x^2)/d^2))/(3d^4(1+(b^2x^2)/a)^p) - (b^2d^2e^2(a+bx^2)^{1+p} \text{Hypergeometric2F1}[2, 1+p, 2+p, (e^2(a+bx^2))/(b^2d^2+a^2e^2)])/((b^2d^2+a^2e^2)^2(1+p))$

**Rubi in Sympy [A]** time = 18.2297, size = 144, normalized size = 0.59

$$\frac{\left(\frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} \left(-\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} (a+bx^2)^p \left(\frac{1}{d+ex}\right)^{2p} \left(\frac{1}{d+ex}\right)^{-2p+1} \text{appellf1}\left(-2p+1, -p, -p, -2p+2, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}\right)}{e(-2p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((b\*x\*\*2+a)\*\*p/(e\*x+d)\*\*2, x)

[Out]  $-(e*(\text{sqrt}(b)*x + \text{sqrt}(-a))/(\text{sqrt}(b)*(d + e*x)))**(-p)*(-e*(-\text{sqrt}(b)*x + \text{sqrt}(-a))/(\text{sqrt}(b)*(d + e*x)))**(-p)*(a + b*x**2)**p*(1/(d$

$+ e^x)^{(2p)} \cdot (1/(d + e^x))^{(-2p + 1)} \cdot \text{appellf1}(-2p + 1, -p, -p, -2p + 2, (d - e \cdot \sqrt{-a}/\sqrt{b})/(d + e^x), (d + e \cdot \sqrt{-a}/\sqrt{b})/(d + e^x))/(e^{(-2p + 1)})$

**Mathematica [A]** time = 0.131194, size = 141, normalized size = 0.58

$$\frac{(a + bx^2)^p \left( \frac{e^{(x - \sqrt{-a/b})}}{d + ex} \right)^{-p} \left( \frac{e^{(\sqrt{-a/b} + x)}}{d + ex} \right)^{-p} F_1 \left( 1 - 2p; -p, -p; 2 - 2p; \frac{d - \sqrt{-a/b} e}{d + ex}, \frac{d + \sqrt{-a/b} e}{d + ex} \right)}{e(2p - 1)(d + ex)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b\*x^2)^p/(d + e\*x)^2, x]

[Out] ((a + b\*x^2)^p\*AppellF1[1 - 2\*p, -p, -p, 2 - 2\*p, (d - Sqrt[-(a/b)]\*e)/(d + e\*x), (d + Sqrt[-(a/b)]\*e)/(d + e\*x)]/(e\*(-1 + 2\*p)\*(e\*(-Sqrt[-(a/b)] + x))/(d + e\*x))^p\*((e\*(Sqrt[-(a/b)] + x))/(d + e\*x))^p\*(d + e\*x))

**Maple [F]** time = 0.069, size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b\*x^2+a)^p/(e\*x+d)^2, x)

[Out] int((b\*x^2+a)^p/(e\*x+d)^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^p/(e\*x + d)^2, x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^p/(e\*x + d)^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{(bx^2 + a)^p}{e^2x^2 + 2dex + d^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^p/(e\*x + d)^2, x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^p/(e^2\*x^2 + 2\*d\*e\*x + d^2), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x\*\*2+a)\*\*p/(e\*x+d)\*\*2,x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^p/(e\*x + d)^2,x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^p/(e\*x + d)^2, x)

$$3.421 \quad \int \frac{(a+bx^2)^p}{x(d+ex)^2} dx$$

**Optimal.** Leaf size=368

$$\begin{aligned} & \frac{e^3 x^3 (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{3}{2}; -p, 2; \frac{5}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{3d^5} \\ & - \frac{ex (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^3} \\ & - \frac{ex (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; -p, 2; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^3} \\ & + \frac{e^2 (a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d^2(p+1)(ae^2+bd^2)} \\ & + \frac{be^2 (a+bx^2)^{p+1} {}_2F_1\left(2, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{(p+1)(ae^2+bd^2)^2} - \frac{(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a} + 1\right)}{2ad^2(p+1)} \end{aligned}$$

[Out]  $-\left(\frac{e^3 x^3 (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\left(\frac{bx^2}{a}\right), \left(\frac{e^2 x^2}{d^2}\right)\right]}{d^3} - \frac{ex (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left[\frac{1}{2}, -p, 2, \frac{3}{2}, -\left(\frac{bx^2}{a}\right), \left(\frac{e^2 x^2}{d^2}\right)\right]}{d^3} - \frac{ex (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\left(\frac{bx^2}{a}\right), \left(\frac{e^2 x^2}{d^2}\right)\right]}{3d^5} + \frac{e^2 (a+bx^2)^{p+1} \text{Hypergeometric2F1}\left[1, 1+p, 2+p, \frac{e^2 (a+bx^2)}{bd^2+ae^2}\right]}{2d^2(p+1)(ae^2+bd^2)} + \frac{be^2 (a+bx^2)^{p+1} \text{Hypergeometric2F1}\left[2, 1+p, 2+p, \frac{e^2 (a+bx^2)}{bd^2+ae^2}\right]}{(p+1)(ae^2+bd^2)^2} - \frac{(a+bx^2)^{p+1} \text{Hypergeometric2F1}\left[1, p+1, p+2, \frac{bx^2}{a} + 1\right]}{2ad^2(p+1)}\right)$

**Rubi [A]** time = 0.900731, antiderivative size = 368, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 11, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.55$

$$\begin{aligned} & \frac{e^3 x^3 (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{3}{2}; -p, 2; \frac{5}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{3d^5} \\ & - \frac{ex (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^3} \\ & - \frac{ex (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; -p, 2; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^3} \\ & + \frac{e^2 (a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d^2(p+1)(ae^2+bd^2)} \\ & + \frac{be^2 (a+bx^2)^{p+1} {}_2F_1\left(2, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{(p+1)(ae^2+bd^2)^2} - \frac{(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a} + 1\right)}{2ad^2(p+1)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^p/(x\*(d + e\*x)^2), x]

[Out]  $-\left(\frac{e^3 x^3 (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left[\frac{1}{2}, -p, 1, \frac{3}{2}, -\left(\frac{bx^2}{a}\right), \left(\frac{e^2 x^2}{d^2}\right)\right]}{d^3} - \frac{ex (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left[\frac{1}{2}, -p, 2, \frac{3}{2}, -\left(\frac{bx^2}{a}\right), \left(\frac{e^2 x^2}{d^2}\right)\right]}{d^3} - \frac{ex (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left[\frac{3}{2}, -p, 2, \frac{5}{2}, -\left(\frac{bx^2}{a}\right), \left(\frac{e^2 x^2}{d^2}\right)\right]}{3d^5} + \frac{e^2 (a+bx^2)^{p+1} \text{Hypergeometric2F1}\left[1, 1+p, 2+p, \frac{e^2 (a+bx^2)}{bd^2+ae^2}\right]}{2d^2(p+1)(ae^2+bd^2)} + \frac{be^2 (a+bx^2)^{p+1} \text{Hypergeometric2F1}\left[2, 1+p, 2+p, \frac{e^2 (a+bx^2)}{bd^2+ae^2}\right]}{(p+1)(ae^2+bd^2)^2} - \frac{(a+bx^2)^{p+1} \text{Hypergeometric2F1}\left[1, p+1, p+2, \frac{bx^2}{a} + 1\right]}{2ad^2(p+1)}\right)$



$$p, 2 + p, (e^{2(a + b x^2)} / (b d^2 + a e^2)) / ((b d^2 + a e^2)^2 (1 + p))$$

**Rubi in Sympy [A]** time = 55.0968, size = 299, normalized size = 0.81

$$\frac{\left(\frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} \left(-\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} (a+bx^2)^p \left(\frac{1}{d+ex}\right)^{2p} \left(\frac{1}{d+ex}\right)^{-2p+1} \operatorname{appellf}_1\left(-2p+1, -p, -p, -2p+2, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}\right)}{d(-2p+1)} - \frac{\left(\frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} \left(-\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} (a+bx^2)^p \operatorname{appellf}_1\left(-2p, -p, -p, -2p+1, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}\right)}{2d^2p} - \frac{(a+bx^2)^{p+1} {}_2F_1\left(1, p+1 \middle| p+2 \middle| 1+\frac{bx^2}{a}\right)}{2ad^2(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**p/x/(e*x+d)**2,x)`

[Out]  $(e(\sqrt{b}x + \sqrt{-a})/(\sqrt{b}(d + e^x)))^{(-p)} (-e(-\sqrt{b}x + \sqrt{-a})/(\sqrt{b}(d + e^x)))^{(-p)} (a + b x^2)^p (1/(d + e^x))^{(2p)} (1/(d + e^x))^{(-2p + 1)} \operatorname{appellf}_1(-2p + 1, -p, -p, -2p + 2, (d - e\sqrt{-a}/\sqrt{b})/(d + e^x), (d + e\sqrt{-a}/\sqrt{b})/(d + e^x))/(d^{(-2p + 1)}) - (e(\sqrt{b}x + \sqrt{-a})/(\sqrt{b}(d + e^x)))^{(-p)} (-e(-\sqrt{b}x + \sqrt{-a})/(\sqrt{b}(d + e^x)))^{(-p)} (a + b x^2)^p \operatorname{appellf}_1(-2p, -p, -p, -2p + 1, (d - e\sqrt{-a}/\sqrt{b})/(d + e^x), (d + e\sqrt{-a}/\sqrt{b})/(d + e^x))/(2d^{2p}) - (a + b x^2)^{p+1} \operatorname{hyper}((1, p + 1), (p + 2), 1 + b x^2/a)/(2a d^{2p+1})$

**Mathematica [A]** time = 0.760674, size = 303, normalized size = 0.82

$$(a+bx^2)^p \left( \frac{\left(\frac{a}{bx^2}+1\right)^{-p} {}_2F_1\left(-p, -p; 1-p; -\frac{a}{bx^2}\right) - \left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p} \left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p} F_1\left(-2p; -p, -p; 1-2p; \frac{d-\sqrt{-\frac{a}{b}}e}{d+ex}, \frac{d+\sqrt{-\frac{a}{b}}e}{d+ex}\right)}{p} - 2d \left(\frac{e\left(x-\sqrt{-\frac{a}{b}}\right)}{d+ex}\right)^{-p} \left(\frac{e\left(\sqrt{-\frac{a}{b}}+x\right)}{d+ex}\right)^{-p} \right) / 2d^2$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(a + b*x^2)^p/(x*(d + e*x)^2),x]`

[Out]  $((a + b x^2)^p ((-2d \operatorname{AppellF1}[1 - 2p, -p, -p, 2 - 2p, (d - \operatorname{Sqrt}[-(a/b)]*e)/(d + e^x), (d + \operatorname{Sqrt}[-(a/b)]*e)/(d + e^x)]/((-1 + 2p) * ((e(-\operatorname{Sqrt}[-(a/b)] + x))/(d + e^x))^p ((e(\operatorname{Sqrt}[-(a/b)] + x))/(d + e^x))^p (d + e^x) + (-\operatorname{AppellF1}[-2p, -p, -p, 1 - 2p, (d - \operatorname{Sqrt}[-(a/b)]*e)/(d + e^x), (d + \operatorname{Sqrt}[-(a/b)]*e)/(d + e^x)]/(((e(-\operatorname{Sqrt}[-(a/b)] + x))/(d + e^x))^p ((e(\operatorname{Sqrt}[-(a/b)] + x))/(d + e^x))^p) + \operatorname{Hypergeometric2F1}[-p, -p, 1 - p, -(a/(b x^2))]/(1 + a/(b x^2))^p)/p))/2d^2$

**Maple [F]** time = 0.067, size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^p/x/(e*x+d)^2,x)`

[Out] `int((b*x^2+a)^p/x/(e*x+d)^2,x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^P}{(ex + d)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/((e*x + d)^2*x),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p/((e*x + d)^2*x), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^P}{e^2x^3 + 2dex^2 + d^2x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/((e*x + d)^2*x),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p/(e^2*x^3 + 2*d*e*x^2 + d^2*x), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**p/x/(e*x+d)**2,x)`

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^P}{(ex + d)^2 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/((e*x + d)^2*x),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p/((e*x + d)^2*x), x)`

$$3.422 \quad \int \frac{(a+bx^2)^p}{x^2(d+ex)^2} dx$$

**Optimal.** Leaf size=421

$$\frac{e^4 x^3 (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{3}{2}; -p, 2; \frac{5}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{3d^6} + \frac{2e^2 x (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^4} + \frac{e^2 x (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; -p, 2; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^4} + \frac{e (a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a} + 1\right) - be^3 (a+bx^2)^{p+1} {}_2F_1\left(2, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{ad^3(p+1) - \frac{d(p+1)(ae^2+bd^2)^2}{d^3(p+1)(ae^2+bd^2)}} + \frac{(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; -\frac{bx^2}{a}\right) - e^3 (a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{d^2 x - \frac{d^3(p+1)(ae^2+bd^2)}{d^3(p+1)(ae^2+bd^2)}}$$

[Out]  $(2 * e^2 * x^3 * (a + b * x^2)^p * \text{AppellF1}[1/2, -p, 1, 3/2, -((b * x^2)/a), (e^2 * x^2)/d^2]) / (d^4 * (1 + (b * x^2)/a)^p) + (e^2 * x * (a + b * x^2)^p * \text{AppellF1}[1/2, -p, 2, 3/2, -((b * x^2)/a), (e^2 * x^2)/d^2]) / (d^4 * (1 + (b * x^2)/a)^p) + (e^2 * x * (a + b * x^2)^p * \text{AppellF1}[3/2, -p, 2, 5/2, -((b * x^2)/a), (e^2 * x^2)/d^2]) / (3 * d^6 * (1 + (b * x^2)/a)^p) - ((a + b * x^2)^{p+1} * \text{Hypergeometric2F1}[-1/2, -p, 1/2, -((b * x^2)/a)]) / (d^2 * x * (1 + (b * x^2)/a)^p) - (e^3 * (a + b * x^2)^{p+1} * \text{Hypergeometric2F1}[1, 1 + p, 2 + p, (e^2 * (a + b * x^2)) / (b * d^2 + a * e^2)]) / (d^3 * (b * d^2 + a * e^2) * (1 + p)) + (e * (a + b * x^2)^{p+1} * \text{Hypergeometric2F1}[1, 1 + p, 2 + p, 1 + (b * x^2)/a]) / (a * d^3 * (1 + p)) - (b * e^3 * (a + b * x^2)^{p+1} * \text{Hypergeometric2F1}[2, 1 + p, 2 + p, (e^2 * (a + b * x^2)) / (b * d^2 + a * e^2)]) / (d * (b * d^2 + a * e^2)^2 * (1 + p))$

**Rubi [A]** time = 0.9679, antiderivative size = 421, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 13, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.65$

$$\frac{e^4 x^3 (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{3}{2}; -p, 2; \frac{5}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{3d^6} + \frac{2e^2 x (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^4} + \frac{e^2 x (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; -p, 2; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^4} + \frac{e (a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a} + 1\right) - be^3 (a+bx^2)^{p+1} {}_2F_1\left(2, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{ad^3(p+1) - \frac{d(p+1)(ae^2+bd^2)^2}{d^3(p+1)(ae^2+bd^2)}} + \frac{(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} {}_2F_1\left(-\frac{1}{2}, -p; \frac{1}{2}; -\frac{bx^2}{a}\right) - e^3 (a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{d^2 x - \frac{d^3(p+1)(ae^2+bd^2)}{d^3(p+1)(ae^2+bd^2)}}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^p/(x^2\*(d + e\*x)^2), x]

[Out]  $(2 * e^2 * x^3 * (a + b * x^2)^p * \text{AppellF1}[1/2, -p, 1, 3/2, -((b * x^2)/a), (e^2 * x^2)/d^2]) / (d^4 * (1 + (b * x^2)/a)^p) + (e^2 * x * (a + b * x^2)^p * \text{AppellF1}[1/2, -p, 2, 3/2, -((b * x^2)/a), (e^2 * x^2)/d^2]) / (d^4 * (1 + (b * x^2)/a)^p) + (e^2 * x * (a + b * x^2)^p * \text{AppellF1}[3/2, -p, 2, 5/2, -((b * x^2)/a), (e^2 * x^2)/d^2]) / (3 * d^6 * (1 + (b * x^2)/a)^p) - ((a + b * x^2)^{p+1} * \text{Hypergeometric2F1}[-1/2, -p, 1/2, -((b * x^2)/a)]) / (d^2 * x * (1 + (b * x^2)/a)^p) - (e^3 * (a + b * x^2)^{p+1} * \text{Hypergeometric2F1}[1, 1 + p, 2 + p, (e^2 * (a + b * x^2)) / (b * d^2 + a * e^2)]) / (d^3 * (b * d^2 + a * e^2) * (1 + p)) + (e * (a + b * x^2)^{p+1} * \text{Hypergeometric2F1}[1, 1 + p, 2 + p, 1 + (b * x^2)/a]) / (a * d^3 * (1 + p)) - (b * e^3 * (a + b * x^2)^{p+1} * \text{Hypergeometric2F1}[2, 1 + p, 2 + p, (e^2 * (a + b * x^2)) / (b * d^2 + a * e^2)]) / (d * (b * d^2 + a * e^2)^2 * (1 + p))$

$$2)^*(1+p)) + (e*(a+b*x^2)^(1+p)*Hypergeometric2F1[1, 1+p, 2+p, 1+(b*x^2)/a])/(a*d^3*(1+p)) - (b*e^3*(a+b*x^2)^(1+p)*Hypergeometric2F1[2, 1+p, 2+p, (e^2*(a+b*x^2))/(b*d^2+a*e^2)])/(d*(b*d^2+a*e^2)^(2*(1+p)))$$

**Rubi in Sympy [A]** time = 68.1964, size = 343, normalized size = 0.81

$$\frac{e\left(\frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p}\left(-\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p}(a+bx^2)^p\left(\frac{1}{d+ex}\right)^{2p}\left(\frac{1}{d+ex}\right)^{-2p+1}\text{appellf}_1\left(-2p+1, -p, -p, -2p+2, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}\right)}{d^2(-2p+1)}$$

$$-\frac{\left(1+\frac{bx^2}{a}\right)^{-p}(a+bx^2)^p{}_2F_1\left(-p, -\frac{1}{2}\left|\frac{bx^2}{a}\right.\right)}{\frac{d^2x}{d^3p}}$$

$$+\frac{e\left(\frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p}\left(-\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p}(a+bx^2)^p\text{appellf}_1\left(-2p, -p, -p, -2p+1, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}\right)}{ad^3(p+1)}$$

$$+\frac{e(a+bx^2)^{p+1}{}_2F_1\left(1, p+1\left|\frac{bx^2}{a}\right.\right)}{ad^3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**p/x**2/(e*x+d)**2, x)`

[Out]  $-e*(e*(\text{sqrt}(b)*x + \text{sqrt}(-a))/(\text{sqrt}(b)*(d + e*x)))^{**}(-p)*(-e*(-\text{sqrt}(b)*x + \text{sqrt}(-a))/(\text{sqrt}(b)*(d + e*x)))^{**}(-p)*(a + b*x**2)**p*(1/(d + e*x))^{**}(2*p)*(1/(d + e*x))^{**}(-2*p + 1)*\text{appellf}_1(-2*p + 1, -p, -p, -2*p + 2, (d - e*\text{sqrt}(-a)/\text{sqrt}(b))/(\text{sqrt}(b)*(d + e*x)), (d + e*\text{sqrt}(-a)/\text{sqrt}(b))/(\text{sqrt}(b)*(d + e*x)))/(d**2*(-2*p + 1)) - (1 + b*x**2/a)**(-p)*(a + b*x**2)**p*\text{hyper}((-p, -1/2), (1/2, ), -b*x**2/a)/(d**2*x) + e*(e*(\text{sqrt}(b)*x + \text{sqrt}(-a))/(\text{sqrt}(b)*(d + e*x)))^{**}(-p)*(-e*(-\text{sqrt}(b)*x + \text{sqrt}(-a))/(\text{sqrt}(b)*(d + e*x)))^{**}(-p)*(a + b*x**2)**p*\text{appellf}_1(-2*p, -p, -p, -2*p + 1, (d - e*\text{sqrt}(-a)/\text{sqrt}(b))/(\text{sqrt}(b)*(d + e*x)), (d + e*\text{sqrt}(-a)/\text{sqrt}(b))/(\text{sqrt}(b)*(d + e*x)))/(d**3*p) + e*(a + b*x**2)**(p + 1)*\text{hyper}((1, p + 1), (p + 2, ), 1 + b*x**2/a)/(a*d**3*(p + 1))$

**Mathematica [A]** time = 0.0980102, size = 0, normalized size = 0.

$$\int \frac{(a+bx^2)^p}{x^2(d+ex)^2} dx$$

Verification is Not applicable to the result.

[In] `Integrate[(a + b*x^2)^p/(x^2*(d + e*x)^2), x]`

[Out] `Integrate[(a + b*x^2)^p/(x^2*(d + e*x)^2), x]`

**Maple [F]** time = 0.091, size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x^2(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^p/x^2/(e*x+d)^2,x)`

[Out] `int((b*x^2+a)^p/x^2/(e*x+d)^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{(ex + d)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/((e*x + d)^2*x^2),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p/((e*x + d)^2*x^2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^p}{e^2x^4 + 2dex^3 + d^2x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/((e*x + d)^2*x^2),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p/(e^2*x^4 + 2*d*e*x^3 + d^2*x^2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**p/x**2/(e*x+d)**2,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{(ex + d)^2 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/((e*x + d)^2*x^2),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p/((e*x + d)^2*x^2), x)`

$$3.423 \quad \int \frac{x^4(a+bx^2)^p}{(d+ex)^3} dx$$

**Optimal.** Leaf size=449

$$\frac{dx (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (6a^2e^4 + 3abd^2e^2(3p + 4) + b^2d^4(2p^2 + 7p + 6)) F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{e^4(ae^2 + bd^2)^2}$$

$$- \frac{dx (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (3a^2e^4 + 2abd^2e^2(4p + 5) + b^2d^4(2p^2 + 7p + 6)) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{e^4(ae^2 + bd^2)^2}$$

$$- \frac{d^2 (a + bx^2)^{p+1} (6a^2e^4 + 3abd^2e^2(3p + 4) + b^2d^4(2p^2 + 7p + 6)) {}_2F_1\left(1, p + 1; p + 2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e^3(p + 1)(ae^2 + bd^2)^3}$$

$$- \frac{d^4 (a + bx^2)^{p+1}}{2e^3(d + ex)^2 (ae^2 + bd^2)} + \frac{d^3 (a + bx^2)^{p+1} (4ae^2 + bd^2(p + 3))}{e^3(d + ex)(ae^2 + bd^2)^2} + \frac{(a + bx^2)^{p+1}}{2be^3(p + 1)}$$

[Out]  $(a + b*x^2)^{(1 + p)}/(2*b*e^{3*(1 + p)}) - (d^4*(a + b*x^2)^{(1 + p)})/(2*e^{3*(b*d^2 + a*e^2)}*(d + e*x)^2) + (d^3*(4*a*e^2 + b*d^2*(3 + p))*(a + b*x^2)^{(1 + p)})/(e^{3*(b*d^2 + a*e^2)^2*(d + e*x)} + (d*(6*a^2*e^4 + 3*a*b*d^2*e^2*(4 + 3*p) + b^2*d^4*(6 + 7*p + 2*p^2)))*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(e^{4*(b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p} - (d*(3*a^2*e^4 + 2*a*b*d^2*e^2*(5 + 4*p) + b^2*d^4*(6 + 7*p + 2*p^2))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e^{4*(b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p} - (d^2*(6*a^2*e^4 + 3*a*b*d^2*e^2*(4 + 3*p) + b^2*d^4*(6 + 7*p + 2*p^2))*(a + b*x^2)^{(1 + p)}*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*e^{3*(b*d^2 + a*e^2)^3*(1 + p)})$

**Rubi [A]** time = 1.63056, antiderivative size = 449, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{dx (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (6a^2e^4 + 3abd^2e^2(3p + 4) + b^2d^4(2p^2 + 7p + 6)) F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{e^4(ae^2 + bd^2)^2}$$

$$- \frac{dx (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (3a^2e^4 + 2abd^2e^2(4p + 5) + b^2d^4(2p^2 + 7p + 6)) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{e^4(ae^2 + bd^2)^2}$$

$$- \frac{d^2 (a + bx^2)^{p+1} (6a^2e^4 + 3abd^2e^2(3p + 4) + b^2d^4(2p^2 + 7p + 6)) {}_2F_1\left(1, p + 1; p + 2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e^3(p + 1)(ae^2 + bd^2)^3}$$

$$- \frac{d^4 (a + bx^2)^{p+1}}{2e^3(d + ex)^2 (ae^2 + bd^2)} + \frac{d^3 (a + bx^2)^{p+1} (4ae^2 + bd^2(p + 3))}{e^3(d + ex)(ae^2 + bd^2)^2} + \frac{(a + bx^2)^{p+1}}{2be^3(p + 1)}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*(a + b\*x^2)^p)/(d + e\*x)^3, x]

[Out]  $(a + b*x^2)^{(1 + p)}/(2*b*e^{3*(1 + p)}) - (d^4*(a + b*x^2)^{(1 + p)})/(2*e^{3*(b*d^2 + a*e^2)}*(d + e*x)^2) + (d^3*(4*a*e^2 + b*d^2*(3 + p))*(a + b*x^2)^{(1 + p)})/(e^{3*(b*d^2 + a*e^2)^2*(d + e*x)} + (d*(6*a^2*e^4 + 3*a*b*d^2*e^2*(4 + 3*p) + b^2*d^4*(6 + 7*p + 2*p^2)))*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(e^{4*(b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p} - (d*(3*a^2*e^4 + 2*a*b*d^2*e^2*(5 + 4*p) + b^2*d^4*(6 + 7*p + 2*p^2))*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e^{4*(b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p} - (d^2*(6*a^2*e^4 + 3*a*b*d^2*e^2*(4 + 3*p) + b^2*d^4*(6 + 7*p + 2*p^2))*(a + b*x^2)^{(1 + p)}*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*e^{3*(b*d^2 + a*e^2)^3*(1 + p)})$

**Rubi in Sympy [A]** time = 91.2566, size = 488, normalized size = 1.09

$$\frac{d^4 \left( \frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}} \right)^{-p} \left( -\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}} \right)^{-p} (a+bx^2)^p \left( \frac{1}{d+ex} \right)^{2p} \left( \frac{1}{d+ex} \right)^{-2p+2} \operatorname{appellf}_1 \left( -2p+2, -p, -p, -2p+3, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex} \right)}{2e^5(-p+1)}$$

$$+ \frac{4d^3 \left( \frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}} \right)^{-p} \left( -\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}} \right)^{-p} (a+bx^2)^p \left( \frac{1}{d+ex} \right)^{2p} \left( \frac{1}{d+ex} \right)^{-2p+1} \operatorname{appellf}_1 \left( -2p+1, -p, -p, -2p+2, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex} \right)}{e^5(-2p+1)}$$

$$+ \frac{3d^2 \left( \frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}} \right)^{-p} \left( -\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}} \right)^{-p} (a+bx^2)^p \operatorname{appellf}_1 \left( -2p, -p, -p, -2p+1, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex} \right)}{e^5 p}$$

$$- \frac{3dx \left( 1 + \frac{bx^2}{a} \right)^{-p} (a+bx^2)^p {}_2F_1 \left( -p, \frac{1}{2} \middle| -\frac{bx^2}{a} \right)}{e^4} + \frac{(a+bx^2)^{p+1}}{2be^3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**4*(b*x**2+a)**p/(e*x+d)**3,x)`

[Out]  $-d^{*4}*(e*(\operatorname{sqrt}(b)*x + \operatorname{sqrt}(-a))/(\operatorname{sqrt}(b)*(d + e*x)))^{*}(-p)*(-e*(-\operatorname{sqrt}(b)*x + \operatorname{sqrt}(-a))/(\operatorname{sqrt}(b)*(d + e*x)))^{*}(-p)*(a + b*x^{*2})^{*}p*(1/(d + e*x))^{*}(2*p)*(1/(d + e*x))^{*}(-2*p + 2)*\operatorname{appellf}_1(-2*p + 2, -p, -p, -2*p + 3, (d - e*\operatorname{sqrt}(-a)/\operatorname{sqrt}(b))/(d + e*x), (d + e*\operatorname{sqrt}(-a)/\operatorname{sqrt}(b))/(d + e*x))/(2*e^{*5}*(-p + 1)) + 4*d^{*3}*(e*(\operatorname{sqrt}(b)*x + \operatorname{sqrt}(-a))/(\operatorname{sqrt}(b)*(d + e*x)))^{*}(-p)*(-e*(-\operatorname{sqrt}(b)*x + \operatorname{sqrt}(-a))/(\operatorname{sqrt}(b)*(d + e*x)))^{*}(-p)*(a + b*x^{*2})^{*}p*(1/(d + e*x))^{*}(2*p)*(1/(d + e*x))^{*}(-2*p + 1)*\operatorname{appellf}_1(-2*p + 1, -p, -p, -2*p + 2, (d - e*\operatorname{sqrt}(-a)/\operatorname{sqrt}(b))/(d + e*x), (d + e*\operatorname{sqrt}(-a)/\operatorname{sqrt}(b))/(d + e*x))/(e^{*5}*(-2*p + 1)) + 3*d^{*2}*(e*(\operatorname{sqrt}(b)*x + \operatorname{sqrt}(-a))/(\operatorname{sqrt}(b)*(d + e*x)))^{*}(-p)*(-e*(-\operatorname{sqrt}(b)*x + \operatorname{sqrt}(-a))/(\operatorname{sqrt}(b)*(d + e*x)))^{*}(-p)*(a + b*x^{*2})^{*}p*\operatorname{appellf}_1(-2*p, -p, -p, -2*p + 1, (d - e*\operatorname{sqrt}(-a)/\operatorname{sqrt}(b))/(d + e*x), (d + e*\operatorname{sqrt}(-a)/\operatorname{sqrt}(b))/(d + e*x))/(e^{*5}*p) - 3*d*x*(1 + b*x^{*2}/a)^{-p}*(a + b*x^{*2})^{*}p*\operatorname{hyper}((-p, 1/2), (3/2, ), -b*x^{*2}/a)/e^{*4} + (a + b*x^{*2})^{*}(p + 1)/(2*b*e^{*3}*(p + 1))$

**Mathematica [A]** time = 1.37435, size = 0, normalized size = 0.

$$\int \frac{x^4 (a + bx^2)^p}{(d + ex)^3} dx$$

Verification is Not applicable to the result.

[In] `Integrate[(x^4*(a + b*x^2)^p)/(d + e*x)^3,x]`

[Out] `Integrate[(x^4*(a + b*x^2)^p)/(d + e*x)^3, x]`

**Maple [F]** time = 0.116, size = 0, normalized size = 0.

$$\int \frac{x^4 (bx^2 + a)^p}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(b*x^2+a)^p/(e*x+d)^3,x)`

[Out] `int(x^4*(b*x^2+a)^p/(e*x+d)^3,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p x^4}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^4/(e*x + d)^3,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p*x^4/(e*x + d)^3, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^p x^4}{e^3 x^3 + 3 d e^2 x^2 + 3 d^2 e x + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^4/(e*x + d)^3,x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p*x^4/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4*(b*x**2+a)**p/(e*x+d)**3,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p x^4}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^4/(e*x + d)^3,x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*x^4/(e*x + d)^3, x)`



$$3.424 \quad \int \frac{x^3(a+bx^2)^p}{(d+ex)^3} dx$$

**Optimal.** Leaf size=416

$$\begin{aligned} & - \frac{x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (3a^2e^4 + abd^2e^2(7p+6) + b^2d^4(2p^2+5p+3)) F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{e^3(ae^2+bd^2)^2} \\ & + \frac{d(a+bx^2)^{p+1} (3a^2e^4 + abd^2e^2(7p+6) + b^2d^4(2p^2+5p+3)) {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e^2(p+1)(ae^2+bd^2)^3} \\ & + \frac{x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (a^2e^4 + abd^2e^2(6p+5) + b^2d^4(2p^2+5p+3)) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{e^3(ae^2+bd^2)^2} \\ & - \frac{d^2(a+bx^2)^{p+1} (3ae^2+bd^2(p+2))}{e^2(d+ex)(ae^2+bd^2)^2} + \frac{d^3(a+bx^2)^{p+1}}{2e^2(d+ex)^2(ae^2+bd^2)} \end{aligned}$$

[Out]  $(d^3*(a+b*x^2)^(1+p))/(2*e^2*(b*d^2+a*e^2)*(d+e*x)^2) - (d^2*(3*a^2*e^2+b*d^2*(2+p))*(a+b*x^2)^(1+p))/(e^2*(b*d^2+a*e^2)^2*(d+e*x)) - ((3*a^2*e^4+a*b*d^2*e^2*(6+7*p)+b^2*d^4*(3+5*p+2*p^2))*x*(a+b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(e^3*(b*d^2+a*e^2)^2*(1+(b*x^2)/a)^p) + ((a^2*e^4+a*b*d^2*e^2*(5+6*p)+b^2*d^4*(3+5*p+2*p^2))*x*(a+b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e^3*(b*d^2+a*e^2)^2*(1+(b*x^2)/a)^p) + (d*(3*a^2*e^4+a*b*d^2*e^2*(6+7*p)+b^2*d^4*(3+5*p+2*p^2))*(a+b*x^2)^(1+p)*Hypergeometric2F1[1, 1+p, 2+p, (e^2*(a+b*x^2))/(b*d^2+a*e^2)])/(2*e^2*(b*d^2+a*e^2)^3*(1+p))$

**Rubi [A]** time = 1.08218, antiderivative size = 416, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.45$

$$\begin{aligned} & - \frac{x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (3a^2e^4 + abd^2e^2(7p+6) + b^2d^4(2p^2+5p+3)) F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{e^3(ae^2+bd^2)^2} \\ & + \frac{d(a+bx^2)^{p+1} (3a^2e^4 + abd^2e^2(7p+6) + b^2d^4(2p^2+5p+3)) {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e^2(p+1)(ae^2+bd^2)^3} \\ & + \frac{x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (a^2e^4 + abd^2e^2(6p+5) + b^2d^4(2p^2+5p+3)) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{e^3(ae^2+bd^2)^2} \\ & - \frac{d^2(a+bx^2)^{p+1} (3ae^2+bd^2(p+2))}{e^2(d+ex)(ae^2+bd^2)^2} + \frac{d^3(a+bx^2)^{p+1}}{2e^2(d+ex)^2(ae^2+bd^2)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(a+b\*x^2)^p)/(d+e\*x)^3,x]

[Out]  $(d^3*(a+b*x^2)^(1+p))/(2*e^2*(b*d^2+a*e^2)*(d+e*x)^2) - (d^2*(3*a^2*e^2+b*d^2*(2+p))*(a+b*x^2)^(1+p))/(e^2*(b*d^2+a*e^2)^2*(d+e*x)) - ((3*a^2*e^4+a*b*d^2*e^2*(6+7*p)+b^2*d^4*(3+5*p+2*p^2))*x*(a+b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(e^3*(b*d^2+a*e^2)^2*(1+(b*x^2)/a)^p) + ((a^2*e^4+a*b*d^2*e^2*(5+6*p)+b^2*d^4*(3+5*p+2*p^2))*x*(a+b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e^3*(b*d^2+a*e^2)^2*(1+(b*x^2)/a)^p) + (d*(3*a^2*e^4+a*b*d^2*e^2*(6+7*p)+b^2*d^4*(3+5*p+2*p^2))*(a+b*x^2)^(1+p)*Hypergeometric2F1[1, 1+p, 2+p, (e^2*(a+b*x^2))/(b*d^2+a*e^2)])/(2*e^2*(b*d^2+a*e^2)^3*(1+p))$

**Rubi in Sympy [A]** time = 86.0886, size = 464, normalized size = 1.12

$$\frac{d^3 \left( \frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}} \right)^{-p} \left( -\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}} \right)^{-p} (a+bx^2)^p \left( \frac{1}{d+ex} \right)^{2p} \left( \frac{1}{d+ex} \right)^{-2p+2} \operatorname{appellf}_1 \left( -2p+2, -p, -p, -2p+3, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex} \right)}{2e^4(-p+1)}$$

$$\frac{3d^2 \left( \frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}} \right)^{-p} \left( -\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}} \right)^{-p} (a+bx^2)^p \left( \frac{1}{d+ex} \right)^{2p} \left( \frac{1}{d+ex} \right)^{-2p+1} \operatorname{appellf}_1 \left( -2p+1, -p, -p, -2p+2, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex} \right)}{e^4(-2p+1)}$$

$$\frac{3d \left( \frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}} \right)^{-p} \left( -\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}} \right)^{-p} (a+bx^2)^p \operatorname{appellf}_1 \left( -2p, -p, -p, -2p+1, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex} \right)}{2e^4p}$$

$$+ \frac{x \left( 1 + \frac{bx^2}{a} \right)^{-p} (a+bx^2)^p {}_2F_1 \left( -p, \frac{1}{2} \middle| -\frac{bx^2}{a} \right)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**3*(b*x**2+a)**p/(e*x+d)**3,x)`

[Out]  $d^{*3}*(e*(\operatorname{sqrt}(b)*x + \operatorname{sqrt}(-a))/(\operatorname{sqrt}(b)*(d + e*x)))^{*(-p)}*(-e*(-\operatorname{sqrt}(b)*x + \operatorname{sqrt}(-a))/(\operatorname{sqrt}(b)*(d + e*x)))^{*(-p)}*(a + b*x^{*2})^{*p}*(1/(d + e*x))^{*(2*p)}*(1/(d + e*x))^{*(-2*p + 2)}*\operatorname{appellf}_1(-2*p + 2, -p, -p, -2*p + 3, (d - e*\operatorname{sqrt}(-a)/\operatorname{sqrt}(b))/(d + e*x), (d + e*\operatorname{sqrt}(-a)/\operatorname{sqrt}(b))/(d + e*x))/(2*e^{*4}*(-p + 1)) - 3*d^{*2}*(e*(\operatorname{sqrt}(b)*x + \operatorname{sqrt}(-a))/(\operatorname{sqrt}(b)*(d + e*x)))^{*(-p)}*(-e*(-\operatorname{sqrt}(b)*x + \operatorname{sqrt}(-a))/(\operatorname{sqrt}(b)*(d + e*x)))^{*(-p)}*(a + b*x^{*2})^{*p}*(1/(d + e*x))^{*(2*p)}*(1/(d + e*x))^{*(-2*p + 1)}*\operatorname{appellf}_1(-2*p + 1, -p, -p, -2*p + 2, (d - e*\operatorname{sqrt}(-a)/\operatorname{sqrt}(b))/(d + e*x), (d + e*\operatorname{sqrt}(-a)/\operatorname{sqrt}(b))/(d + e*x))/(e^{*4}*(-2*p + 1)) - 3*d*(e*(\operatorname{sqrt}(b)*x + \operatorname{sqrt}(-a))/(\operatorname{sqrt}(b)*(d + e*x)))^{*(-p)}*(-e*(-\operatorname{sqrt}(b)*x + \operatorname{sqrt}(-a))/(\operatorname{sqrt}(b)*(d + e*x)))^{*(-p)}*(a + b*x^{*2})^{*p}*\operatorname{appellf}_1(-2*p, -p, -p, -2*p + 1, (d - e*\operatorname{sqrt}(-a)/\operatorname{sqrt}(b))/(d + e*x), (d + e*\operatorname{sqrt}(-a)/\operatorname{sqrt}(b))/(d + e*x))/(2*e^{*4}*p) + x*(1 + b*x^{*2}/a)^{*(-p)}*(a + b*x^{*2})^{*p}*\operatorname{hyper}((-p, 1/2), (3/2, ), -b*x^{*2}/a)/e^{*3}$

**Mathematica [A]** time = 1.06905, size = 0, normalized size = 0.

$$\int \frac{x^3 (a + bx^2)^p}{(d + ex)^3} dx$$

Verification is Not applicable to the result.

[In] `Integrate[(x^3*(a + b*x^2)^p)/(d + e*x)^3,x]`

[Out] `Integrate[(x^3*(a + b*x^2)^p)/(d + e*x)^3, x]`

**Maple [F]** time = 0.117, size = 0, normalized size = 0.

$$\int \frac{x^3 (bx^2 + a)^p}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(b*x^2+a)^p/(e*x+d)^3,x)`

[Out] `int(x^3*(b*x^2+a)^p/(e*x+d)^3,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p x^3}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^3/(e*x + d)^3,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p*x^3/(e*x + d)^3, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^p x^3}{e^3 x^3 + 3 d e^2 x^2 + 3 d^2 e x + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^3/(e*x + d)^3,x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p*x^3/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(b*x**2+a)**p/(e*x+d)**3,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p x^3}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^3/(e*x + d)^3,x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*x^3/(e*x + d)^3, x)`

$$3.425 \quad \int \frac{x^2(a+bx^2)^p}{(d+ex)^3} dx$$

**Optimal.** Leaf size=396

$$\frac{x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (a^2e^4 + abd^2e^2(5p+2) + b^2d^4(2p^2+3p+1)) F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{de^2(ae^2+bd^2)^2} \\ - \frac{(a+bx^2)^{p+1} (a^2e^4 + abd^2e^2(5p+2) + b^2d^4(2p^2+3p+1)) {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e(p+1)(ae^2+bd^2)^3} \\ - \frac{bd(2p+1)x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (2ae^2 + bd^2(p+1)) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{e^2(ae^2+bd^2)^2} \\ + \frac{d(a+bx^2)^{p+1} (2ae^2 + bd^2(p+1))}{e(d+ex)(ae^2+bd^2)^2} - \frac{d^2(a+bx^2)^{p+1}}{2e(d+ex)^2(ae^2+bd^2)}$$

[Out]  $-(d^2(a+bx^2)^{(1+p)})/(2e(bd^2+ae^2)(d+ex)^2) + (d^2(2ae^2+bd^2(1+p))(a+bx^2)^{(1+p)})/(e(bd^2+ae^2)^2(d+ex)) + ((a^2e^4+abd^2e^2(2+5p)+b^2d^4(1+3p+2p^2))x(a+bx^2)^p \text{AppellF1}[1/2, -p, 1, 3/2, -(bx^2/a), (e^2x^2/d^2)]/(d^2e^2(bd^2+ae^2)^2(1+(bx^2/a)^p) - (bd(1+2p)(2ae^2+bd^2(1+p))x(a+bx^2)^p \text{Hypergeometric2F1}[1/2, -p, 3/2, -(bx^2/a)]/(e^2(bd^2+ae^2)^2(1+(bx^2/a)^p) - ((a^2e^4+abd^2e^2(2+5p)+b^2d^4(1+3p+2p^2))(a+bx^2)^{(1+p)} \text{Hypergeometric2F1}[1, 1+p, 2+p, (e^2(a+bx^2))/(bd^2+ae^2)]/(2e(bd^2+ae^2))^3(1+p)))$

**Rubi [A]** time = 1.10001, antiderivative size = 396, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 10, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\frac{x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (a^2e^4 + abd^2e^2(5p+2) + b^2d^4(2p^2+3p+1)) F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{de^2(ae^2+bd^2)^2} \\ - \frac{(a+bx^2)^{p+1} (a^2e^4 + abd^2e^2(5p+2) + b^2d^4(2p^2+3p+1)) {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2e(p+1)(ae^2+bd^2)^3} \\ - \frac{bd(2p+1)x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (2ae^2 + bd^2(p+1)) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{e^2(ae^2+bd^2)^2} \\ + \frac{d(a+bx^2)^{p+1} (2ae^2 + bd^2(p+1))}{e(d+ex)(ae^2+bd^2)^2} - \frac{d^2(a+bx^2)^{p+1}}{2e(d+ex)^2(ae^2+bd^2)}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(a+bx^2)^p)/(d+ex)^3, x]

[Out]  $-(d^2(a+bx^2)^{(1+p)})/(2e(bd^2+ae^2)(d+ex)^2) + (d^2(2ae^2+bd^2(1+p))(a+bx^2)^{(1+p)})/(e(bd^2+ae^2)^2(d+ex)) + ((a^2e^4+abd^2e^2(2+5p)+b^2d^4(1+3p+2p^2))x(a+bx^2)^p \text{AppellF1}[1/2, -p, 1, 3/2, -(bx^2/a), (e^2x^2/d^2)]/(d^2e^2(bd^2+ae^2)^2(1+(bx^2/a)^p) - (bd(1+2p)(2ae^2+bd^2(1+p))x(a+bx^2)^p \text{Hypergeometric2F1}[1/2, -p, 3/2, -(bx^2/a)]/(e^2(bd^2+ae^2)^2(1+(bx^2/a)^p) - ((a^2e^4+abd^2e^2(2+5p)+b^2d^4(1+3p+2p^2))(a+bx^2)^{(1+p)} \text{Hypergeometric2F1}[1, 1+p, 2+p, (e^2(a+bx^2))/(bd^2+ae^2)]/(2e(bd^2+ae^2))^3(1+p)))$

**Rubi in Sympy [A]** time = 73.939, size = 420, normalized size = 1.06

$$\frac{d^2 \left( \frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b}(d+ex)} \right)^{-p} \left( -\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b}(d+ex)} \right)^{-p} (a+bx^2)^p \left( \frac{1}{d+ex} \right)^{2p} \left( \frac{1}{d+ex} \right)^{-2p+2} \operatorname{appellf}_1 \left( -2p+2, -p, -p, -2p+3, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex} \right)}{2e^3(-p+1)} + \frac{2d \left( \frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b}(d+ex)} \right)^{-p} \left( -\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b}(d+ex)} \right)^{-p} (a+bx^2)^p \left( \frac{1}{d+ex} \right)^{2p} \left( \frac{1}{d+ex} \right)^{-2p+1} \operatorname{appellf}_1 \left( -2p+1, -p, -p, -2p+2, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex} \right)}{e^3(-2p+1)} + \frac{\left( \frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b}(d+ex)} \right)^{-p} \left( -\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b}(d+ex)} \right)^{-p} (a+bx^2)^p \operatorname{appellf}_1 \left( -2p, -p, -p, -2p+1, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex} \right)}{2e^3p}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**2*(b*x**2+a)**p/(e*x+d)**3,x)`

[Out]  $-d^{**2}*(e*(\operatorname{sqrt}(b)*x + \operatorname{sqrt}(-a))/(\operatorname{sqrt}(b)*(d + e*x)))^{**}(-p)*(-e*(-\operatorname{sqrt}(b)*x + \operatorname{sqrt}(-a))/(\operatorname{sqrt}(b)*(d + e*x)))^{**}(-p)*(a + b*x**2)**p*(1/(d + e*x))^{**}(2*p)*(1/(d + e*x))^{**}(-2*p + 2)*\operatorname{appellf}_1(-2*p + 2, -p, -p, -2*p + 3, (d - e*\operatorname{sqrt}(-a)/\operatorname{sqrt}(b))/(d + e*x), (d + e*\operatorname{sqrt}(-a)/\operatorname{sqrt}(b))/(d + e*x))/(2*e**3*(-p + 1)) + 2*d*(e*(\operatorname{sqrt}(b)*x + \operatorname{sqrt}(-a))/(\operatorname{sqrt}(b)*(d + e*x)))^{**}(-p)*(-e*(-\operatorname{sqrt}(b)*x + \operatorname{sqrt}(-a))/(\operatorname{sqrt}(b)*(d + e*x)))^{**}(-p)*(a + b*x**2)**p*(1/(d + e*x))^{**}(2*p)*(1/(d + e*x))^{**}(-2*p + 1)*\operatorname{appellf}_1(-2*p + 1, -p, -p, -2*p + 2, (d - e*\operatorname{sqrt}(-a)/\operatorname{sqrt}(b))/(d + e*x), (d + e*\operatorname{sqrt}(-a)/\operatorname{sqrt}(b))/(d + e*x))/(e**3*(-2*p + 1)) + (e*(\operatorname{sqrt}(b)*x + \operatorname{sqrt}(-a))/(\operatorname{sqrt}(b)*(d + e*x)))^{**}(-p)*(-e*(-\operatorname{sqrt}(b)*x + \operatorname{sqrt}(-a))/(\operatorname{sqrt}(b)*(d + e*x)))^{**}(-p)*(a + b*x**2)**p*\operatorname{appellf}_1(-2*p, -p, -p, -2*p + 1, (d - e*\operatorname{sqrt}(-a)/\operatorname{sqrt}(b))/(d + e*x), (d + e*\operatorname{sqrt}(-a)/\operatorname{sqrt}(b))/(d + e*x))/(2*e**3*p)$

**Mathematica [A]** time = 0.392362, size = 290, normalized size = 0.73

$$\frac{(a+bx^2)^p \left( \frac{e(x-\sqrt{-\frac{a}{b}})}{d+ex} \right)^{-p} \left( \frac{e(\sqrt{-\frac{a}{b}}+x)}{d+ex} \right)^{-p} \left( \frac{d^2 F_1 \left( 2-2p; -p, -p; 3-2p; \frac{d-\sqrt{-\frac{a}{b}}e}{d+ex}, \frac{d+\sqrt{-\frac{a}{b}}e}{d+ex} \right)}{(p-1)(d+ex)^2} - \frac{4d F_1 \left( 1-2p; -p, -p; 2-2p; \frac{d-\sqrt{-\frac{a}{b}}e}{d+ex}, \frac{d+\sqrt{-\frac{a}{b}}e}{d+ex} \right)}{(2p-1)(d+ex)} + F_1 \left( -1-2p; -p, -p; -2p; \frac{d-\sqrt{-\frac{a}{b}}e}{d+ex}, \frac{d+\sqrt{-\frac{a}{b}}e}{d+ex} \right)}{2e^3}$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(x^2*(a+b*x^2)^p)/(d+e*x)^3,x]`

[Out]  $((a + b*x^2)^p * ((-4*d*\operatorname{AppellF}_1[1 - 2*p, -p, -p, 2 - 2*p, (d - \operatorname{Sqrt}[-(a/b)]*e)/(d + e*x), (d + \operatorname{Sqrt}[-(a/b)]*e)/(d + e*x)]/((-1 + 2*p)*(d + e*x)) + (d^2*\operatorname{AppellF}_1[2 - 2*p, -p, -p, 3 - 2*p, (d - \operatorname{Sqrt}[-(a/b)]*e)/(d + e*x), (d + \operatorname{Sqrt}[-(a/b)]*e)/(d + e*x)]/((-1 + p)*(d + e*x)^2) + \operatorname{AppellF}_1[-2*p, -p, -p, 1 - 2*p, (d - \operatorname{Sqrt}[-(a/b)]*e)/(d + e*x), (d + \operatorname{Sqrt}[-(a/b)]*e)/(d + e*x)]/p))/2*e^3*((e*(\operatorname{Sqrt}[-(a/b)] + x))/(d + e*x))^p$

**Maple [F]** time = 0.106, size = 0, normalized size = 0.

$$\int \frac{x^2 (bx^2 + a)^p}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(b*x^2+a)^p/(e*x+d)^3,x)`

[Out] `int(x^2*(b*x^2+a)^p/(e*x+d)^3,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p x^2}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^2/(e*x + d)^3,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p*x^2/(e*x + d)^3, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^p x^2}{e^3 x^3 + 3 d e^2 x^2 + 3 d^2 e x + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^2/(e*x + d)^3,x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p*x^2/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(b*x**2+a)**p/(e*x+d)**3,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p x^2}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p*x^2/(e*x + d)^3,x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p*x^2/(e*x + d)^3, x)`

$$3.426 \quad \int \frac{x(a+bx^2)^p}{(d+ex)^3} dx$$

**Optimal.** Leaf size=336

$$\begin{aligned} & \frac{bpx(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (3ae^2 + bd^2(2p+1)) F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{e(ae^2 + bd^2)^2} \\ & + \frac{b(2p+1)x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (ae^2 + bd^2p) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{e(ae^2 + bd^2)^2} \\ & + \frac{bdp(a+bx^2)^{p+1} (3ae^2 + bd^2(2p+1)) {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2 + bd^2)^3} \\ & - \frac{(a+bx^2)^{p+1} (ae^2 + bd^2p)}{(d+ex)(ae^2 + bd^2)^2} + \frac{d(a+bx^2)^{p+1}}{2(d+ex)^2(ae^2 + bd^2)} \end{aligned}$$

[Out]  $(d*(a + b*x^2)^(1 + p))/(2*(b*d^2 + a*e^2)*(d + e*x)^2) - ((a*e^2 + b*d^2*p)*(a + b*x^2)^(1 + p))/((b*d^2 + a*e^2)^2*(d + e*x)) - (b*p*(3*a*e^2 + b*d^2*(1 + 2*p))*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(e*(b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p) + (b*(1 + 2*p)*(a*e^2 + b*d^2*p)*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e*(b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p) + (b*d*p*(3*a*e^2 + b*d^2*(1 + 2*p))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)^3*(1 + p))$

**Rubi [A]** time = 0.832627, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 18,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.5$

$$\begin{aligned} & \frac{bpx(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (3ae^2 + bd^2(2p+1)) F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2x^2}{d^2}\right)}{e(ae^2 + bd^2)^2} \\ & + \frac{b(2p+1)x(a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} (ae^2 + bd^2p) {}_2F_1\left(\frac{1}{2}, -p; \frac{3}{2}; -\frac{bx^2}{a}\right)}{e(ae^2 + bd^2)^2} \\ & + \frac{bdp(a+bx^2)^{p+1} (3ae^2 + bd^2(2p+1)) {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2 + bd^2)^3} \\ & - \frac{(a+bx^2)^{p+1} (ae^2 + bd^2p)}{(d+ex)(ae^2 + bd^2)^2} + \frac{d(a+bx^2)^{p+1}}{2(d+ex)^2(ae^2 + bd^2)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(x\*(a + b\*x^2)^p)/(d + e\*x)^3, x]

[Out]  $(d*(a + b*x^2)^(1 + p))/(2*(b*d^2 + a*e^2)*(d + e*x)^2) - ((a*e^2 + b*d^2*p)*(a + b*x^2)^(1 + p))/((b*d^2 + a*e^2)^2*(d + e*x)) - (b*p*(3*a*e^2 + b*d^2*(1 + 2*p))*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(e*(b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p) + (b*(1 + 2*p)*(a*e^2 + b*d^2*p)*x*(a + b*x^2)^p*Hypergeometric2F1[1/2, -p, 3/2, -((b*x^2)/a)])/(e*(b*d^2 + a*e^2)^2*(1 + (b*x^2)/a)^p) + (b*d*p*(3*a*e^2 + b*d^2*(1 + 2*p))*(a + b*x^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(2*(b*d^2 + a*e^2)^3*(1 + p))$

**Rubi in Sympy [A]** time = 87.0114, size = 292, normalized size = 0.87

$$\frac{bd \left( \frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}} \right)^{-p} \left( -\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}} \right)^{-p} (a+bx^2)^p (3ae^2+2bd^2p+bd^2) \operatorname{appellf}_1 \left( -2p, -p, -p, -2p+1, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex} \right)}{2e^2(ae^2+bd^2)^2} \\ + \frac{bx \left( 1 + \frac{bx^2}{a} \right)^{-p} (a+bx^2)^p (2p+1) (ae^2+bd^2p) {}_2F_1 \left( -p, \frac{1}{2} \middle| -\frac{bx^2}{a} \right)}{e(ae^2+bd^2)^2} \\ + \frac{d(a+bx^2)^{p+1}}{2(d+ex)^2(ae^2+bd^2)} - \frac{(a+bx^2)^{p+1}(ae^2+bd^2p)}{(d+ex)(ae^2+bd^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x*(b*x**2+a)**p/(e*x+d)**3,x)`

[Out] `-b*d*(e*(sqrt(b)*x+sqrt(-a))/(sqrt(b)*(d+e*x)))**(-p)*(-e*(-sqrt(b)*x+sqrt(-a))/(sqrt(b)*(d+e*x)))**(-p)*(a+b*x**2)**p*(3*a*e**2+2*b*d**2*p+b*d**2)*appellf1(-2*p,-p,-p,-2*p+1,(d-e*sqrt(-a)/sqrt(b))/(d+e*x),(d+e*sqrt(-a)/sqrt(b))/(d+e*x))/(2*e**2*(a*e**2+b*d**2)**2)+b*x*(1+b*x**2/a)**(-p)*(a+b*x**2)**p*(2*p+1)*(a*e**2+b*d**2*p)*hyper((-p,1/2),(3/2,)-b*x**2/a)/(e*(a*e**2+b*d**2)**2)+d*(a+b*x**2)**(p+1)/(2*(d+e*x)**2*(a*e**2+b*d**2))- (a+b*x**2)**(p+1)*(a*e**2+b*d**2*p)/((d+e*x)*(a*e**2+b*d**2)**2)`

**Mathematica [A]** time = 0.29078, size = 229, normalized size = 0.68

$$\frac{(a+bx^2)^p \left( \frac{e(x-\sqrt{-\frac{a}{b}})}{d+ex} \right)^{-p} \left( \frac{e(\sqrt{-\frac{a}{b}}+x)}{d+ex} \right)^{-p} \left( 2(p-1)(d+ex)F_1 \left( 1-2p; -p, -p; 2-2p; \frac{d-\sqrt{-\frac{a}{b}}e}{d+ex}, \frac{d+\sqrt{-\frac{a}{b}}e}{d+ex} \right) + d(1-2p)F_1 \left( 2-2p; -p, -p; 3-2p; \frac{d-\sqrt{-\frac{a}{b}}e}{d+ex}, \frac{d+\sqrt{-\frac{a}{b}}e}{d+ex} \right) \right)}{2e^2(p-1)(2p-1)(d+ex)^2}$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(x*(a+b*x^2)^p)/(d+e*x)^3,x]`

[Out] `((a+b*x^2)^p*(2*(-1+p)*(d+e*x)*AppellF1[1-2*p,-p,-p,2-2*p,(d-Sqrt[-(a/b)]*e)/(d+e*x),(d+Sqrt[-(a/b)]*e)/(d+e*x]) + d*(1-2*p)*AppellF1[2-2*p,-p,-p,3-2*p,(d-Sqrt[-(a/b)]*e)/(d+e*x),(d+Sqrt[-(a/b)]*e)/(d+e*x])]/(2*e^2*(-1+p)*(-1+2*p)*((e*(-Sqrt[-(a/b)]+x))/(d+e*x))^p*((e*(Sqrt[-(a/b)]+x))/(d+e*x))^p*(d+e*x)^2)`

**Maple [F]** time = 0.1, size = 0, normalized size = 0.

$$\int \frac{x(bx^2+a)^p}{(ex+d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(b*x^2+a)^p/(e*x+d)^3,x)`

[Out] `int(x*(b*x^2+a)^p/(e*x+d)^3,x)`



**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p x}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^p\*x/(e\*x + d)^3,x, algorithm="maxima")

[Out] integrate((b\*x^2 + a)^p\*x/(e\*x + d)^3, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^p x}{e^3 x^3 + 3 d e^2 x^2 + 3 d^2 e x + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^p\*x/(e\*x + d)^3,x, algorithm="fricas")

[Out] integral((b\*x^2 + a)^p\*x/(e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(b\*x\*\*2+a)\*\*p/(e\*x+d)\*\*3,x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p x}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b\*x^2 + a)^p\*x/(e\*x + d)^3,x, algorithm="giac")

[Out] integrate((b\*x^2 + a)^p\*x/(e\*x + d)^3, x)

$$3.427 \quad \int \frac{(a+bx^2)^p}{(d+ex)^3} dx$$

**Optimal.** Leaf size=322

$$\frac{e^2 x^3 (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{3}{2}; -p, 3; \frac{5}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^5} + \frac{x (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; -p, 3; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^3} - \frac{3b^2 d^2 e (a+bx^2)^{p+1} {}_2F_1\left(3, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2+bd^2)^3} + \frac{be (a+bx^2)^{p+1} (2ae^2+bd^2(p+1)) {}_2F_1\left(2, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{4(p+1)(ae^2+bd^2)^3} - \frac{d^2 e (a+bx^2)^{p+1}}{4(d^2-e^2x^2)^2 (ae^2+bd^2)}$$

[Out]  $-(d^2 e^*(a+b*x^2)^(1+p))/(4*(b*d^2+a*e^2)*(d^2-e^2*x^2)^2) + (x*(a+b*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -(b*x^2)/a], (e^2*x^2)/d^2)]/(d^3*(1+(b*x^2)/a)^p) + (e^2*x^3*(a+b*x^2)^p*AppellF1[3/2, -p, 3, 5/2, -(b*x^2)/a], (e^2*x^2)/d^2)]/(d^5*(1+(b*x^2)/a)^p) + (b*e*(2*a*e^2+b*d^2*(1+p))*(a+b*x^2)^(1+p)*Hypergeometric2F1[2, 1+p, 2+p, (e^2*(a+b*x^2))/(b*d^2+a*e^2)])/((4*(b*d^2+a*e^2)^3*(1+p)) - (3*b^2*d^2*e*(a+b*x^2)^(1+p)*Hypergeometric2F1[3, 1+p, 2+p, (e^2*(a+b*x^2))/(b*d^2+a*e^2)])/(2*(b*d^2+a*e^2)^3*(1+p)))$

**Rubi [A]** time = 0.723602, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$

$$\frac{e^2 x^3 (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{3}{2}; -p, 3; \frac{5}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^5} + \frac{x (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; -p, 3; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^3} - \frac{3b^2 d^2 e (a+bx^2)^{p+1} {}_2F_1\left(3, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2+bd^2)^3} + \frac{be (a+bx^2)^{p+1} (2ae^2+bd^2(p+1)) {}_2F_1\left(2, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{4(p+1)(ae^2+bd^2)^3} - \frac{d^2 e (a+bx^2)^{p+1}}{4(d^2-e^2x^2)^2 (ae^2+bd^2)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^p/(d + e\*x)^3, x]

[Out]  $-(d^2 e^*(a+b*x^2)^(1+p))/(4*(b*d^2+a*e^2)*(d^2-e^2*x^2)^2) + (x*(a+b*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -(b*x^2)/a], (e^2*x^2)/d^2)]/(d^3*(1+(b*x^2)/a)^p) + (e^2*x^3*(a+b*x^2)^p*AppellF1[3/2, -p, 3, 5/2, -(b*x^2)/a], (e^2*x^2)/d^2)]/(d^5*(1+(b*x^2)/a)^p) + (b*e*(2*a*e^2+b*d^2*(1+p))*(a+b*x^2)^(1+p)*Hypergeometric2F1[2, 1+p, 2+p, (e^2*(a+b*x^2))/(b*d^2+a*e^2)])/((4*(b*d^2+a*e^2)^3*(1+p)) - (3*b^2*d^2*e*(a+b*x^2)^(1+p)*Hypergeometric2F1[3, 1+p, 2+p, (e^2*(a+b*x^2))/(b*d^2+a*e^2)])/(2*(b*d^2+a*e^2)^3*(1+p)))$

**Rubi in Sympy [A]** time = 18.4177, size = 144, normalized size = 0.45

$$\frac{\left(\frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} \left(-\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} (a+bx^2)^p \left(\frac{1}{d+ex}\right)^{2p} \left(\frac{1}{d+ex}\right)^{-2p+2} \text{appellf1}\left(-2p+2, -p, -p, -2p+3, \frac{d-\sqrt{-a}}{d+ex}, \frac{d+\sqrt{-a}}{d+ex}\right)}{2e(-p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**p/(e*x+d)**3,x)`

[Out]  $-(e*(\sqrt{b}x + \sqrt{-a})/(\sqrt{b}(d + ex)))^{(-p)}(-e*(-\sqrt{b}x + \sqrt{-a})/(\sqrt{b}(d + ex)))^{(-p)}(a + bx^2)^p(1/(d + ex))^{(2p)}(1/(d + ex))^{(-2p + 2)}\text{appellf1}(-2p + 2, -p, -p, -2p + 3, (d - e\sqrt{-a})/\sqrt{b})/(d + ex), (d + e\sqrt{-a})/\sqrt{b})/(d + ex)/(2e^{(-p + 1)})$

**Mathematica [A]** time = 0.181312, size = 142, normalized size = 0.44

$$\frac{(a + bx^2)^p \left( \frac{e^{(x - \sqrt{-a/b})}}{d + ex} \right)^{-p} \left( \frac{e^{(\sqrt{-a/b} + x)}}{d + ex} \right)^{-p} F_1 \left( 2 - 2p; -p, -p; 3 - 2p; \frac{d - \sqrt{-a/b}e}{d + ex}, \frac{d + \sqrt{-a/b}e}{d + ex} \right)}{2e^{(p - 1)}(d + ex)^2}$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(a + b*x^2)^p/(d + e*x)^3,x]`

[Out]  $((a + b*x^2)^p \text{AppellF1}[2 - 2p, -p, -p, 3 - 2p, (d - \text{Sqrt}[-(a/b)])e]/(d + e*x), (d + \text{Sqrt}[-(a/b)])e/(d + e*x)]/(2e^{(-1 + p)}(e^{(-\text{Sqrt}[-(a/b)] + x))/(d + e*x)})^p((e^{(\text{Sqrt}[-(a/b)] + x))/(d + e*x)})^p(d + e*x)^2)$

**Maple [F]** time = 0.087, size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^p/(e*x+d)^3,x)`

[Out] `int((b*x^2+a)^p/(e*x+d)^3,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/(e*x + d)^3,x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p/(e*x + d)^3, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{(bx^2 + a)^p}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/(e*x + d)^3,x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**p/(e*x+d)**3,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/(e*x + d)^3,x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p/(e*x + d)^3, x)`

$$3.428 \quad \int \frac{(a+bx^2)^p}{x(d+ex)^3} dx$$

**Optimal.** Leaf size=700

$$\begin{aligned} & \frac{e^3 x^3 (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{3}{2}; -p, 2; \frac{5}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{3d^6} \\ & - \frac{e^3 x^3 (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{3}{2}; -p, 3; \frac{5}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^6} \\ & - \frac{ex (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^4} \\ & - \frac{ex (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; -p, 2; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^4} \\ & - \frac{ex (a+bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; -p, 3; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^4} \\ & + \frac{3b^2 de^2 (a+bx^2)^{p+1} {}_2F_1\left(3, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2+bd^2)^3} \\ & - \frac{(a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a} + 1\right)}{2ad^3(p+1)} \\ & - \frac{be^2 (a+bx^2)^{p+1} (2ae^2+bd^2(p+1)) {}_2F_1\left(2, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{4d(p+1)(ae^2+bd^2)^3} \\ & + \frac{be^2 (a+bx^2)^{p+1} {}_2F_1\left(2, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{d(p+1)(ae^2+bd^2)^2} \\ & + \frac{de^2 (a+bx^2)^{p+1}}{4(d^2-e^2x^2)^2(ae^2+bd^2)} + \frac{e^2 (a+bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d^3(p+1)(ae^2+bd^2)} \end{aligned}$$

[Out]  $(d^*e^2*(a+b*x^2)^(1+p))/(4*(b*d^2+a*e^2)*(d^2-e^2*x^2)^2) - (e*x*(a+b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^4*(1+(b*x^2)/a)^p) - (e*x*(a+b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^4*(1+(b*x^2)/a)^p) - (e*x*(a+b*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^4*(1+(b*x^2)/a)^p) - (e^3*x^3*(a+b*x^2)^p*AppellF1[3/2, -p, 2, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(3*d^6*(1+(b*x^2)/a)^p) - (e^3*x^3*(a+b*x^2)^p*AppellF1[3/2, -p, 3, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^6*(1+(b*x^2)/a)^p) + (e^2*(a+b*x^2)^(1+p)*Hypergeometric2F1[1, 1+p, 2+p, (e^2*(a+b*x^2))/(b*d^2+a*e^2)])/(2*d^3*(b*d^2+a*e^2)*(1+p)) - ((a+b*x^2)^(1+p)*Hypergeometric2F1[1, 1+p, 2+p, 1+(b*x^2)/a])/(2*a*d^3*(1+p)) + (b*e^2*(a+b*x^2)^(1+p)*Hypergeometric2F1[2, 1+p, 2+p, (e^2*(a+b*x^2))/(b*d^2+a*e^2)])/(d*(b*d^2+a*e^2)^2*(1+p)) - (b*e^2*(2*a*e^2+b*d^2*(1+p))*(a+b*x^2)^(1+p)*Hypergeometric2F1[2, 1+p, 2+p, (e^2*(a+b*x^2))/(b*d^2+a*e^2)])/(4*d*(b*d^2+a*e^2)^3*(1+p)) + (3*b^2*d*e^2*(a+b*x^2)^(1+p)*Hypergeometric2F1[3, 1+p, 2+p, (e^2*(a+b*x^2))/(b*d^2+a*e^2)])/(2*(b*d^2+a*e^2)^3*(1+p))$

**Rubi [A]** time = 1.75947, antiderivative size = 700, normalized size of antiderivative = 1., number of

steps used = 29, number of rules used = 13, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.65$

$$\begin{aligned}
& \frac{e^3 x^3 (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{3}{2}; -p, 2; \frac{5}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{3d^6} \\
& - \frac{e^3 x^3 (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{3}{2}; -p, 3; \frac{5}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^6} \\
& - \frac{ex (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; -p, 1; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^4} \\
& - \frac{ex (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; -p, 2; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^4} \\
& - \frac{ex (a + bx^2)^p \left(\frac{bx^2}{a} + 1\right)^{-p} F_1\left(\frac{1}{2}; -p, 3; \frac{3}{2}; -\frac{bx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^4} \\
& + \frac{3b^2 de^2 (a + bx^2)^{p+1} {}_2F_1\left(3, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2 + bd^2)^3} \\
& - \frac{(a + bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{bx^2}{a} + 1\right)}{2ad^3(p+1)} \\
& - \frac{be^2 (a + bx^2)^{p+1} (2ae^2 + bd^2(p+1)) {}_2F_1\left(2, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{4d(p+1)(ae^2 + bd^2)^3} \\
& + \frac{be^2 (a + bx^2)^{p+1} {}_2F_1\left(2, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{d(p+1)(ae^2 + bd^2)^2} \\
& + \frac{de^2 (a + bx^2)^{p+1}}{4(d^2 - e^2 x^2)^2 (ae^2 + bd^2)} + \frac{e^2 (a + bx^2)^{p+1} {}_2F_1\left(1, p+1; p+2; \frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d^3(p+1)(ae^2 + bd^2)}
\end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^p/(x\*(d + e\*x)^3), x]

[Out] (d\*e^2\*(a + b\*x^2)^(1 + p))/(4\*(b\*d^2 + a\*e^2)\*(d^2 - e^2\*x^2)^2) - (e\*x\*(a + b\*x^2)^p\*AppellF1[1/2, -p, 1, 3/2, -(b\*x^2)/a], (e^2\*x^2)/d^2])/(d^4\*(1 + (b\*x^2)/a)^p) - (e\*x\*(a + b\*x^2)^p\*AppellF1[1/2, -p, 2, 3/2, -(b\*x^2)/a], (e^2\*x^2)/d^2])/(d^4\*(1 + (b\*x^2)/a)^p) - (e\*x\*(a + b\*x^2)^p\*AppellF1[1/2, -p, 3, 3/2, -(b\*x^2)/a], (e^2\*x^2)/d^2])/(d^4\*(1 + (b\*x^2)/a)^p) - (e^3\*x^3\*(a + b\*x^2)^p\*AppellF1[3/2, -p, 2, 5/2, -(b\*x^2)/a], (e^2\*x^2)/d^2])/(3\*d^6\*(1 + (b\*x^2)/a)^p) - (e^3\*x^3\*(a + b\*x^2)^p\*AppellF1[3/2, -p, 3, 5/2, -(b\*x^2)/a], (e^2\*x^2)/d^2])/(d^6\*(1 + (b\*x^2)/a)^p) + (e^2\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[1, 1 + p, 2 + p, (e^2\*(a + b\*x^2))/(b\*d^2 + a\*e^2)])/(2\*d^3\*(b\*d^2 + a\*e^2)\*(1 + p)) - ((a + b\*x^2)^(1 + p)\*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b\*x^2)/a])/(2\*a\*d^3\*(1 + p)) + (b\*e^2\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2\*(a + b\*x^2))/(b\*d^2 + a\*e^2)])/(d\*(b\*d^2 + a\*e^2)^2\*(1 + p)) - (b\*e^2\*(2\*a\*e^2 + b\*d^2\*(1 + p))\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[2, 1 + p, 2 + p, (e^2\*(a + b\*x^2))/(b\*d^2 + a\*e^2)])/(4\*d\*(b\*d^2 + a\*e^2)^3\*(1 + p)) + (3\*b^2\*d\*e^2\*(a + b\*x^2)^(1 + p)\*Hypergeometric2F1[3, 1 + p, 2 + p, (e^2\*(a + b\*x^2))/(b\*d^2 + a\*e^2)])/(2\*(b\*d^2 + a\*e^2)^3\*(1 + p))

**Rubi in Sympy [A]** time = 77.7613, size = 445, normalized size = 0.64

$$\frac{\left(\frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} \left(-\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} (a+bx^2)^p \left(\frac{1}{d+ex}\right)^{2p} \left(\frac{1}{d+ex}\right)^{-2p+2} \operatorname{appellf}_1\left(-2p+2, -p, -p, -2p+3, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}\right)}{2d(-p+1)}$$

$$+ \frac{\left(\frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} \left(-\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} (a+bx^2)^p \left(\frac{1}{d+ex}\right)^{2p} \left(\frac{1}{d+ex}\right)^{-2p+1} \operatorname{appellf}_1\left(-2p+1, -p, -p, -2p+2, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}\right)}{d^2(-2p+1)}$$

$$- \frac{\left(\frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} \left(-\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p} (a+bx^2)^p \operatorname{appellf}_1\left(-2p, -p, -p, -2p+1, \frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}, \frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}\right)}{2d^3p}$$

$$- \frac{(a+bx^2)^{p+1} {}_2F_1\left(1, p+1 \middle| 1 + \frac{bx^2}{a}\right)}{2ad^3(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**p/x/(e*x+d)**3,x)`

[Out]  $(e^{(\sqrt{b}x + \sqrt{-a})/(\sqrt{b}(d + ex))})^{(-p)} (-e^{(-\sqrt{b}x + \sqrt{-a})/(\sqrt{b}(d + ex))})^{(-p)} (a + b^2x^2)^p (1/(d + ex))^{(2p)} (1/(d + ex))^{(-2p + 2)} \operatorname{appellf}_1(-2p + 2, -p, -p, -2p + 3, (d - e\sqrt{-a}/\sqrt{b})/(d + ex), (d + e\sqrt{-a}/\sqrt{b})/(d + ex)) / (2d^{(-p + 1)} + (e^{(\sqrt{b}x + \sqrt{-a})/(\sqrt{b}(d + ex))})^{(-p)} (-e^{(-\sqrt{b}x + \sqrt{-a})/(\sqrt{b}(d + ex))})^{(-p)} (a + b^2x^2)^p (1/(d + ex))^{(2p)} (1/(d + ex))^{(-2p + 1)} \operatorname{appellf}_1(-2p + 1, -p, -p, -2p + 2, (d - e\sqrt{-a}/\sqrt{b})/(d + ex), (d + e\sqrt{-a}/\sqrt{b})/(d + ex)) / (d^{(-2p + 1)} - (e^{(\sqrt{b}x + \sqrt{-a})/(\sqrt{b}(d + ex))})^{(-p)} (-e^{(-\sqrt{b}x + \sqrt{-a})/(\sqrt{b}(d + ex))})^{(-p)} (a + b^2x^2)^p \operatorname{appellf}_1(-2p, -p, -p, -2p + 1, (d - e\sqrt{-a}/\sqrt{b})/(d + ex), (d + e\sqrt{-a}/\sqrt{b})/(d + ex)) / (2d^{3p}) - (a + b^2x^2)^{p+1} \operatorname{hyper}((1, p + 1), (p + 2, ), 1 + b^2x^2/a) / (2a^d)^{3(p + 1)})$

**Mathematica [A]** time = 0.0688373, size = 0, normalized size = 0.

$$\int \frac{(a + bx^2)^p}{x(d + ex)^3} dx$$

Verification is Not applicable to the result.

[In] `Integrate[(a + b*x^2)^p/(x*(d + e*x)^3), x]`

[Out] `Integrate[(a + b*x^2)^p/(x*(d + e*x)^3), x]`

**Maple [F]** time = 0.084, size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^p/x/(e*x+d)^3,x)`

[Out] `int((b*x^2+a)^p/x/(e*x+d)^3,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{(ex + d)^3 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/((e*x + d)^3*x),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p/((e*x + d)^3*x), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^p}{e^3x^4 + 3de^2x^3 + 3d^2ex^2 + d^3x}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/((e*x + d)^3*x),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p/(e^3*x^4 + 3*d*e^2*x^3 + 3*d^2*e*x^2 + d^3*x), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**p/x/(e*x+d)**3,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{(ex + d)^3 x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/((e*x + d)^3*x),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p/((e*x + d)^3*x), x)`



$$3.429 \quad \int \frac{(a+bx^2)^p}{x^2(d+ex)^3} dx$$

**Optimal.** Leaf size=754

$$\begin{aligned} & \frac{2e^4x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^7} \\ & + \frac{e^4x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,3;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^7} \\ & + \frac{3e^2x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5} \\ & + \frac{2e^2x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5} \\ & + \frac{e^2x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5} \\ & - \frac{3b^2e^3(a+bx^2)^{p+1}{}_2F_1\left(3,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2+bd^2)^3} \\ & + \frac{3e(a+bx^2)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2ad^4(p+1)} \\ & - \frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{d^3x} \\ & + \frac{be^3(a+bx^2)^{p+1}(2ae^2+bd^2(p+1)){}_2F_1\left(2,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{4d^2(p+1)(ae^2+bd^2)^3} \\ & - \frac{2be^3(a+bx^2)^{p+1}{}_2F_1\left(2,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{d^2(p+1)(ae^2+bd^2)^2} \\ & - \frac{e^3(a+bx^2)^{p+1}}{4(d^2-e^2x^2)^2(ae^2+bd^2)} - \frac{3e^3(a+bx^2)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d^4(p+1)(ae^2+bd^2)} \end{aligned}$$

```
[Out] -(e^3*(a + b*x^2)^(1 + p))/(4*(b*d^2 + a*e^2)*(d^2 - e^2*x^2)^2)
+ (3*e^2*x*(a + b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -((b*x^2)/a),
(e^2*x^2)/d^2])/(d^5*(1 + (b*x^2)/a)^p) + (2*e^2*x*(a + b*x^2)^p*
AppellF1[1/2, -p, 2, 3/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^5*(1 +
(b*x^2)/a)^p) + (e^2*x*(a + b*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -
((b*x^2)/a), (e^2*x^2)/d^2])/(d^5*(1 + (b*x^2)/a)^p) + (2*e^4*x^3
*(a + b*x^2)^p*AppellF1[3/2, -p, 2, 5/2, -((b*x^2)/a), (e^2*x^2)/
d^2])/(3*d^7*(1 + (b*x^2)/a)^p) + (e^4*x^3*(a + b*x^2)^p*AppellF1
[3/2, -p, 3, 5/2, -((b*x^2)/a), (e^2*x^2)/d^2])/(d^7*(1 + (b*x^2)
/a)^p) - ((a + b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -((b*x^2)
/a)])/(d^3*x*(1 + (b*x^2)/a)^p) - (3*e^3*(a + b*x^2)^(1 + p)*Hyp
ergeometric2F1[1, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)
])/(2*d^4*(b*d^2 + a*e^2)*(1 + p)) + (3*e*(a + b*x^2)^(1 + p)*Hyp
ergeometric2F1[1, 1 + p, 2 + p, 1 + (b*x^2)/a])/(2*a*d^4*(1 + p))
- (2*b*e^3*(a + b*x^2)^(1 + p)*Hypergeometric2F1[2, 1 + p, 2 + p
, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])/(d^2*(b*d^2 + a*e^2)^2*(1 +
p)) + (b*e^3*(2*a*e^2 + b*d^2*(1 + p))*(a + b*x^2)^(1 + p)*Hyper
geometric2F1[2, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 + a*e^2)])
/(4*d^2*(b*d^2 + a*e^2)^3*(1 + p)) - (3*b^2*e^3*(a + b*x^2)^(1 +
p)*Hypergeometric2F1[3, 1 + p, 2 + p, (e^2*(a + b*x^2))/(b*d^2 +
a*e^2)])/(2*(b*d^2 + a*e^2)^3*(1 + p))
```

**Rubi [A]** time = 1.82706, antiderivative size = 754, normalized size of antiderivative = 1., number of

steps used = 31, number of rules used = 15, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.75$

$$\begin{aligned}
& \frac{2e^4x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,2;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{3d^7} \\
& + \frac{e^4x^3(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{3}{2};-p,3;\frac{5}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^7} \\
& + \frac{3e^2x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,1;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5} \\
& + \frac{2e^2x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,2;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5} \\
& + \frac{e^2x(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-p,3;\frac{3}{2};-\frac{bx^2}{a},\frac{e^2x^2}{d^2}\right)}{d^5} \\
& - \frac{3b^2e^3(a+bx^2)^{p+1}{}_2F_1\left(3,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2(p+1)(ae^2+bd^2)^3} \\
& + \frac{3e(a+bx^2)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{bx^2}{a}+1\right)}{2ad^4(p+1)} \\
& - \frac{(a+bx^2)^p\left(\frac{bx^2}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};-\frac{bx^2}{a}\right)}{d^3x} \\
& + \frac{be^3(a+bx^2)^{p+1}(2ae^2+bd^2(p+1)){}_2F_1\left(2,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{4d^2(p+1)(ae^2+bd^2)^3} \\
& - \frac{2be^3(a+bx^2)^{p+1}{}_2F_1\left(2,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{d^2(p+1)(ae^2+bd^2)^2} \\
& - \frac{e^3(a+bx^2)^{p+1}}{4(d^2-e^2x^2)^2(ae^2+bd^2)} - \frac{3e^3(a+bx^2)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{e^2(bx^2+a)}{bd^2+ae^2}\right)}{2d^4(p+1)(ae^2+bd^2)}
\end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x^2)^p/(x^2\*(d + e\*x)^3), x]

[Out]  $-(e^3*(a+b*x^2)^{(1+p)})/(4*(b*d^2+a*e^2)*(d^2-e^2*x^2)^2)$   
 $+ (3*e^2*x*(a+b*x^2)^p*AppellF1[1/2, -p, 1, 3/2, -(b*x^2)/a, (e^2*x^2)/d^2])/(d^5*(1+(b*x^2)/a)^p) + (2*e^2*x*(a+b*x^2)^p*AppellF1[1/2, -p, 2, 3/2, -(b*x^2)/a, (e^2*x^2)/d^2])/(d^5*(1+(b*x^2)/a)^p) + (e^2*x*(a+b*x^2)^p*AppellF1[1/2, -p, 3, 3/2, -(b*x^2)/a, (e^2*x^2)/d^2])/(d^5*(1+(b*x^2)/a)^p) + (e^2*x^2*(a+b*x^2)^p*AppellF1[3/2, -p, 2, 5/2, -(b*x^2)/a, (e^2*x^2)/d^2])/(3*d^7*(1+(b*x^2)/a)^p) + (e^4*x^3*(a+b*x^2)^p*AppellF1[3/2, -p, 3, 5/2, -(b*x^2)/a, (e^2*x^2)/d^2])/(d^7*(1+(b*x^2)/a)^p) - ((a+b*x^2)^p*Hypergeometric2F1[-1/2, -p, 1/2, -(b*x^2)/a])/(d^3*x*(1+(b*x^2)/a)^p) - (3*e^3*(a+b*x^2)^{(1+p)}*Hypergeometric2F1[1, 1+p, 2+p, (e^2*(a+b*x^2))/(b*d^2+a*e^2)])/(2*d^4*(b*d^2+a*e^2)*(1+p)) + (3*e*(a+b*x^2)^{(1+p)}*Hypergeometric2F1[1, 1+p, 2+p, 1+(b*x^2)/a])/(2*a*d^4*(1+p)) - (2*b*e^3*(a+b*x^2)^{(1+p)}*Hypergeometric2F1[2, 1+p, 2+p, (e^2*(a+b*x^2))/(b*d^2+a*e^2)])/(d^2*(b*d^2+a*e^2)^2*(1+p)) + (b*e^3*(2*a*e^2+b*d^2*(1+p))*(a+b*x^2)^{(1+p)}*Hypergeometric2F1[2, 1+p, 2+p, (e^2*(a+b*x^2))/(b*d^2+a*e^2)])/(4*d^2*(b*d^2+a*e^2)^3*(1+p)) - (3*b^2*e^3*(a+b*x^2)^{(1+p)}*Hypergeometric2F1[3, 1+p, 2+p, (e^2*(a+b*x^2))/(b*d^2+a*e^2)])/(2*(b*d^2+a*e^2)^3*(1+p))$

**Rubi in Sympy [A]** time = 96.0753, size = 500, normalized size = 0.66

$$\frac{e\left(\frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p}\left(-\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p}(a+bx^2)^p\left(\frac{1}{d+ex}\right)^{2p}\left(\frac{1}{d+ex}\right)^{-2p+2}\operatorname{appellf}_1\left(-2p+2,-p,-p,-2p+3,\frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex},\frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}\right)}{2d^2(-p+1)}$$

$$\frac{2e\left(\frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p}\left(-\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p}(a+bx^2)^p\left(\frac{1}{d+ex}\right)^{2p}\left(\frac{1}{d+ex}\right)^{-2p+1}\operatorname{appellf}_1\left(-2p+1,-p,-p,-2p+2,\frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex},\frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}\right)}{d^3(-2p+1)}$$

$$\frac{\left(1+\frac{bx^2}{a}\right)^{-p}(a+bx^2)^p{}_2F_1\left(-p,-\frac{1}{2}\middle|\frac{bx^2}{a}\right)}{d^3x}$$

$$+ \frac{3e\left(\frac{e(\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p}\left(-\frac{e(-\sqrt{bx+\sqrt{-a}})}{\sqrt{b(d+ex)}}\right)^{-p}(a+bx^2)^p\operatorname{appellf}_1\left(-2p,-p,-p,-2p+1,\frac{d-\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex},\frac{d+\frac{e\sqrt{-a}}{\sqrt{b}}}{d+ex}\right)}{2d^4p}$$

$$+ \frac{3e(a+bx^2)^{p+1}{}_2F_1\left(1,p+1\middle|1+\frac{bx^2}{a}\right)}{2ad^4(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((b*x**2+a)**p/x**2/(e*x+d)**3,x)`

[Out]  $-e^*(e*(\sqrt{b}*x + \sqrt{-a})/(\sqrt{b}*(d + e*x)))^{(-p)}*(-e*(-\sqrt{b}*x + \sqrt{-a})/(\sqrt{b}*(d + e*x)))^{(-p)}*(a + b*x**2)**p*(1/(d + e*x))^{(2*p)}*(1/(d + e*x))^{(-2*p + 2)}*\operatorname{appellf}_1(-2*p + 2, -p, -p, -2*p + 3, (d - e*\sqrt{-a})/\sqrt{b})/(d + e*x), (d + e*\sqrt{-a})/\sqrt{b})/(d + e*x)/(2*d**2*(-p + 1)) - 2*e*(e*(\sqrt{b}*x + \sqrt{-a})/(\sqrt{b}*(d + e*x)))^{(-p)}*(-e*(-\sqrt{b}*x + \sqrt{-a})/(\sqrt{b}*(d + e*x)))^{(-p)}*(a + b*x**2)**p*(1/(d + e*x))^{(2*p)}*(1/(d + e*x))^{(-2*p + 1)}*\operatorname{appellf}_1(-2*p + 1, -p, -p, -2*p + 2, (d - e*\sqrt{-a})/\sqrt{b})/(d + e*x), (d + e*\sqrt{-a})/\sqrt{b})/(d + e*x)/(d**3*(-2*p + 1)) - (1 + b*x**2/a)^{(-p)}*(a + b*x**2)**p*\operatorname{hyper}((-p, -1/2), (1/2, ), -b*x**2/a)/(d**3*x) + 3*e*(e*(\sqrt{b}*x + \sqrt{-a})/(\sqrt{b}*(d + e*x)))^{(-p)}*(-e*(-\sqrt{b}*x + \sqrt{-a})/(\sqrt{b}*(d + e*x)))^{(-p)}*(a + b*x**2)**p*\operatorname{appellf}_1(-2*p, -p, -p, -2*p + 1, (d - e*\sqrt{-a})/\sqrt{b})/(d + e*x), (d + e*\sqrt{-a})/\sqrt{b})/(d + e*x)/(2*d**4*p) + 3*e*(a + b*x**2)**(p + 1)*\operatorname{hyper}((1, p + 1), (p + 2, ), 1 + b*x**2/a)/(2*a*d**4*(p + 1))$

**Mathematica [A]** time = 0.156269, size = 0, normalized size = 0.

$$\int \frac{(a + bx^2)^p}{x^2(d + ex)^3} dx$$

Verification is Not applicable to the result.

[In] `Integrate[(a + b*x^2)^p/(x^2*(d + e*x)^3),x]`

[Out] `Integrate[(a + b*x^2)^p/(x^2*(d + e*x)^3), x]`

**Maple [F]** time = 0.14, size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{x^2(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*x^2+a)^p/x^2/(e*x+d)^3,x)`

[Out] `int((b*x^2+a)^p/x^2/(e*x+d)^3,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{(ex + d)^3 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/((e*x + d)^3*x^2),x, algorithm="maxima")`

[Out] `integrate((b*x^2 + a)^p/((e*x + d)^3*x^2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(bx^2 + a)^p}{e^3x^5 + 3de^2x^4 + 3d^2ex^3 + d^3x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/((e*x + d)^3*x^2),x, algorithm="fricas")`

[Out] `integral((b*x^2 + a)^p/(e^3*x^5 + 3*d*e^2*x^4 + 3*d^2*e*x^3 + d^3*x^2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x**2+a)**p/x**2/(e*x+d)**3,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(bx^2 + a)^p}{(ex + d)^3 x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*x^2 + a)^p/((e*x + d)^3*x^2),x, algorithm="giac")`

[Out] `integrate((b*x^2 + a)^p/((e*x + d)^3*x^2), x)`

### 3.430 $\int (gx)^m (d + ex)^3 (a + cx^2)^p dx$

**Optimal.** Leaf size=276

$$\frac{e(gx)^{m+2} (a + cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} (ae^2(m+2) - 3cd^2(m+2p+4)) {}_2F_1\left(\frac{m+2}{2}, -p; \frac{m+4}{2}; -\frac{cx^2}{a}\right)}{cg^2(m+2)(m+2p+4)} - \frac{d(gx)^{m+1} (a + cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} (3ae^2(m+1) - cd^2(m+2p+3)) {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+3}{2}; -\frac{cx^2}{a}\right)}{cg(m+1)(m+2p+3)} + \frac{3de^2(gx)^{m+1} (a + cx^2)^{p+1}}{cg(m+2p+3)} + \frac{e^3(gx)^{m+2} (a + cx^2)^{p+1}}{cg^2(m+2p+4)}$$

[Out]  $(3*d*e^2*(g*x)^(1+m)*(a+c*x^2)^(1+p))/(c*g*(3+m+2*p)) + (e^3*(g*x)^(2+m)*(a+c*x^2)^(1+p))/(c*g^2*(4+m+2*p)) - (d*(3*a*e^2*(1+m) - c*d^2*(3+m+2*p))*(g*x)^(1+m)*(a+c*x^2)^p*Hypergeometric2F1[(1+m)/2, -p, (3+m)/2, -((c*x^2)/a)])/(c*g*(1+m)*(3+m+2*p)*(1+(c*x^2)/a)^p) - (e*(a*e^2*(2+m) - 3*c*d^2*(4+m+2*p))*(g*x)^(2+m)*(a+c*x^2)^p*Hypergeometric2F1[(2+m)/2, -p, (4+m)/2, -((c*x^2)/a)])/(c*g^2*(2+m)*(4+m+2*p)*(1+(c*x^2)/a)^p)$

**Rubi [A]** time = 0.851224, antiderivative size = 254, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{e(gx)^{m+2} (a + cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} \left(\frac{3d^2}{m+2} - \frac{ae^2}{c(m+2p+4)}\right) {}_2F_1\left(\frac{m+2}{2}, -p; \frac{m+4}{2}; -\frac{cx^2}{a}\right)}{g^2} + \frac{d(gx)^{m+1} (a + cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} \left(\frac{d^2}{m+1} - \frac{3ae^2}{c(m+2p+3)}\right) {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+3}{2}; -\frac{cx^2}{a}\right)}{g} + \frac{3de^2(gx)^{m+1} (a + cx^2)^{p+1}}{cg(m+2p+3)} + \frac{e^3(gx)^{m+2} (a + cx^2)^{p+1}}{cg^2(m+2p+4)}$$

Antiderivative was successfully verified.

[In] Int[(g\*x)^m\*(d + e\*x)^3\*(a + c\*x^2)^p, x]

[Out]  $(3*d*e^2*(g*x)^(1+m)*(a+c*x^2)^(1+p))/(c*g*(3+m+2*p)) + (e^3*(g*x)^(2+m)*(a+c*x^2)^(1+p))/(c*g^2*(4+m+2*p)) + (d*(d^2/(1+m) - (3*a*e^2)/(c*(3+m+2*p)))*(g*x)^(1+m)*(a+c*x^2)^p*Hypergeometric2F1[(1+m)/2, -p, (3+m)/2, -((c*x^2)/a)])/(g*(1+(c*x^2)/a)^p) + (e*((3*d^2)/(2+m) - (a*e^2)/(c*(4+m+2*p)))*(g*x)^(2+m)*(a+c*x^2)^p*Hypergeometric2F1[(2+m)/2, -p, (4+m)/2, -((c*x^2)/a)])/(g^2*(1+(c*x^2)/a)^p)$

**Rubi in Sympy [A]** time = 65.7692, size = 233, normalized size = 0.84

$$\frac{3de^2(gx)^{m+1} (a + cx^2)^{p+1}}{cg(m+2p+3)} - \frac{d(gx)^{m+1} \left(1 + \frac{cx^2}{a}\right)^{-p} (a + cx^2)^p (3ae^2(m+1) - cd^2(m+2p+3)) {}_2F_1\left(-p, \frac{m}{2} + \frac{1}{2}; \frac{m}{2} + \frac{3}{2}; -\frac{cx^2}{a}\right)}{cg(m+1)(m+2p+3)} + \frac{e^3(gx)^{m+2} (a + cx^2)^{p+1}}{cg^2(m+2p+4)} - \frac{e(gx)^{m+2} \left(1 + \frac{cx^2}{a}\right)^{-p} (a + cx^2)^p (ae^2(m+2) - 3cd^2(m+2p+4)) {}_2F_1\left(-p, \frac{m}{2} + 1; \frac{m}{2} + 2; -\frac{cx^2}{a}\right)}{cg^2(m+2)(m+2p+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x)**m*(e*x+d)**3*(c*x**2+a)**p,x)`

[Out]  $3*d*e^{2*(g*x)^{m+1}*(a+c*x^2)^{p+1}}/(c*g^{m+2*p+3}) - d*(g*x)^{m+1}*(1+c*x^2/a)^{-p}*(a+c*x^2)^p*(3*a*e^{2*(m+1)-c*d^2*(m+2*p+3)}*\text{hyper}((-p, m/2+1/2), (m/2+3/2, ), -c*x^2/a)/(c*g^{m+1}*(m+2*p+3)) + e^{3*(g*x)^{m+2}*(a+c*x^2)^{p+1}}/(c*g^{2*(m+2*p+4)}) - e*(g*x)^{m+2}*(1+c*x^2/a)^{-p}*(a+c*x^2)^p*(a*e^{2*(m+2)-3*c*d^2*(m+2*p+4)}*\text{hyper}((-p, m/2+1), (m/2+2, ), -c*x^2/a)/(c*g^{2*(m+2)}*(m+2*p+4))$

**Mathematica [A]** time = 0.253318, size = 186, normalized size = 0.67

$$x(gx)^m (a+cx^2)^p \left( \frac{cx^2}{a} + 1 \right)^{-p} \left( \frac{d^3 {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+3}{2}; -\frac{cx^2}{a}\right)}{m+1} + \frac{3d^2 ex {}_2F_1\left(\frac{m}{2}+1, -p; \frac{m}{2}+2; -\frac{cx^2}{a}\right)}{m+2} + \frac{3de^2 x^2 {}_2F_1\left(\frac{m+3}{2}, -p; \frac{m+5}{2}; -\frac{cx^2}{a}\right)}{m+3} + \frac{e^3 x^3 {}_2F_1\left(\frac{m}{2}+2, -p; \frac{m}{2}+3; -\frac{cx^2}{a}\right)}{m+4} \right)$$

Antiderivative was successfully verified.

[In] `Integrate[(g*x)^m*(d+e*x)^3*(a+c*x^2)^p,x]`

[Out]  $(x*(g*x)^m*(a+c*x^2)^p*((3*d^2*e*x*\text{Hypergeometric2F1}[1+m/2, -p, 2+m/2, -((c*x^2)/a)]/(2+m) + (e^3*x^3*\text{Hypergeometric2F1}[2+m/2, -p, 3+m/2, -((c*x^2)/a)]/(4+m) + (d^3*\text{Hypergeometric2F1}[(1+m)/2, -p, (3+m)/2, -((c*x^2)/a)]/(1+m) + (3*d*e^2*x^2*\text{Hypergeometric2F1}[(3+m)/2, -p, (5+m)/2, -((c*x^2)/a)]/(3+m)))/(1+(c*x^2)/a)^p$

**Maple [F]** time = 0.086, size = 0, normalized size = 0.

$$\int (gx)^m (ex+d)^3 (cx^2+a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x)^m*(e*x+d)^3*(c*x^2+a)^p,x)`

[Out] `int((g*x)^m*(e*x+d)^3*(c*x^2+a)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex+d)^3 (cx^2+a)^p (gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^3*(c*x^2+a)^p*(g*x)^m,x, algorithm="maxima")`

[Out] `integrate((e*x+d)^3*(c*x^2+a)^p*(g*x)^m,x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3\right)(cx^2 + a)^p(gx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(c\*x^2 + a)^p\*(g\*x)^m,x, algorithm="fricas")

[Out] integral((e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3)\*(c\*x^2 + a)^p\*(g\*x)^m, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x)\*\*m\*(e\*x+d)\*\*3\*(c\*x\*\*2+a)\*\*p,x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^3(cx^2 + a)^p(gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(c\*x^2 + a)^p\*(g\*x)^m,x, algorithm="giac")

[Out] integrate((e\*x + d)^3\*(c\*x^2 + a)^p\*(g\*x)^m, x)

### 3.431 $\int (gx)^m (d + ex)^2 (a + cx^2)^p dx$

**Optimal.** Leaf size=205

$$\frac{(gx)^{m+1} (a + cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} (ae^2(m+1) - cd^2(m+2p+3)) {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+3}{2}; -\frac{cx^2}{a}\right)}{cg(m+1)(m+2p+3)} + \frac{2de(gx)^{m+2} (a + cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+2}{2}, -p; \frac{m+4}{2}; -\frac{cx^2}{a}\right)}{g^2(m+2)} + \frac{e^2(gx)^{m+1} (a + cx^2)^{p+1}}{cg(m+2p+3)}$$

[Out]  $(e^{2*(g*x)^{(1+m)*(a+c*x^2)^{(1+p)}}}/(c*g*(3+m+2*p))) - ((a^{2*(1+m)} - c*d^{2*(3+m+2*p)})*(g*x)^{(1+m)*(a+c*x^2)^p} \text{Hypergeometric2F1}[(1+m)/2, -p, (3+m)/2, -((c*x^2)/a)]/(c*g*(1+m)*(3+m+2*p)*(1+(c*x^2)/a)^p) + (2*d*e*(g*x)^{(2+m)*(a+c*x^2)^p} \text{Hypergeometric2F1}[(2+m)/2, -p, (4+m)/2, -((c*x^2)/a)]/(g^{2*(2+m)*(1+(c*x^2)/a)^p}))$

**Rubi [A]** time = 0.401063, antiderivative size = 194, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{(gx)^{m+1} (a + cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} \left(\frac{d^2}{m+1} - \frac{ae^2}{c(m+2p+3)}\right) {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+3}{2}; -\frac{cx^2}{a}\right)}{g} + \frac{2de(gx)^{m+2} (a + cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+2}{2}, -p; \frac{m+4}{2}; -\frac{cx^2}{a}\right)}{g^2(m+2)} + \frac{e^2(gx)^{m+1} (a + cx^2)^{p+1}}{cg(m+2p+3)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(g*x)^m*(d + e*x)^2*(a + c*x^2)^p, x]$

[Out]  $(e^{2*(g*x)^{(1+m)*(a+c*x^2)^{(1+p)}}}/(c*g*(3+m+2*p))) + ((d^{2*(1+m)} - (a*e^2)/(c*(3+m+2*p)))*(g*x)^{(1+m)*(a+c*x^2)^p} \text{Hypergeometric2F1}[(1+m)/2, -p, (3+m)/2, -((c*x^2)/a)]/(g*(1+(c*x^2)/a)^p) + (2*d*e*(g*x)^{(2+m)*(a+c*x^2)^p} \text{Hypergeometric2F1}[(2+m)/2, -p, (4+m)/2, -((c*x^2)/a)]/(g^{2*(2+m)*(1+(c*x^2)/a)^p}))$

**Rubi in Sympy [A]** time = 35.3942, size = 167, normalized size = 0.81

$$\frac{2de(gx)^{m+2} \left(1 + \frac{cx^2}{a}\right)^{-p} (a + cx^2)^p {}_2F_1\left(-p, \frac{m}{2} + 1; \frac{m}{2} + 2; -\frac{cx^2}{a}\right)}{g^2(m+2)} + \frac{e^2(gx)^{m+1} (a + cx^2)^{p+1}}{cg(m+2p+3)} - \frac{(gx)^{m+1} \left(1 + \frac{cx^2}{a}\right)^{-p} (a + cx^2)^p (ae^2(m+1) - cd^2(m+2p+3)) {}_2F_1\left(-p, \frac{m}{2} + \frac{1}{2}; \frac{m}{2} + \frac{3}{2}; -\frac{cx^2}{a}\right)}{cg(m+1)(m+2p+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((g*x)**m*(e*x+d)**2*(c*x**2+a)**p, x)$

[Out]  $2*d*e*(g*x)**(m+2)*(1+c*x**2/a)**(-p)*(a+c*x**2)**p*\text{hyper}((-p, m/2+1), (m/2+2, ), -c*x**2/a)/(g**2*(m+2)) + e**2*(g*x)**(m+1)*(a+c*x**2)**(p+1)/(c*g*(m+2*p+3)) - (g*x)**(m+1)*(1+c*x**2/a)**(-p)*(a+c*x**2)**p*(a*e**2*(m+1) - c*d**2*(m+2*p+3))*\text{hyper}((-p, m/2+1/2), (m/2+3/2, ), -c*x**2/a)/(c$



\*g\*(m + 1)\*(m + 2\*p + 3))

**Mathematica [A]** time = 0.168019, size = 162, normalized size = 0.79

$$\frac{x(gx)^m (a + cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} \left( (m+2) \left( d^2(m+3) {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+3}{2}; -\frac{cx^2}{a}\right) + e^2(m+1)x^2 {}_2F_1\left(\frac{m+3}{2}, -p; \frac{m+5}{2}; -\frac{cx^2}{a}\right) \right) + 2d \right)}{(m+1)(m+2)(m+3)}$$

Antiderivative was successfully verified.

[In] Integrate[(g\*x)^m\*(d + e\*x)^2\*(a + c\*x^2)^p,x]

[Out] (x\*(g\*x)^m\*(a + c\*x^2)^p\*(2\*d\*e\*(3 + 4\*m + m^2)\*x\*Hypergeometric2F1[1 + m/2, -p, 2 + m/2, -((c\*x^2)/a)] + (2 + m)\*(d^2\*(3 + m)\*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, -((c\*x^2)/a)] + e^2\*(1 + m)\*x^2\*Hypergeometric2F1[(3 + m)/2, -p, (5 + m)/2, -((c\*x^2)/a)]))/((1 + m)\*(2 + m)\*(3 + m)\*(1 + (c\*x^2)/a)^p)

**Maple [F]** time = 0.079, size = 0, normalized size = 0.

$$\int (gx)^m (ex + d)^2 (cx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x)^m\*(e\*x+d)^2\*(c\*x^2+a)^p,x)

[Out] int((g\*x)^m\*(e\*x+d)^2\*(c\*x^2+a)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2 (cx^2 + a)^p (gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(c\*x^2 + a)^p\*(g\*x)^m,x, algorithm="maxima")

[Out] integrate((e\*x + d)^2\*(c\*x^2 + a)^p\*(g\*x)^m, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^2 + 2dex + d^2\right)(cx^2 + a)^p (gx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*(c\*x^2 + a)^p\*(g\*x)^m,x, algorithm="fricas")

[Out] integral((e^2\*x^2 + 2\*d\*e\*x + d^2)\*(c\*x^2 + a)^p\*(g\*x)^m, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x)**m*(e*x+d)**2*(c*x**2+a)**p,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)^2 (cx^2 + a)^p (gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^2*(c*x^2 + a)^p*(g*x)^m,x, algorithm="giac")`

[Out] `integrate((e*x + d)^2*(c*x^2 + a)^p*(g*x)^m, x)`

### 3.432 $\int (gx)^m (d + ex) (a + cx^2)^p dx$

**Optimal.** Leaf size=135

$$\frac{d(gx)^{m+1} (a + cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+3}{2}; -\frac{cx^2}{a}\right)}{g(m+1)} + \frac{e(gx)^{m+2} (a + cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+2}{2}, -p; \frac{m+4}{2}; -\frac{cx^2}{a}\right)}{g^2(m+2)}$$

[Out]  $(d*(g*x)^{(1+m)}*(a+c*x^2)^p*Hypergeometric2F1[(1+m)/2, -p, (3+m)/2, -((c*x^2)/a)]/(g*(1+m)*(1+(c*x^2)/a)^p) + (e*(g*x)^{(2+m)}*(a+c*x^2)^p*Hypergeometric2F1[(2+m)/2, -p, (4+m)/2, -((c*x^2)/a)]/(g^2*(2+m)*(1+(c*x^2)/a)^p)$

**Rubi [A]** time = 0.150914, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{d(gx)^{m+1} (a + cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+3}{2}; -\frac{cx^2}{a}\right)}{g(m+1)} + \frac{e(gx)^{m+2} (a + cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+2}{2}, -p; \frac{m+4}{2}; -\frac{cx^2}{a}\right)}{g^2(m+2)}$$

Antiderivative was successfully verified.

[In] Int[(g\*x)^m\*(d + e\*x)\*(a + c\*x^2)^p, x]

[Out]  $(d*(g*x)^{(1+m)}*(a+c*x^2)^p*Hypergeometric2F1[(1+m)/2, -p, (3+m)/2, -((c*x^2)/a)]/(g*(1+m)*(1+(c*x^2)/a)^p) + (e*(g*x)^{(2+m)}*(a+c*x^2)^p*Hypergeometric2F1[(2+m)/2, -p, (4+m)/2, -((c*x^2)/a)]/(g^2*(2+m)*(1+(c*x^2)/a)^p)$

**Rubi in Sympy [A]** time = 20.8313, size = 105, normalized size = 0.78

$$\frac{d(gx)^{m+1} \left(1 + \frac{cx^2}{a}\right)^{-p} (a + cx^2)^p {}_2F_1\left(-p, \frac{m}{2} + \frac{1}{2} \middle| -\frac{cx^2}{a}\right)}{g(m+1)} + \frac{e(gx)^{m+2} \left(1 + \frac{cx^2}{a}\right)^{-p} (a + cx^2)^p {}_2F_1\left(-p, \frac{m}{2} + 1 \middle| -\frac{cx^2}{a}\right)}{g^2(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x)\*\*m\*(e\*x+d)\*(c\*x\*\*2+a)\*\*p, x)

[Out]  $d*(g*x)**(m+1)*(1+c*x**2/a)**(-p)*(a+c*x**2)**p*hyper((-p, m/2+1/2), (m/2+3/2, ), -c*x**2/a)/(g*(m+1)) + e*(g*x)**(m+2)*(1+c*x**2/a)**(-p)*(a+c*x**2)**p*hyper((-p, m/2+1), (m/2+2, ), -c*x**2/a)/(g**2*(m+2))$

**Mathematica [A]** time = 0.0996037, size = 106, normalized size = 0.79

$$\frac{x(gx)^m (a + cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} \left(d(m+2) {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+3}{2}; -\frac{cx^2}{a}\right) + e(m+1)x {}_2F_1\left(\frac{m}{2} + 1, -p; \frac{m}{2} + 2; -\frac{cx^2}{a}\right)\right)}{(m+1)(m+2)}$$

Antiderivative was successfully verified.

[In] Integrate[(g\*x)^m\*(d + e\*x)\*(a + c\*x^2)^p,x]

[Out] (x\*(g\*x)^m\*(a + c\*x^2)^p\*(e\*(1 + m)\*x\*Hypergeometric2F1[1 + m/2, -p, 2 + m/2, -((c\*x^2)/a)] + d\*(2 + m)\*Hypergeometric2F1[(1 + m)/2, -p, (3 + m)/2, -((c\*x^2)/a)])/((1 + m)\*(2 + m)\*(1 + (c\*x^2)/a)^p)

**Maple [F]** time = 0.067, size = 0, normalized size = 0.

$$\int (gx)^m (ex + d) (cx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x)^m\*(e\*x+d)\*(c\*x^2+a)^p,x)

[Out] int((g\*x)^m\*(e\*x+d)\*(c\*x^2+a)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(cx^2 + a)^p (gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(c\*x^2 + a)^p\*(g\*x)^m,x, algorithm="maxima")

[Out] integrate((e\*x + d)\*(c\*x^2 + a)^p\*(g\*x)^m, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ex + d)(cx^2 + a)^p (gx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(c\*x^2 + a)^p\*(g\*x)^m,x, algorithm="fricas")

[Out] integral((e\*x + d)\*(c\*x^2 + a)^p\*(g\*x)^m, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x)\*\*m\*(e\*x+d)\*(c\*x\*\*2+a)\*\*p,x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (ex + d)(cx^2 + a)^p (gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*(c\*x^2 + a)^p\*(g\*x)^m,x, algorithm="giac")

[Out] integrate((e\*x + d)\*(c\*x^2 + a)^p\*(g\*x)^m, x)

### 3.433 $\int (gx)^m (a + cx^2)^p dx$

**Optimal.** Leaf size=66

$$\frac{(gx)^{m+1} (a + cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+3}{2}; -\frac{cx^2}{a}\right)}{g(m+1)}$$

[Out]  $((g*x)^{(1+m)}*(a+c*x^2)^p*Hypergeometric2F1[(1+m)/2, -p, (3+m)/2, -((c*x^2)/a)])/(g*(1+m)*(1+(c*x^2)/a)^p)$

**Rubi [A]** time = 0.0479034, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{(gx)^{m+1} (a + cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+3}{2}; -\frac{cx^2}{a}\right)}{g(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(g\*x)^m\*(a+c\*x^2)^p,x]

[Out]  $((g*x)^{(1+m)}*(a+c*x^2)^p*Hypergeometric2F1[(1+m)/2, -p, (3+m)/2, -((c*x^2)/a)])/(g*(1+m)*(1+(c*x^2)/a)^p)$

**Rubi in Sympy [A]** time = 8.98148, size = 51, normalized size = 0.77

$$\frac{(gx)^{m+1} \left(1 + \frac{cx^2}{a}\right)^{-p} (a + cx^2)^p {}_2F_1\left(-p, \frac{m}{2} + \frac{1}{2} \middle| \frac{m}{2} + \frac{3}{2}; -\frac{cx^2}{a}\right)}{g(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x)\*\*m\*(c\*x\*\*2+a)\*\*p,x)

[Out]  $(g*x)**(m+1)*(1+c*x**2/a)**(-p)*(a+c*x**2)**p*hyper((-p, m/2+1/2), (m/2+3/2, ), -c*x**2/a)/(g*(m+1))$

**Mathematica [A]** time = 0.0356605, size = 64, normalized size = 0.97

$$\frac{x(gx)^m (a + cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} {}_2F_1\left(\frac{m+1}{2}, -p; \frac{m+1}{2} + 1; -\frac{cx^2}{a}\right)}{m+1}$$

Antiderivative was successfully verified.

[In] Integrate[(g\*x)^m\*(a+c\*x^2)^p,x]

[Out]  $(x*(g*x)^m*(a+c*x^2)^p*Hypergeometric2F1[(1+m)/2, -p, 1+(1+m)/2, -((c*x^2)/a)])/((1+m)*(1+(c*x^2)/a)^p)$

**Maple [F]** time = 0.08, size = 0, normalized size = 0.

$$\int (gx)^m (cx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x)^m*(c*x^2+a)^p,x)`

[Out] `int((g*x)^m*(c*x^2+a)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (cx^2 + a)^p (gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)^p*(g*x)^m,x, algorithm="maxima")`

[Out] `integrate((c*x^2 + a)^p*(g*x)^m, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((cx^2 + a)^p (gx)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)^p*(g*x)^m,x, algorithm="fricas")`

[Out] `integral((c*x^2 + a)^p*(g*x)^m, x)`

**Sympy [A]** time = 107.458, size = 54, normalized size = 0.82

$$\frac{a^p g^m x x^m \left(\frac{m}{2} + \frac{1}{2}\right) {}_2F_1\left(-p, \frac{m}{2} + \frac{1}{2} \middle| \frac{cx^2 e^{i\pi}}{a}\right)}{2 \left(\frac{m}{2} + \frac{3}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x)**m*(c*x**2+a)**p,x)`

[Out] `a**p*g**m*x*x**m*gamma(m/2 + 1/2)*hyper((-p, m/2 + 1/2), (m/2 + 3/2, ), c*x**2*exp_polar(I*pi)/a)/(2*gamma(m/2 + 3/2))`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (cx^2 + a)^p (gx)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)^p*(g*x)^m,x, algorithm="giac")`

[Out] `integrate((c*x^2 + a)^p*(g*x)^m, x)`

$$3.434 \quad \int \frac{(gx)^m (a+cx^2)^P}{d+ex} dx$$

**Optimal.** Leaf size=157

$$\frac{x(gx)^m (a+cx^2)^P \left(\frac{cx^2}{a} + 1\right)^{-P} F_1\left(\frac{m+1}{2}; -p, 1; \frac{m+3}{2}; -\frac{cx^2}{a}, \frac{e^2x^2}{d^2}\right)}{d(m+1)} - \frac{ex^2(gx)^m (a+cx^2)^P \left(\frac{cx^2}{a} + 1\right)^{-P} F_1\left(\frac{m+2}{2}; -p, 1; \frac{m+4}{2}; -\frac{cx^2}{a}, \frac{e^2x^2}{d^2}\right)}{d^2(m+2)}$$

[Out] (x\*(g\*x)^m\*(a+c\*x^2)^p\*AppellF1[(1+m)/2, -p, 1, (3+m)/2, -(c\*x^2)/a, (e^2\*x^2)/d^2])/(d\*(1+m)\*(1+(c\*x^2)/a)^p) - (e\*x^2\*(g\*x)^m\*(a+c\*x^2)^p\*AppellF1[(2+m)/2, -p, 1, (4+m)/2, -(c\*x^2)/a, (e^2\*x^2)/d^2])/(d^2\*(2+m)\*(1+(c\*x^2)/a)^p)

**Rubi [A]** time = 0.36437, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$\frac{x(gx)^m (a+cx^2)^P \left(\frac{cx^2}{a} + 1\right)^{-P} F_1\left(\frac{m+1}{2}; -p, 1; \frac{m+3}{2}; -\frac{cx^2}{a}, \frac{e^2x^2}{d^2}\right)}{d(m+1)} - \frac{ex^2(gx)^m (a+cx^2)^P \left(\frac{cx^2}{a} + 1\right)^{-P} F_1\left(\frac{m+2}{2}; -p, 1; \frac{m+4}{2}; -\frac{cx^2}{a}, \frac{e^2x^2}{d^2}\right)}{d^2(m+2)}$$

Antiderivative was successfully verified.

[In] Int[((g\*x)^m\*(a+c\*x^2)^p)/(d+e\*x), x]

[Out] (x\*(g\*x)^m\*(a+c\*x^2)^p\*AppellF1[(1+m)/2, -p, 1, (3+m)/2, -(c\*x^2)/a, (e^2\*x^2)/d^2])/(d\*(1+m)\*(1+(c\*x^2)/a)^p) - (e\*x^2\*(g\*x)^m\*(a+c\*x^2)^p\*AppellF1[(2+m)/2, -p, 1, (4+m)/2, -(c\*x^2)/a, (e^2\*x^2)/d^2])/(d^2\*(2+m)\*(1+(c\*x^2)/a)^p)

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx)^m (a+cx^2)^P}{d+ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x)\*\*m\*(c\*x\*\*2+a)\*\*p/(e\*x+d), x)

[Out] Integral((g\*x)\*\*m\*(a+c\*x\*\*2)\*\*p/(d+e\*x), x)

**Mathematica [A]** time = 0.0936008, size = 0, normalized size = 0.

$$\int \frac{(gx)^m (a+cx^2)^P}{d+ex} dx$$

Verification is Not applicable to the result.

[In] Integrate[((g\*x)^m\*(a+c\*x^2)^p)/(d+e\*x), x]



[Out] Integrate[((g\*x)^m\*(a + c\*x^2)^p)/(d + e\*x), x]

**Maple** [F] time = 0.076, size = 0, normalized size = 0.

$$\int \frac{(gx)^m (cx^2 + a)^p}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x)^m\*(c\*x^2+a)^p/(e\*x+d), x)

[Out] int((g\*x)^m\*(c\*x^2+a)^p/(e\*x+d), x)

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + a)^p (gx)^m}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)^p\*(g\*x)^m/(e\*x + d), x, algorithm="maxima")

[Out] integrate((c\*x^2 + a)^p\*(g\*x)^m/(e\*x + d), x)

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^2 + a)^p (gx)^m}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)^p\*(g\*x)^m/(e\*x + d), x, algorithm="fricas")

[Out] integral((c\*x^2 + a)^p\*(g\*x)^m/(e\*x + d), x)

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x)\*\*m\*(c\*x\*\*2+a)\*\*p/(e\*x+d), x)

[Out] Timed out

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + a)^p (gx)^m}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d), x, algorithm="giac")
```

```
[Out] integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d), x)
```

$$3.435 \quad \int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^2} dx$$

**Optimal.** Leaf size=238

$$\frac{x(gx)^m (a+cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} F_1\left(\frac{m+1}{2}; -p, 2; \frac{m+3}{2}; -\frac{cx^2}{a}, \frac{e^2x^2}{d^2}\right)}{d^2(m+1)} + \frac{e^2x^3(gx)^m (a+cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} F_1\left(\frac{m+3}{2}; -p, 2; \frac{m+5}{2}; -\frac{cx^2}{a}, \frac{e^2x^2}{d^2}\right)}{d^4(m+3)} - \frac{2ex^2(gx)^m (a+cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} F_1\left(\frac{m+2}{2}; -p, 2; \frac{m+4}{2}; -\frac{cx^2}{a}, \frac{e^2x^2}{d^2}\right)}{d^3(m+2)}$$

[Out]  $(x*(g*x)^m*(a+c*x^2)^p*AppellF1[(1+m)/2, -p, 2, (3+m)/2, -(c*x^2)/a, (e^2*x^2)/d^2])/(d^2*(1+m)*(1+(c*x^2)/a)^p) - (2*e*x^2*(g*x)^m*(a+c*x^2)^p*AppellF1[(2+m)/2, -p, 2, (4+m)/2, -(c*x^2)/a, (e^2*x^2)/d^2])/(d^3*(2+m)*(1+(c*x^2)/a)^p) + (e^2*x^3*(g*x)^m*(a+c*x^2)^p*AppellF1[(3+m)/2, -p, 2, (5+m)/2, -(c*x^2)/a, (e^2*x^2)/d^2])/(d^4*(3+m)*(1+(c*x^2)/a)^p)$

**Rubi [A]** time = 0.65451, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$\frac{x(gx)^m (a+cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} F_1\left(\frac{m+1}{2}; -p, 2; \frac{m+3}{2}; -\frac{cx^2}{a}, \frac{e^2x^2}{d^2}\right)}{d^2(m+1)} + \frac{e^2x^3(gx)^m (a+cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} F_1\left(\frac{m+3}{2}; -p, 2; \frac{m+5}{2}; -\frac{cx^2}{a}, \frac{e^2x^2}{d^2}\right)}{d^4(m+3)} - \frac{2ex^2(gx)^m (a+cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} F_1\left(\frac{m+2}{2}; -p, 2; \frac{m+4}{2}; -\frac{cx^2}{a}, \frac{e^2x^2}{d^2}\right)}{d^3(m+2)}$$

Antiderivative was successfully verified.

[In] Int[((g\*x)^m\*(a+c\*x^2)^p)/(d+e\*x)^2,x]

[Out]  $(x*(g*x)^m*(a+c*x^2)^p*AppellF1[(1+m)/2, -p, 2, (3+m)/2, -(c*x^2)/a, (e^2*x^2)/d^2])/(d^2*(1+m)*(1+(c*x^2)/a)^p) - (2*e*x^2*(g*x)^m*(a+c*x^2)^p*AppellF1[(2+m)/2, -p, 2, (4+m)/2, -(c*x^2)/a, (e^2*x^2)/d^2])/(d^3*(2+m)*(1+(c*x^2)/a)^p) + (e^2*x^3*(g*x)^m*(a+c*x^2)^p*AppellF1[(3+m)/2, -p, 2, (5+m)/2, -(c*x^2)/a, (e^2*x^2)/d^2])/(d^4*(3+m)*(1+(c*x^2)/a)^p)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x)\*\*m\*(c\*x\*\*2+a)\*\*p/(e\*x+d)\*\*2,x)

[Out] Integral((g\*x)\*\*m\*(a+c\*x\*\*2)\*\*p/(d+e\*x)\*\*2,x)

**Mathematica [A]** time = 0.109583, size = 0, normalized size = 0.

$$\int \frac{(gx)^m (a + cx^2)^p}{(d + ex)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[((g\*x)^m\*(a + c\*x^2)^p)/(d + e\*x)^2, x]

[Out] Integrate[((g\*x)^m\*(a + c\*x^2)^p)/(d + e\*x)^2, x]

**Maple [F]** time = 0.081, size = 0, normalized size = 0.

$$\int \frac{(gx)^m (cx^2 + a)^p}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x)^m\*(c\*x^2+a)^p/(e\*x+d)^2, x)

[Out] int((g\*x)^m\*(c\*x^2+a)^p/(e\*x+d)^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + a)^p (gx)^m}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)^p\*(g\*x)^m/(e\*x + d)^2, x, algorithm="maxima")

[Out] integrate((c\*x^2 + a)^p\*(g\*x)^m/(e\*x + d)^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^2 + a)^p (gx)^m}{e^2x^2 + 2dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)^p\*(g\*x)^m/(e\*x + d)^2, x, algorithm="fricas")

[Out] integral((c\*x^2 + a)^p\*(g\*x)^m/(e^2\*x^2 + 2\*d\*e\*x + d^2), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x)\*\*m\*(c\*x\*\*2+a)\*\*p/(e\*x+d)\*\*2,x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + a)^p (gx)^m}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)^p\*(g\*x)^m/(e\*x + d)^2,x, algorithm="giac")

[Out] integrate((c\*x^2 + a)^p\*(g\*x)^m/(e\*x + d)^2, x)

$$3.436 \quad \int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^3} dx$$

**Optimal.** Leaf size=321

$$\begin{aligned} & \frac{e^3 x^4 (gx)^m (a+cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} F_1\left(\frac{m+4}{2}; -p, 3; \frac{m+6}{2}; -\frac{cx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^6(m+4)} \\ & + \frac{3e^2 x^3 (gx)^m (a+cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} F_1\left(\frac{m+3}{2}; -p, 3; \frac{m+5}{2}; -\frac{cx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^5(m+3)} \\ & - \frac{3ex^2 (gx)^m (a+cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} F_1\left(\frac{m+2}{2}; -p, 3; \frac{m+4}{2}; -\frac{cx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^4(m+2)} \\ & + \frac{x(gx)^m (a+cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} F_1\left(\frac{m+1}{2}; -p, 3; \frac{m+3}{2}; -\frac{cx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^3(m+1)} \end{aligned}$$

[Out]  $(x*(g*x)^m*(a+c*x^2)^p*AppellF1[(1+m)/2, -p, 3, (3+m)/2, -(c*x^2)/a, (e^2*x^2)/d^2])/(d^3*(1+m)*(1+(c*x^2)/a)^p) - (3*e*x^2*(g*x)^m*(a+c*x^2)^p*AppellF1[(2+m)/2, -p, 3, (4+m)/2, -(c*x^2)/a, (e^2*x^2)/d^2])/(d^4*(2+m)*(1+(c*x^2)/a)^p) + (3*e^2*x^3*(g*x)^m*(a+c*x^2)^p*AppellF1[(3+m)/2, -p, 3, (5+m)/2, -(c*x^2)/a, (e^2*x^2)/d^2])/(d^5*(3+m)*(1+(c*x^2)/a)^p) - (e^3*x^4*(g*x)^m*(a+c*x^2)^p*AppellF1[(4+m)/2, -p, 3, (6+m)/2, -(c*x^2)/a, (e^2*x^2)/d^2])/(d^6*(4+m)*(1+(c*x^2)/a)^p)$

**Rubi [A]** time = 0.876187, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$\begin{aligned} & \frac{e^3 x^4 (gx)^m (a+cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} F_1\left(\frac{m+4}{2}; -p, 3; \frac{m+6}{2}; -\frac{cx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^6(m+4)} \\ & + \frac{3e^2 x^3 (gx)^m (a+cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} F_1\left(\frac{m+3}{2}; -p, 3; \frac{m+5}{2}; -\frac{cx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^5(m+3)} \\ & - \frac{3ex^2 (gx)^m (a+cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} F_1\left(\frac{m+2}{2}; -p, 3; \frac{m+4}{2}; -\frac{cx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^4(m+2)} \\ & + \frac{x(gx)^m (a+cx^2)^p \left(\frac{cx^2}{a} + 1\right)^{-p} F_1\left(\frac{m+1}{2}; -p, 3; \frac{m+3}{2}; -\frac{cx^2}{a}, \frac{e^2 x^2}{d^2}\right)}{d^3(m+1)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[((g\*x)^m\*(a+c\*x^2)^p)/(d+e\*x)^3,x]

[Out]  $(x*(g*x)^m*(a+c*x^2)^p*AppellF1[(1+m)/2, -p, 3, (3+m)/2, -(c*x^2)/a, (e^2*x^2)/d^2])/(d^3*(1+m)*(1+(c*x^2)/a)^p) - (3*e*x^2*(g*x)^m*(a+c*x^2)^p*AppellF1[(2+m)/2, -p, 3, (4+m)/2, -(c*x^2)/a, (e^2*x^2)/d^2])/(d^4*(2+m)*(1+(c*x^2)/a)^p) + (3*e^2*x^3*(g*x)^m*(a+c*x^2)^p*AppellF1[(3+m)/2, -p, 3, (5+m)/2, -(c*x^2)/a, (e^2*x^2)/d^2])/(d^5*(3+m)*(1+(c*x^2)/a)^p) - (e^3*x^4*(g*x)^m*(a+c*x^2)^p*AppellF1[(4+m)/2, -p, 3, (6+m)/2, -(c*x^2)/a, (e^2*x^2)/d^2])/(d^6*(4+m)*(1+(c*x^2)/a)^p)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx)^m (a+cx^2)^p}{(d+ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x)**m*(c*x**2+a)**p/(e*x+d)**3,x)`

[Out] `Integral((g*x)**m*(a + c*x**2)**p/(d + e*x)**3, x)`

**Mathematica [A]** time = 0.268244, size = 0, normalized size = 0.

$$\int \frac{(gx)^m (a + cx^2)^p}{(d + ex)^3} dx$$

Verification is Not applicable to the result.

[In] `Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^3,x]`

[Out] `Integrate[((g*x)^m*(a + c*x^2)^p)/(d + e*x)^3, x]`

**Maple [F]** time = 0.098, size = 0, normalized size = 0.

$$\int \frac{(gx)^m (cx^2 + a)^p}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x)^m*(c*x^2+a)^p/(e*x+d)^3,x)`

[Out] `int((g*x)^m*(c*x^2+a)^p/(e*x+d)^3,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + a)^p (gx)^m}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d)^3,x, algorithm="maxima")`

[Out] `integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d)^3, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^2 + a)^p (gx)^m}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)^p*(g*x)^m/(e*x + d)^3,x, algorithm="fricas")`

[Out] `integral((c*x^2 + a)^p*(g*x)^m/(e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x)\*\*m\*(c\*x\*\*2+a)\*\*p/(e\*x+d)\*\*3,x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + a)^p (gx)^m}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)^p\*(g\*x)^m/(e\*x + d)^3,x, algorithm="giac")

[Out] integrate((c\*x^2 + a)^p\*(g\*x)^m/(e\*x + d)^3, x)



$$3.437 \quad \int \frac{x^3 \sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{d + ex} dx$$

**Optimal.** Leaf size=345

$$\frac{(-15a^3e^6 - 2cdex(-5a^2e^4 - 6acd^2e^2 + 35c^2d^4) - 17a^2cd^2e^4 - 25ac^2d^4e^2 + 105c^3d^6) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{192c^3d^3e^4} \\ + \frac{(cd^2 - ae^2)(5a^3e^6 + 9a^2cd^2e^4 + 15ac^2d^4e^2 + 35c^3d^6) \tanh^{-1}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{128c^{7/2}d^{7/2}e^{9/2}} \\ + \frac{1}{24}x^2\left(\frac{a}{cd} - \frac{7d}{e^2}\right)\sqrt{x(ae^2 + cd^2) + ade + cdex^2} + \frac{x^3\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{4e}$$

[Out] ((a/(c\*d) - (7\*d)/e^2)\*x^2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/24 + (x^3\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(4\*e) - ((105\*c^3\*d^6 - 25\*a\*c^2\*d^4\*e^2 - 17\*a^2\*c\*d^2\*e^4 - 15\*a^3\*e^6 - 2\*c\*d\*e\*(35\*c^2\*d^4 - 6\*a\*c\*d^2\*e^2 - 5\*a^2\*e^4)\*x)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(192\*c^3\*d^3\*e^4) + ((c\*d^2 - a\*e^2)\*(35\*c^3\*d^6 + 15\*a\*c^2\*d^4\*e^2 + 9\*a^2\*c\*d^2\*e^4 + 5\*a^3\*e^6)\*ArcTanh[(c\*d^2 + a\*e^2 + 2\*c\*d\*e\*x)/(2\*Sqrt[c]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])])/(128\*c^(7/2)\*d^(7/2)\*e^(9/2))

**Rubi [A]** time = 1.35247, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{(-15a^3e^6 - 2cdex(-5a^2e^4 - 6acd^2e^2 + 35c^2d^4) - 17a^2cd^2e^4 - 25ac^2d^4e^2 + 105c^3d^6) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{192c^3d^3e^4} \\ + \frac{(cd^2 - ae^2)(5a^3e^6 + 9a^2cd^2e^4 + 15ac^2d^4e^2 + 35c^3d^6) \tanh^{-1}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{128c^{7/2}d^{7/2}e^{9/2}} \\ + \frac{1}{24}x^2\left(\frac{a}{cd} - \frac{7d}{e^2}\right)\sqrt{x(ae^2 + cd^2) + ade + cdex^2} + \frac{x^3\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{4e}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(d + e\*x), x]

[Out] ((a/(c\*d) - (7\*d)/e^2)\*x^2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/24 + (x^3\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(4\*e) - ((105\*c^3\*d^6 - 25\*a\*c^2\*d^4\*e^2 - 17\*a^2\*c\*d^2\*e^4 - 15\*a^3\*e^6 - 2\*c\*d\*e\*(35\*c^2\*d^4 - 6\*a\*c\*d^2\*e^2 - 5\*a^2\*e^4)\*x)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(192\*c^3\*d^3\*e^4) + ((c\*d^2 - a\*e^2)\*(35\*c^3\*d^6 + 15\*a\*c^2\*d^4\*e^2 + 9\*a^2\*c\*d^2\*e^4 + 5\*a^3\*e^6)\*ArcTanh[(c\*d^2 + a\*e^2 + 2\*c\*d\*e\*x)/(2\*Sqrt[c]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])])/(128\*c^(7/2)\*d^(7/2)\*e^(9/2))

**Rubi in Sympy [A]** time = 144.696, size = 348, normalized size = 1.01

$$-x^2\left(-\frac{a}{24cd} + \frac{7d}{24e^2}\right)\sqrt{ade + cdex^2 + x(ae^2 + cd^2)} + \frac{x^3\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{4e} \\ + \frac{\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}\left(\frac{15a^3e^6}{8} + \frac{17a^2cd^2e^4}{8} + \frac{25ac^2d^4e^2}{8} - \frac{105c^3d^6}{8} - \frac{cdex(5a^2e^4 + 6acd^2e^2 - 35c^2d^4)}{4}\right)}{24c^3d^3e^4} \\ - \frac{(ae^2 - cd^2)(5a^3e^6 + 9a^2cd^2e^4 + 15ac^2d^4e^2 + 35c^3d^6) \operatorname{atanh}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}\right)}{128c^{\frac{7}{2}}d^{\frac{7}{2}}e^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)`

[Out]  $-x^{**2}*(-a/(24*c*d) + 7*d/(24*e**2))*\sqrt{a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)}/(4*e) + \sqrt{a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)}*(15*a**3*e**6/8 + 17*a**2*c*d**2*e**4/8 + 25*a*c**2*d**4*e**2/8 - 10*5*c**3*d**6/8 - c*d*e*x*(5*a**2*e**4 + 6*a*c*d**2*e**2 - 35*c**2*d**4)/4)/(24*c**3*d**3*e**4) - (a*e**2 - c*d**2)*(5*a**3*e**6 + 9*a**2*c*d**2*e**4 + 15*a*c**2*d**4*e**2 + 35*c**3*d**6)*\operatorname{atanh}((a*e**2 + c*d**2 + 2*c*d*e*x)/(2*\sqrt{c}*\sqrt{d}*\sqrt{e}*\sqrt{a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)}))/((128*c**(7/2)*d**(7/2)*e**(9/2))$

**Mathematica [A]** time = 0.497908, size = 276, normalized size = 0.8

$$\frac{1}{384}\sqrt{(d+ex)(ae+cdx)}\left(\frac{30a^3e^2}{c^3d^3} + \frac{4x\left(-\frac{5a^2e^4}{c^2d^2} - \frac{6ae^2}{c} + 35d^2\right)}{e^3} + \frac{34a^2}{c^2d}\right) + \frac{3(cd^2 - ae^2)(5a^3e^6 + 9a^2cd^2e^4 + 15ac^2d^4e^2 + 35c^3d^6)\log\left(2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx} + ae^2 + cd(d+2ex)\right)}{c^{7/2}d^{7/2}e^{9/2}\sqrt{d+ex}\sqrt{ae+cdx}} + \frac{16x^2\left(\frac{a}{c} - \frac{7d^2}{e^2}\right)}{d} + \frac{50ad}{ce^2} - \frac{210d^3}{e^4} + \frac{96x^3}{e}$$

Antiderivative was successfully verified.

[In] `Integrate[(x^3*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x),x]`

[Out]  $(\sqrt{(a*e + c*d*x)*(d + e*x)}*((34*a^2)/(c^2*d) - (210*d^3)/e^4 + (50*a*d)/(c*e^2) + (30*a^3*e^2)/(c^3*d^3) + (4*(35*d^2 - (6*a*e^2)/c - (5*a^2*e^4)/(c^2*d^2))*x)/e^3 + (16*(a/c - (7*d^2)/e^2)*x^2)/d + (96*x^3)/e + (3*(c*d^2 - a*e^2)*(35*c^3*d^6 + 15*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 + 5*a^3*e^6)*\operatorname{Log}[a*e^2 + 2*\sqrt{c}*\sqrt{d}*\sqrt{e}*\sqrt{d+e*x} + c*d*(d+2*e*x)])/(c^{7/2}*d^{7/2}*e^{9/2}*\sqrt{a*e + c*d*x}*\sqrt{d + e*x}))/384$

**Maple [B]** time = 0.031, size = 946, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x)`

[Out]  $19/64/c^2/d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2+29/32*d^2/e^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+5/32*e/c^2/d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x^2-5/128*e^4/c^3/d^3*\ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)*a^4-1/32*e^2/c^2*a^3/d*\ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)+1/4/e^2*x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/c/d-5/24/e/c^2/d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*a+7/16/e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x^2+a+5/64*e^2/c^3/d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3-13/24/e^3/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+29/64*d^3/e^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/64*d/c*\ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))$

$$\frac{\begin{aligned} & / (c*d*e)^{(1/2)} * a^2 + 11/32 * d^3 / e^2 * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x) \\ & / (c*d*e)^{(1/2)} + (a*d*e + (a*e^2 + c*d^2) * x + c*d*e*x^2)^{(1/2)}) / (c*d*e)^{(1/2)} \\ & * a - 29/128 * d^5 / e^4 * c * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x) / (c*d*e)^{(1/2)} \\ & + (a*d*e + (a*e^2 + c*d^2) * x + c*d*e*x^2)^{(1/2)}) / (c*d*e)^{(1/2)} + 43/6 \\ & 4 * d / e^2 / c * (a*d*e + (a*e^2 + c*d^2) * x + c*d*e*x^2)^{(1/2)} * a - d^3 / e^4 * (c*d * \\ & e * (x+d/e)^2 + (a*e^2 - c*d^2) * (x+d/e))^{(1/2)} - 1/2 * d^3 / e^2 * \ln((1/2 * a * e^2 \\ & - 1/2 * c * d^2 + (x+d/e) * c * d * e) / (c*d*e)^{(1/2)} + (c*d*e * (x+d/e)^2 + (a*e^2 - \\ & c*d^2) * (x+d/e))^{(1/2)}) / (c*d*e)^{(1/2)} * a + 1/2 * d^5 / e^4 * \ln((1/2 * a * e^2 - \\ & 1/2 * c * d^2 + (x+d/e) * c * d * e) / (c*d*e)^{(1/2)} + (c*d*e * (x+d/e)^2 + (a*e^2 - c * \\ & d^2) * (x+d/e))^{(1/2)}) / (c*d*e)^{(1/2)} * c \end{aligned}}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*x^3/(e\*x + d), x, algorithm=

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.375192, size = 1, normalized size = 0.

$$\left[ \frac{4(48c^3d^3e^3x^3 - 105c^3d^6 + 25ac^2d^4e^2 + 17a^2cd^2e^4 + 15a^3e^6 - 8(7c^3d^4e^2 - ac^2d^2e^4)x^2 + 2(35c^3d^5e - 6ac^2d^3e^3 - 5a^2e^5)x - 4a^4e^7)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*x^3/(e\*x + d), x, algorithm=

[Out] [1/768\*(4\*(48\*c^3\*d^3\*e^3\*x^3 - 105\*c^3\*d^6 + 25\*a\*c^2\*d^4\*e^2 + 17\*a^2\*c\*d^2\*e^4 + 15\*a^3\*e^6 - 8\*(7\*c^3\*d^4\*e^2 - a\*c^2\*d^2\*e^4)\*x^2 + 2\*(35\*c^3\*d^5\*e - 6\*a\*c^2\*d^3\*e^3 - 5\*a^2\*e^5)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*e) - 3\*(35\*c^4\*d^8 - 20\*a\*c^3\*d^6\*e^2 - 6\*a^2\*c^2\*d^4\*e^4 - 4\*a^3\*c\*d^2\*e^6 - 5\*a^4\*e^8)\*log(-4\*(2\*c^2\*d^2\*e^2\*x + c^2\*d^3\*e + a\*c\*d\*e^3)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) + (8\*c^2\*d^2\*e^2\*x^2 + c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4 + 8\*(c^2\*d^3\*e + a\*c\*d\*e^3)\*x)\*sqrt(c\*d\*e)))/(sqrt(c\*d\*e)\*c^3\*d^3\*e^4), 1/384\*(2\*(48\*c^3\*d^3\*e^3\*x^3 - 105\*c^3\*d^6 + 25\*a\*c^2\*d^4\*e^2 + 17\*a^2\*c\*d^2\*e^4 + 15\*a^3\*e^6 - 8\*(7\*c^3\*d^4\*e^2 - a\*c^2\*d^2\*e^4)\*x^2 + 2\*(35\*c^3\*d^5\*e - 6\*a\*c^2\*d^3\*e^3 - 5\*a^2\*c\*d^2\*e^4)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*e) + 3\*(35\*c^4\*d^8 - 20\*a\*c^3\*d^6\*e^2 - 6\*a^2\*c^2\*d^4\*e^4 - 4\*a^3\*c\*d^2\*e^6 - 5\*a^4\*e^8)\*arctan(1/2\*(2\*c\*d\*e\*x + c\*d^2 + a\*e^2)\*sqrt(-c\*d\*e)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*c\*d\*e)))/(sqrt(-c\*d\*e)\*c^3\*d^3\*e^4)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/(e\*x+d), x)

[Out] Timed out

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*x^3/(e*x + d), x, algorithm=`

[Out] Exception raised: TypeError

$$3.438 \quad \int \frac{x^2 \sqrt{ade + (cd^2 + ae^2)x + cdex^2}}{d + ex} dx$$

**Optimal.** Leaf size=251

$$\frac{(cd^2 - ae^2)(a^2e^4 + 2acd^2e^2 + 5c^2d^4) \tanh^{-1}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{16c^{5/2}d^{5/2}e^{7/2}} + \frac{((5cd^2 - 3ae^2)(ae^2 + 3cd^2) - 2cdex(5cd^2 - ae^2))\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{24c^2d^2e^3} + \frac{x^2\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{3e}$$

[Out] (x^2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(3\*e) + (((5\*c\*d^2 - 3\*a\*e^2)\*(3\*c\*d^2 + a\*e^2) - 2\*c\*d\*e\*(5\*c\*d^2 - a\*e^2)\*x)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(24\*c^2\*d^2\*e^3) - ((c\*d^2 - a\*e^2)\*(5\*c^2\*d^4 + 2\*a\*c\*d^2\*e^2 + a^2\*e^4)\*ArcTanh[(c\*d^2 + a\*e^2 + 2\*c\*d\*e\*x)/(2\*Sqrt[c]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])])/(16\*c^(5/2)\*d^(5/2)\*e^(7/2))

**Rubi [A]** time = 0.83013, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{(cd^2 - ae^2)(a^2e^4 + 2acd^2e^2 + 5c^2d^4) \tanh^{-1}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{16c^{5/2}d^{5/2}e^{7/2}} + \frac{((5cd^2 - 3ae^2)(ae^2 + 3cd^2) - 2cdex(5cd^2 - ae^2))\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{24c^2d^2e^3} + \frac{x^2\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{3e}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(d + e\*x), x]

[Out] (x^2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(3\*e) + (((5\*c\*d^2 - 3\*a\*e^2)\*(3\*c\*d^2 + a\*e^2) - 2\*c\*d\*e\*(5\*c\*d^2 - a\*e^2)\*x)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(24\*c^2\*d^2\*e^3) - ((c\*d^2 - a\*e^2)\*(5\*c^2\*d^4 + 2\*a\*c\*d^2\*e^2 + a^2\*e^4)\*ArcTanh[(c\*d^2 + a\*e^2 + 2\*c\*d\*e\*x)/(2\*Sqrt[c]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])])/(16\*c^(5/2)\*d^(5/2)\*e^(7/2))

**Rubi in Sympy [A]** time = 85.9153, size = 245, normalized size = 0.98

$$\frac{x^2\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{3e} - \frac{\left(-\frac{cdex(ae^2 - 5cd^2)}{2} + \left(\frac{ae^2}{4} + \frac{3cd^2}{4}\right)(3ae^2 - 5cd^2)\right)\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{6c^2d^2e^3} + \frac{(ae^2 - cd^2)(a^2e^4 + 2acd^2e^2 + 5c^2d^4) \operatorname{atanh}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}\right)}{16c^{5/2}d^{5/2}e^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/(e\*x+d), x)

[Out]  $x^{2}\sqrt{a^{2}d^{2}e + c^{2}d^{2}e^{2}x^{2} + x^{2}(a^{2}e^{2} + c^{2}d^{2})}/(3e) - (-c^{2}d^{2}e^{2}x^{2}(a^{2}e^{2} - 5c^{2}d^{2})/2 + (a^{2}e^{2}/4 + 3c^{2}d^{2}/4)(3a^{2}e^{2} - 5c^{2}d^{2}))\sqrt{a^{2}d^{2}e + c^{2}d^{2}e^{2}x^{2} + x^{2}(a^{2}e^{2} + c^{2}d^{2})}/(6c^{2}d^{2}e^{2}e^{3}) + (a^{2}e^{2} - c^{2}d^{2})(a^{2}e^{4} + 2a^{2}c^{2}d^{2}e^{2} + 5c^{2}d^{2}e^{4})\operatorname{atanh}((a^{2}e^{2} + c^{2}d^{2} + 2c^{2}d^{2}e^{2}x)/(2\sqrt{c}\sqrt{d})\sqrt{e})\sqrt{a^{2}d^{2}e + c^{2}d^{2}e^{2}x^{2} + x^{2}(a^{2}e^{2} + c^{2}d^{2})})/(16c^{2}d^{2}e^{2}e^{7/2})$

**Mathematica [A]** time = 0.366337, size = 207, normalized size = 0.82

$$\frac{1}{48}\sqrt{(d+ex)(ae+cdx)}\left(-\frac{6a^2e}{c^2d^2} + \frac{3(ae^2 - cd^2)(a^2e^4 + 2acd^2e^2 + 5c^2d^4)\log\left(2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx} + ae^2 + cd(d+2ex)\right)}{c^{5/2}d^{5/2}e^{7/2}\sqrt{d+ex}\sqrt{ae+cdx}} + \frac{4ax}{cd} - \frac{8a}{ce} + \frac{30d^2}{e^3} - \frac{20dx}{e^2} + \frac{16x^2}{e}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(x^2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(d + e\*x), x]

[Out]  $(\operatorname{Sqrt}[(a^2e + c^2d^2x)(d + e^2x)]^2((30d^2)/e^3 - (8a)/(c^2e) - (6a^2e)/(c^2d^2) + (4a^2x)/(c^2d) - (20d^2x)/e^2 + (16x^2)/e + (3(-c^2d^2 + a^2e^2)(5c^2d^4 + 2a^2c^2d^2e^2 + a^2e^4)\operatorname{Log}[a^2e^2 + 2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx} + ae^2 + cd(d+2ex)])/(c^{5/2}d^{5/2}e^{7/2}\sqrt{d+ex}\sqrt{ae+cdx}))/48$

**Maple [B]** time = 0.019, size = 713, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)/(e\*x+d), x)

[Out]  $1/3/e^2*(a^2d^2e+(a^2e^2+c^2d^2)x+c^2d^2e^2x^2)^{3/2}/c/d-1/4/c/d*(a^2d^2e+(a^2e^2+c^2d^2)x+c^2d^2e^2x^2)^{1/2}*x^2a^{-3/4}/e^2*d^2*(a^2d^2e+(a^2e^2+c^2d^2)x+c^2d^2e^2x^2)^{1/2}*x-1/8*e/c^2/d^2*(a^2d^2e+(a^2e^2+c^2d^2)x+c^2d^2e^2x^2)^{1/2}*a^2-1/2/e/c*(a^2d^2e+(a^2e^2+c^2d^2)x+c^2d^2e^2x^2)^{1/2})*a^{-3/8}/e^3*d^2*(a^2d^2e+(a^2e^2+c^2d^2)x+c^2d^2e^2x^2)^{1/2}+1/16*e^3/c^2/d^2*\ln((1/2*a^2e^2+1/2*c^2d^2+c^2d^2e^2x)/(c^2d^2e)^{1/2}+(a^2d^2e+(a^2e^2+c^2d^2)x+c^2d^2e^2x^2)^{1/2}))/c^2d^2e^{1/2}*a^3+1/16*e/c*\ln((1/2*a^2e^2+1/2*c^2d^2+c^2d^2e^2x)/(c^2d^2e)^{1/2}+(a^2d^2e+(a^2e^2+c^2d^2)x+c^2d^2e^2x^2)^{1/2}))/c^2d^2e^{1/2}*a^2-5/16/e*d^2*\ln((1/2*a^2e^2+1/2*c^2d^2+c^2d^2e^2x)/(c^2d^2e)^{1/2}+(a^2d^2e+(a^2e^2+c^2d^2)x+c^2d^2e^2x^2)^{1/2}))/c^2d^2e^{1/2}*a^3+1/16/e^3*c^2d^4*\ln((1/2*a^2e^2+1/2*c^2d^2+c^2d^2e^2x)/(c^2d^2e)^{1/2}+(a^2d^2e+(a^2e^2+c^2d^2)x+c^2d^2e^2x^2)^{1/2}))/c^2d^2e^{1/2}+d^2/e^3*(c^2d^2e*(x+d/e)^2+(a^2e^2-c^2d^2)*(x+d/e))^{1/2}+1/2*d^2/e*\ln((1/2*a^2e^2-1/2*c^2d^2+(x+d/e)*c^2d^2e)/(c^2d^2e)^{1/2}+(c^2d^2e*(x+d/e)^2+(a^2e^2-c^2d^2)*(x+d/e))^{1/2}))/c^2d^2e^{1/2}*a-1/2*d^4/e^3*\ln((1/2*a^2e^2-1/2*c^2d^2+(x+d/e)*c^2d^2e)/(c^2d^2e)^{1/2}+(c^2d^2e*(x+d/e)^2+(a^2e^2-c^2d^2)*(x+d/e))^{1/2}))/c^2d^2e^{1/2}*c$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*x^2/(e*x + d), x, algorithm=
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 0.332803, size = 1, normalized size = 0.

$$\frac{4(8c^2d^2e^2x^2 + 15c^2d^4 - 4acd^2e^2 - 3a^2e^4 - 2(5c^2d^3e - acde^3)x)\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{cde} - 3(5c^3d^6 - 3ac^2d^4e^2)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*x^2/(e*x + d), x, algorithm=
```

```
[Out] [1/96*(4*(8*c^2*d^2*e^2*x^2 + 15*c^2*d^4 - 4*a*c*d^2*e^2 - 3*a^2*e^4 - 2*(5*c^2*d^3*e - a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*e) - 3*(5*c^3*d^6 - 3*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6)*log(4*(2*c^2*d^2*e^2*x + c^2*d^3*e + a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x) + (8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 8*(c^2*d^3*e + a*c*d*e^3)*x)*sqrt(c*d*e))/(sqrt(c*d*e)*c^2*d^2*e^3), 1/48*(2*(8*c^2*d^2*e^2*x^2 + 15*c^2*d^4 - 4*a*c*d^2*e^2 - 3*a^2*e^4 - 2*(5*c^2*d^3*e - a*c*d*e^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*e) - 3*(5*c^3*d^6 - 3*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - a^3*e^6)*arctan(1/2*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d*e)))/(sqrt(-c*d*e)*c^2*d^2*e^3)]
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d), x)
```

```
[Out] Timed out
```

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*x^2/(e*x + d), x, algorithm=
```

```
[Out] Exception raised: TypeError
```

$$3.439 \quad \int \frac{x\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex} dx$$

**Optimal.** Leaf size=207

$$\frac{(cd^2 - ae^2)(ae^2 + 3cd^2) \tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{5/2}} + \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{2cde(d + ex)} - \frac{1}{4} \left(\frac{a}{cd} + \frac{3d}{e^2}\right) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}$$

[Out]  $-\left(\frac{a}{c*d} + \frac{3*d}{e^2}\right)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/4 + (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/(2*c*d*e*(d + e*x)) + ((c*d^2 - a*e^2)*(3*c*d^2 + a*e^2)*\text{ArcTanh}[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(8*c^{(3/2)}*d^{(3/2)}*e^{(5/2)})$

**Rubi [A]** time = 0.485473, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{(cd^2 - ae^2)(ae^2 + 3cd^2) \tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8c^{3/2}d^{3/2}e^{5/2}} + \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{2cde(d + ex)} - \frac{1}{4} \left(\frac{a}{cd} + \frac{3d}{e^2}\right) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(d + e*x), x]$

[Out]  $-\left(\frac{a}{c*d} + \frac{3*d}{e^2}\right)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/4 + (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/(2*c*d*e*(d + e*x)) + ((c*d^2 - a*e^2)*(3*c*d^2 + a*e^2)*\text{ArcTanh}[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(8*c^{(3/2)}*d^{(3/2)}*e^{(5/2)})$

**Rubi in Sympy [A]** time = 44.3452, size = 189, normalized size = 0.91

$$-\left(\frac{a}{4cd} + \frac{3d}{4e^2}\right) \sqrt{ade + cdex^2 + x(ae^2 + cd^2)} + \frac{(ade + cdex^2 + x(ae^2 + cd^2))^{3/2}}{2cde(d + ex)} - \frac{(ae^2 - cd^2)(ae^2 + 3cd^2) \text{atanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}\right)}{8c^{3/2}d^{3/2}e^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d), x)$

[Out]  $-(a/(4*c*d) + 3*d/(4*e**2))*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)) + (a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))** (3/2)/(2*c*d*e*(d + e*x)) - (a*e**2 - c*d**2)*(a*e**2 + 3*c*d**2)*\text{atanh}((a*e**2 + c*d**2 + 2*c*d*e*x)/(2*\text{sqrt}(c)*\text{sqrt}(d)*\text{sqrt}(e)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))))/(8*c** (3/2)*d** (3/2)*e** (5/2))$



**Mathematica [A]** time = 0.382126, size = 159, normalized size = 0.77

$$\frac{1}{8} \sqrt{(d+ex)(ae+cdx)} \left( \frac{(cd^2 - ae^2)(ae^2 + 3cd^2) \log\left(2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx} + ae^2 + cd(d+2ex)\right)}{c^{3/2}d^{3/2}e^{5/2}\sqrt{d+ex}\sqrt{ae+cdx}} \right. \\ \left. + \frac{2a}{cd} - \frac{6d}{e^2} + \frac{4x}{e} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(d + e\*x), x]

[Out] (Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)]\*((2\*a)/(c\*d) - (6\*d)/e^2 + (4\*x)/e + ((c\*d^2 - a\*e^2)\*(3\*c\*d^2 + a\*e^2)\*Log[a\*e^2 + 2\*Sqrt[c]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x] + c\*d\*(d + 2\*e\*x))]/(c^(3/2)\*d^(3/2)\*e^(5/2)\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]))/8

**Maple [B]** time = 0.013, size = 516, normalized size = 2.5

$$\frac{x}{2e} \sqrt{ade + (ae^2 + cd^2)x + cdex^2} + \frac{a}{4cd} \sqrt{ade + (ae^2 + cd^2)x + cdex^2} + \frac{d}{4e^2} \sqrt{ade + (ae^2 + cd^2)x + cdex^2} \\ - \frac{e^2 a^2}{8cd} \ln\left(1\left(\frac{ae^2}{2} + \frac{cd^2}{2} + cdex\right) \frac{1}{\sqrt{cde}} + \sqrt{ade + (ae^2 + cd^2)x + cdex^2}\right) \frac{1}{\sqrt{cde}} \\ + \frac{ad}{4} \ln\left(1\left(\frac{ae^2}{2} + \frac{cd^2}{2} + cdex\right) \frac{1}{\sqrt{cde}} + \sqrt{ade + (ae^2 + cd^2)x + cdex^2}\right) \frac{1}{\sqrt{cde}} \\ - \frac{cd^3}{8e^2} \ln\left(1\left(\frac{ae^2}{2} + \frac{cd^2}{2} + cdex\right) \frac{1}{\sqrt{cde}} + \sqrt{ade + (ae^2 + cd^2)x + cdex^2}\right) \frac{1}{\sqrt{cde}} \\ - \frac{d}{e^2} \sqrt{cde \left(x + \frac{d}{e}\right)^2 + (ae^2 - cd^2) \left(x + \frac{d}{e}\right)} \\ - \frac{ad}{2} \ln\left(1\left(\frac{ae^2}{2} - \frac{cd^2}{2} + \left(x + \frac{d}{e}\right) cde\right) \frac{1}{\sqrt{cde}} + \sqrt{cde \left(x + \frac{d}{e}\right)^2 + (ae^2 - cd^2) \left(x + \frac{d}{e}\right)}\right) \frac{1}{\sqrt{cde}} \\ + \frac{cd^3}{2e^2} \ln\left(1\left(\frac{ae^2}{2} - \frac{cd^2}{2} + \left(x + \frac{d}{e}\right) cde\right) \frac{1}{\sqrt{cde}} + \sqrt{cde \left(x + \frac{d}{e}\right)^2 + (ae^2 - cd^2) \left(x + \frac{d}{e}\right)}\right) \frac{1}{\sqrt{cde}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)/(e\*x+d), x)

[Out] 1/2/e\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)\*x+1/4/c/d\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)\*a+1/4/e^2\*d\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)-1/8\*e^2/c/d\*ln((1/2\*a\*e^2+1/2\*c\*d^2+c\*d\*e\*x)/(c\*d\*e)^(1/2)+(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/(c\*d\*e)^(1/2)+(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/(c\*d\*e)^(1/2)\*a-1/8/e^2\*c\*d^3\*ln((1/2\*a\*e^2+1/2\*c\*d^2+c\*d\*e\*x)/(c\*d\*e)^(1/2)+(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/(c\*d\*e)^(1/2)-d/e^2\*(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e)^(1/2)-1/2\*d\*ln((1/2\*a\*e^2-1/2\*c\*d^2+(x+d/e)\*c\*d\*e)/(c\*d\*e)^(1/2)+(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e)^(1/2)))/(c\*d\*e)^(1/2)\*a+1/2\*d^3/e^2\*ln((1/2\*a\*e^2-1/2\*c\*d^2+(x+d/e)\*c\*d\*e)/(c\*d\*e)^(1/2)+(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e)^(1/2)))/(c\*d\*e)^(1/2)\*c

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*x/(e*x + d), x, algorithm="m`

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.307921, size = 1, normalized size = 0.

$$\left[ \frac{4 \sqrt{cdex^2 + ade + (cd^2 + ae^2)x} (2cdex - 3cd^2 + ae^2) \sqrt{cde} - (3c^2d^4 - 2acd^2e^2 - a^2e^4) \log\left(-4(2c^2d^2e^2x + c^2d^3e + acde^3)\right)}{16 \sqrt{cdecde^2}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*x/(e*x + d), x, algorithm="f`

[Out] `[1/16*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x - 3*c*d^2 + a*e^2)*sqrt(c*d*e) - (3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4)*log(-4*(2*c^2*d^2*e^2*x + c^2*d^3*e + a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x) + (8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 8*(c^2*d^3*e + a*c*d*e^3)*x)*sqrt(c*d*e)))/(sqrt(c*d*e)*c*d*e^2), 1/8*(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*e*x - 3*c*d^2 + a*e^2)*sqrt(-c*d*e) + (3*c^2*d^4 - 2*a*c*d^2*e^2 - a^2*e^4)*arctan(1/2*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d*e)))/(sqrt(-c*d*e)*c*d*e^2)]`

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{x \sqrt{(d + ex)(ae + cdx)}}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d), x)`

[Out] `Integral(x*sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)`

**GIAC/XCAS** [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*x/(e*x + d), x, algorithm="g`

[Out] Exception raised: TypeError

$$3.440 \quad \int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{d+ex} dx$$

**Optimal.** Leaf size=131

$$\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e} - \frac{(cd^2-ae^2) \tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}e^{3/2}}$$

[Out] Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/e - ((c\*d^2 - a\*e^2)\*ArcTanh[(c\*d^2 + a\*e^2 + 2\*c\*d\*e\*x)/(2\*Sqrt[c]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])]/(2\*Sqrt[c]\*Sqrt[d]\*e^(3/2))

**Rubi [A]** time = 0.16514, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 37,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.081$

$$\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e} - \frac{(cd^2-ae^2) \tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\sqrt{c}\sqrt{d}e^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(d + e\*x), x]

[Out] Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/e - ((c\*d^2 - a\*e^2)\*ArcTanh[(c\*d^2 + a\*e^2 + 2\*c\*d\*e\*x)/(2\*Sqrt[c]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])]/(2\*Sqrt[c]\*Sqrt[d]\*e^(3/2))

**Rubi in Sympy [A]** time = 27.0271, size = 122, normalized size = 0.93

$$\frac{\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{e} + \frac{(ae^2-cd^2) \operatorname{atanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}\right)}{2\sqrt{c}\sqrt{d}e^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/(e\*x+d), x)

[Out] sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/e + (a\*e\*\*2 - c\*d\*\*2)\*atanh((a\*e\*\*2 + c\*d\*\*2 + 2\*c\*d\*e\*x)/(2\*sqrt(c)\*sqrt(d)\*sqrt(e)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2)))/(2\*sqrt(c)\*sqrt(d)\*e\*\*(3/2))

**Mathematica [A]** time = 0.250237, size = 133, normalized size = 1.02

$$\frac{\sqrt{(d+ex)(ae+cdx)} \left( \frac{(ae^2-cd^2) \log(2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx+ae^2+cd(d+2ex)})}{\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{ae+cdx}} + 2\sqrt{e} \right)}{2e^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(d + e\*x), x]

[Out]  $(\text{Sqrt}[(a^*e + c^*d^*x)^*(d + e^*x)]^*(2^*\text{Sqrt}[e] + ((-(c^*d^{\wedge}2) + a^*e^{\wedge}2)^*\text{Log}[a^*e^{\wedge}2 + 2^*\text{Sqrt}[c]^*\text{Sqrt}[d]^*\text{Sqrt}[e]^*\text{Sqrt}[a^*e + c^*d^*x]^*\text{Sqrt}[d + e^*x] + c^*d^*(d + 2^*e^*x)])/(\text{Sqrt}[c]^*\text{Sqrt}[d]^*\text{Sqrt}[a^*e + c^*d^*x]^*\text{Sqrt}[d + e^*x]))) / (2^*e^{\wedge}(3/2))$

**Maple [A]** time = 0.008, size = 205, normalized size = 1.6

$$\frac{1}{e} \sqrt{cde \left(x + \frac{d}{e}\right)^2 + (ae^2 - cd^2) \left(x + \frac{d}{e}\right)}$$

$$+ \frac{ae}{2} \ln \left( 1 \left( \frac{ae^2}{2} - \frac{cd^2}{2} + \left(x + \frac{d}{e}\right) cde \right) \frac{1}{\sqrt{cde}} + \sqrt{cde \left(x + \frac{d}{e}\right)^2 + (ae^2 - cd^2) \left(x + \frac{d}{e}\right)} \right) \frac{1}{\sqrt{cde}}$$

$$- \frac{cd^2}{2e} \ln \left( 1 \left( \frac{ae^2}{2} - \frac{cd^2}{2} + \left(x + \frac{d}{e}\right) cde \right) \frac{1}{\sqrt{cde}} + \sqrt{cde \left(x + \frac{d}{e}\right)^2 + (ae^2 - cd^2) \left(x + \frac{d}{e}\right)} \right) \frac{1}{\sqrt{cde}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d),x)`

[Out]  $1/e^*(c^*d^*e^*(x+d/e)^2+(a^*e^2-c^*d^2)^*(x+d/e))^{1/2}+1/2^*e^*\ln((1/2^*a^*e^2-1/2^*c^*d^2+(x+d/e)^*c^*d^*e)/(c^*d^*e)^{1/2}+(c^*d^*e^*(x+d/e)^2+(a^*e^2-c^*d^2)^*(x+d/e))^{1/2})/(c^*d^*e)^{1/2}+1/2^*e^*\ln((1/2^*a^*e^2-1/2^*c^*d^2+(x+d/e)^*c^*d^*e)/(c^*d^*e)^{1/2}+(c^*d^*e^*(x+d/e)^2+(a^*e^2-c^*d^2)^*(x+d/e))^{1/2})/(c^*d^*e)^{1/2}^*c^*d^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(e*x + d),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.296127, size = 1, normalized size = 0.01

$$\left[ \frac{(cd^2 - ae^2) \log \left( 4 \left( 2c^2d^2e^2x + c^2d^3e + acde^3 \right) \sqrt{cdex^2 + ade + (cd^2 + ae^2)x} + (8c^2d^2e^2x^2 + c^2d^4 + 6acd^2e^2 + a^2e^4 + 8(c^2d^2e^2x + c^2d^3e + acde^3) \sqrt{cdex^2 + ade + (cd^2 + ae^2)x}) \right)}{4\sqrt{cdee}} \right.$$

$$\left. \frac{(cd^2 - ae^2) \arctan \left( \frac{(2cdex + cd^2 + ae^2)\sqrt{-cde}}{2\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}cde} \right) - 2\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{-cde}}{2\sqrt{-cdee}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(e*x + d),x, algorithm="fricas")`

[Out]  $[-1/4^*((c^*d^{\wedge}2 - a^*e^{\wedge}2)^*\log(4^*(2^*c^{\wedge}2^*d^{\wedge}2^*e^{\wedge}2^*x + c^{\wedge}2^*d^{\wedge}3^*e + a^*c^*d^*e^{\wedge}3)^*\text{sqrt}(c^*d^*e^*x^2 + a^*d^*e + (c^*d^{\wedge}2 + a^*e^{\wedge}2)^*x) + (8^*c^{\wedge}2^*d^{\wedge}2^*e^{\wedge}2^*x^2 + c^{\wedge}2^*d^{\wedge}4 + 6^*a^*c^*d^{\wedge}2^*e^{\wedge}2 + a^{\wedge}2^*e^{\wedge}4 + 8^*(c^{\wedge}2^*d^{\wedge}3^*e + a^*c^*d^*e^{\wedge}3)^*x)^*\text{sqrt}(c^*d^*e)) - 4^*\text{sqrt}(c^*d^*e^*x^2 + a^*d^*e + (c^*d^{\wedge}2 + a^*e^{\wedge}2)^*x)^*\text{sqrt}(-cde)]$

```
*x)*sqrt(c*d*e))/(sqrt(c*d*e)*e), -1/2*((c*d^2 - a*e^2)*arctan(1/
2*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(sqrt(c*d*e*x^2 + a*d*
e + (c*d^2 + a*e^2)*x)*c*d*e) - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^
2 + a*e^2)*x)*sqrt(-c*d*e))/(sqrt(-c*d*e)*e)]
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{d+ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d),x)
```

```
[Out] Integral(sqrt((d + e*x)*(a*e + c*d*x))/(d + e*x), x)
```

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(e*x + d),x, algorithm="gia
```

```
[Out] Exception raised: TypeError
```

$$3.441 \quad \int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x(d+ex)} dx$$

**Optimal.** Leaf size=168

$$\frac{\sqrt{c}\sqrt{d} \tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{e}} - \frac{\sqrt{a}\sqrt{e} \tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{d}}$$

[Out] (Sqrt[c]\*Sqrt[d]\*ArcTanh[(c\*d^2 + a\*e^2 + 2\*c\*d\*e\*x)/(2\*Sqrt[c]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])]/Sqrt[e] - (Sqrt[a]\*Sqrt[e]\*ArcTanh[(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(2\*Sqrt[a]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])]/Sqrt[d])

**Rubi [A]** time = 0.554071, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{\sqrt{c}\sqrt{d} \tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{e}} - \frac{\sqrt{a}\sqrt{e} \tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{d}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(x\*(d + e\*x)), x]

[Out] (Sqrt[c]\*Sqrt[d]\*ArcTanh[(c\*d^2 + a\*e^2 + 2\*c\*d\*e\*x)/(2\*Sqrt[c]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])]/Sqrt[e] - (Sqrt[a]\*Sqrt[e]\*ArcTanh[(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(2\*Sqrt[a]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])]/Sqrt[d])

**Rubi in Sympy [A]** time = 49.8846, size = 162, normalized size = 0.96

$$-\frac{\sqrt{a}\sqrt{e} \operatorname{atanh}\left(\frac{2ade+x(ae^2+cd^2)}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}\right)}{\sqrt{d}} + \frac{\sqrt{c}\sqrt{d} \operatorname{atanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}\right)}{\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/x/(e\*x+d), x)

[Out] -sqrt(a)\*sqrt(e)\*atanh((2\*a\*d\*e + x\*(a\*e\*\*2 + c\*d\*\*2))/(2\*sqrt(a)\*sqrt(d)\*sqrt(e)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))))/sqrt(d) + sqrt(c)\*sqrt(d)\*atanh((a\*e\*\*2 + c\*d\*\*2 + 2\*c\*d\*e\*x)/(2\*sqrt(c)\*sqrt(d)\*sqrt(e)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))))/sqrt(e)

**Mathematica [A]** time = 0.226393, size = 185, normalized size = 1.1

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(-\sqrt{ae}\log\left(2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}+ae(2d+ex)+cd^2x\right)+\sqrt{cd}\log\left(2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}\right)\right)}{\sqrt{d}\sqrt{e}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(x\*(d + e\*x)), x]

[Out] (Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*(Sqrt[a]\*e\*Log[x] - Sqrt[a]\*e\*Log[c\*d^2\*x + 2\*Sqrt[a]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x] + a\*e\*(2\*d + e\*x)) + Sqrt[c]\*d\*Log[a\*e^2 + 2\*Sqrt[c]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x] + c\*d\*(d + 2\*e\*x)))/(Sqrt[d]\*Sqrt[e]\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)])

**Maple [B]** time = 0.017, size = 439, normalized size = 2.6

$$\begin{aligned} & \frac{1}{d} \sqrt{ade + (ae^2 + cd^2)x + cdex^2} \\ & + \frac{ae^2}{2d} \ln \left( 1 \left( \frac{ae^2}{2} + \frac{cd^2}{2} + cdex \right) \frac{1}{\sqrt{cde}} + \sqrt{ade + (ae^2 + cd^2)x + cdex^2} \right) \frac{1}{\sqrt{cde}} \\ & + \frac{cd}{2} \ln \left( 1 \left( \frac{ae^2}{2} + \frac{cd^2}{2} + cdex \right) \frac{1}{\sqrt{cde}} + \sqrt{ade + (ae^2 + cd^2)x + cdex^2} \right) \frac{1}{\sqrt{cde}} \\ & - ae \ln \left( \frac{1}{x} \left( 2ade + (ae^2 + cd^2)x + 2\sqrt{ade}\sqrt{ade + (ae^2 + cd^2)x + cdex^2} \right) \right) \frac{1}{\sqrt{ade}} \\ & - \frac{1}{d} \sqrt{cde \left( x + \frac{d}{e} \right)^2 + (ae^2 - cd^2) \left( x + \frac{d}{e} \right)} \\ & - \frac{ae^2}{2d} \ln \left( 1 \left( \frac{ae^2}{2} - \frac{cd^2}{2} + \left( x + \frac{d}{e} \right) cde \right) \frac{1}{\sqrt{cde}} + \sqrt{cde \left( x + \frac{d}{e} \right)^2 + (ae^2 - cd^2) \left( x + \frac{d}{e} \right)} \right) \frac{1}{\sqrt{cde}} \\ & + \frac{cd}{2} \ln \left( 1 \left( \frac{ae^2}{2} - \frac{cd^2}{2} + \left( x + \frac{d}{e} \right) cde \right) \frac{1}{\sqrt{cde}} + \sqrt{cde \left( x + \frac{d}{e} \right)^2 + (ae^2 - cd^2) \left( x + \frac{d}{e} \right)} \right) \frac{1}{\sqrt{cde}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)/x/(e\*x+d), x)

[Out] 1/d\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)+1/2/d\*ln((1/2\*a\*e^2+1/2\*c\*d^2+c\*d\*e\*x)/(c\*d\*e)^(1/2)+(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/(c\*d\*e)^(1/2)\*a\*e^2+1/2\*d\*ln((1/2\*a\*e^2+1/2\*c\*d^2+c\*d\*e\*x)/(c\*d\*e)^(1/2)+(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/(c\*d\*e)^(1/2)\*c-a\*e/(a\*d\*e)^(1/2)\*ln((2\*a\*d\*e+(a\*e^2+c\*d^2)\*x+2\*(a\*d\*e)^(1/2)\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/x)-1/d\*(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2)-1/2/d\*ln((1/2\*a\*e^2-1/2\*c\*d^2+(x+d/e)\*c\*d\*e)/(c\*d\*e)^(1/2)+(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2))/(c\*d\*e)^(1/2)\*a\*e^2+1/2\*d\*ln((1/2\*a\*e^2-1/2\*c\*d^2+(x+d/e)\*c\*d\*e)/(c\*d\*e)^(1/2)+(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2))/(c\*d\*e)^(1/2)\*c

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/((e\*x + d)\*x), x, algorithm=

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.498645, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/((e\*x + d)\*x), x, algorithm=

[Out] [1/2\*sqrt(c\*d/e)\*log(8\*c^2\*d^2\*e^2\*x^2 + c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4 + 4\*(2\*c\*d\*e^2\*x + c\*d^2\*e + a\*e^3)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d/e) + 8\*(c^2\*d^3\*e + a\*c\*d^3\*e^3)\*x) + 1/2\*sqrt(a\*e/d)\*log((8\*a^2\*d^2\*e^2 + (c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4)\*x^2 - 4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(2\*a\*d^2\*e + (c\*d^3 + a\*d\*e^2)\*x)\*sqrt(a\*e/d) + 8\*(a\*c\*d^3\*e + a^2\*d^3\*e^3)\*x)/x^2), sqrt(-c\*d/e)\*arctan(1/2\*(2\*c\*d\*e\*x + c\*d^2 + a\*e^2)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d/e)\*e)) + 1/2\*sqrt(a\*e/d)\*log((8\*a^2\*d^2\*e^2 + (c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4)\*x^2 - 4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(2\*a\*d^2\*e + (c\*d^3 + a\*d\*e^2)\*x)\*sqrt(a\*e/d) + 8\*(a\*c\*d^3\*e + a^2\*d^3\*e^3)\*x)/x^2), -sqrt(-a\*e/d)\*arctan(1/2\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*d\*sqrt(-a\*e/d))) + 1/2\*sqrt(c\*d/e)\*log(8\*c^2\*d^2\*e^2\*x^2 + c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4 + 4\*(2\*c\*d\*e^2\*x + c\*d^2\*e + a\*e^3)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d/e) + 8\*(c^2\*d^3\*e + a\*c\*d^3\*e^3)\*x), -sqrt(-a\*e/d)\*arctan(1/2\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*d\*sqrt(-a\*e/d))) + sqrt(-c\*d/e)\*arctan(1/2\*(2\*c\*d\*e\*x + c\*d^2 + a\*e^2)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d/e)\*e))] ]

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**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{x(d+ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/x/(e\*x+d), x)

[Out] Integral(sqrt((d + e\*x)\*(a\*e + c\*d\*x))/(x\*(d + e\*x)), x)

---

**GIAC/XCAS [A]** time = 2.25542, size = 4, normalized size = 0.02

sage0\*x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/((e\*x + d)\*x), x, algorithm=

[Out] sage0\*x



$$3.442 \quad \int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^2(d+ex)} dx$$

**Optimal.** Leaf size=137

$$-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{dx} - \frac{(cd^2-ae^2) \tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\sqrt{ad}^{3/2}\sqrt{e}}$$

[Out]  $-(\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d*x)) - ((c*d^2 - a*e^2)*\text{ArcTanh}[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(2*\text{Sqrt}[a]*d^{3/2}*\text{Sqrt}[e])$

**Rubi [A]** time = 0.516473, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$-\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{dx} - \frac{(cd^2-ae^2) \tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2\sqrt{ad}^{3/2}\sqrt{e}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^2*(d + e*x)), x]$

[Out]  $-(\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(d*x)) - ((c*d^2 - a*e^2)*\text{ArcTanh}[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(2*\text{Sqrt}[a]*d^{3/2}*\text{Sqrt}[e])$

**Rubi in Sympy [A]** time = 41.4231, size = 124, normalized size = 0.91

$$-\frac{\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{dx} + \frac{(ae^2-cd^2) \text{atanh}\left(\frac{2ade+x(ae^2+cd^2)}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}\right)}{2\sqrt{ad}^{3/2}\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**2/(e*x+d), x)$

[Out]  $-\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(d*x) + (a*e**2 - c*d**2)*\text{atanh}(((2*a*d*e + x*(a*e**2 + c*d**2))/(2*\text{sqrt}(a)*\text{sqrt}(d)*\text{sqrt}(e)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))))/(2*\text{sqrt}(a)*d**(3/2)*\text{sqrt}(e))$

**Mathematica [A]** time = 0.301953, size = 187, normalized size = 1.36

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(x\log(x)(cd^2-ae^2)+(ae^2x-cd^2x)\log\left(2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}+ae(2d+ex)+cd^2x\right)-2\sqrt{a}\sqrt{d+ex}\sqrt{ae+cdx}\right)}{2\sqrt{ad}^{3/2}\sqrt{ex}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^2*(d + e*x)), x]$

[Out]  $(\sqrt{a^*e + c^*d^*x} * \sqrt{d + e^*x} * (-2 * \sqrt{a} * \sqrt{d} * \sqrt{e} * \sqrt{a^*e + c^*d^*x} * \sqrt{d + e^*x} + (c^*d^{\wedge}2 - a^*e^{\wedge}2) * x * \text{Log}[x] + (- (c^*d^{\wedge}2 * x) + a^*e^{\wedge}2 * x) * \text{Log}[c^*d^{\wedge}2 * x + 2 * \sqrt{a} * \sqrt{d} * \sqrt{e} * \sqrt{a^*e + c^*d^*x} * \sqrt{d + e^*x} + a^*e * (2 * d + e^*x)])) / (2 * \sqrt{a} * d^{\wedge}(3/2) * \sqrt{e} * x * \sqrt{(a^*e + c^*d^*x) * (d + e^*x)})$

**Maple [B]** time = 0.018, size = 594, normalized size = 4.3

$$\begin{aligned} & -\frac{1}{ad^2ex} (ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}} + \frac{c}{ae} \sqrt{ade + (ae^2 + cd^2)x + cdex^2} \\ & + \frac{ce}{2} \ln \left( 1 \left( \frac{ae^2}{2} + \frac{cd^2}{2} + cdex \right) \frac{1}{\sqrt{cde}} + \sqrt{ade + (ae^2 + cd^2)x + cdex^2} \right) \frac{1}{\sqrt{cde}} \\ & + \frac{ae^2}{2d} \ln \left( \frac{1}{x} \left( 2ade + (ae^2 + cd^2)x + 2\sqrt{ade}\sqrt{ade + (ae^2 + cd^2)x + cdex^2} \right) \right) \frac{1}{\sqrt{ade}} \\ & - \frac{cd}{2} \ln \left( \frac{1}{x} \left( 2ade + (ae^2 + cd^2)x + 2\sqrt{ade}\sqrt{ade + (ae^2 + cd^2)x + cdex^2} \right) \right) \frac{1}{\sqrt{ade}} \\ & + \frac{cx}{ad} \sqrt{ade + (ae^2 + cd^2)x + cdex^2} + \frac{e}{d^2} \sqrt{cde \left( x + \frac{d}{e} \right)^2 + (ae^2 - cd^2) \left( x + \frac{d}{e} \right)} \\ & + \frac{e^3a}{2d^2} \ln \left( 1 \left( \frac{ae^2}{2} - \frac{cd^2}{2} + \left( x + \frac{d}{e} \right) cde \right) \frac{1}{\sqrt{cde}} + \sqrt{cde \left( x + \frac{d}{e} \right)^2 + (ae^2 - cd^2) \left( x + \frac{d}{e} \right)} \right) \frac{1}{\sqrt{cde}} \\ & - \frac{ce}{2} \ln \left( 1 \left( \frac{ae^2}{2} - \frac{cd^2}{2} + \left( x + \frac{d}{e} \right) cde \right) \frac{1}{\sqrt{cde}} + \sqrt{cde \left( x + \frac{d}{e} \right)^2 + (ae^2 - cd^2) \left( x + \frac{d}{e} \right)} \right) \frac{1}{\sqrt{cde}} \\ & - \frac{e^3a}{2d^2} \ln \left( 1 \left( \frac{ae^2}{2} + \frac{cd^2}{2} + cdex \right) \frac{1}{\sqrt{cde}} + \sqrt{ade + (ae^2 + cd^2)x + cdex^2} \right) \frac{1}{\sqrt{cde}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^2/(e*x+d), x)`

[Out]  $-1/d^{\wedge}2/a/e/x * (a^*d^*e+(a^*e^{\wedge}2+c^*d^{\wedge}2) * x+c^*d^*e * x^{\wedge}2)^{\wedge}(3/2)+1/a/e * (a^*d^*e+(a^*e^{\wedge}2+c^*d^{\wedge}2) * x+c^*d^*e * x^{\wedge}2)^{\wedge}(1/2) * c+1/2 * e * \ln(((1/2 * a^*e^{\wedge}2+1/2 * c^*d^{\wedge}2+c^*d^*e * x)/(c^*d^*e)^{\wedge}(1/2)+(a^*d^*e+(a^*e^{\wedge}2+c^*d^{\wedge}2) * x+c^*d^*e * x^{\wedge}2)^{\wedge}(1/2)))/(c^*d^*e)^{\wedge}(1/2) * c+1/2/d * a^*e^{\wedge}2/(a^*d^*e)^{\wedge}(1/2) * \ln((2 * a^*d^*e+(a^*e^{\wedge}2+c^*d^{\wedge}2) * x+2 * (a^*d^*e)^{\wedge}(1/2) * (a^*d^*e+(a^*e^{\wedge}2+c^*d^{\wedge}2) * x+c^*d^*e * x^{\wedge}2)^{\wedge}(1/2))/x)-1/2 * d/(a^*d^*e)^{\wedge}(1/2) * \ln((2 * a^*d^*e+(a^*e^{\wedge}2+c^*d^{\wedge}2) * x+2 * (a^*d^*e)^{\wedge}(1/2) * (a^*d^*e+(a^*e^{\wedge}2+c^*d^{\wedge}2) * x+c^*d^*e * x^{\wedge}2)^{\wedge}(1/2))/x) * c+1/d * c/a * (a^*d^*e+(a^*e^{\wedge}2+c^*d^{\wedge}2) * x+c^*d^*e * x^{\wedge}2)^{\wedge}(1/2) * x+e/d^{\wedge}2 * (c^*d^*e * (x+d/e)^{\wedge}2+(a^*e^{\wedge}2-c^*d^{\wedge}2) * (x+d/e))^{\wedge}(1/2)+1/2 * e^{\wedge}3/d^{\wedge}2 * \ln(((1/2 * a^*e^{\wedge}2-1/2 * c^*d^{\wedge}2+(x+d/e) * c^*d^*e)/(c^*d^*e)^{\wedge}(1/2)+(c^*d^*e * (x+d/e)^{\wedge}2+(a^*e^{\wedge}2-c^*d^{\wedge}2) * (x+d/e))^{\wedge}(1/2)))/(c^*d^*e)^{\wedge}(1/2) * a-1/2 * e * \ln(((1/2 * a^*e^{\wedge}2-1/2 * c^*d^{\wedge}2+(x+d/e) * c^*d^*e)/(c^*d^*e)^{\wedge}(1/2)+(c^*d^*e * (x+d/e)^{\wedge}2+(a^*e^{\wedge}2-c^*d^{\wedge}2) * (x+d/e))^{\wedge}(1/2)))/(c^*d^*e)^{\wedge}(1/2) * c-1/2 * e^{\wedge}3/d^{\wedge}2 * \ln(((1/2 * a^*e^{\wedge}2+1/2 * c^*d^{\wedge}2+c^*d^*e * x)/(c^*d^*e)^{\wedge}(1/2)+(a^*d^*e+(a^*e^{\wedge}2+c^*d^{\wedge}2) * x+c^*d^*e * x^{\wedge}2)^{\wedge}(1/2)))/(c^*d^*e)^{\wedge}(1/2) * a$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^2), x, algorithm="maxima")`

[Out] `integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^2), x)`

---

**Fricas [A]** time = 0.33728, size = 1, normalized size = 0.01

$$\left[ \frac{(cd^2 - ae^2)x \log\left(\frac{4(2a^2d^2e^2 + (acd^3e + a^2de^3)x)\sqrt{cdex^2 + ade + (cd^2 + ae^2)x} + (8a^2d^2e^2 + (c^2d^4 + 6acd^2e^2 + a^2e^4)x^2 + 8(acd^3e + a^2de^3)x)\sqrt{ade}}{x^2}\right) + 4\sqrt{adedx}}{4\sqrt{adedx}} \right. \\ \left. \frac{(cd^2 - ae^2)x \arctan\left(\frac{(2ade + (cd^2 + ae^2)x)\sqrt{-ade}}{2\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}ade}\right) + 2\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{-ade}}{2\sqrt{-adedx}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/((e\*x + d)\*x^2), x, algorithm

[Out] [-1/4\*((c\*d^2 - a\*e^2)\*x\*log((4\*(2\*a^2\*d^2\*e^2 + (a\*c\*d^3\*e + a^2\*d\*e^3)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) + (8\*a^2\*d^2\*e^2 + (c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4)\*x^2 + 8\*(a\*c\*d^3\*e + a^2\*d\*e^3)\*x)\*sqrt(a\*d\*e))/x^2) + 4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(a\*d\*e))/(sqrt(a\*d\*e)\*d\*x), -1/2\*((c\*d^2 - a\*e^2)\*x\*arctan(1/2\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*a\*d\*e)) + 2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e))/(sqrt(-a\*d\*e)\*d\*x)]

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{x^2(d+ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/x\*\*2/(e\*x+d), x)

[Out] Integral(sqrt((d + e\*x)\*(a\*e + c\*d\*x))/(x\*\*2\*(d + e\*x)), x)

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/((e\*x + d)\*x^2), x, algorithm

[Out] Exception raised: NotImplementedError

$$3.443 \quad \int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^3(d+ex)} dx$$

**Optimal.** Leaf size=202

$$\frac{(cd^2 - ae^2) (3ae^2 + cd^2) \tanh^{-1} \left( \frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \right)}{8a^{3/2}d^{5/2}e^{3/2}} - \frac{\left(\frac{c}{ae} - \frac{3e}{d^2}\right) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{4x} - \frac{\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{2dx^2}$$

[Out] -Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(2\*d\*x^2) - ((c/(a\*e) - (3\*e)/d^2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(4\*x) + ((c\*d^2 - a\*e^2)\*(c\*d^2 + 3\*a\*e^2)\*ArcTanh[(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(2\*Sqrt[a]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])]/(8\*a^(3/2)\*d^(5/2)\*e^(3/2)))

**Rubi [A]** time = 0.832965, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{(cd^2 - ae^2) (3ae^2 + cd^2) \tanh^{-1} \left( \frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \right)}{8a^{3/2}d^{5/2}e^{3/2}} - \frac{\left(\frac{c}{ae} - \frac{3e}{d^2}\right) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{4x} - \frac{\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{2dx^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(x^3\*(d + e\*x)),x]

[Out] -Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(2\*d\*x^2) - ((c/(a\*e) - (3\*e)/d^2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(4\*x) + ((c\*d^2 - a\*e^2)\*(c\*d^2 + 3\*a\*e^2)\*ArcTanh[(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(2\*Sqrt[a]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])]/(8\*a^(3/2)\*d^(5/2)\*e^(3/2)))

**Rubi in Sympy [A]** time = 80.796, size = 185, normalized size = 0.92

$$\frac{\left(\frac{3e}{4d^2} - \frac{c}{4ae}\right) \sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{x} - \frac{\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{2dx^2} - \frac{(ae^2 - cd^2) (3ae^2 + cd^2) \operatorname{atanh} \left( \frac{2ade+x(ae^2+cd^2)}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+cdex^2+x(ae^2+cd^2)}} \right)}{8a^{\frac{3}{2}}d^{\frac{5}{2}}e^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/x\*\*3/(e\*x+d),x)

[Out] (3\*e/(4\*d\*\*2) - c/(4\*a\*e))\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/x - sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(2\*d\*x\*\*2) - (a\*e\*\*2 - c\*d\*\*2)\*(3\*a\*e\*\*2 + c\*d\*\*2)\*atanh((2\*a\*d\*e + x\*(a\*e\*\*2 + c\*d\*\*2))/(2\*sqrt(a)\*sqrt(d)\*sqrt(e)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))))/(8\*a\*\*3/2\*d\*\*5/2\*e\*\*3/2)

**Mathematica [A]** time = 0.417965, size = 238, normalized size = 1.18

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(x^2\log(x)\left(-(-3a^2e^4+2acd^2e^2+c^2d^4)\right)+x^2\left(-3a^2e^4+2acd^2e^2+c^2d^4\right)\log\left(2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}\right)\right)}{8a^{3/2}d^{5/2}e^{3/2}x^2\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(x^3\*(d + e\*x)), x]

[Out] (Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*(2\*Sqrt[a]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*(-(c\*d^2\*x) + a\*e\*(-2\*d + 3\*e\*x)) - (c^2\*d^4 + 2\*a\*c\*d^2\*e^2 - 3\*a^2\*e^4)\*x^2\*Log[x] + (c^2\*d^4 + 2\*a\*c\*d^2\*e^2 - 3\*a^2\*e^4)\*x^2\*Log[c\*d^2\*x + 2\*Sqrt[a]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x] + a\*e\*(2\*d + e\*x)))/(8\*a^(3/2)\*d^(5/2)\*e^(3/2)\*x^2\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)])

**Maple [B]** time = 0.022, size = 882, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)/x^3/(e\*x+d), x)

[Out] -1/2/d^2/a/e/x^2\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)+5/4/d^3/a/x\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)+1/4/d/a^2/e^2/x\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)\*c-1/4/d^3\*e^2\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)-1/4\*d/a^2/e^2\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)\*c^2-3/8/d^2\*a\*e^3/(a\*d\*e)^(1/2)\*ln((2\*a\*d\*e+(a\*e^2+c\*d^2)\*x+2\*(a\*d\*e)^(1/2)\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/x)+1/4\*e/(a\*d\*e)^(1/2)\*ln((2\*a\*d\*e+(a\*e^2+c\*d^2)\*x+2\*(a\*d\*e)^(1/2)\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/x)\*c+1/8\*d^2/a/e/(a\*d\*e)^(1/2)\*ln((2\*a\*d\*e+(a\*e^2+c\*d^2)\*x+2\*(a\*d\*e)^(1/2)\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/x)\*c^2-5/4/d^2/a\*e\*c\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)\*x-1/4/a^2/e\*c^2\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)\*x+1/2/d^3\*e^4\*ln((1/2\*a\*e^2+1/2\*c\*d^2+c\*d\*e\*x)/(c\*d\*e)^(1/2)+(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/(c\*d\*e)^(1/2))\*a-1/2/d\*e^2\*ln((1/2\*a\*e^2+1/2\*c\*d^2+c\*d\*e\*x)/(c\*d\*e)^(1/2)+(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/(c\*d\*e)^(1/2)\*c-1/d^3\*e^2\*(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2)-1/2/d^3\*e^4\*ln((1/2\*a\*e^2-1/2\*c\*d^2+(x+d/e)\*c\*d\*e)/(c\*d\*e)^(1/2)+(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2))/(c\*d\*e)^(1/2)\*a+1/2/d\*e^2\*ln((1/2\*a\*e^2-1/2\*c\*d^2+(x+d/e)\*c\*d\*e)/(c\*d\*e)^(1/2)+(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2))/(c\*d\*e)^(1/2)\*c-1/d/a\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)\*c

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/((e\*x + d)\*x^3), x, algorithm=

[Out] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/((e\*x + d)\*x^3), x)

**Fricas [A]** time = 0.402843, size = 1, normalized size = 0.

$$\left[ \frac{(c^2 d^4 + 2 a c d^2 e^2 - 3 a^2 e^4) x^2 \log\left(-\frac{4(2 a^2 d^2 e^2 + (a c d^3 e + a^2 d e^3) x) \sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x} - (8 a^2 d^2 e^2 + (c^2 d^4 + 6 a c d^2 e^2 + a^2 e^4) x^2 + 8 (a c d^3 e + a^2 d e^3) x)}{x^2}\right)}{16 \sqrt{a d e a d^2 e x^2}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/((e\*x + d)\*x^3), x, algorithm

[Out] [-1/16\*((c^2\*d^4 + 2\*a\*c\*d^2\*e^2 - 3\*a^2\*e^4)\*x^2\*log(-(4\*(2\*a^2\*d^2\*e^2 + (a\*c\*d^3\*e + a^2\*d\*e^3)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) - (8\*a^2\*d^2\*e^2 + (c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4)\*x^2 + 8\*(a\*c\*d^3\*e + a^2\*d\*e^3)\*x)\*sqrt(a\*d\*e))/x^2) + 4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(2\*a\*d\*e + (c\*d^2 - 3\*a\*e^2)\*x)\*sqrt(a\*d\*e)/(sqrt(a\*d\*e)\*a\*d^2\*e\*x^2), 1/8\*((c^2\*d^4 + 2\*a\*c\*d^2\*e^2 - 3\*a^2\*e^4)\*x^2\*arctan(1/2\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*a\*d\*e) - 2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(2\*a\*d\*e + (c\*d^2 - 3\*a\*e^2)\*x)\*sqrt(-a\*d\*e)/(sqrt(-a\*d\*e)\*a\*d^2\*e\*x^2)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{x^3(d+ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/x\*\*3/(e\*x+d), x)

[Out] Integral(sqrt((d + e\*x)\*(a\*e + c\*d\*x))/(x\*\*3\*(d + e\*x)), x)

**GIAC/XCAS [A]** time = 0.524519, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/((e\*x + d)\*x^3), x, algorithm

[Out] Done

$$3.444 \quad \int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^4(d+ex)} dx$$

**Optimal.** Leaf size=286

$$\begin{aligned} & \frac{(3cd^2 - 5ae^2) (3ae^2 + cd^2) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{24a^2d^3e^2x} \\ & - \frac{(cd^2 - ae^2) (5a^2e^4 + 2acd^2e^2 + c^2d^4) \tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{16a^{5/2}d^{7/2}e^{5/2}} \\ & - \frac{\left(\frac{c}{ae} - \frac{5e}{d^2}\right) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{12x^2} - \frac{\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{3dx^3} \end{aligned}$$

[Out] -Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(3\*d\*x^3) - ((c/(a\*e) - (5\*e)/d^2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(12\*x^2) + ((3\*c\*d^2 - 5\*a\*e^2)\*(c\*d^2 + 3\*a\*e^2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(24\*a^2\*d^3\*e^2\*x) - ((c\*d^2 - a\*e^2)\*(c^2\*d^4 + 2\*a\*c\*d^2\*e^2 + 5\*a^2\*e^4)\*ArcTanh[(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(2\*Sqrt[a]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]])/(16\*a^(5/2)\*d^(7/2)\*e^(5/2))

**Rubi [A]** time = 1.18343, antiderivative size = 286, normalized size of antiderivative = 1., number of rules used = 6, number of rules used = 5, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\begin{aligned} & \frac{(3cd^2 - 5ae^2) (3ae^2 + cd^2) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{24a^2d^3e^2x} \\ & - \frac{(cd^2 - ae^2) (5a^2e^4 + 2acd^2e^2 + c^2d^4) \tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{16a^{5/2}d^{7/2}e^{5/2}} \\ & - \frac{\left(\frac{c}{ae} - \frac{5e}{d^2}\right) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{12x^2} - \frac{\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{3dx^3} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(x^4\*(d + e\*x)), x]

[Out] -Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(3\*d\*x^3) - ((c/(a\*e) - (5\*e)/d^2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(12\*x^2) + ((3\*c\*d^2 - 5\*a\*e^2)\*(c\*d^2 + 3\*a\*e^2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(24\*a^2\*d^3\*e^2\*x) - ((c\*d^2 - a\*e^2)\*(c^2\*d^4 + 2\*a\*c\*d^2\*e^2 + 5\*a^2\*e^4)\*ArcTanh[(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(2\*Sqrt[a]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]])/(16\*a^(5/2)\*d^(7/2)\*e^(5/2))

**Rubi in Sympy [A]** time = 147.703, size = 270, normalized size = 0.94

$$\begin{aligned} & \frac{\left(\frac{5e}{12d^2} - \frac{c}{12ae}\right) \sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{x^2} - \frac{\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{3dx^3} \\ & - \frac{(3ae^2 + cd^2) (5ae^2 - 3cd^2) \sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{24a^2d^3e^2x} \\ & + \frac{(ae^2 - cd^2) (5a^2e^4 + 2acd^2e^2 + c^2d^4) \operatorname{atanh}\left(\frac{2ade+x(ae^2+cd^2)}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}\right)}{16a^{\frac{5}{2}}d^{\frac{7}{2}}e^{\frac{5}{2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**4/(e*x+d), x)`

[Out]  $(5e/(12d^2) - c/(12ae))\sqrt{a^2d^2e + c^2d^2e^2x^2 + x(a^2e^2 + c^2d^2)} / (3d^3x^3) - \sqrt{a^2d^2e + c^2d^2e^2x^2 + x(a^2e^2 + c^2d^2)} / (24a^2d^3e^2x) + (a^2e^2 - c^2d^2)(5a^2e^4 + 2ac^2d^2e^2 + c^2d^4)\operatorname{atanh}((2a^2d^2e + x(a^2e^2 + c^2d^2))/(2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{a^2d^2e + c^2d^2e^2x^2 + x(a^2e^2 + c^2d^2)})) / (16a^{5/2}d^{7/2}e^{5/2})$

**Mathematica [A]** time = 0.573724, size = 299, normalized size = 1.05

$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}\left(a^2e^2(-8d^2+10dex-15e^2x^2)-2acd^2ex(d-2ex)+3c^2d^4x^2\right)+3x^3\log\right)}{48a^{5/2}d}$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(x^4*(d + e*x)), x]`

[Out]  $(\sqrt{a^2e + c^2d^2x}\sqrt{d + e^2x}(2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{a^2e + c^2d^2x}\sqrt{d + e^2x}(3c^2d^4x^2 - 2ac^2d^2e^2x(d - 2e^2x) + a^2e^2(-8d^2 + 10dex - 15e^2x^2)) + 3(c^3d^6 + ac^2d^4e^2 + 3a^2c^2d^2e^4 - 5a^3e^6)x^3\operatorname{Log}[x] - 3(c^3d^6 + ac^2d^4e^2 + 3a^2c^2d^2e^4 - 5a^3e^6)x^3\operatorname{Log}[c^2d^2x + 2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{a^2e + c^2d^2x}\sqrt{d + e^2x} + ae^2(2d + e^2x)])) / (48a^{5/2}d^{7/2}e^{5/2}x^3\sqrt{(a^2e + c^2d^2x)(d + e^2x)})$

**Maple [B]** time = 0.024, size = 1165, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^4/(e*x+d), x)`

[Out]  $\frac{3}{4}d^3/a/x^2(a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{3/2} + \frac{3}{8}a^2/e^*(a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{1/2} * c^{2+1/4}/d/a^2/e^2/x^2 * (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{3/2} * c^{-1/2}/d^2/a^2/e/x * (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{3/2} * c^{-1/16}d^3/a^2/e^2/(a^2d^2e)^{1/2} * \ln((2a^2d^2e + (a^2e^2 + c^2d^2)x + 2(a^2d^2e)^{1/2} * (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{1/2})/x) * c^3 + 11/8/d^3/a^2/e^2 * c * (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{1/2} * x + 1/8/d/a^3/e^2 * c^3 * (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{1/2} * x + 1/2/d^2/e^3 * \ln((1/2 * a^2e^2 + 1/2 * c^2d^2 + c^2d^2e^2x)/(c^2d^2e)^{1/2} + (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{1/2})/(c^2d^2e)^{1/2} * c^{-1/2}/d^4 * e^5 * \ln((1/2 * a^2e^2 + 1/2 * c^2d^2 + c^2d^2e^2x)/(c^2d^2e)^{1/2} + (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{1/2})/(c^2d^2e)^{1/2} * a^{-1/3}/d^2/a/e/x^3 * (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{3/2} - 11/8/d^4/a^2/e/x * (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{3/2} * c^2 + 9/8/d^2/a^2/e * (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{1/2} * c + 1/8/d^2/a^3/e^3 * (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{1/2} * c^3 + 5/16/d^3 * a^2e^4/(a^2d^2e)^{1/2} * \ln((2a^2d^2e + (a^2e^2 + c^2d^2)x + 2(a^2d^2e)^{1/2} * (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{1/2})/x) - 3/16/d^2e^2/(a^2d^2e)^{1/2} * \ln((2a^2d^2e + (a^2e^2 + c^2d^2)x + 2(a^2d^2e)^{1/2} * (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{1/2})/x) * c^{-1/16}d/a/(a^2d^2e)^{1/2} * \ln((2a^2d^2e + (a^2e^2 + c^2d^2)x + 2(a^2d^2e)^{1/2} * (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{1/2})/x) * c^2 + 1/2/d/a^2 * c^2 * (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{1/2} * x + 1/2/d^4 * e^5 * \ln((1/2 * a^2e^2 - 1/2 * c^2d^2 + (x+d/e) * c^2d^2e)/(c^2d^2e)^{1/2} + (c^2d^2e * (x+d/e)^2 + (a^2e^2 - c^2d^2) * (x+d/e))^{1/2})/(c^2d^2e)^{1/2} * a^{-1/2}/d^2 * e^3 * \ln((1/2 * a^2e^2 - 1/2 * c^2d^2 + (x+d/e) * c^2$



$$\frac{d^2 e}{(c^2 d^2 e)^{1/2} + (c^2 d^2 e^* (x+d/e)^2 + (a^2 e^2 - c^2 d^2)^* (x+d/e))^{1/2}} \\ / ((c^2 d^2 e)^{1/2} * c + 1/d^4 * e^3 * (c^2 d^2 e^* (x+d/e)^2 + (a^2 e^2 - c^2 d^2)^* (x+d/e))^{1/2} + 3/8/d^4 * e^3 * (a^2 d^2 e + (a^2 e^2 + c^2 d^2)^* x + c^2 d^2 e^* x^2)^{1/2})$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/((e\*x + d)\*x^4), x, algorithm

[Out] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/((e\*x + d)\*x^4), x)

**Fricas [A]** time = 0.811004, size = 1, normalized size = 0.

$$\frac{3(c^3 d^6 + ac^2 d^4 e^2 + 3a^2 cd^2 e^4 - 5a^3 e^6)x^3 \log\left(\frac{4(2a^2 d^2 e^2 + (acd^3 e + a^2 de^3)x)\sqrt{cdex^2 + ade + (cd^2 + ae^2)x} + (8a^2 d^2 e^2 + (c^2 d^4 + 6acd^2 e^2 + a^2 e^4)x)}{x^2}\right)}{96} \\ + \frac{3(c^3 d^6 + ac^2 d^4 e^2 + 3a^2 cd^2 e^4 - 5a^3 e^6)x^3 \arctan\left(\frac{(2ade + (cd^2 + ae^2)x)\sqrt{-ade}}{2\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}ade}\right) + 2(8a^2 d^2 e^2 - (3c^2 d^4 + 4acd^2 e^2 - 15a^2 e^4))}{48\sqrt{-ade}a^2 d^3 e^2 x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/((e\*x + d)\*x^4), x, algorithm

[Out] [-1/96\*(3\*(c^3\*d^6 + a\*c^2\*d^4\*e^2 + 3\*a^2\*c\*d^2\*e^4 - 5\*a^3\*e^6)\*x^3\*log((4\*(2\*a^2\*d^2\*e^2 + (a\*c\*d^3\*e + a^2\*d\*e^3)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) + (8\*a^2\*d^2\*e^2 + (c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4)\*x^2 + 8\*(a\*c\*d^3\*e + a^2\*d\*e^3)\*x)\*sqrt(a\*d\*e))/x^2) + 4\*(8\*a^2\*d^2\*e^2 - (3\*c^2\*d^4 + 4\*a\*c\*d^2\*e^2 - 15\*a^2\*e^4)\*x^2 + 2\*(a\*c\*d^3\*e - 5\*a^2\*d\*e^3)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(a\*d\*e)/(sqrt(a\*d\*e)\*a^2\*d^3\*e^2\*x^3), -1/48\*(3\*(c^3\*d^6 + a\*c^2\*d^4\*e^2 + 3\*a^2\*c\*d^2\*e^4 - 5\*a^3\*e^6)\*x^3\*arctan(1/2\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*a\*d\*e) + 2\*(8\*a^2\*d^2\*e^2 - (3\*c^2\*d^4 + 4\*a\*c\*d^2\*e^2 - 15\*a^2\*e^4)\*x^2 + 2\*(a\*c\*d^3\*e - 5\*a^2\*d\*e^3)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e)/(sqrt(-a\*d\*e)\*a^2\*d^3\*e^2\*x^3)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/x\*\*4/(e\*x+d), x)

[Out] Timed out

GIAC/XCAS [A] time = 1.1782, size = 1, normalized size = 0.

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^4),x, algorithm`

[Out] Done

$$3.445 \quad \int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{x^5(d+ex)} dx$$

**Optimal.** Leaf size=389

$$\begin{aligned} & \frac{(-35a^2e^4 + 6acd^2e^2 + 5c^2d^4) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{96a^2d^3e^2x^2} \\ & - \frac{(-105a^3e^6 + 25a^2cd^2e^4 + 17ac^2d^4e^2 + 15c^3d^6) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{192a^3d^4e^3x} \\ & + \frac{(cd^2 - ae^2) (35a^3e^6 + 15a^2cd^2e^4 + 9ac^2d^4e^2 + 5c^3d^6) \tanh^{-1} \left( \frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \right)}{128a^{7/2}d^{9/2}e^{7/2}} \\ & - \frac{\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae} - \frac{7e}{d^2}\right) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{24x^3} \end{aligned}$$

[Out] -Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(4\*d\*x^4) - ((c/(a\*e) - (7\*e)/d^2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(24\*x^3) + ((5\*c^2\*d^4 + 6\*a\*c\*d^2\*e^2 - 35\*a^2\*e^4)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(96\*a^2\*d^3\*e^2\*x^2) - ((15\*c^3\*d^6 + 17\*a\*c^2\*d^4\*e^2 + 25\*a^2\*c\*d^2\*e^4 - 105\*a^3\*e^6)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(192\*a^3\*d^4\*e^3\*x) + ((c\*d^2 - a\*e^2)\*(5\*c^3\*d^6 + 9\*a\*c^2\*d^4\*e^2 + 15\*a^2\*c\*d^2\*e^4 + 35\*a^3\*e^6)\*ArcTanh[(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(2\*Sqrt[a]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]])/(128\*a^(7/2)\*d^(9/2)\*e^(7/2))

**Rubi [A]** time = 1.64302, antiderivative size = 389, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\begin{aligned} & \frac{(-35a^2e^4 + 6acd^2e^2 + 5c^2d^4) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{96a^2d^3e^2x^2} \\ & - \frac{(-105a^3e^6 + 25a^2cd^2e^4 + 17ac^2d^4e^2 + 15c^3d^6) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{192a^3d^4e^3x} \\ & + \frac{(cd^2 - ae^2) (35a^3e^6 + 15a^2cd^2e^4 + 9ac^2d^4e^2 + 5c^3d^6) \tanh^{-1} \left( \frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \right)}{128a^{7/2}d^{9/2}e^{7/2}} \\ & - \frac{\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{4dx^4} - \frac{\left(\frac{c}{ae} - \frac{7e}{d^2}\right) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{24x^3} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(x^5\*(d + e\*x)), x]

[Out] -Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(4\*d\*x^4) - ((c/(a\*e) - (7\*e)/d^2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(24\*x^3) + ((5\*c^2\*d^4 + 6\*a\*c\*d^2\*e^2 - 35\*a^2\*e^4)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(96\*a^2\*d^3\*e^2\*x^2) - ((15\*c^3\*d^6 + 17\*a\*c^2\*d^4\*e^2 + 25\*a^2\*c\*d^2\*e^4 - 105\*a^3\*e^6)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(192\*a^3\*d^4\*e^3\*x) + ((c\*d^2 - a\*e^2)\*(5\*c^3\*d^6 + 9\*a\*c^2\*d^4\*e^2 + 15\*a^2\*c\*d^2\*e^4 + 35\*a^3\*e^6)\*ArcTanh[(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(2\*Sqrt[a]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]])/(128\*a^(7/2)\*d^(9/2)\*e^(7/2))

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/x**5/(e*x+d),x)`

[Out] Timed out

**Mathematica [A]** time = 0.826194, size = 378, normalized size = 0.97

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}\left(a^3e^3(-48d^3+56d^2ex-70de^2x^2+105e^3x^3)+a^2cd^2e^2x(-8d^2+12dex\right.\right.\right.$$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2]/(x^5*(d+e*x)),x]`

[Out]  $(\text{Sqrt}[a^*e + c^*d^*x]^*\text{Sqrt}[d + e^*x]^*(2^*\text{Sqrt}[a]^*\text{Sqrt}[d]^*\text{Sqrt}[e]^*\text{Sqrt}[a^*e + c^*d^*x]^*\text{Sqrt}[d + e^*x]^*(-15^*c^3*d^6*x^3 + a^*c^2*d^4*e^*x^2*(10^*d - 17^*e^*x) + a^2*c*d^2*e^2*x^*(-8^*d^2 + 12^*d^*e^*x - 25^*e^2*x^2) + a^3*e^3*(-48^*d^3 + 56^*d^2*e^*x - 70^*d^*e^2*x^2 + 105^*e^3*x^3)) - 3^*(5^*c^4*d^8 + 4^*a^*c^3*d^6*e^2 + 6^*a^2*c^2*d^4*e^4 + 20^*a^3*c*d^2*e^6 - 35^*a^4*e^8)^*x^4*\text{Log}[x] + 3^*(5^*c^4*d^8 + 4^*a^*c^3*d^6*e^2 + 6^*a^2*c^2*d^4*e^4 + 20^*a^3*c*d^2*e^6 - 35^*a^4*e^8)^*x^4*\text{Log}[c^*d^2*x + 2^*\text{Sqrt}[a]^*\text{Sqrt}[d]^*\text{Sqrt}[e]^*\text{Sqrt}[a^*e + c^*d^*x]^*\text{Sqrt}[d + e^*x] + a^*e^*(2^*d + e^*x)))/(384^*a^(7/2)^*d^(9/2)^*e^(7/2)^*x^4*\text{Sqrt}[(a^*e + c^*d^*x)^*(d + e^*x)])$

**Maple [B]** time = 0.033, size = 1494, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/x^5/(e*x+d),x)`

[Out]  $13/24/d^3/a/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-17/32/d/a^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^2+1/32*d^2*c^3/a^2/e/(a*d*e)^(1/2)*\ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)+5/64*d/a^4/e^4/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c^3+5/128*d^4/a^3/e^3/(a*d*e)^(1/2)*\ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^4-93/64/d^4/a^3/e^3*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-43/64/d^2/a^2*e^c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-5/64*d^2/a^4/e^3*c^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-7/16/d^2/a^2/e/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c+5/24/d/a^2/e^2/x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c+19/64/d/a^3/e^2/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c^2-7/32*d/a^3/e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^3-5/64*d^3/a^4/e^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^4-5/32/a^3/e^3/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c^2-1/2/d^5*e^6*\ln((1/2*a^e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a^e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*a+1/2/d^3*e^4*\ln((1/2*a^e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a^e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)*c-35/128/d^4*a^e^5/(a*d*e)^(1/2)*\ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)-1/4/d^2/a/e/x^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)+3/64/a^e/(a*d*e)^(1/2)*\ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^2-19/64/a^3/e^c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+5/32/d^2*c^e^3/(a*d*e)^(1/2)*\ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)-3$

$$\begin{aligned} & 9/32/d^3/a^*e^2*(a^*d^*e+(a^*e^2+c^*d^2)*x+c^*d^*e*x^2)^{(1/2)}*c+43/64/d^4 \\ & 3/a^2/x*(a^*d^*e+(a^*e^2+c^*d^2)*x+c^*d^*e*x^2)^{(3/2)}*c-29/32/d^4/a^*e/x \\ & ^2*(a^*d^*e+(a^*e^2+c^*d^2)*x+c^*d^*e*x^2)^{(3/2)}+93/64/d^5/a^*e^2/x*(a^*d^* \\ & *e+(a^*e^2+c^*d^2)*x+c^*d^*e*x^2)^{(3/2)}-1/2/d^3*e^4*\ln((1/2*a^*e^2+1/2 \\ & *c^*d^2+c^*d^*e*x)/(c^*d^*e)^{(1/2)}+(a^*d^*e+(a^*e^2+c^*d^2)*x+c^*d^*e*x^2)^{(1/2)}) \\ & /((c^*d^*e)^{(1/2)}*c+1/2/d^5*e^6*\ln((1/2*a^*e^2+1/2*c^*d^2+c^*d^*e*x) \\ & )/(c^*d^*e)^{(1/2)}+(a^*d^*e+(a^*e^2+c^*d^2)*x+c^*d^*e*x^2)^{(1/2)})/(c^*d^*e)^{(1/2)} \\ & *a-1/d^5*e^4*(c^*d^*e*(x+d/e)^2+(a^*e^2-c^*d^2)*(x+d/e))^{(1/2)}-2 \\ & 9/64/d^5*e^4*(a^*d^*e+(a^*e^2+c^*d^2)*x+c^*d^*e*x^2)^{(1/2)} \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{(ex + d)x^5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/((e\*x + d)\*x^5), x, algorithm

[Out] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/((e\*x + d)\*x^5), x)

**Fricas [A]** time = 1.99548, size = 1, normalized size = 0.

$$\left[ \frac{3(5c^4d^8 + 4ac^3d^6e^2 + 6a^2c^2d^4e^4 + 20a^3cd^2e^6 - 35a^4e^8)x^4 \log\left(-\frac{4(2a^2d^2e^2 + (acd^3e + a^2de^3)x)\sqrt{cdex^2 + ade + (cd^2 + ae^2)x} - (8a^2d^2e^2 + (acd^3e + a^2de^3)x)}{x^2}\right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/((e\*x + d)\*x^5), x, algorithm

[Out] [-1/768\*(3\*(5\*c^4\*d^8 + 4\*a\*c^3\*d^6\*e^2 + 6\*a^2\*c^2\*d^4\*e^4 + 20\*a^3\*c\*d^2\*e^6 - 35\*a^4\*e^8)\*x^4\*log(-4\*(2\*a^2\*d^2\*e^2 + (a\*c\*d^3\*e + a^2\*d^2\*e^3)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) - (8\*a^2\*d^2\*e^2 + (c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4)\*x^2 + 8\*(a\*c\*d^3\*e + a^2\*d^2\*e^3)\*x)\*sqrt(a\*d\*e))/x^2 + 4\*(48\*a^3\*d^3\*e^3 + (15\*c^3\*d^6 + 17\*a\*c^2\*d^4\*e^2 + 25\*a^2\*c\*d^2\*e^4 - 105\*a^3\*e^6)\*x^3 - 2\*(5\*a\*c^2\*d^5\*e + 6\*a^2\*c\*d^3\*e^3 - 35\*a^3\*d^2\*e^5)\*x^2 + 8\*(a^2\*c\*d^4\*e^2 - 7\*a^3\*d^2\*e^4)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(a\*d\*e)/(sqrt(a\*d\*e)\*(sqrt(a^3\*d^4\*e^3\*x^4), 1/384\*(3\*(5\*c^4\*d^8 + 4\*a\*c^3\*d^6\*e^2 + 6\*a^2\*c^2\*d^4\*e^4 + 20\*a^3\*c\*d^2\*e^6 - 35\*a^4\*e^8)\*x^4\*arctan(1/2\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*a\*d\*e)) - 2\*(48\*a^3\*d^3\*e^3 + (15\*c^3\*d^6 + 17\*a\*c^2\*d^4\*e^2 + 25\*a^2\*c\*d^2\*e^4 - 105\*a^3\*e^6)\*x^3 - 2\*(5\*a\*c^2\*d^5\*e + 6\*a^2\*c\*d^3\*e^3 - 35\*a^3\*d^2\*e^5)\*x^2 + 8\*(a^2\*c\*d^4\*e^2 - 7\*a^3\*d^2\*e^4)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e))/(sqrt(-a\*d\*e)\*a^3\*d^4\*e^3\*x^4)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/x\*\*5/(e\*x+d), x)

[Out] Timed out

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**GIAC/XCAS [A]** time = 3.42025, size = 1, normalized size = 0.

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/((e*x + d)*x^5), x, algorithm`

[Out] Done

$$3.446 \quad \int \frac{x^3 (ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{d+ex} dx$$

**Optimal.** Leaf size=449

$$\frac{(-35a^3e^6 - 6cdex(-7a^2e^4 - 6acd^2e^2 + 21c^2d^4) - 33a^2cd^2e^4 - 21ac^2d^4e^2 + 105c^3d^6)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{960c^3d^3e^4} \\ + \frac{(7a^3e^6 + 15a^2cd^2e^4 + 21ac^2d^4e^2 + 21c^3d^6)(cd^2 - ae^2)^3 \tanh^{-1}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{1024c^{9/2}d^{9/2}e^{11/2}} \\ + \frac{(-7a^4e^8 - 8a^3cd^2e^6 - 6a^2c^2d^4e^4 + 21c^4d^8)(ae^2 + cd^2 + 2cdex)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{512c^4d^4e^5} \\ + \frac{1}{20}x^2\left(\frac{a}{cd} - \frac{3d}{e^2}\right)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2} + \frac{x^3(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{6e}$$

[Out]  $((21*c^4*d^8 - 6*a^2*c^2*d^4*e^4 - 8*a^3*c*d^2*e^6 - 7*a^4*e^8) * (c*d^2 + a*e^2 + 2*c*d*e*x) * \text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (512*c^4*d^4*e^5) + ((a/(c*d) - (3*d)/e^2)*x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) / 20 + (x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) / (6*e) - ((105*c^3*d^6 - 21*a*c^2*d^4*e^2 - 33*a^2*c*d^2*e^4 - 35*a^3*e^6 - 6*c*d*e*(21*c^2*d^4 - 6*a*c*d^2*e^2 - 7*a^2*e^4)*x) * (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) / (960*c^3*d^3*e^4) - ((c*d^2 - a*e^2)^3 * (21*c^3*d^6 + 21*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 + 7*a^3*e^6) * \text{ArcTanh}[(c*d^2 + a*e^2 + 2*c*d*e*x) / (2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]) / (1024*c^{(9/2)}*d^{(9/2)}*e^{(11/2)})$

**Rubi [A]** time = 1.52452, antiderivative size = 449, normalized size of antiderivative = 1., number of rules used = 7, number of rules used = 6, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{(-35a^3e^6 - 6cdex(-7a^2e^4 - 6acd^2e^2 + 21c^2d^4) - 33a^2cd^2e^4 - 21ac^2d^4e^2 + 105c^3d^6)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{960c^3d^3e^4} \\ + \frac{(7a^3e^6 + 15a^2cd^2e^4 + 21ac^2d^4e^2 + 21c^3d^6)(cd^2 - ae^2)^3 \tanh^{-1}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{1024c^{9/2}d^{9/2}e^{11/2}} \\ + \frac{(-7a^4e^8 - 8a^3cd^2e^6 - 6a^2c^2d^4e^4 + 21c^4d^8)(ae^2 + cd^2 + 2cdex)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{512c^4d^4e^5} \\ + \frac{1}{20}x^2\left(\frac{a}{cd} - \frac{3d}{e^2}\right)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2} + \frac{x^3(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{6e}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) / (d + e*x), x]$

[Out]  $((21*c^4*d^8 - 6*a^2*c^2*d^4*e^4 - 8*a^3*c*d^2*e^6 - 7*a^4*e^8) * (c*d^2 + a*e^2 + 2*c*d*e*x) * \text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (512*c^4*d^4*e^5) + ((a/(c*d) - (3*d)/e^2)*x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) / 20 + (x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) / (6*e) - ((105*c^3*d^6 - 21*a*c^2*d^4*e^2 - 33*a^2*c*d^2*e^4 - 35*a^3*e^6 - 6*c*d*e*(21*c^2*d^4 - 6*a*c*d^2*e^2 - 7*a^2*e^4)*x) * (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) / (960*c^3*d^3*e^4) - ((c*d^2 - a*e^2)^3 * (21*c^3*d^6 + 21*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 + 7*a^3*e^6) * \text{ArcTanh}[(c*d^2 + a*e^2 + 2*c*d*e*x) / (2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]) / (1024*c^{(9/2)}*d^{(9/2)}*e^{(11/2)})$

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d),x)`

[Out] Timed out

**Mathematica [A]** time = 0.80535, size = 422, normalized size = 0.94

$$\sqrt{(d+ex)(ae+cdx)} \left( \frac{32x^3 \left( \frac{3a^2e^2}{c} + 14ad^2 - \frac{9cd^4}{e^2} \right)}{d} + \frac{16x^2(-7a^3e^6+3a^2cd^2e^4-33ac^2d^4e^2+21c^3d^6)}{c^2d^2e^3} - \frac{15(cd^2-ae^2)^3(7a^3e^6+15a^2cd^2e^4+21ac^2d^4e^2+21c^3d^6)}{c^9/2d^9/2e^{11}} \right)$$

Antiderivative was successfully verified.

[In] `Integrate[(x^3*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^(3/2))/(d+e*x),x]`

[Out]  $(\text{Sqrt}[(a*e + c*d*x)*(d + e*x)] * ((2*(315*c*d^6 - 525*a*d^4*e^2 + (78*a^2*d^2*e^4)/c + (54*a^3*e^6)/c^2 + (55*a^4*e^8)/(c^3*d^2) - (105*a^5*e^{10})/(c^4*d^4)))/e^5 + (4*((-18*a^2*d^4)/c - (105*c*d^8)/e^4 + (168*a*d^6)/e^2 - (16*a^3*d^2*e^2)/c^2 + (35*a^4*e^4)/c^3) * x)/d^3 + (16*(21*c^3*d^6 - 33*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 - 7*a^3*e^6)*x^2)/(c^2*d^2*e^3) + (32*(14*a*d^2 - (9*c*d^4)/e^2 + (3*a^2*e^2)/c)*x^3)/d + (256*(c*d^2 + 13*a*e^2)*x^4)/e + 2560*c*d*x^5 - (15*(c*d^2 - a*e^2)^3*(21*c^3*d^6 + 21*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 + 7*a^3*e^6)*\text{Log}[a*e^2 + 2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x] + c*d*(d + 2*e*x)])/(c^{9/2}*d^{9/2}*e^{11/2})*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]))/15360$

**Maple [B]** time = 0.034, size = 1883, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d),x)`

[Out]  $1/8*d^6/e^5*c*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{1/2}+29/192/c^2/d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{3/2}*a^2+43/96*d^2/e^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{3/2}*x-7/256*e/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{1/2}*a^3+21/512*d^4/e^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{1/2}*a-43/512*d^6/e^5*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{1/2}+1/16*d^2*e*a^3/c*\text{ln}((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^{1/2}+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{1/2})/(c*d*e)^{1/2}+3/16*d^6/e^3*a*c*\text{ln}((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^{1/2}+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{1/2})/(c*d*e)^{1/2}-1/32*e^2/c^2*a^3/d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{1/2}*x+7/96*e/c^2/d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{3/2}*x*a^2-3/512*e^5/c^3/d^2*\text{ln}((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^{1/2}+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{1/2})/(c*d*e)^{1/2})*a^5-7/256*e^4/c^3/d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{1/2}*x*a^4+7/1024*e^7/c^4/d^4*\text{ln}((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^{1/2}+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{1/2})/(c*d*e)^{1/2})*a^6-17/256*d^2*e/c*\text{ln}((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^{1/2}+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{1/2})/(c*d*e)^{1/2})*a^3-75/512*d^6/e^3*c*\text{ln}((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^{1/2}+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{1/2})/(c*d*e)^{1/2})*a-3/16*d^4/e*a^2*\text{ln}((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^{1/2}+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{1/2})/(c*d*e)^{1/2}+1/4*d^5/e^4*c*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{1/2}*x-1/16*d^8/e^5*c^2*\text{ln}((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^{1/2}+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{1/2})/(c*d*e)^{1/2}-7/60/e/c^2/d^2*(a*d*e$



$$\begin{aligned}
& + (a^2 e^2 + c^2 d^2) x + c^2 d^2 e x^2)^{5/2} a + 7/192 e^2 / c^3 / d^3 (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d^2 e x^2)^{3/2} a^3 - 7/512 e^5 / c^4 / d^4 (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d^2 e x^2)^{1/2} a^5 - 15/512 e^3 / c^3 / d^2 (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d^2 e x^2)^{1/2} a^4 + 11/48 e / c (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d^2 e x^2)^{3/2} x^2 a + 1/6 e^2 x (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d^2 e x^2)^{5/2} / c / d + 65/192 d / e^2 / c (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d^2 e x^2)^{3/2} a - 3/128 d / c (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d^2 e x^2)^{1/2} x^2 a^2 + 1/4 d^3 / e^2 (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d^2 e x^2)^{1/2} x^2 a - 43/256 d^5 / e^4 c (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d^2 e x^2)^{1/2} x + 29/256 d^2 / e c (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d^2 e x^2)^{1/2} a^2 - 3/1024 e^3 / c^2 \ln((1/2 a^2 e^2 + 1/2 c^2 d^2 + c^2 d^2 e x) / (c^2 d^2 e)^{1/2}) + (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d^2 e x^2)^{1/2} / (c^2 d^2 e)^{1/2} a^4 + 177/1024 d^4 / e \ln((1/2 a^2 e^2 + 1/2 c^2 d^2 + c^2 d^2 e x) / (c^2 d^2 e)^{1/2}) + (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d^2 e x^2)^{1/2} / (c^2 d^2 e)^{1/2} a^2 + 43/1024 d^8 / e^5 c^2 \ln((1/2 a^2 e^2 + 1/2 c^2 d^2 + c^2 d^2 e x) / (c^2 d^2 e)^{1/2}) + (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d^2 e x^2)^{1/2} / (c^2 d^2 e)^{1/2} - 1/8 d^2 / e^2 a^2 / c (c^2 d^2 e (x+d/e)^2 + (a^2 e^2 - c^2 d^2) (x+d/e))^{1/2} - 1/4 d^3 / e^2 a^2 (c^2 d^2 e (x+d/e)^2 + (a^2 e^2 - c^2 d^2) (x+d/e))^{1/2} x - 1/3 d^3 / e^4 (c^2 d^2 e (x+d/e)^2 + (a^2 e^2 - c^2 d^2) (x+d/e))^{3/2} - 19/60 / e^3 / c (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d^2 e x^2)^{5/2} + 43/192 d^3 / e^4 (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d^2 e x^2)^{3/2}
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*x^3/(e\*x + d), x, algorithm

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.390298, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*x^3/(e\*x + d), x, algorithm

[Out] [1/30720\*(4\*(1280\*c^5\*d^5\*e^5\*x^5 + 315\*c^5\*d^10 - 525\*a\*c^4\*d^8\*e^2 + 78\*a^2\*c^3\*d^6\*e^4 + 54\*a^3\*c^2\*d^4\*e^6 + 55\*a^4\*c\*d^2\*e^8 - 105\*a^5\*e^10 + 128\*(c^5\*d^6\*e^4 + 13\*a\*c^4\*d^4\*e^6)\*x^4 - 16\*(9\*c^5\*d^7\*e^3 - 14\*a\*c^4\*d^5\*e^5 - 3\*a^2\*c^3\*d^3\*e^7)\*x^3 + 8\*(21\*c^5\*d^8\*e^2 - 33\*a\*c^4\*d^6\*e^4 + 3\*a^2\*c^3\*d^4\*e^6 - 7\*a^3\*c^2\*d^2\*e^8)\*x^2 - 2\*(105\*c^5\*d^9\*e - 168\*a\*c^4\*d^7\*e^3 + 18\*a^2\*c^3\*d^5\*e^5 + 16\*a^3\*c^2\*d^3\*e^7 - 35\*a^4\*c\*d\*e^9)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*e) - 15\*(21\*c^6\*d^12 - 42\*a\*c^5\*d^10\*e^2 + 15\*a^2\*c^4\*d^8\*e^4 + 4\*a^3\*c^3\*d^6\*e^6 + 3\*a^4\*c^2\*d^4\*e^8 + 6\*a^5\*c\*d^2\*e^10 - 7\*a^6\*e^12)\*log(4\*(2\*c^2\*d^2\*e^2\*x + c^2\*d^3\*e + a\*c\*d\*e^3)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) + (8\*c^2\*d^2\*e^2\*x^2 + c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4 + 8\*(c^2\*d^3\*e + a\*c\*d\*e^3)\*x)\*sqrt(c\*d\*e)))/(sqrt(c\*d\*e)\*c^4\*d^4\*e^5), 1/15360\*(2\*(1280\*c^5\*d^5\*e^5\*x^5 + 315\*c^5\*d^10 - 525\*a\*c^4\*d^8\*e^2 + 78\*a^2\*c^3\*d^6\*e^4 + 54\*a^3\*c^2\*d^4\*e^6 + 55\*a^4\*c\*d^2\*e^8 - 105\*a^5\*e^10 + 128\*(c^5\*d^6\*e^4 + 13\*a\*c^4\*d^4\*e^6)\*x^4 - 16\*(9\*c^5\*d^7\*e^3 - 14\*a\*c^4\*d^5\*e^5 - 3\*a^2\*c^3\*d^3\*e^7)\*x^3 + 8\*(21\*c^5\*d^8\*e^2 - 33\*a\*c^4\*d^6\*e^4 + 3\*a^2\*c^3\*d^4\*e^6 - 7\*a^3\*c^2\*d^2\*e^8)\*x^2 - 2\*(105\*c^5\*d^9\*e - 168\*a\*c^4\*d^7\*e^3 + 18\*a^2\*c^3\*d^5\*e^5 + 16\*a^3\*c^2\*d^3\*e^7 - 35\*a^4\*c\*d\*e^9)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*e) - 15\*(21\*c^6\*d^12 - 42\*a\*c^5\*d^10\*e^2 + 15\*a^2\*c^4\*d^8\*e^4 + 4\*a^3\*c^3\*d^6\*e^6 + 3\*a^4\*c^2\*d^4\*e^8 + 6\*a^5\*c\*d^2\*e^10 - 7\*a^6\*e^12)\*arctan(1/2\*(2\*c\*d\*e\*x + c\*d^2 + a\*e^2)\*sqrt(-c\*d\*e))/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 +

$$a \cdot e^2 \cdot x \cdot c \cdot d \cdot e) / (\sqrt{-c \cdot d \cdot e} \cdot c^4 \cdot d^4 \cdot e^5)]$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/(e\*x+d),x)

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*x^3/(e\*x + d),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.447 \quad \int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex} dx$$

**Optimal.** Leaf size=352

$$\begin{aligned} & \frac{(-15a^2e^4 - 6cdex(7cd^2 - 3ae^2) - 12acd^2e^2 + 35c^2d^4)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{240c^2d^2e^3} \\ & + \frac{(3a^2e^4 + 6acd^2e^2 + 7c^2d^4)(cd^2 - ae^2)^3 \tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{256c^{7/2}d^{7/2}e^{9/2}} \\ & - \frac{(3a^2e^4 + 6acd^2e^2 + 7c^2d^4)(cd^2 - ae^2)(ae^2 + cd^2 + 2cdex)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{128c^3d^3e^4} \\ & + \frac{x^2(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{5e} \end{aligned}$$

[Out]  $-\left((c*d^2 - a*e^2)*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*(c*d^2 + a*e^2 + 2*c*d*e*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]\right) / (128*c^3*d^3*e^4) + (x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) / (5*e) + \left(\left(35*c^2*d^4 - 12*a*c*d^2*e^2 - 15*a^2*e^4 - 6*c*d*e*(7*c*d^2 - 3*a*e^2)*x\right)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}\right) / (240*c^2*d^2*e^3) + \left(\left(c*d^2 - a*e^2\right)^3*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*\text{ArcTanh}\left[\left(c*d^2 + a*e^2 + 2*c*d*e*x\right) / \left(2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]\right)\right]\right) / (256*c^{(7/2)}*d^{(7/2)}*e^{(9/2)})$

**Rubi [A]** time = 0.958964, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\begin{aligned} & \frac{(-15a^2e^4 - 6cdex(7cd^2 - 3ae^2) - 12acd^2e^2 + 35c^2d^4)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{240c^2d^2e^3} \\ & + \frac{(3a^2e^4 + 6acd^2e^2 + 7c^2d^4)(cd^2 - ae^2)^3 \tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{256c^{7/2}d^{7/2}e^{9/2}} \\ & - \frac{(3a^2e^4 + 6acd^2e^2 + 7c^2d^4)(cd^2 - ae^2)(ae^2 + cd^2 + 2cdex)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{128c^3d^3e^4} \\ & + \frac{x^2(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{5e} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) / (d + e*x), x]$

[Out]  $-\left((c*d^2 - a*e^2)*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*(c*d^2 + a*e^2 + 2*c*d*e*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]\right) / (128*c^3*d^3*e^4) + (x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) / (5*e) + \left(\left(35*c^2*d^4 - 12*a*c*d^2*e^2 - 15*a^2*e^4 - 6*c*d*e*(7*c*d^2 - 3*a*e^2)*x\right)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}\right) / (240*c^2*d^2*e^3) + \left(\left(c*d^2 - a*e^2\right)^3*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*\text{ArcTanh}\left[\left(c*d^2 + a*e^2 + 2*c*d*e*x\right) / \left(2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]\right)\right]\right) / (256*c^{(7/2)}*d^{(7/2)}*e^{(9/2)})$

**Rubi in Sympy [A]** time = 109.759, size = 354, normalized size = 1.01

$$\frac{x^2 (ade + cdex^2 + x (ae^2 + cd^2))^{\frac{3}{2}}}{5e} - \frac{(ade + cdex^2 + x (ae^2 + cd^2))^{\frac{3}{2}} \left( \frac{15a^2e^4}{4} + 3acd^2e^2 - \frac{35c^2d^4}{4} - \frac{3cdex(3ae^2 - 7cd^2)}{2} \right)}{60c^2d^2e^3} + \frac{(ae^2 - cd^2) (ae^2 + cd^2 + 2cdex) (3a^2e^4 + 6acd^2e^2 + 7c^2d^4) \sqrt{ade + cdex^2 + x (ae^2 + cd^2)}}{128c^3d^3e^4} - \frac{(ae^2 - cd^2)^3 (3a^2e^4 + 6acd^2e^2 + 7c^2d^4) \operatorname{atanh} \left( \frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}} \right)}{256c^{\frac{7}{2}}d^{\frac{7}{2}}e^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d), x)`

[Out] `x**2*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2)/(5*e) - (a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2)*(15*a**2*e**4/4 + 3*a*c*d**2*e**2 - 35*c**2*d**4/4 - 3*c*d*e*x*(3*a*e**2 - 7*c*d**2)/2)/(60*c**2*d**2*e**3) + (a*e**2 - c*d**2)*(a*e**2 + c*d**2 + 2*c*d*e*x)*(3*a**2*e**4 + 6*a*c*d**2*e**2 + 7*c**2*d**4)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(128*c**3*d**3*e**4) - (a*e**2 - c*d**2)**3*(3*a**2*e**4 + 6*a*c*d**2*e**2 + 7*c**2*d**4)*a*tanh((a*e**2 + c*d**2 + 2*c*d*e*x)/(2*sqrt(c)*sqrt(d)*sqrt(e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))))/(256*c**(7/2)*d**(7/2)*e**(9/2))`

**Mathematica [A]** time = 0.5892, size = 320, normalized size = 0.91

$$\frac{\sqrt{(d+ex)(ae+cdx)} \left( \frac{90a^4e^4}{c^3d^3} - \frac{60a^3e^2}{c^2d} + \frac{15(cd^2-ae^2)^3(3a^2e^4+6acd^2e^2+7c^2d^4) \log(2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx+ae^2+cd(d+2ex)})}{c^{7/2}d^{7/2}e^{9/2}\sqrt{d+ex}\sqrt{ae+cdx}} \right) + \frac{16x^2 \left( \frac{3a^2e^2}{c} + 1 \right)}{d}}{3840}$$

3840

Antiderivative was successfully verified.

[In] `Integrate[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x), x]`

[Out] `(Sqrt[(a*e + c*d*x)*(d + e*x)]*((-72*a^2*d)/c - (210*c*d^5)/e^4 + (380*a*d^3)/e^2 - (60*a^3*e^2)/(c^2*d) + (90*a^4*e^4)/(c^3*d^3) + (4*(35*c*d^4 - 61*a*d^2*e^2 + (9*a^2*e^4)/c - (15*a^3*e^6)/(c^2*d^2))*x)/e^3 + (16*(12*a*d^2 - (7*c*d^4)/e^2 + (3*a^2*e^2)/c)*x^2)/d + (96*(c*d^2 + 11*a*e^2)*x^3)/e + 768*c*d*x^4 + (15*(c*d^2 - a*e^2)^3*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*Log[a*e^2 + 2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x] + c*d*(d + 2*e*x)]/(c^(7/2)*d^(7/2)*e^(9/2)*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]))/3840`

**Maple [B]** time = 0.022, size = 1560, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d), x)`

[Out] `1/4*d^2/e*a*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+1/8*d*a^2/c*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+3/16*d^3*a^2`

$$\begin{aligned} & * \ln\left(\frac{1}{2} a^* e^2 - \frac{1}{2} c^* d^2 + (x+d/e) * c^* d^* e\right) / (c^* d^* e)^{1/2} + (c^* d^* e * (x+d/e)^2 + (a^* e^2 - c^* d^2) * (x+d/e))^{1/2} / (c^* d^* e)^{1/2} - 1/8 * d^5 / e^4 * c^* (c^* d^* e * (x+d/e)^2 + (a^* e^2 - c^* d^2) * (x+d/e))^{1/2} + 9/128 / e^4 * c^* d^5 * (a^* d^* e + (a^* e^2 + c^* d^2) * x + c^* d^* e * x^2)^{1/2} - 1/4 / e / c^* (a^* d^* e + (a^* e^2 + c^* d^2) * x + c^* d^* e * x^2)^{3/2} * a - 3/64 / e^2 * d^3 * (a^* d^* e + (a^* e^2 + c^* d^2) * x + c^* d^* e * x^2)^{1/2} * a + 1/5 / e^2 * (a^* d^* e + (a^* e^2 + c^* d^2) * x + c^* d^* e * x^2)^{5/2} / c / d - 3/8 / e^2 * d^2 * (a^* d^* e + (a^* e^2 + c^* d^2) * x + c^* d^* e * x^2)^{3/2} * x - 21/128 * d^3 * \ln\left(\frac{1}{2} a^* e^2 + \frac{1}{2} c^* d^2 + c^* d^* e * x\right) / (c^* d^* e)^{1/2} + (a^* d^* e + (a^* e^2 + c^* d^2) * x + c^* d^* e * x^2)^{1/2} / (c^* d^* e)^{1/2} * a^2 - 3/32 / c^* d^* (a^* d^* e + (a^* e^2 + c^* d^2) * x + c^* d^* e * x^2)^{1/2} * a^2 + 3/256 * e^4 / c^2 / d^* \ln\left(\frac{1}{2} a^* e^2 + \frac{1}{2} c^* d^2 + c^* d^* e * x\right) / (c^* d^* e)^{1/2} + (a^* d^* e + (a^* e^2 + c^* d^2) * x + c^* d^* e * x^2)^{1/2} / (c^* d^* e)^{1/2} * a^4 + 9/128 * e^2 / c^* d^* \ln\left(\frac{1}{2} a^* e^2 + \frac{1}{2} c^* d^2 + c^* d^* e * x\right) / (c^* d^* e)^{1/2} + (a^* d^* e + (a^* e^2 + c^* d^2) * x + c^* d^* e * x^2)^{1/2} / (c^* d^* e)^{1/2} * a^3 + 33/256 / e^2 * c^* d^5 * \ln\left(\frac{1}{2} a^* e^2 + \frac{1}{2} c^* d^2 + c^* d^* e * x\right) / (c^* d^* e)^{1/2} + (a^* d^* e + (a^* e^2 + c^* d^2) * x + c^* d^* e * x^2)^{1/2} / (c^* d^* e)^{1/2} * a - 1/16 * d^2 * a^3 / c^* \ln\left(\frac{1}{2} a^* e^2 - \frac{1}{2} c^* d^2 + (x+d/e) * c^* d^* e\right) / (c^* d^* e)^{1/2} + (c^* d^* e * (x+d/e)^2 + (a^* e^2 - c^* d^2) * (x+d/e))^{1/2} / (c^* d^* e)^{1/2} - 3/16 * d^5 / e^2 * a^* c^* \ln\left(\frac{1}{2} a^* e^2 - \frac{1}{2} c^* d^2 + (x+d/e) * c^* d^* e\right) / (c^* d^* e)^{1/2} + (c^* d^* e * (x+d/e)^2 + (a^* e^2 - c^* d^2) * (x+d/e))^{1/2} / (c^* d^* e)^{1/2} + 3/64 * e^3 / c^2 / d^2 * (a^* d^* e + (a^* e^2 + c^* d^2) * x + c^* d^* e * x^2)^{1/2} * x * a^3 - 3/256 * e^6 / c^3 / d^3 * \ln\left(\frac{1}{2} a^* e^2 + \frac{1}{2} c^* d^2 + c^* d^* e * x\right) / (c^* d^* e)^{1/2} + (a^* d^* e + (a^* e^2 + c^* d^2) * x + c^* d^* e * x^2)^{1/2} / (c^* d^* e)^{1/2} * a^5 + 1/16 * d^7 / e^4 * c^2 * \ln\left(\frac{1}{2} a^* e^2 - \frac{1}{2} c^* d^2 + (x+d/e) * c^* d^* e\right) / (c^* d^* e)^{1/2} + (c^* d^* e * (x+d/e)^2 + (a^* e^2 - c^* d^2) * (x+d/e))^{1/2} / (c^* d^* e)^{1/2} - 15/64 / e^2 * d^2 * (a^* d^* e + (a^* e^2 + c^* d^2) * x + c^* d^* e * x^2)^{1/2} * x * a - 1/8 / c / d^* (a^* d^* e + (a^* e^2 + c^* d^2) * x + c^* d^* e * x^2)^{3/2} * x * a + 3/64 * e^2 / c^2 / d^* (a^* d^* e + (a^* e^2 + c^* d^2) * x + c^* d^* e * x^2)^{1/2} * a^3 - 9/256 / e^4 * c^2 * d^7 * \ln\left(\frac{1}{2} a^* e^2 + \frac{1}{2} c^* d^2 + c^* d^* e * x\right) / (c^* d^* e)^{1/2} + (a^* d^* e + (a^* e^2 + c^* d^2) * x + c^* d^* e * x^2)^{1/2} / (c^* d^* e)^{1/2} - 1/16 * e / c^2 / d^2 * (a^* d^* e + (a^* e^2 + c^* d^2) * x + c^* d^* e * x^2)^{3/2} * a^2 + 9/64 / e^3 * c^* d^4 * (a^* d^* e + (a^* e^2 + c^* d^2) * x + c^* d^* e * x^2)^{1/2} * x + 3/128 * e^4 / c^3 / d^3 * (a^* d^* e + (a^* e^2 + c^* d^2) * x + c^* d^* e * x^2)^{1/2} * a^4 + 3/64 * e / c^* (a^* d^* e + (a^* e^2 + c^* d^2) * x + c^* d^* e * x^2)^{1/2} * x * a^2 - 1/4 * d^4 / e^3 * c^* (c^* d^* e * (x+d/e)^2 + (a^* e^2 - c^* d^2) * (x+d/e))^{1/2} * x + 1/3 * d^2 / e^3 * (c^* d^* e * (x+d/e)^2 + (a^* e^2 - c^* d^2) * (x+d/e))^{3/2} - 3/16 / e^3 * d^2 * (a^* d^* e + (a^* e^2 + c^* d^2) * x + c^* d^* e * x^2)^{3/2} \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*x^2/(e\*x + d), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.330387, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*x^2/(e\*x + d), x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [1/7680 * (4 * (384 * c^4 * d^4 * e^4 * x^4 - 105 * c^4 * d^8 + 190 * a * c^3 * d^6 * e^2 \\ & - 36 * a^2 * c^2 * d^4 * e^4 - 30 * a^3 * c * d^2 * e^6 + 45 * a^4 * e^8 + 48 * (c^4 * d^5 * e^3 + 11 * a * c^3 * d^3 * e^5) * x^3 - 8 * (7 * c^4 * d^6 * e^2 - 12 * a * c^3 * d^4 * e^4 - 3 * a^2 * c^2 * d^2 * e^6) * x^2 + 2 * (35 * c^4 * d^7 * e - 61 * a * c^3 * d^5 * e^3 + 9 * a^2 * c^2 * d^3 * e^5 - 15 * a^3 * c * d * e^7) * x) * \sqrt{c^* d^* e * x^2 + a^* d^* e} \\ & + (c^* d^2 + a^* e^2) * x) * \sqrt{c^* d^* e} - 15 * (7 * c^5 * d^{10} - 15 * a * c^4 * d^8 * e^2 + 6 * a^2 * c^3 * d^6 * e^4 + 2 * a^3 * c^2 * d^4 * e^6 + 3 * a^4 * c * d^2 * e^8 - 3 * a^5 * e^{10}) * \log(-4 * (2 * c^2 * d^2 * e^2 * x + c^2 * d^3 * e + a * c * d * e^3) * \sqrt{c^* d^* e * x^2 + a^* d^* e} + (c^* d^2 + a^* e^2) * x) + (8 * c^2 * d^2 * e^2 * x^2 + c^2 \end{aligned}$$

$$\frac{d^4 + 6ac^2d^2e^2 + a^2e^4 + 8(c^2d^3e + ac^2de^3)x \sqrt{cde}}{\sqrt{cde}c^3d^3e^4}, \frac{1}{3840}(2(384c^4d^4e^4x^4 - 105c^4d^8 + 190ac^3d^6e^2 - 36a^2c^2d^4e^4 - 30a^3c^2d^2e^6 + 45a^4e^8 + 48(c^4d^5e^3 + 11ac^3d^3e^5)x^3 - 8(7c^4d^6e^2 - 12ac^3d^4e^4 - 3a^2c^2d^2e^6)x^2 + 2(35c^4d^7e - 61ac^3d^5e^3 + 9a^2c^2d^3e^5 - 15a^3c^2d^2e^7)x) \sqrt{cde}x^2 + ade + (cd^2 + ae^2)x) \sqrt{-cde} + 15(7c^5d^{10} - 15ac^4d^8e^2 + 6a^2c^3d^6e^4 + 2a^3c^2d^4e^6 + 3a^4c^2d^2e^8 - 3a^5e^{10}) \arctan\left(\frac{1}{2}(2c^2de^2x + cd^2 + ae^2) \sqrt{-cde}\right) / (\sqrt{cde}x^2 + ade + (cd^2 + ae^2)x) \sqrt{cde}) / (\sqrt{-cde}c^3d^3e^4]$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/(e\*x+d),x)

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*x^2/(e\*x + d),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.448 \quad \int \frac{x(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex} dx$$

**Optimal.** Leaf size=295

$$\begin{aligned} & \frac{(3ae^2 + 5cd^2) (cd^2 - ae^2)^3 \tanh^{-1} \left( \frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \right)}{128c^{5/2}d^{5/2}e^{7/2}} \\ & + \frac{(3ae^2 + 5cd^2) (cd^2 - ae^2) (ae^2 + cd^2 + 2cdex) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{64c^2d^2e^3} \\ & + \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{4cde(d + ex)} - \frac{1}{24} \left( \frac{3a}{cd} + \frac{5d}{e^2} \right) (x(ae^2 + cd^2) + ade + cdex^2)^{3/2} \end{aligned}$$

[Out]  $((c*d^2 - a*e^2)*(5*c*d^2 + 3*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x) * \text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (64*c^2*d^2*e^3) - ((3*a)/(c*d) + (5*d)/e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)} / 24 + (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)} / (4*c*d*e*(d + e*x)) - ((c*d^2 - a*e^2)^3*(5*c*d^2 + 3*a*e^2)*\text{ArcTanh}[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]]) / (128*c^{(5/2)}*d^{(5/2)}*e^{(7/2)})$

**Rubi [A]** time = 0.6563, antiderivative size = 295, normalized size of antiderivative = 1., number of rules used = 5, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$

$$\begin{aligned} & \frac{(3ae^2 + 5cd^2) (cd^2 - ae^2)^3 \tanh^{-1} \left( \frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \right)}{128c^{5/2}d^{5/2}e^{7/2}} \\ & + \frac{(3ae^2 + 5cd^2) (cd^2 - ae^2) (ae^2 + cd^2 + 2cdex) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{64c^2d^2e^3} \\ & + \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{4cde(d + ex)} - \frac{1}{24} \left( \frac{3a}{cd} + \frac{5d}{e^2} \right) (x(ae^2 + cd^2) + ade + cdex^2)^{3/2} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(d + e*x), x]$

[Out]  $((c*d^2 - a*e^2)*(5*c*d^2 + 3*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x) * \text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (64*c^2*d^2*e^3) - ((3*a)/(c*d) + (5*d)/e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)} / 24 + (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)} / (4*c*d*e*(d + e*x)) - ((c*d^2 - a*e^2)^3*(5*c*d^2 + 3*a*e^2)*\text{ArcTanh}[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]]) / (128*c^{(5/2)}*d^{(5/2)}*e^{(7/2)})$

**Rubi in Sympy [A]** time = 60.847, size = 275, normalized size = 0.93

$$\begin{aligned} & - \left( \frac{a}{8cd} + \frac{5d}{24e^2} \right) (ade + cdex^2 + x(ae^2 + cd^2))^{\frac{3}{2}} + \frac{(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{5}{2}}}{4cde(d + ex)} \\ & - \frac{(ae^2 - cd^2) (3ae^2 + 5cd^2) (ae^2 + cd^2 + 2cdex) \sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{64c^2d^2e^3} \\ & + \frac{(ae^2 - cd^2)^3 (3ae^2 + 5cd^2) \text{atanh} \left( \frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+cdex^2+x(ae^2+cd^2)}} \right)}{128c^{\frac{5}{2}}d^{\frac{5}{2}}e^{\frac{7}{2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x*(a*d*e+(a*e^2+2*c*d^2)*x+c*d*e*x^2)^{(3/2)}/(e*x+d), x)$

[Out]  $-(a/(8*c*d) + 5*d/(24*e**2))*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2) + (a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(5/2)/(4*c*d*e*(d + e*x)) - (a*e**2 - c*d**2)*(3*a*e**2 + 5*c*d**2)*(a*e**2 + c*d**2 + 2*c*d*e*x)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(64*c**2*d**2*e**3) + (a*e**2 - c*d**2)**3*(3*a*e**2 + 5*c*d**2)*atanh((a*e**2 + c*d**2 + 2*c*d*e*x)/(2*sqrt(c)*sqrt(d)*sqrt(e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))))/(128*c**5/2)*d**5/2)*e**7/2)$

**Mathematica [A]** time = 0.461621, size = 240, normalized size = 0.81

$$\frac{1}{384} \sqrt{(d+ex)(ae+cdx)} \left( -\frac{18a^3e^3}{c^2d^2} + \frac{12a^2e^2x}{cd} + \frac{18a^2e}{c} \right) - \frac{3(cd^2 - ae^2)^3 (3ae^2 + 5cd^2) \log\left(2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx} + ae^2 + cd(d+2ex)\right)}{c^{5/2}d^{5/2}e^{7/2}\sqrt{d+ex}\sqrt{ae+cdx}} + \left( \frac{16x^2(9ae^2 + cd^2)}{e} - \frac{62ad^2}{e} + 40adx + \frac{30cd^4}{e^3} - \frac{20cd^3x}{e^2} + 96cdx^3 \right)$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2))/(d + e\*x), x]

[Out]  $(\text{Sqrt}[(a*e + c*d*x)*(d + e*x)]*((30*c*d^4)/e^3 - (62*a*d^2)/e + (18*a^2*e)/c - (18*a^3*e^3)/(c^2*d^2) + 40*a*d*x - (20*c*d^3*x)/e^2 + (12*a^2*e^2*x)/(c*d) + (16*(c*d^2 + 9*a*e^2)*x^2)/e + 96*c*d*x^3 - (3*(c*d^2 - a*e^2)^3*(5*c*d^2 + 3*a*e^2)*\text{Log}[a*e^2 + 2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x] + c*d*(d + 2*e*x)])/(c^(5/2)*d^(5/2)*e^(7/2)*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]))/384$

**Maple [B]** time = 0.016, size = 1279, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)/(e\*x+d), x)

[Out]  $1/4/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x+1/8/c/d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-3/32*e^2/c/d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+a^2+3/16*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-a-3/32/e^2*c*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-3/64*e^3/c^2/d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3+3/64*e/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2+3/64/e*d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a-3/64/e^3*c*d^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+3/128*e^5/c^2/d^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)*a^4-3/32*e^3/c*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)*a^3+9/64*e*d^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)*a^2-3/32/e*c*d^4*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)+a^3/128/e^3*c^2*d^6*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-1/3*d/e^2*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^3/2-1/4*d*a*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^1/2*x-1/8*e*a^2/c*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^1/2$



$$\begin{aligned}
&)+1/16*e^3*a^3/c*\ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+ \\
&(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2) \\
&-3/16*d^2*e*a^2*\ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+ \\
&(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+ \\
&3/16*d^4/e*a*c*\ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+ \\
&(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+1 \\
&/4*d^3/e^2*c*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+1/8* \\
&d^4/e^3*c*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-1/16*d^6/ \\
&e^3*c^2*\ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+(c*d \\
&*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*x/(e\*x + d), x, algorithm=

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.323182, size = 1, normalized size = 0.

$$\left[ \frac{4(48c^3d^3e^3x^3 + 15c^3d^6 - 31ac^2d^4e^2 + 9a^2cd^2e^4 - 9a^3e^6 + 8(c^3d^4e^2 + 9ac^2d^2e^4)x^2 - 2(5c^3d^5e - 10ac^2d^3e^3 - 3a^2cde^5)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*x/(e\*x + d), x, algorithm=

[Out] [1/768\*(4\*(48\*c^3\*d^3\*e^3\*x^3 + 15\*c^3\*d^6 - 31\*a\*c^2\*d^4\*e^2 + 9\*a^2\*c\*d^2\*e^4 - 9\*a^3\*e^6 + 8\*(c^3\*d^4\*e^2 + 9\*a\*c^2\*d^2\*e^4)\*x^2 - 2\*(5\*c^3\*d^5\*e - 10\*a\*c^2\*d^3\*e^3 - 3\*a^2\*c\*d\*e^5)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*e) - 3\*(5\*c^4\*d^8 - 12\*a\*c^3\*d^6\*e^2 + 6\*a^2\*c^2\*d^4\*e^4 + 4\*a^3\*c\*d^2\*e^6 - 3\*a^4\*e^8)\*log(4\*(2\*c^2\*d^2\*e^2\*x + c^2\*d^3\*e + a\*c\*d\*e^3)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) + (8\*c^2\*d^2\*e^2\*x^2 + c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4 + 8\*(c^2\*d^3\*e + a\*c\*d\*e^3)\*x)\*sqrt(c\*d\*e))/(sqrt(c\*d\*e)\*c^2\*d^2\*e^3), 1/384\*(2\*(48\*c^3\*d^3\*e^3\*x^3 + 15\*c^3\*d^6 - 31\*a\*c^2\*d^4\*e^2 + 9\*a^2\*c\*d^2\*e^4 - 9\*a^3\*e^6 + 8\*(c^3\*d^4\*e^2 + 9\*a\*c^2\*d^2\*e^4)\*x^2 - 2\*(5\*c^3\*d^5\*e - 10\*a\*c^2\*d^3\*e^3 - 3\*a^2\*c\*d\*e^5)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*e) - 3\*(5\*c^4\*d^8 - 12\*a\*c^3\*d^6\*e^2 + 6\*a^2\*c^2\*d^4\*e^4 + 4\*a^3\*c\*d^2\*e^6 - 3\*a^4\*e^8)\*arctan(1/2\*(2\*c\*d\*e\*x + c\*d^2 + a\*e^2)\*sqrt(-c\*d\*e)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*c\*d\*e))/(sqrt(-c\*d\*e)\*c^2\*d^2\*e^3)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/(e\*x+d), x)

[Out] Timed out

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*x/(e*x + d), x, algorithm=`

[Out] Exception raised: TypeError

$$3.449 \quad \int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{d+ex} dx$$

**Optimal.** Leaf size=201

$$\frac{(cd^2 - ae^2)^3 \tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{16c^{3/2}d^{3/2}e^{5/2}} + \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{3e} + \frac{1}{8} \left(\frac{a}{cd} - \frac{d}{e^2}\right) (ae^2 + cd^2 + 2cdex) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}$$

[Out] ((a/(c\*d) - d/e^2)\*(c\*d^2 + a\*e^2 + 2\*c\*d\*e\*x)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/8 + (a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/(3\*e) + ((c\*d^2 - a\*e^2)^3\*ArcTanh[(c\*d^2 + a\*e^2 + 2\*c\*d\*e\*x)/(2\*Sqrt[c]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]))/(16\*c^(3/2)\*d^(3/2)\*e^(5/2))

**Rubi [A]** time = 0.263776, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 37,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$

$$\frac{(cd^2 - ae^2)^3 \tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{16c^{3/2}d^{3/2}e^{5/2}} + \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{3e} + \frac{1}{8} \left(\frac{a}{cd} - \frac{d}{e^2}\right) (ae^2 + cd^2 + 2cdex) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}$$

Antiderivative was successfully verified.

[In] Int[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/(d + e\*x), x]

[Out] ((a/(c\*d) - d/e^2)\*(c\*d^2 + a\*e^2 + 2\*c\*d\*e\*x)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/8 + (a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/(3\*e) + ((c\*d^2 - a\*e^2)^3\*ArcTanh[(c\*d^2 + a\*e^2 + 2\*c\*d\*e\*x)/(2\*Sqrt[c]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]))/(16\*c^(3/2)\*d^(3/2)\*e^(5/2))

**Rubi in Sympy [A]** time = 39.1372, size = 187, normalized size = 0.93

$$-\left(-\frac{a}{8cd} + \frac{d}{8e^2}\right) (ae^2 + cd^2 + 2cdex) \sqrt{ade + cdex^2 + x(ae^2 + cd^2)} + \frac{(ade + cdex^2 + x(ae^2 + cd^2))^{3/2}}{3e} - \frac{(ae^2 - cd^2)^3 \operatorname{atanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}\right)}{16c^{3/2}d^{3/2}e^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/(e\*x+d), x)

[Out] -(-a/(8\*c\*d) + d/(8\*e\*\*2))\*(a\*e\*\*2 + c\*d\*\*2 + 2\*c\*d\*e\*x)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2)) + (a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*\* (3/2)/(3\*e) - (a\*e\*\*2 - c\*d\*\*2)\*\*3\*atanh((a\*e\*\*2 + c\*d\*\*2 + 2\*c\*d\*e\*x)/(2\*sqrt(c)\*sqrt(d)\*sqrt(e)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))))/(16\*c\*\* (3/2)\*d\*\* (3/2)\*e\*\* (5/2))

**Mathematica [A]** time = 0.308849, size = 178, normalized size = 0.89

$$\frac{1}{48} \sqrt{(d+ex)(ae+cdx)} \left( \frac{6a^2e^2}{cd} + \frac{3(cd^2 - ae^2)^3 \log\left(2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx} + ae^2 + cd(d+2ex)\right)}{c^{3/2}d^{3/2}e^{5/2}\sqrt{d+ex}\sqrt{ae+cdx}} + 16ad + 28aex - \frac{6cd^3}{e^2} + \frac{4cd^2x}{e} + 16cdx^2 \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/(d + e\*x), x]

[Out] (Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)]\*(16\*a\*d - (6\*c\*d^3)/e^2 + (6\*a^2\*e^2)/(c\*d) + (4\*c\*d^2\*x)/e + 28\*a\*e\*x + 16\*c\*d\*x^2 + (3\*(c\*d^2 - a\*e^2)^3\*Log[a\*e^2 + 2\*Sqrt[c]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x] + c\*d\*(d + 2\*e\*x)])/(c^(3/2)\*d^(3/2)\*e^(5/2)\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]))/48

**Maple [B]** time = 0.009, size = 566, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)/(e\*x+d), x)

[Out] 1/3/e\*(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(3/2)+1/4\*e\*a\*(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2)\*x+1/8\*e^2\*a^2/c/d\*(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2)-1/16\*e^4\*a^3/c/d\*ln((1/2\*a\*e^2-1/2\*c\*d^2+(x+d/e)\*c\*d\*e)/(c\*d\*e)^(1/2)+(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2))/(c\*d\*e)^(1/2)+3/16\*e^2\*a^2\*d\*ln((1/2\*a\*e^2-1/2\*c\*d^2+(x+d/e)\*c\*d\*e)/(c\*d\*e)^(1/2)+(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2))/(c\*d\*e)^(1/2)-3/16\*a\*c\*d^3\*ln((1/2\*a\*e^2-1/2\*c\*d^2+(x+d/e)\*c\*d\*e)/(c\*d\*e)^(1/2)+(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2))/(c\*d\*e)^(1/2)-1/4/e\*c\*d^2\*(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2)\*x-1/8/e^2\*c\*d^3\*(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2)+1/16/e^2\*c^2\*d^5\*ln((1/2\*a\*e^2-1/2\*c\*d^2+(x+d/e)\*c\*d\*e)/(c\*d\*e)^(1/2)+(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2))/(c\*d\*e)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/(e\*x + d), x, algorithm="m

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.303205, size = 1, normalized size = 0.

$$\frac{4(8c^2d^2e^2x^2 - 3c^2d^4 + 8acd^2e^2 + 3a^2e^4 + 2(c^2d^3e + 7acde^3)x)\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{cde} - 3(c^3d^6 - 3ac^2d^4e^2)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/(e\*x + d), x, algorithm="f

[Out] [1/96\*(4\*(8\*c^2\*d^2\*e^2\*x^2 - 3\*c^2\*d^4 + 8\*a\*c\*d^2\*e^2 + 3\*a^2\*e^4 + 2\*(c^2\*d^3\*e + 7\*a\*c\*d\*e^3)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*e) - 3\*(c^3\*d^6 - 3\*a\*c^2\*d^4\*e^2 + 3\*a^2\*c\*d^2\*e^4 - a^3\*e^6)\*log(-4\*(2\*c^2\*d^2\*e^2\*x + c^2\*d^3\*e + a\*c\*d\*e^3)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) + (8\*c^2\*d^2\*e^2\*x^2 + c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4 + 8\*(c^2\*d^3\*e + a\*c\*d\*e^3)\*x)\*sqrt(c\*d\*e)))/(sqrt(c\*d\*e)\*c\*d\*e^2), 1/48\*(2\*(8\*c^2\*d^2\*e^2\*x^2 - 3\*c^2\*d^4 + 8\*a\*c\*d^2\*e^2 + 3\*a^2\*e^4 + 2\*(c^2\*d^3\*e + 7\*a\*c\*d\*e^3)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*e) + 3\*(c^3\*d^6 - 3\*a\*c^2\*d^4\*e^2 + 3\*a^2\*c\*d^2\*e^4 - a^3\*e^6)\*arctan(1/2\*(2\*c\*d\*e\*x + c\*d^2 + a\*e^2)\*sqrt(-c\*d\*e)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*c\*d\*e)))/(sqrt(-c\*d\*e)\*c\*d\*e^2)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/(e\*x+d), x)

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/(e\*x + d), x, algorithm="g

[Out] Exception raised: TypeError

$$3.450 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x(d+ex)} dx$$

**Optimal.** Leaf size=251

$$\begin{aligned} & -a^{3/2}\sqrt{de}^{3/2} \tanh^{-1}\left(\frac{x(ae^2 + cd^2) + 2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right) \\ & - \frac{(-3a^2e^4 - 6acd^2e^2 + c^2d^4) \tanh^{-1}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{8\sqrt{c}\sqrt{de}^{3/2}} \\ & + \frac{(5ae^2 + cd^2 + 2cdex) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{4e} \end{aligned}$$

[Out]  $((c*d^2 + 5*a*e^2 + 2*c*d*e*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*e) - ((c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*\text{ArcTanh}[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(8*\text{Sqrt}[c]*\text{Sqrt}[d]*e^{(3/2)}) - a^{(3/2)}*\text{Sqrt}[d]*e^{(3/2)}*\text{ArcTanh}[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])$

**Rubi [A]** time = 0.859558, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\begin{aligned} & -a^{3/2}\sqrt{de}^{3/2} \tanh^{-1}\left(\frac{x(ae^2 + cd^2) + 2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right) \\ & - \frac{(-3a^2e^4 - 6acd^2e^2 + c^2d^4) \tanh^{-1}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{8\sqrt{c}\sqrt{de}^{3/2}} \\ & + \frac{(5ae^2 + cd^2 + 2cdex) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{4e} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/(x*(d + e*x)), x]$

[Out]  $((c*d^2 + 5*a*e^2 + 2*c*d*e*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*e) - ((c^2*d^4 - 6*a*c*d^2*e^2 - 3*a^2*e^4)*\text{ArcTanh}[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(8*\text{Sqrt}[c]*\text{Sqrt}[d]*e^{(3/2)}) - a^{(3/2)}*\text{Sqrt}[d]*e^{(3/2)}*\text{ArcTanh}[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])$

**Rubi in Sympy [A]** time = 104.539, size = 245, normalized size = 0.98

$$\begin{aligned} & -a^{\frac{3}{2}}\sqrt{de}^{\frac{3}{2}} \operatorname{atanh}\left(\frac{2ade + x(ae^2 + cd^2)}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}\right) \\ & + \frac{\left(\frac{5ae^2}{2} + \frac{cd^2}{2} + cdex\right) \sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{2e} \\ & + \frac{(3a^2e^4 + 6acd^2e^2 - c^2d^4) \operatorname{atanh}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}\right)}{8\sqrt{c}\sqrt{de}^{\frac{3}{2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x/(e*x+d), x)`

[Out] 
$$-a^{3/2} \sqrt{d} e^{3/2} \operatorname{atanh}\left(\frac{2a d e + x(a e^2 + c d^2)}{2\sqrt{a} \sqrt{d} \sqrt{e} \sqrt{a d e + c d^2 e x^2 + x(a e^2 + c d^2)}}\right) + (5a e^{2/2} + c d^{2/2} + c d^2 e x) \sqrt{a d e + c d^2 e x^2 + x(a e^2 + c d^2)} / (2e) + (3a^2 e^4 + 6a c d^2 e^2 - c^2 d^4) \operatorname{atanh}\left(\frac{a e^2 + c d^2 + 2c d^2 e x}{2\sqrt{c} \sqrt{d} \sqrt{e} \sqrt{a d e + c d^2 e x^2 + x(a e^2 + c d^2)}}\right) / (8\sqrt{c} \sqrt{d} e^{3/2})$$

**Mathematica [A]** time = 0.680792, size = 285, normalized size = 1.14

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(-8a^{3/2}\sqrt{cde^3}\log\left(2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}+ae(2d+ex)+cd^2x\right)+8a^{3/2}\sqrt{cde^3}\log(x)-(-3a^2e^4-8\sqrt{c}\sqrt{d}e^{3/2}\sqrt{\dots}\right)}{8\sqrt{c}\sqrt{d}e^{3/2}\sqrt{\dots}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^(3/2)/(x*(d+e*x)), x]`

[Out] 
$$\left(\sqrt{a e + c d^2 x} \sqrt{d + e x} \left(2 \sqrt{c} \sqrt{d} \sqrt{e} \sqrt{a e + c d^2 x} \sqrt{d + e x} \left(5 a^2 e^2 + c^2 d (d + 2 e x)\right) + 8 a^{3/2} \sqrt{c} d^2 e^3 \operatorname{Log}[x] - 8 a^{3/2} \sqrt{c} d^2 e^3 \operatorname{Log}[c d^2 x + 2 \sqrt{a} \sqrt{d} \sqrt{e} \sqrt{a e + c d^2 x} \sqrt{d + e x} + a e^2 (d + e x)] - (c^2 d^4 - 6 a c d^2 e^2 - 3 a^2 e^4) \operatorname{Log}[a e^2 + 2 \sqrt{c} \sqrt{d} \sqrt{e} \sqrt{a e + c d^2 x} \sqrt{d + e x} + c^2 d (d + 2 e x)]\right)\right) / (8 \sqrt{c} \sqrt{d} e^{3/2} \sqrt{(a e + c d^2 x) (d + e x)})$$

**Maple [B]** time = 0.02, size = 1130, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x/(e*x+d), x)`

[Out] 
$$\frac{1}{3} d (a d e + (a e^2 + c d^2) x + c d^2 e x^2)^{3/2} + \frac{1}{4} d a e^2 (a d e + (a e^2 + c d^2) x + c d^2 e x^2)^{1/2} + \frac{1}{8} d^2 a^2 e^3 / c (a d e + (a e^2 + c d^2) x + c d^2 e x^2)^{1/2} - \frac{1}{16} d^2 a^3 e^5 / c \ln\left(\frac{(1/2 a e^2 + 1/2 c d^2 + c d^2 e x) / (c d^2 e)^{1/2} + (a d e + (a e^2 + c d^2) x + c d^2 e x^2)^{1/2}}{(c d^2 e)^{1/2} + (a d e + (a e^2 + c d^2) x + c d^2 e x^2)^{1/2}}\right) + \frac{9}{16} a^2 e^3 \ln\left(\frac{(1/2 a e^2 + 1/2 c d^2 + c d^2 e x) / (c d^2 e)^{1/2} + (a d e + (a e^2 + c d^2) x + c d^2 e x^2)^{1/2}}{(c d^2 e)^{1/2} + (a d e + (a e^2 + c d^2) x + c d^2 e x^2)^{1/2}}\right) + \frac{9}{16} d^2 a e^2 c \ln\left(\frac{(1/2 a e^2 + 1/2 c d^2 + c d^2 e x) / (c d^2 e)^{1/2} + (a d e + (a e^2 + c d^2) x + c d^2 e x^2)^{1/2}}{(c d^2 e)^{1/2} + (a d e + (a e^2 + c d^2) x + c d^2 e x^2)^{1/2}}\right) + \frac{1}{4} d^2 c (a d e + (a e^2 + c d^2) x + c d^2 e x^2)^{1/2} + \frac{1}{8} d^2 c / e (a d e + (a e^2 + c d^2) x + c d^2 e x^2)^{1/2} - \frac{1}{16} d^4 c^2 / e \ln\left(\frac{(1/2 a e^2 + 1/2 c d^2 + c d^2 e x) / (c d^2 e)^{1/2} + (a d e + (a e^2 + c d^2) x + c d^2 e x^2)^{1/2}}{(c d^2 e)^{1/2} + (a d e + (a e^2 + c d^2) x + c d^2 e x^2)^{1/2}}\right) - d^2 a^2 e^2 / (a d e)^{1/2} \ln\left(\frac{(2 a d e + (a e^2 + c d^2) x + 2 (a d e)^{1/2} (a d e + (a e^2 + c d^2) x + c d^2 e x^2)^{1/2}) / x}{(c d^2 e)^{1/2} + (a d e + (a e^2 + c d^2) x + c d^2 e x^2)^{1/2}}\right) - \frac{1}{3} d^2 (c d^2 e (x + d / e)^2 + (a e^2 - c d^2) (x + d / e))^{3/2} - \frac{1}{4} d^2 a e^2 (c d^2 e (x + d / e)^2 + (a e^2 - c d^2) (x + d / e))^{1/2} + \frac{1}{8} d^2 a^2 e^3 / c (c d^2 e (x + d / e)^2 + (a e^2 - c d^2) (x + d / e))^{1/2} + \frac{1}{16} d^2 a^3 e^5 / c \ln\left(\frac{(1/2 a e^2 - 1/2 c d^2 + (x + d / e) c d^2 e) / (c d^2 e)^{1/2} + (c d^2 e (x + d / e)^2 + (a e^2 - c d^2) (x + d / e))^{1/2}}{(c d^2 e)^{1/2} + (c d^2 e (x + d / e)^2 + (a e^2 - c d^2) (x + d / e))^{1/2}}\right) + \frac{3}{16} d^2 a e^2 c \ln\left(\frac{(1/2 a e^2 - 1/2 c d^2 + (x + d / e) c d^2 e) / (c d^2 e)^{1/2} + (c d^2 e (x + d / e)^2 + (a e^2 - c d^2) (x + d / e))^{1/2}}{(c d^2 e)^{1/2} + (c d^2 e (x + d / e)^2 + (a e^2 - c d^2) (x + d / e))^{1/2}}\right) + \frac{1}{4} d^2 c (c d^2 e (x + d / e)^2 + (a e^2 - c d^2) (x + d / e))^{1/2} + \frac{1}{8} d^2 c / e (c d^2 e (x + d / e)^2 + (a e^2 - c d^2) (x + d / e))^{1/2} - \frac{1}{16} d^4 c^2 / e \ln\left(\frac{(1/2 a e^2 - 1/2 c d^2 + (x + d / e) c d^2 e) / (c d^2 e)^{1/2} + (c d^2 e (x + d / e)^2 + (a e^2 - c d^2) (x + d / e))^{1/2}}{(c d^2 e)^{1/2} + (c d^2 e (x + d / e)^2 + (a e^2 - c d^2) (x + d / e))^{1/2}}\right)$$

$$*d*e)^{(1/2)}+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{(1/2)})/(c*d*e)^{(1/2)}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)\*x), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 4.7552, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)\*x), x, algorithm="fricas")

[Out] [1/16\*(8\*sqrt(a\*d\*e)\*sqrt(c\*d\*e)\*a\*e^2\*log((8\*a^2\*d^2\*e^2 + (c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4)\*x^2 - 4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(a\*d\*e) + 8\*(a\*c\*d^3\*e + a^2\*d\*e^3)\*x)/x^2) + 4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(2\*c\*d\*e\*x + c\*d^2 + 5\*a\*e^2)\*sqrt(c\*d\*e) - (c^2\*d^4 - 6\*a\*c\*d^2\*e^2 - 3\*a^2\*e^4)\*log(4\*(2\*c^2\*d^2\*e^2\*x + c^2\*d^3\*e + a\*c\*d\*e^3)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) + (8\*c^2\*d^2\*e^2\*x^2 + c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4 + 8\*(c^2\*d^3\*e + a\*c\*d\*e^3)\*x)\*sqrt(c\*d\*e)))/(sqrt(c\*d\*e)\*e), 1/8\*(4\*sqrt(a\*d\*e)\*sqrt(-c\*d\*e)\*a\*e^2\*log((8\*a^2\*d^2\*e^2 + (c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4)\*x^2 - 4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(a\*d\*e) + 8\*(a\*c\*d^3\*e + a^2\*d\*e^3)\*x)/x^2) + 2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(2\*c\*d\*e\*x + c\*d^2 + 5\*a\*e^2)\*sqrt(-c\*d\*e) - (c^2\*d^4 - 6\*a\*c\*d^2\*e^2 - 3\*a^2\*e^4)\*arctan(1/2\*(2\*c\*d\*e\*x + c\*d^2 + a\*e^2)\*sqrt(-c\*d\*e))/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*c\*d\*e)))/(sqrt(-c\*d\*e)\*e), -1/16\*(16\*sqrt(-a\*d\*e)\*sqrt(c\*d\*e)\*a\*e^2\*arctan(1/2\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e))) - 4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(2\*c\*d\*e\*x + c\*d^2 + 5\*a\*e^2)\*sqrt(c\*d\*e) + (c^2\*d^4 - 6\*a\*c\*d^2\*e^2 - 3\*a^2\*e^4)\*log(4\*(2\*c^2\*d^2\*e^2\*x + c^2\*d^3\*e + a\*c\*d\*e^3)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) + (8\*c^2\*d^2\*e^2\*x^2 + c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4 + 8\*(c^2\*d^3\*e + a\*c\*d\*e^3)\*x)\*sqrt(c\*d\*e)))/(sqrt(c\*d\*e)\*e), -1/8\*(8\*sqrt(-a\*d\*e)\*sqrt(-c\*d\*e)\*a\*e^2\*arctan(1/2\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e))) - 2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(2\*c\*d\*e\*x + c\*d^2 + 5\*a\*e^2)\*sqrt(-c\*d\*e) + (c^2\*d^4 - 6\*a\*c\*d^2\*e^2 - 3\*a^2\*e^4)\*arctan(1/2\*(2\*c\*d\*e\*x + c\*d^2 + a\*e^2)\*sqrt(-c\*d\*e))/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*c\*d\*e)))/(sqrt(-c\*d\*e)\*e)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/x/(e\*x+d), x)



[Out] Timed out

---

GIAC/XCAS [A] time = 0.710723, size = 4, normalized size = 0.02

$sage_0x$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x), x, algorithm="sage0x")`

[Out]  $sage_0x$

$$3.451 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^2(d+ex)} dx$$

**Optimal.** Leaf size=240

$$\begin{aligned} & - \frac{\sqrt{x(ae^2 + cd^2) + ade + cdex^2}(ae - cdx)}{x} \\ & + \frac{\sqrt{c}\sqrt{d}(3ae^2 + cd^2) \tanh^{-1}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{2\sqrt{e}} \\ & - \frac{\sqrt{a}\sqrt{e}(ae^2 + 3cd^2) \tanh^{-1}\left(\frac{x(ae^2 + cd^2) + 2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{2\sqrt{d}} \end{aligned}$$

[Out] -(((a\*e - c\*d\*x)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/x) + (Sqrt[c]\*Sqrt[d]\*(c\*d^2 + 3\*a\*e^2)\*ArcTanh[(c\*d^2 + a\*e^2 + 2\*c\*d\*e\*x)/(2\*Sqrt[c]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])]/(2\*Sqrt[e]) - (Sqrt[a]\*Sqrt[e]\*(3\*c\*d^2 + a\*e^2)\*ArcTanh[(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(2\*Sqrt[a]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])]/(2\*Sqrt[d]))

**Rubi [A]** time = 0.856779, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\begin{aligned} & - \frac{\sqrt{x(ae^2 + cd^2) + ade + cdex^2}(ae - cdx)}{x} \\ & + \frac{\sqrt{c}\sqrt{d}(3ae^2 + cd^2) \tanh^{-1}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{2\sqrt{e}} \\ & - \frac{\sqrt{a}\sqrt{e}(ae^2 + 3cd^2) \tanh^{-1}\left(\frac{x(ae^2 + cd^2) + 2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{2\sqrt{d}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/(x^2\*(d + e\*x)), x]

[Out] -(((a\*e - c\*d\*x)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/x) + (Sqrt[c]\*Sqrt[d]\*(c\*d^2 + 3\*a\*e^2)\*ArcTanh[(c\*d^2 + a\*e^2 + 2\*c\*d\*e\*x)/(2\*Sqrt[c]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])]/(2\*Sqrt[e]) - (Sqrt[a]\*Sqrt[e]\*(3\*c\*d^2 + a\*e^2)\*ArcTanh[(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(2\*Sqrt[a]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])]/(2\*Sqrt[d]))

**Rubi in Sympy [A]** time = 83.9138, size = 228, normalized size = 0.95

$$\begin{aligned} & - \frac{\sqrt{a}\sqrt{e}(ae^2 + 3cd^2) \operatorname{atanh}\left(\frac{2ade + x(ae^2 + cd^2)}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}\right)}{2\sqrt{d}} \\ & + \frac{\sqrt{c}\sqrt{d}(3ae^2 + cd^2) \operatorname{atanh}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}\right)}{2\sqrt{e}} \\ & - \frac{(ae - cdx)\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{x} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**2/(e*x+d), x)`

[Out] 
$$-\sqrt{a} \sqrt{e} (a e^2 + 3 c d^2) \operatorname{atanh}\left(\frac{2 a d e + x(a e^2 + c d^2)}{2 \sqrt{a} \sqrt{d} \sqrt{e} \sqrt{a d e + c d^2 e x^2 + x(a e^2 + c d^2)}}\right) / (2 \sqrt{a} \sqrt{d} \sqrt{e} \sqrt{a d e + c d^2 e x^2 + x(a e^2 + c d^2)}) + \sqrt{c} \sqrt{d} (3 a e^2 + c d^2) \operatorname{atanh}\left(\frac{a e^2 + c d^2 + 2 c d e x}{2 \sqrt{c} \sqrt{d} \sqrt{e} \sqrt{a d e + c d^2 e x^2 + x(a e^2 + c d^2)}}\right) / (2 \sqrt{c} \sqrt{d} \sqrt{e} \sqrt{a d e + c d^2 e x^2 + x(a e^2 + c d^2)}) - (a e - c d x) \sqrt{a d e + c d^2 e x^2 + x(a e^2 + c d^2)} / x$$

**Mathematica [A]** time = 0.561272, size = 356, normalized size = 1.48

$$\frac{\sqrt{a} \sqrt{e} \log(x) (a e^2 + 3 c d^2) \sqrt{(d + e x)(a e + c d x)}}{2 \sqrt{d} \sqrt{d + e x} \sqrt{a e + c d x}} - \frac{\sqrt{a} \sqrt{e} (a e^2 + 3 c d^2) \sqrt{(d + e x)(a e + c d x)} \log\left(2 \sqrt{a} \sqrt{d} \sqrt{e} \sqrt{d + e x} \sqrt{a e + c d x} + 2 a d e + a e^2 x + c d^2 x\right)}{2 \sqrt{d} \sqrt{d + e x} \sqrt{a e + c d x}} + \frac{\sqrt{c} \sqrt{d} (3 a e^2 + c d^2) \sqrt{(d + e x)(a e + c d x)} \log\left(2 \sqrt{c} \sqrt{d} \sqrt{e} \sqrt{d + e x} \sqrt{a e + c d x} + a e^2 + c d^2 + 2 c d e x\right)}{2 \sqrt{e} \sqrt{d + e x} \sqrt{a e + c d x}} + \sqrt{(d + e x)(a e + c d x)} \left(c d - \frac{a e}{x}\right)$$

Antiderivative was successfully verified.

[In] `Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^2*(d + e*x)), x]`

[Out] 
$$(c d - (a e) / x) \operatorname{Sqrt}[a e + c d x] \operatorname{Sqrt}[d + e x] + (\operatorname{Sqrt}[a] \operatorname{Sqrt}[e] (3 c d^2 + a e^2) \operatorname{Sqrt}[a e + c d x] \operatorname{Sqrt}[d + e x] \operatorname{Log}[x]) / (2 \operatorname{Sqrt}[d] \operatorname{Sqrt}[a e + c d x] \operatorname{Sqrt}[d + e x]) - (\operatorname{Sqrt}[a] \operatorname{Sqrt}[e] (3 c d^2 + a e^2) \operatorname{Sqrt}[a e + c d x] \operatorname{Sqrt}[d + e x] \operatorname{Log}[2 a d e + c d^2 x + a e^2 x + 2 \operatorname{Sqrt}[a] \operatorname{Sqrt}[d] \operatorname{Sqrt}[e] \operatorname{Sqrt}[a e + c d x] \operatorname{Sqrt}[d + e x]]) / (2 \operatorname{Sqrt}[d] \operatorname{Sqrt}[a e + c d x] \operatorname{Sqrt}[d + e x]) + (\operatorname{Sqrt}[c] \operatorname{Sqrt}[d] (c d^2 + 3 a e^2) \operatorname{Sqrt}[a e + c d x] \operatorname{Sqrt}[d + e x] \operatorname{Log}[c d^2 + a e^2 x + 2 c d e x + 2 \operatorname{Sqrt}[c] \operatorname{Sqrt}[d] \operatorname{Sqrt}[e] \operatorname{Sqrt}[a e + c d x] \operatorname{Sqrt}[d + e x]]) / (2 \operatorname{Sqrt}[e] \operatorname{Sqrt}[a e + c d x] \operatorname{Sqrt}[d + e x])$$

**Maple [B]** time = 0.023, size = 1310, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^2/(e*x+d), x)`

[Out] 
$$7/16 d^3 c^2 \ln\left(\frac{(1/2 a e^2 + 1/2 c d^2 + c d e x)}{(c d e)^{1/2}} + (a d e + (a e^2 + c d^2) x + c d e x^2)^{1/2}\right) / (c d e)^{1/2} - 1/2 a^2 e^3 / (a d e)^{1/2} \ln\left(\frac{2 a d e + (a e^2 + c d^2) x + 2 (a d e)^{1/2} (a d e + (a e^2 + c d^2) x + c d e x^2)^{1/2}}{x}\right) + 1/a e (a d e + (a e^2 + c d^2) x + c d e x^2)^{3/2} c + 5/4 e (a d e + (a e^2 + c d^2) x + c d e x^2)^{1/2} x c + 1/d a e^2 (a d e + (a e^2 + c d^2) x + c d e x^2)^{1/2} - 1/4 e^2 c (c d e (x+d/e)^2 + (a e^2 - c d^2) (x+d/e))^{1/2} x + 1/16 d^3 c^2 \ln\left(\frac{(1/2 a e^2 - 1/2 c d^2 + (x+d/e) c d e)}{(c d e)^{1/2}} + (c d e (x+d/e)^2 + (a e^2 - c d^2) (x+d/e))^{1/2}\right) / (c d e)^{1/2} + 1/16 e^6 / d^3 a^3 / c \ln\left(\frac{(1/2 a e^2 + 1/2 c d^2 + c d e x)}{(c d e)^{1/2}} + (a d e + (a e^2 + c d^2) x + c d e x^2)^{1/2}\right) / (c d e)^{1/2} - 1/16 e^6 / d^3 a^3 / c \ln\left(\frac{(1/2 a e^2 - 1/2 c d^2 + (x+d/e) c d e)}{(c d e)^{1/2}} + (c d e (x+d/e)^2 + (a e^2 - c d^2) (x+d/e))^{1/2}\right) / (c d e)^{1/2} + (c d e (x+d/e)^2 + (a e^2 - c d^2) (x+d/e))^{1/2} / (c d e)^{1/2} - 3/16 e^2 d^2 a^2 c \ln\left(\frac{(1/2 a e^2 - 1/2 c d^2 + (x+d/e) c d e)}{(c d e)^{1/2}} + (c d e (x+d/e)^2 + (a e^2 - c d^2) (x+d/e))^{1/2}\right) / (c d e)^{1/2} + 27/16 d^2 a e^2 \ln\left(\frac{(1/2 a e^2 + 1/2 c d^2 + c d e x)}{(c d e)^{1/2}} + (a d e + (a e^2 + c d^2) x + c d e x^2)^{1/2}\right) / (c d e)^{1/2} + (a d e + (a e^2 + c d^2) x + c d e x^2)^{1/2} / (c d e)^{1/2} c - 3/2 d^2 a e / (a d e)^{1/2} \ln\left(\frac{2 a d e + (a e^2 + c d^2) x + c d e x^2}{(c d e)^{1/2}}\right)$$

$$\begin{aligned}
& +c*d^2)*x+2*(a*d*e)^{(1/2)}*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)} \\
& )/x)*c-1/4*e^3/d^2*a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)}*x-1/ \\
& 8*e^4/d^3*a^2/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)}+3/16*e^4/ \\
& d*a^2*\ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^{(1/2)}+(c*d*e \\
& *(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{(1/2)})/(c*d*e)^{(1/2)}-3/16/d*a^2 \\
& *e^4*\ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^{(1/2)}+(a*d*e+(a*e^2 \\
& +c*d^2)*x+c*d*e*x^2)^{(1/2)})/(c*d*e)^{(1/2)}-1/d^2/a/e/x*(a*d*e+(a*e \\
& ^2+c*d^2)*x+c*d*e*x^2)^{(5/2)}+1/4*e^3/d^2*a*(c*d*e*(x+d/e)^2+(a*e \\
& ^2-c*d^2)*(x+d/e))^{(1/2)}*x+1/8*e^4/d^3*a^2/c*(c*d*e*(x+d/e)^2+(a*e \\
& ^2-c*d^2)*(x+d/e))^{(1/2)}+1/d*c/a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2 \\
& )^{(3/2)}*x+1/3*e/d^2*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{(3/2)} \\
& -1/8*d*c*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{(1/2)}+2/3/d^2*e \\
& (a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(3/2)}+17/8*d*(a*d*e+(a*e^2+c*d \\
& ^2)*x+c*d*e*x^2)^{(1/2)}*c
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)\*x^2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.70358, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)\*x^2), x, algorithm="fricas")

[Out] [1/4\*((c\*d^2 + 3\*a\*e^2)\*sqrt(c\*d/e)\*x\*log(8\*c^2\*d^2\*e^2\*x^2 + c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4 + 4\*(2\*c\*d\*e^2\*x + c\*d^2\*e + a\*e^3)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d/e) + 8\*(c^2\*d^3\*e + a\*c\*d\*e^3)\*x) + (3\*c\*d^2 + a\*e^2)\*sqrt(a\*e/d)\*x\*log((8\*a^2\*d^2\*e^2 + (c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4)\*x^2 - 4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(2\*a\*d^2\*e + (c\*d^3 + a\*d\*e^2)\*x)\*sqrt(a\*e/d) + 8\*(a\*c\*d^3\*e + a^2\*d\*e^3)\*x)/x^2) + 4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(c\*d\*x - a\*e))/x, 1/4\*(2\*(c\*d^2 + 3\*a\*e^2)\*sqrt(-c\*d/e)\*x\*arctan(1/2\*(2\*c\*d\*e\*x + c\*d^2 + a\*e^2)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d/e)\*e)) + (3\*c\*d^2 + a\*e^2)\*sqrt(a\*e/d)\*x\*log((8\*a^2\*d^2\*e^2 + (c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4)\*x^2 - 4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(2\*a\*d^2\*e + (c\*d^3 + a\*d\*e^2)\*x)\*sqrt(a\*e/d) + 8\*(a\*c\*d^3\*e + a^2\*d\*e^3)\*x)/x^2) + 4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(c\*d\*x - a\*e))/x, -1/4\*(2\*(3\*c\*d^2 + a\*e^2)\*sqrt(-a\*e/d)\*x\*arctan(1/2\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*d\*sqrt(-a\*e/d))) - (c\*d^2 + 3\*a\*e^2)\*sqrt(c\*d/e)\*x\*log(8\*c^2\*d^2\*e^2\*x^2 + c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4 + 4\*(2\*c\*d\*e^2\*x + c\*d^2\*e + a\*e^3)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d/e) + 8\*(c^2\*d^3\*e + a\*c\*d\*e^3)\*x) - 4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(c\*d\*x - a\*e))/x, -1/2\*((3\*c\*d^2 + a\*e^2)\*sqrt(-a\*e/d)\*x\*arctan(1/2\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*d\*sqrt(-a\*e/d))) - (c\*d^2 + 3\*a\*e^2)\*sqrt(-c\*d/e)\*x\*arctan(1/2\*(2\*c\*d\*e\*x + c\*d^2 + a\*e^2)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d/e)\*e)) - 2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(c\*d\*x - a\*e))/x]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**2/(e*x+d),x)`

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^2),x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.452 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^3(d+ex)} dx$$

**Optimal.** Leaf size=256

$$\frac{(-a^2e^4 + 6acd^2e^2 + 3c^2d^4) \tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8\sqrt{ad}^{3/2}\sqrt{e}} + c^{3/2}d^{3/2}\sqrt{e} \tanh^{-1}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right) - \frac{(x(ae^2 + 5cd^2) + 2ade) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{4dx^2}$$

[Out]  $-\left(\frac{(2ad^2e + (5c^2d^2 + a^2e^2)x) \sqrt{ad^2e + (cd^2 + a^2e^2)x + c^2d^2e^2x^2}}{(4d^2x^2) + c^{3/2}d^{3/2}e \operatorname{ArcTanh}\left(\frac{cd^2 + a^2e^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)} - \frac{((3c^2d^4 + 6a^2c^2d^2e^2 - a^4e^4) \operatorname{ArcTanh}\left(\frac{2ad^2e + (cd^2 + a^2e^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right))}{(8\sqrt{a}d^{3/2}e \operatorname{ArcTanh}\left(\frac{cd^2 + a^2e^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right))}\right)$

**Rubi [A]** time = 0.87493, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{(-a^2e^4 + 6acd^2e^2 + 3c^2d^4) \tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8\sqrt{ad}^{3/2}\sqrt{e}} + c^{3/2}d^{3/2}\sqrt{e} \tanh^{-1}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right) - \frac{(x(ae^2 + 5cd^2) + 2ade) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{4dx^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(ad^2e + (cd^2 + a^2e^2)x + c^2d^2e^2x^2)^{3/2}/(x^3(d + ex)), x]$

[Out]  $-\left(\frac{(2ad^2e + (5c^2d^2 + a^2e^2)x) \sqrt{ad^2e + (cd^2 + a^2e^2)x + c^2d^2e^2x^2}}{(4d^2x^2) + c^{3/2}d^{3/2}e \operatorname{ArcTanh}\left(\frac{cd^2 + a^2e^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)} - \frac{((3c^2d^4 + 6a^2c^2d^2e^2 - a^4e^4) \operatorname{ArcTanh}\left(\frac{2ad^2e + (cd^2 + a^2e^2)x}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right))}{(8\sqrt{a}d^{3/2}e \operatorname{ArcTanh}\left(\frac{cd^2 + a^2e^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right))}\right)$

**Rubi in Sympy [A]** time = 105.661, size = 246, normalized size = 0.96

$$c^{3/2}d^{3/2}\sqrt{e} \operatorname{atanh}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}\right) - \frac{\left(ade + \frac{x(ae^2+5cd^2)}{2}\right) \sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{2dx^2} + \frac{(a^2e^4 - 6acd^2e^2 - 3c^2d^4) \operatorname{atanh}\left(\frac{2ade+x(ae^2+cd^2)}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}\right)}{8\sqrt{ad}^{3/2}\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{rubi\_integrate}((ad^2e+(a^2e^2+c^2d^2)x+c^2d^2e^2x^2)^{3/2}/x^3/(e*x+d), x)$

[Out]  $c^{3/2}d^{3/2}\sqrt{e} \operatorname{atanh}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}\right) - \frac{\left(ade + \frac{x(ae^2+5cd^2)}{2}\right) \sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{2dx^2} + \frac{(a^2e^4 - 6acd^2e^2 - 3c^2d^4) \operatorname{atanh}\left(\frac{2ade+x(ae^2+cd^2)}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}\right)}{8\sqrt{ad}^{3/2}\sqrt{e}}$

```

**2)))) - (a*d*e + x*(a*e**2 + 5*c*d**2)/2)*sqrt(a*d*e + c*d*e*x*
*2 + x*(a*e**2 + c*d**2))/(2*d*x**2) + (a**2*e**4 - 6*a*c*d**2*e
**2 - 3*c**2*d**4)*atanh((2*a*d*e + x*(a*e**2 + c*d**2))/(2*sqrt(a
)*sqrt(d)*sqrt(e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))
)/(8*sqrt(a)*d**(3/2)*sqrt(e))

```

**Mathematica [A]** time = 0.633135, size = 310, normalized size = 1.21

$$\sqrt{d+ex}\sqrt{ae+cdx}\left(x^2\log(x)(a^2e^4-6acd^2e^2-3c^2d^4)+x^2(-a^2e^4+6acd^2e^2+3c^2d^4)\log\left(2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^3*(d + e*x)), x]
```

```
[Out] -(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt
[a*e + c*d*x]*Sqrt[d + e*x]*(5*c*d^2*x + a*e*(2*d + e*x)) + (-3*c
^2*d^4 - 6*a*c*d^2*e^2 + a^2*e^4)*x^2*Log[x] + (3*c^2*d^4 + 6*a*c
*d^2*e^2 - a^2*e^4)*x^2*Log[c*d^2*x + 2*Sqrt[a]*Sqrt[d]*Sqrt[e]*S
qrt[a*e + c*d*x]*Sqrt[d + e*x] + a*e*(2*d + e*x)] - 8*Sqrt[a]*c^(
3/2)*d^3*e*x^2*Log[a*e^2 + 2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c
*d*x]*Sqrt[d + e*x] + c*d*(d + 2*e*x)))/(8*Sqrt[a]*d^(3/2)*Sqrt[
e]*x^2*Sqrt[(a*e + c*d*x)*(d + e*x)])

```

**Maple [B]** time = 0.025, size = 1604, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^3/(e*x+d), x)
```

```
[Out] -3/8*d^3/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)
*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c^2+3/4/d^3/a/x*(a*
d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)-1/4/d^2*a*e^3*(a*d*e+(a*e^2+
c*d^2)*x+c*d*e*x^2)^(1/2)+1/16/d^4*e^7*a^3/c*ln((1/2*a*e^2-1/2*c*
d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(
x+d/e))^(1/2))/(c*d*e)^(1/2)-1/16/d^4*e^7*a^3/c*ln((1/2*a*e^2+1/2
*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2))/(c*d*e)^(1/2)-1/4/d/a^2/e^2/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(5/2)*c-3/4/d^2/a*e*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)
)*x-3/4*d*a*e^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*
e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c-3/16/d^2*e
^5*a^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+(c*d*
e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+3/16*e^3*
a*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+(c*d*e*(
x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/4/d^2*e^2*c
*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-1/16*d^2*e*c^2*ln
((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+(c*d*e*(x+d/e)
^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)+1/4/d^3*e^4*a*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+1/8/d^4*e^5*a^2/c*(a*d*e+(a
*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+3/16/d^2*e^5*a^2*ln((1/2*a*e^2+1/2
*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2))/(c*d*e)^(1/2)-1/2/d*e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(
1/2)*x+c+1/8/d*a^2*e^4/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+
2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)+1/4/a
^2/e*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x-3/16*a*e^3*ln(
(1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*
x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)*c+1/4*d/a^2/e^2*(a*d*e+(a*e^2+c
*d^2)*x+c*d*e*x^2)^(3/2)*c^2+3/4*d^2/a/e*(a*d*e+(a*e^2+c*d^2)*x+c
*d*e*x^2)^(1/2)*c^2+17/16*d^2*e*c^2*ln((1/2*a*e^2+1/2*c*d^2+c*d*e
*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e

```

$$\begin{aligned} &)^{(1/2)} + 3/4 * d/a * (a*d*e + (a*e^2 + c*d^2) * x + c*d*e*x^2)^{(1/2)} * x * c^2 - 1/2 \\ &/d^2/a/e/x^2 * (a*d*e + (a*e^2 + c*d^2) * x + c*d*e*x^2)^{(5/2)} - 1/8/d^4 * e^5 * \\ &a^2/c * (c*d*e * (x+d/e)^2 + (a*e^2 - c*d^2) * (x+d/e))^{(1/2)} - 1/4/d^3 * e^4 * a \\ &* (c*d*e * (x+d/e)^2 + (a*e^2 - c*d^2) * (x+d/e))^{(1/2)} * x - 1/3/d^3 * e^2 * (c*d \\ &* e * (x+d/e)^2 + (a*e^2 - c*d^2) * (x+d/e))^{(3/2)} + 1/8 * e * c * (c*d*e * (x+d/e)^ \\ &2 + (a*e^2 - c*d^2) * (x+d/e))^{(1/2)} + 7/8 * e * (a*d*e + (a*e^2 + c*d^2) * x + c*d*e \\ &* x^2)^{(1/2)} * c - 5/12/d^3 * e^2 * (a*d*e + (a*e^2 + c*d^2) * x + c*d*e*x^2)^{(3/2)} \\ &) \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)\*x^3), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.00095, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)\*x^3), x, algorithm="fricas")

[Out] [1/16\*(8\*sqrt(a\*d\*e)\*sqrt(c\*d\*e)\*c\*d^2\*x^2\*log(8\*c^2\*d^2\*e^2\*x^2 + c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4 + 4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(2\*c\*d\*e\*x + c\*d^2 + a\*e^2)\*sqrt(c\*d\*e) + 8\*(c^2\*d^3\*e + a\*c\*d^2\*e^3)\*x) - (3\*c^2\*d^4 + 6\*a\*c\*d^2\*e^2 - a^2\*e^4)\*x^2\*log((4\*(2\*a^2\*d^2\*e^2 + (a\*c\*d^3\*e + a^2\*d^2\*e^3)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) + (8\*a^2\*d^2\*e^2 + (c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4)\*x^2 + 8\*(a\*c\*d^3\*e + a^2\*d^2\*e^3)\*x)\*sqrt(a\*d\*e))/x^2) - 4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(2\*a\*d\*e + (5\*c\*d^2 + a\*e^2)\*x)\*sqrt(a\*d\*e))/(sqrt(a\*d\*e)\*d\*x^2), 1/16\*(16\*sqrt(a\*d\*e)\*sqrt(-c\*d\*e)\*c\*d^2\*x^2\*arctan(1/2\*(2\*c\*d\*e\*x + c\*d^2 + a\*e^2)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*e))) - (3\*c^2\*d^4 + 6\*a\*c\*d^2\*e^2 - a^2\*e^4)\*x^2\*log((4\*(2\*a^2\*d^2\*e^2 + (a\*c\*d^3\*e + a^2\*d^2\*e^3)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) + (8\*a^2\*d^2\*e^2 + (c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4)\*x^2 + 8\*(a\*c\*d^3\*e + a^2\*d^2\*e^3)\*x)\*sqrt(a\*d\*e))/x^2) - 4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(2\*a\*d\*e + (5\*c\*d^2 + a\*e^2)\*x)\*sqrt(a\*d\*e))/(sqrt(a\*d\*e)\*d\*x^2), 1/8\*(4\*sqrt(-a\*d\*e)\*sqrt(c\*d\*e)\*c\*d^2\*x^2\*log(8\*c^2\*d^2\*e^2\*x^2 + c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4 + 4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(2\*c\*d\*e\*x + c\*d^2 + a\*e^2)\*sqrt(c\*d\*e) + 8\*(c^2\*d^3\*e + a\*c\*d^2\*e^3)\*x) - (3\*c^2\*d^4 + 6\*a\*c\*d^2\*e^2 - a^2\*e^4)\*x^2\*arctan(1/2\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*a\*d\*e)) - 2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(2\*a\*d\*e + (5\*c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e))/(sqrt(-a\*d\*e)\*d\*x^2), 1/8\*(8\*sqrt(-a\*d\*e)\*sqrt(-c\*d\*e)\*c\*d^2\*x^2\*arctan(1/2\*(2\*c\*d\*e\*x + c\*d^2 + a\*e^2)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*e))) - (3\*c^2\*d^4 + 6\*a\*c\*d^2\*e^2 - a^2\*e^4)\*x^2\*arctan(1/2\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*a\*d\*e)) - 2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(2\*a\*d\*e + (5\*c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e))/(sqrt(-a\*d\*e)\*d\*x^2)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**3/(e*x+d),x)
```

```
[Out] Timed out
```

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^3),x, algorith
```

```
[Out] Exception raised: TypeError
```

$$3.453 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^4(d+ex)} dx$$

**Optimal.** Leaf size=211

$$\frac{(cd^2 - ae^2)^3 \tanh^{-1}\left(\frac{x(ae^2 + cd^2) + 2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{16a^{3/2}d^{5/2}e^{3/2}} - \frac{\left(\frac{c}{ae} - \frac{e}{d^2}\right)(x(ae^2 + cd^2) + 2ade)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{8x^2} - \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{3dx^3}$$

[Out]  $-\left(\frac{c}{a^*e} - \frac{e}{d^{\wedge}2}\right) * (2 * a^*d^*e + (c^*d^{\wedge}2 + a^*e^{\wedge}2) * x) * \text{Sqrt}[a^*d^*e + (c^*d^{\wedge}2 + a^*e^{\wedge}2) * x + c^*d^*e * x^{\wedge}2] / (8 * x^{\wedge}2) - (a^*d^*e + (c^*d^{\wedge}2 + a^*e^{\wedge}2) * x + c^*d^*e * x^{\wedge}2)^{\wedge}(3/2) / (3 * d^*x^{\wedge}3) + \left(\frac{c^*d^{\wedge}2 - a^*e^{\wedge}2}{2}\right)^3 * \text{ArcTanh}\left[\frac{2 * a^*d^*e + (c^*d^{\wedge}2 + a^*e^{\wedge}2) * x}{2 * \text{Sqrt}[a] * \text{Sqrt}[d] * \text{Sqrt}[e] * \text{Sqrt}[a^*d^*e + (c^*d^{\wedge}2 + a^*e^{\wedge}2) * x + c^*d^*e * x^{\wedge}2]}\right] / (16 * a^{\wedge}(3/2) * d^{\wedge}(5/2) * e^{\wedge}(3/2))$

**Rubi [A]** time = 0.708721, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{(cd^2 - ae^2)^3 \tanh^{-1}\left(\frac{x(ae^2 + cd^2) + 2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{16a^{3/2}d^{5/2}e^{3/2}} - \frac{\left(\frac{c}{ae} - \frac{e}{d^2}\right)(x(ae^2 + cd^2) + 2ade)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{8x^2} - \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{3dx^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a^*d^*e + (c^*d^{\wedge}2 + a^*e^{\wedge}2) * x + c^*d^*e * x^{\wedge}2)^{\wedge}(3/2) / (x^{\wedge}4 * (d + e * x)), x]$

[Out]  $-\left(\frac{c}{a^*e} - \frac{e}{d^{\wedge}2}\right) * (2 * a^*d^*e + (c^*d^{\wedge}2 + a^*e^{\wedge}2) * x) * \text{Sqrt}[a^*d^*e + (c^*d^{\wedge}2 + a^*e^{\wedge}2) * x + c^*d^*e * x^{\wedge}2] / (8 * x^{\wedge}2) - (a^*d^*e + (c^*d^{\wedge}2 + a^*e^{\wedge}2) * x + c^*d^*e * x^{\wedge}2)^{\wedge}(3/2) / (3 * d^*x^{\wedge}3) + \left(\frac{c^*d^{\wedge}2 - a^*e^{\wedge}2}{2}\right)^3 * \text{ArcTanh}\left[\frac{2 * a^*d^*e + (c^*d^{\wedge}2 + a^*e^{\wedge}2) * x}{2 * \text{Sqrt}[a] * \text{Sqrt}[d] * \text{Sqrt}[e] * \text{Sqrt}[a^*d^*e + (c^*d^{\wedge}2 + a^*e^{\wedge}2) * x + c^*d^*e * x^{\wedge}2]}\right] / (16 * a^{\wedge}(3/2) * d^{\wedge}(5/2) * e^{\wedge}(3/2))$

**Rubi in Sympy [A]** time = 61.1806, size = 194, normalized size = 0.92

$$\frac{\left(\frac{e}{8d^2} - \frac{c}{8ae}\right)(2ade + x(ae^2 + cd^2))\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{x^2} - \frac{(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{3}{2}}}{3dx^3} - \frac{(ae^2 - cd^2)^3 \operatorname{atanh}\left(\frac{2ade + x(ae^2 + cd^2)}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}\right)}{16a^{\frac{3}{2}}d^{\frac{5}{2}}e^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a^*d^*e + (a^*e^{\wedge}2 + c^*d^{\wedge}2) * x + c^*d^*e * x^{\wedge}2)^{\wedge}(3/2) / x^{\wedge}4 / (e * x + d), x)$

[Out]  $(e / (8 * d^{\wedge}2) - c / (8 * a * e)) * (2 * a^*d^*e + x * (a^*e^{\wedge}2 + c^*d^{\wedge}2)) * \text{sqrt}(a^*d^*e + c^*d^*e * x^{\wedge}2 + x * (a^*e^{\wedge}2 + c^*d^{\wedge}2)) / x^{\wedge}2 - (a^*d^*e + c^*d^*e * x^{\wedge}2 + x * (a^*e^{\wedge}2 + c^*d^{\wedge}2))^{\wedge}(3/2) / (3 * d^*x^{\wedge}3) - (a^*e^{\wedge}2 - c^*d^{\wedge}2)^{\wedge}3 * \operatorname{atanh}((2 * a^*d^*e + x * (a^*e^{\wedge}2 + c^*d^{\wedge}2)) / (2 * \text{sqrt}(a) * \text{sqrt}(d) * \text{sqrt}(e)))$

$$\sqrt{a^2 d^2 e + c^2 d^2 e^2 x^2 + x(a^2 e^2 + c^2 d^2)} / ((16 a^{3/2} d^{5/2} e^{3/2} x^3 \sqrt{d+ex} \sqrt{ae+cdx} (a^2 e^2 (8d^2 + 2dex - 3e^2 x^2) + 2acd^2 ex(7d + 4ex) + 3c^2 d^4 x^2) - 3x^3 \log(x))$$

**Mathematica [A]** time = 0.361761, size = 251, normalized size = 1.19

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(-2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}\left(a^2e^2(8d^2+2dex-3e^2x^2)+2acd^2ex(7d+4ex)+3c^2d^4x^2\right)-3x^3\log(x)\right)}{48a^{3/2}d^{5/2}e^{3/2}x^3\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/(x^4\*(d + e\*x)), x]

[Out] (Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*(-2\*Sqrt[a]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*(3\*c^2\*d^4\*x^2 + 2\*a\*c\*d^2\*e\*x\*(7\*d + 4\*e\*x) + a^2\*e^2\*(8\*d^2 + 2\*d\*e\*x - 3\*e^2\*x^2)) - 3\*(c\*d^2 - a\*e^2)^3\*x^3\*Log[x] + 3\*(c\*d^2 - a\*e^2)^3\*x^3\*Log[c\*d^2\*x + 2\*Sqrt[a]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x] + a\*e\*(2\*d + e\*x)))/(48\*a^(3/2)\*d^(5/2)\*e^(3/2)\*x^3\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)])

**Maple [B]** time = 0.029, size = 1945, normalized size = 9.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)/x^4/(e\*x+d), x)

[Out] 3/16/d\*e^4\*a\*ln((1/2\*a\*e^2+1/2\*c\*d^2+c\*d\*e\*x)/(c\*d\*e)^(1/2)+(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/(c\*d\*e)^(1/2)\*c+1/16/d^5\*e^8\*a^3/c\*ln((1/2\*a\*e^2+1/2\*c\*d^2+c\*d\*e\*x)/(c\*d\*e)^(1/2)+(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/(c\*d\*e)^(1/2)-1/16/d^5\*e^8\*a^3/c\*ln((1/2\*a\*e^2-1/2\*c\*d^2+(x+d/e)\*c\*d\*e)/(c\*d\*e)^(1/2)+(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2))/(c\*d\*e)^(1/2)-3/16/d^5\*e^4\*a\*c\*ln((1/2\*a\*e^2-1/2\*c\*d^2+(x+d/e)\*c\*d\*e)/(c\*d\*e)^(1/2)+(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2))/(c\*d\*e)^(1/2)+1/16\*d^4/a/e/(a\*d\*e)^(1/2)\*ln((2\*a\*d\*e+(a\*e^2+c\*d^2)\*x+2\*(a\*d\*e)^(1/2)\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/x)\*c^3+17/24/d^3/a\*e^2\*c\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)\*x-1/24\*d/a^3/e^2\*c^3\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)\*x+1/12/d/a^2/e^2/x^2\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2)\*c-1/3/d^2/a^2/e/x\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2)\*c-1/8\*d^2/a^2/e\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)\*x\*c^3-1/8/d^2\*e^2\*c\*(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2)+1/8/d^3\*a\*e^4\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)+7/12/d^3/a/x^2\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2)+1/8\*d/a\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)\*c^2+5/24/a^2/e\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)\*c^2-1/8/d^5\*e^6\*a^2/c\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)+1/3/d^4\*e^3\*(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(3/2)+3/8/d^4\*e^3\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)+1/4/d^4\*e^5\*a\*(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2)\*x+1/8/d^5\*e^6\*a^2/c\*(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2)+3/16/d^3\*e^6\*a^2\*ln((1/2\*a\*e^2-1/2\*c\*d^2+(x+d/e)\*c\*d\*e)/(c\*d\*e)^(1/2)+(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2))/(c\*d\*e)^(1/2)-1/4/d^2\*e^3\*c\*(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2)\*x+1/16\*d^2\*c^2\*ln((1/2\*a\*e^2-1/2\*c\*d^2+(x+d/e)\*c\*d\*e)/(c\*d\*e)^(1/2)+(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2))/(c\*d\*e)^(1/2)+3/16\*a\*e^3/(a\*d\*e)^(1/2)\*ln((2\*a\*d\*e+(a\*e^2+c\*d^2)\*x+2\*(a\*d\*e)^(1/2)\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/x)\*c+1/24/a^3/e^3/x\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2)\*c^2+3/8/d^2\*e^3\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)\*x\*c-3/16\*d^2\*e/(a\*d\*e)^(1/2)\*ln((2\*a\*d\*e+(a\*e^2+c\*d^2)\*x+2\*(a\*d\*e)^(1/2)\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/x)\*c^2+1/3/d/a^2\*c^2\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)\*

$$x - \frac{1}{16} d e^2 \ln\left(\frac{(1/2 a e^2 + 1/2 c d^2 + c d e x)/(c d e)^{1/2} + (a d e + (a e^2 + c d^2) x + c d e x^2)^{1/2}}{(c d e)^{1/2} c^2 - 1/3 d^2/a/e/x^3 (a d e + (a e^2 + c d^2) x + c d e x^2)^{5/2} - 1/24 d^2/a^3/e^3 (a d e + (a e^2 + c d^2) x + c d e x^2)^{3/2} c^3 - 1/8 d^3/a^2/e^2 (a d e + (a e^2 + c d^2) x + c d e x^2)^{1/2} c^3 - 1/16 d^2 a^2 e^5/(a d e)^{1/2}}\right) \ln\left(\frac{(2 a d e + (a e^2 + c d^2) x + 2 (a d e)^{1/2} (a d e + (a e^2 + c d^2) x + c d e x^2)^{1/2})/x + 11/24 d^2/a e (a d e + (a e^2 + c d^2) x + c d e x^2)^{3/2} c - 17/24 d^4/a e/x (a d e + (a e^2 + c d^2) x + c d e x^2)^{5/2} - 1/4 d^4 e^5 a (a d e + (a e^2 + c d^2) x + c d e x^2)^{1/2} x - 3/16 d^3 e^6 a^2 \ln\left(\frac{(1/2 a e^2 + 1/2 c d^2 + c d e x)/(c d e)^{1/2} + (a d e + (a e^2 + c d^2) x + c d e x^2)^{1/2}}{(c d e)^{1/2}}\right)}{(c d e)^{1/2}}\right)$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)\*x^4), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.26075, size = 1, normalized size = 0.

$$\left[ \frac{3(c^3 d^6 - 3ac^2 d^4 e^2 + 3a^2 c d^2 e^4 - a^3 e^6) x^3 \log\left(-\frac{4(2a^2 d^2 e^2 + (acd^3 e + a^2 d e^3) x) \sqrt{cdex^2 + ade + (cd^2 + ae^2)x} - (8a^2 d^2 e^2 + (c^2 d^4 + 6acd^2 e^2 + a^2 e^4) x^2)}{x^2}\right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)\*x^4), x, algorithm="fricas")

[Out] [-1/96\*(3\*(c^3\*d^6 - 3\*a\*c^2\*d^4\*e^2 + 3\*a^2\*c\*d^2\*e^4 - a^3\*e^6)\*x^3\*log(-4\*(2\*a^2\*d^2\*e^2 + (a\*c\*d^3\*e + a^2\*d\*e^3)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) - (8\*a^2\*d^2\*e^2 + (c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4)\*x^2 + 8\*(a\*c\*d^3\*e + a^2\*d\*e^3)\*x)\*sqrt(a\*d\*e)/x^2) + 4\*(8\*a^2\*d^2\*e^2 + (3\*c^2\*d^4 + 8\*a\*c\*d^2\*e^2 - 3\*a^2\*e^4)\*x^2 + 2\*(7\*a\*c\*d^3\*e + a^2\*d\*e^3)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(a\*d\*e)/(sqrt(a\*d\*e)\*a\*d^2\*e\*x^3), 1/48\*(3\*(c^3\*d^6 - 3\*a\*c^2\*d^4\*e^2 + 3\*a^2\*c\*d^2\*e^4 - a^3\*e^6)\*x^3\*arctan(1/2\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*a\*d\*e) - 2\*(8\*a^2\*d^2\*e^2 + (3\*c^2\*d^4 + 8\*a\*c\*d^2\*e^2 - 3\*a^2\*e^4)\*x^2 + 2\*(7\*a\*c\*d^3\*e + a^2\*d\*e^3)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e)/(sqrt(-a\*d\*e)\*a\*d^2\*e\*x^3)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/x\*\*4/(e\*x+d), x, algorithm="sympy")

[Out] Timed out

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**GIAC/XCAS [A]** time = 2.14089, size = 1, normalized size = 0.

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^4),x, algorithm="giac")`

[Out] Done

$$3.454 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^5(d+ex)} dx$$

**Optimal.** Leaf size=295

$$\begin{aligned} & \frac{(5ae^2 + 3cd^2)(cd^2 - ae^2)^3 \tanh^{-1}\left(\frac{x(ae^2 + cd^2) + 2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{128a^{5/2}d^{7/2}e^{5/2}} \\ & + \frac{(5ae^2 + 3cd^2)(cd^2 - ae^2)(x(ae^2 + cd^2) + 2ade)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{64a^2d^3e^2x^2} \\ & - \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{4dx^4} - \frac{\left(\frac{3c}{ae} - \frac{5e}{d^2}\right)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{24x^3} \end{aligned}$$

[Out] ((c\*d^2 - a\*e^2)\*(3\*c\*d^2 + 5\*a\*e^2)\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(64\*a^2\*d^3\*e^2\*x^2) - (a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/(4\*d\*x^4) - ((3\*c)/(a\*e) - (5\*e)/d^2)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/(24\*x^3) - ((c\*d^2 - a\*e^2)^3\*(3\*c\*d^2 + 5\*a\*e^2)\*ArcTanh[(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(2\*Sqrt[a]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]])/(128\*a^(5/2)\*d^(7/2)\*e^(5/2))

**Rubi [A]** time = 1.07445, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\begin{aligned} & \frac{(5ae^2 + 3cd^2)(cd^2 - ae^2)^3 \tanh^{-1}\left(\frac{x(ae^2 + cd^2) + 2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{128a^{5/2}d^{7/2}e^{5/2}} \\ & + \frac{(5ae^2 + 3cd^2)(cd^2 - ae^2)(x(ae^2 + cd^2) + 2ade)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{64a^2d^3e^2x^2} \\ & - \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{4dx^4} - \frac{\left(\frac{3c}{ae} - \frac{5e}{d^2}\right)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{24x^3} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/(x^5\*(d + e\*x)), x]

[Out] ((c\*d^2 - a\*e^2)\*(3\*c\*d^2 + 5\*a\*e^2)\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(64\*a^2\*d^3\*e^2\*x^2) - (a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/(4\*d\*x^4) - ((3\*c)/(a\*e) - (5\*e)/d^2)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/(24\*x^3) - ((c\*d^2 - a\*e^2)^3\*(3\*c\*d^2 + 5\*a\*e^2)\*ArcTanh[(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(2\*Sqrt[a]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]])/(128\*a^(5/2)\*d^(7/2)\*e^(5/2))

**Rubi in Sympy [A]** time = 106.171, size = 274, normalized size = 0.93

$$\begin{aligned} & \frac{(2ade + x(ae^2 + cd^2))\left(\frac{5e^2}{64d^3} - \frac{c}{32ad} - \frac{3e^2d}{64a^2e^2}\right)\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{x^2} \\ & + \frac{\left(\frac{5e}{24d^2} - \frac{c}{8ae}\right)(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{3}{2}}}{x^3} - \frac{(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{3}{2}}}{4dx^4} \\ & + \frac{(ae^2 - cd^2)^3(5ae^2 + 3cd^2) \operatorname{atanh}\left(\frac{2ade + x(ae^2 + cd^2)}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}\right)}{128a^{\frac{5}{2}}d^{\frac{7}{2}}e^{\frac{5}{2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**5/(e*x+d),x)`

[Out] 
$$-(2*a*d*e + x*(a*e**2 + c*d**2))*(5*e**2/(64*d**3) - c/(32*a*d) - 3*c**2*d/(64*a**2*e**2))*\sqrt{a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)}/x**2 + (5*e/(24*d**2) - c/(8*a*e))*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2)/x**3 - (a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2)/(4*d*x**4) + (a*e**2 - c*d**2)**3*(5*a*e**2 + 3*c*d**2)*\operatorname{atanh}((2*a*d*e + x*(a*e**2 + c*d**2))/(2*\sqrt{a}*\sqrt{d}*\sqrt{e}*\sqrt{a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)}))/((128*a**(5/2)*d**(7/2)*e**(5/2))$$

**Mathematica [A]** time = 0.468948, size = 323, normalized size = 1.09

$$\sqrt{d+ex}\sqrt{ae+cdx}\left(-2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}(a^3e^3(48d^3+8d^2ex-10de^2x^2+15e^3x^3))+a^2cd^2e^2x(72d^2+20dex-3\right)$$

Antiderivative was successfully verified.

[In] `Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^5*(d + e*x)),x]`

[Out] 
$$\left(\sqrt{a*e + c*d*x}*\sqrt{d + e*x}*(-2*\sqrt{a}*\sqrt{d}*\sqrt{e}*\sqrt{a*e + c*d*x}*\sqrt{d + e*x}*(-9*c^3*d^6*x^3 + 3*a*c^2*d^4*e*x^2*(2*d + 3*e*x) + a^2*c*d^2*e^2*x*(72*d^2 + 20*d*e*x - 31*e^2*x^2) + a^3*e^3*(48*d^3 + 8*d^2*e*x - 10*d*e^2*x^2 + 15*e^3*x^3)) + 3*(c*d^2 - a*e^2)^3*(3*c*d^2 + 5*a*e^2)*x^4*\operatorname{Log}[x] - 3*(c*d^2 - a*e^2)^3*(3*c*d^2 + 5*a*e^2)*x^4*\operatorname{Log}[c*d^2*x + 2*\sqrt{a}*\sqrt{d}*\sqrt{e}*\sqrt{a*e + c*d*x}*\sqrt{d + e*x} + a*e*(2*d + e*x)])\right)/(384*a^(5/2)*d^(7/2)*e^(5/2)*x^4*\sqrt{(a*e + c*d*x)*(d + e*x)}$$

**Maple [B]** time = 0.036, size = 2427, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^5/(e*x+d),x)`

[Out] 
$$\frac{1}{16d^6e^9a^3c}\ln\left(\frac{1}{2}a^2e^2-1/2c^2d^2+(x+d/e)^2c^2d^2e\right)/(c^2d^2e)^{1/2}+(c^2d^2e(x+d/e)^2+(a^2e^2-c^2d^2)(x+d/e))^{1/2}/(c^2d^2e)^{1/2}+3/16d^2e^5a^3c\ln\left(\frac{1}{2}a^2e^2-1/2c^2d^2+(x+d/e)^2c^2d^2e\right)/(c^2d^2e)^{1/2}+(c^2d^2e(x+d/e)^2+(a^2e^2-c^2d^2)(x+d/e))^{1/2}/(c^2d^2e)^{1/2}-1/16d^6e^9a^3c\ln\left(\frac{1}{2}a^2e^2+1/2c^2d^2+c^2d^2e^2x\right)/(c^2d^2e)^{1/2}+(a^2d^2e+(a^2e^2+c^2d^2)^2x+c^2d^2e^2x^2)^{1/2}/(c^2d^2e)^{1/2}-3/16d^2e^5a^3c\ln\left(\frac{1}{2}a^2e^2+1/2c^2d^2+c^2d^2e^2x\right)/(c^2d^2e)^{1/2}+(a^2d^2e+(a^2e^2+c^2d^2)^2x+c^2d^2e^2x^2)^{1/2}/(c^2d^2e)^{1/2}+1/64d^2/a^4/e^3c^4(a^2d^2e+(a^2e^2+c^2d^2)^2x+c^2d^2e^2x^2)^{3/2}x-1/64d/a^4/e^4/x(a^2d^2e+(a^2e^2+c^2d^2)^2x+c^2d^2e^2x^2)^{5/2}c^3+1/8d/a^2/e^2/x^3(a^2d^2e+(a^2e^2+c^2d^2)^2x+c^2d^2e^2x^2)^{5/2}c-3/64d/a^2e^2(a^2d^2e+(a^2e^2+c^2d^2)^2x+c^2d^2e^2x^2)^{1/2}x^2c^2-13/48d^2/a^2/e/x^2(a^2d^2e+(a^2e^2+c^2d^2)^2x+c^2d^2e^2x^2)^{5/2}c+3/64d^3/a^3/e^2(a^2d^2e+(a^2e^2+c^2d^2)^2x+c^2d^2e^2x^2)^{1/2}x^2c^4+19/192d/a^3/e^2/x(a^2d^2e+(a^2e^2+c^2d^2)^2x+c^2d^2e^2x^2)^{5/2}c^2-3/128d^5/a^2/e^2/(a^2d^2e)^{1/2}\ln\left(\frac{1}{2}a^2d^2e+(a^2e^2+c^2d^2)^2x+2(a^2d^2e)^{1/2}(a^2d^2e+(a^2e^2+c^2d^2)^2x+c^2d^2e^2x^2)^{1/2}\right)/x)^2c^4-3/32d^2/a^2e^4/(a^2d^2e)^{1/2}\ln\left(\frac{1}{2}a^2d^2e+(a^2e^2+c^2d^2)^2x+2(a^2d^2e)^{1/2}(a^2d^2e+(a^2e^2+c^2d^2)^2x+c^2d^2e^2x^2)^{1/2}\right)/x)^2c-133/192d^4/a^2e^3c^2(a^2d^2e+(a^2e^2+c^2d^2)^2x+c^2d^2e^2x^2)^{3/2}x-91/192d^2/a^2e^2c^2(a^2d^2e+(a^2e^2+c^2d^2)^2x+c^2d^2e^2x^2)^{3/2}x-1/16e^3c^2\ln\left(\frac{1}{2}a^2e^2-1/2c^2d^2+(x+d/e)^2c^2d^2e\right)/(c^2d^2e)^{1/2}+(c^2d^2e(x+d/e)^2+(a^2e^2-c^2d^2)(x+d/e))^{1/2}/(c^2d^2e)^{1/2}-5/64d^4/a^2e^5(a^2d^2e+(a^2e^2+c^2d^2)^2x+c^2d^2e^2x^2)^{1/2}-3/32d^2e^3(a^2d^2e+(a^2e^2+c^2d^2)^2x+c^2d^2e^2x^2)^{1/2}c+11/24d^3/a/x^3(a^2d^2e$$

$$\begin{aligned}
 &+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(5/2)}-29/96/d/a^2*(a^*d^*e+(a^*e^2+c^*d^2) \\
 &)^*x+c^*d^*e^*x^2)^{(3/2)}*c^2-1/32/a^*e^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2 \\
 &)^{(1/2)}*c^2+1/16^*e^3*c^2*\ln((1/2^*a^*e^2+1/2^*c^*d^2+c^*d^*e^*x)/(c^*d^*e \\
 &)^{(1/2)}+(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(1/2)})/(c^*d^*e)^{(1/2)}+1/ \\
 &8/d^2^*e^3*c^*(c^*d^*e^*(x+d/e)^2+(a^*e^2-c^*d^2)^*(x+d/e))^{(1/2)}-1/3/d^5 \\
 &^*e^4*(c^*d^*e^*(x+d/e)^2+(a^*e^2-c^*d^2)^*(x+d/e))^{(3/2)}-23/64/d^5^*e^4^* \\
 &(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(3/2)}-1/4/d^5^*e^6^*a^*(c^*d^*e^*(x+d \\
 &/e)^2+(a^*e^2-c^*d^2)^*(x+d/e))^{(1/2)}*x-3/16/d^4^*e^7^*a^2*\ln((1/2^*a^*e \\
 &^2-1/2^*c^*d^2+(x+d/e)^*c^*d^*e)/(c^*d^*e)^{(1/2)}+(c^*d^*e^*(x+d/e)^2+(a^*e^2 \\
 &-c^*d^2)^*(x+d/e))^{(1/2)})/(c^*d^*e)^{(1/2)}+1/4/d^3^*e^4^*c^*(c^*d^*e^*(x+d/e \\
 &)^2+(a^*e^2-c^*d^2)^*(x+d/e))^{(1/2)}*x+1/4/d^5^*e^6^*a^*(a^*d^*e+(a^*e^2+c^* \\
 &d^2)^*x+c^*d^*e^*x^2)^{(1/2)}*x+1/8/d^6^*e^7^*a^2/c^*(a^*d^*e+(a^*e^2+c^*d^2)^* \\
 &x+c^*d^*e^*x^2)^{(1/2)}+3/16/d^4^*e^7^*a^2*\ln((1/2^*a^*e^2+1/2^*c^*d^2+c^*d^*e \\
 &^*x)/(c^*d^*e)^{(1/2)}+(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(1/2)})/(c^*d^*e \\
 &)^{(1/2)}-1/32/a^3/e^3/x^2*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(5/2)}* \\
 &c^2-19/192/a^3/e^3*c^3*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(3/2)}*x+91 \\
 &/192/d^3/a^2/x*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(5/2)}*c-21/64/d^ \\
 &3^*e^4*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(1/2)}*x^c+1/32^*d^3/a/(a^*d \\
 &^*e)^{(1/2)}*\ln((2^*a^*d^*e+(a^*e^2+c^*d^2)^*x+2*(a^*d^*e)^{(1/2)}*(a^*d^*e+(a^*e \\
 &^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(1/2)})/x)^*c^3+3/64^*d^2/(a^*d^*e)^{(1/2)}*\ln( \\
 &(2^*a^*d^*e+(a^*e^2+c^*d^2)^*x+2*(a^*d^*e)^{(1/2)}*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c \\
 &^*d^*e^*x^2)^{(1/2)})/x)^*c^2+5/64^*d/a^2*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x \\
 &^2)^{(1/2)}*x^c+3-53/96/d^3/a^*e^2*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2) \\
 &^{(3/2)}*c+1/32^*d^2/a^2/e^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(1/2)}*c \\
 &^3+133/192/d^5/a^*e^2/x*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(5/2)}+5/ \\
 &128/d^3^*a^2^*e^6/(a^*d^*e)^{(1/2)}*\ln((2^*a^*d^*e+(a^*e^2+c^*d^2)^*x+2*(a^*d^* \\
 &e)^{(1/2)}*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(1/2)})/x)-5/96^*d/a^3/e \\
 &^2*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(3/2)}*c^3+1/64^*d^3/a^4/e^4*( \\
 &a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(3/2)}*c^4+3/64^*d^4/a^3/e^3*(a^*d^* \\
 &e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(1/2)}*c^4-59/96/d^4/a^*e/x^2*(a^*d^*e+( \\
 &a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(5/2)}-1/4/d^2/a/e/x^4*(a^*d^*e+(a^*e^2+c^*d \\
 &^2)^*x+c^*d^*e^*x^2)^{(5/2)}-1/8/d^6^*e^7^*a^2/c^*(c^*d^*e^*(x+d/e)^2+(a^*e^2- \\
 &c^*d^2)^*(x+d/e))^{(1/2)}
 \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)\*x^5), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.93651, size = 1, normalized size = 0.

$$\frac{3(3c^4d^8 - 4ac^3d^6e^2 - 6a^2c^2d^4e^4 + 12a^3cd^2e^6 - 5a^4e^8)x^4 \log\left(\frac{4(2a^2d^2e^2 + (acd^3e + a^2de^3)x)\sqrt{cdex^2 + ade + (cd^2 + ae^2)x} + (8a^2d^2e^2 + (cd^2 + ae^2)x^2)}{x^2}\right) + 3(3c^4d^8 - 4ac^3d^6e^2 - 6a^2c^2d^4e^4 + 12a^3cd^2e^6 - 5a^4e^8)x^4 \arctan\left(\frac{(2ade + (cd^2 + ae^2)x)\sqrt{-ade}}{2\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}ade}\right) + 2(48a^3d^3e^3 - (9c^3d^6 - 3c^2d^4e^2 + 3cd^2e^4 - 3c^2d^4e^4)x^2)}{x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)\*x^5), x, algorithm="fricas")

[Out] [-1/768\*(3\*(3\*c^4\*d^8 - 4\*a\*c^3\*d^6\*e^2 - 6\*a^2\*c^2\*d^4\*e^4 + 12\*a^3\*c\*d^2\*e^6 - 5\*a^4\*e^8)\*x^4\*log((4\*(2\*a^2\*d^2\*e^2 + (a\*c\*d^3\*e + a^2\*d^2\*e^2)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) + (8\*a^2\*d^2\*e^2 + (c\*d^2 + a\*e^2)\*x^2)) + 2\*(48\*a^3\*d^3\*e^3 - (9\*c^3\*d^6 - 3\*c^2\*d^4\*e^2 + 3\*c\*d^2\*e^4 - 3\*c^2\*d^4\*e^4)\*x^2)]/x^2



$$\begin{aligned}
& *a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 8*(a*c*d \\
& ^3*e + a^2*d*e^3)*x)*\sqrt{a*d*e})/x^2) + 4*(48*a^3*d^3*e^3 - (9*c \\
& ^3*d^6 - 9*a*c^2*d^4*e^2 + 31*a^2*c*d^2*e^4 - 15*a^3*e^6)*x^3 + 2 \\
& *(3*a*c^2*d^5*e + 10*a^2*c*d^3*e^3 - 5*a^3*d*e^5)*x^2 + 8*(9*a^2* \\
& c*d^4*e^2 + a^3*d^2*e^4)*x)*\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e \\
& ^2)*x)*\sqrt{a*d*e})/(\sqrt{a*d*e})*a^2*d^3*e^2*x^4), -1/384*(3*(3*c \\
& ^4*d^8 - 4*a*c^3*d^6*e^2 - 6*a^2*c^2*d^4*e^4 + 12*a^3*c*d^2*e^6 - \\
& 5*a^4*e^8)*x^4*\arctan(1/2*(2*a*d*e + (c*d^2 + a*e^2)*x)*\sqrt{-a* \\
& d*e})/(\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x}*a*d*e)) + 2*(48 \\
& *a^3*d^3*e^3 - (9*c^3*d^6 - 9*a*c^2*d^4*e^2 + 31*a^2*c*d^2*e^4 - \\
& 15*a^3*e^6)*x^3 + 2*(3*a*c^2*d^5*e + 10*a^2*c*d^3*e^3 - 5*a^3*d*e \\
& ^5)*x^2 + 8*(9*a^2*c*d^4*e^2 + a^3*d^2*e^4)*x)*\sqrt{c*d*e*x^2 + a \\
& *d*e + (c*d^2 + a*e^2)*x)*\sqrt{-a*d*e})/(\sqrt{-a*d*e})*a^2*d^3*e^2 \\
& *x^4)]
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/x\*\*5/(e\*x+d),x)

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)\*x^5),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.455 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^6(d+ex)} dx$$

**Optimal.** Leaf size=395

$$\begin{aligned} & \frac{(-35a^2e^4 + 12acd^2e^2 + 15c^2d^4)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{240a^2d^3e^2x^3} \\ & + \frac{(7a^2e^4 + 6acd^2e^2 + 3c^2d^4)(cd^2 - ae^2)^3 \tanh^{-1}\left(\frac{x(ae^2 + cd^2) + 2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{256a^{7/2}d^{9/2}e^{7/2}} \\ & - \frac{(7a^2e^4 + 6acd^2e^2 + 3c^2d^4)(cd^2 - ae^2)(x(ae^2 + cd^2) + 2ade)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{128a^3d^4e^3x^2} \\ & - \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{5dx^5} - \frac{\left(\frac{3c}{ae} - \frac{7e}{d^2}\right)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{40x^4} \end{aligned}$$

[Out]  $-\left((c*d^2 - a*e^2)*(3*c^2*d^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*(2*a*d*e + (c*d^2 + a*e^2)*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]\right)/\left(128*a^3*d^4*e^3*x^2 - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/(5*d*x^5) - \left(\left(3*c\right)/\left(a*e\right) - \left(7*e\right)/d^2\right)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/(40*x^4) + \left(\left(15*c^2*d^4 + 12*a*c*d^2*e^2 - 35*a^2*e^4\right)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/(240*a^2*d^3*e^2*x^3) + \left((c*d^2 - a*e^2)^3*(3*c^2*d^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*\text{ArcTanh}\left[\left(2*a*d*e + (c*d^2 + a*e^2)*x\right)/\left(2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]\right)\right]\right)/\left(256*a^{(7/2)}*d^{(9/2)}*e^{(7/2)}\right)$

**Rubi [A]** time = 1.39558, antiderivative size = 395, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\begin{aligned} & \frac{(-35a^2e^4 + 12acd^2e^2 + 15c^2d^4)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{240a^2d^3e^2x^3} \\ & + \frac{(7a^2e^4 + 6acd^2e^2 + 3c^2d^4)(cd^2 - ae^2)^3 \tanh^{-1}\left(\frac{x(ae^2 + cd^2) + 2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{256a^{7/2}d^{9/2}e^{7/2}} \\ & - \frac{(7a^2e^4 + 6acd^2e^2 + 3c^2d^4)(cd^2 - ae^2)(x(ae^2 + cd^2) + 2ade)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{128a^3d^4e^3x^2} \\ & - \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{5dx^5} - \frac{\left(\frac{3c}{ae} - \frac{7e}{d^2}\right)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{40x^4} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/(x^6*(d + e*x)), x]$

[Out]  $-\left((c*d^2 - a*e^2)*(3*c^2*d^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*(2*a*d*e + (c*d^2 + a*e^2)*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]\right)/\left(128*a^3*d^4*e^3*x^2 - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/(5*d*x^5) - \left(\left(3*c\right)/\left(a*e\right) - \left(7*e\right)/d^2\right)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/(40*x^4) + \left(\left(15*c^2*d^4 + 12*a*c*d^2*e^2 - 35*a^2*e^4\right)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/(240*a^2*d^3*e^2*x^3) + \left((c*d^2 - a*e^2)^3*(3*c^2*d^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*\text{ArcTanh}\left[\left(2*a*d*e + (c*d^2 + a*e^2)*x\right)/\left(2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]\right)\right]\right)/\left(256*a^{(7/2)}*d^{(9/2)}*e^{(7/2)}\right)$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**6/(e*x+d),x)`

[Out] Timed out

**Mathematica [A]** time = 0.593417, size = 407, normalized size = 1.03

$$\sqrt{d+ex}\sqrt{ae+cdx} \left( -15x^5 \log(x) (cd^2 - ae^2)^3 (7a^2e^4 + 6acd^2e^2 + 3c^2d^4) + 15x^5 (cd^2 - ae^2)^3 (7a^2e^4 + 6acd^2e^2 + 3c^2d^4) \log \right)$$

Antiderivative was successfully verified.

[In] `Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^6*(d + e*x)),x]`

[Out]  $(\sqrt{a^2e + c^2d^2x} \sqrt{d + ex})^{3/2} (-2\sqrt{a} \sqrt{d} \sqrt{e} \sqrt{a^2e + c^2d^2x} \sqrt{d + ex} (45c^4d^8x^4 - 30a^2c^3d^6e^2x^3 + (d + ex) + 6a^2c^2d^4e^2x^2 (4d^2 + 3d^2ex - 6e^2x^2) + 2a^3c^2d^2e^3x (264d^3 + 48d^2ex - 61d^2e^2x^2 + 95e^3x^3) + a^4e^4 (384d^4 + 48d^3ex - 56d^2e^2x^2 + 70d^2e^3x^3 - 105e^4x^4)) - 15(c^2d^2 - a^2e^2)^3 (3c^2d^4 + 6a^2c^2d^2e^2 + 7a^2e^4)x^5 \log[x] + 15(c^2d^2 - a^2e^2)^3 (3c^2d^4 + 6a^2c^2d^2e^2 + 7a^2e^4)x^5 \log[c^2d^2x + 2\sqrt{a} \sqrt{d} \sqrt{e} \sqrt{a^2e + c^2d^2x} \sqrt{d + ex} + a^2e(2d + ex)]) / (3840 a^{7/2} d^{9/2} e^{7/2} x^5 \sqrt{(a^2e + c^2d^2x)(d + ex)})$

**Maple [B]** time = 0.043, size = 2888, normalized size = 7.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^6/(e*x+d),x)`

[Out]  $-3/128 d^4/a^4/e^3 (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{1/2} x^5 - 1/128 d^2/a^2e (a^2d^2e)^{1/2} \ln((2a^2d^2e + (a^2e^2 + c^2d^2)x + 2(a^2d^2e)^{1/2} (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{1/2})/x) c^3 + 3/256 d^6/a^3/e^3 (a^2d^2e)^{1/2} \ln((2a^2d^2e + (a^2e^2 + c^2d^2)x + 2(a^2d^2e)^{1/2} (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{1/2})/x) c^5 + 1/8 d/a^2/e^2/x^4 (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{5/2} c + 3/64 d/a^4/e^4/x^2 (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{5/2} c^3 - 103/192 d^4/a^2e/x (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{5/2} c - 3/64 e^2d^2/a^3 (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{1/2} x^4 + 1/16 d^7e^{10}a^3/c \ln((1/2 a^2e^2 + 1/2 c^2d^2 + c^2d^2e^2x)/(c^2d^2e)^{1/2} + (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{1/2})/(c^2d^2e)^{1/2} + 3/64 d^2e^3/a (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{1/2} x^2 + 3/16 d^3e^6a^2 \ln((1/2 a^2e^2 + 1/2 c^2d^2 + c^2d^2e^2x)/(c^2d^2e)^{1/2} + (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{1/2})/(c^2d^2e)^{1/2} + (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{1/2})/(c^2d^2e)^{1/2} c - 1/16 d^7e^{10}a^3/c \ln((1/2 a^2e^2 - 1/2 c^2d^2 + (x+d/e) c^2d^2e)/(c^2d^2e)^{1/2} + (c^2d^2e^2(x+d/e)^2 + (a^2e^2 - c^2d^2)(x+d/e))^{1/2})/(c^2d^2e)^{1/2} - 3/16 d^3e^6a^2 \ln((1/2 a^2e^2 - 1/2 c^2d^2 + (x+d/e) c^2d^2e)/(c^2d^2e)^{1/2} + (c^2d^2e^2(x+d/e)^2 + (a^2e^2 - c^2d^2)(x+d/e))^{1/2})/(c^2d^2e)^{1/2} + 9/64 d/a^3/e^2/x^2 (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{5/2} c^2 + 263/384 d^5/a^2e^4c (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{3/2} x + 103/192 d^3/a^2e^2c^2 (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{3/2} x - 1/128 d^3/a^5/e^4c^5 (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{3/2} x - 1/4 d^2/a^2/e/x^3 (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{5/2} c - 3/256 d^4/a^2/e (a^2d^2e)^{1/2} \ln((2a^2d^2e + (a^2e^2 + c^2d^2)x + 2(a^2d^2e)^{1/2} (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{1/2})/x) c^4 + 15/256 d^2a^2e^5/(a^2d^2e)^{1/2} \ln((2a^2d^2e + (a^2e^2 + c^2d^2)x + 2(a^2d^2e)^{1/2} (a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{1/2})/x) c^4$

$$\begin{aligned} & \frac{1}{2} * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} / x * c - 23/96/d^2/a^3/e/x * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(5/2)} * c^2 + 1/128 * d^2/a^5/e^5/x * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(5/2)} * c^4 - 1/32 * d/a^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} * c^3 + 3/8/d^3/a/x^4 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(5/2)} + 7/128/d^5 * a * e^6 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} + 15/128/d^3 * e^4 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} * c + 7/48/a^3/e * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(3/2)} * c^3 - 3/128 * e^3/(a * d * e)^{(1/2)} * \ln((2 * a * d * e + (a * e^2 + c * d^2) * x + 2 * (a * d * e)^{(1/2)}) * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)}) / x * c^2 - 1/8/d^3 * e^4 * c * (c * d * e * (x + d/e)^2 + (a * e^2 - c * d^2) * (x + d/e))^{(1/2)} - 1/8/d^7 * e^8 * a^2/c * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} + 1/32/d * e^2/a * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} * c^2 - 3/16/d^5 * e^8 * a^2 * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x)/(c * d * e)^{(1/2)} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)}) / (c * d * e)^{(1/2)} - 1/16/d * e^4 * c^2 * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x)/(c * d * e)^{(1/2)} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)}) / (c * d * e)^{(1/2)} + 1/4/d^6 * e^7 * a * (c * d * e * (x + d/e)^2 + (a * e^2 - c * d^2) * (x + d/e))^{(1/2)} * x + 1/8/d^7 * e^8 * a^2/c * (c * d * e * (x + d/e)^2 + (a * e^2 - c * d^2) * (x + d/e))^{(1/2)} + 3/16/d^5 * e^8 * a^2 * \ln((1/2 * a * e^2 - 1/2 * c * d^2 + (x + d/e) * c * d * e)/(c * d * e)^{(1/2)} + (c * d * e * (x + d/e)^2 + (a * e^2 - c * d^2) * (x + d/e))^{(1/2)}) / (c * d * e)^{(1/2)} + 1/3/d^6 * e^5 * (c * d * e * (x + d/e)^2 + (a * e^2 - c * d^2) * (x + d/e))^{(3/2)} + 45/128/d^6 * e^5 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(3/2)} - 1/4/d^6 * e^7 * a * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} * x - 1/4/d^4 * e^5 * c * (c * d * e * (x + d/e)^2 + (a * e^2 - c * d^2) * (x + d/e))^{(1/2)} * x + 1/16/d * e^4 * c^2 * \ln((1/2 * a * e^2 - 1/2 * c * d^2 + (x + d/e) * c * d * e)/(c * d * e)^{(1/2)} + (c * d * e * (x + d/e)^2 + (a * e^2 - c * d^2) * (x + d/e))^{(1/2)}) / (c * d * e)^{(1/2)} - 1/32/a^2 * e * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} * x * c^3 - 3/64/a^4/e^3/x * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(5/2)} * c^3 - 1/16/a^3/e^3/x^3 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(5/2)} * c^2 - 1/5/d^2/a/e/x^5 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(5/2)} + 227/384/d^4/a * e^3 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(3/2)} * c + 19/48/d^2/a^2 * e * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(3/2)} * c^2 - 3/128 * d^3/a^3/e^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} * c^4 - 25/48/d^4/a * e/x^3 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(5/2)} - 7/256/d^4 * a^2 * e^7/(a * d * e)^{(1/2)} * \ln((2 * a * d * e + (a * e^2 + c * d^2) * x + 2 * (a * d * e)^{(1/2)}) * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)}) / x - 3/128 * d^5/a^4/e^4 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} * c^5 + 121/192/d^5/a * e^2/x^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(5/2)} - 263/384/d^6/a * e^3/x * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(5/2)} + 3/128 * d^2/a^4/e^3 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(3/2)} * c^4 - 1/128 * d^4/a^5/e^5 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(3/2)} * c^5 + 23/96/d/a^3 * c^3 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(3/2)} * x + 73/192/d^3/a^2/x^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(5/2)} * c + 39/128/d^4 * e^5 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} * x * c \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)\*x^6),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Ericas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)\*x^6),x, algorithm="ericas")

[Out] Exception raised: TypeError

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**6/(e*x+d),x)`

[Out] Timed out

---

**GIAC/XCAS [A]** time = 17.9608, size = 1, normalized size = 0.

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)*x^6),x, algorithm="giac")`

[Out] Done

$$3.456 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{x^7(d+ex)} dx$$

**Optimal.** Leaf size=498

$$\begin{aligned} & \frac{(-21a^2e^4 + 6acd^2e^2 + 7c^2d^4) (x (ae^2 + cd^2) + ade + cdex^2)^{3/2}}{160a^2d^3e^2x^4} \\ & + \frac{(-21a^4e^8 + 6a^2c^2d^4e^4 + 8ac^3d^6e^2 + 7c^4d^8) (x (ae^2 + cd^2) + 2ade) \sqrt{x (ae^2 + cd^2) + ade + cdex^2}}{512a^4d^5e^4x^2} \\ & - \frac{(-105a^3e^6 + 21a^2cd^2e^4 + 33ac^2d^4e^2 + 35c^3d^6) (x (ae^2 + cd^2) + ade + cdex^2)^{3/2}}{960a^3d^4e^3x^3} \\ & - \frac{(21a^3e^6 + 21a^2cd^2e^4 + 15ac^2d^4e^2 + 7c^3d^6) (cd^2 - ae^2)^3 \tanh^{-1} \left( \frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \right)}{1024a^{9/2}d^{11/2}e^{9/2}} \\ & - \frac{(x (ae^2 + cd^2) + ade + cdex^2)^{3/2}}{6dx^6} - \frac{\left(\frac{c}{ae} - \frac{3e}{d^2}\right) (x (ae^2 + cd^2) + ade + cdex^2)^{3/2}}{20x^5} \end{aligned}$$

[Out]  $((7*c^4*d^8 + 8*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 21*a^4*e^8) * (2*a*d*e + (c*d^2 + a*e^2)*x) * \text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (512*a^4*d^5*e^4*x^2) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)} / (6*d*x^6) - ((c/(a*e) - (3*e)/d^2) * (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) / (20*x^5) + ((7*c^2*d^4 + 6*a*c*d^2*e^2 - 21*a^2*e^4) * (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) / (160*a^2*d^3*e^2*x^4) - ((35*c^3*d^6 + 33*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^4 - 105*a^3*e^6) * (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) / (960*a^3*d^4*e^3*x^3) - ((c*d^2 - a*e^2)^3 * (7*c^3*d^6 + 15*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^4 + 21*a^3*e^6) * \text{ArcTanh}[(2*a*d*e + (c*d^2 + a*e^2)*x) / (2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]) / (1024*a^{(9/2)}*d^{(11/2)}*e^{(9/2)})$

**Rubi [A]** time = 1.88759, antiderivative size = 498, normalized size of antiderivative = 1., number of rules used = 8, number of rules used = 6, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\begin{aligned} & \frac{(-21a^2e^4 + 6acd^2e^2 + 7c^2d^4) (x (ae^2 + cd^2) + ade + cdex^2)^{3/2}}{160a^2d^3e^2x^4} \\ & + \frac{(-21a^4e^8 + 6a^2c^2d^4e^4 + 8ac^3d^6e^2 + 7c^4d^8) (x (ae^2 + cd^2) + 2ade) \sqrt{x (ae^2 + cd^2) + ade + cdex^2}}{512a^4d^5e^4x^2} \\ & - \frac{(-105a^3e^6 + 21a^2cd^2e^4 + 33ac^2d^4e^2 + 35c^3d^6) (x (ae^2 + cd^2) + ade + cdex^2)^{3/2}}{960a^3d^4e^3x^3} \\ & - \frac{(21a^3e^6 + 21a^2cd^2e^4 + 15ac^2d^4e^2 + 7c^3d^6) (cd^2 - ae^2)^3 \tanh^{-1} \left( \frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \right)}{1024a^{9/2}d^{11/2}e^{9/2}} \\ & - \frac{(x (ae^2 + cd^2) + ade + cdex^2)^{3/2}}{6dx^6} - \frac{\left(\frac{c}{ae} - \frac{3e}{d^2}\right) (x (ae^2 + cd^2) + ade + cdex^2)^{3/2}}{20x^5} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)} / (x^7*(d + e*x)), x]$

[Out]  $((7*c^4*d^8 + 8*a*c^3*d^6*e^2 + 6*a^2*c^2*d^4*e^4 - 21*a^4*e^8) * (2*a*d*e + (c*d^2 + a*e^2)*x) * \text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (512*a^4*d^5*e^4*x^2) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)} / (6*d*x^6) - ((c/(a*e) - (3*e)/d^2) * (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) / (20*x^5) + ((7*c^2*d^4 + 6*a*c*d^2*e^2 - 21*a^2*e^4) * (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) / (160*a^2*d^3*e^2*x^4) - ((35*c^3*d^6 + 33*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^4 - 105*a^3*e^6) * (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) / (960*a^3*d^4*e^3*x^3) - ((c*d^2 - a*e^2)^3 * (7*c^3*d^6 + 15*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^4 + 21*a^3*e^6) * \text{ArcTanh}[(2*a*d*e + (c*d^2 + a*e^2)*x) / (2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]) / (1024*a^{(9/2)}*d^{(11/2)}*e^{(9/2)})$

$$d^2 + a e^2) x + c d e x^2]] / (1024 a^{9/2} d^{11/2} e^{9/2})$$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/x**7/(e*x+d), x)`

[Out] Timed out

**Mathematica [A]** time = 0.871886, size = 506, normalized size = 1.02

$$\sqrt{d+ex}\sqrt{ae+cdx} \left( 15x^6 \log(x) (cd^2 - ae^2)^3 (21a^3e^6 + 21a^2cd^2e^4 + 15ac^2d^4e^2 + 7c^3d^6) - 15x^6 (cd^2 - ae^2)^3 (21a^3e^6 + 21a^2cd^2e^4 + 15ac^2d^4e^2 + 7c^3d^6) \right)$$

Antiderivative was successfully verified.

[In] `Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/(x^7*(d + e*x)), x]`

[Out]  $(\sqrt{a e + c d x} \sqrt{d + e x} (-2 \sqrt{a} \sqrt{d} \sqrt{e} \sqrt{a e + c d x} \sqrt{d + e x} (-105 c^5 d^{10} x^5 + 5 a^5 c^4 d^8 e^5 x^4 (14 d + 11 e x) - 2 a^2 c^3 d^6 e^2 x^3 (28 d^2 + 16 d e x - 27 e^2 x^2) + 6 a^3 c^2 d^4 e^3 x^2 (8 d^3 + 4 d^2 e x - 6 d e^2 x^2 + 13 e^3 x^3) + a^4 c d^2 e^4 x (1664 d^4 + 224 d^3 e x - 264 d^2 e^2 x^2 + 336 d e^3 x^3 - 525 e^4 x^4) + a^5 e^5 (1280 d^5 + 128 d^4 e x - 144 d^3 e^2 x^2 + 168 d^2 e^3 x^3 - 210 d e^4 x^4 + 315 e^5 x^5)) + 15 (c d^2 - a e^2)^3 (7 c^3 d^6 + 15 a^5 c^2 d^4 e^2 + 21 a^2 c^3 d^2 e^4 + 21 a^3 e^6) x^6 \text{Log}[x] - 15 (c d^2 - a e^2)^3 (7 c^3 d^6 + 15 a^5 c^2 d^4 e^2 + 21 a^2 c^3 d^2 e^4 + 21 a^3 e^6) x^6 \text{Log}[c d^2 x + 2 \sqrt{a} \sqrt{d} \sqrt{e} \sqrt{a e + c d x} \sqrt{d + e x} + a e (2 d + e x)]) / (15360 a^{9/2} d^{11/2} e^{9/2} x^6 \sqrt{(a e + c d x) (d + e x)})$

**Maple [B]** time = 0.058, size = 3387, normalized size = 6.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/x^7/(e*x+d), x)`

[Out]  $1/8/d^4 e^5 c (c d e (x+d/e)^2 + (a e^2 - c d^2) (x+d/e))^{1/2} - 21/512/d^6 a^5 e^7 (a d e + (a e^2 + c d^2) x + c d e x^2)^{1/2} - 1/8/d^4 c^5 e^5 (a d e + (a e^2 + c d^2) x + c d e x^2)^{5/2} - 23/96/d/a^3 (a d e + (a e^2 + c d^2) x + c d e x^2)^{3/2} + 1/64 c^3/a^2 e (a d e + (a e^2 + c d^2) x + c d e x^2)^{1/2} - 1/3/d^7 e^6 (c d e (x+d/e)^2 + (a e^2 - c d^2) (x+d/e))^{3/2} - 533/1536/d^7 e^6 (a d e + (a e^2 + c d^2) x + c d e x^2)^{3/2} - 1/16/d^8 e^{11} a^3/c \ln((1/2 a e^2 + 1/2 c d^2 + c d e x)/(c d e))^{1/2} + (a d e + (a e^2 + c d^2) x + c d e x^2)^{1/2} / (c d e)^{1/2} - 3/16/d^4 e^7 a^5 c \ln((1/2 a e^2 + 1/2 c d^2 + c d e x)/(c d e))^{1/2} + (a d e + (a e^2 + c d^2) x + c d e x^2)^{1/2} / (c d e)^{1/2} + 41/1536 d/a^5 e^4/x (a d e + (a e^2 + c d^2) x + c d e x^2)^{5/2} + c^4 - 7/1536 d^3/a^6 e^6/x (a d e + (a e^2 + c d^2) x + c d e x^2)^{5/2} + c^5 - 7/768 d^2/a^5 e^5/x^2 (a d e + (a e^2 + c d^2) x + c d e x^2)^{5/2} + c^4 + 3/512 d^5/a^3 e^2/(a$

$$\begin{aligned}
& d^*e)^{(1/2)} * \ln((2^*a^*d^*e+(a^*e^2+c^*d^2)^*x+2^*(a^*d^*e)^{(1/2)} * (a^*d^*e+(a^* \\
& e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(1/2)})/x)^*c^5-7/1024^*d^7/a^4/e^4/(a^*d^*e)^{(1/2)} * \ln((2^*a^*d^*e+(a^*e^2+c^*d^2)^*x+2^*(a^*d^*e)^{(1/2)} * (a^*d^*e+(a^*e^2+c^* \\
& d^2)^*x+c^*d^*e^*x^2)^{(1/2)})/x)^*c^6+877/1536/d^5/a^2^*e^2/x^*(a^*d^*e+(a^* \\
& e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(5/2)} * c+15/512^*d^3/a^4/e^2^*(a^*d^*e+(a^*e^2 \\
& +c^*d^2)^*x+c^*d^*e^*x^2)^{(1/2)} * x^*c^5+7/512^*d^5/a^5/e^4^*(a^*d^*e+(a^*e^2+ \\
& c^*d^2)^*x+c^*d^*e^*x^2)^{(1/2)} * x^*c^6+29/192/d/a^3/e^2/x^3^*(a^*d^*e+(a^*e^2 \\
& +c^*d^2)^*x+c^*d^*e^*x^2)^{(5/2)} * c^2+7/192^*d/a^4/e^4/x^3^*(a^*d^*e+(a^*e^2 \\
& +c^*d^2)^*x+c^*d^*e^*x^2)^{(5/2)} * c^3-257/768/d^2/a^3^*e^*c^3^*(a^*d^*e+(a^*e^2 \\
& +c^*d^2)^*x+c^*d^*e^*x^2)^{(3/2)} * x-91/384/d^2/a^3/e/x^2^*(a^*d^*e+(a^*e^2+ \\
& c^*d^2)^*x+c^*d^*e^*x^2)^{(5/2)} * c^2+1/256^*d/a^*e^2/(a^*d^*e)^{(1/2)} * \ln((2^*a^* \\
& d^*e+(a^*e^2+c^*d^2)^*x+2^*(a^*d^*e)^{(1/2)} * (a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^* \\
& x^2)^{(1/2)})/x)^*c^3-21/512/d^3/a^*e^4^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^* \\
& x^2)^{(1/2)} * x^*c^2-11/48/d^2/a^2/e/x^4^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^* \\
& e^*x^2)^{(5/2)} * c-21/512/d^3^*a^*e^6/(a^*d^*e)^{(1/2)} * \ln((2^*a^*d^*e+(a^*e^2+ \\
& c^*d^2)^*x+2^*(a^*d^*e)^{(1/2)} * (a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(1/2)}) \\
& /x)^*c+7/60/d/a^2/e^2/x^5^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(5/2)} * \\
& c+3/256/d/a^2^*e^2^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(1/2)} * x^*c^3-4 \\
& 1/1536^*d^2/a^5/e^3^*c^5^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(3/2)} * x- \\
& 43/96/d^4/a^2^*e/x^2^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(5/2)} * c-104 \\
& 5/1536/d^6/a^*e^5^*c^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(3/2)} * x-877/ \\
& 1536/d^4/a^2^*e^3^*c^2^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(3/2)} * x+7/ \\
& 1536^*d^4/a^6/e^5^*c^6^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(3/2)} * x+10 \\
& 9/768/d/a^4/e^2/x^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(5/2)} * c^3+1/1 \\
& 6/d^8^*e^11^*a^3/c^* \ln((1/2^*a^*e^2-1/2^*c^*d^2+(x+d/e)^*c^*d^*e)/(c^*d^*e)^{(1/2)} \\
& +(c^*d^*e^*(x+d/e)^2+(a^*e^2-c^*d^2)^*(x+d/e))^{(1/2)})/(c^*d^*e)^{(1/2)} \\
& +3/16/d^4^*e^7^*a^*c^* \ln((1/2^*a^*e^2-1/2^*c^*d^2+(x+d/e)^*c^*d^*e)/(c^*d^*e)^{(1/2)} \\
& +(c^*d^*e^*(x+d/e)^2+(a^*e^2-c^*d^2)^*(x+d/e))^{(1/2)})/(c^*d^*e)^{(1/2)} \\
& )+3/16/d^6^*e^9^*a^2^* \ln((1/2^*a^*e^2+1/2^*c^*d^2+c^*d^*e^*x)/(c^*d^*e)^{(1/2)} \\
& +(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(1/2)})/(c^*d^*e)^{(1/2)}+1/16/d^2^* \\
& e^5^*c^2^* \ln((1/2^*a^*e^2+1/2^*c^*d^2+c^*d^*e^*x)/(c^*d^*e)^{(1/2)}+(a^*d^*e+(a^* \\
& e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(1/2)})/(c^*d^*e)^{(1/2)}-7/96/a^3/e^3/x^4^*(a^* \\
& d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(5/2)} * c^2-109/768/a^4/e^*c^4^*(a^*d^*e \\
& +(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(3/2)} * x-1/12/a^4/e^3/x^2^*(a^*d^*e+(a^*e^2 \\
& +c^*d^2)^*x+c^*d^*e^*x^2)^{(5/2)} * c^3+3/1024^*d^3/a^2/(a^*d^*e)^{(1/2)} * \ln(( \\
& 2^*a^*d^*e+(a^*e^2+c^*d^2)^*x+2^*(a^*d^*e)^{(1/2)} * (a^*d^*e+(a^*e^2+c^*d^2)^*x+c^* \\
& d^*e^*x^2)^{(1/2)})/x)^*c^4+65/192/d^3/a^2/x^3^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+ \\
& c^*d^*e^*x^2)^{(5/2)} * c+7/256^*d/a^3^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(1/2)} * x^*c^4+15/1024/d^*e^4/(a^*d^*e)^{(1/2)} * \ln((2^*a^*d^*e+(a^*e^2+c^*d^2)^* \\
& x+2^*(a^*d^*e)^{(1/2)} * (a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(1/2)})/x)^*c^2 \\
& +257/768/d^3/a^3/x^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(5/2)} * c^2-1 \\
& 49/512/d^5^*e^6^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(1/2)} * x^*c-1/6/d^2/a/e/x^6^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(5/2)}+1/64^*d^4^*c^5/a^4/e^3^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(1/2)}-235/384/d^5/a^*e^4^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(3/2)} * c-15/512/d^2/a^*e^3^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(1/2)} * c^2+7/1536^*d^5/a^6/e^6^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(3/2)} * c^6+13/512^*d^2/a^3/e^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(1/2)} * c^4+7/512^*d^6/a^5/e^5^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(1/2)} * c^6-491/768/d^6/a^*e^3/x^2^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(5/2)}+21/1024/d^5^*a^2^*e^8/(a^*d^*e)^{(1/2)} * \ln((2^*a^*d^*e+(a^*e^2+c^*d^2)^*x+2^*(a^*d^*e)^{(1/2)} * (a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(1/2)})/x)+1045/1536/d^7/a^*e^4/x^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(5/2)}+107/192/d^5/a^*e^2/x^3^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(5/2)}-43/96/d^4/a^*e/x^4^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(5/2)}-703/1536/d^3/a^2^*e^2^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(3/2)} * c^2-131/1536^*d/a^4/e^2^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(3/2)} * c^4-5/384^*d^3/a^5/e^4^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(3/2)} * c^5-1/4/d^7^*e^8^*a^*(c^*d^*e^*(x+d/e)^2+(a^*e^2-c^*d^2)^*(x+d/e))^{(1/2)} * x-1/8/d^8^*e^9^*a^2/c^*(c^*d^*e^*(x+d/e)^2+(a^*e^2-c^*d^2)^*(x+d/e))^{(1/2)}-3/16/d^6^*e^9^*a^2^* \ln((1/2^*a^*e^2-1/2^*c^*d^2+(x+d/e)^*c^*d^*e)/(c^*d^*e)^{(1/2)}+(c^*d^*e^*(x+d/e)^2+(a^*e^2-c^*d^2)^*(x+d/e))^{(1/2)})/(c^*d^*e)^{(1/2)}+1/4/d^5^*e^6^*c^*(c^*d^*e^*(x+d/e)^2+(a^*e^2-c^*d^2)^*(x+d/e))^{(1/2)} * x-1/16/d^2^*e^5^*c^2^* \ln((1/2^*a^*e^2-1/2^*c^*d^2+(x+d/e)^*c^*d^*e)/(c^*d^*e)^{(1/2)}+(c^*d^*e^*(x+d/e)^2+(a^*e^2-c^*d^2)^*(x+d/e))^{(1/2)})/(c^*d^*e)^{(1/2)}+1/8/d^8^*e^9^*a^2/c^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(1/2)}+1/4/d^7^*e^8^*a^*(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(1/2)} * x
\end{aligned}$$



**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)\*x^7), x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)\*x^7), x, algorithm="fricas")

[Out] Exception raised: TypeError

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/x\*\*7/(e\*x+d), x, algorithm="sympy")

[Out] Timed out

---

**GIAC/XCAS [A]** time = 50.9192, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)\*x^7), x, algorithm="giac")

[Out] Done

$$3.457 \quad \int \frac{x^3 (ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex} dx$$

**Optimal.** Leaf size=574

$$\frac{(-105a^3e^6 - 10cdex(-15a^2e^4 - 10acd^2e^2 + 33c^2d^4) - 95a^2cd^2e^4 - 15ac^2d^4e^2 + 231c^3d^6)(x(ae^2 + cd^2) + ade + cdex^2)^5}{4480c^3d^3e^4} + \frac{3(15a^3e^6 + 35a^2cd^2e^4 + 45ac^2d^4e^2 + 33c^3d^6)(cd^2 - ae^2)^5 \tanh^{-1}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{32768c^{11/2}d^{11/2}e^{13/2}} + \frac{3(15a^3e^6 + 35a^2cd^2e^4 + 45ac^2d^4e^2 + 33c^3d^6)(cd^2 - ae^2)^3(ae^2 + cd^2 + 2cdex)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{16384c^5d^5e^6} + \frac{(15a^3e^6 + 35a^2cd^2e^4 + 45ac^2d^4e^2 + 33c^3d^6)(cd^2 - ae^2)(ae^2 + cd^2 + 2cdex)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{2048c^4d^4e^5} + \frac{1}{112}x^2\left(\frac{5a}{cd} - \frac{11d}{e^2}\right)(x(ae^2 + cd^2) + ade + cdex^2)^{5/2} + \frac{x^3(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{8e}$$

[Out]  $(-3*(c*d^2 - a*e^2)^3*(33*c^3*d^6 + 45*a*c^2*d^4*e^2 + 35*a^2*c*d^2*e^4 + 15*a^3*e^6)*(c*d^2 + a*e^2 + 2*c*d*e*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(16384*c^5*d^5*e^6) + ((c*d^2 - a*e^2)^2*(33*c^3*d^6 + 45*a*c^2*d^4*e^2 + 35*a^2*c*d^2*e^4 + 15*a^3*e^6)*(c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(2048*c^4*d^4*e^5) + (((5*a)/(c*d) - (11*d)/e^2)*x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/112 + (x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(8*e) - ((231*c^3*d^6 - 15*a*c^2*d^4*e^2 - 95*a^2*c*d^2*e^4 - 105*a^3*e^6 - 10*c*d*e*(33*c^2*d^4 - 10*a*c*d^2*e^2 - 15*a^2*e^4)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(4480*c^3*d^3*e^4) + (3*(c*d^2 - a*e^2)^5*(33*c^3*d^6 + 45*a*c^2*d^4*e^2 + 35*a^2*c*d^2*e^4 + 15*a^3*e^6)*\text{ArcTanh}[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(32768*c^{(11/2)}*d^{(11/2)}*e^{(13/2)})$

**Rubi [A]** time = 1.74236, antiderivative size = 574, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{(-105a^3e^6 - 10cdex(-15a^2e^4 - 10acd^2e^2 + 33c^2d^4) - 95a^2cd^2e^4 - 15ac^2d^4e^2 + 231c^3d^6)(x(ae^2 + cd^2) + ade + cdex^2)^5}{4480c^3d^3e^4} + \frac{3(15a^3e^6 + 35a^2cd^2e^4 + 45ac^2d^4e^2 + 33c^3d^6)(cd^2 - ae^2)^5 \tanh^{-1}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{32768c^{11/2}d^{11/2}e^{13/2}} + \frac{3(15a^3e^6 + 35a^2cd^2e^4 + 45ac^2d^4e^2 + 33c^3d^6)(cd^2 - ae^2)^3(ae^2 + cd^2 + 2cdex)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{16384c^5d^5e^6} + \frac{(15a^3e^6 + 35a^2cd^2e^4 + 45ac^2d^4e^2 + 33c^3d^6)(cd^2 - ae^2)(ae^2 + cd^2 + 2cdex)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{2048c^4d^4e^5} + \frac{1}{112}x^2\left(\frac{5a}{cd} - \frac{11d}{e^2}\right)(x(ae^2 + cd^2) + ade + cdex^2)^{5/2} + \frac{x^3(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{8e}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(d + e*x), x]$

[Out]  $(-3*(c*d^2 - a*e^2)^3*(33*c^3*d^6 + 45*a*c^2*d^4*e^2 + 35*a^2*c*d^2*e^4 + 15*a^3*e^6)*(c*d^2 + a*e^2 + 2*c*d*e*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(16384*c^5*d^5*e^6) + ((c*d^2 - a*e^2)^2*(33*c^3*d^6 + 45*a*c^2*d^4*e^2 + 35*a^2*c*d^2*e^4 + 15*a^3*e^6)*(c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(2048*c^4*d^4*e^5) + (((5*a)/(c*d) - (11*d)/e^2)*x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/112 + (x^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(8*e) - ((231*c^3*d^6 - 15*a*c^2*d^4*e^2 - 95*a^2*c*d^2*e^4 - 105*a^3*e^6 - 10*c*d*e*(33*c^2*d^4 - 10*a*c*d^2*e^2 - 15*a^2*e^4)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(4480*c^3*d^3*e^4) + (3*(c*d^2 - a*e^2)^5*(33*c^3*d^6 + 45*a*c^2*d^4*e^2 + 35*a^2*c*d^2*e^4 + 15*a^3*e^6)*\text{ArcTanh}[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(32768*c^{(11/2)}*d^{(11/2)}*e^{(13/2)})$

$$\frac{(d^2 x^4 - 10 a c d^2 e^2 - 15 a^2 e^4) x (a d e + (c d^2 + a e^2) x + c d e x^2)^{5/2}}{(4480 c^3 d^3 e^4 + (3 (c d^2 - a e^2)^5 + (33 c^3 d^6 + 45 a c^2 d^4 e^2 + 35 a^2 c d^2 e^4 + 15 a^3 e^6) \operatorname{ArcTanh}[(c d^2 + a e^2 + 2 c d e x)/(2 \sqrt{c} \sqrt{d} \sqrt{e} \sqrt{d+ex} \sqrt{ae+cdx+ae^2+cd(d+2ex)})]) / (32768 c^{11/2} d^{11/2} e^{13/2}))} x$$

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)`

[Out] Timed out

**Mathematica [A]** time = 1.62941, size = 579, normalized size = 1.01

$$\frac{(d+ex)(ae+cdx)^{3/2} \left( \frac{3(15a^3e^6+35a^2cd^2e^4+45ac^2d^4e^2+33c^3d^6)(cd^2-ae^2)^5 \log\left(2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx+ae^2+cd(d+2ex)}\right)}{c^{11/2}d^{11/2}e^{13/2}(d+ex)^{3/2}(ae+cdx)^{3/2}} \right) + \frac{2(1575a^7e^{14}-525a^6c^2d^2e^{12}(7d+2ex)+35a^5c^2d^2e^{10}(29d^2+68de^2+24e^2x^2)+5a^4c^3d^3e^8(185d^3-110d^2e^2x-376de^2x^2-144e^3x^3)+5a^3c^4d^4e^6(265d^4-120d^3e^2x+80d^2e^2x^2+320de^3x^3+128e^4x^4)+a^2c^5d^5e^4(-11193d^5+7034d^4e^2x-5488d^3e^2x^2+4640d^2e^3x^3+137600de^4x^4+103680e^5x^5)+ac^6d^6e^2(11445d^6-7476d^5e^2x+5928d^4e^2x^2-5056d^3e^3x^3+4480d^2e^4x^4+212480de^5x^5+168960e^6x^6)+c^7d^7(-3465d^7+2310d^6e^2x-1848d^5e^2x^2+1584d^4e^3x^3-1408d^3e^4x^4+1280d^2e^5x^5+87040de^6x^6+71680e^7x^7))}{(35c^5d^5e^6(ae+cdx)(d+ex)) + (3(c d^2 - a e^2)^5 (33 c^3 d^6 + 45 a c^2 d^4 e^2 + 35 a^2 c d^2 e^4 + 15 a^3 e^6) \operatorname{Log}[a e^2 + 2 \sqrt{c} \sqrt{d} \sqrt{e} \sqrt{d+ex} \sqrt{ae+cdx+ae^2+cd(d+2ex)}] + c d (d + 2 e x)) / (c^{11/2} d^{11/2} e^{13/2} (d+ex)^{3/2} (ae+cdx)^{3/2} (d+ex)^{3/2})}{32768}}$$

Antiderivative was successfully verified.

[In] `Integrate[(x^3*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^(5/2))/(d+e*x),x]`

[Out]  $((a e + c d x) (d + e x))^{3/2} \left( \frac{2 (1575 a^7 e^{14} - 525 a^6 c^2 d^2 e^{12} (7 d + 2 e x) + 35 a^5 c^2 d^2 e^{10} (29 d^2 + 68 d e^2 + 24 e^2 x^2) + 5 a^4 c^3 d^3 e^8 (185 d^3 - 110 d^2 e^2 x - 376 d e^2 x^2 - 144 e^3 x^3) + 5 a^3 c^4 d^4 e^6 (265 d^4 - 120 d^3 e^2 x + 80 d^2 e^2 x^2 + 320 d e^3 x^3 + 128 e^4 x^4) + a^2 c^5 d^5 e^4 (-11193 d^5 + 7034 d^4 e^2 x - 5488 d^3 e^2 x^2 + 4640 d^2 e^3 x^3 + 137600 d e^4 x^4 + 103680 e^5 x^5) + a c^6 d^6 e^2 (11445 d^6 - 7476 d^5 e^2 x + 5928 d^4 e^2 x^2 - 5056 d^3 e^3 x^3 + 4480 d^2 e^4 x^4 + 212480 d e^5 x^5 + 168960 e^6 x^6) + c^7 d^7 (-3465 d^7 + 2310 d^6 e^2 x - 1848 d^5 e^2 x^2 + 1584 d^4 e^3 x^3 - 1408 d^3 e^4 x^4 + 1280 d^2 e^5 x^5 + 87040 d e^6 x^6 + 71680 e^7 x^7)}{(35 c^5 d^5 e^6 (a e + c d x) (d + e x)) + (3 (c d^2 - a e^2)^5 (33 c^3 d^6 + 45 a c^2 d^4 e^2 + 35 a^2 c d^2 e^4 + 15 a^3 e^6) \operatorname{Log}[a e^2 + 2 \sqrt{c} \sqrt{d} \sqrt{e} \sqrt{d+ex} \sqrt{ae+cdx+ae^2+cd(d+2ex)}] + c d (d + 2 e x)) / (c^{11/2} d^{11/2} e^{13/2} (d+ex)^{3/2} (ae+cdx)^{3/2} (d+ex)^{3/2})}{32768} \right)$

**Maple [B]** time = 0.034, size = 3178, normalized size = 5.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x)`

[Out]  $-15/1024 e^4/c^3/d^3 (a d e + (a e^2 + c d^2) x + c d e x^2)^{3/2} x^4 + 45/8192 e^7/c^4/d^4 (a d e + (a e^2 + c d^2) x + c d e x^2)^{1/2} x^4 a^6 - 15/4096 e^5/c^3/d^2 (a d e + (a e^2 + c d^2) x + c d e x^2)^{1/2} x^5 a^5 - 45/32768 e^{10}/c^5/d^5 \ln\left(\frac{(1/2 a e^2 + 1/2 c d^2 + c d e x)^{1/2}}{(c d e)^{1/2}} + \frac{(a d e + (a e^2 + c d^2) x + c d e x^2)^{1/2}}{(c d e)^{1/2}}\right) a^4$

$$\begin{aligned}
& 8+15/4096 * e^8 / c^4 / d^3 * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x) / (c * d * e)^{(1/2)} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)}) / (c * d * e)^{(1/2)} * a^7 - 49 \\
& 5/4096 * d^6 / e^3 * c * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} * x * a - 15/8 \\
& 192 / d * e^6 / c^3 * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x) / (c * d * e)^{(1/2)} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)}) / (c * d * e)^{(1/2)} * a^6 + 45/4096 * d * \\
& e^4 / c^2 * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x) / (c * d * e)^{(1/2)} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)}) / (c * d * e)^{(1/2)} * a^5 - 975/16384 * d^3 * e^4 \\
& 2 / c * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x) / (c * d * e)^{(1/2)} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)}) / (c * d * e)^{(1/2)} * a^4 + 195/4096 * d^9 / e^4 * c^2 \\
& * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x) / (c * d * e)^{(1/2)} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)}) / (c * d * e)^{(1/2)} * a - 855/8192 * d^7 / e^2 * c * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x) / (c * d * e)^{(1/2)} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)}) / (c * d * e)^{(1/2)} * a^2 - 105/2048 * d^2 * e / c * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} * x * a^3 + 3/64 * d^2 * e * a^3 / c * (c * d * e * (x + d / e)^2 + (a * e^2 - c * d^2) * (x + d / e))^{(1/2)} * x + 9/64 * d^6 / e^3 * a * c * (c * d * e * (x + d / e)^2 + (a * e^2 - c * d^2) * (x + d / e))^{(1/2)} * x - 3/256 * d * e^4 * a^5 / c^2 * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x) / (c * d * e)^{(1/2)} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)}) / (c * d * e)^{(1/2)} + (c * d * e * (x + d / e)^2 + (a * e^2 - c * d^2) * (x + d / e))^{(1/2)} / (c * d * e)^{(1/2)} + 15/128 * d^7 / e^2 * a^2 * c * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x) / (c * d * e)^{(1/2)} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)}) / (c * d * e)^{(1/2)} + (c * d * e * (x + d / e)^2 + (a * e^2 - c * d^2) * (x + d / e))^{(1/2)} / (c * d * e)^{(1/2)} - 15/256 * d^9 / e^4 * a * c^2 * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x) / (c * d * e)^{(1/2)} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)}) / (c * d * e)^{(1/2)} + (c * d * e * (x + d / e)^2 + (a * e^2 - c * d^2) * (x + d / e))^{(1/2)} / (c * d * e)^{(1/2)} + 15/256 * d^3 * e^2 * a^4 / c * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x) / (c * d * e)^{(1/2)} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)}) / (c * d * e)^{(1/2)} + (c * d * e * (x + d / e)^2 + (a * e^2 - c * d^2) * (x + d / e))^{(1/2)} / (c * d * e)^{(1/2)} - 5/256 * e^2 / c^2 * a^3 / d * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(3/2)} * x + 3/64 * e / c^2 / d^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(5/2)} * x * a^2 + 1/16 * d^6 / e^5 * c * (c * d * e * (x + d / e)^2 + (a * e^2 - c * d^2) * (x + d / e))^{(3/2)} - 3/128 * d^9 / e^6 * c^2 * (c * d * e * (x + d / e)^2 + (a * e^2 - c * d^2) * (x + d / e))^{(1/2)} - 15/128 * d^5 * a^3 * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x) / (c * d * e)^{(1/2)} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)}) / (c * d * e)^{(1/2)} + (c * d * e * (x + d / e)^2 + (a * e^2 - c * d^2) * (x + d / e))^{(1/2)} / (c * d * e)^{(1/2)} - 3/64 * d^3 * a^3 / c * (c * d * e * (x + d / e)^2 + (a * e^2 - c * d^2) * (x + d / e))^{(1/2)} + 13/128 / c^2 / d * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(5/2)} * a^2 - 95/2048 * d^6 / e^5 * c * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(3/2)} + 285/16384 * d^9 / e^6 * c^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} + 19/64 * d^2 / e^3 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(5/2)} * x + 45/2048 * d^4 / e^3 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(3/2)} * a + 165/16384 * d^5 / e^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} * a^2 + 465/4096 * d^5 * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x) / (c * d * e)^{(1/2)} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)}) / (c * d * e)^{(1/2)} * a^3 - 15/1024 * e / c^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(3/2)} * a^3 + 735/16384 * d^3 / c * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} * a^3 - 1/5 * d^3 / e^4 * (c * d * e * (x + d / e)^2 + (a * e^2 - c * d^2) * (x + d / e))^{(5/2)} - 25/112 / e^3 / c * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(7/2)} + 19/128 * d^3 / e^4 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(5/2)} + 1155/8192 * d^4 / e * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} * x * a^2 + 29/128 * d / e^2 / c * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(5/2)} * a + 65/1024 * d^2 / e / c * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(3/2)} * a^2 + 285/8192 * d^8 / e^5 * c^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} * x - 75/16384 / d * e^4 / c^3 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} * a^5 - 465/16384 * d * e^2 / c^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} * a^4 - 95/1024 * d^5 / e^4 * c * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(3/2)} * x - 705/16384 * d^7 / e^4 * c * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} * a - 285/32768 * d^11 / e^6 * c^3 * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x) / (c * d * e)^{(1/2)} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)}) / (c * d * e)^{(1/2)} - 5/512 * d / c * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(3/2)} * x * a^2 - 45/8192 * e^3 / c^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} * x * a^4 - 1/8 * d^3 / e^2 * a * (c * d * e * (x + d / e)^2 + (a * e^2 - c * d^2) * (x + d / e))^{(3/2)} * x + 1/8 * d^5 / e^4 * c * (c * d * e * (x + d / e)^2 + (a * e^2 - c * d^2) * (x + d / e))^{(3/2)} * x + 3/64 * d^7 / e^4 * a * c * (c * d * e * (x + d / e)^2 + (a * e^2 - c * d^2) * (x + d / e))^{(1/2)} - 3/64 * d^8 / e^5 * c^2 * (c * d * e * (x + d / e)^2 + (a * e^2 - c * d^2) * (x + d / e))^{(1/2)} * x + 3/256 * d^11 / e^6 * c^3 * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x) / (c * d * e)^{(1/2)} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)}) / (c * d * e)^{(1/2)} + (c * d * e * (x + d / e)^2 + (a * e^2 - c * d^2) * (x + d / e))^{(1/2)} / (c * d * e)^{(1/2)} - 1/16 * d^2 / e * a^2 / c * (c * d * e * (x + d / e)^2 + (a * e^2 - c * d^2) * (x + d / e))^{(3/2)} - 9/64 * d^4 / e * a^2 * (c * d * e * (x + d / e)^2 + (a * e^2 - c * d^2) * (x + d / e))^{(1/2)} * x + 3/128 * d * e^2 * a^4 / c^2 * (c * d * e * (x + d / e)^2 + (a * e^2 - c * d^2) * (x + d / e))^{(1/2)} + 1/8 / e^2 * x * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(7/2)} / c / d + 3/128 * e^2 / c^3 / d^3 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(5/2)} * a^3 - 15/2048 * e^5 / c^4 / d^4 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(3/2)} * a^5 - 35/2048 * e^3 / c^3 / d^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(3/2)} * a^4 + 45/16384 * e^8 / c^5 / d^5 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} * a^7 + 15/16384 * e^6 / c^4 / d^3 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} * a^6 + 35/256 * d^3 / e^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(3/2)} * x * a + 5/32 / e / c * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(5/2)} * x * a - 9/112 / e / c^2 / d^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(7/2)} * a
\end{aligned}$$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*x^3/(e\*x + d),x, algorithm

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.426165, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*x^3/(e\*x + d),x, algorithm

[Out] [1/2293760\*(4\*(71680\*c^7\*d^7\*e^7\*x^7 - 3465\*c^7\*d^14 + 11445\*a\*c^6\*d^12\*e^2 - 11193\*a^2\*c^5\*d^10\*e^4 + 1325\*a^3\*c^4\*d^8\*e^6 + 925\*a^4\*c^3\*d^6\*e^8 + 1015\*a^5\*c^2\*d^4\*e^10 - 3675\*a^6\*c\*d^2\*e^12 + 1575\*a^7\*e^14 + 5120\*(17\*c^7\*d^8\*e^6 + 33\*a\*c^6\*d^6\*e^8)\*x^6 + 1280\*(c^7\*d^9\*e^5 + 166\*a\*c^6\*d^7\*e^7 + 81\*a^2\*c^5\*d^5\*e^9)\*x^5 - 128\*(11\*c^7\*d^10\*e^4 - 35\*a\*c^6\*d^8\*e^6 - 1075\*a^2\*c^5\*d^6\*e^8 - 5\*a^3\*c^4\*d^4\*e^10)\*x^4 + 16\*(99\*c^7\*d^11\*e^3 - 316\*a\*c^6\*d^9\*e^5 + 290\*a^2\*c^5\*d^7\*e^7 + 100\*a^3\*c^4\*d^5\*e^9 - 45\*a^4\*c^3\*d^3\*e^11)\*x^3 - 8\*(231\*c^7\*d^12\*e^2 - 741\*a\*c^6\*d^10\*e^4 + 686\*a^2\*c^5\*d^8\*e^6 - 50\*a^3\*c^4\*d^6\*e^8 + 235\*a^4\*c^3\*d^4\*e^10 - 105\*a^5\*c^2\*d^2\*e^12)\*x^2 + 2\*(1155\*c^7\*d^13\*e - 3738\*a\*c^6\*d^11\*e^3 + 3517\*a^2\*c^5\*d^9\*e^5 - 300\*a^3\*c^4\*d^7\*e^7 - 275\*a^4\*c^3\*d^5\*e^9 + 1190\*a^5\*c^2\*d^3\*e^11 - 525\*a^6\*c\*d\*e^13)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*e) + 105\*(33\*c^8\*d^16 - 120\*a\*c^7\*d^14\*e^2 + 140\*a^2\*c^6\*d^12\*e^4 - 40\*a^3\*c^5\*d^10\*e^6 - 10\*a^4\*c^4\*d^8\*e^8 - 8\*a^5\*c^3\*d^6\*e^10 - 20\*a^6\*c^2\*d^4\*e^12 + 40\*a^7\*c\*d^2\*e^14 - 15\*a^8\*e^16)\*log(4\*(2\*c^2\*d^2\*e^2\*x + c^2\*d^3\*e + a\*c\*d\*e^3)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) + (8\*c^2\*d^2\*e^2\*x^2 + c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4 + 8\*(c^2\*d^3\*e + a\*c\*d\*e^3)\*x)\*sqrt(c\*d\*e))/(sqrt(c\*d\*e)\*c^5\*d^5\*e^6), 1/1146880\*(2\*(71680\*c^7\*d^7\*e^7\*x^7 - 3465\*c^7\*d^14 + 11445\*a\*c^6\*d^12\*e^2 - 11193\*a^2\*c^5\*d^10\*e^4 + 1325\*a^3\*c^4\*d^8\*e^6 + 925\*a^4\*c^3\*d^6\*e^8 + 1015\*a^5\*c^2\*d^4\*e^10 - 3675\*a^6\*c\*d^2\*e^12 + 1575\*a^7\*e^14 + 5120\*(17\*c^7\*d^8\*e^6 + 33\*a\*c^6\*d^6\*e^8)\*x^6 + 1280\*(c^7\*d^9\*e^5 + 166\*a\*c^6\*d^7\*e^7 + 81\*a^2\*c^5\*d^5\*e^9)\*x^5 - 128\*(11\*c^7\*d^10\*e^4 - 35\*a\*c^6\*d^8\*e^6 - 1075\*a^2\*c^5\*d^6\*e^8 - 5\*a^3\*c^4\*d^4\*e^10)\*x^4 + 16\*(99\*c^7\*d^11\*e^3 - 316\*a\*c^6\*d^9\*e^5 + 290\*a^2\*c^5\*d^7\*e^7 + 100\*a^3\*c^4\*d^5\*e^9 - 45\*a^4\*c^3\*d^3\*e^11)\*x^3 - 8\*(231\*c^7\*d^12\*e^2 - 741\*a\*c^6\*d^10\*e^4 + 686\*a^2\*c^5\*d^8\*e^6 - 50\*a^3\*c^4\*d^6\*e^8 + 235\*a^4\*c^3\*d^4\*e^10 - 105\*a^5\*c^2\*d^2\*e^12)\*x^2 + 2\*(1155\*c^7\*d^13\*e - 3738\*a\*c^6\*d^11\*e^3 + 3517\*a^2\*c^5\*d^9\*e^5 - 300\*a^3\*c^4\*d^7\*e^7 - 275\*a^4\*c^3\*d^5\*e^9 + 1190\*a^5\*c^2\*d^3\*e^11 - 525\*a^6\*c\*d\*e^13)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*e) + 105\*(33\*c^8\*d^16 - 120\*a\*c^7\*d^14\*e^2 + 140\*a^2\*c^6\*d^12\*e^4 - 40\*a^3\*c^5\*d^10\*e^6 - 10\*a^4\*c^4\*d^8\*e^8 - 8\*a^5\*c^3\*d^6\*e^10 - 20\*a^6\*c^2\*d^4\*e^12 + 40\*a^7\*c\*d^2\*e^14 - 15\*a^8\*e^16)\*arctan(1/2\*(2\*c\*d\*e\*x + c\*d^2 + a\*e^2)\*sqrt(-c\*d\*e)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*c\*d\*e))/(sqrt(-c\*d\*e)\*c^5\*d^5\*e^6)]

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)
```

```
[Out] Timed out
```

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*x^3/(e*x + d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.458 \quad \int \frac{x^2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex} dx$$

**Optimal.** Leaf size=452

$$\begin{aligned} & \frac{(-35a^2e^4 - 10cdex(9cd^2 - 5ae^2) - 20acd^2e^2 + 63c^2d^4)(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{840c^2d^2e^3} \\ & - \frac{(5a^2e^4 + 10acd^2e^2 + 9c^2d^4)(cd^2 - ae^2)^5 \tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2048c^{9/2}d^{9/2}e^{11/2}} \\ & + \frac{(5a^2e^4 + 10acd^2e^2 + 9c^2d^4)(cd^2 - ae^2)^3(ae^2 + cd^2 + 2cdex)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{1024c^4d^4e^5} \\ & - \frac{(5a^2e^4 + 10acd^2e^2 + 9c^2d^4)(cd^2 - ae^2)(ae^2 + cd^2 + 2cdex)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{384c^3d^3e^4} \\ & + \frac{x^2(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{7e} \end{aligned}$$

[Out]  $((c*d^2 - a*e^2)^3*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(c*d^2 + a*e^2 + 2*c*d*e*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (1024*c^4*d^4*e^5) - ((c*d^2 - a*e^2)*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) / (384*c^3*d^3*e^4) + (x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}) / (7*e) + ((63*c^2*d^4 - 20*a*c*d^2*e^2 - 35*a^2*e^4 - 10*c*d*e*(9*c*d^2 - 5*a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}) / (840*c^2*d^2*e^3) - ((c*d^2 - a*e^2)^5*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*\text{ArcTanh}[(c*d^2 + a*e^2 + 2*c*d*e*x) / (2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]) / (2048*c^{(9/2)}*d^{(9/2)}*e^{(11/2)})$

**Rubi [A]** time = 1.12498, antiderivative size = 452, normalized size of antiderivative = 1., number of rules used = 7, number of rules used = 6, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\begin{aligned} & \frac{(-35a^2e^4 - 10cdex(9cd^2 - 5ae^2) - 20acd^2e^2 + 63c^2d^4)(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{840c^2d^2e^3} \\ & - \frac{(5a^2e^4 + 10acd^2e^2 + 9c^2d^4)(cd^2 - ae^2)^5 \tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2048c^{9/2}d^{9/2}e^{11/2}} \\ & + \frac{(5a^2e^4 + 10acd^2e^2 + 9c^2d^4)(cd^2 - ae^2)^3(ae^2 + cd^2 + 2cdex)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{1024c^4d^4e^5} \\ & - \frac{(5a^2e^4 + 10acd^2e^2 + 9c^2d^4)(cd^2 - ae^2)(ae^2 + cd^2 + 2cdex)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{384c^3d^3e^4} \\ & + \frac{x^2(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{7e} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}) / (d + e*x), x]$

[Out]  $((c*d^2 - a*e^2)^3*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(c*d^2 + a*e^2 + 2*c*d*e*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (1024*c^4*d^4*e^5) - ((c*d^2 - a*e^2)*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) / (384*c^3*d^3*e^4) + (x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}) / (7*e) + ((63*c^2*d^4 - 20*a*c*d^2*e^2 - 35*a^2*e^4 - 10*c*d*e*(9*c*d^2 - 5*a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}) / (840*c^2*d^2*e^3) - ((c*d^2 - a*e^2)^5*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*\text{ArcTanh}[(c*d^2 + a*e^2 + 2*c*d*e*x) / (2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]) / (2048*c^{(9/2)}*d^{(9/2)}*e^{(11/2)})$

**Rubi in Sympy [A]** time = 139.745, size = 454, normalized size = 1.

$$\frac{x^2 (ade + cdex^2 + x (ae^2 + cd^2))^{\frac{5}{2}}}{7e} - \frac{(ade + cdex^2 + x (ae^2 + cd^2))^{\frac{5}{2}} \left( \frac{35a^2e^4}{4} + 5acd^2e^2 - \frac{63c^2d^4}{4} - \frac{5cdex(5ae^2 - 9cd^2)}{2} \right)}{210c^2d^2e^3} + \frac{(ae^2 - cd^2) (ae^2 + cd^2 + 2cdex) (5a^2e^4 + 10acd^2e^2 + 9c^2d^4) (ade + cdex^2 + x (ae^2 + cd^2))^{\frac{3}{2}}}{384c^3d^3e^4} - \frac{(ae^2 - cd^2)^3 (ae^2 + cd^2 + 2cdex) (5a^2e^4 + 10acd^2e^2 + 9c^2d^4) \sqrt{ade + cdex^2 + x (ae^2 + cd^2)}}{1024c^4d^4e^5} + \frac{(ae^2 - cd^2)^5 (5a^2e^4 + 10acd^2e^2 + 9c^2d^4) \operatorname{atanh} \left( \frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}} \right)}{2048c^{\frac{9}{2}}d^{\frac{9}{2}}e^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**2*(a*d*e+(a**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)`

[Out] `x**2*(a*d*e + c*d*e*x**2 + x*(a**2 + c*d**2))**(5/2)/(7*e) - (a*d*e + c*d*e*x**2 + x*(a**2 + c*d**2))**(5/2)*(35*a**2*e**4/4 + 5*a*c*d**2*e**2 - 63*c**2*d**4/4 - 5*c*d*e*x*(5*a**2 - 9*c*d**2)/2)/(210*c**2*d**2*e**3) + (a**2 - c*d**2)*(a**2 + c*d**2 + 2*c*d*e*x)*(5*a**2*e**4 + 10*a*c*d**2*e**2 + 9*c**2*d**4)*(a*d*e + c*d*e*x**2 + x*(a**2 + c*d**2))**(3/2)/(384*c**3*d**3*e**4) - (a**2 - c*d**2)**3*(a**2 + c*d**2 + 2*c*d*e*x)*(5*a**2*e**4 + 10*a*c*d**2*e**2 + 9*c**2*d**4)*sqrt(a*d*e + c*d*e*x**2 + x*(a**2 + c*d**2))/(1024*c**4*d**4*e**5) + (a**2 - c*d**2)**5*(5*a**2*e**4 + 10*a*c*d**2*e**2 + 9*c**2*d**4)*atanh((a**2 + c*d**2 + 2*c*d*e*x)/(2*sqrt(c)*sqrt(d)*sqrt(e)*sqrt(a*d*e + c*d*e*x**2 + x*(a**2 + c*d**2))))/(2048*c**(9/2)*d**(9/2)*e**(11/2))`

**Mathematica [A]** time = 1.11471, size = 478, normalized size = 1.06

$$\frac{((d + ex)(ae + cdex))^{3/2} \left( \frac{2(-525a^6e^{12} + 350a^5cde^{10}(4d+ex) - 35a^4c^2d^2e^8(15d^2+26dex+8e^2x^2) - 60a^3c^3d^3e^6(10d^3-5d^2ex-12de^2x^2-4e^3x^3) + a^2c^4d^4e^4(3d^3+5dex+2e^2x^2))}{(d+ex)^{3/2}} \right)}{(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

[In] `Integrate[(x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]`

[Out] `((a*e + c*d*x)*(d + e*x))^(3/2)*((2*(-525*a^6*e^12 + 350*a^5*c*d*e^10*(4*d + e*x) - 35*a^4*c^2*d^2*e^8*(15*d^2 + 26*d*e*x + 8*e^2*x^2) - 60*a^3*c^3*d^3*e^6*(10*d^3 - 5*d^2*e*x - 12*d*e^2*x^2 - 4*e^3*x^3) + a^2*c^4*d^4*e^4*(3689*d^4 - 2332*d^3*e*x + 1824*d^2*e^2*x^2 + 33520*d*e^3*x^3 + 23680*e^4*x^4) + 2*a*c^5*d^5*e^2*(-1680*d^5 + 1099*d^4*e*x - 872*d^3*e^2*x^2 + 744*d^2*e^3*x^3 + 24320*d*e^4*x^4 + 18560*e^5*x^5) + 3*c^6*d^6*(315*d^6 - 210*d^5*e*x + 168*d^4*e^2*x^2 - 144*d^3*e^3*x^3 + 128*d^2*e^4*x^4 + 6400*d*e^5*x^5 + 5120*e^6*x^6)))/(105*c^4*d^4*e^5*(a*e + c*d*x)*(d + e*x)) - ((c*d^2 - a*e^2)^5*(9*c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*Log[a*e^2 + 2*sqrt[c]*sqrt[d]*sqrt[e]*sqrt[a*e + c*d*x]*sqrt[d + e*x] + c*d*(d + 2*e*x)]/(c^(9/2)*d^(9/2)*e^(11/2)*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)))/2048`

**Maple [B]** time = 0.024, size = 2731, normalized size = 6.

result too large to display



Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{5/2} / (e * x + d), x)$

[Out] 
$$\begin{aligned} & 3/256 * e^5 * a^5 / c^2 * \ln((1/2 * a * e^2 - 1/2 * c * d^2 + (x+d/e) * c * d * e) / (c * d * e)^{1/2} \\ & + (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{1/2}) / (c * d * e)^{1/2} \\ & - 15/128 * d^6 / e^5 * a^2 * c * \ln((1/2 * a * e^2 - 1/2 * c * d^2 + (x+d/e) * c * d * e) / (c * d * e)^{1/2} \\ & + (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{1/2}) / (c * d * e)^{1/2} \\ & + 15/256 * d^8 / e^3 * a^2 * c^2 * \ln((1/2 * a * e^2 - 1/2 * c * d^2 + (x+d/e) * c * d * e) / (c * d * e)^{1/2} \\ & + (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{1/2}) / (c * d * e)^{1/2} \\ & - 3/64 * d^4 * e^2 * a^3 / c * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{1/2} * x - 9/64 * d^5 / e^2 * a * c * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{1/2} * x \\ & - 15/256 * d^2 * e^3 * a^4 / c * \ln((1/2 * a * e^2 - 1/2 * c * d^2 + (x+d/e) * c * d * e) / (c * d * e)^{1/2} + (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{1/2}) \\ & / (c * d * e)^{1/2} + 5/512 * e^4 / c^2 / d * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2} * x * a^4 + 15/256 * e^2 / c * d * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2} * x * a^3 + 195/2048 / e * c * d^6 * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x) / (c * d * e)^{1/2} \\ & + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2}) / (c * d * e)^{1/2} * a^2 - 85/2048 / e^3 * c^2 * d^8 * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x) / (c * d * e)^{1/2} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2}) / (c * d * e)^{1/2} * a + 5/192 * e^3 / c^2 / d^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{3/2} * x * a^3 + 5/2048 * e^9 / c^4 / d^4 * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x) / (c * d * e)^{1/2} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2}) / (c * d * e)^{1/2} * a^7 - 15/2048 * e^7 / c^3 / d^2 * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x) / (c * d * e)^{1/2} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2}) / (c * d * e)^{1/2} * a^6 + 125/2048 * e^3 / c * d^2 * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x) / (c * d * e)^{1/2} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2}) / (c * d * e)^{1/2} * a^4 + 55/512 / e^2 * c * d^5 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2} * x * a - 5/512 * e^6 / c^3 / d^3 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2} * x * a^5 + 1/7 / e^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{7/2} / c / d + 35/1024 * e^3 / c^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2} * a^4 - 1/6 / e / c * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{5/2} * a - 1/4 / e^2 * d * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{5/2} * x - 15/1024 / e * d^4 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2} * a^2 + 5/128 / e^4 * c * d^5 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{3/2} + 3/128 * d^8 / e^5 * c^2 * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{1/2} + 1/16 * d^2 * a^2 / c * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{3/2} + 9/64 * d^3 * a^2 * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{1/2} * x - 3/128 * e^3 * a^4 / c^2 * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{1/2} - 5/192 / e^2 * d^3 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{3/2} * a - 35/256 * d^3 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2} * x * a^2 - 5/96 / c * d * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{3/2} * a^2 - 1/16 * d^5 / e^4 * c * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{3/2} - 15/1024 / e^5 * c^2 * d^8 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2} - 3/64 * d^6 / e^3 * a * c * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{1/2} + 15/128 * d^4 * e * a^3 * \ln((1/2 * a * e^2 - 1/2 * c * d^2 + (x+d/e) * c * d * e) / (c * d * e)^{1/2} + (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{1/2}) / (c * d * e)^{1/2} + 3/64 * d^7 / e^4 * c^2 * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{1/2} * x - 3/256 * d^10 / e^5 * c^3 * \ln((1/2 * a * e^2 - 1/2 * c * d^2 + (x+d/e) * c * d * e) / (c * d * e)^{1/2} + (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{1/2}) / (c * d * e)^{1/2} - 5/1024 * e^7 / c^4 / d^4 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2} * a^6 - 225/2048 * e * d^4 * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x) / (c * d * e)^{1/2} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2}) / (c * d * e)^{1/2} * a^3 - 25/192 / e * d^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{3/2} * x * a + 5/192 * e^2 / c^2 / d * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{3/2} * a^3 - 1/12 / c / d * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{5/2} * x * a + 5/192 * e / c * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{3/2} * x * a^2 - 15/2048 * e^5 / c^2 * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x) / (c * d * e)^{1/2} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2}) / (c * d * e)^{1/2} + 5/128 / e^3 * c * d^6 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2} * a - 5/128 * e / c * d^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2} * a^3 + 5/64 / e^3 * c * d^4 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{3/2} * x - 1/24 * e / c^2 / d^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{5/2} * a^2 + 15/2048 / e^5 * c^3 * d^10 * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x) / (c * d * e)^{1/2} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2}) / (c * d * e)^{1/2} + 5/384 * e^4 / c^3 / d^3 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{3/2} * a^4 - 15/512 / e^4 * c^2 * d^7 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2} * x + 3/64 * d^2 * e * a^3 / c * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{1/2} - 1/8 * d^4 / e^3 * c * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{3/2} * x + 1/8 * d^2 / e * a * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{3/2} * x + 1/5 * d^2 / e^3 * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{5/2} - 1/8 / e^3 * d^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{5/2} \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*x^2/(e*x + d), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.385909, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*x^2/(e*x + d), x, algorithm="fricas")`

[Out] 
$$\left[ \frac{1}{430080} \left( 4 \left( 15360 c^6 d^6 e^6 x^6 + 945 c^6 d^{12} - 3360 a c^5 d^{10} e^2 + 3689 a^2 c^4 d^8 e^4 - 600 a^3 c^3 d^6 e^6 - 525 a^4 c^2 d^4 e^8 + 1400 a^5 c d^2 e^{10} - 525 a^6 e^{12} + 1280 \left( 15 c^6 d^7 e^5 + 29 a c^5 d^5 e^7 \right) x^5 + 128 \left( 3 c^6 d^8 e^4 + 380 a c^5 d^6 e^6 + 185 a^2 c^4 d^4 e^8 \right) x^4 - 16 \left( 27 c^6 d^9 e^3 - 93 a c^5 d^7 e^5 - 2095 a^2 c^4 d^5 e^7 - 15 a^3 c^3 d^3 e^9 \right) x^3 + 8 \left( 63 c^6 d^{10} e^2 - 218 a c^5 d^8 e^4 + 228 a^2 c^4 d^6 e^6 + 90 a^3 c^3 d^4 e^8 - 35 a^4 c^2 d^2 e^{10} \right) x^2 - 2 \left( 315 c^6 d^{11} e - 1099 a c^5 d^9 e^3 + 1166 a^2 c^4 d^7 e^5 - 150 a^3 c^3 d^5 e^7 + 455 a^4 c^2 d^3 e^9 - 175 a^5 c d e^{11} \right) x \right) \sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x} \sqrt{c d e} - 105 \left( 9 c^7 d^{14} - 35 a c^6 d^{12} e^2 + 45 a^2 c^5 d^{10} e^4 - 15 a^3 c^4 d^8 e^6 - 5 a^4 c^3 d^6 e^8 - 9 a^5 c^2 d^4 e^{10} + 15 a^6 c d^2 e^{12} - 5 a^7 e^{14} \right) \log \left( 4 \left( 2 c^2 d^2 e^2 x + c^2 d^3 e + a c d e^3 \right) \sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x} + \left( 8 c^2 d^2 e^2 x^2 + c^2 d^4 + 6 a c d^2 e^2 + a^2 e^4 + 8 \left( c^2 d^3 e + a c d e^3 \right) x \right) \sqrt{c d e} \right) \right) / \left( \sqrt{c d e} c^4 d^4 e^5 \right), \frac{1}{215040} \left( 2 \left( 15360 c^6 d^6 e^6 x^6 + 945 c^6 d^{12} - 3360 a c^5 d^{10} e^2 + 3689 a^2 c^4 d^8 e^4 - 600 a^3 c^3 d^6 e^6 - 525 a^4 c^2 d^4 e^8 + 1400 a^5 c d^2 e^{10} - 525 a^6 e^{12} + 1280 \left( 15 c^6 d^7 e^5 + 29 a c^5 d^5 e^7 \right) x^5 + 128 \left( 3 c^6 d^8 e^4 + 380 a c^5 d^6 e^6 + 185 a^2 c^4 d^4 e^8 \right) x^4 - 16 \left( 27 c^6 d^9 e^3 - 93 a c^5 d^7 e^5 - 2095 a^2 c^4 d^5 e^7 - 15 a^3 c^3 d^3 e^9 \right) x^3 + 8 \left( 63 c^6 d^{10} e^2 - 218 a c^5 d^8 e^4 + 228 a^2 c^4 d^6 e^6 + 90 a^3 c^3 d^4 e^8 - 35 a^4 c^2 d^2 e^{10} \right) x^2 - 2 \left( 315 c^6 d^{11} e - 1099 a c^5 d^9 e^3 + 1166 a^2 c^4 d^7 e^5 - 150 a^3 c^3 d^5 e^7 + 455 a^4 c^2 d^3 e^9 - 175 a^5 c d e^{11} \right) x \right) \sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x} \sqrt{-c d e} - 105 \left( 9 c^7 d^{14} - 35 a c^6 d^{12} e^2 + 45 a^2 c^5 d^{10} e^4 - 15 a^3 c^4 d^8 e^6 - 5 a^4 c^3 d^6 e^8 - 9 a^5 c^2 d^4 e^{10} + 15 a^6 c d^2 e^{12} - 5 a^7 e^{14} \right) \arctan \left( \frac{1}{2} \left( 2 c d e x + c d^2 + a e^2 \right) \sqrt{-c d e} / \left( \sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x} \sqrt{c d e} \right) \right) \right) / \left( \sqrt{-c d e} c^4 d^4 e^5 \right) \right]$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d), x)`

[Out] Timed out

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**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*x^2/(e*x + d), x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.459 \quad \int \frac{x(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{d+ex} dx$$

**Optimal.** Leaf size=381

$$\begin{aligned} & \frac{(5ae^2 + 7cd^2) (cd^2 - ae^2)^5 \tanh^{-1} \left( \frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \right)}{1024c^{7/2}d^{7/2}e^{9/2}} \\ & - \frac{(5ae^2 + 7cd^2) (cd^2 - ae^2)^3 (ae^2 + cd^2 + 2cdex) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{512c^3d^3e^4} \\ & + \frac{(5ae^2 + 7cd^2) (cd^2 - ae^2) (ae^2 + cd^2 + 2cdex) (x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{192c^2d^2e^3} \\ & + \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{6cde(d+ex)} - \frac{1}{60} \left( \frac{5a}{cd} + \frac{7d}{e^2} \right) (x(ae^2 + cd^2) + ade + cdex^2)^{5/2} \end{aligned}$$

[Out]  $-\left((c*d^2 - a*e^2)^3*(7*c*d^2 + 5*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]\right)/(512*c^3*d^3*e^4)$   
 $+ \left((c*d^2 - a*e^2)*(7*c*d^2 + 5*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*\left(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2\right)^{3/2}\right)/(192*c^2*d^2*e^3)$   
 $- \left(\left(\frac{5*a}{c*d} + \frac{7*d}{e^2}\right)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}\right)/60 + \left(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2\right)^{7/2}/\left(6*c*d*e*(d + e*x)\right) + \left((c*d^2 - a*e^2)^5*(7*c*d^2 + 5*a*e^2)*\text{ArcTanh}\left[\frac{c*d^2 + a*e^2 + 2*c*d*e*x}{2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]}\right]\right)/(1024*c^{7/2}*d^{7/2}*e^{9/2})$

**Rubi [A]** time = 0.852181, antiderivative size = 381, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$

$$\begin{aligned} & \frac{(5ae^2 + 7cd^2) (cd^2 - ae^2)^5 \tanh^{-1} \left( \frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \right)}{1024c^{7/2}d^{7/2}e^{9/2}} \\ & - \frac{(5ae^2 + 7cd^2) (cd^2 - ae^2)^3 (ae^2 + cd^2 + 2cdex) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{512c^3d^3e^4} \\ & + \frac{(5ae^2 + 7cd^2) (cd^2 - ae^2) (ae^2 + cd^2 + 2cdex) (x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{192c^2d^2e^3} \\ & + \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{6cde(d+ex)} - \frac{1}{60} \left( \frac{5a}{cd} + \frac{7d}{e^2} \right) (x(ae^2 + cd^2) + ade + cdex^2)^{5/2} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2})/(d + e*x), x]$

[Out]  $-\left((c*d^2 - a*e^2)^3*(7*c*d^2 + 5*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]\right)/(512*c^3*d^3*e^4)$   
 $+ \left((c*d^2 - a*e^2)*(7*c*d^2 + 5*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*\left(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2\right)^{3/2}\right)/(192*c^2*d^2*e^3)$   
 $- \left(\left(\frac{5*a}{c*d} + \frac{7*d}{e^2}\right)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}\right)/60 + \left(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2\right)^{7/2}/\left(6*c*d*e*(d + e*x)\right) + \left((c*d^2 - a*e^2)^5*(7*c*d^2 + 5*a*e^2)*\text{ArcTanh}\left[\frac{c*d^2 + a*e^2 + 2*c*d*e*x}{2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]}\right]\right)/(1024*c^{7/2}*d^{7/2}*e^{9/2})$

**Rubi in Sympy [A]** time = 86.2871, size = 360, normalized size = 0.94

$$\begin{aligned}
 & - \left( \frac{a}{12cd} + \frac{7d}{60e^2} \right) (ade + cdex^2 + x(ae^2 + cd^2))^{\frac{5}{2}} + \frac{(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{7}{2}}}{6cde(d + ex)} \\
 & - \frac{(ae^2 - cd^2)(5ae^2 + 7cd^2)(ae^2 + cd^2 + 2cdex)(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{3}{2}}}{192c^2d^2e^3} \\
 & + \frac{(ae^2 - cd^2)^3(5ae^2 + 7cd^2)(ae^2 + cd^2 + 2cdex)\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{512c^3d^3e^4} \\
 & - \frac{(ae^2 - cd^2)^5(5ae^2 + 7cd^2)\operatorname{atanh}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}\right)}{1024c^{\frac{7}{2}}d^{\frac{7}{2}}e^{\frac{9}{2}}}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)`

[Out]  $-(a/(12*c*d) + 7*d/(60*e**2))*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(5/2) + (a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(7/2)/(6*c*d*e*(d + e*x)) - (a*e**2 - c*d**2)*(5*a*e**2 + 7*c*d**2)*(a*e**2 + c*d**2 + 2*c*d*e*x)*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2)/(192*c**2*d**2*e**3) + (a*e**2 - c*d**2)**3*(5*a*e**2 + 7*c*d**2)*(a*e**2 + c*d**2 + 2*c*d*e*x)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(512*c**3*d**3*e**4) - (a*e**2 - c*d**2)**5*(5*a*e**2 + 7*c*d**2)*atanh((a*e**2 + c*d**2 + 2*c*d*e*x)/(2*sqrt(c)*sqrt(d)*sqrt(e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))))/(1024*c**(7/2)*d**(7/2)*e**(9/2))$

**Mathematica [A]** time = 0.921525, size = 386, normalized size = 1.01

$$((d + ex)(ae + cdex))^{3/2} \left( \frac{2(75a^5e^{10} - 5a^4cde^8(49d + 10ex) + 10a^3c^2d^2e^6(15d^2 + 16dex + 4e^2x^2) - 6a^2c^3d^3e^4(91d^3 - 58d^2ex - 564de^2x^2 - 360e^3x^3) + ac^4d^4e^2(4d^4 + 10dex + 5e^2x^2) - 5c^5d^5e^0)}{15c^3d^3e^4(d+ex)} \right)$$

Antiderivative was successfully verified.

[In] `Integrate[(x*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x),x]`

[Out]  $((a*e + c*d*x)*(d + e*x))^{3/2} * ((2*(75*a^5*e^{10} - 5*a^4*c*d*e^8*(49*d + 10*e*x) + 10*a^3*c^2*d^2*e^6*(15*d^2 + 16*d*e*x + 4*e^2*x^2) - 6*a^2*c^3*d^3*e^4*(91*d^3 - 58*d^2*e*x - 564*d*e^2*x^2 - 360*e^3*x^3) + a*c^4*d^4*e^2*(4*d^4 + 10*dex + 5*e^2*x^2) - 5*c^5*d^5*e^0)/(15*c^3*d^3*e^4*(d+ex))) / ((c*d^2 - a*e^2)^{5/2} * (7*c*d^2 + 5*a*e^2) * Log[a*e^2 + 2*sqrt[c]*sqrt[d]*sqrt[e]*sqrt[a*e + c*d*x]*sqrt[d + e*x] + c*d*(d + 2*e*x)]) / (c^{7/2}*d^{7/2}*e^{9/2}*(a*e + c*d*x)^{3/2}*(d + e*x)^{3/2})) / 1024$

**Maple [B]** time = 0.017, size = 2411, normalized size = 6.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x)`

[Out]  $-3/128*d^7/e^4*c^2*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{1/2}-1/16*e*a^2/c*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{3/2}-5/192/$

$$\begin{aligned}
& e^3 c^3 d^4 (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d e x^2)^{3/2} + 5/512 e^4 c^2 d^7 \\
& (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d e x^2)^{1/2} + 5/192 e/c (a^2 d e + (a^2 e^2 \\
& + c^2 d^2) x + c^2 d e x^2)^{3/2} a^2 + 5/192 e/d^2 (a^2 d e + (a^2 e^2 + c^2 d^2) x \\
& + c^2 d e x^2)^{3/2} a + 5/48 d (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d e x^2)^{3/2} \\
& x^2 a + 1/12/c/d (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d e x^2)^{5/2} a - 1/8 d^2 a \\
& (c^2 d e (x+d/e)^2 + (a^2 e^2 - c^2 d^2) (x+d/e))^{3/2} x + 1/16 d^4/e^3 c^2 ( \\
& c^2 d e (x+d/e)^2 + (a^2 e^2 - c^2 d^2) (x+d/e))^{3/2} - 3/256/d^2 e^6 a^5/c^2 \\
& \ln((1/2 a^2 e^2 - 1/2 c^2 d^2 + (x+d/e) c^2 d e)/(c^2 d e)^{1/2} + (c^2 d e (x+d/ \\
& e)^2 + (a^2 e^2 - c^2 d^2) (x+d/e))^{1/2})/(c^2 d e)^{1/2} - 15/256 d^7/e^2 a \\
& c^2 \ln((1/2 a^2 e^2 - 1/2 c^2 d^2 + (x+d/e) c^2 d e)/(c^2 d e)^{1/2} + (c^2 d e (x+d/ \\
& e)^2 + (a^2 e^2 - c^2 d^2) (x+d/e))^{1/2})/(c^2 d e)^{1/2} + 15/256 d^2 e^4 \\
& a^4/c \ln((1/2 a^2 e^2 - 1/2 c^2 d^2 + (x+d/e) c^2 d e)/(c^2 d e)^{1/2} + (c^2 d \\
& e (x+d/e)^2 + (a^2 e^2 - c^2 d^2) (x+d/e))^{1/2})/(c^2 d e)^{1/2} + 9/64 d^4 \\
& /e^2 a^2 c^2 (c^2 d e (x+d/e)^2 + (a^2 e^2 - c^2 d^2) (x+d/e))^{1/2} x + 5/256 e^5/ \\
& c^2/d^2 (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d e x^2)^{1/2} x^2 a^4 - 5/64/e^2 c^2 d^4 \\
& (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d e x^2)^{1/2} x^2 a + 15/512/e^2 c^2 d^7 \\
& \ln((1/2 a^2 e^2 + 1/2 c^2 d^2 + c^2 d e x)/(c^2 d e)^{1/2} + (a^2 d e + (a^2 e^2 + c^2 d^2) \\
& x + c^2 d e x^2)^{1/2})/(c^2 d e)^{1/2} a - 5/96 e^2/c/d (a^2 d e + (a^2 e^2 \\
& + c^2 d^2) x + c^2 d e x^2)^{3/2} x^2 a^2 - 5/1024 e^8/c^3/d^3 \ln((1/2 a^2 e^2 \\
& + 1/2 c^2 d^2 + c^2 d e x)/(c^2 d e)^{1/2} + (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d e x^2 \\
& )^{1/2})/(c^2 d e)^{1/2} a^6 + 15/512 e^6/c^2/d \ln((1/2 a^2 e^2 + 1/2 c^2 \\
& d^2 + c^2 d e x)/(c^2 d e)^{1/2} + (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d e x^2)^{1/2} \\
& )/(c^2 d e)^{1/2} a^5 - 75/1024 e^4/c^2 d \ln((1/2 a^2 e^2 + 1/2 c^2 d^2 + c^2 d \\
& e x)/(c^2 d e)^{1/2} + (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d e x^2)^{1/2})/(c^2 d \\
& e)^{1/2} a^4 + 3/64 d^5/e^2 a^2 c^2 (c^2 d e (x+d/e)^2 + (a^2 e^2 - c^2 d^2) (x+d \\
& /e))^{1/2} - 15/128 d^3 e^2 a^3 \ln((1/2 a^2 e^2 - 1/2 c^2 d^2 + (x+d/e) c^2 d \\
& e)/(c^2 d e)^{1/2} + (c^2 d e (x+d/e)^2 + (a^2 e^2 - c^2 d^2) (x+d/e))^{1/2})/ \\
& (c^2 d e)^{1/2} - 3/64 d^6/e^3 c^2 (c^2 d e (x+d/e)^2 + (a^2 e^2 - c^2 d^2) (x+ \\
& d/e))^{1/2} x + 3/256 d^9/e^4 c^3 \ln((1/2 a^2 e^2 - 1/2 c^2 d^2 + (x+d/e) c^2 \\
& d e)/(c^2 d e)^{1/2} + (c^2 d e (x+d/e)^2 + (a^2 e^2 - c^2 d^2) (x+d/e))^{1/2} \\
& )/(c^2 d e)^{1/2} - 9/64 d^2 e^2 a^2 (c^2 d e (x+d/e)^2 + (a^2 e^2 - c^2 d^2) (x+ \\
& d/e))^{1/2} x - 3/64 d^2 e^2 a^3/c (c^2 d e (x+d/e)^2 + (a^2 e^2 - c^2 d^2) (x+ \\
& d/e))^{1/2} + 1/8 d^3/e^2 c^2 (c^2 d e (x+d/e)^2 + (a^2 e^2 - c^2 d^2) (x+d/e)) \\
& ^{3/2} x + 3/64 e^3 a^3/c (c^2 d e (x+d/e)^2 + (a^2 e^2 - c^2 d^2) (x+d/e))^{1/2} \\
& x - 5/1024/e^4 c^3 d^9 \ln((1/2 a^2 e^2 + 1/2 c^2 d^2 + c^2 d e x)/(c^2 d e \\
& )^{1/2} + (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d e x^2)^{1/2})/(c^2 d e)^{1/2} - 5/ \\
& 192 e^3/c^2/d^2 (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d e x^2)^{3/2} a^3 + 5/256 \\
& e^2/c^2 d (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d e x^2)^{1/2} a^3 - 5/96/e^2 c^2 d \\
& a^3 (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d e x^2)^{3/2} x + 5/256/e^3 c^2 d^6 (a^2 \\
& d e + (a^2 e^2 + c^2 d^2) x + c^2 d e x^2)^{1/2} x + 5/512 e^6/c^3/d^3 (a^2 d e + \\
& (a^2 e^2 + c^2 d^2) x + c^2 d e x^2)^{1/2} a^5 - 15/512 e^4/c^2/d (a^2 d e + (a^2 e^2 + c^2 \\
& d^2) x + c^2 d e x^2)^{1/2} a^4 - 15/512/e^2 c^2 d^5 (a^2 d e + (a^2 e^2 + c^2 \\
& d^2) x + c^2 d e x^2)^{1/2} a - 5/64 e^3/c (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d \\
& e x^2)^{1/2} x^2 a^3 + 15/128 e^2 d^2 (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d e x^2) \\
& ^{1/2} x^2 a^2 + 25/256 e^2 d^3 \ln((1/2 a^2 e^2 + 1/2 c^2 d^2 + c^2 d e x)/(c^2 d \\
& e)^{1/2} + (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d e x^2)^{1/2})/(c^2 d e)^{1/2} a^3 - 75/1024 c^2 d^5 \\
& \ln((1/2 a^2 e^2 + 1/2 c^2 d^2 + c^2 d e x)/(c^2 d e)^{1/2} + (a^2 d e + (a^2 e^2 + c^2 d^2) \\
& x + c^2 d e x^2)^{1/2})/(c^2 d e)^{1/2} a^2 + 3/128/ \\
& d^2 e^4 a^4/c^2 (c^2 d e (x+d/e)^2 + (a^2 e^2 - c^2 d^2) (x+d/e))^{1/2} + 15/12 \\
& 8 d^5 a^2 c^2 \ln((1/2 a^2 e^2 - 1/2 c^2 d^2 + (x+d/e) c^2 d e)/(c^2 d e)^{1/2} + \\
& (c^2 d e (x+d/e)^2 + (a^2 e^2 - c^2 d^2) (x+d/e))^{1/2})/(c^2 d e)^{1/2} - 1/5^2 \\
& d/e^2 (c^2 d e (x+d/e)^2 + (a^2 e^2 - c^2 d^2) (x+d/e))^{5/2} + 1/6/e (a^2 d e + \\
& (a^2 e^2 + c^2 d^2) x + c^2 d e x^2)^{5/2} x + 5/256 d^3 (a^2 d e + (a^2 e^2 + c^2 d^2) \\
& x + c^2 d e x^2)^{1/2} a^2 + 1/12/e^2 d (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d e x \\
& ^2)^{5/2}
\end{aligned}$$


---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2 d^2 e^3 x^2 + a^2 d e + (c^2 d^2 + a^2 e^2) x)^{5/2} x / (e^3 x + d), x, algorithm=

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.355523, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*x/(e\*x + d), x, algorithm=

[Out] [1/30720\*(4\*(1280\*c^5\*d^5\*e^5\*x^5 - 105\*c^5\*d^10 + 415\*a\*c^4\*d^8\*e^2 - 546\*a^2\*c^3\*d^6\*e^4 + 150\*a^3\*c^2\*d^4\*e^6 - 245\*a^4\*c\*d^2\*e^8 + 75\*a^5\*e^10 + 128\*(13\*c^5\*d^6\*e^4 + 25\*a\*c^4\*d^4\*e^6)\*x^4 + 16\*(3\*c^5\*d^7\*e^3 + 278\*a\*c^4\*d^5\*e^5 + 135\*a^2\*c^3\*d^3\*e^7)\*x^3 - 8\*(7\*c^5\*d^8\*e^2 - 27\*a\*c^4\*d^6\*e^4 - 423\*a^2\*c^3\*d^4\*e^6 - 5\*a^3\*c^2\*d^2\*e^8)\*x^2 + 2\*(35\*c^5\*d^9\*e - 136\*a\*c^4\*d^7\*e^3 + 174\*a^2\*c^3\*d^5\*e^5 + 80\*a^3\*c^2\*d^3\*e^7 - 25\*a^4\*c\*d\*e^9)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*e) - 15\*(7\*c^6\*d^12 - 30\*a\*c^5\*d^10\*e^2 + 45\*a^2\*c^4\*d^8\*e^4 - 20\*a^3\*c^3\*d^6\*e^6 - 15\*a^4\*c^2\*d^4\*e^8 + 18\*a^5\*c\*d^2\*e^10 - 5\*a^6\*e^12)\*log(-4\*(2\*c^2\*d^2\*e^2\*x + c^2\*d^3\*e + a\*c\*d\*e^3)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) + (8\*c^2\*d^2\*e^2\*x^2 + c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4 + 8\*(c^2\*d^3\*e + a\*c\*d\*e^3)\*x)\*sqrt(c\*d\*e)))/(sqrt(c\*d\*e)\*c^3\*d^3\*e^4), 1/15360\*(2\*(1280\*c^5\*d^5\*e^5\*x^5 - 105\*c^5\*d^10 + 415\*a\*c^4\*d^8\*e^2 - 546\*a^2\*c^3\*d^6\*e^4 + 150\*a^3\*c^2\*d^4\*e^6 - 245\*a^4\*c\*d^2\*e^8 + 75\*a^5\*e^10 + 128\*(13\*c^5\*d^6\*e^4 + 25\*a\*c^4\*d^4\*e^6)\*x^4 + 16\*(3\*c^5\*d^7\*e^3 + 278\*a\*c^4\*d^5\*e^5 + 135\*a^2\*c^3\*d^3\*e^7)\*x^3 - 8\*(7\*c^5\*d^8\*e^2 - 27\*a\*c^4\*d^6\*e^4 - 423\*a^2\*c^3\*d^4\*e^6 - 5\*a^3\*c^2\*d^2\*e^8)\*x^2 + 2\*(35\*c^5\*d^9\*e - 136\*a\*c^4\*d^7\*e^3 + 174\*a^2\*c^3\*d^5\*e^5 + 80\*a^3\*c^2\*d^3\*e^7 - 25\*a^4\*c\*d\*e^9)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*e) + 15\*(7\*c^6\*d^12 - 30\*a\*c^5\*d^10\*e^2 + 45\*a^2\*c^4\*d^8\*e^4 - 20\*a^3\*c^3\*d^6\*e^6 - 15\*a^4\*c^2\*d^4\*e^8 + 18\*a^5\*c\*d^2\*e^10 - 5\*a^6\*e^12)\*arctan(1/2\*(2\*c\*d\*e\*x + c\*d^2 + a\*e^2)\*sqrt(-c\*d\*e)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*c\*d\*e)))/(sqrt(-c\*d\*e)\*c^3\*d^3\*e^4)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)/(e\*x+d), x)

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*x/(e\*x + d), x, algorithm=

[Out] Exception raised: TypeError

$$3.460 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{d + ex} dx$$

**Optimal.** Leaf size=274

$$\begin{aligned} & \frac{3(cd^2 - ae^2)^5 \tanh^{-1}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{256c^{5/2}d^{5/2}e^{7/2}} \\ & + \frac{3(cd^2 - ae^2)^3 (ae^2 + cd^2 + 2cdex) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{128c^2d^2e^3} \\ & + \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{5e} \\ & + \frac{1}{16} \left(\frac{a}{cd} - \frac{d}{e^2}\right) (ae^2 + cd^2 + 2cdex) (x(ae^2 + cd^2) + ade + cdex^2)^{3/2} \end{aligned}$$

[Out]  $(3*(c*d^2 - a*e^2)^3*(c*d^2 + a*e^2 + 2*c*d*e*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(128*c^2*d^2*e^3) + ((a/(c*d) - d/e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/16 + (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}/(5*e) - (3*(c*d^2 - a*e^2)^5*\text{ArcTanh}[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(256*c^{(5/2)}*d^{(5/2)}*e^{(7/2)})$

**Rubi [A]** time = 0.377339, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 37,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$

$$\begin{aligned} & \frac{3(cd^2 - ae^2)^5 \tanh^{-1}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{256c^{5/2}d^{5/2}e^{7/2}} \\ & + \frac{3(cd^2 - ae^2)^3 (ae^2 + cd^2 + 2cdex) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{128c^2d^2e^3} \\ & + \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{5e} \\ & + \frac{1}{16} \left(\frac{a}{cd} - \frac{d}{e^2}\right) (ae^2 + cd^2 + 2cdex) (x(ae^2 + cd^2) + ade + cdex^2)^{3/2} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}/(d + e*x), x]$

[Out]  $(3*(c*d^2 - a*e^2)^3*(c*d^2 + a*e^2 + 2*c*d*e*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(128*c^2*d^2*e^3) + ((a/(c*d) - d/e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/16 + (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}/(5*e) - (3*(c*d^2 - a*e^2)^5*\text{ArcTanh}[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(256*c^{(5/2)}*d^{(5/2)}*e^{(7/2)})$

**Rubi in Sympy [A]** time = 59.8112, size = 262, normalized size = 0.96

$$\begin{aligned} & - \left(-\frac{a}{16cd} + \frac{d}{16e^2}\right) (ae^2 + cd^2 + 2cdex) (ade + cdex^2 + x(ae^2 + cd^2))^{\frac{3}{2}} \\ & + \frac{(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{5}{2}}}{5e} \\ & - \frac{3(ae^2 - cd^2)^3 (ae^2 + cd^2 + 2cdex) \sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{128c^2d^2e^3} \\ & + \frac{3(ae^2 - cd^2)^5 \operatorname{atanh}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}\right)}{256c^{\frac{5}{2}}d^{\frac{5}{2}}e^{\frac{7}{2}}} \end{aligned}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d),x)`

[Out] 
$$-\left(-\frac{a}{16cd} + \frac{d}{16e^2}\right) (a^2e^2 + c^2d^2 + 2cde^2x) (a^2de + c^2d^2e^2x^2 + x^2(a^2e^2 + c^2d^2))^{3/2} + (a^2de + c^2d^2e^2x^2 + x^2(a^2e^2 + c^2d^2))^{5/2} / (5e) - 3(a^2e^2 - c^2d^2)^{3/2} (a^2e^2 + c^2d^2 + 2cde^2x) \sqrt{a^2de + c^2d^2e^2x^2 + x^2(a^2e^2 + c^2d^2)} / (128c^2d^2e^3) + 3(a^2e^2 - c^2d^2)^{5/2} \operatorname{atanh}\left(\frac{a^2e^2 + c^2d^2 + 2cde^2x}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{a^2de + c^2d^2e^2x^2 + x^2(a^2e^2 + c^2d^2)}}\right) / (256c^{5/2}d^{5/2}e^{7/2})$$

**Mathematica [A]** time = 0.704413, size = 309, normalized size = 1.13

$$\frac{1}{256}((d + ex)(ae + cdx))^{3/2} \left( \frac{-30a^4e^8 + 20a^3cde^6(7d + ex) + 4a^2c^2d^2e^4(64d^2 + 233dex + 124e^2x^2) + 4ac^3d^3e^2(-35d^3 + 23d^2ex + 256de^2x^2 + 5c^2d^2e^3(d + ex)(ae + cdx))}{c^{5/2}d^{5/2}e^{7/2}(d + ex)^{3/2}(ae + cdx)^{3/2}} \right)$$

Antiderivative was successfully verified.

[In] `Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(d + e*x),x]`

[Out] 
$$\left(\frac{(a^2e + c^2d^2x)(d + e^2x)^{3/2}((-30a^4e^8 + 20a^3c^2d^2e^6(7d + e^2x) + 4a^2c^2d^2e^4(64d^2 + 233d^2e^2x + 124e^2x^2) + 4a^3c^3d^3e^2(-35d^3 + 23d^2e^2x + 256d^2e^2x^2 + 168e^3x^3) + 2c^4d^4(15d^4 - 10d^3e^2x + 8d^2e^2x^2 + 176d^2e^3x^3 + 128e^4x^4))}{5c^2d^2e^3(a^2e + c^2d^2x)(d + e^2x)} - (3(c^2d^2 - a^2e^2)^{5/2} \operatorname{Log}[a^2e^2 + 2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{a^2e^2 + c^2d^2x}]\sqrt{d + e^2x} + c^2d^2(d + 2e^2x))}{c^{5/2}d^{5/2}e^{7/2}(d + e^2x)^{3/2}}\right) / 256$$

**Maple [B]** time = 0.011, size = 1123, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d),x)`

[Out] 
$$\frac{1}{5}e^{5/2}(c^2d^2e^2(x+d/e)^2 + (a^2e^2 - c^2d^2)(x+d/e))^{5/2} - \frac{3}{128}e^{5/2}a^4/c^2d^2 \left( (c^2d^2e^2(x+d/e)^2 + (a^2e^2 - c^2d^2)(x+d/e))^{1/2} - \frac{3}{64}e^{5/2}a^2/c^2d^4 \left( (c^2d^2e^2(x+d/e)^2 + (a^2e^2 - c^2d^2)(x+d/e))^{1/2} + \frac{1}{16}e^{5/2}a^2/c^2d^4 \left( (c^2d^2e^2(x+d/e)^2 + (a^2e^2 - c^2d^2)(x+d/e))^{3/2} + \frac{9}{64}e^{5/2}a^2d^2 \left( (c^2d^2e^2(x+d/e)^2 + (a^2e^2 - c^2d^2)(x+d/e))^{1/2} \right) x + \frac{3}{64}e^{5/2}a^3/c^2 \left( (c^2d^2e^2(x+d/e)^2 + (a^2e^2 - c^2d^2)(x+d/e))^{1/2} - \frac{1}{8}e^{5/2}c^2d^2 \left( (c^2d^2e^2(x+d/e)^2 + (a^2e^2 - c^2d^2)(x+d/e))^{3/2} \right) x - \frac{1}{16}e^{5/2}c^2d^3 \left( (c^2d^2e^2(x+d/e)^2 + (a^2e^2 - c^2d^2)(x+d/e))^{3/2} + \frac{3}{128}e^{5/2}c^2d^6 \left( (c^2d^2e^2(x+d/e)^2 + (a^2e^2 - c^2d^2)(x+d/e))^{1/2} + \frac{1}{8}e^{5/2}a^2 \left( (c^2d^2e^2(x+d/e)^2 + (a^2e^2 - c^2d^2)(x+d/e))^{3/2} \right) x - \frac{3}{64}e^{5/2}a^4/c^2d^4 \left( (c^2d^2e^2(x+d/e)^2 + (a^2e^2 - c^2d^2)(x+d/e))^{1/2} \right) x - \frac{9}{64}a^2c^2d^3 \left( (c^2d^2e^2(x+d/e)^2 + (a^2e^2 - c^2d^2)(x+d/e))^{1/2} \right) x + \frac{3}{256}e^{5/2}a^5/c^2d^2 \operatorname{Ln}\left(\frac{(1/2)a^2e^2 - 1/2c^2d^2 + (x+d/e)c^2d^2e^2}{(c^2d^2e^2)^{1/2}} + \frac{(c^2d^2e^2(x+d/e)^2 + (a^2e^2 - c^2d^2)(x+d/e))^{1/2}}{(c^2d^2e^2)^{1/2}}\right) - \frac{15}{128}e^{5/2}a^2c^2d^4 \operatorname{Ln}\left(\frac{(1/2)a^2e^2 - 1/2c^2d^2 + (x+d/e)c^2d^2e^2}{(c^2d^2e^2)^{1/2}} + \frac{(c^2d^2e^2(x+d/e)^2 + (a^2e^2 - c^2d^2)(x+d/e))^{1/2}}{(c^2d^2e^2)^{1/2}}\right) \right) \right)$$

$$\frac{(x+d/e)^{1/2}}{(c*d*e)^{1/2}} + \frac{15}{256} \frac{e*a*c^2*d^6 \ln((1/2*a*e^2 - 1/2*c*d^2 + (x+d/e)*c*d*e)/(c*d*e)^{1/2} + (c*d*e*(x+d/e)^2 + (a*e^2 - c*d^2)*(x+d/e)^{1/2}))}{(c*d*e)^{1/2}} - \frac{3}{256} \frac{e^3*c^3*d^8 \ln((1/2*a*e^2 - 1/2*c*d^2 + (x+d/e)*c*d*e)/(c*d*e)^{1/2} + (c*d*e*(x+d/e)^2 + (a*e^2 - c*d^2)*(x+d/e)^{1/2}))}{(c*d*e)^{1/2}} + \frac{3}{64} \frac{e^2*c^2*d^5*(c*d*e*(x+d/e)^2 + (a*e^2 - c*d^2)*(x+d/e)^{1/2})}{(c*d*e)^{1/2}} + \frac{15}{128} \frac{e^3*a^3*d^2 \ln((1/2*a*e^2 - 1/2*c*d^2 + (x+d/e)*c*d*e)/(c*d*e)^{1/2} + (c*d*e*(x+d/e)^2 + (a*e^2 - c*d^2)*(x+d/e)^{1/2}))}{(c*d*e)^{1/2}} - \frac{15}{256} \frac{e^5*a^4/c \ln((1/2*a*e^2 - 1/2*c*d^2 + (x+d/e)*c*d*e)/(c*d*e)^{1/2} + (c*d*e*(x+d/e)^2 + (a*e^2 - c*d^2)*(x+d/e)^{1/2}))}{(c*d*e)^{1/2}}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/(e\*x + d), x, algorithm="m

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.321004, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/(e\*x + d), x, algorithm="f

[Out] 
$$\left[ \frac{1}{2560} \left( 4 \left( 128 c^4 d^4 e^4 x^4 + 15 c^4 d^8 - 70 a c^3 d^6 e^2 + 128 a^2 c^2 d^4 e^4 + 70 a^3 c d^2 e^6 - 15 a^4 e^8 + 16 (11 c^4 d^5 e^3 + 21 a c^3 d^3 e^5) x^3 + 8 (c^4 d^6 e^2 + 64 a c^3 d^4 e^4 + 31 a^2 c^2 d^2 e^6) x^2 - 2 (5 c^4 d^7 e - 23 a c^3 d^5 e^3 - 233 a^2 c^2 d^3 e^5 - 5 a^3 c d e^7) x \right) \sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x} \sqrt{c d e} + 15 (c^5 d^{10} - 5 a c^4 d^8 e^2 + 10 a^2 c^3 d^6 e^4 - 10 a^3 c^2 d^4 e^6 + 5 a^4 c d^2 e^8 - a^5 e^{10}) \log(-4 (2 c^2 d^2 e^2 x + c^2 d^3 e + a c d e^3) \sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x}) + (8 c^2 d^2 e^2 x^2 + c^2 d^4 + 6 a c d^2 e^2 + a^2 e^4 + 8 (c^2 d^3 e + a c d e^3) x) \sqrt{c d e} \right) / (\sqrt{c d e} c^2 d^2 e^3), \frac{1}{1280} \left( 2 (128 c^4 d^4 e^4 x^4 + 15 c^4 d^8 - 70 a c^3 d^6 e^2 + 128 a^2 c^2 d^4 e^4 + 70 a^3 c d^2 e^6 - 15 a^4 e^8 + 16 (11 c^4 d^5 e^3 + 21 a c^3 d^3 e^5) x^3 + 8 (c^4 d^6 e^2 + 64 a c^3 d^4 e^4 + 31 a^2 c^2 d^2 e^6) x^2 - 2 (5 c^4 d^7 e - 23 a c^3 d^5 e^3 - 233 a^2 c^2 d^3 e^5 - 5 a^3 c d e^7) x \right) \sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x} \sqrt{-c d e} - 15 (c^5 d^{10} - 5 a c^4 d^8 e^2 + 10 a^2 c^3 d^6 e^4 - 10 a^3 c^2 d^4 e^6 + 5 a^4 c d^2 e^8 - a^5 e^{10}) \arctan(1/2 (2 c d e x + c d^2 + a e^2) \sqrt{-c d e}) / (\sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x} c d e) \right) / (\sqrt{-c d e} c^2 d^2 e^3) \right]$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)/(e\*x+d), x)

[Out] Timed out

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**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/(e*x + d), x, algorithm="g`

[Out] Exception raised: TypeError

$$3.461 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x(d+ex)} dx$$

**Optimal.** Leaf size=394

$$-a^{5/2}d^{3/2}e^{5/2} \tanh^{-1}\left(\frac{x(ae^2 + cd^2) + 2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right) - \frac{(-5a^3e^6 - 83a^2cd^2e^4 - 11ac^2d^4e^2 + 2cdex(cd^2 - 5ae^2))}{64cde^2}$$

[Out]  $-\left((3c^3d^6 - 11a^2c^2d^4e^2 - 83a^2c^2d^2e^4 - 5a^3e^6 + 2c^2d^2e^2(c^2d^2 - 5a^2e^2)(3c^2d^2 + a^2e^2)x\right) \sqrt{a^2d^2e^2 + (c^2d^2 + a^2e^2)x + c^2d^2e^2x^2} / (64c^2d^2e^2) + ((3c^2d^2 + 11a^2e^2 + 6c^2d^2e^2x)(a^2d^2e^2 + (c^2d^2 + a^2e^2)x + c^2d^2e^2x^2)^{3/2}) / (24e^2) + ((3c^4d^8 - 20a^2c^3d^6e^2 + 90a^2c^2d^4e^4 + 60a^3c^2d^2e^6 - 5a^4e^8) \operatorname{ArcTanh}[(c^2d^2 + a^2e^2 + 2c^2d^2e^2x) / (2\sqrt{c} \sqrt{d} \sqrt{e} \sqrt{a^2d^2e^2 + (c^2d^2 + a^2e^2)x + c^2d^2e^2x^2})]) / (128c^{3/2}d^{3/2}e^{5/2}) - a^{5/2}d^{3/2}e^{5/2} \operatorname{ArcTanh}[(2a^2d^2e^2 + (c^2d^2 + a^2e^2)x) / (2\sqrt{a} \sqrt{d} \sqrt{e} \sqrt{a^2d^2e^2 + (c^2d^2 + a^2e^2)x + c^2d^2e^2x^2})])$

**Rubi [A]** time = 1.27514, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$-a^{5/2}d^{3/2}e^{5/2} \tanh^{-1}\left(\frac{x(ae^2 + cd^2) + 2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right) - \frac{(-5a^3e^6 - 83a^2cd^2e^4 - 11ac^2d^4e^2 + 2cdex(cd^2 - 5ae^2))}{64cde^2}$$

Antiderivative was successfully verified.

[In] Int[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2)/(x\*(d + e\*x)), x]

[Out]  $-\left((3c^3d^6 - 11a^2c^2d^4e^2 - 83a^2c^2d^2e^4 - 5a^3e^6 + 2c^2d^2e^2(c^2d^2 - 5a^2e^2)(3c^2d^2 + a^2e^2)x\right) \sqrt{a^2d^2e^2 + (c^2d^2 + a^2e^2)x + c^2d^2e^2x^2} / (64c^2d^2e^2) + ((3c^2d^2 + 11a^2e^2 + 6c^2d^2e^2x)(a^2d^2e^2 + (c^2d^2 + a^2e^2)x + c^2d^2e^2x^2)^{3/2}) / (24e^2) + ((3c^4d^8 - 20a^2c^3d^6e^2 + 90a^2c^2d^4e^4 + 60a^3c^2d^2e^6 - 5a^4e^8) \operatorname{ArcTanh}[(c^2d^2 + a^2e^2 + 2c^2d^2e^2x) / (2\sqrt{c} \sqrt{d} \sqrt{e} \sqrt{a^2d^2e^2 + (c^2d^2 + a^2e^2)x + c^2d^2e^2x^2})]) / (128c^{3/2}d^{3/2}e^{5/2}) - a^{5/2}d^{3/2}e^{5/2} \operatorname{ArcTanh}[(2a^2d^2e^2 + (c^2d^2 + a^2e^2)x) / (2\sqrt{a} \sqrt{d} \sqrt{e} \sqrt{a^2d^2e^2 + (c^2d^2 + a^2e^2)x + c^2d^2e^2x^2})])$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)/x/(e\*x+d), x)

[Out] Timed out

**Mathematica [A]** time = 1.44648, size = 397, normalized size = 1.01

$$\frac{((d+ex)(ae+cdx))^{3/2} \left(384a^{5/2}c^{3/2}d^3e^5 \log(x) - 384a^{5/2}c^{3/2}d^3e^5 \log\left(2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx+ae(2d+ex)+cd^2x}\right)\right)}{64cde^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2)/(x\*(d + e\*x)), x]

[Out] (((a\*e + c\*d\*x)\*(d + e\*x))^(3/2)\*(2\*Sqrt[c]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*(15\*a^3\*e^6 + a^2\*c\*d\*e^4\*(337\*d + 118\*e\*x) + a\*c^2\*d^2\*e^2\*(57\*d^2 + 244\*d\*e\*x + 136\*e^2\*x^2) + c^3\*(-9\*d^6 + 6\*d^5\*e\*x + 72\*d^4\*e^2\*x^2 + 48\*d^3\*e^3\*x^3)) + 384\*a^(5/2)\*c^(3/2)\*d^3\*e^5\*Log[x] - 384\*a^(5/2)\*c^(3/2)\*d^3\*e^5\*Log[c\*d^2\*x + 2\*Sqrt[a]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x] + a\*e\*(2\*d + e\*x)] + 3\*(3\*c^4\*d^8 - 20\*a\*c^3\*d^6\*e^2 + 90\*a^2\*c^2\*d^4\*e^4 + 60\*a^3\*c\*d^2\*e^6 - 5\*a^4\*e^8)\*Log[a\*e^2 + 2\*Sqrt[c]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x] + c\*d\*(d + 2\*e\*x)))/(384\*c^(3/2)\*d^(3/2)\*e^(5/2)\*(a\*e + c\*d\*x)^(3/2)\*(d + e\*x)^(3/2))

**Maple [B]** time = 0.024, size = 2180, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2)/x/(e\*x+d), x)

[Out] 1/8\*d\*c\*(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(3/2)\*x+1/16\*d^2\*c/e\*(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(3/2)-3/128\*d^5\*c^2/e^2\*(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2)-9/64\*a^2\*e^3\*(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2)\*x+3/64\*d^3\*a\*c\*(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2)+1/8\*d^3\*a\*c\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)-3/128\*d^5\*c^2/e^2\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)+83/64\*d\*a^2\*e^2\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)+1/8\*d\*c\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)\*x+1/16\*d^2\*c/e\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)+19/64\*a^2\*e^3\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)\*x+19/64\*d^2\*a\*e\*c\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)\*x+3/256/d^3\*a^5\*e^8/c^2\*ln((1/2\*a\*e^2+1/2\*c\*d^2+c\*d\*e\*x)/(c\*d\*e)^(1/2)+(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/(c\*d\*e)^(1/2)+75/128\*d^3\*a^2\*e^2\*c\*ln((1/2\*a\*e^2+1/2\*c\*d^2+c\*d\*e\*x)/(c\*d\*e)^(1/2)+(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/(c\*d\*e)^(1/2)-25/256/d\*a^4\*e^6/c\*ln((1/2\*a\*e^2+1/2\*c\*d^2+c\*d\*e\*x)/(c\*d\*e)^(1/2)+(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/(c\*d\*e)^(1/2)+3/64/d^2\*a^3\*e^5/c\*(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2)\*x+9/64\*d^2\*a\*e\*c\*(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2)\*x-3/256/d^3\*a^5\*e^8/c^2\*ln((1/2\*a\*e^2-1/2\*c\*d^2+(x+d/e)\*c\*d\*e)/(c\*d\*e)^(1/2)+(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2))/(c\*d\*e)^(1/2)+15/128\*d^3\*a^2\*e^2\*c\*ln((1/2\*a\*e^2-1/2\*c\*d^2+(x+d/e)\*c\*d\*e)/(c\*d\*e)^(1/2)+(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2))/(c\*d\*e)^(1/2)+15/256/d\*a^4\*e^6/c\*ln((1/2\*a\*e^2-1/2\*c\*d^2+(x+d/e)\*c\*d\*e)/(c\*d\*e)^(1/2)+(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2))/(c\*d\*e)^(1/2)-3/64/d^2\*a^3\*e^5/c\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)\*x-1/5/d\*(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(5/2)+1/5/d\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2)-15/256\*d^5\*a\*c^2\*ln((1/2\*a\*e^2-1/2\*c\*d^2+(x+d/e)\*c\*d\*e)/(c\*d\*e)^(1/2)+(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2))/(c\*d\*e)^(1/2)+3/256\*d^7\*c^3/e^2\*ln((1/2\*a\*e^2-1/2\*c\*d^2+(x+d/e)\*c\*d\*e)/(c\*d\*e)^(1/2)+(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2))/(c\*d\*e)^(1/2)+3/128/d^3\*a^4\*e^6/c^2\*(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2)-15/128\*d\*a^3\*e^4\*ln((1/2\*a\*e^2-1/2\*c\*d^2+(x+d/e)\*c\*d\*e)/(c\*d\*e)^(1/2)+(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2))/(c\*d\*e)^(1/2)-3/64\*d^4\*c^2/e\*(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2)\*x-1/16/d^2\*a^2\*e^3/c\*(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(3/2)+3/256\*d^7\*c^3/e^2\*ln((1/2\*a\*e^2+1/2\*c\*d^2+c\*d\*e\*x)/(c\*d\*e)^(1/2)+(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/(c\*d\*e)^(1/2)+1/16/d^2\*a^2\*e^3/c\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)-d^2\*a^3\*e^3/(a\*d\*e)^(1/2)\*ln((2\*a\*d\*e+(a\*e^2+c\*d^2)\*x+2\*(a\*d\*e)^(1/2)\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/x)+75/128\*d\*a^3\*e^4\*ln((1/2\*a\*e^2+1/2\*c\*d^2+c\*d\*e\*x)/(c\*d\*e)^(1/2)+(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/(c\*d\*e)^(1/2)-3/64\*d^4\*c^2/e\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(

$$\frac{1}{2}x - \frac{3}{128}d^3 a^4 e^6 / c^2 (a d e + (a e^2 + c d^2) x + c d e x^2)^{(1/2)} + \frac{1}{8}d^3 a^3 e^4 / c (a d e + (a e^2 + c d^2) x + c d e x^2)^{(1/2)} + \frac{1}{8}d^3 a^2 e^2 (a d e + (a e^2 + c d^2) x + c d e x^2)^{(3/2)} x - \frac{25}{256}d^5 a^2 c^2 \ln\left(\frac{(1/2 a e^2 + 1/2 c d^2 + c d e x)}{(c d e)^{(1/2)} + (a d e + (a e^2 + c d^2) x + c d e x^2)^{(1/2)}}\right) / (c d e)^{(1/2)} - \frac{3}{64}d^3 a^3 e^4 / c (c d e (x + d/e)^2 + (a e^2 - c d^2) (x + d/e))^{(1/2)} - \frac{1}{8}d^3 a^2 e^2 (c d e (x + d/e)^2 + (a e^2 - c d^2) (x + d/e))^{(3/2)} x + \frac{11}{24}a^2 e (a d e + (a e^2 + c d^2) x + c d e x^2)^{(3/2)}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)\*x), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 48.2647, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)\*x), x, algorithm="fricas")

[Out] [1/768\*(384\*sqrt(a\*d\*e)\*sqrt(c\*d\*e)\*a^2\*c\*d^2\*e^4\*log((8\*a^2\*d^2\*e^2 + (c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4)\*x^2 - 4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(a\*d\*e) + 8\*(a\*c\*d^3\*e + a^2\*d\*e^3)\*x)/x^2) + 4\*(48\*c^3\*d^3\*e^3\*x^3 - 9\*c^3\*d^6 + 57\*a\*c^2\*d^4\*e^2 + 337\*a^2\*c\*d^2\*e^4 + 15\*a^3\*e^6 + 8\*(9\*c^3\*d^4\*e^2 + 17\*a\*c^2\*d^2\*e^4)\*x^2 + 2\*(3\*c^3\*d^5\*e + 122\*a\*c^2\*d^3\*e^3 + 59\*a^2\*c\*d\*e^5)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*e) - 3\*(3\*c^4\*d^8 - 20\*a\*c^3\*d^6\*e^2 + 90\*a^2\*c^2\*d^4\*e^4 + 60\*a^3\*c\*d^2\*e^6 - 5\*a^4\*e^8)\*log(-4\*(2\*c^2\*d^2\*e^2\*x + c^2\*d^3\*e + a\*c\*d\*e^3)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) + (8\*c^2\*d^2\*e^2\*x^2 + c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4 + 8\*(c^2\*d^3\*e + a\*c\*d\*e^3)\*x)\*sqrt(c\*d\*e)))/(sqrt(c\*d\*e)\*c\*d\*e^2), 1/384\*(192\*sqrt(a\*d\*e)\*sqrt(-c\*d\*e)\*a^2\*c\*d^2\*e^4\*log((8\*a^2\*d^2\*e^2 + (c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4)\*x^2 - 4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(a\*d\*e) + 8\*(a\*c\*d^3\*e + a^2\*d\*e^3)\*x)/x^2) + 2\*(48\*c^3\*d^3\*e^3\*x^3 - 9\*c^3\*d^6 + 57\*a\*c^2\*d^4\*e^2 + 337\*a^2\*c\*d^2\*e^4 + 15\*a^3\*e^6 + 8\*(9\*c^3\*d^4\*e^2 + 17\*a\*c^2\*d^2\*e^4)\*x^2 + 2\*(3\*c^3\*d^5\*e + 122\*a\*c^2\*d^3\*e^3 + 59\*a^2\*c\*d\*e^5)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*e) + 3\*(3\*c^4\*d^8 - 20\*a\*c^3\*d^6\*e^2 + 90\*a^2\*c^2\*d^4\*e^4 + 60\*a^3\*c\*d^2\*e^6 - 5\*a^4\*e^8)\*arctan(1/2\*(2\*c\*d\*e\*x + c\*d^2 + a\*e^2)\*sqrt(-c\*d\*e)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*c\*d\*e)))/(sqrt(-c\*d\*e)\*c\*d\*e^2), -1/768\*(768\*sqrt(-a\*d\*e)\*sqrt(c\*d\*e)\*a^2\*c\*d^2\*e^4\*arctan(1/2\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e))) - 4\*(48\*c^3\*d^3\*e^3\*x^3 - 9\*c^3\*d^6 + 57\*a\*c^2\*d^4\*e^2 + 337\*a^2\*c\*d^2\*e^4 + 15\*a^3\*e^6 + 8\*(9\*c^3\*d^4\*e^2 + 17\*a\*c^2\*d^2\*e^4)\*x^2 + 2\*(3\*c^3\*d^5\*e + 122\*a\*c^2\*d^3\*e^3 + 59\*a^2\*c\*d\*e^5)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*e) + 3\*(3\*c^4\*d^8 - 20\*a\*c^3\*d^6\*e^2 + 90\*a^2\*c^2\*d^4\*e^4 + 60\*a^3\*c\*d^2\*e^6 - 5\*a^4\*e^8)\*log(-4\*(2\*c^2\*d^2\*e^2\*x + c^2\*d^3\*e + a\*c\*d\*e^3)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) + (8\*c^2\*d^2\*e^2\*x^2 + c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4 + 8\*(c^2\*d^3\*e + a\*c\*d\*e^3)\*x)\*sqrt(c\*d\*e)))/(sqrt(c\*d\*e)\*c\*d\*e^2), -1/384\*(384\*sqrt(-a\*d\*e)\*sqrt(-c\*d\*e)\*a^2\*c\*d^2\*e^4\*arctan(1/2\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e)))

$$) - 2*(48*c^3*d^3*e^3*x^3 - 9*c^3*d^6 + 57*a*c^2*d^4*e^2 + 337*a^2*c*d^2*e^4 + 15*a^3*e^6 + 8*(9*c^3*d^4*e^2 + 17*a*c^2*d^2*e^4)*x^2 + 2*(3*c^3*d^5*e + 122*a*c^2*d^3*e^3 + 59*a^2*c*d*e^5)*x)*\sqrt{(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*\sqrt{-c*d*e}} - 3*(3*c^4*d^8 - 20*a*c^3*d^6*e^2 + 90*a^2*c^2*d^4*e^4 + 60*a^3*c*d^2*e^6 - 5*a^4*e^8)*\arctan(1/2*(2*c*d*e*x + c*d^2 + a*e^2)*\sqrt{-c*d*e})/(\sqrt{(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d*e}))/(\sqrt{-c*d*e}*c*d*e^2)]$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)/x/(e\*x+d),x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.736287, size = 4, normalized size = 0.01

$sage_0x$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)\*x),x, algorithm="sage")

[Out] sage0\*x

$$3.462 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^2(d+ex)} dx$$

**Optimal.** Leaf size=352

$$\begin{aligned} & -\frac{1}{2}a^{3/2}\sqrt{de}^{3/2}(3ae^2 + 5cd^2) \tanh^{-1}\left(\frac{x(ae^2 + cd^2) + 2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right) \\ & + \frac{(19a^2e^4 + 2cdex(7ae^2 + cd^2) + 28acd^2e^2 + c^2d^4)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{8e} \\ & - \frac{(-5a^3e^6 - 45a^2cd^2e^4 - 15ac^2d^4e^2 + c^3d^6) \tanh^{-1}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{16\sqrt{c}\sqrt{de}^{3/2}} \\ & - \frac{(3ae - cdx)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{3x} \end{aligned}$$

[Out]  $((c^2d^4 + 28ac^2d^2e^2 + 19a^2e^4 + 2c^2d^2e^2(c^2d^2 + 7a^2e^2)x) \sqrt{a^2d^2e + (c^2d^2 + a^2e^2)x + c^2d^2e^2x^2}) / (8e) - ((3a^2e - c^2d^2x) (a^2d^2e + (c^2d^2 + a^2e^2)x + c^2d^2e^2x^2)^{3/2}) / (3x) - ((c^3d^6 - 15a^2c^2d^4e^2 - 45a^2c^2d^2e^4 - 5a^3e^6) \operatorname{ArcTanh}[(c^2d^2 + a^2e^2 + 2c^2d^2e^2x) / (2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2})]) / (16\sqrt{c}\sqrt{de}^{3/2}) - (a^{3/2}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}) (5c^2d^2 + 3a^2e^2) \operatorname{ArcTanh}[(2a^2d^2e + (c^2d^2 + a^2e^2)x) / (2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2})]) / 2$

**Rubi [A]** time = 1.22396, antiderivative size = 352, normalized size of antiderivative = 1., number of rules used = 8, number of rules used = 7, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$

$$\begin{aligned} & -\frac{1}{2}a^{3/2}\sqrt{de}^{3/2}(3ae^2 + 5cd^2) \tanh^{-1}\left(\frac{x(ae^2 + cd^2) + 2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right) \\ & + \frac{(19a^2e^4 + 2cdex(7ae^2 + cd^2) + 28acd^2e^2 + c^2d^4)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{8e} \\ & - \frac{(-5a^3e^6 - 45a^2cd^2e^4 - 15ac^2d^4e^2 + c^3d^6) \tanh^{-1}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{16\sqrt{c}\sqrt{de}^{3/2}} \\ & - \frac{(3ae - cdx)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{3x} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a^2d^2e + (c^2d^2 + a^2e^2)x + c^2d^2e^2x^2)^{5/2} / (x^2(d + ex)), x]$

[Out]  $((c^2d^4 + 28ac^2d^2e^2 + 19a^2e^4 + 2c^2d^2e^2(c^2d^2 + 7a^2e^2)x) \sqrt{a^2d^2e + (c^2d^2 + a^2e^2)x + c^2d^2e^2x^2}) / (8e) - ((3a^2e - c^2d^2x) (a^2d^2e + (c^2d^2 + a^2e^2)x + c^2d^2e^2x^2)^{3/2}) / (3x) - ((c^3d^6 - 15a^2c^2d^4e^2 - 45a^2c^2d^2e^4 - 5a^3e^6) \operatorname{ArcTanh}[(c^2d^2 + a^2e^2 + 2c^2d^2e^2x) / (2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2})]) / (16\sqrt{c}\sqrt{de}^{3/2}) - (a^{3/2}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}) (5c^2d^2 + 3a^2e^2) \operatorname{ArcTanh}[(2a^2d^2e + (c^2d^2 + a^2e^2)x) / (2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2})]) / 2$



**Rubi in Sympy [A]** time = 144.324, size = 345, normalized size = 0.98

$$\frac{a^{\frac{3}{2}}\sqrt{de}^{\frac{3}{2}}(3ae^2 + 5cd^2) \operatorname{atanh}\left(\frac{2ade+x(ae^2+cd^2)}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}\right)}{2} - \frac{(3ae - cdx)(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{3}{2}}}{3x} + \frac{\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}\left(\frac{19a^2e^4}{2} + 14acd^2e^2 + \frac{c^2d^4}{2} + cdex(7ae^2 + cd^2)\right)}{4e} + \frac{(5a^3e^6 + 45a^2cd^2e^4 + 15ac^2d^4e^2 - c^3d^6) \operatorname{atanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}\right)}{16\sqrt{c}\sqrt{de}^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**2/(e*x+d), x)`

[Out] `-a**(3/2)*sqrt(d)*e**(3/2)*(3*a*e**2 + 5*c*d**2)*atanh((2*a*d*e + x*(a*e**2 + c*d**2))/(2*sqrt(a)*sqrt(d)*sqrt(e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))))/2 - (3*a*e - c*d*x)*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2)/(3*x) + sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))*(19*a**2*e**4/2 + 14*a*c*d**2*e**2 + c**2*d**4/2 + c*d*e*x*(7*a*e**2 + c*d**2))/(4*e) + (5*a**3*e**6 + 45*a**2*c*d**2*e**4 + 15*a*c**2*d**4*e**2 - c**3*d**6)*atanh((a*e**2 + c*d**2 + 2*c*d*e*x)/(2*sqrt(c)*sqrt(d)*sqrt(e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))))/(16*sqrt(c)*sqrt(d)*e**(3/2))`

**Mathematica [A]** time = 1.17545, size = 376, normalized size = 1.07

$$\frac{((d + ex)(ae + cdx))^{3/2} \left( 24a^{3/2}\sqrt{cde^3x} \log(x) (3ae^2 + 5cd^2) - 24a^{3/2}\sqrt{cde^3x} (3ae^2 + 5cd^2) \log\left(2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d + ex}\sqrt{ae + cdx}\right) \right)}{16\sqrt{c}\sqrt{de}^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^2*(d + e*x)), x]`

[Out] `((a*e + c*d*x)*(d + e*x))^(3/2)*(2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(3*a^2*e^3*(-8*d + 11*e*x) + 2*a*c*d*e^2*x*(34*d + 13*e*x) + c^2*d^2*x*(3*d^2 + 14*d*e*x + 8*e^2*x^2)) + 24*a^(3/2)*Sqrt[c]*d*e^3*(5*c*d^2 + 3*a*e^2)*x*Log[x] - 24*a^(3/2)*Sqrt[c]*d*e^3*(5*c*d^2 + 3*a*e^2)*x*Log[c*d^2*x + 2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x] + a*e*(2*d + e*x)] - 3*(c^3*d^6 - 15*a*c^2*d^4*e^2 - 45*a^2*c*d^2*e^4 - 5*a^3*e^6)*x*Log[a*e^2 + 2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x] + c*d*(d + 2*e*x)))/(48*Sqrt[c]*Sqrt[d]*e^(3/2)*x*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2))`

**Maple [B]** time = 0.025, size = 2364, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^2/(e*x+d), x)`

```
[Out] -15/128*e^3*d^2*a^2*c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d
*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(
(1/2)+15/256*e*d^4*a*c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(
c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*
e)^(1/2)-15/256*e^7/d^2*a^4/c*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d
*e)/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/
(c*d*e)^(1/2)+225/256*d^4*e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d
*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)*
a*c^2+121/64*d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a*c*e^2+
15/256/d^2/c*e^7*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)*a^4+375/128
*d^2*c*e^3*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+
(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)*a^2-5/2*d^3*a^2*e
^2/(a*d*e)^(1/2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d
*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)-3/256*e^9/d^4*a^5/c^2*ln
((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2
)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)+3/64*e^6/d^3*a^3/c*(a*d*e+(a*
e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-3/64*e^6/d^3*a^3/c*(c*d*e*(x+d/e)
^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-9/64*e^2*d*a*c*(c*d*e*(x+d/e)^2
+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+3/256*e^9/d^4*a^5/c^2*ln((1/2*a*e
^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*
e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)-1/8*e*c*(c*d*e*(x+d/e)^2+(a
*e^2-c*d^2)*(x+d/e))^(3/2)*x+3/128/e*d^4*c^2*(c*d*e*(x+d/e)^2+(a*
e^2-c*d^2)*(x+d/e))^(1/2)+15/128*e^5*a^3*ln((1/2*a*e^2-1/2*c*d^2+
(x+d/e)*c*d*e)/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/
e))^(1/2))/(c*d*e)^(1/2)+3/64*d^3*c^2*(c*d*e*(x+d/e)^2+(a*e^2-c*d
^2)*(x+d/e))^(1/2)*x+13/64*d^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2)*x*c^2+13/128*d^4/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*
c^2+1/a/e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*c+9/8*e*c*(a*d*
e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*x+25/128*e^5*ln((1/2*a*e^2+1/2
*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^
(1/2))/(c*d*e)^(1/2)*a^3+1/d*a*e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^
2)^(3/2)+1/5*e/d^2*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(5/2)-
1/16*d*c*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)+19/8*e^3*(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2+4/5/d^2*e*(a*d*e+(a*e^
2+c*d^2)*x+c*d*e*x^2)^(5/2)+67/48*d*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*
e*x^2)^(3/2)-1/d^2/a/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)-
13/256*d^6*c^3/e*ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(
a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)-3/64/d^2/c*
e^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3+227/64*d^2*c*e*(a
*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a-3/2*d*a^3*e^4/(a*d*e)^(1/
2)*ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^
2)*x+c*d*e*x^2)^(1/2))/x)-1/8*e^3/d^2*a*(a*d*e+(a*e^2+c*d^2)*x+c*
d*e*x^2)^(3/2)*x-1/16*e^4/d^3*a^2/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*
x^2)^(3/2)+3/128*e^7/d^4*a^4/c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2
)^(1/2)+3/64*e^5/d^2*a^3/c*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e)
)^(1/2)+1/8*e^3/d^2*a*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/
2)*x+1/16*e^4/d^3*a^2/c*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(
3/2)+9/64*e^4/d*a^2*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)
*x-3/128*e^7/d^4*a^4/c^2*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(
1/2)-3/64*e*d^2*a*c*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)
)-3/256/e*d^6*c^3*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(
1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d*e)^(1/2)
)-9/64/d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^2*e^4+1/d*c/
a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)*x
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)\*x^2), x, algo

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 18.817, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)\*x^2), x, algorithm="fricas")

[Out] [1/96\*(24\*(5\*a\*c\*d^2\*e^2 + 3\*a^2\*e^4)\*sqrt(a\*d\*e)\*sqrt(c\*d\*e)\*x\*log((8\*a^2\*d^2\*e^2 + (c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4)\*x^2 - 4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(a\*d\*e) + 8\*(a\*c\*d^3\*e + a^2\*d\*e^3)\*x)/x^2) - 3\*(c^3\*d^6 - 15\*a\*c^2\*d^4\*e^2 - 45\*a^2\*c\*d^2\*e^4 - 5\*a^3\*e^6)\*x\*log(4\*(2\*c^2\*d^2\*e^2\*x + c^2\*d^3\*e + a\*c\*d\*e^3)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) + (8\*c^2\*d^2\*e^2\*x^2 + c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4 + 8\*(c^2\*d^3\*e + a\*c\*d\*e^3)\*x)\*sqrt(c\*d\*e)) + 4\*(8\*c^2\*d^2\*e^2\*x^3 - 24\*a^2\*d\*e^3 + 2\*(7\*c^2\*d^3\*e + 13\*a\*c\*d\*e^3)\*x^2 + (3\*c^2\*d^4 + 68\*a\*c\*d^2\*e^2 + 33\*a^2\*e^4)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*e))/(sqrt(c\*d\*e)\*e\*x), 1/48\*(12\*(5\*a\*c\*d^2\*e^2 + 3\*a^2\*e^4)\*sqrt(a\*d\*e)\*sqrt(-c\*d\*e)\*x\*log((8\*a^2\*d^2\*e^2 + (c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4)\*x^2 - 4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(a\*d\*e) + 8\*(a\*c\*d^3\*e + a^2\*d\*e^3)\*x)/x^2) - 3\*(c^3\*d^6 - 15\*a\*c^2\*d^4\*e^2 - 45\*a^2\*c\*d^2\*e^4 - 5\*a^3\*e^6)\*x\*arctan(1/2\*(2\*c\*d\*e\*x + c\*d^2 + a\*e^2)\*sqrt(-c\*d\*e)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*c\*d\*e)) + 2\*(8\*c^2\*d^2\*e^2\*x^3 - 24\*a^2\*d\*e^3 + 2\*(7\*c^2\*d^3\*e + 13\*a\*c\*d\*e^3)\*x^2 + (3\*c^2\*d^4 + 68\*a\*c\*d^2\*e^2 + 33\*a^2\*e^4)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*e))/(sqrt(-c\*d\*e)\*e\*x), -1/96\*(48\*(5\*a\*c\*d^2\*e^2 + 3\*a^2\*e^4)\*sqrt(-a\*d\*e)\*sqrt(c\*d\*e)\*x\*arctan(1/2\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e))) + 3\*(c^3\*d^6 - 15\*a\*c^2\*d^4\*e^2 - 45\*a^2\*c\*d^2\*e^4 - 5\*a^3\*e^6)\*x\*log(4\*(2\*c^2\*d^2\*e^2\*x + c^2\*d^3\*e + a\*c\*d\*e^3)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) + (8\*c^2\*d^2\*e^2\*x^2 + c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4 + 8\*(c^2\*d^3\*e + a\*c\*d\*e^3)\*x)\*sqrt(c\*d\*e)) - 4\*(8\*c^2\*d^2\*e^2\*x^3 - 24\*a^2\*d\*e^3 + 2\*(7\*c^2\*d^3\*e + 13\*a\*c\*d\*e^3)\*x^2 + (3\*c^2\*d^4 + 68\*a\*c\*d^2\*e^2 + 33\*a^2\*e^4)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*e))/(sqrt(c\*d\*e)\*e\*x), -1/48\*(24\*(5\*a\*c\*d^2\*e^2 + 3\*a^2\*e^4)\*sqrt(-a\*d\*e)\*sqrt(-c\*d\*e)\*x\*arctan(1/2\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e))) + 3\*(c^3\*d^6 - 15\*a\*c^2\*d^4\*e^2 - 45\*a^2\*c\*d^2\*e^4 - 5\*a^3\*e^6)\*x\*arctan(1/2\*(2\*c\*d\*e\*x + c\*d^2 + a\*e^2)\*sqrt(-c\*d\*e)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*c\*d\*e)) - 2\*(8\*c^2\*d^2\*e^2\*x^3 - 24\*a^2\*d\*e^3 + 2\*(7\*c^2\*d^3\*e + 13\*a\*c\*d\*e^3)\*x^2 + (3\*c^2\*d^4 + 68\*a\*c\*d^2\*e^2 + 33\*a^2\*e^4)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*e))/(sqrt(-c\*d\*e)\*e\*x)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)/x\*\*2/(e\*x+d), x)

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^2), x, algori
```

```
[Out] Exception raised: TypeError
```

$$3.463 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^3(d+ex)} dx$$

**Optimal.** Leaf size=339

$$\begin{aligned} & \frac{3\sqrt{c}\sqrt{d}(5a^2e^4 + 10acd^2e^2 + c^2d^4) \tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8\sqrt{e}} \\ & - \frac{3\sqrt{a}\sqrt{e}(a^2e^4 + 10acd^2e^2 + 5c^2d^4) \tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8\sqrt{d}} \\ & - \frac{(ae - cdx)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{2x^2} \\ & - \frac{3(ae(ae^2 + 3cd^2) - cdx(3ae^2 + cd^2))\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{4x} \end{aligned}$$

[Out]  $(-3*(a*e*(3*c*d^2 + a*e^2) - c*d*(c*d^2 + 3*a*e^2)*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(4*x) - ((a*e - c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(2*x^2) + (3*\text{Sqrt}[c]*\text{Sqrt}[d]*(c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*\text{ArcTanh}[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(8*\text{Sqrt}[e]) - (3*\text{Sqrt}[a]*\text{Sqrt}[e]*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*\text{ArcTanh}[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(8*\text{Sqrt}[d])$

**Rubi [A]** time = 1.14973, antiderivative size = 339, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\begin{aligned} & \frac{3\sqrt{c}\sqrt{d}(5a^2e^4 + 10acd^2e^2 + c^2d^4) \tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8\sqrt{e}} \\ & - \frac{3\sqrt{a}\sqrt{e}(a^2e^4 + 10acd^2e^2 + 5c^2d^4) \tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8\sqrt{d}} \\ & - \frac{(ae - cdx)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{2x^2} \\ & - \frac{3(ae(ae^2 + 3cd^2) - cdx(3ae^2 + cd^2))\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{4x} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}/(x^3*(d + e*x)), x]$

[Out]  $(-3*(a*e*(3*c*d^2 + a*e^2) - c*d*(c*d^2 + 3*a*e^2)*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(4*x) - ((a*e - c*d*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(2*x^2) + (3*\text{Sqrt}[c]*\text{Sqrt}[d]*(c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*\text{ArcTanh}[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(8*\text{Sqrt}[e]) - (3*\text{Sqrt}[a]*\text{Sqrt}[e]*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*\text{ArcTanh}[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(8*\text{Sqrt}[d])$

**Rubi in Sympy [A]** time = 127.867, size = 338, normalized size = 1.

$$\frac{3\sqrt{a}\sqrt{e}(a^2e^4 + 10acd^2e^2 + 5c^2d^4) \operatorname{atanh}\left(\frac{2ade+x(ae^2+cd^2)}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}\right)}{8\sqrt{d}} + \frac{3\sqrt{c}\sqrt{d}(5a^2e^4 + 10acd^2e^2 + c^2d^4) \operatorname{atanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}\right)}{8\sqrt{e}} - \frac{3(2ae(ae^2 + 3cd^2) - 2cdx(3ae^2 + cd^2))\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{8x} - \frac{(2ae - 2cdx)(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}{4x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**3/(e*x+d), x)`

[Out] `-3*sqrt(a)*sqrt(e)*(a**2*e**4 + 10*a*c*d**2*e**2 + 5*c**2*d**4)*a tanh((2*a*d*e + x*(a*e**2 + c*d**2))/(2*sqrt(a)*sqrt(d)*sqrt(e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/(8*sqrt(d)) + 3*sqrt(c)*sqrt(d)*(5*a**2*e**4 + 10*a*c*d**2*e**2 + c**2*d**4)*atanh((a*e**2 + c*d**2 + 2*c*d*e*x)/(2*sqrt(c)*sqrt(d)*sqrt(e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/(8*sqrt(e)) - 3*(2*a*e*(a*e**2 + 3*c*d**2) - 2*c*d*x*(3*a*e**2 + c*d**2))*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(8*x) - (2*a*e - 2*c*d*x)*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2)/(4*x**2)`

**Mathematica [A]** time = 0.969937, size = 361, normalized size = 1.06

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(-2\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}(a^2e^2(2d+5ex)+9acdex(d-ex)-c^2d^2x^2(5d+2ex))+3\sqrt{aex^2}\log(x)(a^2\right)}{8\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^3*(d + e*x)), x]`

[Out] `(Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(-2*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*(9*a*c*d*e*x*(d - e*x) - c^2*d^2*x^2*(5*d + 2*e*x) + a^2*e^2*(2*d + 5*e*x)) + 3*Sqrt[a]*e*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*x^2*Log[x] - 3*Sqrt[a]*e*(5*c^2*d^4 + 10*a*c*d^2*e^2 + a^2*e^4)*x^2*Log[c*d^2*x + 2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x] + a*e*(2*d + e*x)] + 3*Sqrt[c]*d*(c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*x^2*Log[a*e^2 + 2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x] + c*d*(d + 2*e*x)))/(8*Sqrt[d]*Sqrt[e]*x^2*Sqrt[(a*e + c*d*x)*(d + e*x)])`

**Maple [B]** time = 0.029, size = 2688, normalized size = 7.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^3/(e*x+d), x)`

[Out] `3/64/d^4*e^7*a^3/c*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x-3/256/d^5*e^10*a^5/c^2*ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^(1/2)+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2))/(c*d`

$$\begin{aligned}
& e^{(1/2)} + 15/128 * d * e^4 * a^2 * c * \ln((1/2 * a * e^2 - 1/2 * c * d^2 + (x+d/e) * c * d * e) \\
& ) / (c * d * e)^{(1/2)} + (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{(1/2)} / (c \\
& * d * e)^{(1/2)} - 15/256 * d^3 * e^2 * a * c^2 * \ln((1/2 * a * e^2 - 1/2 * c * d^2 + (x+d/e) * \\
& c * d * e) / (c * d * e)^{(1/2)} + (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{(1/2)} \\
& ) / (c * d * e)^{(1/2)} + 15/256 / d^3 * e^8 * a^4 / c * \ln((1/2 * a * e^2 - 1/2 * c * d^2 + (x+ \\
& d/e) * c * d * e) / (c * d * e)^{(1/2)} + (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e)) \\
& ^{(1/2)}) / (c * d * e)^{(1/2)} - 3/64 / d^4 * e^7 * a^3 / c * (a * d * e + (a * e^2 + c * d^2) * x + c \\
& * d * e * x^2)^{(1/2)} * x + 3/256 / d^5 * e^{10} * a^5 / c^2 * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + \\
& c * d * e * x) / (c * d * e)^{(1/2)} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)}) / ( \\
& c * d * e)^{(1/2)} - 15/256 / d^3 * e^8 * a^4 / c * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x \\
& ) / (c * d * e)^{(1/2)} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)}) / (c * d * e)^{ \\
& (1/2)} - 15/4 * d^2 * a^2 * e^3 / (a * d * e)^{(1/2)} * \ln((2 * a * d * e + (a * e^2 + c * d^2) * x + \\
& 2 * (a * d * e)^{(1/2)} * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)}) / x) * c - 15/ \\
& 8 * d^4 * a * e / (a * d * e)^{(1/2)} * \ln((2 * a * d * e + (a * e^2 + c * d^2) * x + 2 * (a * d * e)^{(1/ \\
& 2)} * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)}) / x) * c^2 + 225/128 * d * a^2 * \\
& e^4 * c * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x) / (c * d * e)^{(1/2)} + (a * d * e + (a * e^ \\
& 2 + c * d^2) * x + c * d * e * x^2)^{(1/2)}) / (c * d * e)^{(1/2)} + 975/256 * d^3 * a * e^2 * c^2 * \\
& \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x) / (c * d * e)^{(1/2)} + (a * d * e + (a * e^2 + c * d^ \\
& 2) * x + c * d * e * x^2)^{(1/2)}) / (c * d * e)^{(1/2)} - 3/4 / d / a^2 / e^2 / x * (a * d * e + (a * e^ \\
& 2 + c * d^2) * x + c * d * e * x^2)^{(7/2)} * c - 1/4 / d^2 / a * e * c * (a * d * e + (a * e^2 + c * d^2) * \\
& x + c * d * e * x^2)^{(5/2)} * x + 1/4 / d^3 / a / x * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2 \\
& )^{(7/2)} + 3/256 * d^5 * c^3 * \ln((1/2 * a * e^2 - 1/2 * c * d^2 + (x+d/e) * c * d * e) / (c * d \\
& * e)^{(1/2)} + (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{(1/2)}) / (c * d * e)^{ \\
& (1/2)} + 1/d / a * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(5/2)} * c - 3/8 * a^3 * e^5 \\
& / (a * d * e)^{(1/2)} * \ln((2 * a * d * e + (a * e^2 + c * d^2) * x + 2 * (a * d * e)^{(1/2)} * (a * d * e \\
& + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)}) / x) + 93/256 * d^5 * \ln((1/2 * a * e^2 + 1/ \\
& 2 * c * d^2 + c * d * e * x) / (c * d * e)^{(1/2)} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{ \\
& (1/2)}) / (c * d * e)^{(1/2)} * c^3 + 3/4 / d * a^2 * e^4 * (a * d * e + (a * e^2 + c * d^2) * x + c * d \\
& * e * x^2)^{(1/2)} + 1/4 / d^2 * a * e^3 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(3/ \\
& 2)} - 1/20 / d^3 * e^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(5/2)} + 387/128 * d \\
& ^3 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} * c^2 - 1/5 / d^3 * e^2 * (c * d * e \\
& * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{(5/2)} + 1/16 * e * c * (c * d * e * (x+d/e)^2 \\
& + (a * e^2 - c * d^2) * (x+d/e))^{(3/2)} - 3/128 * d^3 * c^2 * (c * d * e * (x+d/e)^2 + (a * e \\
& ^2 - c * d^2) * (x+d/e))^{(1/2)} + 31/16 * e * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2 \\
& )^{(3/2)} * c - 3/64 / d^3 * e^6 * a^3 / c * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/ \\
& e))^{(1/2)} + 1/8 / d * e^2 * c * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{(3/ \\
& 2)} * x - 1/16 / d^4 * e^5 * a^2 / c * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{( \\
& 3/2)} - 9/64 / d^2 * e^5 * a^2 * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{(1/ \\
& 2)} * x + 3/128 / d^5 * e^8 * a^4 / c^2 * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e) \\
& )^{(1/2)} + 3/64 * d * e^2 * a * c * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{(1 \\
& /2)} - 15/128 / d * e^6 * a^3 * \ln((1/2 * a * e^2 - 1/2 * c * d^2 + (x+d/e) * c * d * e) / (c * d * \\
& e)^{(1/2)} + (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{(1/2)}) / (c * d * e)^{ \\
& (1/2)} - 3/64 * d^2 * e * c^2 * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{(1/2)} \\
& * x + 3/64 / d^3 * e^6 * a^3 / c * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} + 1/8 \\
& / d^3 * e^4 * a * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(3/2)} * x + 1/16 / d^4 * e^5 \\
& * a^2 / c * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(3/2)} + 9/64 / d^2 * e^5 * a^2 * ( \\
& a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} * x - 3/128 / d^5 * e^8 * a^4 / c^2 * (a \\
& * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} + 39/64 * a * e^3 * (a * d * e + (a * e^2 + c \\
& * d^2) * x + c * d * e * x^2)^{(1/2)} * x + 3/4 / a^2 / e * c^2 * (a * d * e + (a * e^2 + c * d^2) * x \\
& + c * d * e * x^2)^{(5/2)} * x + 333/64 * d * a * e^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x \\
& ^2)^{(1/2)} * c + 5/4 * d^2 / a / e * c^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(3/ \\
& 2)} + 1/8 / d * e^2 * c * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(3/2)} * x + 147/64 * d \\
& ^2 * e * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} * x * c^2 + 5/4 * d / a * c^2 * (a \\
& * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(3/2)} * x + 15/128 / d * a^3 * e^6 * \ln((1/2 * \\
& a * e^2 + 1/2 * c * d^2 + c * d * e * x) / (c * d * e)^{(1/2)} + (a * d * e + (a * e^2 + c * d^2) * x + c * d \\
& * e * x^2)^{(1/2)}) / (c * d * e)^{(1/2)} + 3/4 * d / a^2 / e^2 * (a * d * e + (a * e^2 + c * d^2) * x \\
& + c * d * e * x^2)^{(5/2)} * c^2 - 1/2 / d^2 / a / e / x^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d \\
& * e * x^2)^{(7/2)} - 1/8 / d^3 * e^4 * a * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e) \\
& )^{(3/2)} * x + 9/64 * e^3 * a * c * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{(1 \\
& /2)} * x
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)\*x^3), x, algori

[Out] Exception raised: ValueError

**Fricas** [A] time = 7.65457, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)\*x^3), x, algorithm="fricas")

[Out] 
$$\begin{aligned} & \left[ \frac{1}{16} (3 (c^2 d^4 + 10 a c d^2 e^2 + 5 a^2 e^4) \sqrt{c d / e}) x^2 \log(8 c^2 d^2 e^2 x^2 + c^2 d^4 + 6 a c d^2 e^2 + a^2 e^4 + 4 (2 c d e^2 x + c d^2 e + a e^3) \sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x}) \sqrt{c d / e} \right. \\ & + 8 (c^2 d^3 e + a c d e^3) x + 3 (5 c^2 d^4 + 10 a c d^2 e^2 + a^2 e^4) \sqrt{a e / d} x^2 \log((8 a^2 d^2 e^2 + (c^2 d^4 + 6 a c d^2 e^2 + a^2 e^4) x^2 - 4 \sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x}) \\ & (2 a d^2 e + (c d^3 + a d e^2) x) \sqrt{a e / d} + 8 (a c d^3 e + a^2 d e^3) x) / x^2 + 4 (2 c^2 d^2 e x^3 - 2 a^2 d e^2 + (5 c^2 d^3 + 9 a c d e^2) x^2 - (9 a c d^2 e + 5 a^2 e^3) x) \sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x} \\ & \left. \right] / x^2, \frac{1}{16} (6 (c^2 d^4 + 10 a c d^2 e^2 + 5 a^2 e^4) \sqrt{-c d / e}) x^2 \arctan(1 / 2 (2 c d e x + c d^2 + a e^2) / (\sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x} \sqrt{-c d / e})) \\ & + 3 (5 c^2 d^4 + 10 a c d^2 e^2 + a^2 e^4) \sqrt{a e / d} x^2 \log((8 a^2 d^2 e^2 + (c^2 d^4 + 6 a c d^2 e^2 + a^2 e^4) x^2 - 4 \sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x}) \\ & (2 a d^2 e + (c d^3 + a d e^2) x) \sqrt{a e / d} + 8 (a c d^3 e + a^2 d e^3) x) / x^2 + 4 (2 c^2 d^2 e x^3 - 2 a^2 d e^2 + (5 c^2 d^3 + 9 a c d e^2) x^2 - (9 a c d^2 e + 5 a^2 e^3) x) \sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x} \\ & \left. \right] / x^2, -1 / 16 (6 (5 c^2 d^4 + 10 a c d^2 e^2 + a^2 e^4) \sqrt{-a e / d}) x^2 \arctan(1 / 2 (2 a d e + (c d^2 + a e^2) x) / (\sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x} \sqrt{-a e / d})) \\ & - 3 (c^2 d^4 + 10 a c d^2 e^2 + 5 a^2 e^4) \sqrt{c d / e} x^2 \log(8 c^2 d^2 e^2 x^2 + c^2 d^4 + 6 a c d^2 e^2 + a^2 e^4 + 4 (2 c d e^2 x + c d^2 e + a e^3) \sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x}) \sqrt{c d / e} \\ & + 8 (c^2 d^3 e + a c d e^3) x - 4 (2 c^2 d^2 e x^3 - 2 a^2 d e^2 + (5 c^2 d^3 + 9 a c d e^2) x^2 - (9 a c d^2 e + 5 a^2 e^3) x) \sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x} \\ & \left. \right] / x^2, -1 / 8 (3 (5 c^2 d^4 + 10 a c d^2 e^2 + a^2 e^4) \sqrt{-a e / d}) x^2 \arctan(1 / 2 (2 a d e + (c d^2 + a e^2) x) / (\sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x} \sqrt{-a e / d})) \\ & - 3 (c^2 d^4 + 10 a c d^2 e^2 + 5 a^2 e^4) \sqrt{-c d / e} x^2 \arctan(1 / 2 (2 c d e x + c d^2 + a e^2) / (\sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x} \sqrt{-c d / e})) \\ & - 2 (2 c^2 d^2 e x^3 - 2 a^2 d e^2 + (5 c^2 d^3 + 9 a c d e^2) x^2 - (9 a c d^2 e + 5 a^2 e^3) x) \sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x} \\ & \left. \right] / x^2 \end{aligned}$$

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)/x\*\*3/(e\*x+d), x)

[Out] Timed out

**GIAC/XCAS** [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^3), x, algori
```

```
[Out] Exception raised: TypeError
```

$$3.464 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^4(d+ex)} dx$$

**Optimal.** Leaf size=371

$$\frac{(-a^2e^4 - 2cdex(ae^2 + 7cd^2) + 12acd^2e^2 + 5c^2d^4) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{8dx} - \frac{(-a^3e^6 + 15a^2cd^2e^4 + 45ac^2d^4e^2 + 5c^3d^6) \tanh^{-1}\left(\frac{x(ae^2 + cd^2) + 2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{16\sqrt{ad}^{3/2}\sqrt{e}} + \frac{1}{2}c^{3/2}d^{3/2}\sqrt{e}(5ae^2 + 3cd^2) \tanh^{-1}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right) - \frac{(3x(ae^2 + 3cd^2) + 4ade)(x(ae^2 + cd^2) + ade)}{12dx^3}$$

[Out]  $-\left(\left(5c^2d^4 + 12a^2cd^2e^2 - a^2e^4 - 2c^2d^2e^2\right)\sqrt{x(ae^2 + cd^2) + ade + cdex^2} - \left(4a^3e^6 + 15a^2cd^2e^4 + 45ac^2d^4e^2 + 5c^3d^6\right)\tanh^{-1}\left(\frac{x(ae^2 + cd^2) + 2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)\right) - \left(\left(5c^3d^6 + 45a^2cd^2e^4 + 15a^2c^2d^4e^2 - a^3e^6\right)\tanh^{-1}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)\right) - \frac{(3x(ae^2 + 3cd^2) + 4ade)(x(ae^2 + cd^2) + ade)}{12dx^3}$

**Rubi [A]** time = 1.29497, antiderivative size = 371, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$

$$\frac{(-a^2e^4 - 2cdex(ae^2 + 7cd^2) + 12acd^2e^2 + 5c^2d^4) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{8dx} - \frac{(-a^3e^6 + 15a^2cd^2e^4 + 45ac^2d^4e^2 + 5c^3d^6) \tanh^{-1}\left(\frac{x(ae^2 + cd^2) + 2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{16\sqrt{ad}^{3/2}\sqrt{e}} + \frac{1}{2}c^{3/2}d^{3/2}\sqrt{e}(5ae^2 + 3cd^2) \tanh^{-1}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right) - \frac{(3x(ae^2 + 3cd^2) + 4ade)(x(ae^2 + cd^2) + ade)}{12dx^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a^2d^2e + (cd^2 + ae^2)x + cdex^2)^{5/2}/(x^4(d + ex)), x]$

[Out]  $-\left(\left(5c^2d^4 + 12a^2cd^2e^2 - a^2e^4 - 2c^2d^2e^2\right)\sqrt{x(ae^2 + cd^2) + ade + cdex^2} - \left(4a^3e^6 + 15a^2cd^2e^4 + 45ac^2d^4e^2 + 5c^3d^6\right)\tanh^{-1}\left(\frac{x(ae^2 + cd^2) + 2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)\right) - \left(\left(5c^3d^6 + 45a^2cd^2e^4 + 15a^2c^2d^4e^2 - a^3e^6\right)\tanh^{-1}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)\right) - \frac{(3x(ae^2 + 3cd^2) + 4ade)(x(ae^2 + cd^2) + ade)}{12dx^3}$

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a^2d^2e + (a^2e^2 + c^2d^2)x + cdex^2)^{5/2}/x^4/(e^2x + d), x)$

[Out] Timed out

---

**Mathematica [A]** time = 1.11588, size = 399, normalized size = 1.08

$$\sqrt{d+ex}\sqrt{ae+cdx}\left(-2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}(a^2e^2(8d^2+14dex+3e^2x^2)+2acd^2ex(13d+34ex))+3c^2d^3x^2(11d-8e)\right)$$


---

Antiderivative was successfully verified.

[In] Integrate[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2)/(x^4\*(d + e\*x)), x]

[Out] (Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*(-2\*Sqrt[a]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*(3\*c^2\*d^3\*x^2\*(11\*d - 8\*e\*x) + 2\*a\*c\*d^2\*e\*x\*(13\*d + 34\*e\*x) + a^2\*e^2\*(8\*d^2 + 14\*d\*e\*x + 3\*e^2\*x^2)) + 3\*(5\*c^3\*d^6 + 45\*a\*c^2\*d^4\*e^2 + 15\*a^2\*c\*d^2\*e^4 - a^3\*e^6)\*x^3\*Log[x] - 3\*(5\*c^3\*d^6 + 45\*a\*c^2\*d^4\*e^2 + 15\*a^2\*c\*d^2\*e^4 - a^3\*e^6)\*x^3\*Log[c\*d^2\*x + 2\*Sqrt[a]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x] + a\*e\*(2\*d + e\*x)] + 24\*Sqrt[a]\*c^(3/2)\*d^3\*e\*(3\*c\*d^2 + 5\*a\*e^2)\*x^3\*Log[a\*e^2 + 2\*Sqrt[c]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x] + c\*d\*(d + 2\*e\*x)))/(48\*Sqrt[a]\*d^(3/2)\*Sqrt[e]\*x^3\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)])

---

**Maple [B]** time = 0.034, size = 3144, normalized size = 8.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2)/x^4/(e\*x+d), x)

[Out] 
$$-15/16*d*a^2*e^4/(a*d*e)^{(1/2)}*\ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^{(1/2)}*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)})/x)*c-45/16*d^3*a*e^2/(a*d*e)^{(1/2)}*\ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^{(1/2)}*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)})/x)*c^2+3/64/d^5*e^8*a^3/c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)}*x-3/256/d^6*e^11*a^5/c^2*\ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^{(1/2)}+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)})/(c*d*e)^{(1/2)}+15/256/d^4*e^9/c*\ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^{(1/2)}+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)})/(c*d*e)^{(1/2)}*a^4-3/64/d^5*e^8*a^3/c*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{(1/2)}*x-9/64/d*e^4*a*c*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{(1/2)}*x+3/256/d^6*e^11*a^5/c^2*\ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^{(1/2)}+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{(1/2)})/(c*d*e)^{(1/2)}+15/256*d^2*e^3*a*c^2*\ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^{(1/2)}+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{(1/2)})/(c*d*e)^{(1/2)}-15/256/d^4*e^9*a^4/c*\ln((1/2*a*e^2-1/2*c*d^2+(x+d/e)*c*d*e)/(c*d*e)^{(1/2)}+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{(1/2)})/(c*d*e)^{(1/2)}+625/256*d^2*a*e^3*c^2*\ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^{(1/2)}+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)})/(c*d*e)^{(1/2)}-5/6/d^2/a^2/e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(7/2)}*c+5/24*d^2/a^2/e*c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(3/2)}*x-1/12/d/a^2/e^2/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(7/2)}*c+1/64/d*a^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)}*x^c+3/8/d^3/a^2*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(5/2)}*x+1/8*d/a^3/e^2*c^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(5/2)}*x-1/16/d^2*e^2*c*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{(3/2)}+3/128*d^2*e*c^2*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{(1/2)}-3/64*e^3*a*c*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{(1/2)}-5/16*d^5/(a*d*e)^{(1/2)}*\ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^{(1/2)}*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)})/x)*c^3-1/24/d^3*a^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(3/2)}-1/8/d^2*a^2*e^5*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)}+37/48/d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(3/2)}*c+5/12/d^3/a/x^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(7/2)}$$

$$\begin{aligned}
& 2)+35/24*d/a*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(3/2)}+493/128* \\
& d^2*e*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)*c^2+25/24/a^2/e*(a* \\
& d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(5/2)*c^2+107/64*a*e^3*(a*d*e+(a*e \\
& ^2+c*d^2)*x+c*d*e*x^2)^{(1/2)*c+7/40/d^4*e^3*(a*d*e+(a*e^2+c*d^2)* \\
& x+c*d*e*x^2)^{(5/2)}+1/5/d^4*e^3*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+ \\
& d/e))^{(5/2)}-1/8/d^4*e^5*a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(3/2)} \\
& *x+93/64*d*e^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)*x*c^2+5/6/ \\
& d/a^2*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(5/2)*x-1/3/d^2/a/e/x \\
& ^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(7/2)}+19/24/d^2/a*e*(a*d*e+( \\
& a*e^2+c*d^2)*x+c*d*e*x^2)^{(5/2)*c+5/8*d^4/a/e*(a*d*e+(a*e^2+c*d^2) \\
& )*x+c*d*e*x^2)^{(1/2)*c^3+5/24*d^3/a^2/e^2*c^3*(a*d*e+(a*e^2+c*d^2) \\
& )*x+c*d*e*x^2)^{(3/2)}-3/8/d^4/a*e/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x \\
& ^2)^{(7/2)}+1/8*d^2/a^3/e^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(5/2)} \\
& *c^3+1/16/d*a^3*e^6/(a*d*e)^{(1/2)*\ln((2*a*d*e+(a*e^2+c*d^2)*x+2*( \\
& a*d*e)^{(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)})/x)+15/128*a \\
& ^2*e^5*c*\ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^{(1/2)}+(a*d*e+(a \\
& *e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)})/(c*d*e)^{(1/2)}+3/128/d^6*e^9*a^4/c \\
& ^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)}-9/64/d^3*e^6*(a*d*e+(a \\
& *e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)*x*a^2-3/64/d^4*e^7/c*(a*d*e+(a*e^2 \\
& +c*d^2)*x+c*d*e*x^2)^{(1/2)*a^3-15/128/d^2*e^7*\ln((1/2*a*e^2+1/2*c \\
& *d^2+c*d*e*x)/(c*d*e)^{(1/2)}+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/ \\
& 2)})/(c*d*e)^{(1/2)*a^3-1/8/d^2*e^3*c*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2 \\
& )*(x+d/e))^{(3/2)*x-3/128/d^6*e^9*a^4/c^2*(c*d*e*(x+d/e)^2+(a*e^2- \\
& c*d^2)*(x+d/e))^{(1/2)}+15/128/d^2*e^7*a^3*\ln((1/2*a*e^2-1/2*c*d^2+ \\
& (x+d/e)*c*d*e)/(c*d*e)^{(1/2)}+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/ \\
& e))^{(1/2)})/(c*d*e)^{(1/2)}+3/64*d*e^2*c^2*(c*d*e*(x+d/e)^2+(a*e^2-c \\
& *d^2)*(x+d/e))^{(1/2)*x-3/256*d^4*e*c^3*\ln((1/2*a*e^2-1/2*c*d^2+(x \\
& +d/e)*c*d*e)/(c*d*e)^{(1/2)}+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e) \\
& )^{(1/2)})/(c*d*e)^{(1/2)}+1/16/d^5*e^6*a^2/c*(c*d*e*(x+d/e)^2+(a*e^2 \\
& -c*d^2)*(x+d/e))^{(3/2)}+9/64/d^3*e^6*a^2*(c*d*e*(x+d/e)^2+(a*e^2-c \\
& *d^2)*(x+d/e))^{(1/2)*x-15/128*e^5*a^2*c*\ln((1/2*a*e^2-1/2*c*d^2+(x \\
& +d/e)*c*d*e)/(c*d*e)^{(1/2)}+(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e) \\
& ))^{(1/2)})/(c*d*e)^{(1/2)}+1/8/d^4*e^5*a*(c*d*e*(x+d/e)^2+(a*e^2-c*d \\
& ^2)*(x+d/e))^{(3/2)*x+3/64/d^4*e^7*a^3/c*(c*d*e*(x+d/e)^2+(a*e^2-c \\
& *d^2)*(x+d/e))^{(1/2)}+5/6/a*e*c^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2 \\
& )^{(3/2)*x-1/8/a^3/e^3/x*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(7/2)*c \\
& ^2+1/12/d^2*e^3*c*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(3/2)*x+5/8*d \\
& ^3/a*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)*x*c^3+387/256*d^4*e* \\
& c^3*\ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^{(1/2)}+(a*d*e+(a*e^2+ \\
& c*d^2)*x+c*d*e*x^2)^{(1/2)})/(c*d*e)^{(1/2)}-1/16/d^5*e^6*a^2/c*(a*d* \\
& e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(3/2)}
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)\*x^4), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Ericas [A]** time = 9.08583, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)\*x^4), x, algorithm="ericas")

[Out] [1/96\*(24\*(3\*c^2\*d^4 + 5\*a\*c\*d^2\*e^2)\*sqrt(a\*d\*e)\*sqrt(c\*d\*e)\*x^3 + log(8\*c^2\*d^2\*e^2\*x^2 + c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4 + 4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(2\*c\*d\*e\*x + c\*d^2 + a

$$\begin{aligned}
& e^2) \sqrt{c^2 d^3 e} + 8(c^2 d^3 e + a^2 c^2 d^3 e^3) x - 3(5c^3 d^6 + 45a^2 c^2 d^4 e^2 + 15a^2 c^2 d^2 e^4 - a^3 e^6) x^3 \log((4(2a^2 d^2 e^2 + (a^2 c^2 d^3 e + a^2 d^2 e^3) x) \sqrt{c^2 d^2 e^2 + a^2 d^2 e + (c^2 d^2 + a^2 e^2) x} + (8a^2 d^2 e^2 + (c^2 d^4 + 6a^2 c^2 d^2 e^2 + a^2 e^4) x^2 + 8(a^2 c^2 d^3 e + a^2 d^2 e^3) x) \sqrt{a^2 d^2 e}) / x^2) + 4(24c^2 d^3 e^2 x^3 - 8a^2 d^2 e^2 - (33c^2 d^4 + 68a^2 c^2 d^2 e^2 + 3a^2 e^4) x^2 - 2(13a^2 c^2 d^3 e + 7a^2 d^2 e^3) x) \sqrt{c^2 d^2 e^2 + a^2 d^2 e + (c^2 d^2 + a^2 e^2) x} \sqrt{a^2 d^2 e}) / (\sqrt{a^2 d^2 e} d^2 x^3), 1/96(48(3c^2 d^4 + 5a^2 c^2 d^2 e^2) \sqrt{a^2 d^2 e} \sqrt{-c^2 d^2 e} x^3 \arctan(1/2(2c^2 d^2 e x + c^2 d^2 + a^2 e^2) / (\sqrt{c^2 d^2 e^2 + a^2 d^2 e + (c^2 d^2 + a^2 e^2) x} \sqrt{-c^2 d^2 e})) - 3(5c^3 d^6 + 45a^2 c^2 d^4 e^2 + 15a^2 c^2 d^2 e^4 - a^3 e^6) x^3 \log((4(2a^2 d^2 e^2 + (a^2 c^2 d^3 e + a^2 d^2 e^3) x) \sqrt{c^2 d^2 e^2 + a^2 d^2 e + (c^2 d^2 + a^2 e^2) x} + (8a^2 d^2 e^2 + (c^2 d^4 + 6a^2 c^2 d^2 e^2 + a^2 e^4) x^2 + 8(a^2 c^2 d^3 e + a^2 d^2 e^3) x) \sqrt{a^2 d^2 e}) / x^2) + 4(24c^2 d^3 e^2 x^3 - 8a^2 d^2 e^2 - (33c^2 d^4 + 68a^2 c^2 d^2 e^2 + 3a^2 e^4) x^2 - 2(13a^2 c^2 d^3 e + 7a^2 d^2 e^3) x) \sqrt{c^2 d^2 e^2 + a^2 d^2 e + (c^2 d^2 + a^2 e^2) x} \sqrt{a^2 d^2 e}) / (\sqrt{a^2 d^2 e} d^2 x^3), 1/48(12(3c^2 d^4 + 5a^2 c^2 d^2 e^2) \sqrt{-a^2 d^2 e} \sqrt{c^2 d^2 e} x^3 \log(8c^2 d^2 e^2 x^2 + c^2 d^4 + 6a^2 c^2 d^2 e^2 + a^2 e^4 + 4\sqrt{c^2 d^2 e^2 + a^2 d^2 e + (c^2 d^2 + a^2 e^2) x} (2c^2 d^2 e x + c^2 d^2 + a^2 e^2) \sqrt{c^2 d^2 e} + 8(c^2 d^3 e + a^2 c^2 d^3 e^3) x) - 3(5c^3 d^6 + 45a^2 c^2 d^4 e^2 + 15a^2 c^2 d^2 e^4 - a^3 e^6) x^3 \arctan(1/2(2a^2 d^2 e + (c^2 d^2 + a^2 e^2) x) \sqrt{-a^2 d^2 e}) / (\sqrt{c^2 d^2 e^2 + a^2 d^2 e + (c^2 d^2 + a^2 e^2) x} \sqrt{a^2 d^2 e})) + 2(24c^2 d^3 e^2 x^3 - 8a^2 d^2 e^2 - (33c^2 d^4 + 68a^2 c^2 d^2 e^2 + 3a^2 e^4) x^2 - 2(13a^2 c^2 d^3 e + 7a^2 d^2 e^3) x) \sqrt{c^2 d^2 e^2 + a^2 d^2 e + (c^2 d^2 + a^2 e^2) x} \sqrt{-a^2 d^2 e}) / (\sqrt{-a^2 d^2 e} d^2 x^3), 1/48(24(3c^2 d^4 + 5a^2 c^2 d^2 e^2) \sqrt{-a^2 d^2 e} \sqrt{-c^2 d^2 e} x^3 \arctan(1/2(2c^2 d^2 e x + c^2 d^2 + a^2 e^2) / (\sqrt{c^2 d^2 e^2 + a^2 d^2 e + (c^2 d^2 + a^2 e^2) x} \sqrt{-c^2 d^2 e})) - 3(5c^3 d^6 + 45a^2 c^2 d^4 e^2 + 15a^2 c^2 d^2 e^4 - a^3 e^6) x^3 \arctan(1/2(2a^2 d^2 e + (c^2 d^2 + a^2 e^2) x) \sqrt{-a^2 d^2 e}) / (\sqrt{c^2 d^2 e^2 + a^2 d^2 e + (c^2 d^2 + a^2 e^2) x} \sqrt{a^2 d^2 e})) + 2(24c^2 d^3 e^2 x^3 - 8a^2 d^2 e^2 - (33c^2 d^4 + 68a^2 c^2 d^2 e^2 + 3a^2 e^4) x^2 - 2(13a^2 c^2 d^3 e + 7a^2 d^2 e^3) x) \sqrt{c^2 d^2 e^2 + a^2 d^2 e + (c^2 d^2 + a^2 e^2) x} \sqrt{-a^2 d^2 e}) / (\sqrt{-a^2 d^2 e} d^2 x^3)]
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)/x\*\*4/(e\*x+d),x)

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)\*x^4),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.465 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^5(d+ex)} dx$$

**Optimal.** Leaf size=404

$$\frac{(x(-3a^3e^6 + 11a^2cd^2e^4 + 83ac^2d^4e^2 + 5c^3d^6) + 2ade(5cd^2 - ae^2)(3ae^2 + cd^2))\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{64ad^2ex^2} + \frac{(-3a^4e^8 + 20a^3cd^2e^6 - 90a^2c^2d^4e^4 - 60ac^3d^6e^2 + 5c^4d^8) \tanh^{-1}\left(\frac{x(ae^2 + cd^2) + 2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{128a^{3/2}d^{5/2}e^{3/2}} + c^{5/2}d^{5/2}e^{3/2} \tanh^{-1}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right) - \frac{(x(3ae^2 + 11cd^2) + 6ade)(x(ae^2 + cd^2) + ade + cdex^2)^3}{24dx^4}$$

[Out]  $-\left((2*a*d*e*(5*c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2) + (5*c^3*d^6 + 83*a*c^2*d^4*e^2 + 11*a^2*c*d^2*e^4 - 3*a^3*e^6)*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]\right)/(64*a*d^2*e*x^2) - \left(\left(6*a*d*e + (11*c*d^2 + 3*a*e^2)*x\right)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}\right)/(24*d*x^4) + c^{(5/2)}*d^{(5/2)}*e^{(3/2)}*\text{ArcTanh}\left[\frac{c*d^2 + a*e^2 + 2*c*d*e*x}{2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]}\right] + \left(\left(5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 90*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 3*a^4*e^8\right)*\text{ArcTanh}\left[\frac{2*a*d*e + (c*d^2 + a*e^2)*x}{2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]}\right]\right)/(128*a^{(3/2)}*d^{(5/2)}*e^{(3/2)})$

**Rubi [A]** time = 1.31196, antiderivative size = 404, normalized size of antiderivative = 1., number of rules used = 8, number of rules used = 6, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{(x(-3a^3e^6 + 11a^2cd^2e^4 + 83ac^2d^4e^2 + 5c^3d^6) + 2ade(5cd^2 - ae^2)(3ae^2 + cd^2))\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{64ad^2ex^2} + \frac{(-3a^4e^8 + 20a^3cd^2e^6 - 90a^2c^2d^4e^4 - 60ac^3d^6e^2 + 5c^4d^8) \tanh^{-1}\left(\frac{x(ae^2 + cd^2) + 2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{128a^{3/2}d^{5/2}e^{3/2}} + c^{5/2}d^{5/2}e^{3/2} \tanh^{-1}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right) - \frac{(x(3ae^2 + 11cd^2) + 6ade)(x(ae^2 + cd^2) + ade + cdex^2)^3}{24dx^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}/(x^5*(d + e*x)), x]$

[Out]  $-\left((2*a*d*e*(5*c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2) + (5*c^3*d^6 + 83*a*c^2*d^4*e^2 + 11*a^2*c*d^2*e^4 - 3*a^3*e^6)*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]\right)/(64*a*d^2*e*x^2) - \left(\left(6*a*d*e + (11*c*d^2 + 3*a*e^2)*x\right)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}\right)/(24*d*x^4) + c^{(5/2)}*d^{(5/2)}*e^{(3/2)}*\text{ArcTanh}\left[\frac{c*d^2 + a*e^2 + 2*c*d*e*x}{2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]}\right] + \left(\left(5*c^4*d^8 - 60*a*c^3*d^6*e^2 - 90*a^2*c^2*d^4*e^4 + 20*a^3*c*d^2*e^6 - 3*a^4*e^8\right)*\text{ArcTanh}\left[\frac{2*a*d*e + (c*d^2 + a*e^2)*x}{2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]}\right]\right)/(128*a^{(3/2)}*d^{(5/2)}*e^{(3/2)})$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a*d*e + (a*e^2 + c*d^2)*x + c*d*e*x^2)^{(5/2)}/x^5/(e*x + d), x)$



$$\begin{aligned}
& 2) * x + c * d * e * x^2)^{(1/2)} * c^4 - 13/32/d^4/a * e/x^2 * (a * d * e + (a * e^2 + c * d^2) * \\
& x + c * d * e * x^2)^{(7/2)} - 5/192 * d^4/a^3/e^3 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e \\
& * x^2)^{(3/2)} * c^4 - 1/8/d^5 * e^6 * a * (c * d * e * (x + d/e)^2 + (a * e^2 - c * d^2) * (x + d \\
& /e))^3 * x - 3/64/d^5 * e^8 * a^3/c * (c * d * e * (x + d/e)^2 + (a * e^2 - c * d^2) * (x \\
& + d/e))^2 + 1/8/d^3 * e^4 * c * (c * d * e * (x + d/e)^2 + (a * e^2 - c * d^2) * (x + d/e) \\
& )^3 * x - 1/16/d^6 * e^7 * a^2/c * (c * d * e * (x + d/e)^2 + (a * e^2 - c * d^2) * (x + d/e) \\
& )^3 * x - 9/64/d^4 * e^7 * a^2 * (c * d * e * (x + d/e)^2 + (a * e^2 - c * d^2) * (x + d/e) \\
& )^2 + 1/2 * x + 3/128/d^7 * e^10 * a^4/c^2 * (c * d * e * (x + d/e)^2 + (a * e^2 - c * d^2) * (x + d/e) \\
& )^2 + 3/64/d * e^4 * a * c * (c * d * e * (x + d/e)^2 + (a * e^2 - c * d^2) * (x + d/e) \\
& )^2 - 15/128/d^3 * e^8 * a^3 * \ln((1/2 * a * e^2 - 1/2 * c * d^2 + (x + d/e) * c * d * \\
& e)/(c * d * e)^{(1/2)} + (c * d * e * (x + d/e)^2 + (a * e^2 - c * d^2) * (x + d/e))^2)^{(1/2)}) / ( \\
& c * d * e)^{(1/2)} + 3/256 * d^3 * e^2 * c^3 * \ln((1/2 * a * e^2 - 1/2 * c * d^2 + (x + d/e) * c * \\
& d * e)/(c * d * e)^{(1/2)} + (c * d * e * (x + d/e)^2 + (a * e^2 - c * d^2) * (x + d/e))^2)^{(1/2)}) / ( \\
& c * d * e)^{(1/2)} + 3/64/d^5 * e^8 * a^3/c * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2) \\
& )^2)^{(1/2)} + 1/8/d^5 * e^6 * a * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^3 * x + \\
& 1/16/d^6 * e^7 * a^2/c * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^3 + 9/64/d \\
& ^4 * e^7 * a^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^2 + 1/2 * x - 3/128/d^7 * e^10 \\
& * a^4/c^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^2 + 15/128/d^3 * e^8 \\
& * a^3 * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x)/(c * d * e)^{(1/2)} + (a * d * e + (a * e^2 \\
& + c * d^2) * x + c * d * e * x^2)^2)^{(1/2)}) / (c * d * e)^{(1/2)} + 1/96/a^3/e^3/x^2 * (a * d * e \\
& + (a * e^2 + c * d^2) * x + c * d * e * x^2)^2)^{(7/2)} * c^2 - 45/64 * d^2 * a * e^3/(a * d * e)^{(1/2)} \\
& * \ln((2 * a * d * e + (a * e^2 + c * d^2) * x + 2 * (a * d * e)^2)^{(1/2)} * (a * d * e + (a * e^2 + c * d^2) \\
& * x + c * d * e * x^2)^2)^{(1/2)}) / x * c^2 - 43/192/d/a^3/e^2/x * (a * d * e + (a * e^2 + c * \\
& d^2) * x + c * d * e * x^2)^2)^{(7/2)} * c^2 + 1/64 * d/a^4/e^4/x * (a * d * e + (a * e^2 + c * d^2) \\
& * x + c * d * e * x^2)^2)^{(7/2)} * c^3 - 25/64/d^4/a * e^3 * c * (a * d * e + (a * e^2 + c * d^2) * x + \\
& c * d * e * x^2)^2)^{(5/2)} * x - 31/64/d^2/a^2 * e * c^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d \\
& * e * x^2)^2)^{(5/2)} * x - 1/64 * d^2/a^4/e^3 * c^4 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e \\
& * x^2)^2)^{(5/2)} * x - 5/192 * d^3/a^3/e^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2) \\
& )^2)^{(3/2)} * x * c^4 + 1/24/d/a^2/e^2/x^3 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2) \\
& )^2)^{(7/2)} * c + 15/256 * d * a * e^4 * c^2 * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x)/(c * d \\
& * e)^{(1/2)} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^2)^{(1/2)}) / (c * d * e)^{(1/2)} + \\
& 5/128 * d^6/a/e/(a * d * e)^{(1/2)} * \ln((2 * a * d * e + (a * e^2 + c * d^2) * x + 2 * (a * d * e) \\
& )^2)^{(1/2)} * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^2)^{(1/2)}) / x * c^4 - 5/64 * d^4/a \\
& ^2/e * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^2)^{(1/2)} * x * c^4 - 3/32/d^2 * a * e^5 \\
& * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^2)^{(1/2)} * x * c + 45/64 * d^2/a * e * (a * d * e \\
& + (a * e^2 + c * d^2) * x + c * d * e * x^2)^2)^{(1/2)} * x * c^3 - 13/48/d^2/a^2/e/x^2 * (a * d * \\
& e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^2)^{(7/2)} * c + 15/256/d^5 * e^10 * a^4/c * \ln((1/ \\
& 2 * a * e^2 - 1/2 * c * d^2 + (x + d/e) * c * d * e)/(c * d * e)^{(1/2)} + (c * d * e * (x + d/e)^2 + ( \\
& a * e^2 - c * d^2) * (x + d/e))^2)^{(1/2)}) / (c * d * e)^{(1/2)} + 3/64/d^6 * e^9 * a^3/c * (c * \\
& d * e * (x + d/e)^2 + (a * e^2 - c * d^2) * (x + d/e))^2)^{(1/2)} * x + 9/64/d^2 * e^5 * a * c * (c * \\
& d * e * (x + d/e)^2 + (a * e^2 - c * d^2) * (x + d/e))^2)^{(1/2)} * x - 3/256/d^7 * e^12 * a^5/c \\
& ^2 * \ln((1/2 * a * e^2 - 1/2 * c * d^2 + (x + d/e) * c * d * e)/(c * d * e)^{(1/2)} + (c * d * e * (x \\
& + d/e)^2 + (a * e^2 - c * d^2) * (x + d/e))^2)^{(1/2)}) / (c * d * e)^{(1/2)} + 15/128/d * e^6 * \\
& a^2 * c * \ln((1/2 * a * e^2 - 1/2 * c * d^2 + (x + d/e) * c * d * e)/(c * d * e)^{(1/2)} + (c * d * e \\
& * (x + d/e)^2 + (a * e^2 - c * d^2) * (x + d/e))^2)^{(1/2)}) / (c * d * e)^{(1/2)} - 15/256 * d * e \\
& ^4 * a * c^2 * \ln((1/2 * a * e^2 - 1/2 * c * d^2 + (x + d/e) * c * d * e)/(c * d * e)^{(1/2)} + (c * \\
& d * e * (x + d/e)^2 + (a * e^2 - c * d^2) * (x + d/e))^2)^{(1/2)}) / (c * d * e)^{(1/2)} - 3/64/d^6 \\
& * e^9 * a^3/c * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^2)^{(1/2)} * x + 3/256/d^7 * e \\
& ^12 * a^5/c^2 * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x)/(c * d * e)^{(1/2)} + (a * d * e \\
& + (a * e^2 + c * d^2) * x + c * d * e * x^2)^2)^{(1/2)}) / (c * d * e)^{(1/2)} - 15/128/d * e^6 * a^2 \\
& * c * \ln((1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x)/(c * d * e)^{(1/2)} + (a * d * e + (a * e^2 + c \\
& * d^2) * x + c * d * e * x^2)^2)^{(1/2)}) / (c * d * e)^{(1/2)} - 15/256/d^5 * e^10 * a^4/c * \ln( \\
& (1/2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x)/(c * d * e)^{(1/2)} + (a * d * e + (a * e^2 + c * d^2) * \\
& x + c * d * e * x^2)^2)^{(1/2)}) / (c * d * e)^{(1/2)} - 35/192/d/a * e^2 * (a * d * e + (a * e^2 + c * \\
& d^2) * x + c * d * e * x^2)^2)^{(3/2)} * x * c^2
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)\*x^5), x, algorithm="maxima")

[Out] Exception raised: ValueError



**Fricas [A]** time = 18.4224, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)\*x^5), x, algorithm="fricas")

[Out] [1/768\*(384\*sqrt(a\*d\*e)\*sqrt(c\*d\*e)\*a\*c^2\*d^4\*e^2\*x^4\*log(8\*c^2\*d^2\*e^2\*x^2 + c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4 + 4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(2\*c\*d\*e\*x + c\*d^2 + a\*e^2)\*sqrt(c\*d\*e) + 8\*(c^2\*d^3\*e + a\*c\*d^2\*e^3)\*x) - 3\*(5\*c^4\*d^8 - 60\*a\*c^3\*d^6\*e^2 - 90\*a^2\*c^2\*d^4\*e^4 + 20\*a^3\*c\*d^2\*e^6 - 3\*a^4\*e^8)\*x^4\*log(-4\*(2\*a^2\*d^2\*e^2 + (a\*c\*d^3\*e + a^2\*d^2\*e^3)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) - (8\*a^2\*d^2\*e^2 + (c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4)\*x^2 + 8\*(a\*c\*d^3\*e + a^2\*d^2\*e^3)\*x)\*sqrt(a\*d\*e))/x^2) - 4\*(48\*a^3\*d^3\*e^3 + (15\*c^3\*d^6 + 337\*a\*c^2\*d^4\*e^2 + 57\*a^2\*c\*d^2\*e^4 - 9\*a^3\*e^6)\*x^3 + 2\*(59\*a\*c^2\*d^5\*e + 122\*a^2\*c\*d^3\*e^3 + 3\*a^3\*d^2\*e^5)\*x^2 + 8\*(17\*a^2\*c\*d^4\*e^2 + 9\*a^3\*d^2\*e^4)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(a\*d\*e))/(sqrt(a\*d\*e)\*a\*d^2\*e\*x^4), 1/768\*(768\*sqrt(a\*d\*e)\*sqrt(-c\*d\*e)\*a\*c^2\*d^4\*e^2\*x^4\*arctan(1/2\*(2\*c\*d\*e\*x + c\*d^2 + a\*e^2)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*e))) - 3\*(5\*c^4\*d^8 - 60\*a\*c^3\*d^6\*e^2 - 90\*a^2\*c^2\*d^4\*e^4 + 20\*a^3\*c\*d^2\*e^6 - 3\*a^4\*e^8)\*x^4\*log(-4\*(2\*a^2\*d^2\*e^2 + (a\*c\*d^3\*e + a^2\*d^2\*e^3)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) - (8\*a^2\*d^2\*e^2 + (c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4)\*x^2 + 8\*(a\*c\*d^3\*e + a^2\*d^2\*e^3)\*x)\*sqrt(a\*d\*e))/x^2) - 4\*(48\*a^3\*d^3\*e^3 + (15\*c^3\*d^6 + 337\*a\*c^2\*d^4\*e^2 + 57\*a^2\*c\*d^2\*e^4 - 9\*a^3\*e^6)\*x^3 + 2\*(59\*a\*c^2\*d^5\*e + 122\*a^2\*c\*d^3\*e^3 + 3\*a^3\*d^2\*e^5)\*x^2 + 8\*(17\*a^2\*c\*d^4\*e^2 + 9\*a^3\*d^2\*e^4)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(a\*d\*e))/(sqrt(a\*d\*e)\*a\*d^2\*e\*x^4), 1/384\*(192\*sqrt(-a\*d\*e)\*sqrt(c\*d\*e)\*a\*c^2\*d^4\*e^2\*x^4\*log(8\*c^2\*d^2\*e^2\*x^2 + c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4 + 4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(2\*c\*d\*e\*x + c\*d^2 + a\*e^2)\*sqrt(c\*d\*e) + 8\*(c^2\*d^3\*e + a\*c\*d^2\*e^3)\*x) + 3\*(5\*c^4\*d^8 - 60\*a\*c^3\*d^6\*e^2 - 90\*a^2\*c^2\*d^4\*e^4 + 20\*a^3\*c\*d^2\*e^6 - 3\*a^4\*e^8)\*x^4\*arctan(1/2\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*a\*d\*e)) - 2\*(48\*a^3\*d^3\*e^3 + (15\*c^3\*d^6 + 337\*a\*c^2\*d^4\*e^2 + 57\*a^2\*c\*d^2\*e^4 - 9\*a^3\*e^6)\*x^3 + 2\*(59\*a\*c^2\*d^5\*e + 122\*a^2\*c\*d^3\*e^3 + 3\*a^3\*d^2\*e^5)\*x^2 + 8\*(17\*a^2\*c\*d^4\*e^2 + 9\*a^3\*d^2\*e^4)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e))/(sqrt(-a\*d\*e)\*a\*d^2\*e\*x^4), 1/384\*(384\*sqrt(-a\*d\*e)\*sqrt(-c\*d\*e)\*a\*c^2\*d^4\*e^2\*x^4\*arctan(1/2\*(2\*c\*d\*e\*x + c\*d^2 + a\*e^2)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*e))) + 3\*(5\*c^4\*d^8 - 60\*a\*c^3\*d^6\*e^2 - 90\*a^2\*c^2\*d^4\*e^4 + 20\*a^3\*c\*d^2\*e^6 - 3\*a^4\*e^8)\*x^4\*arctan(1/2\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*a\*d\*e)) - 2\*(48\*a^3\*d^3\*e^3 + (15\*c^3\*d^6 + 337\*a\*c^2\*d^4\*e^2 + 57\*a^2\*c\*d^2\*e^4 - 9\*a^3\*e^6)\*x^3 + 2\*(59\*a\*c^2\*d^5\*e + 122\*a^2\*c\*d^3\*e^3 + 3\*a^3\*d^2\*e^5)\*x^2 + 8\*(17\*a^2\*c\*d^4\*e^2 + 9\*a^3\*d^2\*e^4)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e))/(sqrt(-a\*d\*e)\*a\*d^2\*e\*x^4)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)/x\*\*5/(e\*x+d), x)

[Out] Timed out

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)\*x^5), x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.466 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^6(d+ex)} dx$$

**Optimal.** Leaf size=289

$$\begin{aligned} & \frac{3(cd^2 - ae^2)^5 \tanh^{-1}\left(\frac{x(ae^2 + cd^2) + 2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{256a^{5/2}d^{7/2}e^{5/2}} \\ & + \frac{3(cd^2 - ae^2)^3(x(ae^2 + cd^2) + 2ade)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{128a^2d^3e^2x^2} \\ & - \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{5dx^5} \\ & - \frac{\left(\frac{c}{ae} - \frac{e}{d^2}\right)(x(ae^2 + cd^2) + 2ade)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{16x^4} \end{aligned}$$

[Out] (3\*(c\*d^2 - a\*e^2)^3\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(128\*a^2\*d^3\*e^2\*x^2) - ((c/(a\*e) - e/d^2)\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2))/(16\*x^4) - (a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2)/(5\*d\*x^5) - (3\*(c\*d^2 - a\*e^2)^5\*ArcTanh[(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(2\*Sqrt[a]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]])/(256\*a^(5/2)\*d^(7/2)\*e^(5/2))

**Rubi [A]** time = 0.91404, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\begin{aligned} & \frac{3(cd^2 - ae^2)^5 \tanh^{-1}\left(\frac{x(ae^2 + cd^2) + 2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{256a^{5/2}d^{7/2}e^{5/2}} \\ & + \frac{3(cd^2 - ae^2)^3(x(ae^2 + cd^2) + 2ade)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{128a^2d^3e^2x^2} \\ & - \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{5dx^5} \\ & - \frac{\left(\frac{c}{ae} - \frac{e}{d^2}\right)(x(ae^2 + cd^2) + 2ade)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{16x^4} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2)/(x^6\*(d + e\*x)), x]

[Out] (3\*(c\*d^2 - a\*e^2)^3\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(128\*a^2\*d^3\*e^2\*x^2) - ((c/(a\*e) - e/d^2)\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2))/(16\*x^4) - (a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2)/(5\*d\*x^5) - (3\*(c\*d^2 - a\*e^2)^5\*ArcTanh[(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(2\*Sqrt[a]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]])/(256\*a^(5/2)\*d^(7/2)\*e^(5/2))

**Rubi in Sympy [A]** time = 91.441, size = 272, normalized size = 0.94

$$\frac{\left(\frac{e}{16d^2} - \frac{c}{16ae}\right) (2ade + x (ae^2 + cd^2)) (ade + cdex^2 + x (ae^2 + cd^2))^{\frac{3}{2}}}{\frac{x^4}{(ade + cdex^2 + x (ae^2 + cd^2))^{\frac{5}{2}}}} - \frac{5dx^5}{3 (ae^2 - cd^2)^3 (2ade + x (ae^2 + cd^2)) \sqrt{ade + cdex^2 + x (ae^2 + cd^2)}} - \frac{128a^2d^3e^2x^2}{3 (ae^2 - cd^2)^5 \operatorname{atanh}\left(\frac{2ade+x(ae^2+cd^2)}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}\right)}} + \frac{256a^{\frac{5}{2}}d^{\frac{7}{2}}e^{\frac{5}{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**6/(e*x+d), x)`

[Out] `(e/(16*d**2) - c/(16*a*e))*(2*a*d*e + x*(a*e**2 + c*d**2))*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2)/x**4 - (a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(5/2)/(5*d*x**5) - 3*(a*e**2 - c*d**2)**3*(2*a*d*e + x*(a*e**2 + c*d**2))*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(128*a**2*d**3*e**2*x**2) + 3*(a*e**2 - c*d**2)**5*atanh((2*a*d*e + x*(a*e**2 + c*d**2))/(2*sqrt(a)*sqrt(d)*sqrt(e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))))/(256*a**(5/2)*d**(7/2)*e**(5/2))`

**Mathematica [A]** time = 0.786729, size = 354, normalized size = 1.22

$$\frac{((d + ex)(ae + cdx))^{3/2} \left(-2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d + ex}\sqrt{ae + cdx} (a^4e^4 (128d^4 + 176d^3ex + 8d^2e^2x^2 - 10de^3x^3 + 15e^4x^4) + 2a^3cd^2e^3x^3)\right)}{}$$

Antiderivative was successfully verified.

[In] `Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^6*(d + e*x)), x]`

[Out] `((a*e + c*d*x)*(d + e*x))^(3/2)*(-2*sqrt[a]*sqrt[d]*sqrt[e]*sqrt[a*e + c*d*x]*sqrt[d + e*x]*(-15*c^4*d^8*x^4 + 10*a*c^3*d^6*e*x^3*(d + 7*e*x) + 2*a^2*c^2*d^4*e^2*x^2*(124*d^2 + 233*d*e*x + 64*e^2*x^2) + 2*a^3*c*d^2*e^3*x*(168*d^3 + 256*d^2*e*x + 23*d*e^2*x^2 - 35*e^3*x^3) + a^4*e^4*(128*d^4 + 176*d^3*e*x + 8*d^2*e^2*x^2 - 10*d*e^3*x^3 + 15*e^4*x^4)) + 15*(c*d^2 - a*e^2)^5*x^5*Log[x] - 15*(c*d^2 - a*e^2)^5*x^5*Log[c*d^2*x + 2*sqrt[a]*sqrt[d]*sqrt[e]*sqrt[a*e + c*d*x]*sqrt[d + e*x] + a*e*(2*d + e*x)))/(1280*a^(5/2)*d^(7/2)*e^(5/2)*x^5*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2))`

**Maple [B]** time = 0.057, size = 3991, normalized size = 13.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^6/(e*x+d), x)`

[Out] `-1/16/d^3*e^4*c*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(3/2)-1/128/d^5*a*e^6*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)-3/128/d^4*a^2*e^7*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+15/128/d^3*e^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)*c+13/40/d^3/a/x^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2)+17/160/a^3/e*(a*d*e+(a*e^2+c*d^2)*x`

$$\begin{aligned}
& c^*d^*e^*x^2)^{(5/2)} * c^3 + 1/5/d^6 * e^5 * (c^*d^*e^*(x+d/e)^2 + (a^*e^2 - c^*d^2) * (x+d/e))^{(5/2)} + 3/128 * e^3 * c^2 * (c^*d^*e^*(x+d/e)^2 + (a^*e^2 - c^*d^2) * (x+d/e))^{(1/2)} - 15/128 * e^3 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(1/2)} * c^2 + 25/128/d^6 * e^5 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(5/2)} - 9/64/d^5 * e^8 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(1/2)} * x^2 - 15/256 * e^5 * \ln((1/2 * a^*e^2 + 1/2 * c^*d^2 + c^*d^*e^*x) / (c^*d^*e)^{(1/2)} + (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(1/2)}) / (c^*d^*e)^{(1/2)} * a^3 - 15/256/d^6 * e^{11} * a^4 / c * \ln((1/2 * a^*e^2 - 1/2 * c^*d^2 + (x+d/e) * c^*d^*e) / (c^*d^*e)^{(1/2)} + (c^*d^*e * (x+d/e)^2 + (a^*e^2 - c^*d^2) * (x+d/e))^{(1/2)}) / (c^*d^*e)^{(1/2)} - 3/64/d^7 * e^{10} * a^3 / c * (c^*d^*e * (x+d/e)^2 + (a^*e^2 - c^*d^2) * (x+d/e))^{(1/2)} * x - 9/64/d^3 * e^6 * a^3 * (c^*d^*e * (x+d/e)^2 + (a^*e^2 - c^*d^2) * (x+d/e))^{(1/2)} * x + 3/256/d^8 * e^{13} * a^5 / c^2 * \ln((1/2 * a^*e^2 - 1/2 * c^*d^2 + (x+d/e) * c^*d^*e) / (c^*d^*e)^{(1/2)} + (c^*d^*e * (x+d/e)^2 + (a^*e^2 - c^*d^2) * (x+d/e))^{(1/2)}) / (c^*d^*e)^{(1/2)} - 15/128/d^2 * e^7 * a^2 * c * \ln((1/2 * a^*e^2 - 1/2 * c^*d^2 + (x+d/e) * c^*d^*e) / (c^*d^*e)^{(1/2)} + (c^*d^*e * (x+d/e)^2 + (a^*e^2 - c^*d^2) * (x+d/e))^{(1/2)}) / (c^*d^*e)^{(1/2)} + 3/64/d^7 * e^{10} * a^3 / c * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(1/2)} * x - 3/256/d^8 * e^{13} * a^5 / c^2 * \ln((1/2 * a^*e^2 + 1/2 * c^*d^2 + c^*d^*e^*x) / (c^*d^*e)^{(1/2)} + (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(1/2)}) / (c^*d^*e)^{(1/2)} + 15/256/d^6 * e^{11} / c * \ln((1/2 * a^*e^2 + 1/2 * c^*d^2 + c^*d^*e^*x) / (c^*d^*e)^{(1/2)} + (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(1/2)}) / (c^*d^*e)^{(1/2)} * a^4 + 15/128/d^2 * e^7 * c * \ln((1/2 * a^*e^2 + 1/2 * c^*d^2 + c^*d^*e^*x) / (c^*d^*e)^{(1/2)} + (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(1/2)}) / (c^*d^*e)^{(1/2)} * a^2 - 3/640 * d^2 / a^5 / e^5 / x * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(7/2)} * c^4 - 15/256/d^2 * a^2 * e^6 / (a^*d^*e)^{(1/2)} * \ln((2 * a^*d^*e + (a^*e^2 + c^*d^2) * x + 2 * (a^*d^*e)^{(1/2)} * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(1/2)}) / x) * c + 15/128 * d^2 * a^4 / (a^*d^*e)^{(1/2)} * \ln((2 * a^*d^*e + (a^*e^2 + c^*d^2) * x + 2 * (a^*d^*e)^{(1/2)} * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(1/2)}) / x) * c^2 + 3/40/d/a^2/e^2/x^4 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(7/2)} * c - 3/256 * d^7 / a^2 / e^2 / (a^*d^*e)^{(1/2)} * \ln((2 * a^*d^*e + (a^*e^2 + c^*d^2) * x + 2 * (a^*d^*e)^{(1/2)} * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(1/2)}) / x) * c^5 - 11/320 * d/a^4/e^2 * c^4 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(5/2)} * x + 15/128/d^3 * a^6 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(1/2)} * x * c + 19/320/d/a^3/e^2/x^2 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(7/2)} * c^2 + 3/128 * d^5/a^3/e^2 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(1/2)} * x * c^5 - 5/64 * d^2/a^3/e * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(3/2)} * x * c^4 + 1/128 * d^4/a^4/e^3 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(3/2)} * x * c^5 - 139/320/d^4/a^2 * e/x * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(7/2)} * c - 1/320 * d/a^4/e^4/x^2 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(7/2)} * c^3 + 253/640/d^5/a^4 * c * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(5/2)} * x + 139/320/d^3/a^2 * e^2 * c^2 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(5/2)} * x + 3/640 * d^3/a^5/e^4 * c^5 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(5/2)} * x - 1/5/d^2/a^3/e/x * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(7/2)} * c^2 + 5/64/d^2/a^3 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(3/2)} * x * c^2 - 1/5/d^2/a^2/e/x^3 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(7/2)} * c - 15/128/d^4 * e^9 * \ln((1/2 * a^*e^2 + 1/2 * c^*d^2 + c^*d^*e^*x) / (c^*d^*e)^{(1/2)} + (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(1/2)}) / (c^*d^*e)^{(1/2)} * a^3 - 3/64/d^6 * e^9/c * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(1/2)} * a^3 - 1/8/d^4 * e^5 * c * (c^*d^*e * (x+d/e)^2 + (a^*e^2 - c^*d^2) * (x+d/e))^{(3/2)} * x + 1/16/d^7 * e^8 * a^2/c * (c^*d^*e * (x+d/e)^2 + (a^*e^2 - c^*d^2) * (x+d/e))^{(3/2)} + 9/64/d^5 * e^8 * a^2 * (c^*d^*e * (x+d/e)^2 + (a^*e^2 - c^*d^2) * (x+d/e))^{(1/2)} * x - 3/128/d^8 * e^{11} * a^4/c^2 * (c^*d^*e * (x+d/e)^2 + (a^*e^2 - c^*d^2) * (x+d/e))^{(1/2)} - 3/64/d^2 * e^5 * a^3 * c * (c^*d^*e * (x+d/e)^2 + (a^*e^2 - c^*d^2) * (x+d/e))^{(1/2)} + 15/128/d^4 * e^9 * a^3 * \ln((1/2 * a^*e^2 - 1/2 * c^*d^2 + (x+d/e) * c^*d^*e) / (c^*d^*e)^{(1/2)} + (c^*d^*e * (x+d/e)^2 + (a^*e^2 - c^*d^2) * (x+d/e))^{(1/2)}) / (c^*d^*e)^{(1/2)} + 3/64/d^2 * e^4 * c^2 * (c^*d^*e * (x+d/e)^2 + (a^*e^2 - c^*d^2) * (x+d/e))^{(1/2)} * x - 3/256 * d^2 * e^3 * c^3 * \ln((1/2 * a^*e^2 - 1/2 * c^*d^2 + (x+d/e) * c^*d^*e) / (c^*d^*e)^{(1/2)} + (c^*d^*e * (x+d/e)^2 + (a^*e^2 - c^*d^2) * (x+d/e))^{(1/2)}) / (c^*d^*e)^{(1/2)} + 15/256 * e^5 * a^3 * c^2 * \ln((1/2 * a^*e^2 - 1/2 * c^*d^2 + (x+d/e) * c^*d^*e) / (c^*d^*e)^{(1/2)} + (c^*d^*e * (x+d/e)^2 + (a^*e^2 - c^*d^2) * (x+d/e))^{(1/2)}) / (c^*d^*e)^{(1/2)} + 15/128/d^4 * e^5 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(3/2)} * x * c - 15/128 * d^3 * e^2 / (a^*d^*e)^{(1/2)} * \ln((2 * a^*d^*e + (a^*e^2 + c^*d^2) * x + 2 * (a^*d^*e)^{(1/2)} * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(1/2)}) / x) * c^3 + 3/256 * d^2 * e^3 * c^3 * \ln((1/2 * a^*e^2 + 1/2 * c^*d^2 + c^*d^*e^*x) / (c^*d^*e)^{(1/2)} + (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(1/2)}) / (c^*d^*e)^{(1/2)} + 15/256 * d^5/a / (a^*d^*e)^{(1/2)} * \ln((2 * a^*d^*e + (a^*e^2 + c^*d^2) * x + 2 * (a^*d^*e)^{(1/2)} * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(1/2)}) / x) * c^4 + 1/5/d/a^3 * c^3 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(5/2)} * x - 3/64 * d^3/a^2 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(1/2)} * x * c^4 + 109/320/d^3/a^2/x^2 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(7/2)} * c - 1/5/d^2/a/e/x^5 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(7/2)} + 273/640/d^4/a^3 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(5/2)} * c + 47/160/d^2/a^2 * e * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(5/2)} * c^2 - 9/128 * d^4/a^2/e * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(1/2)} * c^4 + 15/128/d^2 * a^5 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(1/2)} * c - 7/128 * d^3/a^3/e^2 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e^*x^2)^{(3/2)}
\end{aligned}$$

$$2) * c^4 - 31/80/d^4/a * e/x^3 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(7/2)} - 17/640 * d^2/a^4/e^3 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(5/2)} * c^4 + 3/32 * d^2/a * e * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} * c^3 + 3/128 * d^6/a^3/e^3 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)} * c^5 + 3/640 * d^4/a^5/e^5 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(5/2)} * c^5 + 129/320/d^5/a * e^2/x^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(7/2)} + 1/128 * d^5/a^4/e^4 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(3/2)} * c^5 - 253/640/d^6/a * e^3/x * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(7/2)} + 3/256/d^3 * a^3 * e^8/(a * d * e)^{(1/2)} * \ln((2 * a * d * e + (a * e^2 + c * d^2) * x + 2 * (a * d * e)^{(1/2)} * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)})/x) - 1/80/a^3/e^3/x^3 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(7/2)} * c^2 + 11/320/a^4/e^3/x * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(7/2)} * c^3 + 1/8/d^6 * e^7 * a * (c * d * e * (x + d/e)^2 + (a * e^2 - c * d^2) * (x + d/e))^{(3/2)} * x + 3/64/d^6 * e^9 * a^3/c * (c * d * e * (x + d/e)^2 + (a * e^2 - c * d^2) * (x + d/e))^{(1/2)} - 1/8/d^6 * e^7 * a * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(3/2)} * x - 1/16/d^7 * e^8 * a^2/c * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(3/2)} + 3/128/d^8 * e^11 * a^4/c^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{(1/2)}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)\*x^6), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)\*x^6), x, algorithm="fricas")

[Out] Exception raised: TypeError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)/x\*\*6/(e\*x+d), x, algorithm="sympy")

[Out] Timed out

**GIAC/XCAS [A]** time = 29.5335, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)\*x^6), x, algorithm="giac")

[Out] Done

$$3.467 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^7(d+ex)} dx$$

**Optimal.** Leaf size=386

$$\frac{(7ae^2 + 5cd^2)(cd^2 - ae^2)^5 \tanh^{-1}\left(\frac{x(ae^2 + cd^2) + 2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{1024a^{7/2}d^{9/2}e^{7/2}} - \frac{(7ae^2 + 5cd^2)(cd^2 - ae^2)^3(x(ae^2 + cd^2) + 2ade)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{512a^3d^4e^3x^2} + \frac{(7ae^2 + 5cd^2)(cd^2 - ae^2)(x(ae^2 + cd^2) + 2ade)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{192a^2d^3e^2x^4} - \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{6dx^6} - \frac{\left(\frac{5c}{ae} - \frac{7e}{d^2}\right)(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{60x^5}$$

[Out]  $-\left((c*d^2 - a*e^2)^3*(5*c*d^2 + 7*a*e^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]\right)/(512*a^3*d^4*e^3*x^2) + \left((c*d^2 - a*e^2)*(5*c*d^2 + 7*a*e^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}\right)/(192*a^2*d^3*e^2*x^4) - \left(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2\right)^{(5/2)}/(6*d*x^6) - \left(\left((5*c)/(a*e) - (7*e)/d^2\right)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}\right)/(60*x^5) + \left((c*d^2 - a*e^2)^5*(5*c*d^2 + 7*a*e^2)*\text{ArcTanh}\left[\frac{(2*a*d*e + (c*d^2 + a*e^2)*x)}{(2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])}\right]\right)/(1024*a^{(7/2)}*d^{(9/2)}*e^{(7/2)})$

**Rubi [A]** time = 1.31555, antiderivative size = 386, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{(7ae^2 + 5cd^2)(cd^2 - ae^2)^5 \tanh^{-1}\left(\frac{x(ae^2 + cd^2) + 2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{1024a^{7/2}d^{9/2}e^{7/2}} - \frac{(7ae^2 + 5cd^2)(cd^2 - ae^2)^3(x(ae^2 + cd^2) + 2ade)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{512a^3d^4e^3x^2} + \frac{(7ae^2 + 5cd^2)(cd^2 - ae^2)(x(ae^2 + cd^2) + 2ade)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{192a^2d^3e^2x^4} - \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{6dx^6} - \frac{\left(\frac{5c}{ae} - \frac{7e}{d^2}\right)(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{60x^5}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}/(x^7*(d + e*x)), x]$

[Out]  $-\left((c*d^2 - a*e^2)^3*(5*c*d^2 + 7*a*e^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]\right)/(512*a^3*d^4*e^3*x^2) + \left((c*d^2 - a*e^2)*(5*c*d^2 + 7*a*e^2)*(2*a*d*e + (c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}\right)/(192*a^2*d^3*e^2*x^4) - \left(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2\right)^{(5/2)}/(6*d*x^6) - \left(\left((5*c)/(a*e) - (7*e)/d^2\right)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}\right)/(60*x^5) + \left((c*d^2 - a*e^2)^5*(5*c*d^2 + 7*a*e^2)*\text{ArcTanh}\left[\frac{(2*a*d*e + (c*d^2 + a*e^2)*x)}{(2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])}\right]\right)/(1024*a^{(7/2)}*d^{(9/2)}*e^{(7/2)})$



**Rubi in Sympy [A]** time = 141.543, size = 362, normalized size = 0.94

$$\frac{(2ade + x(ae^2 + cd^2)) \left( \frac{7e^2}{192d^3} - \frac{c}{96ad} - \frac{5c^2d}{192a^2e^2} \right) (ade + cdex^2 + x(ae^2 + cd^2))^{\frac{3}{2}}}{x^4} + \frac{\left( \frac{7e}{60d^2} - \frac{c}{12ae} \right) (ade + cdex^2 + x(ae^2 + cd^2))^{\frac{5}{2}}}{x^5} - \frac{(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{5}{2}}}{6dx^6} + \frac{(ae^2 - cd^2)^3 (7ae^2 + 5cd^2) (2ade + x(ae^2 + cd^2)) \sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{512a^3d^4e^3x^2} - \frac{(ae^2 - cd^2)^5 (7ae^2 + 5cd^2) \operatorname{atanh}\left(\frac{2ade+x(ae^2+cd^2)}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}\right)}{1024a^{\frac{7}{2}}d^{\frac{9}{2}}e^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**7/(e*x+d), x)`

[Out] `-(2*a*d*e + x*(a*e**2 + c*d**2))*(7*e**2/(192*d**3) - c/(96*a*d) - 5*c**2*d/(192*a**2*e**2))*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**3/2/x**4 + (7*e/(60*d**2) - c/(12*a*e))*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**5/2/x**5 - (a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**5/2/(6*d*x**6) + (a*e**2 - c*d**2)**3*(7*a*e**2 + 5*c*d**2)*(2*a*d*e + x*(a*e**2 + c*d**2))*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(512*a**3*d**4*e**3*x**2) - (a*e**2 - c*d**2)**5*(7*a*e**2 + 5*c*d**2)*atanh((2*a*d*e + x*(a*e**2 + c*d**2))/(2*sqrt(a)*sqrt(d)*sqrt(e)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))))/(1024*a**7/2*d**9/2*e**7/2)`

**Mathematica [A]** time = 0.897661, size = 450, normalized size = 1.17

$$\frac{((d + ex)(ae + cdx))^{3/2} \left( -2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d + ex}\sqrt{ae + cdx} (a^5e^5 (1280d^5 + 1664d^4ex + 48d^3e^2x^2 - 56d^2e^3x^3 + 70de^4x^4 - 105e^5x^5)) \right)}{1024a^{7/2}d^{9/2}e^{7/2}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^7*(d + e*x)), x]`

[Out] `((a*e + c*d*x)*(d + e*x))^(3/2)*(-2*sqrt[a]*sqrt[d]*sqrt[e]*sqrt[a*e + c*d*x]*sqrt[d + e*x]*(75*c^5*d^10*x^5 - 5*a*c^4*d^8*e*x^4*(10*d + 49*e*x) + 10*a^2*c^3*d^6*e^2*x^3*(4*d^2 + 16*d*e*x + 15*e^2*x^2) + 6*a^3*c^2*d^4*e^3*x^2*(360*d^3 + 564*d^2*e*x + 58*d*e^2*x^2 - 91*e^3*x^3) + a^4*c*d^2*e^4*x*(3200*d^4 + 4448*d^3*e*x + 216*d^2*e^2*x^2 - 272*d*e^3*x^3 + 415*e^4*x^4) + a^5*e^5*(1280*d^5 + 1664*d^4*e*x + 48*d^3*e^2*x^2 - 56*d^2*e^3*x^3 + 70*d*e^4*x^4 - 105*e^5*x^5)) - 15*(c*d^2 - a*e^2)^5*(5*c*d^2 + 7*a*e^2)*x^6*Log[x] + 15*(c*d^2 - a*e^2)^5*(5*c*d^2 + 7*a*e^2)*x^6*Log[c*d^2*x + 2*sqrt[a]*sqrt[d]*sqrt[e]*sqrt[a*e + c*d*x]*sqrt[d + e*x] + a*e*(2*d + e*x)))/(15360*a^(7/2)*d^(9/2)*e^(7/2)*x^6*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2))`

**Maple [B]** time = 0.081, size = 4735, normalized size = 12.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^7/(e*x+d), x)`

[Out]  $1/16/d^4 * e^5 * c * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{3/2} - 3/12$   
 $8/d * e^4 * c^2 * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{1/2} + 7/1536/$   
 $d^6 * a * e^7 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{3/2} + 7/512/d^5 * a^2 * e$   
 $^8 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2} - 35/384/d^4 * e^5 * (a * d * e +$   
 $(a * e^2 + c * d^2) * x + c * d * e * x^2)^{3/2} * c + 17/60/d^3/a/x^5 * (a * d * e + (a * e^2 +$   
 $c * d^2) * x + c * d * e * x^2)^{7/2} - 59/320/d/a^3 * (a * d * e + (a * e^2 + c * d^2) * x + c * d$   
 $* e * x^2)^{5/2} * c^3 + 1/512 * d^3/a^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)$   
 $^{1/2} * c^4 + 25/512/d * e^4 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2} * c$   
 $^2 - 1/5/d^7 * e^6 * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{5/2} - 101/$   
 $512/d^7 * e^6 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{5/2} + 3/64/d^3 * e^6 *$   
 $a * c * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{1/2} - 15/128/d^5 * e^{10}$   
 $* a^3 * \ln((1/2 * a * e^2 - 1/2 * c * d^2 + (x+d/e) * c * d * e) / (c * d * e)^{1/2} + (c * d * e *$   
 $(x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{1/2}) / (c * d * e)^{1/2} - 3/64/d^2 * e^4$   
 $5 * c^2 * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{1/2} * x + 3/256 * d * e^4$   
 $* c^3 * \ln((1/2 * a * e^2 - 1/2 * c * d^2 + (x+d/e) * c * d * e) / (c * d * e)^{1/2} + (c * d * e *$   
 $(x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{1/2}) / (c * d * e)^{1/2} + 3/64/d^7 * e^4$   
 $10 * a^3/c * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2} + 1/8/d^7 * e^8 * a * (a$   
 $* d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{3/2} * x + 15/128/d^5 * e^{10} * a^3 * \ln((1/$   
 $2 * a * e^2 + 1/2 * c * d^2 + c * d * e * x) / (c * d * e)^{1/2} + (a * d * e + (a * e^2 + c * d^2) * x +$   
 $c * d * e * x^2)^{1/2}) / (c * d * e)^{1/2} - 3/256 * d * e^4 * c^3 * \ln((1/2 * a * e^2 + 1/2$   
 $* c * d^2 + c * d * e * x) / (c * d * e)^{1/2} + (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2}) /$   
 $(c * d * e)^{1/2} + 1/16/d^8 * e^9 * a^2/c * (a * d * e + (a * e^2 + c * d^2) * x + c * d$   
 $* e * x^2)^{3/2} + 9/64/d^6 * e^9 * a^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2} * x -$   
 $3/128/d^9 * e^{12} * a^4/c^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2} -$   
 $5/1536 * d^6/a^5/e^5 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{3/2} * c^6 -$   
 $1/512 * d^5/a^6/e^6 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{5/2} * c^6$   
 $+ 1017/2560/d^7/a * e^4/x * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{7/2} - 7/$   
 $1024/d^4 * a^3 * e^9 / (a * d * e)^{1/2} * \ln((2 * a * d * e + (a * e^2 + c * d^2) * x + 2 * (a * d$   
 $* e)^{1/2} * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2}) / x) - 185/1536/d^4$   
 $5 * e^6 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{3/2} * x * c + 15/512/d^2 * e^5 *$   
 $(a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2} * x * c^2 + 5/256 * d^2 * e^3 / (a * d$   
 $* e)^{1/2} * \ln((2 * a * d * e + (a * e^2 + c * d^2) * x + 2 * (a * d * e)^{1/2} * (a * d * e + (a * e^2 +$   
 $c * d^2) * x + c * d * e * x^2)^{1/2}) / x) * c^3 + 89/320/d^3/a^2/x^3 * (a * d * e + (a * e^2 +$   
 $c * d^2) * x + c * d * e * x^2)^{7/2} * c + 381/1280/d^3/a^3/x * (a * d * e + (a * e^2 +$   
 $c * d^2) * x + c * d * e * x^2)^{7/2} * c^2 + 35/768 * d/a^3 * (a * d * e + (a * e^2 + c * d^2) * x$   
 $+ c * d * e * x^2)^{3/2} * x * c^4 - 397/960/d^5/a * e^4 * (a * d * e + (a * e^2 + c * d^2) * x +$   
 $c * d * e * x^2)^{5/2} * c - 35/1536/d^2/a * e^3 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e$   
 $* x^2)^{3/2} * c^2 - 1/64 * d/a * e^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2} * c -$   
 $2681/7680/d^3/a^2 * e^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{5/2} * c^2$   
 $- 221/7680 * d/a^4/e^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{5/2} * c^4 + 1/$   
 $64 * d^5/a^3/e^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2} * c^5 - 57/16$   
 $0/d^4/a * e/x^4 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{7/2} + 377/960/d^5$   
 $/a * e^2/x^3 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{7/2} - 5/256/a * e^3 * (a$   
 $* d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2} * x * c^3 - 81/1280/a^4/e * c^4 * (a$   
 $* d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{5/2} * x - 45/1024 * a * e^5 / (a * d * e)^{1/2}$   
 $) * \ln((2 * a * d * e + (a * e^2 + c * d^2) * x + 2 * (a * d * e)^{1/2} * (a * d * e + (a * e^2 + c * d^2)$   
 $* x + c * d * e * x^2)^{1/2}) / x) * c^2 - 11/480/a^4/e^3/x^2 * (a * d * e + (a * e^2 + c * d$   
 $^2) * x + c * d * e * x^2)^{7/2} * c^3 - 1/32/a^3/e^3/x^4 * (a * d * e + (a * e^2 + c * d^2) * x$   
 $+ c * d * e * x^2)^{7/2} * c^2 - 1/6/d^2/a/e/x^6 * (a * d * e + (a * e^2 + c * d^2) * x + c * d$   
 $* e * x^2)^{7/2} + 1/120 * d^3/a^5/e^4 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)$   
 $^{5/2} * c^5 - 5/512 * d^7/a^4/e^4 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2} * c^6 -$   
 $1543/3840/d^6/a * e^3/x^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2} * c^4 + 7/384 * d^4/a^4/e^3 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{3/2} * c^5$   
 $+ 1/8/d^5 * e^6 * c * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{3/2} * x - 1/$   
 $16/d^8 * e^9 * a^2/c * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{3/2} - 9/$   
 $64/d^6 * e^9 * a^2 * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{1/2} * x + 3/$   
 $128/d^9 * e^{12} * a^4/c^2 * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{1/2}$   
 $) - 5/1536 * d^5/a^5/e^4 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{3/2} * x * c^6 + 3211/7680/d^5/a^2 * e^2/x * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{7/2} * c + 7/256 * d^2/a^2 * e * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2} * x * c^4 - 5/512 * d^6/a^4/e^3 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{1/2} * x * c^6 - 25/768/d/a^2 * e^2 * (a * d * e + (a * e^2 + c * d^2) * x + c * d * e * x^2)^{3/2} * x * c^3 + 3/64/d^8 * e^{11} * a^3/c * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{1/2} * x + 9/64/d^4 * e^7 * a * c * (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{1/2} * x - 3/256/d^9 * e^{14} * a^5/c^2 * \ln((1/2 * a * e^2 - 1/2 * c * d^2 + (x+d/e) * c * d * e) / (c * d * e)^{1/2} + (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{1/2}) / (c * d * e)^{1/2} + 15/128/d^3 * e^8 * a^2 * c * \ln((1/2 * a * e^2 - 1/2 * c * d^2 + (x+d/e) * c * d * e) / (c * d * e)^{1/2} + (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{1/2}) / (c * d * e)^{1/2} - 15/256/d * e^6 * a * c^2 * \ln((1/2 * a * e^2 - 1/2 * c * d^2 + (x+d/e) * c * d * e) / (c * d * e)^{1/2} + (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{1/2}) / (c * d * e)^{1/2} + (c * d * e * (x+d/e)^2 + (a * e^2 - c * d^2) * (x+d/e))^{1/2})$

$$\begin{aligned} & / (c^*d^*e)^{(1/2)} + 15/256/d^7 * e^{12} * a^4 / c * \ln((1/2 * a * e^2 - 1/2 * c * d^2 + (x+d/e) * c^*d^*e) / (c^*d^*e)^{(1/2)} + (c^*d^*e * (x+d/e)^2 + (a * e^2 - c^*d^2) * (x+d/e))^{(1/2)}) / (c^*d^*e)^{(1/2)} + 3/256/d^9 * e^{14} * a^5 / c^2 * \ln((1/2 * a * e^2 + 1/2 * c^*d^2 + c^*d^*e * x) / (c^*d^*e)^{(1/2)} + (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(1/2)}) / (c^*d^*e)^{(1/2)} - 15/128/d^3 * e^8 * a^2 * c * \ln((1/2 * a * e^2 + 1/2 * c^*d^2 + c^*d^*e * x) / (c^*d^*e)^{(1/2)} + (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(1/2)}) / (c^*d^*e)^{(1/2)} - 3/64/d^8 * e^{11} * a^3 / c * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(1/2)} * x + 15/256/d * e^6 * a^3 * c^2 * \ln((1/2 * a * e^2 + 1/2 * c^*d^2 + c^*d^*e * x) / (c^*d^*e)^{(1/2)} + (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(1/2)}) / (c^*d^*e)^{(1/2)} - 15/256/d^7 * e^{12} * a^4 / c * \ln((1/2 * a * e^2 + 1/2 * c^*d^2 + c^*d^*e * x) / (c^*d^*e)^{(1/2)} + (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(1/2)}) / (c^*d^*e)^{(1/2)} + 81/1280/d / a^4 / e^2 / x * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(7/2)} * c^3 - 89/7680 * d / a^5 / e^4 / x * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(7/2)} * c^4 + 1/512 * d^3 / a^6 / e^6 / x * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(7/2)} * c^5 - 11/30/d^4 / a^2 * e / x^2 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(7/2)} * c - 65/512/d^4 * a * e^7 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(1/2)} * x^c + 29/320/d / a^3 / e^2 / x^3 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(7/2)} * c^2 + 1/192 * d / a^4 / e^4 / x^3 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(7/2)} * c^3 + 89/7680 * d^2 / a^5 / e^3 * c^5 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(5/2)} * x - 43/240/d^2 / a^2 / e / x^4 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(7/2)} * c + 3/512 * d^4 / a^3 / e * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(1/2)} * x^c^5 - 113/640/d^2 / a^3 / e / x^2 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(7/2)} * c^2 - 381/1280/d^2 / a^3 * e^c^3 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(5/2)} * x - 65/1536/d^3 / a^e^4 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(3/2)} * x^c^2 + 15/512/d^2 * a^2 * e^7 / (a^*d^*e)^{(1/2)} * \ln((2 * a^*d^*e + (a^*e^2 + c^*d^2) * x + 2 * (a^*d^*e)^{(1/2)} * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(1/2)}) / x) * c + 15/1024 * d^4 / a^e / (a^*d^*e)^{(1/2)} * \ln((2 * a^*d^*e + (a^*e^2 + c^*d^2) * x + 2 * (a^*d^*e)^{(1/2)} * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(1/2)}) / x) * c^4 - 1/512 * d^4 / a^6 / e^5 * c^6 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(5/2)} * x + 1/768 * d^2 / a^5 / e^5 / x^2 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(7/2)} * c^4 - 9/512 * d^6 / a^2 / e / (a^*d^*e)^{(1/2)} * \ln((2 * a^*d^*e + (a^*e^2 + c^*d^2) * x + 2 * (a^*d^*e)^{(1/2)} * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(1/2)}) / x) * c^5 + 5/1024 * d^8 / a^3 / e^3 / (a^*d^*e)^{(1/2)} * \ln((2 * a^*d^*e + (a^*e^2 + c^*d^2) * x + 2 * (a^*d^*e)^{(1/2)} * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(1/2)}) / x) * c^6 + 1/12/d / a^2 / e^2 / x^5 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(7/2)} * c - 1017/2560/d^6 / a^e^5 * c * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(5/2)} * x - 3211/7680/d^4 / a^2 * e^3 * c^2 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(5/2)} * x + 43/1536 * d^3 / a^4 / e^2 * (a^*d^*e + (a^*e^2 + c^*d^2) * x + c^*d^*e * x^2)^{(3/2)} * x^c^5 - 1/8/d^7 * e^8 * a * (c^*d^*e * (x+d/e)^2 + (a^*e^2 - c^*d^2) * (x+d/e))^{(3/2)} * x - 3/64/d^7 * e^{10} * a^3 / c * (c^*d^*e * (x+d/e)^2 + (a^*e^2 - c^*d^2) * (x+d/e))^{(1/2)} \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)\*x^7), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)\*x^7), x, algorithm="fricas")

[Out] Exception raised: TypeError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**7/(e*x+d),x)`

[Out] Timed out

**GIAC/XCAS [A]** time = 77.5188, size = 1, normalized size = 0.

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^7),x, algorithm="giac")`

[Out] Done

$$3.468 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^8(d+ex)} dx$$

**Optimal.** Leaf size=500

$$\begin{aligned} & \frac{(-63a^2e^4 + 20acd^2e^2 + 35c^2d^4) (x (ae^2 + cd^2) + ade + cdex^2)^{5/2}}{840a^2d^3e^2x^5} \\ & - \frac{(9a^2e^4 + 10acd^2e^2 + 5c^2d^4) (cd^2 - ae^2)^5 \tanh^{-1} \left( \frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \right)}{2048a^{9/2}d^{11/2}e^{9/2}} \\ & + \frac{(9a^2e^4 + 10acd^2e^2 + 5c^2d^4) (cd^2 - ae^2)^3 (x (ae^2 + cd^2) + 2ade) \sqrt{x (ae^2 + cd^2) + ade + cdex^2}}{1024a^4d^5e^4x^2} \\ & - \frac{(9a^2e^4 + 10acd^2e^2 + 5c^2d^4) (cd^2 - ae^2) (x (ae^2 + cd^2) + 2ade) (x (ae^2 + cd^2) + ade + cdex^2)^{3/2}}{384a^3d^4e^3x^4} \\ & - \frac{(x (ae^2 + cd^2) + ade + cdex^2)^{5/2}}{7dx^7} - \frac{\left( \frac{5c}{ae} - \frac{9e}{d^2} \right) (x (ae^2 + cd^2) + ade + cdex^2)^{5/2}}{84x^6} \end{aligned}$$

[Out]  $((c*d^2 - a*e^2)^3*(5*c^2*d^4 + 10*a*c*d^2*e^2 + 9*a^2*e^4)*(2*a*d*e + (c*d^2 + a*e^2)*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(1024*a^4*d^5*e^4*x^2) - ((c*d^2 - a*e^2)*(5*c^2*d^4 + 10*a*c*d^2*e^2 + 9*a^2*e^4)*(2*a*d*e + (c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(384*a^3*d^4*e^3*x^4) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}/(7*d*x^7) - (((5*c)/(a*e) - (9*e)/d^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(84*x^6) + ((35*c^2*d^4 + 20*a*c*d^2*e^2 - 63*a^2*e^4)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(840*a^2*d^3*e^2*x^5) - ((c*d^2 - a*e^2)^5*(5*c^2*d^4 + 10*a*c*d^2*e^2 + 9*a^2*e^4)*\text{ArcTanh}[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(2048*a^{(9/2)}*d^{(11/2)}*e^{(9/2)})$

**Rubi [A]** time = 1.68289, antiderivative size = 500, normalized size of antiderivative = 1., number of rules used = 8, number of rules used = 6, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\begin{aligned} & \frac{(-63a^2e^4 + 20acd^2e^2 + 35c^2d^4) (x (ae^2 + cd^2) + ade + cdex^2)^{5/2}}{840a^2d^3e^2x^5} \\ & - \frac{(9a^2e^4 + 10acd^2e^2 + 5c^2d^4) (cd^2 - ae^2)^5 \tanh^{-1} \left( \frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \right)}{2048a^{9/2}d^{11/2}e^{9/2}} \\ & + \frac{(9a^2e^4 + 10acd^2e^2 + 5c^2d^4) (cd^2 - ae^2)^3 (x (ae^2 + cd^2) + 2ade) \sqrt{x (ae^2 + cd^2) + ade + cdex^2}}{1024a^4d^5e^4x^2} \\ & - \frac{(9a^2e^4 + 10acd^2e^2 + 5c^2d^4) (cd^2 - ae^2) (x (ae^2 + cd^2) + 2ade) (x (ae^2 + cd^2) + ade + cdex^2)^{3/2}}{384a^3d^4e^3x^4} \\ & - \frac{(x (ae^2 + cd^2) + ade + cdex^2)^{5/2}}{7dx^7} - \frac{\left( \frac{5c}{ae} - \frac{9e}{d^2} \right) (x (ae^2 + cd^2) + ade + cdex^2)^{5/2}}{84x^6} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}/(x^8*(d + e*x)), x]$

[Out]  $((c*d^2 - a*e^2)^3*(5*c^2*d^4 + 10*a*c*d^2*e^2 + 9*a^2*e^4)*(2*a*d*e + (c*d^2 + a*e^2)*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(1024*a^4*d^5*e^4*x^2) - ((c*d^2 - a*e^2)*(5*c^2*d^4 + 10*a*c*d^2*e^2 + 9*a^2*e^4)*(2*a*d*e + (c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(384*a^3*d^4*e^3*x^4) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}/(7*d*x^7) - (((5*c)/(a*e) - (9*e)/d^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(84*x^6) + ((35*c^2*d^4 + 20*a*c*d^2*e^2 - 63*a^2*e^4)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(840*a^2*d^3*e^2*x^5) - ((c*d^2 - a*e^2)^5*(5*c^2*d^4 + 10*a*c*d^2*e^2 + 9*a^2*e^4)*\text{ArcTanh}[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e +$

$$(c*d^2 + a*e^2)*x + c*d*e*x^2)]])/(2048*a^(9/2)*d^(11/2)*e^(9/2))$$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**8/(e*x+d), x)`

[Out] Timed out

**Mathematica [A]** time = 1.23816, size = 557, normalized size = 1.11

$$((d + ex)(ae + cdx))^{3/2} \left( 105x^7 \log(x) (cd^2 - ae^2)^5 (9a^2e^4 + 10acd^2e^2 + 5c^2d^4) - 105x^7 (cd^2 - ae^2)^5 (9a^2e^4 + 10acd^2e^2 + 5c^2d^4) \right)$$

Antiderivative was successfully verified.

[In] `Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^8*(d + e*x)), x]`

[Out] 
$$\frac{\left( (a*e + c*d*x)^3 (d + e*x)^{3/2} \left( -2*\sqrt{a}*\sqrt{d}*\sqrt{e}*\sqrt{a*e + c*d*x}*\sqrt{d + e*x} \right. \right. \right. \\ \left. \left. \left. (-525*c^6*d^12*x^6 + 350*a*c^5*d^10*e*x^5*(d + 4*e*x) - 35*a^2*c^4*d^8*e^2*x^4*(8*d^2 + 26*d*e*x + 15*e^2*x^2) \right. \right. \right. \\ \left. \left. \left. + 60*a^3*c^3*d^6*e^3*x^3*(4*d^3 + 12*d^2*e*x + 5*d*e^2*x^2 - 10*e^3*x^3) \right. \right. \right. \\ \left. \left. \left. + a^4*c^2*d^4*e^4*x^2*(23680*d^4 + 33520*d^3*e*x + 1824*d^2*e^2*x^2 - 2332*d*e^3*x^3 + 3689*e^4*x^4) \right. \right. \right. \\ \left. \left. \left. + 2*a^5*c*d^2*e^5*x*(18560*d^5 + 24320*d^4*e*x + 744*d^3*e^2*x^2 - 872*d^2*e^3*x^3 + 1099*d*e^4*x^4 - 1680*e^5*x^5) \right. \right. \right. \\ \left. \left. \left. + 3*a^6*e^6*(5120*d^6 + 6400*d^5*e*x + 128*d^4*e^2*x^2 - 144*d^3*e^3*x^3 + 168*d^2*e^4*x^4 - 210*d*e^5*x^5 + 315*e^6*x^6) \right) \right. \right. \\ \left. \left. + 105*(c*d^2 - a*e^2)^5*(5*c^2*d^4 + 10*a*c*d^2*e^2 + 9*a^2*e^4)*x^7*\text{Log}[x] - 105*(c*d^2 - a*e^2)^5*(5*c^2*d^4 \right. \right. \\ \left. \left. + 10*a*c*d^2*e^2 + 9*a^2*e^4)*x^7*\text{Log}[c*d^2*x + 2*\sqrt{a}*\sqrt{d}*\sqrt{e}*\sqrt{a*e + c*d*x}*\sqrt{d + e*x} \right. \right. \\ \left. \left. + a*e*(2*d + e*x)] \right) \right) / (215040*a^(9/2)*d^(11/2)*e^(9/2)*x^7*(a*e + c*d*x)^(3/2) * (d + e*x)^(3/2))$$

**Maple [B]** time = 0.107, size = 5353, normalized size = 10.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^8/(e*x+d), x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^8), x, algo`

[Out] Exception raised: ValueError

---

**Fricas** [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^8), x, algo`

[Out] Exception raised: TypeError

---

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**8/(e*x+d), x)`

[Out] Timed out

---

**GIAC/XCAS** [A] time = 176.329, size = 1, normalized size = 0.

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^8), x, algo`

[Out] Done

$$3.469 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{x^9(d+ex)} dx$$

**Optimal.** Leaf size=628

$$\begin{aligned} & \frac{(-33a^2e^4 + 10acd^2e^2 + 15c^2d^4)(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{448a^2d^3e^2x^6} \\ & - \frac{(-231a^3e^6 + 15a^2cd^2e^4 + 95ac^2d^4e^2 + 105c^3d^6)(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{4480a^3d^4e^3x^5} \\ & + \frac{3(33a^3e^6 + 45a^2cd^2e^4 + 35ac^2d^4e^2 + 15c^3d^6)(cd^2 - ae^2)^5 \tanh^{-1}\left(\frac{x(ae^2 + cd^2) + 2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{32768a^{11/2}d^{13/2}e^{11/2}} \\ & - \frac{3(33a^3e^6 + 45a^2cd^2e^4 + 35ac^2d^4e^2 + 15c^3d^6)(cd^2 - ae^2)^3(x(ae^2 + cd^2) + 2ade)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{16384a^5d^6e^5x^2} \\ & + \frac{(33a^3e^6 + 45a^2cd^2e^4 + 35ac^2d^4e^2 + 15c^3d^6)(cd^2 - ae^2)(x(ae^2 + cd^2) + 2ade)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{2048a^4d^5e^4x^4} \\ & - \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{8dx^8} - \frac{\left(\frac{5c}{ae} - \frac{11e}{d^2}\right)(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{112x^7} \end{aligned}$$

[Out]  $(-3*(c*d^2 - a*e^2)^3*(15*c^3*d^6 + 35*a*c^2*d^4*e^2 + 45*a^2*c*d^2*e^4 + 33*a^3*e^6)*(2*a*d*e + (c*d^2 + a*e^2)*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(16384*a^5*d^6*e^5*x^2) + ((c*d^2 - a*e^2)*(15*c^3*d^6 + 35*a*c^2*d^4*e^2 + 45*a^2*c*d^2*e^4 + 33*a^3*e^6)*(2*a*d*e + (c*d^2 + a*e^2)*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(2048*a^4*d^5*e^4*x^4) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(8*d*x^8) - (((5*c)/(a*e) - (11*e)/d^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(112*x^7) + ((15*c^2*d^4 + 10*a*c*d^2*e^2 - 33*a^2*e^4)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(448*a^2*d^3*e^2*x^6) - ((105*c^3*d^6 + 95*a*c^2*d^4*e^2 + 15*a^2*c*d^2*e^4 - 231*a^3*e^6)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(4480*a^3*d^4*e^3*x^5) + (3*(c*d^2 - a*e^2)^5*(15*c^3*d^6 + 35*a*c^2*d^4*e^2 + 45*a^2*c*d^2*e^4 + 33*a^3*e^6)*\text{ArcTanh}[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(32768*a^(11/2)*d^(13/2)*e^(11/2))$

**Rubi [A]** time = 2.24911, antiderivative size = 628, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\begin{aligned} & \frac{(-33a^2e^4 + 10acd^2e^2 + 15c^2d^4)(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{448a^2d^3e^2x^6} \\ & - \frac{(-231a^3e^6 + 15a^2cd^2e^4 + 95ac^2d^4e^2 + 105c^3d^6)(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{4480a^3d^4e^3x^5} \\ & + \frac{3(33a^3e^6 + 45a^2cd^2e^4 + 35ac^2d^4e^2 + 15c^3d^6)(cd^2 - ae^2)^5 \tanh^{-1}\left(\frac{x(ae^2 + cd^2) + 2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{32768a^{11/2}d^{13/2}e^{11/2}} \\ & - \frac{3(33a^3e^6 + 45a^2cd^2e^4 + 35ac^2d^4e^2 + 15c^3d^6)(cd^2 - ae^2)^3(x(ae^2 + cd^2) + 2ade)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{16384a^5d^6e^5x^2} \\ & + \frac{(33a^3e^6 + 45a^2cd^2e^4 + 35ac^2d^4e^2 + 15c^3d^6)(cd^2 - ae^2)(x(ae^2 + cd^2) + 2ade)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{2048a^4d^5e^4x^4} \\ & - \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{8dx^8} - \frac{\left(\frac{5c}{ae} - \frac{11e}{d^2}\right)(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{112x^7} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)), x]$

[Out]  $(-3*(c*d^2 - a*e^2)^3*(15*c^3*d^6 + 35*a*c^2*d^4*e^2 + 45*a^2*c*d^2*e^4 + 33*a^3*e^6)*(2*a*d*e + (c*d^2 + a*e^2)*x)*\text{Sqrt}[a*d*e + ($



$$\frac{c^2 d^2 + a^2 e^2}{x} + \frac{c^2 d^2 e^2 x^2}{(16384 a^5 d^6 e^5 x^2)} + \frac{((c^2 d^2 - a^2 e^2) (15 c^3 d^6 + 35 a^2 c^2 d^4 e^2 + 45 a^2 c^2 d^2 e^4 + 33 a^3 e^6) (2 a^2 d e + (c^2 d^2 + a^2 e^2) x) (a^2 d e + (c^2 d^2 + a^2 e^2) x + c^2 d^2 e^2 x^2)^{3/2}}{(2048 a^4 d^5 e^4 x^4)} - \frac{(a^2 d e + (c^2 d^2 + a^2 e^2) x + c^2 d^2 e^2 x^2)^{5/2}}{(8 d^2 x^8)} - \frac{((5 c)/(a e) - (11 e)/d^2) (a^2 d e + (c^2 d^2 + a^2 e^2) x + c^2 d^2 e^2 x^2)^{5/2}}{(112 x^7)} + \frac{((15 c^2 d^4 + 10 a^2 c^2 d^2 e^2 - 33 a^2 e^4) (a^2 d e + (c^2 d^2 + a^2 e^2) x + c^2 d^2 e^2 x^2)^{5/2}}{(448 a^2 d^3 e^2 x^6)} - \frac{((105 c^3 d^6 + 95 a^2 c^2 d^4 e^2 + 15 a^2 c^2 d^2 e^4 - 231 a^3 e^6) (a^2 d e + (c^2 d^2 + a^2 e^2) x + c^2 d^2 e^2 x^2)^{5/2}}{(4480 a^3 d^4 e^3 x^5)} + \frac{3 (c^2 d^2 - a^2 e^2)^{5/2} (15 c^3 d^6 + 35 a^2 c^2 d^4 e^2 + 45 a^2 c^2 d^2 e^4 + 33 a^3 e^6) \operatorname{ArcTanh}[(2 a^2 d e + (c^2 d^2 + a^2 e^2) x)/(2 \sqrt{a} \sqrt{d} \sqrt{e} \sqrt{a^2 d e + (c^2 d^2 + a^2 e^2) x + c^2 d^2 e^2 x^2})]}{(32768 a^{11/2} d^{13/2} e^{11/2})}$$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**9/(e*x+d),x)`

[Out] Timed out

**Mathematica [A]** time = 1.40307, size = 676, normalized size = 1.08

$$\frac{(d + ex)(ae + cdx)^{3/2} \left( -105x^8 \log(x) (cd^2 - ae^2)^5 (33a^3e^6 + 45a^2cd^2e^4 + 35ac^2d^4e^2 + 15c^3d^6) + 105x^8 (cd^2 - ae^2)^5 (33a^3 \right.$$

Antiderivative was successfully verified.

[In] `Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(x^9*(d + e*x)),x]`

[Out]  $((a^2 e + c^2 d^2 x) (d + e x))^{3/2} (-2 \sqrt{a} \sqrt{d} \sqrt{e} \sqrt{a^2 e + c^2 d^2 x} \sqrt{d + e x} (1575 c^7 d^{14} x^7 - 525 a^2 c^6 d^{12} e x^6 (2 d + 7 e x) + 35 a^2 c^5 d^{10} e^2 x^5 (24 d^2 + 68 d^2 e x + 29 e^2 x^2) - 5 a^3 c^4 d^8 e^3 x^4 (144 d^3 + 376 d^2 e x + 110 d^2 e^2 x^2 - 185 e^3 x^3) + 5 a^4 c^3 d^6 e^4 x^3 (128 d^4 + 320 d^3 e x + 80 d^2 e^2 x^2 - 120 d^2 e^3 x^3 + 265 e^4 x^4) + a^5 c^2 d^4 e^5 x^2 (103680 d^5 + 137600 d^4 e x + 4640 d^3 e^2 x^2 - 5488 d^2 e^3 x^3 + 7034 d^2 e^4 x^4 - 11193 e^5 x^5) + a^6 c^2 d^2 e^6 x^3 (168960 d^6 + 212480 d^5 e x + 4480 d^4 e^2 x^2 - 5056 d^3 e^3 x^3 + 5928 d^2 e^4 x^4 - 7476 d^2 e^5 x^5 + 11445 e^6 x^6) + a^7 e^7 (71680 d^7 + 87040 d^6 e x + 1280 d^5 e^2 x^2 - 1408 d^4 e^3 x^3 + 1584 d^3 e^4 x^4 - 1848 d^2 e^5 x^5 + 2310 d^2 e^6 x^6 - 3465 e^7 x^7)) - 105 (c^2 d^2 - a^2 e^2)^{5/2} (15 c^3 d^6 + 35 a^2 c^2 d^4 e^2 + 45 a^2 c^2 d^2 e^4 + 33 a^3 e^6) x^8 \operatorname{Log}[x] + 105 (c^2 d^2 - a^2 e^2)^{5/2} (15 c^3 d^6 + 35 a^2 c^2 d^4 e^2 + 45 a^2 c^2 d^2 e^4 + 33 a^3 e^6) x^8 \operatorname{Log}[c^2 d^2 x + 2 \sqrt{a} \sqrt{d} \sqrt{e} \sqrt{a^2 e + c^2 d^2 x} \sqrt{d + e x} + a^2 e (2 d + e x)]) / (1146880 a^{11/2} d^{13/2} e^{11/2} x^8 (a^2 e + c^2 d^2 x)^{3/2} (d + e x)^{3/2})$

**Maple [B]** time = 0.154, size = 6030, normalized size = 9.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/x^9/(e*x+d), x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^9), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^9), x, algorithm="fricas")`

[Out] Exception raised: TypeError

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/x**9/(e*x+d), x)`

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)*x^9), x, algorithm="giac")`

[Out] Timed out

$$3.470 \quad \int \frac{x^3}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=271

$$\frac{3(a^2e^4 + 2acd^2e^2 + 5c^2d^4) \tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8c^{5/2}d^{5/2}e^{7/2}} - \frac{3(ae^2 + 3cd^2) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{4c^2d^2e^3} + \frac{(d+ex)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{2cde^3} - \frac{2d^3 \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{e^3(d+ex)(cd^2 - ae^2)}$$

[Out]  $(-3*(3*c*d^2 + a*e^2)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (4*c^2*d^2*e^3) - (2*d^3*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (e^3*(c*d^2 - a*e^2)*(d + e*x)) + ((d + e*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (2*c*d*e^3) + (3*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*\text{ArcTanh}[(c*d^2 + a*e^2 + 2*c*d*e*x) / (2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]) / (8*c^{5/2}*d^{5/2}*e^{7/2})$

**Rubi [A]** time = 0.999006, antiderivative size = 298, normalized size of antiderivative = 1.1, number of steps used = 5, number of rules used = 5, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{3(a^2e^4 + 2acd^2e^2 + 5c^2d^4) \tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8c^{5/2}d^{5/2}e^{7/2}} - \frac{((5cd^2 - 3ae^2)(ae^2 + 3cd^2) - 2cdex(5cd^2 - ae^2)) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{4c^2d^2e^3(cd^2 - ae^2)} - \frac{2dx^2(ae + cdx)}{e(cd^2 - ae^2)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3/((d + e*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]), x]$

[Out]  $(-2*d*x^2*(a*e + c*d*x)) / (e*(c*d^2 - a*e^2)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (((5*c*d^2 - 3*a*e^2)*(3*c*d^2 + a*e^2) - 2*c*d*e*(5*c*d^2 - a*e^2)*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (4*c^2*d^2*e^3*(c*d^2 - a*e^2)) + (3*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*\text{ArcTanh}[(c*d^2 + a*e^2 + 2*c*d*e*x) / (2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]) / (8*c^{5/2}*d^{5/2}*e^{7/2})$

**Rubi in Sympy [A]** time = 118.094, size = 461, normalized size = 1.7

$$\frac{2d^3\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{e^3(d+ex)(ae^2 - cd^2)} - \frac{\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{ce^3} + \frac{x\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{2cde^2} - \frac{3(ae^2 + cd^2)\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{4c^2d^2e^3} + \frac{d^{\frac{3}{2}} \operatorname{atanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}\right)}{\sqrt{ce^{\frac{7}{2}}}} + \frac{(ae^2 + cd^2) \operatorname{atanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}\right)}{2c^{\frac{3}{2}}\sqrt{de^{\frac{7}{2}}}} - \frac{(4acd^2e^2 - 3(ae^2 + cd^2)^2) \operatorname{atanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}\right)}{8c^{\frac{5}{2}}d^{\frac{5}{2}}e^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)`

[Out]  $2*d**3*\sqrt{a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)}/(e**3*(d + e*x)*(a*e**2 - c*d**2)) - \sqrt{a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)}/(c*e**3) + x*\sqrt{a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)}/(2*c*d*e**2) - 3*(a*e**2 + c*d**2)*\sqrt{a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)}/(4*c**2*d**2*e**3) + d**(3/2)*\operatorname{atanh}((a*e**2 + c*d**2 + 2*c*d*e*x)/(2*\sqrt{c}*\sqrt{d}*\sqrt{e}*\sqrt{a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)}))/(\sqrt{c}*e**(7/2)) + (a*e**2 + c*d**2)*\operatorname{atanh}((a*e**2 + c*d**2 + 2*c*d*e*x)/(2*\sqrt{c}*\sqrt{d}*\sqrt{e}*\sqrt{a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)}))/(\sqrt{c}*e**(7/2)) - (4*a*c*d**2*e**2 - 3*(a*e**2 + c*d**2)**2)*\operatorname{atanh}((a*e**2 + c*d**2 + 2*c*d*e*x)/(2*\sqrt{c}*\sqrt{d}*\sqrt{e}*\sqrt{a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)}))/(8*c**(5/2)*d**(5/2)*e**(7/2))$

**Mathematica [A]** time = 0.454964, size = 211, normalized size = 0.78

$$\frac{3\sqrt{d+ex}(a^2e^4+2acd^2e^2+5c^2d^4)\sqrt{ae+cdx}\log\left(2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx+ae^2+cd(d+2ex)}\right)}{c^{5/2}d^{5/2}e^{7/2}} + \frac{2(d+ex)(ae+cdx)\left(-\frac{3ae^2}{c^2} + \frac{8d^5}{(d+ex)(ae^2-cd^2)} + \frac{d(2ex-7d)}{c}\right)}{d^2e^3}$$

$$8\sqrt{(d+ex)(ae+cdx)}$$

Antiderivative was successfully verified.

[In] `Integrate[x^3/((d + e*x)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]),x]`

[Out]  $((2*(a*e + c*d*x)*(d + e*x)*((-3*a*e^2)/c^2 + (8*d^5)/((-c*d^2) + a*e^2)*(d + e*x)) + (d*(-7*d + 2*e*x)/c))/((d^2*e^3) + (3*(5*c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*\sqrt{a*e + c*d*x}*\sqrt{d + e*x}*\operatorname{Log}[a*e^2 + 2*\sqrt{c}*\sqrt{d}*\sqrt{e}*\sqrt{a*e + c*d*x}*\sqrt{d + e*x} + c*d*(d + 2*e*x)]))/(c^{5/2}*d^{5/2}*e^{7/2}))/((8*\sqrt{c}*(a*e + c*d*x)*(d + e*x))$

**Maple [A]** time = 0.028, size = 391, normalized size = 1.4

$$\frac{15d^2}{8e^3}\ln\left(1\left(\frac{ae^2}{2} + \frac{cd^2}{2} + cdex\right)\frac{1}{\sqrt{cde}} + \sqrt{ade + (ae^2 + cd^2)x + cdex^2}\right)\frac{1}{\sqrt{cde}}$$

$$+ \frac{x}{2ce^2d}\sqrt{ade + (ae^2 + cd^2)x + cdex^2}$$

$$- \frac{3a}{4c^2ed^2}\sqrt{ade + (ae^2 + cd^2)x + cdex^2} - \frac{7}{4e^3c}\sqrt{ade + (ae^2 + cd^2)x + cdex^2}$$

$$+ \frac{3a^2e}{8c^2d^2}\ln\left(1\left(\frac{ae^2}{2} + \frac{cd^2}{2} + cdex\right)\frac{1}{\sqrt{cde}} + \sqrt{ade + (ae^2 + cd^2)x + cdex^2}\right)\frac{1}{\sqrt{cde}}$$

$$+ \frac{3a}{4ce}\ln\left(1\left(\frac{ae^2}{2} + \frac{cd^2}{2} + cdex\right)\frac{1}{\sqrt{cde}} + \sqrt{ade + (ae^2 + cd^2)x + cdex^2}\right)\frac{1}{\sqrt{cde}}$$

$$+ 2\frac{d^3}{e^4(ae^2 - cd^2)}\sqrt{cde\left(x + \frac{d}{e}\right)^2 + (ae^2 - cd^2)\left(x + \frac{d}{e}\right)\left(x + \frac{d}{e}\right)^{-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)`

[Out]  $15/8*d^2/e^3*\ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)+1/2/e^2*x/c/d*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/4/e/c^2/d^2*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a-7/4/e^3/c*(a*d*e+(a*e^2+c*d^2)*x+c$

$$d^*e^*x^2)^{(1/2)}+3/8^*e/c^2/d^2*\ln((1/2^*a^*e^2+1/2^*c^*d^2+c^*d^*e^*x)/(c^*d^*e)^{(1/2)}+(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(1/2)})/(c^*d^*e)^{(1/2)}+a^2+3/4/e/c*\ln((1/2^*a^*e^2+1/2^*c^*d^2+c^*d^*e^*x)/(c^*d^*e)^{(1/2)}+(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(1/2)})/(c^*d^*e)^{(1/2)}+a+2^*d^3/e^4/(a^*e^2-c^*d^2)/(x+d/e)^*(c^*d^*e^*(x+d/e)^2+(a^*e^2-c^*d^2)^*(x+d/e))^{(1/2)}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(e\*x + d)),x, algorithm

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.604162, size = 1, normalized size = 0.

$$\frac{4(15c^2d^5 - 4acd^3e^2 - 3a^2de^4 - 2(c^2d^3e^2 - acde^4)x^2 + (5c^2d^4e - 2acd^2e^3 - 3a^2e^5)x)\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{-}}{2(15c^2d^5 - 4acd^3e^2 - 3a^2de^4 - 2(c^2d^3e^2 - acde^4)x^2 + (5c^2d^4e - 2acd^2e^3 - 3a^2e^5)x)\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{-}}$$

$8(c^3d^5e^3 - ac^2d^3e^5 + ($

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(e\*x + d)),x, algorithm

[Out]  $[-1/16*(4*(15*c^2*d^5 - 4*a*c*d^3*e^2 - 3*a^2*d^4*e^4 - 2*(c^2*d^3*e^2 - a*c*d^3*e^2 - acde^4)*x^2 + (5*c^2*d^4*e - 2*a*c*d^2*e^3 - 3*a^2*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*e) - 3*(5*c^3*d^7 - 3*a*c^2*d^5*e^2 - a^2*c*d^3*e^4 - a^3*d^2*e^6 + (5*c^3*d^6*e - 3*a*c^2*d^4*e^3 - a^2*c*d^2*e^5 - a^3*e^7)*x)*log(4*(2*c^2*d^2*e^2*x + c^2*d^3*e + a*c*d^2*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x) + (8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 8*(c^2*d^3*e + a*c*d^2*e^3)*x)*sqrt(c*d*e))/((c^3*d^5*e^3 - a*c^2*d^3*e^5 + (c^3*d^4*e^4 - a*c^2*d^2*e^6)*x)*sqrt(c*d*e)), -1/8*(2*(15*c^2*d^5 - 4*a*c*d^3*e^2 - 3*a^2*d^4*e^4 - 2*(c^2*d^3*e^2 - a*c*d^3*e^2 - acde^4)*x^2 + (5*c^2*d^4*e - 2*a*c*d^2*e^3 - 3*a^2*e^5)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*e) - 3*(5*c^3*d^7 - 3*a*c^2*d^5*e^2 - a^2*c*d^3*e^4 - a^3*d^2*e^6 + (5*c^3*d^6*e - 3*a*c^2*d^4*e^3 - a^2*c*d^2*e^5 - a^3*e^7)*x)*arctan(1/2*(2*c*d^2*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(c*d*e)))/((c^3*d^5*e^3 - a*c^2*d^3*e^5 + (c^3*d^4*e^4 - a*c^2*d^2*e^6)*x)*sqrt(-c*d*e))]$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(e\*x+d)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2),x)

[Out] Timed out

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**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)),x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.471 \quad \int \frac{x^2}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=195

$$\frac{(ae^2 + 3cd^2) \tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2c^{3/2}d^{3/2}e^{5/2}} + \frac{2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^2(d+ex)(cd^2-ae^2)} + \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cde^2}$$

[Out] Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(c\*d\*e^2) + (2\*d^2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(e^2\*(c\*d^2 - a\*e^2)\*(d + e\*x)) - ((3\*c\*d^2 + a\*e^2)\*ArcTanh[(c\*d^2 + a\*e^2 + 2\*c\*d\*e\*x)/(2\*Sqrt[c]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])]/(2\*c^(3/2)\*d^(3/2)\*e^(5/2))

**Rubi [A]** time = 0.68325, antiderivative size = 195, normalized size of antiderivative = 1., number of rules used = 4, number of rules used = 4, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{(ae^2 + 3cd^2) \tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2c^{3/2}d^{3/2}e^{5/2}} + \frac{2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e^2(d+ex)(cd^2-ae^2)} + \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cde^2}$$

Antiderivative was successfully verified.

[In] Int[x^2/((d + e\*x)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]), x]

[Out] Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(c\*d\*e^2) + (2\*d^2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(e^2\*(c\*d^2 - a\*e^2)\*(d + e\*x)) - ((3\*c\*d^2 + a\*e^2)\*ArcTanh[(c\*d^2 + a\*e^2 + 2\*c\*d\*e\*x)/(2\*Sqrt[c]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])]/(2\*c^(3/2)\*d^(3/2)\*e^(5/2))

**Rubi in Sympy [A]** time = 67.3713, size = 262, normalized size = 1.34

$$\frac{-\frac{2d^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{e^2(d+ex)(ae^2-cd^2)} + \frac{\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{cde^2}}{\sqrt{ce}^{\frac{5}{2}}} - \frac{(ae^2+cd^2)\operatorname{atanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}\right)}{2c^{\frac{3}{2}}d^{\frac{3}{2}}e^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(e\*x+d)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2), x)

[Out] -2\*d\*\*2\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(e\*\*2\*(d + e\*x)\*(a\*e\*\*2 - c\*d\*\*2)) + sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(c\*d\*e\*\*2) - sqrt(d)\*atanh((a\*e\*\*2 + c\*d\*\*2 + 2\*c\*d\*e\*x)/(2\*sqrt(c)\*sqrt(d)\*sqrt(e)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))))/(sqrt(c)\*e\*\*(5/2)) - (a\*e\*\*2 + c\*d\*\*2)\*atanh((a\*e\*\*2 + c\*d\*\*2 + 2\*c\*d\*e\*x)/(2\*sqrt(c)\*sqrt(d)\*sqrt(e)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))))/(2\*c\*\*(3/2)\*d\*\*(3/2)\*e\*\*(5/2))

**Mathematica [A]** time = 0.327766, size = 178, normalized size = 0.91

$$\frac{2(d+ex)(ae+cdx)\left(\frac{2d^3}{(d+ex)(cd^2-ae^2)}+\frac{1}{c}\right) - \frac{\sqrt{d+ex}(ae^2+3cd^2)\sqrt{ae+cdx}\log\left(2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx+ae^2+cd(d+2ex)}\right)}{c^{3/2}d^{3/2}e^{5/2}}}{de^2} - \frac{\sqrt{d+ex}(ae+cdx)}{2\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((d + e\*x)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]), x]

[Out] ((2\*(a\*e + c\*d\*x)\*(d + e\*x)\*(c^(-1) + (2\*d^3)/((c\*d^2 - a\*e^2)\*(d + e\*x))))/(d\*e^2) - ((3\*c\*d^2 + a\*e^2)\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*Log[a\*e^2 + 2\*Sqrt[c]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x] + c\*d\*(d + 2\*e\*x)))/(c^(3/2)\*d^(3/2)\*e^(5/2)))/(2\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)])

**Maple [A]** time = 0.017, size = 241, normalized size = 1.2

$$\begin{aligned} & \frac{1}{cde^2}\sqrt{ade + (ae^2 + cd^2)x + cdex^2} \\ & - \frac{a}{2cd}\ln\left(1\left(\frac{ae^2}{2} + \frac{cd^2}{2} + cdex\right)\frac{1}{\sqrt{cde}} + \sqrt{ade + (ae^2 + cd^2)x + cdex^2}\right)\frac{1}{\sqrt{cde}} \\ & - \frac{3d}{2e^2}\ln\left(1\left(\frac{ae^2}{2} + \frac{cd^2}{2} + cdex\right)\frac{1}{\sqrt{cde}} + \sqrt{ade + (ae^2 + cd^2)x + cdex^2}\right)\frac{1}{\sqrt{cde}} \\ & - 2\frac{d^2}{e^3(ae^2 - cd^2)}\sqrt{cde\left(x + \frac{d}{e}\right)^2 + (ae^2 - cd^2)\left(x + \frac{d}{e}\right)\left(x + \frac{d}{e}\right)^{-1}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(e\*x+d)/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2), x)

[Out] (a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)/c/d/e^2-1/2/c/d\*e\*ln((1/2\*a\*e^2+1/2\*c\*d^2+c\*d\*e\*x)/(c\*d\*e)^(1/2)+(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/(c\*d\*e)^(1/2)\*a-3/2/e^2\*d\*ln((1/2\*a\*e^2+1/2\*c\*d^2+c\*d\*e\*x)/(c\*d\*e)^(1/2)+(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/(c\*d\*e)^(1/2)-2\*d^2/e^3/(a\*e^2-c\*d^2)/(x+d/e)\*(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(e\*x + d)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.433189, size = 1, normalized size = 0.01

$$\left[ \frac{4\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(3cd^3 - ade^2 + (cd^2e - ae^3)x)\sqrt{cde} + (3c^2d^5 - 2acd^3e^2 - a^2de^4 + (3c^2d^4e - 2acd^2e^3 - a^3d^2e^2)x^2 + (3cd^3e^2 - a^2de^4)x + a^3d^2e^2)}{4(c^2d^4e^2 - a^3d^2e^2 - a^2de^4)x^2 + (3cd^3e^2 - a^2de^4)x + a^3d^2e^2} \right]$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)),x, algorithm`

[Out] `[1/4*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(3*c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)*sqrt(c*d*e) + (3*c^2*d^5 - 2*a*c*d^3*e^2 - a^2*d*e^4 + (3*c^2*d^4*e - 2*a*c*d^2*e^3 - a^2*e^5)*x)*log(-4*(2*c^2*d^2*e^2*x + c^2*d^3*e + a*c*d*e^3)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x) + (8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 8*(c^2*d^3*e + a*c*d*e^3)*x)*sqrt(c*d*e)))/((c^2*d^4*e^2 - a*c*d^2*e^4 + (c^2*d^3*e^3 - a*c*d*e^5)*x)*sqrt(c*d*e)), 1/2*(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(3*c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)*sqrt(-c*d*e) - (3*c^2*d^5 - 2*a*c*d^3*e^2 - a^2*d*e^4 + (3*c^2*d^4*e - 2*a*c*d^2*e^3 - a^2*e^5)*x)*arctan(1/2*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c*d*e)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d*e)))/((c^2*d^4*e^2 - a*c*d^2*e^4 + (c^2*d^3*e^3 - a*c*d*e^5)*x)*sqrt(-c*d*e))]`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{(d+ex)(ae+cdx)}(d+ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)`

[Out] `Integral(x**2/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)`

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)),x, algorithm`

[Out] `Exception raised: TypeError`

$$3.472 \quad \int \frac{x}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=139

$$\frac{\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}e^{3/2}} - \frac{2d\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e(d+ex)(cd^2-ae^2)}$$

[Out]  $(-2*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e*(c*d^2 - a*e^2)*(d + e*x)) + \text{ArcTanh}[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(\text{Sqrt}[c]*\text{Sqrt}[d]*e^{(3/2)})$

**Rubi [A]** time = 0.304866, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$

$$\frac{\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{c}\sqrt{d}e^{3/2}} - \frac{2d\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{e(d+ex)(cd^2-ae^2)}$$

Antiderivative was successfully verified.

[In] Int[x/((d + e\*x)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]), x]

[Out]  $(-2*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(e*(c*d^2 - a*e^2)*(d + e*x)) + \text{ArcTanh}[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(\text{Sqrt}[c]*\text{Sqrt}[d]*e^{(3/2)})$

**Rubi in Sympy [A]** time = 23.267, size = 129, normalized size = 0.93

$$\frac{2d\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{e(d+ex)(ae^2-cd^2)} + \frac{\text{atanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}\right)}{\sqrt{c}\sqrt{d}e^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x/(e\*x+d)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2), x)

[Out]  $2*d*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(e*(d + e*x)*(a*e**2 - c*d**2)) + \text{atanh}((a*e**2 + c*d**2 + 2*c*d*e*x)/(2*\text{sqrt}(c)*\text{sqrt}(d)*\text{sqrt}(e)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))))/(\text{sqrt}(c)*\text{sqrt}(d)*e^{(3/2)})$

**Mathematica [A]** time = 0.338025, size = 145, normalized size = 1.04

$$\frac{\frac{2d^{3/2}\sqrt{e}(ae+cdx)}{ae^2-cd^2} + \frac{\sqrt{d+ex}\sqrt{ae+cdx}\log(2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx+ae^2+cd(d+2ex)})}{\sqrt{c}}}{\sqrt{d}e^{3/2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[x/((d + e\*x)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]), x]

[Out]  $((2*d^{(3/2)}*Sqrt[e]*(a*e + c*d*x))/(-(c*d^2) + a*e^2) + (Sqrt[a*e + c*d*x]*Sqrt[d + e*x]*Log[a*e^2 + 2*Sqrt[c]*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x] + c*d*(d + 2*e*x)])/Sqrt[c])/(Sqrt[d]*e^{(3/2)}*Sqrt[(a*e + c*d*x)*(d + e*x)])$

**Maple [A]** time = 0.014, size = 131, normalized size = 0.9

$$\frac{1}{e} \ln \left( 1 \left( \frac{ae^2}{2} + \frac{cd^2}{2} + cdex \right) \frac{1}{\sqrt{cde}} + \sqrt{ade + (ae^2 + cd^2)x + cdex^2} \right) \frac{1}{\sqrt{cde}} + 2 \frac{d}{e^2 (ae^2 - cd^2)} \sqrt{cde \left( x + \frac{d}{e} \right)^2 + (ae^2 - cd^2) \left( x + \frac{d}{e} \right) \left( x + \frac{d}{e} \right)^{-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)`

[Out]  $1/e * \ln \left( \left( \frac{1}{2} * a * e^2 + \frac{1}{2} * c * d^2 + c * d * e * x \right) / \left( c * d * e \right)^{(1/2)} + \left( a * d * e + \left( a * e^2 + c * d^2 \right) * x + c * d * e * x^2 \right)^{(1/2)} \right) / \left( c * d * e \right)^{(1/2)} + 2 * d / e^2 / \left( a * e^2 - c * d^2 \right) / \left( x + d / e \right) * \left( c * d * e * \left( x + d / e \right)^2 + \left( a * e^2 - c * d^2 \right) * \left( x + d / e \right) \right)^{(1/2)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)),x, algorithm=`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.384118, size = 1, normalized size = 0.01

$$\left[ \frac{4 \sqrt{cdex^2 + ade + (cd^2 + ae^2)x} \sqrt{cde} - (cd^3 - ade^2 + (cd^2e - ae^3)x) \log \left( 4 (2c^2d^2e^2x + c^2d^3e + acde^3) \sqrt{cdex^2 + ade + (cd^2 + ae^2)x} \sqrt{cde} \right)}{2 (cd^3e - ade^3 + (cd^2e^2 - ae^4)x) \sqrt{cde}} \right. \\ \left. \frac{2 \sqrt{cdex^2 + ade + (cd^2 + ae^2)x} \sqrt{-cde} - (cd^3 - ade^2 + (cd^2e - ae^3)x) \arctan \left( \frac{(2cdex + cd^2 + ae^2) \sqrt{-cde}}{2 \sqrt{cdex^2 + ade + (cd^2 + ae^2)x} \sqrt{cde}} \right)}{(cd^3e - ade^3 + (cd^2e^2 - ae^4)x) \sqrt{-cde}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)),x, algorithm=`

[Out]  $[-1/2 * (4 * \sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x} * \sqrt{c*d*e} * d - (c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x) * \log(4 * (2*c^2*d^2*e^2*x + c^2*d^3*e + a*c*d*e^3) * \sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x} + (8*c^2*d^2*e^2*x^2 + c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4 + 8*(c^2*d^3*e + a*c*d*e^3)*x) * \sqrt{c*d*e}) / ((c*d^3*e - a*d*e^3 + (c*d^2*e^2 - a*e^4)*x) * \sqrt{c*d*e}), -(2 * \sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x} * \sqrt{-c*d*e} * d - (c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x) * \arctan(1/2 * (2*c*d*e*x + c*d^2 + a*e^2) * \sqrt{-c*d*e} / (\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x} * \sqrt{c*d*e})) / ((c*d^3*e - a*d*e^3 + (c*d^2*e^2 - a*e^4)*x) * \sqrt{-c*d*e})]$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{(d+ex)(ae+cdx)}(d+ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(e\*x+d)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2),x)

[Out] Integral(x/(sqrt((d + e\*x)\*(a\*e + c\*d\*x))\*(d + e\*x)), x)

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(e\*x + d)),x, algorithm=

[Out] Exception raised: TypeError

$$3.473 \quad \int \frac{1}{(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=52

$$\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{(d+ex)(cd^2-ae^2)}$$

[Out] (2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/((c\*d^2 - a\*e^2)\*(d + e\*x))

**Rubi [A]** time = 0.0774359, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 37,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.027$

$$\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{(d+ex)(cd^2-ae^2)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]),x]

[Out] (2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/((c\*d^2 - a\*e^2)\*(d + e\*x))

**Rubi in Sympy [A]** time = 18.4113, size = 46, normalized size = 0.88

$$-\frac{2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{(d+ex)(ae^2-cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(e\*x+d)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2),x)

[Out] -2\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/((d + e\*x)\*(a\*e\*\*2 - c\*d\*\*2))

**Mathematica [A]** time = 0.0573582, size = 42, normalized size = 0.81

$$-\frac{2(ae+cdx)}{(ae^2-cd^2)\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]),x]

[Out] (-2\*(a\*e + c\*d\*x))/((-c\*d^2 + a\*e^2)\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)])

**Maple [A]** time = 0.01, size = 51, normalized size = 1.

$$-2 \frac{cdx + ae}{(ae^2 - cd^2)\sqrt{cdex^2 + ae^2x + cd^2x + ade}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)`

[Out] `-2*(c*d*x+a*e)/(a*e^2-c*d^2)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)),x, algorithm=`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.307171, size = 80, normalized size = 1.54

$$\frac{2\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{cd^3 - ade^2 + (cd^2e - ae^3)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)),x, algorithm=`

[Out] `2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(c*d^3 - a*d*e^2 + (c*d^2*e - a*e^3)*x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{(d+ex)(ae+cdx)}(d+ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)`

[Out] `Integral(1/(sqrt((d + e*x)*(a*e + c*d*x))*(d + e*x)), x)`

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)),x, algorithm=`

[Out] Exception raised: NotImplementedError

$$3.474 \quad \int \frac{1}{x(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=143

$$\frac{2e(ae+cdx)}{d(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{ad}^{3/2}\sqrt{e}}$$

[Out]  $(-2*e*(a*e + c*d*x))/(d*(c*d^2 - a*e^2)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - \text{ArcTanh}[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(\text{Sqrt}[a]*d^{(3/2)}*\text{Sqrt}[e])$

**Rubi [A]** time = 0.573941, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{2e(ae+cdx)}{d(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{\sqrt{ad}^{3/2}\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(d + e\*x)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]),x]

[Out]  $(-2*e*(a*e + c*d*x))/(d*(c*d^2 - a*e^2)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - \text{ArcTanh}[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(\text{Sqrt}[a]*d^{(3/2)}*\text{Sqrt}[e])$

**Rubi in Sympy [A]** time = 55.199, size = 133, normalized size = 0.93

$$\frac{2e(ae+cdx)}{d(ae^2-cd^2)\sqrt{ade+cdex^2+x(ae^2+cd^2)}} - \frac{\text{atanh}\left(\frac{2ade+x(ae^2+cd^2)}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}\right)}{\sqrt{ad}^{3/2}\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(e\*x+d)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2),x)

[Out]  $2*e*(a*e + c*d*x)/(d*(a*e**2 - c*d**2)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))) - \text{atanh}(((2*a*d*e + x*(a*e**2 + c*d**2))/(2*\text{sqrt}(a)*\text{sqrt}(d)*\text{sqrt}(e)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))))/(\text{sqrt}(a)*d**(3/2)*\text{sqrt}(e))$

**Mathematica [A]** time = 0.359109, size = 200, normalized size = 1.4

$$\frac{\log(x)\sqrt{d+ex}(cd^2-ae^2)\sqrt{ae+cdx}+\sqrt{d+ex}(ae^2-cd^2)\sqrt{ae+cdx}\log\left(2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}+ae(2d+ex)+cdex^2\right)}{\sqrt{ad}^{3/2}\sqrt{e}(cd^2-ae^2)\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(d + e\*x)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]),x]

[Out]  $(-2\sqrt{a}\sqrt{d}e^{3/2}(ae + cd^2x) + (cd^2 - ae^2)\sqrt{ae + cd^2x})\sqrt{d + ex}\log[x] + (-cd^2 + ae^2)\sqrt{ae + cd^2x}\sqrt{d + ex}\log[cd^2x + 2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ae + cd^2x}\sqrt{d + ex} + ae(2d + ex)] / (\sqrt{a}d^{3/2}\sqrt{e}(cd^2 - ae^2)\sqrt{(ae + cd^2x)(d + ex)})$

**Maple [A]** time = 0.018, size = 136, normalized size = 1.

$$-\frac{1}{d}\ln\left(\frac{1}{x}\left(2ade + (ae^2 + cd^2)x + 2\sqrt{ade}\sqrt{ade + (ae^2 + cd^2)x + cdex^2}\right)\right)\frac{1}{\sqrt{ade}} + 2\frac{1}{d(ae^2 - cd^2)}\sqrt{cde\left(x + \frac{d}{e}\right)^2 + (ae^2 - cd^2)\left(x + \frac{d}{e}\right)\left(x + \frac{d}{e}\right)^{-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)`

[Out]  $-1/d/(a*d*e)^{1/2}\ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^{1/2}*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{1/2})/x+2/d/(a*e^2-c*d^2)/(x+d/e)*(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)*x),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.372042, size = 1, normalized size = 0.01

$$\left[ \frac{4\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{ade} - (cd^3 - ade^2 + (cd^2e - ae^3)x)\log\left(-\frac{4(2a^2d^2e^2 + (acd^3e + a^2de^3)x)\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{2(cd^4 - ad^2e^2 + (cd^3e - ade^3)x)\sqrt{ade}}\right)}{2\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{-ade} + (cd^3 - ade^2 + (cd^2e - ae^3)x)\arctan\left(\frac{(2ade + (cd^2 + ae^2)x)\sqrt{-ade}}{2\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}ade}\right)}{(cd^4 - ad^2e^2 + (cd^3e - ade^3)x)\sqrt{-ade}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(e*x + d)*x),x, algorithm="fricas")`

[Out]  $[-1/2*(4*\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x}*\sqrt{a*d*e})*e - (c*d^3 - a*d^2*e^2 + (c*d^2*e - a*e^3)*x)*\log(-(4*(2*a^2*d^2*e^2 + (a*c*d^3*e + a^2*d^2*e^3)*x)*\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x} - (8*a^2*d^2*e^2 + (c^2*d^4 + 6*a*c*d^2*e^2 + a^2*e^4)*x^2 + 8*(a*c*d^3*e + a^2*d^2*e^3)*x)*\sqrt{a*d*e})/x^2)/((c*d^4 - a*d^2*e^2 + (c*d^3*e - a*d^2*e^3)*x)*\sqrt{a*d*e}), -(2*\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x}*\sqrt{-a*d*e})*e + (c*d^3 - a*d^2*e^2 + (c*d^2*e - a*e^3)*x)*\arctan(1/2*(2*a*d*e + (c*d^2 + a*e^2)*x)*\sqrt{-a*d*e})/(\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x}*a*d*e)]$



$$/((c*d^4 - a*d^2*e^2 + (c*d^3*e - a*d*e^3)*x)*\sqrt{-a*d*e})]$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e\*x+d)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2),x)

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(e\*x + d)\*x),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.475 \quad \int \frac{1}{x^2(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=229

$$\frac{(3ae^2 + cd^2) \tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2a^{3/2}d^{5/2}e^{3/2}} - \frac{(cd^2 - 3ae^2) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{ad^2ex(cd^2 - ae^2)} - \frac{2e(ae + cdx)}{dx(cd^2 - ae^2)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}$$

[Out]  $(-2*e*(a*e + c*d*x))/(d*(c*d^2 - a*e^2)*x*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((c*d^2 - 3*a*e^2)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a*d^2*e*(c*d^2 - a*e^2)*x) + ((c*d^2 + 3*a*e^2)*\text{ArcTanh}[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(2*a^{(3/2)}*d^{(5/2)}*e^{(3/2)})$

**Rubi [A]** time = 0.894965, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{(3ae^2 + cd^2) \tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{2a^{3/2}d^{5/2}e^{3/2}} - \frac{(cd^2 - 3ae^2) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{ad^2ex(cd^2 - ae^2)} - \frac{2e(ae + cdx)}{dx(cd^2 - ae^2)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^2*(d + e*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]), x]$

[Out]  $(-2*e*(a*e + c*d*x))/(d*(c*d^2 - a*e^2)*x*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((c*d^2 - 3*a*e^2)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(a*d^2*e*(c*d^2 - a*e^2)*x) + ((c*d^2 + 3*a*e^2)*\text{ArcTanh}[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(2*a^{(3/2)}*d^{(5/2)}*e^{(3/2)})$

**Rubi in Sympy [A]** time = 105.608, size = 207, normalized size = 0.9

$$\frac{2e(ae + cdx)}{dx(ae^2 - cd^2)\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}} - \frac{(3ae^2 - cd^2) \sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{ad^2ex(ae^2 - cd^2)} + \frac{(3ae^2 + cd^2) \operatorname{atanh}\left(\frac{2ade+x(ae^2+cd^2)}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}\right)}{2a^{3/2}d^{5/2}e^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x^{**2}/(e*x+d)/(a*d*e+(a*e^{**2}+c*d^{**2})*x+c*d*e*x^{**2})^{**}(1/2), x)$

[Out]  $2*e*(a*e + c*d*x)/(d*x*(a*e^{**2} - c*d^{**2})*\text{sqrt}(a*d*e + c*d*e*x^{**2} + x*(a*e^{**2} + c*d^{**2}))) - (3*a*e^{**2} - c*d^{**2})*\text{sqrt}(a*d*e + c*d*e*x^{**2} + x*(a*e^{**2} + c*d^{**2}))/(a*d^{**2}*e*x*(a*e^{**2} - c*d^{**2})) + (3*a*e^{**2} + c*d^{**2})*\text{atanh}((2*a*d*e + x*(a*e^{**2} + c*d^{**2}))/(2*\text{sqrt}(a)*\text{sqrt}(d)*\text{sqrt}(e)*\text{sqrt}(a*d*e + c*d*e*x^{**2} + x*(a*e^{**2} + c*d^{**2}))))/(2*a^{**}(3/2)*d^{**}(5/2)*e^{**}(3/2))$

**Mathematica [A]** time = 0.740895, size = 276, normalized size = 1.21

$$\begin{aligned} & \frac{\log(x)\sqrt{d+ex}(3ae^2+cd^2)\sqrt{ae+cdx}}{2a^{3/2}d^{5/2}e^{3/2}\sqrt{(d+ex)(ae+cdx)}} \\ & + \frac{\sqrt{d+ex}(3ae^2+cd^2)\sqrt{ae+cdx}\log\left(2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}+2ade+ae^2x+cd^2x\right)}{2a^{3/2}d^{5/2}e^{3/2}\sqrt{(d+ex)(ae+cdx)}} \\ & + \frac{(d+ex)(ae+cdx)\left(\frac{2e^2}{d^2(d+ex)(cd^2-ae^2)}-\frac{1}{ad^2ex}\right)}{\sqrt{(d+ex)(ae+cdx)}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^2\*(d+e\*x)\*Sqrt[a\*d\*e+(c\*d^2+a\*e^2)\*x+c\*d\*e\*x^2]),x]

[Out] ((a\*e+c\*d\*x)\*(d+e\*x)\*(-1/(a\*d^2\*e\*x))+(2\*e^2)/(d^2\*(c\*d^2-a\*e^2)\*(d+e\*x)))/Sqrt[(a\*e+c\*d\*x)\*(d+e\*x)]-((c\*d^2+3\*a\*e^2)\*Sqrt[a\*e+c\*d\*x]\*Sqrt[d+e\*x]\*Log[x])/(2\*a^(3/2)\*d^(5/2)\*e^(3/2)\*Sqrt[(a\*e+c\*d\*x)\*(d+e\*x)])+((c\*d^2+3\*a\*e^2)\*Sqrt[a\*e+c\*d\*x]\*Sqrt[d+e\*x]\*Log[2\*a\*d\*e+c\*d^2\*x+a\*e^2\*x+2\*Sqrt[a]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*e+c\*d\*x]\*Sqrt[d+e\*x]])/(2\*a^(3/2)\*d^(5/2)\*e^(3/2)\*Sqrt[(a\*e+c\*d\*x)\*(d+e\*x)])

**Maple [A]** time = 0.02, size = 253, normalized size = 1.1

$$\begin{aligned} & -\frac{1}{ad^2ex}\sqrt{ade+(ae^2+cd^2)x+cdex^2} \\ & +\frac{3e}{2d^2}\ln\left(\frac{1}{x}\left(2ade+(ae^2+cd^2)x+2\sqrt{ade}\sqrt{ade+(ae^2+cd^2)x+cdex^2}\right)\right)\frac{1}{\sqrt{ade}} \\ & +\frac{c}{2ae}\ln\left(\frac{1}{x}\left(2ade+(ae^2+cd^2)x+2\sqrt{ade}\sqrt{ade+(ae^2+cd^2)x+cdex^2}\right)\right)\frac{1}{\sqrt{ade}} \\ & -2\frac{e}{d^2(ae^2-cd^2)}\sqrt{cde\left(x+\frac{d}{e}\right)^2+(ae^2-cd^2)\left(x+\frac{d}{e}\right)\left(x+\frac{d}{e}\right)^{-1}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(e\*x+d)/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2),x)

[Out] -1/d^2/a/e/x\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)+3/2/d^2\*e/(a\*d\*e)^(1/2)\*ln((2\*a\*d\*e+(a\*e^2+c\*d^2)\*x+2\*(a\*d\*e)^(1/2)\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/x)+1/2/a/e/(a\*d\*e)^(1/2)\*ln((2\*a\*d\*e+(a\*e^2+c\*d^2)\*x+2\*(a\*d\*e)^(1/2)\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/x)\*c-2/d^2\*e/(a\*e^2-c\*d^2)/(x+d/e)\*(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(ex+d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*d\*e\*x^2+a\*d\*e+(c\*d^2+a\*e^2)\*x)\*(e\*x+d)\*x^2),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(e\*x + d)^\*x^2), x)

**Fricas [A]** time = 0.432973, size = 1, normalized size = 0.

$$\frac{4\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(cd^3 - ade^2 + (cd^2e - 3ae^3)x)\sqrt{ade} - ((c^2d^4e + 2acd^2e^3 - 3a^2e^5)x^2 + (c^2d^5 + 2acd^3e^2))}{4\sqrt{ade}((acd^4e^2 - a^2d^2e^4)x^2 + (acd^5e - a^2d^3e^3)x)}$$

$$\frac{2\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(cd^3 - ade^2 + (cd^2e - 3ae^3)x)\sqrt{-ade} - ((c^2d^4e + 2acd^2e^3 - 3a^2e^5)x^2 + (c^2d^5 + 2acd^3e^2))}{2\sqrt{-ade}((acd^4e^2 - a^2d^2e^4)x^2 + (acd^5e - a^2d^3e^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(e\*x + d)^\*x^2), x, algorithm="fricas")

[Out] [-1/4\*(4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(c\*d^3 - a\*d^2\*e^2 + (c\*d^2\*e - 3\*a\*e^3)\*x)\*sqrt(a\*d\*e) - ((c^2\*d^4\*e + 2\*a\*c\*d^2\*e^3 - 3\*a^2\*e^5)\*x^2 + (c^2\*d^5 + 2\*a\*c\*d^3\*e^2)\*x)\*log((4\*(2\*a^2\*d^2\*e^2 + (a\*c\*d^3\*e + a^2\*d^2\*e^3)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) + (8\*a^2\*d^2\*e^2 + (c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4)\*x^2 + 8\*(a\*c\*d^3\*e + a^2\*d^2\*e^3)\*x)\*sqrt(a\*d\*e))/x^2)/(sqrt(a\*d\*e)\*((a\*c\*d^4\*e^2 - a^2\*d^2\*e^4)\*x^2 + (a\*c\*d^5\*e - a^2\*d^3\*e^3)\*x)), -1/2\*(2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(c\*d^3 - a\*d^2\*e^2 + (c\*d^2\*e - 3\*a\*e^3)\*x)\*sqrt(-a\*d\*e) - ((c^2\*d^4\*e + 2\*a\*c\*d^2\*e^3 - 3\*a^2\*e^5)\*x^2 + (c^2\*d^5 + 2\*a\*c\*d^3\*e^2)\*x)\*arctan(1/2\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*a\*d\*e))/(sqrt(-a\*d\*e)\*((a\*c\*d^4\*e^2 - a^2\*d^2\*e^4)\*x^2 + (a\*c\*d^5\*e - a^2\*d^3\*e^3)\*x))]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt{(d+ex)(ae+cdx)(d+ex)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(e\*x+d)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2), x)

[Out] Integral(1/(x\*\*2\*sqrt((d + e\*x)\*(a\*e + c\*d\*x))\*(d + e\*x)), x)

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(e\*x + d)^\*x^2), x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.476 \quad \int \frac{1}{x^3(d+ex)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=329

$$\frac{(3cd^2 - 5ae^2)(3ae^2 + cd^2)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{4a^2d^3e^2x(cd^2 - ae^2)} - \frac{3(5a^2e^4 + 2acd^2e^2 + c^2d^4)\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8a^{5/2}d^{7/2}e^{5/2}} - \frac{(cd^2 - 5ae^2)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{2ad^2ex^2(cd^2 - ae^2)} - \frac{2e(ae + cdx)}{dx^2(cd^2 - ae^2)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}$$

[Out]  $(-2*e*(a*e + c*d*x))/(d*(c*d^2 - a*e^2)*x^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((c*d^2 - 5*a*e^2)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*a*d^2*e*(c*d^2 - a*e^2)*x^2) + ((3*c*d^2 - 5*a*e^2)*(c*d^2 + 3*a*e^2)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*a^2*d^3*e^2*(c*d^2 - a*e^2)*x) - (3*(c^2*d^4 + 2*a*c*d^2*e^2 + 5*a^2*e^4)*\text{ArcTanh}[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(8*a^{(5/2)}*d^{(7/2)}*e^{(5/2)})$

**Rubi [A]** time = 1.42798, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{(3cd^2 - 5ae^2)(3ae^2 + cd^2)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{4a^2d^3e^2x(cd^2 - ae^2)} - \frac{3(5a^2e^4 + 2acd^2e^2 + c^2d^4)\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8a^{5/2}d^{7/2}e^{5/2}} - \frac{(cd^2 - 5ae^2)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{2ad^2ex^2(cd^2 - ae^2)} - \frac{2e(ae + cdx)}{dx^2(cd^2 - ae^2)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*(d + e\*x)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]), x]

[Out]  $(-2*e*(a*e + c*d*x))/(d*(c*d^2 - a*e^2)*x^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((c*d^2 - 5*a*e^2)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*a*d^2*e*(c*d^2 - a*e^2)*x^2) + ((3*c*d^2 - 5*a*e^2)*(c*d^2 + 3*a*e^2)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*a^2*d^3*e^2*(c*d^2 - a*e^2)*x) - (3*(c^2*d^4 + 2*a*c*d^2*e^2 + 5*a^2*e^4)*\text{ArcTanh}[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(8*a^{(5/2)}*d^{(7/2)}*e^{(5/2)})$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*3/(e\*x+d)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2), x)

[Out] Timed out

**Mathematica [A]** time = 0.638065, size = 279, normalized size = 0.85

$$\frac{2\sqrt{d}\sqrt{e}(d+ex)(ae+cdx)\left(\frac{3cd^2}{a^2x} + \frac{8e^5}{(d+ex)(ae^2-cd^2)} + \frac{e(7ex-2d)}{ax^2}\right) + \frac{3\log(x)\sqrt{d+ex}(5a^2e^4+2acd^2e^2+c^2d^4)\sqrt{ae+cdx}}{a^{5/2}} - \frac{3\sqrt{d+ex}(5a^2e^4+2acd^2e^2)}{8d^{7/2}e^{5/2}\sqrt{(d+ex)(ae+cdx)}}}{8d^{7/2}e^{5/2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(d+e\*x)\*Sqrt[a\*d\*e+(c\*d^2+a\*e^2)\*x+c\*d\*e\*x^2]),x]

[Out] (2\*Sqrt[d]\*Sqrt[e]\*(a\*e+c\*d\*x)\*(d+e\*x)\*((3\*c\*d^2)/(a^2\*x)+(8\*e^5)/((-c\*d^2)+a\*e^2)\*(d+e\*x))+(e\*(-2\*d+7\*e\*x))/(a\*x^2))+((3\*(c^2\*d^4+2\*a\*c\*d^2\*e^2+5\*a^2\*e^4)\*Sqrt[a\*e+c\*d\*x]\*Sqrt[d+e\*x]\*Log[x])/a^(5/2)-(3\*(c^2\*d^4+2\*a\*c\*d^2\*e^2+5\*a^2\*e^4)\*Sqrt[a\*e+c\*d\*x]\*Sqrt[d+e\*x]\*Log[c\*d^2\*x+2\*Sqrt[a]\*Sqrt[d]\*Sqrt[e]\*Sqrt[a\*e+c\*d\*x]\*Sqrt[d+e\*x]+a\*e\*(2\*d+e\*x)]/a^(5/2))/(8\*d^(7/2)\*e^(5/2)\*Sqrt[(a\*e+c\*d\*x)\*(d+e\*x)])

**Maple [A]** time = 0.024, size = 414, normalized size = 1.3

$$\begin{aligned} & -\frac{1}{2ad^2ex^2}\sqrt{ade+(ae^2+cd^2)x+cdex^2} \\ & +\frac{7}{4d^3ax}\sqrt{ade+(ae^2+cd^2)x+cdex^2}+\frac{3c}{4a^2de^2x}\sqrt{ade+(ae^2+cd^2)x+cdex^2} \\ & -\frac{15e^2}{8d^3}\ln\left(\frac{1}{x}\left(2ade+(ae^2+cd^2)x+2\sqrt{ade}\sqrt{ade+(ae^2+cd^2)x+cdex^2}\right)\right)\frac{1}{\sqrt{ade}} \\ & -\frac{3c}{4ad}\ln\left(\frac{1}{x}\left(2ade+(ae^2+cd^2)x+2\sqrt{ade}\sqrt{ade+(ae^2+cd^2)x+cdex^2}\right)\right)\frac{1}{\sqrt{ade}} \\ & -\frac{3dc^2}{8a^2e^2}\ln\left(\frac{1}{x}\left(2ade+(ae^2+cd^2)x+2\sqrt{ade}\sqrt{ade+(ae^2+cd^2)x+cdex^2}\right)\right)\frac{1}{\sqrt{ade}} \\ & +2\frac{e^2}{d^3(ae^2-cd^2)}\sqrt{cde\left(x+\frac{d}{e}\right)^2+(ae^2-cd^2)\left(x+\frac{d}{e}\right)\left(x+\frac{d}{e}\right)^{-1}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(e\*x+d)/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2),x)

[Out] -1/2/d^2/a/e/x^2\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)+7/4/d^3/a/x\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)+3/4/d/a^2/e^2/x\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)\*c-15/8/d^3\*e^2/(a\*d\*e)^(1/2)\*ln((2\*a\*d\*e+(a\*e^2+c\*d^2)\*x+2\*(a\*d\*e)^(1/2)\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/x)-3/4/d/a/(a\*d\*e)^(1/2)\*ln((2\*a\*d\*e+(a\*e^2+c\*d^2)\*x+2\*(a\*d\*e)^(1/2)\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/x)\*c-3/8\*d/a^2/e^2/(a\*d\*e)^(1/2)\*ln((2\*a\*d\*e+(a\*e^2+c\*d^2)\*x+2\*(a\*d\*e)^(1/2)\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2))/x)\*c^2+2/d^3\*e^2/(a\*e^2-c\*d^2)/(x+d/e)\*(c\*d\*e\*(x+d/e)^2+(a\*e^2-c\*d^2)\*(x+d/e))^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(ex+d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*d\*e\*x^2+a\*d\*e+(c\*d^2+a\*e^2)\*x)\*(e\*x+d)\*x^3),x, algorithm=

[Out] integrate(1/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(e\*x + d)^\*x^3), x)

**Fricas [A]** time = 0.628823, size = 1, normalized size = 0.

$$\frac{4(2acd^4e - 2a^2d^2e^3 - (3c^2d^4e + 4acd^2e^3 - 15a^2e^5)x^2 - (3c^2d^5 + 2acd^3e^2 - 5a^2de^4)x)\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{2(2acd^4e - 2a^2d^2e^3 - (3c^2d^4e + 4acd^2e^3 - 15a^2e^5)x^2 - (3c^2d^5 + 2acd^3e^2 - 5a^2de^4)x)\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}} - \frac{8((a^2cd^5e^3 - a^3d^3e^5)x)}{8((a^2cd^5e^3 - a^3d^3e^5)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(e\*x + d)^\*x^3), x, algorithm="fricas")

[Out] [-1/16\*(4\*(2\*a\*c\*d^4\*e - 2\*a^2\*d^2\*e^3 - (3\*c^2\*d^4\*e + 4\*a\*c\*d^2\*e^3 - 15\*a^2\*e^5)\*x^2 - (3\*c^2\*d^5 + 2\*a\*c\*d^3\*e^2 - 5\*a^2\*d\*e^4)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(a\*d\*e) - 3\*((c^3\*d^6\*e + a\*c^2\*d^4\*e^3 + 3\*a^2\*c\*d^2\*e^5 - 5\*a^3\*e^7)\*x^3 + (c^3\*d^7 + a\*c^2\*d^5\*e^2 + 3\*a^2\*c\*d^3\*e^4 - 5\*a^3\*d\*e^6)\*x^2)\*log(-(4\*(2\*a^2\*d^2\*e^2 + (a\*c\*d^3\*e + a^2\*d\*e^3)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) - (8\*a^2\*d^2\*e^2 + (c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4)\*x^2 + 8\*(a\*c\*d^3\*e + a^2\*d\*e^3)\*x)\*sqrt(a\*d\*e))/x^2))/((a^2\*c\*d^5\*e^3 - a^3\*d^3\*e^5)\*x^3 + (a^2\*c\*d^6\*e^2 - a^3\*d^4\*e^4)\*x^2)\*sqrt(a\*d\*e), -1/8\*(2\*(2\*a\*c\*d^4\*e - 2\*a^2\*d^2\*e^3 - (3\*c^2\*d^4\*e + 4\*a\*c\*d^2\*e^3 - 15\*a^2\*e^5)\*x^2 - (3\*c^2\*d^5 + 2\*a\*c\*d^3\*e^2 - 5\*a^2\*d\*e^4)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e) + 3\*((c^3\*d^6\*e + a\*c^2\*d^4\*e^3 + 3\*a^2\*c\*d^2\*e^5 - 5\*a^3\*e^7)\*x^3 + (c^3\*d^7 + a\*c^2\*d^5\*e^2 + 3\*a^2\*c\*d^3\*e^4 - 5\*a^3\*d\*e^6)\*x^2)\*arctan(1/2\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*a\*d\*e))/((a^2\*c\*d^5\*e^3 - a^3\*d^3\*e^5)\*x^3 + (a^2\*c\*d^6\*e^2 - a^3\*d^4\*e^4)\*x^2)\*sqrt(-a\*d\*e)]]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/(e\*x+d)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2), x)

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(e\*x + d)^\*x^3), x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.477 \quad \int \frac{x^5}{(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx$$

**Optimal.** Leaf size=515

$$\frac{5(3a^2e^4 + 6acd^2e^2 + 7c^2d^4) \tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8c^{7/2}d^{7/2}e^{9/2}} \\ \frac{2x^2(ade(cd^2 - ae^2)(-3a^2e^4 - 12acd^2e^2 + 7c^2d^4) + x(cd^2 - ae^2)(-3a^3e^6 - a^2cd^2e^4 - 11ac^2d^4e^2 + 7c^3d^6))}{3cde^2(cd^2 - ae^2)^4\sqrt{x(ae^2 + cd^2) + ade + cdex^2}} \\ \frac{(-45a^4e^8 + 30a^3cd^2e^6 + 36a^2c^2d^4e^4 - 2cdex(-15a^3e^6 + 9a^2cd^2e^4 - 61ac^2d^4e^2 + 35c^3d^6) - 190ac^3d^6e^2 + 105c^4d^8)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{12c^3d^3e^4(cd^2 - ae^2)^3} \\ \frac{2dx^4(cdx(cd^2 - ae^2) + ae(cd^2 - ae^2))}{3e(cd^2 - ae^2)^2(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}$$

[Out]  $(-2*d*x^4*(a*e*(c*d^2 - a*e^2) + c*d*(c*d^2 - a*e^2)*x))/(3*e*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) - (2*x^2*(a*d*e*(c*d^2 - a*e^2)*(7*c^2*d^4 - 12*a*c*d^2*e^2 - 3*a^2*e^4) + (c*d^2 - a*e^2)*(7*c^3*d^6 - 11*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3*a^3*e^6)*x))/(3*c*d*e^2*(c*d^2 - a*e^2)^4*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((105*c^4*d^8 - 190*a*c^3*d^6*e^2 + 36*a^2*c^2*d^4*e^4 + 30*a^3*c*d^2*e^6 - 45*a^4*e^8 - 2*c*d*e*(35*c^3*d^6 - 61*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 15*a^3*e^6)*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*c^3*d^3*e^4*(c*d^2 - a*e^2)^3) + (5*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*\text{ArcTanh}[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(8*c^{(7/2)}*d^{(7/2)}*e^{(9/2)})$

**Rubi [A]** time = 1.6607, antiderivative size = 515, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{5(3a^2e^4 + 6acd^2e^2 + 7c^2d^4) \tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8c^{7/2}d^{7/2}e^{9/2}} \\ \frac{2x^2(ade(cd^2 - ae^2)(-3a^2e^4 - 12acd^2e^2 + 7c^2d^4) + x(cd^2 - ae^2)(-3a^3e^6 - a^2cd^2e^4 - 11ac^2d^4e^2 + 7c^3d^6))}{3cde^2(cd^2 - ae^2)^4\sqrt{x(ae^2 + cd^2) + ade + cdex^2}} \\ \frac{(-45a^4e^8 + 30a^3cd^2e^6 + 36a^2c^2d^4e^4 - 2cdex(-15a^3e^6 + 9a^2cd^2e^4 - 61ac^2d^4e^2 + 35c^3d^6) - 190ac^3d^6e^2 + 105c^4d^8)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{12c^3d^3e^4(cd^2 - ae^2)^3} \\ \frac{2dx^4(ae + cdx)}{3e(cd^2 - ae^2)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[x^5/((d + e\*x)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)), x]

[Out]  $(-2*d*x^4*(a*e + c*d*x))/(3*e*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) - (2*x^2*(a*d*e*(c*d^2 - a*e^2)*(7*c^2*d^4 - 12*a*c*d^2*e^2 - 3*a^2*e^4) + (c*d^2 - a*e^2)*(7*c^3*d^6 - 11*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3*a^3*e^6)*x))/(3*c*d*e^2*(c*d^2 - a*e^2)^4*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((105*c^4*d^8 - 190*a*c^3*d^6*e^2 + 36*a^2*c^2*d^4*e^4 + 30*a^3*c*d^2*e^6 - 45*a^4*e^8 - 2*c*d*e*(35*c^3*d^6 - 61*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 15*a^3*e^6)*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*c^3*d^3*e^4*(c*d^2 - a*e^2)^3) + (5*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*\text{ArcTanh}[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(8*c^{(7/2)}*d^{(7/2)}*e^{(9/2)})$



**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**5/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)`

[Out] Timed out

**Mathematica [A]** time = 1.08267, size = 296, normalized size = 0.57

$$\frac{2(d+ex)^2(ae+cdx)^2 \left( \frac{24a^5e^9}{c^3(cd^2-ae^2)^3(ae+cdx)} - \frac{3(7ae^2+11cd^2)}{c^3} + \frac{8d^8}{(d+ex)^2(cd^2-ae^2)^2} + \frac{40(3ad^7e^2-2cd^9)}{(d+ex)(cd^2-ae^2)^3} + \frac{6dex}{c^2} \right) + \frac{5(d+ex)^{3/2}(3a^2e^4+6acd^2e^2+7c^2d^4)(ae+cdx)^{3/2}}{c^{7/2}d^7}}{8((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

[In] `Integrate[x^5/((d+e*x)*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^(3/2)),x]`

[Out]  $((2*(a*e + c*d*x)^2*(d + e*x)^2*((-3*(11*c*d^2 + 7*a*e^2))/c^3 + (6*d*e*x)/c^2 + (24*a^5*e^9)/(c^3*(c*d^2 - a*e^2)^3*(a*e + c*d*x)) + (8*d^8)/((c*d^2 - a*e^2)^2*(d + e*x)^2) + (40*(-2*c*d^9 + 3*a*d^7*e^2))/((c*d^2 - a*e^2)^3*(d + e*x)))/(3*d^3*e^4) + (5*(7*c^2*d^4 + 6*a*c*d^2*e^2 + 3*a^2*e^4)*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*Log[a*e^2 + 2*sqrt[c]*sqrt[d]*sqrt[e]*sqrt[a*e + c*d*x]*sqrt[d + e*x] + c*d*(d + 2*e*x)]/(c^(7/2)*d^(7/2)*e^(9/2)))/(8*((a*e + c*d*x)*(d + e*x))^(3/2))$

**Maple [B]** time = 0.035, size = 1680, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^5/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)`

[Out]  $-1/4/c*d/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^2+11/2/e^2*d^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a+15/4/e^2/c^2/d*\ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)*a-15/4/e^2/c^2/d*x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a+51/8/e^4*c*d^5/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+15/16*e^5/c^4/d^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^5+35/16*e^3/c^3/d^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^4+21/8/e/c*d^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2-5/4/e/c^2/d^2*x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a-16/3*d^7/e^4*c^2/(a*e^2-c*d^2)^3/(c*d*e*(x+d/e))^2+(a*e^2-c*d^2)*(x+d/e)^(1/2)*x-8/3*d^6/e^3*c/(a*e^2-c*d^2)^3/(c*d*e*(x+d/e))^2+(a*e^2-c*d^2)*(x+d/e)^(1/2)*a+51/16/e^5/c*d^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-7/16/e^3/c^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a-9/4/e^3/c*x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+15/8*e^4/c^3/d^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^4+5/2*e^2/c^2/d/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*a^3+95/16/e^3*d^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a+51/16/e^5*c*d^6/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)$

$$2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)}+35/8/e^4/c*d*\ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^{(1/2)}+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)})/(c*d*e)^{(1/2)}+15/16*e/c^4/d^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)}*a^3-35/8/e^4/c*d*x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)}+1/2/e^2*x^3/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)}+5/16/e/c^3/d^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)}*a^2-15/8/c^3/d^3*x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)}*a^2+2*d^4/e^5*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)}+2/3*d^5/e^6/(a*e^2-c*d^2)/(x+d/e)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{(1/2)}-8/3*d^8/e^5*c^2/(a*e^2-c*d^2)^3/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{(1/2)}+15/8/c^3/d^3*\ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^{(1/2)}+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)})/(c*d*e)^{(1/2)}*a^2+9/8*e/c^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)}*a^3$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(e\*x + d)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 3.47214, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(e\*x + d)),x, algorithm="fricas")

[Out] [-1/48\*(4\*(105\*a\*c^4\*d^10\*e - 190\*a^2\*c^3\*d^8\*e^3 + 36\*a^3\*c^2\*d^6\*e^5 + 30\*a^4\*c\*d^4\*e^7 - 45\*a^5\*d^2\*e^9 - 6\*(c^5\*d^8\*e^3 - 3\*a\*c^4\*d^6\*e^5 + 3\*a^2\*c^3\*d^4\*e^7 - a^3\*c^2\*d^2\*e^9)\*x^4 + 3\*(7\*c^5\*d^9\*e^2 - 16\*a\*c^4\*d^7\*e^4 + 6\*a^2\*c^3\*d^5\*e^6 + 8\*a^3\*c^2\*d^3\*e^8 - 5\*a^4\*c\*d\*e^10)\*x^3 + (140\*c^5\*d^10\*e - 237\*a\*c^4\*d^8\*e^3 + 12\*a^2\*c^3\*d^6\*e^5 + 66\*a^3\*c^2\*d^4\*e^7 - 45\*a^5\*e^11)\*x^2 + (105\*c^5\*d^11 - 50\*a\*c^4\*d^9\*e^2 - 222\*a^2\*c^3\*d^7\*e^4 + 84\*a^3\*c^2\*d^5\*e^6 + 45\*a^4\*c\*d^3\*e^8 - 90\*a^5\*d\*e^10)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*e) - 15\*(7\*a\*c^5\*d^12\*e - 15\*a^2\*c^4\*d^10\*e^3 + 6\*a^3\*c^3\*d^8\*e^5 + 2\*a^4\*c^2\*d^6\*e^7 + 3\*a^5\*c\*d^4\*e^9 - 3\*a^6\*d^2\*e^11 + (7\*c^6\*d^11\*e^2 - 15\*a\*c^5\*d^9\*e^4 + 6\*a^2\*c^4\*d^7\*e^6 + 2\*a^3\*c^3\*d^5\*e^8 + 3\*a^4\*c^2\*d^3\*e^10 - 3\*a^5\*c\*d\*e^12)\*x^3 + (14\*c^6\*d^12\*e - 23\*a\*c^5\*d^10\*e^3 - 3\*a^2\*c^4\*d^8\*e^5 + 10\*a^3\*c^3\*d^6\*e^7 + 8\*a^4\*c^2\*d^4\*e^9 - 3\*a^5\*c\*d^2\*e^11 - 3\*a^6\*e^13)\*x^2 + (7\*c^6\*d^13 - a\*c^5\*d^11\*e^2 - 24\*a^2\*c^4\*d^9\*e^4 + 14\*a^3\*c^3\*d^7\*e^6 + 7\*a^4\*c^2\*d^5\*e^8 + 3\*a^5\*c\*d^3\*e^10 - 6\*a^6\*d\*e^12)\*x)\*log(4\*(2\*c^2\*d^2\*e^2\*x + c^2\*d^3\*e + a\*c\*d\*e^3)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) + (8\*c^2\*d^2\*e^2\*x^2 + c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4 + 8\*(c^2\*d^3\*e + a\*c\*d\*e^3)\*x)\*sqrt(c\*d\*e))/((a\*c^6\*d^11\*e^5 - 3\*a^2\*c^5\*d^9\*e^7 + 3\*a^3\*c^4\*d^7\*e^9 - a^4\*c^3\*d^5\*e^11 + (c^7\*d^10\*e^6 - 3\*a\*c^6\*d^8\*e^8 + 3\*a^2\*c^5\*d^6\*e^10 - a^3\*c^4\*d^4\*e^12)\*x^3 + (2\*c^7\*d^11\*e^5 - 5\*a\*c^6\*d^9\*e^7 + 3\*a^2\*c^5\*d^7\*e^9 + a^3\*c^4\*d^5\*e^11 - a^4\*c^3\*d^3\*e^13)\*x^2 + (c^7\*d^12\*e^4 - a\*c^6\*d^10\*e^6 - 3\*a^2\*c^5\*d^8\*e^8 + 5\*a^3\*c^4\*d^6\*e^10 - 2\*a^4\*c^3\*d^4\*e^12)\*x)\*sqrt(c\*d\*e)), -1/24\*(2\*(105\*a\*c^4\*d^10\*e - 190\*a^2\*c^3\*d^8\*e^3 + 36\*a^3\*c^2\*d^6\*e^5 + 30\*a^4\*c\*d^4\*e^7 - 45\*a^5\*d^2\*e^9 - 6\*(c^5\*d^8\*e^3 - 3\*a\*c^4\*d^6\*e^5 + 3\*a^2\*c^3\*d^4\*e^7 - a^3\*c^2\*d^2\*e^9)\*x^4 + 3\*(7\*c^5\*d^9\*e^2 - 16\*a\*c^4\*d^7\*e^4 + 6\*a^2\*c^3\*d^5\*e^6 + 8\*a^3\*c^2\*d^3\*e^8 -

$$\begin{aligned}
& 5*a^4*c*d*e^{10})*x^3 + (140*c^5*d^{10}*e - 237*a*c^4*d^8*e^3 + 12*a \\
& ^2*c^3*d^6*e^5 + 66*a^3*c^2*d^4*e^7 - 45*a^5*e^{11})*x^2 + (105*c^5 \\
& *d^{11} - 50*a*c^4*d^9*e^2 - 222*a^2*c^3*d^7*e^4 + 84*a^3*c^2*d^5*e \\
& ^6 + 45*a^4*c*d^3*e^8 - 90*a^5*d*e^{10})*x)*sqrt(c*d*e*x^2 + a*d*e \\
& + (c*d^2 + a*e^2)*x)*sqrt(-c*d*e) - 15*(7*a*c^5*d^{12}*e - 15*a^2*c \\
& ^4*d^{10}*e^3 + 6*a^3*c^3*d^8*e^5 + 2*a^4*c^2*d^6*e^7 + 3*a^5*c*d^4 \\
& *e^9 - 3*a^6*d^2*e^{11} + (7*c^6*d^{11}*e^2 - 15*a*c^5*d^9*e^4 + 6*a^ \\
& 2*c^4*d^7*e^6 + 2*a^3*c^3*d^5*e^8 + 3*a^4*c^2*d^3*e^{10} - 3*a^5*c* \\
& d*e^{12})*x^3 + (14*c^6*d^{12}*e - 23*a*c^5*d^{10}*e^3 - 3*a^2*c^4*d^8* \\
& e^5 + 10*a^3*c^3*d^6*e^7 + 8*a^4*c^2*d^4*e^9 - 3*a^5*c*d^2*e^{11} - \\
& 3*a^6*e^{13})*x^2 + (7*c^6*d^{13} - a*c^5*d^{11}*e^2 - 24*a^2*c^4*d^9* \\
& e^4 + 14*a^3*c^3*d^7*e^6 + 7*a^4*c^2*d^5*e^8 + 3*a^5*c*d^3*e^{10} - \\
& 6*a^6*d*e^{12})*x)*arctan(1/2*(2*c*d*e*x + c*d^2 + a*e^2)*sqrt(-c* \\
& d*e)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*c*d*e)))/((a*c^ \\
& 6*d^{11}*e^5 - 3*a^2*c^5*d^9*e^7 + 3*a^3*c^4*d^7*e^9 - a^4*c^3*d^5* \\
& e^{11} + (c^7*d^{10}*e^6 - 3*a*c^6*d^8*e^8 + 3*a^2*c^5*d^6*e^{10} - a^3 \\
& *c^4*d^4*e^{12})*x^3 + (2*c^7*d^{11}*e^5 - 5*a*c^6*d^9*e^7 + 3*a^2*c^ \\
& 5*d^7*e^9 + a^3*c^4*d^5*e^{11} - a^4*c^3*d^3*e^{13})*x^2 + (c^7*d^{12}* \\
& e^4 - a*c^6*d^{10}*e^6 - 3*a^2*c^5*d^8*e^8 + 5*a^3*c^4*d^6*e^{10} - 2 \\
& *a^4*c^3*d^4*e^{12})*x)*sqrt(-c*d*e))
\end{aligned}$$


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**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^5}{((d+ex)(ae+cdx))^{\frac{3}{2}}(d+ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*5/(e\*x+d)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2),x)

[Out] Integral(x\*\*5/(((d + e\*x)\*(a\*e + c\*d\*x))\*\*(3/2)\*(d + e\*x)), x)

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(e\*x + d)),x, algorithm=

[Out] [undef, undef, undef, undef, undef, 1]

$$3.478 \quad \int \frac{x^4}{(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx$$

**Optimal.** Leaf size=438

$$\frac{2x(ade(cd^2 - ae^2)(-3a^2e^4 - 10acd^2e^2 + 5c^2d^4) + x(cd^2 - ae^2)(-3a^3e^6 - a^2cd^2e^4 - 9ac^2d^4e^2 + 5c^3d^6))}{3cde^2(cd^2 - ae^2)^4 \sqrt{x(ae^2 + cd^2) + ade + cdex^2}} + \frac{(-9a^3e^6 + 9a^2cd^2e^4 - 31ac^2d^4e^2 + 15c^3d^6) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{3c^2d^2e^3(cd^2 - ae^2)^3} - \frac{(3ae^2 + 5cd^2) \tanh^{-1}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{2c^{5/2}d^{5/2}e^{7/2}} - \frac{2dx^3(cd^2 - ae^2) + ae(cd^2 - ae^2)}{3e(cd^2 - ae^2)^2(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}$$

[Out]  $(-2*d*x^3*(a*e*(c*d^2 - a*e^2) + c*d*(c*d^2 - a*e^2)*x))/(3*e*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) - (2*x*(a*d*e*(c*d^2 - a*e^2)*(5*c^2*d^4 - 10*a*c*d^2*e^2 - 3*a^2*e^4) + (c*d^2 - a*e^2)*(5*c^3*d^6 - 9*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3*a^3*e^6)*x))/(3*c*d*e^2*(c*d^2 - a*e^2)^4*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + ((15*c^3*d^6 - 31*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 9*a^3*e^6)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^2*d^2*e^3*(c*d^2 - a*e^2)^3) - ((5*c*d^2 + 3*a*e^2)*\text{ArcTanh}[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(2*c^{(5/2)}*d^{(5/2)}*e^{(7/2)})$

**Rubi [A]** time = 1.46161, antiderivative size = 438, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{2x(ade(cd^2 - ae^2)(-3a^2e^4 - 10acd^2e^2 + 5c^2d^4) + x(cd^2 - ae^2)(-3a^3e^6 - a^2cd^2e^4 - 9ac^2d^4e^2 + 5c^3d^6))}{3cde^2(cd^2 - ae^2)^4 \sqrt{x(ae^2 + cd^2) + ade + cdex^2}} + \frac{(-9a^3e^6 + 9a^2cd^2e^4 - 31ac^2d^4e^2 + 15c^3d^6) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{3c^2d^2e^3(cd^2 - ae^2)^3} - \frac{(3ae^2 + 5cd^2) \tanh^{-1}\left(\frac{ae^2 + cd^2 + 2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}\right)}{2c^{5/2}d^{5/2}e^{7/2}} - \frac{2dx^3(ae + cdx)}{3e(cd^2 - ae^2)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^4/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}), x]$

[Out]  $(-2*d*x^3*(a*e + c*d*x))/(3*e*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) - (2*x*(a*d*e*(c*d^2 - a*e^2)*(5*c^2*d^4 - 10*a*c*d^2*e^2 - 3*a^2*e^4) + (c*d^2 - a*e^2)*(5*c^3*d^6 - 9*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3*a^3*e^6)*x))/(3*c*d*e^2*(c*d^2 - a*e^2)^4*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + ((15*c^3*d^6 - 31*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4 - 9*a^3*e^6)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^2*d^2*e^3*(c*d^2 - a*e^2)^3) - ((5*c*d^2 + 3*a*e^2)*\text{ArcTanh}[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(2*c^{(5/2)}*d^{(5/2)}*e^{(7/2)})$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**4/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)`

[Out] Timed out

**Mathematica [A]** time = 1.16008, size = 261, normalized size = 0.6

$$\frac{2(d+ex)^2(ae+cdx)^2 \left( -\frac{6a^4e^7}{c^2(cd^2-ae^2)^3(ae+cdx)} - \frac{2d^6}{(d+ex)^2(cd^2-ae^2)^2} + \frac{2d^5(7cd^2-12ae^2)}{(d+ex)(cd^2-ae^2)^3} + \frac{3}{c^2} \right)}{3d^2e^3} - \frac{(d+ex)^{3/2}(3ae^2+5cd^2)(ae+cdx)^{3/2} \log\left(2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}\right)}{c^{5/2}d^{5/2}e^{7/2}}$$

$$2((d+ex)(ae+cdx))^{3/2}$$

Antiderivative was successfully verified.

[In] `Integrate[x^4/((d+e*x)*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^(3/2)),x]`

[Out]  $((2*(a*e + c*d*x)^2*(d + e*x)^2*(3/c^2 - (6*a^4*e^7)/(c^2*(c*d^2 - a*e^2)^3*(a*e + c*d*x)) - (2*d^6)/((c*d^2 - a*e^2)^2*(d + e*x)^2) + (2*d^5*(7*c*d^2 - 12*a*e^2))/((c*d^2 - a*e^2)^3*(d + e*x)))/(3*d^2*e^3) - ((5*c*d^2 + 3*a*e^2)*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*\text{Log}[a*e^2 + 2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x] + c*d*(d + 2*e*x)]/(c^(5/2)*d^(5/2)*e^(7/2)))/(2*((a*e + c*d*x)*(d + e*x))^(3/2))$

**Maple [B]** time = 0.018, size = 1266, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)`

[Out]  $-3/2*e^3/c^2/d^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x^3+5/2/e^3/c*x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/4/c^3/d^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2-9/4/e^4/c*d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-5/2/e^3/c*\text{ln}((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)+16/3*d^6/e^3*c^2/(a*e^2-c*d^2)^3/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+8/3*d^5/e^2*c/(a*e^2-c*d^2)^3/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)*a+3/2/e/c^2/d^2*x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a-3/2*e/c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x^2-9/2/e*d^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x^2-9/2/e^3*c*d^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x-3/4*e^4/c^3/d^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^4-3/2*e^2/c^2/d/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^3-3/2/e/c^2/d^2*\text{ln}((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^(1/2)+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/(c*d*e)^(1/2)*a-2/3*d^4/e^5/(a*e^2-c*d^2)/(x+d/e)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+8/3*d^7/e^4*c^2/(a*e^2-c*d^2)^3/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)-2*d^3/e^4*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+1/e^2*x^2/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/c*d/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a^2-9/2/e^2*d^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*a-9/4/e^4*c*d^5/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(e\*x + d)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.6216, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(e\*x + d)),x, algorithm="fricas")

[Out] [1/12\*(4\*(15\*a\*c^3\*d^8\*e - 31\*a^2\*c^2\*d^6\*e^3 + 9\*a^3\*c\*d^4\*e^5 - 9\*a^4\*d^2\*e^7 + 3\*(c^4\*d^7\*e^2 - 3\*a\*c^3\*d^5\*e^4 + 3\*a^2\*c^2\*d^3\*e^6 - a^3\*c\*d\*e^8)\*x^3 + (20\*c^4\*d^8\*e - 39\*a\*c^3\*d^6\*e^3 + 9\*a^2\*c^2\*d^4\*e^5 + 3\*a^3\*c\*d^2\*e^7 - 9\*a^4\*e^9)\*x^2 + (15\*c^4\*d^9 - 11\*a\*c^3\*d^7\*e^2 - 33\*a^2\*c^2\*d^5\*e^4 + 15\*a^3\*c\*d^3\*e^6 - 18\*a^4\*d\*e^8)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*e) + 3\*(5\*a\*c^4\*d^10\*e - 12\*a^2\*c^3\*d^8\*e^3 + 6\*a^3\*c^2\*d^6\*e^5 + 4\*a^4\*c\*d^4\*e^7 - 3\*a^5\*d^2\*e^9 + (5\*c^5\*d^9\*e^2 - 12\*a\*c^4\*d^7\*e^4 + 6\*a^2\*c^3\*d^5\*e^6 + 4\*a^3\*c^2\*d^3\*e^8 - 3\*a^4\*c\*d\*e^10)\*x^3 + (10\*c^5\*d^10\*e - 19\*a\*c^4\*d^8\*e^3 + 14\*a^3\*c^2\*d^4\*e^7 - 2\*a^4\*c\*d^2\*e^9 - 3\*a^5\*e^11)\*x^2 + (5\*c^5\*d^11 - 2\*a\*c^4\*d^9\*e^2 - 18\*a^2\*c^3\*d^7\*e^4 + 16\*a^3\*c^2\*d^5\*e^6 + 5\*a^4\*c\*d^3\*e^8 - 6\*a^5\*d\*e^10)\*x)\*log(-4\*(2\*c^2\*d^2\*e^2\*x + c^2\*d^3\*e + a\*c\*d\*e^3)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) + (8\*c^2\*d^2\*e^2\*x^2 + c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4 + 8\*(c^2\*d^3\*e + a\*c\*d\*e^3)\*x)\*sqrt(c\*d\*e))/((a\*c^5\*d^10\*e^4 - 3\*a^2\*c^4\*d^8\*e^6 + 3\*a^3\*c^3\*d^6\*e^8 - a^4\*c^2\*d^4\*e^10 + (c^6\*d^9\*e^5 - 3\*a\*c^5\*d^7\*e^7 + 3\*a^2\*c^4\*d^5\*e^9 - a^3\*c^3\*d^3\*e^11)\*x^3 + (2\*c^6\*d^10\*e^4 - 5\*a\*c^5\*d^8\*e^6 + 3\*a^2\*c^4\*d^6\*e^8 + a^3\*c^3\*d^4\*e^10 - a^4\*c^2\*d^2\*e^12)\*x^2 + (c^6\*d^11\*e^3 - a\*c^5\*d^9\*e^5 - 3\*a^2\*c^4\*d^7\*e^7 + 5\*a^3\*c^3\*d^5\*e^9 - 2\*a^4\*c^2\*d^3\*e^11)\*x)\*sqrt(c\*d\*e)), 1/6\*(2\*(15\*a\*c^3\*d^8\*e - 31\*a^2\*c^2\*d^6\*e^3 + 9\*a^3\*c\*d^4\*e^5 - 9\*a^4\*d^2\*e^7 + 3\*(c^4\*d^7\*e^2 - 3\*a\*c^3\*d^5\*e^4 + 3\*a^2\*c^2\*d^3\*e^6 - a^3\*c\*d\*e^8)\*x^3 + (20\*c^4\*d^8\*e - 39\*a\*c^3\*d^6\*e^3 + 9\*a^2\*c^2\*d^4\*e^5 + 3\*a^3\*c\*d^2\*e^7 - 9\*a^4\*e^9)\*x^2 + (15\*c^4\*d^9 - 11\*a\*c^3\*d^7\*e^2 - 33\*a^2\*c^2\*d^5\*e^4 + 15\*a^3\*c\*d^3\*e^6 - 18\*a^4\*d\*e^8)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*e) - 3\*(5\*a\*c^4\*d^10\*e - 12\*a^2\*c^3\*d^8\*e^3 + 6\*a^3\*c^2\*d^6\*e^5 + 4\*a^4\*c\*d^4\*e^7 - 3\*a^5\*d^2\*e^9 + (5\*c^5\*d^9\*e^2 - 12\*a\*c^4\*d^7\*e^4 + 6\*a^2\*c^3\*d^5\*e^6 + 4\*a^3\*c^2\*d^3\*e^8 - 3\*a^4\*c\*d\*e^10)\*x^3 + (10\*c^5\*d^10\*e - 19\*a\*c^4\*d^8\*e^3 + 14\*a^3\*c^2\*d^4\*e^7 - 2\*a^4\*c\*d^2\*e^9 - 3\*a^5\*e^11)\*x^2 + (5\*c^5\*d^11 - 2\*a\*c^4\*d^9\*e^2 - 18\*a^2\*c^3\*d^7\*e^4 + 16\*a^3\*c^2\*d^5\*e^6 + 5\*a^4\*c\*d^3\*e^8 - 6\*a^5\*d\*e^10)\*x)\*arctan(1/2\*(2\*c\*d\*e\*x + c\*d^2 + a\*e^2)\*sqrt(-c\*d\*e)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*c\*d\*e))/((a\*c^5\*d^10\*e^4 - 3\*a^2\*c^4\*d^8\*e^6 + 3\*a^3\*c^3\*d^6\*e^8 - a^4\*c^2\*d^4\*e^10 + (c^6\*d^9\*e^5 - 3\*a\*c^5\*d^7\*e^7 + 3\*a^2\*c^4\*d^5\*e^9 - a^3\*c^3\*d^3\*e^11)\*x^3 + (2\*c^6\*d^10\*e^4 - 5\*a\*c^5\*d^8\*e^6 + 3\*a^2\*c^4\*d^6\*e^8 + a^3\*c^3\*d^4\*e^10 - a^4\*c^2\*d^2\*e^12)\*x^2 + (c^6\*d^11\*e^3 - a\*c^5\*d^9\*e^5 - 3\*a^2\*c^4\*d^7\*e^7 + 5\*a^3\*c^3\*d^5\*e^9 - 2\*a^4\*c^2\*d^3\*e^11)\*x)\*sqrt(-c\*d\*e))]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^4}{((d + ex)(ae + cdx))^{\frac{3}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**4/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)`

[Out] `Integral(x**4/(((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

`[undef, undef, undef, undef, 1]`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^4/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)),x, algorithm="giac")`

[Out] `[undef, undef, undef, undef, 1]`

$$3.479 \quad \int \frac{x^3}{(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx$$

**Optimal.** Leaf size=297

$$\frac{2(x(-3a^3e^6 - a^2cd^2e^4 - 7ac^2d^4e^2 + 3c^3d^6) + ade(cd^2 - 3ae^2)(ae^2 + 3cd^2))}{3cde^2(cd^2 - ae^2)^3\sqrt{x(ae^2 + cd^2) + ade + cdex^2}} + \frac{\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{c^{3/2}d^{3/2}e^{5/2}} - \frac{2dx^2(cd^2 - ae^2) + ae(cd^2 - ae^2)}{3e(cd^2 - ae^2)^2(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}$$

[Out]  $(-2*d*x^2*(a*e*(c*d^2 - a*e^2) + c*d*(c*d^2 - a*e^2)*x))/(3*e*(c*d^2 - a*e^2)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) - (2*(a*d*e*(c*d^2 - 3*a*e^2)*(3*c*d^2 + a*e^2) + (3*c^3*d^6 - 7*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3*a^3*e^6)*x))/(3*c*d^2*e^2*(c*d^2 - a*e^2)^3*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + \text{ArcTanh}[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(c^{(3/2)}*d^{(3/2)}*e^{(5/2)})$

**Rubi [A]** time = 0.897572, antiderivative size = 297, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{2(x(-3a^3e^6 - a^2cd^2e^4 - 7ac^2d^4e^2 + 3c^3d^6) + ade(cd^2 - 3ae^2)(ae^2 + 3cd^2))}{3cde^2(cd^2 - ae^2)^3\sqrt{x(ae^2 + cd^2) + ade + cdex^2}} + \frac{\tanh^{-1}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{c^{3/2}d^{3/2}e^{5/2}} - \frac{2dx^2(ae + cd^2)}{3e(cd^2 - ae^2)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}), x]$

[Out]  $(-2*d*x^2*(a*e + c*d*x))/(3*e*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) - (2*(a*d*e*(c*d^2 - 3*a*e^2)*(3*c*d^2 + a*e^2) + (3*c^3*d^6 - 7*a*c^2*d^4*e^2 - a^2*c*d^2*e^4 - 3*a^3*e^6)*x))/(3*c*d^2*e^2*(c*d^2 - a*e^2)^3*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + \text{ArcTanh}[(c*d^2 + a*e^2 + 2*c*d*e*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(c^{(3/2)}*d^{(3/2)}*e^{(5/2)})$

**Rubi in Sympy [A]** time = 127.145, size = 471, normalized size = 1.59

$$\frac{4cd^4(2ae^2 + 2cd^2 + 4cdex)}{3e^3(ae^2 - cd^2)^3\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}} + \frac{2d^3}{3e^3(d + ex)(ae^2 - cd^2)\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}} - \frac{d^2(2ae^2 + 2cd^2 + 4cdex)}{e^3(ae^2 - cd^2)^2\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}} + \frac{2x(2ade + x(ae^2 + cd^2))}{e^2(ae^2 - cd^2)^2\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}} - \frac{d(4ade + x(2ae^2 + 2cd^2))}{e^2(ae^2 - cd^2)^2\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}} + \frac{2(ae^2 + cd^2)\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{cde^2(ae^2 - cd^2)^2} + \frac{\text{atanh}\left(\frac{ae^2+cd^2+2cdex}{2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}\right)}{c^{3/2}d^{3/2}e^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `rubi_integrate(x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)`

[Out] 
$$\begin{aligned} & -4*c*d**4*(2*a*e**2 + 2*c*d**2 + 4*c*d*e*x)/(3*e**3*(a*e**2 - c*d**2)**3*\sqrt{a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)}) + 2*d**3/ \\ & (3*e**3*(d + e*x)*(a*e**2 - c*d**2)*\sqrt{a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)}) - d**2*(2*a*e**2 + 2*c*d**2 + 4*c*d*e*x)/(e**3 \\ & *(a*e**2 - c*d**2)**2*\sqrt{a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)}) - d*(4*a*d*e + x*(2*a*e**2 + 2*c*d**2))/(e**2*(a*e**2 - c*d**2) \\ & **2*\sqrt{a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)}) + 2*x*(2*a*d*e + x*(a*e**2 + c*d**2))/(e*(a*e**2 - c*d**2)**2*\sqrt{a*d*e + \\ & c*d*e*x**2 + x*(a*e**2 + c*d**2)}) - 2*(a*e**2 + c*d**2)*\sqrt{a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)}/(c*d*e**2*(a*e**2 - c*d**2) \\ & **2) + \operatorname{atanh}((a*e**2 + c*d**2 + 2*c*d*e*x)/(2*\sqrt{c}*\sqrt{d}*\sqrt{e}*\sqrt{a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)})))/(c**(3/2)* \\ & d**(3/2)*e**(5/2)) \end{aligned}$$

**Mathematica [A]** time = 0.597055, size = 219, normalized size = 0.74

$$\frac{2(ae+cdx)(3a^3e^5(d+ex)^2+cd^4(cd^2-ae^2)(ae+cdx)-cd^3(d+ex)(4cd^2-9ae^2)(ae+cdx))}{3cde^2(cd^2-ae^2)^3} + \frac{(d+ex)^{3/2}(ae+cdx)^{3/2} \log\left(2\sqrt{c}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx+ae^2+cd(d+ex)}\right)}{c^{3/2}d^{3/2}e^{5/2}}$$

$$((d+ex)(ae+cdx))^{3/2}$$

Antiderivative was successfully verified.

[In] `Integrate[x^3/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]`

[Out] 
$$\begin{aligned} & ((2*(a*e + c*d*x)*(c*d^4*(c*d^2 - a*e^2)*(a*e + c*d*x) - c*d^3*(4 \\ & *c*d^2 - 9*a*e^2)*(a*e + c*d*x)*(d + e*x) + 3*a^3*e^5*(d + e*x)^2 \\ & ))/(3*c*d*e^2*(c*d^2 - a*e^2)^3) + ((a*e + c*d*x)^(3/2)*(d + e*x) \\ & ^{(3/2)*\operatorname{Log}[a*e^2 + 2*\sqrt{c}*\sqrt{d}*\sqrt{e}*\sqrt{a*e + c*d*x}]*\sqrt{d + e*x} + c*d*(d + 2*e*x)}}/(c^{(3/2)*d^{(3/2)*e^{(5/2)}}}))/((a*e \\ & + c*d*x)*(d + e*x))^{(3/2)} \end{aligned}$$

**Maple [B]** time = 0.017, size = 977, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)`

[Out] 
$$\begin{aligned} & 2*d^2/e^3*(2*c*d*e*x+a*e^2+c*d^2)/(4*a*c*d^2*e^2-(a*e^2+c*d^2)^2) \\ & / (a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)} - 1/e^2*x/c/d/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)} + 1/2/e/c^2/d^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)} * a^{3/2}/e^3/c/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)} + e^2/c/d/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)} * x^a+4*d/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)} * x^a+3/e^2*c*d^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)} * x+1/2*e^3/c^2/d^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)} * a^3+5/2*e/c/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)} * a^2+7/2/e*d^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)} * a+3/2/e^3*c*d^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)} + 1/e^2/c/d*\ln((1/2*a*e^2+1/2*c*d^2+c*d*e*x)/(c*d*e)^{(1/2)}+(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)})/(c*d*e)^{(1/2)}+2/3*d^3/e^4/(a*e^2-c*d^2)/(x+d/e)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{(1/2)}-16/3*d^5/e^2*c^2/(a*e^2-c*d^2)^3/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{(1/2)}*x-8/3*d^4/e*c/(a*e^2-c*d^2)^3/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{(1/2)}*a-8/3*d^6/e^3*c^2/(a*e^2-c*d^2)^3/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{(1/2)} \end{aligned}$$

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**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(e\*x + d)),x, algorithm="maxima")

[Out] Exception raised: ValueError

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**Fricas [A]** time = 0.894304, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(e\*x + d)),x, algorithm="fricas")

[Out] [-1/6\*(4\*(3\*a\*c^2\*d^6\*e - 8\*a^2\*c\*d^4\*e^3 - 3\*a^3\*d^2\*e^5 + (4\*c^3\*d^6\*e - 9\*a\*c^2\*d^4\*e^3 - 3\*a^3\*e^7)\*x^2 + (3\*c^3\*d^7 - 4\*a\*c^2\*d^5\*e^2 - 9\*a^2\*c\*d^3\*e^4 - 6\*a^3\*d\*e^6)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*e) - 3\*(a\*c^3\*d^8\*e - 3\*a^2\*c^2\*d^6\*e^3 + 3\*a^3\*c\*d^4\*e^5 - a^4\*d^2\*e^7 + (c^4\*d^7\*e^2 - 3\*a\*c^3\*d^5\*e^4 + 3\*a^2\*c^2\*d^3\*e^6 - a^3\*c\*d\*e^8)\*x^3 + (2\*c^4\*d^8\*e - 5\*a\*c^3\*d^6\*e^3 + 3\*a^2\*c^2\*d^4\*e^5 + a^3\*c\*d^2\*e^7 - a^4\*e^9)\*x^2 + (c^4\*d^9 - a\*c^3\*d^7\*e^2 - 3\*a^2\*c^2\*d^5\*e^4 + 5\*a^3\*c\*d^3\*e^6 - 2\*a^4\*d\*e^8)\*x)\*log(4\*(2\*c^2\*d^2\*e^2\*x + c^2\*d^3\*e + a\*c\*d\*e^3)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) + (8\*c^2\*d^2\*e^2\*x^2 + c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4 + 8\*(c^2\*d^3\*e + a\*c\*d\*e^3)\*x)\*sqrt(c\*d\*e))/((a\*c^4\*d^9\*e^3 - 3\*a^2\*c^3\*d^7\*e^5 + 3\*a^3\*c^2\*d^5\*e^7 - a^4\*c\*d^3\*e^9 + (c^5\*d^8\*e^4 - 3\*a\*c^4\*d^6\*e^6 + 3\*a^2\*c^3\*d^4\*e^8 - a^3\*c^2\*d^2\*e^10)\*x^3 + (2\*c^5\*d^9\*e^3 - 5\*a\*c^4\*d^7\*e^5 + 3\*a^2\*c^3\*d^5\*e^7 + a^3\*c^2\*d^3\*e^9 - a^4\*c\*d\*e^11)\*x^2 + (c^5\*d^10\*e^2 - a\*c^4\*d^8\*e^4 - 3\*a^2\*c^3\*d^6\*e^6 + 5\*a^3\*c^2\*d^4\*e^8 - 2\*a^4\*c\*d^2\*e^10)\*x)\*sqrt(c\*d\*e)), -1/3\*(2\*(3\*a\*c^2\*d^6\*e - 8\*a^2\*c\*d^4\*e^3 - 3\*a^3\*d^2\*e^5 + (4\*c^3\*d^6\*e - 9\*a\*c^2\*d^4\*e^3 - 3\*a^3\*e^7)\*x^2 + (3\*c^3\*d^7 - 4\*a\*c^2\*d^5\*e^2 - 9\*a^2\*c\*d^3\*e^4 - 6\*a^3\*d\*e^6)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*e) - 3\*(a\*c^3\*d^8\*e - 3\*a^2\*c^2\*d^6\*e^3 + 3\*a^3\*c\*d^4\*e^5 - a^4\*d^2\*e^7 + (c^4\*d^7\*e^2 - 3\*a\*c^3\*d^5\*e^4 + 3\*a^2\*c^2\*d^3\*e^6 - a^3\*c\*d\*e^8)\*x^3 + (2\*c^4\*d^8\*e - 5\*a\*c^3\*d^6\*e^3 + 3\*a^2\*c^2\*d^4\*e^5 + a^3\*c\*d^2\*e^7 - a^4\*e^9)\*x^2 + (c^4\*d^9 - a\*c^3\*d^7\*e^2 - 3\*a^2\*c^2\*d^5\*e^4 + 5\*a^3\*c\*d^3\*e^6 - 2\*a^4\*d\*e^8)\*x)\*arctan(1/2\*(2\*c\*d\*e\*x + c\*d^2 + a\*e^2)\*sqrt(-c\*d\*e)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*e)))/((a\*c^4\*d^9\*e^3 - 3\*a^2\*c^3\*d^7\*e^5 + 3\*a^3\*c^2\*d^5\*e^7 - a^4\*c\*d^3\*e^9 + (c^5\*d^8\*e^4 - 3\*a\*c^4\*d^6\*e^6 + 3\*a^2\*c^3\*d^4\*e^8 - a^3\*c^2\*d^2\*e^10)\*x^3 + (2\*c^5\*d^9\*e^3 - 5\*a\*c^4\*d^7\*e^5 + 3\*a^2\*c^3\*d^5\*e^7 + a^3\*c^2\*d^3\*e^9 - a^4\*c\*d\*e^11)\*x^2 + (c^5\*d^10\*e^2 - a\*c^4\*d^8\*e^4 - 3\*a^2\*c^3\*d^6\*e^6 + 5\*a^3\*c^2\*d^4\*e^8 - 2\*a^4\*c\*d^2\*e^10)\*x)\*sqrt(-c\*d\*e)]]

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{((d + ex)(ae + cdx))^{\frac{3}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(e\*x+d)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2),x, algorithm="sympy")

[Out]  $\text{Integral}(x^{**3}/(((d + e*x)*(a*e + c*d*x))^{** (3/2)}*(d + e*x)), x)$

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

[*undef, undef, undef, 1*]

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(x^3/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^{(3/2)}*(e*x + d)), x, \text{algori}$

[Out] [*undef, undef, undef, 1*]

$$3.480 \quad \int \frac{x^2}{(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx$$

**Optimal.** Leaf size=126

$$\frac{2x^2}{3(d+ex)(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{8ae(x(ae^2+cd^2)+2ade)}{3(cd^2-ae^2)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

[Out]  $(2*x^2)/(3*(c*d^2 - a*e^2)*(d + e*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (8*a*e*(2*a*d*e + (c*d^2 + a*e^2)*x))/(3*(c*d^2 - a*e^2)^3*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi [A]** time = 0.355511, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$

$$\frac{2x^2}{3(d+ex)(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{8ae(x(ae^2+cd^2)+2ade)}{3(cd^2-ae^2)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}), x]$

[Out]  $(2*x^2)/(3*(c*d^2 - a*e^2)*(d + e*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (8*a*e*(2*a*d*e + (c*d^2 + a*e^2)*x))/(3*(c*d^2 - a*e^2)^3*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi in Sympy [A]** time = 38.5827, size = 122, normalized size = 0.97

$$\frac{4ae(4ade + x(2ae^2 + 2cd^2))}{3(ae^2 - cd^2)^3\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}} - \frac{2x^2(ae + cdx)}{3(ae^2 - cd^2)(ade + cdex^2 + x(ae^2 + cd^2))^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**2}/(e*x+d)/(a*d*e+(a*e^{**2}+c*d^{**2})*x+c*d*e*x^{**2})^{**}(3/2), x)$

[Out]  $4*a*e*(4*a*d*e + x*(2*a*e^{**2} + 2*c*d^{**2}))/((3*(a*e^{**2} - c*d^{**2}))^{**3}*\text{sqrt}(a*d*e + c*d*e*x^{**2} + x*(a*e^{**2} + c*d^{**2}))) - 2*x^{**2}*(a*e + c*d*x)/((3*(a*e^{**2} - c*d^{**2})*(a*d*e + c*d*e*x^{**2} + x*(a*e^{**2} + c*d^{**2}))^{**}(3/2)))$

**Mathematica [A]** time = 0.207294, size = 99, normalized size = 0.79

$$\frac{-2a^2e^2(8d^2 + 12dex + 3e^2x^2) - 4acd^2ex(2d + 3ex) + 2c^2d^4x^2}{3(d+ex)(cd^2-ae^2)^3\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}), x]$

[Out]  $(2*c^2*d^4*x^2 - 4*a*c*d^2*e*x*(2*d + 3*e*x) - 2*a^2*e^2*(8*d^2 + 12*d*e*x + 3*e^2*x^2))/(3*(c*d^2 - a*e^2)^3*(d + e*x)*\text{Sqrt}[(a*e + c*d*x)*(d + e*x)])$

**Maple [A]** time = 0.013, size = 145, normalized size = 1.2

$$\frac{(2cdx + 2ae)(3a^2e^4x^2 + 6acd^2e^2x^2 - c^2d^4x^2 + 12a^2de^3x + 4acd^3ex + 8a^2d^2e^2)}{3a^3e^6 - 9a^2cd^2e^4 + 9ac^2d^4e^2 - 3c^3d^6} (cdex^2 + ae^2x + cd^2x + ade)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(e\*x+d)/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2), x)

[Out] 2/3\*(c\*d\*x+a\*e)\*(3\*a^2\*e^4\*x^2+6\*a\*c\*d^2\*e^2\*x^2-c^2\*d^4\*x^2+12\*a^2\*d\*e^3\*x+4\*a\*c\*d^3\*e\*x+8\*a^2\*d^2\*e^2)/(a^3\*e^6-3\*a^2\*c\*d^2\*e^4+3\*a\*c^2\*d^4\*e^2-c^3\*d^6)/(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(3/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(e\*x + d)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.580485, size = 416, normalized size = 3.3

$$\frac{2(8a^2d^2e^2 - (c^2d^4 - 6acd^2e^2 - 3a^2e^4)x^2 + 4(acd^3e + 3a^2de^3)x)}{3(ac^3d^8e - 3a^2c^2d^6e^3 + 3a^3cd^4e^5 - a^4d^2e^7 + (c^4d^7e^2 - 3ac^3d^5e^4 + 3a^2c^2d^3e^6 - a^3cde^8)x^3 + (2c^4d^8e - 5ac^3d^6e^3 + 3a^2c^2d^4e^5 - a^3c^2d^2e^7)x^2 + (c^4d^7e^2 - 3ac^3d^5e^4 + 3a^2c^2d^3e^6 - a^3cde^8)x + (2c^4d^8e - 5ac^3d^6e^3 + 3a^2c^2d^4e^5 - a^3c^2d^2e^7))} \sqrt{c^2d^4 - 6acd^2e^2 - 3a^2e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(e\*x + d)), x, algorithm="fricas")

[Out] -2/3\*(8\*a^2\*d^2\*e^2 - (c^2\*d^4 - 6\*a\*c\*d^2\*e^2 - 3\*a^2\*e^4)\*x^2 + 4\*(a\*c\*d^3\*e + 3\*a^2\*d\*e^3)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(a\*c^3\*d^8\*e - 3\*a^2\*c^2\*d^6\*e^3 + 3\*a^3\*c\*d^4\*e^5 - a^4\*d^2\*e^7 + (c^4\*d^7\*e^2 - 3\*a\*c^3\*d^5\*e^4 + 3\*a^2\*c^2\*d^3\*e^6 - a^3\*c\*d\*e^8)\*x^3 + (2\*c^4\*d^8\*e - 5\*a\*c^3\*d^6\*e^3 + 3\*a^2\*c^2\*d^4\*e^5 - a^3\*c^2\*d^2\*e^7)\*x^2 + (c^4\*d^7\*e^2 - 3\*a\*c^3\*d^5\*e^4 + 3\*a^2\*c^2\*d^3\*e^6 - a^3\*c\*d\*e^8)\*x + (2\*c^4\*d^8\*e - 5\*a\*c^3\*d^6\*e^3 + 3\*a^2\*c^2\*d^4\*e^5 - a^3\*c^2\*d^2\*e^7))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{((d + ex)(ae + cdx))^{\frac{3}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(e\*x+d)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2), x)

[Out] Integral(x\*\*2/(((d + e\*x)\*(a\*e + c\*d\*x))\*\*(3/2)\*(d + e\*x)), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

[undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)),x, algori
```

```
[Out] [undef, undef, undef, 1]
```

$$3.481 \quad \int \frac{x}{(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx$$

**Optimal.** Leaf size=138

$$\frac{2(3ae^2 + cd^2)(ae^2 + cd^2 + 2cdex)}{3e(cd^2 - ae^2)^3 \sqrt{x(ae^2 + cd^2) + ade + cdex^2}} - \frac{2d}{3e(d+ex)(cd^2 - ae^2) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}$$

[Out]  $(-2*d)/(3*e*(c*d^2 - a*e^2)*(d + e*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (2*(c*d^2 + 3*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*e*(c*d^2 - a*e^2)^3*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi [A]** time = 0.285649, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$

$$\frac{2(3ae^2 + cd^2)(ae^2 + cd^2 + 2cdex)}{3e(cd^2 - ae^2)^3 \sqrt{x(ae^2 + cd^2) + ade + cdex^2}} - \frac{2d}{3e(d+ex)(cd^2 - ae^2) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}$$

Antiderivative was successfully verified.

[In] Int[x/((d + e\*x)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)), x]

[Out]  $(-2*d)/(3*e*(c*d^2 - a*e^2)*(d + e*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (2*(c*d^2 + 3*a*e^2)*(c*d^2 + a*e^2 + 2*c*d*e*x))/(3*e*(c*d^2 - a*e^2)^3*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi in Sympy [A]** time = 21.6109, size = 128, normalized size = 0.93

$$\frac{2d}{3e(d+ex)(ae^2 - cd^2) \sqrt{ade + cdex^2 + x(ae^2 + cd^2)}} - \frac{(3ae^2 + cd^2)(2ae^2 + 2cd^2 + 4cdex)}{3e(ae^2 - cd^2)^3 \sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x/(e\*x+d)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2), x)

[Out]  $2*d/(3*e*(d + e*x)*(a*e**2 - c*d**2)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))) - (3*a*e**2 + c*d**2)*(2*a*e**2 + 2*c*d**2 + 4*c*d*e*x)/(3*e*(a*e**2 - c*d**2)**3*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))$

**Mathematica [A]** time = 0.159038, size = 100, normalized size = 0.72

$$\frac{2(a^2e^3(2d + 3ex) + 2acde(3d^2 + 5dex + 3e^2x^2) + c^2d^3x(3d + 2ex))}{3(d+ex)(cd^2 - ae^2)^3 \sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[x/((d + e\*x)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)), x]

[Out]  $(2*(c^2*d^3*x*(3*d + 2*e*x) + a^2*e^3*(2*d + 3*e*x) + 2*a*c*d*e*(3*d^2 + 5*d*e*x + 3*e^2*x^2)))/(3*(c*d^2 - a*e^2)^3*(d + e*x)*\text{Sqrt}[(a*e + c*d*x)*(d + e*x)])$

---

**Maple [A]** time = 0.015, size = 149, normalized size = 1.1

$$\frac{(2cdx + 2ae)(6acde^3x^2 + 2c^2d^3ex^2 + 3a^2e^4x + 10acd^2e^2x + 3c^2d^4x + 2a^2de^3 + 6acd^3e)}{3a^3e^6 - 9a^2cd^2e^4 + 9ac^2d^4e^2 - 3c^3d^6} (cdex^2 + ae^2x + cd^2x + ade)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)`

[Out] `-2/3*(c*d*x+a*e)*(6*a*c*d*e^3*x^2+2*c^2*d^3*e*x^2+3*a^2*e^4*x+10*a*c*d^2*e^2*x+3*c^2*d^4*x+2*a^2*d*e^3+6*a*c*d^3*e)/(a^3*e^6-3*a^2*c*d^2*e^4+3*a*c^2*d^4*e^2-3*c^3*d^6)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.585711, size = 424, normalized size = 3.07

$$\frac{2(6acd^3e + 2a^2de^3 + 2(c^2d^3e + 3acde^3)x^2 + (3c^2d^4 + 10acd^2e^2 + 3a^2e^4)x + 3ac^3d^8e - 3a^2c^2d^6e^3 + 3a^3cd^4e^5 - a^4d^2e^7 + (c^4d^7e^2 - 3ac^3d^5e^4 + 3a^2c^2d^3e^6 - a^3cde^8)x^3 + (2c^4d^8e - 5ac^3d^6e^3 + 3a^2c^2d^4e^5)x^2 + (3c^4d^9e - 5ac^3d^7e^3 + 3a^2c^2d^5e^5)x + a^4d^9e^4)}{(c^4d^7e^2 - 3ac^3d^5e^4 + 3a^2c^2d^3e^6 - a^3cde^8)^{3/2} * (e*x + d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)),x, algorithm="fricas")`

[Out] `2/3*(6*a*c*d^3*e + 2*a^2*d*e^3 + 2*(c^2*d^3*e + 3*a*c*d*e^3)*x^2 + (3*c^2*d^4 + 10*a*c*d^2*e^2 + 3*a^2*e^4)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(a*c^3*d^8*e - 3*a^2*c^2*d^6*e^3 + 3*a^3*c*d^4*e^5 - a^4*d^2*e^7 + (c^4*d^7*e^2 - 3*a*c^3*d^5*e^4 + 3*a^2*c^2*d^3*e^6 - a^3*c*d*e^8)*x^3 + (2*c^4*d^8*e - 5*a*c^3*d^6*e^3 + 3*a^2*c^2*d^4*e^5)*x^2 + (3*c^4*d^9*e - 5*a*c^3*d^7*e^3 + 3*a^2*c^2*d^5*e^5)*x + a^4*d^9*e^4)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{((d + ex)(ae + cdx))^{\frac{3}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)`

[Out] `Integral(x/(((d + e*x)*(a*e + c*d*x))^(3/2)*(d + e*x)), x)`



---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

[*undef*, *undef*, *undef*, 1]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(e\*x + d)),x, algorithm

[Out] [*undef*, *undef*, *undef*, 1]

$$3.482 \quad \int \frac{1}{(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx$$

**Optimal.** Leaf size=121

$$\frac{2}{3(d+ex)(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{8cd(ae^2+cd^2+2cdex)}{3(cd^2-ae^2)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

[Out] 2/(3\*(c\*d^2 - a\*e^2)\*(d + e\*x)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]) - (8\*c\*d\*(c\*d^2 + a\*e^2 + 2\*c\*d\*e\*x))/(3\*(c\*d^2 - a\*e^2)^2)^3\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])

**Rubi [A]** time = 0.123342, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 37,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.054$

$$\frac{2}{3(d+ex)(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{8cd(ae^2+cd^2+2cdex)}{3(cd^2-ae^2)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)), x]

[Out] 2/(3\*(c\*d^2 - a\*e^2)\*(d + e\*x)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]) - (8\*c\*d\*(c\*d^2 + a\*e^2 + 2\*c\*d\*e\*x))/(3\*(c\*d^2 - a\*e^2)^2)^3\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])

**Rubi in Sympy [A]** time = 23.4028, size = 116, normalized size = 0.96

$$\frac{4cd(2ae^2 + 2cd^2 + 4cdex)}{3(ae^2 - cd^2)^3\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}} - \frac{2}{3(d+ex)(ae^2 - cd^2)\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(e\*x+d)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2), x)

[Out] 4\*c\*d\*(2\*a\*e\*\*2 + 2\*c\*d\*\*2 + 4\*c\*d\*e\*x)/(3\*(a\*e\*\*2 - c\*d\*\*2)\*\*3\*sqr(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))) - 2/(3\*(d + e\*x)\*(a\*e\*\*2 - c\*d\*\*2)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2)))

**Mathematica [A]** time = 0.140232, size = 95, normalized size = 0.79

$$\frac{2a^2e^4 - 4acde^2(3d + 2ex) - 2c^2d^2(3d^2 + 12dex + 8e^2x^2)}{3(d+ex)(cd^2-ae^2)^3\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)), x]

[Out] (2\*a^2\*e^4 - 4\*a\*c\*d\*e^2\*(3\*d + 2\*e\*x) - 2\*c^2\*d^2\*(3\*d^2 + 12\*d\*e\*x + 8\*e^2\*x^2))/(3\*(c\*d^2 - a\*e^2)^3\*(d + e\*x)\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)])

**Maple [A]** time = 0.013, size = 138, normalized size = 1.1

$$\frac{(2cdx + 2ae)(-8c^2d^2e^2x^2 - 4acde^3x - 12c^2d^3ex + a^2e^4 - 6acd^2e^2 - 3c^2d^4)}{3a^3e^6 - 9a^2cd^2e^4 + 9ac^2d^4e^2 - 3c^3d^6} (cdex^2 + ae^2x + cd^2x + ade)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x+d)/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2),x)

[Out] -2/3\*(c\*d\*x+a\*e)\*(-8\*c^2\*d^2\*e^2\*x^2-4\*a\*c\*d\*e^3\*x-12\*c^2\*d^3\*e\*x+a^2\*e^4-6\*a\*c\*d^2\*e^2-3\*c^2\*d^4)/(a^3\*e^6-3\*a^2\*c\*d^2\*e^4+3\*a\*c^2\*d^4e^2-3\*c^3\*d^6)/(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(3/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(e\*x + d)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.584955, size = 413, normalized size = 3.41

$$\frac{2(8c^2d^2e^2x^2 + 3c^2d^4 + 6acd^2e^2 - a^2e^4 + 4(3c^2d^3e + acde^3)x)\sqrt{cdex^2 + ae^2x + cd^2x + ade}}{3(ac^3d^8e - 3a^2c^2d^6e^3 + 3a^3cd^4e^5 - a^4d^2e^7 + (c^4d^7e^2 - 3ac^3d^5e^4 + 3a^2c^2d^3e^6 - a^3cde^8)x^3 + (2c^4d^8e - 5ac^3d^6e^3 + 3a^2c^2d^4e^5 + a^3cd^2e^7 - a^4d^2e^9)x^2 + (c^4d^9 - a^3c^3d^7e^2 - 3a^2c^2d^5e^4 + 5a^3cd^3e^6 - 2a^4d^2e^8)x + a^4d^2e^9)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(e\*x + d)),x, algorithm="fricas")

[Out] -2/3\*(8\*c^2\*d^2\*e^2\*x^2 + 3\*c^2\*d^4 + 6\*a\*c\*d^2\*e^2 - a^2\*e^4 + 4\*(3\*c^2\*d^3\*e + a\*c\*d^2\*e^3)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(a\*c^3\*d^8\*e - 3\*a^2\*c^2\*d^6\*e^3 + 3\*a^3\*c\*d^4\*e^5 - a^4\*d^2\*e^7 + (c^4\*d^7\*e^2 - 3\*a\*c^3\*d^5\*e^4 + 3\*a^2\*c^2\*d^3\*e^6 - a^3\*c\*d^2\*e^7 + (2\*c^4\*d^8\*e - 5\*a\*c^3\*d^6\*e^3 + 3\*a^2\*c^2\*d^4\*e^5 + a^3\*c\*d^2\*e^7 - a^4\*d^2\*e^9)\*x^2 + (c^4\*d^9 - a^3\*c^3\*d^7\*e^2 - 3\*a^2\*c^2\*d^5\*e^4 + 5\*a^3\*c\*d^3\*e^6 - 2\*a^4\*d^2\*e^8)\*x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{((d + ex)(ae + cdx))^{\frac{3}{2}}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x+d)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2),x)

[Out] Integral(1/(((d + e\*x)\*(a\*e + c\*d\*x))\*\*(3/2)\*(d + e\*x)), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

[undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)),x, algorithm="default")
```

```
[Out] [undef, undef, undef, 1]
```

$$3.483 \quad \int \frac{1}{x(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx$$

**Optimal.** Leaf size=271

$$\frac{\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{a^{3/2}d^{5/2}e^{3/2}} + \frac{2(-3a^3e^6 + 7a^2cd^2e^4 + ac^2d^4e^2 + cdex(3cd^2 - ae^2)(3ae^2 + cd^2) + 3c^3d^6)}{3ad^2e(cd^2 - ae^2)^3\sqrt{x(ae^2 + cd^2) + ade + cdex^2}} - \frac{2e(ae + cdx)}{3d(cd^2 - ae^2)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}$$

[Out]  $(-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 7*a^2*c*d^2*e^4 - 3*a^3*e^6 + c*d*e*(3*c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2)*x))/(3*a*d^2*e*(c*d^2 - a*e^2)^3*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - \text{ArcTanh}[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(a^{(3/2)}*d^{(5/2)}*e^{(3/2)})$

**Rubi [A]** time = 1.0034, antiderivative size = 271, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{\tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{a^{3/2}d^{5/2}e^{3/2}} + \frac{2(-3a^3e^6 + 7a^2cd^2e^4 + ac^2d^4e^2 + cdex(3cd^2 - ae^2)(3ae^2 + cd^2) + 3c^3d^6)}{3ad^2e(cd^2 - ae^2)^3\sqrt{x(ae^2 + cd^2) + ade + cdex^2}} - \frac{2e(ae + cdx)}{3d(cd^2 - ae^2)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}), x]$

[Out]  $(-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 7*a^2*c*d^2*e^4 - 3*a^3*e^6 + c*d*e*(3*c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2)*x))/(3*a*d^2*e*(c*d^2 - a*e^2)^3*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - \text{ArcTanh}[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(a^{(3/2)}*d^{(5/2)}*e^{(3/2)})$

**Rubi in Sympy [A]** time = 138.43, size = 260, normalized size = 0.96

$$\frac{2e(ae + cdx)}{3d(ae^2 - cd^2)(ade + cdex^2 + x(ae^2 + cd^2))^{3/2}} + \frac{2(3a^3e^6 - 7a^2cd^2e^4 - ac^2d^4e^2 - 3c^3d^6 + cdex(ae^2 - 3cd^2)(3ae^2 + cd^2))}{3ad^2e(ae^2 - cd^2)^3\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}} - \frac{\text{atanh}\left(\frac{2ade+x(ae^2+cd^2)}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}\right)}{a^{3/2}d^{5/2}e^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(3/2)}, x)$

[Out]  $2 * e * (a * e + c * d * x) / (3 * d * (a * e^{**2} - c * d^{**2}) * (a * d * e + c * d * e * x^{**2} + x * (a * e^{**2} + c * d^{**2}))^{**3/2}) + 2 * (3 * a^{**3} * e^{**6} - 7 * a^{**2} * c * d^{**2} * e^{**4} - a * c^{**2} * d^{**4} * e^{**2} - 3 * c^{**3} * d^{**6} + c * d * e * x * (a * e^{**2} - 3 * c * d^{**2}) * (3 * a * e^{**2} + c * d^{**2})) / (3 * a * d^{**2} * e * (a * e^{**2} - c * d^{**2})^{**3} * \text{sqrt}(a * d * e + c * d * e * x^{**2} + x * (a * e^{**2} + c * d^{**2}))) - \text{atanh}((2 * a * d * e + x * (a * e^{**2} + c * d^{**2})) / (2 * \text{sqrt}(a) * \text{sqrt}(d) * \text{sqrt}(e) * \text{sqrt}(a * d * e + c * d * e * x^{**2} + x * (a * e^{**2} + c * d^{**2})))) / (a^{**3/2} * d^{**5/2} * e^{**3/2})$

**Mathematica [A]** time = 0.810185, size = 254, normalized size = 0.94

$$\frac{2\sqrt{a}\sqrt{d}\sqrt{e}(ae+cdx)(ade^3(cd^2-ae^2)(ae+cdx)+ae^3(d+ex)(8cd^2-3ae^2)(ae+cdx)+3c^3d^5(d+ex)^2)}{(cd^2-ae^2)^3} - 3(d+ex)^{3/2}(ae+cdx)^{3/2} \log\left(\frac{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d}}{3a^{3/2}d^{5/2}e^{3/2}((d+ex)(ae+cdx))^{3/2}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(d+e\*x)\*(a\*d\*e+(c\*d^2+a\*e^2)\*x+c\*d\*e\*x^2)^(3/2)),x]

[Out]  $((2 * \text{Sqrt}[a] * \text{Sqrt}[d] * \text{Sqrt}[e] * (a * e + c * d * x) * (a * d * e^{**3} * (c * d^{**2} - a * e^{**2}) * (a * e + c * d * x) + a * e^{**3} * (8 * c * d^{**2} - 3 * a * e^{**2}) * (a * e + c * d * x) * (d + e * x) + 3 * c^{**3} * d^{**5} * (d + e * x)^{**2})) / (c * d^{**2} - a * e^{**2})^{**3} + 3 * (a * e + c * d * x)^{**3/2} * (d + e * x)^{**3/2} * \text{Log}[x] - 3 * (a * e + c * d * x)^{**3/2} * (d + e * x)^{**3/2} * \text{Log}[c * d^{**2} * x + 2 * \text{Sqrt}[a] * \text{Sqrt}[d] * \text{Sqrt}[e] * \text{Sqrt}[a * e + c * d * x] * \text{Sqrt}[d + e * x] + a * e * (2 * d + e * x)]) / (3 * a^{**3/2} * d^{**5/2} * e^{**3/2}) * (a * e + c * d * x) * (d + e * x))^{**3/2})$

**Maple [B]** time = 0.02, size = 682, normalized size = 2.5

$$\begin{aligned} & \frac{1}{ad^2e} \frac{1}{\sqrt{ade + (ae^2 + cd^2)x + cdex^2}} \\ & - 2 \frac{xe^{2c}}{d(-a^2e^4 + 2acd^2e^2 - c^2d^4)\sqrt{ade + (ae^2 + cd^2)x + cdex^2}} \\ & - 2 \frac{dxc^2}{a(-a^2e^4 + 2acd^2e^2 - c^2d^4)\sqrt{ade + (ae^2 + cd^2)x + cdex^2}} \\ & - \frac{ae^3}{d^2(-a^2e^4 + 2acd^2e^2 - c^2d^4)} \frac{1}{ce\sqrt{ade + (ae^2 + cd^2)x + cdex^2}} \\ & - 2 \frac{c^2d^2}{(-a^2e^4 + 2acd^2e^2 - c^2d^4)\sqrt{ade + (ae^2 + cd^2)x + cdex^2}} \\ & - \frac{ae(-a^2e^4 + 2acd^2e^2 - c^2d^4)}{ae^3} \frac{1}{\sqrt{ade + (ae^2 + cd^2)x + cdex^2}} \\ & - \frac{1}{ad^2e} \ln\left(\frac{1}{x} \left(2ade + (ae^2 + cd^2)x + 2\sqrt{ade}\sqrt{ade + (ae^2 + cd^2)x + cdex^2}\right)\right) \frac{1}{\sqrt{ade}} \\ & + \frac{2}{3d(ae^2 - cd^2)} \left(x + \frac{d}{e}\right)^{-1} \frac{1}{\sqrt{cde\left(x + \frac{d}{e}\right)^2 + (ae^2 - cd^2)\left(x + \frac{d}{e}\right)}} \\ & - \frac{16dc^2e^2x}{3(ae^2 - cd^2)^3} \frac{1}{\sqrt{cde\left(x + \frac{d}{e}\right)^2 + (ae^2 - cd^2)\left(x + \frac{d}{e}\right)}} \\ & - \frac{8ce^3a}{3(ae^2 - cd^2)^3} \frac{1}{\sqrt{cde\left(x + \frac{d}{e}\right)^2 + (ae^2 - cd^2)\left(x + \frac{d}{e}\right)}} \\ & - \frac{8c^2d^2e}{3(ae^2 - cd^2)^3} \frac{1}{\sqrt{cde\left(x + \frac{d}{e}\right)^2 + (ae^2 - cd^2)\left(x + \frac{d}{e}\right)}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)`

[Out] 
$$\frac{1}{d^2} \frac{a}{e} \frac{1}{(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{1/2}} - \frac{2}{d} \frac{e^2}{(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)} \frac{1}{(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{1/2}} * x^2 - \frac{2*d}{a} \frac{1}{(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)} \frac{1}{(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{1/2}} * x^2 - \frac{1}{d^2} \frac{a^3}{(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)} \frac{1}{(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{1/2}} - \frac{2*e}{(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)} \frac{1}{(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{1/2}} * c - \frac{d^2}{a} \frac{1}{(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)} \frac{1}{(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{1/2}} * c^2 - \frac{1}{d^2} \frac{a}{e} \frac{1}{(a*d*e)^{1/2}} * \ln\left(\frac{(2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^{1/2}*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{1/2}}{x}\right) + \frac{2}{3} \frac{d}{(a*e^2-c*d^2)} \frac{1}{(x+d/e)} \frac{1}{(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{1/2}} - \frac{16}{3} \frac{d^2*c^2*e^2}{(a*e^2-c*d^2)^3} \frac{1}{(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{1/2}} * x - \frac{8}{3} \frac{c^3*e^3}{(a*e^2-c*d^2)^3} \frac{1}{(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{1/2}} * a - \frac{8}{3} \frac{d^2*c^2*e}{(a*e^2-c*d^2)^3} \frac{1}{(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{1/2}}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}(ex + d)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x),x, algorithm="maxima")`

[Out] `integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x), x)`

**Fricas [A]** time = 0.72245, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x),x, algorithm="fricas")`

[Out] 
$$\left[ \frac{1}{6} \left( 4 \left( 3c^3d^7 + 9a^2c^2d^3e^4 - 4a^3d^2e^6 + (3c^3d^5e^2 + 8a^2c^2d^3e^4 - 3a^2c^2d^2e^6) \right) x^2 + (6c^3d^6e + 9a^2c^2d^4e^3 + 4a^2c^2d^2e^5 - 3a^3e^7) \right) x \right] \sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x} \sqrt{a*d*e} + 3 \left( a^3c^3d^8e - 3a^2c^2d^6e^3 + 3a^3c^2d^4e^5 - a^4d^2e^7 + (c^4d^7e^2 - 3a^2c^3d^5e^4 + 3a^2c^2d^3e^6 - a^3c^2d^2e^8) \right) x^3 + (2c^4d^8e - 5a^2c^3d^6e^3 + 3a^2c^2d^4e^5 + a^3c^2d^2e^7 - a^4e^9) x^2 + (c^4d^9 - a^2c^3d^7e^2 - 3a^2c^2d^5e^4 + 5a^3c^2d^3e^6 - 2a^4d^2e^8) x \right) \log\left(-\frac{4 \left( 2a^2d^2e^2 + (a^2c^3d^3e + a^2d^2e^3) \right) x \sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x} - (8a^2d^2e^2 + (c^2d^4 + 6a^2c^2d^2e^2 + a^2e^4) x^2 + 8(a^2c^3d^3e + a^2d^2e^3) x) \sqrt{a*d*e}}{x^2}\right) / \left( (a^2c^3d^{10}e^2 - 3a^3c^2d^8e^4 + 3a^4c^2d^6e^6 - a^5d^4e^8 + (a^2c^4d^9e^3 - 3a^2c^3d^7e^5 + 3a^3c^2d^5e^7 - a^4c^2d^3e^9) x^3 + (2a^2c^4d^{10}e^2 - 5a^2c^3d^8e^4 + 3a^3c^2d^6e^6 + a^4c^2d^4e^8 - a^5d^2e^{10}) x^2 + (a^2c^4d^{11}e - a^2c^3d^9e^3 - 3a^3c^2d^7e^5 + 5a^4c^2d^5e^7 - 2a^5d^3e^9) x \right) \sqrt{a*d*e} \right), \frac{1}{3} \left( 2 \left( 3c^3d^7 + 9a^2c^2d^3e^4 - 4a^3d^2e^6 + (3c^3d^5e^2 + 8a^2c^2d^3e^4 - 3a^2c^2d^2e^6) \right) x^2 + (6c^3d^6e + 9a^2c^2d^4e^3 + 4a^2c^2d^2e^5 - 3a^3e^7) \right) x \right) \sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x} \sqrt{-a*d*e} - 3 \left( a^3c^3d^8e - 3a^2c^2d^6e^3 + 3a^3c^2d^4e^5 - a^4d^2e^7 + (c^4d^7e^2 - 3a^2c^3d^5e^4 + 3a^2c^2d^3e^6 - a^3c^2d^2e^8) \right) x^3 + (2c^4d^8e - 5a^2c^3d^6e^3 + 3a^2c^2d^4e^5 + a^3c^2d^2e^7 - a^4e^9) x^2 + (c^4d^9 - a^2c^3d^7e^2 - 3a^2c^2d^5e^4 + 5a^3c^2d^3e^6 - 2a^4d^2e^8) x \right)$$

$$^8)x) \cdot \arctan\left(\frac{1}{2} \cdot (2 \cdot a \cdot d \cdot e + (c \cdot d^2 + a \cdot e^2) \cdot x) \cdot \sqrt{-a \cdot d \cdot e} / (\sqrt{c \cdot d \cdot e \cdot x^2 + a \cdot d \cdot e + (c \cdot d^2 + a \cdot e^2) \cdot x} \cdot a \cdot d \cdot e)}{(a^2 \cdot c^3 \cdot d^{10} \cdot e^2 - 3 \cdot a^3 \cdot c^2 \cdot d^8 \cdot e^4 + 3 \cdot a^4 \cdot c \cdot d^6 \cdot e^6 - a^5 \cdot d^4 \cdot e^8 + (a \cdot c^4 \cdot d^9 \cdot e^3 - 3 \cdot a^2 \cdot c^3 \cdot d^7 \cdot e^5 + 3 \cdot a^3 \cdot c^2 \cdot d^5 \cdot e^7 - a^4 \cdot c \cdot d^3 \cdot e^9) \cdot x^3 + (2 \cdot a \cdot c^4 \cdot d^{10} \cdot e^2 - 5 \cdot a^2 \cdot c^3 \cdot d^8 \cdot e^4 + 3 \cdot a^3 \cdot c^2 \cdot d^6 \cdot e^6 + a^4 \cdot c \cdot d^4 \cdot e^8 - a^5 \cdot d^2 \cdot e^{10}) \cdot x^2 + (a \cdot c^4 \cdot d^{11} \cdot e - a^2 \cdot c^3 \cdot d^9 \cdot e^3 - 3 \cdot a^3 \cdot c^2 \cdot d^7 \cdot e^5 + 5 \cdot a^4 \cdot c \cdot d^5 \cdot e^7 - 2 \cdot a^5 \cdot d^3 \cdot e^9) \cdot x) \cdot \sqrt{-a \cdot d \cdot e}}\right)$$


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**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x((d+ex)(ae+cdx))^{\frac{3}{2}}(d+ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e\*x+d)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2),x)

[Out] Integral(1/(x\*((d+e\*x)\*(a\*e+c\*d\*x))\*\*(3/2)\*(d+e\*x)), x)

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

[undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*d\*e\*x^2+a\*d\*e+(c\*d^2+a\*e^2)\*x)^(3/2)\*(e\*x+d)\*x),x,algori

[Out] [undef, undef, undef, 1]



$$3.484 \quad \int \frac{1}{x^2(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx$$

**Optimal.** Leaf size=394

$$\begin{aligned} & \frac{(5ae^2 + 3cd^2) \tanh^{-1} \left( \frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \right)}{2a^{5/2}d^{7/2}e^{5/2}} \\ & + \frac{2(-5a^3e^6 + cdex(-5a^2e^4 + 10acd^2e^2 + 3c^2d^4) + 9a^2cd^2e^4 + ac^2d^4e^2 + 3c^3d^6)}{3ad^2ex(cd^2 - ae^2)^3 \sqrt{x(ae^2 + cd^2) + ade + cdex^2}} \\ & - \frac{(-15a^3e^6 + 31a^2cd^2e^4 - 9ac^2d^4e^2 + 9c^3d^6) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{3a^2d^3e^2x(cd^2 - ae^2)^3} \\ & - \frac{2e(ae + cdx)}{3dx(cd^2 - ae^2)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}} \end{aligned}$$

[Out]  $(-2 * e * (a * e + c * d * x)) / (3 * d * (c * d^2 - a * e^2) * x * (a * d * e + (c * d^2 + a * e^2) * x + c * d * e * x^2)^{(3/2)}) + (2 * (3 * c^3 * d^6 + a * c^2 * d^4 * e^2 + 9 * a^2 * c * d^2 * e^4 - 5 * a^3 * e^6 + c * d * e * (3 * c^2 * d^4 + 10 * a * c * d^2 * e^2 - 5 * a^2 * e^4) * x)) / (3 * a * d^2 * e * (c * d^2 - a * e^2)^3 * x * \text{Sqrt}[a * d * e + (c * d^2 + a * e^2) * x + c * d * e * x^2]) - ((9 * c^3 * d^6 - 9 * a * c^2 * d^4 * e^2 + 31 * a^2 * c * d^2 * e^4 - 15 * a^3 * e^6) * \text{Sqrt}[a * d * e + (c * d^2 + a * e^2) * x + c * d * e * x^2]) / (3 * a^2 * d^3 * e^2 * x * (c * d^2 - a * e^2)^3) + ((3 * c * d^2 + 5 * a * e^2) * \text{ArcTanh}[(2 * a * d * e + (c * d^2 + a * e^2) * x) / (2 * \text{Sqrt}[a] * \text{Sqrt}[d] * \text{Sqrt}[e] * \text{Sqrt}[a * d * e + (c * d^2 + a * e^2) * x + c * d * e * x^2])]) / (2 * a^{(5/2)} * d^{(7/2)} * e^{(5/2)})$

**Rubi [A]** time = 1.5022, antiderivative size = 394, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\begin{aligned} & \frac{(5ae^2 + 3cd^2) \tanh^{-1} \left( \frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \right)}{2a^{5/2}d^{7/2}e^{5/2}} \\ & + \frac{2(-5a^3e^6 + cdex(-5a^2e^4 + 10acd^2e^2 + 3c^2d^4) + 9a^2cd^2e^4 + ac^2d^4e^2 + 3c^3d^6)}{3ad^2ex(cd^2 - ae^2)^3 \sqrt{x(ae^2 + cd^2) + ade + cdex^2}} \\ & - \frac{(-15a^3e^6 + 31a^2cd^2e^4 - 9ac^2d^4e^2 + 9c^3d^6) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{3a^2d^3e^2x(cd^2 - ae^2)^3} \\ & - \frac{2e(ae + cdx)}{3dx(cd^2 - ae^2)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^2 * (d + e * x) * (a * d * e + (c * d^2 + a * e^2) * x + c * d * e * x^2)^{(3/2)}), x]$

[Out]  $(-2 * e * (a * e + c * d * x)) / (3 * d * (c * d^2 - a * e^2) * x * (a * d * e + (c * d^2 + a * e^2) * x + c * d * e * x^2)^{(3/2)}) + (2 * (3 * c^3 * d^6 + a * c^2 * d^4 * e^2 + 9 * a^2 * c * d^2 * e^4 - 5 * a^3 * e^6 + c * d * e * (3 * c^2 * d^4 + 10 * a * c * d^2 * e^2 - 5 * a^2 * e^4) * x)) / (3 * a * d^2 * e * (c * d^2 - a * e^2)^3 * x * \text{Sqrt}[a * d * e + (c * d^2 + a * e^2) * x + c * d * e * x^2]) - ((9 * c^3 * d^6 - 9 * a * c^2 * d^4 * e^2 + 31 * a^2 * c * d^2 * e^4 - 15 * a^3 * e^6) * \text{Sqrt}[a * d * e + (c * d^2 + a * e^2) * x + c * d * e * x^2]) / (3 * a^2 * d^3 * e^2 * x * (c * d^2 - a * e^2)^3) + ((3 * c * d^2 + 5 * a * e^2) * \text{ArcTanh}[(2 * a * d * e + (c * d^2 + a * e^2) * x) / (2 * \text{Sqrt}[a] * \text{Sqrt}[d] * \text{Sqrt}[e] * \text{Sqrt}[a * d * e + (c * d^2 + a * e^2) * x + c * d * e * x^2])]) / (2 * a^{(5/2)} * d^{(7/2)} * e^{(5/2)})$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)`

[Out] Timed out

**Mathematica [A]** time = 1.50179, size = 312, normalized size = 0.79

$$\frac{-\frac{3\log(x)(d+ex)^{3/2}(5ae^2+3cd^2)(ae+cdx)^{3/2}}{a^{5/2}} + \frac{3(d+ex)^{3/2}(5ae^2+3cd^2)(ae+cdx)^{3/2}\log\left(2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx}+ae(2d+ex)+cd^2x\right)}{a^{5/2}}}{6d^{7/2}e^{5/2}((d+ex)(ae+cdx))^{3/2}} + 2\sqrt{d}\sqrt{e}(d+ex)$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x^2*(d+e*x)*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^(3/2)),x]`

[Out]  $(2*\text{Sqrt}[d]*\text{Sqrt}[e]*(a*e+c*d*x)^2*(d+e*x)^2*(-3/(a^2*x)+(6*c^4*d^7)/(a^2*(-(c*d^2)+a*e^2)^3*(a*e+c*d*x))-(2*d^5)/(c*d^2-a*e^2)^2*(d+e*x)^2)+(2*e^5*(-11*c*d^2+6*a*e^2))/((c*d^2-a*e^2)^3*(d+e*x))-(3*(3*c*d^2+5*a*e^2)*(a*e+c*d*x)^(3/2)*(d+e*x)^(3/2)*\text{Log}[x])/a^(5/2)+(3*(3*c*d^2+5*a*e^2)*(a*e+c*d*x)^(3/2)*(d+e*x)^(3/2)*\text{Log}[c*d^2*x+2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*e+c*d*x]*\text{Sqrt}[d+e*x]+a*e*(2*d+e*x)])/a^(5/2))/(6*d^(7/2)*e^(5/2)*((a*e+c*d*x)*(d+e*x))^(3/2))$

**Maple [B]** time = 0.022, size = 912, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)`

[Out]  $-1/d^2/a/e/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-5/2/d^3/a/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)-3/2/d/a^2/e^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c+5/d^2*e^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x+c+3*d^2/a^2/e/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x*c^3+5/2/d^3*a*e^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+5/2/d*e^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c+3/2*d/a/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^2+3/2*d^3/a^2/e^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^3+5/2/d^3/a/(a*d*e)^(1/2)*\text{ln}((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)+3/2/d/a^2/e^2/(a*d*e)^(1/2)*\text{ln}((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c-2/3*e/d^2/(a*e^2-c*d^2)/(x+d/e)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)+16/3*e^3*c^2/(a*e^2-c*d^2)^3/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)*x+8/3*e^4/d*c/(a*e^2-c*d^2)^3/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)*a+8/3*e^2*d*c^2/(a*e^2-c*d^2)^3/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^(1/2)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}(ex + d)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(e\*x + d)\*x^2), x, algo

[Out] integrate(1/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(e\*x + d)\*x^2), x)

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**Fricas** [A] time = 1.42976, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(e\*x + d)\*x^2), x, algo

[Out] [-1/12\*(4\*(3\*a\*c^3\*d^8\*e - 9\*a^2\*c^2\*d^6\*e^3 + 9\*a^3\*c\*d^4\*e^5 - 3\*a^4\*d^2\*e^7 + (9\*c^4\*d^7\*e^2 - 9\*a\*c^3\*d^5\*e^4 + 31\*a^2\*c^2\*d^3\*e^6 - 15\*a^3\*c\*d\*e^8)\*x^3 + (18\*c^4\*d^8\*e - 15\*a\*c^3\*d^6\*e^3 + 33\*a^2\*c^2\*d^4\*e^5 + 11\*a^3\*c\*d^2\*e^7 - 15\*a^4\*e^9)\*x^2 + (9\*c^4\*d^9 - 3\*a\*c^3\*d^7\*e^2 - 9\*a^2\*c^2\*d^5\*e^4 + 39\*a^3\*c\*d^3\*e^6 - 20\*a^4\*d\*e^8)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(a\*d\*e) - 3\*((3\*c^5\*d^9\*e^2 - 4\*a\*c^4\*d^7\*e^4 - 6\*a^2\*c^3\*d^5\*e^6 + 12\*a^3\*c^2\*d^3\*e^8 - 5\*a^4\*c\*d\*e^10)\*x^4 + (6\*c^5\*d^10\*e - 5\*a\*c^4\*d^8\*e^3 - 16\*a^2\*c^3\*d^6\*e^5 + 18\*a^3\*c^2\*d^4\*e^7 + 2\*a^4\*c\*d^2\*e^9 - 5\*a^5\*e^11)\*x^3 + (3\*c^5\*d^11 + 2\*a\*c^4\*d^9\*e^2 - 14\*a^2\*c^3\*d^7\*e^4 + 19\*a^4\*c\*d^3\*e^8 - 10\*a^5\*d\*e^10)\*x^2 + (3\*a\*c^4\*d^10\*e - 4\*a^2\*c^3\*d^8\*e^3 - 6\*a^3\*c^2\*d^6\*e^5 + 12\*a^4\*c\*d^4\*e^7 - 5\*a^5\*d^2\*e^9)\*x)\*log((4\*(2\*a^2\*d^2\*e^2 + (a\*c\*d^3\*e + a^2\*d\*e^3)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) + (8\*a^2\*d^2\*e^2 + (c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4)\*x^2 + 8\*(a\*c\*d^3\*e + a^2\*d\*e^3)\*x)\*sqrt(a\*d\*e))/x^2))/(((a^2\*c^4\*d^10\*e^4 - 3\*a^3\*c^3\*d^8\*e^6 + 3\*a^4\*c^2\*d^6\*e^8 - a^5\*c\*d^4\*e^10)\*x^4 + (2\*a^2\*c^4\*d^11\*e^3 - 5\*a^3\*c^3\*d^9\*e^5 + 3\*a^4\*c^2\*d^7\*e^7 + a^5\*c\*d^5\*e^9 - a^6\*d^3\*e^11)\*x^3 + (a^2\*c^4\*d^12\*e^2 - a^3\*c^3\*d^10\*e^4 - 3\*a^4\*c^2\*d^8\*e^6 + 5\*a^5\*c\*d^6\*e^8 - 2\*a^6\*d^4\*e^10)\*x^2 + (a^3\*c^3\*d^11\*e^3 - 3\*a^4\*c^2\*d^9\*e^5 + 3\*a^5\*c\*d^7\*e^7 - a^6\*d^5\*e^9)\*x)\*sqrt(a\*d\*e)), -1/6\*(2\*(3\*a\*c^3\*d^8\*e - 9\*a^2\*c^2\*d^6\*e^3 + 9\*a^3\*c\*d^4\*e^5 - 3\*a^4\*d^2\*e^7 + (9\*c^4\*d^7\*e^2 - 9\*a\*c^3\*d^5\*e^4 + 31\*a^2\*c^2\*d^3\*e^6 - 15\*a^3\*c\*d\*e^8)\*x^3 + (18\*c^4\*d^8\*e - 15\*a\*c^3\*d^6\*e^3 + 33\*a^2\*c^2\*d^4\*e^5 + 11\*a^3\*c\*d^2\*e^7 - 15\*a^4\*e^9)\*x^2 + (9\*c^4\*d^9 - 3\*a\*c^3\*d^7\*e^2 - 9\*a^2\*c^2\*d^5\*e^4 + 39\*a^3\*c\*d^3\*e^6 - 20\*a^4\*d\*e^8)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e) - 3\*((3\*c^5\*d^9\*e^2 - 4\*a\*c^4\*d^7\*e^4 - 6\*a^2\*c^3\*d^5\*e^6 + 12\*a^3\*c^2\*d^3\*e^8 - 5\*a^4\*c\*d\*e^10)\*x^4 + (6\*c^5\*d^10\*e - 5\*a\*c^4\*d^8\*e^3 - 16\*a^2\*c^3\*d^6\*e^5 + 18\*a^3\*c^2\*d^4\*e^7 + 2\*a^4\*c\*d^2\*e^9 - 5\*a^5\*e^11)\*x^3 + (3\*c^5\*d^11 + 2\*a\*c^4\*d^9\*e^2 - 14\*a^2\*c^3\*d^7\*e^4 + 19\*a^4\*c\*d^3\*e^8 - 10\*a^5\*d\*e^10)\*x^2 + (3\*a\*c^4\*d^10\*e - 4\*a^2\*c^3\*d^8\*e^3 - 6\*a^3\*c^2\*d^6\*e^5 + 12\*a^4\*c\*d^4\*e^7 - 5\*a^5\*d^2\*e^9)\*x)\*arctan(1/2\*(2\*a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-a\*d\*e)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*a\*d\*e)))/(((a^2\*c^4\*d^10\*e^4 - 3\*a^3\*c^3\*d^8\*e^6 + 3\*a^4\*c^2\*d^6\*e^8 - a^5\*c\*d^4\*e^10)\*x^4 + (2\*a^2\*c^4\*d^11\*e^3 - 5\*a^3\*c^3\*d^9\*e^5 + 3\*a^4\*c^2\*d^7\*e^7 + a^5\*c\*d^5\*e^9 - a^6\*d^3\*e^11)\*x^3 + (a^2\*c^4\*d^12\*e^2 - a^3\*c^3\*d^10\*e^4 - 3\*a^4\*c^2\*d^8\*e^6 + 5\*a^5\*c\*d^6\*e^8 - 2\*a^6\*d^4\*e^10)\*x^2 + (a^3\*c^3\*d^11\*e^3 - 3\*a^4\*c^2\*d^9\*e^5 + 3\*a^5\*c\*d^7\*e^7 - a^6\*d^5\*e^9)\*x)\*sqrt(-a\*d\*e))]

---

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 ((d + ex)(ae + cdx))^{\frac{3}{2}} (d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)`

[Out] `Integral(1/(x**2*((d + e*x)*(a*e + c*d*x))**(3/2)*(d + e*x)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

`[undef, undef, undef, 1]`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^2),x, algo`

[Out] `[undef, undef, undef, 1]`

$$3.485 \quad \int \frac{1}{x^3(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx$$

**Optimal.** Leaf size=522

$$\begin{aligned} & \frac{5(7a^2e^4 + 6acd^2e^2 + 3c^2d^4) \tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8a^{7/2}d^{9/2}e^{7/2}} \\ & + \frac{2(-7a^3e^6 + cdex(-7a^2e^4 + 12acd^2e^2 + 3c^2d^4) + 11a^2cd^2e^4 + ac^2d^4e^2 + 3c^3d^6)}{3ad^2ex^2(cd^2 - ae^2)^3\sqrt{x(ae^2 + cd^2) + ade + cdex^2}} \\ & - \frac{(-35a^3e^6 + 61a^2cd^2e^4 - 9ac^2d^4e^2 + 15c^3d^6)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{6a^2d^3e^2x^2(cd^2 - ae^2)^3} \\ & + \frac{(-105a^4e^8 + 190a^3cd^2e^6 - 36a^2c^2d^4e^4 - 30ac^3d^6e^2 + 45c^4d^8)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{12a^3d^4e^3x(cd^2 - ae^2)^3} \\ & - \frac{2e(ae + cdx)}{3dx^2(cd^2 - ae^2)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}} \end{aligned}$$

[Out]  $(-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 11*a^2*c*d^2*e^4 - 7*a^3*e^6 + c*d*e*(3*c^2*d^4 + 12*a*c*d^2*e^2 - 7*a^2*e^4)*x))/(3*a*d^2*e*(c*d^2 - a*e^2)^3*x^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((15*c^3*d^6 - 9*a*c^2*d^4*e^2 + 61*a^2*c*d^2*e^4 - 35*a^3*e^6)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(6*a^2*d^3*e^2*(c*d^2 - a*e^2)^3*x^2) + ((45*c^4*d^8 - 30*a*c^3*d^6*e^2 - 36*a^2*c^2*d^4*e^4 + 190*a^3*c*d^2*e^6 - 105*a^4*e^8)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*a^3*d^4*e^3*(c*d^2 - a*e^2)^3*x) - (5*(3*c^2*d^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*\text{ArcTanh}[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(8*a^{(7/2)}*d^{(9/2)}*e^{(7/2)})$

**Rubi [A]** time = 2.03881, antiderivative size = 522, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\begin{aligned} & \frac{5(7a^2e^4 + 6acd^2e^2 + 3c^2d^4) \tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{8a^{7/2}d^{9/2}e^{7/2}} \\ & + \frac{2(-7a^3e^6 + cdex(-7a^2e^4 + 12acd^2e^2 + 3c^2d^4) + 11a^2cd^2e^4 + ac^2d^4e^2 + 3c^3d^6)}{3ad^2ex^2(cd^2 - ae^2)^3\sqrt{x(ae^2 + cd^2) + ade + cdex^2}} \\ & - \frac{(-35a^3e^6 + 61a^2cd^2e^4 - 9ac^2d^4e^2 + 15c^3d^6)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{6a^2d^3e^2x^2(cd^2 - ae^2)^3} \\ & + \frac{(-105a^4e^8 + 190a^3cd^2e^6 - 36a^2c^2d^4e^4 - 30ac^3d^6e^2 + 45c^4d^8)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{12a^3d^4e^3x(cd^2 - ae^2)^3} \\ & - \frac{2e(ae + cdx)}{3dx^2(cd^2 - ae^2)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^3*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}), x]$

[Out]  $(-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*x^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 11*a^2*c*d^2*e^4 - 7*a^3*e^6 + c*d*e*(3*c^2*d^4 + 12*a*c*d^2*e^2 - 7*a^2*e^4)*x))/(3*a*d^2*e*(c*d^2 - a*e^2)^3*x^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - ((15*c^3*d^6 - 9*a*c^2*d^4*e^2 + 61*a^2*c*d^2*e^4 - 35*a^3*e^6)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(6*a^2*d^3*e^2*(c*d^2 - a*e^2)^3*x^2) + ((45*c^4*d^8 - 30*a*c^3*d^6*e^2 - 36*a^2*c^2*d^4*e^4 + 190*a^3*c*d^2*e^6 - 105*a^4*e^8)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*a^3*d^4*e^3*x(ae^2 + cd^2) + ade + cdex^2)^{3/2}$

$$3*(c*d^2 - a*e^2)^3*x - (5*(3*c^2*d^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*\text{ArcTanh}[(2*a*d*e + (c*d^2 + a*e^2)*x)/(2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(8*a^(7/2)*d^(9/2)*e^(7/2))$$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x**3/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)`

[Out] Timed out

**Mathematica [A]** time = 1.9389, size = 362, normalized size = 0.69

$$\frac{15 \log(x)(d+ex)^{3/2}(7a^2e^4+6acd^2e^2+3c^2d^4)(ae+cdx)^{3/2}}{a^{7/2}} - \frac{15(d+ex)^{3/2}(7a^2e^4+6acd^2e^2+3c^2d^4)(ae+cdx)^{3/2} \log\left(2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{d+ex}\sqrt{ae+cdx+ae(2d+ex)+cd}\right)}{a^{7/2}}$$

$$24d^{9/2}e^{7/2}((d+ex)(ae+cdx+ae(2d+ex)+cd))^{3/2}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x^3*(d+e*x)*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^(3/2)),x]`

$$\begin{aligned} & (2*\text{Sqrt}[d]*\text{Sqrt}[e]*(a*e + c*d*x)^2*(d + e*x)^2*((-6*d*e)/(a^2*x^2) \\ & + (21*c*d^2 + 33*a*e^2)/(a^3*x) - (24*c^5*d^9)/(a^3*(-(c*d^2) + \\ & a*e^2)^3*(a*e + c*d*x)) + (8*d^7*e^7)/((c*d^2 - a*e^2)^2*(d + e*x) \\ & ^2) + (8*e^7*(14*c*d^2 - 9*a*e^2))/((c*d^2 - a*e^2)^3*(d + e*x))) \\ & + (15*(3*c^2*d^4 + 6*a*c*d^2*e^2 + 7*a^2*e^4)*(a*e + c*d*x)^(3/2) \\ & )*(d + e*x)^(3/2)*\text{Log}[x])/a^(7/2) - (15*(3*c^2*d^4 + 6*a*c*d^2*e^2 \\ & + 7*a^2*e^4)*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*\text{Log}[c*d^2*x + \\ & 2*\text{Sqrt}[a]*\text{Sqrt}[d]*\text{Sqrt}[e]*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x] + a*e*( \\ & 2*d + e*x)])/a^(7/2))/(24*d^(9/2)*e^(7/2)*((a*e + c*d*x)*(d + e*x) \\ & )^(3/2)) \end{aligned}$$

**Maple [B]** time = 0.025, size = 1319, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^3/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)`

$$\begin{aligned} & -15/4*d^3/a^3/e^2/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+ \\ & c*d^2)*x+c*d*e*x^2)^(1/2)*x^c^4+7/4/d/a^2/e/(-a^2*e^4+2*a*c*d^2*e \\ & ^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x^c^2+9/4/d^3 \\ & /a/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+35/8/d^4/a^2/e/(a*d*e+ \\ & (a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)+15/8/a^3/e^3/(a*d*e+(a*e^2+c*d^2) \\ & )*x+c*d*e*x^2)^(1/2)*c^2-35/4/d^3*e^4/(-a^2*e^4+2*a*c*d^2*e^2-c^2 \\ & *d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*x^c-5/4*d/a^2/(-a^2 \\ & *e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/ \\ & 2)*x^c^3-5/2*d^2/a^2/e/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a \\ & *e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^3-15/8*d^4/a^3/e^3/(-a^2*e^4+2*a \\ & *c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)*c^4-1 \\ & 5/4/d^2/a^2/e/(a*d*e)^(1/2)*\ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e) \\ & ^2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2))/x)*c+5/4/d/a^2/e \end{aligned}$$

$$\begin{aligned} & 2/x/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)}*c+1/4/a*e/(-a^2*e^4+2 \\ & *a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)}*c^2 \\ & -15/8/a^3/e^3/(a*d*e)^{(1/2)}*\ln((2*a*d*e+(a*e^2+c*d^2)*x+2*(a*d*e) \\ & ^{(1/2)}*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)})/x)^*c^2+15/4/d^2/a \\ & ^2/e/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)}*c-35/8/d^4*a*e^5/(-a \\ & ^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)} \\ & -7/2/d^2*e^3/(-a^2*e^4+2*a*c*d^2*e^2-c^2*d^4)/(a*d*e+(a*e^2+c \\ & *d^2)*x+c*d*e*x^2)^{(1/2)}*c-35/8/d^4/a*e/(a*d*e)^{(1/2)}*\ln((2*a*d*e \\ & +(a*e^2+c*d^2)*x+2*(a*d*e)^{(1/2)}*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2 \\ & )^{(1/2)})/x)-1/2/d^2/a/e/x^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)} \\ & +2/3/d^3*e^2/(a*e^2-c*d^2)/(x+d/e)/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2) \\ & *(x+d/e))^{(1/2)}-16/3/d*e^4*c^2/(a*e^2-c*d^2)^3/(c*d*e*(x+d/e)^2 \\ & +(a*e^2-c*d^2)*(x+d/e))^{(1/2)}*x-8/3/d^2*e^5*c/(a*e^2-c*d^2)^3/(c* \\ & d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{(1/2)}*a-8/3*e^3*c^2/(a*e^2-c \\ & *d^2)^3/(c*d*e*(x+d/e)^2+(a*e^2-c*d^2)*(x+d/e))^{(1/2)} \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}(ex + d)x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(e\*x + d)\*x^3), x, algo

[Out] integrate(1/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(e\*x + d)\*x^3), x)

**Fricas [A]** time = 3.66088, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(e\*x + d)\*x^3), x, algo

[Out] [-1/48\*(4\*(6\*a^2\*c^3\*d^9\*e^2 - 18\*a^3\*c^2\*d^7\*e^4 + 18\*a^4\*c\*d^5\*e^6 - 6\*a^5\*d^3\*e^8 - (45\*c^5\*d^9\*e^2 - 30\*a\*c^4\*d^7\*e^4 - 36\*a^2\*c^3\*d^5\*e^6 + 190\*a^3\*c^2\*d^3\*e^8 - 105\*a^4\*c\*d^10)\*x^4 - (90\*c^5\*d^10\*e - 45\*a\*c^4\*d^8\*e^3 - 84\*a^2\*c^3\*d^6\*e^5 + 222\*a^3\*c^2\*d^4\*e^7 + 50\*a^4\*c\*d^2\*e^9 - 105\*a^5\*e^11)\*x^3 - (45\*c^5\*d^11 - 6\*6\*a^2\*c^3\*d^7\*e^4 - 12\*a^3\*c^2\*d^5\*e^6 + 237\*a^4\*c\*d^3\*e^8 - 140\*a^5\*d^10)\*x^2 - 3\*(5\*a\*c^4\*d^10\*e - 8\*a^2\*c^3\*d^8\*e^3 - 6\*a^3\*c^2\*d^6\*e^5 + 16\*a^4\*c\*d^4\*e^7 - 7\*a^5\*d^2\*e^9)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(a\*d\*e) - 15\*((3\*c^6\*d^11\*e^2 - 3\*a\*c^5\*d^9\*e^4 - 2\*a^2\*c^4\*d^7\*e^6 - 6\*a^3\*c^3\*d^5\*e^8 + 15\*a^4\*c^2\*d^3\*e^10 - 7\*a^5\*c\*d^12)\*x^5 + (6\*c^6\*d^12\*e - 3\*a\*c^5\*d^10\*e^3 - 7\*a^2\*c^4\*d^8\*e^5 - 14\*a^3\*c^3\*d^6\*e^7 + 24\*a^4\*c^2\*d^4\*e^9 + a^5\*c\*d^2\*e^11 - 7\*a^6\*e^13)\*x^4 + (3\*c^6\*d^13 + 3\*a\*c^5\*d^11\*e^2 - 8\*a^2\*c^4\*d^9\*e^4 - 10\*a^3\*c^3\*d^7\*e^6 + 3\*a^4\*c^2\*d^5\*e^8 + 23\*a^5\*c\*d^3\*e^10 - 14\*a^6\*d^12)\*x^3 + (3\*a\*c^5\*d^12\*e - 3\*a^2\*c^4\*d^10\*e^3 - 2\*a^3\*c^3\*d^8\*e^5 - 6\*a^4\*c^2\*d^6\*e^7 + 15\*a^5\*c\*d^4\*e^9 - 7\*a^6\*d^2\*e^11)\*x^2)\*log(-(4\*(2\*a^2\*d^2\*e^2 + (a\*c\*d^3\*e + a^2\*d^3\*e^3)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x) - (8\*a^2\*d^2\*e^2 + (c^2\*d^4 + 6\*a\*c\*d^2\*e^2 + a^2\*e^4)\*x^2 + 8\*(a\*c\*d^3\*e + a^2\*d^3\*e^3)\*x)\*sqrt(a\*d\*e))/x^2)/((a^3\*c^4\*d^11\*e^5 - 3\*a^4\*c^3\*d^9\*e^7 + 3\*a^5\*c^2\*d^7\*e^9 - a^6\*c\*d^5\*e^11)\*x^5 + (2\*a^3\*c^4\*d^12\*e^4 - 5\*a^4\*c^3\*d^10\*e^6 + 3\*a^5\*c^2\*d^8\*e^8 + a^6\*c\*d^6\*e^10 - a^7\*d^4\*e^12)\*x^4 + (a^3\*c^4\*d^13\*e^3 - a^4\*c^3\*d^11\*e^5 - 3\*a^5\*c^2\*d^9\*e^7 + 5\*a^6\*c\*d^7\*e^9 - 2\*a^7\*d^5\*e^11)\*x^3 + (a^4\*c^3\*d^12\*e^4 - 3\*a^5\*c^2\*d^10\*e^6 + 3\*a^6\*c\*d^8\*e^8 - a^7\*d^6\*e^10)\*x^2)\*sqrt(a\*d\*e)), -1/24\*(2\*(6\*a^2\*c^3\*d^9\*e^2 - 18\*a^3\*c^2\*d^7\*e^4 + 18\*a^4\*c\*d^5\*e^6 - 6\*a^5\*d^3\*e^8 - (45\*c^5\*d^9\*e^2

$$\begin{aligned}
& - 30*a*c^4*d^7*e^4 - 36*a^2*c^3*d^5*e^6 + 190*a^3*c^2*d^3*e^8 - 105*a^4*c*d*e^{10})*x^4 - (90*c^5*d^{10}*e - 45*a*c^4*d^8*e^3 - 84*a^2*c^3*d^6*e^5 + 222*a^3*c^2*d^4*e^7 + 50*a^4*c*d^2*e^9 - 105*a^5*e^{11})*x^3 - (45*c^5*d^{11} - 66*a^2*c^3*d^7*e^4 - 12*a^3*c^2*d^5*e^6 + 237*a^4*c*d^3*e^8 - 140*a^5*d*e^{10})*x^2 - 3*(5*a*c^4*d^{10}*e - 8*a^2*c^3*d^8*e^3 - 6*a^3*c^2*d^6*e^5 + 16*a^4*c*d^4*e^7 - 7*a^5*d^2*e^9)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e) + 15*((3*c^6*d^{11}*e^2 - 3*a*c^5*d^9*e^4 - 2*a^2*c^4*d^7*e^6 - 6*a^3*c^3*d^5*e^8 + 15*a^4*c^2*d^3*e^{10} - 7*a^5*c*d*e^{12})*x^5 + (6*c^6*d^{12}*e - 3*a*c^5*d^{10}*e^3 - 7*a^2*c^4*d^8*e^5 - 14*a^3*c^3*d^6*e^7 + 24*a^4*c^2*d^4*e^9 + a^5*c*d^2*e^{11} - 7*a^6*e^{13})*x^4 + (3*c^6*d^{13} + 3*a*c^5*d^{11}*e^2 - 8*a^2*c^4*d^9*e^4 - 10*a^3*c^3*d^7*e^6 + 3*a^4*c^2*d^5*e^8 + 23*a^5*c*d^3*e^{10} - 14*a^6*d*e^{12})*x^3 + (3*a*c^5*d^{12}*e - 3*a^2*c^4*d^{10}*e^3 - 2*a^3*c^3*d^8*e^5 - 6*a^4*c^2*d^6*e^7 + 15*a^5*c*d^4*e^9 - 7*a^6*d^2*e^{11})*x^2)*arctan(1/2*(2*a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-a*d*e)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*a*d*e))/(((a^3*c^4*d^{11}*e^5 - 3*a^4*c^3*d^9*e^7 + 3*a^5*c^2*d^7*e^9 - a^6*c*d^5*e^{11})*x^5 + (2*a^3*c^4*d^{12}*e^4 - 5*a^4*c^3*d^{10}*e^6 + 3*a^5*c^2*d^8*e^8 + a^6*c*d^6*e^{10} - a^7*d^4*e^{12})*x^4 + (a^3*c^4*d^{13}*e^3 - a^4*c^3*d^{11}*e^5 - 3*a^5*c^2*d^9*e^7 + 5*a^6*c*d^7*e^9 - 2*a^7*d^5*e^{11})*x^3 + (a^4*c^3*d^{12}*e^4 - 3*a^5*c^2*d^{10}*e^6 + 3*a^6*c*d^8*e^8 - a^7*d^6*e^{10})*x^2)*sqrt(-a*d*e)]
\end{aligned}$$

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*3/(e\*x+d)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2),x)

[Out] Timed out

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

[undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(e\*x + d)\*x^3),x, algo

[Out] [undef, undef, undef, 1]



$$3.486 \quad \int \frac{1}{x^4(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx$$

**Optimal.** Leaf size=664

$$\begin{aligned} & \frac{2(-9a^3e^6 + cdex(-9a^2e^4 + 14acd^2e^2 + 3c^2d^4) + 13a^2cd^2e^4 + ac^2d^4e^2 + 3c^3d^6)}{3ad^2ex^3(cd^2 - ae^2)^3 \sqrt{x(ae^2 + cd^2) + ade + cdex^2}} \\ & - \frac{(-21a^3e^6 + 33a^2cd^2e^4 - 3ac^2d^4e^2 + 7c^3d^6) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{3a^2d^3e^2x^3(cd^2 - ae^2)^3} \\ & + \frac{5(21a^3e^6 + 21a^2cd^2e^4 + 15ac^2d^4e^2 + 7c^3d^6) \tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{16a^{9/2}d^{11/2}e^{9/2}} \\ & + \frac{(-105a^4e^8 + 168a^3cd^2e^6 - 18a^2c^2d^4e^4 - 16ac^3d^6e^2 + 35c^4d^8) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{12a^3d^4e^3x^2(cd^2 - ae^2)^3} \\ & - \frac{(-315a^5e^{10} + 525a^4cd^2e^8 - 78a^3c^2d^4e^6 - 54a^2c^3d^6e^4 - 55ac^4d^8e^2 + 105c^5d^{10}) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{24a^4d^5e^4x(cd^2 - ae^2)^3} \\ & - \frac{2e(ae + cdx)}{3dx^3(cd^2 - ae^2)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}} \end{aligned}$$

[Out]  $(-2 * e * (a * e + c * d * x)) / (3 * d * (c * d^2 - a * e^2) * x^3 * (a * d * e + (c * d^2 + a * e^2) * x + c * d * e * x^2)^{(3/2)}) + (2 * (3 * c^3 * d^6 + a * c^2 * d^4 * e^2 + 13 * a^2 * c * d^2 * e^4 - 9 * a^3 * e^6 + c * d * e * (3 * c^2 * d^4 + 14 * a * c * d^2 * e^2 - 9 * a^2 * e^4) * x)) / (3 * a * d^2 * e * (c * d^2 - a * e^2)^3 * x^3 * \text{Sqrt}[a * d * e + (c * d^2 + a * e^2) * x + c * d * e * x^2]) - ((7 * c^3 * d^6 - 3 * a * c^2 * d^4 * e^2 + 33 * a^2 * c * d^2 * e^4 - 21 * a^3 * e^6) * \text{Sqrt}[a * d * e + (c * d^2 + a * e^2) * x + c * d * e * x^2]) / (3 * a^2 * d^3 * e^2 * x^3 * (c * d^2 - a * e^2)^3) + ((35 * c^4 * d^8 - 16 * a * c^3 * d^6 * e^2 - 18 * a^2 * c^2 * d^4 * e^4 + 168 * a^3 * c * d^2 * e^6 - 105 * a^4 * e^8) * \text{Sqrt}[a * d * e + (c * d^2 + a * e^2) * x + c * d * e * x^2]) / (12 * a^3 * d^4 * e^3 * (c * d^2 - a * e^2)^3 * x^2) - ((105 * c^5 * d^{10} - 55 * a * c^4 * d^8 * e^2 - 54 * a^2 * c^3 * d^6 * e^4 - 78 * a^3 * c^2 * d^4 * e^6 + 525 * a^4 * c * d^2 * e^8 - 315 * a^5 * e^{10}) * \text{Sqrt}[a * d * e + (c * d^2 + a * e^2) * x + c * d * e * x^2]) / (24 * a^4 * d^5 * e^4 * x * (c * d^2 - a * e^2)^3) + (5 * (7 * c^3 * d^6 + 15 * a * c^2 * d^4 * e^2 + 21 * a^2 * c * d^2 * e^4 + 21 * a^3 * e^6) * \text{ArcTanh}[(2 * a * d * e + (c * d^2 + a * e^2) * x) / (2 * \text{Sqrt}[a] * \text{Sqrt}[d] * \text{Sqrt}[e] * \text{Sqrt}[a * d * e + (c * d^2 + a * e^2) * x + c * d * e * x^2])]) / (16 * a^{(9/2)} * d^{(11/2)} * e^{(9/2)})$

**Rubi [A]** time = 2.7994, antiderivative size = 664, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\begin{aligned} & \frac{2(-9a^3e^6 + cdex(-9a^2e^4 + 14acd^2e^2 + 3c^2d^4) + 13a^2cd^2e^4 + ac^2d^4e^2 + 3c^3d^6)}{3ad^2ex^3(cd^2 - ae^2)^3 \sqrt{x(ae^2 + cd^2) + ade + cdex^2}} \\ & - \frac{(-21a^3e^6 + 33a^2cd^2e^4 - 3ac^2d^4e^2 + 7c^3d^6) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{3a^2d^3e^2x^3(cd^2 - ae^2)^3} \\ & + \frac{5(21a^3e^6 + 21a^2cd^2e^4 + 15ac^2d^4e^2 + 7c^3d^6) \tanh^{-1}\left(\frac{x(ae^2+cd^2)+2ade}{2\sqrt{a}\sqrt{d}\sqrt{e}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}\right)}{16a^{9/2}d^{11/2}e^{9/2}} \\ & + \frac{(-105a^4e^8 + 168a^3cd^2e^6 - 18a^2c^2d^4e^4 - 16ac^3d^6e^2 + 35c^4d^8) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{12a^3d^4e^3x^2(cd^2 - ae^2)^3} \\ & - \frac{(-315a^5e^{10} + 525a^4cd^2e^8 - 78a^3c^2d^4e^6 - 54a^2c^3d^6e^4 - 55ac^4d^8e^2 + 105c^5d^{10}) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{24a^4d^5e^4x(cd^2 - ae^2)^3} \\ & - \frac{2e(ae + cdx)}{3dx^3(cd^2 - ae^2)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x^4 * (d + e * x) * (a * d * e + (c * d^2 + a * e^2) * x + c * d * e * x^2)^{(3/2)}), x]$

```
[Out] (-2*e*(a*e + c*d*x))/(3*d*(c*d^2 - a*e^2)*x^3*(a*d*e + (c*d^2 + a
*e^2)*x + c*d*e*x^2)^(3/2)) + (2*(3*c^3*d^6 + a*c^2*d^4*e^2 + 13*
a^2*c*d^2*e^4 - 9*a^3*e^6 + c*d*e*(3*c^2*d^4 + 14*a*c*d^2*e^2 - 9
*a^2*e^4)*x))/(3*a*d^2*e*(c*d^2 - a*e^2)^3*x^3*Sqrt[a*d*e + (c*d^
2 + a*e^2)*x + c*d*e*x^2]) - ((7*c^3*d^6 - 3*a*c^2*d^4*e^2 + 33*a
^2*c*d^2*e^4 - 21*a^3*e^6)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e
*x^2])/(3*a^2*d^3*e^2*(c*d^2 - a*e^2)^3*x^3) + ((35*c^4*d^8 - 16*
a*c^3*d^6*e^2 - 18*a^2*c^2*d^4*e^4 + 168*a^3*c*d^2*e^6 - 105*a^4*
e^8)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*a^3*d^4*e^3
*(c*d^2 - a*e^2)^3*x^2) - ((105*c^5*d^10 - 55*a*c^4*d^8*e^2 - 54*
a^2*c^3*d^6*e^4 - 78*a^3*c^2*d^4*e^6 + 525*a^4*c*d^2*e^8 - 315*a^
5*e^10)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(24*a^4*d^5*
e^4*(c*d^2 - a*e^2)^3*x) + (5*(7*c^3*d^6 + 15*a*c^2*d^4*e^2 + 21*
a^2*c*d^2*e^4 + 21*a^3*e^6)*ArcTanh[(2*a*d*e + (c*d^2 + a*e^2)*x)
/(2*Sqrt[a]*Sqrt[d]*Sqrt[e]*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*
e*x^2])])/(16*a^(9/2)*d^(11/2)*e^(9/2))
```

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate(1/x**4/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)
```

[Out] Timed out

**Mathematica [A]** time = 2.5358, size = 429, normalized size = 0.65

$$-\frac{15 \log(x)(d+ex)^{3/2}(21a^3e^6+21a^2cd^2e^4+15ac^2d^4e^2+7c^3d^6)(ae+cdx)^{3/2}}{a^{9/2}} + \frac{15(d+ex)^{3/2}(21a^3e^6+21a^2cd^2e^4+15ac^2d^4e^2+7c^3d^6)(ae+cdx)^{3/2} \log(2\sqrt{a}\sqrt{d}\sqrt{e})}{a^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)),x]
```

```
[Out] (2*Sqrt[d]*Sqrt[e]*(a*e + c*d*x)^2*(d + e*x)^2*((2*c*d^2*e*(11*d
- 58*e*x))/(a^3*x^2) + (192*a*e^11)/((c*d^2 - a*e^2)^3*(d + e*x))
+ (e^2*(-8*d^2 + 34*d*e*x - 123*e^2*x^2))/(a^2*x^3) + ((-57*c^2*
d^4)/x + (48*c^6*d^11)/((-c*d^2) + a*e^2)^3*(a*e + c*d*x))/a^4
+ (16*d*e^9*(a*e^2 - c*d*(18*d + 17*e*x)))/((c*d^2 - a*e^2)^3*(d
+ e*x)^2) - (15*(7*c^3*d^6 + 15*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^4
+ 21*a^3*e^6)*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*Log[x])/a^(9/2
) + (15*(7*c^3*d^6 + 15*a*c^2*d^4*e^2 + 21*a^2*c*d^2*e^4 + 21*a^3
*e^6)*(a*e + c*d*x)^(3/2)*(d + e*x)^(3/2)*Log[c*d^2*x + 2*Sqrt[a]
*Sqrt[d]*Sqrt[e]*Sqrt[a*e + c*d*x]*Sqrt[d + e*x] + a*e*(2*d + e*x
)])/a^(9/2))/(48*d^(11/2)*e^(9/2)*((a*e + c*d*x)*(d + e*x))^(3/2)
)
```

**Maple [B]** time = 0.03, size = 1705, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)
```

[Out] 
$$\frac{8}{3} \frac{d^3 e^6 c}{(a^2 e^2 - c^2 d^2)^3} \frac{1}{(c d e (x+d/e)^2 + (a^2 e^2 - c^2 d^2) (x+d/e))^{1/2}} + \frac{16}{3} \frac{d^2 e^5 c^2}{(a^2 e^2 - c^2 d^2)^3} \frac{1}{(c d e (x+d/e)^2 + (a^2 e^2 - c^2 d^2) (x+d/e))^{1/2}} x - \frac{1}{6} \frac{e}{(-a^2 e^4 + 2 a^2 c^2 d^2 e^2 - c^2 d^4)} \frac{1}{(a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}} x^3 + \frac{105}{8} \frac{d^4 e^5}{(-a^2 e^4 + 2 a^2 c^2 d^2 e^2 - c^2 d^4)} \frac{1}{(a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}} x^5 + \frac{7}{12} \frac{d}{a^2 e^2} \frac{1}{(-a^2 e^4 + 2 a^2 c^2 d^2 e^2 - c^2 d^4)} \frac{1}{(a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}} x^7 + \frac{43}{24} \frac{d}{a^2 e^2} \frac{1}{(-a^2 e^4 + 2 a^2 c^2 d^2 e^2 - c^2 d^4)} \frac{1}{(a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}} x^9 - \frac{43}{24} \frac{d}{a^2 e^2} \frac{1}{(-a^2 e^4 + 2 a^2 c^2 d^2 e^2 - c^2 d^4)} \frac{1}{(a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}} x^{11} + \frac{155}{48} \frac{d^3}{a^3 e^2} \frac{1}{(-a^2 e^4 + 2 a^2 c^2 d^2 e^2 - c^2 d^4)} \frac{1}{(a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}} x^{13} + \frac{35}{16} \frac{d^5}{a^4 e^4} \frac{1}{(-a^2 e^4 + 2 a^2 c^2 d^2 e^2 - c^2 d^4)} \frac{1}{(a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}} x^{15} + \frac{75}{16} \frac{d}{a^3 e^2} \frac{1}{(-a^2 e^4 + 2 a^2 c^2 d^2 e^2 - c^2 d^4)} \frac{1}{(a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}} \ln\left(\frac{2 a d e + (a^2 e^2 + c^2 d^2) x + 2 (a d e)^{1/2} (a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}}{x}\right) + \frac{35}{16} \frac{d}{a^4 e^4} \frac{1}{(-a^2 e^4 + 2 a^2 c^2 d^2 e^2 - c^2 d^4)} \frac{1}{(a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}} \ln\left(\frac{2 a d e + (a^2 e^2 + c^2 d^2) x + 2 (a d e)^{1/2} (a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}}{x}\right) x^3 - \frac{17}{6} \frac{d^2}{a^2 e} \frac{1}{(-a^2 e^4 + 2 a^2 c^2 d^2 e^2 - c^2 d^4)} \frac{1}{(a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}} x^5 - \frac{105}{16} \frac{d^5}{a^5 e^2} \frac{1}{(-a^2 e^4 + 2 a^2 c^2 d^2 e^2 - c^2 d^4)} \frac{1}{(a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}} x^7 - \frac{105}{16} \frac{d^3}{a^2 e} \frac{1}{(-a^2 e^4 + 2 a^2 c^2 d^2 e^2 - c^2 d^4)} \frac{1}{(a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}} x^9 + \frac{13}{12} \frac{d^3}{a x^2} \frac{1}{(-a^2 e^4 + 2 a^2 c^2 d^2 e^2 - c^2 d^4)} \frac{1}{(a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}} + \frac{35}{8} \frac{d^4}{a^4 e^3} \frac{1}{(-a^2 e^4 + 2 a^2 c^2 d^2 e^2 - c^2 d^4)} \frac{1}{(a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}} x^5 - \frac{41}{12} \frac{d^2}{a^2 e^3} \frac{1}{(-a^2 e^4 + 2 a^2 c^2 d^2 e^2 - c^2 d^4)} \frac{1}{(a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}} x^7 + \frac{25}{12} \frac{d^2}{a^3 e} \frac{1}{(-a^2 e^4 + 2 a^2 c^2 d^2 e^2 - c^2 d^4)} \frac{1}{(a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}} x^9 + \frac{105}{16} \frac{d^5}{a^5 e^2} \frac{1}{(-a^2 e^4 + 2 a^2 c^2 d^2 e^2 - c^2 d^4)} \frac{1}{(a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}} \ln\left(\frac{2 a d e + (a^2 e^2 + c^2 d^2) x + 2 (a d e)^{1/2} (a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}}{x}\right) - \frac{1}{3} \frac{d^2}{a e} \frac{1}{(-a^2 e^4 + 2 a^2 c^2 d^2 e^2 - c^2 d^4)} \frac{1}{(a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}} x^3 - \frac{35}{24} \frac{d^3}{a^3 e^3} \frac{1}{(-a^2 e^4 + 2 a^2 c^2 d^2 e^2 - c^2 d^4)} \frac{1}{(a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}} x^5 + \frac{23}{24} \frac{d}{a^2 e} \frac{1}{(-a^2 e^4 + 2 a^2 c^2 d^2 e^2 - c^2 d^4)} \frac{1}{(a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}} x^7 + \frac{105}{16} \frac{d^3}{a^2 e} \frac{1}{(-a^2 e^4 + 2 a^2 c^2 d^2 e^2 - c^2 d^4)} \frac{1}{(a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}} \ln\left(\frac{2 a d e + (a^2 e^2 + c^2 d^2) x + 2 (a d e)^{1/2} (a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}}{x}\right) x^9 + \frac{33}{48} \frac{d^3 e^4}{(-a^2 e^4 + 2 a^2 c^2 d^2 e^2 - c^2 d^4)} \frac{1}{(a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}} x^{11} - \frac{89}{24} \frac{d^4}{a^4 e} \frac{1}{(-a^2 e^4 + 2 a^2 c^2 d^2 e^2 - c^2 d^4)} \frac{1}{(a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}} x^{13} - \frac{75}{16} \frac{d}{a^3 e^2} \frac{1}{(-a^2 e^4 + 2 a^2 c^2 d^2 e^2 - c^2 d^4)} \frac{1}{(a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}} x^{15} + \frac{105}{16} \frac{d^5}{a^5 e^6} \frac{1}{(-a^2 e^4 + 2 a^2 c^2 d^2 e^2 - c^2 d^4)} \frac{1}{(a d e + (a^2 e^2 + c^2 d^2) x + c d e x^2)^{1/2}} x^{17} + \frac{8}{3} \frac{d^4 e^4 c^2}{(a^2 e^2 - c^2 d^2)^3} \frac{1}{(c d e (x+d/e)^2 + (a^2 e^2 - c^2 d^2) (x+d/e))^{1/2}} - \frac{2}{3} \frac{d^4 e^3}{(a^2 e^2 - c^2 d^2)} \frac{1}{(x+d/e)} \frac{1}{(c d e (x+d/e)^2 + (a^2 e^2 - c^2 d^2) (x+d/e))^{1/2}}$$

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**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}(ex + d)x^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(e\*x + d)\*x^4), x, algo

[Out] integrate(1/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(e\*x + d)\*x^4), x)

---

**Ericas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(e\*x + d)\*x^4), x, algo

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**4/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

`[undef, undef, undef, 1]`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(e*x + d)*x^4),x, algo`

[Out] `[undef, undef, undef, 1]`

$$3.487 \quad \int \frac{x^2}{(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}} dx$$

**Optimal.** Leaf size=259

$$\frac{8(5a^2e^4 + 10acd^2e^2 + c^2d^4)(ae^2 + cd^2 + 2cdex)}{15e(cd^2 - ae^2)^5 \sqrt{x(ae^2 + cd^2) + ade + cdex^2}} - \frac{8(x(-2a^3e^6 + a^2cd^2e^4 + c^3d^6) + ade(cd^2 - ae^2)(3ae^2 + cd^2))}{15e(cd^2 - ae^2)^4(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}} + \frac{2x^2}{5(d+ex)(cd^2 - ae^2)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}$$

[Out]  $(2*x^2)/(5*(c*d^2 - a*e^2)*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) - (8*(a*d*e*(c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2) + (c^3*d^6 + a^2*c*d^2*e^4 - 2*a^3*e^6)*x))/(15*e*(c*d^2 - a*e^2)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) + (8*(c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(c*d^2 + a*e^2 + 2*c*d*e*x))/(15*e*(c*d^2 - a*e^2)^5*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi [A]** time = 0.670368, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$

$$\frac{8(5a^2e^4 + 10acd^2e^2 + c^2d^4)(ae^2 + cd^2 + 2cdex)}{15e(cd^2 - ae^2)^5 \sqrt{x(ae^2 + cd^2) + ade + cdex^2}} - \frac{8(x(-2a^3e^6 + a^2cd^2e^4 + c^3d^6) + ade(cd^2 - ae^2)(3ae^2 + cd^2))}{15e(cd^2 - ae^2)^4(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}} + \frac{2x^2}{5(d+ex)(cd^2 - ae^2)(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}), x]$

[Out]  $(2*x^2)/(5*(c*d^2 - a*e^2)*(d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) - (8*(a*d*e*(c*d^2 - a*e^2)*(c*d^2 + 3*a*e^2) + (c^3*d^6 + a^2*c*d^2*e^4 - 2*a^3*e^6)*x))/(15*e*(c*d^2 - a*e^2)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) + (8*(c^2*d^4 + 10*a*c*d^2*e^2 + 5*a^2*e^4)*(c*d^2 + a*e^2 + 2*c*d*e*x))/(15*e*(c*d^2 - a*e^2)^5*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi in Sympy [A]** time = 76.0519, size = 252, normalized size = 0.97

$$\frac{2x^2(ae + cdx)}{5(ae^2 - cd^2)(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{5}{2}}} + \frac{8(ade(ae^2 - cd^2)(3ae^2 + cd^2) + x(2a^3e^6 - a^2cd^2e^4 - c^3d^6))}{15e(ae^2 - cd^2)^4(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{3}{2}}} - \frac{4(2ae^2 + 2cd^2 + 4cdex)(5a^2e^4 + 10acd^2e^2 + c^2d^4)}{15e(ae^2 - cd^2)^5 \sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2), x)$

[Out]  $-2*x**2*(a*e + c*d*x)/(5*(a*e**2 - c*d**2)*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(5/2)) + 8*(a*d*e*(a*e**2 - c*d**2)*(3*a*e$

$$^2 + c*d^{**2}) + x*(2*a^{**3}*e^{**6} - a^{**2}*c*d^{**2}*e^{**4} - c^{**3}*d^{**6}))/((15*e*(a*e^{**2} - c*d^{**2}))^{**4}*(a*d*e + c*d*e*x^{**2} + x*(a*e^{**2} + c*d^{**2}))^{**3/2}) - 4*(2*a*e^{**2} + 2*c*d^{**2} + 4*c*d*e*x)*(5*a^{**2}*e^{**4} + 10*a*c*d^{**2}*e^{**2} + c^{**2}*d^{**4}))/((15*e*(a*e^{**2} - c*d^{**2}))^{**5}*sqrt(a*d*e + c*d*e*x^{**2} + x*(a*e^{**2} + c*d^{**2})))$$

**Mathematica [A]** time = 1.27308, size = 210, normalized size = 0.81

$$\frac{2(d+ex)^3(ae+cdx)^3 \left( -\frac{15a^2e^4+50acd^2e^2+8c^2d^4}{d+ex} - \frac{5a^2cde^2(ae^2-cd^2)}{(ae+cdx)^2} - \frac{3(cd^3-ade^2)^2}{(d+ex)^3} - \frac{5acde(5ae^2+6cd^2)}{ae+cdx} + \frac{2d(ae^2-cd^2)(5ae^2+2cd^2)}{(d+ex)^2} \right)}{15(ae^2-cd^2)^5((d+ex)(ae+cdx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((d+e\*x)\*(a\*d\*e+(c\*d^2+a\*e^2)\*x+c\*d\*e\*x^2)^(5/2)),x]

[Out] (2\*(a\*e+c\*d\*x)^3\*(d+e\*x)^3\*((-5\*a^2\*c\*d\*e^2\*(-(c\*d^2)+a\*e^2))/((a\*e+c\*d\*x)^2 - (5\*a\*c\*d\*e\*(6\*c\*d^2+5\*a\*e^2))/((a\*e+c\*d\*x) - (3\*(c\*d^3-a\*d\*e^2)^2)/(d+e\*x)^3 + (2\*d\*(-(c\*d^2)+a\*e^2)\*(2\*c\*d^2+5\*a\*e^2))/(d+e\*x)^2 - (8\*c^2\*d^4+50\*a\*c\*d^2\*e^2+15\*a^2\*e^4)/(d+e\*x)))/(15\*(-(c\*d^2)+a\*e^2)^5\*((a\*e+c\*d\*x)\*(d+e\*x))^(5/2))

**Maple [A]** time = 0.019, size = 366, normalized size = 1.4

$$\frac{(2cdx+2ae)(40a^2c^2d^2e^6x^4+80ac^3d^4e^4x^4+8c^4d^6e^2x^4+60a^3cde^7x^3+220a^2c^2d^3e^5x^3+212ac^3d^5e^3x^3+20c^4d^7ex^3+15a^5e^{10}-75a^4e^8x^2+15a^3e^6x^2+15a^2e^4x^2+15ae^2x^2+15e^2x^2)}{15a^5e^{10}-75a^4e^8x^2+15a^3e^6x^2+15a^2e^4x^2+15ae^2x^2+15e^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(e\*x+d)/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2),x)

[Out] -2/15\*(c\*d\*x+a\*e)\*(40\*a^2\*c^2\*d^2\*e^6\*x^4+80\*a\*c^3\*d^4\*e^4\*x^4+8\*c^4\*d^6\*e^2\*x^4+60\*a^3\*c\*d\*e^7\*x^3+220\*a^2\*c^2\*d^3\*e^5\*x^3+212\*a\*c^3\*d^5\*e^3\*x^3+20\*c^4\*d^7\*e\*x^3+15\*a^4\*e^8\*x^2+180\*a^3\*c\*d^2\*e^6\*x^2+378\*a^2\*c^2\*d^4\*e^4\*x^2+180\*a\*c^3\*d^6\*e^2\*x^2+15\*c^4\*d^8\*x^2+20\*a^4\*d^7\*e\*x+212\*a^3\*c\*d^3\*e^5\*x+220\*a^2\*c^2\*d^5\*e^3\*x+60\*a\*c^3\*d^7\*e\*x+8\*a^4\*d^2\*e^6+80\*a^3\*c\*d^4\*e^4+40\*a^2\*c^2\*d^6\*e^2)/(a^5\*e^10-5\*a^4\*c\*d^2\*e^8+10\*a^3\*c^2\*d^4\*e^6-10\*a^2\*c^3\*d^6\*e^4+5\*a\*c^4\*d^8\*e^2-c^5\*d^10)/(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(5/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((c\*d\*e\*x^2+a\*d\*e+(c\*d^2+a\*e^2)\*x)^(5/2)\*(e\*x+d)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 5.85, size = 1107, normalized size = 4.27

$$15(a^2c^5d^{13}e^2 - 5a^3c^4d^{11}e^4 + 10a^4c^3d^9e^6 - 10a^5c^2d^7e^8 + 5a^6cd^5e^{10} - a^7d^3e^{12} + (c^7d^{12}e^3 - 5ac^6d^{10}e^5 + 10a^2c^5d^8e^7 - 10a^3c^4d^6e^5 + 5a^4c^3d^4e^3 - 5a^5c^2d^2e^1 + 5a^6cd^2e^1 - 5a^7d^2e^1))^{5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(e*x + d)),x, algo`

[Out] 
$$\frac{2}{15} \cdot (40 \cdot a^2 \cdot c^2 \cdot d^6 \cdot e^2 + 80 \cdot a^3 \cdot c \cdot d^4 \cdot e^4 + 8 \cdot a^4 \cdot d^2 \cdot e^6 + 8 \cdot (c^4 \cdot d^6 \cdot e^2 + 10 \cdot a \cdot c^3 \cdot d^4 \cdot e^4 + 5 \cdot a^2 \cdot c^2 \cdot d^2 \cdot e^6) \cdot x^4 + 4 \cdot (5 \cdot c^4 \cdot d^7 \cdot e + 53 \cdot a \cdot c^3 \cdot d^5 \cdot e^3 + 55 \cdot a^2 \cdot c^2 \cdot d^3 \cdot e^5 + 15 \cdot a^3 \cdot c \cdot d \cdot e^7) \cdot x^3 + 3 \cdot (5 \cdot c^4 \cdot d^8 + 60 \cdot a \cdot c^3 \cdot d^6 \cdot e^2 + 126 \cdot a^2 \cdot c^2 \cdot d^4 \cdot e^4 + 60 \cdot a^3 \cdot c \cdot d^2 \cdot e^6 + 5 \cdot a^4 \cdot e^8) \cdot x^2 + 4 \cdot (15 \cdot a \cdot c^3 \cdot d^7 \cdot e + 55 \cdot a^2 \cdot c^2 \cdot d^5 \cdot e^3 + 53 \cdot a^3 \cdot c \cdot d^3 \cdot e^5 + 5 \cdot a^4 \cdot d \cdot e^7) \cdot x) \cdot \sqrt{(c \cdot d \cdot e \cdot x^2 + a \cdot d \cdot e + (c \cdot d^2 + a \cdot e^2) \cdot x)} / (a^2 \cdot c^5 \cdot d^{13} \cdot e^2 - 5 \cdot a^3 \cdot c^4 \cdot d^{11} \cdot e^4 + 10 \cdot a^4 \cdot c^3 \cdot d^9 \cdot e^6 - 10 \cdot a^5 \cdot c^2 \cdot d^7 \cdot e^8 + 5 \cdot a^6 \cdot c \cdot d^5 \cdot e^{10} - a^7 \cdot d^3 \cdot e^{12} + (c^7 \cdot d^{12} \cdot e^3 - 5 \cdot a \cdot c^6 \cdot d^{10} \cdot e^5 + 10 \cdot a^2 \cdot c^5 \cdot d^8 \cdot e^7 - 10 \cdot a^3 \cdot c^4 \cdot d^6 \cdot e^9 + 5 \cdot a^4 \cdot c^3 \cdot d^4 \cdot e^{11} - a^5 \cdot c^2 \cdot d^2 \cdot e^{13}) \cdot x^5 + (3 \cdot c^7 \cdot d^{13} \cdot e^2 - 13 \cdot a \cdot c^6 \cdot d^{11} \cdot e^4 + 20 \cdot a^2 \cdot c^5 \cdot d^9 \cdot e^6 - 10 \cdot a^3 \cdot c^4 \cdot d^7 \cdot e^8 - 5 \cdot a^4 \cdot c^3 \cdot d^5 \cdot e^{10} + 7 \cdot a^5 \cdot c^2 \cdot d^3 \cdot e^{12} - 2 \cdot a^6 \cdot c \cdot d \cdot e^{14}) \cdot x^4 + (3 \cdot c^7 \cdot d^{14} \cdot e - 9 \cdot a \cdot c^6 \cdot d^{12} \cdot e^3 + a^2 \cdot c^5 \cdot d^{10} \cdot e^5 + 25 \cdot a^3 \cdot c^4 \cdot d^8 \cdot e^7 - 35 \cdot a^4 \cdot c^3 \cdot d^6 \cdot e^9 + 17 \cdot a^5 \cdot c^2 \cdot d^4 \cdot e^{11} - a^6 \cdot c \cdot d^2 \cdot e^{13} - a^7 \cdot e^{15}) \cdot x^3 + (c^7 \cdot d^{15} + a \cdot c^6 \cdot d^{13} \cdot e^2 - 17 \cdot a^2 \cdot c^5 \cdot d^{11} \cdot e^4 + 35 \cdot a^3 \cdot c^4 \cdot d^9 \cdot e^6 - 25 \cdot a^4 \cdot c^3 \cdot d^7 \cdot e^8 - a^5 \cdot c^2 \cdot d^5 \cdot e^{10} + 9 \cdot a^6 \cdot c \cdot d^3 \cdot e^{12} - 3 \cdot a^7 \cdot d \cdot e^{14}) \cdot x^2 + (2 \cdot a \cdot c^6 \cdot d^{14} \cdot e - 7 \cdot a^2 \cdot c^5 \cdot d^{12} \cdot e^3 + 5 \cdot a^3 \cdot c^4 \cdot d^{10} \cdot e^5 + 10 \cdot a^4 \cdot c^3 \cdot d^8 \cdot e^7 - 20 \cdot a^5 \cdot c^2 \cdot d^6 \cdot e^9 + 13 \cdot a^6 \cdot c \cdot d^4 \cdot e^{11} - 3 \cdot a^7 \cdot d^2 \cdot e^{13}) \cdot x)$$

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**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)`

[Out] Timed out

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

`[undef, undef, undef, undef, undef, 1]`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(e*x + d)),x, algo`

[Out] `[undef, undef, undef, undef, undef, 1]`

$$3.488 \quad \int \frac{x^2}{(d+ex)(ade+(cd^2+ae^2)x+cdex^2)^{7/2}} dx$$

Optimal. Leaf size=341

$$\begin{aligned} & \frac{128cd(7a^2e^4 + 14acd^2e^2 + 3c^2d^4)(ae^2 + cd^2 + 2cdex)}{105(cd^2 - ae^2)^7 \sqrt{x(ae^2 + cd^2) + ade + cdex^2}} \\ & + \frac{16(7a^2e^4 + 14acd^2e^2 + 3c^2d^4)(ae^2 + cd^2 + 2cdex)}{105e(cd^2 - ae^2)^5 (x(ae^2 + cd^2) + ade + cdex^2)^{3/2}} \\ & - \frac{8(x(3a^2e^4 + acd^2e^2 + 2c^2d^4) + 2ade(2ae^2 + cd^2))}{35e(cd^2 - ae^2)^3 (x(ae^2 + cd^2) + ade + cdex^2)^{5/2}} \\ & + \frac{2x^2}{7(d+ex)(cd^2 - ae^2)(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}} \end{aligned}$$

[Out] (2\*x^2)/(7\*(c\*d^2 - a\*e^2)\*(d + e\*x)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2)) - (8\*(2\*a\*d\*e\*(c\*d^2 + 2\*a\*e^2) + (2\*c^2\*d^4 + a\*c\*d^2\*e^2 + 3\*a^2\*e^4)\*x))/(35\*e\*(c\*d^2 - a\*e^2)^3\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2)) + (16\*(3\*c^2\*d^4 + 14\*a\*c\*d^2\*e^2 + 7\*a^2\*e^4)\*(c\*d^2 + a\*e^2 + 2\*c\*d\*e\*x))/(105\*e\*(c\*d^2 - a\*e^2)^5\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)) - (128\*c\*d\*(3\*c^2\*d^4 + 14\*a\*c\*d^2\*e^2 + 7\*a^2\*e^4)\*(c\*d^2 + a\*e^2 + 2\*c\*d\*e\*x))/(105\*(c\*d^2 - a\*e^2)^7\*sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])

**Rubi [A]** time = 0.766494, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 40,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\begin{aligned} & \frac{128cd(7a^2e^4 + 14acd^2e^2 + 3c^2d^4)(ae^2 + cd^2 + 2cdex)}{105(cd^2 - ae^2)^7 \sqrt{x(ae^2 + cd^2) + ade + cdex^2}} \\ & + \frac{16(7a^2e^4 + 14acd^2e^2 + 3c^2d^4)(ae^2 + cd^2 + 2cdex)}{105e(cd^2 - ae^2)^5 (x(ae^2 + cd^2) + ade + cdex^2)^{3/2}} \\ & - \frac{8(x(3a^2e^4 + acd^2e^2 + 2c^2d^4) + 2ade(2ae^2 + cd^2))}{35e(cd^2 - ae^2)^3 (x(ae^2 + cd^2) + ade + cdex^2)^{5/2}} \\ & + \frac{2x^2}{7(d+ex)(cd^2 - ae^2)(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x^2/((d + e\*x)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(7/2)), x]

[Out] (2\*x^2)/(7\*(c\*d^2 - a\*e^2)\*(d + e\*x)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2)) - (8\*(2\*a\*d\*e\*(c\*d^2 + 2\*a\*e^2) + (2\*c^2\*d^4 + a\*c\*d^2\*e^2 + 3\*a^2\*e^4)\*x))/(35\*e\*(c\*d^2 - a\*e^2)^3\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2)) + (16\*(3\*c^2\*d^4 + 14\*a\*c\*d^2\*e^2 + 7\*a^2\*e^4)\*(c\*d^2 + a\*e^2 + 2\*c\*d\*e\*x))/(105\*e\*(c\*d^2 - a\*e^2)^5\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)) - (128\*c\*d\*(3\*c^2\*d^4 + 14\*a\*c\*d^2\*e^2 + 7\*a^2\*e^4)\*(c\*d^2 + a\*e^2 + 2\*c\*d\*e\*x))/(105\*(c\*d^2 - a\*e^2)^7\*sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])



**Rubi in Sympy [A]** time = 94.5367, size = 340, normalized size = 1.

$$\frac{64cd(2ae^2 + 2cd^2 + 4cdex)(7a^2e^4 + 14acd^2e^2 + 3c^2d^4)}{105(ae^2 - cd^2)^7 \sqrt{ade + cdex^2 + x(ae^2 + cd^2)}} - \frac{2x^2(ae + cdx)}{7(ae^2 - cd^2)(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{7}{2}}} + \frac{4(4ade(2ae^2 + cd^2) + x(6a^2e^4 + 2acd^2e^2 + 4c^2d^4))}{35e(ae^2 - cd^2)^3(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{5}{2}}} - \frac{16(ae^2 + cd^2 + 2cdex)(7a^2e^4 + 14acd^2e^2 + 3c^2d^4)}{105e(ae^2 - cd^2)^5(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**2/(e*x+d)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(7/2),x)`

[Out] `64*c*d*(2*a*e**2 + 2*c*d**2 + 4*c*d*e*x)*(7*a**2*e**4 + 14*a*c*d*  
*2*e**2 + 3*c**2*d**4)/(105*(a*e**2 - c*d**2)**7*sqrt(a*d*e + c*d  
*e*x**2 + x*(a*e**2 + c*d**2))) - 2*x**2*(a*e + c*d*x)/(7*(a*e**2  
- c*d**2)*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(7/2)) + 4  
*(4*a*d*e*(2*a*e**2 + c*d**2) + x*(6*a**2*e**4 + 2*a*c*d**2*e**2  
+ 4*c**2*d**4))/(35*e*(a*e**2 - c*d**2)**3*(a*d*e + c*d*e*x**2 +  
x*(a*e**2 + c*d**2))**(5/2)) - 16*(a*e**2 + c*d**2 + 2*c*d*e*x)*(  
7*a**2*e**4 + 14*a*c*d**2*e**2 + 3*c**2*d**4)/(105*e*(a*e**2 - c*  
d**2)**5*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2))`

**Mathematica [A]** time = 2.43307, size = 317, normalized size = 0.93

$$\frac{2\sqrt{(d+ex)(ae+cdx)}\left(\frac{21a^2c^2d^2e^2(cd^2-ae^2)^2}{(ae+cdx)^3} + \frac{7c^2d^2(73a^2e^4+110acd^2e^2+15c^2d^4)}{ae+cdx} + \frac{cde(385a^2e^4+1022acd^2e^2+279c^2d^4)}{d+ex} - \frac{e(ae^2-cd^2)(35a^2e^4}{(d+ex)}\right)}{105(ae^2-cd^2)^7}$$

Antiderivative was successfully verified.

[In] `Integrate[x^2/((d + e*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2)),x]`

[Out] `(2*sqrt[(a*e + c*d*x)*(d + e*x)]*((21*a^2*c^2*d^2*e^2*(c*d^2 - a*  
e^2)^2)/(a*e + c*d*x)^3 + (14*a*c^2*d^2*e*(-(c*d^2) + a*e^2)*(5*c  
*d^2 + 7*a*e^2))/(a*e + c*d*x)^2 + (7*c^2*d^2*(15*c^2*d^4 + 110*a  
*c*d^2*e^2 + 73*a^2*e^4))/(a*e + c*d*x) + (15*d^2*e*(c*d^2 - a*e^2  
^2)^3)/(d + e*x)^4 + (3*d*e*(c*d^2 - a*e^2)^2*(13*c*d^2 + 14*a*e^2  
))/((d + e*x)^3 - (e*(-(c*d^2) + a*e^2)*(87*c^2*d^4 + 196*a*c*d^2*  
e^2 + 35*a^2*e^4))/(d + e*x)^2 + (c*d*e*(279*c^2*d^4 + 1022*a*c*d  
^2*e^2 + 385*a^2*e^4))/(d + e*x)))/(105*(-(c*d^2) + a*e^2)^7)`

**Maple [B]** time = 0.025, size = 663, normalized size = 1.9

$$\frac{(2cdx + 2ae)(-896a^2c^4d^4e^8x^6 - 1792ac^5d^6e^6x^6 - 384c^6d^8e^4x^6 - 2240a^3c^3d^3e^9x^5 - 7616a^2c^4d^5e^7x^5 - 7232ac^5d^7e^5x^5)}{105(ae^2 - cd^2)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2/(e*x+d)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(7/2),x)`

[Out] `-2/105*(c*d*x+a*e)*(-896*a^2*c^4*d^4*e^8*x^6-1792*a*c^5*d^6*e^6*x  
^6-384*c^6*d^8*e^4*x^6-2240*a^3*c^3*d^3*e^9*x^5-7616*a^2*c^4*d^5*  
e^7*x^5-7232*a*c^5*d^7*e^5*x^5-1344*c^6*d^9*e^3*x^5-1680*a^4*c^2*  
d^2*e^10*x^4-11200*a^3*c^3*d^4*e^8*x^4-20320*a^2*c^4*d^6*e^6*x^4-`

$$\frac{11200*a*c^5*d^8*e^4*x^4-1680*c^6*d^10*e^2*x^4-280*a^5*c*d*e^{11}*x^3-6440*a^4*c^2*d^3*e^9*x^3-21680*a^3*c^3*d^5*e^7*x^3-24080*a^2*c^4*d^7*e^5*x^3-8120*a*c^5*d^9*e^3*x^3-840*c^6*d^{11}*e*x^3+35*a^6*e^{12}*x^2-910*a^5*c*d^2*e^{10}*x^2-9295*a^4*c^2*d^4*e^8*x^2-20020*a^3*c^3*d^6*e^6*x^2-13195*a^2*c^4*d^8*e^4*x^2-2590*a*c^5*d^{10}*e^2*x^2-105*c^6*d^{12}*x^2+28*a^6*d*e^{11}*x-764*a^5*c*d^3*e^9*x-6440*a^4*c^2*d^5*e^7*x-8120*a^3*c^3*d^7*e^5*x-2996*a^2*c^4*d^9*e^3*x-140*a*c^5*d^{11}*e*x+8*a^6*d^2*e^{10}-224*a^5*c*d^4*e^8-1680*a^4*c^2*d^6*e^6-1120*a^3*c^3*d^8*e^4-56*a^2*c^4*d^{10}*e^2)/(a^7*e^{14}-7*a^6*c*d^2*e^{12}+21*a^5*c^2*d^4*e^{10}-35*a^4*c^3*d^6*e^8+35*a^3*c^4*d^8*e^6-21*a^2*c^5*d^{10}*e^4+7*a*c^6*d^{12}*e^2-c^7*d^{14})/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^{(7/2)}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(7/2)\*(e\*x + d)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(7/2)\*(e\*x + d)),x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2/(e\*x+d)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(7/2),x, algorithm="sympy")

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

[undef, undef, undef, undef, undef, undef, undef, 1]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(7/2)\*(e\*x + d)),x, algorithm="giac")

[Out] [undef, undef, undef, undef, undef, undef, undef, 1]

### 3.489 $\int x^3 \sqrt{1+x} \sqrt{1-x+x^2} dx$

**Optimal.** Leaf size=170

$$\frac{\frac{6}{55} \sqrt{x+1} \sqrt{x^2-x+1} x + \frac{2}{11} \sqrt{x+1} \sqrt{x^2-x+1} x^4}{4 \cdot 3^{3/4} \sqrt{2+\sqrt{3}} (x+1)^{3/2} \sqrt{x^2-x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)} - \frac{55 \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} (x^3+1)}{55 \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} (x^3+1)}$$

[Out] (6\*x\*Sqrt[1+x]\*Sqrt[1-x+x^2])/55 + (2\*x^4\*Sqrt[1+x]\*Sqrt[1-x+x^2])/11 - (4\*3^(3/4)\*Sqrt[2+Sqrt[3]]\*(1+x)^(3/2)\*Sqrt[1-x+x^2]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(55\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*(1+x^3))

**Rubi [A]** time = 0.157675, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$\frac{\frac{6}{55} \sqrt{x+1} \sqrt{x^2-x+1} x + \frac{2}{11} \sqrt{x+1} \sqrt{x^2-x+1} x^4}{4 \cdot 3^{3/4} \sqrt{2+\sqrt{3}} (x+1)^{3/2} \sqrt{x^2-x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)} - \frac{55 \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} (x^3+1)}{55 \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} (x^3+1)}$$

Antiderivative was successfully verified.

[In] Int[x^3\*Sqrt[1+x]\*Sqrt[1-x+x^2],x]

[Out] (6\*x\*Sqrt[1+x]\*Sqrt[1-x+x^2])/55 + (2\*x^4\*Sqrt[1+x]\*Sqrt[1-x+x^2])/11 - (4\*3^(3/4)\*Sqrt[2+Sqrt[3]]\*(1+x)^(3/2)\*Sqrt[1-x+x^2]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(55\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*(1+x^3))

**Rubi in Sympy [A]** time = 12.4889, size = 151, normalized size = 0.89

$$\frac{\frac{2x^4 \sqrt{x+1} \sqrt{x^2-x+1}}{11} + \frac{6x \sqrt{x+1} \sqrt{x^2-x+1}}{55}}{4 \cdot 3^{3/4} \sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}} \sqrt{\sqrt{3}+2} (x+1)^{3/2} \sqrt{x^2-x+1} F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right) \mid -7-4\sqrt{3}\right)} - \frac{55 \sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}} (x^3+1)}{55 \sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}} (x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(1+x)\*\*(1/2)\*(x\*\*2-x+1)\*\*(1/2),x)

[Out] 2\*x\*\*4\*sqrt(x+1)\*sqrt(x\*\*2-x+1)/11 + 6\*x\*sqrt(x+1)\*sqrt(x\*\*2-x+1)/55 - 4\*3\*\*(3/4)\*sqrt((x\*\*2-x+1)/(x+1+sqrt(3)))\*\*2\*sqrt(sqrt(3)+2)\*(x+1)\*\*(3/2)\*sqrt(x\*\*2-x+1)\*elliptic\_f(asin((x-sqrt(3)+1)/(x+1+sqrt(3))), -7-4\*sqrt(3))/(55\*sqrt((x+1)/(x+1+sqrt(3)))\*\*2)\*(x\*\*3+1)

**Mathematica [C]** time = 1.43432, size = 221, normalized size = 1.3

$$\frac{2 \left( x \sqrt{x+1} (5x^5 - 5x^4 + 5x^3 + 3x^2 - 3x + 3) + \sqrt{-\frac{6i}{\sqrt{3}+3i}} (\sqrt{3} + 3i) (x+1) \sqrt{\frac{(\sqrt{3}-3i)x+\sqrt{3}+3i}{(\sqrt{3}-3i)(x+1)}} \sqrt{\frac{(\sqrt{3}+3i)x+\sqrt{3}-3i}{(\sqrt{3}+3i)(x+1)}} F \left( i \sinh^{-1} \left( \frac{\sqrt{x^2-x+1}}{\sqrt{x+1}} \right) \right) \right)}{55\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*Sqrt[1 + x]\*Sqrt[1 - x + x^2],x]

[Out] (2\*(x\*Sqrt[1 + x]\*(3 - 3\*x + 3\*x^2 + 5\*x^3 - 5\*x^4 + 5\*x^5) + Sqrt[(-6\*I)/(3\*I + Sqrt[3])]\*(3\*I + Sqrt[3])\*(1 + x)\*Sqrt[(3\*I + Sqrt[3] + (-3\*I + Sqrt[3])\*x)/((-3\*I + Sqrt[3])\*(1 + x))]\*Sqrt[(-3\*I + Sqrt[3] + (3\*I + Sqrt[3])\*x)/((3\*I + Sqrt[3])\*(1 + x))]\*EllipticF[I\*ArcSinh[Sqrt[(-6\*I)/(3\*I + Sqrt[3])]/Sqrt[1 + x]], (3\*I + Sqrt[3])/(3\*I - Sqrt[3])])/((55\*Sqrt[1 - x + x^2]))

**Maple [A]** time = 0.225, size = 257, normalized size = 1.5

$$\frac{2}{55x^3 + 55} \sqrt{1+x} \sqrt{x^2-x+1} \left( 5x^7 + 3i\sqrt{3} \sqrt{-2\frac{1+x}{-3+i\sqrt{3}}} \sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}} \sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}} \text{EllipticF} \left( \sqrt{-2\frac{1+x}{-3+i\sqrt{3}}}, \sqrt{-\frac{3}{i\sqrt{3}}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(1+x)^(1/2)\*(x^2-x+1)^(1/2),x)

[Out] 2/55\*(1+x)^(1/2)\*(x^2-x+1)^(1/2)\*(5\*x^7+3\*I\*3^(1/2)\*(-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2),(-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))-9\*(-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2),(-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))+8\*x^4+3\*x)/(x^3+1)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2-x+1} \sqrt{x+1} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x^2 - x + 1)\*sqrt(x + 1)\*x^3,x, algorithm="maxima")

[Out] integrate(sqrt(x^2 - x + 1)\*sqrt(x + 1)\*x^3, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \sqrt{x^2-x+1} \sqrt{x+1} x^3, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x^2 - x + 1)\*sqrt(x + 1)\*x^3,x, algorithm="fricas")

[Out] `integral(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 \sqrt{x+1} \sqrt{x^2-x+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(1+x)**(1/2)*(x**2-x+1)**(1/2), x)`

[Out] `Integral(x**3*sqrt(x + 1)*sqrt(x**2 - x + 1), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2-x+1} \sqrt{x+1} x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3,x, algorithm="giac")`

[Out] `integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3, x)`

$$3.490 \quad \int x^2 \sqrt{1+x} \sqrt{1-x+x^2} dx$$

**Optimal.** Leaf size=23

$$\frac{2}{9}(x+1)^{3/2} (x^2-x+1)^{3/2}$$

[Out] (2\*(1+x)^(3/2)\*(1-x+x^2)^(3/2))/9

**Rubi [A]** time = 0.0169745, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$\frac{2}{9}(x+1)^{3/2} (x^2-x+1)^{3/2}$$

Antiderivative was successfully verified.

[In] Int[x^2\*sqrt[1+x]\*sqrt[1-x+x^2],x]

[Out] (2\*(1+x)^(3/2)\*(1-x+x^2)^(3/2))/9

**Rubi in Sympy [A]** time = 7.23945, size = 24, normalized size = 1.04

$$\frac{2\sqrt{x+1}(x^3+1)\sqrt{x^2-x+1}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2\*(1+x)\*\*(1/2)\*(x\*\*2-x+1)\*\*(1/2),x)

[Out] 2\*sqrt(x+1)\*(x\*\*3+1)\*sqrt(x\*\*2-x+1)/9

**Mathematica [A]** time = 0.0282215, size = 23, normalized size = 1.

$$\frac{2}{9}(x+1)^{3/2} (x^2-x+1)^{3/2}$$

Antiderivative was successfully verified.

[In] Integrate[x^2\*sqrt[1+x]\*sqrt[1-x+x^2],x]

[Out] (2\*(1+x)^(3/2)\*(1-x+x^2)^(3/2))/9

**Maple [A]** time = 0.006, size = 18, normalized size = 0.8

$$\frac{2}{9}(1+x)^{\frac{3}{2}}(x^2-x+1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2\*(1+x)^(1/2)\*(x^2-x+1)^(1/2),x)

[Out]  $2/9 * (1+x)^{(3/2)} * (x^2-x+1)^{(3/2)}$

**Maxima [A]** time = 0.775584, size = 30, normalized size = 1.3

$$\frac{2}{9} (x^3 + 1) \sqrt{x^2 - x + 1} \sqrt{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2,x, algorithm="maxima")`

[Out]  $2/9 * (x^3 + 1) * \sqrt{x^2 - x + 1} * \sqrt{x + 1}$

**Fricas [A]** time = 0.280616, size = 30, normalized size = 1.3

$$\frac{2}{9} (x^3 + 1) \sqrt{x^2 - x + 1} \sqrt{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2,x, algorithm="fricas")`

[Out]  $2/9 * (x^3 + 1) * \sqrt{x^2 - x + 1} * \sqrt{x + 1}$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 \sqrt{x + 1} \sqrt{x^2 - x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(1+x)**(1/2)*(x**2-x+1)**(1/2),x)`

[Out] `Integral(x**2*sqrt(x + 1)*sqrt(x**2 - x + 1), x)`

**GIAC/XCAS [A]** time = 0.269317, size = 36, normalized size = 1.57

$$\frac{2}{9} \sqrt{(x + 1)^2 - 3x((x + 1)(x - 2) + 3)} (x + 1)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2,x, algorithm="giac")`

[Out]  $2/9 * \sqrt{(x + 1)^2 - 3 * x} * ((x + 1) * (x - 2) + 3) * (x + 1)^{(3/2)}$

$$3.491 \quad \int x\sqrt{1+x}\sqrt{1-x+x^2} dx$$

**Optimal.** Leaf size=294

$$\begin{aligned} & \frac{2}{7}\sqrt{x+1}\sqrt{x^2-x+1}x^2 + \frac{6\sqrt{x+1}\sqrt{x^2-x+1}}{7(x+\sqrt{3}+1)} \\ & + \frac{2\sqrt{2}3^{3/4}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\mid-7-4\sqrt{3}\right)}{7\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}(x^3+1)} \\ & - \frac{3^4\sqrt{3}\sqrt{2-\sqrt{3}}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\mid-7-4\sqrt{3}\right)}{7\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}(x^3+1)} \end{aligned}$$

[Out] (2\*x^2\*Sqrt[1+x]\*Sqrt[1-x+x^2])/7 + (6\*Sqrt[1+x]\*Sqrt[1-x+x^2])/(7\*(1+Sqrt[3]+x)) - (3\*3^(1/4)\*Sqrt[2-Sqrt[3]]\*(1+x)^(3/2)\*Sqrt[1-x+x^2]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticE[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(7\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*(1+x^3)) + (2\*Sqrt[2]\*3^(3/4)\*(1+x)^(3/2)\*Sqrt[1-x+x^2]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(7\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*(1+x^3))

**Rubi [A]** time = 0.200344, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$\begin{aligned} & \frac{2}{7}\sqrt{x+1}\sqrt{x^2-x+1}x^2 + \frac{6\sqrt{x+1}\sqrt{x^2-x+1}}{7(x+\sqrt{3}+1)} \\ & + \frac{2\sqrt{2}3^{3/4}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\mid-7-4\sqrt{3}\right)}{7\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}(x^3+1)} \\ & - \frac{3^4\sqrt{3}\sqrt{2-\sqrt{3}}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\mid-7-4\sqrt{3}\right)}{7\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}(x^3+1)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x\*Sqrt[1+x]\*Sqrt[1-x+x^2],x]

[Out] (2\*x^2\*Sqrt[1+x]\*Sqrt[1-x+x^2])/7 + (6\*Sqrt[1+x]\*Sqrt[1-x+x^2])/(7\*(1+Sqrt[3]+x)) - (3\*3^(1/4)\*Sqrt[2-Sqrt[3]]\*(1+x)^(3/2)\*Sqrt[1-x+x^2]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticE[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(7\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*(1+x^3)) + (2\*Sqrt[2]\*3^(3/4)\*(1+x)^(3/2)\*Sqrt[1-x+x^2]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(7\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*(1+x^3))



**Rubi in Sympy [A]** time = 17.1909, size = 262, normalized size = 0.89

$$\frac{2x^2\sqrt{x+1}\sqrt{x^2-x+1}}{7} + \frac{6\sqrt{x+1}\sqrt{x^2-x+1}}{7(x+1+\sqrt{3})}$$

$$- \frac{3\sqrt[3]{3}\sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}}\sqrt{-\sqrt{3}+2}(x+1)^{\frac{3}{2}}\sqrt{x^2-x+1}E\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle| -7-4\sqrt{3}\right)}{7\sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}}(x^3+1)}$$

$$+ \frac{2\sqrt{2}\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}}(x+1)^{\frac{3}{2}}\sqrt{x^2-x+1}F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle| -7-4\sqrt{3}\right)}{7\sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}}(x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x*(1+x)**(1/2)*(x**2-x+1)**(1/2),x)`

[Out]  $2*x**2*\sqrt{x+1}*\sqrt{x**2-x+1}/7 + 6*\sqrt{x+1}*\sqrt{x**2-x+1}/(7*(x+1+\sqrt{3})) - 3*3**(1/4)*\sqrt{(x**2-x+1)/(x+1+\sqrt{3})**2}*\sqrt{-\sqrt{3}+2}*(x+1)**(3/2)*\sqrt{x**2-x+1}*\operatorname{elliptic}_e(\operatorname{asin}((x-\sqrt{3}+1)/(x+1+\sqrt{3})), -7-4*\sqrt{3})/(7*\sqrt{(x+1)/(x+1+\sqrt{3})**2}*(x**3+1)) + 2*\sqrt{2}*3**(3/4)*\sqrt{(x**2-x+1)/(x+1+\sqrt{3})**2}*(x+1)**(3/2)*\sqrt{x**2-x+1}*\operatorname{elliptic}_f(\operatorname{asin}((x-\sqrt{3}+1)/(x+1+\sqrt{3})), -7-4*\sqrt{3})/(7*\sqrt{(x+1)/(x+1+\sqrt{3})**2}*(x**3+1))$

**Mathematica [C]** time = 0.843102, size = 347, normalized size = 1.18

$$\frac{\sqrt{x+1}\left(4\sqrt{-\frac{i(x+1)}{\sqrt{3}+3i}}(x^2-x+1)x^2+3\sqrt{2}(\sqrt{3}-i)\sqrt{\frac{-2ix+\sqrt{3}+i}{\sqrt{3}+3i}}\sqrt{\frac{2ix+\sqrt{3}-i}{\sqrt{3}-3i}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(x+1)}{3i+\sqrt{3}}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)-3\sqrt{2}(\sqrt{3}-i)\sqrt{-\frac{i(x+1)}{\sqrt{3}+3i}}\sqrt{x^2-x+1}\right)}{14\sqrt{-\frac{i(x+1)}{\sqrt{3}+3i}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In] `Integrate[x*Sqrt[1+x]*Sqrt[1-x+x^2],x]`

[Out]  $(\operatorname{Sqrt}[1+x]*(4*x^2*\operatorname{Sqrt}[((-1)*(1+x))/(3*I+\operatorname{Sqrt}[3])])*(1-x+x^2)-3*\operatorname{Sqrt}[2]*(-3*I+\operatorname{Sqrt}[3])*\operatorname{Sqrt}[(I+\operatorname{Sqrt}[3]-(2*I)*x)/(3*I+\operatorname{Sqrt}[3])]*\operatorname{Sqrt}[(-I+\operatorname{Sqrt}[3]+(2*I)*x)/(-3*I+\operatorname{Sqrt}[3])]*\operatorname{EllipticE}[I*\operatorname{ArcSinh}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[((-1)*(1+x))/(3*I+\operatorname{Sqrt}[3])]]],(3*I+\operatorname{Sqrt}[3])/(3*I-\operatorname{Sqrt}[3])]+3*\operatorname{Sqrt}[2]*(-I+\operatorname{Sqrt}[3])*\operatorname{Sqrt}[(I+\operatorname{Sqrt}[3]-(2*I)*x)/(3*I+\operatorname{Sqrt}[3])]*\operatorname{Sqrt}[(-I+\operatorname{Sqrt}[3]+(2*I)*x)/(-3*I+\operatorname{Sqrt}[3])]*\operatorname{EllipticF}[I*\operatorname{ArcSinh}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[((-1)*(1+x))/(3*I+\operatorname{Sqrt}[3])]]],(3*I+\operatorname{Sqrt}[3])/(3*I-\operatorname{Sqrt}[3])]))/(14*\operatorname{Sqrt}[((-1)*(1+x))/(3*I+\operatorname{Sqrt}[3])]*\operatorname{Sqrt}[1-x+x^2])$

**Maple [A]** time = 0.033, size = 361, normalized size = 1.2

$$\frac{1}{7x^3+7}\sqrt{1+x}\sqrt{x^2-x+1}\left(3i\sqrt{3}\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\operatorname{EllipticF}\left(\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(1+x)^(1/2)*(x^2-x+1)^(1/2),x)`

```
[Out] 1/7*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(3*I*3^(1/2)*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))+2*x^5+9*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-18*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticE((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))+2*x^2/(x^3+1)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2 - x + 1} \sqrt{x + 1} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x, x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{x^2 - x + 1} \sqrt{x + 1} x, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)*x, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x \sqrt{x + 1} \sqrt{x^2 - x + 1} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)**(1/2)*(x**2-x+1)**(1/2),x)
```

```
[Out] Integral(x*sqrt(x + 1)*sqrt(x**2 - x + 1), x)
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2 - x + 1} \sqrt{x + 1} x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)*x, x)
```

$$3.492 \quad \int \sqrt{1+x} \sqrt{1-x+x^2} dx$$

**Optimal.** Leaf size=144

$$\frac{2}{5}x\sqrt{x^2-x+1}\sqrt{x+1} + \frac{2 \cdot 3^{3/4} \sqrt{2+\sqrt{3}} \sqrt{x^2-x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} (x+1)^{3/2} F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \middle| -7-4\sqrt{3}\right)}{5 \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} (x^3+1)}$$

[Out] (2\*x\*Sqrt[1+x]\*Sqrt[1-x+x^2])/5 + (2\*3^(3/4)\*Sqrt[2+Sqrt[3]]\*(1+x)^(3/2)\*Sqrt[1-x+x^2]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(5\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*(1+x^3))

**Rubi [A]** time = 0.0822216, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{2}{5}x\sqrt{x^2-x+1}\sqrt{x+1} + \frac{2 \cdot 3^{3/4} \sqrt{2+\sqrt{3}} \sqrt{x^2-x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} (x+1)^{3/2} F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \middle| -7-4\sqrt{3}\right)}{5 \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} (x^3+1)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1+x]\*Sqrt[1-x+x^2],x]

[Out] (2\*x\*Sqrt[1+x]\*Sqrt[1-x+x^2])/5 + (2\*3^(3/4)\*Sqrt[2+Sqrt[3]]\*(1+x)^(3/2)\*Sqrt[1-x+x^2]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(5\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*(1+x^3))

**Rubi in Sympy [A]** time = 7.17304, size = 128, normalized size = 0.89

$$\frac{2x\sqrt{x+1}\sqrt{x^2-x+1}}{5} + \frac{2 \cdot 3^{3/4} \sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}} \sqrt{\sqrt{3}+2} (x+1)^{3/2} \sqrt{x^2-x+1} F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right) \middle| -7-4\sqrt{3}\right)}{5 \sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}} (x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((1+x)\*\*(1/2)\*(x\*\*2-x+1)\*\*(1/2),x)

[Out] 2\*x\*sqrt(x+1)\*sqrt(x\*\*2-x+1)/5 + 2\*3\*\*(3/4)\*sqrt((x\*\*2-x+1)/(x+1+sqrt(3))\*\*2)\*sqrt(sqrt(3)+2)\*(x+1)\*\*(3/2)\*sqrt(x\*\*2-x+1)\*elliptic\_f(asin((x-sqrt(3)+1)/(x+1+sqrt(3))), -7-4\*sqrt(3))/(5\*sqrt((x+1)/(x+1+sqrt(3))\*\*2)\*(x\*\*3+1))

**Mathematica [C]** time = 0.839915, size = 169, normalized size = 1.17

$$2x\sqrt{x+1}(x^2-x+1) + \frac{i(x+1) \sqrt{1+\frac{6i}{(\sqrt{3}-3i)(x+1)}} \sqrt{6-\frac{36i}{(\sqrt{3}+3i)(x+1)}} F\left(i \sinh^{-1}\left(\frac{\sqrt{\frac{-6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right) \middle| \frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{\frac{-i}{\sqrt{3}+3i}}}$$

$$5\sqrt{x^2-x+1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + x]*Sqrt[1 - x + x^2],x]
```

```
[Out] (2*x*Sqrt[1 + x]*(1 - x + x^2) + (I*(1 + x)*Sqrt[1 + (6*I)/((-3*I + Sqrt[3])*(1 + x))]*Sqrt[6 - (36*I)/((3*I + Sqrt[3])*(1 + x))])*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3]))/Sqrt[(-I)/(3*I + Sqrt[3])]/(5*Sqrt[1 - x + x^2])
```

**Maple [B]** time = 0.031, size = 252, normalized size = 1.8

$$-\frac{1}{5x^3+5}\sqrt{1+x}\sqrt{x^2-x+1}\left(3i\sqrt{3}\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\text{EllipticF}\left(\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(1/2)*(x^2-x+1)^(1/2),x)
```

```
[Out] -1/5*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(3*I*3^(1/2)*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),((-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-9*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),((-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-2*x^4-2*x)/(x^3+1)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2 - x + 1}\sqrt{x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{x^2 - x + 1}\sqrt{x + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1),x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x + 1}\sqrt{x^2 - x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x)**(1/2)*(x**2-x+1)**(1/2),x)`

[Out] `Integral(sqrt(x + 1)*sqrt(x**2 - x + 1), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{x^2 - x + 1} \sqrt{x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(x^2 - x + 1)*sqrt(x + 1),x, algorithm="giac")`

[Out] `integrate(sqrt(x^2 - x + 1)*sqrt(x + 1), x)`

$$3.493 \quad \int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x} dx$$

**Optimal.** Leaf size=66

$$\frac{2}{3}\sqrt{x+1}\sqrt{x^2-x+1} - \frac{2\sqrt{x+1}\sqrt{x^2-x+1} \tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

[Out] (2\*Sqrt[1 + x]\*Sqrt[1 - x + x^2])/3 - (2\*Sqrt[1 + x]\*Sqrt[1 - x + x^2]\*ArcTanh[Sqrt[1 + x^3]])/(3\*Sqrt[1 + x^3])

**Rubi [A]** time = 0.0891741, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$

$$\frac{2}{3}\sqrt{x+1}\sqrt{x^2-x+1} - \frac{2\sqrt{x+1}\sqrt{x^2-x+1} \tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[1 + x]\*Sqrt[1 - x + x^2])/x, x]

[Out] (2\*Sqrt[1 + x]\*Sqrt[1 - x + x^2])/3 - (2\*Sqrt[1 + x]\*Sqrt[1 - x + x^2]\*ArcTanh[Sqrt[1 + x^3]])/(3\*Sqrt[1 + x^3])

**Rubi in Sympy [A]** time = 9.78869, size = 58, normalized size = 0.88

$$\frac{2\sqrt{x+1}\sqrt{x^2-x+1}}{3} - \frac{2\sqrt{x+1}\sqrt{x^2-x+1} \operatorname{atanh}\left(\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((1+x)\*\*(1/2)\*(x\*\*2-x+1)\*\*(1/2)/x, x)

[Out] 2\*sqrt(x + 1)\*sqrt(x\*\*2 - x + 1)/3 - 2\*sqrt(x + 1)\*sqrt(x\*\*2 - x + 1)\*atanh(sqrt(x\*\*3 + 1))/(3\*sqrt(x\*\*3 + 1))

**Mathematica [C]** time = 0.625561, size = 197, normalized size = 2.98

$$\frac{\sqrt{x+1} \left( 2(x^2-x+1) + \frac{3i\sqrt{2} \sqrt{\frac{-2ix+\sqrt{3}+i}{\sqrt{3}+3i}} \sqrt{\frac{2ix+\sqrt{3}-i}{\sqrt{3}-3i}} \left(\frac{3}{2} - \frac{i\sqrt{3}}{2}\right); i \sinh^{-1}\left(\sqrt{2} \sqrt{\frac{i(x+1)}{3i+\sqrt{3}}}\right) \frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}} \right)}{3\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[1 + x]\*Sqrt[1 - x + x^2])/x, x]

[Out] (Sqrt[1 + x]\*(2\*(1 - x + x^2) + ((3\*I)\*Sqrt[2]\*Sqrt[(I + Sqrt[3] - (2\*I)\*x)/(3\*I + Sqrt[3]])\*Sqrt[(-I + Sqrt[3] + (2\*I)\*x)/(-3\*I + Sqrt[3])])\*EllipticPi[3/2 - (I/2)\*Sqrt[3], I\*ArcSinh[Sqrt[2]\*Sqrt[((-I)\*(1 + x))/(3\*I + Sqrt[3])]]], (3\*I + Sqrt[3])/(3\*I - Sqrt[3])])/Sqrt[((-I)\*(1 + x))/(3\*I + Sqrt[3])])/(3\*Sqrt[1 - x + x^2])

---

**Maple [A]** time = 0.031, size = 43, normalized size = 0.7

$$-\frac{2}{3}\sqrt{1+x}\sqrt{x^2-x+1}\left(-\sqrt{x^3+1}+\operatorname{Artanh}\left(\sqrt{x^3+1}\right)\right)\frac{1}{\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x)^(1/2)\*(x^2-x+1)^(1/2)/x,x)

[Out] -2/3\*(1+x)^(1/2)\*(x^2-x+1)^(1/2)\*(-(x^3+1)^(1/2)+arctanh((x^3+1)^(1/2)))/(x^3+1)^(1/2)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x^2 - x + 1)\*sqrt(x + 1)/x,x, algorithm="maxima")

[Out] integrate(sqrt(x^2 - x + 1)\*sqrt(x + 1)/x, x)

---

**Fricas [A]** time = 0.286542, size = 81, normalized size = 1.23

$$\frac{2}{3}\sqrt{x^2-x+1}\sqrt{x+1}-\frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}+1\right)+\frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x^2 - x + 1)\*sqrt(x + 1)/x,x, algorithm="fricas")

[Out] 2/3\*sqrt(x^2 - x + 1)\*sqrt(x + 1) - 1/3\*log(sqrt(x^2 - x + 1)\*sqrt(x + 1) + 1) + 1/3\*log(sqrt(x^2 - x + 1)\*sqrt(x + 1) - 1)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+x)\*\*(1/2)\*(x\*\*2-x+1)\*\*(1/2)/x,x)

[Out] Integral(sqrt(x + 1)\*sqrt(x\*\*2 - x + 1)/x, x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x, x)
```



$$3.494 \quad \int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^2} dx$$

**Optimal.** Leaf size=287

$$\begin{aligned} & -\frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x} + \frac{3\sqrt{x^2-x+1}\sqrt{x+1}}{x+\sqrt{3}+1} \\ & + \frac{\sqrt{23}^{3/4}\sqrt{x^2-x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}(x+1)^{3/2}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}(x^3+1)} \\ & - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{x^2-x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}(x+1)^{3/2}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{2\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}(x^3+1)} \end{aligned}$$

[Out] -((Sqrt[1 + x]\*Sqrt[1 - x + x^2])/x) + (3\*Sqrt[1 + x]\*Sqrt[1 - x + x^2])/(1 + Sqrt[3] + x) - (3\*3^(1/4)\*Sqrt[2 - Sqrt[3]]\*(1 + x)^(3/2)\*Sqrt[1 - x + x^2]\*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]\*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4\*Sqrt[3]])/(2\*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]\*(1 + x^3)) + (Sqrt[2]\*3^(3/4)\*(1 + x)^(3/2)\*Sqrt[1 - x + x^2]\*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]\*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4\*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]\*(1 + x^3))

**Rubi [A]** time = 0.21022, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$

$$\begin{aligned} & -\frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x} + \frac{3\sqrt{x^2-x+1}\sqrt{x+1}}{x+\sqrt{3}+1} \\ & + \frac{\sqrt{23}^{3/4}\sqrt{x^2-x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}(x+1)^{3/2}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}(x^3+1)} \\ & - \frac{3\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{x^2-x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}(x+1)^{3/2}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{2\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}(x^3+1)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[1 + x]\*Sqrt[1 - x + x^2])/x^2, x]

[Out] -((Sqrt[1 + x]\*Sqrt[1 - x + x^2])/x) + (3\*Sqrt[1 + x]\*Sqrt[1 - x + x^2])/(1 + Sqrt[3] + x) - (3\*3^(1/4)\*Sqrt[2 - Sqrt[3]]\*(1 + x)^(3/2)\*Sqrt[1 - x + x^2]\*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]\*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4\*Sqrt[3]])/(2\*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]\*(1 + x^3)) + (Sqrt[2]\*3^(3/4)\*(1 + x)^(3/2)\*Sqrt[1 - x + x^2]\*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]\*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4\*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]\*(1 + x^3))

**Rubi in Sympy [A]** time = 18.3855, size = 250, normalized size = 0.87

$$\frac{3\sqrt{x+1}\sqrt{x^2-x+1}}{x+1+\sqrt{3}} - \frac{3\sqrt[3]{3}\sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}}\sqrt{-\sqrt{3}+2}(x+1)^{\frac{3}{2}}\sqrt{x^2-x+1}E\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\right)\bigg|_{-7-4\sqrt{3}}}{2\sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}}(x^3+1)}$$

$$+ \frac{\sqrt{2}\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}}(x+1)^{\frac{3}{2}}\sqrt{x^2-x+1}F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\right)\bigg|_{-7-4\sqrt{3}}}{\sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}}(x^3+1)} - \frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((1+x)**(1/2)*(x**2-x+1)**(1/2)/x**2,x)`

[Out] `3*sqrt(x + 1)*sqrt(x**2 - x + 1)/(x + 1 + sqrt(3)) - 3*3**(1/4)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*sqrt(-sqrt(3) + 2)*(x + 1)**(3/2)*sqrt(x**2 - x + 1)*elliptic_e(asin((x - sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(2*sqrt((x + 1)/(x + 1 + sqrt(3))**2)*(x**3 + 1)) + sqrt(2)*3**(3/4)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*(x + 1)**(3/2)*sqrt(x**2 - x + 1)*elliptic_f(asin((x - sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(sqrt((x + 1)/(x + 1 + sqrt(3))**2)*(x**3 + 1)) - sqrt(x + 1)*sqrt(x**2 - x + 1)/x`

**Mathematica [C]** time = 0.684009, size = 349, normalized size = 1.22

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x} + \frac{3\sqrt{1+\frac{2i(x+1)}{\sqrt{3}-3i}}\sqrt{1-\frac{2i(x+1)}{\sqrt{3}+3i}}\left(\frac{(\sqrt{3}-i)\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{x+1}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(x+1)}{3i+\sqrt{3}}}\right)\bigg|_{\frac{3i+\sqrt{3}}{3i-\sqrt{3}}}\right)}{\sqrt{-\frac{i(x+1)}{\sqrt{3}+3i}}}-\frac{(\sqrt{3}-3i)\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{x+1}E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{-\frac{i(x+1)}{3i+\sqrt{3}}}\right)\bigg|_{\frac{3i+\sqrt{3}}{3i-\sqrt{3}}}\right)}{\sqrt{-\frac{i(x+1)}{\sqrt{3}+3i}}}\right)}{2\sqrt{2}\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{(x+1)^2-3(x+1)+3}}$$

Antiderivative was successfully verified.

[In] `Integrate[(Sqrt[1 + x]*Sqrt[1 - x + x^2])/x^2,x]`

[Out] `-((Sqrt[1 + x]*Sqrt[1 - x + x^2])/x) + (3*Sqrt[1 + ((2*I)*(1 + x))]/(-3*I + Sqrt[3]))*Sqrt[1 - ((2*I)*(1 + x))/(3*I + Sqrt[3])]*(-( (-3*I + Sqrt[3])*Sqrt[(-I)/(3*I + Sqrt[3])] * Sqrt[1 + x]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]) + ((-I + Sqrt[3])*Sqrt[(-I)/(3*I + Sqrt[3])] * Sqrt[1 + x]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[((-I)*(1 + x))/(3*I + Sqrt[3])])/(2*Sqrt[2]*Sqrt[(-I)/(3*I + Sqrt[3])] * Sqrt[3 - 3*(1 + x) + (1 + x)^2])`

**Maple [A]** time = 0.036, size = 363, normalized size = 1.3

$$\frac{1}{2x(x^3+1)}\sqrt{1+x}\sqrt{x^2-x+1}\left(3i\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\operatorname{EllipticF}\left(\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}},\sqrt{\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+x)^(1/2)*(x^2-x+1)^(1/2)/x^2,x)`

```
[Out] 1/2*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(3*I*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*3^(1/2)*x+9*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x-18*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticE((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*x-2*x^3-2)/x/(x^3+1)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 - x + 1}\sqrt{x + 1}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^2, x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2 - x + 1}\sqrt{x + 1}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^2, x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x + 1}\sqrt{x^2 - x + 1}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(1/2)*(x**2-x+1)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(x + 1)*sqrt(x**2 - x + 1)/x**2, x)
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 - x + 1}\sqrt{x + 1}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^2, x)
```

$$3.495 \quad \int \frac{\sqrt{1+x}\sqrt{1-x+x^2}}{x^3} dx$$

**Optimal.** Leaf size=146

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle|-7-4\sqrt{3}\right)}{2\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}(x^3+1)} - \frac{\sqrt{x+1}\sqrt{x^2-x+1}}{2x^2}$$

[Out] -(Sqrt[1 + x]\*Sqrt[1 - x + x^2])/(2\*x^2) + (3^(3/4)\*Sqrt[2 + Sqrt[3]]\*(1 + x)^(3/2)\*Sqrt[1 - x + x^2]\*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]\*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4\*Sqrt[3]])/(2\*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]\*(1 + x^3))

**Rubi [A]** time = 0.124963, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$

$$\frac{3^{3/4}\sqrt{2+\sqrt{3}}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle|-7-4\sqrt{3}\right)}{2\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}(x^3+1)} - \frac{\sqrt{x+1}\sqrt{x^2-x+1}}{2x^2}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[1 + x]\*Sqrt[1 - x + x^2])/x^3, x]

[Out] -(Sqrt[1 + x]\*Sqrt[1 - x + x^2])/(2\*x^2) + (3^(3/4)\*Sqrt[2 + Sqrt[3]]\*(1 + x)^(3/2)\*Sqrt[1 - x + x^2]\*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]\*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4\*Sqrt[3]])/(2\*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]\*(1 + x^3))

**Rubi in Sympy [A]** time = 10.0345, size = 126, normalized size = 0.86

$$\frac{3^{3/4}\sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}}\sqrt{\sqrt{3}+2}(x+1)^{3/2}\sqrt{x^2-x+1}F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle|-7-4\sqrt{3}\right)}{2\sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}}(x^3+1)} - \frac{\sqrt{x+1}\sqrt{x^2-x+1}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((1+x)\*\*(1/2)\*(x\*\*2-x+1)\*\*(1/2)/x\*\*3, x)

[Out] 3\*\*(3/4)\*sqrt((x\*\*2 - x + 1)/(x + 1 + sqrt(3))\*\*2)\*sqrt(sqrt(3) + 2)\*(x + 1)\*\*(3/2)\*sqrt(x\*\*2 - x + 1)\*elliptic\_f(asin((x - sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4\*sqrt(3))/(2\*sqrt((x + 1)/(x + 1 + sqrt(3))\*\*2)\*(x\*\*3 + 1)) - sqrt(x + 1)\*sqrt(x\*\*2 - x + 1)/(2\*x\*\*2)

**Mathematica [C]** time = 0.675258, size = 185, normalized size = 1.27

$$\frac{\sqrt{x+1}\left(-\frac{2(x^2-x+1)}{x^2} - \frac{3i\sqrt{2}\sqrt{\frac{-2ix+\sqrt{3}+i}{\sqrt{3}+3i}}\sqrt{\frac{2ix+\sqrt{3}-i}{\sqrt{3}-3i}}F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{i(x+1)}{3i+\sqrt{3}}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{\frac{-i(x+1)}{\sqrt{3}+3i}}}\right)}{4\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[1 + x]\*Sqrt[1 - x + x^2])/x^3, x]

[Out] (Sqrt[1 + x]\*((-2\*(1 - x + x^2))/x^2 - ((3\*I)\*Sqrt[2]\*Sqrt[(I + Sqrt[3] - (2\*I)\*x)/(3\*I + Sqrt[3]])\*Sqrt[(-I + Sqrt[3] + (2\*I)\*x)/(-3\*I + Sqrt[3])])\*EllipticF[I\*ArcSinh[Sqrt[2]\*Sqrt[((-I)\*(1 + x))/(3\*I + Sqrt[3])]], (3\*I + Sqrt[3])/(3\*I - Sqrt[3])])/Sqrt[((-I)\*(1 + x))/(3\*I + Sqrt[3])])/(4\*Sqrt[1 - x + x^2])

**Maple [B]** time = 0.036, size = 259, normalized size = 1.8

$$-\frac{1}{(4x^3+4)x^2}\sqrt{1+x}\sqrt{x^2-x+1}\left(3i\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\text{EllipticF}\left(\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x)^(1/2)\*(x^2-x+1)^(1/2)/x^3, x)

[Out] -1/4\*(1+x)^(1/2)\*(x^2-x+1)^(1/2)\*(3\*I\*(-2\*(1+x)/(-3+I\*3^(1/2))))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2), (-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2)\*3^(1/2)\*x^2-9\*(-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2), (-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2)\*x^2+2\*x^3+2)/(x^3+1)/x^2

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x^2 - x + 1)\*sqrt(x + 1)/x^3, x, algorithm="maxima")

[Out] integrate(sqrt(x^2 - x + 1)\*sqrt(x + 1)/x^3, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{x^2-x+1}\sqrt{x+1}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x^2 - x + 1)\*sqrt(x + 1)/x^3, x, algorithm="fricas")

[Out] integral(sqrt(x^2 - x + 1)\*sqrt(x + 1)/x^3, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x)**(1/2)*(x**2-x+1)**(1/2)/x**3,x)`

[Out] `Integral(sqrt(x + 1)*sqrt(x**2 - x + 1)/x**3, x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x^2 - x + 1}\sqrt{x + 1}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^3,x, algorithm="giac")`

[Out] `integrate(sqrt(x^2 - x + 1)*sqrt(x + 1)/x^3, x)`

$$3.496 \quad \int x^3(1+x)^{3/2}(1-x+x^2)^{3/2} dx$$

**Optimal.** Leaf size=201

$$\frac{\frac{54}{935}\sqrt{x+1}\sqrt{x^2-x+1}x + \frac{18}{187}\sqrt{x+1}\sqrt{x^2-x+1}x^4}{36 \cdot 3^{3/4}\sqrt{2+\sqrt{3}}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle| -7-4\sqrt{3}\right)} - \frac{935\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}(x^3+1)}{935\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}(x^3+1)} + \frac{2}{17}\sqrt{x+1}\sqrt{x^2-x+1}(x^3+1)x^4$$

[Out] (54\*x\*Sqrt[1+x]\*Sqrt[1-x+x^2])/935 + (18\*x^4\*Sqrt[1+x]\*Sqrt[1-x+x^2])/187 + (2\*x^4\*Sqrt[1+x]\*Sqrt[1-x+x^2]\*(1+x^3))/17 - (36\*3^(3/4)\*Sqrt[2+Sqrt[3]]\*(1+x)^(3/2)\*Sqrt[1-x+x^2]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(935\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*(1+x^3))

**Rubi [A]** time = 0.176159, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$\frac{\frac{54}{935}\sqrt{x+1}\sqrt{x^2-x+1}x + \frac{18}{187}\sqrt{x+1}\sqrt{x^2-x+1}x^4}{36 \cdot 3^{3/4}\sqrt{2+\sqrt{3}}(x+1)^{3/2}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle| -7-4\sqrt{3}\right)} - \frac{935\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}(x^3+1)}{935\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}(x^3+1)} + \frac{2}{17}\sqrt{x+1}\sqrt{x^2-x+1}(x^3+1)x^4$$

Antiderivative was successfully verified.

[In] Int[x^3\*(1+x)^(3/2)\*(1-x+x^2)^(3/2),x]

[Out] (54\*x\*Sqrt[1+x]\*Sqrt[1-x+x^2])/935 + (18\*x^4\*Sqrt[1+x]\*Sqrt[1-x+x^2])/187 + (2\*x^4\*Sqrt[1+x]\*Sqrt[1-x+x^2]\*(1+x^3))/17 - (36\*3^(3/4)\*Sqrt[2+Sqrt[3]]\*(1+x)^(3/2)\*Sqrt[1-x+x^2]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(935\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*(1+x^3))

**Rubi in Sympy [A]** time = 14.8239, size = 180, normalized size = 0.9

$$\frac{\frac{2x^4\sqrt{x+1}(x^3+1)\sqrt{x^2-x+1}}{17} + \frac{18x^4\sqrt{x+1}\sqrt{x^2-x+1}}{187} + \frac{54x\sqrt{x+1}\sqrt{x^2-x+1}}{935}}{36 \cdot 3^{3/4}\sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}}\sqrt{\sqrt{3}+2}(x+1)^{3/2}\sqrt{x^2-x+1}F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle| -7-4\sqrt{3}\right)} - \frac{935\sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}}(x^3+1)}{935\sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}}(x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3\*(1+x)\*\*(3/2)\*(x\*\*2-x+1)\*\*(3/2),x)

[Out] 2\*x\*\*4\*sqrt(x+1)\*(x\*\*3+1)\*sqrt(x\*\*2-x+1)/17 + 18\*x\*\*4\*sqrt(x+1)\*sqrt(x\*\*2-x+1)/187 + 54\*x\*sqrt(x+1)\*sqrt(x\*\*2-x+1)/935 - (36\*3^(3/4)\*sqrt((x^2-x+1)/(x+1+sqrt(3))^2)\*sqrt(sqrt(3)+2)\*(x+1)\*\*(3/2)\*sqrt(x^2-x+1)\*elliptic\_f(asin((x-sqrt(3)+1)/(x+1+sqrt(3))), -7-4\*sqrt(3)))/(935\*sqrt((x+1)/(x+1+sqrt(3))^2)\*(x^3+1))



+ 1)/935 - 36\*3\*\*(3/4)\*sqrt((x\*\*2 - x + 1)/(x + 1 + sqrt(3))\*\*2)\*  
 sqrt(sqrt(3) + 2)\*(x + 1)\*\*(3/2)\*sqrt(x\*\*2 - x + 1)\*elliptic\_f(as  
 in((x - sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4\*sqrt(3))/(935\*sqrt  
 t((x + 1)/(x + 1 + sqrt(3))\*\*2)\*(x\*\*3 + 1))

**Mathematica [C]** time = 1.42532, size = 235, normalized size = 1.17

$$2 \left( x\sqrt{x+1} (55x^8 - 55x^7 + 55x^6 + 100x^5 - 100x^4 + 100x^3 + 27x^2 - 27x + 27) - \frac{9i\sqrt{6}(x+1)\sqrt{\frac{(\sqrt{3}-3i)x+\sqrt{3}+3i}{(\sqrt{3}-3i)(x+1)}}\sqrt{\frac{(\sqrt{3}+3i)x+\sqrt{3}-3i}{(\sqrt{3}+3i)(x+1)}}F\left(i\sinh^{-1}\sqrt{\frac{i}{\sqrt{3}+3i}}\right)}{\sqrt{\frac{i}{\sqrt{3}+3i}}}\right) \\ \hline 935\sqrt{x^2-x+1}$$

Antiderivative was successfully verified.

[In] Integrate[x^3\*(1+x)^(3/2)\*(1-x+x^2)^(3/2),x]

[Out] (2\*(x\*Sqrt[1+x]\*(27-27\*x+27\*x^2+100\*x^3-100\*x^4+100\*x^5+55\*x^6-55\*x^7+55\*x^8)-((9\*I)\*Sqrt[6]\*(1+x)\*Sqrt[(3\*I+Sqrt[3]+(-3\*I+Sqrt[3])\*x]/((-3\*I+Sqrt[3])\*(1+x))]\*Sqrt[(-3\*I+Sqrt[3]+(3\*I+Sqrt[3])\*x)/((3\*I+Sqrt[3])\*(1+x))]\*EllipticF[I\*ArcSinh[Sqrt[(-6\*I)/(3\*I+Sqrt[3])]/Sqrt[1+x]],(3\*I+Sqrt[3])/(3\*I-Sqrt[3])]/Sqrt[(-I)/(3\*I+Sqrt[3])])))/(935\*Sqrt[1-x+x^2])

**Maple [A]** time = 0.035, size = 262, normalized size = 1.3

$$\frac{2}{935x^3+935}\sqrt{1+x}\sqrt{x^2-x+1}\left(55x^{10}+155x^7+27i\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\text{EllipticF}\left(\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3\*(1+x)^(3/2)\*(x^2-x+1)^(3/2),x)

[Out] 2/935\*(1+x)^(1/2)\*(x^2-x+1)^(1/2)\*(55\*x^10+155\*x^7+27\*I\*(-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2),(-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))\*3^(1/2)-81\*(-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2),(-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))+127\*x^4+27\*x)/(x^3+1)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (x^2 - x + 1)^{\frac{3}{2}}(x + 1)^{\frac{3}{2}}x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2 - x + 1)^(3/2)\*(x + 1)^(3/2)\*x^3,x, algorithm="maxima")

[Out] integrate((x^2 - x + 1)^(3/2)\*(x + 1)^(3/2)\*x^3, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((x^6 + x^3)\sqrt{x^2 - x + 1}\sqrt{x + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3,x, algorithm="fricas")`

[Out] `integral((x^6 + x^3)*sqrt(x^2 - x + 1)*sqrt(x + 1), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^3 (x + 1)^{\frac{3}{2}} (x^2 - x + 1)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(1+x)**(3/2)*(x**2-x+1)**(3/2), x)`

[Out] `Integral(x**3*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (x^2 - x + 1)^{\frac{3}{2}}(x + 1)^{\frac{3}{2}}x^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3,x, algorithm="giac")`

[Out] `integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3, x)`

$$3.497 \quad \int x^2(1+x)^{3/2} (1-x+x^2)^{3/2} dx$$

**Optimal.** Leaf size=23

$$\frac{2}{15}(x+1)^{5/2} (x^2-x+1)^{5/2}$$

[Out]  $(2*(1+x)^{(5/2)}*(1-x+x^2)^{(5/2)})/15$

**Rubi [A]** time = 0.0172855, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$\frac{2}{15}(x+1)^{5/2} (x^2-x+1)^{5/2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2*(1+x)^{(3/2)}*(1-x+x^2)^{(3/2)}, x]$

[Out]  $(2*(1+x)^{(5/2)}*(1-x+x^2)^{(5/2)})/15$

**Rubi in Sympy [A]** time = 7.31615, size = 26, normalized size = 1.13

$$\frac{2\sqrt{x+1} (x^3+1)^2 \sqrt{x^2-x+1}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**2*(1+x)**(3/2)*(x**2-x+1)**(3/2), x)$

[Out]  $2*\text{sqrt}(x+1)*(x**3+1)**2*\text{sqrt}(x**2-x+1)/15$

**Mathematica [A]** time = 0.0363037, size = 23, normalized size = 1.

$$\frac{2}{15}(x+1)^{5/2} (x^2-x+1)^{5/2}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^2*(1+x)^{(3/2)}*(1-x+x^2)^{(3/2)}, x]$

[Out]  $(2*(1+x)^{(5/2)}*(1-x+x^2)^{(5/2)})/15$

**Maple [A]** time = 0.006, size = 18, normalized size = 0.8

$$\frac{2}{15}(1+x)^{\frac{5}{2}} (x^2-x+1)^{\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^2*(1+x)^{(3/2)}*(x^2-x+1)^{(3/2)}, x)$

[Out]  $2/15 * (1+x)^{(5/2)} * (x^2-x+1)^{(5/2)}$

**Maxima [A]** time = 0.779158, size = 36, normalized size = 1.57

$$\frac{2}{15} (x^6 + 2x^3 + 1) \sqrt{x^2 - x + 1} \sqrt{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2 - x + 1)^(3/2) * (x + 1)^(3/2) * x^2, x, algorithm="maxima")`

[Out]  $2/15 * (x^6 + 2 * x^3 + 1) * \text{sqrt}(x^2 - x + 1) * \text{sqrt}(x + 1)$

**Fricas [A]** time = 0.281954, size = 36, normalized size = 1.57

$$\frac{2}{15} (x^6 + 2x^3 + 1) \sqrt{x^2 - x + 1} \sqrt{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2 - x + 1)^(3/2) * (x + 1)^(3/2) * x^2, x, algorithm="fricas")`

[Out]  $2/15 * (x^6 + 2 * x^3 + 1) * \text{sqrt}(x^2 - x + 1) * \text{sqrt}(x + 1)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x^2 (x + 1)^{\frac{3}{2}} (x^2 - x + 1)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(1+x)**(3/2)*(x**2-x+1)**(3/2), x)`

[Out] `Integral(x**2*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)`

**GIAC/XCAS [A]** time = 0.293956, size = 100, normalized size = 4.35

$$\frac{2}{45} (((3((x + 1)(x - 5) + 15)(x + 1) - 59)(x + 1) + 42)(x + 1) - 15) \sqrt{(x + 1)^2 - 3x(x + 1)}^{\frac{3}{2}} + \frac{2}{9} \sqrt{(x + 1)^2 - 3x((x + 1)(x - 2) + 3)(x + 1)}^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2 - x + 1)^(3/2) * (x + 1)^(3/2) * x^2, x, algorithm="giac")`

[Out]  $2/45 * (((3 * ((x + 1) * (x - 5) + 15) * (x + 1) - 59) * (x + 1) + 42) * (x + 1) - 15) * \text{sqrt}((x + 1)^2 - 3 * x) * (x + 1)^{(3/2)} + 2/9 * \text{sqrt}((x + 1)^2 - 3 * x) * ((x + 1) * (x - 2) + 3) * (x + 1)^{(3/2)}$

$$3.498 \quad \int x(1+x)^{3/2} (1-x+x^2)^{3/2} dx$$

**Optimal.** Leaf size=325

$$\begin{aligned} & \frac{18}{91} \sqrt{x+1} \sqrt{x^2-x+1} x^2 + \frac{54\sqrt{x+1}\sqrt{x^2-x+1}}{91(x+\sqrt{3}+1)} + \frac{2}{13} \sqrt{x+1} \sqrt{x^2-x+1} (x^3+1) x^2 \\ & + \frac{18\sqrt{2}3^{3/4}(x+1)^{3/2}\sqrt{x^2-x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{91 \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} (x^3+1)} \\ & - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)^{3/2}\sqrt{x^2-x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{91 \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} (x^3+1)} \end{aligned}$$

[Out] (18\*x^2\*Sqrt[1+x]\*Sqrt[1-x+x^2])/91 + (54\*Sqrt[1+x]\*Sqrt[1-x+x^2])/(91\*(1+Sqrt[3]+x)) + (2\*x^2\*Sqrt[1+x]\*Sqrt[1-x+x^2]\*(1+x^3))/13 - (27\*3^(1/4)\*Sqrt[2-Sqrt[3]]\*(1+x)^(3/2)\*Sqrt[1-x+x^2]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticE[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(91\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*(1+x^3)) + (18\*Sqrt[2]\*3^(3/4)\*(1+x)^(3/2)\*Sqrt[1-x+x^2]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(91\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*(1+x^3))

**Rubi [A]** time = 0.230249, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$\begin{aligned} & \frac{18}{91} \sqrt{x+1} \sqrt{x^2-x+1} x^2 + \frac{54\sqrt{x+1}\sqrt{x^2-x+1}}{91(x+\sqrt{3}+1)} + \frac{2}{13} \sqrt{x+1} \sqrt{x^2-x+1} (x^3+1) x^2 \\ & + \frac{18\sqrt{2}3^{3/4}(x+1)^{3/2}\sqrt{x^2-x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{91 \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} (x^3+1)} \\ & - \frac{27\sqrt[4]{3}\sqrt{2-\sqrt{3}}(x+1)^{3/2}\sqrt{x^2-x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{91 \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} (x^3+1)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x\*(1+x)^(3/2)\*(1-x+x^2)^(3/2),x]

[Out] (18\*x^2\*Sqrt[1+x]\*Sqrt[1-x+x^2])/91 + (54\*Sqrt[1+x]\*Sqrt[1-x+x^2])/(91\*(1+Sqrt[3]+x)) + (2\*x^2\*Sqrt[1+x]\*Sqrt[1-x+x^2]\*(1+x^3))/13 - (27\*3^(1/4)\*Sqrt[2-Sqrt[3]]\*(1+x)^(3/2)\*Sqrt[1-x+x^2]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticE[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(91\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*(1+x^3)) + (18\*Sqrt[2]\*3^(3/4)\*(1+x)^(3/2)\*Sqrt[1-x+x^2]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(91\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*(1+x^3))

**Rubi in Sympy [A]** time = 19.7817, size = 291, normalized size = 0.9

$$\frac{2x^2\sqrt{x+1}(x^3+1)\sqrt{x^2-x+1}}{13} + \frac{18x^2\sqrt{x+1}\sqrt{x^2-x+1}}{91} + \frac{54\sqrt{x+1}\sqrt{x^2-x+1}}{91(x+1+\sqrt{3})}$$

$$- \frac{27\sqrt[3]{3}\sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}}\sqrt{-\sqrt{3}+2}(x+1)^{\frac{3}{2}}\sqrt{x^2-x+1}E\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\right|-7-4\sqrt{3})}{91\sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}}(x^3+1)}$$

$$+ \frac{18\sqrt{2}\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}}(x+1)^{\frac{3}{2}}\sqrt{x^2-x+1}F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\right|-7-4\sqrt{3})}{91\sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}}(x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x*(1+x)**(3/2)*(x**2-x+1)**(3/2),x)`

[Out] `2*x**2*sqrt(x+1)*(x**3+1)*sqrt(x**2-x+1)/13 + 18*x**2*sqrt(x+1)*sqrt(x**2-x+1)/91 + 54*sqrt(x+1)*sqrt(x**2-x+1)/(91*(x+1+sqrt(3))) - 27*3**(1/4)*sqrt((x**2-x+1)/(x+1+sqrt(3)))**2*sqrt(-sqrt(3)+2)*(x+1)**(3/2)*sqrt(x**2-x+1)*elliptic_e(asin((x-sqrt(3)+1)/(x+1+sqrt(3))), -7-4*sqrt(3))/(91*sqrt((x+1)/(x+1+sqrt(3)))**2)*(x**3+1) + 18*sqrt(2)*3**(3/4)*sqrt((x**2-x+1)/(x+1+sqrt(3)))**2*(x+1)**(3/2)*sqrt(x**2-x+1)*elliptic_f(asin((x-sqrt(3)+1)/(x+1+sqrt(3))), -7-4*sqrt(3))/(91*sqrt((x+1)/(x+1+sqrt(3)))**2)*(x**3+1)`

**Mathematica [C]** time = 0.832645, size = 244, normalized size = 0.75

$$\frac{\sqrt{x+1}\left(4x^2(x^2-x+1)(7x^3+16) - \frac{27\sqrt{2}\sqrt{\frac{2ix+\sqrt{3}-i}{\sqrt{3}-3i}}\left(\left(\sqrt{3}-3i\right)E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{-i(x+1)}{3i+\sqrt{3}}}\right)\right)\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right) - \left(\sqrt{3}-i\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{-i(x+1)}{3i+\sqrt{3}}}\right)\right)\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{\frac{-i(x+1)}{-2ix+\sqrt{3}+i}}}\right)}{182\sqrt{x^2-x+1}}$$

Warning: Unable to verify antiderivative.

[In] `Integrate[x*(1+x)^(3/2)*(1-x+x^2)^(3/2),x]`

[Out] `(Sqrt[1+x]*(4*x^2*(1-x+x^2)*(16+7*x^3) - (27*Sqrt[2]*Sqrt[(-I+Sqrt[3]+(2*I)*x)/(-3*I+Sqrt[3])]*((-3*I+Sqrt[3])*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1+x))/(3*I+Sqrt[3]])], (3*I+Sqrt[3])/(3*I-Sqrt[3])] - (-I+Sqrt[3])*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1+x))/(3*I+Sqrt[3]])], (3*I+Sqrt[3])/(3*I-Sqrt[3])]))/Sqrt[((-I)*(1+x))/(I+Sqrt[3]-(2*I)*x)])/(182*Sqrt[1-x+x^2])`

**Maple [A]** time = 0.034, size = 366, normalized size = 1.1

$$\frac{1}{91x^3+91}\sqrt{1+x}\sqrt{x^2-x+1}\left(14x^8+27i\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\operatorname{EllipticF}\left(\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}},\sqrt{-\frac{3}{i\sqrt{3}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(1+x)^(3/2)*(x^2-x+1)^(3/2),x)`

```
[Out] 1/91*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(14*x^8+27*I*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))*3^(1/2)+46*x^5+81*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticF((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))-162*(-2*(1+x)/(-3+I*3^(1/2)))^(1/2)*((I*3^(1/2)-2*x+1)/(I*3^(1/2)+3))^(1/2)*((I*3^(1/2)+2*x-1)/(-3+I*3^(1/2)))^(1/2)*EllipticE((-2*(1+x)/(-3+I*3^(1/2)))^(1/2),(-(-3+I*3^(1/2))/(I*3^(1/2)+3))^(1/2))+32*x^2)/(x^3+1)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (x^2 - x + 1)^{\frac{3}{2}}(x + 1)^{\frac{3}{2}}x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x,x, algorithm="maxima")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x, x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((x^4 + x)\sqrt{x^2 - x + 1}\sqrt{x + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x,x, algorithm="fricas")
```

```
[Out] integral((x^4 + x)*sqrt(x^2 - x + 1)*sqrt(x + 1), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int x(x + 1)^{\frac{3}{2}}(x^2 - x + 1)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(1+x)**(3/2)*(x**2-x+1)**(3/2),x)
```

```
[Out] Integral(x*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (x^2 - x + 1)^{\frac{3}{2}}(x + 1)^{\frac{3}{2}}x \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x,x, algorithm="giac")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x, x)
```

$$3.499 \quad \int (1+x)^{3/2} (1-x+x^2)^{3/2} dx$$

**Optimal.** Leaf size=173

$$\frac{18}{55}x\sqrt{x^2-x+1}\sqrt{x+1} + \frac{2}{11}x\sqrt{x^2-x+1}(x^3+1)\sqrt{x+1} \\ + \frac{18 \cdot 3^{3/4} \sqrt{2+\sqrt{3}} \sqrt{x^2-x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} (x+1)^{3/2} F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \middle| -7-4\sqrt{3}\right)}{55 \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} (x^3+1)}$$

[Out] (18\*x\*Sqrt[1+x]\*Sqrt[1-x+x^2])/55 + (2\*x\*Sqrt[1+x]\*Sqrt[1-x+x^2]\*(1+x^3))/11 + (18\*3^(3/4)\*Sqrt[2+Sqrt[3]]\*(1+x)^(3/2)\*Sqrt[1-x+x^2]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(55\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*(1+x^3))

**Rubi [A]** time = 0.102063, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{18}{55}x\sqrt{x^2-x+1}\sqrt{x+1} + \frac{2}{11}x\sqrt{x^2-x+1}(x^3+1)\sqrt{x+1} \\ + \frac{18 \cdot 3^{3/4} \sqrt{2+\sqrt{3}} \sqrt{x^2-x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} (x+1)^{3/2} F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \middle| -7-4\sqrt{3}\right)}{55 \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} (x^3+1)}$$

Antiderivative was successfully verified.

[In] Int[(1+x)^(3/2)\*(1-x+x^2)^(3/2),x]

[Out] (18\*x\*Sqrt[1+x]\*Sqrt[1-x+x^2])/55 + (2\*x\*Sqrt[1+x]\*Sqrt[1-x+x^2]\*(1+x^3))/11 + (18\*3^(3/4)\*Sqrt[2+Sqrt[3]]\*(1+x)^(3/2)\*Sqrt[1-x+x^2]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(55\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*(1+x^3))

**Rubi in Sympy [A]** time = 8.07967, size = 155, normalized size = 0.9

$$\frac{2x\sqrt{x+1}(x^3+1)\sqrt{x^2-x+1}}{11} + \frac{18x\sqrt{x+1}\sqrt{x^2-x+1}}{55} \\ + \frac{18 \cdot 3^{3/4} \sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}} \sqrt{\sqrt{3}+2} (x+1)^{3/2} \sqrt{x^2-x+1} F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right) \middle| -7-4\sqrt{3}\right)}{55 \sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}} (x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((1+x)\*\*(3/2)\*(x\*\*2-x+1)\*\*(3/2),x)

[Out] 2\*x\*sqrt(x+1)\*(x\*\*3+1)\*sqrt(x\*\*2-x+1)/11 + 18\*x\*sqrt(x+1)\*sqrt(x\*\*2-x+1)/55 + 18\*3\*\*(3/4)\*sqrt((x\*\*2-x+1)/(x+1+sqrt(3)))\*\*2)\*sqrt(sqrt(3)+2)\*(x+1)\*\*(3/2)\*sqrt(x\*\*2-x+1)\*elliptic\_f(asin((x-sqrt(3)+1)/(x+1+sqrt(3))), -7-4\*sqrt(3))/(55\*sqrt((x+1)/(x+1+sqrt(3)))\*\*2)\*(x\*\*3+1)



**Mathematica [C]** time = 1.07934, size = 176, normalized size = 1.02

$$\frac{2x\sqrt{x+1}(x^2-x+1)(5x^3+14) + \frac{9i(x+1)\sqrt{1+\frac{6i}{(\sqrt{3}-3i)(x+1)}}\sqrt{6-\frac{36i}{(\sqrt{3}+3i)(x+1)}}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{-6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)\right)\frac{3i+\sqrt{3}}{3i-\sqrt{3}}}{\sqrt{\frac{-i}{\sqrt{3}+3i}}}}{55\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + x)^(3/2)\*(1 - x + x^2)^(3/2), x]

[Out] (2\*x\*Sqrt[1 + x]\*(1 - x + x^2)\*(14 + 5\*x^3) + ((9\*I)\*(1 + x)\*Sqrt[1 + (6\*I)/((-3\*I + Sqrt[3])\*(1 + x))]\*Sqrt[6 - (36\*I)/((3\*I + Sqrt[3])\*(1 + x))]\*EllipticF[I\*ArcSinh[Sqrt[(-6\*I)/(3\*I + Sqrt[3])]]/Sqrt[1 + x]], (3\*I + Sqrt[3])/(3\*I - Sqrt[3])])/Sqrt[(-I)/(3\*I + Sqrt[3])])/(55\*Sqrt[1 - x + x^2])

**Maple [A]** time = 0.016, size = 257, normalized size = 1.5

$$-\frac{1}{55x^3+55}\sqrt{1+x}\sqrt{x^2-x+1}\left(-10x^7+27i\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\text{EllipticF}\left(\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}},\sqrt{-\frac{1+x}{-3+i\sqrt{3}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x)^(3/2)\*(x^2-x+1)^(3/2), x)

[Out] -1/55\*(1+x)^(1/2)\*(x^2-x+1)^(1/2)\*(-10\*x^7+27\*I\*(-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2), (-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))\*3^(1/2)-81\*(-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2), (-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))-38\*x^4-28\*x)/(x^3+1)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (x^2 - x + 1)^{\frac{3}{2}}(x + 1)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2 - x + 1)^(3/2)\*(x + 1)^(3/2), x, algorithm="maxima")

[Out] integrate((x^2 - x + 1)^(3/2)\*(x + 1)^(3/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(x^3+1\right)\sqrt{x^2-x+1}\sqrt{x+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2 - x + 1)^(3/2)\*(x + 1)^(3/2), x, algorithm="fricas")

[Out] `integral((x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1), x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (x + 1)^{\frac{3}{2}} (x^2 - x + 1)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x)**(3/2)*(x**2-x+1)**(3/2), x)`

[Out] `Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2), x)`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (x^2 - x + 1)^{\frac{3}{2}} (x + 1)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2), x, algorithm="giac")`

[Out] `integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2), x)`

$$3.500 \quad \int \frac{(1+x)^{3/2}(1-x+x^2)^{3/2}}{x} dx$$

**Optimal.** Leaf size=94

$$\frac{2}{3}\sqrt{x+1}\sqrt{x^2-x+1} + \frac{2}{9}\sqrt{x+1}\sqrt{x^2-x+1}(x^3+1) - \frac{2\sqrt{x+1}\sqrt{x^2-x+1}\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

[Out] (2\*Sqrt[1 + x]\*Sqrt[1 - x + x^2])/3 + (2\*Sqrt[1 + x]\*Sqrt[1 - x + x^2]\*(1 + x^3))/9 - (2\*Sqrt[1 + x]\*Sqrt[1 - x + x^2]\*ArcTanh[Sqrt[1 + x^3]])/(3\*Sqrt[1 + x^3])

**Rubi [A]** time = 0.10512, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$

$$\frac{2}{3}\sqrt{x+1}\sqrt{x^2-x+1} + \frac{2}{9}\sqrt{x+1}\sqrt{x^2-x+1}(x^3+1) - \frac{2\sqrt{x+1}\sqrt{x^2-x+1}\tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

Antiderivative was successfully verified.

[In] Int[((1 + x)^(3/2)\*(1 - x + x^2)^(3/2))/x, x]

[Out] (2\*Sqrt[1 + x]\*Sqrt[1 - x + x^2])/3 + (2\*Sqrt[1 + x]\*Sqrt[1 - x + x^2]\*(1 + x^3))/9 - (2\*Sqrt[1 + x]\*Sqrt[1 - x + x^2]\*ArcTanh[Sqrt[1 + x^3]])/(3\*Sqrt[1 + x^3])

**Rubi in Sympy [A]** time = 10.7139, size = 83, normalized size = 0.88

$$\frac{2\sqrt{x+1}(x^3+1)\sqrt{x^2-x+1}}{9} + \frac{2\sqrt{x+1}\sqrt{x^2-x+1}}{3} - \frac{2\sqrt{x+1}\sqrt{x^2-x+1}\operatorname{atanh}\left(\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((1+x)\*\*(3/2)\*(x\*\*2-x+1)\*\*(3/2)/x, x)

[Out] 2\*sqrt(x + 1)\*(x\*\*3 + 1)\*sqrt(x\*\*2 - x + 1)/9 + 2\*sqrt(x + 1)\*sqrt(x\*\*2 - x + 1)/3 - 2\*sqrt(x + 1)\*sqrt(x\*\*2 - x + 1)\*atanh(sqrt(x\*\*3 + 1))/(3\*sqrt(x\*\*3 + 1))

**Mathematica [C]** time = 0.467033, size = 201, normalized size = 2.14

$$\frac{\sqrt{x+1}\left(\frac{2}{9}(x^2-x+1)(x^3+4) + \frac{i\sqrt{2}\sqrt{\frac{-2ix+\sqrt{3}+i}{\sqrt{3}+3i}}\sqrt{\frac{2ix+\sqrt{3}-i}{\sqrt{3}-3i}}\left(\frac{3}{2}-\frac{i\sqrt{3}}{2}\right);i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{i(x+1)}{3i+\sqrt{3}}}\right)\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{\frac{i(x+1)}{\sqrt{3}+3i}}}\right)}{\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In] Integrate[((1 + x)^(3/2)\*(1 - x + x^2)^(3/2))/x, x]

[Out] (Sqrt[1 + x]\*((2\*(1 - x + x^2)\*(4 + x^3))/9 + (I\*Sqrt[2]\*Sqrt[(1 + Sqrt[3] - (2\*I)\*x)/(3\*I + Sqrt[3]])\*Sqrt[(-I + Sqrt[3] + (2\*I)\*x)/(-3\*I + Sqrt[3])])\*EllipticPi[3/2 - (I/2)\*Sqrt[3], I\*ArcSinh[Sq

```
rt[2]*Sqrt[((-I)*(1+x))/(3*I+Sqrt[3])], (3*I+Sqrt[3])/(3*I
-Sqrt[3]))/Sqrt[((-I)*(1+x))/(3*I+Sqrt[3]))/Sqrt[1-x+x
x^2]
```

**Maple [A]** time = 0.012, size = 57, normalized size = 0.6

$$-\frac{2}{9}\sqrt{1+x}\sqrt{x^2-x+1}\left(-x^3\sqrt{x^3+1}+3\operatorname{Artanh}\left(\sqrt{x^3+1}\right)-4\sqrt{x^3+1}\right)\frac{1}{\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+x)^(3/2)*(x^2-x+1)^(3/2)/x,x)
```

```
[Out] -2/9*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(-x^3*(x^3+1)^(1/2)+3*arctanh((x
^3+1)^(1/2))-4*(x^3+1)^(1/2))/(x^3+1)^(1/2)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)^(3/2)*(x+1)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((x^2-x+1)^(3/2)*(x+1)^(3/2)/x, x)
```

**Fricas [A]** time = 0.289817, size = 88, normalized size = 0.94

$$\frac{2}{9}(x^3+4)\sqrt{x^2-x+1}\sqrt{x+1}-\frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}+1\right)+\frac{1}{3}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2-x+1)^(3/2)*(x+1)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 2/9*(x^3+4)*sqrt(x^2-x+1)*sqrt(x+1)-1/3*log(sqrt(x^2-x+1)*sqrt(x+1)+1)+1/3*log(sqrt(x^2-x+1)*sqrt(x+1)-1)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(x+1)^{\frac{3}{2}}(x^2-x+1)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+x)**(3/2)*(x**2-x+1)**(3/2)/x,x)
```

```
[Out] Integral((x+1)**(3/2)*(x**2-x+1)**(3/2)/x, x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2 - x + 1)^{\frac{3}{2}}(x + 1)^{\frac{3}{2}}}{x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x,x, algorithm="giac")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x, x)
```

$$3.501 \quad \int \frac{(1+x)^{3/2}(1-x+x^2)^{3/2}}{x^2} dx$$

**Optimal.** Leaf size=323

$$\begin{aligned} & \frac{9}{7} \sqrt{x+1} \sqrt{x^2-x+1} x^2 + \frac{27 \sqrt{x+1} \sqrt{x^2-x+1}}{7(x+\sqrt{3}+1)} - \frac{\sqrt{x+1} \sqrt{x^2-x+1} (x^3+1)}{x} \\ & + \frac{9 \sqrt{23}^{3/4} (x+1)^{3/2} \sqrt{x^2-x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{7 \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} (x^3+1)} \\ & - \frac{27 \sqrt[4]{3} \sqrt{2-\sqrt{3}} (x+1)^{3/2} \sqrt{x^2-x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{14 \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} (x^3+1)} \end{aligned}$$

[Out] (9\*x^2\*Sqrt[1+x]\*Sqrt[1-x+x^2])/7 + (27\*Sqrt[1+x]\*Sqrt[1-x+x^2])/(7\*(1+Sqrt[3]+x)) - (Sqrt[1+x]\*Sqrt[1-x+x^2]\*(1+x^3))/x - (27\*3^(1/4)\*Sqrt[2-Sqrt[3]]\*(1+x)^(3/2)\*Sqrt[1-x+x^2]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticE[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(14\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*(1+x^3)) + (9\*Sqrt[2]\*3^(3/4)\*(1+x)^(3/2)\*Sqrt[1-x+x^2]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(7\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*(1+x^3))

**Rubi [A]** time = 0.248497, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$

$$\begin{aligned} & \frac{9}{7} \sqrt{x+1} \sqrt{x^2-x+1} x^2 + \frac{27 \sqrt{x+1} \sqrt{x^2-x+1}}{7(x+\sqrt{3}+1)} - \frac{\sqrt{x+1} \sqrt{x^2-x+1} (x^3+1)}{x} \\ & + \frac{9 \sqrt{23}^{3/4} (x+1)^{3/2} \sqrt{x^2-x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{7 \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} (x^3+1)} \\ & - \frac{27 \sqrt[4]{3} \sqrt{2-\sqrt{3}} (x+1)^{3/2} \sqrt{x^2-x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{14 \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} (x^3+1)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[((1+x)^(3/2)\*(1-x+x^2)^(3/2))/x^2,x]

[Out] (9\*x^2\*Sqrt[1+x]\*Sqrt[1-x+x^2])/7 + (27\*Sqrt[1+x]\*Sqrt[1-x+x^2])/(7\*(1+Sqrt[3]+x)) - (Sqrt[1+x]\*Sqrt[1-x+x^2]\*(1+x^3))/x - (27\*3^(1/4)\*Sqrt[2-Sqrt[3]]\*(1+x)^(3/2)\*Sqrt[1-x+x^2]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticE[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(14\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*(1+x^3)) + (9\*Sqrt[2]\*3^(3/4)\*(1+x)^(3/2)\*Sqrt[1-x+x^2]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(7\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*(1+x^3))

**Rubi in Sympy [A]** time = 20.9339, size = 286, normalized size = 0.89

$$\frac{9x^2\sqrt{x+1}\sqrt{x^2-x+1}}{7} + \frac{27\sqrt{x+1}\sqrt{x^2-x+1}}{7(x+1+\sqrt{3})}$$

$$- \frac{27\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}}\sqrt{-\sqrt{3}+2}(x+1)^{\frac{3}{2}}\sqrt{x^2-x+1}E\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle| -7-4\sqrt{3}\right)}{14\sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}}(x^3+1)}$$

$$+ \frac{9\sqrt{2}\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}}(x+1)^{\frac{3}{2}}\sqrt{x^2-x+1}F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle| -7-4\sqrt{3}\right)}{7\sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}}(x^3+1)}$$

$$- \frac{\sqrt{x+1}(x^3+1)\sqrt{x^2-x+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((1+x)**(3/2)*(x**2-x+1)**(3/2)/x**2,x)`

[Out] `9*x**2*sqrt(x+1)*sqrt(x**2-x+1)/7 + 27*sqrt(x+1)*sqrt(x**2-x+1)/(7*(x+1+sqrt(3))) - 27*3**(1/4)*sqrt((x**2-x+1)/(x+1+sqrt(3))**2)*sqrt(-sqrt(3)+2)*(x+1)**(3/2)*sqrt(x**2-x+1)*elliptic_e(asin((x-sqrt(3)+1)/(x+1+sqrt(3))), -7-4*sqrt(3))/(14*sqrt((x+1)/(x+1+sqrt(3))**2)*(x**3+1)) + 9*sqrt(2)*3**(3/4)*sqrt((x**2-x+1)/(x+1+sqrt(3))**2)*(x+1)**(3/2)*sqrt(x**2-x+1)*elliptic_f(asin((x-sqrt(3)+1)/(x+1+sqrt(3))), -7-4*sqrt(3))/(7*sqrt((x+1)/(x+1+sqrt(3))**2)*(x**3+1)) - sqrt(x+1)*(x**3+1)*sqrt(x**2-x+1)/x`

**Mathematica [C]** time = 0.845893, size = 244, normalized size = 0.76

$$\frac{\sqrt{x+1}\left(\frac{4(x^2-x+1)(2x^3-7)}{x} - \frac{27\sqrt{2}\sqrt{\frac{2ix+\sqrt{3}-i}{\sqrt{3}-3i}}\left(\left(\sqrt{3}-3i\right)E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{i(x+1)}{3i+\sqrt{3}}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)-\left(\sqrt{3}-i\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{i(x+1)}{3i+\sqrt{3}}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)\right)}{\sqrt{\frac{i(x+1)}{-2ix+\sqrt{3}+i}}}\right)}{28\sqrt{x^2-x+1}}$$

Warning: Unable to verify antiderivative.

[In] `Integrate[((1+x)^(3/2)*(1-x+x^2)^(3/2))/x^2,x]`

[Out] `(Sqrt[1+x]*((4*(1-x+x^2)*(-7+2*x^3))/x - (27*Sqrt[2]*Sqrt[(-I+Sqrt[3]+(2*I)*x)/(-3*I+Sqrt[3])]*((-3*I+Sqrt[3])*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1+x))/(3*I+Sqrt[3])]]), (3*I+Sqrt[3])/(3*I-Sqrt[3])]) - (-I+Sqrt[3])*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[((-I)*(1+x))/(3*I+Sqrt[3])]]], (3*I+Sqrt[3])/(3*I-Sqrt[3])])/Sqrt[((-I)*(1+x))/(I+Sqrt[3]-(2*I)*x)))/(28*Sqrt[1-x+x^2])`

**Maple [A]** time = 0.022, size = 368, normalized size = 1.1

$$\frac{1}{14x(x^3+1)}\sqrt{1+x}\sqrt{x^2-x+1}\left(27i\sqrt{3}\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\operatorname{EllipticF}\left(\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}},\sqrt{\frac{-3+}{i\sqrt{3}}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+x)^(3/2)*(x^2-x+1)^(3/2)/x^2,x)`

[Out]  $\frac{1}{14}(1+x)^{1/2}(x^2-x+1)^{1/2}(27I^3)^{1/2}(-2(1+x)/(-3+I^3)^{1/2})^{1/2}((I^3)^{1/2}-2x+1)/(I^3)^{1/2}+3)^{1/2}((I^3)^{1/2}+2x-1)/(-3+I^3)^{1/2})^{1/2}\text{EllipticF}((-2(1+x)/(-3+I^3)^{1/2})^{1/2},(-(-3+I^3)^{1/2})/(I^3)^{1/2}+3)^{1/2})^x+4x^6-162(-2(1+x)/(-3+I^3)^{1/2})^{1/2}((I^3)^{1/2}-2x+1)/(I^3)^{1/2}+3)^{1/2}((I^3)^{1/2}+2x-1)/(-3+I^3)^{1/2})^{1/2}\text{EllipticE}((-2(1+x)/(-3+I^3)^{1/2})^{1/2},(-(-3+I^3)^{1/2})/(I^3)^{1/2}+3)^{1/2})^x+81(-2(1+x)/(-3+I^3)^{1/2})^{1/2}((I^3)^{1/2}-2x+1)/(I^3)^{1/2}+3)^{1/2}((I^3)^{1/2}+2x-1)/(-3+I^3)^{1/2})^{1/2}\text{EllipticF}((-2(1+x)/(-3+I^3)^{1/2})^{1/2},(-(-3+I^3)^{1/2})/(I^3)^{1/2}+3)^{1/2})^x-10x^3-14)/x/(x^3+1)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2 - x + 1)^{\frac{3}{2}}(x + 1)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^2,x, algorithm="maxima")`

[Out] `integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^2, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(x^3 + 1)\sqrt{x^2 - x + 1}\sqrt{x + 1}}{x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^2,x, algorithm="fricas")`

[Out] `integral((x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)/x^2, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(x + 1)^{\frac{3}{2}}(x^2 - x + 1)^{\frac{3}{2}}}{x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x)**(3/2)*(x**2-x+1)**(3/2)/x**2,x)`

[Out] `Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)/x**2, x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2 - x + 1)^{\frac{3}{2}}(x + 1)^{\frac{3}{2}}}{x^2} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^2, x)
```

$$3.502 \quad \int \frac{(1+x)^{3/2}(1-x+x^2)^{3/2}}{x^3} dx$$

**Optimal.** Leaf size=175

$$\frac{9}{10}x\sqrt{x^2-x+1}\sqrt{x+1} - \frac{\sqrt{x^2-x+1}(x^3+1)\sqrt{x+1}}{2x^2} + \frac{9 \cdot 3^{3/4}\sqrt{2+\sqrt{3}}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}(x+1)^{3/2}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle| -7-4\sqrt{3}\right)}{10\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}(x^3+1)}$$

[Out] (9\*x\*Sqrt[1+x]\*Sqrt[1-x+x^2])/10 - (Sqrt[1+x]\*Sqrt[1-x+x^2]\*(1+x^3))/(2\*x^2) + (9\*3^(3/4)\*Sqrt[2+Sqrt[3]]\*(1+x)^(3/2)\*Sqrt[1-x+x^2]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(10\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*(1+x^3))

**Rubi [A]** time = 0.144037, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$\frac{9}{10}x\sqrt{x^2-x+1}\sqrt{x+1} - \frac{\sqrt{x^2-x+1}(x^3+1)\sqrt{x+1}}{2x^2} + \frac{9 \cdot 3^{3/4}\sqrt{2+\sqrt{3}}\sqrt{x^2-x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}(x+1)^{3/2}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle| -7-4\sqrt{3}\right)}{10\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}(x^3+1)}$$

Antiderivative was successfully verified.

[In] Int[((1+x)^(3/2)\*(1-x+x^2)^(3/2))/x^3,x]

[Out] (9\*x\*Sqrt[1+x]\*Sqrt[1-x+x^2])/10 - (Sqrt[1+x]\*Sqrt[1-x+x^2]\*(1+x^3))/(2\*x^2) + (9\*3^(3/4)\*Sqrt[2+Sqrt[3]]\*(1+x)^(3/2)\*Sqrt[1-x+x^2]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(10\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*(1+x^3))

**Rubi in Sympy [A]** time = 10.9406, size = 155, normalized size = 0.89

$$\frac{9x\sqrt{x+1}\sqrt{x^2-x+1}}{10} + \frac{9 \cdot 3^{3/4}\sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}}\sqrt{\sqrt{3}+2}(x+1)^{3/2}\sqrt{x^2-x+1}F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle| -7-4\sqrt{3}\right)}{10\sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}}(x^3+1)} - \frac{\sqrt{x+1}(x^3+1)\sqrt{x^2-x+1}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((1+x)\*\*(3/2)\*(x\*\*2-x+1)\*\*(3/2)/x\*\*3,x)

[Out] 9\*x\*sqrt(x+1)\*sqrt(x\*\*2-x+1)/10 + 9\*3\*\*(3/4)\*sqrt((x\*\*2-x+1)/(x+1+sqrt(3)))\*\*2)\*sqrt(sqrt(3)+2)\*(x+1)\*\*(3/2)\*sqrt(x\*\*2-x+1)\*elliptic\_f(asin((x-sqrt(3)+1)/(x+1+sqrt(3))), -7-4\*sqrt(3))/(10\*sqrt((x+1)/(x+1+sqrt(3)))\*\*2)\*(x\*\*3+1) - sqrt(x+1)\*(x\*\*3+1)\*sqrt(x\*\*2-x+1)/(2\*x\*\*2)

---

**Mathematica [C]** time = 0.646539, size = 192, normalized size = 1.1

$$\frac{\sqrt{x+1} \left( \frac{2(x^2-x+1)(4x^3-5)}{x^2} - \frac{27i\sqrt{2}\sqrt{\frac{-2ix+\sqrt{3}+i}{\sqrt{3+3i}}}\sqrt{\frac{2ix+\sqrt{3}-i}{\sqrt{3-3i}}} F\left(i \sinh^{-1}\left(\sqrt{2}\sqrt{\frac{i(x+1)}{3i+\sqrt{3}}}\right)\right)}{\sqrt{\frac{i(x+1)}{\sqrt{3+3i}}}} \right)}{20\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In] Integrate[((1 + x)^(3/2) \* (1 - x + x^2)^(3/2))/x^3, x]

[Out] (Sqrt[1 + x] \* ((2 \* (1 - x + x^2) \* (-5 + 4 \* x^3))/x^2 - ((27 \* I) \* Sqrt[2] \* Sqrt[(I + Sqrt[3] - (2 \* I) \* x)/(3 \* I + Sqrt[3])] \* Sqrt[(-I + Sqrt[3] + (2 \* I) \* x)/(-3 \* I + Sqrt[3])] \* EllipticF[I \* ArcSinh[Sqrt[2] \* Sqrt[(-I \* (1 + x))/(3 \* I + Sqrt[3])]]], (3 \* I + Sqrt[3])/(3 \* I - Sqrt[3])]) / Sqrt[(-I \* (1 + x))/(3 \* I + Sqrt[3])])) / (20 \* Sqrt[1 - x + x^2]))

---

**Maple [A]** time = 0.019, size = 264, normalized size = 1.5

$$-\frac{1}{(20x^3+20)x^2} \sqrt{1+x} \sqrt{x^2-x+1} \left( 27i\sqrt{3} \sqrt{-2\frac{1+x}{-3+i\sqrt{3}}} \sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}} \sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}} \text{EllipticF}\left(\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}}, \sqrt{-\frac{1}{i}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+x)^(3/2) \* (x^2-x+1)^(3/2)/x^3, x)

[Out] -1/20\*(1+x)^(1/2)\*(x^2-x+1)^(1/2)\*(27\*I\*3^(1/2)\*(-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2), (-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))\*x^2-81\*(-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2), (-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))\*x^2-8\*x^6+2\*x^3+10)/(x^3+1)/x^2

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2 - x + 1)^{\frac{3}{2}} (x + 1)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((x^2 - x + 1)^(3/2) \* (x + 1)^(3/2)/x^3, x, algorithm="maxima")

[Out] integrate((x^2 - x + 1)^(3/2) \* (x + 1)^(3/2)/x^3, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(x^3+1)\sqrt{x^2-x+1}\sqrt{x+1}}{x^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^3,x, algorithm="fricas")`

[Out] `integral((x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)/x^3, x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(x+1)^{\frac{3}{2}}(x^2-x+1)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+x)**(3/2)*(x**2-x+1)**(3/2)/x**3,x)`

[Out] `Integral((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)/x**3, x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(x^2-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}}{x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^3,x, algorithm="giac")`

[Out] `integrate((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)/x^3, x)`

$$3.503 \quad \int \frac{x^3}{\sqrt{1+x}\sqrt{1-x+x^2}} dx$$

**Optimal.** Leaf size=142

$$\frac{2x(x^3+1)}{5\sqrt{x+1}\sqrt{x^2-x+1}} - \frac{4\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle| -7-4\sqrt{3}\right)}{5\sqrt[3]{3}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^2-x+1}}$$

[Out] (2\*x\*(1+x^3))/(5\*Sqrt[1+x]\*Sqrt[1-x+x^2]) - (4\*Sqrt[2+Sqrt[3]]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(5\*3^(1/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2])

**Rubi [A]** time = 0.1304, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$

$$\frac{2x(x^3+1)}{5\sqrt{x+1}\sqrt{x^2-x+1}} - \frac{4\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle| -7-4\sqrt{3}\right)}{5\sqrt[3]{3}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In] Int[x^3/(Sqrt[1+x]\*Sqrt[1-x+x^2]),x]

[Out] (2\*x\*(1+x^3))/(5\*Sqrt[1+x]\*Sqrt[1-x+x^2]) - (4\*Sqrt[2+Sqrt[3]]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(5\*3^(1/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2])

**Rubi in Sympy [A]** time = 10.4375, size = 128, normalized size = 0.9

$$\frac{2x\sqrt{x+1}\sqrt{x^2-x+1}}{5} - \frac{4 \cdot 3^{\frac{3}{4}} \sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}} \sqrt{\sqrt{3}+2} (x+1)^{\frac{3}{2}} \sqrt{x^2-x+1} F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle| -7-4\sqrt{3}\right)}{15 \sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}} (x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3/(1+x)\*\*(1/2)/(x\*\*2-x+1)\*\*(1/2),x)

[Out] 2\*x\*sqrt(x+1)\*sqrt(x\*\*2-x+1)/5 - 4\*3\*\*(3/4)\*sqrt((x\*\*2-x+1)/(x+1+sqrt(3))\*\*2)\*sqrt(sqrt(3)+2)\*(x+1)\*\*(3/2)\*sqrt(x\*\*2-x+1)\*elliptic\_f(asin((x-sqrt(3)+1)/(x+1+sqrt(3))), -7-4\*sqrt(3))/(15\*sqrt((x+1)/(x+1+sqrt(3))\*\*2)\*(x\*\*3+1))

**Mathematica [C]** time = 0.94886, size = 169, normalized size = 1.19

$$\frac{6x\sqrt{x+1}(x^2-x+1) - \frac{2i(x+1)\sqrt{1+\frac{6i}{(\sqrt{3}-3i)(x+1)}}\sqrt{6-\frac{36i}{(\sqrt{3}+3i)(x+1)}}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{-6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)\middle| \frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{\frac{i}{\sqrt{3}+3i}}}}{15\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/(Sqrt[1 + x]\*Sqrt[1 - x + x^2]),x]

[Out] (6\*x\*Sqrt[1 + x]\*(1 - x + x^2) - ((2\*I)\*(1 + x)\*Sqrt[1 + (6\*I)/((-3\*I + Sqrt[3])\*(1 + x))]\*Sqrt[6 - (36\*I)/((3\*I + Sqrt[3])\*(1 + x))])\*EllipticF[I\*ArcSinh[Sqrt[(-6\*I)/(3\*I + Sqrt[3])]/Sqrt[1 + x]], (3\*I + Sqrt[3])/(3\*I - Sqrt[3])]/Sqrt[(-I)/(3\*I + Sqrt[3])])/(15\*Sqrt[1 - x + x^2])

**Maple [B]** time = 0.05, size = 248, normalized size = 1.8

$$\frac{2}{5x^3 + 5} \sqrt{1+x} \sqrt{x^2 - x + 1} \left( i \sqrt{-2 \frac{1+x}{-3+i\sqrt{3}}} \sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}} \sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}} \operatorname{EllipticF} \left( \sqrt{-2 \frac{1+x}{-3+i\sqrt{3}}}, \sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}} \right) \sqrt{3} - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x)

[Out] 2/5\*(1+x)^(1/2)\*(x^2-x+1)^(1/2)\*(I\*(-2\*(1+x)/(-3+I\*3^(1/2))))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2),(-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))\*3^(1/2)-3\*(-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2),(-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))+x^4+x)/(x^3+1)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{x^2 - x + 1} \sqrt{x + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(sqrt(x^2 - x + 1)\*sqrt(x + 1)),x, algorithm="maxima")

[Out] integrate(x^3/(sqrt(x^2 - x + 1)\*sqrt(x + 1)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{x^3}{\sqrt{x^2 - x + 1} \sqrt{x + 1}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/(sqrt(x^2 - x + 1)\*sqrt(x + 1)),x, algorithm="fricas")

[Out] integral(x^3/(sqrt(x^2 - x + 1)\*sqrt(x + 1)), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{x+1} \sqrt{x^2 - x + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)`

[Out] `Integral(x**3/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{\sqrt{x^2 - x + 1}\sqrt{x + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/(sqrt(x^2 - x + 1)*sqrt(x + 1)),x, algorithm="giac")`

[Out] `integrate(x^3/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)`

$$3.504 \quad \int \frac{x^2}{\sqrt{1+x}\sqrt{1-x+x^2}} dx$$

**Optimal.** Leaf size=23

$$\frac{2}{3}\sqrt{x+1}\sqrt{x^2-x+1}$$

[Out] (2\*Sqrt[1 + x]\*Sqrt[1 - x + x^2])/3

---

**Rubi [A]** time = 0.0185421, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$\frac{2}{3}\sqrt{x+1}\sqrt{x^2-x+1}$$

Antiderivative was successfully verified.

[In] Int[x^2/(Sqrt[1 + x]\*Sqrt[1 - x + x^2]), x]

[Out] (2\*Sqrt[1 + x]\*Sqrt[1 - x + x^2])/3

---

**Rubi in Sympy [A]** time = 7.3994, size = 19, normalized size = 0.83

$$\frac{2\sqrt{x+1}\sqrt{x^2-x+1}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(1+x)\*\*(1/2)/(x\*\*2-x+1)\*\*(1/2), x)

[Out] 2\*sqrt(x + 1)\*sqrt(x\*\*2 - x + 1)/3

---

**Mathematica [A]** time = 0.0285527, size = 23, normalized size = 1.

$$\frac{2}{3}\sqrt{x+1}\sqrt{x^2-x+1}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/(Sqrt[1 + x]\*Sqrt[1 - x + x^2]), x]

[Out] (2\*Sqrt[1 + x]\*Sqrt[1 - x + x^2])/3

---

**Maple [A]** time = 0.006, size = 18, normalized size = 0.8

$$\frac{2}{3}\sqrt{1+x}\sqrt{x^2-x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2), x)



[Out]  $\frac{2}{3} (1+x)^{1/2} (x^2-x+1)^{1/2}$

**Maxima [A]** time = 0.774214, size = 30, normalized size = 1.3

$$\frac{2(x^3 + 1)}{3\sqrt{x^2 - x + 1}\sqrt{x + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(sqrt(x^2 - x + 1)*sqrt(x + 1)),x, algorithm="maxima")`

[Out]  $\frac{2}{3} (x^3 + 1)/(\sqrt{x^2 - x + 1}\sqrt{x + 1})$

**Fricas [A]** time = 0.287209, size = 23, normalized size = 1.

$$\frac{2}{3} \sqrt{x^2 - x + 1}\sqrt{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(sqrt(x^2 - x + 1)*sqrt(x + 1)),x, algorithm="fricas")`

[Out]  $\frac{2}{3} \sqrt{x^2 - x + 1}\sqrt{x + 1}$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{\sqrt{x + 1}\sqrt{x^2 - x + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)`

[Out] `Integral(x**2/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)`

**GIAC/XCAS [A]** time = 0.274158, size = 24, normalized size = 1.04

$$\frac{2}{3} \sqrt{(x + 1)^2 - 3x}\sqrt{x + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/(sqrt(x^2 - x + 1)*sqrt(x + 1)),x, algorithm="giac")`

[Out]  $\frac{2}{3} \sqrt{(x + 1)^2 - 3x}\sqrt{x + 1}$

$$3.505 \quad \int \frac{x}{\sqrt{1+x}\sqrt{1-x+x^2}} dx$$

**Optimal.** Leaf size=253

$$\frac{2\sqrt{2}\sqrt{x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{\sqrt[4]{3} \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} \sqrt{x^2-x+1}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} \sqrt{x^2-x+1}} + \frac{2(x^3+1)}{\sqrt{x+1}(x+\sqrt{3}+1)\sqrt{x^2-x+1}}$$

[Out] (2\*(1 + x^3))/(Sqrt[1 + x]\*(1 + Sqrt[3] + x)\*Sqrt[1 - x + x^2]) - (3^(1/4)\*Sqrt[2 - Sqrt[3]]\*Sqrt[1 + x]\*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]\*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4\*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]\*Sqrt[1 - x + x^2]) + (2\*Sqrt[2]\*Sqrt[1 + x]\*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]\*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4\*Sqrt[3]])/(3^(1/4)\*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]\*Sqrt[1 - x + x^2])

**Rubi [A]** time = 0.160837, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.19$

$$\frac{2\sqrt{2}\sqrt{x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{\sqrt[4]{3} \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} \sqrt{x^2-x+1}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} \sqrt{x^2-x+1}} + \frac{2(x^3+1)}{\sqrt{x+1}(x+\sqrt{3}+1)\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In] Int[x/(Sqrt[1 + x]\*Sqrt[1 - x + x^2]), x]

[Out] (2\*(1 + x^3))/(Sqrt[1 + x]\*(1 + Sqrt[3] + x)\*Sqrt[1 - x + x^2]) - (3^(1/4)\*Sqrt[2 - Sqrt[3]]\*Sqrt[1 + x]\*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]\*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4\*Sqrt[3]])/(Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]\*Sqrt[1 - x + x^2]) + (2\*Sqrt[2]\*Sqrt[1 + x]\*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]\*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4\*Sqrt[3]])/(3^(1/4)\*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]\*Sqrt[1 - x + x^2])

**Rubi in Sympy [A]** time = 14.7091, size = 231, normalized size = 0.91

$$\frac{2\sqrt{x+1}\sqrt{x^2-x+1}}{x+1+\sqrt{3}} - \frac{\sqrt[4]{3} \sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}} \sqrt{-\sqrt{3}+2}(x+1)^{\frac{3}{2}} \sqrt{x^2-x+1} E\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right) \mid -7-4\sqrt{3}\right)}{\sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}} (x^3+1)} + \frac{2\sqrt{2} \cdot 3^{\frac{3}{4}} \sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}} (x+1)^{\frac{3}{2}} \sqrt{x^2-x+1} F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right) \mid -7-4\sqrt{3}\right)}{3 \sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}} (x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)`

[Out]  $2\sqrt{x+1}\sqrt{x^2-x+1}/(x+1+\sqrt{3}) - 3^{1/4}\sqrt{\sqrt{x^2-x+1}/(x+1+\sqrt{3})} \sqrt{-\sqrt{3}+2}\sqrt{x+1}^{3/2}\sqrt{x^2-x+1}\operatorname{elliptic}_e(\operatorname{asin}((x-\sqrt{3}+1)/(x+1+\sqrt{3})), -7-4\sqrt{3})/\sqrt{(x+1)/(x+1+\sqrt{3})}^{2(x^3+1)} + 2\sqrt{2}\sqrt{3}^{3/4}\sqrt{x^2-x+1}/(x+1+\sqrt{3})^{2(x+1)^{3/2}}\sqrt{x^2-x+1}\operatorname{elliptic}_f(\operatorname{asin}((x-\sqrt{3}+1)/(x+1+\sqrt{3})), -7-4\sqrt{3})/(3\sqrt{(x+1)/(x+1+\sqrt{3})}^{2(x^3+1)})$

**Mathematica [C]** time = 1.73202, size = 375, normalized size = 1.48

$$(x+1)^{3/2} \left( \frac{12\sqrt{-\frac{i}{\sqrt{3}+3i}}(x^2-x+1)}{(x+1)^2} + \frac{i\sqrt{2}(\sqrt{3}+3i)\sqrt{-\frac{6i}{x+1}+\sqrt{3}+3i}\sqrt{\frac{6i}{x+1}+\sqrt{3}-3i}}{\sqrt{x+1}} F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{x+1}}}{\sqrt{x+1}}\right), \frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right) + \frac{3\sqrt{2}(1-i\sqrt{3})\sqrt{-\frac{6i}{x+1}+\sqrt{3}+3i}\sqrt{\frac{6i}{x+1}+\sqrt{3}-3i}}{\sqrt{x+1}} E\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{x+1}}}{\sqrt{x+1}}\right), \frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right) \right) \\ \hline 6\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{x^2-x+1}$$

Antiderivative was successfully verified.

[In] `Integrate[x/(Sqrt[1+x]*Sqrt[1-x+x^2]),x]`

[Out]  $((1+x)^{3/2}((12\sqrt{-1}/(3I+\sqrt{3}))\sqrt{1-x+x^2})/(1+x)^2 + (3\sqrt{2}\sqrt{1-I\sqrt{3}}\sqrt{3}\sqrt{(3I+\sqrt{3})-(6I)/(1+x)})/(3I+\sqrt{3})\sqrt{(-3I+\sqrt{3}+(6I)/(1+x))}/(-3I+\sqrt{3}))\operatorname{EllipticE}[I\operatorname{ArcSinh}[\sqrt{(-6I)/(3I+\sqrt{3})}]/\sqrt{1+x}], (3I+\sqrt{3})/(3I-\sqrt{3}))/\sqrt{1+x} + (I\sqrt{2}\sqrt{3}\sqrt{3}\sqrt{(3I+\sqrt{3})-(6I)/(1+x)})/(3I+\sqrt{3})\sqrt{(-3I+\sqrt{3}+(6I)/(1+x))}/(-3I+\sqrt{3})\operatorname{EllipticF}[I\operatorname{ArcSinh}[\sqrt{(-6I)/(3I+\sqrt{3})}]/\sqrt{1+x}], (3I+\sqrt{3})/(3I-\sqrt{3}))/\sqrt{1+x})/(6\sqrt{-1}/(3I+\sqrt{3}))\sqrt{1-x+x^2}$

**Maple [A]** time = 0.037, size = 275, normalized size = 1.1

$$\frac{-3+i\sqrt{3}}{2x^3+2}\sqrt{1+x}\sqrt{x^2-x+1}\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\left(i\operatorname{EllipticE}\left(\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}}, \sqrt{\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x/(1+x)^(1/2)/(x^2-x+1)^(1/2),x)`

[Out]  $1/2(1+x)^{1/2}(x^2-x+1)^{1/2}(-3+I\sqrt{3})^{1/2}(-2(1+x)/(-3+I\sqrt{3})^{1/2})^{1/2}((I\sqrt{3})^{1/2}-2x+1)/(I\sqrt{3})^{1/2}+3)^{1/2}((I\sqrt{3})^{1/2}+2x-1)/(-3+I\sqrt{3})^{1/2})^{1/2}(I\operatorname{EllipticE}((-2(1+x)/(-3+I\sqrt{3})^{1/2}))^{1/2}, (-(-3+I\sqrt{3})^{1/2})/(I\sqrt{3})^{1/2}+3)^{1/2})^{1/2}-I\operatorname{EllipticF}((-2(1+x)/(-3+I\sqrt{3})^{1/2})^{1/2}, (-(-3+I\sqrt{3})^{1/2})/(I\sqrt{3})^{1/2}+3)^{1/2})^{1/2}+3\operatorname{EllipticE}((-2(1+x)/(-3+I\sqrt{3})^{1/2})^{1/2}, (-(-3+I\sqrt{3})^{1/2})/(I\sqrt{3})^{1/2}+3)^{1/2})^{1/2}-\operatorname{EllipticF}((-2(1+x)/(-3+I\sqrt{3})^{1/2})^{1/2}, (-(-3+I\sqrt{3})^{1/2})/(I\sqrt{3})^{1/2}+3)^{1/2})^{1/2})/(x^3+1)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^2 - x + 1}\sqrt{x + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(sqrt(x^2 - x + 1)\*sqrt(x + 1)),x, algorithm="maxima")

[Out] integrate(x/(sqrt(x^2 - x + 1)\*sqrt(x + 1)), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{\sqrt{x^2 - x + 1}\sqrt{x + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(sqrt(x^2 - x + 1)\*sqrt(x + 1)),x, algorithm="fricas")

[Out] integral(x/(sqrt(x^2 - x + 1)\*sqrt(x + 1)), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x + 1}\sqrt{x^2 - x + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)\*\*(1/2)/(x\*\*2-x+1)\*\*(1/2),x)

[Out] Integral(x/(sqrt(x + 1)\*sqrt(x\*\*2 - x + 1)), x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{\sqrt{x^2 - x + 1}\sqrt{x + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(sqrt(x^2 - x + 1)\*sqrt(x + 1)),x, algorithm="giac")

[Out] integrate(x/(sqrt(x^2 - x + 1)\*sqrt(x + 1)), x)

$$3.506 \quad \int \frac{1}{\sqrt{1+x}\sqrt{1-x+x^2}} dx$$

**Optimal.** Leaf size=110

$$\frac{2\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle| -7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^2-x+1}}$$

[Out] (2\*Sqrt[2 + Sqrt[3]]\*Sqrt[1 + x]\*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]\*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4\*Sqrt[3]])/(3^(1/4)\*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]\*Sqrt[1 - x + x^2])

**Rubi [A]** time = 0.0631854, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{2\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle| -7-4\sqrt{3}\right)}{\sqrt[4]{3}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[1 + x]\*Sqrt[1 - x + x^2]), x]

[Out] (2\*Sqrt[2 + Sqrt[3]]\*Sqrt[1 + x]\*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]\*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4\*Sqrt[3]])/(3^(1/4)\*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]\*Sqrt[1 - x + x^2])

**Rubi in Sympy [A]** time = 6.37117, size = 105, normalized size = 0.95

$$\frac{2 \cdot 3^{\frac{3}{4}} \sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}} \sqrt{\sqrt{3}+2} (x+1)^{\frac{3}{2}} \sqrt{x^2-x+1} F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle| -7-4\sqrt{3}\right)}{3 \sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}} (x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(1+x)\*\*(1/2)/(x\*\*2-x+1)\*\*(1/2), x)

[Out] 2\*3\*\*(3/4)\*sqrt((x\*\*2 - x + 1)/(x + 1 + sqrt(3))\*\*2)\*sqrt(sqrt(3) + 2)\*(x + 1)\*\*(3/2)\*sqrt(x\*\*2 - x + 1)\*elliptic\_f(asin((x - sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4\*sqrt(3))/(3\*sqrt((x + 1)/(x + 1 + sqrt(3))\*\*2)\*(x\*\*3 + 1))

**Mathematica [C]** time = 0.248078, size = 148, normalized size = 1.35

$$\frac{i(x+1)\sqrt{1+\frac{6i}{(\sqrt{3}-3i)(x+1)}}\sqrt{\frac{2}{3}-\frac{4i}{(\sqrt{3}+3i)(x+1)}}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{-6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)\middle| \frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[1 + x]\*Sqrt[1 - x + x^2]),x]

[Out] (I\*(1 + x)\*Sqrt[1 + (6\*I)/((-3\*I + Sqrt[3])\*(1 + x))]\*Sqrt[2/3 - (4\*I)/((3\*I + Sqrt[3])\*(1 + x))]\*EllipticF[I\*ArcSinh[Sqrt[(-6\*I)/(3\*I + Sqrt[3])]/Sqrt[1 + x]], (3\*I + Sqrt[3])/(3\*I - Sqrt[3])])/(Sqrt[(-I)/(3\*I + Sqrt[3])]\*Sqrt[1 - x + x^2])

**Maple [A]** time = 0.02, size = 137, normalized size = 1.3

$$\frac{3 - i\sqrt{3}}{x^3 + 1} \sqrt{1+x} \sqrt{x^2 - x + 1} \sqrt{-2 \frac{1+x}{-3+i\sqrt{3}}} \sqrt{\frac{i\sqrt{3} - 2x + 1}{i\sqrt{3} + 3}} \sqrt{\frac{i\sqrt{3} + 2x - 1}{-3+i\sqrt{3}}} \text{EllipticF}\left(\sqrt{-2 \frac{1+x}{-3+i\sqrt{3}}}, \sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+x)^(1/2)/(x^2-x+1)^(1/2),x)

[Out] (3-I\*3^(1/2))\*(1+x)^(1/2)\*(x^2-x+1)^(1/2)\*(-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2), (-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))/(x^3+1)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2 - x + 1}\sqrt{x + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(x^2 - x + 1)\*sqrt(x + 1)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(x^2 - x + 1)\*sqrt(x + 1)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{x^2 - x + 1}\sqrt{x + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(x^2 - x + 1)\*sqrt(x + 1)),x, algorithm="fricas")

[Out] integral(1/(sqrt(x^2 - x + 1)\*sqrt(x + 1)), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x + 1}\sqrt{x^2 - x + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)`

[Out] `Integral(1/(sqrt(x + 1)*sqrt(x**2 - x + 1)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2 - x + 1}\sqrt{x + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)),x, algorithm="giac")`

[Out] `integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)), x)`

$$3.507 \quad \int \frac{1}{x\sqrt{1+x}\sqrt{1-x+x^2}} dx$$

**Optimal.** Leaf size=42

$$-\frac{2\sqrt{x^3+1} \tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

[Out] (-2\*Sqrt[1 + x^3]\*ArcTanh[Sqrt[1 + x^3]])/(3\*Sqrt[1 + x]\*Sqrt[1 - x + x^2])

**Rubi [A]** time = 0.0853295, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$-\frac{2\sqrt{x^3+1} \tanh^{-1}\left(\sqrt{x^3+1}\right)}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*Sqrt[1 + x]\*Sqrt[1 - x + x^2]),x]

[Out] (-2\*Sqrt[1 + x^3]\*ArcTanh[Sqrt[1 + x^3]])/(3\*Sqrt[1 + x]\*Sqrt[1 - x + x^2])

**Rubi in Sympy [A]** time = 8.97279, size = 39, normalized size = 0.93

$$-\frac{2\sqrt{x+1}\sqrt{x^2-x+1} \operatorname{atanh}\left(\sqrt{x^3+1}\right)}{3\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(1+x)\*\*(1/2)/(x\*\*2-x+1)\*\*(1/2),x)

[Out] -2\*sqrt(x + 1)\*sqrt(x\*\*2 - x + 1)\*atanh(sqrt(x\*\*3 + 1))/(3\*sqrt(x\*\*3 + 1))

**Mathematica [C]** time = 0.405498, size = 68, normalized size = 1.62

$$-\frac{2\sqrt{x+1} \left(1 + \sqrt[3]{-1}; \sin^{-1}\left(\sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}}\right) \sqrt[3]{-1}\right)}{\sqrt{3} \sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*Sqrt[1 + x]\*Sqrt[1 - x + x^2]),x]

[Out] (-2\*Sqrt[1 + x]\*EllipticPi[1 + (-1)^(1/3), ArcSin[Sqrt[(1 + (-1)^(2/3)\*x)/(1 + (-1)^(1/3))]], (-1)^(1/3)]/(Sqrt[3]\*Sqrt[(1 + x)/(1 + (-1)^(1/3))]))



**Maple [A]** time = 0.033, size = 33, normalized size = 0.8

$$-\frac{2}{3} \operatorname{Artanh}\left(\sqrt{x^3+1}\right) \sqrt{1+x} \sqrt{x^2-x+1} \frac{1}{\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(1+x)^(1/2)/(x^2-x+1)^(1/2), x)`

[Out] `-2/3*arctanh((x^3+1)^(1/2))*(1+x)^(1/2)*(x^2-x+1)^(1/2)/(x^3+1)^(1/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x), x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x), x)`

**Fricas [A]** time = 0.285613, size = 58, normalized size = 1.38

$$-\frac{1}{3} \log\left(\sqrt{x^2-x+1}\sqrt{x+1}+1\right) + \frac{1}{3} \log\left(\sqrt{x^2-x+1}\sqrt{x+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x), x, algorithm="fricas")`

[Out] `-1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) + 1) + 1/3*log(sqrt(x^2 - x + 1)*sqrt(x + 1) - 1)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x\sqrt{x+1}\sqrt{x^2-x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(1+x)**(1/2)/(x**2-x+1)**(1/2), x)`

[Out] `Integral(1/(x*sqrt(x + 1)*sqrt(x**2 - x + 1)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x), x)
```

$$3.508 \quad \int \frac{1}{x^2 \sqrt{1+x} \sqrt{1-x+x^2}} dx$$

Optimal. Leaf size=282

$$\frac{\sqrt{2}\sqrt{x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{\sqrt[4]{3} \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} \sqrt{x^2-x+1}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{2 \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} \sqrt{x^2-x+1}} - \frac{x^3+1}{x\sqrt{x+1}\sqrt{x^2-x+1}} + \frac{x^3+1}{\sqrt{x+1}(x+\sqrt{3}+1)\sqrt{x^2-x+1}}$$

[Out] -((1 + x^3)/(x\*Sqrt[1 + x]\*Sqrt[1 - x + x^2])) + (1 + x^3)/(Sqrt[1 + x]\*(1 + Sqrt[3] + x)\*Sqrt[1 - x + x^2]) - (3^(1/4)\*Sqrt[2 - Sqrt[3]]\*Sqrt[1 + x]\*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]\*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4\*Sqrt[3]])/(2\*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]\*Sqrt[1 - x + x^2]) + (Sqrt[2]\*Sqrt[1 + x]\*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]\*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4\*Sqrt[3]])/(3^(1/4)\*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]\*Sqrt[1 - x + x^2])

**Rubi [A]** time = 0.227452, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$

$$\frac{\sqrt{2}\sqrt{x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{\sqrt[4]{3} \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} \sqrt{x^2-x+1}} - \frac{\sqrt[4]{3}\sqrt{2-\sqrt{3}}\sqrt{x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{2 \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} \sqrt{x^2-x+1}} - \frac{x^3+1}{x\sqrt{x+1}\sqrt{x^2-x+1}} + \frac{x^3+1}{\sqrt{x+1}(x+\sqrt{3}+1)\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*Sqrt[1 + x]\*Sqrt[1 - x + x^2]),x]

[Out] -((1 + x^3)/(x\*Sqrt[1 + x]\*Sqrt[1 - x + x^2])) + (1 + x^3)/(Sqrt[1 + x]\*(1 + Sqrt[3] + x)\*Sqrt[1 - x + x^2]) - (3^(1/4)\*Sqrt[2 - Sqrt[3]]\*Sqrt[1 + x]\*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]\*EllipticE[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4\*Sqrt[3]])/(2\*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]\*Sqrt[1 - x + x^2]) + (Sqrt[2]\*Sqrt[1 + x]\*Sqrt[(1 - x + x^2)/(1 + Sqrt[3] + x)^2]\*EllipticF[ArcSin[(1 - Sqrt[3] + x)/(1 + Sqrt[3] + x)], -7 - 4\*Sqrt[3]])/(3^(1/4)\*Sqrt[(1 + x)/(1 + Sqrt[3] + x)^2]\*Sqrt[1 - x + x^2])

**Rubi in Sympy [A]** time = 18.1912, size = 248, normalized size = 0.88

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x+1+\sqrt{3}} - \frac{\sqrt[3]{3} \sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}} \sqrt{-\sqrt{3}+2} (x+1)^{\frac{3}{2}} \sqrt{x^2-x+1} E\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle| -7-4\sqrt{3}\right)}{2 \sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}} (x^3+1)}$$

$$+ \frac{\sqrt{2} \cdot 3^{\frac{3}{4}} \sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}} (x+1)^{\frac{3}{2}} \sqrt{x^2-x+1} F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle| -7-4\sqrt{3}\right)}{3 \sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}} (x^3+1)} - \frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x**2/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)`

[Out] `sqrt(x + 1)*sqrt(x**2 - x + 1)/(x + 1 + sqrt(3)) - 3**(1/4)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*sqrt(-sqrt(3) + 2)*(x + 1)**(3/2)*sqrt(x**2 - x + 1)*elliptic_e(asin((x - sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(2*sqrt((x + 1)/(x + 1 + sqrt(3))**2)*(x**3 + 1)) + sqrt(2)*3**(3/4)*sqrt((x**2 - x + 1)/(x + 1 + sqrt(3))**2)*(x + 1)**(3/2)*sqrt(x**2 - x + 1)*elliptic_f(asin((x - sqrt(3) + 1)/(x + 1 + sqrt(3))), -7 - 4*sqrt(3))/(3*sqrt((x + 1)/(x + 1 + sqrt(3))**2)*(x**3 + 1)) - sqrt(x + 1)*sqrt(x**2 - x + 1)/x`

**Mathematica [C]** time = 1.14972, size = 400, normalized size = 1.42

$$\frac{\sqrt{x+1}\sqrt{x^2-x+1}}{x} + \frac{(x+1)^{3/2} \left( \frac{12\sqrt{-\frac{i}{\sqrt{3}+3i}}(x^2-x+1)}{(x+1)^2} + \frac{i\sqrt{2}(\sqrt{3}+3i)\sqrt{-\frac{6i}{x+1}+\sqrt{3}+3i}\sqrt{\frac{6i}{x+1}+\sqrt{3}-3i}}{\sqrt{3}+3i} F\left(i \sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)\middle| \frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{x+1}} + \frac{3\sqrt{2}(1-i\sqrt{3})\sqrt{-\frac{6i}{x+1}+\sqrt{3}+3i}\sqrt{\frac{6i}{x+1}+\sqrt{3}-3i}}{\sqrt{3}+3i} E\left(i \sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)\middle| \frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{x+1}} \right)}{12\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x^2*Sqrt[1 + x]*Sqrt[1 - x + x^2]),x]`

[Out] `-((Sqrt[1 + x]*Sqrt[1 - x + x^2])/x) + ((1 + x)^(3/2)*((12*Sqrt[(-1)/(3*I + Sqrt[3])]^(1 - x + x^2))/(1 + x)^2 + (3*Sqrt[2]^(1 - I)*Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3]])*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticE[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x] + (I*Sqrt[2]^(3*I + Sqrt[3])*Sqrt[(3*I + Sqrt[3] - (6*I)/(1 + x))/(3*I + Sqrt[3]])*Sqrt[(-3*I + Sqrt[3] + (6*I)/(1 + x))/(-3*I + Sqrt[3])]*EllipticF[I*ArcSinh[Sqrt[(-6*I)/(3*I + Sqrt[3])]/Sqrt[1 + x]], (3*I + Sqrt[3])/(3*I - Sqrt[3])])/Sqrt[1 + x]))/(12*Sqrt[(-1)/(3*I + Sqrt[3])]*Sqrt[1 - x + x^2])`

**Maple [A]** time = 0.025, size = 363, normalized size = 1.3

$$\frac{1}{2x(x^3+1)} \sqrt{1+x}\sqrt{x^2-x+1} \left( i \sqrt{-2\frac{1+x}{-3+i\sqrt{3}}} \sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}} \sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}} \operatorname{EllipticF}\left(\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}}, \sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(1+x)^(1/2)/(x^2-x+1)^(1/2),x)`

[Out]  $\frac{1}{2} (1+x)^{1/2} (x^2-x+1)^{1/2} (I^* (-2*(1+x)/(-3+I^*3^{1/2})))^{1/2} ((I^*3^{1/2}-2*x+1)/(I^*3^{1/2}+3))^{1/2} ((I^*3^{1/2}+2*x-1)/(-3+I^*3^{1/2}))^{1/2} \text{EllipticF}((-2*(1+x)/(-3+I^*3^{1/2}))^{1/2}, (-(-3+I^*3^{1/2})/(I^*3^{1/2}+3))^{1/2}) * 3^{1/2} * x + 3 * (-2*(1+x)/(-3+I^*3^{1/2}))^{1/2} ((I^*3^{1/2}-2*x+1)/(I^*3^{1/2}+3))^{1/2} ((I^*3^{1/2}+2*x-1)/(-3+I^*3^{1/2}))^{1/2} \text{EllipticF}((-2*(1+x)/(-3+I^*3^{1/2}))^{1/2}, (-(-3+I^*3^{1/2})/(I^*3^{1/2}+3))^{1/2}) * x - 6 * (-2*(1+x)/(-3+I^*3^{1/2}))^{1/2} ((I^*3^{1/2}-2*x+1)/(I^*3^{1/2}+3))^{1/2} ((I^*3^{1/2}+2*x-1)/(-3+I^*3^{1/2}))^{1/2} \text{EllipticE}((-2*(1+x)/(-3+I^*3^{1/2}))^{1/2}, (-(-3+I^*3^{1/2})/(I^*3^{1/2}+3))^{1/2}) * x - 2 * x^3 - 2) / x / (x^3 + 1)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}x^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2),x, algorithm="fricas")`

[Out] `integral(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2\sqrt{x+1}\sqrt{x^2-x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)`

[Out] `Integral(1/(x**2*sqrt(x + 1)*sqrt(x**2 - x + 1)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2-x+1}\sqrt{x+1}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^2), x)
```

$$3.509 \quad \int \frac{1}{x^3 \sqrt{1+x} \sqrt{1-x+x^2}} dx$$

**Optimal.** Leaf size=144

$$-\frac{\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle| -7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^2-x+1}} - \frac{x^3+1}{2x^2\sqrt{x+1}\sqrt{x^2-x+1}}$$

[Out]  $-(1+x^3)/((2*x^2*\text{Sqrt}[1+x]*\text{Sqrt}[1-x+x^2]) - (\text{Sqrt}[2+\text{Sqrt}[3]]*\text{Sqrt}[1+x]*\text{Sqrt}[(1-x+x^2)/(1+\text{Sqrt}[3]+x)^2]*\text{EllipticF}[\text{ArcSin}[(1-\text{Sqrt}[3]+x)/(1+\text{Sqrt}[3]+x)], -7-4*\text{Sqrt}[3]])/(2*3^{(1/4)}*\text{Sqrt}[(1+x)/(1+\text{Sqrt}[3]+x)^2]*\text{Sqrt}[1-x+x^2]))$

**Rubi [A]** time = 0.130906, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$

$$-\frac{\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle| -7-4\sqrt{3}\right)}{2\sqrt[4]{3}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^2-x+1}} - \frac{x^3+1}{2x^2\sqrt{x+1}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In] `Int[1/(x^3*Sqrt[1+x]*Sqrt[1-x+x^2]),x]`

[Out]  $-(1+x^3)/((2*x^2*\text{Sqrt}[1+x]*\text{Sqrt}[1-x+x^2]) - (\text{Sqrt}[2+\text{Sqrt}[3]]*\text{Sqrt}[1+x]*\text{Sqrt}[(1-x+x^2)/(1+\text{Sqrt}[3]+x)^2]*\text{EllipticF}[\text{ArcSin}[(1-\text{Sqrt}[3]+x)/(1+\text{Sqrt}[3]+x)], -7-4*\text{Sqrt}[3]])/(2*3^{(1/4)}*\text{Sqrt}[(1+x)/(1+\text{Sqrt}[3]+x)^2]*\text{Sqrt}[1-x+x^2]))$

**Rubi in Sympy [A]** time = 10.3742, size = 128, normalized size = 0.89

$$-\frac{3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}}\sqrt{\sqrt{3}+2}(x+1)^{\frac{3}{2}}\sqrt{x^2-x+1}F\left(\text{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle| -7-4\sqrt{3}\right)}{6\sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}}(x^3+1)} - \frac{\sqrt{x+1}\sqrt{x^2-x+1}}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x**3/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)`

[Out]  $-3^{(3/4)}*\text{sqrt}((x^2-x+1)/(x+1+\text{sqrt}(3)))^2)*\text{sqrt}(\text{sqrt}(3)+2)*(x+1)^{(3/2)}*\text{sqrt}(x^2-x+1)*\text{elliptic\_f}(\text{asin}((x-\text{sqrt}(3)+1)/(x+1+\text{sqrt}(3))), -7-4*\text{sqrt}(3))/(6*\text{sqrt}((x+1)/(x+1+\text{sqrt}(3)))^2)*(x^3+1)) - \text{sqrt}(x+1)*\text{sqrt}(x^2-x+1)/(2*x^2)$

**Mathematica [C]** time = 1.15098, size = 171, normalized size = 1.19

$$-\frac{6\sqrt{x+1}(x^2-x+1)}{x^2} - \frac{i(x+1)\sqrt{1+\frac{6i}{(\sqrt{3}-3i)(x+1)}}\sqrt{6-\frac{36i}{(\sqrt{3}+3i)(x+1)}}F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)\middle| \frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{-\frac{i}{\sqrt{3}+3i}}}$$

$$12\sqrt{x^2-x+1}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*Sqrt[1 + x]\*Sqrt[1 - x + x^2]),x]

[Out] ((-6\*Sqrt[1 + x]\*(1 - x + x^2))/x^2 - (I\*(1 + x)\*Sqrt[1 + (6\*I)/((-3\*I + Sqrt[3])\*(1 + x))]\*Sqrt[6 - (36\*I)/((3\*I + Sqrt[3])\*(1 + x))])\*EllipticF[I\*ArcSinh[Sqrt[(-6\*I)/(3\*I + Sqrt[3])]/Sqrt[1 + x]], (3\*I + Sqrt[3])/(3\*I - Sqrt[3])]/Sqrt[(-I)/(3\*I + Sqrt[3])])/(12\*Sqrt[1 - x + x^2])

**Maple [B]** time = 0.022, size = 259, normalized size = 1.8

$$\frac{1}{(4x^3 + 4)x^2} \sqrt{1+x} \sqrt{x^2 - x + 1} \left( i \sqrt{-2 \frac{1+x}{-3+i\sqrt{3}}} \sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}} \sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}} \text{EllipticF} \left( \sqrt{-2 \frac{1+x}{-3+i\sqrt{3}}}, \sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(1+x)^(1/2)/(x^2-x+1)^(1/2),x)

[Out] 1/4\*(1+x)^(1/2)\*(x^2-x+1)^(1/2)\*(I\*(-2\*(1+x)/(-3+I\*3^(1/2))))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2),(-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))\*3^(1/2)\*x^2-3\*(-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2),(-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))\*x^2-2\*x^3-2)/(x^3+1)/x^2

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2 - x + 1} \sqrt{x + 1} x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(x^2 - x + 1)\*sqrt(x + 1)\*x^3),x, algorithm="maxima")

[Out] integrate(1/(sqrt(x^2 - x + 1)\*sqrt(x + 1)\*x^3), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{\sqrt{x^2 - x + 1} \sqrt{x + 1} x^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(x^2 - x + 1)\*sqrt(x + 1)\*x^3),x, algorithm="fricas")

[Out] integral(1/(sqrt(x^2 - x + 1)\*sqrt(x + 1)\*x^3), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 \sqrt{x + 1} \sqrt{x^2 - x + 1}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(1+x)**(1/2)/(x**2-x+1)**(1/2),x)`

[Out] `Integral(1/(x**3*sqrt(x + 1)*sqrt(x**2 - x + 1)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x^2 - x + 1}\sqrt{x + 1}x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3),x, algorithm="giac")`

[Out] `integrate(1/(sqrt(x^2 - x + 1)*sqrt(x + 1)*x^3), x)`

$$3.510 \quad \int \frac{x^3}{(1+x)^{3/2}(1-x+x^2)^{3/2}} dx$$

**Optimal.** Leaf size=137

$$\frac{4\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle| -7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^2-x+1}} - \frac{2x}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

[Out]  $(-2*x)/(3*\text{Sqrt}[1+x]*\text{Sqrt}[1-x+x^2]) + (4*\text{Sqrt}[2+\text{Sqrt}[3]]*\text{Sqrt}[1+x]*\text{Sqrt}[(1-x+x^2)/(1+\text{Sqrt}[3]+x)^2]*\text{EllipticF}[\text{ArcSin}[(1-\text{Sqrt}[3]+x)/(1+\text{Sqrt}[3]+x)], -7-4*\text{Sqrt}[3]])/(3*3^{1/4}*\text{Sqrt}[(1+x)/(1+\text{Sqrt}[3]+x)^2]*\text{Sqrt}[1-x+x^2])$

**Rubi [A]** time = 0.129551, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$

$$\frac{4\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle| -7-4\sqrt{3}\right)}{3\sqrt[4]{3}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^2-x+1}} - \frac{2x}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^3/((1+x)^{(3/2)}*(1-x+x^2)^{(3/2)}), x]$

[Out]  $(-2*x)/(3*\text{Sqrt}[1+x]*\text{Sqrt}[1-x+x^2]) + (4*\text{Sqrt}[2+\text{Sqrt}[3]]*\text{Sqrt}[1+x]*\text{Sqrt}[(1-x+x^2)/(1+\text{Sqrt}[3]+x)^2]*\text{EllipticF}[\text{ArcSin}[(1-\text{Sqrt}[3]+x)/(1+\text{Sqrt}[3]+x)], -7-4*\text{Sqrt}[3]])/(3*3^{1/4}*\text{Sqrt}[(1+x)/(1+\text{Sqrt}[3]+x)^2]*\text{Sqrt}[1-x+x^2])$

**Rubi in Sympy [A]** time = 10.2793, size = 133, normalized size = 0.97

$$-\frac{2x\sqrt{x+1}\sqrt{x^2-x+1}}{3(x^3+1)} + \frac{4 \cdot 3^{\frac{3}{4}} \sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}} \sqrt{\sqrt{3}+2} (x+1)^{\frac{3}{2}} \sqrt{x^2-x+1} F\left(\text{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle| -7-4\sqrt{3}\right)}{9 \sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}} (x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x^{**3}/(1+x)^{(3/2)}/(x^{**2}-x+1)^{(3/2)}, x)$

[Out]  $-2*x*\text{sqrt}(x+1)*\text{sqrt}(x^2-x+1)/(3*(x^3+1)) + 4*3^{3/4}* \text{sqrt}((x^2-x+1)/(x+1+\text{sqrt}(3)))^{**2}*\text{sqrt}(\text{sqrt}(3)+2)*(x+1)^{(3/2)}*\text{sqrt}(x^2-x+1)*\text{elliptic\_f}(\text{asin}((x-\text{sqrt}(3)+1)/(x+1+\text{sqrt}(3))), -7-4*\text{sqrt}(3))/(9*\text{sqrt}((x+1)/(x+1+\text{sqrt}(3)))^{**2}*(x^3+1))$

**Mathematica [C]** time = 0.974688, size = 161, normalized size = 1.18

$$-\frac{6x}{\sqrt{x+1}} + \frac{2i(x+1)\sqrt{1+\frac{6i}{(\sqrt{3}-3i)(x+1)}}\sqrt{6-\frac{36i}{(\sqrt{3}+3i)(x+1)}}F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)\middle| \frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{9\sqrt{x^2-x+1}\sqrt{-\frac{i}{\sqrt{3}+3i}}}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((1 + x)^(3/2)\*(1 - x + x^2)^(3/2)),x]

[Out]  $\frac{(-6x)/\sqrt{1+x} + ((2I)^*(1+x)*\sqrt{1+(6I)/((-3I+\sqrt{3})^*(1+x))})*\sqrt{6-(36I)/((3I+\sqrt{3})^*(1+x))}*\text{EllipticF}[I*\text{ArcSinh}[\sqrt{(-6I)/(3I+\sqrt{3})}]/\sqrt{1+x}], (3I+\sqrt{3})/(3I-\sqrt{3})]/\sqrt{(-I)/(3I+\sqrt{3})})/(9*\sqrt{1-x+x^2})$

**Maple [B]** time = 0.075, size = 245, normalized size = 1.8

$$-\frac{2}{3x^3+3}\sqrt{1+x}\sqrt{x^2-x+1}\left(i\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\text{EllipticF}\left(\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x)

[Out]  $-2/3*(1+x)^{(1/2)}*(x^2-x+1)^{(1/2)}*(I*(-2*(1+x)/(-3+I*3^{(1/2)})))^{(1/2)}*((I*3^{(1/2)}-2*x+1)/(I*3^{(1/2)}+3))^{(1/2)}*((I*3^{(1/2)}+2*x-1)/(-3+I*3^{(1/2)}))^{(1/2)}*\text{EllipticF}((-2*(1+x)/(-3+I*3^{(1/2)}))^{(1/2)},(-(-3+I*3^{(1/2)})/(I*3^{(1/2)}+3))^{(1/2)})*3^{(1/2)}-3*(-2*(1+x)/(-3+I*3^{(1/2)}))^{(1/2)}*((I*3^{(1/2)}-2*x+1)/(I*3^{(1/2)}+3))^{(1/2)}*((I*3^{(1/2)}+2*x-1)/(-3+I*3^{(1/2)}))^{(1/2)}*\text{EllipticF}((-2*(1+x)/(-3+I*3^{(1/2)}))^{(1/2)},(-(-3+I*3^{(1/2)})/(I*3^{(1/2)}+3))^{(1/2)})+x)/(x^3+1)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(x^2-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/((x^2 - x + 1)^(3/2)\*(x + 1)^(3/2)),x, algorithm="maxima")

[Out] integrate(x^3/((x^2 - x + 1)^(3/2)\*(x + 1)^(3/2)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3}{(x^3+1)\sqrt{x^2-x+1}\sqrt{x+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/((x^2 - x + 1)^(3/2)\*(x + 1)^(3/2)),x, algorithm="fricas")

[Out] integral(x^3/((x^3 + 1)\*sqrt(x^2 - x + 1)\*sqrt(x + 1)), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(x+1)^{\frac{3}{2}}(x^2-x+1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)`

[Out] `Integral(x**3/((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(x^2 - x + 1)^{\frac{3}{2}}(x + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^3/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)),x, algorithm="giac")`

[Out] `integrate(x^3/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)`

$$3.511 \quad \int \frac{x^2}{(1+x)^{3/2}(1-x+x^2)^{3/2}} dx$$

**Optimal.** Leaf size=23

$$-\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

[Out] -2/(3\*Sqrt[1 + x]\*Sqrt[1 - x + x^2])

**Rubi [A]** time = 0.0185888, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$-\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In] Int[x^2/((1 + x)^(3/2)\*(1 - x + x^2)^(3/2)), x]

[Out] -2/(3\*Sqrt[1 + x]\*Sqrt[1 - x + x^2])

**Rubi in Sympy [A]** time = 7.29109, size = 26, normalized size = 1.13

$$-\frac{2\sqrt{x+1}\sqrt{x^2-x+1}}{3(x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*2/(1+x)\*\*(3/2)/(x\*\*2-x+1)\*\*(3/2), x)

[Out] -2\*sqrt(x + 1)\*sqrt(x\*\*2 - x + 1)/(3\*(x\*\*3 + 1))

**Mathematica [A]** time = 0.0344814, size = 23, normalized size = 1.

$$-\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In] Integrate[x^2/((1 + x)^(3/2)\*(1 - x + x^2)^(3/2)), x]

[Out] -2/(3\*Sqrt[1 + x]\*Sqrt[1 - x + x^2])

**Maple [A]** time = 0.006, size = 18, normalized size = 0.8

$$-\frac{2}{3} \frac{1}{\sqrt{1+x}} \frac{1}{\sqrt{x^2-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2), x)

[Out]  $-2/3/(1+x)^{1/2}/(x^2-x+1)^{1/2}$

**Maxima [A]** time = 0.772464, size = 23, normalized size = 1.

$$-\frac{2}{3\sqrt{x^2-x+1}\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/((x^2 - x + 1)^(3/2) * (x + 1)^(3/2)),x, algorithm="maxima")`

[Out]  $-2/3/(\text{sqrt}(x^2 - x + 1) * \text{sqrt}(x + 1))$

**Fricas [A]** time = 0.280318, size = 23, normalized size = 1.

$$-\frac{2}{3\sqrt{x^2-x+1}\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/((x^2 - x + 1)^(3/2) * (x + 1)^(3/2)),x, algorithm="fricas")`

[Out]  $-2/3/(\text{sqrt}(x^2 - x + 1) * \text{sqrt}(x + 1))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(x+1)^{\frac{3}{2}}(x^2-x+1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)`

[Out] `Integral(x**2/((x + 1)**(3/2) * (x**2 - x + 1)**(3/2)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(x^2-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/((x^2 - x + 1)^(3/2) * (x + 1)^(3/2)),x, algorithm="giac")`

[Out] `integrate(x^2/((x^2 - x + 1)^(3/2) * (x + 1)^(3/2)), x)`

$$3.512 \quad \int \frac{x}{(1+x)^{3/2}(1-x+x^2)^{3/2}} dx$$

**Optimal.** Leaf size=282

$$\frac{2x^2}{3\sqrt{x+1}\sqrt{x^2-x+1}} - \frac{2\sqrt{2}\sqrt{x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{3\sqrt[3]{3} \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} \sqrt{x^2-x+1}}$$

$$+ \frac{\sqrt{2-\sqrt{3}}\sqrt{x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{3^{3/4} \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} \sqrt{x^2-x+1}} - \frac{2(x^3+1)}{3\sqrt{x+1}(x+\sqrt{3}+1)\sqrt{x^2-x+1}}$$

[Out] (2\*x^2)/(3\*Sqrt[1+x]\*Sqrt[1-x+x^2]) - (2\*(1+x^3))/(3\*Sqrt[1+x]\*(1+Sqrt[3]+x)\*Sqrt[1-x+x^2]) + (Sqrt[2-Sqrt[3]]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticE[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(3^(3/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2]) - (2\*Sqrt[2]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(3\*3^(1/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2])

**Rubi [A]** time = 0.194509, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$\frac{2x^2}{3\sqrt{x+1}\sqrt{x^2-x+1}} - \frac{2\sqrt{2}\sqrt{x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{3\sqrt[3]{3} \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} \sqrt{x^2-x+1}}$$

$$+ \frac{\sqrt{2-\sqrt{3}}\sqrt{x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{3^{3/4} \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} \sqrt{x^2-x+1}} - \frac{2(x^3+1)}{3\sqrt{x+1}(x+\sqrt{3}+1)\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In] Int[x/((1+x)^(3/2)\*(1-x+x^2)^(3/2)),x]

[Out] (2\*x^2)/(3\*Sqrt[1+x]\*Sqrt[1-x+x^2]) - (2\*(1+x^3))/(3\*Sqrt[1+x]\*(1+Sqrt[3]+x)\*Sqrt[1-x+x^2]) + (Sqrt[2-Sqrt[3]]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticE[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(3^(3/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2]) - (2\*Sqrt[2]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(3\*3^(1/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2])

**Rubi in Sympy [A]** time = 17.2559, size = 265, normalized size = 0.94

$$\frac{2x^2\sqrt{x+1}\sqrt{x^2-x+1}}{3(x^3+1)} - \frac{2\sqrt{x+1}\sqrt{x^2-x+1}}{3(x+1+\sqrt{3})} + \frac{\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}}\sqrt{-\sqrt{3}+2}(x+1)^{\frac{3}{2}}\sqrt{x^2-x+1}E\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle| -7-4\sqrt{3}\right)}{3\sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}}(x^3+1)} - \frac{2\sqrt{2}\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}}(x+1)^{\frac{3}{2}}\sqrt{x^2-x+1}F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle| -7-4\sqrt{3}\right)}{9\sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}}(x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rub_i_integrate(x/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)`

[Out]  $2x^2\sqrt{x+1}\sqrt{x^2-x+1}/(3(x^3+1)) - 2\sqrt{x+1}\sqrt{x^2-x+1}/(3(x+1+\sqrt{3})) + 3^{1/4}\sqrt{(x^2-x+1)/(x+1+\sqrt{3})^2}\sqrt{-\sqrt{3}+2}(x+1)^{3/2}\sqrt{x^2-x+1}\operatorname{elliptic}_e(\operatorname{asin}((x-\sqrt{3}+1)/(x+1+\sqrt{3})), -7-4\sqrt{3})/(3\sqrt{(x+1)/(x+1+\sqrt{3})^2})(x^3+1) - 2\sqrt{2}\cdot 3^{3/4}\sqrt{(x^2-x+1)/(x+1+\sqrt{3})^2}(x+1)^{3/2}\sqrt{x^2-x+1}\operatorname{elliptic}_f(\operatorname{asin}((x-\sqrt{3}+1)/(x+1+\sqrt{3})), -7-4\sqrt{3})/(9\sqrt{(x+1)/(x+1+\sqrt{3})^2})(x^3+1)$

**Mathematica [C]** time = 1.45613, size = 402, normalized size = 1.43

$$\frac{2x^2}{3\sqrt{x+1}\sqrt{x^2-x+1}} + \frac{(x+1)^{3/2}\left(\frac{12\sqrt{-\frac{i}{\sqrt{3}+3i}}(x^2-x+1)}{(x+1)^2} + \frac{i\sqrt{2}(\sqrt{3}+3i)\sqrt{\frac{-6i}{x+1}+\sqrt{3}+3i}\sqrt{\frac{6i}{x+1}+\sqrt{3}-3i}}{\sqrt{3}+3i}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{-6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)\middle| \frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{x+1}} + \frac{3\sqrt{2}(1-i\sqrt{3})\sqrt{\frac{-6i}{x+1}+\sqrt{3}+3i}\sqrt{\frac{6i}{x+1}+\sqrt{3}-3i}}{\sqrt{3}+3i}E\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{-6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)\middle| \frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{x+1}}\right)}{18\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In] `Integrate[x/((1+x)^(3/2)*(1-x+x^2)^(3/2)),x]`

[Out]  $(2x^2)/(3\sqrt{1+x}\sqrt{1-x+x^2}) - ((1+x)^{3/2}((12\sqrt{-i/(3+3i\sqrt{3})}}(1-x+x^2))/(1+x)^2 + (3\sqrt{2}(1-i\sqrt{3})\sqrt{-6i/(x+1)+\sqrt{3}+3i}\sqrt{6i/(x+1)+\sqrt{3}-3i}/(3+3i)E(i\sinh^{-1}(\sqrt{-6i/(3i+\sqrt{3})}/\sqrt{x+1})| (3i+\sqrt{3})/(3i-\sqrt{3})))/\sqrt{x+1} + (12\sqrt{-i/(3+3i\sqrt{3})}}(1-x+x^2))/(1+x)^2 + (3\sqrt{2}(1-i\sqrt{3})\sqrt{-6i/(x+1)+\sqrt{3}+3i}\sqrt{6i/(x+1)+\sqrt{3}-3i}/(3+3i)E(i\sinh^{-1}(\sqrt{-6i/(3i+\sqrt{3})}/\sqrt{x+1})| (3i+\sqrt{3})/(3i-\sqrt{3})))/\sqrt{x+1}))/18\sqrt{-i/(3+3i\sqrt{3})}\sqrt{x^2-x+1}$



**Maple [A]** time = 0.058, size = 356, normalized size = 1.3

$$-\frac{1}{3x^3+3}\sqrt{1+x}\sqrt{x^2-x+1}\left(i\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\text{EllipticF}\left(\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}},\sqrt{\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x)

[Out] 
$$-1/3*(1+x)^{(1/2)}*(x^2-x+1)^{(1/2)}*(I*(-2*(1+x)/(-3+I*3^{(1/2)})))^{(1/2)}*((I*3^{(1/2)}-2*x+1)/(I*3^{(1/2)}+3))^{(1/2)}*((I*3^{(1/2)}+2*x-1)/(-3+I*3^{(1/2)}))^{(1/2)}*\text{EllipticF}((-2*(1+x)/(-3+I*3^{(1/2)}))^{(1/2)},(-(-3+I*3^{(1/2)})/(I*3^{(1/2)}+3))^{(1/2)})*3^{(1/2)}+3*(-2*(1+x)/(-3+I*3^{(1/2)}))^{(1/2)}*((I*3^{(1/2)}-2*x+1)/(I*3^{(1/2)}+3))^{(1/2)}*((I*3^{(1/2)}+2*x-1)/(-3+I*3^{(1/2)}))^{(1/2)}*\text{EllipticF}((-2*(1+x)/(-3+I*3^{(1/2)}))^{(1/2)},(-(-3+I*3^{(1/2)})/(I*3^{(1/2)}+3))^{(1/2)})-6*(-2*(1+x)/(-3+I*3^{(1/2)}))^{(1/2)}*((I*3^{(1/2)}-2*x+1)/(I*3^{(1/2)}+3))^{(1/2)}*((I*3^{(1/2)}+2*x-1)/(-3+I*3^{(1/2)}))^{(1/2)}*\text{EllipticE}((-2*(1+x)/(-3+I*3^{(1/2)}))^{(1/2)},(-(-3+I*3^{(1/2)})/(I*3^{(1/2)}+3))^{(1/2)})-2*x^2/(x^3+1)$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^2-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((x^2 - x + 1)^(3/2)\*(x + 1)^(3/2)),x, algorithm="maxima")

[Out] integrate(x/((x^2 - x + 1)^(3/2)\*(x + 1)^(3/2)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x}{(x^3+1)\sqrt{x^2-x+1}\sqrt{x+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((x^2 - x + 1)^(3/2)\*(x + 1)^(3/2)),x, algorithm="fricas")

[Out] integral(x/((x^3 + 1)\*sqrt(x^2 - x + 1)\*sqrt(x + 1)), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x+1)^{\frac{3}{2}}(x^2-x+1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(1+x)\*\*(3/2)/(x\*\*2-x+1)\*\*(3/2),x)

[Out] Integral(x/((x + 1)\*\*(3/2)\*(x\*\*2 - x + 1)\*\*(3/2)), x)

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^2 - x + 1)^{\frac{3}{2}}(x + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)),x, algorithm="giac")`

[Out] `integrate(x/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)`

$$3.513 \quad \int \frac{1}{(1+x)^{3/2}(1-x+x^2)^{3/2}} dx$$

**Optimal.** Leaf size=137

$$\frac{2x}{3\sqrt{x+1}\sqrt{x^2-x+1}} + \frac{2\sqrt{2+\sqrt{3}}\sqrt{x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \middle| -7-4\sqrt{3}\right)}{3\sqrt[3]{3} \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} \sqrt{x^2-x+1}}$$

[Out] (2\*x)/(3\*Sqrt[1+x]\*Sqrt[1-x+x^2]) + (2\*Sqrt[2+Sqrt[3]]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(3^3^(1/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2])

**Rubi [A]** time = 0.0833021, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{2x}{3\sqrt{x+1}\sqrt{x^2-x+1}} + \frac{2\sqrt{2+\sqrt{3}}\sqrt{x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \middle| -7-4\sqrt{3}\right)}{3\sqrt[3]{3} \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} \sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In] Int[1/((1+x)^(3/2)\*(1-x+x^2)^(3/2)),x]

[Out] (2\*x)/(3\*Sqrt[1+x]\*Sqrt[1-x+x^2]) + (2\*Sqrt[2+Sqrt[3]]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(3^3^(1/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2])

**Rubi in Sympy [A]** time = 7.20472, size = 133, normalized size = 0.97

$$\frac{2x\sqrt{x+1}\sqrt{x^2-x+1}}{3(x^3+1)} + \frac{2 \cdot 3^{\frac{3}{4}} \sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}} \sqrt{\sqrt{3}+2} (x+1)^{\frac{3}{2}} \sqrt{x^2-x+1} F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right) \middle| -7-4\sqrt{3}\right)}{9 \sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}} (x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(1+x)\*\*(3/2)/(x\*\*2-x+1)\*\*(3/2),x)

[Out] 2\*x\*sqrt(x+1)\*sqrt(x\*\*2-x+1)/(3\*(x\*\*3+1)) + 2\*3\*\*(3/4)\*sqrt((x\*\*2-x+1)/(x+1+sqrt(3)))\*\*2\*sqrt(sqrt(3)+2)\*(x+1)\*\*(3/2)\*sqrt(x\*\*2-x+1)\*elliptic\_f(asin((x-sqrt(3)+1)/(x+1+sqrt(3))), -7-4\*sqrt(3))/(9\*sqrt((x+1)/(x+1+sqrt(3)))\*\*2\*(x\*\*3+1))

**Mathematica [C]** time = 0.627814, size = 216, normalized size = 1.58

$$\sqrt{(x+1)^2-3(x+1)+3} \left( \frac{2(x+1)^{3/2}}{9((x+1)^2-3(x+1)+3)} - \frac{2}{9\sqrt{x+1}} \right) + \frac{i\sqrt{\frac{2}{3}}(x+1) \sqrt{1-\frac{6}{(3-i\sqrt{3})(x+1)}} \sqrt{1-\frac{6}{(3+i\sqrt{3})(x+1)}} F\left(i \sinh^{-1}\left(\frac{\sqrt{\frac{6}{3-i\sqrt{3}}}}{\sqrt{x+1}}\right) \middle| \frac{3-i\sqrt{3}}{3+i\sqrt{3}}\right)}{3\sqrt{-\frac{1}{3-i\sqrt{3}}}\sqrt{(x+1)^2-3(x+1)+3}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 + x)^(3/2) \* (1 - x + x^2)^(3/2)), x]

[Out] Sqrt[3 - 3\*(1 + x) + (1 + x)^2]\*(-2/(9\*Sqrt[1 + x]) + (2\*(1 + x)^(3/2))/(9\*(3 - 3\*(1 + x) + (1 + x)^2))) + ((I/3)\*Sqrt[2/3]\*(1 + x)\*Sqrt[1 - 6/((3 - I\*Sqrt[3])\*(1 + x))]\*Sqrt[1 - 6/((3 + I\*Sqrt[3])\*(1 + x))])\*EllipticF[I\*ArcSinh[Sqrt[-6/(3 - I\*Sqrt[3])]/Sqrt[1 + x]], (3 - I\*Sqrt[3])/(3 + I\*Sqrt[3])]/(Sqrt[-(3 - I\*Sqrt[3])^(1 - 1)]\*Sqrt[3 - 3\*(1 + x) + (1 + x)^2])

**Maple [B]** time = 0.042, size = 247, normalized size = 1.8

$$-\frac{1}{3x^3+3}\sqrt{1+x}\sqrt{x^2-x+1}\left(i\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\text{EllipticF}\left(\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+x)^(3/2)/(x^2-x+1)^(3/2), x)

[Out] -1/3\*(1+x)^(1/2)\*(x^2-x+1)^(1/2)\*(I\*(-2\*(1+x)/(-3+I\*3^(1/2))))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2), (-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))\*3^(1/2)-3\*(-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2), (-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))-2\*x/(x^3+1)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2 - x + 1)^{\frac{3}{2}}(x + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 - x + 1)^(3/2) \* (x + 1)^(3/2)), x, algorithm="maxima")

[Out] integrate(1/((x^2 - x + 1)^(3/2) \* (x + 1)^(3/2)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(x^3 + 1)\sqrt{x^2 - x + 1}\sqrt{x + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 - x + 1)^(3/2) \* (x + 1)^(3/2)), x, algorithm="fricas")

[Out] integral(1/((x^3 + 1)\*sqrt(x^2 - x + 1)\*sqrt(x + 1)), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x + 1)^{\frac{3}{2}}(x^2 - x + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)`

[Out] `Integral(1/((x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2 - x + 1)^{\frac{3}{2}}(x + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)),x, algorithm="giac")`

[Out] `integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)), x)`

$$3.514 \quad \int \frac{1}{x(1+x)^{3/2}(1-x+x^2)^{3/2}} dx$$

**Optimal.** Leaf size=66

$$\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}} - \frac{2\sqrt{x^3+1} \tanh^{-1}(\sqrt{x^3+1})}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

[Out] 2/(3\*Sqrt[1+x]\*Sqrt[1-x+x^2]) - (2\*Sqrt[1+x^3]\*ArcTanh[Sqrt[1+x^3]])/(3\*Sqrt[1+x]\*Sqrt[1-x+x^2])

**Rubi [A]** time = 0.0958263, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$

$$\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}} - \frac{2\sqrt{x^3+1} \tanh^{-1}(\sqrt{x^3+1})}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In] Int[1/(x\*(1+x)^(3/2)\*(1-x+x^2)^(3/2)),x]

[Out] 2/(3\*Sqrt[1+x]\*Sqrt[1-x+x^2]) - (2\*Sqrt[1+x^3]\*ArcTanh[Sqrt[1+x^3]])/(3\*Sqrt[1+x]\*Sqrt[1-x+x^2])

**Rubi in Sympy [A]** time = 9.79367, size = 63, normalized size = 0.95

$$\frac{2\sqrt{x+1}\sqrt{x^2-x+1}}{3(x^3+1)} - \frac{2\sqrt{x+1}\sqrt{x^2-x+1} \operatorname{atanh}(\sqrt{x^3+1})}{3\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x/(1+x)\*\*(3/2)/(x\*\*2-x+1)\*\*(3/2),x)

[Out] 2\*sqrt(x+1)\*sqrt(x\*\*2-x+1)/(3\*(x\*\*3+1)) - 2\*sqrt(x+1)\*sqrt(x\*\*2-x+1)\*atanh(sqrt(x\*\*3+1))/(3\*sqrt(x\*\*3+1))

**Mathematica [C]** time = 0.343846, size = 88, normalized size = 1.33

$$2 \left( \frac{1}{\sqrt{x^2-x+1}} - \frac{\sqrt{3}(x+1) \left( 1 + \sqrt[3]{-1}; \sin^{-1} \left( \sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}} \right) \sqrt[3]{-1} \right)}{\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}} \right) \frac{1}{3\sqrt{x+1}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x\*(1+x)^(3/2)\*(1-x+x^2)^(3/2)),x]

[Out] (2\*(1/Sqrt[1-x+x^2]) - (Sqrt[3]\*(1+x)\*EllipticPi[1+(-1)^(1/3)], ArcSin[Sqrt[(1+(-1)^(2/3)\*x)/(1+(-1)^(1/3)]]], (-1)^(1/3)))/Sqrt[(1+x)/(1+(-1)^(1/3)))]/(3\*Sqrt[1+x])

---

**Maple [A]** time = 0.05, size = 43, normalized size = 0.7

$$-\frac{2}{3x^3+3}\sqrt{1+x}\sqrt{x^2-x+1}\left(\operatorname{Artanh}\left(\sqrt{x^3+1}\right)\sqrt{x^3+1}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x/(1+x)^(3/2)/(x^2-x+1)^(3/2),x)`

[Out] `-2/3*(1+x)^(1/2)*(x^2-x+1)^(1/2)*(arctanh((x^3+1)^(1/2))*(x^3+1)^(1/2)-1)/(x^3+1)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((x^2-x+1)^(3/2)*(x+1)^(3/2)*x),x,algorithm="maxima")`

[Out] `integrate(1/((x^2-x+1)^(3/2)*(x+1)^(3/2)*x),x)`

---

**Fricas [A]** time = 0.287374, size = 122, normalized size = 1.85

$$\frac{\sqrt{x^2-x+1}\sqrt{x+1}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}+1\right)-\sqrt{x^2-x+1}\sqrt{x+1}\log\left(\sqrt{x^2-x+1}\sqrt{x+1}-1\right)-2}{3\sqrt{x^2-x+1}\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((x^2-x+1)^(3/2)*(x+1)^(3/2)*x),x,algorithm="fricas")`

[Out] `-1/3*(sqrt(x^2-x+1)*sqrt(x+1)*log(sqrt(x^2-x+1)*sqrt(x+1)+1)-sqrt(x^2-x+1)*sqrt(x+1)*log(sqrt(x^2-x+1)*sqrt(x+1)-1)-2)/(sqrt(x^2-x+1)*sqrt(x+1))`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x(x+1)^{\frac{3}{2}}(x^2-x+1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)`

[Out] `Integral(1/(x*(x+1)**(3/2)*(x**2-x+1)**(3/2)),x)`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x), x, algorithm="giac")
```

```
[Out] integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x), x)
```



$$3.515 \quad \int \frac{1}{x^2(1+x)^{3/2}(1-x+x^2)^{3/2}} dx$$

**Optimal.** Leaf size=316

$$\begin{aligned} & \frac{2}{3x\sqrt{x+1}\sqrt{x^2-x+1}} + \frac{5\sqrt{2}\sqrt{x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{3\sqrt[4]{3} \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} \sqrt{x^2-x+1}} \\ & - \frac{5\sqrt{2-\sqrt{3}}\sqrt{x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{2 \cdot 3^{3/4} \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} \sqrt{x^2-x+1}} \\ & - \frac{5(x^3+1)}{3x\sqrt{x+1}\sqrt{x^2-x+1}} + \frac{5(x^3+1)}{3\sqrt{x+1}(x+\sqrt{3}+1)\sqrt{x^2-x+1}} \end{aligned}$$

[Out] 2/(3\*x\*Sqrt[1+x]\*Sqrt[1-x+x^2]) - (5\*(1+x^3))/(3\*x\*Sqrt[1+x]\*Sqrt[1-x+x^2]) + (5\*(1+x^3))/(3\*Sqrt[1+x]\*(1+Sqrt[3]+x)\*Sqrt[1-x+x^2]) - (5\*Sqrt[2-Sqrt[3]]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticE[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(2\*3^(3/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2]) + (5\*Sqrt[2]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(3\*3^(1/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2])

**Rubi [A]** time = 0.252133, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$

$$\begin{aligned} & \frac{2}{3x\sqrt{x+1}\sqrt{x^2-x+1}} + \frac{5\sqrt{2}\sqrt{x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{3\sqrt[4]{3} \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} \sqrt{x^2-x+1}} \\ & - \frac{5\sqrt{2-\sqrt{3}}\sqrt{x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{2 \cdot 3^{3/4} \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} \sqrt{x^2-x+1}} \\ & - \frac{5(x^3+1)}{3x\sqrt{x+1}\sqrt{x^2-x+1}} + \frac{5(x^3+1)}{3\sqrt{x+1}(x+\sqrt{3}+1)\sqrt{x^2-x+1}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(1+x)^(3/2)\*(1-x+x^2)^(3/2)),x]

[Out] 2/(3\*x\*Sqrt[1+x]\*Sqrt[1-x+x^2]) - (5\*(1+x^3))/(3\*x\*Sqrt[1+x]\*Sqrt[1-x+x^2]) + (5\*(1+x^3))/(3\*Sqrt[1+x]\*(1+Sqrt[3]+x)\*Sqrt[1-x+x^2]) - (5\*Sqrt[2-Sqrt[3]]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticE[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(2\*3^(3/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2]) + (5\*Sqrt[2]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(3\*3^(1/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2])

**Rubi in Sympy [A]** time = 20.8172, size = 287, normalized size = 0.91

$$\frac{5\sqrt{x+1}\sqrt{x^2-x+1}}{3(x+1+\sqrt{3})} - \frac{5\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}}\sqrt{-\sqrt{3}+2}(x+1)^{\frac{3}{2}}\sqrt{x^2-x+1}E\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle| -7-4\sqrt{3}\right)}{6\sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}}(x^3+1)} + \frac{5\sqrt{2}\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}}(x+1)^{\frac{3}{2}}\sqrt{x^2-x+1}F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle| -7-4\sqrt{3}\right)}{9\sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}}(x^3+1)} - \frac{5\sqrt{x+1}\sqrt{x^2-x+1}}{3x} + \frac{2\sqrt{x+1}\sqrt{x^2-x+1}}{3x(x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x**2/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)`

[Out]  $5\sqrt{x+1}\sqrt{x^2-x+1}/(3(x+1+\sqrt{3})) - 5\cdot 3^{3/4}/4\sqrt{(x^2-x+1)/(x+1+\sqrt{3})^2}\sqrt{-\sqrt{3}+2}\sqrt{(x+1)^{3/2}\sqrt{x^2-x+1}\operatorname{elliptic}_e(\operatorname{asin}(x-\sqrt{3}+1)/(x+1+\sqrt{3})), -7-4\sqrt{3})}/(6\sqrt{(x+1)/(x+1+\sqrt{3})^2}(x^3+1)) + 5\sqrt{2}\cdot 3^{3/4}\sqrt{(x^2-x+1)/(x+1+\sqrt{3})^2}(x+1)^{3/2}\sqrt{x^2-x+1}\operatorname{elliptic}_f(\operatorname{asin}(x-\sqrt{3}+1)/(x+1+\sqrt{3})), -7-4\sqrt{3})}/(9\sqrt{(x+1)/(x+1+\sqrt{3})^2}(x^3+1)) - 5\sqrt{x+1}\sqrt{x^2-x+1}/(3x) + 2\sqrt{x+1}\sqrt{x^2-x+1}/(3x(x^3+1))$

**Mathematica [C]** time = 1.44661, size = 409, normalized size = 1.29

$$\frac{5x^3+3}{3x\sqrt{x+1}\sqrt{x^2-x+1}} + \frac{5(x+1)^{3/2}\left(\frac{12\sqrt{-\frac{i}{\sqrt{3}+3i}}(x^2-x+1)}{(x+1)^2} + \frac{i\sqrt{2}(\sqrt{3}+3i)\sqrt{-\frac{6i}{x+1}+\sqrt{3}+3i}}{\sqrt{3}+3i}\sqrt{\frac{6i}{x+1}+\sqrt{3}-3i}}{\sqrt{3}-3i}F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)\middle| \frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{3\sqrt{2}(1-i\sqrt{3})\sqrt{-\frac{6i}{x+1}+\sqrt{3}+3i}}\sqrt{\frac{6i}{x+1}+\sqrt{3}-3i}}{\sqrt{3}-3i}E\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)\middle| \frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{3\sqrt{2}(1-i\sqrt{3})}\right)}{36\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x^2*(1+x)^(3/2)*(1-x+x^2)^(3/2)),x]`

[Out]  $-(3+5x^3)/(3x\sqrt{1+x}\sqrt{1-x+x^2}) + (5(1+x)^{3/2}((12\sqrt{(-1)/(3I+\sqrt{3})})\sqrt{(1-x+x^2)}/(1+x)^2 + (3\sqrt{2}\sqrt{(1-I\sqrt{3})}\sqrt{(3I+\sqrt{3}-(6I)/(1+x))}/(3I+\sqrt{3}))\sqrt{((-3I+\sqrt{3}+(6I)/(1+x)))/(-3I+\sqrt{3})})\operatorname{EllipticE}[I\operatorname{ArcSinh}[\sqrt{(-6I)/(3I+\sqrt{3})}]/\sqrt{1+x}], (3I+\sqrt{3})/(3I-\sqrt{3}))/\sqrt{1+x} + (I\sqrt{2}\sqrt{(3I+\sqrt{3})}\sqrt{(3I+\sqrt{3}-(6I)/(1+x))}/(3I+\sqrt{3}))\sqrt{((-3I+\sqrt{3}+(6I)/(1+x)))/(-3I+\sqrt{3})})\operatorname{EllipticF}[I\operatorname{ArcSinh}[\sqrt{(-6I)/(3I+\sqrt{3})}]/\sqrt{1+x}], (3I+\sqrt{3})/(3I-\sqrt{3}))/\sqrt{1+x}))/36\sqrt{(-1)/(3I+\sqrt{3})}\sqrt{1-x+x^2})$

**Maple [A]** time = 0.046, size = 363, normalized size = 1.2

$$\frac{1}{(6x^3+6)x} \sqrt{1+x} \sqrt{x^2-x+1} \left( 5 \operatorname{EllipticF} \left( \sqrt{-2 \frac{1+x}{-3+i\sqrt{3}}}, \sqrt{\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}} \right) \sqrt{3x} \sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}} \sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}} \sqrt{-2 \frac{1+x}{-3+i\sqrt{3}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(1+x)^(3/2)/(x^2-x+1)^(3/2), x)

[Out]  $\frac{1}{6} (1+x)^{1/2} (x^2-x+1)^{1/2} (5 \operatorname{EllipticF}((-2(1+x)/(-3+I^3)^{1/2}))^{1/2}, (-(-3+I^3)^{1/2})/(I^3)^{1/2}+3)^{1/2})^{3/2} x ((I^3)^{1/2}-2x+1)/(I^3)^{1/2}+3)^{1/2} ((I^3)^{1/2}+2x-1)/(-3+I^3)^{1/2})^{1/2} (-2(1+x)/(-3+I^3)^{1/2})^{1/2}+15(-2(1+x)/(-3+I^3)^{1/2})^{1/2} ((I^3)^{1/2}-2x+1)/(I^3)^{1/2}+3)^{1/2} ((I^3)^{1/2}+2x-1)/(-3+I^3)^{1/2})^{1/2} \operatorname{EllipticF}((-2(1+x)/(-3+I^3)^{1/2}))^{1/2}, (-(-3+I^3)^{1/2})/(I^3)^{1/2}+3)^{1/2} x-30(-2(1+x)/(-3+I^3)^{1/2})^{1/2} ((I^3)^{1/2}-2x+1)/(I^3)^{1/2}+3)^{1/2} ((I^3)^{1/2}+2x-1)/(-3+I^3)^{1/2})^{1/2} \operatorname{EllipticE}((-2(1+x)/(-3+I^3)^{1/2}))^{1/2}, (-(-3+I^3)^{1/2})/(I^3)^{1/2}+3)^{1/2} x-10x^3-6)/(x^3+1)/x$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 - x + 1)^(3/2)\*(x + 1)^(3/2)\*x^2), x, algorithm="maxima")

[Out] integrate(1/((x^2 - x + 1)^(3/2)\*(x + 1)^(3/2)\*x^2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{1}{(x^5+x^2)\sqrt{x^2-x+1}\sqrt{x+1}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 - x + 1)^(3/2)\*(x + 1)^(3/2)\*x^2), x, algorithm="fricas")

[Out] integral(1/((x^5 + x^2)\*sqrt(x^2 - x + 1)\*sqrt(x + 1)), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2(x+1)^{\frac{3}{2}}(x^2-x+1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x\*\*2/(1+x)\*\*(3/2)/(x\*\*2-x+1)\*\*(3/2), x)

[Out] Integral(1/(x\*\*2\*(x + 1)\*\*(3/2)\*(x\*\*2 - x + 1)\*\*(3/2)), x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2 - x + 1)^{\frac{3}{2}}(x + 1)^{\frac{3}{2}}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^2),x, algorithm="giac")`

[Out] `integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^2), x)`

$$3.516 \quad \int \frac{1}{x^3(1+x)^{3/2}(1-x+x^2)^{3/2}} dx$$

**Optimal.** Leaf size=170

$$\frac{2}{3x^2\sqrt{x+1}\sqrt{x^2-x+1}} - \frac{7\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle| -7-4\sqrt{3}\right)}{6\sqrt[4]{3}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^2-x+1}} - \frac{7(x^3+1)}{6x^2\sqrt{x+1}\sqrt{x^2-x+1}}$$

[Out] 2/(3\*x^2\*Sqrt[1+x]\*Sqrt[1-x+x^2]) - (7\*(1+x^3))/(6\*x^2\*Sqrt[1+x]\*Sqrt[1-x+x^2]) - (7\*Sqrt[2+Sqrt[3]]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(6\*3^(1/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2])

**Rubi [A]** time = 0.159164, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$\frac{2}{3x^2\sqrt{x+1}\sqrt{x^2-x+1}} - \frac{7\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle| -7-4\sqrt{3}\right)}{6\sqrt[4]{3}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^2-x+1}} - \frac{7(x^3+1)}{6x^2\sqrt{x+1}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*(1+x)^(3/2)\*(1-x+x^2)^(3/2)),x]

[Out] 2/(3\*x^2\*Sqrt[1+x]\*Sqrt[1-x+x^2]) - (7\*(1+x^3))/(6\*x^2\*Sqrt[1+x]\*Sqrt[1-x+x^2]) - (7\*Sqrt[2+Sqrt[3]]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(6\*3^(1/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2])

**Rubi in Sympy [A]** time = 12.2867, size = 158, normalized size = 0.93

$$\frac{7 \cdot 3^{\frac{3}{4}} \sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}} \sqrt{\sqrt{3}+2} (x+1)^{\frac{3}{2}} \sqrt{x^2-x+1} F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle| -7-4\sqrt{3}\right)}{18 \sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}} (x^3+1)} - \frac{7\sqrt{x+1}\sqrt{x^2-x+1}}{6x^2} + \frac{2\sqrt{x+1}\sqrt{x^2-x+1}}{3x^2(x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*3/(1+x)\*\*(3/2)/(x\*\*2-x+1)\*\*(3/2),x)

[Out] -7\*3\*\*(3/4)\*sqrt((x\*\*2-x+1)/(x+1+sqrt(3))\*\*2)\*sqrt(sqrt(3)+2)\*(x+1)\*\*(3/2)\*sqrt(x\*\*2-x+1)\*elliptic\_f(asin((x-sqrt(3)+1)/(x+1+sqrt(3))), -7-4\*sqrt(3))/(18\*sqrt((x+1)/(x+1+sqrt(3))\*\*2)\*(x\*\*3+1)) - 7\*sqrt(x+1)\*sqrt(x\*\*2-x+1)/(6\*x\*\*2) + 2\*sqrt(x+1)\*sqrt(x\*\*2-x+1)/(3\*x\*\*2\*(x\*\*3+1))

**Mathematica [C]** time = 0.675265, size = 170, normalized size = 1.

$$\frac{\frac{6(7x^3+3)}{x^2\sqrt{x+1}} - \frac{7i(x+1)\sqrt{1+\frac{6i}{(\sqrt{3}-3i)(x+1)}}\sqrt{6-\frac{36i}{(\sqrt{3}+3i)(x+1)}}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{-6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{\frac{-i}{\sqrt{3}+3i}}}}{36\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(1+x)^(3/2)\*(1-x+x^2)^(3/2)),x]

[Out] ((-6\*(3+7\*x^3))/(x^2\*Sqrt[1+x]) - ((7\*I)\*(1+x)\*Sqrt[1+(6\*I)/((-3\*I+Sqrt[3])\*(1+x))]\*Sqrt[6-(36\*I)/((3\*I+Sqrt[3])\*(1+x))]\*EllipticF[I\*ArcSinh[Sqrt[(-6\*I)/(3\*I+Sqrt[3])]/Sqrt[1+x]],(3\*I+Sqrt[3])/(3\*I-Sqrt[3])])/Sqrt[(-I)/(3\*I+Sqrt[3])])/Sqrt[1-x+x^2])

**Maple [A]** time = 0.045, size = 259, normalized size = 1.5

$$\frac{1}{(12x^3+12)x^2}\sqrt{1+x}\sqrt{x^2-x+1}\left(7i\text{EllipticF}\left(\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\sqrt{3}x^2\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}}{-3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(1+x)^(3/2)/(x^2-x+1)^(3/2),x)

[Out] 1/12\*(1+x)^(1/2)\*(x^2-x+1)^(1/2)\*(7\*I\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2),(-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))\*3^(1/2)\*x^2\*(-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)-21\*(-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2),(-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))\*x^2-14\*x^3-6)/(x^3+1)/x^2

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2-x+1)^{\frac{3}{2}}(x+1)^{\frac{3}{2}}x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2-x+1)^(3/2)\*(x+1)^(3/2)\*x^3),x,algorithm="maxima")

[Out] integrate(1/((x^2-x+1)^(3/2)\*(x+1)^(3/2)\*x^3),x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(x^6+x^3)\sqrt{x^2-x+1}\sqrt{x+1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3),x, algorithm="fricas")`

[Out] `integral(1/((x^6 + x^3)*sqrt(x^2 - x + 1)*sqrt(x + 1)), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^3 (x + 1)^{\frac{3}{2}} (x^2 - x + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(1+x)**(3/2)/(x**2-x+1)**(3/2),x)`

[Out] `Integral(1/(x**3*(x + 1)**(3/2)*(x**2 - x + 1)**(3/2)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2 - x + 1)^{\frac{3}{2}} (x + 1)^{\frac{3}{2}} x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3),x, algorithm="giac")`

[Out] `integrate(1/((x^2 - x + 1)^(3/2)*(x + 1)^(3/2)*x^3), x)`

$$3.517 \quad \int \frac{x^3}{(1+x)^{5/2}(1-x+x^2)^{5/2}} dx$$

**Optimal.** Leaf size=168

$$\frac{4x}{27\sqrt{x+1}\sqrt{x^2-x+1}} + \frac{4\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^2-x+1}} - \frac{2x}{9\sqrt{x+1}\sqrt{x^2-x+1}(x^3+1)}$$

[Out] (4\*x)/(27\*Sqrt[1+x]\*Sqrt[1-x+x^2]) - (2\*x)/(9\*Sqrt[1+x]\*Sqrt[1-x+x^2]\*(1+x^3)) + (4\*Sqrt[2+Sqrt[3]]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(27\*3^(1/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2])

**Rubi [A]** time = 0.149086, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$\frac{4x}{27\sqrt{x+1}\sqrt{x^2-x+1}} + \frac{4\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^2-x+1}} - \frac{2x}{9\sqrt{x+1}\sqrt{x^2-x+1}(x^3+1)}$$

Antiderivative was successfully verified.

[In] Int[x^3/((1+x)^(5/2)\*(1-x+x^2)^(5/2)),x]

[Out] (4\*x)/(27\*Sqrt[1+x]\*Sqrt[1-x+x^2]) - (2\*x)/(9\*Sqrt[1+x]\*Sqrt[1-x+x^2]\*(1+x^3)) + (4\*Sqrt[2+Sqrt[3]]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(27\*3^(1/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2])

**Rubi in Sympy [A]** time = 11.1705, size = 162, normalized size = 0.96

$$\frac{4x\sqrt{x+1}\sqrt{x^2-x+1}}{27(x^3+1)} - \frac{2x\sqrt{x+1}\sqrt{x^2-x+1}}{9(x^3+1)^2} + \frac{4 \cdot 3^{\frac{3}{4}} \sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}} \sqrt{\sqrt{3}+2} (x+1)^{\frac{3}{2}} \sqrt{x^2-x+1} F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right) \mid -7-4\sqrt{3}\right)}{81 \sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}} (x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*3/(1+x)\*\*(5/2)/(x\*\*2-x+1)\*\*(5/2),x)

[Out] 4\*x\*sqrt(x+1)\*sqrt(x\*\*2-x+1)/(27\*(x\*\*3+1)) - 2\*x\*sqrt(x+1)\*sqrt(x\*\*2-x+1)/(9\*(x\*\*3+1)\*\*2) + 4\*3\*\*(3/4)\*sqrt((x\*\*2-x+1)/(x+1+sqrt(3))\*\*2)\*sqrt(sqrt(3)+2)\*(x+1)\*\*(3/2)\*sqrt(x\*\*2-x+1)\*elliptic\_f(asin((x-sqrt(3)+1)/(x+1+sqrt(3))), -7-4\*sqrt(3))/(81\*sqrt((x+1)/(x+1+sqrt(3))\*\*2)\*(x



\* 3 + 1))

**Mathematica [C]** time = 0.761217, size = 178, normalized size = 1.06

$$\frac{6x(2x^3-1)}{(x+1)^{3/2}(x^2-x+1)} + \frac{2i(x+1)\sqrt{1+\frac{6i}{(\sqrt{3}-3i)(x+1)}}\sqrt{6-\frac{36i}{(\sqrt{3}+3i)(x+1)}}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{-6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)\right)}{\sqrt{\frac{-i}{\sqrt{3}+3i}}}$$

$$81\sqrt{x^2-x+1}$$

Antiderivative was successfully verified.

[In] Integrate[x^3/((1+x)^(5/2)\*(1-x+x^2)^(5/2)),x]

[Out] ((6\*x\*(-1+2\*x^3))/((1+x)^(3/2)\*(1-x+x^2)) + ((2\*I)\*(1+x)\*Sqrt[1+(6\*I)/((-3\*I+Sqrt[3])\*(1+x))]\*Sqrt[6-(36\*I)/((3\*I+Sqrt[3])\*(1+x))])\*EllipticF[I\*ArcSinh[Sqrt[(-6\*I)/(3\*I+Sqrt[3])]/Sqrt[1+x]],(3\*I+Sqrt[3])/(3\*I-Sqrt[3])])/Sqrt[(-I)/(3\*I+Sqrt[3])])/(81\*Sqrt[1-x+x^2])

**Maple [B]** time = 0.084, size = 467, normalized size = 2.8

$$-\frac{2}{27}\left(i\sqrt{3}\text{EllipticF}\left(\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)x^3\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}-3\text{EllipticF}\left(\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}},\sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x)

[Out] -2/27\*(I\*3^(1/2)\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2),(-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))\*x^3\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*(-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)-3\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2),(-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))\*x^3\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*(-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)+I\*(-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2),(-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))\*3^(1/2)-3\*(-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2),(-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))-2\*x^4+x)/(x^2-x+1)^(3/2)/(1+x)^(3/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(x^2-x+1)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/((x^2-x+1)^(5/2)\*(x+1)^(5/2)),x,algorithm="maxima")

[Out] integrate(x^3/((x^2-x+1)^(5/2)\*(x+1)^(5/2)),x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{x^3}{(x^6 + 2x^3 + 1)\sqrt{x^2 - x + 1}\sqrt{x + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/((x^2 - x + 1)^(5/2)\*(x + 1)^(5/2)),x, algorithm="fricas")

[Out] integral(x^3/((x^6 + 2\*x^3 + 1)\*sqrt(x^2 - x + 1)\*sqrt(x + 1)), x  
)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(x + 1)^{\frac{5}{2}}(x^2 - x + 1)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3/(1+x)\*\*(5/2)/(x\*\*2-x+1)\*\*(5/2),x)

[Out] Integral(x\*\*3/((x + 1)\*\*(5/2)\*(x\*\*2 - x + 1)\*\*(5/2)), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^3}{(x^2 - x + 1)^{\frac{5}{2}}(x + 1)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^3/((x^2 - x + 1)^(5/2)\*(x + 1)^(5/2)),x, algorithm="giac")

[Out] integrate(x^3/((x^2 - x + 1)^(5/2)\*(x + 1)^(5/2)), x)

$$3.518 \quad \int \frac{x^2}{(1+x)^{5/2}(1-x+x^2)^{5/2}} dx$$

**Optimal.** Leaf size=23

$$-\frac{2}{9(x+1)^{3/2}(x^2-x+1)^{3/2}}$$

[Out]  $-2/(9*(1+x)^{(3/2)}*(1-x+x^2)^{(3/2)})$

**Rubi [A]** time = 0.0186099, antiderivative size = 23, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$-\frac{2}{9(x+1)^{3/2}(x^2-x+1)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[x^2/((1+x)^{(5/2)}*(1-x+x^2)^{(5/2)}), x]$

[Out]  $-2/(9*(1+x)^{(3/2)}*(1-x+x^2)^{(3/2)})$

**Rubi in Sympy [A]** time = 7.2097, size = 27, normalized size = 1.17

$$-\frac{2\sqrt{x+1}\sqrt{x^2-x+1}}{9(x^3+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(x**2/(1+x)**(5/2)/(x**2-x+1)**(5/2), x)$

[Out]  $-2*\text{sqrt}(x+1)*\text{sqrt}(x**2-x+1)/(9*(x**3+1)**2)$

**Mathematica [A]** time = 0.044109, size = 23, normalized size = 1.

$$-\frac{2}{9(x+1)^{3/2}(x^2-x+1)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[x^2/((1+x)^{(5/2)}*(1-x+x^2)^{(5/2)}), x]$

[Out]  $-2/(9*(1+x)^{(3/2)}*(1-x+x^2)^{(3/2)})$

**Maple [A]** time = 0.006, size = 18, normalized size = 0.8

$$-\frac{2}{9}(1+x)^{-\frac{3}{2}}(x^2-x+1)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(x^2/(1+x)^{(5/2)}/(x^2-x+1)^{(5/2)}, x)$

[Out]  $-2/9/(1+x)^{3/2}/(x^2-x+1)^{3/2}$

**Maxima [A]** time = 0.771442, size = 32, normalized size = 1.39

$$-\frac{2}{9(x^3+1)\sqrt{x^2-x+1}\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/((x^2 - x + 1)^(5/2) * (x + 1)^(5/2)),x, algorithm="maxima")`

[Out]  $-2/9/((x^3 + 1)*\text{sqrt}(x^2 - x + 1)*\text{sqrt}(x + 1))$

**Fricas [A]** time = 0.271938, size = 32, normalized size = 1.39

$$-\frac{2}{9(x^3+1)\sqrt{x^2-x+1}\sqrt{x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/((x^2 - x + 1)^(5/2) * (x + 1)^(5/2)),x, algorithm="fricas")`

[Out]  $-2/9/((x^3 + 1)*\text{sqrt}(x^2 - x + 1)*\text{sqrt}(x + 1))$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(x+1)^{\frac{5}{2}}(x^2-x+1)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)`

[Out] `Integral(x**2/((x + 1)**(5/2) * (x**2 - x + 1)**(5/2)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x^2}{(x^2-x+1)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x^2/((x^2 - x + 1)^(5/2) * (x + 1)^(5/2)),x, algorithm="giac")`

[Out] `integrate(x^2/((x^2 - x + 1)^(5/2) * (x + 1)^(5/2)), x)`

$$3.519 \quad \int \frac{x}{(1+x)^{5/2}(1-x+x^2)^{5/2}} dx$$

**Optimal.** Leaf size=318

$$\begin{aligned} & \frac{10x^2}{27\sqrt{x+1}\sqrt{x^2-x+1}} - \frac{10\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle| -7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^2-x+1}} \\ & + \frac{5\sqrt{2-\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle| -7-4\sqrt{3}\right)}{9\cdot 3^{3/4}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^2-x+1}} \\ & + \frac{2x^2}{9\sqrt{x+1}\sqrt{x^2-x+1}(x^3+1)} - \frac{10(x^3+1)}{27\sqrt{x+1}(x+\sqrt{3}+1)\sqrt{x^2-x+1}} \end{aligned}$$

[Out] (10\*x^2)/(27\*Sqrt[1+x]\*Sqrt[1-x+x^2]) + (2\*x^2)/(9\*Sqrt[1+x]\*Sqrt[1-x+x^2]\*(1+x^3)) - (10\*(1+x^3))/(27\*Sqrt[1+x]\*(1+Sqrt[3]+x)\*Sqrt[1-x+x^2]) + (5\*Sqrt[2-Sqrt[3]]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticE[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(9\*3^(3/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2]) - (10\*Sqrt[2]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(27\*3^(1/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2])

**Rubi [A]** time = 0.233651, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$\begin{aligned} & \frac{10x^2}{27\sqrt{x+1}\sqrt{x^2-x+1}} - \frac{10\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle| -7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^2-x+1}} \\ & + \frac{5\sqrt{2-\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle| -7-4\sqrt{3}\right)}{9\cdot 3^{3/4}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^2-x+1}} \\ & + \frac{2x^2}{9\sqrt{x+1}\sqrt{x^2-x+1}(x^3+1)} - \frac{10(x^3+1)}{27\sqrt{x+1}(x+\sqrt{3}+1)\sqrt{x^2-x+1}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[x/((1+x)^(5/2)\*(1-x+x^2)^(5/2)),x]

[Out] (10\*x^2)/(27\*Sqrt[1+x]\*Sqrt[1-x+x^2]) + (2\*x^2)/(9\*Sqrt[1+x]\*Sqrt[1-x+x^2]\*(1+x^3)) - (10\*(1+x^3))/(27\*Sqrt[1+x]\*(1+Sqrt[3]+x)\*Sqrt[1-x+x^2]) + (5\*Sqrt[2-Sqrt[3]]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticE[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(9\*3^(3/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2]) - (10\*Sqrt[2]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(27\*3^(1/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2])

**Rubi in Sympy [A]** time = 19.913, size = 298, normalized size = 0.94

$$\frac{10x^2\sqrt{x+1}\sqrt{x^2-x+1}}{27(x^3+1)} + \frac{2x^2\sqrt{x+1}\sqrt{x^2-x+1}}{9(x^3+1)^2} - \frac{10\sqrt{x+1}\sqrt{x^2-x+1}}{27(x+1+\sqrt{3})}$$

$$+ \frac{5^4\sqrt{3}\sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}}\sqrt{-\sqrt{3}+2}(x+1)^{\frac{3}{2}}\sqrt{x^2-x+1}E\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle| -7-4\sqrt{3}\right)}{27\sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}}(x^3+1)}$$

$$- \frac{10\sqrt{2}\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}}(x+1)^{\frac{3}{2}}\sqrt{x^2-x+1}F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle| -7-4\sqrt{3}\right)}{81\sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}}(x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)`

[Out]  $10x^{**2}\sqrt{x+1}\sqrt{x^{**2}-x+1}/(27*(x^{**3}+1)) + 2x^{**2}\sqrt{x+1}\sqrt{x^{**2}-x+1}/(9*(x^{**3}+1)**2) - 10\sqrt{x+1}\sqrt{x^{**2}-x+1}/(27*(x+1+\sqrt{3})) + 5*3^{**}(1/4)\sqrt{x^{**2}-x+1}/(x+1+\sqrt{3})^{**2}\sqrt{-\sqrt{3}+2}(x+1)^{**}(3/2)\sqrt{x^{**2}-x+1}\operatorname{elliptic}_e(\operatorname{asin}((x-\sqrt{3}+1)/(x+1+\sqrt{3})), -7-4\sqrt{3})/(27\sqrt{(x+1)/(x+1+\sqrt{3})}^{**2})(x^{**3}+1) - 10\sqrt{2}\cdot 3^{**}(3/4)\sqrt{(x^{**2}-x+1)/(x+1+\sqrt{3})}^{**2}(x+1)^{**}(3/2)\sqrt{x^{**2}-x+1}\operatorname{elliptic}_f(\operatorname{asin}((x-\sqrt{3}+1)/(x+1+\sqrt{3})), -7-4\sqrt{3})/(81\sqrt{(x+1)/(x+1+\sqrt{3})}^{**2})(x^{**3}+1)$

**Mathematica [C]** time = 1.3609, size = 409, normalized size = 1.29

$$\frac{2x^2(5x^3+8)}{27(x+1)^{3/2}(x^2-x+1)^{3/2}}$$

$$5(x+1)^{3/2}\left(\frac{12\sqrt{-\frac{i}{\sqrt{3}+3i}}(x^2-x+1)}{(x+1)^2} + \frac{i\sqrt{2}(\sqrt{3}+3i)\sqrt{\frac{6i}{x+1}+\sqrt{3}+3i}}{\sqrt{3}+3i}\sqrt{\frac{6i}{x+1}+\sqrt{3}-3i}}{\sqrt{x+1}}F\left(i\sinh^{-1}\left(\sqrt{\frac{-6i}{3i+\sqrt{3}}}\right)\middle| \frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right) + \frac{3\sqrt{2}(1-i\sqrt{3})\sqrt{\frac{6i}{x+1}+\sqrt{3}+3i}}{\sqrt{3}+3i}\sqrt{\frac{6i}{x+1}+\sqrt{3}-3i}}{\sqrt{x+1}}E\left(\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)\right)$$

$$- \frac{162\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{x^2-x+1}}{162\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In] `Integrate[x/((1+x)^(5/2)*(1-x+x^2)^(5/2)),x]`

[Out]  $(2x^2(8+5x^3))/(27(1+x)^{3/2}(1-x+x^2)^{3/2}) - (5(1+x)^{3/2}((12\sqrt{(-1)/(3I+\sqrt{3})})(1-x+x^2))/(1+x)^2 + (3\sqrt{2}(1-I\sqrt{3})\sqrt{(3I+\sqrt{3})-(6I)/(1+x)})/(3I+\sqrt{3})\sqrt{(-3I+\sqrt{3}+(6I)/(1+x))}/(-3I+\sqrt{3}))\operatorname{EllipticE}[I\operatorname{ArcSinh}[\sqrt{(-6I)/(3I+\sqrt{3})}]/\sqrt{1+x}], (3I+\sqrt{3})/(3I-\sqrt{3}))/\sqrt{1+x} + (I\sqrt{2}(1-I\sqrt{3})\sqrt{(3I+\sqrt{3})-(6I)/(1+x)})/(3I+\sqrt{3})\sqrt{(-3I+\sqrt{3}+(6I)/(1+x))}/(-3I+\sqrt{3}))\operatorname{EllipticF}[I\operatorname{ArcSinh}[\sqrt{(-6I)/(3I+\sqrt{3})}]/\sqrt{1+x}], (3I+\sqrt{3})/(3I-\sqrt{3}))/\sqrt{1+x}))/162\sqrt{(-1)/(3I+\sqrt{3})}\sqrt{1-x+x^2})$

**Maple [B]** time = 0.066, size = 688, normalized size = 2.2

$$-\frac{1}{27} \left( 5 i \sqrt{3} \operatorname{EllipticF} \left( \sqrt{-2 \frac{1+x}{-3+i\sqrt{3}}}, \sqrt{\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}} \right) x^3 \sqrt{-2 \frac{1+x}{-3+i\sqrt{3}}} \sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}} \sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}} + 15 \operatorname{EllipticF} \left( \sqrt{-2 \frac{1+x}{-3+i\sqrt{3}}}, \sqrt{\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}} \right) x^3 \sqrt{-2 \frac{1+x}{-3+i\sqrt{3}}} \sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}} \sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(1+x)^(5/2)/(x^2-x+1)^(5/2), x)

[Out] 
$$-1/27*(5*I^3^{1/2}*\operatorname{EllipticF}((-2*(1+x)/(-3+I^3^{1/2}))^{1/2}, (-(-3+I^3^{1/2})/(I^3^{1/2}+3))^{1/2})*x^3*(-2*(1+x)/(-3+I^3^{1/2}))^{1/2}*((I^3^{1/2}-2*x+1)/(I^3^{1/2}+3))^{1/2}*((I^3^{1/2}+2*x-1)/(-3+I^3^{1/2}))^{1/2}+15*\operatorname{EllipticF}((-2*(1+x)/(-3+I^3^{1/2}))^{1/2}, (-(-3+I^3^{1/2})/(I^3^{1/2}+3))^{1/2})*x^3*((I^3^{1/2}+2*x-1)/(-3+I^3^{1/2}))^{1/2}*(-2*(1+x)/(-3+I^3^{1/2}))^{1/2}*((I^3^{1/2}-2*x+1)/(I^3^{1/2}+3))^{1/2}-30*\operatorname{EllipticE}((-2*(1+x)/(-3+I^3^{1/2}))^{1/2}, (-(-3+I^3^{1/2})/(I^3^{1/2}+3))^{1/2})*x^3*(-2*(1+x)/(-3+I^3^{1/2}))^{1/2}*((I^3^{1/2}-2*x+1)/(I^3^{1/2}+3))^{1/2}*((I^3^{1/2}+2*x-1)/(-3+I^3^{1/2}))^{1/2}+5*I^3^{1/2}*\operatorname{EllipticF}((-2*(1+x)/(-3+I^3^{1/2}))^{1/2}, (-(-3+I^3^{1/2})/(I^3^{1/2}+3))^{1/2})*(-2*(1+x)/(-3+I^3^{1/2}))^{1/2}*((I^3^{1/2}-2*x+1)/(I^3^{1/2}+3))^{1/2}*((I^3^{1/2}+2*x-1)/(-3+I^3^{1/2}))^{1/2}-10*x^5+15*(-2*(1+x)/(-3+I^3^{1/2}))^{1/2}*((I^3^{1/2}-2*x+1)/(I^3^{1/2}+3))^{1/2}*((I^3^{1/2}+2*x-1)/(-3+I^3^{1/2}))^{1/2}*\operatorname{EllipticF}((-2*(1+x)/(-3+I^3^{1/2}))^{1/2}, (-(-3+I^3^{1/2})/(I^3^{1/2}+3))^{1/2})-30*(-2*(1+x)/(-3+I^3^{1/2}))^{1/2}*((I^3^{1/2}-2*x+1)/(I^3^{1/2}+3))^{1/2}*((I^3^{1/2}+2*x-1)/(-3+I^3^{1/2}))^{1/2}*\operatorname{EllipticE}((-2*(1+x)/(-3+I^3^{1/2}))^{1/2}, (-(-3+I^3^{1/2})/(I^3^{1/2}+3))^{1/2})-16*x^2/(x^2-x+1)^{3/2}/(1+x)^{3/2}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^2-x+1)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((x^2 - x + 1)^(5/2)\*(x + 1)^(5/2)), x, algorithm="maxima")

[Out] integrate(x/((x^2 - x + 1)^(5/2)\*(x + 1)^(5/2)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{x}{(x^6 + 2x^3 + 1)\sqrt{x^2 - x + 1}\sqrt{x + 1}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((x^2 - x + 1)^(5/2)\*(x + 1)^(5/2)), x, algorithm="fricas")

[Out] integral(x/((x^6 + 2\*x^3 + 1)\*sqrt(x^2 - x + 1)\*sqrt(x + 1)), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x+1)^{\frac{5}{2}}(x^2-x+1)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)`

[Out] `Integral(x/((x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x}{(x^2 - x + 1)^{\frac{5}{2}}(x + 1)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)),x, algorithm="giac")`

[Out] `integrate(x/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)`



$$3.520 \quad \int \frac{1}{(1+x)^{5/2}(1-x+x^2)^{5/2}} dx$$

**Optimal.** Leaf size=168

$$\frac{14x}{27\sqrt{x+1}\sqrt{x^2-x+1}} + \frac{14\sqrt{2+\sqrt{3}}\sqrt{x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{27\sqrt[4]{3} \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} \sqrt{x^2-x+1}} + \frac{2x}{9\sqrt{x+1}\sqrt{x^2-x+1}(x^3+1)}$$

[Out] (14\*x)/(27\*Sqrt[1+x]\*Sqrt[1-x+x^2]) + (2\*x)/(9\*Sqrt[1+x]\*Sqrt[1-x+x^2]\*(1+x^3)) + (14\*Sqrt[2+Sqrt[3]]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(27\*3^(1/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2])

**Rubi [A]** time = 0.101846, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{14x}{27\sqrt{x+1}\sqrt{x^2-x+1}} + \frac{14\sqrt{2+\sqrt{3}}\sqrt{x+1} \sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}} F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right) \mid -7-4\sqrt{3}\right)}{27\sqrt[4]{3} \sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}} \sqrt{x^2-x+1}} + \frac{2x}{9\sqrt{x+1}\sqrt{x^2-x+1}(x^3+1)}$$

Antiderivative was successfully verified.

[In] Int[1/((1+x)^(5/2)\*(1-x+x^2)^(5/2)),x]

[Out] (14\*x)/(27\*Sqrt[1+x]\*Sqrt[1-x+x^2]) + (2\*x)/(9\*Sqrt[1+x]\*Sqrt[1-x+x^2]\*(1+x^3)) + (14\*Sqrt[2+Sqrt[3]]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(27\*3^(1/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2])

**Rubi in Sympy [A]** time = 8.15758, size = 162, normalized size = 0.96

$$\frac{14x\sqrt{x+1}\sqrt{x^2-x+1}}{27(x^3+1)} + \frac{2x\sqrt{x+1}\sqrt{x^2-x+1}}{9(x^3+1)^2} + \frac{14 \cdot 3^{\frac{3}{4}} \sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}} \sqrt{\sqrt{3}+2} (x+1)^{\frac{3}{2}} \sqrt{x^2-x+1} F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right) \mid -7-4\sqrt{3}\right)}{81 \sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}} (x^3+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(1+x)\*\*(5/2)/(x\*\*2-x+1)\*\*(5/2),x)

[Out] 14\*x\*sqrt(x+1)\*sqrt(x\*\*2-x+1)/(27\*(x\*\*3+1)) + 2\*x\*sqrt(x+1)\*sqrt(x\*\*2-x+1)/(9\*(x\*\*3+1)\*\*2) + 14\*3\*\*(3/4)\*sqrt((x\*\*2-x+1)/(x+1+sqrt(3))\*\*2)\*sqrt(sqrt(3)+2)\*(x+1)\*\*(3/2)\*sqrt(x\*\*2-x+1)\*elliptic\_f(asin((x-sqrt(3)+1)/(x+1+sqrt(3))), -7-4\*sqrt(3))/(81\*sqrt((x+1)/(x+1+sqrt(3))\*\*2)\*

$x^3 + 1$ )

**Mathematica [C]** time = 0.754859, size = 178, normalized size = 1.06

$$\frac{6x(7x^3+10)}{(x+1)^{3/2}(x^2-x+1)} + \frac{7i(x+1)\sqrt{1+\frac{6i}{(\sqrt{3}-3i)(x+1)}}\sqrt{6-\frac{36i}{(\sqrt{3}+3i)(x+1)}}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{-6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)\middle|\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{\frac{-i}{\sqrt{3}+3i}}}$$

$$81\sqrt{x^2-x+1}$$

Antiderivative was successfully verified.

[In] Integrate[1/((1 + x)^(5/2)\*(1 - x + x^2)^(5/2)), x]

[Out] ((6\*x\*(10 + 7\*x^3))/((1 + x)^(3/2)\*(1 - x + x^2))) + ((7\*I)\*(1 + x)\*Sqrt[1 + (6\*I)/((-3\*I + Sqrt[3])\*(1 + x))]\*Sqrt[6 - (36\*I)/((3\*I + Sqrt[3])\*(1 + x))])\*EllipticF[I\*ArcSinh[Sqrt[(-6\*I)/(3\*I + Sqrt[3])]]/Sqrt[1 + x]], (3\*I + Sqrt[3])/(3\*I - Sqrt[3])]/Sqrt[(-I)/(3\*I + Sqrt[3])]/(81\*Sqrt[1 - x + x^2])

**Maple [B]** time = 0.061, size = 469, normalized size = 2.8

$$-\frac{1}{27}\left(7i\sqrt{3}\text{EllipticF}\left(\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}}, \sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)x^3\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}}\sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}}\sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}}-21\text{EllipticF}\left(\sqrt{-2\frac{1+x}{-3+i\sqrt{3}}}, \sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(1+x)^(5/2)/(x^2-x+1)^(5/2), x)

[Out] -1/27\*(7\*I\*3^(1/2)\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2), (-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))\*x^3\*(-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)-21\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2), (-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))\*x^3\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*(-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)+7\*I\*3^(1/2)\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2), (-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))\*(-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)-21\*(-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2), (-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))-14\*x^4-20\*x)/(x^2-x+1)^(3/2)/(1+x)^(3/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2-x+1)^{5/2}(x+1)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 - x + 1)^(5/2)\*(x + 1)^(5/2)), x, algorithm="maxima")

[Out] integrate(1/((x^2 - x + 1)^(5/2)\*(x + 1)^(5/2)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(x^6 + 2x^3 + 1)\sqrt{x^2 - x + 1}\sqrt{x + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)),x, algorithm="fricas")`

[Out] `integral(1/((x^6 + 2*x^3 + 1)*sqrt(x^2 - x + 1)*sqrt(x + 1)), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x + 1)^{\frac{5}{2}}(x^2 - x + 1)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)`

[Out] `Integral(1/((x + 1)**(5/2)*(x**2 - x + 1)**(5/2)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2 - x + 1)^{\frac{5}{2}}(x + 1)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)),x, algorithm="giac")`

[Out] `integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)), x)`

$$3.521 \quad \int \frac{1}{x(1+x)^{5/2}(1-x+x^2)^{5/2}} dx$$

**Optimal.** Leaf size=96

$$\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}} + \frac{2}{9\sqrt{x+1}\sqrt{x^2-x+1}(x^3+1)} - \frac{2\sqrt{x^3+1} \tanh^{-1}(\sqrt{x^3+1})}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

[Out]  $2/(3*\text{Sqrt}[1+x]*\text{Sqrt}[1-x+x^2]) + 2/(9*\text{Sqrt}[1+x]*\text{Sqrt}[1-x+x^2]*(1+x^3)) - (2*\text{Sqrt}[1+x^3]*\text{ArcTanh}[\text{Sqrt}[1+x^3]])/(3*\text{Sqrt}[1+x]*\text{Sqrt}[1-x+x^2])$

**Rubi [A]** time = 0.112605, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$

$$\frac{2}{3\sqrt{x+1}\sqrt{x^2-x+1}} + \frac{2}{9\sqrt{x+1}\sqrt{x^2-x+1}(x^3+1)} - \frac{2\sqrt{x^3+1} \tanh^{-1}(\sqrt{x^3+1})}{3\sqrt{x+1}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(x*(1+x)^{(5/2)}*(1-x+x^2)^{(5/2)}), x]$

[Out]  $2/(3*\text{Sqrt}[1+x]*\text{Sqrt}[1-x+x^2]) + 2/(9*\text{Sqrt}[1+x]*\text{Sqrt}[1-x+x^2]*(1+x^3)) - (2*\text{Sqrt}[1+x^3]*\text{ArcTanh}[\text{Sqrt}[1+x^3]])/(3*\text{Sqrt}[1+x]*\text{Sqrt}[1-x+x^2])$

**Rubi in Sympy [A]** time = 10.8541, size = 90, normalized size = 0.94

$$\frac{2\sqrt{x+1}\sqrt{x^2-x+1}}{3(x^3+1)} + \frac{2\sqrt{x+1}\sqrt{x^2-x+1}}{9(x^3+1)^2} - \frac{2\sqrt{x+1}\sqrt{x^2-x+1} \operatorname{atanh}(\sqrt{x^3+1})}{3\sqrt{x^3+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/x/(1+x)^{(5/2)}/(x^2-x+1)^{(5/2)}, x)$

[Out]  $2*\text{sqrt}(x+1)*\text{sqrt}(x^2-x+1)/(3*(x^3+1)) + 2*\text{sqrt}(x+1)*\text{sqrt}(x^2-x+1)/(9*(x^3+1)^2) - 2*\text{sqrt}(x+1)*\text{sqrt}(x^2-x+1)*\text{atanh}(\text{sqrt}(x^3+1))/(3*\text{sqrt}(x^3+1))$

**Mathematica [C]** time = 0.369333, size = 98, normalized size = 1.02

$$\frac{2 \left( \frac{3x^3+4}{(x^2-x+1)^{3/2}} - \frac{3\sqrt{3}(x+1)^2 \left( 1 + \sqrt[3]{-1}; \sin^{-1} \left( \sqrt{\frac{(-1)^{2/3}x+1}{1+\sqrt[3]{-1}}} \right) \sqrt[3]{-1} \right)}{\sqrt{\frac{x+1}{1+\sqrt[3]{-1}}}} \right)}{9(x+1)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[1/(x*(1+x)^{(5/2)}*(1-x+x^2)^{(5/2)}), x]$

[Out]  $(2*((4+3*x^3)/(1-x+x^2)^{(3/2)} - (3*\text{Sqrt}[3]*(1+x)^2*\text{EllipticPi}[1+(-1)^{(1/3)}, \text{ArcSin}[\text{Sqrt}[(1+(-1)^{(2/3)}*x)/(1+(-1)^{(1/3)}]]$

3))]], (-1)^(1/3)]/Sqrt[(1 + x)/(1 + (-1)^(1/3))])/(9\*(1 + x)^(3/2))

**Maple [A]** time = 0.071, size = 69, normalized size = 0.7

$$-\frac{2}{9x^3+9} \left( 3 \operatorname{Artanh}(\sqrt{x^3+1}) \sqrt{x^3+1} x^3 - 3x^3 + 3 \operatorname{Artanh}(\sqrt{x^3+1}) \sqrt{x^3+1} - 4 \right) \frac{1}{\sqrt{1+x}} \frac{1}{\sqrt{x^2-x+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(1+x)^(5/2)/(x^2-x+1)^(5/2), x)

[Out] -2/9\*(3\*arctanh((x^3+1)^(1/2))\* (x^3+1)^(1/2)\*x^3-3\*x^3+3\*arctanh((x^3+1)^(1/2))\* (x^3+1)^(1/2)-4)/(x^3+1)/(x^2-x+1)^(1/2)/(1+x)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2-x+1)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 - x + 1)^(5/2)\*(x + 1)^(5/2)\*x), x, algorithm="maxima")

[Out] integrate(1/((x^2 - x + 1)^(5/2)\*(x + 1)^(5/2)\*x), x)

**Fricas [A]** time = 0.277998, size = 153, normalized size = 1.59

$$\frac{6x^3 - 3(x^3 + 1)\sqrt{x^2 - x + 1}\sqrt{x + 1} \log(\sqrt{x^2 - x + 1}\sqrt{x + 1} + 1) + 3(x^3 + 1)\sqrt{x^2 - x + 1}\sqrt{x + 1} \log(\sqrt{x^2 - x + 1}\sqrt{x + 1} - 1)}{9(x^3 + 1)\sqrt{x^2 - x + 1}\sqrt{x + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 - x + 1)^(5/2)\*(x + 1)^(5/2)\*x), x, algorithm="fricas")

[Out] 1/9\*(6\*x^3 - 3\*(x^3 + 1)\*sqrt(x^2 - x + 1)\*sqrt(x + 1)\*log(sqrt(x^2 - x + 1)\*sqrt(x + 1) + 1) + 3\*(x^3 + 1)\*sqrt(x^2 - x + 1)\*sqrt(x + 1)\*log(sqrt(x^2 - x + 1)\*sqrt(x + 1) - 1) + 8)/((x^3 + 1)\*sqrt(x^2 - x + 1)\*sqrt(x + 1))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(1+x)\*\*(5/2)/(x\*\*2-x+1)\*\*(5/2), x)

[Out] Timed out

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2 - x + 1)^{\frac{5}{2}}(x + 1)^{\frac{5}{2}}x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x), x, algorithm="giac")`

[Out] `integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x), x)`

$$3.522 \quad \int \frac{1}{x^2(1+x)^{5/2}(1-x+x^2)^{5/2}} dx$$

**Optimal.** Leaf size=349

$$\begin{aligned} & \frac{22}{27x\sqrt{x+1}\sqrt{x^2-x+1}} + \frac{55\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\mid -7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^2-x+1}} \\ & - \frac{55\sqrt{2-\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\mid -7-4\sqrt{3}\right)}{18\cdot 3^{3/4}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^2-x+1}} - \frac{55(x^3+1)}{27x\sqrt{x+1}\sqrt{x^2-x+1}} \\ & + \frac{55(x^3+1)}{27\sqrt{x+1}(x+\sqrt{3}+1)\sqrt{x^2-x+1}} + \frac{2}{9x\sqrt{x+1}\sqrt{x^2-x+1}(x^3+1)} \end{aligned}$$

[Out] 22/(27\*x\*Sqrt[1+x]\*Sqrt[1-x+x^2]) + 2/(9\*x\*Sqrt[1+x]\*Sqrt[1-x+x^2]\*(1+x^3)) - (55\*(1+x^3))/(27\*x\*Sqrt[1+x]\*Sqrt[1-x+x^2]) + (55\*(1+x^3))/(27\*Sqrt[1+x]\*(1+Sqrt[3]+x)\*Sqrt[1-x+x^2]) - (55\*Sqrt[2-Sqrt[3]]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticE[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(18\*3^(3/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2]) + (55\*Sqrt[2]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(27\*3^(1/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2])

**Rubi [A]** time = 0.294704, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$

$$\begin{aligned} & \frac{22}{27x\sqrt{x+1}\sqrt{x^2-x+1}} + \frac{55\sqrt{2}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\mid -7-4\sqrt{3}\right)}{27\sqrt[4]{3}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^2-x+1}} \\ & - \frac{55\sqrt{2-\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}E\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\mid -7-4\sqrt{3}\right)}{18\cdot 3^{3/4}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^2-x+1}} - \frac{55(x^3+1)}{27x\sqrt{x+1}\sqrt{x^2-x+1}} \\ & + \frac{55(x^3+1)}{27\sqrt{x+1}(x+\sqrt{3}+1)\sqrt{x^2-x+1}} + \frac{2}{9x\sqrt{x+1}\sqrt{x^2-x+1}(x^3+1)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2\*(1+x)^(5/2)\*(1-x+x^2)^(5/2)),x]

[Out] 22/(27\*x\*Sqrt[1+x]\*Sqrt[1-x+x^2]) + 2/(9\*x\*Sqrt[1+x]\*Sqrt[1-x+x^2]\*(1+x^3)) - (55\*(1+x^3))/(27\*x\*Sqrt[1+x]\*Sqrt[1-x+x^2]) + (55\*(1+x^3))/(27\*Sqrt[1+x]\*(1+Sqrt[3]+x)\*Sqrt[1-x+x^2]) - (55\*Sqrt[2-Sqrt[3]]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticE[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(18\*3^(3/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2]) + (55\*Sqrt[2]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(27\*3^(1/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2])

**Rubi in Sympy [A]** time = 23.8582, size = 316, normalized size = 0.91

$$\frac{55\sqrt{x+1}\sqrt{x^2-x+1}}{27(x+1+\sqrt{3})} - \frac{55\sqrt[4]{3}\sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}}\sqrt{-\sqrt{3}+2}(x+1)^{\frac{3}{2}}\sqrt{x^2-x+1}E\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\right)\Big|_{-7-4\sqrt{3}}}{54\sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}}(x^3+1)}$$

$$+ \frac{55\sqrt{2}\cdot 3^{\frac{3}{4}}\sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}}(x+1)^{\frac{3}{2}}\sqrt{x^2-x+1}F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\right)\Big|_{-7-4\sqrt{3}}}{81\sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}}(x^3+1)}$$

$$- \frac{55\sqrt{x+1}\sqrt{x^2-x+1}}{27x} + \frac{22\sqrt{x+1}\sqrt{x^2-x+1}}{27x(x^3+1)} + \frac{2\sqrt{x+1}\sqrt{x^2-x+1}}{9x(x^3+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/x**2/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)`

[Out]  $55*\sqrt{x+1}*\sqrt{x^2-x+1}/(27*(x+1+\sqrt{3})) - 55*3^{3/4}*(1/4)*\sqrt{(x^2-x+1)/(x+1+\sqrt{3})^2}*\sqrt{-\sqrt{3}+2}*(x+1)^{3/2}*\sqrt{x^2-x+1}*\operatorname{elliptic}_e(\operatorname{asin}((x-\sqrt{3}+1)/(x+1+\sqrt{3})), -7-4*\sqrt{3})/(54*\sqrt{(x+1)/(x+1+\sqrt{3})^2}*(x^3+1)) + 55*\sqrt{2}*3^{3/4}*\sqrt{(x^2-x+1)/(x+1+\sqrt{3})^2}*(x+1)^{3/2}*\sqrt{x^2-x+1}*\operatorname{elliptic}_f(\operatorname{asin}((x-\sqrt{3}+1)/(x+1+\sqrt{3})), -7-4*\sqrt{3})/(81*\sqrt{(x+1)/(x+1+\sqrt{3})^2}*(x^3+1)) - 55*\sqrt{x+1}*\sqrt{x^2-x+1}/(27*x) + 22*\sqrt{x+1}*\sqrt{x^2-x+1}/(27*x*(x^3+1)) + 2*\sqrt{x+1}*\sqrt{x^2-x+1}/(9*x*(x^3+1)^2)$

**Mathematica [C]** time = 1.69747, size = 414, normalized size = 1.19

$$\frac{55x^6 + 88x^3 + 27}{27x(x+1)^{3/2}(x^2-x+1)^{3/2}}$$

$$+ \frac{55(x+1)^{3/2} \left( \frac{12\sqrt{-\frac{i}{\sqrt{3}+3i}}(x^2-x+1)}{(x+1)^2} + \frac{i\sqrt{2}(\sqrt{3}+3i)\sqrt{\frac{6i}{x+1}+\sqrt{3}+3i}\sqrt{\frac{6i}{x+1}+\sqrt{3}-3i}}{\sqrt{3}+3i} F\left(i \sinh^{-1}\left(\frac{\sqrt{\frac{-6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)\Big|_{\frac{3i+\sqrt{3}}{3i-\sqrt{3}}}\right)}{\sqrt{x+1}} + \frac{3\sqrt{2}(1-i\sqrt{3})\sqrt{\frac{6i}{x+1}+\sqrt{3}+3i}\sqrt{\frac{6i}{x+1}+\sqrt{3}-3i}}{\sqrt{3}+3i} E\left(\frac{3i+\sqrt{3}}{3i-\sqrt{3}}\right)}{\sqrt{x+1}} \right)}{324\sqrt{-\frac{i}{\sqrt{3}+3i}}\sqrt{x^2-x+1}}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(x^2*(1+x)^(5/2)*(1-x+x^2)^(5/2)),x]`

[Out]  $-(27+88*x^3+55*x^6)/(27*x*(1+x)^{3/2}*(1-x+x^2)^{3/2}) + (55*(1+x)^{3/2}*((12*\sqrt{-I}/(3*I+\sqrt{3}))*(1-x+x^2))/(1+x)^2 + (3*\sqrt{2}*(1-I*\sqrt{3})*\sqrt{(3*I+\sqrt{3})-6*I}/(1+x))/(-3*I+\sqrt{3})*\sqrt{(-3*I+\sqrt{3}+(6*I)/(1+x))}/(-3*I+\sqrt{3}))*\operatorname{EllipticE}[I*\operatorname{ArcSinh}[\sqrt{(-6*I)/(3*I+\sqrt{3})}]/\sqrt{1+x}], (3*I+\sqrt{3})/(3*I-\sqrt{3}))/\sqrt{1+x} + (I*\sqrt{2}*(3*I+\sqrt{3})*\sqrt{(3*I+\sqrt{3})-6*I}/(1+x))/(-3*I+\sqrt{3}))*\sqrt{(-3*I+\sqrt{3}+(6*I)/(1+x))}/(-3*I+\sqrt{3}))*\operatorname{EllipticF}[I*\operatorname{ArcSinh}[\sqrt{(-6*I)/(3*I+\sqrt{3})}]/\sqrt{1+x}], (3*I+\sqrt{3})/(3*I-\sqrt{3}))/\sqrt{1+x}))/324*\sqrt{-I}/(3*I+\sqrt{3}))*\sqrt{1-x+x^2})$



**Maple [B]** time = 0.067, size = 695, normalized size = 2.

$$\frac{1}{54x} \left( 55 i\sqrt{3} \text{EllipticF} \left( \sqrt{-2 \frac{1+x}{-3+i\sqrt{3}}}, \sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}} \right) x^4 \sqrt{-2 \frac{1+x}{-3+i\sqrt{3}}} \sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}} \sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}} + 165 \text{EllipticF} \left( \sqrt{\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^2/(1+x)^(5/2)/(x^2-x+1)^(5/2), x)

[Out] 1/54\*(55\*I^3^(1/2)\*EllipticF((-2\*(1+x)/(-3+I^3^(1/2)))^(1/2), (-(-3+I^3^(1/2))/(I^3^(1/2)+3))^(1/2))\*x^4\*(-2\*(1+x)/(-3+I^3^(1/2)))^(1/2)\*((I^3^(1/2)-2\*x+1)/(I^3^(1/2)+3))^(1/2)\*((I^3^(1/2)+2\*x-1)/(-3+I^3^(1/2)))^(1/2)+165\*EllipticF((-2\*(1+x)/(-3+I^3^(1/2)))^(1/2), (-(-3+I^3^(1/2))/(I^3^(1/2)+3))^(1/2))\*x^4\*(-2\*(1+x)/(-3+I^3^(1/2)))^(1/2)\*((I^3^(1/2)-2\*x+1)/(I^3^(1/2)+3))^(1/2)\*((I^3^(1/2)+2\*x-1)/(-3+I^3^(1/2)))^(1/2)-330\*EllipticE((-2\*(1+x)/(-3+I^3^(1/2)))^(1/2), (-(-3+I^3^(1/2))/(I^3^(1/2)+3))^(1/2))\*x^4\*(-2\*(1+x)/(-3+I^3^(1/2)))^(1/2)\*((I^3^(1/2)-2\*x+1)/(I^3^(1/2)+3))^(1/2)\*((I^3^(1/2)+2\*x-1)/(-3+I^3^(1/2)))^(1/2)+55\*I^3^(1/2)\*EllipticF((-2\*(1+x)/(-3+I^3^(1/2)))^(1/2), (-(-3+I^3^(1/2))/(I^3^(1/2)+3))^(1/2))\*x\*(-2\*(1+x)/(-3+I^3^(1/2)))^(1/2)\*((I^3^(1/2)-2\*x+1)/(I^3^(1/2)+3))^(1/2)\*((I^3^(1/2)+2\*x-1)/(-3+I^3^(1/2)))^(1/2)-110\*x^6+165\*(-2\*(1+x)/(-3+I^3^(1/2)))^(1/2)\*((I^3^(1/2)-2\*x+1)/(I^3^(1/2)+3))^(1/2)\*((I^3^(1/2)+2\*x-1)/(-3+I^3^(1/2)))^(1/2)\*EllipticF((-2\*(1+x)/(-3+I^3^(1/2)))^(1/2), (-(-3+I^3^(1/2))/(I^3^(1/2)+3))^(1/2))\*x-330\*(-2\*(1+x)/(-3+I^3^(1/2)))^(1/2)\*((I^3^(1/2)-2\*x+1)/(I^3^(1/2)+3))^(1/2)\*((I^3^(1/2)+2\*x-1)/(-3+I^3^(1/2)))^(1/2)\*EllipticE((-2\*(1+x)/(-3+I^3^(1/2)))^(1/2), (-(-3+I^3^(1/2))/(I^3^(1/2)+3))^(1/2))\*x-176\*x^3-54)/x/(x^2-x+1)^(3/2)/(1+x)^(3/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2 - x + 1)^{\frac{5}{2}}(x + 1)^{\frac{5}{2}}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 - x + 1)^(5/2)\*(x + 1)^(5/2)\*x^2), x, algorithm="maxima")

[Out] integrate(1/((x^2 - x + 1)^(5/2)\*(x + 1)^(5/2)\*x^2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{(x^8 + 2x^5 + x^2)\sqrt{x^2 - x + 1}\sqrt{x + 1}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2 - x + 1)^(5/2)\*(x + 1)^(5/2)\*x^2), x, algorithm="fricas")

[Out] integral(1/((x^8 + 2\*x^5 + x^2)\*sqrt(x^2 - x + 1)\*sqrt(x + 1)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(1+x)**(5/2)/(x**2-x+1)**(5/2),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2 - x + 1)^{\frac{5}{2}}(x + 1)^{\frac{5}{2}}x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^2),x, algorithm="giac")`

[Out] `integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^2), x)`

$$3.523 \quad \int \frac{1}{x^3(1+x)^{5/2}(1-x+x^2)^{5/2}} dx$$

**Optimal.** Leaf size=203

$$\frac{26}{27x^2\sqrt{x+1}\sqrt{x^2-x+1}} - \frac{91\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle| -7-4\sqrt{3}\right)}{54\sqrt[4]{3}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^2-x+1}} - \frac{91(x^3+1)}{54x^2\sqrt{x+1}\sqrt{x^2-x+1}} + \frac{2}{9x^2\sqrt{x+1}\sqrt{x^2-x+1}(x^3+1)}$$

[Out] 26/(27\*x^2\*Sqrt[1+x]\*Sqrt[1-x+x^2]) + 2/(9\*x^2\*Sqrt[1+x]\*Sqrt[1-x+x^2]\*(1+x^3)) - (91\*(1+x^3))/(54\*x^2\*Sqrt[1+x]\*Sqrt[1-x+x^2]) - (91\*Sqrt[2+Sqrt[3]]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(54\*3^(1/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2])

**Rubi [A]** time = 0.18377, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$\frac{26}{27x^2\sqrt{x+1}\sqrt{x^2-x+1}} - \frac{91\sqrt{2+\sqrt{3}}\sqrt{x+1}\sqrt{\frac{x^2-x+1}{(x+\sqrt{3}+1)^2}}F\left(\sin^{-1}\left(\frac{x-\sqrt{3}+1}{x+\sqrt{3}+1}\right)\middle| -7-4\sqrt{3}\right)}{54\sqrt[4]{3}\sqrt{\frac{x+1}{(x+\sqrt{3}+1)^2}}\sqrt{x^2-x+1}} - \frac{91(x^3+1)}{54x^2\sqrt{x+1}\sqrt{x^2-x+1}} + \frac{2}{9x^2\sqrt{x+1}\sqrt{x^2-x+1}(x^3+1)}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3\*(1+x)^(5/2)\*(1-x+x^2)^(5/2)),x]

[Out] 26/(27\*x^2\*Sqrt[1+x]\*Sqrt[1-x+x^2]) + 2/(9\*x^2\*Sqrt[1+x]\*Sqrt[1-x+x^2]\*(1+x^3)) - (91\*(1+x^3))/(54\*x^2\*Sqrt[1+x]\*Sqrt[1-x+x^2]) - (91\*Sqrt[2+Sqrt[3]]\*Sqrt[1+x]\*Sqrt[(1-x+x^2)/(1+Sqrt[3]+x)^2]\*EllipticF[ArcSin[(1-Sqrt[3]+x)/(1+Sqrt[3]+x)], -7-4\*Sqrt[3]])/(54\*3^(1/4)\*Sqrt[(1+x)/(1+Sqrt[3]+x)^2]\*Sqrt[1-x+x^2])

**Rubi in Sympy [A]** time = 14.4477, size = 189, normalized size = 0.93

$$\frac{91 \cdot 3^{\frac{3}{4}} \sqrt{\frac{x^2-x+1}{(x+1+\sqrt{3})^2}} \sqrt{\sqrt{3}+2} (x+1)^{\frac{3}{2}} \sqrt{x^2-x+1} F\left(\operatorname{asin}\left(\frac{x-\sqrt{3}+1}{x+1+\sqrt{3}}\right)\middle| -7-4\sqrt{3}\right)}{162 \sqrt{\frac{x+1}{(x+1+\sqrt{3})^2}} (x^3+1)} - \frac{91\sqrt{x+1}\sqrt{x^2-x+1}}{54x^2} + \frac{26\sqrt{x+1}\sqrt{x^2-x+1}}{27x^2(x^3+1)} + \frac{2\sqrt{x+1}\sqrt{x^2-x+1}}{9x^2(x^3+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*3/(1+x)\*\*(5/2)/(x\*\*2-x+1)\*\*(5/2),x)

[Out] -91\*3\*\*(3/4)\*sqrt((x\*\*2-x+1)/(x+1+sqrt(3)))\*\*2)\*sqrt(sqrt(3)+2)\*(x+1)\*\*(3/2)\*sqrt(x\*\*2-x+1)\*elliptic\_f(asin((x-sqrt(3)+1)/(x+1+sqrt(3))), -7-4\*sqrt(3))/(162\*sqrt((x+1)/

$$(x + 1 + \sqrt{3})^{*2} * (x^{*3} + 1) - 91 * \sqrt{x + 1} * \sqrt{x^{*2} - x + 1} / (54 * x^{*2}) + 26 * \sqrt{x + 1} * \sqrt{x^{*2} - x + 1} / (27 * x^{*2} * (x^{*3} + 1)) + 2 * \sqrt{x + 1} * \sqrt{x^{*2} - x + 1} / (9 * x^{*2} * (x^{*3} + 1)^{*2})$$

**Mathematica [C]** time = 1.08172, size = 183, normalized size = 0.9

$$\frac{-\frac{6(91x^6+130x^3+27)}{x^2(x+1)^{3/2}} - \frac{91i(x+1)(x^2-x+1)\sqrt{6+\frac{36i}{(\sqrt{3}-3i)(x+1)}}\sqrt{1-\frac{6i}{(\sqrt{3}+3i)(x+1)}}F\left(i\sinh^{-1}\left(\frac{\sqrt{\frac{6i}{3i+\sqrt{3}}}}{\sqrt{x+1}}\right)\Big|_{\frac{3i+\sqrt{3}}{3i-\sqrt{3}}}\right)}{\sqrt{-\frac{i}{\sqrt{3}+3i}}}}{324(x^2-x+1)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(x^3\*(1+x)^(5/2)\*(1-x+x^2)^(5/2)),x]

[Out] ((-6\*(27+130\*x^3+91\*x^6))/(x^2\*(1+x)^(3/2)) - ((91\*I)\*(1+x)\*(1-x+x^2)\*Sqrt[6+(36\*I)/((-3\*I+Sqrt[3])\*(1+x))]\*Sqrt[1-(6\*I)/((3\*I+Sqrt[3])\*(1+x))])\*EllipticF[I\*ArcSinh[Sqrt[(-6\*I)/(3\*I+Sqrt[3])]/Sqrt[1+x]],(3\*I+Sqrt[3])/(3\*I-Sqrt[3])])/Sqrt[(-I)/(3\*I+Sqrt[3])])/(324\*(1-x+x^2)^(3/2))

**Maple [B]** time = 0.063, size = 481, normalized size = 2.4

$$\frac{1}{108x^2} \left( 91i\sqrt{3}\text{EllipticF} \left( \sqrt{-2\frac{1+x}{-3+i\sqrt{3}}}, \sqrt{-\frac{-3+i\sqrt{3}}{i\sqrt{3}+3}} \right) x^5 \sqrt{\frac{i\sqrt{3}+2x-1}{-3+i\sqrt{3}}} \sqrt{-2\frac{1+x}{-3+i\sqrt{3}}} \sqrt{\frac{i\sqrt{3}-2x+1}{i\sqrt{3}+3}} - 273\text{EllipticF} \left( \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^3/(1+x)^(5/2)/(x^2-x+1)^(5/2),x)

[Out] 1/108\*(91\*I\*3^(1/2)\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2),(-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))\*x^5\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*(-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)-273\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2),(-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))\*x^5\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*(-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)+91\*I\*3^(1/2)\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2),(-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))\*x^2\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*(-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)-273\*(-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2)\*((I\*3^(1/2)-2\*x+1)/(I\*3^(1/2)+3))^(1/2)\*((I\*3^(1/2)+2\*x-1)/(-3+I\*3^(1/2)))^(1/2)\*EllipticF((-2\*(1+x)/(-3+I\*3^(1/2)))^(1/2),(-(-3+I\*3^(1/2))/(I\*3^(1/2)+3))^(1/2))\*x^2-182\*x^6-260\*x^3-54)/x^2/(x^2-x+1)^(3/2)/(1+x)^(3/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2-x+1)^{\frac{5}{2}}(x+1)^{\frac{5}{2}}x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((x^2-x+1)^(5/2)\*(x+1)^(5/2)\*x^3),x,algorithm="maxima")

[Out] integrate(1/((x^2-x+1)^(5/2)\*(x+1)^(5/2)\*x^3),x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{(x^9 + 2x^6 + x^3)\sqrt{x^2 - x + 1}\sqrt{x + 1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^3), x, algorithm="fricas")`

[Out] `integral(1/((x^9 + 2*x^6 + x^3)*sqrt(x^2 - x + 1)*sqrt(x + 1)), x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**3/(1+x)**(5/2)/(x**2-x+1)**(5/2), x)`

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(x^2 - x + 1)^{\frac{5}{2}}(x + 1)^{\frac{5}{2}}x^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^3), x, algorithm="giac")`

[Out] `integrate(1/((x^2 - x + 1)^(5/2)*(x + 1)^(5/2)*x^3), x)`

$$3.524 \quad \int \frac{x}{(-1+x)^3(3+5x+4x^2)^2} dx$$

**Optimal.** Leaf size=97

$$\frac{44x+39}{276(1-x)^2(4x^2+5x+3)} - \frac{11 \log(4x^2+5x+3)}{4608} - \frac{97}{4416(1-x)} \\ - \frac{21}{736(1-x)^2} + \frac{11 \log(1-x)}{2304} + \frac{6023 \tan^{-1}\left(\frac{8x+5}{\sqrt{23}}\right)}{52992\sqrt{23}}$$

[Out] -21/(736\*(1-x)^2) - 97/(4416\*(1-x)) + (39+44\*x)/(276\*(1-x)^2\*(3+5\*x+4\*x^2)) + (6023\*ArcTan[(5+8\*x)/Sqrt[23]])/(52992\*Sqrt[23]) + (11\*Log[1-x])/2304 - (11\*Log[3+5\*x+4\*x^2])/4608

**Rubi [A]** time = 0.151769, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$

$$\frac{44x+39}{276(1-x)^2(4x^2+5x+3)} - \frac{11 \log(4x^2+5x+3)}{4608} - \frac{97}{4416(1-x)} \\ - \frac{21}{736(1-x)^2} + \frac{11 \log(1-x)}{2304} + \frac{6023 \tan^{-1}\left(\frac{8x+5}{\sqrt{23}}\right)}{52992\sqrt{23}}$$

Antiderivative was successfully verified.

[In] Int[x/((-1+x)^3\*(3+5\*x+4\*x^2)^2),x]

[Out] -21/(736\*(1-x)^2) - 97/(4416\*(1-x)) + (39+44\*x)/(276\*(1-x)^2\*(3+5\*x+4\*x^2)) + (6023\*ArcTan[(5+8\*x)/Sqrt[23]])/(52992\*Sqrt[23]) + (11\*Log[1-x])/2304 - (11\*Log[3+5\*x+4\*x^2])/4608

**Rubi in Sympy [A]** time = 21.3727, size = 83, normalized size = 0.86

$$\frac{11 \log(-x+1)}{2304} - \frac{11 \log(4x^2+5x+3)}{4608} + \frac{6023\sqrt{23} \operatorname{atan}\left(\sqrt{23}\left(\frac{8x}{23} + \frac{5}{23}\right)\right)}{1218816} \\ - \frac{97}{4416(-x+1)} + \frac{44x+39}{276(-x+1)^2(4x^2+5x+3)} - \frac{21}{736(-x+1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x/(-1+x)\*\*3/(4\*x\*\*2+5\*x+3)\*\*2,x)

[Out] 11\*log(-x+1)/2304 - 11\*log(4\*x\*\*2+5\*x+3)/4608 + 6023\*sqrt(23)\*atan(sqrt(23)\*(8\*x/23+5/23))/1218816 - 97/(4416\*(-x+1)) + (44\*x+39)/(276\*(-x+1)\*\*2\*(4\*x\*\*2+5\*x+3)) - 21/(736\*(-x+1)\*\*2)

**Mathematica [A]** time = 0.0697265, size = 78, normalized size = 0.8

$$\frac{184(2204x+975)}{4x^2+5x+3} - 17457 \log(4x^2+5x+3) + \frac{59248}{x-1} - \frac{25392}{(x-1)^2} + 34914 \log(1-x) + 36138\sqrt{23} \tan^{-1}\left(\frac{8x+5}{\sqrt{23}}\right) \\ \frac{7312896}{7312896}$$

Antiderivative was successfully verified.

[In] Integrate[x/((-1 + x)^3\*(3 + 5\*x + 4\*x^2)^2), x]

[Out] (-25392/(-1 + x)^2 + 59248/(-1 + x) + (184\*(975 + 2204\*x))/(3 + 5\*x + 4\*x^2) + 36138\*sqrt(23)\*ArcTan[(5 + 8\*x)/sqrt(23)] + 34914\*Log[1 - x] - 17457\*Log[3 + 5\*x + 4\*x^2])/7312896

**Maple [A]** time = 0.017, size = 68, normalized size = 0.7

$$-\frac{1}{288(-1+x)^2} + \frac{7}{-864+864x} + \frac{11 \ln(-1+x)}{2304} - \frac{1}{6912} \left( -\frac{2204x}{23} - \frac{975}{23} \right) \left( x^2 + \frac{5x}{4} + \frac{3}{4} \right)^{-1} - \frac{11 \ln(16x^2 + 20x + 12)}{4608} + \frac{6023 \sqrt{23}}{1218816} \arctan \left( \frac{(32x + 20) \sqrt{23}}{92} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x/(-1+x)^3/(4\*x^2+5\*x+3)^2, x)

[Out] -1/288/(-1+x)^2+7/864/(-1+x)+11/2304\*ln(-1+x)-1/6912\*(-2204/23\*x-975/23)/(x^2+5/4\*x+3/4)-11/4608\*ln(16\*x^2+20\*x+12)+6023/1218816\*2\*3^(1/2)\*arctan(1/92\*(32\*x+20)\*23^(1/2))

**Maxima [A]** time = 0.773856, size = 101, normalized size = 1.04

$$\frac{6023}{1218816} \sqrt{23} \arctan \left( \frac{1}{23} \sqrt{23}(8x + 5) \right) + \frac{388x^3 - 407x^2 - 120x - 45}{4416(4x^4 - 3x^3 - 3x^2 - x + 3)} - \frac{11}{4608} \log(4x^2 + 5x + 3) + \frac{11}{2304} \log(x - 1)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((4\*x^2 + 5\*x + 3)^2\*(x - 1)^3), x, algorithm="maxima")

[Out] 6023/1218816\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(8\*x + 5)) + 1/4416\*(388\*x^3 - 407\*x^2 - 120\*x - 45)/(4\*x^4 - 3\*x^3 - 3\*x^2 - x + 3) - 11/4608\*log(4\*x^2 + 5\*x + 3) + 11/2304\*log(x - 1)

**Fricas [A]** time = 0.271904, size = 197, normalized size = 2.03

$$\frac{\sqrt{23} \left( 253 \sqrt{23} (4x^4 - 3x^3 - 3x^2 - x + 3) \log(4x^2 + 5x + 3) - 506 \sqrt{23} (4x^4 - 3x^3 - 3x^2 - x + 3) \log(x - 1) - 12046 (4x^4 - 3x^3 - 3x^2 - x + 3) \arctan \left( \frac{1}{23} \sqrt{23} (8x + 5) \right) - 24 (388x^3 - 407x^2 - 120x - 45) \right)}{2437632 (4x^4 - 3x^3 - 3x^2 - x + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((4\*x^2 + 5\*x + 3)^2\*(x - 1)^3), x, algorithm="fricas")

[Out] -1/2437632\*sqrt(23)\*(253\*sqrt(23)\*(4\*x^4 - 3\*x^3 - 3\*x^2 - x + 3)\*log(4\*x^2 + 5\*x + 3) - 506\*sqrt(23)\*(4\*x^4 - 3\*x^3 - 3\*x^2 - x + 3)\*log(x - 1) - 12046\*(4\*x^4 - 3\*x^3 - 3\*x^2 - x + 3)\*arctan(1/23\*sqrt(23)\*(8\*x + 5)) - 24\*sqrt(23)\*(388\*x^3 - 407\*x^2 - 120\*x - 45))/(4\*x^4 - 3\*x^3 - 3\*x^2 - x + 3)

**Sympy [A]** time = 0.605728, size = 88, normalized size = 0.91

$$\frac{388x^3 - 407x^2 - 120x - 45}{17664x^4 - 13248x^3 - 13248x^2 - 4416x + 13248} + \frac{11 \log(x - 1)}{2304} - \frac{11 \log\left(x^2 + \frac{5x}{4} + \frac{3}{4}\right)}{4608} + \frac{6023\sqrt{23} \operatorname{atan}\left(\frac{8\sqrt{23}x}{23} + \frac{5\sqrt{23}}{23}\right)}{1218816}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/(-1+x)\*\*3/(4\*x\*\*2+5\*x+3)\*\*2,x)

[Out] (388\*x\*\*3 - 407\*x\*\*2 - 120\*x - 45)/(17664\*x\*\*4 - 13248\*x\*\*3 - 13248\*x\*\*2 - 4416\*x + 13248) + 11\*log(x - 1)/2304 - 11\*log(x\*\*2 + 5\*x/4 + 3/4)/4608 + 6023\*sqrt(23)\*atan(8\*sqrt(23)\*x/23 + 5\*sqrt(23)/23)/1218816

**GIAC/XCAS [A]** time = 0.26709, size = 96, normalized size = 0.99

$$\frac{6023}{1218816} \sqrt{23} \arctan\left(\frac{1}{23} \sqrt{23}(8x + 5)\right) + \frac{388x^3 - 407x^2 - 120x - 45}{4416(4x^2 + 5x + 3)(x - 1)^2} - \frac{11}{4608} \ln(4x^2 + 5x + 3) + \frac{11}{2304} \ln(|x - 1|)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x/((4\*x^2 + 5\*x + 3)^2\*(x - 1)^3),x, algorithm="giac")

[Out] 6023/1218816\*sqrt(23)\*arctan(1/23\*sqrt(23)\*(8\*x + 5)) + 1/4416\*(388\*x^3 - 407\*x^2 - 120\*x - 45)/((4\*x^2 + 5\*x + 3)\*(x - 1)^2) - 11/4608\*ln(4\*x^2 + 5\*x + 3) + 11/2304\*ln(abs(x - 1))



$$3.525 \quad \int \frac{x^4 \sqrt{d+ex}}{a+bx+cx^2} dx$$

**Optimal.** Leaf size=490

$$\frac{\sqrt{2} \left( -\frac{5a^2bc^2e+2a^2c^3d+5ab^3ce-4ab^2c^2d+b^5(-e)+b^4cd}{\sqrt{b^2-4ac}} - a^2c^2e + 3ab^2ce - 2abc^2d + b^4(-e) + b^3cd \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} \right)}{c^{9/2} \sqrt{2cd-e(b-\sqrt{b^2-4ac})}} + \frac{\sqrt{2} \left( -\frac{5a^2bc^2e+2a^2c^3d+5ab^3ce-4ab^2c^2d+b^5(-e)+b^4cd}{\sqrt{b^2-4ac}} - a^2c^2e + 3ab^2ce - 2abc^2d + b^4(-e) + b^3cd \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} \right)}{c^{9/2} \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} - \frac{2b(b^2-2ac)\sqrt{d+ex}}{c^4} + \frac{2(d+ex)^{3/2}(ce(bd-ae)+b^2e^2+c^2d^2)}{3c^3e^3} - \frac{2(d+ex)^{5/2}(be+2cd)}{5c^2e^3} + \frac{2(d+ex)^{7/2}}{7ce^3}$$

[Out]  $(-2*b*(b^2 - 2*a*c)*\text{Sqrt}[d + e*x])/c^4 + (2*(c^2*d^2 + b^2*e^2 + c*e*(b*d - a*e))*(d + e*x)^{(3/2)})/(3*c^3*e^3) - (2*(2*c*d + b*e)*(d + e*x)^{(5/2)})/(5*c^2*e^3) + (2*(d + e*x)^{(7/2)})/(7*c*e^3) + (\text{Sqrt}[2]*(b^3*c*d - 2*a*b*c^2*d - b^4*e + 3*a*b^2*c*e - a^2*c^2*e - (b^4*c*d - 4*a*b^2*c^2*d + 2*a^2*c^3*d - b^5*e + 5*a*b^3*c*e - 5*a^2*b*c^2*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]])/(c^{(9/2)}*\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]) + (\text{Sqrt}[2]*(b^3*c*d - 2*a*b*c^2*d - b^4*e + 3*a*b^2*c*e - a^2*c^2*e + (b^4*c*d - 4*a*b^2*c^2*d + 2*a^2*c^3*d - b^5*e + 5*a*b^3*c*e - 5*a^2*b*c^2*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]])/(c^{(9/2)}*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e])$

**Rubi [A]** time = 21.7071, antiderivative size = 490, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{\sqrt{2} \left( -\frac{5a^2bc^2e+2a^2c^3d+5ab^3ce-4ab^2c^2d+b^5(-e)+b^4cd}{\sqrt{b^2-4ac}} - a^2c^2e + 3ab^2ce - 2abc^2d + b^4(-e) + b^3cd \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} \right)}{c^{9/2} \sqrt{2cd-e(b-\sqrt{b^2-4ac})}} + \frac{\sqrt{2} \left( -\frac{5a^2bc^2e+2a^2c^3d+5ab^3ce-4ab^2c^2d+b^5(-e)+b^4cd}{\sqrt{b^2-4ac}} - a^2c^2e + 3ab^2ce - 2abc^2d + b^4(-e) + b^3cd \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} \right)}{c^{9/2} \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} - \frac{2b(b^2-2ac)\sqrt{d+ex}}{c^4} + \frac{2(d+ex)^{3/2}(ce(bd-ae)+b^2e^2+c^2d^2)}{3c^3e^3} - \frac{2(d+ex)^{5/2}(be+2cd)}{5c^2e^3} + \frac{2(d+ex)^{7/2}}{7ce^3}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*Sqrt[d + e\*x])/(a + b\*x + c\*x^2), x]

[Out]  $(-2*b*(b^2 - 2*a*c)*\text{Sqrt}[d + e*x])/c^4 + (2*(c^2*d^2 + b^2*e^2 + c*e*(b*d - a*e))*(d + e*x)^{(3/2)})/(3*c^3*e^3) - (2*(2*c*d + b*e)*(d + e*x)^{(5/2)})/(5*c^2*e^3) + (2*(d + e*x)^{(7/2)})/(7*c*e^3) + (\text{Sqrt}[2]*(b^3*c*d - 2*a*b*c^2*d - b^4*e + 3*a*b^2*c*e - a^2*c^2*e - (b^4*c*d - 4*a*b^2*c^2*d + 2*a^2*c^3*d - b^5*e + 5*a*b^3*c*e - 5*a^2*b*c^2*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]])/(c^{(9/2)}*\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]) + (\text{Sqrt}[2]*(b^3*c*d - 2*a*b*c^2*d - b^4*e + 3*a*b^2*c*e - a^2*c^2*e + (b^4*c*d - 4*a*b^2*c^2*d + 2*a^2*c^3*d - b^5*e + 5*a*b^3*c*e - 5*a^2*b*c^2*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]])/(c^{(9/2)}*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e])$

$$+ 2*a^2*c^3*d - b^5*e + 5*a*b^3*c*e - 5*a^2*b*c^2*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]])/(c^(9/2)*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e])$$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**4*(e*x+d)**(1/2)/(c*x**2+b*x+a), x)`

[Out] Timed out

**Mathematica [A]** time = 1.24065, size = 568, normalized size = 1.16

$$\frac{\sqrt{2} \left( a^2 c^2 \left( e \sqrt{b^2 - 4ac} + 2cd \right) + abc^2 \left( 2d \sqrt{b^2 - 4ac} - 5ae \right) - ab^2 c \left( 3e \sqrt{b^2 - 4ac} + 4cd \right) + b^4 \left( e \sqrt{b^2 - 4ac} + cd \right) + b^3 c \left( 5ae - 2bd + 3bex \right) \right)}{c^{9/2} \sqrt{b^2 - 4ac} \sqrt{e \left( \sqrt{b^2 - 4ac} - b \right) + 2cd}}$$

$$\frac{\sqrt{2} \left( a^2 c^2 \left( e \sqrt{b^2 - 4ac} - 2cd \right) + abc^2 \left( 2d \sqrt{b^2 - 4ac} + 5ae \right) + ab^2 c \left( 4cd - 3e \sqrt{b^2 - 4ac} \right) + b^4 \left( e \sqrt{b^2 - 4ac} - cd \right) - b^3 c \left( 5ae - 2bd + 3bex \right) \right)}{c^{9/2} \sqrt{b^2 - 4ac} \sqrt{2cd - e \left( \sqrt{b^2 - 4ac} + b \right)}}$$

$$+ \frac{2\sqrt{d+ex} \left( -7c^2 e(d+ex)(5ae - 2bd + 3bex) + 35bce^2(6ae + b(d+ex)) - 105b^3e^3 + c^3(8d^3 - 4d^2ex + 3de^2x^2 + 15e^3x^3) \right)}{105c^4e^3}$$

Antiderivative was successfully verified.

[In] `Integrate[(x^4*Sqrt[d + e*x])/(a + b*x + c*x^2), x]`

[Out]  $(2*\text{Sqrt}[d + e*x]*(-105*b^3*e^3 - 7*c^2*e*(d + e*x)*(-2*b*d + 5*a*e + 3*b*e*x) + c^3*(8*d^3 - 4*d^2*e*x + 3*d*e^2*x^2 + 15*e^3*x^3) + 35*b*c*e^2*(6*a*e + b*(d + e*x)))/(105*c^4*e^3) - (\text{Sqrt}[2]*(-b^5*e) + a*b*c^2*(2*\text{Sqrt}[b^2 - 4*a*c]*d - 5*a*e) + b^3*c*(-(\text{Sqrt}[b^2 - 4*a*c]*d) + 5*a*e) + b^4*(c*d + \text{Sqrt}[b^2 - 4*a*c]*e) + a^2*c^2*(2*c*d + \text{Sqrt}[b^2 - 4*a*c]*e) - a*b^2*c*(4*c*d + 3*\text{Sqrt}[b^2 - 4*a*c]*e))*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - b*e + \text{Sqrt}[b^2 - 4*a*c]*e]])/(c^(9/2)*\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c])*e]) - (\text{Sqrt}[2]*(b^5*e - b^3*c*(\text{Sqrt}[b^2 - 4*a*c]*d + 5*a*e) + a*b*c^2*(2*\text{Sqrt}[b^2 - 4*a*c]*d + 5*a*e) + a*b^2*c*(4*c*d - 3*\text{Sqrt}[b^2 - 4*a*c]*e) + a^2*c^2*(-2*c*d + \text{Sqrt}[b^2 - 4*a*c]*e) + b^4*(-(c*d) + \text{Sqrt}[b^2 - 4*a*c]*e))*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]])/(c^(9/2)*\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e])$

**Maple [B]** time = 0.114, size = 2218, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^4*(e*x+d)^(1/2)/(c*x^2+b*x+a), x)`

```
[Out] 2/7*(e*x+d)^(7/2)/c/e^3+e/c^2*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^2+e/c^4*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^4-e/c^2*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^2-e/c^4*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^4-1/c^3*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^3*d+1/c^3*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^3*d-2/5/e^2/c^2*(e*x+d)^(5/2)*b-2/3/e/c^2*(e*x+d)^(3/2)*a+2/3/e/c^3*(e*x+d)^(3/2)*b^2-4/5/e^3/c*(e*x+d)^(5/2)*d+2/3/e^3/c*(e*x+d)^(3/2)*d^2+4/c^3*a*b*(e*x+d)^(1/2)+4*e/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b^2*d+4*e/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b^2*d+e^2/c^4/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^5-3*e/c^3*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b^2+e^2/c^4/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^5+3*e/c^3*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b^2+2/c^2*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b*d-2/c^2*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b*d-2/c^4*b^3*(e*x+d)^(1/2)+2/3/e^2/c^2*(e*x+d)^(3/2)*b*d-2*e/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^2*d-5*e^2/c^3/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b^3-e/c^3/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^4*d+5*e^2/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^2*b-2*e/c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^2*d-5*e^2/c^3/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a*b^3-e/c^3/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b^4*d+5*e^2/c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*a^2*b
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+dx^4}}{cx^2+bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*x^4/(c\*x^2 + b\*x + a),x, algorithm="maxima")

[Out] integrate(sqrt(e\*x + d)\*x^4/(c\*x^2 + b\*x + a), x)

**Fricas [A]** time = 0.842421, size = 7434, normalized size = 15.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*x^4/(c\*x^2 + b\*x + a),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & \frac{1}{210} \cdot (105 \sqrt{2}) \cdot c^4 \cdot e^3 \cdot \sqrt{\left( (b^8 c - 8 a b^6 c^2 + 20 a^2 b^4 c^3 - 16 a^3 b^2 c^4 + 2 a^4 c^5) d - (b^9 - 9 a b^7 c + 27 a^2 b^5 c^2 - 30 a^3 b^3 c^3 + 9 a^4 b c^4) e + (b^2 c^9 - 4 a c^{10}) \right)} \\ & \cdot \sqrt{\left( (b^{14} c^2 - 12 a b^{12} c^3 + 56 a^2 b^{10} c^4 - 128 a^3 b^8 c^5 + 148 a^4 b^6 c^6 - 80 a^5 b^4 c^7 + 16 a^6 b^2 c^8) d^2 - 2 \right.} \\ & \cdot (b^{15} c - 13 a b^{13} c^2 + 67 a^2 b^{11} c^3 - 174 a^3 b^9 c^4 + 239 a^4 b^7 c^5 - 166 a^5 b^5 c^6 + 50 a^6 b^3 c^7 - 4 a^7 b c^8) d \cdot \\ & \left. e + (b^{16} - 14 a b^{14} c + 79 a^2 b^{12} c^2 - 230 a^3 b^{10} c^3 + 367 a^4 b^8 c^4 - 314 a^5 b^6 c^5 + 130 a^6 b^4 c^6 - 20 a^7 b^2 c^7 + a^8 c^8) e^2 \right) / (b^2 c^9 - 4 a c^{10}) \cdot \log(\sqrt{2}) \cdot \left( (b^{12} c - 12 a b^{10} c^2 + 54 a^2 b^8 c^3 - 112 a^3 b^6 c^4 + 104 a^4 b^4 c^5 - 32 a^5 b^2 c^6) d - (b^{13} - 13 a b^{11} c + 65 a^2 b^9 c^2 - 156 a^3 b^7 c^3 + 181 a^4 b^5 c^4 - 86 a^5 b^3 c^5 + 8 a^6 b c^6) e - (b^6 c^9 - 8 a b^4 c^{10} + 18 a^2 b^2 c^{11} - 8 a^3 c^{12}) \right) \cdot \sqrt{\left( (b^{14} c^2 - 12 a b^{12} c^3 + 56 a^2 b^{10} c^4 - 128 a^3 b^8 c^5 + 148 a^4 b^6 c^6 - 80 a^5 b^4 c^7 + 16 a^6 b^2 c^8) d^2 - 2 \right.} \\ & \cdot (b^{15} c - 13 a b^{13} c^2 + 67 a^2 b^{11} c^3 - 174 a^3 b^9 c^4 + 239 a^4 b^7 c^5 - 166 a^5 b^5 c^6 + 50 a^6 b^3 c^7 - 4 a^7 b c^8) d \cdot e + (b^{16} - 14 a b^{14} c + 79 a^2 b^{12} c^2 - 230 a^3 b^{10} c^3 + 367 a^4 b^8 c^4 - 314 a^5 b^6 c^5 + 130 a^6 b^4 c^6 - 20 a^7 b^2 c^7 + a^8 c^8) e^2 \Big) / (b^2 c^9 - 4 a c^{10}) - 4 \cdot \left( (a^4 b^7 c - 6 a^5 b^5 c^2 + 10 a^6 b^3 c^3 - 4 a^7 b c^4) d - (a^4 b^8 - 7 a^5 b^6 c + 15 a^6 b^4 c^2 - 10 a^7 b^2 c^3 + a^8 c^4) e \right) \cdot \sqrt{e x + d} - 105 \sqrt{2} \cdot c^4 \cdot e^3 \cdot \sqrt{\left( (b^8 c - 8 a b^6 c^2 + 20 a^2 b^4 c^3 - 16 a^3 b^2 c^4 + 2 a^4 c^5) d - (b^9 - 9 a b^7 c + 27 a^2 b^5 c^2 - 30 a^3 b^3 c^3 + 9 a^4 b c^4) e + (b^2 c^9 - 4 a c^{10}) \right)} \cdot \sqrt{\left( (b^{14} c^2 - 12 a b^{12} c^3 + 56 a^2 b^{10} c^4 - 128 a^3 b^8 c^5 + 148 a^4 b^6 c^6 - 80 a^5 b^4 c^7 + 16 a^6 b^2 c^8) d^2 - 2 \right.} \\ & \cdot (b^{15} c - 13 a b^{13} c^2 + 67 a^2 b^{11} c^3 - 174 a^3 b^9 c^4 + 239 a^4 b^7 c^5 - 166 a^5 b^5 c^6 + 50 a^6 b^3 c^7 - 4 a^7 b c^8) d \cdot e + (b^{16} - 14 a b^{14} c + 79 a^2 b^{12} c^2 - 230 a^3 b^{10} c^3 + 367 a^4 b^8 c^4 - 314 a^5 b^6 c^5 + 130 a^6 b^4 c^6 - 20 a^7 b^2 c^7 + a^8 c^8) e^2 \Big) / (b^2 c^9 - 4 a c^{10}) \cdot \log(-\sqrt{2}) \cdot \left( (b^{12} c - 12 a b^{10} c^2 + 54 a^2 b^8 c^3 - 112 a^3 b^6 c^4 + 104 a^4 b^4 c^5 - 32 a^5 b^2 c^6) d - (b^{13} - 13 a b^{11} c + 65 a^2 b^9 c^2 - 156 a^3 b^7 c^3 + 181 a^4 b^5 c^4 - 86 a^5 b^3 c^5 + 8 a^6 b c^6) e - (b^6 c^9 - 8 a b^4 c^{10} + 18 a^2 b^2 c^{11} - 8 a^3 c^{12}) \right) \cdot \sqrt{\left( (b^{14} c^2 - 12 a b^{12} c^3 + 56 a^2 b^{10} c^4 - 128 a^3 b^8 c^5 + 148 a^4 b^6 c^6 - 80 a^5 b^4 c^7 + 16 a^6 b^2 c^8) d^2 - 2 \right.} \\ & \cdot (b^{15} c - 13 a b^{13} c^2 + 67 a^2 b^{11} c^3 - 174 a^3 b^9 c^4 + 239 a^4 b^7 c^5 - 166 a^5 b^5 c^6 + 50 a^6 b^3 c^7 - 4 a^7 b c^8) d \cdot e \\ & \left. + (b^{16} - 14 a b^{14} c + 79 a^2 b^{12} c^2 - 230 a^3 b^{10} c^3 + 367 a^4 b^8 c^4 - 314 a^5 b^6 c^5 + 130 a^6 b^4 c^6 - 20 a^7 b^2 c^7 + a^8 c^8) e^2 \right) / (b^2 c^9 - 4 a c^{10}) \cdot \sqrt{\left( (b^8 c - 8 a b^6 c^2 + 20 a^2 b^4 c^3 - 16 a^3 b^2 c^4 + 2 a^4 c^5) d - (b^9 - 9 a b^7 c + 27 a^2 b^5 c^2 - 30 a^3 b^3 c^3 + 9 a^4 b c^4) e + (b^2 c^9 - 4 a c^{10}) \right)} \end{aligned}$$

$$\begin{aligned}
& a^9 - 4*a*c^{10}) * \text{sqrt}(((b^{14}*c^2 - 12*a*b^{12}*c^3 + 56*a^2*b^{10}*c^4 \\
& - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2 \\
& *c^8)*d^2 - 2*(b^{15}*c - 13*a*b^{13}*c^2 + 67*a^2*b^{11}*c^3 - 174*a^3 \\
& *b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4 \\
& *a^7*b*c^8)*d*e + (b^{16} - 14*a*b^{14}*c + 79*a^2*b^{12}*c^2 - 230*a^3 \\
& *b^{10}*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - \\
& 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^{18} - 4*a*c^{19}))/((b^2*c^9 \\
& - 4*a*c^{10})) - 4*((a^4*b^7*c - 6*a^5*b^5*c^2 + 10*a^6*b^3*c^3 - 4 \\
& *a^7*b*c^4)*d - (a^4*b^8 - 7*a^5*b^6*c + 15*a^6*b^4*c^2 - 10*a^7* \\
& b^2*c^3 + a^8*c^4)*e)*\text{sqrt}(e*x + d)) + 105*\text{sqrt}(2)*c^4*e^3*\text{sqrt}(( \\
& (b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5) \\
& *d - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4 \\
& *b*c^4)*e - (b^2*c^9 - 4*a*c^{10})*\text{sqrt}(((b^{14}*c^2 - 12*a*b^{12}*c^3 \\
& + 56*a^2*b^{10}*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4 \\
& *c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^{15}*c - 13*a*b^{13}*c^2 + 67*a^2* \\
& b^{11}*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + \\
& 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^{16} - 14*a*b^{14}*c + 79*a^2* \\
& b^{12}*c^2 - 230*a^3*b^{10}*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + \\
& 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^{18} - 4*a \\
& *c^{19}))/((b^2*c^9 - 4*a*c^{10}))*\log(\text{sqrt}(2)*((b^{12}*c - 12*a*b^{10}*c \\
& ^2 + 54*a^2*b^8*c^3 - 112*a^3*b^6*c^4 + 104*a^4*b^4*c^5 - 32*a^5* \\
& b^2*c^6)*d - (b^{13} - 13*a*b^{11}*c + 65*a^2*b^9*c^2 - 156*a^3*b^7*c \\
& ^3 + 181*a^4*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e + (b^6*c^9 \\
& - 8*a*b^4*c^{10} + 18*a^2*b^2*c^{11} - 8*a^3*c^{12})*\text{sqrt}(((b^{14}*c^2 - \\
& 12*a*b^{12}*c^3 + 56*a^2*b^{10}*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6* \\
& c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^{15}*c - 13*a*b^{13} \\
& *c^2 + 67*a^2*b^{11}*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166 \\
& *a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^{16} - 14*a*b \\
& ^{14}*c + 79*a^2*b^{12}*c^2 - 230*a^3*b^{10}*c^3 + 367*a^4*b^8*c^4 - 31 \\
& 4*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/ \\
& (b^2*c^{18} - 4*a*c^{19}))*\text{sqrt}(((b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 \\
& - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d - (b^9 - 9*a*b^7*c + 27*a^2*b^5 \\
& *c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e - (b^2*c^9 - 4*a*c^{10})*\text{sq} \\
& \text{rt}(((b^{14}*c^2 - 12*a*b^{12}*c^3 + 56*a^2*b^{10}*c^4 - 128*a^3*b^8*c^5 \\
& + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^ \\
& 15*c - 13*a*b^{13}*c^2 + 67*a^2*b^{11}*c^3 - 174*a^3*b^9*c^4 + 239*a^4 \\
& *b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + \\
& (b^{16} - 14*a*b^{14}*c + 79*a^2*b^{12}*c^2 - 230*a^3*b^{10}*c^3 + 367*a \\
& ^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + \\
& a^8*c^8)*e^2)/(b^2*c^{18} - 4*a*c^{19}))/((b^2*c^9 - 4*a*c^{10})) - 4* \\
& ((a^4*b^7*c - 6*a^5*b^5*c^2 + 10*a^6*b^3*c^3 - 4*a^7*b*c^4)*d - ( \\
& a^4*b^8 - 7*a^5*b^6*c + 15*a^6*b^4*c^2 - 10*a^7*b^2*c^3 + a^8*c^4) \\
& *e)*\text{sqrt}(e*x + d)) - 105*\text{sqrt}(2)*c^4*e^3*\text{sqrt}(((b^8*c - 8*a*b^6* \\
& c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d - (b^9 - 9*a \\
& *b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e - (b^2* \\
& c^9 - 4*a*c^{10})*\text{sqrt}(((b^{14}*c^2 - 12*a*b^{12}*c^3 + 56*a^2*b^{10}*c^4 \\
& - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2 \\
& *c^8)*d^2 - 2*(b^{15}*c - 13*a*b^{13}*c^2 + 67*a^2*b^{11}*c^3 - 174*a^3 \\
& *b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - \\
& 4*a^7*b*c^8)*d*e + (b^{16} - 14*a*b^{14}*c + 79*a^2*b^{12}*c^2 - 230*a^3 \\
& *b^{10}*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 \\
& - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^{18} - 4*a*c^{19}))/((b^2*c^9 \\
& - 4*a*c^{10}))*\log(-\text{sqrt}(2)*((b^{12}*c - 12*a*b^{10}*c^2 + 54*a^2*b^8* \\
& c^3 - 112*a^3*b^6*c^4 + 104*a^4*b^4*c^5 - 32*a^5*b^2*c^6)*d - (b^ \\
& 13 - 13*a*b^{11}*c + 65*a^2*b^9*c^2 - 156*a^3*b^7*c^3 + 181*a^4*b^5 \\
& *c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e + (b^6*c^9 - 8*a*b^4*c^{10} \\
& + 18*a^2*b^2*c^{11} - 8*a^3*c^{12})*\text{sqrt}(((b^{14}*c^2 - 12*a*b^{12}*c^3 + \\
& 56*a^2*b^{10}*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c^6 - 80*a^5*b^4 \\
& *c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^{15}*c - 13*a*b^{13}*c^2 + 67*a^2*b \\
& ^{11}*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 5 \\
& 0*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^{16} - 14*a*b^{14}*c + 79*a^2*b \\
& ^{12}*c^2 - 230*a^3*b^{10}*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + \\
& 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(b^2*c^{18} - 4*a \\
& *c^{19}))*\text{sqrt}(((b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2* \\
& c^4 + 2*a^4*c^5)*d - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b \\
& ^3*c^3 + 9*a^4*b*c^4)*e - (b^2*c^9 - 4*a*c^{10})*\text{sqrt}(((b^{14}*c^2 - \\
& 12*a*b^{12}*c^3 + 56*a^2*b^{10}*c^4 - 128*a^3*b^8*c^5 + 148*a^4*b^6*c \\
& ^6 - 80*a^5*b^4*c^7 + 16*a^6*b^2*c^8)*d^2 - 2*(b^{15}*c - 13*a*b^{13} \\
& *c^2 + 67*a^2*b^{11}*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166* \\
& a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e + (b^{16} - 14*a*b^ \\
& ^{14}*c + 79*a^2*b^{12}*c^2 - 230*a^3*b^{10}*c^3 + 367*a^4*b^8*c^4 - 314 \\
& *a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^2)/(
\end{aligned}$$

$$\frac{b^2 c^{18} - 4 a^2 c^{19}}{b^2 c^9 - 4 a^2 c^{10}} - 4 \left( (a^4 b^7 c - 6 a^5 b^5 c^2 + 10 a^6 b^3 c^3 - 4 a^7 b c^4) d - (a^4 b^8 - 7 a^5 b^6 c + 15 a^6 b^4 c^2 - 10 a^7 b^2 c^3 + a^8 c^4) e \right) \sqrt{e x + d} + 4 \left( 15 c^3 e^3 x^3 + 8 c^3 d^3 + 14 b c^2 d^2 e + 35 (b^2 c - a c^2) d e^2 - 105 (b^3 - 2 a b c) e^3 + 3 (c^3 d e^2 - 7 b c^2 e^3) x^2 - (4 c^3 d^2 e + 7 b c^2 d e^2 - 35 (b^2 c - a c^2) e^3) x \right) \sqrt{e x + d} / (c^4 e^3)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(e\*x+d)\*\*(1/2)/(c\*x\*\*2+b\*x+a),x)

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*x^4/(c\*x^2 + b\*x + a),x, algorithm="giac")

[Out] Timed out

$$3.526 \quad \int \frac{x^3 \sqrt{d+ex}}{a+bx+cx^2} dx$$

**Optimal.** Leaf size=326

$$\frac{2(b^2 - ac) \sqrt{d+ex}}{c^3} + \frac{\left(-\sqrt{b^2 - 4ac}(b^2 - ac) - 3abc + b^3\right) \sqrt{2cd - e(b - \sqrt{b^2 - 4ac})} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd - e(b - \sqrt{b^2 - 4ac})}}\right)}{\sqrt{2}c^{7/2}\sqrt{b^2 - 4ac}} - \frac{\left(\sqrt{b^2 - 4ac}(b^2 - ac) - 3abc + b^3\right) \sqrt{2cd - e(\sqrt{b^2 - 4ac} + b)} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd - e(\sqrt{b^2 - 4ac} + b)}}\right)}{\sqrt{2}c^{7/2}\sqrt{b^2 - 4ac}} - \frac{2(d+ex)^{3/2}(be+cd)}{3c^2e^2} + \frac{2(d+ex)^{5/2}}{5ce^2}$$

[Out]  $(2*(b^2 - a*c)*\text{Sqrt}[d + e*x])/c^3 - (2*(c*d + b*e)*(d + e*x)^{(3/2)})/(3*c^2*e^2) + (2*(d + e*x)^{(5/2)})/(5*c*e^2) + ((b^3 - 3*a*b*c - \text{Sqrt}[b^2 - 4*a*c]*(b^2 - a*c))*\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/(\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e])]) / (\text{Sqrt}[2]*c^{(7/2)}*\text{Sqrt}[b^2 - 4*a*c]) - ((b^3 - 3*a*b*c + \text{Sqrt}[b^2 - 4*a*c]*(b^2 - a*c))*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/(\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e])]) / (\text{Sqrt}[2]*c^{(7/2)}*\text{Sqrt}[b^2 - 4*a*c])$

**Rubi [A]** time = 14.1294, antiderivative size = 397, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{\sqrt{2} \left( -\frac{2a^2c^2e+4ab^2ce-3abc^2d+b^4(-e)+b^3cd}{\sqrt{b^2-4ac}} + 2abce - ac^2d + b^3(-e) + b^2cd \right) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}\right)}{c^{7/2}\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} - \frac{\sqrt{2} \left( -\frac{2a^2c^2e+4ab^2ce-3abc^2d+b^4(-e)+b^3cd}{\sqrt{b^2-4ac}} + 2abce - ac^2d + b^3(-e) + b^2cd \right) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}\right)}{c^{7/2}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} + \frac{2(b^2-ac)\sqrt{d+ex}}{c^3} - \frac{2(d+ex)^{3/2}(be+cd)}{3c^2e^2} + \frac{2(d+ex)^{5/2}}{5ce^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^3*\text{Sqrt}[d + e*x])/(a + b*x + c*x^2), x]$

[Out]  $(2*(b^2 - a*c)*\text{Sqrt}[d + e*x])/c^3 - (2*(c*d + b*e)*(d + e*x)^{(3/2)})/(3*c^2*e^2) + (2*(d + e*x)^{(5/2)})/(5*c*e^2) - (\text{Sqrt}[2]*(b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e - (b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/(\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e])]) / (c^{(7/2)}*\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]) - (\text{Sqrt}[2]*(b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e + (b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/(\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e])]) / (c^{(7/2)}*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e])$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**3*(e*x+d)**(1/2)/(c*x**2+b*x+a), x)`

[Out] Timed out

**Mathematica [A]** time = 0.876965, size = 466, normalized size = 1.43

$$\frac{2\sqrt{d+ex}(-5ce(3ae+b(d+ex))+15b^2e^2+c^2(-2d^2+dex+3e^2x^2))}{15c^3e^2} + \frac{\sqrt{2}\left(ac^2\left(d\sqrt{b^2-4ac}-2ae\right)+b^2c\left(4ae-d\sqrt{b^2-4ac}\right)-abc\left(2e\sqrt{b^2-4ac}+3cd\right)+b^3\left(e\sqrt{b^2-4ac}+cd\right)+b^4(-e)\right)\tanh^{-1}\left(\frac{c^{7/2}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}{c^{7/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{c^{7/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

Antiderivative was successfully verified.

[In] `Integrate[(x^3*Sqrt[d + e*x])/(a + b*x + c*x^2), x]`

[Out]  $(2\sqrt{d+ex}(15b^2e^2+c^2(-2d^2+dex+3e^2x^2))-5c^3e(3ae+b(d+ex)))/(15c^3e^2)+(\sqrt{2}(-b^4e+a^2c^2(\sqrt{b^2-4ac}d-2ae)+b^2c(-(\sqrt{b^2-4ac}d+4ae)+b^3(c^2d+\sqrt{b^2-4ac}e)-ab^2c(3cd+2\sqrt{b^2-4ac}e))\text{ArcTanh}[\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-b^2e+\sqrt{b^2-4ac}e}}])/(c^{7/2}\sqrt{b^2-4ac}c^2\sqrt{2cd+(-b+\sqrt{b^2-4ac}e)}+(\sqrt{2}(b^4e+a^2c^2(\sqrt{b^2-4ac}d+2ae)-b^2c(\sqrt{b^2-4ac}d+4ae)+4ae)+ab^2c(3cd-2\sqrt{b^2-4ac}e)+b^3(-cd+\sqrt{b^2-4ac}e))\text{ArcTanh}[\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-(b+\sqrt{b^2-4ac}e)}}])/(c^{7/2}\sqrt{b^2-4ac}c^2\sqrt{2cd-(b+\sqrt{b^2-4ac}e)}))$

**Maple [B]** time = 0.061, size = 1764, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(e*x+d)^(1/2)/(c*x^2+b*x+a), x)`

[Out]  $2/5*(e*x+d)^{5/2}/c/e^{2-2/3}/e/c^2*(e*x+d)^{3/2}*b-2/3/e^2/c*(e*x+d)^{3/2}*d-2/c^2*a*(e*x+d)^{1/2}+2/c^3*b^2*(e*x+d)^{1/2}-2*e^2/c/(-e^2*(4*a*c-b^2))^{1/2}*2^{1/2}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2}*\arctan(c*(e*x+d)^{1/2}*2^{1/2}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2})*a^2+4*e^2/c^2/(-e^2*(4*a*c-b^2))^{1/2}*2^{1/2}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2}*\arctan(c*(e*x+d)^{1/2}*2^{1/2}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2})*a*b^2-3*e/c/(-e^2*(4*a*c-b^2))^{1/2}*2^{1/2}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2}*\arctan(c*(e*x+d)^{1/2}*2^{1/2}/$



$$\begin{aligned} & ((b^2 e^{-2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2} a^2 b^2 d - e^{2/c^3} / (-e^{2(4ac - b^2)})^{1/2} \\ & \cdot 2^{1/2} / ((b^2 e^{-2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2} \arctan(c^2 (e^x + d)^{1/2} / ((b^2 e^{-2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2}) \\ & \cdot b^4 + e/c^2 / (-e^{2(4ac - b^2)})^{1/2} \cdot 2^{1/2} / ((b^2 e^{-2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2} \arctan(c^2 (e^x + d)^{1/2} / ((b^2 e^{-2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2}) \\ & \cdot b^3 d + 2e/c^2 \cdot 2^{1/2} / ((b^2 e^{-2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2} \arctan(c^2 (e^x + d)^{1/2} / ((b^2 e^{-2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2}) \\ & \cdot a^2 b - 1/c^2 \cdot 2^{1/2} / ((b^2 e^{-2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2} \arctan(c^2 (e^x + d)^{1/2} / ((b^2 e^{-2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2}) \\ & \cdot a^2 d - e/c^3 \cdot 2^{1/2} / ((b^2 e^{-2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2} \arctan(c^2 (e^x + d)^{1/2} / ((b^2 e^{-2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2}) \\ & \cdot b^3 + 1/c^2 \cdot 2^{1/2} / ((b^2 e^{-2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2} \arctan(c^2 (e^x + d)^{1/2} / ((b^2 e^{-2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2}) \\ & \cdot b^2 d - 2e^{2/c} / (-e^{2(4ac - b^2)})^{1/2} \cdot 2^{1/2} / ((-b^2 e^{2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2} \operatorname{arctanh}(c^2 (e^x + d)^{1/2} / ((-b^2 e^{2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2}) \\ & \cdot a^2 + 4e^{2/c^2} / (-e^{2(4ac - b^2)})^{1/2} \cdot 2^{1/2} / ((-b^2 e^{2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2} \operatorname{arctanh}(c^2 (e^x + d)^{1/2} / ((-b^2 e^{2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2}) \\ & \cdot a^2 + 4e^{2/c^2} / (-e^{2(4ac - b^2)})^{1/2} \cdot 2^{1/2} / ((-b^2 e^{2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2} \operatorname{arctanh}(c^2 (e^x + d)^{1/2} / ((-b^2 e^{2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2}) \\ & \cdot a^2 b^2 - 3e/c / (-e^{2(4ac - b^2)})^{1/2} \cdot 2^{1/2} / ((-b^2 e^{2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2} \operatorname{arctanh}(c^2 (e^x + d)^{1/2} / ((-b^2 e^{2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2}) \\ & \cdot a^2 b d - e^{2/c^3} / (-e^{2(4ac - b^2)})^{1/2} \cdot 2^{1/2} / ((-b^2 e^{2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2} \operatorname{arctanh}(c^2 (e^x + d)^{1/2} / ((-b^2 e^{2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2}) \\ & \cdot b^4 + e/c^2 / (-e^{2(4ac - b^2)})^{1/2} \cdot 2^{1/2} / ((-b^2 e^{2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2} \operatorname{arctanh}(c^2 (e^x + d)^{1/2} / ((-b^2 e^{2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2}) \\ & \cdot b^3 d - 2e/c^2 \cdot 2^{1/2} / ((-b^2 e^{2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2} \operatorname{arctanh}(c^2 (e^x + d)^{1/2} / ((-b^2 e^{2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2}) \\ & \cdot a^2 b + 1/c^2 \cdot 2^{1/2} / ((-b^2 e^{2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2} \operatorname{arctanh}(c^2 (e^x + d)^{1/2} / ((-b^2 e^{2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2}) \\ & \cdot a^2 d + e/c^3 \cdot 2^{1/2} / ((-b^2 e^{2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2} \operatorname{arctanh}(c^2 (e^x + d)^{1/2} / ((-b^2 e^{2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2}) \\ & \cdot b^3 - 1/c^2 \cdot 2^{1/2} / ((-b^2 e^{2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2} \operatorname{arctanh}(c^2 (e^x + d)^{1/2} / ((-b^2 e^{2c} d + (-e^{2(4ac - b^2)})^{1/2})^c)^{1/2}) \\ & \cdot b^2 d \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex + d} x^3}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*x^3/(c\*x^2 + b\*x + a), x, algorithm="maxima")

[Out] integrate(sqrt(e\*x + d)\*x^3/(c\*x^2 + b\*x + a), x)

**Fricas [A]** time = 0.534732, size = 5731, normalized size = 17.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*x^3/(c\*x^2 + b\*x + a), x, algorithm="fricas")

[Out] 1/30\*(15\*sqrt(2)\*c^3\*e^2\*sqrt(((b^6\*c - 6\*a\*b^4\*c^2 + 9\*a^2\*b^2\*c^3 - 2\*a^3\*c^4)\*d - (b^7 - 7\*a\*b^5\*c + 14\*a^2\*b^3\*c^2 - 7\*a^3\*b\*c^3)\*e + (b^2\*c^7 - 4\*a\*c^8)\*sqrt(((b^10\*c^2 - 8\*a\*b^8\*c^3 + 22\*a^2\*b^6\*c^4 - 24\*a^3\*b^4\*c^5 + 9\*a^4\*b^2\*c^6)\*d^2 - 2\*(b^11\*c - 9\*a\*b^9\*c^2 + 29\*a^2\*b^7\*c^3 - 40\*a^3\*b^5\*c^4 + 22\*a^4\*b^3\*c^5 - 3\*a



$$\begin{aligned}
& ^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^{11}*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 \\
& 3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e + (b^{12} - \\
& 10*a*b^{10}*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - \\
& 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^{14} - 4*a*c^{15}))/((b^2*c^7 - \\
& 4*a*c^8))*\log(-\sqrt{2}*((b^9*c - 9*a*b^7*c^2 + 27*a^2*b^5*c^3 - \\
& 31*a^3*b^3*c^4 + 12*a^4*b*c^5)*d - (b^{10} - 10*a*b^8*c + 35*a^2*b^6 \\
& 6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*e + (b^5*c^7 \\
& - 7*a*b^3*c^8 + 12*a^2*b*c^9))*\sqrt{((b^{10}*c^2 - 8*a*b^8*c^3 + 22 \\
& *a^2*b^6*c^4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^{11}*c - \\
& 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - \\
& 3*a^5*b*c^6)*d*e + (b^{12} - 10*a*b^{10}*c + 37*a^2*b^8*c^2 - 62*a^3* \\
& b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^{14} \\
& 14 - 4*a*c^{15}))*\sqrt{((b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a \\
& ^3*c^4)*d - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e - \\
& (b^2*c^7 - 4*a*c^8))*\sqrt{((b^{10}*c^2 - 8*a*b^8*c^3 + 22*a^2*b^6*c^4 \\
& 4 - 24*a^3*b^4*c^5 + 9*a^4*b^2*c^6)*d^2 - 2*(b^{11}*c - 9*a*b^9*c^2 \\
& + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6 \\
& ))*d*e + (b^{12} - 10*a*b^{10}*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 4 \\
& 6*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^2)/(b^2*c^{14} - 4*a*c^{15} \\
& 15}))/((b^2*c^7 - 4*a*c^8)) + 4*((a^3*b^5*c - 4*a^4*b^3*c^2 + 3*a^5 \\
& *b*c^3)*d - (a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*e) \\
& *\sqrt{e*x + d}) + 4*(3*c^2*e^2*x^2 - 2*c^2*d^2 - 5*b*c*d*e + 15*( \\
& b^2 - a*c)*e^2 + (c^2*d*e - 5*b*c*e^2)*x)*\sqrt{e*x + d))/(c^3*e^2 \\
& )
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(e\*x+d)\*\*(1/2)/(c\*x\*\*2+b\*x+a), x)

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*x^3/(c\*x^2 + b\*x + a), x, algorithm="giac")

[Out] Timed out

$$3.527 \quad \int \frac{x^2 \sqrt{d+ex}}{a+bx+cx^2} dx$$

**Optimal.** Leaf size=316

$$\frac{\sqrt{2} \left( -\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}} + ace + b^2(-e) + bcd \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} \right)}{c^{5/2} \sqrt{2cd-e(b-\sqrt{b^2-4ac})}} + \frac{\sqrt{2} \left( \frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}} + ace + b^2(-e) + bcd \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} \right)}{c^{5/2} \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} - \frac{2b\sqrt{d+ex}}{c^2} + \frac{2(d+ex)^{3/2}}{3ce}$$

[Out]  $(-2*b*\text{Sqrt}[d + e*x])/c^2 + (2*(d + e*x)^{(3/2)})/(3*c*e) + (\text{Sqrt}[2]*(b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]])/(c^{(5/2)}*\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]) + (\text{Sqrt}[2]*(b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]])/(c^{(5/2)}*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e])$

**Rubi [A]** time = 5.98178, antiderivative size = 316, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{\sqrt{2} \left( -\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}} + ace + b^2(-e) + bcd \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} \right)}{c^{5/2} \sqrt{2cd-e(b-\sqrt{b^2-4ac})}} + \frac{\sqrt{2} \left( \frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}} + ace + b^2(-e) + bcd \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} \right)}{c^{5/2} \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} - \frac{2b\sqrt{d+ex}}{c^2} + \frac{2(d+ex)^{3/2}}{3ce}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^2*\text{Sqrt}[d + e*x])/(a + b*x + c*x^2), x]$

[Out]  $(-2*b*\text{Sqrt}[d + e*x])/c^2 + (2*(d + e*x)^{(3/2)})/(3*c*e) + (\text{Sqrt}[2]*(b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]])/(c^{(5/2)}*\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]) + (\text{Sqrt}[2]*(b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]])/(c^{(5/2)}*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e])$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**2*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)`

[Out] Timed out

**Mathematica [A]** time = 0.683912, size = 375, normalized size = 1.19

$$\frac{\sqrt{2} \left( -b^2 \left( e\sqrt{b^2 - 4ac} + cd \right) + bc \left( d\sqrt{b^2 - 4ac} - 3ae \right) + ac \left( e\sqrt{b^2 - 4ac} + 2cd \right) + b^3e \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}} \right)}{c^{5/2}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} + \frac{\sqrt{2} \left( b^2 \left( e\sqrt{b^2 - 4ac} - cd \right) - bc \left( d\sqrt{b^2 - 4ac} + 3ae \right) + ac \left( 2cd - e\sqrt{b^2 - 4ac} \right) + b^3e \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} \right)}{c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} + \frac{2\sqrt{d+ex}(c(d+ex)-3be)}{3c^2e}$$

Antiderivative was successfully verified.

[In] `Integrate[(x^2*Sqrt[d + e*x])/(a + b*x + c*x^2),x]`

[Out] 
$$\frac{(2\sqrt{d+ex}(-3be+cd)+bc(d\sqrt{b^2-4ac}-3ae)+ac(e\sqrt{b^2-4ac}+2cd)+b^3e)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)+(b^2(e\sqrt{b^2-4ac}-cd)-bc(d\sqrt{b^2-4ac}+3ae)+ac(2cd-e\sqrt{b^2-4ac})+b^3e)\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}\right)+\frac{2\sqrt{d+ex}(c(d+ex)-3be)}{3c^2e}}{c^{5/2}\sqrt{b^2-4ac}\sqrt{e(\sqrt{b^2-4ac}-b)+2cd}+c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}$$

**Maple [B]** time = 0.051, size = 1329, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(e*x+d)^(1/2)/(c*x^2+b*x+a),x)`

[Out] 
$$\frac{2}{3} \frac{(e*x+d)^{3/2}}{c} \frac{e-2*b*(e*x+d)^{1/2}}{c^2-3*e^2/c} \frac{1}{(-e^2*(4*a*c-b^2))^{1/2}} \frac{2^{1/2}}{((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2}} \operatorname{arctan}\left(\frac{c*(e*x+d)^{1/2}}{((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2}}\right) + \frac{a*b+2*e}{(-e^2*(4*a*c-b^2))^{1/2}} \frac{2^{1/2}}{((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2}} \operatorname{arctan}\left(\frac{c*(e*x+d)^{1/2}}{((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2}}\right) + \frac{a*d+e^2/c^2}{(-e^2*(4*a*c-b^2))^{1/2}} \frac{2^{1/2}}{((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2}} \operatorname{arctan}\left(\frac{c*(e*x+d)^{1/2}}{((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2}}\right) + \frac{b^3-e/c}{(-e^2*(4*a*c-b^2))^{1/2}} \frac{2^{1/2}}{((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2}} \operatorname{arctan}\left(\frac{c*(e*x+d)^{1/2}}{((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2}}\right) + \frac{b^2*d-e/c^2}{(-e^2*(4*a*c-b^2))^{1/2}} \frac{2^{1/2}}{((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2}} \operatorname{arctan}\left(\frac{c*(e*x+d)^{1/2}}{((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2}}\right) + \frac{a+e/c^2}{(-e^2*(4*a*c-b^2))^{1/2}} \frac{2^{1/2}}{((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2}} \operatorname{arctan}\left(\frac{c*(e*x+d)^{1/2}}{((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2}}\right) + \frac{b^2-1/c^2}{(-e^2*(4*a*c-b^2))^{1/2}} \frac{2^{1/2}}{((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2}} \operatorname{arctan}\left(\frac{c*(e*x+d)^{1/2}}{((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2}}\right)$$

$$\begin{aligned} & (c-b^2)^{1/2})^{1/2})^c)^{1/2} \arctan(c*(e*x+d)^{1/2}*2^{1/2}/((b*e-2*c \\ & *d+(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2})^b*d-3*e^2/c/(-e^2*(4*a*c-b \\ & ^2))^{1/2}*2^{1/2}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2} \\ & )^* \operatorname{arctanh}(c*(e*x+d)^{1/2}*2^{1/2}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2)) \\ & ^{1/2})^c)^{1/2})^*a*b+2*e/(-e^2*(4*a*c-b^2))^{1/2}*2^{1/2}/((-b*e \\ & +2*c*d+(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2} \operatorname{arctanh}(c*(e*x+d)^{1/2} \\ & *2^{1/2}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2})^*a*d+e^2 \\ & /c^2/(-e^2*(4*a*c-b^2))^{1/2}*2^{1/2}/((-b*e+2*c*d+(-e^2*(4*a*c-b \\ & ^2))^{1/2})^c)^{1/2} \operatorname{arctanh}(c*(e*x+d)^{1/2}*2^{1/2}/((-b*e+2*c*d \\ & +(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2})^*b^3-e/c/(-e^2*(4*a*c-b^2))^{1/2} \\ & *2^{1/2}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2} \operatorname{arct} \\ & \operatorname{anh}(c*(e*x+d)^{1/2}*2^{1/2}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{1/2} \\ & )^c)^{1/2})^*b^2*d+e/c*2^{1/2}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{1/2} \\ & )^c)^{1/2} \operatorname{arctanh}(c*(e*x+d)^{1/2}*2^{1/2}/((-b*e+2*c*d+(-e^2*( \\ & 4*a*c-b^2))^{1/2})^c)^{1/2})^*a-e/c^2*2^{1/2}/((-b*e+2*c*d+(-e^2*( \\ & 4*a*c-b^2))^{1/2})^c)^{1/2} \operatorname{arctanh}(c*(e*x+d)^{1/2}*2^{1/2}/((-b* \\ & e+2*c*d+(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2})^*b^2+1/c*2^{1/2}/((-b* \\ & e+2*c*d+(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2} \operatorname{arctanh}(c*(e*x+d)^{1/2} \\ & )^*2^{1/2}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{1/2})^c)^{1/2})^*b*d \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+dx^2}}{cx^2+bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*x^2/(c\*x^2 + b\*x + a),x, algorithm="maxima")

[Out] integrate(sqrt(e\*x + d)\*x^2/(c\*x^2 + b\*x + a), x)

**Fricas [A]** time = 0.374641, size = 4004, normalized size = 12.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*x^2/(c\*x^2 + b\*x + a),x, algorithm="fricas")

[Out]  $\frac{1}{6}*(3*\sqrt{2})^*c^2*e*\sqrt{((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e + (b^2*c^5 - 4*a*c^6)*\sqrt{((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^{10} - 4*a*c^{11})))/(b^2*c^5 - 4*a*c^6)}*\log(\sqrt{2})*((b^6*c - 6*a*b^4*c^2 + 8*a^2*b^2*c^3)*d - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*\sqrt{((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^{10} - 4*a*c^{11})))*\sqrt{((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e + (b^2*c^5 - 4*a*c^6)*\sqrt{((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^{10} - 4*a*c^{11})))/(b^2*c^5 - 4*a*c^6)} - 4*((a^2*b^3*c - 2*a^3*b*c^2)*d - (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*e)*\sqrt{e*x + d}) - 3*\sqrt{2})^*c^2*e*\sqrt{((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e + (b^2*c^5 - 4*a*c^6)*\sqrt{((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^{10} - 4*a*c^{11})))/(b^2*c^5 - 4*a*c^6)}*\log(-\sqrt{2})*((b^6*c - 6*a*b^4*c^2 + 8*a^2*b^2*c^3)*d - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e - (b^4*c^5 -$

$$\begin{aligned}
& 6*a*b^2*c^6 + 8*a^2*c^7)*\sqrt{((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^{10} - 4*a*c^{11}))}\sqrt{((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e + (b^2*c^5 - 4*a*c^6)*\sqrt{((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^{10} - 4*a*c^{11}))})/(b^2*c^5 - 4*a*c^6)) - 4*((a^2*b^3*c - 2*a^3*b*c^2)*d - (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*e)*\sqrt{e*x + d}) + 3*\sqrt{2)*c^2*e*\sqrt{((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*\sqrt{((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^{10} - 4*a*c^{11}))})/(b^2*c^5 - 4*a*c^6)))*\log(\sqrt{2)*((b^6*c - 6*a*b^4*c^2 + 8*a^2*b^2*c^3)*d - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e + (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*\sqrt{((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^{10} - 4*a*c^{11}))})/(b^2*c^5 - 4*a*c^6)) - 4*((a^2*b^3*c - 2*a^3*b*c^2)*d - (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*e)*\sqrt{e*x + d}) - 3*\sqrt{2)*c^2*e*\sqrt{((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*\sqrt{((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^{10} - 4*a*c^{11}))})/(b^2*c^5 - 4*a*c^6)))*\log(-\sqrt{2)*((b^6*c - 6*a*b^4*c^2 + 8*a^2*b^2*c^3)*d - (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e + (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*\sqrt{((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^{10} - 4*a*c^{11}))})/(b^2*c^5 - 4*a*c^6)) - 4*((a^2*b^3*c - 2*a^3*b*c^2)*d - (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*e)*\sqrt{e*x + d}) + 4*(c*e*x + c*d - 3*b*e)*\sqrt{e*x + d})/(c^2*e)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*2\*(e\*x+d)\*\*(1/2)/(c\*x\*\*2+b\*x+a), x)

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*x^2/(c\*x^2 + b\*x + a), x, algorithm="giac")

[Out] Timed out



$$3.528 \quad \int \frac{x\sqrt{d+ex}}{a+bx+cx^2} dx$$

**Optimal.** Leaf size=287

$$\frac{\sqrt{2} \left( -\sqrt{b^2 - 4ac}(cd - be) + 2ace + b^2(-e) + bcd \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e}(b-\sqrt{b^2-4ac})}} \right)}{c^{3/2}\sqrt{b^2 - 4ac}\sqrt{2cd - e} \left( b - \sqrt{b^2 - 4ac} \right)}$$

$$- \frac{\sqrt{2} \left( \sqrt{b^2 - 4ac}(cd - be) + 2ace + b^2(-e) + bcd \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e}(\sqrt{b^2-4ac}+b)} \right)}{c^{3/2}\sqrt{b^2 - 4ac}\sqrt{2cd - e} \left( \sqrt{b^2 - 4ac} + b \right)} + \frac{2\sqrt{d+ex}}{c}$$

```
[Out] (2*Sqrt[d + e*x])/c + (Sqrt[2]*(b*c*d - b^2*e + 2*a*c*e - Sqrt[b^2 - 4*a*c]*(c*d - b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*(b*c*d - b^2*e + 2*a*c*e + Sqrt[b^2 - 4*a*c]*(c*d - b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

**Rubi [A]** time = 5.73249, antiderivative size = 287, normalized size of antiderivative = 1., number of rules used = 5, number of rules used = 4, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$\frac{\sqrt{2} \left( -\sqrt{b^2 - 4ac}(cd - be) + 2ace + b^2(-e) + bcd \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e}(b-\sqrt{b^2-4ac})}} \right)}{c^{3/2}\sqrt{b^2 - 4ac}\sqrt{2cd - e} \left( b - \sqrt{b^2 - 4ac} \right)}$$

$$- \frac{\sqrt{2} \left( \sqrt{b^2 - 4ac}(cd - be) + 2ace + b^2(-e) + bcd \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e}(\sqrt{b^2-4ac}+b)} \right)}{c^{3/2}\sqrt{b^2 - 4ac}\sqrt{2cd - e} \left( \sqrt{b^2 - 4ac} + b \right)} + \frac{2\sqrt{d+ex}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[d + e*x])/(a + b*x + c*x^2), x]
```

```
[Out] (2*Sqrt[d + e*x])/c + (Sqrt[2]*(b*c*d - b^2*e + 2*a*c*e - Sqrt[b^2 - 4*a*c]*(c*d - b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*(b*c*d - b^2*e + 2*a*c*e + Sqrt[b^2 - 4*a*c]*(c*d - b*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x*(e*x+d)**(1/2)/(c*x**2+b*x+a), x)`

[Out] Timed out

**Mathematica [A]** time = 0.542808, size = 301, normalized size = 1.05

$$\frac{\sqrt{2}(-cd\sqrt{b^2-4ac}+be\sqrt{b^2-4ac}+2ace+b^2(-e)+bcd) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{b^2-4ac}\sqrt{e(\sqrt{b^2-4ac}-b)+2cd}} + \frac{\sqrt{2}(-cd\sqrt{b^2-4ac}+be\sqrt{b^2-4ac}-2ace+b^2e-bcd) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{e}\sqrt{d+ex}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}$$

$c^{3/2}$

Antiderivative was successfully verified.

[In] `Integrate[(x*Sqrt[d + e*x])/(a + b*x + c*x^2), x]`

[Out]  $(2*\text{Sqrt}[c]*\text{Sqrt}[d + e*x] + (\text{Sqrt}[2]*(b*c*d - c*\text{Sqrt}[b^2 - 4*a*c])^* d - b^2*e + 2*a*c*e + b*\text{Sqrt}[b^2 - 4*a*c]*e)*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - b*e + \text{Sqrt}[b^2 - 4*a*c]*e]])/(\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c])*e]) + (\text{Sqrt}[2]*(-b*c*d) - c*\text{Sqrt}[b^2 - 4*a*c]*d + b^2*e - 2*a*c*e + b*\text{Sqrt}[b^2 - 4*a*c]*e)*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]])/(\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]))/c^{3/2}$

**Maple [B]** time = 0.043, size = 926, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(e*x+d)^(1/2)/(c*x^2+b*x+a), x)`

[Out]  $2*(e*x+d)^{1/2}/c+2/(-e^{1/2}*(4*a*c-b^2))^{1/2}*2^{1/2}/((b*e-2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2}*\arctan(c*(e*x+d)^{1/2}*2^{1/2})/((b*e-2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2})*a*e^{1/2}-1/c/(-e^{1/2}*(4*a*c-b^2))^{1/2}*2^{1/2}/((b*e-2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2}*\arctan(c*(e*x+d)^{1/2}*2^{1/2})/((b*e-2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2})*b^2*e^{1/2}+1/(-e^{1/2}*(4*a*c-b^2))^{1/2}*2^{1/2}/((b*e-2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2}*\arctan(c*(e*x+d)^{1/2}*2^{1/2})/((b*e-2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2})*b*d*e-1/c*2^{1/2}/((b*e-2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2})*\arctan(c*(e*x+d)^{1/2}*2^{1/2})/((b*e-2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2})*b*e+2^{1/2}/((b*e-2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2})*\arctan(c*(e*x+d)^{1/2}*2^{1/2})/((b*e-2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2})*d+2/(-e^{1/2}*(4*a*c-b^2))^{1/2}*2^{1/2}/((-b*e+2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2})*\operatorname{arctanh}(c*(e*x+d)^{1/2}*2^{1/2})/((-b*e+2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2})*a*e^{1/2}-1/c/(-e^{1/2}*(4*a*c-b^2))^{1/2}*2^{1/2}/((-b*e+2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2})*\operatorname{arctanh}(c*(e*x+d)^{1/2}*2^{1/2})/((-b*e+2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2})*b^2*e^{1/2}+1/(-e^{1/2}*(4*a*c-b^2))^{1/2}*2^{1/2}/((-b*e+2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2})*\operatorname{arctanh}(c*(e*x+d)^{1/2}*2^{1/2})/((-b*e+2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2})*b*d*e+1/c*2^{1/2}/((-b*e+2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2})*\operatorname{arctanh}(c*(e*x+d)^{1/2}*2^{1/2})/((-b*e+2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2})*b*e-2^{1/2}/((-b*e+2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2})*\operatorname{arctanh}(c*(e*x+d)^{1/2}*2^{1/2})/((-b*e+2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2})*d$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex + dx}}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*x/(c\*x^2 + b\*x + a),x, algorithm="maxima")

[Out] integrate(sqrt(e\*x + d)\*x/(c\*x^2 + b\*x + a), x)

**Fricas [A]** time = 0.314283, size = 2323, normalized size = 8.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*x/(c\*x^2 + b\*x + a),x, algorithm="fricas")

[Out]  $\frac{1}{2} \cdot (\sqrt{2}) \cdot c \cdot \sqrt{((b^2 \cdot c - 2 \cdot a \cdot c^2) \cdot d - (b^3 - 3 \cdot a \cdot b \cdot c) \cdot e + (b^4 \cdot c^3 - 4 \cdot a \cdot c^4) \cdot \sqrt{(b^2 \cdot c^2 \cdot d^2 - 2 \cdot (b^3 \cdot c - a \cdot b \cdot c^2) \cdot d \cdot e + (b^4 - 2 \cdot a \cdot b^2 \cdot c + a^2 \cdot c^2) \cdot e^2)}) / (b^2 \cdot c^3 - 4 \cdot a \cdot c^4))} \cdot \log(\sqrt{2}) \cdot ((b^3 \cdot c - 4 \cdot a \cdot b \cdot c^2) \cdot d - (b^4 - 5 \cdot a \cdot b^2 \cdot c + 4 \cdot a^2 \cdot c^2) \cdot e - (b^3 \cdot c^3 - 4 \cdot a \cdot b \cdot c^4) \cdot \sqrt{(b^2 \cdot c^2 \cdot d^2 - 2 \cdot (b^3 \cdot c - a \cdot b \cdot c^2) \cdot d \cdot e + (b^4 - 2 \cdot a \cdot b^2 \cdot c + a^2 \cdot c^2) \cdot e^2)}) / (b^2 \cdot c^6 - 4 \cdot a \cdot c^7))} + 4 \cdot (a \cdot b \cdot c \cdot d - (a \cdot b^2 - a^2 \cdot c) \cdot e) \cdot \sqrt{e \cdot x + d} - \sqrt{2} \cdot c \cdot \sqrt{((b^2 \cdot c - 2 \cdot a \cdot c^2) \cdot d - (b^3 - 3 \cdot a \cdot b \cdot c) \cdot e + (b^4 - 2 \cdot a \cdot b^2 \cdot c + a^2 \cdot c^2) \cdot e^2)}) / (b^2 \cdot c^3 - 4 \cdot a \cdot c^4)} \cdot \log(-\sqrt{2}) \cdot ((b^3 \cdot c - 4 \cdot a \cdot b \cdot c^2) \cdot d - (b^4 - 5 \cdot a \cdot b^2 \cdot c + 4 \cdot a^2 \cdot c^2) \cdot e - (b^3 \cdot c^3 - 4 \cdot a \cdot b \cdot c^4) \cdot \sqrt{(b^2 \cdot c^2 \cdot d^2 - 2 \cdot (b^3 \cdot c - a \cdot b \cdot c^2) \cdot d \cdot e + (b^4 - 2 \cdot a \cdot b^2 \cdot c + a^2 \cdot c^2) \cdot e^2)}) / (b^2 \cdot c^6 - 4 \cdot a \cdot c^7))} + 4 \cdot (a \cdot b \cdot c \cdot d - (a \cdot b^2 - a^2 \cdot c) \cdot e) \cdot \sqrt{e \cdot x + d} + \sqrt{2} \cdot c \cdot \sqrt{((b^2 \cdot c - 2 \cdot a \cdot c^2) \cdot d - (b^3 - 3 \cdot a \cdot b \cdot c) \cdot e - (b^2 \cdot c^3 - 4 \cdot a \cdot c^4) \cdot \sqrt{(b^2 \cdot c^2 \cdot d^2 - 2 \cdot (b^3 \cdot c - a \cdot b \cdot c^2) \cdot d \cdot e + (b^4 - 2 \cdot a \cdot b^2 \cdot c + a^2 \cdot c^2) \cdot e^2)}) / (b^2 \cdot c^6 - 4 \cdot a \cdot c^7))} / (b^2 \cdot c^3 - 4 \cdot a \cdot c^4)} + 4 \cdot (a \cdot b \cdot c \cdot d - (a \cdot b^2 - a^2 \cdot c) \cdot e) \cdot \sqrt{e \cdot x + d} - \sqrt{2} \cdot c \cdot \sqrt{((b^2 \cdot c - 2 \cdot a \cdot c^2) \cdot d - (b^3 - 3 \cdot a \cdot b \cdot c) \cdot e - (b^2 \cdot c^3 - 4 \cdot a \cdot c^4) \cdot \sqrt{(b^2 \cdot c^2 \cdot d^2 - 2 \cdot (b^3 \cdot c - a \cdot b \cdot c^2) \cdot d \cdot e + (b^4 - 2 \cdot a \cdot b^2 \cdot c + a^2 \cdot c^2) \cdot e^2)}) / (b^2 \cdot c^6 - 4 \cdot a \cdot c^7))} / (b^2 \cdot c^3 - 4 \cdot a \cdot c^4)} \cdot \log(\sqrt{2}) \cdot ((b^3 \cdot c - 4 \cdot a \cdot b \cdot c^2) \cdot d - (b^4 - 5 \cdot a \cdot b^2 \cdot c + 4 \cdot a^2 \cdot c^2) \cdot e + (b^3 \cdot c^3 - 4 \cdot a \cdot b \cdot c^4) \cdot \sqrt{(b^2 \cdot c^2 \cdot d^2 - 2 \cdot (b^3 \cdot c - a \cdot b \cdot c^2) \cdot d \cdot e + (b^4 - 2 \cdot a \cdot b^2 \cdot c + a^2 \cdot c^2) \cdot e^2)}) / (b^2 \cdot c^6 - 4 \cdot a \cdot c^7))} + 4 \cdot (a \cdot b \cdot c \cdot d - (a \cdot b^2 - a^2 \cdot c) \cdot e) \cdot \sqrt{e \cdot x + d} - \sqrt{2} \cdot c \cdot \sqrt{((b^2 \cdot c - 2 \cdot a \cdot c^2) \cdot d - (b^3 - 3 \cdot a \cdot b \cdot c) \cdot e - (b^2 \cdot c^3 - 4 \cdot a \cdot c^4) \cdot \sqrt{(b^2 \cdot c^2 \cdot d^2 - 2 \cdot (b^3 \cdot c - a \cdot b \cdot c^2) \cdot d \cdot e + (b^4 - 2 \cdot a \cdot b^2 \cdot c + a^2 \cdot c^2) \cdot e^2)}) / (b^2 \cdot c^6 - 4 \cdot a \cdot c^7))} / (b^2 \cdot c^3 - 4 \cdot a \cdot c^4)} + 4 \cdot (a \cdot b \cdot c \cdot d - (a \cdot b^2 - a^2 \cdot c) \cdot e) \cdot \sqrt{e \cdot x + d} + 4 \cdot \sqrt{2} \cdot c \cdot \sqrt{e \cdot x + d} / c$

**Sympy [A]** time = 138.83, size = 1435, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(e*x+d)**(1/2)/(c*x**2+b*x+a),x)`

[Out] 
$$-2*a*e**2*RootSum(_t**4*(256*a**3*c**2*e**6 - 128*a**2*b**2*c*e**6 - 256*a**2*b*c**2*d*e**5 + 256*a**2*c**3*d**2*e**4 + 16*a*b**4*e**6 + 128*a*b**3*c*d*e**5 - 128*a*b**2*c**2*d**2*e**4 - 16*b**5*d*e**5 + 16*b**4*c*d**2*e**4) + _t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*e**2) + c, Lambda(_t, _t*log(32*_t**3*a**2*b*e**5 - 64*_t**3*a**2*c*d*e**4 - 8*_t**3*a*b**3*e**5/c - 16*_t**3*a*b**2*d*e**4 + 96*_t**3*a*b*c*d**2*e**3 - 64*_t**3*a*c**2*d**3*e**2 + 8*_t**3*b**4*d*e**4/c - 24*_t**3*b**3*d**2*e**3 + 16*_t**3*b**2*c*d**3*e**2 + 4*_t*a*e**2 - 2*_t*b**2*e**2/c + 4*_t*b*d*e - 4*_t*c*d**2 + sqrt(d + e*x)))/c + 2*b*d*e*RootSum(_t**4*(256*a**3*c**2*e**6 - 128*a**2*b**2*c*e**6 - 256*a**2*b*c**2*d*e**5 + 256*a**2*c**3*d**2*e**4 + 16*a*b**4*e**6 + 128*a*b**3*c*d*e**5 - 128*a*b**2*c**2*d**2*e**4 - 16*b**5*d*e**5 + 16*b**4*c*d**2*e**4) + _t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*e**2) + c, Lambda(_t, _t*log(32*_t**3*a**2*b*e**5 - 64*_t**3*a**2*c*d*e**4 - 8*_t**3*a*b**3*e**5/c - 16*_t**3*a*b**2*d*e**4 + 96*_t**3*a*b*c*d**2*e**3 - 64*_t**3*a*c**2*d**3*e**2 + 8*_t**3*b**4*d*e**4/c - 24*_t**3*b**3*d**2*e**3 + 16*_t**3*b**2*c*d**3*e**2 + 4*_t*a*e**2 - 2*_t*b**2*e**2/c + 4*_t*b*d*e - 4*_t*c*d**2 + sqrt(d + e*x)))/c - 2*b*e*RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4 + 16*b**4*c*e**4) + _t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*e**2) + a*e**2 - b*d*e + c*d**2, Lambda(_t, _t*log(64*_t**3*a*c**2*e**2 - 16*_t**3*b**2*c*e**2 - 2*_t*b*e + 4*_t*c*d + sqrt(d + e*x)))/c - 2*d**2*RootSum(_t**4*(256*a**3*c**2*e**6 - 128*a**2*b**2*c*e**6 - 256*a**2*b*c**2*d*e**5 + 256*a**2*c**3*d**2*e**4 + 16*a*b**4*e**6 + 128*a*b**3*c*d*e**5 - 128*a*b**2*c**2*d**2*e**4 - 16*b**5*d*e**5 + 16*b**4*c*d**2*e**4) + _t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*e**2) + c, Lambda(_t, _t*log(32*_t**3*a**2*b*e**5 - 64*_t**3*a**2*c*d*e**4 - 8*_t**3*a*b**3*e**5/c - 16*_t**3*a*b**2*d*e**4 + 96*_t**3*a*b*c*d**2*e**3 - 64*_t**3*a*c**2*d**3*e**2 + 8*_t**3*b**4*d*e**4/c - 24*_t**3*b**3*d**2*e**3 + 16*_t**3*b**2*c*d**3*e**2 + 4*_t*a*e**2 - 2*_t*b**2*e**2/c + 4*_t*b*d*e - 4*_t*c*d**2 + sqrt(d + e*x))) + 2*d*RootSum(_t**4*(256*a**2*c**3*e**4 - 128*a*b**2*c**2*e**4 + 16*b**4*c*e**4) + _t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*e**2) + a*e**2 - b*d*e + c*d**2, Lambda(_t, _t*log(64*_t**3*a*c**2*e**2 - 16*_t**3*b**2*c*e**2 - 2*_t*b*e + 4*_t*c*d + sqrt(d + e*x)))) + 2*sqrt(d + e*x)/c$$

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**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(e*x + d)*x/(c*x^2 + b*x + a),x, algorithm="giac")`

[Out] Timed out

$$3.529 \quad \int \frac{\sqrt{d+ex}}{a+bx+cx^2} dx$$

Optimal. Leaf size=198

$$\frac{\sqrt{2}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}\right)}{\sqrt{c}\sqrt{b^2-4ac}} - \frac{\sqrt{2}\sqrt{2cd-e(b-\sqrt{b^2-4ac})} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}$$

[Out] -((Sqrt[2]\*Sqrt[2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e]\*ArcTanh[(Sqrt[2]\*Sqrt[c]\*Sqrt[d + e\*x])/Sqrt[2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e]]/(Sqrt[c]\*Sqrt[b^2 - 4\*a\*c])) + (Sqrt[2]\*Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e]\*ArcTanh[(Sqrt[2]\*Sqrt[c]\*Sqrt[d + e\*x])/Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e]]/(Sqrt[c]\*Sqrt[b^2 - 4\*a\*c]))

**Rubi [A]** time = 0.677084, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$\frac{\sqrt{2}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}\right)}{\sqrt{c}\sqrt{b^2-4ac}} - \frac{\sqrt{2}\sqrt{2cd-e(b-\sqrt{b^2-4ac})} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}\right)}{\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e\*x]/(a + b\*x + c\*x^2), x]

[Out] -((Sqrt[2]\*Sqrt[2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e]\*ArcTanh[(Sqrt[2]\*Sqrt[c]\*Sqrt[d + e\*x])/Sqrt[2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e]]/(Sqrt[c]\*Sqrt[b^2 - 4\*a\*c])) + (Sqrt[2]\*Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e]\*ArcTanh[(Sqrt[2]\*Sqrt[c]\*Sqrt[d + e\*x])/Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e]]/(Sqrt[c]\*Sqrt[b^2 - 4\*a\*c]))

**Rubi in Sympy [A]** time = 82.6199, size = 185, normalized size = 0.93

$$\frac{\sqrt{2}\sqrt{be-2cd-e\sqrt{-4ac+b^2}} \operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{be-2cd-e\sqrt{-4ac+b^2}}}\right)}{\sqrt{c}\sqrt{-4ac+b^2}} + \frac{\sqrt{2}\sqrt{be-2cd+e\sqrt{-4ac+b^2}} \operatorname{atan}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{be-2cd+e\sqrt{-4ac+b^2}}}\right)}{\sqrt{c}\sqrt{-4ac+b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*(1/2)/(c\*x\*\*2+b\*x+a), x)

[Out] -sqrt(2)\*sqrt(b\*e - 2\*c\*d - e\*sqrt(-4\*a\*c + b\*\*2))\*atan(sqrt(2)\*sqrt(c)\*sqrt(d + e\*x)/sqrt(b\*e - 2\*c\*d - e\*sqrt(-4\*a\*c + b\*\*2)))/(

$$\sqrt{c} \sqrt{-4ac + b^2} + \sqrt{2} \sqrt{be - 2cd + e \sqrt{-4ac + b^2}} \operatorname{atan}\left(\frac{\sqrt{2} \sqrt{c} \sqrt{d+ex}}{\sqrt{2cd - e(\sqrt{b^2 - 4ac} + b)}}\right) / \left(\sqrt{c} \sqrt{-4ac + b^2}\right)$$

**Mathematica [A]** time = 0.750661, size = 175, normalized size = 0.88

$$\frac{\sqrt{2} \left( \sqrt{2cd - e(\sqrt{b^2 - 4ac} + b)} \operatorname{tanh}^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd - e(\sqrt{b^2 - 4ac} + b)}}\right) - \sqrt{e\sqrt{b^2 - 4ac} - be + 2cd} \operatorname{tanh}^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2 - 4ac} - be + 2cd}}\right) \right)}{\sqrt{c}\sqrt{b^2 - 4ac}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e\*x]/(a + b\*x + c\*x^2), x]

[Out] (Sqrt[2]\*(-(Sqrt[2\*c\*d - b\*e + Sqrt[b^2 - 4\*a\*c]\*e)\*ArcTanh[(Sqrt[2]\*Sqrt[c]\*Sqrt[d + e\*x])/Sqrt[2\*c\*d - b\*e + Sqrt[b^2 - 4\*a\*c]\*e]]) + Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e]\*ArcTanh[(Sqrt[2]\*Sqrt[c]\*Sqrt[d + e\*x])/Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e]]))/Sqrt[c]\*Sqrt[b^2 - 4\*a\*c])

**Maple [B]** time = 0.032, size = 545, normalized size = 2.8

$$\begin{aligned} & e^2 \sqrt{2} b \operatorname{arctan}\left(c \sqrt{2} \sqrt{ex + d} \frac{1}{\sqrt{(be - 2cd + \sqrt{-e^2(4ac - b^2)})c}}\right) \frac{1}{\sqrt{-e^2(4ac - b^2)}} \frac{1}{\sqrt{(be - 2cd + \sqrt{-e^2(4ac - b^2)})c}} \\ & - 2 \frac{ce\sqrt{2}d}{\sqrt{-e^2(4ac - b^2)} \sqrt{(be - 2cd + \sqrt{-e^2(4ac - b^2)})c}} \operatorname{arctan}\left(\frac{c\sqrt{ex + d}\sqrt{2}}{\sqrt{(be - 2cd + \sqrt{-e^2(4ac - b^2)})c}}\right) \\ & + e\sqrt{2} \operatorname{arctan}\left(c \sqrt{2} \sqrt{ex + d} \frac{1}{\sqrt{(be - 2cd + \sqrt{-e^2(4ac - b^2)})c}}\right) \frac{1}{\sqrt{(be - 2cd + \sqrt{-e^2(4ac - b^2)})c}} \\ & + e^2 \sqrt{2} b \operatorname{Artanh}\left(c \sqrt{2} \sqrt{ex + d} \frac{1}{\sqrt{(-be + 2cd + \sqrt{-e^2(4ac - b^2)})c}}\right) \frac{1}{\sqrt{-e^2(4ac - b^2)}} \frac{1}{\sqrt{(-be + 2cd + \sqrt{-e^2(4ac - b^2)})c}} \\ & - 2 \frac{ce\sqrt{2}d}{\sqrt{-e^2(4ac - b^2)} \sqrt{(-be + 2cd + \sqrt{-e^2(4ac - b^2)})c}} \operatorname{Artanh}\left(\frac{c\sqrt{ex + d}\sqrt{2}}{\sqrt{(-be + 2cd + \sqrt{-e^2(4ac - b^2)})c}}\right) \\ & - e\sqrt{2} \operatorname{Artanh}\left(c \sqrt{2} \sqrt{ex + d} \frac{1}{\sqrt{(-be + 2cd + \sqrt{-e^2(4ac - b^2)})c}}\right) \frac{1}{\sqrt{(-be + 2cd + \sqrt{-e^2(4ac - b^2)})c}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(1/2)/(c\*x^2+b\*x+a), x)

[Out] e^2/(-e^2\*(4\*a\*c-b^2))^(1/2)\*2^(1/2)/((b\*e-2\*c\*d+(-e^2\*(4\*a\*c-b^2))^(1/2))\*c)^(1/2)\*arctan(c\*(e\*x+d)^(1/2)\*2^(1/2)/((b\*e-2\*c\*d+(-e^2\*(4\*a\*c-b^2))^(1/2))\*c)^(1/2))\*b-2\*c\*e/(-e^2\*(4\*a\*c-b^2))^(1/2)\*2^(1/2)/((b\*e-2\*c\*d+(-e^2\*(4\*a\*c-b^2))^(1/2))\*c)^(1/2)\*arctan(c\*

$$\begin{aligned} & (e^x+d)^{1/2} \cdot 2^{1/2} / ((b^2 e - 2^2 c^2 d + (-e^2 (4^2 a^2 c - b^2))^2)^{1/2}) \cdot c^{1/2} \\ & \cdot d + e^2 \cdot 2^{1/2} / ((b^2 e - 2^2 c^2 d + (-e^2 (4^2 a^2 c - b^2))^2)^{1/2}) \cdot c^{1/2} \cdot a \\ & \cdot \operatorname{rctan}(c \cdot (e^x+d)^{1/2} \cdot 2^{1/2} / ((b^2 e - 2^2 c^2 d + (-e^2 (4^2 a^2 c - b^2))^2)^{1/2}) \cdot c^{1/2}) \\ & + e^2 / (-e^2 (4^2 a^2 c - b^2))^2)^{1/2} \cdot 2^{1/2} / ((-b^2 e + 2^2 c^2 d + (-e^2 (4^2 a^2 c - b^2))^2)^{1/2}) \cdot c^{1/2} \\ & \cdot \operatorname{arctanh}(c \cdot (e^x+d)^{1/2} \cdot 2^{1/2} / ((-b^2 e + 2^2 c^2 d + (-e^2 (4^2 a^2 c - b^2))^2)^{1/2}) \cdot c^{1/2}) \\ & \cdot b - 2^2 c^2 e / (-e^2 (4^2 a^2 c - b^2))^2)^{1/2} \cdot 2^{1/2} / ((-b^2 e + 2^2 c^2 d + (-e^2 (4^2 a^2 c - b^2))^2)^{1/2}) \cdot c^{1/2} \\ & \cdot \operatorname{arctanh}(c \cdot (e^x+d)^{1/2} \cdot 2^{1/2} / ((-b^2 e + 2^2 c^2 d + (-e^2 (4^2 a^2 c - b^2))^2)^{1/2}) \cdot c^{1/2}) \\ & \cdot d - e^2 \cdot 2^{1/2} / ((-b^2 e + 2^2 c^2 d + (-e^2 (4^2 a^2 c - b^2))^2)^{1/2}) \cdot c^{1/2} \\ & \cdot \operatorname{arctanh}(c \cdot (e^x+d)^{1/2} \cdot 2^{1/2} / ((-b^2 e + 2^2 c^2 d + (-e^2 (4^2 a^2 c - b^2))^2)^{1/2}) \cdot c^{1/2}) \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{cx^2+bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/(c\*x^2 + b\*x + a), x, algorithm="maxima")

[Out] integrate(sqrt(e\*x + d)/(c\*x^2 + b\*x + a), x)

**Fricas [A]** time = 0.294236, size = 965, normalized size = 4.87

$$\begin{aligned} & -\frac{1}{2} \sqrt{2} \sqrt{\frac{2cd - be + (b^2c - 4ac^2) \sqrt{\frac{e^2}{b^2c^2 - 4ac^3}}}{b^2c - 4ac^2}} \log \left( \sqrt{2} (b^2c - 4ac^2) \sqrt{\frac{e^2}{b^2c^2 - 4ac^3}} \sqrt{\frac{2cd - be + (b^2c - 4ac^2) \sqrt{\frac{e^2}{b^2c^2 - 4ac^3}}}{b^2c - 4ac^2}} \right. \\ & \left. + 2 \sqrt{ex + de} \right) \\ & + \frac{1}{2} \sqrt{2} \sqrt{\frac{2cd - be + (b^2c - 4ac^2) \sqrt{\frac{e^2}{b^2c^2 - 4ac^3}}}{b^2c - 4ac^2}} \log \left( -\sqrt{2} (b^2c - 4ac^2) \sqrt{\frac{e^2}{b^2c^2 - 4ac^3}} \sqrt{\frac{2cd - be + (b^2c - 4ac^2) \sqrt{\frac{e^2}{b^2c^2 - 4ac^3}}}{b^2c - 4ac^2}} \right. \\ & \left. + 2 \sqrt{ex + de} \right) \\ & + \frac{1}{2} \sqrt{2} \sqrt{\frac{2cd - be - (b^2c - 4ac^2) \sqrt{\frac{e^2}{b^2c^2 - 4ac^3}}}{b^2c - 4ac^2}} \log \left( \sqrt{2} (b^2c - 4ac^2) \sqrt{\frac{e^2}{b^2c^2 - 4ac^3}} \sqrt{\frac{2cd - be - (b^2c - 4ac^2) \sqrt{\frac{e^2}{b^2c^2 - 4ac^3}}}{b^2c - 4ac^2}} \right. \\ & \left. + 2 \sqrt{ex + de} \right) \\ & - \frac{1}{2} \sqrt{2} \sqrt{\frac{2cd - be - (b^2c - 4ac^2) \sqrt{\frac{e^2}{b^2c^2 - 4ac^3}}}{b^2c - 4ac^2}} \log \left( -\sqrt{2} (b^2c - 4ac^2) \sqrt{\frac{e^2}{b^2c^2 - 4ac^3}} \sqrt{\frac{2cd - be - (b^2c - 4ac^2) \sqrt{\frac{e^2}{b^2c^2 - 4ac^3}}}{b^2c - 4ac^2}} \right. \\ & \left. + 2 \sqrt{ex + de} \right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/(c\*x^2 + b\*x + a),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/2*\sqrt{2}*\sqrt{(2*c*d - b*e + (b^2*c - 4*a*c^2)*\sqrt{e^2/(b^2*c^2 - 4*a*c^3)})}/(b^2*c - 4*a*c^2)*\log(\sqrt{2}*(b^2*c - 4*a*c^2) \\ & *\sqrt{e^2/(b^2*c^2 - 4*a*c^3)})*\sqrt{(2*c*d - b*e + (b^2*c - 4*a*c^2)*\sqrt{e^2/(b^2*c^2 - 4*a*c^3)})}/(b^2*c - 4*a*c^2) + 2*\sqrt{e*x + d}*e) \\ & + 1/2*\sqrt{2}*\sqrt{(2*c*d - b*e + (b^2*c - 4*a*c^2)*\sqrt{e^2/(b^2*c^2 - 4*a*c^3)})}/(b^2*c - 4*a*c^2)*\log(-\sqrt{2}*(b^2*c - 4*a*c^2) \\ & *\sqrt{e^2/(b^2*c^2 - 4*a*c^3)})*\sqrt{(2*c*d - b*e + (b^2*c - 4*a*c^2)*\sqrt{e^2/(b^2*c^2 - 4*a*c^3)})}/(b^2*c - 4*a*c^2) \\ & + 2*\sqrt{e*x + d}*e) + 1/2*\sqrt{2}*\sqrt{(2*c*d - b*e - (b^2*c - 4*a*c^2)*\sqrt{e^2/(b^2*c^2 - 4*a*c^3)})}/(b^2*c - 4*a*c^2)*\log(\sqrt{2}*(b^2*c - 4*a*c^2) \\ & *\sqrt{e^2/(b^2*c^2 - 4*a*c^3)})*\sqrt{(2*c*d - b*e - (b^2*c - 4*a*c^2)*\sqrt{e^2/(b^2*c^2 - 4*a*c^3)})}/(b^2*c - 4*a*c^2) \\ & + 2*\sqrt{e*x + d}*e) - 1/2*\sqrt{2}*\sqrt{(2*c*d - b*e - (b^2*c - 4*a*c^2)*\sqrt{e^2/(b^2*c^2 - 4*a*c^3)})}/(b^2*c - 4*a*c^2) \\ & *\log(-\sqrt{2}*(b^2*c - 4*a*c^2)*\sqrt{e^2/(b^2*c^2 - 4*a*c^3)})*\sqrt{(2*c*d - b*e - (b^2*c - 4*a*c^2)*\sqrt{e^2/(b^2*c^2 - 4*a*c^3)})}/(b^2*c - 4*a*c^2) \\ & + 2*\sqrt{e*x + d}*e) \end{aligned}$$

**Sympy [A]** time = 21.9676, size = 155, normalized size = 0.78

$$2e \operatorname{RootSum}\left(t^4 (256a^2c^3e^4 - 128ab^2c^2e^4 + 16b^4ce^4) + t^2 (-16abce^3 + 32ac^2de^2 + 4b^3e^3 - 8b^2cde^2) + ae^2 - bde + cd^2, (t + \dots)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(1/2)/(c\*x\*\*2+b\*x+a),x)

[Out] 
$$\begin{aligned} & 2*e*\operatorname{RootSum}(\_t^{**4}*(256*a^{**2}*c^{**3}*e^{**4} - 128*a*b^{**2}*c^{**2}*e^{**4} + 16 \\ & *b^{**4}*c*e^{**4}) + \_t^{**2}*(-16*a*b*c*e^{**3} + 32*a*c^{**2}*d*e^{**2} + 4*b^{**3} \\ & *e^{**3} - 8*b^{**2}*c*d*e^{**2}) + a*e^{**2} - b*d*e + c*d^{**2}, \operatorname{Lambda}(\_t, \_t \\ & *\log(64*\_t^{**3}*a*c^{**2}*e^{**2} - 16*\_t^{**3}*b^{**2}*c*e^{**2} - 2*\_t*b*e + 4*\_t \\ & *c*d + \sqrt{d + e*x}))) \end{aligned}$$

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/(c\*x^2 + b\*x + a),x, algorithm="giac")

[Out] Timed out



$$3.530 \quad \int \frac{\sqrt{d+ex}}{x(a+bx+cx^2)} dx$$

**Optimal.** Leaf size=275

$$\frac{\sqrt{2}\sqrt{c} \left( d\sqrt{b^2 - 4ac} - 2ae + bd \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} \right)}{a\sqrt{b^2 - 4ac}\sqrt{2cd - e} \left( b - \sqrt{b^2 - 4ac} \right)} - \frac{\sqrt{2}\sqrt{c} \left( -d\sqrt{b^2 - 4ac} - 2ae + bd \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} \right)}{a\sqrt{b^2 - 4ac}\sqrt{2cd - e} \left( \sqrt{b^2 - 4ac} + b \right)} - \frac{2\sqrt{d} \tanh^{-1} \left( \frac{\sqrt{d+ex}}{\sqrt{d}} \right)}{a}$$

[Out]  $(-2*\text{Sqrt}[d]*\text{ArcTanh}[\text{Sqrt}[d + e*x]/\text{Sqrt}[d]])/a + (\text{Sqrt}[2]*\text{Sqrt}[c] * (b*d + \text{Sqrt}[b^2 - 4*a*c]*d - 2*a*e)*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]])/(a*\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]) - (\text{Sqrt}[2]*\text{Sqrt}[c]*(b*d - \text{Sqrt}[b^2 - 4*a*c]*d - 2*a*e)*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]])/(a*\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e])$

**Rubi [A]** time = 2.23391, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{\sqrt{2}\sqrt{c} \left( d\sqrt{b^2 - 4ac} - 2ae + bd \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} \right)}{a\sqrt{b^2 - 4ac}\sqrt{2cd - e} \left( b - \sqrt{b^2 - 4ac} \right)} - \frac{\sqrt{2}\sqrt{c} \left( -d\sqrt{b^2 - 4ac} - 2ae + bd \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} \right)}{a\sqrt{b^2 - 4ac}\sqrt{2cd - e} \left( \sqrt{b^2 - 4ac} + b \right)} - \frac{2\sqrt{d} \tanh^{-1} \left( \frac{\sqrt{d+ex}}{\sqrt{d}} \right)}{a}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e\*x]/(x\*(a + b\*x + c\*x^2)), x]

[Out]  $(-2*\text{Sqrt}[d]*\text{ArcTanh}[\text{Sqrt}[d + e*x]/\text{Sqrt}[d]])/a + (\text{Sqrt}[2]*\text{Sqrt}[c] * (b*d + \text{Sqrt}[b^2 - 4*a*c]*d - 2*a*e)*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]])/(a*\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]) - (\text{Sqrt}[2]*\text{Sqrt}[c]*(b*d - \text{Sqrt}[b^2 - 4*a*c]*d - 2*a*e)*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]])/(a*\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e])$

**Rubi in Sympy [F-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*(1/2)/x/(c\*x\*\*2+b\*x+a), x)

[Out] Timed out

**Mathematica [A]** time = 0.432707, size = 267, normalized size = 0.97

$$\frac{\sqrt{2}\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right) + \sqrt{2}\sqrt{c}\left(d\sqrt{b^2-4ac}+2ae-bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right) - 2\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd} + \sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}} \cdot a$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e\*x]/(x\*(a + b\*x + c\*x^2)), x]

[Out]  $(-2*\text{Sqrt}[d]*\text{ArcTanh}[\text{Sqrt}[d + e*x]/\text{Sqrt}[d]] + (\text{Sqrt}[2]*\text{Sqrt}[c]*(b*d + \text{Sqrt}[b^2 - 4*a*c]*d - 2*a*e)*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - b*e + \text{Sqrt}[b^2 - 4*a*c]*e]])/(\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c])*e]) + (\text{Sqrt}[2]*\text{Sqrt}[c]*(-b*d + \text{Sqrt}[b^2 - 4*a*c]*d + 2*a*e)*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]])/(\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]))/a$

**Maple [B]** time = 0.045, size = 581, normalized size = 2.1

$$\begin{aligned} & -2 \frac{e^2 c \sqrt{2}}{\sqrt{-e^2(4ac - b^2)} \sqrt{(be - 2cd + \sqrt{-e^2(4ac - b^2)})c}} \arctan\left(\frac{c\sqrt{ex + d}\sqrt{2}}{\sqrt{(be - 2cd + \sqrt{-e^2(4ac - b^2)})c}}\right) \\ & + \frac{ce\sqrt{2}bd}{a} \arctan\left(c\sqrt{2}\sqrt{ex + d} \frac{1}{\sqrt{(be - 2cd + \sqrt{-e^2(4ac - b^2)})c}}\right) \frac{1}{\sqrt{-e^2(4ac - b^2)}} \frac{1}{\sqrt{(be - 2cd + \sqrt{-e^2(4ac - b^2)})c}} \\ & - \frac{c\sqrt{2}d}{a} \arctan\left(c\sqrt{2}\sqrt{ex + d} \frac{1}{\sqrt{(be - 2cd + \sqrt{-e^2(4ac - b^2)})c}}\right) \frac{1}{\sqrt{(be - 2cd + \sqrt{-e^2(4ac - b^2)})c}} \\ & - 2 \frac{e^2 c \sqrt{2}}{\sqrt{-e^2(4ac - b^2)} \sqrt{(-be + 2cd + \sqrt{-e^2(4ac - b^2)})c}} \text{Artanh}\left(\frac{c\sqrt{ex + d}\sqrt{2}}{\sqrt{(-be + 2cd + \sqrt{-e^2(4ac - b^2)})c}}\right) \\ & + \frac{ce\sqrt{2}bd}{a} \text{Artanh}\left(c\sqrt{2}\sqrt{ex + d} \frac{1}{\sqrt{(-be + 2cd + \sqrt{-e^2(4ac - b^2)})c}}\right) \frac{1}{\sqrt{-e^2(4ac - b^2)}} \frac{1}{\sqrt{(-be + 2cd + \sqrt{-e^2(4ac - b^2)})c}} \\ & + \frac{c\sqrt{2}d}{a} \text{Artanh}\left(c\sqrt{2}\sqrt{ex + d} \frac{1}{\sqrt{(-be + 2cd + \sqrt{-e^2(4ac - b^2)})c}}\right) \frac{1}{\sqrt{(-be + 2cd + \sqrt{-e^2(4ac - b^2)})c}} \\ & - 2 \frac{\sqrt{d}}{a} \text{Artanh}\left(\frac{\sqrt{ex + d}}{\sqrt{d}}\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(1/2)/x/(c\*x^2+b\*x+a), x)

```
[Out] -2*e^2*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))+e/a*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d-1/a*c*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d-2*e^2*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))+e/a*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*b*d+1/a*c*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))*c)^(1/2))*d-2*arctanh((e*x+d)^(1/2)/d^(1/2))*d^(1/2)/a
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)*x),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 0.64712, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)*x),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4*a^3*b*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)) - sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4*a^3*b*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)) + sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e - (a^2*b^3 - 4*a^3*b*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)) - sqrt(2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-sqrt(2)*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e - (a^2*b^3 - 4*a^3*b*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*sqrt(e*x + d)) + 2*sqrt(d)*log((e
```

$$\begin{aligned} & *x - 2*\sqrt{e*x + d}*\sqrt{d} + 2*d)/x)/a, 1/2*(\sqrt{2}*a*\sqrt{-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*\sqrt{(b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))}/(a^2*b^2 - 4*a^3*c))*\log(\sqrt{2}*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4*a^3*b*c)*\sqrt{(b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)})))*\sqrt{-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*\sqrt{(b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))}/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*\sqrt{e*x + d}) - \sqrt{2}*a*\sqrt{-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*\sqrt{(b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))}/(a^2*b^2 - 4*a^3*c))*\log(-\sqrt{2}*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e + (a^2*b^3 - 4*a^3*b*c)*\sqrt{(b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)}))*\sqrt{-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*\sqrt{(b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))}/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*\sqrt{e*x + d}) + \sqrt{2}*a*\sqrt{-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*\sqrt{(b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))}/(a^2*b^2 - 4*a^3*c))*\log(\sqrt{2}*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e - (a^2*b^3 - 4*a^3*b*c)*\sqrt{(b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)}))*\sqrt{-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*\sqrt{(b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))}/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*\sqrt{e*x + d}) - \sqrt{2}*a*\sqrt{-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*\sqrt{(b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))}/(a^2*b^2 - 4*a^3*c))*\log(-\sqrt{2}*((b^3 - 4*a*b*c)*d - (a*b^2 - 4*a^2*c)*e - (a^2*b^3 - 4*a^3*b*c)*\sqrt{(b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)}))*\sqrt{-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*\sqrt{(b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))}/(a^2*b^2 - 4*a^3*c)) - 4*(b*c*d - a*c*e)*\sqrt{e*x + d}) - 4*\sqrt{-d}*\arctan(\sqrt{e*x + d}/\sqrt{-d}))/a \end{aligned}$$

**Sympy [A]** time = 125.775, size = 1352, normalized size = 4.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(1/2)/x/(c\*x\*\*2+b\*x+a), x)

[Out]  $2*e**2*\text{RootSum}(\_t**4*(256*a**3*c**2*e**6 - 128*a**2*b**2*c*e**6 - 256*a**2*b*c**2*d*e**5 + 256*a**2*c**3*d**2*e**4 + 16*a*b**4*e**6 + 128*a*b**3*c*d*e**5 - 128*a*b**2*c**2*d**2*e**4 - 16*b**5*d*e**5 + 16*b**4*c*d**2*e**4) + \_t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*e**2) + c, \text{Lambda}(\_t, \_t*\log(32*\_t**3*a**2*b*e**5 - 64*\_t**3*a**2*c*d*e**4 - 8*\_t**3*a*b**3*e**5/c - 16*\_t**3*a*b**2*d*e**4 + 96*\_t**3*a*b*c*d**2*e**3 - 64*\_t**3*a*c**2*d**3*e**2 + 8*\_t**3*b**4*d*e**4/c - 24*\_t**3*b**3*d**2*e**3 + 16*\_t**3*b**2*c*d**3*e**2 + 4*\_t*a*e**2 - 2*\_t*b**2*e**2/c + 4*\_t*b*d*e - 4*\_t*c*d**2 + \sqrt{d + e*x}))) - 2*b*d*e*\text{RootSum}(\_t**4*(256*a**3*c**2*e**6 - 128*a**2*b**2*c*e**6 - 256*a**2*b*c**2*d*e**5 + 256*a**2*c**3*d**2*e**4 + 16*a*b**4*e**6 + 128*a*b**3*c*d*e**5 - 128*a*b**2*c**2*d**2*e**4 - 16*b**5*d*e**5 + 16*b**4*c*d**2*e**4) + \_t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*e**2) + c, \text{Lambda}(\_t, \_t*\log(32*\_t**3*a**2*b*e**5 - 64*\_t**3*a**2*c*d*e**4 - 8*\_t**3*a*b**3*e**5/c - 16*\_t**3*a*b**2*d*e**4 + 96*\_t**3*a*b*c*d**2*e**3 - 64*\_t**3*a*c**2*d**3*e**2 + 8*\_t**3*b**4*d*e**4/c - 24*\_t**3*b**3*d**2*e**3 + 16*\_t**3*b**2*c*d**3*e**2 + 4*\_t*a*e**2 - 2*\_t*b**2*e**2/c + 4*\_t*b*d*e - 4*\_t*c*d**2 + \sqrt{d + e*x}))) / a + 2*c*d**2*\text{RootSum}(\_t**4*(256*a**3*c**2*e**6 - 128*a**2*b**2*c*e**6 - 256*a**2*b*c**2*d*e**5 + 256*a**2*c**3*d**2*e**4 + 16*a*b**4*e**6 + 128*a*b**3*c*d*e**5 - 128*a*b**2*c**2*d**2*e**4 - 16*b**5*d*e**5 + 16*b**4*c*d**2*e**4) + \_t**2*(-16*a*b*c*e**3 + 32*a*c**2*d*e**2 + 4*b**3*e**3 - 8*b**2*c*d*e**2) + c, \text{Lambda}(\_t, \_t*\log(32*\_t**3*a**2*b*e**5 - 64*\_t**3*a**2*c*d*e**4 - 8*\_t**3*a*b**3*e**5/c - 16*\_t**3*a*b**2*d*e**4 + 96*\_t**3*a*b*c*d**2*e**3 - 64*\_t**3*a*c**2*d**3*e**2 + 8*\_t**3*b**4*d*e**4/c - 24*\_t**3*b**3*d**2*e**3 + 16*\_t**3*b**2*c*d**3*e**2 + 4*\_t$

$$\begin{aligned} & *a*e^{**2} - 2*_t*b^{**2}*e^{**2}/c + 4*_t*b*d*e - 4*_t*c*d^{**2} + \sqrt{d + e*x} \\ & ))/a - 2*c*d*\text{RootSum}(_t^{**4}(256*a^{**2}*c^{**3}*e^{**4} - 128*a*b^{**2}* \\ & c^{**2}*e^{**4} + 16*b^{**4}*c*e^{**4}) + _t^{**2}(-16*a*b*c*e^{**3} + 32*a*c^{**2}*d \\ & *e^{**2} + 4*b^{**3}*e^{**3} - 8*b^{**2}*c*d*e^{**2}) + a*e^{**2} - b*d*e + c*d^{**2}, \\ & \text{Lambda}(_t, _t*\log(64*_t^{**3}*a*c^{**2}*e^{**2} - 16*_t^{**3}*b^{**2}*c*e^{**2} - \\ & 2*_t*b*e + 4*_t*c*d + \sqrt{d + e*x}))/a - 2*d*\text{Piecewise}((-atan(\sqrt{d + e*x}/\sqrt{-d})/\sqrt{-d}, -d > 0), \\ & (\text{acoth}(\sqrt{d + e*x}/\sqrt{d})/\sqrt{d}, (-d < 0) \& (d < d + e*x)), (\text{atanh}(\sqrt{d + e*x}/\sqrt{d})/\sqrt{d}, \\ & (-d < 0) \& (d > d + e*x)))/a \end{aligned}$$

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/((c\*x^2 + b\*x + a)\*x),x, algorithm="giac")

[Out] Timed out

$$3.531 \quad \int \frac{\sqrt{d+ex}}{x^2(a+bx+cx^2)} dx$$

**Optimal.** Leaf size=368

$$\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}(bd-ae)-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} + \frac{\sqrt{2}\sqrt{c}\left(-b(d\sqrt{b^2-4ac}+ae)-a(2cd-e\sqrt{b^2-4ac})+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} + \frac{2(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}} - \frac{\sqrt{d+ex}}{ax} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a\sqrt{d}}$$

[Out]  $-(\text{Sqrt}[d + e*x]/(a*x)) + (e*\text{ArcTanh}[\text{Sqrt}[d + e*x]/\text{Sqrt}[d]])/(a*\text{Sqrt}[d]) + (2*(b*d - a*e)*\text{ArcTanh}[\text{Sqrt}[d + e*x]/\text{Sqrt}[d]])/(a^2*\text{Sqrt}[d]) - (\text{Sqrt}[2]*\text{Sqrt}[c]*(b^2*d - 2*a*c*d - a*b*e + \text{Sqrt}[b^2 - 4*a*c])*(b*d - a*e))*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/(\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e])]/(a^2*\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]) + (\text{Sqrt}[2]*\text{Sqrt}[c]*(b^2*d - b*(\text{Sqrt}[b^2 - 4*a*c]*d + a*e) - a*(2*c*d - \text{Sqrt}[b^2 - 4*a*c]*e))*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/(\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e])]/(a^2*\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e])$

**Rubi [A]** time = 6.8136, antiderivative size = 356, normalized size of antiderivative = 0.97, number of steps used = 9, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{\sqrt{2}\sqrt{c}\left(\sqrt{b^2-4ac}(bd-ae)-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} + \frac{\sqrt{2}\sqrt{c}\left(-\sqrt{b^2-4ac}(bd-ae)-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} + \frac{2(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}} - \frac{\sqrt{d+ex}}{ax} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a\sqrt{d}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[d + e*x]/(x^2*(a + b*x + c*x^2)), x]$

[Out]  $-(\text{Sqrt}[d + e*x]/(a*x)) + (e*\text{ArcTanh}[\text{Sqrt}[d + e*x]/\text{Sqrt}[d]])/(a*\text{Sqrt}[d]) + (2*(b*d - a*e)*\text{ArcTanh}[\text{Sqrt}[d + e*x]/\text{Sqrt}[d]])/(a^2*\text{Sqrt}[d]) - (\text{Sqrt}[2]*\text{Sqrt}[c]*(b^2*d - 2*a*c*d - a*b*e + \text{Sqrt}[b^2 - 4*a*c])*(b*d - a*e))*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/(\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e])]/(a^2*\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]) + (\text{Sqrt}[2]*\text{Sqrt}[c]*(b^2*d - 2*a*c*d - a*b*e - \text{Sqrt}[b^2 - 4*a*c])*(b*d - a*e))*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/(\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e])]/(a^2*\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e])$

---

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**(1/2)/x**2/(c*x**2+b*x+a), x)`

[Out] Timed out

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**Mathematica [A]** time = 0.69396, size = 340, normalized size = 0.92

$$\frac{\sqrt{2}\sqrt{c}\left(-bd\sqrt{b^2-4ac}+ae\sqrt{b^2-4ac}+abe+2acd+b^2(-d)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}-\frac{\sqrt{2}\sqrt{c}\left(bd\sqrt{b^2-4ac}-ae\sqrt{b^2-4ac}+abe+2acd+b^2(-d)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{2cd-e}}\right)}{\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}{a^2}$$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[d + e*x]/(x^2*(a + b*x + c*x^2)), x]`

[Out] 
$$\left(-\left(\frac{a\sqrt{d+ex}}{x}\right)+\frac{\left(2bd-a^2\right)\operatorname{ArcTanh}\left[\frac{\sqrt{d+ex}}{\sqrt{d}}\right]+\sqrt{2}e\sqrt{c}\sqrt{d+ex}+\left(2ac-d\right)\sqrt{d}}{\sqrt{d}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\left(2bd-a^2\right)\operatorname{ArcTanh}\left[\frac{\sqrt{d+ex}}{\sqrt{d}}\right]+\sqrt{2}e\sqrt{c}\sqrt{d+ex}+\left(2ac-d\right)\sqrt{d}}{\sqrt{d}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}}{a^2}\right)$$

---

**Maple [B]** time = 0.052, size = 999, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^(1/2)/x^2/(c*x^2+b*x+a), x)`

[Out] 
$$\frac{e^2/a^2c/(-e^2(4a^2c-b^2))^{1/2}2^{1/2}/((b^2e-2c^2d+(-e^2(4a^2c-b^2))^{1/2})c)^{1/2}\arctan(c(e^2x+d)^{1/2})2^{1/2}/((b^2e-2c^2d+(-e^2(4a^2c-b^2))^{1/2})c)^{1/2}+(-e^2(4a^2c-b^2))^{1/2})c)^{1/2}b+2e/a^2c^2/(-e^2(4a^2c-b^2))^{1/2}2^{1/2}/((b^2e-2c^2d+(-e^2(4a^2c-b^2))^{1/2})c)^{1/2}a\arctan(c(e^2x+d)^{1/2})2^{1/2}/((b^2e-2c^2d+(-e^2(4a^2c-b^2))^{1/2})c)^{1/2}+(-e^2(4a^2c-b^2))^{1/2})c)^{1/2}d-e/a^2c/(-e^2(4a^2c-b^2))^{1/2}2^{1/2}/((b^2e-2c^2d+(-e^2(4a^2c-b^2))^{1/2})c)^{1/2}+(-e^2(4a^2c-b^2))^{1/2})c)^{1/2}\arctan(c(e^2x+d)^{1/2})2^{1/2}/((b^2e-2c^2d+(-e^2(4a^2c-b^2))^{1/2})c)^{1/2}+(-e^2(4a^2c-b^2))^{1/2})c)^{1/2}b^2d-e/a^2c^2/(-e^2(4a^2c-b^2))^{1/2}2^{1/2}/((b^2e-2c^2d+(-e^2(4a^2c-b^2))^{1/2})c)^{1/2}\arctan(c(e^2x+d)^{1/2})2^{1/2}/((b^2e-2c^2d+(-e^2(4a^2c-b^2))^{1/2})c)^{1/2}+(-e^2(4a^2c-b^2))^{1/2})c)^{1/2}+1/a^2c^2/(-e^2(4a^2c-b^2))^{1/2}2^{1/2}/((b^2e-2c^2d+(-e^2(4a^2c-b^2))^{1/2})c)^{1/2}+(-e^2(4a^2c-b^2))^{1/2})c)^{1/2}\arctan(c(e^2x+d)^{1/2})2^{1/2}/((b^2e-2c^2d+(-e^2(4a^2c-b^2))^{1/2})c)^{1/2}+(-e^2(4a^2c-b^2))^{1/2})c)^{1/2}b^2d+e^2/a^2c/(-e^2(4a^2c-b^2))^{1/2}2^{1/2}/((-b^2e+2c^2d+(-e^2(4a^2c-b^2))^{1/2})c)^{1/2}\operatorname{arctanh}(c(e^2x+d)^{1/2})2^{1/2}/((-b^2e+2c^2d+(-e^2(4a^2c-b^2))^{1/2})c)^{1/2}+(-e^2(4a^2c-b^2))^{1/2})c)^{1/2}\operatorname{arctanh}(c(e^2x+d)^{1/2})2^{1/2}/((-b^2e+2c^2d+(-e^2(4a^2c-b^2))^{1/2})c)^{1/2}+(-e^2(4a^2c-b^2))^{1/2})c)^{1/2}d-e/a^2c/(-e^2(4a^2c-b^2))^{1/2}2^{1/2}/((-b^2e+2c^2d+(-e^2(4a^2c-b^2))^{1/2})c)^{1/2}+(-e^2(4a^2c-b^2))^{1/2})c)^{1/2}d-e/a^2c/(-e^2(4a^2c-b^2))^{1/2}2^{1/2}/((-b^2e+2c^2d+(-e^2(4a^2c-b^2))^{1/2})c)^{1/2}+(-e^2(4a^2c-b^2))^{1/2})c)^{1/2}}$$

$$\begin{aligned} & )^{1/2} \cdot 2^{1/2} / ((-b^*e+2^*c*d+(-e^{1/2} \cdot (4^*a^*c-b^2))^{1/2})^*c)^{1/2} \cdot a \\ & \operatorname{rctanh}(c^*(e^*x+d)^{1/2} \cdot 2^{1/2} / ((-b^*e+2^*c*d+(-e^{1/2} \cdot (4^*a^*c-b^2))^{1/2})^*c)^{1/2}) \cdot b^2 \cdot d + e/a^*c^2 \cdot 2^{1/2} / ((-b^*e+2^*c*d+(-e^{1/2} \cdot (4^*a^*c-b^2))^{1/2})^*c)^{1/2}) \cdot \operatorname{arctanh}(c^*(e^*x+d)^{1/2} \cdot 2^{1/2} / ((-b^*e+2^*c*d+(-e^{1/2} \cdot (4^*a^*c-b^2))^{1/2})^*c)^{1/2}) - 1/a^2 \cdot c^2 \cdot 2^{1/2} / ((-b^*e+2^*c*d+(-e^{1/2} \cdot (4^*a^*c-b^2))^{1/2})^*c)^{1/2}) \cdot \operatorname{arctanh}(c^*(e^*x+d)^{1/2} \cdot 2^{1/2} / ((-b^*e+2^*c*d+(-e^{1/2} \cdot (4^*a^*c-b^2))^{1/2})^*c)^{1/2}) \cdot b \cdot d - (e^*x+d)^{1/2} / a/x - e^* \operatorname{arctanh}((e^*x+d)^{1/2} / d^{1/2}) / a/d^{1/2} + 2/a^2 \cdot d^{1/2} \cdot \operatorname{arctanh}((e^*x+d)^{1/2} / d^{1/2}) \cdot b \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)*x^2), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 10.3774, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)*x^2), x, algorithm="fricas")`

[Out] 
$$\begin{aligned} & [1/2^*(\operatorname{sqrt}(2)^*a^2 \cdot \operatorname{sqrt}(d)^*x \cdot \operatorname{sqrt}((b^4 - 4^*a^*b^2 \cdot c + 2^*a^2 \cdot c^2)^*d \\ & - (a^*b^3 - 3^*a^2 \cdot b^*c)^*e + (a^4 \cdot b^2 - 4^*a^5 \cdot c)^* \operatorname{sqrt}((b^6 - 4^*a^*b^4 \cdot c + 4^*a^2 \cdot b^2 \cdot c^2)^*d^2 - 2^*(a^*b^5 - 3^*a^2 \cdot b^3 \cdot c + 2^*a^3 \cdot b^*c^2) \\ & \cdot d^*e + (a^2 \cdot b^4 - 2^*a^3 \cdot b^2 \cdot c + a^4 \cdot c^2)^*e^2) / (a^8 \cdot b^2 - 4^*a^9 \cdot c) \\ & )) / (a^4 \cdot b^2 - 4^*a^5 \cdot c)^* \log(\operatorname{sqrt}(2)^*((b^6 - 6^*a^*b^4 \cdot c + 8^*a^2 \cdot b^2 \cdot c^2)^*d - (a^*b^5 - 5^*a^2 \cdot b^3 \cdot c + 4^*a^3 \cdot b^*c^2)^*e - (a^4 \cdot b^4 - 6^*a^5 \cdot b^2 \cdot c + 8^*a^6 \cdot c^2)^* \operatorname{sqrt}((b^6 - 4^*a^*b^4 \cdot c + 4^*a^2 \cdot b^2 \cdot c^2)^*d^2 \\ & - 2^*(a^*b^5 - 3^*a^2 \cdot b^3 \cdot c + 2^*a^3 \cdot b^*c^2)^*d^*e + (a^2 \cdot b^4 - 2^*a^3 \cdot b^2 \cdot c + a^4 \cdot c^2)^*e^2) / (a^8 \cdot b^2 - 4^*a^9 \cdot c)) \cdot \operatorname{sqrt}((b^4 - 4^*a^*b^2 \cdot c \\ & + 2^*a^2 \cdot c^2)^*d - (a^*b^3 - 3^*a^2 \cdot b^*c)^*e + (a^4 \cdot b^2 - 4^*a^5 \cdot c)^* \operatorname{sqrt} \\ & ((b^6 - 4^*a^*b^4 \cdot c + 4^*a^2 \cdot b^2 \cdot c^2)^*d^2 - 2^*(a^*b^5 - 3^*a^2 \cdot b^3 \cdot c + 2^*a^3 \cdot b^*c^2)^*d^*e + (a^2 \cdot b^4 - 2^*a^3 \cdot b^2 \cdot c + a^4 \cdot c^2)^*e^2) / (a^8 \cdot b^2 - 4^*a^9 \cdot c)) / (a^4 \cdot b^2 - 4^*a^5 \cdot c) + 4^*((b^3 \cdot c^2 - 2^*a^*b^*c^3)^* \\ & d - (a^*b^2 \cdot c^2 - a^2 \cdot c^3)^*e)^* \operatorname{sqrt}(e^*x + d) - \operatorname{sqrt}(2)^*a^2 \cdot \operatorname{sqrt}(d) \\ & \cdot x \cdot \operatorname{sqrt}((b^4 - 4^*a^*b^2 \cdot c + 2^*a^2 \cdot c^2)^*d - (a^*b^3 - 3^*a^2 \cdot b^*c)^*e \\ & + (a^4 \cdot b^2 - 4^*a^5 \cdot c)^* \operatorname{sqrt}((b^6 - 4^*a^*b^4 \cdot c + 4^*a^2 \cdot b^2 \cdot c^2)^*d^2 \\ & - 2^*(a^*b^5 - 3^*a^2 \cdot b^3 \cdot c + 2^*a^3 \cdot b^*c^2)^*d^*e + (a^2 \cdot b^4 - 2^*a^3 \cdot b^2 \cdot c + a^4 \cdot c^2)^*e^2) / (a^8 \cdot b^2 - 4^*a^9 \cdot c)) / (a^4 \cdot b^2 - 4^*a^5 \cdot c)^* \log(-\operatorname{sqrt}(2)^*((b^6 - 6^*a^*b^4 \cdot c + 8^*a^2 \cdot b^2 \cdot c^2)^*d - (a^*b^5 - 5^*a^2 \cdot b^3 \cdot c + 4^*a^3 \cdot b^*c^2)^*e - (a^4 \cdot b^4 - 6^*a^5 \cdot b^2 \cdot c + 8^*a^6 \cdot c^2)^* \operatorname{sqrt} \\ & ((b^6 - 4^*a^*b^4 \cdot c + 4^*a^2 \cdot b^2 \cdot c^2)^*d^2 - 2^*(a^*b^5 - 3^*a^2 \cdot b^3 \cdot c + 2^*a^3 \cdot b^*c^2)^*d^*e + (a^2 \cdot b^4 - 2^*a^3 \cdot b^2 \cdot c + a^4 \cdot c^2)^*e^2) / (a^8 \cdot b^2 - 4^*a^9 \cdot c)) \cdot \operatorname{sqrt}((b^4 - 4^*a^*b^2 \cdot c + 2^*a^2 \cdot c^2)^*d - (a^*b^3 \\ & - 3^*a^2 \cdot b^*c)^*e + (a^4 \cdot b^2 - 4^*a^5 \cdot c)^* \operatorname{sqrt}((b^6 - 4^*a^*b^4 \cdot c + 4^*a^2 \cdot b^2 \cdot c^2)^*d^2 - 2^*(a^*b^5 - 3^*a^2 \cdot b^3 \cdot c + 2^*a^3 \cdot b^*c^2)^*d^*e + (a^2 \cdot b^4 - 2^*a^3 \cdot b^2 \cdot c + a^4 \cdot c^2)^*e^2) / (a^8 \cdot b^2 - 4^*a^9 \cdot c)) / (a^4 \cdot b^2 - 4^*a^5 \cdot c) + 4^*((b^3 \cdot c^2 - 2^*a^*b^*c^3)^* \\ & d - (a^*b^2 \cdot c^2 - a^2 \cdot c^3)^*e)^* \operatorname{sqrt}(e^*x + d) + \operatorname{sqrt}(2)^*a^2 \cdot \operatorname{sqrt}(d)^*x \cdot \operatorname{sqrt}((b^4 - 4^*a^*b^2 \cdot c \\ & + 2^*a^2 \cdot c^2)^*d - (a^*b^3 - 3^*a^2 \cdot b^*c)^*e - (a^4 \cdot b^2 - 4^*a^5 \cdot c)^* \operatorname{sqrt} \\ & ((b^6 - 4^*a^*b^4 \cdot c + 4^*a^2 \cdot b^2 \cdot c^2)^*d^2 - 2^*(a^*b^5 - 3^*a^2 \cdot b^3 \cdot c + 2^*a^3 \cdot b^*c^2)^*d^*e + (a^2 \cdot b^4 - 2^*a^3 \cdot b^2 \cdot c + a^4 \cdot c^2)^*e^2) / (a^8 \cdot b^2 - 4^*a^9 \cdot c)) / (a^4 \cdot b^2 - 4^*a^5 \cdot c)^* \log(\operatorname{sqrt}(2)^*((b^6 - 6^*a^*b^4 \cdot c + 8^*a^2 \cdot b^2 \cdot c^2)^*d - (a^*b^5 - 5^*a^2 \cdot b^3 \cdot c + 4^*a^3 \cdot b^*c^2)^*e + \\ & (a^4 \cdot b^4 - 6^*a^5 \cdot b^2 \cdot c + 8^*a^6 \cdot c^2)^* \operatorname{sqrt}((b^6 - 4^*a^*b^4 \cdot c + 4^*a^2 \cdot b^2 \cdot c^2)^*d^2 - 2^*(a^*b^5 - 3^*a^2 \cdot b^3 \cdot c + 2^*a^3 \cdot b^*c^2)^*d^*e + (a^2 \cdot b^4 - 2^*a^3 \cdot b^2 \cdot c + a^4 \cdot c^2)^*e^2) / (a^8 \cdot b^2 - 4^*a^9 \cdot c)) / (a^4 \cdot b^2 - 4^*a^5 \cdot c) \end{aligned}$$





$$4*a^5*c)*\sqrt{((b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^2 - 2*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d*e + (a^2*b^4 - 2*a^3*b^2*c + a^4*c^2)*e^2)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)} + 4*((b^3*c^2 - 2*a*b*c^3)*d - (a*b^2*c^2 - a^2*c^3)*e)*\sqrt{e*x + d)} - 2*(2*b*d - a*e)*x*\arctan(d/(\sqrt{e*x + d}*\sqrt{-d})) - 2*\sqrt{e*x + d} * a*\sqrt{-d})/(a^2*\sqrt{-d}*x)}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(1/2)/x\*\*2/(c\*x\*\*2+b\*x+a),x)

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/((c\*x^2 + b\*x + a)\*x^2),x, algorithm="giac")

[Out] Timed out

$$3.532 \quad \int \frac{\sqrt{d+ex}}{x^3(a+bx+cx^2)} dx$$

**Optimal.** Leaf size=531

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right) (-abe - acd + b^2d)}{a^3\sqrt{d}}$$

$$+ \frac{\sqrt{2}\sqrt{c} \left( b^2 \left( d\sqrt{b^2 - 4ac} - ae \right) - ab \left( e\sqrt{b^2 - 4ac} + 3cd \right) - ac \left( d\sqrt{b^2 - 4ac} - 2ae \right) + b^3d \right) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}\right)}{a^3\sqrt{b^2 - 4ac}\sqrt{2cd - e(b - \sqrt{b^2 - 4ac})}}$$

$$+ \frac{\sqrt{2}\sqrt{c} \left( -b^2 \left( d\sqrt{b^2 - 4ac} + ae \right) - ab \left( 3cd - e\sqrt{b^2 - 4ac} \right) + ac \left( d\sqrt{b^2 - 4ac} + 2ae \right) + b^3d \right) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}\right)}{a^3\sqrt{b^2 - 4ac}\sqrt{2cd - e(\sqrt{b^2 - 4ac} + b)}}$$

$$- \frac{e(bd - ae) \tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2d^{3/2}} + \frac{\sqrt{d+ex}(bd - ae)}{a^2dx} - \frac{3e^2 \tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4ad^{3/2}} - \frac{\sqrt{d+ex}}{2ax^2} + \frac{3e\sqrt{d+ex}}{4adx}$$

[Out] -Sqrt[d + e\*x]/(2\*a\*x^2) + (3\*e\*Sqrt[d + e\*x])/(4\*a\*d\*x) + ((b\*d - a\*e)\*Sqrt[d + e\*x])/(a^2\*d\*x) - (3\*e^2\*ArcTanh[Sqrt[d + e\*x]/Sqrt[d]])/(4\*a\*d^(3/2)) - (e\*(b\*d - a\*e)\*ArcTanh[Sqrt[d + e\*x]/Sqrt[d]])/(a^2\*d^(3/2)) - (2\*(b^2\*d - a\*c\*d - a\*b\*e)\*ArcTanh[Sqrt[d + e\*x]/Sqrt[d]])/(a^3\*Sqrt[d]) + (Sqrt[2]\*Sqrt[c]\*(b^3\*d - a\*c\*(Sqrt[b^2 - 4\*a\*c]\*d - 2\*a\*e) + b^2\*(Sqrt[b^2 - 4\*a\*c]\*d - a\*e) - a\*b\*(3\*c\*d + Sqrt[b^2 - 4\*a\*c]\*e))\*ArcTanh[(Sqrt[2]\*Sqrt[c]\*Sqrt[d + e\*x])/Sqrt[2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c]\*e)])/ (a^3\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c]\*e)] - (Sqrt[2]\*Sqrt[c]\*(b^3\*d - b^2\*(Sqrt[b^2 - 4\*a\*c]\*d + a\*e) + a\*c\*(Sqrt[b^2 - 4\*a\*c]\*d + 2\*a\*e) - a\*b\*(3\*c\*d - Sqrt[b^2 - 4\*a\*c]\*e))\*ArcTanh[(Sqrt[2]\*Sqrt[c]\*Sqrt[d + e\*x])/Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c]\*e)])/ (a^3\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c]\*e)]

**Rubi [A]** time = 7.12883, antiderivative size = 531, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right) (-abe - acd + b^2d)}{a^3\sqrt{d}}$$

$$+ \frac{\sqrt{2}\sqrt{c} \left( b^2 \left( d\sqrt{b^2 - 4ac} - ae \right) - ab \left( e\sqrt{b^2 - 4ac} + 3cd \right) - ac \left( d\sqrt{b^2 - 4ac} - 2ae \right) + b^3d \right) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}\right)}{a^3\sqrt{b^2 - 4ac}\sqrt{2cd - e(b - \sqrt{b^2 - 4ac})}}$$

$$+ \frac{\sqrt{2}\sqrt{c} \left( -b^2 \left( d\sqrt{b^2 - 4ac} + ae \right) - ab \left( 3cd - e\sqrt{b^2 - 4ac} \right) + ac \left( d\sqrt{b^2 - 4ac} + 2ae \right) + b^3d \right) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}\right)}{a^3\sqrt{b^2 - 4ac}\sqrt{2cd - e(\sqrt{b^2 - 4ac} + b)}}$$

$$- \frac{e(bd - ae) \tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2d^{3/2}} + \frac{\sqrt{d+ex}(bd - ae)}{a^2dx} - \frac{3e^2 \tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4ad^{3/2}} - \frac{\sqrt{d+ex}}{2ax^2} + \frac{3e\sqrt{d+ex}}{4adx}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e\*x]/(x^3\*(a + b\*x + c\*x^2)), x]

```
[Out] -Sqrt[d + e*x]/(2*a*x^2) + (3*e*Sqrt[d + e*x])/(4*a*d*x) + ((b*d
- a*e)*Sqrt[d + e*x])/(a^2*d*x) - (3*e^2*ArcTanh[Sqrt[d + e*x]/Sqr
rt[d]])/(4*a*d^(3/2)) - (e*(b*d - a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt
[d]])/(a^2*d^(3/2)) - (2*(b^2*d - a*c*d - a*b*e)*ArcTanh[Sqrt[d +
e*x]/Sqrt[d]])/(a^3*Sqrt[d]) + (Sqrt[2]*Sqrt[c]*(b^3*d - a*c*(Sqr
rt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(Sqrt[b^2 - 4*a*c]*d - a*e) - a*
b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
+ e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a^3*Sqrt[b^2 -
4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*Sqrt[
c]*(b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a
*c]*d + 2*a*e) - a*b*(3*c*d - Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt
[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
]])/(a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
])
```

**Rubi in Sympy [F-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((e*x+d)**(1/2)/x**3/(c*x**2+b*x+a), x)
```

[Out] Timed out

**Mathematica [A]** time = 1.39466, size = 435, normalized size = 0.82

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(4abde+a(ae^2+8cd^2)-8b^2d^2)}{d^{3/2}} - \frac{4\sqrt{2}\sqrt{c}\left(b^2(ae-d\sqrt{b^2-4ac})+ab\left(e\sqrt{b^2-4ac}+3cd\right)+ac\left(d\sqrt{b^2-4ac}-2ae\right)+b^3(-d)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}$$

$4a^3$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x]/(x^3*(a + b*x + c*x^2)), x]
```

```
[Out] ((a*Sqrt[d + e*x]*(4*b*d*x - a*(2*d + e*x)))/(d*x^2) + ((-8*b^2*d
^2 + 4*a*b*d*e + a*(8*c*d^2 + a*e^2))*ArcTanh[Sqrt[d + e*x]/Sqrt[
d]])/d^(3/2) - (4*Sqrt[2]*Sqrt[c]*(-(b^3*d) + a*c*(Sqrt[b^2 - 4*a
*c]*d - 2*a*e) + b^2*(-(Sqrt[b^2 - 4*a*c]*d) + a*e) + a*b*(3*c*d
+ Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/S
qrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[
2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) - (4*Sqrt[2]*Sqrt[c]*(b^3*d
- b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c]*d + 2*
a*e) + a*b*(-3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[
c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt
[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(4*a^3)
```

**Maple [B]** time = 0.057, size = 1486, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(1/2)/x^3/(c*x^2+b*x+a), x)
```

```
[Out] 2*e^2/a*c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2))-e^2/a^2*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2))*b^2-3*e/a^2*c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2))*b*d+e/a^3*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2))*b^3*d+e/a^2*c^2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2))*b+1/a^2*c^2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2))*d-1/a^3*c^2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2)*arctan(c*(e*x+d)^(1/2)*2^(1/2)/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2))*b^2*d+2*e^2/a*c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2))-e^2/a^2*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2))*b^2-3*e/a^2*c^2/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2))*b*d+e/a^3*c/(-e^2*(4*a*c-b^2))^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2))*b^3*d-e/a^2*c^2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2))*b-1/a^2*c^2*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2))*d+1/a^3*c^2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2)*arctanh(c*(e*x+d)^(1/2)*2^(1/2)/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^(1/2))^c)^(1/2))*b^2*d-1/4/a/x^2/d*(e*x+d)^(3/2)+1/e/a^2/x^2*(e*x+d)^(3/2)*b-1/e/a^2/x^2*(e*x+d)^(1/2)*b*d-1/4*(e*x+d)^(1/2)/a/x^2+1/4*e^2*arctanh((e*x+d)^(1/2)/d^(1/2))/a/d^(3/2)+e/a^2/d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2))*b+2/a^2*d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2))*c-2/a^3*d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2))*b^2
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)*x^3),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 162.592, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x + d)/((c*x^2 + b*x + a)*x^3),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(2)*a^3*d^(3/2)*x^2*sqrt((b^6 - 6*a*b^4*c + 9*a^2*b^4*2*c^2 - 2*a^3*c^3)*d - (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e + (a^6*b^2 - 4*a^7*c)*sqrt((b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3
```

$$\begin{aligned}
& *b^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b \\
& ^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^{12}*b^2 - \\
& 4*a^{13}*c))/(a^6*b^2 - 4*a^7*c))*\log(\sqrt{2}*((b^9 - 9*a*b^7*c + \\
& 27*a^2*b^5*c^2 - 31*a^3*b^3*c^3 + 12*a^4*b*c^4)*d - (a*b^8 - 8*a \\
& ^2*b^6*c + 20*a^3*b^4*c^2 - 17*a^4*b^2*c^3 + 4*a^5*c^4)*e - (a^6* \\
& b^5 - 7*a^7*b^3*c + 12*a^8*b*c^2)*\sqrt{((b^{10} - 8*a*b^8*c + 22*a^2 \\
& ^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2 \\
& ^2*b^7*c + 16*a^3*b^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a \\
& ^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)* \\
& e^2)/(a^{12}*b^2 - 4*a^{13}*c)))*\sqrt{((b^6 - 6*a*b^4*c + 9*a^2*b^2*c \\
& ^2 - 2*a^3*c^3)*d - (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e + (a^6* \\
& b^2 - 4*a^7*c)*\sqrt{((b^{10} - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3* \\
& b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5 \\
& ^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6* \\
& c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^{12}*b^2 - 4* \\
& a^{13}*c)))/(a^6*b^2 - 4*a^7*c)) - 4*((b^5*c^3 - 4*a*b^3*c^4 + 3*a^2 \\
& ^2*b*c^5)*d - (a*b^4*c^3 - 3*a^2*b^2*c^4 + a^3*c^5)*e)*\sqrt{e*x + \\
& d)} - 4*\sqrt{2}*a^3*d^{(3/2)}*x^2*\sqrt{((b^6 - 6*a*b^4*c + 9*a^2*b^2 \\
& ^2*c^2 - 2*a^3*c^3)*d - (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e + (a \\
& ^6*b^2 - 4*a^7*c)*\sqrt{((b^{10} - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3 \\
& ^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3* \\
& ^3*b^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6 \\
& ^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^{12}*b^2 - 4* \\
& a^{13}*c)))/(a^6*b^2 - 4*a^7*c))*\log(-\sqrt{2}*((b^9 - 9*a*b^7*c + \\
& 27*a^2*b^5*c^2 - 31*a^3*b^3*c^3 + 12*a^4*b*c^4)*d - (a*b^8 - 8* \\
& a^2*b^6*c + 20*a^3*b^4*c^2 - 17*a^4*b^2*c^3 + 4*a^5*c^4)*e - (a^6* \\
& ^6*b^5 - 7*a^7*b^3*c + 12*a^8*b*c^2)*\sqrt{((b^{10} - 8*a*b^8*c + 22*a^2 \\
& ^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2 \\
& ^2*b^7*c + 16*a^3*b^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + ( \\
& a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4) \\
& ^*e^2)/(a^{12}*b^2 - 4*a^{13}*c)))*\sqrt{((b^6 - 6*a*b^4*c + 9*a^2*b^2* \\
& ^2*c^2 - 2*a^3*c^3)*d - (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e + (a^6 \\
& ^6*b^2 - 4*a^7*c)*\sqrt{((b^{10} - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3 \\
& ^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5 \\
& ^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6 \\
& ^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^{12}*b^2 - 4 \\
& ^4*a^{13}*c)))/(a^6*b^2 - 4*a^7*c)) - 4*((b^5*c^3 - 4*a*b^3*c^4 + 3*a^2 \\
& ^2*b*c^5)*d - (a*b^4*c^3 - 3*a^2*b^2*c^4 + a^3*c^5)*e)*\sqrt{e*x + \\
& d)} + 4*\sqrt{2}*a^3*d^{(3/2)}*x^2*\sqrt{((b^6 - 6*a*b^4*c + 9*a^2*b^2 \\
& ^2*c^2 - 2*a^3*c^3)*d - (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e - ( \\
& a^6*b^2 - 4*a^7*c)*\sqrt{((b^{10} - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24* \\
& ^4*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3 \\
& ^3*b^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3* \\
& ^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^{12}*b^2 - 4 \\
& ^4*a^{13}*c)))/(a^6*b^2 - 4*a^7*c))*\log(\sqrt{2}*((b^9 - 9*a*b^7*c + \\
& 27*a^2*b^5*c^2 - 31*a^3*b^3*c^3 + 12*a^4*b*c^4)*d - (a*b^8 - 8* \\
& a^2*b^6*c + 20*a^3*b^4*c^2 - 17*a^4*b^2*c^3 + 4*a^5*c^4)*e + (a^6* \\
& ^6*b^5 - 7*a^7*b^3*c + 12*a^8*b*c^2)*\sqrt{((b^{10} - 8*a*b^8*c + 22*a^2 \\
& ^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2 \\
& ^2*b^7*c + 16*a^3*b^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + ( \\
& a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4) \\
& ^*e^2)/(a^{12}*b^2 - 4*a^{13}*c)))*\sqrt{((b^6 - 6*a*b^4*c + 9*a^2*b^2* \\
& ^2*c^2 - 2*a^3*c^3)*d - (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e - (a^6 \\
& ^6*b^2 - 4*a^7*c)*\sqrt{((b^{10} - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3 \\
& ^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3*b^5 \\
& ^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3*b^6 \\
& ^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^{12}*b^2 - 4 \\
& ^4*a^{13}*c)))/(a^6*b^2 - 4*a^7*c)) - 4*((b^5*c^3 - 4*a*b^3*c^4 + 3*a^2 \\
& ^2*b*c^5)*d - (a*b^4*c^3 - 3*a^2*b^2*c^4 + a^3*c^5)*e)*\sqrt{e*x + \\
& d)} - 4*\sqrt{2}*a^3*d^{(3/2)}*x^2*\sqrt{((b^6 - 6*a*b^4*c + 9*a^2*b^2 \\
& ^2*c^2 - 2*a^3*c^3)*d - (a*b^5 - 5*a^2*b^3*c + 5*a^3*b*c^2)*e - ( \\
& a^6*b^2 - 4*a^7*c)*\sqrt{((b^{10} - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24* \\
& ^4*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7*a^2*b^7*c + 16*a^3 \\
& ^3*b^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + (a^2*b^8 - 6*a^3* \\
& ^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)*e^2)/(a^{12}*b^2 - 4 \\
& ^4*a^{13}*c)))/(a^6*b^2 - 4*a^7*c))*\log(-\sqrt{2}*((b^9 - 9*a*b^7*c + \\
& 27*a^2*b^5*c^2 - 31*a^3*b^3*c^3 + 12*a^4*b*c^4)*d - (a*b^8 - 8* \\
& a^2*b^6*c + 20*a^3*b^4*c^2 - 17*a^4*b^2*c^3 + 4*a^5*c^4)*e + (a^6 \\
& ^6*b^5 - 7*a^7*b^3*c + 12*a^8*b*c^2)*\sqrt{((b^{10} - 8*a*b^8*c + 22* \\
& ^2*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*d^2 - 2*(a*b^9 - 7* \\
& ^7*a^2*b^7*c + 16*a^3*b^5*c^2 - 13*a^4*b^3*c^3 + 3*a^5*b*c^4)*d*e + \\
& (a^2*b^8 - 6*a^3*b^6*c + 11*a^4*b^4*c^2 - 6*a^5*b^2*c^3 + a^6*c^4)
\end{aligned}$$



$$\begin{aligned} & c)) / (a^6 b^2 - 4 a^7 c) - 4 \left( (b^5 c^3 - 4 a b^3 c^4 + 3 a^2 b^2 c^5) d - (a b^4 c^3 - 3 a^2 b^2 c^4 + a^3 c^5) e \right) \sqrt{e x + d} - \\ & 2 \sqrt{2} a^3 \sqrt{-d} d x^2 \sqrt{\left( (b^6 - 6 a b^4 c + 9 a^2 b^2 c^2 - 2 a^3 c^3) d - (a b^5 - 5 a^2 b^3 c + 5 a^3 b^2 c^2) e - (a^6 b^2 - 4 a^7 c) \right) \sqrt{\left( (b^{10} - 8 a b^8 c + 22 a^2 b^6 c^2 - 24 a^3 b^4 c^3 + 9 a^4 b^2 c^4) d^2 - 2 (a b^9 - 7 a^2 b^7 c + 16 a^3 b^5 c^2 - 13 a^4 b^3 c^3 + 3 a^5 b^2 c^4) d e + (a^2 b^8 - 6 a^3 b^6 c + 11 a^4 b^4 c^2 - 6 a^5 b^2 c^3 + a^6 c^4) e^2 \right)} / (a^{12} b^2 - 4 a^{13} c) \Big) / (a^6 b^2 - 4 a^7 c) \Big) \log(-\sqrt{2} \left( (b^9 - 9 a b^7 c + 27 a^2 b^5 c^2 - 31 a^3 b^3 c^3 + 12 a^4 b^2 c^4) d - (a b^8 - 8 a^2 b^6 c + 20 a^3 b^4 c^2 - 17 a^4 b^2 c^3 + 4 a^5 c^4) e + (a^6 b^5 - 7 a^7 b^3 c + 12 a^8 b^2 c^2) \sqrt{\left( (b^{10} - 8 a b^8 c + 22 a^2 b^6 c^2 - 24 a^3 b^4 c^3 + 9 a^4 b^2 c^4) d^2 - 2 (a b^9 - 7 a^2 b^7 c + 16 a^3 b^5 c^2 - 13 a^4 b^3 c^3 + 3 a^5 b^2 c^4) d e + (a^2 b^8 - 6 a^3 b^6 c + 11 a^4 b^4 c^2 - 6 a^5 b^2 c^3 + a^6 c^4) e^2 \right)} / (a^{12} b^2 - 4 a^{13} c) \right) \sqrt{\left( (b^6 - 6 a b^4 c + 9 a^2 b^2 c^2 - 2 a^3 c^3) d - (a b^5 - 5 a^2 b^3 c + 5 a^3 b^2 c^2) e - (a^6 b^2 - 4 a^7 c) \right) \sqrt{\left( (b^{10} - 8 a b^8 c + 22 a^2 b^6 c^2 - 24 a^3 b^4 c^3 + 9 a^4 b^2 c^4) d^2 - 2 (a b^9 - 7 a^2 b^7 c + 16 a^3 b^5 c^2 - 13 a^4 b^3 c^3 + 3 a^5 b^2 c^4) d e + (a^2 b^8 - 6 a^3 b^6 c + 11 a^4 b^4 c^2 - 6 a^5 b^2 c^3 + a^6 c^4) e^2 \right)} / (a^{12} b^2 - 4 a^{13} c) \Big) / (a^6 b^2 - 4 a^7 c) - 4 \left( (b^5 c^3 - 4 a b^3 c^4 + 3 a^2 b^2 c^5) d - (a b^4 c^3 - 3 a^2 b^2 c^4 + a^3 c^5) e \right) \sqrt{e x + d} - (4 a b^4 d e + a^2 e^2 - 8 (b^2 - a c) d^2) x^2 \arctan(d / (\sqrt{e x + d} \sqrt{-d})) - (2 a^2 d - (4 a b d - a^2 e) x) \sqrt{e x + d} \sqrt{-d} / (a^3 \sqrt{-d} d x^2) \Big] \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(1/2)/x\*\*3/(c\*x\*\*2+b\*x+a),x)

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/((c\*x^2 + b\*x + a)\*x^3),x, algorithm="giac")

[Out] Timed out



$$3.533 \quad \int \frac{x^4(d+ex)^{3/2}}{a+bx+cx^2} dx$$

Optimal. Leaf size=650

$$\frac{2\sqrt{d+ex}(-a^2c^2e+3ab^2ce-2abc^2d+b^4(-e)+b^3cd)}{c^5}$$

$$+\frac{\sqrt{2}\left(\frac{10a^2bc^3de-2a^2c^3(cd^2-ae^2)-b^4c(cd^2-6ae^2)-10ab^3c^2de+ab^2c^2(4cd^2-9ae^2)+b^6(-e^2)+2b^5cde}{\sqrt{b^2-4ac}}+(ace+b^2(-e)+bcd)(3abce-2ac^2d)\right)}{c^{11/2}\sqrt{2cd-e}\left(b-\sqrt{b^2-4ac}\right)}$$

$$+\frac{\sqrt{2}\left((ace+b^2(-e)+bcd)(3abce-2ac^2d+b^3(-e)+b^2cd)-\frac{10a^2bc^3de-2a^2c^3(cd^2-ae^2)-b^4c(cd^2-6ae^2)-10ab^3c^2de+ab^2c^2(4cd^2-9ae^2)+b^6(-e^2)+2b^5cde}{\sqrt{b^2-4ac}}\right)}{c^{11/2}\sqrt{2cd-e}\left(\sqrt{b^2-4ac}+b\right)}$$

$$-\frac{2b(b^2-2ac)(d+ex)^{3/2}}{3c^4}+\frac{2(d+ex)^{5/2}(ce(bd-ae)+b^2e^2+c^2d^2)}{5c^3e^3}$$

$$-\frac{2(d+ex)^{7/2}(be+2cd)}{7c^2e^3}+\frac{2(d+ex)^{9/2}}{9ce^3}$$

[Out]  $(-2*(b^3*c*d - 2*a*b*c^2*d - b^4*e + 3*a*b^2*c*e - a^2*c^2*e)*\text{Sqrt}[d + e*x])/c^5 - (2*b*(b^2 - 2*a*c)*(d + e*x)^{(3/2)})/(3*c^4) + (2*(c^2*d^2 + b^2*e^2 + c*e*(b*d - a*e))*(d + e*x)^{(5/2)})/(5*c^3*e^3) - (2*(2*c*d + b*e)*(d + e*x)^{(7/2)})/(7*c^2*e^3) + (2*(d + e*x)^{(9/2)})/(9*c*e^3) + (\text{Sqrt}[2]*((b*c*d - b^2*e + a*c*e)*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e) + (2*b^5*c*d*e - 10*a*b^3*c^2*d*e + 10*a^2*b*c^3*d*e - b^6*e^2 + a*b^2*c^2*(4*c*d^2 - 9*a*e^2) - b^4*c*(c*d^2 - 6*a*e^2) - 2*a^2*c^3*(c*d^2 - a*e^2))/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/(\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e])]/(c^{(11/2)}*\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]) + (\text{Sqrt}[2]*((b*c*d - b^2*e + a*c*e)*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e) - (2*b^5*c*d*e - 10*a*b^3*c^2*d*e + 10*a^2*b*c^3*d*e - b^6*e^2 + a*b^2*c^2*(4*c*d^2 - 9*a*e^2) - b^4*c*(c*d^2 - 6*a*e^2) - 2*a^2*c^3*(c*d^2 - a*e^2))/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/(\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e])]/(c^{(11/2)}*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e])$

Rubi [A] time = 8.42859, antiderivative size = 650, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{2\sqrt{d+ex}(-a^2c^2e+3ab^2ce-2abc^2d+b^4(-e)+b^3cd)}{c^5}$$

$$+\frac{\sqrt{2}\left(\frac{10a^2bc^3de-2a^2c^3(cd^2-ae^2)-b^4c(cd^2-6ae^2)-10ab^3c^2de+ab^2c^2(4cd^2-9ae^2)+b^6(-e^2)+2b^5cde}{\sqrt{b^2-4ac}}+(ace+b^2(-e)+bcd)(3abce-2ac^2d)\right)}{c^{11/2}\sqrt{2cd-e}\left(b-\sqrt{b^2-4ac}\right)}$$

$$+\frac{\sqrt{2}\left((ace+b^2(-e)+bcd)(3abce-2ac^2d+b^3(-e)+b^2cd)-\frac{10a^2bc^3de-2a^2c^3(cd^2-ae^2)-b^4c(cd^2-6ae^2)-10ab^3c^2de+ab^2c^2(4cd^2-9ae^2)+b^6(-e^2)+2b^5cde}{\sqrt{b^2-4ac}}\right)}{c^{11/2}\sqrt{2cd-e}\left(\sqrt{b^2-4ac}+b\right)}$$

$$-\frac{2b(b^2-2ac)(d+ex)^{3/2}}{3c^4}+\frac{2(d+ex)^{5/2}(ce(bd-ae)+b^2e^2+c^2d^2)}{5c^3e^3}$$

$$-\frac{2(d+ex)^{7/2}(be+2cd)}{7c^2e^3}+\frac{2(d+ex)^{9/2}}{9ce^3}$$

Antiderivative was successfully verified.

[In] Int[(x^4\*(d + e\*x)^(3/2))/(a + b\*x + c\*x^2), x]

[Out] 
$$\begin{aligned} & (-2*(b^3*c*d - 2*a*b*c^2*d - b^4*e + 3*a*b^2*c*e - a^2*c^2*e)*\text{Sqrt}[d + e*x])/c^5 - (2*b*(b^2 - 2*a*c)*(d + e*x)^{(3/2)})/(3*c^4) + \\ & (2*(c^2*d^2 + b^2*e^2 + c*e*(b*d - a*e))*(d + e*x)^{(5/2)})/(5*c^3*e^3) - (2*(2*c*d + b*e)*(d + e*x)^{(7/2)})/(7*c^2*e^3) + (2*(d + e*x)^{(9/2)})/(9*c*e^3) + \\ & (\text{Sqrt}[2]*((b*c*d - b^2*e + a*c*e)*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e) + (2*b^5*c*d*e - 10*a*b^3*c^2*d*e + 10*a^2*b*c^3*d*e - b^6*e^2 + a*b^2*c^2*(4*c*d^2 - 9*a*e^2) - b^4*c*(c*d^2 - 6*a*e^2) - 2*a^2*c^3*(c*d^2 - a*e^2))/\text{Sqrt}[b^2 - 4*a*c]) * \text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/ \text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]])/(c^{(11/2)}*\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]) + \\ & (\text{Sqrt}[2]*((b*c*d - b^2*e + a*c*e)*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e) - (2*b^5*c*d*e - 10*a*b^3*c^2*d*e + 10*a^2*b*c^3*d*e - b^6*e^2 + a*b^2*c^2*(4*c*d^2 - 9*a*e^2) - b^4*c*(c*d^2 - 6*a*e^2) - 2*a^2*c^3*(c*d^2 - a*e^2))/\text{Sqrt}[b^2 - 4*a*c]) * \text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/ \text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]])/(c^{(11/2)}*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]) \end{aligned}$$

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*4\*(e\*x+d)\*\*(3/2)/(c\*x\*\*2+b\*x+a), x)

[Out] Timed out

**Mathematica [A]** time = 2.82945, size = 808, normalized size = 1.24

$$\frac{2\sqrt{d+ex}((d+ex)^2(8d^2-20exd+35e^2x^2)c^4-9e(d+ex)^2(-2bd+7ae+5bex)c^3+21e^2(15a^2e^2+10ab(4d+ex)e+3b^2))}{315c^5e^3} + \frac{\sqrt{2}\left(-e^2b^6+e\left(2cd+\sqrt{b^2-4ace}\right)b^5-c\left(cd^2+2e\left(\sqrt{b^2-4acd}-3ae\right)\right)b^4+c\left(cd\left(\sqrt{b^2-4acd}-10ae\right)-4a\sqrt{b^2-4ace}\right)\right)}{+}$$

$$\frac{\sqrt{2}\left(e^2b^6+e\left(\sqrt{b^2-4ace}-2cd\right)b^5+c\left(cd^2-2e\left(\sqrt{b^2-4acd}+3ae\right)\right)b^4+c\left(cd\left(\sqrt{b^2-4acd}+10ae\right)-4a\sqrt{b^2-4ace}\right)\right)}{+}$$

Antiderivative was successfully verified.

[In] Integrate[(x^4\*(d + e\*x)^(3/2))/(a + b\*x + c\*x^2), x]

[Out] 
$$\begin{aligned} & (2*\text{Sqrt}[d + e*x]*(315*b^4*e^4 - 105*b^2*c*e^3*(4*b*d + 9*a*e + b*e*x) - 9*c^3*e*(d + e*x)^2*(-2*b*d + 7*a*e + 5*b*e*x) + c^4*(d + e*x)^2*(8*d^2 - 20*d*e*x + 35*e^2*x^2) + 21*c^2*e^2*(15*a^2*e^2 + 3*b^2*(d + e*x)^2 + 10*a*b*e*(4*d + e*x)))/(315*c^5*e^3) + \\ & (\text{Sqrt}[2]*(-b^6*e^2) + b^5*e*(2*c*d + \text{Sqrt}[b^2 - 4*a*c]*e) + a*b^2*c^2*(4*c*d^2 + 6*\text{Sqrt}[b^2 - 4*a*c]*d*e - 9*a*e^2) + b^3*c*(-4*a*\text{Sqrt}[b^2 - 4*a*c]*e^2 + c*d*(\text{Sqrt}[b^2 - 4*a*c]*d - 10*a*e)) + a*b*c^2*(3*a*\text{Sqrt}[b^2 - 4*a*c]*e^2 - 2*c*d*(\text{Sqrt}[b^2 - 4*a*c]*d - 5*a*e)) - b^4*c*(c*d^2 + 2*e*(\text{Sqrt}[b^2 - 4*a*c]*d - 3*a*e)) + 2*a^2*c^3*(-c*d^2) + e*(-(\text{Sqrt}[b^2 - 4*a*c]*d) + a*e))*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/ \text{Sqrt}[2*c*d - b*e + \text{Sqrt}[b^2 - 4*a*c]*e]])/(c^{(11/2)}*\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c])]) \end{aligned}$$

$$\begin{aligned} & )^*e]) + (\text{Sqrt}[2]^*(b^6*e^2 + b^5*e*(-2*c*d + \text{Sqrt}[b^2 - 4*a*c]^*e) \\ & + a*b^2*c^2*(-4*c*d^2 + 6*\text{Sqrt}[b^2 - 4*a*c]^*d*e + 9*a*e^2) - 2*a^2 \\ & *c^3*(-(c*d^2) + e*(\text{Sqrt}[b^2 - 4*a*c]^*d + a*e)) + b^4*c*(c*d^2 - \\ & 2*e*(\text{Sqrt}[b^2 - 4*a*c]^*d + 3*a*e)) + a*b*c^2*(3*a*\text{Sqrt}[b^2 - 4*a \\ & *c]^*e^2 - 2*c*d*(\text{Sqrt}[b^2 - 4*a*c]^*d + 5*a*e)) + b^3*c*(-4*a*\text{Sqrt} \\ & [b^2 - 4*a*c]^*e^2 + c*d*(\text{Sqrt}[b^2 - 4*a*c]^*d + 10*a*e))) * \text{ArcTanh}[ \\ & (\text{Sqrt}[2]^*\text{Sqrt}[c]^*\text{Sqrt}[d + e*x]) / \text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a \\ & c]^*e))] / (c^{(11/2)} * \text{Sqrt}[b^2 - 4*a*c]^*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - \\ & 4*a*c]^*e)]) \end{aligned}$$

**Maple [B]** time = 0.091, size = 3685, normalized size = 5.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4\*(e\*x+d)^(3/2)/(c\*x^2+b\*x+a), x)

[Out] 
$$\begin{aligned} & 2/9*(e*x+d)^{(9/2)}/c/e^3+4*e/c^2/(-e^2*(4*a*c-b^2))^{(1/2)}*2^{(1/2)}/ \\ & ((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^*c)^{(1/2)}*\text{arctanh}(c*(e*x+d) \\ & ^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^*c)^{(1/2)})^*a \\ & *b^2*d^2-10*e^2/c^3/(-e^2*(4*a*c-b^2))^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d \\ & +(-e^2*(4*a*c-b^2))^{(1/2)})^*c)^{(1/2)}*\text{arctanh}(c*(e*x+d)^{(1/2)}*2^{(1/2)}/ \\ & ((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^*c)^{(1/2)})^*a*b^3*d+10*e^2 \\ & /c^2/(-e^2*(4*a*c-b^2))^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b \\ & ^2))^{(1/2)})^*c)^{(1/2)}*\text{arctan}(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+ \\ & (-e^2*(4*a*c-b^2))^{(1/2)})^*c)^{(1/2)})^*a^2*b*d-10*e^2/c^3/(-e^2*(4*a \\ & c-b^2))^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^*c)^{(1/2)} \\ & *\text{arctan}(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2)) \\ & ^{(1/2)})^*c)^{(1/2)})^*a*b^3*d+4*e/c^2/(-e^2*(4*a*c-b^2))^{(1/2)}*2^{(1/2)}/ \\ & ((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^*c)^{(1/2)}*\text{arctan}(c*(e*x+d) \\ & ^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^*c)^{(1/2)})^*a \\ & *b^2*d^2+10*e^2/c^2/(-e^2*(4*a*c-b^2))^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+ \\ & (-e^2*(4*a*c-b^2))^{(1/2)})^*c)^{(1/2)}*\text{arctanh}(c*(e*x+d)^{(1/2)}*2^{(1/2)}/ \\ & ((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^*c)^{(1/2)})^*a^2*b*d-2/7/e^2 \\ & /c^2*(e*x+d)^{(7/2)}*b-2/5/e/c^2*(e*x+d)^{(5/2)}*a+2/5/e/c^3*(e*x+d) \\ & ^{(5/2)}*b^2+2*e/c^3*a^2*(e*x+d)^{(1/2)}+2*e/c^5*b^4*(e*x+d)^{(1/2)}-4/ \\ & 7/e^3/c*(e*x+d)^{(7/2)}*d+2/5/e^3/c*(e*x+d)^{(5/2)}*d^2+4/3/c^3*(e*x+ \\ & d)^{(3/2)}*a*b-2/c^4*b^3*d*(e*x+d)^{(1/2)}-e/c^3/(-e^2*(4*a*c-b^2))^{(1/2)} \\ & *2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^*c)^{(1/2)}*\text{arctan} \\ & (c*(e*x+d)^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^*c \\ & )^{(1/2)})^*b^4*d^2-6*e/c^3*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)} \\ & )^*c)^{(1/2)}*\text{arctan}(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4 \\ & *a*c-b^2))^{(1/2)})^*c)^{(1/2)})^*a*b^2*d+6*e/c^3*2^{(1/2)}/((-b*e+2*c*d \\ & +(-e^2*(4*a*c-b^2))^{(1/2)})^*c)^{(1/2)}*\text{arctanh}(c*(e*x+d)^{(1/2)}*2^{(1/2)}/ \\ & ((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^*c)^{(1/2)})^*a*b^2*d-9*e^3 \\ & /c^3/(-e^2*(4*a*c-b^2))^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b \\ & ^2))^{(1/2)})^*c)^{(1/2)}*\text{arctanh}(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d \\ & +(-e^2*(4*a*c-b^2))^{(1/2)})^*c)^{(1/2)})^*a^2*b^2-2*e/c/(-e^2*(4*a*c-b \\ & ^2))^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^*c)^{(1/2)} \\ & *\text{arctanh}(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2)) \\ & ^{(1/2)})^*c)^{(1/2)})^*a^2*d^2+6*e^3/c^4/(-e^2*(4*a*c-b^2))^{(1/2)}*2^{(1/2)}/ \\ & ((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^*c)^{(1/2)}*\text{arctanh}(c*(e* \\ & x+d)^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^*c)^{(1/2)} \\ & ))^*a*b^4+2*e^2/c^4/(-e^2*(4*a*c-b^2))^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+ \\ & (-e^2*(4*a*c-b^2))^{(1/2)})^*c)^{(1/2)}*\text{arctanh}(c*(e*x+d)^{(1/2)}*2^{(1/2)}/ \\ & ((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^*c)^{(1/2)})^*b^5*d-e/c^3/(- \\ & e^2*(4*a*c-b^2))^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)} \\ & )^*c)^{(1/2)}*\text{arctanh}(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4 \\ & *a*c-b^2))^{(1/2)})^*c)^{(1/2)})^*b^4*d^2-9*e^3/c^3/(-e^2*(4*a*c-b^2) \\ & )^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^*c)^{(1/2)}*\text{ar} \\ & \text{ctan}(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)} \\ & )^*c)^{(1/2)})^*a^2*b^2-2*e/c/(-e^2*(4*a*c-b^2))^{(1/2)}*2^{(1/2)}/((b*e- \\ & 2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^*c)^{(1/2)}*\text{arctan}(c*(e*x+d)^{(1/2)}*2 \\ & ^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^*c)^{(1/2)})^*a^2*d^2+6* \\ & e^3/c^4/(-e^2*(4*a*c-b^2))^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c \\ & -b^2))^{(1/2)})^*c)^{(1/2)}*\text{arctan}(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((b*e-2*c*d \end{aligned}$$

$$\begin{aligned}
& +(-e^{2*(4*a*c-b^2)})^{(1/2)}*c)^{(1/2)}*a*b^4-6*e/c^4*a*b^2*(e*x+d)^{1/2}+4/c^3*a*b*d*(e*x+d)^{(1/2)}-2/3/c^4*(e*x+d)^{(3/2)}*b^3+2/5/e^2/c^2*(e*x+d)^{(5/2)}*b*d+3*e^2/c^3*2^{(1/2)}/((-b*e+2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)}*arctanh(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)})*a^2*b-2*e/c^2*2^{(1/2)}/((-b*e+2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)}*arctanh(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)})*a^2*d-4*e^2/c^4*2^{(1/2)}/((-b*e+2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)}*arctanh(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)})*a*b^3-2*e/c^4*2^{(1/2)}/((-b*e+2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)}*arctanh(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)})*b^4*d+2*e^3/c^2/(-e^{2*(4*a*c-b^2)})^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)}*arctan(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)})*a^3+2/c^2*2^{(1/2)}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)}*arctan(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)})*a*b*d^2-2/c^2*2^{(1/2)}/((-b*e+2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)}*arctanh(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)})*a*b*d^2-e^3/c^5/(-e^{2*(4*a*c-b^2)})^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)}*arctan(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)})*b^6+2*e^3/c^2/(-e^{2*(4*a*c-b^2)})^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)}*arctanh(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)})*a^3-e^3/c^5/(-e^{2*(4*a*c-b^2)})^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)}*arctanh(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)})*b^6-3*e^2/c^3*2^{(1/2)}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)}*arctan(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)})*a^2*b+2*e/c^2*2^{(1/2)}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)}*arctan(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)})*a^2*d+4*e^2/c^4*2^{(1/2)}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)}*arctan(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)})*a*b^3+2*e/c^4*2^{(1/2)}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)}*arctan(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)})*b^4*d-1/c^3*2^{(1/2)}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)}*arctan(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)})*b^3*d^2+1/c^3*2^{(1/2)}/((-b*e+2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)}*arctanh(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)})*b^3*d^2-e^2/c^5*2^{(1/2)}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)}*arctan(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)})*b^5+e^2/c^5*2^{(1/2)}/((-b*e+2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)}*arctanh(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)})*b^5+2*e^2/c^4/(-e^{2*(4*a*c-b^2)})^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)}*arctan(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)})^{(1/2)})*c)^{(1/2)})*b^5*d
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^{\frac{3}{2}}x^4}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*x^4/(c\*x^2 + b\*x + a), x, algorithm="maxima")

[Out] integrate((e\*x + d)^(3/2)\*x^4/(c\*x^2 + b\*x + a), x)

**Fricas [A]** time = 10.8565, size = 19359, normalized size = 29.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*x^4/(c\*x^2 + b\*x + a),x, algorithm="fricas")

[Out] 
$$\frac{-1/630 \cdot (315 \cdot \sqrt{2}) \cdot c^5 \cdot e^3 \cdot \sqrt{((b^8 \cdot c^3 - 8 \cdot a \cdot b^6 \cdot c^4 + 20 \cdot a^2 \cdot b^4 \cdot c^5 - 16 \cdot a^3 \cdot b^2 \cdot c^6 + 2 \cdot a^4 \cdot c^7) \cdot d^3 - 3 \cdot (b^9 \cdot c^2 - 9 \cdot a \cdot b^7 \cdot c^3 + 27 \cdot a^2 \cdot b^5 \cdot c^4 - 30 \cdot a^3 \cdot b^3 \cdot c^5 + 9 \cdot a^4 \cdot b \cdot c^6) \cdot d^2 \cdot e + 3 \cdot (b^{10} \cdot c - 10 \cdot a \cdot b^8 \cdot c^2 + 35 \cdot a^2 \cdot b^6 \cdot c^3 - 50 \cdot a^3 \cdot b^4 \cdot c^4 + 25 \cdot a^4 \cdot b^2 \cdot c^5 - 2 \cdot a^5 \cdot c^6) \cdot d \cdot e^2 - (b^{11} - 11 \cdot a \cdot b^9 \cdot c + 44 \cdot a^2 \cdot b^7 \cdot c^2 - 77 \cdot a^3 \cdot b^5 \cdot c^3 + 55 \cdot a^4 \cdot b^3 \cdot c^4 - 11 \cdot a^5 \cdot b \cdot c^5) \cdot e^3 + (b^2 \cdot c^{11} - 4 \cdot a \cdot c^{12}) \cdot \sqrt{((b^{14} \cdot c^6 - 12 \cdot a \cdot b^{12} \cdot c^7 + 56 \cdot a^2 \cdot b^{10} \cdot c^8 - 128 \cdot a^3 \cdot b^8 \cdot c^9 + 148 \cdot a^4 \cdot b^6 \cdot c^{10} - 80 \cdot a^5 \cdot b^4 \cdot c^{11} + 16 \cdot a^6 \cdot b^2 \cdot c^{12}) \cdot d^6 - 6 \cdot (b^{15} \cdot c^5 - 13 \cdot a \cdot b^{13} \cdot c^6 + 67 \cdot a^2 \cdot b^{11} \cdot c^7 - 174 \cdot a^3 \cdot b^9 \cdot c^8 + 239 \cdot a^4 \cdot b^7 \cdot c^9 - 166 \cdot a^5 \cdot b^5 \cdot c^{10} + 50 \cdot a^6 \cdot b^3 \cdot c^{11} - 4 \cdot a^7 \cdot b \cdot c^{12}) \cdot d^5 \cdot e + 3 \cdot (5 \cdot b^{16} \cdot c^4 - 70 \cdot a \cdot b^{14} \cdot c^5 + 395 \cdot a^2 \cdot b^{12} \cdot c^6 - 1150 \cdot a^3 \cdot b^{10} \cdot c^7 + 1835 \cdot a^4 \cdot b^8 \cdot c^8 - 1570 \cdot a^5 \cdot b^6 \cdot c^9 + 650 \cdot a^6 \cdot b^4 \cdot c^{10} - 100 \cdot a^7 \cdot b^2 \cdot c^{11} + 3 \cdot a^8 \cdot c^{12}) \cdot d^4 \cdot e^2 - 2 \cdot (10 \cdot b^{17} \cdot c^3 - 150 \cdot a \cdot b^{15} \cdot c^4 + 920 \cdot a^2 \cdot b^{13} \cdot c^5 - 2970 \cdot a^3 \cdot b^{11} \cdot c^6 + 5410 \cdot a^4 \cdot b^9 \cdot c^7 - 5530 \cdot a^5 \cdot b^7 \cdot c^8 + 2960 \cdot a^6 \cdot b^5 \cdot c^9 - 700 \cdot a^7 \cdot b^3 \cdot c^{10} + 49 \cdot a^8 \cdot b \cdot c^{11}) \cdot d^3 \cdot e^3 + 3 \cdot (5 \cdot b^{18} \cdot c^2 - 80 \cdot a \cdot b^{16} \cdot c^3 + 530 \cdot a^2 \cdot b^{14} \cdot c^4 - 1880 \cdot a^3 \cdot b^{12} \cdot c^5 + 3855 \cdot a^4 \cdot b^{10} \cdot c^6 - 4600 \cdot a^5 \cdot b^8 \cdot c^7 + 3050 \cdot a^6 \cdot b^6 \cdot c^8 - 1000 \cdot a^7 \cdot b^4 \cdot c^9 + 125 \cdot a^8 \cdot b^2 \cdot c^{10} - 2 \cdot a^9 \cdot c^{11}) \cdot d^2 \cdot e^4 - 6 \cdot (b^{19} \cdot c - 17 \cdot a \cdot b^{17} \cdot c^2 + 121 \cdot a^2 \cdot b^{15} \cdot c^3 - 468 \cdot a^3 \cdot b^{13} \cdot c^4 + 1068 \cdot a^4 \cdot b^{11} \cdot c^5 - 1461 \cdot a^5 \cdot b^9 \cdot c^6 + 1163 \cdot a^6 \cdot b^7 \cdot c^7 - 496 \cdot a^7 \cdot b^5 \cdot c^8 + 95 \cdot a^8 \cdot b^3 \cdot c^9 - 5 \cdot a^9 \cdot b \cdot c^{10}) \cdot d \cdot e^5 + (b^{20} - 18 \cdot a \cdot b^{18} \cdot c + 137 \cdot a^2 \cdot b^{16} \cdot c^2 - 574 \cdot a^3 \cdot b^{14} \cdot c^3 + 1444 \cdot a^4 \cdot b^{12} \cdot c^4 - 2232 \cdot a^5 \cdot b^{10} \cdot c^5 + 2083 \cdot a^6 \cdot b^8 \cdot c^6 - 1106 \cdot a^7 \cdot b^6 \cdot c^7 + 295 \cdot a^8 \cdot b^4 \cdot c^8 - 30 \cdot a^9 \cdot b^2 \cdot c^9 + a^{10} \cdot c^{10}) \cdot e^6) / (b^2 \cdot c^{22} - 4 \cdot a \cdot c^{23})) / (b^2 \cdot c^{11} - 4 \cdot a \cdot c^{12})) \cdot \log(\sqrt{2}) \cdot ((b^{12} \cdot c^4 - 12 \cdot a \cdot b^{10} \cdot c^5 + 54 \cdot a^2 \cdot b^8 \cdot c^6 - 112 \cdot a^3 \cdot b^6 \cdot c^7 + 104 \cdot a^4 \cdot b^4 \cdot c^8 - 32 \cdot a^5 \cdot b^2 \cdot c^9) \cdot d^4 - (4 \cdot b^{13} \cdot c^3 - 52 \cdot a \cdot b^{11} \cdot c^4 + 260 \cdot a^2 \cdot b^9 \cdot c^5 - 624 \cdot a^3 \cdot b^7 \cdot c^6 + 725 \cdot a^4 \cdot b^5 \cdot c^7 - 350 \cdot a^5 \cdot b^3 \cdot c^8 + 40 \cdot a^6 \cdot b \cdot c^9) \cdot d^3 \cdot e + 3 \cdot (2 \cdot b^{14} \cdot c^2 - 28 \cdot a \cdot b^{12} \cdot c^3 + 154 \cdot a^2 \cdot b^{10} \cdot c^4 - 420 \cdot a^3 \cdot b^8 \cdot c^5 + 587 \cdot a^4 \cdot b^6 \cdot c^6 - 387 \cdot a^5 \cdot b^4 \cdot c^7 + 93 \cdot a^6 \cdot b^2 \cdot c^8 - 4 \cdot a^7 \cdot c^9) \cdot d^2 \cdot e^2 - (4 \cdot b^{15} \cdot c - 60 \cdot a \cdot b^{13} \cdot c^2 + 360 \cdot a^2 \cdot b^{11} \cdot c^3 - 1100 \cdot a^3 \cdot b^9 \cdot c^4 + 1799 \cdot a^4 \cdot b^7 \cdot c^5 - 1508 \cdot a^5 \cdot b^5 \cdot c^6 + 561 \cdot a^6 \cdot b^3 \cdot c^7 - 68 \cdot a^7 \cdot b \cdot c^8) \cdot d \cdot e^3 + (b^{16} - 16 \cdot a \cdot b^{14} \cdot c + 104 \cdot a^2 \cdot b^{12} \cdot c^2 - 352 \cdot a^3 \cdot b^{10} \cdot c^3 + 660 \cdot a^4 \cdot b^8 \cdot c^4 - 673 \cdot a^5 \cdot b^6 \cdot c^5 + 342 \cdot a^6 \cdot b^4 \cdot c^6 - 73 \cdot a^7 \cdot b^2 \cdot c^7 + 4 \cdot a^8 \cdot c^8) \cdot e^4 - ((b^6 \cdot c^{12} - 8 \cdot a \cdot b^4 \cdot c^{13} + 18 \cdot a^2 \cdot b^2 \cdot c^{14} - 8 \cdot a^3 \cdot c^{15}) \cdot d - (b^7 \cdot c^{11} - 9 \cdot a \cdot b^5 \cdot c^{12} + 25 \cdot a^2 \cdot b^3 \cdot c^{13} - 20 \cdot a^3 \cdot b \cdot c^{14}) \cdot e) \cdot \sqrt{((b^{14} \cdot c^6 - 12 \cdot a \cdot b^{12} \cdot c^7 + 56 \cdot a^2 \cdot b^{10} \cdot c^8 - 128 \cdot a^3 \cdot b^8 \cdot c^9 + 148 \cdot a^4 \cdot b^6 \cdot c^{10} - 80 \cdot a^5 \cdot b^4 \cdot c^{11} + 16 \cdot a^6 \cdot b^2 \cdot c^{12}) \cdot d^6 - 6 \cdot (b^{15} \cdot c^5 - 13 \cdot a \cdot b^{13} \cdot c^6 + 67 \cdot a^2 \cdot b^{11} \cdot c^7 - 174 \cdot a^3 \cdot b^9 \cdot c^8 + 239 \cdot a^4 \cdot b^7 \cdot c^9 - 166 \cdot a^5 \cdot b^5 \cdot c^{10} + 50 \cdot a^6 \cdot b^3 \cdot c^{11} - 4 \cdot a^7 \cdot b \cdot c^{12}) \cdot d^5 \cdot e + 3 \cdot (5 \cdot b^{16} \cdot c^4 - 70 \cdot a \cdot b^{14} \cdot c^5 + 395 \cdot a^2 \cdot b^{12} \cdot c^6 - 1150 \cdot a^3 \cdot b^{10} \cdot c^7 + 1835 \cdot a^4 \cdot b^8 \cdot c^8 - 1570 \cdot a^5 \cdot b^6 \cdot c^9 + 650 \cdot a^6 \cdot b^4 \cdot c^{10} - 100 \cdot a^7 \cdot b^2 \cdot c^{11} + 3 \cdot a^8 \cdot c^{12}) \cdot d^4 \cdot e^2 - 2 \cdot (10 \cdot b^{17} \cdot c^3 - 150 \cdot a \cdot b^{15} \cdot c^4 + 920 \cdot a^2 \cdot b^{13} \cdot c^5 - 2970 \cdot a^3 \cdot b^{11} \cdot c^6 + 5410 \cdot a^4 \cdot b^9 \cdot c^7 - 5530 \cdot a^5 \cdot b^7 \cdot c^8 + 2960 \cdot a^6 \cdot b^5 \cdot c^9 - 700 \cdot a^7 \cdot b^3 \cdot c^{10} + 49 \cdot a^8 \cdot b \cdot c^{11}) \cdot d^3 \cdot e^3 + 3 \cdot (5 \cdot b^{18} \cdot c^2 - 80 \cdot a \cdot b^{16} \cdot c^3 + 530 \cdot a^2 \cdot b^{14} \cdot c^4 - 1880 \cdot a^3 \cdot b^{12} \cdot c^5 + 3855 \cdot a^4 \cdot b^{10} \cdot c^6 - 4600 \cdot a^5 \cdot b^8 \cdot c^7 + 3050 \cdot a^6 \cdot b^6 \cdot c^8 - 1000 \cdot a^7 \cdot b^4 \cdot c^9 + 125 \cdot a^8 \cdot b^2 \cdot c^{10} - 2 \cdot a^9 \cdot c^{11}) \cdot d^2 \cdot e^4 - 6 \cdot (b^{19} \cdot c - 17 \cdot a \cdot b^{17} \cdot c^2 + 121 \cdot a^2 \cdot b^{15} \cdot c^3 - 468 \cdot a^3 \cdot b^{13} \cdot c^4 + 1068 \cdot a^4 \cdot b^{11} \cdot c^5 - 1461 \cdot a^5 \cdot b^9 \cdot c^6 + 1163 \cdot a^6 \cdot b^7 \cdot c^7 - 496 \cdot a^7 \cdot b^5 \cdot c^8 + 95 \cdot a^8 \cdot b^3 \cdot c^9 - 5 \cdot a^9 \cdot b \cdot c^{10}) \cdot d \cdot e^5 + (b^{20} - 18 \cdot a \cdot b^{18} \cdot c + 137 \cdot a^2 \cdot b^{16} \cdot c^2 - 574 \cdot a^3 \cdot b^{14} \cdot c^3 + 1444 \cdot a^4 \cdot b^{12} \cdot c^4 - 2232 \cdot a^5 \cdot b^{10} \cdot c^5 + 2083 \cdot a^6 \cdot b^8 \cdot c^6 - 1106 \cdot a^7 \cdot b^6 \cdot c^7 + 295 \cdot a^8 \cdot b^4 \cdot c^8 - 30 \cdot a^9 \cdot b^2 \cdot c^9 + a^{10} \cdot c^{10}) \cdot e^6) / (b^2 \cdot c^{22} - 4 \cdot a \cdot c^{23})) \cdot \sqrt{((b^8 \cdot c^3 - 8 \cdot a \cdot b^6 \cdot c^4 + 20 \cdot a^2 \cdot b^4 \cdot c^5 - 16 \cdot a^3 \cdot b^2 \cdot c^6 + 2 \cdot a^4 \cdot c^7) \cdot d^3 - 3 \cdot (b^9 \cdot c^2 - 9 \cdot a \cdot b^7 \cdot c^3 + 27 \cdot a^2 \cdot b^5 \cdot c^4 - 30 \cdot a^3 \cdot b^3 \cdot c^5 + 9 \cdot a^4 \cdot b \cdot c^6) \cdot d^2 \cdot e + 3 \cdot (b^{10} \cdot c - 10 \cdot a \cdot b^8 \cdot c^2 + 35 \cdot a^2 \cdot b^6 \cdot c^3 - 50 \cdot a^3 \cdot b^4 \cdot c^4 + 25 \cdot a^4 \cdot b^2 \cdot c^5 - 2 \cdot a^5 \cdot c^6) \cdot d \cdot e^2 - (b^{11} - 11 \cdot a \cdot b^9 \cdot c + 44 \cdot a^2 \cdot b^7 \cdot c^2 - 77 \cdot a^3 \cdot b^5 \cdot c^3 + 55 \cdot a^4 \cdot b^3 \cdot c^4 - 11 \cdot a^5 \cdot b \cdot c^5) \cdot e^3 + (b^2 \cdot c^{11} - 4 \cdot a \cdot c^{12}) \cdot \sqrt{((b^{14} \cdot c^6 - 12 \cdot a \cdot b^{12} \cdot c^7 + 56 \cdot a^2 \cdot b^{10} \cdot c^8 - 128 \cdot a^3 \cdot b^8 \cdot c^9 + 148 \cdot a^4 \cdot b^6 \cdot c^{10} - 80 \cdot a^5 \cdot b^4 \cdot c^{11} + 16 \cdot a^6 \cdot b^2 \cdot c^{12}) \cdot d^6 - 6 \cdot (b^{15} \cdot c^5 - 13 \cdot a \cdot b^{13} \cdot c^6 + 67 \cdot a^2 \cdot b^{11} \cdot c^7 - 174 \cdot a^3 \cdot b^9 \cdot c^8 + 239 \cdot a^4 \cdot b^7 \cdot c^9 - 166 \cdot a^5 \cdot b^5 \cdot c^{10} + 50 \cdot a^6 \cdot b^3 \cdot c^{11} - 4 \cdot a^7 \cdot b \cdot c^{12}) \cdot d^5 \cdot e + 3 \cdot (5 \cdot b^{16} \cdot c^4 - 70 \cdot a \cdot b^{14} \cdot c^5 + 395 \cdot a^2 \cdot b^{12} \cdot c^6 - 1150 \cdot a^3 \cdot b^{10} \cdot c^7 + 1835 \cdot a^4 \cdot b^8 \cdot c^8 - 1570 \cdot a^5 \cdot b^6 \cdot c^9 + 650 \cdot a^6 \cdot b^4 \cdot c^{10} - 100 \cdot a^7 \cdot b^2 \cdot c^{11} + 3 \cdot a^8 \cdot c^{12}) \cdot d^4 \cdot e^2 - 2 \cdot (10 \cdot b^{17} \cdot c^3 - 150 \cdot a \cdot b^{15} \cdot c^4 + 920 \cdot a^2 \cdot b^{13} \cdot c^5 - 2970 \cdot a^3 \cdot b^{11} \cdot c^6 + 5410 \cdot a^4 \cdot b^9 \cdot c^7 - 5530 \cdot a^5 \cdot b^7 \cdot c^8 + 2960 \cdot a^6 \cdot b^5 \cdot c^9 - 700 \cdot a^7 \cdot b^3 \cdot c^{10} + 49 \cdot a^8 \cdot b \cdot c^{11}) \cdot d^3 \cdot e^3 + 3 \cdot (5 \cdot b^{18} \cdot c^2 - 80 \cdot a \cdot b^{16} \cdot c^3 + 530 \cdot a^2 \cdot b^{14} \cdot c^4 - 1880 \cdot a^3 \cdot b^{12} \cdot c^5 + 3855 \cdot a^4 \cdot b^{10} \cdot c^6 - 4600 \cdot a^5 \cdot b^8 \cdot c^7 + 3050 \cdot a^6 \cdot b^6 \cdot c^8 - 1000 \cdot a^7 \cdot b^4 \cdot c^9 + 125 \cdot a^8 \cdot b^2 \cdot c^{10} - 2 \cdot a^9 \cdot c^{11}) \cdot d^2 \cdot e^4 - 6 \cdot (b^{19} \cdot c - 17 \cdot a \cdot b^{17} \cdot c^2 + 121 \cdot a^2 \cdot b^{15} \cdot c^3 - 468 \cdot a^3 \cdot b^{13} \cdot c^4 + 1068 \cdot a^4 \cdot b^{11} \cdot c^5 - 1461 \cdot a^5 \cdot b^9 \cdot c^6 + 1163 \cdot a^6 \cdot b^7 \cdot c^7 - 496 \cdot a^7 \cdot b^5 \cdot c^8 + 95 \cdot a^8 \cdot b^3 \cdot c^9 - 5 \cdot a^9 \cdot b \cdot c^{10}) \cdot d \cdot e^5 + (b^{20} - 18 \cdot a \cdot b^{18} \cdot c + 137 \cdot a^2 \cdot b^{16} \cdot c^2 - 574 \cdot a^3 \cdot b^{14} \cdot c^3 + 1444 \cdot a^4 \cdot b^{12} \cdot c^4 - 2232 \cdot a^5 \cdot b^{10} \cdot c^5 + 2083 \cdot a^6 \cdot b^8 \cdot c^6 - 1106 \cdot a^7 \cdot b^6 \cdot c^7 + 295 \cdot a^8 \cdot b^4 \cdot c^8 - 30 \cdot a^9 \cdot b^2 \cdot c^9 + a^{10} \cdot c^{10}) \cdot e^6) / (b^2 \cdot c^{22} - 4 \cdot a \cdot c^{23}))$$

$$\begin{aligned}
& 13*c^5 - 2970*a^3*b^{11}*c^6 + 5410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 \\
& + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^{10} + 49*a^8*b*c^{11})*d^3*e^3 + \\
& 3*(5*b^{18}*c^2 - 80*a*b^{16}*c^3 + 530*a^2*b^{14}*c^4 - 1880*a^3*b^{12}* \\
& c^5 + 3855*a^4*b^{10}*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1 \\
& 000*a^7*b^4*c^9 + 125*a^8*b^2*c^{10} - 2*a^9*c^{11})*d^2*e^4 - 6*(b^{19}*c \\
& - 17*a*b^{17}*c^2 + 121*a^2*b^{15}*c^3 - 468*a^3*b^{13}*c^4 + 1068* \\
& a^4*b^{11}*c^5 - 1461*a^5*b^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5* \\
& c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^{10})*d*e^5 + (b^{20} - 18*a*b^{18}*c \\
& + 137*a^2*b^{16}*c^2 - 574*a^3*b^{14}*c^3 + 1444*a^4*b^{12}*c^4 - 2232* \\
& a^5*b^{10}*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4* \\
& c^8 - 30*a^9*b^2*c^9 + a^{10}*c^{10})*e^6)/(b^2*c^{22} - 4*a*c^{23}))/ (b \\
& ^2*c^{11} - 4*a*c^{12})) + 4*((a^4*b^7*c^4 - 6*a^5*b^5*c^5 + 10*a^6*b \\
& ^3*c^6 - 4*a^7*b*c^7)*d^5 - (4*a^4*b^8*c^3 - 27*a^5*b^6*c^4 + 55* \\
& a^6*b^4*c^5 - 34*a^7*b^2*c^6 + 3*a^8*c^7)*d^4*e + 2*(3*a^4*b^9*c^2 \\
& - 22*a^5*b^7*c^3 + 51*a^6*b^5*c^4 - 40*a^7*b^3*c^5 + 7*a^8*b*c^6) \\
& *d^3*e^2 - 2*(2*a^4*b^{10}*c - 15*a^5*b^8*c^2 + 35*a^6*b^6*c^3 - \\
& 25*a^7*b^4*c^4 + a^9*c^6)*d^2*e^3 + (a^4*b^{11} - 6*a^5*b^9*c + 4*a \\
& ^6*b^7*c^2 + 28*a^7*b^5*c^3 - 45*a^8*b^3*c^4 + 14*a^9*b*c^5)*d*e^4 \\
& - (a^5*b^{10} - 9*a^6*b^8*c + 28*a^7*b^6*c^2 - 35*a^8*b^4*c^3 + 1 \\
& 5*a^9*b^2*c^4 - a^{10}*c^5)*e^5)*sqrt(e*x + d)) - 315*sqrt(2)*c^5*e \\
& ^3*sqrt(((b^8*c^3 - 8*a*b^6*c^4 + 20*a^2*b^4*c^5 - 16*a^3*b^2*c^6 \\
& + 2*a^4*c^7)*d^3 - 3*(b^9*c^2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^4 - 3 \\
& 0*a^3*b^3*c^5 + 9*a^4*b*c^6)*d^2*e + 3*(b^{10}*c - 10*a*b^8*c^2 + 3 \\
& 5*a^2*b^6*c^3 - 50*a^3*b^4*c^4 + 25*a^4*b^2*c^5 - 2*a^5*c^6)*d*e^2 \\
& - (b^{11} - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 55*a^4 \\
& *b^3*c^4 - 11*a^5*b*c^5)*e^3 + (b^2*c^{11} - 4*a*c^{12})*sqrt(((b^{14}* \\
& c^6 - 12*a*b^{12}*c^7 + 56*a^2*b^{10}*c^8 - 128*a^3*b^8*c^9 + 148*a^4 \\
& *b^6*c^{10} - 80*a^5*b^4*c^{11} + 16*a^6*b^2*c^{12})*d^6 - 6*(b^{15}*c^5 \\
& - 13*a*b^{13}*c^6 + 67*a^2*b^{11}*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7 \\
& *c^9 - 166*a^5*b^5*c^{10} + 50*a^6*b^3*c^{11} - 4*a^7*b*c^{12})*d^5*e + \\
& 3*(5*b^{16}*c^4 - 70*a*b^{14}*c^5 + 395*a^2*b^{12}*c^6 - 1150*a^3*b^{10} \\
& *c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4*c^{10} - 1 \\
& 00*a^7*b^2*c^{11} + 3*a^8*c^{12})*d^4*e^2 - 2*(10*b^{17}*c^3 - 150*a*b^{15} \\
& *c^4 + 920*a^2*b^{13}*c^5 - 2970*a^3*b^{11}*c^6 + 5410*a^4*b^9*c^7 \\
& - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^{10} + 49*a^8 \\
& *b*c^{11})*d^3*e^3 + 3*(5*b^{18}*c^2 - 80*a*b^{16}*c^3 + 530*a^2*b^{14}*c \\
& ^4 - 1880*a^3*b^{12}*c^5 + 3855*a^4*b^{10}*c^6 - 4600*a^5*b^8*c^7 + 3 \\
& 050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 125*a^8*b^2*c^{10} - 2*a^9*c^{11} \\
& )*d^2*e^4 - 6*(b^{19}*c - 17*a*b^{17}*c^2 + 121*a^2*b^{15}*c^3 - 468*a \\
& ^3*b^{13}*c^4 + 1068*a^4*b^{11}*c^5 - 1461*a^5*b^9*c^6 + 1163*a^6*b^7 \\
& *c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^{10})*d*e^5 + ( \\
& b^{20} - 18*a*b^{18}*c + 137*a^2*b^{16}*c^2 - 574*a^3*b^{14}*c^3 + 1444*a \\
& ^4*b^{12}*c^4 - 2232*a^5*b^{10}*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6 \\
& *c^7 + 295*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^{10}*c^{10})*e^6)/(b^2*c^ \\
& ^{22} - 4*a*c^{23}))/ (b^2*c^{11} - 4*a*c^{12}))*log(-sqrt(2)*((b^{12}*c^4 - \\
& 12*a*b^{10}*c^5 + 54*a^2*b^8*c^6 - 112*a^3*b^6*c^7 + 104*a^4*b^4*c \\
& ^8 - 32*a^5*b^2*c^9)*d^4 - (4*b^{13}*c^3 - 52*a*b^{11}*c^4 + 260*a^2* \\
& b^9*c^5 - 624*a^3*b^7*c^6 + 725*a^4*b^5*c^7 - 350*a^5*b^3*c^8 + 4 \\
& 0*a^6*b*c^9)*d^3*e + 3*(2*b^{14}*c^2 - 28*a*b^{12}*c^3 + 154*a^2*b^{10} \\
& *c^4 - 420*a^3*b^8*c^5 + 587*a^4*b^6*c^6 - 387*a^5*b^4*c^7 + 93*a \\
& ^6*b^2*c^8 - 4*a^7*c^9)*d^2*e^2 - (4*b^{15}*c - 60*a*b^{13}*c^2 + 360 \\
& *a^2*b^{11}*c^3 - 1100*a^3*b^9*c^4 + 1799*a^4*b^7*c^5 - 1508*a^5*b^5 \\
& *c^6 + 561*a^6*b^3*c^7 - 68*a^7*b*c^8)*d*e^3 + (b^{16} - 16*a*b^{14} \\
& *c + 104*a^2*b^{12}*c^2 - 352*a^3*b^{10}*c^3 + 660*a^4*b^8*c^4 - 673* \\
& a^5*b^6*c^5 + 342*a^6*b^4*c^6 - 73*a^7*b^2*c^7 + 4*a^8*c^8)*e^4 - \\
& ((b^6*c^{12} - 8*a*b^4*c^{13} + 18*a^2*b^2*c^{14} - 8*a^3*c^{15})*d - (b \\
& ^7*c^{11} - 9*a*b^5*c^{12} + 25*a^2*b^3*c^{13} - 20*a^3*b*c^{14})*e)*sqrt \\
& (((b^{14}*c^6 - 12*a*b^{12}*c^7 + 56*a^2*b^{10}*c^8 - 128*a^3*b^8*c^9 + \\
& 148*a^4*b^6*c^{10} - 80*a^5*b^4*c^{11} + 16*a^6*b^2*c^{12})*d^6 - 6*(b \\
& ^{15}*c^5 - 13*a*b^{13}*c^6 + 67*a^2*b^{11}*c^7 - 174*a^3*b^9*c^8 + 239 \\
& *a^4*b^7*c^9 - 166*a^5*b^5*c^{10} + 50*a^6*b^3*c^{11} - 4*a^7*b*c^{12} \\
& )*d^5*e + 3*(5*b^{16}*c^4 - 70*a*b^{14}*c^5 + 395*a^2*b^{12}*c^6 - 1150* \\
& a^3*b^{10}*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4* \\
& c^{10} - 100*a^7*b^2*c^{11} + 3*a^8*c^{12})*d^4*e^2 - 2*(10*b^{17}*c^3 - \\
& 150*a*b^{15}*c^4 + 920*a^2*b^{13}*c^5 - 2970*a^3*b^{11}*c^6 + 5410*a^4* \\
& b^9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^{10} \\
& + 49*a^8*b*c^{11})*d^3*e^3 + 3*(5*b^{18}*c^2 - 80*a*b^{16}*c^3 + 530*a^2 \\
& *b^{14}*c^4 - 1880*a^3*b^{12}*c^5 + 3855*a^4*b^{10}*c^6 - 4600*a^5*b^8 \\
& *c^7 + 3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 125*a^8*b^2*c^{10} - 2 \\
& *a^9*c^{11})*d^2*e^4 - 6*(b^{19}*c - 17*a*b^{17}*c^2 + 121*a^2*b^{15}*c^3 \\
& - 468*a^3*b^{13}*c^4 + 1068*a^4*b^{11}*c^5 - 1461*a^5*b^9*c^6 + 1163 \\
& *a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^{10})*d
\end{aligned}$$

$$\begin{aligned}
& *e^5 + (b^{20} - 18*a*b^{18}*c + 137*a^2*b^{16}*c^2 - 574*a^3*b^{14}*c^3 \\
& + 1444*a^4*b^{12}*c^4 - 2232*a^5*b^{10}*c^5 + 2083*a^6*b^8*c^6 - 1106 \\
& *a^7*b^6*c^7 + 295*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^{10}*c^{10})*e^6) \\
& /((b^2*c^{22} - 4*a*c^{23}))*\sqrt{((b^8*c^3 - 8*a*b^6*c^4 + 20*a^2*b^4 \\
& *c^5 - 16*a^3*b^2*c^6 + 2*a^4*c^7)*d^3 - 3*(b^9*c^2 - 9*a*b^7*c^3 \\
& + 27*a^2*b^5*c^4 - 30*a^3*b^3*c^5 + 9*a^4*b*c^6)*d^2*e + 3*(b^{10} \\
& *c - 10*a*b^8*c^2 + 35*a^2*b^6*c^3 - 50*a^3*b^4*c^4 + 25*a^4*b^2 \\
& *c^5 - 2*a^5*c^6)*d*e^2 - (b^{11} - 11*a*b^9*c + 44*a^2*b^7*c^2 - 7 \\
& 7*a^3*b^5*c^3 + 55*a^4*b^3*c^4 - 11*a^5*b*c^5)*e^3 + (b^2*c^{11} - \\
& 4*a*c^{12})*\sqrt{((b^{14}*c^6 - 12*a*b^{12}*c^7 + 56*a^2*b^{10}*c^8 - 128 \\
& *a^3*b^8*c^9 + 148*a^4*b^6*c^{10} - 80*a^5*b^4*c^{11} + 16*a^6*b^2*c^{12} \\
& )*d^6 - 6*(b^{15}*c^5 - 13*a*b^{13}*c^6 + 67*a^2*b^{11}*c^7 - 174*a^3 \\
& *b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^{10} + 50*a^6*b^3*c^{11} - \\
& 4*a^7*b*c^{12})*d^5*e + 3*(5*b^{16}*c^4 - 70*a*b^{14}*c^5 + 395*a^2*b^{12} \\
& *c^6 - 1150*a^3*b^{10}*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 \\
& + 650*a^6*b^4*c^{10} - 100*a^7*b^2*c^{11} + 3*a^8*c^{12})*d^4*e^2 - 2*( \\
& 10*b^{17}*c^3 - 150*a*b^{15}*c^4 + 920*a^2*b^{13}*c^5 - 2970*a^3*b^{11}*c^6 \\
& + 5410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700 \\
& *a^7*b^3*c^{10} + 49*a^8*b*c^{11})*d^3*e^3 + 3*(5*b^{18}*c^2 - 80*a*b^{16} \\
& *c^3 + 530*a^2*b^{14}*c^4 - 1880*a^3*b^{12}*c^5 + 3855*a^4*b^{10}*c^6 \\
& - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 125*a^8 \\
& *b^2*c^{10} - 2*a^9*c^{11})*d^2*e^4 - 6*(b^{19}*c - 17*a*b^{17}*c^2 + 12 \\
& 1*a^2*b^{15}*c^3 - 468*a^3*b^{13}*c^4 + 1068*a^4*b^{11}*c^5 - 1461*a^5 \\
& *b^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5 \\
& *a^9*b*c^{10})*d*e^5 + (b^{20} - 18*a*b^{18}*c + 137*a^2*b^{16}*c^2 - 574 \\
& *a^3*b^{14}*c^3 + 1444*a^4*b^{12}*c^4 - 2232*a^5*b^{10}*c^5 + 2083*a^6 \\
& *b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^{10} \\
& *c^{10})*e^6)/(b^2*c^{22} - 4*a*c^{23}))/((b^2*c^{11} - 4*a*c^{12}))*4 \\
& *((a^4*b^7*c^4 - 6*a^5*b^5*c^5 + 10*a^6*b^3*c^6 - 4*a^7*b*c^7)*d^5 \\
& - (4*a^4*b^8*c^3 - 27*a^5*b^6*c^4 + 55*a^6*b^4*c^5 - 34*a^7*b^2 \\
& *c^6 + 3*a^8*c^7)*d^4*e + 2*(3*a^4*b^9*c^2 - 22*a^5*b^7*c^3 + 51* \\
& a^6*b^5*c^4 - 40*a^7*b^3*c^5 + 7*a^8*b*c^6)*d^3*e^2 - 2*(2*a^4*b^{10} \\
& *c - 15*a^5*b^8*c^2 + 35*a^6*b^6*c^3 - 25*a^7*b^4*c^4 + a^9*c^6) \\
& )*d^2*e^3 + (a^4*b^{11} - 6*a^5*b^9*c + 4*a^6*b^7*c^2 + 28*a^7*b^5 \\
& *c^3 - 45*a^8*b^3*c^4 + 14*a^9*b*c^5)*d*e^4 - (a^5*b^{10} - 9*a^6*b^8 \\
& *c + 28*a^7*b^6*c^2 - 35*a^8*b^4*c^3 + 15*a^9*b^2*c^4 - a^{10}*c^5) \\
& )*e^5)*\sqrt{e*x + d)} + 315*\sqrt{2}*c^5*e^3*\sqrt{((b^8*c^3 - 8*a* \\
& b^6*c^4 + 20*a^2*b^4*c^5 - 16*a^3*b^2*c^6 + 2*a^4*c^7)*d^3 - 3*(b^9 \\
& *c^2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^4 - 30*a^3*b^3*c^5 + 9*a^4*b* \\
& c^6)*d^2*e + 3*(b^{10}*c - 10*a*b^8*c^2 + 35*a^2*b^6*c^3 - 50*a^3*b^4 \\
& *c^4 + 25*a^4*b^2*c^5 - 2*a^5*c^6)*d*e^2 - (b^{11} - 11*a*b^9*c + \\
& 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 55*a^4*b^3*c^4 - 11*a^5*b*c^5) \\
& )*e^3 - (b^2*c^{11} - 4*a*c^{12})*\sqrt{((b^{14}*c^6 - 12*a*b^{12}*c^7 + 56 \\
& *a^2*b^{10}*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b^6*c^{10} - 80*a^5*b^4*c^{11} \\
& + 16*a^6*b^2*c^{12})*d^6 - 6*(b^{15}*c^5 - 13*a*b^{13}*c^6 + 67*a^2 \\
& *b^{11}*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^{10} \\
& + 50*a^6*b^3*c^{11} - 4*a^7*b*c^{12})*d^5*e + 3*(5*b^{16}*c^4 - 70*a*b^{14} \\
& *c^5 + 395*a^2*b^{12}*c^6 - 1150*a^3*b^{10}*c^7 + 1835*a^4*b^8*c^8 \\
& - 1570*a^5*b^6*c^9 + 650*a^6*b^4*c^{10} - 100*a^7*b^2*c^{11} + 3*a^8 \\
& *c^{12})*d^4*e^2 - 2*(10*b^{17}*c^3 - 150*a*b^{15}*c^4 + 920*a^2*b^{13}*c^5 \\
& - 2970*a^3*b^{11}*c^6 + 5410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 + 2960 \\
& *a^6*b^5*c^9 - 700*a^7*b^3*c^{10} + 49*a^8*b*c^{11})*d^3*e^3 + 3*(5* \\
& b^{18}*c^2 - 80*a*b^{16}*c^3 + 530*a^2*b^{14}*c^4 - 1880*a^3*b^{12}*c^5 + \\
& 3855*a^4*b^{10}*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a^7 \\
& *b^4*c^9 + 125*a^8*b^2*c^{10} - 2*a^9*c^{11})*d^2*e^4 - 6*(b^{19}*c - \\
& 17*a*b^{17}*c^2 + 121*a^2*b^{15}*c^3 - 468*a^3*b^{13}*c^4 + 1068*a^4*b^{11} \\
& *c^5 - 1461*a^5*b^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5*c^8 + \\
& 95*a^8*b^3*c^9 - 5*a^9*b*c^{10})*d*e^5 + (b^{20} - 18*a*b^{18}*c + 137 \\
& *a^2*b^{16}*c^2 - 574*a^3*b^{14}*c^3 + 1444*a^4*b^{12}*c^4 - 2232*a^5*b^{10} \\
& *c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4*c^8 - \\
& 30*a^9*b^2*c^9 + a^{10}*c^{10})*e^6)/(b^2*c^{22} - 4*a*c^{23}))/((b^2*c^{11} \\
& - 4*a*c^{12}))*\log(\sqrt{2})*((b^{12}*c^4 - 12*a*b^{10}*c^5 + 54*a^2*b^8 \\
& *c^6 - 112*a^3*b^6*c^7 + 104*a^4*b^4*c^8 - 32*a^5*b^2*c^9)*d^4 \\
& - (4*b^{13}*c^3 - 52*a*b^{11}*c^4 + 260*a^2*b^9*c^5 - 624*a^3*b^7*c^6 \\
& + 725*a^4*b^5*c^7 - 350*a^5*b^3*c^8 + 40*a^6*b*c^9)*d^3*e + 3*(2 \\
& *b^{14}*c^2 - 28*a*b^{12}*c^3 + 154*a^2*b^{10}*c^4 - 420*a^3*b^8*c^5 + \\
& 587*a^4*b^6*c^6 - 387*a^5*b^4*c^7 + 93*a^6*b^2*c^8 - 4*a^7*c^9)*d^2 \\
& *e^2 - (4*b^{15}*c - 60*a*b^{13}*c^2 + 360*a^2*b^{11}*c^3 - 1100*a^3* \\
& b^9*c^4 + 1799*a^4*b^7*c^5 - 1508*a^5*b^5*c^6 + 561*a^6*b^3*c^7 - \\
& 68*a^7*b*c^8)*d*e^3 + (b^{16} - 16*a*b^{14}*c + 104*a^2*b^{12}*c^2 - 3 \\
& 52*a^3*b^{10}*c^3 + 660*a^4*b^8*c^4 - 673*a^5*b^6*c^5 + 342*a^6*b^4 \\
& *c^6 - 73*a^7*b^2*c^7 + 4*a^8*c^8)*e^4 + ((b^6*c^{12} - 8*a*b^4*c^{11}
\end{aligned}$$

$$\begin{aligned}
& 3 + 18*a^2*b^2*c^{14} - 8*a^3*c^{15})*d - (b^7*c^{11} - 9*a*b^5*c^{12} + \\
& 25*a^2*b^3*c^{13} - 20*a^3*b*c^{14})*e)*\text{sqrt}(((b^{14}*c^6 - 12*a*b^{12}*c \\
& ^7 + 56*a^2*b^{10}*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b^6*c^{10} - 80*a^5 \\
& *b^4*c^{11} + 16*a^6*b^2*c^{12})*d^6 - 6*(b^{15}*c^5 - 13*a*b^{13}*c^6 + \\
& 67*a^2*b^{11}*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5 \\
& *c^{10} + 50*a^6*b^3*c^{11} - 4*a^7*b*c^{12})*d^5*e + 3*(5*b^{16}*c^4 - \\
& 70*a*b^{14}*c^5 + 395*a^2*b^{12}*c^6 - 1150*a^3*b^{10}*c^7 + 1835*a^4*b \\
& ^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4*c^{10} - 100*a^7*b^2*c^{11} + \\
& 3*a^8*c^{12})*d^4*e^2 - 2*(10*b^{17}*c^3 - 150*a*b^{15}*c^4 + 920*a^2* \\
& b^{13}*c^5 - 2970*a^3*b^{11}*c^6 + 5410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 \\
& + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^{10} + 49*a^8*b*c^{11})*d^3*e^3 \\
& + 3*(5*b^{18}*c^2 - 80*a*b^{16}*c^3 + 530*a^2*b^{14}*c^4 - 1880*a^3*b^{12} \\
& *c^5 + 3855*a^4*b^{10}*c^6 - 4600*a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - \\
& 1000*a^7*b^4*c^9 + 125*a^8*b^2*c^{10} - 2*a^9*c^{11})*d^2*e^4 - 6*(b \\
& ^{19}*c - 17*a*b^{17}*c^2 + 121*a^2*b^{15}*c^3 - 468*a^3*b^{13}*c^4 + 106 \\
& 8*a^4*b^{11}*c^5 - 1461*a^5*b^9*c^6 + 1163*a^6*b^7*c^7 - 496*a^7*b^5 \\
& *c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^{10})*d*e^5 + (b^{20} - 18*a*b^{18}* \\
& c + 137*a^2*b^{16}*c^2 - 574*a^3*b^{14}*c^3 + 1444*a^4*b^{12}*c^4 - 223 \\
& 2*a^5*b^{10}*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 + 295*a^8*b^4 \\
& *c^8 - 30*a^9*b^2*c^9 + a^{10}*c^{10})*e^6)/(b^2*c^{22} - 4*a*c^{23})))* \\
& \text{sqrt}(((b^8*c^3 - 8*a*b^6*c^4 + 20*a^2*b^4*c^5 - 16*a^3*b^2*c^6 + \\
& 2*a^4*c^7)*d^3 - 3*(b^9*c^2 - 9*a*b^7*c^3 + 27*a^2*b^5*c^4 - 30*a \\
& ^3*b^3*c^5 + 9*a^4*b*c^6)*d^2*e + 3*(b^{10}*c - 10*a*b^8*c^2 + 35*a \\
& ^2*b^6*c^3 - 50*a^3*b^4*c^4 + 25*a^4*b^2*c^5 - 2*a^5*c^6)*d*e^2 - \\
& (b^{11} - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 55*a^4*b^3 \\
& *c^4 - 11*a^5*b*c^5)*e^3 - (b^2*c^{11} - 4*a*c^{12})*\text{sqrt}(((b^{14}*c^6 \\
& - 12*a*b^{12}*c^7 + 56*a^2*b^{10}*c^8 - 128*a^3*b^8*c^9 + 148*a^4*b^6 \\
& *c^{10} - 80*a^5*b^4*c^{11} + 16*a^6*b^2*c^{12})*d^6 - 6*(b^{15}*c^5 - 1 \\
& 3*a*b^{13}*c^6 + 67*a^2*b^{11}*c^7 - 174*a^3*b^9*c^8 + 239*a^4*b^7*c^9 \\
& - 166*a^5*b^5*c^{10} + 50*a^6*b^3*c^{11} - 4*a^7*b*c^{12})*d^5*e + 3* \\
& (5*b^{16}*c^4 - 70*a*b^{14}*c^5 + 395*a^2*b^{12}*c^6 - 1150*a^3*b^{10}*c^7 \\
& + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650*a^6*b^4*c^{10} - 100* \\
& a^7*b^2*c^{11} + 3*a^8*c^{12})*d^4*e^2 - 2*(10*b^{17}*c^3 - 150*a*b^{15}* \\
& c^4 + 920*a^2*b^{13}*c^5 - 2970*a^3*b^{11}*c^6 + 5410*a^4*b^9*c^7 - 5 \\
& 530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3*c^{10} + 49*a^8*b* \\
& c^{11})*d^3*e^3 + 3*(5*b^{18}*c^2 - 80*a*b^{16}*c^3 + 530*a^2*b^{14}*c^4 \\
& - 1880*a^3*b^{12}*c^5 + 3855*a^4*b^{10}*c^6 - 4600*a^5*b^8*c^7 + 3050 \\
& *a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 125*a^8*b^2*c^{10} - 2*a^9*c^{11})* \\
& d^2*e^4 - 6*(b^{19}*c - 17*a*b^{17}*c^2 + 121*a^2*b^{15}*c^3 - 468*a^3* \\
& b^{13}*c^4 + 1068*a^4*b^{11}*c^5 - 1461*a^5*b^9*c^6 + 1163*a^6*b^7*c^7 \\
& - 496*a^7*b^5*c^8 + 95*a^8*b^3*c^9 - 5*a^9*b*c^{10})*d*e^5 + (b^2 \\
& 0 - 18*a*b^{18}*c + 137*a^2*b^{16}*c^2 - 574*a^3*b^{14}*c^3 + 1444*a^4* \\
& b^{12}*c^4 - 2232*a^5*b^{10}*c^5 + 2083*a^6*b^8*c^6 - 1106*a^7*b^6*c^7 \\
& + 295*a^8*b^4*c^8 - 30*a^9*b^2*c^9 + a^{10}*c^{10})*e^6)/(b^2*c^{22} \\
& - 4*a*c^{23}))/((b^2*c^{11} - 4*a*c^{12})) + 4*((a^4*b^7*c^4 - 6*a^5*b^5 \\
& *c^5 + 10*a^6*b^3*c^6 - 4*a^7*b*c^7)*d^5 - (4*a^4*b^8*c^3 - 27*a \\
& ^5*b^6*c^4 + 55*a^6*b^4*c^5 - 34*a^7*b^2*c^6 + 3*a^8*c^7)*d^4*e + \\
& 2*(3*a^4*b^9*c^2 - 22*a^5*b^7*c^3 + 51*a^6*b^5*c^4 - 40*a^7*b^3* \\
& c^5 + 7*a^8*b*c^6)*d^3*e^2 - 2*(2*a^4*b^{10}*c - 15*a^5*b^8*c^2 + 3 \\
& 5*a^6*b^6*c^3 - 25*a^7*b^4*c^4 + a^9*c^6)*d^2*e^3 + (a^4*b^{11} - 6 \\
& *a^5*b^9*c + 4*a^6*b^7*c^2 + 28*a^7*b^5*c^3 - 45*a^8*b^3*c^4 + 14 \\
& *a^9*b*c^5)*d*e^4 - (a^5*b^{10} - 9*a^6*b^8*c + 28*a^7*b^6*c^2 - 35 \\
& *a^8*b^4*c^3 + 15*a^9*b^2*c^4 - a^{10}*c^5)*e^5)*\text{sqrt}(e*x + d)) - 3 \\
& 15*\text{sqrt}(2)*c^5*e^3*\text{sqrt}(((b^8*c^3 - 8*a*b^6*c^4 + 20*a^2*b^4*c^5 \\
& - 16*a^3*b^2*c^6 + 2*a^4*c^7)*d^3 - 3*(b^9*c^2 - 9*a*b^7*c^3 + 27 \\
& *a^2*b^5*c^4 - 30*a^3*b^3*c^5 + 9*a^4*b*c^6)*d^2*e + 3*(b^{10}*c - \\
& 10*a*b^8*c^2 + 35*a^2*b^6*c^3 - 50*a^3*b^4*c^4 + 25*a^4*b^2*c^5 - \\
& 2*a^5*c^6)*d*e^2 - (b^{11} - 11*a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3* \\
& b^5*c^3 + 55*a^4*b^3*c^4 - 11*a^5*b*c^5)*e^3 - (b^2*c^{11} - 4*a*c^ \\
& ^{12})*\text{sqrt}(((b^{14}*c^6 - 12*a*b^{12}*c^7 + 56*a^2*b^{10}*c^8 - 128*a^3*b \\
& ^8*c^9 + 148*a^4*b^6*c^{10} - 80*a^5*b^4*c^{11} + 16*a^6*b^2*c^{12})*d^6 \\
& - 6*(b^{15}*c^5 - 13*a*b^{13}*c^6 + 67*a^2*b^{11}*c^7 - 174*a^3*b^9*c \\
& ^8 + 239*a^4*b^7*c^9 - 166*a^5*b^5*c^{10} + 50*a^6*b^3*c^{11} - 4*a^7 \\
& *b*c^{12})*d^5*e + 3*(5*b^{16}*c^4 - 70*a*b^{14}*c^5 + 395*a^2*b^{12}*c^6 \\
& - 1150*a^3*b^{10}*c^7 + 1835*a^4*b^8*c^8 - 1570*a^5*b^6*c^9 + 650* \\
& a^6*b^4*c^{10} - 100*a^7*b^2*c^{11} + 3*a^8*c^{12})*d^4*e^2 - 2*(10*b^{17} \\
& *c^3 - 150*a*b^{15}*c^4 + 920*a^2*b^{13}*c^5 - 2970*a^3*b^{11}*c^6 + 5 \\
& 410*a^4*b^9*c^7 - 5530*a^5*b^7*c^8 + 2960*a^6*b^5*c^9 - 700*a^7*b^3 \\
& *c^{10} + 49*a^8*b*c^{11})*d^3*e^3 + 3*(5*b^{18}*c^2 - 80*a*b^{16}*c^3 \\
& + 530*a^2*b^{14}*c^4 - 1880*a^3*b^{12}*c^5 + 3855*a^4*b^{10}*c^6 - 4600 \\
& *a^5*b^8*c^7 + 3050*a^6*b^6*c^8 - 1000*a^7*b^4*c^9 + 125*a^8*b^2* \\
& c^{10} - 2*a^9*c^{11})*d^2*e^4 - 6*(b^{19}*c - 17*a*b^{17}*c^2 + 121*a^2*
\end{aligned}$$



$$\begin{aligned}
& b^{15}c^3 - 468a^3b^{13}c^4 + 1068a^4b^{11}c^5 - 1461a^5b^9c^6 + 1163a^6b^7c^7 - 496a^7b^5c^8 + 95a^8b^3c^9 - 5a^9b \\
& *c^{10}) * d^e^5 + (b^{20} - 18a^*b^{18}c + 137a^2b^{16}c^2 - 574a^3b \\
& ^{14}c^3 + 1444a^4b^{12}c^4 - 2232a^5b^{10}c^5 + 2083a^6b^8c^6 - 1106a^7b^6c^7 + 295a^8b^4c^8 - 30a^9b^2c^9 + a^{10}c^{10} \\
& ) * e^6) / (b^2c^{22} - 4a^*c^{23})) / (b^2c^{11} - 4a^*c^{12})) * \log(-\sqrt{ \\
& (2) * ((b^{12}c^4 - 12a^*b^{10}c^5 + 54a^2b^8c^6 - 112a^3b^6c^7 \\
& + 104a^4b^4c^8 - 32a^5b^2c^9) * d^4 - (4b^{13}c^3 - 52a^*b^{11} \\
& c^4 + 260a^2b^9c^5 - 624a^3b^7c^6 + 725a^4b^5c^7 - 350 \\
& *a^5b^3c^8 + 40a^6b^*c^9) * d^3 * e + 3 * (2b^{14}c^2 - 28a^*b^{12}c^3 \\
& + 154a^2b^{10}c^4 - 420a^3b^8c^5 + 587a^4b^6c^6 - 387a^5 \\
& b^4c^7 + 93a^6b^2c^8 - 4a^7c^9) * d^2 * e^2 - (4b^{15}c - 60a^* \\
& b^{13}c^2 + 360a^2b^{11}c^3 - 1100a^3b^9c^4 + 1799a^4b^7c^5 - 1508a^5b^5c^6 + 561a^6b^3c^7 - 68a^7b^*c^8) * d^e^3 + ( \\
& b^{16} - 16a^*b^{14}c + 104a^2b^{12}c^2 - 352a^3b^{10}c^3 + 660a^4 \\
& b^8c^4 - 673a^5b^6c^5 + 342a^6b^4c^6 - 73a^7b^2c^7 + 4a^8c^8) * e^4 + ((b^6c^{12} - 8a^*b^4c^{13} + 18a^2b^2c^{14} - 8a^3 \\
& c^{15}) * d - (b^7c^{11} - 9a^*b^5c^{12} + 25a^2b^3c^{13} - 20a^3 \\
& *b^*c^{14}) * e) * \sqrt{((b^{14}c^6 - 12a^*b^{12}c^7 + 56a^2b^{10}c^8 - 1 \\
& 28a^3b^8c^9 + 148a^4b^6c^{10} - 80a^5b^4c^{11} + 16a^6b^2c^{12} \\
& ) * d^6 - 6 * (b^{15}c^5 - 13a^*b^{13}c^6 + 67a^2b^{11}c^7 - 174a^3 \\
& b^9c^8 + 239a^4b^7c^9 - 166a^5b^5c^{10} + 50a^6b^3c^{11} \\
& - 4a^7b^*c^{12}) * d^5 * e + 3 * (5b^{16}c^4 - 70a^*b^{14}c^5 + 395a^2 * \\
& b^{12}c^6 - 1150a^3b^{10}c^7 + 1835a^4b^8c^8 - 1570a^5b^6c^9 + 650a^6b^4c^{10} - 100a^7b^2c^{11} + 3a^8c^{12}) * d^4 * e^2 - 2 \\
& * (10b^{17}c^3 - 150a^*b^{15}c^4 + 920a^2b^{13}c^5 - 2970a^3b^{11} \\
& *c^6 + 5410a^4b^9c^7 - 5530a^5b^7c^8 + 2960a^6b^5c^9 - 7 \\
& 00a^7b^3c^{10} + 49a^8b^*c^{11}) * d^3 * e^3 + 3 * (5b^{18}c^2 - 80a^*b \\
& ^{16}c^3 + 530a^2b^{14}c^4 - 1880a^3b^{12}c^5 + 3855a^4b^{10}c^6 - 4600a^5b^8c^7 + 3050a^6b^6c^8 - 1000a^7b^4c^9 + 125a^8 \\
& b^2c^{10} - 2a^9c^{11}) * d^2 * e^4 - 6 * (b^{19}c - 17a^*b^{17}c^2 + \\
& 121a^2b^{15}c^3 - 468a^3b^{13}c^4 + 1068a^4b^{11}c^5 - 1461a^5 \\
& b^9c^6 + 1163a^6b^7c^7 - 496a^7b^5c^8 + 95a^8b^3c^9 - 5a^9b^*c^{10}) * d^e^5 + (b^{20} - 18a^*b^{18}c + 137a^2b^{16}c^2 - 5 \\
& 74a^3b^{14}c^3 + 1444a^4b^{12}c^4 - 2232a^5b^{10}c^5 + 2083a^6 \\
& b^8c^6 - 1106a^7b^6c^7 + 295a^8b^4c^8 - 30a^9b^2c^9 + a^{10}c^{10} \\
& ) * e^6) / (b^2c^{22} - 4a^*c^{23})) * \sqrt{((b^8c^3 - 8a^*b^6 \\
& *c^4 + 20a^2b^4c^5 - 16a^3b^2c^6 + 2a^4c^7) * d^3 - 3 * (b^9c^2 - 9a^*b^7c^3 + 27a^2b^5c^4 - 30a^3b^3c^5 + 9a^4b^*c^6 \\
& ) * d^2 * e + 3 * (b^{10}c - 10a^*b^8c^2 + 35a^2b^6c^3 - 50a^3b^4c^4 + 25a^4b^2c^5 - 2a^5c^6) * d^e^2 - (b^{11} - 11a^*b^9c + 44 \\
& *a^2b^7c^2 - 77a^3b^5c^3 + 55a^4b^3c^4 - 11a^5b^*c^5) * e^3 - (b^2c^{11} - 4a^*c^{12}) * \sqrt{((b^{14}c^6 - 12a^*b^{12}c^7 + 56a^2 \\
& b^{10}c^8 - 128a^3b^8c^9 + 148a^4b^6c^{10} - 80a^5b^4c^{11} \\
& + 16a^6b^2c^{12}) * d^6 - 6 * (b^{15}c^5 - 13a^*b^{13}c^6 + 67a^2b^{11} \\
& c^7 - 174a^3b^9c^8 + 239a^4b^7c^9 - 166a^5b^5c^{10} + 5 \\
& 0a^6b^3c^{11} - 4a^7b^*c^{12}) * d^5 * e + 3 * (5b^{16}c^4 - 70a^*b^{14} \\
& c^5 + 395a^2b^{12}c^6 - 1150a^3b^{10}c^7 + 1835a^4b^8c^8 - 1 \\
& 570a^5b^6c^9 + 650a^6b^4c^{10} - 100a^7b^2c^{11} + 3a^8c^{12} \\
& ) * d^4 * e^2 - 2 * (10b^{17}c^3 - 150a^*b^{15}c^4 + 920a^2b^{13}c^5 - \\
& 2970a^3b^{11}c^6 + 5410a^4b^9c^7 - 5530a^5b^7c^8 + 2960a^6 \\
& b^5c^9 - 700a^7b^3c^{10} + 49a^8b^*c^{11}) * d^3 * e^3 + 3 * (5b^{18} \\
& c^2 - 80a^*b^{16}c^3 + 530a^2b^{14}c^4 - 1880a^3b^{12}c^5 + 38 \\
& 55a^4b^{10}c^6 - 4600a^5b^8c^7 + 3050a^6b^6c^8 - 1000a^7b^4 \\
& c^9 + 125a^8b^2c^{10} - 2a^9c^{11}) * d^2 * e^4 - 6 * (b^{19}c - 17 \\
& *a^*b^{17}c^2 + 121a^2b^{15}c^3 - 468a^3b^{13}c^4 + 1068a^4b^{11} \\
& *c^5 - 1461a^5b^9c^6 + 1163a^6b^7c^7 - 496a^7b^5c^8 + 95 \\
& *a^8b^3c^9 - 5a^9b^*c^{10}) * d^e^5 + (b^{20} - 18a^*b^{18}c + 137a^2 \\
& b^{16}c^2 - 574a^3b^{14}c^3 + 1444a^4b^{12}c^4 - 2232a^5b^{10} \\
& *c^5 + 2083a^6b^8c^6 - 1106a^7b^6c^7 + 295a^8b^4c^8 - 30 \\
& *a^9b^2c^9 + a^{10}c^{10}) * e^6) / (b^2c^{22} - 4a^*c^{23})) / (b^2c^{11} \\
& - 4a^*c^{12})) + 4 * ((a^4b^7c^4 - 6a^5b^5c^5 + 10a^6b^3c^6 - \\
& 4a^7b^*c^7) * d^5 - (4a^4b^8c^3 - 27a^5b^6c^4 + 55a^6b^4c^5 - \\
& 34a^7b^2c^6 + 3a^8c^7) * d^4 * e + 2 * (3a^4b^9c^2 - 22a^5 \\
& b^7c^3 + 51a^6b^5c^4 - 40a^7b^3c^5 + 7a^8b^*c^6) * d^3 * e^2 - 2 * (2a^4b^{10}c - 15a^5b^8c^2 + 35a^6b^6c^3 - 25a^7b^4 \\
& c^4 + a^9c^6) * d^2 * e^3 + (a^4b^{11} - 6a^5b^9c + 4a^6b^7c^2 + 28a^7b^5c^3 - 45a^8b^3c^4 + 14a^9b^*c^5) * d^e^4 - (a^5 \\
& b^{10} - 9a^6b^8c + 28a^7b^6c^2 - 35a^8b^4c^3 + 15a^9b^2 \\
& c^4 - a^{10}c^5) * e^5) * \sqrt{(e^*x + d)} - 4 * (35c^4e^4x^4 + 8c^4 \\
& *d^4 + 18b^*c^3d^3e + 63 * (b^2c^2 - a^*c^3) * d^2 * e^2 - 420 * (b^3c \\
& - 2a^*b^*c^2) * d^e^3 + 315 * (b^4 - 3a^*b^2c + a^2c^2) * e^4 + 5 * (10
\end{aligned}$$

$$\frac{(c^4 d e^3 - 9 b c^3 e^4) x^3 + 3(c^4 d^2 e^2 - 24 b c^3 d e^3 + 21(b^2 c^2 - a c^3) e^4) x^2 - (4 c^4 d^3 e + 9 b c^3 d^2 e^2 - 126(b^2 c^2 - a c^3) d e^3 + 105(b^3 c - 2 a b c^2) e^4) x}{c^5 e^3} \sqrt{e x + d}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*4\*(e\*x+d)\*\*(3/2)/(c\*x\*\*2+b\*x+a),x)

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*x^4/(c\*x^2 + b\*x + a),x, algorithm="giac")

[Out] Timed out

$$3.534 \quad \int \frac{x^3(d+ex)^{3/2}}{a+bx+cx^2} dx$$

Optimal. Leaf size=581

$$\sqrt{2} \left( -\frac{4a^2c^3de - b^3c(cd^2 - 5ae^2) - 8ab^2c^2de + abc^2(3cd^2 - 5ae^2) + b^5(-e^2) + 2b^4cde}{\sqrt{b^2 - 4ac}} - b^2c(cd^2 - 3ae^2) - 4abc^2de + ac^2(cd^2 - ae^2) + b^4(-e^2) \right)$$


---


$$c^{9/2} \sqrt{2cd - e} \left( b - \sqrt{b^2 - 4ac} \right)$$


---


$$\sqrt{2} \left( \frac{4a^2c^3de - b^3c(cd^2 - 5ae^2) - 8ab^2c^2de + abc^2(3cd^2 - 5ae^2) + b^5(-e^2) + 2b^4cde}{\sqrt{b^2 - 4ac}} - b^2c(cd^2 - 3ae^2) - 4abc^2de + ac^2(cd^2 - ae^2) + b^4(-e^2) \right)$$


---


$$c^{9/2} \sqrt{2cd - e} \left( \sqrt{b^2 - 4ac} + b \right)$$


---


$$+ \frac{2(b^2 - ac)(d + ex)^{3/2}}{3c^3} + \frac{2\sqrt{d + ex}(2abce - ac^2d + b^3(-e) + b^2cd)}{c^4} - \frac{2(d + ex)^{5/2}(be + cd)}{5c^2e^2} + \frac{2(d + ex)^{7/2}}{7ce^2}$$

[Out] (2\*(b^2\*c\*d - a\*c^2\*d - b^3\*e + 2\*a\*b\*c\*e)\*Sqrt[d + e\*x])/c^4 + (2\*(b^2 - a\*c)\*(d + e\*x)^(3/2))/(3\*c^3) - (2\*(c\*d + b\*e)\*(d + e\*x)^(5/2))/(5\*c^2\*e^2) + (2\*(d + e\*x)^(7/2))/(7\*c\*e^2) + (Sqrt[2]\*(2\*b^3\*c\*d\*e - 4\*a\*b\*c^2\*d\*e - b^4\*e^2 - b^2\*c\*(c\*d^2 - 3\*a\*e^2) + a\*c^2\*(c\*d^2 - a\*e^2) - (2\*b^4\*c\*d\*e - 8\*a\*b^2\*c^2\*d\*e + 4\*a^2\*c^3\*d\*e - b^5\*e^2 - b^3\*c\*(c\*d^2 - 5\*a\*e^2) + a\*b\*c^2\*(3\*c\*d^2 - 5\*a\*e^2))/Sqrt[b^2 - 4\*a\*c])\*ArcTanh[(Sqrt[2]\*Sqrt[c]\*Sqrt[d + e\*x])/Sqrt[2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e]]/(c^(9/2)\*Sqrt[2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e]) + (Sqrt[2]\*(2\*b^3\*c\*d\*e - 4\*a\*b\*c^2\*d\*e - b^4\*e^2 - b^2\*c\*(c\*d^2 - 3\*a\*e^2) + a\*c^2\*(c\*d^2 - a\*e^2) + (2\*b^4\*c\*d\*e - 8\*a\*b^2\*c^2\*d\*e + 4\*a^2\*c^3\*d\*e - b^5\*e^2 - b^3\*c\*(c\*d^2 - 5\*a\*e^2) + a\*b\*c^2\*(3\*c\*d^2 - 5\*a\*e^2))/Sqrt[b^2 - 4\*a\*c])\*ArcTanh[(Sqrt[2]\*Sqrt[c]\*Sqrt[d + e\*x])/Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e]]/(c^(9/2)\*Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e])

Rubi [A] time = 27.4465, antiderivative size = 581, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\sqrt{2} \left( -\frac{4a^2c^3de - b^3c(cd^2 - 5ae^2) - 8ab^2c^2de + abc^2(3cd^2 - 5ae^2) + b^5(-e^2) + 2b^4cde}{\sqrt{b^2 - 4ac}} - b^2c(cd^2 - 3ae^2) - 4abc^2de + ac^2(cd^2 - ae^2) + b^4(-e^2) \right)$$


---


$$c^{9/2} \sqrt{2cd - e} \left( b - \sqrt{b^2 - 4ac} \right)$$


---


$$\sqrt{2} \left( \frac{4a^2c^3de - b^3c(cd^2 - 5ae^2) - 8ab^2c^2de + abc^2(3cd^2 - 5ae^2) + b^5(-e^2) + 2b^4cde}{\sqrt{b^2 - 4ac}} - b^2c(cd^2 - 3ae^2) - 4abc^2de + ac^2(cd^2 - ae^2) + b^4(-e^2) \right)$$


---


$$c^{9/2} \sqrt{2cd - e} \left( \sqrt{b^2 - 4ac} + b \right)$$


---


$$+ \frac{2(b^2 - ac)(d + ex)^{3/2}}{3c^3} + \frac{2\sqrt{d + ex}(2abce - ac^2d + b^3(-e) + b^2cd)}{c^4} - \frac{2(d + ex)^{5/2}(be + cd)}{5c^2e^2} + \frac{2(d + ex)^{7/2}}{7ce^2}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(d + e\*x)^(3/2))/(a + b\*x + c\*x^2), x]

[Out] (2\*(b^2\*c\*d - a\*c^2\*d - b^3\*e + 2\*a\*b\*c\*e)\*Sqrt[d + e\*x])/c^4 + (2\*(b^2 - a\*c)\*(d + e\*x)^(3/2))/(3\*c^3) - (2\*(c\*d + b\*e)\*(d + e\*x)^(5/2))/(5\*c^2\*e^2) + (2\*(d + e\*x)^(7/2))/(7\*c\*e^2) + (Sqrt[2]\*(2\*b^3\*c\*d\*e - 4\*a\*b\*c^2\*d\*e - b^4\*e^2 - b^2\*c\*(c\*d^2 - 3\*a\*e^2) + a\*c^2\*(c\*d^2 - a\*e^2) - (2\*b^4\*c\*d\*e - 8\*a\*b^2\*c^2\*d\*e + 4\*a^2\*c^3\*d\*e - b^5\*e^2 - b^3\*c\*(c\*d^2 - 5\*a\*e^2) + a\*b\*c^2\*(3\*c\*d^2 - 5\*a\*e^2))/Sqrt[b^2 - 4\*a\*c])\*ArcTanh[(Sqrt[2]\*Sqrt[c]\*Sqrt[d + e\*x])/Sqrt[2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e]]/(c^(9/2)\*Sqrt[2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e]) + (Sqrt[2]\*(2\*b^3\*c\*d\*e - 4\*a\*b\*c^2\*d\*e - b^4\*e^2 - b^2\*c\*(c\*d^2 - 3\*a\*e^2) + a\*c^2\*(c\*d^2 - a\*e^2) + (2\*b^4\*c\*d\*e - 8\*a\*b^2\*c^2\*d\*e + 4\*a^2\*c^3\*d\*e - b^5\*e^2 - b^3\*c\*(c\*d^2 - 5\*a\*e^2) + a\*b\*c^2\*(3\*c\*d^2 - 5\*a\*e^2))/Sqrt[b^2 - 4\*a\*c])\*ArcTanh[(Sqrt[2]\*Sqrt[c]\*Sqrt[d + e\*x])/Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e]]/(c^(9/2)\*Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e])

$$\frac{1}{\sqrt{2cd - (b - \sqrt{b^2 - 4ac})e}} \Big/ (c^{9/2} \sqrt{2cd - (b - \sqrt{b^2 - 4ac})e}) + (\sqrt{2} (2b^3cd^2e - 4ab^2c^2d^2e - b^4e^2 - b^2c(c^2d^2 - 3a^2e^2) + ac^2(c^2d^2 - a^2e^2) + (2b^4cd^2e - 8ab^2c^2d^2e + 4a^2c^3d^2e - b^5e^2 - b^3c(c^2d^2 - 5a^2e^2) + ab^2c^2(3c^2d^2 - 5a^2e^2)) / \sqrt{b^2 - 4ac}) \operatorname{ArcTanh}(\frac{\sqrt{2} \sqrt{c} \sqrt{d + ex}}{\sqrt{2cd - (b + \sqrt{b^2 - 4ac})e}}) \Big/ (c^{9/2} \sqrt{2cd - (b + \sqrt{b^2 - 4ac})e})$$

**Rubi in Sympy** [F-1] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**3*(e*x+d)**(3/2)/(c*x**2+b*x+a), x)`

[Out] Timed out

**Mathematica** [A] time = 1.81059, size = 680, normalized size = 1.17

$$\frac{2\sqrt{d+ex} (7c^2e (5ae(4d+ex) + 3b(d+ex)^2) - 35bce^2(6ae + 4bd + bex) + 105b^3e^3 + 3c^3(2d - 5ex)(d+ex)^2)}{105c^4e^2}$$

$$\sqrt{2} \left( abc^2 \left( e \left( 4d\sqrt{b^2 - 4ac} - 5ae \right) + 3cd^2 \right) + ac^2 \left( cd \left( 4ae - d\sqrt{b^2 - 4ac} \right) + ae^2\sqrt{b^2 - 4ac} \right) + b^2c \left( cd \left( d\sqrt{b^2 - 4ac} - 8ae \right) \right. \right.$$

$$\left. \left. c^{9/2}\sqrt{b^2 - 4ac}\sqrt{e} \right) \right)$$

$$\sqrt{2} \left( abc^2 \left( e \left( 4d\sqrt{b^2 - 4ac} + 5ae \right) - 3cd^2 \right) + ac^2 \left( ae^2\sqrt{b^2 - 4ac} - cd \left( d\sqrt{b^2 - 4ac} + 4ae \right) \right) + b^2c \left( cd \left( d\sqrt{b^2 - 4ac} + 8ae \right) \right. \right.$$

$$\left. \left. c^{9/2}\sqrt{b^2 - 4ac}\sqrt{2cd} \right) \right)$$

Antiderivative was successfully verified.

[In] `Integrate[(x^3*(d + e*x)^(3/2))/(a + b*x + c*x^2), x]`

[Out]  $(-2\sqrt{d+ex} (105b^3e^3 + 3c^3(2d - 5ex)(d+ex)^2 - 35b^2c^2e^2(4bd + 6ae + b^2e^2) + 7c^2e^2(3b^2(d+ex)^2 + 5a^2e^2(4d+ex)))) / (105c^4e^2) - (\sqrt{2} (-(b^5e^2) + b^4e^2(2cd + \sqrt{b^2 - 4ac})e) + b^2c^2(-3a\sqrt{b^2 - 4ac})e^2 + c^2d(\sqrt{b^2 - 4ac})d - 8a^2e) - b^3c^2(c^2d^2 + e(2\sqrt{b^2 - 4ac})d - 5a^2e) + ab^2c^2(3c^2d^2 + e(4\sqrt{b^2 - 4ac})d - 5a^2e) + ac^2(a\sqrt{b^2 - 4ac})e^2 + c^2d(-(\sqrt{b^2 - 4ac})d + 4a^2e)) \operatorname{ArcTanh}(\frac{\sqrt{2} \sqrt{c} \sqrt{d + ex}}{\sqrt{2cd - b^2e + \sqrt{b^2 - 4ac})e}}) \Big/ (c^{9/2} \sqrt{b^2 - 4ac}) \sqrt{2cd - b^2e + \sqrt{b^2 - 4ac})e} - (\sqrt{2} (b^5e^2 + b^4e^2(-2cd + \sqrt{b^2 - 4ac})e) + ac^2(a\sqrt{b^2 - 4ac})e^2 - c^2d(\sqrt{b^2 - 4ac})d + 4a^2e) + b^3c^2(c^2d^2 - e(2\sqrt{b^2 - 4ac})d + 5a^2e) + ab^2c^2(-3c^2d^2 + e(4\sqrt{b^2 - 4ac})d + 5a^2e) + b^2c^2(-3a\sqrt{b^2 - 4ac})e^2 + c^2d(\sqrt{b^2 - 4ac})d + 8a^2e)) \operatorname{ArcTanh}(\frac{\sqrt{2} \sqrt{c} \sqrt{d + ex}}{\sqrt{2cd - (b + \sqrt{b^2 - 4ac})e}}) \Big/ (c^{9/2} \sqrt{b^2 - 4ac}) \sqrt{2cd - (b + \sqrt{b^2 - 4ac})e}$

**Maple** [B] time = 0.076, size = 2988, normalized size = 5.1

output too large to display



$$e^{-2cx+d}(-e^{2(4ac-b^2)^{1/2}})^{1/2}c^{1/2})^{1/2})b^3d^2+5e^3/c^2/(-e^{2(4ac-b^2)^{1/2}})^{1/2}2^{1/2}/((-b^2e+2c^2d+(-e^{2(4ac-b^2)^{1/2}})^{1/2})^{1/2})c^{1/2})\operatorname{arctanh}(c(e^x+d)^{1/2}2^{1/2}/((-b^2e+2c^2d+(-e^{2(4ac-b^2)^{1/2}})^{1/2})^{1/2})^{1/2})^{1/2})^{1/2})a^2b-4e^2/c/(-e^{2(4ac-b^2)^{1/2}})^{1/2}2^{1/2}/((-b^2e+2c^2d+(-e^{2(4ac-b^2)^{1/2}})^{1/2})^{1/2})^{1/2})c^{1/2})\operatorname{arctanh}(c(e^x+d)^{1/2}2^{1/2}/((-b^2e+2c^2d+(-e^{2(4ac-b^2)^{1/2}})^{1/2})^{1/2})^{1/2})^{1/2})^{1/2})^{1/2})a^2d-5e^3/c^3/(-e^{2(4ac-b^2)^{1/2}})^{1/2}2^{1/2}/((-b^2e+2c^2d+(-e^{2(4ac-b^2)^{1/2}})^{1/2})^{1/2})^{1/2})^{1/2})c^{1/2})\operatorname{arctanh}(c(e^x+d)^{1/2}2^{1/2}/((-b^2e+2c^2d+(-e^{2(4ac-b^2)^{1/2}})^{1/2})^{1/2})^{1/2})^{1/2})^{1/2})^{1/2})^{1/2})^{1/2})a^2b^3-2e^2/c^3/(-e^{2(4ac-b^2)^{1/2}})^{1/2}2^{1/2}/((-b^2e+2c^2d+(-e^{2(4ac-b^2)^{1/2}})^{1/2})^{1/2})^{1/2})^{1/2})^{1/2})^{1/2})^{1/2})c^{1/2})\operatorname{arctanh}(c(e^x+d)^{1/2}2^{1/2}/((-b^2e+2c^2d+(-e^{2(4ac-b^2)^{1/2}})^{1/2})^{1/2})^{1/2})^{1/2})^{1/2})^{1/2})^{1/2})^{1/2})^{1/2})^{1/2})b^4d$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}x^3}{cx^2+bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*x^3/(c\*x^2 + b\*x + a), x, algorithm="maxima")

[Out] integrate((e\*x + d)^(3/2)\*x^3/(c\*x^2 + b\*x + a), x)

**Fricas [A]** time = 5.8051, size = 15470, normalized size = 26.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*x^3/(c\*x^2 + b\*x + a), x, algorithm="fricas")

[Out] 
$$-1/210*(105*\sqrt{2})c^4e^2\sqrt{((b^6c^3 - 6ab^4c^4 + 9a^2b^2c^5 - 2a^3c^6)d^3 - 3(b^7c^2 - 7ab^5c^3 + 14a^2b^3c^4 - 7a^3b^2c^5)d^2e + 3(b^8c - 8ab^6c^2 + 20a^2b^4c^3 - 16a^3b^2c^4 + 2a^4c^5)d^2e^2 - (b^9 - 9ab^7c + 27a^2b^5c^2 - 30a^3b^3c^3 + 9a^4b^2c^4)e^3 + (b^2c^9 - 4ac^10)\sqrt{((b^{10}c^6 - 8ab^8c^7 + 22a^2b^6c^8 - 24a^3b^4c^9 + 9a^4b^2c^{10})d^6 - 6(b^{11}c^5 - 9ab^9c^6 + 29a^2b^7c^7 - 40a^3b^5c^8 + 22a^4b^3c^9 - 3a^5b^2c^{10})d^5e + 3(5b^{12}c^4 - 50ab^{10}c^5 + 185a^2b^8c^6 - 310a^3b^6c^7 + 230a^4b^4c^8 - 60a^5b^2c^9 + 3a^6c^{10})d^4e^2 - 2(10b^{13}c^3 - 110ab^{11}c^4 + 460a^2b^9c^5 - 910a^3b^7c^6 + 860a^4b^5c^7 - 340a^5b^3c^8 + 39a^6b^2c^9)d^3e^3 + 3(5b^{14}c^2 - 60ab^{12}c^3 + 280a^2b^{10}c^4 - 640a^3b^8c^5 + 740a^4b^6c^6 - 400a^5b^4c^7 + 80a^6b^2c^8 - 2a^7c^9)d^2e^4 - 6(b^{15}c - 13ab^{13}c^2 + 67a^2b^{11}c^3 - 174a^3b^9c^4 + 239a^4b^7c^5 - 166a^5b^5c^6 + 50a^6b^3c^7 - 4a^7b^2c^8)d^2e^5 + (b^{16} - 14ab^{14}c + 79a^2b^{12}c^2 - 230a^3b^{10}c^3 + 367a^4b^8c^4 - 314a^5b^6c^5 + 130a^6b^4c^6 - 20a^7b^2c^7 + a^8c^8)e^6)/((b^2c^{18} - 4a^2c^{19}))/((b^2c^9 - 4a^2c^{10})\log(\sqrt{2})((b^9c^4 - 9ab^7c^5 + 27a^2b^5c^6 - 31a^3b^3c^7 + 12a^4b^2c^8)d^4 - (4b^{10}c^3 - 40ab^8c^4 + 140a^2b^6c^5 - 203a^3b^4c^6 + 111a^4b^2c^7 - 12a^5c^8)d^3e + 3(2b^{11}c^2 - 22ab^9c^3 + 88a^2b^7c^4 - 155a^3b^5c^5 + 114a^4b^3c^6 - 24a^5b^2c^7)d^2e^2 - (4b^{12}c - 48ab^{10}c^2 + 216a^2b^8c^3 - 449a^3b^6c^4 + 423a^4b^4c^5 - 141a^5b^2c^6 + 4a^6c^7)d^2e^3 + (b^{13} - 13ab^{11}c + 65a^2b^9c^2 - 156a^3b^7c^3 + 181a^4b^5c^4 - 86a^5b^3c^5 + 8a^6b^2c^6)e^4 - ((b^5c^{10} - 7ab^3c^{11} + 12a^2b^2c^{12})d - (b^6c^9 - 8ab^4c^{10} + 18a^2b^2c^{11} - 8a^3c^{12})e)\sqrt{((b^{10}c^6 - 8ab^8c^7 + 22a^2b^6c^8 - 24a^3b^4c^9 + 9a^4b^2c^{10})d^6 - 6(b^{11}c^5 - 9ab^9c^6 + 29a^2b^7c^7 -$$

$$\begin{aligned}
& 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^{10})*d^5*e + 3*(5*b^{12} \\
& *c^4 - 50*a*b^{10}*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4 \\
& *b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^{10})*d^4*e^2 - 2*(10*b^{13}*c^3 \\
& - 110*a*b^{11}*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b \\
& ^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^{14}*c^2 \\
& - 60*a*b^{12}*c^3 + 280*a^2*b^{10}*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6 \\
& *c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6 \\
& *(b^{15}*c - 13*a*b^{13}*c^2 + 67*a^2*b^{11}*c^3 - 174*a^3*b^9*c^4 + 23 \\
& 9*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d \\
& *e^5 + (b^{16} - 14*a*b^{14}*c + 79*a^2*b^{12}*c^2 - 230*a^3*b^{10}*c^3 + \\
& 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2 \\
& *c^7 + a^8*c^8)*e^6)/(b^2*c^{18} - 4*a*c^{19}))*sqrt(((b^6*c^3 - 6*a \\
& *b^4*c^4 + 9*a^2*b^2*c^5 - 2*a^3*c^6)*d^3 - 3*(b^7*c^2 - 7*a*b^5* \\
& c^3 + 14*a^2*b^3*c^4 - 7*a^3*b*c^5)*d^2*e + 3*(b^8*c - 8*a*b^6*c^2 \\
& + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d*e^2 - (b^9 - 9 \\
& *a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e^3 + ( \\
& b^2*c^9 - 4*a*c^{10})*sqrt(((b^{10}*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 \\
& - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^{10})*d^6 - 6*(b^{11}*c^5 - 9*a*b^9* \\
& c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b* \\
& c^{10})*d^5*e + 3*(5*b^{12}*c^4 - 50*a*b^{10}*c^5 + 185*a^2*b^8*c^6 - 3 \\
& 10*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^{10})*d \\
& ^4*e^2 - 2*(10*b^{13}*c^3 - 110*a*b^{11}*c^4 + 460*a^2*b^9*c^5 - 910* \\
& a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d \\
& ^3*e^3 + 3*(5*b^{14}*c^2 - 60*a*b^{12}*c^3 + 280*a^2*b^{10}*c^4 - 640*a \\
& ^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - \\
& 2*a^7*c^9)*d^2*e^4 - 6*(b^{15}*c - 13*a*b^{13}*c^2 + 67*a^2*b^{11}*c^3 \\
& - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b \\
& ^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^{16} - 14*a*b^{14}*c + 79*a^2*b^{12}*c \\
& ^2 - 230*a^3*b^{10}*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a \\
& ^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^{18} - 4*a*c^{19} \\
& ))/(b^2*c^9 - 4*a*c^{10})) - 4*((a^3*b^5*c^4 - 4*a^4*b^3*c^5 + 3*a^5 \\
& *b*c^6)*d^5 - (4*a^3*b^6*c^3 - 19*a^4*b^4*c^4 + 21*a^5*b^2*c^5 - \\
& 3*a^6*c^6)*d^4*e + 2*(3*a^3*b^7*c^2 - 16*a^4*b^5*c^3 + 22*a^5*b^3 \\
& *c^4 - 6*a^6*b*c^5)*d^3*e^2 - 2*(2*a^3*b^8*c - 11*a^4*b^6*c^2 + \\
& 15*a^5*b^4*c^3 - 2*a^6*b^2*c^4 - a^7*c^5)*d^2*e^3 + (a^3*b^9 - 4* \\
& a^4*b^7*c - 3*a^5*b^5*c^2 + 20*a^6*b^3*c^3 - 11*a^7*b*c^4)*d*e^4 \\
& - (a^4*b^8 - 7*a^5*b^6*c + 15*a^6*b^4*c^2 - 10*a^7*b^2*c^3 + a^8* \\
& c^4)*e^5)*sqrt(e*x + d) - 105*sqrt(2)*c^4*e^2*sqrt(((b^6*c^3 - 6 \\
& *a*b^4*c^4 + 9*a^2*b^2*c^5 - 2*a^3*c^6)*d^3 - 3*(b^7*c^2 - 7*a*b^5 \\
& *c^3 + 14*a^2*b^3*c^4 - 7*a^3*b*c^5)*d^2*e + 3*(b^8*c - 8*a*b^6* \\
& c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c^5)*d*e^2 - (b^9 - \\
& 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e^3 + \\
& (b^2*c^9 - 4*a*c^{10})*sqrt(((b^{10}*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 \\
& - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^{10})*d^6 - 6*(b^{11}*c^5 - 9*a*b^9 \\
& *c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5* \\
& b*c^{10})*d^5*e + 3*(5*b^{12}*c^4 - 50*a*b^{10}*c^5 + 185*a^2*b^8*c^6 - \\
& 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^{10}) \\
& *d^4*e^2 - 2*(10*b^{13}*c^3 - 110*a*b^{11}*c^4 + 460*a^2*b^9*c^5 - 91 \\
& 0*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9) \\
& *d^3*e^3 + 3*(5*b^{14}*c^2 - 60*a*b^{12}*c^3 + 280*a^2*b^{10}*c^4 - 640 \\
& *a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 \\
& - 2*a^7*c^9)*d^2*e^4 - 6*(b^{15}*c - 13*a*b^{13}*c^2 + 67*a^2*b^{11}*c \\
& ^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6 \\
& *b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^{16} - 14*a*b^{14}*c + 79*a^2*b^{12} \\
& *c^2 - 230*a^3*b^{10}*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130 \\
& *a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^{18} - 4*a*c^{19} \\
& 9)))/(b^2*c^9 - 4*a*c^{10}))*log(-sqrt(2)*((b^9*c^4 - 9*a*b^7*c^5 + \\
& 27*a^2*b^5*c^6 - 31*a^3*b^3*c^7 + 12*a^4*b*c^8)*d^4 - (4*b^{10}*c^3 \\
& - 40*a*b^8*c^4 + 140*a^2*b^6*c^5 - 203*a^3*b^4*c^6 + 111*a^4*b^2 \\
& *c^7 - 12*a^5*c^8)*d^3*e + 3*(2*b^{11}*c^2 - 22*a*b^9*c^3 + 88*a^2 \\
& *b^7*c^4 - 155*a^3*b^5*c^5 + 114*a^4*b^3*c^6 - 24*a^5*b*c^7)*d^2* \\
& e^2 - (4*b^{12}*c - 48*a*b^{10}*c^2 + 216*a^2*b^8*c^3 - 449*a^3*b^6*c \\
& ^4 + 423*a^4*b^4*c^5 - 141*a^5*b^2*c^6 + 4*a^6*c^7)*d*e^3 + (b^{13} \\
& - 13*a*b^{11}*c + 65*a^2*b^9*c^2 - 156*a^3*b^7*c^3 + 181*a^4*b^5*c \\
& ^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e^4 - ((b^5*c^{10} - 7*a*b^3*c^{11} \\
& + 12*a^2*b*c^{12})*d - (b^6*c^9 - 8*a*b^4*c^{10} + 18*a^2*b^2*c^{11} \\
& - 8*a^3*c^{12})*e)*sqrt(((b^{10}*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - \\
& 24*a^3*b^4*c^9 + 9*a^4*b^2*c^{10})*d^6 - 6*(b^{11}*c^5 - 9*a*b^9*c^6 \\
& + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^{10} \\
& )*d^5*e + 3*(5*b^{12}*c^4 - 50*a*b^{10}*c^5 + 185*a^2*b^8*c^6 - 310* \\
& a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^{10})*d^4* \\
& e^2 - 2*(10*b^{13}*c^3 - 110*a*b^{11}*c^4 + 460*a^2*b^9*c^5 - 910*a^3
\end{aligned}$$

$$\begin{aligned}
& *b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3* \\
& e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2*b^10*c^4 - 640*a^3* \\
& b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2* \\
& a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 + 67*a^2*b^11*c^3 - \\
& 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3* \\
& c^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14*c + 79*a^2*b^12*c^2 \\
& - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6* \\
& b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2*c^18 - 4*a*c^19))) * \\
& \text{sqrt}(((b^6*c^3 - 6*a*b^4*c^4 + 9*a^2*b^2*c^5 - 2*a^3*c^6)*d^3 - 3 \\
& *(b^7*c^2 - 7*a*b^5*c^3 + 14*a^2*b^3*c^4 - 7*a^3*b*c^5)*d^2*e + 3 \\
& *(b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4*c \\
& ^5)*d*e^2 - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 + \\
& 9*a^4*b*c^4)*e^3 + (b^2*c^9 - 4*a*c^10)*\text{sqrt}(((b^10*c^6 - 8*a*b^8 \\
& *c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6* \\
& (b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4 \\
& *b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5 + \\
& 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2 \\
& *c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 46 \\
& 0*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c \\
& ^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280* \\
& a^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 \\
& + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c \\
& ^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5 \\
& *b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14*a*b^14 \\
& *c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 314 \\
& *a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(( \\
& b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10)) - 4*((a^3*b^5*c^4 - \\
& 4*a^4*b^3*c^5 + 3*a^5*b*c^6)*d^5 - (4*a^3*b^6*c^3 - 19*a^4*b^4*c^4 \\
& + 21*a^5*b^2*c^5 - 3*a^6*c^6)*d^4*e + 2*(3*a^3*b^7*c^2 - 16*a^4 \\
& *b^5*c^3 + 22*a^5*b^3*c^4 - 6*a^6*b*c^5)*d^3*e^2 - 2*(2*a^3*b^8*c \\
& - 11*a^4*b^6*c^2 + 15*a^5*b^4*c^3 - 2*a^6*b^2*c^4 - a^7*c^5)*d^2 \\
& *e^3 + (a^3*b^9 - 4*a^4*b^7*c - 3*a^5*b^5*c^2 + 20*a^6*b^3*c^3 - \\
& 11*a^7*b*c^4)*d*e^4 - (a^4*b^8 - 7*a^5*b^6*c + 15*a^6*b^4*c^2 - 1 \\
& 0*a^7*b^2*c^3 + a^8*c^4)*e^5)*\text{sqrt}(e*x + d)) + 105*\text{sqrt}(2)*c^4*e^4 \\
& 2*\text{sqrt}(((b^6*c^3 - 6*a*b^4*c^4 + 9*a^2*b^2*c^5 - 2*a^3*c^6)*d^3 - 3 \\
& *(b^7*c^2 - 7*a*b^5*c^3 + 14*a^2*b^3*c^4 - 7*a^3*b*c^5)*d^2*e + 3 \\
& *(b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^3*b^2*c^4 + 2*a^4 \\
& *c^5)*d*e^2 - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 - 30*a^3*b^3*c^3 \\
& + 9*a^4*b*c^4)*e^3 - (b^2*c^9 - 4*a*c^10)*\text{sqrt}(((b^10*c^6 - 8*a*b^8 \\
& *c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6* \\
& (b^11*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22* \\
& a^4*b^3*c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5 \\
& + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2 \\
& *c^9 + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + \\
& 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3 \\
& *c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 28 \\
& 0*a^2*b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4* \\
& c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13 \\
& *c^2 + 67*a^2*b^11*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166* \\
& a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d*e^5 + (b^16 - 14*a* \\
& b^14*c + 79*a^2*b^12*c^2 - 230*a^3*b^10*c^3 + 367*a^4*b^8*c^4 - 3 \\
& 14*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6) / \\
& (b^2*c^18 - 4*a*c^19)))/(b^2*c^9 - 4*a*c^10))*\text{log}(\text{sqrt}(2))*((b^9* \\
& c^4 - 9*a*b^7*c^5 + 27*a^2*b^5*c^6 - 31*a^3*b^3*c^7 + 12*a^4*b*c^8) \\
& *d^4 - (4*b^10*c^3 - 40*a*b^8*c^4 + 140*a^2*b^6*c^5 - 203*a^3*b^4 \\
& *c^6 + 111*a^4*b^2*c^7 - 12*a^5*c^8)*d^3*e + 3*(2*b^11*c^2 - 22 \\
& *a*b^9*c^3 + 88*a^2*b^7*c^4 - 155*a^3*b^5*c^5 + 114*a^4*b^3*c^6 - \\
& 24*a^5*b*c^7)*d^2*e^2 - (4*b^12*c - 48*a*b^10*c^2 + 216*a^2*b^8* \\
& c^3 - 449*a^3*b^6*c^4 + 423*a^4*b^4*c^5 - 141*a^5*b^2*c^6 + 4*a^6 \\
& *c^7)*d*e^3 + (b^13 - 13*a*b^11*c + 65*a^2*b^9*c^2 - 156*a^3*b^7* \\
& c^3 + 181*a^4*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6*b*c^6)*e^4 + ((b^5 \\
& *c^10 - 7*a*b^3*c^11 + 12*a^2*b*c^12)*d - (b^6*c^9 - 8*a*b^4*c^10 \\
& + 18*a^2*b^2*c^11 - 8*a^3*c^12)*e)*\text{sqrt}(((b^10*c^6 - 8*a*b^8*c^7 \\
& + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2*c^10)*d^6 - 6*(b^11 \\
& *c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3*b^5*c^8 + 22*a^4*b^3 \\
& *c^9 - 3*a^5*b*c^10)*d^5*e + 3*(5*b^12*c^4 - 50*a*b^10*c^5 + 185 \\
& *a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4*c^8 - 60*a^5*b^2*c^9 \\
& + 3*a^6*c^10)*d^4*e^2 - 2*(10*b^13*c^3 - 110*a*b^11*c^4 + 460*a^2 \\
& *b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 - 340*a^5*b^3*c^8 + \\
& 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^14*c^2 - 60*a*b^12*c^3 + 280*a^2* \\
& b^10*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 - 400*a^5*b^4*c^7 + \\
& 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^15*c - 13*a*b^13*c^2 +
\end{aligned}$$



$$\begin{aligned}
& 67*a^2*b^{11}*c^3 - 174*a^3*b^9*c^4 + 239*a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d^e^5 + (b^{16} - 14*a*b^{14}*c \\
& + 79*a^2*b^{12}*c^2 - 230*a^3*b^{10}*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 + a^8*c^8)*e^6)/(b^2* \\
& c^{18} - 4*a*c^{19})))*sqrt(((b^6*c^3 - 6*a*b^4*c^4 + 9*a^2*b^2*c^5 - \\
& 2*a^3*c^6)*d^3 - 3*(b^7*c^2 - 7*a*b^5*c^3 + 14*a^2*b^3*c^4 - 7*a \\
& ^3*b*c^5)*d^2*e + 3*(b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16*a^ \\
& ^3*b^2*c^4 + 2*a^4*c^5)*d^e^2 - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 \\
& - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e^3 - (b^2*c^9 - 4*a*c^{10})*sqrt((( \\
& (b^{10}*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4 \\
& *b^2*c^{10})*d^6 - 6*(b^{11}*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40* \\
& a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^{10})*d^5*e + 3*(5*b^{12}*c^4 \\
& - 50*a*b^{10}*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b \\
& ^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^{10})*d^4*e^2 - 2*(10*b^{13}*c^3 - \\
& 110*a*b^{11}*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5* \\
& c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^{14}*c^2 - 6 \\
& 0*a*b^{12}*c^3 + 280*a^2*b^{10}*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c \\
& ^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b \\
& ^{15}*c - 13*a*b^{13}*c^2 + 67*a^2*b^{11}*c^3 - 174*a^3*b^9*c^4 + 239*a \\
& ^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d^e^5 + (b^{16} - 14*a*b^{14}*c \\
& + 79*a^2*b^{12}*c^2 - 230*a^3*b^{10}*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 \\
& + a^8*c^8)*e^6)/(b^2*c^{18} - 4*a*c^{19}))/((b^2*c^9 - 4*a*c^{10})) - \\
& 4*((a^3*b^5*c^4 - 4*a^4*b^3*c^5 + 3*a^5*b*c^6)*d^5 - (4*a^3*b^6* \\
& c^3 - 19*a^4*b^4*c^4 + 21*a^5*b^2*c^5 - 3*a^6*c^6)*d^4*e + 2*(3*a \\
& ^3*b^7*c^2 - 16*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 6*a^6*b*c^5)*d^3*e^2 - 2*(2*a^3*b^8*c - 11*a^4*b^6*c^2 + 15*a^5*b^4*c^3 - 2*a^6*b^2 \\
& *c^4 - a^7*c^5)*d^2*e^3 + (a^3*b^9 - 4*a^4*b^7*c - 3*a^5*b^5*c^2 \\
& + 20*a^6*b^3*c^3 - 11*a^7*b*c^4)*d^e^4 - (a^4*b^8 - 7*a^5*b^6*c + \\
& 15*a^6*b^4*c^2 - 10*a^7*b^2*c^3 + a^8*c^4)*e^5)*sqrt(e*x + d)) - \\
& 105*sqrt(2)*c^4*e^2*sqrt(((b^6*c^3 - 6*a*b^4*c^4 + 9*a^2*b^2*c^5 - \\
& 2*a^3*c^6)*d^3 - 3*(b^7*c^2 - 7*a*b^5*c^3 + 14*a^2*b^3*c^4 - 7 \\
& *a^3*b*c^5)*d^2*e + 3*(b^8*c - 8*a*b^6*c^2 + 20*a^2*b^4*c^3 - 16* \\
& a^3*b^2*c^4 + 2*a^4*c^5)*d^e^2 - (b^9 - 9*a*b^7*c + 27*a^2*b^5*c^2 \\
& - 30*a^3*b^3*c^3 + 9*a^4*b*c^4)*e^3 - (b^2*c^9 - 4*a*c^{10})*sqrt((( \\
& (b^{10}*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4 \\
& *b^2*c^{10})*d^6 - 6*(b^{11}*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 4 \\
& 0*a^3*b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^{10})*d^5*e + 3*(5*b^{12}* \\
& c^4 - 50*a*b^{10}*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4 \\
& *b^4*c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^{10})*d^4*e^2 - 2*(10*b^{13}*c^3 - \\
& 110*a*b^{11}*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5* \\
& c^7 - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^{14}*c^2 - \\
& 60*a*b^{12}*c^3 + 280*a^2*b^{10}*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6* \\
& c^6 - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6* \\
& (b^{15}*c - 13*a*b^{13}*c^2 + 67*a^2*b^{11}*c^3 - 174*a^3*b^9*c^4 + 239 \\
& *a^4*b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d^e^5 + (b^{16} - 14*a*b^{14}*c \\
& + 79*a^2*b^{12}*c^2 - 230*a^3*b^{10}*c^3 + 367*a^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 \\
& c^7 + a^8*c^8)*e^6)/(b^2*c^{18} - 4*a*c^{19}))/((b^2*c^9 - 4*a*c^{10})) \\
& *log(-sqrt(2)*((b^9*c^4 - 9*a*b^7*c^5 + 27*a^2*b^5*c^6 - 31*a^3*b \\
& ^3*c^7 + 12*a^4*b*c^8)*d^4 - (4*b^{10}*c^3 - 40*a*b^8*c^4 + 140*a^2 \\
& *b^6*c^5 - 203*a^3*b^4*c^6 + 111*a^4*b^2*c^7 - 12*a^5*c^8)*d^3*e \\
& + 3*(2*b^{11}*c^2 - 22*a*b^9*c^3 + 88*a^2*b^7*c^4 - 155*a^3*b^5*c^5 \\
& + 114*a^4*b^3*c^6 - 24*a^5*b*c^7)*d^2*e^2 - (4*b^{12}*c - 48*a*b^{11} \\
& *c^2 + 216*a^2*b^8*c^3 - 449*a^3*b^6*c^4 + 423*a^4*b^4*c^5 - 141 \\
& *a^5*b^2*c^6 + 4*a^6*c^7)*d^e^3 + (b^{13} - 13*a*b^{11}*c + 65*a^2*b^9 \\
& *c^2 - 156*a^3*b^7*c^3 + 181*a^4*b^5*c^4 - 86*a^5*b^3*c^5 + 8*a^6 \\
& *b*c^6)*e^4 + ((b^5*c^{10} - 7*a*b^3*c^{11} + 12*a^2*b*c^{12})*d - (b^6 \\
& *c^9 - 8*a*b^4*c^{10} + 18*a^2*b^2*c^{11} - 8*a^3*c^{12})*e)*sqrt(((b^ \\
& ^{10}*c^6 - 8*a*b^8*c^7 + 22*a^2*b^6*c^8 - 24*a^3*b^4*c^9 + 9*a^4*b^2 \\
& *c^{10})*d^6 - 6*(b^{11}*c^5 - 9*a*b^9*c^6 + 29*a^2*b^7*c^7 - 40*a^3 \\
& *b^5*c^8 + 22*a^4*b^3*c^9 - 3*a^5*b*c^{10})*d^5*e + 3*(5*b^{12}*c^4 - \\
& 50*a*b^{10}*c^5 + 185*a^2*b^8*c^6 - 310*a^3*b^6*c^7 + 230*a^4*b^4* \\
& c^8 - 60*a^5*b^2*c^9 + 3*a^6*c^{10})*d^4*e^2 - 2*(10*b^{13}*c^3 - 110 \\
& *a*b^{11}*c^4 + 460*a^2*b^9*c^5 - 910*a^3*b^7*c^6 + 860*a^4*b^5*c^7 \\
& - 340*a^5*b^3*c^8 + 39*a^6*b*c^9)*d^3*e^3 + 3*(5*b^{14}*c^2 - 60*a \\
& *b^{12}*c^3 + 280*a^2*b^{10}*c^4 - 640*a^3*b^8*c^5 + 740*a^4*b^6*c^6 \\
& - 400*a^5*b^4*c^7 + 80*a^6*b^2*c^8 - 2*a^7*c^9)*d^2*e^4 - 6*(b^{15} \\
& *c - 13*a*b^{13}*c^2 + 67*a^2*b^{11}*c^3 - 174*a^3*b^9*c^4 + 239*a^4* \\
& b^7*c^5 - 166*a^5*b^5*c^6 + 50*a^6*b^3*c^7 - 4*a^7*b*c^8)*d^e^5 + \\
& (b^{16} - 14*a*b^{14}*c + 79*a^2*b^{12}*c^2 - 230*a^3*b^{10}*c^3 + 367*a \\
& ^4*b^8*c^4 - 314*a^5*b^6*c^5 + 130*a^6*b^4*c^6 - 20*a^7*b^2*c^7 +
\end{aligned}$$

$$\frac{a^8 c^8 e^6}{(b^2 c^{18} - 4 a c^{19})} \sqrt{\left( (b^6 c^3 - 6 a b^4 c^4 + 9 a^2 b^2 c^5 - 2 a^3 c^6) d^3 - 3 (b^7 c^2 - 7 a b^5 c^3 + 14 a^2 b^3 c^4 - 7 a^3 b c^5) d^2 e + 3 (b^8 c - 8 a b^6 c^2 + 20 a^2 b^4 c^3 - 16 a^3 b^2 c^4 + 2 a^4 c^5) d e^2 - (b^9 - 9 a b^7 c + 27 a^2 b^5 c^2 - 30 a^3 b^3 c^3 + 9 a^4 b c^4) e^3 - (b^2 c^9 - 4 a c^{10}) \sqrt{\left( (b^{10} c^6 - 8 a b^8 c^7 + 22 a^2 b^6 c^8 - 24 a^3 b^4 c^9 + 9 a^4 b^2 c^{10}) d^6 - 6 (b^{11} c^5 - 9 a b^9 c^6 + 29 a^2 b^7 c^7 - 40 a^3 b^5 c^8 + 22 a^4 b^3 c^9 - 3 a^5 b c^{10}) d^5 e + 3 (5 b^{12} c^4 - 50 a b^{10} c^5 + 185 a^2 b^8 c^6 - 310 a^3 b^6 c^7 + 230 a^4 b^4 c^8 - 60 a^5 b^2 c^9 + 3 a^6 c^{10}) d^4 e^2 - 2 (10 b^{13} c^3 - 110 a b^{11} c^4 + 460 a^2 b^9 c^5 - 910 a^3 b^7 c^6 + 860 a^4 b^5 c^7 - 340 a^5 b^3 c^8 + 39 a^6 b c^9) d^3 e^3 + 3 (5 b^{14} c^2 - 60 a b^{12} c^3 + 280 a^2 b^{10} c^4 - 640 a^3 b^8 c^5 + 740 a^4 b^6 c^6 - 400 a^5 b^4 c^7 + 80 a^6 b^2 c^8 - 2 a^7 c^9) d^2 e^4 - 6 (b^{15} c - 13 a b^{13} c^2 + 67 a^2 b^{11} c^3 - 174 a^3 b^9 c^4 + 239 a^4 b^7 c^5 - 166 a^5 b^5 c^6 + 50 a^6 b^3 c^7 - 4 a^7 b c^8) d e^5 + (b^{16} - 14 a b^{14} c + 79 a^2 b^{12} c^2 - 230 a^3 b^{10} c^3 + 367 a^4 b^8 c^4 - 314 a^5 b^6 c^5 + 130 a^6 b^4 c^6 - 20 a^7 b^2 c^7 + a^8 c^8) e^6 \right) / (b^2 c^9 - 4 a c^{10}) - 4 \left( (a^3 b^5 c^4 - 4 a^4 b^3 c^5 + 3 a^5 b c^6) d^5 - (4 a^3 b^6 c^3 - 19 a^4 b^4 c^4 + 21 a^5 b^2 c^5 - 3 a^6 c^6) d^4 e + 2 (3 a^3 b^7 c^2 - 16 a^4 b^5 c^3 + 22 a^5 b^3 c^4 - 6 a^6 b c^5) d^3 e^2 - 2 (2 a^3 b^8 c - 11 a^4 b^6 c^2 + 15 a^5 b^4 c^3 - 2 a^6 b^2 c^4 - a^7 c^5) d^2 e^3 + (a^3 b^9 - 4 a^4 b^7 c - 3 a^5 b^5 c^2 + 20 a^6 b^3 c^3 - 11 a^7 b c^4) d e^4 - (a^4 b^8 - 7 a^5 b^6 c + 15 a^6 b^4 c^2 - 10 a^7 b^2 c^3 + a^8 c^4) e^5 \right) \sqrt{e x + d} - 4 (15 c^3 e^3 x^3 - 6 c^3 d^3 - 21 b c^2 d^2 e + 140 (b^2 c - a c^2) d e^2 - 105 (b^3 - 2 a b c) e^3 + 3 (8 c^3 d e^2 - 7 b c^2 e^3) x^2 + (3 c^3 d^2 e - 42 b c^2 d e^2 + 35 (b^2 c - a c^2) e^3) x) \sqrt{e x + d} / (c^4 e^2)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*3\*(e\*x+d)\*\*(3/2)/(c\*x\*\*2+b\*x+a),x)

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*x^3/(c\*x^2 + b\*x + a),x, algorithm="giac")

[Out] Timed out

$$3.535 \quad \int \frac{x^2(d+ex)^{3/2}}{a+bx+cx^2} dx$$

**Optimal.** Leaf size=441

$$\frac{2\sqrt{d+ex}(ace+b^2(-e)+bcd)}{c^3} + \frac{\sqrt{2}\left((cd-be)(2ace+b^2(-e)+bcd) + \frac{-b^2c(cd^2-4ae^2)-6abc^2de+2ac^2(cd^2-ae^2)+b^4(-e^2)+2b^3cde}{\sqrt{b^2-4ac}}\right) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}\right)}{c^{7/2}\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} + \frac{\sqrt{2}\left((cd-be)(2ace+b^2(-e)+bcd) - \frac{-b^2c(cd^2-4ae^2)-6abc^2de+2ac^2(cd^2-ae^2)+b^4(-e^2)+2b^3cde}{\sqrt{b^2-4ac}}\right) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}\right)}{c^{7/2}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} - \frac{2b(d+ex)^{3/2}}{3c^2} + \frac{2(d+ex)^{5/2}}{5ce}$$

[Out]  $(-2*(b*c*d - b^2*e + a*c*e)*\text{Sqrt}[d + e*x])/c^3 - (2*b*(d + e*x)^(3/2))/(3*c^2) + (2*(d + e*x)^(5/2))/(5*c*e) + (\text{Sqrt}[2]*((c*d - b*e)*(b*c*d - b^2*e + 2*a*c*e) + (2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) + 2*a*c^2*(c*d^2 - a*e^2))/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]])/(c^(7/2)*\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]) + (\text{Sqrt}[2]*((c*d - b*e)*(b*c*d - b^2*e + 2*a*c*e) - (2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) + 2*a*c^2*(c*d^2 - a*e^2))/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]])/(c^(7/2)*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e])$

**Rubi [A]** time = 7.09688, antiderivative size = 441, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{2\sqrt{d+ex}(ace+b^2(-e)+bcd)}{c^3} + \frac{\sqrt{2}\left((cd-be)(2ace+b^2(-e)+bcd) + \frac{-b^2c(cd^2-4ae^2)-6abc^2de+2ac^2(cd^2-ae^2)+b^4(-e^2)+2b^3cde}{\sqrt{b^2-4ac}}\right) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}\right)}{c^{7/2}\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} + \frac{\sqrt{2}\left((cd-be)(2ace+b^2(-e)+bcd) - \frac{-b^2c(cd^2-4ae^2)-6abc^2de+2ac^2(cd^2-ae^2)+b^4(-e^2)+2b^3cde}{\sqrt{b^2-4ac}}\right) \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}\right)}{c^{7/2}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} - \frac{2b(d+ex)^{3/2}}{3c^2} + \frac{2(d+ex)^{5/2}}{5ce}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(x^2*(d + e*x)^(3/2))/(a + b*x + c*x^2), x]$

[Out]  $(-2*(b*c*d - b^2*e + a*c*e)*\text{Sqrt}[d + e*x])/c^3 - (2*b*(d + e*x)^(3/2))/(3*c^2) + (2*(d + e*x)^(5/2))/(5*c*e) + (\text{Sqrt}[2]*((c*d - b*e)*(b*c*d - b^2*e + 2*a*c*e) + (2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) + 2*a*c^2*(c*d^2 - a*e^2))/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]])/(c^(7/2)*\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]) + (\text{Sqrt}[2]*((c*d - b*e)*(b*c*d - b^2*e + 2*a*c*e) - (2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) + 2*a*c^2*(c*d^2 - a*e^2))/\text{Sqrt}[b^2 - 4*a*c])*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]])/(c^(7/2)*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e])$

$$- 4*a*c)) * e)) + (\text{Sqrt}[2] * ((c*d - b*e) * (b*c*d - b^2*e + 2*a*c*e) - (2*b^3*c*d*e - 6*a*b*c^2*d*e - b^4*e^2 - b^2*c*(c*d^2 - 4*a*e^2) + 2*a*c^2*(c*d^2 - a*e^2)) / \text{Sqrt}[b^2 - 4*a*c]) * \text{ArcTanh}[(\text{Sqrt}[2] * \text{Sqrt}[c] * \text{Sqrt}[d + e*x]) / \text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]) * e]]) / (c^{7/2} * \text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]) * e])$$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**2*(e*x+d)**(3/2)/(c*x**2+b*x+a), x)`

[Out] Timed out

**Mathematica [A]** time = 1.22447, size = 538, normalized size = 1.22

$$\frac{2\sqrt{d+ex}(-5ce(3ae+4bd+bex)+15b^2e^2+3c^2(d+ex)^2)}{15c^3e}$$

$$+ \frac{\sqrt{2}\left(2ac^2\left(e\left(d\sqrt{b^2-4ac}-ae\right)+cd^2\right)-b^2c\left(2e\left(d\sqrt{b^2-4ac}-2ae\right)+cd^2\right)+bc\left(cd\left(d\sqrt{b^2-4ac}-6ae\right)-2ae^2\sqrt{b^2-4ac}\right)\right)}{c^{7/2}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}$$

$$+ \frac{\sqrt{2}\left(2ac^2\left(e\left(d\sqrt{b^2-4ac}+ae\right)-cd^2\right)+b^2c\left(cd^2-2e\left(d\sqrt{b^2-4ac}+2ae\right)\right)+bc\left(cd\left(d\sqrt{b^2-4ac}+6ae\right)-2ae^2\sqrt{b^2-4ac}\right)\right)}{c^{7/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

Antiderivative was successfully verified.

[In] `Integrate[(x^2*(d + e*x)^(3/2))/(a + b*x + c*x^2), x]`

[Out]  $(2*\text{Sqrt}[d + e*x] * (15*b^2*e^2 + 3*c^2*(d + e*x)^2 - 5*c*e*(4*b*d + 3*a*e + b*e*x)) / (15*c^3*e) + (\text{Sqrt}[2] * (-(b^4*e^2) + b^3*e*(2*c*d + \text{Sqrt}[b^2 - 4*a*c]*e) + b*c*(-2*a*\text{Sqrt}[b^2 - 4*a*c]*e^2 + c*d*(\text{Sqrt}[b^2 - 4*a*c]*d - 6*a*e)) - b^2*c*(c*d^2 + 2*e*(\text{Sqrt}[b^2 - 4*a*c]*d - 2*a*e)) + 2*a*c^2*(c*d^2 + e*(\text{Sqrt}[b^2 - 4*a*c]*d - a*e))) * \text{ArcTanh}[(\text{Sqrt}[2] * \text{Sqrt}[c] * \text{Sqrt}[d + e*x]) / \text{Sqrt}[2*c*d - b*e + \text{Sqrt}[b^2 - 4*a*c]*e]]) / (c^{7/2} * \text{Sqrt}[b^2 - 4*a*c] * \text{Sqrt}[2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c]) * e]) + (\text{Sqrt}[2] * (b^4*e^2 + b^3*e*(-2*c*d + \text{Sqrt}[b^2 - 4*a*c]*e) + 2*a*c^2*(-(c*d^2) + e*(\text{Sqrt}[b^2 - 4*a*c]*d + a*e)) + b^2*c*(c*d^2 - 2*e*(\text{Sqrt}[b^2 - 4*a*c]*d + 2*a*e)) + b*c*(-2*a*\text{Sqrt}[b^2 - 4*a*c]*e^2 + c*d*(\text{Sqrt}[b^2 - 4*a*c]*d + 6*a*e))) * \text{ArcTanh}[(\text{Sqrt}[2] * \text{Sqrt}[c] * \text{Sqrt}[d + e*x]) / \text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]) * e]]) / (c^{7/2} * \text{Sqrt}[b^2 - 4*a*c] * \text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]) * e])$

**Maple [B]** time = 0.063, size = 2358, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(e*x+d)^(3/2)/(c*x^2+b*x+a), x)`

[Out] 
$$-2/3*b*(e*x+d)^{(3/2)}/c^2+2/5*(e*x+d)^{(5/2)}/c/e-e^2/c^3*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}*\arctan(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)})^b^3+e^2/c^3*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}*\arctanh(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)})^b^3+1/c*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}*\arctanh(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)})^b*d^2-1/c*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}*\arctan(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)})^b*d^2-2*e/c^2*a*(e*x+d)^{(1/2)}+2*e/c^3*b^2*(e*x+d)^{(1/2)}-2/c^2*b*d*(e*x+d)^{(1/2)}-6*e^2/c/(-e^2*(4*a*c-b^2))^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}*\arctan(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)})^a*b*d-6*e^2/c/(-e^2*(4*a*c-b^2))^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}*\arctanh(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)})^a*b*d+4*e^3/c^2/(-e^2*(4*a*c-b^2))^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}*\arctan(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)})^a*b^2+2*e/c*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}*\arctanh(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)})^a*d-2*e/c^2*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}*\arctanh(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)})^b^2*d+2*e/(-e^2*(4*a*c-b^2))^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}*\arctan(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)})^a*d^2+2*e/(-e^2*(4*a*c-b^2))^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}*\arctanh(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)})^a*d^2-2*e^3/c/(-e^2*(4*a*c-b^2))^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}*\arctan(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)})^b^4+2*e^2/c^2*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}*\arctan(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)})^a*b-2*e/c*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}*\arctan(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)})^a*d+2*e/c^2*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}*\arctan(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)})^b^2*d-2*e^3/c/(-e^2*(4*a*c-b^2))^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}*\arctanh(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)})^a^2-e^3/c^3/(-e^2*(4*a*c-b^2))^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}*\arctanh(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)})^b^4-2*e^2/c^2*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}*\arctanh(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)})^a^2-e^3/c^3/(-e^2*(4*a*c-b^2))^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}*\arctanh(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)})^a*b+2*e^2/c^2/(-e^2*(4*a*c-b^2))^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}*\arctan(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)})^b^3*d-e/c/(-e^2*(4*a*c-b^2))^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}*\arctan(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)})^b^2*d^2+4*e^3/c^2/(-e^2*(4*a*c-b^2))^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}*\arctanh(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)})^a*b^2+2*e^2/c^2/(-e^2*(4*a*c-b^2))^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}*\arctanh(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)})^b^3*d-e/c/(-e^2*(4*a*c-b^2))^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}*\arctanh(c*(e*x+d)^{(1/2)}*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)})^b^2*d^2$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}x^2}{cx^2+bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*x^2/(c\*x^2 + b\*x + a),x, algorithm="maxima")

[Out] integrate((e\*x + d)^(3/2)\*x^2/(c\*x^2 + b\*x + a), x)

**Fricas [A]** time = 2.25939, size = 11516, normalized size = 26.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*x^2/(c\*x^2 + b\*x + a),x, algorithm="fricas")

[Out] 
$$-1/30*(15*\sqrt{2})^c*c^3*e*\sqrt{((b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d^3 - 3*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d*e^2 - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e^3 + (b^2*c^7 - 4*a*c^8)*\sqrt{((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*(5*b^{10}*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^{11}*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^{12} - 10*a*b^{10}*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^6)/(b^2*c^{14} - 4*a*c^{15})))/(b^2*c^7 - 4*a*c^8))*\log(\sqrt{2})*((b^6*c^4 - 6*a*b^4*c^5 + 8*a^2*b^2*c^6)*d^4 - (4*b^7*c^3 - 28*a*b^5*c^4 + 53*a^2*b^3*c^5 - 20*a^3*b*c^6)*d^3*e + 3*(2*b^8*c^2 - 16*a*b^6*c^3 + 39*a^2*b^4*c^4 - 29*a^3*b^2*c^5 + 4*a^4*c^6)*d^2*e^2 - (4*b^9*c - 36*a*b^7*c^2 + 107*a^2*b^5*c^3 - 118*a^3*b^3*c^4 + 40*a^4*b*c^5)*d*e^3 + (b^{10} - 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4*b^2*c^4 - 4*a^5*c^5)*e^4 - ((b^4*c^8 - 6*a*b^2*c^9 + 8*a^2*c^{10})*d - (b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*e)*\sqrt{((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*(5*b^{10}*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^{11}*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^{12} - 10*a*b^{10}*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^6)/(b^2*c^{14} - 4*a*c^{15})))*\sqrt{((b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d^3 - 3*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d*e^2 - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e^3 + (b^2*c^7 - 4*a*c^8)*\sqrt{((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*(5*b^{10}*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^{11}*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^{12} - 10*a*b^{10}*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^6)/(b^2*c^{14} - 4*a*c^{15})))/(b^2*c^7 - 4*a*c^8)) + 4*((a^2*b^3*c^4 - 2*a^3*b*c^5)*d^4 - (4*a^2*b^4*c^3 - 11*a^3*b^2*c^4 + 3*a^4*c^5)*d^4*e + 2*(3*a^2*b^5*c^2 - 10*a^3*b^3*c^3 + 5*a^4*b*c^4)*d^3*e^2 - 2*(2*a^2*b^6*c - 7*a^3*b^4*c^2 + 3*a^4*b^2*c^3 + a^5*c^4)*d^2*e^3 + (a^2*b^7 - 2*a^3*b^5*c - 6*a^4*b^3*c^2 + 8*a^5*b*c^3)*d*e^4 - (a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*e^5)*\sqrt{e*x + d}) - 15*\sqrt{2})^c*c^3*e*\sqrt{((b^4*c^3 - 4*a*b^2*c^4 + 2*a^2*c^5)*d^3 - 3*(b^5*c^2 - 5*a*b^3*c^3 + 5*a^2*b*c^4)*d^2*e + 3*(b^6*c - 6*a*b^4*c^2 + 9*a^2*b^2*c^3 - 2*a^3*c^4)*d*e^2 - (b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*e^3 + (b^2*c^7 - 4*a*c^8)*\sqrt{((b^6*c^6 - 4*a*b^4*c^7 + 4*a^2*b^2*c^8)*d^6 - 6*(b^7*c^5 - 5*a*b^5*c^6 + 7*a^2*b^3*c^7 - 2*a^3*b*c^8)*d^5*e + 3*(5*b^8*c^4 - 30*a*b^6*c^5 + 55*a^2*b^4*c^6 - 30*a^3*b^2*c^7 + 3*a^4*c^8)*d^4*e^2 - 2*(10*b^9*c^3 - 70*a*b^7*c^4 + 160*a^2*b^5*c^5 - 130*a^3*b^3*c^6 + 29*a^4*b*c^7)*d^3*e^3 + 3*(5*b^{10}*c^2 - 40*a*b^8*c^3 + 110*a^2*b^6*c^4 - 120*a^3*b^4*c^5 + 45*a^4*b^2*c^6 - 2*a^5*c^7)*d^2*e^4 - 6*(b^{11}*c - 9*a*b^9*c^2 + 29*a^2*b^7*c^3 - 40*a^3*b^5*c^4 + 22*a^4*b^3*c^5 - 3*a^5*b*c^6)*d*e^5 + (b^{12} - 10*a*b^{10}*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*e^6)/(b^2*c^{14} - 4*a*c^{15})))/(b^2*c^7 - 4*a*c^8))$$

$$\begin{aligned}
& a^5 e + 3(5b^8 c^4 - 30a^2 b^6 c^5 + 55a^2 b^4 c^6 - 30a^3 b^2 c^7 + 3a^4 c^8) d^4 e^2 - 2(10b^9 c^3 - 70a^2 b^7 c^4 + 160a^2 b^5 c^5 - 130a^3 b^3 c^6 + 29a^4 b^2 c^7) d^3 e^3 + 3(5b^{10} c^2 - 40a^2 b^8 c^3 + 110a^2 b^6 c^4 - 120a^3 b^4 c^5 + 45a^4 b^2 c^6 - 2a^5 c^7) d^2 e^4 - 6(b^{11} c - 9a^2 b^9 c^2 + 29a^2 b^7 c^3 - 40a^3 b^5 c^4 + 22a^4 b^3 c^5 - 3a^5 b^2 c^6) d e^5 + (b^{12} - 10a^2 b^{10} c + 37a^2 b^8 c^2 - 62a^3 b^6 c^3 + 46a^4 b^4 c^4 - 12a^5 b^2 c^5 + a^6 c^6) e^6 / (b^2 c^{14} - 4a^2 c^{15}) / (b^2 c^7 - 4a^2 c^8) \log(-\sqrt{2}) \left( (b^6 c^4 - 6a^2 b^4 c^5 + 8a^2 b^2 c^6) d^4 - (4b^7 c^3 - 28a^2 b^5 c^4 + 53a^2 b^3 c^5 - 20a^3 b^2 c^6) d^3 e + 3(2b^8 c^2 - 16a^2 b^6 c^3 + 39a^2 b^4 c^4 - 29a^3 b^2 c^5 + 4a^4 c^6) d^2 e^2 - (4b^9 c - 36a^2 b^7 c^2 + 107a^2 b^5 c^3 - 118a^3 b^3 c^4 + 40a^4 b^2 c^5) d e^3 + (b^{10} - 10a^2 b^8 c + 35a^2 b^6 c^2 - 51a^3 b^4 c^3 + 29a^4 b^2 c^4 - 4a^5 c^5) e^4 - ((b^4 c^8 - 6a^2 b^2 c^9 + 8a^2 c^{10}) d - (b^5 c^7 - 7a^2 b^3 c^8 + 12a^2 b^2 c^9) e) \sqrt{((b^6 c^6 - 4a^2 b^4 c^7 + 4a^2 b^2 c^8) d^6 - 6(b^7 c^5 - 5a^2 b^5 c^6 + 7a^2 b^3 c^7 - 2a^3 b^2 c^8) d^5 e + 3(5b^8 c^4 - 30a^2 b^6 c^5 + 55a^2 b^4 c^6 - 30a^3 b^2 c^7 + 3a^4 c^8) d^4 e^2 - 2(10b^9 c^3 - 70a^2 b^7 c^4 + 160a^2 b^5 c^5 - 130a^3 b^3 c^6 + 29a^4 b^2 c^7) d^3 e^3 + 3(5b^{10} c^2 - 40a^2 b^8 c^3 + 110a^2 b^6 c^4 - 120a^3 b^4 c^5 + 45a^4 b^2 c^6 - 2a^5 c^7) d^2 e^4 - 6(b^{11} c - 9a^2 b^9 c^2 + 29a^2 b^7 c^3 - 40a^3 b^5 c^4 + 22a^4 b^3 c^5 - 3a^5 b^2 c^6) d e^5 + (b^{12} - 10a^2 b^{10} c + 37a^2 b^8 c^2 - 62a^3 b^6 c^3 + 46a^4 b^4 c^4 - 12a^5 b^2 c^5 + a^6 c^6) e^6 \right) / (b^2 c^{14} - 4a^2 c^{15}) \sqrt{((b^4 c^3 - 4a^2 b^2 c^4 + 2a^2 c^5) d^3 - 3(b^5 c^2 - 5a^2 b^3 c^3 + 5a^2 b^2 c^4) d^2 e + 3(b^6 c - 6a^2 b^4 c^2 + 9a^2 b^2 c^3 - 2a^3 c^4) d e^2 - (b^7 - 7a^2 b^5 c + 14a^2 b^3 c^2 - 7a^3 b^2 c^3) e^3 + (b^2 c^7 - 4a^2 c^8) \sqrt{((b^6 c^6 - 4a^2 b^4 c^7 + 4a^2 b^2 c^8) d^6 - 6(b^7 c^5 - 5a^2 b^5 c^6 + 7a^2 b^3 c^7 - 2a^3 b^2 c^8) d^5 e + 3(5b^8 c^4 - 30a^2 b^6 c^5 + 55a^2 b^4 c^6 - 30a^3 b^2 c^7 + 3a^4 c^8) d^4 e^2 - 2(10b^9 c^3 - 70a^2 b^7 c^4 + 160a^2 b^5 c^5 - 130a^3 b^3 c^6 + 29a^4 b^2 c^7) d^3 e^3 + 3(5b^{10} c^2 - 40a^2 b^8 c^3 + 110a^2 b^6 c^4 - 120a^3 b^4 c^5 + 45a^4 b^2 c^6 - 2a^5 c^7) d^2 e^4 - 6(b^{11} c - 9a^2 b^9 c^2 + 29a^2 b^7 c^3 - 40a^3 b^5 c^4 + 22a^4 b^3 c^5 - 3a^5 b^2 c^6) d e^5 + (b^{12} - 10a^2 b^{10} c + 37a^2 b^8 c^2 - 62a^3 b^6 c^3 + 46a^4 b^4 c^4 - 12a^5 b^2 c^5 + a^6 c^6) e^6} / (b^2 c^{14} - 4a^2 c^{15})) / (b^2 c^7 - 4a^2 c^8) + 4((a^2 b^3 c^4 - 2a^3 b^2 c^5) d^5 - (4a^2 b^4 c^3 - 11a^3 b^2 c^4 + 3a^4 c^5) d^4 e + 2(3a^2 b^5 c^2 - 10a^3 b^3 c^3 + 5a^4 b^2 c^4) d^3 e^2 - 2(2a^2 b^6 c - 7a^3 b^4 c^2 + 3a^4 b^2 c^3 + a^5 c^4) d^2 e^3 + (a^2 b^7 - 2a^3 b^5 c - 6a^4 b^3 c^2 + 8a^5 b^2 c^3) d e^4 - (a^3 b^6 - 5a^4 b^4 c + 6a^5 b^2 c^2 - a^6 c^3) e^5) \sqrt{e x + d} + 15 \sqrt{2} c^3 e \sqrt{((b^4 c^3 - 4a^2 b^2 c^4 + 2a^2 c^5) d^3 - 3(b^5 c^2 - 5a^2 b^3 c^3 + 5a^2 b^2 c^4) d^2 e + 3(b^6 c - 6a^2 b^4 c^2 + 9a^2 b^2 c^3 - 2a^3 c^4) d e^2 - (b^7 - 7a^2 b^5 c + 14a^2 b^3 c^2 - 7a^3 b^2 c^3) e^3 - (b^2 c^7 - 4a^2 c^8) \sqrt{((b^6 c^6 - 4a^2 b^4 c^7 + 4a^2 b^2 c^8) d^6 - 6(b^7 c^5 - 5a^2 b^5 c^6 + 7a^2 b^3 c^7 - 2a^3 b^2 c^8) d^5 e + 3(5b^8 c^4 - 30a^2 b^6 c^5 + 55a^2 b^4 c^6 - 30a^3 b^2 c^7 + 3a^4 c^8) d^4 e^2 - 2(10b^9 c^3 - 70a^2 b^7 c^4 + 160a^2 b^5 c^5 - 130a^3 b^3 c^6 + 29a^4 b^2 c^7) d^3 e^3 + 3(5b^{10} c^2 - 40a^2 b^8 c^3 + 110a^2 b^6 c^4 - 120a^3 b^4 c^5 + 45a^4 b^2 c^6 - 2a^5 c^7) d^2 e^4 - 6(b^{11} c - 9a^2 b^9 c^2 + 29a^2 b^7 c^3 - 40a^3 b^5 c^4 + 22a^4 b^3 c^5 - 3a^5 b^2 c^6) d e^5 + (b^{12} - 10a^2 b^{10} c + 37a^2 b^8 c^2 - 62a^3 b^6 c^3 + 46a^4 b^4 c^4 - 12a^5 b^2 c^5 + a^6 c^6) e^6} / (b^2 c^{14} - 4a^2 c^{15})) / (b^2 c^7 - 4a^2 c^8) \log(\sqrt{2}) \left( (b^6 c^4 - 6a^2 b^4 c^5 + 8a^2 b^2 c^6) d^4 - (4b^7 c^3 - 28a^2 b^5 c^4 + 53a^2 b^3 c^5 - 20a^3 b^2 c^6) d^3 e + 3(2b^8 c^2 - 16a^2 b^6 c^3 + 39a^2 b^4 c^4 - 29a^3 b^2 c^5 + 4a^4 c^6) d^2 e^2 - (4b^9 c - 36a^2 b^7 c^2 + 107a^2 b^5 c^3 - 118a^3 b^3 c^4 + 40a^4 b^2 c^5) d e^3 + (b^{10} - 10a^2 b^8 c + 35a^2 b^6 c^2 - 51a^3 b^4 c^3 + 29a^4 b^2 c^4 - 4a^5 c^5) e^4 + ((b^4 c^8 - 6a^2 b^2 c^9 + 8a^2 c^{10}) d - (b^5 c^7 - 7a^2 b^3 c^8 + 12a^2 b^2 c^9) e) \sqrt{((b^6 c^6 - 4a^2 b^4 c^7 + 4a^2 b^2 c^8) d^6 - 6(b^7 c^5 - 5a^2 b^5 c^6 + 7a^2 b^3 c^7 - 2a^3 b^2 c^8) d^5 e + 3(5b^8 c^4 - 30a^2 b^6 c^5 + 55a^2 b^4 c^6 - 30a^3 b^2 c^7 + 3a^4 c^8) d^4 e^2 - 2(10b^9 c^3 - 70a^2 b^7 c^4 + 160a^2 b^5 c^5 - 130a^3 b^3 c^6 + 29a^4 b^2 c^7) d^3 e^3 + 3(5b^{10} c^2 - 40a^2 b^8 c^3 + 110a^2 b^6 c^4 - 120a^3 b^4 c^5 + 45a^4 b^2 c^6 - 2a^5 c^7) d^2 e^4 - 6(b^{11} c - 9a^2 b^9 c^2 + 29a^2 b^7 c^3 - 40a^3 b^5 c^4 + 22a^4 b^3 c^5 - 3a^5 b^2 c^6) d e^5 + (b^{12} - 10a^2 b^{10} c + 37a^2 b^8 c^2 - 62a^3 b^6 c^3 + 46a^4 b^4 c^4 - 12a^5 b^2 c^5 + a^6 c^6) e^6 \right)
\end{aligned}$$

$$\begin{aligned}
& a^3c^5 - 3a^5b^2c^6) * d^e^5 + (b^{12} - 10a^2b^{10}c + 37a^4b^8c^2 - 62a^6b^6c^3 + 46a^8b^4c^4 - 12a^{10}b^2c^5 + a^{12}c^6) * e^6 / (b^2c^{14} - 4a^2c^{15})) * \sqrt{((b^4c^3 - 4a^2b^2c^4 + 2a^4c^5) * d^3 - 3(b^5c^2 - 5a^3b^3c^3 + 5a^5b^4c^4) * d^2 * e + 3(b^6c - 6a^4b^4c^2 + 9a^6b^5c^3 - 2a^8b^6c^4) * d * e^2 - (b^7 - 7a^5b^5c + 14a^7b^6c^2 - 7a^9b^7c^3) * e^3 - (b^2c^7 - 4a^4c^8) * \sqrt{((b^6c^6 - 4a^4b^4c^7 + 4a^6b^5c^8) * d^6 - 6(b^7c^5 - 5a^5b^5c^6 + 7a^7b^6c^7 - 2a^9b^7c^8) * d^5 * e + 3(5b^8c^4 - 30a^2b^6c^5 + 55a^4b^7c^6 - 30a^6b^8c^7 + 3a^8b^9c^8) * d^4 * e^2 - 2(10b^9c^3 - 70a^2b^7c^4 + 160a^4b^8c^5 - 130a^6b^9c^6 + 29a^8b^{10}c^7) * d^3 * e^3 + 3(5b^{10}c^2 - 40a^2b^8c^3 + 110a^4b^9c^4 - 120a^6b^{10}c^5 + 45a^8b^{11}c^6 - 2a^{10}b^{12}c^7) * d^2 * e^4 - 6(b^{11}c - 9a^3b^9c^2 + 29a^5b^{10}c^3 - 40a^7b^{11}c^4 + 22a^9b^{12}c^5 - 3a^{11}b^{13}c^6) * d * e^5 + (b^{12} - 10a^2b^{10}c + 37a^4b^8c^2 - 62a^6b^6c^3 + 46a^8b^4c^4 - 12a^{10}b^2c^5 + a^{12}c^6) * e^6) / (b^2c^{14} - 4a^2c^{15})) / (b^2c^7 - 4a^4c^8)) + 4((a^2b^3c^4 - 2a^4b^5c^5) * d^5 - (4a^2b^4c^3 - 11a^4b^6c^4 + 3a^6b^8c^5) * d^4 * e + 2(3a^2b^5c^2 - 10a^4b^7c^3 + 5a^6b^9c^4) * d^3 * e^2 - 2(2a^2b^6c - 7a^4b^8c^2 + 3a^6b^{10}c^3 + a^8b^{12}c^4) * d^2 * e^3 + (a^2b^7 - 2a^4b^9c - 6a^6b^{11}c^2 + 8a^8b^{13}c^3) * d * e^4 - (a^3b^6 - 5a^5b^8c + 6a^7b^{10}c^2 - a^9b^{12}c^3) * e^5) * \sqrt{e * x + d} - 15 * \sqrt{2} * c^3 * e * \sqrt{((b^4c^3 - 4a^2b^2c^4 + 2a^4c^5) * d^3 - 3(b^5c^2 - 5a^3b^3c^3 + 5a^5b^4c^4) * d^2 * e + 3(b^6c - 6a^4b^4c^2 + 9a^6b^5c^3 - 2a^8b^6c^4) * d * e^2 - (b^7 - 7a^5b^5c + 14a^7b^6c^2 - 7a^9b^7c^3) * e^3 - (b^2c^7 - 4a^4c^8) * \sqrt{((b^6c^6 - 4a^4b^4c^7 + 4a^6b^5c^8) * d^6 - 6(b^7c^5 - 5a^5b^5c^6 + 7a^7b^6c^7 - 2a^9b^7c^8) * d^5 * e + 3(5b^8c^4 - 30a^2b^6c^5 + 55a^4b^7c^6 - 30a^6b^8c^7 + 3a^8b^9c^8) * d^4 * e^2 - 2(10b^9c^3 - 70a^2b^7c^4 + 160a^4b^8c^5 - 130a^6b^9c^6 + 29a^8b^{10}c^7) * d^3 * e^3 + 3(5b^{10}c^2 - 40a^2b^8c^3 + 110a^4b^9c^4 - 120a^6b^{10}c^5 + 45a^8b^{11}c^6 - 2a^{10}b^{12}c^7) * d^2 * e^4 - 6(b^{11}c - 9a^3b^9c^2 + 29a^5b^{10}c^3 - 40a^7b^{11}c^4 + 22a^9b^{12}c^5 - 3a^{11}b^{13}c^6) * d * e^5 + (b^{12} - 10a^2b^{10}c + 37a^4b^8c^2 - 62a^6b^6c^3 + 46a^8b^4c^4 - 12a^{10}b^2c^5 + a^{12}c^6) * e^6) / (b^2c^{14} - 4a^2c^{15})) / (b^2c^7 - 4a^4c^8)) * \log(-\sqrt{2} * ((b^6c^4 - 6a^4b^4c^5 + 8a^6b^5c^6) * d^4 - (4b^7c^3 - 28a^2b^5c^4 + 53a^4b^6c^5 - 20a^6b^7c^6) * d^3 * e + 3(2b^8c^2 - 16a^2b^6c^3 + 39a^4b^7c^4 - 29a^6b^8c^5 + 4a^8b^9c^6) * d^2 * e^2 - (4b^9c - 36a^2b^7c^2 + 107a^4b^8c^3 - 118a^6b^9c^4 + 40a^8b^{10}c^5) * d * e^3 + (b^{10} - 10a^2b^8c + 35a^4b^6c^2 - 51a^6b^4c^3 + 29a^8b^2c^4 - 4a^{10}c^5) * e^4 + ((b^4c^8 - 6a^2b^2c^9 + 8a^4c^{10}) * d - (b^5c^7 - 7a^3b^3c^8 + 12a^5b^4c^9) * e) * \sqrt{((b^6c^6 - 4a^4b^4c^7 + 4a^6b^5c^8) * d^6 - 6(b^7c^5 - 5a^5b^5c^6 + 7a^7b^6c^7 - 2a^9b^7c^8) * d^5 * e + 3(5b^8c^4 - 30a^2b^6c^5 + 55a^4b^7c^6 - 30a^6b^8c^7 + 3a^8b^9c^8) * d^4 * e^2 - 2(10b^9c^3 - 70a^2b^7c^4 + 160a^4b^8c^5 - 130a^6b^9c^6 + 29a^8b^{10}c^7) * d^3 * e^3 + 3(5b^{10}c^2 - 40a^2b^8c^3 + 110a^4b^9c^4 - 120a^6b^{10}c^5 + 45a^8b^{11}c^6 - 2a^{10}b^{12}c^7) * d^2 * e^4 - 6(b^{11}c - 9a^3b^9c^2 + 29a^5b^{10}c^3 - 40a^7b^{11}c^4 + 22a^9b^{12}c^5 - 3a^{11}b^{13}c^6) * d * e^5 + (b^{12} - 10a^2b^{10}c + 37a^4b^8c^2 - 62a^6b^6c^3 + 46a^8b^4c^4 - 12a^{10}b^2c^5 + a^{12}c^6) * e^6) / (b^2c^{14} - 4a^2c^{15})) / (b^2c^7 - 4a^4c^8)) + 4((a^2b^3c^4 - 2a^4b^5c^5) * d^5 - (4a^2b^4c^3 - 11a^4b^6c^4 + 3a^6b^8c^5) * d^4 * e + 2(3a^2b^5c^2 - 10a^4b^7c^3 + 5a^6b^9c^4) * d^3 * e^2 - 2(2a^2b^6c - 7a^4b^8c^2 + 3a^6b^{10}c^3 + a^8b^{12}c^4) * d^2 * e^3 + (a^2b^7 - 2a^4b^9c - 6a^6b^{11}c^2 + 8a^8b^{13}c^3) * d * e^4 - (a^3b^6 - 5a^5b^8c + 6a^7b^{10}c^2 - a^9b^{12}c^3) * e^5) * \sqrt{e * x + d} - 4(3c^2 * e^2 * x^2 + 3c^2 * d^2 - 20b * c * d * e + 15(b^2 - a * c) * e^2 + (6c^2 * d * e - 5b *
\end{aligned}$$



$$c \cdot e^{2x} \cdot \sqrt{e^x + d} / (c^3 \cdot e)$$

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**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(e*x+d)**(3/2)/(c*x**2+b*x+a),x)`

[Out] Timed out

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**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(3/2)*x^2/(c*x^2 + b*x + a),x, algorithm="giac")`

[Out] Timed out

$$3.536 \quad \int \frac{x(d+ex)^{3/2}}{a+bx+cx^2} dx$$

Optimal. Leaf size=453

$$\sqrt{2} \left( bc \left( e \left( 2d\sqrt{b^2 - 4ac} - 3ae \right) + cd^2 \right) + c \left( ae^2\sqrt{b^2 - 4ac} - cd \left( d\sqrt{b^2 - 4ac} - 4ae \right) \right) - b^2e \left( e\sqrt{b^2 - 4ac} + 2cd \right) + b^3e^2 \right) \tan^{-1} \left( \frac{c^{5/2}\sqrt{b^2 - 4ac}\sqrt{2cd - e \left( b - \sqrt{b^2 - 4ac} \right)}}{\sqrt{2} \left( bc \left( cd^2 - e \left( 2d\sqrt{b^2 - 4ac} + 3ae \right) \right) - c \left( ae^2\sqrt{b^2 - 4ac} - cd \left( d\sqrt{b^2 - 4ac} + 4ae \right) \right) - b^2e \left( 2cd - e\sqrt{b^2 - 4ac} \right) + b^3e^2 \right)} \right)$$

$$\frac{c^{5/2}\sqrt{b^2 - 4ac}\sqrt{2cd - e \left( b - \sqrt{b^2 - 4ac} \right)}}{\sqrt{2} \left( bc \left( cd^2 - e \left( 2d\sqrt{b^2 - 4ac} + 3ae \right) \right) - c \left( ae^2\sqrt{b^2 - 4ac} - cd \left( d\sqrt{b^2 - 4ac} + 4ae \right) \right) - b^2e \left( 2cd - e\sqrt{b^2 - 4ac} \right) + b^3e^2 \right)}$$


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$$\frac{c^{5/2}\sqrt{b^2 - 4ac}\sqrt{2cd - e \left( \sqrt{b^2 - 4ac} + b \right)}}{\sqrt{2} \left( bc \left( cd^2 - e \left( 2d\sqrt{b^2 - 4ac} + 3ae \right) \right) - c \left( ae^2\sqrt{b^2 - 4ac} - cd \left( d\sqrt{b^2 - 4ac} + 4ae \right) \right) - b^2e \left( 2cd - e\sqrt{b^2 - 4ac} \right) + b^3e^2 \right)}$$


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$$+ \frac{2\sqrt{d+ex}(cd-be)}{c^2} + \frac{2(d+ex)^{3/2}}{3c}$$

[Out] (2\*(c\*d - b\*e)\*Sqrt[d + e\*x])/c^2 + (2\*(d + e\*x)^(3/2))/(3\*c) + (Sqrt[2]\*(b^3\*e^2 - b^2\*e\*(2\*c\*d + Sqrt[b^2 - 4\*a\*c]\*e) + c\*(a\*Sqrt[b^2 - 4\*a\*c]\*e^2 - c\*d\*(Sqrt[b^2 - 4\*a\*c]\*d - 4\*a\*e)) + b\*c\*(c\*d^2 + e\*(2\*Sqrt[b^2 - 4\*a\*c]\*d - 3\*a\*e)))\*ArcTanh[(Sqrt[2]\*Sqrt[c]\*Sqrt[d + e\*x])/Sqrt[2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c]\*e)]]/(c^(5/2)\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c]\*e)]) - (Sqrt[2]\*(b^3\*e^2 - b^2\*e\*(2\*c\*d - Sqrt[b^2 - 4\*a\*c]\*e) + b\*c\*(c\*d^2 - e\*(2\*Sqrt[b^2 - 4\*a\*c]\*d + 3\*a\*e)) - c\*(a\*Sqrt[b^2 - 4\*a\*c]\*e^2 - c\*d\*(Sqrt[b^2 - 4\*a\*c]\*d + 4\*a\*e)))\*ArcTanh[(Sqrt[2]\*Sqrt[c]\*Sqrt[d + e\*x])/Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c]\*e)]]/(c^(5/2)\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c]\*e)])

**Rubi [A]** time = 8.74588, antiderivative size = 453, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$\sqrt{2} \left( bc \left( e \left( 2d\sqrt{b^2 - 4ac} - 3ae \right) + cd^2 \right) + c \left( ae^2\sqrt{b^2 - 4ac} - cd \left( d\sqrt{b^2 - 4ac} - 4ae \right) \right) - b^2e \left( e\sqrt{b^2 - 4ac} + 2cd \right) + b^3e^2 \right) \tan^{-1} \left( \frac{c^{5/2}\sqrt{b^2 - 4ac}\sqrt{2cd - e \left( b - \sqrt{b^2 - 4ac} \right)}}{\sqrt{2} \left( bc \left( cd^2 - e \left( 2d\sqrt{b^2 - 4ac} + 3ae \right) \right) - c \left( ae^2\sqrt{b^2 - 4ac} - cd \left( d\sqrt{b^2 - 4ac} + 4ae \right) \right) - b^2e \left( 2cd - e\sqrt{b^2 - 4ac} \right) + b^3e^2 \right)} \right)$$

$$\frac{c^{5/2}\sqrt{b^2 - 4ac}\sqrt{2cd - e \left( b - \sqrt{b^2 - 4ac} \right)}}{\sqrt{2} \left( bc \left( cd^2 - e \left( 2d\sqrt{b^2 - 4ac} + 3ae \right) \right) - c \left( ae^2\sqrt{b^2 - 4ac} - cd \left( d\sqrt{b^2 - 4ac} + 4ae \right) \right) - b^2e \left( 2cd - e\sqrt{b^2 - 4ac} \right) + b^3e^2 \right)}$$


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$$\frac{c^{5/2}\sqrt{b^2 - 4ac}\sqrt{2cd - e \left( \sqrt{b^2 - 4ac} + b \right)}}{\sqrt{2} \left( bc \left( cd^2 - e \left( 2d\sqrt{b^2 - 4ac} + 3ae \right) \right) - c \left( ae^2\sqrt{b^2 - 4ac} - cd \left( d\sqrt{b^2 - 4ac} + 4ae \right) \right) - b^2e \left( 2cd - e\sqrt{b^2 - 4ac} \right) + b^3e^2 \right)}$$


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$$+ \frac{2\sqrt{d+ex}(cd-be)}{c^2} + \frac{2(d+ex)^{3/2}}{3c}$$

Antiderivative was successfully verified.

[In] Int[(x\*(d + e\*x)^(3/2))/(a + b\*x + c\*x^2), x]

[Out] (2\*(c\*d - b\*e)\*Sqrt[d + e\*x])/c^2 + (2\*(d + e\*x)^(3/2))/(3\*c) + (Sqrt[2]\*(b^3\*e^2 - b^2\*e\*(2\*c\*d + Sqrt[b^2 - 4\*a\*c]\*e) + c\*(a\*Sqrt[b^2 - 4\*a\*c]\*e^2 - c\*d\*(Sqrt[b^2 - 4\*a\*c]\*d - 4\*a\*e)) + b\*c\*(c\*d^2 + e\*(2\*Sqrt[b^2 - 4\*a\*c]\*d - 3\*a\*e)))\*ArcTanh[(Sqrt[2]\*Sqrt[c]\*Sqrt[d + e\*x])/Sqrt[2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c]\*e)]]/(c^(5/2)\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c]\*e)]) - (Sqrt[2]\*(b^3\*e^2 - b^2\*e\*(2\*c\*d - Sqrt[b^2 - 4\*a\*c]\*e) + b\*c\*(c\*d^2 - e\*(2\*Sqrt[b^2 - 4\*a\*c]\*d + 3\*a\*e)) - c\*(a\*Sqrt[b^2 - 4\*a\*c]\*e^2 - c\*d\*(Sqrt[b^2 - 4\*a\*c]\*d + 4\*a\*e)))\*ArcTanh[(Sqrt[2]\*Sqrt[c]\*Sqrt[d + e\*x])/Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c]\*e)]]/(c^(5/2)\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c]\*e)])

2)\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e])

**Rubi in Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x*(e*x+d)**(3/2)/(c*x**2+b*x+a), x)`

[Out] Timed out

**Mathematica** [A] time = 0.891702, size = 441, normalized size = 0.97

$$\frac{\sqrt{2} \left( -bc \left( e \left( 2d\sqrt{b^2 - 4ac} - 3ae \right) + cd^2 \right) + c \left( cd \left( d\sqrt{b^2 - 4ac} - 4ae \right) - ae^2\sqrt{b^2 - 4ac} \right) + b^2e \left( e\sqrt{b^2 - 4ac} + 2cd \right) + b^3 \left( - \right) \right)}{c^{5/2}\sqrt{b^2 - 4ac}\sqrt{e \left( \sqrt{b^2 - 4ac} - b \right) + 2cd}}$$

$$\frac{\sqrt{2} \left( bc \left( cd^2 - e \left( 2d\sqrt{b^2 - 4ac} + 3ae \right) \right) + c \left( cd \left( d\sqrt{b^2 - 4ac} + 4ae \right) - ae^2\sqrt{b^2 - 4ac} \right) + b^2e \left( e\sqrt{b^2 - 4ac} - 2cd \right) + b^3e^2 \right)}{c^{5/2}\sqrt{b^2 - 4ac}\sqrt{2cd - e \left( \sqrt{b^2 - 4ac} + b \right)}}$$

$$+ \frac{2\sqrt{d + ex(-3be + 4cd + cex)}}{3c^2}$$

Antiderivative was successfully verified.

[In] `Integrate[(x*(d + e*x)^(3/2))/(a + b*x + c*x^2), x]`

[Out]  $(2*\text{Sqrt}[d + e*x]*(4*c*d - 3*b*e + c*e*x))/(3*c^2) - (\text{Sqrt}[2]*(-b^3*e^2 + b^2*e*(2*c*d + \text{Sqrt}[b^2 - 4*a*c]*e) + c*(-(a*\text{Sqrt}[b^2 - 4*a*c]*e^2) + c*d*(\text{Sqrt}[b^2 - 4*a*c]*d - 4*a*e)) - b*c*(c*d^2 + e*(2*\text{Sqrt}[b^2 - 4*a*c]*d - 3*a*e)))*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - b*e + \text{Sqrt}[b^2 - 4*a*c]*e]]/(c^{5/2}*\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c])*e]) - (\text{Sqrt}[2]*(b^3*e^2 + b^2*e*(-2*c*d + \text{Sqrt}[b^2 - 4*a*c]*e) + b*c*(c*d^2 - e*(2*\text{Sqrt}[b^2 - 4*a*c]*d + 3*a*e)) + c*(-(a*\text{Sqrt}[b^2 - 4*a*c]*e^2) + c*d*(\text{Sqrt}[b^2 - 4*a*c]*d + 4*a*e)))*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]]/(c^{5/2}*\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e])$

**Maple** [B] time = 0.053, size = 1714, normalized size = 3.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x*(e*x+d)^(3/2)/(c*x^2+b*x+a), x)`

[Out]  $2/3*(e*x+d)^{3/2}/c - 2/c^2*b*e*(e*x+d)^{1/2} + 2/c*d*(e*x+d)^{1/2} - 3/c/(-e^2*(4*a*c - b^2))^{1/2}*2^{1/2}/((b*e - 2*c*d + (-e^2*(4*a*c - b^2))^{1/2})^{1/2})^c)^{1/2}*\arctan(c*(e*x+d)^{1/2}*2^{1/2}/((b*e - 2*c*d + (-e^2*(4*a*c - b^2))^{1/2})^{1/2})^c)^{1/2})^a*b*e^3 + 4/(-e^2*(4*a*c - b^2))^{1/2})^c)^{1/2}*\arctan(c$

$$\begin{aligned} & * (e^*x+d)^{(1/2)} * 2^{(1/2)} / ((b^*e-2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)} \\ & * a^*d^*e^{*2}+1/c^{*2} / (-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)} * 2^{(1/2)} / ((b^*e-2^*c*d+ \\ & (-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)} * \arctan(c^*(e^*x+d)^{(1/2)} * 2^{(1/2)} \\ & / ((b^*e-2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)}) * b^{*3} * e^{*3} - 2/c / (-e^{*2} \\ & * (4^*a^*c-b^{*2}))^{(1/2)} * 2^{(1/2)} / ((b^*e-2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)} \\ & * \arctan(c^*(e^*x+d)^{(1/2)} * 2^{(1/2)} / ((b^*e-2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)}) \\ & * b^{*2} * d^*e^{*2}+1 / (-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)} * 2^{(1/2)} / ((b^*e-2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)} \\ & * \arctan(c^*(e^*x+d)^{(1/2)} * 2^{(1/2)} / ((b^*e-2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)}) \\ & * b^*d^{*2} * e - 1/c^{*2} / ((b^*e-2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)} \\ & * \arctan(c^*(e^*x+d)^{(1/2)} * 2^{(1/2)} / ((b^*e-2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)}) \\ & * a^*e^{*2}+1/c^{*2} * 2^{(1/2)} / ((b^*e-2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)} \\ & * \arctan(c^*(e^*x+d)^{(1/2)} * 2^{(1/2)} / ((b^*e-2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)}) \\ & * b^{*2} * e^{*2} - 2/c^{*2} / ((b^*e-2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)} \\ & * \arctan(c^*(e^*x+d)^{(1/2)} * 2^{(1/2)} / ((b^*e-2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)}) \\ & * b^*d^*e^{*2}+1/c^{*2} * 2^{(1/2)} / ((b^*e-2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)} \\ & * \arctan(c^*(e^*x+d)^{(1/2)} * 2^{(1/2)} / ((b^*e-2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)}) \\ & * a^*e^{*2}+1/c^{*2} * 2^{(1/2)} / ((b^*e-2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)} \\ & * \arctan(c^*(e^*x+d)^{(1/2)} * 2^{(1/2)} / ((b^*e-2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)}) \\ & * b^{*2} * e^{*2} - 2/c^{*2} / ((b^*e-2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)} \\ & * \arctan(c^*(e^*x+d)^{(1/2)} * 2^{(1/2)} / ((b^*e-2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)}) \\ & * a^*b^*e^{*3}+4 / (-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)} * 2^{(1/2)} / ((-b^*e+2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)} \\ & * \operatorname{arctanh}(c^*(e^*x+d)^{(1/2)} * 2^{(1/2)} / ((-b^*e+2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)}) \\ & * a^*d^*e^{*2}+1/c^{*2} / (-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)} * 2^{(1/2)} / ((-b^*e+2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)} \\ & * \operatorname{arctanh}(c^*(e^*x+d)^{(1/2)} * 2^{(1/2)} / ((-b^*e+2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)}) \\ & * b^{*3} * e^{*3} - 2/c / (-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)} * 2^{(1/2)} / ((-b^*e+2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)} \\ & * \operatorname{arctanh}(c^*(e^*x+d)^{(1/2)} * 2^{(1/2)} / ((-b^*e+2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)}) \\ & * b^{*2} * d^*e^{*2}+1 / (-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)} * 2^{(1/2)} / ((-b^*e+2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)} \\ & * \operatorname{arctanh}(c^*(e^*x+d)^{(1/2)} * 2^{(1/2)} / ((-b^*e+2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)}) \\ & * a^*e^{*2} - 1/c^{*2} * 2^{(1/2)} / ((-b^*e+2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)} \\ & * \operatorname{arctanh}(c^*(e^*x+d)^{(1/2)} * 2^{(1/2)} / ((-b^*e+2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)}) \\ & * b^{*2} * e^{*2} + 2/c^{*2} / ((-b^*e+2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)} \\ & * \operatorname{arctanh}(c^*(e^*x+d)^{(1/2)} * 2^{(1/2)} / ((-b^*e+2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)}) \\ & * b^*d^*e - 2^{(1/2)} / ((-b^*e+2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)} \\ & * \operatorname{arctanh}(c^*(e^*x+d)^{(1/2)} * 2^{(1/2)} / ((-b^*e+2^*c*d+(-e^{*2} * (4^*a^*c-b^{*2}))^{(1/2)})^*c)^{(1/2)}) \\ & * d^{*2} \end{aligned}$$


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**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}x}{cx^2+bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*x/(c\*x^2 + b\*x + a), x, algorithm="maxima")

[Out] integrate((e\*x + d)^(3/2)\*x/(c\*x^2 + b\*x + a), x)

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**Fricas [A]** time = 0.940821, size = 7522, normalized size = 16.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*x/(c\*x^2 + b\*x + a), x, algorithm="fricas")

[Out] -1/6\*(3\*sqrt(2)\*c^2\*sqrt(((b^2\*c^3 - 2\*a\*c^4)\*d^3 - 3\*(b^3\*c^2 - 3\*a\*b\*c^3)\*d^2\*e + 3\*(b^4\*c - 4\*a\*b^2\*c^2 + 2\*a^2\*c^3)\*d\*e^2 - (b

$$\begin{aligned}
& a^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^3 + (b^2*c^5 - 4*a*c^6)*\sqrt{(b^2 \\
& *c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2* \\
& c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2* \\
& b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2 \\
& *a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^ \\
& 3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^ \\
& 3 + a^4*c^4)*e^6)/(b^2*c^{10} - 4*a*c^{11}))/ (b^2*c^5 - 4*a*c^6))*\log \\
& (\sqrt{2})*((b^3*c^4 - 4*a*b*c^5)*d^4 - (4*b^4*c^3 - 19*a*b^2*c^4 \\
& + 12*a^2*c^5)*d^3*e + 3*(2*b^5*c^2 - 11*a*b^3*c^3 + 12*a^2*b*c^4) \\
& *d^2*e^2 - (4*b^6*c - 25*a*b^4*c^2 + 37*a^2*b^2*c^3 - 4*a^3*c^4)* \\
& d*e^3 + (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e^4 - (( \\
& b^3*c^6 - 4*a*b*c^7)*d - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*e)*\sqrt{ \\
& \sqrt{(b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 1 \\
& 0*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + \\
& 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2 \\
& *c^4 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^ \\
& 3 - 2*a^3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^ \\
& 3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^{10} - 4*a*c^{11}))*\sqrt{((b^2*c^3 \\
& - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3*(b^4*c - 4*a*b \\
& ^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^3 + \\
& (b^2*c^5 - 4*a*c^6)*\sqrt{(b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^ \\
& 5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^ \\
& 5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20* \\
& a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a* \\
& b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*c + \\
& 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^{10} - 4*a*c \\
& ^{11}))/ (b^2*c^5 - 4*a*c^6)) - 4*(a*b*c^4*d^5 - (4*a*b^2*c^3 - 3*a \\
& ^2*c^4)*d^4*e + 2*(3*a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^2 - 2*(2*a*b^ \\
& 4*c - 3*a^2*b^2*c^2 - a^3*c^3)*d^2*e^3 + (a*b^5 - 5*a^3*b*c^2)*d* \\
& e^4 - (a^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*e^5)*\sqrt{e*x + d}) - 3*\sqrt{ \\
& \sqrt{2})*c^2*\sqrt{((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3) \\
& )*d^2*e + 3*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a* \\
& b^3*c + 5*a^2*b*c^2)*e^3 + (b^2*c^5 - 4*a*c^6)*\sqrt{(b^2*c^6*d^6 \\
& - 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a \\
& ^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^ \\
& 3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5) \\
& *d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)* \\
& d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^ \\
& ^4)*e^6)/(b^2*c^{10} - 4*a*c^{11}))/ (b^2*c^5 - 4*a*c^6))*\log(-\sqrt{2} \\
& )*((b^3*c^4 - 4*a*b*c^5)*d^4 - (4*b^4*c^3 - 19*a*b^2*c^4 + 12*a^2 \\
& *c^5)*d^3*e + 3*(2*b^5*c^2 - 11*a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e^2 \\
& - (4*b^6*c - 25*a*b^4*c^2 + 37*a^2*b^2*c^3 - 4*a^3*c^4)*d*e^3 + \\
& (b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e^4 - ((b^3*c^6 \\
& - 4*a*b*c^7)*d - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*e)*\sqrt{(b^2 \\
& *c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2* \\
& c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2* \\
& b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2 \\
& *a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^ \\
& 3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^ \\
& 3 + a^4*c^4)*e^6)/(b^2*c^{10} - 4*a*c^{11}))*\sqrt{((b^2*c^3 - 2*a*c^ \\
& 4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3*(b^4*c - 4*a*b^2*c^2 + \\
& 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^3 + (b^2*c^ \\
& 5 - 4*a*c^6)*\sqrt{(b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3* \\
& (5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - \\
& 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^ \\
& 3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 \\
& + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2* \\
& b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^{10} - 4*a*c^{11}))/ ( \\
& b^2*c^5 - 4*a*c^6)) - 4*(a*b*c^4*d^5 - (4*a*b^2*c^3 - 3*a^2*c^4)* \\
& d^4*e + 2*(3*a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^2 - 2*(2*a*b^4*c - 3* \\
& a^2*b^2*c^2 - a^3*c^3)*d^2*e^3 + (a*b^5 - 5*a^3*b*c^2)*d*e^4 - (a \\
& ^2*b^4 - 3*a^3*b^2*c + a^4*c^2)*e^5)*\sqrt{e*x + d}) + 3*\sqrt{2}*c \\
& ^2*\sqrt{((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e \\
& + 3*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + \\
& 5*a^2*b*c^2)*e^3 - (b^2*c^5 - 4*a*c^6)*\sqrt{(b^2*c^6*d^6 - 6*(b^3 \\
& *c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)* \\
& d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3*e^3 + \\
& 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^2*e^4 \\
& - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + \\
& (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6) \\
& / (b^2*c^{10} - 4*a*c^{11}))/ (b^2*c^5 - 4*a*c^6))*\log(\sqrt{2})*((b^3*c \\
& ^4 - 4*a*b*c^5)*d^4 - (4*b^4*c^3 - 19*a*b^2*c^4 + 12*a^2*c^5)*d^3
\end{aligned}$$

$$\begin{aligned}
& *e + 3*(2*b^5*c^2 - 11*a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e^2 - (4*b^6 \\
& *c - 25*a*b^4*c^2 + 37*a^2*b^2*c^3 - 4*a^3*c^4)*d*e^3 + (b^7 - 7* \\
& a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e^4 + ((b^3*c^6 - 4*a*b*c \\
& ^7)*d - (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*e)*\text{sqrt}((b^2*c^6*d^6 \\
& - 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a \\
& ^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3 \\
& *e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5) \\
& *d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)* \\
& d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c \\
& ^4)*e^6)/(b^2*c^10 - 4*a*c^11))*\text{sqrt}(((b^2*c^3 - 2*a*c^4)*d^3 - \\
& 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3) \\
& *d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^3 - (b^2*c^5 - 4*a*c^6) \\
& *e^4)*\text{sqrt}((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 \\
& - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 \\
& + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2 \\
& *b^2*c^4 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3 \\
& *c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - \\
& 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - \\
& 4*a*c^6) - 4*(a*b*c^4*d^5 - (4*a*b^2*c^3 - 3*a^2*c^4)*d^4*e + 2 \\
& *(3*a*b^3*c^2 - 4*a^2*b*c^3)*d^3*e^2 - 2*(2*a*b^4*c - 3*a^2*b^2*c^2 \\
& - a^3*c^3)*d^2*e^3 + (a*b^5 - 5*a^3*b*c^2)*d*e^4 - (a^2*b^4 - \\
& 3*a^3*b^2*c + a^4*c^2)*e^5)*\text{sqrt}(e*x + d) - 3*\text{sqrt}(2)*c^2*\text{sqrt}(( \\
& (b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 - 3*a*b*c^3)*d^2*e + 3*(b^4*c \\
& - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2) \\
& *e^3 - (b^2*c^5 - 4*a*c^6)*\text{sqrt}((b^2*c^6*d^6 - 6*(b^3*c^5 - a* \\
& b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - \\
& 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 \\
& - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c \\
& - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + (b^8 - 6* \\
& a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6)/(b^2*c^10 \\
& - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*\text{log}(-\text{sqrt}(2))*((b^3*c^4 - 4*a \\
& *b*c^5)*d^4 - (4*b^4*c^3 - 19*a*b^2*c^4 + 12*a^2*c^5)*d^3*e + 3*( \\
& 2*b^5*c^2 - 11*a*b^3*c^3 + 12*a^2*b*c^4)*d^2*e^2 - (4*b^6*c - 25* \\
& a*b^4*c^2 + 37*a^2*b^2*c^3 - 4*a^3*c^4)*d*e^3 + (b^7 - 7*a*b^5*c \\
& + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*e^4 + ((b^3*c^6 - 4*a*b*c^7)*d - \\
& (b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*e)*\text{sqrt}((b^2*c^6*d^6 - 6*(b^3 \\
& *c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*a*b^2*c^5 + 3*a^2*c^6)* \\
& d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19*a^2*b*c^5)*d^3*e^3 + \\
& 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 - 2*a^3*c^5)*d^2*e^4 \\
& - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^5 + \\
& (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^6) \\
& /((b^2*c^10 - 4*a*c^11))*\text{sqrt}(((b^2*c^3 - 2*a*c^4)*d^3 - 3*(b^3*c^2 \\
& - 3*a*b*c^3)*d^2*e + 3*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e^2 \\
& - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^3 - (b^2*c^5 - 4*a*c^6)*\text{sqrt} \\
& ((b^2*c^6*d^6 - 6*(b^3*c^5 - a*b*c^6)*d^5*e + 3*(5*b^4*c^4 - 10*a \\
& *b^2*c^5 + 3*a^2*c^6)*d^4*e^2 - 2*(10*b^5*c^3 - 30*a*b^3*c^4 + 19 \\
& *a^2*b*c^5)*d^3*e^3 + 3*(5*b^6*c^2 - 20*a*b^4*c^3 + 20*a^2*b^2*c^4 \\
& - 2*a^3*c^5)*d^2*e^4 - 6*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - \\
& 2*a^3*b*c^4)*d*e^5 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2 \\
& *c^3 + a^4*c^4)*e^6)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6) \\
& )) - 4*(a*b*c^4*d^5 - (4*a*b^2*c^3 - 3*a^2*c^4)*d^4*e + 2*(3*a*b^3 \\
& *c^2 - 4*a^2*b*c^3)*d^3*e^2 - 2*(2*a*b^4*c - 3*a^2*b^2*c^2 - a^3 \\
& *c^3)*d^2*e^3 + (a*b^5 - 5*a^3*b*c^2)*d*e^4 - (a^2*b^4 - 3*a^3*b^2 \\
& *c + a^4*c^2)*e^5)*\text{sqrt}(e*x + d) - 4*(c*e*x + 4*c*d - 3*b*e)*\text{sq} \\
& \text{rt}(e*x + d))/c^2
\end{aligned}$$


---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*(e\*x+d)\*\*(3/2)/(c\*x\*\*2+b\*x+a), x)

[Out] Timed out

---

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^(3/2)*x/(c*x^2 + b*x + a),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.537 \quad \int \frac{(d+ex)^{3/2}}{a+bx+cx^2} dx$$

**Optimal.** Leaf size=322

$$\frac{\sqrt{2} \left( -2ce \left( -d\sqrt{b^2 - 4ac} + ae + bd \right) + be^2 \left( b - \sqrt{b^2 - 4ac} \right) + 2c^2d^2 \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} \right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} + \frac{\sqrt{2} \left( -2ce \left( d\sqrt{b^2 - 4ac} + ae + bd \right) + be^2 \left( \sqrt{b^2 - 4ac} + b \right) + 2c^2d^2 \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} \right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} + \frac{2e\sqrt{d+ex}}{c}$$

[Out] (2\*e\*Sqrt[d + e\*x])/c - (Sqrt[2]\*(2\*c^2\*d^2 + b\*(b - Sqrt[b^2 - 4\*a\*c]))\*e^2 - 2\*c\*e\*(b\*d - Sqrt[b^2 - 4\*a\*c]\*d + a\*e))\*ArcTanh[(Sqrt[2]\*Sqrt[c]\*Sqrt[d + e\*x])/Sqrt[2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e]]/(c^(3/2)\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e]) + (Sqrt[2]\*(2\*c^2\*d^2 + b\*(b + Sqrt[b^2 - 4\*a\*c]))\*e^2 - 2\*c\*e\*(b\*d + Sqrt[b^2 - 4\*a\*c]\*d + a\*e))\*ArcTanh[(Sqrt[2]\*Sqrt[c]\*Sqrt[d + e\*x])/Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e]]/(c^(3/2)\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e])

**Rubi [A]** time = 2.47086, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{\sqrt{2} \left( -2ce \left( -d\sqrt{b^2 - 4ac} + ae + bd \right) + be^2 \left( b - \sqrt{b^2 - 4ac} \right) + 2c^2d^2 \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} \right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} + \frac{\sqrt{2} \left( -2ce \left( d\sqrt{b^2 - 4ac} + ae + bd \right) + be^2 \left( \sqrt{b^2 - 4ac} + b \right) + 2c^2d^2 \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} \right)}{c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} + \frac{2e\sqrt{d+ex}}{c}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^(3/2)/(a + b\*x + c\*x^2), x]

[Out] (2\*e\*Sqrt[d + e\*x])/c - (Sqrt[2]\*(2\*c^2\*d^2 + b\*(b - Sqrt[b^2 - 4\*a\*c]))\*e^2 - 2\*c\*e\*(b\*d - Sqrt[b^2 - 4\*a\*c]\*d + a\*e))\*ArcTanh[(Sqrt[2]\*Sqrt[c]\*Sqrt[d + e\*x])/Sqrt[2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e]]/(c^(3/2)\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e]) + (Sqrt[2]\*(2\*c^2\*d^2 + b\*(b + Sqrt[b^2 - 4\*a\*c]))\*e^2 - 2\*c\*e\*(b\*d + Sqrt[b^2 - 4\*a\*c]\*d + a\*e))\*ArcTanh[(Sqrt[2]\*Sqrt[c]\*Sqrt[d + e\*x])/Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e]]/(c^(3/2)\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e])

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out



Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**(3/2)/(c*x**2+b*x+a),x)`

[Out] Timed out

**Mathematica [A]** time = 0.729373, size = 317, normalized size = 0.98

$$\frac{\sqrt{2}\left(2ce\left(-d\sqrt{b^2-4ac+ae+bd}\right)+be^2\left(\sqrt{b^2-4ac-b}\right)-2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac-be+2cd}}}\right)+\sqrt{2}\left(-2ce\left(d\sqrt{b^2-4ac+ae+bd}\right)+be^2\left(\sqrt{b^2-4ac+b}\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac-be+2cd}}}\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac-b}\right)+2cd}+c^{3/2}}$$

Antiderivative was successfully verified.

[In] `Integrate[(d + e*x)^(3/2)/(a + b*x + c*x^2),x]`

[Out]  $(2*\text{Sqrt}[c]*e*\text{Sqrt}[d + e*x] + (\text{Sqrt}[2]*(-2*c^2*d^2 + b*(-b + \text{Sqrt}[b^2 - 4*a*c]))*e^2 + 2*c*e*(b*d - \text{Sqrt}[b^2 - 4*a*c]*d + a*e))*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - b*e + \text{Sqrt}[b^2 - 4*a*c]*e]])/(\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c]*e)] + (\text{Sqrt}[2]*(2*c^2*d^2 + b*(b + \text{Sqrt}[b^2 - 4*a*c]))*e^2 - 2*c*e*(b*d + \text{Sqrt}[b^2 - 4*a*c]*d + a*e))*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]*e)])/(\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]*e)))/c^{3/2}$

**Maple [B]** time = 0.041, size = 1138, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^(3/2)/(c*x^2+b*x+a),x)`

[Out]  $2*e*(e*x+d)^{1/2}/c+2/(-e^{1/2}*(4*a*c-b^2))^{1/2}*2^{1/2}/((b*e-2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2}*\arctan(c*(e*x+d)^{1/2}*2^{1/2}/((b*e-2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2})*a*e^3-1/c/(-e^{1/2}*(4*a*c-b^2))^{1/2}*2^{1/2}/((b*e-2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2}*\arctan(c*(e*x+d)^{1/2}*2^{1/2}/((b*e-2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2})*b^2*e^3+2/(-e^{1/2}*(4*a*c-b^2))^{1/2}*2^{1/2}/((b*e-2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2}*\arctan(c*(e*x+d)^{1/2}*2^{1/2}/((b*e-2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2})*b*d*e^2-2*e*c/(-e^{1/2}*(4*a*c-b^2))^{1/2}*2^{1/2}/((b*e-2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2}*\arctan(c*(e*x+d)^{1/2}*2^{1/2}/((b*e-2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2})*d^2-1/c*2^{1/2}/((b*e-2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2}*\arctan(c*(e*x+d)^{1/2}*2^{1/2}/((b*e-2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2})*b^2*e^2+2*e^2/((b*e-2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2}*\arctan(c*(e*x+d)^{1/2}*2^{1/2}/((b*e-2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2})*d+2/(-e^{1/2}*(4*a*c-b^2))^{1/2}*2^{1/2}/((-b*e+2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2}*\operatorname{arctanh}(c*(e*x+d)^{1/2}*2^{1/2}/((-b*e+2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2})*a*e^3-1/c/(-e^{1/2}*(4*a*c-b^2))^{1/2}*2^{1/2}/((-b*e+2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2}*\operatorname{arctanh}(c*(e*x+d)^{1/2}*2^{1/2}/((-b*e+2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2})*b*d*e^2-2*e*c/(-e^{1/2}*(4*a*c-b^2))^{1/2}*2^{1/2}/((-b*e+2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2}*\operatorname{arctanh}(c*(e*x+d)^{1/2}*2^{1/2}/((-b*e+2*c*d+(-e^{1/2}*(4*a*c-b^2))^{1/2})*c)^{1/2})*d^2+1/c*2^{1/2}/((-b*e+2*c$

$$d+(-e^{2*(4*a*c-b^2)})^{1/2})^c)^{1/2}*\operatorname{arctanh}(c*(e*x+d)^{1/2})^{2*(1/2)/((-b*e+2*c*d+(-e^{2*(4*a*c-b^2)})^{1/2})^c)^{1/2}}*b*e^{2-2*e^{2*(1/2)/((-b*e+2*c*d+(-e^{2*(4*a*c-b^2)})^{1/2})^c)^{1/2}}*\operatorname{arctanh}(c*(e*x+d)^{1/2})^{2*(1/2)/((-b*e+2*c*d+(-e^{2*(4*a*c-b^2)})^{1/2})^c)^{1/2}})*d$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^{\frac{3}{2}}}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)/(c\*x^2 + b\*x + a),x, algorithm="maxima")

[Out] integrate((e\*x + d)^(3/2)/(c\*x^2 + b\*x + a), x)

**Fricas [A]** time = 0.401202, size = 3740, normalized size = 11.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)/(c\*x^2 + b\*x + a),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/2*(\sqrt{2}*c*\sqrt{((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*\sqrt{(9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7))})/(b^2*c^3 - 4*a*c^4))*\log(\sqrt{2}*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^3 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 - (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e)*\sqrt{(9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))*\sqrt{((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*\sqrt{(9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7))})/(b^2*c^3 - 4*a*c^4)) - 4*(3*c^3*d^4*e - 6*b*c^2*d^3*e^2 + 2*(2*b^2*c + a*c^2)*d^2*e^3 - (b^3 + 2*a*b*c)*d*e^4 + (a*b^2 - a^2*c)*e^5)*\sqrt{e*x + d}) - \sqrt{2}*c*\sqrt{((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*\sqrt{(9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7))})/(b^2*c^3 - 4*a*c^4))*\log(-\sqrt{2}*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^3 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 - (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e)*\sqrt{(9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))*\sqrt{((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*\sqrt{(9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7))})/(b^2*c^3 - 4*a*c^4)) - 4*(3*c^3*d^4*e - 6*b*c^2*d^3*e^2 + 2*(2*b^2*c + a*c^2)*d^2*e^3 - (b^3 + 2*a*b*c)*d*e^4 + (a*b^2 - a^2*c)*e^5)*\sqrt{e*x + d}) + \sqrt{2}*c*\sqrt{((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*\sqrt{(9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7))})/(b^2*c^3 - 4*a*c^4))*\log(\sqrt{2}*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^3 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 + (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e)*\sqrt{(9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7))})/(b^2*c^3 - 4*a*c^4))*\log(\sqrt{2}*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^3 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 + (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e)*\sqrt{(9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7))})/(b^2*c^3 - 4*a*c^4))*\log(\sqrt{2}*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^3 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 + (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e)*\sqrt{(9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7))})/(b^2*c^3 - 4*a*c^4))*\log(\sqrt{2}*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^3 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 + (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e)*\sqrt{(9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7))})/(b^2*c^3 - 4*a*c^4))$$

$$\begin{aligned}
& *c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e)*\sqrt{(9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))*\sqrt{(2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*\sqrt{(9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 4*(3*c^3*d^4*e - 6*b*c^2*d^3*e^2 + 2*(2*b^2*c + a*c^2)*d^2*e^3 - (b^3 + 2*a*b*c)*d*e^4 + (a*b^2 - a^2*c)*e^5)*\sqrt{e*x + d)} - \sqrt{2)*c*\sqrt{(2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*\sqrt{(9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*\log(-\sqrt{2)*(3*(b^2*c^2 - 4*a*c^3)*d^2*e^2 - 3*(b^3*c - 4*a*b*c^2)*d*e^3 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^4 + (2*(b^2*c^4 - 4*a*c^5)*d - (b^3*c^3 - 4*a*b*c^4)*e)*\sqrt{(9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))*\sqrt{(2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*\sqrt{(9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 4*(3*c^3*d^4*e - 6*b*c^2*d^3*e^2 + 2*(2*b^2*c + a*c^2)*d^2*e^3 - (b^3 + 2*a*b*c)*d*e^4 + (a*b^2 - a^2*c)*e^5)*\sqrt{e*x + d)} - 4*\sqrt{e*x + d)*e)/c
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(3/2)/(c\*x\*\*2+b\*x+a),x)

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)/(c\*x^2 + b\*x + a),x, algorithm="giac")

[Out] Timed out

$$3.538 \quad \int \frac{(d+ex)^{3/2}}{x(a+bx+cx^2)} dx$$

**Optimal.** Leaf size=340

$$\frac{\sqrt{2} \left( -cd \left( d\sqrt{b^2 - 4ac} - 4ae \right) + ae^2\sqrt{b^2 - 4ac} - b \left( ae^2 + cd^2 \right) \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} \right)}{a\sqrt{c}\sqrt{b^2 - 4ac}\sqrt{2cd - e \left( b - \sqrt{b^2 - 4ac} \right)}} - \frac{\sqrt{2} \left( -cd \left( d\sqrt{b^2 - 4ac} + 4ae \right) + ae^2\sqrt{b^2 - 4ac} + b \left( ae^2 + cd^2 \right) \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} \right)}{a\sqrt{c}\sqrt{b^2 - 4ac}\sqrt{2cd - e \left( \sqrt{b^2 - 4ac} + b \right)}} - \frac{2d^{3/2} \tanh^{-1} \left( \frac{\sqrt{d+ex}}{\sqrt{d}} \right)}{a}$$

[Out]  $(-2*d^{(3/2)}*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a - (Sqrt[2]*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) - b*(c*d^2 + a*e^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + b*(c*d^2 + a*e^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])$

**Rubi [A]** time = 3.16591, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{\sqrt{2} \left( -cd \left( d\sqrt{b^2 - 4ac} - 4ae \right) + ae^2\sqrt{b^2 - 4ac} - b \left( ae^2 + cd^2 \right) \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} \right)}{a\sqrt{c}\sqrt{b^2 - 4ac}\sqrt{2cd - e \left( b - \sqrt{b^2 - 4ac} \right)}} - \frac{\sqrt{2} \left( -cd \left( d\sqrt{b^2 - 4ac} + 4ae \right) + ae^2\sqrt{b^2 - 4ac} + b \left( ae^2 + cd^2 \right) \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} \right)}{a\sqrt{c}\sqrt{b^2 - 4ac}\sqrt{2cd - e \left( \sqrt{b^2 - 4ac} + b \right)}} - \frac{2d^{3/2} \tanh^{-1} \left( \frac{\sqrt{d+ex}}{\sqrt{d}} \right)}{a}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^(3/2)/(x\*(a + b\*x + c\*x^2)), x]

[Out]  $(-2*d^{(3/2)}*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/a - (Sqrt[2]*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) - b*(c*d^2 + a*e^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + b*(c*d^2 + a*e^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**(3/2)/x/(c*x**2+b*x+a), x)`

[Out] Timed out

**Mathematica [A]** time = 0.629174, size = 331, normalized size = 0.97

$$\frac{\sqrt{2}\left(cd\left(d\sqrt{b^2-4ac}-4ae\right)-ae^2\sqrt{b^2-4ac}+b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right)}{\sqrt{c}\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} + \frac{\sqrt{2}\left(cd\left(d\sqrt{b^2-4ac}+4ae\right)-ae^2\sqrt{b^2-4ac}-b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{c}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

*a*

Antiderivative was successfully verified.

[In] `Integrate[(d + e*x)^(3/2)/(x*(a + b*x + c*x^2)), x]`

[Out]  $(-2*d^{3/2}*ArcTanh[Sqrt[d + e*x]/Sqrt[d]] + (Sqrt[2]*(-a*Sqrt[b^2 - 4*a*c]*e^2) + c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) + b*(c*d^2 + a*e^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*(-a*Sqrt[b^2 - 4*a*c]*e^2) + c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) - b*(c*d^2 + a*e^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])/a$

**Maple [B]** time = 0.046, size = 944, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^(3/2)/x/(c*x^2+b*x+a), x)`

[Out]  $e^3/(-e^{2*(4*a*c-b^2)})^{1/2}*2^{1/2}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)}))^{1/2})*c^{1/2}*arctan(c*(e*x+d)^{1/2}*2^{1/2}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)}))^{1/2})*c^{1/2})*b-4*e^2*c/(-e^{2*(4*a*c-b^2)})^{1/2}*2^{1/2}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)}))^{1/2})*c^{1/2}*arctan(c*(e*x+d)^{1/2}*2^{1/2}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)}))^{1/2})*c^{1/2})*d+e/a*c/(-e^{2*(4*a*c-b^2)})^{1/2}*2^{1/2}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)}))^{1/2})*c^{1/2}*arctan(c*(e*x+d)^{1/2}*2^{1/2}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)}))^{1/2})*c^{1/2})*b*d^2+e^2*2^{1/2}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)}))^{1/2})*c^{1/2}*arctan(c*(e*x+d)^{1/2}*2^{1/2}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)}))^{1/2})*c^{1/2})*2^{1/2}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)}))^{1/2})*c^{1/2})*-1/a*c^2^{1/2}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)}))^{1/2})*c^{1/2}*arctan(c*(e*x+d)^{1/2}*2^{1/2}/((b*e-2*c*d+(-e^{2*(4*a*c-b^2)}))^{1/2})*c^{1/2})*d^2+e^3/(-e^{2*(4*a*c-b^2)})^{1/2}*2^{1/2}/((-b*e+2*c*d+(-e^{2*(4*a*c-b^2)}))^{1/2})*c^{1/2}*arctanh(c*(e*x+d)^{1/2}*2^{1/2}/((-b*e+2*c*d+(-e^{2*(4*a*c-b^2)}))^{1/2})*c^{1/2})*b-4*e^2*c/(-e^{2*(4*a*c-b^2)})^{1/2}*2^{1/2}/((-b*e+2*c*d+(-e^{2*(4*a*c-b^2)}))^{1/2})*c^{1/2}*arctanh(c*(e*x+d)^{1/2}*2^{1/2}/((-b*e+2*c*d+(-e^{2*(4*a*c-b^2)}))^{1/2})*c^{1/2})*d+e/a*c/(-e^{2*(4*a*c-b^2)})^{1/2}*2^{1/2}/((-b*e+2*c*d+(-e^{2*(4*a*c-b^2)}))^{1/2})*c^{1/2}*arctanh(c*(e*x+d)^{1/2}*2^{1/2}/((-b*e+2*c*d+(-e^{2*(4*a*c-b^2)}))^{1/2})*c^{1/2})*b*d^2$

$$-e^{2\sqrt{2}}/((-b^2e+2^2cd+(-e^2(4ac-b^2))^{1/2})^2c)^{1/2}\operatorname{arctanh}(c(e^2x+d)^{1/2})^{2\sqrt{2}}/((-b^2e+2^2cd+(-e^2(4ac-b^2))^{1/2})^2c)^{1/2})+1/a^2c^{2\sqrt{2}}/((-b^2e+2^2cd+(-e^2(4ac-b^2))^{1/2})^2c)^{1/2}\operatorname{arctanh}(c(e^2x+d)^{1/2})^{2\sqrt{2}}/((-b^2e+2^2cd+(-e^2(4ac-b^2))^{1/2})^2c)^{1/2})d^2-2d^{3/2}\operatorname{arctanh}((e^2x+d)^{1/2}/d^{1/2})/a$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e^x + d)^(3/2)/((c\*x^2 + b\*x + a)\*x), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 8.62231, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e^x + d)^(3/2)/((c\*x^2 + b\*x + a)\*x), x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/2 * (\sqrt{2} * a * \sqrt{-3 * a * b * c * d^2 * e - 6 * a^2 * c * d * e^2 + a^2 * b * e^3} \\ & - (b^2 * c - 2 * a * c^2) * d^3 + (a^2 * b^2 * c - 4 * a^3 * c^2) * \sqrt{(b^2 * c^2 * \\ & d^6 - 6 * a * b * c^2 * d^5 * e + 9 * a^2 * c^2 * d^4 * e^2 + 2 * a^2 * b * c * d^3 * e^3 - 6 * \\ & a^3 * c * d^2 * e^4 + a^4 * e^6) / (a^4 * b^2 * c^2 - 4 * a^5 * c^3)) / (a^2 * b^2 * c \\ & - 4 * a^3 * c^2) * \log(\sqrt{2} * ((b^3 * c - 4 * a * b * c^2) * d^4 - 3 * (a * b^2 * c - \\ & 4 * a^2 * c^2) * d^3 * e + (a^2 * b^2 - 4 * a^3 * c) * d * e^3 + ((a^2 * b^3 * c - 4 * a \\ & a^3 * b * c^2) * d - 2 * (a^3 * b^2 * c - 4 * a^4 * c^2) * e) * \sqrt{(b^2 * c^2 * d^6 - 6 * \\ & a * b * c^2 * d^5 * e + 9 * a^2 * c^2 * d^4 * e^2 + 2 * a^2 * b * c * d^3 * e^3 - 6 * a^3 * c * d \\ & a^2 * e^4 + a^4 * e^6) / (a^4 * b^2 * c^2 - 4 * a^5 * c^3)) * \sqrt{-3 * a * b * c * d^2 * \\ & e - 6 * a^2 * c * d * e^2 + a^2 * b * e^3 - (b^2 * c - 2 * a * c^2) * d^3 + (a^2 * b^2 * \\ & c - 4 * a^3 * c^2) * \sqrt{(b^2 * c^2 * d^6 - 6 * a * b * c^2 * d^5 * e + 9 * a^2 * c^2 * d^4 * \\ & 4 * e^2 + 2 * a^2 * b * c * d^3 * e^3 - 6 * a^3 * c * d^2 * e^4 + a^4 * e^6) / (a^4 * b^2 * c^2 \\ & a^2 - 4 * a^5 * c^3)) / (a^2 * b^2 * c - 4 * a^3 * c^2) + 4 * (b * c^2 * d^5 + 4 * a * b \\ & * c * d^3 * e^2 - 2 * a^2 * c * d^2 * e^3 - a^2 * b * d * e^4 + a^3 * e^5 - (b^2 * c + 3 \\ & * a * c^2) * d^4 * e) * \sqrt{e * x + d}) - \sqrt{2} * a * \sqrt{-3 * a * b * c * d^2 * e - \\ & 6 * a^2 * c * d * e^2 + a^2 * b * e^3 - (b^2 * c - 2 * a * c^2) * d^3 + (a^2 * b^2 * c - \\ & 4 * a^3 * c^2) * \sqrt{(b^2 * c^2 * d^6 - 6 * a * b * c^2 * d^5 * e + 9 * a^2 * c^2 * d^4 * e^2 \\ & 2 + 2 * a^2 * b * c * d^3 * e^3 - 6 * a^3 * c * d^2 * e^4 + a^4 * e^6) / (a^4 * b^2 * c^2 - \\ & 4 * a^5 * c^3)) / (a^2 * b^2 * c - 4 * a^3 * c^2) * \log(-\sqrt{2} * ((b^3 * c - 4 * a \\ & * b * c^2) * d^4 - 3 * (a * b^2 * c - 4 * a^2 * c^2) * d^3 * e + (a^2 * b^2 - 4 * a^3 * c) \\ & * d * e^3 + ((a^2 * b^3 * c - 4 * a^3 * b * c^2) * d - 2 * (a^3 * b^2 * c - 4 * a^4 * c^2) \\ & * e) * \sqrt{(b^2 * c^2 * d^6 - 6 * a * b * c^2 * d^5 * e + 9 * a^2 * c^2 * d^4 * e^2 + 2 * a \\ & a^2 * b * c * d^3 * e^3 - 6 * a^3 * c * d^2 * e^4 + a^4 * e^6) / (a^4 * b^2 * c^2 - 4 * a^5 * \\ & c^3)) * \sqrt{-3 * a * b * c * d^2 * e - 6 * a^2 * c * d * e^2 + a^2 * b * e^3 - (b^2 * c \\ & - 2 * a * c^2) * d^3 + (a^2 * b^2 * c - 4 * a^3 * c^2) * \sqrt{(b^2 * c^2 * d^6 - 6 * a * \\ & b * c^2 * d^5 * e + 9 * a^2 * c^2 * d^4 * e^2 + 2 * a^2 * b * c * d^3 * e^3 - 6 * a^3 * c * d^2 \\ & * e^4 + a^4 * e^6) / (a^4 * b^2 * c^2 - 4 * a^5 * c^3)) / (a^2 * b^2 * c - 4 * a^3 * c^2) \\ & + 4 * (b * c^2 * d^5 + 4 * a * b * c * d^3 * e^2 - 2 * a^2 * c * d^2 * e^3 - a^2 * b * d * \\ & e^4 + a^3 * e^5 - (b^2 * c + 3 * a * c^2) * d^4 * e) * \sqrt{e * x + d}) + \sqrt{2} \\ & * a * \sqrt{-3 * a * b * c * d^2 * e - 6 * a^2 * c * d * e^2 + a^2 * b * e^3 - (b^2 * c - 2 * \\ & a * c^2) * d^3 - (a^2 * b^2 * c - 4 * a^3 * c^2) * \sqrt{(b^2 * c^2 * d^6 - 6 * a * b * c^2 \\ & 2 * d^5 * e + 9 * a^2 * c^2 * d^4 * e^2 + 2 * a^2 * b * c * d^3 * e^3 - 6 * a^3 * c * d^2 * e^4 \\ & + a^4 * e^6) / (a^4 * b^2 * c^2 - 4 * a^5 * c^3)) / (a^2 * b^2 * c - 4 * a^3 * c^2) * \\ & \log(\sqrt{2} * ((b^3 * c - 4 * a * b * c^2) * d^4 - 3 * (a * b^2 * c - 4 * a^2 * c^2) * d^3 \\ & 3 * e + (a^2 * b^2 - 4 * a^3 * c) * d * e^3 - ((a^2 * b^3 * c - 4 * a^3 * b * c^2) * d - \\ & 2 * (a^3 * b^2 * c - 4 * a^4 * c^2) * e) * \sqrt{(b^2 * c^2 * d^6 - 6 * a * b * c^2 * d^5 * e \\ & + 9 * a^2 * c^2 * d^4 * e^2 + 2 * a^2 * b * c * d^3 * e^3 - 6 * a^3 * c * d^2 * e^4 + a^4 * e^6) \\ & a^6) / (a^4 * b^2 * c^2 - 4 * a^5 * c^3)) * \sqrt{-3 * a * b * c * d^2 * e - 6 * a^2 * c * d * \\ & e^2 + a^2 * b * e^3 - (b^2 * c - 2 * a * c^2) * d^3 + (a^2 * b^2 * c - 4 * a^3 * c^2) * \sqrt{(b^2 * c^2 * d^6 - 6 * a * b * c^2 * d^5 * e + 9 * a^2 * c^2 * d^4 * e^2 + 2 * a^2 * b * c * d^3 * e^3 - 6 * a^3 * c * d^2 * e^4 + a^4 * e^6) / (a^4 * b^2 * c^2 - 4 * a^5 * c^3)) / (a^2 * b^2 * c - 4 * a^3 * c^2) \end{aligned}$$

$$\begin{aligned}
 & e^2 + a^2*b^e^3 - (b^2*c - 2*a*c^2)*d^3 - (a^2*b^2*c - 4*a^3*c^2) \\
 & * \sqrt{(b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2* \\
 & b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3) \\
 & ))/(a^2*b^2*c - 4*a^3*c^2) + 4*(b*c^2*d^5 + 4*a*b*c*d^3*e^2 - 2 \\
 & *a^2*c*d^2*e^3 - a^2*b*d*e^4 + a^3*e^5 - (b^2*c + 3*a*c^2)*d^4*e) \\
 & * \sqrt(e*x + d) - \sqrt(2)*a*\sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d^2*e^2 \\
 & + a^2*b^e^3 - (b^2*c - 2*a*c^2)*d^3 - (a^2*b^2*c - 4*a^3*c^2)*\sqrt{ \\
 & t((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c* \\
 & d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/ \\
 & (a^2*b^2*c - 4*a^3*c^2))*\log(-\sqrt(2)*((b^3*c - 4*a*b*c^2)*d^4 - \\
 & 3*(a*b^2*c - 4*a^2*c^2)*d^3*e + (a^2*b^2 - 4*a^3*c)*d^e^3 - ((a^2 \\
 & *b^3*c - 4*a^3*b*c^2)*d - 2*(a^3*b^2*c - 4*a^4*c^2)*e)*\sqrt((b^2* \\
 & c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 \\
 & - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3))*\sqrt(-( \\
 & 3*a*b*c*d^2*e - 6*a^2*c*d^2*e^2 + a^2*b^e^3 - (b^2*c - 2*a*c^2)*d^3 \\
 & - (a^2*b^2*c - 4*a^3*c^2)*\sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + \\
 & 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6) \\
 & )/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2) + 4*(b*c^2 \\
 & *d^5 + 4*a*b*c*d^3*e^2 - 2*a^2*c*d^2*e^3 - a^2*b*d^e^4 + a^3*e^5 \\
 & - (b^2*c + 3*a*c^2)*d^4*e)*\sqrt(e*x + d) - 2*d^{3/2}*\log((e*x - \\
 & 2*\sqrt(e*x + d)*\sqrt(d) + 2*d)/x)/a, -1/2*(\sqrt(2)*a*\sqrt(-(3*a* \\
 & b*c*d^2*e - 6*a^2*c*d^2*e^2 + a^2*b^e^3 - (b^2*c - 2*a*c^2)*d^3 + ( \\
 & a^2*b^2*c - 4*a^3*c^2)*\sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2* \\
 & c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a \\
 & ^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2))*\log(\sqrt(2)*(( \\
 & b^3*c - 4*a*b*c^2)*d^4 - 3*(a*b^2*c - 4*a^2*c^2)*d^3*e + (a^2*b^2 \\
 & - 4*a^3*c)*d^e^3 + ((a^2*b^3*c - 4*a^3*b*c^2)*d - 2*(a^3*b^2*c - \\
 & 4*a^4*c^2)*e)*\sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4 \\
 & *e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c \\
 & ^2 - 4*a^5*c^3))*\sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d^2*e^2 + a^2*b^e^3 \\
 & - (b^2*c - 2*a*c^2)*d^3 + (a^2*b^2*c - 4*a^3*c^2)*\sqrt((b^2*c^2 \\
 & *d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - \\
 & 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c \\
 & - 4*a^3*c^2) + 4*(b*c^2*d^5 + 4*a*b*c*d^3*e^2 - 2*a^2*c*d^2*e^3 \\
 & - a^2*b*d^e^4 + a^3*e^5 - (b^2*c + 3*a*c^2)*d^4*e)*\sqrt(e*x + d) \\
 & ) - \sqrt(2)*a*\sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d^2*e^2 + a^2*b^e^3 - \\
 & (b^2*c - 2*a*c^2)*d^3 + (a^2*b^2*c - 4*a^3*c^2)*\sqrt((b^2*c^2*d^6 \\
 & - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3 \\
 & *c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4 \\
 & *a^3*c^2))*\log(-\sqrt(2)*((b^3*c - 4*a*b*c^2)*d^4 - 3*(a*b^2*c - 4 \\
 & *a^2*c^2)*d^3*e + (a^2*b^2 - 4*a^3*c)*d^e^3 + ((a^2*b^3*c - 4*a^3 \\
 & *b*c^2)*d - 2*(a^3*b^2*c - 4*a^4*c^2)*e)*\sqrt((b^2*c^2*d^6 - 6*a* \\
 & b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2 \\
 & *e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3))*\sqrt(-(3*a*b*c*d^2*e \\
 & - 6*a^2*c*d^2*e^2 + a^2*b^e^3 - (b^2*c - 2*a*c^2)*d^3 + (a^2*b^2*c \\
 & - 4*a^3*c^2)*\sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4* \\
 & e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 \\
 & - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2) + 4*(b*c^2*d^5 + 4*a*b*c \\
 & *d^3*e^2 - 2*a^2*c*d^2*e^3 - a^2*b*d^e^4 + a^3*e^5 - (b^2*c + 3*a \\
 & *c^2)*d^4*e)*\sqrt(e*x + d) + \sqrt(2)*a*\sqrt(-(3*a*b*c*d^2*e - 6* \\
 & a^2*c*d^2*e^2 + a^2*b^e^3 - (b^2*c - 2*a*c^2)*d^3 - (a^2*b^2*c - 4* \\
 & a^3*c^2)*\sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 \\
 & + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4 \\
 & *a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2))*\log(\sqrt(2)*((b^3*c - 4*a*b* \\
 & c^2)*d^4 - 3*(a*b^2*c - 4*a^2*c^2)*d^3*e + (a^2*b^2 - 4*a^3*c)*d^ \\
 & e^3 - ((a^2*b^3*c - 4*a^3*b*c^2)*d - 2*(a^3*b^2*c - 4*a^4*c^2)*e) \\
 & *\sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2* \\
 & b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3 \\
 & ))*\sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d^2*e^2 + a^2*b^e^3 - (b^2*c - 2 \\
 & *a*c^2)*d^3 - (a^2*b^2*c - 4*a^3*c^2)*\sqrt((b^2*c^2*d^6 - 6*a*b*c \\
 & ^2*d^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 \\
 & + a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2)) \\
 & + 4*(b*c^2*d^5 + 4*a*b*c*d^3*e^2 - 2*a^2*c*d^2*e^3 - a^2*b*d^e^4 \\
 & + a^3*e^5 - (b^2*c + 3*a*c^2)*d^4*e)*\sqrt(e*x + d) - \sqrt(2)*a* \\
 & \sqrt(-(3*a*b*c*d^2*e - 6*a^2*c*d^2*e^2 + a^2*b^e^3 - (b^2*c - 2*a*c \\
 & ^2)*d^3 - (a^2*b^2*c - 4*a^3*c^2)*\sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d \\
 & ^5*e + 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + \\
 & a^4*e^6)/(a^4*b^2*c^2 - 4*a^5*c^3)))/(a^2*b^2*c - 4*a^3*c^2))*\log \\
 & (-\sqrt(2)*((b^3*c - 4*a*b*c^2)*d^4 - 3*(a*b^2*c - 4*a^2*c^2)*d^3* \\
 & e + (a^2*b^2 - 4*a^3*c)*d^e^3 - ((a^2*b^3*c - 4*a^3*b*c^2)*d - 2* \\
 & (a^3*b^2*c - 4*a^4*c^2)*e)*\sqrt((b^2*c^2*d^6 - 6*a*b*c^2*d^5*e + \\
 & 9*a^2*c^2*d^4*e^2 + 2*a^2*b*c*d^3*e^3 - 6*a^3*c*d^2*e^4 + a^4*e^6
 \end{aligned}$$

$$\begin{aligned} & )/(a^4 b^2 c^2 - 4 a^5 c^3)) * \sqrt{-(3 a b c d^2 e - 6 a^2 c d e^2 + a^2 b e^3 - (b^2 c - 2 a c^2) d^3 - (a^2 b^2 c - 4 a^3 c^2) s} \\ & \sqrt{(b^2 c^2 d^6 - 6 a b c^2 d^5 e + 9 a^2 c^2 d^4 e^2 + 2 a^2 b c d^3 e^3 - 6 a^3 c d^2 e^4 + a^4 e^6)/(a^4 b^2 c^2 - 4 a^5 c^3))} \\ & )/(a^2 b^2 c - 4 a^3 c^2)) + 4 (b c^2 d^5 + 4 a b c d^3 e^2 - 2 a^2 c d^2 e^3 - a^2 b d e^4 + a^3 e^5 - (b^2 c + 3 a c^2) d^4 e) s \\ & \sqrt{e x + d} + 4 \sqrt{-d} d \arctan(\sqrt{e x + d}/\sqrt{-d})/a \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(3/2)/x/(c\*x\*\*2+b\*x+a),x)

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)/((c\*x^2 + b\*x + a)\*x),x, algorithm="giac")

[Out] Timed out



$$3.539 \quad \int \frac{(d+ex)^{3/2}}{x^2(a+bx+cx^2)} dx$$

**Optimal.** Leaf size=403

$$\frac{\sqrt{2}\sqrt{c} \left( -2a \left( e \left( d\sqrt{b^2-4ac} - ae \right) + cd^2 \right) + bd \left( d\sqrt{b^2-4ac} - 2ae \right) + b^2d^2 \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} \right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} + \frac{\sqrt{2}\sqrt{c} \left( -2a \left( cd^2 - e \left( d\sqrt{b^2-4ac} + ae \right) \right) - bd \left( d\sqrt{b^2-4ac} + 2ae \right) + b^2d^2 \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} \right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} + \frac{2\sqrt{d}(bd-2ae) \tanh^{-1} \left( \frac{\sqrt{d+ex}}{\sqrt{d}} \right)}{a^2} - \frac{d\sqrt{d+ex}}{ax} + \frac{\sqrt{de} \tanh^{-1} \left( \frac{\sqrt{d+ex}}{\sqrt{d}} \right)}{a}$$

[Out] -((d\*Sqrt[d + e\*x])/(a\*x)) + (Sqrt[d]\*e\*ArcTanh[Sqrt[d + e\*x]/Sqrt[d]])/a + (2\*Sqrt[d]\*(b\*d - 2\*a\*e)\*ArcTanh[Sqrt[d + e\*x]/Sqrt[d]])/a^2 - (Sqrt[2]\*Sqrt[c]\*(b^2\*d^2 + b\*d\*(Sqrt[b^2 - 4\*a\*c]\*d - 2\*a\*e) - 2\*a\*(c\*d^2 + e\*(Sqrt[b^2 - 4\*a\*c]\*d - a\*e)))\*ArcTanh[(Sqrt[2]\*Sqrt[c]\*Sqrt[d + e\*x])/Sqrt[2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e]])/(a^2\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e]) + (Sqrt[2]\*Sqrt[c]\*(b^2\*d^2 - b\*d\*(Sqrt[b^2 - 4\*a\*c]\*d + 2\*a\*e) - 2\*a\*(c\*d^2 - e\*(Sqrt[b^2 - 4\*a\*c]\*d + a\*e)))\*ArcTanh[(Sqrt[2]\*Sqrt[c]\*Sqrt[d + e\*x])/Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e]])/(a^2\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e])

**Rubi [A]** time = 6.0059, antiderivative size = 402, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{\sqrt{2}\sqrt{c} \left( bd \left( d\sqrt{b^2-4ac} - 2ae \right) - 2ae \left( d\sqrt{b^2-4ac} - ae \right) - 2acd^2 + b^2d^2 \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} \right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} + \frac{\sqrt{2}\sqrt{c} \left( -bd \left( d\sqrt{b^2-4ac} + 2ae \right) + 2ae \left( d\sqrt{b^2-4ac} + ae \right) - 2acd^2 + b^2d^2 \right) \tanh^{-1} \left( \frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} \right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} + \frac{2\sqrt{d}(bd-2ae) \tanh^{-1} \left( \frac{\sqrt{d+ex}}{\sqrt{d}} \right)}{a^2} - \frac{d\sqrt{d+ex}}{ax} + \frac{\sqrt{de} \tanh^{-1} \left( \frac{\sqrt{d+ex}}{\sqrt{d}} \right)}{a}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^(3/2)/(x^2\*(a + b\*x + c\*x^2)), x]

[Out] -((d\*Sqrt[d + e\*x])/(a\*x)) + (Sqrt[d]\*e\*ArcTanh[Sqrt[d + e\*x]/Sqrt[d]])/a + (2\*Sqrt[d]\*(b\*d - 2\*a\*e)\*ArcTanh[Sqrt[d + e\*x]/Sqrt[d]])/a^2 - (Sqrt[2]\*Sqrt[c]\*(b^2\*d^2 - 2\*a\*c\*d^2 + b\*d\*(Sqrt[b^2 - 4\*a\*c]\*d - 2\*a\*e) - 2\*a\*e\*(Sqrt[b^2 - 4\*a\*c]\*d - a\*e)))\*ArcTanh[(Sqrt[2]\*Sqrt[c]\*Sqrt[d + e\*x])/Sqrt[2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e]])/(a^2\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e]) + (Sqrt[2]\*Sqrt[c]\*(b^2\*d^2 - 2\*a\*c\*d^2 + 2\*a\*e\*(Sqrt[b^2 - 4\*a\*c]\*d + a\*e) - b\*d\*(Sqrt[b^2 - 4\*a\*c]\*d + 2\*a\*e)))\*ArcTanh[(Sqrt[2]\*Sqrt[c]\*Sqrt[d + e\*x])/Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e]])/(a^2\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e])

\* e])

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**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**(3/2)/x**2/(c*x**2+b*x+a), x)`

[Out] Timed out

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**Mathematica [A]** time = 0.896975, size = 369, normalized size = 0.92

$$\frac{\sqrt{2}\sqrt{c}\left(2a\left(d\sqrt{b^2-4ac}-ae\right)+cd^2\right)+bd\left(2ae-d\sqrt{b^2-4ac}\right)-b^2d^2}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}} \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex}}{\sqrt{e\sqrt{b^2-4ac}-be+2cd}}\right) - \frac{\sqrt{2}\sqrt{c}\left(bd\left(d\sqrt{b^2-4ac}+2ae\right)-2ae\left(d\sqrt{b^2-4ac}+ae\right)+2acd^2\right)}{a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}-b\right)+2cd}}$$

Antiderivative was successfully verified.

[In] `Integrate[(d + e*x)^(3/2)/(x^2*(a + b*x + c*x^2)), x]`

[Out] 
$$\begin{aligned} & -\left(\frac{a*d*\text{Sqrt}[d + e*x]}{x}\right) + \text{Sqrt}[d]*(2*b*d - 3*a*e)*\text{ArcTanh}\left[\frac{\text{Sqrt}[d + e*x]}{\text{Sqrt}[d]}\right] + (\text{Sqrt}[2]*\text{Sqrt}[c]*(-(b^2*d^2) + b*d*(-(\text{Sqrt}[b^2 - 4*a*c]*d) + 2*a*e) + 2*a*(c*d^2 + e*(\text{Sqrt}[b^2 - 4*a*c]*d - a*e))))*\text{ArcTanh}\left[\frac{\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x]}{\text{Sqrt}[2*c*d - b*e + \text{Sqrt}[b^2 - 4*a*c]*e]}\right] / (\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*d + (-b + \text{Sqrt}[b^2 - 4*a*c])*e]) - (\text{Sqrt}[2]*\text{Sqrt}[c]*(-(b^2*d^2) + 2*a*c*d^2 - 2*a*e*(\text{Sqrt}[b^2 - 4*a*c]*d + a*e) + b*d*(\text{Sqrt}[b^2 - 4*a*c]*d + 2*a*e))*\text{ArcTanh}\left[\frac{\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x]}{\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]}\right] / (\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]))/a^2 \end{aligned}$$

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**Maple [B]** time = 0.055, size = 1215, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^(3/2)/x^2/(c*x^2+b*x+a), x)`

[Out] 
$$\begin{aligned} & -2*e^3*c/(-e^2*(4*a*c-b^2))^{(1/2)*2^{(1/2)}}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)*\arctan(c*(e*x+d)^{(1/2)*2^{(1/2)}}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}} + 2*e^2/a*c/(-e^2*(4*a*c-b^2))^{(1/2)*2^{(1/2)}}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)*\arctan(c*(e*x+d)^{(1/2)*2^{(1/2)}}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}} + b*d+2*e/a*c^2/(-e^2*(4*a*c-b^2))^{(1/2)*2^{(1/2)}}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)*\arctan(c*(e*x+d)^{(1/2)*2^{(1/2)}}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}} + d^2-e/a^2*c/(-e^2*(4*a*c-b^2))^{(1/2)*2^{(1/2)}}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)*\arctan(c*(e*x+d)^{(1/2)*2^{(1/2)}}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}} + b^2*d^2-2*e/a*c^2/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)*\arctan(c*(e*x+d)^{(1/2)*2^{(1/2)}}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}} + d+1/a^2*c^2/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)*\arctan(c*(e*x+d)^{(1/2)*2^{(1/2)}}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}} \end{aligned}$$

$$d^{1/2} \cdot 2^{1/2} / ((b^2 e - 2^2 c^2 d + (-e^2 (4^2 a^2 c - b^2))^{1/2})^2 c)^{1/2} \cdot b^2 d^2 - 2^2 e^3 c / (-e^2 (4^2 a^2 c - b^2))^{1/2} \cdot 2^{1/2} / ((-b^2 e + 2^2 c^2 d + (-e^2 (4^2 a^2 c - b^2))^{1/2})^2 c)^{1/2} \cdot \operatorname{arctanh}(c (e^2 x + d)^{1/2} \cdot 2^{1/2} / ((-b^2 e + 2^2 c^2 d + (-e^2 (4^2 a^2 c - b^2))^{1/2})^2 c)^{1/2}) + 2^2 e^2 / a^2 c / (-e^2 (4^2 a^2 c - b^2))^{1/2} \cdot 2^{1/2} / ((-b^2 e + 2^2 c^2 d + (-e^2 (4^2 a^2 c - b^2))^{1/2})^2 c)^{1/2} \cdot \operatorname{arctanh}(c (e^2 x + d)^{1/2} \cdot 2^{1/2} / ((-b^2 e + 2^2 c^2 d + (-e^2 (4^2 a^2 c - b^2))^{1/2})^2 c)^{1/2}) \cdot b^2 d + 2^2 e / a^2 c^2 / (-e^2 (4^2 a^2 c - b^2))^{1/2} \cdot 2^{1/2} / ((-b^2 e + 2^2 c^2 d + (-e^2 (4^2 a^2 c - b^2))^{1/2})^2 c)^{1/2} \cdot \operatorname{arctanh}(c (e^2 x + d)^{1/2} \cdot 2^{1/2} / ((-b^2 e + 2^2 c^2 d + (-e^2 (4^2 a^2 c - b^2))^{1/2})^2 c)^{1/2}) \cdot d^2 - e / a^2 c / (-e^2 (4^2 a^2 c - b^2))^{1/2} \cdot 2^{1/2} / ((-b^2 e + 2^2 c^2 d + (-e^2 (4^2 a^2 c - b^2))^{1/2})^2 c)^{1/2} \cdot \operatorname{arctanh}(c (e^2 x + d)^{1/2} \cdot 2^{1/2} / ((-b^2 e + 2^2 c^2 d + (-e^2 (4^2 a^2 c - b^2))^{1/2})^2 c)^{1/2}) \cdot b^2 d^2 + 2^2 e / a^2 c^2 \cdot d^{1/2} / ((-b^2 e + 2^2 c^2 d + (-e^2 (4^2 a^2 c - b^2))^{1/2})^2 c)^{1/2} \cdot \operatorname{arctanh}(c (e^2 x + d)^{1/2} \cdot 2^{1/2} / ((-b^2 e + 2^2 c^2 d + (-e^2 (4^2 a^2 c - b^2))^{1/2})^2 c)^{1/2}) \cdot d - 1 / a^2 c^2 \cdot 2^{1/2} / ((-b^2 e + 2^2 c^2 d + (-e^2 (4^2 a^2 c - b^2))^{1/2})^2 c)^{1/2} \cdot \operatorname{arctanh}(c (e^2 x + d)^{1/2} \cdot 2^{1/2} / ((-b^2 e + 2^2 c^2 d + (-e^2 (4^2 a^2 c - b^2))^{1/2})^2 c)^{1/2}) \cdot b^2 d^2 - d^2 (e^2 x + d)^{1/2} / a^2 x - 3^2 e^2 \operatorname{arctanh}((e^2 x + d)^{1/2} / d^{1/2}) \cdot d^{1/2} / a^2 \cdot 2^{1/2} / a^2 \cdot \operatorname{arctanh}((e^2 x + d)^{1/2} / d^{1/2}) \cdot b$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*x^2), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 39.0698, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*x^2), x, algorithm="fricas")`

[Out] 
$$\begin{aligned} & [-1/2 \cdot (\sqrt{2}) \cdot a^2 \cdot x \cdot \sqrt{-(a^3 \cdot b \cdot e^3 - (b^4 - 4 \cdot a \cdot b^2 \cdot c + 2 \cdot a^2 \cdot c^2) \cdot d^3 + 3 \cdot (a \cdot b^3 - 3 \cdot a^2 \cdot b \cdot c) \cdot d^2 \cdot e - 3 \cdot (a^2 \cdot b^2 - 2 \cdot a^3 \cdot c) \cdot d \cdot e^2 + (a^4 \cdot b^2 - 4 \cdot a^5 \cdot c) \cdot \sqrt{-(6 \cdot a^5 \cdot b \cdot d \cdot e^5 - a^6 \cdot e^6 - (b^6 - 4 \cdot a \cdot b^4 \cdot c + 4 \cdot a^2 \cdot b^2 \cdot c^2) \cdot d^6 + 6 \cdot (a \cdot b^5 - 3 \cdot a^2 \cdot b^3 \cdot c + 2 \cdot a^3 \cdot b \cdot c^2) \cdot d^5 \cdot e - 3 \cdot (5 \cdot a^2 \cdot b^4 - 10 \cdot a^3 \cdot b^2 \cdot c + 3 \cdot a^4 \cdot c^2) \cdot d^4 \cdot e^2 + 2 \cdot (10 \cdot a^3 \cdot b^3 - 11 \cdot a^4 \cdot b \cdot c) \cdot d^3 \cdot e^3 - 3 \cdot (5 \cdot a^4 \cdot b^2 - 2 \cdot a^5 \cdot c) \cdot d^2 \cdot e^4) / (a^8 \cdot b^2 - 4 \cdot a^9 \cdot c))} / (a^4 \cdot b^2 - 4 \cdot a^5 \cdot c) \cdot \log(\sqrt{2}) \cdot ((b^6 - 6 \cdot a \cdot b^4 \cdot c + 8 \cdot a^2 \cdot b^2 \cdot c^2) \cdot d^4 - (4 \cdot a \cdot b^5 - 21 \cdot a^2 \cdot b^3 \cdot c + 2 \cdot a^3 \cdot b \cdot c^2) \cdot d^3 \cdot e + 3 \cdot (2 \cdot a^2 \cdot b^4 - 9 \cdot a^3 \cdot b^2 \cdot c + 4 \cdot a^4 \cdot c^2) \cdot d^2 \cdot e^2 - 4 \cdot (a^3 \cdot b^3 - 4 \cdot a^4 \cdot b \cdot c) \cdot d \cdot e^3 + (a^4 \cdot b^2 - 4 \cdot a^5 \cdot c) \cdot e^4 + (a^4 \cdot b^4 - 6 \cdot a^5 \cdot b^2 \cdot c + 8 \cdot a^6 \cdot c^2) \cdot d - (a^5 \cdot b^3 - 4 \cdot a^6 \cdot b \cdot c) \cdot e) \cdot \sqrt{-(6 \cdot a^5 \cdot b \cdot d \cdot e^5 - a^6 \cdot e^6 - (b^6 - 4 \cdot a \cdot b^4 \cdot c + 4 \cdot a^2 \cdot b^2 \cdot c^2) \cdot d^6 + 6 \cdot (a \cdot b^5 - 3 \cdot a^2 \cdot b^3 \cdot c + 2 \cdot a^3 \cdot b \cdot c^2) \cdot d^5 \cdot e - 3 \cdot (5 \cdot a^2 \cdot b^4 - 10 \cdot a^3 \cdot b^2 \cdot c + 3 \cdot a^4 \cdot c^2) \cdot d^4 \cdot e^2 + 2 \cdot (10 \cdot a^3 \cdot b^3 - 11 \cdot a^4 \cdot b \cdot c) \cdot d^3 \cdot e^3 - 3 \cdot (5 \cdot a^4 \cdot b^2 - 2 \cdot a^5 \cdot c) \cdot d^2 \cdot e^4) / (a^8 \cdot b^2 - 4 \cdot a^9 \cdot c))} \cdot \sqrt{-(a^3 \cdot b \cdot e^3 - (b^4 - 4 \cdot a \cdot b^2 \cdot c + 2 \cdot a^2 \cdot c^2) \cdot d^3 + 3 \cdot (a \cdot b^3 - 3 \cdot a^2 \cdot b \cdot c) \cdot d^2 \cdot e - 3 \cdot (a^2 \cdot b^2 - 2 \cdot a^3 \cdot c) \cdot d \cdot e^2 + (a^4 \cdot b^2 - 4 \cdot a^5 \cdot c) \cdot \sqrt{-(6 \cdot a^5 \cdot b \cdot d \cdot e^5 - a^6 \cdot e^6 - (b^6 - 4 \cdot a \cdot b^4 \cdot c + 4 \cdot a^2 \cdot b^2 \cdot c^2) \cdot d^6 + 6 \cdot (a \cdot b^5 - 3 \cdot a^2 \cdot b^3 \cdot c + 2 \cdot a^3 \cdot b \cdot c^2) \cdot d^5 \cdot e - 3 \cdot (5 \cdot a^2 \cdot b^4 - 10 \cdot a^3 \cdot b^2 \cdot c + 3 \cdot a^4 \cdot c^2) \cdot d^4 \cdot e^2 + 2 \cdot (10 \cdot a^3 \cdot b^3 - 11 \cdot a^4 \cdot b \cdot c) \cdot d^3 \cdot e^3 - 3 \cdot (5 \cdot a^4 \cdot b^2 - 2 \cdot a^5 \cdot c) \cdot d^2 \cdot e^4) / (a^8 \cdot b^2 - 4 \cdot a^9 \cdot c))} / (a^4 \cdot b^2 - 4 \cdot a^5 \cdot c) - 4 \cdot (4 \cdot a^3 \cdot b \cdot c \cdot d \cdot e^4 - a^4 \cdot c \cdot e^5 + (b^3 \cdot c^2 - 2 \cdot a \cdot b \cdot c^3) \cdot d^5 - (b^4 \cdot c + a \cdot b^2 \cdot c^2 - 3 \cdot a^2 \cdot c^3) \cdot d^4 \cdot e + 2 \cdot (2 \cdot a \cdot b^3 \cdot c - a^2 \cdot b \cdot c^2) \cdot d^3 \cdot e^2 - 2 \cdot (3 \cdot a^2 \cdot b^2 \cdot c - a^3 \cdot c^2) \cdot d^2 \cdot e^3) \cdot \sqrt{(e \cdot x + d)} - \sqrt{2} \cdot a^2 \cdot x \cdot \sqrt{-(a^3 \cdot b \cdot e^3 - (b^4 - 4 \cdot a \cdot b^2 \cdot c + 2 \cdot a^2 \cdot c^2) \cdot d^3 + 3 \cdot (a \cdot b^3 - 3 \cdot a^2 \cdot b \cdot c) \cdot d^2 \cdot e - 3 \cdot (a^2 \cdot b^2 - 2 \cdot a^3 \cdot c) \cdot d \cdot e^2 + (a^4 \cdot b^2 - 4 \cdot a^5 \cdot c) \cdot \sqrt{-(6 \cdot a^5 \cdot b \cdot d \cdot e^5 - a^6 \cdot e^6 - (b^6 - 4 \cdot a \cdot b^4 \cdot c + 4 \cdot a^2 \cdot b^2 \cdot c^2) \cdot d^6 + 6 \cdot (a \cdot b^5 - 3 \cdot a^2 \cdot b^3 \cdot c + 2 \cdot a^3 \cdot b \cdot c^2) \cdot d^5 \cdot e - 3 \cdot (5 \cdot a^2 \cdot b^4 - 10 \cdot a^3 \cdot b^2 \cdot c + 3 \cdot a^4 \cdot c^2) \cdot d^4 \cdot e^2 + 2 \cdot (10 \cdot a^3 \cdot b^3 - 11 \cdot a^4 \cdot b \cdot c) \cdot d^3 \cdot e^3 - 3 \cdot (5 \cdot a^4 \cdot b^2 - 2 \cdot a^5 \cdot c) \cdot d^2 \cdot e^4) / (a^8 \cdot b^2 - 4 \cdot a^9 \cdot c))} \end{aligned}$$

$$\begin{aligned}
& - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*( \\
& a^2*b^2 - 2*a^3*c)*d*e^2 + (a^4*b^2 - 4*a^5*c)*\sqrt{-(6*a^5*b*d*e \\
& ^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - \\
& 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + \\
& 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5* \\
& a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c))/((a^4*b^2 - 4*a^5 \\
& *c))*\log(-\sqrt{2}*((b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d^4 - (4*a* \\
& b^5 - 21*a^2*b^3*c + 20*a^3*b*c^2)*d^3*e + 3*(2*a^2*b^4 - 9*a^3*b \\
& ^2*c + 4*a^4*c^2)*d^2*e^2 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4* \\
& b^2 - 4*a^5*c)*e^4 + ((a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*d - (a^ \\
& 5*b^3 - 4*a^6*b*c)*e)*\sqrt{-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a \\
& *b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2 \\
& )*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*( \\
& 10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4 \\
& )/(a^8*b^2 - 4*a^9*c))*\sqrt{-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2* \\
& a^2*c^2)*d^3 + 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c \\
& )*d*e^2 + (a^4*b^2 - 4*a^5*c)*\sqrt{-(6*a^5*b*d*e^5 - a^6*e^6 - (b \\
& ^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2* \\
& a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e \\
& ^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c \\
& )*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 4*(4*a^3* \\
& b*c*d*e^4 - a^4*c*e^5 + (b^3*c^2 - 2*a*b*c^3)*d^5 - (b^4*c + a*b^ \\
& 2*c^2 - 3*a^2*c^3)*d^4*e + 2*(2*a*b^3*c - a^2*b*c^2)*d^3*e^2 - 2* \\
& (3*a^2*b^2*c - a^3*c^2)*d^2*e^3)*\sqrt{e*x + d}) + \sqrt{2}*a^2*x*s \\
& \sqrt{-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^3 - \\
& 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 - (a^4*b^2 - 4*a^5 \\
& *c)*\sqrt{-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2 \\
& *c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^ \\
& 2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4 \\
& *b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9 \\
& *c)))/(a^4*b^2 - 4*a^5*c))*\log(\sqrt{2}*((b^6 - 6*a*b^4*c + 8*a^2 \\
& *b^2*c^2)*d^4 - (4*a*b^5 - 21*a^2*b^3*c + 20*a^3*b*c^2)*d^3*e + 3 \\
& *(2*a^2*b^4 - 9*a^3*b^2*c + 4*a^4*c^2)*d^2*e^2 - 4*(a^3*b^3 - 4*a \\
& ^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4 - ((a^4*b^4 - 6*a^5*b^2*c \\
& + 8*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)*\sqrt{-(6*a^5*b*d*e^5 - \\
& a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a \\
& ^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a \\
& ^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4* \\
& b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c))*\sqrt{-(a^3*b*e^3 - \\
& (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^3 - 3*a^2*b*c)*d^2*e - \\
& 3*(a^2*b^2 - 2*a^3*c)*d*e^2 - (a^4*b^2 - 4*a^5*c)*\sqrt{-(6*a^5*b \\
& *d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b \\
& ^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2 \\
& *c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3 \\
& *(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - \\
& 4*a^5*c)) - 4*(4*a^3*b*c*d*e^4 - a^4*c*e^5 + (b^3*c^2 - 2*a*b*c^3 \\
& )*d^5 - (b^4*c + a*b^2*c^2 - 3*a^2*c^3)*d^4*e + 2*(2*a*b^3*c - a^ \\
& 2*b*c^2)*d^3*e^2 - 2*(3*a^2*b^2*c - a^3*c^2)*d^2*e^3)*\sqrt{e*x + \\
& d}) - \sqrt{2}*a^2*x*\sqrt{-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c \\
& ^2)*d^3 + 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e \\
& ^2 - (a^4*b^2 - 4*a^5*c)*\sqrt{-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - \\
& 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b \\
& *c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + \\
& 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2 \\
& *e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*\log(-\sqrt{2}*((b \\
& ^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d^4 - (4*a*b^5 - 21*a^2*b^3*c + 2 \\
& 0*a^3*b*c^2)*d^3*e + 3*(2*a^2*b^4 - 9*a^3*b^2*c + 4*a^4*c^2)*d^2* \\
& e^2 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4 - ( \\
& (a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)* \\
& \sqrt{-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2 \\
& )*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^ \\
& 4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b* \\
& c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c \\
& ))*\sqrt{-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^ \\
& 3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 - (a^4*b^2 - 4 \\
& *a^5*c)*\sqrt{-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2 \\
& *b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*( \\
& 5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 1 \\
& 1*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - \\
& 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 4*(4*a^3*b*c*d*e^4 - a^4*c*e^5 \\
& + (b^3*c^2 - 2*a*b*c^3)*d^5 - (b^4*c + a*b^2*c^2 - 3*a^2*c^3)*d^4 \\
& *e + 2*(2*a*b^3*c - a^2*b*c^2)*d^3*e^2 - 2*(3*a^2*b^2*c - a^3*c^2
\end{aligned}$$

$$\begin{aligned}
& ) * d^2 * e^3) * \sqrt{e * x + d}) + (2 * b * d - 3 * a * e) * \sqrt{d} * x * \log((e * x - \\
& 2 * \sqrt{e * x + d} * \sqrt{d} + 2 * d) / x) + 2 * \sqrt{e * x + d} * a * d / (a^2 * x), \\
& - 1 / 2 * (\sqrt{2} * a^2 * x * \sqrt{-(a^3 * b * e^3 - (b^4 - 4 * a * b^2 * c + 2 * a^2 * \\
& c^2) * d^3 + 3 * (a * b^3 - 3 * a^2 * b * c) * d^2 * e - 3 * (a^2 * b^2 - 2 * a^3 * c) * d * \\
& e^2 + (a^4 * b^2 - 4 * a^5 * c) * \sqrt{-(6 * a^5 * b * d * e^5 - a^6 * e^6 - (b^6 - \\
& 4 * a * b^4 * c + 4 * a^2 * b^2 * c^2) * d^6 + 6 * (a * b^5 - 3 * a^2 * b^3 * c + 2 * a^3 * \\
& b * c^2) * d^5 * e - 3 * (5 * a^2 * b^4 - 10 * a^3 * b^2 * c + 3 * a^4 * c^2) * d^4 * e^2 + \\
& 2 * (10 * a^3 * b^3 - 11 * a^4 * b * c) * d^3 * e^3 - 3 * (5 * a^4 * b^2 - 2 * a^5 * c) * d^2 * \\
& e^4) / (a^8 * b^2 - 4 * a^9 * c))) / (a^4 * b^2 - 4 * a^5 * c)) * \log(\sqrt{2} * ((b \\
& ^6 - 6 * a * b^4 * c + 8 * a^2 * b^2 * c^2) * d^4 - (4 * a * b^5 - 21 * a^2 * b^3 * c + 2 \\
& 0 * a^3 * b * c^2) * d^3 * e + 3 * (2 * a^2 * b^4 - 9 * a^3 * b^2 * c + 4 * a^4 * c^2) * d^2 * \\
& e^2 - 4 * (a^3 * b^3 - 4 * a^4 * b * c) * d * e^3 + (a^4 * b^2 - 4 * a^5 * c) * e^4 + ( \\
& (a^4 * b^4 - 6 * a^5 * b^2 * c + 8 * a^6 * c^2) * d - (a^5 * b^3 - 4 * a^6 * b * c) * e) * \\
& \sqrt{-(6 * a^5 * b * d * e^5 - a^6 * e^6 - (b^6 - 4 * a * b^4 * c + 4 * a^2 * b^2 * c^2 \\
& ) * d^6 + 6 * (a * b^5 - 3 * a^2 * b^3 * c + 2 * a^3 * b * c^2) * d^5 * e - 3 * (5 * a^2 * b^4 \\
& - 10 * a^3 * b^2 * c + 3 * a^4 * c^2) * d^4 * e^2 + 2 * (10 * a^3 * b^3 - 11 * a^4 * b * \\
& c) * d^3 * e^3 - 3 * (5 * a^4 * b^2 - 2 * a^5 * c) * d^2 * e^4) / (a^8 * b^2 - 4 * a^9 * c) \\
& )) * \sqrt{-(a^3 * b * e^3 - (b^4 - 4 * a * b^2 * c + 2 * a^2 * c^2) * d^3 + 3 * (a * b^3 \\
& - 3 * a^2 * b * c) * d^2 * e - 3 * (a^2 * b^2 - 2 * a^3 * c) * d * e^2 + (a^4 * b^2 - 4 * \\
& a^5 * c) * \sqrt{-(6 * a^5 * b * d * e^5 - a^6 * e^6 - (b^6 - 4 * a * b^4 * c + 4 * a^2 * \\
& b^2 * c^2) * d^6 + 6 * (a * b^5 - 3 * a^2 * b^3 * c + 2 * a^3 * b * c^2) * d^5 * e - 3 * ( \\
& 5 * a^2 * b^4 - 10 * a^3 * b^2 * c + 3 * a^4 * c^2) * d^4 * e^2 + 2 * (10 * a^3 * b^3 - 1 \\
& 1 * a^4 * b * c) * d^3 * e^3 - 3 * (5 * a^4 * b^2 - 2 * a^5 * c) * d^2 * e^4) / (a^8 * b^2 - \\
& 4 * a^9 * c))) / (a^4 * b^2 - 4 * a^5 * c)) - 4 * (4 * a^3 * b * c * d * e^4 - a^4 * c * e^5 \\
& + (b^3 * c^2 - 2 * a * b * c^3) * d^5 - (b^4 * c + a * b^2 * c^2 - 3 * a^2 * c^3) * d^4 \\
& * e + 2 * (2 * a * b^3 * c - a^2 * b * c^2) * d^3 * e^2 - 2 * (3 * a^2 * b^2 * c - a^3 * c^2 \\
& ) * d^2 * e^3) * \sqrt{e * x + d}) - \sqrt{2} * a^2 * x * \sqrt{-(a^3 * b * e^3 - (b^4 \\
& - 4 * a * b^2 * c + 2 * a^2 * c^2) * d^3 + 3 * (a * b^3 - 3 * a^2 * b * c) * d^2 * e - 3 * ( \\
& a^2 * b^2 - 2 * a^3 * c) * d * e^2 + (a^4 * b^2 - 4 * a^5 * c) * \sqrt{-(6 * a^5 * b * d * e \\
& ^5 - a^6 * e^6 - (b^6 - 4 * a * b^4 * c + 4 * a^2 * b^2 * c^2) * d^6 + 6 * (a * b^5 - \\
& 3 * a^2 * b^3 * c + 2 * a^3 * b * c^2) * d^5 * e - 3 * (5 * a^2 * b^4 - 10 * a^3 * b^2 * c + \\
& 3 * a^4 * c^2) * d^4 * e^2 + 2 * (10 * a^3 * b^3 - 11 * a^4 * b * c) * d^3 * e^3 - 3 * (5 * \\
& a^4 * b^2 - 2 * a^5 * c) * d^2 * e^4) / (a^8 * b^2 - 4 * a^9 * c))) / (a^4 * b^2 - 4 * a^5 \\
& * c)) * \log(-\sqrt{2} * ((b^6 - 6 * a * b^4 * c + 8 * a^2 * b^2 * c^2) * d^4 - (4 * a * \\
& b^5 - 21 * a^2 * b^3 * c + 20 * a^3 * b * c^2) * d^3 * e + 3 * (2 * a^2 * b^4 - 9 * a^3 * b \\
& ^2 * c + 4 * a^4 * c^2) * d^2 * e^2 - 4 * (a^3 * b^3 - 4 * a^4 * b * c) * d * e^3 + (a^4 * \\
& b^2 - 4 * a^5 * c) * e^4 + ((a^4 * b^4 - 6 * a^5 * b^2 * c + 8 * a^6 * c^2) * d - (a^5 * \\
& b^3 - 4 * a^6 * b * c) * e) * \sqrt{-(6 * a^5 * b * d * e^5 - a^6 * e^6 - (b^6 - 4 * a * \\
& b^4 * c + 4 * a^2 * b^2 * c^2) * d^6 + 6 * (a * b^5 - 3 * a^2 * b^3 * c + 2 * a^3 * b * c^2 \\
& ) * d^5 * e - 3 * (5 * a^2 * b^4 - 10 * a^3 * b^2 * c + 3 * a^4 * c^2) * d^4 * e^2 + 2 * ( \\
& 10 * a^3 * b^3 - 11 * a^4 * b * c) * d^3 * e^3 - 3 * (5 * a^4 * b^2 - 2 * a^5 * c) * d^2 * e^4 \\
& ) / (a^8 * b^2 - 4 * a^9 * c))) * \sqrt{-(a^3 * b * e^3 - (b^4 - 4 * a * b^2 * c + 2 * \\
& a^2 * c^2) * d^3 + 3 * (a * b^3 - 3 * a^2 * b * c) * d^2 * e - 3 * (a^2 * b^2 - 2 * a^3 * c \\
& ) * d * e^2 + (a^4 * b^2 - 4 * a^5 * c) * \sqrt{-(6 * a^5 * b * d * e^5 - a^6 * e^6 - (b \\
& ^6 - 4 * a * b^4 * c + 4 * a^2 * b^2 * c^2) * d^6 + 6 * (a * b^5 - 3 * a^2 * b^3 * c + 2 * \\
& a^3 * b * c^2) * d^5 * e - 3 * (5 * a^2 * b^4 - 10 * a^3 * b^2 * c + 3 * a^4 * c^2) * d^4 * e \\
& ^2 + 2 * (10 * a^3 * b^3 - 11 * a^4 * b * c) * d^3 * e^3 - 3 * (5 * a^4 * b^2 - 2 * a^5 * c \\
& ) * d^2 * e^4) / (a^8 * b^2 - 4 * a^9 * c))) / (a^4 * b^2 - 4 * a^5 * c)) - 4 * (4 * a^3 * \\
& b * c * d * e^4 - a^4 * c * e^5 + (b^3 * c^2 - 2 * a * b * c^3) * d^5 - (b^4 * c + a * b^2 \\
& * c^2 - 3 * a^2 * c^3) * d^4 * e + 2 * (2 * a * b^3 * c - a^2 * b * c^2) * d^3 * e^2 - 2 * \\
& (3 * a^2 * b^2 * c - a^3 * c^2) * d^2 * e^3) * \sqrt{e * x + d}) + \sqrt{2} * a^2 * x * \sqrt{ \\
& \sqrt{-(a^3 * b * e^3 - (b^4 - 4 * a * b^2 * c + 2 * a^2 * c^2) * d^3 + 3 * (a * b^3 - \\
& 3 * a^2 * b * c) * d^2 * e - 3 * (a^2 * b^2 - 2 * a^3 * c) * d * e^2 - (a^4 * b^2 - 4 * a^5 \\
& * c) * \sqrt{-(6 * a^5 * b * d * e^5 - a^6 * e^6 - (b^6 - 4 * a * b^4 * c + 4 * a^2 * b^2 * \\
& c^2) * d^6 + 6 * (a * b^5 - 3 * a^2 * b^3 * c + 2 * a^3 * b * c^2) * d^5 * e - 3 * (5 * a^2 \\
& * b^4 - 10 * a^3 * b^2 * c + 3 * a^4 * c^2) * d^4 * e^2 + 2 * (10 * a^3 * b^3 - 11 * a^4 \\
& * b * c) * d^3 * e^3 - 3 * (5 * a^4 * b^2 - 2 * a^5 * c) * d^2 * e^4) / (a^8 * b^2 - 4 * a^9 \\
& * c))) / (a^4 * b^2 - 4 * a^5 * c)) * \log(\sqrt{2} * ((b^6 - 6 * a * b^4 * c + 8 * a^2 \\
& * b^2 * c^2) * d^4 - (4 * a * b^5 - 21 * a^2 * b^3 * c + 20 * a^3 * b * c^2) * d^3 * e + 3 \\
& * (2 * a^2 * b^4 - 9 * a^3 * b^2 * c + 4 * a^4 * c^2) * d^2 * e^2 - 4 * (a^3 * b^3 - 4 * a^4 \\
& * b * c) * d * e^3 + (a^4 * b^2 - 4 * a^5 * c) * e^4 - ((a^4 * b^4 - 6 * a^5 * b^2 * c \\
& + 8 * a^6 * c^2) * d - (a^5 * b^3 - 4 * a^6 * b * c) * e) * \sqrt{-(6 * a^5 * b * d * e^5 - \\
& a^6 * e^6 - (b^6 - 4 * a * b^4 * c + 4 * a^2 * b^2 * c^2) * d^6 + 6 * (a * b^5 - 3 * a^2 \\
& * b^3 * c + 2 * a^3 * b * c^2) * d^5 * e - 3 * (5 * a^2 * b^4 - 10 * a^3 * b^2 * c + 3 * a^4 \\
& * c^2) * d^4 * e^2 + 2 * (10 * a^3 * b^3 - 11 * a^4 * b * c) * d^3 * e^3 - 3 * (5 * a^4 * \\
& b^2 - 2 * a^5 * c) * d^2 * e^4) / (a^8 * b^2 - 4 * a^9 * c))) * \sqrt{-(a^3 * b * e^3 - \\
& (b^4 - 4 * a * b^2 * c + 2 * a^2 * c^2) * d^3 + 3 * (a * b^3 - 3 * a^2 * b * c) * d^2 * e - \\
& 3 * (a^2 * b^2 - 2 * a^3 * c) * d * e^2 - (a^4 * b^2 - 4 * a^5 * c) * \sqrt{-(6 * a^5 * b \\
& * d * e^5 - a^6 * e^6 - (b^6 - 4 * a * b^4 * c + 4 * a^2 * b^2 * c^2) * d^6 + 6 * (a * b \\
& ^5 - 3 * a^2 * b^3 * c + 2 * a^3 * b * c^2) * d^5 * e - 3 * (5 * a^2 * b^4 - 10 * a^3 * b^2 \\
& * c + 3 * a^4 * c^2) * d^4 * e^2 + 2 * (10 * a^3 * b^3 - 11 * a^4 * b * c) * d^3 * e^3 - 3 \\
& * (5 * a^4 * b^2 - 2 * a^5 * c) * d^2 * e^4) / (a^8 * b^2 - 4 * a^9 * c))) / (a^4 * b^2 -
\end{aligned}$$

$$\begin{aligned}
& 4*a^5*c)) - 4*(4*a^3*b*c*d*e^4 - a^4*c*e^5 + (b^3*c^2 - 2*a*b*c^3) \\
& )*d^5 - (b^4*c + a*b^2*c^2 - 3*a^2*c^3)*d^4*e + 2*(2*a*b^3*c - a^2 \\
& *b*c^2)*d^3*e^2 - 2*(3*a^2*b^2*c - a^3*c^2)*d^2*e^3)*\sqrt{e*x + \\
& d) - \sqrt{2}*a^2*x*\sqrt{-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c \\
& ^2)*d^3 + 3*(a*b^3 - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e \\
& ^2 - (a^4*b^2 - 4*a^5*c)*\sqrt{-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - \\
& 4*a*b^4*c + 4*a^2*b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b \\
& *c^2)*d^5*e - 3*(5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + \\
& 2*(10*a^3*b^3 - 11*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2 \\
& *e^4)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*\log(-\sqrt{2}*((b \\
& ^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*d^4 - (4*a*b^5 - 21*a^2*b^3*c + 2 \\
& 0*a^3*b*c^2)*d^3*e + 3*(2*a^2*b^4 - 9*a^3*b^2*c + 4*a^4*c^2)*d^2* \\
& e^2 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4 - ( \\
& (a^4*b^4 - 6*a^5*b^2*c + 8*a^6*c^2)*d - (a^5*b^3 - 4*a^6*b*c)*e)* \\
& \sqrt{-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2) \\
& )*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*(5*a^2*b^4 \\
& - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 11*a^4*b* \\
& c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - 4*a^9*c) \\
& ))*\sqrt{-(a^3*b*e^3 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*d^3 + 3*(a*b^3 \\
& - 3*a^2*b*c)*d^2*e - 3*(a^2*b^2 - 2*a^3*c)*d*e^2 - (a^4*b^2 - 4 \\
& *a^5*c)*\sqrt{-(6*a^5*b*d*e^5 - a^6*e^6 - (b^6 - 4*a*b^4*c + 4*a^2 \\
& *b^2*c^2)*d^6 + 6*(a*b^5 - 3*a^2*b^3*c + 2*a^3*b*c^2)*d^5*e - 3*( \\
& 5*a^2*b^4 - 10*a^3*b^2*c + 3*a^4*c^2)*d^4*e^2 + 2*(10*a^3*b^3 - 1 \\
& 1*a^4*b*c)*d^3*e^3 - 3*(5*a^4*b^2 - 2*a^5*c)*d^2*e^4)/(a^8*b^2 - \\
& 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 4*(4*a^3*b*c*d*e^4 - a^4*c*e^5 \\
& + (b^3*c^2 - 2*a*b*c^3)*d^5 - (b^4*c + a*b^2*c^2 - 3*a^2*c^3)*d^4 \\
& *e + 2*(2*a*b^3*c - a^2*b*c^2)*d^3*e^2 - 2*(3*a^2*b^2*c - a^3*c^2) \\
& )*d^2*e^3)*\sqrt{e*x + d) - 2*(2*b*d - 3*a*e)*\sqrt{-d}*x*\arctan(s \\
& \sqrt{e*x + d)/\sqrt{-d)) + 2*\sqrt{e*x + d)*a*d)/(a^2*x)]
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(3/2)/x\*\*2/(c\*x\*\*2+b\*x+a),x)

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)/((c\*x^2 + b\*x + a)\*x^2),x, algorithm="giac")

[Out] Timed out

$$3.540 \quad \int \frac{(d+ex)^{3/2}}{x^3(a+bx+cx^2)} dx$$

**Optimal.** Leaf size=607

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right) (-2abde - a(cd^2 - ae^2) + b^2d^2)}{a^3\sqrt{d}}$$

$$+ \frac{\sqrt{2}\sqrt{c} \left(-ab \left(e \left(2d\sqrt{b^2 - 4ac} - ae\right) + 3cd^2\right) + a \left(ae^2\sqrt{b^2 - 4ac} - cd \left(d\sqrt{b^2 - 4ac} - 4ae\right)\right) + b^2d \left(d\sqrt{b^2 - 4ac} - 2ae\right) + b^3\right)}{a^3\sqrt{b^2 - 4ac}\sqrt{2cd - e \left(b - \sqrt{b^2 - 4ac}\right)}}$$

$$+ \frac{\sqrt{2}\sqrt{c} \left(-ab \left(3cd^2 - e \left(2d\sqrt{b^2 - 4ac} + ae\right)\right) - a \left(ae^2\sqrt{b^2 - 4ac} - cd \left(d\sqrt{b^2 - 4ac} + 4ae\right)\right) - b^2d \left(d\sqrt{b^2 - 4ac} + 2ae\right) + b^3\right)}{a^3\sqrt{b^2 - 4ac}\sqrt{2cd - e \left(\sqrt{b^2 - 4ac} + b\right)}}$$

$$+ \frac{\sqrt{d+ex}(bd - 2ae)}{a^2x} - \frac{e(bd - 2ae) \tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}} - \frac{3e^2 \tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4a\sqrt{d}} - \frac{d\sqrt{d+ex}}{2ax^2} + \frac{3e\sqrt{d+ex}}{4ax}$$

[Out]  $-(d*\text{Sqrt}[d + e*x])/(2*a*x^2) + (3*e*\text{Sqrt}[d + e*x])/(4*a*x) + ((b*d - 2*a*e)*\text{Sqrt}[d + e*x])/(a^2*x) - (3*e^2*\text{ArcTanh}[\text{Sqrt}[d + e*x]/\text{Sqrt}[d]])/(4*a*\text{Sqrt}[d]) - (e*(b*d - 2*a*e)*\text{ArcTanh}[\text{Sqrt}[d + e*x]/\text{Sqrt}[d]])/(a^2*\text{Sqrt}[d]) - (2*(b^2*d^2 - 2*a*b*d*e - a*(c*d^2 - a*e^2))*\text{ArcTanh}[\text{Sqrt}[d + e*x]/\text{Sqrt}[d]])/(a^3*\text{Sqrt}[d]) + (\text{Sqrt}[2]*\text{Sqrt}[c]*(b^3*d^2 + b^2*d*(\text{Sqrt}[b^2 - 4*a*c]*d - 2*a*e) + a*(a*\text{Sqrt}[b^2 - 4*a*c]*e^2 - c*d*(\text{Sqrt}[b^2 - 4*a*c]*d - 4*a*e)) - a*b*(3*c*d^2 + e*(2*\text{Sqrt}[b^2 - 4*a*c]*d - a*e)))*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]])/(a^3*\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]) - (\text{Sqrt}[2]*\text{Sqrt}[c]*(b^3*d^2 - b^2*d*(\text{Sqrt}[b^2 - 4*a*c]*d + 2*a*e) - a*b*(3*c*d^2 - e*(2*\text{Sqrt}[b^2 - 4*a*c]*d + a*e)) - a*(a*\text{Sqrt}[b^2 - 4*a*c]*e^2 - c*d*(\text{Sqrt}[b^2 - 4*a*c]*d + 4*a*e)))*\text{ArcTanh}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[d + e*x])/\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]])/(a^3*\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e])$

**Rubi [A]** time = 7.43062, antiderivative size = 607, normalized size of antiderivative = 1., number of rules used = 12, number of rules used = 6, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.24$

$$\frac{2 \tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right) (-2abde - a(cd^2 - ae^2) + b^2d^2)}{a^3\sqrt{d}}$$

$$+ \frac{\sqrt{2}\sqrt{c} \left(-ab \left(e \left(2d\sqrt{b^2 - 4ac} - ae\right) + 3cd^2\right) + a \left(ae^2\sqrt{b^2 - 4ac} - cd \left(d\sqrt{b^2 - 4ac} - 4ae\right)\right) + b^2d \left(d\sqrt{b^2 - 4ac} - 2ae\right) + b^3\right)}{a^3\sqrt{b^2 - 4ac}\sqrt{2cd - e \left(b - \sqrt{b^2 - 4ac}\right)}}$$

$$+ \frac{\sqrt{2}\sqrt{c} \left(-ab \left(3cd^2 - e \left(2d\sqrt{b^2 - 4ac} + ae\right)\right) - a \left(ae^2\sqrt{b^2 - 4ac} - cd \left(d\sqrt{b^2 - 4ac} + 4ae\right)\right) - b^2d \left(d\sqrt{b^2 - 4ac} + 2ae\right) + b^3\right)}{a^3\sqrt{b^2 - 4ac}\sqrt{2cd - e \left(\sqrt{b^2 - 4ac} + b\right)}}$$

$$+ \frac{\sqrt{d+ex}(bd - 2ae)}{a^2x} - \frac{e(bd - 2ae) \tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{a^2\sqrt{d}} - \frac{3e^2 \tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4a\sqrt{d}} - \frac{d\sqrt{d+ex}}{2ax^2} + \frac{3e\sqrt{d+ex}}{4ax}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^(3/2)/(x^3\*(a + b\*x + c\*x^2)), x]

```
[Out] -(d*Sqrt[d + e*x])/(2*a*x^2) + (3*e*Sqrt[d + e*x])/(4*a*x) + ((b*d - 2*a*e)*Sqrt[d + e*x])/(a^2*x) - (3*e^2*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(4*a*Sqrt[d]) - (e*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^2*Sqrt[d]) - (2*(b^2*d^2 - 2*a*b*d*e - a*(c*d^2 - a*e^2))*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(a^3*Sqrt[d]) + (Sqrt[2]*Sqrt[c]*(b^3*d^2 + b^2*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + a*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e)) - a*b*(3*c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]]/(a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[2]*Sqrt[c]*(b^3*d^2 - b^2*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - a*b*(3*c*d^2 - e*(2*Sqrt[b^2 - 4*a*c]*d + a*e)) - a*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]]/(a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((e*x+d)**(3/2)/x**3/(c*x**2+b*x+a), x)
```

[Out] Timed out

**Mathematica [A]** time = 1.72831, size = 499, normalized size = 0.82

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(-12abde+a(3ae^2-8cd^2)+8b^2d^2)}{\sqrt{d}} + \frac{4\sqrt{2}\sqrt{c}\left(ab\left(e\left(ae-2d\sqrt{b^2-4ac}\right)-3cd^2\right)+a\left(cd\left(4ae-d\sqrt{b^2-4ac}\right)+ae^2\sqrt{b^2-4ac}\right)+b^2d\left(d\sqrt{b^2-4ac}-2a\right)\right)}{\sqrt{b^2-4ac}\sqrt{e\left(\sqrt{b^2-4ac}-b\right)+2cd}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x)^(3/2)/(x^3*(a + b*x + c*x^2)), x]
```

```
[Out] ((a*Sqrt[d + e*x]*(-2*a*d + 4*b*d*x - 5*a*e*x))/x^2 - ((8*b^2*d^2 - 12*a*b*d*e + a*(-8*c*d^2 + 3*a*e^2))*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d] + (4*Sqrt[2]*Sqrt[c]*(b^3*d^2 + b^2*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + a*b*(-3*c*d^2 + e*(-2*Sqrt[b^2 - 4*a*c]*d + a*e)) + a*(a*Sqrt[b^2 - 4*a*c]*e^2 + c*d*(-(Sqrt[b^2 - 4*a*c]*d) + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]]/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) - (4*Sqrt[2]*Sqrt[c]*(b^3*d^2 - b^2*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) + a*b*(-3*c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d + a*e)) + a*(-(a*Sqrt[b^2 - 4*a*c]*e^2) + c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]]/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/(4*a^3)
```

**Maple [B]** time = 0.067, size = 1880, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.



[In] `int((e*x+d)^(3/2)/x^3/(c*x^2+b*x+a),x)`

[Out] 
$$\begin{aligned} & -1/a^2*c^2*2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)} \\ & * \operatorname{arctanh}(c*(e*x+d)^{(1/2)*2^{(1/2)}}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}) \\ & * d^2-e^2/a*c^2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)} * \operatorname{arctan}(c*(e*x+d)^{(1/2)*2^{(1/2)}}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}) \\ & + e^2/a*c^2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)} * \operatorname{arctanh}(c*(e*x+d)^{(1/2)*2^{(1/2)}}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}) \\ & + 1/e/a^2/x^2*(e*x+d)^{(3/2)} * b*d-1/e/a^2/x^2*(e*x+d)^{(1/2)} * b*d^2+1/a^2*c^2*2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)} * \operatorname{arctan}(c*(e*x+d)^{(1/2)*2^{(1/2)}}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}) \\ & * d^2+2/a^2*d^{(3/2)} * \operatorname{arctanh}((e*x+d)^{(1/2)}/d^{(1/2)}) * b^2+e^3/a*c/(-e^2*(4*a*c-b^2))^{(1/2)*2^{(1/2)}}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)} * \operatorname{arctan}(c*(e*x+d)^{(1/2)*2^{(1/2)}}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}) \\ & * b+4*e^2/a*c^2/(-e^2*(4*a*c-b^2))^{(1/2)*2^{(1/2)}}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)} * \operatorname{arctan}(c*(e*x+d)^{(1/2)*2^{(1/2)}}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}) \\ & * d-3/4*e^2 * \operatorname{arctanh}((e*x+d)^{(1/2)}/d^{(1/2)})/a/d^{(1/2)}+3/4*d*(e*x+d)^{(1/2)}/a/x^2-1/a^3*c^2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)} * \operatorname{arctan}(c*(e*x+d)^{(1/2)*2^{(1/2)}}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}) \\ & * b^2*d^2+1/a^3*c^2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)} * \operatorname{arctanh}(c*(e*x+d)^{(1/2)*2^{(1/2)}}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}) \\ & * b^2*d^2+3*e/a^2*d^{(1/2)} * \operatorname{arctanh}((e*x+d)^{(1/2)}/d^{(1/2)}) * b-5/4/a/x^2*(e*x+d)^{(3/2)}-3*e/a^2*c^2/(-e^2*(4*a*c-b^2))^{(1/2)*2^{(1/2)}}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)} * \operatorname{arctanh}(c*(e*x+d)^{(1/2)*2^{(1/2)}}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}) \\ & * b^3*d^2-2*e^2/a^2*c/(-e^2*(4*a*c-b^2))^{(1/2)*2^{(1/2)}}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)} * \operatorname{arctan}(c*(e*x+d)^{(1/2)*2^{(1/2)}}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}) \\ & * b^2*d-3*e/a^2*c^2/(-e^2*(4*a*c-b^2))^{(1/2)*2^{(1/2)}}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)} * \operatorname{arctan}(c*(e*x+d)^{(1/2)*2^{(1/2)}}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}) \\ & * b^2*d^2+e/a^3*c/(-e^2*(4*a*c-b^2))^{(1/2)*2^{(1/2)}}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)} * \operatorname{arctan}(c*(e*x+d)^{(1/2)*2^{(1/2)}}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}) \\ & * b^3*d^2-2*e^2/a^2*c/(-e^2*(4*a*c-b^2))^{(1/2)*2^{(1/2)}}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)} * \operatorname{arctanh}(c*(e*x+d)^{(1/2)*2^{(1/2)}}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}) \\ & * b^2*d+2*e/a^2*c^2^{(1/2)}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)} * \operatorname{arctan}(c*(e*x+d)^{(1/2)*2^{(1/2)}}/((b*e-2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}) \\ & * b*d+e^3/a*c/(-e^2*(4*a*c-b^2))^{(1/2)*2^{(1/2)}}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)} * \operatorname{arctanh}(c*(e*x+d)^{(1/2)*2^{(1/2)}}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}) \\ & * b+4*e^2/a*c^2/(-e^2*(4*a*c-b^2))^{(1/2)*2^{(1/2)}}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)} * \operatorname{arctanh}(c*(e*x+d)^{(1/2)*2^{(1/2)}}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}) \\ & * d-2*e/a^2*c^2^{(1/2)}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)} * \operatorname{arctanh}(c*(e*x+d)^{(1/2)*2^{(1/2)}}/((-b*e+2*c*d+(-e^2*(4*a*c-b^2))^{(1/2)})^c)^{(1/2)}) \\ & * b*d \end{aligned}$$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*x^3),x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*x^3), x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/x**3/(c*x**2+b*x+a), x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*x^3), x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.541 \quad \int \frac{x^m(e+fx)^n}{a+bx+cx^2} dx$$

**Optimal.** Leaf size=201

$$\frac{2cx^{m+1}(e+fx)^n \left(\frac{fx}{e} + 1\right)^{-n} F_1\left(m+1; -n, 1; m+2; -\frac{fx}{e}, -\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac} \left(b - \sqrt{b^2-4ac}\right)} - \frac{2cx^{m+1}(e+fx)^n \left(\frac{fx}{e} + 1\right)^{-n} F_1\left(m+1; -n, 1; m+2; -\frac{fx}{e}, -\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac} \left(\sqrt{b^2-4ac} + b\right)}$$

[Out]  $(2*c*x^{(1+m)}*(e+f*x)^n*AppellF1[1+m, -n, 1, 2+m, -((f*x)/e), (-2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c]))*(1+m)*(1+(f*x)/e)^n - (2*c*x^{(1+m)}*(e+f*x)^n*AppellF1[1+m, -n, 1, 2+m, -((f*x)/e), (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c]))*(1+m)*(1+(f*x)/e)^n$

**Rubi [A]** time = 0.786145, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.13$

$$\frac{2cx^{m+1}(e+fx)^n \left(\frac{fx}{e} + 1\right)^{-n} F_1\left(m+1; -n, 1; m+2; -\frac{fx}{e}, -\frac{2cx}{b-\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac} \left(b - \sqrt{b^2-4ac}\right)} - \frac{2cx^{m+1}(e+fx)^n \left(\frac{fx}{e} + 1\right)^{-n} F_1\left(m+1; -n, 1; m+2; -\frac{fx}{e}, -\frac{2cx}{b+\sqrt{b^2-4ac}}\right)}{(m+1)\sqrt{b^2-4ac} \left(\sqrt{b^2-4ac} + b\right)}$$

Antiderivative was successfully verified.

[In] Int[(x^m\*(e+f\*x)^n)/(a+b\*x+c\*x^2),x]

[Out]  $(2*c*x^{(1+m)}*(e+f*x)^n*AppellF1[1+m, -n, 1, 2+m, -((f*x)/e), (-2*c*x)/(b - Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt[b^2 - 4*a*c]))*(1+m)*(1+(f*x)/e)^n - (2*c*x^{(1+m)}*(e+f*x)^n*AppellF1[1+m, -n, 1, 2+m, -((f*x)/e), (-2*c*x)/(b + Sqrt[b^2 - 4*a*c])])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c]))*(1+m)*(1+(f*x)/e)^n$

**Rubi in Sympy [A]** time = 70.7704, size = 168, normalized size = 0.84

$$\frac{2cx^{m+1} \left(1 + \frac{fx}{e}\right)^{-n} (e+fx)^n \operatorname{appellf}_1\left(m+1, 1, -n, m+2, -\frac{2cx}{b+\sqrt{-4ac+b^2}}, -\frac{fx}{e}\right)}{\left(b + \sqrt{-4ac + b^2}\right) (m+1) \sqrt{-4ac + b^2}} + \frac{2cx^{m+1} \left(1 + \frac{fx}{e}\right)^{-n} (e+fx)^n \operatorname{appellf}_1\left(m+1, 1, -n, m+2, -\frac{2cx}{b-\sqrt{-4ac+b^2}}, -\frac{fx}{e}\right)}{\left(b - \sqrt{-4ac + b^2}\right) (m+1) \sqrt{-4ac + b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*\*m\*(f\*x+e)\*\*n/(c\*x\*\*2+b\*x+a),x)

[Out]  $-2*c*x^{(m+1)}*(1+f*x/e)**(-n)*(e+f*x)**n*appellf1(m+1, 1, -n, m+2, -2*c*x/(b + sqrt(-4*a*c + b**2)), -f*x/e)/((b + sqrt(-4*a*c + b**2))**2) + 2*c*x^{(m+1)}*(1+f*x/e)**(-n)*(e+f*x)**n*appellf1(m+1, 1, -n, m+2, -2*c*x/(b - sqrt(-4*a*c + b**2)), -f*x/e)/((b - sqrt(-4*a*c + b**2))**2)$

$$+ f*x/e)^{(-n)}*(e + f*x)^n*\text{appellf1}(m + 1, 1, -n, m + 2, -2*c*x/(b - \sqrt{-4*a*c + b^2}), -f*x/e)/((b - \sqrt{-4*a*c + b^2})^m + 1)*\sqrt{-4*a*c + b^2})$$

**Mathematica [A]** time = 0.103834, size = 0, normalized size = 0.

$$\int \frac{x^m (e + fx)^n}{a + bx + cx^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(x^m\*(e + f\*x)^n)/(a + b\*x + c\*x^2), x]

[Out] Integrate[(x^m\*(e + f\*x)^n)/(a + b\*x + c\*x^2), x]

**Maple [F]** time = 0.223, size = 0, normalized size = 0.

$$\int \frac{x^m (fx + e)^n}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^m\*(f\*x+e)^n/(c\*x^2+b\*x+a), x)

[Out] int(x^m\*(f\*x+e)^n/(c\*x^2+b\*x+a), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^n x^m}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x + e)^n\*x^m/(c\*x^2 + b\*x + a), x, algorithm="maxima")

[Out] integrate((f\*x + e)^n\*x^m/(c\*x^2 + b\*x + a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx + e)^n x^m}{cx^2 + bx + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x + e)^n\*x^m/(c\*x^2 + b\*x + a), x, algorithm="fricas")

[Out] integral((f\*x + e)^n\*x^m/(c\*x^2 + b\*x + a), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**m*(f*x+e)**n/(c*x**2+b*x+a),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^n x^m}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x + e)^n*x^m/(c*x^2 + b*x + a),x, algorithm="giac")`

[Out] `integrate((f*x + e)^n*x^m/(c*x^2 + b*x + a), x)`

$$3.542 \quad \int \frac{x^3(e+fx)^n}{a+bx+cx^2} dx$$

Optimal. Leaf size=289

$$\frac{\left(\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}} + a - \frac{b^2}{c}\right) (e+fx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{2c(e+fx)}{2ce-bf+\sqrt{b^2-4ac}f}\right)}{c(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)} \\ + \frac{\left(-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}} + a - \frac{b^2}{c}\right) (e+fx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{2c(e+fx)}{2ce-(b+\sqrt{b^2-4ac})f}\right)}{c(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)} \\ - \frac{(bf+ce)(e+fx)^{n+1}}{c^2f^2(n+1)} + \frac{(e+fx)^{n+2}}{cf^2(n+2)}$$

[Out] -(((c\*e + b\*f)\*(e + f\*x)^(1 + n))/(c^2\*f^2\*(1 + n))) + (e + f\*x)^(2 + n)/(c\*f^2\*(2 + n)) + ((a - b^2/c + (b\*(b^2 - 3\*a\*c))/(c\*Sqrt[b^2 - 4\*a\*c]))\*(e + f\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (2\*c\*(e + f\*x))/(2\*c\*e - b\*f + Sqrt[b^2 - 4\*a\*c]\*f)]/(c\*(2\*c\*e - (b - Sqrt[b^2 - 4\*a\*c])\*f)\*(1 + n)) + ((a - b^2/c - (b\*(b^2 - 3\*a\*c))/(c\*Sqrt[b^2 - 4\*a\*c]))\*(e + f\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (2\*c\*(e + f\*x))/(2\*c\*e - (b + Sqrt[b^2 - 4\*a\*c])\*f)]/(c\*(2\*c\*e - (b + Sqrt[b^2 - 4\*a\*c])\*f)\*(1 + n))

Rubi [A] time = 1.24327, antiderivative size = 289, normalized size of antiderivative = 1., number of rules used = 4, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{\left(\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}} + a - \frac{b^2}{c}\right) (e+fx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{2c(e+fx)}{2ce-bf+\sqrt{b^2-4ac}f}\right)}{c(n+1)\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)} \\ + \frac{\left(-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}} + a - \frac{b^2}{c}\right) (e+fx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{2c(e+fx)}{2ce-(b+\sqrt{b^2-4ac})f}\right)}{c(n+1)\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)} \\ - \frac{(bf+ce)(e+fx)^{n+1}}{c^2f^2(n+1)} + \frac{(e+fx)^{n+2}}{cf^2(n+2)}$$

Antiderivative was successfully verified.

[In] Int[(x^3\*(e + f\*x)^n)/(a + b\*x + c\*x^2), x]

[Out] -(((c\*e + b\*f)\*(e + f\*x)^(1 + n))/(c^2\*f^2\*(1 + n))) + (e + f\*x)^(2 + n)/(c\*f^2\*(2 + n)) + ((a - b^2/c + (b\*(b^2 - 3\*a\*c))/(c\*Sqrt[b^2 - 4\*a\*c]))\*(e + f\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (2\*c\*(e + f\*x))/(2\*c\*e - b\*f + Sqrt[b^2 - 4\*a\*c]\*f)]/(c\*(2\*c\*e - (b - Sqrt[b^2 - 4\*a\*c])\*f)\*(1 + n)) + ((a - b^2/c - (b\*(b^2 - 3\*a\*c))/(c\*Sqrt[b^2 - 4\*a\*c]))\*(e + f\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (2\*c\*(e + f\*x))/(2\*c\*e - (b + Sqrt[b^2 - 4\*a\*c])\*f)]/(c\*(2\*c\*e - (b + Sqrt[b^2 - 4\*a\*c])\*f)\*(1 + n))

**Rubi in Sympy [A]** time = 72.3735, size = 272, normalized size = 0.94

$$\frac{(e+fx)^{n+2}}{cf^2(n+2)} + \frac{(e+fx)^{n+1} \left( b(-3ac+b^2) - \sqrt{-4ac+b^2}(-ac+b^2) \right) {}_2F_1 \left( \begin{matrix} 1, n+1 \\ n+2 \end{matrix} \middle| \frac{c(-2e-2fx)}{bf-2ce-f\sqrt{-4ac+b^2}} \right)}{c^2(n+1)\sqrt{-4ac+b^2} \left( 2ce-f \left( b - \sqrt{-4ac+b^2} \right) \right)} - \frac{(e+fx)^{n+1} \left( b(-3ac+b^2) + \sqrt{-4ac+b^2}(-ac+b^2) \right) {}_2F_1 \left( \begin{matrix} 1, n+1 \\ n+2 \end{matrix} \middle| \frac{c(-2e-2fx)}{bf-2ce+f\sqrt{-4ac+b^2}} \right)}{c^2(n+1)\sqrt{-4ac+b^2} \left( 2ce-f \left( b + \sqrt{-4ac+b^2} \right) \right)} - \frac{(e+fx)^{n+1}(bf+ce)}{c^2f^2(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**3*(f*x+e)**n/(c*x**2+b*x+a), x)`

[Out]  $(e+f*x)^{(n+2)}/(c*f^2*(n+2)) + (e+f*x)^{(n+1)}*(b*(-3*a*c+b^2) - \text{sqrt}(-4*a*c+b^2)*(-a*c+b^2))*\text{hyper}((1, n+1), (n+2, ), c*(-2*e-2*f*x)/(b*f-2*c*e-f*\text{sqrt}(-4*a*c+b^2)))/(c^2*(n+1)*\text{sqrt}(-4*a*c+b^2)*(2*c*e-f*(b-\text{sqrt}(-4*a*c+b^2)))) - (e+f*x)^{(n+1)}*(b*(-3*a*c+b^2) + \text{sqrt}(-4*a*c+b^2)*(-a*c+b^2))*\text{hyper}((1, n+1), (n+2, ), c*(-2*e-2*f*x)/(b*f-2*c*e+f*\text{sqrt}(-4*a*c+b^2)))/(c^2*(n+1)*\text{sqrt}(-4*a*c+b^2)*(2*c*e-f*(b+\text{sqrt}(-4*a*c+b^2)))) - (e+f*x)^{(n+1)}*(b*f+c*e)/(c^2*f^2*(n+1))$

**Mathematica [A]** time = 1.18899, size = 353, normalized size = 1.22

$$2^{-n-1}(e+fx)^n \left( \left( b^2\sqrt{f^2(b^2-4ac)} - ac\sqrt{f^2(b^2-4ac)} + 3abcf + b^3(-f) \right) \left( \frac{c(e+fx)}{-\sqrt{f^2(b^2-4ac)}+bf+2cfx} \right)^{-n} {}_2F_1 \left( -n, -n; 1-n; \frac{2}{-b} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(x^3*(e+f*x)^n)/(a+b*x+c*x^2), x]`

[Out]  $(2^{(-1-n)}*(e+f*x)^n*(((b^3*f) + 3*a*b*c*f + b^2*\text{Sqrt}[(b^2 - 4*a*c)*f^2] - a*c*\text{Sqrt}[(b^2 - 4*a*c)*f^2]))*\text{Hypergeometric2F1}[-n, -n, 1-n, (2*c*e - b*f + \text{Sqrt}[(b^2 - 4*a*c)*f^2])/(-b*f) + \text{Sqrt}[(b^2 - 4*a*c)*f^2] - 2*c*f*x)]/((c*(e+f*x))/(b*f - \text{Sqrt}[(b^2 - 4*a*c)*f^2] + 2*c*f*x))^n + ((b^3*f - 3*a*b*c*f + b^2*\text{Sqrt}[(b^2 - 4*a*c)*f^2] - a*c*\text{Sqrt}[(b^2 - 4*a*c)*f^2]))*\text{Hypergeometric2F1}[-n, -n, 1-n, (-2*c*e + b*f + \text{Sqrt}[(b^2 - 4*a*c)*f^2])/b*f + \text{Sqrt}[(b^2 - 4*a*c)*f^2] + 2*c*f*x)]/((c*(e+f*x))/(b*f + \text{Sqrt}[(b^2 - 4*a*c)*f^2] + 2*c*f*x))^n)/(c^3*\text{Sqrt}[(b^2 - 4*a*c)*f^2]^n)$

**Maple [F]** time = 0.135, size = 0, normalized size = 0.

$$\int \frac{x^3 (fx + e)^n}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^3*(f*x+e)^n/(c*x^2+b*x+a), x)`

[Out] `int(x^3*(f*x+e)^n/(c*x^2+b*x+a), x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^n x^3}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x + e)^n*x^3/(c*x^2 + b*x + a), x, algorithm="maxima")`

[Out] `integrate((f*x + e)^n*x^3/(c*x^2 + b*x + a), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx + e)^n x^3}{cx^2 + bx + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x + e)^n*x^3/(c*x^2 + b*x + a), x, algorithm="fricas")`

[Out] `integral((f*x + e)^n*x^3/(c*x^2 + b*x + a), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**3*(f*x+e)**n/(c*x**2+b*x+a), x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^n x^3}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x + e)^n*x^3/(c*x^2 + b*x + a), x, algorithm="giac")`

[Out] `integrate((f*x + e)^n*x^3/(c*x^2 + b*x + a), x)`



$$3.543 \quad \int \frac{x^2(e+fx)^n}{a+bx+cx^2} dx$$

Optimal. Leaf size=236

$$\frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) (e+fx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{2c(e+fx)}{2ce-bf+\sqrt{b^2-4ac}f}\right)}{c(n+1) \left(2ce - f \left(b - \sqrt{b^2-4ac}\right)\right)} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) (e+fx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{2c(e+fx)}{2ce-(b+\sqrt{b^2-4ac})f}\right)}{c(n+1) \left(2ce - f \left(\sqrt{b^2-4ac} + b\right)\right)} + \frac{(e+fx)^{n+1}}{cf(n+1)}$$

[Out]  $(e + f*x)^{(1 + n)}/(c*f*(1 + n)) + ((b - (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c])*(e + f*x)^{(1 + n)}*\text{Hypergeometric2F1}[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - b*f + \text{Sqrt}[b^2 - 4*a*c]*f)]/(c*(2*c*e - (b - \text{Sqrt}[b^2 - 4*a*c])*f)*(1 + n)) + ((b + (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c])*(e + f*x)^{(1 + n)}*\text{Hypergeometric2F1}[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + \text{Sqrt}[b^2 - 4*a*c])*f)]/(c*(2*c*e - (b + \text{Sqrt}[b^2 - 4*a*c])*f)*(1 + n))$

Rubi [A] time = 0.707815, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{\left(b - \frac{b^2-2ac}{\sqrt{b^2-4ac}}\right) (e+fx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{2c(e+fx)}{2ce-bf+\sqrt{b^2-4ac}f}\right)}{c(n+1) \left(2ce - f \left(b - \sqrt{b^2-4ac}\right)\right)} + \frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}} + b\right) (e+fx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{2c(e+fx)}{2ce-(b+\sqrt{b^2-4ac})f}\right)}{c(n+1) \left(2ce - f \left(\sqrt{b^2-4ac} + b\right)\right)} + \frac{(e+fx)^{n+1}}{cf(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(x^2\*(e + f\*x)^n)/(a + b\*x + c\*x^2), x]

[Out]  $(e + f*x)^{(1 + n)}/(c*f*(1 + n)) + ((b - (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c])*(e + f*x)^{(1 + n)}*\text{Hypergeometric2F1}[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - b*f + \text{Sqrt}[b^2 - 4*a*c]*f)]/(c*(2*c*e - (b - \text{Sqrt}[b^2 - 4*a*c])*f)*(1 + n)) + ((b + (b^2 - 2*a*c)/\text{Sqrt}[b^2 - 4*a*c])*(e + f*x)^{(1 + n)}*\text{Hypergeometric2F1}[1, 1 + n, 2 + n, (2*c*(e + f*x))/(2*c*e - (b + \text{Sqrt}[b^2 - 4*a*c])*f)]/(c*(2*c*e - (b + \text{Sqrt}[b^2 - 4*a*c])*f)*(1 + n))$

Rubi in Sympy [A] time = 66.5773, size = 228, normalized size = 0.97

$$\frac{(e+fx)^{n+1} \left(-2ac + b^2 + b\sqrt{-4ac + b^2}\right) {}_2F_1\left(1, n+1; n+2; \frac{c(-2e-2fx)}{bf-2ce+f\sqrt{-4ac+b^2}}\right)}{c(n+1) \sqrt{-4ac + b^2} \left(2ce - f \left(b + \sqrt{-4ac + b^2}\right)\right)} - \frac{(e+fx)^{n+1} \left(-2ac + b^2 - b\sqrt{-4ac + b^2}\right) {}_2F_1\left(1, n+1; n+2; \frac{c(-2e-2fx)}{bf-2ce-f\sqrt{-4ac+b^2}}\right)}{c(n+1) \sqrt{-4ac + b^2} \left(2ce - f \left(b - \sqrt{-4ac + b^2}\right)\right)} + \frac{(e+fx)^{n+1}}{cf(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**2*(f*x+e)**n/(c*x**2+b*x+a),x)`

[Out]  $(e + f*x)^{(n + 1)} * (-2*a*c + b**2 + b*sqrt(-4*a*c + b**2)) * hyper((1, n + 1), (n + 2, ), c*(-2*e - 2*f*x)/(b*f - 2*c*e + f*sqrt(-4*a*c + b**2)))/(c*(n + 1)*sqrt(-4*a*c + b**2)*(2*c*e - f*(b + sqrt(-4*a*c + b**2)))) - (e + f*x)^{(n + 1)} * (-2*a*c + b**2 - b*sqrt(-4*a*c + b**2)) * hyper((1, n + 1), (n + 2, ), c*(-2*e - 2*f*x)/(b*f - 2*c*e - f*sqrt(-4*a*c + b**2)))/(c*(n + 1)*sqrt(-4*a*c + b**2)*(2*c*e - f*(b - sqrt(-4*a*c + b**2)))) + (e + f*x)^{(n + 1)}/(c*f*(n + 1))$

**Mathematica [A]** time = 1.90567, size = 346, normalized size = 1.47

$$2^{-n-1}(e + fx)^n \left( - \left( b\sqrt{f^2(b^2 - 4ac)} + 2acf + b^2(-f) \right) \left( \frac{c(e+fx)}{-\sqrt{f^2(b^2-4ac)+bf+2cfx}} \right)^{-n} {}_2F_1 \left( -n, -n; 1 - n; \frac{2ce-bf+\sqrt{(b^2-4ac)f^2}}{-bf-2cxf+\sqrt{(b^2-4ac)f^2}} \right) \right) / c^2 n \sqrt{f^2}$$

Antiderivative was successfully verified.

[In] `Integrate[(x^2*(e + f*x)^n)/(a + b*x + c*x^2),x]`

[Out]  $(2^{(-1 - n)} * (e + f*x)^n * ((2^{(1 + n)} * c * Sqrt[(b^2 - 4*a*c)] * f^2)^n * (e + f*x)) / (f * (1 + n)) - ((-(b^2*f) + 2*a*c*f + b*Sqrt[(b^2 - 4*a*c)] * f^2)) * Hypergeometric2F1[-n, -n, 1 - n, (2*c*e - b*f + Sqrt[(b^2 - 4*a*c)] * f^2)] / (-(b*f) + Sqrt[(b^2 - 4*a*c)] * f^2 - 2*c*f*x)) / ((c * (e + f*x)) / (b*f - Sqrt[(b^2 - 4*a*c)] * f^2 + 2*c*f*x))^n - ((b^2*f - 2*a*c*f + b*Sqrt[(b^2 - 4*a*c)] * f^2)) * Hypergeometric2F1[-n, -n, 1 - n, (-2*c*e + b*f + Sqrt[(b^2 - 4*a*c)] * f^2)] / (b*f + Sqrt[(b^2 - 4*a*c)] * f^2 + 2*c*f*x)) / ((c * (e + f*x)) / (b*f + Sqrt[(b^2 - 4*a*c)] * f^2 + 2*c*f*x))^n) / (c^2 * Sqrt[(b^2 - 4*a*c)] * f^2)^n$

**Maple [F]** time = 0.136, size = 0, normalized size = 0.

$$\int \frac{x^2 (fx + e)^n}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(x^2*(f*x+e)^n/(c*x^2+b*x+a),x)`

[Out] `int(x^2*(f*x+e)^n/(c*x^2+b*x+a),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^n x^2}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x + e)^n*x^2/(c*x^2 + b*x + a),x, algorithm="maxima")`

[Out] `integrate((f*x + e)^n*x^2/(c*x^2 + b*x + a), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx + e)^n x^2}{cx^2 + bx + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x + e)^n*x^2/(c*x^2 + b*x + a),x, algorithm="fricas")`

[Out] `integral((f*x + e)^n*x^2/(c*x^2 + b*x + a), x)`

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**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x**2*(f*x+e)**n/(c*x**2+b*x+a),x)`

[Out] Timed out

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^n x^2}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x + e)^n*x^2/(c*x^2 + b*x + a),x, algorithm="giac")`

[Out] `integrate((f*x + e)^n*x^2/(c*x^2 + b*x + a), x)`

$$3.544 \quad \int \frac{x(e+fx)^n}{a+bx+cx^2} dx$$

**Optimal.** Leaf size=197

$$\frac{\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) (e+fx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{2c(e+fx)}{2ce-bf+\sqrt{b^2-4ac}f}\right)}{(n+1) \left(2ce - f \left(b - \sqrt{b^2-4ac}\right)\right)} - \frac{\left(\frac{b}{\sqrt{b^2-4ac}} + 1\right) (e+fx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{2c(e+fx)}{2ce-(b+\sqrt{b^2-4ac})f}\right)}{(n+1) \left(2ce - f \left(\sqrt{b^2-4ac} + b\right)\right)}$$

[Out] -(((1 - b/Sqrt[b^2 - 4\*a\*c])\*(e + f\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (2\*c\*(e + f\*x))/(2\*c\*e - b\*f + Sqrt[b^2 - 4\*a\*c]\*f)]/((2\*c\*e - (b - Sqrt[b^2 - 4\*a\*c])\*f)\*(1 + n))) - ((1 + b/Sqrt[b^2 - 4\*a\*c])\*(e + f\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (2\*c\*(e + f\*x))/(2\*c\*e - (b + Sqrt[b^2 - 4\*a\*c])\*f)]/((2\*c\*e - (b + Sqrt[b^2 - 4\*a\*c])\*f)\*(1 + n)))

**Rubi [A]** time = 0.399191, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$\frac{\left(1 - \frac{b}{\sqrt{b^2-4ac}}\right) (e+fx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{2c(e+fx)}{2ce-bf+\sqrt{b^2-4ac}f}\right)}{(n+1) \left(2ce - f \left(b - \sqrt{b^2-4ac}\right)\right)} - \frac{\left(\frac{b}{\sqrt{b^2-4ac}} + 1\right) (e+fx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{2c(e+fx)}{2ce-(b+\sqrt{b^2-4ac})f}\right)}{(n+1) \left(2ce - f \left(\sqrt{b^2-4ac} + b\right)\right)}$$

Antiderivative was successfully verified.

[In] Int[(x\*(e + f\*x)^n)/(a + b\*x + c\*x^2), x]

[Out] -(((1 - b/Sqrt[b^2 - 4\*a\*c])\*(e + f\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (2\*c\*(e + f\*x))/(2\*c\*e - b\*f + Sqrt[b^2 - 4\*a\*c]\*f)]/((2\*c\*e - (b - Sqrt[b^2 - 4\*a\*c])\*f)\*(1 + n))) - ((1 + b/Sqrt[b^2 - 4\*a\*c])\*(e + f\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (2\*c\*(e + f\*x))/(2\*c\*e - (b + Sqrt[b^2 - 4\*a\*c])\*f)]/((2\*c\*e - (b + Sqrt[b^2 - 4\*a\*c])\*f)\*(1 + n)))

**Rubi in Sympy [A]** time = 47.9636, size = 196, normalized size = 0.99

$$\frac{\left(b - \sqrt{-4ac + b^2}\right) (e+fx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{c(-2e-2fx)}{bf-2ce-f\sqrt{-4ac+b^2}}\right)}{(n+1) \sqrt{-4ac + b^2} \left(bf - 2ce - f\sqrt{-4ac + b^2}\right)} - \frac{\left(b + \sqrt{-4ac + b^2}\right) (e+fx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{c(-2e-2fx)}{bf-2ce+f\sqrt{-4ac+b^2}}\right)}{(n+1) \sqrt{-4ac + b^2} \left(2ce - f \left(b + \sqrt{-4ac + b^2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(x\*(f\*x+e)\*\*n/(c\*x\*\*2+b\*x+a), x)

[Out]  $-(b - \sqrt{-4ac + b^2})^n (e + fx)^{n+1} \operatorname{hyper}((1, n+1), (n+2, ), c(-2e - 2fx)/(bf - 2ce - f\sqrt{-4ac + b^2})) / ((n+1)\sqrt{-4ac + b^2}(bf - 2ce - f\sqrt{-4ac + b^2})) - (b + \sqrt{-4ac + b^2})^n (e + fx)^{n+1} \operatorname{hyper}((1, n+1), (n+2, ), c(-2e - 2fx)/(bf - 2ce + f\sqrt{-4ac + b^2})) / ((n+1)\sqrt{-4ac + b^2}(2ce - f(b + \sqrt{-4ac + b^2})))$

**Mathematica [A]** time = 0.696193, size = 289, normalized size = 1.47

$$\frac{2^{-n-1}(e+fx)^n \left( \left( \sqrt{f^2(b^2-4ac)} - bf \right) \left( \frac{c(e+fx)}{-\sqrt{f^2(b^2-4ac)+bf+2cfx}} \right)^{-n} {}_2F_1 \left( -n, -n; 1-n; \frac{2ce-bf+\sqrt{(b^2-4ac)f^2}}{-bf-2cxf+\sqrt{(b^2-4ac)f^2}} \right) + \left( \sqrt{f^2(b^2-4ac)} + bf \right) \left( \frac{c(e+fx)}{-\sqrt{f^2(b^2-4ac)+bf+2cfx}} \right)^{-n} {}_2F_1 \left( -n, -n; 1-n; \frac{2ce-bf-\sqrt{(b^2-4ac)f^2}}{-bf-2cxf+\sqrt{(b^2-4ac)f^2}} \right) \right)}{cn\sqrt{f^2(b^2-4ac)}}$$

Antiderivative was successfully verified.

[In] Integrate[(x\*(e + f\*x)^n)/(a + b\*x + c\*x^2), x]

[Out]  $(2^{(-1-n)}(e+fx)^n (((-bf) + \operatorname{Sqrt}[(b^2-4ac)f^2]) \operatorname{Hypergeometric2F1}[-n, -n, 1-n, (2ce-bf+\sqrt{(b^2-4ac)f^2})/(-bf-2cxf+\sqrt{(b^2-4ac)f^2})]) / ((c(e+fx))/(bf - \operatorname{Sqrt}[(b^2-4ac)f^2] + 2cfx))^n + ((bf + \operatorname{Sqrt}[(b^2-4ac)f^2]) \operatorname{Hypergeometric2F1}[-n, -n, 1-n, (-2ce-bf-\sqrt{(b^2-4ac)f^2})/(-bf-2cxf+\sqrt{(b^2-4ac)f^2})]) / ((c(e+fx))/(bf + \operatorname{Sqrt}[(b^2-4ac)f^2] + 2cfx))^n) / (c \operatorname{Sqrt}[(b^2-4ac)f^2]^n)$

**Maple [F]** time = 0.123, size = 0, normalized size = 0.

$$\int \frac{x(fx+e)^n}{cx^2+bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x\*(f\*x+e)^n/(c\*x^2+b\*x+a), x)

[Out] int(x\*(f\*x+e)^n/(c\*x^2+b\*x+a), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx+e)^n x}{cx^2+bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x + e)^n\*x/(c\*x^2 + b\*x + a), x, algorithm="maxima")

[Out] integrate((f\*x + e)^n\*x/(c\*x^2 + b\*x + a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{(fx+e)^n x}{cx^2+bx+a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x + e)^n*x/(c*x^2 + b*x + a),x, algorithm="fricas")`

[Out] `integral((f*x + e)^n*x/(c*x^2 + b*x + a), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{x(e + fx)^n}{a + bx + cx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(x*(f*x+e)**n/(c*x**2+b*x+a),x)`

[Out] `Integral(x*(e + f*x)**n/(a + b*x + c*x**2), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^n x}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x + e)^n*x/(c*x^2 + b*x + a),x, algorithm="giac")`

[Out] `integrate((f*x + e)^n*x/(c*x^2 + b*x + a), x)`

$$3.545 \quad \int \frac{(e+fx)^n}{a+bx+cx^2} dx$$

**Optimal.** Leaf size=190

$$\frac{2c(e+fx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{2c(e+fx)}{2ce-(b+\sqrt{b^2-4ac})f}\right)}{(n+1)\sqrt{b^2-4ac}\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)} - \frac{2c(e+fx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{2c(e+fx)}{2ce-bf+\sqrt{b^2-4ac}f}\right)}{(n+1)\sqrt{b^2-4ac}\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}$$

[Out]  $(-2*c*(e+f*x)^{(1+n)}*Hypergeometric2F1[1, 1+n, 2+n, (2*c*(e+f*x))/(2*c*e-b*f+Sqrt[b^2-4*a*c]*f)]/(Sqrt[b^2-4*a*c]*(2*c*e-(b-Sqrt[b^2-4*a*c])*f)*(1+n)) + (2*c*(e+f*x)^{(1+n)}*Hypergeometric2F1[1, 1+n, 2+n, (2*c*(e+f*x))/(2*c*e-(b+Sqrt[b^2-4*a*c])*f)]/(Sqrt[b^2-4*a*c]*(2*c*e-(b+Sqrt[b^2-4*a*c])*f)*(1+n))$

**Rubi [A]** time = 0.478302, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{2c(e+fx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{2c(e+fx)}{2ce-(b+\sqrt{b^2-4ac})f}\right)}{(n+1)\sqrt{b^2-4ac}\left(2ce-f\left(\sqrt{b^2-4ac}+b\right)\right)} - \frac{2c(e+fx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{2c(e+fx)}{2ce-bf+\sqrt{b^2-4ac}f}\right)}{(n+1)\sqrt{b^2-4ac}\left(2ce-f\left(b-\sqrt{b^2-4ac}\right)\right)}$$

Antiderivative was successfully verified.

[In] Int[(e + f\*x)^n/(a + b\*x + c\*x^2), x]

[Out]  $(-2*c*(e+f*x)^{(1+n)}*Hypergeometric2F1[1, 1+n, 2+n, (2*c*(e+f*x))/(2*c*e-b*f+Sqrt[b^2-4*a*c]*f)]/(Sqrt[b^2-4*a*c]*(2*c*e-(b-Sqrt[b^2-4*a*c])*f)*(1+n)) + (2*c*(e+f*x)^{(1+n)}*Hypergeometric2F1[1, 1+n, 2+n, (2*c*(e+f*x))/(2*c*e-(b+Sqrt[b^2-4*a*c])*f)]/(Sqrt[b^2-4*a*c]*(2*c*e-(b+Sqrt[b^2-4*a*c])*f)*(1+n))$

**Rubi in Sympy [A]** time = 58.5569, size = 172, normalized size = 0.91

$$\frac{2c(e+fx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{c(-2e-2fx)}{bf-2ce+f\sqrt{-4ac+b^2}}\right)}{(n+1)\sqrt{-4ac+b^2}\left(2ce-f\left(b+\sqrt{-4ac+b^2}\right)\right)} - \frac{2c(e+fx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{c(-2e-2fx)}{bf-2ce-f\sqrt{-4ac+b^2}}\right)}{(n+1)\sqrt{-4ac+b^2}\left(2ce-f\left(b-\sqrt{-4ac+b^2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((f\*x+e)\*\*n/(c\*x\*\*2+b\*x+a), x)

[Out]  $2*c*(e+f*x)**(n+1)*hyper((1, n+1), (n+2, ), c*(-2*e-2*f*x)/(b*f-2*c*e+f*sqrt(-4*a*c+b**2)))/((n+1)*sqrt(-4*a*c+b**2)*(2*c*e-f*(b+sqrt(-4*a*c+b**2)))) - 2*c*(e+f*x)**(n+1)*hyper((1, n+1), (n+2, ), c*(-2*e-2*f*x)/(b*f-2*c*e-f*sqrt(-4*a*c+b**2)))/((n+1)*sqrt(-4*a*c+b**2)*(2*c*e-f*(b-sqrt(-4*a*c+b**2))))$

**Mathematica [A]** time = 0.646766, size = 245, normalized size = 1.29

$$\frac{f 2^{-n} (e + f x)^n \left( \left( \frac{c(e+fx)}{-\sqrt{f^2(b^2-4ac)+bf+2cfx}} \right)^{-n} {}_2F_1 \left( -n, -n; 1-n; \frac{2ce-bf+\sqrt{(b^2-4ac)f^2}}{-bf-2cfx+\sqrt{(b^2-4ac)f^2}} \right) - \left( \frac{c(e+fx)}{\sqrt{f^2(b^2-4ac)+bf+2cfx}} \right)^{-n} {}_2F_1 \left( -n, -n; 1-n; \frac{2ce-bf-\sqrt{(b^2-4ac)f^2}}{-bf-2cfx+\sqrt{(b^2-4ac)f^2}} \right) \right)}{n \sqrt{f^2(b^2-4ac)}}$$

Antiderivative was successfully verified.

[In] Integrate[(e + f\*x)^n/(a + b\*x + c\*x^2), x]

[Out] (f\*(e + f\*x)^n\*(Hypergeometric2F1[-n, -n, 1 - n, (2\*c\*e - b\*f + Sqrt[(b^2 - 4\*a\*c)\*f^2])/(-b\*f + Sqrt[(b^2 - 4\*a\*c)\*f^2] - 2\*c\*f\*x)]/((c\*(e + f\*x))/(b\*f - Sqrt[(b^2 - 4\*a\*c)\*f^2] + 2\*c\*f\*x))^n - Hypergeometric2F1[-n, -n, 1 - n, (-2\*c\*e + b\*f + Sqrt[(b^2 - 4\*a\*c)\*f^2])/(-b\*f + Sqrt[(b^2 - 4\*a\*c)\*f^2] + 2\*c\*f\*x)]/((c\*(e + f\*x))/(b\*f + Sqrt[(b^2 - 4\*a\*c)\*f^2] + 2\*c\*f\*x))^n)/(2^n\*Sqrt[(b^2 - 4\*a\*c)\*f^2]^n)

**Maple [F]** time = 0.154, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^n}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f\*x+e)^n/(c\*x^2+b\*x+a), x)

[Out] int((f\*x+e)^n/(c\*x^2+b\*x+a), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^n}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x + e)^n/(c\*x^2 + b\*x + a), x, algorithm="maxima")

[Out] integrate((f\*x + e)^n/(c\*x^2 + b\*x + a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{(fx + e)^n}{cx^2 + bx + a}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x + e)^n/(c\*x^2 + b\*x + a), x, algorithm="fricas")

[Out] integral((f\*x + e)^n/(c\*x^2 + b\*x + a), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(e + fx)^n}{a + bx + cx^2} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)**n/(c*x**2+b*x+a),x)`

[Out] `Integral((e + f*x)**n/(a + b*x + c*x**2), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^n}{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x + e)^n/(c*x^2 + b*x + a),x, algorithm="giac")`

[Out] `integrate((f*x + e)^n/(c*x^2 + b*x + a), x)`

$$3.546 \quad \int \frac{(e+fx)^n}{x(a+bx+cx^2)} dx$$

Optimal. Leaf size=241

$$\frac{c \left( \frac{b}{\sqrt{b^2-4ac}} + 1 \right) (e+fx)^{n+1} {}_2F_1 \left( 1, n+1; n+2; \frac{2c(e+fx)}{2ce-bf+\sqrt{b^2-4ac}f} \right)}{a(n+1) \left( 2ce - f \left( b - \sqrt{b^2-4ac} \right) \right)} + \frac{c \left( 1 - \frac{b}{\sqrt{b^2-4ac}} \right) (e+fx)^{n+1} {}_2F_1 \left( 1, n+1; n+2; \frac{2c(e+fx)}{2ce-(b+\sqrt{b^2-4ac})f} \right)}{a(n+1) \left( 2ce - f \left( \sqrt{b^2-4ac} + b \right) \right)} - \frac{(e+fx)^{n+1} {}_2F_1 \left( 1, n+1; n+2; \frac{fx}{e} + 1 \right)}{ae(n+1)}$$

[Out] (c\*(1 + b/Sqrt[b^2 - 4\*a\*c])\*(e + f\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (2\*c\*(e + f\*x))/(2\*c\*e - b\*f + Sqrt[b^2 - 4\*a\*c]\*f)]/(a\*(2\*c\*e - (b - Sqrt[b^2 - 4\*a\*c])\*f)\*(1 + n)) + (c\*(1 - b/Sqrt[b^2 - 4\*a\*c])\*(e + f\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (2\*c\*(e + f\*x))/(2\*c\*e - (b + Sqrt[b^2 - 4\*a\*c])\*f)]/(a\*(2\*c\*e - (b + Sqrt[b^2 - 4\*a\*c])\*f)\*(1 + n)) - ((e + f\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (f\*x)/e])/ (a\*e\*(1 + n))

**Rubi [A]** time = 0.797336, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$

$$\frac{c \left( \frac{b}{\sqrt{b^2-4ac}} + 1 \right) (e+fx)^{n+1} {}_2F_1 \left( 1, n+1; n+2; \frac{2c(e+fx)}{2ce-bf+\sqrt{b^2-4ac}f} \right)}{a(n+1) \left( 2ce - f \left( b - \sqrt{b^2-4ac} \right) \right)} + \frac{c \left( 1 - \frac{b}{\sqrt{b^2-4ac}} \right) (e+fx)^{n+1} {}_2F_1 \left( 1, n+1; n+2; \frac{2c(e+fx)}{2ce-(b+\sqrt{b^2-4ac})f} \right)}{a(n+1) \left( 2ce - f \left( \sqrt{b^2-4ac} + b \right) \right)} - \frac{(e+fx)^{n+1} {}_2F_1 \left( 1, n+1; n+2; \frac{fx}{e} + 1 \right)}{ae(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(e + f\*x)^n/(x\*(a + b\*x + c\*x^2)),x]

[Out] (c\*(1 + b/Sqrt[b^2 - 4\*a\*c])\*(e + f\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (2\*c\*(e + f\*x))/(2\*c\*e - b\*f + Sqrt[b^2 - 4\*a\*c]\*f)]/(a\*(2\*c\*e - (b - Sqrt[b^2 - 4\*a\*c])\*f)\*(1 + n)) + (c\*(1 - b/Sqrt[b^2 - 4\*a\*c])\*(e + f\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (2\*c\*(e + f\*x))/(2\*c\*e - (b + Sqrt[b^2 - 4\*a\*c])\*f)]/(a\*(2\*c\*e - (b + Sqrt[b^2 - 4\*a\*c])\*f)\*(1 + n)) - ((e + f\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (f\*x)/e])/ (a\*e\*(1 + n))

**Rubi in Sympy [A]** time = 86.3901, size = 228, normalized size = 0.95

$$\frac{c \left( b - \sqrt{-4ac + b^2} \right) (e + fx)^{n+1} {}_2F_1 \left( \begin{matrix} 1, n+1 \\ n+2 \end{matrix} \middle| \frac{c(-2e-2fx)}{bf-2ce+f\sqrt{-4ac+b^2}} \right)}{a(n+1)\sqrt{-4ac+b^2} \left( bf - 2ce + f\sqrt{-4ac+b^2} \right)} + \frac{c \left( b + \sqrt{-4ac + b^2} \right) (e + fx)^{n+1} {}_2F_1 \left( \begin{matrix} 1, n+1 \\ n+2 \end{matrix} \middle| \frac{c(-2e-2fx)}{bf-2ce-f\sqrt{-4ac+b^2}} \right)}{a(n+1)\sqrt{-4ac+b^2} \left( 2ce - f \left( b - \sqrt{-4ac+b^2} \right) \right)} - \frac{(e + fx)^{n+1} {}_2F_1 \left( \begin{matrix} 1, n+1 \\ n+2 \end{matrix} \middle| 1 + \frac{fx}{e} \right)}{ae(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((f*x+e)**n/x/(c*x**2+b*x+a), x)`

[Out] `c*(b - sqrt(-4*a*c + b**2))*(e + f*x)**(n + 1)*hyper((1, n + 1), (n + 2, ), c*(-2*e - 2*f*x)/(b*f - 2*c*e + f*sqrt(-4*a*c + b**2)))/(a*(n + 1)*sqrt(-4*a*c + b**2)*(b*f - 2*c*e + f*sqrt(-4*a*c + b**2))) + c*(b + sqrt(-4*a*c + b**2))*(e + f*x)**(n + 1)*hyper((1, n + 1), (n + 2, ), c*(-2*e - 2*f*x)/(b*f - 2*c*e - f*sqrt(-4*a*c + b**2)))/(a*(n + 1)*sqrt(-4*a*c + b**2)*(2*c*e - f*(b - sqrt(-4*a*c + b**2)))) - (e + f*x)**(n + 1)*hyper((1, n + 1), (n + 2, ), 1 + f*x/e)/(a*e*(n + 1))`

**Mathematica [A]** time = 4.25474, size = 331, normalized size = 1.37

$$(e + fx)^n \left( \frac{2^{-n} \left( -\left( \sqrt{f^2(b^2-4ac)} + bf \right) \left( \frac{c(e+fx)}{-\sqrt{f^2(b^2-4ac)} + bf + 2cfx} \right)^{-n} {}_2F_1 \left( -n, -n; 1-n; \frac{2ce-bf+\sqrt{(b^2-4ac)}f^2}{-bf-2cxf+\sqrt{(b^2-4ac)}f^2} \right) - \left( \sqrt{f^2(b^2-4ac)} - bf \right) \left( \frac{c(e+fx)}{\sqrt{f^2(b^2-4ac)} + bf + 2cfx} \right)^{-n}}{\sqrt{f^2(b^2-4ac)}} \right)$$

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Antiderivative was successfully verified.

[In] `Integrate[(e + f*x)^n/(x*(a + b*x + c*x^2)), x]`

[Out] `((e + f*x)^n*((2*Hypergeometric2F1[-n, -n, 1 - n, -(e/(f*x))])/(1 + e/(f*x))^n + (-(((b*f + Sqrt[(b^2 - 4*a*c)]*f^2)*Hypergeometric2F1[-n, -n, 1 - n, (2*c*e - b*f + Sqrt[(b^2 - 4*a*c)]*f^2)]/(-(b*f) + Sqrt[(b^2 - 4*a*c)]*f^2) - 2*c*f*x)]/((c*(e + f*x))/(b*f - Sqrt[(b^2 - 4*a*c)]*f^2) + 2*c*f*x))^n - (((-b*f) + Sqrt[(b^2 - 4*a*c)]*f^2)*Hypergeometric2F1[-n, -n, 1 - n, (-2*c*e + b*f + Sqrt[(b^2 - 4*a*c)]*f^2)]/(b*f + Sqrt[(b^2 - 4*a*c)]*f^2) + 2*c*f*x)]/((c*(e + f*x))/(b*f + Sqrt[(b^2 - 4*a*c)]*f^2) + 2*c*f*x))^n)/(2^n*Sqrt[(b^2 - 4*a*c)]*f^2)))/(2*a^n)`

**Maple [F]** time = 0.113, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^n}{x(cx^2 + bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)^n/x/(c*x^2+b*x+a), x)`

[Out] `int((f*x+e)^n/x/(c*x^2+b*x+a), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^n}{(cx^2 + bx + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x + e)^n/((c\*x^2 + b\*x + a)\*x), x, algorithm="maxima")

[Out] integrate((f\*x + e)^n/((c\*x^2 + b\*x + a)\*x), x)

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**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx + e)^n}{cx^3 + bx^2 + ax}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x + e)^n/((c\*x^2 + b\*x + a)\*x), x, algorithm="fricas")

[Out] integral((f\*x + e)^n/(c\*x^3 + b\*x^2 + a\*x), x)

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**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x+e)\*\*n/x/(c\*x\*\*2+b\*x+a), x)

[Out] Timed out

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^n}{(cx^2 + bx + a)x} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f\*x + e)^n/((c\*x^2 + b\*x + a)\*x), x, algorithm="giac")

[Out] integrate((f\*x + e)^n/((c\*x^2 + b\*x + a)\*x), x)

$$3.547 \quad \int \frac{(e+fx)^n}{x^2(a+bx+cx^2)} dx$$

**Optimal.** Leaf size=295

$$\frac{c \left( \frac{b^2-2ac}{\sqrt{b^2-4ac}} + b \right) (e+fx)^{n+1} {}_2F_1 \left( 1, n+1; n+2; \frac{2c(e+fx)}{2ce-bf+\sqrt{b^2-4ac}f} \right)}{a^2(n+1) \left( 2ce - f \left( b - \sqrt{b^2-4ac} \right) \right)} - \frac{c \left( b - \frac{b^2-2ac}{\sqrt{b^2-4ac}} \right) (e+fx)^{n+1} {}_2F_1 \left( 1, n+1; n+2; \frac{2c(e+fx)}{2ce-(b+\sqrt{b^2-4ac})f} \right)}{a^2(n+1) \left( 2ce - f \left( \sqrt{b^2-4ac} + b \right) \right)} + \frac{b(e+fx)^{n+1} {}_2F_1 \left( 1, n+1; n+2; \frac{fx}{e} + 1 \right)}{a^2e(n+1)} + \frac{f(e+fx)^{n+1} {}_2F_1 \left( 2, n+1; n+2; \frac{fx}{e} + 1 \right)}{ae^2(n+1)}$$

[Out] -((c\*(b + (b^2 - 2\*a\*c)/Sqrt[b^2 - 4\*a\*c])\*(e + f\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (2\*c\*(e + f\*x))/(2\*c\*e - b\*f + Sqrt[b^2 - 4\*a\*c]\*f)]/(a^2\*(2\*c\*e - (b - Sqrt[b^2 - 4\*a\*c])\*f)\*(1 + n))) - (c\*(b - (b^2 - 2\*a\*c)/Sqrt[b^2 - 4\*a\*c])\*(e + f\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (2\*c\*(e + f\*x))/(2\*c\*e - (b + Sqrt[b^2 - 4\*a\*c]\*f)]/(a^2\*(2\*c\*e - (b + Sqrt[b^2 - 4\*a\*c])\*f)\*(1 + n))) + (b\*(e + f\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (f\*x)/e])/(a^2\*e\*(1 + n)) + (f\*(e + f\*x)^(1 + n)\*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (f\*x)/e])/(a\*e^2\*(1 + n))

**Rubi [A]** time = 1.00389, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$

$$\frac{c \left( \frac{b^2-2ac}{\sqrt{b^2-4ac}} + b \right) (e+fx)^{n+1} {}_2F_1 \left( 1, n+1; n+2; \frac{2c(e+fx)}{2ce-bf+\sqrt{b^2-4ac}f} \right)}{a^2(n+1) \left( 2ce - f \left( b - \sqrt{b^2-4ac} \right) \right)} - \frac{c \left( b - \frac{b^2-2ac}{\sqrt{b^2-4ac}} \right) (e+fx)^{n+1} {}_2F_1 \left( 1, n+1; n+2; \frac{2c(e+fx)}{2ce-(b+\sqrt{b^2-4ac})f} \right)}{a^2(n+1) \left( 2ce - f \left( \sqrt{b^2-4ac} + b \right) \right)} + \frac{b(e+fx)^{n+1} {}_2F_1 \left( 1, n+1; n+2; \frac{fx}{e} + 1 \right)}{a^2e(n+1)} + \frac{f(e+fx)^{n+1} {}_2F_1 \left( 2, n+1; n+2; \frac{fx}{e} + 1 \right)}{ae^2(n+1)}$$

Antiderivative was successfully verified.

[In] Int[(e + f\*x)^n/(x^2\*(a + b\*x + c\*x^2)), x]

[Out] -((c\*(b + (b^2 - 2\*a\*c)/Sqrt[b^2 - 4\*a\*c])\*(e + f\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (2\*c\*(e + f\*x))/(2\*c\*e - b\*f + Sqrt[b^2 - 4\*a\*c]\*f)]/(a^2\*(2\*c\*e - (b - Sqrt[b^2 - 4\*a\*c])\*f)\*(1 + n))) - (c\*(b - (b^2 - 2\*a\*c)/Sqrt[b^2 - 4\*a\*c])\*(e + f\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (2\*c\*(e + f\*x))/(2\*c\*e - (b + Sqrt[b^2 - 4\*a\*c]\*f)]/(a^2\*(2\*c\*e - (b + Sqrt[b^2 - 4\*a\*c])\*f)\*(1 + n))) + (b\*(e + f\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, 1 + (f\*x)/e])/(a^2\*e\*(1 + n)) + (f\*(e + f\*x)^(1 + n)\*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (f\*x)/e])/(a\*e^2\*(1 + n))

**Rubi in Sympy [A]** time = 116.988, size = 282, normalized size = 0.96

$$\frac{f(e+fx)^{n+1} {}_2F_1\left(2, n+1 \middle| 1 + \frac{fx}{e}\right)}{ae^2(n+1)} + \frac{b(e+fx)^{n+1} {}_2F_1\left(1, n+1 \middle| 1 + \frac{fx}{e}\right)}{a^2e(n+1)}$$

$$- \frac{c(e+fx)^{n+1} \left(-2ac + b^2 - b\sqrt{-4ac + b^2}\right) {}_2F_1\left(1, n+1 \middle| \frac{c(-2e-2fx)}{bf-2ce+f\sqrt{-4ac+b^2}}\right)}{a^2(n+1)\sqrt{-4ac+b^2}\left(bf-2ce+f\sqrt{-4ac+b^2}\right)}$$

$$- \frac{c(e+fx)^{n+1} \left(-2ac + b^2 + b\sqrt{-4ac + b^2}\right) {}_2F_1\left(1, n+1 \middle| \frac{c(-2e-2fx)}{bf-2ce-f\sqrt{-4ac+b^2}}\right)}{a^2(n+1)\sqrt{-4ac+b^2}\left(2ce-f\left(b-\sqrt{-4ac+b^2}\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((f*x+e)**n/x**2/(c*x**2+b*x+a), x)`

[Out]  $f*(e + f*x)**(n + 1)*\text{hyper}((2, n + 1), (n + 2, ), 1 + f*x/e)/(a*e**2*(n + 1)) + b*(e + f*x)**(n + 1)*\text{hyper}((1, n + 1), (n + 2, ), 1 + f*x/e)/(a**2*e*(n + 1)) - c*(e + f*x)**(n + 1)*(-2*a*c + b**2 - b*\text{sqrt}(-4*a*c + b**2))*\text{hyper}((1, n + 1), (n + 2, ), c*(-2*e - 2*f*x)/(b*f - 2*c*e + f*\text{sqrt}(-4*a*c + b**2)))/(a**2*(n + 1)*\text{sqrt}(-4*a*c + b**2)*(b*f - 2*c*e + f*\text{sqrt}(-4*a*c + b**2))) - c*(e + f*x)**(n + 1)*(-2*a*c + b**2 + b*\text{sqrt}(-4*a*c + b**2))*\text{hyper}((1, n + 1), (n + 2, ), c*(-2*e - 2*f*x)/(b*f - 2*c*e - f*\text{sqrt}(-4*a*c + b**2)))/(a**2*(n + 1)*\text{sqrt}(-4*a*c + b**2)*(2*c*e - f*(b - \text{sqrt}(-4*a*c + b**2))))$

**Mathematica [A]** time = 3.73591, size = 431, normalized size = 1.46

$$\left(\frac{e}{fx} + 1\right)^{-n} (e + fx)^n \left( \frac{2^{-n} (b\sqrt{f^2(b^2-4ac)} - 2acf + b^2f) \left(\frac{e}{fx} + 1\right)^n \left(\frac{c(e+fx)}{-\sqrt{f^2(b^2-4ac)} + bf + 2cfx}\right)^{-n} {}_2F_1\left(-n, -n; 1-n; \frac{2ce-bf+\sqrt{(b^2-4ac)f^2}}{-bf-2cxf+\sqrt{(b^2-4ac)f^2}}\right)}{n\sqrt{f^2(b^2-4ac)}} + \frac{2^{-n} (b\sqrt{f^2(b^2-4ac)} + 2acf + b^2f) \left(\frac{e}{fx} + 1\right)^n \left(\frac{c(e+fx)}{\sqrt{f^2(b^2-4ac)} + bf + 2cfx}\right)^{-n} {}_2F_1\left(-n, -n; 1-n; \frac{2ce-bf-\sqrt{(b^2-4ac)f^2}}{-bf-2cxf-\sqrt{(b^2-4ac)f^2}}\right)}{n\sqrt{f^2(b^2-4ac)}} \right)$$

Antiderivative was successfully verified.

[In] `Integrate[(e + f*x)^n/(x^2*(a + b*x + c*x^2)), x]`

[Out]  $((e + f*x)^n*((2*a*Hypergeometric2F1[1 - n, -n, 2 - n, -(e/(f*x))])/((-1 + n)*x) - (2*b*Hypergeometric2F1[-n, -n, 1 - n, -(e/(f*x))])/n + ((b^2*f - 2*a*c*f + b*\text{Sqrt}[(b^2 - 4*a*c)*f^2])*(1 + e/(f*x))^n*Hypergeometric2F1[-n, -n, 1 - n, (2*c*e - b*f + \text{Sqrt}[(b^2 - 4*a*c)*f^2])/(-b*f) + \text{Sqrt}[(b^2 - 4*a*c)*f^2] - 2*c*f*x)])/((2^n*\text{Sqrt}[(b^2 - 4*a*c)*f^2]*n*((c*(e + f*x))/(b*f - \text{Sqrt}[(b^2 - 4*a*c)*f^2] + 2*c*f*x))^n) + ((-b^2*f) + 2*a*c*f + b*\text{Sqrt}[(b^2 - 4*a*c)*f^2])*(1 + e/(f*x))^n*Hypergeometric2F1[-n, -n, 1 - n, (-2*c*e + b*f + \text{Sqrt}[(b^2 - 4*a*c)*f^2])/(b*f + \text{Sqrt}[(b^2 - 4*a*c)*f^2] + 2*c*f*x)])/((2^n*\text{Sqrt}[(b^2 - 4*a*c)*f^2]*n*((c*(e + f*x))/(b*f + \text{Sqrt}[(b^2 - 4*a*c)*f^2] + 2*c*f*x))^n)))/(2*a^2*(1 + e/(f*x))^n)$

**Maple [F]** time = 0.143, size = 0, normalized size = 0.

$$\int \frac{(fx + e)^n}{x^2(cx^2 + bx + a)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((f*x+e)^n/x^2/(c*x^2+b*x+a),x)`

[Out] `int((f*x+e)^n/x^2/(c*x^2+b*x+a),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^n}{(cx^2 + bx + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x + e)^n/((c*x^2 + b*x + a)*x^2),x, algorithm="maxima")`

[Out] `integrate((f*x + e)^n/((c*x^2 + b*x + a)*x^2), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(fx + e)^n}{cx^4 + bx^3 + ax^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x + e)^n/((c*x^2 + b*x + a)*x^2),x, algorithm="fricas")`

[Out] `integral((f*x + e)^n/(c*x^4 + b*x^3 + a*x^2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x+e)**n/x**2/(c*x**2+b*x+a),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(fx + e)^n}{(cx^2 + bx + a)x^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((f*x + e)^n/((c*x^2 + b*x + a)*x^2),x, algorithm="giac")`

[Out] `integrate((f*x + e)^n/((c*x^2 + b*x + a)*x^2), x)`

$$3.548 \quad \int \frac{(d+ex)^4(f+gx)^2}{d^2-e^2x^2} dx$$

**Optimal.** Leaf size=141

$$\frac{8d^3(dg+ef)^2 \log(d-ex)}{e^3} - \frac{dx^2(4d^2g^2+7defg+2e^2f^2)}{e} - \frac{d^2x(8d^2g^2+16defg+7e^2f^2)}{e^2} - \frac{1}{2}egx^4(2dg+ef) - \frac{1}{3}x^3(dg+ef)(7dg+ef) - \frac{1}{5}e^2g^2x^5$$

[Out]  $-\left(\frac{d^2(7e^2f^2+16d^2efg+8d^2g^2)x}{e^2}\right) - \frac{d(2e^2f^2+7d^2efg+4d^2g^2)x^2}{e} - \frac{(ef+dg)(ef+7d^2g)x^3}{3} - \frac{e^2g^2x^5}{5} - \frac{e^2g^2x^5}{5} - \frac{(8d^3(ef+dg)^2 \text{Log}[d-ex])}{e^3}$

**Rubi [A]** time = 0.322101, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$\frac{8d^3(dg+ef)^2 \log(d-ex)}{e^3} - \frac{dx^2(4d^2g^2+7defg+2e^2f^2)}{e} - \frac{d^2x(8d^2g^2+16defg+7e^2f^2)}{e^2} - \frac{1}{2}egx^4(2dg+ef) - \frac{1}{3}x^3(dg+ef)(7dg+ef) - \frac{1}{5}e^2g^2x^5$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^4\*(f + g\*x)^2)/(d^2 - e^2\*x^2), x]

[Out]  $-\left(\frac{d^2(7e^2f^2+16d^2efg+8d^2g^2)x}{e^2}\right) - \frac{d(2e^2f^2+7d^2efg+4d^2g^2)x^2}{e} - \frac{(ef+dg)(ef+7d^2g)x^3}{3} - \frac{e^2g^2x^5}{5} - \frac{e^2g^2x^5}{5} - \frac{(8d^3(ef+dg)^2 \text{Log}[d-ex])}{e^3}$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{8d^3(dg+ef)^2 \log(d-ex)}{e^3} - \frac{7d^2x\left(\frac{8dg(dg+2ef)}{7} + e^2f^2\right)}{e^2} - \frac{2d(4d^2g^2+7defg+2e^2f^2) \int x dx}{e} - \frac{e^2g^2x^5}{5} - \frac{egx^4(2dg+ef)}{2} - \frac{x^3(dg+ef)(7dg+ef)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*4\*(g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2), x)

[Out]  $-8d^3(dg+ef)^2 \log(d-ex)/e^3 - 7d^2x(8d^2g^2+16d^2efg+7e^2f^2)/e^2 - 2d(4d^2g^2+7defg+2e^2f^2) \int x dx/e - e^2g^2x^5/5 - egx^4(2dg+ef)/2 - x^3(dg+ef)(7dg+ef)/3$

**Mathematica [A]** time = 0.130076, size = 134, normalized size = 0.95

$$\frac{8d^3(dg+ef)^2 \log(d-ex)}{e^3} - \frac{x(240d^4g^2+120d^3eg(4f+gx)+70d^2e^2(3f^2+3fgx+g^2x^2)+10de^3x(6f^2+8fgx+3g^2x^2)+e^4x^2(10f^2+15fgx+6g^2x^2))}{30e^2}$$

Antiderivative was successfully verified.



[In] Integrate[((d + e\*x)^4\*(f + g\*x)^2)/(d^2 - e^2\*x^2), x]

[Out]  $-(x*(240*d^4*g^2 + 120*d^3*e*g*(4*f + g*x) + 70*d^2*e^2*(3*f^2 + 3*f*g*x + g^2*x^2) + 10*d*e^3*x*(6*f^2 + 8*f*g*x + 3*g^2*x^2) + e^4*x^2*(10*f^2 + 15*f*g*x + 6*g^2*x^2)))/(30*e^2) - (8*d^3*(e*f + d*g)^2*\text{Log}[d - e*x])/e^3$

**Maple [A]** time = 0.007, size = 186, normalized size = 1.3

$$-\frac{e^2 g^2 x^5}{5} - ex^4 dg^2 - \frac{e^2 x^4 fg}{2} - \frac{7 x^3 d^2 g^2}{3} - \frac{8 ex^3 dfg}{3} - \frac{e^2 x^3 f^2}{3} - 4 \frac{x^2 d^3 g^2}{e} - 7 x^2 d^2 fg - 2 ex^2 df^2 - 8 \frac{d^4 g^2 x}{e^2} - 16 \frac{d^3 fgx}{e} - 7 d^2 f^2 x - 8 \frac{d^5 \ln(ex-d) g^2}{e^3} - 16 \frac{d^4 \ln(ex-d) fg}{e^2} - 8 \frac{d^3 \ln(ex-d) f^2}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^4\*(g\*x+f)^2/(-e^2\*x^2+d^2), x)

[Out]  $-1/5*e^2*g^2*x^5 - e*x^4*d*g^2 - 1/2*e^2*x^4*f*g - 7/3*x^3*d^2*g^2 - 8/3*e*x^3*d*f*g - 1/3*e^2*x^3*f^2 - 4/e*x^2*d^3*g^2 - 7*x^2*d^2*f*g - 2*e*x^2*d*f^2 - 8/e^2*d^4*g^2*x - 16/e*d^3*f*g*x - 7*d^2*f^2*x - 8*d^5/e^3*\ln(e*x-d)*g^2 - 16*d^4/e^2*\ln(e*x-d)*f*g - 8*d^3/e*\ln(e*x-d)*f^2$

**Maxima [A]** time = 0.714868, size = 236, normalized size = 1.67

$$\frac{6 e^4 g^2 x^5 + 15 (e^4 fg + 2 d e^3 g^2) x^4 + 10 (e^4 f^2 + 8 d e^3 fg + 7 d^2 e^2 g^2) x^3 + 30 (2 d e^3 f^2 + 7 d^2 e^2 fg + 4 d^3 e g^2) x^2 + 30 (7 d^2 e^3 f^2 + 8 d^3 e^2 f^2 + 2 d^4 e fg + d^5 g^2) \log(ex-d)}{30 e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(e\*x + d)^4\*(g\*x + f)^2/(e^2\*x^2 - d^2), x, algorithm="maxima")

[Out]  $-1/30*(6*e^4*g^2*x^5 + 15*(e^4*f*g + 2*d*e^3*g^2)*x^4 + 10*(e^4*f^2 + 8*d*e^3*f*g + 7*d^2*e^2*g^2)*x^3 + 30*(2*d*e^3*f^2 + 7*d^2*e^2*fg + 4*d^3*e*g^2)*x^2 + 30*(7*d^2*e^3*f^2 + 16*d^3*e^2*f*g + 8*d^4*g^2)*x)/e^2 - 8*(d^3*e^2*f^2 + 2*d^4*e*fg + d^5*g^2)*\log(e*x - d)/e^3$

**Fricas [A]** time = 0.280468, size = 238, normalized size = 1.69

$$\frac{6 e^5 g^2 x^5 + 15 (e^5 fg + 2 d e^4 g^2) x^4 + 10 (e^5 f^2 + 8 d e^4 fg + 7 d^2 e^3 g^2) x^3 + 30 (2 d e^4 f^2 + 7 d^2 e^3 fg + 4 d^3 e^2 g^2) x^2 + 30 (7 d^2 e^3 f^2 + 8 d^3 e^2 f^2 + 2 d^4 e fg + d^5 g^2) \log(ex-d)}{30 e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(e\*x + d)^4\*(g\*x + f)^2/(e^2\*x^2 - d^2), x, algorithm="fricas")

[Out]  $-1/30*(6*e^5*g^2*x^5 + 15*(e^5*f*g + 2*d*e^4*g^2)*x^4 + 10*(e^5*f^2 + 8*d*e^4*f*g + 7*d^2*e^3*g^2)*x^3 + 30*(2*d*e^4*f^2 + 7*d^2*e^3*fg + 4*d^3*e^2*g^2)*x^2 + 30*(7*d^2*e^3*f^2 + 16*d^3*e^2*f*g + 8*d^4*e*g^2)*x + 240*(d^3*e^2*f^2 + 2*d^4*e*fg + d^5*g^2)*\log(e*x - d)/e^3$

**Sympy [A]** time = 2.60131, size = 156, normalized size = 1.11

$$\frac{\frac{8d^3 (dg + ef)^2 \log(-d + ex)}{e^3} - \frac{e^2 g^2 x^5}{5} - x^4 \left( deg^2 + \frac{e^2 fg}{2} \right) - x^3 \left( \frac{7d^2 g^2}{3} + \frac{8defg}{3} + \frac{e^2 f^2}{3} \right)}{\frac{x^2 (4d^3 g^2 + 7d^2 efg + 2de^2 f^2)}{e} - \frac{x (8d^4 g^2 + 16d^3 efg + 7d^2 e^2 f^2)}{e^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*4\*(g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2), x)

[Out]  $-8*d^{**3}*(d*g + e*f)^{**2}*\log(-d + e*x)/e^{**3} - e^{**2}*g^{**2}*x^{**5}/5 - x^{**4}*(d*e*g^{**2} + e^{**2}*f*g/2) - x^{**3}*(7*d^{**2}*g^{**2}/3 + 8*d*e*f*g/3 + e^{**2}*f^{**2}/3) - x^{**2}*(4*d^{**3}*g^{**2} + 7*d^{**2}*e*f*g + 2*d*e^{**2}*f^{**2})/e - x*(8*d^{**4}*g^{**2} + 16*d^{**3}*e*f*g + 7*d^{**2}*e^{**2}*f^{**2})/e^{**2}$

**GIAC/XCAS [A]** time = 0.275882, size = 336, normalized size = 2.38

$$\frac{-4(d^5 g^2 e^3 + 2d^4 f g e^4 + d^3 f^2 e^5) e^{(-6)} \ln(|x^2 e^2 - d^2|) - \frac{1}{30} (6g^2 x^5 e^{12} + 30dg^2 x^4 e^{11} + 70d^2 g^2 x^3 e^{10} + 120d^3 g^2 x^2 e^9 + 240d^4 g^2 x e^8 + 15fgx^4 e^{12} + 80dfgx^3 e^{11} + 210d^2 fgx^2 e^{10} + 4(d^6 g^2 e^4 + 2d^5 f g e^5 + d^4 f^2 e^6) e^{(-7)} \ln\left(\frac{|2xe^{2-2}|d|e|}{|2xe^{2+2}|d|e|}\right)}{|d|}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(e\*x + d)^4\*(g\*x + f)^2/(e^2\*x^2 - d^2), x, algorithm="giac")

[Out]  $-4*(d^5*g^2*e^3 + 2*d^4*f*g*e^4 + d^3*f^2*e^5)*e^{(-6)}*\ln(\text{abs}(x^2*e^2 - d^2)) - 1/30*(6*g^2*x^5*e^{12} + 30*d*g^2*x^4*e^{11} + 70*d^2*g^2*x^3*e^{10} + 120*d^3*g^2*x^2*e^9 + 240*d^4*g^2*x*e^8 + 15*f*g*x^4*e^{12} + 80*d*f*g*x^3*e^{11} + 210*d^2*f*g*x^2*e^{10} + 480*d^3*f*g*x^2*e^9 + 10*f^2*x^3*e^{12} + 60*d*f^2*x^2*e^{11} + 210*d^2*f^2*x*e^{10})*e^{(-10)} - 4*(d^6*g^2*e^4 + 2*d^5*f*g*e^5 + d^4*f^2*e^6)*e^{(-7)}*\ln(\text{abs}(2*x*e^2 - 2*\text{abs}(d)*e)/\text{abs}(2*x*e^2 + 2*\text{abs}(d)*e))/\text{abs}(d)$

$$3.549 \quad \int \frac{(d+ex)^3(f+gx)^2}{d^2-e^2x^2} dx$$

**Optimal.** Leaf size=109

$$\frac{4d^2(dg+ef)^2 \log(d-ex)}{e^3} - \frac{x^2(4d^2g^2+6defg+e^2f^2)}{2e} - \frac{dx(2dg+ef)(2dg+3ef)}{e^2} - \frac{1}{3}gx^3(3dg+2ef) - \frac{1}{4}eg^2x^4$$

[Out]  $-\left(\frac{d(e^2f+2d^2g)(3e^2f+2d^2g)x}{e^2} - \frac{(e^2f^2+6d^2efg+4d^2g^2)x^2}{2e} - \frac{g(2e^2f+3d^2g)x^3}{3} - \frac{(e^2g^2x^4+4d^2(e^2f+d^2g)^2 \text{Log}[d-ex])}{e^3}\right)$

**Rubi [A]** time = 0.242567, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$\frac{4d^2(dg+ef)^2 \log(d-ex)}{e^3} - \frac{x^2(4d^2g^2+6defg+e^2f^2)}{2e} - \frac{dx(2dg+ef)(2dg+3ef)}{e^2} - \frac{1}{3}gx^3(3dg+2ef) - \frac{1}{4}eg^2x^4$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(f + g\*x)^2)/(d^2 - e^2\*x^2), x]

[Out]  $-\left(\frac{d(e^2f+2d^2g)(3e^2f+2d^2g)x}{e^2} - \frac{(e^2f^2+6d^2efg+4d^2g^2)x^2}{2e} - \frac{g(2e^2f+3d^2g)x^3}{3} - \frac{(e^2g^2x^4+4d^2(e^2f+d^2g)^2 \text{Log}[d-ex])}{e^3}\right)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{4d^2(dg+ef)^2 \log(d-ex)}{e^3} - \frac{eg^2x^4}{4} - \frac{gx^3(3dg+2ef)}{3} - \frac{(4d^2g^2+6defg+e^2f^2) \int x dx}{e} - \frac{(2dg+ef)(2dg+3ef) \int d dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3\*(g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2), x)

[Out]  $-4d^2(dg+ef)^2 \log(d-ex)/e^3 - e^2g^2x^4/4 - g^2x^3(3d^2g+2e^2f)/3 - (4d^2g^2+6d^2efg+e^2f^2) \text{Integral}(x, x)/e - (2d^2g+e^2f)(2d^2g+3e^2f) \text{Integral}(d, x)/e^2$

**Mathematica [A]** time = 0.0952753, size = 103, normalized size = 0.94

$$\frac{48d^2(dg+ef)^2 \log(d-ex) + ex(48d^3g^2 + 24d^2eg(4f+gx) + 12de^2(3f^2 + 3fgx + g^2x^2) + e^3x(6f^2 + 8fgx + 3g^2x^2))}{12e^3}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(f + g\*x)^2)/(d^2 - e^2\*x^2), x]

[Out]  $-\left(\frac{e^2x(48d^3g^2 + 24d^2e^2g(4f+gx) + 12d^2e^2(3f^2 + 3fgx + g^2x^2) + 6f^2 + 8fgx + 3g^2x^2)}{12e^3} + 48d^2g^2 \log(d-ex) + \frac{eg^2x^4}{4} + \frac{gx^3(3dg+2ef)}{3} + \frac{(4d^2g^2+6defg+e^2f^2) \int x dx}{e} + \frac{(2dg+ef)(2dg+3ef) \int d dx}{e^2}\right)$

$$(e^*f + d^*g)^2 * \text{Log}[d - e^*x] / (12^*e^3)$$

**Maple [A]** time = 0.006, size = 145, normalized size = 1.3

$$\begin{aligned} & -\frac{eg^2x^4}{4} - x^3dg^2 - \frac{2ex^3fg}{3} - 2\frac{x^2d^2g^2}{e} - 3x^2dfg - \frac{ex^2f^2}{2} - 4\frac{d^3g^2x}{e^2} - 8\frac{d^2fgx}{e} \\ & - 3df^2x - 4\frac{d^4\ln(ex-d)g^2}{e^3} - 8\frac{d^3\ln(ex-d)fg}{e^2} - 4\frac{d^2\ln(ex-d)f^2}{e} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(g\*x+f)^2/(-e^2\*x^2+d^2), x)

[Out] -1/4\*e\*g^2\*x^4-x^3\*d\*g^2-2/3\*e\*x^3\*f\*g-2/e\*x^2\*d^2\*g^2-3\*x^2\*d\*f\*g-1/2\*e\*x^2\*f^2-4/e^2\*d^3\*g^2\*x-8/e\*d^2\*f\*g\*x-3\*d\*f^2\*x-4\*d^4/e^3\*ln(e\*x-d)\*g^2-8\*d^3/e^2\*ln(e\*x-d)\*f\*g-4\*d^2/e\*ln(e\*x-d)\*f^2

**Maxima [A]** time = 0.708116, size = 186, normalized size = 1.71

$$\begin{aligned} & \frac{3e^3g^2x^4 + 4(2e^3fg + 3de^2g^2)x^3 + 6(e^3f^2 + 6de^2fg + 4d^2eg^2)x^2 + 12(3de^2f^2 + 8d^2efg + 4d^3g^2)x}{12e^2} \\ & - \frac{4(d^2e^2f^2 + 2d^3efg + d^4g^2)\log(ex-d)}{e^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(e\*x + d)^3\*(g\*x + f)^2/(e^2\*x^2 - d^2), x, algorithm="maxima")

[Out] -1/12\*(3\*e^3\*g^2\*x^4 + 4\*(2\*e^3\*f\*g + 3\*d\*e^2\*g^2)\*x^3 + 6\*(e^3\*f^2 + 6\*d\*e^2\*f\*g + 4\*d^2\*e\*g^2)\*x^2 + 12\*(3\*d\*e^2\*f^2 + 8\*d^2\*e\*f\*g + 4\*d^3\*g^2)\*x)/e^2 - 4\*(d^2\*e^2\*f^2 + 2\*d^3\*e\*f\*g + d^4\*g^2)\*log(e\*x - d)/e^3

**Fricas [A]** time = 0.281944, size = 188, normalized size = 1.72

$$\frac{3e^4g^2x^4 + 4(2e^4fg + 3de^3g^2)x^3 + 6(e^4f^2 + 6de^3fg + 4d^2e^2g^2)x^2 + 12(3de^3f^2 + 8d^2e^2fg + 4d^3eg^2)x + 48(d^2e^2f^2 + 4d^3fg + d^4g^2)}{12e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(e\*x + d)^3\*(g\*x + f)^2/(e^2\*x^2 - d^2), x, algorithm="fricas")

[Out] -1/12\*(3\*e^4\*g^2\*x^4 + 4\*(2\*e^4\*f\*g + 3\*d\*e^3\*g^2)\*x^3 + 6\*(e^4\*f^2 + 6\*d\*e^3\*f\*g + 4\*d^2\*e^2\*g^2)\*x^2 + 12\*(3\*d\*e^3\*f^2 + 8\*d^2\*e^2\*f\*g + 4\*d^3\*e\*g^2)\*x + 48\*(d^2\*e^2\*f^2 + 2\*d^3\*e\*f\*g + d^4\*g^2)\*log(e\*x - d))/e^3

**Sympy [A]** time = 2.2243, size = 116, normalized size = 1.06

$$\begin{aligned} & -\frac{4d^2(dg + ef)^2 \log(-d + ex)}{e^3} - \frac{eg^2x^4}{4} - x^3\left(dg^2 + \frac{2efg}{3}\right) \\ & - \frac{x^2(4d^2g^2 + 6defg + e^2f^2)}{2e} - \frac{x(4d^3g^2 + 8d^2efg + 3de^2f^2)}{e^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2),x)

[Out]  $-4*d**2*(d*g + e*f)**2*\log(-d + e*x)/e**3 - e*g**2*x**4/4 - x**3*(d*g**2 + 2*e*f*g/3) - x**2*(4*d**2*g**2 + 6*d*e*f*g + e**2*f**2)/(2*e) - x*(4*d**3*g**2 + 8*d**2*e*f*g + 3*d*e**2*f**2)/e**2$

**GIAC/XCAS [A]** time = 0.273402, size = 285, normalized size = 2.61

$$-2(d^4g^2e^3 + 2d^3fge^4 + d^2f^2e^5)e^{(-6)}\ln(|x^2e^2 - d^2|) - \frac{1}{12}(3g^2x^4e^9 + 12dg^2x^3e^8 + 24d^2g^2x^2e^7 + 48d^3g^2xe^6 + 8fgx^3e^9 + 36dfgx^2e^8 + 96d^2fgxe^7 + 6f^2x^2e^9 + 36df^2xe^8)e^{(-8)} - \frac{2(d^5g^2e^2 + 2d^4fge^3 + d^3f^2e^4)e^{(-5)}\ln\left(\frac{|2xe^2-2|d|e|}{|2xe^2+2|d|e|}\right)}{|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(e\*x + d)^3\*(g\*x + f)^2/(e^2\*x^2 - d^2),x, algorithm="giac")

[Out]  $-2*(d^4*g^2*e^3 + 2*d^3*f*g*e^4 + d^2*f^2*e^5)*e^{(-6)}*\ln(\text{abs}(x^2*e^2 - d^2)) - 1/12*(3*g^2*x^4*e^9 + 12*d*g^2*x^3*e^8 + 24*d^2*g^2*x^2*e^7 + 48*d^3*g^2*x*e^6 + 8*f*g*x^3*e^9 + 36*d*f*g*x^2*e^8 + 96*d^2*f*g*x*e^7 + 6*f^2*x^2*e^9 + 36*d*f^2*x*e^8)*e^{(-8)} - 2*(d^5*g^2*e^2 + 2*d^4*f*g*e^3 + d^3*f^2*e^4)*e^{(-5)}*\ln(\text{abs}(2*x*e^2 - 2*\text{abs}(d)*e)/\text{abs}(2*x*e^2 + 2*\text{abs}(d)*e))/\text{abs}(d)$

$$3.550 \quad \int \frac{(d+ex)^2(f+gx)^2}{d^2-e^2x^2} dx$$

**Optimal.** Leaf size=65

$$-\frac{2d(dg+ef)^2 \log(d-ex)}{e^3} - \frac{2dgx(dg+ef)}{e^2} - \frac{d(f+gx)^2}{e} - \frac{(f+gx)^3}{3g}$$

[Out]  $(-2*d*g*(e*f + d*g)*x)/e^2 - (d*(f + g*x)^2)/e - (f + g*x)^3/(3*g) - (2*d*(e*f + d*g)^2*\text{Log}[d - e*x])/e^3$

**Rubi [A]** time = 0.110778, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$-\frac{2d(dg+ef)^2 \log(d-ex)}{e^3} - \frac{2dgx(dg+ef)}{e^2} - \frac{d(f+gx)^2}{e} - \frac{(f+gx)^3}{3g}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^2\*(f + g\*x)^2)/(d^2 - e^2\*x^2), x]

[Out]  $(-2*d*g*(e*f + d*g)*x)/e^2 - (d*(f + g*x)^2)/e - (f + g*x)^3/(3*g) - (2*d*(e*f + d*g)^2*\text{Log}[d - e*x])/e^3$

**Rubi in Sympy [A]** time = 18.7836, size = 60, normalized size = 0.92

$$-\frac{d(f+gx)^2}{e} - \frac{2dgx(dg+ef)}{e^2} - \frac{2d(dg+ef)^2 \log(d-ex)}{e^3} - \frac{(f+gx)^3}{3g}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*2\*(g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2), x)

[Out]  $-d*(f + g*x)**2/e - 2*d*g*x*(d*g + e*f)/e**2 - 2*d*(d*g + e*f)**2*\log(d - e*x)/e**3 - (f + g*x)**3/(3*g)$

**Mathematica [A]** time = 0.0662406, size = 73, normalized size = 1.12

$$-\frac{ex(6d^2g^2 + 3deg(4f + gx) + e^2(3f^2 + 3fgx + g^2x^2)) + 6d(dg + ef)^2 \log(d - ex)}{3e^3}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^2\*(f + g\*x)^2)/(d^2 - e^2\*x^2), x]

[Out]  $-(e*x*(6*d^2*g^2 + 3*d*e*g*(4*f + g*x) + e^2*(3*f^2 + 3*f*g*x + g^2*x^2)) + 6*d*(e*f + d*g)^2*\text{Log}[d - e*x])/(3*e^3)$

**Maple [A]** time = 0.005, size = 110, normalized size = 1.7

$$-\frac{g^2x^3}{3} - \frac{dx^2g^2}{e} - x^2fg - 2\frac{d^2g^2x}{e^2} - 4\frac{dfgx}{e} - f^2x - 2\frac{d^3 \ln(ex-d)g^2}{e^3} - 4\frac{d^2 \ln(ex-d)fg}{e^2} - 2\frac{d \ln(ex-d)f^2}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2),x)`

[Out]  $-1/3*g^2*x^3-1/e*x^2*d*g^2-x^2*f*g-2/e^2*d^2*g^2*x-4/e*d*f*g*x-f^2*x-2*d^3/e^3*\ln(e*x-d)*g^2-4*d^2/e^2*\ln(e*x-d)*f*g-2*d/e*\ln(e*x-d)*f^2$

**Maxima [A]** time = 0.69207, size = 131, normalized size = 2.02

$$\frac{e^2 g^2 x^3 + 3(e^2 f g + d e g^2) x^2 + 3(e^2 f^2 + 4 d e f g + 2 d^2 g^2) x}{3 e^2} - \frac{2(d e^2 f^2 + 2 d^2 e f g + d^3 g^2) \log(e x - d)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(e*x+d)^2*(g*x+f)^2/(e^2*x^2-d^2),x,algorithm="maxima")`

[Out]  $-1/3*(e^2*g^2*x^3 + 3*(e^2*f*g + d*e*g^2)*x^2 + 3*(e^2*f^2 + 4*d*e*f*g + 2*d^2*g^2)*x)/e^2 - 2*(d*e^2*f^2 + 2*d^2*e*f*g + d^3*g^2)*\log(e*x - d)/e^3$

**Fricas [A]** time = 0.277888, size = 132, normalized size = 2.03

$$\frac{e^3 g^2 x^3 + 3(e^3 f g + d e^2 g^2) x^2 + 3(e^3 f^2 + 4 d e^2 f g + 2 d^2 e g^2) x + 6(d e^2 f^2 + 2 d^2 e f g + d^3 g^2) \log(e x - d)}{3 e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(e*x+d)^2*(g*x+f)^2/(e^2*x^2-d^2),x,algorithm="fricas")`

[Out]  $-1/3*(e^3*g^2*x^3 + 3*(e^3*f*g + d*e^2*g^2)*x^2 + 3*(e^3*f^2 + 4*d*e^2*f*g + 2*d^2*e*g^2)*x + 6*(d*e^2*f^2 + 2*d^2*e*f*g + d^3*g^2)*\log(e*x - d))/e^3$

**Sympy [A]** time = 1.91627, size = 75, normalized size = 1.15

$$\frac{2d(dg+ef)^2 \log(-d+ex)}{e^3} - \frac{g^2 x^3}{3} - \frac{x^2(dg^2+efg)}{e} - \frac{x(2d^2g^2+4defg+e^2f^2)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**2*(g*x+f)**2/(-e**2*x**2+d**2),x)`

[Out]  $-2*d*(d*g + e*f)**2*\log(-d + e*x)/e**3 - g**2*x**3/3 - x**2*(d*g**2 + e*f*g)/e - x*(2*d**2*g**2 + 4*d*e*f*g + e**2*f**2)/e**2$

**GIAC/XCAS [A]** time = 0.271417, size = 232, normalized size = 3.57

$$\frac{-(d^3 g^2 e + 2 d^2 f g e^2 + d f^2 e^3) e^{(-4)} \ln(|x^2 e^2 - d^2|) - \frac{1}{3} (g^2 x^3 e^6 + 3 d g^2 x^2 e^5 + 6 d^2 g^2 x e^4 + 3 f g x^2 e^6 + 12 d f g x e^5 + 3 f^2 x e^6) e^{(-6)} + (d^4 g^2 e^2 + 2 d^3 f g e^3 + d^2 f^2 e^4) e^{(-5)} \ln\left(\frac{|2 x e^2 - 2 |d| e|}{|2 x e^2 + 2 |d| e|}\right)}{|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(e*x + d)^2*(g*x + f)^2/(e^2*x^2 - d^2),x, algorithm="giac")`

[Out]  $-(d^3 g^2 e + 2 d^2 f g e^2 + d f^2 e^3) e^{-4} \ln(\operatorname{abs}(x^2 e^2 - d^2)) - \frac{1}{3} (g^2 x^3 e^6 + 3 d g^2 x^2 e^5 + 6 d^2 g^2 x e^4 + 3 f g^2 x^2 e^6 + 12 d f g x e^5 + 3 f^2 x e^6) e^{-6} - (d^4 g^2 e^2 + 2 d^3 f g e^3 + d^2 f^2 e^4) e^{-5} \ln(\operatorname{abs}(2 x e^2 - 2 \operatorname{abs}(d) e) / \operatorname{abs}(2 x e^2 + 2 \operatorname{abs}(d) e)) / \operatorname{abs}(d)$



$$3.551 \quad \int \frac{(d+ex)(f+gx)^2}{d^2-e^2x^2} dx$$

**Optimal.** Leaf size=50

$$-\frac{(dg+ef)^2 \log(d-ex)}{e^3} - \frac{gx(dg+ef)}{e^2} - \frac{(f+gx)^2}{2e}$$

[Out]  $-\left(\frac{g(e^2f+dx)}{e^2} - \frac{(f+gx)^2}{2e} - \frac{(e^2f+dx)^2 \operatorname{Log}[d-ex]}{e^3}\right)$

**Rubi [A]** time = 0.0676834, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$

$$-\frac{(dg+ef)^2 \log(d-ex)}{e^3} - \frac{gx(dg+ef)}{e^2} - \frac{(f+gx)^2}{2e}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\frac{(d+ex)(f+gx)^2}{d^2-e^2x^2}, x\right]$

[Out]  $-\left(\frac{g(e^2f+dx)}{e^2} - \frac{(f+gx)^2}{2e} - \frac{(e^2f+dx)^2 \operatorname{Log}[d-ex]}{e^3}\right)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(f+gx)^2}{2e} - \frac{(dg+ef) \int g dx}{e^2} - \frac{(dg+ef)^2 \log(d-ex)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{rubi\_integrate}\left(\frac{(ex+d)(gx+f)^2}{-e^2x^2+d^2}, x\right)$

[Out]  $-\frac{(f+gx)^2}{2e} - \frac{(dg+ef) \operatorname{Integral}(g, x)}{e^2} - \frac{(dg+ef)^2 \log(d-ex)}{e^3}$

**Mathematica [A]** time = 0.032685, size = 43, normalized size = 0.86

$$-\frac{egx(2dg+4ef+egx) + 2(dg+ef)^2 \log(d-ex)}{2e^3}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Integrate}\left[\frac{(d+ex)(f+gx)^2}{d^2-e^2x^2}, x\right]$

[Out]  $-\frac{e^2gx^2(4e^2f+2dg+egx) + 2(e^2f+dx)^2 \operatorname{Log}[d-ex]}{2e^3}$

**Maple [A]** time = 0.005, size = 82, normalized size = 1.6

$$-\frac{g^2x^2}{2e} - \frac{g^2dx}{e^2} - 2\frac{gfx}{e} - \frac{\ln(ex-d)d^2g^2}{e^3} - 2\frac{\ln(ex-d)dfg}{e^2} - \frac{\ln(ex-d)f^2}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2),x)`

[Out]  $-1/2*g^2*x^2/e-g^2/e^2*d*x-2*g/e*f*x-1/e^3*\ln(e*x-d)*d^2*g^2-2/e^2*\ln(e*x-d)*d*f*g-1/e*\ln(e*x-d)*f^2$

**Maxima [A]** time = 0.691315, size = 85, normalized size = 1.7

$$-\frac{eg^2x^2 + 2(2efg + dg^2)x}{2e^2} - \frac{(e^2f^2 + 2defg + d^2g^2)\log(ex - d)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(e*x + d)*(g*x + f)^2/(e^2*x^2 - d^2),x, algorithm="maxima")`

[Out]  $-1/2*(e*g^2*x^2 + 2*(2*e*f*g + d*g^2)*x)/e^2 - (e^2*f^2 + 2*d*e*f*g + d^2*g^2)*\log(e*x - d)/e^3$

**Fricas [A]** time = 0.273369, size = 86, normalized size = 1.72

$$-\frac{e^2g^2x^2 + 2(2e^2fg + deg^2)x + 2(e^2f^2 + 2defg + d^2g^2)\log(ex - d)}{2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(e*x + d)*(g*x + f)^2/(e^2*x^2 - d^2),x, algorithm="fricas")`

[Out]  $-1/2*(e^2*g^2*x^2 + 2*(2*e^2*f*g + d*e*g^2)*x + 2*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*\log(e*x - d))/e^3$

**Sympy [A]** time = 1.5712, size = 46, normalized size = 0.92

$$-\frac{g^2x^2}{2e} - \frac{x(dg^2 + 2efg)}{e^2} - \frac{(dg + ef)^2\log(-d + ex)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(g*x+f)**2/(-e**2*x**2+d**2),x)`

[Out]  $-g**2*x**2/(2*e) - x*(d*g**2 + 2*e*f*g)/e**2 - (d*g + e*f)**2*\log(-d + e*x)/e**3$

**GIAC/XCAS [A]** time = 0.275754, size = 181, normalized size = 3.62

$$-\frac{1}{2}(d^2g^2e + 2dfge^2 + f^2e^3)e^{(-4)}\ln(|x^2e^2 - d^2|) - \frac{1}{2}(g^2x^2e^3 + 2dg^2xe^2 + 4fgxe^3)e^{(-4)} - \frac{(d^3g^2 + 2d^2fge + df^2e^2)e^{(-3)}\ln\left(\frac{|2xe^2-2|d|e|}{|2xe^2+2|d|e|}\right)}{2|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(e*x + d)*(g*x + f)^2/(e^2*x^2 - d^2),x, algorithm="giac")`

```
[Out] -1/2*(d^2*g^2*e + 2*d*f*g*e^2 + f^2*e^3)*e^(-4)*ln(abs(x^2*e^2 -
d^2)) - 1/2*(g^2*x^2*e^3 + 2*d*g^2*x*e^2 + 4*f*g*x*e^3)*e^(-4) -
1/2*(d^3*g^2 + 2*d^2*f*g*e + d*f^2*e^2)*e^(-3)*ln(abs(2*x*e^2 - 2
*abs(d)*e)/abs(2*x*e^2 + 2*abs(d)*e))/abs(d)
```

$$3.552 \quad \int \frac{(f+gx)^2}{d^2-e^2x^2} dx$$

**Optimal.** Leaf size=62

$$-\frac{(dg+ef)^2 \log(d-ex)}{2de^3} + \frac{(ef-dg)^2 \log(d+ex)}{2de^3} - \frac{g^2x}{e^2}$$

[Out]  $-\frac{(g^2x)/e^2}{e^2} - \frac{((ef+d^2g)^2 \text{Log}[d-ex])/(2d^2e^3)}{e^2} + \frac{((ef-d^2g)^2 \text{Log}[d+ex])/(2d^2e^3)}{e^2}$

**Rubi [A]** time = 0.158385, antiderivative size = 62, normalized size of antiderivative = 1., number of rules used = 5, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$-\frac{(dg+ef)^2 \log(d-ex)}{2de^3} + \frac{(ef-dg)^2 \log(d+ex)}{2de^3} - \frac{g^2x}{e^2}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)^2/(d^2 - e^2\*x^2), x]

[Out]  $-\frac{(g^2x)/e^2}{e^2} - \frac{((ef+d^2g)^2 \text{Log}[d-ex])/(2d^2e^3)}{e^2} + \frac{((ef-d^2g)^2 \text{Log}[d+ex])/(2d^2e^3)}{e^2}$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-g^2 \int \frac{1}{e^2} dx + \frac{(dg-ef)^2 \log(d+ex)}{2de^3} - \frac{(dg+ef)^2 \log(d-ex)}{2de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2), x)

[Out]  $-g^2 \text{Integral}(e^{(-2)}, x) + \frac{(d^2g - e^2f)^2 \log(d+ex)}{(2d^2e^3)} - \frac{(d^2g + e^2f)^2 \log(d-ex)}{(2d^2e^3)}$

**Mathematica [A]** time = 0.0381196, size = 55, normalized size = 0.89

$$\frac{(d^2g^2 + e^2f^2) \tanh^{-1}\left(\frac{ex}{d}\right) - deg(f \log(d^2 - e^2x^2) + gx)}{de^3}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^2/(d^2 - e^2\*x^2), x]

[Out]  $\frac{((e^2f^2 + d^2g^2) \text{ArcTanh}[ex/d] - d^2e^2g^2(gx + f \text{Log}[d^2 - e^2x^2]))}{(d^2e^3)}$

**Maple [A]** time = 0.01, size = 107, normalized size = 1.7

$$\frac{g^2x}{e^2} - \frac{d \ln(ex-d)g^2}{2e^3} - \frac{\ln(ex-d)fg}{e^2} - \frac{\ln(ex-d)f^2}{2de} + \frac{d \ln(ex+d)g^2}{2e^3} - \frac{\ln(ex+d)fg}{e^2} + \frac{\ln(ex+d)f^2}{2de}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^2/(-e^2*x^2+d^2),x)`

[Out]  $-\frac{g^2x}{e^2} + \frac{(e^2f^2 - 2defg + d^2g^2) \log(ex + d)}{2de^3} - \frac{(e^2f^2 + 2defg + d^2g^2) \log(ex - d)}{2de^3}$

**Maxima [A]** time = 0.689586, size = 111, normalized size = 1.79

$$-\frac{g^2x}{e^2} + \frac{(e^2f^2 - 2defg + d^2g^2) \log(ex + d)}{2de^3} - \frac{(e^2f^2 + 2defg + d^2g^2) \log(ex - d)}{2de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(g*x + f)^2/(e^2*x^2 - d^2),x, algorithm="maxima")`

[Out]  $-\frac{g^2x}{e^2} + \frac{1}{2} \frac{(e^2f^2 - 2d^2efg + d^2g^2) \log(ex + d)}{e^3} - \frac{1}{2} \frac{(e^2f^2 + 2d^2efg + d^2g^2) \log(ex - d)}{e^3}$

**Fricas [A]** time = 0.277806, size = 103, normalized size = 1.66

$$-\frac{2deg^2x - (e^2f^2 - 2defg + d^2g^2) \log(ex + d) + (e^2f^2 + 2defg + d^2g^2) \log(ex - d)}{2de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(g*x + f)^2/(e^2*x^2 - d^2),x, algorithm="fricas")`

[Out]  $-\frac{1}{2} \frac{(2d^2efg - (e^2f^2 - 2d^2efg + d^2g^2) \log(ex + d) + (e^2f^2 + 2d^2efg + d^2g^2) \log(ex - d))}{e^3}$

**Sympy [A]** time = 2.68079, size = 112, normalized size = 1.81

$$-\frac{g^2x}{e^2} + \frac{(dg - ef)^2 \log\left(x + \frac{2d^2fg + \frac{d(dg-ef)^2}{e}}{d^2g^2 + e^2f^2}\right)}{2de^3} - \frac{(dg + ef)^2 \log\left(x + \frac{2d^2fg - \frac{d(dg+ef)^2}{e}}{d^2g^2 + e^2f^2}\right)}{2de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)**2/(-e**2*x**2+d**2),x)`

[Out]  $-\frac{g^2x}{e^2} + \frac{(dg - ef)^2 \log(x + \frac{2d^2fg + d(dg - ef)^2}{e^2})}{2de^3} - \frac{(dg + ef)^2 \log(x + \frac{2d^2fg - d(dg + ef)^2}{e^2})}{2de^3}$

**GIAC/XCAS [A]** time = 0.27282, size = 109, normalized size = 1.76

$$-g^2xe^{(-2)} - fge^{(-2)} \ln(|x^2e^2 - d^2|) - \frac{(d^2g^2 + f^2e^2)e^{(-3)} \ln\left(\frac{|2xe^2 - 2|d|e|}{|2xe^2 + 2|d|e|}\right)}{2|d|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(g*x + f)^2/(e^2*x^2 - d^2),x, algorithm="giac")
```

```
[Out] -g^2*x*e^(-2) - f*g*e^(-2)*ln(abs(x^2*e^2 - d^2)) - 1/2*(d^2*g^2  
+ f^2*e^2)*e^(-3)*ln(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*ab  
s(d)*e))/abs(d)
```

$$3.553 \quad \int \frac{(f+gx)^2}{(d+ex)(d^2-e^2x^2)} dx$$

**Optimal.** Leaf size=86

$$\frac{(3dg+ef)(ef-dg)\log(d+ex)}{4d^2e^3} - \frac{(dg+ef)^2\log(d-ex)}{4d^2e^3} - \frac{(ef-dg)^2}{2de^3(d+ex)}$$

[Out]  $-(e^*f - d^*g)^2/(2^*d^*e^3*(d + e^*x)) - ((e^*f + d^*g)^2*\text{Log}[d - e^*x])/(4^*d^2*e^3) + ((e^*f - d^*g)*(e^*f + 3^*d^*g)*\text{Log}[d + e^*x])/(4^*d^2*e^3)$

**Rubi [A]** time = 0.203801, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$\frac{(3dg+ef)(ef-dg)\log(d+ex)}{4d^2e^3} - \frac{(dg+ef)^2\log(d-ex)}{4d^2e^3} - \frac{(ef-dg)^2}{2de^3(d+ex)}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)^2/((d + e\*x)\*(d^2 - e^2\*x^2)), x]

[Out]  $-(e^*f - d^*g)^2/(2^*d^*e^3*(d + e^*x)) - ((e^*f + d^*g)^2*\text{Log}[d - e^*x])/(4^*d^2*e^3) + ((e^*f - d^*g)*(e^*f + 3^*d^*g)*\text{Log}[d + e^*x])/(4^*d^2*e^3)$

**Rubi in Sympy [A]** time = 31.9988, size = 75, normalized size = 0.87

$$-\frac{(dg-ef)^2}{2de^3(d+ex)} - \frac{(dg-ef)(3dg+ef)\log(d+ex)}{4d^2e^3} - \frac{(dg+ef)^2\log(d-ex)}{4d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*2/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2), x)

[Out]  $-(d^*g - e^*f)**2/(2^*d^*e**3*(d + e^*x)) - (d^*g - e^*f)*(3^*d^*g + e^*f)*\log(d + e^*x)/(4^*d**2*e**3) - (d^*g + e^*f)**2*\log(d - e^*x)/(4^*d**2*e**3)$

**Mathematica [A]** time = 0.080469, size = 82, normalized size = 0.95

$$\frac{(ef-dg)((d+ex)(3dg+ef)\log(d+ex)+2d(dg-ef))-(d+ex)(dg+ef)^2\log(d-ex)}{4d^2e^3(d+ex)}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^2/((d + e\*x)\*(d^2 - e^2\*x^2)), x]

[Out]  $(-((e^*f + d^*g)^2*(d + e^*x)*\text{Log}[d - e^*x]) + (e^*f - d^*g)*(2^*d*(-(e^*f) + d^*g) + (e^*f + 3^*d^*g)*(d + e^*x)*\text{Log}[d + e^*x]))/(4^*d^2*e^3*(d + e^*x))$

**Maple [A]** time = 0.017, size = 149, normalized size = 1.7

$$\begin{aligned} & -\frac{\ln(ex-d)g^2}{4e^3} - \frac{\ln(ex-d)fg}{2e^2d} - \frac{\ln(ex-d)f^2}{4d^2e} - \frac{3\ln(ex+d)g^2}{4e^3} \\ & + \frac{\ln(ex+d)fg}{2e^2d} + \frac{\ln(ex+d)f^2}{4d^2e} - \frac{g^2d}{2e^3(ex+d)} + \frac{fg}{e^2(ex+d)} - \frac{f^2}{2de(ex+d)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2),x)`

[Out] `-1/4/e^3*ln(e*x-d)*g^2-1/2/e^2/d*ln(e*x-d)*f*g-1/4/e/d^2*ln(e*x-d)*f^2-3/4/e^3*ln(e*x+d)*g^2+1/2/e^2/d*ln(e*x+d)*f*g+1/4/e/d^2*ln(e*x+d)*f^2-1/2/e^3*d/(e*x+d)*g^2+1/e^2/(e*x+d)*f*g-1/2/e/d/(e*x+d)*f^2`

**Maxima [A]** time = 0.69311, size = 153, normalized size = 1.78

$$-\frac{e^2f^2-2defg+d^2g^2}{2(de^4x+d^2e^3)} + \frac{(e^2f^2+2defg-3d^2g^2)\log(ex+d)}{4d^2e^3} - \frac{(e^2f^2+2defg+d^2g^2)\log(ex-d)}{4d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(g*x+f)^2/((e^2*x^2-d^2)*(e*x+d)),x,algorithm="maxima")`

[Out] `-1/2*(e^2*f^2-2*d*e*f*g+d^2*g^2)/(d*e^4*x+d^2*e^3)+1/4*(e^2*f^2+2*d*e*f*g-3*d^2*g^2)*log(e*x+d)/(d^2*e^3)-1/4*(e^2*f^2+2*d*e*f*g+d^2*g^2)*log(e*x-d)/(d^2*e^3)`

**Fricas [A]** time = 0.279178, size = 223, normalized size = 2.59

$$\frac{2de^2f^2-4d^2efg+2d^3g^2-(de^2f^2+2d^2efg-3d^3g^2+(e^3f^2+2de^2fg-3d^2eg^2)x)\log(ex+d)+(de^2f^2+2d^2efg-3d^3g^2)\log(ex-d)}{4(d^2e^4x+d^3e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(g*x+f)^2/((e^2*x^2-d^2)*(e*x+d)),x,algorithm="fricas")`

[Out] `-1/4*(2*d*e^2*f^2-4*d^2*e*f*g+2*d^3*g^2-(d*e^2*f^2+2*d^2*e*f*g-3*d^3*g^2+(e^3*f^2+2*d*e^2*f*g-3*d^2*e*g^2)*x)*log(e*x+d)+(d*e^2*f^2+2*d^2*e*f*g+d^3*g^2+(e^3*f^2+2*d*e^2*f*g-3*d^2*e*g^2)*x)*log(e*x-d))/(d^2*e^4*x+d^3*e^3)`

**Sympy [A]** time = 4.10636, size = 182, normalized size = 2.12

$$\begin{aligned} & -\frac{d^2g^2-2defg+e^2f^2}{2d^2e^3+2de^4x} - \frac{(dg-ef)(3dg+ef)\log\left(x+\frac{-2d^3g^2+d(dg-ef)(3dg+ef)}{d^2eg^2-2de^2fg-e^3f^2}\right)}{4d^2e^3} \\ & - \frac{(dg+ef)^2\log\left(x+\frac{-2d^3g^2+d(dg+ef)^2}{d^2eg^2-2de^2fg-e^3f^2}\right)}{4d^2e^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)**2/(e*x+d)/(-e**2*x**2+d**2),x)`



```
[Out] -(d**2*g**2 - 2*d*e*f*g + e**2*f**2)/(2*d**2*e**3 + 2*d*e**4*x) -
(d*g - e*f)*(3*d*g + e*f)*log(x + (-2*d**3*g**2 + d*(d*g - e*f)*
(3*d*g + e*f)))/(d**2*e*g**2 - 2*d*e**2*f*g - e**3*f**2)/(4*d**2*
e**3) - (d*g + e*f)**2*log(x + (-2*d**3*g**2 + d*(d*g + e*f)**2)/
(d**2*e*g**2 - 2*d*e**2*f*g - e**3*f**2))/(4*d**2*e**3)
```

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(g*x + f)^2/((e^2*x^2 - d^2)*(e*x + d)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.554 \quad \int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)} dx$$

**Optimal.** Leaf size=87

$$\frac{(dg+ef)^2 \tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3} - \frac{(3dg+ef)(ef-dg)}{4d^2e^3(d+ex)} - \frac{(ef-dg)^2}{4de^3(d+ex)^2}$$

[Out]  $-(e^*f - d^*g)^2/(4^*d^*e^3*(d + e^*x)^2) - ((e^*f - d^*g)*(e^*f + 3^*d^*g))/(4^*d^2*e^3*(d + e^*x)) + ((e^*f + d^*g)^2*ArcTanh[(e^*x)/d])/(4^*d^3*e^3)$

**Rubi [A]** time = 0.21826, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$\frac{(dg+ef)^2 \tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3} - \frac{(3dg+ef)(ef-dg)}{4d^2e^3(d+ex)} - \frac{(ef-dg)^2}{4de^3(d+ex)^2}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)^2/((d + e\*x)^2\*(d^2 - e^2\*x^2)), x]

[Out]  $-(e^*f - d^*g)^2/(4^*d^*e^3*(d + e^*x)^2) - ((e^*f - d^*g)*(e^*f + 3^*d^*g))/(4^*d^2*e^3*(d + e^*x)) + ((e^*f + d^*g)^2*ArcTanh[(e^*x)/d])/(4^*d^3*e^3)$

**Rubi in Sympy [A]** time = 43.5138, size = 73, normalized size = 0.84

$$-\frac{(dg-ef)^2}{4de^3(d+ex)^2} + \frac{(dg-ef)(3dg+ef)}{4d^2e^3(d+ex)} + \frac{(dg+ef)^2 \operatorname{atanh}\left(\frac{ex}{d}\right)}{4d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*2/(e\*x+d)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2), x)

[Out]  $-(d^*g - e^*f)**2/(4^*d^*e^3*(d + e^*x)**2) + (d^*g - e^*f)*(3^*d^*g + e^*f)/(4^*d^2*e^3*(d + e^*x)) + (d^*g + e^*f)**2*atanh(e^*x/d)/(4^*d^3*e^3)$

**Mathematica [A]** time = 0.127547, size = 87, normalized size = 1.

$$\frac{2d(dg-ef)(2d^2g+de(2f+3gx)+e^2fx)}{(d+ex)^2} + \frac{(dg+ef)^2(-\log(d-ex)) + (dg+ef)^2 \log(d+ex)}{8d^3e^3}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^2/((d + e\*x)^2\*(d^2 - e^2\*x^2)), x]

[Out]  $((2^*d^*(-e^*f) + d^*g)*(2^*d^2*g + e^2*f*x + d^*e*(2*f + 3^*g*x)))/(d + e^*x)^2 - (e^*f + d^*g)^2*Log[d - e^*x] + (e^*f + d^*g)^2*Log[d + e^*x])/(8^*d^3*e^3)$

**Maple [B]** time = 0.015, size = 206, normalized size = 2.4

$$\begin{aligned} & -\frac{\ln(ex-d)g^2}{8de^3} - \frac{\ln(ex-d)fg}{4d^2e^2} - \frac{\ln(ex-d)f^2}{8d^3e} + \frac{3g^2}{4e^3(ex+d)} - \frac{fg}{2e^2d(ex+d)} - \frac{f^2}{4d^2e(ex+d)} \\ & - \frac{g^2d}{4e^3(ex+d)^2} + \frac{fg}{2e^2(ex+d)^2} - \frac{f^2}{4de(ex+d)^2} + \frac{\ln(ex+d)g^2}{8de^3} + \frac{\ln(ex+d)fg}{4d^2e^2} + \frac{\ln(ex+d)f^2}{8d^3e} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^2/(e*x+d)^2/(-e^2*x^2+d^2),x)`

[Out]  $-1/8/e^3/d*\ln(e*x-d)*g^2-1/4/e^2/d^2*\ln(e*x-d)*f*g-1/8/e/d^3*\ln(e*x-d)*f^2+3/4/e^3/(e*x+d)*g^2-1/2/d/e^2/(e*x+d)*f*g-1/4/d^2/e/(e*x+d)*f^2-1/4/e^3*d/(e*x+d)^2*g^2+1/2/e^2/(e*x+d)^2*f*g-1/4/e/d/(e*x+d)^2*f^2+1/8/e^3/d*\ln(e*x+d)*g^2+1/4/e^2/d^2*\ln(e*x+d)*f*g+1/8/e/d^3*\ln(e*x+d)*f^2$

**Maxima [A]** time = 0.692041, size = 201, normalized size = 2.31

$$\begin{aligned} & -\frac{2de^2f^2-2d^3g^2+(e^3f^2+2de^2fg-3d^2eg^2)x}{4(d^2e^5x^2+2d^3e^4x+d^4e^3)} \\ & + \frac{(e^2f^2+2defg+d^2g^2)\log(ex+d)}{8d^3e^3} - \frac{(e^2f^2+2defg+d^2g^2)\log(ex-d)}{8d^3e^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(g*x+f)^2/((e^2*x^2-d^2)*(e*x+d)^2),x,algorithm="maxima")`

[Out]  $-1/4*(2*d*e^2*f^2-2*d^3*g^2+(e^3*f^2+2*d*e^2*f*g-3*d^2*e*g^2)*x)/(d^2*e^5*x^2+2*d^3*e^4*x+d^4*e^3)+1/8*(e^2*f^2+2*d*e*f*g+d^2*g^2)*\log(e*x+d)/(d^3*e^3)-1/8*(e^2*f^2+2*d*e*f*g+d^2*g^2)*\log(e*x-d)/(d^3*e^3)$

**Fricas [A]** time = 0.280624, size = 366, normalized size = 4.21

$$\frac{4d^2e^2f^2-4d^4g^2+2(de^3f^2+2d^2e^2fg-3d^3eg^2)x-(d^2e^2f^2+2d^3efg+d^4g^2+(e^4f^2+2de^3fg+d^2e^2g^2)x^2+2(de^3f^2+2d^2e^2fg-3d^3eg^2)x)}{8(d^2e^5x^2+2d^3e^4x+d^4e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(g*x+f)^2/((e^2*x^2-d^2)*(e*x+d)^2),x,algorithm="fricas")`

[Out]  $-1/8*(4*d^2*e^2*f^2-4*d^4*g^2+2*(d^3*e^3*f^2+2*d^2*e^2*f*g-3*d^3*e*g^2)*x-(d^2*e^2*f^2+2*d^3*efg+d^4*g^2+(e^4*f^2+2*d^2*e^2*g^2)*x^2+2*(d^3*f^2+2*d^2*e^2*fg-3*d^3*eg^2)*x)/((d^2*e^5*x^2+2*d^3*e^4*x+d^4*e^3))+1/8*(e^2*f^2+2*d*e*f*g+d^2*g^2)*\log(e*x+d)/(d^3*e^3)-1/8*(e^2*f^2+2*d*e*f*g+d^2*g^2)*\log(e*x-d)/(d^3*e^3)$

**Sympy [A]** time = 4.06054, size = 185, normalized size = 2.13

$$\begin{aligned} & \frac{2d^3g^2-2de^2f^2+x(3d^2eg^2-2de^2fg-e^3f^2)}{4d^4e^3+8d^3e^4x+4d^2e^5x^2} \\ & - \frac{(dg+ef)^2\log\left(-\frac{d(dg+ef)^2}{e(d^2g^2+2defg+e^2f^2)}+x\right)}{8d^3e^3} + \frac{(dg+ef)^2\log\left(\frac{d(dg+ef)^2}{e(d^2g^2+2defg+e^2f^2)}+x\right)}{8d^3e^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**2/(-e**2*x**2+d**2),x)
```

```
[Out] (2*d**3*g**2 - 2*d*e**2*f**2 + x*(3*d**2*e*g**2 - 2*d*e**2*f*g -
e**3*f**2))/(4*d**4*e**3 + 8*d**3*e**4*x + 4*d**2*e**5*x**2) - (d
*g + e*f)**2*log(-d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*d*e*f*g + e
**2*f**2)) + x)/(8*d**3*e**3) + (d*g + e*f)**2*log(d*(d*g + e*f)**
2/(e*(d**2*g**2 + 2*d*e*f*g + e**2*f**2)) + x)/(8*d**3*e**3)
```

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(g*x + f)^2/((e^2*x^2 - d^2)*(e*x + d)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.555 \quad \int \frac{(f+gx)^2}{(d+ex)^3(d^2-e^2x^2)} dx$$

**Optimal.** Leaf size=113

$$\frac{(dg+ef)^2 \tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3} - \frac{(dg+ef)^2}{8d^3e^3(d+ex)} - \frac{(3dg+ef)(ef-dg)}{8d^2e^3(d+ex)^2} - \frac{(ef-dg)^2}{6de^3(d+ex)^3}$$

[Out]  $-(e^*f - d^*g)^2/(6*d^*e^3*(d + e^*x)^3) - ((e^*f - d^*g)*(e^*f + 3*d^*g))/(8*d^2*e^3*(d + e^*x)^2) - (e^*f + d^*g)^2/(8*d^3*e^3*(d + e^*x)) + ((e^*f + d^*g)^2*ArcTanh[(e^*x)/d])/(8*d^4*e^3)$

**Rubi [A]** time = 0.25704, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$\frac{(dg+ef)^2 \tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3} - \frac{(dg+ef)^2}{8d^3e^3(d+ex)} - \frac{(3dg+ef)(ef-dg)}{8d^2e^3(d+ex)^2} - \frac{(ef-dg)^2}{6de^3(d+ex)^3}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)^2/((d + e\*x)^3\*(d^2 - e^2\*x^2)), x]

[Out]  $-(e^*f - d^*g)^2/(6*d^*e^3*(d + e^*x)^3) - ((e^*f - d^*g)*(e^*f + 3*d^*g))/(8*d^2*e^3*(d + e^*x)^2) - (e^*f + d^*g)^2/(8*d^3*e^3*(d + e^*x)) + ((e^*f + d^*g)^2*ArcTanh[(e^*x)/d])/(8*d^4*e^3)$

**Rubi in Sympy [A]** time = 50.8643, size = 97, normalized size = 0.86

$$-\frac{(dg-ef)^2}{6de^3(d+ex)^3} + \frac{(dg-ef)(3dg+ef)}{8d^2e^3(d+ex)^2} - \frac{(dg+ef)^2}{8d^3e^3(d+ex)} + \frac{(dg+ef)^2 \operatorname{atanh}\left(\frac{ex}{d}\right)}{8d^4e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*2/(e\*x+d)\*\*3/(-e\*\*2\*x\*\*2+d\*\*2), x)

[Out]  $-(d^*g - e^*f)**2/(6*d^*e**3*(d + e^*x)**3) + (d^*g - e^*f)*(3*d^*g + e^*f)/(8*d**2*e**3*(d + e^*x)**2) - (d^*g + e^*f)**2/(8*d**3*e**3*(d + e^*x)) + (d^*g + e^*f)**2*atanh(e^*x/d)/(8*d**4*e**3)$

**Mathematica [A]** time = 0.0948727, size = 122, normalized size = 1.08

$$\frac{-\frac{8d^3(ef-dg)^2}{(d+ex)^3} + \frac{6d^2(3d^2g^2-2defg-e^2f^2)}{(d+ex)^2} - \frac{6d(dg+ef)^2}{d+ex} - 3(dg+ef)^2 \log(d-ex) + 3(dg+ef)^2 \log(d+ex)}{48d^4e^3}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^2/((d + e\*x)^3\*(d^2 - e^2\*x^2)), x]

[Out]  $((-8*d^3*(e^*f - d^*g)^2)/(d + e^*x)^3 + (6*d^2*(-(e^2*f^2) - 2*d^*e^*f^*g + 3*d^2*g^2))/(d + e^*x)^2 - (6*d*(e^*f + d^*g)^2)/(d + e^*x) - 3*(e^*f + d^*g)^2*Log[d - e^*x] + 3*(e^*f + d^*g)^2*Log[d + e^*x])/(48*d^4*e^3)$

**Maple [B]** time = 0.016, size = 259, normalized size = 2.3

$$\begin{aligned} & -\frac{\ln(ex-d)g^2}{16d^2e^3} - \frac{\ln(ex-d)fg}{8d^3e^2} - \frac{\ln(ex-d)f^2}{16d^4e} + \frac{3g^2}{8e^3(ex+d)^2} - \frac{fg}{4e^2d(ex+d)^2} \\ & - \frac{f^2}{8d^2e(ex+d)^2} - \frac{g^2d}{6e^3(ex+d)^3} + \frac{fg}{3e^2(ex+d)^3} - \frac{f^2}{6de(ex+d)^3} + \frac{\ln(ex+d)g^2}{16d^2e^3} \\ & + \frac{\ln(ex+d)fg}{8d^3e^2} + \frac{\ln(ex+d)f^2}{16d^4e} - \frac{g^2}{8de^3(ex+d)} - \frac{fg}{4d^2e^2(ex+d)} - \frac{f^2}{8d^3e(ex+d)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^2/(e*x+d)^3/(-e^2*x^2+d^2),x)`

[Out] 
$$-1/16/e^3/d^2*\ln(e*x-d)*g^2-1/8/e^2/d^3*\ln(e*x-d)*f*g-1/16/e/d^4*\ln(e*x-d)*f^2+3/8/e^3/(e*x+d)^2*g^2-1/4/d/e^2/(e*x+d)^2*f*g-1/8/d^2/e/(e*x+d)^2*f^2-1/6/e^3*d/(e*x+d)^3*g^2+1/3/e^2/(e*x+d)^3*f*g-1/6/e/d/(e*x+d)^3*f^2+1/16/e^3/d^2*\ln(e*x+d)*g^2+1/8/e^2/d^3*\ln(e*x+d)*f*g+1/16/e/d^4*\ln(e*x+d)*f^2-1/8/d/e^3/(e*x+d)*g^2-1/4/d^2/e^2/(e*x+d)*f*g-1/8/d^3/e/(e*x+d)*f^2$$

**Maxima [A]** time = 0.695729, size = 278, normalized size = 2.46

$$\begin{aligned} & -\frac{10d^2e^2f^2+4d^3efg-2d^4g^2+3(e^4f^2+2de^3fg+d^2e^2g^2)x^2+3(3de^3f^2+6d^2e^2fg-d^3eg^2)x}{24(d^3e^6x^3+3d^4e^5x^2+3d^5e^4x+d^6e^3)} \\ & + \frac{(e^2f^2+2defg+d^2g^2)\log(ex+d)}{16d^4e^3} - \frac{(e^2f^2+2defg+d^2g^2)\log(ex-d)}{16d^4e^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(g*x+f)^2/((e^2*x^2-d^2)*(e*x+d)^3),x,algorithm="maxima")`

[Out] 
$$-1/24*(10*d^2*e^2*f^2+4*d^3*e*f*g-2*d^4*g^2+3*(e^4*f^2+2*d*e^3*f*g+d^2*e^2*g^2)*x^2+3*(3*d^3*e^3*f^2+6*d^2*e^2*f*g-d^3*d^3*e^3*g^2)*x)/(d^3*e^6*x^3+3*d^4*e^5*x^2+3*d^5*e^4*x+d^6*e^3)+1/16*(e^2*f^2+2*d*e*f*g+d^2*g^2)*\log(e*x+d)/(d^4*e^3)-1/16*(e^2*f^2+2*d*e*f*g+d^2*g^2)*\log(e*x-d)/(d^4*e^3)$$

**Fricas [A]** time = 0.282725, size = 540, normalized size = 4.78

$$-\frac{20d^3e^2f^2+8d^4efg-4d^5g^2+6(de^4f^2+2d^2e^3fg+d^3e^2g^2)x^2+6(3d^2e^3f^2+6d^3e^2fg-d^4eg^2)x-3(d^3e^2f^2+2d^4efg-d^5g^2)}{24(d^3e^6x^3+3d^4e^5x^2+3d^5e^4x+d^6e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(g*x+f)^2/((e^2*x^2-d^2)*(e*x+d)^3),x,algorithm="fricas")`

[Out] 
$$-1/48*(20*d^3*e^2*f^2+8*d^4*e*f*g-4*d^5*g^2+6*(d^2*e^4*f^2+2*d^2*e^3*f*g+d^3*e^2*g^2)*x^2+6*(3*d^2*e^3*f^2+6*d^3*e^2*f*g-d^4*d^3*e^3*g^2)*x-3*(d^3*e^2*f^2+2*d^4*e*f*g+d^5*d^4*e^4*g^2)*x^3+3*(d^2*e^4*f^2+2*d^2*e^3*f*g+d^3*d^3*e^3*g^2)*x^2+3*(d^2*e^3*f^2+2*d^3*e^2*f*g+d^4*d^4*e^4*g^2)*x)*\log(e*x+d)+3*(d^3*e^2*f^2+2*d^4*e*f*g+d^5*d^5*e^5*g^2+(e^5*f^2+2*d*e^4*f*g+d^2*e^3*g^2)*x^3+3*(d^2*e^4*f^2+2*d^2*e^3*f*g+d^3*d^3*e^3*g^2)*x^2+3*(d^2*e^3*f^2+2*d^3*e^2*f*g+d^4*d^4*e^4*g^2)*x)*\log(e*x-d))/(d^4*e^6*x^3+3*d^5*e^5*x^2+3*d^6*e^4*x+d^7*e^3)$$

**Sympy [A]** time = 5.23426, size = 248, normalized size = 2.19

$$\frac{-2d^4g^2 + 4d^3efg + 10d^2e^2f^2 + x^2(3d^2e^2g^2 + 6de^3fg + 3e^4f^2) + x(-3d^3eg^2 + 18d^2e^2fg + 9de^3f^2)}{24d^6e^3 + 72d^5e^4x + 72d^4e^5x^2 + 24d^3e^6x^3} \\ - \frac{(dg + ef)^2 \log\left(-\frac{d(dg+ef)^2}{e(d^2g^2+2defg+e^2f^2)} + x\right)}{16d^4e^3} + \frac{(dg + ef)^2 \log\left(\frac{d(dg+ef)^2}{e(d^2g^2+2defg+e^2f^2)} + x\right)}{16d^4e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*2/(e\*x+d)\*\*3/(-e\*\*2\*x\*\*2+d\*\*2),x)

[Out] 
$$-\frac{(-2d^4g^2 + 4d^3efg + 10d^2e^2f^2 + x^2(3d^2e^2g^2 + 6de^3fg + 3e^4f^2) + x(-3d^3eg^2 + 18d^2e^2fg + 9de^3f^2))}{(24d^6e^3 + 72d^5e^4x + 72d^4e^5x^2 + 24d^3e^6x^3)} - \frac{(dg + ef)^2 \log(-d(dg + ef)^2/(e(d^2g^2 + 2defg + e^2f^2)) + x)}{(16d^4e^3)} + \frac{(dg + ef)^2 \log(d(dg + ef)^2/(e(d^2g^2 + 2defg + e^2f^2)) + x)}{(16d^4e^3)}$$

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(g\*x + f)^2/((e^2\*x^2 - d^2)\*(e\*x + d)^3),x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.556 \quad \int \frac{(f+gx)^2}{(d+ex)^4(d^2-e^2x^2)} dx$$

**Optimal.** Leaf size=139

$$\frac{(dg+ef)^2 \tanh^{-1}\left(\frac{ex}{d}\right)}{16d^5e^3} - \frac{(dg+ef)^2}{16d^4e^3(d+ex)} - \frac{(dg+ef)^2}{16d^3e^3(d+ex)^2} - \frac{(3dg+ef)(ef-dg)}{12d^2e^3(d+ex)^3} - \frac{(ef-dg)^2}{8de^3(d+ex)^4}$$

[Out]  $-(e^*f - d^*g)^2/(8^*d^*e^{\wedge}3^*(d + e^*x)^{\wedge}4) - ((e^*f - d^*g)^*(e^*f + 3^*d^*g))/(12^*d^{\wedge}2^*e^{\wedge}3^*(d + e^*x)^{\wedge}3) - (e^*f + d^*g)^2/(16^*d^{\wedge}3^*e^{\wedge}3^*(d + e^*x)^{\wedge}2) - (e^*f + d^*g)^2/(16^*d^{\wedge}4^*e^{\wedge}3^*(d + e^*x)) + ((e^*f + d^*g)^2^*ArcTan h[(e^*x)/d])/(16^*d^{\wedge}5^*e^{\wedge}3)$

**Rubi [A]** time = 0.293004, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$\frac{(dg+ef)^2 \tanh^{-1}\left(\frac{ex}{d}\right)}{16d^5e^3} - \frac{(dg+ef)^2}{16d^4e^3(d+ex)} - \frac{(dg+ef)^2}{16d^3e^3(d+ex)^2} - \frac{(3dg+ef)(ef-dg)}{12d^2e^3(d+ex)^3} - \frac{(ef-dg)^2}{8de^3(d+ex)^4}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)^2/((d + e\*x)^4\*(d^2 - e^2\*x^2)), x]

[Out]  $-(e^*f - d^*g)^2/(8^*d^*e^{\wedge}3^*(d + e^*x)^{\wedge}4) - ((e^*f - d^*g)^*(e^*f + 3^*d^*g))/(12^*d^{\wedge}2^*e^{\wedge}3^*(d + e^*x)^{\wedge}3) - (e^*f + d^*g)^2/(16^*d^{\wedge}3^*e^{\wedge}3^*(d + e^*x)^{\wedge}2) - (e^*f + d^*g)^2/(16^*d^{\wedge}4^*e^{\wedge}3^*(d + e^*x)) + ((e^*f + d^*g)^2^*ArcTan h[(e^*x)/d])/(16^*d^{\wedge}5^*e^{\wedge}3)$

**Rubi in Sympy [A]** time = 58.5063, size = 121, normalized size = 0.87

$$-\frac{(dg-ef)^2}{8de^3(d+ex)^4} + \frac{(dg-ef)(3dg+ef)}{12d^2e^3(d+ex)^3} - \frac{(dg+ef)^2}{16d^3e^3(d+ex)^2} - \frac{(dg+ef)^2}{16d^4e^3(d+ex)} + \frac{(dg+ef)^2 \operatorname{atanh}\left(\frac{ex}{d}\right)}{16d^5e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*2/(e\*x+d)\*\*4/(-e\*\*2\*x\*\*2+d\*\*2), x)

[Out]  $-(d^*g - e^*f)^2/(8^*d^*e^{\wedge}3^*(d + e^*x)^{\wedge}4) + (d^*g - e^*f)^*(3^*d^*g + e^*f)/(12^*d^{\wedge}2^*e^{\wedge}3^*(d + e^*x)^{\wedge}3) - (d^*g + e^*f)^2/(16^*d^{\wedge}3^*e^{\wedge}3^*(d + e^*x)^{\wedge}2) - (d^*g + e^*f)^2/(16^*d^{\wedge}4^*e^{\wedge}3^*(d + e^*x)) + (d^*g + e^*f)^2^*atanh(e^*x/d)/(16^*d^{\wedge}5^*e^{\wedge}3)$

**Mathematica [A]** time = 0.150246, size = 142, normalized size = 1.02

$$\frac{12d^4(ef-dg)^2}{(d+ex)^4} + \frac{6d^2(dg+ef)^2}{(d+ex)^2} + \frac{8d^3(-3d^2g^2+2defg+e^2f^2)}{(d+ex)^3} + \frac{6d(dg+ef)^2}{d+ex} + 3(dg+ef)^2 \log(d-ex) - 3(dg+ef)^2 \log(d+ex)$$


---


$$96d^5e^3$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^2/((d + e\*x)^4\*(d^2 - e^2\*x^2)), x]

[Out]  $-(12^*d^{\wedge}4^*(e^*f - d^*g)^2)/(d + e^*x)^{\wedge}4 + (8^*d^{\wedge}3^*(e^{\wedge}2^*f^{\wedge}2 + 2^*d^*e^*f^*g - 3^*d^{\wedge}2^*g^{\wedge}2))/(d + e^*x)^{\wedge}3 + (6^*d^{\wedge}2^*(e^*f + d^*g)^2)/(d + e^*x)^{\wedge}2 + (6^*d^*(e^*f + d^*g)^2)/(d + e^*x) + 3^*(e^*f + d^*g)^2^*Log[d - e^*x] - 3$



$$*(e*f + d*g)^2*\text{Log}[d + e*x]]/(96*d^5*e^3)$$

**Maple [B]** time = 0.016, size = 312, normalized size = 2.2

$$\begin{aligned} & -\frac{\ln(ex-d)g^2}{32d^3e^3} - \frac{\ln(ex-d)fg}{16d^4e^2} - \frac{\ln(ex-d)f^2}{32d^5e} + \frac{g^2}{4e^3(ex+d)^3} - \frac{fg}{6e^2d(ex+d)^3} \\ & - \frac{f^2}{12d^2e(ex+d)^3} - \frac{g^2d}{8e^3(ex+d)^4} + \frac{fg}{4e^2(ex+d)^4} - \frac{f^2}{8de(ex+d)^4} + \frac{\ln(ex+d)g^2}{32d^3e^3} \\ & + \frac{\ln(ex+d)fg}{16d^4e^2} + \frac{\ln(ex+d)f^2}{32d^5e} - \frac{g^2}{16d^2e^3(ex+d)} - \frac{fg}{8d^3e^2(ex+d)} \\ & - \frac{f^2}{16d^4e(ex+d)} - \frac{g^2}{16de^3(ex+d)^2} - \frac{fg}{8d^2e^2(ex+d)^2} - \frac{f^2}{16d^3e(ex+d)^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^2/(e*x+d)^4/(-e^2*x^2+d^2),x)`

[Out] `-1/32/e^3/d^3*ln(e*x-d)*g^2-1/16/e^2/d^4*ln(e*x-d)*f*g-1/32/e/d^5*ln(e*x-d)*f^2+1/4/e^3/(e*x+d)^3*g^2-1/6/d/e^2/(e*x+d)^3*f*g-1/12/d^2/e/(e*x+d)^3*f^2-1/8/e^3*d/(e*x+d)^4*g^2+1/4/e^2/(e*x+d)^4*f*g-1/8/e/d/(e*x+d)^4*f^2+1/32/e^3/d^3*ln(e*x+d)*g^2+1/16/e^2/d^4*ln(e*x+d)*f*g+1/32/e/d^5*ln(e*x+d)*f^2-1/16/d^2/e^3/(e*x+d)*g^2-1/8/d^3/e^2/(e*x+d)*f*g-1/16/d^4/e/(e*x+d)*f^2-1/16/d/e^3/(e*x+d)^2*g^2-1/8/d^2/e^2/(e*x+d)^2*f*g-1/16/d^3/e/(e*x+d)^2*f^2`

**Maxima [A]** time = 0.701227, size = 319, normalized size = 2.29

$$\begin{aligned} & \frac{16d^3ef^2 + 8d^4fg + 3(e^4f^2 + 2de^3fg + d^2e^2g^2)x^3 + 12(de^3f^2 + 2d^2e^2fg + d^3eg^2)x^2 + (19d^2e^2f^2 + 38d^3efg + 3d^4g^2)}{48(d^4e^6x^4 + 4d^5e^5x^3 + 6d^6e^4x^2 + 4d^7e^3x + d^8e^2)} \\ & + \frac{(e^2f^2 + 2defg + d^2g^2)\log(ex+d)}{32d^5e^3} - \frac{(e^2f^2 + 2defg + d^2g^2)\log(ex-d)}{32d^5e^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(g*x+f)^2/((e^2*x^2-d^2)*(e*x+d)^4),x,algorithm="maxima")`

[Out] `-1/48*(16*d^3*e*f^2 + 8*d^4*f*g + 3*(e^4*f^2 + 2*d*e^3*f*g + d^2*e^2*g^2)*x^3 + 12*(d*e^3*f^2 + 2*d^2*e^2*f*g + d^3*e*g^2)*x^2 + (19*d^2*e^2*f^2 + 38*d^3*e*f*g + 3*d^4*g^2)*x)/(d^4*e^6*x^4 + 4*d^5*e^5*x^3 + 6*d^6*e^4*x^2 + 4*d^7*e^3*x + d^8*e^2) + 1/32*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(e*x+d)/(d^5*e^3) - 1/32*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)*log(e*x-d)/(d^5*e^3)`

**Fricas [A]** time = 0.282478, size = 690, normalized size = 4.96

$$\frac{32d^4e^2f^2 + 16d^5efg + 6(de^5f^2 + 2d^2e^4fg + d^3e^3g^2)x^3 + 24(d^2e^4f^2 + 2d^3e^3fg + d^4e^2g^2)x^2 + 2(19d^3e^3f^2 + 38d^4e^2fg + 3d^5e^2g^2)x}{48(d^4e^6x^4 + 4d^5e^5x^3 + 6d^6e^4x^2 + 4d^7e^3x + d^8e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(g*x+f)^2/((e^2*x^2-d^2)*(e*x+d)^4),x,algorithm="fricas")`

[Out] `-1/96*(32*d^4*e^2*f^2 + 16*d^5*e*f*g + 6*(d*e^5*f^2 + 2*d^2*e^4*f*g + d^3*e^3*g^2)*x^3 + 24*(d^2*e^4*f^2 + 2*d^3*e^3*f*g + d^4*e^2*g^2)*x^2 + 2*(19*d^3*e^3*f^2 + 38*d^4*e^2*f*g + 3*d^5*e^2*g^2)*x -`

$$3*(d^4*e^2*f^2 + 2*d^5*e*f*g + d^6*g^2 + (e^6*f^2 + 2*d*e^5*f*g + d^2*e^4*g^2)*x^4 + 4*(d*e^5*f^2 + 2*d^2*e^4*f*g + d^3*e^3*g^2)*x^3 + 6*(d^2*e^4*f^2 + 2*d^3*e^3*f*g + d^4*e^2*g^2)*x^2 + 4*(d^3*e^3*f^2 + 2*d^4*e^2*f*g + d^5*e*g^2)*x)*\log(e*x + d) + 3*(d^4*e^2*f^2 + 2*d^5*e*f*g + d^6*g^2 + (e^6*f^2 + 2*d*e^5*f*g + d^2*e^4*g^2)*x^4 + 4*(d*e^5*f^2 + 2*d^2*e^4*f*g + d^3*e^3*g^2)*x^3 + 6*(d^2*e^4*f^2 + 2*d^3*e^3*f*g + d^4*e^2*g^2)*x^2 + 4*(d^3*e^3*f^2 + 2*d^4*e^2*f*g + d^5*e*g^2)*x)*\log(e*x - d))/(d^5*e^7*x^4 + 4*d^6*e^6*x^3 + 6*d^7*e^5*x^2 + 4*d^8*e^4*x + d^9*e^3)$$

**Sympy [A]** time = 6.09852, size = 282, normalized size = 2.03

$$\frac{8d^4fg + 16d^3ef^2 + x^3(3d^2e^2g^2 + 6de^3fg + 3e^4f^2) + x^2(12d^3eg^2 + 24d^2e^2fg + 12de^3f^2) + x(3d^4g^2 + 38d^3efg + 19d^2e^2f^2)}{48d^8e^2 + 192d^7e^3x + 288d^6e^4x^2 + 192d^5e^5x^3 + 48d^4e^6x^4} + \frac{(dg + ef)^2 \log\left(-\frac{d(dg+ef)^2}{e(d^2g^2+2defg+e^2f^2)} + x\right)}{32d^5e^3} + \frac{(dg + ef)^2 \log\left(\frac{d(dg+ef)^2}{e(d^2g^2+2defg+e^2f^2)} + x\right)}{32d^5e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*2/(e\*x+d)\*\*4/(-e\*\*2\*x\*\*2+d\*\*2), x)

[Out]  $-(8*d**4*f*g + 16*d**3*e*f**2 + x**3*(3*d**2*e**2*g**2 + 6*d*e**3*f*g + 3*e**4*f**2) + x**2*(12*d**3*e*g**2 + 24*d**2*e**2*f*g + 12*d*e**3*f**2) + x*(3*d**4*g**2 + 38*d**3*e*f*g + 19*d**2*e**2*f**2))/(48*d**8*e**2 + 192*d**7*e**3*x + 288*d**6*e**4*x**2 + 192*d**5*e**5*x**3 + 48*d**4*e**6*x**4) - (d*g + e*f)**2*log(-d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*d*e*f*g + e**2*f**2)) + x)/(32*d**5*e**3) + (d*g + e*f)**2*log(d*(d*g + e*f)**2/(e*(d**2*g**2 + 2*d*e*f*g + e**2*f**2)) + x)/(32*d**5*e**3)$

**GIAC/XCAS [A]** time = 0.281945, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(g\*x + f)^2/((e^2\*x^2 - d^2)\*(e\*x + d)^4), x, algorithm="giac")

[Out] Done

$$3.557 \quad \int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^2} dx$$

**Optimal.** Leaf size=218

$$\begin{aligned} & \frac{32d^5(dg+ef)^2}{e^3(d-ex)} + \frac{16d^4(dg+ef)(9dg+5ef)\log(d-ex)}{e^3} + \frac{1}{4}ex^4(23d^2g^2+14defg+e^2f^2) \\ & + \frac{1}{3}dx^3(49d^2g^2+46defg+7e^2f^2) + \frac{d^2x^2(80d^2g^2+98defg+23e^2f^2)}{2e} \\ & + \frac{d^3x(112d^2g^2+160defg+49e^2f^2)}{e^2} + \frac{1}{5}e^2gx^5(7dg+2ef) + \frac{1}{6}e^3g^2x^6 \end{aligned}$$

[Out]  $(d^3*(49*e^2*f^2 + 160*d*e*f*g + 112*d^2*g^2)*x)/e^2 + (d^2*(23*e^2*f^2 + 98*d*e*f*g + 80*d^2*g^2)*x^2)/(2*e) + (d*(7*e^2*f^2 + 46*d*e*f*g + 49*d^2*g^2)*x^3)/3 + (e*(e^2*f^2 + 14*d*e*f*g + 23*d^2*g^2)*x^4)/4 + (e^2*g*(2*e*f + 7*d*g)*x^5)/5 + (e^3*g^2*x^6)/6 + (32*d^5*(e*f + d*g)^2)/(e^3*(d - e*x)) + (16*d^4*(e*f + d*g)*(5*e*f + 9*d*g)*\text{Log}[d - e*x])/e^3$

**Rubi [A]** time = 0.539032, antiderivative size = 218, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$\begin{aligned} & \frac{32d^5(dg+ef)^2}{e^3(d-ex)} + \frac{16d^4(dg+ef)(9dg+5ef)\log(d-ex)}{e^3} + \frac{1}{4}ex^4(23d^2g^2+14defg+e^2f^2) \\ & + \frac{1}{3}dx^3(49d^2g^2+46defg+7e^2f^2) + \frac{d^2x^2(80d^2g^2+98defg+23e^2f^2)}{2e} \\ & + \frac{d^3x(112d^2g^2+160defg+49e^2f^2)}{e^2} + \frac{1}{5}e^2gx^5(7dg+2ef) + \frac{1}{6}e^3g^2x^6 \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d+e*x)^7*(f+g*x)^2/(d^2-e^2*x^2)^2, x]$

[Out]  $(d^3*(49*e^2*f^2 + 160*d*e*f*g + 112*d^2*g^2)*x)/e^2 + (d^2*(23*e^2*f^2 + 98*d*e*f*g + 80*d^2*g^2)*x^2)/(2*e) + (d*(7*e^2*f^2 + 46*d*e*f*g + 49*d^2*g^2)*x^3)/3 + (e*(e^2*f^2 + 14*d*e*f*g + 23*d^2*g^2)*x^4)/4 + (e^2*g*(2*e*f + 7*d*g)*x^5)/5 + (e^3*g^2*x^6)/6 + (32*d^5*(e*f + d*g)^2)/(e^3*(d - e*x)) + (16*d^4*(e*f + d*g)*(5*e*f + 9*d*g)*\text{Log}[d - e*x])/e^3$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\begin{aligned} & \frac{32d^5(dg+ef)^2}{e^3(d-ex)} + \frac{16d^4(dg+ef)(9dg+5ef)\log(d-ex)}{e^3} + \frac{d^2(80d^2g^2+98defg+23e^2f^2)}{e} \int x dx \\ & + \frac{dx^3(49d^2g^2+46defg+7e^2f^2)}{3} + \frac{e^3g^2x^6}{6} + \frac{e^2gx^5(7dg+2ef)}{5} \\ & + \frac{ex^4(23d^2g^2+14defg+e^2f^2)}{4} + \frac{(112d^2g^2+160defg+49e^2f^2)}{e^2} \int d^3 dx \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)**7*(g*x+f)**2/(-e**2*x**2+d**2)**2, x)$

[Out]  $32*d**5*(d*g + e*f)**2/(e**3*(d - e*x)) + 16*d**4*(d*g + e*f)*(9*d*g + 5*e*f)*\log(d - e*x)/e**3 + d**2*(80*d**2*g**2 + 98*d*e*f*g + 23*e**2*f**2)*\text{Integral}(x, x)/e + d*x**3*(49*d**2*g**2 + 46*d*e*f*g + 7*e**2*f**2)/3 + e**3*g**2*x**6/6 + e**2*g*x**5*(7*d*g + 2*e*f)/5 + e*x**4*(23*d**2*g**2 + 14*d*e*f*g + e**2*f**2)/4 + (112*$

$$d^{**2}g^{**2} + 160*d*e*f*g + 49*e^{**2}*f^{**2}) * \text{Integral}(d^{**3}, x)/e^{**2}$$

**Mathematica [A]** time = 0.249684, size = 226, normalized size = 1.04

$$\begin{aligned} & -\frac{32d^5(dg + ef)^2}{e^3(ex - d)} + \frac{1}{4}ex^4(23d^2g^2 + 14defg + e^2f^2) + \frac{1}{3}dx^3(49d^2g^2 + 46defg + 7e^2f^2) \\ & + \frac{d^2x^2(80d^2g^2 + 98defg + 23e^2f^2)}{2e} + \frac{16d^4(9d^2g^2 + 14defg + 5e^2f^2)\log(d - ex)}{e^3} \\ & + \frac{d^3x(112d^2g^2 + 160defg + 49e^2f^2)}{e^2} + \frac{1}{5}e^2gx^5(7dg + 2ef) + \frac{1}{6}e^3g^2x^6 \end{aligned}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^7\*(f + g\*x)^2)/(d^2 - e^2\*x^2)^2, x]

[Out] (d^3\*(49\*e^2\*f^2 + 160\*d\*e\*f\*g + 112\*d^2\*g^2)\*x)/e^2 + (d^2\*(23\*e^2\*f^2 + 98\*d\*e\*f\*g + 80\*d^2\*g^2)\*x^2)/(2\*e) + (d\*(7\*e^2\*f^2 + 46\*d\*e\*f\*g + 49\*d^2\*g^2)\*x^3)/3 + (e\*(e^2\*f^2 + 14\*d\*e\*f\*g + 23\*d^2\*g^2)\*x^4)/4 + (e^2\*g\*(2\*e\*f + 7\*d\*g)\*x^5)/5 + (e^3\*g^2\*x^6)/6 - (32\*d^5\*(e\*f + d\*g)^2)/(e^3\*(-d + e\*x)) + (16\*d^4\*(5\*e^2\*f^2 + 14\*d\*e\*f\*g + 9\*d^2\*g^2)\*Log[d - e\*x])/e^3

**Maple [A]** time = 0.014, size = 286, normalized size = 1.3

$$\begin{aligned} & \frac{e^3g^2x^6}{6} + \frac{7e^2x^5dg^2}{5} + \frac{2e^3x^5fg}{5} + \frac{23ex^4d^2g^2}{4} + \frac{7e^2x^4dfg}{2} + \frac{e^3x^4f^2}{4} + \frac{49x^3d^3g^2}{3} \\ & + \frac{46ex^3d^2fg}{3} + \frac{7e^2x^3df^2}{3} + 40\frac{x^2d^4g^2}{e} + 49x^2d^3fg + \frac{23ex^2d^2f^2}{2} + 112\frac{d^5g^2x}{e^2} \\ & + 160\frac{d^4fgx}{e} + 49d^3f^2x + 144\frac{d^6\ln(ex - d)g^2}{e^3} + 224\frac{d^5\ln(ex - d)fg}{e^2} \\ & + 80\frac{d^4\ln(ex - d)f^2}{e} - 32\frac{d^7g^2}{e^3(ex - d)} - 64\frac{d^6fg}{e^2(ex - d)} - 32\frac{d^5f^2}{e(ex - d)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^7\*(g\*x+f)^2/(-e^2\*x^2+d^2)^2, x)

[Out] 1/6\*e^3\*g^2\*x^6+7/5\*e^2\*x^5\*d\*g^2+2/5\*e^3\*x^5\*f\*g+23/4\*e\*x^4\*d^2\*g^2+7/2\*e^2\*x^4\*d\*f\*g+1/4\*e^3\*x^4\*f^2+49/3\*x^3\*d^3\*g^2+46/3\*e\*x^3\*d^2\*f\*g+7/3\*e^2\*x^3\*d\*f^2+40/e\*x^2\*d^4\*g^2+49\*x^2\*d^3\*f\*g+23/2\*e\*x^2\*d^2\*f^2+112/e^2\*d^5\*g^2\*x+160/e\*d^4\*f\*g\*x+49\*d^3\*f^2\*x+144\*d^6/e^3\*ln(e\*x-d)\*g^2+224\*d^5/e^2\*ln(e\*x-d)\*f\*g+80\*d^4/e\*ln(e\*x-d)\*f^2-32\*d^7/e^3/(e\*x-d)\*g^2-64\*d^6/e^2/(e\*x-d)\*f\*g-32\*d^5/e/(e\*x-d)\*f^2

**Maxima [A]** time = 0.697253, size = 348, normalized size = 1.6

$$\begin{aligned} & \frac{32(d^5e^2f^2 + 2d^6efg + d^7g^2)}{e^4x - de^3} \\ & + \frac{10e^5g^2x^6 + 12(2e^5fg + 7de^4g^2)x^5 + 15(e^5f^2 + 14de^4fg + 23d^2e^3g^2)x^4 + 20(7de^4f^2 + 46d^2e^3fg + 49d^3e^2g^2)x^3 + 30e^3d^5g^2x^2 + 12e^2d^4fgx + 12e^2d^4f^2x + 12e^2d^4g^2x + 12e^2d^4fg + 12e^2d^4f^2 + 12e^2d^4g^2}{60e^2} \\ & + \frac{16(5d^4e^2f^2 + 14d^5efg + 9d^6g^2)\log(ex - d)}{e^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^7\*(g\*x + f)^2/(e^2\*x^2 - d^2)^2,x, algorithm="maxima")

[Out]  $-32*(d^5*e^2*f^2 + 2*d^6*e*f*g + d^7*g^2)/(e^4*x - d*e^3) + 1/60*(10*e^5*g^2*x^6 + 12*(2*e^5*f*g + 7*d*e^4*g^2)*x^5 + 15*(e^5*f^2 + 14*d*e^4*f*g + 23*d^2*e^3*g^2)*x^4 + 20*(7*d*e^4*f^2 + 46*d^2*e^3*f*g + 49*d^3*e^2*g^2)*x^3 + 30*(23*d^2*e^3*f^2 + 98*d^3*e^2*f*g + 80*d^4*e*g^2)*x^2 + 60*(49*d^3*e^2*f^2 + 160*d^4*e*f*g + 112*d^5*g^2)*x)/e^2 + 16*(5*d^4*e^2*f^2 + 14*d^5*e*f*g + 9*d^6*g^2)*\log(e*x - d)/e^3$

**Fricas [A]** time = 0.270051, size = 443, normalized size = 2.03

$$10 e^7 g^2 x^7 - 1920 d^5 e^2 f^2 - 3840 d^6 e f g - 1920 d^7 g^2 + 2 (12 e^7 f g + 37 d e^6 g^2) x^6 + 3 (5 e^7 f^2 + 62 d e^6 f g + 87 d^2 e^5 g^2) x^5 + 5 (23 d^2 e^3 f^2 + 98 d^3 e^2 f g + 80 d^4 e g^2) x^4 + 60 (49 d^3 e^2 f^2 + 160 d^4 e f g + 112 d^5 g^2) x^3 + 30 (23 d^2 e^3 f^2 + 98 d^3 e^2 f g + 80 d^4 e g^2) x^2 + 60 (49 d^3 e^2 f^2 + 160 d^4 e f g + 112 d^5 g^2) x + 16 (5 d^4 e^2 f^2 + 14 d^5 e f g + 9 d^6 g^2) \log(e x - d)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^7\*(g\*x + f)^2/(e^2\*x^2 - d^2)^2,x, algorithm="fricas")

[Out]  $1/60*(10*e^7*g^2*x^7 - 1920*d^5*e^2*f^2 - 3840*d^6*e*f*g - 1920*d^7*g^2 + 2*(12*e^7*f*g + 37*d*e^6*g^2)*x^6 + 3*(5*e^7*f^2 + 62*d*e^6*f*g + 87*d^2*e^5*g^2)*x^5 + 5*(25*d^2*e^3*f^2 + 142*d^3*e^2*f*g + 127*d^4*e*g^2)*x^4 + 10*(55*d^2*e^3*f^2 + 202*d^3*e^2*f*g + 142*d^4*e*g^2)*x^3 + 90*(25*d^3*e^2*f^2 + 74*d^4*e*f*g + 48*d^5*e*g^2)*x^2 - 60*(49*d^4*e^2*f^2 + 160*d^5*e*f*g + 112*d^6*e*g^2)*x - 960*(5*d^5*e^2*f^2 + 14*d^6*e*f*g + 9*d^7*g^2 - (5*d^4*e^3*f^2 + 14*d^5*e^2*f*g + 9*d^6*e*g^2)*x)*\log(e*x - d)/(e^4*x - d*e^3)$

**Sympy [A]** time = 4.57074, size = 255, normalized size = 1.17

$$\frac{16d^4(dg + ef)(9dg + 5ef)\log(-d + ex)}{e^3} + \frac{e^3g^2x^6}{6} + x^5\left(\frac{7de^2g^2}{5} + \frac{2e^3fg}{5}\right) + x^4\left(\frac{23d^2eg^2}{4} + \frac{7de^2fg}{2} + \frac{e^3f^2}{4}\right) + x^3\left(\frac{49d^3g^2}{3} + \frac{46d^2efg}{3} + \frac{7de^2f^2}{3}\right) - \frac{32d^7g^2 + 64d^6efg + 32d^5e^2f^2}{-de^3 + e^4x} + \frac{x^2(80d^4g^2 + 98d^3efg + 23d^2e^2f^2)}{2e} + \frac{x(112d^5g^2 + 160d^4efg + 49d^3e^2f^2)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*7\*(g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*2,x)

[Out]  $16*d^4*(d*g + e*f)*(9*d*g + 5*e*f)*\log(-d + e*x)/e^3 + e^3*g^2*x^6/6 + x^5*(7*d*e^2*g^2/5 + 2*e^3*f*g/5) + x^4*(23*d^2*e*g^2/4 + 7*d*e^2*f*g/2 + e^3*f^2/4) + x^3*(49*d^3*g^2/3 + 46*d^2*e*f*g/3 + 7*d*e^2*f^2/3) - (32*d^7*g^2 + 64*d^6*e*f*g + 32*d^5*e^2*f^2)/(-d*e^3 + e^4*x) + x^2*(80*d^4*g^2 + 98*d^3*e*f*g + 23*d^2*e^2*f^2)/(2*e) + x*(112*d^5*g^2 + 160*d^4*e*f*g + 49*d^3*e^2*f^2)/e^2$

**GIAC/XCAS [A]** time = 0.30777, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^7\*(g\*x + f)^2/(e^2\*x^2 - d^2)^2,x, algorithm="giac")

[Out] Done

$$3.558 \quad \int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^2} dx$$

**Optimal.** Leaf size=177

$$\frac{16d^4(dg+ef)^2}{e^3(d-ex)} + \frac{32d^3(dg+ef)(2dg+ef)\log(d-ex)}{e^3} + \frac{1}{3}x^3(17d^2g^2+12defg+e^2f^2) \\ + \frac{dx^2(16d^2g^2+17defg+3e^2f^2)}{e} + \frac{d^2x(48d^2g^2+64defg+17e^2f^2)}{e^2} + \frac{1}{2}egx^4(3dg+ef) + \frac{1}{5}e^2g^2x^5$$

[Out]  $(d^2*(17*e^2*f^2 + 64*d*e*f*g + 48*d^2*g^2)*x)/e^2 + (d*(3*e^2*f^2 + 17*d*e*f*g + 16*d^2*g^2)*x^2)/e + ((e^2*f^2 + 12*d*e*f*g + 17*d^2*g^2)*x^3)/3 + (e*g*(e*f + 3*d*g)*x^4)/2 + (e^2*g^2*x^5)/5 + (16*d^4*(e*f + d*g)^2)/(e^3*(d - e*x)) + (32*d^3*(e*f + d*g)*(e*f + 2*d*g)*\text{Log}[d - e*x])/e^3$

**Rubi [A]** time = 0.458731, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$\frac{16d^4(dg+ef)^2}{e^3(d-ex)} + \frac{32d^3(dg+ef)(2dg+ef)\log(d-ex)}{e^3} + \frac{1}{3}x^3(17d^2g^2+12defg+e^2f^2) \\ + \frac{dx^2(16d^2g^2+17defg+3e^2f^2)}{e} + \frac{d^2x(48d^2g^2+64defg+17e^2f^2)}{e^2} + \frac{1}{2}egx^4(3dg+ef) + \frac{1}{5}e^2g^2x^5$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^6*(f + g*x)^2/(d^2 - e^2*x^2)^2, x]$

[Out]  $(d^2*(17*e^2*f^2 + 64*d*e*f*g + 48*d^2*g^2)*x)/e^2 + (d*(3*e^2*f^2 + 17*d*e*f*g + 16*d^2*g^2)*x^2)/e + ((e^2*f^2 + 12*d*e*f*g + 17*d^2*g^2)*x^3)/3 + (e*g*(e*f + 3*d*g)*x^4)/2 + (e^2*g^2*x^5)/5 + (16*d^4*(e*f + d*g)^2)/(e^3*(d - e*x)) + (32*d^3*(e*f + d*g)*(e*f + 2*d*g)*\text{Log}[d - e*x])/e^3$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{16d^4(dg+ef)^2}{e^3(d-ex)} + \frac{32d^3(dg+ef)(2dg+ef)\log(d-ex)}{e^3} + \frac{2d(16d^2g^2+17defg+3e^2f^2)}{e} \int x dx \\ + \frac{e^2g^2x^5}{5} + \frac{egx^4(3dg+ef)}{2} + x^3 \left( \frac{17d^2g^2}{3} + 4defg + \frac{e^2f^2}{3} \right) + \frac{(48d^2g^2+64defg+17e^2f^2)}{e^2} \int d^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)**6*(g*x+f)**2/(-e**2*x**2+d**2)**2, x)$

[Out]  $16*d**4*(d*g + e*f)**2/(e**3*(d - e*x)) + 32*d**3*(d*g + e*f)*(2*d*g + e*f)*\log(d - e*x)/e**3 + 2*d*(16*d**2*g**2 + 17*d*e*f*g + 3*e**2*f**2)*\text{Integral}(x, x)/e + e**2*g**2*x**5/5 + e*g*x**4*(3*d*g + e*f)/2 + x**3*(17*d**2*g**2/3 + 4*d*e*f*g + e**2*f**2/3) + (48*d**2*g**2 + 64*d*e*f*g + 17*e**2*f**2)*\text{Integral}(d**2, x)/e**2$

**Mathematica [A]** time = 0.278589, size = 185, normalized size = 1.05

$$\begin{aligned}
 & -\frac{16d^4(dg+ef)^2}{e^3(ex-d)} + \frac{1}{3}x^3(17d^2g^2+12defg+e^2f^2) \\
 & + \frac{dx^2(16d^2g^2+17defg+3e^2f^2)}{e} + \frac{d^2x(48d^2g^2+64defg+17e^2f^2)}{e^2} \\
 & + \frac{32d^3(2d^2g^2+3defg+e^2f^2)\log(d-ex)}{e^3} + \frac{1}{2}egx^4(3dg+ef) + \frac{1}{5}e^2g^2x^5
 \end{aligned}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^6\*(f + g\*x)^2)/(d^2 - e^2\*x^2)^2,x]

[Out] (d^2\*(17\*e^2\*f^2 + 64\*d\*e\*f\*g + 48\*d^2\*g^2)\*x)/e^2 + (d\*(3\*e^2\*f^2 + 17\*d\*e\*f\*g + 16\*d^2\*g^2)\*x^2)/e + ((e^2\*f^2 + 12\*d\*e\*f\*g + 17\*d^2\*g^2)\*x^3)/3 + (e\*g\*(e\*f + 3\*d\*g)\*x^4)/2 + (e^2\*g^2\*x^5)/5 - (16\*d^4\*(e\*f + d\*g)^2)/(e^3\*(-d + e\*x)) + (32\*d^3\*(e^2\*f^2 + 3\*d\*e\*f\*g + 2\*d^2\*g^2)\*Log[d - e\*x])/e^3

**Maple [A]** time = 0.013, size = 245, normalized size = 1.4

$$\begin{aligned}
 & \frac{e^2g^2x^5}{5} + \frac{3ex^4dg^2}{2} + \frac{e^2x^4fg}{2} + \frac{17x^3d^2g^2}{3} + 4ex^3dfg + \frac{e^2x^3f^2}{3} + 16\frac{x^2d^3g^2}{e} \\
 & + 17x^2d^2fg + 3ex^2df^2 + 48\frac{d^4g^2x}{e^2} + 64\frac{d^3fgx}{e} + 17d^2f^2x + 64\frac{d^5\ln(ex-d)g^2}{e^3} \\
 & + 96\frac{d^4\ln(ex-d)fg}{e^2} + 32\frac{d^3\ln(ex-d)f^2}{e} - 16\frac{d^6g^2}{e^3(ex-d)} - 32\frac{d^5fg}{e^2(ex-d)} - 16\frac{d^4f^2}{e(ex-d)}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^6\*(g\*x+f)^2/(-e^2\*x^2+d^2)^2,x)

[Out] 1/5\*e^2\*g^2\*x^5+3/2\*e\*x^4\*d\*g^2+1/2\*e^2\*x^4\*f\*g+17/3\*x^3\*d^2\*g^2+4\*e\*x^3\*d\*f\*g+1/3\*e^2\*x^3\*f^2+16/e\*x^2\*d^3\*g^2+17\*x^2\*d^2\*f\*g+3\*e\*x^2\*d\*f^2+48/e^2\*d^4\*g^2\*x+64/e\*d^3\*f\*g\*x+17\*d^2\*f^2\*x+64\*d^5/e^3\*ln(e\*x-d)\*g^2+96\*d^4/e^2\*ln(e\*x-d)\*f\*g+32\*d^3/e\*ln(e\*x-d)\*f^2-16\*d^6/e^3/(e\*x-d)\*g^2-32\*d^5/e^2/(e\*x-d)\*f\*g-16\*d^4/e/(e\*x-d)\*f^2

**Maxima [A]** time = 0.701551, size = 294, normalized size = 1.66

$$\begin{aligned}
 & \frac{16(d^4e^2f^2+2d^5efg+d^6g^2)}{e^4x-de^3} \\
 & + \frac{6e^4g^2x^5+15(e^4fg+3de^3g^2)x^4+10(e^4f^2+12de^3fg+17d^2e^2g^2)x^3+30(3de^3f^2+17d^2e^2fg+16d^3eg^2)x^2+30(17d^2fg+16d^3eg^2)x+30e^4f^2}{30e^2} \\
 & + \frac{32(d^3e^2f^2+3d^4efg+2d^5g^2)\log(ex-d)}{e^3}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^6\*(g\*x + f)^2/(e^2\*x^2 - d^2)^2,x, algorithm="maxima")

[Out] -16\*(d^4\*e^2\*f^2 + 2\*d^5\*e\*f\*g + d^6\*g^2)/(e^4\*x - d\*e^3) + 1/30\*(6\*e^4\*g^2\*x^5 + 15\*(e^4\*f\*g + 3\*d\*e^3\*g^2)\*x^4 + 10\*(e^4\*f^2 + 12\*d\*e^3\*f\*g + 17\*d^2\*e^2\*g^2)\*x^3 + 30\*(3\*d\*e^3\*f^2 + 17\*d^2\*e^2\*f\*g + 16\*d^3\*e\*g^2)\*x^2 + 30\*(17\*d^2\*f\*g + 16\*d^3\*e\*g^2)\*x)/e^2 + 32\*(d^3\*e^2\*f^2 + 3\*d^4\*e\*f\*g + 2\*d^5\*g^2)\*log(e\*x - d)/e^3



**Fricas [A]** time = 0.275103, size = 389, normalized size = 2.2

$$\frac{6e^6g^2x^6 - 480d^4e^2f^2 - 960d^5efg - 480d^6g^2 + 3(5e^6fg + 13de^5g^2)x^5 + 5(2e^6f^2 + 21de^5fg + 25d^2e^4g^2)x^4 + 10(8de^5f^2 + 12d^2e^4fg + 12d^3e^3g^2)x^3 + 30(14d^2e^4f^2 + 47d^3e^3fg + 32d^4e^2g^2)x^2 - 30(17d^3e^3f^2 + 64d^4e^2fg + 48d^5e^2g^2)x - 960(d^4e^2f^2 + 3d^5efg + 2d^6g^2 - (d^3e^3f^2 + 3d^4e^2fg + 2d^5efg^2)x) \log(ex - d)}{(e^2x^2 - d^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^6\*(g\*x + f)^2/(e^2\*x^2 - d^2)^2,x, algorithm="fricas")

[Out] 1/30\*(6\*e^6\*g^2\*x^6 - 480\*d^4\*e^2\*f^2 - 960\*d^5\*e\*f\*g - 480\*d^6\*g^2 + 3\*(5\*e^6\*f\*g + 13\*d\*e^5\*g^2)\*x^5 + 5\*(2\*e^6\*f^2 + 21\*d\*e^5\*f\*g + 25\*d^2\*e^4\*g^2)\*x^4 + 10\*(8\*d\*e^5\*f^2 + 39\*d^2\*e^4\*f\*g + 31\*d^3\*e^3\*g^2)\*x^3 + 30\*(14\*d^2\*e^4\*f^2 + 47\*d^3\*e^3\*f\*g + 32\*d^4\*e^2\*g^2)\*x^2 - 30\*(17\*d^3\*e^3\*f^2 + 64\*d^4\*e^2\*f\*g + 48\*d^5\*e^2\*g^2)\*x - 960\*(d^4\*e^2\*f^2 + 3\*d^5\*e\*f\*g + 2\*d^6\*g^2 - (d^3\*e^3\*f^2 + 3\*d^4\*e^2\*f\*g + 2\*d^5\*e\*f\*g^2)\*x) \* log(e\*x - d))/(e^4\*x - d\*e^3)

**Sympy [A]** time = 4.09398, size = 204, normalized size = 1.15

$$\frac{32d^3(dg + ef)(2dg + ef)\log(-d + ex)}{e^3} + \frac{e^2g^2x^5}{5} + x^4\left(\frac{3deg^2}{2} + \frac{e^2fg}{2}\right) + x^3\left(\frac{17d^2g^2}{3} + 4defg + \frac{e^2f^2}{3}\right) - \frac{16d^6g^2 + 32d^5efg + 16d^4e^2f^2}{-de^3 + e^4x} + \frac{x^2(16d^3g^2 + 17d^2efg + 3de^2f^2)}{e} + \frac{x(48d^4g^2 + 64d^3efg + 17d^2e^2f^2)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*6\*(g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*2,x)

[Out] 32\*d\*\*3\*(d\*g + e\*f)\*(2\*d\*g + e\*f)\*log(-d + e\*x)/e\*\*3 + e\*\*2\*g\*\*2\*x\*\*5/5 + x\*\*4\*(3\*d\*e\*g\*\*2/2 + e\*\*2\*f\*g/2) + x\*\*3\*(17\*d\*\*2\*g\*\*2/3 + 4\*d\*e\*f\*g + e\*\*2\*f\*\*2/3) - (16\*d\*\*6\*g\*\*2 + 32\*d\*\*5\*e\*f\*g + 16\*d\*\*4\*e\*\*2\*f\*\*2)/(-d\*e\*\*3 + e\*\*4\*x) + x\*\*2\*(16\*d\*\*3\*g\*\*2 + 17\*d\*\*2\*e\*f\*g + 3\*d\*e\*\*2\*f\*\*2)/e + x\*(48\*d\*\*4\*g\*\*2 + 64\*d\*\*3\*e\*f\*g + 17\*d\*\*2\*e\*\*2\*f\*\*2)/e\*\*2

**GIAC/XCAS [A]** time = 0.303288, size = 441, normalized size = 2.49

$$16(2d^5g^2e^5 + 3d^4fge^6 + d^3f^2e^7)e^{(-8)}\ln(|x^2e^2 - d^2|) + \frac{1}{30}(6g^2x^5e^{22} + 45dg^2x^4e^{21} + 170d^2g^2x^3e^{20} + 480d^3g^2x^2e^{19} + 1440d^4g^2xe^{18} + 15fgx^4e^{22} + 120dfgx^3e^{21} + 510d^2fgx^2e^{20} + 1920d^3fgxe^{19} + 90d^4fg^2e^{18} + 15d^5fg^2e^{17} + 120d^6fg^2e^{16} + 15d^7fg^2e^{15}) + \frac{16(2d^6g^2e^6 + 3d^5fge^7 + d^4f^2e^8)e^{(-9)}\ln\left(\frac{|2xe^2 - 2d|e|}{|2xe^2 + 2d|e|}\right)}{|d|} - \frac{16(d^7g^2e^5 + 2d^6fge^6 + d^5f^2e^7 + (d^6g^2e^6 + 2d^5fge^7 + d^4f^2e^8)x)e^{(-8)}}{x^2e^2 - d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^6\*(g\*x + f)^2/(e^2\*x^2 - d^2)^2,x, algorithm="giac")

[Out] 16\*(2\*d^5\*g^2\*e^5 + 3\*d^4\*f\*g\*e^6 + d^3\*f^2\*e^7)\*e^(-8)\*ln(abs(x^2\*e^2 - d^2)) + 1/30\*(6\*g^2\*x^5\*e^22 + 45\*d\*g^2\*x^4\*e^21 + 170\*d^2\*g^2\*x^3\*e^20 + 480\*d^3\*g^2\*x^2\*e^19 + 1440\*d^4\*g^2\*x\*e^18 + 15\*f\*g\*x^4\*e^22 + 120\*d\*f\*g\*x^3\*e^21 + 510\*d^2\*f\*g\*x^2\*e^20 + 1920\*d^3\*f\*g\*x\*e^19 + 90\*d^4\*f\*g^2\*e^18 + 15\*d^5\*f\*g^2\*e^17 + 120\*d^6\*f\*g^2\*e^16 + 15\*d^7\*f\*g^2\*e^15)

$$\begin{aligned}
 & *x^*e^{20})^*e^{(-20)} + 16^*(2^*d^6^*g^2^*e^6 + 3^*d^5^*f^*g^*e^7 + d^4^*f^2^*e^8) \\
 & *e^{(-9)}^*\ln(\text{abs}(2^*x^*e^2 - 2^*\text{abs}(d)^*e)/\text{abs}(2^*x^*e^2 + 2^*\text{abs}(d)^*e)) \\
 & / \text{abs}(d) - 16^*(d^7^*g^2^*e^5 + 2^*d^6^*f^*g^*e^6 + d^5^*f^2^*e^7 + (d^6^*g^2^*e^6 + 2^*d^5^*f^*g^*e^7 + d^4^*f^2^*e^8)^*x) \\
 & *e^{(-8)}/(x^2^*e^2 - d^2)
 \end{aligned}$$

$$3.559 \quad \int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^2} dx$$

**Optimal.** Leaf size=146

$$\frac{8d^3(dg+ef)^2}{e^3(d-ex)} + \frac{4d^2(dg+ef)(7dg+3ef)\log(d-ex)}{e^3} + \frac{x^2(12d^2g^2+10defg+e^2f^2)}{2e} \\ + \frac{dx(20d^2g^2+24defg+5e^2f^2)}{e^2} + \frac{1}{3}gx^3(5dg+2ef) + \frac{1}{4}eg^2x^4$$

[Out]  $(d*(5*e^2*f^2 + 24*d*e*f*g + 20*d^2*g^2)*x)/e^2 + ((e^2*f^2 + 10*d*e*f*g + 12*d^2*g^2)*x^2)/(2*e) + (g*(2*e*f + 5*d*g)*x^3)/3 + (e*g^2*x^4)/4 + (8*d^3*(e*f + d*g)^2)/(e^3*(d - e*x)) + (4*d^2*(e*f + d*g)*(3*e*f + 7*d*g)*\text{Log}[d - e*x])/e^3$

**Rubi [A]** time = 0.38107, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$\frac{8d^3(dg+ef)^2}{e^3(d-ex)} + \frac{4d^2(dg+ef)(7dg+3ef)\log(d-ex)}{e^3} + \frac{x^2(12d^2g^2+10defg+e^2f^2)}{2e} \\ + \frac{dx(20d^2g^2+24defg+5e^2f^2)}{e^2} + \frac{1}{3}gx^3(5dg+2ef) + \frac{1}{4}eg^2x^4$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^5*(f + g*x)^2/(d^2 - e^2*x^2)^2, x]$

[Out]  $(d*(5*e^2*f^2 + 24*d*e*f*g + 20*d^2*g^2)*x)/e^2 + ((e^2*f^2 + 10*d*e*f*g + 12*d^2*g^2)*x^2)/(2*e) + (g*(2*e*f + 5*d*g)*x^3)/3 + (e*g^2*x^4)/4 + (8*d^3*(e*f + d*g)^2)/(e^3*(d - e*x)) + (4*d^2*(e*f + d*g)*(3*e*f + 7*d*g)*\text{Log}[d - e*x])/e^3$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{8d^3(dg+ef)^2}{e^3(d-ex)} + \frac{4d^2(dg+ef)(7dg+3ef)\log(d-ex)}{e^3} + \frac{eg^2x^4}{4} + \frac{gx^3(5dg+2ef)}{3} \\ + \frac{(12d^2g^2+10defg+e^2f^2)\int x dx}{e} + \frac{(20d^2g^2+24defg+5e^2f^2)\int d dx}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)**5*(g*x+f)**2/(-e**2*x**2+d**2)**2, x)$

[Out]  $8*d**3*(d*g + e*f)**2/(e**3*(d - e*x)) + 4*d**2*(d*g + e*f)*(7*d*g + 3*e*f)*\log(d - e*x)/e**3 + e*g**2*x**4/4 + g*x**3*(5*d*g + 2*e*f)/3 + (12*d**2*g**2 + 10*d*e*f*g + e**2*f**2)*\text{Integral}(x, x)/e + (20*d**2*g**2 + 24*d*e*f*g + 5*e**2*f**2)*\text{Integral}(d, x)/e**2$

**Mathematica [A]** time = 0.176843, size = 154, normalized size = 1.05

$$-\frac{8d^3(dg+ef)^2}{e^3(ex-d)} + \frac{x^2(12d^2g^2+10defg+e^2f^2)}{2e} + \frac{dx(20d^2g^2+24defg+5e^2f^2)}{e^2} \\ + \frac{4d^2(7d^2g^2+10defg+3e^2f^2)\log(d-ex)}{e^3} + \frac{1}{3}gx^3(5dg+2ef) + \frac{1}{4}eg^2x^4$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^5\*(f + g\*x)^2)/(d^2 - e^2\*x^2)^2, x]

[Out] (d\*(5\*e^2\*f^2 + 24\*d\*e\*f\*g + 20\*d^2\*g^2)\*x)/e^2 + ((e^2\*f^2 + 10\*d\*e\*f\*g + 12\*d^2\*g^2)\*x^2)/(2\*e) + (g\*(2\*e\*f + 5\*d\*g)\*x^3)/3 + (e\*g^2\*x^4)/4 - (8\*d^3\*(e\*f + d\*g)^2)/(e^3\*(-d + e\*x)) + (4\*d^2\*(3\*e^2\*f^2 + 10\*d\*e\*f\*g + 7\*d^2\*g^2)\*Log[d - e\*x])/e^3

**Maple [A]** time = 0.014, size = 204, normalized size = 1.4

$$\begin{aligned} & \frac{eg^2x^4}{4} + \frac{5x^3dg^2}{3} + \frac{2ex^3fg}{3} + 6\frac{x^2d^2g^2}{e} + 5x^2dfg + \frac{ex^2f^2}{2} + 20\frac{d^3g^2x}{e^2} \\ & + 24\frac{d^2fgx}{e} + 5df^2x + 28\frac{d^4\ln(ex-d)g^2}{e^3} + 40\frac{d^3\ln(ex-d)fg}{e^2} \\ & + 12\frac{d^2\ln(ex-d)f^2}{e} - 8\frac{d^5g^2}{e^3(ex-d)} - 16\frac{d^4fg}{e^2(ex-d)} - 8\frac{d^3f^2}{e(ex-d)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^5\*(g\*x+f)^2/(-e^2\*x^2+d^2)^2, x)

[Out] 1/4\*e\*g^2\*x^4+5/3\*x^3\*d\*g^2+2/3\*e\*x^3\*f\*g+6/e\*x^2\*d^2\*g^2+5\*x^2\*d\*f\*g+1/2\*e\*x^2\*f^2+20/e^2\*d^3\*g^2\*x+24/e\*d^2\*f\*g\*x+5\*d\*f^2\*x+28\*d^4/e^3\*ln(e\*x-d)\*g^2+40\*d^3/e^2\*ln(e\*x-d)\*f\*g+12\*d^2/e\*ln(e\*x-d)\*f^2-8\*d^5/e^3/(e\*x-d)\*g^2-16\*d^4/e^2/(e\*x-d)\*f\*g-8\*d^3/e/(e\*x-d)\*f^2

**Maxima [A]** time = 0.689217, size = 246, normalized size = 1.68

$$\begin{aligned} & \frac{8(d^3e^2f^2 + 2d^4efg + d^5g^2)}{e^4x - de^3} \\ & + \frac{3e^3g^2x^4 + 4(2e^3fg + 5de^2g^2)x^3 + 6(e^3f^2 + 10de^2fg + 12d^2eg^2)x^2 + 12(5de^2f^2 + 24d^2efg + 20d^3g^2)x}{12e^2} \\ & + \frac{4(3d^2e^2f^2 + 10d^3efg + 7d^4g^2)\log(ex-d)}{e^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^5\*(g\*x + f)^2/(e^2\*x^2 - d^2)^2, x, algorithm="maxima")

[Out] -8\*(d^3\*e^2\*f^2 + 2\*d^4\*e\*f\*g + d^5\*g^2)/(e^4\*x - d\*e^3) + 1/12\*(3\*e^3\*g^2\*x^4 + 4\*(2\*e^3\*f\*g + 5\*d\*e^2\*g^2)\*x^3 + 6\*(e^3\*f^2 + 10\*d\*e^2\*f\*g + 12\*d^2\*e\*g^2)\*x^2 + 12\*(5\*d\*e^2\*f^2 + 24\*d^2\*e\*f\*g + 20\*d^3\*g^2)\*x)/e^2 + 4\*(3\*d^2\*e^2\*f^2 + 10\*d^3\*e\*f\*g + 7\*d^4\*g^2)\*log(e\*x - d)/e^3

**Fricas [A]** time = 0.275875, size = 339, normalized size = 2.32

$$3e^5g^2x^5 - 96d^3e^2f^2 - 192d^4efg - 96d^5g^2 + (8e^5fg + 17de^4g^2)x^4 + 2(3e^5f^2 + 26de^4fg + 26d^2e^3g^2)x^3 + 6(9de^4f^2 + 3e^5fg + 17de^4g^2)x^2 + 12(5de^2f^2 + 24d^2efg + 20d^3g^2)x + 4(3d^2e^2f^2 + 10d^3efg + 7d^4g^2)\log(ex-d)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^5\*(g\*x + f)^2/(e^2\*x^2 - d^2)^2, x, algorithm="fricas")

[Out] 1/12\*(3\*e^5\*g^2\*x^5 - 96\*d^3\*e^2\*f^2 - 192\*d^4\*e\*f\*g - 96\*d^5\*g^2 + (8\*e^5\*f\*g + 17\*d\*e^4\*g^2)\*x^4 + 2\*(3\*e^5\*f^2 + 26\*d\*e^4\*f\*g +

$$26*d^2*e^3*g^2)*x^3 + 6*(9*d*e^4*f^2 + 38*d^2*e^3*f*g + 28*d^3*e^2*g^2)*x^2 - 12*(5*d^2*e^3*f^2 + 24*d^3*e^2*f*g + 20*d^4*e*g^2)*x - 48*(3*d^3*e^2*f^2 + 10*d^4*e*f*g + 7*d^5*g^2 - (3*d^2*e^3*f^2 + 10*d^3*e^2*f*g + 7*d^4*e*g^2)*x)*\log(e*x - d)/(e^4*x - d*e^3)$$

**Sympy [A]** time = 3.69083, size = 167, normalized size = 1.14

$$\frac{4d^2(dg + ef)(7dg + 3ef)\log(-d + ex)}{e^3} + \frac{eg^2x^4}{4} + x^3\left(\frac{5dg^2}{3} + \frac{2efg}{3}\right) - \frac{8d^5g^2 + 16d^4efg + 8d^3e^2f^2}{-de^3 + e^4x} + \frac{x^2(12d^2g^2 + 10defg + e^2f^2)}{2e} + \frac{x(20d^3g^2 + 24d^2efg + 5de^2f^2)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*5\*(g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*2,x)

[Out] 4\*d\*\*2\*(d\*g + e\*f)\*(7\*d\*g + 3\*e\*f)\*log(-d + e\*x)/e\*\*3 + e\*g\*\*2\*x\*\*4/4 + x\*\*3\*(5\*d\*g\*\*2/3 + 2\*e\*f\*g/3) - (8\*d\*\*5\*g\*\*2 + 16\*d\*\*4\*e\*f\*g + 8\*d\*\*3\*e\*\*2\*f\*\*2)/(-d\*e\*\*3 + e\*\*4\*x) + x\*\*2\*(12\*d\*\*2\*g\*\*2 + 10\*d\*e\*f\*g + e\*\*2\*f\*\*2)/(2\*e) + x\*(20\*d\*\*3\*g\*\*2 + 24\*d\*\*2\*e\*f\*g + 5\*d\*e\*\*2\*f\*\*2)/e\*\*2

**GIAC/XCAS [A]** time = 0.286695, size = 393, normalized size = 2.69

$$2(7d^4g^2e^5 + 10d^3fge^6 + 3d^2f^2e^7)e^{(-8)}\ln(|x^2e^2 - d^2|) + \frac{1}{12}(3g^2x^4e^{17} + 20dg^2x^3e^{16} + 72d^2g^2x^2e^{15} + 240d^3g^2xe^{14} + 8fgx^3e^{17} + 60dfgx^2e^{16} + 288d^2fgxe^{15} + 6f^2x^2e^{17} + 60df^2xe^{16} + 288d^2f^2xe^{15} + 6f^2x^2e^{17} + 60df^2xe^{16}) + \frac{2(7d^5g^2e^4 + 10d^4fge^5 + 3d^3f^2e^6)e^{(-7)}\ln\left(\frac{|2xe^2-2|d|e|}{|2xe^2+2|d|e|}\right) + \frac{|d|}{x^2e^2 - d^2}8(d^6g^2e^5 + 2d^5fge^6 + d^4f^2e^7 + (d^5g^2e^6 + 2d^4fge^7 + d^3f^2e^8)x)e^{(-8)}}{x^2e^2 - d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^5\*(g\*x + f)^2/(e^2\*x^2 - d^2)^2,x, algorithm="giac")

[Out] 2\*(7\*d^4\*g^2\*e^5 + 10\*d^3\*f\*g\*e^6 + 3\*d^2\*f^2\*e^7)\*e^(-8)\*ln(abs(x^2\*e^2 - d^2)) + 1/12\*(3\*g^2\*x^4\*e^17 + 20\*d\*g^2\*x^3\*e^16 + 72\*d^2\*g^2\*x^2\*e^15 + 240\*d^3\*g^2\*x\*e^14 + 8\*f\*g\*x^3\*e^17 + 60\*d\*f\*g\*x^2\*e^16 + 288\*d^2\*f\*g\*x\*e^15 + 6\*f^2\*x^2\*e^17 + 60\*d\*f^2\*x\*e^16)\*e^(-16) + 2\*(7\*d^5\*g^2\*e^4 + 10\*d^4\*f\*g\*e^5 + 3\*d^3\*f^2\*e^6)\*e^(-7)\*ln(abs(2\*x\*e^2 - 2\*abs(d)\*e)/abs(2\*x\*e^2 + 2\*abs(d)\*e))/abs(d) - 8\*(d^6\*g^2\*e^5 + 2\*d^5\*f\*g\*e^6 + d^4\*f^2\*e^7 + (d^5\*g^2\*e^6 + 2\*d^4\*f\*g\*e^7 + d^3\*f^2\*e^8)\*x)\*e^(-8)/(x^2\*e^2 - d^2)

$$3.560 \quad \int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^2} dx$$

**Optimal.** Leaf size=107

$$\frac{4d^2(dg+ef)^2}{e^3(d-ex)} + \frac{x(8d^2g^2+8defg+e^2f^2)}{e^2} + \frac{4d(dg+ef)(3dg+ef)\log(d-ex)}{e^3} + \frac{gx^2(2dg+ef)}{e} + \frac{g^2x^3}{3}$$

[Out]  $((e^2f^2 + 8d^2efg + 8d^2g^2)x)/e^2 + (g(e^2f + 2d^2g)x^2)/e + (g^2x^3)/3 + (4d^2(e^2f + d^2g)^2)/(e^3(d - ex)) + (4d^2(e^2f + d^2g)(e^2f + 3d^2g)\text{Log}[d - ex])/e^3$

**Rubi [A]** time = 0.291335, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$\frac{4d^2(dg+ef)^2}{e^3(d-ex)} + \frac{x(8d^2g^2+8defg+e^2f^2)}{e^2} + \frac{4d(dg+ef)(3dg+ef)\log(d-ex)}{e^3} + \frac{gx^2(2dg+ef)}{e} + \frac{g^2x^3}{3}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^4\*(f + g\*x)^2)/(d^2 - e^2\*x^2)^2, x]

[Out]  $((e^2f^2 + 8d^2efg + 8d^2g^2)x)/e^2 + (g(e^2f + 2d^2g)x^2)/e + (g^2x^3)/3 + (4d^2(e^2f + d^2g)^2)/(e^3(d - ex)) + (4d^2(e^2f + d^2g)(e^2f + 3d^2g)\text{Log}[d - ex])/e^3$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{4d^2(dg+ef)^2}{e^3(d-ex)} + \frac{4d(dg+ef)(3dg+ef)\log(d-ex)}{e^3} + \frac{g^2x^3}{3} + \frac{2g(2dg+ef)\int x dx}{e} + \frac{(8dg(dg+ef)+e^2f^2)\int f^2 dx}{e^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*4\*(g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*2,x)

[Out]  $4d^2(dg+ef)^2/(e^3(d-ex)) + 4d^2(dg+ef)(3d^2g+e^2f)\log(d-ex)/e^3 + g^2x^3/3 + 2g(2dg+ef)\text{Integral}(x, x)/e + (8dg(dg+ef)+e^2f^2)\text{Integral}(f^2, x)/(e^2f^2)$

**Mathematica [A]** time = 0.157385, size = 115, normalized size = 1.07

$$-\frac{4d^2(dg+ef)^2}{e^3(ex-d)} + \frac{x(8d^2g^2+8defg+e^2f^2)}{e^2} + \frac{4d(3d^2g^2+4defg+e^2f^2)\log(d-ex)}{e^3} + \frac{gx^2(2dg+ef)}{e} + \frac{g^2x^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^4\*(f + g\*x)^2)/(d^2 - e^2\*x^2)^2, x]

[Out]  $((e^2f^2 + 8d^2efg + 8d^2g^2)x)/e^2 + (g(e^2f + 2d^2g)x^2)/e + (g^2x^3)/3 - (4d^2(e^2f + d^2g)^2)/(e^3(-d + ex)) + (4d^2(e^2f + d^2g)(e^2f + 3d^2g)\text{Log}[d - ex])/e^3$

$$(e^2 f^2 + 4 d e f g + 3 d^2 g^2) \operatorname{Log}[d - e x] / e^3$$

**Maple [A]** time = 0.012, size = 167, normalized size = 1.6

$$\frac{g^2 x^3}{3} + 2 \frac{d x^2 g^2}{e} + x^2 f g + 8 \frac{d^2 g^2 x}{e^2} + 8 \frac{d f g x}{e} + f^2 x + 12 \frac{d^3 \ln(ex-d) g^2}{e^3} + 16 \frac{d^2 \ln(ex-d) f g}{e^2} + 4 \frac{d \ln(ex-d) f^2}{e} - 4 \frac{d^4 g^2}{e^3 (ex-d)} - 8 \frac{d^3 f g}{e^2 (ex-d)} - 4 \frac{d^2 f^2}{e (ex-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^2,x)`

[Out]  $1/3 * g^2 * x^3 + 2/e * x^2 * d * g^2 + x^2 * f * g + 8/e^2 * d^2 * g^2 * x + 8/e * d * f * g * x + f^2 * x + 12 * d^3 / e^3 * \ln(e * x - d) * g^2 + 16 * d^2 / e^2 * \ln(e * x - d) * f * g + 4 * d / e * \ln(e * x - d) * f^2 - 4 * d^4 / e^3 / (e * x - d) * g^2 - 8 * d^3 / e^2 / (e * x - d) * f * g - 4 * d^2 / e / (e * x - d) * f^2$

**Maxima [A]** time = 0.690636, size = 190, normalized size = 1.78

$$-\frac{4(d^2 e^2 f^2 + 2 d^3 e f g + d^4 g^2)}{e^4 x - d e^3} + \frac{e^2 g^2 x^3 + 3(e^2 f g + 2 d e g^2) x^2 + 3(e^2 f^2 + 8 d e f g + 8 d^2 g^2) x}{3 e^2} + \frac{4(d e^2 f^2 + 4 d^2 e f g + 3 d^3 g^2) \log(ex-d)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^4*(g*x+f)^2/(e^2*x^2-d^2)^2,x,algorithm="maxima")`

[Out]  $-4 * (d^2 * e^2 * f^2 + 2 * d^3 * e * f * g + d^4 * g^2) / (e^4 * x - d * e^3) + 1/3 * (e^2 * g^2 * x^3 + 3 * (e^2 * f * g + 2 * d * e * g^2) * x^2 + 3 * (e^2 * f^2 + 8 * d * e * f * g + 8 * d^2 * g^2) * x) / e^2 + 4 * (d * e^2 * f^2 + 4 * d^2 * e * f * g + 3 * d^3 * g^2) * \log(e * x - d) / e^3$

**Fricas [A]** time = 0.27675, size = 278, normalized size = 2.6

$$\frac{e^4 g^2 x^4 - 12 d^2 e^2 f^2 - 24 d^3 e f g - 12 d^4 g^2 + (3 e^4 f g + 5 d e^3 g^2) x^3 + 3(e^4 f^2 + 7 d e^3 f g + 6 d^2 e^2 g^2) x^2 - 3(d e^3 f^2 + 8 d^2 e^2 f g - 3(e^4 x - d e^3))}{3(e^4 x - d e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^4*(g*x+f)^2/(e^2*x^2-d^2)^2,x,algorithm="fricas")`

[Out]  $1/3 * (e^4 * g^2 * x^4 - 12 * d^2 * e^2 * f^2 - 24 * d^3 * e * f * g - 12 * d^4 * g^2 + (3 * e^4 * f * g + 5 * d * e^3 * g^2) * x^3 + 3 * (e^4 * f^2 + 7 * d * e^3 * f * g + 6 * d^2 * e^2 * g^2) * x^2 - 3 * (d * e^3 * f^2 + 8 * d^2 * e^2 * f * g + 8 * d^3 * e * g^2) * x - 12 * (d^2 * e^2 * f^2 + 4 * d^3 * e * f * g + 3 * d^4 * g^2 - (d * e^3 * f^2 + 4 * d^2 * e^2 * f * g + 3 * d^3 * e * g^2) * x) * \log(e * x - d)) / (e^4 * x - d * e^3)$

**Sympy [A]** time = 3.14097, size = 122, normalized size = 1.14

$$\frac{4d(dg+ef)(3dg+ef)\log(-d+ex)}{e^3} + \frac{g^2 x^3}{3} - \frac{4d^4 g^2 + 8d^3 e f g + 4d^2 e^2 f^2}{-d e^3 + e^4 x} + \frac{x^2(2d g^2 + e f g)}{e} + \frac{x(8d^2 g^2 + 8d e f g + e^2 f^2)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*4\*(g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*2,x)

[Out]  $4*d*(d*g + e*f)*(3*d*g + e*f)*\log(-d + e*x)/e**3 + g**2*x**3/3 - (4*d**4*g**2 + 8*d**3*e*f*g + 4*d**2*e**2*f**2)/(-d*e**3 + e**4*x) + x**2*(2*d*g**2 + e*f*g)/e + x*(8*d**2*g**2 + 8*d*e*f*g + e**2*f**2)/e**2$

**GIAC/XCAS [A]** time = 0.281117, size = 338, normalized size = 3.16

$$\begin{aligned} & 2(3d^3g^2e^3 + 4d^2fge^4 + df^2e^5)e^{(-6)}\ln(|x^2e^2 - d^2|) \\ & + \frac{1}{3}(g^2x^3e^{12} + 6dg^2x^2e^{11} + 24d^2g^2xe^{10} + 3fgx^2e^{12} + 24dfgxe^{11} + 3f^2xe^{12})e^{(-12)} \\ & + \frac{2(3d^4g^2e^4 + 4d^3fge^5 + d^2f^2e^6)e^{(-7)}\ln\left(\frac{|2xe^2-2|d|e|}{|2xe^2+2|d|e|}\right)}{|d|} \\ & - \frac{4(d^5g^2e^3 + 2d^4fge^4 + d^3f^2e^5 + (d^4g^2e^4 + 2d^3fge^5 + d^2f^2e^6)x)e^{(-6)}}{x^2e^2 - d^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^4\*(g\*x + f)^2/(e^2\*x^2 - d^2)^2,x, algorithm="giac")

[Out]  $2*(3*d^3*g^2*e^3 + 4*d^2*f*g*e^4 + d*f^2*e^5)*e^{(-6)}*\ln(\text{abs}(x^2*e^2 - d^2)) + 1/3*(g^2*x^3*e^{12} + 6*d*g^2*x^2*e^{11} + 24*d^2*g^2*x*e^{10} + 3*f*g*x^2*e^{12} + 24*d*f*g*x*e^{11} + 3*f^2*x*e^{12})*e^{(-12)} + 2*(3*d^4*g^2*e^4 + 4*d^3*f*g*e^5 + d^2*f^2*e^6)*e^{(-7)}*\ln(\text{abs}(2*x*e^2 - 2*\text{abs}(d)*e)/\text{abs}(2*x*e^2 + 2*\text{abs}(d)*e))/\text{abs}(d) - 4*(d^5*g^2*e^3 + 2*d^4*f*g*e^4 + d^3*f^2*e^5 + (d^4*g^2*e^4 + 2*d^3*f*g*e^5 + d^2*f^2*e^6)*x)*e^{(-6)}/(x^2*e^2 - d^2)$



$$3.561 \quad \int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^2} dx$$

**Optimal.** Leaf size=78

$$\frac{2d(dg+ef)^2}{e^3(d-ex)} + \frac{(5dg+ef)(dg+ef)\log(d-ex)}{e^3} + \frac{gx(3dg+2ef)}{e^2} + \frac{g^2x^2}{2e}$$

[Out] (g\*(2\*e\*f + 3\*d\*g)\*x)/e^2 + (g^2\*x^2)/(2\*e) + (2\*d\*(e\*f + d\*g)^2)/(e^3\*(d - e\*x)) + ((e\*f + d\*g)\*(e\*f + 5\*d\*g)\*Log[d - e\*x])/e^3

**Rubi [A]** time = 0.208224, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$\frac{2d(dg+ef)^2}{e^3(d-ex)} + \frac{(5dg+ef)(dg+ef)\log(d-ex)}{e^3} + \frac{gx(3dg+2ef)}{e^2} + \frac{g^2x^2}{2e}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(f + g\*x)^2)/(d^2 - e^2\*x^2)^2, x]

[Out] (g\*(2\*e\*f + 3\*d\*g)\*x)/e^2 + (g^2\*x^2)/(2\*e) + (2\*d\*(e\*f + d\*g)^2)/(e^3\*(d - e\*x)) + ((e\*f + d\*g)\*(e\*f + 5\*d\*g)\*Log[d - e\*x])/e^3

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2d(dg+ef)^2}{e^3(d-ex)} + \frac{g^2 \int x dx}{e} + \frac{(3dg+2ef) \int g dx}{e^2} + \frac{(dg+ef)(5dg+ef)\log(d-ex)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3\*(g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*2, x)

[Out] 2\*d\*(d\*g + e\*f)\*\*2/(e\*\*3\*(d - e\*x)) + g\*\*2\*Integral(x, x)/e + (3\*d\*g + 2\*e\*f)\*Integral(g, x)/e\*\*2 + (d\*g + e\*f)\*(5\*d\*g + e\*f)\*log(d - e\*x)/e\*\*3

**Mathematica [A]** time = 0.116785, size = 83, normalized size = 1.06

$$\frac{2(5d^2g^2 + 6defg + e^2f^2)\log(d-ex) + \frac{4d(dg+ef)^2}{d-ex} + 2egx(3dg+2ef) + e^2g^2x^2}{2e^3}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(f + g\*x)^2)/(d^2 - e^2\*x^2)^2, x]

[Out] (2\*e\*g\*(2\*e\*f + 3\*d\*g)\*x + e^2\*g^2\*x^2 + (4\*d\*(e\*f + d\*g)^2)/(d - e\*x) + 2\*(e^2\*f^2 + 6\*d\*e\*f\*g + 5\*d^2\*g^2)\*Log[d - e\*x])/(2\*e^3)

**Maple [A]** time = 0.012, size = 138, normalized size = 1.8

$$\frac{g^2x^2}{2e} + 3\frac{dg^2x}{e^2} + 2\frac{fgx}{e} + 5\frac{\ln(ex-d)d^2g^2}{e^3} + 6\frac{\ln(ex-d)dfg}{e^2} + \frac{\ln(ex-d)f^2}{e} - 2\frac{d^3g^2}{e^3(ex-d)} - 4\frac{d^2fg}{e^2(ex-d)} - 2\frac{df^2}{e(ex-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^2,x)`

[Out]  $\frac{1}{2} \frac{g^2 x^2}{e^3} + \frac{g^2}{e^2} \frac{x}{d} + \frac{2g}{e^3} \ln(e^2 x - d) + \frac{d^2 g^2 + 6}{e^2} \ln(e^2 x - d) + \frac{d^2 f^2 g + 1}{e^2} \ln(e^2 x - d) + \frac{f^2 - 2d^3}{e^3} \frac{1}{(e^2 x - d)} + \frac{g^2 - 4d^2}{e^2} \frac{1}{(e^2 x - d)} + \frac{f^2 g - 2d}{e^2} \frac{1}{(e^2 x - d)}$

**Maxima [A]** time = 0.687276, size = 140, normalized size = 1.79

$$-\frac{2(de^2 f^2 + 2d^2 efg + d^3 g^2)}{e^4 x - de^3} + \frac{eg^2 x^2 + 2(2efg + 3dg^2)x}{2e^2} + \frac{(e^2 f^2 + 6defg + 5d^2 g^2) \log(ex - d)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^3*(g*x + f)^2/(e^2*x^2 - d^2)^2,x, algorithm="maxima")`

[Out]  $-\frac{2(d^2 e^2 f^2 + 2d^2 e f g + d^3 g^2)}{(e^4 x - d^2 e^3)} + \frac{1}{2} \frac{(e^2 g^2 x^2 + 2(2e f g + 3d g^2)x)}{e^2} + \frac{(e^2 f^2 + 6d^2 e f g + 5d^2 g^2) \log(e^2 x - d)}{e^3}$

**Fricas [A]** time = 0.275545, size = 212, normalized size = 2.72

$$\frac{e^3 g^2 x^3 - 4de^2 f^2 - 8d^2 efg - 4d^3 g^2 + (4e^3 fg + 5de^2 g^2)x^2 - 2(2de^2 fg + 3d^2 eg^2)x - 2(de^2 f^2 + 6d^2 efg + 5d^3 g^2 - (e^3 fg^2 + 5d^2 e^2 f^2)) \log(e^2 x - d)}{2(e^4 x - de^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^3*(g*x + f)^2/(e^2*x^2 - d^2)^2,x, algorithm="fricas")`

[Out]  $\frac{1}{2} \frac{(e^3 g^2 x^3 - 4d^2 e^2 f^2 - 8d^2 e f g - 4d^3 g^2 + (4e^3 fg + 5de^2 g^2)x^2 - 2(2de^2 fg + 3d^2 eg^2)x - 2(d^2 e^2 f^2 + 6d^2 e f g + 5d^3 g^2 - (e^3 fg^2 + 5d^2 e^2 f^2)) \log(e^2 x - d))}{(e^4 x - d^2 e^3)}$

**Sympy [A]** time = 2.76637, size = 92, normalized size = 1.18

$$-\frac{2d^3 g^2 + 4d^2 efg + 2de^2 f^2}{-de^3 + e^4 x} + \frac{g^2 x^2}{2e} + \frac{x(3dg^2 + 2efg)}{e^2} + \frac{(dg + ef)(5dg + ef) \log(-d + ex)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**3*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)`

[Out]  $-\frac{(2d^3 g^2 + 4d^2 e f g + 2d^2 e^2 f^2)}{(-d^2 e^3 + e^4 x)} + \frac{g^2 x^2}{2e} + \frac{x(3d g^2 + 2e f g)}{e^2} + \frac{(d g + e f)^2 (5d g + e f) \log(-d + e x)}{e^3}$

**GIAC/XCAS [A]** time = 0.291678, size = 286, normalized size = 3.67

$$\frac{1}{2} (5 d^2 g^2 e^3 + 6 d f g e^4 + f^2 e^5) e^{(-6)} \ln(|x^2 e^2 - d^2|) + \frac{1}{2} (g^2 x^2 e^7 + 6 d g^2 x e^6 + 4 f g x e^7) e^{(-8)}$$

$$+ \frac{(5 d^3 g^2 e^2 + 6 d^2 f g e^3 + d f^2 e^4) e^{(-5)} \ln\left(\frac{|2 x e^2 - 2 |d| e|}{|2 x e^2 + 2 |d| e|}\right)}{2 |d|}$$

$$- \frac{2 (d^4 g^2 e^3 + 2 d^3 f g e^4 + d^2 f^2 e^5 + (d^3 g^2 e^4 + 2 d^2 f g e^5 + d f^2 e^6) x) e^{(-6)}}{x^2 e^2 - d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(g\*x + f)^2/(e^2\*x^2 - d^2)^2,x, algorithm="giac")

[Out] 1/2\*(5\*d^2\*g^2\*e^3 + 6\*d\*f\*g\*e^4 + f^2\*e^5)\*e^(-6)\*ln(abs(x^2\*e^2 - d^2)) + 1/2\*(g^2\*x^2\*e^7 + 6\*d\*g^2\*x\*e^6 + 4\*f\*g\*x\*e^7)\*e^(-8) + 1/2\*(5\*d^3\*g^2\*e^2 + 6\*d^2\*f\*g\*e^3 + d\*f^2\*e^4)\*e^(-5)\*ln(abs(2\*x\*e^2 - 2\*abs(d)\*e)/abs(2\*x\*e^2 + 2\*abs(d)\*e))/abs(d) - 2\*(d^4\*g^2\*e^3 + 2\*d^3\*f\*g\*e^4 + d^2\*f^2\*e^5 + (d^3\*g^2\*e^4 + 2\*d^2\*f\*g\*e^5 + d\*f^2\*e^6)\*x)\*e^(-6)/(x^2\*e^2 - d^2)

$$3.562 \quad \int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^2} dx$$

**Optimal.** Leaf size=50

$$\frac{(dg+ef)^2}{e^3(d-ex)} + \frac{2g(dg+ef)\log(d-ex)}{e^3} + \frac{g^2x}{e^2}$$

[Out] (g^2\*x)/e^2 + (e\*f + d\*g)^2/(e^3\*(d - e\*x)) + (2\*g\*(e\*f + d\*g)\*Log[d - e\*x])/e^3

**Rubi [A]** time = 0.120194, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$\frac{(dg+ef)^2}{e^3(d-ex)} + \frac{2g(dg+ef)\log(d-ex)}{e^3} + \frac{g^2x}{e^2}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^2\*(f + g\*x)^2)/(d^2 - e^2\*x^2)^2, x]

[Out] (g^2\*x)/e^2 + (e\*f + d\*g)^2/(e^3\*(d - e\*x)) + (2\*g\*(e\*f + d\*g)\*Log[d - e\*x])/e^3

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$g^2 \int \frac{1}{e^2} dx + \frac{2g(dg+ef)\log(d-ex)}{e^3} + \frac{(dg+ef)^2}{e^3(d-ex)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*2\*(g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*2, x)

[Out] g\*\*2\*Integral(e\*\*(-2), x) + 2\*g\*(d\*g + e\*f)\*log(d - e\*x)/e\*\*3 + (d\*g + e\*f)\*\*2/(e\*\*3\*(d - e\*x))

**Mathematica [A]** time = 0.0697832, size = 46, normalized size = 0.92

$$\frac{\frac{(dg+ef)^2}{d-ex} + 2g(dg+ef)\log(d-ex) + eg^2x}{e^3}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^2\*(f + g\*x)^2)/(d^2 - e^2\*x^2)^2, x]

[Out] (e\*g^2\*x + (e\*f + d\*g)^2/(d - e\*x) + 2\*g\*(e\*f + d\*g)\*Log[d - e\*x])/e^3

**Maple [A]** time = 0.01, size = 96, normalized size = 1.9

$$\frac{g^2x}{e^2} + 2 \frac{d \ln(ex-d)g^2}{e^3} + 2 \frac{\ln(ex-d)fg}{e^2} - \frac{d^2g^2}{e^3(ex-d)} - 2 \frac{dfg}{e^2(ex-d)} - \frac{f^2}{e(ex-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^2,x)`

[Out]  $g^2x/e^2+2/e^3*d*\ln(e*x-d)*g^2+2/e^2*\ln(e*x-d)*f*g-1/e^3/(e*x-d)*d^2*g^2-2/e^2/(e*x-d)*d*f*g-1/e/(e*x-d)*f^2$

**Maxima [A]** time = 0.691849, size = 93, normalized size = 1.86

$$\frac{g^2x}{e^2} - \frac{e^2f^2 + 2defg + d^2g^2}{e^4x - de^3} + \frac{2(efg + dg^2)\log(ex - d)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^2*(g*x+f)^2/(e^2*x^2-d^2)^2,x,algorithm="maxima")`

[Out]  $g^2x/e^2 - (e^2f^2 + 2*d*e*f*g + d^2g^2)/(e^4x - d*e^3) + 2*(e*f*g + d*g^2)*\log(e*x - d)/e^3$

**Fricas [A]** time = 0.267389, size = 128, normalized size = 2.56

$$\frac{e^2g^2x^2 - deg^2x - e^2f^2 - 2defg - d^2g^2 - 2(defg + d^2g^2 - (e^2fg + deg^2)x)\log(ex - d)}{e^4x - de^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^2*(g*x+f)^2/(e^2*x^2-d^2)^2,x,algorithm="fricas")`

[Out]  $(e^2g^2x^2 - d*e*g^2*x - e^2f^2 - 2*d*e*f*g - d^2g^2 - 2*(d*e*f*g + d^2g^2 - (e^2f*g + d*e*g^2)*x)*\log(e*x - d))/(e^4x - d*e^3)$

**Sympy [A]** time = 2.1251, size = 60, normalized size = 1.2

$$-\frac{d^2g^2 + 2defg + e^2f^2}{-de^3 + e^4x} + \frac{g^2x}{e^2} + \frac{2g(dg + ef)\log(-d + ex)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**2*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)`

[Out]  $-(d**2*g**2 + 2*d*e*f*g + e**2*f**2)/(-d*e**3 + e**4*x) + g**2*x/e**2 + 2*g*(d*g + e*f)*\log(-d + e*x)/e**3$

**GIAC/XCAS [A]** time = 0.279072, size = 216, normalized size = 4.32

$$g^2xe^{(-2)} + (dg^2e + fge^2)e^{(-4)}\ln(|x^2e^2 - d^2|) + \frac{(d^2g^2e^2 + dfge^3)e^{(-5)}\ln\left(\frac{|2xe^2-2|d|e|}{|2xe^2+2|d|e|}\right)}{|d|} - \frac{(d^3g^2e + 2d^2fge^2 + df^2e^3 + (d^2g^2e^2 + 2dfge^3 + f^2e^4)x)e^{(-4)}}{x^2e^2 - d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^2*(g*x + f)^2/(e^2*x^2 - d^2)^2,x, algorithm="giac")
```

```
[Out] g^2*x*e^(-2) + (d*g^2*e + f*g*e^2)*e^(-4)*ln(abs(x^2*e^2 - d^2))
+ (d^2*g^2*e^2 + d*f*g*e^3)*e^(-5)*ln(abs(2*x*e^2 - 2*abs(d)*e)/a
bs(2*x*e^2 + 2*abs(d)*e))/abs(d) - (d^3*g^2*e + 2*d^2*f*g*e^2 + d
*f^2*e^3 + (d^2*g^2*e^2 + 2*d*f*g*e^3 + f^2*e^4)*x)*e^(-4)/(x^2*e
^2 - d^2)
```

$$3.563 \quad \int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^2} dx$$

**Optimal.** Leaf size=86

$$\frac{(ef-dg)^2 \log(d+ex)}{4d^2e^3} - \frac{(ef-3dg)(dg+ef) \log(d-ex)}{4d^2e^3} + \frac{(dg+ef)^2}{2de^3(d-ex)}$$

[Out] (e\*f + d\*g)^2/(2\*d\*e^3\*(d - e\*x)) - ((e\*f - 3\*d\*g)\*(e\*f + d\*g)\*Log[d - e\*x])/(4\*d^2\*e^3) + ((e\*f - d\*g)^2\*Log[d + e\*x])/(4\*d^2\*e^3)

**Rubi [A]** time = 0.175445, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$

$$\frac{(ef-dg)^2 \log(d+ex)}{4d^2e^3} - \frac{(ef-3dg)(dg+ef) \log(d-ex)}{4d^2e^3} + \frac{(dg+ef)^2}{2de^3(d-ex)}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*(f + g\*x)^2)/(d^2 - e^2\*x^2)^2, x]

[Out] (e\*f + d\*g)^2/(2\*d\*e^3\*(d - e\*x)) - ((e\*f - 3\*d\*g)\*(e\*f + d\*g)\*Log[d - e\*x])/(4\*d^2\*e^3) + ((e\*f - d\*g)^2\*Log[d + e\*x])/(4\*d^2\*e^3)

**Rubi in Sympy [A]** time = 29.9269, size = 73, normalized size = 0.85

$$\frac{(dg+ef)^2}{2de^3(d-ex)} + \frac{(dg-ef)^2 \log(d+ex)}{4d^2e^3} + \frac{(dg+ef)(3dg-ef) \log(d-ex)}{4d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*(g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*2, x)

[Out] (d\*g + e\*f)\*\*2/(2\*d\*e\*\*3\*(d - e\*x)) + (d\*g - e\*f)\*\*2\*log(d + e\*x)/(4\*d\*\*2\*e\*\*3) + (d\*g + e\*f)\*(3\*d\*g - e\*f)\*log(d - e\*x)/(4\*d\*\*2\*e\*\*3)

**Mathematica [A]** time = 0.0799993, size = 91, normalized size = 1.06

$$\frac{(d-ex)(3d^2g^2 + 2defg - e^2f^2) \log(d-ex) + (d-ex)(ef-dg)^2 \log(d+ex) + 2d(dg+ef)^2}{4d^2e^3(d-ex)}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(f + g\*x)^2)/(d^2 - e^2\*x^2)^2, x]

[Out] (2\*d\*(e\*f + d\*g)^2 + (-e^2\*f^2) + 2\*d\*e\*f\*g + 3\*d^2\*g^2)\*(d - e\*x)\*Log[d - e\*x] + (e\*f - d\*g)^2\*(d - e\*x)\*Log[d + e\*x]/(4\*d^2\*e^3\*(d - e\*x))

**Maple [A]** time = 0.016, size = 156, normalized size = 1.8

$$\begin{aligned} & -\frac{g^2 d}{2e^3(ex-d)} - \frac{fg}{e^2(ex-d)} - \frac{f^2}{2de(ex-d)} + \frac{3\ln(ex-d)g^2}{4e^3} + \frac{\ln(ex-d)fg}{2e^2d} \\ & - \frac{\ln(ex-d)f^2}{4d^2e} + \frac{\ln(ex+d)g^2}{4e^3} - \frac{\ln(ex+d)fg}{2e^2d} + \frac{\ln(ex+d)f^2}{4d^2e} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)*(g*x+f)^2/(-e^2*x^2+d^2)^2,x)`

[Out] `-1/2/e^3*d/(e*x-d)*g^2-1/e^2/(e*x-d)*f*g-1/2/e/d/(e*x-d)*f^2+3/4/e^3*ln(e*x-d)*g^2+1/2/e^2/d*ln(e*x-d)*f*g-1/4/e/d^2*ln(e*x-d)*f^2+1/4/e^3*ln(e*x+d)*g^2-1/2/e^2/d*ln(e*x+d)*f*g+1/4/e/d^2*ln(e*x+d)*f^2`

**Maxima [A]** time = 0.688366, size = 154, normalized size = 1.79

$$-\frac{e^2 f^2 + 2 d e f g + d^2 g^2}{2(d e^4 x - d^2 e^3)} + \frac{(e^2 f^2 - 2 d e f g + d^2 g^2) \log(ex + d)}{4 d^2 e^3} - \frac{(e^2 f^2 - 2 d e f g - 3 d^2 g^2) \log(ex - d)}{4 d^2 e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)*(g*x + f)^2/(e^2*x^2 - d^2)^2,x, algorithm="maxima")`

[Out] `-1/2*(e^2*f^2 + 2*d*e*f*g + d^2*g^2)/(d*e^4*x - d^2*e^3) + 1/4*(e^2*f^2 - 2*d*e*f*g + d^2*g^2)*log(e*x + d)/(d^2*e^3) - 1/4*(e^2*f^2 - 2*d*e*f*g - 3*d^2*g^2)*log(e*x - d)/(d^2*e^3)`

**Fricas [A]** time = 0.278749, size = 227, normalized size = 2.64

$$\frac{2de^2f^2 + 4d^2efg + 2d^3g^2 + (de^2f^2 - 2d^2efg + d^3g^2 - (e^3f^2 - 2de^2fg + d^2eg^2)x) \log(ex + d) - (de^2f^2 - 2d^2efg - 3d^3g^2)x \log(ex - d)}{4(d^2e^4x - d^3e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)*(g*x + f)^2/(e^2*x^2 - d^2)^2,x, algorithm="fricas")`

[Out] `-1/4*(2*d*e^2*f^2 + 4*d^2*e*f*g + 2*d^3*g^2 + (d*e^2*f^2 - 2*d^2*e*f*g + d^3*g^2 - (e^3*f^2 - 2*d*e^2*f*g + d^2*e*g^2)*x)*log(e*x + d) - (d*e^2*f^2 - 2*d^2*e*f*g - 3*d^3*g^2 - (e^3*f^2 - 2*d*e^2*f*g - 3*d^2*e*g^2)*x)*log(e*x - d))/(d^2*e^4*x - d^3*e^3)`

**Sympy [A]** time = 4.13262, size = 180, normalized size = 2.09

$$\begin{aligned} & \frac{d^2 g^2 + 2 d e f g + e^2 f^2}{-2 d^2 e^3 + 2 d e^4 x} + \frac{(d g - e f)^2 \log\left(x + \frac{2 d^3 g^2 - d(d g - e f)^2}{d^2 e g^2 + 2 d e^2 f g - e^3 f^2}\right)}{4 d^2 e^3} \\ & + \frac{(d g + e f)(3 d g - e f) \log\left(x + \frac{2 d^3 g^2 - d(d g + e f)(3 d g - e f)}{d^2 e g^2 + 2 d e^2 f g - e^3 f^2}\right)}{4 d^2 e^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(g*x+f)**2/(-e**2*x**2+d**2)**2,x)`



```
[Out] -(d**2*g**2 + 2*d*e*f*g + e**2*f**2)/(-2*d**2*e**3 + 2*d*e**4*x)
+ (d*g - e*f)**2*log(x + (2*d**3*g**2 - d*(d*g - e*f)**2)/(d**2*e
*g**2 + 2*d*e**2*f*g - e**3*f**2))/(4*d**2*e**3) + (d*g + e*f)*(3
*d*g - e*f)*log(x + (2*d**3*g**2 - d*(d*g + e*f)*(3*d*g - e*f))/(
d**2*e*g**2 + 2*d*e**2*f*g - e**3*f**2))/(4*d**2*e**3)
```

**GIAC/XCAS [A]** time = 0.276982, size = 215, normalized size = 2.5

$$\frac{1}{2} g^2 e^{(-3)} \ln(|x^2 e^2 - d^2|) + \frac{(d^2 g^2 + 2 d f g e - f^2 e^2) e^{(-3)} \ln\left(\frac{|2 x e^2 - 2 |d| e|}{|2 x e^2 + 2 |d| e|}\right)}{4 d |d|}$$

$$- \frac{\left((d^2 g^2 + 2 d f g e + f^2 e^2) x + (d^3 g^2 e + 2 d^2 f g e^2 + d f^2 e^3) e^{(-2)}\right) e^{(-2)}}{2 (x^2 e^2 - d^2) d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)*(g*x + f)^2/(e^2*x^2 - d^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*g^2*e^(-3)*ln(abs(x^2*e^2 - d^2)) + 1/4*(d^2*g^2 + 2*d*f*g*e
- f^2*e^2)*e^(-3)*ln(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*ab
s(d)*e))/(d*abs(d)) - 1/2*((d^2*g^2 + 2*d*f*g*e + f^2*e^2)*x + (d
^3*g^2*e + 2*d^2*f*g*e^2 + d*f^2*e^3)*e^(-2))*e^(-2)/((x^2*e^2 -
d^2)*d)
```

$$3.564 \quad \int \frac{(f+gx)^2}{(d^2-e^2x^2)^2} dx$$

**Optimal.** Leaf size=74

$$\frac{(ef-dg)(dg+ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{2d^3e^3} + \frac{(f+gx)(d^2g+e^2fx)}{2d^2e^2(d^2-e^2x^2)}$$

[Out]  $((d^2g + e^2fx)(f + gx)) / (2d^2e^2(d^2 - e^2x^2)) + ((ef - dg)(ef + dg) \operatorname{ArcTanh}[ex/d]) / (2d^3e^3)$

**Rubi [A]** time = 0.0735644, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{(ef-dg)(dg+ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{2d^3e^3} + \frac{(f+gx)(d^2g+e^2fx)}{2d^2e^2(d^2-e^2x^2)}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)^2/(d^2 - e^2\*x^2)^2, x]

[Out]  $((d^2g + e^2fx)(f + gx)) / (2d^2e^2(d^2 - e^2x^2)) + ((ef - dg)(ef + dg) \operatorname{ArcTanh}[ex/d]) / (2d^3e^3)$

**Rubi in Sympy [A]** time = 13.4728, size = 63, normalized size = 0.85

$$\frac{(f+gx)(d^2g+e^2fx)}{2d^2e^2(d^2-e^2x^2)} - \frac{(d^2g^2-e^2f^2)\operatorname{atanh}\left(\frac{ex}{d}\right)}{2d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*2, x)

[Out]  $(f + gx)(d^2g + e^2fx) / (2d^2e^2(d^2 - e^2x^2)) - (d^2g^2 - e^2f^2) \operatorname{atanh}(ex/d) / (2d^3e^3)$

**Mathematica [A]** time = 0.0632949, size = 85, normalized size = 1.15

$$\frac{-2d^2fg - d^2g^2x - e^2f^2x}{2d^2e^2(e^2x^2 - d^2)} - \frac{(d^2g^2 - e^2f^2)\tanh^{-1}\left(\frac{ex}{d}\right)}{2d^3e^3}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^2/(d^2 - e^2\*x^2)^2, x]

[Out]  $(-2d^2fg - e^2f^2x - d^2g^2x) / (2d^2e^2(-d^2 + e^2x^2)) - ((-e^2f^2) + d^2g^2) \operatorname{ArcTanh}[ex/d] / (2d^3e^3)$

**Maple [B]** time = 0.017, size = 180, normalized size = 2.4

$$\begin{aligned} & \frac{\ln(ex-d)g^2}{4e^3d} - \frac{\ln(ex-d)f^2}{4ed^3} - \frac{g^2}{4e^3(ex-d)} - \frac{fg}{2e^2d(ex-d)} - \frac{f^2}{4ed^2(ex-d)} \\ & - \frac{\ln(ex+d)g^2}{4e^3d} + \frac{\ln(ex+d)f^2}{4ed^3} - \frac{g^2}{4e^3(ex+d)} + \frac{fg}{2e^2d(ex+d)} - \frac{f^2}{4ed^2(ex+d)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^2/(-e^2*x^2+d^2)^2,x)`

[Out]  $\frac{1}{4} \frac{e^3}{d} \ln(e^x - d) g^2 - \frac{1}{4} \frac{e}{d^3} \ln(e^x - d) f^2 - \frac{1}{4} \frac{e^3}{(e^x - d)} g^2 - \frac{1}{2} \frac{e^2}{d} \frac{1}{(e^x - d)} f g - \frac{1}{4} \frac{e}{d^2} \frac{1}{(e^x - d)} f^2 - \frac{1}{4} \frac{e^3}{d} \ln(e^x + d) g^2 + \frac{1}{4} \frac{e}{d^3} \ln(e^x + d) f^2 - \frac{1}{4} \frac{e^3}{(e^x + d)} g^2 + \frac{1}{2} \frac{d}{e^2} \frac{1}{(e^x + d)} f g - \frac{1}{4} \frac{d^2}{e} \frac{1}{(e^x + d)} f^2$

**Maxima [A]** time = 0.691804, size = 150, normalized size = 2.03

$$-\frac{2d^2fg + (e^2f^2 + d^2g^2)x}{2(d^2e^4x^2 - d^4e^2)} + \frac{(e^2f^2 - d^2g^2)\log(ex + d)}{4d^3e^3} - \frac{(e^2f^2 - d^2g^2)\log(ex - d)}{4d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x + f)^2/(e^2*x^2 - d^2)^2,x, algorithm="maxima")`

[Out]  $-\frac{1}{2} \frac{(2d^2fg + (e^2f^2 + d^2g^2)x)}{(d^2e^4x^2 - d^4e^2)} + \frac{1}{4} \frac{(e^2f^2 - d^2g^2)\log(e^x + d)}{(d^3e^3)} - \frac{1}{4} \frac{(e^2f^2 - d^2g^2)\log(e^x - d)}{(d^3e^3)}$

**Fricas [A]** time = 0.278029, size = 209, normalized size = 2.82

$$\frac{4d^3efg + 2(de^3f^2 + d^3eg^2)x + (d^2e^2f^2 - d^4g^2 - (e^4f^2 - d^2e^2g^2)x^2)\log(ex + d) - (d^2e^2f^2 - d^4g^2 - (e^4f^2 - d^2e^2g^2)x^2)\log(ex - d)}{4(d^3e^5x^2 - d^5e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x + f)^2/(e^2*x^2 - d^2)^2,x, algorithm="fricas")`

[Out]  $-\frac{1}{4} \frac{(4d^3efg + 2(d^2e^2f^2 + d^3eg^2)x + (d^2e^2f^2 - d^4g^2 - (e^4f^2 - d^2e^2g^2)x^2)\log(e^x + d) - (d^2e^2f^2 - d^4g^2 - (e^4f^2 - d^2e^2g^2)x^2)\log(e^x - d))}{(d^3e^5x^2 - d^5e^3)}$

**Sympy [A]** time = 2.96306, size = 155, normalized size = 2.09

$$-\frac{2d^2fg + x(d^2g^2 + e^2f^2)}{-2d^4e^2 + 2d^2e^4x^2} + \frac{(dg - ef)(dg + ef)\log\left(-\frac{d(dg-ef)(dg+ef)}{e(d^2g^2-e^2f^2)} + x\right)}{4d^3e^3} - \frac{(dg - ef)(dg + ef)\log\left(\frac{d(dg-ef)(dg+ef)}{e(d^2g^2-e^2f^2)} + x\right)}{4d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)**2/(-e**2*x**2+d**2)**2,x)`

[Out]  $-\frac{(2d^2fg + x(d^2g^2 + e^2f^2))}{(-2d^4e^2 + 2d^2e^4x^2)} + \frac{(dg - ef)(dg + ef)\log(-d(dg - ef)(dg + ef)/(e(d^2g^2 - e^2f^2)) + x)}{(4d^3e^3)} - \frac{(dg - ef)(dg + ef)\log(d(dg - ef)(dg + ef)/(e(d^2g^2 - e^2f^2)) + x)}{(4d^3e^3)}$

**GIAC/XCAS [A]** time = 0.300546, size = 136, normalized size = 1.84

$$\frac{(d^2 g^2 - f^2 e^2) e^{(-3)} \ln\left(\frac{|2 x e^2 - 2 |d| e|}{|2 x e^2 + 2 |d| e|}\right)}{4 d^2 |d|} - \frac{(d^2 g^2 x + 2 d^2 f g + f^2 x e^2) e^{(-2)}}{2 (x^2 e^2 - d^2) d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^2/(e^2\*x^2 - d^2)^2,x, algorithm="giac")

[Out] 1/4\*(d^2\*g^2 - f^2\*e^2)\*e^(-3)\*ln(abs(2\*x\*e^2 - 2\*abs(d)\*e)/abs(2\*x\*e^2 + 2\*abs(d)\*e))/(d^2\*abs(d)) - 1/2\*(d^2\*g^2\*x + 2\*d^2\*f\*g + f^2\*x\*e^2)\*e^(-2)/((x^2\*e^2 - d^2)\*d^2)

$$3.565 \quad \int \frac{(f+gx)^2}{(d+ex)(d^2-e^2x^2)^2} dx$$

**Optimal.** Leaf size=121

$$\frac{(3ef-dg)(dg+ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3} + \frac{(dg+ef)^2}{8d^3e^3(d-ex)} - \frac{(ef-dg)^2}{8d^2e^3(d+ex)^2} - \frac{e^2f^2-d^2g^2}{4d^3e^3(d+ex)}$$

[Out]  $(e^*f + d^*g)^2/(8^*d^3e^3(d - e^*x)) - (e^*f - d^*g)^2/(8^*d^2e^3(d + e^*x)^2) - (e^2f^2 - d^2g^2)/(4^*d^3e^3(d + e^*x)) + ((3^*e^*f - d^*g)^*(e^*f + d^*g)^*\text{ArcTanh}[(e^*x)/d])/(8^*d^4e^3)$

**Rubi [A]** time = 0.29482, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$\frac{(3ef-dg)(dg+ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3} + \frac{(dg+ef)^2}{8d^3e^3(d-ex)} - \frac{(ef-dg)^2}{8d^2e^3(d+ex)^2} - \frac{e^2f^2-d^2g^2}{4d^3e^3(d+ex)}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)^2/((d + e\*x)\*(d^2 - e^2\*x^2)^2), x]

[Out]  $(e^*f + d^*g)^2/(8^*d^3e^3(d - e^*x)) - (e^*f - d^*g)^2/(8^*d^2e^3(d + e^*x)^2) - (e^2f^2 - d^2g^2)/(4^*d^3e^3(d + e^*x)) + ((3^*e^*f - d^*g)^*(e^*f + d^*g)^*\text{ArcTanh}[(e^*x)/d])/(8^*d^4e^3)$

**Rubi in Sympy [A]** time = 47.8624, size = 133, normalized size = 1.1

$$\begin{aligned} &-\frac{(dg-ef)^2}{8d^2e^3(d+ex)^2} + \frac{(dg-ef)(dg+ef)}{4d^3e^3(d+ex)} + \frac{(dg+ef)^2}{8d^3e^3(d-ex)} \\ &+ \frac{(dg-3ef)(dg+ef)\log(d-ex)}{16d^4e^3} - \frac{(dg-3ef)(dg+ef)\log(d+ex)}{16d^4e^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*2/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*2,x)

[Out]  $-(d^*g - e^*f)^2/(8^*d^2e^3(d + e^*x)^2) + (d^*g - e^*f)^*(d^*g + e^*f)/(4^*d^3e^3(d + e^*x)) + (d^*g + e^*f)^2/(8^*d^3e^3(d - e^*x)) + (d^*g - 3^*e^*f)^*(d^*g + e^*f)^*\log(d - e^*x)/(16^*d^4e^3) - (d^*g - 3^*e^*f)^*(d^*g + e^*f)^*\log(d + e^*x)/(16^*d^4e^3)$

**Mathematica [A]** time = 0.18215, size = 139, normalized size = 1.15

$$\frac{\frac{4d(d^2g^2-e^2f^2)}{d+ex} + (d^2g^2 - 2defg - 3e^2f^2)\log(d-ex) + (-d^2g^2 + 2defg + 3e^2f^2)\log(d+ex) - \frac{2d^2(ef-dg)^2}{(d+ex)^2} + \frac{2d(dg+ef)^2}{d-ex}}{16d^4e^3}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^2/((d + e\*x)\*(d^2 - e^2\*x^2)^2), x]

[Out]  $((2^*d^*(e^*f + d^*g)^2)/(d - e^*x) - (2^*d^2*(e^*f - d^*g)^2)/(d + e^*x)^2 + (4^*d^*(-(e^2f^2) + d^2g^2))/(d + e^*x) + (-3^*e^2f^2 - 2^*d^*e^*f^*g + d^2g^2)^*\text{Log}[d - e^*x] + (3^*e^2f^2 + 2^*d^*e^*f^*g - d^2g^2)^*\text{Log}[d + e^*x])/16d^4e^3$

$\log[d + e^*x])/(16*d^4*e^3)$

**Maple [B]** time = 0.019, size = 253, normalized size = 2.1

$$\begin{aligned} & \frac{\ln(ex-d)g^2}{16e^3d^2} - \frac{\ln(ex-d)fg}{8e^2d^3} - \frac{3\ln(ex-d)f^2}{16ed^4} - \frac{g^2}{8de^3(ex-d)} - \frac{fg}{4e^2d^2(ex-d)} \\ & - \frac{f^2}{8ed^3(ex-d)} + \frac{g^2}{4de^3(ex+d)} - \frac{f^2}{4ed^3(ex+d)} - \frac{\ln(ex+d)g^2}{16e^3d^2} + \frac{\ln(ex+d)fg}{8e^2d^3} \\ & + \frac{3\ln(ex+d)f^2}{16ed^4} - \frac{g^2}{8e^3(ex+d)^2} + \frac{fg}{4de^2(ex+d)^2} - \frac{f^2}{8ed^2(ex+d)^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^2/(e*x+d)/(-e^2*x^2+d^2)^2,x)`

[Out]  $1/16/e^3/d^2*\ln(e*x-d)*g^2-1/8/e^2/d^3*\ln(e*x-d)*f*g-3/16/e/d^4*\ln(e*x-d)*f^2-1/8/e^3/d/(e*x-d)*g^2-1/4/e^2/d^2/(e*x-d)*f*g-1/8/e/d^3/(e*x-d)*f^2+1/4/d/e^3/(e*x+d)*g^2-1/4/d^3/e/(e*x+d)*f^2-1/16/e^3/d^2*\ln(e*x+d)*g^2+1/8/e^2/d^3*\ln(e*x+d)*f*g+3/16/e/d^4*\ln(e*x+d)*f^2-1/8/e^3/(e*x+d)^2*g^2+1/4/d/e^2/(e*x+d)^2*f*g-1/8/d^2/e/(e*x+d)^2*f^2$

**Maxima [A]** time = 0.704462, size = 286, normalized size = 2.36

$$\begin{aligned} & \frac{2d^2e^2f^2 - 4d^3efg - 2d^4g^2 - (3e^4f^2 + 2de^3fg - d^2e^2g^2)x^2 - (3de^3f^2 + 2d^2e^2fg + 3d^3eg^2)x}{8(d^3e^6x^3 + d^4e^5x^2 - d^5e^4x - d^6e^3)} \\ & + \frac{(3e^2f^2 + 2defg - d^2g^2)\log(ex+d)}{16d^4e^3} - \frac{(3e^2f^2 + 2defg - d^2g^2)\log(ex-d)}{16d^4e^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x + f)^2/((e^2*x^2 - d^2)^2*(e*x + d)),x, algorithm="maxima")`

[Out]  $1/8*(2*d^2*e^2*f^2 - 4*d^3*e*f*g - 2*d^4*g^2 - (3*e^4*f^2 + 2*d*e^3*f*g - d^2*e^2*g^2)*x^2 - (3*d*e^3*f^2 + 2*d^2*e^2*f*g + 3*d^3*e*g^2)*x)/(d^3*e^6*x^3 + d^4*e^5*x^2 - d^5*e^4*x - d^6*e^3) + 1/16*(3*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*\log(e*x + d)/(d^4*e^3) - 1/16*(3*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*\log(e*x - d)/(d^4*e^3)$

**Fricas [A]** time = 0.277431, size = 563, normalized size = 4.65

$$\frac{4d^3e^2f^2 - 8d^4efg - 4d^5g^2 - 2(3de^4f^2 + 2d^2e^3fg - d^3e^2g^2)x^2 - 2(3d^2e^3f^2 + 2d^3e^2fg + 3d^4eg^2)x - (3d^3e^2f^2 + 2d^4efg - 2d^5g^2)}{8(d^3e^6x^3 + d^4e^5x^2 - d^5e^4x - d^6e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x + f)^2/((e^2*x^2 - d^2)^2*(e*x + d)),x, algorithm="fricas")`

[Out]  $1/16*(4*d^3*e^2*f^2 - 8*d^4*e*f*g - 4*d^5*g^2 - 2*(3*d*e^4*f^2 + 2*d^2*e^3*f*g - d^3*e^2*g^2)*x^2 - 2*(3*d^2*e^3*f^2 + 2*d^3*e^2*f*g + 3*d^4*e*g^2)*x - (3*d^3*e^2*f^2 + 2*d^4*e*f*g - d^5*g^2) * \log(e*x + d) + (3*d^3*e^2*f^2 + 2*d^4*e*f*g - d^5*g^2 - (3*d^2*e^3*f^2 + 2*d^3*e^2*f*g + 3*d^4*e*g^2)*x) * \log(e*x - d))/(d^3*e^6*x^3 + d^4*e^5*x^2 - d^5*e^4*x - d^6*e^3)$

$$d^4 e^g x^2 \log(e^x - d) / (d^4 e^6 x^3 + d^5 e^5 x^2 - d^6 e^4 x - d^7 e^3)$$

**Sympy [A]** time = 5.12669, size = 279, normalized size = 2.31

$$\frac{-2d^4 g^2 - 4d^3 efg + 2d^2 e^2 f^2 + x^2 (d^2 e^2 g^2 - 2de^3 fg - 3e^4 f^2) + x (-3d^3 eg^2 - 2d^2 e^2 fg - 3de^3 f^2)}{-8d^6 e^3 - 8d^5 e^4 x + 8d^4 e^5 x^2 + 8d^3 e^6 x^3} + \frac{(dg - 3ef)(dg + ef) \log\left(-\frac{d(dg-3ef)(dg+ef)}{e(d^2 g^2 - 2defg - 3e^2 f^2)} + x\right)}{16d^4 e^3} - \frac{(dg - 3ef)(dg + ef) \log\left(\frac{d(dg-3ef)(dg+ef)}{e(d^2 g^2 - 2defg - 3e^2 f^2)} + x\right)}{16d^4 e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*2/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*2,x)

[Out]  $(-2*d**4*g**2 - 4*d**3*e*f*g + 2*d**2*e**2*f**2 + x**2*(d**2*e**2*g**2 - 2*d*e**3*f*g - 3*e**4*f**2) + x*(-3*d**3*e*g**2 - 2*d**2*e**2*f*g - 3*d*e**3*f**2))/(-8*d**6*e**3 - 8*d**5*e**4*x + 8*d**4*e**5*x**2 + 8*d**3*e**6*x**3) + (d*g - 3*e*f)*(d*g + e*f)*\log(-d*(d*g - 3*e*f)*(d*g + e*f)/(e*(d**2*g**2 - 2*d*e*f*g - 3*e**2*f**2)) + x)/(16*d**4*e**3) - (d*g - 3*e*f)*(d*g + e*f)*\log(d*(d*g - 3*e*f)*(d*g + e*f)/(e*(d**2*g**2 - 2*d*e*f*g - 3*e**2*f**2)) + x)/(16*d**4*e**3)$

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^2/((e^2\*x^2 - d^2)^2\*(e\*x + d)),x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.566 \quad \int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)^2} dx$$

**Optimal.** Leaf size=146

$$\frac{f(dg+ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^5e^2} + \frac{(dg+ef)^2}{16d^4e^3(d-ex)} - \frac{(3ef-dg)(dg+ef)}{16d^4e^3(d+ex)} - \frac{(ef-dg)^2}{12d^2e^3(d+ex)^3} - \frac{e^2f^2-d^2g^2}{8d^3e^3(d+ex)^2}$$

[Out]  $(e^*f + d^*g)^2/(16*d^4*e^3*(d - e^*x)) - (e^*f - d^*g)^2/(12*d^2*e^3*(d + e^*x)^3) - (e^2*f^2 - d^2*g^2)/(8*d^3*e^3*(d + e^*x)^2) - ((3*e^*f - d^*g)*(e^*f + d^*g))/(16*d^4*e^3*(d + e^*x)) + (f*(e^*f + d^*g)*ArcTanh[(e^*x)/d])/(4*d^5*e^2)$

**Rubi [A]** time = 0.339463, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$\frac{f(dg+ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{4d^5e^2} + \frac{(dg+ef)^2}{16d^4e^3(d-ex)} - \frac{(3ef-dg)(dg+ef)}{16d^4e^3(d+ex)} - \frac{(ef-dg)^2}{12d^2e^3(d+ex)^3} - \frac{e^2f^2-d^2g^2}{8d^3e^3(d+ex)^2}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)^2/((d + e\*x)^2\*(d^2 - e^2\*x^2)^2), x]

[Out]  $(e^*f + d^*g)^2/(16*d^4*e^3*(d - e^*x)) - (e^*f - d^*g)^2/(12*d^2*e^3*(d + e^*x)^3) - (e^2*f^2 - d^2*g^2)/(8*d^3*e^3*(d + e^*x)^2) - ((3*e^*f - d^*g)*(e^*f + d^*g))/(16*d^4*e^3*(d + e^*x)) + (f*(e^*f + d^*g)*ArcTanh[(e^*x)/d])/(4*d^5*e^2)$

**Rubi in Sympy [A]** time = 53.2411, size = 150, normalized size = 1.03

$$-\frac{(dg-ef)^2}{12d^2e^3(d+ex)^3} + \frac{(dg-ef)(dg+ef)}{8d^3e^3(d+ex)^2} + \frac{(dg-3ef)(dg+ef)}{16d^4e^3(d+ex)} + \frac{(dg+ef)^2}{16d^4e^3(d-ex)} - \frac{f(dg+ef)\log(d-ex)}{8d^5e^2} + \frac{f(dg+ef)\log(d+ex)}{8d^5e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*2/(e\*x+d)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*2, x)

[Out]  $-(d^*g - e^*f)**2/(12*d**2*e**3*(d + e^*x)**3) + (d^*g - e^*f)*(d^*g + e^*f)/(8*d**3*e**3*(d + e^*x)**2) + (d^*g - 3*e^*f)*(d^*g + e^*f)/(16*d**4*e**3*(d + e^*x)) + (d^*g + e^*f)**2/(16*d**4*e**3*(d - e^*x)) - f*(d^*g + e^*f)*log(d - e^*x)/(8*d**5*e**2) + f*(d^*g + e^*f)*log(d + e^*x)/(8*d**5*e**2)$

**Mathematica [A]** time = 0.163659, size = 171, normalized size = 1.17

$$\frac{2d(2d^5g^2 + 2d^4eg(f + 2gx) + d^3e^2f(gx - 4f) + d^2e^3fx(f + 6gx) + 3de^4fx^2(2f + gx) + 3e^5f^2x^3) + 3ef(ex - d)(d + ex)^3(d + ex)^2}{24d^5e^3(d - ex)(d + ex)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^2/((d + e\*x)^2\*(d^2 - e^2\*x^2)^2), x]





$$\frac{(d^3 e^3 f^2 + d^4 e^2 f g) x \log(e x + d) + 3(d^4 e^2 f^2 + d^5 e f g - (e^6 f^2 + d e^5 f g) x^4 - 2(d e^5 f^2 + d^2 e^4 f g) x^3 + 2(d^3 e^3 f^2 + d^4 e^2 f g) x) \log(e x - d)}{(d^5 e^7 x^4 + 2 d^6 e^6 x^3 - 2 d^8 e^4 x - d^9 e^3)}$$

**Sympy [A]** time = 5.84698, size = 236, normalized size = 1.62

$$\frac{2d^5 g^2 + 2d^4 e f g - 4d^3 e^2 f^2 + x^3 (3d e^4 f g + 3e^5 f^2) + x^2 (6d^2 e^3 f g + 6d e^4 f^2) + x (4d^4 e g^2 + d^3 e^2 f g + d^2 e^3 f^2) - 12d^8 e^3 - 24d^7 e^4 x + 24d^5 e^6 x^3 + 12d^4 e^7 x^4}{8d^5 e^2} \frac{f(dg + ef) \log\left(-\frac{df(dg+ef)}{e(dfg+ef^2)} + x\right)}{8d^5 e^2} + \frac{f(dg + ef) \log\left(\frac{df(dg+ef)}{e(dfg+ef^2)} + x\right)}{8d^5 e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*2/(e\*x+d)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*2,x)

[Out]  $-(2*d**5*g**2 + 2*d**4*e*f*g - 4*d**3*e**2*f**2 + x**3*(3*d*e**4*f*g + 3*e**5*f**2) + x**2*(6*d**2*e**3*f*g + 6*d*e**4*f**2) + x*(4*d**4*e*g**2 + d**3*e**2*f*g + d**2*e**3*f**2))/(-12*d**8*e**3 - 24*d**7*e**4*x + 24*d**5*e**6*x**3 + 12*d**4*e**7*x**4) - f*(d*g + e*f)*\log(-d*f*(d*g + e*f)/(e*(d*f*g + e*f**2)) + x)/(8*d**5*e**2) + f*(d*g + e*f)*\log(d*f*(d*g + e*f)/(e*(d*f*g + e*f**2)) + x)/(8*d**5*e**2)$

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^2/((e^2\*x^2 - d^2)^2\*(e\*x + d)^2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.567 \quad \int \frac{(f+gx)^2}{(d+ex)^3(d^2-e^2x^2)^2} dx$$

**Optimal.** Leaf size=178

$$\frac{(dg+ef)(dg+5ef) \tanh^{-1}\left(\frac{ex}{d}\right)}{32d^6e^3} + \frac{(dg+ef)^2}{32d^5e^3(d-ex)} - \frac{f(dg+ef)}{8d^5e^2(d+ex)} - \frac{(3ef-dg)(dg+ef)}{32d^4e^3(d+ex)^2} - \frac{(ef-dg)^2}{16d^2e^3(d+ex)^4} - \frac{e^2f^2-d^2g^2}{12d^3e^3(d+ex)^3}$$

[Out]  $(e^*f + d^*g)^2 / (32^*d^{\wedge}5^*e^{\wedge}3^*(d - e^*x)) - (e^*f - d^*g)^2 / (16^*d^{\wedge}2^*e^{\wedge}3^*(d + e^*x)^4) - (e^{\wedge}2^*f^{\wedge}2 - d^{\wedge}2^*g^{\wedge}2) / (12^*d^{\wedge}3^*e^{\wedge}3^*(d + e^*x)^3) - ((3^*e^*f - d^*g)^*(e^*f + d^*g)) / (32^*d^{\wedge}4^*e^{\wedge}3^*(d + e^*x)^2) - (f^*(e^*f + d^*g)) / (8^*d^{\wedge}5^*e^{\wedge}2^*(d + e^*x)) + ((e^*f + d^*g)^*(5^*e^*f + d^*g)^*ArcTanh[(e^*x)/d]) / (32^*d^{\wedge}6^*e^{\wedge}3)$

**Rubi [A]** time = 0.429567, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$\frac{(dg+ef)(dg+5ef) \tanh^{-1}\left(\frac{ex}{d}\right)}{32d^6e^3} + \frac{(dg+ef)^2}{32d^5e^3(d-ex)} - \frac{f(dg+ef)}{8d^5e^2(d+ex)} - \frac{(3ef-dg)(dg+ef)}{32d^4e^3(d+ex)^2} - \frac{(ef-dg)^2}{16d^2e^3(d+ex)^4} - \frac{e^2f^2-d^2g^2}{12d^3e^3(d+ex)^3}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)^2/((d + e\*x)^3\*(d^2 - e^2\*x^2)^2), x]

[Out]  $(e^*f + d^*g)^2 / (32^*d^{\wedge}5^*e^{\wedge}3^*(d - e^*x)) - (e^*f - d^*g)^2 / (16^*d^{\wedge}2^*e^{\wedge}3^*(d + e^*x)^4) - (e^{\wedge}2^*f^{\wedge}2 - d^{\wedge}2^*g^{\wedge}2) / (12^*d^{\wedge}3^*e^{\wedge}3^*(d + e^*x)^3) - ((3^*e^*f - d^*g)^*(e^*f + d^*g)) / (32^*d^{\wedge}4^*e^{\wedge}3^*(d + e^*x)^2) - (f^*(e^*f + d^*g)) / (8^*d^{\wedge}5^*e^{\wedge}2^*(d + e^*x)) + ((e^*f + d^*g)^*(5^*e^*f + d^*g)^*ArcTanh[(e^*x)/d]) / (32^*d^{\wedge}6^*e^{\wedge}3)$

**Rubi in Sympy [A]** time = 64.3836, size = 187, normalized size = 1.05

$$-\frac{(dg-ef)^2}{16d^2e^3(d+ex)^4} + \frac{(dg-ef)(dg+ef)}{12d^3e^3(d+ex)^3} + \frac{(dg-3ef)(dg+ef)}{32d^4e^3(d+ex)^2} - \frac{f(dg+ef)}{8d^5e^2(d+ex)} + \frac{(dg+ef)^2}{32d^5e^3(d-ex)} - \frac{(dg+ef)(dg+5ef)\log(d-ex)}{64d^6e^3} + \frac{(dg+ef)(dg+5ef)\log(d+ex)}{64d^6e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*2/(e\*x+d)\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*2, x)

[Out]  $-(d^*g - e^*f)^2 / (16^*d^{\wedge}2^*e^{\wedge}3^*(d + e^*x)^4) + (d^*g - e^*f)^*(d^*g + e^*f) / (12^*d^{\wedge}3^*e^{\wedge}3^*(d + e^*x)^3) + (d^*g - 3^*e^*f)^*(d^*g + e^*f) / (32^*d^{\wedge}4^*e^{\wedge}3^*(d + e^*x)^2) - f^*(d^*g + e^*f) / (8^*d^{\wedge}5^*e^{\wedge}2^*(d + e^*x)) + (d^*g + e^*f)^2 / (32^*d^{\wedge}5^*e^{\wedge}3^*(d - e^*x)) - (d^*g + e^*f)^*(d^*g + 5^*e^*f)^*\log(d - e^*x) / (64^*d^{\wedge}6^*e^{\wedge}3) + (d^*g + e^*f)^*(d^*g + 5^*e^*f)^*\log(d + e^*x) / (64^*d^{\wedge}6^*e^{\wedge}3)$

**Mathematica [A]** time = 0.247167, size = 195, normalized size = 1.1

$$\frac{-\frac{12d^4(ef-dg)^2}{(d+ex)^4} + \frac{6d^2(d^2g^2-2defg-3e^2f^2)}{(d+ex)^2}}{192d^6e^3} - 3(d^2g^2 + 6defg + 5e^2f^2)\log(d-ex) + 3(d^2g^2 + 6defg + 5e^2f^2)\log(d+ex) + \frac{16d^3}{192d^6e^3}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^2/((d + e\*x)^3\*(d^2 - e^2\*x^2)^2), x]

[Out] 
$$\frac{((6*d*(e*f + d*g)^2)/(d - e*x) - (12*d^4*(e*f - d*g)^2)/(d + e*x)^4 + (16*d^3*(-(e^2*f^2) + d^2*g^2))/(d + e*x)^3 + (6*d^2*(-3*e^2*f^2 - 2*d*e*f*g + d^2*g^2))/(d + e*x)^2 - (24*d*e*f*(e*f + d*g))/(d + e*x) - 3*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)*\text{Log}[d - e*x] + 3*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)*\text{Log}[d + e*x])/(192*d^6*e^3)}$$

**Maple [B]** time = 0.022, size = 341, normalized size = 1.9

$$\begin{aligned} & -\frac{\ln(ex-d)g^2}{64e^3d^4} - \frac{3\ln(ex-d)fg}{32e^2d^5} - \frac{5\ln(ex-d)f^2}{64ed^6} - \frac{g^2}{32d^3e^3(ex-d)} - \frac{fg}{16e^2d^4(ex-d)} \\ & - \frac{f^2}{32ed^5(ex-d)} + \frac{\ln(ex+d)g^2}{64e^3d^4} + \frac{3\ln(ex+d)fg}{32e^2d^5} + \frac{5\ln(ex+d)f^2}{64ed^6} + \frac{g^2}{12e^3d(ex+d)^3} \\ & - \frac{f^2}{12ed^3(ex+d)^3} + \frac{g^2}{32e^3d^2(ex+d)^2} - \frac{fg}{16e^2d^3(ex+d)^2} - \frac{3f^2}{32ed^4(ex+d)^2} \\ & - \frac{g^2}{16e^3(ex+d)^4} + \frac{fg}{8de^2(ex+d)^4} - \frac{f^2}{16ed^2(ex+d)^4} - \frac{fg}{8e^2d^4(ex+d)} - \frac{f^2}{8ed^5(ex+d)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^2/(e\*x+d)^3/(-e^2\*x^2+d^2)^2, x)

[Out] 
$$\begin{aligned} & -1/64/e^3/d^4*\ln(e*x-d)*g^2-3/32/e^2/d^5*\ln(e*x-d)*f*g-5/64/e/d^6 \\ & * \ln(e*x-d)*f^2-1/32/e^3/d^3/(e*x-d)*g^2-1/16/e^2/d^4/(e*x-d)*f*g- \\ & 1/32/e/d^5/(e*x-d)*f^2+1/64/e^3/d^4*\ln(e*x+d)*g^2+3/32/e^2/d^5*\ln \\ & (e*x+d)*f*g+5/64/e/d^6*\ln(e*x+d)*f^2+1/12/e^3/d/(e*x+d)^3*g^2-1/1 \\ & 2/e/d^3/(e*x+d)^3*f^2+1/32/e^3/d^2/(e*x+d)^2*g^2-1/16/e^2/d^3/(e* \\ & x+d)^2*f*g-3/32/e/d^4/(e*x+d)^2*f^2-1/16/e^3/(e*x+d)^4*g^2+1/8/e^ \\ & 2/d/(e*x+d)^4*f*g-1/16/e/d^2/(e*x+d)^4*f^2-1/8/e^2*f/d^4/(e*x+d)* \\ & g-1/8/e*f^2/d^5/(e*x+d) \end{aligned}$$

**Maxima [A]** time = 0.70641, size = 402, normalized size = 2.26

$$\begin{aligned} & \frac{32d^4e^2f^2 - 8d^6g^2 - 3(5e^6f^2 + 6de^5fg + d^2e^4g^2)x^4 - 9(5de^5f^2 + 6d^2e^4fg + d^3e^3g^2)x^3 - 7(5d^2e^4f^2 + 6d^3e^3fg + d^4e^2g^2)x^2 - 5d^2e^4f^2 - 6d^3e^3fg + d^4e^2g^2}{96(d^5e^8x^5 + 3d^6e^7x^4 + 2d^7e^6x^3 - 2d^8e^5x^2 - 3d^9e^4x - d^{10}e^3)} \\ & + \frac{(5e^2f^2 + 6defg + d^2g^2)\log(ex+d)}{64d^6e^3} - \frac{(5e^2f^2 + 6defg + d^2g^2)\log(ex-d)}{64d^6e^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^2/((e^2\*x^2 - d^2)^2\*(e\*x + d)^3), x, algorithm="maxima")

[Out] 
$$\begin{aligned} & 1/96*(32*d^4*e^2*f^2 - 8*d^6*g^2 - 3*(5*e^6*f^2 + 6*d*e^5*f*g + d \\ & ^2*e^4*g^2)*x^4 - 9*(5*d^2*e^5*f^2 + 6*d^3*e^4*f*g + d^3*e^3*g^2)*x \\ & ^3 - 7*(5*d^2*e^4*f^2 + 6*d^3*e^3*f*g + d^4*e^2*g^2)*x^2 + 3*(5*d \\ & ^3*e^3*f^2 + 6*d^4*e^2*f*g - 7*d^5*e*g^2)*x)/(d^5*e^8*x^5 + 3*d^6 \\ & *e^7*x^4 + 2*d^7*e^6*x^3 - 2*d^8*e^5*x^2 - 3*d^9*e^4*x - d^{10}*e^3 \\ & ) + 1/64*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)*\log(e*x + d)/(d^6*e^3) \\ & - 1/64*(5*e^2*f^2 + 6*d*e*f*g + d^2*g^2)*\log(e*x - d)/(d^6*e^3) \end{aligned}$$

**Fricas [A]** time = 0.27959, size = 875, normalized size = 4.92

$$\frac{64d^5e^2f^2 - 16d^7g^2 - 6(5de^6f^2 + 6d^2e^5fg + d^3e^4g^2)x^4 - 18(5d^2e^5f^2 + 6d^3e^4fg + d^4e^3g^2)x^3 - 14(5d^3e^4f^2 + 6d^4e^3fg + d^5e^2g^2)x^2 - 5d^2e^4f^2 - 6d^3e^3fg + d^4e^2g^2}{96(d^5e^8x^5 + 3d^6e^7x^4 + 2d^7e^6x^3 - 2d^8e^5x^2 - 3d^9e^4x - d^{10}e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x + f)^2/((e^2*x^2 - d^2)^2*(e*x + d)^3),x, algorithm="fricas")`

[Out] 
$$\frac{1}{192} (64d^5e^2f^2 - 16d^7g^2 - 6(5d^2e^6f^2 + 6d^2e^5fg + d^3e^4g^2)x^4 - 18(5d^2e^5f^2 + 6d^3e^4fg + d^4e^3g^2)x^3 - 14(5d^3e^4f^2 + 6d^4e^3fg + d^5e^2g^2)x^2 + 6(5d^4e^3f^2 + 6d^5e^2fg - 7d^6e^2g^2)x - 3(5d^5e^2f^2 + 6d^6e^2fg + d^7g^2 - (5e^7f^2 + 6d^2e^6fg + d^2e^5g^2)x^5 - 3(5d^2e^6f^2 + 6d^2e^5fg + d^3e^4g^2)x^4 - 2(5d^2e^5f^2 + 6d^3e^4fg + d^4e^3g^2)x^3 + 2(5d^3e^4f^2 + 6d^4e^3fg + d^5e^2g^2)x^2 + 3(5d^4e^3f^2 + 6d^5e^2fg + d^6e^2g^2)x) \log(e*x + d) + 3(5d^5e^2f^2 + 6d^6e^2fg + d^7g^2 - (5e^7f^2 + 6d^2e^6fg + d^2e^5g^2)x^5 - 3(5d^2e^6f^2 + 6d^2e^5fg + d^3e^4g^2)x^4 - 2(5d^2e^5f^2 + 6d^3e^4fg + d^4e^3g^2)x^3 + 2(5d^3e^4f^2 + 6d^4e^3fg + d^5e^2g^2)x^2 + 3(5d^4e^3f^2 + 6d^5e^2fg + d^6e^2g^2)x) \log(e*x - d)) / (d^6e^8x^5 + 3d^7e^7x^4 + 2d^8e^6x^3 - 2d^9e^5x^2 - 3d^{10}e^4x - d^{11}e^3)$$

**Sympy [A]** time = 7.46259, size = 371, normalized size = 2.08

$$\frac{8d^6g^2 - 32d^4e^2f^2 + x^4(3d^2e^4g^2 + 18de^5fg + 15e^6f^2) + x^3(9d^3e^3g^2 + 54d^2e^4fg + 45de^5f^2) + x^2(7d^4e^2g^2 + 42d^3e^3fg + 3d^4e^2f^2) - 96d^{10}e^3 - 288d^9e^4x - 192d^8e^5x^2 + 192d^7e^6x^3 + 288d^6e^7x^4 + 96d^5e^8x^5}{64d^6e^3} \log\left(\frac{d(dg+ef)(dg+5ef)}{e(d^2g^2+6defg+5e^2f^2)} + x\right) + \frac{(dg+ef)(dg+5ef) \log\left(\frac{d(dg+ef)(dg+5ef)}{e(d^2g^2+6defg+5e^2f^2)} + x\right)}{64d^6e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)**2/(e*x+d)**3/(-e**2*x**2+d**2)**2,x)`

[Out] 
$$\frac{-(8d^6g^2 - 32d^4e^2f^2 + x^4(3d^2e^4g^2 + 18d^3e^3fg + 15e^6f^2) + x^3(9d^3e^3g^2 + 54d^2e^4fg + 45de^5f^2) + x^2(7d^4e^2g^2 + 42d^3e^3fg + 3d^4e^2f^2) + x(21d^5e^2g^2 - 18d^4e^2fg - 15d^3e^3f^2)) / (-96d^{10}e^3 - 288d^9e^4x - 192d^8e^5x^2 + 192d^7e^6x^3 + 288d^6e^7x^4 + 96d^5e^8x^5) - (d^2g + e^2f)(d^2g + 5e^2f) \log(-d(d^2g + e^2f)(d^2g + 5e^2f)) / (e(d^2g^2 + 6d^2e^2fg + 5e^2f^2)) + x) / (64d^6e^3) + (d^2g + e^2f)(d^2g + 5e^2f) \log(d^2(d^2g + e^2f)(d^2g + 5e^2f)) / (e(d^2g^2 + 6d^2e^2fg + 5e^2f^2)) + x) / (64d^6e^3)}$$

**GIAC/XCAS [A]** time = 0.27226, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x + f)^2/((e^2*x^2 - d^2)^2*(e*x + d)^3),x, algorithm="giac")`

[Out] Done

$$3.568 \quad \int \frac{(f+gx)^2}{(d+ex)^4(d^2-e^2x^2)^2} dx$$

**Optimal.** Leaf size=210

$$\frac{(dg+ef)(dg+3ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{32d^7e^3} + \frac{(dg+ef)^2}{64d^6e^3(d-ex)} - \frac{(dg+ef)(dg+5ef)}{64d^6e^3(d+ex)} \\ - \frac{f(dg+ef)}{16d^5e^2(d+ex)^2} - \frac{(3ef-dg)(dg+ef)}{48d^4e^3(d+ex)^3} - \frac{(ef-dg)^2}{20d^2e^3(d+ex)^5} - \frac{e^2f^2-d^2g^2}{16d^3e^3(d+ex)^4}$$

[Out]  $(e^*f + d^*g)^2 / (64*d^6*e^3*(d - e^*x)) - (e^*f - d^*g)^2 / (20*d^2*e^3*(d + e^*x)^5) - (e^2*f^2 - d^2*g^2) / (16*d^3*e^3*(d + e^*x)^4) - ((3*e^*f - d^*g)*(e^*f + d^*g)) / (48*d^4*e^3*(d + e^*x)^3) - (f*(e^*f + d^*g)) / (16*d^5*e^2*(d + e^*x)^2) - ((e^*f + d^*g)*(5*e^*f + d^*g)) / (64*d^6*e^3*(d + e^*x)) + ((e^*f + d^*g)*(3*e^*f + d^*g)*ArcTanh[(e^*x)/d]) / (32*d^7*e^3)$

**Rubi [A]** time = 0.521798, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$\frac{(dg+ef)(dg+3ef)\tanh^{-1}\left(\frac{ex}{d}\right)}{32d^7e^3} + \frac{(dg+ef)^2}{64d^6e^3(d-ex)} - \frac{(dg+ef)(dg+5ef)}{64d^6e^3(d+ex)} \\ - \frac{f(dg+ef)}{16d^5e^2(d+ex)^2} - \frac{(3ef-dg)(dg+ef)}{48d^4e^3(d+ex)^3} - \frac{(ef-dg)^2}{20d^2e^3(d+ex)^5} - \frac{e^2f^2-d^2g^2}{16d^3e^3(d+ex)^4}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)^2/((d + e\*x)^4\*(d^2 - e^2\*x^2)^2), x]

[Out]  $(e^*f + d^*g)^2 / (64*d^6*e^3*(d - e^*x)) - (e^*f - d^*g)^2 / (20*d^2*e^3*(d + e^*x)^5) - (e^2*f^2 - d^2*g^2) / (16*d^3*e^3*(d + e^*x)^4) - ((3*e^*f - d^*g)*(e^*f + d^*g)) / (48*d^4*e^3*(d + e^*x)^3) - (f*(e^*f + d^*g)) / (16*d^5*e^2*(d + e^*x)^2) - ((e^*f + d^*g)*(5*e^*f + d^*g)) / (64*d^6*e^3*(d + e^*x)) + ((e^*f + d^*g)*(3*e^*f + d^*g)*ArcTanh[(e^*x)/d]) / (32*d^7*e^3)$

**Rubi in Sympy [A]** time = 75.9566, size = 218, normalized size = 1.04

$$-\frac{(dg-ef)^2}{20d^2e^3(d+ex)^5} + \frac{(dg-ef)(dg+ef)}{16d^3e^3(d+ex)^4} + \frac{(dg-3ef)(dg+ef)}{48d^4e^3(d+ex)^3} \\ - \frac{f(dg+ef)}{16d^5e^2(d+ex)^2} - \frac{(dg+ef)(dg+5ef)}{64d^6e^3(d+ex)} + \frac{(dg+ef)^2}{64d^6e^3(d-ex)} \\ - \frac{(dg+ef)(dg+3ef)\log(d-ex)}{64d^7e^3} + \frac{(dg+ef)(dg+3ef)\log(d+ex)}{64d^7e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*2/(e\*x+d)\*\*4/(-e\*\*2\*x\*\*2+d\*\*2)\*\*2, x)

[Out]  $-(d^*g - e^*f)**2 / (20*d**2*e**3*(d + e^*x)**5) + (d^*g - e^*f)*(d^*g + e^*f) / (16*d**3*e**3*(d + e^*x)**4) + (d^*g - 3*e^*f)*(d^*g + e^*f) / (48*d**4*e**3*(d + e^*x)**3) - f*(d^*g + e^*f) / (16*d**5*e**2*(d + e^*x)**2) - (d^*g + e^*f)*(d^*g + 5*e^*f) / (64*d**6*e**3*(d + e^*x)) + (d^*g + e^*f)**2 / (64*d**6*e**3*(d - e^*x)) - (d^*g + e^*f)*(d^*g + 3*e^*f)*log(d - e^*x) / (64*d**7*e**3) + (d^*g + e^*f)*(d^*g + 3*e^*f)*log(d + e^*x) / (64*d**7*e**3)$

**Mathematica [A]** time = 0.348466, size = 229, normalized size = 1.09

$$\frac{-\frac{48d^5(e^2f-dg)^2}{(d+ex)^5} - \frac{15d(d^2g^2+6defg+5e^2f^2)}{d+ex} - 15(d^2g^2+4defg+3e^2f^2)\log(d-ex) + 15(d^2g^2+4defg+3e^2f^2)\log(d+ex) - 6}{960d^7e^3}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^2/((d + e\*x)^4\*(d^2 - e^2\*x^2)^2), x]

[Out] ((15\*d\*(e\*f + d\*g)^2)/(d - e\*x) - (48\*d^5\*(e\*f - d\*g)^2)/(d + e\*x)^5 + (60\*d^4\*(-(e^2\*f^2) + d^2\*g^2))/(d + e\*x)^4 + (20\*d^3\*(-3\*e^2\*f^2 - 2\*d\*e\*f\*g + d^2\*g^2))/(d + e\*x)^3 - (60\*d^2\*e\*f\*(e\*f + d\*g))/(d + e\*x)^2 - (15\*d\*(5\*e^2\*f^2 + 6\*d\*e\*f\*g + d^2\*g^2))/(d + e\*x) - 15\*(3\*e^2\*f^2 + 4\*d\*e\*f\*g + d^2\*g^2)\*Log[d - e\*x] + 15\*(3\*e^2\*f^2 + 4\*d\*e\*f\*g + d^2\*g^2)\*Log[d + e\*x])/(960\*d^7\*e^3)

**Maple [B]** time = 0.026, size = 394, normalized size = 1.9

$$\begin{aligned} & -\frac{\ln(ex-d)g^2}{64e^3d^5} - \frac{\ln(ex-d)fg}{16e^2d^6} - \frac{3\ln(ex-d)f^2}{64ed^7} - \frac{g^2}{64d^4e^3(ex-d)} - \frac{fg}{32e^2d^5(ex-d)} \\ & - \frac{f^2}{64ed^6(ex-d)} + \frac{\ln(ex+d)g^2}{64e^3d^5} + \frac{\ln(ex+d)fg}{16e^2d^6} + \frac{3\ln(ex+d)f^2}{64ed^7} - \frac{g^2}{64d^4e^3(ex+d)} \\ & - \frac{3fg}{32e^2d^5(ex+d)} - \frac{5f^2}{64ed^6(ex+d)} + \frac{g^2}{16e^3d(ex+d)^4} - \frac{f^2}{16ed^3(ex+d)^4} \\ & + \frac{g^2}{48e^3d^2(ex+d)^3} - \frac{fg}{24e^2d^3(ex+d)^3} - \frac{f^2}{16ed^4(ex+d)^3} - \frac{g^2}{20e^3(ex+d)^5} \\ & + \frac{fg}{10de^2(ex+d)^5} - \frac{f^2}{20ed^2(ex+d)^5} - \frac{fg}{16e^2d^4(ex+d)^2} - \frac{f^2}{16ed^5(ex+d)^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^2/(e\*x+d)^4/(-e^2\*x^2+d^2)^2, x)

[Out] -1/64/e^3/d^5\*ln(e\*x-d)\*g^2-1/16/e^2/d^6\*ln(e\*x-d)\*f\*g-3/64/e/d^7\*ln(e\*x-d)\*f^2-1/64/e^3/d^4/(e\*x-d)\*g^2-1/32/e^2/d^5/(e\*x-d)\*f\*g-1/64/e/d^6/(e\*x-d)\*f^2+1/64/e^3/d^5\*ln(e\*x+d)\*g^2+1/16/e^2/d^6\*ln(e\*x+d)\*f\*g+3/64/e/d^7\*ln(e\*x+d)\*f^2-1/64/e^3/d^4/(e\*x+d)\*g^2-3/32/e^2/d^5/(e\*x+d)\*f\*g-5/64/e/d^6/(e\*x+d)\*f^2+1/16/e^3/d/(e\*x+d)^4\*g^2-1/16/e/d^3/(e\*x+d)^4\*f^2+1/48/e^3/d^2/(e\*x+d)^3\*g^2-1/24/e^2/d^3/(e\*x+d)^3\*f\*g-1/16/e/d^4/(e\*x+d)^3\*f^2-1/20/e^3/(e\*x+d)^5\*g^2+1/10/e^2/d/(e\*x+d)^5\*f\*g-1/20/e/d^2/(e\*x+d)^5\*f^2-1/16/e^2\*f/d^4/(e\*x+d)^2\*g-1/16/e\*f^2/d^5/(e\*x+d)^2

**Maxima [A]** time = 0.718927, size = 462, normalized size = 2.2

$$\frac{144d^5e^2f^2 + 32d^6efg - 16d^7g^2 - 15(3e^7f^2 + 4de^6fg + d^2e^5g^2)x^5 - 60(3de^6f^2 + 4d^2e^5fg + d^3e^4g^2)x^4 - 80(3d^2e^5f^2 + 4de^4fg + d^3e^4g^2)x^3 - 80(3d^2e^5f^2 + 4de^4fg + d^3e^4g^2)x^2 - 80(3d^2e^5f^2 + 4de^4fg + d^3e^4g^2)x - 80(3d^2e^5f^2 + 4de^4fg + d^3e^4g^2)}{480(d^6e^9x^6 + 4d^7e^8x^5 + 5d^8e^7x^4 - 5d^{10}e^6)} + \frac{(3e^2f^2 + 4defg + d^2g^2)\log(ex+d)}{64d^7e^3} - \frac{(3e^2f^2 + 4defg + d^2g^2)\log(ex-d)}{64d^7e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^2/((e^2\*x^2 - d^2)^2\*(e\*x + d)^4), x, algorithm="maxima")

[Out] 1/480\*(144\*d^5\*e^2\*f^2 + 32\*d^6\*e\*f\*g - 16\*d^7\*g^2 - 15\*(3\*e^7\*f^2 + 4\*d\*e^6\*f\*g + d^2\*e^5\*g^2)\*x^5 - 60\*(3\*d^2\*e^5\*f^2 + 4\*d^2\*e^5\*f\*g + d^3\*e^4\*g^2)\*x^4 - 80\*(3\*d^2\*e^5\*f^2 + 4\*d^2\*e^5\*f\*g + d^3\*e^4\*g^2)\*x^3 - 80\*(3\*d^2\*e^5\*f^2 + 4\*d^2\*e^5\*f\*g + d^3\*e^4\*g^2)\*x^2 - 80\*(3\*d^2\*e^5\*f^2 + 4\*d^2\*e^5\*f\*g + d^3\*e^4\*g^2)\*x - 80\*(3\*d^2\*e^5\*f^2 + 4\*d^2\*e^5\*f\*g + d^3\*e^4\*g^2))

$$e^3 g^2) x^3 - 20(3 d^3 e^4 f^2 + 4 d^4 e^3 f g + d^5 e^2 g^2) x^2 + (141 d^4 e^3 f^2 + 188 d^5 e^2 f g - 49 d^6 e g^2) x) / (d^6 e^9 x^6 + 4 d^7 e^8 x^5 + 5 d^8 e^7 x^4 - 5 d^{10} e^5 x^2 - 4 d^{11} e^4 x - d^{12} e^3) + 1/64(3 e^2 f^2 + 4 d e f g + d^2 g^2) \log(e x + d) / (d^7 e^3) - 1/64(3 e^2 f^2 + 4 d e f g + d^2 g^2) \log(e x - d) / (d^7 e^3)$$

**Fricas [A]** time = 0.283642, size = 936, normalized size = 4.46

$$288 d^6 e^2 f^2 + 64 d^7 e f g - 32 d^8 g^2 - 30(3 d e^7 f^2 + 4 d^2 e^6 f g + d^3 e^5 g^2) x^5 - 120(3 d^2 e^6 f^2 + 4 d^3 e^5 f g + d^4 e^4 g^2) x^4 - 160(3 d^3 e^5 f^2 + 4 d^4 e^4 f g + d^5 e^3 g^2) x^3 - 40(3 d^4 e^4 f^2 + 4 d^5 e^3 f g + d^6 e^2 g^2) x^2 + 2(141 d^5 e^3 f^2 + 188 d^6 e^2 f g - 49 d^7 e g^2) x - 15(3 d^6 e^2 f^2 + 4 d^7 e f g + d^8 g^2 - (3 e^8 f^2 + 4 d e^7 f g + d^2 e^6 g^2) x^6 - 4(3 d e^7 f^2 + 4 d^2 e^6 f g + d^3 e^5 g^2) x^5 - 5(3 d^2 e^6 f^2 + 4 d^3 e^5 f g + d^4 e^4 g^2) x^4 + 5(3 d^4 e^4 f^2 + 4 d^5 e^3 f g + d^6 e^2 g^2) x^2 + 4(3 d^5 e^3 f^2 + 4 d^6 e^2 f g + d^7 e g^2) x) \log(e x + d) + 15(3 d^6 e^2 f^2 + 4 d^7 e f g + d^8 g^2 - (3 e^8 f^2 + 4 d e^7 f g + d^2 e^6 g^2) x^6 - 4(3 d e^7 f^2 + 4 d^2 e^6 f g + d^3 e^5 g^2) x^5 - 5(3 d^2 e^6 f^2 + 4 d^3 e^5 f g + d^4 e^4 g^2) x^4 + 5(3 d^4 e^4 f^2 + 4 d^5 e^3 f g + d^6 e^2 g^2) x^2 + 4(3 d^5 e^3 f^2 + 4 d^6 e^2 f g + d^7 e g^2) x) \log(e x - d) / (d^7 e^9 x^6 + 4 d^8 e^8 x^5 + 5 d^9 e^7 x^4 - 5 d^{11} e^5 x^2 - 4 d^{12} e^4 x - d^{13} e^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^2/((e^2\*x^2 - d^2)^2\*(e\*x + d)^4), x, algorithm="fricas")

[Out] 1/960\*(288\*d^6\*e^2\*f^2 + 64\*d^7\*e\*f\*g - 32\*d^8\*g^2 - 30\*(3\*d\*e^7\*f^2 + 4\*d^2\*e^6\*f\*g + d^3\*e^5\*g^2)\*x^5 - 120\*(3\*d^2\*e^6\*f^2 + 4\*d^3\*e^5\*f\*g + d^4\*e^4\*g^2)\*x^4 - 160\*(3\*d^3\*e^5\*f^2 + 4\*d^4\*e^4\*f\*g + d^5\*e^3\*g^2)\*x^3 - 40\*(3\*d^4\*e^4\*f^2 + 4\*d^5\*e^3\*f\*g + d^6\*e^2\*g^2)\*x^2 + 2\*(141\*d^5\*e^3\*f^2 + 188\*d^6\*e^2\*f\*g - 49\*d^7\*e\*g^2)\*x - 15\*(3\*d^6\*e^2\*f^2 + 4\*d^7\*e\*f\*g + d^8\*g^2 - (3\*e^8\*f^2 + 4\*d\*e^7\*f\*g + d^2\*e^6\*g^2)\*x^6 - 4\*(3\*d\*e^7\*f^2 + 4\*d^2\*e^6\*f\*g + d^3\*e^5\*g^2)\*x^5 - 5\*(3\*d^2\*e^6\*f^2 + 4\*d^3\*e^5\*f\*g + d^4\*e^4\*g^2)\*x^4 + 5\*(3\*d^4\*e^4\*f^2 + 4\*d^5\*e^3\*f\*g + d^6\*e^2\*g^2)\*x^2 + 4\*(3\*d^5\*e^3\*f^2 + 4\*d^6\*e^2\*f\*g + d^7\*e\*g^2)\*x)\*log(e\*x + d) + 15\*(3\*d^6\*e^2\*f^2 + 4\*d^7\*e\*f\*g + d^8\*g^2 - (3\*e^8\*f^2 + 4\*d\*e^7\*f\*g + d^2\*e^6\*g^2)\*x^6 - 4\*(3\*d\*e^7\*f^2 + 4\*d^2\*e^6\*f\*g + d^3\*e^5\*g^2)\*x^5 - 5\*(3\*d^2\*e^6\*f^2 + 4\*d^3\*e^5\*f\*g + d^4\*e^4\*g^2)\*x^4 + 5\*(3\*d^4\*e^4\*f^2 + 4\*d^5\*e^3\*f\*g + d^6\*e^2\*g^2)\*x^2 + 4\*(3\*d^5\*e^3\*f^2 + 4\*d^6\*e^2\*f\*g + d^7\*e\*g^2)\*x)\*log(e\*x - d))/(d^7\*e^9\*x^6 + 4\*d^8\*e^8\*x^5 + 5\*d^9\*e^7\*x^4 - 5\*d^11\*e^5\*x^2 - 4\*d^12\*e^4\*x - d^13\*e^3)

**Sympy [A]** time = 8.98622, size = 420, normalized size = 2.

$$\frac{16d^7g^2 - 32d^6efg - 144d^5e^2f^2 + x^5(15d^2e^5g^2 + 60de^6fg + 45e^7f^2) + x^4(60d^3e^4g^2 + 240d^2e^5fg + 180de^6f^2) + x^3(80d^4e^3g^2 + 240d^3e^4fg + 180d^2e^5f^2) + x^2(20d^5e^2g^2 + 80d^4e^3fg + 60d^3e^4f^2) + x(49d^6e^2g^2 - 188d^5e^2fg - 141d^4e^3f^2)}{64d^7e^3} + \frac{(dg + ef)(dg + 3ef) \log\left(-\frac{d(dg+ef)(dg+3ef)}{e(d^2g^2+4defg+3e^2f^2)} + x\right)}{64d^7e^3} + \frac{(dg + ef)(dg + 3ef) \log\left(\frac{d(dg+ef)(dg+3ef)}{e(d^2g^2+4defg+3e^2f^2)} + x\right)}{64d^7e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*2/(e\*x+d)\*\*4/(-e\*\*2\*x\*\*2+d\*\*2)\*\*2,x)

[Out] -(16\*d\*\*7\*g\*\*2 - 32\*d\*\*6\*e\*f\*g - 144\*d\*\*5\*e\*\*2\*f\*\*2 + x\*\*5\*(15\*d\*\*2\*e\*\*5\*g\*\*2 + 60\*d\*\*6\*e\*f\*g + 45\*e\*\*7\*f\*\*2) + x\*\*4\*(60\*d\*\*3\*e\*\*4\*g\*\*2 + 240\*d\*\*2\*e\*\*5\*f\*g + 180\*d\*\*6\*f\*\*2) + x\*\*3\*(80\*d\*\*4\*e\*\*3\*g\*\*2 + 320\*d\*\*3\*e\*\*4\*f\*g + 240\*d\*\*2\*e\*\*5\*f\*\*2) + x\*\*2\*(20\*d\*\*5\*e\*\*2\*g\*\*2 + 80\*d\*\*4\*e\*\*3\*f\*g + 60\*d\*\*3\*e\*\*4\*f\*\*2) + x\*(49\*d\*\*6\*e\*\*2\*g\*\*2 - 188\*d\*\*5\*e\*\*2\*f\*g - 141\*d\*\*4\*e\*\*3\*f\*\*2))/(-480\*d\*\*12\*e\*\*3 - 1920\*d\*\*11\*e\*\*4\*x - 2400\*d\*\*10\*e\*\*5\*x\*\*2 + 2400\*d\*\*8\*e\*\*7\*x\*\*4 + 1920\*d\*\*7\*e\*\*8\*x\*\*5 + 480\*d\*\*6\*e\*\*9\*x\*\*6) - (d\*g + e\*f)\*(d\*g + 3\*e\*f)\*log(-d\*(d\*g + e\*f)\*(d\*g + 3\*e\*f)/(e\*(d\*\*2\*g\*\*2 + 4\*d\*e\*f\*g + 3\*e\*\*2\*f\*\*2)) + x)/(64\*d\*\*7\*e\*\*3) + (d\*g + e\*f)\*(d\*g + 3\*e\*f)\*log(d\*(d\*g + e\*f)\*(d\*g + 3\*e\*f)/(e\*(d\*\*2\*g\*\*2 + 4\*d\*e\*f\*g + 3\*e\*\*2\*f\*\*2)) + x)/(64\*d\*\*7\*e\*\*3)



GIAC/XCAS [A] time = 0.28349, size = 1, normalized size = 0.

*Done*

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x + f)^2/((e^2*x^2 - d^2)^2*(e*x + d)^4),x, algorithm="giac")
```

```
[Out] Done
```

$$3.569 \quad \int \frac{(d+ex)^7(f+gx)^2}{(d^2-e^2x^2)^3} dx$$

**Optimal.** Leaf size=179

$$\frac{8d^4(dg+ef)^2}{e^3(d-ex)^2} - \frac{32d^3(dg+ef)(2dg+ef)}{e^3(d-ex)} - \frac{dx(56d^2g^2+48defg+7e^2f^2)}{e^2} - \frac{8d^2(13d^2g^2+14defg+3e^2f^2)\log(d-ex)}{e^3} - \frac{1}{3}gx^3(7dg+2ef) - \frac{x^2(2dg+ef)(12dg+ef)}{2e} - \frac{1}{4}eg^2x^4$$

[Out]  $-\left(\frac{d(7e^2f^2+48d^2efg+56d^2g^2)x}{e^2}\right) - \left(\frac{(ef+2dg)(ef+12d^2g)x^2}{2e}\right) - \frac{g(2e^2f+7d^2g)x^3}{3} - \frac{e^2g^2x^4}{4} + \frac{8d^4(ef+dg)^2}{e^3(d-ex)^2} - \frac{32d^3(ef+dg)(2dg+ef)}{e^3(d-ex)} - \frac{32d^3(ef+dg)(2dg+ef)}{e^3} - \frac{8d^2(13d^2g^2+14defg+3e^2f^2)\log(d-ex)}{e^3} - \frac{1}{3}gx^3(7dg+2ef) - \frac{x^2(2dg+ef)(12dg+ef)}{2e} - \frac{1}{4}eg^2x^4$

**Rubi [A]** time = 0.499029, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$\frac{8d^4(dg+ef)^2}{e^3(d-ex)^2} - \frac{32d^3(dg+ef)(2dg+ef)}{e^3(d-ex)} - \frac{dx(56d^2g^2+48defg+7e^2f^2)}{e^2} - \frac{8d^2(13d^2g^2+14defg+3e^2f^2)\log(d-ex)}{e^3} - \frac{1}{3}gx^3(7dg+2ef) - \frac{x^2(2dg+ef)(12dg+ef)}{2e} - \frac{1}{4}eg^2x^4$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^7\*(f + g\*x)^2)/(d^2 - e^2\*x^2)^3, x]

[Out]  $-\left(\frac{d(7e^2f^2+48d^2efg+56d^2g^2)x}{e^2}\right) - \left(\frac{(ef+2dg)(ef+12d^2g)x^2}{2e}\right) - \frac{g(2e^2f+7d^2g)x^3}{3} - \frac{e^2g^2x^4}{4} + \frac{8d^4(ef+dg)^2}{e^3(d-ex)^2} - \frac{32d^3(ef+dg)(2dg+ef)}{e^3(d-ex)} - \frac{32d^3(ef+dg)(2dg+ef)}{e^3} - \frac{8d^2(13d^2g^2+14defg+3e^2f^2)\log(d-ex)}{e^3} - \frac{1}{3}gx^3(7dg+2ef) - \frac{x^2(2dg+ef)(12dg+ef)}{2e} - \frac{1}{4}eg^2x^4$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{8d^4(dg+ef)^2}{e^3(d-ex)^2} - \frac{32d^3(dg+ef)(2dg+ef)}{e^3(d-ex)} - \frac{8d^2(13d^2g^2+14defg+3e^2f^2)\log(d-ex)}{e^3} - \frac{dx(8dg(7dg+6ef)+7e^2f^2)}{e^2} - \frac{eg^2x^4}{4} - \frac{gx^3(7dg+2ef)}{3} - \frac{(2dg+ef)(12dg+ef)\int x dx}{e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*7\*(g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*3, x)

[Out]  $8d^4(dg+ef)^2/(e^3(d-ex)^2) - 32d^3(dg+ef)(2dg+ef)/(e^3(d-ex)) - 8d^2(13d^2g^2+14defg+3e^2f^2)\log(d-ex)/e^3 - dx(8dg(7dg+6ef)+7e^2f^2)/e^2 - eg^2x^4/4 - gx^3(7dg+2ef)/3 - (2dg+ef)(12dg+ef)\int x dx/e$

**Mathematica [A]** time = 0.161405, size = 193, normalized size = 1.08

$$\frac{8d^4(dg + ef)^2}{e^3(d - ex)^2} - \frac{x^2(24d^2g^2 + 14defg + e^2f^2)}{2e} - \frac{dx(56d^2g^2 + 48defg + 7e^2f^2)}{e^2}$$

$$- \frac{8d^2(13d^2g^2 + 14defg + 3e^2f^2)\log(d - ex)}{e^3}$$

$$+ \frac{32d^3(2d^2g^2 + 3defg + e^2f^2)}{e^3(ex - d)} - \frac{1}{3}gx^3(7dg + 2ef) - \frac{1}{4}eg^2x^4$$

Antiderivative was successfully verified.

[In] Integrate[(((d + e\*x)^7\*(f + g\*x)^2)/(d^2 - e^2\*x^2)^3,x]

[Out] -((d\*(7\*e^2\*f^2 + 48\*d\*e\*f\*g + 56\*d^2\*g^2)\*x)/e^2) - ((e^2\*f^2 + 14\*d\*e\*f\*g + 24\*d^2\*g^2)\*x^2)/(2\*e) - (g\*(2\*e\*f + 7\*d\*g)\*x^3)/3 - (e\*g^2\*x^4)/4 + (8\*d^4\*(e\*f + d\*g)^2)/(e^3\*(d - e\*x)^2) + (32\*d^3\*(e^2\*f^2 + 3\*d\*e\*f\*g + 2\*d^2\*g^2))/(e^3\*(-d + e\*x)) - (8\*d^2\*(3\*e^2\*f^2 + 14\*d\*e\*f\*g + 13\*d^2\*g^2)\*Log[d - e\*x])/e^3

**Maple [A]** time = 0.014, size = 263, normalized size = 1.5

$$-\frac{eg^2x^4}{4} - \frac{7x^3dg^2}{3} - \frac{2ex^3fg}{3} - 12\frac{x^2d^2g^2}{e} - 7x^2dfg - \frac{ex^2f^2}{2} - 56\frac{d^3g^2x}{e^2} - 48\frac{d^2fgx}{e}$$

$$- 7df^2x - 104\frac{d^4\ln(ex-d)g^2}{e^3} - 112\frac{d^3\ln(ex-d)fg}{e^2} - 24\frac{d^2\ln(ex-d)f^2}{e} + 64\frac{d^5g^2}{e^3(ex-d)}$$

$$+ 96\frac{d^4fg}{e^2(ex-d)} + 32\frac{d^3f^2}{e(ex-d)} + 8\frac{d^6g^2}{e^3(ex-d)^2} + 16\frac{d^5fg}{e^2(ex-d)^2} + 8\frac{d^4f^2}{e(ex-d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^7\*(g\*x+f)^2/(-e^2\*x^2+d^2)^3,x)

[Out] -1/4\*e\*g^2\*x^4-7/3\*x^3\*d\*g^2-2/3\*e\*x^3\*f\*g-12/e\*x^2\*d^2\*g^2-7\*x^2\*d\*f\*g-1/2\*e\*x^2\*f^2-56/e^2\*d^3\*g^2\*x-48/e\*d^2\*f\*g\*x-7\*d\*f^2\*x-104\*d^4/e^3\*ln(e\*x-d)\*g^2-112\*d^3/e^2\*ln(e\*x-d)\*f\*g-24\*d^2/e\*ln(e\*x-d)\*f^2+64\*d^5/e^3/(e\*x-d)\*g^2+96\*d^4/e^2/(e\*x-d)\*f\*g+32\*d^3/e/(e\*x-d)\*f^2+8\*d^6/e^3/(e\*x-d)^2\*g^2+16\*d^5/e^2/(e\*x-d)^2\*f\*g+8\*d^4/e/(e\*x-d)^2\*f^2

**Maxima [A]** time = 0.698915, size = 306, normalized size = 1.71

$$\frac{8(3d^4e^2f^2 + 10d^5efg + 7d^6g^2 - 4(d^3e^3f^2 + 3d^4e^2fg + 2d^5eg^2)x)}{e^5x^2 - 2de^4x + d^2e^3}$$

$$\frac{3e^3g^2x^4 + 4(2e^3fg + 7de^2g^2)x^3 + 6(e^3f^2 + 14de^2fg + 24d^2eg^2)x^2 + 12(7de^2f^2 + 48d^2efg + 56d^3g^2)x}{12e^2}$$

$$- \frac{8(3d^2e^2f^2 + 14d^3efg + 13d^4g^2)\log(ex - d)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(e\*x + d)^7\*(g\*x + f)^2/(e^2\*x^2 - d^2)^3,x, algorithm="maxima")

[Out] -8\*(3\*d^4\*e^2\*f^2 + 10\*d^5\*e\*f\*g + 7\*d^6\*g^2 - 4\*(d^3\*e^3\*f^2 + 3\*d^4\*e^2\*f\*g + 2\*d^5\*e\*g^2)\*x)/(e^5\*x^2 - 2\*d\*e^4\*x + d^2\*e^3) - 1/12\*(3\*e^3\*g^2\*x^4 + 4\*(2\*e^3\*f\*g + 7\*d\*e^2\*g^2)\*x^3 + 6\*(e^3\*f^2 + 14\*d\*e^2\*f\*g + 24\*d^2\*e\*g^2)\*x^2 + 12\*(7\*d\*e^2\*f^2 + 48\*d^2\*e\*f\*g + 56\*d^3\*g^2)\*x)/e^2 - 8\*(3\*d^2\*e^2\*f^2 + 14\*d^3\*e\*f\*g + 13\*d^4\*g^2)\*Log[d - e\*x]/e^3

$$d^4 g^2) \cdot \log(e^x - d) / e^3$$

**Fricas [A]** time = 0.274277, size = 454, normalized size = 2.54

$$\frac{3e^6 g^2 x^6 + 288d^4 e^2 f^2 + 960d^5 efg + 672d^6 g^2 + 2(4e^6 fg + 11de^5 g^2)x^5 + (6e^6 f^2 + 68de^5 fg + 91d^2 e^4 g^2)x^4 + 4(18de^5 f^2 + 14d^4 e^2 fg + 13d^5 efg + 6d^6 g^2)x^3 + (27d^4 e^2 f^2 + 178d^3 e^3 fg + 200d^4 e^2 g^2)x^2 - 12(25d^3 e^3 f^2 + 48d^4 e^2 fg + 8d^5 e^2 g^2)x + 96(3d^4 e^2 f^2 + 14d^5 efg + 13d^6 g^2 + (3d^2 e^4 f^2 + 14d^3 e^3 fg + 13d^4 e^2 g^2)x^2 - 2(3d^3 e^3 f^2 + 14d^4 e^2 fg + 13d^5 efg + 6d^6 g^2)x) \cdot \log(e^x - d)}{(e^5 x^2 - 2d^2 e^4 x + d^2 e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(e\*x + d)^7\*(g\*x + f)^2/(e^2\*x^2 - d^2)^3,x, algorithm="fricas")

[Out] -1/12\*(3\*e^6\*g^2\*x^6 + 288\*d^4\*e^2\*f^2 + 960\*d^5\*e\*f\*g + 672\*d^6\*g^2 + 2\*(4\*e^6\*f^2 + 11\*d\*e^5\*g^2)\*x^5 + (6\*e^6\*f^2 + 68\*d\*e^5\*f\*g + 91\*d^2\*e^4\*g^2)\*x^4 + 4\*(18\*d\*e^5\*f^2 + 104\*d^2\*e^4\*f\*g + 103\*d^3\*e^3\*g^2)\*x^3 - 6\*(27\*d^2\*e^4\*f^2 + 178\*d^3\*e^3\*f\*g + 200\*d^4\*e^2\*g^2)\*x^2 - 12\*(25\*d^3\*e^3\*f^2 + 48\*d^4\*e^2\*f\*g + 8\*d^5\*e^2\*g^2)\*x + 96\*(3\*d^4\*e^2\*f^2 + 14\*d^5\*e\*f\*g + 13\*d^6\*g^2 + (3\*d^2\*e^4\*f^2 + 14\*d^3\*e^3\*f\*g + 13\*d^4\*e^2\*g^2)\*x^2 - 2\*(3\*d^3\*e^3\*f^2 + 14\*d^4\*e^2\*f\*g + 13\*d^5\*e\*f\*g + 6\*d^6\*g^2)\*x) \* log(e\*x - d) / (e^5\*x^2 - 2\*d^2\*e^4\*x + d^2\*e^3)

**Sympy [A]** time = 6.18843, size = 223, normalized size = 1.25

$$\frac{8d^2(13d^2g^2 + 14defg + 3e^2f^2) \log(-d + ex)}{e^3} - \frac{eg^2x^4}{4} - x^3 \left( \frac{7dg^2}{3} + \frac{2efg}{3} \right) + \frac{-56d^6g^2 - 80d^5efg - 24d^4e^2f^2 + x(64d^5eg^2 + 96d^4e^2fg + 32d^3e^3f^2)}{d^2e^3 - 2de^4x + e^5x^2} - \frac{x^2(24d^2g^2 + 14defg + e^2f^2)}{2e} - \frac{x(56d^3g^2 + 48d^2efg + 7de^2f^2)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*7\*(g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*3,x)

[Out] -8\*d\*\*2\*(13\*d\*\*2\*g\*\*2 + 14\*d\*e\*f\*g + 3\*e\*\*2\*f\*\*2)\*log(-d + e\*x)/e\*\*3 - e\*g\*\*2\*x\*\*4/4 - x\*\*3\*(7\*d\*g\*\*2/3 + 2\*e\*f\*g/3) + (-56\*d\*\*6\*g\*\*2 - 80\*d\*\*5\*e\*f\*g - 24\*d\*\*4\*e\*\*2\*f\*\*2 + x\*(64\*d\*\*5\*e\*g\*\*2 + 96\*d\*\*4\*e\*\*2\*f\*g + 32\*d\*\*3\*e\*\*3\*f\*\*2))/(d\*\*2\*e\*\*3 - 2\*d\*e\*\*4\*x + e\*\*5\*x\*\*2) - x\*\*2\*(24\*d\*\*2\*g\*\*2 + 14\*d\*e\*f\*g + e\*\*2\*f\*\*2)/(2\*e) - x\*(56\*d\*\*3\*g\*\*2 + 48\*d\*\*2\*e\*f\*g + 7\*d\*e\*\*2\*f\*\*2)/e\*\*2

**GIAC/XCAS [A]** time = 0.268887, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(e\*x + d)^7\*(g\*x + f)^2/(e^2\*x^2 - d^2)^3,x, algorithm="giac")

[Out] Done

$$3.570 \quad \int \frac{(d+ex)^6(f+gx)^2}{(d^2-e^2x^2)^3} dx$$

**Optimal.** Leaf size=149

$$\frac{4d^3(dg+ef)^2}{e^3(d-ex)^2} - \frac{4d^2(dg+ef)(7dg+3ef)}{e^3(d-ex)} - \frac{x(18d^2g^2+12defg+e^2f^2)}{e^2} - \frac{2d(19d^2g^2+18defg+3e^2f^2)\log(d-ex)}{e^3} - \frac{gx^2(3dg+ef)}{e} - \frac{g^2x^3}{3}$$

[Out]  $-\left(\frac{(e^2f^2 + 12d^*e^*f^*g + 18d^2g^2)x}{e^2} - \frac{g^*(e^*f + 3d^*g)x^2}{e} - \frac{g^2x^3}{3} + \frac{4d^3(e^*f + d^*g)^2}{e^3(d - e^*x)^2} - \frac{4d^2(e^*f + d^*g)(3e^*f + 7d^*g)}{e^3(d - e^*x)} - \frac{2d^*(3e^2f^2 + 18d^*e^*f^*g + 19d^2g^2)\text{Log}[d - e^*x]}{e^3}\right)$

**Rubi [A]** time = 0.407899, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$\frac{4d^3(dg+ef)^2}{e^3(d-ex)^2} - \frac{4d^2(dg+ef)(7dg+3ef)}{e^3(d-ex)} - \frac{x(18d^2g^2+12defg+e^2f^2)}{e^2} - \frac{2d(19d^2g^2+18defg+3e^2f^2)\log(d-ex)}{e^3} - \frac{gx^2(3dg+ef)}{e} - \frac{g^2x^3}{3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(d + e^*x)^6 * (f + g^*x)^2}{(d^2 - e^2 * x^2)^3}, x]$

[Out]  $-\left(\frac{(e^2f^2 + 12d^*e^*f^*g + 18d^2g^2)x}{e^2} - \frac{g^*(e^*f + 3d^*g)x^2}{e} - \frac{g^2x^3}{3} + \frac{4d^3(e^*f + d^*g)^2}{e^3(d - e^*x)^2} - \frac{4d^2(e^*f + d^*g)(3e^*f + 7d^*g)}{e^3(d - e^*x)} - \frac{2d^*(3e^2f^2 + 18d^*e^*f^*g + 19d^2g^2)\text{Log}[d - e^*x]}{e^3}\right)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{4d^3(dg+ef)^2}{e^3(d-ex)^2} - \frac{4d^2(dg+ef)(7dg+3ef)}{e^3(d-ex)} - \frac{2d(19d^2g^2+18defg+3e^2f^2)\log(d-ex)}{e^3} - \frac{g^2x^3}{3} - \frac{2g(3dg+ef)\int x dx}{e} - \frac{(6dg(3dg+2ef)+e^2f^2)\int f^2 dx}{e^2f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e^*x+d)^6 * (g^*x+f)^2 / (-e^2 * x^2 + d^2)^3, x)$

[Out]  $4d^3(dg+ef)^2/(e^3(d-ex)^2) - 4d^2(dg+ef)(7dg+3ef)/(e^3(d-ex)) - 2d^*(19d^2g^2+18d^*e^*f^*g+3e^2f^2)\log(d-ex)/e^3 - g^2x^3/3 - 2g^*(3d^*g+ef)\int x dx/e - (6d^*g^*(3d^*g+2e^*f)+e^2f^2)\int f^2 dx/(e^2f^2)$

**Mathematica [A]** time = 0.144051, size = 157, normalized size = 1.05

$$\frac{4d^3(dg+ef)^2}{e^3(d-ex)^2} - \frac{x(18d^2g^2+12defg+e^2f^2)}{e^2} + \frac{4d^2(7d^2g^2+10defg+3e^2f^2)}{e^3(ex-d)} - \frac{2d(19d^2g^2+18defg+3e^2f^2)\log(d-ex)}{e^3} - \frac{gx^2(3dg+ef)}{e} - \frac{g^2x^3}{3}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^6\*(f + g\*x)^2)/(d^2 - e^2\*x^2)^3,x]

[Out] -(((e^2\*f^2 + 12\*d\*e\*f\*g + 18\*d^2\*g^2)\*x)/e^2) - (g\*(e\*f + 3\*d\*g)\*x^2)/e - (g^2\*x^3)/3 + (4\*d^3\*(e\*f + d\*g)^2)/(e^3\*(d - e\*x)^2) + (4\*d^2\*(3\*e^2\*f^2 + 10\*d\*e\*f\*g + 7\*d^2\*g^2))/(e^3\*(-d + e\*x)) - (2\*d\*(3\*e^2\*f^2 + 18\*d\*e\*f\*g + 19\*d^2\*g^2)\*Log[d - e\*x])/e^3

**Maple [A]** time = 0.015, size = 228, normalized size = 1.5

$$\begin{aligned} & -\frac{g^2x^3}{3} - 3\frac{dx^2g^2}{e} - x^2fg - 18\frac{d^2g^2x}{e^2} - 12\frac{dfgx}{e} - f^2x - 38\frac{d^3\ln(ex-d)g^2}{e^3} \\ & - 36\frac{d^2\ln(ex-d)fg}{e^2} - 6\frac{d\ln(ex-d)f^2}{e} + 28\frac{d^4g^2}{e^3(ex-d)} + 40\frac{d^3fg}{e^2(ex-d)} \\ & + 12\frac{d^2f^2}{e(ex-d)} + 4\frac{d^5g^2}{e^3(ex-d)^2} + 8\frac{d^4fg}{e^2(ex-d)^2} + 4\frac{d^3f^2}{e(ex-d)^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^6\*(g\*x+f)^2/(-e^2\*x^2+d^2)^3,x)

[Out] -1/3\*g^2\*x^3-3/e\*x^2\*d\*g^2-x^2\*f\*g-18/e^2\*d^2\*g^2\*x-12/e\*d\*f\*g\*x-f^2\*x-38\*d^3/e^3\*ln(e\*x-d)\*g^2-36\*d^2/e^2\*ln(e\*x-d)\*f\*g-6\*d/e\*ln(e\*x-d)\*f^2+28\*d^4/e^3/(e\*x-d)\*g^2+40\*d^3/e^2/(e\*x-d)\*f\*g+12\*d^2/e/(e\*x-d)\*f^2+4\*d^5/e^3/(e\*x-d)^2\*g^2+8\*d^4/e^2/(e\*x-d)^2\*f\*g+4\*d^3/e/(e\*x-d)^2\*f^2

**Maxima [A]** time = 0.699083, size = 254, normalized size = 1.7

$$\begin{aligned} & \frac{4(2d^3e^2f^2 + 8d^4efg + 6d^5g^2 - (3d^2e^3f^2 + 10d^3e^2fg + 7d^4eg^2)x)}{e^5x^2 - 2de^4x + d^2e^3} \\ & - \frac{e^2g^2x^3 + 3(e^2fg + 3deg^2)x^2 + 3(e^2f^2 + 12defg + 18d^2g^2)x}{3e^2} \\ & - \frac{2(3de^2f^2 + 18d^2efg + 19d^3g^2)\log(ex-d)}{e^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(e\*x + d)^6\*(g\*x + f)^2/(e^2\*x^2 - d^2)^3,x, algorithm="maxima")

[Out] -4\*(2\*d^3\*e^2\*f^2 + 8\*d^4\*e\*f\*g + 6\*d^5\*g^2 - (3\*d^2\*e^3\*f^2 + 10\*d^3\*e^2\*f\*g + 7\*d^4\*e\*g^2)\*x)/(e^5\*x^2 - 2\*d\*e^4\*x + d^2\*e^3) - 1/3\*(e^2\*g^2\*x^3 + 3\*(e^2\*f\*g + 3\*d\*e\*g^2)\*x^2 + 3\*(e^2\*f^2 + 12\*d\*e\*f\*g + 18\*d^2\*g^2)\*x)/e^2 - 2\*(3\*d\*e^2\*f^2 + 18\*d^2\*e\*f\*g + 19\*d^3\*g^2)\*log(e\*x - d)/e^3

**Fricas [A]** time = 0.276292, size = 397, normalized size = 2.66

$$\frac{e^5g^2x^5 + 24d^3e^2f^2 + 96d^4efg + 72d^5g^2 + (3e^5fg + 7de^4g^2)x^4 + (3e^5f^2 + 30de^4fg + 37d^2e^3g^2)x^3 - 3(2de^4f^2 + 23d^2e^3fg + 19d^3g^2)x^2 + 3(e^2f^2 + 12defg + 18d^2g^2)x + 3e^2\log(ex-d)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(e\*x + d)^6\*(g\*x + f)^2/(e^2\*x^2 - d^2)^3,x, algorithm="fricas")

[Out] 
$$-1/3*(e^{5g^2x^5} + 24d^3e^{2f^2} + 96d^4efg + 72d^5g^2 + (3e^{5fg} + 7d^4g^2)x^4 + (3e^{5f^2} + 30d^4efg + 37d^2e^{3g^2})x^3 - 3(2d^4f^2 + 23d^2e^{3fg} + 33d^3e^{2g^2})x^2 - 3(11d^2e^{3f^2} + 28d^3e^{2fg} + 10d^4eg^2)x + 6(3d^3e^{2f^2} + 18d^4efg + 19d^5g^2 + (3d^4f^2 + 18d^2e^{3fg} + 19d^3e^{2g^2})x^2 - 2(3d^2e^{3f^2} + 18d^3e^{2fg} + 19d^4eg^2)x) \log(ex - d)) / (e^{5x^2} - 2d^4ex + d^2e^3)$$

**Sympy [A]** time = 5.49979, size = 180, normalized size = 1.21

$$\frac{2d(19d^2g^2 + 18defg + 3e^2f^2) \log(-d + ex)}{e^3} - \frac{g^2x^3}{3} + \frac{-24d^5g^2 - 32d^4efg - 8d^3e^2f^2 + x(28d^4eg^2 + 40d^3e^2fg + 12d^2e^3f^2)}{d^2e^3 - 2de^4x + e^5x^2} - \frac{x^2(3dg^2 + efg)}{e} - \frac{x(18d^2g^2 + 12defg + e^2f^2)}{e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**6*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)`

[Out] 
$$-2*d*(19*d**2*g**2 + 18*d*e*f*g + 3*e**2*f**2)*\log(-d + e*x)/e**3 - g**2*x**3/3 + (-24*d**5*g**2 - 32*d**4*e*f*g - 8*d**3*e**2*f**2 + x*(28*d**4*e*g**2 + 40*d**3*e**2*f*g + 12*d**2*e**3*f**2))/(d**2*e**3 - 2*d*e**4*x + e**5*x**2) - x**2*(3*d*g**2 + e*f*g)/e - x*(18*d**2*g**2 + 12*d*e*f*g + e**2*f**2)/e**2$$

**GIAC/XCAS [A]** time = 0.266332, size = 1, normalized size = 0.01

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(e*x + d)^6*(g*x + f)^2/(e^2*x^2 - d^2)^3,x, algorithm="giac")`

[Out] Done

$$3.571 \quad \int \frac{(d+ex)^5(f+gx)^2}{(d^2-e^2x^2)^3} dx$$

**Optimal.** Leaf size=118

$$\frac{2d^2(dg+ef)^2}{e^3(d-ex)^2} - \frac{(13d^2g^2+10defg+e^2f^2)\log(d-ex)}{e^3} - \frac{4d(3dg+ef)(dg+ef)}{e^3(d-ex)} - \frac{gx(5dg+2ef)}{e^2} - \frac{g^2x^2}{2e}$$

[Out]  $-\left(\frac{g(2e^2f+5d^2g)x}{e^2} - \frac{g^2x^2}{2e}\right) - \frac{(2d^2(e^2f+d^2g)^2)/(e^3(d-ex)^2) - (4d^2(e^2f+d^2g)(e^2f+3d^2g))/(e^3(d-ex)) - ((e^2f^2+10d^2efg+13d^2g^2)\text{Log}[d-ex])}{e^3}$

**Rubi [A]** time = 0.316602, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$\frac{2d^2(dg+ef)^2}{e^3(d-ex)^2} - \frac{(13d^2g^2+10defg+e^2f^2)\log(d-ex)}{e^3} - \frac{4d(3dg+ef)(dg+ef)}{e^3(d-ex)} - \frac{gx(5dg+2ef)}{e^2} - \frac{g^2x^2}{2e}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^5\*(f + g\*x)^2)/(d^2 - e^2\*x^2)^3, x]

[Out]  $-\left(\frac{g(2e^2f+5d^2g)x}{e^2} - \frac{g^2x^2}{2e}\right) - \frac{(2d^2(e^2f+d^2g)^2)/(e^3(d-ex)^2) - (4d^2(e^2f+d^2g)(e^2f+3d^2g))/(e^3(d-ex)) - ((e^2f^2+10d^2efg+13d^2g^2)\text{Log}[d-ex])}{e^3}$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2d^2(dg+ef)^2}{e^3(d-ex)^2} - \frac{4d(dg+ef)(3dg+ef)}{e^3(d-ex)} - \frac{g^2 \int x dx}{e} - \frac{(5dg+2ef) \int g dx}{e^2} - \frac{(13d^2g^2+10defg+e^2f^2)\log(d-ex)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*5\*(g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*3, x)

[Out]  $2*d^2*(d*g+e*f)**2/(e**3*(d-e*x)**2) - 4*d*(d*g+e*f)*(3*d*g+e*f)/(e**3*(d-e*x)) - g**2*Integral(x, x)/e - (5*d*g+2*e*f)*Integral(g, x)/e**2 - (13*d**2*g**2+10*d*e*f*g+e**2*f**2)*log(d-e*x)/e**3$

**Mathematica [A]** time = 0.160402, size = 118, normalized size = 1.

$$\frac{\frac{8d(3d^2g^2+4defg+e^2f^2)}{d-ex} + 2(13d^2g^2+10defg+e^2f^2)\log(d-ex) - \frac{4d^2(dg+ef)^2}{(d-ex)^2} + 2egx(5dg+2ef) + e^2g^2x^2}{2e^3}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^5\*(f + g\*x)^2)/(d^2 - e^2\*x^2)^3, x]

[Out]  $-\frac{(2e^2g(2e^2f+5d^2g)x + e^2g^2x^2 - (4d^2(e^2f+d^2g)^2)/(d-e*x)^2 + (8d^2(e^2f^2+4d^2efg+3d^2g^2))/(d-e*x) + 2(e^2f^2+10d^2efg+13d^2g^2)\text{Log}[d-ex])}{2e^3}$



**Maple [A]** time = 0.013, size = 198, normalized size = 1.7

$$-\frac{g^2 x^2}{2e} - 5 \frac{dg^2 x}{e^2} - 2 \frac{fgx}{e} - 13 \frac{\ln(ex-d)d^2 g^2}{e^3} - 10 \frac{\ln(ex-d)dfg}{e^2} - \frac{\ln(ex-d)f^2}{e} + 12 \frac{d^3 g^2}{e^3(ex-d)} + 16 \frac{d^2 fg}{e^2(ex-d)} + 4 \frac{df^2}{e(ex-d)} + 2 \frac{d^4 g^2}{e^3(ex-d)^2} + 4 \frac{d^3 fg}{e^2(ex-d)^2} + 2 \frac{d^2 f^2}{e(ex-d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^5*(g*x+f)^2/(-e^2*x^2+d^2)^3,x)`

[Out]  $-1/2 * g^2 * x^2 / e - 5 * g^2 / e^2 * d * x - 2 * g / e * f * x - 13 / e^3 * \ln(e * x - d) * d^2 * g^2 - 10 / e^2 * \ln(e * x - d) * d * f * g - 1 / e * \ln(e * x - d) * f^2 + 12 * d^3 / e^3 / (e * x - d) * g^2 + 16 * d^2 / e^2 / (e * x - d) * f * g + 4 * d / e / (e * x - d) * f^2 + 2 * d^4 / e^3 / (e * x - d)^2 * g^2 + 4 * d^3 / e^2 / (e * x - d)^2 * f * g + 2 * d^2 / e / (e * x - d)^2 * f^2$

**Maxima [A]** time = 0.698158, size = 201, normalized size = 1.7

$$\frac{2(d^2 e^2 f^2 + 6 d^3 e f g + 5 d^4 g^2 - 2(d e^3 f^2 + 4 d^2 e^2 f g + 3 d^3 e g^2)x)}{e^5 x^2 - 2 d e^4 x + d^2 e^3} - \frac{e g^2 x^2 + 2(2 e f g + 5 d g^2)x}{2 e^2} - \frac{(e^2 f^2 + 10 d e f g + 13 d^2 g^2) \log(ex-d)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(e*x+d)^5*(g*x+f)^2/(e^2*x^2-d^2)^3,x,algorithm="maxima")`

[Out]  $-2 * (d^2 * e^2 * f^2 + 6 * d^3 * e * f * g + 5 * d^4 * g^2 - 2 * (d * e^3 * f^2 + 4 * d^2 * e^2 * f * g + 3 * d^3 * e * g^2) * x) / (e^5 * x^2 - 2 * d * e^4 * x + d^2 * e^3) - 1/2 * (e * g^2 * x^2 + 2 * (2 * e * f * g + 5 * d * g^2) * x) / e^2 - (e^2 * f^2 + 10 * d * e * f * g + 13 * d^2 * g^2) * \log(e * x - d) / e^3$

**Fricas [A]** time = 0.280647, size = 325, normalized size = 2.75

$$\frac{e^4 g^2 x^4 + 4 d^2 e^2 f^2 + 24 d^3 e f g + 20 d^4 g^2 + 4(e^4 f g + 2 d e^3 g^2)x^3 - (8 d e^3 f g + 19 d^2 e^2 g^2)x^2 - 2(4 d e^3 f^2 + 14 d^2 e^2 f g + 7 d^3 e g^2)x - 2 d^4 f^2}{2(e^5 x^2 - 2 d e^4 x + d^2 e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(e*x+d)^5*(g*x+f)^2/(e^2*x^2-d^2)^3,x,algorithm="fricas")`

[Out]  $-1/2 * (e^4 * g^2 * x^4 + 4 * d^2 * e^2 * f^2 + 24 * d^3 * e * f * g + 20 * d^4 * g^2 + 4 * (e^4 * f * g + 2 * d * e^3 * g^2) * x^3 - (8 * d * e^3 * f * g + 19 * d^2 * e^2 * g^2) * x^2 - 2 * (4 * d * e^3 * f^2 + 14 * d^2 * e^2 * f * g + 7 * d^3 * e * g^2) * x + 2 * (d^2 * e^2 * f^2 + 10 * d^3 * e * f * g + 13 * d^4 * g^2 + (e^4 * f^2 + 10 * d * e^3 * f * g + 13 * d^2 * e^2 * g^2) * x^2 - 2 * (d * e^3 * f^2 + 10 * d^2 * e^2 * f * g + 13 * d^3 * e * g^2) * x) * \log(e * x - d)) / (e^5 * x^2 - 2 * d * e^4 * x + d^2 * e^3)$

**Sympy [A]** time = 4.95416, size = 148, normalized size = 1.25

$$-\frac{10 d^4 g^2 - 12 d^3 e f g - 2 d^2 e^2 f^2 + x(12 d^3 e g^2 + 16 d^2 e^2 f g + 4 d e^3 f^2)}{e^2} - \frac{d^2 e^3 - 2 d e^4 x + e^5 x^2}{e^2} - \frac{x(5 d g^2 + 2 e f g) + (13 d^2 g^2 + 10 d e f g + e^2 f^2) \log(-d + e x)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*5\*(g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*3,x)

[Out] (-10\*d\*\*4\*g\*\*2 - 12\*d\*\*3\*e\*f\*g - 2\*d\*\*2\*e\*\*2\*f\*\*2 + x\*(12\*d\*\*3\*e\*g\*\*2 + 16\*d\*\*2\*e\*\*2\*f\*g + 4\*d\*e\*\*3\*f\*\*2))/(d\*\*2\*e\*\*3 - 2\*d\*e\*\*4\*x + e\*\*5\*x\*\*2) - g\*\*2\*x\*\*2/(2\*e) - x\*(5\*d\*g\*\*2 + 2\*e\*f\*g)/e\*\*2 - (13\*d\*\*2\*g\*\*2 + 10\*d\*e\*f\*g + e\*\*2\*f\*\*2)\*log(-d + e\*x)/e\*\*3

**GIAC/XCAS [A]** time = 0.274908, size = 369, normalized size = 3.13

$$-\frac{1}{2} (13 d^2 g^2 e^5 + 10 d f g e^6 + f^2 e^7) e^{(-8)} \ln(|x^2 e^2 - d^2|) - \frac{1}{2} (g^2 x^2 e^{11} + 10 d g^2 x e^{10} + 4 f g x e^{11}) e^{(-12)}$$

$$-\frac{(13 d^3 g^2 e^4 + 10 d^2 f g e^5 + d f^2 e^6) e^{(-7)} \ln\left(\frac{|2 x e^2 - 2 |d| e|}{|2 x e^2 + 2 |d| e|}\right)}{2 |d|}$$

$$-\frac{2 (5 d^6 g^2 e^5 + 6 d^5 f g e^6 + d^4 f^2 e^7 - 2 (3 d^3 g^2 e^8 + 4 d^2 f g e^9 + d f^2 e^{10}) x^3 - (7 d^4 g^2 e^7 + 10 d^3 f g e^8 + 3 d^2 f^2 e^9) x^2 + 4 (d^5 g^2 e^6 - (x^2 e^2 - d^2)^2))}{(x^2 e^2 - d^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(e\*x + d)^5\*(g\*x + f)^2/(e^2\*x^2 - d^2)^3,x, algorithm="giac")

[Out] -1/2\*(13\*d^2\*g^2\*e^5 + 10\*d\*f\*g\*e^6 + f^2\*e^7)\*e^(-8)\*ln(abs(x^2\*e^2 - d^2)) - 1/2\*(g^2\*x^2\*e^11 + 10\*d\*g^2\*x\*e^10 + 4\*f\*g\*x\*e^11)\*e^(-12) - 1/2\*(13\*d^3\*g^2\*e^4 + 10\*d^2\*f\*g\*e^5 + d\*f^2\*e^6)\*e^(-7)\*ln(abs(2\*x\*e^2 - 2\*abs(d)\*e)/abs(2\*x\*e^2 + 2\*abs(d)\*e))/abs(d) - 2\*(5\*d^6\*g^2\*e^5 + 6\*d^5\*f\*g\*e^6 + d^4\*f^2\*e^7 - 2\*(3\*d^3\*g^2\*e^8 + 4\*d^2\*f\*g\*e^9 + d\*f^2\*e^10)\*x^3 - (7\*d^4\*g^2\*e^7 + 10\*d^3\*f\*g\*e^8 + 3\*d^2\*f^2\*e^9)\*x^2 + 4\*(d^5\*g^2\*e^6 - (x^2\*e^2 - d^2)^2))/e^(-8)

$$3.572 \quad \int \frac{(d+ex)^4(f+gx)^2}{(d^2-e^2x^2)^3} dx$$

**Optimal.** Leaf size=81

$$-\frac{(dg+ef)(5dg+ef)}{e^3(d-ex)} + \frac{d(dg+ef)^2}{e^3(d-ex)^2} - \frac{2g(2dg+ef)\log(d-ex)}{e^3} - \frac{g^2x}{e^2}$$

[Out]  $-\frac{(g^2x)/e^2}{e^3} + \frac{d(e^2f + d^2g)^2}{(e^3(d - ex)^2)} - \frac{(e^2f + d^2g)(e^2f + 5d^2g)}{(e^3(d - ex))} - \frac{2g^2(e^2f + 2d^2g)\text{Log}[d - ex]}{e^3}$

**Rubi [A]** time = 0.219449, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$-\frac{(dg+ef)(5dg+ef)}{e^3(d-ex)} + \frac{d(dg+ef)^2}{e^3(d-ex)^2} - \frac{2g(2dg+ef)\log(d-ex)}{e^3} - \frac{g^2x}{e^2}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^4\*(f + g\*x)^2)/(d^2 - e^2\*x^2)^3, x]

[Out]  $-\frac{(g^2x)/e^2}{e^3} + \frac{d(e^2f + d^2g)^2}{(e^3(d - ex)^2)} - \frac{(e^2f + d^2g)(e^2f + 5d^2g)}{(e^3(d - ex))} - \frac{2g^2(e^2f + 2d^2g)\text{Log}[d - ex]}{e^3}$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{d(dg+ef)^2}{e^3(d-ex)^2} - g^2 \int \frac{1}{e^2} dx - \frac{2g(2dg+ef)\log(d-ex)}{e^3} - \frac{(dg+ef)(5dg+ef)}{e^3(d-ex)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*4\*(g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*3, x)

[Out]  $\frac{d^2(d^2g + e^2f)^2}{(e^3(d - e^2x)^2)} - \frac{g^2 \text{Integral}(e^{-2}, x)}{e^3} - \frac{2g^2(2d^2g + e^2f)\log(d - e^2x)}{e^3} - \frac{(d^2g + e^2f)(5d^2g + e^2f)}{(e^3(d - e^2x))}$

**Mathematica [A]** time = 0.0656663, size = 93, normalized size = 1.15

$$\frac{-4d^3g^2 + 4d^2eg(gx - f) + 2de^2gx(3f + gx) - 2g(d - ex)^2(2dg + ef)\log(d - ex) + e^3x(f^2 - g^2x^2)}{e^3(d - ex)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(((d + e\*x)^4\*(f + g\*x)^2)/(d^2 - e^2\*x^2)^3, x]

[Out]  $\frac{(-4d^3g^2 + 4d^2e^2g^2(-f + gx) + 2d^2e^2g^2x(3f + gx) + e^3x(f^2 - g^2x^2) - 2g^2(e^2f + 2d^2g)(d - e^2x)^2\text{Log}[d - e^2x])}{(e^3(d - e^2x)^2)}$

**Maple [A]** time = 0.012, size = 151, normalized size = 1.9

$$-\frac{g^2x}{e^2} - 4\frac{d\ln(ex-d)g^2}{e^3} - 2\frac{\ln(ex-d)fg}{e^2} + 5\frac{d^2g^2}{e^3(ex-d)} + 6\frac{dfg}{e^2(ex-d)} + \frac{f^2}{e(ex-d)} + \frac{d^3g^2}{e^3(ex-d)^2} + 2\frac{d^2fg}{e^2(ex-d)^2} + \frac{df^2}{e(ex-d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^4*(g*x+f)^2/(-e^2*x^2+d^2)^3,x)`

[Out] `-g^2*x/e^2-4/e^3*d*ln(e*x-d)*g^2-2/e^2*ln(e*x-d)*f*g+5/e^3/(e*x-d)*d^2*g^2+6/e^2/(e*x-d)*d*f*g+1/e/(e*x-d)*f^2+d^3/e^3/(e*x-d)^2*g^2+2*d^2/e^2/(e*x-d)^2*f*g+d/e/(e*x-d)^2*f^2`

**Maxima [A]** time = 0.703187, size = 142, normalized size = 1.75

$$-\frac{g^2x}{e^2} - \frac{4d^2efg + 4d^3g^2 - (e^3f^2 + 6de^2fg + 5d^2eg^2)x}{e^5x^2 - 2de^4x + d^2e^3} - \frac{2(efg + 2dg^2)\log(ex-d)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(e*x+d)^4*(g*x+f)^2/(e^2*x^2-d^2)^3,x,algorithm="maxima")`

[Out] `-g^2*x/e^2 - (4*d^2*e*f*g + 4*d^3*g^2 - (e^3*f^2 + 6*d*e^2*f*g + 5*d^2*e*g^2)*x)/(e^5*x^2 - 2*d*e^4*x + d^2*e^3) - 2*(e*f*g + 2*d*g^2)*log(e*x - d)/e^3`

**Fricas [A]** time = 0.275255, size = 215, normalized size = 2.65

$$-\frac{e^3g^2x^3 - 2de^2g^2x^2 + 4d^2efg + 4d^3g^2 - (e^3f^2 + 6de^2fg + 4d^2eg^2)x + 2(d^2efg + 2d^3g^2 + (e^3fg + 2de^2g^2)x^2 - 2(de^2fg + 2d^3g^2))\log(ex-d)}{e^5x^2 - 2de^4x + d^2e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(e*x+d)^4*(g*x+f)^2/(e^2*x^2-d^2)^3,x,algorithm="fricas")`

[Out] `-(e^3*g^2*x^3 - 2*d*e^2*g^2*x^2 + 4*d^2*e*f*g + 4*d^3*g^2 - (e^3*f^2 + 6*d*e^2*f*g + 4*d^2*e*g^2)*x + 2*(d^2*e*f*g + 2*d^3*g^2 + (e^3*f*g + 2*d*e^2*g^2)*x^2 - 2*(d^2*efg + 2*d^3*g^2 + (e^3*fg + 2*de^2*g^2)*x^2 - 2*(de^2*fg + 2*d^3*g^2))*log(e*x - d))/(e^5*x^2 - 2*d*e^4*x + d^2*e^3)`

**Sympy [A]** time = 3.94117, size = 99, normalized size = 1.22

$$-\frac{-4d^3g^2 - 4d^2efg + x(5d^2eg^2 + 6de^2fg + e^3f^2)}{d^2e^3 - 2de^4x + e^5x^2} - \frac{g^2x}{e^2} - \frac{2g(2dg + ef)\log(-d + ex)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**4*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)`

[Out] `(-4*d**3*g**2 - 4*d**2*e*f*g + x*(5*d**2*e*g**2 + 6*d*e**2*f*g + e**3*f**2))/(d**2*e**3 - 2*d*e**4*x + e**5*x**2) - g**2*x/e**2 - 2*g*(2*d*g + e*f)*log(-d + e*x)/e**3`

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**GIAC/XCAS [A]** time = 0.270494, size = 306, normalized size = 3.78

$$-g^2xe^{(-2)} - (2dg^2e^3 + fge^4)e^{(-6)}\ln(|x^2e^2 - d^2|) - \frac{(2d^2g^2e^4 + dfge^5)e^{(-7)}\ln\left(\frac{|2xe^2-2|d|e|}{|2xe^2+2|d|e|}\right)}{|d|}$$


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$$\frac{(4d^5g^2e^3 + 4d^4fge^4 - (5d^2g^2e^6 + 6dfge^7 + f^2e^8)x^3 - 2(3d^3g^2e^5 + 4d^2fge^6 + df^2e^7)x^2 + (3d^4g^2e^4 + 2d^3fge^5 - d^2f^2e^6)x - d^4f^2e^6)}{(x^2e^2 - d^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(e\*x + d)^4\*(g\*x + f)^2/(e^2\*x^2 - d^2)^3,x, algorithm="giac")

[Out] -g^2\*x\*e^(-2) - (2\*d\*g^2\*e^3 + f\*g\*e^4)\*e^(-6)\*ln(abs(x^2\*e^2 - d^2)) - (2\*d^2\*g^2\*e^4 + d\*f\*g\*e^5)\*e^(-7)\*ln(abs(2\*x\*e^2 - 2\*abs(d)\*e)/abs(2\*x\*e^2 + 2\*abs(d)\*e))/abs(d) - (4\*d^5\*g^2\*e^3 + 4\*d^4\*f\*g\*e^4 - (5\*d^2\*g^2\*e^6 + 6\*d\*f\*g\*e^7 + f^2\*e^8)\*x^3 - 2\*(3\*d^3\*g^2\*e^5 + 4\*d^2\*f\*g\*e^6 + d\*f^2\*e^7)\*x^2 + (3\*d^4\*g^2\*e^4 + 2\*d^3\*f\*g\*e^5 - d^2\*f^2\*e^6)\*x - d^4\*f^2\*e^6)/(x^2\*e^2 - d^2)^2

$$3.573 \quad \int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^3} dx$$

**Optimal.** Leaf size=61

$$-\frac{2g(dg+ef)}{e^3(d-ex)} + \frac{(dg+ef)^2}{2e^3(d-ex)^2} - \frac{g^2 \log(d-ex)}{e^3}$$

[Out] (e\*f + d\*g)^2/(2\*e^3\*(d - e\*x)^2) - (2\*g\*(e\*f + d\*g))/(e^3\*(d - e\*x)) - (g^2\*Log[d - e\*x])/e^3

**Rubi [A]** time = 0.11208, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$

$$-\frac{2g(dg+ef)}{e^3(d-ex)} + \frac{(dg+ef)^2}{2e^3(d-ex)^2} - \frac{g^2 \log(d-ex)}{e^3}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(f + g\*x)^2)/(d^2 - e^2\*x^2)^3, x]

[Out] (e\*f + d\*g)^2/(2\*e^3\*(d - e\*x)^2) - (2\*g\*(e\*f + d\*g))/(e^3\*(d - e\*x)) - (g^2\*Log[d - e\*x])/e^3

**Rubi in Sympy [A]** time = 20.7184, size = 51, normalized size = 0.84

$$-\frac{g^2 \log(d-ex)}{e^3} - \frac{2g(dg+ef)}{e^3(d-ex)} + \frac{(dg+ef)^2}{2e^3(d-ex)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3\*(g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*3, x)

[Out] -g\*\*2\*log(d - e\*x)/e\*\*3 - 2\*g\*(d\*g + e\*f)/(e\*\*3\*(d - e\*x)) + (d\*g + e\*f)\*\*2/(2\*e\*\*3\*(d - e\*x)\*\*2)

**Mathematica [A]** time = 0.0489859, size = 49, normalized size = 0.8

$$\frac{\frac{(dg+ef)(ef+4gx)-3dg}{(d-ex)^2} - 2g^2 \log(d-ex)}{2e^3}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(f + g\*x)^2)/(d^2 - e^2\*x^2)^3, x]

[Out] (((e\*f + d\*g)\*(-3\*d\*g + e\*(f + 4\*g\*x)))/(d - e\*x)^2 - 2\*g^2\*Log[d - e\*x])/(2\*e^3)

**Maple [A]** time = 0.01, size = 105, normalized size = 1.7

$$-\frac{\ln(ex-d)g^2}{e^3} + 2\frac{g^2d}{e^3(ex-d)} + 2\frac{fg}{e^2(ex-d)} + \frac{d^2g^2}{2e^3(ex-d)^2} + \frac{fgd}{e^2(ex-d)^2} + \frac{f^2}{2e(ex-d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^3*(g*x+f)^2/(-e^2*x^2+d^2)^3,x)`

[Out]  $-1/e^3 \ln(e*x-d) * g^2 + 2/e^3 * d / (e*x-d) * g^2 + 2/e^2 / (e*x-d) * f * g + 1/2/e^3 / (e*x-d)^2 * d^2 * g^2 + 1/e^2 / (e*x-d)^2 * d * f * g + 1/2/e / (e*x-d)^2 * f^2$

**Maxima [A]** time = 0.694079, size = 109, normalized size = 1.79

$$\frac{e^2 f^2 - 2 d e f g - 3 d^2 g^2 + 4 (e^2 f g + d e g^2) x}{2 (e^5 x^2 - 2 d e^4 x + d^2 e^3)} - \frac{g^2 \log (e x - d)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(e*x + d)^3*(g*x + f)^2/(e^2*x^2 - d^2)^3,x, algorithm="maxima")`

[Out]  $1/2*(e^2*f^2 - 2*d*e*f*g - 3*d^2*g^2 + 4*(e^2*f*g + d*e*g^2)*x) / (e^5*x^2 - 2*d*e^4*x + d^2*e^3) - g^2*log(e*x - d)/e^3$

**Fricas [A]** time = 0.272137, size = 135, normalized size = 2.21

$$\frac{e^2 f^2 - 2 d e f g - 3 d^2 g^2 + 4 (e^2 f g + d e g^2) x - 2 (e^2 g^2 x^2 - 2 d e g^2 x + d^2 g^2) \log (e x - d)}{2 (e^5 x^2 - 2 d e^4 x + d^2 e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(e*x + d)^3*(g*x + f)^2/(e^2*x^2 - d^2)^3,x, algorithm="fricas")`

[Out]  $1/2*(e^2*f^2 - 2*d*e*f*g - 3*d^2*g^2 + 4*(e^2*f*g + d*e*g^2)*x - 2*(e^2*g^2*x^2 - 2*d*e*g^2*x + d^2*g^2)*log(e*x - d)) / (e^5*x^2 - 2*d*e^4*x + d^2*e^3)$

**Sympy [A]** time = 2.72868, size = 80, normalized size = 1.31

$$\frac{-3d^2g^2 - 2defg + e^2f^2 + x(4deg^2 + 4e^2fg)}{2d^2e^3 - 4de^4x + 2e^5x^2} - \frac{g^2 \log(-d + ex)}{e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**3*(g*x+f)**2/(-e**2*x**2+d**2)**3,x)`

[Out]  $(-3*d**2*g**2 - 2*d*e*f*g + e**2*f**2 + x*(4*d*e*g**2 + 4*e**2*f*g)) / (2*d**2*e**3 - 4*d*e**4*x + 2*e**5*x**2) - g**2*log(-d + e*x) / e**3$

**GIAC/XCAS [A]** time = 0.271113, size = 263, normalized size = 4.31

$$-\frac{d g^2 e^{(-3)} \ln\left(\frac{|2 x e^2 - 2 |d| e|}{|2 x e^2 + 2 |d| e|}\right)}{2 |d|} - \frac{1}{2} g^2 e^{(-3)} \ln(|x^2 e^2 - d^2|)$$

$$+ \frac{\left(4(d^2 g^2 e^4 + d f g e^5) x^3 + (5 d^3 g^2 e^3 + 6 d^2 f g e^4 + d f^2 e^5) x^2 - 2(d^4 g^2 e^2 - d^2 f^2 e^4) x - (3 d^5 g^2 e^3 + 2 d^4 f g e^4 - d^3 f^2 e^5) e^{(-2)}\right)}{2(x^2 e^2 - d^2)^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(e*x + d)^3*(g*x + f)^2/(e^2*x^2 - d^2)^3,x, algorithm="giac")
```

```
[Out] -1/2*d*g^2*e^(-3)*ln(abs(2*x*e^2 - 2*abs(d)*e)/abs(2*x*e^2 + 2*abs(d)*e))/abs(d) - 1/2*g^2*e^(-3)*ln(abs(x^2*e^2 - d^2)) + 1/2*(4*(d^2*g^2*e^4 + d*f*g*e^5)*x^3 + (5*d^3*g^2*e^3 + 6*d^2*f*g*e^4 + d*f^2*e^5)*x^2 - 2*(d^4*g^2*e^2 - d^2*f^2*e^4)*x - (3*d^5*g^2*e^3 + 2*d^4*f*g*e^4 - d^3*f^2*e^5)*e^(-2))*e^(-4)/((x^2*e^2 - d^2)^2*d)
```



$$3.574 \quad \int \frac{(d+ex)^2(f+gx)^2}{(d^2-e^2x^2)^3} dx$$

**Optimal.** Leaf size=88

$$\frac{(ef-dg)^2 \tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3} + \frac{(ef-3dg)(dg+ef)}{4d^2e^3(d-ex)} + \frac{(dg+ef)^2}{4de^3(d-ex)^2}$$

[Out]  $(e^*f + d^*g)^2/(4^*d^*e^3*(d - e^*x)^2) + ((e^*f - 3^*d^*g)*(e^*f + d^*g))/(4^*d^2*e^3*(d - e^*x)) + ((e^*f - d^*g)^2*ArcTanh[(e^*x)/d])/(4^*d^3*e^3)$

**Rubi [A]** time = 0.213714, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$\frac{(ef-dg)^2 \tanh^{-1}\left(\frac{ex}{d}\right)}{4d^3e^3} + \frac{(ef-3dg)(dg+ef)}{4d^2e^3(d-ex)} + \frac{(dg+ef)^2}{4de^3(d-ex)^2}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^2\*(f + g\*x)^2)/(d^2 - e^2\*x^2)^3, x]

[Out]  $(e^*f + d^*g)^2/(4^*d^*e^3*(d - e^*x)^2) + ((e^*f - 3^*d^*g)*(e^*f + d^*g))/(4^*d^2*e^3*(d - e^*x)) + ((e^*f - d^*g)^2*ArcTanh[(e^*x)/d])/(4^*d^3*e^3)$

**Rubi in Sympy [A]** time = 43.458, size = 73, normalized size = 0.83

$$\frac{(dg+ef)^2}{4de^3(d-ex)^2} - \frac{(dg+ef)(3dg-ef)}{4d^2e^3(d-ex)} + \frac{(dg-ef)^2 \operatorname{atanh}\left(\frac{ex}{d}\right)}{4d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*2\*(g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*3, x)

[Out]  $(d^*g + e^*f)^2/(4^*d^*e^3*(d - e^*x)^2) - (d^*g + e^*f)*(3^*d^*g - e^*f)/(4^*d^2*e^3*(d - e^*x)) + (d^*g - e^*f)^2*\operatorname{atanh}(e^*x/d)/(4^*d^3*e^3)$

**Mathematica [A]** time = 0.133155, size = 90, normalized size = 1.02

$$\frac{-\frac{2d(dg+ef)(2d^2g-de(2f+3gx)+e^2fx)}{(d-ex)^2} + (ef-dg)^2(-\log(d-ex)) + (ef-dg)^2 \log(d+ex)}{8d^3e^3}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^2\*(f + g\*x)^2)/(d^2 - e^2\*x^2)^3, x]

[Out]  $((-2^*d^*(e^*f + d^*g)*(2^*d^2*g + e^2*f*x - d^*e*(2^*f + 3^*g*x)))/(d - e^*x)^2 - (e^*f - d^*g)^2*\operatorname{Log}[d - e^*x] + (e^*f - d^*g)^2*\operatorname{Log}[d + e^*x])/(8^*d^3*e^3)$

**Maple [B]** time = 0.015, size = 218, normalized size = 2.5

$$\frac{3g^2}{4e^3(ex-d)} + \frac{fg}{2e^2d(ex-d)} - \frac{f^2}{4d^2e(ex-d)} + \frac{g^2d}{4e^3(ex-d)^2} + \frac{fg}{2e^2(ex-d)^2} + \frac{f^2}{4de(ex-d)^2}$$

$$- \frac{\ln(ex-d)g^2}{8de^3} + \frac{\ln(ex-d)fg}{4d^2e^2} - \frac{\ln(ex-d)f^2}{8d^3e} + \frac{\ln(ex+d)g^2}{8de^3} - \frac{\ln(ex+d)fg}{4d^2e^2} + \frac{\ln(ex+d)f^2}{8d^3e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^2*(g*x+f)^2/(-e^2*x^2+d^2)^3,x)`

[Out]  $\frac{3}{4} \frac{e^3}{(e^2x+d)^2} \frac{g^2+1/2}{e^2/d} \frac{1}{(e^2x-d)} f^2 g - \frac{1}{4} \frac{e}{d^2} \frac{1}{(e^2x-d)} f^2 + \frac{1}{4} \frac{e^3}{d} \frac{1}{(e^2x-d)^2} \frac{g^2+1/2}{e^2/d} \frac{1}{(e^2x-d)^2} f^2 g + \frac{1}{4} \frac{e}{d} \frac{1}{(e^2x-d)^2} f^2$   
 $2 - \frac{1}{8} \frac{e^3}{d} \ln(e^2x-d) \frac{g^2+1/4}{e^2/d^2} \ln(e^2x-d) f^2 g - \frac{1}{8} \frac{e}{d^3} \ln(e^2x-d) f^2 + \frac{1}{8} \frac{e^3}{d} \ln(e^2x+d) \frac{g^2-1/4}{e^2/d^2} \ln(e^2x+d) f^2 g + \frac{1}{8} \frac{e}{d^3} \ln(e^2x+d) f^2$

**Maxima [A]** time = 0.709806, size = 203, normalized size = 2.31

$$\frac{2de^2f^2 - 2d^3g^2 - (e^3f^2 - 2de^2fg - 3d^2eg^2)x}{4(d^2e^5x^2 - 2d^3e^4x + d^4e^3)}$$

$$+ \frac{(e^2f^2 - 2defg + d^2g^2) \log(ex+d)}{8d^3e^3} - \frac{(e^2f^2 - 2defg + d^2g^2) \log(ex-d)}{8d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(e*x+d)^2*(g*x+f)^2/(e^2*x^2-d^2)^3,x, algorithm="maxima")`

[Out]  $\frac{1}{4} \frac{(2d^2e^2f^2 - 2d^3g^2 - (e^3f^2 - 2de^2fg - 3d^2eg^2)x)}{(d^2e^5x^2 - 2d^3e^4x + d^4e^3)} + \frac{1}{8} \frac{(e^2f^2 - 2defg + d^2g^2) \log(ex+d)}{(d^3e^3)} - \frac{1}{8} \frac{(e^2f^2 - 2defg + d^2g^2) \log(ex-d)}{(d^3e^3)}$

**Fricas [A]** time = 0.279323, size = 366, normalized size = 4.16

$$\frac{4d^2e^2f^2 - 4d^4g^2 - 2(de^3f^2 - 2d^2e^2fg - 3d^3eg^2)x + (d^2e^2f^2 - 2d^3efg + d^4g^2 + (e^4f^2 - 2de^3fg + d^2e^2g^2)x^2 - 2(de^3fg - 2d^2eg^2)x + d^4e^3)}{8(d^2e^5x^2 - 2d^3e^4x + d^4e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(e*x+d)^2*(g*x+f)^2/(e^2*x^2-d^2)^3,x, algorithm="fricas")`

[Out]  $\frac{1}{8} \frac{(4d^2e^2f^2 - 4d^4g^2 - 2(d^2e^2f^2 - 2d^3efg + d^4g^2 + (e^4f^2 - 2de^3fg + d^2e^2g^2)x^2 - 2(de^3fg - 2d^2eg^2)x + d^4e^3)) \log(ex+d) - (d^2e^2f^2 - 2d^3efg + d^4g^2 + (e^4f^2 - 2de^3fg + d^2e^2g^2)x^2 - 2(de^3fg - 2d^2eg^2)x + d^4e^3) \log(ex-d)}{(d^3e^5x^2 - 2d^4e^4x + d^5e^3)}$

**Sympy [A]** time = 4.24326, size = 185, normalized size = 2.1

$$\frac{-2d^3g^2 + 2de^2f^2 + x(3d^2eg^2 + 2de^2fg - e^3f^2)}{4d^4e^3 - 8d^3e^4x + 4d^2e^5x^2}$$

$$+ \frac{(dg - ef)^2 \log\left(-\frac{d(dg-ef)^2}{e(d^2g^2-2defg+e^2f^2)} + x\right)}{8d^3e^3} + \frac{(dg - ef)^2 \log\left(\frac{d(dg-ef)^2}{e(d^2g^2-2defg+e^2f^2)} + x\right)}{8d^3e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2\*(g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*3,x)

[Out]  $(-2*d**3*g**2 + 2*d*e**2*f**2 + x*(3*d**2*e*g**2 + 2*d*e**2*f*g - e**3*f**2))/(4*d**4*e**3 - 8*d**3*e**4*x + 4*d**2*e**5*x**2) - (d*g - e*f)**2*\log(-d*(d*g - e*f)**2/(e*(d**2*g**2 - 2*d*e*f*g + e**2*f**2))) + x)/(8*d**3*e**3) + (d*g - e*f)**2*\log(d*(d*g - e*f)**2/(e*(d**2*g**2 - 2*d*e*f*g + e**2*f**2))) + x)/(8*d**3*e**3)$

**GIAC/XCAS [A]** time = 0.276341, size = 266, normalized size = 3.02

$$\frac{(d^2g^2e^2 - 2dfge^3 + f^2e^4)e^{(-5)}\ln\left(\frac{|2xe^2-2|d|e|}{|2xe^2+2|d|e|}\right)}{8d^2|d|} + \frac{(3d^2g^2x^3e^4 + 4d^3g^2x^2e^3 - d^4g^2xe^2 - 2d^5g^2e + 2dfgx^3e^5 + 4d^2fgx^2e^4 + 2d^3fgxe^3 - f^2x^3e^6 + 3d^2f^2xe^4 + 2d^3f^2e^3)e^{(-5)}}{4(x^2e^2 - d^2)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(e\*x + d)^2\*(g\*x + f)^2/(e^2\*x^2 - d^2)^3,x, algorithm="giac")

[Out]  $-1/8*(d^2*g^2*e^2 - 2*d*f*g*e^3 + f^2*e^4)*e^{(-5)}*\ln(\text{abs}(2*x*e^2 - 2*\text{abs}(d)*e)/\text{abs}(2*x*e^2 + 2*\text{abs}(d)*e))/d^2*\text{abs}(d) + 1/4*(3*d^2*g^2*x^3*e^4 + 4*d^3*g^2*x^2*e^3 - d^4*g^2*x*e^2 - 2*d^5*g^2*e + 2*d*f*g*x^3*e^5 + 4*d^2*f*g*x^2*e^4 + 2*d^3*f*g*x*e^3 - f^2*x^3*e^6 + 3*d^2*f^2*x*e^4 + 2*d^3*f^2*e^3)*e^{(-5)}/((x^2*e^2 - d^2)^2*d^2)$

$$3.575 \quad \int \frac{(d+ex)(f+gx)^2}{(d^2-e^2x^2)^3} dx$$

**Optimal.** Leaf size=122

$$\frac{(dg+3ef)(ef-dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3} - \frac{(ef-dg)^2}{8d^3e^3(d+ex)} + \frac{(dg+ef)^2}{8d^2e^3(d-ex)^2} + \frac{e^2f^2-d^2g^2}{4d^3e^3(d-ex)}$$

[Out]  $(e^*f + d^*g)^2/(8^*d^2^*e^3^*(d - e^*x)^2) + (e^2^*f^2 - d^2^*g^2)/(4^*d^3^*e^3^*(d - e^*x)) - (e^*f - d^*g)^2/(8^*d^3^*e^3^*(d + e^*x)) + ((e^*f - d^*g)^*(3^*e^*f + d^*g)^*ArcTanh[(e^*x)/d])/(8^*d^4^*e^3)$

**Rubi [A]** time = 0.263918, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{(dg+3ef)(ef-dg)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^4e^3} - \frac{(ef-dg)^2}{8d^3e^3(d+ex)} + \frac{(dg+ef)^2}{8d^2e^3(d-ex)^2} + \frac{e^2f^2-d^2g^2}{4d^3e^3(d-ex)}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*(f + g\*x)^2)/(d^2 - e^2\*x^2)^3, x]

[Out]  $(e^*f + d^*g)^2/(8^*d^2^*e^3^*(d - e^*x)^2) + (e^2^*f^2 - d^2^*g^2)/(4^*d^3^*e^3^*(d - e^*x)) - (e^*f - d^*g)^2/(8^*d^3^*e^3^*(d + e^*x)) + ((e^*f - d^*g)^*(3^*e^*f + d^*g)^*ArcTanh[(e^*x)/d])/(8^*d^4^*e^3)$

**Rubi in Sympy [A]** time = 46.6515, size = 133, normalized size = 1.09

$$\frac{(dg+ef)^2}{8d^2e^3(d-ex)^2} - \frac{(dg-ef)^2}{8d^3e^3(d+ex)} - \frac{(dg-ef)(dg+ef)}{4d^3e^3(d-ex)} + \frac{(dg-ef)(dg+3ef)\log(d-ex)}{16d^4e^3} - \frac{(dg-ef)(dg+3ef)\log(d+ex)}{16d^4e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*(g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*3, x)

[Out]  $(d^*g + e^*f)^2/(8^*d^2^*e^3^*(d - e^*x)^2) - (d^*g - e^*f)^2/(8^*d^3^*e^3^*(d + e^*x)) - (d^*g - e^*f)^*(d^*g + e^*f)/(4^*d^3^*e^3^*(d - e^*x)) + (d^*g - e^*f)^*(d^*g + 3^*e^*f)^*\log(d - e^*x)/(16^*d^4^*e^3) - (d^*g - e^*f)^*(d^*g + 3^*e^*f)^*\log(d + e^*x)/(16^*d^4^*e^3)$

**Mathematica [A]** time = 0.212078, size = 140, normalized size = 1.15

$$\frac{\frac{4de^2f^2-4d^3g^2}{d-ex} + (d^2g^2 + 2defg - 3e^2f^2)\log(d-ex) + (-d^2g^2 - 2defg + 3e^2f^2)\log(d+ex) + \frac{2d^2(dg+ef)^2}{(d-ex)^2} - \frac{2d(ef-dg)^2}{d+ex}}{16d^4e^3}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(f + g\*x)^2)/(d^2 - e^2\*x^2)^3, x]

[Out]  $((2^*d^2^*(e^*f + d^*g)^2)/(d - e^*x)^2 + (4^*d^2^*e^2^*f^2 - 4^*d^3^*g^2)/(d - e^*x) - (2^*d^2^*(e^*f - d^*g)^2)/(d + e^*x) + (-3^*e^2^*f^2 + 2^*d^2^*e^*f^*g + d^2^*g^2)^*\text{Log}[d - e^*x] + (3^*e^2^*f^2 - 2^*d^2^*e^*f^*g - d^2^*g^2)^*\text{Log}[$

$$d + e^x)]/(16*d^4*e^3)$$

**Maple [B]** time = 0.019, size = 257, normalized size = 2.1

$$\begin{aligned} & \frac{g^2}{4de^3(ex-d)} - \frac{f^2}{4ed^3(ex-d)} + \frac{g^2}{8e^3(ex-d)^2} + \frac{fg}{4de^2(ex-d)^2} + \frac{f^2}{8ed^2(ex-d)^2} \\ & + \frac{\ln(ex-d)g^2}{16e^3d^2} + \frac{\ln(ex-d)fg}{8e^2d^3} - \frac{3\ln(ex-d)f^2}{16ed^4} - \frac{\ln(ex+d)g^2}{16e^3d^2} - \frac{\ln(ex+d)fg}{8e^2d^3} \\ & + \frac{3\ln(ex+d)f^2}{16ed^4} - \frac{g^2}{8de^3(ex+d)} + \frac{fg}{4e^2d^2(ex+d)} - \frac{f^2}{8ed^3(ex+d)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)\*(g\*x+f)^2/(-e^2\*x^2+d^2)^3,x)

[Out] 1/4/e^3/d/(e\*x-d)\*g^2-1/4/e/d^3/(e\*x-d)\*f^2+1/8/e^3/(e\*x-d)^2\*g^2+1/4/e^2/d/(e\*x-d)^2\*f\*g+1/8/e/d^2/(e\*x-d)^2\*f^2+1/16/e^3/d^2\*ln(e\*x-d)\*g^2+1/8/e^2/d^3\*ln(e\*x-d)\*f\*g-3/16/e/d^4\*ln(e\*x-d)\*f^2-1/16/e^3/d^2\*ln(e\*x+d)\*g^2-1/8/e^2/d^3\*ln(e\*x+d)\*f\*g+3/16/e/d^4\*ln(e\*x+d)\*f^2-1/8/d/e^3/(e\*x+d)\*g^2+1/4/d^2/e^2/(e\*x+d)\*f\*g-1/8/d^3/e/(e\*x+d)\*f^2

**Maxima [A]** time = 0.703857, size = 285, normalized size = 2.34

$$\begin{aligned} & \frac{2d^2e^2f^2 + 4d^3efg - 2d^4g^2 - (3e^4f^2 - 2de^3fg - d^2e^2g^2)x^2 + (3de^3f^2 - 2d^2e^2fg + 3d^3eg^2)x}{8(d^3e^6x^3 - d^4e^5x^2 - d^5e^4x + d^6e^3)} \\ & + \frac{(3e^2f^2 - 2defg - d^2g^2)\log(ex+d)}{16d^4e^3} - \frac{(3e^2f^2 - 2defg - d^2g^2)\log(ex-d)}{16d^4e^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(e\*x+d)\*(g\*x+f)^2/(e^2\*x^2-d^2)^3,x, algorithm="maxima")

[Out] 1/8\*(2\*d^2\*e^2\*f^2 + 4\*d^3\*e\*f\*g - 2\*d^4\*g^2 - (3\*e^4\*f^2 - 2\*d\*e^3\*f\*g - d^2\*e^2\*g^2)\*x^2 + (3\*d\*e^3\*f^2 - 2\*d^2\*e^2\*f\*g + 3\*d^3\*e\*g^2)\*x)/(d^3\*e^6\*x^3 - d^4\*e^5\*x^2 - d^5\*e^4\*x + d^6\*e^3) + 1/16\*(3\*e^2\*f^2 - 2\*d\*e\*f\*g - d^2\*g^2)\*log(e\*x+d)/(d^4\*e^3) - 1/16\*(3\*e^2\*f^2 - 2\*d\*e\*f\*g - d^2\*g^2)\*log(e\*x-d)/(d^4\*e^3)

**Fricas [A]** time = 0.278086, size = 563, normalized size = 4.61

$$\frac{4d^3e^2f^2 + 8d^4efg - 4d^5g^2 - 2(3de^4f^2 - 2d^2e^3fg - d^3e^2g^2)x^2 + 2(3d^2e^3f^2 - 2d^3e^2fg + 3d^4eg^2)x + (3d^3e^2f^2 - 2d^4efg - d^5g^2)}{8(d^3e^6x^3 - d^4e^5x^2 - d^5e^4x + d^6e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(e\*x+d)\*(g\*x+f)^2/(e^2\*x^2-d^2)^3,x, algorithm="fricas")

[Out] 1/16\*(4\*d^3\*e^2\*f^2 + 8\*d^4\*e\*f\*g - 4\*d^5\*g^2 - 2\*(3\*d\*e^4\*f^2 - 2\*d^2\*e^3\*f\*g - d^3\*e^2\*g^2)\*x^2 + 2\*(3\*d^2\*e^3\*f^2 - 2\*d^3\*e^2\*f\*g + 3\*d^4\*e\*g^2)\*x + (3\*d^3\*e^2\*f^2 - 2\*d^4\*e\*f\*g - d^5\*g^2)\*log(e\*x+d) - (3\*d^3\*e^2\*f^2 - 2\*d^4\*e\*f\*g - d^5\*g^2)\*log(e\*x-d)/(d^4\*e^3) - 1/16\*(3\*d^2\*e^3\*f^2 - 2\*d^3\*e^2\*f\*g + 3\*d^4\*e\*g^2)\*x + (3\*d^3\*e^2\*f^2 - 2\*d^4\*e\*f\*g - d^5\*g^2)\*log(e\*x+d) - (3\*d^3\*e^2\*f^2 - 2\*d^4\*e\*f\*g - d^5\*g^2)\*log(e\*x-d)/(d^4\*e^3)

$$d^4 e^g x^2 \log(e^x - d) / (d^4 e^6 x^3 - d^5 e^5 x^2 - d^6 e^4 x + d^7 e^3)$$

**Sympy [A]** time = 5.28868, size = 277, normalized size = 2.27

$$\frac{-2d^4 g^2 + 4d^3 efg + 2d^2 e^2 f^2 + x^2 (d^2 e^2 g^2 + 2de^3 fg - 3e^4 f^2) + x (3d^3 eg^2 - 2d^2 e^2 fg + 3de^3 f^2)}{8d^6 e^3 - 8d^5 e^4 x - 8d^4 e^5 x^2 + 8d^3 e^6 x^3} + \frac{(dg - ef)(dg + 3ef) \log\left(-\frac{d(dg-ef)(dg+3ef)}{e(d^2 g^2 + 2defg - 3e^2 f^2)} + x\right)}{16d^4 e^3} - \frac{(dg - ef)(dg + 3ef) \log\left(\frac{d(dg-ef)(dg+3ef)}{e(d^2 g^2 + 2defg - 3e^2 f^2)} + x\right)}{16d^4 e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*3,x)

[Out]  $(-2*d**4*g**2 + 4*d**3*e*f*g + 2*d**2*e**2*f**2 + x**2*(d**2*e**2*g**2 + 2*d*e**3*f*g - 3*e**4*f**2) + x*(3*d**3*e*g**2 - 2*d**2*e**2*f*g + 3*d*e**3*f**2))/(8*d**6*e**3 - 8*d**5*e**4*x - 8*d**4*e**5*x**2 + 8*d**3*e**6*x**3) + (d*g - e*f)*(d*g + 3*e*f)*\log(-d*(d*g - e*f)*(d*g + 3*e*f)/(e*(d**2*g**2 + 2*d*e*f*g - 3*e**2*f**2)) + x)/(16*d**4*e**3) - (d*g - e*f)*(d*g + 3*e*f)*\log(d*(d*g - e*f)*(d*g + 3*e*f)/(e*(d**2*g**2 + 2*d*e*f*g - 3*e**2*f**2)) + x)/(16*d**4*e**3)$

**GIAC/XCAS [A]** time = 0.270168, size = 258, normalized size = 2.11

$$\frac{(d^2 g^2 + 2dfge - 3f^2 e^2) e^{(-3)} \ln\left(\frac{|2xe^{2-2}|d|e|}{|2xe^{2+2}|d|e|}\right)}{16d^3|d|} + \frac{(d^2 g^2 x^3 e^4 + 4d^3 g^2 x^2 e^3 + d^4 g^2 x e^2 - 2d^5 g^2 e + 2dfgx^3 e^5 + 2d^3 fgxe^3 + 4d^4 fge^2 - 3f^2 x^3 e^6 + 5d^2 f^2 x e^4 + 2d^3 f^2 e^3) e^{(-4)}}{8(x^2 e^2 - d^2)^2 d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(e\*x + d)\*(g\*x + f)^2/(e^2\*x^2 - d^2)^3,x, algorithm="giac")

[Out]  $1/16*(d^2*g^2 + 2*d*f*g*e - 3*f^2*e^2)*e^{(-3)}*\ln(\text{abs}(2*x*e^2 - 2*\text{abs}(d)*e)/\text{abs}(2*x*e^2 + 2*\text{abs}(d)*e))/(d^3*\text{abs}(d)) + 1/8*(d^2*g^2*x^3*e^4 + 4*d^3*g^2*x^2*e^3 + d^4*g^2*x*e^2 - 2*d^5*g^2*e + 2*d*f*g*x^3*e^5 + 2*d^3*f*g*x^2*e^3 + 4*d^4*f*g*e^2 - 3*f^2*x^3*e^6 + 5*d^2*f^2*x*e^4 + 2*d^3*f^2*e^3)*e^{(-4)}/((x^2*e^2 - d^2)^2*d^3)$

$$3.576 \quad \int \frac{(f+gx)^2}{(d^2-e^2x^2)^3} dx$$

**Optimal.** Leaf size=127

$$\frac{(f+gx)(d^2g+e^2fx)}{4d^2e^2(d^2-e^2x^2)^2} + \frac{(3e^2f^2-d^2g^2)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^5e^3} + \frac{x(3e^2f^2-d^2g^2)+2d^2fg}{8d^4e^2(d^2-e^2x^2)}$$

[Out]  $((d^2g + e^2fx)(f + gx))/(4d^2e^2(d^2 - e^2x^2)^2) + (2d^2fg + (3e^2f^2 - d^2g^2)x)/(8d^4e^2(d^2 - e^2x^2)) + ((3e^2f^2 - d^2g^2) \operatorname{ArcTanh}[ex/d])/(8d^5e^3)$

**Rubi [A]** time = 0.134243, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$

$$\frac{(f+gx)(d^2g+e^2fx)}{4d^2e^2(d^2-e^2x^2)^2} + \frac{(3e^2f^2-d^2g^2)\tanh^{-1}\left(\frac{ex}{d}\right)}{8d^5e^3} + \frac{x(3e^2f^2-d^2g^2)+2d^2fg}{8d^4e^2(d^2-e^2x^2)}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)^2/(d^2 - e^2\*x^2)^3, x]

[Out]  $((d^2g + e^2fx)(f + gx))/(4d^2e^2(d^2 - e^2x^2)^2) + (2d^2fg + (3e^2f^2 - d^2g^2)x)/(8d^4e^2(d^2 - e^2x^2)) + ((3e^2f^2 - d^2g^2) \operatorname{ArcTanh}[ex/d])/(8d^5e^3)$

**Rubi in Sympy [A]** time = 30.4908, size = 110, normalized size = 0.87

$$\frac{(f+gx)(d^2g+e^2fx)}{4d^2e^2(d^2-e^2x^2)^2} - \frac{-2d^2fg+x(d^2g^2-3e^2f^2)}{8d^4e^2(d^2-e^2x^2)} - \frac{(d^2g^2-3e^2f^2)\operatorname{atanh}\left(\frac{ex}{d}\right)}{8d^5e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*3, x)

[Out]  $(f + gx)(d^2g + e^2fx)/(4d^2e^2(d^2 - e^2x^2)^2) - (-2d^2fg + x(d^2g^2 - 3e^2f^2))/(8d^4e^2(d^2 - e^2x^2)) - (d^2g^2 - 3e^2f^2) \operatorname{atanh}(ex/d)/(8d^5e^3)$

**Mathematica [A]** time = 0.0727657, size = 110, normalized size = 0.87

$$\frac{d^5eg(4f+gx)+d^3e^3x(5f^2+g^2x^2)+(d^2-e^2x^2)^2(3e^2f^2-d^2g^2)\tanh^{-1}\left(\frac{ex}{d}\right)-3de^5f^2x^3}{8d^5e^3(d^2-e^2x^2)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^2/(d^2 - e^2\*x^2)^3, x]

[Out]  $(-3d^5e^5f^2x^3 + d^5e^5g(4f + gx) + d^3e^3x(5f^2 + g^2x^2) + (d^2 - e^2x^2)^2 \operatorname{ArcTanh}[ex/d])/(8d^5e^3(d^2 - e^2x^2)^2)$

**Maple [B]** time = 0.019, size = 298, normalized size = 2.4

$$\begin{aligned} & \frac{\ln(ex-d)g^2}{16e^3d^3} - \frac{3\ln(ex-d)f^2}{16ed^5} + \frac{g^2}{16e^3d(ex-d)^2} + \frac{fg}{8e^2d^2(ex-d)^2} \\ & + \frac{f^2}{16ed^3(ex-d)^2} + \frac{g^2}{16e^3d^2(ex-d)} - \frac{fg}{8e^2d^3(ex-d)} - \frac{3f^2}{16ed^4(ex-d)} \\ & - \frac{\ln(ex+d)g^2}{16e^3d^3} + \frac{3\ln(ex+d)f^2}{16ed^5} + \frac{g^2}{16e^3d^2(ex+d)} + \frac{fg}{8e^2d^3(ex+d)} \\ & - \frac{3f^2}{16ed^4(ex+d)} - \frac{g^2}{16e^3d(ex+d)^2} + \frac{fg}{8e^2d^2(ex+d)^2} - \frac{f^2}{16ed^3(ex+d)^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^2/(-e^2*x^2+d^2)^3,x)`

[Out]  $1/16/e^3/d^3*\ln(e*x-d)*g^2-3/16/e/d^5*\ln(e*x-d)*f^2+1/16/e^3/d/(e*x-d)^2*g^2+1/8/e^2/d^2/(e*x-d)^2*f*g+1/16/e/d^3/(e*x-d)^2*f^2+1/16/e^3/d^2/(e*x-d)*g^2-1/8/e^2/d^3/(e*x-d)*f*g-3/16/e/d^4/(e*x-d)*f^2-1/16/e^3/d^3*\ln(e*x+d)*g^2+3/16/e/d^5*\ln(e*x+d)*f^2+1/16/d^2/e^3/(e*x+d)*g^2+1/8/d^3/e^2/(e*x+d)*f*g-3/16/d^4/e/(e*x+d)*f^2-1/16/d/e^3/(e*x+d)^2*g^2+1/8/d^2/e^2/(e*x+d)^2*f*g-1/16/d^3/e/(e*x+d)^2*f^2$

**Maxima [A]** time = 0.697985, size = 205, normalized size = 1.61

$$\begin{aligned} & \frac{4d^4fg - (3e^4f^2 - d^2e^2g^2)x^3 + (5d^2e^2f^2 + d^4g^2)x}{8(d^4e^6x^4 - 2d^6e^4x^2 + d^8e^2)} \\ & + \frac{(3e^2f^2 - d^2g^2)\log(ex+d)}{16d^5e^3} - \frac{(3e^2f^2 - d^2g^2)\log(ex-d)}{16d^5e^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(g*x + f)^2/(e^2*x^2 - d^2)^3,x, algorithm="maxima")`

[Out]  $1/8*(4*d^4*f*g - (3*e^4*f^2 - d^2*e^2*g^2)*x^3 + (5*d^2*e^2*f^2 + d^4*g^2)*x)/(d^4*e^6*x^4 - 2*d^6*e^4*x^2 + d^8*e^2) + 1/16*(3*e^2*f^2 - d^2*g^2)*\log(e*x + d)/(d^5*e^3) - 1/16*(3*e^2*f^2 - d^2*g^2)*\log(e*x - d)/(d^5*e^3)$

**Fricas [A]** time = 0.274451, size = 340, normalized size = 2.68

$$\frac{8d^5efg - 2(3de^5f^2 - d^3e^3g^2)x^3 + 2(5d^3e^3f^2 + d^5eg^2)x + (3d^4e^2f^2 - d^6g^2 + (3e^6f^2 - d^2e^4g^2)x^4 - 2(3d^2e^4f^2 - d^4e^2g^2)x^2 - 2(3d^2e^4f^2 - d^4e^2g^2)x^2)\log(ex+d) - (3d^4e^2f^2 - d^6g^2 + (3e^6f^2 - d^2e^4g^2)x^4 - 2(3d^2e^4f^2 - d^4e^2g^2)x^2)\log(ex-d)}{16(d^5e^7x^4 - 2d^7e^5x^2 + d^9e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(-(g*x + f)^2/(e^2*x^2 - d^2)^3,x, algorithm="fricas")`

[Out]  $1/16*(8*d^5*e*f*g - 2*(3*d*e^5*f^2 - d^3*e^3*g^2)*x^3 + 2*(5*d^3*e^3*f^2 + d^5*e*g^2)*x + (3*d^4*e^2*f^2 - d^6*g^2 + (3*e^6*f^2 - d^2*e^4*g^2)*x^4 - 2*(3*d^2*e^4*f^2 - d^4*e^2*g^2)*x^2)\log(e*x + d) - (3*d^4*e^2*f^2 - d^6*g^2 + (3*e^6*f^2 - d^2*e^4*g^2)*x^4 - 2*(3*d^2*e^4*f^2 - d^4*e^2*g^2)*x^2)\log(e*x - d)/(d^5*e^7*x^4 - 2*d^7*e^5*x^2 + d^9*e^3)$



**Sympy [A]** time = 4.07097, size = 143, normalized size = 1.13

$$\frac{4d^4fg + x^3(d^2e^2g^2 - 3e^4f^2) + x(d^4g^2 + 5d^2e^2f^2)}{8d^8e^2 - 16d^6e^4x^2 + 8d^4e^6x^4} + \frac{(d^2g^2 - 3e^2f^2) \log\left(-\frac{d}{e} + x\right)}{16d^5e^3} - \frac{(d^2g^2 - 3e^2f^2) \log\left(\frac{d}{e} + x\right)}{16d^5e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*3,x)

[Out] (4\*d\*\*4\*f\*g + x\*\*3\*(d\*\*2\*e\*\*2\*g\*\*2 - 3\*e\*\*4\*f\*\*2) + x\*(d\*\*4\*g\*\*2 + 5\*d\*\*2\*e\*\*2\*f\*\*2))/(8\*d\*\*8\*e\*\*2 - 16\*d\*\*6\*e\*\*4\*x\*\*2 + 8\*d\*\*4\*e\*\*6\*x\*\*4) + (d\*\*2\*g\*\*2 - 3\*e\*\*2\*f\*\*2)\*log(-d/e + x)/(16\*d\*\*5\*e\*\*3) - (d\*\*2\*g\*\*2 - 3\*e\*\*2\*f\*\*2)\*log(d/e + x)/(16\*d\*\*5\*e\*\*3)

**GIAC/XCAS [A]** time = 0.275215, size = 171, normalized size = 1.35

$$\frac{(d^2g^2 - 3f^2e^2)e^{(-3)}\ln\left(\frac{|2xe^2-2|d|e|}{|2xe^2+2|d|e|}\right)}{16d^4|d|} + \frac{(d^2g^2x^3e^2 + d^4g^2x + 4d^4fg - 3f^2x^3e^4 + 5d^2f^2xe^2)e^{(-2)}}{8(x^2e^2 - d^2)^2d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(g\*x + f)^2/(e^2\*x^2 - d^2)^3,x, algorithm="giac")

[Out] 1/16\*(d^2\*g^2 - 3\*f^2\*e^2)\*e^(-3)\*ln(abs(2\*x\*e^2 - 2\*abs(d)\*e)/abs(2\*x\*e^2 + 2\*abs(d)\*e))/(d^4\*abs(d)) + 1/8\*(d^2\*g^2\*x^3\*e^2 + d^4\*g^2\*x + 4\*d^4\*f\*g - 3\*f^2\*x^3\*e^4 + 5\*d^2\*f^2\*x\*e^2)\*e^(-2)/((x^2\*e^2 - d^2)^2\*d^4)

$$3.577 \quad \int \frac{(f+gx)^2}{(d+ex)(d^2-e^2x^2)^3} dx$$

**Optimal.** Leaf size=188

$$\frac{f(dg+ef)}{8d^5e^2(d-ex)} - \frac{(dg+3ef)(ef-dg)}{32d^4e^3(d+ex)^2} + \frac{(dg+ef)^2}{32d^4e^3(d-ex)^2} - \frac{(ef-dg)^2}{24d^3e^3(d+ex)^3} \\ + \frac{(-d^2g^2+2defg+5e^2f^2) \tanh^{-1}\left(\frac{ex}{d}\right)}{16d^6e^3} - \frac{3e^2f^2-d^2g^2}{16d^5e^3(d+ex)}$$

[Out]  $(e^*f + d^*g)^2 / (32*d^4*e^3*(d - e^*x)^2) + (f*(e^*f + d^*g)) / (8*d^5*e^2*(d - e^*x)) - (e^*f - d^*g)^2 / (24*d^3*e^3*(d + e^*x)^3) - ((e^*f - d^*g)*(3*e^*f + d^*g)) / (32*d^4*e^3*(d + e^*x)^2) - (3*e^2*f^2 - d^2*g^2) / (16*d^5*e^3*(d + e^*x)) + ((5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*ArcTanh[(e^*x)/d]) / (16*d^6*e^3)$

**Rubi [A]** time = 0.457558, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$\frac{f(dg+ef)}{8d^5e^2(d-ex)} - \frac{(dg+3ef)(ef-dg)}{32d^4e^3(d+ex)^2} + \frac{(dg+ef)^2}{32d^4e^3(d-ex)^2} - \frac{(ef-dg)^2}{24d^3e^3(d+ex)^3} \\ + \frac{(-d^2g^2+2defg+5e^2f^2) \tanh^{-1}\left(\frac{ex}{d}\right)}{16d^6e^3} - \frac{3e^2f^2-d^2g^2}{16d^5e^3(d+ex)}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)^2/((d + e\*x)\*(d^2 - e^2\*x^2)^3), x]

[Out]  $(e^*f + d^*g)^2 / (32*d^4*e^3*(d - e^*x)^2) + (f*(e^*f + d^*g)) / (8*d^5*e^2*(d - e^*x)) - (e^*f - d^*g)^2 / (24*d^3*e^3*(d + e^*x)^3) - ((e^*f - d^*g)*(3*e^*f + d^*g)) / (32*d^4*e^3*(d + e^*x)^2) - (3*e^2*f^2 - d^2*g^2) / (16*d^5*e^3*(d + e^*x)) + ((5*e^2*f^2 + 2*d*e*f*g - d^2*g^2)*ArcTanh[(e^*x)/d]) / (16*d^6*e^3)$

**Rubi in Sympy [A]** time = 68.3837, size = 206, normalized size = 1.1

$$-\frac{(dg-ef)^2}{24d^3e^3(d+ex)^3} + \frac{(dg-ef)(dg+3ef)}{32d^4e^3(d+ex)^2} + \frac{(dg+ef)^2}{32d^4e^3(d-ex)^2} + \frac{f(dg+ef)}{8d^5e^2(d-ex)} + \frac{d^2g^2-3e^2f^2}{16d^5e^3(d+ex)} \\ + \frac{(d^2g^2-2defg-5e^2f^2) \log(d-ex)}{32d^6e^3} - \frac{(d^2g^2-2defg-5e^2f^2) \log(d+ex)}{32d^6e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*2/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*3, x)

[Out]  $-(d^*g - e^*f)^2 / (24*d^3*e^3*(d + e^*x)^3) + (d^*g - e^*f)*(d^*g + 3*e^*f) / (32*d^4*e^3*(d + e^*x)^2) + (d^*g + e^*f)^2 / (32*d^4*e^3*(d - e^*x)^2) + f*(d^*g + e^*f) / (8*d^5*e^2*(d - e^*x)) + (d^2*g^2 - 3*e^2*f^2) / (16*d^5*e^3*(d + e^*x)) + (d^2*g^2 - 2*d*e*f*g - 5*e^2*f^2) * \log(d - e^*x) / (32*d^6*e^3) - (d^2*g^2 - 2*d*e*f*g - 5*e^2*f^2) * \log(d + e^*x) / (32*d^6*e^3)$

**Mathematica [A]** time = 0.29076, size = 197, normalized size = 1.05

$$\frac{-4d^3(ef-dg)^2}{(d+ex)^3} + \frac{3d^2(d^2g^2+2defg-3e^2f^2)}{(d+ex)^2} + \frac{6d(d^2g^2-3e^2f^2)}{d+ex} + 3(d^2g^2-2defg-5e^2f^2) \log(d-ex) + 3(-d^2g^2+2defg+5e^2f^2) \operatorname{ArcTanh}\left(\frac{ex}{d}\right) \\ \frac{96d^6e^3}{96d^6e^3}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^2/((d + e\*x)\*(d^2 - e^2\*x^2)^3), x]

[Out] ((3\*d^2\*(e\*f + d\*g)^2)/(d - e\*x)^2 + (12\*d\*e\*f\*(e\*f + d\*g))/(d - e\*x) - (4\*d^3\*(e\*f - d\*g)^2)/(d + e\*x)^3 + (3\*d^2\*(-3\*e^2\*f^2 + 2\*d\*e\*f\*g + d^2\*g^2))/(d + e\*x)^2 + (6\*d\*(-3\*e^2\*f^2 + d^2\*g^2))/(d + e\*x) + 3\*(-5\*e^2\*f^2 - 2\*d\*e\*f\*g + d^2\*g^2)\*Log[d - e\*x] + 3\*(5\*e^2\*f^2 + 2\*d\*e\*f\*g - d^2\*g^2)\*Log[d + e\*x])/(96\*d^6\*e^3)

**Maple [A]** time = 0.021, size = 348, normalized size = 1.9

$$\begin{aligned} & \frac{g^2}{32 d^2 e^3 (ex - d)^2} + \frac{fg}{16 e^2 d^3 (ex - d)^2} + \frac{f^2}{32 ed^4 (ex - d)^2} + \frac{\ln(ex - d) g^2}{32 e^3 d^4} - \frac{\ln(ex - d) fg}{16 e^2 d^5} \\ & - \frac{5 \ln(ex - d) f^2}{32 ed^6} - \frac{fg}{8 e^2 d^4 (ex - d)} - \frac{f^2}{8 ed^5 (ex - d)} + \frac{g^2}{16 e^3 d^3 (ex + d)} - \frac{3 f^2}{16 ed^5 (ex + d)} \\ & + \frac{g^2}{32 d^2 e^3 (ex + d)^2} + \frac{fg}{16 e^2 d^3 (ex + d)^2} - \frac{3 f^2}{32 ed^4 (ex + d)^2} - \frac{\ln(ex + d) g^2}{32 e^3 d^4} + \frac{\ln(ex + d) fg}{16 e^2 d^5} \\ & + \frac{5 \ln(ex + d) f^2}{32 ed^6} - \frac{g^2}{24 de^3 (ex + d)^3} + \frac{fg}{12 d^2 e^2 (ex + d)^3} - \frac{f^2}{24 ed^3 (ex + d)^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^2/(e\*x+d)/(-e^2\*x^2+d^2)^3, x)

[Out] 1/32/e^3/d^2/(e\*x-d)^2\*g^2+1/16/e^2/d^3/(e\*x-d)^2\*f\*g+1/32/e/d^4/(e\*x-d)^2\*f^2+1/32/e^3/d^4\*ln(e\*x-d)\*g^2-1/16/e^2/d^5\*ln(e\*x-d)\*f\*g-5/32/e/d^6\*ln(e\*x-d)\*f^2-1/8/e^2/d^4/(e\*x-d)\*f\*g-1/8/e/d^5/(e\*x-d)\*f^2+1/16/e^3/d^3/(e\*x+d)\*g^2-3/16/e\*f^2/d^5/(e\*x+d)+1/32/e^3/d^2/(e\*x+d)^2\*g^2+1/16/e^2/d^3/(e\*x+d)^2\*f\*g-3/32/e/d^4/(e\*x+d)^2\*f^2-1/32/e^3/d^4\*ln(e\*x+d)\*g^2+1/16/e^2/d^5\*ln(e\*x+d)\*f\*g+5/32/e/d^6\*ln(e\*x+d)\*f^2-1/24/e^3/d/(e\*x+d)^3\*g^2+1/12/e^2/d^2/(e\*x+d)^3\*f\*g-1/24/e/d^3/(e\*x+d)^3\*f^2

**Maxima [A]** time = 0.717695, size = 416, normalized size = 2.21

$$\begin{aligned} & \frac{8 d^4 e^2 f^2 - 16 d^5 e f g - 4 d^6 g^2 + 3 (5 e^6 f^2 + 2 d e^5 f g - d^2 e^4 g^2) x^4 + 3 (5 d e^5 f^2 + 2 d^2 e^4 f g - d^3 e^3 g^2) x^3 - 5 (5 d^2 e^4 f^2 + 2 d^3 e^3 f g - d^4 e^2 f^2 - 2 d^5 e f g + d^6 g^2) x^2 + 5 (5 e^2 f^2 + 2 d e f g - d^2 g^2) \log(ex + d) - (5 e^2 f^2 + 2 d e f g - d^2 g^2) \log(ex - d)}{48 (d^5 e^8 x^5 + d^6 e^7 x^4 - 2 d^7 e^6 x^3 - 2 d^8 e^5 x^2 + d^9 e^4 x + d^{10} e^3)} \\ & + \frac{(5 e^2 f^2 + 2 d e f g - d^2 g^2) \log(ex + d)}{32 d^6 e^3} - \frac{(5 e^2 f^2 + 2 d e f g - d^2 g^2) \log(ex - d)}{32 d^6 e^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(g\*x + f)^2/((e^2\*x^2 - d^2)^3\*(e\*x + d)), x, algorithm="maxima")

[Out] -1/48\*(8\*d^4\*e^2\*f^2 - 16\*d^5\*e\*f\*g - 4\*d^6\*g^2 + 3\*(5\*e^6\*f^2 + 2\*d\*e^5\*f\*g - d^2\*e^4\*g^2)\*x^4 + 3\*(5\*d^2\*e^5\*f^2 + 2\*d^3\*e^4\*f\*g - d^4\*e^3\*g^2)\*x^3 - 5\*(5\*d^2\*e^4\*f^2 + 2\*d^3\*e^3\*f\*g - d^4\*e^2\*f^2 - 2\*d^5\*e\*f\*g + d^6\*g^2)\*x^2 - (25\*d^3\*e^3\*f^2 + 10\*d^4\*e^2\*f\*g + 7\*d^5\*e\*g^2)\*x)/(d^5\*e^8\*x^5 + d^6\*e^7\*x^4 - 2\*d^7\*e^6\*x^3 - 2\*d^8\*e^5\*x^2 + d^9\*e^4\*x + d^10\*e^3) + 1/32\*(5\*e^2\*f^2 + 2\*d\*e\*f\*g - d^2\*g^2)\*log(e\*x + d)/(d^6\*e^3) - 1/32\*(5\*e^2\*f^2 + 2\*d\*e\*f\*g - d^2\*g^2)\*log(e\*x - d)/(d^6\*e^3)

**Fricas [A]** time = 0.284789, size = 894, normalized size = 4.76

$$\frac{16 d^5 e^2 f^2 - 32 d^6 e f g - 8 d^7 g^2 + 6 (5 d e^6 f^2 + 2 d^2 e^5 f g - d^3 e^4 g^2) x^4 + 6 (5 d^2 e^5 f^2 + 2 d^3 e^4 f g - d^4 e^3 g^2) x^3 - 10 (5 d^3 e^4 f^2 + 2 d^4 e^3 f g - d^5 e^2 f^2 - 2 d^6 e f g + d^7 g^2) x^2 + 10 (5 e^2 f^2 + 2 d e f g - d^2 g^2) \log(ex + d) - 10 (5 e^2 f^2 + 2 d e f g - d^2 g^2) \log(ex - d)}{48 (d^5 e^8 x^5 + d^6 e^7 x^4 - 2 d^7 e^6 x^3 - 2 d^8 e^5 x^2 + d^9 e^4 x + d^{10} e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(g\*x + f)^2/((e^2\*x^2 - d^2)^3\*(e\*x + d)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/96*(16*d^5*e^2*f^2 - 32*d^6*e*f*g - 8*d^7*g^2 + 6*(5*d*e^6*f^2 \\ & + 2*d^2*e^5*f*g - d^3*e^4*g^2)*x^4 + 6*(5*d^2*e^5*f^2 + 2*d^3*e^4*f*g - d^4*e^3*g^2)*x^3 - 10*(5*d^3*e^4*f^2 + 2*d^4*e^3*f*g - d^5*e^2*g^2)*x^2 - 2*(25*d^4*e^3*f^2 + 10*d^5*e^2*f*g + 7*d^6*e*g^2) \\ & *x - 3*(5*d^5*e^2*f^2 + 2*d^6*e*f*g - d^7*g^2 + (5*e^7*f^2 + 2*d*e^6*f*g - d^2*e^5*g^2)*x^5 + (5*d*e^6*f^2 + 2*d^2*e^5*f*g - d^3*e^4*g^2)*x^4 - 2*(5*d^2*e^5*f^2 + 2*d^3*e^4*f*g - d^4*e^3*g^2)*x^3 - 2*(5*d^3*e^4*f^2 + 2*d^4*e^3*f*g - d^5*e^2*g^2)*x^2 + (5*d^4*e^3*f^2 + 2*d^5*e^2*f*g - d^6*e*g^2)*x) \\ & *log(e*x + d) + 3*(5*d^5*e^2*f^2 + 2*d^6*e*f*g - d^7*g^2 + (5*e^7*f^2 + 2*d*e^6*f*g - d^2*e^5*g^2)*x^5 + (5*d*e^6*f^2 + 2*d^2*e^5*f*g - d^3*e^4*g^2)*x^4 - 2*(5*d^2*e^5*f^2 + 2*d^3*e^4*f*g - d^4*e^3*g^2)*x^3 - 2*(5*d^3*e^4*f^2 + 2*d^4*e^3*f*g - d^5*e^2*g^2)*x^2 + (5*d^4*e^3*f^2 + 2*d^5*e^2*f*g - d^6*e*g^2)*x) \\ & *log(e*x - d))/(d^6*e^8*x^5 + d^7*e^7*x^4 - 2*d^8*e^6*x^3 - 2*d^9*e^5*x^2 + d^10*e^4*x + d^11*e^3) \end{aligned}$$

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**Sympy [A]** time = 7.60997, size = 320, normalized size = 1.7

$$\frac{4d^6g^2 + 16d^5efg - 8d^4e^2f^2 + x^4(3d^2e^4g^2 - 6de^5fg - 15e^6f^2) + x^3(3d^3e^3g^2 - 6d^2e^4fg - 15de^5f^2) + x^2(-5d^4e^2g^2 + 10d^5e^3fg - 15d^6e^4f^2) + x(-5d^5e^3g^2 + 10d^6e^4fg - 15d^7e^5f^2) + (-5d^6e^4g^2 + 10d^7e^5fg - 15d^8e^6f^2)}{48d^{10}e^3 + 48d^9e^4x - 96d^8e^5x^2 - 96d^7e^6x^3 + 48d^6e^7x^4 + 48d^5e^8x^5} + \frac{(d^2g^2 - 2defg - 5e^2f^2) \log\left(-\frac{d}{e} + x\right)}{32d^6e^3} - \frac{(d^2g^2 - 2defg - 5e^2f^2) \log\left(\frac{d}{e} + x\right)}{32d^6e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*2/(e\*x+d)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*3,x)

[Out] 
$$\begin{aligned} & (4*d**6*g**2 + 16*d**5*e*f*g - 8*d**4*e**2*f**2 + x**4*(3*d**2*e**4*g**2 - 6*d**2*e**4*f*g - 15*e**6*f**2) + x**3*(3*d**3*e**3*g**2 - 6*d**2*e**4*f*g - 15*d**5*f**2) + x**2*(-5*d**4*e**2*g**2 + 10*d**3*e**3*f*g + 25*d**2*e**4*f**2) + x*(7*d**5*e*g**2 + 10*d**4*e**2*f*g + 25*d**3*e**3*f**2))/(48*d**10*e**3 + 48*d**9*e**4*x - 96*d**8*e**5*x**2 - 96*d**7*e**6*x**3 + 48*d**6*e**7*x**4 + 48*d**5*e**8*x**5) + (d**2*g**2 - 2*d*e*f*g - 5*e**2*f**2)*log(-d/e + x)/(32*d**6*e**3) - (d**2*g**2 - 2*d*e*f*g - 5*e**2*f**2)*log(d/e + x)/(32*d**6*e**3) \end{aligned}$$

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(g\*x + f)^2/((e^2\*x^2 - d^2)^3\*(e\*x + d)),x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.578 \quad \int \frac{(f+gx)^2}{(d+ex)^2(d^2-e^2x^2)^3} dx$$

**Optimal.** Leaf size=235

$$\begin{aligned} & \frac{(dg+ef)(dg+5ef)}{64d^6e^3(d-ex)} + \frac{(dg+ef)^2}{64d^5e^3(d-ex)^2} - \frac{(dg+3ef)(ef-dg)}{48d^4e^3(d+ex)^3} - \frac{(ef-dg)^2}{32d^3e^3(d+ex)^4} \\ & + \frac{(-d^2g^2+10defg+15e^2f^2)\tanh^{-1}\left(\frac{ex}{d}\right)}{64d^7e^3} - \frac{-d^2g^2+2defg+5e^2f^2}{32d^6e^3(d+ex)} - \frac{3e^2f^2-d^2g^2}{32d^5e^3(d+ex)^2} \end{aligned}$$

[Out]  $(e^*f + d^*g)^2 / (64^*d^5^*e^3^*(d - e^*x)^2) + ((e^*f + d^*g)^*(5^*e^*f + d^*g)) / (64^*d^6^*e^3^*(d - e^*x)) - (e^*f - d^*g)^2 / (32^*d^3^*e^3^*(d + e^*x)^4) - ((e^*f - d^*g)^*(3^*e^*f + d^*g)) / (48^*d^4^*e^3^*(d + e^*x)^3) - (3^*e^2^*f^2 - d^2^*g^2) / (32^*d^5^*e^3^*(d + e^*x)^2) - (5^*e^2^*f^2 + 2^*d^*e^*f^*g - d^2^*g^2) / (32^*d^6^*e^3^*(d + e^*x)) + ((15^*e^2^*f^2 + 10^*d^*e^*f^*g - d^2^*g^2)^*ArcTanh[(e^*x)/d]) / (64^*d^7^*e^3)$

**Rubi [A]** time = 0.590926, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$\begin{aligned} & \frac{(dg+ef)(dg+5ef)}{64d^6e^3(d-ex)} + \frac{(dg+ef)^2}{64d^5e^3(d-ex)^2} - \frac{(dg+3ef)(ef-dg)}{48d^4e^3(d+ex)^3} - \frac{(ef-dg)^2}{32d^3e^3(d+ex)^4} \\ & + \frac{(-d^2g^2+10defg+15e^2f^2)\tanh^{-1}\left(\frac{ex}{d}\right)}{64d^7e^3} - \frac{-d^2g^2+2defg+5e^2f^2}{32d^6e^3(d+ex)} - \frac{3e^2f^2-d^2g^2}{32d^5e^3(d+ex)^2} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)^2/((d + e\*x)^2\*(d^2 - e^2\*x^2)^3), x]

[Out]  $(e^*f + d^*g)^2 / (64^*d^5^*e^3^*(d - e^*x)^2) + ((e^*f + d^*g)^*(5^*e^*f + d^*g)) / (64^*d^6^*e^3^*(d - e^*x)) - (e^*f - d^*g)^2 / (32^*d^3^*e^3^*(d + e^*x)^4) - ((e^*f - d^*g)^*(3^*e^*f + d^*g)) / (48^*d^4^*e^3^*(d + e^*x)^3) - (3^*e^2^*f^2 - d^2^*g^2) / (32^*d^5^*e^3^*(d + e^*x)^2) - (5^*e^2^*f^2 + 2^*d^*e^*f^*g - d^2^*g^2) / (32^*d^6^*e^3^*(d + e^*x)) + ((15^*e^2^*f^2 + 10^*d^*e^*f^*g - d^2^*g^2)^*ArcTanh[(e^*x)/d]) / (64^*d^7^*e^3)$

**Rubi in Sympy [A]** time = 86.4509, size = 252, normalized size = 1.07

$$\begin{aligned} & -\frac{(dg-ef)^2}{32d^3e^3(d+ex)^4} + \frac{(dg-ef)(dg+3ef)}{48d^4e^3(d+ex)^3} + \frac{d^2g^2-3e^2f^2}{32d^5e^3(d+ex)^2} \\ & + \frac{(dg+ef)^2}{64d^5e^3(d-ex)^2} + \frac{d^2g^2-2defg-5e^2f^2}{32d^6e^3(d+ex)} + \frac{(dg+ef)(dg+5ef)}{64d^6e^3(d-ex)} \\ & + \frac{(d^2g^2-10defg-15e^2f^2)\log(d-ex)}{128d^7e^3} - \frac{(d^2g^2-10defg-15e^2f^2)\log(d+ex)}{128d^7e^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*2/(e\*x+d)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*3,x)

[Out]  $-(d^*g - e^*f)^2 / (32^*d^3^*e^3^*(d + e^*x)^4) + (d^*g - e^*f)^*(d^*g + 3^*e^*f) / (48^*d^4^*e^3^*(d + e^*x)^3) + (d^2^*g^2 - 3^*e^2^*f^2) / (32^*d^5^*e^3^*(d + e^*x)^2) + (d^*g + e^*f)^2 / (64^*d^5^*e^3^*(d - e^*x)^2) + (d^2^*g^2 - 2^*d^*e^*f^*g - 5^*e^2^*f^2) / (32^*d^6^*e^3^*(d + e^*x)) + (d^*g + e^*f)^*(d^*g + 5^*e^*f) / (64^*d^6^*e^3^*(d - e^*x)) + (d^2^*g^2 - 10^*d^*e^*f^*g - 15^*e^2^*f^2)^*log(d - e^*x) / (128^*d^7^*e^3) - (d^2^*g^2 - 10^*d^*e^*f^*g - 15^*e^2^*f^2)^*log(d + e^*x) / (128^*d^7^*e^3)$

**Mathematica [A]** time = 0.301928, size = 244, normalized size = 1.04

$$\frac{-\frac{12d^4(ef-dg)^2}{(d+ex)^4} + \frac{12d^2(d^2g^2-3e^2f^2)}{(d+ex)^2} + \frac{6d(d^2g^2+6defg+5e^2f^2)}{d-ex} + \frac{12d(d^2g^2-2defg-5e^2f^2)}{d+ex} + 3(d^2g^2 - 10defg - 15e^2f^2) \log(d-ex) + 3}{384d^7e^3}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^2/((d + e\*x)^2\*(d^2 - e^2\*x^2)^3),x]

[Out] ((6\*d^2\*(e\*f + d\*g)^2)/(d - e\*x)^2 + (6\*d\*(5\*e^2\*f^2 + 6\*d\*e\*f\*g + d^2\*g^2))/(d - e\*x) - (12\*d^4\*(e\*f - d\*g)^2)/(d + e\*x)^4 + (8\*d^3\*(-3\*e^2\*f^2 + 2\*d\*e\*f\*g + d^2\*g^2))/(d + e\*x)^3 + (12\*d^2\*(-3\*e^2\*f^2 + d^2\*g^2))/(d + e\*x)^2 + (12\*d\*(-5\*e^2\*f^2 - 2\*d\*e\*f\*g + d^2\*g^2))/(d + e\*x) + 3\*(-15\*e^2\*f^2 - 10\*d\*e\*f\*g + d^2\*g^2)\*Log[d - e\*x] + 3\*(15\*e^2\*f^2 + 10\*d\*e\*f\*g - d^2\*g^2)\*Log[d + e\*x])/(384\*d^7\*e^3)

**Maple [A]** time = 0.022, size = 421, normalized size = 1.8

$$\begin{aligned} & -\frac{g^2}{64e^3d^4(ex-d)} - \frac{3fg}{32e^2d^5(ex-d)} - \frac{5f^2}{64ed^6(ex-d)} + \frac{g^2}{64d^3e^3(ex-d)^2} + \frac{fg}{32e^2d^4(ex-d)^2} \\ & + \frac{f^2}{64ed^5(ex-d)^2} + \frac{\ln(ex-d)g^2}{128e^3d^5} - \frac{5\ln(ex-d)fg}{64e^2d^6} - \frac{15\ln(ex-d)f^2}{128ed^7} + \frac{g^2}{32d^3e^3(ex+d)^2} \\ & - \frac{3f^2}{32ed^5(ex+d)^2} + \frac{g^2}{48e^3d^2(ex+d)^3} + \frac{fg}{24e^2d^3(ex+d)^3} - \frac{f^2}{16ed^4(ex+d)^3} \\ & + \frac{g^2}{32e^3d^4(ex+d)} - \frac{fg}{16e^2d^5(ex+d)} - \frac{5f^2}{32ed^6(ex+d)} - \frac{\ln(ex+d)g^2}{128e^3d^5} + \frac{5\ln(ex+d)fg}{64e^2d^6} \\ & + \frac{15\ln(ex+d)f^2}{128ed^7} - \frac{g^2}{32de^3(ex+d)^4} + \frac{fg}{16d^2e^2(ex+d)^4} - \frac{f^2}{32ed^3(ex+d)^4} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^2/(e\*x+d)^2/(-e^2\*x^2+d^2)^3,x)

[Out] -1/64/e^3/d^4/(e\*x-d)\*g^2-3/32/e^2/d^5/(e\*x-d)\*f\*g-5/64/e/d^6/(e\*x-d)\*f^2+1/64/e^3/d^3/(e\*x-d)^2\*g^2+1/32/e^2/d^4/(e\*x-d)^2\*f\*g+1/64/e/d^5/(e\*x-d)^2\*f^2+1/128/e^3/d^5\*ln(e\*x-d)\*g^2-5/64/e^2/d^6\*ln(e\*x-d)\*f\*g-15/128/e/d^7\*ln(e\*x-d)\*f^2+1/32/e^3/d^3/(e\*x+d)^2\*g^2-3/32/e\*f^2/d^5/(e\*x+d)^2+1/48/e^3/d^2/(e\*x+d)^3\*g^2+1/24/e^2/d^3/(e\*x+d)^3\*f\*g-1/16/e/d^4/(e\*x+d)^3\*f^2+1/32/e^3/d^4/(e\*x+d)\*g^2-1/16/e^2/d^5/(e\*x+d)\*f\*g-5/32/e/d^6/(e\*x+d)\*f^2-1/128/e^3/d^5\*ln(e\*x+d)\*g^2+5/64/e^2/d^6\*ln(e\*x+d)\*f\*g+15/128/e/d^7\*ln(e\*x+d)\*f^2-1/32/e^3/d/(e\*x+d)^4\*g^2+1/16/e^2/d^2/(e\*x+d)^4\*f\*g-1/32/e/d^3/(e\*x+d)^4\*f^2

**Maxima [A]** time = 0.710225, size = 485, normalized size = 2.06

$$\frac{48d^5e^2f^2 - 32d^6efg - 16d^7g^2 + 3(15e^7f^2 + 10de^6fg - d^2e^5g^2)x^5 + 6(15de^6f^2 + 10d^2e^5fg - d^3e^4g^2)x^4 - 2(15d^2e^5f^2 + 10de^4efg - d^2e^3g^2)x^3 + (15e^2f^2 + 10defg - d^2g^2) \log(ex+d) - (15e^2f^2 + 10defg - d^2g^2) \log(ex-d)}{192(d^6e^9x^6 + 2d^7e^8x^5 - d^8e^7x^4 - 4d^9e^6x^3) + 128d^7e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-(g\*x + f)^2/((e^2\*x^2 - d^2)^3\*(e\*x + d)^2),x, algorithm="maxima")

```
[Out] -1/192*(48*d^5*e^2*f^2 - 32*d^6*e*f*g - 16*d^7*g^2 + 3*(15*e^7*f^2 + 10*d*e^6*f*g - d^2*e^5*g^2)*x^5 + 6*(15*d^2*e^6*f^2 + 10*d^2*e^5*f*g - d^3*e^4*g^2)*x^4 - 2*(15*d^2*e^5*f^2 + 10*d^3*e^4*f*g - d^4*e^3*g^2)*x^3 - 10*(15*d^3*e^4*f^2 + 10*d^4*e^3*f*g - d^5*e^2*g^2)*x^2 - (51*d^4*e^3*f^2 + 34*d^5*e^2*f*g + 35*d^6*e*g^2)*x)/(d^6*e^9*x^6 + 2*d^7*e^8*x^5 - d^8*e^7*x^4 - 4*d^9*e^6*x^3 - d^10*e^5*x^2 + 2*d^11*e^4*x + d^12*e^3) + 1/128*(15*e^2*f^2 + 10*d*e*f*g - d^2*g^2)*log(e*x + d)/(d^7*e^3) - 1/128*(15*e^2*f^2 + 10*d*e*f*g - d^2*g^2)*log(e*x - d)/(d^7*e^3)
```

**Fricas [A]** time = 0.286354, size = 1071, normalized size = 4.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(g*x + f)^2/((e^2*x^2 - d^2)^3*(e*x + d)^2), x, algorithm="fricas")
```

```
[Out] -1/384*(96*d^6*e^2*f^2 - 64*d^7*e*f*g - 32*d^8*g^2 + 6*(15*d*e^7*f^2 + 10*d^2*e^6*f*g - d^3*e^5*g^2)*x^5 + 12*(15*d^2*e^6*f^2 + 10*d^3*e^5*f*g - d^4*e^4*g^2)*x^4 - 4*(15*d^3*e^5*f^2 + 10*d^4*e^4*f*g - d^5*e^3*g^2)*x^3 - 20*(15*d^4*e^4*f^2 + 10*d^5*e^3*f*g - d^6*e^2*g^2)*x^2 - 2*(51*d^5*e^3*f^2 + 34*d^6*e^2*f*g + 35*d^7*e*g^2)*x - 3*(15*d^6*e^2*f^2 + 10*d^7*e*f*g - d^8*g^2 + (15*e^8*f^2 + 10*d*e^7*f*g - d^2*e^6*g^2)*x^6 + 2*(15*d*e^7*f^2 + 10*d^2*e^6*f*g - d^3*e^5*g^2)*x^5 - (15*d^2*e^6*f^2 + 10*d^3*e^5*f*g - d^4*e^4*g^2)*x^4 - 4*(15*d^3*e^5*f^2 + 10*d^4*e^4*f*g - d^5*e^3*g^2)*x^3 - (15*d^4*e^4*f^2 + 10*d^5*e^3*f*g - d^6*e^2*g^2)*x^2 + 2*(15*d^5*e^3*f^2 + 10*d^6*e^2*f*g - d^7*e*g^2)*x)*log(e*x + d) + 3*(15*d^6*e^2*f^2 + 10*d^7*e*f*g - d^8*g^2 + (15*e^8*f^2 + 10*d*e^7*f*g - d^2*e^6*g^2)*x^6 + 2*(15*d*e^7*f^2 + 10*d^2*e^6*f*g - d^3*e^5*g^2)*x^5 - (15*d^2*e^6*f^2 + 10*d^3*e^5*f*g - d^4*e^4*g^2)*x^4 - 4*(15*d^3*e^5*f^2 + 10*d^4*e^4*f*g - d^5*e^3*g^2)*x^3 - (15*d^4*e^4*f^2 + 10*d^5*e^3*f*g - d^6*e^2*g^2)*x^2 + 2*(15*d^5*e^3*f^2 + 10*d^6*e^2*f*g - d^7*e*g^2)*x)*log(e*x - d))/(d^7*e^9*x^6 + 2*d^8*e^8*x^5 - d^9*e^7*x^4 - 4*d^10*e^6*x^3 - d^11*e^5*x^2 + 2*d^12*e^4*x + d^13*e^3)
```

**Sympy [A]** time = 8.92703, size = 371, normalized size = 1.58

$$\frac{16d^7g^2 + 32d^6efg - 48d^5e^2f^2 + x^5(3d^2e^5g^2 - 30de^6fg - 45e^7f^2) + x^4(6d^3e^4g^2 - 60d^2e^5fg - 90de^6f^2) + x^3(-2d^4e^3g^2 + 192d^{12}e^3 + 384d^{11}e^4x - 192d^{10}e^5x^2 - 768d^9e^6x^3 - (d^2g^2 - 10defg - 15e^2f^2) \log\left(-\frac{d}{e} + x\right) - (d^2g^2 - 10defg - 15e^2f^2) \log\left(\frac{d}{e} + x\right))}{128d^7e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)**2/(-e**2*x**2+d**2)**3,x)
```

```
[Out] (16*d**7*g**2 + 32*d**6*e*f*g - 48*d**5*e**2*f**2 + x**5*(3*d**2*e**5*g**2 - 30*d**2*e**5*f*g - 45*e**7*f**2) + x**4*(6*d**3*e**4*g**2 - 60*d**2*e**5*f*g - 90*d**2*e**6*f**2) + x**3*(-2*d**4*e**3*g**2 + 20*d**3*e**4*f*g + 30*d**2*e**5*f**2) + x**2*(-10*d**5*e**2*g**2 + 100*d**4*e**3*f*g + 150*d**3*e**4*f**2) + x*(35*d**6*e*g**2 + 34*d**5*e**2*f*g + 51*d**4*e**3*f**2))/(192*d**12*e**3 + 384*d**11*e**4*x - 192*d**10*e**5*x**2 - 768*d**9*e**6*x**3 - 192*d**8*e**7*x**4 + 384*d**7*e**8*x**5 + 192*d**6*e**9*x**6) + (d**2*g**2 - 10*d*e*f*g - 15*e**2*f**2)*log(-d/e + x)/(128*d**7*e**3) - (d**2*g**2 - 10*d*e*f*g - 15*e**2*f**2)*log(d/e + x)/(128*d**7*e**3)
```

GIAC/XCAS [A] time = 0.34375, size = 1, normalized size = 0.

*Done*

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(-(g*x + f)^2/((e^2*x^2 - d^2)^3*(e*x + d)^2),x, algorithm="giac")
```

```
[Out] Done
```



$$3.579 \quad \int \frac{(d+ex)^3(f+gx)^5}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=269

$$\begin{aligned} & -\frac{g^3(13d^2g^2+30defg+20e^2f^2)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6} + \frac{g^4\sqrt{d^2-e^2x^2}(3dg+5ef)}{e^6} \\ & + \frac{(d+ex)^2(2ef-23dg)(dg+ef)^4}{15d^2e^6(d^2-e^2x^2)^{3/2}} + \frac{(d+ex)^3(dg+ef)^5}{5de^6(d^2-e^2x^2)^{5/2}} \\ & + \frac{g^5x\sqrt{d^2-e^2x^2}}{2e^5} + \frac{(d+ex)(dg+ef)^3(127d^2g^2-21defg+2e^2f^2)}{15d^3e^6\sqrt{d^2-e^2x^2}} \end{aligned}$$

[Out]  $((e*f + d*g)^5*(d + e*x)^3)/(5*d*e^6*(d^2 - e^2*x^2)^{(5/2)}) + ((2*e*f - 23*d*g)*(e*f + d*g)^4*(d + e*x)^2)/(15*d^2*e^6*(d^2 - e^2*x^2)^{(3/2)}) + ((e*f + d*g)^3*(2*e^2*f^2 - 21*d*e*f*g + 127*d^2*g^2)*(d + e*x))/(15*d^3*e^6*\text{Sqrt}[d^2 - e^2*x^2]) + (g^4*(5*e*f + 3*d*g)*\text{Sqrt}[d^2 - e^2*x^2])/e^6 + (g^5*x*\text{Sqrt}[d^2 - e^2*x^2])/(2*e^5) - (g^3*(20*e^2*f^2 + 30*d*e*f*g + 13*d^2*g^2)*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(2*e^6)$

**Rubi [A]** time = 1.42872, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$

$$\begin{aligned} & -\frac{g^3(13d^2g^2+30defg+20e^2f^2)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6} + \frac{g^4\sqrt{d^2-e^2x^2}(3dg+5ef)}{e^6} \\ & + \frac{(d+ex)^2(2ef-23dg)(dg+ef)^4}{15d^2e^6(d^2-e^2x^2)^{3/2}} + \frac{(d+ex)^3(dg+ef)^5}{5de^6(d^2-e^2x^2)^{5/2}} \\ & + \frac{g^5x\sqrt{d^2-e^2x^2}}{2e^5} + \frac{(d+ex)(dg+ef)^3(127d^2g^2-21defg+2e^2f^2)}{15d^3e^6\sqrt{d^2-e^2x^2}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}(((d + e*x)^3*(f + g*x)^5)/(d^2 - e^2*x^2)^{(7/2}), x)$

[Out]  $((e*f + d*g)^5*(d + e*x)^3)/(5*d*e^6*(d^2 - e^2*x^2)^{(5/2)}) + ((2*e*f - 23*d*g)*(e*f + d*g)^4*(d + e*x)^2)/(15*d^2*e^6*(d^2 - e^2*x^2)^{(3/2)}) + ((e*f + d*g)^3*(2*e^2*f^2 - 21*d*e*f*g + 127*d^2*g^2)*(d + e*x))/(15*d^3*e^6*\text{Sqrt}[d^2 - e^2*x^2]) + (g^4*(5*e*f + 3*d*g)*\text{Sqrt}[d^2 - e^2*x^2])/e^6 + (g^5*x*\text{Sqrt}[d^2 - e^2*x^2])/(2*e^5) - (g^3*(20*e^2*f^2 + 30*d*e*f*g + 13*d^2*g^2)*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/(2*e^6)$

**Rubi in Sympy [A]** time = 117.876, size = 411, normalized size = 1.53

$$\begin{aligned} & -\frac{3d^2g^5\text{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{2e^6} - \frac{3dg^5\sqrt{d^2-e^2x^2}}{2e^6} + \frac{5dg^4(dg+ef)\text{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6} \\ & - \frac{g^5(d-ex)\sqrt{d^2-e^2x^2}}{2e^6} + \frac{5g^4\sqrt{d^2-e^2x^2}(dg+ef)}{e^6} - \frac{10g^3(dg+ef)^2\text{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^6} \\ & + \frac{10g^2\sqrt{d^2-e^2x^2}(dg+ef)^3}{de^6(d-ex)} - \frac{5g\sqrt{d^2-e^2x^2}(dg+ef)^4}{3de^6(d-ex)^2} + \frac{\sqrt{d^2-e^2x^2}(dg+ef)^5}{5de^6(d-ex)^3} \\ & - \frac{5g\sqrt{d^2-e^2x^2}(dg+ef)^4}{3d^2e^6(d-ex)} + \frac{2\sqrt{d^2-e^2x^2}(dg+ef)^5}{15d^2e^6(d-ex)^2} + \frac{2\sqrt{d^2-e^2x^2}(dg+ef)^5}{15d^3e^6(d-ex)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**3*(g*x+f)**5/(-e**2*x**2+d**2)**(7/2),x)`

[Out]  $-3d^{**2}g^{**5}\operatorname{atan}(e*x/\sqrt{d^{**2}-e^{**2}x^{**2}})/(2e^{**6})-3d^*g^{**5}\sqrt{d^{**2}-e^{**2}x^{**2}}/(2e^{**6})+5d^*g^{**4}(d^*g+e^*f)\operatorname{atan}(e*x/\sqrt{d^{**2}-e^{**2}x^{**2}})/e^{**6}-g^{**5}(d-e^*x)\sqrt{d^{**2}-e^{**2}x^{**2}}/(2e^{**6})+5g^{**4}\sqrt{d^{**2}-e^{**2}x^{**2}}(d^*g+e^*f)/e^{**6}-10g^{**3}(d^*g+e^*f)^{**2}\operatorname{atan}(e*x/\sqrt{d^{**2}-e^{**2}x^{**2}})/e^{**6}+10g^{**2}\sqrt{d^{**2}-e^{**2}x^{**2}}(d^*g+e^*f)^{**3}/(d^*e^{**6}(d-e^*x))-5g^*\sqrt{d^{**2}-e^{**2}x^{**2}}(d^*g+e^*f)^{**4}/(3d^*e^{**6}(d-e^*x)^{**2})+\sqrt{d^{**2}-e^{**2}x^{**2}}(d^*g+e^*f)^{**5}/(5d^*e^{**6}(d-e^*x)^{**3})-5g^*\sqrt{d^{**2}-e^{**2}x^{**2}}(d^*g+e^*f)^{**4}/(3d^{**2}e^{**6}(d-e^*x))+2\sqrt{d^{**2}-e^{**2}x^{**2}}(d^*g+e^*f)^{**5}/(15d^{**2}e^{**6}(d-e^*x)^{**2})+2\sqrt{d^{**2}-e^{**2}x^{**2}}(d^*g+e^*f)^{**5}/(15d^{**3}e^{**6}(d-e^*x))$

**Mathematica [A]** time = 0.571026, size = 193, normalized size = 0.72

$$\frac{\sqrt{d^2 - e^2 x^2} \left( \frac{2(2ef - 23dg)(dg + ef)^4}{d^2(d - ex)^2} + \frac{2(dg + ef)^3(127d^2g^2 - 21defg + 2e^2f^2)}{d^3(d - ex)} + 30g^4(3dg + 5ef) + \frac{6(dg + ef)^5}{d(d - ex)^3} + 15eg^5x \right) - 15g^3(13d^2g^2 + 30e^2g^2)}{30e^6}$$

Antiderivative was successfully verified.

[In] `Integrate[((d + e*x)^3*(f + g*x)^5)/(d^2 - e^2*x^2)^(7/2),x]`

[Out]  $(\sqrt{d^2 - e^2 x^2} (30g^4(5ef + 3dg) + 15e^2g^5x + (6(e^2f + dg)^5)/(d(d - ex)^3) + (2(2ef - 23dg)(ef + dg)^4)/(d^2(d - ex)^2) + (2(ef + dg)^3(2e^2f^2 - 21d^2efg + 127d^2g^2))/(d^3(d - ex))) - 15g^3(20e^2f^2 + 30d^2efg + 13d^2g^2) \operatorname{ArcTan}[e*x/\sqrt{d^2 - e^2 x^2}])/(30e^6)$

**Maple [B]** time = 0.021, size = 1308, normalized size = 4.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^3*(g*x+f)^5/(-e^2*x^2+d^2)^(7/2),x)`

[Out]  $\frac{1}{3}x^2e/(-e^2x^2+d^2)^{(5/2)}f^5+7/15e/(-e^2x^2+d^2)^{(5/2)}d^2f^5-3x^6/(-e^2x^2+d^2)^{(5/2)}d^2g^5+152/15d^7/e^6/(-e^2x^2+d^2)^{(5/2)}g^5-1/2e^2g^5x^7/(-e^2x^2+d^2)^{(5/2)}+4/5d^2f^5x/(-e^2x^2+d^2)^{(5/2)}+1/15d^2f^5x/(-e^2x^2+d^2)^{(3/2)}+2/15d^3f^5x/(-e^2x^2+d^2)^{(1/2)}-110/3d^3/e^2x^2/(-e^2x^2+d^2)^{(5/2)}f^2g^3-10/3d^2/e^2x^2/(-e^2x^2+d^2)^{(5/2)}f^3g^2-7/e^2x/(-e^2x^2+d^2)^{(5/2)}d^3f^3g^2+3/2e^2x/(-e^2x^2+d^2)^{(5/2)}d^2f^4g+7/3d/e^2x/(-e^2x^2+d^2)^{(3/2)}f^3g^2+14/3d/e^2x/(-e^2x^2+d^2)^{(1/2)}f^3g^2-1/d^2/e^2x/(-e^2x^2+d^2)^{(1/2)}f^4g-60d^4/e^3x^2/(-e^2x^2+d^2)^{(5/2)}f^2g^4-15/e^4/(e^2)^{(1/2)}\arctan((e^2)^{(1/2)}x/(-e^2x^2+d^2)^{(1/2)})d^2f^2g^4+45d^2/e^2x^4/(-e^2x^2+d^2)^{(5/2)}f^2g^4-5/e^2x^3/(-e^2x^2+d^2)^{(3/2)}d^2f^2g^4+16/e^4x/(-e^2x^2+d^2)^{(1/2)}d^2f^2g^4+3/e^3x/(-e^2x^2+d^2)^{(3/2)}d^2f^2g^3+15x^3/e/(-e^2x^2+d^2)^{(5/2)}d^2f^2g^3-3/2d^5/e^4x/(-e^2x^2+d^2)^{(5/2)}f^2g^4-9d^4/e^3x/(-e^2x^2+d^2)^{(5/2)}f^2g^3+1/2/e^4x/(-e^2x^2+d^2)^{(3/2)}d^3f^2g^4+5/2x^3/e^2/(-e^2x^2+d^2)^{(5/2)}d^3f^2g^4+19d^3/e^2x^4/(-e^2x^2+d^2)^{(5/2)}g^5-5x^6e/(-e^2x^2+d^2)^{(5/2)}f^2g^4+10x^4e/(-e^2x^2+d^2)^{(5/2)}f^3g^2+44/3d^5/e^4/(-e^2x^2+d^2)^{(5/2)}f^2g^3+4/3d^4/e^3/(-e^2x^2+d^2)^{(5/2)}f^3g^2+15x^3/(-e^2x^2+d^2)^{(5/2)}d^2f^3g^2-1/2e^2x^2+d^2)^{(3/2)}f^4g+30x^4/(-e^2x^2+d^2)^{(5/2)}d^2f^2g^3+13/2/e^5x/(-e^2x^2+d^2)^{(1/2)}d^2g^5+16/e^3x/(-e^2x^2+d^2)^{(1/2)}f^2g^3-13/2/e^5/(e^2)^{(1/2)}\arctan((e^2)^{(1/2)}x/(-e^2x^2+d^2)^{(1/2)})$

$$\begin{aligned} & /2)) * d^2 * g^5 - 10/e^3 / (e^2)^{(1/2)} * \arctan((e^2)^{(1/2)} * x / (-e^2 * x^2 + d^2)^{(1/2)}) * f^2 * g^3 + 5/2 * x^3 * e / (-e^2 * x^2 + d^2)^{(5/2)} * f^4 * g + 5 * x^2 / (-e^2 * x^2 + d^2)^{(5/2)} * d * f^4 * g - 76/3 * d^5 / e^4 * x^2 / (-e^2 * x^2 + d^2)^{(5/2)} * g^5 + 24 * d^6 / e^5 / (-e^2 * x^2 + d^2)^{(5/2)} * f * g^4 - 1/e^2 / (-e^2 * x^2 + d^2)^{(5/2)} * d^3 * f^4 * g + 13/10 * x^5 / e / (-e^2 * x^2 + d^2)^{(5/2)} * d^2 * g^5 + 3 * x^5 / (-e^2 * x^2 + d^2)^{(5/2)} * d * f * g^4 + 2 * x^5 * e / (-e^2 * x^2 + d^2)^{(5/2)} * f^2 * g^3 - 13/6 / e^3 * x^3 / (-e^2 * x^2 + d^2)^{(3/2)} * d^2 * g^5 - 10/3 / e * x^3 / (-e^2 * x^2 + d^2)^{(3/2)} * f^2 * g^3 \end{aligned}$$

**Maxima [A]** time = 0.78757, size = 2167, normalized size = 8.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(g\*x + f)^5/(-e^2\*x^2 + d^2)^(7/2),x, algorithm="maxima")

[Out] 
$$\begin{aligned} & -1/2 * e * g^5 * x^7 / (-e^2 * x^2 + d^2)^{(5/2)} + 7/30 * d^2 * e * g^5 * x * (15 * x^4 / ((-e^2 * x^2 + d^2)^{(5/2)} * e^2) - 20 * d^2 * x^2 / ((-e^2 * x^2 + d^2)^{(5/2)} * e^4) + 8 * d^4 / ((-e^2 * x^2 + d^2)^{(5/2)} * e^6)) - 7/6 * d^2 * g^5 * x * (3 * x^2 / ((-e^2 * x^2 + d^2)^{(3/2)} * e^2) - 2 * d^2 / ((-e^2 * x^2 + d^2)^{(3/2)} * e^4)) / e + 1/5 * d * f^5 * x / (-e^2 * x^2 + d^2)^{(5/2)} + 3/5 * d^2 * f^5 / ((-e^2 * x^2 + d^2)^{(5/2)} * e) + d^3 * f^4 * g / ((-e^2 * x^2 + d^2)^{(5/2)} * e^2) + 4/15 * f^5 * x / ((-e^2 * x^2 + d^2)^{(3/2)} * d) + 14/15 * d^4 * g^5 * x / ((-e^2 * x^2 + d^2)^{(3/2)} * e^5) + 1/15 * (10 * e^3 * f^2 * g^3 + 15 * d * e^2 * f * g^4 + 3 * d^2 * e * g^5) * x * (15 * x^4 / ((-e^2 * x^2 + d^2)^{(5/2)} * e^2) - 20 * d^2 * x^2 / ((-e^2 * x^2 + d^2)^{(5/2)} * e^4) + 8 * d^4 / ((-e^2 * x^2 + d^2)^{(5/2)} * e^6)) + 8/15 * f^5 * x / (\sqrt{-e^2 * x^2 + d^2}) * d^3 - 49/30 * d^2 * g^5 * x / (\sqrt{-e^2 * x^2 + d^2}) * e^5 - (5 * e^3 * f * g^4 + 3 * d * e^2 * g^5) * x^6 / ((-e^2 * x^2 + d^2)^{(5/2)} * e^2) - 7/2 * d^2 * g^5 * \arcsin(e^2 * x / \sqrt{d^2 * e^2}) / (\sqrt{e^2}) * e^5 - 1/3 * (10 * e^3 * f^2 * g^3 + 15 * d * e^2 * f * g^4 + 3 * d^2 * e * g^5) * x * (3 * x^2 / ((-e^2 * x^2 + d^2)^{(3/2)} * e^2) - 2 * d^2 / ((-e^2 * x^2 + d^2)^{(3/2)} * e^4)) / e^2 + 6 * (5 * e^3 * f * g^4 + 3 * d * e^2 * g^5) * d^2 * x^4 / ((-e^2 * x^2 + d^2)^{(5/2)} * e^4) + (10 * e^3 * f^3 * g^2 + 30 * d * e^2 * f^2 * g^3 + 15 * d^2 * e * f * g^4 + d^3 * g^5) * x^4 / ((-e^2 * x^2 + d^2)^{(5/2)} * e^2) + 5/2 * (e^3 * f^4 * g + 6 * d * e^2 * f^3 * g^2 + 6 * d^2 * e * f^2 * g^3 + d^3 * f * g^4) * x^3 / ((-e^2 * x^2 + d^2)^{(5/2)} * e^2) - 8 * (5 * e^3 * f * g^4 + 3 * d * e^2 * g^5) * d^4 * x^2 / ((-e^2 * x^2 + d^2)^{(5/2)} * e^6) - 4/3 * (10 * e^3 * f^3 * g^2 + 30 * d * e^2 * f^2 * g^3 + 15 * d^2 * e * f * g^4 + d^3 * g^5) * d^2 * x^2 / ((-e^2 * x^2 + d^2)^{(5/2)} * e^4) + 1/3 * (e^3 * f^5 + 15 * d * e^2 * f^4 * g + 30 * d^2 * e * f^3 * g^2 + 10 * d^3 * f^2 * g^3) * x^2 / ((-e^2 * x^2 + d^2)^{(5/2)} * e^2) - 3/2 * (e^3 * f^4 * g + 6 * d * e^2 * f^3 * g^2 + 6 * d^2 * e * f^2 * g^3 + d^3 * f * g^4) * d^2 * x / ((-e^2 * x^2 + d^2)^{(5/2)} * e^4) + 1/5 * (3 * d * e^2 * f^5 + 15 * d^2 * e * f^4 * g + 10 * d^3 * f^3 * g^2) * x / ((-e^2 * x^2 + d^2)^{(5/2)} * e^2) + 16/5 * (5 * e^3 * f * g^4 + 3 * d * e^2 * g^5) * d^6 / ((-e^2 * x^2 + d^2)^{(5/2)} * e^8) + 8/15 * (10 * e^3 * f^3 * g^2 + 30 * d * e^2 * f^2 * g^3 + 15 * d^2 * e * f * g^4 + d^3 * g^5) * d^4 / ((-e^2 * x^2 + d^2)^{(5/2)} * e^6) - 2/15 * (e^3 * f^5 + 15 * d * e^2 * f^4 * g + 30 * d^2 * e * f^3 * g^2 + 10 * d^3 * f^2 * g^3) * d^2 / ((-e^2 * x^2 + d^2)^{(5/2)} * e^4) + 4/15 * (10 * e^3 * f^2 * g^3 + 15 * d * e^2 * f * g^4 + 3 * d^2 * e * g^5) * d^2 * x / ((-e^2 * x^2 + d^2)^{(3/2)} * e^6) + 1/2 * (e^3 * f^4 * g + 6 * d * e^2 * f^3 * g^2 + 6 * d^2 * e * f^2 * g^3 + d^3 * f * g^4) * x / ((-e^2 * x^2 + d^2)^{(3/2)} * e^4) - 1/15 * (3 * d * e^2 * f^5 + 15 * d^2 * e * f^4 * g + 10 * d^3 * f^3 * g^2) * x / ((-e^2 * x^2 + d^2)^{(3/2)} * d^2 * e^2) - 7/15 * (10 * e^3 * f^2 * g^3 + 15 * d * e^2 * f * g^4 + 3 * d^2 * e * g^5) * x / (\sqrt{-e^2 * x^2 + d^2}) * e^6 + (e^3 * f^4 * g + 6 * d * e^2 * f^3 * g^2 + 6 * d^2 * e * f^2 * g^3 + d^3 * f * g^4) * x / (\sqrt{-e^2 * x^2 + d^2}) * d^2 * e^4 - 2/15 * (3 * d * e^2 * f^5 + 15 * d^2 * e * f^4 * g + 10 * d^3 * f^3 * g^2) * x / (\sqrt{-e^2 * x^2 + d^2}) * d^4 * e^2 - (10 * e^3 * f^2 * g^3 + 15 * d * e^2 * f * g^4 + 3 * d^2 * e * g^5) * \arcsin(e^2 * x / \sqrt{d^2 * e^2}) / (\sqrt{e^2}) * e^6 \end{aligned}$$

**Fricas [A]** time = 0.318411, size = 2519, normalized size = 9.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(g\*x + f)^5/(-e^2\*x^2 + d^2)^(7/2),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/30*(15*d^3*e^9*g^5*x^9 + 30*(5*d^3*e^9*f*g^4 + 4*d^4*e^8*g^5)* \\ & x^8 - 6*(3*e^{12}*f^5 - 10*d*e^{11}*f^4*g + 30*d^2*e^{10}*f^3*g^2 + 180 \\ & *d^3*e^9*f^2*g^3 + 190*d^4*e^8*f*g^4 + 113*d^5*e^7*g^5)*x^7 + 30* \\ & (3*d*e^{11}*f^5 - 5*d^2*e^{10}*f^4*g - 10*d^3*e^9*f^3*g^2 + 30*d^4*e^8 \\ & *f^2*g^3 - 10*d^5*e^7*f*g^4 - 7*d^6*e^6*g^5)*x^6 + 3*(12*d^2*e^{10} \\ & *f^5 - 190*d^3*e^9*f^4*g + 520*d^4*e^8*f^3*g^2 + 2820*d^5*e^7*f^2 \\ & *g^3 + 4010*d^6*e^6*f*g^4 + 1807*d^7*e^5*g^5)*x^5 - 20*(26*d^3*e^9 \\ & *f^5 - 60*d^4*e^8*f^4*g - 20*d^5*e^7*f^3*g^2 + 560*d^6*e^6*f^2* \\ & g^3 + 750*d^7*e^5*f*g^4 + 325*d^8*e^4*g^5)*x^4 + 20*(20*d^4*e^8*f \\ & ^5 + 30*d^5*e^7*f^4*g - 80*d^6*e^6*f^3*g^2 - 220*d^7*e^5*f^2*g^3 \\ & - 330*d^8*e^4*f*g^4 - 143*d^9*e^3*g^5)*x^3 + 120*(4*d^5*e^7*f^5 - \\ & 10*d^6*e^6*f^4*g + 100*d^8*e^4*f^2*g^3 + 150*d^9*e^3*f*g^4 + 65* \\ & d^{10}*e^2*g^5)*x^2 - 240*(2*d^6*e^6*f^5 + 20*d^9*e^3*f^2*g^3 + 30* \\ & d^{10}*e^2*f*g^4 + 13*d^{11}*e*g^5)*x + 30*(320*d^{10}*e^2*f^2*g^3 + 48 \\ & 0*d^{11}*e*f*g^4 + 208*d^{12}*g^5 - (20*d^3*e^9*f^2*g^3 + 30*d^4*e^8* \\ & f*g^4 + 13*d^5*e^7*g^5)*x^7 + 7*(20*d^4*e^8*f^2*g^3 + 30*d^5*e^7* \\ & f*g^4 + 13*d^6*e^6*g^5)*x^6 - 3*(20*d^5*e^7*f^2*g^3 + 30*d^6*e^6* \\ & f*g^4 + 13*d^7*e^5*g^5)*x^5 - 31*(20*d^6*e^6*f^2*g^3 + 30*d^7*e^5 \\ & *f*g^4 + 13*d^8*e^4*g^5)*x^4 + 40*(20*d^7*e^5*f^2*g^3 + 30*d^8*e^4 \\ & *f*g^4 + 13*d^9*e^3*g^5)*x^3 + 12*(20*d^8*e^4*f^2*g^3 + 30*d^9*e^3 \\ & *f*g^4 + 13*d^{10}*e^2*g^5)*x^2 - 40*(20*d^9*e^3*f^2*g^3 + 30*d^{10} \\ & *e^2*f*g^4 + 13*d^{11}*e*g^5)*x - (320*d^9*e^2*f^2*g^3 + 480*d^{10} \\ & *e*f*g^4 + 208*d^{11}*g^5 + (20*d^3*e^8*f^2*g^3 + 30*d^4*e^7*f*g^4 + \\ & 13*d^5*e^6*g^5)*x^6 + 2*(20*d^4*e^7*f^2*g^3 + 30*d^5*e^6*f*g^4 + \\ & 13*d^6*e^5*g^5)*x^5 - 19*(20*d^5*e^6*f^2*g^3 + 30*d^6*e^5*f*g^4 \\ & + 13*d^7*e^4*g^5)*x^4 + 20*(20*d^6*e^5*f^2*g^3 + 30*d^7*e^4*f*g^4 \\ & + 13*d^8*e^3*g^5)*x^3 + 20*(20*d^7*e^4*f^2*g^3 + 30*d^8*e^3*f*g^4 \\ & + 13*d^9*e^2*g^5)*x^2 - 40*(20*d^8*e^3*f^2*g^3 + 30*d^9*e^2*f*g^4 \\ & + 13*d^{10}*e*g^5)*x)*sqrt(-e^2*x^2 + d^2))*arctan(-(d - sqrt(-e \\ & ^2*x^2 + d^2))/(e*x)) - (15*d^3*e^8*g^5*x^8 + 15*(10*d^3*e^8*f*g^4 \\ & - d^4*e^7*g^5)*x^7 + 5*(2*e^{11}*f^5 - 20*d^2*e^9*f^3*g^2 - 40*d^3 \\ & *e^8*f^2*g^3 - 210*d^4*e^7*f*g^4 - 107*d^5*e^6*g^5)*x^6 + (56*d* \\ & e^{10}*f^5 - 270*d^2*e^9*f^4*g + 760*d^3*e^8*f^3*g^2 + 4460*d^4*e^7 \\ & *f^2*g^3 + 6030*d^5*e^6*f*g^4 + 2821*d^6*e^5*g^5)*x^5 - 40*(7*d^2 \\ & *e^9*f^5 - 15*d^3*e^8*f^4*g - 10*d^4*e^7*f^3*g^2 + 130*d^5*e^6*f^2 \\ & *g^3 + 150*d^6*e^5*f*g^4 + 65*d^7*e^4*g^5)*x^4 + 20*(8*d^3*e^8*f \\ & ^5 + 30*d^4*e^7*f^4*g - 80*d^5*e^6*f^3*g^2 - 340*d^6*e^5*f^2*g^3 \\ & - 510*d^7*e^4*f*g^4 - 221*d^8*e^3*g^5)*x^3 + 120*(4*d^4*e^7*f^5 - \\ & 10*d^5*e^6*f^4*g + 100*d^7*e^4*f^2*g^3 + 150*d^8*e^3*f*g^4 + 65* \\ & d^9*e^2*g^5)*x^2 - 240*(2*d^5*e^6*f^5 + 20*d^8*e^3*f^2*g^3 + 30*d \\ & ^9*e^2*f*g^4 + 13*d^{10}*e*g^5)*x)*sqrt(-e^2*x^2 + d^2))/(d^3*e^{13} \\ & x^7 - 7*d^4*e^{12}*x^6 + 3*d^5*e^{11}*x^5 + 31*d^6*e^{10}*x^4 - 40*d^7* \\ & e^9*x^3 - 12*d^8*e^8*x^2 + 40*d^9*e^7*x - 16*d^{10}*e^6 + (d^3*e^{12} \\ & *x^6 + 2*d^4*e^{11}*x^5 - 19*d^5*e^{10}*x^4 + 20*d^6*e^9*x^3 + 20*d^7 \\ & *e^8*x^2 - 40*d^8*e^7*x + 16*d^9*e^6)*sqrt(-e^2*x^2 + d^2)) \end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^3 (f + gx)^5}{(-(-d + ex)(d + ex))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(g\*x+f)\*\*5/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2),x)

[Out] Integral((d + e\*x)\*\*3\*(f + g\*x)\*\*5/((-(-d + e\*x)\*(d + e\*x))\*\*(7/2), x)

**GIAC/XCAS [A]** time = 0.295557, size = 725, normalized size = 2.7

$$-\frac{1}{2} (13 d^2 g^5 + 30 d f g^4 e + 20 f^2 g^3 e^2) \arcsin\left(\frac{x e}{d}\right) e^{(-6) \operatorname{sign}(d)} \\ \sqrt{-x^2 e^2 + d^2} \left( \left( \left( \left( \left( 15 \left( g^5 x e + \frac{2(3 d^5 g^5 e^{12} + 5 d^4 f g^4 e^{13}) e^{(-12)}}{d^4} \right) x - \frac{(299 d^6 g^5 e^{11} + 720 d^5 f g^4 e^{12} + 640 d^4 f^2 g^3 e^{13} + 140 d^3 f^3 g^2 e^{14} - 30 d^2 f^4 g e^{15} + 4 d f^5 e^{16})}{d^4} \right) \right) \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(g\*x + f)^5/(-e^2\*x^2 + d^2)^(7/2),x, algorithm="giac")

[Out] 
$$-1/2*(13*d^2*g^5 + 30*d*f*g^4*e + 20*f^2*g^3*e^2)*\arcsin(x*e/d)*e^{(-6)*\operatorname{sign}(d)} + 1/30*\sqrt{-x^2*e^2 + d^2}*(((((15*(g^5*x*e + 2*(3*d^5*g^5*e^{12} + 5*d^4*f*g^4*e^{13})*e^{(-12)}/d^4)*x - (299*d^6*g^5*e^{11} + 720*d^5*f*g^4*e^{12} + 640*d^4*f^2*g^3*e^{13} + 140*d^3*f^3*g^2*e^{14} - 30*d^2*f^4*g*e^{15} + 4*d*f^5*e^{16})*e^{(-12)}/d^4)*x - 30*(19*d^7*g^5*e^{10} + 45*d^6*f*g^4*e^{11} + 30*d^5*f^2*g^3*e^{12} + 10*d^4*f^3*g^2*e^{13})*e^{(-12)}/d^4)*x + 5*(91*d^8*g^5*e^9 + 210*d^7*f*g^4*e^{10} + 140*d^6*f^2*g^3*e^{11} - 20*d^5*f^3*g^2*e^{12} - 30*d^4*f^4*g*e^{13} + 2*d^3*f^5*e^{14})*e^{(-12)}/d^4)*x + 10*(76*d^9*g^5*e^8 + 180*d^8*f*g^4*e^9 + 110*d^7*f^2*g^3*e^{10} + 10*d^6*f^3*g^2*e^{11} - 15*d^5*f^4*g*e^{12} - d^4*f^5*e^{13})*e^{(-12)}/d^4)*x - 15*(13*d^{10}*g^5*e^7 + 30*d^9*f*g^4*e^8 + 20*d^8*f^2*g^3*e^9 + 2*d^5*f^5*e^{12})*e^{(-12)}/d^4)*x - 2*(152*d^{11}*g^5*e^6 + 360*d^{10}*f*g^4*e^7 + 220*d^9*f^2*g^3*e^8 + 20*d^8*f^3*g^2*e^9 - 15*d^7*f^4*g*e^{10} + 7*d^6*f^5*e^{11})*e^{(-12)}/d^4)/(x^2*e^2 - d^2)^3$$

$$3.580 \quad \int \frac{(d+ex)^3(f+gx)^4}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=215

$$\begin{aligned} & -\frac{g^3(3dg+4ef)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5} + \frac{2(d+ex)^2(ef-9dg)(dg+ef)^3}{15d^2e^5(d^2-e^2x^2)^{3/2}} \\ & + \frac{(d+ex)^3(dg+ef)^4}{5de^5(d^2-e^2x^2)^{5/2}} + \frac{g^4\sqrt{d^2-e^2x^2}}{e^5} + \frac{2(d+ex)(dg+ef)^2(36d^2g^2-8defg+e^2f^2)}{15d^3e^5\sqrt{d^2-e^2x^2}} \end{aligned}$$

[Out]  $((e*f + d*g)^4*(d + e*x)^3)/(5*d*e^5*(d^2 - e^2*x^2)^{(5/2)}) + (2*(e*f - 9*d*g)*(e*f + d*g)^3*(d + e*x)^2)/(15*d^2*e^5*(d^2 - e^2*x^2)^{(3/2)}) + (2*(e*f + d*g)^2*(e^2*f^2 - 8*d*e*f*g + 36*d^2*g^2)*(d + e*x))/(15*d^3*e^5*\text{Sqrt}[d^2 - e^2*x^2]) + (g^4*\text{Sqrt}[d^2 - e^2*x^2])/e^5 - (g^3*(4*e*f + 3*d*g)*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/e^5$

**Rubi [A]** time = 0.981534, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$

$$\begin{aligned} & -\frac{g^3(3dg+4ef)\tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5} + \frac{2(d+ex)^2(ef-9dg)(dg+ef)^3}{15d^2e^5(d^2-e^2x^2)^{3/2}} \\ & + \frac{(d+ex)^3(dg+ef)^4}{5de^5(d^2-e^2x^2)^{5/2}} + \frac{g^4\sqrt{d^2-e^2x^2}}{e^5} + \frac{2(d+ex)(dg+ef)^2(36d^2g^2-8defg+e^2f^2)}{15d^3e^5\sqrt{d^2-e^2x^2}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^3*(f + g*x)^4/(d^2 - e^2*x^2)^{(7/2)}, x]$

[Out]  $((e*f + d*g)^4*(d + e*x)^3)/(5*d*e^5*(d^2 - e^2*x^2)^{(5/2)}) + (2*(e*f - 9*d*g)*(e*f + d*g)^3*(d + e*x)^2)/(15*d^2*e^5*(d^2 - e^2*x^2)^{(3/2)}) + (2*(e*f + d*g)^2*(e^2*f^2 - 8*d*e*f*g + 36*d^2*g^2)*(d + e*x))/(15*d^3*e^5*\text{Sqrt}[d^2 - e^2*x^2]) + (g^4*\text{Sqrt}[d^2 - e^2*x^2])/e^5 - (g^3*(4*e*f + 3*d*g)*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/e^5$

**Rubi in Sympy [A]** time = 88.5424, size = 282, normalized size = 1.31

$$\begin{aligned} & \frac{g^4\sqrt{d^2-e^2x^2}}{e^5} - \frac{g^3(3dg+4ef)\text{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^5} + \frac{6g^2\sqrt{d^2-e^2x^2}(dg+ef)^2}{de^5(d-ex)} \\ & - \frac{4g\sqrt{d^2-e^2x^2}(dg+ef)^3}{3de^5(d-ex)^2} + \frac{\sqrt{d^2-e^2x^2}(dg+ef)^4}{5de^5(d-ex)^3} - \frac{4g\sqrt{d^2-e^2x^2}(dg+ef)^3}{3d^2e^5(d-ex)} \\ & + \frac{2\sqrt{d^2-e^2x^2}(dg+ef)^4}{15d^2e^5(d-ex)^2} + \frac{2\sqrt{d^2-e^2x^2}(dg+ef)^4}{15d^3e^5(d-ex)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)**3*(g*x+f)**4/(-e**2*x**2+d**2)**(7/2), x)$

[Out]  $g**4*\text{sqrt}(d**2 - e**2*x**2)/e**5 - g**3*(3*d*g + 4*e*f)*\text{atan}(e*x/\text{sqrt}(d**2 - e**2*x**2))/e**5 + 6*g**2*\text{sqrt}(d**2 - e**2*x**2)*(d*g + e*f)**2/(d*e**5*(d - e*x)) - 4*g*\text{sqrt}(d**2 - e**2*x**2)*(d*g + e*f)**3/(3*d*e**5*(d - e*x)**2) + \text{sqrt}(d**2 - e**2*x**2)*(d*g + e*f)**4/(5*d*e**5*(d - e*x)**3) - 4*g*\text{sqrt}(d**2 - e**2*x**2)*(d*g + e*f)**3/(3*d**2*e**5*(d - e*x)) + 2*\text{sqrt}(d**2 - e**2*x**2)*(d*$

$$(g + e*f)^{**4}/(15*d^{**2}*e^{**5}*(d - e*x)^{**2}) + 2*sqrt(d^{**2} - e^{**2}*x^{**2})*(d*g + e*f)^{**4}/(15*d^{**3}*e^{**5}*(d - e*x))$$

**Mathematica [A]** time = 0.339964, size = 168, normalized size = 0.78

$$\frac{\sqrt{d^2 - e^2 x^2} (15 d^3 g^4 (d - e x)^3 + 2 (d - e x)^2 (d g + e f)^2 (36 d^2 g^2 - 8 d e f g + e^2 f^2) + 3 d^2 (d g + e f)^4 + 2 d (d - e x) (e f - 9 d g) (d g + e f)^3)}{d^3 (d - e x)^3} - 15 g^3 (3 d g + 4 e f) \tan^{-1} \left( \frac{e x}{\sqrt{d^2 - e^2 x^2}} \right)$$

15e<sup>5</sup>

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(f + g\*x)^4)/(d^2 - e^2\*x^2)^(7/2), x]

[Out] ((Sqrt[d^2 - e^2\*x^2])\*(3\*d^2\*(e\*f + d\*g)^4 + 2\*d\*(e\*f - 9\*d\*g)\*(e\*f + d\*g)^3\*(d - e\*x) + 2\*(e\*f + d\*g)^2\*(e^2\*f^2 - 8\*d\*e\*f\*g + 36\*d^2\*g^2)\*(d - e\*x)^2 + 15\*d^3\*g^4\*(d - e\*x)^3)/(d^3\*(d - e\*x)^3) - 15\*g^3\*(4\*e\*f + 3\*d\*g)\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]]/(15\*e^5)

**Maple [B]** time = 0.017, size = 1030, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(g\*x+f)^4/(-e^2\*x^2+d^2)^(7/2), x)

[Out] -e\*g^4\*x^6/(-e^2\*x^2+d^2)^(5/2)+1/3\*x^2\*e/(-e^2\*x^2+d^2)^(5/2)\*f^4+7/15/e/(-e^2\*x^2+d^2)^(5/2)\*d^2\*f^4+24/5\*d^6/e^5/(-e^2\*x^2+d^2)^(5/2)\*g^4+3/5\*x^5/(-e^2\*x^2+d^2)^(5/2)\*d\*g^4+4/5\*d\*f^4\*x/(-e^2\*x^2+d^2)^(5/2)+1/15/d\*f^4\*x/(-e^2\*x^2+d^2)^(3/2)+2/15/d^3\*f^4\*x/(-e^2\*x^2+d^2)^(1/2)+7/5\*d/e^2\*x/(-e^2\*x^2+d^2)^(3/2)\*f^2\*g^2+14/5/d/e^2\*x/(-e^2\*x^2+d^2)^(1/2)\*f^2\*g^2-4/5/d^2/e\*x/(-e^2\*x^2+d^2)^(1/2)\*f^3\*g-44/3\*d^3/e^2\*x^2/(-e^2\*x^2+d^2)^(5/2)\*f\*g^3-2\*d^2/e\*x^2/(-e^2\*x^2+d^2)^(5/2)\*f^2\*g^2+6/5/e^3\*x/(-e^2\*x^2+d^2)^(3/2)\*d^2\*f\*g^3+6\*x^3/e/(-e^2\*x^2+d^2)^(5/2)\*d^2\*f\*g^3-18/5\*d^4/e^3\*x/(-e^2\*x^2+d^2)^(5/2)\*f\*g^3-21/5/e^2\*x/(-e^2\*x^2+d^2)^(5/2)\*d^3\*f^2\*g^2+6/5/e\*x/(-e^2\*x^2+d^2)^(5/2)\*d^2\*f^3\*g-4/3/e\*x^3/(-e^2\*x^2+d^2)^(3/2)\*f\*g^3+16/5/e^4\*x/(-e^2\*x^2+d^2)^(1/2)\*d\*g^4+32/5/e^3\*x/(-e^2\*x^2+d^2)^(1/2)\*f\*g^3-3/e^4/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))\*d\*g^4-4/e^3/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))\*f\*g^3-12\*d^4/e^3\*x^2/(-e^2\*x^2+d^2)^(5/2)\*g^4+88/15\*d^5/e^4/(-e^2\*x^2+d^2)^(5/2)\*f\*g^3-4/5/e^2/(-e^2\*x^2+d^2)^(5/2)\*d^3\*f^3\*g+9\*x^4/e/(-e^2\*x^2+d^2)^(5/2)\*d^2\*g^4+12\*x^4/(-e^2\*x^2+d^2)^(5/2)\*d\*f\*g^3+4/5\*x^5/e/(-e^2\*x^2+d^2)^(5/2)\*f\*g^3-1/e^2\*x^3/(-e^2\*x^2+d^2)^(3/2)\*d\*g^4-2/5/e\*x/(-e^2\*x^2+d^2)^(3/2)\*f^3\*g+6\*x^4/e/(-e^2\*x^2+d^2)^(5/2)\*f^2\*g^2+1/10/e^4\*x/(-e^2\*x^2+d^2)^(3/2)\*d^3\*g^4+1/2\*x^3/e^2/(-e^2\*x^2+d^2)^(5/2)\*d^3\*g^4+9\*x^3/(-e^2\*x^2+d^2)^(5/2)\*d\*f^2\*g^2+2\*x^3\*e/(-e^2\*x^2+d^2)^(5/2)\*f^3\*g+4/5\*d^4/e^3/(-e^2\*x^2+d^2)^(5/2)\*f^2\*g^2-3/10\*d^5/e^4\*x/(-e^2\*x^2+d^2)^(5/2)\*g^4+4\*x^2/(-e^2\*x^2+d^2)^(5/2)\*d\*f^3\*g

**Maxima [A]** time = 0.784423, size = 1608, normalized size = 7.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(g\*x + f)^4/(-e^2\*x^2 + d^2)^(7/2), x, algorithm="maxima")

```
[Out] -e*g^4*x^6/(-e^2*x^2 + d^2)^(5/2) + 6*d^2*g^4*x^4/((-e^2*x^2 + d^2)^(5/2)*e) - 8*d^4*g^4*x^2/((-e^2*x^2 + d^2)^(5/2)*e^3) + 1/5*d*f^4*x/(-e^2*x^2 + d^2)^(5/2) + 1/15*(4*e^3*f*g^3 + 3*d*e^2*g^4)*x*(15*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) - 20*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 8*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6)) + 3/5*d^2*f^4/((-e^2*x^2 + d^2)^(5/2)*e) + 4/5*d^3*f^3*g/((-e^2*x^2 + d^2)^(5/2)*e^2) + 16/5*d^6*g^4/((-e^2*x^2 + d^2)^(5/2)*e^5) + 4/15*f^4*x/((-e^2*x^2 + d^2)^(3/2)*d) + 8/15*f^4*x/(sqrt(-e^2*x^2 + d^2)*d^3) - 1/3*(4*e^3*f*g^3 + 3*d*e^2*g^4)*x*(3*x^2/((-e^2*x^2 + d^2)^(3/2)*e^2) - 2*d^2/((-e^2*x^2 + d^2)^(3/2)*e^4))/e^2 + 3*(2*e^3*f^2*g^2 + 4*d*e^2*f*g^3 + d^2*e*g^4)*x^4/((-e^2*x^2 + d^2)^(5/2)*e^2) + 1/2*(4*e^3*f^3*g + 18*d*e^2*f^2*g^2 + 12*d^2*e*f*g^3 + d^3*g^4)*x^3/((-e^2*x^2 + d^2)^(5/2)*e^2) - 4*(2*e^3*f^2*g^2 + 4*d*e^2*f*g^3 + d^2*e*g^4)*d^2*x^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 1/3*(e^3*f^4 + 12*d*e^2*f^3*g + 18*d^2*e*f^2*g^2 + 4*d^3*f*g^3)*x^2/((-e^2*x^2 + d^2)^(5/2)*e^2) - 3/10*(4*e^3*f^3*g + 18*d*e^2*f^2*g^2 + 12*d^2*e*f*g^3 + d^3*g^4)*d^2*x/((-e^2*x^2 + d^2)^(5/2)*e^4) + 3/5*(d*e^2*f^4 + 4*d^2*e*f^3*g + 2*d^3*f^2*g^2)*x/((-e^2*x^2 + d^2)^(5/2)*e^2) + 8/5*(2*e^3*f^2*g^2 + 4*d*e^2*f*g^3 + d^2*e*g^4)*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6) - 2/15*(e^3*f^4 + 12*d*e^2*f^3*g + 18*d^2*e*f^2*g^2 + 4*d^3*f*g^3)*d^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 4/15*(4*e^3*f*g^3 + 3*d*e^2*g^4)*d^2*x/((-e^2*x^2 + d^2)^(3/2)*e^6) + 1/10*(4*e^3*f^3*g + 18*d*e^2*f^2*g^2 + 12*d^2*e*f*g^3 + d^3*g^4)*x/((-e^2*x^2 + d^2)^(3/2)*e^4) - 1/5*(d*e^2*f^4 + 4*d^2*e*f^3*g + 2*d^3*f^2*g^2)*x/((-e^2*x^2 + d^2)^(3/2)*d^2*e^2) - 7/15*(4*e^3*f*g^3 + 3*d*e^2*g^4)*x/(sqrt(-e^2*x^2 + d^2)*e^6) + 1/5*(4*e^3*f^3*g + 18*d*e^2*f^2*g^2 + 12*d^2*e*f*g^3 + d^3*g^4)*x/(sqrt(-e^2*x^2 + d^2)*d^2*e^4) - 2/5*(d*e^2*f^4 + 4*d^2*e*f^3*g + 2*d^3*f^2*g^2)*x/(sqrt(-e^2*x^2 + d^2)*d^4*e^2) - (4*e^3*f*g^3 + 3*d*e^2*g^4)*arcsin(e^2*x/sqrt(d^2*e^2))/sqrt(e^2)*e^6
```

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**Fricas [A]** time = 0.310466, size = 1616, normalized size = 7.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^3*(g*x + f)^4/(-e^2*x^2 + d^2)^(7/2),x, algorithm="fricas")
```

```
[Out] 1/15*(15*d^3*e^7*g^4*x^7 + 5*(e^10*f^4 - 6*d^2*e^8*f^2*g^2 - 8*d^3*e^7*f*g^3 - 18*d^4*e^6*g^4)*x^6 + (19*d*e^9*f^4 - 84*d^2*e^8*f^3*g + 174*d^3*e^7*f^2*g^2 + 676*d^4*e^6*f*g^3 + 474*d^5*e^5*g^4)*x^5 - 5*(20*d^2*e^8*f^4 - 36*d^3*e^7*f^3*g - 12*d^4*e^6*f^2*g^2 + 164*d^5*e^5*f*g^3 + 105*d^6*e^4*g^4)*x^4 + 10*(7*d^3*e^7*f^4 + 12*d^4*e^6*f^3*g - 24*d^5*e^5*f^2*g^2 - 56*d^6*e^4*f*g^3 - 42*d^7*e^3*g^4)*x^3 + 60*(2*d^4*e^6*f^4 - 4*d^5*e^5*f^3*g + 20*d^7*e^3*f*g^3 + 15*d^8*e^2*g^4)*x^2 - 120*(d^5*e^5*f^4 + 4*d^8*e^2*f*g^3 + 3*d^9*e*g^4)*x + 30*(32*d^9*e*f*g^3 + 24*d^10*g^4 + (4*d^3*e^7*f*g^3 + 3*d^4*e^6*g^4)*x^6 + (4*d^4*e^6*f*g^3 + 3*d^5*e^5*g^4)*x^5 - 13*(4*d^5*e^5*f*g^3 + 3*d^6*e^4*g^4)*x^4 + 15*(4*d^6*e^4*f*g^3 + 3*d^7*e^3*g^4)*x^3 + 8*(4*d^7*e^3*f*g^3 + 3*d^8*e^2*g^4)*x^2 - 20*(4*d^8*e^2*f*g^3 + 3*d^9*e*g^4)*x - (32*d^8*e*f*g^3 + 24*d^9*g^4 + (4*d^3*e^6*f*g^3 + 3*d^4*e^5*g^4)*x^5 - 6*(4*d^4*e^5*f*g^3 + 3*d^5*e^4*g^4)*x^4 + 5*(4*d^5*e^4*f*g^3 + 3*d^6*e^3*g^4)*x^3 + 12*(4*d^6*e^3*f*g^3 + 3*d^7*e^2*g^4)*x^2 - 20*(4*d^7*e^2*f*g^3 + 3*d^8*e*g^4)*x)*sqrt(-e^2*x^2 + d^2))*arctan(-(d - sqrt(-e^2*x^2 + d^2))/(e*x)) + (15*d^3*e^6*g^4*x^6 - 3*(3*e^9*f^4 - 8*d*e^8*f^3*g + 18*d^2*e^7*f^2*g^2 + 72*d^3*e^6*f*g^3 + 43*d^4*e^5*g^4)*x^5 + 5*(8*d*e^8*f^4 - 12*d^2*e^7*f^3*g - 12*d^3*e^6*f^2*g^2 + 44*d^4*e^5*f*g^3 + 15*d^5*e^4*g^4)*x^4 - 10*(d^2*e^7*f^4 + 12*d^3*e^6*f^3*g - 24*d^4*e^5*f^2*g^2 - 80*d^5*e^4*f*g^3 - 60*d^6*e^3*g^4)*x^3 - 60*(2*d^3*e^6*f^4 - 4*d^4*e^5*f^3*g + 20*d^6*e^3*f*g^3 + 15*d^7*e^2*g^4)*x^2 + 120*(d^4*e^5*f^4 + 4*d^7*e^2*f*g^3 + 3*d^8*e*g^4)*x)*sqrt(-e^2*x^2 + d^2))/(d^3*e^11*x^6 + d^4*e^10*x^5 - 13*d^5*e^9*x^4 + 15*d^6*e^8*x^3 + 8*d^7*e^7*x^2 - 20*d^8*e^6*x + 8*d^9*e^5 - (d^3*e^10*x^5 - 6*d^4*e^9*x^4 + 5*d^5*e^8*x^3 + 12*d^6*e^7*x^2 - 20*d^7*e^6*x + 8*d^8*e^5)*sqrt(-e^2*x^2 + d^2))
```



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**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^3 (f+gx)^4}{(-(-d+ex)(d+ex))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(g\*x+f)\*\*4/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2),x)

[Out] Integral((d + e\*x)\*\*3\*(f + g\*x)\*\*4/((-d + e\*x)\*(d + e\*x))\*\*(7/2), x)

---

**GIAC/XCAS [A]** time = 0.294212, size = 555, normalized size = 2.58

$$-(3dg^4 + 4fg^3e) \arcsin\left(\frac{xe}{d}\right) e^{(-5)} \text{sign}(d) + \frac{\sqrt{-x^2e^2 + d^2} \left( \left( \left( \left( \left( 15g^4xe - \frac{2(36d^5g^4e^{10} + 64d^4fg^3e^{11} + 21d^3f^2g^2e^{12} - 6d^2f^3ge^{13} + df^4e^{14})e^{(-10)}}{d^4} \right) x - \frac{45(3d^6g^4e^9 + 4d^5fg^3e^{10} + 2d^4f^2g^2e^{11})e^{(-10)}}{d^4} \right) \right) \right) \right) x - \frac{45(3d^6g^4e^9 + 4d^5fg^3e^{10} + 2d^4f^2g^2e^{11})e^{(-10)}}{d^4}}{\sqrt{-x^2e^2 + d^2}}$$


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Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(g\*x + f)^4/(-e^2\*x^2 + d^2)^(7/2),x, algorithm="giac")

[Out]  $-(3*d*g^4 + 4*f*g^3*e)*\arcsin(x*e/d)*e^{(-5)}*\text{sign}(d) + 1/15*\text{sqrt}(-x^2*e^2 + d^2)*\left(\left(\left(\left(\left(15*g^4*x*e - 2*(36*d^5*g^4*e^{10} + 64*d^4*f*g^3*e^{11} + 21*d^3*f^2*g^2*e^{12} - 6*d^2*f^3*g*e^{13} + d*f^4*e^{14})\right)*e^{(-10)}/d^4\right)*x - 45*(3*d^6*g^4*e^9 + 4*d^5*f*g^3*e^{10} + 2*d^4*f^2*g^2*e^{11})\right)*e^{(-10)}/d^4\right)*x + 5*(21*d^7*g^4*e^8 + 28*d^6*f*g^3*e^9 - 6*d^5*f^2*g^2*e^{10} - 12*d^4*f^3*g*e^{11} + d^3*f^4*e^{12})\right)*e^{(-10)}/d^4\right)*x + 5*(36*d^8*g^4*e^7 + 44*d^7*f*g^3*e^8 + 6*d^6*f^2*g^2*e^9 - 12*d^5*f^3*g*e^{10} - d^4*f^4*e^{11})\right)*e^{(-10)}/d^4\right)*x - 15*(3*d^9*g^4*e^6 + 4*d^8*f*g^3*e^7 + d^5*f^4*e^{10})\right)*e^{(-10)}/d^4\right)*x - (72*d^{10}*g^4*e^5 + 88*d^9*f*g^3*e^6 + 12*d^8*f^2*g^2*e^7 - 12*d^7*f^3*g*e^8 + 7*d^6*f^4*e^9)*e^{(-10)}/d^4)/(x^2*e^2 - d^2)^3$

$$3.581 \quad \int \frac{(d+ex)^3(f+gx)^3}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=183

$$\frac{(d+ex)^2(2ef-13dg)(dg+ef)^2}{15d^2e^4(d^2-e^2x^2)^{3/2}} + \frac{(d+ex)^3(dg+ef)^3}{5de^4(d^2-e^2x^2)^{5/2}} - \frac{g^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4} + \frac{(d+ex)(dg+ef)(32d^2g^2-11defg+2e^2f^2)}{15d^3e^4\sqrt{d^2-e^2x^2}}$$

[Out]  $((e*f + d*g)^3*(d + e*x)^3)/(5*d*e^4*(d^2 - e^2*x^2)^{(5/2)}) + ((2*e*f - 13*d*g)*(e*f + d*g)^2*(d + e*x)^2)/(15*d^2*e^4*(d^2 - e^2*x^2)^{(3/2)}) + ((e*f + d*g)*(2*e^2*f^2 - 11*d*e*f*g + 32*d^2*g^2)*(d + e*x))/(15*d^3*e^4*\text{Sqrt}[d^2 - e^2*x^2]) - (g^3*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/e^4$

**Rubi [A]** time = 0.634761, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$

$$\frac{(d+ex)^2(2ef-13dg)(dg+ef)^2}{15d^2e^4(d^2-e^2x^2)^{3/2}} + \frac{(d+ex)^3(dg+ef)^3}{5de^4(d^2-e^2x^2)^{5/2}} - \frac{g^3 \tan^{-1}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4} + \frac{(d+ex)(dg+ef)(32d^2g^2-11defg+2e^2f^2)}{15d^3e^4\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(f + g\*x)^3)/(d^2 - e^2\*x^2)^(7/2), x]

[Out]  $((e*f + d*g)^3*(d + e*x)^3)/(5*d*e^4*(d^2 - e^2*x^2)^{(5/2)}) + ((2*e*f - 13*d*g)*(e*f + d*g)^2*(d + e*x)^2)/(15*d^2*e^4*(d^2 - e^2*x^2)^{(3/2)}) + ((e*f + d*g)*(2*e^2*f^2 - 11*d*e*f*g + 32*d^2*g^2)*(d + e*x))/(15*d^3*e^4*\text{Sqrt}[d^2 - e^2*x^2]) - (g^3*\text{ArcTan}[(e*x)/\text{Sqrt}[d^2 - e^2*x^2]])/e^4$

**Rubi in Sympy [A]** time = 77.108, size = 243, normalized size = 1.33

$$-\frac{g^3 \operatorname{atan}\left(\frac{ex}{\sqrt{d^2-e^2x^2}}\right)}{e^4} + \frac{3g^2\sqrt{d^2-e^2x^2}(dg+ef)}{de^4(d-ex)} - \frac{g\sqrt{d^2-e^2x^2}(dg+ef)^2}{de^4(d-ex)^2} + \frac{\sqrt{d^2-e^2x^2}(dg+ef)^3}{5de^4(d-ex)^3} - \frac{g\sqrt{d^2-e^2x^2}(dg+ef)^2}{d^2e^4(d-ex)} + \frac{2\sqrt{d^2-e^2x^2}(dg+ef)^3}{15d^2e^4(d-ex)^2} + \frac{2\sqrt{d^2-e^2x^2}(dg+ef)^3}{15d^3e^4(d-ex)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3\*(g\*x+f)\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out]  $-g^3*\operatorname{atan}(e*x/\operatorname{sqrt}(d^2 - e^2*x^2))/e^4 + 3*g^2*\operatorname{sqrt}(d^2 - e^2*x^2)*(d*g + e*f)/(d*e^4*(d - e*x)) - g*\operatorname{sqrt}(d^2 - e^2*x^2)*(d*g + e*f)**2/(d*e^4*(d - e*x)**2) + \operatorname{sqrt}(d^2 - e^2*x^2)*(d*g + e*f)**3/(5*d*e^4*(d - e*x)**3) - g*\operatorname{sqrt}(d^2 - e^2*x^2)*(d*g + e*f)**2/(d^2*e^4*(d - e*x)) + 2*\operatorname{sqrt}(d^2 - e^2*x^2)*(d*g + e*f)**3/(15*d^2*e^4*(d - e*x)**2) + 2*\operatorname{sqrt}(d^2 - e^2*x^2)*(d*g + e*f)**3/(15*d^3*e^4*(d - e*x))$

**Mathematica [A]** time = 0.307271, size = 145, normalized size = 0.79

$$\frac{\sqrt{d^2 - e^2 x^2} (dg + ef) (22d^4 g^2 - d^3 eg(16f + 51gx) + d^2 e^2 (7f^2 + 33fgx + 32g^2 x^2) - de^3 f x(6f + 11gx) + 2e^4 f^2 x^2)}{d^3 (d - ex)^3} - 15g^3 \tan^{-1} \left( \frac{ex}{\sqrt{d^2 - e^2 x^2}} \right)}{15e^4}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(f + g\*x)^3)/(d^2 - e^2\*x^2)^(7/2), x]

[Out] (((e\*f + d\*g)\*Sqrt[d^2 - e^2\*x^2]\*(22\*d^4\*g^2 + 2\*e^4\*f^2\*x^2 - d\*e^3\*f\*x\*(6\*f + 11\*g\*x) - d^3\*e\*g\*(16\*f + 51\*g\*x) + d^2\*e^2\*(7\*f^2 + 33\*f\*g\*x + 32\*g^2\*x^2)))/(d^3\*(d - e\*x)^3) - 15\*g^3\*ArcTan[(e\*x)/Sqrt[d^2 - e^2\*x^2]])/(15\*e^4)

**Maple [B]** time = 0.016, size = 713, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(g\*x+f)^3/(-e^2\*x^2+d^2)^(7/2), x)

[Out] 1/3\*x^2\*e/(-e^2\*x^2+d^2)^(5/2)\*f^3+3\*x^4/(-e^2\*x^2+d^2)^(5/2)\*d\*g^3+22/15\*d^5/e^4/(-e^2\*x^2+d^2)^(5/2)\*g^3+7/15/e/(-e^2\*x^2+d^2)^(5/2)\*d^2\*f^3+4/5\*d\*f^3\*x/(-e^2\*x^2+d^2)^(5/2)+1/15/d\*f^3\*x/(-e^2\*x^2+d^2)^(3/2)+2/15/d^3\*f^3\*x/(-e^2\*x^2+d^2)^(1/2)+1/5\*e\*g^3\*x^5/(-e^2\*x^2+d^2)^(5/2)-1/3/e\*g^3\*x^3/(-e^2\*x^2+d^2)^(3/2)+8/5/e^3\*g^3\*x/(-e^2\*x^2+d^2)^(1/2)-1/e^3\*g^3/(e^2)^(1/2)\*arctan((e^2)^(1/2)\*x/(-e^2\*x^2+d^2)^(1/2))-d^2/e\*x^2/(-e^2\*x^2+d^2)^(5/2)\*f\*g^2-21/10\*d^3/e^2\*x/(-e^2\*x^2+d^2)^(5/2)\*f\*g^2+9/10\*d^2/e\*x/(-e^2\*x^2+d^2)^(5/2)\*f^2\*g+7/10/e^2\*x/(-e^2\*x^2+d^2)^(3/2)\*d\*f\*g^2+7/5/d/e^2\*x/(-e^2\*x^2+d^2)^(1/2)\*f\*g^2-3/5/d^2/e\*x/(-e^2\*x^2+d^2)^(1/2)\*f^2\*g+3\*x^4\*e/(-e^2\*x^2+d^2)^(5/2)\*f\*g^2-11/3\*d^3/e^2\*x^2/(-e^2\*x^2+d^2)^(5/2)\*g^3+2/5\*d^4/e^3/(-e^2\*x^2+d^2)^(5/2)\*f\*g^2+3/10/e^3\*x/(-e^2\*x^2+d^2)^(3/2)\*d^2\*g^3-3/10/e\*x/(-e^2\*x^2+d^2)^(3/2)\*f^2\*g-3/5/e^2/(-e^2\*x^2+d^2)^(5/2)\*d^3\*f^2\*g+9/2\*x^3/(-e^2\*x^2+d^2)^(5/2)\*d\*f\*g^2+3/2\*x^3\*e/(-e^2\*x^2+d^2)^(5/2)\*f^2\*g-9/10\*d^4/e^3\*x/(-e^2\*x^2+d^2)^(5/2)\*g^3+3\*x^2/(-e^2\*x^2+d^2)^(5/2)\*d\*f^2\*g+3/2\*x^3/e/(-e^2\*x^2+d^2)^(5/2)\*d^2\*g^3

**Maxima [A]** time = 0.781927, size = 1220, normalized size = 6.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(g\*x + f)^3/(-e^2\*x^2 + d^2)^(7/2), x, algorithm="maxima")

[Out] 1/15\*e^3\*g^3\*x\*(15\*x^4/((-e^2\*x^2 + d^2)^(5/2)\*e^2) - 20\*d^2\*x^2/((-e^2\*x^2 + d^2)^(5/2)\*e^4) + 8\*d^4/((-e^2\*x^2 + d^2)^(5/2)\*e^6)) - 1/3\*e\*g^3\*x\*(3\*x^2/((-e^2\*x^2 + d^2)^(3/2)\*e^2) - 2\*d^2/((-e^2\*x^2 + d^2)^(3/2)\*e^4)) + 1/5\*d\*f^3\*x/(-e^2\*x^2 + d^2)^(5/2) + 3/5\*d^2\*f^3/((-e^2\*x^2 + d^2)^(5/2)\*e) + 3/5\*d^3\*f^2\*g/((-e^2\*x^2 + d^2)^(5/2)\*e^2) + 4/15\*f^3\*x/((-e^2\*x^2 + d^2)^(3/2)\*d) + 4/15\*d^2\*g^3\*x/((-e^2\*x^2 + d^2)^(3/2)\*e^3) + 8/15\*f^3\*x/(sqrt(-e^2\*x^2 + d^2)\*d^3) - 7/15\*g^3\*x/(sqrt(-e^2\*x^2 + d^2)\*e^3) + 3\*(e^3\*f\*g^2 + d\*e^2\*g^3)\*x^4/((-e^2\*x^2 + d^2)^(5/2)\*e^2) - g^3\*arcsin(e^2\*x/sqrt(d^2\*e^2))/sqrt(e^2)\*e^3 + 3/2\*(e^3\*f^2\*g + 3\*d\*e^2\*f\*g^2 + d^2\*e\*g^3)\*x^3/((-e^2\*x^2 + d^2)^(5/2)\*e^2) - 4\*(e^3\*f\*g^2 + d\*e^2\*g^3)\*d^2\*x^2/((-e^2\*x^2 + d^2)^(5/2)\*e^4) + 1/3\*(e^3\*f^3 +

$$9*d^2*e^2*f^2*g + 9*d^2*e*f*g^2 + d^3*g^3)*x^2/((-e^2*x^2 + d^2)^(5/2)*e^2) - 9/10*(e^3*f^2*g + 3*d*e^2*f*g^2 + d^2*e*g^3)*d^2*x/((-e^2*x^2 + d^2)^(5/2)*e^4) + 3/5*(d*e^2*f^3 + 3*d^2*e*f^2*g + d^3*f*g^2)*x/((-e^2*x^2 + d^2)^(5/2)*e^2) + 8/5*(e^3*f*g^2 + d*e^2*g^3)*d^4/((-e^2*x^2 + d^2)^(5/2)*e^6) - 2/15*(e^3*f^3 + 9*d*e^2*f^2*g + 9*d^2*e*f*g^2 + d^3*g^3)*d^2/((-e^2*x^2 + d^2)^(5/2)*e^4) + 3/10*(e^3*f^2*g + 3*d*e^2*f*g^2 + d^2*e*g^3)*x/((-e^2*x^2 + d^2)^(3/2)*e^4) - 1/5*(d*e^2*f^3 + 3*d^2*e*f^2*g + d^3*f*g^2)*x/((-e^2*x^2 + d^2)^(3/2)*d^2*e^2) + 3/5*(e^3*f^2*g + 3*d*e^2*f*g^2 + d^2*e*g^3)*x/(sqrt(-e^2*x^2 + d^2)*d^2*e^4) - 2/5*(d*e^2*f^3 + 3*d^2*e*f^2*g + d^3*f*g^2)*x/(sqrt(-e^2*x^2 + d^2)*d^4*e^2)$$

**Fricas [A]** time = 0.309537, size = 887, normalized size = 4.85

$$9(e^8 f^3 - 2de^7 f^2 g + 3d^2 e^6 f g^2 + 6d^3 e^5 g^3)x^5 - 5(7de^7 f^3 - 9d^2 e^6 f^2 g - 3d^3 e^5 f g^2 + 13d^4 e^4 g^3)x^4 + 5(4d^2 e^6 f^3 + 9d^3 e^5 f^2 g - 12d^4 e^4 f g^2 + 13d^5 e^3 g^3)x^3 - 5(4d^3 e^5 f^3 + 9d^4 e^4 f^2 g - 12d^5 e^3 f g^2 + 13d^6 e^2 g^3)x^2 - 5(4d^4 e^4 f^3 + 9d^5 e^3 f^2 g - 12d^6 e^2 f g^2 + 13d^7 e g^3)x - 5(4d^5 e^3 f^3 + 9d^6 e^2 f^2 g - 12d^7 e f g^2 + 13d^8 e g^3)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(g\*x + f)^3/(-e^2\*x^2 + d^2)^(7/2), x, algorithm="fricas")

[Out] 1/15\*(9\*(e^8\*f^3 - 2\*d\*e^7\*f^2\*g + 3\*d^2\*e^6\*f\*g^2 + 6\*d^3\*e^5\*g^3)\*x^5 - 5\*(7\*d\*e^7\*f^3 - 9\*d^2\*e^6\*f^2\*g - 3\*d^3\*e^5\*f\*g^2 + 13\*d^4\*e^4\*g^3)\*x^4 + 5\*(4\*d^2\*e^6\*f^3 + 9\*d^3\*e^5\*f^2\*g - 12\*d^4\*e^4\*f\*g^2 - 17\*d^5\*e^3\*g^3)\*x^3 + 30\*(2\*d^3\*e^5\*f^3 - 3\*d^4\*e^4\*f^2\*g + 5\*d^5\*e^3\*g^3)\*x^2 - 60\*(d^4\*e^4\*f^3 + d^7\*e\*g^3)\*x + 30\*(d^3\*e^5\*g^3\*x^5 - 5\*d^4\*e^4\*g^3\*x^4 + 5\*d^5\*e^3\*g^3\*x^3 + 5\*d^6\*e^2\*g^3\*x^2 - 10\*d^7\*e\*g^3\*x + 4\*d^8\*g^3 + (d^3\*e^4\*g^3\*x^4 - 7\*d^5\*e^2\*g^3\*x^2 + 10\*d^6\*e\*g^3\*x - 4\*d^7\*g^3)\*sqrt(-e^2\*x^2 + d^2))\*arctan(-(d - sqrt(-e^2\*x^2 + d^2))/(e\*x)) + 5\*((e^7\*f^3 - 3\*d^2\*e^5\*f\*g^2 - 2\*d^3\*e^4\*g^3)\*x^4 + (2\*d\*e^6\*f^3 - 9\*d^2\*e^5\*f^2\*g + 12\*d^3\*e^4\*f\*g^2 + 23\*d^4\*e^3\*g^3)\*x^3 - 6\*(2\*d^2\*e^5\*f^3 - 3\*d^3\*e^4\*f^2\*g + 5\*d^5\*e^2\*g^3)\*x^2 + 12\*(d^3\*e^4\*f^3 + d^6\*e\*g^3)\*x)\*sqrt(-e^2\*x^2 + d^2))/(d^3\*e^9\*x^5 - 5\*d^4\*e^8\*x^4 + 5\*d^5\*e^7\*x^3 + 5\*d^6\*e^6\*x^2 - 10\*d^7\*e^5\*x + 4\*d^8\*e^4 + (d^3\*e^8\*x^4 - 7\*d^5\*e^6\*x^2 + 10\*d^6\*e^5\*x - 4\*d^7\*e^4)\*sqrt(-e^2\*x^2 + d^2))

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^3 (f + gx)^3}{(-(-d + ex)(d + ex))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(g\*x+f)\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out] Integral((d + e\*x)\*\*3\*(f + g\*x)\*\*3/((-(-d + e\*x)\*(d + e\*x))\*\*(7/2), x)

**GIAC/XCAS [A]** time = 0.294936, size = 417, normalized size = 2.28

$$-g^3 \arcsin\left(\frac{xe}{d}\right) e^{(-4)} \operatorname{sign}(d) - \frac{\sqrt{-x^2 e^2 + d^2} \left( \left( \left( x \left( \frac{(32d^4 g^3 e^8 + 21d^3 f g^2 e^9 - 9d^2 f^2 g e^{10} + 2df^3 e^{11}) x e^{(-7)}}{d^4} + \frac{45(d^5 g^3 e^7 + d^4 f g^2 e^8) e^{(-7)}}{d^4} \right) \right) - \frac{5(7d^6 g^3 e^6 - 3d^5 f g^2 e^7 - 9d^4 f^2 g e^8 + d^3 f^3 e^9) e^{(-7)}}{d^4} \right)}{d^4}$$

15(x<sup>2</sup>e

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(g\*x + f)^3/(-e^2\*x^2 + d^2)^(7/2),x, algorithm="giac")

[Out]  $-g^3 \arcsin(xe/d) e^{-4} \operatorname{sign}(d) - \frac{1}{15} \sqrt{-x^2 e^2 + d^2} \left( \left( x \left( (32d^4 g^3 e^8 + 21d^3 f g^2 e^9 - 9d^2 f^2 g e^{10} + 2d f^3 e^{11}) x e^{-7} / d^4 + 45(d^5 g^3 e^7 + d^4 f g^2 e^8) e^{-7} / d^4 \right) - 5(7d^6 g^3 e^6 - 3d^5 f g^2 e^7 - 9d^4 f^2 g e^8 + d^3 f^3 e^9) e^{-7} / d^4 \right) x - 5(11d^7 g^3 e^5 + 3d^6 f g^2 e^6 - 9d^5 f^2 g e^7 - d^4 f^3 e^8) e^{-7} / d^4 \right) x + 15(d^8 g^3 e^4 + d^5 f^3 e^7) e^{-7} / d^4 \right) x + (22d^9 g^3 e^3 + 6d^8 f g^2 e^4 - 9d^7 f^2 g e^5 + 7d^6 f^3 e^6) e^{-7} / d^4 \right) / (x^2 e^2 - d^2)^3$

$$3.582 \quad \int \frac{(d+ex)^3(f+gx)^2}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=145

$$\frac{(d+ex)^3(dg+ef)^2}{5de^3(d^2-e^2x^2)^{5/2}} + \frac{2(d+ex)^2(ef-4dg)(dg+ef)}{15d^2e^3(d^2-e^2x^2)^{3/2}} + \frac{(d+ex)(7d^2g^2-6defg+2e^2f^2)}{15d^3e^3\sqrt{d^2-e^2x^2}}$$

[Out]  $((e*f + d*g)^2*(d + e*x)^3)/(5*d*e^3*(d^2 - e^2*x^2)^{(5/2)}) + (2*(e*f - 4*d*g)*(e*f + d*g)*(d + e*x)^2)/(15*d^2*e^3*(d^2 - e^2*x^2)^{(3/2)}) + ((2*e^2*f^2 - 6*d*e*f*g + 7*d^2*g^2)*(d + e*x))/(15*d^3*e^3*\text{Sqrt}[d^2 - e^2*x^2])$

**Rubi [A]** time = 0.435677, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$

$$\frac{(d+ex)^3(dg+ef)^2}{5de^3(d^2-e^2x^2)^{5/2}} + \frac{2(d+ex)^2(ef-4dg)(dg+ef)}{15d^2e^3(d^2-e^2x^2)^{3/2}} + \frac{(d+ex)(7d^2g^2-6defg+2e^2f^2)}{15d^3e^3\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(f + g\*x)^2)/(d^2 - e^2\*x^2)^(7/2), x]

[Out]  $((e*f + d*g)^2*(d + e*x)^3)/(5*d*e^3*(d^2 - e^2*x^2)^{(5/2)}) + (2*(e*f - 4*d*g)*(e*f + d*g)*(d + e*x)^2)/(15*d^2*e^3*(d^2 - e^2*x^2)^{(3/2)}) + ((2*e^2*f^2 - 6*d*e*f*g + 7*d^2*g^2)*(d + e*x))/(15*d^3*e^3*\text{Sqrt}[d^2 - e^2*x^2])$

**Rubi in Sympy [A]** time = 59.1491, size = 212, normalized size = 1.46

$$\frac{g^2\sqrt{d^2-e^2x^2}}{de^3(d-ex)} - \frac{2g\sqrt{d^2-e^2x^2}(dg+ef)}{3de^3(d-ex)^2} + \frac{\sqrt{d^2-e^2x^2}(dg+ef)^2}{5de^3(d-ex)^3} - \frac{2g\sqrt{d^2-e^2x^2}(dg+ef)}{3d^2e^3(d-ex)} + \frac{2\sqrt{d^2-e^2x^2}(dg+ef)^2}{15d^2e^3(d-ex)^2} + \frac{2\sqrt{d^2-e^2x^2}(dg+ef)^2}{15d^3e^3(d-ex)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3\*(g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out]  $g**2*\text{sqrt}(d**2 - e**2*x**2)/(d*e**3*(d - e*x)) - 2*g*\text{sqrt}(d**2 - e**2*x**2)*(d*g + e*f)/(3*d*e**3*(d - e*x)**2) + \text{sqrt}(d**2 - e**2*x**2)*(d*g + e*f)**2/(5*d*e**3*(d - e*x)**3) - 2*g*\text{sqrt}(d**2 - e**2*x**2)*(d*g + e*f)/(3*d**2*e**3*(d - e*x)) + 2*\text{sqrt}(d**2 - e**2*x**2)*(d*g + e*f)**2/(15*d**2*e**3*(d - e*x)**2) + 2*\text{sqrt}(d**2 - e**2*x**2)*(d*g + e*f)**2/(15*d**3*e**3*(d - e*x))$

**Mathematica [A]** time = 0.124361, size = 105, normalized size = 0.72

$$\frac{\sqrt{d^2-e^2x^2}(2d^4g^2-6d^3eg(f+gx)+d^2e^2(7f^2+18fgx+7g^2x^2))-6de^3fx(f+gx)+2e^4f^2x^2}{15d^3e^3(d-ex)^3}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(f + g\*x)^2)/(d^2 - e^2\*x^2)^(7/2), x]

[Out]  $(\text{Sqrt}[d^2 - e^2 x^2]) \cdot (2 d^4 g^2 + 2 e^4 f^2 x^2 - 6 d^3 e g^2 (f + g x) - 6 d^2 e^3 f^2 x (f + g x) + d^2 e^2 (7 f^2 + 18 f g x + 7 g^2 x^2)) / (15 d^3 e^3 (d - e x)^3)$

**Maple [A]** time = 0.012, size = 131, normalized size = 0.9

$$\frac{(-ex + d)(ex + d)^4 (7 d^2 e^2 g^2 x^2 - 6 d e^3 f g x^2 + 2 e^4 f^2 x^2 - 6 d^3 e g^2 x + 18 d^2 e^2 f g x - 6 d e^3 f^2 x + 2 d^4 g^2 - 6 d^3 e f g + 7 d^2 e^2 f^2)}{15 d^3 e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((e^x + d)^3 (g x + f)^2 / (-e^2 x^2 + d^2)^{7/2}, x)$

[Out]  $1/15 \cdot (-e^x + d) \cdot (e^x + d)^4 \cdot (7 d^2 e^2 g^2 x^2 - 6 d^3 e g^2 x + 18 d^2 e^2 f g x - 6 d e^3 f^2 x + 2 d^4 g^2 - 6 d^3 e f g + 7 d^2 e^2 f^2) / d^3 e^3 / (-e^2 x^2 + d^2)^{7/2}$

**Maxima [A]** time = 0.701287, size = 787, normalized size = 5.43

$$\begin{aligned} & \frac{eg^2 x^4}{(-e^2 x^2 + d^2)^{5/2}} - \frac{4 d^2 g^2 x^2}{3(-e^2 x^2 + d^2)^{5/2} e} + \frac{d f^2 x}{5(-e^2 x^2 + d^2)^{5/2}} + \frac{3 d^2 f^2}{5(-e^2 x^2 + d^2)^{5/2} e} \\ & + \frac{2 d^3 f g}{5(-e^2 x^2 + d^2)^{5/2} e^2} + \frac{8 d^4 g^2}{15(-e^2 x^2 + d^2)^{5/2} e^3} + \frac{4 f^2 x}{15(-e^2 x^2 + d^2)^{3/2} d} + \frac{8 f^2 x}{15 \sqrt{-e^2 x^2 + d^2} d^3} \\ & + \frac{(2 e^3 f g + 3 d e^2 g^2) x^3}{2(-e^2 x^2 + d^2)^{5/2} e^2} + \frac{(e^3 f^2 + 6 d e^2 f g + 3 d^2 e g^2) x^2}{3(-e^2 x^2 + d^2)^{5/2} e^2} - \frac{3(2 e^3 f g + 3 d e^2 g^2) d^2 x}{10(-e^2 x^2 + d^2)^{5/2} e^4} \\ & + \frac{(3 d e^2 f^2 + 6 d^2 e f g + d^3 g^2) x}{5(-e^2 x^2 + d^2)^{5/2} e^2} - \frac{2(e^3 f^2 + 6 d e^2 f g + 3 d^2 e g^2) d^2}{15(-e^2 x^2 + d^2)^{5/2} e^4} + \frac{(2 e^3 f g + 3 d e^2 g^2) x}{10(-e^2 x^2 + d^2)^{3/2} e^4} \\ & - \frac{(3 d e^2 f^2 + 6 d^2 e f g + d^3 g^2) x}{15(-e^2 x^2 + d^2)^{3/2} d^2 e^2} + \frac{(2 e^3 f g + 3 d e^2 g^2) x}{5 \sqrt{-e^2 x^2 + d^2} d^2 e^4} - \frac{2(3 d e^2 f^2 + 6 d^2 e f g + d^3 g^2) x}{15 \sqrt{-e^2 x^2 + d^2} d^4 e^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^x + d)^3 (g x + f)^2 / (-e^2 x^2 + d^2)^{7/2}, x, \text{algorithm}="maxima")$

[Out]  $e^x g^2 x^4 / (-e^2 x^2 + d^2)^{5/2} - 4/3 d^2 g^2 x^2 / ((-e^2 x^2 + d^2)^{5/2} e) + 1/5 d^2 f^2 x / (-e^2 x^2 + d^2)^{5/2} + 3/5 d^2 f^2 / ((-e^2 x^2 + d^2)^{5/2} e) + 2/5 d^3 f^2 g / ((-e^2 x^2 + d^2)^{5/2} e^2) + 8/15 d^4 g^2 / ((-e^2 x^2 + d^2)^{5/2} e^3) + 4/15 f^2 x / ((-e^2 x^2 + d^2)^{3/2} d) + 8/15 f^2 x / (\text{sqrt}(-e^2 x^2 + d^2) d^3) + 1/2 (2 e^3 f^2 g + 3 d^2 e^2 g^2) x^3 / ((-e^2 x^2 + d^2)^{5/2} e^2) + 1/3 (e^3 f^2 + 6 d e^2 f g + 3 d^2 e g^2) x^2 / ((-e^2 x^2 + d^2)^{5/2} e^2) - 3/10 (2 e^3 f^2 g + 3 d^2 e^2 g^2) d^2 x / ((-e^2 x^2 + d^2)^{5/2} e^4) + 1/5 (3 d^2 e^2 f^2 + 6 d^2 e f g + d^3 g^2) x / ((-e^2 x^2 + d^2)^{5/2} e^2) - 2/15 (e^3 f^2 + 6 d e^2 f g + 3 d^2 e g^2) d^2 / ((-e^2 x^2 + d^2)^{5/2} e^4) + 1/10 (2 e^3 f^2 g + 3 d^2 e^2 g^2) x / ((-e^2 x^2 + d^2)^{3/2} e^4) - 1/15 (3 d^2 e^2 f^2 + 6 d^2 e f g + d^3 g^2) x / ((-e^2 x^2 + d^2)^{3/2} d^2 e^2) + 1/5 (2 e^3 f^2 g + 3 d^2 e^2 g^2) x / (\text{sqrt}(-e^2 x^2 + d^2) d^2 e^4) - 2/15 (3 d^2 e^2 f^2 + 6 d^2 e f g + d^3 g^2) x / (\text{sqrt}(-e^2 x^2 + d^2) d^4 e^2)$

**Fricas [A]** time = 0.286283, size = 456, normalized size = 3.14

$$\frac{60 d^4 f^2 x - 3(3 e^4 f^2 - 4 d e^3 f g + 3 d^2 e^2 g^2) x^5 + 5(7 d e^3 f^2 - 6 d^2 e^2 f g - d^3 e g^2) x^4 - 10(2 d^2 e^2 f^2 + 3 d^3 e f g - 2 d^4 g^2) x^3 - 10(2 d^2 e^2 f^2 + 3 d^3 e f g - 2 d^4 g^2) x^3 - 10(d^3 e^5 x^5 - 5 d^4 e^4 x^4 + 5 d^5 e^3 x^3 + 5 d^6 e^2 x^2 - 10 d^7 e)}{15(d^3 e^5 x^5 - 5 d^4 e^4 x^4 + 5 d^5 e^3 x^3 + 5 d^6 e^2 x^2 - 10 d^7 e)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^3*(g*x + f)^2/(-e^2*x^2 + d^2)^(7/2),x, algorithm="fricas")`

[Out] 
$$\frac{-1/15*(60*d^4*f^2*x - 3*(3*e^4*f^2 - 4*d*e^3*f*g + 3*d^2*e^2*g^2)*x^5 + 5*(7*d^2*e^3*f^2 - 6*d^2*e^2*f*g - d^3*e*g^2)*x^4 - 10*(2*d^2*e^2*f^2 + 3*d^3*e*f*g - 2*d^4*g^2)*x^3 - 60*(d^3*e*f^2 - d^4*f*g)*x^2 - 5*(12*d^3*f^2*x + (e^3*f^2 - d^2*e*g^2)*x^4 + 2*(d^2*e^2*f^2 - 3*d^2*e*f*g + 2*d^3*g^2)*x^3 - 12*(d^2*e*f^2 - d^3*f*g)*x^2)*\sqrt{-e^2*x^2 + d^2}}{(d^3*e^5*x^5 - 5*d^4*e^4*x^4 + 5*d^5*e^3*x^3 + 5*d^6*e^2*x^2 - 10*d^7*e*x + 4*d^8 + (d^3*e^4*x^4 - 7*d^5*e^2*x^2 + 10*d^6*e*x - 4*d^7)*\sqrt{-e^2*x^2 + d^2})}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^3 (f + gx)^2}{(-(-d + ex)(d + ex))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**3*(g*x+f)**2/(-e**2*x**2+d**2)**(7/2),x)`

[Out] `Integral((d + e*x)**3*(f + g*x)**2/(-(-d + e*x)*(d + e*x))**(7/2), x)`

**GIAC/XCAS [A]** time = 0.28964, size = 267, normalized size = 1.84

$$\frac{\sqrt{-x^2e^2 + d^2} \left( \left( 15df^2 + \left( \left( 15g^2e + \frac{(7d^3g^2e^6 - 6d^2fge^7 + 2df^2e^8)xe^{(-4)}}{d^4} \right) x + \frac{5(d^5g^2e^4 + 6d^4fge^5 - d^3f^2e^6)e^{(-4)}}{d^4} \right) x - \frac{5(d^6g^2e^3 - 6d^5fge^4 - 6d^4f^2e^5 + 7d^3g^2e^6 - 6d^2fge^7 + 2df^2e^8)xe^{(-4)}}{d^4} \right) \right)}{15(x^2e^2 - d^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^3*(g*x + f)^2/(-e^2*x^2 + d^2)^(7/2),x, algorithm="giac")`

[Out] 
$$\frac{-1/15*\sqrt{-x^2*e^2 + d^2}*((15*d*f^2 + ((15*g^2*e + (7*d^3*g^2*e^6 - 6*d^2*f*g*e^7 + 2*d*f^2*e^8)*x*e^{(-4)})/d^4)*x + 5*(d^5*g^2*e^4 + 6*d^4*f*g*e^5 - d^3*f^2*e^6)*e^{(-4)}/d^4)*x - 5*(d^6*g^2*e^3 - 6*d^5*f*g*e^4 - d^4*f^2*e^5)*e^{(-4)}/d^4)*x + (2*d^8*g^2*e^3 - 6*d^7*f*g*e^2 + 7*d^6*f^2*e^3)*e^{(-4)}/d^4)/(x^2*e^2 - d^2)^3}$$



$$3.583 \quad \int \frac{(d+ex)^3(f+gx)}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=117

$$\frac{(d+ex)^3(dg+ef)}{5de^2(d^2-e^2x^2)^{5/2}} + \frac{2(d+ex)(2ef-3dg)}{15de^2(d^2-e^2x^2)^{3/2}} + \frac{x(2ef-3dg)}{15d^3e\sqrt{d^2-e^2x^2}}$$

[Out]  $((e*f + d*g)*(d + e*x)^3)/(5*d*e^2*(d^2 - e^2*x^2)^{(5/2)}) + (2*(2*e*f - 3*d*g)*(d + e*x))/(15*d*e^2*(d^2 - e^2*x^2)^{(3/2)}) + ((2*e*f - 3*d*g)*x)/(15*d^3*e*\text{Sqrt}[d^2 - e^2*x^2])$

**Rubi [A]** time = 0.18492, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$\frac{(d+ex)^3(dg+ef)}{5de^2(d^2-e^2x^2)^{5/2}} + \frac{2(d+ex)(2ef-3dg)}{15de^2(d^2-e^2x^2)^{3/2}} + \frac{x(2ef-3dg)}{15d^3e\sqrt{d^2-e^2x^2}}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(f + g\*x))/(d^2 - e^2\*x^2)^(7/2), x]

[Out]  $((e*f + d*g)*(d + e*x)^3)/(5*d*e^2*(d^2 - e^2*x^2)^{(5/2)}) + (2*(2*e*f - 3*d*g)*(d + e*x))/(15*d*e^2*(d^2 - e^2*x^2)^{(3/2)}) + ((2*e*f - 3*d*g)*x)/(15*d^3*e*\text{Sqrt}[d^2 - e^2*x^2])$

**Rubi in Sympy [A]** time = 15.6559, size = 102, normalized size = 0.87

$$\frac{(d+ex)^3(dg+ef)}{5de^2(d^2-e^2x^2)^{5/2}} - \frac{2(d+ex)(3dg-2ef)}{15de^2(d^2-e^2x^2)^{3/2}} - \frac{x(3dg-2ef)}{15d^3e\sqrt{d^2-e^2x^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3\*(g\*x+f)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out]  $(d + e*x)**3*(d*g + e*f)/(5*d*e**2*(d**2 - e**2*x**2)**(5/2)) - 2*(d + e*x)*(3*d*g - 2*e*f)/(15*d*e**2*(d**2 - e**2*x**2)**(3/2)) - x*(3*d*g - 2*e*f)/(15*d**3*e*\text{sqrt}(d**2 - e**2*x**2))$

**Mathematica [A]** time = 0.0851478, size = 77, normalized size = 0.66

$$\frac{\sqrt{d^2 - e^2x^2}(-3d^3g + d^2e(7f + 9gx) - 3de^2x(2f + gx) + 2e^3fx^2)}{15d^3e^2(d - ex)^3}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(f + g\*x))/(d^2 - e^2\*x^2)^(7/2), x]

[Out]  $(\text{Sqrt}[d^2 - e^2*x^2]*(-3*d^3*g + 2*e^3*f*x^2 - 3*d*e^2*x*(2*f + g*x) + d^2*e*(7*f + 9*g*x)))/(15*d^3*e^2*(d - e*x)^3)$

**Maple [A]** time = 0.011, size = 85, normalized size = 0.7

$$-\frac{(-ex+d)(ex+d)^4(3de^2gx^2-2e^3fx^2-9d^2egx+6de^2fx+3d^3g-7d^2ef)}{15d^3e^2}(-e^2x^2+d^2)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(g\*x+f)/(-e^2\*x^2+d^2)^(7/2),x)

[Out] -1/15\*(-e\*x+d)\*(e\*x+d)^4\*(3\*d\*e^2\*g\*x^2-2\*e^3\*f\*x^2-9\*d^2\*e\*g\*x+6\*d\*e^2\*f\*x+3\*d^3\*g-7\*d^2\*e\*f)/d^3/e^2/(-e^2\*x^2+d^2)^(7/2)

**Maxima [A]** time = 0.69434, size = 504, normalized size = 4.31

$$\begin{aligned} & \frac{egx^3}{2(-e^2x^2+d^2)^{\frac{5}{2}}} + \frac{dfx}{5(-e^2x^2+d^2)^{\frac{5}{2}}} - \frac{3d^2gx}{10(-e^2x^2+d^2)^{\frac{5}{2}}e} + \frac{3d^2f}{5(-e^2x^2+d^2)^{\frac{5}{2}}e} \\ & + \frac{d^3g}{5(-e^2x^2+d^2)^{\frac{5}{2}}e^2} + \frac{4fx}{15(-e^2x^2+d^2)^{\frac{3}{2}}d} + \frac{gx}{10(-e^2x^2+d^2)^{\frac{3}{2}}e} \\ & + \frac{8fx}{15\sqrt{-e^2x^2+d^2}d^3} + \frac{gx}{5\sqrt{-e^2x^2+d^2}d^2e} + \frac{(e^3f+3de^2g)x^2}{3(-e^2x^2+d^2)^{\frac{5}{2}}e^2} + \frac{3(de^2f+d^2eg)x}{5(-e^2x^2+d^2)^{\frac{5}{2}}e^2} \\ & - \frac{2(e^3f+3de^2g)d^2}{15(-e^2x^2+d^2)^{\frac{5}{2}}e^4} - \frac{(de^2f+d^2eg)x}{5(-e^2x^2+d^2)^{\frac{3}{2}}d^2e^2} - \frac{2(de^2f+d^2eg)x}{5\sqrt{-e^2x^2+d^2}d^4e^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(g\*x+f)/(-e^2\*x^2+d^2)^(7/2),x, algorithm="maxima")

[Out] 1/2\*e\*g\*x^3/(-e^2\*x^2+d^2)^(5/2) + 1/5\*d\*f\*x/(-e^2\*x^2+d^2)^(5/2) - 3/10\*d^2\*g\*x/((-e^2\*x^2+d^2)^(5/2)\*e) + 3/5\*d^2\*f/((-e^2\*x^2+d^2)^(5/2)\*e) + 1/5\*d^3\*g/((-e^2\*x^2+d^2)^(5/2)\*e^2) + 4/15\*f\*x/((-e^2\*x^2+d^2)^(3/2)\*d) + 1/10\*g\*x/((-e^2\*x^2+d^2)^(3/2)\*e) + 8/15\*f\*x/(sqrt(-e^2\*x^2+d^2)\*d^3) + 1/5\*g\*x/(sqrt(-e^2\*x^2+d^2)\*d^2\*e) + 1/3\*(e^3\*f+3\*d\*e^2\*g)\*x^2/((-e^2\*x^2+d^2)^(5/2)\*e^2) + 3/5\*(d\*e^2\*f+d^2\*e\*g)\*x/((-e^2\*x^2+d^2)^(5/2)\*e^2) - 2/15\*(e^3\*f+3\*d\*e^2\*g)\*d^2/((-e^2\*x^2+d^2)^(5/2)\*e^4) - 1/5\*(d\*e^2\*f+d^2\*e\*g)\*x/((-e^2\*x^2+d^2)^(3/2)\*d^2\*e^2) - 2/5\*(d\*e^2\*f+d^2\*e\*g)\*x/(sqrt(-e^2\*x^2+d^2)\*d^4\*e^2)

**Fricas [A]** time = 0.283317, size = 363, normalized size = 3.1

$$\frac{60d^4fx-3(3e^4f-2de^3g)x^5+5(7de^3f-3d^2e^2g)x^4-5(4d^2e^2f+3d^3eg)x^3-30(2d^3ef-d^4g)x^2-5(e^3fx^4+12d^3efx-3d^4g)}{15(d^3e^5x^5-5d^4e^4x^4+5d^5e^3x^3+5d^6e^2x^2-10d^7ex+4d^8+(d^3e^4x^4-7d^5e^2x^2+12d^6ex-3d^7e^2x^2+4d^8))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^3\*(g\*x+f)/(-e^2\*x^2+d^2)^(7/2),x, algorithm="fricas")

[Out] -1/15\*(60\*d^4\*f\*x-3\*(3\*e^4\*f-2\*d\*e^3\*g)\*x^5+5\*(7\*d\*e^3\*f-3\*d^2\*e^2\*g)\*x^4-5\*(4\*d^2\*e^2\*f+3\*d^3\*e\*g)\*x^3-30\*(2\*d^3\*e\*f-d^4\*g)\*x^2-5\*(e^3\*f\*x^4+12\*d^3\*e\*f\*x-3\*d^4\*g))/d^3/e^2/(-e^2\*x^2+d^2)^(7/2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^3 (f + gx)}{(-(-d + ex)(d + ex))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(g\*x+f)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2),x)

[Out] Integral((d + e\*x)\*\*3\*(f + g\*x)/(-(-d + e\*x)\*(d + e\*x))\*\*(7/2), x)

**GIAC/XCAS [A]** time = 0.293895, size = 188, normalized size = 1.61

$$\frac{\sqrt{-x^2e^2 + d^2} \left( \left( 15df - \left( x \left( \frac{3d^2ge^7 - 2dfe^8}{d^4} x^2e^{(-4)} - \frac{5(3d^4ge^5 - d^3fe^6)e^{(-4)}}{d^4} - \frac{5(3d^5ge^4 + d^4fe^5)e^{(-4)}}{d^4} \right) x \right) x - \frac{(3d^7ge^2 - 7d^6fe^3)e^{(-4)}}{d^4} \right)}{15(x^2e^2 - d^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*(g\*x + f)/(-e^2\*x^2 + d^2)^(7/2),x, algorithm="giac")

[Out] -1/15\*sqrt(-x^2\*e^2 + d^2)\*((15\*d\*f - (x\*((3\*d^2\*g\*e^7 - 2\*d\*f\*e^8)\*x^2\*e^(-4)/d^4 - 5\*(3\*d^4\*g\*e^5 - d^3\*f\*e^6)\*e^(-4)/d^4) - 5\*(3\*d^5\*g\*e^4 + d^4\*f\*e^5)\*e^(-4)/d^4)\*x)\*x - (3\*d^7\*g\*e^2 - 7\*d^6\*f\*e^3)\*e^(-4)/d^4)/(x^2\*e^2 - d^2)^3

$$3.584 \quad \int \frac{(d+ex)^3}{(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=103

$$\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2} + \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}$$

[Out] Sqrt[d^2 - e^2\*x^2]/(5\*d\*e\*(d - e\*x)^3) + (2\*Sqrt[d^2 - e^2\*x^2])/(15\*d^2\*e\*(d - e\*x)^2) + (2\*Sqrt[d^2 - e^2\*x^2])/(15\*d^3\*e\*(d - e\*x))

**Rubi [A]** time = 0.136913, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2} + \frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^3/(d^2 - e^2\*x^2)^(7/2), x]

[Out] Sqrt[d^2 - e^2\*x^2]/(5\*d\*e\*(d - e\*x)^3) + (2\*Sqrt[d^2 - e^2\*x^2])/(15\*d^2\*e\*(d - e\*x)^2) + (2\*Sqrt[d^2 - e^2\*x^2])/(15\*d^3\*e\*(d - e\*x))

**Rubi in Sympy [A]** time = 21.2142, size = 80, normalized size = 0.78

$$\frac{\sqrt{d^2-e^2x^2}}{5de(d-ex)^3} + \frac{2\sqrt{d^2-e^2x^2}}{15d^2e(d-ex)^2} + \frac{2\sqrt{d^2-e^2x^2}}{15d^3e(d-ex)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out] sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(5\*d\*e\*(d - e\*x)\*\*3) + 2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*d\*\*2\*e\*(d - e\*x)\*\*2) + 2\*sqrt(d\*\*2 - e\*\*2\*x\*\*2)/(15\*d\*\*3\*e\*(d - e\*x))

**Mathematica [A]** time = 0.0425405, size = 53, normalized size = 0.51

$$\frac{\sqrt{d^2-e^2x^2}(7d^2-6dex+2e^2x^2)}{15d^3e(d-ex)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^3/(d^2 - e^2\*x^2)^(7/2), x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*(7\*d^2 - 6\*d\*e\*x + 2\*e^2\*x^2))/(15\*d^3\*e\*(d - e\*x)^3)

**Maple [A]** time = 0., size = 55, normalized size = 0.5

$$\frac{(-ex + d)(ex + d)^4 (2e^2x^2 - 6dex + 7d^2)}{15d^3e} (-e^2x^2 + d^2)^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^3/(-e^2*x^2+d^2)^(7/2), x)`

[Out] `1/15*(-e*x+d)*(e*x+d)^4*(2*e^2*x^2-6*d*e*x+7*d^2)/d^3/e/(-e^2*x^2+d^2)^(7/2)`

**Maxima [A]** time = 0.694023, size = 136, normalized size = 1.32

$$\frac{ex^2}{3(-e^2x^2 + d^2)^{\frac{5}{2}}} + \frac{4dx}{5(-e^2x^2 + d^2)^{\frac{5}{2}}} + \frac{7d^2}{15(-e^2x^2 + d^2)^{\frac{5}{2}}e} + \frac{x}{15(-e^2x^2 + d^2)^{\frac{3}{2}}d} + \frac{2x}{15\sqrt{-e^2x^2 + d^2}d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^3/(-e^2*x^2 + d^2)^(7/2), x, algorithm="maxima")`

[Out] `1/3*e*x^2/(-e^2*x^2 + d^2)^(5/2) + 4/5*d*x/(-e^2*x^2 + d^2)^(5/2) + 7/15*d^2/((-e^2*x^2 + d^2)^(5/2)*e) + 1/15*x/((-e^2*x^2 + d^2)^(3/2)*d) + 2/15*x/(sqrt(-e^2*x^2 + d^2)*d^3)`

**Fricas [A]** time = 0.28108, size = 271, normalized size = 2.63

$$\frac{9e^4x^5 - 35de^3x^4 + 20d^2e^2x^3 + 60d^3ex^2 - 60d^4x + 5(e^3x^4 + 2de^2x^3 - 12d^2ex^2 + 12d^3x)\sqrt{-e^2x^2 + d^2}}{15(d^3e^5x^5 - 5d^4e^4x^4 + 5d^5e^3x^3 + 5d^6e^2x^2 - 10d^7ex + 4d^8 + (d^3e^4x^4 - 7d^5e^2x^2 + 10d^6ex - 4d^7)\sqrt{-e^2x^2 + d^2})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^3/(-e^2*x^2 + d^2)^(7/2), x, algorithm="fricas")`

[Out] `1/15*(9*e^4*x^5 - 35*d*e^3*x^4 + 20*d^2*e^2*x^3 + 60*d^3*e*x^2 - 60*d^4*x + 5*(e^3*x^4 + 2*d*e^2*x^3 - 12*d^2*e*x^2 + 12*d^3*x)*sqrt(-e^2*x^2 + d^2))/(d^3*e^5*x^5 - 5*d^4*e^4*x^4 + 5*d^5*e^3*x^3 + 5*d^6*e^2*x^2 - 10*d^7*e*x + 4*d^8 + (d^3*e^4*x^4 - 7*d^5*e^2*x^2 + 10*d^6*ex - 4*d^7)*sqrt(-e^2*x^2 + d^2))`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^3}{(-(-d + ex)(d + ex))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**3/(-e**2*x**2+d**2)**(7/2), x)`

[Out] `Integral((d + e*x)**3/(-(-d + e*x)*(d + e*x))**(7/2), x)`

**GIAC/XCAS [A]** time = 0.298221, size = 95, normalized size = 0.92

$$\frac{\sqrt{-x^2e^2 + d^2} \left( 7d^2e^{(-1)} + \left( x \left( \frac{2x^2e^4}{d^3} - \frac{5e^2}{d} \right) + 5e \right) x + 15d \right) x}{15(x^2e^2 - d^2)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3/(-e^2\*x^2 + d^2)^(7/2),x, algorithm="giac")

[Out] -1/15\*sqrt(-x^2\*e^2 + d^2)\*(7\*d^2\*e^(-1) + ((x\*(2\*x^2\*e^4/d^3 - 5\*e^2/d) + 5\*e)\*x + 15\*d)\*x)/(x^2\*e^2 - d^2)^3

$$3.585 \quad \int \frac{(d+ex)^3}{(f+gx)(d^2-e^2x^2)^{7/2}} dx$$

Optimal. Leaf size=242

$$\frac{g^3 \tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{(dg+ef)^3\sqrt{e^2f^2-d^2g^2}} - \frac{5d(ef-dg)-ex(11dg+ef)}{15d(d^2-e^2x^2)^{3/2}(dg+ef)^2} + \frac{4d(d+ex)}{5(d^2-e^2x^2)^{5/2}(dg+ef)} + \frac{15d^3g^2+ex(22d^2g^2+9defg+2e^2f^2)}{15d^3\sqrt{d^2-e^2x^2}(dg+ef)^3}$$

[Out]  $(4*d*(d+e*x))/(5*(e*f+d*g)*(d^2-e^2*x^2)^(5/2)) - (5*d*(e*f-d*g) - e*(e*f+11*d*g)*x)/(15*d*(e*f+d*g)^2*(d^2-e^2*x^2)^(3/2)) + (15*d^3*g^2 + e*(2*e^2*f^2 + 9*d*e*f*g + 22*d^2*g^2)*x)/(15*d^3*(e*f+d*g)^3*\text{Sqrt}[d^2 - e^2*x^2]) + (g^3*\text{ArcTan}[(d^2*g + e^2*f*x)/(\text{Sqrt}[e^2*f^2 - d^2*g^2]*\text{Sqrt}[d^2 - e^2*x^2])])/(e*f + d*g)^3*\text{Sqrt}[e^2*f^2 - d^2*g^2]$

Rubi [A] time = 1.1902, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$

$$\frac{g^3 \tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{(dg+ef)^3\sqrt{e^2f^2-d^2g^2}} - \frac{5d(ef-dg)-ex(11dg+ef)}{15d(d^2-e^2x^2)^{3/2}(dg+ef)^2} + \frac{4d(d+ex)}{5(d^2-e^2x^2)^{5/2}(dg+ef)} + \frac{15d^3g^2+ex(22d^2g^2+9defg+2e^2f^2)}{15d^3\sqrt{d^2-e^2x^2}(dg+ef)^3}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^3/((f + g\*x)\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out]  $(4*d*(d+e*x))/(5*(e*f+d*g)*(d^2-e^2*x^2)^(5/2)) - (5*d*(e*f-d*g) - e*(e*f+11*d*g)*x)/(15*d*(e*f+d*g)^2*(d^2-e^2*x^2)^(3/2)) + (15*d^3*g^2 + e*(2*e^2*f^2 + 9*d*e*f*g + 22*d^2*g^2)*x)/(15*d^3*(e*f+d*g)^3*\text{Sqrt}[d^2 - e^2*x^2]) + (g^3*\text{ArcTan}[(d^2*g + e^2*f*x)/(\text{Sqrt}[e^2*f^2 - d^2*g^2]*\text{Sqrt}[d^2 - e^2*x^2])])/(e*f + d*g)^3*\text{Sqrt}[e^2*f^2 - d^2*g^2]$

Rubi in Sympy [A] time = 82.6724, size = 267, normalized size = 1.1

$$-\frac{g^3 \operatorname{atanh}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{dg-ef}\sqrt{dg+ef}}\right)}{\sqrt{dg-ef}(dg+ef)^{7/2}} + \frac{g^2\sqrt{d^2-e^2x^2}}{d(d-ex)(dg+ef)^3} + \frac{g\sqrt{d^2-e^2x^2}}{3d(d-ex)^2(dg+ef)^2} + \frac{\sqrt{d^2-e^2x^2}}{5d(d-ex)^3(dg+ef)} + \frac{g\sqrt{d^2-e^2x^2}}{3d^2(d-ex)(dg+ef)^2} + \frac{2\sqrt{d^2-e^2x^2}}{15d^2(d-ex)^2(dg+ef)} + \frac{2\sqrt{d^2-e^2x^2}}{15d^3(d-ex)(dg+ef)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3/(g\*x+f)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out]  $-g**3*\operatorname{atanh}((d**2*g + e**2*f*x)/(\text{sqrt}(d**2 - e**2*x**2)*\text{sqrt}(d*g - e*f)*\text{sqrt}(d*g + e*f)))/(\text{sqrt}(d*g - e*f)*(d*g + e*f)**(7/2)) + g**2*\text{sqrt}(d**2 - e**2*x**2)/(d*(d - e*x)*(d*g + e*f)**3) + g*\text{sqrt}(d**2 - e**2*x**2)/(3*d*(d - e*x)**2*(d*g + e*f)**2) + \text{sqrt}(d**2 - e**2*x**2)/(5*d*(d - e*x)**3*(d*g + e*f)) + g*\text{sqrt}(d**2 - e**2*x**2)/(3*d**2*(d - e*x)*(d*g + e*f)**2) + 2*\text{sqrt}(d**2 - e**2*x**2)$

$$\frac{1}{(15d^2(d - ex)^2(dg + ef) + 2\sqrt{d^2 - e^2x^2})} + \frac{2\sqrt{d^2 - e^2x^2}}{15d^3(d - ex)(dg + ef)}$$

**Mathematica [A]** time = 0.355943, size = 219, normalized size = 0.9

$$\frac{15g^3 \log\left(\frac{\sqrt{d^2 - e^2x^2}\sqrt{d^2g^2 - e^2f^2 + d^2g + e^2fx}}{\sqrt{d^2g^2 - e^2f^2}}\right) + \frac{15g^3 \log(f+gx)}{\sqrt{d^2g^2 - e^2f^2}} + \frac{\sqrt{d^2 - e^2x^2}((d-ex)^2(22d^2g^2 + 9defg + 2e^2f^2) + 3d^2(dg+ef)^2 + d(d-ex)(7dg+2ef)(dg+ef))}{d^3(d-ex)^3}}{15(dg + ef)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^3/((f + g\*x)\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out] ((Sqrt[d^2 - e^2\*x^2])\*(3\*d^2\*(e\*f + d\*g)^2 + d\*(e\*f + d\*g)\*(2\*e\*f + 7\*d\*g)\*(d - e\*x) + (2\*e^2\*f^2 + 9\*d\*e\*f\*g + 22\*d^2\*g^2)\*(d - e\*x)^2))/((d^3\*(d - e\*x)^3) + (15\*g^3\*Log[f + g\*x])/Sqrt[-(e^2\*f^2) + d^2\*g^2] - (15\*g^3\*Log[d^2\*g + e^2\*f\*x + Sqrt[-(e^2\*f^2) + d^2\*g^2])\*Sqrt[d^2 - e^2\*x^2]])/Sqrt[-(e^2\*f^2) + d^2\*g^2]/(15\*(e\*f + d\*g)^3)

**Maple [B]** time = 0.048, size = 3961, normalized size = 16.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3/(g\*x+f)/(-e^2\*x^2+d^2)^(7/2), x)

[Out]  $\frac{1}{5} \frac{e^3}{g^3} \frac{f^2 x}{d^2} / (-e^2 x^2 + d^2)^{5/2} + \frac{4}{15} \frac{e^3}{g^3} \frac{f^2}{d^4} \frac{x}{(-e^2 x^2 + d^2)^{3/2}} + \frac{8}{15} \frac{e^3}{g^3} \frac{f^2}{d^6} \frac{x}{(-e^2 x^2 + d^2)^{1/2}} - \frac{3}{5} \frac{e^2}{g^2} \frac{d^2 f x}{(-e^2 x^2 + d^2)^{5/2}} - \frac{4}{5} \frac{e^2}{g^2} \frac{d^3 f x}{(-e^2 x^2 + d^2)^{3/2}} - \frac{8}{5} \frac{e^2}{g^2} \frac{d^5 f x}{(-e^2 x^2 + d^2)^{1/2}} + \frac{g}{(d^2 g^2 - e^2 f^2)^2} / (-\frac{x+f}{g})^2 \frac{e^2 + 2e^2 f/g * (x+f/g) + (d^2 g^2 - e^2 f^2)}{g^2} / (d^2 g^2 - e^2 f^2)^{3/2} * d^2 \frac{e^2 f^2 - 3g^4}{(d^2 g^2 - e^2 f^2)^3} / (-\frac{x+f}{g})^2 \frac{e^2 + 2e^2 f/g * (x+f/g) + (d^2 g^2 - e^2 f^2)}{g^2} / (d^2 g^2 - e^2 f^2)^{1/2} * d^2 \frac{e^2 f + 3g^3}{(d^2 g^2 - e^2 f^2)^3} / (-\frac{x+f}{g})^2 \frac{e^2 + 2e^2 f/g * (x+f/g) + (d^2 g^2 - e^2 f^2)}{g^2} / (d^2 g^2 - e^2 f^2)^{1/2} * d^2 \frac{e^2 f^2 + g^2}{(d^2 g^2 - e^2 f^2)^3} / ((\frac{d^2 g^2 - e^2 f^2}{g^2})^{1/2} * \ln((2 * (\frac{d^2 g^2 - e^2 f^2}{g^2}) / (d^2 g^2 - e^2 f^2) + 2 * ((\frac{d^2 g^2 - e^2 f^2}{g^2})^{1/2} * (-\frac{x+f}{g})^2 \frac{e^2 + 2e^2 f/g * (x+f/g) + (d^2 g^2 - e^2 f^2)}{g^2})^{1/2}) / (x+f/g)) * e^3 f^3 + 3/5 g^3 e^3 f^2 / (d^2 g^2 - e^2 f^2) / (-\frac{x+f}{g})^2 \frac{e^2 + 2e^2 f/g * (x+f/g) + (d^2 g^2 - e^2 f^2)}{g^2} / (d^2 g^2 - e^2 f^2)^{5/2} * x + 3g^3 / (d^2 g^2 - e^2 f^2)^3 f^2 / (-\frac{x+f}{g})^2 \frac{e^2 + 2e^2 f/g * (x+f/g) + (d^2 g^2 - e^2 f^2)}{g^2} / (d^2 g^2 - e^2 f^2)^{1/2} * e^3 x + g / (d^2 g^2 - e^2 f^2)^2 * e^3 f^2 / (-\frac{x+f}{g})^2 \frac{e^2 + 2e^2 f/g * (x+f/g) + (d^2 g^2 - e^2 f^2)}{g^2} / (d^2 g^2 - e^2 f^2)^{3/2} * x - 1/5 \frac{e^2 f}{(d^2 g^2 - e^2 f^2)} * d / (-\frac{x+f}{g})^2 \frac{e^2 + 2e^2 f/g * (x+f/g) + (d^2 g^2 - e^2 f^2)}{g^2} / (d^2 g^2 - e^2 f^2)^{5/2} * x - 4/15 \frac{e^2 f}{(d^2 g^2 - e^2 f^2)} / d / (-\frac{x+f}{g})^2 \frac{e^2 + 2e^2 f/g * (x+f/g) + (d^2 g^2 - e^2 f^2)}{g^2} / (d^2 g^2 - e^2 f^2)^{3/2} * x - 8/15 \frac{e^2 f}{(d^2 g^2 - e^2 f^2)} / d^3 / (-\frac{x+f}{g})^2 \frac{e^2 + 2e^2 f/g * (x+f/g) + (d^2 g^2 - e^2 f^2)}{g^2} / (d^2 g^2 - e^2 f^2)^{1/2} * x - 1 / (d^2 g^2 - e^2 f^2)^2 * e^4 f^3 / d / (-\frac{x+f}{g})^2 \frac{e^2 + 2e^2 f/g * (x+f/g) + (d^2 g^2 - e^2 f^2)}{g^2} / (d^2 g^2 - e^2 f^2)^{3/2} * x - 2 / (d^2 g^2 - e^2 f^2)^2 * e^4 f^3 / d^3 / (-\frac{x+f}{g})^2 \frac{e^2 + 2e^2 f/g * (x+f/g) + (d^2 g^2 - e^2 f^2)}{g^2} / (d^2 g^2 - e^2 f^2)^{1/2} * x + 3/5 g / (d^2 g^2 - e^2 f^2) / (-\frac{x+f}{g})^2 \frac{e^2 + 2e^2 f/g * (x+f/g) + (d^2 g^2 - e^2 f^2)}{g^2} / (d^2 g^2 - e^2 f^2)^{5/2} * d^2 \frac{e^2 f^2 - g^2}{(d^2 g^2 - e^2 f^2)^2} / (-\frac{x+f}{g})^2 \frac{e^2 + 2e^2 f/g * (x+f/g) + (d^2 g^2 - e^2 f^2)}{g^2} / (d^2 g^2 - e^2 f^2)^{3/2} * d^2 \frac{e^2 f - 1/5 e/g^2}{(-e^2 x^2 + d^2)^{5/2}} * f + 4/5 \frac{e}{g} \frac{x}{(-e^2 x^2 + d^2)^{5/2}} + 1/5 \frac{g}{(d^2 g^2 - e^2 f^2)} / (-\frac{x+f}{g})^2 \frac{e^2 + 2e^2 f/g * (x+f/g) + (d^2 g^2 - e^2 f^2)}{g^2} / (d^2 g^2 - e^2 f^2)^{5/2} * d^3 + 1/3 \frac{g^3}{(d^2 g^2 - e^2 f^2)^2} / (-\frac{x+f}{g})^2 \frac{e^2 + 2e^2 f/g * (x+f/g) + (d^2 g^2 - e^2 f^2)}{g^2} / (d^2 g^2 - e^2 f^2)^{3/2} * d^3 + g^5 / (d^2 g^2 - e^2 f^2)^3 / (-\frac{x+f}{g})^2 \frac{e^2 + 2e^2 f/g * (x+f/g) + (d^2 g^2 - e^2 f^2)}{g^2} / (d^2 g^2 - e^2 f^2)^2 * e^2 f / g * (x+f/g) + (d^2 g^2 - e^2 f^2) / g^2 / (d^2 g^2 - e^2 f^2)^{1/2} * d^3 - 1/3 / (d^2 g^2 - e^2 f^2)^2 / (-\frac{x+f}{g})^2 \frac{e^2 + 2e^2 f/g * (x+f/g) + (d^2 g^2 - e^2 f^2)}{g^2} / (d^2 g^2 - e^2 f^2)^{3/2} * e^3 f^3 + 3/5 g / (-e^2 x^2 + d^2)^{5/2} * d - 3/5 g^2 * e^4 f^3 / (d^2 g^2 - e^2 f^2)^2$



$$\frac{g^2 - e^2 f^2}{d} \frac{1}{(-x+f/g)^2 e^{2+2} e^{2f/g} (x+f/g) + (d^2 g^2 - e^2 f^2) / g^2} \frac{1}{g^2} \frac{1}{x+1/5/g^3 e^5 f^4} \frac{1}{(d^2 g^2 - e^2 f^2) / d^2} \frac{1}{(-x+f/g)^2 e^{2+2} e^{2f/g} (x+f/g) + (d^2 g^2 - e^2 f^2) / g^2} \frac{1}{g^2} \frac{1}{x+4/5/g e^3 f^2} \frac{1}{(d^2 g^2 - e^2 f^2) / d^2} \frac{1}{(-x+f/g)^2 e^{2+2} e^{2f/g} (x+f/g) + (d^2 g^2 - e^2 f^2) / g^2} \frac{1}{g^2} \frac{1}{x-4/5/g^2 e^4 f^3} \frac{1}{(d^2 g^2 - e^2 f^2) / d^3} \frac{1}{(-x+f/g)^2 e^{2+2} e^{2f/g} (x+f/g) + (d^2 g^2 - e^2 f^2) / g^2} \frac{1}{g^2} \frac{1}{x-g^4} \frac{1}{(d^2 g^2 - e^2 f^2)^3 f^4} \frac{1}{(-x+f/g)^2 e^{2+2} e^{2f/g} (x+f/g) + (d^2 g^2 - e^2 f^2) / g^2} \frac{1}{g^2} \frac{1}{x+2} \frac{1}{(d^2 g^2 - e^2 f^2)^3 f^4 / d^2} \frac{1}{(-x+f/g)^2 e^{2+2} e^{2f/g} (x+f/g) + (d^2 g^2 - e^2 f^2) / g^2} \frac{1}{g^2} \frac{1}{x+4/15/g^3 e^5 f^4} \frac{1}{(d^2 g^2 - e^2 f^2) / d^4} \frac{1}{(-x+f/g)^2 e^{2+2} e^{2f/g} (x+f/g) + (d^2 g^2 - e^2 f^2) / g^2} \frac{1}{g^2} \frac{1}{x+1/3/g} \frac{1}{(d^2 g^2 - e^2 f^2)^2 e^5 f^4 / d^2} \frac{1}{(-x+f/g)^2 e^{2+2} e^{2f/g} (x+f/g) + (d^2 g^2 - e^2 f^2) / g^2} \frac{1}{g^2} \frac{1}{x-2/3/g^2} \frac{1}{(d^2 g^2 - e^2 f^2)^2 e^{2f/d} (-x+f/g)^2 e^{2+2} e^{2f/g} (x+f/g) + (d^2 g^2 - e^2 f^2) / g^2} \frac{1}{g^2} \frac{1}{x+2} \frac{1}{(d^2 g^2 - e^2 f^2)^2 e^3 f^2 / d^2} \frac{1}{(-x+f/g)^2 e^{2+2} e^{2f/g} (x+f/g) + (d^2 g^2 - e^2 f^2) / g^2} \frac{1}{g^2} \frac{1}{x+8/5/g e^3 f^2} \frac{1}{(d^2 g^2 - e^2 f^2) / d^4} \frac{1}{(-x+f/g)^2 e^{2+2} e^{2f/g} (x+f/g) + (d^2 g^2 - e^2 f^2) / g^2} \frac{1}{g^2} \frac{1}{x-8/5/g^2 e^4 f^3} \frac{1}{(d^2 g^2 - e^2 f^2) / d^5} \frac{1}{(-x+f/g)^2 e^{2+2} e^{2f/g} (x+f/g) + (d^2 g^2 - e^2 f^2) / g^2} \frac{1}{g^2} \frac{1}{x-1/3/g^2} \frac{1}{(d^2 g^2 - e^2 f^2)^2 e^{2f/d} (-x+f/g)^2 e^{2+2} e^{2f/g} (x+f/g) + (d^2 g^2 - e^2 f^2) / g^2} \frac{1}{g^2} \frac{1}{x+2/3/g} \frac{1}{(d^2 g^2 - e^2 f^2)^2 e^5 f^4 / d^4} \frac{1}{(-x+f/g)^2 e^{2+2} e^{2f/g} (x+f/g) + (d^2 g^2 - e^2 f^2) / g^2} \frac{1}{g^2} \frac{1}{x-3/g^2} \frac{1}{(d^2 g^2 - e^2 f^2)^3 f^3 / d} \frac{1}{(-x+f/g)^2 e^{2+2} e^{2f/g} (x+f/g) + (d^2 g^2 - e^2 f^2) / g^2} \frac{1}{g^2} \frac{1}{x+3} \frac{1}{(d^2 g^2 - e^2 f^2)^3} \frac{1}{((d^2 g^2 - e^2 f^2) / g^2)^{1/2} \ln((2 * (d^2 g^2 - e^2 f^2) / g^2 + 2 * e^{2f/g} (x+f/g) + 2 * ((d^2 g^2 - e^2 f^2) / g^2)^{1/2} * (-x+f/g)^2 e^{2+2} e^{2f/g} (x+f/g) + (d^2 g^2 - e^2 f^2) / g^2)^{1/2}) / (x+f/g)) * d^2 * e^f - 3 * g^3 / (d^2 g^2 - e^2 f^2)^3} \frac{1}{((d^2 g^2 - e^2 f^2) / g^2)^{1/2} \ln((2 * (d^2 g^2 - e^2 f^2) / g^2 + 2 * e^{2f/g} (x+f/g) + 2 * ((d^2 g^2 - e^2 f^2) / g^2)^{1/2} * (-x+f/g)^2 e^{2+2} e^{2f/g} (x+f/g) + (d^2 g^2 - e^2 f^2) / g^2)^{1/2}) / (x+f/g)) * d * e^{2f^2 - 3/5} / (d^2 g^2 - e^2 f^2)} \frac{1}{(-x+f/g)^2 e^{2+2} e^{2f/g} (x+f/g) + (d^2 g^2 - e^2 f^2) / g^2} \frac{1}{g^2} \frac{1}{x+1/5/g^3 e^5 f^4} \frac{1}{(d^2 g^2 - e^2 f^2)^3} \frac{1}{((d^2 g^2 - e^2 f^2) / g^2)^{1/2} \ln((2 * (d^2 g^2 - e^2 f^2) / g^2 + 2 * e^{2f/g} (x+f/g) + 2 * ((d^2 g^2 - e^2 f^2) / g^2)^{1/2} * (-x+f/g)^2 e^{2+2} e^{2f/g} (x+f/g) + (d^2 g^2 - e^2 f^2) / g^2)^{1/2}) / (x+f/g)) * d^3 - 1/5/g^2} \frac{1}{(d^2 g^2 - e^2 f^2)} \frac{1}{(-x+f/g)^2 e^{2+2} e^{2f/g} (x+f/g) + (d^2 g^2 - e^2 f^2) / g^2} \frac{1}{g^2} \frac{1}{x+22/15/g^2 e^4 f^3} \frac{1}{(d^2 g^2 - e^2 f^2)^{5/2}} \frac{1}{e^3 f^3 + 22/15 * e/g / d^4 * x / (-e^2 x^2 + d^2)^{1/2} + 11/15 * e/g / d^2 * x / (-e^2 x^2 + d^2)^{3/2}}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3/((-e^2\*x^2 + d^2)^(7/2)\*(g\*x + f)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.329276, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3/((-e^2\*x^2 + d^2)^(7/2)\*(g\*x + f)),x, algorithm="fricas")

[Out] [1/15\*(15\*(d^3\*e^5\*g^3\*x^5 - 5\*d^4\*e^4\*g^3\*x^4 + 5\*d^5\*e^3\*g^3\*x^3 + 5\*d^6\*e^2\*g^3\*x^2 - 10\*d^7\*e\*g^3\*x + 4\*d^8\*g^3 + (d^3\*e^4\*g^3\*x^4 - 7\*d^5\*e^2\*g^3\*x^2 + 10\*d^6\*e\*g^3\*x - 4\*d^7\*g^3)\*sqrt(-e^2\*

$$\begin{aligned}
& x^2 + d^2)) * \log(((d^2 e^2 f^2 g - d^3 g^3) x^2 - (e^2 f^3 - d^2 f g^2) \sqrt{-e^2 x^2 + d^2}) x + (d^2 e^2 f^3 - d^3 f g^2) x + (d^2 f g^2 x + d^2 f^2 - (e^2 f^2 - d^2 g^2) x^2 - (d f g x + d f^2) \sqrt{-e^2 x^2 + d^2})) \sqrt{-e^2 f^2 + d^2 g^2}) / (d g x + d f - \sqrt{-e^2 x^2 + d^2}) (g x + f)) + (3 (3 e^7 f^2 + 11 d e^6 f g + 18 d^2 e^5 g^2) x^5 - 5 (7 d e^6 f^2 + 24 d^2 e^5 f g + 29 d^3 e^4 g^2) x^4 + 5 (4 d^2 e^5 f^2 + 9 d^3 e^4 f g - d^4 e^3 g^2) x^3 + 30 (2 d^3 e^4 f^2 + 7 d^4 e^3 f g + 9 d^5 e^2 g^2) x^2 - 60 (d^4 e^3 f^2 + 3 d^5 e^2 f g + 3 d^6 e g^2) x + 5 ((e^6 f^2 + 3 d e^5 f g + 2 d^2 e^4 g^2) x^4 + (2 d e^5 f^2 + 9 d^2 e^4 f g + 19 d^3 e^3 g^2) x^3 - 6 (2 d^2 e^4 f^2 + 7 d^3 e^3 f g + 9 d^4 e^2 g^2) x^2 + 12 (d^3 e^3 f^2 + 3 d^4 e^2 f g + 3 d^5 e g^2) x) \sqrt{-e^2 x^2 + d^2}) \sqrt{-e^2 f^2 + d^2 g^2}) / ((4 d^8 e^3 f^3 + 12 d^9 e^2 f^2 g + 12 d^{10} e f g^2 + 4 d^{11} g^3 + (d^3 e^8 f^3 + 3 d^4 e^7 f^2 g + 3 d^5 e^6 f g^2 + d^6 e^5 g^3) x^5 - 5 (d^4 e^7 f^3 + 3 d^5 e^6 f^2 g + 3 d^6 e^5 f g^2 + d^7 e^4 g^3) x^4 + 5 (d^5 e^6 f^3 + 3 d^6 e^5 f^2 g + 3 d^7 e^4 f g^2 + d^8 e^3 g^3) x^3 + 5 (d^6 e^5 f^3 + 3 d^7 e^4 f^2 g + 3 d^8 e^3 f g^2 + d^9 e^2 g^3) x^2 - 10 (d^7 e^4 f^3 + 3 d^8 e^3 f^2 g + 3 d^9 e^2 f g^2 + d^{10} e g^3) x - (4 d^7 e^3 f^3 + 12 d^8 e^2 f^2 g + 12 d^9 e f g^2 + 4 d^{10} g^3 - (d^3 e^7 f^3 + 3 d^4 e^6 f^2 g + 3 d^5 e^5 f g^2 + d^6 e^4 g^3) x^4 + 7 (d^5 e^5 f^3 + 3 d^6 e^4 f^2 g + 3 d^7 e^3 f g^2 + d^8 e^2 g^3) x^2 - 10 (d^6 e^4 f^3 + 3 d^7 e^3 f^2 g + 3 d^8 e^2 f g^2 + d^9 e g^3) x) \sqrt{-e^2 x^2 + d^2}) \sqrt{-e^2 f^2 + d^2 g^2}), 1/15 (30 (d^3 e^5 g^3 x^5 - 5 d^4 e^4 g^3 x^4 + 5 d^5 e^3 g^3 x^3 + 5 d^6 e^2 g^3 x^2 - 10 d^7 e g^3 x + 4 d^8 g^3 + (d^3 e^4 g^3 x^4 - 7 d^5 e^2 g^3 x^2 + 10 d^6 e g^3 x - 4 d^7 g^3) \sqrt{-e^2 x^2 + d^2})) \arctan((d g x + d f - \sqrt{-e^2 x^2 + d^2}) f) / (\sqrt{(e^2 f^2 - d^2 g^2) x}) + (3 (3 e^7 f^2 + 11 d e^6 f g + 18 d^2 e^5 g^2) x^5 - 5 (7 d e^6 f^2 + 24 d^2 e^5 f g + 29 d^3 e^4 g^2) x^4 + 5 (4 d^2 e^5 f^2 + 9 d^3 e^4 f g - d^4 e^3 g^2) x^3 + 30 (2 d^3 e^4 f^2 + 7 d^4 e^3 f g + 9 d^5 e^2 g^2) x^2 - 60 (d^4 e^3 f^2 + 3 d^5 e^2 f g + 3 d^6 e g^2) x + 5 ((e^6 f^2 + 3 d e^5 f g + 2 d^2 e^4 g^2) x^4 + (2 d e^5 f^2 + 9 d^2 e^4 f g + 19 d^3 e^3 g^2) x^3 - 6 (2 d^2 e^4 f^2 + 7 d^3 e^3 f g + 9 d^4 e^2 g^2) x^2 + 12 (d^3 e^3 f^2 + 3 d^4 e^2 f g + 3 d^5 e g^2) x) \sqrt{-e^2 x^2 + d^2}) \sqrt{(e^2 f^2 - d^2 g^2)}) / ((4 d^8 e^3 f^3 + 12 d^9 e^2 f^2 g + 12 d^{10} e f g^2 + 4 d^{11} g^3 + (d^3 e^8 f^3 + 3 d^4 e^7 f^2 g + 3 d^5 e^6 f g^2 + d^6 e^5 g^3) x^5 - 5 (d^4 e^7 f^3 + 3 d^5 e^6 f^2 g + 3 d^6 e^5 f g^2 + d^7 e^4 g^3) x^4 + 5 (d^5 e^6 f^3 + 3 d^6 e^5 f^2 g + 3 d^7 e^4 f g^2 + d^8 e^3 g^3) x^3 + 5 (d^6 e^5 f^3 + 3 d^7 e^4 f^2 g + 3 d^8 e^3 f g^2 + d^9 e^2 g^3) x^2 - 10 (d^7 e^4 f^3 + 3 d^8 e^3 f^2 g + 3 d^9 e^2 f g^2 + d^{10} e g^3) x - (4 d^7 e^3 f^3 + 12 d^8 e^2 f^2 g + 12 d^9 e f g^2 + 4 d^{10} g^3 - (d^3 e^7 f^3 + 3 d^4 e^6 f^2 g + 3 d^5 e^5 f g^2 + d^6 e^4 g^3) x^4 + 7 (d^5 e^5 f^3 + 3 d^6 e^4 f^2 g + 3 d^7 e^3 f g^2 + d^8 e^2 g^3) x^2 - 10 (d^6 e^4 f^3 + 3 d^7 e^3 f^2 g + 3 d^8 e^2 f g^2 + d^9 e g^3) x) \sqrt{-e^2 x^2 + d^2}) \sqrt{(e^2 f^2 - d^2 g^2)})]
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3/(g\*x+f)/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2),x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.3068, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^3/((-e^2*x^2 + d^2)^(7/2)*(g*x + f)),x, algorithm="giac")
```

```
[Out] Done
```

$$3.586 \quad \int \frac{(d+ex)^3}{(f+gx)^2(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=311

$$\frac{eg^3(4ef-3dg)\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{(ef-dg)(dg+ef)^4\sqrt{e^2f^2-d^2g^2}} + \frac{g^4\sqrt{d^2-e^2x^2}}{(f+gx)(ef-dg)(dg+ef)^4}$$

$$- \frac{e(5d(ef-3dg)-ex(21dg+ef))}{15d(d^2-e^2x^2)^{3/2}(dg+ef)^3} + \frac{4de(d+ex)}{5(d^2-e^2x^2)^{5/2}(dg+ef)^2}$$

$$+ \frac{e(45d^3g^2+ex(57d^2g^2+14defg+2e^2f^2))}{15d^3\sqrt{d^2-e^2x^2}(dg+ef)^4}$$

[Out]  $(4*d*e*(d+e*x))/(5*(e*f+d*g)^2*(d^2-e^2*x^2)^(5/2)) - (e*(5*d*(e*f-3*d*g) - e*(e*f+21*d*g)*x))/(15*d*(e*f+d*g)^3*(d^2-e^2*x^2)^(3/2)) + (e*(45*d^3*g^2 + e*(2*e^2*f^2 + 14*d*e*f*g + 57*d^2*g^2)*x))/(15*d^3*(e*f+d*g)^4*sqrt[d^2-e^2*x^2]) + (g^4*sqrt[d^2-e^2*x^2])/((e*f-d*g)*(e*f+d*g)^4*(f+g*x)) + (e*g^3*(4*e*f-3*d*g)*ArcTan[(d^2*g+e^2*f*x)/(sqrt[e^2*f^2-d^2*g^2]*sqrt[d^2-e^2*x^2])])/((e*f-d*g)*(e*f+d*g)^4*sqrt[e^2*f^2-d^2*g^2])$

**Rubi [A]** time = 2.41898, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$

$$\frac{eg^3(4ef-3dg)\tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{(ef-dg)(dg+ef)^4\sqrt{e^2f^2-d^2g^2}} + \frac{g^4\sqrt{d^2-e^2x^2}}{(f+gx)(ef-dg)(dg+ef)^4}$$

$$- \frac{e(5d(ef-3dg)-ex(21dg+ef))}{15d(d^2-e^2x^2)^{3/2}(dg+ef)^3} + \frac{4de(d+ex)}{5(d^2-e^2x^2)^{5/2}(dg+ef)^2}$$

$$+ \frac{e(45d^3g^2+ex(57d^2g^2+14defg+2e^2f^2))}{15d^3\sqrt{d^2-e^2x^2}(dg+ef)^4}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^3/((f + g\*x)^2\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out]  $(4*d*e*(d+e*x))/(5*(e*f+d*g)^2*(d^2-e^2*x^2)^(5/2)) - (e*(5*d*(e*f-3*d*g) - e*(e*f+21*d*g)*x))/(15*d*(e*f+d*g)^3*(d^2-e^2*x^2)^(3/2)) + (e*(45*d^3*g^2 + e*(2*e^2*f^2 + 14*d*e*f*g + 57*d^2*g^2)*x))/(15*d^3*(e*f+d*g)^4*sqrt[d^2-e^2*x^2]) + (g^4*sqrt[d^2-e^2*x^2])/((e*f-d*g)*(e*f+d*g)^4*(f+g*x)) + (e*g^3*(4*e*f-3*d*g)*ArcTan[(d^2*g+e^2*f*x)/(sqrt[e^2*f^2-d^2*g^2]*sqrt[d^2-e^2*x^2])])/((e*f-d*g)*(e*f+d*g)^4*sqrt[e^2*f^2-d^2*g^2])$

**Rubi in Sympy [A]** time = 129.43, size = 405, normalized size = 1.3

$$\frac{e^2fg^3\operatorname{atanh}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{dg-ef}\sqrt{dg+ef}}\right)}{(dg-ef)^{\frac{3}{2}}(dg+ef)^{\frac{9}{2}}} - \frac{3eg^3\operatorname{atanh}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{dg-ef}\sqrt{dg+ef}}\right)}{\sqrt{dg-ef}(dg+ef)^{\frac{9}{2}}}$$

$$- \frac{g^4\sqrt{d^2-e^2x^2}}{(f+gx)(dg-ef)(dg+ef)^4} + \frac{3eg^2\sqrt{d^2-e^2x^2}}{d(d-ex)(dg+ef)^4}$$

$$+ \frac{2eg\sqrt{d^2-e^2x^2}}{3d(d-ex)^2(dg+ef)^3} + \frac{e\sqrt{d^2-e^2x^2}}{5d(d-ex)^3(dg+ef)^2} + \frac{2eg\sqrt{d^2-e^2x^2}}{3d^2(d-ex)(dg+ef)^3}$$

$$+ \frac{2e\sqrt{d^2-e^2x^2}}{15d^2(d-ex)^2(dg+ef)^2} + \frac{2e\sqrt{d^2-e^2x^2}}{15d^3(d-ex)(dg+ef)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**3/(g*x+f)**2/(-e**2*x**2+d**2)**(7/2),x)`

[Out] 
$$e^{2f}g^3 \operatorname{atanh}\left(\frac{d^2g + e^{2f}x}{\sqrt{d^2 - e^{2f}x^2}}\right) \sqrt{d^2g - e^f} \sqrt{d^2g + e^f} \left/ \left( (d^2g - e^f)^{3/2} (d^2g + e^f)^{9/2} \right) \right. - 3e^fg^3 \operatorname{atanh}\left(\frac{d^2g + e^{2f}x}{\sqrt{d^2 - e^{2f}x^2}}\right) \sqrt{d^2g - e^f} \sqrt{d^2g + e^f} \left/ \left( \sqrt{d^2g - e^f} (d^2g + e^f)^{9/2} \right) \right. - g^4 \sqrt{d^2 - e^{2f}x^2} \left/ \left( (f + gx)(d^2g - e^f)(d^2g + e^f)^4 \right) \right. + 3e^g \sqrt{d^2 - e^{2f}x^2} \left/ \left( d(d - ex)(d^2g + e^f)^4 \right) \right. + 2e^g \sqrt{d^2 - e^{2f}x^2} \left/ \left( 3d(d - ex)^2(d^2g + e^f)^3 \right) \right. + e \sqrt{d^2 - e^{2f}x^2} \left/ \left( 5d(d - ex)^3(d^2g + e^f)^2 \right) \right. + 2e^g \sqrt{d^2 - e^{2f}x^2} \left/ \left( 3d^2(d - ex)(d^2g + e^f)^3 \right) \right. + 2e \sqrt{d^2 - e^{2f}x^2} \left/ \left( 15d^2(d - ex)^2(d^2g + e^f)^2 \right) \right. + 2e \sqrt{d^2 - e^{2f}x^2} \left/ \left( 15d^3(d - ex)(d^2g + e^f)^2 \right) \right.$$

**Mathematica [C]** time = 1.00998, size = 308, normalized size = 0.99

$$\frac{\sqrt{d^2 - e^{2f}x^2} \left( \frac{2e(dg+ef)(6dg+ef)}{d^2(d-ex)^2} + \frac{e(57d^2g^2+14defg+2e^2f^2)}{d^3(d-ex)} + \frac{15g^4}{(f+gx)(ef-dg)} + \frac{3e(dg+ef)^2}{d(d-ex)^3} \right) - \frac{15ieg^3(4ef-3dg) \log\left(\frac{2(ef-dg)(dg+ef)^4 \sqrt{d^2 - e^{2f}x^2}}{eg^2(f+gx)(4ef-3dg)}\right)}{(ef-dg)\sqrt{e^2f^2 - d^2g^2}}}{15(dg + ef)^4}$$

Antiderivative was successfully verified.

[In] `Integrate[(d + e*x)^3/((f + g*x)^2*(d^2 - e^2*x^2)^(7/2)),x]`

[Out] 
$$\left( \sqrt{d^2 - e^2x^2} \left( \frac{3e^f(e^f + d^2g)^2}{d(d - ex)^3} + \frac{2e^f(e^f + d^2g)(e^f + 6d^2g)}{d^2(d - ex)^2} + \frac{e^f(2e^2f^2 + 14d^2e^fg + 57d^2g^2)}{d^3(d - ex)} + \frac{15g^4}{(e^f - d^2g)(f + gx)} \right) - \frac{(15I)e^fg^3(4e^f - 3d^2g) \operatorname{Log}\left[\frac{2(e^f - d^2g)(e^f + d^2g)^4(I^2d^2g + I^2e^2fx + \sqrt{e^2f^2 - d^2g^2}) \sqrt{d^2 - e^2x^2}}{(e^fg^2(4e^f - 3d^2g) \sqrt{e^2f^2 - d^2g^2})(f + gx)}\right]}{(e^f - d^2g) \sqrt{e^2f^2 - d^2g^2}} \right) \left/ \left( 15(e^f + d^2g)^4 \right) \right.$$

**Maple [B]** time = 0.038, size = 6760, normalized size = 21.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^3/(g*x+f)^2/(-e^2*x^2+d^2)^(7/2),x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^3/((-e^2*x^2 + d^2)^(7/2)*(g*x + f)^2),x, algorithm="maxima"`

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.471573, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3/((-e^2\*x^2 + d^2)^(7/2)\*(g\*x + f)^2),x, algorithm="fricas"

[Out] [1/15\*(15\*(32\*d^9\*e^2\*f^3\*g^3 - 24\*d^10\*e\*f^2\*g^4 + (4\*d^3\*e^8\*f^2\*g^2\*g^4 - 3\*d^4\*e^7\*f\*g^5)\*x^7 + (4\*d^3\*e^8\*f^3\*g^3 + d^4\*e^7\*f^2\*g^4 - 3\*d^5\*e^6\*f\*g^5)\*x^6 + (4\*d^4\*e^7\*f^3\*g^3 - 55\*d^5\*e^6\*f^2\*g^4 + 39\*d^6\*e^5\*f\*g^5)\*x^5 - (52\*d^5\*e^6\*f^3\*g^3 - 99\*d^6\*e^5\*f^2\*g^4 + 45\*d^7\*e^4\*f\*g^5)\*x^4 + (60\*d^6\*e^5\*f^3\*g^3 - 13\*d^7\*e^4\*f^2\*g^4 - 24\*d^8\*e^3\*f\*g^5)\*x^3 + 4\*(8\*d^7\*e^4\*f^3\*g^3 - 26\*d^8\*e^3\*f^2\*g^4 + 15\*d^9\*e^2\*f\*g^5)\*x^2 - 4\*(20\*d^8\*e^3\*f^3\*g^3 - 23\*d^9\*e^2\*f^2\*g^4 + 6\*d^10\*e\*f\*g^5)\*x - (32\*d^8\*e^2\*f^3\*g^3 - 24\*d^9\*e\*f^2\*g^4 + (4\*d^3\*e^7\*f^2\*g^4 - 3\*d^4\*e^6\*f\*g^5)\*x^6 + (4\*d^3\*e^7\*f^3\*g^3 - 27\*d^4\*e^6\*f^2\*g^4 + 18\*d^5\*e^5\*f\*g^5)\*x^5 - (24\*d^4\*e^6\*f^3\*g^3 - 38\*d^5\*e^5\*f^2\*g^4 + 15\*d^6\*e^4\*f\*g^5)\*x^4 + (20\*d^5\*e^5\*f^3\*g^3 + 33\*d^6\*e^4\*f^2\*g^4 - 36\*d^7\*e^3\*f\*g^5)\*x^3 + 4\*(12\*d^6\*e^4\*f^3\*g^3 - 29\*d^7\*e^3\*f^2\*g^4 + 15\*d^8\*e^2\*f\*g^5)\*x^2 - 4\*(20\*d^7\*e^3\*f^3\*g^3 - 23\*d^8\*e^2\*f^2\*g^4 + 6\*d^9\*e\*f\*g^5)\*x)\*sqrt(-e^2\*x^2 + d^2))\*log(((d\*e^2\*f^2\*g - d^3\*g^3)\*x^2 - (e^2\*f^3 - d^2\*f\*g^2)\*sqrt(-e^2\*x^2 + d^2)\*x + (d\*e^2\*f^3 - d^3\*f\*g^2)\*x + (d^2\*f\*g\*x + d^2\*f^2 - (e^2\*f^2 - d^2\*g^2)\*x^2 - (d\*f\*g\*x + d\*f^2)\*sqrt(-e^2\*x^2 + d^2))\*sqrt(-e^2\*f^2 + d^2\*g^2))/(d\*g\*x + d\*f - sqrt(-e^2\*x^2 + d^2)\*(g\*x + f))) + (5\*(e^10\*f^4\*g + 3\*d\*e^9\*f^3\*g^2 - d^2\*e^8\*f^2\*g^3 + 3\*d^4\*e^6\*g^5)\*x^7 + (5\*e^10\*f^5 + 34\*d\*e^9\*f^4\*g + 94\*d^2\*e^8\*f^3\*g^2 + 236\*d^3\*e^7\*f^2\*g^3 - 444\*d^4\*e^6\*f^2\*g^4 + 15\*d^5\*e^5\*g^5)\*x^6 + (19\*d\*e^9\*f^5 - d^2\*e^8\*f^4\*g - 139\*d^3\*e^7\*f^3\*g^2 - 809\*d^4\*e^6\*f^2\*g^3 + 1005\*d^5\*e^5\*f\*g^4 - 195\*d^6\*e^4\*g^5)\*x^5 - 5\*(20\*d^2\*e^8\*f^5 + 64\*d^3\*e^7\*f^4\*g + 58\*d^4\*e^6\*f^3\*g^2 - 154\*d^5\*e^5\*f^2\*g^3 - 3\*d^6\*e^4\*f\*g^4 - 45\*d^7\*e^3\*g^5)\*x^4 + 10\*(7\*d^3\*e^7\*f^5 + 27\*d^4\*e^6\*f^4\*g + 32\*d^5\*e^5\*f^3\*g^2 + 54\*d^6\*e^4\*f^2\*g^3 - 135\*d^7\*e^3\*f\*g^4 + 12\*d^8\*e^2\*g^5)\*x^3 + 60\*(2\*d^4\*e^6\*f^5 + 6\*d^5\*e^5\*f^4\*g + 4\*d^6\*e^4\*f^3\*g^2 - 24\*d^7\*e^3\*f^2\*g^3 + 13\*d^8\*e^2\*f\*g^4 - 5\*d^9\*e\*g^5)\*x^2 - 120\*(d^5\*e^5\*f^5 + 3\*d^6\*e^4\*f^4\*g + 2\*d^7\*e^3\*f^3\*g^2 - 6\*d^8\*e^2\*f^2\*g^3 - d^10\*g^5)\*x - (3\*(3\*e^9\*f^4\*g + 13\*d\*e^8\*f^3\*g^2 + 27\*d^2\*e^7\*f^2\*g^3 - 48\*d^3\*e^6\*f\*g^4 + 5\*d^4\*e^5\*g^5)\*x^6 + (9\*e^9\*f^5 - d\*e^8\*f^4\*g - 69\*d^2\*e^7\*f^3\*g^2 - 269\*d^3\*e^6\*f^2\*g^3 + 330\*d^4\*e^5\*f\*g^4 - 90\*d^5\*e^4\*g^5)\*x^5 - 5\*(8\*d\*e^8\*f^5 + 28\*d^2\*e^7\*f^4\*g + 34\*d^3\*e^6\*f^3\*g^2 - 10\*d^4\*e^5\*f^2\*g^3 - 81\*d^5\*e^4\*f\*g^4 - 15\*d^6\*e^3\*g^5)\*x^4 + 10\*(d^2\*e^7\*f^5 + 9\*d^3\*e^6\*f^4\*g + 20\*d^4\*e^5\*f^3\*g^2 + 90\*d^5\*e^4\*f^2\*g^3 - 135\*d^6\*e^3\*f\*g^4 + 18\*d^7\*e^2\*g^5)\*x^3 + 60\*(2\*d^3\*e^6\*f^5 + 6\*d^4\*e^5\*f^4\*g + 4\*d^5\*e^4\*f^3\*g^2 - 24\*d^6\*e^3\*f^2\*g^3 + 13\*d^7\*e^2\*f\*g^4 - 5\*d^8\*e\*g^5)\*x^2 - 120\*(d^4\*e^5\*f^5 + 3\*d^5\*e^4\*f^4\*g + 2\*d^6\*e^3\*f^3\*g^2 - 6\*d^7\*e^2\*f^2\*g^3 - d^9\*g^5)\*x)\*sqrt(-e^2\*x^2 + d^2))\*sqrt(-e^2\*f^2 + d^2\*g^2))/((8\*d^9\*e^5\*f^7 + 24\*d^10\*e^4\*f^6\*g + 16\*d^11\*e^3\*f^5\*g^2 - 16\*d^12\*e^2\*f^4\*g^3 - 24\*d^13\*e\*f^3\*g^4 - 8\*d^14\*f^2\*g^5 + (d^3\*e^11\*f^6\*g + 3\*d^4\*e^10\*f^5\*g^2 + 2\*d^5\*e^9\*f^4\*g^3 - 2\*d^6\*e^8\*f^3\*g^4 - 3\*d^7\*e^7\*f^2\*g^5 - d^8\*e^6\*f\*g^6)\*x^7 + (d^3\*e^11\*f^7 + 4\*d^4\*e^10\*f^6\*g + 5\*d^5\*e^9\*f^5\*g^2 - 5\*d^7\*e^7\*f^3\*g^4 - 4\*d^8\*e^6\*f^2\*g^5 - d^9\*e^5\*f\*g^6)\*x^6 + (d^4\*e^10\*f^7 - 10\*d^5\*e^9\*f^6\*g - 37\*d^6\*e^8\*f^5\*g^2 - 28\*d^7\*e^7\*f^4\*g^3 + 23\*d^8\*e^6\*f^3\*g^4 + 38\*d^9\*e^5\*f^2\*g^5 + 13\*d^10\*e^4\*f\*g^6)\*x^5 - (13\*d^5\*e^9\*f^7 + 24\*d^6\*e^8\*f^6\*g - 19\*d^7\*e^7\*f^5\*g^2 - 56\*d^8\*e^6\*f^4\*g^3 - 9\*d^9\*e^5\*f^3\*g^4 + 32\*d^10\*e^4\*f^2\*g^5 + 15\*d^11\*e^3\*f\*g^6)\*x^4 + (15\*d^6\*e^8\*f^7 + 53\*d^7\*e^7\*f^6\*g + 54\*d^8\*e^6\*f^5\*g^2 - 14\*d^9\*e^5\*f^4\*g^3 - 61\*d^10\*e^4\*f^3\*g^4 - 39\*d^11\*e^3\*f^2\*g^5 - 8\*d^12\*e^2\*f\*g^6)\*x^3 + 4\*(2\*d^7\*e^7\*f^7 + d^8\*e^6\*f^6\*g - 11\*d^9\*e^5\*f^5\*g^2 - 14\*d^10\*e^4\*f^4\*g^3 + 4\*d^11\*e^3\*f^3\*g^4 + 13\*d^12\*e^2\*f^2\*g^5 + 5\*d^13\*e\*f\*g^6)\*x^2 - 4\*(5\*d^8\*e^6\*f^7 + 13\*d^9\*e^5\*f^6\*g + 4\*d^10\*e^4\*f^5\*g^2 - 14\*d^11\*e^3\*f^4\*g^3 - 11\*d^12\*e^2\*f^3\*g^4 + d^13\*e\*f^2\*g^5 + 2\*d^14\*f\*g^6)\*x - (8\*d^8\*e^5\*f^7 + 24\*d^9\*e^4\*f^6\*g + 16\*d^10\*e^3\*f^5\*g^2 - 16\*d^11\*e^2\*f^4\*g^3 - 24\*d^12\*e\*f^3\*g^4 - 8\*d^13\*f^2\*g^5 + (d^3\*e^10\*f^6\*g + 3\*d^4\*e^9\*f^5\*g^2 + 2



$$\begin{aligned} & ^4g^3 + 9d^7e^6f^3g^4 + 17d^8e^5f^2g^5 + 6d^9e^4f^1g^6 \\ & )x^5 - (6d^4e^9f^7 + 13d^5e^8f^6g - 3d^6e^7f^5g^2 - 2 \\ & 2d^7e^6f^4g^3 - 8d^8e^5f^3g^4 + 9d^9e^4f^2g^5 + 5d^{10} \\ & 0e^3f^1g^6)x^4 + (5d^5e^8f^7 + 27d^6e^7f^6g + 46d^7e^6 \\ & f^5g^2 + 14d^8e^5f^4g^3 - 39d^9e^4f^3g^4 - 41d^{10}e^3 \\ & f^2g^5 - 12d^{11}e^2f^1g^6)x^3 + 4(3d^6e^7f^7 + 4d^7e^6f^6 \\ & g - 9d^8e^5f^5g^2 - 16d^9e^4f^4g^3 + d^{10}e^3f^3g^4 \\ & + 12d^{11}e^2f^2g^5 + 5d^{12}e^1f^1g^6)x^2 - 4(5d^7e^6f^7 + \\ & 13d^8e^5f^6g + 4d^9e^4f^5g^2 - 14d^{10}e^3f^4g^3 - 11d^{11} \\ & e^2f^3g^4 + d^{12}e^1f^2g^5 + 2d^{13}f^1g^6)x \sqrt{-e^2x^2 + d^2} \\ & ) \sqrt{e^2f^2 - d^2g^2} \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3/(g\*x+f)\*\*2/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2),x)

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3/((-e^2\*x^2 + d^2)^(7/2)\*(g\*x + f)^2),x, algorithm="giac")

[Out] Exception raised: TypeError



$$3.587 \quad \int \frac{(d+ex)^3}{(f+gx)^3(d^2-e^2x^2)^{7/2}} dx$$

**Optimal.** Leaf size=398

$$\frac{e^2g^3(13d^2g^2 - 30defg + 20e^2f^2) \tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{2(ef-dg)^2(dg+ef)^5\sqrt{e^2f^2-d^2g^2}} + \frac{3eg^4\sqrt{d^2-e^2x^2}(3ef-2dg)}{2(f+gx)(ef-dg)^2(dg+ef)^5}$$

$$+ \frac{g^4\sqrt{d^2-e^2x^2}}{2(f+gx)^2(ef-dg)(dg+ef)^4} - \frac{e^2(5d(ef-5dg) - ex(31dg+ef))}{15d(d^2-e^2x^2)^{3/2}(dg+ef)^4}$$

$$+ \frac{4de^2(d+ex)}{5(d^2-e^2x^2)^{5/2}(dg+ef)^3} + \frac{e^2(90d^3g^2 + ex(107d^2g^2 + 19defg + 2e^2f^2))}{15d^3\sqrt{d^2-e^2x^2}(dg+ef)^5}$$

[Out]  $(4*d*e^2*(d+e*x))/(5*(e*f+d*g)^3*(d^2-e^2*x^2)^(5/2)) - (e^2*(5*d*(e*f-5*d*g) - e*(e*f+31*d*g)*x))/(15*d*(e*f+d*g)^4*(d^2-e^2*x^2)^(3/2)) + (e^2*(90*d^3*g^2 + e*(2*e^2*f^2 + 19*d*e*f*g + 107*d^2*g^2)*x))/(15*d^3*(e*f+d*g)^5*sqrt[d^2-e^2*x^2]) + (g^4*sqrt[d^2-e^2*x^2])/(2*(e*f-d*g)*(e*f+d*g)^4*(f+g*x)^2) + (3*e*g^4*(3*e*f-2*d*g)*sqrt[d^2-e^2*x^2])/(2*(e*f-d*g)^2*(e*f+d*g)^5*(f+g*x)) + (e^2*g^3*(20*e^2*f^2 - 30*d*e*f*g + 13*d^2*g^2)*ArcTan[(d^2*g + e^2*f*x)/(sqrt[e^2*f^2 - d^2*g^2]*sqrt[d^2-e^2*x^2])])/(2*(e*f-d*g)^2*(e*f+d*g)^5*sqrt[e^2*f^2 - d^2*g^2])$

**Rubi [A]** time = 5.42682, antiderivative size = 398, normalized size of antiderivative = 1., number of rules used = 7, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$

$$\frac{e^2g^3(13d^2g^2 - 30defg + 20e^2f^2) \tan^{-1}\left(\frac{d^2g+e^2fx}{\sqrt{d^2-e^2x^2}\sqrt{e^2f^2-d^2g^2}}\right)}{2(ef-dg)^2(dg+ef)^5\sqrt{e^2f^2-d^2g^2}} + \frac{3eg^4\sqrt{d^2-e^2x^2}(3ef-2dg)}{2(f+gx)(ef-dg)^2(dg+ef)^5}$$

$$+ \frac{g^4\sqrt{d^2-e^2x^2}}{2(f+gx)^2(ef-dg)(dg+ef)^4} - \frac{e^2(5d(ef-5dg) - ex(31dg+ef))}{15d(d^2-e^2x^2)^{3/2}(dg+ef)^4}$$

$$+ \frac{4de^2(d+ex)}{5(d^2-e^2x^2)^{5/2}(dg+ef)^3} + \frac{e^2(90d^3g^2 + ex(107d^2g^2 + 19defg + 2e^2f^2))}{15d^3\sqrt{d^2-e^2x^2}(dg+ef)^5}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^3/((f + g\*x)^3\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out]  $(4*d*e^2*(d+e*x))/(5*(e*f+d*g)^3*(d^2-e^2*x^2)^(5/2)) - (e^2*(5*d*(e*f-5*d*g) - e*(e*f+31*d*g)*x))/(15*d*(e*f+d*g)^4*(d^2-e^2*x^2)^(3/2)) + (e^2*(90*d^3*g^2 + e*(2*e^2*f^2 + 19*d*e*f*g + 107*d^2*g^2)*x))/(15*d^3*(e*f+d*g)^5*sqrt[d^2-e^2*x^2]) + (g^4*sqrt[d^2-e^2*x^2])/(2*(e*f-d*g)*(e*f+d*g)^4*(f+g*x)^2) + (3*e*g^4*(3*e*f-2*d*g)*sqrt[d^2-e^2*x^2])/(2*(e*f-d*g)^2*(e*f+d*g)^5*(f+g*x)) + (e^2*g^3*(20*e^2*f^2 - 30*d*e*f*g + 13*d^2*g^2)*ArcTan[(d^2*g + e^2*f*x)/(sqrt[e^2*f^2 - d^2*g^2]*sqrt[d^2-e^2*x^2])])/(2*(e*f-d*g)^2*(e*f+d*g)^5*sqrt[e^2*f^2 - d^2*g^2])$

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3/(g\*x+f)\*\*3/(-e\*\*2\*x\*\*2+d\*\*2)\*\*(7/2), x)

[Out] Timed out

**Mathematica [C]** time = 1.65175, size = 387, normalized size = 0.97

$$\frac{\sqrt{d^2 - e^2 x^2} \left( \frac{2e^2(dg+ef)(17dg+2ef)}{d^2(d-ex)^2} + \frac{2e^2(107d^2g^2+19defg+2e^2f^2)}{d^3(d-ex)} + \frac{6e^2(dg+ef)^2}{d(d-ex)^3} + \frac{45eg^4(3ef-2dg)}{(f+gx)(ef-dg)^2} + \frac{15g^4(dg+ef)}{(f+gx)^2(ef-dg)} \right) - \frac{15ie^2g^3(13d^2g^2-30d^2g+ef)}{30(dg+ef)^5}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^3/((f + g\*x)^3\*(d^2 - e^2\*x^2)^(7/2)), x]

[Out] (Sqrt[d^2 - e^2\*x^2]\*((6\*e^2\*(e\*f + d\*g)^2)/(d\*(d - e\*x)^3) + (2\*e^2\*(e\*f + d\*g)\*(2\*e\*f + 17\*d\*g))/(d^2\*(d - e\*x)^2) + (2\*e^2\*(2\*e^2\*f^2 + 19\*d\*e\*f\*g + 107\*d^2\*g^2))/(d^3\*(d - e\*x)) + (15\*g^4\*(e\*f + d\*g))/((e\*f - d\*g)\*(f + g\*x)^2) + (45\*e\*g^4\*(3\*e\*f - 2\*d\*g))/((e\*f - d\*g)^2\*(f + g\*x))) - ((15\*I)\*e^2\*g^3\*(20\*e^2\*f^2 - 30\*d\*e\*f\*g + 13\*d^2\*g^2)\*Log[(4\*(e\*f - d\*g)^2\*(e\*f + d\*g)^5\*(I\*d^2\*g + I\*e^2\*f\*x + Sqrt[e^2\*f^2 - d^2\*g^2]\*Sqrt[d^2 - e^2\*x^2])]/(e^2\*g^2\*Sqrt[e^2\*f^2 - d^2\*g^2]\*(20\*e^2\*f^2 - 30\*d\*e\*f\*g + 13\*d^2\*g^2)\*(f + g\*x)))/((e\*f - d\*g)^2\*Sqrt[e^2\*f^2 - d^2\*g^2]))/(30\*(e\*f + d\*g)^5)

**Maple [B]** time = 0.046, size = 9593, normalized size = 24.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3/(g\*x+f)^3/(-e^2\*x^2+d^2)^(7/2), x)

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3/((-e^2\*x^2 + d^2)^(7/2)\*(g\*x + f)^3), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.905889, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3/((-e^2\*x^2 + d^2)^(7/2)\*(g\*x + f)^3), x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [1/30*(15*(320*d^{10}*e^4*f^6*g^3 - 480*d^{11}*e^3*f^5*g^4 + 208*d^{12} \\ & *e^2*f^4*g^5 - (20*d^3*e^{11}*f^4*g^5 - 30*d^4*e^{10}*f^3*g^6 + 13*d^5 \\ & *e^9*f^2*g^7)*x^9 - (40*d^3*e^{11}*f^5*g^4 - 200*d^4*e^{10}*f^4*g^5 \\ & + 236*d^5*e^9*f^3*g^6 - 91*d^6*e^8*f^2*g^7)*x^8 - (20*d^3*e^{11}*f^6 \\ & *g^3 - 310*d^4*e^{10}*f^5*g^4 + 493*d^5*e^9*f^4*g^5 - 272*d^6*e^8* \\ & f^3*g^6 + 39*d^7*e^7*f^2*g^7)*x^7 + (140*d^4*e^{10}*f^6*g^3 - 330*d \\ & ^5*e^9*f^5*g^4 - 349*d^6*e^8*f^4*g^5 + 852*d^7*e^7*f^3*g^6 - 403* \\ & d^8*e^6*f^2*g^7)*x^6 - (60*d^5*e^9*f^6*g^3 + 1150*d^6*e^8*f^5*g^4 \\ & - 2621*d^7*e^7*f^4*g^5 + 2006*d^8*e^6*f^3*g^6 - 520*d^9*e^5*f^2* \\ & g^7)*x^5 - (620*d^6*e^8*f^6*g^3 - 2530*d^7*e^7*f^5*g^4 + 2563*d^8 \\ & *e^6*f^4*g^5 - 680*d^9*e^5*f^3*g^6 - 156*d^{10}*e^4*f^2*g^7)*x^4 + \\ & 8*(100*d^7*e^7*f^6*g^3 - 90*d^8*e^6*f^5*g^4 - 125*d^9*e^5*f^4*g^5 \\ & + 189*d^{10}*e^4*f^3*g^6 - 65*d^{11}*e^3*f^2*g^7)*x^3 + 4*(60*d^8*e^6 \\ & *f^6*g^3 - 490*d^9*e^5*f^5*g^4 + 719*d^{10}*e^4*f^4*g^5 - 380*d^{11} \\ & *e^3*f^3*g^6 + 52*d^{12}*e^2*f^2*g^7)*x^2 - 8*(100*d^9*e^5*f^6*g^3 \\ & - 230*d^{10}*e^4*f^5*g^4 + 185*d^{11}*e^3*f^4*g^5 - 52*d^{12}*e^2*f^3*g \\ & ^6)*x - (320*d^9*e^4*f^6*g^3 - 480*d^{10}*e^3*f^5*g^4 + 208*d^{11}*e^2 \\ & *f^4*g^5 + (20*d^3*e^{10}*f^4*g^5 - 30*d^4*e^9*f^3*g^6 + 13*d^5*e^8 \\ & *f^2*g^7)*x^8 + 2*(20*d^3*e^{10}*f^5*g^4 - 10*d^4*e^9*f^4*g^5 - 17 \\ & *d^5*e^8*f^3*g^6 + 13*d^6*e^7*f^2*g^7)*x^7 + (20*d^3*e^{10}*f^6*g^3 \\ & + 50*d^4*e^9*f^5*g^4 - 487*d^5*e^8*f^4*g^5 + 622*d^6*e^7*f^3*g^6 \\ & - 247*d^7*e^6*f^2*g^7)*x^6 + 2*(20*d^4*e^9*f^6*g^3 - 410*d^5*e^8 \\ & *f^5*g^4 + 783*d^6*e^7*f^4*g^5 - 547*d^7*e^6*f^3*g^6 + 130*d^8*e^5 \\ & *f^2*g^7)*x^5 - (380*d^5*e^8*f^6*g^3 - 1370*d^6*e^7*f^5*g^4 + 10 \\ & 47*d^7*e^6*f^4*g^5 + 80*d^8*e^5*f^3*g^6 - 260*d^9*e^4*f^2*g^7)*x^4 \\ & + 20*(20*d^6*e^7*f^6*g^3 + 10*d^7*e^6*f^5*g^4 - 87*d^8*e^5*f^4* \\ & g^5 + 86*d^9*e^4*f^3*g^6 - 26*d^{10}*e^3*f^2*g^7)*x^3 + 4*(100*d^7* \\ & e^6*f^6*g^3 - 550*d^8*e^5*f^5*g^4 + 745*d^9*e^4*f^4*g^5 - 380*d^{10} \\ & *e^3*f^3*g^6 + 52*d^{11}*e^2*f^2*g^7)*x^2 - 8*(100*d^8*e^5*f^6*g^3 \\ & - 230*d^9*e^4*f^5*g^4 + 185*d^{10}*e^3*f^4*g^5 - 52*d^{11}*e^2*f^3*g \\ & ^6)*x)*\text{sqrt}(-e^2*x^2 + d^2))*\text{log}(((d*e^2*f^2*g - d^3*g^3)*x^2 - ( \\ & e^2*f^3 - d^2*f*g^2)*\text{sqrt}(-e^2*x^2 + d^2))*x + (d*e^2*f^3 - d^3*f* \\ & g^2)*x + (d^2*f*g*x + d^2*f^2 - (e^2*f^2 - d^2*g^2)*x^2 - (d*f*g* \\ & x + d*f^2)*\text{sqrt}(-e^2*x^2 + d^2))*\text{sqrt}(-e^2*f^2 + d^2*g^2))/(d*g*x \\ & + d*f - \text{sqrt}(-e^2*x^2 + d^2))*(g*x + f))) - (3*(6*e^{13}*f^6*g^2 + \\ & 30*d*e^{12}*f^5*g^3 + 78*d^2*e^{11}*f^4*g^4 - 315*d^3*e^{10}*f^3*g^5 + \\ & 236*d^4*e^9*f^2*g^6 - 30*d^5*e^8*f*g^7 - 5*d^6*e^7*g^8)*x^9 + 3*( \\ & 12*e^{13}*f^7*g + 30*d*e^{12}*f^6*g^2 + 36*d^2*e^{11}*f^5*g^3 - 650*d^3 \\ & *e^{10}*f^4*g^4 + 952*d^4*e^9*f^3*g^5 - 735*d^5*e^8*f^2*g^6 + 200*d \\ & ^6*e^7*f*g^7 + 35*d^7*e^6*g^8)*x^8 + 3*(6*e^{13}*f^8 - 30*d*e^{12}*f^7 \\ & *g - 174*d^2*e^{11}*f^6*g^2 - 540*d^3*e^{10}*f^5*g^3 + 640*d^4*e^9*f^4 \\ & *g^4 + 1280*d^5*e^8*f^3*g^5 - 1027*d^6*e^7*f^2*g^6 - 20*d^7*e^6 \\ & *f*g^7 - 15*d^8*e^5*g^8)*x^7 - (90*d*e^{12}*f^8 + 432*d^2*e^{11}*f^7* \\ & g + 560*d^3*e^{10}*f^6*g^2 - 84*d^4*e^9*f^5*g^3 - 14200*d^5*e^8*f^4 \\ & *g^4 + 22512*d^6*e^7*f^3*g^5 - 14095*d^7*e^6*f^2*g^6 + 2880*d^8*e \\ & ^5*f*g^7 + 465*d^9*e^4*g^8)*x^6 - (36*d^2*e^{11}*f^8 - 590*d^3*e^{10} \\ & *f^7*g - 2162*d^4*e^9*f^6*g^2 - 9530*d^5*e^8*f^5*g^3 + 31216*d^6* \\ & e^7*f^4*g^4 - 23345*d^7*e^6*f^3*g^5 + 9085*d^8*e^5*f^2*g^6 - 2670 \\ & *d^9*e^4*f*g^7 - 600*d^{10}*e^3*g^8)*x^5 + 20*(26*d^3*e^{10}*f^8 + 74 \\ & *d^4*e^9*f^7*g + 14*d^5*e^8*f^6*g^2 - 634*d^6*e^7*f^5*g^3 + 320*d \\ & ^7*e^6*f^4*g^4 + 401*d^8*e^5*f^3*g^5 - 414*d^9*e^4*f^2*g^6 + 114* \\ & d^{10}*e^3*f*g^7 + 9*d^{11}*e^2*g^8)*x^4 - 20*(20*d^4*e^9*f^8 + 90*d^5 \\ & *e^8*f^7*g + 110*d^6*e^7*f^6*g^2 + 150*d^7*e^6*f^5*g^3 - 1480*d^8 \\ & *e^5*f^4*g^4 + 1443*d^9*e^4*f^3*g^5 - 615*d^{10}*e^3*f^2*g^6 + 162 \\ & *d^{11}*e^2*f*g^7 + 30*d^{12}*e*g^8)*x^3 - 120*(4*d^5*e^8*f^8 + 10*d^6 \\ & *e^7*f^7*g - 2*d^7*e^6*f^6*g^2 - 122*d^8*e^5*f^5*g^3 + 200*d^9*e \\ & ^4*f^4*g^4 - 141*d^{10}*e^3*f^3*g^5 + 49*d^{11}*e^2*f^2*g^6 - 2*d^{12}* \\ & e*f*g^7 - 2*d^{13}*g^8)*x^2 + 240*(2*d^6*e^7*f^8 + 6*d^7*e^6*f^7*g \\ & + 2*d^8*e^5*f^6*g^2 - 30*d^9*e^4*f^5*g^3 + 20*d^{10}*e^3*f^4*g^4 - \\ & 11*d^{11}*e^2*f^3*g^5 + 6*d^{12}*e*f^2*g^6 + 2*d^{13}*f*g^7)*x + (5*(2* \\ & e^{12}*f^6*g^2 + 6*d*e^{11}*f^5*g^3 - 10*d^2*e^{10}*f^4*g^4 + 21*d^3*e^9 \\ & *f^3*g^5 + 20*d^4*e^8*f^2*g^6 - 18*d^5*e^7*f*g^7 - 3*d^6*e^6*g^8) \\ & )*x^8 + (20*e^{12}*f^7*g + 116*d*e^{11}*f^6*g^2 + 230*d^2*e^{10}*f^5*g^3 \\ & + 1088*d^3*e^9*f^4*g^4 - 3865*d^4*e^8*f^3*g^5 + 2561*d^5*e^7*f^2 \\ & *g^6 - 210*d^6*e^6*f*g^7 - 30*d^7*e^5*g^8)*x^7 + (10*e^{12}*f^8 + \\ & 142*d*e^{11}*f^7*g + 330*d^2*e^{10}*f^6*g^2 + 766*d^3*e^9*f^5*g^3 - 8 \\ & 400*d^4*e^8*f^4*g^4 + 12157*d^5*e^7*f^3*g^5 - 7750*d^6*e^6*f^2*g^6 \\ & + 1650*d^7*e^5*f*g^7 + 285*d^8*e^4*g^8)*x^6 + (56*d*e^{11}*f^8 - \\ & 230*d^2*e^{10}*f^7*g - 1242*d^3*e^9*f^6*g^2 - 5330*d^4*e^8*f^5*g^3 \\ & + 14616*d^5*e^7*f^4*g^4 - 7925*d^6*e^6*f^3*g^5 + 2395*d^7*e^5*f^2 \\ & *g^6 - 1230*d^8*e^4*f*g^7 - 300*d^9*e^3*g^8)*x^5 - 20*(14*d^2*e^{10} \\ & *f^8 + 44*d^3*e^9*f^7*g + 20*d^4*e^8*f^6*g^2 - 268*d^5*e^7*f^5*g \end{aligned}$$

$$\begin{aligned}
& \wedge^3 - 280*d^6*e^6*f^4*g^4 + 824*d^7*e^5*f^3*g^5 - 561*d^8*e^4*f^2* \\
& g^6 + 120*d^9*e^3*f^1*g^7 + 15*d^{10}*e^2*g^8)*x^4 + 20*(8*d^3*e^9*f^8 \\
& + 54*d^4*e^8*f^7*g + 98*d^5*e^7*f^6*g^2 + 330*d^6*e^6*f^5*g^3 - \\
& 1600*d^7*e^5*f^4*g^4 + 1509*d^8*e^4*f^3*g^5 - 651*d^9*e^3*f^2*g^6 \\
& + 150*d^{10}*e^2*f^1*g^7 + 30*d^{11}*e*g^8)*x^3 + 120*(4*d^4*e^8*f^8 \\
& + 10*d^5*e^7*f^7*g - 2*d^6*e^6*f^6*g^2 - 122*d^7*e^5*f^5*g^3 + 20 \\
& 0*d^8*e^4*f^4*g^4 - 141*d^9*e^3*f^3*g^5 + 49*d^{10}*e^2*f^2*g^6 - 2 \\
& *d^{11}*e*f^1*g^7 - 2*d^{12}*g^8)*x^2 - 240*(2*d^5*e^7*f^8 + 6*d^6*e^6* \\
& f^7*g + 2*d^7*e^5*f^6*g^2 - 30*d^8*e^4*f^5*g^3 + 20*d^9*e^3*f^4*g \\
& ^4 - 11*d^{10}*e^2*f^3*g^5 + 6*d^{11}*e*f^2*g^6 + 2*d^{12}*f^1*g^7)*x)*sq \\
& rt(-e^2*x^2 + d^2))*sqrt(-e^2*f^2 + d^2*g^2))/((16*d^{10}*e^7*f^{11} \\
& + 48*d^{11}*e^6*f^{10}*g + 16*d^{12}*e^5*f^9*g^2 - 80*d^{13}*e^4*f^8*g^3 \\
& - 80*d^{14}*e^3*f^7*g^4 + 16*d^{15}*e^2*f^6*g^5 + 48*d^{16}*e*f^5*g^6 + \\
& 16*d^{17}*f^4*g^7 - (d^3*e^{14}*f^9*g^2 + 3*d^4*e^{13}*f^8*g^3 + d^5*e \\
& ^{12}*f^7*g^4 - 5*d^6*e^{11}*f^6*g^5 - 5*d^7*e^{10}*f^5*g^6 + d^8*e^9*f \\
& ^4*g^7 + 3*d^9*e^8*f^3*g^8 + d^{10}*e^7*f^2*g^9)*x^9 - (2*d^3*e^{14}* \\
& f^{10}*g - d^4*e^{13}*f^9*g^2 - 19*d^5*e^{12}*f^8*g^3 - 17*d^6*e^{11}*f^7 \\
& *g^4 + 25*d^7*e^{10}*f^6*g^5 + 37*d^8*e^9*f^5*g^6 - d^9*e^8*f^4*g^7 \\
& - 19*d^{10}*e^7*f^3*g^8 - 7*d^{11}*e^6*f^2*g^9)*x^8 - (d^3*e^{14}*f^{11} \\
& - 11*d^4*e^{13}*f^{10}*g - 38*d^5*e^{12}*f^9*g^2 - 10*d^6*e^{11}*f^8*g^3 \\
& + 68*d^7*e^{10}*f^7*g^4 + 56*d^8*e^9*f^6*g^5 - 26*d^9*e^8*f^5*g^6 \\
& - 38*d^{10}*e^7*f^4*g^7 - 5*d^{11}*e^6*f^3*g^8 + 3*d^{12}*e^5*f^2*g^9)* \\
& x^7 + (7*d^4*e^{13}*f^{11} + 15*d^5*e^{12}*f^{10}*g - 42*d^6*e^{11}*f^9*g^2 \\
& - 134*d^7*e^{10}*f^8*g^3 - 36*d^8*e^9*f^7*g^4 + 192*d^9*e^8*f^6*g^5 \\
& + 170*d^{10}*e^7*f^5*g^6 - 42*d^{11}*e^6*f^4*g^7 - 99*d^{12}*e^5*f^3* \\
& g^8 - 31*d^{13}*e^4*f^2*g^9)*x^6 - (3*d^5*e^{12}*f^{11} + 71*d^6*e^{11}*f \\
& ^{10}*g + 149*d^7*e^{10}*f^9*g^2 - 73*d^8*e^9*f^8*g^3 - 365*d^9*e^8*f \\
& ^7*g^4 - 107*d^{10}*e^7*f^6*g^5 + 271*d^{11}*e^6*f^5*g^6 + 149*d^{12}*e \\
& ^5*f^4*g^7 - 58*d^{13}*e^4*f^3*g^8 - 40*d^{14}*e^3*f^2*g^9)*x^5 - (31 \\
& *d^6*e^{11}*f^{11} + 13*d^7*e^{10}*f^{10}*g - 221*d^8*e^9*f^9*g^2 - 271*d \\
& ^9*e^8*f^8*g^3 + 233*d^{10}*e^7*f^7*g^4 + 491*d^{11}*e^6*f^6*g^5 + 73 \\
& *d^{12}*e^5*f^5*g^6 - 221*d^{13}*e^4*f^4*g^7 - 116*d^{14}*e^3*f^3*g^8 - \\
& 12*d^{15}*e^2*f^2*g^9)*x^4 + 8*(5*d^7*e^{10}*f^{11} + 18*d^8*e^9*f^{10}* \\
& g + 9*d^9*e^8*f^9*g^2 - 37*d^{10}*e^7*f^8*g^3 - 45*d^{11}*e^6*f^7*g^4 \\
& + 15*d^{12}*e^5*f^6*g^5 + 43*d^{13}*e^4*f^5*g^6 + 9*d^{14}*e^3*f^4*g^7 \\
& - 12*d^{15}*e^2*f^3*g^8 - 5*d^{16}*e*f^2*g^9)*x^3 + 4*(3*d^8*e^9*f^{11} \\
& - 11*d^9*e^8*f^{10}*g - 53*d^{10}*e^7*f^9*g^2 - 23*d^{11}*e^6*f^8*g^3 \\
& + 89*d^{12}*e^5*f^7*g^4 + 83*d^{13}*e^4*f^6*g^5 - 31*d^{14}*e^3*f^5*g^6 \\
& - 53*d^{15}*e^2*f^4*g^7 - 8*d^{16}*e*f^3*g^8 + 4*d^{17}*f^2*g^9)*x^2 \\
& - 8*(5*d^9*e^8*f^{11} + 11*d^{10}*e^7*f^{10}*g - 7*d^{11}*e^6*f^9*g^2 - 2 \\
& 9*d^{12}*e^5*f^8*g^3 - 5*d^{13}*e^4*f^7*g^4 + 25*d^{14}*e^3*f^6*g^5 + 1 \\
& 1*d^{15}*e^2*f^5*g^6 - 7*d^{16}*e*f^4*g^7 - 4*d^{17}*f^3*g^8)*x - (16*d \\
& ^9*e^7*f^{11} + 48*d^{10}*e^6*f^{10}*g + 16*d^{11}*e^5*f^9*g^2 - 80*d^{12}* \\
& e^4*f^8*g^3 - 80*d^{13}*e^3*f^7*g^4 + 16*d^{14}*e^2*f^6*g^5 + 48*d^{15} \\
& *e*f^5*g^6 + 16*d^{16}*f^4*g^7 + (d^3*e^{13}*f^9*g^2 + 3*d^4*e^{12}*f^8 \\
& *g^3 + d^5*e^{11}*f^7*g^4 - 5*d^6*e^{10}*f^6*g^5 - 5*d^7*e^9*f^5*g^6 \\
& + d^8*e^8*f^4*g^7 + 3*d^9*e^7*f^3*g^8 + d^{10}*e^6*f^2*g^9)*x^8 + 2 \\
& *(d^3*e^{13}*f^{10}*g + 4*d^4*e^{12}*f^9*g^2 + 4*d^5*e^{11}*f^8*g^3 - 4*d \\
& ^6*e^{10}*f^7*g^4 - 10*d^7*e^9*f^6*g^5 - 4*d^8*e^8*f^5*g^6 + 4*d^9* \\
& e^7*f^4*g^7 + 4*d^{10}*e^6*f^3*g^8 + d^{11}*e^5*f^2*g^9)*x^7 + (d^3*e \\
& ^{13}*f^{11} + 7*d^4*e^{12}*f^{10}*g - 6*d^5*e^{11}*f^9*g^2 - 58*d^6*e^{10}*f \\
& ^8*g^3 - 44*d^7*e^9*f^7*g^4 + 76*d^8*e^8*f^6*g^5 + 102*d^9*e^7*f^5 \\
& *g^6 - 6*d^{10}*e^6*f^4*g^7 - 53*d^{11}*e^5*f^3*g^8 - 19*d^{12}*e^4*f^2 \\
& *g^9)*x^6 + 2*(d^4*e^{12}*f^{11} - 16*d^5*e^{11}*f^{10}*g - 46*d^6*e^{10}* \\
& f^9*g^2 + 6*d^7*e^9*f^8*g^3 + 100*d^8*e^8*f^7*g^4 + 46*d^9*e^7*f^6 \\
& *g^5 - 66*d^{10}*e^6*f^5*g^6 - 46*d^{11}*e^5*f^4*g^7 + 11*d^{12}*e^4*f^3 \\
& *g^8 + 10*d^{13}*e^3*f^2*g^9)*x^5 - (19*d^5*e^{11}*f^{11} + 17*d^6*e^ \\
& ^{10}*f^{10}*g - 121*d^7*e^9*f^9*g^2 - 195*d^8*e^8*f^8*g^3 + 85*d^9*e^ \\
& ^7*f^7*g^4 + 319*d^{10}*e^6*f^6*g^5 + 117*d^{11}*e^5*f^5*g^6 - 121*d^{11} \\
& ^2*e^4*f^4*g^7 - 100*d^{13}*e^3*f^3*g^8 - 20*d^{14}*e^2*f^2*g^9)*x^4 + \\
& 20*(d^6*e^{10}*f^{11} + 5*d^7*e^9*f^{10}*g + 5*d^8*e^8*f^9*g^2 - 9*d^9 \\
& *e^7*f^8*g^3 - 17*d^{10}*e^6*f^7*g^4 + d^{11}*e^5*f^6*g^5 + 15*d^{12}*e \\
& ^4*f^5*g^6 + 5*d^{13}*e^3*f^4*g^7 - 4*d^{14}*e^2*f^3*g^8 - 2*d^{15}*e*f \\
& ^2*g^9)*x^3 + 4*(5*d^7*e^9*f^{11} - 5*d^8*e^8*f^{10}*g - 51*d^9*e^7*f \\
& ^9*g^2 - 33*d^{10}*e^6*f^8*g^3 + 79*d^{11}*e^5*f^7*g^4 + 85*d^{12}*e^4* \\
& f^6*g^5 - 25*d^{13}*e^3*f^5*g^6 - 51*d^{14}*e^2*f^4*g^7 - 8*d^{15}*e*f^3 \\
& *g^8 + 4*d^{16}*f^2*g^9)*x^2 - 8*(5*d^8*e^8*f^{11} + 11*d^9*e^7*f^{10} \\
& *g - 7*d^{10}*e^6*f^9*g^2 - 29*d^{11}*e^5*f^8*g^3 - 5*d^{12}*e^4*f^7*g^4 \\
& + 25*d^{13}*e^3*f^6*g^5 + 11*d^{14}*e^2*f^5*g^6 - 7*d^{15}*e*f^4*g^7 \\
& - 4*d^{16}*f^3*g^8)*x)*sqrt(-e^2*x^2 + d^2))*sqrt(-e^2*f^2 + d^2*g^2)), \\
& 1/30*(30*(320*d^{10}*e^4*f^6*g^3 - 480*d^{11}*e^3*f^5*g^4 + 208* \\
& d^{12}*e^2*f^4*g^5 - (20*d^3*e^{11}*f^4*g^5 - 30*d^4*e^{10}*f^3*g^6 + 1
\end{aligned}$$

$$\begin{aligned}
& 3*d^5*e^9*f^2*g^7)*x^9 - (40*d^3*e^{11}*f^5*g^4 - 200*d^4*e^{10}*f^4* \\
& g^5 + 236*d^5*e^9*f^3*g^6 - 91*d^6*e^8*f^2*g^7)*x^8 - (20*d^3*e^{11} \\
& *f^6*g^3 - 310*d^4*e^{10}*f^5*g^4 + 493*d^5*e^9*f^4*g^5 - 272*d^6* \\
& e^8*f^3*g^6 + 39*d^7*e^7*f^2*g^7)*x^7 + (140*d^4*e^{10}*f^6*g^3 - 3 \\
& 30*d^5*e^9*f^5*g^4 - 349*d^6*e^8*f^4*g^5 + 852*d^7*e^7*f^3*g^6 - \\
& 403*d^8*e^6*f^2*g^7)*x^6 - (60*d^5*e^9*f^6*g^3 + 1150*d^6*e^8*f^5* \\
& *g^4 - 2621*d^7*e^7*f^4*g^5 + 2006*d^8*e^6*f^3*g^6 - 520*d^9*e^5* \\
& f^2*g^7)*x^5 - (620*d^6*e^8*f^6*g^3 - 2530*d^7*e^7*f^5*g^4 + 2563 \\
& *d^8*e^6*f^4*g^5 - 680*d^9*e^5*f^3*g^6 - 156*d^{10}*e^4*f^2*g^7)*x^4 \\
& + 8*(100*d^7*e^7*f^6*g^3 - 90*d^8*e^6*f^5*g^4 - 125*d^9*e^5*f^4* \\
& *g^5 + 189*d^{10}*e^4*f^3*g^6 - 65*d^{11}*e^3*f^2*g^7)*x^3 + 4*(60*d^8 \\
& *e^6*f^6*g^3 - 490*d^9*e^5*f^5*g^4 + 719*d^{10}*e^4*f^4*g^5 - 380* \\
& d^{11}*e^3*f^3*g^6 + 52*d^{12}*e^2*f^2*g^7)*x^2 - 8*(100*d^9*e^5*f^6* \\
& g^3 - 230*d^{10}*e^4*f^5*g^4 + 185*d^{11}*e^3*f^4*g^5 - 52*d^{12}*e^2*f \\
& ^3*g^6)*x - (320*d^9*e^4*f^6*g^3 - 480*d^{10}*e^3*f^5*g^4 + 208*d^{11} \\
& *e^2*f^4*g^5 + (20*d^3*e^{10}*f^4*g^5 - 30*d^4*e^9*f^3*g^6 + 13*d^5 \\
& *e^8*f^2*g^7)*x^8 + 2*(20*d^3*e^{10}*f^5*g^4 - 10*d^4*e^9*f^4*g^5 \\
& - 17*d^5*e^8*f^3*g^6 + 13*d^6*e^7*f^2*g^7)*x^7 + (20*d^3*e^{10}*f^6 \\
& *g^3 + 50*d^4*e^9*f^5*g^4 - 487*d^5*e^8*f^4*g^5 + 622*d^6*e^7*f^3 \\
& *g^6 - 247*d^7*e^6*f^2*g^7)*x^6 + 2*(20*d^4*e^9*f^6*g^3 - 410*d^5 \\
& *e^8*f^5*g^4 + 783*d^6*e^7*f^4*g^5 - 547*d^7*e^6*f^3*g^6 + 130*d^8 \\
& *e^5*f^2*g^7)*x^5 - (380*d^5*e^8*f^6*g^3 - 1370*d^6*e^7*f^5*g^4 \\
& + 1047*d^7*e^6*f^4*g^5 + 80*d^8*e^5*f^3*g^6 - 260*d^9*e^4*f^2*g^7) \\
& *x^4 + 20*(20*d^6*e^7*f^6*g^3 + 10*d^7*e^6*f^5*g^4 - 87*d^8*e^5* \\
& f^4*g^5 + 86*d^9*e^4*f^3*g^6 - 26*d^{10}*e^3*f^2*g^7)*x^3 + 4*(100* \\
& d^7*e^6*f^6*g^3 - 550*d^8*e^5*f^5*g^4 + 745*d^9*e^4*f^4*g^5 - 380 \\
& *d^{10}*e^3*f^3*g^6 + 52*d^{11}*e^2*f^2*g^7)*x^2 - 8*(100*d^8*e^5*f^6 \\
& *g^3 - 230*d^9*e^4*f^5*g^4 + 185*d^{10}*e^3*f^4*g^5 - 52*d^{11}*e^2*f \\
& ^3*g^6)*x)*sqrt(-e^2*x^2 + d^2))*arctan((d*g*x + d*f - sqrt(-e^2* \\
& x^2 + d^2)*f)/(sqrt(e^2*f^2 - d^2*g^2)*x)) - (3*(6*e^{13}*f^6*g^2 + \\
& 30*d*e^{12}*f^5*g^3 + 78*d^2*e^{11}*f^4*g^4 - 315*d^3*e^{10}*f^3*g^5 + \\
& 236*d^4*e^9*f^2*g^6 - 30*d^5*e^8*f*g^7 - 5*d^6*e^7*g^8)*x^9 + 3* \\
& (12*e^{13}*f^7*g + 30*d*e^{12}*f^6*g^2 + 36*d^2*e^{11}*f^5*g^3 - 650*d^3 \\
& *e^{10}*f^4*g^4 + 952*d^4*e^9*f^3*g^5 - 735*d^5*e^8*f^2*g^6 + 200* \\
& d^6*e^7*f*g^7 + 35*d^7*e^6*g^8)*x^8 + 3*(6*e^{13}*f^8 - 30*d*e^{12}*f \\
& ^7*g - 174*d^2*e^{11}*f^6*g^2 - 540*d^3*e^{10}*f^5*g^3 + 640*d^4*e^9* \\
& f^4*g^4 + 1280*d^5*e^8*f^3*g^5 - 1027*d^6*e^7*f^2*g^6 - 20*d^7*e^6 \\
& *f*g^7 - 15*d^8*e^5*g^8)*x^7 - (90*d*e^{12}*f^8 + 432*d^2*e^{11}*f^7 \\
& *g + 560*d^3*e^{10}*f^6*g^2 - 84*d^4*e^9*f^5*g^3 - 14200*d^5*e^8*f^4 \\
& *g^4 + 22512*d^6*e^7*f^3*g^5 - 14095*d^7*e^6*f^2*g^6 + 2880*d^8* \\
& e^5*f*g^7 + 465*d^9*e^4*g^8)*x^6 - (36*d^2*e^{11}*f^8 - 590*d^3*e^{10} \\
& *f^7*g - 2162*d^4*e^9*f^6*g^2 - 9530*d^5*e^8*f^5*g^3 + 31216*d^6 \\
& *e^7*f^4*g^4 - 23345*d^7*e^6*f^3*g^5 + 9085*d^8*e^5*f^2*g^6 - 267 \\
& 0*d^9*e^4*f*g^7 - 600*d^{10}*e^3*g^8)*x^5 + 20*(26*d^3*e^{10}*f^8 + 7 \\
& 4*d^4*e^9*f^7*g + 14*d^5*e^8*f^6*g^2 - 634*d^6*e^7*f^5*g^3 + 320* \\
& d^7*e^6*f^4*g^4 + 401*d^8*e^5*f^3*g^5 - 414*d^9*e^4*f^2*g^6 + 114 \\
& *d^{10}*e^3*f*g^7 + 9*d^{11}*e^2*g^8)*x^4 - 20*(20*d^4*e^9*f^8 + 90*d \\
& ^5*e^8*f^7*g + 110*d^6*e^7*f^6*g^2 + 150*d^7*e^6*f^5*g^3 - 1480*d \\
& ^8*e^5*f^4*g^4 + 1443*d^9*e^4*f^3*g^5 - 615*d^{10}*e^3*f^2*g^6 + 16 \\
& 2*d^{11}*e^2*f*g^7 + 30*d^{12}*e*g^8)*x^3 - 120*(4*d^5*e^8*f^8 + 10*d \\
& ^6*e^7*f^7*g - 2*d^7*e^6*f^6*g^2 - 122*d^8*e^5*f^5*g^3 + 200*d^9* \\
& e^4*f^4*g^4 - 141*d^{10}*e^3*f^3*g^5 + 49*d^{11}*e^2*f^2*g^6 - 2*d^{12} \\
& *e*f*g^7 - 2*d^{13}*g^8)*x^2 + 240*(2*d^6*e^7*f^8 + 6*d^7*e^6*f^7*g \\
& + 2*d^8*e^5*f^6*g^2 - 30*d^9*e^4*f^5*g^3 + 20*d^{10}*e^3*f^4*g^4 - \\
& 11*d^{11}*e^2*f^3*g^5 + 6*d^{12}*e*f^2*g^6 + 2*d^{13}*f*g^7)*x + (5*(2 \\
& *e^{12}*f^6*g^2 + 6*d*e^{11}*f^5*g^3 - 10*d^2*e^{10}*f^4*g^4 + 21*d^3*e \\
& ^9*f^3*g^5 + 20*d^4*e^8*f^2*g^6 - 18*d^5*e^7*f*g^7 - 3*d^6*e^6*g^8) \\
& *x^8 + (20*e^{12}*f^7*g + 116*d*e^{11}*f^6*g^2 + 230*d^2*e^{10}*f^5*g \\
& ^3 + 1088*d^3*e^9*f^4*g^4 - 3865*d^4*e^8*f^3*g^5 + 2561*d^5*e^7*f^2 \\
& *g^6 - 210*d^6*e^6*f*g^7 - 30*d^7*e^5*g^8)*x^7 + (10*e^{12}*f^8 + \\
& 142*d*e^{11}*f^7*g + 330*d^2*e^{10}*f^6*g^2 + 766*d^3*e^9*f^5*g^3 - \\
& 8400*d^4*e^8*f^4*g^4 + 12157*d^5*e^7*f^3*g^5 - 7750*d^6*e^6*f^2*g^6 \\
& + 1650*d^7*e^5*f*g^7 + 285*d^8*e^4*g^8)*x^6 + (56*d*e^{11}*f^8 - \\
& 230*d^2*e^{10}*f^7*g - 1242*d^3*e^9*f^6*g^2 - 5330*d^4*e^8*f^5*g^3 \\
& + 14616*d^5*e^7*f^4*g^4 - 7925*d^6*e^6*f^3*g^5 + 2395*d^7*e^5*f^2 \\
& *g^6 - 1230*d^8*e^4*f*g^7 - 300*d^9*e^3*g^8)*x^5 - 20*(14*d^2*e^{10} \\
& *f^8 + 44*d^3*e^9*f^7*g + 20*d^4*e^8*f^6*g^2 - 268*d^5*e^7*f^5* \\
& g^3 - 280*d^6*e^6*f^4*g^4 + 824*d^7*e^5*f^3*g^5 - 561*d^8*e^4*f^2 \\
& *g^6 + 120*d^9*e^3*f*g^7 + 15*d^{10}*e^2*g^8)*x^4 + 20*(8*d^3*e^9*f^8 \\
& + 54*d^4*e^8*f^7*g + 98*d^5*e^7*f^6*g^2 + 330*d^6*e^6*f^5*g^3 \\
& - 1600*d^7*e^5*f^4*g^4 + 1509*d^8*e^4*f^3*g^5 - 651*d^9*e^3*f^2*g \\
& ^6 + 150*d^{10}*e^2*f*g^7 + 30*d^{11}*e*g^8)*x^3 + 120*(4*d^4*e^8*f^8
\end{aligned}$$

$$\begin{aligned}
& + 10*d^5*e^7*f^7*g - 2*d^6*e^6*f^6*g^2 - 122*d^7*e^5*f^5*g^3 + 2 \\
& 00*d^8*e^4*f^4*g^4 - 141*d^9*e^3*f^3*g^5 + 49*d^{10}*e^2*f^2*g^6 - \\
& 2*d^{11}*e*f*g^7 - 2*d^{12}*g^8)*x^2 - 240*(2*d^5*e^7*f^8 + 6*d^6*e^6 \\
& *f^7*g + 2*d^7*e^5*f^6*g^2 - 30*d^8*e^4*f^5*g^3 + 20*d^9*e^3*f^4* \\
& g^4 - 11*d^{10}*e^2*f^3*g^5 + 6*d^{11}*e*f^2*g^6 + 2*d^{12}*f*g^7)*x)*s \\
& \text{qrt}(-e^2*x^2 + d^2))*\text{sqrt}(e^2*f^2 - d^2*g^2))/((16*d^{10}*e^7*f^{11} \\
& + 48*d^{11}*e^6*f^{10}*g + 16*d^{12}*e^5*f^9*g^2 - 80*d^{13}*e^4*f^8*g^3 \\
& - 80*d^{14}*e^3*f^7*g^4 + 16*d^{15}*e^2*f^6*g^5 + 48*d^{16}*e*f^5*g^6 + \\
& 16*d^{17}*f^4*g^7 - (d^3*e^{14}*f^9*g^2 + 3*d^4*e^{13}*f^8*g^3 + d^5*e \\
& ^{12}*f^7*g^4 - 5*d^6*e^{11}*f^6*g^5 - 5*d^7*e^{10}*f^5*g^6 + d^8*e^9*f \\
& ^4*g^7 + 3*d^9*e^8*f^3*g^8 + d^{10}*e^7*f^2*g^9)*x^9 - (2*d^3*e^{14}* \\
& f^{10}*g - d^4*e^{13}*f^9*g^2 - 19*d^5*e^{12}*f^8*g^3 - 17*d^6*e^{11}*f^7 \\
& *g^4 + 25*d^7*e^{10}*f^6*g^5 + 37*d^8*e^9*f^5*g^6 - d^9*e^8*f^4*g^7 \\
& - 19*d^{10}*e^7*f^3*g^8 - 7*d^{11}*e^6*f^2*g^9)*x^8 - (d^3*e^{14}*f^{11} \\
& - 11*d^4*e^{13}*f^{10}*g - 38*d^5*e^{12}*f^9*g^2 - 10*d^6*e^{11}*f^8*g^3 \\
& + 68*d^7*e^{10}*f^7*g^4 + 56*d^8*e^9*f^6*g^5 - 26*d^9*e^8*f^5*g^6 \\
& - 38*d^{10}*e^7*f^4*g^7 - 5*d^{11}*e^6*f^3*g^8 + 3*d^{12}*e^5*f^2*g^9)* \\
& x^7 + (7*d^4*e^{13}*f^{11} + 15*d^5*e^{12}*f^{10}*g - 42*d^6*e^{11}*f^9*g^2 \\
& - 134*d^7*e^{10}*f^8*g^3 - 36*d^8*e^9*f^7*g^4 + 192*d^9*e^8*f^6*g^5 \\
& + 170*d^{10}*e^7*f^5*g^6 - 42*d^{11}*e^6*f^4*g^7 - 99*d^{12}*e^5*f^3* \\
& g^8 - 31*d^{13}*e^4*f^2*g^9)*x^6 - (3*d^5*e^{12}*f^{11} + 71*d^6*e^{11}*f \\
& ^{10}*g + 149*d^7*e^{10}*f^9*g^2 - 73*d^8*e^9*f^8*g^3 - 365*d^9*e^8*f \\
& ^7*g^4 - 107*d^{10}*e^7*f^6*g^5 + 271*d^{11}*e^6*f^5*g^6 + 149*d^{12}*e \\
& ^5*f^4*g^7 - 58*d^{13}*e^4*f^3*g^8 - 40*d^{14}*e^3*f^2*g^9)*x^5 - (31 \\
& *d^6*e^{11}*f^{11} + 13*d^7*e^{10}*f^{10}*g - 221*d^8*e^9*f^9*g^2 - 271*d \\
& ^9*e^8*f^8*g^3 + 233*d^{10}*e^7*f^7*g^4 + 491*d^{11}*e^6*f^6*g^5 + 73 \\
& *d^{12}*e^5*f^5*g^6 - 221*d^{13}*e^4*f^4*g^7 - 116*d^{14}*e^3*f^3*g^8 - \\
& 12*d^{15}*e^2*f^2*g^9)*x^4 + 8*(5*d^7*e^{10}*f^{11} + 18*d^8*e^9*f^{10}* \\
& g + 9*d^9*e^8*f^9*g^2 - 37*d^{10}*e^7*f^8*g^3 - 45*d^{11}*e^6*f^7*g^4 \\
& + 15*d^{12}*e^5*f^6*g^5 + 43*d^{13}*e^4*f^5*g^6 + 9*d^{14}*e^3*f^4*g^7 \\
& - 12*d^{15}*e^2*f^3*g^8 - 5*d^{16}*e*f^2*g^9)*x^3 + 4*(3*d^8*e^9*f^{11} \\
& 1 - 11*d^9*e^8*f^{10}*g - 53*d^{10}*e^7*f^9*g^2 - 23*d^{11}*e^6*f^8*g^3 \\
& + 89*d^{12}*e^5*f^7*g^4 + 83*d^{13}*e^4*f^6*g^5 - 31*d^{14}*e^3*f^5*g^6 \\
& - 53*d^{15}*e^2*f^4*g^7 - 8*d^{16}*e*f^3*g^8 + 4*d^{17}*f^2*g^9)*x^2 \\
& - 8*(5*d^9*e^8*f^{11} + 11*d^{10}*e^7*f^{10}*g - 7*d^{11}*e^6*f^9*g^2 - 2 \\
& 9*d^{12}*e^5*f^8*g^3 - 5*d^{13}*e^4*f^7*g^4 + 25*d^{14}*e^3*f^6*g^5 + 1 \\
& 1*d^{15}*e^2*f^5*g^6 - 7*d^{16}*e*f^4*g^7 - 4*d^{17}*f^3*g^8)*x - (16*d \\
& ^9*e^7*f^{11} + 48*d^{10}*e^6*f^{10}*g + 16*d^{11}*e^5*f^9*g^2 - 80*d^{12}* \\
& e^4*f^8*g^3 - 80*d^{13}*e^3*f^7*g^4 + 16*d^{14}*e^2*f^6*g^5 + 48*d^{15} \\
& *e*f^5*g^6 + 16*d^{16}*f^4*g^7 + (d^3*e^{13}*f^9*g^2 + 3*d^4*e^{12}*f^8 \\
& *g^3 + d^5*e^{11}*f^7*g^4 - 5*d^6*e^{10}*f^6*g^5 - 5*d^7*e^9*f^5*g^6 \\
& + d^8*e^8*f^4*g^7 + 3*d^9*e^7*f^3*g^8 + d^{10}*e^6*f^2*g^9)*x^8 + 2 \\
& *(d^3*e^{13}*f^{10}*g + 4*d^4*e^{12}*f^9*g^2 + 4*d^5*e^{11}*f^8*g^3 - 4*d \\
& ^6*e^{10}*f^7*g^4 - 10*d^7*e^9*f^6*g^5 - 4*d^8*e^8*f^5*g^6 + 4*d^9* \\
& e^7*f^4*g^7 + 4*d^{10}*e^6*f^3*g^8 + d^{11}*e^5*f^2*g^9)*x^7 + (d^3*e \\
& ^{13}*f^{11} + 7*d^4*e^{12}*f^{10}*g - 6*d^5*e^{11}*f^9*g^2 - 58*d^6*e^{10}*f \\
& ^8*g^3 - 44*d^7*e^9*f^7*g^4 + 76*d^8*e^8*f^6*g^5 + 102*d^9*e^7*f^5* \\
& g^6 - 6*d^{10}*e^6*f^4*g^7 - 53*d^{11}*e^5*f^3*g^8 - 19*d^{12}*e^4*f^2* \\
& g^9)*x^6 + 2*(d^4*e^{12}*f^{11} - 16*d^5*e^{11}*f^{10}*g - 46*d^6*e^{10}* \\
& f^9*g^2 + 6*d^7*e^9*f^8*g^3 + 100*d^8*e^8*f^7*g^4 + 46*d^9*e^7*f^6* \\
& g^5 - 66*d^{10}*e^6*f^5*g^6 - 46*d^{11}*e^5*f^4*g^7 + 11*d^{12}*e^4*f^3* \\
& g^8 + 10*d^{13}*e^3*f^2*g^9)*x^5 - (19*d^5*e^{11}*f^{11} + 17*d^6*e^ \\
& ^{10}*f^{10}*g - 121*d^7*e^9*f^9*g^2 - 195*d^8*e^8*f^8*g^3 + 85*d^9*e^ \\
& ^7*f^7*g^4 + 319*d^{10}*e^6*f^6*g^5 + 117*d^{11}*e^5*f^5*g^6 - 121*d^{11} \\
& 2*e^4*f^4*g^7 - 100*d^{13}*e^3*f^3*g^8 - 20*d^{14}*e^2*f^2*g^9)*x^4 + \\
& 20*(d^6*e^{10}*f^{11} + 5*d^7*e^9*f^{10}*g + 5*d^8*e^8*f^9*g^2 - 9*d^9* \\
& e^7*f^8*g^3 - 17*d^{10}*e^6*f^7*g^4 + d^{11}*e^5*f^6*g^5 + 15*d^{12}*e \\
& ^4*f^5*g^6 + 5*d^{13}*e^3*f^4*g^7 - 4*d^{14}*e^2*f^3*g^8 - 2*d^{15}*e*f^2* \\
& g^9)*x^3 + 4*(5*d^7*e^9*f^{11} - 5*d^8*e^8*f^{10}*g - 51*d^9*e^7*f^9* \\
& g^2 - 33*d^{10}*e^6*f^8*g^3 + 79*d^{11}*e^5*f^7*g^4 + 85*d^{12}*e^4* \\
& f^6*g^5 - 25*d^{13}*e^3*f^5*g^6 - 51*d^{14}*e^2*f^4*g^7 - 8*d^{15}*e*f^3* \\
& g^8 + 4*d^{16}*f^2*g^9)*x^2 - 8*(5*d^8*e^8*f^{11} + 11*d^9*e^7*f^{10} \\
& *g - 7*d^{10}*e^6*f^9*g^2 - 29*d^{11}*e^5*f^8*g^3 - 5*d^{12}*e^4*f^7*g^4 \\
& + 25*d^{13}*e^3*f^6*g^5 + 11*d^{14}*e^2*f^5*g^6 - 7*d^{15}*e*f^4*g^7 \\
& - 4*d^{16}*f^3*g^8)*x)*\text{sqrt}(-e^2*x^2 + d^2))*\text{sqrt}(e^2*f^2 - d^2*g^2 \\
& ))]
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3/(g*x+f)**3/(-e**2*x**2+d**2)**(7/2),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [A]** time = 1.15921, size = 1, normalized size = 0.

*Done*

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^3/((-e^2*x^2 + d^2)^(7/2)*(g*x + f)^3),x, algorithm="giac")
```

```
[Out] Done
```

$$3.588 \quad \int \frac{a+cx^2}{(d+ex)^{3/2}(f+gx)} dx$$

**Optimal.** Leaf size=112

$$-\frac{2(ae^2 + cd^2)}{e^2\sqrt{d+ex}(ef-dg)} - \frac{2(ag^2 + cf^2) \tan^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{3/2}(ef-dg)^{3/2}} + \frac{2c\sqrt{d+ex}}{e^2g}$$

[Out]  $(-2*(c*d^2 + a*e^2))/(e^2*(e*f - d*g)*\text{Sqrt}[d + e*x]) + (2*c*\text{Sqrt}[d + e*x])/(e^2*g) - (2*(c*f^2 + a*g^2)*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[d + e*x])/\text{Sqrt}[e*f - d*g]])/(g^{3/2}*(e*f - d*g)^{3/2})$

**Rubi [A]** time = 0.456118, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$-\frac{2(ae^2 + cd^2)}{e^2\sqrt{d+ex}(ef-dg)} - \frac{2(ag^2 + cf^2) \tan^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{ef-dg}}\right)}{g^{3/2}(ef-dg)^{3/2}} + \frac{2c\sqrt{d+ex}}{e^2g}$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^2)/((d + e\*x)^(3/2)\*(f + g\*x)), x]

[Out]  $(-2*(c*d^2 + a*e^2))/(e^2*(e*f - d*g)*\text{Sqrt}[d + e*x]) + (2*c*\text{Sqrt}[d + e*x])/(e^2*g) - (2*(c*f^2 + a*g^2)*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[d + e*x])/\text{Sqrt}[e*f - d*g]])/(g^{3/2}*(e*f - d*g)^{3/2})$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{2(ag^2 + cf^2) \operatorname{atanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{dg-ef}}\right)}{g^{\frac{3}{2}}(dg-ef)^{\frac{3}{2}}} + \frac{2(ae^2 + cd^2)}{e^2\sqrt{d+ex}(dg-ef)} + \frac{2\int\sqrt{d+ex} c dx}{e^2g}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+a)/(e\*x+d)\*\*(3/2)/(g\*x+f), x)

[Out]  $-2*(a*g^2 + c*f^2)*\operatorname{atanh}(\text{sqrt}(g)*\text{sqrt}(d + e*x)/\text{sqrt}(d*g - e*f))/((g^{3/2}*(d*g - e*f)^{3/2})) + 2*(a*e^2 + c*d^2)/(e^2*\text{sqrt}(d + e*x)*(d*g - e*f)) + 2*\text{Integral}(c, (x, \text{sqrt}(d + e*x)))/(e^2*g)$

**Mathematica [A]** time = 0.333416, size = 108, normalized size = 0.96

$$\frac{2\sqrt{d+ex}\left(\frac{ae^2+cd^2}{(d+ex)(dg-ef)} + \frac{c}{g}\right)}{e^2} - \frac{2(ag^2 + cf^2) \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{dg-ef}}\right)}{g^{3/2}(dg-ef)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c\*x^2)/((d + e\*x)^(3/2)\*(f + g\*x)), x]

[Out]  $(2*\text{Sqrt}[d + e*x]*(c/g + (c*d^2 + a*e^2)/((-e*f) + d*g)*(d + e*x)))/e^2 - (2*(c*f^2 + a*g^2)*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[d + e*x])/\text{Sqrt}$



$[-(e^*f) + d^*g]]/(g^{(3/2)}*(-(e^*f) + d^*g)^{(3/2)})$

**Maple [A]** time = 0.022, size = 114, normalized size = 1.

$$2 \frac{1}{e^2} \left( \frac{c\sqrt{ex+d}}{g} - \frac{e^2(ag^2 + cf^2)}{(dg - ef)g\sqrt{(dg - ef)g}} \operatorname{Artanh} \left( \frac{g\sqrt{ex+d}}{\sqrt{(dg - ef)g}} \right) - \frac{-ae^2 - cd^2}{(dg - ef)\sqrt{ex+d}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+a)/(e*x+d)^(3/2)/(g*x+f), x)`

[Out]  $2/e^2*(c/g*(e*x+d)^{(1/2)}-e^2*(a*g^2+c*f^2)/(d*g-e*f)/g/((d*g-e*f)*g)^{(1/2)}*\operatorname{arctanh}(g*(e*x+d)^{(1/2)/((d*g-e*f)*g)^{(1/2)})-(a*e^2-c*d^2)/(d*g-e*f)/(e*x+d)^{(1/2)})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)/((e*x + d)^(3/2)*(g*x + f)), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.290642, size = 1, normalized size = 0.01

$$\left[ \frac{(ce^2f^2 + ae^2g^2)\sqrt{ex+d} \log\left(\frac{\sqrt{-efg+dg^2}(egx-ef+2dg)+2(efg-dg^2)\sqrt{ex+d}}{gx+f}\right) - 2(cdef - (2cd^2 + ae^2)g + (ce^2f - cdeg)x)\sqrt{-efg+dg^2}\sqrt{ex+d}}{(e^3fg - de^2g^2)\sqrt{-efg+dg^2}\sqrt{ex+d}} \right. \\ \left. \frac{2\left((ce^2f^2 + ae^2g^2)\sqrt{ex+d} \arctan\left(-\frac{ef-dg}{\sqrt{efg-dg^2}\sqrt{ex+d}}\right) - (cdef - (2cd^2 + ae^2)g + (ce^2f - cdeg)x)\sqrt{efg-dg^2}\right)}{(e^3fg - de^2g^2)\sqrt{efg-dg^2}\sqrt{ex+d}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)/((e*x + d)^(3/2)*(g*x + f)), x, algorithm="fricas")`

[Out]  $[-((c^*e^2*f^2 + a^*e^2*g^2)*\operatorname{sqrt}(e^*x + d)*\log((\operatorname{sqrt}(-e^*f*g + d^*g^2)*(e^*g*x - e^*f + 2*d^*g) + 2*(e^*f*g - d^*g^2)*\operatorname{sqrt}(e^*x + d)))/(g^*x + f)) - 2*(c^*d^*e^*f - (2*c^*d^2 + a^*e^2)*g + (c^*e^2*f - c^*d^*e^*g)*x)*\operatorname{sqrt}(-e^*f*g + d^*g^2))/((e^3*f*g - d^*e^2*g^2)*\operatorname{sqrt}(-e^*f*g + d^*g^2)*\operatorname{sqrt}(e^*x + d)), -2*((c^*e^2*f^2 + a^*e^2*g^2)*\operatorname{sqrt}(e^*x + d)*\operatorname{arctan}((-e^*f - d^*g)/(\operatorname{sqrt}(e^*f*g - d^*g^2)*\operatorname{sqrt}(e^*x + d))) - (c^*d^*e^*f - (2*c^*d^2 + a^*e^2)*g + (c^*e^2*f - c^*d^*e^*g)*x)*\operatorname{sqrt}(e^*f*g - d^*g^2)))/((e^3*f*g - d^*e^2*g^2)*\operatorname{sqrt}(e^*f*g - d^*g^2)*\operatorname{sqrt}(e^*x + d))]$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + cx^2}{(d + ex)^{\frac{3}{2}}(f + gx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+a)/(e*x+d)**(3/2)/(g*x+f),x)`

[Out] `Integral((a + c*x**2)/((d + e*x)**(3/2)*(f + g*x)), x)`

**GIAC/XCAS [A]** time = 0.288194, size = 157, normalized size = 1.4

$$\frac{2\sqrt{xe+d}ce^{(-2)}}{g} + \frac{2(cf^2 + ag^2) \arctan\left(\frac{\sqrt{xe+dg}}{\sqrt{-dg^2+fge}}\right)}{(dg^2 - fge)\sqrt{-dg^2 + fge}} + \frac{2(cd^2 + ae^2)}{(dge^2 - fe^3)\sqrt{xe+d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)/((e*x + d)^(3/2)*(g*x + f)),x, algorithm="giac")`

[Out] `2*sqrt(x*e + d)*c*e^(-2)/g + 2*(c*f^2 + a*g^2)*arctan(sqrt(x*e + d)*g/sqrt(-d*g^2 + f*g*e))/((d*g^2 - f*g*e)*sqrt(-d*g^2 + f*g*e)) + 2*(c*d^2 + a*e^2)/((d*g*e^2 - f*e^3)*sqrt(x*e + d))`

$$3.589 \quad \int \frac{(d+ex)^3(a+cx^2)}{\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=240

$$\frac{2e(f+gx)^{7/2}(ae^2g^2+c(3d^2g^2-12defg+10e^2f^2))}{7g^6} - \frac{2(f+gx)^{5/2}(ef-dg)(3ae^2g^2+c(d^2g^2-8defg+10e^2f^2))}{5g^6} - \frac{2\sqrt{f+gx}(ag^2+cf^2)(ef-dg)^3}{g^6} + \frac{2(f+gx)^{3/2}(ef-dg)^2(3aeg^2+cf(5ef-2dg))}{3g^6} - \frac{2ce^2(f+gx)^{9/2}(5ef-3dg)}{9g^6} + \frac{2ce^3(f+gx)^{11/2}}{11g^6}$$

[Out]  $(-2*(e*f - d*g)^3*(c*f^2 + a*g^2)*\text{Sqrt}[f + g*x])/g^6 + (2*(e*f - d*g)^2*(3*a*e*g^2 + c*f*(5*e*f - 2*d*g))*(f + g*x)^{(3/2)})/(3*g^6) - (2*(e*f - d*g)*(3*a*e^2*g^2 + c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^{(5/2)})/(5*g^6) + (2*e*(a*e^2*g^2 + c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^{(7/2)})/(7*g^6) - (2*c*e^2*(5*e*f - 3*d*g)*(f + g*x)^{(9/2)})/(9*g^6) + (2*c*e^3*(f + g*x)^{(11/2)})/(11*g^6)$

**Rubi [A]** time = 0.683848, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{2e(f+gx)^{7/2}(ae^2g^2+c(3d^2g^2-12defg+10e^2f^2))}{7g^6} - \frac{2(f+gx)^{5/2}(ef-dg)(3ae^2g^2+c(d^2g^2-8defg+10e^2f^2))}{5g^6} - \frac{2\sqrt{f+gx}(ag^2+cf^2)(ef-dg)^3}{g^6} + \frac{2(f+gx)^{3/2}(ef-dg)^2(3aeg^2+cf(5ef-2dg))}{3g^6} - \frac{2ce^2(f+gx)^{9/2}(5ef-3dg)}{9g^6} + \frac{2ce^3(f+gx)^{11/2}}{11g^6}$$

Antiderivative was successfully verified.

[In]  $\text{Int}(((d + e*x)^3*(a + c*x^2))/\text{Sqrt}[f + g*x], x)$

[Out]  $(-2*(e*f - d*g)^3*(c*f^2 + a*g^2)*\text{Sqrt}[f + g*x])/g^6 + (2*(e*f - d*g)^2*(3*a*e*g^2 + c*f*(5*e*f - 2*d*g))*(f + g*x)^{(3/2)})/(3*g^6) - (2*(e*f - d*g)*(3*a*e^2*g^2 + c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^{(5/2)})/(5*g^6) + (2*e*(a*e^2*g^2 + c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^{(7/2)})/(7*g^6) - (2*c*e^2*(5*e*f - 3*d*g)*(f + g*x)^{(9/2)})/(9*g^6) + (2*c*e^3*(f + g*x)^{(11/2)})/(11*g^6)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2ce^3(f+gx)^{\frac{11}{2}}}{11g^6} + \frac{2ce^2(f+gx)^{\frac{9}{2}}(3dg-5ef)}{9g^6} + \frac{2e(f+gx)^{\frac{7}{2}}(ae^2g^2+3cd^2g^2-12cdefg+10ce^2f^2)}{7g^6} + \frac{2(ag^2+cf^2)(dg-ef)^3 \int \sqrt{f+gx} \frac{1}{g^5} dx}{g} + \frac{2(f+gx)^{\frac{5}{2}}(dg-ef)(3ae^2g^2+cd^2g^2-8cdefg+10ce^2f^2)}{5g^6} + \frac{2(f+gx)^{\frac{3}{2}}(dg-ef)^2(3aeg^2-2cdfg+5cef^2)}{3g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**3*(c*x**2+a)/(g*x+f)**(1/2),x)`

[Out]  $2*c*e^{3*(f+g*x)^{11/2}}/(11*g^6) + 2*c*e^{2*(f+g*x)^{9/2}}*(3*d*g - 5*e*f)/(9*g^6) + 2*e*(f+g*x)^{7/2}*(a*e^{2*g^2} + 3*c*d^2*g^2 - 12*c*d*e*f*g + 10*c*e^{2*f^2})/(7*g^6) + 2*(a*g^2 + c*f^2)*(d*g - e*f)^3*\text{Integral}(g^{-5}, (x, \text{sqrt}(f+g*x)))/g + 2*(f+g*x)^{5/2}*(d*g - e*f)*(3*a*e^{2*g^2} + c*d^2*g^2 - 8*c*d*e*f*g + 10*c*e^{2*f^2})/(5*g^6) + 2*(f+g*x)^{3/2}*(d*g - e*f)^2*(3*a*e*g^2 - 2*c*d*f*g + 5*c*e*f^2)/(3*g^6)$

**Mathematica [A]** time = 0.485726, size = 282, normalized size = 1.18

$2\sqrt{f+gx}(99ag^2(35d^3g^3+35d^2eg^2(gx-2f))+7de^2g(8f^2-4fgx+3g^2x^2))+e^3(-16f^3+8f^2gx-6fg^2x^2+5g^3x^3))+c$

Antiderivative was successfully verified.

[In] `Integrate[((d + e*x)^3*(a + c*x^2))/Sqrt[f + g*x],x]`

[Out]  $(2*\text{Sqrt}[f+g*x]*(99*a*g^2*(35*d^3*g^3+35*d^2*e*g^2*(-2*f+g*x))+7*d*e^2*g*(8*f^2-4*f*g*x+3*g^2*x^2))+e^3*(-16*f^3+8*f^2*g*x-6*f*g^2*x^2+5*g^3*x^3))+c*(231*d^3*g^3*(8*f^2-4*f*g*x+3*g^2*x^2))+297*d^2*e*g^2*(-16*f^3+8*f^2*g*x-6*f*g^2*x^2+5*g^3*x^3))+33*d*e^2*g*(128*f^4-64*f^3*g*x+48*f^2*g^2*x^2-40*f*g^3*x^3+35*g^4*x^4)-5*e^3*(256*f^5-128*f^4*g*x+96*f^3*g^2*x^2-80*f^2*g^3*x^3+70*f*g^4*x^4-63*g^5*x^5)))/(3465*g^6)$

**Maple [A]** time = 0.01, size = 365, normalized size = 1.5

$630 e^3 c x^5 g^5 + 2310 c d e^2 g^5 x^4 - 700 c e^3 f g^4 x^4 + 990 a e^3 g^5 x^3 + 2970 c d^2 e g^5 x^3 - 2640 c d e^2 f g^4 x^3 + 800 c e^3 f^2 g^3 x^3 + 4158 a d e^3$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^3*(c*x^2+a)/(g*x+f)^(1/2),x)`

[Out]  $2/3465*(g*x+f)^{1/2}*(315*c*e^3*g^5*x^5+1155*c*d*e^2*g^5*x^4-350*c*e^3*f*g^4*x^4+495*a*e^3*g^5*x^3+1485*c*d^2*e*g^5*x^3-1320*c*d*e^2*f*g^4*x^3+400*c*e^3*f^2*g^3*x^3+2079*a*d*e^2*g^5*x^2-594*a*e^3*f*g^4*x^2+693*c*d^3*g^5*x^2-1782*c*d^2*e*f*g^4*x^2+1584*c*d*e^2*f^2*g^3*x^2-480*c*e^3*f^3*g^2*x^2+3465*a*d^2*e*g^5*x-2772*a*d*e^2*f*g^4*x+792*a*e^3*f^2*g^3*x-924*c*d^3*f*g^4*x+2376*c*d^2*e*f^2*g^3*x-2112*c*d*e^2*f^3*g^2*x+640*c*e^3*f^4*g*x+3465*a*d^3*g^5-6930*a*d^2*e*f*g^4+5544*a*d*e^2*f^2*g^3-1584*a*e^3*f^3*g^2+1848*c*d^3*f^2*g^3-4752*c*d^2*e*f^3*g^2+4224*c*d*e^2*f^4*g-1280*c*e^3*f^5)/g^6$

**Maxima [A]** time = 0.700273, size = 440, normalized size = 1.83

$2\left(315(gx+f)^{\frac{11}{2}}ce^3-385(5ce^3f-3cde^2g)(gx+f)^{\frac{9}{2}}+495(10ce^3f^2-12cde^2fg+(3cd^2e+ae^3)g^2)(gx+f)^{\frac{7}{2}}-693(1$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)*(e*x + d)^3/sqrt(g*x + f),x, algorithm="maxima")`

```
[Out] 2/3465*(315*(g*x + f)^(11/2)*c*e^3 - 385*(5*c*e^3*f - 3*c*d*e^2*g)
)*(g*x + f)^(9/2) + 495*(10*c*e^3*f^2 - 12*c*d*e^2*f*g + (3*c*d^2
*e + a*e^3)*g^2)*(g*x + f)^(7/2) - 693*(10*c*e^3*f^3 - 18*c*d*e^2
*f^2*g + 3*(3*c*d^2*e + a*e^3)*f*g^2 - (c*d^3 + 3*a*d*e^2)*g^3)*(
g*x + f)^(5/2) + 1155*(5*c*e^3*f^4 - 12*c*d*e^2*f^3*g + 3*a*d^2*e
*g^4 + 3*(3*c*d^2*e + a*e^3)*f^2*g^2 - 2*(c*d^3 + 3*a*d*e^2)*f*g^
3)*(g*x + f)^(3/2) - 3465*(c*e^3*f^5 - 3*c*d*e^2*f^4*g + 3*a*d^2*
e*f*g^4 - a*d^3*g^5 + (3*c*d^2*e + a*e^3)*f^3*g^2 - (c*d^3 + 3*a*
d*e^2)*f^2*g^3)*sqrt(g*x + f))/g^6
```

---

**Fricas [A]** time = 0.276816, size = 437, normalized size = 1.82

$$2(315ce^3g^5x^5 - 1280ce^3f^5 + 4224cde^2f^4g - 6930ad^2efg^4 + 3465ad^3g^5 - 1584(3cd^2e + ae^3)f^3g^2 + 1848(cd^3 + 3ade^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + a)*(e*x + d)^3/sqrt(g*x + f), x, algorithm="fricas")
```

```
[Out] 2/3465*(315*c*e^3*g^5*x^5 - 1280*c*e^3*f^5 + 4224*c*d*e^2*f^4*g -
6930*a*d^2*e*f*g^4 + 3465*a*d^3*g^5 - 1584*(3*c*d^2*e + a*e^3)*f
^3*g^2 + 1848*(c*d^3 + 3*a*d*e^2)*f^2*g^3 - 35*(10*c*e^3*f*g^4 -
33*c*d*e^2*g^5)*x^4 + 5*(80*c*e^3*f^2*g^3 - 264*c*d*e^2*f*g^4 + 9
9*(3*c*d^2*e + a*e^3)*g^5)*x^3 - 3*(160*c*e^3*f^3*g^2 - 528*c*d*e
^2*f^2*g^3 + 198*(3*c*d^2*e + a*e^3)*f*g^4 - 231*(c*d^3 + 3*a*d*e
^2)*g^5)*x^2 + (640*c*e^3*f^4*g - 2112*c*d*e^2*f^3*g^2 + 3465*a*d
^2*e*g^5 + 792*(3*c*d^2*e + a*e^3)*f^2*g^3 - 924*(c*d^3 + 3*a*d*e
^2)*f*g^4)*x)*sqrt(g*x + f)/g^6
```

---

**Sympy [A]** time = 76.1024, size = 1040, normalized size = 4.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+a)/(g*x+f)**(1/2), x)
```

```
[Out] Piecewise((- (2*a*d**3*f/sqrt(f + g*x) + 2*a*d**3*(-f/sqrt(f + g*x)
) - sqrt(f + g*x)) + 6*a*d**2*e*f*(-f/sqrt(f + g*x) - sqrt(f + g*
x))/g + 6*a*d**2*e*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f +
g*x)**(3/2)/3)/g + 6*a*d*e**2*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f
+ g*x) - (f + g*x)**(3/2)/3)/g**2 + 6*a*d*e**2*(-f**3/sqrt(f + g
*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2
)/5)/g**2 + 2*a*e**3*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x
) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**3 + 2*a*e**3*(f**
4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2)
+ 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**3 + 2*c*d**3*f*
(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**
2 + 2*c*d**3*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f +
g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 + 6*c*d**2*e*f*(-f**3/sqr
t(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x
)**(5/2)/5)/g**3 + 6*c*d**2*e*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f
+ g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f +
g*x)**(7/2)/7)/g**3 + 6*c*d*e**2*f*(f**4/sqrt(f + g*x) + 4*f**3*
sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5
- (f + g*x)**(7/2)/7)/g**4 + 6*c*d*e**2*(-f**5/sqrt(f + g*x) - 5*
f**4*sqrt(f + g*x) + 10*f**3*(f + g*x)**(3/2)/3 - 2*f**2*(f + g*x
)**(5/2) + 5*f*(f + g*x)**(7/2)/7 - (f + g*x)**(9/2)/9)/g**4 + 2*
c*e**3*f*(-f**5/sqrt(f + g*x) - 5*f**4*sqrt(f + g*x) + 10*f**3*(f
+ g*x)**(3/2)/3 - 2*f**2*(f + g*x)**(5/2) + 5*f*(f + g*x)**(7/2)
/7 - (f + g*x)**(9/2)/9)/g**5 + 2*c*e**3*(f**6/sqrt(f + g*x) + 6*
f**5*sqrt(f + g*x) - 5*f**4*(f + g*x)**(3/2) + 4*f**3*(f + g*x)**
```

$(5/2) - 15*f**2*(f + g*x)**(7/2)/7 + 2*f*(f + g*x)**(9/2)/3 - (f + g*x)**(11/2)/11)/g**5)/g, Ne(g, 0)), ((a*d**3*x + 3*a*d**2*e*x**2/2 + 3*c*d*e**2*x**5/5 + c*e**3*x**6/6 + x**4*(a*e**3 + 3*c*d**2*e)/4 + x**3*(3*a*d*e**2 + c*d**3)/3)/sqrt(f), True))$

**GIAC/XCAS [A]** time = 0.270841, size = 612, normalized size = 2.55

$$2 \left( 3465 \sqrt{gx+f} ad^3 + \frac{3465 \left( (gx+f)^{\frac{3}{2}} - 3 \sqrt{gx+ff} \right) ad^2 e}{g} + \frac{231 \left( 3(gx+f)^{\frac{5}{2}} g^8 - 10(gx+f)^{\frac{3}{2}} f g^8 + 15 \sqrt{gx+ff} f^2 g^8 \right) cd^3}{g^{10}} + \frac{693 \left( 3(gx+f)^{\frac{5}{2}} g^8 - 10(gx+f)^{\frac{3}{2}} f g^8 \right)}{g^{10}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)\*(e\*x + d)^3/sqrt(g\*x + f),x, algorithm="giac")

[Out]  $2/3465*(3465*\sqrt{g*x + f}*a*d^3 + 3465*((g*x + f)^{(3/2)} - 3*\sqrt{g*x + f}*f)*a*d^2*e/g + 231*(3*(g*x + f)^{(5/2)}*g^8 - 10*(g*x + f)^{(3/2)}*f*g^8 + 15*\sqrt{g*x + f}*f^2*g^8)*c*d^3/g^{10} + 693*(3*(g*x + f)^{(5/2)}*g^8 - 10*(g*x + f)^{(3/2)}*f*g^8 + 15*\sqrt{g*x + f}*f^2*g^8)*a*d^2*e/g^{10} + 297*(5*(g*x + f)^{(7/2)}*g^{18} - 21*(g*x + f)^{(5/2)}*f*g^{18} + 35*(g*x + f)^{(3/2)}*f^2*g^{18} - 35*\sqrt{g*x + f}*f^3*g^{18})*c*d^2*e/g^{21} + 99*(5*(g*x + f)^{(7/2)}*g^{18} - 21*(g*x + f)^{(5/2)}*f*g^{18} + 35*(g*x + f)^{(3/2)}*f^2*g^{18} - 35*\sqrt{g*x + f}*f^3*g^{18})*a*e^3/g^{21} + 33*(35*(g*x + f)^{(9/2)}*g^{32} - 180*(g*x + f)^{(7/2)}*f*g^{32} + 378*(g*x + f)^{(5/2)}*f^2*g^{32} - 420*(g*x + f)^{(3/2)}*f^3*g^{32} + 315*\sqrt{g*x + f}*f^4*g^{32})*c*d^2*e^2/g^{36} + 5*(63*(g*x + f)^{(11/2)}*g^{50} - 385*(g*x + f)^{(9/2)}*f*g^{50} + 990*(g*x + f)^{(7/2)}*f^2*g^{50} - 1386*(g*x + f)^{(5/2)}*f^3*g^{50} + 1155*(g*x + f)^{(3/2)}*f^4*g^{50} - 693*\sqrt{g*x + f}*f^5*g^{50})*c*e^3/g^{55})/g$

$$3.590 \quad \int \frac{(d+ex)^2(a+cx^2)}{\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=175

$$\frac{2(f+gx)^{5/2}(ae^2g^2+c(d^2g^2-6defg+6e^2f^2))}{5g^5} + \frac{2\sqrt{f+gx}(ag^2+cf^2)(ef-dg)^2}{g^5} - \frac{4(f+gx)^{3/2}(ef-dg)(aeg^2+cf(2ef-dg))}{3g^5} - \frac{4ce(f+gx)^{7/2}(2ef-dg)}{7g^5} + \frac{2ce^2(f+gx)^{9/2}}{9g^5}$$

[Out]  $(2*(e*f - d*g)^2*(c*f^2 + a*g^2)*\text{Sqrt}[f + g*x])/g^5 - (4*(e*f - d*g)*(a*e*g^2 + c*f*(2*e*f - d*g))*(f + g*x)^{(3/2)})/(3*g^5) + (2*(a*e^2*g^2 + c*(6*e^2*f^2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^{(5/2)})/(5*g^5) - (4*c*e*(2*e*f - d*g)*(f + g*x)^{(7/2)})/(7*g^5) + (2*c*e^2*(f + g*x)^{(9/2)})/(9*g^5)$

**Rubi [A]** time = 0.49055, antiderivative size = 175, normalized size of antiderivative = 1., number of rules used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{2(f+gx)^{5/2}(ae^2g^2+c(d^2g^2-6defg+6e^2f^2))}{5g^5} + \frac{2\sqrt{f+gx}(ag^2+cf^2)(ef-dg)^2}{g^5} - \frac{4(f+gx)^{3/2}(ef-dg)(aeg^2+cf(2ef-dg))}{3g^5} - \frac{4ce(f+gx)^{7/2}(2ef-dg)}{7g^5} + \frac{2ce^2(f+gx)^{9/2}}{9g^5}$$

Antiderivative was successfully verified.

[In]  $\text{Int}(((d + e*x)^2*(a + c*x^2))/\text{Sqrt}[f + g*x], x)$

[Out]  $(2*(e*f - d*g)^2*(c*f^2 + a*g^2)*\text{Sqrt}[f + g*x])/g^5 - (4*(e*f - d*g)*(a*e*g^2 + c*f*(2*e*f - d*g))*(f + g*x)^{(3/2)})/(3*g^5) + (2*(a*e^2*g^2 + c*(6*e^2*f^2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^{(5/2)})/(5*g^5) - (4*c*e*(2*e*f - d*g)*(f + g*x)^{(7/2)})/(7*g^5) + (2*c*e^2*(f + g*x)^{(9/2)})/(9*g^5)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2ce^2(f+gx)^{\frac{9}{2}}}{9g^5} + \frac{4ce(f+gx)^{\frac{7}{2}}(dg-2ef)}{7g^5} + \frac{2(ag^2+cf^2)(dg-ef)^2 \int^{\sqrt{f+gx}} \frac{1}{g^4} dx}{g} + \frac{2(f+gx)^{\frac{5}{2}}(ae^2g^2+cd^2g^2-6cdefg+6ce^2f^2)}{5g^5} + \frac{4(f+gx)^{\frac{3}{2}}(dg-ef)(aeg^2-cdfg+2cef^2)}{3g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)**2*(c*x**2+a)/(g*x+f)**(1/2), x)$

[Out]  $2*c*e**2*(f + g*x)**(9/2)/(9*g**5) + 4*c*e*(f + g*x)**(7/2)*(d*g - 2*e*f)/(7*g**5) + 2*(a*g**2 + c*f**2)*(d*g - e*f)**2*\text{Integral}(g**(-4), (x, \text{sqrt}(f + g*x)))/g + 2*(f + g*x)**(5/2)*(a*e**2*g**2 + c*d**2*g**2 - 6*c*d*e*f*g + 6*c*e**2*f**2)/(5*g**5) + 4*(f + g*x)**(3/2)*(d*g - e*f)*(a*e*g**2 - c*d*f*g + 2*c*e*f**2)/(3*g**5)$

**Mathematica [A]** time = 0.186395, size = 177, normalized size = 1.01

$$\frac{2\sqrt{f+gx}(21ag^2(15d^2g^2+10deg(gx-2f))+e^2(8f^2-4fgx+3g^2x^2))+c(21d^2g^2(8f^2-4fgx+3g^2x^2)+18deg(-16f+315g^5))}{315g^5}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^2\*(a + c\*x^2))/Sqrt[f + g\*x],x]

[Out] (2\*Sqrt[f + g\*x]\*(21\*a\*g^2\*(15\*d^2\*g^2 + 10\*d\*e\*g\*(-2\*f + g\*x) + e^2\*(8\*f^2 - 4\*f\*g\*x + 3\*g^2\*x^2)) + c\*(21\*d^2\*g^2\*(8\*f^2 - 4\*f\*g\*x + 3\*g^2\*x^2) + 18\*d\*e\*g\*(-16\*f^3 + 8\*f^2\*g\*x - 6\*f\*g^2\*x^2 + 5\*g^3\*x^3) + e^2\*(128\*f^4 - 64\*f^3\*g\*x + 48\*f^2\*g^2\*x^2 - 40\*f\*g^3\*x^3 + 35\*g^4\*x^4)))/(315\*g^5)

**Maple [A]** time = 0.01, size = 215, normalized size = 1.2

$$\frac{70 e^2 c x^4 g^4 + 180 c d e g^4 x^3 - 80 c e^2 f g^3 x^3 + 126 a e^2 g^4 x^2 + 126 c d^2 g^4 x^2 - 216 c d e f g^3 x^2 + 96 c e^2 f^2 g^2 x^2 + 420 a d e g^4 x - 168 a d e^2 g^4}{315 g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^2\*(c\*x^2+a)/(g\*x+f)^(1/2),x)

[Out] 2/315\*(g\*x+f)^(1/2)\*(35\*c\*e^2\*g^4\*x^4+90\*c\*d\*e\*g^4\*x^3-40\*c\*e^2\*f\*g^3\*x^3+63\*a\*e^2\*g^4\*x^2+63\*c\*d^2\*g^4\*x^2-108\*c\*d\*e\*f\*g^3\*x^2+48\*c\*e^2\*f^2\*g^2\*x^2+210\*a\*d\*e\*g^4\*x-84\*a\*e^2\*f\*g^3\*x-84\*c\*d^2\*f\*g^3\*x+144\*c\*d\*e\*f^2\*g^2\*x-64\*c\*e^2\*f^3\*g\*x+315\*a\*d^2\*g^4-420\*a\*d\*e\*f\*g^3+168\*a\*e^2\*f^2\*g^2+168\*c\*d^2\*f^2\*g^2-288\*c\*d\*e\*f^3\*g+128\*c\*e^2\*f^4)/g^5

**Maxima [A]** time = 0.704692, size = 266, normalized size = 1.52

$$\frac{2 \left( 35 (g x + f)^{\frac{9}{2}} c e^2 - 90 (2 c e^2 f - c d e g) (g x + f)^{\frac{7}{2}} + 63 (6 c e^2 f^2 - 6 c d e f g + (c d^2 + a e^2) g^2) (g x + f)^{\frac{5}{2}} - 210 (2 c e^2 f^3 - 3 c d e f^2 g + 3 a e^2 f^2) (g x + f)^{\frac{3}{2}} + 315 a d^2 g^4 - 420 a d e f^3 g + 168 a e^2 f^2 g^2 - 168 c d^2 f^2 g^2 - 288 c d e f^3 g + 128 c e^2 f^4 \right)}{315 g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)\*(e\*x + d)^2/sqrt(g\*x + f),x, algorithm="maxima")

[Out] 2/315\*(35\*(g\*x + f)^(9/2)\*c\*e^2 - 90\*(2\*c\*e^2\*f - c\*d\*e\*g)\*(g\*x + f)^(7/2) + 63\*(6\*c\*e^2\*f^2 - 6\*c\*d\*e\*f\*g + (c\*d^2 + a\*e^2)\*g^2)\*(g\*x + f)^(5/2) - 210\*(2\*c\*e^2\*f^3 - 3\*c\*d\*e\*f^2\*g - a\*d\*e\*g^3 + (c\*d^2 + a\*e^2)\*f\*g^2)\*(g\*x + f)^(3/2) + 315\*(c\*e^2\*f^4 - 2\*c\*d\*e\*f^3\*g - 2\*a\*d\*e\*f\*g^2 + a\*d^2\*g^4 + (c\*d^2 + a\*e^2)\*f^2\*g^2)\*sqrt(g\*x + f))/g^5

**Fricas [A]** time = 0.271521, size = 266, normalized size = 1.52

$$\frac{2 \left( 35 c e^2 g^4 x^4 + 128 c e^2 f^4 - 288 c d e f^3 g - 420 a d e f g^3 + 315 a d^2 g^4 + 168 (c d^2 + a e^2) f^2 g^2 - 10 (4 c e^2 f g^3 - 9 c d e g^4) x^3 + 3 (16 c e^2 f^2 g^2 - 36 c d e f g^3 + 21 (c d^2 + a e^2) g^4) x^2 - 2 (32 c e^2 f^3 g - 72 c d e f^2 g^2 + 35 c e^2 f^4 - 128 c d e f^3 g + 168 a d e f g^3 - 168 c d^2 f^2 g^2 - 288 c d e f^3 g + 128 c e^2 f^4) \right)}{315 g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)\*(e\*x + d)^2/sqrt(g\*x + f),x, algorithm="fricas")

[Out] 2/315\*(35\*c\*e^2\*g^4\*x^4 + 128\*c\*e^2\*f^4 - 288\*c\*d\*e\*f^3\*g - 420\*a\*d\*e\*f\*g^3 + 315\*a\*d^2\*g^4 + 168\*(c\*d^2 + a\*e^2)\*f^2\*g^2 - 10\*(4\*c\*e^2\*f\*g^3 - 9\*c\*d\*e\*g^4)\*x^3 + 3\*(16\*c\*e^2\*f^2\*g^2 - 36\*c\*d\*e\*f\*g^3 + 21\*(c\*d^2 + a\*e^2)\*g^4)\*x^2 - 2\*(32\*c\*e^2\*f^3\*g - 72\*c\*d\*e\*f^2\*g^2 + 35\*c\*e^2\*f^4 - 128\*c\*d\*e\*f^3\*g + 168\*a\*d\*e\*f\*g^3 - 168\*c\*d^2\*f^2\*g^2 - 288\*c\*d\*e\*f^3\*g + 128\*c\*e^2\*f^4)/g^5



$$*f^2*g^2 - 105*a*d*e*g^4 + 42*(c*d^2 + a*e^2)*f*g^3)*x)*\sqrt{g*x + f}/g^5$$

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**Sympy [A]** time = 44.9736, size = 673, normalized size = 3.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2\*(c\*x\*\*2+a)/(g\*x+f)\*\*(1/2),x)

[Out] Piecewise(((2\*a\*d\*\*2\*f/sqrt(f + g\*x) + 2\*a\*d\*\*2\*(-f/sqrt(f + g\*x) - sqrt(f + g\*x)))/g + 4\*a\*d\*e\*(f\*\*2/sqrt(f + g\*x) + 2\*f\*sqrt(f + g\*x) - (f + g\*x)\*\*(3/2)/3)/g + 2\*a\*e\*\*2\*f\*(f\*\*2/sqrt(f + g\*x) + 2\*f\*sqrt(f + g\*x) - (f + g\*x)\*\*(3/2)/3)/g\*\*2 + 2\*a\*e\*\*2\*(-f\*\*3/sqrt(f + g\*x) - 3\*f\*\*2\*sqrt(f + g\*x) + f\*(f + g\*x)\*\*(3/2) - (f + g\*x)\*\*(5/2)/5)/g\*\*2 + 2\*c\*d\*\*2\*f\*(f\*\*2/sqrt(f + g\*x) + 2\*f\*sqrt(f + g\*x) - (f + g\*x)\*\*(3/2)/3)/g\*\*2 + 2\*c\*d\*\*2\*(-f\*\*3/sqrt(f + g\*x) - 3\*f\*\*2\*sqrt(f + g\*x) + f\*(f + g\*x)\*\*(3/2) - (f + g\*x)\*\*(5/2)/5)/g\*\*2 + 4\*c\*d\*e\*f\*(-f\*\*3/sqrt(f + g\*x) - 3\*f\*\*2\*sqrt(f + g\*x) + f\*(f + g\*x)\*\*(3/2) - (f + g\*x)\*\*(5/2)/5)/g\*\*3 + 4\*c\*d\*e\*(f\*\*4/sqrt(f + g\*x) + 4\*f\*\*3\*sqrt(f + g\*x) - 2\*f\*\*2\*(f + g\*x)\*\*(3/2) + 4\*f\*(f + g\*x)\*\*(5/2)/5 - (f + g\*x)\*\*(7/2)/7)/g\*\*3 + 2\*c\*e\*\*2\*f\*(f\*\*4/sqrt(f + g\*x) + 4\*f\*\*3\*sqrt(f + g\*x) - 2\*f\*\*2\*(f + g\*x)\*\*(3/2) + 4\*f\*(f + g\*x)\*\*(5/2)/5 - (f + g\*x)\*\*(7/2)/7)/g\*\*4 + 2\*c\*e\*\*2\*(-f\*\*5/sqrt(f + g\*x) - 5\*f\*\*4\*sqrt(f + g\*x) + 10\*f\*\*3\*(f + g\*x)\*\*(3/2)/3 - 2\*f\*\*2\*(f + g\*x)\*\*(5/2) + 5\*f\*(f + g\*x)\*\*(7/2)/7 - (f + g\*x)\*\*(9/2)/9)/g\*\*4)/g, Ne(g, 0)), ((a\*d\*\*2\*x + a\*d\*e\*x\*\*2 + c\*d\*e\*x\*\*4/2 + c\*e\*\*2\*x\*\*5/5 + x\*\*3\*(a\*e\*\*2 + c\*d\*\*2)/3)/sqrt(f), True))

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**GIAC/XCAS [A]** time = 0.270121, size = 389, normalized size = 2.22

$$2 \left( 315 \sqrt{gx + f} ad^2 + \frac{210 \left( (gx+f)^{\frac{3}{2}} - 3 \sqrt{gx+ff} \right) ade}{g} + \frac{21 \left( 3(gx+f)^{\frac{5}{2}} g^8 - 10(gx+f)^{\frac{3}{2}} f g^8 + 15 \sqrt{gx+ff} f^2 g^8 \right) cd^2}{g^{10}} + \frac{21 \left( 3(gx+f)^{\frac{5}{2}} g^8 - 10(gx+f)^{\frac{3}{2}} f g^8 + 15 \sqrt{gx+ff} f^2 g^8 \right)}{g^{10}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)\*(e\*x + d)^2/sqrt(g\*x + f),x, algorithm="giac")

[Out] 2/315\*(315\*sqrt(g\*x + f)\*a\*d^2 + 210\*((g\*x + f)^(3/2) - 3\*sqrt(g\*x + f)\*f)\*a\*d\*e/g + 21\*(3\*(g\*x + f)^(5/2)\*g^8 - 10\*(g\*x + f)^(3/2)\*f\*g^8 + 15\*sqrt(g\*x + f)\*f^2\*g^8)\*c\*d^2/g^10 + 21\*(3\*(g\*x + f)^(5/2)\*g^8 - 10\*(g\*x + f)^(3/2)\*f\*g^8 + 15\*sqrt(g\*x + f)\*f^2\*g^8)\*a\*e^2/g^10 + 18\*(5\*(g\*x + f)^(7/2)\*g^18 - 21\*(g\*x + f)^(5/2)\*f\*g^18 + 35\*(g\*x + f)^(3/2)\*f^2\*g^18 - 35\*sqrt(g\*x + f)\*f^3\*g^18)\*c\*d\*e/g^21 + (35\*(g\*x + f)^(9/2)\*g^32 - 180\*(g\*x + f)^(7/2)\*f\*g^32 + 378\*(g\*x + f)^(5/2)\*f^2\*g^32 - 420\*(g\*x + f)^(3/2)\*f^3\*g^32 + 315\*sqrt(g\*x + f)\*f^4\*g^32)\*c\*e^2/g^36)/g

$$3.591 \quad \int \frac{(d+ex)(a+cx^2)}{\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=113

$$\begin{aligned} & -\frac{2\sqrt{f+gx}(ag^2+cf^2)(ef-dg)}{g^4} + \frac{2(f+gx)^{3/2}(aeg^2+cf(3ef-2dg))}{3g^4} \\ & -\frac{2c(f+gx)^{5/2}(3ef-dg)}{5g^4} + \frac{2ce(f+gx)^{7/2}}{7g^4} \end{aligned}$$

[Out]  $(-2*(e*f - d*g)*(c*f^2 + a*g^2)*\text{Sqrt}[f + g*x])/g^4 + (2*(a*e*g^2 + c*f*(3*e*f - 2*d*g))*(f + g*x)^{(3/2)})/(3*g^4) - (2*c*(3*e*f - d*g)*(f + g*x)^{(5/2)})/(5*g^4) + (2*c*e*(f + g*x)^{(7/2)})/(7*g^4)$

**Rubi [A]** time = 0.176672, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\begin{aligned} & -\frac{2\sqrt{f+gx}(ag^2+cf^2)(ef-dg)}{g^4} + \frac{2(f+gx)^{3/2}(aeg^2+cf(3ef-2dg))}{3g^4} \\ & -\frac{2c(f+gx)^{5/2}(3ef-dg)}{5g^4} + \frac{2ce(f+gx)^{7/2}}{7g^4} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)*(a + c*x^2)/\text{Sqrt}[f + g*x], x]$

[Out]  $(-2*(e*f - d*g)*(c*f^2 + a*g^2)*\text{Sqrt}[f + g*x])/g^4 + (2*(a*e*g^2 + c*f*(3*e*f - 2*d*g))*(f + g*x)^{(3/2)})/(3*g^4) - (2*c*(3*e*f - d*g)*(f + g*x)^{(5/2)})/(5*g^4) + (2*c*e*(f + g*x)^{(7/2)})/(7*g^4)$

**Rubi in Sympy [A]** time = 23.0326, size = 112, normalized size = 0.99

$$\begin{aligned} & \frac{2ce(f+gx)^{\frac{7}{2}}}{7g^4} + \frac{2c(f+gx)^{\frac{5}{2}}(dg-3ef)}{5g^4} \\ & + \frac{2(f+gx)^{\frac{3}{2}}(aeg^2-2cdfg+3cef^2)}{3g^4} + \frac{2\sqrt{f+gx}(ag^2+cf^2)(dg-ef)}{g^4} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)*(c*x^2+a)/(g*x+f)^{(1/2)}, x)$

[Out]  $2*c*e*(f + g*x)^{(7/2)}/(7*g^4) + 2*c*(f + g*x)^{(5/2)}*(d*g - 3*e*f)/(5*g^4) + 2*(f + g*x)^{(3/2)}*(a*e*g^2 - 2*c*d*f*g + 3*c*e*f^2)/(3*g^4) + 2*\text{sqrt}(f + g*x)*(a*g^2 + c*f^2)*(d*g - e*f)/g^4$

**Mathematica [A]** time = 0.0994581, size = 94, normalized size = 0.83

$$\frac{2\sqrt{f+gx}(35ag^2(3dg-2ef+egx)+7cdg(8f^2-4fgx+3g^2x^2)-3ce(16f^3-8f^2gx+6fg^2x^2-5g^3x^3))}{105g^4}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(a + c\*x^2))/Sqrt[f + g\*x], x]

[Out] (2\*Sqrt[f + g\*x]\*(35\*a\*g^2\*(-2\*e\*f + 3\*d\*g + e\*g\*x) + 7\*c\*d\*g\*(8\*f^2 - 4\*f\*g\*x + 3\*g^2\*x^2) - 3\*c\*e\*(16\*f^3 - 8\*f^2\*g\*x + 6\*f\*g^2\*x^2 - 5\*g^3\*x^3)))/(105\*g^4)

**Maple [A]** time = 0.007, size = 101, normalized size = 0.9

$$\frac{30 cex^3g^3 + 42 cdg^3x^2 - 36 cefg^2x^2 + 70 aeg^3x - 56 cdfg^2x + 48 cef^2gx + 210 adg^3 - 140 aefg^2 + 112 cdf^2g - 96 cef^3}{105g^4} \sqrt{g}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)\*(c\*x^2+a)/(g\*x+f)^(1/2), x)

[Out] 2/105\*(g\*x+f)^(1/2)\*(15\*c\*e\*g^3\*x^3+21\*c\*d\*g^3\*x^2-18\*c\*e\*f\*g^2\*x^2+35\*a\*e\*g^3\*x-28\*c\*d\*f\*g^2\*x+24\*c\*e\*f^2\*g\*x+105\*a\*d\*g^3-70\*a\*e\*f\*g^2+56\*c\*d\*f^2\*g-48\*c\*e\*f^3)/g^4

**Maxima [A]** time = 0.694818, size = 140, normalized size = 1.24

$$\frac{2 \left( 15(gx + f)^{\frac{7}{2}}ce - 21(3cef - cdg)(gx + f)^{\frac{5}{2}} + 35(3cef^2 - 2cdfg + aeg^2)(gx + f)^{\frac{3}{2}} - 105(cef^3 - cdf^2g + aefg^2 - adg^3) \right)}{105g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)\*(e\*x + d)/sqrt(g\*x + f), x, algorithm="maxima")

[Out] 2/105\*(15\*(g\*x + f)^(7/2)\*c\*e - 21\*(3\*c\*e\*f - c\*d\*g)\*(g\*x + f)^(5/2) + 35\*(3\*c\*e\*f^2 - 2\*c\*d\*f\*g + a\*e\*g^2)\*(g\*x + f)^(3/2) - 105\*(c\*e\*f^3 - c\*d\*f^2\*g + a\*e\*f\*g^2 - a\*d\*g^3)\*sqrt(g\*x + f))/g^4

**Fricas [A]** time = 0.272993, size = 135, normalized size = 1.19

$$\frac{2(15ceg^3x^3 - 48cef^3 + 56cdf^2g - 70aefg^2 + 105adg^3 - 3(6cef^2g - 7cdg^3)x^2 + (24cef^2g - 28cdfg^2 + 35aeg^3)x)\sqrt{g}}{105g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)\*(e\*x + d)/sqrt(g\*x + f), x, algorithm="fricas")

[Out] 2/105\*(15\*c\*e\*g^3\*x^3 - 48\*c\*e\*f^3 + 56\*c\*d\*f^2\*g - 70\*a\*e\*f\*g^2 + 105\*a\*d\*g^3 - 3\*(6\*c\*e\*f\*g^2 - 7\*c\*d\*g^3)\*x^2 + (24\*c\*e\*f^2\*g - 28\*c\*d\*f\*g^2 + 35\*a\*e\*g^3)\*x)\*sqrt(g\*x + f)/g^4

**Sympy [A]** time = 23.5858, size = 374, normalized size = 3.31

$$\left\{ \begin{array}{l} \frac{2adf}{\sqrt{f+gx}} + 2ad \left( -\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx} \right) + \frac{2aef \left( -\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx} \right)}{g} + \frac{2ae \left( \frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3} \right)}{g} + \frac{2cdf \left( \frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3} \right)}{g^2} + \frac{2cd \left( -\frac{f^3}{\sqrt{f+gx}} - 3f^2\sqrt{f+gx} + f(f+gx) \right)}{g^2} \\ \frac{adx + \frac{aex^2}{2} + \frac{cdx^3}{3} + \frac{cex^4}{4}}{\sqrt{f}} \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(c*x**2+a)/(g*x+f)**(1/2),x)`

[Out] `Piecewise((-2*a*d*f/sqrt(f + g*x) + 2*a*d*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g + 2*a*e*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g + 2*c*d*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 + 2*c*d*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 + 2*c*e*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**3 + 2*c*e*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**3/g, Ne(g, 0)), ((a*d*x + a*e*x**2/2 + c*d*x**3/3 + c*e*x**4/4)/sqrt(f), True))`

---

**GIAC/XCAS [A]** time = 0.265687, size = 209, normalized size = 1.85

$$2 \left( 105 \sqrt{gx + f} ad + \frac{35 \left( (gx+f)^{\frac{3}{2}} - 3 \sqrt{gx+ff} \right) ae}{g} + \frac{7 \left( 3 (gx+f)^{\frac{5}{2}} g^8 - 10 (gx+f)^{\frac{3}{2}} f g^8 + 15 \sqrt{gx+ff} f^2 g^8 \right) cd}{g^{10}} + \frac{3 \left( 5 (gx+f)^{\frac{7}{2}} g^{18} - 21 (gx+f)^{\frac{5}{2}} f g^{18} + 35 (gx+f)^{\frac{3}{2}} f^2 g^{18} - 35 \sqrt{gx+ff} f^3 g^{18} \right) ce}{g^{21}} \right) / 105 g$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)*(e*x + d)/sqrt(g*x + f),x, algorithm="giac")`

[Out] `2/105*(105*sqrt(g*x + f)*a*d + 35*((g*x + f)^(3/2) - 3*sqrt(g*x + f)*f)*a*e/g + 7*(3*(g*x + f)^(5/2)*g^8 - 10*(g*x + f)^(3/2)*f*g^8 + 15*sqrt(g*x + f)*f^2*g^8)*c*d/g^10 + 3*(5*(g*x + f)^(7/2)*g^18 - 21*(g*x + f)^(5/2)*f*g^18 + 35*(g*x + f)^(3/2)*f^2*g^18 - 35*sqrt(g*x + f)*f^3*g^18)*c*e/g^21)/g`

$$3.592 \quad \int \frac{a+cx^2}{\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=61

$$\frac{2\sqrt{f+gx}(ag^2+cf^2)}{g^3} + \frac{2c(f+gx)^{5/2}}{5g^3} - \frac{4cf(f+gx)^{3/2}}{3g^3}$$

[Out]  $(2*(c*f^2 + a*g^2)*\text{Sqrt}[f + g*x])/g^3 - (4*c*f*(f + g*x)^{(3/2)})/(3*g^3) + (2*c*(f + g*x)^{(5/2)})/(5*g^3)$

**Rubi [A]** time = 0.0624908, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$\frac{2\sqrt{f+gx}(ag^2+cf^2)}{g^3} + \frac{2c(f+gx)^{5/2}}{5g^3} - \frac{4cf(f+gx)^{3/2}}{3g^3}$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^2)/Sqrt[f + g\*x], x]

[Out]  $(2*(c*f^2 + a*g^2)*\text{Sqrt}[f + g*x])/g^3 - (4*c*f*(f + g*x)^{(3/2)})/(3*g^3) + (2*c*(f + g*x)^{(5/2)})/(5*g^3)$

**Rubi in Sympy [A]** time = 10.9287, size = 58, normalized size = 0.95

$$-\frac{4cf(f+gx)^{3/2}}{3g^3} + \frac{2c(f+gx)^{5/2}}{5g^3} + \frac{2\sqrt{f+gx}(ag^2+cf^2)}{g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+a)/(g\*x+f)\*\*(1/2), x)

[Out]  $-4*c*f*(f + g*x)^{(3/2)}/(3*g^{**3}) + 2*c*(f + g*x)^{(5/2)}/(5*g^{**3}) + 2*\text{sqrt}(f + g*x)*(a*g^{**2} + c*f^{**2})/g^{**3}$

**Mathematica [A]** time = 0.0351767, size = 44, normalized size = 0.72

$$\frac{2\sqrt{f+gx}(15ag^2+c(8f^2-4fgx+3g^2x^2))}{15g^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c\*x^2)/Sqrt[f + g\*x], x]

[Out]  $(2*\text{Sqrt}[f + g*x]*(15*a*g^2 + c*(8*f^2 - 4*f*g*x + 3*g^2*x^2)))/(15*g^3)$

**Maple [A]** time = 0.005, size = 41, normalized size = 0.7

$$\frac{6cx^2g^2 - 8cfxg + 30ag^2 + 16cf^2}{15g^3} \sqrt{gx+f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+a)/(g*x+f)^(1/2),x)`

[Out]  $2/15*(g*x+f)^{(1/2)}*(3*c*g^2*x^2-4*c*f*g*x+15*a*g^2+8*c*f^2)/g^3$

**Maxima [A]** time = 0.688643, size = 72, normalized size = 1.18

$$\frac{2 \left( 15 \sqrt{gx+f} a + \frac{(3(gx+f)^{\frac{5}{2}} - 10(gx+f)^{\frac{3}{2}} f + 15 \sqrt{gx+f} f^2) c}{g^2} \right)}{15 g}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)/sqrt(g*x + f),x, algorithm="maxima")`

[Out]  $2/15*(15*\sqrt{g*x + f}*a + (3*(g*x + f)^{(5/2)} - 10*(g*x + f)^{(3/2)})*f + 15*\sqrt{g*x + f}*f^2)*c/g^2)/g$

**Fricas [A]** time = 0.274226, size = 54, normalized size = 0.89

$$\frac{2(3cg^2x^2 - 4cf gx + 8cf^2 + 15ag^2)\sqrt{gx+f}}{15g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)/sqrt(g*x + f),x, algorithm="fricas")`

[Out]  $2/15*(3*c*g^2*x^2 - 4*c*f*g*x + 8*c*f^2 + 15*a*g^2)*\sqrt{g*x + f}/g^3$

**Sympy [A]** time = 6.20861, size = 150, normalized size = 2.46

$$\begin{cases} \frac{\frac{2af}{\sqrt{f+gx}} + 2a\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right) + \frac{2cf\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g^2} + \frac{2c\left(-\frac{f^3}{\sqrt{f+gx}} - 3f^2\sqrt{f+gx} + f(f+gx)^{\frac{3}{2}} - \frac{(f+gx)^{\frac{5}{2}}}{5}\right)}{g^2}}{g} & \text{for } g \neq 0 \\ \frac{ax + \frac{ex^3}{3}}{\sqrt{f}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+a)/(g*x+f)**(1/2),x)`

[Out] `Piecewise((-2*a*f/sqrt(f + g*x) + 2*a*(-f/sqrt(f + g*x) - sqrt(f + g*x)) + 2*c*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 + 2*c*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2)/g, Ne(g, 0)), ((a*x + c*x**3/3)/sqrt(f), True))`

**GIAC/XCAS [A]** time = 0.299768, size = 84, normalized size = 1.38

$$\frac{2 \left( 15 \sqrt{gx+f} a + \frac{(3(gx+f)^{\frac{5}{2}} g^8 - 10(gx+f)^{\frac{3}{2}} f g^8 + 15 \sqrt{gx+f} f^2 g^8) c}{g^{10}} \right)}{15 g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + a)/sqrt(g*x + f),x, algorithm="giac")
```

```
[Out] 2/15*(15*sqrt(g*x + f)*a + (3*(g*x + f)^(5/2)*g^8 - 10*(g*x + f)^(3/2)*f*g^8 + 15*sqrt(g*x + f)*f^2*g^8)*c/g^10)/g
```

$$3.593 \quad \int \frac{a+cx^2}{(d+ex)\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=104

$$-\frac{2(ae^2 + cd^2) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}} - \frac{2c\sqrt{f+gx}(dg+ef)}{e^2g^2} + \frac{2c(f+gx)^{3/2}}{3eg^2}$$

[Out]  $(-2*c*(e*f + d*g)*\text{Sqrt}[f + g*x])/(e^2*g^2) + (2*c*(f + g*x)^{(3/2)})/(3*e*g^2) - (2*(c*d^2 + a*e^2)*\text{ArcTanh}[(\text{Sqrt}[e]*\text{Sqrt}[f + g*x])/\text{Sqrt}[e*f - d*g]])/(e^{(5/2)}*\text{Sqrt}[e*f - d*g])$

**Rubi [A]** time = 0.262637, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$-\frac{2(ae^2 + cd^2) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}} - \frac{2c\sqrt{f+gx}(dg+ef)}{e^2g^2} + \frac{2c(f+gx)^{3/2}}{3eg^2}$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^2)/((d + e\*x)\*Sqrt[f + g\*x]), x]

[Out]  $(-2*c*(e*f + d*g)*\text{Sqrt}[f + g*x])/(e^2*g^2) + (2*c*(f + g*x)^{(3/2)})/(3*e*g^2) - (2*(c*d^2 + a*e^2)*\text{ArcTanh}[(\text{Sqrt}[e]*\text{Sqrt}[f + g*x])/\text{Sqrt}[e*f - d*g]])/(e^{(5/2)}*\text{Sqrt}[e*f - d*g])$

**Rubi in Sympy [A]** time = 63.4458, size = 95, normalized size = 0.91

$$\frac{2c(f+gx)^{3/2}}{3eg^2} - \frac{2c\sqrt{f+gx}(dg+ef)}{e^2g^2} + \frac{2(ae^2 + cd^2) \operatorname{atan}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{dg-ef}}\right)}{e^{5/2}\sqrt{dg-ef}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+a)/(e\*x+d)/(g\*x+f)\*\*(1/2), x)

[Out]  $2*c*(f + g*x)^{(3/2)}/(3*e*g^2) - 2*c*\text{sqrt}(f + g*x)*(d*g + e*f)/(e^2*g^2) + 2*(a*e^2 + c*d^2)*\text{atan}(\text{sqrt}(e)*\text{sqrt}(f + g*x)/\text{sqrt}(d*g - e*f))/(e^{(5/2)}*\text{sqrt}(d*g - e*f))$

**Mathematica [A]** time = 0.228189, size = 92, normalized size = 0.88

$$\frac{2c\sqrt{f+gx}(-3dg-2ef+egx)}{3e^2g^2} - \frac{2(ae^2 + cd^2) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c\*x^2)/((d + e\*x)\*Sqrt[f + g\*x]), x]

[Out]  $(2*c*\text{Sqrt}[f + g*x]*(-2*e*f - 3*d*g + e*g*x))/(3*e^2*g^2) - (2*(c*d^2 + a*e^2)*\text{ArcTanh}[(\text{Sqrt}[e]*\text{Sqrt}[f + g*x])/\text{Sqrt}[e*f - d*g]])/(e$



$$^{(5/2)} \sqrt{e \cdot f - d \cdot g}$$

**Maple [A]** time = 0.02, size = 132, normalized size = 1.3

$$\frac{2c}{3eg^2} (gx+f)^{\frac{3}{2}} - 2 \frac{cd\sqrt{gx+f}}{ge^2} - 2 \frac{cf\sqrt{gx+f}}{eg^2} + 2 \frac{a}{\sqrt{(dg-ef)e}} \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{(dg-ef)e}}\right) + 2 \frac{cd^2}{e^2\sqrt{(dg-ef)e}} \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{(dg-ef)e}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+a)/(e\*x+d)/(g\*x+f)^(1/2), x)

[Out]  $\frac{2}{3}c(gx+f)^{3/2}/e/g^2 - 2/g^2c/e^2d(gx+f)^{1/2} - 2/g^2c/e^2f(gx+f)^{1/2} + 2/((dg-ef)e)^{1/2} \arctan((gx+f)^{1/2}e/(dg-ef)^{1/2}) + 2/e^2/((dg-ef)e)^{1/2} \arctan((gx+f)^{1/2}e/(dg-ef)^{1/2})c/d^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)/((e\*x + d)\*sqrt(g\*x + f)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.291061, size = 1, normalized size = 0.01

$$\left[ \frac{3(cd^2 + ae^2)g^2 \log\left(\frac{\sqrt{e^2f - deg}(egx + 2ef - dg) - 2(e^2f - deg)\sqrt{gx+f}}{ex+d}\right) + 2(cegx - 2cef - 3cdg)\sqrt{e^2f - deg}\sqrt{gx+f}}{3\sqrt{e^2f - dege^2g^2}}, \right. \\ \left. \frac{2\left(3(cd^2 + ae^2)g^2 \arctan\left(-\frac{ef-dg}{\sqrt{-e^2f+deg}\sqrt{gx+f}}\right) - (cegx - 2cef - 3cdg)\sqrt{-e^2f + deg}\sqrt{gx+f}\right)}{3\sqrt{-e^2f + dege^2g^2}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)/((e\*x + d)\*sqrt(g\*x + f)), x, algorithm="fricas")

[Out]  $\left[ \frac{1}{3} \left( 3(c^2d^2 + a^2e^2)g^2 \log\left(\frac{\sqrt{e^2f - d^2e^2g}(e^2g^2x + 2e^2f - d^2g) - 2(e^2f - d^2e^2g)\sqrt{gx+f}}{(e^2x + d)}\right) + 2(c^2e^2g^2x - 2c^2e^2f - 3c^2d^2g)\sqrt{e^2f - d^2e^2g}\sqrt{gx+f}}{\sqrt{e^2f - d^2e^2g}e^2g^2}, -\frac{2}{3} \left( 3(c^2d^2 + a^2e^2)g^2 \arctan\left(-\frac{e^2f - d^2g}{\sqrt{-e^2f + d^2e^2g}\sqrt{gx+f}}\right) - (cegx - 2cef - 3cdg)\sqrt{-e^2f + deg}\sqrt{gx+f} \right) - \frac{(c^2e^2g^2x - 2c^2e^2f - 3c^2d^2g)\sqrt{-e^2f + d^2e^2g}\sqrt{gx+f}}{\sqrt{-e^2f + d^2e^2g}e^2g^2} \right) \right]$

**Sympy [A]** time = 17.4424, size = 245, normalized size = 2.36

$$\frac{2c(f+gx)^{\frac{3}{2}}}{3eg^2} - \frac{2c\sqrt{f+gx}(dg+ef)}{e^2g^2} - \frac{2(ae^2+cd^2)}{e^2} \left( \begin{array}{l} \frac{\operatorname{atan}\left(\frac{1}{\sqrt{\frac{e}{dg-ef}}\sqrt{f+gx}}\right)}{\sqrt{\frac{e}{dg-ef}}(dg-ef)} \quad \text{for } \frac{e}{dg-ef} > 0 \\ -\frac{\operatorname{acoth}\left(\frac{1}{\sqrt{\frac{e}{dg-ef}}\sqrt{f+gx}}\right)}{\sqrt{-\frac{e}{dg-ef}}(dg-ef)} \quad \text{for } \frac{1}{f+gx} > -\frac{e}{dg-ef} \wedge \frac{e}{dg-ef} < 0 \\ -\frac{\operatorname{atanh}\left(\frac{1}{\sqrt{-\frac{e}{dg-ef}}\sqrt{f+gx}}\right)}{\sqrt{-\frac{e}{dg-ef}}(dg-ef)} \quad \text{for } \frac{e}{dg-ef} < 0 \wedge \frac{1}{f+gx} < -\frac{e}{dg-ef} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+a)/(e\*x+d)/(g\*x+f)\*\*(1/2),x)

[Out]  $2*c*(f+g*x)^{(3/2)}/(3*e*g**2) - 2*c*\operatorname{sqrt}(f+g*x)*(d*g+e*f)/(e**2*g**2) - 2*(a*e**2+c*d**2)*\operatorname{Piecewise}((\operatorname{atan}(1/(\operatorname{sqrt}(e/(d*g-e*f))*\operatorname{sqrt}(f+g*x)))/(\operatorname{sqrt}(e/(d*g-e*f))*(d*g-e*f))), e/(d*g-e*f) > 0), (-\operatorname{acoth}(1/(\operatorname{sqrt}(-e/(d*g-e*f))*\operatorname{sqrt}(f+g*x)))/(\operatorname{sqrt}(-e/(d*g-e*f))*(d*g-e*f))), (e/(d*g-e*f) < 0) \& (1/(f+g*x) > -e/(d*g-e*f))), (-\operatorname{atanh}(1/(\operatorname{sqrt}(-e/(d*g-e*f))*\operatorname{sqrt}(f+g*x)))/(\operatorname{sqrt}(-e/(d*g-e*f))*(d*g-e*f))), (e/(d*g-e*f) < 0) \& (1/(f+g*x) < -e/(d*g-e*f))))/e**2$

**GIAC/XCAS [A]** time = 0.301089, size = 144, normalized size = 1.38

$$\frac{2(cd^2+ae^2)\arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{dge-fe^2}}\right)e^{(-2)}}{\sqrt{dge-fe^2}} - \frac{2\left(3\sqrt{gx+fc}dg^5e - (gx+f)^{\frac{3}{2}}cg^4e^2 + 3\sqrt{gx+fc}fg^4e^2\right)e^{(-3)}}{3g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2+a)/((e\*x+d)\*sqrt(g\*x+f)),x, algorithm="giac")

[Out]  $2*(c*d^2+a*e^2)*\arctan(\operatorname{sqrt}(g*x+f)*e/\operatorname{sqrt}(d*g*e-f*e^2))*e^{(-2)}/\operatorname{sqrt}(d*g*e-f*e^2) - 2/3*(3*\operatorname{sqrt}(g*x+f)*c*d*g^5*e - (g*x+f)^{(3/2)}*c*g^4*e^2 + 3*\operatorname{sqrt}(g*x+f)*c*f*g^4*e^2)*e^{(-3)}/g^6$

$$3.594 \quad \int \frac{a+cx^2}{(d+ex)^2\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=122

$$-\frac{\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{(d+ex)(ef-dg)} + \frac{(ae^2g+cd(4ef-3dg))\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}(ef-dg)^{3/2}} + \frac{2c\sqrt{f+gx}}{e^2g}$$

[Out] (2\*c\*Sqrt[f + g\*x])/(e^2\*g) - ((a + (c\*d^2)/e^2)\*Sqrt[f + g\*x])/((e\*f - d\*g)\*(d + e\*x)) + ((a\*e^2\*g + c\*d\*(4\*e\*f - 3\*d\*g))\*ArcTanh[(Sqrt[e]\*Sqrt[f + g\*x])/Sqrt[e\*f - d\*g]])/(e^(5/2)\*(e\*f - d\*g)^(3/2))

**Rubi [A]** time = 0.445049, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{\sqrt{f+gx}\left(a+\frac{cd^2}{e^2}\right)}{(d+ex)(ef-dg)} + \frac{(ae^2g+cd(4ef-3dg))\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}(ef-dg)^{3/2}} + \frac{2c\sqrt{f+gx}}{e^2g}$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^2)/((d + e\*x)^2\*Sqrt[f + g\*x]), x]

[Out] (2\*c\*Sqrt[f + g\*x])/(e^2\*g) - ((a + (c\*d^2)/e^2)\*Sqrt[f + g\*x])/((e\*f - d\*g)\*(d + e\*x)) + ((a\*e^2\*g + c\*d\*(4\*e\*f - 3\*d\*g))\*ArcTanh[(Sqrt[e]\*Sqrt[f + g\*x])/Sqrt[e\*f - d\*g]])/(e^(5/2)\*(e\*f - d\*g)^(3/2))

**Rubi in Sympy [A]** time = 83.965, size = 124, normalized size = 1.02

$$\frac{2c\sqrt{f+gx}}{e^2g} + \frac{g\sqrt{f+gx}(ae^2+cd^2)}{e^2(dg-ef)(dg-ef+e(f+gx))} + \frac{(ae^2g-3cd^2g+4cdef)\operatorname{atan}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{dg-ef}}\right)}{e^{\frac{5}{2}}(dg-ef)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+a)/(e\*x+d)\*\*2/(g\*x+f)\*\*(1/2), x)

[Out] 2\*c\*sqrt(f + g\*x)/(e\*\*2\*g) + g\*sqrt(f + g\*x)\*(a\*e\*\*2 + c\*d\*\*2)/(e\*\*2\*(d\*g - e\*f)\*(d\*g - e\*f + e\*(f + g\*x))) + (a\*e\*\*2\*g - 3\*c\*d\*\*2\*g + 4\*c\*d\*e\*f)\*atan(sqrt(e)\*sqrt(f + g\*x)/sqrt(d\*g - e\*f))/(e\*\*5/2\*(d\*g - e\*f)\*\*(3/2))

**Mathematica [A]** time = 0.349434, size = 115, normalized size = 0.94

$$\frac{\sqrt{f+gx}\left(\frac{ae^2+cd^2}{(d+ex)(dg-ef)} + \frac{2c}{g}\right)}{e^2} + \frac{(ae^2g+cd(4ef-3dg))\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}(ef-dg)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c\*x^2)/((d + e\*x)^2\*Sqrt[f + g\*x]), x]

[Out]  $(\text{Sqrt}[f + g*x] * ((2*c)/g + (c*d^2 + a*e^2)/((-e*f) + d*g) * (d + e*x))) / e^2 + ((a*e^2*g + c*d*(4*e*f - 3*d*g)) * \text{ArcTanh}[\text{Sqrt}[e] * \text{Sqrt}[f + g*x]) / \text{Sqrt}[e*f - d*g]]) / (e^{5/2} * (e*f - d*g)^{3/2})$

**Maple [B]** time = 0.022, size = 237, normalized size = 1.9

$$2 \frac{c\sqrt{gx+f}}{e^2g} + \frac{ag}{(dg-ef)(egx+dg)}\sqrt{gx+f} + \frac{cd^2g}{e^2(dg-ef)(egx+dg)}\sqrt{gx+f}$$

$$+ \frac{ag}{dg-ef} \arctan\left(e\sqrt{gx+f} \frac{1}{\sqrt{(dg-ef)e}}\right) \frac{1}{\sqrt{(dg-ef)e}}$$

$$- 3 \frac{cd^2g}{e^2(dg-ef)\sqrt{(dg-ef)e}} \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{(dg-ef)e}}\right)$$

$$+ 4 \frac{cdf}{(dg-ef)e\sqrt{(dg-ef)e}} \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{(dg-ef)e}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+a)/(e*x+d)^2/(g*x+f)^(1/2),x)`

[Out]  $2*c*(g*x+f)^{1/2}/e^2/g+g/(d*g-e*f)*(g*x+f)^{1/2}/(e*g*x+d*g)^*a+g/e^2/(d*g-e*f)*(g*x+f)^{1/2}/(e*g*x+d*g)^*c*d^2+g/(d*g-e*f)/((d*g-e*f)*e)^{1/2}*\arctan((g*x+f)^{1/2}*e/((d*g-e*f)*e)^{1/2})^*a-3*g/e^2/(d*g-e*f)/((d*g-e*f)*e)^{1/2}*\arctan((g*x+f)^{1/2}*e/((d*g-e*f)*e)^{1/2})^*c*d^2+4/e/(d*g-e*f)/((d*g-e*f)*e)^{1/2}*\arctan((g*x+f)^{1/2}*e/((d*g-e*f)*e)^{1/2})^*c*d*f$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)/((e*x + d)^2*sqrt(g*x + f)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.289926, size = 1, normalized size = 0.01

$$\frac{2(2cdef - (3cd^2 + ae^2)g + 2(ce^2f - cdeg)x)\sqrt{e^2f - deg}\sqrt{gx+f} - (4cd^2efg - (3cd^3 - ade^2)g^2 + (4cde^2fg - (3cd^2e^2f - d^2e^2g^2 + (e^4fg - de^3g^2)x)\sqrt{e^2f - deg}}}{2(de^3fg - d^2e^2g^2 + (e^4fg - de^3g^2)x)\sqrt{e^2f - deg}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)/((e*x + d)^2*sqrt(g*x + f)),x, algorithm="fricas")`

[Out]  $[1/2*(2*(2*c*d*e*f - (3*c*d^2 + a*e^2)*g + 2*(c*e^2*f - c*d*e*g)*x)*\text{sqrt}(e^2*f - d*e*g)*\text{sqrt}(g*x + f) - (4*c*d^2*e*f*g - (3*c*d^3 - a*d*e^2)*g^2 + (4*c*d*e^2*f*g - (3*c*d^2e^2f - d^2e^2g^2 + (e^4fg - de^3g^2)x)\sqrt{e^2f - deg}))/((d*e^3*f*g - d^2*e^2*g^2 + (e^4*f*g - d*e^3*g^2)*x)*\text{sqrt}(e^2*f - d*e*g)), ((2*c*d*e*f - (3*c*d^2 + a*e^2)*g + 2*(c*e^2*f - c*d*e*g)*x)*\text{sqrt}(-e^2*f + d*e*g)*\text{sqrt}(g*x + f)$

$$\left. \begin{aligned} & + (4c^2d^2efg - (3c^2d^3 - a^2de^2)g^2 + (4c^2de^2f^2g - (3c^2d^2e - a^2e^3)g^2)x) \arctan\left(\frac{ef - dg}{\sqrt{-e^2f + d^2eg}}\right) \\ & + (d^3efg - d^2e^2g^2 + (e^4f^2g - d^2e^3g^2)x) \sqrt{-e^2f + d^2eg} \end{aligned} \right] / ((d^3efg - d^2e^2g^2 + (e^4f^2g - d^2e^3g^2)x) \sqrt{-e^2f + d^2eg})$$

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+a)/(e\*x+d)\*\*2/(g\*x+f)\*\*(1/2),x)

[Out] Exception raised: ValueError

**GIAC/XCAS [A]** time = 0.319369, size = 200, normalized size = 1.64

$$\frac{2\sqrt{gx+fe}ce^{-2}}{g} - \frac{(3cd^2g - 4cdfe - age^2) \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{dge-fe^2}}\right)}{(dge^2 - fe^3)\sqrt{dge-fe^2}} + \frac{\sqrt{gx+fe}cd^2g + \sqrt{gx+fe}age^2}{(dge^2 - fe^3)(dg + (gx+f)e - fe)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)/((e\*x + d)^2\*sqrt(g\*x + f)),x, algorithm="giac")

[Out]  $2\sqrt{gx+f}c^2e^{-2}/g - (3c^2d^2g - 4c^2df^2e - a^2g^2e^2) \arctan(\sqrt{gx+f}e/\sqrt{d^2g^2e - f^2e^2}) / ((d^2g^2e^2 - f^2e^3) \sqrt{d^2g^2e - f^2e^2}) + (\sqrt{gx+f}c^2d^2g + \sqrt{gx+f}a^2g^2e^2) / ((d^2g^2e^2 - f^2e^3) (d^2g + (gx+f)e - f^2e))$

$$3.595 \quad \int \frac{a+cx^2}{(d+ex)^3\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=178

$$\frac{\sqrt{f+gx} \left( a + \frac{cd^2}{e^2} \right)}{2(d+ex)^2(e f - dg)} - \frac{(3ae^2g^2 + c(3d^2g^2 - 8defg + 8e^2f^2)) \tanh^{-1} \left( \frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}} \right)}{4e^{5/2}(ef-dg)^{5/2}} + \frac{\sqrt{f+gx}(3ae^2g + cd(8ef - 5dg))}{4e^2(d+ex)(ef-dg)^2}$$

[Out]  $-\left(\left(a + \frac{c \cdot d^2}{e^2}\right) \cdot \text{Sqrt}[f + g \cdot x]\right) / \left(2 \cdot (e \cdot f - d \cdot g) \cdot (d + e \cdot x)^2\right) + \left(\left(3 \cdot a \cdot e^2 \cdot g + c \cdot d \cdot (8 \cdot e \cdot f - 5 \cdot d \cdot g)\right) \cdot \text{Sqrt}[f + g \cdot x]\right) / \left(4 \cdot e^2 \cdot (e \cdot f - d \cdot g)^2 \cdot (d + e \cdot x)\right) - \left(\left(3 \cdot a \cdot e^2 \cdot g^2 + c \cdot (8 \cdot e^2 \cdot f^2 - 8 \cdot d \cdot e \cdot f \cdot g + 3 \cdot d^2 \cdot g^2)\right) \cdot \text{ArcTanh}\left[\frac{\text{Sqrt}[e] \cdot \text{Sqrt}[f + g \cdot x]}{\text{Sqrt}[e \cdot f - d \cdot g]}\right]\right) / \left(4 \cdot e^{5/2} \cdot (e \cdot f - d \cdot g)^{5/2}\right)$

**Rubi [A]** time = 0.635283, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$\frac{\sqrt{f+gx} \left( a + \frac{cd^2}{e^2} \right)}{2(d+ex)^2(e f - dg)} - \frac{(3ae^2g^2 + c(3d^2g^2 - 8defg + 8e^2f^2)) \tanh^{-1} \left( \frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}} \right)}{4e^{5/2}(ef-dg)^{5/2}} + \frac{\sqrt{f+gx}(3ae^2g + cd(8ef - 5dg))}{4e^2(d+ex)(ef-dg)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^2)/((d + e\*x)^3\*Sqrt[f + g\*x]),x]

[Out]  $-\left(\left(a + \frac{c \cdot d^2}{e^2}\right) \cdot \text{Sqrt}[f + g \cdot x]\right) / \left(2 \cdot (e \cdot f - d \cdot g) \cdot (d + e \cdot x)^2\right) + \left(\left(3 \cdot a \cdot e^2 \cdot g + c \cdot d \cdot (8 \cdot e \cdot f - 5 \cdot d \cdot g)\right) \cdot \text{Sqrt}[f + g \cdot x]\right) / \left(4 \cdot e^2 \cdot (e \cdot f - d \cdot g)^2 \cdot (d + e \cdot x)\right) - \left(\left(3 \cdot a \cdot e^2 \cdot g^2 + c \cdot (8 \cdot e^2 \cdot f^2 - 8 \cdot d \cdot e \cdot f \cdot g + 3 \cdot d^2 \cdot g^2)\right) \cdot \text{ArcTanh}\left[\frac{\text{Sqrt}[e] \cdot \text{Sqrt}[f + g \cdot x]}{\text{Sqrt}[e \cdot f - d \cdot g]}\right]\right) / \left(4 \cdot e^{5/2} \cdot (e \cdot f - d \cdot g)^{5/2}\right)$

**Rubi in Sympy [A]** time = 106.987, size = 194, normalized size = 1.09

$$\frac{g^2\sqrt{f+gx}(ae^2+cd^2)}{2e^2(dg-ef)(dg-ef+e(f+gx))^2} + \frac{g\sqrt{f+gx}(3ae^2g-5cd^2g+8cdef)}{4e^2(dg-ef)^2(dg-ef+e(f+gx))} + \frac{(3ae^2g^2+3cd^2g^2-8cdefg+8ce^2f^2) \operatorname{atan}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{dg-ef}}\right)}{4e^{5/2}(dg-ef)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+a)/(e\*x+d)\*\*3/(g\*x+f)\*\*(1/2),x)

[Out]  $g^{**2} \cdot \text{sqrt}(f + g \cdot x) \cdot (a \cdot e^{**2} + c \cdot d^{**2}) / (2 \cdot e^{**2} \cdot (d \cdot g - e \cdot f) \cdot (d \cdot g - e \cdot f + e \cdot (f + g \cdot x))^{**2}) + g \cdot \text{sqrt}(f + g \cdot x) \cdot (3 \cdot a \cdot e^{**2} \cdot g - 5 \cdot c \cdot d^{**2} \cdot g + 8 \cdot c \cdot d \cdot e \cdot f) / (4 \cdot e^{**2} \cdot (d \cdot g - e \cdot f)^{**2} \cdot (d \cdot g - e \cdot f + e \cdot (f + g \cdot x))) + (3 \cdot a \cdot e^{**2} \cdot g^{**2} + 3 \cdot c \cdot d^{**2} \cdot g^{**2} - 8 \cdot c \cdot d \cdot e \cdot f \cdot g + 8 \cdot c \cdot e^{**2} \cdot f^{**2}) \cdot \operatorname{atan}(\text{sqrt}(e) \cdot \text{sqrt}(f + g \cdot x) / \text{sqrt}(d \cdot g - e \cdot f)) / (4 \cdot e^{**5/2} \cdot (d \cdot g - e \cdot f)^{**5/2})$

**Mathematica [A]** time = 0.409367, size = 166, normalized size = 0.93

$$\frac{\sqrt{f+gx} (ae^2(5dg - 2ef + 3egx) + cd(-3d^2g + de(6f - 5gx) + 8e^2fx))}{4e^2(d+ex)^2(ef-dg)^2} - \frac{(3ae^2g^2 + c(3d^2g^2 - 8defg + 8e^2f^2)) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{5/2}(ef-dg)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c\*x^2)/((d + e\*x)^3\*Sqrt[f + g\*x]), x]

[Out] (Sqrt[f + g\*x]\*(a\*e^2\*(-2\*e\*f + 5\*d\*g + 3\*e\*g\*x) + c\*d\*(-3\*d^2\*g + 8\*e^2\*f\*x + d\*e\*(6\*f - 5\*g\*x)))/(4\*e^2\*(e\*f - d\*g)^2\*(d + e\*x)^2) - ((3\*a\*e^2\*g^2 + c\*(8\*e^2\*f^2 - 8\*d\*e\*f\*g + 3\*d^2\*g^2))\*ArcTanh[(Sqrt[e]\*Sqrt[f + g\*x])/Sqrt[e\*f - d\*g]])/(4\*e^(5/2)\*(e\*f - d\*g)^(5/2))

**Maple [B]** time = 0.026, size = 384, normalized size = 2.2

$$2 \frac{1}{(e(gx+f) + dg - ef)^2} \left( \frac{1}{8} \frac{g(3ae^2g - 5cd^2g + 8cdef)(gx+f)^{3/2}}{e(d^2g^2 - 2defg + e^2f^2)} + \frac{1}{8} \frac{(5ae^2g - 3cd^2g + 8cdef)g\sqrt{gx+f}}{e^2(dg-ef)} \right) + \frac{3ag^2}{4d^2g^2 - 8defg + 4e^2f^2} \arctan\left(e\sqrt{gx+f} \frac{1}{\sqrt{(dg-ef)e}}\right) \frac{1}{\sqrt{(dg-ef)e}} + \frac{3cd^2g^2}{(4d^2g^2 - 8defg + 4e^2f^2)e^2} \arctan\left(e\sqrt{gx+f} \frac{1}{\sqrt{(dg-ef)e}}\right) \frac{1}{\sqrt{(dg-ef)e}} - 2 \frac{cdfg}{e(d^2g^2 - 2defg + e^2f^2)\sqrt{(dg-ef)e}} \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{(dg-ef)e}}\right) + 2 \frac{cf^2}{(d^2g^2 - 2defg + e^2f^2)\sqrt{(dg-ef)e}} \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{(dg-ef)e}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+a)/(e\*x+d)^3/(g\*x+f)^(1/2), x)

[Out] 2\*(1/8\*g\*(3\*a\*e^2\*g-5\*c\*d^2\*g+8\*c\*d\*e\*f)/e/(d^2\*g^2-2\*d\*e\*f\*g+e^2\*f^2)\*(g\*x+f)^(3/2)+1/8\*(5\*a\*e^2\*g-3\*c\*d^2\*g+8\*c\*d\*e\*f)/e^2\*g/(d\*g-e\*f)\*(g\*x+f)^(1/2))/(e\*(g\*x+f)+d\*g-e\*f)^2+3/4/(d^2\*g^2-2\*d\*e\*f\*g+e^2\*f^2)/((d\*g-e\*f)\*e)^(1/2)\*arctan((g\*x+f)^(1/2)\*e/((d\*g-e\*f)\*e)^(1/2))\*a\*g^2+3/4/(d^2\*g^2-2\*d\*e\*f\*g+e^2\*f^2)/e^2/((d\*g-e\*f)\*e)^(1/2)\*arctan((g\*x+f)^(1/2)\*e/((d\*g-e\*f)\*e)^(1/2))\*c\*d^2\*g^2-2/(d^2\*g^2-2\*d\*e\*f\*g+e^2\*f^2)/e/((d\*g-e\*f)\*e)^(1/2)\*arctan((g\*x+f)^(1/2)\*e/((d\*g-e\*f)\*e)^(1/2))\*c\*d\*f\*g+2/(d^2\*g^2-2\*d\*e\*f\*g+e^2\*f^2)/((d\*g-e\*f)\*e)^(1/2)\*arctan((g\*x+f)^(1/2)\*e/((d\*g-e\*f)\*e)^(1/2))\*c\*f^2

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)/((e\*x + d)^3\*sqrt(g\*x + f)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.296683, size = 1, normalized size = 0.01

$$\frac{2\sqrt{e^2f - deg}(2(3cd^2e - ae^3)f - (3cd^3 - 5ade^2)g + (8cde^2f - (5cd^2e - 3ae^3)g)x)\sqrt{gx + f} + (8cd^2e^2f^2 - 8cd^3efg)}{8(d^2e^4f^2 - 2d^3e^3fg + d^4e^2g^2 + ($$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)/((e\*x + d)^3\*sqrt(g\*x + f)),x, algorithm="fricas")

[Out] [1/8\*(2\*sqrt(e^2\*f - d\*e\*g)\*(2\*(3\*c\*d^2\*e - a\*e^3)\*f - (3\*c\*d^3 - 5\*a\*d\*e^2)\*g + (8\*c\*d\*e^2\*f - (5\*c\*d^2\*e - 3\*a\*e^3)\*g)\*x)\*sqrt(g\*x + f) + (8\*c\*d^2\*e^2\*f^2 - 8\*c\*d^3\*e\*f\*g + 3\*(c\*d^4 + a\*d^2\*e^2)\*g^2 + (8\*c\*e^4\*f^2 - 8\*c\*d\*e^3\*f\*g + 3\*(c\*d^2\*e^2 + a\*e^4)\*g^2)\*x^2 + 2\*(8\*c\*d\*e^3\*f^2 - 8\*c\*d^2\*e^2\*f\*g + 3\*(c\*d^3\*e + a\*d\*e^3)\*g^2)\*x)\*log((sqrt(e^2\*f - d\*e\*g)\*(e\*g\*x + 2\*e\*f - d\*g) - 2\*(e^2\*f - d\*e\*g)\*sqrt(g\*x + f))/(e\*x + d))/((d^2\*e^4\*f^2 - 2\*d^3\*e^3\*f\*g + d^4\*e^2\*g^2 + (e^6\*f^2 - 2\*d\*e^5\*f\*g + d^2\*e^4\*g^2)\*x^2 + 2\*(d\*e^5\*f^2 - 2\*d^2\*e^4\*f\*g + d^3\*e^3\*g^2)\*x)\*sqrt(e^2\*f - d\*e\*g)), 1/4\*(sqrt(-e^2\*f + d\*e\*g)\*(2\*(3\*c\*d^2\*e - a\*e^3)\*f - (3\*c\*d^3 - 5\*a\*d\*e^2)\*g + (8\*c\*d\*e^2\*f - (5\*c\*d^2\*e - 3\*a\*e^3)\*g)\*x)\*sqrt(g\*x + f) - (8\*c\*d^2\*e^2\*f^2 - 8\*c\*d^3\*e\*f\*g + 3\*(c\*d^4 + a\*d^2\*e^2)\*g^2 + (8\*c\*e^4\*f^2 - 8\*c\*d\*e^3\*f\*g + 3\*(c\*d^2\*e^2 + a\*e^4)\*g^2)\*x^2 + 2\*(8\*c\*d\*e^3\*f^2 - 8\*c\*d^2\*e^2\*f\*g + 3\*(c\*d^3\*e + a\*d\*e^3)\*g^2)\*x)\*arctan(-(e\*f - d\*g)/(sqrt(-e^2\*f + d\*e\*g)\*sqrt(g\*x + f)))/((d^2\*e^4\*f^2 - 2\*d^3\*e^3\*f\*g + d^4\*e^2\*g^2 + (e^6\*f^2 - 2\*d\*e^5\*f\*g + d^2\*e^4\*g^2)\*x^2 + 2\*(d\*e^5\*f^2 - 2\*d^2\*e^4\*f\*g + d^3\*e^3\*g^2)\*x)\*sqrt(-e^2\*f + d\*e\*g))]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+a)/(e\*x+d)\*\*3/(g\*x+f)\*\*(1/2),x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.287413, size = 375, normalized size = 2.11

$$\frac{(3cd^2g^2 - 8cdfge + 8cf^2e^2 + 3ag^2e^2) \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{dge-fe^2}}\right)}{4(d^2g^2e^2 - 2dfge^3 + f^2e^4)\sqrt{dge - fe^2}} - \frac{3\sqrt{gx+fc}d^3g^3 + 5(gx+f)^{\frac{3}{2}}cd^2g^2e - 11\sqrt{gx+fc}d^2fg^2e - 8(gx+f)^{\frac{3}{2}}cdfge^2 + 8\sqrt{gx+fc}cdf^2ge^2 - 5\sqrt{gx+fc}adg^3e^2}{4(d^2g^2e^2 - 2dfge^3 + f^2e^4)(dg + (gx+f)e - fe^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)/((e\*x + d)^3\*sqrt(g\*x + f)),x, algorithm="giac")

[Out] 1/4\*(3\*c\*d^2\*g^2 - 8\*c\*d\*f\*g\*e + 8\*c\*f^2\*e^2 + 3\*a\*g^2\*e^2)\*arctan(sqrt(g\*x + f)\*e/sqrt(d\*g\*e - f\*e^2))/((d^2\*g^2\*e^2 - 2\*d\*f\*g\*e^2



$$\frac{(3 + f^2 e^4) \sqrt{d g e - f e^2} - \frac{1}{4} (3 \sqrt{g x + f} c d^3 g^3 + 5 (g x + f)^{3/2} c d^2 g^2 e - 11 \sqrt{g x + f} c d^2 f g^2 e - 8 (g x + f)^{3/2} c d f g e^2 + 8 \sqrt{g x + f} c d f^2 g e^2 - 5 \sqrt{g x + f} a d g^3 e^2 - 3 (g x + f)^{3/2} a g^2 e^3 + 5 \sqrt{g x + f} a f g^2 e^3)}{(d^2 g^2 e^2 - 2 d f g e^3 + f^2 e^4) (d g + (g x + f) e - f e)^2}$$

$$3.596 \quad \int \frac{(d+ex)^3(a+cx^2)}{(f+gx)^{3/2}} dx$$

**Optimal.** Leaf size=238

$$\frac{2e(f+gx)^{5/2}(ae^2g^2+c(3d^2g^2-12defg+10e^2f^2))}{5g^6} - \frac{2(f+gx)^{3/2}(ef-dg)(3ae^2g^2+c(d^2g^2-8defg+10e^2f^2))}{3g^6} + \frac{2(ag^2+cf^2)(ef-dg)^3}{g^6\sqrt{f+gx}} + \frac{2\sqrt{f+gx}(ef-dg)^2(3aeg^2+cf(5ef-2dg))}{g^6} - \frac{2ce^2(f+gx)^{7/2}(5ef-3dg)}{7g^6} + \frac{2ce^3(f+gx)^{9/2}}{9g^6}$$

[Out] (2\*(e\*f - d\*g)^3\*(c\*f^2 + a\*g^2))/(g^6\*Sqrt[f + g\*x]) + (2\*(e\*f - d\*g)^2\*(3\*a\*e\*g^2 + c\*f\*(5\*e\*f - 2\*d\*g))\*Sqrt[f + g\*x])/g^6 - (2\*(e\*f - d\*g)\*(3\*a\*e^2\*g^2 + c\*(10\*e^2\*f^2 - 8\*d\*e\*f\*g + d^2\*g^2))\*(f + g\*x)^(3/2))/(3\*g^6) + (2\*e\*(a\*e^2\*g^2 + c\*(10\*e^2\*f^2 - 12\*d\*e\*f\*g + 3\*d^2\*g^2))\*(f + g\*x)^(5/2))/(5\*g^6) - (2\*c\*e^2\*(5\*e\*f - 3\*d\*g)\*(f + g\*x)^(7/2))/(7\*g^6) + (2\*c\*e^3\*(f + g\*x)^(9/2))/(9\*g^6)

**Rubi [A]** time = 0.683078, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{2e(f+gx)^{5/2}(ae^2g^2+c(3d^2g^2-12defg+10e^2f^2))}{5g^6} - \frac{2(f+gx)^{3/2}(ef-dg)(3ae^2g^2+c(d^2g^2-8defg+10e^2f^2))}{3g^6} + \frac{2(ag^2+cf^2)(ef-dg)^3}{g^6\sqrt{f+gx}} + \frac{2\sqrt{f+gx}(ef-dg)^2(3aeg^2+cf(5ef-2dg))}{g^6} - \frac{2ce^2(f+gx)^{7/2}(5ef-3dg)}{7g^6} + \frac{2ce^3(f+gx)^{9/2}}{9g^6}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(a + c\*x^2))/(f + g\*x)^(3/2), x]

[Out] (2\*(e\*f - d\*g)^3\*(c\*f^2 + a\*g^2))/(g^6\*Sqrt[f + g\*x]) + (2\*(e\*f - d\*g)^2\*(3\*a\*e\*g^2 + c\*f\*(5\*e\*f - 2\*d\*g))\*Sqrt[f + g\*x])/g^6 - (2\*(e\*f - d\*g)\*(3\*a\*e^2\*g^2 + c\*(10\*e^2\*f^2 - 8\*d\*e\*f\*g + d^2\*g^2))\*(f + g\*x)^(3/2))/(3\*g^6) + (2\*e\*(a\*e^2\*g^2 + c\*(10\*e^2\*f^2 - 12\*d\*e\*f\*g + 3\*d^2\*g^2))\*(f + g\*x)^(5/2))/(5\*g^6) - (2\*c\*e^2\*(5\*e\*f - 3\*d\*g)\*(f + g\*x)^(7/2))/(7\*g^6) + (2\*c\*e^3\*(f + g\*x)^(9/2))/(9\*g^6)

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2ce^3(f+gx)^{\frac{5}{2}}}{9g^6} + \frac{2ce^2(f+gx)^{\frac{7}{2}}(3dg-5ef)}{7g^6} + \frac{2e(f+gx)^{\frac{5}{2}}(ae^2g^2+3cd^2g^2-12cdefg+10ce^2f^2)}{5g^6} + \frac{2(dg-ef)^2(3aeg^2-2cdfg+5cef^2)\int^{\sqrt{f+gx}}\frac{1}{g^5}dx}{g} + \frac{2(f+gx)^{\frac{3}{2}}(dg-ef)(3ae^2g^2+cd^2g^2-8cdefg+10ce^2f^2)}{3g^6} - \frac{2(ag^2+cf^2)(dg-ef)^3}{g^6\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3\*(c\*x\*\*2+a)/(g\*x+f)\*\*(3/2), x)

[Out]  $2*c*e^{3*(f+g*x)^{(9/2)}}/(9*g^{**6}) + 2*c*e^{2*(f+g*x)^{(7/2)}}*(3*d*g - 5*e*f)/(7*g^{**6}) + 2*e*(f+g*x)^{(5/2)}*(a*e^{2*g^{**2}} + 3*c*d^{**2}*g^{**2} - 12*c*d*e*f*g + 10*c*e^{2*f^{**2}})/(5*g^{**6}) + 2*(d*g - e*f)^{**2}*(3*a*e*g^{**2} - 2*c*d*f*g + 5*c*e*f^{**2})*Integral(g^{**(-5)}, (x, sqrt(f+g*x)))/g + 2*(f+g*x)^{(3/2)}*(d*g - e*f)*(3*a*e^{2*g^{**2}} + c*d^{**2}*g^{**2} - 8*c*d*e*f*g + 10*c*e^{2*f^{**2}})/(3*g^{**6}) - 2*(a*g^{**2} + c*f^{**2})*(d*g - e*f)^{**3}/(g^{**6}*sqrt(f+g*x))$

**Mathematica [A]** time = 0.3283, size = 278, normalized size = 1.17

$2(63ag^2(-5d^3g^3 + 15d^2eg^2(2f + gx) + 5de^2g(-8f^2 - 4fgx + g^2x^2) + e^3(16f^3 + 8f^2gx - 2fg^2x^2 + g^3x^3)) + c(105d^3g^3$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*(a + c\*x^2))/(f + g\*x)^(3/2), x]

[Out]  $(2*(63*a*g^2*(-5*d^3*g^3 + 15*d^2*e*g^2*(2*f + g*x) + 5*d*e^2*g*(-8*f^2 - 4*f*g*x + g^2*x^2) + e^3*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3)) + c*(105*d^3*g^3*(-8*f^2 - 4*f*g*x + g^2*x^2) + 18*9*d^2*e*g^2*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3) + 27*d*e^2*g*(-128*f^4 - 64*f^3*g*x + 16*f^2*g^2*x^2 - 8*f*g^3*x^3 + 5*g^4*x^4) + 5*e^3*(256*f^5 + 128*f^4*g*x - 32*f^3*g^2*x^2 + 16*f^2*g^3*x^3 - 10*f*g^4*x^4 + 7*g^5*x^5)))/(315*g^6*sqrt(f + g*x))$

**Maple [A]** time = 0.011, size = 365, normalized size = 1.5

$-70e^3cx^5g^5 - 270cde^2g^5x^4 + 100ce^3fg^4x^4 - 126ae^3g^5x^3 - 378cd^2eg^5x^3 + 432cde^2fg^4x^3 - 160ce^3f^2g^3x^3 - 630ade^2g$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(c\*x^2+a)/(g\*x+f)^(3/2), x)

[Out]  $-2/315/(g*x+f)^{(1/2)}*(-35*c*e^3*g^5*x^5-135*c*d*e^2*g^5*x^4+50*c*e^3*f*g^4*x^4-63*a*e^3*g^5*x^3-189*c*d^2*e*g^5*x^3+216*c*d*e^2*f*g^4*x^3-80*c*e^3*f^2*g^3*x^3-315*a*d*e^2*g^5*x^2+126*a*e^3*f*g^4*x^2-105*c*d^3*g^5*x^2+378*c*d^2*e*f*g^4*x^2-432*c*d*e^2*f^2*g^3*x^2+160*c*e^3*f^3*g^2*x^2-945*a*d^2*e*g^5*x+1260*a*d*e^2*f*g^4*x-504*a*e^3*f^2*g^3*x+420*c*d^3*f*g^4*x-1512*c*d^2*e*f^2*g^3*x+1728*c*d*e^2*f^3*g^2*x-640*c*e^3*f^4*g*x+315*a*d^3*g^5-1890*a*d^2*e*f*g^4+2520*a*d*e^2*f^2*g^3-1008*a*e^3*f^3*g^2+840*c*d^3*f^2*g^3-3024*c*d^2*e*f^3*g^2+3456*c*d*e^2*f^4*g-1280*c*e^3*f^5)/g^6$

**Maxima [A]** time = 0.697628, size = 451, normalized size = 1.89

$2\left(\frac{35(gx+f)^{\frac{9}{2}}ce^3-45(5ce^3f-3cde^2g)(gx+f)^{\frac{7}{2}}+63(10ce^3f^2-12cde^2fg+(3cd^2e+ae^3)g^2)(gx+f)^{\frac{5}{2}}-105(10ce^3f^3-18cde^2f^2g+3(3cd^2e+ae^3)fg^2-(cd^3+ae^3)g^3)}{g^5}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a) \* (e\*x + d)^3/(g\*x + f)^(3/2), x, algorithm="maxima")

[Out]  $2/315*((35*(g*x + f)^{(9/2)}*c*e^3 - 45*(5*c*e^3*f - 3*c*d*e^2*g)*(g*x + f)^{(7/2)} + 63*(10*c*e^3*f^2 - 12*c*d*e^2*f*g + (3*c*d^2*e + a*e^3)*g^2)*(g*x + f)^{(5/2)} - 105*(10*c*e^3*f^3 - 18*c*d*e^2*f^2$

$$\frac{g + 3(3cd^2e + ae^3)fg^2 - (cd^3 + 3ad^2e)g^3}{(gx + f)^{3/2}} + \frac{315(5c^3e^3f^4 - 12cd^2e^2f^3g + 3ad^2e^2fg^4 + 3(3cd^2e + ae^3)f^2g^2 - 2(cd^3 + 3ad^2e)fg^3) \operatorname{sqrt}(gx + f)}{g^5} + \frac{315(c^3e^3f^5 - 3cd^2e^2f^4g + 3ad^2e^2fg^4 - ad^3g^5 + (3cd^2e + ae^3)f^3g^2 - (cd^3 + 3ad^2e)f^2g^3)}{(\operatorname{sqrt}(gx + f)g^5)} / g$$

**Fricas [A]** time = 0.276658, size = 436, normalized size = 1.83

$$2(35ce^3g^5x^5 + 1280ce^3f^5 - 3456cde^2fg^4 + 1890ad^2efg^4 - 315ad^3g^5 + 1008(3cd^2e + ae^3)f^3g^2 - 840(cd^3 + 3ade^2)f^2g^2)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)*(e*x + d)^3/(g*x + f)^(3/2), x, algorithm="fricas")`

[Out]  $\frac{2}{315} \cdot (35c^3e^3g^5x^5 + 1280c^3e^3f^5 - 3456c^2d^2e^2f^4g + 1890a^2d^2e^2fg^4 - 315a^2d^3g^5 + 1008(3cd^2e + ae^3)f^3g^2 - 840(cd^3 + 3ad^2e)f^2g^3 - 5(10c^3e^3fg^4 - 27c^2d^2e^2g^5)x^4 + (80c^3e^3f^2g^3 - 216c^2d^2e^2fg^4 + 63(3cd^2e + ae^3)g^5)x^3 - (160c^3e^3f^3g^2 - 432c^2d^2e^2f^2g^3 + 126(3cd^2e + ae^3)fg^4 - 105(cd^3 + 3ad^2e)g^5)x^2 + (640c^3e^3f^4g - 1728c^2d^2e^2f^3g^2 + 945a^2d^2e^2g^5 + 504(3cd^2e + ae^3)f^2g^3 - 420(cd^3 + 3ad^2e)fg^4)x) / (\operatorname{sqrt}(gx + f)g^6)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + cx^2)(d + ex)^3}{(f + gx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**3*(c*x**2+a)/(g*x+f)**(3/2), x)`

[Out] `Integral((a + c*x**2)*(d + e*x)**3/(f + g*x)**(3/2), x)`

**GIAC/XCAS [A]** time = 0.286452, size = 612, normalized size = 2.57

$$\frac{2(cd^3f^2g^3 + ad^3g^5 - 3cd^2f^3g^2e - 3ad^2fg^4e + 3cdf^4ge^2 + 3adf^2g^3e^2 - cf^5e^3 - af^3g^2e^3)}{\sqrt{gx + fg^6}} + \frac{2\left(105(gx + f)^{3/2}cd^3g^{51} - 630\sqrt{gx + f}cd^3fg^{51} + 189(gx + f)^{5/2}cd^2g^{50}e - 945(gx + f)^{3/2}cd^2fg^{50}e + 2835\sqrt{gx + f}cd^2f^2g^{50}e\right)}{+}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)*(e*x + d)^3/(g*x + f)^(3/2), x, algorithm="giac")`

[Out]  $-2(c^2d^3f^2g^3 + a^2d^3g^5 - 3c^2d^2f^3g^2e - 3a^2d^2fg^4e + 3c^2d^2f^4ge^2 + 3ad^2f^2g^3e^2 - cf^5e^3 - af^3g^2e^3) / (\operatorname{sqrt}(gx + f)g^6) + \frac{2}{315} \cdot (105(gx + f)^{3/2}cd^3g^{51} - 630\operatorname{sqrt}(gx + f)c^2d^3fg^{51} + 189(gx + f)^{5/2}c^2d^2g^{50}e - 945(gx + f)^{3/2}c^2d^2fg^{50}e + 2835\operatorname{sqrt}(gx + f)c^2d^2f^2g^{50}e + 945\operatorname{sqrt}(gx + f)a^2d^2g^{52}e + 135(gx + f)^{7/2}c^2d^2fg^{49}e^2 - 756(gx + f)^{5/2}c^2d^2fg^{49}e^2 + 1890(gx$

$$\begin{aligned}
& + f)^{(3/2)} * c * d * f^2 * g^{49} * e^2 - 3780 * \text{sqrt}(g * x + f) * c * d * f^3 * g^{49} * e^2 \\
& + 315 * (g * x + f)^{(3/2)} * a * d * g^{51} * e^2 - 1890 * \text{sqrt}(g * x + f) * a * d * f * g^{51} * e^2 \\
& + 35 * (g * x + f)^{(9/2)} * c * g^{48} * e^3 - 225 * (g * x + f)^{(7/2)} * c * f * g^{48} * e^3 \\
& + 630 * (g * x + f)^{(5/2)} * c * f^2 * g^{48} * e^3 - 1050 * (g * x + f)^{(3/2)} * c * f^3 * g^{48} * e^3 \\
& + 1575 * \text{sqrt}(g * x + f) * c * f^4 * g^{48} * e^3 + 63 * (g * x + f)^{(5/2)} * a * g^{50} * e^3 \\
& - 315 * (g * x + f)^{(3/2)} * a * f * g^{50} * e^3 + 945 * \text{sqrt}(g * x + f) * a * f^2 * g^{50} * e^3 / g^{54}
\end{aligned}$$

$$3.597 \quad \int \frac{(d+ex)^2(a+cx^2)}{(f+gx)^{3/2}} dx$$

**Optimal.** Leaf size=173

$$\frac{2(f+gx)^{3/2}(ae^2g^2+c(d^2g^2-6defg+6e^2f^2))}{3g^5} - \frac{2(ag^2+cf^2)(ef-dg)^2}{g^5\sqrt{f+gx}} - \frac{4\sqrt{f+gx}(ef-dg)(aeg^2+cf(2ef-dg))}{g^5} - \frac{4ce(f+gx)^{5/2}(2ef-dg)}{5g^5} + \frac{2ce^2(f+gx)^{7/2}}{7g^5}$$

[Out]  $(-2*(e*f - d*g)^2*(c*f^2 + a*g^2))/(g^5*\text{Sqrt}[f + g*x]) - (4*(e*f - d*g)*(a*e*g^2 + c*f*(2*e*f - d*g))*\text{Sqrt}[f + g*x])/g^5 + (2*(a*e^2*g^2 + c*(6*e^2*f^2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^{(3/2)})/(3*g^5) - (4*c*e*(2*e*f - d*g)*(f + g*x)^{(5/2)})/(5*g^5) + (2*c*e^2*(f + g*x)^{(7/2)})/(7*g^5)$

**Rubi [A]** time = 0.492887, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{2(f+gx)^{3/2}(ae^2g^2+c(d^2g^2-6defg+6e^2f^2))}{3g^5} - \frac{2(ag^2+cf^2)(ef-dg)^2}{g^5\sqrt{f+gx}} - \frac{4\sqrt{f+gx}(ef-dg)(aeg^2+cf(2ef-dg))}{g^5} - \frac{4ce(f+gx)^{5/2}(2ef-dg)}{5g^5} + \frac{2ce^2(f+gx)^{7/2}}{7g^5}$$

Antiderivative was successfully verified.

[In]  $\text{Int}(((d + e*x)^2*(a + c*x^2))/(f + g*x)^{(3/2}), x)$

[Out]  $(-2*(e*f - d*g)^2*(c*f^2 + a*g^2))/(g^5*\text{Sqrt}[f + g*x]) - (4*(e*f - d*g)*(a*e*g^2 + c*f*(2*e*f - d*g))*\text{Sqrt}[f + g*x])/g^5 + (2*(a*e^2*g^2 + c*(6*e^2*f^2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^{(3/2)})/(3*g^5) - (4*c*e*(2*e*f - d*g)*(f + g*x)^{(5/2)})/(5*g^5) + (2*c*e^2*(f + g*x)^{(7/2)})/(7*g^5)$

**Rubi in Sympy [A]** time = 79.7398, size = 173, normalized size = 1.

$$\frac{2ce^2(f+gx)^{7/2}}{7g^5} + \frac{4ce(f+gx)^{5/2}(dg-2ef)}{5g^5} + \frac{2(f+gx)^{3/2}(ae^2g^2+cd^2g^2-6cdefg+6ce^2f^2)}{3g^5} + \frac{4\sqrt{f+gx}(dg-ef)(aeg^2-cdfg+2cef^2)}{g^5} - \frac{2(ag^2+cf^2)(dg-ef)^2}{g^5\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)**2*(c*x**2+a)/(g*x+f)**(3/2), x)$

[Out]  $2*c*e**2*(f + g*x)**(7/2)/(7*g**5) + 4*c*e*(f + g*x)**(5/2)*(d*g - 2*e*f)/(5*g**5) + 2*(f + g*x)**(3/2)*(a*e**2*g**2 + c*d**2*g**2 - 6*c*d*e*f*g + 6*c*e**2*f**2)/(3*g**5) + 4*\text{sqrt}(f + g*x)*(d*g - e*f)*(a*e*g**2 - c*d*f*g + 2*c*e*f**2)/g**5 - 2*(a*g**2 + c*f**2)*(d*g - e*f)**2/(g**5*\text{sqrt}(f + g*x))$

**Mathematica [A]** time = 0.279657, size = 177, normalized size = 1.02

$$\frac{2c(35d^2g^2(-8f^2-4fgx+g^2x^2)+42deg(16f^3+8f^2gx-2fg^2x^2+g^3x^3)-3e^2(128f^4+64f^3gx-16f^2g^2x^2+8fg^3x^3-4f^2g^2x^2+4fg^3x^3-4f^2g^2x^2+4fg^3x^3-4f^2g^2x^2+4fg^3x^3))}{105g^5\sqrt{f+gx}}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^2\*(a + c\*x^2))/(f + g\*x)^(3/2), x]

[Out]  $(-70*a*g^2*(3*d^2*g^2 - 6*d*e*g*(2*f + g*x) + e^2*(8*f^2 + 4*f*g*x - g^2*x^2)) + 2*c*(35*d^2*g^2*(-8*f^2 - 4*f*g*x + g^2*x^2) + 42*d*e*g*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3) - 3*e^2*(128*f^4 + 64*f^3*g*x - 16*f^2*g^2*x^2 + 8*f*g^3*x^3 - 5*g^4*x^4)))/(105*g^5*\text{Sqrt}[f + g*x])$

**Maple [A]** time = 0.01, size = 215, normalized size = 1.2

$$\frac{-30 e^2 c x^4 g^4 - 84 c d e g^4 x^3 + 48 c e^2 f g^3 x^3 - 70 a e^2 g^4 x^2 - 70 c d^2 g^4 x^2 + 168 c d e f g^3 x^2 - 96 c e^2 f^2 g^2 x^2 - 420 a d e g^4 x + 280 a^2 e^2 g^4}{105 g^5 \sqrt{f + g x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^2\*(c\*x^2+a)/(g\*x+f)^(3/2), x)

[Out]  $-2/105/(g*x+f)^{(1/2)}*(-15*c*e^2*g^4*x^4-42*c*d*e*g^4*x^3+24*c*e^2*f*g^3*x^3-35*a*e^2*g^4*x^2-35*c*d^2*g^4*x^2+84*c*d*e*f*g^3*x^2-4*8*c*e^2*f^2*g^2*x^2-210*a*d*e*g^4*x+140*a*e^2*f*g^3*x+140*c*d^2*f*g^3*x-336*c*d*e*f^2*g^2*x+192*c*e^2*f^3*g*x+105*a*d^2*g^4-420*a*d*e*f*g^3+280*a*e^2*f^2*g^2+280*c*d^2*f^2*g^2-672*c*d*e*f^3*g+384*c*e^2*f^4)/g^5$

**Maxima [A]** time = 0.700947, size = 277, normalized size = 1.6

$$2 \left( \frac{15(gx+f)^{7/2}ce^2-42(2ce^2f-cdeg)(gx+f)^{5/2}+35(6ce^2f^2-6cdefg+(cd^2+ae^2)g^2)(gx+f)^{3/2}-210(2ce^2f^3-3cdef^2g-ade^2g^3+(cd^2+ae^2)fg^2)\sqrt{gx+f}}{g^4} - \frac{105}{105g} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)\*(e\*x + d)^2/(g\*x + f)^(3/2), x, algorithm="maxima")

[Out]  $2/105*((15*(g*x + f)^{(7/2)}*c*e^2 - 42*(2*c*e^2*f - c*d*e*g)*(g*x + f)^{(5/2)} + 35*(6*c*e^2*f^2 - 6*c*d*e*f*g + (c*d^2 + a*e^2)*g^2)*(g*x + f)^{(3/2)} - 210*(2*c*e^2*f^3 - 3*c*d*e*f^2*g - a*d*e*g^3 + (c*d^2 + a*e^2)*f*g^2)*\text{sqrt}(g*x + f))/g^4 - 105*(c*e^2*f^4 - 2*c*d*e*f^3*g - 2*a*d*e*f*g^3 + a*d^2*g^4 + (c*d^2 + a*e^2)*f^2*g^2)/(g*\text{sqrt}(g*x + f)*g^4)/g$

**Fricas [A]** time = 0.274904, size = 265, normalized size = 1.53

$$\frac{2(15ce^2g^4x^4 - 384ce^2f^4 + 672cdef^3g + 420adefg^3 - 105ad^2g^4 - 280(cd^2 + ae^2)f^2g^2 - 6(4ce^2fg^3 - 7cdeg^4)x^3 + (48e^2fg^3 - 7c^2d^2e^2g^4)x^2 + (48c^2d^2e^2fg^3 - 7c^2d^2e^2g^4)x + 48c^2d^2e^2fg^3 - 7c^2d^2e^2g^4)}{105\sqrt{gx + fg}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)\*(e\*x + d)^2/(g\*x + f)^(3/2), x, algorithm="fricas")

[Out]  $2/105*(15*c*e^2*g^4*x^4 - 384*c*e^2*f^4 + 672*c*d*e*f^3*g + 420*a*d*e*f*g^3 - 105*a*d^2*g^4 - 280*(c*d^2 + a*e^2)*f^2*g^2 - 6*(4*c*e^2*f*g^3 - 7*c^2*d^2*e^2*g^4)*x^3 + (48*c^2*d^2*e^2*f*g^3 - 7*c^2*d^2*e^2*g^4)*x^2 + (48*c^2*d^2*e^2*f*g^3 - 7*c^2*d^2*e^2*g^4)*x + 48*c^2*d^2*e^2*f*g^3 - 7*c^2*d^2*e^2*g^4)$

$$\frac{3 + 35(c^2d^2 + a^2e^2)g^4x^2 - 2(96c^2e^2f^3g - 168c^2d^2e^2f^2g^2 - 105a^2d^2e^2g^4 + 70(c^2d^2 + a^2e^2)f^2g^3)x}{(g^2x + f)^2g^5}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + cx^2)(d + ex)^2}{(f + gx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2\*(c\*x\*\*2+a)/(g\*x+f)\*\*(3/2), x)

[Out] Integral((a + c\*x\*\*2)\*(d + e\*x)\*\*2/(f + g\*x)\*\*(3/2), x)

**GIAC/XCAS [A]** time = 0.282372, size = 371, normalized size = 2.14

$$\frac{2(cd^2f^2g^2 + ad^2g^4 - 2cdf^3ge - 2adf^3e + cf^4e^2 + af^2g^2e^2)}{\sqrt{gx + f}g^5} + \frac{2\left(35(gx + f)^{\frac{3}{2}}cd^2g^3e - 210\sqrt{gx + f}cd^2fg^3e + 42(gx + f)^{\frac{5}{2}}cdg^3e - 210(gx + f)^{\frac{3}{2}}cdfg^3e + 630\sqrt{gx + f}cdf^2g^3e + 210\right)}{g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)\*(e\*x + d)^2/(g\*x + f)^(3/2), x, algorithm="giac")

[Out] 
$$\frac{-2(c^2d^2f^2g^2 + a^2d^2g^4 - 2c^2d^2f^3ge - 2a^2d^2f^3e + c^2f^4e^2 + a^2f^2g^2e^2)}{(g^2x + f)^2g^5} + \frac{2}{105} \left( 35(g^2x + f)^{\frac{3}{2}}c^2d^2g^3e - 210\sqrt{g^2x + f}c^2d^2fg^3e + 42(g^2x + f)^{\frac{5}{2}}cdg^3e - 210(g^2x + f)^{\frac{3}{2}}cdfg^3e + 630\sqrt{g^2x + f}cdf^2g^3e + 210 \right) / g^5$$



$$3.598 \quad \int \frac{(d+ex)(a+cx^2)}{(f+gx)^{3/2}} dx$$

**Optimal.** Leaf size=111

$$\frac{2(ag^2 + cf^2)(ef - dg)}{g^4\sqrt{f+gx}} + \frac{2\sqrt{f+gx}(aeg^2 + cf(3ef - 2dg))}{g^4} - \frac{2c(f+gx)^{3/2}(3ef - dg)}{3g^4} + \frac{2ce(f+gx)^{5/2}}{5g^4}$$

[Out]  $(2*(e*f - d*g)*(c*f^2 + a*g^2))/(g^4*\text{Sqrt}[f + g*x]) + (2*(a*e*g^2 + c*f*(3*e*f - 2*d*g))*\text{Sqrt}[f + g*x])/g^4 - (2*c*(3*e*f - d*g)*(f + g*x)^{(3/2)})/(3*g^4) + (2*c*e*(f + g*x)^{(5/2)})/(5*g^4)$

**Rubi [A]** time = 0.171267, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{2(ag^2 + cf^2)(ef - dg)}{g^4\sqrt{f+gx}} + \frac{2\sqrt{f+gx}(aeg^2 + cf(3ef - 2dg))}{g^4} - \frac{2c(f+gx)^{3/2}(3ef - dg)}{3g^4} + \frac{2ce(f+gx)^{5/2}}{5g^4}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*(a + c\*x^2))/(f + g\*x)^(3/2), x]

[Out]  $(2*(e*f - d*g)*(c*f^2 + a*g^2))/(g^4*\text{Sqrt}[f + g*x]) + (2*(a*e*g^2 + c*f*(3*e*f - 2*d*g))*\text{Sqrt}[f + g*x])/g^4 - (2*c*(3*e*f - d*g)*(f + g*x)^{(3/2)})/(3*g^4) + (2*c*e*(f + g*x)^{(5/2)})/(5*g^4)$

**Rubi in Sympy [A]** time = 23.2014, size = 110, normalized size = 0.99

$$\frac{2ce(f+gx)^{\frac{5}{2}}}{5g^4} + \frac{2c(f+gx)^{\frac{3}{2}}(dg - 3ef)}{3g^4} + \frac{2\sqrt{f+gx}(aeg^2 - 2cdfg + 3cef^2)}{g^4} - \frac{2(ag^2 + cf^2)(dg - ef)}{g^4\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*(c\*x\*\*2+a)/(g\*x+f)\*\*(3/2), x)

[Out]  $2*c*e*(f + g*x)**(5/2)/(5*g**4) + 2*c*(f + g*x)**(3/2)*(d*g - 3*e*f)/(3*g**4) + 2*\text{sqrt}(f + g*x)*(a*e*g**2 - 2*c*d*f*g + 3*c*e*f**2)/g**4 - 2*(a*g**2 + c*f**2)*(d*g - e*f)/(g**4*\text{sqrt}(f + g*x))$

**Mathematica [A]** time = 0.125521, size = 92, normalized size = 0.83

$$\frac{30ag^2(-dg + 2ef + egx) + 10cdg(-8f^2 - 4fgx + g^2x^2) + 6ce(16f^3 + 8f^2gx - 2fg^2x^2 + g^3x^3)}{15g^4\sqrt{f+gx}}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(a + c\*x^2))/(f + g\*x)^(3/2), x]

[Out]  $(30*a*g^2*(2*e*f - d*g + e*g*x) + 10*c*d*g*(-8*f^2 - 4*f*g*x + g^2*x^2) + 6*c*e*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3))/(15*g^4*\text{Sqrt}[f + g*x])$

**Maple [A]** time = 0.006, size = 101, normalized size = 0.9

$$\frac{-6cex^3g^3 - 10cdg^3x^2 + 12cef^2g^2x^2 - 30aeg^3x + 40cdfg^2x - 48cef^2gx + 30adg^3 - 60aefg^2 + 80cdf^2g - 96cef^3}{15g^4} \sqrt{g^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)*(c*x^2+a)/(g*x+f)^(3/2),x)`

[Out] 
$$-2/15/(g*x+f)^{(1/2)} * (-3*c*e*g^3*x^3 - 5*c*d*g^3*x^2 + 6*c*e*f*g^2*x^2 - 15*a*e*g^3*x + 20*c*d*f*g^2*x - 24*c*e*f^2*g*x + 15*a*d*g^3 - 30*a*e*f*g^2 + 40*c*d*f^2*g - 48*c*e*f^3)/g^4$$

**Maxima [A]** time = 0.701284, size = 151, normalized size = 1.36

$$\frac{2 \left( \frac{3(gx+f)^{\frac{5}{2}}ce - 5(3cef - cdg)(gx+f)^{\frac{3}{2}} + 15(3cef^2 - 2cdfg + aeg^2)\sqrt{gx+f}}{g^3} + \frac{15(cef^3 - cdf^2g + aefg^2 - adg^3)}{\sqrt{gx+f}g^3} \right)}{15g}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)*(e*x + d)/(g*x + f)^(3/2),x, algorithm="maxima")`

[Out] 
$$2/15 * ((3*(g*x + f)^{(5/2)} * c*e - 5*(3*c*e*f - c*d*g)*(g*x + f)^{(3/2)} + 15*(3*c*e*f^2 - 2*c*d*f*g + a*e*g^2)*\sqrt{g*x + f})/g^3 + 15*(c*e*f^3 - c*d*f^2*g + a*e*f*g^2 - a*d*g^3)/(\sqrt{g*x + f}*g^3))/g$$

**Fricas [A]** time = 0.273172, size = 135, normalized size = 1.22

$$\frac{2(3ceg^3x^3 + 48cef^3 - 40cdf^2g + 30aefg^2 - 15adg^3 - (6cef^2g - 5cdg^3)x^2 + (24cef^2g - 20cdfg^2 + 15aeg^3)x)}{15\sqrt{gx + fg^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)*(e*x + d)/(g*x + f)^(3/2),x, algorithm="fricas")`

[Out] 
$$2/15 * (3*c*e*g^3*x^3 + 48*c*e*f^3 - 40*c*d*f^2*g + 30*a*e*f*g^2 - 15*a*d*g^3 - (6*c*e*f^2g - 5*c*d*g^3)*x^2 + (24*c*e*f^2g - 20*c*d*f*g^2 + 15*a*e*g^3)*x)/(\sqrt{g*x + f}*g^4)$$

**Sympy [A]** time = 20.2027, size = 2139, normalized size = 19.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(c*x**2+a)/(g*x+f)**(3/2),x)`

[Out] 
$$-2*a*d/(g*\sqrt{f + g*x}) + a*e*\text{Piecewise}((4*f/(g**2*\sqrt{f + g*x})) + 2*x/(g*\sqrt{f + g*x}), \text{Ne}(g, 0)), (x**2/(2*f**(3/2)), \text{True})) + c*d*(-16*f**(19/2)*\sqrt{1 + g*x/f}/(3*f**8*g**3 + 9*f**7*g**4*x + 9*f**6*g**5*x**2 + 3*f**5*g**6*x**3) + 16*f**(19/2)/(3*f**8*g**3 + 9*f**7*g**4*x + 9*f**6*g**5*x**2 + 3*f**5*g**6*x**3) - 40*f*$$

$$\begin{aligned}
& * (17/2) * g * x * \sqrt{1 + g * x / f} / (3 * f ** 8 * g ** 3 + 9 * f ** 7 * g ** 4 * x + 9 * f ** 6 * \\
& * g ** 5 * x ** 2 + 3 * f ** 5 * g ** 6 * x ** 3) + 48 * f ** (17/2) * g * x / (3 * f ** 8 * g ** 3 + \\
& 9 * f ** 7 * g ** 4 * x + 9 * f ** 6 * g ** 5 * x ** 2 + 3 * f ** 5 * g ** 6 * x ** 3) - 30 * f ** (15/ \\
& 2) * g ** 2 * x ** 2 * \sqrt{1 + g * x / f} / (3 * f ** 8 * g ** 3 + 9 * f ** 7 * g ** 4 * x + 9 * f ** 6 * \\
& 6 * g ** 5 * x ** 2 + 3 * f ** 5 * g ** 6 * x ** 3) + 48 * f ** (15/2) * g ** 2 * x ** 2 / (3 * f ** 8 * \\
& g ** 3 + 9 * f ** 7 * g ** 4 * x + 9 * f ** 6 * g ** 5 * x ** 2 + 3 * f ** 5 * g ** 6 * x ** 3) - 4 * f \\
& ** (13/2) * g ** 3 * x ** 3 * \sqrt{1 + g * x / f} / (3 * f ** 8 * g ** 3 + 9 * f ** 7 * g ** 4 * x + \\
& 9 * f ** 6 * g ** 5 * x ** 2 + 3 * f ** 5 * g ** 6 * x ** 3) + 16 * f ** (13/2) * g ** 3 * x ** 3 / (3 \\
& * f ** 8 * g ** 3 + 9 * f ** 7 * g ** 4 * x + 9 * f ** 6 * g ** 5 * x ** 2 + 3 * f ** 5 * g ** 6 * x ** 3) \\
& + 2 * f ** (11/2) * g ** 4 * x ** 4 * \sqrt{1 + g * x / f} / (3 * f ** 8 * g ** 3 + 9 * f ** 7 * g ** 4 * x + \\
& 9 * f ** 6 * g ** 5 * x ** 2 + 3 * f ** 5 * g ** 6 * x ** 3) + c * e * (32 * f ** (45/2) * \\
& \sqrt{1 + g * x / f} / (5 * f ** 20 * g ** 4 + 30 * f ** 19 * g ** 5 * x + 75 * f ** 18 * g ** 6 * x \\
& ** 2 + 100 * f ** 17 * g ** 7 * x ** 3 + 75 * f ** 16 * g ** 8 * x ** 4 + 30 * f ** 15 * g ** 9 * x \\
& * 5 + 5 * f ** 14 * g ** 10 * x ** 6) - 32 * f ** (45/2) / (5 * f ** 20 * g ** 4 + 30 * f ** 19 * \\
& g ** 5 * x + 75 * f ** 18 * g ** 6 * x ** 2 + 100 * f ** 17 * g ** 7 * x ** 3 + 75 * f ** 16 * g ** 8 * \\
& * x ** 4 + 30 * f ** 15 * g ** 9 * x ** 5 + 5 * f ** 14 * g ** 10 * x ** 6) + 176 * f ** (43/2) * \\
& g * x * \sqrt{1 + g * x / f} / (5 * f ** 20 * g ** 4 + 30 * f ** 19 * g ** 5 * x + 75 * f ** 18 * g ** 6 * \\
& * x ** 2 + 100 * f ** 17 * g ** 7 * x ** 3 + 75 * f ** 16 * g ** 8 * x ** 4 + 30 * f ** 15 * g ** 9 * \\
& * x ** 5 + 5 * f ** 14 * g ** 10 * x ** 6) - 192 * f ** (43/2) * g * x / (5 * f ** 20 * g ** 4 + \\
& 30 * f ** 19 * g ** 5 * x + 75 * f ** 18 * g ** 6 * x ** 2 + 100 * f ** 17 * g ** 7 * x ** 3 + 75 * f \\
& ** 16 * g ** 8 * x ** 4 + 30 * f ** 15 * g ** 9 * x ** 5 + 5 * f ** 14 * g ** 10 * x ** 6) + 396 * f \\
& ** (41/2) * g ** 2 * x ** 2 * \sqrt{1 + g * x / f} / (5 * f ** 20 * g ** 4 + 30 * f ** 19 * g ** 5 * \\
& x + 75 * f ** 18 * g ** 6 * x ** 2 + 100 * f ** 17 * g ** 7 * x ** 3 + 75 * f ** 16 * g ** 8 * x ** 4 \\
& + 30 * f ** 15 * g ** 9 * x ** 5 + 5 * f ** 14 * g ** 10 * x ** 6) - 480 * f ** (41/2) * g ** 2 * \\
& x ** 2 / (5 * f ** 20 * g ** 4 + 30 * f ** 19 * g ** 5 * x + 75 * f ** 18 * g ** 6 * x ** 2 + 100 * f \\
& ** 17 * g ** 7 * x ** 3 + 75 * f ** 16 * g ** 8 * x ** 4 + 30 * f ** 15 * g ** 9 * x ** 5 + 5 * f ** 1 \\
& 4 * g ** 10 * x ** 6) + 462 * f ** (39/2) * g ** 3 * x ** 3 * \sqrt{1 + g * x / f} / (5 * f ** 20 * \\
& g ** 4 + 30 * f ** 19 * g ** 5 * x + 75 * f ** 18 * g ** 6 * x ** 2 + 100 * f ** 17 * g ** 7 * x ** 3 \\
& + 75 * f ** 16 * g ** 8 * x ** 4 + 30 * f ** 15 * g ** 9 * x ** 5 + 5 * f ** 14 * g ** 10 * x ** 6) \\
& - 640 * f ** (39/2) * g ** 3 * x ** 3 / (5 * f ** 20 * g ** 4 + 30 * f ** 19 * g ** 5 * x + 75 * f * \\
& * 18 * g ** 6 * x ** 2 + 100 * f ** 17 * g ** 7 * x ** 3 + 75 * f ** 16 * g ** 8 * x ** 4 + 30 * f ** \\
& 15 * g ** 9 * x ** 5 + 5 * f ** 14 * g ** 10 * x ** 6) + 290 * f ** (37/2) * g ** 4 * x ** 4 * \sqrt{ \\
& (1 + g * x / f) / (5 * f ** 20 * g ** 4 + 30 * f ** 19 * g ** 5 * x + 75 * f ** 18 * g ** 6 * x ** 2 \\
& + 100 * f ** 17 * g ** 7 * x ** 3 + 75 * f ** 16 * g ** 8 * x ** 4 + 30 * f ** 15 * g ** 9 * x ** 5 + \\
& 5 * f ** 14 * g ** 10 * x ** 6) - 480 * f ** (37/2) * g ** 4 * x ** 4 / (5 * f ** 20 * g ** 4 + 30 \\
& * f ** 19 * g ** 5 * x + 75 * f ** 18 * g ** 6 * x ** 2 + 100 * f ** 17 * g ** 7 * x ** 3 + 75 * f ** \\
& 16 * g ** 8 * x ** 4 + 30 * f ** 15 * g ** 9 * x ** 5 + 5 * f ** 14 * g ** 10 * x ** 6) + 92 * f ** ( \\
& 35/2) * g ** 5 * x ** 5 * \sqrt{1 + g * x / f} / (5 * f ** 20 * g ** 4 + 30 * f ** 19 * g ** 5 * x + \\
& 75 * f ** 18 * g ** 6 * x ** 2 + 100 * f ** 17 * g ** 7 * x ** 3 + 75 * f ** 16 * g ** 8 * x ** 4 + \\
& 30 * f ** 15 * g ** 9 * x ** 5 + 5 * f ** 14 * g ** 10 * x ** 6) - 192 * f ** (35/2) * g ** 5 * x ** \\
& 5 / (5 * f ** 20 * g ** 4 + 30 * f ** 19 * g ** 5 * x + 75 * f ** 18 * g ** 6 * x ** 2 + 100 * f ** 1 \\
& 7 * g ** 7 * x ** 3 + 75 * f ** 16 * g ** 8 * x ** 4 + 30 * f ** 15 * g ** 9 * x ** 5 + 5 * f ** 14 * g \\
& ** 10 * x ** 6) + 16 * f ** (33/2) * g ** 6 * x ** 6 * \sqrt{1 + g * x / f} / (5 * f ** 20 * g ** 4 \\
& + 30 * f ** 19 * g ** 5 * x + 75 * f ** 18 * g ** 6 * x ** 2 + 100 * f ** 17 * g ** 7 * x ** 3 + 7 \\
& 5 * f ** 16 * g ** 8 * x ** 4 + 30 * f ** 15 * g ** 9 * x ** 5 + 5 * f ** 14 * g ** 10 * x ** 6) - 32 \\
& * f ** (33/2) * g ** 6 * x ** 6 / (5 * f ** 20 * g ** 4 + 30 * f ** 19 * g ** 5 * x + 75 * f ** 18 * g \\
& ** 6 * x ** 2 + 100 * f ** 17 * g ** 7 * x ** 3 + 75 * f ** 16 * g ** 8 * x ** 4 + 30 * f ** 15 * g * \\
& * 9 * x ** 5 + 5 * f ** 14 * g ** 10 * x ** 6) + 6 * f ** (31/2) * g ** 7 * x ** 7 * \sqrt{1 + g * \\
& x / f} / (5 * f ** 20 * g ** 4 + 30 * f ** 19 * g ** 5 * x + 75 * f ** 18 * g ** 6 * x ** 2 + 100 * f \\
& ** 17 * g ** 7 * x ** 3 + 75 * f ** 16 * g ** 8 * x ** 4 + 30 * f ** 15 * g ** 9 * x ** 5 + 5 * f ** 1 \\
& 4 * g ** 10 * x ** 6) + 2 * f ** (29/2) * g ** 8 * x ** 8 * \sqrt{1 + g * x / f} / (5 * f ** 20 * g * \\
& * 4 + 30 * f ** 19 * g ** 5 * x + 75 * f ** 18 * g ** 6 * x ** 2 + 100 * f ** 17 * g ** 7 * x ** 3 + \\
& 75 * f ** 16 * g ** 8 * x ** 4 + 30 * f ** 15 * g ** 9 * x ** 5 + 5 * f ** 14 * g ** 10 * x ** 6)
\end{aligned}$$

**GIAC/XCAS [A]** time = 0.270497, size = 193, normalized size = 1.74

$$\frac{2(cdf^2g + adg^3 - cf^3e - afg^2e)}{\sqrt{gx + fg^4}} + \frac{2\left(5(gx + f)^{\frac{3}{2}}cdg^{17} - 30\sqrt{gx + f}cdfg^{17} + 3(gx + f)^{\frac{5}{2}}cg^{16}e - 15(gx + f)^{\frac{3}{2}}cfg^{16}e + 45\sqrt{gx + f}cf^2g^{16}e + 15\sqrt{gx + f}ag^{18}\right)}{15g^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)\*(e\*x + d)/(g\*x + f)^(3/2), x, algorithm="giac")

[Out] -2\*(c\*d\*f^2\*g + a\*d\*g^3 - c\*f^3\*e - a\*f\*g^2\*e)/(sqrt(g\*x + f)\*g^4

$$\begin{aligned}
 & ) + 2/15*(5*(g*x + f)^{(3/2)}*c*d*g^{17} - 30*\text{sqrt}(g*x + f)*c*d*f*g^{17} \\
 & + 3*(g*x + f)^{(5/2)}*c*g^{16}*e - 15*(g*x + f)^{(3/2)}*c*f*g^{16}*e + \\
 & 45*\text{sqrt}(g*x + f)*c*f^2*g^{16}*e + 15*\text{sqrt}(g*x + f)*a*g^{18}*e)/g^{20}
 \end{aligned}$$

$$3.599 \quad \int \frac{a+cx^2}{(f+gx)^{3/2}} dx$$

**Optimal.** Leaf size=59

$$-\frac{2(ag^2 + cf^2)}{g^3\sqrt{f+gx}} + \frac{2c(f+gx)^{3/2}}{3g^3} - \frac{4cf\sqrt{f+gx}}{g^3}$$

[Out]  $(-2*(c*f^2 + a*g^2))/(g^3*\text{Sqrt}[f + g*x]) - (4*c*f*\text{Sqrt}[f + g*x])/g^3 + (2*c*(f + g*x)^{(3/2)})/(3*g^3)$

**Rubi [A]** time = 0.061942, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 17,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$

$$-\frac{2(ag^2 + cf^2)}{g^3\sqrt{f+gx}} + \frac{2c(f+gx)^{3/2}}{3g^3} - \frac{4cf\sqrt{f+gx}}{g^3}$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^2)/(f + g\*x)^(3/2), x]

[Out]  $(-2*(c*f^2 + a*g^2))/(g^3*\text{Sqrt}[f + g*x]) - (4*c*f*\text{Sqrt}[f + g*x])/g^3 + (2*c*(f + g*x)^{(3/2)})/(3*g^3)$

**Rubi in Sympy [A]** time = 11.3242, size = 58, normalized size = 0.98

$$-\frac{4cf\sqrt{f+gx}}{g^3} + \frac{2c(f+gx)^{3/2}}{3g^3} - \frac{2(ag^2 + cf^2)}{g^3\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+a)/(g\*x+f)\*\*(3/2), x)

[Out]  $-4*c*f*\text{sqrt}(f + g*x)/g**3 + 2*c*(f + g*x)**(3/2)/(3*g**3) - 2*(a*g**2 + c*f**2)/(g**3*\text{sqrt}(f + g*x))$

**Mathematica [A]** time = 0.0388053, size = 43, normalized size = 0.73

$$\frac{2(c(-8f^2 - 4fgx + g^2x^2) - 3ag^2)}{3g^3\sqrt{f+gx}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c\*x^2)/(f + g\*x)^(3/2), x]

[Out]  $(2*(-3*a*g^2 + c*(-8*f^2 - 4*f*g*x + g^2*x^2)))/(3*g^3*\text{Sqrt}[f + g*x])$

**Maple [A]** time = 0.006, size = 41, normalized size = 0.7

$$-\frac{-2cx^2g^2 + 8cfxg + 6ag^2 + 16cf^2}{3g^3} \frac{1}{\sqrt{gx+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+a)/(g*x+f)^(3/2),x)`

[Out]  $-2/3/(g*x+f)^{(1/2)}*(-c*g^2*x^2+4*c*f*g*x+3*a*g^2+8*c*f^2)/g^3$

**Maxima [A]** time = 0.693281, size = 73, normalized size = 1.24

$$\frac{2 \left( \frac{(gx+f)^{\frac{3}{2}} c - 6 \sqrt{gx+f} cf}{g^2} - \frac{3(cf^2+ag^2)}{\sqrt{gx+f} g^2} \right)}{3g}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)/(g*x + f)^(3/2),x, algorithm="maxima")`

[Out]  $2/3*((g*x + f)^{(3/2)}*c - 6*\sqrt{g*x + f}*c*f)/g^2 - 3*(c*f^2 + a*g^2)/(\sqrt{g*x + f}*g^2)/g$

**Fricas [A]** time = 0.276173, size = 53, normalized size = 0.9

$$\frac{2(cg^2x^2 - 4cfgx - 8cf^2 - 3ag^2)}{3\sqrt{gx + f}g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)/(g*x + f)^(3/2),x, algorithm="fricas")`

[Out]  $2/3*(c*g^2*x^2 - 4*c*f*g*x - 8*c*f^2 - 3*a*g^2)/(\sqrt{g*x + f}*g^3)$

**Sympy [A]** time = 9.19211, size = 549, normalized size = 9.31

$$\begin{aligned} & -\frac{2a}{g\sqrt{f+gx}} + c \left( -\frac{16f^{\frac{19}{2}}\sqrt{1+\frac{gx}{f}}}{3f^8g^3+9f^7g^4x+9f^6g^5x^2+3f^5g^6x^3} \right. \\ & + \frac{16f^{\frac{19}{2}}}{3f^8g^3+9f^7g^4x+9f^6g^5x^2+3f^5g^6x^3} - \frac{40f^{\frac{17}{2}}gx\sqrt{1+\frac{gx}{f}}}{3f^8g^3+9f^7g^4x+9f^6g^5x^2+3f^5g^6x^3} \\ & + \frac{48f^{\frac{17}{2}}gx}{3f^8g^3+9f^7g^4x+9f^6g^5x^2+3f^5g^6x^3} - \frac{30f^{\frac{15}{2}}g^2x^2\sqrt{1+\frac{gx}{f}}}{3f^8g^3+9f^7g^4x+9f^6g^5x^2+3f^5g^6x^3} \\ & + \frac{48f^{\frac{15}{2}}g^2x^2}{3f^8g^3+9f^7g^4x+9f^6g^5x^2+3f^5g^6x^3} - \frac{4f^{\frac{13}{2}}g^3x^3\sqrt{1+\frac{gx}{f}}}{3f^8g^3+9f^7g^4x+9f^6g^5x^2+3f^5g^6x^3} \\ & \left. + \frac{16f^{\frac{13}{2}}g^3x^3}{3f^8g^3+9f^7g^4x+9f^6g^5x^2+3f^5g^6x^3} + \frac{2f^{\frac{11}{2}}g^4x^4\sqrt{1+\frac{gx}{f}}}{3f^8g^3+9f^7g^4x+9f^6g^5x^2+3f^5g^6x^3} \right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+a)/(g*x+f)**(3/2),x)`

```
[Out] -2*a/(g*sqrt(f + g*x)) + c*(-16*f**(19/2)*sqrt(1 + g*x/f)/(3*f**8
*g**3 + 9*f**7*g**4*x + 9*f**6*g**5*x**2 + 3*f**5*g**6*x**3) + 16
*f**(19/2)/(3*f**8*g**3 + 9*f**7*g**4*x + 9*f**6*g**5*x**2 + 3*f
*5*g**6*x**3) - 40*f**(17/2)*g*x*sqrt(1 + g*x/f)/(3*f**8*g**3 + 9
*f**7*g**4*x + 9*f**6*g**5*x**2 + 3*f**5*g**6*x**3) + 48*f**(17/2
)*g*x/(3*f**8*g**3 + 9*f**7*g**4*x + 9*f**6*g**5*x**2 + 3*f**5*g
*6*x**3) - 30*f**(15/2)*g**2*x**2*sqrt(1 + g*x/f)/(3*f**8*g**3 +
9*f**7*g**4*x + 9*f**6*g**5*x**2 + 3*f**5*g**6*x**3) + 48*f**(15/
2)*g**2*x**2/(3*f**8*g**3 + 9*f**7*g**4*x + 9*f**6*g**5*x**2 + 3*
f**5*g**6*x**3) - 4*f**(13/2)*g**3*x**3*sqrt(1 + g*x/f)/(3*f**8*g
**3 + 9*f**7*g**4*x + 9*f**6*g**5*x**2 + 3*f**5*g**6*x**3) + 16*f
**(13/2)*g**3*x**3/(3*f**8*g**3 + 9*f**7*g**4*x + 9*f**6*g**5*x**
2 + 3*f**5*g**6*x**3) + 2*f**(11/2)*g**4*x**4*sqrt(1 + g*x/f)/(3*
f**8*g**3 + 9*f**7*g**4*x + 9*f**6*g**5*x**2 + 3*f**5*g**6*x**3))
```

**GIAC/XCAS [A]** time = 0.269809, size = 76, normalized size = 1.29

$$-\frac{2(cf^2 + ag^2)}{\sqrt{gx + fg^3}} + \frac{2\left((gx + f)^{\frac{3}{2}}cg^6 - 6\sqrt{gx + f}cfg^6\right)}{3g^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + a)/(g*x + f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*f^2 + a*g^2)/(sqrt(g*x + f)*g^3) + 2/3*((g*x + f)^(3/2)*c*g
^6 - 6*sqrt(g*x + f)*c*f*g^6)/g^9
```

$$3.600 \quad \int \frac{a+cx^2}{(d+ex)(f+gx)^{3/2}} dx$$

**Optimal.** Leaf size=112

$$-\frac{2(ae^2 + cd^2) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}} + \frac{2(ag^2 + cf^2)}{g^2\sqrt{f+gx}(ef-dg)} + \frac{2c\sqrt{f+gx}}{eg^2}$$

[Out] (2\*(c\*f^2 + a\*g^2))/(g^2\*(e\*f - d\*g)\*Sqrt[f + g\*x]) + (2\*c\*Sqrt[f + g\*x))/(e\*g^2) - (2\*(c\*d^2 + a\*e^2)\*ArcTanh[(Sqrt[e]\*Sqrt[f + g\*x])/Sqrt[e\*f - d\*g]])/(e^(3/2)\*(e\*f - d\*g)^(3/2))

**Rubi [A]** time = 0.371492, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$-\frac{2(ae^2 + cd^2) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}} + \frac{2(ag^2 + cf^2)}{g^2\sqrt{f+gx}(ef-dg)} + \frac{2c\sqrt{f+gx}}{eg^2}$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^2)/((d + e\*x)\*(f + g\*x)^(3/2)), x]

[Out] (2\*(c\*f^2 + a\*g^2))/(g^2\*(e\*f - d\*g)\*Sqrt[f + g\*x]) + (2\*c\*Sqrt[f + g\*x))/(e\*g^2) - (2\*(c\*d^2 + a\*e^2)\*ArcTanh[(Sqrt[e]\*Sqrt[f + g\*x])/Sqrt[e\*f - d\*g]])/(e^(3/2)\*(e\*f - d\*g)^(3/2))

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{2(ag^2 + cf^2)}{g^2\sqrt{f+gx}(dg-ef)} + \frac{2\int\sqrt{f+gx}c dx}{eg^2} - \frac{2(ae^2 + cd^2) \operatorname{atan}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{dg-ef}}\right)}{e^{3/2}(dg-ef)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+a)/(e\*x+d)/(g\*x+f)\*\*(3/2), x)

[Out] -2\*(a\*g\*\*2 + c\*f\*\*2)/(g\*\*2\*sqrt(f + g\*x)\*(d\*g - e\*f)) + 2\*Integral(c, (x, sqrt(f + g\*x)))/(e\*g\*\*2) - 2\*(a\*e\*\*2 + c\*d\*\*2)\*atan(sqrt(e)\*sqrt(f + g\*x)/sqrt(d\*g - e\*f))/(e\*\*(3/2)\*(d\*g - e\*f)\*\*(3/2))

**Mathematica [A]** time = 0.341282, size = 108, normalized size = 0.96

$$\frac{2\sqrt{f+gx}\left(\frac{ag^2+cf^2}{(f+gx)(ef-dg)} + \frac{c}{e}\right)}{g^2} - \frac{2(ae^2 + cd^2) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c\*x^2)/((d + e\*x)\*(f + g\*x)^(3/2)), x]

[Out] (2\*Sqrt[f + g\*x]\*(c/e + (c\*f^2 + a\*g^2)/((e\*f - d\*g)\*(f + g\*x))))/g^2 - (2\*(c\*d^2 + a\*e^2)\*ArcTanh[(Sqrt[e]\*Sqrt[f + g\*x])/Sqrt[e\*f - d\*g]])/(e^(3/2)\*(e\*f - d\*g)^(3/2))



$f - d \cdot g]] / (e^{3/2} \cdot (e \cdot f - d \cdot g)^{3/2})$

**Maple [A]** time = 0.017, size = 165, normalized size = 1.5

$$2 \frac{c\sqrt{gx+f}}{eg^2} - 2 \frac{a}{(dg-ef)\sqrt{gx+f}} - 2 \frac{cf^2}{g^2(dg-ef)\sqrt{gx+f}}$$

$$- 2 \frac{ae}{(dg-ef)\sqrt{(dg-ef)e}} \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{(dg-ef)e}}\right)$$

$$- 2 \frac{cd^2}{(dg-ef)e\sqrt{(dg-ef)e}} \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{(dg-ef)e}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+a)/(e*x+d)/(g*x+f)^(3/2),x)`

[Out]  $2 \cdot c \cdot (g \cdot x + f)^{1/2} / e / g^2 - 2 / (d \cdot g - e \cdot f) / (g \cdot x + f)^{1/2} \cdot a - 2 / g^2 / (d \cdot g - e \cdot f) / (g \cdot x + f)^{1/2} \cdot c \cdot f^2 - 2 / (d \cdot g - e \cdot f) \cdot e / ((d \cdot g - e \cdot f) \cdot e)^{1/2} \cdot \arctan((g \cdot x + f)^{1/2} \cdot e / ((d \cdot g - e \cdot f) \cdot e)^{1/2}) \cdot a - 2 / (d \cdot g - e \cdot f) / e / ((d \cdot g - e \cdot f) \cdot e)^{1/2} \cdot \arctan((g \cdot x + f)^{1/2} \cdot e / ((d \cdot g - e \cdot f) \cdot e)^{1/2}) \cdot c \cdot d^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)/((e*x + d)*(g*x + f)^(3/2)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.289716, size = 1, normalized size = 0.01

$$\left[ \frac{(cd^2 + ae^2)\sqrt{gx+f}g^2 \log\left(\frac{\sqrt{e^2f-deg}(egx+2ef-dg)+2(e^2f-deg)\sqrt{gx+f}}{ex+d}\right) - 2(2cef^2 - cdfg + aeg^2 + (cefg - cdg^2)x)\sqrt{e^2f-deg}}{(e^2fg^2 - deg^3)\sqrt{e^2f-deg}\sqrt{gx+f}} \right.$$

$$\left. - \frac{2\left((cd^2 + ae^2)\sqrt{gx+f}g^2 \arctan\left(-\frac{ef-dg}{\sqrt{-e^2f+deg}\sqrt{gx+f}}\right) - (2cef^2 - cdfg + aeg^2 + (cefg - cdg^2)x)\sqrt{-e^2f+deg}\right)}{(e^2fg^2 - deg^3)\sqrt{-e^2f+deg}\sqrt{gx+f}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)/((e*x + d)*(g*x + f)^(3/2)),x, algorithm="fricas")`

[Out]  $[-((c \cdot d^2 + a \cdot e^2) \cdot \sqrt{g \cdot x + f}) \cdot g^2 \cdot \log((\sqrt{e^2 \cdot f - d \cdot e \cdot g}) \cdot (e \cdot g \cdot x + 2 \cdot e \cdot f - d \cdot g) + 2 \cdot (e^2 \cdot f - d \cdot e \cdot g) \cdot \sqrt{g \cdot x + f}) / (e \cdot x + d)) - 2 \cdot (2 \cdot c \cdot e \cdot f^2 - c \cdot d \cdot f \cdot g + a \cdot e \cdot g^2 + (c \cdot e \cdot f \cdot g - c \cdot d \cdot g^2) \cdot x) \cdot \sqrt{e^2 \cdot f - d \cdot e \cdot g} / ((e^2 \cdot f \cdot g^2 - d \cdot e \cdot g^3) \cdot \sqrt{e^2 \cdot f - d \cdot e \cdot g}) \cdot \sqrt{g \cdot x + f}), -2 \cdot ((c \cdot d^2 + a \cdot e^2) \cdot \sqrt{g \cdot x + f}) \cdot g^2 \cdot \arctan(-(e \cdot f - d \cdot g) / (\sqrt{-e^2 \cdot f + d \cdot e \cdot g} \cdot \sqrt{g \cdot x + f})) - (2 \cdot c \cdot e \cdot f^2 - c \cdot d \cdot f \cdot g + a \cdot e \cdot g^2 + (c \cdot e \cdot f \cdot g - c \cdot d \cdot g^2) \cdot x) \cdot \sqrt{-e^2 \cdot f + d \cdot e \cdot g} / ((e^2 \cdot f \cdot g^2 - d \cdot e \cdot g^3) \cdot \sqrt{-e^2 \cdot f + d \cdot e \cdot g}) \cdot \sqrt{g \cdot x + f})]$

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + cx^2}{(d + ex)(f + gx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+a)/(e\*x+d)/(g\*x+f)\*\*(3/2),x)

[Out] Integral((a + c\*x\*\*2)/((d + e\*x)\*(f + g\*x)\*\*(3/2)), x)

---

**GIAC/XCAS [A]** time = 0.278422, size = 136, normalized size = 1.21

$$-\frac{2(cd^2 + ae^2) \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{dge-fe^2}}\right)}{(dge - fe^2)^{\frac{3}{2}}} + \frac{2\sqrt{gx+fe}ce^{(-1)}}{g^2} - \frac{2(cf^2 + ag^2)}{(dg^3 - fg^2e)\sqrt{gx+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)/((e\*x + d)\*(g\*x + f)^(3/2)),x, algorithm="giac")

[Out] -2\*(c\*d^2 + a\*e^2)\*arctan(sqrt(g\*x + f)\*e/sqrt(d\*g\*e - f\*e^2))/(d\*g\*e - f\*e^2)^(3/2) + 2\*sqrt(g\*x + f)\*c\*e^(-1)/g^2 - 2\*(c\*f^2 + a\*g^2)/((d\*g^3 - f\*g^2\*e)\*sqrt(g\*x + f))

$$3.601 \quad \int \frac{a+cx^2}{(d+ex)^2(f+gx)^{3/2}} dx$$

**Optimal.** Leaf size=144

$$-\frac{\sqrt{f+gx}(ae^2+cd^2)}{e(d+ex)(ef-dg)^2} + \frac{(3ae^2g+cd(4ef-dg)) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{5/2}} - \frac{2(ag^2+cf^2)}{g\sqrt{f+gx}(ef-dg)^2}$$

[Out]  $(-2*(c*f^2 + a*g^2))/(g*(e*f - d*g)^2*\text{Sqrt}[f + g*x]) - ((c*d^2 + a*e^2)*\text{Sqrt}[f + g*x])/(e*(e*f - d*g)^2*(d + e*x)) + ((3*a*e^2*g + c*d*(4*e*f - d*g))*\text{ArcTanh}[(\text{Sqrt}[e]*\text{Sqrt}[f + g*x])/\text{Sqrt}[e*f - d*g]])/(e^{3/2}*(e*f - d*g)^{5/2})$

**Rubi [A]** time = 0.610172, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$

$$-\frac{\sqrt{f+gx}(ae^2+cd^2)}{e(d+ex)(ef-dg)^2} + \frac{(3ae^2g+cd(4ef-dg)) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{5/2}} - \frac{2(ag^2+cf^2)}{g\sqrt{f+gx}(ef-dg)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^2)/((d + e\*x)^2\*(f + g\*x)^(3/2)), x]

[Out]  $(-2*(c*f^2 + a*g^2))/(g*(e*f - d*g)^2*\text{Sqrt}[f + g*x]) - ((c*d^2 + a*e^2)*\text{Sqrt}[f + g*x])/(e*(e*f - d*g)^2*(d + e*x)) + ((3*a*e^2*g + c*d*(4*e*f - d*g))*\text{ArcTanh}[(\text{Sqrt}[e]*\text{Sqrt}[f + g*x])/\text{Sqrt}[e*f - d*g]])/(e^{3/2}*(e*f - d*g)^{5/2})$

**Rubi in Sympy [A]** time = 174.434, size = 141, normalized size = 0.98

$$-\frac{2(ag^2+cf^2)}{g\sqrt{f+gx}(dg-ef)^2} - \frac{g\sqrt{f+gx}(ae^2+cd^2)}{e(dg-ef)^2(dg-ef+e(f+gx))} - \frac{(3ae^2g-cd^2g+4cdef) \operatorname{atan}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{dg-ef}}\right)}{e^{3/2}(dg-ef)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+a)/(e\*x+d)\*\*2/(g\*x+f)\*\*(3/2), x)

[Out]  $-2*(a*g^2 + c*f^2)/(g*\text{sqrt}(f + g*x)*(d*g - e*f)^2) - g*\text{sqrt}(f + g*x)*(a*e^2 + c*d^2)/(e*(d*g - e*f)^2*(d*g - e*f + e*(f + g*x))) - (3*a*e^2*g - c*d^2*g + 4*c*d*e*f)*\text{atan}(\text{sqrt}(e)*\text{sqrt}(f + g*x)/\text{sqrt}(d*g - e*f))/(e^{3/2}*(d*g - e*f)^{5/2})$

**Mathematica [A]** time = 0.572277, size = 135, normalized size = 0.94

$$\frac{(3ae^2g+cd(4ef-dg)) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{5/2}} - \frac{\sqrt{f+gx}\left(\frac{ae^2+cd^2}{de+e^2x} + \frac{2(ag^2+cf^2)}{g(f+gx)}\right)}{(ef-dg)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c\*x^2)/((d + e\*x)^2\*(f + g\*x)^(3/2)), x]

[Out]  $-\left(\frac{\sqrt{f+gx} \left(\frac{c^2 d^2 + a^2 e^2}{d^2 e + e^2 x} + \frac{2(c^2 f^2 + a^2 g^2)}{g(f+gx)}\right)}{(e^2 f - d^2 g)^2} + \frac{\left(3a^2 e^2 g + c^2 d^2 (4e^2 f - d^2 g)\right) \operatorname{ArcTanh}\left(\frac{\sqrt{e} \sqrt{f+gx}}{\sqrt{e^2 f - d^2 g}}\right)}{(e^{3/2} (e^2 f - d^2 g)^{5/2})}\right)$

**Maple [B]** time = 0.03, size = 269, normalized size = 1.9

$$\begin{aligned} & -2 \frac{ag}{(dg-ef)^2 \sqrt{gx+f}} - 2 \frac{cf^2}{g(dg-ef)^2 \sqrt{gx+f}} - \frac{aeg}{(dg-ef)^2 (egx+dg)} \sqrt{gx+f} \\ & - \frac{cd^2 g}{(dg-ef)^2 e (egx+dg)} \sqrt{gx+f} - 3 \frac{aeg}{(dg-ef)^2 \sqrt{(dg-ef)e}} \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{(dg-ef)e}}\right) \\ & + \frac{cd^2 g}{(dg-ef)^2 e} \arctan\left(e \sqrt{gx+f} \frac{1}{\sqrt{(dg-ef)e}}\right) \frac{1}{\sqrt{(dg-ef)e}} \\ & - 4 \frac{cdf}{(dg-ef)^2 \sqrt{(dg-ef)e}} \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{(dg-ef)e}}\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+a)/(e*x+d)^2/(g*x+f)^(3/2), x)`

[Out]  $-2g/(d^2 g - e^2 f)^2 / (g^2 x + f)^{1/2} * a - 2/g / (d^2 g - e^2 f)^2 / (g^2 x + f)^{1/2} * c^2 f^2 - g / (d^2 g - e^2 f)^2 * e^2 (g^2 x + f)^{1/2} / (e^2 g^2 x + d^2 g) * a - g / (d^2 g - e^2 f)^2 / e^2 (g^2 x + f)^{1/2} / (e^2 g^2 x + d^2 g) * c^2 d^2 - 3g / (d^2 g - e^2 f)^2 * e / ((d^2 g - e^2 f)^2 * e)^{1/2} * \arctan((g^2 x + f)^{1/2} * e / ((d^2 g - e^2 f)^2 * e)^{1/2}) * a + g / (d^2 g - e^2 f)^2 / e / ((d^2 g - e^2 f)^2 * e)^{1/2} * \arctan((g^2 x + f)^{1/2} * e / ((d^2 g - e^2 f)^2 * e)^{1/2}) * c^2 d^2 - 4 / (d^2 g - e^2 f)^2 / ((d^2 g - e^2 f)^2 * e)^{1/2} * \arctan((g^2 x + f)^{1/2} * e / ((d^2 g - e^2 f)^2 * e)^{1/2}) * c^2 d^2 f$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)/((e*x + d)^2*(g*x + f)^(3/2)), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.296011, size = 1, normalized size = 0.01

$$\left[ \frac{(4cd^2efg - (cd^3 - 3ade^2)g^2 + (4cde^2fg - (cd^2e - 3ae^3)g^2)x) \sqrt{gx+f} \log\left(\frac{\sqrt{e^2f-deg}(egx+2ef-dg)+2(e^2f-deg)\sqrt{gx+f}}{ex+d}\right)}{2(de^3f^2g - 2d^2e^2fg^2 + d^3eg^3 + (e^4f^2g - 2de^3fg^2 + d^2e^2g^3)x)} \right] -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)/((e*x + d)^2*(g*x + f)^(3/2)), x, algorithm="fricas")`

[Out]  $[1/2 * ((4c^2 d^2 e^2 f^2 g - (c^2 d^3 - 3a^2 d^2 e^2) * g^2 + (4c^2 d^2 e^2 f^2 g - (c^2 d^2 e - 3a^2 e^3) * g^2) * x) * \sqrt{g^2 x + f} * \log((\sqrt{e^2 f - d^2 e^2 g} * (e^2 g^2 x + 2e^2 f - d^2 g) + 2 * (e^2 f - d^2 e^2 g) * \sqrt{g^2 x + f}) / (e^2 x + d)) - 2 * (2 * c^2 d^2 e^2 f^2 + 2 * a^2 d^2 e^2 g^2 + (c^2 d^2 + a^2 e^2) * f^2 g + (2 * c^2 e^2 f^2 + (c^2 d^2 + 3 * a^2 e^2) * g^2) * x) * \sqrt{e^2 f - d^2 e^2 g}) / ((d^2 e^2$

```
*f^2*g - 2*d^2*e^2*f*g^2 + d^3*e*g^3 + (e^4*f^2*g - 2*d*e^3*f*g^2
+ d^2*e^2*g^3)*x)*sqrt(e^2*f - d*e*g)*sqrt(g*x + f)), ((4*c*d^2*
e*f*g - (c*d^3 - 3*a*d*e^2)*g^2 + (4*c*d*e^2*f*g - (c*d^2*e - 3*a
*e^3)*g^2)*x)*sqrt(g*x + f)*arctan(-(e*f - d*g)/(sqrt(-e^2*f + d*
e*g)*sqrt(g*x + f))) - (2*c*d*e*f^2 + 2*a*d*e*g^2 + (c*d^2 + a*e^
2)*f*g + (2*c*e^2*f^2 + (c*d^2 + 3*a*e^2)*g^2)*x)*sqrt(-e^2*f + d
*e*g))/((d*e^3*f^2*g - 2*d^2*e^2*f*g^2 + d^3*e*g^3 + (e^4*f^2*g -
2*d*e^3*f*g^2 + d^2*e^2*g^3)*x)*sqrt(-e^2*f + d*e*g)*sqrt(g*x +
f))]
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+a)/(e\*x+d)\*\*2/(g\*x+f)\*\*(3/2), x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.27459, size = 304, normalized size = 2.11

$$\frac{(cd^2g - 4cdfe - 3age^2) \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{dge-fe^2}}\right)}{(d^2g^2e - 2dfge^2 + f^2e^3)\sqrt{dge - fe^2}} - \frac{(gx + f)cd^2g^2 + 2cdf^2ge + 2adg^3e + 2(gx + f)cf^2e^2 - 2cf^3e^2 + 3(gx + f)ag^2e^2 - 2afg^2e^2}{(d^2g^3e - 2dfg^2e^2 + f^2ge^3)\left(\sqrt{gx + fdg} + (gx + f)^{\frac{3}{2}}e - \sqrt{gx + ffe}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)/((e\*x + d)^2\*(g\*x + f)^(3/2)), x, algorithm="giac")

[Out] (c\*d^2\*g - 4\*c\*d\*f\*e - 3\*a\*g\*e^2)\*arctan(sqrt(g\*x + f)\*e/sqrt(d\*g\*e - f\*e^2))/((d^2\*g^2\*e - 2\*d\*f\*g\*e^2 + f^2\*e^3)\*sqrt(d\*g\*e - f\*e^2)) - ((g\*x + f)\*c\*d^2\*g^2 + 2\*c\*d\*f^2\*g\*e + 2\*a\*d\*g^3\*e + 2\*(g\*x + f)\*c\*f^2\*e^2 - 2\*c\*f^3\*e^2 + 3\*(g\*x + f)\*a\*g^2\*e^2 - 2\*a\*f\*g^2\*e^2)/((d^2\*g^3\*e - 2\*d\*f\*g^2\*e^2 + f^2\*g\*e^3)\*(sqrt(g\*x + f)\*d\*g + (g\*x + f)^(3/2)\*e - sqrt(g\*x + f)\*f\*e))

$$3.602 \quad \int \frac{a+cx^2}{(d+ex)^3(f+gx)^{3/2}} dx$$

**Optimal.** Leaf size=214

$$\frac{\sqrt{f+gx}(ae^2+cd^2)}{2e(d+ex)^2(ef-dg)^2} - \frac{(15ae^2g^2+c(-d^2g^2+8defg+8e^2f^2))\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{3/2}(ef-dg)^{7/2}} + \frac{\sqrt{f+gx}(7ae^2g+cd(8ef-dg))}{4e(d+ex)(ef-dg)^3} + \frac{2(ag^2+cf^2)}{\sqrt{f+gx}(ef-dg)^3}$$

[Out] (2\*(c\*f^2 + a\*g^2))/((e\*f - d\*g)^3\*Sqrt[f + g\*x]) - ((c\*d^2 + a\*e^2)\*Sqrt[f + g\*x])/(2\*e\*(e\*f - d\*g)^2\*(d + e\*x)^2) + ((7\*a\*e^2\*g + c\*d\*(8\*e\*f - d\*g))\*Sqrt[f + g\*x])/(4\*e\*(e\*f - d\*g)^3\*(d + e\*x)) - ((15\*a\*e^2\*g^2 + c\*(8\*e^2\*f^2 + 8\*d\*e\*f\*g - d^2\*g^2))\*ArcTanh[(Sqrt[e]\*Sqrt[f + g\*x])/Sqrt[e\*f - d\*g]])/(4\*e^(3/2)\*(e\*f - d\*g)^(7/2))

**Rubi [A]** time = 1.00825, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$\frac{\sqrt{f+gx}(ae^2+cd^2)}{2e(d+ex)^2(ef-dg)^2} - \frac{(15ae^2g^2+c(-d^2g^2+8defg+8e^2f^2))\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{4e^{3/2}(ef-dg)^{7/2}} + \frac{\sqrt{f+gx}(7ae^2g+cd(8ef-dg))}{4e(d+ex)(ef-dg)^3} + \frac{2(ag^2+cf^2)}{\sqrt{f+gx}(ef-dg)^3}$$

Antiderivative was successfully verified.

[In] Int[(a + c\*x^2)/((d + e\*x)^3\*(f + g\*x)^(3/2)), x]

[Out] (2\*(c\*f^2 + a\*g^2))/((e\*f - d\*g)^3\*Sqrt[f + g\*x]) - ((c\*d^2 + a\*e^2)\*Sqrt[f + g\*x])/(2\*e\*(e\*f - d\*g)^2\*(d + e\*x)^2) + ((7\*a\*e^2\*g + c\*d\*(8\*e\*f - d\*g))\*Sqrt[f + g\*x])/(4\*e\*(e\*f - d\*g)^3\*(d + e\*x)) - ((15\*a\*e^2\*g^2 + c\*(8\*e^2\*f^2 + 8\*d\*e\*f\*g - d^2\*g^2))\*ArcTanh[(Sqrt[e]\*Sqrt[f + g\*x])/Sqrt[e\*f - d\*g]])/(4\*e^(3/2)\*(e\*f - d\*g)^(7/2))

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+a)/(e\*x+d)\*\*3/(g\*x+f)\*\*(3/2), x)

[Out] Timed out

**Mathematica [A]** time = 1.37627, size = 190, normalized size = 0.89

$$\frac{1}{4} \left( \frac{\sqrt{f+gx} \left( \frac{2(ae^2+cd^2)(dg-ef)}{e(d+ex)^2} + \frac{7ae^2g+cd(8ef-dg)}{e(d+ex)} + \frac{8(ag^2+cf^2)}{f+gx} \right)}{(ef-dg)^3} \right. \\ \left. - \frac{(15ae^2g^2 + c(-d^2g^2 + 8defg + 8e^2f^2)) \tanh^{-1} \left( \frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}} \right)}{e^{3/2}(ef-dg)^{7/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + c\*x^2)/((d + e\*x)^3\*(f + g\*x)^(3/2)), x]

[Out] ((Sqrt[f + g\*x]\*((2\*(c\*d^2 + a\*e^2)\*(-e\*f) + d\*g))/(e\*(d + e\*x)^2) + (7\*a\*e^2\*g + c\*d\*(8\*e\*f - d\*g))/(e\*(d + e\*x)) + (8\*(c\*f^2 + a\*g^2))/(f + g\*x))/(e\*f - d\*g)^3 - ((15\*a\*e^2\*g^2 + c\*(8\*e^2\*f^2 + 8\*d\*e\*f\*g - d^2\*g^2))\*ArcTanh[(Sqrt[e]\*Sqrt[f + g\*x])/Sqrt[e\*f - d\*g]])/(e^(3/2)\*(e\*f - d\*g)^(7/2)))/4

**Maple [B]** time = 0.033, size = 546, normalized size = 2.6

$$\begin{aligned} & -2 \frac{ag^2}{(dg-ef)^3 \sqrt{gx+f}} - 2 \frac{cf^2}{(dg-ef)^3 \sqrt{gx+f}} - \frac{7ae^2g^2}{4(dg-ef)^3 (egx+dg)^2} (gx+f)^{\frac{3}{2}} \\ & + \frac{cd^2g^2}{4(dg-ef)^3 (egx+dg)^2} (gx+f)^{\frac{3}{2}} - 2 \frac{(gx+f)^{3/2} cdefg}{(dg-ef)^3 (egx+dg)^2} \\ & - \frac{9g^3ead}{4(dg-ef)^3 (egx+dg)^2} \sqrt{gx+f} + \frac{9ae^2g^2f}{4(dg-ef)^3 (egx+dg)^2} \sqrt{gx+f} \\ & - \frac{cd^3g^3}{4(dg-ef)^3 (egx+dg)^2} e \sqrt{gx+f} - \frac{7cd^2g^2f}{4(dg-ef)^3 (egx+dg)^2} \sqrt{gx+f} \\ & + 2 \frac{eg\sqrt{gx+f}cdf^2}{(dg-ef)^3 (egx+dg)^2} - \frac{15aeg^2}{4(dg-ef)^3} \arctan\left(e\sqrt{gx+f} \frac{1}{\sqrt{(dg-ef)e}}\right) \frac{1}{\sqrt{(dg-ef)e}} \\ & + \frac{cd^2g^2}{4(dg-ef)^3 e} \arctan\left(e\sqrt{gx+f} \frac{1}{\sqrt{(dg-ef)e}}\right) \frac{1}{\sqrt{(dg-ef)e}} \\ & - 2 \frac{cdfg}{(dg-ef)^3 \sqrt{(dg-ef)e}} \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{(dg-ef)e}}\right) \\ & - 2 \frac{cef^2}{(dg-ef)^3 \sqrt{(dg-ef)e}} \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{(dg-ef)e}}\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+a)/(e\*x+d)^3/(g\*x+f)^(3/2), x)

[Out] -2/(d\*g-e\*f)^3/(g\*x+f)^(1/2)\*a\*g^2-2/(d\*g-e\*f)^3/(g\*x+f)^(1/2)\*c\*f^2-7/4/(d\*g-e\*f)^3/(e\*g\*x+d\*g)^2\*(g\*x+f)^(3/2)\*a\*e^2\*g^2+1/4/(d\*g-e\*f)^3/(e\*g\*x+d\*g)^2\*(g\*x+f)^(3/2)\*c\*d^2\*g^2-2/(d\*g-e\*f)^3/(e\*g\*x+d\*g)^2\*(g\*x+f)^(3/2)\*c\*d\*e\*f\*g-9/4/(d\*g-e\*f)^3/(e\*g\*x+d\*g)^2\*g^3\*e\*(g\*x+f)^(1/2)\*a\*d+9/4/(d\*g-e\*f)^3/(e\*g\*x+d\*g)^2\*g^2\*e^2\*(g\*x+f)^(1/2)\*a\*f-1/4/(d\*g-e\*f)^3/(e\*g\*x+d\*g)^2\*g^3/e\*(g\*x+f)^(1/2)\*c\*d^3-7/4/(d\*g-e\*f)^3/(e\*g\*x+d\*g)^2\*g^2\*(g\*x+f)^(1/2)\*c\*d^2\*f+2/(d\*g-e\*f)^3/(e\*g\*x+d\*g)^2\*g\*e\*(g\*x+f)^(1/2)\*c\*d\*f^2-15/4/(d\*g-e\*f)^3\*e/((d\*g-e\*f)\*e)^(1/2)\*arctan((g\*x+f)^(1/2)\*e/((d\*g-e\*f)\*e)^(1/2))\*a\*g^2+1/4/(d\*g-e\*f)^3/e/((d\*g-e\*f)\*e)^(1/2)\*arctan((g\*x+f)^(1/2)\*e/((d\*g-e\*f)\*e)^(1/2))\*c\*d^2\*g^2-2/(d\*g-e\*f)^3/((d\*g-e\*f)\*e)^(1/2)\*arctan((g\*x+f)^(1/2)\*e/((d\*g-e\*f)\*e)^(1/2))\*c\*d\*f\*g-2/(d\*g-e\*f)^3\*e/((d\*g-e\*f)\*e)^(1/2)\*arctan((g\*x+f)^(1/2)\*e/((d\*g-e\*f)\*e)^(1/2))

$(1/2) * c * f^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)/((e\*x + d)^3\*(g\*x + f)^(3/2)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.311973, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)/((e\*x + d)^3\*(g\*x + f)^(3/2)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/8 * ((8 * c * d^2 * e^2 * f^2 + 8 * c * d^3 * e * f * g - (c * d^4 - 15 * a * d^2 * e^2) * \\ & g^2 + (8 * c * e^4 * f^2 + 8 * c * d * e^3 * f * g - (c * d^2 * e^2 - 15 * a * e^4) * g^2) * \\ & x^2 + 2 * (8 * c * d * e^3 * f^2 + 8 * c * d^2 * e^2 * f * g - (c * d^3 * e - 15 * a * d * e^3) \\ & * g^2) * x) * \sqrt{g * x + f} * \log((\sqrt{e^2 * f - d * e * g}) * (e * g * x + 2 * e * f - \\ & d * g) + 2 * (e^2 * f - d * e * g) * \sqrt{g * x + f})) / (e * x + d) - 2 * (8 * a * d^2 * e \\ & * g^2 + 2 * (7 * c * d^2 * e - a * e^3) * f^2 + (c * d^3 + 9 * a * d * e^2) * f * g + (8 * c \\ & * e^3 * f^2 + 8 * c * d * e^2 * f * g - (c * d^2 * e - 15 * a * e^3) * g^2) * x^2 + (24 * c * \\ & d * e^2 * f^2 + 5 * (c * d^2 * e + a * e^3) * f * g + (c * d^3 + 25 * a * d * e^2) * g^2) * x \\ & ) * \sqrt{e^2 * f - d * e * g}) / ((d^2 * e^4 * f^3 - 3 * d^3 * e^3 * f^2 * g + 3 * d^4 * e^2 * f * g^2 \\ & - d^5 * e * g^3 + (e^6 * f^3 - 3 * d * e^5 * f^2 * g + 3 * d^2 * e^4 * f * g^2 \\ & - d^3 * e^3 * g^3) * x^2 + 2 * (d * e^5 * f^3 - 3 * d^2 * e^4 * f^2 * g + 3 * d^3 * e^3 * f \\ & * g^2 - d^4 * e^2 * g^3) * x) * \sqrt{e^2 * f - d * e * g} * \sqrt{g * x + f}), -1/4 * ( \\ & (8 * c * d^2 * e^2 * f^2 + 8 * c * d^3 * e * f * g - (c * d^4 - 15 * a * d^2 * e^2) * g^2 + ( \\ & 8 * c * e^4 * f^2 + 8 * c * d * e^3 * f * g - (c * d^2 * e^2 - 15 * a * e^4) * g^2) * x^2 + 2 \\ & * (8 * c * d * e^3 * f^2 + 8 * c * d^2 * e^2 * f * g - (c * d^3 * e - 15 * a * d * e^3) * g^2) * x \\ & ) * \sqrt{g * x + f} * \arctan(-(e * f - d * g) / (\sqrt{-e^2 * f + d * e * g}) * \sqrt{g * \\ & x + f})) - (8 * a * d^2 * e * g^2 + 2 * (7 * c * d^2 * e - a * e^3) * f^2 + (c * d^3 + \\ & 9 * a * d * e^2) * f * g + (8 * c * e^3 * f^2 + 8 * c * d * e^2 * f * g - (c * d^2 * e - 15 * a * e \\ & ^3) * g^2) * x^2 + (24 * c * d * e^2 * f^2 + 5 * (c * d^2 * e + a * e^3) * f * g + (c * d^3 \\ & + 25 * a * d * e^2) * g^2) * x) * \sqrt{-e^2 * f + d * e * g}) / ((d^2 * e^4 * f^3 - 3 * d^3 \\ & * e^3 * f^2 * g + 3 * d^4 * e^2 * f * g^2 - d^5 * e * g^3 + (e^6 * f^3 - 3 * d * e^5 * f^2 * \\ & g + 3 * d^2 * e^4 * f * g^2 - d^3 * e^3 * g^3) * x^2 + 2 * (d * e^5 * f^3 - 3 * d^2 * e \\ & ^4 * f^2 * g + 3 * d^3 * e^3 * f * g^2 - d^4 * e^2 * g^3) * x) * \sqrt{-e^2 * f + d * e * g} \\ & * \sqrt{g * x + f})] \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+a)/(e\*x+d)\*\*3/(g\*x+f)\*\*(3/2),x)

[Out] Timed out



**GIAC/XCAS [A]** time = 0.282456, size = 487, normalized size = 2.28

$$\frac{(cd^2g^2 - 8cdfge - 8cf^2e^2 - 15ag^2e^2) \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{dge-fe^2}}\right)}{4(d^3g^3e - 3d^2fg^2e^2 + 3df^2ge^3 - f^3e^4)\sqrt{dge - fe^2} - \frac{2(cf^2 + ag^2)}{(d^3g^3 - 3d^2fg^2e + 3df^2ge^2 - f^3e^3)\sqrt{gx+f}}}{\sqrt{gx+f}cd^3g^3 - (gx+f)^{\frac{3}{2}}cd^2g^2e + 7\sqrt{gx+f}cd^2fg^2e + 8(gx+f)^{\frac{3}{2}}cdfge^2 - 8\sqrt{gx+f}cdf^2ge^2 + 9\sqrt{gx+f}adg^3e^2 + 7(4(d^3g^3e - 3d^2fg^2e^2 + 3df^2ge^3 - f^3e^4)(dg + (gx+f)e - fe)^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)/((e\*x + d)^3\*(g\*x + f)^(3/2)),x, algorithm="giac")

[Out] 1/4\*(c\*d^2\*g^2 - 8\*c\*d\*f\*g\*e - 8\*c\*f^2\*e^2 - 15\*a\*g^2\*e^2)\*arctan(sqrt(g\*x + f)\*e/sqrt(d\*g\*e - f\*e^2))/((d^3\*g^3\*e - 3\*d^2\*f\*g^2\*e^2 + 3\*d\*f^2\*g\*e^3 - f^3\*e^4)\*sqrt(d\*g\*e - f\*e^2)) - 2\*(c\*f^2 + a\*g^2)/((d^3\*g^3 - 3\*d^2\*f\*g^2\*e + 3\*d\*f^2\*g\*e^2 - f^3\*e^3)\*sqrt(g\*x + f)) - 1/4\*(sqrt(g\*x + f)\*c\*d^3\*g^3 - (g\*x + f)^(3/2)\*c\*d^2\*g^2\*e + 7\*sqrt(g\*x + f)\*c\*d^2\*f\*g^2\*e + 8\*(g\*x + f)^(3/2)\*c\*d\*f\*g\*e^2 - 8\*sqrt(g\*x + f)\*c\*d\*f^2\*g\*e^2 + 9\*sqrt(g\*x + f)\*a\*d\*g^3\*e^2 + 7\*(g\*x + f)^(3/2)\*a\*g^2\*e^3 - 9\*sqrt(g\*x + f)\*a\*f\*g^2\*e^3)/((d^3\*g^3\*e - 3\*d^2\*f\*g^2\*e^2 + 3\*d\*f^2\*g\*e^3 - f^3\*e^4)\*(d\*g + (g\*x + f)\*e - f\*e)^2)

$$3.603 \quad \int \frac{a+cx^2}{\sqrt{d+ex}\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=147

$$\frac{(8ae^2g^2 + c(3d^2g^2 + 2defg + 3e^2f^2)) \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{4e^{5/2}g^{5/2}} - \frac{c\sqrt{d+ex}\sqrt{f+gx}(5dg + 3ef)}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

[Out]  $-(c*(3*e*f + 5*d*g)*\text{Sqrt}[d + e*x]*\text{Sqrt}[f + g*x])/(4*e^{5/2}*g^{5/2}) + (c*(d + e*x)^{3/2}*\text{Sqrt}[f + g*x])/(2*e^2*g) + ((8*a*e^2*g^2 + c*(3*e^2*f^2 + 2*d*e*f*g + 3*d^2*g^2))*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[d + e*x])]/(\text{Sqrt}[e]*\text{Sqrt}[f + g*x]))/(4*e^{5/2}*g^{5/2})$

**Rubi [A]** time = 0.323546, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{(8ae^2g^2 + c(3d^2g^2 + 2defg + 3e^2f^2)) \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{4e^{5/2}g^{5/2}} - \frac{c\sqrt{d+ex}\sqrt{f+gx}(5dg + 3ef)}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + c*x^2)/(\text{Sqrt}[d + e*x]*\text{Sqrt}[f + g*x]), x]$

[Out]  $-(c*(3*e*f + 5*d*g)*\text{Sqrt}[d + e*x]*\text{Sqrt}[f + g*x])/(4*e^{5/2}*g^{5/2}) + (c*(d + e*x)^{3/2}*\text{Sqrt}[f + g*x])/(2*e^2*g) + ((8*a*e^2*g^2 + c*(3*e^2*f^2 + 2*d*e*f*g + 3*d^2*g^2))*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[d + e*x])]/(\text{Sqrt}[e]*\text{Sqrt}[f + g*x]))/(4*e^{5/2}*g^{5/2})$

**Rubi in Sympy [A]** time = 30.3004, size = 163, normalized size = 1.11

$$\frac{2a \operatorname{atanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{\sqrt{e}\sqrt{g}} + \frac{cx\sqrt{d+ex}\sqrt{f+gx}}{2eg} - \frac{3c\sqrt{d+ex}\sqrt{f+gx}(dg + ef)}{4e^2g^2} - \frac{c(4defg - 3(dg + ef)^2) \operatorname{atanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{4e^{5/2}g^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((c*x**2+a)/(e*x+d)**(1/2)/(g*x+f)**(1/2), x)$

[Out]  $2*a*\operatorname{atanh}(\text{sqrt}(e)*\text{sqrt}(f + g*x)/(\text{sqrt}(g)*\text{sqrt}(d + e*x)))/(\text{sqrt}(e)*\text{sqrt}(g)) + c*x*\text{sqrt}(d + e*x)*\text{sqrt}(f + g*x)/(2*e*g) - 3*c*\text{sqrt}(d + e*x)*\text{sqrt}(f + g*x)*(d*g + e*f)/(4*e^{5/2}*g^{5/2}) - c*(4*d*e*f*g - 3*(d*g + e*f)**2)*\operatorname{atanh}(\text{sqrt}(e)*\text{sqrt}(f + g*x)/(\text{sqrt}(g)*\text{sqrt}(d + e*x)))/4*e^{5/2}*g^{5/2}$

**Mathematica [A]** time = 0.165486, size = 136, normalized size = 0.93

$$\frac{(8ae^2g^2 + c(3d^2g^2 + 2defg + 3e^2f^2)) \log\left(2\sqrt{e}\sqrt{g}\sqrt{d+ex}\sqrt{f+gx} + dg + ef + 2egx\right)}{8e^{5/2}g^{5/2}} + \frac{c\sqrt{d+ex}\sqrt{f+gx}(-3dg - 3ef + 2egx)}{4e^2g^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + c\*x^2)/(Sqrt[d + e\*x]\*Sqrt[f + g\*x]), x]

[Out] (c\*Sqrt[d + e\*x]\*Sqrt[f + g\*x]\*(-3\*e\*f - 3\*d\*g + 2\*e\*g\*x))/(4\*e^2\*g^2) + ((8\*a\*e^2\*g^2 + c\*(3\*e^2\*f^2 + 2\*d\*e\*f\*g + 3\*d^2\*g^2))\*Log[g[e\*f + d\*g + 2\*e\*g\*x + 2\*Sqrt[e]\*Sqrt[g]\*Sqrt[d + e\*x]\*Sqrt[f + g\*x]])/(8\*e^(5/2)\*g^(5/2))

**Maple [B]** time = 0.044, size = 306, normalized size = 2.1

$$\frac{1}{8e^2g^2} \left( 8 \ln \left( \frac{1}{2} \frac{2egx + 2\sqrt{(ex+d)(gx+f)}\sqrt{eg} + dg + ef}{\sqrt{eg}} \right) ae^2g^2 + 3 \ln \left( \frac{1}{2} \frac{2egx + 2\sqrt{(ex+d)(gx+f)}\sqrt{eg} + dg + ef}{\sqrt{eg}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+a)/(e\*x+d)^(1/2)/(g\*x+f)^(1/2), x)

[Out] 1/8\*(8\*ln(1/2\*(2\*e\*g\*x+2\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+d\*g+e\*f)/(e\*g)^(1/2))\*a\*e^2\*g^2+3\*ln(1/2\*(2\*e\*g\*x+2\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+d\*g+e\*f)/(e\*g)^(1/2))\*c\*d^2\*g^2+2\*c\*ln(1/2\*(2\*e\*g\*x+2\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+d\*g+e\*f)/(e\*g)^(1/2))\*d\*f\*e\*g+3\*ln(1/2\*(2\*e\*g\*x+2\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+d\*g+e\*f)/(e\*g)^(1/2))\*c\*e^2\*f^2+4\*c\*((e\*x+d)\*(g\*x+f))^(1/2)\*x\*e\*g\*(e\*g)^(1/2)-6\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)\*c\*d\*g-6\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)\*c\*e\*f\*(e\*x+d)^(1/2)\*(g\*x+f)^(1/2)/(e\*g)^(1/2)/g^2/e^2/((e\*x+d)\*(g\*x+f))^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)/(sqrt(e\*x + d)\*sqrt(g\*x + f)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.608631, size = 1, normalized size = 0.01

$$\left[ \frac{4(2cegx - 3cef - 3cdg)\sqrt{eg}\sqrt{ex+d}\sqrt{gx+f} + (3ce^2f^2 + 2cdefg + (3cd^2 + 8ae^2)g^2) \log\left(4(2e^2g^2x + e^2fg + deg^2)\sqrt{\dots}\right)}{16\sqrt{eg}e^2g^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)/(sqrt(e\*x + d)\*sqrt(g\*x + f)),x, algorithm="fricas")

[Out] [1/16\*(4\*(2\*c\*e\*g\*x - 3\*c\*e\*f - 3\*c\*d\*g)\*sqrt(e\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f) + (3\*c\*e^2\*f^2 + 2\*c\*d\*e\*f\*g + (3\*c\*d^2 + 8\*a\*e^2)\*g^2)\*log(4\*(2\*e^2\*g^2\*x + e^2\*f\*g + d\*e\*g^2)\*sqrt(e\*x + d)\*sqrt(g\*x + f) + (8\*e^2\*g^2\*x^2 + e^2\*f^2 + 6\*d\*e\*f\*g + d^2\*g^2 + 8\*(e^2\*f\*g + d\*e\*g^2)\*x)\*sqrt(e\*g)))/(sqrt(e\*g)\*e^2\*g^2), 1/8\*(2\*(2\*c\*e\*g\*x - 3\*c\*e\*f - 3\*c\*d\*g)\*sqrt(-e\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f) + (3\*c\*e^2\*f^2 + 2\*c\*d\*e\*f\*g + (3\*c\*d^2 + 8\*a\*e^2)\*g^2)\*arctan(1/2\*(2\*e\*g\*x + e\*f + d\*g)\*sqrt(-e\*g)/(sqrt(e\*x + d)\*sqrt(g\*x + f)\*e\*g)))/(sqrt(-e\*g)\*e^2\*g^2)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + cx^2}{\sqrt{d + ex}\sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+a)/(e\*x+d)\*\*(1/2)/(g\*x+f)\*\*(1/2),x)

[Out] Integral((a + c\*x\*\*2)/(sqrt(d + e\*x)\*sqrt(f + g\*x)), x)

**GIAC/XCAS [A]** time = 0.288106, size = 209, normalized size = 1.42

$$\frac{1}{4} \sqrt{(xe + d)ge - dge + fe^2} \sqrt{xe + d} \left( \frac{2(xe + d)ce^{(-3)}}{g} - \frac{(5cdg^2e^5 + 3cfge^6)e^{(-8)}}{g^3} \right) - \frac{(3cd^2g^2 + 2cdfge + 3cf^2e^2 + 8ag^2e^2)e^{(-\frac{5}{2})} \ln \left( \left| -\sqrt{xe + d}\sqrt{ge}^{\frac{1}{2}} + \sqrt{(xe + d)ge - dge + fe^2} \right| \right)}{4g^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)/(sqrt(e\*x + d)\*sqrt(g\*x + f)),x, algorithm="giac")

[Out] 1/4\*sqrt((x\*e + d)\*g\*e - d\*g\*e + f\*e^2)\*sqrt(x\*e + d)\*(2\*(x\*e + d)\*c\*e^(-3)/g - (5\*c\*d\*g^2\*e^5 + 3\*c\*f\*g\*e^6)\*e^(-8)/g^3) - 1/4\*(3\*c\*d^2\*g^2 + 2\*c\*d\*f\*g\*e + 3\*c\*f^2\*e^2 + 8\*a\*g^2\*e^2)\*e^(-5/2)\*ln(abs(-sqrt(x\*e + d)\*sqrt(g)\*e^(1/2) + sqrt((x\*e + d)\*g\*e - d\*g\*e + f\*e^2)))/g^(5/2)

$$3.604 \quad \int \frac{-1+2x^2}{\sqrt{-1+x}\sqrt{1+x}} dx$$

**Optimal.** Leaf size=16

$$\sqrt{x-1}x\sqrt{x+1}$$

[Out] Sqrt[-1 + x]\*x\*Sqrt[1 + x]

**Rubi [A]** time = 0.0114458, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\sqrt{x-1}x\sqrt{x+1}$$

Antiderivative was successfully verified.

[In] Int[(-1 + 2\*x^2)/(Sqrt[-1 + x]\*Sqrt[1 + x]), x]

[Out] Sqrt[-1 + x]\*x\*Sqrt[1 + x]

**Rubi in Sympy [A]** time = 5.63188, size = 14, normalized size = 0.88

$$x\sqrt{x-1}\sqrt{x+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((2\*x\*\*2-1)/(-1+x)\*\*(1/2)/(1+x)\*\*(1/2), x)

[Out] x\*sqrt(x - 1)\*sqrt(x + 1)

**Mathematica [A]** time = 0.0235895, size = 16, normalized size = 1.

$$\sqrt{x-1}x\sqrt{x+1}$$

Antiderivative was successfully verified.

[In] Integrate[(-1 + 2\*x^2)/(Sqrt[-1 + x]\*Sqrt[1 + x]), x]

[Out] Sqrt[-1 + x]\*x\*Sqrt[1 + x]

**Maple [A]** time = 0.005, size = 13, normalized size = 0.8

$$x\sqrt{-1+x}\sqrt{1+x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2\*x^2-1)/(-1+x)^(1/2)/(1+x)^(1/2), x)

[Out] x\*(-1+x)^(1/2)\*(1+x)^(1/2)

---

**Maxima [A]** time = 0.684832, size = 12, normalized size = 0.75

$$\sqrt{x^2 - 1}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2 - 1)/(sqrt(x + 1)\*sqrt(x - 1)),x, algorithm="maxima")

[Out] sqrt(x^2 - 1)\*x

---

**Fricas [A]** time = 0.276767, size = 74, normalized size = 4.62

$$\frac{2x^4 - (2x^3 - x)\sqrt{x+1}\sqrt{x-1} - 2x^2}{2\sqrt{x+1}\sqrt{x-1} - 2x^2 + 1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2 - 1)/(sqrt(x + 1)\*sqrt(x - 1)),x, algorithm="fricas")

[Out] (2\*x^4 - (2\*x^3 - x)\*sqrt(x + 1)\*sqrt(x - 1) - 2\*x^2)/(2\*sqrt(x + 1)\*sqrt(x - 1)\*x - 2\*x^2 + 1)

---

**Sympy [A]** time = 26.2927, size = 129, normalized size = 8.06

$$-\begin{cases} 2 \operatorname{acosh}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) & \text{for } \frac{|x+1|}{2} > 1 \\ -2i \operatorname{asin}\left(\frac{\sqrt{2}\sqrt{x+1}}{2}\right) & \text{otherwise} \end{cases} + \frac{G_{6,6}^{6,2}\left(-1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 \mid \frac{1}{x^2}\right)}{2\pi^{\frac{3}{2}}} - \frac{iG_{6,6}^{2,6}\left(-\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 1 \mid \frac{e^{2i\pi}}{x^2}\right)}{2\pi^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x\*\*2-1)/(-1+x)\*\*(1/2)/(1+x)\*\*(1/2),x)

[Out] -Piecewise((2\*acosh(sqrt(2)\*sqrt(x + 1)/2), Abs(x + 1)/2 > 1), (-2\*I\*asin(sqrt(2)\*sqrt(x + 1)/2), True)) + meijerg((( -3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), x\*\*(-2)) / (2\*pi\*\*(3/2)) - I\*meijerg((( -3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1, 0)), exp\_polar(2\*I\*pi)/x\*\*2)/(2\*pi\*\*(3/2))

---

**GIAC/XCAS [A]** time = 0.266865, size = 16, normalized size = 1.

$$\sqrt{x+1}\sqrt{x-1}x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2\*x^2 - 1)/(sqrt(x + 1)\*sqrt(x - 1)),x, algorithm="giac")

[Out] sqrt(x + 1)\*sqrt(x - 1)\*x

$$3.605 \quad \int \frac{(d+ex)^{3/2} \sqrt{f+gx}}{a+cx^2} dx$$

**Optimal.** Leaf size=411

$$\begin{aligned} & \frac{\left(\frac{a(ae^2g-cd(dg+2ef))}{\sqrt{c}} - \sqrt{-a}(cd^2f - ae(2dg + ef))\right) \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f - \sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d - \sqrt{-a}e}}\right)}{ac\sqrt{\sqrt{c}d - \sqrt{-a}e}\sqrt{\sqrt{c}f - \sqrt{-a}g}} \\ & + \frac{\left(\sqrt{-a}(cd^2f - ae(2dg + ef)) + \frac{a(ae^2g-cd(dg+2ef))}{\sqrt{c}}\right) \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g + \sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e + \sqrt{c}d}}\right)}{ac\sqrt{\sqrt{-a}e + \sqrt{c}d}\sqrt{\sqrt{-a}g + \sqrt{c}f}} \\ & + \frac{e\sqrt{d+ex}\sqrt{f+gx}}{c} + \frac{\sqrt{e}(3dg + ef) \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}} \end{aligned}$$

[Out] (e\*Sqrt[d + e\*x]\*Sqrt[f + g\*x])/c + (Sqrt[e]\*(e\*f + 3\*d\*g)\*ArcTan h[(Sqrt[g]\*Sqrt[d + e\*x])/(Sqrt[e]\*Sqrt[f + g\*x])])/(c\*Sqrt[g]) + (((a\*(a\*e^2\*g - c\*d\*(2\*e\*f + d\*g)))/Sqrt[c] - Sqrt[-a]\*(c\*d^2\*f - a\*e\*(e\*f + 2\*d\*g)))\*ArcTanh[(Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*Sqrt[f + g\*x])])/(a\*c\*Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]) + (((a\*(a\*e^2\*g - c\*d\*(2\*e\*f + d\*g)))/Sqrt[c] + Sqrt[-a]\*(c\*d^2\*f - a\*e\*(e\*f + 2\*d\*g)))\*ArcTanh[(Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*Sqrt[f + g\*x])])/(a\*c\*Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g])

**Rubi [A]** time = 4.64825, antiderivative size = 411, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$\begin{aligned} & \frac{\left(\frac{a(ae^2g-cd(dg+2ef))}{\sqrt{c}} - \sqrt{-a}(cd^2f - ae(2dg + ef))\right) \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f - \sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d - \sqrt{-a}e}}\right)}{ac\sqrt{\sqrt{c}d - \sqrt{-a}e}\sqrt{\sqrt{c}f - \sqrt{-a}g}} \\ & + \frac{\left(\sqrt{-a}(cd^2f - ae(2dg + ef)) + \frac{a(ae^2g-cd(dg+2ef))}{\sqrt{c}}\right) \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g + \sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e + \sqrt{c}d}}\right)}{ac\sqrt{\sqrt{-a}e + \sqrt{c}d}\sqrt{\sqrt{-a}g + \sqrt{c}f}} \\ & + \frac{e\sqrt{d+ex}\sqrt{f+gx}}{c} + \frac{\sqrt{e}(3dg + ef) \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^(3/2)\*Sqrt[f + g\*x])/(a + c\*x^2), x]

[Out] (e\*Sqrt[d + e\*x]\*Sqrt[f + g\*x])/c + (Sqrt[e]\*(e\*f + 3\*d\*g)\*ArcTan h[(Sqrt[g]\*Sqrt[d + e\*x])/(Sqrt[e]\*Sqrt[f + g\*x])])/(c\*Sqrt[g]) + (((a\*(a\*e^2\*g - c\*d\*(2\*e\*f + d\*g)))/Sqrt[c] - Sqrt[-a]\*(c\*d^2\*f - a\*e\*(e\*f + 2\*d\*g)))\*ArcTanh[(Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*Sqrt[f + g\*x])])/(a\*c\*Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]) + (((a\*(a\*e^2\*g - c\*d\*(2\*e\*f + d\*g)))/Sqrt[c] + Sqrt[-a]\*(c\*d^2\*f - a\*e\*(e\*f + 2\*d\*g)))\*ArcTanh[(Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*Sqrt[f + g\*x])])/(a\*c\*Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g])

**Rubi in Sympy [F(1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**(3/2)*(g*x+f)**(1/2)/(c*x**2+a),x)`

[Out] Timed out

**Mathematica [C]** time = 8.30699, size = 1076, normalized size = 2.62

$$\frac{\sqrt{d+ex}\sqrt{f+gxe}}{c} + \frac{(ef+3dg)\log\left(ef+dg+2egx+2\sqrt{e}\sqrt{g}\sqrt{d+ex}\sqrt{f+gx}\right)\sqrt{e}}{2c\sqrt{g}}$$

$$+ \frac{(-ic^2fd^2 - \sqrt{ac}^{3/2}gd^2 - 2\sqrt{ac}^{3/2}efd + 2iacegd + iace^2f + a^{3/2}\sqrt{ce^2g})\log\left(\frac{2i\sqrt{a}\sqrt{d+ex}\sqrt{f+gxc}^{5/2}}{(-ic^2fd^2 - \sqrt{ac}^{3/2}gd^2 - 2\sqrt{ac}^{3/2}efd + 2iacegd + iace^2f + a^{3/2}\sqrt{ce^2g})}\right)}{2\sqrt{ac}^2\sqrt{\sqrt{cd} - i\sqrt{ae}\sqrt{cg}}}$$

$$+ \frac{(ic^2fd^2 - \sqrt{ac}^{3/2}gd^2 - 2\sqrt{ac}^{3/2}efd - 2iacegd - iace^2f + a^{3/2}\sqrt{ce^2g})\log\left(-\frac{2i\sqrt{a}\sqrt{d+ex}\sqrt{f+gxc}^{5/2}}{(ic^2fd^2 - \sqrt{ac}^{3/2}gd^2 - 2\sqrt{ac}^{3/2}efd - 2iacegd - iace^2f + a^{3/2}\sqrt{ce^2g})}\right)}{2\sqrt{ac}^2\sqrt{\sqrt{cd} + i\sqrt{ae}\sqrt{cg}}}$$

Antiderivative was successfully verified.

[In] `Integrate[((d + e*x)^(3/2)*Sqrt[f + g*x])/(a + c*x^2),x]`

$$\begin{aligned} & \text{[Out] } (e\sqrt{d+e*x})\sqrt{f+g*x}/c + (\sqrt{e}(e*f+3*d*g)\text{Log}[e* \\ & f+d*g+2*e*g*x+2*\sqrt{e}\sqrt{g}\sqrt{d+e*x}\sqrt{f+g*x} \\ & ])/(2*c*\sqrt{g}) + (((-I)*c^2*d^2*f-2*\sqrt{a}*c^{3/2}*d*e*f+I \\ & *a*c*e^2*f-\sqrt{a}*c^{3/2}*d^2*g+(2*I)*a*c*d*e*g+a^{3/2}*\sqrt{c} \\ & *e^2*g)\text{Log}[((2*I)*\sqrt{a}*c^{5/2}\sqrt{d+e*x}\sqrt{f+g*x})/((-I)*c^2*d^2*f-2*\sqrt{a}*c^{3/2}*d*e*f+I*a*c*e^2*f-\sqrt{a}*c^{3/2}*d^2*g+(2*I)*a*c*d*e*g+a^{3/2}*\sqrt{c}*e^2*g)*(Sqrt[a]*Sqrt[c]-I*c*x)) + (I*\sqrt{c}*(2*\sqrt{a}*c^{5/2}*d*f-I*a*c^2*e*f-I*a*c^2*d*g+\sqrt{a}*c^{5/2}*e*f*x+\sqrt{a}*c^{5/2}*d*g*x-(2*I)*a*c^2*e*g*x))/(\sqrt{Sqrt[c]*d-I*\sqrt{a}*e}\sqrt{Sqrt[c]*f-I*\sqrt{a}*g})*((-I)*c^2*d^2*f-2*\sqrt{a}*c^{3/2}*d*e*f+I*a*c*e^2*f-\sqrt{a}*c^{3/2}*d^2*g+(2*I)*a*c*d*e*g+a^{3/2}*\sqrt{c}*e^2*g)*(Sqrt[a]*Sqrt[c]-I*c*x))]/(2*\sqrt{a}*c^2*\sqrt{Sqrt[c]*d-I*\sqrt{a}*e}\sqrt{Sqrt[c]*f-I*\sqrt{a}*g}) + ((I*c^2*d^2*f-2*\sqrt{a}*c^{3/2}*d*e*f-I*a*c*e^2*f-\sqrt{a}*c^{3/2}*d^2*g-(2*I)*a*c*d*e*g+a^{3/2}*\sqrt{c}*e^2*g)\text{Log}[((-2*I)*\sqrt{a}*c^{5/2}\sqrt{d+e*x}\sqrt{f+g*x})/((I*c^2*d^2*f-2*\sqrt{a}*c^{3/2}*d*e*f-I*a*c*e^2*f-\sqrt{a}*c^{3/2}*d^2*g-(2*I)*a*c*d*e*g+a^{3/2}*\sqrt{c}*e^2*g)*(Sqrt[a]*Sqrt[c]+I*c*x)) - (I*\sqrt{c}*(2*\sqrt{a}*c^{5/2}*d*f+I*a*c^2*e*f+I*a*c^2*d*g+\sqrt{a}*c^{5/2}*e*f*x+\sqrt{a}*c^{5/2}*d*g*x+(2*I)*a*c^2*e*g*x))/(\sqrt{Sqrt[c]*d+I*\sqrt{a}*e}\sqrt{Sqrt[c]*f+I*\sqrt{a}*g})*(I*c^2*d^2*f-2*\sqrt{a}*c^{3/2}*d*e*f-I*a*c*e^2*f-\sqrt{a}*c^{3/2}*d^2*g-(2*I)*a*c*d*e*g+a^{3/2}*\sqrt{c}*e^2*g)*(Sqrt[a]*Sqrt[c]+I*c*x))]/(2*\sqrt{a}*c^2*\sqrt{Sqrt[c]*d+I*\sqrt{a}*e}\sqrt{Sqrt[c]*f+I*\sqrt{a}*g}) \end{aligned}$$

**Maple [B]** time = 0.153, size = 2497, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^(3/2)*(g*x+f)^(1/2)/(c*x^2+a),x)`

$$\text{[Out] } 1/2*(e*x+d)^{1/2}*(g*x+f)^{1/2}*(\ln((2*(-a*c)^{1/2}*x*e*g+x*c*d*g+x*c*e*f+(-a*c)^{1/2}*d*g+(-a*c)^{1/2}*e*f+2*(e*g*x^2+d*g*x+e*f*x$$





Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(3/2)*sqrt(g*x + f)/(c*x^2 + a), x, algorithm="maxima")`

[Out] `integrate((e*x + d)^(3/2)*sqrt(g*x + f)/(c*x^2 + a), x)`

---

**Fricas** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(3/2)*sqrt(g*x + f)/(c*x^2 + a), x, algorithm="fricas")`

[Out] Timed out

---

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**(3/2)*(g*x+f)**(1/2)/(c*x**2+a), x)`

[Out] Timed out

---

**GIAC/XCAS** [A] time = 3.55089, size = 4, normalized size = 0.01

*sage<sub>0</sub>x*

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(3/2)*sqrt(g*x + f)/(c*x^2 + a), x, algorithm="giac")`

[Out] `sage0*x`

$$3.606 \quad \int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2} dx$$

**Optimal.** Leaf size=342

$$\frac{(-\sqrt{-a}\sqrt{c}(dg+ef) - aeg + cdf) \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{cf-\sqrt{-ag}}}{\sqrt{f+gx}\sqrt{cd-\sqrt{-ae}}}\right)}{\sqrt{-ac}\sqrt{cd} - \sqrt{-ae}\sqrt{cf} - \sqrt{-ag}} - \frac{(\sqrt{-a}\sqrt{c}(dg+ef) - aeg + cdf) \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-ag+\sqrt{cf}}}{\sqrt{f+gx}\sqrt{-ae+\sqrt{cd}}}\right)}{\sqrt{-ac}\sqrt{-ae} + \sqrt{cd}\sqrt{-ag} + \sqrt{cf}} + \frac{2\sqrt{e}\sqrt{g} \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c}$$

[Out] (2\*Sqrt[e]\*Sqrt[g]\*ArcTanh[(Sqrt[g]\*Sqrt[d + e\*x])/(Sqrt[e]\*Sqrt[f + g\*x]))/c + ((c\*d\*f - a\*e\*g - Sqrt[-a]\*Sqrt[c]\*(e\*f + d\*g))\*ArcTanh[(Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*Sqrt[f + g\*x])])/(Sqrt[-a]\*c\*Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]) - ((c\*d\*f - a\*e\*g + Sqrt[-a]\*Sqrt[c]\*(e\*f + d\*g))\*ArcTanh[(Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*Sqrt[f + g\*x])])/(Sqrt[-a]\*c\*Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g])

**Rubi [A]** time = 3.86892, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 5, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$

$$\frac{(-\sqrt{-a}\sqrt{c}(dg+ef) - aeg + cdf) \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{cf-\sqrt{-ag}}}{\sqrt{f+gx}\sqrt{cd-\sqrt{-ae}}}\right)}{\sqrt{-ac}\sqrt{cd} - \sqrt{-ae}\sqrt{cf} - \sqrt{-ag}} - \frac{(\sqrt{-a}\sqrt{c}(dg+ef) - aeg + cdf) \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-ag+\sqrt{cf}}}{\sqrt{f+gx}\sqrt{-ae+\sqrt{cd}}}\right)}{\sqrt{-ac}\sqrt{-ae} + \sqrt{cd}\sqrt{-ag} + \sqrt{cf}} + \frac{2\sqrt{e}\sqrt{g} \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[d + e\*x]\*Sqrt[f + g\*x])/(a + c\*x^2), x]

[Out] (2\*Sqrt[e]\*Sqrt[g]\*ArcTanh[(Sqrt[g]\*Sqrt[d + e\*x])/(Sqrt[e]\*Sqrt[f + g\*x]))/c + ((c\*d\*f - a\*e\*g - Sqrt[-a]\*Sqrt[c]\*(e\*f + d\*g))\*ArcTanh[(Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*Sqrt[f + g\*x])])/(Sqrt[-a]\*c\*Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]) - ((c\*d\*f - a\*e\*g + Sqrt[-a]\*Sqrt[c]\*(e\*f + d\*g))\*ArcTanh[(Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*Sqrt[f + g\*x])])/(Sqrt[-a]\*c\*Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g])

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*(1/2)\*(g\*x+f)\*\*(1/2)/(c\*x\*\*2+a), x)

[Out] Timed out



$$-a^*c)^{(1/2)}*e^*f-a^*e^*g+c^*d^*f)/c)^{(1/2)}*c+2^*c^*d^*f)/(c^*x-(-a^*c)^{(1/2)})))*a^*e^*g*(e^*g)^{(1/2)}*(-((-a^*c)^{(1/2)}*d^*g+(-a^*c)^{(1/2)}*e^*f+a^*e^*g-c^*d^*f)/c)^{(1/2)}-\ln((2^*(-a^*c)^{(1/2)}*x^*e^*g+x^*c^*d^*g+x^*c^*e^*f+(-a^*c)^{(1/2)}*d^*g+(-a^*c)^{(1/2)}*e^*f+2^*(e^*g^*x^2+d^*g^*x+e^*f^*x+d^*f)^{(1/2)}*((-a^*c)^{(1/2)}*d^*g+(-a^*c)^{(1/2)}*e^*f-a^*e^*g+c^*d^*f)/c)^{(1/2)}*c+2^*c^*d^*f)/(c^*x-(-a^*c)^{(1/2)}))*d^*f*c^*(e^*g)^{(1/2)}*(-((-a^*c)^{(1/2)}*d^*g+(-a^*c)^{(1/2)}*e^*f+a^*e^*g-c^*d^*f)/c)^{(1/2)})/(e^*g^*x^2+d^*g^*x+e^*f^*x+d^*f)^{(1/2)}/c/(e^*g)^{(1/2)}/(((a^*c)^{(1/2)}*d^*g+(-a^*c)^{(1/2)}*e^*f-a^*e^*g+c^*d^*f)/c)^{(1/2)}/(-a^*c)^{(1/2)}/(-((-a^*c)^{(1/2)}*d^*g+(-a^*c)^{(1/2)}*e^*f+a^*e^*g-c^*d^*f)/c)^{(1/2)}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}\sqrt{gx+f}}{cx^2+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*sqrt(g\*x + f)/(c\*x^2 + a), x, algorithm="maxima")

[Out] integrate(sqrt(e\*x + d)\*sqrt(g\*x + f)/(c\*x^2 + a), x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*sqrt(g\*x + f)/(c\*x^2 + a), x, algorithm="fricas")

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}\sqrt{f+gx}}{a+cx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(1/2)\*(g\*x+f)\*\*(1/2)/(c\*x\*\*2+a), x)

[Out] Integral(sqrt(d + e\*x)\*sqrt(f + g\*x)/(a + c\*x\*\*2), x)

**GIAC/XCAS [A]** time = 3.64717, size = 4, normalized size = 0.01

sage0\*x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*sqrt(g\*x + f)/(c\*x^2 + a), x, algorithm="giac")

[Out] sage0\*x

$$3.607 \quad \int \frac{\sqrt{f+gx}}{\sqrt{d+ex}(a+cx^2)} dx$$

**Optimal.** Leaf size=240

$$\frac{\sqrt{\sqrt{c}f - \sqrt{-a}g} \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f - \sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d - \sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}d - \sqrt{-a}e}} - \frac{\sqrt{\sqrt{-a}g + \sqrt{c}f} \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g + \sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e + \sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{-a}e + \sqrt{c}d}}$$

[Out] (Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]\*ArcTanh[(Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*Sqrt[f + g\*x])])/(Sqrt[-a]\*Sqrt[c]\*Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]) - (Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g]\*ArcTanh[(Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*Sqrt[f + g\*x])])/(Sqrt[-a]\*Sqrt[c]\*Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e])

**Rubi [A]** time = 0.918239, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$

$$\frac{\sqrt{\sqrt{c}f - \sqrt{-a}g} \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f - \sqrt{-a}g}}{\sqrt{f+gx}\sqrt{\sqrt{c}d - \sqrt{-a}e}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{c}d - \sqrt{-a}e}} - \frac{\sqrt{\sqrt{-a}g + \sqrt{c}f} \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-a}g + \sqrt{c}f}}{\sqrt{f+gx}\sqrt{\sqrt{-a}e + \sqrt{c}d}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{-a}e + \sqrt{c}d}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[f + g\*x]/(Sqrt[d + e\*x]\*(a + c\*x^2)), x]

[Out] (Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]\*ArcTanh[(Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*Sqrt[f + g\*x])])/(Sqrt[-a]\*Sqrt[c]\*Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]) - (Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g]\*ArcTanh[(Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*Sqrt[f + g\*x])])/(Sqrt[-a]\*Sqrt[c]\*Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e])

**Rubi in Sympy [A]** time = 77.3828, size = 209, normalized size = 0.87

$$-\frac{\sqrt{\sqrt{c}f + g\sqrt{-a}} \operatorname{atanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f + g\sqrt{-a}}}{\sqrt{f+gx}\sqrt{\sqrt{c}d + e\sqrt{-a}}}\right)}{\sqrt{c}\sqrt{-a}\sqrt{\sqrt{c}d + e\sqrt{-a}}} + \frac{\sqrt{\sqrt{c}f - g\sqrt{-a}} \operatorname{atanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f - g\sqrt{-a}}}{\sqrt{f+gx}\sqrt{\sqrt{c}d - e\sqrt{-a}}}\right)}{\sqrt{c}\sqrt{-a}\sqrt{\sqrt{c}d - e\sqrt{-a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*(1/2)/(e\*x+d)\*\*(1/2)/(c\*x\*\*2+a), x)

[Out] -sqrt(sqrt(c)\*f + g\*sqrt(-a))\*atanh(sqrt(d + e\*x)\*sqrt(sqrt(c)\*f + g\*sqrt(-a))/(sqrt(f + g\*x)\*sqrt(sqrt(c)\*d + e\*sqrt(-a))))/(sqrt(c)\*sqrt(-a)\*sqrt(sqrt(c)\*d + e\*sqrt(-a))) + sqrt(sqrt(c)\*f - g\*sqrt(-a))\*atanh(sqrt(d + e\*x)\*sqrt(sqrt(c)\*f - g\*sqrt(-a))/(sqrt(f + g\*x)\*sqrt(sqrt(c)\*d - e\*sqrt(-a))))/(sqrt(c)\*sqrt(-a)\*sqrt(sqrt(c)\*d - e\*sqrt(-a)))

**Mathematica [C]** time = 2.74332, size = 496, normalized size = 2.07

$$i \left( \frac{(cf + i\sqrt{a}\sqrt{cg}) \log\left(\frac{i\sqrt{a}\sqrt{c}(2\sqrt{d+ex}\sqrt{f+gx}\sqrt{\sqrt{c}d+i\sqrt{a}e}\sqrt{\sqrt{c}f+i\sqrt{a}g}+i\sqrt{a}(dg+e(f+2gx))+\sqrt{c}(2df+dgx+efx))}{(\sqrt{c}x-i\sqrt{a})\sqrt{\sqrt{c}d+i\sqrt{a}e}(\sqrt{c}f+i\sqrt{a}g)^{3/2}}\right)}{\sqrt{\sqrt{c}d+i\sqrt{a}e}\sqrt{\sqrt{c}f+i\sqrt{a}g}} \right) - \frac{\sqrt{c}\sqrt{\sqrt{c}f-i\sqrt{a}g} \log\left(-\frac{\sqrt{a}\sqrt{c}(2i\sqrt{d+ex}\sqrt{f+gx}\sqrt{\sqrt{c}d-i\sqrt{a}e}\sqrt{\sqrt{c}f-i\sqrt{a}g}}{(\sqrt{c}x+i\sqrt{a})\sqrt{\sqrt{c}d-i\sqrt{a}e}}\right)}{2\sqrt{ac}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[f + g\*x]/(Sqrt[d + e\*x]\*(a + c\*x^2)),x]

[Out] 
$$\left(\frac{1}{2}\right) \left( \frac{\left( \left( c \cdot f + I \cdot \sqrt{a} \cdot \sqrt{c} \cdot g \right) \cdot \text{Log} \left[ \left( I \cdot \sqrt{a} \cdot \sqrt{c} \right) \cdot \left( 2 \cdot \sqrt{\sqrt{c} \cdot d + I \cdot \sqrt{a} \cdot e} \cdot \sqrt{\sqrt{c} \cdot f + I \cdot \sqrt{a} \cdot g} \right) \cdot \sqrt{d + e \cdot x} \cdot \sqrt{f + g \cdot x} + \sqrt{c} \cdot \left( 2 \cdot d \cdot f + e \cdot f \cdot x + d \cdot g \cdot x \right) + I \cdot \sqrt{a} \cdot \left( d \cdot g + e \cdot \left( f + 2 \cdot g \cdot x \right) \right) \right]}{\left( \sqrt{\sqrt{c} \cdot d + I \cdot \sqrt{a} \cdot e} \right) \cdot \left( \sqrt{c} \cdot f + I \cdot \sqrt{a} \cdot g \right)^{3/2} \cdot \left( -I \cdot \sqrt{a} + \sqrt{c} \cdot x \right)} \right) - \left( \sqrt{c} \cdot \sqrt{\sqrt{c} \cdot f - I \cdot \sqrt{a} \cdot g} \cdot \text{Log} \left[ - \left( \sqrt{a} \cdot \sqrt{c} \right) \cdot \left( \left( 2 \cdot I \right) \cdot \sqrt{\sqrt{c} \cdot d - I \cdot \sqrt{a} \cdot e} \cdot \sqrt{\sqrt{c} \cdot f - I \cdot \sqrt{a} \cdot g} \right) \cdot \sqrt{d + e \cdot x} \cdot \sqrt{f + g \cdot x} + I \cdot \sqrt{c} \cdot \left( 2 \cdot d \cdot f + e \cdot f \cdot x + d \cdot g \cdot x \right) + \sqrt{a} \cdot \left( d \cdot g + e \cdot \left( f + 2 \cdot g \cdot x \right) \right) \right]}{\left( \sqrt{\sqrt{c} \cdot d - I \cdot \sqrt{a} \cdot e} \right) \cdot \left( \sqrt{c} \cdot f - I \cdot \sqrt{a} \cdot g \right)^{3/2} \cdot \left( I \cdot \sqrt{a} + \sqrt{c} \cdot x \right)} \right) \right) / \sqrt{\sqrt{c} \cdot d - I \cdot \sqrt{a} \cdot e} \right) / \left( \sqrt{a} \cdot c \right)$$

**Maple [B]** time = 0.045, size = 1383, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^(1/2)/(e\*x+d)^(1/2)/(c\*x^2+a),x)

[Out] 
$$\begin{aligned} & -1/2 \cdot (g \cdot x + f)^{1/2} \cdot (e \cdot x + d)^{1/2} \cdot \left( \ln \left( \left( 2 \cdot (-a \cdot c)^{1/2} \cdot x \cdot e^g + x^c \cdot d^g + x^c \cdot e^f + (-a \cdot c)^{1/2} \cdot d^g + (-a \cdot c)^{1/2} \cdot e^f + 2 \cdot \left( (e \cdot x + d) \cdot (g \cdot x + f) \right)^{1/2} \cdot \left( \left( (-a \cdot c)^{1/2} \cdot d^g + (-a \cdot c)^{1/2} \cdot e^f - a \cdot e^g + c \cdot d^f \right) / c \right)^{1/2} \cdot c + 2 \cdot c \cdot d^f \right) / \left( c \cdot x - (-a \cdot c)^{1/2} \right) \right) \cdot a \cdot c \cdot e^{2 \cdot f} \cdot \left( - \left( (-a \cdot c)^{1/2} \cdot d^g + (-a \cdot c)^{1/2} \cdot e^f + a \cdot e^g - c \cdot d^f \right) / c \right)^{1/2} + \ln \left( \left( 2 \cdot (-a \cdot c)^{1/2} \cdot x \cdot e^g + x^c \cdot d^g + x^c \cdot e^f + (-a \cdot c)^{1/2} \cdot d^g + (-a \cdot c)^{1/2} \cdot e^f + 2 \cdot \left( (e \cdot x + d) \cdot (g \cdot x + f) \right)^{1/2} \cdot \left( \left( (-a \cdot c)^{1/2} \cdot d^g + (-a \cdot c)^{1/2} \cdot e^f - a \cdot e^g + c \cdot d^f \right) / c \right)^{1/2} \cdot c + 2 \cdot c \cdot d^f \right) / \left( c \cdot x - (-a \cdot c)^{1/2} \right) \right) \cdot a \cdot e^{2 \cdot g} \cdot \left( - \left( (-a \cdot c)^{1/2} \cdot d^g + (-a \cdot c)^{1/2} \cdot e^f + a \cdot e^g - c \cdot d^f \right) / c \right)^{1/2} \cdot (-a \cdot c)^{1/2} + \ln \left( \left( 2 \cdot (-a \cdot c)^{1/2} \cdot x \cdot e^g + x^c \cdot d^g + x^c \cdot e^f + (-a \cdot c)^{1/2} \cdot d^g + (-a \cdot c)^{1/2} \cdot e^f + 2 \cdot \left( (e \cdot x + d) \cdot (g \cdot x + f) \right)^{1/2} \cdot \left( \left( (-a \cdot c)^{1/2} \cdot d^g + (-a \cdot c)^{1/2} \cdot e^f - a \cdot e^g + c \cdot d^f \right) / c \right)^{1/2} \cdot c + 2 \cdot c \cdot d^f \right) / \left( c \cdot x - (-a \cdot c)^{1/2} \right) \right) \cdot c^2 \cdot d^2 \cdot f \cdot \left( - \left( (-a \cdot c)^{1/2} \cdot d^g + (-a \cdot c)^{1/2} \cdot e^f + a \cdot e^g - c \cdot d^f \right) / c \right)^{1/2} + \ln \left( \left( 2 \cdot (-a \cdot c)^{1/2} \cdot x \cdot e^g + x^c \cdot d^g + x^c \cdot e^f + (-a \cdot c)^{1/2} \cdot d^g + (-a \cdot c)^{1/2} \cdot e^f + 2 \cdot \left( (e \cdot x + d) \cdot (g \cdot x + f) \right)^{1/2} \cdot \left( \left( (-a \cdot c)^{1/2} \cdot d^g + (-a \cdot c)^{1/2} \cdot e^f - a \cdot e^g + c \cdot d^f \right) / c \right)^{1/2} \cdot c - (-a \cdot c)^{1/2} \cdot d^g - (-a \cdot c)^{1/2} \cdot e^f + 2 \cdot c \cdot d^f \right) / \left( c \cdot x + (-a \cdot c)^{1/2} \right) \right) \cdot a \cdot c \cdot e^{2 \cdot f} \cdot \left( \left( (-a \cdot c)^{1/2} \cdot d^g + (-a \cdot c)^{1/2} \cdot e^f - a \cdot e^g + c \cdot d^f \right) / c \right)^{1/2} + \ln \left( \left( -2 \cdot (-a \cdot c)^{1/2} \cdot x \cdot e^g + x^c \cdot d^g + x^c \cdot e^f + 2 \cdot \left( - \left( (-a \cdot c)^{1/2} \cdot d^g + (-a \cdot c)^{1/2} \cdot e^f + a \cdot e^g - c \cdot d^f \right) / c \right)^{1/2} \cdot \left( (e \cdot x + d) \cdot (g \cdot x + f) \right)^{1/2} \cdot c - (-a \cdot c)^{1/2} \cdot d^g - (-a \cdot c)^{1/2} \cdot e^f + 2 \cdot c \cdot d^f \right) / \left( c \cdot x + (-a \cdot c)^{1/2} \right) \right) \cdot c^2 \cdot d^2 \cdot f \cdot \left( \left( (-a \cdot c)^{1/2} \cdot d^g + (-a \cdot c)^{1/2} \cdot e^f - a \cdot e^g + c \cdot d^f \right) / c \right)^{1/2} + \ln \left( \left( -2 \cdot (-a \cdot c)^{1/2} \cdot x \cdot e^g + x^c \cdot d^g + x^c \cdot e^f + 2 \cdot \left( - \left( (-a \cdot c)^{1/2} \cdot d^g + (-a \cdot c)^{1/2} \cdot e^f + a \cdot e^g - c \cdot d^f \right) / c \right)^{1/2} \cdot \left( (e \cdot x + d) \cdot (g \cdot x + f) \right)^{1/2} \cdot c - (-a \cdot c)^{1/2} \cdot d^g - (-a \cdot c)^{1/2} \cdot e^f + 2 \cdot c \cdot d^f \right) / \left( c \cdot x + (-a \cdot c)^{1/2} \right) \right) \cdot c \cdot d^2 \cdot g \cdot \left( \left( (-a \cdot c)^{1/2} \cdot d^g + (-a \cdot c)^{1/2} \cdot e^f - a \cdot e^g + c \cdot d^f \right) / c \right)^{1/2} \cdot (-a \cdot c)^{1/2} - \ln \left( \left( -2 \cdot (-a \cdot c)^{1/2} \cdot x \cdot e^g + x^c \cdot d^g + x^c \cdot e^f + 2 \cdot \left( - \left( (-a \cdot c)^{1/2} \cdot d^g + (-a \cdot c)^{1/2} \cdot e^f + a \cdot e^g - c \cdot d^f \right) / c \right)^{1/2} \cdot \left( (e \cdot x + d) \cdot (g \cdot x + f) \right)^{1/2} \cdot c - (-a \cdot c)^{1/2} \cdot d^g - (-a \cdot c)^{1/2} \cdot e^f + 2 \cdot c \cdot d^f \right) / \left( c \cdot x + (-a \cdot c)^{1/2} \right) \right) \cdot c^2 \cdot d^2 \cdot f \cdot \left( \left( (-a \cdot c)^{1/2} \cdot d^g + (-a \cdot c)^{1/2} \cdot e^f - a \cdot e^g + c \cdot d^f \right) / c \right)^{1/2} + \ln \left( \left( -2 \cdot (-a \cdot c)^{1/2} \cdot x \cdot e^g + x^c \cdot d^g + x^c \cdot e^f + 2 \cdot \left( - \left( (-a \cdot c)^{1/2} \cdot d^g + (-a \cdot c)^{1/2} \cdot e^f + a \cdot e^g - c \cdot d^f \right) / c \right)^{1/2} \cdot \left( (e \cdot x + d) \cdot (g \cdot x + f) \right)^{1/2} \cdot c - (-a \cdot c)^{1/2} \cdot d^g - (-a \cdot c)^{1/2} \cdot e^f + 2 \cdot c \cdot d^f \right) / \left( c \cdot x + (-a \cdot c)^{1/2} \right) \right) \cdot c \cdot d^2 \cdot g \cdot \left( \left( (-a \cdot c)^{1/2} \cdot d^g + (-a \cdot c)^{1/2} \cdot e^f - a \cdot e^g + c \cdot d^f \right) / c \right)^{1/2} \cdot (-a \cdot c)^{1/2} \right) / \left( (e \cdot x + d) \cdot (g \cdot x + f) \right)^{1/2} / \left( (-a \cdot c)^{1/2} \cdot e + c \cdot d \right) / (-a \cdot c)^{1/2} / \left( \left( (-a \cdot c)^{1/2} \cdot d^g + (-a \cdot c)^{1/2} \cdot e^f - a \cdot e^g + c \cdot d^f \right) / c \right)^{1/2} / \left( c \cdot d - (-a \cdot c)^{1/2} \cdot e \right) / \left( - \left( (-a \cdot c)^{1/2} \cdot d^g + (-a \cdot c)^{1/2} \cdot e^f + a \cdot e^g - c \cdot d^f \right) / c \right)^{1/2} \right) \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}}{(cx^2+a)\sqrt{ex+d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(g\*x + f)/((c\*x^2 + a)\*sqrt(e\*x + d)),x, algorithm="maxima")

[Out] integrate(sqrt(g\*x + f)/((c\*x^2 + a)\*sqrt(e\*x + d)), x)

**Fricas [A]** time = 9.80736, size = 2593, normalized size = 10.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(g\*x + f)/((c\*x^2 + a)\*sqrt(e\*x + d)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/4*\sqrt{-(c*d*f + a*e*g + (a*c^2*d^2 + a^2*c*e^2)*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)})/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)} \\ & + (a*c^2*d^2 + a^2*c*e^2)*\log(-(e^2*f^2 - d^2*g^2 + 2*(c*d*e*f - c*d^2*g - (a*c^2*d^2*e + a^2*c*e^3)*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)})/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))*\sqrt{e*x + d} \\ & + \sqrt{g*x + f}*\sqrt{-(c*d*f + a*e*g + (a*c^2*d^2 + a^2*c*e^2)*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)})/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)} \\ & + (e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((a*c^2*d^2 + a^2*c*e^2)) + 2*(e^2*f*g - d*e*g^2)*x + (2*(c^2*d^3 + a*c*d*e^2)*f + ((c^2*d^2*e + a*c*e^3)*f + (c^2*d^3 + a*c*d*e^2)*g)*x) \\ & *\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)}/x + 1/4*\sqrt{-(c*d*f + a*e*g + (a*c^2*d^2 + a^2*c*e^2)*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)})/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)} \\ & + (a*c^2*d^2 + a^2*c*e^2)*\log(-(e^2*f^2 - d^2*g^2 - 2*(c*d*e*f - c*d^2*g - (a*c^2*d^2*e + a^2*c*e^3)*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)})/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))*\sqrt{e*x + d} \\ & + \sqrt{g*x + f}*\sqrt{-(c*d*f + a*e*g + (a*c^2*d^2 + a^2*c*e^2)*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)})/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)} \\ & + (e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((a*c^2*d^2 + a^2*c*e^2)) + 2*(e^2*f*g - d*e*g^2)*x + (2*(c^2*d^3 + a*c*d*e^2)*f + ((c^2*d^2*e + a*c*e^3)*f + (c^2*d^3 + a*c*d*e^2)*g)*x) \\ & *\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)}/x - 1/4*\sqrt{-(c*d*f + a*e*g - (a*c^2*d^2 + a^2*c*e^2)*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)})/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)} \\ & + (a*c^2*d^2 + a^2*c*e^2)*\log(-(e^2*f^2 - d^2*g^2 + 2*(c*d*e*f - c*d^2*g + (a*c^2*d^2*e + a^2*c*e^3)*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)})/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))*\sqrt{e*x + d} \\ & + \sqrt{g*x + f}*\sqrt{-(c*d*f + a*e*g - (a*c^2*d^2 + a^2*c*e^2)*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)})/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)} \\ & + (e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((a*c^2*d^2 + a^2*c*e^2)) + 2*(e^2*f*g - d*e*g^2)*x - (2*(c^2*d^3 + a*c*d*e^2)*f + ((c^2*d^2*e + a*c*e^3)*f + (c^2*d^3 + a*c*d*e^2)*g)*x) \\ & *\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)}/x + 1/4*\sqrt{-(c*d*f + a*e*g - (a*c^2*d^2 + a^2*c*e^2)*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)})/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)} \\ & + (a*c^2*d^2 + a^2*c*e^2)*\log(-(e^2*f^2 - d^2*g^2 - 2*(c*d*e*f - c*d^2*g + (a*c^2*d^2*e + a^2*c*e^3)*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)})/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)))*\sqrt{e*x + d} \\ & + \sqrt{g*x + f}*\sqrt{-(c*d*f + a*e*g - (a*c^2*d^2 + a^2*c*e^2)*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)})/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)} \\ & + (e^2*f^2 - 2*d*e*f*g + d^2*g^2)/((a*c^2*d^2 + a^2*c*e^2)) + 2*(e^2*f*g - d*e*g^2)*x - (2*(c^2*d^3 + a*c*d*e^2)*f + ((c^2*d^2*e + a*c*e^3)*f + (c^2*d^3 + a*c*d*e^2)*g)*x) \\ & *\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*e^4)}/x \end{aligned}$$



**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{f+gx}}{(a+cx^2)\sqrt{d+ex}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*(1/2)/(e\*x+d)\*\*(1/2)/(c\*x\*\*2+a),x)

[Out] Integral(sqrt(f + g\*x)/((a + c\*x\*\*2)\*sqrt(d + e\*x)), x)

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(g\*x + f)/((c\*x^2 + a)\*sqrt(e\*x + d)),x, algorithm="giac")

[Out] Exception raised: RuntimeError

$$3.608 \quad \int \frac{\sqrt{f+gx}}{(d+ex)^{3/2}(a+cx^2)} dx$$

**Optimal.** Leaf size=351

$$\begin{aligned} & -\frac{2e\sqrt{f+gx}}{\sqrt{d+ex}(ae^2+cd^2)} + \frac{(\sqrt{-a}\sqrt{c}(ef-dg) + aeg + cdf) \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{cf}-\sqrt{-ag}}{\sqrt{f+gx}\sqrt{cd}-\sqrt{-ae}}\right)}{\sqrt{-a}\sqrt{cd}-\sqrt{-ae}(ae^2+cd^2)\sqrt{cf}-\sqrt{-ag}} \\ & - \frac{(-\sqrt{-a}\sqrt{c}(ef-dg) + aeg + cdf) \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-ag+\sqrt{cf}}}{\sqrt{f+gx}\sqrt{-ae+\sqrt{cd}}}\right)}{\sqrt{-a}\sqrt{-ae+\sqrt{cd}}(ae^2+cd^2)\sqrt{-ag+\sqrt{cf}}} \end{aligned}$$

[Out]  $(-2*e*\text{Sqrt}[f + g*x])/((c*d^2 + a*e^2)*\text{Sqrt}[d + e*x]) + ((c*d*f + a*e*g + \text{Sqrt}[-a]*\text{Sqrt}[c]*(e*f - d*g))*\text{ArcTanh}[(\text{Sqrt}[\text{Sqrt}[c]*f - \text{Sqrt}[-a]*g]*\text{Sqrt}[d + e*x])/(\text{Sqrt}[\text{Sqrt}[c]*d - \text{Sqrt}[-a]*e]*\text{Sqrt}[f + g*x])]) / (\text{Sqrt}[-a]*\text{Sqrt}[\text{Sqrt}[c]*d - \text{Sqrt}[-a]*e]*(c*d^2 + a*e^2)*\text{Sqrt}[\text{Sqrt}[c]*f - \text{Sqrt}[-a]*g]) - ((c*d*f + a*e*g - \text{Sqrt}[-a]*\text{Sqrt}[c]*(e*f - d*g))*\text{ArcTanh}[(\text{Sqrt}[\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g]*\text{Sqrt}[d + e*x])/(\text{Sqrt}[\text{Sqrt}[c]*d + \text{Sqrt}[-a]*e]*\text{Sqrt}[f + g*x])]) / (\text{Sqrt}[-a]*\text{Sqrt}[\text{Sqrt}[c]*d + \text{Sqrt}[-a]*e]*(c*d^2 + a*e^2)*\text{Sqrt}[\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g])$

**Rubi [A]** time = 4.0677, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$

$$\begin{aligned} & -\frac{2e\sqrt{f+gx}}{\sqrt{d+ex}(ae^2+cd^2)} + \frac{(\sqrt{-a}\sqrt{c}(ef-dg) + aeg + cdf) \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{cf}-\sqrt{-ag}}{\sqrt{f+gx}\sqrt{cd}-\sqrt{-ae}}\right)}{\sqrt{-a}\sqrt{cd}-\sqrt{-ae}(ae^2+cd^2)\sqrt{cf}-\sqrt{-ag}} \\ & - \frac{(-\sqrt{-a}\sqrt{c}(ef-dg) + aeg + cdf) \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-ag+\sqrt{cf}}}{\sqrt{f+gx}\sqrt{-ae+\sqrt{cd}}}\right)}{\sqrt{-a}\sqrt{-ae+\sqrt{cd}}(ae^2+cd^2)\sqrt{-ag+\sqrt{cf}}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[f + g*x]/((d + e*x)^{(3/2)}*(a + c*x^2)), x]$

[Out]  $(-2*e*\text{Sqrt}[f + g*x])/((c*d^2 + a*e^2)*\text{Sqrt}[d + e*x]) + ((c*d*f + a*e*g + \text{Sqrt}[-a]*\text{Sqrt}[c]*(e*f - d*g))*\text{ArcTanh}[(\text{Sqrt}[\text{Sqrt}[c]*f - \text{Sqrt}[-a]*g]*\text{Sqrt}[d + e*x])/(\text{Sqrt}[\text{Sqrt}[c]*d - \text{Sqrt}[-a]*e]*\text{Sqrt}[f + g*x])]) / (\text{Sqrt}[-a]*\text{Sqrt}[\text{Sqrt}[c]*d - \text{Sqrt}[-a]*e]*(c*d^2 + a*e^2)*\text{Sqrt}[\text{Sqrt}[c]*f - \text{Sqrt}[-a]*g]) - ((c*d*f + a*e*g - \text{Sqrt}[-a]*\text{Sqrt}[c]*(e*f - d*g))*\text{ArcTanh}[(\text{Sqrt}[\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g]*\text{Sqrt}[d + e*x])/(\text{Sqrt}[\text{Sqrt}[c]*d + \text{Sqrt}[-a]*e]*\text{Sqrt}[f + g*x])]) / (\text{Sqrt}[-a]*\text{Sqrt}[\text{Sqrt}[c]*d + \text{Sqrt}[-a]*e]*(c*d^2 + a*e^2)*\text{Sqrt}[\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g])$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((g*x+f)**(1/2)/(e*x+d)**(3/2)/(c*x**2+a), x)$

[Out] Timed out

**Mathematica [C]** time = 4.4792, size = 531, normalized size = 1.51

$$\frac{(\sqrt{ae+i\sqrt{cd}})\sqrt{\sqrt{cf+i\sqrt{ag}}}\log\left(\frac{i\sqrt{a}\sqrt{\sqrt{cd}+i\sqrt{ae}}(2\sqrt{d+ex}\sqrt{f+gx}\sqrt{\sqrt{cd}+i\sqrt{ae}}\sqrt{\sqrt{cf+i\sqrt{ag}}+i\sqrt{a}(dg+ef+2egx)}+\sqrt{c(2df+dgx+efx)})}{(\sqrt{cx-i\sqrt{a}})(\sqrt{cf+i\sqrt{ag}})^{3/2}}\right)}{\sqrt{a}\sqrt{\sqrt{cd}+i\sqrt{ae}}} + \frac{(\sqrt{ae-i\sqrt{cd}})\sqrt{\sqrt{cf-i\sqrt{ag}}}\log\left(-\frac{i\sqrt{a}\sqrt{\sqrt{cd}-i\sqrt{ae}}(2\sqrt{d+ex}\sqrt{f+gx}\sqrt{\sqrt{cd}-i\sqrt{ae}}\sqrt{\sqrt{cf-i\sqrt{ag}}-i\sqrt{a}(dg+ef+2egx)}+\sqrt{c(2df+dgx+efx)})}{(\sqrt{cx+i\sqrt{a}})(\sqrt{cf-i\sqrt{ag}})^{3/2}}\right)}{\sqrt{a}\sqrt{\sqrt{cd}-i\sqrt{ae}}}}{2(ae^2 + cd^2)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[f + g\*x]/((d + e\*x)^(3/2)\*(a + c\*x^2)),x]

[Out]  $((-4*e*\text{Sqrt}[f + g*x])/ \text{Sqrt}[d + e*x] + ((I*\text{Sqrt}[c]*d + \text{Sqrt}[a]*e)* \text{Sqrt}[\text{Sqrt}[c]*f + I*\text{Sqrt}[a]*g]* \text{Log}[(I*\text{Sqrt}[a]*\text{Sqrt}[\text{Sqrt}[c]*d + I*\text{Sqrt}[a]*e]*(2*\text{Sqrt}[\text{Sqrt}[c]*d + I*\text{Sqrt}[a]*e)*\text{Sqrt}[\text{Sqrt}[c]*f + I*\text{Sqrt}[a]*g]*\text{Sqrt}[d + e*x]*\text{Sqrt}[f + g*x] + \text{Sqrt}[c]*(2*d*f + e*f*x + d*g*x) + I*\text{Sqrt}[a]*(e*f + d*g + 2*e*g*x)))/((\text{Sqrt}[c]*f + I*\text{Sqrt}[a]*g)^{(3/2)}*((-I)*\text{Sqrt}[a] + \text{Sqrt}[c]*x)))/(\text{Sqrt}[a]*\text{Sqrt}[\text{Sqrt}[c]*d + I*\text{Sqrt}[a]*e]) + (((-I)*\text{Sqrt}[c]*d + \text{Sqrt}[a]*e)*\text{Sqrt}[\text{Sqrt}[c]*f - I*\text{Sqrt}[a]*g]* \text{Log}[( (-I)*\text{Sqrt}[a]*\text{Sqrt}[\text{Sqrt}[c]*d - I*\text{Sqrt}[a]*e]*(2*\text{Sqrt}[\text{Sqrt}[c]*d - I*\text{Sqrt}[a]*e)*\text{Sqrt}[\text{Sqrt}[c]*f - I*\text{Sqrt}[a]*g]*\text{Sqrt}[d + e*x]*\text{Sqrt}[f + g*x] + \text{Sqrt}[c]*(2*d*f + e*f*x + d*g*x) - I*\text{Sqrt}[a]*(d*g + e*(f + 2*g*x)))/((\text{Sqrt}[c]*f - I*\text{Sqrt}[a]*g)^{(3/2)}*(I*\text{Sqrt}[a] + \text{Sqrt}[c]*x)))/(\text{Sqrt}[a]*\text{Sqrt}[\text{Sqrt}[c]*d - I*\text{Sqrt}[a]*e]))/(2*(c*d^2 + a*e^2))$

**Maple [B]** time = 0.1, size = 5383, normalized size = 15.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^(1/2)/(e\*x+d)^(3/2)/(c\*x^2+a),x)

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}}{(cx^2+a)(ex+d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(g\*x + f)/((c\*x^2 + a)\*(e\*x + d)^(3/2)),x, algorithm="maxima")

[Out] integrate(sqrt(g\*x + f)/((c\*x^2 + a)\*(e\*x + d)^(3/2)), x)

**Fricas [A]** time = 54.5613, size = 7852, normalized size = 22.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(g\*x + f)/((c\*x^2 + a)\*(e\*x + d)^(3/2)),x, algorithm="fricas")

[Out]  $-1/4*((c*d^3 + a*d*e^2 + (c*d^2*e + a*e^3)*x)*\text{sqrt}(-((c^2*d^3 - 3*a*c*d*e^2)*f + (3*a*c*d^2*e - a^2*e^3)*g + (a*c^3*d^6 + 3*a^2*c^4$

$$\begin{aligned}
& 2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*\sqrt{-((9*c^3*d^4*e^2 - 6* \\
& a*c^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 \\
& + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2 \\
& *e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 \\
& + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7 \\
& *e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4* \\
& e^6))*\log(((3*c*d^2*e^2 - a*e^4)*f^2 + 2*(c*d^3*e + a*d*e^3)*f*g \\
& - (c*d^4 - 3*a*d^2*e^2)*g^2 + 2*((3*c^2*d^4*e - 4*a*c*d^2*e^3 + a \\
& ^2*e^5)*f - (c^2*d^5 - 4*a*c*d^3*e^2 + 3*a^2*d*e^4)*g - 2*(a*c^3* \\
& d^7*e + 3*a^2*c^2*d^5*e^3 + 3*a^3*c*d^3*e^5 + a^4*d*e^7)*\sqrt{-(( \\
& 9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e \\
& - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4 \\
& *e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 1 \\
& 5*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a \\
& ^6*c*d^2*e^10 + a^7*e^12))*\sqrt{e*x + d)*\sqrt{g*x + f)*\sqrt{-((c \\
& ^2*d^3 - 3*a*c*d*e^2)*f + (3*a*c*d^2*e - a^2*e^3)*g + (a*c^3*d^6 \\
& + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*\sqrt{-((9*c^3*d^4 \\
& *e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e - 10*a* \\
& c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9 \\
& *a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4 \\
& *d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2 \\
& *e^10 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2* \\
& e^4 + a^4*e^6)) + 2*((3*c*d^2*e^2 - a*e^4)*f*g - (c*d^3*e - 3*a*d \\
& *e^3)*g^2)*x + (2*(c^3*d^7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + \\
& a^3*d*e^6)*f + ((c^3*d^6*e + 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 + \\
& a^3*e^7)*f + (c^3*d^7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d \\
& *e^6)*g)*x)*\sqrt{-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)* \\
& f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c \\
& ^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6* \\
& a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a \\
& ^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12))/x) - (c*d^3 + a*d \\
& *e^2 + (c*d^2*e + a*e^3)*x)*\sqrt{-((c^2*d^3 - 3*a*c*d*e^2)*f + (3 \\
& *a*c*d^2*e - a^2*e^3)*g + (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3* \\
& c*d^2*e^4 + a^4*e^6)*\sqrt{-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2 \\
& *c*e^6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5) \\
& *f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6* \\
& d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e \\
& ^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a*c^3*d \\
& ^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6))*\log(((3*c*d^2 \\
& *e^2 - a*e^4)*f^2 + 2*(c*d^3*e + a*d*e^3)*f*g - (c*d^4 - 3*a*d^2 \\
& *e^2)*g^2 - 2*((3*c^2*d^4*e - 4*a*c*d^2*e^3 + a^2*e^5)*f - (c^2*d \\
& ^5 - 4*a*c*d^3*e^2 + 3*a^2*d*e^4)*g - 2*(a*c^3*d^7*e + 3*a^2*c^2* \\
& d^5*e^3 + 3*a^3*c*d^3*e^5 + a^4*d*e^7)*\sqrt{-((9*c^3*d^4*e^2 - 6* \\
& a*c^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 \\
& + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2 \\
& *e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 \\
& + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7 \\
& *e^12))*\sqrt{e*x + d)*\sqrt{g*x + f)*\sqrt{-((c^2*d^3 - 3*a*c*d*e \\
& ^2)*f + (3*a*c*d^2*e - a^2*e^3)*g + (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 \\
& + 3*a^3*c*d^2*e^4 + a^4*e^6)*\sqrt{-((9*c^3*d^4*e^2 - 6*a*c^2*d^2 \\
& *e^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2 \\
& *c*d*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^ \\
& ^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4 \\
& *c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)) \\
& )/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)) + \\
& 2*((3*c*d^2*e^2 - a*e^4)*f*g - (c*d^3*e - 3*a*d*e^3)*g^2)*x + (2* \\
& (c^3*d^7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d*e^6)*f + ((c \\
& ^3*d^6*e + 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 + a^3*e^7)*f + (c^3* \\
& d^7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d*e^6)*g)*x)*\sqrt{- \\
& ((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*(3*c^3*d^5 \\
& *e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d \\
& ^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + \\
& 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6 \\
& *a^6*c*d^2*e^10 + a^7*e^12))/x) + (c*d^3 + a*d*e^2 + (c*d^2*e + \\
& a*e^3)*x)*\sqrt{-((c^2*d^3 - 3*a*c*d*e^2)*f + (3*a*c*d^2*e - a^2*e \\
& ^3)*g - (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^ \\
& ^6)*\sqrt{-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c*e^6)*f^2 - 2*( \\
& 3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d*e^5)*f*g + (c^3*d^6 - \\
& 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d \\
& ^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4 \\
& *e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4 \\
& *e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6))*\log(((3*c*d^2*e^2 - a*e^4)*f^2
\end{aligned}$$

$$\begin{aligned}
& + 2*(c*d^3*e + a*d^2*e^3)*f*g - (c*d^4 - 3*a*d^2*e^2)*g^2 + 2*((3*c^2*d^4*e - 4*a*c*d^2*e^3 + a^2*e^5)*f - (c^2*d^5 - 4*a*c*d^3*e^2 + 3*a^2*d^4*e^4)*g + 2*(a*c^3*d^7*e + 3*a^2*c^2*d^5*e^3 + 3*a^3*c*d^3*e^5 + a^4*d^4*e^7)*\sqrt{-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c^3*d^5*e^5 + a^4*d^4*e^7)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d^4*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)}))*\sqrt{e*x + d}*\sqrt{g*x + f}*\sqrt{-((c^2*d^3 - 3*a*c*d^2*e^2)*f + (3*a*c*d^2*e - a^2*e^3)*g - (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*\sqrt{-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c^3*d^5*e^5 + a^4*d^4*e^7)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d^4*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)} + 2*((3*c*d^2*e^2 - a*e^4)*f*g - (c*d^3*e - 3*a*d^2*e^3)*g^2)*x - (2*(c^3*d^7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d^4*e^6)*f + ((c^3*d^6*e + 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 + a^3*e^7)*f + (c^3*d^7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d^4*e^6)*g)*x)*\sqrt{-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c^3*d^5*e^5 + a^4*d^4*e^7)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d^4*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/x) - (c*d^3 + a*d^2*e^2 + (c*d^2*e + a*e^3)*x)*\sqrt{-((c^2*d^3 - 3*a*c*d^2*e^2)*f + (3*a*c*d^2*e - a^2*e^3)*g - (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*\sqrt{-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c^3*d^5*e^5 + a^4*d^4*e^7)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d^4*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)}*\log(((3*c*d^2*e^2 - a*e^4)*f^2 + 2*(c*d^3*e + a*d^2*e^3)*f*g - (c*d^4 - 3*a*d^2*e^2)*g^2 - 2*((3*c^2*d^4*e - 4*a*c*d^2*e^3 + a^2*e^5)*f - (c^2*d^5 - 4*a*c*d^3*e^2 + 3*a^2*d^4*e^4)*g + 2*(a*c^3*d^7*e + 3*a^2*c^2*d^5*e^3 + 3*a^3*c*d^3*e^5 + a^4*d^4*e^7)*\sqrt{-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c^3*d^5*e^5 + a^4*d^4*e^7)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d^4*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)})))*\sqrt{e*x + d}*\sqrt{g*x + f})*\sqrt{-((c^2*d^3 - 3*a*c*d^2*e^2)*f + (3*a*c*d^2*e - a^2*e^3)*g - (a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)*\sqrt{-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c^3*d^5*e^5 + a^4*d^4*e^7)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d^4*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/(a*c^3*d^6 + 3*a^2*c^2*d^4*e^2 + 3*a^3*c*d^2*e^4 + a^4*e^6)} + 2*((3*c*d^2*e^2 - a*e^4)*f*g - (c*d^3*e - 3*a*d^2*e^3)*g^2)*x - (2*(c^3*d^7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d^4*e^6)*f + ((c^3*d^6*e + 3*a*c^2*d^4*e^3 + 3*a^2*c*d^2*e^5 + a^3*e^7)*f + (c^3*d^7 + 3*a*c^2*d^5*e^2 + 3*a^2*c*d^3*e^4 + a^3*d^4*e^6)*g)*x)*\sqrt{-((9*c^3*d^4*e^2 - 6*a*c^2*d^2*e^4 + a^2*c^3*d^5*e^5 + a^4*d^4*e^7)*f^2 - 2*(3*c^3*d^5*e - 10*a*c^2*d^3*e^3 + 3*a^2*c*d^4*e^5)*f*g + (c^3*d^6 - 6*a*c^2*d^4*e^2 + 9*a^2*c*d^2*e^4)*g^2)/(a*c^6*d^12 + 6*a^2*c^5*d^10*e^2 + 15*a^3*c^4*d^8*e^4 + 20*a^4*c^3*d^6*e^6 + 15*a^5*c^2*d^4*e^8 + 6*a^6*c*d^2*e^10 + a^7*e^12)))/x) + 8*\sqrt{e*x + d}*\sqrt{g*x + f}*e)/(c*d^3 + a*d^2*e^2 + (c*d^2*e + a*e^3)*x)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*(1/2)/(e\*x+d)\*\*(3/2)/(c\*x\*\*2+a),x)

[Out] Timed out

---

**GIAC/XCAS [A]** time = 6.67416, size = 4, normalized size = 0.01

$sage_0x$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(g*x + f)/((c*x^2 + a)*(e*x + d)^(3/2)), x, algorithm="giac")`

[Out]  $sage_0x$

$$3.609 \quad \int \frac{\sqrt{f+gx}}{(d+ex)^{5/2}(a+cx^2)} dx$$

**Optimal.** Leaf size=613

$$\begin{aligned} & \frac{e\sqrt{f+gx}(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf)}{\sqrt{-a}\sqrt{d+ex}(\sqrt{-ae}+\sqrt{cd})(ae^2+cd^2)(ef-dg)} \\ & - \frac{e\sqrt{f+gx}(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf)}{\sqrt{-a}\sqrt{d+ex}(\sqrt{cd}-\sqrt{-ae})(ae^2+cd^2)(ef-dg)} + \frac{4eg\sqrt{f+gx}}{3\sqrt{d+ex}(ae^2+cd^2)(ef-dg)} \\ & - \frac{2e\sqrt{f+gx}}{3(d+ex)^{3/2}(ae^2+cd^2)} + \frac{\sqrt{c}(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{cf-\sqrt{-ag}}}{\sqrt{f+gx}\sqrt{cd-\sqrt{-ae}}}\right)}{\sqrt{-a}(\sqrt{cd}-\sqrt{-ae})^{3/2}(ae^2+cd^2)\sqrt{\sqrt{c}f-\sqrt{-ag}}} \\ & + \frac{\sqrt{c}(a\sqrt{c}(ef-dg)+\sqrt{-a}cdf+\sqrt{-aaeg})\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-ag+\sqrt{cf}}}{\sqrt{f+gx}\sqrt{-ae+\sqrt{cd}}}\right)}{a(\sqrt{-ae}+\sqrt{cd})^{3/2}(ae^2+cd^2)\sqrt{\sqrt{-ag}+\sqrt{cf}}} \end{aligned}$$

[Out]  $(-2*e*\text{Sqrt}[f + g*x])/(3*(c*d^2 + a*e^2)*(d + e*x)^{(3/2)}) + (4*e*g*\text{Sqrt}[f + g*x])/(3*(c*d^2 + a*e^2)*(e*f - d*g)*\text{Sqrt}[d + e*x]) + (e*(c*d*f + a*e*g - \text{Sqrt}[-a]*\text{Sqrt}[c]*(e*f - d*g))*\text{Sqrt}[f + g*x])/(\text{Sqrt}[-a]*(\text{Sqrt}[c]*d + \text{Sqrt}[-a]*e)*(c*d^2 + a*e^2)*(e*f - d*g)*\text{Sqrt}[d + e*x]) - (e*(c*d*f + a*e*g + \text{Sqrt}[-a]*\text{Sqrt}[c]*(e*f - d*g))*\text{Sqrt}[f + g*x])/(\text{Sqrt}[-a]*(\text{Sqrt}[c]*d - \text{Sqrt}[-a]*e)*(c*d^2 + a*e^2)*(e*f - d*g)*\text{Sqrt}[d + e*x]) + (\text{Sqrt}[c]*(c*d*f + a*e*g + \text{Sqrt}[-a]*\text{Sqrt}[c]*(e*f - d*g))*\text{ArcTanh}[(\text{Sqrt}[\text{Sqrt}[c]*f - \text{Sqrt}[-a]*g]*\text{Sqrt}[d + e*x])]/(\text{Sqrt}[\text{Sqrt}[c]*d - \text{Sqrt}[-a]*e]*\text{Sqrt}[f + g*x]))/(\text{Sqrt}[-a]*(\text{Sqrt}[c]*d - \text{Sqrt}[-a]*e)^{(3/2)}*(c*d^2 + a*e^2)*\text{Sqrt}[\text{Sqrt}[c]*f - \text{Sqrt}[-a]*g]) + (\text{Sqrt}[c]*(\text{Sqrt}[-a]*c*d*f + \text{Sqrt}[-a]*a*e*g + a*\text{Sqrt}[c]*(e*f - d*g))*\text{ArcTanh}[(\text{Sqrt}[\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g]*\text{Sqrt}[d + e*x])]/(\text{Sqrt}[\text{Sqrt}[c]*d + \text{Sqrt}[-a]*e]*\text{Sqrt}[f + g*x]))/(a*(\text{Sqrt}[c]*d + \text{Sqrt}[-a]*e)^{(3/2)}*(c*d^2 + a*e^2)*\text{Sqrt}[\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g])$

**Rubi [A]** time = 6.53762, antiderivative size = 613, normalized size of antiderivative = 1., number of rules used = 11, number of rules used = 7, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\begin{aligned} & \frac{e\sqrt{f+gx}(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf)}{\sqrt{-a}\sqrt{d+ex}(\sqrt{-ae}+\sqrt{cd})(ae^2+cd^2)(ef-dg)} \\ & - \frac{e\sqrt{f+gx}(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf)}{\sqrt{-a}\sqrt{d+ex}(\sqrt{cd}-\sqrt{-ae})(ae^2+cd^2)(ef-dg)} + \frac{4eg\sqrt{f+gx}}{3\sqrt{d+ex}(ae^2+cd^2)(ef-dg)} \\ & - \frac{2e\sqrt{f+gx}}{3(d+ex)^{3/2}(ae^2+cd^2)} + \frac{\sqrt{c}(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{cf-\sqrt{-ag}}}{\sqrt{f+gx}\sqrt{cd-\sqrt{-ae}}}\right)}{\sqrt{-a}(\sqrt{cd}-\sqrt{-ae})^{3/2}(ae^2+cd^2)\sqrt{\sqrt{c}f-\sqrt{-ag}}} \\ & + \frac{\sqrt{c}(a\sqrt{c}(ef-dg)+\sqrt{-a}cdf+\sqrt{-aaeg})\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-ag+\sqrt{cf}}}{\sqrt{f+gx}\sqrt{-ae+\sqrt{cd}}}\right)}{a(\sqrt{-ae}+\sqrt{cd})^{3/2}(ae^2+cd^2)\sqrt{\sqrt{-ag}+\sqrt{cf}}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[f + g*x]/((d + e*x)^{(5/2)}*(a + c*x^2)), x]$

[Out]  $(-2*e*\text{Sqrt}[f + g*x])/(3*(c*d^2 + a*e^2)*(d + e*x)^{(3/2)}) + (4*e*g*\text{Sqrt}[f + g*x])/(3*(c*d^2 + a*e^2)*(e*f - d*g)*\text{Sqrt}[d + e*x]) + (e*(c*d*f + a*e*g - \text{Sqrt}[-a]*\text{Sqrt}[c]*(e*f - d*g))*\text{Sqrt}[f + g*x])/(\text{Sqrt}[-a]*(\text{Sqrt}[c]*d + \text{Sqrt}[-a]*e)*(c*d^2 + a*e^2)*(e*f - d*g)*\text{Sqrt}[d + e*x]) - (e*(c*d*f + a*e*g + \text{Sqrt}[-a]*\text{Sqrt}[c]*(e*f - d*g))*\text{Sqrt}[f + g*x])/(\text{Sqrt}[-a]*(\text{Sqrt}[c]*d - \text{Sqrt}[-a]*e)*(c*d^2 + a*e^2)*(e*f - d*g)*\text{Sqrt}[d + e*x]) + (\text{Sqrt}[c]*(c*d*f + a*e*g + \text{Sqrt}[-a]*\text{Sqrt}[c]*(e*f - d*g))*\text{ArcTanh}[(\text{Sqrt}[\text{Sqrt}[c]*f - \text{Sqrt}[-a]*g]*\text{Sqrt}[d + e*x])]/(\text{Sqrt}[\text{Sqrt}[c]*d - \text{Sqrt}[-a]*e]*\text{Sqrt}[f + g*x]))/(\text{Sqrt}[-a]*(\text{Sqrt}[c]*d - \text{Sqrt}[-a]*e)^{(3/2)}*(c*d^2 + a*e^2)*\text{Sqrt}[\text{Sqrt}[c]*f - \text{Sqrt}[-a]*g]) + (\text{Sqrt}[c]*(\text{Sqrt}[-a]*c*d*f + \text{Sqrt}[-a]*a*e*g + a*\text{Sqrt}[c]*(e*f - d*g))*\text{ArcTanh}[(\text{Sqrt}[\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g]*\text{Sqrt}[d + e*x])]/(\text{Sqrt}[\text{Sqrt}[c]*d + \text{Sqrt}[-a]*e]*\text{Sqrt}[f + g*x]))/(a*(\text{Sqrt}[c]*d + \text{Sqrt}[-a]*e)^{(3/2)}*(c*d^2 + a*e^2)*\text{Sqrt}[\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g])$

$$(\text{Sqrt}[c]*d - \text{Sqrt}[-a]*e)^{(3/2)}*(c*d^2 + a*e^2)*\text{Sqrt}[\text{Sqrt}[c]*f - \text{Sqrt}[-a]*g] + (\text{Sqrt}[c]*(\text{Sqrt}[-a]*c*d*f + \text{Sqrt}[-a]*a*e*g + a*\text{Sqrt}[c]*(e*f - d*g))*\text{ArcTanh}[(\text{Sqrt}[\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g]*\text{Sqrt}[d + e*x])/(\text{Sqrt}[\text{Sqrt}[c]*d + \text{Sqrt}[-a]*e]*\text{Sqrt}[f + g*x])])/(a*(\text{Sqrt}[c]*d + \text{Sqrt}[-a]*e)^{(3/2)}*(c*d^2 + a*e^2)*\text{Sqrt}[\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g])$$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x+f)**(1/2)/(e*x+d)**(5/2)/(c*x**2+a), x)`

[Out] Timed out

**Mathematica [C]** time = 5.72531, size = 600, normalized size = 0.98

$$\frac{1}{2} \left( \frac{4e\sqrt{f+gx}(ae^3(f+gx) + cd(-6d^2g + de(7f - 5gx) + 6e^2fx))}{3(d+ex)^{3/2}(ae^2 + cd^2)^2(e f - dg)} \right. \\ \left. i(cf - i\sqrt{a}\sqrt{cg}) \log \left( \frac{i\sqrt{a}(\sqrt{cd} - i\sqrt{ae})^{3/2}(2\sqrt{d+ex}\sqrt{f+gx}\sqrt{\sqrt{cd} - i\sqrt{ae}}\sqrt{\sqrt{cf} - i\sqrt{ag}} - i\sqrt{a}(dg+ef+2egx) + \sqrt{c}(2df+dgx+efx))}{\sqrt{c}(\sqrt{cx} + i\sqrt{a})(\sqrt{cf} - i\sqrt{ag})^{3/2}} \right) \right. \\ \left. + \frac{i(cf + i\sqrt{a}\sqrt{cg}) \log \left( \frac{i\sqrt{a}(\sqrt{cd} + i\sqrt{ae})^{3/2}(2\sqrt{d+ex}\sqrt{f+gx}\sqrt{\sqrt{cd} + i\sqrt{ae}}\sqrt{\sqrt{cf} + i\sqrt{ag}} + i\sqrt{a}(dg+ef+2egx) + \sqrt{c}(2df+dgx+efx))}{\sqrt{c}(\sqrt{cx} - i\sqrt{a})(\sqrt{cf} + i\sqrt{ag})^{3/2}} \right)}{\sqrt{a}(\sqrt{cd} + i\sqrt{ae})^{5/2}\sqrt{\sqrt{cf} + i\sqrt{ag}}} \right)$$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[f + g*x]/((d + e*x)^(5/2)*(a + c*x^2)), x]`

[Out]  $((-4*e*\text{Sqrt}[f + g*x]*(a*e^3*(f + g*x) + c*d*(-6*d^2*g + 6*e^2*f*x + d*e*(7*f - 5*g*x)))/(3*(c*d^2 + a*e^2)^2*(e*f - d*g)*(d + e*x)^{(3/2)}) - (I*(c*f - I*\text{Sqrt}[a]*\text{Sqrt}[c]*g)*\text{Log}[((-I)*\text{Sqrt}[a]*(\text{Sqrt}[c]*d - I*\text{Sqrt}[a]*e)^{(3/2)}*(2*\text{Sqrt}[\text{Sqrt}[c]*d - I*\text{Sqrt}[a]*e]*\text{Sqrt}[\text{Sqrt}[c]*f - I*\text{Sqrt}[a]*g]*\text{Sqrt}[d + e*x]*\text{Sqrt}[f + g*x] + \text{Sqrt}[c]*(2*d*f + e*f*x + d*g*x) - I*\text{Sqrt}[a]*(e*f + d*g + 2*e*g*x)))/(\text{Sqrt}[c]*(\text{Sqrt}[c]*f - I*\text{Sqrt}[a]*g)^{(3/2)}*(I*\text{Sqrt}[a] + \text{Sqrt}[c]*x)))]/(\text{Sqrt}[a]*(\text{Sqrt}[c]*d - I*\text{Sqrt}[a]*e)^{(5/2)}*\text{Sqrt}[\text{Sqrt}[c]*f - I*\text{Sqrt}[a]*g]) + (I*(c*f + I*\text{Sqrt}[a]*\text{Sqrt}[c]*g)*\text{Log}[(I*\text{Sqrt}[a]*(\text{Sqrt}[c]*d + I*\text{Sqrt}[a]*e)^{(3/2)}*(2*\text{Sqrt}[\text{Sqrt}[c]*d + I*\text{Sqrt}[a]*e]*\text{Sqrt}[\text{Sqrt}[c]*f + I*\text{Sqrt}[a]*g]*\text{Sqrt}[d + e*x]*\text{Sqrt}[f + g*x] + \text{Sqrt}[c]*(2*d*f + e*f*x + d*g*x) + I*\text{Sqrt}[a]*(e*f + d*g + 2*e*g*x)))/(\text{Sqrt}[c]*(\text{Sqrt}[c]*f + I*\text{Sqrt}[a]*g)^{(3/2)}*((-I)*\text{Sqrt}[a] + \text{Sqrt}[c]*x)))]/(\text{Sqrt}[a]*(\text{Sqrt}[c]*d + I*\text{Sqrt}[a]*e)^{(5/2)}*\text{Sqrt}[\text{Sqrt}[c]*f + I*\text{Sqrt}[a]*g]))/2$

**Maple [B]** time = 0.108, size = 14861, normalized size = 24.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.



[In] `int((g*x+f)^(1/2)/(e*x+d)^(5/2)/(c*x^2+a),x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}}{(cx^2+a)(ex+d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(g*x+f)/((c*x^2+a)*(e*x+d)^(5/2)),x,algorithm="maxima")`

[Out] `integrate(sqrt(g*x+f)/((c*x^2+a)*(e*x+d)^(5/2)),x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(g*x+f)/((c*x^2+a)*(e*x+d)^(5/2)),x,algorithm="fricas")`

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)**(1/2)/(e*x+d)**(5/2)/(c*x**2+a),x)`

[Out] Timed out

**GIAC/XCAS [A]** time = 6.51322, size = 4, normalized size = 0.01

*sage<sub>0</sub>x*

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(g*x+f)/((c*x^2+a)*(e*x+d)^(5/2)),x,algorithm="giac")`

[Out] `sage0*x`

$$3.610 \quad \int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}(a+cx^2)} dx$$

**Optimal.** Leaf size=337

$$\frac{(-2\sqrt{-a}\sqrt{cde} - ae^2 + cd^2) \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{cf}-\sqrt{-ag}}}{\sqrt{f+gx}\sqrt{\sqrt{cd}-\sqrt{-ae}}}\right)}{\sqrt{-ac}\sqrt{\sqrt{cd}-\sqrt{-ae}}\sqrt{\sqrt{cf}-\sqrt{-ag}}} - \frac{(2\sqrt{-a}\sqrt{cde} - ae^2 + cd^2) \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-ag}+\sqrt{cf}}}{\sqrt{f+gx}\sqrt{\sqrt{-ae}+\sqrt{cd}}}\right)}{\sqrt{-ac}\sqrt{\sqrt{-ae}+\sqrt{cd}}\sqrt{\sqrt{-ag}+\sqrt{cf}}} + \frac{2e^{3/2} \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}}$$

[Out] (2\*e^(3/2)\*ArcTanh[(Sqrt[g]\*Sqrt[d + e\*x])/(Sqrt[e]\*Sqrt[f + g\*x])])/(c\*Sqrt[g]) + ((c\*d^2 - 2\*Sqrt[-a]\*Sqrt[c]\*d\*e - a\*e^2)\*ArcTanh[(Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*Sqrt[f + g\*x])])/(Sqrt[-a]\*c\*Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]) - ((c\*d^2 + 2\*Sqrt[-a]\*Sqrt[c]\*d\*e - a\*e^2)\*ArcTanh[(Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*Sqrt[f + g\*x])])/(Sqrt[-a]\*c\*Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g])

**Rubi [A]** time = 4.70677, antiderivative size = 337, normalized size of antiderivative = 1., number of rules used = 10, number of rules used = 5, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$

$$\frac{(-2\sqrt{-a}\sqrt{cde} - ae^2 + cd^2) \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{cf}-\sqrt{-ag}}}{\sqrt{f+gx}\sqrt{\sqrt{cd}-\sqrt{-ae}}}\right)}{\sqrt{-ac}\sqrt{\sqrt{cd}-\sqrt{-ae}}\sqrt{\sqrt{cf}-\sqrt{-ag}}} - \frac{(2\sqrt{-a}\sqrt{cde} - ae^2 + cd^2) \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-ag}+\sqrt{cf}}}{\sqrt{f+gx}\sqrt{\sqrt{-ae}+\sqrt{cd}}}\right)}{\sqrt{-ac}\sqrt{\sqrt{-ae}+\sqrt{cd}}\sqrt{\sqrt{-ag}+\sqrt{cf}}} + \frac{2e^{3/2} \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{c\sqrt{g}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^(3/2)/(Sqrt[f + g\*x]\*(a + c\*x^2)), x]

[Out] (2\*e^(3/2)\*ArcTanh[(Sqrt[g]\*Sqrt[d + e\*x])/(Sqrt[e]\*Sqrt[f + g\*x])])/(c\*Sqrt[g]) + ((c\*d^2 - 2\*Sqrt[-a]\*Sqrt[c]\*d\*e - a\*e^2)\*ArcTanh[(Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*Sqrt[f + g\*x])])/(Sqrt[-a]\*c\*Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]) - ((c\*d^2 + 2\*Sqrt[-a]\*Sqrt[c]\*d\*e - a\*e^2)\*ArcTanh[(Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*Sqrt[f + g\*x])])/(Sqrt[-a]\*c\*Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g])

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*(3/2)/(c\*x\*\*2+a)/(g\*x+f)\*\*(1/2), x)

[Out] Timed out

**Mathematica [C]** time = 3.30889, size = 527, normalized size = 1.56

$$\frac{i(\sqrt{cd+i\sqrt{ae}})^{3/2} \log\left(\frac{i\sqrt{ac}(2\sqrt{d+ex}\sqrt{f+gx}\sqrt{\sqrt{cd+i\sqrt{ae}}\sqrt{cf+i\sqrt{ag}}+i\sqrt{a}(dg+ef+2egx)+\sqrt{c(2df+dgx+efx)}})}{(\sqrt{cx-i\sqrt{a}})(\sqrt{cd+i\sqrt{ae}})^{5/2}\sqrt{cf+i\sqrt{ag}}}\right)}{\sqrt{a}\sqrt{\sqrt{cf+i\sqrt{ag}}}} - \frac{i(\sqrt{cd-i\sqrt{ae}})^{3/2} \log\left(-\frac{\sqrt{ac}(2i\sqrt{d+ex}\sqrt{f+gx}\sqrt{\sqrt{cd-i\sqrt{ae}}\sqrt{cf-i\sqrt{ag}}-i\sqrt{a}(dg+ef+2egx)+\sqrt{c(2df+dgx+efx)}})}{(\sqrt{cx+i\sqrt{a}})(\sqrt{cd-i\sqrt{ae}})^{5/2}\sqrt{cf-i\sqrt{ag}}}\right)}{\sqrt{a}\sqrt{\sqrt{cf-i\sqrt{ag}}}}$$

2c

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^(3/2)/(Sqrt[f + g\*x]\*(a + c\*x^2)), x]

[Out] 
$$\frac{((2e^{3/2}) \text{Log}[e^f + d^*g + 2e^*g^*x + 2\text{Sqrt}[e]^*\text{Sqrt}[g]^*\text{Sqrt}[d + e^*x]^*\text{Sqrt}[f + g^*x]])/\text{Sqrt}[g] + (I^*(\text{Sqrt}[c]^*d + I^*\text{Sqrt}[a]^*e)^{3/2}) \text{Log}[(I^*\text{Sqrt}[a]^*c^*(2^*\text{Sqrt}[\text{Sqrt}[c]^*d + I^*\text{Sqrt}[a]^*e]^*\text{Sqrt}[\text{Sqrt}[c]^*f + I^*\text{Sqrt}[a]^*g]^*\text{Sqrt}[d + e^*x]^*\text{Sqrt}[f + g^*x] + \text{Sqrt}[c]^*(2^*d^*f + e^*f^*x + d^*g^*x)) + I^*\text{Sqrt}[a]^*(e^*f + d^*g + 2e^*g^*x)))/((\text{Sqrt}[c]^*d + I^*\text{Sqrt}[a]^*e)^{5/2}) \text{Sqrt}[\text{Sqrt}[c]^*f + I^*\text{Sqrt}[a]^*g]^*((-I)^*\text{Sqrt}[a] + \text{Sqrt}[c]^*x)))]/(\text{Sqrt}[a]^*\text{Sqrt}[\text{Sqrt}[c]^*f + I^*\text{Sqrt}[a]^*g]) - (I^*(\text{Sqrt}[c]^*d - I^*\text{Sqrt}[a]^*e)^{3/2}) \text{Log}[-(\text{Sqrt}[a]^*c^*((2^*I)^*\text{Sqrt}[\text{Sqrt}[c]^*d - I^*\text{Sqrt}[a]^*e]^*\text{Sqrt}[\text{Sqrt}[c]^*f - I^*\text{Sqrt}[a]^*g]^*\text{Sqrt}[d + e^*x]^*\text{Sqrt}[f + g^*x] + I^*\text{Sqrt}[c]^*(2^*d^*f + e^*f^*x + d^*g^*x)) + \text{Sqrt}[a]^*(e^*f + d^*g + 2e^*g^*x)))/((\text{Sqrt}[c]^*d - I^*\text{Sqrt}[a]^*e)^{5/2}) \text{Sqrt}[\text{Sqrt}[c]^*f - I^*\text{Sqrt}[a]^*g]^*(I^*\text{Sqrt}[a] + \text{Sqrt}[c]^*x)))]/(\text{Sqrt}[a]^*\text{Sqrt}[\text{Sqrt}[c]^*f - I^*\text{Sqrt}[a]^*g])]/(2^*c)$$

**Maple [B]** time = 0.05, size = 2336, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(3/2)/(c\*x^2+a)/(g\*x+f)^(1/2), x)

[Out] 
$$\frac{1/2*(e^*x+d)^{(1/2)}*(g^*x+f)^{(1/2)}*(2^*\ln(1/2^*(2^*e^*g^*x+2^*((e^*x+d)^*(g^*x+f))^{(1/2)}*(e^*g)^{(1/2)+d^*g+e^*f)/(e^*g)^{(1/2)})^*a^*e^{2^*g^2^*(-((-a^*c)^{(1/2)}*d^*g+(-a^*c)^{(1/2)}*e^*f+a^*e^*g-c^*d^*f)/c)^{(1/2)}*(-a^*c)^{(1/2)}*(((-a^*c)^{(1/2)}*d^*g+(-a^*c)^{(1/2)}*e^*f-a^*e^*g+c^*d^*f)/c)^{(1/2)+2^*\ln(1/2^*(2^*e^*g^*x+2^*((e^*x+d)^*(g^*x+f))^{(1/2)}*(e^*g)^{(1/2)+d^*g+e^*f)/(e^*g)^{(1/2)})^*c^*e^{2^*f^2^*(-((-a^*c)^{(1/2)}*d^*g+(-a^*c)^{(1/2)}*e^*f+a^*e^*g-c^*d^*f)/c)^{(1/2)}*(-a^*c)^{(1/2)}*(((-a^*c)^{(1/2)}*d^*g+(-a^*c)^{(1/2)}*e^*f-a^*e^*g+c^*d^*f)/c)^{(1/2)}-\ln((-2^*(-a^*c)^{(1/2)}*x^*e^*g+x^*c^*d^*g+x^*c^*e^*f+2^*(-((-a^*c)^{(1/2)}*d^*g+(-a^*c)^{(1/2)}*e^*f+a^*e^*g-c^*d^*f)/c)^{(1/2)}*((e^*x+d)^*(g^*x+f))^{(1/2)}*c-((-a^*c)^{(1/2)}*d^*g-(-a^*c)^{(1/2)}*e^*f+2^*c^*d^*f)/(c^*x+(-a^*c)^{(1/2)})))*a^{2^*e^{2^*g^2^*(e^*g)^{(1/2)}*(((-a^*c)^{(1/2)}*d^*g+(-a^*c)^{(1/2)}*e^*f-a^*e^*g+c^*d^*f)/c)^{(1/2)+\ln((-2^*(-a^*c)^{(1/2)}*x^*e^*g+x^*c^*d^*g+x^*c^*e^*f+2^*(-((-a^*c)^{(1/2)}*d^*g+(-a^*c)^{(1/2)}*e^*f+a^*e^*g-c^*d^*f)/c)^{(1/2)}*((e^*x+d)^*(g^*x+f))^{(1/2)}*c-((-a^*c)^{(1/2)}*d^*g-(-a^*c)^{(1/2)}*e^*f+2^*c^*d^*f)/(c^*x+(-a^*c)^{(1/2)})))*a^*c^*d^{2^*g^2^*(e^*g)^{(1/2)}*(((-a^*c)^{(1/2)}*d^*g+(-a^*c)^{(1/2)}*e^*f-a^*e^*g+c^*d^*f)/c)^{(1/2)}-\ln((-2^*(-a^*c)^{(1/2)}*x^*e^*g+x^*c^*d^*g+x^*c^*e^*f+2^*(-((-a^*c)^{(1/2)}*d^*g+(-a^*c)^{(1/2)}*e^*f+a^*e^*g-c^*d^*f)/c)^{(1/2)}*((e^*x+d)^*(g^*x+f))^{(1/2)}*c-((-a^*c)^{(1/2)}*d^*g-(-a^*c)^{(1/2)}*e^*f+2^*c^*d^*f)/(c^*x+(-a^*c)^{(1/2)})))*a^*c^*e^{2^*f^2^*(e^*g)^{(1/2)}*(((-a^*c)^{(1/2)}*d^*g+(-a^*c)^{(1/2)}*e^*f-a^*e^*g+c^*d^*f)/c)^{(1/2)}-2^*\ln((-2^*(-a^*c)^{(1/2)}*x^*e^*g+x^*c^*d^*g+x^*c^*e^*f+2^*(-((-a^*c)^{(1/2)}*d^*g+(-a^*c)^{(1/2)}*e^*f+a^*e^*g-c^*d^*f)/c)^{(1/2)}*((e^*x+d)^*(g^*x+f))^{(1/2)}*c-((-a^*c)^{(1/2)}*d^*g-(-a^*c)^{(1/2)}*e^*f+2^*c^*d^*f)/(c^*x+(-a^*c)^{(1/2)})))*c^{2^*d^2^*f^2^*(e^*g)^{(1/2)}*(((-a^*c)^{(1/2)}*d^*g+(-a^*c)^{(1/2)}*e^*f-a^*e^*g+c^*d^*f)/c)^{(1/2)}-2^*\ln((-2^*(-a^*c)^{(1/2)}*x^*e^*g+x^*c^*d^*g+x^*c^*e^*f+2^*(-((-a^*c)^{(1/2)}*d^*g+(-a^*c)^{(1/2)}*e^*f+a^*e^*g-c^*d^*f)/c)^{(1/2)}*((e^*x+d)^*(g^*x+f))^{(1/2)}*c-((-a^*c)^{(1/2)}*d^*g-(-a^*c)^{(1/2)}*e^*f+2^*c^*d^*f)/(c^*x+(-a^*c)^{(1/2)})))*c^*d^*e^*f^{2^*(-a^*c)^{(1/2)}*(e^*g)^{(1/2)}$$

$$\begin{aligned}
& * (((-a^*c)^{(1/2)} * d^*g + (-a^*c)^{(1/2)} * e^*f - a^*e^*g + c^*d^*f) / c)^{(1/2)} + \ln((2^* \\
& (-a^*c)^{(1/2)} * x^*e^*g + x^*c^*d^*g + x^*c^*e^*f + (-a^*c)^{(1/2)} * d^*g + (-a^*c)^{(1/2)} * \\
& e^*f + 2^*((e^*x + d)^*(g^*x + f))^{(1/2)} * (((-a^*c)^{(1/2)} * d^*g + (-a^*c)^{(1/2)} * e^*f \\
& - a^*e^*g + c^*d^*f) / c)^{(1/2)} * c + 2^*c^*d^*f) / (c^*x - (-a^*c)^{(1/2)})) * a^2 * e^2 * g^2 \\
& * (-((-a^*c)^{(1/2)} * d^*g + (-a^*c)^{(1/2)} * e^*f + a^*e^*g - c^*d^*f) / c)^{(1/2)} * (e^*g) \\
& ^{(1/2)} - \ln((2^*(-a^*c)^{(1/2)} * x^*e^*g + x^*c^*d^*g + x^*c^*e^*f + (-a^*c)^{(1/2)} * d^*g + \\
& (-a^*c)^{(1/2)} * e^*f + 2^*((e^*x + d)^*(g^*x + f))^{(1/2)} * (((-a^*c)^{(1/2)} * d^*g + (-a^* \\
& c)^{(1/2)} * e^*f - a^*e^*g + c^*d^*f) / c)^{(1/2)} * c + 2^*c^*d^*f) / (c^*x - (-a^*c)^{(1/2)})) \\
& ) * a^*c^*d^2 * g^2 * (-((-a^*c)^{(1/2)} * d^*g + (-a^*c)^{(1/2)} * e^*f + a^*e^*g - c^*d^*f) / c \\
& )^{(1/2)} * (e^*g)^{(1/2)} + \ln((2^*(-a^*c)^{(1/2)} * x^*e^*g + x^*c^*d^*g + x^*c^*e^*f + (-a^* \\
& c)^{(1/2)} * d^*g + (-a^*c)^{(1/2)} * e^*f + 2^*((e^*x + d)^*(g^*x + f))^{(1/2)} * (((-a^*c)^{(1/2)} \\
& ^{(1/2)} * d^*g + (-a^*c)^{(1/2)} * e^*f - a^*e^*g + c^*d^*f) / c)^{(1/2)} * c + 2^*c^*d^*f) / (c^*x - \\
& (-a^*c)^{(1/2)})) * a^*c^*e^2 * f^2 * (-((-a^*c)^{(1/2)} * d^*g + (-a^*c)^{(1/2)} * e^*f + a^* \\
& e^*g - c^*d^*f) / c)^{(1/2)} * (e^*g)^{(1/2)} - 2^*\ln((2^*(-a^*c)^{(1/2)} * x^*e^*g + x^*c^*d^* \\
& g + x^*c^*e^*f + (-a^*c)^{(1/2)} * d^*g + (-a^*c)^{(1/2)} * e^*f + 2^*((e^*x + d)^*(g^*x + f))^{(1/2)} * \\
& (((-a^*c)^{(1/2)} * d^*g + (-a^*c)^{(1/2)} * e^*f - a^*e^*g + c^*d^*f) / c)^{(1/2)} * c \\
& + 2^*c^*d^*f) / (c^*x - (-a^*c)^{(1/2)})) * a^*d^*e^*g^2 * (-((-a^*c)^{(1/2)} * d^*g + (-a^*c) \\
& )^{(1/2)} * e^*f + a^*e^*g - c^*d^*f) / c)^{(1/2)} * (-a^*c)^{(1/2)} * (e^*g)^{(1/2)} - \ln((2^* \\
& (-a^*c)^{(1/2)} * x^*e^*g + x^*c^*d^*g + x^*c^*e^*f + (-a^*c)^{(1/2)} * d^*g + (-a^*c)^{(1/2)} * \\
& e^*f + 2^*((e^*x + d)^*(g^*x + f))^{(1/2)} * (((-a^*c)^{(1/2)} * d^*g + (-a^*c)^{(1/2)} * e^*f \\
& - a^*e^*g + c^*d^*f) / c)^{(1/2)} * c + 2^*c^*d^*f) / (c^*x - (-a^*c)^{(1/2)})) * c^2 * d^2 * f^2 \\
& * (-((-a^*c)^{(1/2)} * d^*g + (-a^*c)^{(1/2)} * e^*f + a^*e^*g - c^*d^*f) / c)^{(1/2)} * (e^*g) \\
& ^{(1/2)} - 2^*\ln((2^*(-a^*c)^{(1/2)} * x^*e^*g + x^*c^*d^*g + x^*c^*e^*f + (-a^*c)^{(1/2)} * d^* \\
& g + (-a^*c)^{(1/2)} * e^*f + 2^*((e^*x + d)^*(g^*x + f))^{(1/2)} * (((-a^*c)^{(1/2)} * d^*g + ( \\
& -a^*c)^{(1/2)} * e^*f - a^*e^*g + c^*d^*f) / c)^{(1/2)} * c + 2^*c^*d^*f) / (c^*x - (-a^*c)^{(1/2)} \\
& )) * c^*d^*e^*f^2 * (-((-a^*c)^{(1/2)} * d^*g + (-a^*c)^{(1/2)} * e^*f + a^*e^*g - c^*d^*f) / c \\
& )^{(1/2)} * (-a^*c)^{(1/2)} * (e^*g)^{(1/2)} / ((e^*x + d)^*(g^*x + f))^{(1/2)} / (c^*f - g^* \\
& (-a^*c)^{(1/2)}) / (e^*g)^{(1/2)} / (-((-a^*c)^{(1/2)} * d^*g + (-a^*c)^{(1/2)} * e^*f + a^* \\
& e^*g - c^*d^*f) / c)^{(1/2)} / (g^*(-a^*c)^{(1/2)} + c^*f) / (((-a^*c)^{(1/2)} * d^*g + (-a^*c) \\
& )^{(1/2)} * e^*f - a^*e^*g + c^*d^*f) / c)^{(1/2)} / (-a^*c)^{(1/2)}
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^{\frac{3}{2}}}{(cx^2 + a)\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)/((c\*x^2 + a)\*sqrt(g\*x + f)), x, algorithm="maxima")

[Out] integrate((e\*x + d)^(3/2)/((c\*x^2 + a)\*sqrt(g\*x + f)), x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)/((c\*x^2 + a)\*sqrt(g\*x + f)), x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(3/2)/(c\*x\*\*2+a)/(g\*x+f)\*\*(1/2), x)

[Out] Timed out

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**GIAC/XCAS [A]** time = 0.644023, size = 4, normalized size = 0.01

$sage_0x$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(3/2)/((c*x^2 + a)*sqrt(g*x + f)),x, algorithm="giac")`

[Out] `sage0*x`

$$3.611 \quad \int \frac{\sqrt{d+ex}}{\sqrt{f+gx}(a+cx^2)} dx$$

**Optimal.** Leaf size=240

$$\frac{\sqrt{\sqrt{cd}-\sqrt{-ae}} \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{cf}-\sqrt{-ag}}}{\sqrt{f+gx}\sqrt{\sqrt{cd}-\sqrt{-ae}}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{cf}-\sqrt{-ag}}} - \frac{\sqrt{\sqrt{-ae}+\sqrt{cd}} \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-ag}+\sqrt{cf}}}{\sqrt{f+gx}\sqrt{\sqrt{-ae}+\sqrt{cd}}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{-ag}+\sqrt{cf}}}$$

[Out] (Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*ArcTanh[(Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*Sqrt[f + g\*x])])/(Sqrt[-a]\*Sqrt[c]\*Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]) - (Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*ArcTanh[(Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*Sqrt[f + g\*x])])/(Sqrt[-a]\*Sqrt[c]\*Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g])

**Rubi [A]** time = 0.845938, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$

$$\frac{\sqrt{\sqrt{cd}-\sqrt{-ae}} \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{cf}-\sqrt{-ag}}}{\sqrt{f+gx}\sqrt{\sqrt{cd}-\sqrt{-ae}}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{cf}-\sqrt{-ag}}} - \frac{\sqrt{\sqrt{-ae}+\sqrt{cd}} \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-ag}+\sqrt{cf}}}{\sqrt{f+gx}\sqrt{\sqrt{-ae}+\sqrt{cd}}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{-ag}+\sqrt{cf}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e\*x]/(Sqrt[f + g\*x]\*(a + c\*x^2)), x]

[Out] (Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*ArcTanh[(Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*Sqrt[f + g\*x])])/(Sqrt[-a]\*Sqrt[c]\*Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]) - (Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*ArcTanh[(Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*Sqrt[f + g\*x])])/(Sqrt[-a]\*Sqrt[c]\*Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g])

**Rubi in Sympy [A]** time = 80.0216, size = 209, normalized size = 0.87

$$\frac{\sqrt{\sqrt{cd}-e\sqrt{-a}} \operatorname{atanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{cf}-g\sqrt{-a}}}{\sqrt{f+gx}\sqrt{\sqrt{cd}-e\sqrt{-a}}}\right)}{\sqrt{c}\sqrt{-a}\sqrt{\sqrt{cf}-g\sqrt{-a}}} - \frac{\sqrt{\sqrt{cd}+e\sqrt{-a}} \operatorname{atanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{cf}+g\sqrt{-a}}}{\sqrt{f+gx}\sqrt{\sqrt{cd}+e\sqrt{-a}}}\right)}{\sqrt{c}\sqrt{-a}\sqrt{\sqrt{cf}+g\sqrt{-a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*(1/2)/(c\*x\*\*2+a)/(g\*x+f)\*\*(1/2), x)

[Out] sqrt(sqrt(c)\*d - e\*sqrt(-a))\*atanh(sqrt(d + e\*x)\*sqrt(sqrt(c)\*f - g\*sqrt(-a))/(sqrt(f + g\*x)\*sqrt(sqrt(c)\*d - e\*sqrt(-a))))/(sqrt(c)\*sqrt(-a)\*sqrt(sqrt(c)\*f - g\*sqrt(-a))) - sqrt(sqrt(c)\*d + e\*sqrt(-a))\*atanh(sqrt(d + e\*x)\*sqrt(sqrt(c)\*f + g\*sqrt(-a))/(sqrt(f + g\*x)\*sqrt(sqrt(c)\*d + e\*sqrt(-a))))/(sqrt(c)\*sqrt(-a)\*sqrt(sqrt(c)\*f + g\*sqrt(-a)))

**Mathematica [C]** time = 2.52612, size = 496, normalized size = 2.07

$$i \left( \frac{(cd+i\sqrt{a}\sqrt{ce}) \log\left(\frac{i\sqrt{a}\sqrt{c}(2\sqrt{d+ex}\sqrt{f+gx}\sqrt{\sqrt{cd}+i\sqrt{ae}}\sqrt{\sqrt{cf}+i\sqrt{ag}}+i\sqrt{a}(dg+e(f+2gx))+\sqrt{c}(2df+dgx+efx))}{(\sqrt{cx-i\sqrt{a}})(\sqrt{cd+i\sqrt{ae}})^{3/2}\sqrt{\sqrt{cf}+i\sqrt{ag}}}\right)}{\sqrt{\sqrt{cd}+i\sqrt{ae}}\sqrt{\sqrt{cf}+i\sqrt{ag}}}\right) - \frac{\sqrt{c}\sqrt{\sqrt{cd}-i\sqrt{ae}} \log\left(-\frac{\sqrt{a}\sqrt{c}(2i\sqrt{d+ex}\sqrt{f+gx}\sqrt{\sqrt{cd}-i\sqrt{ae}}\sqrt{\sqrt{cf}-i\sqrt{ag}}+(\sqrt{cx+i\sqrt{a}})(\sqrt{cd-i\sqrt{ae}})^{3/2}\sqrt{\sqrt{cf}-i\sqrt{ag}})}{\sqrt{\sqrt{cf}-i\sqrt{ag}}}\right)}{2\sqrt{ac}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e\*x]/(Sqrt[f + g\*x]\*(a + c\*x^2)),x]

[Out] 
$$\left(\frac{1}{2}\right) \left( \frac{\left( (c*d + I*\text{Sqrt}[a]*\text{Sqrt}[c]*e) * \text{Log}\left[ \left( I*\text{Sqrt}[a]*\text{Sqrt}[c] * (2*\text{Sqrt}[\text{Sqrt}[c]*d + I*\text{Sqrt}[a]*e) * \text{Sqrt}[\text{Sqrt}[c]*f + I*\text{Sqrt}[a]*g] * \text{Sqrt}[d + e*x] * \text{Sqrt}[f + g*x] + \text{Sqrt}[c] * (2*d*f + e*f*x + d*g*x) + I*\text{Sqrt}[a] * (d*g + e*(f + 2*g*x)) \right) \right]}{\left( \text{Sqrt}[c]*d + I*\text{Sqrt}[a]*e \right)^{3/2} * \text{Sqrt}[\text{Sqrt}[c]*f + I*\text{Sqrt}[a]*g] * \left( (-I)*\text{Sqrt}[a] + \text{Sqrt}[c]*x \right)} \right) - \left( \text{Sqrt}[c]*\text{Sqrt}[\text{Sqrt}[c]*d - I*\text{Sqrt}[a]*e] * \text{Log}\left[ - \left( \text{Sqrt}[a]*\text{Sqrt}[c] * \left( (2*I)*\text{Sqrt}[\text{Sqrt}[c]*d - I*\text{Sqrt}[a]*e \right) * \text{Sqrt}[\text{Sqrt}[c]*f - I*\text{Sqrt}[a]*g] * \text{Sqrt}[d + e*x] * \text{Sqrt}[f + g*x] + I*\text{Sqrt}[c] * (2*d*f + e*f*x + d*g*x) + \text{Sqrt}[a] * (d*g + e*(f + 2*g*x)) \right) \right]}{\left( \text{Sqrt}[c]*d - I*\text{Sqrt}[a]*e \right)^{3/2} * \text{Sqrt}[\text{Sqrt}[c]*f - I*\text{Sqrt}[a]*g] * \left( I*\text{Sqrt}[a] + \text{Sqrt}[c]*x \right)} \right) \right) / \text{Sqrt}[\text{Sqrt}[c]*f - I*\text{Sqrt}[a]*g] \right) / \left( \text{Sqrt}[a]*c \right)$$

**Maple [B]** time = 0.045, size = 1383, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(1/2)/(c\*x^2+a)/(g\*x+f)^(1/2),x)

[Out] 
$$\begin{aligned} & -1/2 * (e*x+d)^{1/2} * (g*x+f)^{1/2} * \left( \ln\left( (2 * (-a*c)^{1/2} * x * e^{g*x+c*d} * \right. \right. \\ & g*x*c * e^{f+(-a*c)^{1/2} * d * g + (-a*c)^{1/2} * e * f + 2 * ((e*x+d) * (g*x+f))^{1/2} * \\ & \left. \left. \left( (-a*c)^{1/2} * d * g + (-a*c)^{1/2} * e * f - a * e * g + c * d * f \right) / c \right)^{1/2} * c + \right. \\ & \left. 2 * c * d * f \right) / (c*x - (-a*c)^{1/2}) \left. \right) * a * c * d * g^2 * \left( - \left( (-a*c)^{1/2} * d * g + (-a*c)^{1/2} * \right. \right. \\ & \left. \left. e * f + a * e * g - c * d * f \right) / c \right)^{1/2} + \ln\left( (2 * (-a*c)^{1/2} * x * e^{g*x+c*d} * g \right. \\ & + x * c * e^{f+(-a*c)^{1/2} * d * g + (-a*c)^{1/2} * e * f + 2 * ((e*x+d) * (g*x+f))^{1/2} * \\ & \left. \left. \left( (-a*c)^{1/2} * d * g + (-a*c)^{1/2} * e * f - a * e * g + c * d * f \right) / c \right)^{1/2} * c + \right. \\ & \left. 2 * c * d * f \right) / (c*x - (-a*c)^{1/2}) \left. \right) * a * e * g^2 * \left( - \left( (-a*c)^{1/2} * d * g + (-a*c)^{1/2} * \right. \right. \\ & \left. \left. e * f + a * e * g - c * d * f \right) / c \right)^{1/2} * \left( -a*c \right)^{1/2} + \ln\left( (2 * (-a*c)^{1/2} * x * e \right. \\ & * g + x * c * d * g + x * c * e^{f+(-a*c)^{1/2} * d * g + (-a*c)^{1/2} * e * f + 2 * ((e*x+d) * (g*x+f))^{1/2} * \\ & \left. \left. \left( (-a*c)^{1/2} * d * g + (-a*c)^{1/2} * e * f - a * e * g + c * d * f \right) / c \right)^{1/2} * c + \right. \\ & \left. 2 * c * d * f \right) / (c*x - (-a*c)^{1/2}) \left. \right) * c^2 * d * f^2 * \left( - \left( (-a*c)^{1/2} * d * \right. \right. \\ & \left. \left. g + (-a*c)^{1/2} * e * f + a * e * g - c * d * f \right) / c \right)^{1/2} + \ln\left( (2 * (-a*c)^{1/2} * x * e \right. \\ & * g + x * c * d * g + x * c * e^{f+(-a*c)^{1/2} * d * g + (-a*c)^{1/2} * e * f + 2 * ((e*x+d) * (g*x+f))^{1/2} * \\ & \left. \left. \left( (-a*c)^{1/2} * d * g + (-a*c)^{1/2} * e * f - a * e * g + c * d * f \right) / c \right)^{1/2} * c + \right. \\ & \left. 2 * c * d * f \right) / (c*x - (-a*c)^{1/2}) \left. \right) * c * e * f^2 * \left( - \left( (-a*c)^{1/2} * d * g + \right. \right. \\ & \left. \left. (-a*c)^{1/2} * e * f + a * e * g - c * d * f \right) / c \right)^{1/2} * \left( -a*c \right)^{1/2} - \ln\left( (-2 * (-a*c) \right. \\ & \left. \left. \right)^{1/2} * x * e^{g*x+c*d} * g + x * c * e^{f+(-a*c)^{1/2} * d * g + (-a*c)^{1/2} * e * f + 2 * ((e*x+d) * (g*x+f))^{1/2} * \right. \right. \\ & \left. \left. \left( (-a*c)^{1/2} * d * g + (-a*c)^{1/2} * e * f - a * e * g + c * d * f \right) / c \right)^{1/2} * c - \left( -a*c \right)^{1/2} * d \right. \\ & \left. * g - (-a*c)^{1/2} * e * f + 2 * c * d * f \right) / (c*x + (-a*c)^{1/2}) \left. \right) * a * c * d * g^2 * \left( \left( -a*c \right)^{1/2} * \right. \right. \\ & \left. \left. d * g + (-a*c)^{1/2} * e * f - a * e * g + c * d * f \right) / c \right)^{1/2} + \ln\left( (-2 * (-a*c) \right. \\ & \left. \left. \right)^{1/2} * x * e^{g*x+c*d} * g + x * c * e^{f+(-a*c)^{1/2} * d * g + (-a*c)^{1/2} * e * f + 2 * ((e*x+d) * (g*x+f))^{1/2} * \right. \right. \\ & \left. \left. \left( (-a*c)^{1/2} * d * g + (-a*c)^{1/2} * e * f - a * e * g + c * d * f \right) / c \right)^{1/2} * c - \left( -a*c \right)^{1/2} * d \right. \\ & \left. * g - (-a*c)^{1/2} * e * f + 2 * c * d * f \right) / (c*x + (-a*c)^{1/2}) \left. \right) * a * e * g^2 * \left( -a*c \right)^{1/2} * \left( \left( -a*c \right)^{1/2} * \right. \right. \\ & \left. \left. d * g + (-a*c)^{1/2} * e * f - a * e * g + c * d * f \right) / c \right)^{1/2} - \ln\left( \left( (-2 * (-a*c) \right)^{1/2} * x * e^{g*x+c*d} * g + x * c * e^{f+(-a*c)^{1/2} * d * g + (-a*c)^{1/2} * e * f + 2 * ((e*x+d) * (g*x+f))^{1/2} * \right. \right. \\ & \left. \left. \left( (-a*c)^{1/2} * d * g + (-a*c)^{1/2} * e * f - a * e * g + c * d * f \right) / c \right)^{1/2} * c - \left( -a*c \right)^{1/2} * d \right. \\ & \left. * g - (-a*c)^{1/2} * e * f + 2 * c * d * f \right) / (c*x + (-a*c)^{1/2}) \left. \right) * c^2 * d * f^2 * \left( \left( -a*c \right)^{1/2} * \right. \right. \\ & \left. \left. d * g + (-a*c)^{1/2} * e * f - a * e * g + c * d * f \right) / c \right)^{1/2} + \ln\left( \left( (-2 * (-a*c) \right)^{1/2} * x * e^{g*x+c*d} * g + x * c * e^{f+(-a*c)^{1/2} * d * g + (-a*c)^{1/2} * e * f + 2 * ((e*x+d) * (g*x+f))^{1/2} * \right. \right. \\ & \left. \left. \left( (-a*c)^{1/2} * d * g + (-a*c)^{1/2} * e * f - a * e * g + c * d * f \right) / c \right)^{1/2} * c - \left( -a*c \right)^{1/2} * d \right. \\ & \left. * g - (-a*c)^{1/2} * e * f + 2 * c * d * f \right) / (c*x + (-a*c)^{1/2}) \left. \right) * c * e * f^2 * \left( -a*c \right)^{1/2} * \left( \left( -a*c \right)^{1/2} * \right. \right. \\ & \left. \left. d * g + (-a*c)^{1/2} * e * f - a * e * g + c * d * f \right) / c \right)^{1/2} \left. \right) / \left( (e*x+d) * (g*x+f) \right)^{1/2} / \left( g * \left( -a*c \right)^{1/2} + c * f \right) / \left( -a*c \right)^{1/2} \right. \\ & \left. / \left( \left( -a*c \right)^{1/2} * d * g + (-a*c)^{1/2} * e * f - a * e * g + c * d * f \right) / c \right)^{1/2} / \left( c * f - g * \left( -a*c \right)^{1/2} \right) / \left( - \left( -a*c \right)^{1/2} * d * g + (-a*c)^{1/2} * e * f + a * e * g - c * d * f \right) / c \right)^{1/2} \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{(cx^2+a)\sqrt{gx+f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/((c\*x^2 + a)\*sqrt(g\*x + f)),x, algorithm="maxima")

[Out] integrate(sqrt(e\*x + d)/((c\*x^2 + a)\*sqrt(g\*x + f)), x)

**Fricas [A]** time = 10.4801, size = 2583, normalized size = 10.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/((c\*x^2 + a)\*sqrt(g\*x + f)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/4*\sqrt{-(c*d*f + a*e*g + (a*c^2*f^2 + a^2*c*g^2))*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)}}/(a*c^2*f^2 + a^2*c*g^2))*\log(-(e^2*f^2 - d^2*g^2 + 2*(c*e*f^2 - c*d*f*g + (a*c^2*f^2*g + a^2*c*g^3))*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)})))*\sqrt{(e*x + d)*\sqrt{g*x + f}*\sqrt{-(c*d*f + a*e*g + (a*c^2*f^2 + a^2*c*g^2))*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)}})/(a*c^2*f^2 + a^2*c*g^2))} \\ & + 2*(e^2*f*g - d*e*g^2)*x - (2*c^2*d*f^3 + 2*a*c*d*f*g^2 + (c^2*e*f^3 + c^2*d*f^2*g + a*c*e*f*g^2 + a*c*d*g^3)*x)*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)}}/x + 1/4*\sqrt{-(c*d*f + a*e*g + (a*c^2*f^2 + a^2*c*g^2))*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)}}/(a*c^2*f^2 + a^2*c*g^2))*\log(-(e^2*f^2 - d^2*g^2 - 2*(c*e*f^2 - c*d*f*g + (a*c^2*f^2*g + a^2*c*g^3))*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)})))*\sqrt{(e*x + d)*\sqrt{g*x + f}*\sqrt{-(c*d*f + a*e*g + (a*c^2*f^2 + a^2*c*g^2))*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)}})/(a*c^2*f^2 + a^2*c*g^2))} \\ & + 2*(e^2*f*g - d*e*g^2)*x - (2*c^2*d*f^3 + 2*a*c*d*f*g^2 + (c^2*e*f^3 + c^2*d*f^2*g + a*c*e*f*g^2 + a*c*d*g^3)*x)*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)}}/x - 1/4*\sqrt{-(c*d*f + a*e*g - (a*c^2*f^2 + a^2*c*g^2))*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)}}/(a*c^2*f^2 + a^2*c*g^2))*\log(-(e^2*f^2 - d^2*g^2 + 2*(c*e*f^2 - c*d*f*g - (a*c^2*f^2*g + a^2*c*g^3))*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)})))*\sqrt{(e*x + d)*\sqrt{g*x + f}*\sqrt{-(c*d*f + a*e*g - (a*c^2*f^2 + a^2*c*g^2))*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)}})/(a*c^2*f^2 + a^2*c*g^2))} \\ & + 2*(e^2*f*g - d*e*g^2)*x + (2*c^2*d*f^3 + 2*a*c*d*f*g^2 + (c^2*e*f^3 + c^2*d*f^2*g + a*c*e*f*g^2 + a*c*d*g^3)*x)*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)}}/x + 1/4*\sqrt{-(c*d*f + a*e*g - (a*c^2*f^2 + a^2*c*g^2))*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)}}/(a*c^2*f^2 + a^2*c*g^2))*\log(-(e^2*f^2 - d^2*g^2 - 2*(c*e*f^2 - c*d*f*g - (a*c^2*f^2*g + a^2*c*g^3))*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)})))*\sqrt{(e*x + d)*\sqrt{g*x + f}*\sqrt{-(c*d*f + a*e*g - (a*c^2*f^2 + a^2*c*g^2))*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)}})/(a*c^2*f^2 + a^2*c*g^2))} \\ & + 2*(e^2*f*g - d*e*g^2)*x + (2*c^2*d*f^3 + 2*a*c*d*f*g^2 + (c^2*e*f^3 + c^2*d*f^2*g + a*c*e*f*g^2 + a*c*d*g^3)*x)*\sqrt{-(e^2*f^2 - 2*d*e*f*g + d^2*g^2)/(a*c^3*f^4 + 2*a^2*c^2*f^2*g^2 + a^3*c*g^4)}}/x \end{aligned}$$



**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}}{(a+cx^2)\sqrt{f+gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(1/2)/(c\*x\*\*2+a)/(g\*x+f)\*\*(1/2),x)

[Out] Integral(sqrt(d + e\*x)/((a + c\*x\*\*2)\*sqrt(f + g\*x)), x)

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/((c\*x^2 + a)\*sqrt(g\*x + f)),x, algorithm="giac")

[Out] Exception raised: RuntimeError

$$3.612 \quad \int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}(a+cx^2)} dx$$

**Optimal.** Leaf size=230

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{cf-\sqrt{-ag}}}{\sqrt{f+gx}\sqrt{cd-\sqrt{-ae}}}\right)}{\sqrt{-a}\sqrt{cd-\sqrt{-ae}}\sqrt{cf-\sqrt{-ag}}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-ag+\sqrt{cf}}}{\sqrt{f+gx}\sqrt{-ae+\sqrt{cd}}}\right)}{\sqrt{-a}\sqrt{-ae+\sqrt{cd}}\sqrt{-ag+\sqrt{cf}}}$$

[Out] ArcTanh[(Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*Sqrt[f + g\*x])]/(Sqrt[-a]\*Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]) - ArcTanh[(Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*Sqrt[f + g\*x])]/(Sqrt[-a]\*Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g])

**Rubi [A]** time = 0.644941, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{cf-\sqrt{-ag}}}{\sqrt{f+gx}\sqrt{cd-\sqrt{-ae}}}\right)}{\sqrt{-a}\sqrt{cd-\sqrt{-ae}}\sqrt{cf-\sqrt{-ag}}} - \frac{\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-ag+\sqrt{cf}}}{\sqrt{f+gx}\sqrt{-ae+\sqrt{cd}}}\right)}{\sqrt{-a}\sqrt{-ae+\sqrt{cd}}\sqrt{-ag+\sqrt{cf}}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[d + e\*x]\*Sqrt[f + g\*x]\*(a + c\*x^2)), x]

[Out] ArcTanh[(Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*Sqrt[f + g\*x])]/(Sqrt[-a]\*Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]) - ArcTanh[(Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*Sqrt[f + g\*x])]/(Sqrt[-a]\*Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g])

**Rubi in Sympy [A]** time = 77.0667, size = 199, normalized size = 0.87

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{d+ex}\sqrt{cf+g\sqrt{-a}}}{\sqrt{f+gx}\sqrt{cd+e\sqrt{-a}}}\right)}{\sqrt{-a}\sqrt{cd+e\sqrt{-a}}\sqrt{cf+g\sqrt{-a}}} + \frac{\operatorname{atanh}\left(\frac{\sqrt{d+ex}\sqrt{cf-g\sqrt{-a}}}{\sqrt{f+gx}\sqrt{cd-e\sqrt{-a}}}\right)}{\sqrt{-a}\sqrt{cd-e\sqrt{-a}}\sqrt{cf-g\sqrt{-a}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(e\*x+d)\*\*(1/2)/(c\*x\*\*2+a)/(g\*x+f)\*\*(1/2), x)

[Out] -atanh(sqrt(d + e\*x)\*sqrt(sqrt(c)\*f + g\*sqrt(-a))/(sqrt(f + g\*x)\*sqrt(sqrt(c)\*d + e\*sqrt(-a))))/(sqrt(-a)\*sqrt(sqrt(c)\*d + e\*sqrt(-a))\*sqrt(sqrt(c)\*f + g\*sqrt(-a))) + atanh(sqrt(d + e\*x)\*sqrt(sqrt(c)\*f - g\*sqrt(-a))/(sqrt(f + g\*x)\*sqrt(sqrt(c)\*d - e\*sqrt(-a))))/(sqrt(-a)\*sqrt(sqrt(c)\*d - e\*sqrt(-a))\*sqrt(sqrt(c)\*f - g\*sqrt(-a)))

**Mathematica [C]** time = 3.85713, size = 451, normalized size = 1.96

$$i \left( \frac{\log\left(\frac{-a(dg+e(f+2gx))-i\sqrt{a}(\sqrt{c}(2df+dgx+efx)+2\sqrt{d+ex}\sqrt{f+gx}\sqrt{cd-i\sqrt{ae}}\sqrt{cf-i\sqrt{ag}})}{(\sqrt{cx+i\sqrt{a}})\sqrt{cd-i\sqrt{ae}}\sqrt{cf-i\sqrt{ag}}}\right)}{\sqrt{cd-i\sqrt{ae}}\sqrt{cf-i\sqrt{ag}}}\right) - \frac{\log\left(\frac{-a(dg+e(f+2gx))+i\sqrt{a}(\sqrt{c}(2df+dgx+efx)+2\sqrt{d+ex}\sqrt{f+gx}\sqrt{cd+i\sqrt{ae}}\sqrt{cf+i\sqrt{ag}})}{(\sqrt{cx-i\sqrt{a}})\sqrt{cd+i\sqrt{ae}}\sqrt{cf+i\sqrt{ag}}}\right)}{\sqrt{cd+i\sqrt{ae}}\sqrt{cf+i\sqrt{ag}}}$$

$2\sqrt{a}$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[d + e\*x]\*Sqrt[f + g\*x]\*(a + c\*x^2)),x]

[Out] 
$$\frac{(-1/2) \cdot (\text{Log}[-(a \cdot (d \cdot g + e \cdot (f + 2 \cdot g \cdot x)))] - I \cdot \text{Sqrt}[a] \cdot (2 \cdot \text{Sqrt}[\text{Sqrt}[c] \cdot d - I \cdot \text{Sqrt}[a] \cdot e] \cdot \text{Sqrt}[\text{Sqrt}[c] \cdot f - I \cdot \text{Sqrt}[a] \cdot g] \cdot \text{Sqrt}[d + e \cdot x] \cdot \text{Sqrt}[f + g \cdot x] + \text{Sqrt}[c] \cdot (2 \cdot d \cdot f + e \cdot f \cdot x + d \cdot g \cdot x)))/(\text{Sqrt}[\text{Sqrt}[c] \cdot d - I \cdot \text{Sqrt}[a] \cdot e] \cdot \text{Sqrt}[\text{Sqrt}[c] \cdot f - I \cdot \text{Sqrt}[a] \cdot g] \cdot (I \cdot \text{Sqrt}[a] + \text{Sqrt}[c] \cdot x)))/(\text{Sqrt}[\text{Sqrt}[c] \cdot d - I \cdot \text{Sqrt}[a] \cdot e] \cdot \text{Sqrt}[\text{Sqrt}[c] \cdot f - I \cdot \text{Sqrt}[a] \cdot g]) - \text{Log}[-(a \cdot (d \cdot g + e \cdot (f + 2 \cdot g \cdot x)))] + I \cdot \text{Sqrt}[a] \cdot (2 \cdot \text{Sqrt}[\text{Sqrt}[c] \cdot d + I \cdot \text{Sqrt}[a] \cdot e] \cdot \text{Sqrt}[\text{Sqrt}[c] \cdot f + I \cdot \text{Sqrt}[a] \cdot g] \cdot \text{Sqrt}[d + e \cdot x] \cdot \text{Sqrt}[f + g \cdot x] + \text{Sqrt}[c] \cdot (2 \cdot d \cdot f + e \cdot f \cdot x + d \cdot g \cdot x)))/(\text{Sqrt}[\text{Sqrt}[c] \cdot d + I \cdot \text{Sqrt}[a] \cdot e] \cdot \text{Sqrt}[\text{Sqrt}[c] \cdot f + I \cdot \text{Sqrt}[a] \cdot g] \cdot ((-I) \cdot \text{Sqrt}[a] + \text{Sqrt}[c] \cdot x)))/(\text{Sqrt}[\text{Sqrt}[c] \cdot d + I \cdot \text{Sqrt}[a] \cdot e] \cdot \text{Sqrt}[\text{Sqrt}[c] \cdot f + I \cdot \text{Sqrt}[a] \cdot g]))/\text{Sqrt}[a]$$

**Maple [B]** time = 0.055, size = 1415, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x+d)^(1/2)/(c\*x^2+a)/(g\*x+f)^(1/2),x)

[Out] 
$$\begin{aligned} & -1/2 \cdot c^2 \cdot (\ln((2 \cdot (-a \cdot c)^{(1/2)} \cdot x \cdot e^{g \cdot x} \cdot c \cdot d \cdot g + x \cdot c \cdot e^{f \cdot x} + (-a \cdot c)^{(1/2)} \cdot d \cdot g + (-a \cdot c)^{(1/2)} \cdot e^{f \cdot x} + 2 \cdot ((e \cdot x + d) \cdot (g \cdot x + f))^{(1/2)} \cdot (((-a \cdot c)^{(1/2)} \cdot d \cdot g + (-a \cdot c)^{(1/2)} \cdot e^{f \cdot x} - a \cdot e^{g \cdot x} \cdot c \cdot d \cdot f)/c)^{(1/2)} \cdot c + 2 \cdot c \cdot d \cdot f)/(c \cdot x - (-a \cdot c)^{(1/2)})) \cdot a^2 \cdot e^2 \cdot g^2 \cdot (-((-a \cdot c)^{(1/2)} \cdot d \cdot g + (-a \cdot c)^{(1/2)} \cdot e^{f \cdot x} + a \cdot e^{g \cdot x} \cdot c \cdot d \cdot f)/c)^{(1/2)} + \ln((2 \cdot (-a \cdot c)^{(1/2)} \cdot x \cdot e^{g \cdot x} \cdot c \cdot d \cdot g + x \cdot c \cdot e^{f \cdot x} + (-a \cdot c)^{(1/2)} \cdot d \cdot g + (-a \cdot c)^{(1/2)} \cdot e^{f \cdot x} + 2 \cdot ((e \cdot x + d) \cdot (g \cdot x + f))^{(1/2)} \cdot (((-a \cdot c)^{(1/2)} \cdot d \cdot g + (-a \cdot c)^{(1/2)} \cdot e^{f \cdot x} - a \cdot e^{g \cdot x} \cdot c \cdot d \cdot f)/c)^{(1/2)} \cdot c + 2 \cdot c \cdot d \cdot f)/(c \cdot x - (-a \cdot c)^{(1/2)})) \cdot a \cdot c \cdot d^2 \cdot g^2 \cdot (-((-a \cdot c)^{(1/2)} \cdot d \cdot g + (-a \cdot c)^{(1/2)} \cdot e^{f \cdot x} + a \cdot e^{g \cdot x} \cdot c \cdot d \cdot f)/c)^{(1/2)} + \ln((2 \cdot (-a \cdot c)^{(1/2)} \cdot x \cdot e^{g \cdot x} \cdot c \cdot d \cdot g + x \cdot c \cdot e^{f \cdot x} + (-a \cdot c)^{(1/2)} \cdot d \cdot g + (-a \cdot c)^{(1/2)} \cdot e^{f \cdot x} + 2 \cdot ((e \cdot x + d) \cdot (g \cdot x + f))^{(1/2)} \cdot (((-a \cdot c)^{(1/2)} \cdot d \cdot g + (-a \cdot c)^{(1/2)} \cdot e^{f \cdot x} - a \cdot e^{g \cdot x} \cdot c \cdot d \cdot f)/c)^{(1/2)} \cdot c + 2 \cdot c \cdot d \cdot f)/(c \cdot x - (-a \cdot c)^{(1/2)})) \cdot a \cdot c \cdot e^2 \cdot f^2 \cdot (-((-a \cdot c)^{(1/2)} \cdot d \cdot g + (-a \cdot c)^{(1/2)} \cdot e^{f \cdot x} + a \cdot e^{g \cdot x} \cdot c \cdot d \cdot f)/c)^{(1/2)} + \ln((2 \cdot (-a \cdot c)^{(1/2)} \cdot x \cdot e^{g \cdot x} \cdot c \cdot d \cdot g + x \cdot c \cdot e^{f \cdot x} + (-a \cdot c)^{(1/2)} \cdot d \cdot g + (-a \cdot c)^{(1/2)} \cdot e^{f \cdot x} + 2 \cdot ((e \cdot x + d) \cdot (g \cdot x + f))^{(1/2)} \cdot (((-a \cdot c)^{(1/2)} \cdot d \cdot g + (-a \cdot c)^{(1/2)} \cdot e^{f \cdot x} - a \cdot e^{g \cdot x} \cdot c \cdot d \cdot f)/c)^{(1/2)} \cdot c - (-a \cdot c)^{(1/2)} \cdot d \cdot g - (-a \cdot c)^{(1/2)} \cdot e^{f \cdot x} + 2 \cdot c \cdot d \cdot f)/(c \cdot x + (-a \cdot c)^{(1/2)})) \cdot a^2 \cdot e^2 \cdot g^2 \cdot (((-a \cdot c)^{(1/2)} \cdot d \cdot g + (-a \cdot c)^{(1/2)} \cdot e^{f \cdot x} - a \cdot e^{g \cdot x} \cdot c \cdot d \cdot f)/c)^{(1/2)} - \ln((2 \cdot (-a \cdot c)^{(1/2)} \cdot x \cdot e^{g \cdot x} \cdot c \cdot d \cdot g + x \cdot c \cdot e^{f \cdot x} + 2 \cdot ((e \cdot x + d) \cdot (g \cdot x + f))^{(1/2)} \cdot (((-a \cdot c)^{(1/2)} \cdot d \cdot g + (-a \cdot c)^{(1/2)} \cdot e^{f \cdot x} - a \cdot e^{g \cdot x} \cdot c \cdot d \cdot f)/c)^{(1/2)} \cdot c - (-a \cdot c)^{(1/2)} \cdot d \cdot g - (-a \cdot c)^{(1/2)} \cdot e^{f \cdot x} + 2 \cdot c \cdot d \cdot f)/(c \cdot x + (-a \cdot c)^{(1/2)})) \cdot a \cdot c \cdot d^2 \cdot g^2 \cdot (((-a \cdot c)^{(1/2)} \cdot d \cdot g + (-a \cdot c)^{(1/2)} \cdot e^{f \cdot x} - a \cdot e^{g \cdot x} \cdot c \cdot d \cdot f)/c)^{(1/2)} - \ln((2 \cdot (-a \cdot c)^{(1/2)} \cdot x \cdot e^{g \cdot x} \cdot c \cdot d \cdot g + x \cdot c \cdot e^{f \cdot x} + 2 \cdot ((e \cdot x + d) \cdot (g \cdot x + f))^{(1/2)} \cdot (((-a \cdot c)^{(1/2)} \cdot d \cdot g + (-a \cdot c)^{(1/2)} \cdot e^{f \cdot x} - a \cdot e^{g \cdot x} \cdot c \cdot d \cdot f)/c)^{(1/2)} \cdot c - (-a \cdot c)^{(1/2)} \cdot d \cdot g - (-a \cdot c)^{(1/2)} \cdot e^{f \cdot x} + 2 \cdot c \cdot d \cdot f)/(c \cdot x + (-a \cdot c)^{(1/2)})) \cdot a \cdot c \cdot e^2 \cdot f^2 \cdot (((-a \cdot c)^{(1/2)} \cdot d \cdot g + (-a \cdot c)^{(1/2)} \cdot e^{f \cdot x} - a \cdot e^{g \cdot x} \cdot c \cdot d \cdot f)/c)^{(1/2)} - \ln((2 \cdot (-a \cdot c)^{(1/2)} \cdot x \cdot e^{g \cdot x} \cdot c \cdot d \cdot g + x \cdot c \cdot e^{f \cdot x} + 2 \cdot ((e \cdot x + d) \cdot (g \cdot x + f))^{(1/2)} \cdot (((-a \cdot c)^{(1/2)} \cdot d \cdot g + (-a \cdot c)^{(1/2)} \cdot e^{f \cdot x} - a \cdot e^{g \cdot x} \cdot c \cdot d \cdot f)/c)^{(1/2)} \cdot c - (-a \cdot c)^{(1/2)} \cdot d \cdot g - (-a \cdot c)^{(1/2)} \cdot e^{f \cdot x} + 2 \cdot c \cdot d \cdot f)/(c \cdot x + (-a \cdot c)^{(1/2)})) \cdot c^2 \cdot d^2 \cdot f^2 \cdot (((-a \cdot c)^{(1/2)} \cdot d \cdot g + (-a \cdot c)^{(1/2)} \cdot e^{f \cdot x} - a \cdot e^{g \cdot x} \cdot c \cdot d \cdot f)/c)^{(1/2)} \cdot (g \cdot x + f)^{(1/2)} \cdot (e \cdot x + d)^{(1/2)} / (-((-a \cdot c)^{(1/2)} \cdot d \cdot g + (-a \cdot c)^{(1/2)} \cdot e^{f \cdot x} + a \cdot e^{g \cdot x} \cdot c \cdot d \cdot f)/c)^{(1/2)} / (c \cdot f - g \cdot (-a \cdot c)^{(1/2)}) / (((-a \cdot c)^{(1/2)} \cdot d \cdot g + (-a \cdot c)^{(1/2)} \cdot e^{f \cdot x} - a \cdot e^{g \cdot x} \cdot c \cdot d \cdot f)/c)^{(1/2)} / (-a \cdot c)^{(1/2)} / (g \cdot (-a \cdot c)^{(1/2)} + c \cdot f) / (c \cdot d - (-a \cdot c)^{(1/2)} \cdot e) / (((-a \cdot c)^{(1/2)} \cdot e + c \cdot d) / ((e \cdot x + d) \cdot (g \cdot x + f))^{(1/2)}) \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + a)\sqrt{ex + d}\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + a)\*sqrt(e\*x + d)\*sqrt(g\*x + f)),x, algorithm="maxima")

[Out] integrate(1/((c\*x^2 + a)\*sqrt(e\*x + d)\*sqrt(g\*x + f)), x)

**Fricas [A]** time = 24.5565, size = 5839, normalized size = 25.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + a)\*sqrt(e\*x + d)\*sqrt(g\*x + f)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & -1/4*\sqrt{-(c*d*f - a*e*g + ((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c \\ & *d^2 + a^3*e^2)*g^2)*\sqrt{-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)}/ \\ & ((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d \\ & ^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4 \\ & *c*d^2*e^2 + a^5*e^4)*g^4))/((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2 \\ & *c*d^2 + a^3*e^2)*g^2))*\log((e^2*f^2 + 2*d*e*f*g + d^2*g^2 + 2*(c \\ & *d*e*f^2 - a*d*e*g^2 + (c*d^2 - a*e^2)*f*g - ((a*c^2*d^2*e + a^2*c \\ & *e^3)*f^3 + (a*c^2*d^3 + a^2*c*d*e^2)*f^2*g + (a^2*c*d^2*e + a^3 \\ & *e^3)*f*g^2 + (a^2*c*d^3 + a^3*d*e^2)*g^3)*\sqrt{-(c*e^2*f^2 + 2*c \\ & *d*e*f*g + c*d^2*g^2)}/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e \\ & ^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 \\ & + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4))*\sqrt{e*x + d} \\ & *\sqrt{g*x + f)*\sqrt{-(c*d*f - a*e*g + ((a*c^2*d^2 + a^2*c*e^2)*f^2 + \\ & (a^2*c*d^2 + a^3*e^2)*g^2)*\sqrt{-(c*e^2*f^2 + 2*c*d*e*f*g + c \\ & *d^2*g^2)}/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2* \\ & (a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d \\ & ^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4))/((a*c^2*d^2 + a^2*c*e^2)* \\ & f^2 + (a^2*c*d^2 + a^3*e^2)*g^2)) + 2*(e^2*f*g + d*e*g^2)*x + (2* \\ & (c^2*d^3 + a*c*d*e^2)*f^3 + 2*(a*c*d^3 + a^2*d*e^2)*f*g^2 + ((c^2 \\ & *d^2*e + a*c*e^3)*f^3 + (c^2*d^3 + a*c*d*e^2)*f^2*g + (a*c*d^2*e \\ & + a^2*e^3)*f*g^2 + (a*c*d^3 + a^2*d*e^2)*g^3)*x)*\sqrt{-(c*e^2*f^2 \\ & + 2*c*d*e*f*g + c*d^2*g^2)}/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3 \\ & *c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f \\ & ^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4))/x) + 1/ \\ & 4*\sqrt{-(c*d*f - a*e*g + ((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 \\ & + a^3*e^2)*g^2)*\sqrt{-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2)}/((a \\ & *c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 \\ & + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c \\ & *d^2*e^2 + a^5*e^4)*g^4))/((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c* \\ & d^2 + a^3*e^2)*g^2))*\log((e^2*f^2 + 2*d*e*f*g + d^2*g^2 - 2*(c*d \\ & *e*f^2 - a*d*e*g^2 + (c*d^2 - a*e^2)*f*g - ((a*c^2*d^2*e + a^2*c*e \\ & ^3)*f^3 + (a*c^2*d^3 + a^2*c*d*e^2)*f^2*g + (a^2*c*d^2*e + a^3*e^ \\ & ^3)*f*g^2 + (a^2*c*d^3 + a^3*d*e^2)*g^3)*\sqrt{-(c*e^2*f^2 + 2*c*d \\ & *e*f*g + c*d^2*g^2)}/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4) \\ & *f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + \\ & (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4))*\sqrt{e*x + d}*sq \\ & rt(g*x + f)*\sqrt{-(c*d*f - a*e*g + ((a*c^2*d^2 + a^2*c*e^2)*f^2 + \\ & (a^2*c*d^2 + a^3*e^2)*g^2)*\sqrt{-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2 \\ & *g^2)}/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2 \\ & *c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 \\ & + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4))/((a*c^2*d^2 + a^2*c*e^2)*f^2 \\ & + (a^2*c*d^2 + a^3*e^2)*g^2)) + 2*(e^2*f*g + d*e*g^2)*x + (2*(c^2 \\ & *d^3 + a*c*d*e^2)*f^3 + 2*(a*c*d^3 + a^2*d*e^2)*f*g^2 + ((c^2*d^2 \\ & *e + a*c*e^3)*f^3 + (c^2*d^3 + a*c*d*e^2)*f^2*g + (a*c*d^2*e + a \\ & ^2*e^3)*f*g^2 + (a*c*d^3 + a^2*d*e^2)*g^3)*x)*\sqrt{-(c*e^2*f^2 + \\ & 2*c*d*e*f*g + c*d^2*g^2)}/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2 \\ & *e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + \\ & (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4)) \end{aligned}$$

$$\begin{aligned}
& 2^*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2* \\
& g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4))/x) - 1/4*s \\
& \text{qrt}(-(c*d*f - a*e*g - ((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + \\
& a^3*e^2)*g^2))*\text{sqrt}(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2))/((a*c^4 \\
& d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2 \\
& *a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2 \\
& *e^2 + a^5*e^4)*g^4))/((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 \\
& + a^3*e^2)*g^2))*\text{log}((e^2*f^2 + 2*d*e*f*g + d^2*g^2 + 2*(c*d*e*f \\
& ^2 - a*d*e*g^2 + (c*d^2 - a*e^2)*f*g + ((a*c^2*d^2*e + a^2*c*e^3) \\
& *f^3 + (a*c^2*d^3 + a^2*c*d*e^2)*f^2*g + (a^2*c*d^2*e + a^3*e^3)* \\
& f*g^2 + (a^2*c*d^3 + a^3*d*e^2)*g^3))*\text{sqrt}(-(c*e^2*f^2 + 2*c*d*e*f \\
& *g + c*d^2*g^2))/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 \\
& + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3 \\
& *c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4))*\text{sqrt}(e*x + d)*\text{sqrt}( \\
& g*x + f)*\text{sqrt}(-(c*d*f - a*e*g - ((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a \\
& ^2*c*d^2 + a^3*e^2)*g^2))*\text{sqrt}(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g \\
& ^2))/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c \\
& ^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + \\
& 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4))/((a*c^2*d^2 + a^2*c*e^2)*f^2 + \\
& (a^2*c*d^2 + a^3*e^2)*g^2)) + 2*(e^2*f*g + d*e*g^2)*x - (2*(c^2*d \\
& ^3 + a*c*d*e^2)*f^3 + 2*(a*c*d^3 + a^2*d*e^2)*f*g^2 + ((c^2*d^2*e \\
& + a*c*e^3)*f^3 + (c^2*d^3 + a*c*d*e^2)*f^2*g + (a*c*d^2*e + a^2* \\
& e^3)*f*g^2 + (a*c*d^3 + a^2*d*e^2)*g^3)*x)*\text{sqrt}(-(c*e^2*f^2 + 2*c \\
& *d*e*f*g + c*d^2*g^2))/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e \\
& ^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 \\
& + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4))/x) + 1/4*\text{sqrt} \\
& (-(c*d*f - a*e*g - ((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + a^3 \\
& *e^2)*g^2))*\text{sqrt}(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2))/((a*c^4*d \\
& ^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3*d^4 + 2*a^3 \\
& *c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a^4*c*d^2*e \\
& ^2 + a^5*e^4)*g^4))/((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2*c*d^2 + \\
& a^3*e^2)*g^2))*\text{log}((e^2*f^2 + 2*d*e*f*g + d^2*g^2 - 2*(c*d*e*f^2 \\
& - a*d*e*g^2 + (c*d^2 - a*e^2)*f*g + ((a*c^2*d^2*e + a^2*c*e^3)*f^3 \\
& + (a*c^2*d^3 + a^2*c*d*e^2)*f^2*g + (a^2*c*d^2*e + a^3*e^3)*f*g \\
& ^2 + (a^2*c*d^3 + a^3*d*e^2)*g^3))*\text{sqrt}(-(c*e^2*f^2 + 2*c*d*e*f*g \\
& + c*d^2*g^2))/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + \\
& 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c \\
& ^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4))*\text{sqrt}(e*x + d)*\text{sqrt}(g*x \\
& + f)*\text{sqrt}(-(c*d*f - a*e*g - ((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2* \\
& c*d^2 + a^3*e^2)*g^2))*\text{sqrt}(-(c*e^2*f^2 + 2*c*d*e*f*g + c*d^2*g^2) \\
& /((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4)*f^4 + 2*(a^2*c^3* \\
& d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + (a^3*c^2*d^4 + 2*a \\
& ^4*c*d^2*e^2 + a^5*e^4)*g^4))/((a*c^2*d^2 + a^2*c*e^2)*f^2 + (a^2 \\
& *c*d^2 + a^3*e^2)*g^2)) + 2*(e^2*f*g + d*e*g^2)*x - (2*(c^2*d^3 \\
& + a*c*d*e^2)*f^3 + 2*(a*c*d^3 + a^2*d*e^2)*f*g^2 + ((c^2*d^2*e + \\
& a*c*e^3)*f^3 + (c^2*d^3 + a*c*d*e^2)*f^2*g + (a*c*d^2*e + a^2*e^3) \\
& )*f*g^2 + (a*c*d^3 + a^2*d*e^2)*g^3)*x)*\text{sqrt}(-(c*e^2*f^2 + 2*c*d \\
& *e*f*g + c*d^2*g^2))/((a*c^4*d^4 + 2*a^2*c^3*d^2*e^2 + a^3*c^2*e^4) \\
& *f^4 + 2*(a^2*c^3*d^4 + 2*a^3*c^2*d^2*e^2 + a^4*c*e^4)*f^2*g^2 + \\
& (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*g^4))/x)
\end{aligned}$$


---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + cx^2)\sqrt{d + ex}\sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x+d)\*\*(1/2)/(c\*x\*\*2+a)/(g\*x+f)\*\*(1/2),x)

[Out] Integral(1/((a + c\*x\*\*2)\*sqrt(d + e\*x)\*sqrt(f + g\*x)), x)

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^2 + a)*sqrt(e*x + d)*sqrt(g*x + f)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

$$3.613 \quad \int \frac{1}{(d+ex)^{3/2} \sqrt{f+gx} (a+cx^2)} dx$$

**Optimal.** Leaf size=354

$$\begin{aligned} & -\frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}(\sqrt{cd}-\sqrt{-ae})(ef-dg)} + \frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}(\sqrt{-ae}+\sqrt{cd})(ef-dg)} \\ & + \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{cf}-\sqrt{-ag}}{\sqrt{f+gx}\sqrt{cd}-\sqrt{-ae}}\right)}{\sqrt{-a}(\sqrt{cd}-\sqrt{-ae})^{3/2}\sqrt{\sqrt{cf}-\sqrt{-ag}}} - \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-ag}+\sqrt{cf}}{\sqrt{f+gx}\sqrt{-ae}+\sqrt{cd}}\right)}{\sqrt{-a}(\sqrt{-ae}+\sqrt{cd})^{3/2}\sqrt{\sqrt{-ag}+\sqrt{cf}}} \end{aligned}$$

[Out]  $-\left(\frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}(\sqrt{cd}-\sqrt{-ae})(ef-dg)} + \frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}(\sqrt{-ae}+\sqrt{cd})(ef-dg)}\right) + \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{cf}-\sqrt{-ag}}{\sqrt{f+gx}\sqrt{cd}-\sqrt{-ae}}\right)}{\sqrt{-a}(\sqrt{cd}-\sqrt{-ae})^{3/2}\sqrt{\sqrt{cf}-\sqrt{-ag}}} - \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-ag}+\sqrt{cf}}{\sqrt{f+gx}\sqrt{-ae}+\sqrt{cd}}\right)}{\sqrt{-a}(\sqrt{-ae}+\sqrt{cd})^{3/2}\sqrt{\sqrt{-ag}+\sqrt{cf}}}$

**Rubi [A]** time = 1.59736, antiderivative size = 354, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\begin{aligned} & -\frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}(\sqrt{cd}-\sqrt{-ae})(ef-dg)} + \frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}(\sqrt{-ae}+\sqrt{cd})(ef-dg)} \\ & + \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{cf}-\sqrt{-ag}}{\sqrt{f+gx}\sqrt{cd}-\sqrt{-ae}}\right)}{\sqrt{-a}(\sqrt{cd}-\sqrt{-ae})^{3/2}\sqrt{\sqrt{cf}-\sqrt{-ag}}} - \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-ag}+\sqrt{cf}}{\sqrt{f+gx}\sqrt{-ae}+\sqrt{cd}}\right)}{\sqrt{-a}(\sqrt{-ae}+\sqrt{cd})^{3/2}\sqrt{\sqrt{-ag}+\sqrt{cf}}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)^(3/2)\*Sqrt[f + g\*x]\*(a + c\*x^2)), x]

[Out]  $-\left(\frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}(\sqrt{cd}-\sqrt{-ae})(ef-dg)} + \frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}(\sqrt{-ae}+\sqrt{cd})(ef-dg)}\right) + \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{cf}-\sqrt{-ag}}{\sqrt{f+gx}\sqrt{cd}-\sqrt{-ae}}\right)}{\sqrt{-a}(\sqrt{cd}-\sqrt{-ae})^{3/2}\sqrt{\sqrt{cf}-\sqrt{-ag}}} - \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-ag}+\sqrt{cf}}{\sqrt{f+gx}\sqrt{-ae}+\sqrt{cd}}\right)}{\sqrt{-a}(\sqrt{-ae}+\sqrt{cd})^{3/2}\sqrt{\sqrt{-ag}+\sqrt{cf}}}$

**Rubi in Sympy [A]** time = 143.349, size = 304, normalized size = 0.86

$$\begin{aligned} & -\frac{\sqrt{c} \operatorname{atanh}\left(\frac{\sqrt{d+ex}\sqrt{cf+g\sqrt{-a}}}{\sqrt{f+gx}\sqrt{cd+e\sqrt{-a}}}\right)}{\sqrt{-a}(\sqrt{cd+e\sqrt{-a}})^{3/2}\sqrt{\sqrt{cf+g\sqrt{-a}}}} + \frac{\sqrt{c} \operatorname{atanh}\left(\frac{\sqrt{d+ex}\sqrt{cf-g\sqrt{-a}}}{\sqrt{f+gx}\sqrt{cd-e\sqrt{-a}}}\right)}{\sqrt{-a}(\sqrt{cd-e\sqrt{-a}})^{3/2}\sqrt{\sqrt{cf-g\sqrt{-a}}}} \\ & - \frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}(\sqrt{cd+e\sqrt{-a}})(dg-ef)} + \frac{e\sqrt{f+gx}}{\sqrt{-a}\sqrt{d+ex}(\sqrt{cd-e\sqrt{-a}})(dg-ef)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(e\*x+d)\*\*(3/2)/(c\*x\*\*2+a)/(g\*x+f)\*\*(1/2), x)

[Out]  $-\sqrt{c} \operatorname{atanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f+g\sqrt{-a}}}{\sqrt{d+ex}\sqrt{\sqrt{c}d+e\sqrt{-a}}}\right) / \left(\sqrt{-a}\sqrt{\sqrt{c}d+e\sqrt{-a}}\right)^{3/2} \sqrt{\sqrt{c}f+g\sqrt{-a}} + \sqrt{c} \operatorname{atanh}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{c}f-g\sqrt{-a}}}{\sqrt{d+ex}\sqrt{\sqrt{c}d-e\sqrt{-a}}}\right) / \left(\sqrt{-a}\sqrt{\sqrt{c}d-e\sqrt{-a}}\right)^{3/2} \sqrt{\sqrt{c}f-g\sqrt{-a}} - e\sqrt{f+gx} / \left(\sqrt{-a}\sqrt{d+ex}\sqrt{\sqrt{c}d+e\sqrt{-a}}\right) (dg - ef) + e\sqrt{f+gx} / \left(\sqrt{-a}\sqrt{d+ex}\sqrt{\sqrt{c}d-e\sqrt{-a}}\right) (dg - ef)$

**Mathematica [C]** time = 3.41567, size = 555, normalized size = 1.57

$$\frac{(\sqrt{a}\sqrt{ce-icd}) \log\left(-\frac{i\sqrt{a}\sqrt{cd-i\sqrt{ae}}(2\sqrt{d+ex}\sqrt{f+gx}\sqrt{cd-i\sqrt{ae}}\sqrt{cf-i\sqrt{ag}}-i\sqrt{a}(dg+ef+2egx)+\sqrt{c}(2df+dgx+efx))}{\sqrt{c}(\sqrt{cx+i\sqrt{a}})\sqrt{cf-i\sqrt{ag}}}\right)}{\sqrt{a}\sqrt{cd-i\sqrt{ae}}\sqrt{cf-i\sqrt{ag}}} + \frac{(\sqrt{a}\sqrt{ce+icd}) \log\left(\frac{i\sqrt{a}\sqrt{cd+i\sqrt{ae}}(2\sqrt{d+ex}\sqrt{f+gx}\sqrt{cd+i\sqrt{ae}}\sqrt{cf+i\sqrt{ag}}+i\sqrt{a}(dg+ef+2egx)+\sqrt{c}(2df+dgx+efx))}{\sqrt{c}(\sqrt{cx+i\sqrt{a}})\sqrt{cf+i\sqrt{ag}}}\right)}{\sqrt{a}\sqrt{cd+i\sqrt{ae}}\sqrt{cf+i\sqrt{ag}}}}{2(ae^2 + cd^2)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x)^(3/2)\*Sqrt[f + g\*x]\*(a + c\*x^2)),x]

[Out]  $\left(\frac{-4e^2\sqrt{f+gx}}{(ef-dg)\sqrt{d+ex}}\right) + \left(\frac{(-I)^c d + \sqrt{a}\sqrt{c}e \operatorname{Log}\left[\frac{(-I)\sqrt{a}\sqrt{\sqrt{c}d - I\sqrt{a}e} \sqrt{\sqrt{c}f - I\sqrt{a}g} \sqrt{d+ex} \sqrt{f+gx} + \sqrt{c}(2df+dgx+efx) - I\sqrt{a}(ef+dg+2egx)}{\sqrt{c}\sqrt{\sqrt{c}f - I\sqrt{a}g} \sqrt{d+ex} \sqrt{f+gx} + \sqrt{c}(2df+dgx+efx) - I\sqrt{a}(ef+dg+2egx)}\right]}{\sqrt{a}\sqrt{c}e \operatorname{Log}\left[\frac{I\sqrt{a}\sqrt{\sqrt{c}d + I\sqrt{a}e} \sqrt{\sqrt{c}f + I\sqrt{a}g} \sqrt{d+ex} \sqrt{f+gx} + \sqrt{c}(2df+dgx+efx) + I\sqrt{a}(ef+dg+2egx)}{\sqrt{c}\sqrt{\sqrt{c}f + I\sqrt{a}g} \sqrt{d+ex} \sqrt{f+gx} + \sqrt{c}(2df+dgx+efx) + I\sqrt{a}(ef+dg+2egx)}\right]}\right) / (2(c d^2 + a e^2))$

**Maple [B]** time = 0.11, size = 10977, normalized size = 31.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x+d)^(3/2)/(c\*x^2+a)/(g\*x+f)^(1/2),x)

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + a)(ex + d)^{3/2}\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + a)\*(e\*x + d)^(3/2)\*sqrt(g\*x + f)),x, algorithm="maxima")

[Out] integrate(1/((c\*x^2 + a)\*(e\*x + d)^(3/2)\*sqrt(g\*x + f)), x)



**Fricas [A]** time = 84.3266, size = 15992, normalized size = 45.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + a)\*(e\*x + d)^(3/2)\*sqrt(g\*x + f)),x, algorithm="fricas")

[Out] 
$$-1/4*(8*\sqrt{e*x + d}*\sqrt{g*x + f}*e^2 + ((c*d^3*e + a*d^2*e^3)*f - (c*d^4 + a*d^2*e^2)*g + ((c*d^2*e^2 + a*e^4)*f - (c*d^3*e + a*d^2*e^3)*g)*x)*\sqrt{-((c^3*d^3 - 3*a*c^2*d^2*e^2)*f - (3*a*c^2*d^2*e - a^2*c^2*e^3)*g + ((a*c^4*d^6 + 3*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4 + a^4*c^2*e^6)*f^2 + (a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c^2*d^2*e^4 + a^5*e^6)*g^2)}*\sqrt{-((9*c^5*d^4*e^2 - 6*a*c^4*d^2*e^4 + a^2*c^3*e^6)*f^2 + 2*(3*c^5*d^5*e - 10*a*c^4*d^3*e^3 + 3*a^2*c^3*d^2*e^5)*f*g + (c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2*e^4)*g^2)}/((a*c^8*d^12 + 6*a^2*c^7*d^10*e^2 + 15*a^3*c^6*d^8*e^4 + 20*a^4*c^5*d^6*e^6 + 15*a^5*c^4*d^4*e^8 + 6*a^6*c^3*d^2*e^10 + a^7*c^2*e^12)*f^4 + 2*(a^2*c^7*d^12 + 6*a^3*c^6*d^10*e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5*c^4*d^6*e^6 + 15*a^6*c^3*d^4*e^8 + 6*a^7*c^2*d^2*e^10 + a^8*c^2*e^12)*f^2*g^2 + (a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c^2*d^2*e^10 + a^9*e^12)*g^4))/((a*c^4*d^6 + 3*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4 + a^4*c^2*e^6)*f^2 + (a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c^2*d^2*e^4 + a^5*e^6)*g^2))*\log(-((3*c^3*d^2*e^2 - a*c^2*e^4)*f^2 + 4*(c^3*d^3*e - a*c^2*d^2*e^3)*f*g + (c^3*d^4 - 3*a*c^2*d^2*e^2)*g^2 + 2*((3*c^4*d^4*e - 4*a*c^3*d^2*e^3 + a^2*c^2*e^5)*f^2 + (c^4*d^5 - 10*a*c^3*d^3*e^2 + 5*a^2*c^2*d^2*e^4)*f*g - 2*(a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*g^2 - (2*(a*c^5*d^7*e + 3*a^2*c^4*d^5*e^3 + 3*a^3*c^3*d^3*e^5 + a^4*c^2*d^2*e^7)*f^3 + (a*c^5*d^8 + 2*a^2*c^4*d^6*e^2 - 2*a^4*c^2*d^2*e^6 - a^5*c^2*e^8)*f^2*g + 2*(a^2*c^4*d^7*e + 3*a^3*c^3*d^5*e^3 + 3*a^4*c^2*d^3*e^5 + a^5*c^2*d^2*e^7)*f*g^2 + (a^2*c^4*d^8 + 2*a^3*c^3*d^6*e^2 - 2*a^5*c^2*d^2*e^6 - a^6*e^8)*g^3))*\sqrt{-((9*c^5*d^4*e^2 - 6*a*c^4*d^2*e^4 + a^2*c^3*e^6)*f^2 + 2*(3*c^5*d^5*e - 10*a*c^4*d^3*e^3 + 3*a^2*c^3*d^2*e^5)*f*g + (c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2*e^4)*g^2)}/((a*c^8*d^12 + 6*a^2*c^7*d^10*e^2 + 15*a^3*c^6*d^8*e^4 + 20*a^4*c^5*d^6*e^6 + 15*a^5*c^4*d^4*e^8 + 6*a^6*c^3*d^2*e^10 + a^7*c^2*e^12)*f^4 + 2*(a^2*c^7*d^12 + 6*a^3*c^6*d^10*e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5*c^4*d^6*e^6 + 15*a^6*c^3*d^4*e^8 + 6*a^7*c^2*d^2*e^10 + a^8*c^2*e^12)*f^2*g^2 + (a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c^2*d^2*e^10 + a^9*e^12)*g^4))*\sqrt{e*x + d}*\sqrt{g*x + f}*\sqrt{-((c^3*d^3 - 3*a*c^2*d^2*e^2)*f - (3*a*c^2*d^2*e - a^2*c^2*e^3)*g + ((a*c^4*d^6 + 3*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4 + a^4*c^2*e^6)*f^2 + (a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c^2*d^2*e^4 + a^5*e^6)*g^2)}*\sqrt{-((9*c^5*d^4*e^2 - 6*a*c^4*d^2*e^4 + a^2*c^3*e^6)*f^2 + 2*(3*c^5*d^5*e - 10*a*c^4*d^3*e^3 + 3*a^2*c^3*d^2*e^5)*f*g + (c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2*e^4)*g^2)}/((a*c^8*d^12 + 6*a^2*c^7*d^10*e^2 + 15*a^3*c^6*d^8*e^4 + 20*a^4*c^5*d^6*e^6 + 15*a^5*c^4*d^4*e^8 + 6*a^6*c^3*d^2*e^10 + a^7*c^2*e^12)*f^4 + 2*(a^2*c^7*d^12 + 6*a^3*c^6*d^10*e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5*c^4*d^6*e^6 + 15*a^6*c^3*d^4*e^8 + 6*a^7*c^2*d^2*e^10 + a^8*c^2*e^12)*f^2*g^2 + (a^3*c^6*d^12 + 6*a^4*c^5*d^10*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c^2*d^2*e^10 + a^9*e^12)*g^4))/((a*c^4*d^6 + 3*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4 + a^4*c^2*e^6)*f^2 + (a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c^2*d^2*e^4 + a^5*e^6)*g^2)) + 2*((3*c^3*d^2*e^2 - a*c^2*e^4)*f*g + (c^3*d^3*e - 3*a*c^2*d^2*e^3)*g^2)*x + (2*(c^5*d^7 + 3*a*c^4*d^5*e^2 + 3*a^2*c^3*d^3*e^4 + a^3*c^2*d^2*e^6)*f^3 + 2*(a*c^4*d^7 + 3*a^2*c^3*d^5*e^2 + 3*a^3*c^2*d^3*e^4 + a^4*c^2*d^2*e^6)*f*g^2 + ((c^5*d^6*e + 3*a*c^4*d^4*e^3 + 3*a^2*c^3*d^2*e^5 + a^3*c^2*e^7)*f^3 + (c^5*d^7 + 3*a*c^4*d^5*e^2 + 3*a^2*c^3*d^3*e^4 + a^3*c^2*d^2*e^6)*f^2*g + (a*c^4*d^6*e + 3*a^2*c^3*d^4*e^3 + 3*a^3*c^2*d^2*e^5 + a^4*c^2*e^7)*f*g^2 + (a*c^4*d^7 + 3*a^2*c^3*d^5*e^2 + 3*a^3*c^2*d^3*e^4 + a^4*c^2*d^2*e^6)*g^3)*x)*\sqrt{-((9*c^5*d^4*e^2 - 6*a*c^4*d^2*e^4 + a^2*c^3*e^6)*f^2 + 2*(3*c^5*d^5*e - 10*a*c^4*d^3*e^3 + 3*a^2*c^3*d^2*e^5)*f*g + (c^5*d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2*e^4)*g^2)}/((a*c^8*d^12 + 6*a^2*c^7*d^10*e^2 + 15*a^3*c^6*d^8*e^4 + 20*a^4*c^5*d^6*e^6 + 15*a^5*c^4*d^4*e^8 + 6*a^6*c^3*d^2*e^10 + a^7*c^2*e^12)*f^4 + 2$$

$$\begin{aligned}
& * (a^2*c^7*d^{12} + 6*a^3*c^6*d^{10}*e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5 \\
& *c^4*d^6*e^6 + 15*a^6*c^3*d^4*e^8 + 6*a^7*c^2*d^2*e^{10} + a^8*c*e^{12}) \\
& *f^2*g^2 + (a^3*c^6*d^{12} + 6*a^4*c^5*d^{10}*e^2 + 15*a^5*c^4*d^8 \\
& *e^4 + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^{10} \\
& + a^9*e^{12})*g^4))/x - ((c*d^3*e + a*d*e^3)*f - (c*d^4 + a*d^2* \\
& e^2)*g + ((c*d^2*e^2 + a*e^4)*f - (c*d^3*e + a*d*e^3)*g)*x)*\text{sqrt}( \\
& -((c^3*d^3 - 3*a*c^2*d*e^2)*f - (3*a*c^2*d^2*e - a^2*c*e^3)*g + ( \\
& (a*c^4*d^6 + 3*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4 + a^4*c*e^6)*f \\
& ^2 + (a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6) \\
& )*g^2)*\text{sqrt}(-((9*c^5*d^4*e^2 - 6*a*c^4*d^2*e^4 + a^2*c^3*e^6)*f^2 \\
& + 2*(3*c^5*d^5*e - 10*a*c^4*d^3*e^3 + 3*a^2*c^3*d*e^5)*f*g + (c^5 \\
& *d^6 - 6*a*c^4*d^4*e^2 + 9*a^2*c^3*d^2*e^4)*g^2)/((a*c^8*d^{12} + \\
& 6*a^2*c^7*d^{10}*e^2 + 15*a^3*c^6*d^8*e^4 + 20*a^4*c^5*d^6*e^6 + 15 \\
& *a^5*c^4*d^4*e^8 + 6*a^6*c^3*d^2*e^{10} + a^7*c^2*e^{12})*f^4 + 2*(a^2 \\
& *c^7*d^{12} + 6*a^3*c^6*d^{10}*e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5*c^4 \\
& *d^6*e^6 + 15*a^6*c^3*d^4*e^8 + 6*a^7*c^2*d^2*e^{10} + a^8*c*e^{12})* \\
& f^2*g^2 + (a^3*c^6*d^{12} + 6*a^4*c^5*d^{10}*e^2 + 15*a^5*c^4*d^8*e^4 \\
& + 20*a^6*c^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^{10} + a \\
& ^9*e^{12})*g^4))/((a*c^4*d^6 + 3*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e \\
& ^4 + a^4*c*e^6)*f^2 + (a^2*c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c* \\
& d^2*e^4 + a^5*e^6)*g^2))*\text{log}(-((3*c^3*d^2*e^2 - a*c^2*e^4)*f^2 + \\
& 4*(c^3*d^3*e - a*c^2*d*e^3)*f*g + (c^3*d^4 - 3*a*c^2*d^2*e^2)*g^2 \\
& - 2*((3*c^4*d^4*e - 4*a*c^3*d^2*e^3 + a^2*c^2*e^5)*f^2 + (c^4*d^5 \\
& - 10*a*c^3*d^3*e^2 + 5*a^2*c^2*d*e^4)*f*g - 2*(a*c^3*d^4*e - 3* \\
& a^2*c^2*d^2*e^3)*g^2 - (2*(a*c^5*d^7*e + 3*a^2*c^4*d^5*e^3 + 3*a^1 \\
& *c^3*d^3*e^5 + a^4*c^2*d*e^7)*f^3 + (a*c^5*d^8 + 2*a^2*c^4*d^6*e \\
& ^2 - 2*a^4*c^2*d^2*e^6 - a^5*c*e^8)*f^2*g + 2*(a^2*c^4*d^7*e + 3* \\
& a^3*c^3*d^5*e^3 + 3*a^4*c^2*d^3*e^5 + a^5*c*d*e^7)*f*g^2 + (a^2*c \\
& ^4*d^8 + 2*a^3*c^3*d^6*e^2 - 2*a^5*c*d^2*e^6 - a^6*e^8)*g^3)*\text{sqrt} \\
& (-((9*c^5*d^4*e^2 - 6*a*c^4*d^2*e^4 + a^2*c^3*e^6)*f^2 + 2*(3*c^5 \\
& *d^5*e - 10*a*c^4*d^3*e^3 + 3*a^2*c^3*d*e^5)*f*g + (c^5*d^6 - 6*a \\
& *c^4*d^4*e^2 + 9*a^2*c^3*d^2*e^4)*g^2)/((a*c^8*d^{12} + 6*a^2*c^7*d \\
& ^{10}*e^2 + 15*a^3*c^6*d^8*e^4 + 20*a^4*c^5*d^6*e^6 + 15*a^5*c^4*d^4 \\
& *e^8 + 6*a^6*c^3*d^2*e^{10} + a^7*c^2*e^{12})*f^4 + 2*(a^2*c^7*d^{12} \\
& + 6*a^3*c^6*d^{10}*e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5*c^4*d^6*e^6 + \\
& 15*a^6*c^3*d^4*e^8 + 6*a^7*c^2*d^2*e^{10} + a^8*c*e^{12})*f^2*g^2 + ( \\
& a^3*c^6*d^{12} + 6*a^4*c^5*d^{10}*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c \\
& ^3*d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^{10} + a^9*e^{12})*g^4 \\
& ))*\text{sqrt}(e*x + d)*\text{sqrt}(g*x + f)*\text{sqrt}(-((c^3*d^3 - 3*a*c^2*d*e^2) \\
& *f - (3*a*c^2*d^2*e - a^2*c*e^3)*g + ((a*c^4*d^6 + 3*a^2*c^3*d^4* \\
& e^2 + 3*a^3*c^2*d^2*e^4 + a^4*c*e^6)*f^2 + (a^2*c^3*d^6 + 3*a^3*c \\
& ^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6)*g^2)*\text{sqrt}(-((9*c^5*d^4*e^2 \\
& - 6*a*c^4*d^2*e^4 + a^2*c^3*e^6)*f^2 + 2*(3*c^5*d^5*e - 10*a*c^4 \\
& *d^3*e^3 + 3*a^2*c^3*d*e^5)*f*g + (c^5*d^6 - 6*a*c^4*d^4*e^2 + 9 \\
& *a^2*c^3*d^2*e^4)*g^2)/((a*c^8*d^{12} + 6*a^2*c^7*d^{10}*e^2 + 15*a^3 \\
& *c^6*d^8*e^4 + 20*a^4*c^5*d^6*e^6 + 15*a^5*c^4*d^4*e^8 + 6*a^6*c^3 \\
& *d^2*e^{10} + a^7*c^2*e^{12})*f^4 + 2*(a^2*c^7*d^{12} + 6*a^3*c^6*d^{10} \\
& *e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5*c^4*d^6*e^6 + 15*a^6*c^3*d^4*e \\
& ^8 + 6*a^7*c^2*d^2*e^{10} + a^8*c*e^{12})*f^2*g^2 + (a^3*c^6*d^{12} + 6 \\
& *a^4*c^5*d^{10}*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3*d^6*e^6 + 15* \\
& a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^{10} + a^9*e^{12})*g^4))/((a*c^4*d^6 \\
& + 3*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4 + a^4*c*e^6)*f^2 + (a^2* \\
& c^3*d^6 + 3*a^3*c^2*d^4*e^2 + 3*a^4*c*d^2*e^4 + a^5*e^6)*g^2)) + \\
& 2*((3*c^3*d^2*e^2 - a*c^2*e^4)*f*g + (c^3*d^3*e - 3*a*c^2*d*e^3)* \\
& g^2)*x + (2*(c^5*d^7 + 3*a*c^4*d^5*e^2 + 3*a^2*c^3*d^3*e^4 + a^3* \\
& c^2*d*e^6)*f^3 + 2*(a*c^4*d^7 + 3*a^2*c^3*d^5*e^2 + 3*a^3*c^2*d^3 \\
& *e^4 + a^4*c*d*e^6)*f*g^2 + ((c^5*d^6*e + 3*a*c^4*d^4*e^3 + 3*a^2 \\
& *c^3*d^2*e^5 + a^3*c^2*e^7)*f^3 + (c^5*d^7 + 3*a*c^4*d^5*e^2 + 3* \\
& a^2*c^3*d^3*e^4 + a^3*c^2*d*e^6)*f^2*g + (a*c^4*d^6*e + 3*a^2*c^3 \\
& *d^4*e^3 + 3*a^3*c^2*d^2*e^5 + a^4*c*e^7)*f*g^2 + (a*c^4*d^7 + 3* \\
& a^2*c^3*d^5*e^2 + 3*a^3*c^2*d^3*e^4 + a^4*c*d*e^6)*g^3)*x)*\text{sqrt}(- \\
& ((9*c^5*d^4*e^2 - 6*a*c^4*d^2*e^4 + a^2*c^3*e^6)*f^2 + 2*(3*c^5*d \\
& ^5*e - 10*a*c^4*d^3*e^3 + 3*a^2*c^3*d*e^5)*f*g + (c^5*d^6 - 6*a*c \\
& ^4*d^4*e^2 + 9*a^2*c^3*d^2*e^4)*g^2)/((a*c^8*d^{12} + 6*a^2*c^7*d^{10} \\
& *e^2 + 15*a^3*c^6*d^8*e^4 + 20*a^4*c^5*d^6*e^6 + 15*a^5*c^4*d^4* \\
& e^8 + 6*a^6*c^3*d^2*e^{10} + a^7*c^2*e^{12})*f^4 + 2*(a^2*c^7*d^{12} + \\
& 6*a^3*c^6*d^{10}*e^2 + 15*a^4*c^5*d^8*e^4 + 20*a^5*c^4*d^6*e^6 + 15 \\
& *a^6*c^3*d^4*e^8 + 6*a^7*c^2*d^2*e^{10} + a^8*c*e^{12})*f^2*g^2 + (a^3 \\
& *c^6*d^{12} + 6*a^4*c^5*d^{10}*e^2 + 15*a^5*c^4*d^8*e^4 + 20*a^6*c^3 \\
& *d^6*e^6 + 15*a^7*c^2*d^4*e^8 + 6*a^8*c*d^2*e^{10} + a^9*e^{12})*g^4) \\
& ))/x + ((c*d^3*e + a*d*e^3)*f - (c*d^4 + a*d^2*e^2)*g + ((c*d^2* \\
& e^2 + a*e^4)*f - (c*d^3*e + a*d*e^3)*g)*x)*\text{sqrt}(-((c^3*d^3 - 3*a*
\end{aligned}$$



$$\begin{aligned}
& 8e^{4} + 20a^{4}c^{5}d^{6}e^{6} + 15a^{5}c^{4}d^{4}e^{8} + 6a^{6}c^{3}d^{2}e^{10} + a^{7}c^{2}e^{12})f^{4} + 2(a^{2}c^{7}d^{12} + 6a^{3}c^{6}d^{10}e^{2} + 15a^{4}c^{5}d^{8}e^{4} + 20a^{5}c^{4}d^{6}e^{6} + 15a^{6}c^{3}d^{4}e^{8} + 6a^{7}c^{2}d^{2}e^{10} + a^{8}c^{1}e^{12})f^{2}g^{2} + (a^{3}c^{6}d^{12} + 6a^{4}c^{5}d^{10}e^{2} + 15a^{5}c^{4}d^{8}e^{4} + 20a^{6}c^{3}d^{6}e^{6} + 15a^{7}c^{2}d^{4}e^{8} + 6a^{8}c^{1}d^{2}e^{10} + a^{9}e^{12})g^{4}) / ((a^{2}c^{3}d^{4}e^{2} + 3a^{3}c^{2}d^{2}e^{4} + a^{4}c^{1}e^{6})f^{2} + (a^{2}c^{3}d^{6} + 3a^{3}c^{2}d^{4}e^{2} + 3a^{4}c^{1}d^{2}e^{4} + a^{5}e^{6})g^{2}) * \log(-((3c^{3}d^{2}e^{2} - a^{2}c^{1}e^{4})f^{2} + 4(c^{3}d^{3}e - a^{2}c^{1}d^{1}e^{3})f^{1}g + (c^{3}d^{4} - 3a^{2}c^{1}d^{2}e^{2})g^{2} - 2((3c^{4}d^{4}e - 4a^{3}c^{3}d^{2}e^{3} + a^{2}c^{2}e^{5})f^{2} + (c^{4}d^{5} - 10a^{3}c^{3}d^{3}e^{2} + 5a^{2}c^{2}d^{1}e^{4})f^{1}g - 2(a^{2}c^{3}d^{4}e - 3a^{1}c^{2}d^{2}e^{3})g^{2} + (2(a^{2}c^{5}d^{7}e + 3a^{1}c^{4}d^{5}e^{3} + 3a^{3}c^{3}d^{3}e^{5} + a^{4}c^{2}d^{1}e^{7})f^{3} + (a^{2}c^{5}d^{8} + 2a^{1}c^{4}d^{6}e^{2} - 2a^{3}c^{2}d^{2}e^{6} - a^{5}c^{1}e^{8})f^{2}g + 2(a^{2}c^{4}d^{7}e + 3a^{1}c^{3}d^{5}e^{3} + 3a^{3}c^{2}d^{3}e^{5} + a^{5}c^{1}d^{1}e^{7})f^{1}g^{2} + (a^{2}c^{4}d^{8} + 2a^{1}c^{3}d^{6}e^{2} - 2a^{5}c^{1}d^{2}e^{6} - a^{6}e^{8})g^{3}) * \sqrt{-((9c^{5}d^{4}e^{2} - 6a^{4}c^{3}d^{2}e^{4} + a^{2}c^{3}e^{6})f^{2} + 2(3c^{5}d^{5}e - 10a^{4}c^{3}d^{3}e^{3} + 3a^{2}c^{3}d^{1}e^{5})f^{1}g + (c^{5}d^{6} - 6a^{4}c^{3}d^{4}e^{2} + 9a^{2}c^{3}d^{2}e^{4})g^{2}) / ((a^{2}c^{8}d^{12} + 6a^{3}c^{7}d^{10}e^{2} + 15a^{4}c^{6}d^{8}e^{4} + 20a^{5}c^{5}d^{6}e^{6} + 15a^{6}c^{4}d^{4}e^{8} + 6a^{7}c^{3}d^{2}e^{10} + a^{8}c^{2}e^{12})f^{4} + 2(a^{2}c^{7}d^{12} + 6a^{3}c^{6}d^{10}e^{2} + 15a^{4}c^{5}d^{8}e^{4} + 20a^{5}c^{4}d^{6}e^{6} + 15a^{6}c^{3}d^{4}e^{8} + 6a^{7}c^{2}d^{2}e^{10} + a^{8}c^{1}e^{12})f^{2}g^{2} + (a^{3}c^{6}d^{12} + 6a^{4}c^{5}d^{10}e^{2} + 15a^{5}c^{4}d^{8}e^{4} + 20a^{6}c^{3}d^{6}e^{6} + 15a^{7}c^{2}d^{4}e^{8} + 6a^{8}c^{1}d^{2}e^{10} + a^{9}e^{12})g^{4})) * \sqrt{e^{2}x + d} * \sqrt{g^{2}x + f} * \sqrt{-((c^{3}d^{3}e - 3a^{2}c^{1}d^{1}e^{3})f - (3a^{3}c^{2}d^{2}e - a^{2}c^{1}e^{3})g - ((a^{2}c^{4}d^{6} + 3a^{1}c^{3}d^{4}e^{2} + 3a^{3}c^{2}d^{2}e^{4} + a^{4}c^{1}e^{6})f^{2} + (a^{2}c^{3}d^{6} + 3a^{1}c^{2}d^{4}e^{2} + 3a^{3}c^{1}d^{2}e^{4} + a^{5}e^{6})g^{2}) * \sqrt{-((9c^{5}d^{4}e^{2} - 6a^{4}c^{3}d^{2}e^{4} + a^{2}c^{3}e^{6})f^{2} + 2(3c^{5}d^{5}e - 10a^{4}c^{3}d^{3}e^{3} + 3a^{2}c^{3}d^{1}e^{5})f^{1}g + (c^{5}d^{6} - 6a^{4}c^{3}d^{4}e^{2} + 9a^{2}c^{3}d^{2}e^{4})g^{2}) / ((a^{2}c^{8}d^{12} + 6a^{3}c^{7}d^{10}e^{2} + 15a^{4}c^{6}d^{8}e^{4} + 20a^{5}c^{5}d^{6}e^{6} + 15a^{6}c^{4}d^{4}e^{8} + 6a^{7}c^{3}d^{2}e^{10} + a^{8}c^{2}e^{12})f^{4} + 2(a^{2}c^{7}d^{12} + 6a^{3}c^{6}d^{10}e^{2} + 15a^{4}c^{5}d^{8}e^{4} + 20a^{5}c^{4}d^{6}e^{6} + 15a^{6}c^{3}d^{4}e^{8} + 6a^{7}c^{2}d^{2}e^{10} + a^{8}c^{1}e^{12})f^{2}g^{2} + (a^{3}c^{6}d^{12} + 6a^{4}c^{5}d^{10}e^{2} + 15a^{5}c^{4}d^{8}e^{4} + 20a^{6}c^{3}d^{6}e^{6} + 15a^{7}c^{2}d^{4}e^{8} + 6a^{8}c^{1}d^{2}e^{10} + a^{9}e^{12})g^{4})) / ((a^{2}c^{4}d^{6} + 3a^{1}c^{3}d^{4}e^{2} + 3a^{3}c^{2}d^{2}e^{4} + a^{4}c^{1}e^{6})f^{2} + (a^{2}c^{3}d^{6} + 3a^{1}c^{2}d^{4}e^{2} + 3a^{3}c^{1}d^{2}e^{4} + a^{5}e^{6})g^{2})) + 2(((3c^{3}d^{2}e^{2} - a^{2}c^{1}e^{4})f^{1}g + (c^{3}d^{3}e - 3a^{2}c^{1}d^{1}e^{3})g^{2}) * x - (2(c^{5}d^{7} + 3a^{4}c^{3}d^{5}e^{2} + 3a^{2}c^{3}d^{3}e^{4} + a^{3}c^{2}d^{1}e^{6})f^{3} + 2(a^{2}c^{4}d^{7} + 3a^{1}c^{3}d^{5}e^{2} + 3a^{3}c^{2}d^{3}e^{4} + a^{4}c^{1}d^{1}e^{6})f^{2}g^{2} + ((c^{5}d^{6}e + 3a^{4}c^{3}d^{4}e^{3} + 3a^{2}c^{3}d^{2}e^{5} + a^{3}c^{2}d^{1}e^{7})f^{3} + (c^{5}d^{7} + 3a^{4}c^{3}d^{5}e^{2} + 3a^{2}c^{3}d^{3}e^{4} + a^{3}c^{2}d^{1}e^{6})f^{2}g + (a^{2}c^{4}d^{6}e + 3a^{1}c^{3}d^{4}e^{2} + 3a^{3}c^{2}d^{2}e^{4} + a^{5}e^{6})f^{1}g^{2} + (a^{2}c^{4}d^{7} + 3a^{1}c^{3}d^{5}e^{2} + 3a^{3}c^{2}d^{3}e^{4} + a^{4}c^{1}d^{1}e^{6})g^{3}) * x) * \sqrt{-((9c^{5}d^{4}e^{2} - 6a^{4}c^{3}d^{2}e^{4} + a^{2}c^{3}e^{6})f^{2} + 2(3c^{5}d^{5}e - 10a^{4}c^{3}d^{3}e^{3} + 3a^{2}c^{3}d^{1}e^{5})f^{1}g + (c^{5}d^{6} - 6a^{4}c^{3}d^{4}e^{2} + 9a^{2}c^{3}d^{2}e^{4})g^{2}) / ((a^{2}c^{8}d^{12} + 6a^{3}c^{7}d^{10}e^{2} + 15a^{4}c^{6}d^{8}e^{4} + 20a^{5}c^{5}d^{6}e^{6} + 15a^{6}c^{4}d^{4}e^{8} + 6a^{7}c^{3}d^{2}e^{10} + a^{8}c^{2}e^{12})f^{4} + 2(a^{2}c^{7}d^{12} + 6a^{3}c^{6}d^{10}e^{2} + 15a^{4}c^{5}d^{8}e^{4} + 20a^{5}c^{4}d^{6}e^{6} + 15a^{6}c^{3}d^{4}e^{8} + 6a^{7}c^{2}d^{2}e^{10} + a^{8}c^{1}e^{12})f^{2}g^{2} + (a^{3}c^{6}d^{12} + 6a^{4}c^{5}d^{10}e^{2} + 15a^{5}c^{4}d^{8}e^{4} + 20a^{6}c^{3}d^{6}e^{6} + 15a^{7}c^{2}d^{4}e^{8} + 6a^{8}c^{1}d^{2}e^{10} + a^{9}e^{12})g^{4})) / x) / ((c^{3}d^{3}e + a^{2}d^{1}e^{3})f - (c^{3}d^{4} + a^{2}d^{2}e^{2})g + ((c^{2}d^{2}e^{2} + a^{1}e^{4})f - (c^{2}d^{3}e + a^{1}d^{1}e^{3})g) * x)
\end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + cx^2)(d + ex)^{\frac{3}{2}} \sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(3/2)/(c*x**2+a)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(1/((a + c*x**2)*(d + e*x)**(3/2)*sqrt(f + g*x)), x)
```

---

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^2 + a)*(e*x + d)^(3/2)*sqrt(g*x + f)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.614 \quad \int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}(a+cx^2)} dx$$

**Optimal.** Leaf size=625

$$\begin{aligned} & \frac{2\sqrt{d+ex}(ef-dg)}{\sqrt{f+gx}(ag^2+cf^2)} - \frac{2\sqrt{e}(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{g}(ag^2+cf^2)} \\ & - \frac{\sqrt{e}(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{g}(ag^2+cf^2)} \\ & + \frac{\sqrt{e}(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{g}(ag^2+cf^2)} \\ & + \frac{\sqrt{\sqrt{cd}-\sqrt{-ae}}(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{cd}-\sqrt{-ag}}}{\sqrt{f+gx}\sqrt{\sqrt{cd}-\sqrt{-ae}}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{cd}-\sqrt{-ag}}(ag^2+cf^2)} \\ & - \frac{\sqrt{\sqrt{-ae}+\sqrt{cd}}(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-ag}+\sqrt{cd}}}{\sqrt{f+gx}\sqrt{\sqrt{-ae}+\sqrt{cd}}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{-ag}+\sqrt{cd}}(ag^2+cf^2)} \end{aligned}$$

[Out] (2\*(e\*f - d\*g)\*Sqrt[d + e\*x])/((c\*f^2 + a\*g^2)\*Sqrt[f + g\*x]) - (2\*Sqrt[e]\*(e\*f - d\*g)\*ArcTanh[(Sqrt[g]\*Sqrt[d + e\*x])/(Sqrt[e]\*Sqrt[f + g\*x])])/(Sqrt[g]\*(c\*f^2 + a\*g^2)) - (Sqrt[e]\*(c\*d\*f + a\*e\*g - Sqrt[-a]\*Sqrt[c]\*(e\*f - d\*g))\*ArcTanh[(Sqrt[g]\*Sqrt[d + e\*x])/(Sqrt[e]\*Sqrt[f + g\*x])])/(Sqrt[-a]\*Sqrt[c]\*Sqrt[g]\*(c\*f^2 + a\*g^2)) + (Sqrt[e]\*(c\*d\*f + a\*e\*g + Sqrt[-a]\*Sqrt[c]\*(e\*f - d\*g))\*ArcTanh[(Sqrt[g]\*Sqrt[d + e\*x])/(Sqrt[e]\*Sqrt[f + g\*x])])/(Sqrt[-a]\*Sqrt[c]\*Sqrt[g]\*(c\*f^2 + a\*g^2)) + (Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*(c\*d\*f + a\*e\*g - Sqrt[-a]\*Sqrt[c]\*(e\*f - d\*g))\*ArcTanh[(Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*Sqrt[f + g\*x])])/(Sqrt[-a]\*Sqrt[c]\*Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]\*(c\*f^2 + a\*g^2)) - (Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*(c\*d\*f + a\*e\*g + Sqrt[-a]\*Sqrt[c]\*(e\*f - d\*g))\*ArcTanh[(Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*Sqrt[f + g\*x])])/(Sqrt[-a]\*Sqrt[c]\*Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g]\*(c\*f^2 + a\*g^2))

**Rubi [A]** time = 4.86496, antiderivative size = 625, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\begin{aligned} & \frac{2\sqrt{d+ex}(ef-dg)}{\sqrt{f+gx}(ag^2+cf^2)} - \frac{2\sqrt{e}(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{g}(ag^2+cf^2)} \\ & - \frac{\sqrt{e}(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{g}(ag^2+cf^2)} \\ & + \frac{\sqrt{e}(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{g}(ag^2+cf^2)} \\ & + \frac{\sqrt{\sqrt{cd}-\sqrt{-ae}}(-\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{cd}-\sqrt{-ag}}}{\sqrt{f+gx}\sqrt{\sqrt{cd}-\sqrt{-ae}}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{cd}-\sqrt{-ag}}(ag^2+cf^2)} \\ & - \frac{\sqrt{\sqrt{-ae}+\sqrt{cd}}(\sqrt{-a}\sqrt{c}(ef-dg)+aeg+cdf)\tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{\sqrt{-ag}+\sqrt{cd}}}{\sqrt{f+gx}\sqrt{\sqrt{-ae}+\sqrt{cd}}}\right)}{\sqrt{-a}\sqrt{c}\sqrt{\sqrt{-ag}+\sqrt{cd}}(ag^2+cf^2)} \end{aligned}$$

Antiderivative was successfully verified.



[In] `int((e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a),x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+a)(gx+f)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^(3/2)/((c*x^2+a)*(g*x+f)^(3/2)),x,algorithm="maxima")`

[Out] `integrate((e*x+d)^(3/2)/((c*x^2+a)*(g*x+f)^(3/2)),x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^(3/2)/((c*x^2+a)*(g*x+f)^(3/2)),x,algorithm="fricas")`

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**(3/2)/(g*x+f)**(3/2)/(c*x**2+a),x)`

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^(3/2)/((c*x^2+a)*(g*x+f)^(3/2)),x,algorithm="giac")`

[Out] Timed out



$$3.615 \quad \int \frac{\sqrt{d+ex}}{(f+gx)^{3/2}(a+cx^2)} dx$$

**Optimal.** Leaf size=351

$$\frac{2g\sqrt{d+ex}}{\sqrt{f+gx}(ag^2+cf^2)} + \frac{(-\sqrt{-a}\sqrt{c}(ef-dg) + aeg + cdf) \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{cf}-\sqrt{-ag}}{\sqrt{f+gx}\sqrt{cd}-\sqrt{-ae}}\right)}{\sqrt{-a}\sqrt{cd} - \sqrt{-ae}\sqrt{cf} - \sqrt{-ag}(ag^2+cf^2)}$$

$$\frac{(\sqrt{-a}\sqrt{c}(ef-dg) + aeg + cdf) \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-ag+\sqrt{cf}}}{\sqrt{f+gx}\sqrt{-ae+\sqrt{cd}}}\right)}{\sqrt{-a}\sqrt{-ae} + \sqrt{cd}\sqrt{\sqrt{-ag} + \sqrt{cf}}(ag^2+cf^2)}$$

[Out]  $(-2*g*\text{Sqrt}[d + e*x])/((c*f^2 + a*g^2)*\text{Sqrt}[f + g*x]) + ((c*d*f + a*e*g - \text{Sqrt}[-a]*\text{Sqrt}[c]*(e*f - d*g))*\text{ArcTanh}[(\text{Sqrt}[\text{Sqrt}[c]*f - \text{Sqrt}[-a]*g]*\text{Sqrt}[d + e*x])/(\text{Sqrt}[\text{Sqrt}[c]*d - \text{Sqrt}[-a]*e]*\text{Sqrt}[f + g*x])])/(\text{Sqrt}[-a]*\text{Sqrt}[\text{Sqrt}[c]*d - \text{Sqrt}[-a]*e]*\text{Sqrt}[\text{Sqrt}[c]*f - \text{Sqrt}[-a]*g]*(c*f^2 + a*g^2)) - ((c*d*f + a*e*g + \text{Sqrt}[-a]*\text{Sqrt}[c]*(e*f - d*g))*\text{ArcTanh}[(\text{Sqrt}[\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g]*\text{Sqrt}[d + e*x])/(\text{Sqrt}[\text{Sqrt}[c]*d + \text{Sqrt}[-a]*e]*\text{Sqrt}[f + g*x])])/(\text{Sqrt}[-a]*\text{Sqrt}[\text{Sqrt}[c]*d + \text{Sqrt}[-a]*e]*\text{Sqrt}[\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g]*(c*f^2 + a*g^2)))$

**Rubi [A]** time = 3.75604, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$

$$\frac{2g\sqrt{d+ex}}{\sqrt{f+gx}(ag^2+cf^2)} + \frac{(-\sqrt{-a}\sqrt{c}(ef-dg) + aeg + cdf) \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{cf}-\sqrt{-ag}}{\sqrt{f+gx}\sqrt{cd}-\sqrt{-ae}}\right)}{\sqrt{-a}\sqrt{cd} - \sqrt{-ae}\sqrt{cf} - \sqrt{-ag}(ag^2+cf^2)}$$

$$\frac{(\sqrt{-a}\sqrt{c}(ef-dg) + aeg + cdf) \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-ag+\sqrt{cf}}}{\sqrt{f+gx}\sqrt{-ae+\sqrt{cd}}}\right)}{\sqrt{-a}\sqrt{-ae} + \sqrt{cd}\sqrt{\sqrt{-ag} + \sqrt{cf}}(ag^2+cf^2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[d + e*x]/((f + g*x)^{(3/2)}*(a + c*x^2)), x]$

[Out]  $(-2*g*\text{Sqrt}[d + e*x])/((c*f^2 + a*g^2)*\text{Sqrt}[f + g*x]) + ((c*d*f + a*e*g - \text{Sqrt}[-a]*\text{Sqrt}[c]*(e*f - d*g))*\text{ArcTanh}[(\text{Sqrt}[\text{Sqrt}[c]*f - \text{Sqrt}[-a]*g]*\text{Sqrt}[d + e*x])/(\text{Sqrt}[\text{Sqrt}[c]*d - \text{Sqrt}[-a]*e]*\text{Sqrt}[f + g*x])])/(\text{Sqrt}[-a]*\text{Sqrt}[\text{Sqrt}[c]*d - \text{Sqrt}[-a]*e]*\text{Sqrt}[\text{Sqrt}[c]*f - \text{Sqrt}[-a]*g]*(c*f^2 + a*g^2)) - ((c*d*f + a*e*g + \text{Sqrt}[-a]*\text{Sqrt}[c]*(e*f - d*g))*\text{ArcTanh}[(\text{Sqrt}[\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g]*\text{Sqrt}[d + e*x])/(\text{Sqrt}[\text{Sqrt}[c]*d + \text{Sqrt}[-a]*e]*\text{Sqrt}[f + g*x])])/(\text{Sqrt}[-a]*\text{Sqrt}[\text{Sqrt}[c]*d + \text{Sqrt}[-a]*e]*\text{Sqrt}[\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g]*(c*f^2 + a*g^2)))$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)**(1/2)/(g*x+f)**(3/2)/(c*x**2+a), x)$

[Out] Timed out

**Mathematica [C]** time = 4.34368, size = 531, normalized size = 1.51

$$\frac{\sqrt{\sqrt{cd+i\sqrt{ae}}(\sqrt{ag+i\sqrt{cf}})\log\left(\frac{i\sqrt{a}\sqrt{\sqrt{cf}+i\sqrt{ag}}(2\sqrt{d+ex}\sqrt{f+gx}\sqrt{\sqrt{cd+i\sqrt{ae}}\sqrt{\sqrt{cf}+i\sqrt{ag}}+i\sqrt{a}(dg+ef+2egx)+\sqrt{c}(2df+dgx+efx))}}{(\sqrt{ex-i\sqrt{a}})(\sqrt{cd+i\sqrt{ae}})^{3/2}}\right)}}{\sqrt{a}\sqrt{\sqrt{cf}+i\sqrt{ag}}} + \frac{\sqrt{\sqrt{cd-i\sqrt{ae}}(\sqrt{ag-i\sqrt{cf}})\log\left(-\frac{i\sqrt{a}\sqrt{\sqrt{cf}-i\sqrt{ag}}(2\sqrt{d+ex}\sqrt{f+gx}\sqrt{\sqrt{cd-i\sqrt{ae}}\sqrt{\sqrt{cf}-i\sqrt{ag}}+i\sqrt{a}(dg+ef+2egx)+\sqrt{c}(2df+dgx+efx))}}{(\sqrt{ex+i\sqrt{a}})(\sqrt{cd-i\sqrt{ae}})^{3/2}}\right)}}{\sqrt{a}\sqrt{\sqrt{cf}-i\sqrt{ag}}}}{2(ag^2 + cf^2)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e\*x]/((f + g\*x)^(3/2)\*(a + c\*x^2)),x]

[Out] ((-4\*g\*Sqrt[d + e\*x])/Sqrt[f + g\*x] + (Sqrt[Sqrt[c]\*d + I\*Sqrt[a]\*e]\*(I\*Sqrt[c]\*f + Sqrt[a]\*g)\*Log[(I\*Sqrt[a]\*Sqrt[Sqrt[c]\*f + I\*Sqrt[a]\*g]\*(2\*Sqrt[Sqrt[c]\*d + I\*Sqrt[a]\*e]\*Sqrt[Sqrt[c]\*f + I\*Sqrt[a]\*g]\*Sqrt[d + e\*x]\*Sqrt[f + g\*x] + Sqrt[c]\*(2\*d\*f + e\*f\*x + d\*g\*x) + I\*Sqrt[a]\*(e\*f + d\*g + 2\*e\*g\*x)))/((Sqrt[c]\*d + I\*Sqrt[a]\*e)^(3/2)\*((-I)\*Sqrt[a] + Sqrt[c]\*x)))/(Sqrt[a]\*Sqrt[Sqrt[c]\*f + I\*Sqrt[a]\*g]) + (Sqrt[Sqrt[c]\*d - I\*Sqrt[a]\*e]\*((-I)\*Sqrt[c]\*f + Sqrt[a]\*g)\*Log[((-I)\*Sqrt[a]\*Sqrt[Sqrt[c]\*f - I\*Sqrt[a]\*g]\*(2\*Sqrt[Sqrt[c]\*d - I\*Sqrt[a]\*e]\*Sqrt[Sqrt[c]\*f - I\*Sqrt[a]\*g]\*Sqrt[d + e\*x]\*Sqrt[f + g\*x] + Sqrt[c]\*(2\*d\*f + e\*f\*x + d\*g\*x) - I\*Sqrt[a]\*(d\*g + e\*(f + 2\*g\*x)))/((Sqrt[c]\*d - I\*Sqrt[a]\*e)^(3/2)\*(I\*Sqrt[a] + Sqrt[c]\*x)))/(Sqrt[a]\*Sqrt[Sqrt[c]\*f - I\*Sqrt[a]\*g])/(2\*(c\*f^2 + a\*g^2))

**Maple [B]** time = 0.07, size = 5383, normalized size = 15.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(1/2)/(g\*x+f)^(3/2)/(c\*x^2+a),x)

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{(cx^2+a)(gx+f)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/((c\*x^2 + a)\*(g\*x + f)^(3/2)),x, algorithm="maxima")

[Out] integrate(sqrt(e\*x + d)/((c\*x^2 + a)\*(g\*x + f)^(3/2)), x)

**Fricas [A]** time = 74.9854, size = 7889, normalized size = 22.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/((c\*x^2 + a)\*(g\*x + f)^(3/2)),x, algorithm="fricas")

[Out] -1/4\*((c\*f^3 + a\*f\*g^2 + (c\*f^2\*g + a\*g^3)\*x)\*sqrt(-(c^2\*d\*f^3 + 3\*a\*c\*e\*f^2\*g - 3\*a\*c\*d\*f\*g^2 - a^2\*e\*g^3 + (a\*c^3\*f^6 + 3\*a^2\*c^

$$\begin{aligned}
& 2*f^4*g^2 + 3*a^3*c*f^2*g^4 + a^4*g^6)*\sqrt{-(c^3*e^2*f^6 - 6*c^3 \\
& *d*e*f^5*g + 20*a*c^2*d*e*f^3*g^3 - 6*a^2*c*d*e*f*g^5 + a^2*c*d^2 \\
& *g^6 + 3*(3*c^3*d^2 - 2*a*c^2*e^2)*f^4*g^2 - 3*(2*a*c^2*d^2 - 3*a \\
& ^2*c*e^2)*f^2*g^4)/(a*c^6*f^12 + 6*a^2*c^5*f^10*g^2 + 15*a^3*c^4* \\
& f^8*g^4 + 20*a^4*c^3*f^6*g^6 + 15*a^5*c^2*f^4*g^8 + 6*a^6*c*f^2*g \\
& ^10 + a^7*g^12)))/(a*c^3*f^6 + 3*a^2*c^2*f^4*g^2 + 3*a^3*c*f^2*g^4 \\
& + a^4*g^6))*\log((c*e^2*f^4 - 2*c*d*e*f^3*g - 2*a*d*e*f*g^3 + a* \\
& d^2*g^4 - 3*(c*d^2 + a*e^2)*f^2*g^2 + 2*(c^2*e*f^5 - 3*c^2*d*f^4* \\
& g - 4*a*c*e*f^3*g^2 + 4*a*c*d*f^2*g^3 + 3*a^2*e*f*g^4 - a^2*d*g^5 \\
& + 2*(a*c^3*f^7*g + 3*a^2*c^2*f^5*g^3 + 3*a^3*c*f^3*g^5 + a^4*f*g \\
& ^7)*\sqrt{-(c^3*e^2*f^6 - 6*c^3*d*e*f^5*g + 20*a*c^2*d*e*f^3*g^3 - \\
& 6*a^2*c*d*e*f*g^5 + a^2*c*d^2*g^6 + 3*(3*c^3*d^2 - 2*a*c^2*e^2)* \\
& f^4*g^2 - 3*(2*a*c^2*d^2 - 3*a^2*c*e^2)*f^2*g^4)/(a*c^6*f^12 + 6* \\
& a^2*c^5*f^10*g^2 + 15*a^3*c^4*f^8*g^4 + 20*a^4*c^3*f^6*g^6 + 15*a \\
& ^5*c^2*f^4*g^8 + 6*a^6*c*f^2*g^10 + a^7*g^12)))*\sqrt{e*x + d)*\sqrt{ \\
& t(g*x + f)*\sqrt{-(c^2*d*f^3 + 3*a*c*e*f^2*g - 3*a*c*d*f*g^2 - a^2 \\
& *e*g^3 + (a*c^3*f^6 + 3*a^2*c^2*f^4*g^2 + 3*a^3*c*f^2*g^4 + a^4*g \\
& ^6)*\sqrt{-(c^3*e^2*f^6 - 6*c^3*d*e*f^5*g + 20*a*c^2*d*e*f^3*g^3 - \\
& 6*a^2*c*d*e*f*g^5 + a^2*c*d^2*g^6 + 3*(3*c^3*d^2 - 2*a*c^2*e^2)* \\
& f^4*g^2 - 3*(2*a*c^2*d^2 - 3*a^2*c*e^2)*f^2*g^4)/(a*c^6*f^12 + 6* \\
& a^2*c^5*f^10*g^2 + 15*a^3*c^4*f^8*g^4 + 20*a^4*c^3*f^6*g^6 + 15*a \\
& ^5*c^2*f^4*g^8 + 6*a^6*c*f^2*g^10 + a^7*g^12)))/(a*c^3*f^6 + 3*a^2 \\
& *c^2*f^4*g^2 + 3*a^3*c*f^2*g^4 + a^4*g^6)) + 2*(c*e^2*f^3*g - 3* \\
& c*d*e*f^2*g^2 - 3*a*e^2*f*g^3 + a*d*e*g^4)*x - (2*c^3*d*f^7 + 6*a \\
& *c^2*d*f^5*g^2 + 6*a^2*c*d*f^3*g^4 + 2*a^3*d*f*g^6 + (c^3*e*f^7 + \\
& c^3*d*f^6*g + 3*a*c^2*e*f^5*g^2 + 3*a*c^2*d*f^4*g^3 + 3*a^2*c*e* \\
& f^3*g^4 + 3*a^2*c*d*f^2*g^5 + a^3*e*f*g^6 + a^3*d*g^7)*x)*\sqrt{-( \\
& c^3*e^2*f^6 - 6*c^3*d*e*f^5*g + 20*a*c^2*d*e*f^3*g^3 - 6*a^2*c*d* \\
& e*f*g^5 + a^2*c*d^2*g^6 + 3*(3*c^3*d^2 - 2*a*c^2*e^2)*f^4*g^2 - 3 \\
& *(2*a*c^2*d^2 - 3*a^2*c*e^2)*f^2*g^4)/(a*c^6*f^12 + 6*a^2*c^5*f^1 \\
& 0*g^2 + 15*a^3*c^4*f^8*g^4 + 20*a^4*c^3*f^6*g^6 + 15*a^5*c^2*f^4* \\
& g^8 + 6*a^6*c*f^2*g^10 + a^7*g^12)))/x) - (c*f^3 + a*f*g^2 + (c*f \\
& ^2*g + a*g^3)*x)*\sqrt{-(c^2*d*f^3 + 3*a*c*e*f^2*g - 3*a*c*d*f*g^2 \\
& - a^2*e*g^3 + (a*c^3*f^6 + 3*a^2*c^2*f^4*g^2 + 3*a^3*c*f^2*g^4 + \\
& a^4*g^6)*\sqrt{-(c^3*e^2*f^6 - 6*c^3*d*e*f^5*g + 20*a*c^2*d*e*f^3* \\
& *g^3 - 6*a^2*c*d*e*f*g^5 + a^2*c*d^2*g^6 + 3*(3*c^3*d^2 - 2*a*c^2 \\
& *e^2)*f^4*g^2 - 3*(2*a*c^2*d^2 - 3*a^2*c*e^2)*f^2*g^4)/(a*c^6*f^1 \\
& 2 + 6*a^2*c^5*f^10*g^2 + 15*a^3*c^4*f^8*g^4 + 20*a^4*c^3*f^6*g^6 \\
& + 15*a^5*c^2*f^4*g^8 + 6*a^6*c*f^2*g^10 + a^7*g^12)))/(a*c^3*f^6 \\
& + 3*a^2*c^2*f^4*g^2 + 3*a^3*c*f^2*g^4 + a^4*g^6))*\log((c*e^2*f^4 \\
& - 2*c*d*e*f^3*g - 2*a*d*e*f*g^3 + a*d^2*g^4 - 3*(c*d^2 + a*e^2)*f \\
& ^2*g^2 - 2*(c^2*e*f^5 - 3*c^2*d*f^4*g - 4*a*c*e*f^3*g^2 + 4*a*c*d \\
& *f^2*g^3 + 3*a^2*e*f*g^4 - a^2*d*g^5 + 2*(a*c^3*f^7*g + 3*a^2*c^2 \\
& *f^5*g^3 + 3*a^3*c*f^3*g^5 + a^4*f*g^7)*\sqrt{-(c^3*e^2*f^6 - 6*c^3 \\
& *d*e*f^5*g + 20*a*c^2*d*e*f^3*g^3 - 6*a^2*c*d*e*f*g^5 + a^2*c*d^2 \\
& *g^6 + 3*(3*c^3*d^2 - 2*a*c^2*e^2)*f^4*g^2 - 3*(2*a*c^2*d^2 - 3* \\
& a^2*c*e^2)*f^2*g^4)/(a*c^6*f^12 + 6*a^2*c^5*f^10*g^2 + 15*a^3*c^4 \\
& *f^8*g^4 + 20*a^4*c^3*f^6*g^6 + 15*a^5*c^2*f^4*g^8 + 6*a^6*c*f^2* \\
& g^10 + a^7*g^12)))*\sqrt{e*x + d)*\sqrt{g*x + f)*\sqrt{-(c^2*d*f^3 + \\
& 3*a*c*e*f^2*g - 3*a*c*d*f*g^2 - a^2*e*g^3 + (a*c^3*f^6 + 3*a^2*c \\
& ^2*f^4*g^2 + 3*a^3*c*f^2*g^4 + a^4*g^6)*\sqrt{-(c^3*e^2*f^6 - 6*c^3 \\
& *d*e*f^5*g + 20*a*c^2*d*e*f^3*g^3 - 6*a^2*c*d*e*f*g^5 + a^2*c*d^2 \\
& *g^6 + 3*(3*c^3*d^2 - 2*a*c^2*e^2)*f^4*g^2 - 3*(2*a*c^2*d^2 - 3* \\
& a^2*c*e^2)*f^2*g^4)/(a*c^6*f^12 + 6*a^2*c^5*f^10*g^2 + 15*a^3*c^4 \\
& *f^8*g^4 + 20*a^4*c^3*f^6*g^6 + 15*a^5*c^2*f^4*g^8 + 6*a^6*c*f^2* \\
& g^10 + a^7*g^12)))/(a*c^3*f^6 + 3*a^2*c^2*f^4*g^2 + 3*a^3*c*f^2*g \\
& ^4 + a^4*g^6)) + 2*(c*e^2*f^3*g - 3*c*d*e*f^2*g^2 - 3*a*e^2*f*g^3 \\
& + a*d*e*g^4)*x - (2*c^3*d*f^7 + 6*a*c^2*d*f^5*g^2 + 6*a^2*c*d*f^ \\
& 3*g^4 + 2*a^3*d*f*g^6 + (c^3*e*f^7 + c^3*d*f^6*g + 3*a*c^2*e*f^5* \\
& g^2 + 3*a*c^2*d*f^4*g^3 + 3*a^2*c*e*f^3*g^4 + 3*a^2*c*d*f^2*g^5 + \\
& a^3*e*f*g^6 + a^3*d*g^7)*x)*\sqrt{-(c^3*e^2*f^6 - 6*c^3*d*e*f^5*g \\
& + 20*a*c^2*d*e*f^3*g^3 - 6*a^2*c*d*e*f*g^5 + a^2*c*d^2*g^6 + 3*( \\
& 3*c^3*d^2 - 2*a*c^2*e^2)*f^4*g^2 - 3*(2*a*c^2*d^2 - 3*a^2*c*e^2)* \\
& f^2*g^4)/(a*c^6*f^12 + 6*a^2*c^5*f^10*g^2 + 15*a^3*c^4*f^8*g^4 + \\
& 20*a^4*c^3*f^6*g^6 + 15*a^5*c^2*f^4*g^8 + 6*a^6*c*f^2*g^10 + a^7* \\
& g^12)))/x) + (c*f^3 + a*f*g^2 + (c*f^2*g + a*g^3)*x)*\sqrt{-(c^2*d \\
& *f^3 + 3*a*c*e*f^2*g - 3*a*c*d*f*g^2 - a^2*e*g^3 - (a*c^3*f^6 + 3 \\
& *a^2*c^2*f^4*g^2 + 3*a^3*c*f^2*g^4 + a^4*g^6)*\sqrt{-(c^3*e^2*f^6 \\
& - 6*c^3*d*e*f^5*g + 20*a*c^2*d*e*f^3*g^3 - 6*a^2*c*d*e*f*g^5 + a^2 \\
& *c*d^2*g^6 + 3*(3*c^3*d^2 - 2*a*c^2*e^2)*f^4*g^2 - 3*(2*a*c^2*d^2 \\
& - 3*a^2*c*e^2)*f^2*g^4)/(a*c^6*f^12 + 6*a^2*c^5*f^10*g^2 + 15*a \\
& ^3*c^4*f^8*g^4 + 20*a^4*c^3*f^6*g^6 + 15*a^5*c^2*f^4*g^8 + 6*a^6*
\end{aligned}$$

$$\begin{aligned}
& c^2 f^2 g^{10} + a^7 g^{12}))) / (a^3 c^3 f^6 + 3 a^2 c^2 f^4 g^2 + 3 a^3 c \\
& f^2 g^4 + a^4 g^6)) * \log((c^2 e^2 f^4 - 2 c^2 d^2 e^2 f^3 g - 2 a^2 d^2 e^2 f^3 g \\
& ^3 + a^2 d^2 g^4 - 3 (c^2 d^2 + a^2 e^2) f^2 g^2 + 2 (c^2 e^2 f^5 - 3 c^2 \\
& d^2 f^4 g - 4 a^2 c^2 e^2 f^3 g^2 + 4 a^2 c^2 d^2 f^2 g^3 + 3 a^2 e^2 f^3 g^4 - a^2 \\
& d^2 g^5 - 2 (a^2 c^3 f^7 g + 3 a^2 c^2 f^5 g^3 + 3 a^3 c^2 f^3 g^5 + \\
& a^4 f^2 g^7) * \sqrt{-(c^3 e^2 f^6 - 6 c^3 d^2 e^2 f^5 g + 20 a^2 c^2 d^2 e^2 f^4 \\
& g^3 - 6 a^2 c^2 d^2 e^2 f^5 g + a^2 c^2 d^2 g^6 + 3 (3 c^3 d^2 - 2 a^2 c^2 \\
& e^2) f^4 g^2 - 3 (2 a^2 c^2 d^2 - 3 a^2 c^2 e^2) f^2 g^4) / (a^2 c^6 f^{12} + 6 a^2 c^5 f^{10} g^2 + 15 a^3 c^4 f^8 g^4 + 20 a^4 c^3 f^6 g^6 \\
& + 15 a^5 c^2 f^4 g^8 + 6 a^6 c^2 f^2 g^{10} + a^7 g^{12})) * \sqrt{e^2 x + \\
& d} * \sqrt{g^2 x + f} * \sqrt{-(c^2 d^2 f^3 + 3 a^2 c^2 e^2 f^2 g - 3 a^2 c^2 d^2 f^2 g^2 - a^2 e^2 g^3 - (a^2 c^3 f^6 + 3 a^2 c^2 f^4 g^2 + 3 a^3 c^2 f^2 g^4 \\
& + a^4 g^6) * \sqrt{-(c^3 e^2 f^6 - 6 c^3 d^2 e^2 f^5 g + 20 a^2 c^2 d^2 e^2 f^4 g^3 - 6 a^2 c^2 d^2 e^2 f^5 g + a^2 c^2 d^2 g^6 + 3 (3 c^3 d^2 - 2 a^2 c^2 e^2) f^4 g^2 - 3 (2 a^2 c^2 d^2 - 3 a^2 c^2 e^2) f^2 g^4) / (a^2 c^6 f^{12} + 6 a^2 c^5 f^{10} g^2 + 15 a^3 c^4 f^8 g^4 + 20 a^4 c^3 f^6 g^6 + 15 a^5 c^2 f^4 g^8 + 6 a^6 c^2 f^2 g^{10} + a^7 g^{12})) / (a^2 c^3 f^6 + 3 a^2 c^2 f^4 g^2 + 3 a^3 c^2 f^2 g^4 + a^4 g^6)) + 2 (c^2 e^2 f^3 g - 3 c^2 d^2 e^2 f^2 g^2 - 3 a^2 e^2 f^2 g^3 + a^2 d^2 e^2 g^4) * x + (2 c^3 d^2 f^7 + 6 a^2 c^2 d^2 f^5 g^2 + 6 a^2 c^2 d^2 f^3 g^4 + 2 a^3 d^2 f^2 g^6 + (c^3 e^2 f^7 + c^3 d^2 f^6 g + 3 a^2 c^2 e^2 f^5 g^2 + 3 a^2 c^2 d^2 f^4 g^3 + 3 a^2 c^2 e^2 f^3 g^4 + 3 a^2 c^2 d^2 f^2 g^5 + a^3 e^2 f^2 g^6 + a^3 d^2 g^7) * x) * \sqrt{-(c^3 e^2 f^6 - 6 c^3 d^2 e^2 f^5 g + 20 a^2 c^2 d^2 e^2 f^4 g^3 - 6 a^2 c^2 d^2 e^2 f^5 g + a^2 c^2 d^2 g^6 + 3 (3 c^3 d^2 - 2 a^2 c^2 e^2) f^4 g^2 - 3 (2 a^2 c^2 d^2 - 3 a^2 c^2 e^2) f^2 g^4) / (a^2 c^6 f^{12} + 6 a^2 c^5 f^{10} g^2 + 15 a^3 c^4 f^8 g^4 + 20 a^4 c^3 f^6 g^6 + 15 a^5 c^2 f^4 g^8 + 6 a^6 c^2 f^2 g^{10} + a^7 g^{12})) / x) - (c^2 f^3 + a^2 f^2 g^2 + (c^2 f^2 g + a^2 g^3) * x) * \sqrt{-(c^2 d^2 f^3 + 3 a^2 c^2 e^2 f^2 g - 3 a^2 c^2 d^2 f^2 g^2 - a^2 e^2 g^3 - (a^2 c^3 f^6 + 3 a^2 c^2 f^4 g^2 + 3 a^3 c^2 f^2 g^4 + a^4 g^6) * \sqrt{-(c^3 e^2 f^6 - 6 c^3 d^2 e^2 f^5 g + 20 a^2 c^2 d^2 e^2 f^4 g^3 - 6 a^2 c^2 d^2 e^2 f^5 g + a^2 c^2 d^2 g^6 + 3 (3 c^3 d^2 - 2 a^2 c^2 e^2) f^4 g^2 - 3 (2 a^2 c^2 d^2 - 3 a^2 c^2 e^2) f^2 g^4) / (a^2 c^6 f^{12} + 6 a^2 c^5 f^{10} g^2 + 15 a^3 c^4 f^8 g^4 + 20 a^4 c^3 f^6 g^6 + 15 a^5 c^2 f^4 g^8 + 6 a^6 c^2 f^2 g^{10} + a^7 g^{12})) / (a^2 c^3 f^6 + 3 a^2 c^2 f^4 g^2 + 3 a^3 c^2 f^2 g^4 + a^4 g^6)) * \log((c^2 e^2 f^4 - 2 c^2 d^2 e^2 f^3 g - 2 a^2 d^2 e^2 f^3 g^3 + a^2 d^2 g^4 - 3 (c^2 d^2 + a^2 e^2) f^2 g^2 - 2 (c^2 e^2 f^5 - 3 c^2 d^2 f^4 g - 4 a^2 c^2 e^2 f^3 g^2 + 4 a^2 c^2 d^2 f^2 g^3 + 3 a^2 e^2 f^3 g^4 - a^2 d^2 g^5 - 2 (a^2 c^3 f^7 g + 3 a^2 c^2 f^5 g^3 + 3 a^3 c^2 f^3 g^5 + a^4 f^2 g^7) * \sqrt{-(c^3 e^2 f^6 - 6 c^3 d^2 e^2 f^5 g + 20 a^2 c^2 d^2 e^2 f^4 g^3 - 6 a^2 c^2 d^2 e^2 f^5 g + a^2 c^2 d^2 g^6 + 3 (3 c^3 d^2 - 2 a^2 c^2 e^2) f^4 g^2 - 3 (2 a^2 c^2 d^2 - 3 a^2 c^2 e^2) f^2 g^4) / (a^2 c^6 f^{12} + 6 a^2 c^5 f^{10} g^2 + 15 a^3 c^4 f^8 g^4 + 20 a^4 c^3 f^6 g^6 + 15 a^5 c^2 f^4 g^8 + 6 a^6 c^2 f^2 g^{10} + a^7 g^{12})) * \sqrt{e^2 x + d} * \sqrt{g^2 x + f} * \sqrt{-(c^2 d^2 f^3 + 3 a^2 c^2 e^2 f^2 g - 3 a^2 c^2 d^2 f^2 g^2 - a^2 e^2 g^3 - (a^2 c^3 f^6 + 3 a^2 c^2 f^4 g^2 + 3 a^3 c^2 f^2 g^4 + a^4 g^6) * \sqrt{-(c^3 e^2 f^6 - 6 c^3 d^2 e^2 f^5 g + 20 a^2 c^2 d^2 e^2 f^4 g^3 - 6 a^2 c^2 d^2 e^2 f^5 g + a^2 c^2 d^2 g^6 + 3 (3 c^3 d^2 - 2 a^2 c^2 e^2) f^4 g^2 - 3 (2 a^2 c^2 d^2 - 3 a^2 c^2 e^2) f^2 g^4) / (a^2 c^6 f^{12} + 6 a^2 c^5 f^{10} g^2 + 15 a^3 c^4 f^8 g^4 + 20 a^4 c^3 f^6 g^6 + 15 a^5 c^2 f^4 g^8 + 6 a^6 c^2 f^2 g^{10} + a^7 g^{12})) / (a^2 c^3 f^6 + 3 a^2 c^2 f^4 g^2 + 3 a^3 c^2 f^2 g^4 + a^4 g^6)) + 2 (c^2 e^2 f^3 g - 3 c^2 d^2 e^2 f^2 g^2 - 3 a^2 e^2 f^2 g^3 + a^2 d^2 e^2 g^4) * x + (2 c^3 d^2 f^7 + 6 a^2 c^2 d^2 f^5 g^2 + 6 a^2 c^2 d^2 f^3 g^4 + 2 a^3 d^2 f^2 g^6 + (c^3 e^2 f^7 + c^3 d^2 f^6 g + 3 a^2 c^2 e^2 f^5 g^2 + 3 a^2 c^2 d^2 f^4 g^3 + 3 a^2 c^2 e^2 f^3 g^4 + 3 a^2 c^2 d^2 f^2 g^5 + a^3 e^2 f^2 g^6 + a^3 d^2 g^7) * x) * \sqrt{-(c^3 e^2 f^6 - 6 c^3 d^2 e^2 f^5 g + 20 a^2 c^2 d^2 e^2 f^4 g^3 - 6 a^2 c^2 d^2 e^2 f^5 g + a^2 c^2 d^2 g^6 + 3 (3 c^3 d^2 - 2 a^2 c^2 e^2) f^4 g^2 - 3 (2 a^2 c^2 d^2 - 3 a^2 c^2 e^2) f^2 g^4) / (a^2 c^6 f^{12} + 6 a^2 c^5 f^{10} g^2 + 15 a^3 c^4 f^8 g^4 + 20 a^4 c^3 f^6 g^6 + 15 a^5 c^2 f^4 g^8 + 6 a^6 c^2 f^2 g^{10} + a^7 g^{12})) / x) + 8 * \sqrt{e^2 x + d} * \sqrt{g^2 x + f} * g) / (c^2 f^3 + a^2 f^2 g^2 + (c^2 f^2 g + a^2 g^3) * x)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x + d)/((c*x^2 + a)*(g*x + f)^(3/2)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.616 \quad \int \frac{1}{\sqrt{d+ex}(f+gx)^{3/2}(a+cx^2)} dx$$

**Optimal.** Leaf size=354

$$\frac{g\sqrt{d+ex}}{\sqrt{-a}\sqrt{f+gx}(\sqrt{c}f-\sqrt{-ag})(ef-dg)} - \frac{g\sqrt{d+ex}}{\sqrt{-a}\sqrt{f+gx}(\sqrt{-ag}+\sqrt{c}f)(ef-dg)} \\ + \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{c}f-\sqrt{-ag}}{\sqrt{f+gx}\sqrt{c}d-\sqrt{-ae}}\right)}{\sqrt{-a}\sqrt{c}d-\sqrt{-ae}(\sqrt{c}f-\sqrt{-ag})^{3/2}} - \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-ag}+\sqrt{c}f}{\sqrt{f+gx}\sqrt{-ae}+\sqrt{c}d}\right)}{\sqrt{-a}\sqrt{-ae}+\sqrt{c}d(\sqrt{-ag}+\sqrt{c}f)^{3/2}}$$

[Out] (g\*Sqrt[d + e\*x])/(Sqrt[-a]\*(Sqrt[c]\*f - Sqrt[-a]\*g)\*(e\*f - d\*g)\*Sqrt[f + g\*x]) - (g\*Sqrt[d + e\*x])/(Sqrt[-a]\*(Sqrt[c]\*f + Sqrt[-a]\*g)\*(e\*f - d\*g)\*Sqrt[f + g\*x]) + (Sqrt[c]\*ArcTanh[(Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*Sqrt[f + g\*x])])/(Sqrt[-a]\*Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*(Sqrt[c]\*f - Sqrt[-a]\*g)^(3/2)) - (Sqrt[c]\*ArcTanh[(Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*Sqrt[f + g\*x])])/(Sqrt[-a]\*Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*(Sqrt[c]\*f + Sqrt[-a]\*g)^(3/2))

**Rubi [A]** time = 1.71511, antiderivative size = 354, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{g\sqrt{d+ex}}{\sqrt{-a}\sqrt{f+gx}(\sqrt{c}f-\sqrt{-ag})(ef-dg)} - \frac{g\sqrt{d+ex}}{\sqrt{-a}\sqrt{f+gx}(\sqrt{-ag}+\sqrt{c}f)(ef-dg)} \\ + \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{c}f-\sqrt{-ag}}{\sqrt{f+gx}\sqrt{c}d-\sqrt{-ae}}\right)}{\sqrt{-a}\sqrt{c}d-\sqrt{-ae}(\sqrt{c}f-\sqrt{-ag})^{3/2}} - \frac{\sqrt{c} \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-ag}+\sqrt{c}f}{\sqrt{f+gx}\sqrt{-ae}+\sqrt{c}d}\right)}{\sqrt{-a}\sqrt{-ae}+\sqrt{c}d(\sqrt{-ag}+\sqrt{c}f)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[d + e\*x]\*(f + g\*x)^(3/2)\*(a + c\*x^2)), x]

[Out] (g\*Sqrt[d + e\*x])/(Sqrt[-a]\*(Sqrt[c]\*f - Sqrt[-a]\*g)\*(e\*f - d\*g)\*Sqrt[f + g\*x]) - (g\*Sqrt[d + e\*x])/(Sqrt[-a]\*(Sqrt[c]\*f + Sqrt[-a]\*g)\*(e\*f - d\*g)\*Sqrt[f + g\*x]) + (Sqrt[c]\*ArcTanh[(Sqrt[Sqrt[c]\*f - Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*Sqrt[f + g\*x])])/(Sqrt[-a]\*Sqrt[Sqrt[c]\*d - Sqrt[-a]\*e]\*(Sqrt[c]\*f - Sqrt[-a]\*g)^(3/2)) - (Sqrt[c]\*ArcTanh[(Sqrt[Sqrt[c]\*f + Sqrt[-a]\*g]\*Sqrt[d + e\*x])/(Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*Sqrt[f + g\*x])])/(Sqrt[-a]\*Sqrt[Sqrt[c]\*d + Sqrt[-a]\*e]\*(Sqrt[c]\*f + Sqrt[-a]\*g)^(3/2))

**Rubi in Sympy [A]** time = 142.261, size = 304, normalized size = 0.86

$$-\frac{\sqrt{c} \operatorname{atanh}\left(\frac{\sqrt{d+ex}\sqrt{c}f+g\sqrt{-a}}{\sqrt{f+gx}\sqrt{c}d+e\sqrt{-a}}\right)}{\sqrt{-a}\sqrt{c}d+e\sqrt{-a}(\sqrt{c}f+g\sqrt{-a})^{3/2}} + \frac{\sqrt{c} \operatorname{atanh}\left(\frac{\sqrt{d+ex}\sqrt{c}f-g\sqrt{-a}}{\sqrt{f+gx}\sqrt{c}d-e\sqrt{-a}}\right)}{\sqrt{-a}\sqrt{c}d-e\sqrt{-a}(\sqrt{c}f-g\sqrt{-a})^{3/2}} \\ + \frac{g\sqrt{d+ex}}{\sqrt{-a}\sqrt{f+gx}(\sqrt{c}f+g\sqrt{-a})(dg-ef)} - \frac{g\sqrt{d+ex}}{\sqrt{-a}\sqrt{f+gx}(\sqrt{c}f-g\sqrt{-a})(dg-ef)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(e\*x+d)\*\*(1/2)/(g\*x+f)\*\*(3/2)/(c\*x\*\*2+a), x)

[Out]  $-\sqrt{c} \operatorname{atanh}\left(\frac{\sqrt{d+ex} \sqrt{\sqrt{c}f+g\sqrt{-a}}}{\sqrt{f+gx} \sqrt{\sqrt{c}d+e\sqrt{-a}}}\right) / \left(\sqrt{-a} \sqrt{\sqrt{c}d+e\sqrt{-a}}\right)^{3/2} + \sqrt{c} \operatorname{atanh}\left(\frac{\sqrt{d+ex} \sqrt{\sqrt{c}f-g\sqrt{-a}}}{\sqrt{f+gx} \sqrt{\sqrt{c}d-e\sqrt{-a}}}\right) / \left(\sqrt{-a} \sqrt{\sqrt{c}d-e\sqrt{-a}}\right)^{3/2} + g\sqrt{d+ex} / \left(\sqrt{-a} \sqrt{f+gx} \sqrt{\sqrt{c}f+g\sqrt{-a}}\right) - g\sqrt{d+ex} / \left(\sqrt{-a} \sqrt{f+gx} \sqrt{\sqrt{c}f-g\sqrt{-a}}\right) (d^2g - e^2f)$

**Mathematica [C]** time = 4.36252, size = 555, normalized size = 1.57

$$\frac{(\sqrt{a}\sqrt{cg+icf}) \log\left(\frac{i\sqrt{a}\sqrt{cf+i\sqrt{ag}}(2\sqrt{d+ex}\sqrt{f+gx}\sqrt{cd+i\sqrt{ae}}\sqrt{cf+i\sqrt{ag}}+i\sqrt{a}(dg+ef+2egx)+\sqrt{c}(2df+dgx+efx))}{\sqrt{c}(\sqrt{cx-i\sqrt{a}})\sqrt{cd+i\sqrt{ae}}}\right)}{\sqrt{a}\sqrt{cd+i\sqrt{ae}}\sqrt{cf+i\sqrt{ag}}} + \frac{(\sqrt{a}\sqrt{cg-icf}) \log\left(-\frac{i\sqrt{a}\sqrt{cf-i\sqrt{ag}}(2\sqrt{d+ex}\sqrt{f+gx}\sqrt{cd-i\sqrt{ae}}\sqrt{cf-i\sqrt{ag}}-i\sqrt{a}(dg-ef+2egx)+\sqrt{c}(2df+dgx+efx))}{\sqrt{c}(\sqrt{cx+i\sqrt{a}})\sqrt{cd-i\sqrt{ae}}}\right)}{\sqrt{a}\sqrt{cd-i\sqrt{ae}}\sqrt{cf-i\sqrt{ag}}}}{2(ag^2 + cf^2)}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[d + e\*x]\*(f + g\*x)^(3/2)\*(a + c\*x^2)), x]

[Out]  $\left(\frac{4g^2\sqrt{d+ex}}{(ef-dg)\sqrt{f+gx}} + \frac{(Ic^2f + \sqrt{a}\sqrt{c}g)\operatorname{Log}\left[\frac{(I\sqrt{a}\sqrt{c}f + I\sqrt{a}\sqrt{c}g)\sqrt{d+ex} + I\sqrt{a}\sqrt{c}g\sqrt{f+gx} + \sqrt{c}(2df+dgx+efx) + I\sqrt{a}\sqrt{c}g(e^2f+dg+2e^2gx)}{(I\sqrt{a}\sqrt{c}f + I\sqrt{a}\sqrt{c}g)\sqrt{d+ex} + I\sqrt{a}\sqrt{c}g\sqrt{f+gx} + \sqrt{c}(2df+dgx+efx) + I\sqrt{a}\sqrt{c}g(e^2f+dg+2e^2gx)}\right]}{(I\sqrt{a}\sqrt{c}f + I\sqrt{a}\sqrt{c}g)\sqrt{d+ex} + I\sqrt{a}\sqrt{c}g\sqrt{f+gx} + \sqrt{c}(2df+dgx+efx) + I\sqrt{a}\sqrt{c}g(e^2f+dg+2e^2gx)}\right) + \left(\frac{(-I)\sqrt{a}\sqrt{c}f + \sqrt{a}\sqrt{c}g}{(I\sqrt{a}\sqrt{c}f + I\sqrt{a}\sqrt{c}g)\sqrt{d+ex} + I\sqrt{a}\sqrt{c}g\sqrt{f+gx} + \sqrt{c}(2df+dgx+efx) + I\sqrt{a}\sqrt{c}g(e^2f+dg+2e^2gx)}\right) + \left(\frac{(-I)c^2f + \sqrt{a}\sqrt{c}g}{(I\sqrt{a}\sqrt{c}f + I\sqrt{a}\sqrt{c}g)\sqrt{d+ex} + I\sqrt{a}\sqrt{c}g\sqrt{f+gx} + \sqrt{c}(2df+dgx+efx) + I\sqrt{a}\sqrt{c}g(e^2f+dg+2e^2gx)}\right) \operatorname{Log}\left[\frac{(-I)\sqrt{a}\sqrt{c}f - I\sqrt{a}\sqrt{c}g}{(I\sqrt{a}\sqrt{c}f + I\sqrt{a}\sqrt{c}g)\sqrt{d+ex} + I\sqrt{a}\sqrt{c}g\sqrt{f+gx} + \sqrt{c}(2df+dgx+efx) + I\sqrt{a}\sqrt{c}g(e^2f+dg+2e^2gx)}\right] + \left(\frac{(-I)c^2f + \sqrt{a}\sqrt{c}g}{(I\sqrt{a}\sqrt{c}f + I\sqrt{a}\sqrt{c}g)\sqrt{d+ex} + I\sqrt{a}\sqrt{c}g\sqrt{f+gx} + \sqrt{c}(2df+dgx+efx) + I\sqrt{a}\sqrt{c}g(e^2f+dg+2e^2gx)}\right) \operatorname{Log}\left[\frac{(-I)\sqrt{a}\sqrt{c}f - I\sqrt{a}\sqrt{c}g}{(I\sqrt{a}\sqrt{c}f + I\sqrt{a}\sqrt{c}g)\sqrt{d+ex} + I\sqrt{a}\sqrt{c}g\sqrt{f+gx} + \sqrt{c}(2df+dgx+efx) + I\sqrt{a}\sqrt{c}g(e^2f+dg+2e^2gx)}\right] + \left(\frac{(-I)\sqrt{a}\sqrt{c}f + \sqrt{a}\sqrt{c}g}{(I\sqrt{a}\sqrt{c}f + I\sqrt{a}\sqrt{c}g)\sqrt{d+ex} + I\sqrt{a}\sqrt{c}g\sqrt{f+gx} + \sqrt{c}(2df+dgx+efx) + I\sqrt{a}\sqrt{c}g(e^2f+dg+2e^2gx)}\right) \operatorname{Log}\left[\frac{(-I)\sqrt{a}\sqrt{c}f - I\sqrt{a}\sqrt{c}g}{(I\sqrt{a}\sqrt{c}f + I\sqrt{a}\sqrt{c}g)\sqrt{d+ex} + I\sqrt{a}\sqrt{c}g\sqrt{f+gx} + \sqrt{c}(2df+dgx+efx) + I\sqrt{a}\sqrt{c}g(e^2f+dg+2e^2gx)}\right] + \left(\frac{(-I)c^2f + \sqrt{a}\sqrt{c}g}{(I\sqrt{a}\sqrt{c}f + I\sqrt{a}\sqrt{c}g)\sqrt{d+ex} + I\sqrt{a}\sqrt{c}g\sqrt{f+gx} + \sqrt{c}(2df+dgx+efx) + I\sqrt{a}\sqrt{c}g(e^2f+dg+2e^2gx)}\right) \operatorname{Log}\left[\frac{(-I)\sqrt{a}\sqrt{c}f - I\sqrt{a}\sqrt{c}g}{(I\sqrt{a}\sqrt{c}f + I\sqrt{a}\sqrt{c}g)\sqrt{d+ex} + I\sqrt{a}\sqrt{c}g\sqrt{f+gx} + \sqrt{c}(2df+dgx+efx) + I\sqrt{a}\sqrt{c}g(e^2f+dg+2e^2gx)}\right]$

**Maple [B]** time = 0.112, size = 10977, normalized size = 31.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x+d)^(1/2)/(g\*x+f)^(3/2)/(c\*x^2+a), x)

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + a)\sqrt{ex + d}(gx + f)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + a)\*sqrt(e\*x + d)\*(g\*x + f)^(3/2)), x, algorithm="maxima")

[Out] integrate(1/((c\*x^2 + a)\*sqrt(e\*x + d)\*(g\*x + f)^(3/2)), x)

**Fricas** [A] time = 147.838, size = 16238, normalized size = 45.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + a)\*sqrt(e\*x + d)\*(g\*x + f)^(3/2)),x, algorithm="fricas")

[Out] 
$$\frac{1}{4} \cdot (8 \cdot \sqrt{e \cdot x + d} \cdot \sqrt{g \cdot x + f} \cdot g^2 - (c \cdot e \cdot f^4 - c \cdot d \cdot f^3 \cdot g + a \cdot e \cdot f^2 \cdot g^2 - a \cdot d \cdot f \cdot g^3 + (c \cdot e \cdot f^3 \cdot g - c \cdot d \cdot f^2 \cdot g^2 + a \cdot e \cdot f \cdot g^3 - a \cdot d \cdot g^4) \cdot x) \cdot \sqrt{-(c^3 \cdot d \cdot f^3 - 3 \cdot a \cdot c^2 \cdot e \cdot f^2 \cdot g - 3 \cdot a \cdot c^2 \cdot d \cdot f \cdot g^2 + a^2 \cdot c \cdot e \cdot g^3 + ((a \cdot c^4 \cdot d^2 + a^2 \cdot c^3 \cdot e^2) \cdot f^6 + 3 \cdot (a^2 \cdot c^3 \cdot d^2 + a^3 \cdot c^2 \cdot e^2) \cdot f^4 \cdot g^2 + 3 \cdot (a^3 \cdot c^2 \cdot d^2 + a^4 \cdot c \cdot e^2) \cdot f^2 \cdot g^4 + (a^4 \cdot c \cdot d^2 + a^5 \cdot e^2) \cdot g^6) \cdot \sqrt{-(c^5 \cdot e^2 \cdot f^6 + 6 \cdot c^5 \cdot d \cdot e \cdot f^5 \cdot g - 20 \cdot a \cdot c^4 \cdot d \cdot e \cdot f^3 \cdot g^3 + 6 \cdot a^2 \cdot c^3 \cdot d \cdot e \cdot f \cdot g^5 + a^2 \cdot c^3 \cdot d^2 \cdot g^6 + 3 \cdot (3 \cdot c^5 \cdot d^2 - 2 \cdot a \cdot c^4 \cdot e^2) \cdot f^4 \cdot g^2 - 3 \cdot (2 \cdot a \cdot c^4 \cdot d^2 - 3 \cdot a^2 \cdot c^3 \cdot e^2) \cdot f^2 \cdot g^4) / ((a \cdot c^8 \cdot d^4 + 2 \cdot a^2 \cdot c^7 \cdot d^2 \cdot e^2 + a^3 \cdot c^6 \cdot e^4) \cdot f^{12} + 6 \cdot (a^2 \cdot c^7 \cdot d^4 + 2 \cdot a^3 \cdot c^6 \cdot d^2 \cdot e^2 + a^4 \cdot c^5 \cdot e^4) \cdot f^{10} \cdot g^2 + 15 \cdot (a^3 \cdot c^6 \cdot d^4 + 2 \cdot a^4 \cdot c^5 \cdot d^2 \cdot e^2 + a^5 \cdot c^4 \cdot e^4) \cdot f^8 \cdot g^4 + 20 \cdot (a^4 \cdot c^5 \cdot d^4 + 2 \cdot a^5 \cdot c^4 \cdot d^2 \cdot e^2 + a^6 \cdot c^3 \cdot e^4) \cdot f^6 \cdot g^6 + 15 \cdot (a^5 \cdot c^4 \cdot d^4 + 2 \cdot a^6 \cdot c^3 \cdot d^2 \cdot e^2 + a^7 \cdot c^2 \cdot e^4) \cdot f^4 \cdot g^8 + 6 \cdot (a^6 \cdot c^3 \cdot d^4 + 2 \cdot a^7 \cdot c^2 \cdot d^2 \cdot e^2 + a^8 \cdot c \cdot e^4) \cdot f^2 \cdot g^{10} + (a^7 \cdot c^2 \cdot d^4 + 2 \cdot a^8 \cdot c \cdot d^2 \cdot e^2 + a^9 \cdot e^4) \cdot g^{12})) / ((a \cdot c^4 \cdot d^2 + a^2 \cdot c^3 \cdot e^2) \cdot f^6 + 3 \cdot (a^2 \cdot c^3 \cdot d^2 + a^3 \cdot c^2 \cdot e^2) \cdot f^4 \cdot g^2 + 3 \cdot (a^3 \cdot c^2 \cdot d^2 + a^4 \cdot c \cdot e^2) \cdot f^2 \cdot g^4 + (a^4 \cdot c \cdot d^2 + a^5 \cdot e^2) \cdot g^6) \cdot \log(-(c^3 \cdot e^2 \cdot f^4 + 4 \cdot c^3 \cdot d \cdot e \cdot f^3 \cdot g - 4 \cdot a \cdot c^2 \cdot d \cdot e \cdot f \cdot g^3 - a \cdot c^2 \cdot d^2 \cdot g^4 + 3 \cdot (c^3 \cdot d^2 - a \cdot c^2 \cdot e^2) \cdot f^2 \cdot g^2 + 2 \cdot (c^4 \cdot d \cdot e \cdot f^5 - 10 \cdot a \cdot c^3 \cdot d \cdot e \cdot f^3 \cdot g^2 + 5 \cdot a^2 \cdot c^2 \cdot d \cdot e \cdot f \cdot g^4 + a^2 \cdot c^2 \cdot d^2 \cdot g^5 + (3 \cdot c^4 \cdot d^2 - 2 \cdot a \cdot c^3 \cdot e^2) \cdot f^4 \cdot g - 2 \cdot (2 \cdot a \cdot c^3 \cdot d^2 - 3 \cdot a^2 \cdot c^2 \cdot e^2) \cdot f^2 \cdot g^3 - ((a \cdot c^5 \cdot d^2 \cdot e + a^2 \cdot c^4 \cdot e^3) \cdot f^8 + 2 \cdot (a \cdot c^5 \cdot d^3 + a^2 \cdot c^4 \cdot d \cdot e^2) \cdot f^7 \cdot g + 2 \cdot (a^2 \cdot c^4 \cdot d^2 \cdot e + a^3 \cdot c^3 \cdot e^3) \cdot f^6 \cdot g^2 + 6 \cdot (a^2 \cdot c^4 \cdot d^3 + a^3 \cdot c^3 \cdot d \cdot e^2) \cdot f^5 \cdot g^3 + 6 \cdot (a^3 \cdot c^3 \cdot d^3 + a^4 \cdot c^2 \cdot d \cdot e^2) \cdot f^3 \cdot g^5 - 2 \cdot (a^4 \cdot c^2 \cdot d^2 \cdot e + a^5 \cdot c \cdot e^3) \cdot f^2 \cdot g^6 + 2 \cdot (a^4 \cdot c^2 \cdot d^3 + a^5 \cdot c \cdot d \cdot e^2) \cdot f \cdot g^7 - (a^5 \cdot c \cdot d^2 \cdot e + a^6 \cdot e^3) \cdot g^8) \cdot \sqrt{-(c^5 \cdot e^2 \cdot f^6 + 6 \cdot c^5 \cdot d \cdot e \cdot f^5 \cdot g - 20 \cdot a \cdot c^4 \cdot d \cdot e \cdot f^3 \cdot g^3 + 6 \cdot a^2 \cdot c^3 \cdot d \cdot e \cdot f \cdot g^5 + a^2 \cdot c^3 \cdot d^2 \cdot g^6 + 3 \cdot (3 \cdot c^5 \cdot d^2 - 2 \cdot a \cdot c^4 \cdot e^2) \cdot f^4 \cdot g^2 - 3 \cdot (2 \cdot a \cdot c^4 \cdot d^2 - 3 \cdot a^2 \cdot c^3 \cdot e^2) \cdot f^2 \cdot g^4) / ((a \cdot c^8 \cdot d^4 + 2 \cdot a^2 \cdot c^7 \cdot d^2 \cdot e^2 + a^3 \cdot c^6 \cdot e^4) \cdot f^{12} + 6 \cdot (a^2 \cdot c^7 \cdot d^4 + 2 \cdot a^3 \cdot c^6 \cdot d^2 \cdot e^2 + a^4 \cdot c^5 \cdot e^4) \cdot f^{10} \cdot g^2 + 15 \cdot (a^3 \cdot c^6 \cdot d^4 + 2 \cdot a^4 \cdot c^5 \cdot d^2 \cdot e^2 + a^5 \cdot c^4 \cdot e^4) \cdot f^8 \cdot g^4 + 20 \cdot (a^4 \cdot c^5 \cdot d^4 + 2 \cdot a^5 \cdot c^4 \cdot d^2 \cdot e^2 + a^6 \cdot c^3 \cdot e^4) \cdot f^6 \cdot g^6 + 15 \cdot (a^5 \cdot c^4 \cdot d^4 + 2 \cdot a^6 \cdot c^3 \cdot d^2 \cdot e^2 + a^7 \cdot c^2 \cdot e^4) \cdot f^4 \cdot g^8 + 6 \cdot (a^6 \cdot c^3 \cdot d^4 + 2 \cdot a^7 \cdot c^2 \cdot d^2 \cdot e^2 + a^8 \cdot c \cdot e^4) \cdot f^2 \cdot g^{10} + (a^7 \cdot c^2 \cdot d^4 + 2 \cdot a^8 \cdot c \cdot d^2 \cdot e^2 + a^9 \cdot e^4) \cdot g^{12})) \cdot \sqrt{e \cdot x + d} \cdot \sqrt{g \cdot x + f} \cdot \sqrt{-(c^3 \cdot d \cdot f^3 - 3 \cdot a \cdot c^2 \cdot e \cdot f^2 \cdot g - 3 \cdot a \cdot c^2 \cdot d \cdot f \cdot g^2 + a^2 \cdot c \cdot e \cdot g^3 + ((a \cdot c^4 \cdot d^2 + a^2 \cdot c^3 \cdot e^2) \cdot f^6 + 3 \cdot (a^2 \cdot c^3 \cdot d^2 + a^3 \cdot c^2 \cdot e^2) \cdot f^4 \cdot g^2 + 3 \cdot (a^3 \cdot c^2 \cdot d^2 + a^4 \cdot c \cdot e^2) \cdot f^2 \cdot g^4 + (a^4 \cdot c \cdot d^2 + a^5 \cdot e^2) \cdot g^6) \cdot \sqrt{-(c^5 \cdot e^2 \cdot f^6 + 6 \cdot c^5 \cdot d \cdot e \cdot f^5 \cdot g - 20 \cdot a \cdot c^4 \cdot d \cdot e \cdot f^3 \cdot g^3 + 6 \cdot a^2 \cdot c^3 \cdot d \cdot e \cdot f \cdot g^5 + a^2 \cdot c^3 \cdot d^2 \cdot g^6 + 3 \cdot (3 \cdot c^5 \cdot d^2 - 2 \cdot a \cdot c^4 \cdot e^2) \cdot f^4 \cdot g^2 - 3 \cdot (2 \cdot a \cdot c^4 \cdot d^2 - 3 \cdot a^2 \cdot c^3 \cdot e^2) \cdot f^2 \cdot g^4) / ((a \cdot c^8 \cdot d^4 + 2 \cdot a^2 \cdot c^7 \cdot d^2 \cdot e^2 + a^3 \cdot c^6 \cdot e^4) \cdot f^{12} + 6 \cdot (a^2 \cdot c^7 \cdot d^4 + 2 \cdot a^3 \cdot c^6 \cdot d^2 \cdot e^2 + a^4 \cdot c^5 \cdot e^4) \cdot f^{10} \cdot g^2 + 15 \cdot (a^3 \cdot c^6 \cdot d^4 + 2 \cdot a^4 \cdot c^5 \cdot d^2 \cdot e^2 + a^5 \cdot c^4 \cdot e^4) \cdot f^8 \cdot g^4 + 20 \cdot (a^4 \cdot c^5 \cdot d^4 + 2 \cdot a^5 \cdot c^4 \cdot d^2 \cdot e^2 + a^6 \cdot c^3 \cdot e^4) \cdot f^6 \cdot g^6 + 15 \cdot (a^5 \cdot c^4 \cdot d^4 + 2 \cdot a^6 \cdot c^3 \cdot d^2 \cdot e^2 + a^7 \cdot c^2 \cdot e^4) \cdot f^4 \cdot g^8 + 6 \cdot (a^6 \cdot c^3 \cdot d^4 + 2 \cdot a^7 \cdot c^2 \cdot d^2 \cdot e^2 + a^8 \cdot c \cdot e^4) \cdot f^2 \cdot g^{10} + (a^7 \cdot c^2 \cdot d^4 + 2 \cdot a^8 \cdot c \cdot d^2 \cdot e^2 + a^9 \cdot e^4) \cdot g^{12})) / ((a \cdot c^4 \cdot d^2 + a^2 \cdot c^3 \cdot e^2) \cdot f^6 + 3 \cdot (a^2 \cdot c^3 \cdot d^2 + a^3 \cdot c^2 \cdot e^2) \cdot f^4 \cdot g^2 + 3 \cdot (a^3 \cdot c^2 \cdot d^2 + a^4 \cdot c \cdot e^2) \cdot f^2 \cdot g^4 + (a^4 \cdot c \cdot d^2 + a^5 \cdot e^2) \cdot g^6) + 2 \cdot (c^3 \cdot e^2 \cdot f^3 \cdot g + 3 \cdot c^3 \cdot d \cdot e \cdot f^2 \cdot g^2 - 3 \cdot a \cdot c^2 \cdot e^2 \cdot f \cdot g^3 - a \cdot c^2 \cdot d \cdot e \cdot g^4) \cdot x + (2 \cdot (c^5 \cdot d^3 + a \cdot c^4 \cdot d \cdot e^2) \cdot f^7 + 6 \cdot (a \cdot c^4 \cdot d^3 + a^2 \cdot c^3 \cdot d \cdot e^2) \cdot f^5 \cdot g^2 + 6 \cdot (a^2 \cdot c^3 \cdot d^3 + a^3 \cdot c^2 \cdot d \cdot e^2) \cdot f^3 \cdot g^4 + 2 \cdot (a^3 \cdot c^2 \cdot d^3 + a^4 \cdot c \cdot d \cdot e^2) \cdot f \cdot g^6 + ((c^5 \cdot d^2 \cdot e + a \cdot c^4 \cdot e^3) \cdot f^7 + (c^5 \cdot d^3 + a \cdot c^4 \cdot d \cdot e^2) \cdot f^6 \cdot g + 3 \cdot (a \cdot c^4 \cdot d^2 \cdot e + a^2 \cdot c^3 \cdot e^3) \cdot f^5 \cdot g^2 + 3 \cdot (a \cdot c^4 \cdot d^3 + a^2 \cdot c^3 \cdot d \cdot e^2) \cdot f^4 \cdot g^3 + 3 \cdot (a^2 \cdot c^3 \cdot d^2 \cdot e + a^3 \cdot c^2 \cdot e^3) \cdot f^3 \cdot g^4 + 3 \cdot (a^2 \cdot c^3 \cdot d^3 + a^3 \cdot c^2 \cdot d \cdot e^2) \cdot f^2 \cdot g^5 + (a^3 \cdot c^2 \cdot d^2 \cdot e + a^4 \cdot c \cdot e^3) \cdot f \cdot g^6 + (a^3 \cdot c^2 \cdot d^3 + a^4 \cdot c \cdot d \cdot e^2) \cdot g^7) \cdot x) \cdot \sqrt{-(c^5 \cdot e^2 \cdot f^6 + 6 \cdot c^5 \cdot d \cdot e \cdot f^5 \cdot g - 20 \cdot a \cdot c^4 \cdot d \cdot e \cdot f^3 \cdot g^3 + 6 \cdot a^2 \cdot c^3 \cdot d \cdot e \cdot f \cdot g^5 + a^2 \cdot c^3 \cdot d^2 \cdot g^6 + 3 \cdot (3 \cdot c^5 \cdot d^2 - 2 \cdot a \cdot c^4 \cdot e^2) \cdot f^4 \cdot g^2 - 3 \cdot (2 \cdot a \cdot c^4 \cdot d^2 - 3 \cdot a^2 \cdot c^3 \cdot e^2) \cdot f^2 \cdot g^4) / ((a \cdot c^8 \cdot d^4 + 2 \cdot a^2 \cdot c^7 \cdot d^2 \cdot e^2 + a^3 \cdot c^6 \cdot e^4) \cdot f^{12} + 6 \cdot (a^2 \cdot c^7 \cdot d^4 + 2 \cdot a^3 \cdot c^6 \cdot d^2 \cdot e^2 + a^4 \cdot c^5 \cdot e^4) \cdot f^{10} \cdot g^2 + 15 \cdot (a^3 \cdot c^6 \cdot d^4 + 2 \cdot a^4 \cdot c^5 \cdot d^2 \cdot e^2 + a^5 \cdot c^4 \cdot e^4) \cdot f^8 \cdot g^4 + 20 \cdot (a^4 \cdot c^5 \cdot d^4 + 2 \cdot a^5 \cdot c^4 \cdot d^2 \cdot e^2 + a^6 \cdot c^3 \cdot e^4) \cdot f^6 \cdot g^6 + 15 \cdot (a^5 \cdot c^4 \cdot d^4 + 2 \cdot a^6 \cdot c^3 \cdot d^2 \cdot e^2 + a^7 \cdot c^2 \cdot e^4) \cdot f^4 \cdot g^8 + 6 \cdot (a^6 \cdot c^3 \cdot d^4 + 2 \cdot a^7 \cdot c^2 \cdot d^2 \cdot e^2 + a^8 \cdot c \cdot e^4) \cdot f^2 \cdot g^{10} + (a^7 \cdot c^2 \cdot d^4 + 2 \cdot a^8 \cdot c \cdot d^2 \cdot e^2 + a^9 \cdot e^4) \cdot g^{12}))$$



$$\begin{aligned}
& *c^6*d^4 + 2*a^4*c^5*d^2*e^2 + a^5*c^4*e^4)*f^8*g^4 + 20*(a^4*c^5 \\
& *d^4 + 2*a^5*c^4*d^2*e^2 + a^6*c^3*e^4)*f^6*g^6 + 15*(a^5*c^4*d^4 \\
& + 2*a^6*c^3*d^2*e^2 + a^7*c^2*e^4)*f^4*g^8 + 6*(a^6*c^3*d^4 + 2* \\
& a^7*c^2*d^2*e^2 + a^8*c*e^4)*f^2*g^{10} + (a^7*c^2*d^4 + 2*a^8*c*d^2 \\
& *e^2 + a^9*e^4)*g^{12}))/x) + (c*e*f^4 - c*d*f^3*g + a*e*f^2*g^2 \\
& - a*d*f*g^3 + (c*e*f^3*g - c*d*f^2*g^2 + a*e*f*g^3 - a*d*g^4)*x)* \\
& \text{sqrt}(-(c^3*d*f^3 - 3*a*c^2*e*f^2*g - 3*a*c^2*d*f*g^2 + a^2*c*e*g^3 \\
& + ((a*c^4*d^2 + a^2*c^3*e^2)*f^6 + 3*(a^2*c^3*d^2 + a^3*c^2*e^2) \\
& )*f^4*g^2 + 3*(a^3*c^2*d^2 + a^4*c*e^2)*f^2*g^4 + (a^4*c*d^2 + a^5 \\
& *e^2)*g^6)*\text{sqrt}(-(c^5*e^2*f^6 + 6*c^5*d*e*f^5*g - 20*a*c^4*d*e*f \\
& ^3*g^3 + 6*a^2*c^3*d*e*f*g^5 + a^2*c^3*d^2*g^6 + 3*(3*c^5*d^2 - 2 \\
& *a*c^4*e^2)*f^4*g^2 - 3*(2*a*c^4*d^2 - 3*a^2*c^3*e^2)*f^2*g^4)/(( \\
& a*c^8*d^4 + 2*a^2*c^7*d^2*e^2 + a^3*c^6*e^4)*f^{12} + 6*(a^2*c^7*d^4 \\
& + 2*a^3*c^6*d^2*e^2 + a^4*c^5*e^4)*f^{10}*g^2 + 15*(a^3*c^6*d^4 + \\
& 2*a^4*c^5*d^2*e^2 + a^5*c^4*e^4)*f^8*g^4 + 20*(a^4*c^5*d^4 + 2*a \\
& ^5*c^4*d^2*e^2 + a^6*c^3*e^4)*f^6*g^6 + 15*(a^5*c^4*d^4 + 2*a^6*c \\
& ^3*d^2*e^2 + a^7*c^2*e^4)*f^4*g^8 + 6*(a^6*c^3*d^4 + 2*a^7*c^2*d^2 \\
& *e^2 + a^8*c*e^4)*f^2*g^{10} + (a^7*c^2*d^4 + 2*a^8*c*d^2*e^2 + a^9 \\
& *e^4)*g^{12}))/((a*c^4*d^2 + a^2*c^3*e^2)*f^6 + 3*(a^2*c^3*d^2 + \\
& a^3*c^2*e^2)*f^4*g^2 + 3*(a^3*c^2*d^2 + a^4*c*e^2)*f^2*g^4 + (a^4 \\
& *c*d^2 + a^5*e^2)*g^6))*\log(-(c^3*e^2*f^4 + 4*c^3*d*e*f^3*g - 4*a \\
& *c^2*d*e*f*g^3 - a*c^2*d^2*g^4 + 3*(c^3*d^2 - a*c^2*e^2)*f^2*g^2 \\
& - 2*(c^4*d*e*f^5 - 10*a*c^3*d*e*f^3*g^2 + 5*a^2*c^2*d*e*f*g^4 + a \\
& ^2*c^2*d^2*g^5 + (3*c^4*d^2 - 2*a*c^3*e^2)*f^4*g - 2*(2*a*c^3*d^2 \\
& - 3*a^2*c^2*e^2)*f^2*g^3 - ((a*c^5*d^2*e + a^2*c^4*e^3)*f^8 + 2* \\
& (a*c^5*d^3 + a^2*c^4*d*e^2)*f^7*g + 2*(a^2*c^4*d^2*e + a^3*c^3*e^3 \\
& )*f^6*g^2 + 6*(a^2*c^4*d^3 + a^3*c^3*d*e^2)*f^5*g^3 + 6*(a^3*c^3 \\
& *d^3 + a^4*c^2*d*e^2)*f^3*g^5 - 2*(a^4*c^2*d^2*e + a^5*c*e^3)*f^2 \\
& *g^6 + 2*(a^4*c^2*d^3 + a^5*c*d*e^2)*f*g^7 - (a^5*c*d^2*e + a^6*e \\
& ^3)*g^8)*\text{sqrt}(-(c^5*e^2*f^6 + 6*c^5*d*e*f^5*g - 20*a*c^4*d*e*f^3* \\
& g^3 + 6*a^2*c^3*d*e*f*g^5 + a^2*c^3*d^2*g^6 + 3*(3*c^5*d^2 - 2*a* \\
& c^4*e^2)*f^4*g^2 - 3*(2*a*c^4*d^2 - 3*a^2*c^3*e^2)*f^2*g^4)/((a*c \\
& ^8*d^4 + 2*a^2*c^7*d^2*e^2 + a^3*c^6*e^4)*f^{12} + 6*(a^2*c^7*d^4 + \\
& 2*a^3*c^6*d^2*e^2 + a^4*c^5*e^4)*f^{10}*g^2 + 15*(a^3*c^6*d^4 + 2* \\
& a^4*c^5*d^2*e^2 + a^5*c^4*e^4)*f^8*g^4 + 20*(a^4*c^5*d^4 + 2*a^5* \\
& c^4*d^2*e^2 + a^6*c^3*e^4)*f^6*g^6 + 15*(a^5*c^4*d^4 + 2*a^6*c^3* \\
& d^2*e^2 + a^7*c^2*e^4)*f^4*g^8 + 6*(a^6*c^3*d^4 + 2*a^7*c^2*d^2*e \\
& ^2 + a^8*c*e^4)*f^2*g^{10} + (a^7*c^2*d^4 + 2*a^8*c*d^2*e^2 + a^9*e \\
& ^4)*g^{12}))*\text{sqrt}(e*x + d)*\text{sqrt}(g*x + f)*\text{sqrt}(-(c^3*d*f^3 - 3*a*c^2 \\
& *e*f^2*g - 3*a*c^2*d*f*g^2 + a^2*c*e*g^3 + ((a*c^4*d^2 + a^2*c^3 \\
& *e^2)*f^6 + 3*(a^2*c^3*d^2 + a^3*c^2*e^2)*f^4*g^2 + 3*(a^3*c^2*d^2 \\
& + a^4*c*e^2)*f^2*g^4 + (a^4*c*d^2 + a^5*e^2)*g^6)*\text{sqrt}(-(c^5*e^2 \\
& *f^6 + 6*c^5*d*e*f^5*g - 20*a*c^4*d*e*f^3*g^3 + 6*a^2*c^3*d*e*f* \\
& g^5 + a^2*c^3*d^2*g^6 + 3*(3*c^5*d^2 - 2*a*c^4*e^2)*f^4*g^2 - 3*( \\
& 2*a*c^4*d^2 - 3*a^2*c^3*e^2)*f^2*g^4)/((a*c^8*d^4 + 2*a^2*c^7*d^2 \\
& *e^2 + a^3*c^6*e^4)*f^{12} + 6*(a^2*c^7*d^4 + 2*a^3*c^6*d^2*e^2 + a \\
& ^4*c^5*e^4)*f^{10}*g^2 + 15*(a^3*c^6*d^4 + 2*a^4*c^5*d^2*e^2 + a^5* \\
& c^4*e^4)*f^8*g^4 + 20*(a^4*c^5*d^4 + 2*a^5*c^4*d^2*e^2 + a^6*c^3* \\
& e^4)*f^6*g^6 + 15*(a^5*c^4*d^4 + 2*a^6*c^3*d^2*e^2 + a^7*c^2*e^4) \\
& *f^4*g^8 + 6*(a^6*c^3*d^4 + 2*a^7*c^2*d^2*e^2 + a^8*c*e^4)*f^2*g^{10} \\
& + (a^7*c^2*d^4 + 2*a^8*c*d^2*e^2 + a^9*e^4)*g^{12}))/((a*c^4*d^2 \\
& + a^2*c^3*e^2)*f^6 + 3*(a^2*c^3*d^2 + a^3*c^2*e^2)*f^4*g^2 + 3* \\
& (a^3*c^2*d^2 + a^4*c*e^2)*f^2*g^4 + (a^4*c*d^2 + a^5*e^2)*g^6)) + \\
& 2*(c^3*e^2*f^3*g + 3*c^3*d*e*f^2*g^2 - 3*a*c^2*e^2*f*g^3 - a*c^2 \\
& *d*e*g^4)*x + (2*(c^5*d^3 + a*c^4*d*e^2)*f^7 + 6*(a*c^4*d^3 + a^2 \\
& *c^3*d*e^2)*f^5*g^2 + 6*(a^2*c^3*d^3 + a^3*c^2*d*e^2)*f^3*g^4 + 2 \\
& *(a^3*c^2*d^3 + a^4*c*d*e^2)*f*g^6 + ((c^5*d^2*e + a*c^4*e^3)*f^7 \\
& + (c^5*d^3 + a*c^4*d*e^2)*f^6*g + 3*(a*c^4*d^2*e + a^2*c^3*e^3)* \\
& f^5*g^2 + 3*(a*c^4*d^3 + a^2*c^3*d*e^2)*f^4*g^3 + 3*(a^2*c^3*d^2* \\
& e + a^3*c^2*e^3)*f^3*g^4 + 3*(a^2*c^3*d^3 + a^3*c^2*d*e^2)*f^2*g^5 \\
& + (a^3*c^2*d^2*e + a^4*c*e^3)*f*g^6 + (a^3*c^2*d^3 + a^4*c*d*e^3 \\
& )*g^7)*x)*\text{sqrt}(-(c^5*e^2*f^6 + 6*c^5*d*e*f^5*g - 20*a*c^4*d*e*f^3* \\
& g^3 + 6*a^2*c^3*d*e*f*g^5 + a^2*c^3*d^2*g^6 + 3*(3*c^5*d^2 - 2* \\
& a*c^4*e^2)*f^4*g^2 - 3*(2*a*c^4*d^2 - 3*a^2*c^3*e^2)*f^2*g^4)/((a \\
& *c^8*d^4 + 2*a^2*c^7*d^2*e^2 + a^3*c^6*e^4)*f^{12} + 6*(a^2*c^7*d^4 \\
& + 2*a^3*c^6*d^2*e^2 + a^4*c^5*e^4)*f^{10}*g^2 + 15*(a^3*c^6*d^4 + \\
& 2*a^4*c^5*d^2*e^2 + a^5*c^4*e^4)*f^8*g^4 + 20*(a^4*c^5*d^4 + 2*a^5 \\
& *c^4*d^2*e^2 + a^6*c^3*e^4)*f^6*g^6 + 15*(a^5*c^4*d^4 + 2*a^6*c^3 \\
& *d^2*e^2 + a^7*c^2*e^4)*f^4*g^8 + 6*(a^6*c^3*d^4 + 2*a^7*c^2*d^2 \\
& *e^2 + a^8*c*e^4)*f^2*g^{10} + (a^7*c^2*d^4 + 2*a^8*c*d^2*e^2 + a^9 \\
& *e^4)*g^{12}))/x) - (c*e*f^4 - c*d*f^3*g + a*e*f^2*g^2 - a*d*f*g^3 \\
& + (c*e*f^3*g - c*d*f^2*g^2 + a*e*f*g^3 - a*d*g^4)*x)*\text{sqrt}(-(c^3*
\end{aligned}$$

$$\begin{aligned}
& d^2 f^3 - 3 a^2 c^2 e f^2 g - 3 a^2 c^2 d f^2 g^2 + a^2 c^2 e g^3 - ((a^2 c^4 \\
& * d^2 + a^2 c^3 e^2) f^6 + 3 (a^2 c^3 d^2 + a^3 c^2 e^2) f^4 g^2 + \\
& 3 (a^3 c^2 d^2 + a^4 c^2 e^2) f^2 g^4 + (a^4 c^2 d^2 + a^5 e^2) g^6) \\
& * \sqrt{-(c^5 e^2 f^6 + 6 c^5 d^2 e f^5 g - 20 a^2 c^4 d^2 e f^3 g^3 + 6 a^2 \\
& a^2 c^3 d^2 e f^2 g^5 + a^2 c^3 d^2 g^6 + 3 (3 c^5 d^2 - 2 a^2 c^4 e^2) \\
& * f^4 g^2 - 3 (2 a^2 c^4 d^2 - 3 a^2 c^3 e^2) f^2 g^4) / ((a^2 c^8 d^4 + \\
& 2 a^2 c^7 d^2 e^2 + a^3 c^6 e^4) f^{12} + 6 (a^2 c^7 d^4 + 2 a^3 c^6 \\
& d^2 e^2 + a^4 c^5 e^4) f^{10} g^2 + 15 (a^3 c^6 d^4 + 2 a^4 c^5 d^2 \\
& e^2 + a^5 c^4 e^4) f^8 g^4 + 20 (a^4 c^5 d^4 + 2 a^5 c^4 d^2 e^2 \\
& e^2 + a^6 c^3 e^4) f^6 g^6 + 15 (a^5 c^4 d^4 + 2 a^6 c^3 d^2 e^2 + a^7 \\
& c^2 e^4) f^4 g^8 + 6 (a^6 c^3 d^4 + 2 a^7 c^2 d^2 e^2 + a^8 \\
& * c^2 e^4) f^2 g^{10} + (a^7 c^2 d^4 + 2 a^8 c^2 d^2 e^2 + a^9 e^4) g^{12} \\
& )) / ((a^2 c^4 d^2 + a^2 c^3 e^2) f^6 + 3 (a^2 c^3 d^2 + a^3 c^2 e^2) \\
& * f^4 g^2 + 3 (a^3 c^2 d^2 + a^4 c^2 e^2) f^2 g^4 + (a^4 c^2 d^2 + a^5 \\
& e^2) g^6) * \log(-(c^3 e^2 f^4 + 4 c^3 d^2 e f^3 g - 4 a^2 c^2 d^2 e f^2 \\
& g^3 - a^2 c^2 d^2 g^4 + 3 (c^3 d^2 - a^2 c^2 e^2) f^2 g^2 + 2 (c^4 d^2 \\
& e f^5 - 10 a^2 c^3 d^2 e f^3 g^2 + 5 a^2 c^2 d^2 e f^2 g^4 + a^2 c^2 d^2 \\
& g^5 + (3 c^4 d^2 - 2 a^2 c^3 e^2) f^4 g - 2 (2 a^2 c^3 d^2 - 3 a^2 c^2 \\
& e^2) f^2 g^3 + ((a^2 c^5 d^2 e + a^2 c^4 e^3) f^8 + 2 (a^2 c^5 d^3 \\
& + a^2 c^4 d^2 e^2) f^7 g + 2 (a^2 c^4 d^2 e + a^3 c^3 e^3) f^6 g^2 \\
& + 6 (a^2 c^4 d^3 + a^3 c^3 d^2 e^2) f^5 g^3 + 6 (a^3 c^3 d^3 + a^4 \\
& c^2 d^2 e^2) f^4 g^5 - 2 (a^4 c^2 d^2 e + a^5 c^2 e^3) f^2 g^6 + 2 (a^4 \\
& c^2 d^3 + a^5 c^2 d^2 e^2) f^2 g^7 - (a^5 c^2 d^2 e + a^6 e^3) g^8) * \sqrt{ \\
& -(c^5 e^2 f^6 + 6 c^5 d^2 e f^5 g - 20 a^2 c^4 d^2 e f^3 g^3 + 6 a^2 \\
& c^3 d^2 e f^2 g^5 + a^2 c^3 d^2 g^6 + 3 (3 c^5 d^2 - 2 a^2 c^4 e^2) f^4 \\
& g^2 - 3 (2 a^2 c^4 d^2 - 3 a^2 c^3 e^2) f^2 g^4) / ((a^2 c^8 d^4 + 2 a^2 \\
& a^2 c^7 d^2 e^2 + a^3 c^6 e^4) f^{12} + 6 (a^2 c^7 d^4 + 2 a^3 c^6 \\
& d^2 e^2 + a^4 c^5 e^4) f^{10} g^2 + 15 (a^3 c^6 d^4 + 2 a^4 c^5 d^2 \\
& e^2 + a^5 c^4 e^4) f^8 g^4 + 20 (a^4 c^5 d^4 + 2 a^5 c^4 d^2 e^2 \\
& + a^6 c^3 e^4) f^6 g^6 + 15 (a^5 c^4 d^4 + 2 a^6 c^3 d^2 e^2 + a^7 \\
& c^2 e^4) f^4 g^8 + 6 (a^6 c^3 d^4 + 2 a^7 c^2 d^2 e^2 + a^8 c^2 \\
& e^4) f^2 g^{10} + (a^7 c^2 d^4 + 2 a^8 c^2 d^2 e^2 + a^9 e^4) g^{12})) \\
& * \sqrt{e x + d} * \sqrt{g x + f} * \sqrt{-(c^3 d^2 f^3 - 3 a^2 c^2 e f^2 g - \\
& 3 a^2 c^2 d f^2 g^2 + a^2 c^2 e g^3 - ((a^2 c^4 d^2 + a^2 c^3 e^2) f^6 + \\
& 3 (a^2 c^3 d^2 + a^3 c^2 e^2) f^4 g^2 + 3 (a^3 c^2 d^2 + a^4 c^2 e^2) \\
& f^2 g^4 + (a^4 c^2 d^2 + a^5 e^2) g^6) * \sqrt{-(c^5 e^2 f^6 + 6 c^5 \\
& d^2 e f^5 g - 20 a^2 c^4 d^2 e f^3 g^3 + 6 a^2 c^3 d^2 e f^2 g^5 + a^2 c^3 \\
& d^2 g^6 + 3 (3 c^5 d^2 - 2 a^2 c^4 e^2) f^4 g^2 - 3 (2 a^2 c^4 d^2 \\
& - 3 a^2 c^3 e^2) f^2 g^4) / ((a^2 c^8 d^4 + 2 a^2 c^7 d^2 e^2 + a^3 c^6 \\
& e^4) f^{12} + 6 (a^2 c^7 d^4 + 2 a^3 c^6 d^2 e^2 + a^4 c^5 e^4) \\
& f^{10} g^2 + 15 (a^3 c^6 d^4 + 2 a^4 c^5 d^2 e^2 + a^5 c^4 e^4) f^8 \\
& g^4 + 20 (a^4 c^5 d^4 + 2 a^5 c^4 d^2 e^2 + a^6 c^3 e^4) f^6 g^6 + \\
& 15 (a^5 c^4 d^4 + 2 a^6 c^3 d^2 e^2 + a^7 c^2 e^4) f^4 g^8 + \\
& 6 (a^6 c^3 d^4 + 2 a^7 c^2 d^2 e^2 + a^8 c^2 e^4) f^2 g^{10} + (a^7 c^2 \\
& d^4 + 2 a^8 c^2 d^2 e^2 + a^9 e^4) g^{12})) / ((a^2 c^4 d^2 + a^2 c^3 \\
& e^2) f^6 + 3 (a^2 c^3 d^2 + a^3 c^2 e^2) f^4 g^2 + 3 (a^3 c^2 d^2 \\
& + a^4 c^2 e^2) f^2 g^4 + (a^4 c^2 d^2 + a^5 e^2) g^6) + 2 (c^3 e^2 \\
& f^3 g + 3 c^3 d^2 e f^2 g^2 - 3 a^2 c^2 e^2 f^2 g^3 - a^2 c^2 d^2 e g^4) * x \\
& - (2 (c^5 d^3 + a^2 c^4 d^2 e^2) f^7 + 6 (a^2 c^4 d^3 + a^2 c^3 d^2 e^2) \\
& * f^5 g^2 + 6 (a^2 c^3 d^3 + a^3 c^2 d^2 e^2) f^3 g^4 + 2 (a^3 c^2 d^3 \\
& + a^4 c^2 d^2 e^2) f^2 g^6 + ((c^5 d^2 e + a^2 c^4 e^3) f^7 + (c^5 d^3 \\
& + a^2 c^4 d^2 e^2) f^6 g + 3 (a^2 c^4 d^2 e + a^2 c^3 e^3) f^5 g^2 + 3 \\
& (a^2 c^4 d^3 + a^2 c^3 d^2 e^2) f^4 g^3 + 3 (a^2 c^3 d^2 e + a^3 c^2 \\
& e^3) f^3 g^4 + 3 (a^2 c^3 d^3 + a^3 c^2 d^2 e^2) f^2 g^5 + (a^3 c^2 \\
& d^2 e + a^4 c^2 e^3) f^2 g^6 + (a^3 c^2 d^3 + a^4 c^2 d^2 e^2) g^7) * x) * \\
& \sqrt{-(c^5 e^2 f^6 + 6 c^5 d^2 e f^5 g - 20 a^2 c^4 d^2 e f^3 g^3 + 6 a^2 \\
& a^2 c^3 d^2 e f^2 g^5 + a^2 c^3 d^2 g^6 + 3 (3 c^5 d^2 - 2 a^2 c^4 e^2) \\
& f^4 g^2 - 3 (2 a^2 c^4 d^2 - 3 a^2 c^3 e^2) f^2 g^4) / ((a^2 c^8 d^4 + \\
& 2 a^2 c^7 d^2 e^2 + a^3 c^6 e^4) f^{12} + 6 (a^2 c^7 d^4 + 2 a^3 c^6 \\
& d^2 e^2 + a^4 c^5 e^4) f^{10} g^2 + 15 (a^3 c^6 d^4 + 2 a^4 c^5 d^2 \\
& e^2 + a^5 c^4 e^4) f^8 g^4 + 20 (a^4 c^5 d^4 + 2 a^5 c^4 d^2 e^2 \\
& + a^6 c^3 e^4) f^6 g^6 + 15 (a^5 c^4 d^4 + 2 a^6 c^3 d^2 e^2 + a^7 \\
& c^2 e^4) f^4 g^8 + 6 (a^6 c^3 d^4 + 2 a^7 c^2 d^2 e^2 + a^8 c^2 \\
& e^4) f^2 g^{10} + (a^7 c^2 d^4 + 2 a^8 c^2 d^2 e^2 + a^9 e^4) g^{12} \\
& )) / x) + (c^2 e f^4 - c^2 d f^3 g + a^2 e f^2 g^2 - a^2 d f^2 g^3 + (c^2 e f^3 \\
& * g - c^2 d f^2 g^2 + a^2 e f^2 g^3 - a^2 d g^4) * x) * \sqrt{-(c^3 d^2 f^3 - 3 a^2 \\
& c^2 e f^2 g - 3 a^2 c^2 d f^2 g^2 + a^2 c^2 e g^3 - ((a^2 c^4 d^2 + a^2 c^3 \\
& e^2) f^6 + 3 (a^2 c^3 d^2 + a^3 c^2 e^2) f^4 g^2 + 3 (a^3 c^2 d^2 \\
& + a^4 c^2 e^2) f^2 g^4 + (a^4 c^2 d^2 + a^5 e^2) g^6) * \sqrt{-(c^5 \\
& e^2 f^6 + 6 c^5 d^2 e f^5 g - 20 a^2 c^4 d^2 e f^3 g^3 + 6 a^2 c^3 d^2 e \\
& f^2 g^5 + a^2 c^3 d^2 g^6 + 3 (3 c^5 d^2 - 2 a^2 c^4 e^2) f^4 g^2 - \\
& 3 (2 a^2 c^4 d^2 - 3 a^2 c^3 e^2) f^2 g^4) / ((a^2 c^8 d^4 + 2 a^2 c^7
\end{aligned}$$

$$\begin{aligned}
& d^2 e^2 + a^3 c^6 e^4) f^{12} + 6(a^2 c^7 d^4 + 2 a^3 c^6 d^2 e^2 + a^4 c^5 e^4) f^{10} g^2 + 15(a^3 c^6 d^4 + 2 a^4 c^5 d^2 e^2 + a^5 c^4 e^4) f^8 g^4 + 20(a^4 c^5 d^4 + 2 a^5 c^4 d^2 e^2 + a^6 c^3 e^4) f^6 g^6 + 15(a^5 c^4 d^4 + 2 a^6 c^3 d^2 e^2 + a^7 c^2 e^4) f^4 g^8 + 6(a^6 c^3 d^4 + 2 a^7 c^2 d^2 e^2 + a^8 c e^4) f^2 g^{10} + (a^7 c^2 d^4 + 2 a^8 c d^2 e^2 + a^9 e^4) g^{12}))/((a^2 c^4 d^2 + a^2 c^3 e^2) f^6 + 3(a^2 c^3 d^2 + a^3 c^2 e^2) f^4 g^2 + 3(a^3 c^2 d^2 + a^4 c e^2) f^2 g^4 + (a^4 c d^2 + a^5 e^2) g^6) \\
& ) \log(-(c^3 e^2 f^4 + 4 c^3 d^2 e f^3 g - 4 a^2 c^2 d^2 e f^2 g^3 - a^2 c^2 d^2 g^4 + 3(c^3 d^2 - a^2 c^2 e^2) f^2 g^2 - 2(c^4 d^2 e f^5 - 10 a^2 c^3 d^2 e f^3 g^2 + 5 a^2 c^2 d^2 e f^2 g^4 + a^2 c^2 d^2 g^5 + (3 c^4 d^2 - 2 a^2 c^3 e^2) f^4 g - 2(2 a^2 c^3 d^2 - 3 a^2 c^2 e^2) f^2 g^3 + ((a^2 c^5 d^2 e + a^2 c^4 e^3) f^8 + 2(a^2 c^5 d^3 + a^2 c^4 d^2 e^2) f^7 g + 2(a^2 c^4 d^2 e + a^3 c^3 e^3) f^6 g^2 + 6(a^2 c^4 d^3 + a^3 c^3 d^2 e^2) f^5 g^3 + 6(a^3 c^3 d^3 + a^4 c^2 d^2 e^2) f^4 g^4 - 2(a^4 c^2 d^2 e + a^5 c e^3) f^2 g^6 + 2(a^4 c^2 d^3 + a^5 c d^2 e^2) f^2 g^7 - (a^5 c d^2 e + a^6 e^3) g^8) \sqrt{-(c^5 e^2 f^6 + 6 c^5 d^2 e f^5 g - 20 a^2 c^4 d^2 e f^3 g^3 + 6 a^2 c^3 d^2 e f^2 g^5 + a^2 c^3 d^2 g^6 + 3(3 c^5 d^2 - 2 a^2 c^4 e^2) f^4 g^2 - 3(2 a^2 c^4 d^2 - 3 a^2 c^3 e^2) f^2 g^4)} / ((a^2 c^8 d^4 + 2 a^2 c^7 d^2 e^2 + a^3 c^6 e^4) f^{12} + 6(a^2 c^7 d^4 + 2 a^3 c^6 d^2 e^2 + a^4 c^5 e^4) f^{10} g^2 + 15(a^3 c^6 d^4 + 2 a^4 c^5 d^2 e^2 + a^5 c^4 e^4) f^8 g^4 + 20(a^4 c^5 d^4 + 2 a^5 c^4 d^2 e^2 + a^6 c^3 e^4) f^6 g^6 + 15(a^5 c^4 d^4 + 2 a^6 c^3 d^2 e^2 + a^7 c^2 e^4) f^4 g^8 + 6(a^6 c^3 d^4 + 2 a^7 c^2 d^2 e^2 + a^8 c e^4) f^2 g^{10} + (a^7 c^2 d^4 + 2 a^8 c d^2 e^2 + a^9 e^4) g^{12}))) \sqrt{e^2 x + d} \sqrt{g^2 x + f} \sqrt{-(c^3 d^2 f^3 - 3 a^2 c^2 e f^2 g - 3 a^2 c^2 d^2 f^2 g^2 + a^2 c^2 e g^3 - ((a^2 c^4 d^2 + a^2 c^3 e^2) f^6 + 3(a^2 c^3 d^2 + a^3 c^2 e^2) f^4 g^2 + 3(a^3 c^2 d^2 + a^4 c e^2) f^2 g^4 + (a^4 c d^2 + a^5 e^2) g^6) \sqrt{-(c^5 e^2 f^6 + 6 c^5 d^2 e f^5 g - 20 a^2 c^4 d^2 e f^3 g^3 + 6 a^2 c^3 d^2 e f^2 g^5 + a^2 c^3 d^2 g^6 + 3(3 c^5 d^2 - 2 a^2 c^4 e^2) f^4 g^2 - 3(2 a^2 c^4 d^2 - 3 a^2 c^3 e^2) f^2 g^4)} / ((a^2 c^8 d^4 + 2 a^2 c^7 d^2 e^2 + a^3 c^6 e^4) f^{12} + 6(a^2 c^7 d^4 + 2 a^3 c^6 d^2 e^2 + a^4 c^5 e^4) f^{10} g^2 + 15(a^3 c^6 d^4 + 2 a^4 c^5 d^2 e^2 + a^5 c^4 e^4) f^8 g^4 + 20(a^4 c^5 d^4 + 2 a^5 c^4 d^2 e^2 + a^6 c^3 e^4) f^6 g^6 + 15(a^5 c^4 d^4 + 2 a^6 c^3 d^2 e^2 + a^7 c^2 e^4) f^4 g^8 + 6(a^6 c^3 d^4 + 2 a^7 c^2 d^2 e^2 + a^8 c e^4) f^2 g^{10} + (a^7 c^2 d^4 + 2 a^8 c d^2 e^2 + a^9 e^4) g^{12}))) / ((a^2 c^4 d^2 + a^2 c^3 e^2) f^6 + 3(a^2 c^3 d^2 + a^3 c^2 e^2) f^4 g^2 + 3(a^3 c^2 d^2 + a^4 c e^2) f^2 g^4 + (a^4 c d^2 + a^5 e^2) g^6) + 2(c^3 e^2 f^3 g + 3 c^3 d^2 e f^2 g^2 - 3 a^2 c^2 e^2 f^2 g^3 - a^2 c^2 d^2 e g^4) x - (2(c^5 d^3 + a^2 c^4 d^2 e^2) f^7 + 6(a^2 c^4 d^3 + a^2 c^3 d^2 e^2) f^5 g^2 + 6(a^2 c^3 d^3 + a^3 c^2 d^2 e^2) f^3 g^4 + 2(a^3 c^2 d^3 + a^4 c d^2 e^2) f^2 g^6 + ((c^5 d^2 e + a^2 c^4 e^3) f^7 + (c^5 d^3 + a^2 c^4 d^2 e^2) f^6 g + 3(a^2 c^4 d^2 e + a^2 c^3 e^3) f^5 g^2 + 3(a^2 c^4 d^3 + a^2 c^3 d^2 e^2) f^4 g^3 + 3(a^2 c^3 d^3 + a^3 c^2 d^2 e^2) f^2 g^5 + (a^3 c^2 d^2 e + a^4 c e^3) f^2 g^6 + (a^3 c^2 d^3 + a^4 c d^2 e^2) g^7) x) \sqrt{-(c^5 e^2 f^6 + 6 c^5 d^2 e f^5 g - 20 a^2 c^4 d^2 e f^3 g^3 + 6 a^2 c^3 d^2 e f^2 g^5 + a^2 c^3 d^2 g^6 + 3(3 c^5 d^2 - 2 a^2 c^4 e^2) f^4 g^2 - 3(2 a^2 c^4 d^2 - 3 a^2 c^3 e^2) f^2 g^4)} / ((a^2 c^8 d^4 + 2 a^2 c^7 d^2 e^2 + a^3 c^6 e^4) f^{12} + 6(a^2 c^7 d^4 + 2 a^3 c^6 d^2 e^2 + a^4 c^5 e^4) f^{10} g^2 + 15(a^3 c^6 d^4 + 2 a^4 c^5 d^2 e^2 + a^5 c^4 e^4) f^8 g^4 + 20(a^4 c^5 d^4 + 2 a^5 c^4 d^2 e^2 + a^6 c^3 e^4) f^6 g^6 + 15(a^5 c^4 d^4 + 2 a^6 c^3 d^2 e^2 + a^7 c^2 e^4) f^4 g^8 + 6(a^6 c^3 d^4 + 2 a^7 c^2 d^2 e^2 + a^8 c e^4) f^2 g^{10} + (a^7 c^2 d^4 + 2 a^8 c d^2 e^2 + a^9 e^4) g^{12}))) / (c^2 e^2 f^4 - c^2 d^2 f^3 g + a^2 e^2 f^2 g^2 - a^2 d^2 f^2 g^3 + (c^2 e^2 f^3 g - c^2 d^2 f^2 g^2 + a^2 e^2 f^2 g^3 - a^2 d^2 g^4) x)
\end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(a + cx^2) \sqrt{d + ex} (f + gx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**(1/2)/(g*x+f)**(3/2)/(c*x**2+a),x)
```

```
[Out] Integral(1/((a + c*x**2)*sqrt(d + e*x)*(f + g*x)**(3/2)), x)
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^2 + a)*sqrt(e*x + d)*(g*x + f)^(3/2)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

$$3.617 \quad \int \frac{1}{(d+ex)^{3/2}(f+gx)^{3/2}(a+cx^2)} dx$$

**Optimal.** Leaf size=549

$$\begin{aligned} & -\frac{e}{\sqrt{-a}\sqrt{d+ex}\sqrt{f+gx}(\sqrt{cd}-\sqrt{-ae})(ef-dg)} + \frac{e}{\sqrt{-a}\sqrt{d+ex}\sqrt{f+gx}(\sqrt{-ae}+\sqrt{cd})(ef-dg)} \\ & + \frac{g\sqrt{d+ex}(2\sqrt{-aeg}-\sqrt{c}(dg+ef))}{\sqrt{-a}\sqrt{f+gx}(\sqrt{cd}-\sqrt{-ae})(\sqrt{cf}-\sqrt{-ag})(ef-dg)^2} \\ & + \frac{g\sqrt{d+ex}(2\sqrt{-aeg}+\sqrt{c}(dg+ef))}{\sqrt{-a}\sqrt{f+gx}(\sqrt{-ae}+\sqrt{cd})(\sqrt{-ag}+\sqrt{cf})(ef-dg)^2} \\ & + \frac{c \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{cf}-\sqrt{-ag}}{\sqrt{f+gx}\sqrt{cd}-\sqrt{-ae}}\right)}{\sqrt{-a}(\sqrt{cd}-\sqrt{-ae})^{3/2}(\sqrt{cf}-\sqrt{-ag})^{3/2}} - \frac{c \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-ag}+\sqrt{cf}}{\sqrt{f+gx}\sqrt{-ae}+\sqrt{cd}}\right)}{\sqrt{-a}(\sqrt{-ae}+\sqrt{cd})^{3/2}(\sqrt{-ag}+\sqrt{cf})^{3/2}} \end{aligned}$$

[Out]  $-(e/(\text{Sqrt}[-a] * (\text{Sqrt}[c] * d - \text{Sqrt}[-a] * e) * (e * f - d * g) * \text{Sqrt}[d + e * x] * \text{Sqrt}[f + g * x])) + e/(\text{Sqrt}[-a] * (\text{Sqrt}[c] * d + \text{Sqrt}[-a] * e) * (e * f - d * g) * \text{Sqrt}[d + e * x] * \text{Sqrt}[f + g * x]) + (g * (2 * \text{Sqrt}[-a] * e * g - \text{Sqrt}[c] * (e * f + d * g)) * \text{Sqrt}[d + e * x]) / (\text{Sqrt}[-a] * (\text{Sqrt}[c] * d - \text{Sqrt}[-a] * e) * (\text{Sqrt}[c] * f - \text{Sqrt}[-a] * g) * (e * f - d * g) ^ 2 * \text{Sqrt}[f + g * x]) + (g * (2 * \text{Sqrt}[-a] * e * g + \text{Sqrt}[c] * (e * f + d * g)) * \text{Sqrt}[d + e * x]) / (\text{Sqrt}[-a] * (\text{Sqrt}[c] * d + \text{Sqrt}[-a] * e) * (\text{Sqrt}[c] * f + \text{Sqrt}[-a] * g) * (e * f - d * g) ^ 2 * \text{Sqrt}[f + g * x]) + (c * \text{ArcTanh}[(\text{Sqrt}[\text{Sqrt}[c] * f - \text{Sqrt}[-a] * g] * \text{Sqrt}[d + e * x]) / (\text{Sqrt}[\text{Sqrt}[c] * d - \text{Sqrt}[-a] * e] * \text{Sqrt}[f + g * x])]) / (\text{Sqrt}[-a] * (\text{Sqrt}[c] * d - \text{Sqrt}[-a] * e) ^ (3/2) * (\text{Sqrt}[c] * f - \text{Sqrt}[-a] * g) ^ (3/2)) - (c * \text{ArcTanh}[(\text{Sqrt}[\text{Sqrt}[c] * f + \text{Sqrt}[-a] * g] * \text{Sqrt}[d + e * x]) / (\text{Sqrt}[\text{Sqrt}[c] * d + \text{Sqrt}[-a] * e] * \text{Sqrt}[f + g * x])]) / (\text{Sqrt}[-a] * (\text{Sqrt}[c] * d + \text{Sqrt}[-a] * e) ^ (3/2) * (\text{Sqrt}[c] * f + \text{Sqrt}[-a] * g) ^ (3/2))$

**Rubi [A]** time = 3.44717, antiderivative size = 543, normalized size of antiderivative = 0.99, number of steps used = 12, number of rules used = 6, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$\begin{aligned} & -\frac{e}{\sqrt{-a}\sqrt{d+ex}\sqrt{f+gx}(\sqrt{cd}-\sqrt{-ae})(ef-dg)} + \frac{e}{\sqrt{-a}\sqrt{d+ex}\sqrt{f+gx}(\sqrt{-ae}+\sqrt{cd})(ef-dg)} \\ & + \frac{g\sqrt{d+ex}(2aeg-\sqrt{-a}\sqrt{c}(dg+ef))}{a\sqrt{f+gx}(\sqrt{-ae}+\sqrt{cd})(\sqrt{-ag}+\sqrt{cf})(ef-dg)^2} \\ & + \frac{g\sqrt{d+ex}(\sqrt{-a}\sqrt{c}(dg+ef)+2aeg)}{a\sqrt{f+gx}(\sqrt{cd}-\sqrt{-ae})(\sqrt{cf}-\sqrt{-ag})(ef-dg)^2} \\ & + \frac{c \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{cf}-\sqrt{-ag}}{\sqrt{f+gx}\sqrt{cd}-\sqrt{-ae}}\right)}{\sqrt{-a}(\sqrt{cd}-\sqrt{-ae})^{3/2}(\sqrt{cf}-\sqrt{-ag})^{3/2}} - \frac{c \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{-ag}+\sqrt{cf}}{\sqrt{f+gx}\sqrt{-ae}+\sqrt{cd}}\right)}{\sqrt{-a}(\sqrt{-ae}+\sqrt{cd})^{3/2}(\sqrt{-ag}+\sqrt{cf})^{3/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)^(3/2)\*(f + g\*x)^(3/2)\*(a + c\*x^2)), x]

[Out]  $-(e/(\text{Sqrt}[-a] * (\text{Sqrt}[c] * d - \text{Sqrt}[-a] * e) * (e * f - d * g) * \text{Sqrt}[d + e * x] * \text{Sqrt}[f + g * x])) + e/(\text{Sqrt}[-a] * (\text{Sqrt}[c] * d + \text{Sqrt}[-a] * e) * (e * f - d * g) * \text{Sqrt}[d + e * x] * \text{Sqrt}[f + g * x]) + (g * (2 * a * e * g - \text{Sqrt}[-a] * \text{Sqrt}[c] * (e * f + d * g)) * \text{Sqrt}[d + e * x]) / (a * (\text{Sqrt}[c] * d + \text{Sqrt}[-a] * e) * (\text{Sqrt}[c] * f + \text{Sqrt}[-a] * g) * (e * f - d * g) ^ 2 * \text{Sqrt}[f + g * x]) + (g * (2 * a * e * g + \text{Sqrt}[-a] * \text{Sqrt}[c] * (e * f + d * g)) * \text{Sqrt}[d + e * x]) / (a * (\text{Sqrt}[c] * d - \text{Sqrt}[-a] * e) * (\text{Sqrt}[c] * f - \text{Sqrt}[-a] * g) * (e * f - d * g) ^ 2 * \text{Sqrt}[f + g * x]) + (c * \text{ArcTanh}[(\text{Sqrt}[\text{Sqrt}[c] * f - \text{Sqrt}[-a] * g] * \text{Sqrt}[d + e * x]) / (\text{Sqrt}[\text{Sqrt}[c] * d - \text{Sqrt}[-a] * e] * \text{Sqrt}[f + g * x])]) / (\text{Sqrt}[-a] * (\text{Sqrt}[c] * d - \text{Sqrt}[-a] * e) ^ (3/2) * (\text{Sqrt}[c] * f - \text{Sqrt}[-a] * g) ^ (3/2)) - (c * \text{ArcTanh}[(\text{Sqrt}[\text{Sqrt}[c] * f + \text{Sqrt}[-a] * g] * \text{Sqrt}[d + e * x]) / (\text{Sqrt}[\text{Sqrt}[c] * d + \text{Sqrt}[-a] * e] * \text{Sqrt}[f + g * x])]) / (\text{Sqrt}[-a] * (\text{Sqrt}[c] * d + \text{Sqrt}[-a] * e) ^ (3/2) * (\text{Sqrt}[c] * f + \text{Sqrt}[-a] * g) ^ (3/2))$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(e*x+d)**(3/2)/(g*x+f)**(3/2)/(c*x**2+a), x)`

[Out] Timed out

**Mathematica [C]** time = 9.17378, size = 1619, normalized size = 2.95

$$\sqrt{d+ex}\sqrt{f+gx}\left(-\frac{2e^3}{(cd^2+ae^2)(ef-dg)^2(d+ex)}-\frac{2g^3}{(ef-dg)^2(cf^2+ag^2)(f+gx)}\right)$$

$$+ \frac{c(icdf+\sqrt{a}\sqrt{cef}+\sqrt{a}\sqrt{cdg}-iaeg)\log\left(\frac{2i\sqrt{a}(cd^2+ae^2)\sqrt{d+ex}\sqrt{f+gx}(cf^2+ag^2)}{\sqrt{c}(cdf-i\sqrt{a}\sqrt{cef}-i\sqrt{a}\sqrt{cdg}-aeg)(cx-i\sqrt{a}\sqrt{c})}+\frac{-2i\sqrt{ac}^{5/2}f^3d^3+a^2cg^3d^3-2ia^{3/2}c^{3/2}fg^2d^3+a^2c^2f^2gd^3+ia^{3/2}c^{3/2}g^3xd^3+i\sqrt{ac}^{5/2}f^2gxd^3+ac^2ef^3d^2}{\sqrt{c}(cdf-i\sqrt{a}\sqrt{cef}-i\sqrt{a}\sqrt{cdg}-aeg)(cx-i\sqrt{a}\sqrt{c})}\right)}{c(-icdf+\sqrt{a}\sqrt{cef}+\sqrt{a}\sqrt{cdg}+iaeg)\log\left(\frac{2i\sqrt{ac}^{5/2}f^3d^3+a^2cg^3d^3+2ia^{3/2}c^{3/2}fg^2d^3+ac^2f^2gd^3+ia^{3/2}c^{3/2}g^3xd^3+i\sqrt{ac}^{5/2}f^2gxd^3+ac^2ef^3d^2}{\sqrt{c}(cdf-i\sqrt{a}\sqrt{cef}-i\sqrt{a}\sqrt{cdg}-aeg)(cx-i\sqrt{a}\sqrt{c})}\right)}$$

Antiderivative was successfully verified.

[In] `Integrate[1/((d + e*x)^(3/2)*(f + g*x)^(3/2)*(a + c*x^2)), x]`

[Out] 
$$\sqrt{d+e x} \sqrt{f+g x} \left( \frac{(-2 e^3) / ((c d^2+a e^2) (e f-d g)^2 (d+e x)) - (2 g^3) / ((e f-d g)^2 (c f^2+a g^2) (f+g x))}{(c d^2+a e^2) (e f-d g)^2 (d+e x)} + \frac{(c (I c d f + \sqrt{a} \sqrt{c e f} + \sqrt{a} \sqrt{c d g} - I a e g) \log\left(\frac{2 i \sqrt{a} (c d^2+a e^2) \sqrt{d+e x} \sqrt{f+g x} (c f^2+a g^2)}{\sqrt{c} (c d f-i \sqrt{a} \sqrt{c e f}-i \sqrt{a} \sqrt{c d g}-a e g) (c x-i \sqrt{a} \sqrt{c})}+\frac{-2 i \sqrt{a c}^{5 / 2} f^3 d^3+a^2 c g^3 d^3-2 i a^{3 / 2} c^{3 / 2} f g^2 d^3+a^2 c^2 f^2 g d^3+i a^{3 / 2} c^{3 / 2} g^3 x d^3+i \sqrt{a c}^{5 / 2} f^2 g x d^3+a c^2 e f^3 d^2}{\sqrt{c} (c d f-i \sqrt{a} \sqrt{c e f}-i \sqrt{a} \sqrt{c d g}-a e g) (c x-i \sqrt{a} \sqrt{c})}\right)}{c(-i c d f+\sqrt{a} \sqrt{c e f}+\sqrt{a} \sqrt{c d g}+i a e g) \log\left(\frac{2 i \sqrt{a c}^{5 / 2} f^3 d^3+a^2 c g^3 d^3+2 i a^{3 / 2} c^{3 / 2} f g^2 d^3+a c^2 f^2 g d^3+i a^{3 / 2} c^{3 / 2} g^3 x d^3+i \sqrt{a c}^{5 / 2} f^2 g x d^3+a c^2 e f^3 d^2}{\sqrt{c} (c d f-i \sqrt{a} \sqrt{c e f}-i \sqrt{a} \sqrt{c d g}-a e g) (c x-i \sqrt{a} \sqrt{c})}\right)}$$

$\text{Sqrt}[a]*e*(c*d^2 + a*e^2)*\text{Sqrt}[\text{Sqrt}[c]*f - I*\text{Sqrt}[a]*g]*(c*f^2 + a*g^2)$

**Maple [B]** time = 0.231, size = 30648, normalized size = 55.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x+d)^(3/2)/(g*x+f)^(3/2)/(c*x^2+a), x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + a)(ex + d)^{\frac{3}{2}}(gx + f)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((c*x^2 + a)*(e*x + d)^(3/2)*(g*x + f)^(3/2)), x, algorithm="maxima")`

[Out] `integrate(1/((c*x^2 + a)*(e*x + d)^(3/2)*(g*x + f)^(3/2)), x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((c*x^2 + a)*(e*x + d)^(3/2)*(g*x + f)^(3/2)), x, algorithm="fricas")`

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x+d)**(3/2)/(g*x+f)**(3/2)/(c*x**2+a), x)`

[Out] Timed out

**GIAC/XCAS [A]** time = 0.70614, size = 4, normalized size = 0.01

$sage_0x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^2 + a)*(e*x + d)^(3/2)*(g*x + f)^(3/2)),x, algorithm="giac")
```

```
[Out] sage0*x
```



$$3.618 \quad \int \frac{\sqrt{x}}{\sqrt{1+x(1+x^2)}} dx$$

**Optimal.** Leaf size=65

$$-\frac{1}{2}(1-i)^{3/2} \tanh^{-1}\left(\frac{\sqrt{1-i}\sqrt{x}}{\sqrt{x+1}}\right) - \frac{1}{2}(1+i)^{3/2} \tanh^{-1}\left(\frac{\sqrt{1+i}\sqrt{x}}{\sqrt{x+1}}\right)$$

[Out]  $-\left((1-I)^{(3/2)} \operatorname{ArcTanh}\left[\frac{\operatorname{Sqrt}[1-I] \operatorname{Sqrt}[x]}{\operatorname{Sqrt}[1+x]}\right]\right)/2 - \left((1+I)^{(3/2)} \operatorname{ArcTanh}\left[\frac{\operatorname{Sqrt}[1+I] \operatorname{Sqrt}[x]}{\operatorname{Sqrt}[1+x]}\right]\right)/2$

**Rubi [A]** time = 0.126082, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$-\frac{1}{2}(1-i)^{3/2} \tanh^{-1}\left(\frac{\sqrt{1-i}\sqrt{x}}{\sqrt{x+1}}\right) - \frac{1}{2}(1+i)^{3/2} \tanh^{-1}\left(\frac{\sqrt{1+i}\sqrt{x}}{\sqrt{x+1}}\right)$$

Antiderivative was successfully verified.

[In] `Int[Sqrt[x]/(Sqrt[1+x]*(1+x^2)),x]`

[Out]  $-\left((1-I)^{(3/2)} \operatorname{ArcTanh}\left[\frac{\operatorname{Sqrt}[1-I] \operatorname{Sqrt}[x]}{\operatorname{Sqrt}[1+x]}\right]\right)/2 - \left((1+I)^{(3/2)} \operatorname{ArcTanh}\left[\frac{\operatorname{Sqrt}[1+I] \operatorname{Sqrt}[x]}{\operatorname{Sqrt}[1+x]}\right]\right)/2$

**Rubi in Sympy [A]** time = 19.2993, size = 121, normalized size = 1.86

$$\frac{i\left(\sqrt{1+\sqrt{2}}-\sqrt{-\sqrt{2}+1}\right) \operatorname{atanh}\left(\frac{2\sqrt{x}}{\sqrt{x+1}\left(\sqrt{1+\sqrt{2}}-i\sqrt{-1+\sqrt{2}}\right)}\right)}{2} + \frac{i\left(\sqrt{1+\sqrt{2}}+\sqrt{-\sqrt{2}+1}\right) \operatorname{atanh}\left(\frac{2\sqrt{x}}{\sqrt{x+1}\left(\sqrt{1+\sqrt{2}}+i\sqrt{-1+\sqrt{2}}\right)}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(x**(1/2)/(x**2+1)/(1+x)**(1/2),x)`

[Out]  $-I^*(\operatorname{sqrt}(1+\operatorname{sqrt}(2))-\operatorname{sqrt}(-\operatorname{sqrt}(2)+1))^* \operatorname{atanh}(2^* \operatorname{sqrt}(x)/(\operatorname{sqrt}(x+1)^*(\operatorname{sqrt}(1+\operatorname{sqrt}(2))-I^* \operatorname{sqrt}(-1+\operatorname{sqrt}(2)))))/2 + I^*(\operatorname{sqrt}(1+\operatorname{sqrt}(2))+\operatorname{sqrt}(-\operatorname{sqrt}(2)+1))^* \operatorname{atanh}(2^* \operatorname{sqrt}(x)/(\operatorname{sqrt}(x+1)^*(\operatorname{sqrt}(1+\operatorname{sqrt}(2))+I^* \operatorname{sqrt}(-1+\operatorname{sqrt}(2)))))/2$

**Mathematica [A]** time = 0.126005, size = 65, normalized size = 1.

$$\frac{1}{2}\left(\sqrt{2-2i} \tan^{-1}\left((1-i)^{3/2} \sqrt{\frac{x}{2x+2}}\right) + \sqrt{2+2i} \tan^{-1}\left((1+i)^{3/2} \sqrt{\frac{x}{2x+2}}\right)\right)$$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[x]/(Sqrt[1+x]*(1+x^2)),x]`

[Out]  $(\operatorname{Sqrt}[2-2^*I]^* \operatorname{ArcTan}[(1-I)^{(3/2)} \operatorname{Sqrt}[x/(2+2^*x)]] + \operatorname{Sqrt}[2+2^*I]^* \operatorname{ArcTan}[(1+I)^{(3/2)} \operatorname{Sqrt}[x/(2+2^*x)]])/2$

**Maple [B]** time = 0.19, size = 305, normalized size = 4.7

$$\frac{(\sqrt{2}-1+x)\sqrt{2}}{(-16+12\sqrt{2})\sqrt{1+\sqrt{2}}}\sqrt{\frac{x(1+x)}{(\sqrt{2}-1+x)^2}}\left(\sqrt{-2+2\sqrt{2}}\arctan\left(\frac{(\sqrt{2}+1-x)(-4+3\sqrt{2})(3+2\sqrt{2})\sqrt{-2+2\sqrt{2}}(\sqrt{2}-1)}{4x(1+x)}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^(1/2)/(x^2+1)/(1+x)^(1/2), x)

[Out] 1/4/x^(1/2)/(1+x)^(1/2)\*(x\*(1+x)/(2^(1/2)-1+x)^2)^(1/2)\*(2^(1/2)-1+x)\*((-2+2\*2^(1/2))^(1/2)\*arctan(1/4\*(2^(1/2)+1-x)\*(-4+3\*2^(1/2))\* (3+2\*2^(1/2))\*(-2+2\*2^(1/2))^(1/2)\*(x\*(1+x)\*(-4+3\*2^(1/2))\* (3\*2^(1/2)+4)/(2^(1/2)-1+x)^2)^(1/2)\*(2^(1/2)-1+x)/x/(1+x))^(1/2)\*2^(1/2)-2\*(-2+2\*2^(1/2))^(1/2)\*arctan(1/4\*(2^(1/2)+1-x)\* (-4+3\*2^(1/2))\* (3+2\*2^(1/2))\*(-2+2\*2^(1/2))^(1/2)\*(x\*(1+x)\*(-4+3\* 2^(1/2))\* (3\*2^(1/2)+4)/(2^(1/2)-1+x)^2)^(1/2)\*(2^(1/2)-1+x)/x/(1+ x))^(1/2)/(1+2^(1/2))^(1/2)+4\*arctanh(2^(1/2)\*(x\*(1+x)/(2^(1/2)-1+x)^2)^(1/2)/(1+2^(1/2))^(1/2))\*2^(1/2)-6\*arctanh(2^(1/2)\*(x\*(1+x)/(2^(1/2)-1+x)^2)^(1/2)/(1+2^(1/2))^(1/2)))\*2^(1/2)/(-4+3\*2^(1/2))/(1+ 2^(1/2))^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{(x^2+1)\sqrt{x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x)/((x^2 + 1)\*sqrt(x + 1)), x, algorithm="maxima")

[Out] integrate(sqrt(x)/((x^2 + 1)\*sqrt(x + 1)), x)

**Fricas [A]** time = 0.32074, size = 1146, normalized size = 17.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x)/((x^2 + 1)\*sqrt(x + 1)), x, algorithm="fricas")

[Out] -1/4\*sqrt(2)\*(2^(1/4)\*(sqrt(2)-1)\*log(2\*(2^(3/4)\*(sqrt(2)\*(12\*x + 5) - 17\*x - 7)\*sqrt((sqrt(2)-2)/(2\*sqrt(2)-3)) - (2^(3/4)\*(12\*sqrt(2) - 17)\*sqrt((sqrt(2)-2)/(2\*sqrt(2)-3)) + 14\*sqrt(2)\*x - 20\*x)\*sqrt(x+1)\*sqrt(x) - 20\*x^2 + 7\*sqrt(2)\*(2\*x^2 + x + 1) + sqrt(2)\*(7\*sqrt(2)-10) - 10\*x - 10)/(7\*sqrt(2)-10)) - 2^(1/4)\*(sqrt(2)-1)\*log(-2\*(2^(3/4)\*(sqrt(2)\*(12\*x + 5) - 17\*x - 7)\*sqrt((sqrt(2)-2)/(2\*sqrt(2)-3)) - (2^(3/4)\*(12\*sqrt(2) - 17)\*sqrt((sqrt(2)-2)/(2\*sqrt(2)-3)) - 14\*sqrt(2)\*x + 20\*x)\*sqrt(x+1)\*sqrt(x) + 20\*x^2 - 7\*sqrt(2)\*(2\*x^2 + x + 1) - sqrt(2)\*(7\*sqrt(2)-10) + 10\*x + 10)/(7\*sqrt(2)-10)) - 4\*2^(1/4)\*arctan(-(sqrt(2)\*(sqrt(2)-2)\*sqrt((sqrt(2)-2)/(2\*sqrt(2)-3)) + 2^(3/4))/(sqrt(2)\*sqrt(x+1)\*sqrt(x)\*(sqrt(2)-2)\*sqrt((sqrt(2)-2)/(2\*sqrt(2)-3)) - sqrt(2)\*(sqrt(2)\*x - 2\*x)\*sqrt((sqrt(2)-2)/(2\*sqrt(2)-3)) - 2\*(sqrt(2)-1)\*sqrt((2^(3/4)\*(sqrt(2)\*(12\*x + 5) - 17\*x - 7)\*sqrt((sqrt(2)-2)/(2\*sqrt(2)-3)) - (2^(3/4)\*(12\*sqrt(2) - 17)\*sqrt((sqrt(2)-2)/(2\*sqrt(2)-3)) + 14\*sqrt(2)\*x - 20\*x)\*sqrt(x+1)\*sqrt(x) - 20\*x^2 + 7\*sqrt(2)\*(2\*x^2 + x + 1) + sqrt(2)\*(7\*sqrt(2)-10) - 10\*x - 10)/(7\*sqrt(2)-10))

$(2*x - 20*x)*\sqrt{x + 1}*\sqrt{x} - 20*x^2 + 7*\sqrt{2}*(2*x^2 + x + 1) + \sqrt{2}*(7*\sqrt{2} - 10) - 10*x - 10)/(7*\sqrt{2} - 10))*\sqrt{(\sqrt{2} - 2)/(2*\sqrt{2} - 3)} - 2^{(1/4)}*(\sqrt{2} - 2)) + 4*2^{(1/4)}*\arctan(-(\sqrt{2})*(\sqrt{2} - 2)*\sqrt{(\sqrt{2} - 2)/(2*\sqrt{2} - 3)} - 2^{(3/4)})/(\sqrt{2})*\sqrt{x + 1}*\sqrt{x}*(\sqrt{2} - 2)*\sqrt{(\sqrt{2} - 2)/(2*\sqrt{2} - 3)} - \sqrt{2}*(\sqrt{2}*x - 2*x)*\sqrt{(\sqrt{2} - 2)/(2*\sqrt{2} - 3)} - 2*(\sqrt{2} - 1)*\sqrt{-(2^{(3/4)}*(\sqrt{2}*(12*x + 5) - 17*x - 7)*\sqrt{(\sqrt{2} - 2)/(2*\sqrt{2} - 3)} - (2^{(3/4)}*(12*\sqrt{2} - 17)*\sqrt{(\sqrt{2} - 2)/(2*\sqrt{2} - 3)} - 14*\sqrt{2}*x + 20*x)*\sqrt{x + 1}*\sqrt{x} + 20*x^2 - 7*\sqrt{2}*(2*x^2 + x + 1) - \sqrt{2}*(7*\sqrt{2} - 10) + 10*x + 10)/(7*\sqrt{2} - 10))*\sqrt{(\sqrt{2} - 2)/(2*\sqrt{2} - 3)} + 2^{(1/4)}*(\sqrt{2} - 2))))/((\sqrt{2} - 2)*\sqrt{(\sqrt{2} - 2)/(2*\sqrt{2} - 3)})$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{\sqrt{x+1}(x^2+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x\*\*(1/2)/(x\*\*2+1)/(1+x)\*\*(1/2), x)

[Out] Integral(sqrt(x)/(sqrt(x + 1)\*(x\*\*2 + 1)), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{x}}{(x^2+1)\sqrt{x+1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(x)/((x^2 + 1)\*sqrt(x + 1)), x, algorithm="giac")

[Out] integrate(sqrt(x)/((x^2 + 1)\*sqrt(x + 1)), x)

$$3.619 \quad \int \frac{(f+gx)^2 \sqrt{1-x^2}}{(1-x)^4} dx$$

**Optimal.** Leaf size=80

$$\frac{(x+1)^4(f+g)^2}{5(1-x^2)^{5/2}} + \frac{(x+1)^3(f-9g)(f+g)}{15(1-x^2)^{3/2}} + \frac{2g^2(x+1)}{\sqrt{1-x^2}} - g^2 \sin^{-1}(x)$$

[Out] ((f + g)^2\*(1 + x)^4)/(5\*(1 - x^2)^(5/2)) + ((f - 9\*g)\*(f + g)\*(1 + x)^3)/(15\*(1 - x^2)^(3/2)) + (2\*g^2\*(1 + x))/Sqrt[1 - x^2] - g^2\*ArcSin[x]

**Rubi [A]** time = 0.27267, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$

$$\frac{(x+1)^4(f+g)^2}{5(1-x^2)^{5/2}} + \frac{(x+1)^3(f-9g)(f+g)}{15(1-x^2)^{3/2}} + \frac{2g^2(x+1)}{\sqrt{1-x^2}} - g^2 \sin^{-1}(x)$$

Antiderivative was successfully verified.

[In] Int[((f + g\*x)^2\*Sqrt[1 - x^2])/(1 - x)^4, x]

[Out] ((f + g)^2\*(1 + x)^4)/(5\*(1 - x^2)^(5/2)) + ((f - 9\*g)\*(f + g)\*(1 + x)^3)/(15\*(1 - x^2)^(3/2)) + (2\*g^2\*(1 + x))/Sqrt[1 - x^2] - g^2\*ArcSin[x]

**Rubi in Sympy [A]** time = 21.7769, size = 85, normalized size = 1.06

$$-g^2 \operatorname{asin}(x) + \frac{2g^2 \sqrt{-x^2 + 1}}{-x + 1} - \frac{2g(f + g)(-x^2 + 1)^{\frac{3}{2}}}{3(-x + 1)^3} + \frac{(f + g)^2(-x^2 + 1)^{\frac{3}{2}}}{15(-x + 1)^3} + \frac{(f + g)^2(-x^2 + 1)^{\frac{3}{2}}}{5(-x + 1)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*2\*(-x\*\*2+1)\*\*(1/2)/(1-x)\*\*4, x)

[Out] -g\*\*2\*asin(x) + 2\*g\*\*2\*sqrt(-x\*\*2 + 1)/(-x + 1) - 2\*g\*(f + g)\*(-x\*\*2 + 1)\*\*(3/2)/(3\*(-x + 1)\*\*3) + (f + g)\*\*2\*(-x\*\*2 + 1)\*\*(3/2)/(15\*(-x + 1)\*\*3) + (f + g)\*\*2\*(-x\*\*2 + 1)\*\*(3/2)/(5\*(-x + 1)\*\*4)

**Mathematica [A]** time = 0.157479, size = 104, normalized size = 1.3

$$\frac{\sqrt{1-x^2} \left( \sqrt{x+1} (f^2 (x^2 - 3x - 4) - 2fg(4x^2 + 3x - 1) - 3g^2(13x^2 - 19x + 8)) + 30g^2(x-1)^{5/2} \sinh^{-1} \left( \frac{\sqrt{x-1}}{\sqrt{2}} \right) \right)}{15(x-1)^3 \sqrt{x+1}}$$

Antiderivative was successfully verified.

[In] Integrate[((f + g\*x)^2\*Sqrt[1 - x^2])/(1 - x)^4, x]

[Out] (Sqrt[1 - x^2]\*(Sqrt[1 + x]\*(f^2\*(-4 - 3\*x + x^2) - 2\*f\*g\*(-1 + 3\*x + 4\*x^2) - 3\*g^2\*(8 - 19\*x + 13\*x^2)) + 30\*g^2\*(-1 + x)^(5/2))\*ArcSinh[Sqrt[-1 + x]/Sqrt[2]])/(15\*(-1 + x)^3\*Sqrt[1 + x])

**Maple [A]** time = 0.022, size = 125, normalized size = 1.6

$$g^2 \left( \frac{1}{(-1+x)^2} (-(1+x)^2 - 2x + 2)^{\frac{3}{2}} + \sqrt{-(1+x)^2 - 2x + 2} - \arcsin(x) \right) \\ + (f^2 + 2fg + g^2) \left( \frac{1}{5(-1+x)^4} (-(1+x)^2 - 2x + 2)^{\frac{3}{2}} - \frac{1}{15(-1+x)^3} (-(1+x)^2 - 2x + 2)^{\frac{3}{2}} \right) \\ + \frac{2g(f+g)}{3(-1+x)^3} (-(1+x)^2 - 2x + 2)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^2*(-x^2+1)^(1/2)/(1-x)^4,x)`

[Out] `g^2*(1/(-1+x)^2*(-(-1+x)^2-2*x+2)^(3/2)+(-(-1+x)^2-2*x+2)^(1/2)-arcsin(x))+(f^2+2*f*g+g^2)*(1/5/(-1+x)^4*(-(-1+x)^2-2*x+2)^(3/2)-1/15/(-1+x)^3*(-(-1+x)^2-2*x+2)^(3/2))+2/3*g*(f+g)/(-1+x)^3*(-(-1+x)^2-2*x+2)^(3/2)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx+f)^2 \sqrt{-x^2+1}}{(x-1)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)^2*sqrt(-x^2+1)/(x-1)^4,x,algorithm="maxima")`

[Out] `integrate((g*x+f)^2*sqrt(-x^2+1)/(x-1)^4,x)`

**Fricas [A]** time = 0.28936, size = 416, normalized size = 5.2

$$3(f^2 + 2fg + 21g^2)x^5 - 20(f^2 - 2fg + 3g^2)x^4 + 5(7f^2 - 2fg - 21g^2)x^3 + 30(f^2 - 2fg + 5g^2)x^2 - 60(f^2 + g^2)x + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)^2*sqrt(-x^2+1)/(x-1)^4,x,algorithm="fricas")`

[Out] `1/15*(3*(f^2+2*f*g+21*g^2)*x^5-20*(f^2-2*f*g+3*g^2)*x^4+5*(7*f^2-2*f*g-21*g^2)*x^3+30*(f^2-2*f*g+5*g^2)*x^2-60*(f^2+g^2)*x+30*(g^2*x^5-5*g^2*x^4+5*g^2*x^3+5*g^2*x^2-10*g^2*x+4*g^2+(g^2*x^4-7*g^2*x^2+10*g^2*x-4*g^2)*sqrt(-x^2+1))*arctan((sqrt(-x^2+1)-1)/x)+5*((f^2-2*f*g-3*g^2)*x^4-(f^2-2*f*g-27*g^2)*x^3-6*(f^2-2*f*g+5*g^2)*x^2+12*(f^2+g^2)*x)*sqrt(-x^2+1))/(x^5-5*x^4+5*x^3+5*x^2+(x^4-7*x^2+10*x-4)*sqrt(-x^2+1)-10*x+4)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{-(x-1)(x+1)}(f+gx)^2}{(x-1)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*2\*(-x\*\*2+1)\*\*(1/2)/(1-x)\*\*4,x)

[Out] Integral(sqrt(-(x - 1)\*(x + 1))\*(f + g\*x)\*\*2/(x - 1)\*\*4, x)

**GIAC/XCAS [A]** time = 0.296428, size = 359, normalized size = 4.49

$$-g^2 \arcsin(x) + \frac{2 \left( 4f^2 - 2fg + 24g^2 + \frac{5f^2(\sqrt{-x^2+1}-1)}{x} - \frac{10fg(\sqrt{-x^2+1}-1)}{x} + \frac{105g^2(\sqrt{-x^2+1}-1)}{x} + \frac{25f^2(\sqrt{-x^2+1}-1)^2}{x^2} + \frac{10fg(\sqrt{-x^2+1}-1)^2}{x^2} + \frac{165g^2(\sqrt{-x^2+1}-1)^2}{x^2} \right)}{15 \left( \frac{\sqrt{-x^2+1}-1}{x} + 1 \right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^2\*sqrt(-x^2 + 1)/(x - 1)^4,x, algorithm="giac")

[Out] -g^2\*arcsin(x) + 2/15\*(4\*f^2 - 2\*f\*g + 24\*g^2 + 5\*f^2\*(sqrt(-x^2 + 1) - 1)/x - 10\*f\*g\*(sqrt(-x^2 + 1) - 1)/x + 105\*g^2\*(sqrt(-x^2 + 1) - 1)/x + 25\*f^2\*(sqrt(-x^2 + 1) - 1)^2/x^2 + 10\*f\*g\*(sqrt(-x^2 + 1) - 1)^2/x^2 + 165\*g^2\*(sqrt(-x^2 + 1) - 1)^2/x^2 + 15\*f^2\*(sqrt(-x^2 + 1) - 1)^3/x^3 - 30\*f\*g\*(sqrt(-x^2 + 1) - 1)^3/x^3 + 75\*g^2\*(sqrt(-x^2 + 1) - 1)^3/x^3 + 15\*f^2\*(sqrt(-x^2 + 1) - 1)^4/x^4 + 15\*g^2\*(sqrt(-x^2 + 1) - 1)^4/x^4)/((sqrt(-x^2 + 1) - 1)/x + 1)^5

$$3.620 \quad \int \frac{(1-a^2x^2)^{3/2}}{(1-ax)^2(c+dx)} dx$$

**Optimal.** Leaf size=107

$$\frac{(ac-d)^2 \tan^{-1}\left(\frac{a^2cx+d}{\sqrt{1-a^2x^2}\sqrt{a^2c^2-d^2}}\right)}{d^2\sqrt{a^2c^2-d^2}} - \frac{\sqrt{1-a^2x^2}}{d} - \frac{(ac-2d)\sin^{-1}(ax)}{d^2}$$

[Out] -(Sqrt[1 - a^2\*x^2]/d) - ((a\*c - 2\*d)\*ArcSin[a\*x])/d^2 + ((a\*c - d)^2\*ArcTan[(d + a^2\*c\*x)/(Sqrt[a^2\*c^2 - d^2]\*Sqrt[1 - a^2\*x^2]])/(d^2\*Sqrt[a^2\*c^2 - d^2])

**Rubi [A]** time = 0.498534, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$

$$\frac{(ac-d)^2 \tan^{-1}\left(\frac{a^2cx+d}{\sqrt{1-a^2x^2}\sqrt{a^2c^2-d^2}}\right)}{d^2\sqrt{a^2c^2-d^2}} - \frac{\sqrt{1-a^2x^2}}{d} - \frac{(ac-2d)\sin^{-1}(ax)}{d^2}$$

Antiderivative was successfully verified.

[In] Int[(1 - a^2\*x^2)^(3/2)/((1 - a\*x)^2\*(c + d\*x)), x]

[Out] -(Sqrt[1 - a^2\*x^2]/d) - ((a\*c - 2\*d)\*ArcSin[a\*x])/d^2 + ((a\*c - d)^2\*ArcTan[(d + a^2\*c\*x)/(Sqrt[a^2\*c^2 - d^2]\*Sqrt[1 - a^2\*x^2]])/(d^2\*Sqrt[a^2\*c^2 - d^2])

**Rubi in Sympy [A]** time = 43.4141, size = 92, normalized size = 0.86

$$-\frac{\sqrt{-a^2x^2+1}}{d} + \frac{\text{asin}(ax)}{d} - \frac{(-ac+d)^{\frac{3}{2}} \text{atanh}\left(\frac{a^2cx+d}{\sqrt{-ac+d}\sqrt{ac+d}\sqrt{-a^2x^2+1}}\right)}{d^2\sqrt{ac+d}} + \frac{(-ac+d)\text{asin}(ax)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-a\*\*2\*x\*\*2+1)\*\*(3/2)/(-a\*x+1)\*\*2/(d\*x+c), x)

[Out] -sqrt(-a\*\*2\*x\*\*2 + 1)/d + asin(a\*x)/d - (-a\*c + d)\*\*(3/2)\*atanh((a\*\*2\*c\*x + d)/(sqrt(-a\*c + d)\*sqrt(a\*c + d)\*sqrt(-a\*\*2\*x\*\*2 + 1)))/(d\*\*2\*sqrt(a\*c + d)) + (-a\*c + d)\*asin(a\*x)/d\*\*2

**Mathematica [C]** time = 0.262097, size = 148, normalized size = 1.38

$$-\frac{i(d-ac)^2 \log\left(\frac{2d^3(\sqrt{1-a^2x^2}\sqrt{a^2c^2-d^2+ia^2cx+id})}{(d-ac)^2\sqrt{a^2c^2-d^2}(c+dx)}\right)}{\sqrt{a^2c^2-d^2}} + \frac{d\sqrt{1-a^2x^2} + (ac-2d)\sin^{-1}(ax)}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(1 - a^2\*x^2)^(3/2)/((1 - a\*x)^2\*(c + d\*x)), x]

[Out] -((d\*Sqrt[1 - a^2\*x^2] + (a\*c - 2\*d)\*ArcSin[a\*x] + (I\*(-(a\*c) + d)^2\*Log[(2\*d^3\*(I\*d + I\*a^2\*c\*x + Sqrt[a^2\*c^2 - d^2]\*Sqrt[1 - a^2\*x^2]))/((-a\*c) + d)^2\*Sqrt[a^2\*c^2 - d^2]\*(c + d\*x)]))/Sqrt[a^2\*c^2 - d^2]

$$2^*c^2 - d^2])/d^2)$$

**Maple [B]** time = 0.053, size = 1178, normalized size = 11.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-a^2*x^2+1)^(3/2)/(-a*x+1)^2/(d*x+c), x)`

[Out] 
$$\begin{aligned} & -1/a^2/(a*c+d)/(x-1/a)^2 * (- (x-1/a)^2 * a^2 - 2 * (x-1/a) * a)^{(5/2)} - 1/(a * \\ & c+d) * (- (x-1/a)^2 * a^2 - 2 * (x-1/a) * a)^{(3/2)} + 3/2 * a/(a*c+d) * (- (x-1/a)^2 \\ & * a^2 - 2 * (x-1/a) * a)^{(1/2)} * x + 3/2 * a/(a*c+d)/(a^2)^{(1/2)} * \arctan((a^2)^{(1/2)} \\ & * x / (- (x-1/a)^2 * a^2 - 2 * (x-1/a) * a)^{(1/2)}) + 1/3 * d/(a*c+d)^2 * (- (x+ \\ & c/d)^2 * a^2 + 2 * a^2 * c/d * (x+c/d) - (a^2 * c^2 - d^2)/d^2)^{(3/2)} + 1/2/(a*c+d) \\ & ^2 * a^2 * c * (- (x+c/d)^2 * a^2 + 2 * a^2 * c/d * (x+c/d) - (a^2 * c^2 - d^2)/d^2)^{(1/2)} \\ & * x + 3/2/(a*c+d)^2 * a^2 * c/(a^2)^{(1/2)} * \arctan((a^2)^{(1/2)} * x / (- (x+c/ \\ & d)^2 * a^2 + 2 * a^2 * c/d * (x+c/d) - (a^2 * c^2 - d^2)/d^2)^{(1/2)}) - 1/d/(a*c+d)^2 \\ & * (- (x+c/d)^2 * a^2 + 2 * a^2 * c/d * (x+c/d) - (a^2 * c^2 - d^2)/d^2)^{(1/2)} * a^2 * \\ & c^2 + d/(a*c+d)^2 * (- (x+c/d)^2 * a^2 + 2 * a^2 * c/d * (x+c/d) - (a^2 * c^2 - d^2)/d \\ & ^2)^{(1/2)} - 1/d^2/(a*c+d)^2 * a^4 * c^3/(a^2)^{(1/2)} * \arctan((a^2)^{(1/2)} * \\ & x / (- (x+c/d)^2 * a^2 + 2 * a^2 * c/d * (x+c/d) - (a^2 * c^2 - d^2)/d^2)^{(1/2)}) - 1/d \\ & ^3/(a*c+d)^2 / (- (a^2 * c^2 - d^2)/d^2)^{(1/2)} * \ln((-2 * (a^2 * c^2 - d^2)/d^2 + \\ & 2 * a^2 * c/d * (x+c/d) + 2 * (- (a^2 * c^2 - d^2)/d^2)^{(1/2)} * (- (x+c/d)^2 * a^2 + 2 * \\ & a^2 * c/d * (x+c/d) - (a^2 * c^2 - d^2)/d^2)^{(1/2)}) / (x+c/d) * a^4 * c^4 + 2/d/(a \\ & * c+d)^2 / (- (a^2 * c^2 - d^2)/d^2)^{(1/2)} * \ln((-2 * (a^2 * c^2 - d^2)/d^2 + 2 * a^2 \\ & * c/d * (x+c/d) + 2 * (- (a^2 * c^2 - d^2)/d^2)^{(1/2)} * (- (x+c/d)^2 * a^2 + 2 * a^2 * c \\ & /d * (x+c/d) - (a^2 * c^2 - d^2)/d^2)^{(1/2)}) / (x+c/d) * a^2 * c^2 - d/(a*c+d)^2 \\ & / (- (a^2 * c^2 - d^2)/d^2)^{(1/2)} * \ln((-2 * (a^2 * c^2 - d^2)/d^2 + 2 * a^2 * c/d * (x \\ & +c/d) + 2 * (- (a^2 * c^2 - d^2)/d^2)^{(1/2)} * (- (x+c/d)^2 * a^2 + 2 * a^2 * c/d * (x+c \\ & /d) - (a^2 * c^2 - d^2)/d^2)^{(1/2)}) / (x+c/d) - 1/3 * d/(a*c+d)^2 * (- (x-1/a)^2 \\ & * a^2 - 2 * (x-1/a) * a)^{(3/2)} + 1/2 * d/(a*c+d)^2 * a * (- (x-1/a)^2 * a^2 - 2 * (x-1 \\ & /a) * a)^{(1/2)} * x + 1/2 * d/(a*c+d)^2 * a/(a^2)^{(1/2)} * \arctan((a^2)^{(1/2)} * x \\ & / (- (x-1/a)^2 * a^2 - 2 * (x-1/a) * a)^{(1/2)}) \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-a^2*x^2 + 1)^(3/2)/((a*x - 1)^2*(d*x + c)), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.415777, size = 1, normalized size = 0.01

$$\left[ \frac{a^2 dx^2 + \left( ac - \sqrt{-a^2 x^2 + 1}(ac - d) - d \right) \sqrt{-\frac{ac-d}{ac+d}} \log \left( \frac{cdx - (a^2 c^2 - d^2)x^2 + c^2 - \sqrt{-a^2 x^2 + 1}(cdx + c^2) - ((acd + d^2)x^2 - \sqrt{-a^2 x^2 + 1}(ac^2 + cd)x + (acd + d^2))}{dx - \sqrt{-a^2 x^2 + 1}(dx + c) + c} \right)}{\sqrt{-a^2 x^2 + 1}d^2 - d^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-a^2*x^2 + 1)^(3/2)/((a*x - 1)^2*(d*x + c)), x, algorithm="fricas")`



```
[Out] [(a^2*d*x^2 + (a*c - sqrt(-a^2*x^2 + 1))*(a*c - d) - d)*sqrt(-(a*c
- d)/(a*c + d))*log((c*d*x - (a^2*c^2 - d^2)*x^2 + c^2 - sqrt(-a
^2*x^2 + 1)*(c*d*x + c^2) - ((a*c*d + d^2)*x^2 - sqrt(-a^2*x^2 +
1)*(a*c^2 + c*d)*x + (a*c^2 + c*d)*x)*sqrt(-(a*c - d)/(a*c + d)))
/(d*x - sqrt(-a^2*x^2 + 1)*(d*x + c) + c) - 2*(a*c - sqrt(-a^2*x
^2 + 1)*(a*c - 2*d) - 2*d)*arctan((sqrt(-a^2*x^2 + 1) - 1)/(a*x))
)/(sqrt(-a^2*x^2 + 1)*d^2 - d^2), (a^2*d*x^2 + 2*(a*c - sqrt(-a^2
*x^2 + 1)*(a*c - d) - d)*sqrt((a*c - d)/(a*c + d))*arctan(-(d*x -
sqrt(-a^2*x^2 + 1)*c + c)/((a*c + d)*x*sqrt((a*c - d)/(a*c + d))
)) - 2*(a*c - sqrt(-a^2*x^2 + 1)*(a*c - 2*d) - 2*d)*arctan((sqrt(
-a^2*x^2 + 1) - 1)/(a*x)))/(sqrt(-a^2*x^2 + 1)*d^2 - d^2)]
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{-(ax-1)(ax+1)^{\frac{3}{2}}}{(c+dx)(ax-1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a**2*x**2+1)**(3/2)/(-a*x+1)**2/(d*x+c),x)
```

```
[Out] Integral((-a*x - 1)*(a*x + 1)**(3/2)/((c + d*x)*(a*x - 1)**2),
x)
```

**GIAC/XCAS [A]** time = 0.32538, size = 281, normalized size = 2.63

$$\left( \frac{(ax-1)\sqrt{-\frac{2}{ax-1}-1}\operatorname{sign}\left(\frac{1}{ax-1}\right)\operatorname{sign}(a)}{ad} - \frac{2\left(\operatorname{acsign}\left(\frac{1}{ax-1}\right)\operatorname{sign}(a) - 2d\operatorname{sign}\left(\frac{1}{ax-1}\right)\operatorname{sign}(a)\right)\operatorname{arctan}\left(\sqrt{-\frac{2}{ax-1}-1}\right)}{ad^2} \right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-a^2*x^2 + 1)^(3/2)/((a*x - 1)^2*(d*x + c)),x, algorithm="giac")
```

```
[Out] -((a*x - 1)*sqrt(-2/(a*x - 1) - 1)*sign(1/(a*x - 1))*sign(a)/(a*d
) - 2*(a*c*sign(1/(a*x - 1))*sign(a) - 2*d*sign(1/(a*x - 1))*sign
(a))*arctan(sqrt(-2/(a*x - 1) - 1))/(a*d^2) + 2*(a^2*c^2*sign(1/(
a*x - 1))*sign(a) - 2*a*c*d*sign(1/(a*x - 1))*sign(a) + d^2*sign(
1/(a*x - 1))*sign(a))*arctan((a*c*sqrt(-2/(a*x - 1) - 1) + d*sqrt
(-2/(a*x - 1) - 1))/sqrt(a^2*c^2 - d^2))/(sqrt(a^2*c^2 - d^2)*a*d
^2))*abs(a)
```

$$3.621 \quad \int \frac{(1+ax)^2}{(c+dx)\sqrt{1-a^2x^2}} dx$$

**Optimal.** Leaf size=107

$$\frac{(ac-d)^2 \tan^{-1}\left(\frac{a^2cx+d}{\sqrt{1-a^2x^2}\sqrt{a^2c^2-d^2}}\right)}{d^2\sqrt{a^2c^2-d^2}} - \frac{\sqrt{1-a^2x^2}}{d} - \frac{(ac-2d)\sin^{-1}(ax)}{d^2}$$

[Out] -(Sqrt[1 - a^2\*x^2]/d) - ((a\*c - 2\*d)\*ArcSin[a\*x])/d^2 + ((a\*c - d)^2\*ArcTan[(d + a^2\*c\*x)/(Sqrt[a^2\*c^2 - d^2]\*Sqrt[1 - a^2\*x^2]])/(d^2\*Sqrt[a^2\*c^2 - d^2])

**Rubi [A]** time = 0.353255, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$\frac{(ac-d)^2 \tan^{-1}\left(\frac{a^2cx+d}{\sqrt{1-a^2x^2}\sqrt{a^2c^2-d^2}}\right)}{d^2\sqrt{a^2c^2-d^2}} - \frac{\sqrt{1-a^2x^2}}{d} - \frac{(ac-2d)\sin^{-1}(ax)}{d^2}$$

Antiderivative was successfully verified.

[In] Int[(1 + a\*x)^2/((c + d\*x)\*Sqrt[1 - a^2\*x^2]), x]

[Out] -(Sqrt[1 - a^2\*x^2]/d) - ((a\*c - 2\*d)\*ArcSin[a\*x])/d^2 + ((a\*c - d)^2\*ArcTan[(d + a^2\*c\*x)/(Sqrt[a^2\*c^2 - d^2]\*Sqrt[1 - a^2\*x^2]])/(d^2\*Sqrt[a^2\*c^2 - d^2])

**Rubi in Sympy [A]** time = 36.6038, size = 92, normalized size = 0.86

$$-\frac{\sqrt{-a^2x^2+1}}{d} + \frac{\text{asin}(ax)}{d} - \frac{(-ac+d)^{\frac{3}{2}} \operatorname{atanh}\left(\frac{a^2cx+d}{\sqrt{-ac+d}\sqrt{ac+d}\sqrt{-a^2x^2+1}}\right)}{d^2\sqrt{ac+d}} + \frac{(-ac+d)\text{asin}(ax)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a\*x+1)\*\*2/(d\*x+c)/(-a\*\*2\*x\*\*2+1)\*\*(1/2), x)

[Out] -sqrt(-a\*\*2\*x\*\*2 + 1)/d + asin(a\*x)/d - (-a\*c + d)\*\*(3/2)\*atanh((a\*\*2\*c\*x + d)/(sqrt(-a\*c + d)\*sqrt(a\*c + d)\*sqrt(-a\*\*2\*x\*\*2 + 1)))/(d\*\*2\*sqrt(a\*c + d)) + (-a\*c + d)\*asin(a\*x)/d\*\*2

**Mathematica [C]** time = 0.0776976, size = 148, normalized size = 1.38

$$-\frac{i(d-ac)^2 \log\left(\frac{2d^3(\sqrt{1-a^2x^2}\sqrt{a^2c^2-d^2+ia^2cx+id})}{(d-ac)^2\sqrt{a^2c^2-d^2}(c+dx)}\right)}{\sqrt{a^2c^2-d^2}} + \frac{d\sqrt{1-a^2x^2} + (ac-2d)\sin^{-1}(ax)}{d^2}$$

Antiderivative was successfully verified.

[In] Integrate[(1 + a\*x)^2/((c + d\*x)\*Sqrt[1 - a^2\*x^2]), x]

[Out] -((d\*Sqrt[1 - a^2\*x^2] + (a\*c - 2\*d)\*ArcSin[a\*x] + (I\*(-(a\*c) + d)^2\*Log[(2\*d^3\*(I\*d + I\*a^2\*c\*x + Sqrt[a^2\*c^2 - d^2]\*Sqrt[1 - a^2\*x^2]))/((-a\*c) + d)^2\*Sqrt[a^2\*c^2 - d^2]\*(c + d\*x)]))/Sqrt[a^2\*c^2 - d^2]

$$2 * c^2 - d^2) / d^2)$$

**Maple [B]** time = 0.016, size = 524, normalized size = 4.9

$$\begin{aligned} & -\frac{1}{d} \sqrt{-a^2 x^2 + 1} + 2 \frac{a}{d \sqrt{a^2}} \arctan\left(\frac{\sqrt{a^2} x}{\sqrt{-a^2 x^2 + 1}}\right) - \frac{a^2 c}{d^2} \arctan\left(x \sqrt{a^2} \frac{1}{\sqrt{-a^2 x^2 + 1}}\right) \frac{1}{\sqrt{a^2}} \\ & - \frac{a^2 c^2}{d^3} \ln\left(1 \left(-2 \frac{a^2 c^2 - d^2}{d^2} + 2 \frac{a^2 c}{d} \left(x + \frac{c}{d}\right) + 2 \sqrt{-\frac{a^2 c^2 - d^2}{d^2}} \sqrt{-\left(x + \frac{c}{d}\right)^2 a^2 + 2 \frac{a^2 c}{d} \left(x + \frac{c}{d}\right) - \frac{a^2 c^2 - d^2}{d^2}}\right) \left(x + \frac{c}{d}\right)^{-1}\right) \\ & + 2 \frac{ac}{d^2} \ln\left(1 \left(-2 \frac{a^2 c^2 - d^2}{d^2} + 2 \frac{a^2 c}{d} \left(x + \frac{c}{d}\right) + 2 \sqrt{-\frac{a^2 c^2 - d^2}{d^2}} \sqrt{-\left(x + \frac{c}{d}\right)^2 a^2 + 2 \frac{a^2 c}{d} \left(x + \frac{c}{d}\right) - \frac{a^2 c^2 - d^2}{d^2}}\right) \left(x + \frac{c}{d}\right)^{-1}\right) \\ & - \frac{1}{d} \ln\left(1 \left(-2 \frac{a^2 c^2 - d^2}{d^2} + 2 \frac{a^2 c}{d} \left(x + \frac{c}{d}\right) + 2 \sqrt{-\frac{a^2 c^2 - d^2}{d^2}} \sqrt{-\left(x + \frac{c}{d}\right)^2 a^2 + 2 \frac{a^2 c}{d} \left(x + \frac{c}{d}\right) - \frac{a^2 c^2 - d^2}{d^2}}\right) \left(x + \frac{c}{d}\right)^{-1}\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*x+1)^2/(d\*x+c)/(-a^2\*x^2+1)^(1/2), x)

[Out] 
$$\begin{aligned} & -(-a^2 x^2 + 1)^{1/2} / d + 2 * a / d / (a^2)^{1/2} * \arctan((a^2)^{1/2} * x / (-a^2 * x^2 + 1)^{1/2}) - a^2 / d^2 * c / (a^2)^{1/2} * \arctan((a^2)^{1/2} * x / (-a^2 * x^2 + 1)^{1/2}) \\ & - 1 / d^3 / (-a^2 * c^2 - d^2) / d^2)^{1/2} * \ln((-2 * (a^2 * c^2 - d^2) / d^2 + 2 * a^2 * c / d * (x + c / d) + 2 * (-a^2 * c^2 - d^2) / d^2)^{1/2} * (-x + c / d)^2 * a^2 + 2 * a^2 * c / d * (x + c / d) - (a^2 * c^2 - d^2) / d^2) / (x + c / d) * a^2 * c^2 \\ & + 2 / d^2 / (-a^2 * c^2 - d^2) / d^2)^{1/2} * \ln((-2 * (a^2 * c^2 - d^2) / d^2 + 2 * a^2 * c / d * (x + c / d) + 2 * (-a^2 * c^2 - d^2) / d^2)^{1/2} * (-x + c / d)^2 * a^2 + 2 * a^2 * c / d * (x + c / d) - (a^2 * c^2 - d^2) / d^2) / (x + c / d) * a * c - 1 / d / (-a^2 * c^2 - d^2) / d^2)^{1/2} \\ & * \ln((-2 * (a^2 * c^2 - d^2) / d^2 + 2 * a^2 * c / d * (x + c / d) + 2 * (-a^2 * c^2 - d^2) / d^2)^{1/2} * (-x + c / d)^2 * a^2 + 2 * a^2 * c / d * (x + c / d) - (a^2 * c^2 - d^2) / d^2) / (x + c / d) \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*x + 1)^2/(sqrt(-a^2\*x^2 + 1)\*(d\*x + c)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.418532, size = 1, normalized size = 0.01

$$\left[ \frac{a^2 dx^2 + \left( ac - \sqrt{-a^2 x^2 + 1} (ac - d) - d \right) \sqrt{-\frac{ac-d}{ac+d}} \log\left( \frac{cdx - (a^2 c^2 - d^2) x^2 + c^2 - \sqrt{-a^2 x^2 + 1} (cdx + c^2) - ((acd + d^2) x^2 - \sqrt{-a^2 x^2 + 1} (ac^2 + cd) x + (a^2 c^2 - d^2) c)}{dx - \sqrt{-a^2 x^2 + 1} (dx + c) + c} \right)}{\sqrt{-a^2 x^2 + 1} d^2 - d^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*x + 1)^2/(sqrt(-a^2\*x^2 + 1)\*(d\*x + c)), x, algorithm="fricas")

[Out] 
$$\left[ (a^2 * d * x^2 + (a * c - \sqrt{-a^2 * x^2 + 1}) * (a * c - d) - d) * \sqrt{-(a * c - d) / (a * c + d)} * \log((c * d * x - (a^2 * c^2 - d^2) * x^2 + c^2 - \sqrt{-a^2 * x^2 + 1} * (cdx + c^2) - ((acd + d^2) * x^2 - \sqrt{-a^2 * x^2 + 1} * (ac^2 + cd) * x + (a^2 * c^2 - d^2) * c)) / (dx - \sqrt{-a^2 * x^2 + 1} * (dx + c) + c)} \right]$$

$$\begin{aligned} & \sqrt{-a^2x^2 + 1} \cdot (c \cdot d \cdot x + c^2) - ((a \cdot c \cdot d + d^2) \cdot x^2 - \sqrt{-a^2x^2 + 1} \cdot (a \cdot c^2 + c \cdot d) \cdot x + (a \cdot c^2 + c \cdot d) \cdot x) \cdot \sqrt{-(a \cdot c - d)/(a \cdot c + d)} \\ & / ((d \cdot x - \sqrt{-a^2x^2 + 1}) \cdot (d \cdot x + c) + c) - 2 \cdot (a \cdot c - \sqrt{-a^2x^2 + 1}) \cdot (a \cdot c - 2 \cdot d) - 2 \cdot d \cdot \arctan((\sqrt{-a^2x^2 + 1} - 1)/(a \cdot x)) \\ & ) / (\sqrt{-a^2x^2 + 1} \cdot d^2 - d^2), (a^2 \cdot d \cdot x^2 + 2 \cdot (a \cdot c - \sqrt{-a^2x^2 + 1}) \cdot (a \cdot c - d) - d) \cdot \sqrt{(a \cdot c - d)/(a \cdot c + d)} \cdot \arctan(-(d \cdot x - \sqrt{-a^2x^2 + 1}) \cdot c + c) / ((a \cdot c + d) \cdot x \cdot \sqrt{(a \cdot c - d)/(a \cdot c + d)}) \\ & ) - 2 \cdot (a \cdot c - \sqrt{-a^2x^2 + 1}) \cdot (a \cdot c - 2 \cdot d) - 2 \cdot d \cdot \arctan((\sqrt{-a^2x^2 + 1} - 1)/(a \cdot x)) / (\sqrt{-a^2x^2 + 1} \cdot d^2 - d^2) \end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ax + 1)^2}{\sqrt{-(ax - 1)(ax + 1)(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*x+1)\*\*2/(d\*x+c)/(-a\*\*2\*x\*\*2+1)\*\*(1/2),x)

[Out] Integral((a\*x + 1)\*\*2/(sqrt(-(a\*x - 1)\*(a\*x + 1))\*(c + d\*x)), x)

**GIAC/XCAS [A]** time = 0.310594, size = 177, normalized size = 1.65

$$\frac{(a^2c - 2ad) \arcsin(ax) \operatorname{sign}(a)}{d^2|a|} - \frac{\sqrt{-a^2x^2 + 1}}{d} - \frac{2(a^3c^2 - 2a^2cd + ad^2) \arctan\left(\frac{d + \frac{(\sqrt{-a^2x^2 + 1}|a| + a)c}{ax}}{\sqrt{a^2c^2 - d^2}}\right)}{\sqrt{a^2c^2 - d^2}d|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*x + 1)^2/(sqrt(-a^2\*x^2 + 1)\*(d\*x + c)),x, algorithm="giac")

[Out]  $-(a^2 \cdot c - 2 \cdot a \cdot d) \cdot \arcsin(a \cdot x) \cdot \operatorname{sign}(a) / (d^2 \cdot \operatorname{abs}(a)) - \sqrt{-a^2 \cdot x^2 + 1} / d - 2 \cdot (a^3 \cdot c^2 - 2 \cdot a^2 \cdot c \cdot d + a \cdot d^2) \cdot \arctan((d + (\sqrt{-a^2 \cdot x^2 + 1} \cdot \operatorname{abs}(a) + a) \cdot c / (a \cdot x)) / \sqrt{a^2 \cdot c^2 - d^2}) / (\sqrt{a^2 \cdot c^2 - d^2} \cdot d^2 \cdot \operatorname{abs}(a))$

$$3.622 \quad \int (d + ex)^3 \sqrt{f + gx} \sqrt{a + cx^2} dx$$

**Optimal.** Leaf size=851

$$\frac{2\sqrt{f + gx}\sqrt{cx^2 + a}(d + ex)^4}{11e}$$

$$4\sqrt{-a} (3a^2e^2(26ef + 231dg)g^4 - 9ac (6e^3f^3 - 33de^2gf^2 + 88d^2eg^2f + 77d^3g^3) g^2 - c^2f^2 (64e^3f^3 - 264de^2gf^2 + 396d^2eg^2f + 315d^3g^3))$$

+

$$3465c^{3/2}g^5 \sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-a}g}}} \sqrt{cx^2 + a}$$

$$4\sqrt{-a} (cf^2 + ag^2) (75a^2e^3g^4 - 3ace (2e^2f^2 - 33degf + 165d^2g^2) g^2 - c^2f (64e^3f^3 - 264de^2gf^2 + 396d^2eg^2f - 231d^3g^3))$$

$$\frac{2e^2(ef - 3dg)(f + gx)^{7/2}\sqrt{cx^2 + a}}{99g^4} + \frac{2e (18ae^2g^2 - c (29e^2f^2 - 96degf + 81d^2g^2)) (f + gx)^{5/2}\sqrt{cx^2 + a}}{693cg^4}$$

$$\frac{2(2ae^2g^2(74ef - 231dg) - c (233e^3f^3 - 843de^2gf^2 + 1107d^2eg^2f - 567d^3g^3)) (f + gx)^{3/2}\sqrt{cx^2 + a}}{3465cg^4}$$

$$\frac{2(150a^2e^4g^4 - 6ace^2 (2e^2f^2 - 33degf + 165d^2g^2) g^2 + c^2 (187e^4f^4 - 732de^3gf^3 + 1098d^2e^2g^2f^2 - 798d^3eg^3f + 315d^4g^4))}{3465c^2eg^4}$$

[Out]  $(-2*(150*a^2*e^4*g^4 - 6*a*c*e^2*g^2*(2*e^2*f^2 - 33*d*e*f*g + 165*d^2*g^2) + c^2*(187*e^4*f^4 - 732*d*e^3*f^3*g + 1098*d^2*e^2*f^2*g^2 - 798*d^3*e*f*g^3 + 315*d^4*g^4))*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])/(3465*c^2*e*g^4) + (2*(d + e*x)^4*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])/(11*e) - (2*(2*a*e^2*g^2*(74*e*f - 231*d*g) - c*(233*e^3*f^3 - 843*d*e^2*f^2*g + 1107*d^2*e*f*g^2 - 567*d^3*g^3))*(f + g*x)^(3/2)*\text{Sqrt}[a + c*x^2])/(3465*c*g^4) + (2*e*(18*a*e^2*g^2 - c*(29*e^2*f^2 - 96*d*e*f*g + 81*d^2*g^2))*(f + g*x)^(5/2)*\text{Sqrt}[a + c*x^2])/(693*c*g^4) + (2*e^2*(e*f - 3*d*g)*(f + g*x)^(7/2)*\text{Sqrt}[a + c*x^2])/(99*g^4) + (4*\text{Sqrt}[-a]*(3*a^2*e^2*g^4*(26*e*f + 231*d*g) - c^2*f^2*(64*e^3*f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3) - 9*a*c*g^2*(6*e^3*f^3 - 33*d*e^2*f^2*g + 88*d^2*e*f*g^2 + 77*d^3*g^3))*\text{Sqrt}[f + g*x]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)]/(3465*c^(3/2)*g^5*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[a + c*x^2]) - (4*\text{Sqrt}[-a]*(c*f^2 + a*g^2)*(75*a^2*e^3*g^4 - 3*a*c*e*g^2*(2*e^2*f^2 - 33*d*e*f*g + 165*d^2*g^2) - c^2*f*(64*e^3*f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3))*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)]/(3465*c^(5/2)*g^5*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])$

**Rubi [A]** time = 4.74988, antiderivative size = 851, normalized size of antiderivative = 1., number of

steps used = 10, number of rules used = 6, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+a}(d+ex)^4}{11e}$$

$$4\sqrt{-a}(3a^2e^2(26ef+231dg)g^4 - 9ac(6e^3f^3 - 33de^2gf^2 + 88d^2eg^2f + 77d^3g^3)g^2 - c^2f^2(64e^3f^3 - 264de^2gf^2 + 396d^2eg^2f + 396d^3g^3))$$

$$+ \frac{3465c^{3/2}g^5\sqrt{\frac{\sqrt{c(f+gx)}}{\sqrt{cf+\sqrt{-a}g}}}\sqrt{cx^2+a}}{11e}$$

$$4\sqrt{-a}(cf^2+ag^2)(75a^2e^3g^4 - 3ace(2e^2f^2 - 33degf + 165d^2g^2)g^2 - c^2f(64e^3f^3 - 264de^2gf^2 + 396d^2eg^2f - 231d^3g^3))$$

$$+ \frac{2e^2(ef-3dg)(f+gx)^{7/2}\sqrt{cx^2+a}}{99g^4} + \frac{2e(18ae^2g^2 - c(29e^2f^2 - 96degf + 81d^2g^2))(f+gx)^{5/2}\sqrt{cx^2+a}}{693cg^4}$$

$$- \frac{2(2ae^2g^2(74ef-231dg) - c(233e^3f^3 - 843de^2gf^2 + 1107d^2eg^2f - 567d^3g^3))(f+gx)^{3/2}\sqrt{cx^2+a}}{3465cg^4}$$

$$- \frac{2(150a^2e^4g^4 - 6ace^2(2e^2f^2 - 33degf + 165d^2g^2)g^2 + c^2(187e^4f^4 - 732de^3gf^3 + 1098d^2e^2g^2f^2 - 798d^3eg^3f + 315d^4g^4))}{3465c^2eg^4}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^3\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2], x]

[Out]  $(-2*(150*a^2*e^4*g^4 - 6*a*c*e^2*g^2*(2*e^2*f^2 - 33*d*e*f*g + 165*d^2*g^2) + c^2*(187*e^4*f^4 - 732*d*e^3*f^3*g + 1098*d^2*e^2*f^2*g^2 - 798*d^3*e*f*g^3 + 315*d^4*g^4))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3465*c^2*e*g^4) + (2*(d + e*x)^4*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(11*e) - (2*(2*a*e^2*g^2*(74*e*f - 231*d*g) - c*(233*e^3*f^3 - 843*d*e^2*f^2*g + 1107*d^2*e*f*g^2 - 567*d^3*g^3))*(f + g*x)^(3/2)*Sqrt[a + c*x^2])/(3465*c*g^4) + (2*e*(18*a*e^2*g^2 - c*(29*e^2*f^2 - 96*d*e*f*g + 81*d^2*g^2))*(f + g*x)^(5/2)*Sqrt[a + c*x^2])/(693*c*g^4) + (2*e^2*(e*f - 3*d*g)*(f + g*x)^(7/2)*Sqrt[a + c*x^2])/(99*g^4) + (4*Sqrt[-a]*(3*a^2*e^2*g^4*(26*e*f + 231*d*g) - c^2*f^2*(64*e^3*f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3) - 9*a*c*g^2*(6*e^3*f^3 - 33*d*e^2*f^2*g + 88*d^2*e*f*g^2 + 77*d^3*g^3))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)]/(3465*c^(3/2)*g^5*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (4*Sqrt[-a]*(c*f^2 + a*g^2)*(75*a^2*e^3*g^4 - 3*a*c*e*g^2*(2*e^2*f^2 - 33*d*e*f*g + 165*d^2*g^2) - c^2*f*(64*e^3*f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*g^2 - 231*d^3*g^3))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)]/(3465*c^(5/2)*g^5*Sqrt[f + g*x]*Sqrt[a + c*x^2])$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3\*(g\*x+f)\*\*(1/2)\*(c\*x\*\*2+a)\*\*(1/2), x)

[Out] Timed out

**Mathematica [C]** time = 13.7258, size = 6884, normalized size = 8.09

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^3\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2],x]

[Out] Result too large to show

**Maple [B]** time = 0.166, size = 6457, normalized size = 7.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(g\*x+f)^(1/2)\*(c\*x^2+a)^(1/2),x)

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + a}(ex + d)^3 \sqrt{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)\*(e\*x + d)^3\*sqrt(g\*x + f),x, algorithm="maxima")

[Out] integrate(sqrt(c\*x^2 + a)\*(e\*x + d)^3\*sqrt(g\*x + f), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3\right)\sqrt{cx^2 + a}\sqrt{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)\*(e\*x + d)^3\*sqrt(g\*x + f),x, algorithm="fricas")

[Out] integral((e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3)\*sqrt(c\*x^2 + a)\*sqrt(g\*x + f), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + cx^2} (d + ex)^3 \sqrt{f + gx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(g\*x+f)\*\*(1/2)\*(c\*x\*\*2+a)\*\*(1/2),x)

[Out] Integral(sqrt(a + c\*x\*\*2)\*(d + e\*x)\*\*3\*sqrt(f + g\*x), x)

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + a)*(e*x + d)^3*sqrt(g*x + f),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```



### 3.623 $\int (d + ex)^2 \sqrt{f + gx} \sqrt{a + cx^2} dx$

**Optimal.** Leaf size=635

$$\begin{aligned}
 & 4\sqrt{-a}\sqrt{\frac{cx^2}{a}} + 1\sqrt{f+gx} (21a^2e^2g^4 + 3acg^2(-21d^2g^2 - 16defg + 3e^2f^2) + c^2f^2(21d^2g^2 - 24defg + 8e^2f^2)) E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{cx^2}{a}}}{\sqrt{-a}}\right)\right) \\
 & \frac{315c^{3/2}g^4\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}}{315c^{3/2}g^4\sqrt{a+cx^2}\sqrt{f+gx}} \\
 & 4\sqrt{-a}\sqrt{\frac{cx^2}{a}} + 1(ag^2 + cf^2) \sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}} (3aeg^2(ef - 10dg) + cf(21d^2g^2 - 24defg + 8e^2f^2)) F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{cx^2}{a}}}{\sqrt{-a}}\right)\right) \Big|_{-\frac{2}{\sqrt{-a}}} \\
 & + \frac{4\sqrt{a+cx^2}(f+gx)^{3/2}(7ae^2g^2 - c(21d^2g^2 - 24defg + 8e^2f^2))}{315cg^3} \\
 & - \frac{2\sqrt{a+cx^2}\sqrt{f+gx}(6ae^2g^2(ef - 10dg) - c(-35d^3g^3 + 63d^2efg^2 - 57de^2f^2g + 19e^3f^3))}{315ceg^3} \\
 & + \frac{2e\sqrt{a+cx^2}(f+gx)^{5/2}(ef - 3dg)}{63g^3} + \frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9e}
 \end{aligned}$$

[Out]  $(-2*(6*a*e^2*g^2*(e*f - 10*d*g) - c*(19*e^3*f^3 - 57*d*e^2*f^2*g + 63*d^2*e*f*g^2 - 35*d^3*g^3))*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])/(315*c*e*g^3) + (2*(d + e*x)^3*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])/(9*e) + (4*(7*a*e^2*g^2 - c*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2))* (f + g*x)^{3/2}*\text{Sqrt}[a + c*x^2])/(315*c*g^3) + (2*e*(e*f - 3*d*g)*(f + g*x)^{5/2}*\text{Sqrt}[a + c*x^2])/(63*g^3) + (4*\text{Sqrt}[-a]*(21*a^2*e^2*g^4 + 3*a*c*g^2*(3*e^2*f^2 - 16*d*e*f*g - 21*d^2*g^2) + c^2*f^2*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2))*\text{Sqrt}[f + g*x]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)]/(315*c^{3/2}*g^4*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[a + c*x^2]) - (4*\text{Sqrt}[-a]*(c*f^2 + a*g^2)*(3*a*e*g^2*(e*f - 10*d*g) + c*f*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2))*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)]/(315*c^{3/2}*g^4*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])$

**Rubi [A]** time = 2.94972, antiderivative size = 635, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$\begin{aligned}
 & 4\sqrt{-a}\sqrt{\frac{cx^2}{a}} + 1\sqrt{f+gx} (21a^2e^2g^4 + 3acg^2(-21d^2g^2 - 16defg + 3e^2f^2) + c^2f^2(21d^2g^2 - 24defg + 8e^2f^2)) E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{cx^2}{a}}}{\sqrt{-a}}\right)\right) \\
 & \frac{315c^{3/2}g^4\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}}{315c^{3/2}g^4\sqrt{a+cx^2}\sqrt{f+gx}} \\
 & 4\sqrt{-a}\sqrt{\frac{cx^2}{a}} + 1(ag^2 + cf^2) \sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}} (3aeg^2(ef - 10dg) + cf(21d^2g^2 - 24defg + 8e^2f^2)) F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{cx^2}{a}}}{\sqrt{-a}}\right)\right) \Big|_{-\frac{2}{\sqrt{-a}}} \\
 & + \frac{4\sqrt{a+cx^2}(f+gx)^{3/2}(7ae^2g^2 - c(21d^2g^2 - 24defg + 8e^2f^2))}{315cg^3} \\
 & - \frac{2\sqrt{a+cx^2}\sqrt{f+gx}(6ae^2g^2(ef - 10dg) - c(-35d^3g^3 + 63d^2efg^2 - 57de^2f^2g + 19e^3f^3))}{315ceg^3} \\
 & + \frac{2e\sqrt{a+cx^2}(f+gx)^{5/2}(ef - 3dg)}{63g^3} + \frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9e}
 \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^2\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2],x]

[Out] 
$$\begin{aligned} & (-2*(6*a*e^2*g^2*(e*f - 10*d*g) - c*(19*e^3*f^3 - 57*d*e^2*f^2*g \\ & + 63*d^2*e*f*g^2 - 35*d^3*g^3)) * \text{Sqrt}[f + g*x] * \text{Sqrt}[a + c*x^2]) / (3 \\ & 15*c*e*g^3) + (2*(d + e*x)^3 * \text{Sqrt}[f + g*x] * \text{Sqrt}[a + c*x^2]) / (9*e) \\ & + (4*(7*a*e^2*g^2 - c*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2)) * (f \\ & + g*x)^{(3/2)} * \text{Sqrt}[a + c*x^2]) / (315*c*g^3) + (2*e*(e*f - 3*d*g) * (f \\ & + g*x)^{(5/2)} * \text{Sqrt}[a + c*x^2]) / (63*g^3) + (4*\text{Sqrt}[-a] * (21*a^2*e^2 \\ & *g^4 + 3*a*c*g^2*(3*e^2*f^2 - 16*d*e*f*g - 21*d^2*g^2) + c^2*f^2 * \\ & (8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2)) * \text{Sqrt}[f + g*x] * \text{Sqrt}[1 + (c* \\ & x^2)/a] * \text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], \\ & (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)]) / (315*c^{(3/2)}*g^4*\text{Sqrt}[(\text{Sqr} \\ & \text{t}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)] * \text{Sqrt}[a + c*x^2]) - (4*S \\ & \text{qrt}[-a] * (c*f^2 + a*g^2) * (3*a*e*g^2*(e*f - 10*d*g) + c*f*(8*e^2*f^2 \\ & 2 - 24*d*e*f*g + 21*d^2*g^2)) * \text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f \\ & + \text{Sqrt}[-a]*g)] * \text{Sqrt}[1 + (c*x^2)/a] * \text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqr} \\ & \text{t}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)] \\ & ) / (315*c^{(3/2)}*g^4*\text{Sqrt}[f + g*x] * \text{Sqrt}[a + c*x^2]) \end{aligned}$$

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*2\*(g\*x+f)\*\*(1/2)\*(c\*x\*\*2+a)\*\*(1/2),x)

[Out] Timed out

**Mathematica [C]** time = 12.7148, size = 910, normalized size = 1.43

$$2 \left( -2 \sqrt{-f - \frac{i\sqrt{ag}}{\sqrt{c}}} (21a^2e^2g^4 - 3ac(-3e^2f^2 + 16degf + 21d^2g^2)g^2 + c^2f^2(8e^2f^2 - 24degf + 21d^2g^2)) \left( \frac{ag^2}{(f+gx)^2} + c \left( \frac{f}{f+gx} \right. \right. \right. \\ \left. \left. \left. + \sqrt{cx^2 + a} \left( \frac{2e^2x^3}{9} + \frac{2e(ef + 18dg)x^2}{63g} + \frac{2(-6ce^2f^2 + 18cdegf + 63cd^2g^2 + 14ae^2g^2)x}{315cg^2} + \frac{2(8ce^2f^3 - 24cdegf^2 + 21cd^2g^2f + 315cg^3}{315cg^3} \right) \right) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^2\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2],x]

[Out] 
$$\begin{aligned} & \text{Sqrt}[f + g*x] * \text{Sqrt}[a + c*x^2] * ((2*(8*c*e^2*f^3 - 24*c*d*e*f^2*g + \\ & 21*c*d^2*f*g^2 + 8*a*e^2*f*g^2 + 60*a*d*e*g^3)) / (315*c*g^3) + (2 \\ & * (-6*c*e^2*f^2 + 18*c*d*e*f*g + 63*c*d^2*g^2 + 14*a*e^2*g^2) * x) / ( \\ & 315*c*g^2) + (2*e*(e*f + 18*d*g) * x^2) / (63*g) + (2*e^2*x^3) / 9) + ( \\ & 2*(f + g*x)^{(3/2)} * (-2*\text{Sqrt}[-f - (I*\text{Sqrt}[a]*g)/\text{Sqrt}[c]] * (21*a^2*e^2 \\ & 2*g^4 + c^2*f^2*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2) - 3*a*c*g^2 \\ & * (-3*e^2*f^2 + 16*d*e*f*g + 21*d^2*g^2)) * ((a*g^2)/(f + g*x)^2 + c \\ & * (-1 + f/(f + g*x))^2) + ((2*I)*\text{Sqrt}[c] * (\text{Sqrt}[c]*f + I*\text{Sqrt}[a]*g) \\ & * (21*a^2*e^2*g^4 + c^2*f^2*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2) \\ & - 3*a*c*g^2*(-3*e^2*f^2 + 16*d*e*f*g + 21*d^2*g^2)) * \text{Sqrt}[1 - f/(f \\ & + g*x) - (I*\text{Sqrt}[a]*g)/(\text{Sqrt}[c]*(f + g*x))] * \text{Sqrt}[1 - f/(f + g*x) \\ & + (I*\text{Sqrt}[a]*g)/(\text{Sqrt}[c]*(f + g*x))] * \text{EllipticE}[I*\text{ArcSinh}[\text{Sqrt}[-f \\ & - (I*\text{Sqrt}[a]*g)/\text{Sqrt}[c]]/\text{Sqrt}[f + g*x]], (\text{Sqrt}[c]*f - I*\text{Sqrt}[a]* \\ & g)/(\text{Sqrt}[c]*f + I*\text{Sqrt}[a]*g)] / \text{Sqrt}[f + g*x] + (2*\text{Sqrt}[a]*\text{Sqrt}[c] \\ & * g * (\text{Sqrt}[c]*f + I*\text{Sqrt}[a]*g) * ((-21*I)*a^{(3/2)}*e^2*g^3 + 3*a*\text{Sqrt}[ \end{aligned}$$

$$c] * e * g^2 * (e * f - 10 * d * g) + c^{(3/2)} * f * (8 * e^2 * f^2 - 24 * d * e * f * g + 21 * d^2 * g^2) + (3 * I) * \text{Sqrt}[a] * c * g * (-2 * e^2 * f^2 + 6 * d * e * f * g + 21 * d^2 * g^2) * \text{Sqrt}[1 - f / (f + g * x) - (I * \text{Sqrt}[a] * g) / (\text{Sqrt}[c] * (f + g * x))] * \text{Sqrt}[1 - f / (f + g * x) + (I * \text{Sqrt}[a] * g) / (\text{Sqrt}[c] * (f + g * x))] * \text{EllipticF}[I * \text{ArcSinh}[\text{Sqrt}[-f - (I * \text{Sqrt}[a] * g) / \text{Sqrt}[c]] / \text{Sqrt}[f + g * x]], (\text{Sqrt}[c] * f - I * \text{Sqrt}[a] * g) / (\text{Sqrt}[c] * f + I * \text{Sqrt}[a] * g)] / \text{Sqrt}[f + g * x]] / (315 * c^2 * g^5 * \text{Sqrt}[-f - (I * \text{Sqrt}[a] * g) / \text{Sqrt}[c]] * \text{Sqrt}[a + (c * (f + g * x))^2 * (-1 + f / (f + g * x))^2] / g^2)]$$

Maple [B]    time = 0.041, size = 4351, normalized size = 6.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]     $\text{int}((e * x + d)^2 * (g * x + f)^{(1/2)} * (c * x^2 + a)^{(1/2)}, x)$

[Out]     $-2/315 * (g * x + f)^{(1/2)} * (c * x^2 + a)^{(1/2)} * (6 * x^3 * c^3 * d * e * f^2 * g^4 - 7 * x^2 * a * c^2 * e^2 * f^2 * g^4 + 24 * x^2 * c^3 * d * e * f^3 * g^3 - 60 * x * a^2 * c * d * e * g^6 - 22 * x * a^2 * c * e^2 * f * g^5 - 84 * x * a * c^2 * d^2 * f * g^5 - 2 * x * a * c^2 * e^2 * f^3 * g^3 - 108 * x^4 * c^3 * d * e * f * g^5 - 150 * x^3 * a * c^2 * d * e * g^6 - 62 * x^3 * a * c^2 * e^2 * f * g^5 - 60 * a^2 * c * d * e * f * g^5 + 24 * a * c^2 * d * e * f^3 * g^3 - 35 * x^6 * c^3 * e^2 * g^6 - 63 * x^4 * c^3 * d^2 * g^6 - 108 * (- (g * x + f) * c / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)} * ((-c * x + (-a * c)^{(1/2)}) * g / (g * (-a * c)^{(1/2)} + c * f))^{(1/2)} * ((c * x + (-a * c)^{(1/2)}) * g / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)} * \text{EllipticF}((- (g * x + f) * c / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)}, (- (g * (-a * c)^{(1/2)} - c * f) / (g * (-a * c)^{(1/2)} + c * f))^{(1/2)}) * (-a * c)^{(1/2)} * a * c * d * e * f^2 * g^4 - 2 * x^3 * c^3 * e^2 * f^3 * g^3 - 60 * (- (g * x + f) * c / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)} * ((-c * x + (-a * c)^{(1/2)}) * g / (g * (-a * c)^{(1/2)} + c * f))^{(1/2)} * ((c * x + (-a * c)^{(1/2)}) * g / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)} * \text{EllipticE}((- (g * x + f) * c / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)}, (- (g * (-a * c)^{(1/2)} - c * f) / (g * (-a * c)^{(1/2)} + c * f))^{(1/2)}) * a^2 * c * e^2 * f^2 * g^4 + 84 * (- (g * x + f) * c / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)} * ((-c * x + (-a * c)^{(1/2)}) * g / (g * (-a * c)^{(1/2)} + c * f))^{(1/2)} * ((c * x + (-a * c)^{(1/2)}) * g / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)} * \text{EllipticE}((- (g * x + f) * c / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)}, (- (g * (-a * c)^{(1/2)} - c * f) / (g * (-a * c)^{(1/2)} + c * f))^{(1/2)}) * a^2 * c * e^2 * f^2 * g^4 + 34 * (- (g * x + f) * c / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)} * ((-c * x + (-a * c)^{(1/2)}) * g / (g * (-a * c)^{(1/2)} + c * f))^{(1/2)} * ((c * x + (-a * c)^{(1/2)}) * g / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)} * \text{EllipticE}((- (g * x + f) * c / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)}, (- (g * (-a * c)^{(1/2)} - c * f) / (g * (-a * c)^{(1/2)} + c * f))^{(1/2)}) * a * c^2 * e^2 * f^4 * g^2 + 48 * (- (g * x + f) * c / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)} * ((-c * x + (-a * c)^{(1/2)}) * g / (g * (-a * c)^{(1/2)} + c * f))^{(1/2)} * ((c * x + (-a * c)^{(1/2)}) * g / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)} * \text{EllipticE}((- (g * x + f) * c / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)}, (- (g * (-a * c)^{(1/2)} - c * f) / (g * (-a * c)^{(1/2)} + c * f))^{(1/2)}) * c^3 * d * e * f^5 * g - 60 * (- (g * x + f) * c / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)} * ((-c * x + (-a * c)^{(1/2)}) * g / (g * (-a * c)^{(1/2)} + c * f))^{(1/2)} * ((c * x + (-a * c)^{(1/2)}) * g / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)} * \text{EllipticF}((- (g * x + f) * c / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)}, (- (g * (-a * c)^{(1/2)} - c * f) / (g * (-a * c)^{(1/2)} + c * f))^{(1/2)}) * (-a * c)^{(1/2)} * a^2 * d * e * g^6 + 6 * (- (g * x + f) * c / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)} * ((-c * x + (-a * c)^{(1/2)}) * g / (g * (-a * c)^{(1/2)} + c * f))^{(1/2)} * ((c * x + (-a * c)^{(1/2)}) * g / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)} * \text{EllipticF}((- (g * x + f) * c / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)}, (- (g * (-a * c)^{(1/2)} - c * f) / (g * (-a * c)^{(1/2)} + c * f))^{(1/2)}) * (-a * c)^{(1/2)} * a^2 * e^2 * f * g^5 + 42 * (- (g * x + f) * c / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)} * ((-c * x + (-a * c)^{(1/2)}) * g / (g * (-a * c)^{(1/2)} + c * f))^{(1/2)} * ((c * x + (-a * c)^{(1/2)}) * g / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)} * \text{EllipticF}((- (g * x + f) * c / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)}, (- (g * (-a * c)^{(1/2)} - c * f) / (g * (-a * c)^{(1/2)} + c * f))^{(1/2)}) * (-a * c)^{(1/2)} * c^2 * e^2 * f^5 * g + 54 * (- (g * x + f) * c / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)} * ((-c * x + (-a * c)^{(1/2)}) * g / (g * (-a * c)^{(1/2)} + c * f))^{(1/2)} * ((c * x + (-a * c)^{(1/2)}) * g / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)} * \text{EllipticF}((- (g * x + f) * c / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)}, (- (g * (-a * c)^{(1/2)} - c * f) / (g * (-a * c)^{(1/2)} + c * f))^{(1/2)}) * a^2 * c * e^2 * f^2 * g^4 - 126 * (- (g * x + f) * c / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)} * ((-c * x + (-a * c)^{(1/2)}) * g / (g * (-a * c)^{(1/2)} + c * f))^{(1/2)} * ((c * x + (-a * c)^{(1/2)}) * g / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)} * \text{EllipticF}((- (g * x + f) * c / (g * (-a * c)^{(1/2)} - c * f))^{(1/2)}, (- (g * (-a * c)^{(1/2)} - c * f) / (g * (-a * c)^{(1/2)} + c * f))^{(1/2)})$

$$\begin{aligned}
& 2) - c^* f) / (g^* (-a^* c)^{(1/2)} + c^* f))^{(1/2)} * a^* c^2 * d^2 * f^2 * g^4 + 12 * (- (g^* x + \\
& f) * c / (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)} * ((-c^* x + (-a^* c)^{(1/2)}) * g / (g^* (-a^* c) \\
& ^{(1/2)} + c^* f))^{(1/2)} * ((c^* x + (-a^* c)^{(1/2)}) * g / (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)} * \\
& \text{EllipticF}((- (g^* x + f) * c / (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)}, (- (g^* (-a^* c) \\
& ^{(1/2)} - c^* f) / (g^* (-a^* c)^{(1/2)} + c^* f))^{(1/2)}) * a^* c^2 * e^2 * f^4 * g^2 - 14 * x^2 \\
& * a^2 * c^* e^2 * g^6 - 63 * x^2 * a^* c^2 * d^2 * g^6 - 21 * x^2 * c^3 * d^2 * f^2 * g^4 - 8 * x^2 * \\
& c^3 * e^2 * f^4 * g^2 - 90 * x^5 * c^3 * d^* e * g^6 - 40 * x^5 * c^3 * e^2 * f * g^5 - 49 * x^4 * a^* \\
& c^2 * e^2 * g^6 + x^4 * c^3 * e^2 * f^2 * g^4 - 84 * x^3 * c^3 * d^2 * f * g^5 + 6 * x * a^* c^2 * d^* \\
& e * f^2 * g^4 + 42 * (- (g^* x + f) * c / (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)} * ((-c^* x + (-a^* c) \\
& ^{(1/2)}) * g / (g^* (-a^* c)^{(1/2)} + c^* f))^{(1/2)} * ((c^* x + (-a^* c)^{(1/2)}) * g / (g^* (-a^* c) \\
& ^{(1/2)} - c^* f))^{(1/2)} * \text{EllipticF}((- (g^* x + f) * c / (g^* (-a^* c)^{(1/2)} - c^* f) \\
& ))^{(1/2)}, (- (g^* (-a^* c)^{(1/2)} - c^* f) / (g^* (-a^* c)^{(1/2)} + c^* f))^{(1/2)}) * a^3 * \\
& e^2 * g^6 - 42 * (- (g^* x + f) * c / (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)} * ((-c^* x + (-a^* c) \\
& ^{(1/2)}) * g / (g^* (-a^* c)^{(1/2)} + c^* f))^{(1/2)} * ((c^* x + (-a^* c)^{(1/2)}) * g / (g^* (-a^* c) \\
& ^{(1/2)} - c^* f))^{(1/2)} * \text{EllipticE}((- (g^* x + f) * c / (g^* (-a^* c)^{(1/2)} - c^* f)) \\
& ^{(1/2)}, (- (g^* (-a^* c)^{(1/2)} - c^* f) / (g^* (-a^* c)^{(1/2)} + c^* f))^{(1/2)}) * a^3 * e^2 * \\
& 2 * g^6 - 16 * (- (g^* x + f) * c / (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)} * ((-c^* x + (-a^* c) \\
& ^{(1/2)}) * g / (g^* (-a^* c)^{(1/2)} + c^* f))^{(1/2)} * ((c^* x + (-a^* c)^{(1/2)}) * g / (g^* (-a^* c) \\
& ^{(1/2)} - c^* f))^{(1/2)} * \text{EllipticE}((- (g^* x + f) * c / (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)}, \\
& (- (g^* (-a^* c)^{(1/2)} - c^* f) / (g^* (-a^* c)^{(1/2)} + c^* f))^{(1/2)}) * c^3 * e^2 * \\
& f^6 - 168 * x^2 * a^* c^2 * d^* e * f * g^5 + 126 * (- (g^* x + f) * c / (g^* (-a^* c)^{(1/2)} - c^* f)) \\
& ^{(1/2)} * ((-c^* x + (-a^* c)^{(1/2)}) * g / (g^* (-a^* c)^{(1/2)} + c^* f))^{(1/2)} * ((c^* x + \\
& (-a^* c)^{(1/2)}) * g / (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)} * \text{EllipticE}((- (g^* x + f) * c / \\
& (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)}, (- (g^* (-a^* c)^{(1/2)} - c^* f) / (g^* (-a^* c)^{(1/2)} \\
& + c^* f))^{(1/2)}) * a^2 * c^* d^2 * g^6 - 42 * (- (g^* x + f) * c / (g^* (-a^* c)^{(1/2)} - c^* f)) \\
& ^{(1/2)} * ((-c^* x + (-a^* c)^{(1/2)}) * g / (g^* (-a^* c)^{(1/2)} + c^* f))^{(1/2)} * ((c^* x + \\
& (-a^* c)^{(1/2)}) * g / (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)} * \text{EllipticE}((- (g^* x + f) * c / \\
& (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)}, (- (g^* (-a^* c)^{(1/2)} - c^* f) / (g^* (-a^* c)^{(1/2)} \\
& + c^* f))^{(1/2)}) * c^3 * d^2 * f^4 * g^2 + 96 * (- (g^* x + f) * c / (g^* (-a^* c)^{(1/2)} - c^* f) \\
& ))^{(1/2)} * ((-c^* x + (-a^* c)^{(1/2)}) * g / (g^* (-a^* c)^{(1/2)} + c^* f))^{(1/2)} * ((c^* x \\
& + (-a^* c)^{(1/2)}) * g / (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)} * \text{EllipticE}((- (g^* x + f) * \\
& c / (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)}, (- (g^* (-a^* c)^{(1/2)} - c^* f) / (g^* (-a^* c)^{(1/2)} \\
& + c^* f))^{(1/2)}) * a^2 * c^* d^* e * f * g^5 + 144 * (- (g^* x + f) * c / (g^* (-a^* c)^{(1/2)} - \\
& c^* f))^{(1/2)} * ((-c^* x + (-a^* c)^{(1/2)}) * g / (g^* (-a^* c)^{(1/2)} + c^* f))^{(1/2)} * (( \\
& c^* x + (-a^* c)^{(1/2)}) * g / (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)} * \text{EllipticE}((- (g^* x + \\
& f) * c / (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)}, (- (g^* (-a^* c)^{(1/2)} - c^* f) / (g^* (-a^* c) \\
& ^{(1/2)} + c^* f))^{(1/2)}) * a^* c^2 * d^* e * f^3 * g^3 + 42 * (- (g^* x + f) * c / (g^* (-a^* c)^{(1/2)} \\
& - c^* f))^{(1/2)} * ((-c^* x + (-a^* c)^{(1/2)}) * g / (g^* (-a^* c)^{(1/2)} + c^* f))^{(1/2)} * \\
& ((c^* x + (-a^* c)^{(1/2)}) * g / (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)} * \text{EllipticF}((- ( \\
& g^* x + f) * c / (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)}, (- (g^* (-a^* c)^{(1/2)} - c^* f) / (g^* (- \\
& a^* c)^{(1/2)} + c^* f))^{(1/2)}) * (-a^* c)^{(1/2)} * a^* c^* d^2 * f * g^5 + 22 * (- (g^* x + f) * c \\
& / (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)} * ((-c^* x + (-a^* c)^{(1/2)}) * g / (g^* (-a^* c)^{(1/2)} \\
& + c^* f))^{(1/2)} * ((c^* x + (-a^* c)^{(1/2)}) * g / (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)} * \\
& \text{EllipticF}((- (g^* x + f) * c / (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)}, (- (g^* (-a^* c)^{(1/2)} \\
& - c^* f) / (g^* (-a^* c)^{(1/2)} + c^* f))^{(1/2)}) * (-a^* c)^{(1/2)} * a^* c^* e^2 * f^3 * g^3 \\
& - 48 * (- (g^* x + f) * c / (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)} * ((-c^* x + (-a^* c)^{(1/2)}) * \\
& g / (g^* (-a^* c)^{(1/2)} + c^* f))^{(1/2)} * ((c^* x + (-a^* c)^{(1/2)}) * g / (g^* (-a^* c)^{(1/2)} \\
& - c^* f))^{(1/2)} * \text{EllipticF}((- (g^* x + f) * c / (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)}, \\
& (- (g^* (-a^* c)^{(1/2)} - c^* f) / (g^* (-a^* c)^{(1/2)} + c^* f))^{(1/2)}) * (-a^* c)^{(1/2)} * \\
& c^2 * d^* e * f^4 * g^2 - 36 * (- (g^* x + f) * c / (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)} * ((-c^* x \\
& + (-a^* c)^{(1/2)}) * g / (g^* (-a^* c)^{(1/2)} + c^* f))^{(1/2)} * ((c^* x + (-a^* c)^{(1/2)}) * \\
& g / (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)} * \text{EllipticF}((- (g^* x + f) * c / (g^* (-a^* c)^{(1/2)} \\
& - c^* f))^{(1/2)}, (- (g^* (-a^* c)^{(1/2)} - c^* f) / (g^* (-a^* c)^{(1/2)} + c^* f))^{(1/2)}) \\
& * a^2 * c^* d^* e * f * g^5 - 36 * (- (g^* x + f) * c / (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)} * ((-c^* \\
& x + (-a^* c)^{(1/2)}) * g / (g^* (-a^* c)^{(1/2)} + c^* f))^{(1/2)} * ((c^* x + (-a^* c)^{(1/2)} \\
& ) * g / (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)} * \text{EllipticF}((- (g^* x + f) * c / (g^* (-a^* c)^{(1/2)} \\
& - c^* f))^{(1/2)}, (- (g^* (-a^* c)^{(1/2)} - c^* f) / (g^* (-a^* c)^{(1/2)} + c^* f))^{(1/2)}) \\
& * a^* c^2 * d^* e * f^3 * g^3 - 8 * a^2 * c^* e^2 * f^2 * g^4 - 21 * a^* c^2 * d^2 * f^2 * g^4 - 8 * \\
& a^* c^2 * e^2 * f^4 * g^2 - 126 * (- (g^* x + f) * c / (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)} * ((- \\
& c^* x + (-a^* c)^{(1/2)}) * g / (g^* (-a^* c)^{(1/2)} + c^* f))^{(1/2)} * ((c^* x + (-a^* c)^{(1/2)} \\
& )) * g / (g^* (-a^* c)^{(1/2)} - c^* f))^{(1/2)} * \text{EllipticF}((- (g^* x + f) * c / (g^* (-a^* c)^{(1/2)} \\
& - c^* f))^{(1/2)}, (- (g^* (-a^* c)^{(1/2)} - c^* f) / (g^* (-a^* c)^{(1/2)} + c^* f))^{(1/2)}) \\
& * a^2 * c^* d^2 * g^6) / c^2 / (c^* g^* x^3 + c^* f^* x^2 + a^* g^* x + a^* f) / g^5
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + a} (ex + d)^2 \sqrt{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + a)*(e*x + d)^2*sqrt(g*x + f), x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^2 + a)*(e*x + d)^2*sqrt(g*x + f), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^2 + 2dex + d^2\right)\sqrt{cx^2 + a}\sqrt{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + a)*(e*x + d)^2*sqrt(g*x + f), x, algorithm="fricas")`

[Out] `integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(c*x^2 + a)*sqrt(g*x + f), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + cx^2} (d + ex)^2 \sqrt{f + gx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**2*(g*x+f)**(1/2)*(c*x**2+a)**(1/2), x)`

[Out] `Integral(sqrt(a + c*x**2)*(d + e*x)**2*sqrt(f + g*x), x)`

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + a)*(e*x + d)^2*sqrt(g*x + f), x, algorithm="giac")`

[Out] Exception raised: RuntimeError

### 3.624 $\int (d + ex)\sqrt{f + gx}\sqrt{a + cx^2} dx$

**Optimal.** Leaf size=434

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a} + 1} (ag^2 + cf^2) \sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}} (5aeg^2 + cf(4ef - 7dg)) F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{105c^{3/2}g^3\sqrt{a + cx^2}\sqrt{f + gx}} - \frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a} + 1}\sqrt{f + gx} (ag^2(21dg + 8ef) + cf^2(4ef - 7dg)) E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{105\sqrt{cg^3}\sqrt{a + cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}} - \frac{2\sqrt{a + cx^2}\sqrt{f + gx} (5aeg^2 - 3cgx(7dg + ef) + cf(4ef - 7dg))}{105cg^2} + \frac{2e(a + cx^2)^{3/2}\sqrt{f + gx}}{7c}$$

[Out]  $(-2*\text{Sqrt}[f + g*x]*(5*a*e*g^2 + c*f*(4*e*f - 7*d*g) - 3*c*g*(e*f + 7*d*g)*x)*\text{Sqrt}[a + c*x^2])/((105*c*g^2) + (2*e*\text{Sqrt}[f + g*x]*(a + c*x^2)^{(3/2)})/(7*c) - (4*\text{Sqrt}[-a]*(c*f^2*(4*e*f - 7*d*g) + a*g^2*(8*e*f + 21*d*g))*\text{Sqrt}[f + g*x]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)))/(105*\text{Sqrt}[c]*g^3*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[a + c*x^2]) + (4*\text{Sqrt}[-a]*(c*f^2 + a*g^2)*(5*a*e*g^2 + c*f*(4*e*f - 7*d*g))*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)))/(105*c^{(3/2)}*g^3*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])$

**Rubi [A]** time = 1.2024, antiderivative size = 434, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a} + 1} (ag^2 + cf^2) \sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}} (5aeg^2 + cf(4ef - 7dg)) F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{105c^{3/2}g^3\sqrt{a + cx^2}\sqrt{f + gx}} - \frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a} + 1}\sqrt{f + gx} (ag^2(21dg + 8ef) + cf^2(4ef - 7dg)) E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{105\sqrt{cg^3}\sqrt{a + cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}} - \frac{2\sqrt{a + cx^2}\sqrt{f + gx} (5aeg^2 - 3cgx(7dg + ef) + cf(4ef - 7dg))}{105cg^2} + \frac{2e(a + cx^2)^{3/2}\sqrt{f + gx}}{7c}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2], x]$

[Out]  $(-2*\text{Sqrt}[f + g*x]*(5*a*e*g^2 + c*f*(4*e*f - 7*d*g) - 3*c*g*(e*f + 7*d*g)*x)*\text{Sqrt}[a + c*x^2])/((105*c*g^2) + (2*e*\text{Sqrt}[f + g*x]*(a + c*x^2)^{(3/2)})/(7*c) - (4*\text{Sqrt}[-a]*(c*f^2*(4*e*f - 7*d*g) + a*g^2*(8*e*f + 21*d*g))*\text{Sqrt}[f + g*x]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)))/(105*\text{Sqrt}[c]*g^3*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[a + c*x^2]) + (4*\text{Sqrt}[-a]*(c*f^2 + a*g^2)*(5*a*e*g^2 + c*f*(4*e*f - 7*d*g))*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)))/(105*c^{(3/2)}*g^3*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])$

**Rubi in Sympy [A]** time = 174.839, size = 437, normalized size = 1.01

$$\frac{2e(a+cx^2)^{\frac{3}{2}}\sqrt{f+gx}}{7c} - \frac{8\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{5aeg^2}{4} - \frac{7cdfg}{4} + cef^2 - \frac{3cgx(7dg+ef)}{4}\right)}{105cg^2}$$

$$- \frac{4\sqrt{-a}\sqrt{1+\frac{cx^2}{a}}\sqrt{f+gx}(21adg^3 + 8aefg^2 - 7cdf^2g + 4cef^3) E\left(\operatorname{asin}\left(\sqrt{-\frac{\sqrt{cx}}{2\sqrt{-a}} + \frac{1}{2}}\right)\middle|\frac{2ag}{ag-\sqrt{cf}\sqrt{-a}}\right)}{105\sqrt{c}g^3\sqrt{\frac{\sqrt{c}\sqrt{-a}(-f-gx)}{ag-\sqrt{cf}\sqrt{-a}}}\sqrt{a+cx^2}}$$

$$+ \frac{4\sqrt{-a}\sqrt{\frac{\sqrt{c}\sqrt{-a}(-f-gx)}{ag-\sqrt{cf}\sqrt{-a}}}\sqrt{1+\frac{cx^2}{a}}(ag^2+cf^2)(5aeg^2-7cdfg+4cef^2) F\left(\operatorname{asin}\left(\sqrt{-\frac{\sqrt{cx}}{2\sqrt{-a}} + \frac{1}{2}}\right)\middle|\frac{2ag}{ag-\sqrt{cf}\sqrt{-a}}\right)}{105c^{\frac{3}{2}}g^3\sqrt{a+cx^2}\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)*(g*x+f)**(1/2)*(c*x**2+a)**(1/2),x)`

[Out]  $2*e*(a+c*x**2)**(3/2)*\operatorname{sqrt}(f+g*x)/(7*c) - 8*\operatorname{sqrt}(a+c*x**2)*\operatorname{sqrt}(f+g*x)*(5*a*e*g**2/4 - 7*c*d*f*g/4 + c*e*f**2 - 3*c*g*x*(7*d*g + e*f)/4)/(105*c*g**2) - 4*\operatorname{sqrt}(-a)*\operatorname{sqrt}(1+c*x**2/a)*\operatorname{sqrt}(f+g*x)*(21*a*d*g**3 + 8*a*e*f*g**2 - 7*c*d*f**2*g + 4*c*e*f**3)*\operatorname{elliptic}_e(\operatorname{asin}(\operatorname{sqrt}(-\operatorname{sqrt}(c)*x/(2*\operatorname{sqrt}(-a)) + 1/2)), 2*a*g/(a*g - \operatorname{sqrt}(c)*f*\operatorname{sqrt}(-a)))/(105*\operatorname{sqrt}(c)*g**3*\operatorname{sqrt}(\operatorname{sqrt}(c)*\operatorname{sqrt}(-a)*(-f - g*x)/(a*g - \operatorname{sqrt}(c)*f*\operatorname{sqrt}(-a)))*\operatorname{sqrt}(a+c*x**2)) + 4*\operatorname{sqrt}(-a)*\operatorname{sqrt}(\operatorname{sqrt}(c)*\operatorname{sqrt}(-a)*(-f - g*x)/(a*g - \operatorname{sqrt}(c)*f*\operatorname{sqrt}(-a)))*\operatorname{sqrt}(1+c*x**2/a)*(a*g**2 + c*f**2)*(5*a*e*g**2 - 7*c*d*f*g + 4*c*e*f**2)*\operatorname{elliptic}_f(\operatorname{asin}(\operatorname{sqrt}(-\operatorname{sqrt}(c)*x/(2*\operatorname{sqrt}(-a)) + 1/2)), 2*a*g/(a*g - \operatorname{sqrt}(c)*f*\operatorname{sqrt}(-a)))/(105*c**(3/2)*g**3*\operatorname{sqrt}(a+c*x**2)*\operatorname{sqrt}(f+g*x))$

**Mathematica [C]** time = 7.98219, size = 610, normalized size = 1.41

$$\sqrt{f+gx} \left( \frac{2(a+cx^2)(10aeg^2+7cdg(f+3gx)+ce(-4f^2+3fgx+15g^2x^2))}{cg^2} + \frac{4\left(g^2(a+cx^2)\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}(ag^2(21dg+8ef)+cf^2(4ef-7dg))+i\sqrt{c}(f+gx)^{3/2}(\sqrt{cf}+i\sqrt{c}g)\right)}{cg^2} \right)$$

Antiderivative was successfully verified.

[In] `Integrate[(d + e*x)*Sqrt[f + g*x]*Sqrt[a + c*x^2],x]`

[Out]  $(\operatorname{Sqrt}[f+g*x]*((2*(a+c*x^2)*(10*a*e*g^2+7*c*d*g*(f+3*g*x)+c*e*(-4*f^2+3*f*g*x+15*g^2*x^2)))/(c*g^2)+(4*(g^2*\operatorname{Sqrt}[-f-(I*\operatorname{Sqrt}[a]*g)/\operatorname{Sqrt}[c]])*(c*f^2*(4*e*f-7*d*g)+a*g^2*(8*e*f+21*d*g))*(a+c*x^2)+I*\operatorname{Sqrt}[c]*(\operatorname{Sqrt}[c]*f+I*\operatorname{Sqrt}[a]*g)*(c*f^2*(-4*e*f+7*d*g)-a*g^2*(8*e*f+21*d*g))*\operatorname{Sqrt}[(g*((I*\operatorname{Sqrt}[a])/ \operatorname{Sqrt}[c]+x))/(f+g*x))*\operatorname{Sqrt}[-( ((I*\operatorname{Sqrt}[a]*g)/\operatorname{Sqrt}[c]-g*x)/(f+g*x) )]*(f+g*x)^{(3/2)}*\operatorname{EllipticE}[I*\operatorname{ArcSinh}[\operatorname{Sqrt}[-f-(I*\operatorname{Sqrt}[a]*g)/\operatorname{Sqrt}[c]]/\operatorname{Sqrt}[f+g*x]],(\operatorname{Sqrt}[c]*f-I*\operatorname{Sqrt}[a]*g)/(\operatorname{Sqrt}[c]*f+I*\operatorname{Sqrt}[a]*g))+\operatorname{Sqrt}[a]*g*(I*\operatorname{Sqrt}[c]*f-\operatorname{Sqrt}[a]*g)*((5*I)*a*e*g^2+I*c*f*(4*e*f-7*d*g)+3*\operatorname{Sqrt}[a]*\operatorname{Sqrt}[c]*g*(e*f+7*d*g))*\operatorname{Sqrt}[(g*((I*\operatorname{Sqrt}[a])/ \operatorname{Sqrt}[c]+x))/(f+g*x))*\operatorname{Sqrt}[-( ((I*\operatorname{Sqrt}[a]*g)/\operatorname{Sqrt}[c]-g*x)/(f+g*x) )]*(f+g*x)^{(3/2)}*\operatorname{EllipticF}[I*\operatorname{ArcSinh}[\operatorname{Sqrt}[-f-(I*\operatorname{Sqrt}[a]*g)/\operatorname{Sqrt}[c]]/\operatorname{Sqrt}[f+g*x]],(\operatorname{Sqrt}[c]*f-I*\operatorname{Sqrt}[a]*g)/(\operatorname{Sqrt}[c]*f+I*\operatorname{Sqrt}[a]*g)))/(c*g^4*\operatorname{Sqrt}[-f-(I*\operatorname{Sqrt}[a]*g)/\operatorname{Sqrt}[c]]*(f+g*x)))/(105*\operatorname{Sqrt}[a+c*x^2])$

Maple [B] time = 0.03, size = 2549, normalized size = 5.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((e^*x+d)^*(g^*x+f)^{(1/2)}*(c^*x^2+a)^{(1/2)},x)$

[Out] 
$$\begin{aligned} & -2/105*(g^*x+f)^{(1/2)}*(c^*x^2+a)^{(1/2)}*(-28*x^2*a^*c^2*d^*f^*g^4+x^2*a^*c^2 \\ & *e^*f^2*g^3-28*x^2*a^*c^2*e^*f^*g^4-18*(-a^*c)^{(1/2)}*(-(g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*EllipticF((- (g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)},(- (g^*(-a^*c)^{(1/2)}-c^*f) \\ & )/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*a^*c^2*e^*f^2*g^3+14*(-a^*c)^{(1/2)}*(-(g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*EllipticF((- (g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)},(- (g^*(-a^*c)^{(1/2)}-c^*f) \\ & )/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*a^*c^2*d^*f^*g^4-15*x^5*c^3 \\ & *e^*g^5-21*x^4*c^3*d^*g^5-6*a^*c^2*(-(g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*EllipticF((- (g^*x+f)^*c \\ & )/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)},(- (g^*(-a^*c)^{(1/2)}-c^*f) \\ & )/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*e^*f^3*g^2+16*a^2*c^2*(-(g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*EllipticE((- (g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)},(- (g^*(-a^*c)^{(1/2)}-c^*f) \\ & )/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*e^*f^3*g^4+28*a^*c^2*(-(g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*EllipticE((- (g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)},(- (g^*(-a^*c)^{(1/2)}-c^*f) \\ & )/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*d^*f^2*g^3+24*a^*c^2*(-(g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*EllipticE((- (g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)},(- (g^*(-a^*c)^{(1/2)}-c^*f) \\ & )/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*e^*f^3*g^2+14*(-a^*c)^{(1/2)}*(-(g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*EllipticF((- (g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)},(- (g^*(-a^*c)^{(1/2)}-c^*f) \\ & )/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*c^2*d^*f^3*g^2-8*(-a^*c)^{(1/2)}*(-(g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*EllipticF((- (g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)},(- (g^*(-a^*c)^{(1/2)}-c^*f) \\ & )/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*c^2*e^*f^4*g-6*a^2*c^2*(-(g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*EllipticF((- (g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)},(- (g^*(-a^*c)^{(1/2)}-c^*f) \\ & )/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*e^*f^4-42*a^*c^2*(-(g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*EllipticF((- (g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)},(- (g^*(-a^*c)^{(1/2)}-c^*f) \\ & )/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*d^*f^2*g^3+x^3*c^3*e^*f^2 \\ & *g^3-42*a^2*c^2*(-(g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*EllipticF((- (g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)},(- (g^*(-a^*c)^{(1/2)}-c^*f) \\ & )/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*d^*g^5+8*(-(g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*EllipticE((- (g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)},(- (g^*(-a^*c)^{(1/2)}-c^*f) \\ & )/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*c^3*e^*f^5-21*x^2*a^*c^2*d^*g^5-7*x^2*c^3*d^*f^2*g^3+4*x^2*c^3*e^*f^3*g^2-10*x^2*a^*c^2*c^2*e^*g^5-18*x^4*c^3*e^*f^*g^4-25*x^3*a^*c^2*e^*g^5-28*x^3*c^3*d^*f^*g^4-10*a^2*c^2*e^*f^*g^4-7*a^*c^2*d^*f^2*g^3+4*a^*c^2*e^*f^3*g^2+42*a^2*c^2*(-(g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*EllipticE((- (g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)},(- (g^*(-a^*c)^{(1/2)}-c^*f) \\ & )/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*d^*g^5-10*(-a^*c)^{(1/2)}*(-(g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f) \\ & )^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)}*EllipticF((- (g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\ & )^{(1/2)},(- (g^*(-a^*c)^{(1/2)}-c^*f) \\ & )/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*a^2*e^*g^5-14 \end{aligned}$$



$$\frac{(-g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}*((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}*EllipticE((-g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)},(-g^*(-a^*c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*c^3*d*f^4*g)/(c^*g^*x^3+c^*f^*x^2+a^*g^*x+a^*f)/g^4/c^2$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + a}(ex + d)\sqrt{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)\*(e\*x + d)\*sqrt(g\*x + f), x, algorithm="maxima")

[Out] integrate(sqrt(c\*x^2 + a)\*(e\*x + d)\*sqrt(g\*x + f), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^2 + a}(ex + d)\sqrt{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)\*(e\*x + d)\*sqrt(g\*x + f), x, algorithm="fricas")

[Out] integral(sqrt(c\*x^2 + a)\*(e\*x + d)\*sqrt(g\*x + f), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + cx^2}(d + ex)\sqrt{f + gx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(g\*x+f)\*\*(1/2)\*(c\*x\*\*2+a)\*\*(1/2), x)

[Out] Integral(sqrt(a + c\*x\*\*2)\*(d + e\*x)\*sqrt(f + g\*x), x)

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)\*(e\*x + d)\*sqrt(g\*x + f), x, algorithm="giac")

[Out] Exception raised: RuntimeError

### 3.625 $\int \sqrt{f + gx} \sqrt{a + cx^2} dx$

**Optimal.** Leaf size=362

$$\frac{4\sqrt{-a}f\sqrt{\frac{cx^2}{a}+1}(ag^2+cf^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15\sqrt{c}g^2\sqrt{a+cx^2}\sqrt{f+gx}} + \frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}(cf^2-3ag^2)E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15\sqrt{c}g^2\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}} + \frac{2\sqrt{a+cx^2}(f+gx)^{3/2}}{5g} - \frac{4f\sqrt{a+cx^2}\sqrt{f+gx}}{15g}$$

[Out]  $(-4*f*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])/(15*g) + (2*(f + g*x)^{(3/2)}*\text{Sqrt}[a + c*x^2])/(5*g) + (4*\text{Sqrt}[-a]*(c*f^2 - 3*a*g^2)*\text{Sqrt}[f + g*x]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)))/(15*\text{Sqrt}[c]*g^2*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[a + c*x^2]) - (4*\text{Sqrt}[-a]*f*(c*f^2 + a*g^2)*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)))/(15*\text{Sqrt}[c]*g^2*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])$

**Rubi [A]** time = 0.850302, antiderivative size = 362, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{4\sqrt{-a}f\sqrt{\frac{cx^2}{a}+1}(ag^2+cf^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15\sqrt{c}g^2\sqrt{a+cx^2}\sqrt{f+gx}} + \frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}(cf^2-3ag^2)E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{15\sqrt{c}g^2\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}} + \frac{2\sqrt{a+cx^2}(f+gx)^{3/2}}{5g} - \frac{4f\sqrt{a+cx^2}\sqrt{f+gx}}{15g}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2], x]$

[Out]  $(-4*f*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])/(15*g) + (2*(f + g*x)^{(3/2)}*\text{Sqrt}[a + c*x^2])/(5*g) + (4*\text{Sqrt}[-a]*(c*f^2 - 3*a*g^2)*\text{Sqrt}[f + g*x]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)))/(15*\text{Sqrt}[c]*g^2*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[a + c*x^2]) - (4*\text{Sqrt}[-a]*f*(c*f^2 + a*g^2)*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)))/(15*\text{Sqrt}[c]*g^2*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])$

**Rubi in Sympy [A]** time = 144.348, size = 340, normalized size = 0.94

$$\begin{aligned} & -\frac{4f\sqrt{a+cx^2}\sqrt{f+gx}}{15g} + \frac{2\sqrt{a+cx^2}(f+gx)^{\frac{3}{2}}}{5g} \\ & - \frac{4f\sqrt{-a}\sqrt{\frac{\sqrt{c}\sqrt{-a}(-f-gx)}{ag-\sqrt{c}f\sqrt{-a}}}\sqrt{1+\frac{cx^2}{a}}(ag^2+cf^2)F\left(\operatorname{asin}\left(\sqrt{-\frac{\sqrt{c}x}{2\sqrt{-a}}+\frac{1}{2}}\right)\middle|\frac{2ag}{ag-\sqrt{c}f\sqrt{-a}}\right)}{15\sqrt{c}g^2\sqrt{a+cx^2}\sqrt{f+gx}} \\ & - \frac{4\sqrt{-a}\sqrt{1+\frac{cx^2}{a}}\sqrt{f+gx}(3ag^2-cf^2)E\left(\operatorname{asin}\left(\sqrt{-\frac{\sqrt{c}x}{2\sqrt{-a}}+\frac{1}{2}}\right)\middle|\frac{2ag}{ag-\sqrt{c}f\sqrt{-a}}\right)}{15\sqrt{c}g^2\sqrt{\frac{\sqrt{c}\sqrt{-a}(-f-gx)}{ag-\sqrt{c}f\sqrt{-a}}}\sqrt{a+cx^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2),x)`

[Out] `-4*f*sqrt(a + c*x**2)*sqrt(f + g*x)/(15*g) + 2*sqrt(a + c*x**2)*(f + g*x)**(3/2)/(5*g) - 4*f*sqrt(-a)*sqrt(sqrt(c)*sqrt(-a))*(-f - g*x)/(a*g - sqrt(c)*f*sqrt(-a))*sqrt(1 + c*x**2/a)*(a*g**2 + c*f**2)*elliptic_f(asin(sqrt(-sqrt(c)*x/(2*sqrt(-a)) + 1/2)), 2*a*g/(a*g - sqrt(c)*f*sqrt(-a)))/(15*sqrt(c)*g**2*sqrt(a + c*x**2)*sqrt(f + g*x)) - 4*sqrt(-a)*sqrt(1 + c*x**2/a)*sqrt(f + g*x)*(3*a*g**2 - c*f**2)*elliptic_e(asin(sqrt(-sqrt(c)*x/(2*sqrt(-a)) + 1/2)), 2*a*g/(a*g - sqrt(c)*f*sqrt(-a)))/(15*sqrt(c)*g**2*sqrt(sqrt(c)*sqrt(-a))*(-f - g*x)/(a*g - sqrt(c)*f*sqrt(-a))*sqrt(a + c*x**2)`

**Mathematica [C]** time = 4.60113, size = 536, normalized size = 1.48

$$\sqrt{f+gx} \left( \frac{2(a+cx^2)(f+3gx)}{g} - \frac{4 \left( \sqrt{c}(f+gx)^{3/2} \left( -3a^{3/2}g^3 + \sqrt{c}cf^2g + 3ia\sqrt{c}fg^2 - ic^{3/2}f^3 \right) \sqrt{\frac{g(x+\frac{i\sqrt{a}}{\sqrt{c}})}{f+gx}} \sqrt{-\frac{-gx+\frac{i\sqrt{a}g}{\sqrt{c}}}{f+gx}} E \left( i \sinh^{-1} \left( \frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}} \right) \middle| \frac{\sqrt{c}f-i\sqrt{a}g}{\sqrt{c}f+i\sqrt{a}g} \right) + g^2 \right)}{15\sqrt{a+cx^2}} \right)$$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[f + g*x]*Sqrt[a + c*x^2],x]`

[Out] `(Sqrt[f + g*x]*((2*(f + 3*g*x)*(a + c*x^2))/g - (4*(g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(-3*a^2*g^2 + c^2*f^2*x^2 + a*c*(f^2 - 3*g^2*x^2)) + Sqrt[c]*((-I)*c^(3/2)*f^3 + Sqrt[a]*c*f^2*g + (3*I)*a*Sqrt[c]*f*g^2 - 3*a^(3/2)*g^3)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - Sqrt[a]*Sqrt[c]*g*(c*f^2 + (4*I)*Sqrt[a]*Sqrt[c]*f*g - 3*a*g^2)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)))/(c*g^3*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)))/(15*Sqrt[a + c*x^2])`

**Maple [B]** time = 0.025, size = 1162, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^(1/2)*(c*x^2+a)^(1/2),x)`

[Out] 
$$\frac{2}{15} (g^*x+f)^{(1/2)} (c^*x^2+a)^{(1/2)} (6^*a^2 (-g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} ((-c^*x+(-a^*c)^{(1/2)})^*g / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} ((c^*x+(-a^*c)^{(1/2)})^*g / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} \text{EllipticF}((-g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)}, (-g^*(-a^*c)^{(1/2)} - c^*f) / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} g^{4+6} (-g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} ((-c^*x+(-a^*c)^{(1/2)})^*g / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} ((c^*x+(-a^*c)^{(1/2)})^*g / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} \text{EllipticF}((-g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)}, (-g^*(-a^*c)^{(1/2)} - c^*f) / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} a^*c^*f^2 g^2 - 2^* (-a^*c)^{(1/2)} (-g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} ((-c^*x+(-a^*c)^{(1/2)})^*g / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} ((c^*x+(-a^*c)^{(1/2)})^*g / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} \text{EllipticF}((-g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)}, (-g^*(-a^*c)^{(1/2)} - c^*f) / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} a^*f^2 g^3 - 2^* (-a^*c)^{(1/2)} (-g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} ((-c^*x+(-a^*c)^{(1/2)})^*g / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} ((c^*x+(-a^*c)^{(1/2)})^*g / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} \text{EllipticF}((-g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)}, (-g^*(-a^*c)^{(1/2)} - c^*f) / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} c^*f^3 g - 6^*a^2 (-g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} ((-c^*x+(-a^*c)^{(1/2)})^*g / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} ((c^*x+(-a^*c)^{(1/2)})^*g / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} \text{EllipticE}((-g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)}, (-g^*(-a^*c)^{(1/2)} - c^*f) / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} g^{4-4} (-g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} ((-c^*x+(-a^*c)^{(1/2)})^*g / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} ((c^*x+(-a^*c)^{(1/2)})^*g / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} \text{EllipticE}((-g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)}, (-g^*(-a^*c)^{(1/2)} - c^*f) / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} a^*c^*f^2 g^2 + 2^* (-g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} ((-c^*x+(-a^*c)^{(1/2)})^*g / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} ((c^*x+(-a^*c)^{(1/2)})^*g / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} \text{EllipticE}((-g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)}, (-g^*(-a^*c)^{(1/2)} - c^*f) / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} c^2 f^4 + 3^*x^4 c^2 g^4 + 4^*x^3 c^2 f^3 g^3 + 3^*x^2 a^*c^*g^4 + x^2 c^2 f^2 g^2 + 4^*x^*a^*c^*f^2 g^3 + a^*c^*f^2 g^2) / c / (c^*g^*x^3 + c^*f^*x^2 + a^*g^*x + a^*f) / g^3$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + a} \sqrt{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + a)*sqrt(g*x + f),x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^2 + a)*sqrt(g*x + f), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^2 + a}\sqrt{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + a)*sqrt(g*x + f),x, algorithm="fricas")`

[Out] `integral(sqrt(c*x^2 + a)*sqrt(g*x + f), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + cx^2} \sqrt{f + gx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)*sqrt(f + g*x), x)
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

$$3.626 \quad \int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{d+ex} dx$$

**Optimal.** Leaf size=683

$$\begin{aligned} & \frac{2\sqrt{\frac{cx^2}{a}+1}(ae^2+cd^2)(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}\left(\frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}}+e}; \sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle| \frac{2\sqrt{-ag}}{\sqrt{cf}+\sqrt{-ag}}\right)}{e^3\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{cd}}{\sqrt{-a}}+e\right)} \\ & + \frac{2\sqrt{-a}\sqrt{cf}\sqrt{\frac{cx^2}{a}+1}(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf}-ag}\right)}{3e^2g\sqrt{a+cx^2}\sqrt{f+gx}} \\ & - \frac{2\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}(ef-3dg)E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf}-ag}\right)}{3e^2g\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}} \\ & - \frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}(2ae^2g-3cd(ef-dg))F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf}-ag}\right)}{3\sqrt{ce^3}\sqrt{a+cx^2}\sqrt{f+gx}} \\ & + \frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3e} \end{aligned}$$

```
[Out] (2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(3*e) - (2*Sqrt[-a]*Sqrt[c]*(e*f - 3*d*g)*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)]/(3*e^2*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (2*Sqrt[-a]*Sqrt[c]*f*(e*f - 3*d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)]/(3*e^2*g*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (2*Sqrt[-a]*(2*a*e^2*g - 3*c*d*(e*f - d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)]/(3*Sqrt[c]*e^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (2*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g))/(e^3*((Sqrt[c]*d)/Sqrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

**Rubi [A]** time = 4.65332, antiderivative size = 683, normalized size of antiderivative = 1., number of

steps used = 14, number of rules used = 10, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$\begin{aligned}
 & \frac{2\sqrt{\frac{cx^2}{a} + 1} (ae^2 + cd^2) (ef - dg) \sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}} \left( \frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}} + e}; \sin^{-1} \left( \frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}} \right) \Big| \frac{2\sqrt{-ag}}{\sqrt{cf+\sqrt{-ag}}} \right)}{e^3 \sqrt{a + cx^2} \sqrt{f + gx} \left( \frac{\sqrt{cd}}{\sqrt{-a}} + e \right)} \\
 & + \frac{2\sqrt{-a} \sqrt{cf} \sqrt{\frac{cx^2}{a} + 1} (ef - 3dg) \sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}} F \left( \sin^{-1} \left( \frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}} \right) \Big| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}} \right)}{3e^2 g \sqrt{a + cx^2} \sqrt{f + gx}} \\
 & - \frac{2\sqrt{-a} \sqrt{c} \sqrt{\frac{cx^2}{a} + 1} \sqrt{f + gx} (ef - 3dg) E \left( \sin^{-1} \left( \frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}} \right) \Big| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}} \right)}{3e^2 g \sqrt{a + cx^2} \sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}} \\
 & - \frac{2\sqrt{-a} \sqrt{\frac{cx^2}{a} + 1} \sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}} (2ae^2 g - 3cd(ef - dg)) F \left( \sin^{-1} \left( \frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}} \right) \Big| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}} \right)}{3\sqrt{c} e^3 \sqrt{a + cx^2} \sqrt{f + gx}} \\
 & + \frac{2\sqrt{a + cx^2} \sqrt{f + gx}}{3e}
 \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])/(d + e\*x), x]

[Out]  $(2*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])/(3*e) - (2*\text{Sqrt}[-a]*\text{Sqrt}[c]*(e*f - 3*d*g)*\text{Sqrt}[f + g*x]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)]/(3*e^2*g*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[a + c*x^2]) + (2*\text{Sqrt}[-a]*\text{Sqrt}[c]*f*(e*f - 3*d*g)*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)]/(3*e^2*g*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2]) - (2*\text{Sqrt}[-a]*(2*a*e^2*g - 3*c*d*(e*f - d*g))*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)]/(3*\text{Sqrt}[c]*e^3*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2]) - (2*(c*d^2 + a*e^2)*(e*f - d*g)*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticPi}[(2*e)/((\text{Sqrt}[c]*d)/\text{Sqrt}[-a] + e), \text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (2*\text{Sqrt}[-a]*g)/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]/(e^3*((\text{Sqrt}[c]*d)/\text{Sqrt}[-a] + e)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])$

**Rubi in SymPy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2} \sqrt{f + gx}}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*(1/2)\*(c\*x\*\*2+a)\*\*(1/2)/(e\*x+d), x)

[Out] Integral(sqrt(a + c\*x\*\*2)\*sqrt(f + g\*x)/(d + e\*x), x)

**Mathematica [C]** time = 12.6701, size = 1216, normalized size = 1.78

$$\left( \frac{2ce^2\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}f^3}}{(f+gx)^2} - \frac{4ce^2\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}f^2}}{f+gx} - \frac{6cdeg\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}f^2}}{(f+gx)^2} + 2ce^2\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}f} + \frac{12cdeg\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}f}}{f+gx} + \frac{2ae^2g^2\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}f}}{(f+gx)^2} + \frac{2\sqrt{ce}}{(f+gx)^2} \right) + \frac{2\sqrt{cx^2+a}\sqrt{f+gx}}{3e}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])/(d + e\*x),x]

[Out]  $(2*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])/(3*e) + ((f + g*x)^{(3/2)}*(2*c*e^{1/2}*f*\text{Sqrt}[-f - (I*\text{Sqrt}[a]*g)/\text{Sqrt}[c]] - 6*c*d*e*g*\text{Sqrt}[-f - (I*\text{Sqrt}[a]*g)/\text{Sqrt}[c]])/(f + g*x)^2 - (6*c*d*e*f^2*g*\text{Sqrt}[-f - (I*\text{Sqrt}[a]*g)/\text{Sqrt}[c]])/(f + g*x)^2 + (2*a*e^{1/2}*f^2*g*\text{Sqrt}[-f - (I*\text{Sqrt}[a]*g)/\text{Sqrt}[c]])/(f + g*x)^2 - (6*a*d*e*g^3*\text{Sqrt}[-f - (I*\text{Sqrt}[a]*g)/\text{Sqrt}[c]])/(f + g*x)^2 - (4*c*e^{1/2}*f^2*g*\text{Sqrt}[-f - (I*\text{Sqrt}[a]*g)/\text{Sqrt}[c]])/(f + g*x) + (12*c*d*e*f*g*\text{Sqrt}[-f - (I*\text{Sqrt}[a]*g)/\text{Sqrt}[c]])/(f + g*x) + (2*\text{Sqrt}[c]*e*((-I)*\text{Sqrt}[c]*f + \text{Sqrt}[a]*g)*(e*f - 3*d*g)*\text{Sqrt}[1 - f/(f + g*x)] - (I*\text{Sqrt}[a]*g)/(\text{Sqrt}[c]*(f + g*x))*\text{EllipticE}[I*\text{ArcSinh}[\text{Sqrt}[-f - (I*\text{Sqrt}[a]*g)/\text{Sqrt}[c]]/\text{Sqrt}[f + g*x]], (\text{Sqrt}[c]*f - I*\text{Sqrt}[a]*g)/(\text{Sqrt}[c]*f + I*\text{Sqrt}[a]*g)]/\text{Sqrt}[f + g*x] + (2*e*(3*\text{Sqrt}[c]*d - I*\text{Sqrt}[a]*e)*g*((-I)*\text{Sqrt}[c]*f + \text{Sqrt}[a]*g)*\text{Sqrt}[1 - f/(f + g*x)] - (I*\text{Sqrt}[a]*g)/(\text{Sqrt}[c]*(f + g*x))*\text{Sqrt}[1 - f/(f + g*x)] + (I*\text{Sqrt}[a]*g)/(\text{Sqrt}[c]*(f + g*x))*\text{EllipticF}[I*\text{ArcSinh}[\text{Sqrt}[-f - (I*\text{Sqrt}[a]*g)/\text{Sqrt}[c]]/\text{Sqrt}[f + g*x]], (\text{Sqrt}[c]*f - I*\text{Sqrt}[a]*g)/(\text{Sqrt}[c]*f + I*\text{Sqrt}[a]*g)]/\text{Sqrt}[f + g*x] + ((6*I)*c*d^2*g^2*\text{Sqrt}[1 - f/(f + g*x)] - (I*\text{Sqrt}[a]*g)/(\text{Sqrt}[c]*(f + g*x))*\text{Sqrt}[1 - f/(f + g*x)] + (I*\text{Sqrt}[a]*g)/(\text{Sqrt}[c]*(f + g*x))*\text{EllipticPi}[(\text{Sqrt}[c]*(e*f - d*g))/(\text{Sqrt}[c]*f + I*\text{Sqrt}[a]*g)], I*\text{ArcSinh}[\text{Sqrt}[-f - (I*\text{Sqrt}[a]*g)/\text{Sqrt}[c]]/\text{Sqrt}[f + g*x]], (\text{Sqrt}[c]*f - I*\text{Sqrt}[a]*g)/(\text{Sqrt}[c]*f + I*\text{Sqrt}[a]*g)]/\text{Sqrt}[f + g*x] + ((6*I)*a*e^{1/2}*g^2*\text{Sqrt}[1 - f/(f + g*x)] - (I*\text{Sqrt}[a]*g)/(\text{Sqrt}[c]*(f + g*x))*\text{Sqrt}[1 - f/(f + g*x)] + (I*\text{Sqrt}[a]*g)/(\text{Sqrt}[c]*(f + g*x))*\text{EllipticPi}[(\text{Sqrt}[c]*(e*f - d*g))/(\text{Sqrt}[c]*f + I*\text{Sqrt}[a]*g)], I*\text{ArcSinh}[\text{Sqrt}[-f - (I*\text{Sqrt}[a]*g)/\text{Sqrt}[c]]/\text{Sqrt}[f + g*x]], (\text{Sqrt}[c]*f - I*\text{Sqrt}[a]*g)/(\text{Sqrt}[c]*f + I*\text{Sqrt}[a]*g)]/\text{Sqrt}[f + g*x]))/(3*e^{3/2}*g^2*\text{Sqrt}[-f - (I*\text{Sqrt}[a]*g)/\text{Sqrt}[c]]*\text{Sqrt}[a + (c*(f + g*x)^2*(-1 + f/(f + g*x))^2)]/g^2)$

**Maple [B]** time = 0.05, size = 2496, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^(1/2)\*(c\*x^2+a)^(1/2)/(e\*x+d),x)

[Out]  $-2/3*(2*(-(g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}*((-c*x+(-a*c)^{(1/2)})^2)*g/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*((c*x+(-a*c)^{(1/2)})^2)*g/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}*\text{EllipticF}((- (g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (- (g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)})*(-a*c)^{(1/2)}*a*e^{1/2}*g^3+3*(-(g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}*((-c*x+(-a*c)^{(1/2)})^2)*g/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*((c*x+(-a*c)^{(1/2)})^2)*g/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}*\text{EllipticF}((- (g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (- (g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)})*($



$$\begin{aligned}
& -a^*c)^{(1/2)} * c^*d^2 * g^3 - ((g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * ((- \\
& c^*x + (-a^*c)^{(1/2)}) * g / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} * ((c^*x + (-a^*c)^{(1/2)} \\
& )) * g / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * \text{EllipticF}((- (g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)}, \\
& (- (g^*(-a^*c)^{(1/2)} - c^*f) / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)}) * (-a^*c)^{(1/2)} * c^*e^2 * f^2 * g^3 + (- (g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * \\
& ((-c^*x + (-a^*c)^{(1/2)}) * g / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} * ((c^*x + (-a^*c)^{(1/2)}) * g / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * \text{EllipticF}((- (g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)}, \\
& (- (g^*(-a^*c)^{(1/2)} - c^*f) / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)}) * a^*c^*d^*e^*g^3 - 3^* (- (g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * ((-c^*x + (-a^*c)^{(1/2)}) * g / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} * ((c^*x + (-a^*c)^{(1/2)}) * g / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * \text{EllipticF}((- (g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)}, \\
& (- (g^*(-a^*c)^{(1/2)} - c^*f) / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)}) * a^*c^*e^2 * f^2 * g^2 - 3^* (- (g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * ((-c^*x + (-a^*c)^{(1/2)}) * g / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} * ((c^*x + (-a^*c)^{(1/2)}) * g / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * \text{EllipticF}((- (g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)}, \\
& (- (g^*(-a^*c)^{(1/2)} - c^*f) / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)}) * c^2 * d^2 * f^2 * g^2 + 3^* (- (g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * ((-c^*x + (-a^*c)^{(1/2)}) * g / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} * ((c^*x + (-a^*c)^{(1/2)}) * g / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * \text{EllipticF}((- (g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)}, \\
& (- (g^*(-a^*c)^{(1/2)} - c^*f) / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)}) * c^2 * d^*e^*f^2 * g - 3^* (- (g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * ((-c^*x + (-a^*c)^{(1/2)}) * g / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} * ((c^*x + (-a^*c)^{(1/2)}) * g / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * \text{EllipticE}((- (g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)}, \\
& (- (g^*(-a^*c)^{(1/2)} - c^*f) / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)}) * a^*c^*d^*e^*g^3 + (- (g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * ((-c^*x + (-a^*c)^{(1/2)}) * g / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} * ((c^*x + (-a^*c)^{(1/2)}) * g / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * \text{EllipticE}((- (g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)}, \\
& (- (g^*(-a^*c)^{(1/2)} - c^*f) / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)}) * a^*c^*e^2 * f^2 * g^2 - 3^* (- (g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * ((-c^*x + (-a^*c)^{(1/2)}) * g / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} * ((c^*x + (-a^*c)^{(1/2)}) * g / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * \text{EllipticE}((- (g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)}, \\
& (- (g^*(-a^*c)^{(1/2)} - c^*f) / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)}) * c^2 * d^*e^*f^2 * g + (- (g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * ((-c^*x + (-a^*c)^{(1/2)}) * g / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} * ((c^*x + (-a^*c)^{(1/2)}) * g / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * \text{EllipticE}((- (g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)}, \\
& (- (g^*(-a^*c)^{(1/2)} - c^*f) / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)}) * c^2 * e^2 * f^3 - 3^* (- (g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * ((-c^*x + (-a^*c)^{(1/2)}) * g / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} * ((c^*x + (-a^*c)^{(1/2)}) * g / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * \text{EllipticPi}((- (g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)}, \\
& (g^*(-a^*c)^{(1/2)} - c^*f)^*e / c / (d^*g - e^*f), (- (g^*(-a^*c)^{(1/2)} - c^*f) / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)}) * (-a^*c)^{(1/2)} * a^*e^2 * g^3 - 3^* (- (g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * ((-c^*x + (-a^*c)^{(1/2)}) * g / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} * ((c^*x + (-a^*c)^{(1/2)}) * g / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * \text{EllipticPi}((- (g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)}, \\
& (g^*(-a^*c)^{(1/2)} - c^*f)^*e / c / (d^*g - e^*f), (- (g^*(-a^*c)^{(1/2)} - c^*f) / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)}) * (-a^*c)^{(1/2)} * c^*d^2 * g^3 + 3^* (- (g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * ((-c^*x + (-a^*c)^{(1/2)}) * g / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} * ((c^*x + (-a^*c)^{(1/2)}) * g / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * \text{EllipticPi}((- (g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)}, \\
& (g^*(-a^*c)^{(1/2)} - c^*f)^*e / c / (d^*g - e^*f), (- (g^*(-a^*c)^{(1/2)} - c^*f) / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)}) * a^*c^*e^2 * f^2 * g^2 + 3^* (- (g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * ((-c^*x + (-a^*c)^{(1/2)}) * g / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} * ((c^*x + (-a^*c)^{(1/2)}) * g / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * \text{EllipticPi}((- (g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)}, \\
& (g^*(-a^*c)^{(1/2)} - c^*f)^*e / c / (d^*g - e^*f), (- (g^*(-a^*c)^{(1/2)} - c^*f) / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)}) * c^2 * d^2 * f^2 * g^2 - x^3 * c^2 * e^2 * g^3 - x^2 * c^2 * e^2 * f^2 * g^2 - x^2 * a^*c^*e^2 * g^3 - a^*c^*e^2 * f^2 * g^2 * (g^*x+f)^{(1/2)} * (c^*x^2 + a)^{(1/2)} / e^3 / c / g^2 / (c^*g^*x^3 + c^*f^*x^2 + a^*g^*x + a^*f)
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}\sqrt{gx + f}}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)\*sqrt(g\*x + f)/(e\*x + d),x, algorithm="maxima")

[Out] integrate(sqrt(c\*x^2 + a)\*sqrt(g\*x + f)/(e\*x + d), x)

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d), x, algorithm="fricas")`

[Out] Timed out

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2} \sqrt{f + gx}}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2)/(e*x+d), x)`

[Out] `Integral(sqrt(a + c*x**2)*sqrt(f + g*x)/(d + e*x), x)`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a} \sqrt{gx + f}}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d), x, algorithm="giac")`

[Out] `integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d), x)`

$$3.627 \quad \int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^2} dx$$

**Optimal.** Leaf size=650

$$\frac{\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}(2ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}}{\sqrt{-a}}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^3\sqrt{a+cx^2}\sqrt{f+gx}}$$

$$-\frac{\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}(ae^2g-cd(2ef-3dg))\left(\frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}}{\sqrt{-a}}}\right)\middle|\frac{2\sqrt{-ag}}{\sqrt{c}f+\sqrt{-ag}}\right)}{e^3\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{cd}}{\sqrt{-a}}+e\right)}$$

$$-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{e(d+ex)}+\frac{3\sqrt{-a}\sqrt{c}f\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}}{\sqrt{-a}}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2\sqrt{a+cx^2}\sqrt{f+gx}}$$

$$-\frac{3\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}}{\sqrt{-a}}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e^2\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}}$$

```
[Out] -((Sqrt[f + g*x]*Sqrt[a + c*x^2])/(e*(d + e*x))) - (3*Sqrt[-a]*Sqrt[c]*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)]/(e^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (3*Sqrt[-a]*Sqrt[c]*f*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)]/(e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*(2*e*f - 3*d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)]/(e^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - ((a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)]/(e^3*((Sqrt[c]*d)/Sqrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

**Rubi [A]** time = 4.10413, antiderivative size = 650, normalized size of antiderivative = 1., number of

steps used = 14, number of rules used = 10, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$\frac{\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a} + 1}(2ef - 3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}}{\sqrt{-a}}}\right)\Big|_{-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}}\right)}{e^3\sqrt{a+cx^2}\sqrt{f+gx}}$$

$$-\frac{\sqrt{\frac{cx^2}{a} + 1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}(ae^2g - cd(2ef - 3dg))\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e}; \sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}}{\sqrt{-a}}}\right)\Big|_{\frac{2\sqrt{-a}g}{\sqrt{c}f+\sqrt{-a}g}}\right)}{e^3\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)}$$

$$-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{e(d+ex)} + \frac{3\sqrt{-a}\sqrt{c}f\sqrt{\frac{cx^2}{a} + 1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}}{\sqrt{-a}}}\right)\Big|_{-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}}\right)}{e^2\sqrt{a+cx^2}\sqrt{f+gx}}$$

$$-\frac{3\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a} + 1}\sqrt{f+gx}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}}{\sqrt{-a}}}\right)\Big|_{-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}}\right)}{e^2\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])/(d + e\*x)^2, x]

[Out] -((Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])/(e\*(d + e\*x))) - (3\*Sqrt[-a]\*Sqrt[c]\*Sqrt[f + g\*x]\*Sqrt[1 + (c\*x^2)/a]\*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(e^2\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[a + c\*x^2]) + (3\*Sqrt[-a]\*Sqrt[c]\*f\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(e^2\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]) - (Sqrt[-a]\*Sqrt[c]\*(2\*e\*f - 3\*d\*g)\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(e^3\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]) - ((a\*e^2\*g - c\*d\*(2\*e\*f - 3\*d\*g))\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticPi[(2\*e)/((Sqrt[c]\*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (2\*Sqrt[-a]\*g)/(Sqrt[c]\*f + Sqrt[-a]\*g)]/(e^3\*((Sqrt[c]\*d)/Sqrt[-a] + e)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*(1/2)\*(c\*x\*\*2+a)\*\*(1/2)/(e\*x+d)\*\*2, x)

[Out] Integral(sqrt(a + c\*x\*\*2)\*sqrt(f + g\*x)/(d + e\*x)\*\*2, x)

**Mathematica [C]** time = 13.5394, size = 7969, normalized size = 12.26

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])/(d + e\*x)^2,x]

[Out] Result too large to show

**Maple [B]** time = 0.064, size = 6044, normalized size = 9.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^(1/2)\*(c\*x^2+a)^(1/2)/(e\*x+d)^2,x)

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}\sqrt{gx + f}}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)\*sqrt(g\*x + f)/(e\*x + d)^2,x, algorithm="maxima")

[Out] integrate(sqrt(c\*x^2 + a)\*sqrt(g\*x + f)/(e\*x + d)^2, x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)\*sqrt(g\*x + f)/(e\*x + d)^2,x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*(1/2)\*(c\*x\*\*2+a)\*\*(1/2)/(e\*x+d)\*\*2,x)

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d)^2,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

$$3.628 \quad \int \frac{\sqrt{f+gx}\sqrt{a+cx^2}}{(d+ex)^3} dx$$

Optimal. Leaf size=1205

result too large to display

```
[Out] -(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(2*e*(d + e*x)^2) - ((a*e^2*g -
c*d*(2*e*f - 3*d*g))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(4*e*(c*d^2 +
a*e^2)*(e*f - d*g)*(d + e*x)) - (Sqrt[-a]*Sqrt[c]*(a*e^2*g - c*d
*(2*e*f - 3*d*g))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[Arc
Sin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*S
qrt[c]*f - a*g)))/(4*e^2*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[(Sqrt[c
]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) - (3*Sqrt
[-a]*Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]
*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-
a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)))/(2*e^3*Sqrt[f
+ g*x]*Sqrt[a + c*x^2]) + (Sqrt[-a]*Sqrt[c]*f*(a*e^2*g - c*d*(2*
e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*
Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a
]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)))/(4*e^2*(c*d^2
+ a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (Sqrt[-a]*S
qrt[c]*d*g*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x
))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin
[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt
[c]*f - a*g)))/(4*e^3*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*S
qrt[a + c*x^2]) - (c*(e*f - 3*d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt
[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[
c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2
]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)))/(e^3*((Sqrt[c]*d)/S
qrt[-a] + e)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) + ((a*e^2*g - c*d*(2*
e*f - 3*d*g))^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)
]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e)
, ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/
(Sqrt[c]*f + Sqrt[-a]*g)))/(4*e^3*((Sqrt[c]*d)/Sqrt[-a] + e)*(c*d
^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

---

**Rubi [A]** time = 9.71483, antiderivative size = 1205, normalized size of antiderivative = 1., number

of steps used = 23, number of rules used = 11, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$

$$\begin{aligned}
 & \frac{\sqrt{\frac{\sqrt{c(f+gx)}}{\sqrt{cf+\sqrt{-ag}}}} \sqrt{\frac{cx^2}{a} + 1} \left( \frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}} + e}; \sin^{-1} \left( \sqrt{\frac{1-\sqrt{cx}}{\sqrt{-a}}} \right) \middle| \frac{2\sqrt{-ag}}{\sqrt{cf+\sqrt{-ag}}} \right) (ae^2g - cd(2ef - 3dg))^2}{4e^3 \left( \frac{\sqrt{cd}}{\sqrt{-a}} + e \right) (cd^2 + ae^2) (ef - dg) \sqrt{f + gx} \sqrt{cx^2 + a}} \\
 & - \frac{\sqrt{-a} \sqrt{c} \sqrt{f + gx} \sqrt{\frac{cx^2}{a} + 1} E \left( \sin^{-1} \left( \sqrt{\frac{1-\sqrt{cx}}{\sqrt{-a}}} \right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}} \right) (ae^2g - cd(2ef - 3dg))}{4e^2 (cd^2 + ae^2) (ef - dg) \sqrt{\frac{\sqrt{c(f+gx)}}{\sqrt{cf+\sqrt{-ag}}}} \sqrt{cx^2 + a}} \\
 & + \frac{\sqrt{-a} \sqrt{c} f \sqrt{\frac{\sqrt{c(f+gx)}}{\sqrt{cf+\sqrt{-ag}}}} \sqrt{\frac{cx^2}{a} + 1} F \left( \sin^{-1} \left( \sqrt{\frac{1-\sqrt{cx}}{\sqrt{-a}}} \right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}} \right) (ae^2g - cd(2ef - 3dg))}{4e^2 (cd^2 + ae^2) (ef - dg) \sqrt{f + gx} \sqrt{cx^2 + a}} \\
 & - \frac{\sqrt{-a} \sqrt{cdg} \sqrt{\frac{\sqrt{c(f+gx)}}{\sqrt{cf+\sqrt{-ag}}}} \sqrt{\frac{cx^2}{a} + 1} F \left( \sin^{-1} \left( \sqrt{\frac{1-\sqrt{cx}}{\sqrt{-a}}} \right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}} \right) (ae^2g - cd(2ef - 3dg))}{4e^3 (cd^2 + ae^2) (ef - dg) \sqrt{f + gx} \sqrt{cx^2 + a}} \\
 & - \frac{\sqrt{f + gx} \sqrt{cx^2 + a} (ae^2g - cd(2ef - 3dg))}{4e (cd^2 + ae^2) (ef - dg) (d + ex)} \\
 & - \frac{3\sqrt{-a} \sqrt{cg} \sqrt{\frac{\sqrt{c(f+gx)}}{\sqrt{cf+\sqrt{-ag}}}} \sqrt{\frac{cx^2}{a} + 1} F \left( \sin^{-1} \left( \sqrt{\frac{1-\sqrt{cx}}{\sqrt{-a}}} \right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}} \right)}{2e^3 \sqrt{f + gx} \sqrt{cx^2 + a}} \\
 & - \frac{c(ef - 3dg) \sqrt{\frac{\sqrt{c(f+gx)}}{\sqrt{cf+\sqrt{-ag}}}} \sqrt{\frac{cx^2}{a} + 1} \left( \frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}} + e}; \sin^{-1} \left( \sqrt{\frac{1-\sqrt{cx}}{\sqrt{-a}}} \right) \middle| \frac{2\sqrt{-ag}}{\sqrt{cf+\sqrt{-ag}}} \right)}{e^3 \left( \frac{\sqrt{cd}}{\sqrt{-a}} + e \right) \sqrt{f + gx} \sqrt{cx^2 + a}} - \frac{\sqrt{f + gx} \sqrt{cx^2 + a}}{2e(d + ex)^2}
 \end{aligned}$$

Warning: Unable to verify antiderivative.

[In] Int[(Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])/(d + e\*x)^3,x]

[Out]  $-(\text{Sqrt}[f + g*x] * \text{Sqrt}[a + c*x^2]) / (2 * e * (d + e*x)^2) - ((a * e^2 * g - c * d * (2 * e * f - 3 * d * g)) * \text{Sqrt}[f + g*x] * \text{Sqrt}[a + c*x^2]) / (4 * e * (c * d^2 + a * e^2) * (e * f - d * g) * (d + e*x)) - (\text{Sqrt}[-a] * \text{Sqrt}[c] * (a * e^2 * g - c * d * (2 * e * f - 3 * d * g)) * \text{Sqrt}[f + g*x] * \text{Sqrt}[1 + (c*x^2)/a] * \text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2 * a * g) / (\text{Sqrt}[-a] * \text{Sqrt}[c] * f - a * g)]) / (4 * e^2 * (c * d^2 + a * e^2) * (e * f - d * g) * \text{Sqrt}[(\text{Sqrt}[c] * (f + g*x)) / (\text{Sqrt}[c] * f + \text{Sqrt}[-a] * g)]) * \text{Sqrt}[a + c*x^2]) - (3 * \text{Sqrt}[-a] * \text{Sqrt}[c] * g * \text{Sqrt}[(\text{Sqrt}[c] * (f + g*x)) / (\text{Sqrt}[c] * f + \text{Sqrt}[-a] * g)]) * \text{Sqrt}[1 + (c*x^2)/a] * \text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2 * a * g) / (\text{Sqrt}[-a] * \text{Sqrt}[c] * f - a * g)]) / (2 * e^3 * \text{Sqrt}[f + g*x] * \text{Sqrt}[a + c*x^2]) + (\text{Sqrt}[-a] * \text{Sqrt}[c] * f * (a * e^2 * g - c * d * (2 * e * f - 3 * d * g)) * \text{Sqrt}[(\text{Sqrt}[c] * (f + g*x)) / (\text{Sqrt}[c] * f + \text{Sqrt}[-a] * g)]) * \text{Sqrt}[1 + (c*x^2)/a] * \text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2 * a * g) / (\text{Sqrt}[-a] * \text{Sqrt}[c] * f - a * g)]) / (4 * e^2 * (c * d^2 + a * e^2) * (e * f - d * g) * \text{Sqrt}[f + g*x] * \text{Sqrt}[a + c*x^2]) - (\text{Sqrt}[-a] * \text{Sqrt}[c] * d * g * (a * e^2 * g - c * d * (2 * e * f - 3 * d * g)) * \text{Sqrt}[(\text{Sqrt}[c] * (f + g*x)) / (\text{Sqrt}[c] * f + \text{Sqrt}[-a] * g)]) * \text{Sqrt}[1 + (c*x^2)/a] * \text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2 * a * g) / (\text{Sqrt}[-a] * \text{Sqrt}[c] * f - a * g)]) / (4 * e^3 * (c * d^2 + a * e^2) * (e * f - d * g) * \text{Sqrt}[f + g*x] * \text{Sqrt}[a + c*x^2]) - (c * (e * f - 3 * d * g) * \text{Sqrt}[(\text{Sqrt}[c] * (f + g*x)) / (\text{Sqrt}[c] * f + \text{Sqrt}[-a] * g)]) * \text{Sqrt}[1 + (c*x^2)/a] * \text{EllipticPi}[(2 * e) / ((\text{Sqrt}[c] * d) / \text{Sqrt}[-a] + e), \text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (2 * \text{Sqrt}[-a] * g) / (\text{Sqrt}[c] * f + \text{Sqrt}[-a] * g)]) / (e^3 * ((\text{Sqrt}[c] * d) / \text{Sqrt}[-a] + e) * \text{Sqrt}[f + g*x] * \text{Sqrt}[a + c*x^2]) + ((a * e^2 * g - c * d * (2 * e * f - 3 * d * g))^2 * \text{Sqrt}[(\text{Sqrt}[c] * (f + g*x)) / (\text{Sqrt}[c] * f + \text{Sqrt}[-a] * g)]) * \text{Sqrt}[1 + (c*x^2)/a] * \text{EllipticPi}[(2 * e) / ((\text{Sqrt}[c] * d) / \text{Sqrt}[-a] + e), \text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (2 * \text{Sqrt}[-a] * g) / (\text{Sqrt}[c] * f + \text{Sqrt}[-a] * g)]) / (4 * e^3 * ((\text{Sqrt}[c] * d) / \text{Sqrt}[-a] + e) * (c * d$



$$^2 + a * e^2) * (e * f - d * g) * \text{Sqrt}[f + g * x] * \text{Sqrt}[a + c * x^2])$$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2} \sqrt{f + gx}}{(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2)/(e*x+d)**3,x)`

[Out] `Integral(sqrt(a + c*x**2)*sqrt(f + g*x)/(d + e*x)**3, x)`

**Mathematica [C]** time = 14.9315, size = 12364, normalized size = 10.26

Result too large to show

Antiderivative was successfully verified.

[In] `Integrate[(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(d + e*x)^3,x]`

[Out] Result too large to show

**Maple [B]** time = 0.099, size = 19181, normalized size = 15.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(e*x+d)^3,x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a} \sqrt{gx + f}}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d)^3,x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d)^3, x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+a)**(1/2)/(e*x+d)**3,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + a)*sqrt(g*x + f)/(e*x + d)^3,x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

$$3.629 \quad \int \frac{(d+ex)^3 \sqrt{a+cx^2}}{\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=666

$$4\sqrt{-a}\sqrt{\frac{cx^2}{a}} + 1\sqrt{f+gx} (21a^2e^3g^4 - 3aceg^2 (63d^2g^2 - 39defg + 10e^2f^2) - c^2f (-105d^3g^3 + 252d^2efg^2 - 216de^2f^2g + 64e^3f^3))$$

$$\frac{315c^{3/2}g^5\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c(f+gx)}}{\sqrt{-ag+\sqrt{cf}}}}}{4\sqrt{-a}\sqrt{\frac{cx^2}{a}} + 1(ag^2 + cf^2)\sqrt{\frac{\sqrt{c(f+gx)}}{\sqrt{-ag+\sqrt{cf}}}} (9ae^2g^2(2ef - 5dg) - c(-105d^3g^3 + 252d^2efg^2 - 216de^2f^2g + 64e^3f^3))} F\left(\sin^{-1}\left(\frac{315c^{3/2}g^5\sqrt{a+cx^2}\sqrt{f+gx}}{4e\sqrt{a+cx^2}(f+gx)^{3/2}(7ae^2g^2 + c(42d^2g^2 - 111defg + 64e^2f^2))}\right)\right) + \frac{4\sqrt{a+cx^2}\sqrt{f+gx}(9ae^2g^2(2ef - 5dg) + c(-35d^3g^3 + 168d^2efg^2 - 204de^2f^2g + 76e^3f^3))}{315cg^4} - \frac{4e^2\sqrt{a+cx^2}(f+gx)^{5/2}(4ef - 3dg)}{63g^4} + \frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9g}$$

[Out]  $(-4*(9*a*e^2*g^2*(2*e*f - 5*d*g) + c*(76*e^3*f^3 - 204*d*e^2*f^2*g + 168*d^2*e*f*g^2 - 35*d^3*g^3))*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2]) / (315*c*g^4) + (2*(d + e*x)^3*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2]) / (9*g) + (4*e*(7*a*e^2*g^2 + c*(64*e^2*f^2 - 111*d*e*f*g + 42*d^2*g^2)) * (f + g*x)^{(3/2)}*\text{Sqrt}[a + c*x^2]) / (315*c*g^4) - (4*e^2*(4*e*f - 3*d*g) * (f + g*x)^{(5/2)}*\text{Sqrt}[a + c*x^2]) / (63*g^4) + (4*\text{Sqrt}[-a]*(2*1*a^2*e^3*g^4 - 3*a*c*e*g^2*(10*e^2*f^2 - 39*d*e*f*g + 63*d^2*g^2) - c^2*f*(64*e^3*f^3 - 216*d*e^2*f^2*g + 252*d^2*e*f*g^2 - 105*d^3*g^3))*\text{Sqrt}[f + g*x]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)] / (315*c^{(3/2)}*g^5*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[a + c*x^2]) - (4*\text{Sqrt}[-a]*(c*f^2 + a*g^2)*(9*a*e^2*g^2*(2*e*f - 5*d*g) - c*(64*e^3*f^3 - 216*d*e^2*f^2*g + 252*d^2*e*f*g^2 - 105*d^3*g^3))*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)] / (315*c^{(3/2)}*g^5*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])$

**Rubi [A]** time = 3.01197, antiderivative size = 666, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$4\sqrt{-a}\sqrt{\frac{cx^2}{a}} + 1\sqrt{f+gx} (21a^2e^3g^4 - 3aceg^2 (63d^2g^2 - 39defg + 10e^2f^2) - c^2f (-105d^3g^3 + 252d^2efg^2 - 216de^2f^2g + 64e^3f^3))$$

$$\frac{315c^{3/2}g^5\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c(f+gx)}}{\sqrt{-ag+\sqrt{cf}}}}}{4\sqrt{-a}\sqrt{\frac{cx^2}{a}} + 1(ag^2 + cf^2)\sqrt{\frac{\sqrt{c(f+gx)}}{\sqrt{-ag+\sqrt{cf}}}} (9ae^2g^2(2ef - 5dg) - c(-105d^3g^3 + 252d^2efg^2 - 216de^2f^2g + 64e^3f^3))} F\left(\sin^{-1}\left(\frac{315c^{3/2}g^5\sqrt{a+cx^2}\sqrt{f+gx}}{4e\sqrt{a+cx^2}(f+gx)^{3/2}(7ae^2g^2 + c(42d^2g^2 - 111defg + 64e^2f^2))}\right)\right) + \frac{4\sqrt{a+cx^2}\sqrt{f+gx}(9ae^2g^2(2ef - 5dg) + c(-35d^3g^3 + 168d^2efg^2 - 204de^2f^2g + 76e^3f^3))}{315cg^4} - \frac{4e^2\sqrt{a+cx^2}(f+gx)^{5/2}(4ef - 3dg)}{63g^4} + \frac{2\sqrt{a+cx^2}(d+ex)^3\sqrt{f+gx}}{9g}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*Sqrt[a + c\*x^2])/Sqrt[f + g\*x],x]

[Out] 
$$\frac{-4(9a^2e^2g^2(2ef - 5dg) + c(76e^3f^3 - 204d^2e^2f^2g + 168d^2e^2fg^2 - 35d^3g^3))\sqrt{f+gx}\sqrt{a+cx^2}}{(315c^2g^4) + (2(d+ex)^3\sqrt{f+gx}\sqrt{a+cx^2})/(9g)} + \frac{(4e(7a^2e^2g^2 + c(64e^2f^2 - 111d^2efg + 42d^2g^2))\sqrt{f+gx})^{3/2}\sqrt{a+cx^2}}{(315c^2g^4)} - \frac{(4e^2(4ef - 3dg)\sqrt{f+gx})^{5/2}\sqrt{a+cx^2}}{(63g^4)} + \frac{(4\sqrt{-a}(21a^2e^3g^4 - 3ac^2e^2g^2(10e^2f^2 - 39d^2efg + 63d^2g^2) - c^2f(64e^3f^3 - 216d^2e^2f^2g + 252d^2efg^2 - 105d^3g^3))\sqrt{f+gx}\sqrt{1+(cx^2)/a})\text{EllipticE}[\text{ArcSin}[\sqrt{1-(\sqrt{c}x)/\sqrt{-a}}/\sqrt{2}}], (-2ag)/(\sqrt{-a}\sqrt{c}f - ag)]}{(315c^{3/2}g^5\sqrt{(\sqrt{c}(f+gx)/(\sqrt{c}f + \sqrt{-a}g)})\sqrt{a+cx^2}} - (4\sqrt{-a}(c^2f^2 + ag^2)(9a^2e^2g^2(2ef - 5dg) - c(64e^3f^3 - 216d^2e^2f^2g + 252d^2efg^2 - 105d^3g^3))\sqrt{(\sqrt{c}(f+gx)/(\sqrt{c}f + \sqrt{-a}g)})\sqrt{1+(cx^2)/a})\text{EllipticF}[\text{ArcSin}[\sqrt{1-(\sqrt{c}x)/\sqrt{-a}}/\sqrt{2}}], (-2ag)/(\sqrt{-a}\sqrt{c}f - ag)]}{(315c^{3/2}g^5\sqrt{f+gx}\sqrt{a+cx^2})}$$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3\*(c\*x\*\*2+a)\*\*(1/2)/(g\*x+f)\*\*(1/2),x)

[Out] Timed out

**Mathematica [C]** time = 13.5582, size = 5379, normalized size = 8.08

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^3\*Sqrt[a + c\*x^2])/Sqrt[f + g\*x],x]

[Out] Result too large to show

**Maple [B]** time = 0.063, size = 5079, normalized size = 7.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(c\*x^2+a)^(1/2)/(g\*x+f)^(1/2),x)

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}(ex + d)^3}{\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + a)*(e*x + d)^3/sqrt(g*x + f), x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^2 + a)*(e*x + d)^3/sqrt(g*x + f), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)\sqrt{cx^2 + a}}{\sqrt{gx + f}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + a)*(e*x + d)^3/sqrt(g*x + f), x, algorithm="fricas")`

[Out] `integral((e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x + d^3)*sqrt(c*x^2 + a)/sqrt(g*x + f), x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2}(d + ex)^3}{\sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**3*(c*x**2+a)**(1/2)/(g*x+f)**(1/2), x)`

[Out] `Integral(sqrt(a + c*x**2)*(d + e*x)**3/sqrt(f + g*x), x)`

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + a)*(e*x + d)^3/sqrt(g*x + f), x, algorithm="giac")`

[Out] Exception raised: RuntimeError

$$3.630 \quad \int \frac{(d+ex)^2 \sqrt{a+cx^2}}{\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=508

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}(ag^2+cf^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}(5ae^2g^2-c(35d^2g^2-56defg+24e^2f^2))F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{105c^{3/2}g^4\sqrt{a+cx^2}\sqrt{f+gx}} + \frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}(aeg^2(13ef-42dg)+cf(35d^2g^2-56defg+24e^2f^2))E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{105\sqrt{c}g^4\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}} + \frac{4\sqrt{a+cx^2}\sqrt{f+gx}(5ae^2g^2+c(10d^2g^2-34defg+21e^2f^2))}{105cg^3} - \frac{4e\sqrt{a+cx^2}(f+gx)^{3/2}(3ef-2dg)}{35g^3} + \frac{2\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7g}$$

[Out]  $(4*(5*a*e^2*g^2 + c*(21*e^2*f^2 - 34*d*e*f*g + 10*d^2*g^2))*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])/(105*c*g^4) + (2*(d + e*x)^2*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])/(7*g) - (4*e*(3*e*f - 2*d*g)*(f + g*x)^{(3/2)}*\text{Sqrt}[a + c*x^2])/(35*g^3) + (4*\text{Sqrt}[-a]*(a*e*g^2*(13*e*f - 42*d*g) + c*f*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g^2))*\text{Sqrt}[f + g*x]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g))]/(105*\text{Sqrt}[c]*g^4*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[a + c*x^2]) + (4*\text{Sqrt}[-a]*(c*f^2 + a*g^2)*(5*a*e^2*g^2 - c*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g^2))*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g))]/(105*c^{(3/2)}*g^4*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])$

**Rubi [A]** time = 1.9797, antiderivative size = 503, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}(ag^2+cf^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}(5ae^2g^2-c(35d^2g^2-56defg+24e^2f^2))F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{105c^{3/2}g^4\sqrt{a+cx^2}\sqrt{f+gx}} + \frac{4\sqrt{a+cx^2}\sqrt{f+gx}\left(e^2\left(\frac{5a}{c}+\frac{21f^2}{g^2}\right)+10d^2-\frac{34def}{g}\right)}{105g} + \frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}(aeg^2(13ef-42dg)+cf(35d^2g^2-56defg+24e^2f^2))E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{105\sqrt{c}g^4\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}} - \frac{4e\sqrt{a+cx^2}(f+gx)^{3/2}(3ef-2dg)}{35g^3} + \frac{2\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7g}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^2*\text{Sqrt}[a + c*x^2]/\text{Sqrt}[f + g*x], x]$

[Out]  $(4*(10*d^2 + e^2*((5*a)/c + (21*f^2)/g^2) - (34*d*e*f)/g)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])/(105*g) + (2*(d + e*x)^2*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])/(7*g) - (4*e*(3*e*f - 2*d*g)*(f + g*x)^{(3/2)}*\text{Sqrt}[a + c*x^2])/(35*g^3) + (4*\text{Sqrt}[-a]*(a*e*g^2*(13*e*f - 42*d*g) + c$

```
*f*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g^2))*Sqrt[f + g*x]*Sqrt[1 +
(c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[
2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)]/(105*Sqrt[c]*g^4*Sqrt[
(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) +
(4*Sqrt[-a]*(c*f^2 + a*g^2)*(5*a*e^2*g^2 - c*(24*e^2*f^2 - 56*d*e
*f*g + 35*d^2*g^2))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a
]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/S
qrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)]/(105*c^(
3/2)*g^4*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

**Rubi in Sympy [F-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((e*x+d)**2*(c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)
```

[Out] Timed out

**Mathematica [C]** time = 9.45535, size = 712, normalized size = 1.4

$$2\sqrt{f+gx} \left( g^2 (a+cx^2) (10ae^2g^2 + c(35d^2g^2 + 14deg(3gx-4f) + 3e^2(8f^2 - 6fgx + 5g^2x^2))) - \frac{2 \left( g^2 \sqrt{-f - \frac{i\sqrt{ag}}{\sqrt{c}}} (a^2eg^2(13ef - 4d^2g) + c^2f(24e^2f^2 - 56d^2efg + 35d^2g^2)x^2 + a^2c(35d^2f^2g^2 - 14d^2efg(4f^2 + 3g^2x^2) + e^2(24f^3 + 13fg^2x^2)) - I\sqrt{c}(\sqrt{c}f + I\sqrt{a}g)(a^2eg^2(13ef - 42d^2g) + c^2f(24e^2f^2 - 56d^2efg + 35d^2g^2))\sqrt{(g((I\sqrt{a})/\sqrt{c} + x))/(f+gx))\sqrt{-(((I\sqrt{a}g)/\sqrt{c} - g^2x)/(f+gx))} (f+gx)^{3/2} \text{EllipticE}[I\text{ArcSinh}[\sqrt{-f - (I\sqrt{a}g)/\sqrt{c}}]/\sqrt{f+gx}], (\sqrt{c}f - I\sqrt{a}g)/(\sqrt{c}f + I\sqrt{a}g)] + \sqrt{a}g(\sqrt{c}f + I\sqrt{a}g) (5a^2e^2g^2 + (6I)\sqrt{a}\sqrt{c}eg(3ef - 7d^2g) + c(-24e^2f^2 + 56d^2efg - 35d^2g^2))\sqrt{(g((I\sqrt{a})/\sqrt{c} + x))/(f+gx))\sqrt{-(((I\sqrt{a}g)/\sqrt{c} - g^2x)/(f+gx))} (f+gx)^{3/2} \text{EllipticF}[I\text{ArcSinh}[\sqrt{-f - (I\sqrt{a}g)/\sqrt{c}}]/\sqrt{f+gx}], (\sqrt{c}f - I\sqrt{a}g)/(\sqrt{c}f + I\sqrt{a}g)])))/(\sqrt{-f - (I\sqrt{a}g)/\sqrt{c}}(f+gx)))/(105c^5g^5\sqrt{a+cx^2}) \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] (2*Sqrt[f + g*x]*(g^2*(a + c*x^2)*(10*a*e^2*g^2 + c*(35*d^2*g^2 +
14*d*e*g*(-4*f + 3*g*x) + 3*e^2*(8*f^2 - 6*f*g*x + 5*g^2*x^2)))
- (2*(g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(a^2*e*g^2*(13*e*f - 4
2*d*g) + c^2*f*(24*e^2*f^2 - 56*d^2*e*f*g + 35*d^2*g^2)*x^2 + a*c*(
35*d^2*f^2*g^2 - 14*d^2*e*g*(4*f^2 + 3*g^2*x^2) + e^2*(24*f^3 + 13*f*
g^2*x^2))) - I*Sqrt[c]*(Sqrt[c]*f + I*Sqrt[a]*g)*(a^2*e*g^2*(13*e*f
- 42*d*g) + c^2*f*(24*e^2*f^2 - 56*d^2*e*f*g + 35*d^2*g^2))*Sqrt[(g*
((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[
c] - g^2*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f
- (I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*
g)/(Sqrt[c]*f + I*Sqrt[a]*g)] + Sqrt[a]*g*(Sqrt[c]*f + I*Sqrt[a]*
g)*(5*a^2*e^2*g^2 + (6*I)*Sqrt[a]*Sqrt[c]*e*g*(3*e*f - 7*d^2*g) + c*(
-24*e^2*f^2 + 56*d^2*e*f*g - 35*d^2*g^2))*Sqrt[(g*((I*Sqrt[a])/Sqrt
[c] + x))/(f + g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g^2*x)/(f + g*
x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/
Sqrt[c]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I
*Sqrt[a]*g)])))/(\sqrt{-f - (I*Sqrt[a]*g)/Sqrt[c]}(f + g*x)))/(10
5*c^5*g^5*Sqrt[a + c*x^2])
```

**Maple [B]** time = 0.039, size = 3278, normalized size = 6.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.





$$\begin{aligned} & ((1/2 - c*f))^{(1/2)} * ((-c*x + (-a*c)^{(1/2)}) * g / (g * (-a*c)^{(1/2)} + c*f))^{(1/2)} \\ & * ((c*x + (-a*c)^{(1/2)}) * g / (g * (-a*c)^{(1/2)} - c*f))^{(1/2)} * \text{EllipticE}(( \\ & -(g*x + f) * c / (g * (-a*c)^{(1/2)} - c*f))^{(1/2)}, (-g * (-a*c)^{(1/2)} - c*f) / (g * \\ & (-a*c)^{(1/2)} + c*f))^{(1/2)} * a^2 * c * e^2 * f * g^4 + 70 * (-g*x + f) * c / (g * (-a*c) \\ & )^{(1/2)} - c*f))^{(1/2)} * ((-c*x + (-a*c)^{(1/2)}) * g / (g * (-a*c)^{(1/2)} + c*f))^{(1/2)} \\ & * ((c*x + (-a*c)^{(1/2)}) * g / (g * (-a*c)^{(1/2)} - c*f))^{(1/2)} * \text{EllipticE} \\ & ((-g*x + f) * c / (g * (-a*c)^{(1/2)} - c*f))^{(1/2)}, (-g * (-a*c)^{(1/2)} - c*f) / ( \\ & g * (-a*c)^{(1/2)} + c*f))^{(1/2)} * a * c^2 * d^2 * f * g^4 + 74 * (-g*x + f) * c / (g * (-a \\ & *c)^{(1/2)} - c*f))^{(1/2)} * ((-c*x + (-a*c)^{(1/2)}) * g / (g * (-a*c)^{(1/2)} + c*f) \\ & )^{(1/2)} * ((c*x + (-a*c)^{(1/2)}) * g / (g * (-a*c)^{(1/2)} - c*f))^{(1/2)} * \text{EllipticE} \\ & ((-g*x + f) * c / (g * (-a*c)^{(1/2)} - c*f))^{(1/2)}, (-g * (-a*c)^{(1/2)} - c*f) / ( \\ & g * (-a*c)^{(1/2)} + c*f))^{(1/2)} * a * c^2 * e^2 * f^3 * g^2 - 112 * (-g*x + f) * c / ( \\ & g * (-a*c)^{(1/2)} - c*f))^{(1/2)} * ((-c*x + (-a*c)^{(1/2)}) * g / (g * (-a*c)^{(1/2)} \\ & + c*f))^{(1/2)} * ((c*x + (-a*c)^{(1/2)}) * g / (g * (-a*c)^{(1/2)} - c*f))^{(1/2)} * \text{El} \\ & \text{lipticE}((-g*x + f) * c / (g * (-a*c)^{(1/2)} - c*f))^{(1/2)}, (-g * (-a*c)^{(1/2)} \\ & - c*f) / (g * (-a*c)^{(1/2)} + c*f))^{(1/2)} * c^3 * d * e * f^4 * g / g^5 / (c * g * x^3 + c * \\ & f * x^2 + a * g * x + a * f) / c^2 \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}(ex + d)^2}{\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a) \* (e\*x + d)^2 / sqrt(g\*x + f), x, algorithm="maxima")

[Out] integrate(sqrt(c\*x^2 + a) \* (e\*x + d)^2 / sqrt(g\*x + f), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e^2x^2 + 2dex + d^2)\sqrt{cx^2 + a}}{\sqrt{gx + f}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a) \* (e\*x + d)^2 / sqrt(g\*x + f), x, algorithm="fricas")

[Out] integral((e^2\*x^2 + 2\*d\*e\*x + d^2)\*sqrt(c\*x^2 + a)/sqrt(g\*x + f), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2}(d + ex)^2}{\sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2\*(c\*x\*\*2+a)\*\*(1/2)/(g\*x+f)\*\*(1/2), x)

[Out] Integral(sqrt(a + c\*x\*\*2) \* (d + e\*x)\*\*2 / sqrt(f + g\*x), x)

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + a)*(e*x + d)^2/sqrt(g*x + f),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

$$3.631 \quad \int \frac{(d+ex)\sqrt{a+cx^2}}{\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=364

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}(ag^2+cf^2)(4ef-5dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{15\sqrt{c}g^3\sqrt{a+cx^2}\sqrt{f+gx}} - \frac{2\sqrt{a+cx^2}\sqrt{f+gx}(-5dg+4ef-3egx)}{15g^2} - \frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}(3aeg^2+cf(4ef-5dg))E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{15\sqrt{c}g^3\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}}$$

[Out]  $(-2*\text{Sqrt}[f + g*x]*(4*e*f - 5*d*g - 3*e*g*x)*\text{Sqrt}[a + c*x^2])/(15*g^2) - (4*\text{Sqrt}[-a]*(3*a*e*g^2 + c*f*(4*e*f - 5*d*g))*\text{Sqrt}[f + g*x]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)))/(15*\text{Sqrt}[c]*g^3*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[a + c*x^2]) + (4*\text{Sqrt}[-a]*(4*e*f - 5*d*g)*(c*f^2 + a*g^2)*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)))/(15*\text{Sqrt}[c]*g^3*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])$

**Rubi [A]** time = 0.946578, antiderivative size = 364, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}(ag^2+cf^2)(4ef-5dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{15\sqrt{c}g^3\sqrt{a+cx^2}\sqrt{f+gx}} - \frac{2\sqrt{a+cx^2}\sqrt{f+gx}(-5dg+4ef-3egx)}{15g^2} - \frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}(3aeg^2+cf(4ef-5dg))E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{15\sqrt{c}g^3\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*Sqrt[a + c\*x^2])/Sqrt[f + g\*x], x]

[Out]  $(-2*\text{Sqrt}[f + g*x]*(4*e*f - 5*d*g - 3*e*g*x)*\text{Sqrt}[a + c*x^2])/(15*g^2) - (4*\text{Sqrt}[-a]*(3*a*e*g^2 + c*f*(4*e*f - 5*d*g))*\text{Sqrt}[f + g*x]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)))/(15*\text{Sqrt}[c]*g^3*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[a + c*x^2]) + (4*\text{Sqrt}[-a]*(4*e*f - 5*d*g)*(c*f^2 + a*g^2)*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)))/(15*\text{Sqrt}[c]*g^3*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])$

**Rubi in Sympy [A]** time = 131.463, size = 359, normalized size = 0.99

$$\frac{4\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{5dg}{2}-2ef+\frac{3egx}{2}\right)}{15g^2}$$

$$\frac{4\sqrt{-a}\sqrt{\frac{\sqrt{c}\sqrt{-a}(-f-gx)}{ag-\sqrt{c}f\sqrt{-a}}}\sqrt{1+\frac{cx^2}{a}}(ag^2+cf^2)(5dg-4ef)F\left(\operatorname{asin}\left(\sqrt{-\frac{\sqrt{c}x}{2\sqrt{-a}}+\frac{1}{2}}\right)\middle|\frac{2ag}{ag-\sqrt{c}f\sqrt{-a}}\right)}{15\sqrt{c}g^3\sqrt{a+cx^2}\sqrt{f+gx}}$$

$$\frac{4\sqrt{-a}\sqrt{1+\frac{cx^2}{a}}\sqrt{f+gx}(3aeg^2-5cdfg+4cef^2)E\left(\operatorname{asin}\left(\sqrt{-\frac{\sqrt{c}x}{2\sqrt{-a}}+\frac{1}{2}}\right)\middle|\frac{2ag}{ag-\sqrt{c}f\sqrt{-a}}\right)}{15\sqrt{c}g^3\sqrt{\frac{\sqrt{c}\sqrt{-a}(-f-gx)}{ag-\sqrt{c}f\sqrt{-a}}}\sqrt{a+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)*(c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)`

[Out] `4*sqrt(a + c*x**2)*sqrt(f + g*x)*(5*d*g/2 - 2*e*f + 3*e*g*x/2)/(15*g**2) - 4*sqrt(-a)*sqrt(sqrt(c)*sqrt(-a)*(-f - g*x)/(a*g - sqrt(c)*f*sqrt(-a)))*sqrt(1 + c*x**2/a)*(a*g**2 + c*f**2)*(5*d*g - 4*e*f)*elliptic_f(asin(sqrt(-sqrt(c)*x/(2*sqrt(-a))) + 1/2)), 2*a*g/(a*g - sqrt(c)*f*sqrt(-a))/(15*sqrt(c)*g**3*sqrt(a + c*x**2)*sqrt(f + g*x)) - 4*sqrt(-a)*sqrt(1 + c*x**2/a)*sqrt(f + g*x)*(3*a*e*g**2 - 5*c*d*f*g + 4*c*e*f**2)*elliptic_e(asin(sqrt(-sqrt(c)*x/(2*sqrt(-a))) + 1/2)), 2*a*g/(a*g - sqrt(c)*f*sqrt(-a))/(15*sqrt(c)*g**3*sqrt(sqrt(c)*sqrt(-a)*(-f - g*x)/(a*g - sqrt(c)*f*sqrt(-a)))*sqrt(a + c*x**2))`

**Mathematica [C]** time = 7.75054, size = 545, normalized size = 1.5

$$\sqrt{f+gx} \left( \frac{2(a+cx^2)(5dg-4ef+3egx)}{g^2} + \frac{4 \left( g^2(a+cx^2) \sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}} (3aeg^2+cf(4ef-5dg)) - \sqrt{c}(f+gx)^{3/2} (-\sqrt{ag}+i\sqrt{c}f) \sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}} \sqrt{-\frac{-gx+\frac{i\sqrt{ag}}{\sqrt{c}}}{f+gx}} (3aeg^2+cf(4ef-5dg)) \right)}{g^2} \right)$$

Antiderivative was successfully verified.

[In] `Integrate[((d + e*x)*Sqrt[a + c*x^2])/Sqrt[f + g*x],x]`

[Out] `(Sqrt[f + g*x]*((2*(-4*e*f + 5*d*g + 3*e*g*x)*(a + c*x^2))/g^2 + (4*(g^2*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(3*a*e*g^2 + c*f*(4*e*f - 5*d*g))*(a + c*x^2) - Sqrt[c]*(I*Sqrt[c]*f - Sqrt[a]*g)*(3*a*e*g^2 + c*f*(4*e*f - 5*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x])*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g) + Sqrt[a]*Sqrt[c]*g*(Sqrt[c]*f + I*Sqrt[a]*g)*((3*I)*Sqrt[a]*e*g + Sqrt[c]*(-4*e*f + 5*d*g))*Sqrt[(g*((I*Sqrt[a])/Sqrt[c] + x))/(f + g*x])*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c] - g*x)/(f + g*x))]*(f + g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]]/Sqrt[f + g*x]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)))/(c*g^4*Sqrt[-f - (I*Sqrt[a]*g)/Sqrt[c]]*(f + g*x)))/(15*Sqrt[a + c*x^2])`

**Maple [B]** time = 0.03, size = 1828, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)*(c*x^2+a)^(1/2)/(g*x+f)^(1/2),x)`

[Out] 
$$\frac{2}{15} (c x^2 + a)^{1/2} (g x + f)^{1/2} (6 a^2 (-g x + f) c / (g (-a c)^{1/2} - c f))^{1/2} ((-c x + (-a c)^{1/2}) g / (g (-a c)^{1/2} + c f))^{1/2} ((c x + (-a c)^{1/2}) g / (g (-a c)^{1/2} - c f))^{1/2} \text{EllipticF}((-g x + f) c / (g (-a c)^{1/2} - c f))^{1/2}, (-g (-a c)^{1/2} - c f) / (g (-a c)^{1/2} + c f))^{1/2} e^g x^4 + 6 (-g x + f) c / (g (-a c)^{1/2} - c f)^{1/2} ((-c x + (-a c)^{1/2}) g / (g (-a c)^{1/2} + c f))^{1/2} ((c x + (-a c)^{1/2}) g / (g (-a c)^{1/2} - c f))^{1/2} \text{EllipticF}((-g x + f) c / (g (-a c)^{1/2} - c f))^{1/2}, (-g (-a c)^{1/2} - c f) / (g (-a c)^{1/2} + c f))^{1/2} a^c e^g f^2 g^2 - 10 (-a c)^{1/2} (-g x + f) c / (g (-a c)^{1/2} - c f)^{1/2} ((-c x + (-a c)^{1/2}) g / (g (-a c)^{1/2} + c f))^{1/2} ((c x + (-a c)^{1/2}) g / (g (-a c)^{1/2} - c f))^{1/2} \text{EllipticF}((-g x + f) c / (g (-a c)^{1/2} - c f))^{1/2}, (-g (-a c)^{1/2} - c f) / (g (-a c)^{1/2} + c f))^{1/2} a^d g^4 + 8 (-a c)^{1/2} (-g x + f) c / (g (-a c)^{1/2} - c f)^{1/2} ((-c x + (-a c)^{1/2}) g / (g (-a c)^{1/2} + c f))^{1/2} ((c x + (-a c)^{1/2}) g / (g (-a c)^{1/2} - c f))^{1/2} \text{EllipticF}((-g x + f) c / (g (-a c)^{1/2} - c f))^{1/2}, (-g (-a c)^{1/2} - c f) / (g (-a c)^{1/2} + c f))^{1/2} a^e f^3 g^3 - 10 (-a c)^{1/2} (-g x + f) c / (g (-a c)^{1/2} - c f)^{1/2} ((-c x + (-a c)^{1/2}) g / (g (-a c)^{1/2} + c f))^{1/2} ((c x + (-a c)^{1/2}) g / (g (-a c)^{1/2} - c f))^{1/2} \text{EllipticF}((-g x + f) c / (g (-a c)^{1/2} - c f))^{1/2}, (-g (-a c)^{1/2} - c f) / (g (-a c)^{1/2} + c f))^{1/2} c^d f^2 g^2 + 8 (-a c)^{1/2} (-g x + f) c / (g (-a c)^{1/2} - c f)^{1/2} ((-c x + (-a c)^{1/2}) g / (g (-a c)^{1/2} + c f))^{1/2} ((c x + (-a c)^{1/2}) g / (g (-a c)^{1/2} - c f))^{1/2} \text{EllipticF}((-g x + f) c / (g (-a c)^{1/2} - c f))^{1/2}, (-g (-a c)^{1/2} - c f) / (g (-a c)^{1/2} + c f))^{1/2} c^e f^3 g - 6 a^2 (-g x + f) c / (g (-a c)^{1/2} - c f)^{1/2} ((-c x + (-a c)^{1/2}) g / (g (-a c)^{1/2} + c f))^{1/2} ((c x + (-a c)^{1/2}) g / (g (-a c)^{1/2} - c f))^{1/2} \text{EllipticE}((-g x + f) c / (g (-a c)^{1/2} - c f))^{1/2}, (-g (-a c)^{1/2} - c f) / (g (-a c)^{1/2} + c f))^{1/2} e^g x^4 + 10 a^c (-g x + f) c / (g (-a c)^{1/2} - c f)^{1/2} ((-c x + (-a c)^{1/2}) g / (g (-a c)^{1/2} + c f))^{1/2} ((c x + (-a c)^{1/2}) g / (g (-a c)^{1/2} - c f))^{1/2} \text{EllipticE}((-g x + f) c / (g (-a c)^{1/2} - c f))^{1/2}, (-g (-a c)^{1/2} - c f) / (g (-a c)^{1/2} + c f))^{1/2} d^f g^3 - 14 (-g x + f) c / (g (-a c)^{1/2} - c f)^{1/2} ((-c x + (-a c)^{1/2}) g / (g (-a c)^{1/2} + c f))^{1/2} ((c x + (-a c)^{1/2}) g / (g (-a c)^{1/2} - c f))^{1/2} \text{EllipticE}((-g x + f) c / (g (-a c)^{1/2} - c f))^{1/2}, (-g (-a c)^{1/2} - c f) / (g (-a c)^{1/2} + c f))^{1/2} a^c e^g f^2 g^2 + 10 (-g x + f) c / (g (-a c)^{1/2} - c f)^{1/2} ((-c x + (-a c)^{1/2}) g / (g (-a c)^{1/2} + c f))^{1/2} ((c x + (-a c)^{1/2}) g / (g (-a c)^{1/2} - c f))^{1/2} \text{EllipticE}((-g x + f) c / (g (-a c)^{1/2} - c f))^{1/2}, (-g (-a c)^{1/2} - c f) / (g (-a c)^{1/2} + c f))^{1/2} c^2 d^f g^3 - 8 (-g x + f) c / (g (-a c)^{1/2} - c f)^{1/2} ((-c x + (-a c)^{1/2}) g / (g (-a c)^{1/2} + c f))^{1/2} ((c x + (-a c)^{1/2}) g / (g (-a c)^{1/2} - c f))^{1/2} \text{EllipticE}((-g x + f) c / (g (-a c)^{1/2} - c f))^{1/2}, (-g (-a c)^{1/2} - c f) / (g (-a c)^{1/2} + c f))^{1/2} c^2 e^g f^4 + 3 x^4 c^2 e^g x^4 + 5 x^3 c^2 d^g x^4 - x^3 c^2 e^g f^3 + 3 x^2 a^c e^g x^4 + 5 x^2 c^2 d^f g^3 - 4 x^2 c^2 e^g f^2 g^2 + 5 x a^c d^g x^4 - x a^c e^g f^3 + 5 a^c d^f g^3 - 4 a^c e^g f^2 g^2) / (c^g x^3 + c^f x^2 + a^g x + a^f) / g^4$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}(ex + d)}{\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + a)*(e*x + d)/sqrt(g*x + f),x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^2 + a)*(e*x + d)/sqrt(g*x + f), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + a}(ex + d)}{\sqrt{gx + f}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + a)*(e*x + d)/sqrt(g*x + f), x, algorithm="fricas")`

[Out] `integral(sqrt(c*x^2 + a)*(e*x + d)/sqrt(g*x + f), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2}(d + ex)}{\sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(c*x**2+a)**(1/2)/(g*x+f)**(1/2), x)`

[Out] `Integral(sqrt(a + c*x**2)*(d + e*x)/sqrt(f + g*x), x)`

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + a)*(e*x + d)/sqrt(g*x + f), x, algorithm="giac")`

[Out] Exception raised: RuntimeError

$$3.632 \quad \int \frac{\sqrt{a+cx^2}}{\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=322

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}(ag^2+cf^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{3\sqrt{c}g^2\sqrt{a+cx^2}\sqrt{f+gx}} + \frac{4\sqrt{-a}\sqrt{cf}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{3g^2\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}} + \frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3g}$$

[Out] (2\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])/(3\*g) + (4\*Sqrt[-a]\*Sqrt[c]\*f\*Sqrt[f + g\*x]\*Sqrt[1 + (c\*x^2)/a]\*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)))/(3\*g^2\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[a + c\*x^2]) - (4\*Sqrt[-a]\*(c\*f^2 + a\*g^2)\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)))/(3\*Sqrt[c]\*g^2\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])

**Rubi [A]** time = 0.635278, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$

$$\frac{4\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}(ag^2+cf^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{3\sqrt{c}g^2\sqrt{a+cx^2}\sqrt{f+gx}} + \frac{4\sqrt{-a}\sqrt{cf}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{3g^2\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}} + \frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3g}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c\*x^2]/Sqrt[f + g\*x], x]

[Out] (2\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])/(3\*g) + (4\*Sqrt[-a]\*Sqrt[c]\*f\*Sqrt[f + g\*x]\*Sqrt[1 + (c\*x^2)/a]\*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)))/(3\*g^2\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[a + c\*x^2]) - (4\*Sqrt[-a]\*(c\*f^2 + a\*g^2)\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)))/(3\*Sqrt[c]\*g^2\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])

**Rubi in Sympy [A]** time = 112.412, size = 303, normalized size = 0.94

$$\frac{4\sqrt{c}f\sqrt{-a}\sqrt{1+\frac{cx^2}{a}}\sqrt{f+gx}E\left(\operatorname{asin}\left(\sqrt{-\frac{\sqrt{cx}}{2\sqrt{-a}}+\frac{1}{2}}\right)\middle|\frac{2ag}{ag-\sqrt{cf}\sqrt{-a}}\right)}{3g^2\sqrt{\frac{\sqrt{c}\sqrt{-a}(-f-gx)}{ag-\sqrt{cf}\sqrt{-a}}}\sqrt{a+cx^2}} + \frac{2\sqrt{a+cx^2}\sqrt{f+gx}}{3g}$$

$$-\frac{4\sqrt{-a}\sqrt{\frac{\sqrt{c}\sqrt{-a}(-f-gx)}{ag-\sqrt{cf}\sqrt{-a}}}\sqrt{1+\frac{cx^2}{a}}(ag^2+cf^2)F\left(\operatorname{asin}\left(\sqrt{-\frac{\sqrt{cx}}{2\sqrt{-a}}+\frac{1}{2}}\right)\middle|\frac{2ag}{ag-\sqrt{cf}\sqrt{-a}}\right)}{3\sqrt{c}g^2\sqrt{a+cx^2}\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)`

[Out] `4*sqrt(c)*f*sqrt(-a)*sqrt(1+c*x**2/a)*sqrt(f+g*x)*elliptic_e(asin(sqrt(-sqrt(c)*x/(2*sqrt(-a))+1/2)),2*a*g/(a*g-sqrt(c)*f*sqrt(-a)))/(3*g**2*sqrt(sqrt(c)*sqrt(-a)*(-f-g*x)/(a*g-sqrt(c)*f*sqrt(-a)))*sqrt(a+c*x**2))+2*sqrt(a+c*x**2)*sqrt(f+g*x)/(3*g)-4*sqrt(-a)*sqrt(sqrt(c)*sqrt(-a)*(-f-g*x)/(a*g-sqrt(c)*f*sqrt(-a)))*sqrt(1+c*x**2/a)*(a*g**2+c*f**2)*elliptic_f(asin(sqrt(-sqrt(c)*x/(2*sqrt(-a))+1/2)),2*a*g/(a*g-sqrt(c)*f*sqrt(-a)))/(3*sqrt(c)*g**2*sqrt(a+c*x**2)*sqrt(f+g*x))`

**Mathematica [C]** time = 3.7268, size = 456, normalized size = 1.42

$$2\sqrt{f+gx}\left(g^2(a+cx^2)-\frac{2\left(fg^2(a+cx^2)\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}-\sqrt{ag}(f+gx)^{3/2}(\sqrt{cf+i\sqrt{ag}})\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{ag}}{\sqrt{c}}}{f+gx}}F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{cf-i\sqrt{ag}}}{\sqrt{cf+i\sqrt{ag}}}\right)+\sqrt{cf}}{(f+gx)\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}}\right)}{3g^3\sqrt{a+cx^2}}$$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[a+c*x^2]/Sqrt[f+g*x],x]`

[Out] `(2*Sqrt[f+g*x]*(g^2*(a+c*x^2)-(2*(f*g^2*Sqrt[-f-(I*Sqrt[a]*g)/Sqrt[c]]*(a+c*x^2)+Sqrt[c]*f*(-I)*Sqrt[c]*f+Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c]+x))/(f+g*x])*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c]-g*x)/(f+g*x))]*(f+g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f-(I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f+g*x]],(Sqrt[c]*f-I*Sqrt[a]*g)/(Sqrt[c]*f+I*Sqrt[a]*g))-Sqrt[a]*g*(Sqrt[c]*f+I*Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c]+x))/(f+g*x])*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c]-g*x)/(f+g*x))]*(f+g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f-(I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f+g*x]],(Sqrt[c]*f-I*Sqrt[a]*g)/(Sqrt[c]*f+I*Sqrt[a]*g)))/(Sqrt[-f-(I*Sqrt[a]*g)/Sqrt[c]]*(f+g*x)))/(3*g^3*Sqrt[a+c*x^2])`

**Maple [B]** time = 0.024, size = 688, normalized size = 2.1

$$-\frac{2}{3c(cgx^3+cfx^2+agx+fa)g^3}\sqrt{gx+f}\sqrt{cx^2+a}\left(2\sqrt{-ac}\sqrt{\frac{(gx+f)c}{g\sqrt{-ac}-cf}}\sqrt{\frac{(-cx+\sqrt{-ac})g}{g\sqrt{-ac}+cf}}\sqrt{\frac{(cx+\sqrt{-ac})g}{g\sqrt{-ac}-cf}}\operatorname{EllipticF}\left(\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+a)^(1/2)/(g*x+f)^(1/2),x)`



```
[Out] -2/3*(c*x^2+a)^(1/2)*(g*x+f)^(1/2)*(2*(-a*c)^(1/2)*(-(g*x+f)*c/(g
*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+
c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*Ell
ipticF((-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-
c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*g^3+2*(-a*c)^(1/2)*(-(g*x+f)*
c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1
/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)
*EllipticF((-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1
/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c*f^2*g-2*a*c*(-g*x+f)*c/(
g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)
+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*El
lipticE((-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)
-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*f*g^2-2*(-g*x+f)*c/(g*(-a*c)^(
1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1
/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((
-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-(g*(-a*c)^(1/2)-c*f)/(g*
(-a*c)^(1/2)+c*f))^(1/2))*c^2*f^3-x^3*c^2*g^3-x^2*c^2*f*g^2-x*a*c
*g^3-a*c*f*g^2)/c/(c*g*x^3+c*f*x^2+a*g*x+a*f)/g^3
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}}{\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + a)/sqrt(g*x + f),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^2 + a)/sqrt(g*x + f), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + a}}{\sqrt{gx + f}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + a)/sqrt(g*x + f),x, algorithm="fricas")
```

```
[Out] integral(sqrt(c*x^2 + a)/sqrt(g*x + f), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2}}{\sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + c*x**2)/sqrt(f + g*x), x)
```

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + a)/sqrt(g*x + f),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

$$3.633 \quad \int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=473

$$\frac{2\sqrt{\frac{cx^2}{a}+1}(ae^2+cd^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}\left(\frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{-ag}}{\sqrt{cf}+\sqrt{-ag}}\right)}{e^2\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{cd}}{\sqrt{-a}}+e\right)} + \frac{2\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}(dg+ef)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{2ag}{\sqrt{-a}\sqrt{cf}-ag}\right)}{e^2g\sqrt{a+cx^2}\sqrt{f+gx}} - \frac{2\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{2ag}{\sqrt{-a}\sqrt{cf}-ag}\right)}{eg\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}}$$

[Out] (-2\*Sqrt[-a]\*Sqrt[c]\*Sqrt[f + g\*x]\*Sqrt[1 + (c\*x^2)/a]\*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(e\*g\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[a + c\*x^2]) + (2\*Sqrt[-a]\*Sqrt[c]\*(e\*f + d\*g)\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(e^2\*g\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]) - (2\*(c\*d^2 + a\*e^2)\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticPi[(2\*e)/((Sqrt[c]\*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (2\*Sqrt[-a]\*g)/(Sqrt[c]\*f + Sqrt[-a]\*g)]/(e^2\*((Sqrt[c]\*d)/Sqrt[-a] + e)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])

**Rubi [A]** time = 2.23286, antiderivative size = 473, normalized size of antiderivative = 1., number of rules used = 10, number of rules used = 9, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$

$$\frac{2\sqrt{\frac{cx^2}{a}+1}(ae^2+cd^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}\left(\frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{-ag}}{\sqrt{cf}+\sqrt{-ag}}\right)}{e^2\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{cd}}{\sqrt{-a}}+e\right)} + \frac{2\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}(dg+ef)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{2ag}{\sqrt{-a}\sqrt{cf}-ag}\right)}{e^2g\sqrt{a+cx^2}\sqrt{f+gx}} - \frac{2\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{2ag}{\sqrt{-a}\sqrt{cf}-ag}\right)}{eg\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c\*x^2]/((d + e\*x)\*Sqrt[f + g\*x]),x]

[Out] (-2\*Sqrt[-a]\*Sqrt[c]\*Sqrt[f + g\*x]\*Sqrt[1 + (c\*x^2)/a]\*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(e\*g\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[a + c\*x^2]) + (2\*Sqrt[-a]\*Sqrt[c]\*(e\*f + d\*g)\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(e^2\*g\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]) - (2\*(c\*d^2 + a\*e^2)\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticPi[(2\*e)/((Sqrt[c]\*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (2\*Sqrt[-a]\*g)/(Sqrt[c]\*f + Sqrt[-a]\*g)]/(e^2\*((Sqrt[c]\*d)/Sqrt[-a] + e)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])

/a]\*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(e^2\*g\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]) - (2\*(c\*d^2 + a\*e^2)\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticPi[(2\*e)/((Sqrt[c]\*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (2\*Sqrt[-a]\*g)/(Sqrt[c]\*f + Sqrt[-a]\*g)]/(e^2\*((Sqrt[c]\*d)/Sqrt[-a] + e)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)\sqrt{f+gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+a)\*\*(1/2)/(e\*x+d)/(g\*x+f)\*\*(1/2),x)

[Out] Integral(sqrt(a + c\*x\*\*2)/((d + e\*x)\*sqrt(f + g\*x)), x)

**Mathematica [C]** time = 7.46917, size = 1096, normalized size = 2.32

$$2 \left( -ce^2 \sqrt{-f - \frac{i\sqrt{ag}}{\sqrt{c}}} f^3 + 2ce^2 \sqrt{-f - \frac{i\sqrt{ag}}{\sqrt{c}}} (f + gx) f^2 + cdeg \sqrt{-f - \frac{i\sqrt{ag}}{\sqrt{c}}} f^2 - ce^2 \sqrt{-f - \frac{i\sqrt{ag}}{\sqrt{c}}} (f + gx)^2 f - 2cdeg \sqrt{-f - \frac{i\sqrt{ag}}{\sqrt{c}}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + c\*x^2]/((d + e\*x)\*Sqrt[f + g\*x]),x]

[Out] (-2\*(-(c\*e^2\*f^3\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]) + c\*d\*e\*f^2\*g\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]] - a\*e^2\*f\*g^2\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]] + a\*d\*e\*g^3\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]] + 2\*c\*e^2\*f^2\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]\*(f + g\*x) - 2\*c\*d\*e\*f\*g\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]\*(f + g\*x) - c\*e^2\*f\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]\*(f + g\*x)^2 + c\*d\*e\*g\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]\*(f + g\*x)^2 + Sqrt[c]\*e\*((-I)\*Sqrt[c]\*f + Sqrt[a]\*g)\*(-e\*f + d\*g)\*Sqrt[(g\*((I\*Sqrt[a])/Sqrt[c] + x))/(f + g\*x)]\*Sqrt[-(((I\*Sqrt[a]\*g)/Sqrt[c] - g\*x)/(f + g\*x))]\*(f + g\*x)^(3/2)\*EllipticE[I\*ArcSinh[Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]/Sqrt[f + g\*x]], (Sqrt[c]\*f - I\*Sqrt[a]\*g)/(Sqrt[c]\*f + I\*Sqrt[a]\*g)] + e\*(I\*Sqrt[c]\*d + Sqrt[a]\*e)\*g\*(Sqrt[c]\*f + I\*Sqrt[a]\*g)\*Sqrt[(g\*((I\*Sqrt[a])/Sqrt[c] + x))/(f + g\*x)]\*Sqrt[-(((I\*Sqrt[a]\*g)/Sqrt[c] - g\*x)/(f + g\*x))]\*(f + g\*x)^(3/2)\*EllipticF[I\*ArcSinh[Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]/Sqrt[f + g\*x]], (Sqrt[c]\*f - I\*Sqrt[a]\*g)/(Sqrt[c]\*f + I\*Sqrt[a]\*g)] - I\*c\*d^2\*g^2\*Sqrt[(g\*((I\*Sqrt[a])/Sqrt[c] + x))/(f + g\*x)]\*Sqrt[-(((I\*Sqrt[a]\*g)/Sqrt[c] - g\*x)/(f + g\*x))]\*(f + g\*x)^(3/2)\*EllipticPi[(Sqrt[c]\*(e\*f - d\*g))/(e\*(Sqrt[c]\*f + I\*Sqrt[a]\*g))], I\*ArcSinh[Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]/Sqrt[f + g\*x]], (Sqrt[c]\*f - I\*Sqrt[a]\*g)/(Sqrt[c]\*f + I\*Sqrt[a]\*g)] - I\*a\*e^2\*g^2\*Sqrt[(g\*((I\*Sqrt[a])/Sqrt[c] + x))/(f + g\*x)]\*Sqrt[-(((I\*Sqrt[a]\*g)/Sqrt[c] - g\*x)/(f + g\*x))]\*(f + g\*x)^(3/2)\*EllipticPi[(Sqrt[c]\*(e\*f - d\*g))/(e\*(Sqrt[c]\*f + I\*Sqrt[a]\*g))], I\*ArcSinh[Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]/Sqrt[f + g\*x]], (Sqrt[c]\*f - I\*Sqrt[a]\*g)/(Sqrt[c]\*f + I\*Sqrt[a]\*g)))/(e^2\*g^2\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]\*(e\*f - d\*g)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])

**Maple [B]** time = 0.036, size = 1216, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x)`

[Out]  $2 * (\text{EllipticF}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * (-a*c)^{(1/2)} * c*d^2*g^3 - \text{EllipticF}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * (-a*c)^{(1/2)} * c*e^2*f^2*g + \text{EllipticF}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * a*c*d*e*g^3 - \text{EllipticF}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * a*c*e^2*f*g^2 - \text{EllipticF}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * c^2*d^2*f*g^2 + \text{EllipticF}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * c^2*d*e*f^2 * g - \text{EllipticE}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * a*c*d*e*g^3 + \text{EllipticE}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * a*c*e^2*f*g^2 - \text{EllipticE}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * c^2*d*e*f^2 * g + \text{EllipticE}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * c^2*e^2*f^3 - \text{EllipticPi}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (g*(-a*c)^{(1/2)}-c*f)*e/c/(d*g-e*f), (-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * (-a*c)^{(1/2)} * a*e^2*g^3 - \text{EllipticPi}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (g*(-a*c)^{(1/2)}-c*f)*e/c/(d*g-e*f), (-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * (-a*c)^{(1/2)} * c*d^2*g^3 + \text{EllipticPi}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (g*(-a*c)^{(1/2)}-c*f)*e/c/(d*g-e*f), (-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * a*c*e^2*f*g^2 + \text{EllipticPi}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (g*(-a*c)^{(1/2)}-c*f)*e/c/(d*g-e*f), (-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * (-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)} * ((c*x+(-a*c)^{(1/2)}) * g/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)} * ((-c*x+(-a*c)^{(1/2)}) * g/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * (-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)} * (g*x+f)^{(1/2)} * (c*x^2+a)^{(1/2)}/e^2/c/g^2/(d*g-e*f)/(c*g*x^3+c*f*x^2+a*g*x+a*f)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}}{(ex + d)\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + a)/((e*x + d)*sqrt(g*x + f)),x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^2 + a)/((e*x + d)*sqrt(g*x + f)), x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + a)/((e*x + d)*sqrt(g*x + f)),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + cx^2}}{(d + ex)\sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+a)\*\*(1/2)/(e\*x+d)/(g\*x+f)\*\*(1/2),x)

[Out] Integral(sqrt(a + c\*x\*\*2)/((d + e\*x)\*sqrt(f + g\*x)), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}}{(ex + d)\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + a)/((e\*x + d)\*sqrt(g\*x + f)),x, algorithm="giac")

[Out] integrate(sqrt(c\*x^2 + a)/((e\*x + d)\*sqrt(g\*x + f)), x)

$$3.634 \quad \int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=694

$$\frac{\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}(2ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{e^2\sqrt{a+cx^2}\sqrt{f+gx}(ef-dg)} + \frac{\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}(ae^2g+cd(2ef-dg))\left(\frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{-ag}}{\sqrt{cf+\sqrt{-ag}}}\right)}{e^2\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{cd}}{\sqrt{-a}}+e\right)(ef-dg)} - \frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ef-dg)} + \frac{\sqrt{-a}\sqrt{c}f\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{e\sqrt{a+cx^2}\sqrt{f+gx}(ef-dg)} - \frac{\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{e\sqrt{a+cx^2}(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}}$$

```
[Out] -((Sqrt[f + g*x]*Sqrt[a + c*x^2])/((e*f - d*g)*(d + e*x))) - (Sqrt[-a]*Sqrt[c]*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)]/(e*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (Sqrt[-a]*Sqrt[c]*f*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)]/(e*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*(2*e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)]/(e^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) + ((a*e^2*g + c*d*(2*e*f - d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)]/(e^2*((Sqrt[c]*d)/Sqrt[-a] + e)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

**Rubi [A]** time = 4.18081, antiderivative size = 694, normalized size of antiderivative = 1., number of

steps used = 14, number of rules used = 10, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$\frac{\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}(2ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf}-ag}\right)}{e^2\sqrt{a+cx^2}\sqrt{f+gx}(ef-dg)}$$

$$+\frac{\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}(ae^2g+cd(2ef-dg))\left(\frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{-ag}}{\sqrt{cf}+\sqrt{-ag}}\right)}{e^2\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{cd}}{\sqrt{-a}}+e\right)(ef-dg)}$$

$$-\frac{\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ef-dg)}+\frac{\sqrt{-a}\sqrt{c}f\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf}-ag}\right)}{e\sqrt{a+cx^2}\sqrt{f+gx}(ef-dg)}$$

$$-\frac{\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf}-ag}\right)}{e\sqrt{a+cx^2}(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + c\*x^2]/((d + e\*x)^2\*Sqrt[f + g\*x]), x]

[Out] -((Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])/((e\*f - d\*g)\*(d + e\*x))) - (Sqrt[-a]\*Sqrt[c]\*Sqrt[f + g\*x]\*Sqrt[1 + (c\*x^2)/a]\*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(e\*(e\*f - d\*g)\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[a + c\*x^2]) + (Sqrt[-a]\*Sqrt[c]\*f\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(e\*(e\*f - d\*g)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]) - (Sqrt[-a]\*Sqrt[c]\*(2\*e\*f - d\*g)\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(e^2\*(e\*f - d\*g)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]) + ((a\*e^2\*g + c\*d\*(2\*e\*f - d\*g))\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticPi[(2\*e)/((Sqrt[c]\*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (2\*Sqrt[-a]\*g)/(Sqrt[c]\*f + Sqrt[-a]\*g)]/(e^2\*((Sqrt[c]\*d)/Sqrt[-a] + e)\*(e\*f - d\*g)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a+cx^2}}{(d+ex)^2\sqrt{f+gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+a)\*\*(1/2)/(e\*x+d)\*\*2/(g\*x+f)\*\*(1/2), x)

[Out] Integral(sqrt(a + c\*x\*\*2)/((d + e\*x)\*\*2\*sqrt(f + g\*x)), x)



**Mathematica [C]** time = 10.3592, size = 1336, normalized size = 1.93

$$\sqrt{f+gx} \left( \frac{cx^2+a}{d+ex} - \frac{ce^2\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}f^3-2ce^2\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}(f+gx)f^2-cdeg\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}f^2+ce^2\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}(f+gx)^2f+2cdeg\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}(f+gx)f-2icdeg\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}(f+gx)}}{\dots} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + c\*x^2]/((d + e\*x)^2\*Sqrt[f + g\*x]),x]

[Out] (Sqrt[f + g\*x]\*((a + c\*x^2)/(d + e\*x) - (c\*e^2\*f^3\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]] - c\*d\*e\*f^2\*g\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]] + a\*e^2\*f\*g^2\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]] - a\*d\*e\*g^3\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]] - 2\*c\*e^2\*f^2\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]\*(f + g\*x) + 2\*c\*d\*e\*f\*g\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]\*(f + g\*x) + c\*e^2\*f\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]\*(f + g\*x)^2 - c\*d\*e\*g\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]\*(f + g\*x)^2 + I\*Sqrt[c]\*e\*(Sqrt[c]\*f + I\*Sqrt[a]\*g)\*(-(e\*f) + d\*g)\*Sqrt[(g\*((I\*Sqrt[a])/Sqrt[c] + x))/(f + g\*x)]/Sqrt[-(((I\*Sqrt[a]\*g)/Sqrt[c] - g\*x)/(f + g\*x))]\*(f + g\*x)^(3/2)\*EllipticE[I\*ArcSinh[Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]]/Sqrt[f + g\*x]], (Sqrt[c]\*f - I\*Sqrt[a]\*g)/(Sqrt[c]\*f + I\*Sqrt[a]\*g) + e\*(Sqrt[c]\*f + I\*Sqrt[a]\*g)\*(Sqrt[a]\*e\*g + I\*Sqrt[c]\*(2\*e\*f - d\*g))\*Sqrt[(g\*((I\*Sqrt[a])/Sqrt[c] + x))/(f + g\*x)]\*Sqrt[-(((I\*Sqrt[a]\*g)/Sqrt[c] - g\*x)/(f + g\*x))]\*(f + g\*x)^(3/2)\*EllipticF[I\*ArcSinh[Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]]/Sqrt[f + g\*x]], (Sqrt[c]\*f - I\*Sqrt[a]\*g)/(Sqrt[c]\*f + I\*Sqrt[a]\*g) - (2\*I)\*c\*d\*e\*f\*g\*Sqrt[(g\*((I\*Sqrt[a])/Sqrt[c] + x))/(f + g\*x)]\*Sqrt[-(((I\*Sqrt[a]\*g)/Sqrt[c] - g\*x)/(f + g\*x))]\*(f + g\*x)^(3/2)\*EllipticPi[(Sqrt[c]\*(e\*f - d\*g))/(e\*(Sqrt[c]\*f + I\*Sqrt[a]\*g)), I\*ArcSinh[Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]]/Sqrt[f + g\*x]], (Sqrt[c]\*f - I\*Sqrt[a]\*g)/(Sqrt[c]\*f + I\*Sqrt[a]\*g) + I\*c\*d^2\*g^2\*Sqrt[(g\*((I\*Sqrt[a])/Sqrt[c] + x))/(f + g\*x)]\*Sqrt[-(((I\*Sqrt[a]\*g)/Sqrt[c] - g\*x)/(f + g\*x))]\*(f + g\*x)^(3/2)\*EllipticPi[(Sqrt[c]\*(e\*f - d\*g))/(e\*(Sqrt[c]\*f + I\*Sqrt[a]\*g)), I\*ArcSinh[Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]]/Sqrt[f + g\*x]], (Sqrt[c]\*f - I\*Sqrt[a]\*g)/(Sqrt[c]\*f + I\*Sqrt[a]\*g) - I\*a\*e^2\*g^2\*Sqrt[(g\*((I\*Sqrt[a])/Sqrt[c] + x))/(f + g\*x)]\*Sqrt[-(((I\*Sqrt[a]\*g)/Sqrt[c] - g\*x)/(f + g\*x))]\*(f + g\*x)^(3/2)\*EllipticPi[(Sqrt[c]\*(e\*f - d\*g))/(e\*(Sqrt[c]\*f + I\*Sqrt[a]\*g)), I\*ArcSinh[Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]]/Sqrt[f + g\*x]], (Sqrt[c]\*f - I\*Sqrt[a]\*g)/(Sqrt[c]\*f + I\*Sqrt[a]\*g))/(e^2\*g\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]\*(e\*f - d\*g)\*(f + g\*x)))/((- (e\*f) + d\*g)\*Sqrt[a + c\*x^2])

**Maple [B]** time = 0.046, size = 6034, normalized size = 8.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+a)^(1/2)/(e\*x+d)^2/(g\*x+f)^(1/2),x)

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2+a}}{(ex+d)^2\sqrt{gx+f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + a)/((e*x + d)^2*sqrt(g*x + f)),x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^2 + a)/((e*x + d)^2*sqrt(g*x + f)), x)`

---

**Fricas** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + a)/((e*x + d)^2*sqrt(g*x + f)),x, algorithm="fricas")`

[Out] Timed out

---

**Sympy** [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+a)**(1/2)/(e*x+d)**2/(g*x+f)**(1/2),x)`

[Out] Exception raised: ValueError

---

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + a}}{(ex + d)^2 \sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + a)/((e*x + d)^2*sqrt(g*x + f)),x, algorithm="giac")`

[Out] `integrate(sqrt(c*x^2 + a)/((e*x + d)^2*sqrt(g*x + f)), x)`

$$3.635 \quad \int \frac{\sqrt{a+cx^2}}{(d+ex)^3\sqrt{f+gx}} dx$$

Optimal. Leaf size=1241

result too large to display

```
[Out] -(Sqrt[f + g*x]*Sqrt[a + c*x^2])/(2*(e*f - d*g)*(d + e*x)^2) + ((
3*a*e^2*g + c*d*(2*e*f + d*g))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(4*
(c*d^2 + a*e^2)*(e*f - d*g)^2*(d + e*x)) + (Sqrt[-a]*Sqrt[c]*(3*a
*e^2*g + c*d*(2*e*f + d*g))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*Ell
ipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(
Sqrt[-a]*Sqrt[c]*f - a*g)))/(4*e*(c*d^2 + a*e^2)*(e*f - d*g)^2*Sq
rt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2])
+ (Sqrt[-a]*Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt
[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x
)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)))/(2*e^
2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*
f*(3*a*e^2*g + c*d*(2*e*f + d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[
c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 -
(Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a
*g)))/(4*e*(c*d^2 + a*e^2)*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c
*x^2]) + (Sqrt[-a]*Sqrt[c]*d*g*(3*a*e^2*g + c*d*(2*e*f + d*g))*Sq
rt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)
/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2
*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)))/(4*e^2*(c*d^2 + a*e^2)*(e*f -
d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (c*(e*f + d*g)*Sqrt[(Sqrt
[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*Elli
pticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]
*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g))
/(e^2*((Sqrt[c]*d)/Sqrt[-a] + e)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a
+ c*x^2]) - ((a*e^2*g - c*d*(2*e*f - 3*d*g))*(3*a*e^2*g + c*d*(2
*e*f + d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*S
qrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), A
rcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sq
rt[c]*f + Sqrt[-a]*g)))/(4*e^2*((Sqrt[c]*d)/Sqrt[-a] + e)*(c*d^2
+ a*e^2)*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

---

**Rubi [A]** time = 9.70493, antiderivative size = 1241, normalized size of antiderivative = 1., number

of steps used = 23, number of rules used = 11, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$

$$\begin{aligned}
 & \frac{\sqrt{-a}\sqrt{c}\sqrt{f+gx}\sqrt{\frac{cx^2}{a}} + 1E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)(3age^2 + cd(2ef + dg))}{4e(cd^2 + ae^2)(ef - dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-ag}}}\sqrt{cx^2+a}}} \\
 & \frac{\sqrt{-a}\sqrt{c}f\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-ag}}}\sqrt{\frac{cx^2}{a}}} + 1F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)(3age^2 + cd(2ef + dg))}{4e(cd^2 + ae^2)(ef - dg)^2\sqrt{f+gx}\sqrt{cx^2+a}} \\
 & + \frac{\sqrt{-a}\sqrt{cdg}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-ag}}}\sqrt{\frac{cx^2}{a}}} + 1F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)(3age^2 + cd(2ef + dg))}{4e^2(cd^2 + ae^2)(ef - dg)^2\sqrt{f+gx}\sqrt{cx^2+a}} \\
 & \frac{(ae^2g - cd(2ef - 3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-ag}}}\sqrt{\frac{cx^2}{a}}} + 1\left(\frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}}+e}; \sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{-ag}}{\sqrt{cf+\sqrt{-ag}}}\right)(3age^2 + cd(2ef + dg))}{4e^2\left(\frac{\sqrt{cd}}{\sqrt{-a}} + e\right)(cd^2 + ae^2)(ef - dg)^2\sqrt{f+gx}\sqrt{cx^2+a}} \\
 & + \frac{\sqrt{f+gx}\sqrt{cx^2+a}(3age^2 + cd(2ef + dg))}{4(cd^2 + ae^2)(ef - dg)^2(d + ex)} \\
 & + \frac{\sqrt{-a}\sqrt{cg}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-ag}}}\sqrt{\frac{cx^2}{a}}} + 1F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{2e^2(ef - dg)\sqrt{f+gx}\sqrt{cx^2+a}} \\
 & - \frac{c(ef + dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-ag}}}\sqrt{\frac{cx^2}{a}}} + 1\left(\frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}}+e}; \sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{-ag}}{\sqrt{cf+\sqrt{-ag}}}\right)}{e^2\left(\frac{\sqrt{cd}}{\sqrt{-a}} + e\right)(ef - dg)\sqrt{f+gx}\sqrt{cx^2+a}} - \frac{\sqrt{f+gx}\sqrt{cx^2+a}}{2(ef - dg)(d + ex)^2}
 \end{aligned}$$

Warning: Unable to verify antiderivative.

[In] Int[Sqrt[a + c\*x^2]/((d + e\*x)^3\*Sqrt[f + g\*x]),x]

[Out]  $-(\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])/((2*(e*f - d*g)*(d + e*x)^2) + ((3*a*e^2*g + c*d*(2*e*f + d*g))*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])/(4*(c*d^2 + a*e^2)*(e*f - d*g)^2*(d + e*x)) + (\text{Sqrt}[-a]*\text{Sqrt}[c]*(3*a*e^2*g + c*d*(2*e*f + d*g))*\text{Sqrt}[f + g*x]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)))/(4*e*(c*d^2 + a*e^2)*(e*f - d*g)^2*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[a + c*x^2]) + (\text{Sqrt}[-a]*\text{Sqrt}[c]*g*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)))/(2*e^2*(e*f - d*g)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2]) - (\text{Sqrt}[-a]*\text{Sqrt}[c]*f*(3*a*e^2*g + c*d*(2*e*f + d*g))*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)))/(4*e*(c*d^2 + a*e^2)*(e*f - d*g)^2*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2]) + (\text{Sqrt}[-a]*\text{Sqrt}[c]*d*g*(3*a*e^2*g + c*d*(2*e*f + d*g))*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)))/(4*e^2*(c*d^2 + a*e^2)*(e*f - d*g)^2*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2]) - (c*(e*f + d*g)*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticPi}[(2*e)/((\text{Sqrt}[c]*d)/\text{Sqrt}[-a] + e), \text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (2*\text{Sqrt}[-a]*g)/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)))/(e^2*((\text{Sqrt}[c]*d)/\text{Sqrt}[-a] + e)*(e*f - d*g)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2]) - ((a*e^2*g - c*d*(2*e*f - 3*d*g))*(3*a*e^2*g + c*d*(2*e*f + d*g))*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticPi}[(2*e)/((\text{Sqrt}[c]*d)/\text{Sqrt}[-a] + e), \text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (2*\text{Sqrt}[-a]*g)/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)))/(e^2*((\text{Sqrt}[c]*d)/\text{Sqrt}[-a] + e)*(e*f - d*g)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2]) - ((a*e^2*g - c*d*(2*e*f - 3*d*g))*(3*a*e^2*g + c*d*(2*e*f + d*g))*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticPi}[(2*e)/((\text{Sqrt}[c]*d)/\text{Sqrt}[-a] + e), \text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (2*\text{Sqrt}[-a]*g)/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)))/(e^2*((\text{Sqrt}[c]*d)/\text{Sqrt}[-a] + e)*(e*f - d*g)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])$

$$\frac{\sqrt{c} \sqrt{f + \sqrt{-a} g}}{(4 e^{2 \sqrt{c} d} (\sqrt{c} d / \sqrt{-a} + e) (c d^2 + a e^2) (e f - d g)^2 \sqrt{f + g x} \sqrt{a + c x^2})}$$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + c x^2}}{(d + e x)^3 \sqrt{f + g x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x**2+a)**(1/2)/(e*x+d)**3/(g*x+f)**(1/2),x)`

[Out] `Integral(sqrt(a + c*x**2)/((d + e*x)**3*sqrt(f + g*x)), x)`

**Mathematica [C]** time = 14.8152, size = 12365, normalized size = 9.96

Result too large to show

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[a + c*x^2]/((d + e*x)^3*sqrt(f + g*x)),x]`

[Out] Result too large to show

**Maple [B]** time = 0.079, size = 19187, normalized size = 15.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2),x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{c x^2 + a}}{(e x + d)^3 \sqrt{g x + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + a)/((e*x + d)^3*sqrt(g*x + f)),x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^2 + a)/((e*x + d)^3*sqrt(g*x + f)), x)`

**Ericsas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + a)/((e*x + d)^3*sqrt(g*x + f)),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+a)**(1/2)/(e*x+d)**3/(g*x+f)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + a)/((e*x + d)^3*sqrt(g*x + f)),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

$$3.636 \quad \int \frac{(d+ex)^3 \sqrt{f+gx}}{\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=531

$$\frac{2\sqrt{-ae}\sqrt{\frac{cx^2}{a}+1}(ag^2+cf^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}(25ae^2g^2-c(105d^2g^2-42defg+8e^2f^2))F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{105c^{5/2}g^3\sqrt{a+cx^2}\sqrt{f+gx}}$$

$$+\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}(ae^2g^2(189dg+19ef)-c(105d^3g^3+105d^2efg^2-42de^2f^2g+8e^3f^3))E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{-2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{105c^{3/2}g^3\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}}$$

$$-\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}(25ae^2g^2+c(-90d^2g^2+12defg+7e^2f^2))}{105c^2g^2}$$

$$+\frac{2e^2\sqrt{a+cx^2}(f+gx)^{3/2}(11dg+ef)}{35cg^2}+\frac{2e\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7c}$$

[Out]  $(-2 * e * (25 * a * e^2 * g^2 + c * (7 * e^2 * f^2 + 12 * d * e * f * g - 90 * d^2 * g^2)) * \text{Sqrt}[f + g * x] * \text{Sqrt}[a + c * x^2]) / (105 * c^2 * g^2) + (2 * e * (d + e * x)^2 * \text{Sqrt}[f + g * x] * \text{Sqrt}[a + c * x^2]) / (7 * c) + (2 * e^2 * (e * f + 11 * d * g) * (f + g * x)^{(3/2)} * \text{Sqrt}[a + c * x^2]) / (35 * c * g^2) + (2 * \text{Sqrt}[-a] * (a * e^2 * g^2 * (19 * e * f + 189 * d * g) - c * (8 * e^3 * f^3 - 42 * d * e^2 * f^2 * g + 105 * d^2 * e * f * g^2 + 105 * d^3 * g^3)) * \text{Sqrt}[f + g * x] * \text{Sqrt}[1 + (c * x^2) / a]) * \text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c] * x) / \text{Sqrt}[-a]]] / \text{Sqrt}[2]], (-2 * a * g) / (\text{Sqrt}[-a] * \text{Sqrt}[c] * f - a * g)) / (105 * c^{(3/2)} * g^3 * \text{Sqrt}[(\text{Sqrt}[c] * (f + g * x)) / (\text{Sqrt}[c] * f + \text{Sqrt}[-a] * g)]) * \text{Sqrt}[a + c * x^2]) - (2 * \text{Sqrt}[-a] * e * (c * f^2 + a * g^2) * (25 * a * e^2 * g^2 - c * (8 * e^2 * f^2 - 42 * d * e * f * g + 105 * d^2 * g^2)) * \text{Sqrt}[(\text{Sqrt}[c] * (f + g * x)) / (\text{Sqrt}[c] * f + \text{Sqrt}[-a] * g)]) * \text{Sqrt}[1 + (c * x^2) / a]) * \text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c] * x) / \text{Sqrt}[-a]]] / \text{Sqrt}[2]], (-2 * a * g) / (\text{Sqrt}[-a] * \text{Sqrt}[c] * f - a * g)) / (105 * c^{(5/2)} * g^3 * \text{Sqrt}[f + g * x] * \text{Sqrt}[a + c * x^2])$

**Rubi [A]** time = 2.19036, antiderivative size = 527, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 6, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$\frac{2\sqrt{-ae}\sqrt{\frac{cx^2}{a}+1}(ag^2+cf^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}(25ae^2g^2-c(105d^2g^2-42defg+8e^2f^2))F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{105c^{5/2}g^3\sqrt{a+cx^2}\sqrt{f+gx}}$$

$$+\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}(ae^2g^2(189dg+19ef)-c(105d^3g^3+105d^2efg^2-42de^2f^2g+8e^3f^3))E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{-2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{105c^{3/2}g^3\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}}$$

$$+\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}\left(-e^2\left(\frac{25a}{c}+\frac{7f^2}{g^2}\right)+90d^2-\frac{12def}{g}\right)}{105c}$$

$$+\frac{2e^2\sqrt{a+cx^2}(f+gx)^{3/2}(11dg+ef)}{35cg^2}+\frac{2e\sqrt{a+cx^2}(d+ex)^2\sqrt{f+gx}}{7c}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*Sqrt[f + g\*x])/Sqrt[a + c\*x^2], x]

[Out]  $(2 * e * (90 * d^2 - e^2 * ((25 * a) / c + (7 * f^2) / g^2)) - (12 * d * e * f) / g) * \text{Sqrt}[f + g * x] * \text{Sqrt}[a + c * x^2] / (105 * c) + (2 * e * (d + e * x)^2 * \text{Sqrt}[f + g * x] * \text{Sqrt}[a + c * x^2]) / (7 * c) + (2 * e^2 * (e * f + 11 * d * g) * (f + g * x)^{(3/2)} * \text{Sqrt}[a + c * x^2]) / (35 * c * g^2) + (2 * \text{Sqrt}[-a] * (a * e^2 * g^2 * (189 * d * g + 19 * e * f) - c * (105 * d^3 * g^3 + 105 * d^2 * e * f * g^2 - 42 * d * e^2 * f^2 * g + 8 * e^3 * f^3)) * \text{Sqrt}[f + g * x] * \text{Sqrt}[1 + (c * x^2) / a]) * \text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c] * x) / \text{Sqrt}[-a]]] / \text{Sqrt}[2]], (-2 * a * g) / (\text{Sqrt}[-a] * \text{Sqrt}[c] * f - a * g)) / (105 * c^{(3/2)} * g^3 * \text{Sqrt}[(\text{Sqrt}[c] * (f + g * x)) / (\text{Sqrt}[c] * f + \text{Sqrt}[-a] * g)]) * \text{Sqrt}[a + c * x^2] - (2 * \text{Sqrt}[-a] * e * (c * f^2 + a * g^2) * (25 * a * e^2 * g^2 - c * (8 * e^2 * f^2 - 42 * d * e * f * g + 105 * d^2 * g^2)) * \text{Sqrt}[(\text{Sqrt}[c] * (f + g * x)) / (\text{Sqrt}[c] * f + \text{Sqrt}[-a] * g)]) * \text{Sqrt}[1 + (c * x^2) / a]) * \text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c] * x) / \text{Sqrt}[-a]]] / \text{Sqrt}[2]], (-2 * a * g) / (\text{Sqrt}[-a] * \text{Sqrt}[c] * f - a * g)) / (105 * c^{(5/2)} * g^3 * \text{Sqrt}[f + g * x] * \text{Sqrt}[a + c * x^2])$

$$\begin{aligned} & \text{Sqrt}[a + c*x^2])/((35*c*g^2) + (2*\text{Sqrt}[-a]*(a*e^2*g^2*(19*e*f + 18 \\ & 9*d*g) - c*(8*e^3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f*g^2 + 105*d^ \\ & 3*g^3)))*\text{Sqrt}[f + g*x]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 \\ & - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - \\ & a*g))]/(105*c^(3/2)*g^3*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqr} \\ & \text{rt}[-a]*g)]*\text{Sqrt}[a + c*x^2)) - (2*\text{Sqrt}[-a]*e*(c*f^2 + a*g^2)*(25*a \\ & *e^2*g^2 - c*(8*e^2*f^2 - 42*d*e*f*g + 105*d^2*g^2))*\text{Sqrt}[(\text{Sqrt}[c] \\ & ]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{Elliptic} \\ & \text{icF}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqr} \\ & \text{t}[-a]*\text{Sqrt}[c]*f - a*g))]/(105*c^(5/2)*g^3*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + \\ & c*x^2]) \end{aligned}$$

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**3*(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)`

[Out] Timed out

**Mathematica [C]** time = 10.9903, size = 747, normalized size = 1.41

$$2\sqrt{f+gx} \left( \frac{g\sqrt{f+gx}(\sqrt{cf+i\sqrt{ag}})\sqrt{\frac{g(x+\frac{i\sqrt{a}}{\sqrt{c}})}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{ag}}{\sqrt{c}}}{f+gx}}(25a^{3/2}e^3g^2+\sqrt{ace}(-105d^2g^2+42defg-8e^2f^2)+3ia\sqrt{ce^2g(2ef-63dg)+105ic^{3/2}d^3g^2)}{i\sinh}}}{\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}} \right)$$

Antiderivative was successfully verified.

[In] `Integrate[((d + e*x)^3*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]`

$$\begin{aligned} & (2*\text{Sqrt}[f + g*x]*(-(g^2*(a + c*x^2)*(25*a*e^3*g^2 + c*e*(-105*d^2 \\ & *g^2 - 21*d*e*g*(f + 3*g*x) + e^2*(4*f^2 - 3*f*g*x - 15*g^2*x^2)) \\ & )) + (g^2*(-(a^2*e^2*g^2*(19*e*f + 189*d*g)) + c^2*(8*e^3*f^3 - 4 \\ & 2*d*e^2*f^2*g + 105*d^2*e*f*g^2 + 105*d^3*g^3)*x^2 + a*c*(105*d^2 \\ & *e*f*g^2 + 105*d^3*g^3 - 21*d*e^2*g*(2*f^2 + 9*g^2*x^2) + e^3*(8* \\ & f^3 - 19*f*g^2*x^2))))/(f + g*x) + I*c*\text{Sqrt}[-f - (I*\text{Sqrt}[a]*g)/\text{Sqr} \\ & \text{rt}[c]]*(-(a*e^2*g^2*(19*e*f + 189*d*g)) + c*(8*e^3*f^3 - 42*d*e^2 \\ & *f^2*g + 105*d^2*e*f*g^2 + 105*d^3*g^3))*\text{Sqrt}[(g*((I*\text{Sqrt}[a])/ \text{Sqr} \\ & \text{t}[c] + x))/(f + g*x)]*\text{Sqrt}[-(I*\text{Sqrt}[a]*g)/\text{Sqrt}[c] - g*x)/(f + g \\ & *x)]*\text{Sqrt}[f + g*x]*\text{EllipticE}[I*\text{ArcSinh}[\text{Sqrt}[-f - (I*\text{Sqrt}[a]*g)/\text{Sqr} \\ & \text{t}[c]]/\text{Sqrt}[f + g*x]], (\text{Sqrt}[c]*f - I*\text{Sqrt}[a]*g)/(\text{Sqrt}[c]*f + I* \\ & \text{Sqrt}[a]*g)] + (g*(\text{Sqrt}[c]*f + I*\text{Sqrt}[a]*g)*((105*I)*c^(3/2)*d^3*g \\ & ^2 + 25*a^(3/2)*e^3*g^2 + (3*I)*a*\text{Sqrt}[c]*e^2*g*(2*e*f - 63*d*g) \\ & + \text{Sqrt}[a]*c*e*(-8*e^2*f^2 + 42*d*e*f*g - 105*d^2*g^2))*\text{Sqrt}[(g*(( \\ & I*\text{Sqrt}[a])/ \text{Sqrt}[c] + x))/(f + g*x)]*\text{Sqrt}[-(I*\text{Sqrt}[a]*g)/\text{Sqrt}[c] \\ & - g*x)/(f + g*x)]*\text{Sqrt}[f + g*x]*\text{EllipticF}[I*\text{ArcSinh}[\text{Sqrt}[-f - ( \\ & I*\text{Sqrt}[a]*g)/\text{Sqrt}[c]]/\text{Sqrt}[f + g*x]], (\text{Sqrt}[c]*f - I*\text{Sqrt}[a]*g)/ \\ & (\text{Sqrt}[c]*f + I*\text{Sqrt}[a]*g)]/\text{Sqrt}[-f - (I*\text{Sqrt}[a]*g)/\text{Sqrt}[c]]))/((10 \\ & 5*c^2*g^4*\text{Sqrt}[a + c*x^2]) \end{aligned}$$

**Maple [B]** time = 0.059, size = 3922, normalized size = 7.4

output too large to display



Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((e^x+d)^3*(g*x+f)^{(1/2)}/(c*x^2+a)^{(1/2)},x)$

[Out] 
$$\begin{aligned} & -2/105*(g*x+f)^{(1/2)}*(c*x^2+a)^{(1/2)}*(-105*x^2*c^3*d^2*e*f*g^4-21 \\ & *x^2*c^3*d^2*e^2*f^2*g^3-105*x*a*c^2*d^2*e*g^5+x*a*c^2*e^3*f^2*g^3- \\ & 84*x^3*c^3*d^2*e^2*f^2*g^4-63*x^2*a*c^2*d^2*e^2*g^5+7*x^2*a*c^2*e^3*f^2*g \\ & ^4+25*a^2*c^3*d^2*e^3*f^2*g^4+4*a*c^2*e^3*f^3*g^2-15*x^5*c^3*e^3*g^5+8*( \\ & (g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}*((-c*x+(-a*c)^{(1/2)})^*g/(g*( \\ & -a*c)^{(1/2)}+c*f))^{(1/2)}*((c*x+(-a*c)^{(1/2)})^*g/(g*(-a*c)^{(1/2)}-c*f \\ & ))^{(1/2)}*\text{EllipticE}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)},(-g*( \\ & -a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*c^3*e^3*f^5-84*x*a \\ & c^2*d^2*e^2*f^2*g^4+42*(-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}*((-c*x \\ & +(-a*c)^{(1/2)})^*g/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*((c*x+(-a*c)^{(1/2)})^* \\ & g/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}*\text{EllipticF}((-g*x+f)*c/(g*(-a*c)^{(1/2) \\ & }-c*f))^{(1/2)},(-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} \\ & )*(-a*c)^{(1/2)}*a*c^2*d^2*e^2*f^2*g^4+42*(-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f \\ & ))^{(1/2)}*((-c*x+(-a*c)^{(1/2)})^*g/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*((c*x \\ & +(-a*c)^{(1/2)})^*g/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}*\text{EllipticF}((-g*x+f)* \\ & c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)},(-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1 \\ & /2)}+c*f))^{(1/2)}*(-a*c)^{(1/2)}*c^2*d^2*e^2*f^3*g^2-105*(-g*x+f)*c/( \\ & g*(-a*c)^{(1/2)}-c*f))^{(1/2)}*((-c*x+(-a*c)^{(1/2)})^*g/(g*(-a*c)^{(1/2) \\ & }+c*f))^{(1/2)}*((c*x+(-a*c)^{(1/2)})^*g/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}*\text{El \\ & lipticF}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)},(-g*(-a*c)^{(1/2) \\ & }-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*a*c^2*d^3*g^5+25*(-g*x+f)*c/( \\ & g*(-a*c)^{(1/2)}-c*f))^{(1/2)}*((-c*x+(-a*c)^{(1/2)})^*g/(g*(-a*c)^{(1/2) \\ & }+c*f))^{(1/2)}*((c*x+(-a*c)^{(1/2)})^*g/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}*\text{El \\ & lipticF}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)},(-g*(-a*c)^{(1/2) \\ & }-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*(-a*c)^{(1/2)}*a^2*e^3*g^5-105*( \\ & -g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}*((-c*x+(-a*c)^{(1/2)})^*g/(g* \\ & (-a*c)^{(1/2)}+c*f))^{(1/2)}*((c*x+(-a*c)^{(1/2)})^*g/(g*(-a*c)^{(1/2)}-c* \\ & f))^{(1/2)}*\text{EllipticF}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)},(-g* \\ & (-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*c^3*d^3*f^2*g^3+10 \\ & 5*(-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}*((-c*x+(-a*c)^{(1/2)})^*g/ \\ & (g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*((c*x+(-a*c)^{(1/2)})^*g/(g*(-a*c)^{(1/2) \\ & }-c*f))^{(1/2)}*\text{EllipticE}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)},(- \\ & (g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*a*c^2*d^3*g^5+1 \\ & 05*(-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}*((-c*x+(-a*c)^{(1/2)})^*g \\ & /(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*((c*x+(-a*c)^{(1/2)})^*g/(g*(-a*c)^{(1/2) \\ & }-c*f))^{(1/2)}*\text{EllipticE}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)},( \\ & -g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*c^3*d^3*f^2*g^3 \\ & +189*(-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}*((-c*x+(-a*c)^{(1/2) \\ & })*g/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*((c*x+(-a*c)^{(1/2)})^*g/(g*(-a*c)^{( \\ & 1/2)}-c*f))^{(1/2)}*\text{EllipticF}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2) \\ & },(-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*a^2*c^2*d^2 \\ & *g^5-6*(-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}*((-c*x+(-a*c)^{(1/2) \\ & })*g/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*((c*x+(-a*c)^{(1/2)})^*g/(g*(-a*c)^{( \\ & 1/2)}-c*f))^{(1/2)}*\text{EllipticF}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2) \\ & },(-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*a^2*c^2*e^3* \\ & f^2*g^4-6*(-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}*((-c*x+(-a*c)^{(1/2) \\ & })*g/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*((c*x+(-a*c)^{(1/2)})^*g/(g*(-a*c)^ \\ & ^{(1/2)}-c*f))^{(1/2)}*\text{EllipticF}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1 \\ & /2)},(-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*a*c^2*e^3 \\ & *f^3*g^2-8*(-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}*((-c*x+(-a*c)^{( \\ & 1/2)})*g/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*((c*x+(-a*c)^{(1/2)})^*g/(g*(-a \\ & *c)^{(1/2)}-c*f))^{(1/2)}*\text{EllipticF}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f)) \\ & ^{(1/2)},(-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*(-a*c) \\ & ^{(1/2)}*c^2*e^3*f^4*g-189*(-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}* \\ & ((-c*x+(-a*c)^{(1/2)})^*g/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*((c*x+(-a*c)^{( \\ & 1/2)})*g/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}*\text{EllipticE}((-g*x+f)*c/(g*(-a* \\ & c)^{(1/2)}-c*f))^{(1/2)},(-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f)) \\ & ^{(1/2)}*a^2*c^2*d^2*e^2*g^5-19*(-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2) \\ & }*((-c*x+(-a*c)^{(1/2)})^*g/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*((c*x+(-a*c) \\ & ^{(1/2)})^*g/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}*\text{EllipticE}((-g*x+f)*c/(g*(- \\ & a*c)^{(1/2)}-c*f))^{(1/2)},(-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f \\ & ))^{(1/2)}*a^2*c^2*e^3*f^2*g^4-11*(-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1 \\ & /2)}*((-c*x+(-a*c)^{(1/2)})^*g/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*((c*x+(-a* \\ & c)^{(1/2)})^*g/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}*\text{EllipticE}((-g*x+f)*c/(g* \\ & (-a*c)^{(1/2)}-c*f))^{(1/2)},(-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c \\ & *f))^{(1/2)}*a*c^2*e^3*f^3*g^2+105*(-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f) \\ & ))^{(1/2)}*((-c*x+(-a*c)^{(1/2)})^*g/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*((c*x \end{aligned}$$

$$\begin{aligned}
& +(-a^*c)^{(1/2)} * g / (g^* (-a^*c)^{(1/2)} - c^*f)^{(1/2)} * \text{EllipticE}((-g^*x+f)^* \\
& c / (g^* (-a^*c)^{(1/2)} - c^*f)^{(1/2)}, (-g^* (-a^*c)^{(1/2)} - c^*f) / (g^* (-a^*c)^{(1/2)} \\
& + c^*f))^{(1/2)} * c^3 d^2 e^2 f^3 g^2 - 42 * (-g^*x+f)^* c / (g^* (-a^*c)^{(1/2)} \\
& - c^*f)^{(1/2)} * ((-c^*x + (-a^*c)^{(1/2)})^* g / (g^* (-a^*c)^{(1/2)} + c^*f))^{(1/2)} * ( \\
& (c^*x + (-a^*c)^{(1/2)})^* g / (g^* (-a^*c)^{(1/2)} - c^*f))^{(1/2)} * \text{EllipticE}((-g^*x \\
& + f)^* c / (g^* (-a^*c)^{(1/2)} - c^*f)^{(1/2)}, (-g^* (-a^*c)^{(1/2)} - c^*f) / (g^* (-a^*c) \\
& )^{(1/2)} + c^*f))^{(1/2)} * c^3 d^2 e^2 f^4 g - 18 * x^4 c^3 e^3 f^2 g^4 + 10 * x^3 * \\
& a^* c^2 e^3 g^5 - 105 * x^3 c^3 d^2 e^2 g^5 + x^3 c^3 e^3 f^2 g^3 + 4 * x^2 c^3 \\
& * e^3 f^3 g^2 + 25 * x^2 a^2 c^2 e^3 g^5 + 105 * (-g^*x+f)^* c / (g^* (-a^*c)^{(1/2)} - c \\
& * f))^{(1/2)} * ((-c^*x + (-a^*c)^{(1/2)})^* g / (g^* (-a^*c)^{(1/2)} + c^*f))^{(1/2)} * ((c \\
& * x + (-a^*c)^{(1/2)})^* g / (g^* (-a^*c)^{(1/2)} - c^*f))^{(1/2)} * \text{EllipticE}((-g^*x+f) \\
& )^* c / (g^* (-a^*c)^{(1/2)} - c^*f)^{(1/2)}, (-g^* (-a^*c)^{(1/2)} - c^*f) / (g^* (-a^*c)^{(1/2)} \\
& + c^*f))^{(1/2)} * a^* c^2 d^2 e^2 f^2 g^4 - 231 * (-g^*x+f)^* c / (g^* (-a^*c)^{(1/2)} - c^*f) \\
& )^{(1/2)} * ((-c^*x + (-a^*c)^{(1/2)})^* g / (g^* (-a^*c)^{(1/2)} + c^*f))^{(1/2)} * ((c^*x + (-a^*c) \\
& )^{(1/2)})^* g / (g^* (-a^*c)^{(1/2)} - c^*f))^{(1/2)} * \text{EllipticE}((-g^*x+f)^* c / (g^* (-a^*c)^{(1/2)} - c^*f) \\
& )^{(1/2)}, (-g^* (-a^*c)^{(1/2)} - c^*f) / (g^* (-a^*c)^{(1/2)} + c^*f))^{(1/2)} * a^* c^2 d^2 e^2 f^2 g^3 + 189 * (-g^*x+f)^* c / (g^* (- \\
& a^*c)^{(1/2)} - c^*f))^{(1/2)} * ((-c^*x + (-a^*c)^{(1/2)})^* g / (g^* (-a^*c)^{(1/2)} + c^*f) \\
& ))^{(1/2)} * ((c^*x + (-a^*c)^{(1/2)})^* g / (g^* (-a^*c)^{(1/2)} - c^*f))^{(1/2)} * \text{EllipticF}((-g^*x+f)^* c / (g^* (-a^*c)^{(1/2)} - c^*f) \\
& )^{(1/2)}, (-g^* (-a^*c)^{(1/2)} - c^*f) / (g^* (-a^*c)^{(1/2)} + c^*f))^{(1/2)} * a^* c^2 d^2 e^2 f^2 g^3 - 105 * (-g^*x+f)^* \\
& c / (g^* (-a^*c)^{(1/2)} - c^*f))^{(1/2)} * ((-c^*x + (-a^*c)^{(1/2)})^* g / (g^* (-a^*c)^{(1/2)} + c^*f))^{(1/2)} * ((c^*x + (-a^*c) \\
& )^{(1/2)})^* g / (g^* (-a^*c)^{(1/2)} - c^*f))^{(1/2)} * \text{EllipticF}((-g^*x+f)^* c / (g^* (-a^*c)^{(1/2)} - c^*f) \\
& )^{(1/2)}, (-g^* (-a^*c)^{(1/2)} - c^*f) / (g^* (-a^*c)^{(1/2)} + c^*f))^{(1/2)} * (-a^*c)^{(1/2)} * a^* c^2 d^2 e^2 g^5 + \\
& 17 * (-g^*x+f)^* c / (g^* (-a^*c)^{(1/2)} - c^*f))^{(1/2)} * ((-c^*x + (-a^*c)^{(1/2)})^* g \\
& / (g^* (-a^*c)^{(1/2)} + c^*f))^{(1/2)} * ((c^*x + (-a^*c)^{(1/2)})^* g / (g^* (-a^*c)^{(1/2)} \\
& - c^*f))^{(1/2)} * \text{EllipticF}((-g^*x+f)^* c / (g^* (-a^*c)^{(1/2)} - c^*f))^{(1/2)}, ( \\
& -g^* (-a^*c)^{(1/2)} - c^*f) / (g^* (-a^*c)^{(1/2)} + c^*f))^{(1/2)} * (-a^*c)^{(1/2)} * a^* \\
& * c^2 e^3 f^2 g^3 - 105 * (-g^*x+f)^* c / (g^* (-a^*c)^{(1/2)} - c^*f))^{(1/2)} * ((-c^*x \\
& + (-a^*c)^{(1/2)})^* g / (g^* (-a^*c)^{(1/2)} + c^*f))^{(1/2)} * ((c^*x + (-a^*c)^{(1/2)})^* \\
& g / (g^* (-a^*c)^{(1/2)} - c^*f))^{(1/2)} * \text{EllipticF}((-g^*x+f)^* c / (g^* (-a^*c)^{(1/2)} \\
& - c^*f))^{(1/2)}, (-g^* (-a^*c)^{(1/2)} - c^*f) / (g^* (-a^*c)^{(1/2)} + c^*f))^{(1/2)} \\
& ) * (-a^*c)^{(1/2)} * c^2 d^2 e^2 f^2 g^3 - 63 * x^4 c^3 d^2 e^2 g^5 - 21 * a^* c^2 d^2 \\
& e^2 f^2 g^3 - 105 * a^* c^2 d^2 e^2 f^2 g^4) / (c^* g^* x^3 + c^* f^* x^2 + a^* g^* x + a^* f) / c^3 / g^4
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^3 \sqrt{gx + f}}{\sqrt{cx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*sqrt(g\*x + f)/sqrt(c\*x^2 + a), x, algorithm="maxima")

[Out] integrate((e\*x + d)^3\*sqrt(g\*x + f)/sqrt(c\*x^2 + a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)\sqrt{gx + f}}{\sqrt{cx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*sqrt(g\*x + f)/sqrt(c\*x^2 + a), x, algorithm="fricas")

[Out] integral((e^3\*x^3 + 3\*d^2\*e^2\*x^2 + 3\*d^2\*e\*x + d^3)\*sqrt(g\*x + f)/sqrt(c\*x^2 + a), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^3 \sqrt{f + gx}}{\sqrt{a + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(g\*x+f)\*\*(1/2)/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] Integral((d + e\*x)\*\*3\*sqrt(f + g\*x)/sqrt(a + c\*x\*\*2), x)

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*sqrt(g\*x + f)/sqrt(c\*x^2 + a),x, algorithm="giac")

[Out] Exception raised: RuntimeError

$$3.637 \quad \int \frac{(d+ex)^2 \sqrt{f+gx}}{\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=410

$$\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}} + 1\sqrt{f+gx} (9ae^2g^2 + c(-15d^2g^2 - 10defg + 2e^2f^2)) E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{15c^{3/2}g^2\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}}$$

$$- \frac{4\sqrt{-ae}\sqrt{\frac{cx^2}{a}} + 1(ag^2 + cf^2)(ef - 5dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}} F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{15c^{3/2}g^2\sqrt{a+cx^2}\sqrt{f+gx}}$$

$$+ \frac{2e\sqrt{a+cx^2}\sqrt{f+gx}(7dg + ef)}{15cg} + \frac{2e\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5c}$$

[Out] (2\*e\*(e\*f + 7\*d\*g)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])/(15\*c\*g) + (2\*e\*(d + e\*x)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])/(5\*c) + (2\*Sqrt[-a]\*(9\*a\*e^2\*g^2 + c\*(2\*e^2\*f^2 - 10\*d\*e\*f\*g - 15\*d^2\*g^2))\*Sqrt[f + g\*x]\*Sqrt[1 + (c\*x^2)/a]\*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(15\*c^(3/2)\*g^2\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[a + c\*x^2]) - (4\*Sqrt[-a]\*e\*(e\*f - 5\*d\*g)\*(c\*f^2 + a\*g^2)\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(15\*c^(3/2)\*g^2\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])

**Rubi [A]** time = 1.22907, antiderivative size = 410, normalized size of antiderivative = 1., number of rules used = 7, number of rules used = 6, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}} + 1\sqrt{f+gx} (9ae^2g^2 + c(-15d^2g^2 - 10defg + 2e^2f^2)) E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{15c^{3/2}g^2\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}}$$

$$- \frac{4\sqrt{-ae}\sqrt{\frac{cx^2}{a}} + 1(ag^2 + cf^2)(ef - 5dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}} F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{15c^{3/2}g^2\sqrt{a+cx^2}\sqrt{f+gx}}$$

$$+ \frac{2e\sqrt{a+cx^2}\sqrt{f+gx}(7dg + ef)}{15cg} + \frac{2e\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5c}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^2\*Sqrt[f + g\*x])/Sqrt[a + c\*x^2], x]

[Out] (2\*e\*(e\*f + 7\*d\*g)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])/(15\*c\*g) + (2\*e\*(d + e\*x)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])/(5\*c) + (2\*Sqrt[-a]\*(9\*a\*e^2\*g^2 + c\*(2\*e^2\*f^2 - 10\*d\*e\*f\*g - 15\*d^2\*g^2))\*Sqrt[f + g\*x]\*Sqrt[1 + (c\*x^2)/a]\*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(15\*c^(3/2)\*g^2\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[a + c\*x^2]) - (4\*Sqrt[-a]\*e\*(e\*f - 5\*d\*g)\*(c\*f^2 + a\*g^2)\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(15\*c^(3/2)\*g^2\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])

---

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**2*(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)`

[Out] Timed out

---

**Mathematica [C]** time = 7.58755, size = 596, normalized size = 1.45

$$2\sqrt{f+gx} \left( \frac{g^2(-9a^2e^2g^2+ac(15d^2g^2+10defg+e^2(-2f^2+9g^2x^2)))+c^2x^2(15d^2g^2+10defg-2e^2f^2)}{f+gx} - ic\sqrt{f+gx}\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}\sqrt{\frac{g(x+\frac{i\sqrt{a}}{\sqrt{c}})}{f+gx}} \right)$$


---

Antiderivative was successfully verified.

[In] `Integrate[((d + e*x)^2*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]`

[Out]  $(2\sqrt{f+gx})(c^2e^2g^2(a+c^2x^2)(10dg+e(f+3gx))+(g^2(-9a^2e^2g^2+c^2(-2e^2f^2+10d^2efg+15d^2g^2)x^2+a^2c(10d^2efg+15d^2g^2-e^2(2f^2+9g^2x^2))))/(f+gx)-Ic\sqrt{f+gx}\sqrt{-f-(I\sqrt{a}g)/\sqrt{c}}(9a^2e^2g^2+c(2e^2f^2-10d^2efg-15d^2g^2))\sqrt{(g(I\sqrt{a})/\sqrt{c}+x))/(f+gx)}\sqrt{-((I\sqrt{a}g)/\sqrt{c}-gx)/(f+gx))}\sqrt{f+gx}\text{EllipticE}[I\text{ArcSinh}[\sqrt{-f-(I\sqrt{a}g)/\sqrt{c}}]/\sqrt{f+gx}],(\sqrt{c}gf-I\sqrt{a}g)/(\sqrt{c}f+I\sqrt{a}g))+(\sqrt{c}g(\sqrt{c}f+I\sqrt{a}g)((15I)c^2d^2g-(9I)a^2e^2g+2\sqrt{a}\sqrt{c}e^2(e^2f-5d^2g))\sqrt{(g(I\sqrt{a})/\sqrt{c}+x))/(f+gx)}\sqrt{-((I\sqrt{a}g)/\sqrt{c}-gx)/(f+gx))}\sqrt{f+gx}\text{EllipticF}[I\text{ArcSinh}[\sqrt{-f-(I\sqrt{a}g)/\sqrt{c}}]/\sqrt{f+gx}],(\sqrt{c}gf-I\sqrt{a}g)/(\sqrt{c}f+I\sqrt{a}g))/\sqrt{-f-(I\sqrt{a}g)/\sqrt{c}})/(15c^2g^3\sqrt{a+c^2x^2})$

---

**Maple [B]** time = 0.054, size = 2470, normalized size = 6.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)`

[Out]  $-2/15(g*x+f)^{1/2}(c*x^2+a)^{1/2}(-10*x^2*c^2*d*e*f*g^3-10*x*a*c*d*e*g^4-4*x*a*c^2*e^2*f*g^3-10*a*c*d*e*f*g^3-3*x^4*c^2*e^2*g^4+9*(-(g*x+f)*c/(g*(-a*c)^{1/2}-c*f))^{1/2}((c*x+(-a*c)^{1/2})^2g/(g*(-a*c)^{1/2}-c*f))^{1/2}((c*x+(-a*c)^{1/2})^2g/(g*(-a*c)^{1/2}-c*f))^{1/2})\text{EllipticF}((-g*x+f)*c/(g*(-a*c)^{1/2}-c*f))^{1/2},(-(g*(-a*c)^{1/2}-c*f)/(g*(-a*c)^{1/2}+c*f))^{1/2})^2a^2e^2g^4-9*(-(g*x+f)*c/(g*(-a*c)^{1/2}-c*f))^{1/2}((c*x+(-a*c)^{1/2})^2g/(g*(-a*c)^{1/2}-c*f))^{1/2})\text{EllipticE}((-g*x+f)*c/(g*(-a*c)^{1/2}-c*f))^{1/2},(-(g*(-a*c)^{1/2}-c*f)/(g*(-a*c)^{1/2}+c*f))^{1/2})^2a^2e^2g^4-2*(-(g*$

```

x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*
c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(
1/2)*EllipticE((-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-g*(-a*
c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*e^2*f^4+2*(-g*x+f
)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(
1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/
2)*EllipticF((-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-g*(-a*c)^(
1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-a*c)^(1/2)*a*e^2*f*g^3+
2*(-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/
(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)
-c*f))^(1/2)*EllipticF((-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-
g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-a*c)^(1/2)*c*
e^2*f^3*g+9*(-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)
^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-
a*c)^(1/2)-c*f))^(1/2)*EllipticF((-g*x+f)*c/(g*(-a*c)^(1/2)-c*f)
)^(1/2),(-g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c*e
^2*f^2*g^2-11*(-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*
c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*
(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-g*x+f)*c/(g*(-a*c)^(1/2)-c*
f))^(1/2),(-g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c
*e^2*f^2*g^2+10*(-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-
a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(
g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-g*x+f)*c/(g*(-a*c)^(1/2)-
c*f))^(1/2),(-g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c
^2*d*e*f^3*g-15*(-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-
a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(
g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-g*x+f)*c/(g*(-a*c)^(1/2)-
c*f))^(1/2),(-g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c
^2*d^2*f^2*g^2+15*(-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+
(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g
/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-g*x+f)*c/(g*(-a*c)^(1/2)
-c*f))^(1/2),(-g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))
*a*c*d^2*g^4+15*(-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-
a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(
g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-g*x+f)*c/(g*(-a*c)^(1/2)-
c*f))^(1/2),(-g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c
^2*d^2*f^2*g^2-10*(-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+
(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g
/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-g*x+f)*c/(g*(-a*c)^(1/2)
-c*f))^(1/2),(-g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))
*(-a*c)^(1/2)*c*d*e*f^2*g^2+10*(-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(
1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-
a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-g*x+f)*c/(
g*(-a*c)^(1/2)-c*f))^(1/2),(-g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)
+c*f))^(1/2))*a*c*d*e*f*g^3-10*(-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(
1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-
a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-g*x+f)*c/(
g*(-a*c)^(1/2)-c*f))^(1/2),(-g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)
+c*f))^(1/2))*(-a*c)^(1/2)*a*d*e*g^4-10*x^3*c^2*d*e*g^4-15*(-g*x
+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c
)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(
1/2)*EllipticF((-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-g*(-a*c)
)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c*d^2*g^4-4*x^3*c^2*e
^2*f*g^3-3*x^2*a*c*e^2*g^4-x^2*c^2*e^2*f^2*g^2-a*c*e^2*f^2*g^2)/c
^2/(c*g*x^3+c*f*x^2+a*g*x+a*f)/g^3

```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^2 \sqrt{gx + f}}{\sqrt{cx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2\*sqrt(g\*x + f)/sqrt(c\*x^2 + a),x, algorithm="maxima")

[Out] integrate((e\*x + d)^2\*sqrt(g\*x + f)/sqrt(c\*x^2 + a), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e^2x^2 + 2dex + d^2)\sqrt{gx + f}}{\sqrt{cx^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^2*sqrt(g*x + f)/sqrt(c*x^2 + a), x, algorithm="fricas")`

[Out] `integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(g*x + f)/sqrt(c*x^2 + a), x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^2 \sqrt{f + gx}}{\sqrt{a + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**2*(g*x+f)**(1/2)/(c*x**2+a)**(1/2), x)`

[Out] `Integral((d + e*x)**2*sqrt(f + g*x)/sqrt(a + c*x**2), x)`

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^2*sqrt(g*x + f)/sqrt(c*x^2 + a), x, algorithm="giac")`

[Out] `Exception raised: RuntimeError`

$$3.638 \quad \int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=331

$$\frac{2\sqrt{-a}e\sqrt{\frac{cx^2}{a}+1}(ag^2+cf^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}}{\sqrt{-a}}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3c^{3/2}g\sqrt{a+cx^2}\sqrt{f+gx}} - \frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}(3dg+ef)E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}}{\sqrt{-a}}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}g\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}} + \frac{2e\sqrt{a+cx^2}\sqrt{f+gx}}{3c}$$

[Out] (2\*e\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])/(3\*c) - (2\*Sqrt[-a]\*(e\*f + 3\*d\*g)\*Sqrt[f + g\*x]\*Sqrt[1 + (c\*x^2)/a]\*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(3\*Sqrt[c]\*g\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[a + c\*x^2]) + (2\*Sqrt[-a]\*e\*(c\*f^2 + a\*g^2)\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(3\*c^(3/2)\*g\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])

**Rubi [A]** time = 0.736217, antiderivative size = 331, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$

$$\frac{2\sqrt{-a}e\sqrt{\frac{cx^2}{a}+1}(ag^2+cf^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}}{\sqrt{-a}}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3c^{3/2}g\sqrt{a+cx^2}\sqrt{f+gx}} - \frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}(3dg+ef)E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}}{\sqrt{-a}}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{3\sqrt{c}g\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}} + \frac{2e\sqrt{a+cx^2}\sqrt{f+gx}}{3c}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*Sqrt[f + g\*x])/Sqrt[a + c\*x^2], x]

[Out] (2\*e\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])/(3\*c) - (2\*Sqrt[-a]\*(e\*f + 3\*d\*g)\*Sqrt[f + g\*x]\*Sqrt[1 + (c\*x^2)/a]\*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(3\*Sqrt[c]\*g\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[a + c\*x^2]) + (2\*Sqrt[-a]\*e\*(c\*f^2 + a\*g^2)\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(3\*c^(3/2)\*g\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])



**Rubi in Sympy [A]** time = 123.818, size = 309, normalized size = 0.93

$$\frac{2e\sqrt{a+cx^2}\sqrt{f+gx}}{3c} - \frac{2\sqrt{-a}\sqrt{1+\frac{cx^2}{a}}\sqrt{f+gx}(3dg+ef)E\left(\operatorname{asin}\left(\sqrt{-\frac{\sqrt{c}x}{2\sqrt{-a}}+\frac{1}{2}}\right)\middle|\frac{2ag}{ag-\sqrt{c}f\sqrt{-a}}\right)}{3\sqrt{c}g\sqrt{\frac{\sqrt{c}\sqrt{-a}(-f-gx)}{ag-\sqrt{c}f\sqrt{-a}}}\sqrt{a+cx^2}}$$

$$+ \frac{2e\sqrt{-a}\sqrt{\frac{\sqrt{c}\sqrt{-a}(-f-gx)}{ag-\sqrt{c}f\sqrt{-a}}}\sqrt{1+\frac{cx^2}{a}}(ag^2+cf^2)F\left(\operatorname{asin}\left(\sqrt{-\frac{\sqrt{c}x}{2\sqrt{-a}}+\frac{1}{2}}\right)\middle|\frac{2ag}{ag-\sqrt{c}f\sqrt{-a}}\right)}{3c^{\frac{3}{2}}g\sqrt{a+cx^2}\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)*(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)`

[Out]  $2*e*\sqrt{a+c*x**2}*\sqrt{f+g*x}/(3*c) - 2*\sqrt{-a}*\sqrt{1+c*x**2/a}*\sqrt{f+g*x}*(3*d*g+e*f)*\operatorname{elliptic\_e}(\operatorname{asin}(\sqrt{-\sqrt{c}*x/(2*\sqrt{-a})+1/2})), 2*a*g/(a*g-\sqrt{c}*f*\sqrt{-a}))/ (3*\sqrt{c}*g*\sqrt{\sqrt{c}\sqrt{-a}(-f-gx)/(ag-\sqrt{c}f\sqrt{-a})}\sqrt{a+cx^2}) + 2*e*\sqrt{-a}*\sqrt{\sqrt{c}\sqrt{-a}(-f-gx)/(ag-\sqrt{c}f\sqrt{-a})}*\sqrt{1+c*x**2/a}*(a*g**2+c*f**2)*\operatorname{elliptic\_f}(\operatorname{asin}(\sqrt{-\sqrt{c}*x/(2*\sqrt{-a})+1/2})), 2*a*g/(a*g-\sqrt{c}*f*\sqrt{-a}))/ (3*c**(3/2)*g*\sqrt{a+c*x**2}*\sqrt{f+g*x})$

**Mathematica [C]** time = 5.39814, size = 464, normalized size = 1.4

$$2\sqrt{f+gx} \left( \frac{ic\sqrt{f+gx}\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}(3dg+ef)\sqrt{\frac{g(x+\frac{i\sqrt{a}}{\sqrt{c}})}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{ag}}{\sqrt{c}}}{f+gx}}E\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}f-i\sqrt{ag}}{\sqrt{c}f+i\sqrt{ag}}\right)}{g^2} + \frac{(a+cx^2)(3dg+ef)}{f+gx} + \frac{i\sqrt{f+gx}(3\sqrt{cd+i\sqrt{ae}})}{3c\sqrt{a+cx^2}} \right)$$

Antiderivative was successfully verified.

[In] `Integrate[((d + e*x)*Sqrt[f + g*x])/Sqrt[a + c*x^2],x]`

[Out]  $(2*\sqrt{f+g*x}*(e*(a+c*x^2)+((e*f+3*d*g)*(a+c*x^2)))/(f+g*x) + (I*c*\sqrt{-f-(I*\sqrt{a}*g)/\sqrt{c}})/\sqrt{c}*(e*f+3*d*g)*\sqrt{(g*((I*\sqrt{a}))/\sqrt{c})/\sqrt{c}+x))/\sqrt{f+g*x}*\sqrt{-(((I*\sqrt{a})*g)/\sqrt{c}-g*x)/(f+g*x))}*\sqrt{f+g*x}*\operatorname{EllipticE}[I*\operatorname{ArcSinh}[\sqrt{-f-(I*\sqrt{a})*g/\sqrt{c}}]/\sqrt{f+g*x}], (\sqrt{c}*f-I*\sqrt{a})*g)/(\sqrt{c}*f+I*\sqrt{a}*g))/g^2 + (I*(3*\sqrt{c}*d+I*\sqrt{a})*e*(\sqrt{c}*f+I*\sqrt{a}*g)*\sqrt{(g*((I*\sqrt{a}))/\sqrt{c})/\sqrt{c}+x))/\sqrt{f+g*x}*\sqrt{-(((I*\sqrt{a})*g)/\sqrt{c}-g*x)/(f+g*x))}*\sqrt{f+g*x}*\operatorname{EllipticF}[I*\operatorname{ArcSinh}[\sqrt{-f-(I*\sqrt{a})*g/\sqrt{c}}]/\sqrt{f+g*x}], (\sqrt{c}*f-I*\sqrt{a})*g)/(\sqrt{c}*f+I*\sqrt{a}*g))/ (g*\sqrt{-f-(I*\sqrt{a})*g/\sqrt{c}}))/ (3*c*\sqrt{a+c*x^2})$

**Maple [B]** time = 0.042, size = 1286, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)*(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)`

```
[Out] 2/3*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)*((-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-a*c)^(1/2)*a*e*g^3+(-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-a*c)^(1/2)*c*e*f^2*g+3*(-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c*d*g^3+3*(-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticF((-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*d*f^2*g-3*(-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*c*e*f*g^2-3*(-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*d*f^2*g-(-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2)*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2)*EllipticE((-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2),(-g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c^2*e*f^3+x^3*c^2*e*g^3+x^2*c^2*e*f*g^2+x*a*c*e*g^3+a*c*e*f*g^2)/(c*g*x^3+c*f*x^2+a*g*x+a*f)/c^2/g^2
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)\sqrt{gx+f}}{\sqrt{cx^2+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)*sqrt(g*x + f)/sqrt(c*x^2 + a),x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex+d)\sqrt{gx+f}}{\sqrt{cx^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)*sqrt(g*x + f)/sqrt(c*x^2 + a),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)*sqrt(g*x + f)/sqrt(c*x^2 + a), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex) \sqrt{f + gx}}{\sqrt{a + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(g\*x+f)\*\*(1/2)/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] Integral((d + e\*x)\*sqrt(f + g\*x)/sqrt(a + c\*x\*\*2), x)

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*sqrt(g\*x + f)/sqrt(c\*x^2 + a),x, algorithm="giac")

[Out] Exception raised: RuntimeError

$$3.639 \quad \int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=136

$$\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}} + 1\sqrt{f+gx}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{\sqrt{c}\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}}$$

[Out] (-2\*Sqrt[-a]\*Sqrt[f + g\*x]\*Sqrt[1 + (c\*x^2)/a]\*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)))/(Sqrt[c]\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[a + c\*x^2])

**Rubi [A]** time = 0.208611, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}} + 1\sqrt{f+gx}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{\sqrt{c}\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[f + g\*x]/Sqrt[a + c\*x^2], x]

[Out] (-2\*Sqrt[-a]\*Sqrt[f + g\*x]\*Sqrt[1 + (c\*x^2)/a]\*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)))/(Sqrt[c]\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[a + c\*x^2])

**Rubi in Sympy [A]** time = 37.2516, size = 129, normalized size = 0.95

$$\frac{2\sqrt{-a}\sqrt{1 + \frac{cx^2}{a}}\sqrt{f+gx}E\left(\operatorname{asin}\left(\sqrt{-\frac{\sqrt{cx}}{2\sqrt{-a}} + \frac{1}{2}}\right)\middle|\frac{2ag}{ag-\sqrt{cf}\sqrt{-a}}\right)}{\sqrt{c}\sqrt{\frac{\sqrt{c}\sqrt{-a}(-f-gx)}{ag-\sqrt{cf}\sqrt{-a}}}\sqrt{a+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*(1/2)/(c\*x\*\*2+a)\*\*(1/2), x)

[Out] -2\*sqrt(-a)\*sqrt(1 + c\*x\*\*2/a)\*sqrt(f + g\*x)\*elliptic\_e(asin(sqrt(-sqrt(c)\*x/(2\*sqrt(-a)) + 1/2)), 2\*a\*g/(a\*g - sqrt(c)\*f\*sqrt(-a)))/(sqrt(c)\*sqrt(sqrt(c)\*sqrt(-a)\*(-f - g\*x)/(a\*g - sqrt(c)\*f\*sqrt(-a)))\*sqrt(a + c\*x\*\*2))

**Mathematica [C]** time = 0.656388, size = 294, normalized size = 2.16

$$\frac{2i\sqrt{f+gx}(\sqrt{cf} + i\sqrt{ag})\sqrt{\frac{g(\sqrt{a+i\sqrt{cx}})}{\sqrt{ag-i\sqrt{cf}}}}\left(E\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}(f+gx)}}{\sqrt{cf-i\sqrt{ag}}}\right)\middle|\frac{\sqrt{cf-i\sqrt{ag}}}{\sqrt{cf+i\sqrt{ag}}}\right) - F\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}(f+gx)}}{\sqrt{cf-i\sqrt{ag}}}\right)\middle|\frac{\sqrt{cf-i\sqrt{ag}}}{\sqrt{cf+i\sqrt{ag}}}\right)\right)}{\sqrt{cg}\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{g(\sqrt{cx+i\sqrt{a}})}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[f + g\*x]/Sqrt[a + c\*x^2],x]

[Out] 
$$\frac{((2*I)*(Sqrt[c]*f + I*Sqrt[a]*g)*Sqrt[(g*(Sqrt[a] + I*Sqrt[c]*x)) / ((-I)*Sqrt[c]*f + Sqrt[a]*g)]*Sqrt[f + g*x]*(EllipticE[I*ArcSinh[Sqrt[-((Sqrt[c]*(f + g*x))/(Sqrt[c]*f - I*Sqrt[a]*g))]]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)] - EllipticF[I*ArcSinh[Sqrt[-((Sqrt[c]*(f + g*x))/(Sqrt[c]*f - I*Sqrt[a]*g))]]], (Sqrt[c]*f - I*Sqrt[a]*g)/(Sqrt[c]*f + I*Sqrt[a]*g)))/(Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(g*(I*Sqrt[a] + Sqrt[c]*x))]*Sqrt[a + c*x^2])$$

**Maple [B]** time = 0.04, size = 396, normalized size = 2.9

$$2 \frac{\sqrt{gx+f}\sqrt{cx^2+a}(cf-g\sqrt{-ac})}{g(cgx^3+cfx^2+agx+fa)c^2} \sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}} \sqrt{\frac{(-cx+\sqrt{-ac})g}{g\sqrt{-ac}+cf}} \sqrt{\frac{(cx+\sqrt{-ac})g}{g\sqrt{-ac}-cf}} \left( \sqrt{-ac} \text{EllipticF} \left( \sqrt{-\frac{(gx+f)c}{g\sqrt{-ac}-cf}}, \right. \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^(1/2)/(c\*x^2+a)^(1/2),x)

[Out] 
$$2*(g*x+f)^{(1/2)}*(c*x^2+a)^{(1/2)}*(c*f-g*(-a*c)^{(1/2)})*(-(g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}*((-c*x+(-a*c)^{(1/2)})*g/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*((c*x+(-a*c)^{(1/2)})*g/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}*((-a*c)^{(1/2)}*EllipticF((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)},(-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*g+f*EllipticF((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)},(-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*c-EllipticE((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)},(-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*(-a*c)^{(1/2)}*g-EllipticE((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)},(-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*c*f)/g/(c*g*x^3+c*f*x^2+a*g*x+a*f)/c^2$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(g\*x + f)/sqrt(c\*x^2 + a),x, algorithm="maxima")

[Out] integrate(sqrt(g\*x + f)/sqrt(c\*x^2 + a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(g\*x + f)/sqrt(c\*x^2 + a),x, algorithm="fricas")

[Out] integral(sqrt(g\*x + f)/sqrt(c\*x^2 + a), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{f + gx}}{\sqrt{a + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*(1/2)/(c\*x\*\*2+a)\*\*(1/2), x)

[Out] Integral(sqrt(f + g\*x)/sqrt(a + c\*x\*\*2), x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx + f}}{\sqrt{cx^2 + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(g\*x + f)/sqrt(c\*x^2 + a), x, algorithm="giac")

[Out] integrate(sqrt(g\*x + f)/sqrt(c\*x^2 + a), x)

$$3.640 \quad \int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=319

$$\frac{2\sqrt{\frac{cx^2}{a}+1}(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}\left(\frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}}+e}; \sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle| \frac{2\sqrt{-ag}}{\sqrt{cf+\sqrt{-ag}}}\right)}{e\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{cd}}{\sqrt{-a}}+e\right)} - \frac{2\sqrt{-ag}\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{\sqrt{ce}\sqrt{a+cx^2}\sqrt{f+gx}}$$

[Out]  $(-2*\text{Sqrt}[-a]*g*\text{Sqrt}[(\text{Sqrt}[c]*(f+g*x))/(\text{Sqrt}[c]*f+\text{Sqrt}[-a]*g)]*\text{Sqrt}[1+(c*x^2)/a]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1-(\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f-a*g)]/(\text{Sqrt}[c]*e*\text{Sqrt}[f+g*x]*\text{Sqrt}[a+c*x^2]) - (2*(e*f-d*g)*\text{Sqrt}[(\text{Sqrt}[c]*(f+g*x))/(\text{Sqrt}[c]*f+\text{Sqrt}[-a]*g)]*\text{Sqrt}[1+(c*x^2)/a]*\text{EllipticPi}[(2*e)/((\text{Sqrt}[c]*d)/\text{Sqrt}[-a]+e), \text{ArcSin}[\text{Sqrt}[1-(\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (2*\text{Sqrt}[-a]*g)/(\text{Sqrt}[c]*f+\text{Sqrt}[-a]*g)]/(e*((\text{Sqrt}[c]*d)/\text{Sqrt}[-a]+e)*\text{Sqrt}[f+g*x]*\text{Sqrt}[a+c*x^2])$

**Rubi [A]** time = 1.80919, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{2\sqrt{\frac{cx^2}{a}+1}(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}\left(\frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}}+e}; \sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle| \frac{2\sqrt{-ag}}{\sqrt{cf+\sqrt{-ag}}}\right)}{e\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{cd}}{\sqrt{-a}}+e\right)} - \frac{2\sqrt{-ag}\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{\sqrt{ce}\sqrt{a+cx^2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[f+g*x]/((d+e*x)*\text{Sqrt}[a+c*x^2]), x]$

[Out]  $(-2*\text{Sqrt}[-a]*g*\text{Sqrt}[(\text{Sqrt}[c]*(f+g*x))/(\text{Sqrt}[c]*f+\text{Sqrt}[-a]*g)]*\text{Sqrt}[1+(c*x^2)/a]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1-(\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f-a*g)]/(\text{Sqrt}[c]*e*\text{Sqrt}[f+g*x]*\text{Sqrt}[a+c*x^2]) - (2*(e*f-d*g)*\text{Sqrt}[(\text{Sqrt}[c]*(f+g*x))/(\text{Sqrt}[c]*f+\text{Sqrt}[-a]*g)]*\text{Sqrt}[1+(c*x^2)/a]*\text{EllipticPi}[(2*e)/((\text{Sqrt}[c]*d)/\text{Sqrt}[-a]+e), \text{ArcSin}[\text{Sqrt}[1-(\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (2*\text{Sqrt}[-a]*g)/(\text{Sqrt}[c]*f+\text{Sqrt}[-a]*g)]/(e*((\text{Sqrt}[c]*d)/\text{Sqrt}[-a]+e)*\text{Sqrt}[f+g*x]*\text{Sqrt}[a+c*x^2])$

**Rubi in Sympy [A]** time = 95.722, size = 330, normalized size = 1.03

$$\frac{2\sqrt{\frac{g(-\sqrt{cx}-\sqrt{-a})}{\sqrt{cf-g\sqrt{-a}}}}\sqrt{\frac{g(-\sqrt{cx}+\sqrt{-a})}{\sqrt{cf+g\sqrt{-a}}}}\left(-\frac{e(\sqrt{cf+g\sqrt{-a}})}{\sqrt{c}(dg-ef)}; \text{asin}\left(\sqrt{\frac{c}{\sqrt{cg}\sqrt{-a}+cf}}\sqrt{f+gx}\right)\middle| \frac{\sqrt{cf+g\sqrt{-a}}}{\sqrt{cf-g\sqrt{-a}}}\right)}{e\sqrt{\frac{c}{\sqrt{cg}\sqrt{-a}+cf}}\sqrt{a+cx^2}} - \frac{2g\sqrt{-a}\sqrt{\frac{\sqrt{c}\sqrt{-a}(-f-gx)}{ag-\sqrt{cf}\sqrt{-a}}}}{\sqrt{ce}\sqrt{a+cx^2}\sqrt{f+gx}}\sqrt{1+\frac{cx^2}{a}}F\left(\text{asin}\left(\sqrt{-\frac{\sqrt{cx}}{2\sqrt{-a}}+\frac{1}{2}}\right)\middle| \frac{2ag}{ag-\sqrt{cf}\sqrt{-a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x+f)**(1/2)/(e*x+d)/(c*x**2+a)**(1/2),x)`

[Out]  $-2\sqrt{g(-\sqrt{c}x - \sqrt{-a})}/(\sqrt{c}f - g\sqrt{-a})\sqrt{g(-\sqrt{c}x + \sqrt{-a})}/(\sqrt{c}f + g\sqrt{-a})\text{elliptic\_pi}(-e(\sqrt{c}f + g\sqrt{-a})/(\sqrt{c}(d^*g - e^*f)), \text{asin}(\sqrt{c}/(\sqrt{c}g\sqrt{-a} + c^*f))\sqrt{f + g^*x}), (\sqrt{c}f + g\sqrt{-a})/(\sqrt{c}f - g\sqrt{-a})/(e^*\sqrt{c}/(\sqrt{c}g\sqrt{-a} + c^*f))\sqrt{a + c^*x^2}) - 2g\sqrt{-a}\sqrt{\sqrt{c}\sqrt{-a}(-f - g^*x)/(a^*g - \sqrt{c}f\sqrt{-a})}\sqrt{1 + c^*x^2/a}\text{elliptic\_f}(\text{asin}(\sqrt{-\sqrt{c}x}/(2\sqrt{-a}) + 1/2)), 2^*a^*g/(a^*g - \sqrt{c}f\sqrt{-a})/(\sqrt{c}e^*\sqrt{a + c^*x^2})\sqrt{f + g^*x})$

**Mathematica [C]** time = 0.90318, size = 300, normalized size = 0.94

$$\frac{2i\sqrt{f+gx}\sqrt{\frac{g(\sqrt{a+i\sqrt{c}x})}{\sqrt{ag-i\sqrt{c}f}}}\left(F\left(i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}(f+gx)}}{\sqrt{c}f-i\sqrt{ag}}}\right)\middle|\frac{\sqrt{c}f-i\sqrt{ag}}{\sqrt{c}f+i\sqrt{ag}}\right)-\left(\frac{e\left(\frac{f-i\sqrt{ag}}{\sqrt{c}}\right)}{ef-dg};i\sinh^{-1}\left(\sqrt{-\frac{\sqrt{c}(f+gx)}}{\sqrt{c}f-i\sqrt{ag}}}\right)\middle|\frac{\sqrt{c}f-i\sqrt{ag}}{\sqrt{c}f+i\sqrt{ag}}\right)\right)}{e\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{g(\sqrt{c}x+i\sqrt{a})}}}$$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[f + g*x]/((d + e*x)*Sqrt[a + c*x^2]),x]`

[Out]  $((-2^*I)^*\text{Sqrt}[(g^*(\text{Sqrt}[a] + I^*\text{Sqrt}[c]^*x))/((-I)^*\text{Sqrt}[c]^*f + \text{Sqrt}[a]^*g)]^*\text{Sqrt}[f + g^*x]^*(\text{EllipticF}[I^*\text{ArcSinh}[\text{Sqrt}[-((\text{Sqrt}[c]^*(f + g^*x))/(\text{Sqrt}[c]^*f - I^*\text{Sqrt}[a]^*g))]]), (\text{Sqrt}[c]^*f - I^*\text{Sqrt}[a]^*g)/(\text{Sqrt}[c]^*f + I^*\text{Sqrt}[a]^*g)] - \text{EllipticPi}[(e^*(f - (I^*\text{Sqrt}[a]^*g)/\text{Sqrt}[c]))/(e^*f - d^*g), I^*\text{ArcSinh}[\text{Sqrt}[-((\text{Sqrt}[c]^*(f + g^*x))/(\text{Sqrt}[c]^*f - I^*\text{Sqrt}[a]^*g))]]), (\text{Sqrt}[c]^*f - I^*\text{Sqrt}[a]^*g)/(\text{Sqrt}[c]^*f + I^*\text{Sqrt}[a]^*g))]/(e^*\text{Sqrt}[(\text{Sqrt}[c]^*(f + g^*x))/g^*(I^*\text{Sqrt}[a] + \text{Sqrt}[c]^*x)]^*\text{Sqrt}[a + c^*x^2])$

**Maple [A]** time = 0.051, size = 439, normalized size = 1.4

$$2\frac{\sqrt{gx+f}\sqrt{cx^2+a}}{ce(cx^3+cfx^2+agx+fa)}\sqrt{\frac{(gx+f)c}{g\sqrt{-ac}-cf}}\sqrt{\frac{(-cx+\sqrt{-ac})g}{g\sqrt{-ac}+cf}}\sqrt{\frac{(cx+\sqrt{-ac})g}{g\sqrt{-ac}-cf}}\left(f\text{EllipticF}\left(\sqrt{\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\sqrt{\frac{g\sqrt{-ac}}{g\sqrt{-ac}-cf}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^(1/2)/(e*x+d)/(c*x^2+a)^(1/2),x)`

[Out]  $2^*(g^*x+f)^{(1/2)}*(c^*x^2+a)^{(1/2)}*(-(g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}*((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}*(f^*\text{EllipticF}((-g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}, (-g^*(-a^*c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*c-(-a^*c)^{(1/2)}^*\text{EllipticF}((-g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}, (-g^*(-a^*c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*g-\text{EllipticPi}((-g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}, (g^*(-a^*c)^{(1/2)}-c^*f)^*e/c/(d^*g-e^*f), (-g^*(-a^*c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*c^*f+\text{EllipticPi}((-g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}, (g^*(-a^*c)^{(1/2)}-c^*f)^*e/c/(d^*g-e^*f), (-g^*(-a^*c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*(-a^*c)^{(1/2)}^*g/e/c/(c^*g^*x^3+c^*f^*x^2+a^*g^*x+a^*f)$



**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx + f}}{\sqrt{cx^2 + a}(ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(g\*x + f)/(sqrt(c\*x^2 + a)\*(e\*x + d)),x, algorithm="maxima")

[Out] integrate(sqrt(g\*x + f)/(sqrt(c\*x^2 + a)\*(e\*x + d)), x)

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(g\*x + f)/(sqrt(c\*x^2 + a)\*(e\*x + d)),x, algorithm="fricas")

[Out] Timed out

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{f + gx}}{\sqrt{a + cx^2}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*(1/2)/(e\*x+d)/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] Integral(sqrt(f + g\*x)/(sqrt(a + c\*x\*\*2)\*(d + e\*x)), x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx + f}}{\sqrt{cx^2 + a}(ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(g\*x + f)/(sqrt(c\*x^2 + a)\*(e\*x + d)),x, algorithm="giac")

[Out] integrate(sqrt(g\*x + f)/(sqrt(c\*x^2 + a)\*(e\*x + d)), x)

$$3.641 \quad \int \frac{\sqrt{f+gx}}{(d+ex)^2 \sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=698

$$\begin{aligned} & -\frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)} + \frac{\sqrt{-a}\sqrt{c}f\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}}{\sqrt{-a}}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}(ae^2+cd^2)} \\ & -\frac{\sqrt{-a}\sqrt{c}dg\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}}{\sqrt{-a}}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{e\sqrt{a+cx^2}\sqrt{f+gx}(ae^2+cd^2)} \\ & -\frac{\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}}{\sqrt{-a}}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{a+cx^2}(ae^2+cd^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}} \\ & -\frac{\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}(ae^2g+cd(2ef-dg))\left(\frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}}{\sqrt{-a}}}\right)\middle|\frac{2\sqrt{-ag}}{\sqrt{c}f+\sqrt{-ag}}\right)}{e\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{cd}}{\sqrt{-a}}+e\right)(ae^2+cd^2)} \end{aligned}$$

```
[Out] -((e*Sqrt[f + g*x]*Sqrt[a + c*x^2])/((c*d^2 + a*e^2)*(d + e*x)))
- (Sqrt[-a]*Sqrt[c]*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)]/((c*d^2 + a*e^2)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (Sqrt[-a]*Sqrt[c]*f*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)]/((c*d^2 + a*e^2)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*d*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)]/(e*(c*d^2 + a*e^2)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - ((a*e^2*g + c*d*(2*e*f - d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)]/(e*((Sqrt[c]*d)/Sqrt[-a] + e)*(c*d^2 + a*e^2)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

**Rubi [A]** time = 4.72655, antiderivative size = 698, normalized size of antiderivative = 1., number of

steps used = 14, number of rules used = 10, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$\begin{aligned}
 & - \frac{e\sqrt{a+cx^2}\sqrt{f+gx}}{(d+ex)(ae^2+cd^2)} + \frac{\sqrt{-a}\sqrt{c}f\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}}{\sqrt{a+cx^2}\sqrt{f+gx}(ae^2+cd^2)} F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right) \\
 & - \frac{\sqrt{-a}\sqrt{c}dg\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}}{e\sqrt{a+cx^2}\sqrt{f+gx}(ae^2+cd^2)} F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right) \\
 & - \frac{\sqrt{-a}\sqrt{c}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{a+cx^2}(ae^2+cd^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}} \\
 & - \frac{\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}(ae^2g+cd(2ef-dg))\left(\frac{2e}{\frac{\sqrt{c}d}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{-ag}}{\sqrt{c}f+\sqrt{-ag}}\right)}{e\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{c}d}{\sqrt{-a}}+e\right)(ae^2+cd^2)}
 \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[f + g\*x]/((d + e\*x)^2\*Sqrt[a + c\*x^2]), x]

[Out] -((e\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])/((c\*d^2 + a\*e^2)\*(d + e\*x))) - (Sqrt[-a]\*Sqrt[c]\*Sqrt[f + g\*x]\*Sqrt[1 + (c\*x^2)/a]\*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/((c\*d^2 + a\*e^2)\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[a + c\*x^2]) + (Sqrt[-a]\*Sqrt[c]\*f\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/((c\*d^2 + a\*e^2)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]) - (Sqrt[-a]\*Sqrt[c]\*d\*g\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(e\*(c\*d^2 + a\*e^2)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]) - ((a\*e^2\*g + c\*d\*(2\*e\*f - d\*g))\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticPi[(2\*e)/((Sqrt[c]\*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (2\*Sqrt[-a]\*g)/(Sqrt[c]\*f + Sqrt[-a]\*g)]/(e\*((Sqrt[c]\*d)/Sqrt[-a] + e)\*(c\*d^2 + a\*e^2)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{f+gx}}{\sqrt{a+cx^2}(d+ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*(1/2)/(e\*x+d)\*\*2/(c\*x\*\*2+a)\*\*(1/2), x)

[Out] Integral(sqrt(f + g\*x)/(sqrt(a + c\*x\*\*2)\*(d + e\*x)\*\*2), x)

**Mathematica [C]** time = 10.3557, size = 1330, normalized size = 1.91

$$\sqrt{f+gx} \left( -\frac{(cx^2+a)e^2}{d+ex} - \frac{-ce^2\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}f^3+2ce^2\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}(f+gx)f^2+cdeg\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}f^2-ce^2\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}(f+gx)^2f-2cdeg\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}(f+gx)f-2i}}{d+ex}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[f + g\*x]/((d + e\*x)^2\*Sqrt[a + c\*x^2]),x]

[Out] (Sqrt[f + g\*x]\*(-(e^2\*(a + c\*x^2))/(d + e\*x)) - ((c\*e^2\*f^3\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]] + c\*d\*e\*f^2\*g\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]] - a\*e^2\*f\*g^2\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]] + a\*d\*e\*g^3\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]] + 2\*c\*e^2\*f^2\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]\*(f + g\*x) - 2\*c\*d\*e\*f\*g\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]\*(f + g\*x) - c\*e^2\*f\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]\*(f + g\*x)^2 + c\*d\*e\*g\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]\*(f + g\*x)^2 + Sqrt[c]\*e\*((-I)\*Sqrt[c]\*f + Sqrt[a]\*g)\*(-(e\*f) + d\*g)\*Sqrt[(g\*((I\*Sqrt[a])/Sqrt[c] + x))/(f + g\*x)]\*Sqrt[-(((I\*Sqrt[a]\*g)/Sqrt[c] - g\*x)/(f + g\*x))]^(3/2)\*EllipticE[I\*ArcSinh[Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]/Sqrt[f + g\*x]], (Sqrt[c]\*f - I\*Sqrt[a]\*g)/(Sqrt[c]\*f + I\*Sqrt[a]\*g)] + e\*(I\*Sqrt[c]\*d + Sqrt[a]\*e)\*g\*(Sqrt[c]\*f + I\*Sqrt[a]\*g)\*Sqrt[(g\*((I\*Sqrt[a])/Sqrt[c] + x))/(f + g\*x)]\*Sqrt[-(((I\*Sqrt[a]\*g)/Sqrt[c] - g\*x)/(f + g\*x))]^(3/2)\*EllipticF[I\*ArcSinh[Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]/Sqrt[f + g\*x]], (Sqrt[c]\*f - I\*Sqrt[a]\*g)/(Sqrt[c]\*f + I\*Sqrt[a]\*g)] - (2\*I)\*c\*d\*e\*f\*g\*Sqrt[(g\*((I\*Sqrt[a])/Sqrt[c] + x))/(f + g\*x)]\*Sqrt[-(((I\*Sqrt[a]\*g)/Sqrt[c] - g\*x)/(f + g\*x))]^(3/2)\*EllipticPi[(Sqrt[c]\*(e\*f - d\*g))/(e\*(Sqrt[c]\*f + I\*Sqrt[a]\*g)), I\*ArcSinh[Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]/Sqrt[f + g\*x]], (Sqrt[c]\*f - I\*Sqrt[a]\*g)/(Sqrt[c]\*f + I\*Sqrt[a]\*g)] + I\*c\*d^2\*g^2\*Sqrt[(g\*((I\*Sqrt[a])/Sqrt[c] + x))/(f + g\*x)]\*Sqrt[-(((I\*Sqrt[a]\*g)/Sqrt[c] - g\*x)/(f + g\*x))]^(3/2)\*EllipticPi[(Sqrt[c]\*(e\*f - d\*g))/(e\*(Sqrt[c]\*f + I\*Sqrt[a]\*g)), I\*ArcSinh[Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]/Sqrt[f + g\*x]], (Sqrt[c]\*f - I\*Sqrt[a]\*g)/(Sqrt[c]\*f + I\*Sqrt[a]\*g)] - I\*a\*e^2\*g^2\*Sqrt[(g\*((I\*Sqrt[a])/Sqrt[c] + x))/(f + g\*x)]\*Sqrt[-(((I\*Sqrt[a]\*g)/Sqrt[c] - g\*x)/(f + g\*x))]^(3/2)\*EllipticPi[(Sqrt[c]\*(e\*f - d\*g))/(e\*(Sqrt[c]\*f + I\*Sqrt[a]\*g)), I\*ArcSinh[Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]/Sqrt[f + g\*x]], (Sqrt[c]\*f - I\*Sqrt[a]\*g)/(Sqrt[c]\*f + I\*Sqrt[a]\*g)]/(g\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]\*(e\*f - d\*g)\*(f + g\*x)))/((c\*d^2\*e + a\*e^3)\*Sqrt[a + c\*x^2])

**Maple [B]** time = 0.063, size = 5743, normalized size = 8.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^(1/2)/(e\*x+d)^2/(c\*x^2+a)^(1/2),x)

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+a}(ex+d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^2), x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^2), x)
```

---

**Fricas** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^2), x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**2/(c*x**2+a)**(1/2), x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx + f}}{\sqrt{cx^2 + a}(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^2), x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^2), x)
```

$$3.642 \quad \int \frac{\sqrt{f+gx}}{(d+ex)^3 \sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=1246

result too large to display

```
[Out] -(e*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(2*(c*d^2 + a*e^2)*(d + e*x)^2) - (e*(a*e^2*g + c*d*(6*e*f - 5*d*g))*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(4*(c*d^2 + a*e^2)^2*(e*f - d*g)*(d + e*x)) - (Sqrt[-a]*Sqrt[c]*(a*e^2*g + c*d*(6*e*f - 5*d*g))*Sqrt[f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)]/(4*(c*d^2 + a*e^2)^2*(e*f - d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (Sqrt[-a]*Sqrt[c]*g*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(2*e*(c*d^2 + a*e^2)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) + (Sqrt[-a]*Sqrt[c]*f*(a*e^2*g + c*d*(6*e*f - 5*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*(c*d^2 + a*e^2)^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) - (Sqrt[-a]*Sqrt[c]*d*g*(a*e^2*g + c*d*(6*e*f - 5*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)])/(4*e*(c*d^2 + a*e^2)^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) + (c*(e*f - 3*d*g)*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)]/(e*((Sqrt[c]*d)/Sqrt[-a] + e)*(c*d^2 + a*e^2)*Sqrt[f + g*x]*Sqrt[a + c*x^2]) + ((a*e^2*g + c*d*(6*e*f - 5*d*g))*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)]/(4*e*((Sqrt[c]*d)/Sqrt[-a] + e)*(c*d^2 + a*e^2)^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + c*x^2])
```

---

**Rubi [A]** time = 10.169, antiderivative size = 1246, normalized size of antiderivative = 1., number of

steps used = 23, number of rules used = 11, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.393$

$$\begin{aligned}
 & \frac{(age^2 + cd(6ef - 5dg)) \sqrt{f + gx} \sqrt{cx^2 + ae}}{4(cd^2 + ae^2)^2 (ef - dg)(d + ex)} - \frac{\sqrt{f + gx} \sqrt{cx^2 + ae}}{2(cd^2 + ae^2)(d + ex)^2} \\
 & - \frac{\sqrt{-a} \sqrt{c} (age^2 + cd(6ef - 5dg)) \sqrt{f + gx} \sqrt{\frac{cx^2}{a}} + 1E \left( \sin^{-1} \left( \frac{\sqrt{1 - \frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}} \right) \middle| -\frac{2ag}{\sqrt{-a} \sqrt{cf - ag}} \right)}{4(cd^2 + ae^2)^2 (ef - dg) \sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-ag}}}} \sqrt{cx^2 + a}} \\
 & + \frac{\sqrt{-a} \sqrt{c} f (age^2 + cd(6ef - 5dg)) \sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-ag}}}} \sqrt{\frac{cx^2}{a}} + 1F \left( \sin^{-1} \left( \frac{\sqrt{1 - \frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}} \right) \middle| -\frac{2ag}{\sqrt{-a} \sqrt{cf - ag}} \right)}{4(cd^2 + ae^2)^2 (ef - dg) \sqrt{f + gx} \sqrt{cx^2 + a}} \\
 & + \frac{\sqrt{-a} \sqrt{c} g \sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-ag}}}} \sqrt{\frac{cx^2}{a}} + 1F \left( \sin^{-1} \left( \frac{\sqrt{1 - \frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}} \right) \middle| -\frac{2ag}{\sqrt{-a} \sqrt{cf - ag}} \right)}{2(cd^2 + ae^2) \sqrt{f + gx} \sqrt{cx^2 + ae}} \\
 & - \frac{\sqrt{-a} \sqrt{c} dg (age^2 + cd(6ef - 5dg)) \sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-ag}}}} \sqrt{\frac{cx^2}{a}} + 1F \left( \sin^{-1} \left( \frac{\sqrt{1 - \frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}} \right) \middle| -\frac{2ag}{\sqrt{-a} \sqrt{cf - ag}} \right)}{4(cd^2 + ae^2)^2 (ef - dg) \sqrt{f + gx} \sqrt{cx^2 + ae}} \\
 & + \frac{c(ef - 3dg) \sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-ag}}}} \sqrt{\frac{cx^2}{a}} + 1 \left( \frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}} + e}; \sin^{-1} \left( \frac{\sqrt{1 - \frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}} \right) \middle| \frac{2\sqrt{-ag}}{\sqrt{cf+\sqrt{-ag}}} \right)}{(\frac{\sqrt{cd}}{\sqrt{-a}} + e) (cd^2 + ae^2) \sqrt{f + gx} \sqrt{cx^2 + ae}} \\
 & + \frac{(age^2 + cd(6ef - 5dg)) (ae^2 g - cd(2ef - 3dg)) \sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-ag}}}} \sqrt{\frac{cx^2}{a}} + 1 \left( \frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}} + e}; \sin^{-1} \left( \frac{\sqrt{1 - \frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}} \right) \middle| \frac{2\sqrt{-ag}}{\sqrt{cf+\sqrt{-ag}}} \right)}{4 \left( \frac{\sqrt{cd}}{\sqrt{-a}} + e \right) (cd^2 + ae^2)^2 (ef - dg) \sqrt{f + gx} \sqrt{cx^2 + ae}}
 \end{aligned}$$

Warning: Unable to verify antiderivative.

[In] Int[Sqrt[f + g\*x]/((d + e\*x)^3\*Sqrt[a + c\*x^2]),x]

[Out]  $-(e \sqrt{f + gx}) \sqrt{a + cx^2} / (2(c^2 d^2 + a^2 e^2)(d + ex)^2) - (e(a^2 e^2 g + c^2 d(6ef - 5dg)) \sqrt{f + gx} \sqrt{a + cx^2}) / (4(c^2 d^2 + a^2 e^2)^2 (ef - dg)(d + ex)) - (\sqrt{-a} \sqrt{c} (a^2 e^2 g + c^2 d(6ef - 5dg)) \sqrt{f + gx} \sqrt{1 + (cx^2)/a}) \text{EllipticE}[\text{ArcSin}[\sqrt{1 - (\sqrt{c}x)/\sqrt{-a}}]/\sqrt{2}], (-2ag)/(\sqrt{-a} \sqrt{c} f - ag)] / (4(c^2 d^2 + a^2 e^2)^2 (ef - dg) \sqrt{(\sqrt{c}x)(f + gx)} / (\sqrt{c}x f + \sqrt{-a}g)] \sqrt{a + cx^2} + (\sqrt{-a} \sqrt{c} g \sqrt{(\sqrt{c}x)(f + gx)} / (\sqrt{c}x f + \sqrt{-a}g)) \sqrt{1 + (cx^2)/a} \text{EllipticF}[\text{ArcSin}[\sqrt{1 - (\sqrt{c}x)/\sqrt{-a}}]/\sqrt{2}], (-2ag)/(\sqrt{-a} \sqrt{c} f - ag)] / (2e(c^2 d^2 + a^2 e^2) \sqrt{f + gx} \sqrt{a + cx^2}) + (\sqrt{-a} \sqrt{c} f (a^2 e^2 g + c^2 d(6ef - 5dg)) \sqrt{(\sqrt{c}x)(f + gx)} / (\sqrt{c}x f + \sqrt{-a}g)) \sqrt{1 + (cx^2)/a} \text{EllipticF}[\text{ArcSin}[\sqrt{1 - (\sqrt{c}x)/\sqrt{-a}}]/\sqrt{2}], (-2ag)/(\sqrt{-a} \sqrt{c} f - ag)] / (4(c^2 d^2 + a^2 e^2)^2 (ef - dg) \sqrt{f + gx} \sqrt{a + cx^2}) - (\sqrt{-a} \sqrt{c} dg (a^2 e^2 g + c^2 d(6ef - 5dg)) \sqrt{(\sqrt{c}x)(f + gx)} / (\sqrt{c}x f + \sqrt{-a}g)) \sqrt{1 + (cx^2)/a} \text{EllipticF}[\text{ArcSin}[\sqrt{1 - (\sqrt{c}x)/\sqrt{-a}}]/\sqrt{2}], (-2ag)/(\sqrt{-a} \sqrt{c} f - ag)] / (4e(c^2 d^2 + a^2 e^2)^2 (ef - dg) \sqrt{f + gx} \sqrt{a + cx^2}) + (c(ef - 3dg) \sqrt{(\sqrt{c}x)(f + gx)} / (\sqrt{c}x f + \sqrt{-a}g)) \sqrt{1 + (cx^2)/a} \text{EllipticPi}[(2e)/((\sqrt{c}d)/\sqrt{-a} + e), \text{ArcSin}[\sqrt{1 - (\sqrt{c}x)/\sqrt{-a}}]/\sqrt{2}], (2\sqrt{-ag})/(\sqrt{c}x f + \sqrt{-a}g)] / (e((\sqrt{c}d)/\sqrt{-a} + e)(c^2 d^2 + a^2 e^2) \sqrt{f + gx} \sqrt{a + cx^2}) + ((a^2 e^2 g + c^2 d(6ef - 5dg)) (a^2 e^2 g - c^2 d(2ef - 3dg)) \sqrt{(\sqrt{c}x)(f + gx)} / (\sqrt{c}x f + \sqrt{-a}g)) \sqrt{1 + (cx^2)/a} \text{EllipticPi}[(2e)/((\sqrt{c}d)/\sqrt{-a} + e), \text{ArcSin}[\sqrt{1 - (\sqrt{c}x)/\sqrt{-a}}]/\sqrt{2}], (2\sqrt{-a}$

$$\frac{] * g) / (\text{Sqrt}[c] * f + \text{Sqrt}[-a] * g)] / (4 * e * ((\text{Sqrt}[c] * d) / \text{Sqrt}[-a] + e) * (c * d^2 + a * e^2)^{1/2} * (e * f - d * g) * \text{Sqrt}[f + g * x] * \text{Sqrt}[a + c * x^2])}{}$$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{f + gx}}{\sqrt{a + cx^2} (d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x+f)**(1/2)/(e*x+d)**3/(c*x**2+a)**(1/2),x)`

[Out] `Integral(sqrt(f + g*x)/(sqrt(a + c*x**2)*(d + e*x)**3), x)`

**Mathematica [C]** time = 15.1412, size = 12363, normalized size = 9.92

Result too large to show

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[f + g*x]/((d + e*x)^3*Sqrt[a + c*x^2]),x]`

[Out] Result too large to show

**Maple [B]** time = 0.094, size = 20364, normalized size = 16.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+a)^(1/2),x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx + f}}{\sqrt{cx^2 + a} (ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^3),x, algorithm="maxima")`

[Out] `integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^3), x)`

**Ericas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^3), x, algorithm="fricas")`

[Out] Timed out

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)**(1/2)/(e*x+d)**3/(c*x**2+a)**(1/2), x)`

[Out] Timed out

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx + f}}{\sqrt{cx^2 + a}(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^3), x, algorithm="giac")`

[Out] `integrate(sqrt(g*x + f)/(sqrt(c*x^2 + a)*(e*x + d)^3), x)`

$$3.643 \quad \int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=600

$$\frac{2\sqrt{-ag}\sqrt{\frac{cx^2}{a}} + 1\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}} (ae^2g^2 + c(-3d^2g^2 + 6defg - 2e^2f^2)) F\left(\sin^{-1}\left(\frac{\sqrt{\frac{a\sqrt{cx}}{(-a)^{3/2}+1}}}{\sqrt{2}}\right) \middle| \frac{2ag}{ag-\sqrt{-a}\sqrt{cf}}\right)}{3c^{3/2}e^3\sqrt{a+cx^2}\sqrt{f+gx}} \\ - \frac{2(ef-dg)^2\sqrt{\frac{g(\sqrt{-a}-\sqrt{cx})}{\sqrt{-ag+\sqrt{cf}}}}\sqrt{\frac{g(\sqrt{-a}+\sqrt{cx})}{\sqrt{cf}-\sqrt{-ag}}}} \left(\frac{e\left(\frac{f+\sqrt{-ag}}{\sqrt{c}}\right)}{ef-dg}; \sin^{-1}\left(\sqrt{\frac{c}{cf+\sqrt{-a}\sqrt{cg}}}\sqrt{f+gx}\right) \middle| \frac{\sqrt{cf}+\sqrt{-ag}}{\sqrt{cf}-\sqrt{-ag}}\right)}{e^3\sqrt{a+cx^2}\sqrt{\frac{c}{\sqrt{-a}\sqrt{cg}+cf}}}} \\ - \frac{2\sqrt{-ag}\sqrt{\frac{cx^2}{a}} + 1\sqrt{f+gx}(7ef-3dg)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{a\sqrt{cx}}{(-a)^{3/2}+1}}}{\sqrt{2}}\right) \middle| \frac{2ag}{ag-\sqrt{-a}\sqrt{cf}}\right)}{3\sqrt{ce^2}\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}} + \frac{2g^2\sqrt{a+cx^2}\sqrt{f+gx}}{3ce}$$

[Out] (2\*g^2\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])/(3\*c\*e) - (2\*Sqrt[-a]\*g\*(7\*e\*f - 3\*d\*g)\*Sqrt[f + g\*x]\*Sqrt[1 + (c\*x^2)/a]\*EllipticE[ArcSin[Sqrt[1 + (a\*Sqrt[c]\*x)/(-a)^(3/2)]]/Sqrt[2]], (2\*a\*g)/(-(Sqrt[-a]\*Sqrt[c]\*f) + a\*g))/(3\*Sqrt[c]\*e^2\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[a + c\*x^2]) + (2\*Sqrt[-a]\*g\*(a\*e^2\*g^2 + c\*(-2\*e^2\*f^2 + 6\*d\*e\*f\*g - 3\*d^2\*g^2))\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 + (a\*Sqrt[c]\*x)/(-a)^(3/2)]]/Sqrt[2]], (2\*a\*g)/(-(Sqrt[-a]\*Sqrt[c]\*f) + a\*g))/(3\*c^(3/2)\*e^3\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]) - (2\*(e\*f - d\*g)^2\*Sqrt[(g\*(Sqrt[-a] - Sqrt[c]\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[-((g\*(Sqrt[-a] + Sqrt[c]\*x))/(Sqrt[c]\*f - Sqrt[-a]\*g))]\*EllipticPi[(e\*(f + (Sqrt[-a]\*g)/Sqrt[c]))/(e\*f - d\*g), ArcSin[Sqrt[c]/(c\*f + Sqrt[-a]\*Sqrt[c]\*g)]\*Sqrt[f + g\*x]], (Sqrt[c]\*f + Sqrt[-a]\*g)/(Sqrt[c]\*f - Sqrt[-a]\*g))/(e^3\*Sqrt[c]/(c\*f + Sqrt[-a]\*Sqrt[c]\*g))\*Sqrt[a + c\*x^2])

**Rubi [A]** time = 2.89956, antiderivative size = 808, normalized size of antiderivative = 1.35, number

of steps used = 16, number of rules used = 10, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$\begin{aligned}
 & \frac{2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-ag}}}}\sqrt{\frac{cx^2}{a}+1}\left(\frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}}+e}; \sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{-ag}}{\sqrt{cf+\sqrt{-ag}}}\right)(ef-dg)^3}{e^3\left(\frac{\sqrt{cd}}{\sqrt{-a}}+e\right)\sqrt{f+gx}\sqrt{cx^2+a}} \\
 & - \frac{2\sqrt{-ag}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-ag}}}}\sqrt{\frac{cx^2}{a}+1}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)(ef-dg)^2}{\sqrt{ce^3}\sqrt{f+gx}\sqrt{cx^2+a}} \\
 & - \frac{2\sqrt{-ag}\sqrt{f+gx}\sqrt{\frac{cx^2}{a}+1}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)(ef-dg)}{\sqrt{ce^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-ag}}}}\sqrt{cx^2+a}} \\
 & - \frac{8\sqrt{-a}fg\sqrt{f+gx}\sqrt{\frac{cx^2}{a}+1}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{3\sqrt{ce}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-ag}}}}\sqrt{cx^2+a}} \\
 & + \frac{2\sqrt{-ag}(cf^2+ag^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-ag}}}}\sqrt{\frac{cx^2}{a}+1}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{3c^{3/2}e\sqrt{f+gx}\sqrt{cx^2+a}} \\
 & + \frac{2g^2\sqrt{f+gx}\sqrt{cx^2+a}}{3ce}
 \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)^(5/2)/((d + e\*x)\*Sqrt[a + c\*x^2]),x]

[Out] (2\*g^2\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])/(3\*c\*e) - (8\*Sqrt[-a]\*f\*g\*Sqrt[f + g\*x]\*Sqrt[1 + (c\*x^2)/a]\*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(3\*Sqrt[c]\*e\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[a + c\*x^2]) - (2\*Sqrt[-a]\*g\*(e\*f - d\*g)\*Sqrt[f + g\*x]\*Sqrt[1 + (c\*x^2)/a]\*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(Sqrt[c]\*e^2\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[a + c\*x^2]) - (2\*Sqrt[-a]\*g\*(e\*f - d\*g)^2\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(Sqrt[c]\*e^3\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]) + (2\*Sqrt[-a]\*g\*(c\*f^2 + a\*g^2)\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(3\*c^(3/2)\*e\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]) - (2\*(e\*f - d\*g)^3\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticPi[(2\*e)/((Sqrt[c]\*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (2\*Sqrt[-a]\*g)/(Sqrt[c]\*f + Sqrt[-a]\*g)]/(e^3\*((Sqrt[c]\*d)/Sqrt[-a] + e)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*(5/2)/(e\*x+d)/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] Timed out

**Mathematica [C]** time = 13.7764, size = 1440, normalized size = 2.4

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+ag^2}}{3ce} + \frac{2(f+gx)^{3/2} \left( \frac{7ce^2 \sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}}{(f+gx)^2} f^3 + \frac{3ice^2 \sqrt{-\frac{f}{f+gx} - \frac{i\sqrt{ag}}{\sqrt{c}(f+gx)}} + 1} \sqrt{-\frac{f}{f+gx} + \frac{i\sqrt{ag}}{\sqrt{c}(f+gx)}} + 1 \right) \frac{\sqrt{c}(ef-dg)}{e(\sqrt{cf+i\sqrt{ag}})}; i \sinh^{-1} \left( \frac{\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}}{\sqrt{f+gx}} \right) \sqrt{\frac{\sqrt{cf-i\sqrt{ag}}}{\sqrt{cf+i\sqrt{ag}}}} f^2 - \frac{14ce^2 \sqrt{-f}}{f+g}}{\sqrt{f+gx}} - \frac{14ce^2 \sqrt{-f}}{f+g}}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^(5/2)/((d + e\*x)\*Sqrt[a + c\*x^2]),x]

[Out]  $(2g^2\sqrt{f+gx}\sqrt{a+cx^2})/(3ce) + (2(f+gx)^{3/2}) * (7c^2e^2f\sqrt{-f-(I\sqrt{a}g)/\sqrt{c}} - 3cd^2e^2g\sqrt{-f-(I\sqrt{a}g)/\sqrt{c}})/\sqrt{c} + (7c^2e^2f^3\sqrt{-f-(I\sqrt{a}g)/\sqrt{c}})/\sqrt{c}]/(f+gx)^2 - (3cd^2e^2f^2g\sqrt{-f-(I\sqrt{a}g)/\sqrt{c}})/\sqrt{c}]/(f+gx)^2 + (7a^2e^2f^2g^2\sqrt{-f-(I\sqrt{a}g)/\sqrt{c}})/\sqrt{c}]/(f+gx)^2 - (3ad^2e^2g^3\sqrt{-f-(I\sqrt{a}g)/\sqrt{c}})/\sqrt{c}]/(f+gx)^2 - (14c^2e^2f^2\sqrt{-f-(I\sqrt{a}g)/\sqrt{c}})/\sqrt{c}]/(f+gx) + (6cd^2e^2f^2g\sqrt{-f-(I\sqrt{a}g)/\sqrt{c}})/\sqrt{c}]/(f+gx) + (\sqrt{c}e^2((-I)\sqrt{c}f + \sqrt{a}g)^2(7ef - 3d^2g)\sqrt{1-f/(f+gx) - (I\sqrt{a}g)/(\sqrt{c}(f+gx))})\sqrt{1-f/(f+gx) + (I\sqrt{a}g)/(\sqrt{c}(f+gx))} \text{EllipticE}[I\text{ArcSin}[\sqrt{-f-(I\sqrt{a}g)/\sqrt{c}}]/\sqrt{f+gx}], (\sqrt{c}f - I\sqrt{a}g)/(\sqrt{c}f + I\sqrt{a}g))/\sqrt{f+gx} + (Ie^2(\sqrt{c}f + I\sqrt{a}g)^2(I\sqrt{a}g + \sqrt{c}(6ef - 3d^2g))\sqrt{1-f/(f+gx) - (I\sqrt{a}g)/(\sqrt{c}(f+gx))})\sqrt{1-f/(f+gx) + (I\sqrt{a}g)/(\sqrt{c}(f+gx))} \text{EllipticF}[I\text{ArcSinh}[\sqrt{-f-(I\sqrt{a}g)/\sqrt{c}}]/\sqrt{f+gx}], (\sqrt{c}f - I\sqrt{a}g)/(\sqrt{c}f + I\sqrt{a}g))/\sqrt{f+gx} + ((3I)^2c^2e^2f^2\sqrt{1-f/(f+gx) - (I\sqrt{a}g)/(\sqrt{c}(f+gx))})\sqrt{1-f/(f+gx) + (I\sqrt{a}g)/(\sqrt{c}(f+gx))} \text{EllipticPi}[(\sqrt{c}(ef - d^2g))/(e(\sqrt{c}f + I\sqrt{a}g))], I\text{ArcSinh}[\sqrt{-f-(I\sqrt{a}g)/\sqrt{c}}]/\sqrt{f+gx}], (\sqrt{c}f - I\sqrt{a}g)/(\sqrt{c}f + I\sqrt{a}g))/\sqrt{f+gx} - ((6I)^2c^2d^2e^2f^2g\sqrt{1-f/(f+gx) - (I\sqrt{a}g)/(\sqrt{c}(f+gx))})\sqrt{1-f/(f+gx) + (I\sqrt{a}g)/(\sqrt{c}(f+gx))} \text{EllipticPi}[(\sqrt{c}(ef - d^2g))/(e(\sqrt{c}f + I\sqrt{a}g))], I\text{ArcSinh}[\sqrt{-f-(I\sqrt{a}g)/\sqrt{c}}]/\sqrt{f+gx}], (\sqrt{c}f - I\sqrt{a}g)/(\sqrt{c}f + I\sqrt{a}g))/\sqrt{f+gx} + ((3I)^2c^2d^2g^2\sqrt{1-f/(f+gx) - (I\sqrt{a}g)/(\sqrt{c}(f+gx))})\sqrt{1-f/(f+gx) + (I\sqrt{a}g)/(\sqrt{c}(f+gx))} \text{EllipticPi}[(\sqrt{c}(ef - d^2g))/(e(\sqrt{c}f + I\sqrt{a}g))], I\text{ArcSinh}[\sqrt{-f-(I\sqrt{a}g)/\sqrt{c}}]/\sqrt{f+gx}], (\sqrt{c}f - I\sqrt{a}g)/(\sqrt{c}f + I\sqrt{a}g))/\sqrt{f+gx}]/(3c^2e^3\sqrt{-f-(I\sqrt{a}g)/\sqrt{c}})\sqrt{a+(c(f+gx))^2}(-1+f/(f+gx))^2/g^2]$

**Maple [B]** time = 0.057, size = 3164, normalized size = 5.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^(5/2)/(e\*x+d)/(c\*x^2+a)^(1/2),x)

[Out] 
$$\begin{aligned}
& -2/3*(g^*x+f)^{(1/2)}*(c^*x^2+a)^{(1/2)}*(3*(-(g^*x+f)*c/(g^*(-a^*c)^{(1/2)} \\
& -c^*f))^{(1/2)}*((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*( \\
& (c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}*EllipticF((- (g^*x \\
& +f)*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)},(-(g^*(-a^*c)^{(1/2)}-c^*f)/(g^*(-a^*c \\
& )^{(1/2)}+c^*f))^{(1/2)})^*a^*c*d^*e^*g^3-6*(-(g^*x+f)*c/(g^*(-a^*c)^{(1/2)}-c^* \\
& f))^{(1/2)}*((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*((c^* \\
& x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}*EllipticF((- (g^*x+f) \\
& *c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)},(-(g^*(-a^*c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{( \\
& 1/2)}+c^*f))^{(1/2)})^*a^*c^*e^2*f^*g^2-(-(g^*x+f)*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\
& )^{(1/2)}*((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*((c^*x+ \\
& -a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}*EllipticF((- (g^*x+f)*c/ \\
& (g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)},(-(g^*(-a^*c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1/2) \\
& )+c^*f))^{(1/2)})^*(-a^*c)^{(1/2)}*a^*e^2*g^3-3*(-(g^*x+f)*c/(g^*(-a^*c)^{(1/2) \\
& )-c^*f))^{(1/2)}*((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)} \\
& *((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}*EllipticF((- (g^* \\
& x+f)*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)},(-(g^*(-a^*c)^{(1/2)}-c^*f)/(g^*(-a \\
& *c)^{(1/2)}+c^*f))^{(1/2)})^*c^2*d^2*f^*g^2+9*(-(g^*x+f)*c/(g^*(-a^*c)^{(1/2) \\
& )-c^*f))^{(1/2)}*((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}* \\
& ((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}*EllipticF((- (g^* \\
& x+f)*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)},(-(g^*(-a^*c)^{(1/2)}-c^*f)/(g^*(-a^* \\
& c)^{(1/2)}+c^*f))^{(1/2)})^*c^2*d^*e^*f^2*g-9*(-(c^*x+(-a^*c)^{(1/2)})^*g/(g^*( \\
& -a^*c)^{(1/2)}+c^*f))^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f \\
& ))^{(1/2)}*EllipticF((- (g^*x+f)*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)},(-(g^*( \\
& -a^*c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)})^*(-(g^*x+f)*c/(g^*(-a^* \\
& c)^{(1/2)}-c^*f))^{(1/2)}*c^2*e^2*f^3+3*(-(g^*x+f)*c/(g^*(-a^*c)^{(1/2)}-c^* \\
& f))^{(1/2)}*((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*((c^* \\
& x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}*EllipticF((- (g^*x+f) \\
& *c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)},(-(g^*(-a^*c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{( \\
& 1/2)}+c^*f))^{(1/2)})^*(-a^*c)^{(1/2)}*c*d^2*g^3-6*((-c^*x+(-a^*c)^{(1/2)})^*g \\
& / (g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2) \\
& )-c^*f))^{(1/2)}*EllipticF((- (g^*x+f)*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)},( \\
& -(g^*(-a^*c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)})^*(-(g^*x+f)*c/(g \\
& *(-a^*c)^{(1/2)}-c^*f))^{(1/2)}*(-a^*c)^{(1/2)}*c*d^*e^*f^*g^2+2*(-(g^*x+f)*c/ \\
& (g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}*((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2) \\
& )+c^*f))^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}*E \\
& llipticF((- (g^*x+f)*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)},(-(g^*(-a^*c)^{(1/2) \\
& )-c^*f)/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)})^*(-a^*c)^{(1/2)}*c^*e^2*f^2*g-3*(- \\
& (g^*x+f)*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}*((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*( \\
& -a^*c)^{(1/2)}+c^*f))^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f \\
& ))^{(1/2)}*EllipticE((- (g^*x+f)*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)},(-(g^*( \\
& -a^*c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)})^*a^*c*d^*e^*g^3+7*(-(g^* \\
& x+f)*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}*((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^* \\
& c)^{(1/2)}+c^*f))^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f))^{( \\
& 1/2)}*EllipticE((- (g^*x+f)*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)},(-(g^*(-a^* \\
& c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)})^*a^*c^*e^2*f^*g^2-3*(-(g^*x \\
& +f)*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}*((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c \\
& )^{(1/2)}+c^*f))^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f))^{( \\
& 1/2)}*EllipticE((- (g^*x+f)*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)},(-(g^*(-a^*c \\
& )^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)})^*c^2*d^*e^*f^2*g+7*(-(g^*x+ \\
& f)*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}*((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c \\
& )^{(1/2)}+c^*f))^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1 \\
& /2)}*EllipticE((- (g^*x+f)*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)},(-(g^*(-a^*c \\
& )^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)})^*c^2*e^2*f^3+3*(-(g^*x+f)* \\
& c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}*((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1 \\
& /2)}+c^*f))^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2) \\
& }*EllipticPi((- (g^*x+f)*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)},(g^*(-a^*c)^{(1/2) \\
& )-c^*f)*e/c/(d^*g-e^*f),(-(g^*(-a^*c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1/2)}+c^*f) \\
& )^{(1/2)}*c^2*d^2*f^*g^2-6*((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c \\
& *f))^{(1/2)}*((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}*Elli \\
& pticPi((- (g^*x+f)*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)},(g^*(-a^*c)^{(1/2)}-c^* \\
& f)*e/c/(d^*g-e^*f),(-(g^*(-a^*c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/ \\
& 2)})^*(-(g^*x+f)*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}*c^2*d^*e^*f^2*g+3*((-c^* \\
& x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*((c^*x+(-a^*c)^{(1/2)}) \\
& *g/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}*EllipticPi((- (g^*x+f)*c/(g^*(-a^*c)^{( \\
& 1/2)}-c^*f))^{(1/2)},(g^*(-a^*c)^{(1/2)}-c^*f)*e/c/(d^*g-e^*f),(-(g^*(-a^*c)^{( \\
& 1/2)}-c^*f)/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)})^*(-(g^*x+f)*c/(g^*(-a^*c)^{(1/2) \\
& )-c^*f))^{(1/2)}*c^2*e^2*f^3-3*(-(g^*x+f)*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/ \\
& 2)}*((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*((c^*x+(-a^*c \\
& )^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}*EllipticPi((- (g^*x+f)*c/(g^* \\
& (-a^*c)^{(1/2)}-c^*f))^{(1/2)},(g^*(-a^*c)^{(1/2)}-c^*f)*e/c/(d^*g-e^*f),(-(g^* \\
& (-a^*c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)})^*(-a^*c)^{(1/2)}*c*d^2 \\
& *g^3+6*((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)}*((c^*x+(
\end{aligned}$$

$$-a^*c)^{(1/2)} * g / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * \text{EllipticPi}((-g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)}, (g^*(-a^*c)^{(1/2)} - c^*f)^*e/c/(d^*g - e^*f), (-g^*(-a^*c)^{(1/2)} - c^*f) / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} * (-g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * (-a^*c)^{(1/2)} * c^*d^*e^*f * g^2 - 3 * ((-c^*x + (-a^*c)^{(1/2)})^2)^{(1/2)} * g / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} * ((c^*x + (-a^*c)^{(1/2)})^2)^{(1/2)} * g / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * \text{EllipticPi}((-g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)}, (g^*(-a^*c)^{(1/2)} - c^*f)^*e/c/(d^*g - e^*f), (-g^*(-a^*c)^{(1/2)} - c^*f) / (g^*(-a^*c)^{(1/2)} + c^*f))^{(1/2)} * (-g^*x+f)^*c / (g^*(-a^*c)^{(1/2)} - c^*f))^{(1/2)} * (-a^*c)^{(1/2)} * c^*e^2 * f^2 * g - x^3 * c^2 * e^2 * g^3 - x^2 * c^2 * e^2 * f * g^2 - x * a^*c^*e^2 * g^3 - a^*c^*e^2 * f * g^2) / e^3 / c^2 / (c^*g^*x^3 + c^*f^*x^2 + a^*g^*x + a^*f)$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx + f)^{\frac{5}{2}}}{\sqrt{cx^2 + a}(ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^(5/2)/(sqrt(c\*x^2 + a)\*(e\*x + d)), x, algorithm="maxima")

[Out] integrate((g\*x + f)^(5/2)/(sqrt(c\*x^2 + a)\*(e\*x + d)), x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^(5/2)/(sqrt(c\*x^2 + a)\*(e\*x + d)), x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*(5/2)/(e\*x+d)/(c\*x\*\*2+a)\*\*(1/2), x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx + f)^{\frac{5}{2}}}{\sqrt{cx^2 + a}(ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^(5/2)/(sqrt(c\*x^2 + a)\*(e\*x + d)), x, algorithm="giac")

[Out] integrate((g\*x + f)^(5/2)/(sqrt(c\*x^2 + a)\*(e\*x + d)), x)

$$3.644 \quad \int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=469

$$\frac{2\sqrt{-ag}\sqrt{\frac{cx^2}{a}+1}(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{\sqrt{ce^2\sqrt{a+cx^2}}\sqrt{f+gx}}$$

$$\frac{2\sqrt{\frac{cx^2}{a}+1}(ef-dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}\left(\frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{-ag}}{\sqrt{cf}+\sqrt{-ag}}\right)}{e^2\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{cd}}{\sqrt{-a}}+e\right)}$$

$$\frac{2\sqrt{-ag}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{\sqrt{ce}\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}}$$

[Out]  $(-2*\text{Sqrt}[-a]*g*\text{Sqrt}[f + g*x]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)])/(\text{Sqrt}[c]*e*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[a + c*x^2]) - (2*\text{Sqrt}[-a]*g*(e*f - d*g)*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)])/(\text{Sqrt}[c]*e^2*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2]) - (2*(e*f - d*g)^2*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticPi}[(2*e)/((\text{Sqrt}[c]*d)/\text{Sqrt}[-a] + e), \text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (2*\text{Sqrt}[-a]*g)/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)])/(\text{Sqrt}[c]*d)/\text{Sqrt}[-a] + e)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])$

**Rubi [A]** time = 2.14008, antiderivative size = 469, normalized size of antiderivative = 1., number of rules used = 10, number of rules used = 8, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$

$$\frac{2\sqrt{-ag}\sqrt{\frac{cx^2}{a}+1}(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{\sqrt{ce^2\sqrt{a+cx^2}}\sqrt{f+gx}}$$

$$\frac{2\sqrt{\frac{cx^2}{a}+1}(ef-dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}\left(\frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{-ag}}{\sqrt{cf}+\sqrt{-ag}}\right)}{e^2\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{cd}}{\sqrt{-a}}+e\right)}$$

$$\frac{2\sqrt{-ag}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{\sqrt{ce}\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(f + g*x)^{(3/2)} / ((d + e*x)*\text{Sqrt}[a + c*x^2]), x]$

[Out]  $(-2*\text{Sqrt}[-a]*g*\text{Sqrt}[f + g*x]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)])/(\text{Sqrt}[c]*e*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[a + c*x^2]) - (2*\text{Sqrt}[-a]*g*(e*f - d*g)*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)])/(\text{Sqrt}[c]*e^2*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2]) - (2*(e*f - d*g)^2*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticPi}[(2*e)/((\text{Sqrt}[c]*d)/\text{Sqrt}[-a] + e), \text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (2*\text{Sqrt}[-a]*g)/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)])/(\text{Sqrt}[c]*d)/\text{Sqrt}[-a] + e)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])$

lipticF[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(Sqrt[c]\*e^2\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]) - (2\*(e\*f - d\*g)^2\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticPi[(2\*e)/((Sqrt[c]\*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (2\*Sqrt[-a]\*g)/(Sqrt[c]\*f + Sqrt[-a]\*g)]/(e^2\*((Sqrt[c]\*d)/Sqrt[-a] + e)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])

**Rubi in Sympy [A]** time = 150.141, size = 478, normalized size = 1.02

$$2\sqrt{\frac{g(-\sqrt{cx}-\sqrt{-a})}{\sqrt{cf}-g\sqrt{-a}}}\sqrt{\frac{g(-\sqrt{cx}+\sqrt{-a})}{\sqrt{cf}+g\sqrt{-a}}}(dg-ef)\left(-\frac{e(\sqrt{cf}+g\sqrt{-a})}{\sqrt{c}(dg-ef)};\operatorname{asin}\left(\sqrt{\frac{c}{\sqrt{cg}\sqrt{-a}+cf}}\sqrt{f+gx}\right)\left|\frac{\sqrt{cf}+g\sqrt{-a}}{\sqrt{cf}-g\sqrt{-a}}\right.\right)$$

$$\frac{e^2\sqrt{\frac{c}{\sqrt{cg}\sqrt{-a}+cf}}\sqrt{a+cx^2}}{2g\sqrt{-a}\sqrt{1+\frac{cx^2}{a}}\sqrt{f+gx}E\left(\operatorname{asin}\left(\sqrt{-\frac{\sqrt{cx}}{2\sqrt{-a}}+\frac{1}{2}}\right)\left|\frac{2ag}{ag-\sqrt{cf}\sqrt{-a}}\right.\right)}$$

$$\frac{\sqrt{ce}\sqrt{\frac{\sqrt{c}\sqrt{-a}(-f-gx)}{ag-\sqrt{cf}\sqrt{-a}}}\sqrt{a+cx^2}}{2g\sqrt{-a}\sqrt{\frac{\sqrt{c}\sqrt{-a}(-f-gx)}{ag-\sqrt{cf}\sqrt{-a}}}\sqrt{1+\frac{cx^2}{a}}(dg-ef)F\left(\operatorname{asin}\left(\sqrt{-\frac{\sqrt{cx}}{2\sqrt{-a}}+\frac{1}{2}}\right)\left|\frac{2ag}{ag-\sqrt{cf}\sqrt{-a}}\right.\right)}$$

$$+\frac{\sqrt{ce^2}\sqrt{a+cx^2}\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*(3/2)/(e\*x+d)/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] 2\*sqrt(g\*(-sqrt(c)\*x - sqrt(-a))/(sqrt(c)\*f - g\*sqrt(-a)))\*sqrt(g\*(-sqrt(c)\*x + sqrt(-a))/(sqrt(c)\*f + g\*sqrt(-a)))\*(d\*g - e\*f)\*elliptic\_pi(-e\*(sqrt(c)\*f + g\*sqrt(-a))/(sqrt(c)\*(d\*g - e\*f)), asin(sqrt(c)/(sqrt(c)\*g\*sqrt(-a) + c\*f))\*sqrt(f + g\*x), (sqrt(c)\*f + g\*sqrt(-a))/(sqrt(c)\*f - g\*sqrt(-a)))/(e\*\*2\*sqrt(c)/(sqrt(c)\*g\*sqrt(-a) + c\*f))\*sqrt(a + c\*x\*\*2)) - 2\*g\*sqrt(-a)\*sqrt(1 + c\*x\*\*2/a)\*sqrt(f + g\*x)\*elliptic\_e(asin(sqrt(-sqrt(c)\*x/(2\*sqrt(-a)) + 1/2)), 2\*a\*g/(a\*g - sqrt(c)\*f\*sqrt(-a)))/(sqrt(c)\*e\*sqrt(sqrt(c)\*sqrt(-a)\*(-f - g\*x)/(a\*g - sqrt(c)\*f\*sqrt(-a)))\*sqrt(a + c\*x\*\*2)) + 2\*g\*sqrt(-a)\*sqrt(sqrt(c)\*sqrt(-a)\*(-f - g\*x)/(a\*g - sqrt(c)\*f\*sqrt(-a)))\*sqrt(1 + c\*x\*\*2/a)\*(d\*g - e\*f)\*elliptic\_f(asin(sqrt(-sqrt(c)\*x/(2\*sqrt(-a)) + 1/2)), 2\*a\*g/(a\*g - sqrt(c)\*f\*sqrt(-a)))/(sqrt(c)\*e\*\*2\*sqrt(a + c\*x\*\*2)\*sqrt(f + g\*x))

**Mathematica [C]** time = 1.68562, size = 927, normalized size = 1.98

$$2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf}-i\sqrt{ag}}}\left(\frac{\sqrt{a}\sqrt{\frac{cx^2}{a}+1}\left(\frac{2\sqrt{ae}}{i\sqrt{cd}+\sqrt{ae}};\sin^{-1}\left(\frac{\sqrt{1-\frac{i\sqrt{cx}}{\sqrt{a}}}}{\sqrt{2}}\right)\left|\frac{2\sqrt{ag}}{i\sqrt{cf}+\sqrt{ag}}\right.\right)f^2}{i\sqrt{cd}+\sqrt{ae}}+\frac{2i\sqrt{ag}\sqrt{\frac{cx^2}{a}+1}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{i\sqrt{cx}}{\sqrt{a}}}}{\sqrt{2}}\right)\left|\frac{2\sqrt{ag}}{i\sqrt{cf}+\sqrt{ag}}\right.\right)f}{\sqrt{ce}}+\frac{2\sqrt{adg}\sqrt{\frac{cx^2}{a}+1}\left(\frac{2\sqrt{ae}}{i\sqrt{cd}+\sqrt{ae}};\sin^{-1}\left(\frac{\sqrt{1-\frac{i\sqrt{cx}}{\sqrt{a}}}}{\sqrt{2}}\right)\left|\frac{2\sqrt{ag}}{i\sqrt{cf}+\sqrt{ag}}\right.\right)f}{\sqrt{ae}}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^(3/2)/((d + e\*x)\*Sqrt[a + c\*x^2]),x]

[Out] (2\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f - I\*Sqrt[a]\*g)]\*(((2\*I)\*Sqrt[a]\*f\*g\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 - (I\*Sqrt[c]\*x)/Sqrt[a]]/Sqrt[2]], (2\*Sqrt[a]\*g)/(I\*Sqrt[c]\*f + Sqrt[a]\*g)])/(Sqrt[c]\*e) - (I\*Sqrt[a]\*d\*g^2\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 - (I\*Sqrt[c]\*x)/Sqrt[a]]/Sqrt[2]], (2\*Sqrt[a]\*g)/(I\*Sqrt[c]\*f + Sqrt[a]\*g)])/(Sqrt[c]\*e^2) + (g\*Sqrt[(g\*(Sqrt[a] + I\*Sqrt[a]\*g)/Sqrt[c])]/Sqrt[a])



$$\begin{aligned} & \text{rt}[c]*x)/((-I)*\text{Sqrt}[c]*f + \text{Sqrt}[a]*g)]*(I*\text{Sqrt}[a] + \text{Sqrt}[c]*x)* \\ & ((\text{Sqrt}[c]*f + I*\text{Sqrt}[a]*g)*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x) \\ & )/(\text{Sqrt}[c]*f - I*\text{Sqrt}[a]*g)]], (\text{Sqrt}[c]*f - I*\text{Sqrt}[a]*g)/(\text{Sqrt}[c] \\ & ]*f + I*\text{Sqrt}[a]*g)] - I*\text{Sqrt}[a]*g*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(\text{Sqrt}[c]* \\ & (f + g*x))/(\text{Sqrt}[c]*f - I*\text{Sqrt}[a]*g)]], (\text{Sqrt}[c]*f - I*\text{Sqrt}[a]*g) \\ & )/(\text{Sqrt}[c]*f + I*\text{Sqrt}[a]*g)))/(c*e*\text{Sqrt}[(g*(\text{Sqrt}[a] - I*\text{Sqrt}[c]*x \\ & ))/(I*\text{Sqrt}[c]*f + \text{Sqrt}[a]*g))] - (\text{Sqrt}[a]*f^2*\text{Sqrt}[1 + (c*x^2)/a] \\ & )*\text{EllipticPi}[(2*\text{Sqrt}[a]*e)/(I*\text{Sqrt}[c]*d + \text{Sqrt}[a]*e), \text{ArcSin}[\text{Sqrt}[ \\ & 1 - (I*\text{Sqrt}[c]*x)/\text{Sqrt}[a]]/\text{Sqrt}[2]], (2*\text{Sqrt}[a]*g)/(I*\text{Sqrt}[c]*f + \\ & \text{Sqrt}[a]*g)))/(I*\text{Sqrt}[c]*d + \text{Sqrt}[a]*e) + (2*\text{Sqrt}[a]*d*f*g*\text{Sqrt}[1 \\ & + (c*x^2)/a]*\text{EllipticPi}[(2*\text{Sqrt}[a]*e)/(I*\text{Sqrt}[c]*d + \text{Sqrt}[a]*e), \\ & \text{ArcSin}[\text{Sqrt}[1 - (I*\text{Sqrt}[c]*x)/\text{Sqrt}[a]]/\text{Sqrt}[2]], (2*\text{Sqrt}[a]*g)/( \\ & I*\text{Sqrt}[c]*f + \text{Sqrt}[a]*g)))/(I*\text{Sqrt}[c]*d*e + \text{Sqrt}[a]*e^2) - (\text{Sqrt}[ \\ & a]*d^2*g^2*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticPi}[(2*\text{Sqrt}[a]*e)/(I*\text{Sqrt}[c] \\ & ]*d + \text{Sqrt}[a]*e), \text{ArcSin}[\text{Sqrt}[1 - (I*\text{Sqrt}[c]*x)/\text{Sqrt}[a]]/\text{Sqrt}[2]] \\ & , (2*\text{Sqrt}[a]*g)/(I*\text{Sqrt}[c]*f + \text{Sqrt}[a]*g)))/(e^2*(I*\text{Sqrt}[c]*d + \text{S} \\ & \text{qrt}[a]*e)))/(\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2]) \end{aligned}$$

**Maple [B]** time = 0.052, size = 959, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((g*x+f)^{(3/2)}/(e*x+d)/(c*x^2+a)^{(1/2)}, x)$

[Out] 
$$\begin{aligned} & 2*(g*x+f)^{(1/2)}*(c*x^2+a)^{(1/2)}*(-(g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f)) \\ & ^{(1/2)}*((-c*x+(-a*c)^{(1/2)})*g/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}*((c*x+ \\ & -a*c)^{(1/2)})*g/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}/c*(\text{EllipticF}((-(g*x+f) \\ & )*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (-(g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)} \\ & +c*f))^{(1/2)})*(-a*c)^{(1/2)}*d*g^2-\text{EllipticF}((-(g*x+f)*c/(g*(-a \\ & )*c)^{(1/2)}-c*f))^{(1/2)}, (-(g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f) \\ & )^{(1/2)})*(-a*c)^{(1/2)}*e*f*g+\text{EllipticF}((-(g*x+f)*c/(g*(-a*c)^{(1/2)} \\ & -c*f))^{(1/2)}, (-(g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}) \\ & )*a*e*g^2-\text{EllipticF}((-(g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (-(g*(- \\ & )*a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)})*c*d*f*g+2*\text{EllipticF} \\ & ((-(g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (-(g*(-a*c)^{(1/2)}-c*f)/(g \\ & )*(-a*c)^{(1/2)}+c*f))^{(1/2)})*c*e*f^2-\text{EllipticE}((-(g*x+f)*c/(g*(-a*c) \\ & )^{(1/2)}-c*f))^{(1/2)}, (-(g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}) \\ & )*a*e*g^2-\text{EllipticE}((-(g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, \\ & (-(g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)})*c*e*f^2-\text{Ellip} \\ & \text{ticPi}((-(g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (g*(-a*c)^{(1/2)}-c*f \\ & )*e/c/(d*g-e*f), (-(g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} \\ & )*(-a*c)^{(1/2)}*d*g^2+\text{EllipticPi}((-(g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f) \\ & )^{(1/2)}, (g*(-a*c)^{(1/2)}-c*f)*e/c/(d*g-e*f), (-(g*(-a*c)^{(1/2)}-c*f) \\ & )/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)})*(-a*c)^{(1/2)}*e*f*g+\text{EllipticPi}((-(g* \\ & )*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (g*(-a*c)^{(1/2)}-c*f)*e/c/(d*g- \\ & )*e*f), (-(g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)})*c*d*f*g- \\ & \text{EllipticPi}((-(g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (g*(-a*c)^{(1/2)} \\ & )-c*f)*e/c/(d*g-e*f), (-(g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f)) \\ & )^{(1/2)})*c*e*f^2)/e^2/(c*g*x^3+c*f*x^2+a*g*x+a*f) \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx + f)^{\frac{3}{2}}}{\sqrt{cx^2 + a}(ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((g*x + f)^{(3/2)}/(\text{sqrt}(c*x^2 + a)*(e*x + d)), x, \text{algorithm}=\text{"maxima"})$

[Out]  $\text{integrate}((g*x + f)^{(3/2)}/(\text{sqrt}(c*x^2 + a)*(e*x + d)), x)$

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x + f)^(3/2)/(sqrt(c*x^2 + a)*(e*x + d)),x, algorithm="fricas")`

[Out] Timed out

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(f + gx)^{\frac{3}{2}}}{\sqrt{a + cx^2}(d + ex)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)**(3/2)/(e*x+d)/(c*x**2+a)**(1/2),x)`

[Out] `Integral((f + g*x)**(3/2)/(sqrt(a + c*x**2)*(d + e*x)), x)`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx + f)^{\frac{3}{2}}}{\sqrt{cx^2 + a}(ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x + f)^(3/2)/(sqrt(c*x^2 + a)*(e*x + d)),x, algorithm="giac")`

[Out] `integrate((g*x + f)^(3/2)/(sqrt(c*x^2 + a)*(e*x + d)), x)`

$$3.645 \quad \int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=457

$$\frac{2\sqrt{-a}e\sqrt{\frac{cx^2}{a}} + 1\sqrt{f+gx} (9ae^2g^2 - c(45d^2g^2 - 30defg + 8e^2f^2)) E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{15c^{3/2}g^3\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}}$$

$$- \frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}} + 1\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}} (ae^2g^2(7ef - 15dg) - c(-15d^3g^3 + 45d^2efg^2 - 30de^2f^2g + 8e^3f^3)) F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{15c^{3/2}g^3\sqrt{a+cx^2}\sqrt{f+gx}}$$

$$- \frac{8e^2\sqrt{a+cx^2}\sqrt{f+gx}(ef - 3dg)}{15cg^2} + \frac{2e^2\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5cg}$$

[Out]  $(-8*e^2*(e*f - 3*d*g)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])/(15*c*g^2) + (2*e^2*(d + e*x)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])/(5*c*g) + (2*\text{Sqrt}[-a]*e*(9*a*e^2*g^2 - c*(8*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2))*\text{Sqrt}[f + g*x]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)))/(15*c^{3/2}*g^3*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[a + c*x^2]) - (2*\text{Sqrt}[-a]*(a*e^2*g^2*(7*e*f - 15*d*g) - c*(8*e^3*f^3 - 30*d*e^2*f^2*g + 45*d^2*e*f*g^2 - 15*d^3*g^3))*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)))/(15*c^{3/2}*g^3*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])$

**Rubi [A]** time = 1.40465, antiderivative size = 457, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$\frac{2\sqrt{-a}e\sqrt{\frac{cx^2}{a}} + 1\sqrt{f+gx} (9ae^2g^2 - c(45d^2g^2 - 30defg + 8e^2f^2)) E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{15c^{3/2}g^3\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}}$$

$$- \frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}} + 1\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}} (ae^2g^2(7ef - 15dg) - c(-15d^3g^3 + 45d^2efg^2 - 30de^2f^2g + 8e^3f^3)) F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{15c^{3/2}g^3\sqrt{a+cx^2}\sqrt{f+gx}}$$

$$- \frac{8e^2\sqrt{a+cx^2}\sqrt{f+gx}(ef - 3dg)}{15cg^2} + \frac{2e^2\sqrt{a+cx^2}(d+ex)\sqrt{f+gx}}{5cg}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^3/(Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]), x]

[Out]  $(-8*e^2*(e*f - 3*d*g)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])/(15*c*g^2) + (2*e^2*(d + e*x)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])/(5*c*g) + (2*\text{Sqrt}[-a]*e*(9*a*e^2*g^2 - c*(8*e^2*f^2 - 30*d*e*f*g + 45*d^2*g^2))*\text{Sqrt}[f + g*x]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)))/(15*c^{3/2}*g^3*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[a + c*x^2]) - (2*\text{Sqrt}[-a]*(a*e^2*g^2*(7*e*f - 15*d*g) - c*(8*e^3*f^3 - 30*d*e^2*f^2*g + 45*d^2*e*f*g^2 - 15*d^3*g^3))*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)))/(15*c^{3/2}*g^3*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])$

[a + c\*x^2])

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**3/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)`

[Out] Timed out

**Mathematica [C]** time = 7.70356, size = 625, normalized size = 1.37

$$2\sqrt{f+gx} \left( \frac{\sqrt{c}g\sqrt{f+gx}\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{a}g}{\sqrt{c}}}{f+gx}}\left(9a^{3/2}e^3g^2+\sqrt{ace}\left(-45d^2g^2+30defg-8e^2f^2\right)-ia\sqrt{ce^2g(15dg+2ef)+15ic^{3/2}d^3g^2}\right)F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}}{\sqrt{f+gx}}\right)}{\sqrt{-f-\frac{i\sqrt{a}g}{\sqrt{c}}}} \right)$$

Antiderivative was successfully verified.

[In] `Integrate[(d + e*x)^3/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]`

[Out]  $(2\sqrt{f+gx}(c^2e^2g^2(-4ef+15dg+3egx)(a+c^2x^2)+(eg^2(-9a^2e^2g^2+c^2(8e^2f^2-30defg+45d^2g^2)x^2+a^2(-30defg+45d^2g^2+e^2(8f^2-9g^2x^2)))))/(f+gx)+Ic^2e\sqrt{-f-(I\sqrt{a}g)/\sqrt{c}}(-9a^2e^2g^2+c^2(8e^2f^2-30defg+45d^2g^2))\sqrt{(g((I\sqrt{a})/\sqrt{c}+x))/(f+gx))\sqrt{-((I\sqrt{a}g)/\sqrt{c}-gx)/(f+gx))}\sqrt{f+gx}\text{EllipticE}[I\text{ArcSinh}[\sqrt{-f-(I\sqrt{a}g)/\sqrt{c}}]/\sqrt{f+gx}],(\sqrt{c}f-I\sqrt{a}g)/(\sqrt{c}f+I\sqrt{a}g)+(\sqrt{c}g((15I)c^{3/2}d^3g^2+9a^{3/2}e^3g^2-Ia\sqrt{ce^2g(15dg+2ef)+15ic^{3/2}d^3g^2)}\sqrt{(g((I\sqrt{a})/\sqrt{c}+x))/(f+gx))\sqrt{-((I\sqrt{a}g)/\sqrt{c}-gx)/(f+gx))}\sqrt{f+gx}\text{EllipticF}[I\text{ArcSinh}[\sqrt{-f-(I\sqrt{a}g)/\sqrt{c}}]/\sqrt{f+gx}],(\sqrt{c}f-I\sqrt{a}g)/(\sqrt{c}f+I\sqrt{a}g))/\sqrt{-f-(I\sqrt{a}g)/\sqrt{c}})/(15c^2g^4\sqrt{a+c^2x^2})$

**Maple [B]** time = 0.061, size = 2949, normalized size = 6.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)`

[Out]  $-2/15(-15x^2ac^2d^2e^2g^4+x^2ac^2e^3fg^3+4ac^2e^3f^2g^2-3x^4c^2e^3g^4-15x^2c^2d^2e^2fg^3-15ac^2d^2e^2fg^3+15(-(gx+f)^2c/(g^2(-ac)^{1/2}-cf))^{1/2})^{1/2}(((-cx+(-ac)^{1/2})g/(g^2(-ac)^{1/2}+cf))^{1/2})^{1/2}((cx+(-ac)^{1/2})g/(g^2(-ac)^{1/2}-cf))^{1/2})\text{EllipticF}(((-gx+f)^2c/(g^2(-ac)^{1/2}-cf))^{1/2},(-g^2(-ac)^{1/2}-cf)/(g^2(-ac)^{1/2}+cf))^{1/2})(-ac)^{1/2}c^2d^3g^4-$

$$\begin{aligned}
& 8^* (-g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)} * ((-c^*x+(-a^*c)^{(1/2)})^*g/ \\
& (g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)} * ((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)} \\
& -c^*f))^{(1/2)} * \text{EllipticF}((-g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}, (- \\
& (g^*(-a^*c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)} * (-a^*c)^{(1/2)} * c^* \\
& e^3 * f^3 * g - 45^* (-g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)} * ((-c^*x+(-a^*c) \\
& )^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)} * ((c^*x+(-a^*c)^{(1/2)})^*g/(g^* \\
& (-a^*c)^{(1/2)}-c^*f))^{(1/2)} * \text{EllipticF}((-g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\
& ))^{(1/2)}, (-g^*(-a^*c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)} * a^*c^* \\
& d^2 * e^*g^4 - 15^* (-g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)} * ((-c^*x+(-a^*c) \\
& )^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)} * ((c^*x+(-a^*c)^{(1/2)})^*g/(g^* \\
& (-a^*c)^{(1/2)}-c^*f))^{(1/2)} * \text{EllipticF}((-g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\
& ))^{(1/2)}, (-g^*(-a^*c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)} * (-a^* \\
& c)^{(1/2)} * a^*d^2 * e^2 * g^4 + 7^* (-g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)} * (( \\
& -c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)} * ((c^*x+(-a^*c)^{(1/2)} \\
& )^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)} * \text{EllipticF}((-g^*x+f)^*c/(g^*(-a^*c) \\
& )^{(1/2)}-c^*f))^{(1/2)}, (-g^*(-a^*c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)} * ( \\
& 1/2)) * (-a^*c)^{(1/2)} * a^*e^3 * f^3 * g^3 - 6^* (-g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\
& )^{(1/2)} * ((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)} * ((c^*x+ \\
& (-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)} * \text{EllipticF}((-g^*x+f)^*c \\
& / (g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}, (-g^*(-a^*c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1/2) \\
& }+c^*f))^{(1/2)} * a^*c^*e^3 * f^2 * g^2 + 45^* (-g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^* \\
& f))^{(1/2)} * ((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)} * ((c^* \\
& x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)} * \text{EllipticE}((-g^*x+f) \\
& )^*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}, (-g^*(-a^*c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{( \\
& 1/2)}+c^*f))^{(1/2)} * a^*c^*d^2 * e^*g^4 - (-g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\
& )^{(1/2)} * ((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)} * ((c^*x+ \\
& (-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)} * \text{EllipticE}((-g^*x+f)^*c/ \\
& (g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}, (-g^*(-a^*c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1/2) \\
& }+c^*f))^{(1/2)} * a^*c^*e^3 * f^2 * g^2 + 45^* (-g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\
& ))^{(1/2)} * ((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)} * ((c^*x \\
& +(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)} * \text{EllipticE}((-g^*x+f)^* \\
& c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}, (-g^*(-a^*c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1 \\
& /2)}+c^*f))^{(1/2)} * c^2 * d^2 * e^*f^2 * g^2 - 30^* (-g^*x+f)^*c/(g^*(-a^*c)^{(1/2) \\
& }-c^*f))^{(1/2)} * ((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)} * ( \\
& (c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)} * \text{EllipticE}((-g^*x \\
& +f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}, (-g^*(-a^*c)^{(1/2)}-c^*f)/(g^*(-a^*c) \\
& )^{(1/2)}+c^*f))^{(1/2)} * c^2 * d^2 * e^2 * f^3 * g + x^3 * c^2 * e^3 * f^3 * g^3 - 3^* x^2 * a^*c^* \\
& e^3 * g^4 + 4^* x^2 * c^2 * e^3 * f^2 * g^2 - 45^* (-g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f) \\
& )^{(1/2)} * ((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)} * ((c^*x+ \\
& (-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)} * \text{EllipticF}((-g^*x+f)^*c \\
& / (g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}, (-g^*(-a^*c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1/2) \\
& }+c^*f))^{(1/2)} * (-a^*c)^{(1/2)} * c^*d^2 * e^*f^3 * g^3 + 30^* (-g^*x+f)^*c/(g^*(-a^* \\
& c)^{(1/2)}-c^*f))^{(1/2)} * ((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}+c^*f)) \\
& )^{(1/2)} * ((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)} * \text{Elliptic} \\
& \text{F}((-g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}, (-g^*(-a^*c)^{(1/2)}-c^*f)/ \\
& (g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)} * (-a^*c)^{(1/2)} * c^*d^2 * e^2 * f^2 * g^2 + 45^* (-g \\
& *x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)} * ((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a \\
& *c)^{(1/2)}+c^*f))^{(1/2)} * ((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f)) \\
& )^{(1/2)} * \text{EllipticF}((-g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}, (-g^*(-a \\
& *c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)} * a^*c^*d^2 * e^2 * f^3 * g^3 - 30^* (- \\
& (g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)} * ((-c^*x+(-a^*c)^{(1/2)})^*g/(g^* \\
& (-a^*c)^{(1/2)}+c^*f))^{(1/2)} * ((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f) \\
& ))^{(1/2)} * \text{EllipticE}((-g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}, (-g^*( \\
& -a^*c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)} * a^*c^*d^2 * e^2 * f^3 * g^3 + 9^* ( \\
& -g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)} * ((-c^*x+(-a^*c)^{(1/2)})^*g/(g^* \\
& (-a^*c)^{(1/2)}+c^*f))^{(1/2)} * ((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^* \\
& f))^{(1/2)} * \text{EllipticF}((-g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}, (-g^* \\
& (-a^*c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)} * a^2 * e^3 * g^4 - 9^* (-g \\
& *x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)} * ((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a \\
& *c)^{(1/2)}+c^*f))^{(1/2)} * ((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f)) \\
& )^{(1/2)} * \text{EllipticE}((-g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}, (-g^*(-a \\
& *c)^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)} * a^2 * e^3 * g^4 + 8^* (-g^*x+ \\
& f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)} * ((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c) \\
& )^{(1/2)}+c^*f))^{(1/2)} * ((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1 \\
& /2)} * \text{EllipticE}((-g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}, (-g^*(-a^*c) \\
& )^{(1/2)}-c^*f)/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)} * c^2 * e^3 * f^4 - 15^* (-g^*x+f) \\
& )^*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)} * ((-c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{( \\
& 1/2)}+c^*f))^{(1/2)} * ((c^*x+(-a^*c)^{(1/2)})^*g/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2) \\
& )} * \text{EllipticF}((-g^*x+f)^*c/(g^*(-a^*c)^{(1/2)}-c^*f))^{(1/2)}, (-g^*(-a^*c)^{( \\
& 1/2)}-c^*f)/(g^*(-a^*c)^{(1/2)}+c^*f))^{(1/2)} * c^2 * d^3 * f^3 * g^3 - 15^* x^3 * c^2 * d \\
& )^*e^2 * g^4 * (g^*x+f)^{(1/2)} * (c^*x^2+a)^{(1/2)}/c^2/g^4/(c^*g^*x^3+c^*f^*x^2+a \\
& )^*g^*x+a^*f)
\end{aligned}$$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^3}{\sqrt{cx^2 + a}\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3/(sqrt(c\*x^2 + a)\*sqrt(g\*x + f)),x, algorithm="maxima")

[Out] integrate((e\*x + d)^3/(sqrt(c\*x^2 + a)\*sqrt(g\*x + f)), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}{\sqrt{cx^2 + a}\sqrt{gx + f}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3/(sqrt(c\*x^2 + a)\*sqrt(g\*x + f)),x, algorithm="fricas")

[Out] integral((e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3)/(sqrt(c\*x^2 + a)\*sqrt(g\*x + f)), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^3}{\sqrt{a + cx^2}\sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3/(g\*x+f)\*\*(1/2)/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] Integral((d + e\*x)\*\*3/(sqrt(a + c\*x\*\*2)\*sqrt(f + g\*x)), x)

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3/(sqrt(c\*x^2 + a)\*sqrt(g\*x + f)),x, algorithm="giac")

[Out] Exception raised: RuntimeError

$$3.646 \quad \int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=356

$$\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}} + 1\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}} (g^2(3cd^2 - ae^2) + 2cef(ef - 3dg)) F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{3c^{3/2}g^2\sqrt{a+cx^2}\sqrt{f+gx}} + \frac{4\sqrt{-ae}\sqrt{\frac{cx^2}{a}} + 1\sqrt{f+gx}(ef - 3dg)E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{3\sqrt{cg^2}\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}} + \frac{2e^2\sqrt{a+cx^2}\sqrt{f+gx}}{3cg}$$

[Out] (2\*e^2\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])/(3\*c\*g) + (4\*Sqrt[-a]\*e\*(e\*f - 3\*d\*g)\*Sqrt[f + g\*x]\*Sqrt[1 + (c\*x^2)/a]\*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(3\*Sqrt[c]\*g^2\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[a + c\*x^2]) - (2\*Sqrt[-a]\*((3\*c\*d^2 - a\*e^2)\*g^2 + 2\*c\*e\*f\*(e\*f - 3\*d\*g))\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(3\*c^(3/2)\*g^2\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])

**Rubi [A]** time = 1.03231, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$

$$\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}} + 1\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}} (g^2(3cd^2 - ae^2) + 2cef(ef - 3dg)) F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{3c^{3/2}g^2\sqrt{a+cx^2}\sqrt{f+gx}} + \frac{4\sqrt{-ae}\sqrt{\frac{cx^2}{a}} + 1\sqrt{f+gx}(ef - 3dg)E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{3\sqrt{cg^2}\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}} + \frac{2e^2\sqrt{a+cx^2}\sqrt{f+gx}}{3cg}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^2/(Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]), x]

[Out] (2\*e^2\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])/(3\*c\*g) + (4\*Sqrt[-a]\*e\*(e\*f - 3\*d\*g)\*Sqrt[f + g\*x]\*Sqrt[1 + (c\*x^2)/a]\*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(3\*Sqrt[c]\*g^2\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[a + c\*x^2]) - (2\*Sqrt[-a]\*((3\*c\*d^2 - a\*e^2)\*g^2 + 2\*c\*e\*f\*(e\*f - 3\*d\*g))\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(3\*c^(3/2)\*g^2\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])

**Rubi in Sympy [A]** time = 174.557, size = 602, normalized size = 1.69

$$\frac{2e^2\sqrt{a+cx^2}\sqrt{f+gx}}{3cg} - \frac{2e\sqrt{-a}\sqrt{1+\frac{cx^2}{a}}\sqrt{f+gx}(dg-ef)E\left(\operatorname{asin}\left(\sqrt{-\frac{\sqrt{cx}}{2\sqrt{-a}}+\frac{1}{2}}\right)\middle|\frac{2ag}{ag-\sqrt{cf}\sqrt{-a}}\right)}{\sqrt{cg^2}\sqrt{\frac{\sqrt{c}\sqrt{-a}(-f-gx)}{ag-\sqrt{cf}\sqrt{-a}}}\sqrt{a+cx^2}}$$

$$- \frac{2e\sqrt{-a}\sqrt{1+\frac{cx^2}{a}}\sqrt{f+gx}(3dg+ef)E\left(\operatorname{asin}\left(\sqrt{-\frac{\sqrt{cx}}{2\sqrt{-a}}+\frac{1}{2}}\right)\middle|\frac{2ag}{ag-\sqrt{cf}\sqrt{-a}}\right)}{3\sqrt{cg^2}\sqrt{\frac{\sqrt{c}\sqrt{-a}(-f-gx)}{ag-\sqrt{cf}\sqrt{-a}}}\sqrt{a+cx^2}}$$

$$- \frac{2\sqrt{-a}\sqrt{\frac{\sqrt{c}\sqrt{-a}(-f-gx)}{ag-\sqrt{cf}\sqrt{-a}}}\sqrt{1+\frac{cx^2}{a}}(dg-ef)^2F\left(\operatorname{asin}\left(\sqrt{-\frac{\sqrt{cx}}{2\sqrt{-a}}+\frac{1}{2}}\right)\middle|\frac{2ag}{ag-\sqrt{cf}\sqrt{-a}}\right)}{\sqrt{cg^2}\sqrt{a+cx^2}\sqrt{f+gx}}$$

$$+ \frac{2e^2\sqrt{-a}\sqrt{\frac{\sqrt{c}\sqrt{-a}(-f-gx)}{ag-\sqrt{cf}\sqrt{-a}}}\sqrt{1+\frac{cx^2}{a}}(ag^2+cf^2)F\left(\operatorname{asin}\left(\sqrt{-\frac{\sqrt{cx}}{2\sqrt{-a}}+\frac{1}{2}}\right)\middle|\frac{2ag}{ag-\sqrt{cf}\sqrt{-a}}\right)}{3c^{\frac{3}{2}}g^2\sqrt{a+cx^2}\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**2/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)`

[Out]  $2e^{2}\sqrt{a+cx^{2}}\sqrt{f+gx}/(3cg) - 2e\sqrt{-a}\sqrt{1+\frac{cx^{2}}{a}}\sqrt{f+gx}(dg-ef)\operatorname{elliptic}_e(\operatorname{asin}(\sqrt{-\frac{\sqrt{cx}}{2\sqrt{-a}}+\frac{1}{2}}), 2ag/(ag-\sqrt{cf}\sqrt{-a}))/(\sqrt{c}g^{2}\sqrt{\frac{\sqrt{c}\sqrt{-a}(-f-gx)}{ag-\sqrt{cf}\sqrt{-a}}}\sqrt{a+cx^{2}}) - 2e\sqrt{-a}\sqrt{1+\frac{cx^{2}}{a}}\sqrt{f+gx}(3dg+ef)\operatorname{elliptic}_e(\operatorname{asin}(\sqrt{-\frac{\sqrt{cx}}{2\sqrt{-a}}+\frac{1}{2}}), 2ag/(ag-\sqrt{cf}\sqrt{-a}))/(\sqrt{c}g^{2}\sqrt{\frac{\sqrt{c}\sqrt{-a}(-f-gx)}{ag-\sqrt{cf}\sqrt{-a}}}\sqrt{a+cx^{2}}) - 2\sqrt{-a}\sqrt{\frac{\sqrt{c}\sqrt{-a}(-f-gx)}{ag-\sqrt{cf}\sqrt{-a}}}\sqrt{1+\frac{cx^{2}}{a}}(dg-ef)^{2}\operatorname{elliptic}_f(\operatorname{asin}(\sqrt{-\frac{\sqrt{cx}}{2\sqrt{-a}}+\frac{1}{2}}), 2ag/(ag-\sqrt{cf}\sqrt{-a}))/(\sqrt{c}g^{2}\sqrt{a+cx^{2}}\sqrt{f+gx}) + 2e^{2}\sqrt{-a}\sqrt{\frac{\sqrt{c}\sqrt{-a}(-f-gx)}{ag-\sqrt{cf}\sqrt{-a}}}\sqrt{1+\frac{cx^{2}}{a}}(ag^{2}+cf^{2})\operatorname{elliptic}_f(\operatorname{asin}(\sqrt{-\frac{\sqrt{cx}}{2\sqrt{-a}}+\frac{1}{2}}), 2ag/(ag-\sqrt{cf}\sqrt{-a}))/(\sqrt{c}g^{2}\sqrt{a+cx^{2}}\sqrt{f+gx})$

**Mathematica [C]** time = 5.85252, size = 473, normalized size = 1.33

$$2\sqrt{f+gx}\left(\frac{g\sqrt{f+gx}\sqrt{\frac{g(x+\frac{i\sqrt{a}}{\sqrt{c}})}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{ag}}{\sqrt{c}}}{f+gx}}(2\sqrt{a}\sqrt{c}e(ef-3dg)-iae^{2}g+3icd^{2}g)F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{cf-i\sqrt{ag}}}{\sqrt{cf+i\sqrt{ag}}}\right)}{\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}} - \frac{2eg^{2}(a+cx^{2})(ef-3dg)}{f+gx} - 2icd^{2}g\sqrt{a+cx^{2}}\right)$$

Antiderivative was successfully verified.

[In] `Integrate[(d + e*x)^2/(Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]`

[Out]  $(2\sqrt{f+gx})(e^{2}g^{2}(a+cx^{2}) - (2e^{2}g^{2}(ef-3dg) + a + cx^{2}))/(\sqrt{f+gx}) - (2I)c^{2}e\sqrt{-f - (I\sqrt{a}g)/\sqrt{c}}/\sqrt{c} + (ef-3dg)\sqrt{(g((I\sqrt{a}g)/\sqrt{c} + x))/(\sqrt{f+gx})}\sqrt{-(((I\sqrt{a}g)/\sqrt{c} - g)/(\sqrt{f+gx}))}\sqrt{f+gx}\operatorname{EllipticE}[I\operatorname{ArcSinh}[\sqrt{-f - (I\sqrt{a}g)/\sqrt{c}}]/\sqrt{f+gx}], (\sqrt{c}f - I\sqrt{a}g)/(\sqrt{c}f + I\sqrt{a}g) + (g((3I)c^{2}d^{2}g - Iae^{2}g + 2\sqrt{a}\sqrt{c}e(ef-3dg))\sqrt{(g((I\sqrt{a}g)/\sqrt{c} + x))/(\sqrt{f+gx})}\sqrt{-(((I\sqrt{a}g)/\sqrt{c} - g)/(\sqrt{f+gx}))}\sqrt{f+gx}\operatorname{EllipticF}[I\operatorname{ArcSinh}[\sqrt{-f - ($





Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^2/(sqrt(c*x^2 + a)*sqrt(g*x + f)),x, algorithm="maxima")`

[Out] `integrate((e*x + d)^2/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^2x^2 + 2dex + d^2}{\sqrt{cx^2 + a}\sqrt{gx + f}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^2/(sqrt(c*x^2 + a)*sqrt(g*x + f)),x, algorithm="fricas")`

[Out] `integral((e^2*x^2 + 2*d*e*x + d^2)/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^2}{\sqrt{a + cx^2}\sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**2/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)`

[Out] `Integral((d + e*x)**2/(sqrt(a + c*x**2)*sqrt(f + g*x)), x)`

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^2/(sqrt(c*x^2 + a)*sqrt(g*x + f)),x, algorithm="giac")`

[Out] Exception raised: RuntimeError

$$3.647 \quad \int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=288

$$\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}g\sqrt{a+cx^2}\sqrt{f+gx}} - \frac{2\sqrt{-ae}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}g\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}}$$

[Out] (-2\*Sqrt[-a]\*e\*Sqrt[f + g\*x]\*Sqrt[1 + (c\*x^2)/a]\*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(Sqrt[c]\*g\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[a + c\*x^2]) + (2\*Sqrt[-a]\*(e\*f - d\*g)\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(Sqrt[c]\*g\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]))

**Rubi [A]** time = 0.542987, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$

$$\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}+1}(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}g\sqrt{a+cx^2}\sqrt{f+gx}} - \frac{2\sqrt{-ae}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c}x}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{c}f-ag}\right)}{\sqrt{c}g\sqrt{a+cx^2}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{c}f}}}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)/(Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]),x]

[Out] (-2\*Sqrt[-a]\*e\*Sqrt[f + g\*x]\*Sqrt[1 + (c\*x^2)/a]\*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(Sqrt[c]\*g\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[a + c\*x^2]) + (2\*Sqrt[-a]\*(e\*f - d\*g)\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(Sqrt[c]\*g\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]))

**Rubi in Sympy [A]** time = 93.1342, size = 270, normalized size = 0.94

$$\frac{2e\sqrt{-a}\sqrt{1+\frac{cx^2}{a}}\sqrt{f+gx}E\left(\operatorname{asin}\left(\sqrt{-\frac{\sqrt{c}x}{2\sqrt{-a}}+\frac{1}{2}}\right)\middle|\frac{2ag}{ag-\sqrt{c}f\sqrt{-a}}\right)}{\sqrt{cg}\sqrt{\frac{\sqrt{c}\sqrt{-a}(-f-gx)}{ag-\sqrt{c}f\sqrt{-a}}}\sqrt{a+cx^2}}$$

$$\frac{2\sqrt{-a}\sqrt{\frac{\sqrt{c}\sqrt{-a}(-f-gx)}{ag-\sqrt{c}f\sqrt{-a}}}\sqrt{1+\frac{cx^2}{a}}(dg-ef)F\left(\operatorname{asin}\left(\sqrt{-\frac{\sqrt{c}x}{2\sqrt{-a}}+\frac{1}{2}}\right)\middle|\frac{2ag}{ag-\sqrt{c}f\sqrt{-a}}\right)}{\sqrt{cg}\sqrt{a+cx^2}\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)`

[Out] `-2*e*sqrt(-a)*sqrt(1+c*x**2/a)*sqrt(f+g*x)*elliptic_e(asin(sqrt(-sqrt(c)*x/(2*sqrt(-a))+1/2)),2*a*g/(a*g-sqrt(c)*f*sqrt(-a)))/(sqrt(c)*g*sqrt(sqrt(c)*sqrt(-a)*(-f-g*x)/(a*g-sqrt(c)*f*sqrt(-a)))*sqrt(a+c*x**2))-2*sqrt(-a)*sqrt(sqrt(c)*sqrt(-a)*(-f-g*x)/(a*g-sqrt(c)*f*sqrt(-a)))*sqrt(1+c*x**2/a)*(d*g-e*f)*elliptic_f(asin(sqrt(-sqrt(c)*x/(2*sqrt(-a))+1/2)),2*a*g/(a*g-sqrt(c)*f*sqrt(-a)))/(sqrt(c)*g*sqrt(a+c*x**2)*sqrt(f+g*x))`

**Mathematica [C]** time = 2.66158, size = 439, normalized size = 1.52

$$\frac{2\left(\sqrt{cg}(f+gx)^{3/2}(\sqrt{ae}-i\sqrt{cd})\sqrt{\frac{g(x+\frac{i\sqrt{a}}{\sqrt{c}})}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{ag}}{\sqrt{c}}}{f+gx}}F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}f-i\sqrt{ag}}{\sqrt{c}f+i\sqrt{ag}}\right)-eg^2(a+cx^2)\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}\right)}{cg^2\sqrt{a+cx^2}\sqrt{f+gx}\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}}$$

Antiderivative was successfully verified.

[In] `Integrate[(d+e*x)/(Sqrt[f+g*x]*Sqrt[a+c*x^2]),x]`

[Out] `(-2*(-(e*g^2*Sqrt[-f-(I*Sqrt[a]*g)/Sqrt[c]]*(a+c*x^2))+I*Sqrt[c]*e*(Sqrt[c]*f+I*Sqrt[a]*g)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c]+x))/(f+g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c]-g*x)/(f+g*x))]*(f+g*x)^(3/2)*EllipticE[I*ArcSinh[Sqrt[-f-(I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f+g*x]],(Sqrt[c]*f-I*Sqrt[a]*g)/(Sqrt[c]*f+I*Sqrt[a]*g)]+Sqrt[c]*((-I)*Sqrt[c]*d+Sqrt[a]*e)*g*Sqrt[(g*((I*Sqrt[a])/Sqrt[c]+x))/(f+g*x)]*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c]-g*x)/(f+g*x))]*(f+g*x)^(3/2)*EllipticF[I*ArcSinh[Sqrt[-f-(I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f+g*x]],(Sqrt[c]*f-I*Sqrt[a]*g)/(Sqrt[c]*f+I*Sqrt[a]*g)))/(c*g^2*Sqrt[-f-(I*Sqrt[a]*g)/Sqrt[c]]*Sqrt[f+g*x]*Sqrt[a+c*x^2))`

**Maple [B]** time = 0.048, size = 520, normalized size = 1.8

$$2\frac{\sqrt{gx+f}\sqrt{cx^2+a}}{g^2c(cgx^3+cfx^2+agx+fa)}\left(EllipticF\left(\sqrt{\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\sqrt{\frac{g\sqrt{-ac}-cf}{g\sqrt{-ac}+cf}}\right)aeg^2+EllipticF\left(\sqrt{\frac{(gx+f)c}{g\sqrt{-ac}-cf}},\sqrt{\frac{g\sqrt{-ac}-cf}{g\sqrt{-ac}+cf}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)`

```
[Out] 2*(EllipticF((-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2), (-g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*e*g^2+EllipticF((-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2), (-g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c*d*f*g-EllipticF((-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2), (-g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-a*c)^(1/2)*d*g^2+EllipticF((-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2), (-g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-a*c)^(1/2)*e*f*g-EllipticE((-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2), (-g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*a*e*g^2-EllipticE((-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2), (-g*(-a*c)^(1/2)-c*f)/(g*(-a*c)^(1/2)+c*f))^(1/2))*c*e*f^2*((c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)-c*f))^(1/2))*((-c*x+(-a*c)^(1/2))*g/(g*(-a*c)^(1/2)+c*f))^(1/2))*(-g*x+f)*c/(g*(-a*c)^(1/2)-c*f))^(1/2)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/c/g^2/(c*g*x^3+c*f*x^2+a*g*x+a*f)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{ex + d}{\sqrt{cx^2 + a}\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x, algorithm="maxima")
```

```
[Out] integrate((e*x + d)/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex + d}{\sqrt{cx^2 + a}\sqrt{gx + f}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x, algorithm="fricas")
```

```
[Out] integral((e*x + d)/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex}{\sqrt{a + cx^2}\sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(g*x+f)**(1/2)/(c*x**2+a)**(1/2), x)
```

```
[Out] Integral((d + e*x)/(sqrt(a + c*x**2)*sqrt(f + g*x)), x)
```

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)/(sqrt(c*x^2 + a)*sqrt(g*x + f)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.648 \quad \int \frac{1}{\sqrt{f+gx}\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=136

$$\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}} + 1\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}} F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{\sqrt{c}\sqrt{a+cx^2}\sqrt{f+gx}}$$

[Out] (-2\*Sqrt[-a]\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(Sqrt[c]\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])

**Rubi [A]** time = 0.2144, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$

$$\frac{2\sqrt{-a}\sqrt{\frac{cx^2}{a}} + 1\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}} F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{\sqrt{c}\sqrt{a+cx^2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]),x]

[Out] (-2\*Sqrt[-a]\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/(Sqrt[c]\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])

**Rubi in Sympy [A]** time = 44.9692, size = 129, normalized size = 0.95

$$\frac{2\sqrt{-a}\sqrt{\frac{\sqrt{c}\sqrt{-a}(-f-gx)}{ag-\sqrt{cf}\sqrt{-a}}}\sqrt{1+\frac{cx^2}{a}} F\left(\operatorname{asin}\left(\sqrt{-\frac{\sqrt{cx}}{2\sqrt{-a}}+\frac{1}{2}}\right) \middle| \frac{2ag}{ag-\sqrt{cf}\sqrt{-a}}\right)}{\sqrt{c}\sqrt{a+cx^2}\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(g\*x+f)\*\*(1/2)/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] -2\*sqrt(-a)\*sqrt(sqrt(c)\*sqrt(-a)\*(-f - g\*x)/(a\*g - sqrt(c)\*f\*sqrt(-a)))\*sqrt(1 + c\*x\*\*2/a)\*elliptic\_f(asin(sqrt(-sqrt(c)\*x/(2\*sqrt(-a) + 1/2))), 2\*a\*g/(a\*g - sqrt(c)\*f\*sqrt(-a)))/(sqrt(c)\*sqrt(a + c\*x\*\*2)\*sqrt(f + g\*x))

**Mathematica [C]** time = 0.443463, size = 186, normalized size = 1.37

$$\frac{2i(f+gx)\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{ag}}{\sqrt{c}}}{f+gx}} F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}}{\sqrt{f+gx}}}\right) \middle| \frac{\sqrt{cf-i\sqrt{ag}}}{\sqrt{cf+i\sqrt{ag}}}\right)}{g\sqrt{a+cx^2}\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]),x]

[Out] ((2\*I)\*Sqrt[(g\*((I\*Sqrt[a])/Sqrt[c] + x))/(f + g\*x)]\*Sqrt[-(((I\*Sqrt[a]\*g)/Sqrt[c] - g\*x)/(f + g\*x))]\*(f + g\*x)\*EllipticF[I\*ArcSin h[Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]/Sqrt[f + g\*x]], (Sqrt[c]\*f - I\*Sqrt[a]\*g)/(Sqrt[c]\*f + I\*Sqrt[a]\*g)]/(g\*Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]\*Sqrt[a + c\*x^2])

**Maple [A]** time = 0.044, size = 200, normalized size = 1.5

$$2 \frac{(cf - g\sqrt{-ac}) \sqrt{gx + f} \sqrt{cx^2 + a}}{cg(cx^3 + cfx^2 + agx + fa)} \text{EllipticF} \left( \sqrt{\frac{(gx + f)c}{g\sqrt{-ac} - cf}}, \sqrt{\frac{g\sqrt{-ac} - cf}{g\sqrt{-ac} + cf}} \right) \sqrt{\frac{(cx + \sqrt{-ac})g}{g\sqrt{-ac} - cf}} \sqrt{\frac{(-cx + \sqrt{-ac})g}{g\sqrt{-ac} + cf}} \sqrt{\frac{-g}{g}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(g\*x+f)^(1/2)/(c\*x^2+a)^(1/2),x)

[Out] 2\*(c\*f-g\*(-a\*c)^(1/2))\*EllipticF((- (g\*x+f)\*c/(g\*(-a\*c)^(1/2)-c\*f))^(1/2), (- (g\*(-a\*c)^(1/2)-c\*f)/(g\*(-a\*c)^(1/2)+c\*f))^(1/2))\*((c\*x+(-a\*c)^(1/2))\*g/(g\*(-a\*c)^(1/2)-c\*f))^(1/2)\*((-c\*x+(-a\*c)^(1/2))\*g/(g\*(-a\*c)^(1/2)+c\*f))^(1/2)\*(- (g\*x+f)\*c/(g\*(-a\*c)^(1/2)-c\*f))^(1/2)\*(g\*x+f)^(1/2)\*(c\*x^2+a)^(1/2)/g/c/(c\*g\*x^3+c\*f\*x^2+a\*g\*x+a\*f)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + a}\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + a)\*sqrt(g\*x + f)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c\*x^2 + a)\*sqrt(g\*x + f)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{1}{\sqrt{cx^2 + a}\sqrt{gx + f}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + a)\*sqrt(g\*x + f)),x, algorithm="fricas")

[Out] integral(1/(sqrt(c\*x^2 + a)\*sqrt(g\*x + f)), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + cx^2}\sqrt{f + gx}} dx$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)`

[Out] `Integral(1/(sqrt(a + c*x**2)*sqrt(f + g*x)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + a}\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*x^2 + a)*sqrt(g*x + f)),x, algorithm="giac")`

[Out] `integrate(1/(sqrt(c*x^2 + a)*sqrt(g*x + f)), x)`

$$3.649 \quad \int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=167

$$\frac{2\sqrt{\frac{cx^2}{a} + 1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}\left(\frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}}+e}; \sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{-ag}}{\sqrt{cf}+\sqrt{-ag}}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{cd}}{\sqrt{-a}}+e\right)}$$

[Out] (-2\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticPi[(2\*e)/((Sqrt[c]\*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (2\*Sqrt[-a]\*g)/(Sqrt[c]\*f + Sqrt[-a]\*g)]/(((Sqrt[c]\*d)/Sqrt[-a] + e)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])

**Rubi [A]** time = 1.41636, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{2\sqrt{\frac{cx^2}{a} + 1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag}+\sqrt{cf}}}\left(\frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}}+e}; \sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{-ag}}{\sqrt{cf}+\sqrt{-ag}}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{cd}}{\sqrt{-a}}+e\right)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]),x]

[Out] (-2\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticPi[(2\*e)/((Sqrt[c]\*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (2\*Sqrt[-a]\*g)/(Sqrt[c]\*f + Sqrt[-a]\*g)]/(((Sqrt[c]\*d)/Sqrt[-a] + e)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])

**Rubi in Sympy [A]** time = 18.5421, size = 201, normalized size = 1.2

$$\frac{2\sqrt{\frac{g(-\sqrt{cx}-\sqrt{-a})}{\sqrt{cf}-g\sqrt{-a}}}\sqrt{\frac{g(-\sqrt{cx}+\sqrt{-a})}{\sqrt{cf}+g\sqrt{-a}}}\left(-\frac{e(\sqrt{cf}+g\sqrt{-a})}{\sqrt{c}(dg-ef)}; \operatorname{asin}\left(\sqrt{\frac{c}{\sqrt{cg}\sqrt{-a}+cf}}\sqrt{f+gx}\right)\middle|\frac{\sqrt{cf}+g\sqrt{-a}}{\sqrt{cf}-g\sqrt{-a}}\right)}{\sqrt{\frac{c}{\sqrt{cg}\sqrt{-a}+cf}}\sqrt{a+cx^2}(dg-ef)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(e\*x+d)/(g\*x+f)\*\*(1/2)/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] 2\*sqrt(g\*(-sqrt(c)\*x - sqrt(-a))/(sqrt(c)\*f - g\*sqrt(-a)))\*sqrt(g\*(-sqrt(c)\*x + sqrt(-a))/(sqrt(c)\*f + g\*sqrt(-a)))\*elliptic\_pi(-e\*(sqrt(c)\*f + g\*sqrt(-a))/(sqrt(c)\*(d\*g - e\*f)), asin(sqrt(c/(sqrt(c)\*g\*sqrt(-a) + c\*f))\*sqrt(f + g\*x)), (sqrt(c)\*f + g\*sqrt(-a))/(sqrt(c)\*f - g\*sqrt(-a)))/(sqrt(c)/(sqrt(c)\*g\*sqrt(-a) + c\*f))\*sqrt(a + c\*x\*\*2)\*(d\*g - e\*f)

**Mathematica [C]** time = 0.903257, size = 311, normalized size = 1.86

$$\frac{2i(f+gx)\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{ag}}{\sqrt{c}}}{f+gx}}\left(F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{cf-i\sqrt{ag}}}{\sqrt{cf+i\sqrt{ag}}}\right)-\left(\frac{\sqrt{c}(ef-dg)}{e(\sqrt{cf+i\sqrt{ag}})};i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{cf-i\sqrt{ag}}}{\sqrt{cf+i\sqrt{ag}}}\right)\right)}{\sqrt{a+cx^2}\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}(ef-dg)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]),x]

[Out] ((-2\*I)\*Sqrt[(g\*(I\*Sqrt[a])/Sqrt[c] + x)/(f + g\*x)]\*Sqrt[-(((I\*Sqrt[a]\*g)/Sqrt[c] - g\*x)/(f + g\*x))\* (f + g\*x)\*(EllipticF[I\*ArcSinh[Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]/Sqrt[f + g\*x]], (Sqrt[c]\*f - I\*Sqrt[a]\*g)/(Sqrt[c]\*f + I\*Sqrt[a]\*g)] - EllipticPi[(Sqrt[c]\*(e\*f - d\*g))/(e\*(Sqrt[c]\*f + I\*Sqrt[a]\*g)), I\*ArcSinh[Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]/Sqrt[f + g\*x]], (Sqrt[c]\*f - I\*Sqrt[a]\*g)/(Sqrt[c]\*f + I\*Sqrt[a]\*g)))/(Sqrt[-f - (I\*Sqrt[a]\*g)/Sqrt[c]]\*(e\*f - d\*g)\*Sqrt[a + c\*x^2])

**Maple [A]** time = 0.052, size = 235, normalized size = 1.4

$$2\frac{(cf - g\sqrt{-ac})\sqrt{cx^2 + a}\sqrt{gx + f}}{c(dg - ef)(cgx^3 + cf x^2 + agx + fa)}\text{EllipticPi}\left(\sqrt{-\frac{(gx + f)c}{g\sqrt{-ac} - cf}}, \frac{(g\sqrt{-ac} - cf)e}{c(dg - ef)}, \sqrt{-\frac{g\sqrt{-ac} - cf}{g\sqrt{-ac} + cf}}\right)\sqrt{\frac{(cx + \sqrt{-ac})g}{g\sqrt{-ac} - cf}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x+d)/(g\*x+f)^(1/2)/(c\*x^2+a)^(1/2),x)

[Out] 2\*(c\*f-g\*(-a\*c)^(1/2))\*EllipticPi((- (g\*x+f)\*c/(g\*(-a\*c)^(1/2)-c\*f))^(1/2), (g\*(-a\*c)^(1/2)-c\*f)\*e/c/(d\*g-e\*f), -(g\*(-a\*c)^(1/2)-c\*f)/(g\*(-a\*c)^(1/2)+c\*f))^(1/2))\*((c\*x+(-a\*c)^(1/2))\*g/(g\*(-a\*c)^(1/2)-c\*f))^(1/2))\*((-c\*x+(-a\*c)^(1/2))\*g/(g\*(-a\*c)^(1/2)+c\*f))^(1/2))\*(-(g\*x+f)\*c/(g\*(-a\*c)^(1/2)-c\*f))^(1/2)\*(c\*x^2+a)^(1/2)\*(g\*x+f)^(1/2)/c/(d\*g-e\*f)/(c\*g\*x^3+c\*f\*x^2+a\*g\*x+a\*f)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + a}(ex + d)\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)\*sqrt(g\*x + f)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)\*sqrt(g\*x + f)), x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)\*sqrt(g\*x + f)),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + cx^2}(d + ex)\sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x+d)/(g\*x+f)\*\*(1/2)/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] Integral(1/(sqrt(a + c\*x\*\*2)\*(d + e\*x)\*sqrt(f + g\*x)), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + a}(ex + d)\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)\*sqrt(g\*x + f)),x, algorithm="giac")

[Out] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)\*sqrt(g\*x + f)), x)

$$3.650 \quad \int \frac{1}{(d+ex)^2 \sqrt{f+gx} \sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=746

$$\begin{aligned} & \frac{e^2 \sqrt{a+cx^2} \sqrt{f+gx}}{(d+ex)(ae^2+cd^2)(ef-dg)} \\ & + \frac{\sqrt{-a} \sqrt{cef} \sqrt{\frac{cx^2}{a} + 1} \sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}} F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{\sqrt{a+cx^2} \sqrt{f+gx} (ae^2+cd^2)(ef-dg)} \\ & - \frac{\sqrt{-a} \sqrt{cdg} \sqrt{\frac{cx^2}{a} + 1} \sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}} F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{\sqrt{a+cx^2} \sqrt{f+gx} (ae^2+cd^2)(ef-dg)} \\ & - \frac{\sqrt{-a} \sqrt{ce} \sqrt{\frac{cx^2}{a} + 1} \sqrt{f+gx} E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{\sqrt{a+cx^2} (ae^2+cd^2)(ef-dg) \sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}} \\ & + \frac{\sqrt{\frac{cx^2}{a} + 1} \sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}} (ae^2g - cd(2ef - 3dg)) \left(\frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}} + e}; \sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| \frac{2\sqrt{-ag}}{\sqrt{cf+\sqrt{-ag}}}\right)}{\sqrt{a+cx^2} \sqrt{f+gx} \left(\frac{\sqrt{cd}}{\sqrt{-a}} + e\right) (ae^2+cd^2)(ef-dg)} \end{aligned}$$

[Out]  $-\left(\left(e^2 \sqrt{f+g x} \sqrt{a+c x^2}\right) / \left(\left(c d^2+a e^2\right)\left(e f-d g\right)\left(d+e x\right)\right)\right)-\left(\sqrt{-a} \sqrt{c} e \sqrt{f+g x} \sqrt{1+\left(c x^2\right) / a}\right) \operatorname{EllipticE}\left[\operatorname{ArcSin}\left[\sqrt{1-\left(\sqrt{c} x\right) / \sqrt{-a}}\right] / \sqrt{2}\right],\left(-2 a g\right) / \left(\sqrt{-a} \sqrt{c} f-a g\right)\right) / \left(\left(c d^2+a e^2\right)\left(e f-d g\right) \sqrt{\left(\sqrt{c}\left(f+g x\right)\right) / \left(\sqrt{c} f+\sqrt{-a} g\right)} \sqrt{a+c x^2}\right)+\left(\sqrt{-a} \sqrt{c} e f \sqrt{\frac{c x^2}{a}+1} \sqrt{\frac{\sqrt{c}\left(f+g x\right)}{\sqrt{-a g+\sqrt{c} f}}}\right) F\left(\sin ^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c} x}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| -\frac{2 a g}{\sqrt{-a} \sqrt{c} f-a g}\right) / \left(\sqrt{a+c x^2} \sqrt{f+g x}\left(a e^2+c d^2\right)\left(e f-d g\right)\right)+\left(\sqrt{-a} \sqrt{c} d g \sqrt{\frac{c x^2}{a}+1} \sqrt{\frac{\sqrt{c}\left(f+g x\right)}{\sqrt{-a g+\sqrt{c} f}}}\right) F\left(\sin ^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c} x}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| -\frac{2 a g}{\sqrt{-a} \sqrt{c} f-a g}\right) / \left(\sqrt{a+c x^2} \sqrt{f+g x}\left(a e^2+c d^2\right)\left(e f-d g\right)\right)-\left(\sqrt{-a} \sqrt{c} e \sqrt{\frac{c x^2}{a}+1} \sqrt{f+g x} E\left(\sin ^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c} x}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| -\frac{2 a g}{\sqrt{-a} \sqrt{c} f-a g}\right)\right) / \left(\sqrt{a+c x^2}\left(a e^2+c d^2\right)\left(e f-d g\right) \sqrt{\frac{\sqrt{c}\left(f+g x\right)}{\sqrt{-a g+\sqrt{c} f}}}\right)+\left(\sqrt{\frac{c x^2}{a}+1} \sqrt{\frac{\sqrt{c}\left(f+g x\right)}{\sqrt{-a g+\sqrt{c} f}}}\left(a e^2 g-c d\left(2 e f-3 d g\right)\right)\left(\frac{2 e}{\frac{\sqrt{c} d}{\sqrt{-a}}+e} ; \sin ^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{c} x}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| \frac{2 \sqrt{-a} g}{\sqrt{c} f+\sqrt{-a} g}\right)\right) / \left(\sqrt{a+c x^2} \sqrt{f+g x}\left(\frac{\sqrt{c} d}{\sqrt{-a}}+e\right)\left(a e^2+c d^2\right)\left(e f-d g\right)\right)$

**Rubi [A]** time = 5.03619, antiderivative size = 746, normalized size of antiderivative = 1., number of

steps used = 14, number of rules used = 10, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$\begin{aligned} & \frac{e^2 \sqrt{a + cx^2} \sqrt{f + gx}}{(d + ex)(ae^2 + cd^2)(ef - dg)} \\ & + \frac{\sqrt{-a} \sqrt{ce} f \sqrt{\frac{cx^2}{a}} + 1 \sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag} + \sqrt{cf}}} F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf}-ag}\right)}{\sqrt{a + cx^2} \sqrt{f + gx} (ae^2 + cd^2)(ef - dg)} \\ & - \frac{\sqrt{-a} \sqrt{cd} g \sqrt{\frac{cx^2}{a}} + 1 \sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag} + \sqrt{cf}}} F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf}-ag}\right)}{\sqrt{a + cx^2} \sqrt{f + gx} (ae^2 + cd^2)(ef - dg)} \\ & - \frac{\sqrt{-a} \sqrt{ce} \sqrt{\frac{cx^2}{a}} + 1 \sqrt{f + gx} E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf}-ag}\right)}{\sqrt{a + cx^2} (ae^2 + cd^2)(ef - dg) \sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag} + \sqrt{cf}}}} \\ & + \frac{\sqrt{\frac{cx^2}{a}} + 1 \sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag} + \sqrt{cf}}} (ae^2 g - cd(2ef - 3dg)) \left(\frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}} + e}; \sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right) \middle| \frac{2\sqrt{-ag}}{\sqrt{cf} + \sqrt{-ag}}\right)}{\sqrt{a + cx^2} \sqrt{f + gx} \left(\frac{\sqrt{cd}}{\sqrt{-a}} + e\right) (ae^2 + cd^2)(ef - dg)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)^2\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]),x]

[Out] -((e^2\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])/((c\*d^2 + a\*e^2)\*(e\*f - d\*g)\*(d + e\*x))) - (Sqrt[-a]\*Sqrt[c]\*e\*Sqrt[f + g\*x]\*Sqrt[1 + (c\*x^2)/a]\*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/((c\*d^2 + a\*e^2)\*(e\*f - d\*g)\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[a + c\*x^2]) + (Sqrt[-a]\*Sqrt[c]\*e\*f\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/((c\*d^2 + a\*e^2)\*(e\*f - d\*g)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]) - (Sqrt[-a]\*Sqrt[c]\*d\*g\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/((c\*d^2 + a\*e^2)\*(e\*f - d\*g)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]) + ((a\*e^2\*g - c\*d\*(2\*e\*f - 3\*d\*g))\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticPi[(2\*e)/((Sqrt[c]\*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (2\*Sqrt[-a]\*g)/(Sqrt[c]\*f + Sqrt[-a]\*g)]/(((Sqrt[c]\*d)/Sqrt[-a] + e)\*(c\*d^2 + a\*e^2)\*(e\*f - d\*g)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]))

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + cx^2} (d + ex)^2 \sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(e\*x+d)\*\*2/(g\*x+f)\*\*(1/2)/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] Integral(1/(sqrt(a + c\*x\*\*2)\*(d + e\*x)\*\*2\*sqrt(f + g\*x)), x)

**Mathematica [C]** time = 11.1016, size = 1491, normalized size = 2.

$$(f + gx)^{3/2} \left( -\frac{2ce^2 \sqrt{-f - \frac{i\sqrt{ag}}{\sqrt{e}}}}{\sqrt{e}} f^3}{(f+gx)^2} + \frac{4ce^2 \sqrt{-f - \frac{i\sqrt{ag}}{\sqrt{e}}}}{\sqrt{e}} f^2}{f+gx} + \frac{2cdeg \sqrt{-f - \frac{i\sqrt{ag}}{\sqrt{e}}}}{\sqrt{e}} f^2}{(f+gx)^2} - \frac{4icdeg \sqrt{-\frac{f}{f+gx} - \frac{i\sqrt{ag}}{\sqrt{c(f+gx)}}} + 1 \sqrt{-\frac{f}{f+gx} + \frac{i\sqrt{ag}}{\sqrt{c(f+gx)}}} + 1 \left( \frac{\sqrt{c}(ef-dg)}{e(\sqrt{cf+i\sqrt{ag}})} \right)}{\sqrt{f+gx}} \right)$$

$$\frac{e^2 \sqrt{f+gx} \sqrt{cx^2+a}}{(cd^2+ae^2)(ef-dg)(d+ex)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x)^2\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]),x]

[Out]  $-\left(\frac{e^2 \sqrt{f+gx} \sqrt{cx^2+a}}{(cd^2+ae^2)(ef-dg)(d+ex)}\right) + \left(\frac{(f+gx)^{3/2} (-2c^2 e^2 f \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}} + 2c^2 d e^2 g \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}} - (2c^2 e^2 f^3 \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}}) / (f+gx)^2 + (2c^2 d e^2 f^2 g \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}}) / (f+gx)^2 - (2a^2 e^2 f^3 g^2 \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}}) / (f+gx)^2 + (2a^2 d e^2 f^2 g^3 \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}}) / (f+gx)^2 + (4c^2 e^2 f^2 g^2 \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}}) / (f+gx) - (4c^2 d e^2 f g^2 \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}}) / (f+gx) + ((2I) \sqrt{c} e^2 (\sqrt{c} f + I \sqrt{a} g) (e f - d g) \sqrt{1 - f/(f+gx)} - (I \sqrt{a} g) / (\sqrt{c} (f+gx))) \sqrt{1 - f/(f+gx)} + (I \sqrt{a} g) / (\sqrt{c} (f+gx))\right) \text{EllipticE}\left[\frac{I \text{ArcSinh}\left[\frac{\sqrt{-f - (I \sqrt{a} g)/\sqrt{c}}}{\sqrt{f+gx}}\right], (\sqrt{c} f - I \sqrt{a} g) / (\sqrt{c} f + I \sqrt{a} g)}{\sqrt{f+gx}} + (2(\sqrt{c} d - I \sqrt{a} e) g (\sqrt{a} e g + I \sqrt{c} (e f - 2d g)) \sqrt{1 - f/(f+gx)} - (I \sqrt{a} g) / (\sqrt{c} (f+gx))) \sqrt{1 - f/(f+gx)} + (I \sqrt{a} g) / (\sqrt{c} (f+gx))\right) \text{EllipticF}\left[\frac{I \text{ArcSinh}\left[\frac{\sqrt{-f - (I \sqrt{a} g)/\sqrt{c}}}{\sqrt{f+gx}}\right], (\sqrt{c} f - I \sqrt{a} g) / (\sqrt{c} f + I \sqrt{a} g)}{\sqrt{f+gx}} - ((4I) c^2 d e^2 f g^2 \sqrt{1 - f/(f+gx)} - (I \sqrt{a} g) / (\sqrt{c} (f+gx))) \sqrt{1 - f/(f+gx)} + (I \sqrt{a} g) / (\sqrt{c} (f+gx))\right) \text{EllipticPi}\left[\frac{(\sqrt{c} (e f - d g)) / (e (\sqrt{c} f + I \sqrt{a} g))}{I \text{ArcSinh}\left[\frac{\sqrt{-f - (I \sqrt{a} g)/\sqrt{c}}}{\sqrt{f+gx}}\right], (\sqrt{c} f - I \sqrt{a} g) / (\sqrt{c} f + I \sqrt{a} g)}{\sqrt{f+gx}} + ((6I) c^2 d^2 g^2 \sqrt{1 - f/(f+gx)} - (I \sqrt{a} g) / (\sqrt{c} (f+gx))) \sqrt{1 - f/(f+gx)} + (I \sqrt{a} g) / (\sqrt{c} (f+gx))\right) \text{EllipticPi}\left[\frac{(\sqrt{c} (e f - d g)) / (e (\sqrt{c} f + I \sqrt{a} g))}{I \text{ArcSinh}\left[\frac{\sqrt{-f - (I \sqrt{a} g)/\sqrt{c}}}{\sqrt{f+gx}}\right], (\sqrt{c} f - I \sqrt{a} g) / (\sqrt{c} f + I \sqrt{a} g)}{\sqrt{f+gx}} + ((2I) a^2 e^2 g^2 \sqrt{1 - f/(f+gx)} - (I \sqrt{a} g) / (\sqrt{c} (f+gx))) \sqrt{1 - f/(f+gx)} + (I \sqrt{a} g) / (\sqrt{c} (f+gx))\right) \text{EllipticPi}\left[\frac{(\sqrt{c} (e f - d g)) / (e (\sqrt{c} f + I \sqrt{a} g))}{I \text{ArcSinh}\left[\frac{\sqrt{-f - (I \sqrt{a} g)/\sqrt{c}}}{\sqrt{f+gx}}\right], (\sqrt{c} f - I \sqrt{a} g) / (\sqrt{c} f + I \sqrt{a} g)}{\sqrt{f+gx}}\right) / (2(c^2 d^2 + a^2 e^2) g \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}} (e f - d g) (-e f + d g) \sqrt{a + (c(f+gx))^2} (-1 + f/(f+gx))^2) / g^2]$

**Maple [B]** time = 0.072, size = 5738, normalized size = 7.7

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x+d)^2/(g\*x+f)^(1/2)/(c\*x^2+a)^(1/2),x)

[Out] result too large to display

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + a}(ex + d)^2 \sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)^2\*sqrt(g\*x + f)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)^2\*sqrt(g\*x + f)), x)

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)^2\*sqrt(g\*x + f)),x, algorithm="fricas")

[Out] Timed out

---

**Sympy [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x+d)\*\*2/(g\*x+f)\*\*(1/2)/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] Exception raised: ValueError

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + a}(ex + d)^2 \sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)^2\*sqrt(g\*x + f)),x, algorithm="giac")

[Out] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)^2\*sqrt(g\*x + f)), x)



$$3.651 \quad \int \frac{1}{(d+ex)^3 \sqrt{f+gx} \sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=1257

result too large to display

```
[Out] -(e^2*Sqrt[f + g*x]*Sqrt[a + c*x^2])/(2*(c*d^2 + a*e^2)*(e*f - d*
g)*(d + e*x)^2) + (3*e^2*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[f +
g*x]*Sqrt[a + c*x^2])/(4*(c*d^2 + a*e^2)^2*(e*f - d*g)^2*(d + e*
x)) + (3*Sqrt[-a]*Sqrt[c]*e*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[
f + g*x]*Sqrt[1 + (c*x^2)/a]*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]*x
)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)]/(4*(c
*d^2 + a*e^2)^2*(e*f - d*g)^2*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f
+ Sqrt[-a]*g)]*Sqrt[a + c*x^2]) + (Sqrt[-a]*Sqrt[c]*g*Sqrt[(Sqrt
[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*Elli
pticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(S
qrt[-a]*Sqrt[c]*f - a*g)]/(2*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f
+ g*x]*Sqrt[a + c*x^2]) - (3*Sqrt[-a]*Sqrt[c]*e*f*(a*e^2*g - c*d*
(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f + g*x))/(Sqrt[c]*f + Sqrt[-a]*g
)]*Sqrt[1 + (c*x^2)/a]*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt
[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]*Sqrt[c]*f - a*g)]/(4*(c*d^2 +
a*e^2)^2*(e*f - d*g)^2*Sqrt[f + g*x]*Sqrt[a + c*x^2]) + (3*Sqrt[
-a]*Sqrt[c]*d*g*(a*e^2*g - c*d*(2*e*f - 3*d*g))*Sqrt[(Sqrt[c]*(f
+ g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticF[A
rcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/Sqrt[2]], (-2*a*g)/(Sqrt[-a]
*Sqrt[c]*f - a*g)]/(4*(c*d^2 + a*e^2)^2*(e*f - d*g)^2*Sqrt[f + g
*x]*Sqrt[a + c*x^2]) + (c*(e*f - 3*d*g)*Sqrt[(Sqrt[c]*(f + g*x))/
(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(2*e)/((
Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt[-a]]/S
qrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)]/(((Sqrt[c]*d)/
Sqrt[-a] + e)*(c*d^2 + a*e^2)*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a +
c*x^2]) - (3*(a*e^2*g - c*d*(2*e*f - 3*d*g))^2*Sqrt[(Sqrt[c]*(f +
g*x))/(Sqrt[c]*f + Sqrt[-a]*g)]*Sqrt[1 + (c*x^2)/a]*EllipticPi[(
2*e)/((Sqrt[c]*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]*x)/Sqrt
[-a]]/Sqrt[2]], (2*Sqrt[-a]*g)/(Sqrt[c]*f + Sqrt[-a]*g)]/(4*((Sq
rt[c]*d)/Sqrt[-a] + e)*(c*d^2 + a*e^2)^2*(e*f - d*g)^2*Sqrt[f + g
*x]*Sqrt[a + c*x^2])
```

---

**Rubi [A]** time = 10.234, antiderivative size = 1257, normalized size of antiderivative = 1., number of

steps used = 23, number of rules used = 10, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$

$$\begin{aligned}
& \frac{3(ae^2g - cd(2ef - 3dg))\sqrt{f+gx}\sqrt{cx^2+ae^2}}{4(cd^2+ae^2)^2(ef-dg)^2(d+ex)} - \frac{\sqrt{f+gx}\sqrt{cx^2+ae^2}}{2(cd^2+ae^2)(ef-dg)(d+ex)^2} \\
& + \frac{3\sqrt{-a}\sqrt{c}(ae^2g - cd(2ef - 3dg))\sqrt{f+gx}\sqrt{\frac{cx^2}{a}} + 1E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)e}{4(cd^2+ae^2)^2(ef-dg)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-ag}}}}\sqrt{cx^2+a}} \\
& - \frac{3\sqrt{-a}\sqrt{cf}(ae^2g - cd(2ef - 3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-ag}}}}\sqrt{\frac{cx^2}{a}} + 1F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)e}{4(cd^2+ae^2)^2(ef-dg)^2\sqrt{f+gx}\sqrt{cx^2+a}} \\
& + \frac{3\sqrt{-a}\sqrt{cdg}(ae^2g - cd(2ef - 3dg))\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-ag}}}}\sqrt{\frac{cx^2}{a}} + 1F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{4(cd^2+ae^2)^2(ef-dg)^2\sqrt{f+gx}\sqrt{cx^2+a}} \\
& + \frac{\sqrt{-a}\sqrt{cg}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-ag}}}}\sqrt{\frac{cx^2}{a}} + 1F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle| -\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{2(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{cx^2+a}} \\
& - \frac{3(ae^2g - cd(2ef - 3dg))^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-ag}}}}\sqrt{\frac{cx^2}{a}} + 1\left(\frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}}+e}; \sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle| \frac{2\sqrt{-ag}}{\sqrt{cf+\sqrt{-ag}}}\right)}{4\left(\frac{\sqrt{cd}}{\sqrt{-a}}+e\right)(cd^2+ae^2)^2(ef-dg)^2\sqrt{f+gx}\sqrt{cx^2+a}} \\
& + \frac{c(ef-3dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-ag}}}}\sqrt{\frac{cx^2}{a}} + 1\left(\frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}}+e}; \sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle| \frac{2\sqrt{-ag}}{\sqrt{cf+\sqrt{-ag}}}\right)}{\left(\frac{\sqrt{cd}}{\sqrt{-a}}+e\right)(cd^2+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{cx^2+a}}
\end{aligned}$$

Warning: Unable to verify antiderivative.

[In] Int[1/((d + e\*x)^3\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]),x]

[Out]  $-(e^2\sqrt{f+gx}\sqrt{a+c x^2})/(2(c^2d^2+a^2e^2)(e^2f-dg)(d+e x)^2) + (3e^2(a^2e^2g-cd(2e^2f-3d^2g))\sqrt{f+gx}\sqrt{a+c x^2})/(4(c^2d^2+a^2e^2)^2(e^2f-dg)^2(d+e x)) + (3\sqrt{-a}\sqrt{c}e(a^2e^2g-cd(2e^2f-3d^2g))\sqrt{f+gx}\sqrt{1+(c x^2)/a}\text{EllipticE}[\text{ArcSin}[\sqrt{1-(\sqrt{c}x)/\sqrt{-a}}]/\sqrt{2}],(-2ag)/(\sqrt{-a}\sqrt{c}f-ag)]/(4(c^2d^2+a^2e^2)^2(e^2f-dg)^2\sqrt{(\sqrt{c}(f+gx))/(\sqrt{c}f+\sqrt{-a}g)})\sqrt{a+c x^2}) + (\sqrt{-a}\sqrt{c}g\sqrt{(\sqrt{c}(f+gx))/(\sqrt{c}f+\sqrt{-a}g)}\sqrt{1+(c x^2)/a}\text{EllipticF}[\text{ArcSin}[\sqrt{1-(\sqrt{c}x)/\sqrt{-a}}]/\sqrt{2}],(-2ag)/(\sqrt{-a}\sqrt{c}f-ag)]/(2(c^2d^2+a^2e^2)(e^2f-dg)\sqrt{f+gx}\sqrt{a+c x^2}) - (3\sqrt{-a}\sqrt{c}e^2f(a^2e^2g-cd(2e^2f-3d^2g))\sqrt{(\sqrt{c}(f+gx))/(\sqrt{c}f+\sqrt{-a}g)})\sqrt{1+(c x^2)/a}\text{EllipticF}[\text{ArcSin}[\sqrt{1-(\sqrt{c}x)/\sqrt{-a}}]/\sqrt{2}],(-2ag)/(\sqrt{-a}\sqrt{c}f-ag)]/(4(c^2d^2+a^2e^2)^2(e^2f-dg)^2\sqrt{f+gx}\sqrt{a+c x^2}) + (3\sqrt{-a}\sqrt{c}d^2g(a^2e^2g-cd(2e^2f-3d^2g))\sqrt{(\sqrt{c}(f+gx))/(\sqrt{c}f+\sqrt{-a}g)})\sqrt{1+(c x^2)/a}\text{EllipticF}[\text{ArcSin}[\sqrt{1-(\sqrt{c}x)/\sqrt{-a}}]/\sqrt{2}],(-2ag)/(\sqrt{-a}\sqrt{c}f-ag)]/(4(c^2d^2+a^2e^2)^2(e^2f-dg)^2\sqrt{f+gx}\sqrt{a+c x^2}) + (c(e^2f-3d^2g)\sqrt{(\sqrt{c}(f+gx))/(\sqrt{c}f+\sqrt{-a}g)}\sqrt{1+(c x^2)/a}\text{EllipticPi}[(2e)/((\sqrt{c}d)/\sqrt{-a}+e),\text{ArcSin}[\sqrt{1-(\sqrt{c}x)/\sqrt{-a}}]/\sqrt{2}],(2\sqrt{-a}g)/(\sqrt{c}f+\sqrt{-a}g)]/(((\sqrt{c}d)/\sqrt{-a}+e)(c^2d^2+a^2e^2)(e^2f-dg)\sqrt{f+gx}\sqrt{a+c x^2}) - (3(a^2e^2g-cd(2e^2f-3d^2g))^2\sqrt{(\sqrt{c}(f+gx))/(\sqrt{c}f+\sqrt{-a}g)})\sqrt{1+(c x^2)/a}\text{EllipticPi}[(2e)/((\sqrt{c}d)/\sqrt{-a}+e),\text{ArcSin}[\sqrt{1-(\sqrt{c}x)/\sqrt{-a}}]/\sqrt{2}]$

$$\frac{[-a]/\sqrt{2}], (2\sqrt{-a}g)/(\sqrt{c}f + \sqrt{-a}g)]/(4((\sqrt{c}d)/\sqrt{-a} + e)(c^2d^2 + a^2e^2)^2(e^2f - d^2g)^2\sqrt{f + gx})\sqrt{a + cx^2}]$$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + cx^2} (d + ex)^3 \sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(e*x+d)**3/(g*x+f)**(1/2)/(c*x**2+a)**(1/2), x)`

[Out] `Integral(1/(sqrt(a + c*x**2)*(d + e*x)**3*sqrt(f + g*x)), x)`

**Mathematica [C]** time = 15.7331, size = 15233, normalized size = 12.12

Result too large to show

Antiderivative was successfully verified.

[In] `Integrate[1/((d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + c*x^2]), x]`

[Out] Result too large to show

**Maple [B]** time = 0.128, size = 20365, normalized size = 16.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+a)^(1/2), x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + a}(ex + d)^3\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^3*sqrt(g*x + f)), x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^3*sqrt(g*x + f)), x)`

**Ericas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^3*sqrt(g*x + f)),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy** [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)**3/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)
```

```
[Out] Exception raised: ValueError
```

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + a}(ex + d)^3 \sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^3*sqrt(g*x + f)),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(c*x^2 + a)*(e*x + d)^3*sqrt(g*x + f)), x)
```

$$3.652 \quad \int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=387

$$\frac{2g^2\sqrt{a+cx^2}}{\sqrt{f+gx}(ag^2+cf^2)(ef-dg)} + \frac{2\sqrt{-a}\sqrt{cg}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{\sqrt{a+cx^2}(ag^2+cf^2)(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}}$$

$$- \frac{2e\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}\left(\frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}}+e}; \sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{-ag}}{\sqrt{cf+\sqrt{-ag}}}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{cd}}{\sqrt{-a}}+e\right)(ef-dg)}$$

[Out] (2\*g^2\*Sqrt[a + c\*x^2])/((e\*f - d\*g)\*(c\*f^2 + a\*g^2)\*Sqrt[f + g\*x]) + (2\*Sqrt[-a]\*Sqrt[c]\*g\*Sqrt[f + g\*x]\*Sqrt[1 + (c\*x^2)/a]\*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/((e\*f - d\*g)\*(c\*f^2 + a\*g^2)\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[a + c\*x^2]) - (2\*e\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticPi[(2\*e)/((Sqrt[c]\*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (2\*Sqrt[-a]\*g)/(Sqrt[c]\*f + Sqrt[-a]\*g)]/(((Sqrt[c]\*d)/Sqrt[-a] + e)\*(e\*f - d\*g)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]))

**Rubi [A]** time = 2.03397, antiderivative size = 387, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$

$$\frac{2g^2\sqrt{a+cx^2}}{\sqrt{f+gx}(ag^2+cf^2)(ef-dg)} + \frac{2\sqrt{-a}\sqrt{cg}\sqrt{\frac{cx^2}{a}+1}\sqrt{f+gx}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{\sqrt{a+cx^2}(ag^2+cf^2)(ef-dg)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}}$$

$$- \frac{2e\sqrt{\frac{cx^2}{a}+1}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{-ag+\sqrt{cf}}}}\left(\frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}}+e}; \sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{-ag}}{\sqrt{cf+\sqrt{-ag}}}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}\left(\frac{\sqrt{cd}}{\sqrt{-a}}+e\right)(ef-dg)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)\*(f + g\*x)^(3/2)\*Sqrt[a + c\*x^2]),x]

[Out] (2\*g^2\*Sqrt[a + c\*x^2])/((e\*f - d\*g)\*(c\*f^2 + a\*g^2)\*Sqrt[f + g\*x]) + (2\*Sqrt[-a]\*Sqrt[c]\*g\*Sqrt[f + g\*x]\*Sqrt[1 + (c\*x^2)/a]\*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)]/((e\*f - d\*g)\*(c\*f^2 + a\*g^2)\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[a + c\*x^2]) - (2\*e\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticPi[(2\*e)/((Sqrt[c]\*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (2\*Sqrt[-a]\*g)/(Sqrt[c]\*f + Sqrt[-a]\*g)]/(((Sqrt[c]\*d)/Sqrt[-a] + e)\*(e\*f - d\*g)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]))

**Rubi in Sympy [A]** time = 115.104, size = 394, normalized size = 1.02

$$\frac{2\sqrt{c}g\sqrt{-a}\sqrt{1+\frac{cx^2}{a}}\sqrt{f+gx}E\left(\operatorname{asin}\left(\sqrt{-\frac{\sqrt{c}x}{2\sqrt{-a}}+\frac{1}{2}}\right)\middle|\frac{2ag}{ag-\sqrt{c}f\sqrt{-a}}\right)}{\sqrt{\frac{\sqrt{c}\sqrt{-a}(-f-gx)}{ag-\sqrt{c}f\sqrt{-a}}}\sqrt{a+cx^2}(ag^2+cf^2)(dg-ef)}$$

$$-\frac{2e\sqrt{\frac{g(-\sqrt{c}x-\sqrt{-a})}{\sqrt{c}f-g\sqrt{-a}}}\sqrt{\frac{g(-\sqrt{c}x+\sqrt{-a})}{\sqrt{c}f+g\sqrt{-a}}}\left(-\frac{e(\sqrt{c}f+g\sqrt{-a})}{\sqrt{c}(dg-ef)};\operatorname{asin}\left(\sqrt{\frac{c}{\sqrt{c}g\sqrt{-a}+cf}}\sqrt{f+gx}\right)\middle|\frac{\sqrt{c}f+g\sqrt{-a}}{\sqrt{c}f-g\sqrt{-a}}\right)}{\sqrt{\frac{c}{\sqrt{c}g\sqrt{-a}+cf}}\sqrt{a+cx^2}(dg-ef)^2}$$

$$-\frac{2g^2\sqrt{a+cx^2}}{\sqrt{f+gx}(ag^2+cf^2)(dg-ef)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(e*x+d)/(g*x+f)**(3/2)/(c*x**2+a)**(1/2),x)`

[Out] `-2*sqrt(c)*g*sqrt(-a)*sqrt(1+c*x**2/a)*sqrt(f+g*x)*elliptic_e(asin(sqrt(-sqrt(c)*x/(2*sqrt(-a))+1/2)),2*a*g/(a*g-sqrt(c)*f*sqrt(-a))/(sqrt(sqrt(c)*sqrt(-a)*(-f-g*x)/(a*g-sqrt(c)*f*sqrt(-a))*sqrt(a+c*x**2)*(a*g**2+c*f**2)*(d*g-e*f))-2*e*sqrt(g*(-sqrt(c)*x-sqrt(-a))/(sqrt(c)*f-g*sqrt(-a))*sqrt(g*(-sqrt(c)*x+sqrt(-a))/(sqrt(c)*f+g*sqrt(-a))*elliptic_pi(-e*(sqrt(c)*f+g*sqrt(-a))/(sqrt(c)*(d*g-e*f)),asin(sqrt(c/(sqrt(c)*g*sqrt(-a)+c*f))*sqrt(f+g*x)),(sqrt(c)*f+g*sqrt(-a))/(sqrt(c)*f-g*sqrt(-a))/(sqrt(c/(sqrt(c)*g*sqrt(-a)+c*f))*sqrt(a+c*x**2)*(d*g-e*f)**2)-2*g**2*sqrt(a+c*x**2)/(sqrt(f+g*x)*(a*g**2+c*f**2)*(d*g-e*f))`

**Mathematica [C]** time = 6.35817, size = 468, normalized size = 1.21

$$2i(f+gx)\sqrt{\frac{g\left(x+\frac{i\sqrt{a}}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{i\sqrt{ag}}{\sqrt{c}}}{f+gx}}\left(\sqrt{c}(dg-2ef)+i\sqrt{a}eg\right)F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}f-i\sqrt{ag}}{\sqrt{c}f+i\sqrt{ag}}\right)+\sqrt{c}(ef-dg)E\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{i\sqrt{ag}}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{c}f-i\sqrt{ag}}{\sqrt{c}f+i\sqrt{ag}}\right)$$

Antiderivative was successfully verified.

[In] `Integrate[1/((d+e*x)*(f+g*x)^(3/2)*Sqrt[a+c*x^2]),x]`

[Out] `((2*I)*Sqrt[(g*((I*Sqrt[a])/Sqrt[c]+x))/(f+g*x])*Sqrt[-(((I*Sqrt[a]*g)/Sqrt[c]-g*x)/(f+g*x))*(f+g*x)*(Sqrt[c]*(e*f-d*g)*EllipticE[I*ArcSinh[Sqrt[-f-(I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f+g*x]],(Sqrt[c]*f-I*Sqrt[a]*g)/(Sqrt[c]*f+I*Sqrt[a]*g)]+(I*Sqrt[a]*e*g+Sqrt[c]*(-2*e*f+d*g))*EllipticF[I*ArcSinh[Sqrt[-f-(I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f+g*x]],(Sqrt[c]*f-I*Sqrt[a]*g)/(Sqrt[c]*f+I*Sqrt[a]*g)]+e*(Sqrt[c]*f-I*Sqrt[a]*g)*EllipticPi[(Sqrt[c]*(e*f-d*g))/(e*(Sqrt[c]*f+I*Sqrt[a]*g)),I*ArcSinh[Sqrt[-f-(I*Sqrt[a]*g)/Sqrt[c]]/Sqrt[f+g*x]],(Sqrt[c]*f-I*Sqrt[a]*g)/(Sqrt[c]*f+I*Sqrt[a]*g)))/((Sqrt[c]*f-I*Sqrt[a]*g)*Sqrt[-f-(I*Sqrt[a]*g)/Sqrt[c]]*(e*f-d*g)^2*Sqrt[a+c*x^2])`

**Maple [B]** time = 0.093, size = 2011, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x+d)/(g\*x+f)^(3/2)/(c\*x^2+a)^(1/2),x)

[Out]  $-2 * (\text{EllipticPi}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (g*(-a*c)^{(1/2)}-c*f)*e/c/(d*g-e*f), (-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}) * a*c*e*f*g^2 * (-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)} * ((-c*x+(-a*c)^{(1/2)}) * g/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * ((c*x+(-a*c)^{(1/2)}) * g/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)} - \text{EllipticPi}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (g*(-a*c)^{(1/2)}-c*f)*e/c/(d*g-e*f), (-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}) * a*e*g^3 * (-a*c)^{(1/2)} * (-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)} * ((-c*x+(-a*c)^{(1/2)}) * g/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * ((c*x+(-a*c)^{(1/2)}) * g/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)} + \text{EllipticPi}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (g*(-a*c)^{(1/2)}-c*f)*e/c/(d*g-e*f), (-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}) * c^2 * e*f^3 * (-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)} * ((-c*x+(-a*c)^{(1/2)}) * g/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * ((c*x+(-a*c)^{(1/2)}) * g/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)} - \text{EllipticPi}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (g*(-a*c)^{(1/2)}-c*f)*e/c/(d*g-e*f), (-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}) * c*e*f^2 * g*(-a*c)^{(1/2)} * (-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)} * ((-c*x+(-a*c)^{(1/2)}) * g/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * ((c*x+(-a*c)^{(1/2)}) * g/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)} - (-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)} * ((-c*x+(-a*c)^{(1/2)}) * g/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * ((c*x+(-a*c)^{(1/2)}) * g/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)} * \text{EllipticF}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}) * a*c*d*g^3 + \text{EllipticF}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}) * a*c*e*f*g^2 * (-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)} * ((-c*x+(-a*c)^{(1/2)}) * g/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * ((c*x+(-a*c)^{(1/2)}) * g/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)} - (-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)} * ((-c*x+(-a*c)^{(1/2)}) * g/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * ((c*x+(-a*c)^{(1/2)}) * g/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)} * \text{EllipticF}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}) * c^2 * d*f^2 * g + \text{EllipticF}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}) * c^2 * e*f^3 * (-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)} * ((-c*x+(-a*c)^{(1/2)}) * g/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * ((c*x+(-a*c)^{(1/2)}) * g/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)} + (-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)} * ((-c*x+(-a*c)^{(1/2)}) * g/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * ((c*x+(-a*c)^{(1/2)}) * g/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)} * \text{EllipticE}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}) * a*c*d*g^3 - (-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)} * ((-c*x+(-a*c)^{(1/2)}) * g/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * ((c*x+(-a*c)^{(1/2)}) * g/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)} * \text{EllipticE}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)}) * a*c*e*f*g^2 + (-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)} * ((-c*x+(-a*c)^{(1/2)}) * g/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * ((c*x+(-a*c)^{(1/2)}) * g/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)} * \text{EllipticE}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * c^2 * d*f^2 * g - (-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)} * ((-c*x+(-a*c)^{(1/2)}) * g/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * ((c*x+(-a*c)^{(1/2)}) * g/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)} * \text{EllipticE}((-g*x+f)*c/(g*(-a*c)^{(1/2)}-c*f))^{(1/2)}, (-g*(-a*c)^{(1/2)}-c*f)/(g*(-a*c)^{(1/2)}+c*f))^{(1/2)} * c^2 * e*f^3 + x^2 * c^2 * d*g^3 - x^2 * c^2 * e*f*g^2 + a*c*d*g^3 - a*c*e*f*g^2) * (c*x^2+a)^(1/2) * (g*x+f)^(1/2) / (a*g^2+c*f^2) / (d*g-e*f)^2 / (c*g*x^3+c*f*x^2+a*g*x+a*f)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}(ex+d)(gx+f)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)\*(g\*x + f)^(3/2)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c\*x^2 + a)\*(e\*x + d)\*(g\*x + f)^(3/2)), x)

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*(g*x + f)^(3/2)),x, algorithm="fricas")`

[Out] Timed out

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + cx^2}(d + ex)(f + gx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x+d)/(g*x+f)**(3/2)/(c*x**2+a)**(1/2),x)`

[Out] `Integral(1/(sqrt(a + c*x**2)*(d + e*x)*(f + g*x)**(3/2)), x)`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + a}(ex + d)(gx + f)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*(g*x + f)^(3/2)),x, algorithm="giac")`

[Out] `integrate(1/(sqrt(c*x^2 + a)*(e*x + d)*(g*x + f)^(3/2)), x)`



$$3.653 \quad \int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=818

$$\begin{aligned} & \frac{2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf}+\sqrt{-ag}}}\sqrt{\frac{cx^2}{a}+1}\left(\frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}}+e}; \sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{-ag}}{\sqrt{cf}+\sqrt{-ag}}\right)e^2}{\left(\frac{\sqrt{cd}}{\sqrt{-a}}+e\right)(ef-dg)^2\sqrt{f+gx}\sqrt{cx^2+a}} \\ & + \frac{2\sqrt{-a}\sqrt{cg}\sqrt{f+gx}\sqrt{\frac{cx^2}{a}+1}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)e}{(ef-dg)^2(cf^2+ag^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf}+\sqrt{-ag}}}\sqrt{cx^2+a}} \\ & + \frac{2g^2\sqrt{cx^2+ae}}{(ef-dg)^2(cf^2+ag^2)\sqrt{f+gx}} \\ & + \frac{8\sqrt{-a}c^{3/2}fg\sqrt{f+gx}\sqrt{\frac{cx^2}{a}+1}E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{3(ef-dg)(cf^2+ag^2)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf}+\sqrt{-ag}}}\sqrt{cx^2+a}} \\ & - \frac{2\sqrt{-a}\sqrt{cg}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf}+\sqrt{-ag}}}\sqrt{\frac{cx^2}{a}+1}F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\middle|-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}\right)}{3(ef-dg)(cf^2+ag^2)\sqrt{f+gx}\sqrt{cx^2+a}} \\ & + \frac{8cf^2g^2\sqrt{cx^2+a}}{3(ef-dg)(cf^2+ag^2)^2\sqrt{f+gx}} + \frac{2g^2\sqrt{cx^2+a}}{3(ef-dg)(cf^2+ag^2)(f+gx)^{3/2}} \end{aligned}$$

[Out] (2\*g^2\*Sqrt[a + c\*x^2])/(3\*(e\*f - d\*g)\*(c\*f^2 + a\*g^2)\*(f + g\*x)^(3/2)) + (8\*c\*f\*g^2\*Sqrt[a + c\*x^2])/(3\*(e\*f - d\*g)\*(c\*f^2 + a\*g^2)^2\*Sqrt[f + g\*x]) + (2\*e\*g^2\*Sqrt[a + c\*x^2])/((e\*f - d\*g)^2\*(c\*f^2 + a\*g^2)\*Sqrt[f + g\*x]) + (8\*Sqrt[-a]\*c^(3/2)\*f\*g\*Sqrt[f + g\*x]\*Sqrt[1 + (c\*x^2)/a]\*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)))/(3\*(e\*f - d\*g)\*(c\*f^2 + a\*g^2)^2\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[a + c\*x^2]) + (2\*Sqrt[-a]\*Sqrt[c]\*e\*g\*Sqrt[f + g\*x]\*Sqrt[1 + (c\*x^2)/a]\*EllipticE[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)))/((e\*f - d\*g)^2\*(c\*f^2 + a\*g^2)\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[a + c\*x^2]) - (2\*Sqrt[-a]\*Sqrt[c]\*g\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticF[ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (-2\*a\*g)/(Sqrt[-a]\*Sqrt[c]\*f - a\*g)))/(3\*(e\*f - d\*g)\*(c\*f^2 + a\*g^2)\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]) - (2\*e^2\*Sqrt[(Sqrt[c]\*(f + g\*x))/(Sqrt[c]\*f + Sqrt[-a]\*g)]\*Sqrt[1 + (c\*x^2)/a]\*EllipticPi[(2\*e)/((Sqrt[c]\*d)/Sqrt[-a] + e), ArcSin[Sqrt[1 - (Sqrt[c]\*x)/Sqrt[-a]]/Sqrt[2]], (2\*Sqrt[-a]\*g)/(Sqrt[c]\*f + Sqrt[-a]\*g)))/(((Sqrt[c]\*d)/Sqrt[-a] + e)\*(e\*f - d\*g)^2\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2])

**Rubi [A]** time = 3.08598, antiderivative size = 818, normalized size of antiderivative = 1., number of

steps used = 17, number of rules used = 12, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$

$$\begin{aligned}
& \frac{2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-ag}}}}\sqrt{\frac{cx^2}{a}}+1\left(\frac{2e}{\frac{\sqrt{cd}}{\sqrt{-a}}+e};\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\Big|_{\frac{2\sqrt{-ag}}{\sqrt{cf+\sqrt{-ag}}}}\right)e^2}{\left(\frac{\sqrt{cd}}{\sqrt{-a}}+e\right)(ef-dg)^2\sqrt{f+gx}\sqrt{cx^2+a}} \\
& + \frac{2\sqrt{-a}\sqrt{cg}\sqrt{f+gx}\sqrt{\frac{cx^2}{a}}+1E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\Big|_{-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}}\right)e}{(ef-dg)^2(cf^2+ag^2)\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-ag}}}}\sqrt{cx^2+a}} \\
& + \frac{2g^2\sqrt{cx^2+ae}}{(ef-dg)^2(cf^2+ag^2)\sqrt{f+gx}} \\
& + \frac{8\sqrt{-a}c^{3/2}fg\sqrt{f+gx}\sqrt{\frac{cx^2}{a}}+1E\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\Big|_{-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}}\right)}{3(ef-dg)(cf^2+ag^2)^2\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-ag}}}}\sqrt{cx^2+a}} \\
& - \frac{2\sqrt{-a}\sqrt{cg}\sqrt{\frac{\sqrt{c}(f+gx)}{\sqrt{cf+\sqrt{-ag}}}}\sqrt{\frac{cx^2}{a}}+1F\left(\sin^{-1}\left(\frac{\sqrt{1-\frac{\sqrt{cx}}{\sqrt{-a}}}}{\sqrt{2}}\right)\Big|_{-\frac{2ag}{\sqrt{-a}\sqrt{cf-ag}}}\right)}{3(ef-dg)(cf^2+ag^2)\sqrt{f+gx}\sqrt{cx^2+a}} \\
& + \frac{8cf^2g^2\sqrt{cx^2+a}}{3(ef-dg)(cf^2+ag^2)^2\sqrt{f+gx}} + \frac{2g^2\sqrt{cx^2+a}}{3(ef-dg)(cf^2+ag^2)(f+gx)^{3/2}}
\end{aligned}$$

Warning: Unable to verify antiderivative.

[In] Int[1/((d + e\*x)\*(f + g\*x)^(5/2)\*Sqrt[a + c\*x^2]),x]

[Out]  $(2*g^2*\text{Sqrt}[a + c*x^2])/(3*(e*f - d*g)*(c*f^2 + a*g^2)*(f + g*x)^{(3/2)}) + (8*c*f*g^2*\text{Sqrt}[a + c*x^2])/(3*(e*f - d*g)*(c*f^2 + a*g^2)^2*\text{Sqrt}[f + g*x]) + (2*e*g^2*\text{Sqrt}[a + c*x^2])/((e*f - d*g)^2*(c*f^2 + a*g^2)*\text{Sqrt}[f + g*x]) + (8*\text{Sqrt}[-a]*c^{(3/2)}*f*g*\text{Sqrt}[f + g*x]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)))/(3*(e*f - d*g)*(c*f^2 + a*g^2)^2*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[a + c*x^2]) + (2*\text{Sqrt}[-a]*\text{Sqrt}[c]*e*g*\text{Sqrt}[f + g*x]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)))/((e*f - d*g)^2*(c*f^2 + a*g^2)*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[a + c*x^2]) - (2*\text{Sqrt}[-a]*\text{Sqrt}[c]*g*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (-2*a*g)/(\text{Sqrt}[-a]*\text{Sqrt}[c]*f - a*g)))/(3*(e*f - d*g)*(c*f^2 + a*g^2)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2]) - (2*e^2*\text{Sqrt}[(\text{Sqrt}[c]*(f + g*x))/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)]*\text{Sqrt}[1 + (c*x^2)/a]*\text{EllipticPi}[(2*e)/((\text{Sqrt}[c]*d)/\text{Sqrt}[-a] + e), \text{ArcSin}[\text{Sqrt}[1 - (\text{Sqrt}[c]*x)/\text{Sqrt}[-a]]/\text{Sqrt}[2]], (2*\text{Sqrt}[-a]*g)/(\text{Sqrt}[c]*f + \text{Sqrt}[-a]*g)))/(((\text{Sqrt}[c]*d)/\text{Sqrt}[-a] + e)*(e*f - d*g)^2*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + c*x^2])$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(e\*x+d)/(g\*x+f)\*\*(5/2)/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] Timed out

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**Mathematica [C]** time = 14.2351, size = 2069, normalized size = 2.53

Result too large to show

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x)\*(f + g\*x)^(5/2)\*Sqrt[a + c\*x^2]),x]

[Out]  $\sqrt{f + gx} \sqrt{a + cx^2} \left( \frac{(2g^2)}{3(e f - dg)(c f^2 + a g^2)(f + gx)^2} + \frac{2g^2(7c^2 e f^2 - 4c^2 d f g + 3a^2 e g^2)}{3(e f - dg)^2(c f^2 + a g^2)^2(f + gx)} \right) - \frac{2(f + gx)^{3/2}}{(7c^2 e^2 f^3 \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}} - 11c^2 d^2 e f^2 g \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}} + 4c^2 d^2 f g^2 \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}} + 3a^2 c^2 e^2 f g^2 \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}} - 3a^2 c^2 d^2 e g^3 \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}} + (7c^2 e^2 f^5 \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}})/(f + gx)^2 - (11c^2 d^2 e f^4 g \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}})/(f + gx)^2 + (4c^2 d^2 f^3 g^2 \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}})/(f + gx)^2 + (10a^2 c^2 e^2 f^3 g^2 \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}})/(f + gx)^2 - (14a^2 c^2 d^2 e f^2 g^3 \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}})/(f + gx)^2 + (4a^2 c^2 d^2 f^2 g^4 \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}})/(f + gx)^2 + (3a^2 e^2 f^2 g^4 \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}})/(f + gx)^2 - (3a^2 d^2 e g^5 \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}})/(f + gx)^2 - (14c^2 e^2 f^4 g \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}})/(f + gx) + (22c^2 d^2 e f^3 g \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}})/(f + gx) - (8c^2 d^2 f^2 g^2 \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}})/(f + gx) - (6a^2 c^2 e^2 f^2 g^2 \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}})/(f + gx) + (6a^2 c^2 d^2 e f g^3 \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}})/(f + gx) + (\sqrt{c})((-I) \sqrt{c} f + \sqrt{a} g)(e f - dg)(3a^2 e g^2 + c f(7e f - 4d g)) \sqrt{1 - f/(f + gx) - (I \sqrt{a} g)/(\sqrt{c}(f + gx))} \sqrt{1 - f/(f + gx) + (I \sqrt{a} g)/(\sqrt{c}(f + gx))} \text{EllipticE}[I \text{ArcSinh}[\sqrt{-f - (I \sqrt{a} g)/\sqrt{c}}]/\sqrt{f + gx}], (\sqrt{c} f - I \sqrt{a} g)/(\sqrt{c} f + I \sqrt{a} g))/\sqrt{f + gx} + ((\sqrt{c} f + I \sqrt{a} g)(3a^{3/2} e^2 g^3 + (3I) a \sqrt{c} e g^2 (2e f - dg) + \sqrt{a} c g (2e^2 f^2 + 2d^2 e f g - d^2 g^2) + (3I) c^{3/2} f (3e^2 f^2 - 3d^2 e f g + d^2 g^2)) \sqrt{1 - f/(f + gx) - (I \sqrt{a} g)/(\sqrt{c}(f + gx))} \sqrt{1 - f/(f + gx) + (I \sqrt{a} g)/(\sqrt{c}(f + gx))} \text{EllipticF}[I \text{ArcSinh}[\sqrt{-f - (I \sqrt{a} g)/\sqrt{c}}]/\sqrt{f + gx}], (\sqrt{c} f - I \sqrt{a} g)/(\sqrt{c} f + I \sqrt{a} g))/\sqrt{f + gx} - ((3I) c^2 e^2 f^4 \sqrt{1 - f/(f + gx) - (I \sqrt{a} g)/(\sqrt{c}(f + gx))} \sqrt{1 - f/(f + gx) + (I \sqrt{a} g)/(\sqrt{c}(f + gx))} \text{EllipticPi}[(\sqrt{c}(e f - dg))/(e(\sqrt{c} f + I \sqrt{a} g))], I \text{ArcSinh}[\sqrt{-f - (I \sqrt{a} g)/\sqrt{c}}]/\sqrt{f + gx}], (\sqrt{c} f - I \sqrt{a} g)/(\sqrt{c} f + I \sqrt{a} g))/\sqrt{f + gx} - ((6I) a^2 c^2 e^2 f^2 g^2 \sqrt{1 - f/(f + gx) - (I \sqrt{a} g)/(\sqrt{c}(f + gx))} \sqrt{1 - f/(f + gx) + (I \sqrt{a} g)/(\sqrt{c}(f + gx))} \text{EllipticPi}[(\sqrt{c}(e f - dg))/(e(\sqrt{c} f + I \sqrt{a} g))], I \text{ArcSinh}[\sqrt{-f - (I \sqrt{a} g)/\sqrt{c}}]/\sqrt{f + gx}], (\sqrt{c} f - I \sqrt{a} g)/(\sqrt{c} f + I \sqrt{a} g))/\sqrt{f + gx} - ((3I) a^2 e^2 g^4 \sqrt{1 - f/(f + gx) - (I \sqrt{a} g)/(\sqrt{c}(f + gx))} \sqrt{1 - f/(f + gx) + (I \sqrt{a} g)/(\sqrt{c}(f + gx))} \text{EllipticPi}[(\sqrt{c}(e f - dg))/(e(\sqrt{c} f + I \sqrt{a} g))], I \text{ArcSinh}[\sqrt{-f - (I \sqrt{a} g)/\sqrt{c}}]/\sqrt{f + gx}], (\sqrt{c} f - I \sqrt{a} g)/(\sqrt{c} f + I \sqrt{a} g))/\sqrt{f + gx} - ((3 \sqrt{-f - (I \sqrt{a} g)/\sqrt{c}})(e f - dg)(-(e f) + dg)^2 (c f^2 + a g^2)^2 \sqrt{a + (c(f + gx)^2(-1 + f/(f + gx))^2)/g^2})$

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**Maple [B]** time = 0.129, size = 9409, normalized size = 11.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+a)^(1/2),x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}(ex+d)(gx+f)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*x^2+a)*(e*x+d)*(g*x+f)^(5/2)),x,algorithm="maxima")`

[Out] `integrate(1/(sqrt(c*x^2+a)*(e*x+d)*(g*x+f)^(5/2)),x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*x^2+a)*(e*x+d)*(g*x+f)^(5/2)),x,algorithm="fricas")`

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+cx^2}(d+ex)(f+gx)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x+d)/(g*x+f)**(5/2)/(c*x**2+a)**(1/2),x)`

[Out] `Integral(1/(sqrt(a+c*x**2)*(d+e*x)*(f+g*x)**(5/2)),x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+a}(ex+d)(gx+f)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*x^2+a)*(e*x+d)*(g*x+f)^(5/2)),x,algorithm="giac")`

[Out] `integrate(1/(sqrt(c*x^2+a)*(e*x+d)*(g*x+f)^(5/2)),x)`

$$3.654 \quad \int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{1+cx^2}} dx$$

**Optimal.** Leaf size=110

$$\frac{2\sqrt{\frac{\sqrt{-c}(f+gx)}{\sqrt{-cf+g}}}\left(\frac{2e}{\sqrt{-cd+e}}; \sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c}x}}{\sqrt{2}}\right)\middle|\frac{2g}{\sqrt{-cf+g}}\right)}{(\sqrt{-cd+e})\sqrt{f+gx}}$$

[Out] (-2\*Sqrt[(Sqrt[-c]\*(f + g\*x))/(Sqrt[-c]\*f + g)]\*EllipticPi[(2\*e)/(Sqrt[-c]\*d + e), ArcSin[Sqrt[1 - Sqrt[-c]\*x]/Sqrt[2]], (2\*g)/(Sqrt[-c]\*f + g)]/((Sqrt[-c]\*d + e)\*Sqrt[f + g\*x])

**Rubi [A]** time = 1.02136, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{2\sqrt{\frac{\sqrt{-c}(f+gx)}{\sqrt{-cf+g}}}\left(\frac{2e}{\sqrt{-cd+e}}; \sin^{-1}\left(\frac{\sqrt{1-\sqrt{-c}x}}{\sqrt{2}}\right)\middle|\frac{2g}{\sqrt{-cf+g}}\right)}{(\sqrt{-cd+e})\sqrt{f+gx}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)\*Sqrt[f + g\*x]\*Sqrt[1 + c\*x^2]), x]

[Out] (-2\*Sqrt[(Sqrt[-c]\*(f + g\*x))/(Sqrt[-c]\*f + g)]\*EllipticPi[(2\*e)/(Sqrt[-c]\*d + e), ArcSin[Sqrt[1 - Sqrt[-c]\*x]/Sqrt[2]], (2\*g)/(Sqrt[-c]\*f + g)]/((Sqrt[-c]\*d + e)\*Sqrt[f + g\*x])

**Rubi in Sympy [A]** time = 14.2788, size = 162, normalized size = 1.47

$$\frac{2\sqrt{\frac{g(-cx-\sqrt{-c})}{cf-g\sqrt{-c}}}\sqrt{\frac{g(-cx+\sqrt{-c})}{cf+g\sqrt{-c}}}\left(\frac{e(cf+g\sqrt{-c})}{c(-dg+ef)}; \operatorname{asin}\left(\sqrt{\frac{c}{cf+g\sqrt{-c}}}\sqrt{f+gx}\right)\middle|\frac{cf+g\sqrt{-c}}{cf-g\sqrt{-c}}\right)}{\sqrt{\frac{c}{cf+g\sqrt{-c}}}\sqrt{cx^2+1}(dg-ef)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(e\*x+d)/(g\*x+f)\*\*(1/2)/(c\*x\*\*2+1)\*\*(1/2), x)

[Out] 2\*sqrt(g\*(-c\*x - sqrt(-c))/(c\*f - g\*sqrt(-c)))\*sqrt(g\*(-c\*x + sqrt(-c))/(c\*f + g\*sqrt(-c)))\*elliptic\_pi(e\*(c\*f + g\*sqrt(-c))/(c\*(-d\*g + e\*f)), asin(sqrt(c/(c\*f + g\*sqrt(-c)))\*sqrt(f + g\*x)), (c\*f + g\*sqrt(-c))/(c\*f - g\*sqrt(-c)))/(sqrt(c/(c\*f + g\*sqrt(-c)))\*sqrt(c\*x\*\*2 + 1)\*(d\*g - e\*f))

**Mathematica [C]** time = 0.804204, size = 261, normalized size = 2.37

$$\frac{2i(f+gx)\sqrt{\frac{g\left(x+\frac{i}{\sqrt{c}}\right)}{f+gx}}\sqrt{-\frac{-gx+\frac{ig}{\sqrt{c}}}{f+gx}}\left(F\left(i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{ig}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{cf-ig}}{\sqrt{cf+ig}}\right)-\left(\frac{\sqrt{c}(ef-dg)}{e(\sqrt{cf+ig})}; i\sinh^{-1}\left(\frac{\sqrt{-f-\frac{ig}{\sqrt{c}}}}{\sqrt{f+gx}}\right)\middle|\frac{\sqrt{cf-ig}}{\sqrt{cf+ig}}\right)\right)}{\sqrt{cx^2+1}\sqrt{-f-\frac{ig}{\sqrt{c}}}(ef-dg)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x)\*Sqrt[f + g\*x]\*Sqrt[1 + c\*x^2]),x]

[Out]  $((-2*I)*\text{Sqrt}[(g*(I/\text{Sqrt}[c] + x))/(f + g*x)]*\text{Sqrt}[-((I*g)/\text{Sqrt}[c] - g*x)/(f + g*x]))*(f + g*x)*(\text{EllipticF}[I*\text{ArcSinh}[\text{Sqrt}[-f - (I*g)/\text{Sqrt}[c]]/\text{Sqrt}[f + g*x]], (\text{Sqrt}[c]*f - I*g)/(\text{Sqrt}[c]*f + I*g)] - \text{EllipticPi}[(\text{Sqrt}[c]*(e*f - d*g))/(e*(\text{Sqrt}[c]*f + I*g)), I*\text{ArcSinh}[\text{Sqrt}[-f - (I*g)/\text{Sqrt}[c]]/\text{Sqrt}[f + g*x]], (\text{Sqrt}[c]*f - I*g)/(\text{Sqrt}[c]*f + I*g)))/(\text{Sqrt}[-f - (I*g)/\text{Sqrt}[c]]*(e*f - d*g)*\text{Sqrt}[1 + c*x^2])$

**Maple [B]** time = 0.128, size = 215, normalized size = 2.

$$2 \frac{(g + f\sqrt{-c}) \sqrt{cx^2 + 1} \sqrt{gx + f}}{\sqrt{-c}(dg - ef)(cgx^3 + cf x^2 + gx + f)} \text{EllipticPi} \left( \sqrt{\frac{(gx + f)\sqrt{-c}}{g + f\sqrt{-c}}}, -\frac{(g + f\sqrt{-c})e}{\sqrt{-c}(dg - ef)}, \sqrt{\frac{g + f\sqrt{-c}}{f\sqrt{-c} - g}} \right) \sqrt{-\frac{(-1 + x\sqrt{-c})g}{g + f\sqrt{-c}}} \sqrt{-\frac{(-1 + x\sqrt{-c})g}{g + f\sqrt{-c}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x+d)/(g\*x+f)^(1/2)/(c\*x^2+1)^(1/2),x)

[Out]  $2*(g+f*(-c)^{(1/2)})/(-c)^{(1/2)}*\text{EllipticPi}(((g*x+f)*(-c)^{(1/2)})/(g+f*(-c)^{(1/2)}))^{(1/2)}, -(g+f*(-c)^{(1/2)})*e/(-c)^{(1/2)}/(d*g-e*f), ((g+f*(-c)^{(1/2)})/(f*(-c)^{(1/2)}-g))^{(1/2)}*(-(-1+x*(-c)^{(1/2)})*g/(g+f*(-c)^{(1/2)}))^{(1/2)}*(-(x*(-c)^{(1/2)}+1)*g/(f*(-c)^{(1/2)}-g))^{(1/2)}*((g*x+f)*(-c)^{(1/2)})/(g+f*(-c)^{(1/2)})^{(1/2)}*(c*x^2+1)^{(1/2)}*(g*x+f)^{(1/2)}/(d*g-e*f)/(c*g*x^3+c*f*x^2+g*x+f)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + 1}(ex + d)\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + 1)\*(e\*x + d)\*sqrt(g\*x + f)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c\*x^2 + 1)\*(e\*x + d)\*sqrt(g\*x + f)), x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + 1)\*(e\*x + d)\*sqrt(g\*x + f)),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex)\sqrt{f + gx}\sqrt{cx^2 + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x+d)/(g*x+f)**(1/2)/(c*x**2+1)**(1/2),x)`

[Out] `Integral(1/((d + e*x)*sqrt(f + g*x)*sqrt(c*x**2 + 1)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + 1}(ex + d)\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*x^2 + 1)*(e*x + d)*sqrt(g*x + f)),x, algorithm="giac")`

[Out] `integrate(1/(sqrt(c*x^2 + 1)*(e*x + d)*sqrt(g*x + f)), x)`

$$3.655 \quad \int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+cx^2}} dx$$

**Optimal.** Leaf size=454

$$\frac{(d+ex)\sqrt[4]{ag^2+cf^2}\sqrt{\frac{(a+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2+cf^2)}}\left(\frac{(f+gx)\sqrt{ae^2+cd^2}}{(d+ex)\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{\frac{(f+gx)^2(ae^2+cd^2)}{(d+ex)^2(ag^2+cf^2)}-\frac{2(f+gx)(aeg+cdf)}{(d+ex)(ag^2+cf^2)}+1}{\left(\frac{(f+gx)\sqrt{ae^2+cd^2}}{(d+ex)\sqrt{ag^2+cf^2}}+1\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{cd^2+ae^2}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}\sqrt{d+ex}}\right)\right)}{\sqrt{a+cx^2}\sqrt[4]{ae^2+cd^2}(ef-dg)\sqrt{\frac{(f+gx)^2(ae^2+cd^2)}{(d+ex)^2(ag^2+cf^2)}-\frac{2(f+gx)(aeg+cdf)}{(d+ex)(ag^2+cf^2)}+1}}$$

[Out] -(((c\*f^2 + a\*g^2)^(1/4)\*(d + e\*x)\*Sqrt[((e\*f - d\*g)^2\*(a + c\*x^2))/((c\*f^2 + a\*g^2)\*(d + e\*x)^2)]\*(1 + (Sqrt[c\*d^2 + a\*e^2]\*(f + g\*x))/(Sqrt[c\*f^2 + a\*g^2]\*(d + e\*x)))\*Sqrt[(1 - (2\*(c\*d\*f + a\*e\*g)\*(f + g\*x))/((c\*f^2 + a\*g^2)\*(d + e\*x)) + ((c\*d^2 + a\*e^2)\*(f + g\*x)^2)/((c\*f^2 + a\*g^2)\*(d + e\*x)^2))]/(1 + (Sqrt[c\*d^2 + a\*e^2]\*(f + g\*x))/(Sqrt[c\*f^2 + a\*g^2]\*(d + e\*x)))^2)\*EllipticF[2\*ArcTan[((c\*d^2 + a\*e^2)^(1/4)\*Sqrt[f + g\*x])/((c\*f^2 + a\*g^2)^(1/4)\*Sqrt[d + e\*x]), (1 + (c\*d\*f + a\*e\*g)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[c\*f^2 + a\*g^2]))/2])/((c\*d^2 + a\*e^2)^(1/4)\*(e\*f - d\*g)\*Sqrt[a + c\*x^2]\*Sqrt[1 - (2\*(c\*d\*f + a\*e\*g)\*(f + g\*x))/((c\*f^2 + a\*g^2)\*(d + e\*x)) + ((c\*d^2 + a\*e^2)\*(f + g\*x)^2)/((c\*f^2 + a\*g^2)\*(d + e\*x)^2)])]

**Rubi [A]** time = 1.21633, antiderivative size = 454, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$

$$\frac{(d+ex)\sqrt[4]{ag^2+cf^2}\sqrt{\frac{(a+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2+cf^2)}}\left(\frac{(f+gx)\sqrt{ae^2+cd^2}}{(d+ex)\sqrt{ag^2+cf^2}}+1\right)\sqrt{\frac{\frac{(f+gx)^2(ae^2+cd^2)}{(d+ex)^2(ag^2+cf^2)}-\frac{2(f+gx)(aeg+cdf)}{(d+ex)(ag^2+cf^2)}+1}{\left(\frac{(f+gx)\sqrt{ae^2+cd^2}}{(d+ex)\sqrt{ag^2+cf^2}}+1\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{cd^2+ae^2}\sqrt{f+gx}}{\sqrt[4]{cf^2+ag^2}\sqrt{d+ex}}\right)\right)}{\sqrt{a+cx^2}\sqrt[4]{ae^2+cd^2}(ef-dg)\sqrt{\frac{(f+gx)^2(ae^2+cd^2)}{(d+ex)^2(ag^2+cf^2)}-\frac{2(f+gx)(aeg+cdf)}{(d+ex)(ag^2+cf^2)}+1}}$$

Warning: Unable to verify antiderivative.

[In] Int[1/(Sqrt[d + e\*x]\*Sqrt[f + g\*x]\*Sqrt[a + c\*x^2]),x]

[Out] -(((c\*f^2 + a\*g^2)^(1/4)\*(d + e\*x)\*Sqrt[((e\*f - d\*g)^2\*(a + c\*x^2))/((c\*f^2 + a\*g^2)\*(d + e\*x)^2)]\*(1 + (Sqrt[c\*d^2 + a\*e^2]\*(f + g\*x))/(Sqrt[c\*f^2 + a\*g^2]\*(d + e\*x)))\*Sqrt[(1 - (2\*(c\*d\*f + a\*e\*g)\*(f + g\*x))/((c\*f^2 + a\*g^2)\*(d + e\*x)) + ((c\*d^2 + a\*e^2)\*(f + g\*x)^2)/((c\*f^2 + a\*g^2)\*(d + e\*x)^2))]/(1 + (Sqrt[c\*d^2 + a\*e^2]\*(f + g\*x))/(Sqrt[c\*f^2 + a\*g^2]\*(d + e\*x)))^2)\*EllipticF[2\*ArcTan[((c\*d^2 + a\*e^2)^(1/4)\*Sqrt[f + g\*x])/((c\*f^2 + a\*g^2)^(1/4)\*Sqrt[d + e\*x]), (1 + (c\*d\*f + a\*e\*g)/(Sqrt[c\*d^2 + a\*e^2]\*Sqrt[c\*f^2 + a\*g^2]))/2])/((c\*d^2 + a\*e^2)^(1/4)\*(e\*f - d\*g)\*Sqrt[a + c\*x^2]\*Sqrt[1 - (2\*(c\*d\*f + a\*e\*g)\*(f + g\*x))/((c\*f^2 + a\*g^2)\*(d + e\*x)) + ((c\*d^2 + a\*e^2)\*(f + g\*x)^2)/((c\*f^2 + a\*g^2)\*(d + e\*x)^2)])]

**Rubi in Sympy [A]** time = 170.923, size = 400, normalized size = 0.88

$$\frac{\sqrt{\frac{1-\frac{2(f+gx)(aeg+cdf)}{(d+ex)(ag^2+cf^2)}+\frac{(f+gx)^2(ae^2+cd^2)}{(d+ex)^2(ag^2+cf^2)}}{\left(1+\frac{(f+gx)\sqrt{ae^2+cd^2}}{(d+ex)\sqrt{ag^2+cf^2}}\right)^2}}\sqrt{\frac{(a+cx^2)(dg-ef)^2}{(d+ex)^2(ag^2+cf^2)}}\left(1+\frac{(f+gx)\sqrt{ae^2+cd^2}}{(d+ex)\sqrt{ag^2+cf^2}}\right)(d+ex)\sqrt[4]{ag^2+cf^2}F\left(2\operatorname{atan}\left(\frac{\sqrt{f+gx}\sqrt[4]{ae^2+cd^2}}{\sqrt{d+ex}\sqrt[4]{ag^2+cf^2}}\right)\right)}{\sqrt{a+cx^2}\sqrt[4]{ae^2+cd^2}(dg-ef)\sqrt{1-\frac{2(f+gx)(aeg+cdf)}{(d+ex)(ag^2+cf^2)}+\frac{(f+gx)^2(ae^2+cd^2)}{(d+ex)^2(ag^2+cf^2)}}}}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(e*x+d)**(1/2)/(g*x+f)**(1/2)/(c*x**2+a)**(1/2),x)`

[Out] 
$$\frac{\sqrt{2}(\sqrt{cx+i\sqrt{a}})\sqrt{d+ex}\sqrt{\frac{i\sqrt{c}dx-i\sqrt{ae}}{\sqrt{a}}+d+ex}\sqrt{\frac{(f+gx)(\sqrt{ae+i\sqrt{c}d})}{(d+ex)(\sqrt{ag+i\sqrt{c}f})}}F\left(\sin^{-1}\left(\sqrt{\frac{(ef-dg)(\sqrt{cx+i\sqrt{a}})}{(\sqrt{c}f-i\sqrt{ag})(d+ex)}}\right)\middle|-\frac{\frac{i\sqrt{c}df}{\sqrt{a}}-ef+dg+\frac{i\sqrt{ae}g}{\sqrt{c}}}{2ef-2dg}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}(\sqrt{cd-i\sqrt{ae}})\sqrt{\frac{(\sqrt{cx+i\sqrt{a}})(ef-dg)}{(d+ex)(\sqrt{c}f-i\sqrt{ag})}}}$$

**Mathematica [C]** time = 1.95133, size = 344, normalized size = 0.76

$$\frac{\sqrt{2}(\sqrt{cx+i\sqrt{a}})\sqrt{d+ex}\sqrt{\frac{i\sqrt{c}dx-i\sqrt{ae}}{\sqrt{a}}+d+ex}\sqrt{\frac{(f+gx)(\sqrt{ae+i\sqrt{c}d})}{(d+ex)(\sqrt{ag+i\sqrt{c}f})}}F\left(\sin^{-1}\left(\sqrt{\frac{(ef-dg)(\sqrt{cx+i\sqrt{a}})}{(\sqrt{c}f-i\sqrt{ag})(d+ex)}}\right)\middle|-\frac{\frac{i\sqrt{c}df}{\sqrt{a}}-ef+dg+\frac{i\sqrt{ae}g}{\sqrt{c}}}{2ef-2dg}\right)}{\sqrt{a+cx^2}\sqrt{f+gx}(\sqrt{cd-i\sqrt{ae}})\sqrt{\frac{(\sqrt{cx+i\sqrt{a}})(ef-dg)}{(d+ex)(\sqrt{c}f-i\sqrt{ag})}}}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*Sqrt[a + c*x^2]),x]`

[Out] 
$$\frac{(\text{Sqrt}[2])^2(I\text{Sqrt}[a] + \text{Sqrt}[c]x)\text{Sqrt}[d + e*x]\text{Sqrt}[(d - (I\text{Sqrt}[a]e)/\text{Sqrt}[c] + (I\text{Sqrt}[c]d*x)/\text{Sqrt}[a] + e*x)/(d + e*x)]\text{Sqrt}[(I\text{Sqrt}[c]d + \text{Sqrt}[a]e)(f + g*x)/((I\text{Sqrt}[c]f + \text{Sqrt}[a]g)(d + e*x))]\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(ef - d*g)(I\text{Sqrt}[a] + \text{Sqrt}[c]x)/((\text{Sqrt}[c]f - I\text{Sqrt}[a]g)(d + e*x))]], -((I\text{Sqrt}[c]d*f)/\text{Sqrt}[a] - e*f + d*g + (I\text{Sqrt}[a]e*g)/\text{Sqrt}[c])/(2*e*f - 2*d*g)]/((\text{Sqrt}[c]d - I\text{Sqrt}[a]e)\text{Sqrt}[(ef - d*g)(I\text{Sqrt}[a] + \text{Sqrt}[c]x)/((\text{Sqrt}[c]f - I\text{Sqrt}[a]g)(d + e*x))]\text{Sqrt}[f + g*x]\text{Sqrt}[a + c*x^2])$$

**Maple [A]** time = 0.221, size = 433, normalized size = 1.

$$\frac{2\frac{(ce^2fx^2 - \sqrt{-ac}x^2e^2g + 2xcdef - 2\sqrt{-ac}xdg + cd^2f - \sqrt{-acd}^2g)\sqrt{ex+d}\sqrt{gx+f}\sqrt{cx^2+a}}{(cd - \sqrt{-ace})(dg - ef)\sqrt{ceg}x^4 + cdgx^3 + cefx^3 + aegx^2 + cdfx^2 + adgx + aefx + adf}\text{EllipticF}\left(\sqrt{\frac{(\sqrt{-ace} - cd)}{(g\sqrt{-ac} - cf)}}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+a)^(1/2),x)`

[Out] 
$$2*(c*e^2*f*x^2 - (-a*c)^(1/2)*x^2*e^2*g + 2*x*c*d*e*f - 2*(-a*c)^(1/2)*x*d*e*g + c*d^2*f - (-a*c)^(1/2)*d^2*g)\text{EllipticF}(\frac{((-a*c)^(1/2)*e - c*d)*(g*x+f)}{(g*(-a*c)^(1/2) - c*f)/(e*x+d)^(1/2)}, \frac{((-a*c)^(1/2)*e + c*d)*(g*(-a*c)^(1/2) - c*f)}{(g*(-a*c)^(1/2) + c*f)/((-a*c)^(1/2)*e - c*d)^(1/2)}) * \frac{((d*g - e*f)*(c*x + (-a*c)^(1/2)))/(g*(-a*c)^(1/2) - c*f)/(e*x+d)^(1/2)}{((d*g - e*f)*(-c*x + (-a*c)^(1/2)))/(g*(-a*c)^(1/2) + c*f)/(e*x+d)^(1/2)} * \frac{(((-a*c)^(1/2)*e - c*d)*(g*x+f)/(g*(-a*c)^(1/2) - c*f)/(e*x+d)^(1/2)) * ((e*x+d)^(1/2)*(g*x+f)^(1/2)*(c*x^2+a)^(1/2)/(-1/c*(g*x+f)*(e*x+d)*(-c*x + (-a*c)^(1/2))*(c*x + (-a*c)^(1/2)))^(1/2)/(c*d - (-a*c)^(1/2)*e)/(d*g - e*f)/(c*e*g*x^4 + c*d*g*x^3 + c*e*f*x^3 + a*e*g*x^2 + c*d*f*x^2 + a*d*g*x + a*e*f*x + a*d*f)^(1/2)}$$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + a}\sqrt{ex + d}\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + a)\*sqrt(e\*x + d)\*sqrt(g\*x + f)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c\*x^2 + a)\*sqrt(e\*x + d)\*sqrt(g\*x + f)), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{cx^2 + a}\sqrt{ex + d}\sqrt{gx + f}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + a)\*sqrt(e\*x + d)\*sqrt(g\*x + f)),x, algorithm="fricas")

[Out] integral(1/(sqrt(c\*x^2 + a)\*sqrt(e\*x + d)\*sqrt(g\*x + f)), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + cx^2}\sqrt{d + ex}\sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x+d)\*\*(1/2)/(g\*x+f)\*\*(1/2)/(c\*x\*\*2+a)\*\*(1/2),x)

[Out] Integral(1/(sqrt(a + c\*x\*\*2)\*sqrt(d + e\*x)\*sqrt(f + g\*x)), x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + a}\sqrt{ex + d}\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + a)\*sqrt(e\*x + d)\*sqrt(g\*x + f)),x, algorithm="giac")

[Out] integrate(1/(sqrt(c\*x^2 + a)\*sqrt(e\*x + d)\*sqrt(g\*x + f)), x)

$$3.656 \quad \int \frac{1}{\sqrt{-1+x}\sqrt{1+x}\sqrt{-1+2x^2}} dx$$

**Optimal.** Leaf size=52

$$\frac{\sqrt{1-2x^2}\sqrt{1-x^2}F(\sin^{-1}(x)|2)}{\sqrt{x-1}\sqrt{x+1}\sqrt{2x^2-1}}$$

[Out] (Sqrt[1 - 2\*x^2]\*Sqrt[1 - x^2]\*EllipticF[ArcSin[x], 2])/(Sqrt[-1 + x]\*Sqrt[1 + x]\*Sqrt[-1 + 2\*x^2])

**Rubi [A]** time = 0.135364, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$

$$\frac{\sqrt{1-2x^2}\sqrt{1-x^2}F(\sin^{-1}(x)|2)}{\sqrt{x-1}\sqrt{x+1}\sqrt{2x^2-1}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[-1 + x]\*Sqrt[1 + x]\*Sqrt[-1 + 2\*x^2]), x]

[Out] (Sqrt[1 - 2\*x^2]\*Sqrt[1 - x^2]\*EllipticF[ArcSin[x], 2])/(Sqrt[-1 + x]\*Sqrt[1 + x]\*Sqrt[-1 + 2\*x^2])

**Rubi in Sympy [A]** time = 15.2904, size = 26, normalized size = 0.5

$$\frac{\sqrt{x-1}\sqrt{x+1}F(\arccos(x)|2)}{\sqrt{-x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(-1+x)\*\*(1/2)/(1+x)\*\*(1/2)/(2\*x\*\*2-1)\*\*(1/2), x)

[Out] sqrt(x - 1)\*sqrt(x + 1)\*elliptic\_f(arccos(x), 2)/sqrt(-x\*\*2 + 1)

**Mathematica [B]** time = 0.238965, size = 107, normalized size = 2.06

$$\frac{2(x-1)^{3/2}\sqrt{\frac{x+1}{1-x}}\sqrt{\frac{1-2x^2}{(x-1)^2}}F\left(\sin^{-1}\left(\frac{\sqrt{\sqrt{2+2+\frac{1}{x-1}}}}{2^{3/4}}\right)\middle|4\left(-4+3\sqrt{2}\right)\right)}{\sqrt{3+2\sqrt{2}}\sqrt{x+1}\sqrt{2x^2-1}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/(Sqrt[-1 + x]\*Sqrt[1 + x]\*Sqrt[-1 + 2\*x^2]), x]

[Out] (-2\*(-1 + x)^(3/2)\*Sqrt[(1 + x)/(1 - x)]\*Sqrt[(1 - 2\*x^2)/(-1 + x)^2]\*EllipticF[ArcSin[Sqrt[2 + Sqrt[2] + (-1 + x)^(-1)]]/2^(3/4)], 4\*(-4 + 3\*Sqrt[2]))/(Sqrt[3 + 2\*Sqrt[2]]\*Sqrt[1 + x]\*Sqrt[-1 + 2\*x^2])

**Maple [A]** time = 0.151, size = 58, normalized size = 1.1

$$\frac{\text{EllipticF}\left(x, \sqrt{2}\right)}{2x^4 - 3x^2 + 1} \sqrt{-1+x}\sqrt{1+x}\sqrt{2x^2-1}\sqrt{-x^2+1}\sqrt{-2x^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-1+x)^(1/2)/(1+x)^(1/2)/(2*x^2-1)^(1/2),x)`

[Out]  $(-1+x)^{1/2} (1+x)^{1/2} (2x^2-1)^{1/2} / (2x^4-3x^2+1)^{1/2} (-x^2+1)^{1/2} (-2x^2+1)^{1/2} \text{EllipticF}(x, 2^{1/2})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^2-1}\sqrt{x+1}\sqrt{x-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(2*x^2-1)*sqrt(x+1)*sqrt(x-1)),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(2*x^2-1)*sqrt(x+1)*sqrt(x-1)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{2x^2-1}\sqrt{x+1}\sqrt{x-1}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(2*x^2-1)*sqrt(x+1)*sqrt(x-1)),x, algorithm="fricas")`

[Out] `integral(1/(sqrt(2*x^2-1)*sqrt(x+1)*sqrt(x-1)), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{x-1}\sqrt{x+1}\sqrt{2x^2-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-1+x)**(1/2)/(1+x)**(1/2)/(2*x**2-1)**(1/2),x)`

[Out] `Integral(1/(sqrt(x-1)*sqrt(x+1)*sqrt(2*x**2-1)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{2x^2-1}\sqrt{x+1}\sqrt{x-1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(2*x^2-1)*sqrt(x+1)*sqrt(x-1)),x, algorithm="giac")`

[Out] `integrate(1/(sqrt(2*x^2-1)*sqrt(x+1)*sqrt(x-1)), x)`

$$3.657 \quad \int \frac{\sqrt{d+ex}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=269

$$\begin{aligned} & -\frac{16\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2(2ae^2g-cd(3ef-dg))}{35c^4d^4e\sqrt{d+ex}} \\ & + \frac{16g\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2}{35c^3d^3e} \\ & + \frac{12(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}{35c^2d^2\sqrt{d+ex}} + \frac{2(f+gx)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{7cd\sqrt{d+ex}} \end{aligned}$$

[Out]  $(-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(3*e*f - d*g))*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(35*c^4*d^4*e*\text{Sqrt}[d + e*x]) + (16*g*(c*d*f - a*e*g)^2*\text{Sqrt}[d + e*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(35*c^3*d^3*e) + (12*(c*d*f - a*e*g)*(f + g*x)^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(35*c^2*d^2*\text{Sqrt}[d + e*x]) + (2*(f + g*x)^3*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(7*c*d*\text{Sqrt}[d + e*x])$

**Rubi [A]** time = 1.17954, antiderivative size = 269, normalized size of antiderivative = 1., number of rules used = 4, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\begin{aligned} & -\frac{16\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2(2ae^2g-cd(3ef-dg))}{35c^4d^4e\sqrt{d+ex}} \\ & + \frac{16g\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2}{35c^3d^3e} \\ & + \frac{12(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}{35c^2d^2\sqrt{d+ex}} + \frac{2(f+gx)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{7cd\sqrt{d+ex}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Sqrt}[d + e*x])*(f + g*x)^3]/\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2], x]$

[Out]  $(-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(3*e*f - d*g))*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(35*c^4*d^4*e*\text{Sqrt}[d + e*x]) + (16*g*(c*d*f - a*e*g)^2*\text{Sqrt}[d + e*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(35*c^3*d^3*e) + (12*(c*d*f - a*e*g)*(f + g*x)^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(35*c^2*d^2*\text{Sqrt}[d + e*x]) + (2*(f + g*x)^3*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(7*c*d*\text{Sqrt}[d + e*x])$

**Rubi in Sympy [A]** time = 95.1076, size = 262, normalized size = 0.97

$$\begin{aligned} & \frac{2(f+gx)^3\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{7cd\sqrt{d+ex}} - \frac{12(f+gx)^2(aeg-cdf)\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{35c^2d^2\sqrt{d+ex}} \\ & + \frac{16g\sqrt{d+ex}(aeg-cdf)^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{35c^3d^3e} \\ & - \frac{16(aeg-cdf)^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}(2ae^2g+cd^2g-3cdf)}{35c^4d^4e\sqrt{d+ex}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((g*x+f)**3*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)*$

[Out]  $2*(f + g*x)^3*\sqrt{a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2)}/(7*c*d*\sqrt{d + e*x}) - 12*(f + g*x)^2*(a*e*g - c*d*f)*\sqrt{a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2)}/(35*c^2*d^2*\sqrt{d + e*x}) + 16*g*\sqrt{d + e*x}*(a*e*g - c*d*f)^2*\sqrt{a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2)}/(35*c^3*d^3*e) - 16*(a*e*g - c*d*f)^2*\sqrt{a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2)}*(2*a*e^2*g + c*d^2*g - 3*c*d*e*f)/(35*c^4*d^4*e*\sqrt{d + e*x})$

**Mathematica [A]** time = 0.194018, size = 136, normalized size = 0.51

$$\frac{2\sqrt{(d+ex)(ae+cdx)}(-16a^3e^3g^3+8a^2cde^2g^2(7f+gx)-2ac^2d^2eg(35f^2+14fgx+3g^2x^2)+c^3d^3(35f^3+35f^2gx+21fg^2))}{35c^4d^4\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[d + e\*x] \* (f + g\*x)^3)/Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2], x]

[Out]  $(2*\sqrt{(a*e + c*d*x)*(d + e*x)}*(-16*a^3*e^3*g^3 + 8*a^2*c*d*e^2*g^2*(7*f + g*x) - 2*a*c^2*d^2*e*g*(35*f^2 + 14*f*g*x + 3*g^2*x^2) + c^3*d^3*(35*f^3 + 35*f^2*g*x + 21*f*g^2*x^2 + 5*g^3*x^3)))/(35*c^4*d^4*\sqrt{d + e*x})$

**Maple [A]** time = 0.012, size = 188, normalized size = 0.7

$$\frac{(2cdx + 2ae)(-5g^3x^3c^3d^3 + 6ac^2d^2eg^3x^2 - 21c^3d^3fg^2x^2 - 8a^2cde^2g^3x + 28ac^2d^2efg^2x - 35c^3d^3f^2gx + 16a^3e^3g^3 - 5c^4d^4)}{35c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^3\*(e\*x+d)^(1/2)/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2), x)

[Out]  $-2/35*(c*d*x+a*e)*(-5*c^3*d^3*g^3*x^3+6*a*c^2*d^2*e*g^3*x^2-21*c^3*d^3*f*g^2*x^2-8*a^2*c*d*e^2*g^3*x+28*a*c^2*d^2*e*f*g^2*x-35*c^3*d^3*f^2*g*x+16*a^3*e^3*g^3-5*c^4*d^4)/(e*x+d)^(1/2)/c^4/d^4/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)$

**Maxima [A]** time = 0.780034, size = 294, normalized size = 1.09

$$\frac{2\sqrt{cdx+ae}f^3}{cd} + \frac{2(c^2d^2x^2 - acdex - 2a^2e^2)f^2g}{\sqrt{cdx+aec^2d^2}} + \frac{2(3c^3d^3x^3 - ac^2d^2ex^2 + 4a^2cde^2x + 8a^3e^3)fg^2}{5\sqrt{cdx+aec^3d^3}} + \frac{2(5c^4d^4x^4 - ac^3d^3ex^3 + 2a^2c^2d^2e^2x^2 - 8a^3cde^3x - 16a^4e^4)g^3}{35\sqrt{cdx+aec^4d^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d) \* (g\*x + f)^3/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

[Out]  $2*\sqrt{c*d*x + a*e}*f^3/(c*d) + 2*(c^2*d^2*x^2 - a*c*d*e*x - 2*a^2*e^2)*f^2*g/(sqrt(c*d*x + a*e)*c^2*d^2) + 2/5*(3*c^3*d^3*x^3 - a*c^2*d^2*e*x^2 + 4*a^2*c*d*e^2*x + 8*a^3*e^3)*f*g^2/(sqrt(c*d*x + a*e)*c^3*d^3) + 2/35*(5*c^4*d^4*x^4 - a*c^3*d^3*e*x^3 + 2*a^2*c^2*d^2*e^2*x^2 - 8*a^3*c*d*e^3*x - 16*a^4*e^4)*g^3/(sqrt(c*d*x + a*e)*c^4*d^4)$

**Fricas [A]** time = 0.291107, size = 605, normalized size = 2.25

$$2(5c^4d^4eg^3x^5 + 35ac^3d^4ef^3 - 70a^2c^2d^3e^2f^2g + 56a^3cd^2e^3fg^2 - 16a^4de^4g^3 + (21c^4d^4efg^2 + (5c^4d^5 - ac^3d^3e^2)g^3)x^4 + \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*(g\*x + f)^3/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

[Out] 
$$\frac{2}{35} \cdot (5c^4d^4eg^3x^5 + 35a^3c^3d^4ef^3 - 70a^2c^2d^3e^2f^2g + 56a^3cd^2e^3fg^2 - 16a^4de^4g^3 + (21c^4d^4efg^2 + (5c^4d^5 - ac^3d^3e^2)g^3)x^4 + (35c^4d^4ef^2g + 7(3c^4d^5 - ac^3d^3e^2)f^2g - (ac^3d^4e - 2a^2c^2d^2e^3)g^3)x^3 + (35c^4d^4ef^3 + 35(c^4d^5 - ac^3d^3e^2)f^2g - 7(ac^3d^4e - 4a^2c^2d^2e^3)f^2g + 2(a^2c^2d^3e^2 - 4a^3cd^2e^4)g^3)x^2 + (35(c^4d^5 + ac^3d^3e^2)f^3 - 35(ac^3d^4e + 2a^2c^2d^2e^3)f^2g + 28(a^2c^2d^3e^2 + 2a^3cd^2e^4)f^2g - 8(a^3cd^2e^3 + 2a^4e^5)g^3)x) / (\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x} * \sqrt{e*x + d} * c^4*d^4)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*3\*(e\*x+d)\*\*(1/2)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}(gx+f)^3}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*(g\*x + f)^3/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

[Out] integrate(sqrt(e\*x + d)\*(g\*x + f)^3/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

$$3.658 \quad \int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=200

$$\frac{8\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)(2ae^2g-cd(3ef-dg))}{15c^3d^3e\sqrt{d+ex}} + \frac{8g\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}{15c^2d^2e} + \frac{2(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5cd\sqrt{d+ex}}$$

[Out]  $(-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(3*e*f - d*g))*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15*c^3*d^3*e*\text{Sqrt}[d + e*x]) + (8*g*(c*d*f - a*e*g)*\text{Sqrt}[d + e*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15*c^2*d^2*e) + (2*(f + g*x)^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c*d*\text{Sqrt}[d + e*x])$

**Rubi [A]** time = 0.707216, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{8\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)(2ae^2g-cd(3ef-dg))}{15c^3d^3e\sqrt{d+ex}} + \frac{8g\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}{15c^2d^2e} + \frac{2(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Sqrt}[d + e*x])*(f + g*x)^2]/\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2], x]$

[Out]  $(-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(3*e*f - d*g))*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15*c^3*d^3*e*\text{Sqrt}[d + e*x]) + (8*g*(c*d*f - a*e*g)*\text{Sqrt}[d + e*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15*c^2*d^2*e) + (2*(f + g*x)^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c*d*\text{Sqrt}[d + e*x])$

**Rubi in Sympy [A]** time = 61.8147, size = 194, normalized size = 0.97

$$\frac{2(f+gx)^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{5cd\sqrt{d+ex}} - \frac{8g\sqrt{d+ex}(aeg-cdf)\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{15c^2d^2e} + \frac{8(aeg-cdf)\sqrt{ade+cdex^2+x(ae^2+cd^2)}(2ae^2g+cd^2g-3cdef)}{15c^3d^3e\sqrt{d+ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((g*x+f)**2*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)*$

[Out]  $2*(f + g*x)**2*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(5*c*d*\text{sqrt}(d + e*x)) - 8*g*\text{sqrt}(d + e*x)*(a*e*g - c*d*f)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(15*c**2*d**2*e) + 8*(a*e*g - c*d*f)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))*(2*a*e**2*g + c*d**2*g - 3*c*d*e*f)/(15*c**3*d**3*e*\text{sqrt}(d + e*x))$

**Mathematica [A]** time = 0.125982, size = 89, normalized size = 0.44

$$\frac{2\sqrt{(d+ex)(ae+cdx)}(8a^2e^2g^2-4acdeg(5f+gx)+c^2d^2(15f^2+10fgx+3g^2x^2))}{15c^3d^3\sqrt{d+ex}}$$



Antiderivative was successfully verified.

[In] Integrate[(Sqrt[d + e\*x] \* (f + g\*x)^2)/Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]

[Out] (2\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)]\*(8\*a^2\*e^2\*g^2 - 4\*a\*c\*d\*e\*g\*(5\*f + g\*x) + c^2\*d^2\*(15\*f^2 + 10\*f\*g\*x + 3\*g^2\*x^2)))/(15\*c^3\*d^3\*Sqrt[d + e\*x])

**Maple [A]** time = 0.011, size = 116, normalized size = 0.6

$$\frac{(2cdx + 2ae)(3g^2x^2c^2d^2 - 4acdeg^2x + 10c^2d^2fgx + 8a^2e^2g^2 - 20acdefg + 15f^2c^2d^2)}{15c^3d^3} \sqrt{ex + d} \frac{1}{\sqrt{cdex^2 + ae^2x + cd^2x + d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^2\*(e\*x+d)^(1/2)/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2),x)

[Out] 2/15\*(c\*d\*x+a\*e)\*(3\*c^2\*d^2\*g^2\*x^2-4\*a\*c\*d\*e\*g^2\*x+10\*c^2\*d^2\*f\*g\*x+8\*a^2\*e^2\*g^2-20\*a\*c\*d\*e\*f\*g+15\*c^2\*d^2\*f^2)\*(e\*x+d)^(1/2)/c^3/d^3/(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)

**Maxima [A]** time = 0.764394, size = 180, normalized size = 0.9

$$\frac{2\sqrt{cdx+ae}f^2}{cd} + \frac{4(c^2d^2x^2 - acdex - 2a^2e^2)fg}{3\sqrt{cdx+ae}c^2d^2} + \frac{2(3c^3d^3x^3 - ac^2d^2ex^2 + 4a^2cde^2x + 8a^3e^3)g^2}{15\sqrt{cdx+ae}c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d) \* (g\*x + f)^2/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

[Out] 2\*sqrt(c\*d\*x + a\*e)\*f^2/(c\*d) + 4/3\*(c^2\*d^2\*x^2 - a\*c\*d\*e\*x - 2\*a^2\*e^2)\*f\*g/(sqrt(c\*d\*x + a\*e)\*c^2\*d^2) + 2/15\*(3\*c^3\*d^3\*x^3 - a\*c^2\*d^2\*e\*x^2 + 4\*a^2\*c\*d\*e^2\*x + 8\*a^3\*e^3)\*g^2/(sqrt(c\*d\*x + a\*e)\*c^3\*d^3)

**Fricas [A]** time = 0.287774, size = 378, normalized size = 1.89

$$\frac{2(3c^3d^3eg^2x^4 + 15ac^2d^3ef^2 - 20a^2cd^2e^2fg + 8a^3de^3g^2 + (10c^3d^3efg + (3c^3d^4 - ac^2d^2e^2)g^2)x^3 + (15c^3d^3ef^2 + 10(c^3d^4 - ac^2d^2e^2)fg - 20a^2cd^2e^2fg + 8a^3de^3g^2)x^2 + (15c^3d^3ef^2 + 10(c^3d^4 - ac^2d^2e^2)fg - 20a^2cd^2e^2fg + 8a^3de^3g^2)x + 15c^3d^3ef^2 + 10(c^3d^4 - ac^2d^2e^2)fg - 20a^2cd^2e^2fg + 8a^3de^3g^2)}{15\sqrt{cdex^2 + ade + d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d) \* (g\*x + f)^2/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

[Out] 2/15\*(3\*c^3\*d^3\*e\*g^2\*x^4 + 15\*a\*c^2\*d^3\*e\*f^2 - 20\*a^2\*c\*d^2\*e^2\*f\*g + 8\*a^3\*d^3\*e^3\*g^2 + (10\*c^3\*d^3\*e\*f\*g + (3\*c^3\*d^4 - a\*c^2\*d^2\*e^2)\*g^2)\*x^3 + (15\*c^3\*d^3\*e\*f^2 + 10\*(c^3\*d^4 - a\*c^2\*d^2\*e^2)\*f\*g - (a\*c^2\*d^3\*e - 4\*a^2\*c\*d\*e^3)\*g^2)\*x^2 + (15\*(c^3\*d^4 + a\*c^2\*d^2\*e^2)\*f^2 - 10\*(a\*c^2\*d^3\*e + 2\*a^2\*c\*d\*e^3)\*f\*g + 4\*(a^2\*c\*d^2\*e^2 + 2\*a^3\*e^4)\*g^2)\*x)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*c^3\*d^3)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}(f+gx)^2}{\sqrt{(d+ex)(ae+cdx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*2\*(e\*x+d)\*\*(1/2)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)

[Out] Integral(sqrt(d + e\*x)\*(f + g\*x)\*\*2/sqrt((d + e\*x)\*(a\*e + c\*d\*x)), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}(gx+f)^2}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*(g\*x + f)^2/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x

[Out] integrate(sqrt(e\*x + d)\*(g\*x + f)^2/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

$$3.659 \quad \int \frac{\sqrt{d+ex}(f+gx)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=125

$$\frac{2g\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cde} - \frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(2ae^2g-cd(3ef-dg))}{3c^2d^2e\sqrt{d+ex}}$$

[Out]  $(-2*(2*a*e^2*g - c*d*(3*e*f - d*g))*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^2*d^2*e*\text{Sqrt}[d + e*x]) + (2*g*\text{Sqrt}[d + e*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*e)$

**Rubi [A]** time = 0.334967, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 44,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{2g\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3cde} - \frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(2ae^2g-cd(3ef-dg))}{3c^2d^2e\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Sqrt}[d + e*x]*(f + g*x))/\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2], x]$

[Out]  $(-2*(2*a*e^2*g - c*d*(3*e*f - d*g))*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^2*d^2*e*\text{Sqrt}[d + e*x]) + (2*g*\text{Sqrt}[d + e*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*e)$

**Rubi in Sympy [A]** time = 35.6517, size = 119, normalized size = 0.95

$$\frac{2g\sqrt{d+ex}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{3cde} - \frac{2\sqrt{ade+cdex^2+x(ae^2+cd^2)}(2ae^2g+cd^2g-3cdef)}{3c^2d^2e\sqrt{d+ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((g*x+f)*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2))$

[Out]  $2*g*\text{sqrt}(d + e*x)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(3*c*d*e) - 2*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))*(2*a*e**2*g + c*d**2*g - 3*c*d*e*f)/(3*c**2*d**2*e*\text{sqrt}(d + e*x))$

**Mathematica [A]** time = 0.0601866, size = 53, normalized size = 0.42

$$\frac{2\sqrt{(d+ex)(ae+cdx)}(cd(3f+gx)-2aeg)}{3c^2d^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(\text{Sqrt}[d + e*x]*(f + g*x))/\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2], x]$

[Out]  $(2*\text{Sqrt}[(a*e + c*d*x)*(d + e*x)]*(-2*a*e*g + c*d*(3*f + g*x)))/(3*c^2*d^2*\text{Sqrt}[d + e*x])$

**Maple [A]** time = 0.008, size = 67, normalized size = 0.5

$$-\frac{(2cdx + 2ae)(-xcdg + 2aeg - 3cdf)}{3c^2d^2} \sqrt{ex + d} \frac{1}{\sqrt{cdex^2 + ae^2x + cd^2x + ade}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)\*(e\*x+d)^(1/2)/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2),x)

[Out] -2/3\*(c\*d\*x+a\*e)\*(-c\*d\*g\*x+2\*a\*e\*g-3\*c\*d\*f)\*(e\*x+d)^(1/2)/c^2/d^2/(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)

**Maxima [A]** time = 0.761707, size = 88, normalized size = 0.7

$$\frac{2\sqrt{cdx + aef}}{cd} + \frac{2(c^2d^2x^2 - acdex - 2a^2e^2)g}{3\sqrt{cdx + aec^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*(g\*x + f)/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x, a

[Out] 2\*sqrt(c\*d\*x + a\*e)\*f/(c\*d) + 2/3\*(c^2\*d^2\*x^2 - a\*c\*d\*e\*x - 2\*a^2\*e^2)\*g/(sqrt(c\*d\*x + a\*e)\*c^2\*d^2)

**Fricas [A]** time = 0.293412, size = 198, normalized size = 1.58

$$\frac{2(c^2d^2egx^3 + 3acd^2ef - 2a^2de^2g + (3c^2d^2ef + (c^2d^3 - acde^2)g)x^2 + (3(c^2d^3 + acde^2)f - (acd^2e + 2a^2e^3)g)x)}{3\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{ex + d}c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*(g\*x + f)/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x, a

[Out] 2/3\*(c^2\*d^2\*e\*g\*x^3 + 3\*a\*c\*d^2\*e\*f - 2\*a^2\*d\*e^2\*g + (3\*c^2\*d^2\*e\*f + (c^2\*d^3 - a\*c\*d\*e^2)\*g)\*x^2 + (3\*(c^2\*d^3 + a\*c\*d\*e^2)\*f - (a\*c\*d^2\*e + 2\*a^2\*e^3)\*g)\*x)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*c^2\*d^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d + ex}(f + gx)}{\sqrt{(d + ex)(ae + cdx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(e\*x+d)\*\*(1/2)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2),x)

[Out] Integral(sqrt(d + e\*x)\*(f + g\*x)/sqrt((d + e\*x)\*(a\*e + c\*d\*x)), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex + d}(gx + f)}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x + d)*(g*x + f)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x), x, a
```

```
[Out] integrate(sqrt(e*x + d)*(g*x + f)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x), x)
```

$$3.660 \quad \int \frac{\sqrt{d+ex}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=46

$$\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

[Out] (2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(c\*d\*Sqrt[d + e\*x])

**Rubi [A]** time = 0.0721802, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$

$$\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e\*x]/Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2], x]

[Out] (2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(c\*d\*Sqrt[d + e\*x])

**Rubi in Sympy [A]** time = 17.1803, size = 41, normalized size = 0.89

$$\frac{2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{cd\sqrt{d+ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*(1/2)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2), x)

[Out] 2\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(c\*d\*sqrt(d + e\*x))

**Mathematica [A]** time = 0.0288138, size = 35, normalized size = 0.76

$$\frac{2\sqrt{(d+ex)(ae+cdx)}}{cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e\*x]/Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2], x]

[Out] (2\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)])/(c\*d\*Sqrt[d + e\*x])

**Maple [A]** time = 0.007, size = 50, normalized size = 1.1

$$2 \frac{(cdx + ae) \sqrt{ex + d}}{cd\sqrt{cdex^2 + ae^2x + cd^2x + ade}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)`

[Out] `2*(c*d*x+a*e)*(e*x+d)^(1/2)/c/d/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)`

**Maxima [A]** time = 0.751561, size = 24, normalized size = 0.52

$$\frac{2\sqrt{cdx+ae}}{cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(e*x+d)/sqrt(c*d*e*x^2+a*d*e+(c*d^2+a*e^2)*x),x,algorithm=`

[Out] `2*sqrt(c*d*x+a*e)/(c*d)`

**Fricas [A]** time = 0.287553, size = 57, normalized size = 1.24

$$\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{\sqrt{ex+d}cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(e*x+d)/sqrt(c*d*e*x^2+a*d*e+(c*d^2+a*e^2)*x),x,algorithm=`

[Out] `2*sqrt(c*d*e*x^2+a*d*e+(c*d^2+a*e^2)*x)/(sqrt(e*x+d)*c*d)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)(ae+cdx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2),x)`

[Out] `Integral(sqrt(d+e*x)/sqrt((d+e*x)*(a*e+c*d*x)),x)`

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(e*x+d)/sqrt(c*d*e*x^2+a*d*e+(c*d^2+a*e^2)*x),x,algorithm=`

[Out] Timed out

$$3.661 \quad \int \frac{\sqrt{d+ex}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=80

$$\frac{2 \tan^{-1} \left( \frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}} \right)}{\sqrt{g}\sqrt{cdf-aeg}}$$

[Out] (2\*ArcTan[(Sqrt[g]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(Sqrt[c\*d\*f - a\*e\*g]\*Sqrt[d + e\*x])])/(Sqrt[g]\*Sqrt[c\*d\*f - a\*e\*g])

**Rubi [A]** time = 0.377879, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$

$$\frac{2 \tan^{-1} \left( \frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}} \right)}{\sqrt{g}\sqrt{cdf-aeg}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e\*x]/((f + g\*x)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]), x]

[Out] (2\*ArcTan[(Sqrt[g]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(Sqrt[c\*d\*f - a\*e\*g]\*Sqrt[d + e\*x])])/(Sqrt[g]\*Sqrt[c\*d\*f - a\*e\*g])

**Rubi in Sympy [A]** time = 32.8203, size = 78, normalized size = 0.98

$$\frac{2 \operatorname{atanh} \left( \frac{\sqrt{g}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}\sqrt{aeg-cdf}} \right)}{\sqrt{g}\sqrt{aeg-cdf}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*(1/2)/(g\*x+f)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1

[Out] -2\*atanh(sqrt(g)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2)))/(sqrt(d + e\*x)\*sqrt(a\*e\*g - c\*d\*f))/(sqrt(g)\*sqrt(a\*e\*g - c\*d\*f))

**Mathematica [A]** time = 0.090064, size = 93, normalized size = 1.16

$$\frac{2\sqrt{d+ex}\sqrt{ae+cdx} \tanh^{-1} \left( \frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{aeg-cdf}} \right)}{\sqrt{g}\sqrt{(d+ex)(ae+cdx)}\sqrt{aeg-cdf}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e\*x]/((f + g\*x)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]), x]

[Out] (-2\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/Sqrt[-(c\*d\*f) + a\*e\*g]])/(Sqrt[g]\*Sqrt[-(c\*d\*f) + a\*e\*g])\*S



qrt[(a\*e + c\*d\*x)\*(d + e\*x)]

**Maple [A]** time = 0.036, size = 87, normalized size = 1.1

$$-2 \frac{\sqrt{cdex^2 + ae^2x + cd^2x + ade}}{\sqrt{ex + d}\sqrt{cdx + ae}\sqrt{(aeg - cdf)g}} \operatorname{Artanh} \left( \frac{g\sqrt{cdx + ae}}{\sqrt{(aeg - cdf)g}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(1/2)/(g\*x+f)/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2),x)

[Out] -2/(e\*x+d)^(1/2)\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)/((c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2)\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)),x)

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.299598, size = 1, normalized size = 0.01

$$\left[ \frac{\log \left( \frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}(cdfg-ae^2)\sqrt{ex+d}+(cdegx^2-cd^2f+2adeg-(cdf-(cd^2+2ae^2)g)x)\sqrt{-cdfg+ae^2}}{egx^2+df+(ef+dg)x}}{\sqrt{-cdfg+ae^2}} \right)}{2 \arctan \left( \frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{cdfg-ae^2}\sqrt{ex+d}}{cdegx^2+adeg+(cd^2+ae^2)gx}}{\sqrt{cdfg-ae^2}} \right)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)),x)

[Out] [log(-(2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(c\*d\*f\*g - a\*e\*g^2)\*sqrt(e\*x + d) + (c\*d\*e\*g\*x^2 - c\*d^2\*f + 2\*a\*d\*e\*g - (c\*d\*e\*f - (c\*d^2 + 2\*a\*e^2)\*g)\*x)\*sqrt(-c\*d\*f\*g + a\*e\*g^2))/(e\*g\*x^2 + d\*f + (e\*f + d\*g)\*x))/sqrt(-c\*d\*f\*g + a\*e\*g^2), -2\*arctan(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*f\*g - a\*e\*g^2)\*sqrt(e\*x + d)/(c\*d\*e\*g\*x^2 + a\*d\*e\*g + (c\*d^2 + a\*e^2)\*g\*x))/sqrt(c\*d\*f\*g - a\*e\*g^2)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)(ae+cdx)}(f+gx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(1/2)/(g\*x+f)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2), x)

[Out] Integral(sqrt(d + e\*x)/(sqrt((d + e\*x)\*(a\*e + c\*d\*x))\*(f + g\*x)), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex + d}}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(gx + f)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)), x)

[Out] integrate(sqrt(e\*x + d)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)), x)

$$3.662 \quad \int \frac{\sqrt{d+ex}}{(f+gx)^2 \sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=140

$$\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)} + \frac{cd \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}$$

[Out] Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/((c\*d\*f - a\*e\*g)\*Sqrt[d + e\*x]\*(f + g\*x)) + (c\*d\*ArcTan[(Sqrt[g]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(Sqrt[c\*d\*f - a\*e\*g]\*Sqrt[d + e\*x])])/(Sqrt[g]\*(c\*d\*f - a\*e\*g)^(3/2))

**Rubi [A]** time = 0.640196, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)} + \frac{cd \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{\sqrt{g}(cdf-aeg)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e\*x]/((f + g\*x)^2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]), x]

[Out] Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/((c\*d\*f - a\*e\*g)\*Sqrt[d + e\*x]\*(f + g\*x)) + (c\*d\*ArcTan[(Sqrt[g]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(Sqrt[c\*d\*f - a\*e\*g]\*Sqrt[d + e\*x])])/(Sqrt[g]\*(c\*d\*f - a\*e\*g)^(3/2))

**Rubi in Sympy [A]** time = 56.4043, size = 131, normalized size = 0.94

$$\frac{cd \operatorname{atanh}\left(\frac{\sqrt{g}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}\sqrt{aeg-cdf}}\right)}{\sqrt{g}(aeg-cdf)^{3/2}} - \frac{\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}(f+gx)(aeg-cdf)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*(1/2)/(g\*x+f)\*\*2/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2))

[Out] c\*d\*atanh(sqrt(g)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(sqrt(d + e\*x)\*sqrt(a\*e\*g - c\*d\*f)))/(sqrt(g)\*(a\*e\*g - c\*d\*f)\*\*(3/2)) - sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(sqrt(d + e\*x)\*(f + g\*x)\*(a\*e\*g - c\*d\*f))

**Mathematica [A]** time = 0.264266, size = 138, normalized size = 0.99

$$\frac{\sqrt{d+ex}\left(\sqrt{g}(ae+cdx)\sqrt{aeg-cdf}-cd(f+gx)\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{aeg-cdf}}\right)\right)}{\sqrt{g}(f+gx)\sqrt{(d+ex)(ae+cdx)}(aeg-cdf)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e\*x]/((f + g\*x)^2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])

[Out]  $-\left(\sqrt{d+e^*x}\right)\left(\sqrt{g}\sqrt{-(c^*d^*f)+a^*e^*g}\right)\left(a^*e+c^*d^*x\right)-c^*d^*\sqrt{a^*e+c^*d^*x}\left(f+g^*x\right)\operatorname{ArcTanh}\left(\frac{\sqrt{g}\sqrt{a^*e+c^*d^*x}}{\sqrt{-(c^*d^*f)+a^*e^*g}}\right)\right)/\left(\sqrt{g}\left(-\left(c^*d^*f\right)+a^*e^*g\right)^{3/2}\sqrt{\left(a^*e+c^*d^*x\right)\left(d+e^*x\right)\left(f+g^*x\right)}\right)$

**Maple [A]** time = 0.029, size = 168, normalized size = 1.2

$$\frac{1}{(aeg - cdf)(gx + f)}\sqrt{cdex^2 + ae^2x + cd^2x + ade}\left(\operatorname{Artanh}\left(g\sqrt{cdx + ae}\frac{1}{\sqrt{(aeg - cdf)g}}\right)xcdg + \operatorname{Artanh}\left(g\sqrt{cdx + ae}\frac{1}{\sqrt{(aeg - cdf)g}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{int}\left(\left(e^*x+d\right)^{1/2}/\left(g^*x+f\right)^2/\left(a^*d^*e+\left(a^*e^2+c^*d^2\right)^*x+c^*d^*e^*x^2\right)^{1/2},x\right)$

[Out]  $\left(c^*d^*e^*x^2+a^*e^2^*x+c^*d^2^*x+a^*d^*e\right)^{1/2}\left(\operatorname{arctanh}\left(g^*\left(c^*d^*x+a^*e\right)^{1/2}/\left(\left(a^*e^*g-c^*d^*f\right)^*g\right)^{1/2}\right)^*x^2+c^*d^*g+\operatorname{arctanh}\left(g^*\left(c^*d^*x+a^*e\right)^{1/2}/\left(\left(a^*e^*g-c^*d^*f\right)^*g\right)^{1/2}\right)^*c^*d^*f-\left(c^*d^*x+a^*e\right)^{1/2}\left(\left(a^*e^*g-c^*d^*f\right)^*g\right)^{1/2}\right)/\left(e^*x+d\right)^{1/2}/\left(c^*d^*x+a^*e\right)^{1/2}/\left(a^*e^*g-c^*d^*f\right)/\left(g^*x+f\right)/\left(\left(a^*e^*g-c^*d^*f\right)^*g\right)^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}\left(\sqrt{e^*x+d}/\left(\sqrt{c^*d^*e^*x^2+a^*d^*e+\left(c^*d^2+a^*e^2\right)^*x}\right)^*(g^*x+f)^2\right)$

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.30394, size = 1, normalized size = 0.01

$$\frac{\left(cdegx^2 + cd^2f + (cdf + cd^2g)x\right) \log\left(\frac{2\sqrt{cdex^2 + ade + (cd^2 + ae^2)x} \sqrt{cdfg - aeg^2} \sqrt{ex + d} - (cdegx^2 - cd^2f + 2adeg - (cdf - (cd^2 + 2ae^2)g)x)}{egx^2 + df + (ef + dg)x}\right)}{2(cd^2f^2 - adefg + (cdfg - ae^2g^2)x^2 + (cdf^2 - adeg^2 + (cd^2 - ae^2)fg)x)} + \frac{\left(cdegx^2 + cd^2f + (cdf + cd^2g)x\right) \operatorname{arctan}\left(\frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x} \sqrt{cdfg - aeg^2} \sqrt{ex + d}}{cdegx^2 + adeg + (cd^2 + ae^2)gx}\right) - \sqrt{cdex^2 + ade + (cd^2 + ae^2)x} \sqrt{cdfg}}{(cd^2f^2 - adefg + (cdfg - ae^2g^2)x^2 + (cdf^2 - adeg^2 + (cd^2 - ae^2)fg)x) \sqrt{cdfg - aeg^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}\left(\sqrt{e^*x+d}/\left(\sqrt{c^*d^*e^*x^2+a^*d^*e+\left(c^*d^2+a^*e^2\right)^*x}\right)^*(g^*x+f)^2\right)$

[Out]  $\left[-1/2*\left(\left(c^*d^*e^*g^*x^2+c^*d^2^*f+\left(c^*d^*e^*f+c^*d^2^*g\right)^*x\right)*\log\left(\left(2*\sqrt{c^*d^*e^*x^2+a^*d^*e+\left(c^*d^2+a^*e^2\right)^*x}\right)^*\left(c^*d^*f^*g-a^*e^*g^2\right)*\sqrt{\left(e^*x+d\right)-\left(c^*d^*e^*g^*x^2-c^*d^2^*f+2*a^*d^*e^*g-\left(c^*d^*e^*f-\left(c^*d^2+2*a^*e^2\right)^*g\right)^*x\right)*\sqrt{\left(-c^*d^*f^*g+a^*e^*g^2\right)}\right)/\left(\left(e^*g^*x^2+d^*f+\left(e^*f+d^*g\right)^*x\right)-2*\sqrt{c^*d^*e^*x^2+a^*d^*e+\left(c^*d^2+a^*e^2\right)^*x}\sqrt{\left(-c^*d^*f^*g+a^*e^*g^2\right)*\sqrt{\left(e^*x+d\right)}\right)/\left(\left(c^*d^2^*f^2-a^*d^*e^*f^*g+\left(c^*d^*e^*f^*g-a^*e^2^*g^2\right)^*x^2+\left(c^*d^*e^*f^2-a^*d^*e^*g^2+\left(c^*d^2-a^*e^2\right)^*f^*g\right)^*x\right)*\sqrt{\left(-c^*d^*f^*g+a^*e^*g^2\right)}\right),-\left(\left(c^*d^*e^*g^*x^2+c^*d^2^*f+\left(c^*d^*e^*f+c^*d^2^*g\right)^*x\right)*\operatorname{arctan}\left(\sqrt{c^*d^*e^*x^2+a^*d^*e+\left(c^*d^2+a^*e^2\right)^*x}\sqrt{c^*d^*f^*g-a^*e^*g^2}\right)*\sqrt{\left(e^*x+d\right)}\right)/\left(\left(c^*d^*e^*g^*x^2+a^*d^*e+\left(c^*d^2+a^*e^2\right)^*g^*x\right)-\sqrt{c^*d^*e^*x^2+a^*d^*e+\left(c^*d^2+a^*e^2\right)^*g^*x}\right)-\sqrt{c^*d^*e^*x^2+a^*d^*e+\left(c^*d^2+a^*e^2\right)^*g^*x}\right)$

$$a^2 e^x \sqrt{c d f g - a^2 e^x g^2} \sqrt{e^x + d} / ((c d^2 f^2 - a^2 d e f g + (c d e f g - a^2 e^2 g^2) x^2 + (c d e f^2 - a^2 d e g^2 + (c d^2 - a^2 e^2) f g) x) \sqrt{c d f g - a^2 e^x g^2})$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(1/2)/(g\*x+f)\*\*2/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^2)

[Out] Timed out

$$3.663 \quad \int \frac{\sqrt{d+ex}}{(f+gx)^3 \sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=213

$$\frac{3c^2d^2 \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-ae^2}}\right)}{4\sqrt{g}(cdf-ae^2)^{5/2}} + \frac{3cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4\sqrt{d+ex}(f+gx)(cdf-ae^2)^2} + \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-ae^2)}$$

[Out] Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(2\*(c\*d\*f - a\*e\*g)\*Sqrt[d + e\*x]\*(f + g\*x)^2) + (3\*c\*d\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(4\*(c\*d\*f - a\*e\*g)^2\*Sqrt[d + e\*x]\*(f + g\*x)) + (3\*c^2\*d^2\*ArcTan[(Sqrt[g]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(Sqrt[c\*d\*f - a\*e\*g]\*Sqrt[d + e\*x])])/(4\*Sqrt[g]\*(c\*d\*f - a\*e\*g)^(5/2))

**Rubi [A]** time = 1.01549, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{3c^2d^2 \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-ae^2}}\right)}{4\sqrt{g}(cdf-ae^2)^{5/2}} + \frac{3cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4\sqrt{d+ex}(f+gx)(cdf-ae^2)^2} + \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-ae^2)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e\*x]/((f + g\*x)^3\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]), x]

[Out] Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(2\*(c\*d\*f - a\*e\*g)\*Sqrt[d + e\*x]\*(f + g\*x)^2) + (3\*c\*d\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(4\*(c\*d\*f - a\*e\*g)^2\*Sqrt[d + e\*x]\*(f + g\*x)) + (3\*c^2\*d^2\*ArcTan[(Sqrt[g]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(Sqrt[c\*d\*f - a\*e\*g]\*Sqrt[d + e\*x])])/(4\*Sqrt[g]\*(c\*d\*f - a\*e\*g)^(5/2))

**Rubi in Sympy [A]** time = 84.5071, size = 202, normalized size = 0.95

$$\frac{3c^2d^2 \operatorname{atanh}\left(\frac{\sqrt{g}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}\sqrt{aeg-cdf}}\right)}{4\sqrt{g}(aeg-cdf)^{5/2}} + \frac{3cd\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{4\sqrt{d+ex}(f+gx)(aeg-cdf)^2} - \frac{\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{2\sqrt{d+ex}(f+gx)^2(aeg-cdf)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*(1/2)/(g\*x+f)\*\*3/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*

[Out] -3\*c\*\*2\*d\*\*2\*atanh(sqrt(g)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2)))/(sqrt(d + e\*x)\*sqrt(a\*e\*g - c\*d\*f))/(4\*sqrt(g)\*(a\*e\*g - c\*d\*f)\*\*(5/2)) + 3\*c\*d\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(4\*sqrt(d + e\*x)\*(f + g\*x)\*(a\*e\*g - c\*d\*f)\*\*2) - sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(2\*sqrt(d + e\*x)\*(f + g\*x)\*\*2\*(a\*e\*g - c\*d\*f))

**Mathematica [A]** time = 0.367804, size = 163, normalized size = 0.77

$$\frac{\sqrt{d+ex}\left(\sqrt{g}(ae+cdx)\sqrt{aeg-cdf}(cd(5f+3gx)-2aeg)-3c^2d^2(f+gx)^2\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{aeg-cdf}}\right)\right)}{4\sqrt{g}(f+gx)^2\sqrt{(d+ex)(ae+cdx)}(aeg-cdf)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e\*x]/((f + g\*x)^3\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])

[Out] (Sqrt[d + e\*x]\*(Sqrt[g]\*Sqrt[-(c\*d\*f) + a\*e\*g]\*(a\*e + c\*d\*x)^(-2\*a\*e\*g + c\*d\*(5\*f + 3\*g\*x)) - 3\*c^2\*d^2\*Sqrt[a\*e + c\*d\*x]\*(f + g\*x)^2\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/Sqrt[-(c\*d\*f) + a\*e\*g]])) / (4\*Sqrt[g]\*(-(c\*d\*f) + a\*e\*g)^(5/2)\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)]\*(f + g\*x)^2)

**Maple [A]** time = 0.03, size = 285, normalized size = 1.3

$$-\frac{1}{4(aeg - cdf)^2(gx + f)^2} \sqrt{cdex^2 + ae^2x + cd^2x + ade} \left( 3 \operatorname{Artanh} \left( \frac{g\sqrt{cdx + ae}}{\sqrt{(aeg - cdf)g}} \right) x^2 c^2 d^2 g^2 + 6 \operatorname{Artanh} \left( \frac{g\sqrt{cdx + ae}}{\sqrt{(aeg - cdf)g}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(1/2)/(g\*x+f)^3/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2), x)

[Out] -1/4\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)\*(3\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x^2\*c^2\*d^2\*g^2+6\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x\*c^2\*d^2\*f\*g+3\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*c^2\*d^2\*f^2-3\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*x\*c\*d\*g+2\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*a\*e\*g-5\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*c\*d\*f)/(e\*x+d)^(1/2)/(c\*d\*x+a\*e)^(1/2)/(a\*e\*g-c\*d\*f)^2/(g\*x+f)^2/((a\*e\*g-c\*d\*f)\*g)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^3)

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.311114, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^3)

[Out] [1/8\*(2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*f\*g + a\*e\*g^2)\*(3\*c\*d\*g\*x + 5\*c\*d\*f - 2\*a\*e\*g)\*sqrt(e\*x + d) + 3\*(c^2\*d^2\*e\*g^2\*x^3 + c^2\*d^3\*f^2 + (2\*c^2\*d^2\*e\*f\*g + c^2\*d^3\*g^2)\*x^2 + (c^2\*d^2\*e\*f^2 + 2\*c^2\*d^3\*f\*g)\*x)\*log(-(2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(c\*d\*f\*g - a\*e\*g^2)\*sqrt(e\*x + d) + (c\*d\*e\*g\*x^2 - c\*d^2\*f + 2\*a\*d\*e\*g - (c\*d\*e\*f - (c\*d^2 + 2\*a\*e^2)\*g)\*x)\*sqrt(-c\*d\*f\*g + a\*e\*g^2))/(e\*g\*x^2 + d\*f + (e\*f + d\*g)\*x))/((c^2\*d^3\*f^4 - 2\*a\*c\*d^2\*e\*f^3\*g + a^2\*d\*e^2\*f^2\*g^2 + (c^2\*d^2\*e\*f^2\*g^2 - 2\*a\*c\*d\*e^2\*f\*g^3 + a^2\*e^3\*g^4)\*x^3 + (2\*c^2\*d^2\*e\*f^3\*g + a^2\*d\*e^2\*g^4 + (c^2\*d^3 - 4\*a\*c\*d\*e^2)\*f^2\*g^2 - 2\*(a\*c\*d

$$\begin{aligned} &^2e - a^2e^3) * f * g^3) * x^2 + (c^2 * d^2 * e * f^4 + 2 * a^2 * d * e^2 * f * g^3 + \\ &2 * (c^2 * d^3 - a * c * d * e^2) * f^3 * g - (4 * a * c * d^2 * e - a^2 * e^3) * f^2 * g^2) \\ &* x) * \sqrt{-c * d * f * g + a * e * g^2}), 1/4 * (\sqrt{c * d * e * x^2 + a * d * e + (c * d \\ &^2 + a * e^2) * x) * \sqrt{c * d * f * g - a * e * g^2}) * (3 * c * d * g * x + 5 * c * d * f - 2 * a \\ &* e * g) * \sqrt{e * x + d} - 3 * (c^2 * d^2 * e * g^2 * x^3 + c^2 * d^3 * f^2 + (2 * c^2 \\ &* d^2 * e * f * g + c^2 * d^3 * g^2) * x^2 + (c^2 * d^2 * e * f^2 + 2 * c^2 * d^3 * f * g) * x \\ &)* \arctan(\sqrt{c * d * e * x^2 + a * d * e + (c * d^2 + a * e^2) * x) * \sqrt{c * d * f * g \\ &- a * e * g^2}) * \sqrt{e * x + d} / (c * d * e * g * x^2 + a * d * e * g + (c * d^2 + a * e^2 \\ &)* g * x)) / ((c^2 * d^3 * f^4 - 2 * a * c * d^2 * e * f^3 * g + a^2 * d * e^2 * f^2 * g^2 + \\ &(c^2 * d^2 * e * f^2 * g^2 - 2 * a * c * d * e^2 * f * g^3 + a^2 * e^3 * g^4) * x^3 + (2 * c^2 \\ &* d^2 * e * f^3 * g + a^2 * d * e^2 * g^4 + (c^2 * d^3 - 4 * a * c * d * e^2) * f^2 * g^2 - \\ &2 * (a * c * d^2 * e - a^2 * e^3) * f * g^3) * x^2 + (c^2 * d^2 * e * f^4 + 2 * a^2 * d * e^2 \\ &* f * g^3 + 2 * (c^2 * d^3 - a * c * d * e^2) * f^3 * g - (4 * a * c * d^2 * e - a^2 * e^3) \\ &* f^2 * g^2) * x) * \sqrt{c * d * f * g - a * e * g^2})] \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(1/2)/(g\*x+f)\*\*3/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^3)

[Out] Timed out



$$3.664 \quad \int \frac{\sqrt{d+ex}}{(f+gx)^4 \sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=280

$$\frac{5c^3 d^3 \tan^{-1} \left( \frac{\sqrt{g} \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex} \sqrt{cdf-aeg}} \right)}{8\sqrt{g}(cdf-aeg)^{7/2}} + \frac{5c^2 d^2 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{8\sqrt{d+ex}(f+gx)(cdf-aeg)^3} \\ + \frac{5cd \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{12\sqrt{d+ex}(f+gx)^2(cdf-aeg)^2} + \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}$$

[Out] Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(3\*(c\*d\*f - a\*e\*g)\*Sqrt[d + e\*x]\*(f + g\*x)^3) + (5\*c\*d\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(12\*(c\*d\*f - a\*e\*g)^2\*Sqrt[d + e\*x]\*(f + g\*x)^2) + (5\*c^2\*d^2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(8\*(c\*d\*f - a\*e\*g)^3\*Sqrt[d + e\*x]\*(f + g\*x)) + (5\*c^3\*d^3\*ArcTan[(Sqrt[g]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(Sqrt[c\*d\*f - a\*e\*g]\*Sqrt[d + e\*x])])/(8\*Sqrt[g]\*(c\*d\*f - a\*e\*g)^(7/2))

**Rubi [A]** time = 1.37471, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{5c^3 d^3 \tan^{-1} \left( \frac{\sqrt{g} \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex} \sqrt{cdf-aeg}} \right)}{8\sqrt{g}(cdf-aeg)^{7/2}} + \frac{5c^2 d^2 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{8\sqrt{d+ex}(f+gx)(cdf-aeg)^3} \\ + \frac{5cd \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{12\sqrt{d+ex}(f+gx)^2(cdf-aeg)^2} + \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^3(cdf-aeg)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e\*x]/((f + g\*x)^4\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]), x]

[Out] Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(3\*(c\*d\*f - a\*e\*g)\*Sqrt[d + e\*x]\*(f + g\*x)^3) + (5\*c\*d\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(12\*(c\*d\*f - a\*e\*g)^2\*Sqrt[d + e\*x]\*(f + g\*x)^2) + (5\*c^2\*d^2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(8\*(c\*d\*f - a\*e\*g)^3\*Sqrt[d + e\*x]\*(f + g\*x)) + (5\*c^3\*d^3\*ArcTan[(Sqrt[g]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(Sqrt[c\*d\*f - a\*e\*g]\*Sqrt[d + e\*x])])/(8\*Sqrt[g]\*(c\*d\*f - a\*e\*g)^(7/2))

**Rubi in Sympy [A]** time = 116.729, size = 269, normalized size = 0.96

$$\frac{5c^3 d^3 \operatorname{atanh} \left( \frac{\sqrt{g} \sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex} \sqrt{aeg-cdf}} \right)}{8\sqrt{g}(aeg-cdf)^{7/2}} - \frac{5c^2 d^2 \sqrt{ade+cdex^2+x(ae^2+cd^2)}}{8\sqrt{d+ex}(f+gx)(aeg-cdf)^3} \\ + \frac{5cd \sqrt{ade+cdex^2+x(ae^2+cd^2)}}{12\sqrt{d+ex}(f+gx)^2(aeg-cdf)^2} - \frac{\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{3\sqrt{d+ex}(f+gx)^3(aeg-cdf)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*(1/2)/(g\*x+f)\*\*4/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2), x)

[Out] 5\*c\*\*3\*d\*\*3\*atanh(sqrt(g)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2)))/(sqrt(d + e\*x)\*sqrt(a\*e\*g - c\*d\*f))/(8\*sqrt(g)\*(a\*e\*g - c\*d\*f)\*\*(7/2)) - 5\*c\*\*2\*d\*\*2\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(8\*sqrt(d + e\*x)\*(f + g\*x)\*(a\*e\*g - c\*d\*f)\*\*3) + 5\*c\*d

$$\frac{\sqrt{a^2 d^2 e + c^2 d^2 e^2 x^2 + x(a^2 e^2 + c^2 d^2)}}{(12 \sqrt{d + e^2 x} (f + g^2 x)^2 (a^2 e^2 g - c^2 d^2 f)^2) - \sqrt{a^2 d^2 e + c^2 d^2 e^2 x^2 + x(a^2 e^2 + c^2 d^2)}}{(3 \sqrt{d + e^2 x} (f + g^2 x)^3 (a^2 e^2 g - c^2 d^2 f))}$$

**Mathematica [A]** time = 0.666635, size = 188, normalized size = 0.67

$$\frac{\sqrt{d + ex} \left( \frac{(ae+cdx)(8a^2e^2g^2-2acdeg(13f+5gx)+c^2d^2(33f^2+40fgx+15g^2x^2))}{3(f+gx)^3(cdf-aeg)^3} + \frac{5c^3d^3\sqrt{ae+cdx} \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{aeg-cdf}}\right)}{\sqrt{g}(aeg-cdf)^{7/2}} \right)}{8\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e\*x]/((f + g\*x)^4\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])

[Out] (Sqrt[d + e\*x]\*(((a\*e + c\*d\*x)\*(8\*a^2\*e^2\*g^2 - 2\*a\*c\*d\*e\*g\*(13\*f + 5\*g\*x) + c^2\*d^2\*(33\*f^2 + 40\*f\*g\*x + 15\*g^2\*x^2)))/(3\*(c\*d\*f - a\*e\*g)^3\*(f + g\*x)^3) + (5\*c^3\*d^3\*Sqrt[a\*e + c\*d\*x]\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/Sqrt[-(c\*d\*f) + a\*e\*g]])/(Sqrt[g]\*(-(c\*d\*f) + a\*e\*g)^(7/2))))/(8\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)])

**Maple [A]** time = 0.031, size = 450, normalized size = 1.6

$$\frac{1}{24(aeg - cdf)^3(gx + f)^3} \sqrt{cdex^2 + ae^2x + cd^2x + ade} \left( 15 \operatorname{Artanh} \left( \frac{g\sqrt{cdx + ae}}{\sqrt{(aeg - cdf)g}} \right) x^3 c^3 d^3 g^3 + 45 \operatorname{Artanh} \left( \frac{g\sqrt{cdx + ae}}{\sqrt{(aeg - cdf)g}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(1/2)/(g\*x+f)^4/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2), x)

[Out] 1/24\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)\*(15\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x^3\*c^3\*d^3\*g^3+45\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x^2\*c^3\*d^3\*f\*g^2+45\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x\*c^3\*d^3\*f^2\*g+15\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*c^3\*d^3\*f^3-15\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*x^2\*c^2\*d^2\*g^2+10\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*x\*a\*c\*d\*e\*g^2-40\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*x\*c^2\*d^2\*f\*g-8\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*a^2\*e^2\*g^2+26\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*a\*c\*d\*e\*f\*g-33\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*c^2\*d^2\*f^2)/(e\*x+d)^(1/2)/(c\*d\*x+a\*e)^(1/2)/(a\*e\*g-c\*d\*f)^3/(g\*x+f)^3/((a\*e\*g-c\*d\*f)\*g)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^4)

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.320529, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^4)`

[Out] 
$$\left[ \frac{1}{48} \left( 2 \left( 15 c^2 d^2 g^2 x^2 + 33 c^2 d^2 f^2 - 26 a c d e f g + 8 a^2 e^2 g^2 + 10 (4 c^2 d^2 f g - a c d e g^2) x \right) \sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x} \sqrt{-c d f g + a e g^2} \sqrt{e x + d} - 15 (c^3 d^3 e g^3 x^4 + c^3 d^4 f^3 + (3 c^3 d^3 e f g^2 + c^3 d^4 g^3) x^3 + 3 (c^3 d^3 e f^2 g + c^3 d^4 f g^2) x^2 + (c^3 d^3 e f^3 + 3 c^3 d^4 f^2 g) x \right) \log \left( (2 \sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x} (c d f g - a e g^2) \sqrt{e x + d} - (c d e g x^2 - c d^2 f + 2 a d e g - (c d e f - (c d^2 + 2 a e^2) g) x) \sqrt{-c d f g + a e g^2} \right) / (e g x^2 + d f + (e f + d g) x) \right) / \left( (c^3 d^4 f^6 - 3 a c^2 d^3 e f^5 g + 3 a^2 c d^2 e^2 f^4 g^2 - a^3 d e^3 f^3 g^3 + (c^3 d^3 e f^3 g^3 - 3 a c^2 d^2 e^2 f^2 g^4 + 3 a^2 c d e^3 f g^5 - a^3 e^4 g^6) x^4 + (3 c^3 d^3 e f^4 g^2 - a^3 d e^3 g^6 + (c^3 d^4 - 9 a c^2 d^2 e^2) f^3 g^3 - 3 (a c^2 d^3 e - 3 a^2 c d e^3) f^2 g^4 + 3 (a^2 c d^2 e^2 - a^3 e^4) f g^5) x^3 + 3 (c^3 d^3 e f^5 g - a^3 d e^3 f g^5 + (c^3 d^4 - 3 a c^2 d^2 e^2) f^4 g^2 - 3 (a c^2 d^3 e - a^2 c d e^3) f^3 g^3 + (3 a^2 c d^2 e^2 - a^3 e^4) f^2 g^4) x^2 + (c^3 d^3 e f^6 - 3 a^3 d e^3 f^2 g^4 + 3 (c^3 d^4 - a c^2 d^2 e^2) f^5 g - 3 (3 a c^2 d^3 e - a^2 c d e^3) f^4 g^2 + (9 a^2 c d^2 e^2 - a^3 e^4) f^3 g^3) x \right) \sqrt{-c d f g + a e g^2} \right), \frac{1}{24} \left( (15 c^2 d^2 g^2 x^2 + 33 c^2 d^2 f^2 - 26 a c d e f g + 8 a^2 e^2 g^2 + 10 (4 c^2 d^2 f g - a c d e g^2) x \right) \sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x} \sqrt{c d f g - a e g^2} \sqrt{e x + d} - 15 (c^3 d^3 e g^3 x^4 + c^3 d^4 f^3 + (3 c^3 d^3 e f g^2 + c^3 d^4 g^3) x^3 + 3 (c^3 d^3 e f^2 g + c^3 d^4 f g^2) x^2 + (c^3 d^3 e f^3 + 3 c^3 d^4 f^2 g) x \right) \arctan \left( \sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x} \sqrt{c d f g - a e g^2} \sqrt{e x + d} / (c d e g x^2 + a d e g + (c d^2 + a e^2) g x) \right) / \left( (c^3 d^4 f^6 - 3 a c^2 d^3 e f^5 g + 3 a^2 c d^2 e^2 f^4 g^2 - a^3 d e^3 f^3 g^3 + (c^3 d^3 e f^3 g^3 - 3 a c^2 d^2 e^2 f^2 g^4 + 3 a^2 c d e^3 f g^5 - a^3 e^4 g^6) x^4 + (3 c^3 d^3 e f^4 g^2 - a^3 d e^3 g^6 + (c^3 d^4 - 9 a c^2 d^2 e^2) f^3 g^3 - 3 (a c^2 d^3 e - 3 a^2 c d e^3) f^2 g^4 + 3 (a^2 c d^2 e^2 - a^3 e^4) f g^5) x^3 + 3 (c^3 d^3 e f^5 g - a^3 d e^3 f g^5 + (c^3 d^4 - 3 a c^2 d^2 e^2) f^4 g^2 - 3 (a c^2 d^3 e - a^2 c d e^3) f^3 g^3 + (3 a^2 c d^2 e^2 - a^3 e^4) f^2 g^4) x^2 + (c^3 d^3 e f^6 - 3 a^3 d e^3 f^2 g^4 + 3 (c^3 d^4 - a c^2 d^2 e^2) f^5 g - 3 (3 a c^2 d^3 e - a^2 c d e^3) f^4 g^2 + (9 a^2 c d^2 e^2 - a^3 e^4) f^3 g^3) x \right) \sqrt{c d f g - a e g^2} \right) ]$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**(1/2)/(g*x+f)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)`

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^4)`

[Out] Timed out

$$3.665 \quad \int \frac{(d+ex)^{3/2}(f+gx)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx$$

**Optimal.** Leaf size=257

$$\begin{aligned} & -\frac{16g\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)(2ae^2g-cd(3ef-dg))}{5c^4d^4e\sqrt{d+ex}} \\ & +\frac{16g^2\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}{5c^3d^3e} \\ & +\frac{12g(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5c^2d^2\sqrt{d+ex}} - \frac{2\sqrt{d+ex}(f+gx)^3}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \end{aligned}$$

[Out]  $(-2*\text{Sqrt}[d + e*x]^*(f + g*x)^3)/(c*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (16*g*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(3*e*f - d*g))*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c^4*d^4*e*\text{Sqrt}[d + e*x]) + (16*g^2*(c*d*f - a*e*g)*\text{Sqrt}[d + e*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c^3*d^3*e) + (12*g*(f + g*x)^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c^2*d^2*\text{Sqrt}[d + e*x])$

**Rubi [A]** time = 1.01343, antiderivative size = 257, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\begin{aligned} & -\frac{16g\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)(2ae^2g-cd(3ef-dg))}{5c^4d^4e\sqrt{d+ex}} \\ & +\frac{16g^2\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}{5c^3d^3e} \\ & +\frac{12g(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5c^2d^2\sqrt{d+ex}} - \frac{2\sqrt{d+ex}(f+gx)^3}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^{(3/2)}*(f + g*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}, x]$

[Out]  $(-2*\text{Sqrt}[d + e*x]^*(f + g*x)^3)/(c*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (16*g*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(3*e*f - d*g))*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c^4*d^4*e*\text{Sqrt}[d + e*x]) + (16*g^2*(c*d*f - a*e*g)*\text{Sqrt}[d + e*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c^3*d^3*e) + (12*g*(f + g*x)^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c^2*d^2*\text{Sqrt}[d + e*x])$

**Rubi in Sympy [A]** time = 90.5588, size = 252, normalized size = 0.98

$$\begin{aligned} & -\frac{2\sqrt{d+ex}(f+gx)^3}{cd\sqrt{ade+cdex^2+x(ae^2+cd^2)}} + \frac{12g(f+gx)^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{5c^2d^2\sqrt{d+ex}} \\ & -\frac{16g^2\sqrt{d+ex}(aeg-cdf)\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{5c^3d^3e} \\ & +\frac{16g(aeg-cdf)\sqrt{ade+cdex^2+x(ae^2+cd^2)}(2ae^2g+cd^2g-3cdf)}{5c^4d^4e\sqrt{d+ex}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)^{(3/2)}*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(3/2)}$

[Out]  $-2\sqrt{d+ex}(f+gx)^3/(c^2d\sqrt{a^2d^2e+cd^2ex^2+x(a^2e^2+cd^2)})+12g(f+gx)^2\sqrt{a^2d^2e+cd^2ex^2+x(a^2e^2+cd^2)}/(5c^2d^2\sqrt{d+ex})-16g^2\sqrt{d+ex}(a^2eg-c^2df)\sqrt{a^2d^2e+cd^2ex^2+x(a^2e^2+cd^2)}/(5c^3d^3e)+16g(a^2eg-c^2df)\sqrt{a^2d^2e+cd^2ex^2+x(a^2e^2+cd^2)}(2a^2e^2g+cd^2g-3c^2d^2ef)/(5c^4d^4e\sqrt{d+ex})$

**Mathematica [A]** time = 0.186859, size = 134, normalized size = 0.52

$$\frac{2\sqrt{d+ex}(16a^3e^3g^3+8a^2cde^2g^2(gx-5f)-2ac^2d^2eg(-15f^2+10fgx+g^2x^2)+c^3d^3(-5f^3+15f^2gx+5fg^2x^2+g^3x^3))}{5c^4d^4\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((d+e\*x)^(3/2)\*(f+g\*x)^3)/(a\*d\*e+(c\*d^2+a\*e^2)\*x+c\*d\*e\*x^2)^(3/2),x]

[Out]  $(2\sqrt{d+ex}(16a^3e^3g^3+8a^2c^2d^2e^2g^2(-5f+gx)-2a^2c^2d^2e^2g^2(-15f^2+10fgx+g^2x^2)+c^3d^3(-5f^3+15f^2gx+5fg^2x^2+g^3x^3)))/(5c^4d^4\sqrt{(a^2e^2+cd^2)x(d+ex)})$

**Maple [A]** time = 0.012, size = 187, normalized size = 0.7

$$\frac{(2cdx+2ae)(g^3x^3c^3d^3-2ac^2d^2eg^3x^2+5c^3d^3fg^2x^2+8a^2cde^2g^3x-20ac^2d^2efg^2x+15c^3d^3f^2gx+16a^3e^3g^3-40a^2cd^2efg^2)}{5c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(3/2)\*(g\*x+f)^3/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2),x)

[Out]  $2/5*(c^2d^2x+a^2e)*(c^3d^3g^3x^3-2a^2c^2d^2e^2g^3x^2+5c^3d^3f^2g^2x+8a^2c^2d^2e^2g^3x-20a^2c^2d^2e^2f^2g^2x+15c^3d^3f^2gx+16a^3e^3g^3-40a^2c^2d^2e^2f^2g^2+30a^2c^2d^2e^2f^2g-5c^3d^3f^3)*(e*x+d)^(3/2)/c^4/d^4/(c^2d^2e^2x+a^2e^2x+c^2d^2x+a^2e)^(3/2)$

**Maxima [A]** time = 0.786263, size = 223, normalized size = 0.87

$$-\frac{2f^3}{\sqrt{cdx+aecd}}+\frac{6(cdx+2ae)f^2g}{\sqrt{cdx+aec^2d^2}}+\frac{2(c^2d^2x^2-4acdex-8a^2e^2)fg^2}{\sqrt{cdx+aec^3d^3}}+\frac{2(c^3d^3x^3-2ac^2d^2ex^2+8a^2cde^2x+16a^3e^3)g^3}{5\sqrt{cdx+aec^4d^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)^(3/2)\*(g\*x+f)^3/(c\*d\*e\*x^2+a\*d\*e+(c\*d^2+a\*e^2)\*x)^(3/2),x)

[Out]  $-2f^3/(\sqrt{c^2d^2x+a^2e}*c*d)+6*(c^2d^2x+2a^2e)*f^2g/(\sqrt{c^2d^2x+a^2e}*c^2d^2)+2*(c^2d^2x^2-4a^2c^2d^2e*x-8a^2e^2)*f^2g^2/(\sqrt{c^2d^2x+a^2e}*c^3d^3)+2/5*(c^3d^3x^3-2a^2c^2d^2e*x^2+8a^2c^2d^2e^2x+16a^3e^3)*g^3/(\sqrt{c^2d^2x+a^2e}*c^4d^4)$

**Fricas [A]** time = 0.28209, size = 424, normalized size = 1.65

$$\frac{2(c^3 d^3 e g^3 x^4 - 5 c^3 d^4 f^3 + 30 a c^2 d^3 e f^2 g - 40 a^2 c d^2 e^2 f g^2 + 16 a^3 d e^3 g^3 + (5 c^3 d^3 e f g^2 + (c^3 d^4 - 2 a c^2 d^2 e^2) g^3) x^3 + (15 c^3 d^3 e f^2 g + 5 (c^3 d^4 - 4 a c^2 d^2 e^2) f g^2 - 2 (a c^2 d^3 e - 4 a^2 c d^2 e^3) g^3) x^2 - (5 c^3 d^3 e f^3 - 15 (c^3 d^4 + 2 a c^2 d^2 e^2) f^2 g + 20 (a c^2 d^3 e + 2 a^2 c d^2 e^3) f g^2 - 8 (a^2 c d^2 e^2 + 2 a^3 e^4) g^3) x}{\sqrt{(c d e x^2 + a d e + (c d^2 + a e^2) x)^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*(g\*x + f)^3/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2))

[Out] 2/5\*(c^3\*d^3\*e\*g^3\*x^4 - 5\*c^3\*d^4\*f^3 + 30\*a\*c^2\*d^3\*e\*f^2\*g - 40\*a^2\*c\*d^2\*e^2\*f\*g^2 + 16\*a^3\*d\*e^3\*g^3 + (5\*c^3\*d^3\*e\*f\*g^2 + (c^3\*d^4 - 2\*a\*c^2\*d^2\*e^2)\*g^3)\*x^3 + (15\*c^3\*d^3\*e\*f^2\*g + 5\*(c^3\*d^4 - 4\*a\*c^2\*d^2\*e^2)\*f\*g^2 - 2\*(a\*c^2\*d^3\*e - 4\*a^2\*c\*d^2\*e^3)\*g^3)\*x^2 - (5\*c^3\*d^3\*e\*f^3 - 15\*(c^3\*d^4 + 2\*a\*c^2\*d^2\*e^2)\*f^2\*g + 20\*(a\*c^2\*d^3\*e + 2\*a^2\*c\*d^2\*e^3)\*f\*g^2 - 8\*(a^2\*c\*d^2\*e^2 + 2\*a^3\*e^4)\*g^3)\*x)/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*c^4\*d^4)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(3/2)\*(g\*x+f)\*\*3/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2))

[Out] Timed out

**GIAC/XCAS [A]** time = 0.73925, size = 4, normalized size = 0.02

sage0\*x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*(g\*x + f)^3/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2))

[Out] sage0\*x

$$3.666 \quad \int \frac{(d+ex)^{3/2}(f+gx)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx$$

**Optimal.** Leaf size=181

$$\begin{aligned} & -\frac{8g\sqrt{x(ae^2+cd^2)+ade+cdex^2}(2ae^2g-cd(3ef-dg))}{3c^3d^3e\sqrt{d+ex}} \\ & + \frac{8g^2\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3c^2d^2e} - \frac{2\sqrt{d+ex}(f+gx)^2}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \end{aligned}$$

[Out]  $(-2*\text{Sqrt}[d + e*x]*(f + g*x)^2)/(c*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (8*g*(2*a*e^2*g - c*d*(3*e*f - d*g))*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^3*d^3*e*\text{Sqrt}[d + e*x]) + (8*g^2*\text{Sqrt}[d + e*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^2*d^2*e)$

**Rubi [A]** time = 0.606305, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\begin{aligned} & -\frac{8g\sqrt{x(ae^2+cd^2)+ade+cdex^2}(2ae^2g-cd(3ef-dg))}{3c^3d^3e\sqrt{d+ex}} \\ & + \frac{8g^2\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3c^2d^2e} - \frac{2\sqrt{d+ex}(f+gx)^2}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(d + e*x)^{(3/2)}*(f + g*x)^2}{(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}, x]$

[Out]  $(-2*\text{Sqrt}[d + e*x]*(f + g*x)^2)/(c*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (8*g*(2*a*e^2*g - c*d*(3*e*f - d*g))*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^3*d^3*e*\text{Sqrt}[d + e*x]) + (8*g^2*\text{Sqrt}[d + e*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^2*d^2*e)$

**Rubi in Sympy [A]** time = 61.106, size = 175, normalized size = 0.97

$$\begin{aligned} & -\frac{2\sqrt{d+ex}(f+gx)^2}{cd\sqrt{ade+cdex^2+x(ae^2+cd^2)}} + \frac{8g^2\sqrt{d+ex}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{3c^2d^2e} \\ & - \frac{8g\sqrt{ade+cdex^2+x(ae^2+cd^2)}(2ae^2g+cd^2g-3cdf)}{3c^3d^3e\sqrt{d+ex}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)**(3/2)*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2), x)$

[Out]  $-2*\text{sqrt}(d + e*x)*(f + g*x)**2/(c*d*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))) + 8*g**2*\text{sqrt}(d + e*x)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(3*c**2*d**2*e) - 8*g*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))*(2*a*e**2*g + c*d**2*g - 3*c*d*e*f)/(3*c**3*d**3*e*\text{sqrt}(d + e*x))$

**Mathematica [A]** time = 0.118233, size = 88, normalized size = 0.49

$$\frac{2\sqrt{d+ex}(-8a^2e^2g^2-4acdeg(gx-3f)+c^2d^2(-3f^2+6fgx+g^2x^2))}{3c^3d^3\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^(3/2)\*(f + g\*x)^2)/(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2), x]

[Out] (2\*sqrt(d + e\*x)\*(-8\*a^2\*e^2\*g^2 - 4\*a\*c\*d\*e\*g\*(-3\*f + g\*x) + c^2\*d^2\*(-3\*f^2 + 6\*f\*g\*x + g^2\*x^2)))/(3\*c^3\*d^3\*sqrt((a\*e + c\*d\*x)\*(d + e\*x)))

**Maple [A]** time = 0.012, size = 116, normalized size = 0.6

$$\frac{(2cdx + 2ae)(-g^2x^2c^2d^2 + 4acdeg^2x - 6c^2d^2fgx + 8a^2e^2g^2 - 12acdefg + 3c^2d^2f^2)}{3c^3d^3}(ex + d)^{\frac{3}{2}}(cdex^2 + ae^2x + cd^2x + \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(3/2)\*(g\*x+f)^2/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2), x)

[Out] -2/3\*(c\*d\*x+a\*e)\*(-c^2\*d^2\*g^2\*x^2+4\*a\*c\*d\*e\*g^2\*x-6\*c^2\*d^2\*f\*g\*x+8\*a^2\*e^2\*g^2-12\*a\*c\*d\*e\*f\*g+3\*c^2\*d^2\*f^2)\*(e\*x+d)^(3/2)/c^3/d^3/(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(3/2)

**Maxima [A]** time = 0.775449, size = 132, normalized size = 0.73

$$-\frac{2f^2}{\sqrt{cdx + aecd}} + \frac{4(cdx + 2ae)fg}{\sqrt{cdx + aec^2d^2}} + \frac{2(c^2d^2x^2 - 4acdex - 8a^2e^2)g^2}{3\sqrt{cdx + aec^3d^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*(g\*x + f)^2/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2), x)

[Out] -2\*f^2/(sqrt(c\*d\*x + a\*e)\*c\*d) + 4\*(c\*d\*x + 2\*a\*e)\*f\*g/(sqrt(c\*d\*x + a\*e)\*c^2\*d^2) + 2/3\*(c^2\*d^2\*x^2 - 4\*a\*c\*d\*e\*x - 8\*a^2\*e^2)\*g^2/(sqrt(c\*d\*x + a\*e)\*c^3\*d^3)

**Fricas [A]** time = 0.269696, size = 247, normalized size = 1.36

$$\frac{2(c^2d^2eg^2x^3 - 3c^2d^3f^2 + 12acd^2efg - 8a^2de^2g^2 + (6c^2d^2efg + (c^2d^3 - 4acde^2)g^2)x^2 - (3c^2d^2ef^2 - 6(c^2d^3 + 2acde^2)g^2)x + 2a^2e^2f^2)}{3\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{ex + dc^3d^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*(g\*x + f)^2/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2), x)

[Out] 2/3\*(c^2\*d^2\*e\*g^2\*x^3 - 3\*c^2\*d^3\*f^2 + 12\*a\*c\*d^2\*e\*f\*g - 8\*a^2\*d\*e^2\*g^2 + (6\*c^2\*d^2\*e\*f\*g + (c^2\*d^3 - 4\*a\*c\*d\*e^2)\*g^2)\*x^2 - (3\*c^2\*d^2\*e\*f^2 - 6\*(c^2\*d^3 + 2\*a\*c\*d\*e^2)\*g^2)\*x)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*c^3\*d^3)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**(3/2)*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)`

[Out] Timed out

**GIAC/XCAS [A]** time = 0.715027, size = 4, normalized size = 0.02

*sage0\*x*

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(3/2)*(g*x + f)^2/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)`

[Out] *sage0\*x*

$$3.667 \quad \int \frac{(d+ex)^{3/2}(f+gx)}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx$$

**Optimal.** Leaf size=150

$$-\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(2ae^2g-cd(dg+ef))}{c^2d^2\sqrt{d+ex}(cd^2-ae^2)} - \frac{2(d+ex)^{3/2}(cdf-aeg)}{cd(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

[Out]  $(-2*(c*d*f - a*e*g)*(d + e*x)^{(3/2)})/(c*d*(c*d^2 - a*e^2)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (2*(2*a*e^2*g - c*d*(e*f + d*g))*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c^2*d^2*(c*d^2 - a*e^2)*\text{Sqrt}[d + e*x])$

**Rubi [A]** time = 0.429481, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 44,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$-\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(2ae^2g-cd(dg+ef))}{c^2d^2\sqrt{d+ex}(cd^2-ae^2)} - \frac{2(d+ex)^{3/2}(cdf-aeg)}{cd(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^(3/2)\*(f + g\*x))/(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2), x]

[Out]  $(-2*(c*d*f - a*e*g)*(d + e*x)^{(3/2)})/(c*d*(c*d^2 - a*e^2)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (2*(2*a*e^2*g - c*d*(e*f + d*g))*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c^2*d^2*(c*d^2 - a*e^2)*\text{Sqrt}[d + e*x])$

**Rubi in Sympy [A]** time = 46.1415, size = 139, normalized size = 0.93

$$-\frac{2(d+ex)^{\frac{3}{2}}(aeg-cdf)}{cd(ae^2-cd^2)\sqrt{ade+cdex^2+x(ae^2+cd^2)}} + \frac{2\sqrt{ade+cdex^2+x(ae^2+cd^2)}(2ae^2g-cd^2g-cdef)}{c^2d^2\sqrt{d+ex}(ae^2-cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*(3/2)\*(g\*x+f)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2), x)

[Out]  $-2*(d + e*x)**(3/2)*(a*e*g - c*d*f)/(c*d*(a*e**2 - c*d**2)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))) + 2*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))*(2*a*e**2*g - c*d**2*g - c*d*e*f)/(c**2*d**2*\text{sqrt}(d + e*x)*(a*e**2 - c*d**2))$

**Mathematica [A]** time = 0.0642961, size = 51, normalized size = 0.34

$$\frac{2\sqrt{d+ex}(2aeg+cd(gx-f))}{c^2d^2\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^(3/2)\*(f + g\*x))/(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2), x]

[Out]  $(2*\text{Sqrt}[d + e*x]*(2*a*e*g + c*d*(-f + g*x)))/(c^2*d^2*\text{Sqrt}[(a*e + c*d*x)*(d + e*x)])$

---

**Maple [A]** time = 0.007, size = 66, normalized size = 0.4

$$2 \frac{(cdx + ae)(xcdg + 2aeg - cdf)(ex + d)^{3/2}}{c^2 d^2 (cdex^2 + ae^2 x + cd^2 x + ade)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^(3/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)`

[Out] `2*(c*d*x+a*e)*(c*d*g*x+2*a*e*g-c*d*f)*(e*x+d)^(3/2)/c^2/d^2/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)`

---

**Maxima [A]** time = 0.763476, size = 65, normalized size = 0.43

$$-\frac{2f}{\sqrt{cdx + aecd}} + \frac{2(cdx + 2ae)g}{\sqrt{cdx + aec^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(3/2)*(g*x + f)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)`

[Out] `-2*f/(sqrt(c*d*x + a*e)*c*d) + 2*(c*d*x + 2*a*e)*g/(sqrt(c*d*x + a*e)*c^2*d^2)`

---

**Fricas [A]** time = 0.269545, size = 119, normalized size = 0.79

$$\frac{2(cdegx^2 - cd^2f + 2adeg - (cdf - (cd^2 + 2ae^2)g)x)}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{ex + dc^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(3/2)*(g*x + f)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)`

[Out] `2*(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*c^2*d^2)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**(3/2)*(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)`

[Out] Timed out

---

**GIAC/XCAS [A]** time = 0.654246, size = 4, normalized size = 0.03

`sage_0x`

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^(3/2)*(g*x + f)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)
```

```
[Out] sage0*x
```

$$3.668 \quad \int \frac{(d+ex)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx$$

**Optimal.** Leaf size=46

$$-\frac{2\sqrt{d+ex}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

[Out] (-2\*Sqrt[d + e\*x])/(c\*d\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])

**Rubi [A]** time = 0.0697889, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$

$$-\frac{2\sqrt{d+ex}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^(3/2)/(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2), x]

[Out] (-2\*Sqrt[d + e\*x])/(c\*d\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])

**Rubi in Sympy [A]** time = 17.1135, size = 42, normalized size = 0.91

$$-\frac{2\sqrt{d+ex}}{cd\sqrt{ade+cdex^2+x(ae^2+cd^2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*(3/2)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2), x)

[Out] -2\*sqrt(d + e\*x)/(c\*d\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2)))

**Mathematica [A]** time = 0.0208251, size = 35, normalized size = 0.76

$$-\frac{2\sqrt{d+ex}}{cd\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^(3/2)/(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2), x]

[Out] (-2\*Sqrt[d + e\*x])/(c\*d\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)])

**Maple [A]** time = 0.004, size = 50, normalized size = 1.1

$$-2 \frac{(cdx + ae)(ex + d)^{3/2}}{cd(cdex^2 + ae^2x + cd^2x + ade)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^(3/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2),x)`

[Out] `-2*(c*d*x+a*e)*(e*x+d)^(3/2)/c/d/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)`

**Maxima [A]** time = 0.712586, size = 24, normalized size = 0.52

$$-\frac{2}{\sqrt{cdx + aecd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(3/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2),x, algorithm="maxima")`

[Out] `-2/(sqrt(c*d*x + a*e)*c*d)`

**Fricas [A]** time = 0.265459, size = 100, normalized size = 2.17

$$-\frac{2\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{ex + d}}{c^2d^2ex^2 + acd^2e + (c^2d^3 + acde^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(3/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2),x, algorithm="fricas")`

[Out] `-2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)/(c^2*d^2*e*x^2 + a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x)`

[Out] Timed out

**GIAC/XCAS [A]** time = 0.681508, size = 4, normalized size = 0.09

`sage0*x`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(3/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2),x, algorithm="giac")`

[Out] `sage0*x`

$$3.669 \quad \int \frac{(d+ex)^{3/2}}{(f+gx)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx$$

**Optimal.** Leaf size=133

$$-\frac{2\sqrt{d+ex}}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)} - \frac{2\sqrt{g} \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{3/2}}$$

[Out]  $(-2*\text{Sqrt}[d + e*x])/((c*d*f - a*e*g)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (2*\text{Sqrt}[g]*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x])])/(c*d*f - a*e*g)^{(3/2)}$

**Rubi [A]** time = 0.60469, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$-\frac{2\sqrt{d+ex}}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)} - \frac{2\sqrt{g} \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^{(3/2)} / ((f + g*x) * (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}), x]$

[Out]  $(-2*\text{Sqrt}[d + e*x])/((c*d*f - a*e*g)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (2*\text{Sqrt}[g]*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x])])/(c*d*f - a*e*g)^{(3/2)}$

**Rubi in Sympy [A]** time = 57.6079, size = 126, normalized size = 0.95

$$-\frac{2\sqrt{g} \operatorname{atanh}\left(\frac{\sqrt{g}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}\sqrt{aeg-cdf}}\right)}{(aeg-cdf)^{3/2}} + \frac{2\sqrt{d+ex}}{(aeg-cdf)\sqrt{ade+cdex^2+x(ae^2+cd^2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)^{(3/2)} / (g*x+f) / (a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(3/2)})$

[Out]  $-2*\text{sqrt}(g)*\text{atanh}(\text{sqrt}(g)*\text{sqrt}(a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2)) / (\text{sqrt}(d + e*x)*\text{sqrt}(a*e*g - c*d*f))) / (a*e*g - c*d*f)^{(3/2)} + 2*\text{sqrt}(d + e*x) / ((a*e*g - c*d*f)*\text{sqrt}(a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2)))$

**Mathematica [A]** time = 0.171734, size = 110, normalized size = 0.83

$$\frac{2\sqrt{d+ex}\left(\sqrt{aeg-cdf} - \sqrt{g}\sqrt{ae+cdx} \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{aeg-cdf}}\right)\right)}{\sqrt{(d+ex)(ae+cdx)}(aeg-cdf)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(d + e*x)^{(3/2)} / ((f + g*x) * (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})]$

[Out]  $(2\sqrt{d+ex}(\sqrt{-(cdf)+aeg}-\sqrt{g}\sqrt{ae+cdx})\operatorname{ArcTanh}(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{-(cdf)+aeg}})) / ((-(cdf)+aeg)^{3/2}\sqrt{(ae+cdx)(d+ex)})$

**Maple [A]** time = 0.03, size = 128, normalized size = 1.

$$-2 \frac{\sqrt{cdex^2 + ae^2x + cd^2x + ade}}{\sqrt{ex+d}(cdx+ae)(aeg-cdf)\sqrt{(aeg-cdf)g}} \left( g \operatorname{Arctanh} \left( \frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}} \right) \sqrt{cdx+ae} - \sqrt{(aeg-cdf)g} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{int}((ex+d)^{3/2}/(gx+f)/(a^2d^2e+(a^2e^2+cd^2)x+cd^2e^2x^2)^{3/2}, x)$

[Out]  $-2(c^2d^2e^2x^2+a^2e^2x+cd^2x+a^2d^2e)^{1/2}(g\operatorname{arctanh}(g(c^2d^2x+a^2e)^{1/2}/((a^2e^2g-c^2d^2f)g)^{1/2}))(c^2d^2x+a^2e)^{1/2}-((a^2e^2g-c^2d^2f)g)^{1/2})/(ex+d)^{1/2}/(c^2d^2x+a^2e)/(a^2e^2g-c^2d^2f)/((a^2e^2g-c^2d^2f)g)^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}((ex+d)^{3/2}/((c^2d^2e^2x^2+a^2d^2e+(c^2d^2+a^2e^2)x)^{3/2})(gx+f)$

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.284208, size = 1, normalized size = 0.01

$$\left[ \frac{(cdex^2 + ade + (cd^2 + ae^2)x) \sqrt{-\frac{g}{cdf-aeg}} \log \left( -\frac{cdex^2 - cd^2f + 2adeg + 2\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(cdf - aeg)\sqrt{ex+d}\sqrt{-\frac{g}{cdf-aeg}} - (cdf - aeg)}{egx^2 + df + (ef + dg)x} \right)}{acd^2ef - a^2de^2g + (c^2d^2ef - acde^2g)x^2 + ((c^2d^3 + acde^2)f - (acd^2e + a^2de^2g))x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}((ex+d)^{3/2}/((c^2d^2e^2x^2+a^2d^2e+(c^2d^2+a^2e^2)x)^{3/2})(gx+f)$

[Out]  $[-((c^2d^2e^2x^2+a^2d^2e+(c^2d^2+a^2e^2)x)\sqrt{-g/(c^2d^2f-a^2e^2g)})\log(-(c^2d^2e^2g^2x^2-c^2d^2f+2a^2d^2e^2g+2\sqrt{c^2d^2e^2x^2+a^2d^2e+(c^2d^2+a^2e^2)x}(c^2d^2f-a^2e^2g)\sqrt{ex+d})\sqrt{-g/(c^2d^2f-a^2e^2g)})-(c^2d^2ef-(c^2d^2+2a^2e^2)g)x)/(e^2g^2x^2+df+(ef+dg)x)+2\sqrt{c^2d^2e^2x^2+a^2d^2e+(c^2d^2+a^2e^2)x}\sqrt{ex+d})/(a^2c^2d^2e^2f-a^2d^2e^2g+(c^2d^2+a^2e^2)ef-a^2c^2d^2e^2g)x^2+((c^2d^3+a^2c^2d^2e^2)f-(a^2c^2d^2e+a^2e^2g)x)g)x, 2((c^2d^2e^2x^2+a^2d^2e+(c^2d^2+a^2e^2)x)\sqrt{g/(c^2d^2f-a^2e^2g)})\operatorname{arctan}(\sqrt{ex+d}/(\sqrt{c^2d^2e^2x^2+a^2d^2e+(c^2d^2+a^2e^2)x}\sqrt{g/(c^2d^2f-a^2e^2g)})))-\sqrt{c^2d^2e^2x^2+a^2d^2e+(c^2d^2+a^2e^2)x}\sqrt{ex+d})/(a^2c^2d^2e^2f-a^2d^2e^2g+(c^2d^2+a^2e^2)ef-a^2c^2d^2e^2g)x^2+((c^2d^3+a^2c^2d^2e^2)f-(a^2c^2d^2e+a^2e^2g)x)g)x]$



**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**(3/2)/(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2),x`

[Out] Timed out

**GIAC/XCAS [A]** time = 0.781274, size = 4, normalized size = 0.03

$sage_0x$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f`

[Out] `sage0*x`

$$3.670 \quad \int \frac{(d+ex)^{3/2}}{(f+gx)^2(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx$$

**Optimal.** Leaf size=202

$$\frac{3g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)^2} - \frac{2\sqrt{d+ex}}{(f+gx)\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)} - \frac{3cd\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{5/2}}$$

[Out]  $(-2*\text{Sqrt}[d + e*x])/((c*d*f - a*e*g)*(f + g*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (3*g*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d*f - a*e*g)^2*\text{Sqrt}[d + e*x]*(f + g*x)) - (3*c*d*\text{Sqrt}[g]*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x])])/(c*d*f - a*e*g)^{(5/2)}$

**Rubi [A]** time = 0.861758, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{3g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-aeg)^2} - \frac{2\sqrt{d+ex}}{(f+gx)\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)} - \frac{3cd\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^{(3/2)}/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}), x]$

[Out]  $(-2*\text{Sqrt}[d + e*x])/((c*d*f - a*e*g)*(f + g*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (3*g*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d*f - a*e*g)^2*\text{Sqrt}[d + e*x]*(f + g*x)) - (3*c*d*\text{Sqrt}[g]*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x])])/(c*d*f - a*e*g)^{(5/2)}$

**Rubi in Sympy [A]** time = 83.4448, size = 192, normalized size = 0.95

$$\frac{3cd\sqrt{g}\operatorname{atanh}\left(\frac{\sqrt{g}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}\sqrt{aeg-cdf}}\right)}{(aeg-cdf)^{\frac{5}{2}}} - \frac{3g\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}(f+gx)(aeg-cdf)^2} + \frac{2\sqrt{d+ex}}{(f+gx)(aeg-cdf)\sqrt{ade+cdex^2+x(ae^2+cd^2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)^{(3/2)}/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(3/2)}$

[Out]  $3*c*d*\text{sqrt}(g)*\operatorname{atanh}(\text{sqrt}(g)*\text{sqrt}(a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2)))/(\text{sqrt}(d + e*x)*\text{sqrt}(a*e*g - c*d*f))/((a*e*g - c*d*f)^{(5/2)} - 3*g*\text{sqrt}(a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2))/(\text{sqrt}(d + e*x)*(f + g*x)*(a*e*g - c*d*f)^2) + 2*\text{sqrt}(d + e*x)/((f + g*x)*(a*e*g - c*d*f)*\text{sqrt}(a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2))))$

**Mathematica [A]** time = 0.43877, size = 141, normalized size = 0.7

$$\frac{\sqrt{d+ex} \left( 3cd\sqrt{g}(f+gx)\sqrt{ae+cdx} \tanh^{-1} \left( \frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{aeg-cdf}} \right) - \sqrt{aeg-cdf}(aeg+cd(2f+3gx)) \right)}{(f+gx)\sqrt{(d+ex)(ae+cdx)(aeg-cdf)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^(3/2)/((f + g\*x)^2\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2), x)

[Out] (Sqrt[d + e\*x]\*(-(Sqrt[-(c\*d\*f) + a\*e\*g]\*(a\*e\*g + c\*d\*(2\*f + 3\*g\*x))) + 3\*c\*d\*Sqrt[g]\*Sqrt[a\*e + c\*d\*x]\*(f + g\*x)\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/Sqrt[-(c\*d\*f) + a\*e\*g]]))/((- (c\*d\*f) + a\*e\*g)^(5/2)\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)]\*(f + g\*x))

**Maple [A]** time = 0.042, size = 225, normalized size = 1.1

$$\frac{1}{(cdx + ae)(aeg - cdf)^2(gx + f)} \sqrt{cdex^2 + ae^2x + cd^2x + ade} \left( 3 \operatorname{Artanh} \left( \frac{g\sqrt{cdx + ae}}{\sqrt{(aeg - cdf)g}} \right) \sqrt{cdx + ae} xcdg^2 + 3 \operatorname{Artanh} \left( \frac{g}{\sqrt{(aeg - cdf)g}} \right) \sqrt{cdx + ae} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(3/2)/(g\*x+f)^2/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2), x)

[Out] (c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)\*(3\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*(c\*d\*x+a\*e)^(1/2)\*x\*c\*d\*g^2+3\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*(c\*d\*x+a\*e)^(1/2)\*c\*d\*f\*g-3\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*x\*c\*d\*g-((a\*e\*g-c\*d\*f)\*g)^(1/2)\*a\*e\*g-2\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*c\*d\*f)/(e\*x+d)^(1/2)/(c\*d\*x+a\*e)/(a\*e\*g-c\*d\*f)^2/(g\*x+f)/((a\*e\*g-c\*d\*f)\*g)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f)^2), x)

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.29038, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f)^2), x)

[Out] [1/2\*(3\*(c^2\*d^2\*e\*g\*x^3 + a\*c\*d^2\*e\*f + (c^2\*d^2\*e\*f + (c^2\*d^3 + a\*c\*d\*e^2)\*g)\*x^2 + (a\*c\*d^2\*e\*g + (c^2\*d^3 + a\*c\*d\*e^2)\*f)\*x)\*sqrt(-g/(c\*d\*f - a\*e\*g))\*log(-(c\*d\*e\*g\*x^2 - c\*d^2\*f + 2\*a\*d\*e\*g - 2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(c\*d\*f - a\*e\*g))\*sqrt(e\*x + d)\*sqrt(-g/(c\*d\*f - a\*e\*g)) - (c\*d\*e\*f - (c\*d^2 + 2\*a\*e

$$\begin{aligned} &^2) * g) * x) / (e * g * x^2 + d * f + (e * f + d * g) * x) - 2 * \sqrt{c * d * e * x^2 + a} \\ &* d * e + (c * d^2 + a * e^2) * x) * (3 * c * d * g * x + 2 * c * d * f + a * e * g) * \sqrt{e * x} \\ &+ d) / (a * c^2 * d^3 * e * f^3 - 2 * a^2 * c * d^2 * e^2 * f^2 * g + a^3 * d * e^3 * f * g^2 \\ &+ (c^3 * d^3 * e * f^2 * g - 2 * a * c^2 * d^2 * e^2 * f * g^2 + a^2 * c * d * e^3 * g^3) * x^3 \\ &+ (c^3 * d^3 * e * f^3 + (c^3 * d^4 - a * c^2 * d^2 * e^2) * f^2 * g - (2 * a * c^2 * d^4 \\ &3 * e + a^2 * c * d * e^3) * f * g^2 + (a^2 * c * d^2 * e^2 + a^3 * e^4) * g^3) * x^2 + ( \\ &a^3 * d * e^3 * g^3 + (c^3 * d^4 + a * c^2 * d^2 * e^2) * f^3 - (a * c^2 * d^3 * e + 2 * \\ &a^2 * c * d * e^3) * f^2 * g - (a^2 * c * d^2 * e^2 - a^3 * e^4) * f * g^2) * x), (3 * (c^2 \\ &* d^2 * e * g * x^3 + a * c * d^2 * e * f + (c^2 * d^2 * e * f + (c^2 * d^3 + a * c * d * e^2) \\ &* g) * x^2 + (a * c * d^2 * e * g + (c^2 * d^3 + a * c * d * e^2) * f) * x) * \sqrt{g / (c * d * \\ &f - a * e * g)}) * \arctan(\sqrt{e * x + d} / (\sqrt{c * d * e * x^2 + a * d * e + (c * d^2 \\ &+ a * e^2) * x) * \sqrt{g / (c * d * f - a * e * g)})) - \sqrt{c * d * e * x^2 + a * d * e + \\ &(c * d^2 + a * e^2) * x) * (3 * c * d * g * x + 2 * c * d * f + a * e * g) * \sqrt{e * x + d} / \\ &(a * c^2 * d^3 * e * f^3 - 2 * a^2 * c * d^2 * e^2 * f^2 * g + a^3 * d * e^3 * f * g^2 + (c^3 \\ &* d^3 * e * f^2 * g - 2 * a * c^2 * d^2 * e^2 * f * g^2 + a^2 * c * d * e^3 * g^3) * x^3 + (c^3 \\ &* d^3 * e * f^3 + (c^3 * d^4 - a * c^2 * d^2 * e^2) * f^2 * g - (2 * a * c^2 * d^3 * e + \\ &a^2 * c * d * e^3) * f * g^2 + (a^2 * c * d^2 * e^2 + a^3 * e^4) * g^3) * x^2 + (a^3 * d * \\ &e^3 * g^3 + (c^3 * d^4 + a * c^2 * d^2 * e^2) * f^3 - (a * c^2 * d^3 * e + 2 * a^2 * c * \\ &d * e^3) * f^2 * g - (a^2 * c * d^2 * e^2 - a^3 * e^4) * f * g^2) * x] \end{aligned}$$


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**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(3/2)/(g\*x+f)\*\*2/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)

[Out] Timed out

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**GIAC/XCAS [A]** time = 0.768357, size = 4, normalized size = 0.02

$sage_0 x$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f

[Out] sage0\*x

$$3.671 \quad \int \frac{(d+ex)^{3/2}}{(f+gx)^3(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx$$

**Optimal.** Leaf size=274

$$\frac{15c^2d^2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4(cdf-aeg)^{7/2}} - \frac{15cdg\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4\sqrt{d+ex}(f+gx)(cdf-aeg)^3} - \frac{5g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)^2} - \frac{2\sqrt{d+ex}}{(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}$$

[Out]  $(-2*\text{Sqrt}[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (5*g*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*(c*d*f - a*e*g)^2*\text{Sqrt}[d + e*x]*(f + g*x)^2) - (15*c*d*g*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*(c*d*f - a*e*g)^3*\text{Sqrt}[d + e*x]*(f + g*x)) - (15*c^2*d^2*\text{Sqrt}[g]*\text{ArcTan}[\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*(c*d*f - a*e*g)^{7/2})$

**Rubi [A]** time = 1.25028, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{15c^2d^2\sqrt{g}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4(cdf-aeg)^{7/2}} - \frac{15cdg\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4\sqrt{d+ex}(f+gx)(cdf-aeg)^3} - \frac{5g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2\sqrt{d+ex}(f+gx)^2(cdf-aeg)^2} - \frac{2\sqrt{d+ex}}{(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^{3/2}/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{3/2}), x]$

[Out]  $(-2*\text{Sqrt}[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (5*g*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*(c*d*f - a*e*g)^2*\text{Sqrt}[d + e*x]*(f + g*x)^2) - (15*c*d*g*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*(c*d*f - a*e*g)^3*\text{Sqrt}[d + e*x]*(f + g*x)) - (15*c^2*d^2*\text{Sqrt}[g]*\text{ArcTan}[\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*(c*d*f - a*e*g)^{7/2})$

**Rubi in Sympy [A]** time = 116.395, size = 265, normalized size = 0.97

$$\frac{15c^2d^2\sqrt{g}\operatorname{atanh}\left(\frac{\sqrt{g}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}\sqrt{aeg-cdf}}\right)}{4(aeg-cdf)^{7/2}} + \frac{15cdg\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{4\sqrt{d+ex}(f+gx)(aeg-cdf)^3} - \frac{5g\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{2\sqrt{d+ex}(f+gx)^2(aeg-cdf)^2} + \frac{2\sqrt{d+ex}}{(f+gx)^2(aeg-cdf)\sqrt{ade+cdex^2+x(ae^2+cd^2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)^{(3/2)}/(g*x+f)^3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2), x)$

[Out]  $-15*c**2*d**2*\text{sqrt}(g)*\operatorname{atanh}(\text{sqrt}(g)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/(\text{sqrt}(d + e*x)*\text{sqrt}(a*e*g - c*d*f)))/(4*(a*e*g - c*d*f)**(7/2)) + 15*c*d*g*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(4*\text{sqrt}(d + e*x)*(f + g*x)*(a*e*g - c*d*f)**3) - 5*g*\text{sq}$

$$\frac{\sqrt{d+ex} \left( \sqrt{aeg-cdf} (-2a^2e^2g^2 + acdeg(9f+5gx) + c^2d^2(8f^2+25fgx+15g^2x^2)) - 15c^2d^2\sqrt{g}(f+gx)^2\sqrt{ae+cdx} \tanh\left(\frac{\sqrt{d+ex}}{\sqrt{ae+cdx}}\right) \right)}{4(f+gx)^2\sqrt{(d+ex)(ae+cdx)}(aeg-cdf)^{7/2}}$$

**Mathematica [A]** time = 0.491026, size = 185, normalized size = 0.68

$$\frac{\sqrt{d+ex} \left( \sqrt{aeg-cdf} (-2a^2e^2g^2 + acdeg(9f+5gx) + c^2d^2(8f^2+25fgx+15g^2x^2)) - 15c^2d^2\sqrt{g}(f+gx)^2\sqrt{ae+cdx} \tanh\left(\frac{\sqrt{d+ex}}{\sqrt{ae+cdx}}\right) \right)}{4(f+gx)^2\sqrt{(d+ex)(ae+cdx)}(aeg-cdf)^{7/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^(3/2)/((f + g\*x)^3\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2), x)

[Out] (Sqrt[d + e\*x]\*(Sqrt[-(c\*d\*f) + a\*e\*g]\*(-2\*a^2\*e^2\*g^2 + a\*c\*d\*e\*g\*(9\*f + 5\*g\*x) + c^2\*d^2\*(8\*f^2 + 25\*f\*g\*x + 15\*g^2\*x^2)) - 15\*c^2\*d^2\*Sqrt[g]\*Sqrt[a\*e + c\*d\*x]\*(f + g\*x)^2\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/Sqrt[-(c\*d\*f) + a\*e\*g]]))/(4\*(-(c\*d\*f) + a\*e\*g)^(7/2)\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)]\*(f + g\*x)^2)

**Maple [A]** time = 0.043, size = 379, normalized size = 1.4

$$-\frac{1}{(4cdx+4ae)(aeg-cdf)^3(gx+f)^2}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(15\operatorname{Artanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)\sqrt{cdx+ae}x^2c^2d^2g^3+30\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(3/2)/(g\*x+f)^3/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2), x)

[Out] -1/4\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)\*(15\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*(c\*d\*x+a\*e)^(1/2)\*x^2\*c^2\*d^2\*g^3+30\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*(c\*d\*x+a\*e)^(1/2)\*x\*c^2\*d^2\*f\*g^2+15\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*(c\*d\*x+a\*e)^(1/2)\*c^2\*d^2\*f^2\*g-15\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*x^2\*c^2\*d^2\*g^2-5\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*x\*a\*c\*d\*e\*g^2-25\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*x\*c^2\*d^2\*f\*g+2\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*a^2\*e^2\*g^2-9\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*a\*c\*d\*e\*f\*g-8\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*c^2\*d^2\*f^2)/(e\*x+d)^(1/2)/(c\*d\*x+a\*e)/(a\*e\*g-c\*d\*f)^3/(g\*x+f)^2/((a\*e\*g-c\*d\*f)\*g)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f)^3), x)

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.302045, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^x + d)^{3/2}/((c^2 d e^x + a d e + (c^2 d^2 + a^2 e^2) x)^{3/2}) * (g^x + f$

[Out]  $[-1/8 * (15 * (c^3 d^3 e^g x^4 + a^2 c^2 d^3 e^f x^4 + (2 * c^3 d^3 e^f g + (c^3 d^4 + a^2 c^2 d^2 e^2) g^2) x^3 + (c^3 d^3 e^f x^2 + a^2 c^2 d^2 e^2) f g) x^2 + (2 * a^2 c^2 d^3 e^f g + (c^3 d^4 + a^2 c^2 d^2 e^2) f^2) x) * \sqrt{-g/(c^2 d f - a^2 e^g)} * \log(-(c^2 d e^g x^2 - c^2 d^2 f + 2 * a^2 d e^g + 2 * \sqrt{c^2 d e^x + a^2 d e + (c^2 d^2 + a^2 e^2) x}) * (c^2 d f - a^2 e^g) * \sqrt{e^x + d} * \sqrt{-g/(c^2 d f - a^2 e^g)}) - (c^2 d e^f - (c^2 d^2 + 2 * a^2 e^2) g) x) / (e^g x^2 + d^2 f + (e^f + d^2 g) x) + 2 * (15 * c^2 d^2 g^2 x^2 + 8 * c^2 d^2 f^2 + 9 * a^2 c^2 d e^f g - 2 * a^2 e^2 g^2 + 5 * (5 * c^2 d^2 f g + a^2 c^2 d e^g x) * \sqrt{c^2 d e^x + a^2 d e + (c^2 d^2 + a^2 e^2) x} * \sqrt{e^x + d}) / (a^2 c^3 d^4 e^f x^5 - 3 * a^2 c^2 d^3 e^2 f^4 g + 3 * a^3 c^2 d^2 e^3 f^3 g^2 - a^4 d^2 e^4 f^2 g^3 + (c^4 d^4 e^f x^3 g^2 - 3 * a^2 c^3 d^3 e^2 f^2 g^3 + 3 * a^2 c^2 d^2 e^3 f^2 g^4 - a^3 c^2 d e^4 g^5) x^4 + (2 * c^4 d^4 e^f x^4 g + (c^4 d^5 - 5 * a^2 c^3 d^3 e^2) f^3 g^2 - 3 * (a^2 c^3 d^4 e - a^2 c^2 d^2 e^3) f^2 g^3 + (3 * a^2 c^2 d^3 e^2 + a^3 c^2 d e^4) f^2 g^4 - (a^3 c^2 d^2 e^3 + a^4 e^5) g^5) x^3 + (c^4 d^4 e^f x^5 - a^4 d^2 e^4 g^5 + (2 * c^4 d^5 - a^2 c^3 d^3 e^2) f^4 g - (5 * a^2 c^3 d^4 e + 3 * a^2 c^2 d^2 e^3) f^3 g^2 + (3 * a^2 c^2 d^3 e^2 + 5 * a^3 c^2 d e^4) f^2 g^3 + (a^3 c^2 d^2 e^3 - 2 * a^4 e^5) f^2 g^4) x^2 - (2 * a^4 d^2 e^4 f^2 g^4 - (c^4 d^5 + a^2 c^3 d^3 e^2) f^4 g + (a^2 c^3 d^4 e + 3 * a^2 c^2 d^2 e^3) f^4 g + 3 * (a^2 c^2 d^3 e^2 - a^3 c^2 d e^4) f^3 g^2 - (5 * a^3 c^2 d^2 e^3 - a^4 e^5) f^2 g^3) x), 1/4 * (15 * (c^3 d^3 e^g x^4 + a^2 c^2 d^3 e^f x^4 + (2 * c^3 d^3 e^f g + (c^3 d^4 + a^2 c^2 d^2 e^2) g^2) x^3 + (c^3 d^3 e^f x^2 + a^2 c^2 d^2 e^2) f g) x^2 + (2 * a^2 c^2 d^3 e^f g + (c^3 d^4 + a^2 c^2 d^2 e^2) f^2) x) * \sqrt{g/(c^2 d f - a^2 e^g)} * \arctan(\sqrt{e^x + d} / (\sqrt{c^2 d e^x + a^2 d e + (c^2 d^2 + a^2 e^2) x} * \sqrt{g/(c^2 d f - a^2 e^g)})) - (15 * c^2 d^2 g^2 x^2 + 8 * c^2 d^2 f^2 + 9 * a^2 c^2 d e^f g - 2 * a^2 e^2 g^2 + 5 * (5 * c^2 d^2 f g + a^2 c^2 d e^g x) * \sqrt{c^2 d e^x + a^2 d e + (c^2 d^2 + a^2 e^2) x} * \sqrt{e^x + d}) / (a^2 c^3 d^4 e^f x^5 - 3 * a^2 c^2 d^3 e^2 f^4 g + 3 * a^3 c^2 d^2 e^3 f^3 g^2 - a^4 d^2 e^4 f^2 g^3 + (c^4 d^4 e^f x^3 g^2 - 3 * a^2 c^3 d^3 e^2 f^2 g^3 + 3 * a^2 c^2 d^2 e^3 f^2 g^4 - a^3 c^2 d e^4 g^5) x^4 + (2 * c^4 d^4 e^f x^4 g + (c^4 d^5 - 5 * a^2 c^3 d^3 e^2) f^3 g^2 - 3 * (a^2 c^3 d^4 e - a^2 c^2 d^2 e^3) f^2 g^3 + (3 * a^2 c^2 d^3 e^2 + a^3 c^2 d e^4) f^2 g^4 - (a^3 c^2 d^2 e^3 + a^4 e^5) g^5) x^3 + (c^4 d^4 e^f x^5 - a^4 d^2 e^4 g^5 + (2 * c^4 d^5 - a^2 c^3 d^3 e^2) f^4 g - (5 * a^2 c^3 d^4 e + 3 * a^2 c^2 d^2 e^3) f^3 g^2 + (3 * a^2 c^2 d^3 e^2 + 5 * a^3 c^2 d e^4) f^2 g^3 + (a^3 c^2 d^2 e^3 - 2 * a^4 e^5) f^2 g^4) x^2 - (2 * a^4 d^2 e^4 f^2 g^4 - (c^4 d^5 + a^2 c^3 d^3 e^2) f^4 g + (a^2 c^3 d^4 e + 3 * a^2 c^2 d^2 e^3) f^4 g + 3 * (a^2 c^2 d^3 e^2 - a^3 c^2 d e^4) f^3 g^2 - (5 * a^3 c^2 d^2 e^3 - a^4 e^5) f^2 g^3) x)]$

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**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^x + d)^{3/2} / (g^x + f)^3 / (a^2 d e + (a^2 e^2 + c^2 d^2) x + c^2 d e^x)^{3/2})$

[Out] Timed out

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**GIAC/XCAS [A]** time = 0.90778, size = 4, normalized size = 0.01

$\text{sage}_0 x$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^x + d)^{3/2} / ((c^2 d e^x + a d e + (c^2 d^2 + a^2 e^2) x)^{3/2}) * (g^x + f$

[Out] sage0\*x



$$3.672 \quad \int \frac{(d+ex)^{5/2}(f+gx)^3}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}} dx$$

**Optimal.** Leaf size=239

$$\begin{aligned} & - \frac{16g^2 \sqrt{x(ae^2 + cd^2) + ade + cdex^2} (2ae^2g - cd(3ef - dg))}{3c^4d^4e\sqrt{d+ex}} \\ & + \frac{16g^3 \sqrt{d+ex} \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{3c^3d^3e} \\ & - \frac{4g\sqrt{d+ex}(f+gx)^2}{c^2d^2\sqrt{x(ae^2 + cd^2) + ade + cdex^2}} - \frac{2(d+ex)^{3/2}(f+gx)^3}{3cd(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}} \end{aligned}$$

[Out]  $(-2*(d + e*x)^{(3/2)}*(f + g*x)^3)/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) - (4*g*\text{Sqrt}[d + e*x]*(f + g*x)^2)/(c^2*d^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (16*g^2*(2*a*e^2*g - c*d*(3*e*f - d*g))*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^4*d^4*e*\text{Sqrt}[d + e*x]) + (16*g^3*\text{Sqrt}[d + e*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^3*d^3*e)$

**Rubi [A]** time = 0.900248, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\begin{aligned} & - \frac{16g^2 \sqrt{x(ae^2 + cd^2) + ade + cdex^2} (2ae^2g - cd(3ef - dg))}{3c^4d^4e\sqrt{d+ex}} \\ & + \frac{16g^3 \sqrt{d+ex} \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{3c^3d^3e} \\ & - \frac{4g\sqrt{d+ex}(f+gx)^2}{c^2d^2\sqrt{x(ae^2 + cd^2) + ade + cdex^2}} - \frac{2(d+ex)^{3/2}(f+gx)^3}{3cd(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(d + e*x)^{(5/2)}*(f + g*x)^3}{(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}, x]$

[Out]  $(-2*(d + e*x)^{(3/2)}*(f + g*x)^3)/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) - (4*g*\text{Sqrt}[d + e*x]*(f + g*x)^2)/(c^2*d^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (16*g^2*(2*a*e^2*g - c*d*(3*e*f - d*g))*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^4*d^4*e*\text{Sqrt}[d + e*x]) + (16*g^3*\text{Sqrt}[d + e*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^3*d^3*e)$

**Rubi in Sympy [A]** time = 91.85, size = 233, normalized size = 0.97

$$\begin{aligned} & - \frac{2(d+ex)^{\frac{3}{2}}(f+gx)^3}{3cd(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}} - \frac{4g\sqrt{d+ex}(f+gx)^2}{c^2d^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}} \\ & + \frac{16g^3\sqrt{d+ex}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{3c^3d^3e} \\ & - \frac{16g^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}(2ae^2g+cd^2g-3cdef)}{3c^4d^4e\sqrt{d+ex}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)^{(5/2)}*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)$

[Out]  $-2*(d + e*x)^{(3/2)}*(f + g*x)^3/(3*c*d*(a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2)))^{(3/2)} - 4*g*\text{sqrt}(d + e*x)*(f + g*x)^2/(c^2*d^2*$

$$d^{*2} \sqrt{a*d*e + c*d*e*x^{*2} + x*(a*e^{*2} + c*d^{*2})} + 16*g^{*3} \sqrt{d + e*x} \sqrt{a*d*e + c*d*e*x^{*2} + x*(a*e^{*2} + c*d^{*2})} / (3*c^{*3} d^{*3} e - 16*g^{*2} \sqrt{a*d*e + c*d*e*x^{*2} + x*(a*e^{*2} + c*d^{*2})} * (2*a*e^{*2} * g + c*d^{*2} * g - 3*c*d*e*f) / (3*c^{*4} d^{*4} e \sqrt{d + e*x}))$$

**Mathematica [A]** time = 0.210321, size = 131, normalized size = 0.55

$$\frac{2(d + ex)^{3/2} (-16a^3 e^3 g^3 + 24a^2 c d e^2 g^2 (f - gx) - 6ac^2 d^2 e g (f^2 - 6fgx + g^2 x^2) + c^3 d^3 (-f^3 - 9f^2 gx + 9fg^2 x^2 + g^3 x^3))}{3c^4 d^4 ((d + ex)(ae + cdx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^(5/2) \* (f + g\*x)^3) / (a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2), x]

[Out] (2\*(d + e\*x)^(3/2) \* (-16\*a^3\*e^3\*g^3 + 24\*a^2\*c\*d\*e^2\*g^2\*(f - g\*x) - 6\*a\*c^2\*d^2\*e\*g\*(f^2 - 6\*f\*g\*x + g^2\*x^2) + c^3\*d^3\*(-f^3 - 9\*f^2\*g\*x + 9\*f\*g^2\*x^2 + g^3\*x^3))) / (3\*c^4\*d^4\*((a\*e + c\*d\*x)\*(d + e\*x))^(3/2))

**Maple [A]** time = 0.012, size = 187, normalized size = 0.8

$$\frac{(2cdx + 2ae)(-g^3x^3c^3d^3 + 6ac^2d^2eg^3x^2 - 9c^3d^3fg^2x^2 + 24a^2cde^2g^3x - 36ac^2d^2efg^2x + 9c^3d^3f^2gx + 16a^3e^3g^3 - 24a^3e^3g^3)}{3c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(5/2)\*(g\*x+f)^3/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2), x)

[Out] -2/3\*(c\*d\*x+a\*e)\*(-c^3\*d^3\*g^3\*x^3+6\*a\*c^2\*d^2\*e\*g^3\*x^2-9\*c^3\*d^3\*f\*g^2\*x^2+24\*a^2\*c\*d\*e^2\*g^3\*x-36\*a\*c^2\*d^2\*e\*f\*g^2\*x+9\*c^3\*d^3\*f^2\*g\*x+16\*a^3\*e^3\*g^3-24\*a^3\*e^3\*g^3)/(c^4\*d^4\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(5/2))

**Maxima [A]** time = 0.764224, size = 296, normalized size = 1.24

$$\frac{2(3cdx + 2ae)f^2g}{(c^3d^3x + ac^2d^2e)\sqrt{cdx + ae}} + \frac{2(3c^2d^2x^2 + 12acdex + 8a^2e^2)fg^2}{(c^4d^4x + ac^3d^3e)\sqrt{cdx + ae}} + \frac{2(c^3d^3x^3 - 6ac^2d^2ex^2 - 24a^2cde^2x - 16a^3e^3)g^3}{3(c^5d^5x + ac^4d^4e)\sqrt{cdx + ae}} - \frac{2f^3}{3(c^2d^2x + acde)\sqrt{cdx + ae}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(5/2) \* (g\*x + f)^3 / (c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2), x)

[Out] -2\*(3\*c\*d\*x + 2\*a\*e)\*f^2\*g/((c^3\*d^3\*x + a\*c^2\*d^2\*e)\*sqrt(c\*d\*x + a\*e)) + 2\*(3\*c^2\*d^2\*x^2 + 12\*a\*c\*d\*e\*x + 8\*a^2\*e^2)\*f\*g^2/((c^4\*d^4\*x + a\*c^3\*d^3\*e)\*sqrt(c\*d\*x + a\*e)) + 2/3\*(c^3\*d^3\*x^3 - 6\*a\*c^2\*d^2\*e\*x^2 - 24\*a^2\*c\*d\*e^2\*x - 16\*a^3\*e^3)\*g^3/((c^5\*d^5\*x + a\*c^4\*d^4\*e)\*sqrt(c\*d\*x + a\*e)) - 2/3\*f^3/((c^2\*d^2\*x + a\*c\*d\*e)\*sqrt(c\*d\*x + a\*e))

**Fricas [A]** time = 0.274493, size = 447, normalized size = 1.87

$$\frac{2(c^3 d^3 e g^3 x^4 - c^3 d^4 f^3 - 6 a c^2 d^3 e f^2 g + 24 a^2 c d^2 e^2 f g^2 - 16 a^3 d e^3 g^3 + (9 c^3 d^3 e f g^2 + (c^3 d^4 - 6 a c^2 d^2 e^2) g^3) x^3 - 3(3 c^3 d^3 e f g^2 + (c^3 d^4 - 6 a c^2 d^2 e^2) g^3) x^2 - 3(3 c^3 d^3 e f g^2 + (c^3 d^4 - 6 a c^2 d^2 e^2) g^3) x - 3(3 c^3 d^3 e f g^2 + (c^3 d^4 - 6 a c^2 d^2 e^2) g^3)}{3(c^5 d^5 x^5 + (c^4 d^4 + a^2 c^2 d^2 e^2) x^4 + (c^3 d^3 + 4 a^2 c d e^2) x^3 + (c^2 d^2 + 4 a^2 c d e^2) x^2 + (c d + 4 a^2 c d e^2) x + 4 a^2 c d e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(5/2) \* (g\*x + f)^3 / (c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2))

[Out] 2/3\*(c^3\*d^3\*e\*g^3\*x^4 - c^3\*d^4\*f^3 - 6\*a\*c^2\*d^3\*e\*f^2\*g + 24\*a^2\*c\*d^2\*e^2\*f\*g^2 - 16\*a^3\*d\*e^3\*g^3 + (9\*c^3\*d^3\*e\*f\*g^2 + (c^3\*d^4 - 6\*a\*c^2\*d^2\*e^2)\*g^3)\*x^3 - 3\*(3\*c^3\*d^3\*e\*f^2\*g - 3\*(c^3\*d^4 + 4\*a\*c^2\*d^2\*e^2)\*f\*g^2 + 2\*(a\*c^2\*d^3\*e + 4\*a^2\*c\*d\*e^3)\*g^3)\*x^2 - (c^3\*d^3\*e\*f^3 + 3\*(3\*c^3\*d^4 + 2\*a\*c^2\*d^2\*e^2)\*f^2\*g - 12\*(3\*a\*c^2\*d^3\*e + 2\*a^2\*c\*d\*e^3)\*f\*g^2 + 8\*(3\*a^2\*c\*d^2\*e^2 + 2\*a^3\*e^4)\*g^3)\*x) / ((c^5\*d^5\*x + a\*c^4\*d^4\*e)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(5/2) \* (g\*x+f)\*\*3 / (a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2))

[Out] Timed out

**GIAC/XCAS [A]** time = 0.908452, size = 4, normalized size = 0.02

sage0\*x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(5/2) \* (g\*x + f)^3 / (c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2))

[Out] sage0\*x

$$3.673 \quad \int \frac{(d+ex)^{5/2}(f+gx)^2}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}} dx$$

**Optimal.** Leaf size=211

$$\frac{8g\sqrt{x(ae^2+cd^2)+ade+cdex^2}(2ae^2g-cd(dg+ef))}{3c^3d^3\sqrt{d+ex}(cd^2-ae^2)} - \frac{8g(d+ex)^{3/2}(cdf-ae^2)}{3c^2d^2(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{2(d+ex)^{3/2}(f+gx)^2}{3cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

[Out]  $(-2*(d+e*x)^{(3/2)}*(f+g*x)^2)/(3*c*d*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(3/2)} - (8*g*(c*d*f-a*e*g)*(d+e*x)^{(3/2)})/(3*c^2*d^2*(c*d^2-a*e^2)*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2]) - (8*g*(2*a*e^2*g-c*d*(e*f+d*g))*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])/(3*c^3*d^3*(c*d^2-a*e^2)*\text{Sqrt}[d+e*x])$

**Rubi [A]** time = 0.68186, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{8g\sqrt{x(ae^2+cd^2)+ade+cdex^2}(2ae^2g-cd(dg+ef))}{3c^3d^3\sqrt{d+ex}(cd^2-ae^2)} - \frac{8g(d+ex)^{3/2}(cdf-ae^2)}{3c^2d^2(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{2(d+ex)^{3/2}(f+gx)^2}{3cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d+e*x)^{(5/2)}*(f+g*x)^2/(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(5/2)}, x]$

[Out]  $(-2*(d+e*x)^{(3/2)}*(f+g*x)^2)/(3*c*d*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(3/2)} - (8*g*(c*d*f-a*e*g)*(d+e*x)^{(3/2)})/(3*c^2*d^2*(c*d^2-a*e^2)*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2]) - (8*g*(2*a*e^2*g-c*d*(e*f+d*g))*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])/(3*c^3*d^3*(c*d^2-a*e^2)*\text{Sqrt}[d+e*x])$

**Rubi in Sympy [A]** time = 76.6149, size = 201, normalized size = 0.95

$$\frac{2(d+ex)^{\frac{3}{2}}(f+gx)^2}{3cd(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}} - \frac{8g(d+ex)^{\frac{3}{2}}(aeg-cdf)}{3c^2d^2(ae^2-cd^2)\sqrt{ade+cdex^2+x(ae^2+cd^2)}} + \frac{8g\sqrt{ade+cdex^2+x(ae^2+cd^2)}(2ae^2g-cd^2g-cdef)}{3c^3d^3\sqrt{d+ex}(ae^2-cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)^{(5/2)}*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(5/2)}$

[Out]  $-2*(d+e*x)^{(3/2)}*(f+g*x)^2/(3*c*d*(a*d*e+c*d*e*x^2+x*(a*e^2+c*d^2))^{(3/2)} - 8*g*(d+e*x)^{(3/2)}*(a*e*g-c*d*f)/(3*c^2*d^2*(a*e^2-c*d^2)*\text{sqrt}(a*d*e+c*d*e*x^2+x*(a*e^2+c*d^2))) + 8*g*\text{sqrt}(a*d*e+c*d*e*x^2+x*(a*e^2+c*d^2))*(2*a*e^2*g-c*d^2*g-c*d*e*f)/(3*c^3*d^3*\text{sqrt}(d+e*x)*(a*e^2-c*d^2))$

**Mathematica [A]** time = 0.142515, size = 87, normalized size = 0.41

$$\frac{2(d+ex)^{3/2}(8a^2e^2g^2-4acdeg(f-3gx)-c^2d^2(f^2+6fgx-3g^2x^2))}{3c^3d^3((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^(5/2)\*(f + g\*x)^2)/(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2)]

[Out] (2\*(d + e\*x)^(3/2)\*(8\*a^2\*e^2\*g^2 - 4\*a\*c\*d\*e\*g\*(f - 3\*g\*x) - c^2\*d^2\*(f^2 + 6\*f\*g\*x - 3\*g^2\*x^2)))/(3\*c^3\*d^3\*((a\*e + c\*d\*x)\*(d + e\*x))^(3/2))

**Maple [A]** time = 0.012, size = 116, normalized size = 0.6

$$\frac{(2cdx + 2ae)(3g^2x^2c^2d^2 + 12acdeg^2x - 6c^2d^2fgx + 8a^2e^2g^2 - 4acdefg - c^2d^2f^2)}{3c^3d^3}(ex + d)^{\frac{5}{2}}(cdex^2 + ae^2x + cd^2x + ade)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(5/2)\*(g\*x+f)^2/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2), x)

[Out] 2/3\*(c\*d\*x+a\*e)\*(3\*c^2\*d^2\*g^2\*x^2+12\*a\*c\*d\*e\*g^2\*x-6\*c^2\*d^2\*f\*g\*x+8\*a^2\*e^2\*g^2-4\*a\*c\*d\*e\*f\*g-c^2\*d^2\*f^2)\*(e\*x+d)^(5/2)/c^3/d^3/(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(5/2)

**Maxima [A]** time = 0.743962, size = 186, normalized size = 0.88

$$-\frac{4(3cdx + 2ae)fg}{3(c^3d^3x + ac^2d^2e)\sqrt{cdx + ae}} + \frac{2(3c^2d^2x^2 + 12acdex + 8a^2e^2)g^2}{3(c^4d^4x + ac^3d^3e)\sqrt{cdx + ae}} - \frac{2f^2}{3(c^2d^2x + acde)\sqrt{cdx + ae}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(5/2)\*(g\*x + f)^2/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2))

[Out] -4/3\*(3\*c\*d\*x + 2\*a\*e)\*f\*g/((c^3\*d^3\*x + a\*c^2\*d^2\*e)\*sqrt(c\*d\*x + a\*e)) + 2/3\*(3\*c^2\*d^2\*x^2 + 12\*a\*c\*d\*e\*x + 8\*a^2\*e^2)\*g^2/((c^4\*d^4\*x + a\*c^3\*d^3\*e)\*sqrt(c\*d\*x + a\*e)) - 2/3\*f^2/((c^2\*d^2\*x + a\*c\*d\*e)\*sqrt(c\*d\*x + a\*e))

**Fricas [A]** time = 0.271277, size = 271, normalized size = 1.28

$$\frac{2(3c^2d^2eg^2x^3 - c^2d^3f^2 - 4acd^2efg + 8a^2de^2g^2 - 3(2c^2d^2efg - (c^2d^3 + 4acde^2)g^2)x^2 - (c^2d^2ef^2 + 2(3c^2d^3 + 2acde^2)g^2)x - c^2d^3f^2)}{3(c^4d^4x + ac^3d^3e)\sqrt{cdex^2 + ade} + (cd^2 + ae^2)x\sqrt{ex + d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(5/2)\*(g\*x + f)^2/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2))

[Out] 2/3\*(3\*c^2\*d^2\*e\*g^2\*x^3 - c^2\*d^3\*f^2 - 4\*a\*c\*d^2\*e\*f\*g + 8\*a^2\*d^2\*e^2\*g^2 - 3\*(2\*c^2\*d^2\*e\*f\*g - (c^2\*d^3 + 4\*a\*c\*d^2\*e^2)\*g^2)\*x^2 - (c^2\*d^2\*e\*f^2 + 2\*(3\*c^2\*d^3 + 2\*a\*c\*d^2\*e^2)\*g^2)\*x)/((c^4\*d^4\*x + a\*c^3\*d^3\*e)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d))

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**(5/2)*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)`

[Out] Timed out

**GIAC/XCAS [A]** time = 0.83176, size = 4, normalized size = 0.02

*sage0\*x*

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(5/2)*(g*x + f)^2/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)`

[Out] `sage0*x`

$$3.674 \quad \int \frac{(d+ex)^{5/2}(f+gx)}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}} dx$$

**Optimal.** Leaf size=154

$$\frac{2\sqrt{d+ex}(2ae^2g+cd(ef-3dg))}{3c^2d^2(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{2(d+ex)^{5/2}(cdf-aeg)}{3cd(cd^2-ae^2)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

[Out]  $(-2*(c*d*f - a*e*g)*(d + e*x)^{(5/2)})/(3*c*d*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) + (2*(2*a*e^2*g + c*d*(e*f - 3*d*g))*\text{Sqrt}[d + e*x])/(3*c^2*d^2*(c*d^2 - a*e^2)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi [A]** time = 0.410604, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 44,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{2\sqrt{d+ex}(2ae^2g+cd(ef-3dg))}{3c^2d^2(cd^2-ae^2)\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{2(d+ex)^{5/2}(cdf-aeg)}{3cd(cd^2-ae^2)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^{(5/2)}*(f + g*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}, x]$

[Out]  $(-2*(c*d*f - a*e*g)*(d + e*x)^{(5/2)})/(3*c*d*(c*d^2 - a*e^2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) + (2*(2*a*e^2*g + c*d*(e*f - 3*d*g))*\text{Sqrt}[d + e*x])/(3*c^2*d^2*(c*d^2 - a*e^2)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi in Sympy [A]** time = 46.1659, size = 146, normalized size = 0.95

$$-\frac{2(d+ex)^{\frac{5}{2}}(aeg-cdf)}{3cd(ae^2-cd^2)(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}} - \frac{2\sqrt{d+ex}(2ae^2g-3cd^2g+cdef)}{3c^2d^2(ae^2-cd^2)\sqrt{ade+cdex^2+x(ae^2+cd^2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)^{(5/2)}*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(5/2)}$

[Out]  $-2*(d + e*x)^{(5/2)}*(a*e*g - c*d*f)/(3*c*d*(a*e^2 - c*d^2)*(a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2))^{(3/2)}) - 2*\text{sqrt}(d + e*x)*(2*a*e^2*g - 3*c*d^2*g + c*d*e*f)/(3*c^2*d^2*(a*e^2 - c*d^2)*\text{sqrt}(a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2)))$

**Mathematica [A]** time = 0.0698872, size = 52, normalized size = 0.34

$$-\frac{2(d+ex)^{3/2}(2aeg+cd(f+3gx))}{3c^2d^2((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(d + e*x)^{(5/2)}*(f + g*x)/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}$

[Out]  $(-2*(d + e*x)^{(3/2)}*(2*a*e*g + c*d*(f + 3*g*x)))/(3*c^2*d^2*((a*e + c*d*x)*(d + e*x))^{(3/2)})$

---

**Maple [A]** time = 0.008, size = 66, normalized size = 0.4

$$-\frac{(2cdx + 2ae)(3xcdg + 2aeg + cdf)}{3c^2d^2}(ex + d)^{\frac{5}{2}}(cdex^2 + ae^2x + cd^2x + ade)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^(5/2)*(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)`

[Out] `-2/3*(c*d*x+a*e)*(3*c*d*g*x+2*a*e*g+c*d*f)*(e*x+d)^(5/2)/c^2/d^2/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)`

---

**Maxima [A]** time = 0.729525, size = 99, normalized size = 0.64

$$-\frac{2(3cdx + 2ae)g}{3(c^3d^3x + ac^2d^2e)\sqrt{cdx + ae}} - \frac{2f}{3(c^2d^2x + acde)\sqrt{cdx + ae}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(5/2)*(g*x + f)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)`

[Out] `-2/3*(3*c*d*x + 2*a*e)*g/((c^3*d^3*x + a*c^2*d^2*e)*sqrt(c*d*x + a*e)) - 2/3*f/((c^2*d^2*x + a*c*d*e)*sqrt(c*d*x + a*e))`

---

**Fricas [A]** time = 0.27033, size = 174, normalized size = 1.13

$$-\frac{2\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(3cdgx + cdf + 2aeg)\sqrt{ex + d}}{3(c^4d^4ex^3 + a^2c^2d^3e^2 + (c^4d^5 + 2ac^3d^3e^2)x^2 + (2ac^3d^4e + a^2c^2d^2e^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(5/2)*(g*x + f)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)`

[Out] `-2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(3*c*d*g*x + c*d*f + 2*a*e*g)*sqrt(e*x + d)/(c^4*d^4*e*x^3 + a^2*c^2*d^3*e^2 + (c^4*d^5 + 2*a*c^3*d^3*e^2)*x^2 + (2*a*c^3*d^4*e + a^2*c^2*d^2*e^3)*x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**(5/2)*(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)`

[Out] Timed out

---

**GIAC/XCAS [A]** time = 0.835185, size = 4, normalized size = 0.03

`sage_0x`



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^(5/2)*(g*x + f)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)
```

```
[Out] sage0*x
```

$$3.675 \quad \int \frac{(d+ex)^{5/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}} dx$$

**Optimal.** Leaf size=48

$$-\frac{2(d+ex)^{3/2}}{3cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

[Out]  $(-2*(d + e*x)^{(3/2)})/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}$

**Rubi [A]** time = 0.0697854, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$

$$-\frac{2(d+ex)^{3/2}}{3cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^(5/2)/(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2), x]

[Out]  $(-2*(d + e*x)^{(3/2)})/(3*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}$

**Rubi in Sympy [A]** time = 17.0581, size = 44, normalized size = 0.92

$$-\frac{2(d+ex)^{\frac{3}{2}}}{3cd(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*(5/2)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2), x)

[Out]  $-2*(d + e*x)**(3/2)/(3*c*d*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2)$

**Mathematica [A]** time = 0.0471428, size = 37, normalized size = 0.77

$$-\frac{2(d+ex)^{3/2}}{3cd((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^(5/2)/(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2), x]

[Out]  $(-2*(d + e*x)^{(3/2)})/(3*c*d*((a*e + c*d*x)*(d + e*x))^{(3/2)})$

**Maple [A]** time = 0.004, size = 50, normalized size = 1.

$$-\frac{2cdx+2ae}{3cd}(ex+d)^{\frac{5}{2}}(cdex^2+ae^2x+cd^2x+ade)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2),x)`

[Out]  $-\frac{2}{3}*(c*d*x+a*e)*(e*x+d)^(5/2)/c/d/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)$

**Maxima [A]** time = 0.716309, size = 38, normalized size = 0.79

$$-\frac{2}{3(c^2d^2x + acde)\sqrt{cdx + ae}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(5/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2),x, algorithm="maxima")`

[Out]  $-\frac{2}{3}/((c^2*d^2*x + a*c*d*e)*\text{sqrt}(c*d*x + a*e))$

**Fricas [A]** time = 0.267834, size = 144, normalized size = 3.

$$-\frac{2\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{ex + d}}{3(c^3d^3ex^3 + a^2cd^2e^2 + (c^3d^4 + 2ac^2d^2e^2)x^2 + (2ac^2d^3e + a^2cde^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(5/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2),x, algorithm="fricas")`

[Out]  $-\frac{2}{3}*\text{sqrt}(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*\text{sqrt}(e*x + d)/(c^3*d^3*e*x^3 + a^2*c*d^2*e^2 + (c^3*d^4 + 2*a*c^2*d^2*e^2)*x^2 + (2*a*c^2*d^3*e + a^2*c*d*e^3)*x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2),x)`

[Out] Timed out

**GIAC/XCAS [A]** time = 0.755499, size = 4, normalized size = 0.08

$sage_0x$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(5/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2),x, algorithm="giac")`

[Out]  $sage_0*x$

$$3.676 \quad \int \frac{(d+ex)^{5/2}}{(f+gx)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}} dx$$

**Optimal.** Leaf size=188

$$\frac{2g^{3/2} \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{5/2}} + \frac{2g\sqrt{d+ex}}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2} - \frac{2(d+ex)^{3/2}}{3(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)}$$

[Out]  $(-2*(d+e*x)^{(3/2)})/(3*(c*d*f-a*e*g)*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(3/2)}) + (2*g*\text{Sqrt}[d+e*x])/((c*d*f-a*e*g)^2*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2]) + (2*g^{(3/2)}*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])/(\text{Sqrt}[c*d*f-a*e*g]*\text{Sqrt}[d+e*x])])/(c*d*f-a*e*g)^{(5/2)}$

**Rubi [A]** time = 0.896746, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{2g^{3/2} \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{(cdf-aeg)^{5/2}} + \frac{2g\sqrt{d+ex}}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2} - \frac{2(d+ex)^{3/2}}{3(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d+e*x)^{(5/2)}/((f+g*x)*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(5/2)}),x]$

[Out]  $(-2*(d+e*x)^{(3/2)})/(3*(c*d*f-a*e*g)*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(3/2)}) + (2*g*\text{Sqrt}[d+e*x])/((c*d*f-a*e*g)^2*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2]) + (2*g^{(3/2)}*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])/(\text{Sqrt}[c*d*f-a*e*g]*\text{Sqrt}[d+e*x])])/(c*d*f-a*e*g)^{(5/2)}$

**Rubi in Sympy [A]** time = 87.1219, size = 180, normalized size = 0.96

$$-\frac{2g^{\frac{3}{2}} \operatorname{atanh}\left(\frac{\sqrt{g}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}\sqrt{aeg-cdf}}\right)}{(aeg-cdf)^{\frac{5}{2}}} + \frac{2g\sqrt{d+ex}}{(aeg-cdf)^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}} + \frac{2(d+ex)^{\frac{3}{2}}}{3(aeg-cdf)(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)^{(5/2)}/(g*x+f)/(a*d*e+(a*e^{**2}+c*d^{**2})*x+c*d*e*x^{**2})^{**}(5)$

[Out]  $-2*g^{(3/2)}*\operatorname{atanh}(\text{sqrt}(g)*\text{sqrt}(a*d*e+c*d*e*x^{**2}+x*(a*e^{**2}+c*d^{**2}))/(\text{sqrt}(d+e*x)*\text{sqrt}(a*e*g-c*d*f)))/(a*e*g-c*d*f)^{(5/2)} + 2*g*\text{sqrt}(d+e*x)/((a*e*g-c*d*f)^{**2}*\text{sqrt}(a*d*e+c*d*e*x^{**2}+x*(a*e^{**2}+c*d^{**2}))) + 2*(d+e*x)^{(3/2)}/(3*(a*e*g-c*d*f)*(a*d*e+c*d*e*x^{**2}+x*(a*e^{**2}+c*d^{**2}))^{(3/2)})$

**Mathematica [A]** time = 0.340817, size = 128, normalized size = 0.68

$$\frac{2(d+ex)^{3/2} \left( 3g^{3/2}(ae+cdx)^{3/2} \tanh^{-1} \left( \frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{aeg-cdf}} \right) + \sqrt{aeg-cdf}(cd(f-3gx)-4aeg) \right)}{3((d+ex)(ae+cdx))^{3/2}(aeg-cdf)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^(5/2)/((f + g\*x)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2)

[Out] (-2\*(d + e\*x)^(3/2)\*(Sqrt[-(c\*d\*f) + a\*e\*g]\*(-4\*a\*e\*g + c\*d\*(f - 3\*g\*x)) + 3\*g^(3/2)\*(a\*e + c\*d\*x)^(3/2)\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/Sqrt[-(c\*d\*f) + a\*e\*g]])/(3\*(-(c\*d\*f) + a\*e\*g)^(5/2))\*((a\*e + c\*d\*x)\*(d + e\*x))^(3/2))

**Maple [A]** time = 0.033, size = 219, normalized size = 1.2

$$-\frac{2}{3(cdx+ae)^2(aeg-cdf)^2} \sqrt{cdex^2+ae^2x+cd^2x+ade} \left( 3 \operatorname{Artanh} \left( \frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}} \right) \sqrt{cdx+ae} xcdg^2 + 3 \operatorname{Artanh} \left( \frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}} \right) \sqrt{cdx+ae} xcdg^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(5/2)/(g\*x+f)/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2),x)

[Out] -2/3\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)\*(3\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*(c\*d\*x+a\*e)^(1/2)\*x\*c\*d\*g^2+3\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*a\*e\*g^2\*(c\*d\*x+a\*e)^(1/2)-3\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*x\*c\*d\*g-4\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*a\*e\*g+((a\*e\*g-c\*d\*f)\*g)^(1/2)\*c\*d\*f)/(e\*x+d)^(1/2)/(c\*d\*x+a\*e)^2/(a\*e\*g-c\*d\*f)^2/((a\*e\*g-c\*d\*f)\*g)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(5/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*(g\*x + f

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.288565, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(5/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*(g\*x + f

[Out] [1/3\*(3\*(c^2\*d^2\*e\*g\*x^3 + a^2\*d\*e^2\*g + (c^2\*d^3 + 2\*a\*c\*d\*e^2)\*g\*x^2 + (2\*a\*c\*d^2\*e + a^2\*e^3)\*g\*x)\*sqrt(-g/(c\*d\*f - a\*e\*g))\*log(-(c\*d\*e\*g\*x^2 - c\*d^2\*f + 2\*a\*d\*e\*g + 2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(c\*d\*f - a\*e\*g)\*sqrt(e\*x + d)\*sqrt(-g/(c\*d\*f - a\*e\*g)) - (c\*d\*e\*f - (c\*d^2 + 2\*a\*e^2)\*g)\*x)/(e\*g\*x^2 + d\*f + (

$$\begin{aligned}
& e^f + d^g)^x)) + 2\sqrt{c^d e^x^2 + a^d e + (c^d^2 + a^e^2)^x} \cdot (3 \\
& \cdot c^d g^x - c^d f + 4^a e^g) \cdot \sqrt{e^x + d}) / (a^2 c^2 d^3 e^2 f^2 - \\
& 2^a^3 c^d^2 e^3 f^g + a^4 d^e^4 g^2 + (c^4 d^4 e^f^2 - 2^a c^3 d^3 e^2 \\
& f^g + a^2 c^2 d^2 e^3 g^2)^x^3 + ((c^4 d^5 + 2^a c^3 d^3 e^2 \\
& f^2 - 2^a c^3 d^4 e + 2^a^2 c^2 d^2 e^3)^f^g + (a^2 c^2 d^3 e^2 \\
& + 2^a^3 c^d e^4)^g^2)^x^2 + ((2^a c^3 d^4 e + a^2 c^2 d^2 e^3 \\
& )^f^2 - 2^a (2^a^2 c^2 d^3 e^2 + a^3 c^d e^4)^f^g + (2^a^3 c^d^2 e^3 \\
& + a^4 e^5)^g^2)^x), -2/3 \cdot (3^c (c^2 d^2 e^g x^3 + a^2 d^e^2 g + (c \\
& ^2 d^3 + 2^a c^d e^2)^g x^2 + (2^a c^d^2 e + a^2 e^3)^g x) \cdot \sqrt{g \\
& / (c^d f - a^e g)} \cdot \arctan(\sqrt{e^x + d}) / (\sqrt{c^d e^x^2 + a^d e + \\
& (c^d^2 + a^e^2)^x} \cdot \sqrt{g / (c^d f - a^e g)})) - \sqrt{c^d e^x^2 + a^d e + \\
& (c^d^2 + a^e^2)^x} \cdot (3^c d^g x - c^d f + 4^a e^g) \cdot \sqrt{e^x \\
& + d}) / (a^2 c^2 d^3 e^2 f^2 - 2^a^3 c^d^2 e^3 f^g + a^4 d^e^4 g^2 \\
& + (c^4 d^4 e^f^2 - 2^a c^3 d^3 e^2 f^g + a^2 c^2 d^2 e^3 g^2)^x^3 \\
& + ((c^4 d^5 + 2^a c^3 d^3 e^2)^f^2 - 2^a (a^3 c^d^4 e + 2^a^2 c^2 d^2 \\
& e^3)^f^g + (a^2 c^2 d^3 e^2 + 2^a^3 c^d e^4)^g^2)^x^2 + ((2^a \\
& c^3 d^4 e + a^2 c^2 d^2 e^3)^f^2 - 2^a (2^a^2 c^2 d^3 e^2 + a^3 c^d \\
& e^4)^f^g + (2^a^3 c^d^2 e^3 + a^4 e^5)^g^2)^x]
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(5/2)/(g\*x+f)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2),x

[Out] Timed out

**GIAC/XCAS [A]** time = 1.00782, size = 4, normalized size = 0.02

*sage<sub>0</sub>x*

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(5/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*(g\*x + f

[Out] sage0\*x

$$3.677 \quad \int \frac{(d+ex)^{5/2}}{(f+gx)^2(ade+(cd^2+ae^2)x+cdex^2)^{5/2}} dx$$

**Optimal.** Leaf size=268

$$\frac{5cdg^{3/2} \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-ae^2g}}\right)}{(cdf-ae^2g)^{7/2}} + \frac{5g^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-ae^2g)^3}$$

$$+ \frac{10g\sqrt{d+ex}}{3(f+gx)\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-ae^2g)^2}$$

$$- \frac{2(d+ex)^{3/2}}{3(f+gx)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-ae^2g)}$$

[Out]  $(-2*(d + e*x)^{(3/2)})/(3*(c*d*f - a*e*g)*(f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) + (10*g*\text{Sqrt}[d + e*x])/(3*(c*d*f - a*e*g)^2*(f + g*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (5*g^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d*f - a*e*g)^3*\text{Sqrt}[d + e*x]*(f + g*x)) + (5*c*d*g^{(3/2)}*\text{ArcTan}[\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x])]/(c*d*f - a*e*g)^{(7/2)}$

**Rubi [A]** time = 1.18414, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{5cdg^{3/2} \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-ae^2g}}\right)}{(cdf-ae^2g)^{7/2}} + \frac{5g^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}(f+gx)(cdf-ae^2g)^3}$$

$$+ \frac{10g\sqrt{d+ex}}{3(f+gx)\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-ae^2g)^2}$$

$$- \frac{2(d+ex)^{3/2}}{3(f+gx)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-ae^2g)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^{(5/2)}/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}), x]$

[Out]  $(-2*(d + e*x)^{(3/2)})/(3*(c*d*f - a*e*g)*(f + g*x)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) + (10*g*\text{Sqrt}[d + e*x])/(3*(c*d*f - a*e*g)^2*(f + g*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (5*g^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d*f - a*e*g)^3*\text{Sqrt}[d + e*x]*(f + g*x)) + (5*c*d*g^{(3/2)}*\text{ArcTan}[\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]])/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x])]/(c*d*f - a*e*g)^{(7/2)}$

**Rubi in Sympy [A]** time = 115.781, size = 255, normalized size = 0.95

$$\frac{5cdg^{\frac{3}{2}} \operatorname{atanh}\left(\frac{\sqrt{g}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}\sqrt{aeg-cdf}}\right)}{(aeg-cdf)^{\frac{7}{2}}} - \frac{5g^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}(f+gx)(aeg-cdf)^3}$$

$$+ \frac{10g\sqrt{d+ex}}{3(f+gx)(aeg-cdf)^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}$$

$$+ \frac{2(d+ex)^{\frac{3}{2}}}{3(f+gx)(aeg-cdf)(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**(5/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2))`

[Out]  $5*c*d*g^{3/2}*atanh(\sqrt{g}*\sqrt{a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2)})/(\sqrt{d + e*x}*\sqrt{a*e*g - c*d*f})/(a*e*g - c*d*f)^{7/2} - 5*g^2*\sqrt{a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2)}/(\sqrt{d + e*x}*(f + g*x)*(a*e*g - c*d*f)^3) + 10*g*\sqrt{d + e*x}/(3*(f + g*x)*(a*e*g - c*d*f)^2*\sqrt{a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2)}) + 2*(d + e*x)^{3/2}/(3*(f + g*x)*(a*e*g - c*d*f)*(a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2))^{3/2})$

**Mathematica [A]** time = 0.533796, size = 180, normalized size = 0.67

$$\frac{(d + ex)^{3/2} \left( \sqrt{aeg - cdf} (-3a^2e^2g^2 - 2acdeg(7f + 10gx) + c^2d^2(2f^2 - 10fgx - 15g^2x^2)) + 15cdg^{3/2}(f + gx)(ae + cdx)^{3/2} \right)}{3(f + gx)((d + ex)(ae + cdx))^{3/2}(aeg - cdf)^{7/2}}$$

Antiderivative was successfully verified.

[In] `Integrate[(d + e*x)^(5/2)/((f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2), x)`

[Out]  $((d + e*x)^{3/2}*(\text{Sqrt}[-(c*d*f) + a*e*g]*(-3*a^2*e^2*g^2 - 2*a*c*d*e*g*(7*f + 10*g*x) + c^2*d^2*(2*f^2 - 10*f*g*x - 15*g^2*x^2)) + 15*c*d*g^{3/2}*(a*e + c*d*x)^{3/2}*(f + g*x)*\text{ArcTanh}[\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x]/\text{Sqrt}[-(c*d*f) + a*e*g]])/(3*(-(c*d*f) + a*e*g)^{7/2}*((a*e + c*d*x)*(d + e*x))^{3/2}*(f + g*x))$

**Maple [A]** time = 0.042, size = 424, normalized size = 1.6

$$\frac{1}{3(cdx + ae)^2(aeg - cdf)^3(gx + f)} \sqrt{cdex^2 + ae^2x + cd^2x + ade} \left( 15 \text{Artanh} \left( \frac{g\sqrt{cdx + ae}}{\sqrt{(aeg - cdf)g}} \right) \sqrt{cdx + aex^2c^2d^2g^3 + 15Artanh} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^(5/2)/(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2), x)`

[Out]  $1/3*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^{1/2}*(15*\text{arctanh}(g*(c*d*x+a*e)^{1/2}/((a*e*g-c*d*f)*g)^{1/2})*(c*d*x+a*e)^{1/2}*x^2*c^2*d^2*g^3+15*\text{arctanh}(g*(c*d*x+a*e)^{1/2}/((a*e*g-c*d*f)*g)^{1/2})*x*a*c*d*e*g^3*(c*d*x+a*e)^{1/2}+15*\text{arctanh}(g*(c*d*x+a*e)^{1/2}/((a*e*g-c*d*f)*g)^{1/2})*(c*d*x+a*e)^{1/2}*x*c^2*d^2*f*g^2+15*\text{arctanh}(g*(c*d*x+a*e)^{1/2}/((a*e*g-c*d*f)*g)^{1/2})*a*c*d*e*f*g^2*(c*d*x+a*e)^{1/2}-15*((a*e*g-c*d*f)*g)^{1/2}*x^2*c^2*d^2*g^2-20*((a*e*g-c*d*f)*g)^{1/2}*x*a*c*d*e*g^2-10*((a*e*g-c*d*f)*g)^{1/2}*x*c^2*d^2*f*g-3*((a*e*g-c*d*f)*g)^{1/2}*a^2*e^2*g^2-14*((a*e*g-c*d*f)*g)^{1/2}*a*c*d*e*f*g+2*((a*e*g-c*d*f)*g)^{1/2}*c^2*d^2*f^2)/(e*x+d)^{1/2}/(c*d*x+a*e)^2/(a*e*g-c*d*f)^3/(g*x+f)/((a*e*g-c*d*f)*g)^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2))*(g*x + f)`



[Out] Exception raised: ValueError

**Fricas** [A] time = 0.298935, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(5/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2))\*(g\*x + f

[Out] [-1/6\*(15\*(c^3\*d^3\*e\*g^2\*x^4 + a^2\*c\*d^2\*e^2\*f\*g + (c^3\*d^3\*e\*f\*g + (c^3\*d^4 + 2\*a\*c^2\*d^2\*e^2)\*g^2)\*x^3 + ((c^3\*d^4 + 2\*a\*c^2\*d^2\*e^2)\*f\*g + (2\*a\*c^2\*d^3\*e + a^2\*c\*d\*e^3)\*g^2)\*x^2 + (a^2\*c\*d^2\*e^2\*g^2 + (2\*a\*c^2\*d^3\*e + a^2\*c\*d\*e^3)\*f\*g)\*x)\*sqrt(-g/(c\*d\*f - a\*e\*g))\*log(-(c\*d\*e\*g\*x^2 - c\*d^2\*f + 2\*a\*d\*e\*g - 2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(c\*d\*f - a\*e\*g)\*sqrt(e\*x + d)\*sqrt(-g/(c\*d\*f - a\*e\*g)) - (c\*d\*e\*f - (c\*d^2 + 2\*a\*e^2)\*g)\*x)/(e\*g\*x^2 + d\*f + (e\*f + d\*g)\*x)) - 2\*(15\*c^2\*d^2\*g^2\*x^2 - 2\*c^2\*d^2\*f^2 + 14\*a\*c\*d\*e\*f\*g + 3\*a^2\*e^2\*g^2 + 10\*(c^2\*d^2\*f\*g + 2\*a\*c\*d\*e\*g^2)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d))/(a^2\*c^3\*d^4\*e^2\*f^4 - 3\*a^3\*c^2\*d^3\*e^3\*f^3\*g + 3\*a^4\*c\*d^2\*e^4\*f^2\*g^2 - a^5\*d\*e^5\*f\*g^3 + (c^5\*d^5\*e\*f^3\*g - 3\*a\*c^4\*d^4\*e^2\*f^2\*g^2 + 3\*a^2\*c^3\*d^3\*e^3\*f\*g^3 - a^3\*c^2\*d^2\*e^4\*g^4)\*x^4 + (c^5\*d^5\*e\*f^4 + (c^5\*d^6 - a\*c^4\*d^4\*e^2)\*f^3\*g - 3\*(a\*c^4\*d^5\*e + a^2\*c^3\*d^3\*e^3)\*f^2\*g^2 + (3\*a^2\*c^3\*d^4\*e^2 + 5\*a^3\*c^2\*d^2\*e^4)\*f\*g^3 - (a^3\*c^2\*d^3\*e^3 + 2\*a^4\*c\*d\*e^5)\*g^4)\*x^3 + ((c^5\*d^6 + 2\*a\*c^4\*d^4\*e^2)\*f^4 - (a\*c^4\*d^5\*e + 5\*a^2\*c^3\*d^3\*e^3)\*f^3\*g - 3\*(a^2\*c^3\*d^4\*e^2 - a^3\*c^2\*d^2\*e^4)\*f^2\*g^2 + (5\*a^3\*c^2\*d^3\*e^3 + a^4\*c\*d\*e^5)\*f\*g^3 - (2\*a^4\*c\*d^2\*e^4 + a^5\*e^6)\*g^4)\*x^2 - (a^5\*d\*e^5\*g^4 - (2\*a\*c^4\*d^5\*e + a^2\*c^3\*d^3\*e^3)\*f^4 + (5\*a^2\*c^3\*d^4\*e^2 + 3\*a^3\*c^2\*d^2\*e^4)\*f^3\*g - 3\*(a^3\*c^2\*d^3\*e^3 + a^4\*c\*d\*e^5)\*f^2\*g^2 - (a^4\*c\*d^2\*e^4 - a^5\*e^6)\*f\*g^3)\*x), -1/3\*(15\*(c^3\*d^3\*e\*g^2\*x^4 + a^2\*c\*d^2\*e^2\*f\*g + (c^3\*d^3\*e\*f\*g + (c^3\*d^4 + 2\*a\*c^2\*d^2\*e^2)\*g^2)\*x^3 + ((c^3\*d^4 + 2\*a\*c^2\*d^2\*e^2)\*f\*g + (2\*a\*c^2\*d^3\*e + a^2\*c\*d\*e^3)\*g^2)\*x^2 + (a^2\*c\*d^2\*e^2\*g^2 + (2\*a\*c^2\*d^3\*e + a^2\*c\*d\*e^3)\*f\*g)\*x)\*sqrt(g/(c\*d\*f - a\*e\*g))\*arctan(sqrt(e\*x + d)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(g/(c\*d\*f - a\*e\*g)))) - (15\*c^2\*d^2\*g^2\*x^2 - 2\*c^2\*d^2\*f^2 + 14\*a\*c\*d\*e\*f\*g + 3\*a^2\*e^2\*g^2 + 10\*(c^2\*d^2\*f\*g + 2\*a\*c\*d\*e\*g^2)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d))/(a^2\*c^3\*d^4\*e^2\*f^4 - 3\*a^3\*c^2\*d^3\*e^3\*f^3\*g + 3\*a^4\*c\*d^2\*e^4\*f^2\*g^2 - a^5\*d\*e^5\*f\*g^3 + (c^5\*d^5\*e\*f^3\*g - 3\*a\*c^4\*d^4\*e^2\*f^2\*g^2 + 3\*a^2\*c^3\*d^3\*e^3\*f\*g^3 - a^3\*c^2\*d^2\*e^4\*g^4)\*x^4 + (c^5\*d^5\*e\*f^4 + (c^5\*d^6 - a\*c^4\*d^4\*e^2)\*f^3\*g - 3\*(a\*c^4\*d^5\*e + a^2\*c^3\*d^3\*e^3)\*f^2\*g^2 + (3\*a^2\*c^3\*d^4\*e^2 + 5\*a^3\*c^2\*d^2\*e^4)\*f\*g^3 - (a^3\*c^2\*d^3\*e^3 + 2\*a^4\*c\*d\*e^5)\*g^4)\*x^3 + ((c^5\*d^6 + 2\*a\*c^4\*d^4\*e^2)\*f^4 - (a\*c^4\*d^5\*e + 5\*a^2\*c^3\*d^3\*e^3)\*f^3\*g - 3\*(a^2\*c^3\*d^4\*e^2 - a^3\*c^2\*d^2\*e^4)\*f^2\*g^2 + (5\*a^3\*c^2\*d^3\*e^3 + a^4\*c\*d\*e^5)\*f\*g^3 - (2\*a^4\*c\*d^2\*e^4 + a^5\*e^6)\*g^4)\*x^2 - (a^5\*d\*e^5\*g^4 - (2\*a\*c^4\*d^5\*e + a^2\*c^3\*d^3\*e^3)\*f^4 + (5\*a^2\*c^3\*d^4\*e^2 + 3\*a^3\*c^2\*d^2\*e^4)\*f^3\*g - 3\*(a^3\*c^2\*d^3\*e^3 + a^4\*c\*d\*e^5)\*f^2\*g^2 - (a^4\*c\*d^2\*e^4 - a^5\*e^6)\*f\*g^3)\*x]

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(5/2)/(g\*x+f)\*\*2/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)

[Out] Timed out

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**GIAC/XCAS [A]** time = 1.16868, size = 4, normalized size = 0.01

*sage0\*x*

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(5/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*(g\*x + f

[Out] sage0\*x

$$3.678 \quad \int \frac{(d+ex)^{5/2}}{(f+gx)^3(ade+(cd^2+ae^2)x+cdex^2)^{5/2}} dx$$

**Optimal.** Leaf size=342

$$\frac{35c^2d^2g^{3/2} \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4(cdf-aeg)^{9/2}} + \frac{35cdg^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4\sqrt{d+ex}(f+gx)(cdf-aeg)^4}$$

$$+ \frac{35g^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{6\sqrt{d+ex}(f+gx)^2(cdf-aeg)^3} + \frac{14g\sqrt{d+ex}}{3(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2}$$

$$- \frac{2(d+ex)^{3/2}}{3(f+gx)^2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)}$$

[Out]  $(-2*(d+e*x)^{(3/2)})/(3*(c*d*f-a*e*g)*(f+g*x)^2*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(3/2)})+(14*g*\text{Sqrt}[d+e*x])/(3*(c*d*f-a*e*g)^2*(f+g*x)^2*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])+(35*g^2*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])/(6*(c*d*f-a*e*g)^3*\text{Sqrt}[d+e*x]*(f+g*x)^2)+(35*c*d*g^2*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])/(4*(c*d*f-a*e*g)^4*\text{Sqrt}[d+e*x]*(f+g*x))+(35*c^2*d^2*g^{3/2}*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])]/(\text{Sqrt}[c*d*f-a*e*g]*\text{Sqrt}[d+e*x]))/(4*(c*d*f-a*e*g)^{(9/2)})$

**Rubi [A]** time = 1.8413, antiderivative size = 342, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{35c^2d^2g^{3/2} \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4(cdf-aeg)^{9/2}} + \frac{35cdg^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4\sqrt{d+ex}(f+gx)(cdf-aeg)^4}$$

$$+ \frac{35g^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{6\sqrt{d+ex}(f+gx)^2(cdf-aeg)^3} + \frac{14g\sqrt{d+ex}}{3(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2}$$

$$- \frac{2(d+ex)^{3/2}}{3(f+gx)^2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d+e*x)^{(5/2)}]/((f+g*x)^3*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(5/2)}), x$

[Out]  $(-2*(d+e*x)^{(3/2)})/(3*(c*d*f-a*e*g)*(f+g*x)^2*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(3/2)})+(14*g*\text{Sqrt}[d+e*x])/(3*(c*d*f-a*e*g)^2*(f+g*x)^2*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])+(35*g^2*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])/(6*(c*d*f-a*e*g)^3*\text{Sqrt}[d+e*x]*(f+g*x)^2)+(35*c*d*g^2*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])/(4*(c*d*f-a*e*g)^4*\text{Sqrt}[d+e*x]*(f+g*x))+(35*c^2*d^2*g^{3/2}*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])]/(\text{Sqrt}[c*d*f-a*e*g]*\text{Sqrt}[d+e*x]))/(4*(c*d*f-a*e*g)^{(9/2)})$

**Rubi in Sympy [A]** time = 153.982, size = 332, normalized size = 0.97

$$\frac{35c^2d^2g^{\frac{3}{2}} \operatorname{atanh}\left(\frac{\sqrt{g}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}\sqrt{aeg-cdf}}\right)}{4(aeg-cdf)^{\frac{9}{2}}} + \frac{35cdg^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{4\sqrt{d+ex}(f+gx)(aeg-cdf)^4}$$

$$- \frac{35g^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{6\sqrt{d+ex}(f+gx)^2(aeg-cdf)^3} + \frac{14g\sqrt{d+ex}}{3(f+gx)^2(aeg-cdf)^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}$$

$$+ \frac{2(d+ex)^{\frac{3}{2}}}{3(f+gx)^2(aeg-cdf)(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**(5/2)/(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2)`

[Out] 
$$-35*c^{**2}*d^{**2}*g^{**3/2}*atanh(sqrt(g)*sqrt(a*d*e + c*d*e*x^{**2} + x*(a*e^{**2} + c*d^{**2}))/sqrt(d + e*x)*sqrt(a*e*g - c*d*f)))/(4*(a*e*g - c*d*f)**(9/2)) + 35*c*d*g^{**2}*sqrt(a*d*e + c*d*e*x^{**2} + x*(a*e^{**2} + c*d^{**2}))/4*sqrt(d + e*x)*(f + g*x)*(a*e*g - c*d*f)**4 - 35*g^{**2}*sqrt(a*d*e + c*d*e*x^{**2} + x*(a*e^{**2} + c*d^{**2}))/6*sqrt(d + e*x)*(f + g*x)**2*(a*e*g - c*d*f)**3 + 14*g*sqrt(d + e*x)/(3*(f + g*x)**2*(a*e*g - c*d*f)**2*sqrt(a*d*e + c*d*e*x^{**2} + x*(a*e^{**2} + c*d^{**2}))) + 2*(d + e*x)**(3/2)/(3*(f + g*x)**2*(a*e*g - c*d*f)*(a*d*e + c*d*e*x^{**2} + x*(a*e^{**2} + c*d^{**2}))^{(3/2)})$$

**Mathematica [A]** time = 0.985426, size = 212, normalized size = 0.62

$$\frac{\sqrt{d+ex} \left( \frac{(ae+cdx) \left( -\frac{8c^2d^2(cdf-ae g)}{(ae+cdx)^2} + \frac{72c^2d^2g}{ae+cdx} + \frac{6g^2(cdf-ae g)}{(f+gx)^2} + \frac{33cdg^2}{f+gx} \right)}{3(cdf-ae g)^4} - \frac{35c^2d^2g^{3/2}\sqrt{ae+cdx} \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{ae g-cdf}}\right)}{(ae g-cdf)^{9/2}} \right)}{4\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] `Integrate[(d + e*x)^(5/2)/((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))]`

[Out] 
$$\frac{\sqrt{d+e*x} \left( \left( (a*e + c*d*x) \left( (-8*c^2*d^2*(c*d*f - a*e*g))/(a*e + c*d*x)^2 + (72*c^2*d^2*g)/(a*e + c*d*x) + (6*g^2*(c*d*f - a*e*g))/(f + g*x)^2 + (33*c*d*g^2)/(f + g*x) \right) \right) / (3*(c*d*f - a*e*g)^4) - (35*c^2*d^2*g^{3/2}*sqrt[a*e + c*d*x]*ArcTanh[(sqrt[g]*sqrt[a*e + c*d*x])/sqrt[-(c*d*f) + a*e*g]]) / (-c*d*f + a*e*g)^{9/2} \right) / (4*sqrt[(a*e + c*d*x)*(d + e*x)])$$

**Maple [B]** time = 0.047, size = 670, normalized size = 2.

$$-\frac{1}{12(cdx+ae)^2(aeg-cdf)^4(gx+f)^2} \sqrt{cdex^2+ae^2x+cd^2x+ade} \left( 105 \operatorname{Arctanh} \left( \frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}} \right) x^3 c^3 d^3 g^4 \sqrt{cdx+ae} + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^(5/2)/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2), x)`

[Out] 
$$-1/12*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^{1/2}*(105*arctanh(g*(c*d*x+a*e)^{1/2}/((a*e*g-c*d*f)*g)^{1/2})*x^3*c^3*d^3*g^4*(c*d*x+a*e)^{1/2}+105*arctanh(g*(c*d*x+a*e)^{1/2}/((a*e*g-c*d*f)*g)^{1/2})*x^2*a*c^2*d^2*e*g^4*(c*d*x+a*e)^{1/2}+210*arctanh(g*(c*d*x+a*e)^{1/2}/((a*e*g-c*d*f)*g)^{1/2})*x^2*c^3*d^3*f*g^3*(c*d*x+a*e)^{1/2}+210*arctanh(g*(c*d*x+a*e)^{1/2}/((a*e*g-c*d*f)*g)^{1/2})*x*a*c^2*d^2*e*f*g^3*(c*d*x+a*e)^{1/2}+105*arctanh(g*(c*d*x+a*e)^{1/2}/((a*e*g-c*d*f)*g)^{1/2})*x*c^3*d^3*f^2*g^2*(c*d*x+a*e)^{1/2}-105*((a*e*g-c*d*f)*g)^{1/2}*x^3*c^3*d^3*g^3+105*arctanh(g*(c*d*x+a*e)^{1/2}/((a*e*g-c*d*f)*g)^{1/2})*a*c^2*d^2*e*f^2*g^2*(c*d*x+a*e)^{1/2}-140*((a*e*g-c*d*f)*g)^{1/2}*x^2*a*c^2*d^2*e*g^3-175*((a*e*g-c*d*f)*g)^{1/2}*x^2*c^3*d^3*f*g^2-21*((a*e*g-c*d*f)*g)^{1/2}*x*a^2*c*d*e^2*g^3-238*((a*e*g-c*d*f)*g)^{1/2}*x*a*c^2*d^2*e*f*g^2-56*((a*e*g-c*d*f)*g)^{1/2}*x*c^3*d^3*f^2*g+6*((a*e*g-c*d*f)*g)^{1/2}*a^3*e^3*g^3-39*((a*e*g-c*d*f)*g)^{1/2}*a^2*c*d*e^2*f*g^2-80*((a*e*g-c*d*f)*g)^{1/2}*a*c^2*d^2*e*f^2*g+8*((a*e*g-c*d*f)*g)^{1/2}*c^3*d^3*f^3/(e*x+d)^{1/2}/(c*d*x+a*e)^2/(a*e*g-c*d*f)^4/(g*x+f)^2/((a*e*g-c*d*f)*g)^{1/2}$$

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**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(5/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2))\*(g\*x + f

[Out] Exception raised: ValueError

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**Fricas [A]** time = 0.312331, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(5/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2))\*(g\*x + f

[Out] [1/24\*(105\*(c^4\*d^4\*e\*g^3\*x^5 + a^2\*c^2\*d^3\*e^2\*f^2\*g + (2\*c^4\*d^4\*  
4\*e\*f\*g^2 + (c^4\*d^5 + 2\*a\*c^3\*d^3\*e^2)\*g^3)\*x^4 + (c^4\*d^4\*e\*f^2  
\*g + 2\*(c^4\*d^5 + 2\*a\*c^3\*d^3\*e^2)\*f\*g^2 + (2\*a\*c^3\*d^4\*e + a^2\*c  
^2\*d^2\*e^3)\*g^3)\*x^3 + (a^2\*c^2\*d^3\*e^2\*g^3 + (c^4\*d^5 + 2\*a\*c^3\*  
d^3\*e^2)\*f^2\*g + 2\*(2\*a\*c^3\*d^4\*e + a^2\*c^2\*d^2\*e^3)\*f\*g^2)\*x^2 +  
(2\*a^2\*c^2\*d^3\*e^2\*f\*g^2 + (2\*a\*c^3\*d^4\*e + a^2\*c^2\*d^2\*e^3)\*f^2  
\*g)\*x)\*sqrt(-g/(c\*d\*f - a\*e\*g))\*log(-(c\*d\*e\*g\*x^2 - c\*d^2\*f + 2\*a  
\*d\*e\*g + 2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(c\*d\*f - a  
\*e\*g)\*sqrt(e\*x + d)\*sqrt(-g/(c\*d\*f - a\*e\*g)) - (c\*d\*e\*f - (c\*d^2  
+ 2\*a\*e^2)\*g)\*x)/(e\*g\*x^2 + d\*f + (e\*f + d\*g)\*x)) + 2\*(105\*c^3\*d^4  
\*g^3\*x^3 - 8\*c^3\*d^3\*f^3 + 80\*a\*c^2\*d^2\*e\*f^2\*g + 39\*a^2\*c\*d\*e^2  
\*f\*g^2 - 6\*a^3\*e^3\*g^3 + 35\*(5\*c^3\*d^3\*f\*g^2 + 4\*a\*c^2\*d^2\*e\*g^3)  
\*x^2 + 7\*(8\*c^3\*d^3\*f^2\*g + 34\*a\*c^2\*d^2\*e\*f\*g^2 + 3\*a^2\*c\*d\*e^2\*  
g^3)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)  
)/(a^2\*c^4\*d^5\*e^2\*f^6 - 4\*a^3\*c^3\*d^4\*e^3\*f^5\*g + 6\*a^4\*c^2\*d^3\*  
e^4\*f^4\*g^2 - 4\*a^5\*c\*d^2\*e^5\*f^3\*g^3 + a^6\*d\*e^6\*f^2\*g^4 + (c^6\*  
d^6\*e\*f^4\*g^2 - 4\*a\*c^5\*d^5\*e^2\*f^3\*g^3 + 6\*a^2\*c^4\*d^4\*e^3\*f^2\*g  
^4 - 4\*a^3\*c^3\*d^3\*e^4\*f\*g^5 + a^4\*c^2\*d^2\*e^5\*g^6)\*x^5 + (2\*c^6\*  
d^6\*e\*f^5\*g + (c^6\*d^7 - 6\*a\*c^5\*d^5\*e^2)\*f^4\*g^2 - 4\*(a\*c^5\*d^6\*  
e - a^2\*c^4\*d^4\*e^3)\*f^3\*g^3 + 2\*(3\*a^2\*c^4\*d^5\*e^2 + 2\*a^3\*c^3\*d  
^3\*e^4)\*f^2\*g^4 - 2\*(2\*a^3\*c^3\*d^4\*e^3 + 3\*a^4\*c^2\*d^2\*e^5)\*f\*g^5  
+ (a^4\*c^2\*d^3\*e^4 + 2\*a^5\*c\*d\*e^6)\*g^6)\*x^4 + (c^6\*d^6\*e\*f^6 +  
2\*c^6\*d^7\*f^5\*g - 6\*a^4\*c^2\*d^3\*e^4\*f\*g^5 - 3\*(2\*a\*c^5\*d^6\*e + 3\*  
a^2\*c^4\*d^4\*e^3)\*f^4\*g^2 + 4\*(a^2\*c^4\*d^5\*e^2 + 4\*a^3\*c^3\*d^3\*e^4  
)\*f^3\*g^3 + (4\*a^3\*c^3\*d^4\*e^3 - 9\*a^4\*c^2\*d^2\*e^5)\*f^2\*g^4 + (2\*  
a^5\*c\*d^2\*e^5 + a^6\*e^7)\*g^6)\*x^3 - (6\*a^2\*c^4\*d^4\*e^3\*f^5\*g - 2\*  
a^6\*e^7\*f\*g^5 - a^6\*d\*e^6\*g^6 - (c^6\*d^7 + 2\*a\*c^5\*d^5\*e^2)\*f^6 +  
(9\*a^2\*c^4\*d^5\*e^2 - 4\*a^3\*c^3\*d^3\*e^4)\*f^4\*g^2 - 4\*(4\*a^3\*c^3\*d  
^4\*e^3 + a^4\*c^2\*d^2\*e^5)\*f^3\*g^3 + 3\*(3\*a^4\*c^2\*d^3\*e^4 + 2\*a^5\*  
c\*d\*e^6)\*f^2\*g^4)\*x^2 + (2\*a^6\*d\*e^6\*f\*g^5 + (2\*a\*c^5\*d^6\*e + a^2  
\*c^4\*d^4\*e^3)\*f^6 - 2\*(3\*a^2\*c^4\*d^5\*e^2 + 2\*a^3\*c^3\*d^3\*e^4)\*f^5  
\*g + 2\*(2\*a^3\*c^3\*d^4\*e^3 + 3\*a^4\*c^2\*d^2\*e^5)\*f^4\*g^2 + 4\*(a^4\*c  
^2\*d^3\*e^4 - a^5\*c\*d\*e^6)\*f^3\*g^3 - (6\*a^5\*c\*d^2\*e^5 - a^6\*e^7)\*f  
^2\*g^4)\*x), -1/12\*(105\*(c^4\*d^4\*e\*g^3\*x^5 + a^2\*c^2\*d^3\*e^2\*f^2\*g  
+ (2\*c^4\*d^4\*e\*f\*g^2 + (c^4\*d^5 + 2\*a\*c^3\*d^3\*e^2)\*g^3)\*x^4 + (c  
^4\*d^4\*e\*f^2\*g + 2\*(c^4\*d^5 + 2\*a\*c^3\*d^3\*e^2)\*f\*g^2 + (2\*a\*c^3\*d  
^4\*e + a^2\*c^2\*d^2\*e^3)\*g^3)\*x^3 + (a^2\*c^2\*d^3\*e^2\*g^3 + (c^4\*d^5  
+ 2\*a\*c^3\*d^3\*e^2)\*f^2\*g + 2\*(2\*a\*c^3\*d^4\*e + a^2\*c^2\*d^2\*e^3)\*  
f\*g^2)\*x^2 + (2\*a^2\*c^2\*d^3\*e^2\*f\*g^2 + (2\*a\*c^3\*d^4\*e + a^2\*c^2\*  
d^2\*e^3)\*f^2\*g)\*x)\*sqrt(g/(c\*d\*f - a\*e\*g))\*arctan(sqrt(e\*x + d)/(  
sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(g/(c\*d\*f - a\*e\*g  
)))) - (105\*c^3\*d^3\*g^3\*x^3 - 8\*c^3\*d^3\*f^3 + 80\*a\*c^2\*d^2\*e\*f^2\*  
g + 39\*a^2\*c\*d\*e^2\*f\*g^2 - 6\*a^3\*e^3\*g^3 + 35\*(5\*c^3\*d^3\*f\*g^2 +  
4\*a\*c^2\*d^2\*e\*g^3)\*x^2 + 7\*(8\*c^3\*d^3\*f^2\*g + 34\*a\*c^2\*d^2\*e\*f\*g^2  
+ 3\*a^2\*c\*d\*e^2\*g^3)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2  
)\*x)\*sqrt(e\*x + d)/(a^2\*c^4\*d^5\*e^2\*f^6 - 4\*a^3\*c^3\*d^4\*e^3\*f^5\*

$$\begin{aligned}
&g + 6*a^4*c^2*d^3*e^4*f^4*g^2 - 4*a^5*c*d^2*e^5*f^3*g^3 + a^6*d*e \\
&^6*f^2*g^4 + (c^6*d^6*e*f^4*g^2 - 4*a*c^5*d^5*e^2*f^3*g^3 + 6*a^2 \\
&*c^4*d^4*e^3*f^2*g^4 - 4*a^3*c^3*d^3*e^4*f*g^5 + a^4*c^2*d^2*e^5* \\
&g^6)*x^5 + (2*c^6*d^6*e*f^5*g + (c^6*d^7 - 6*a*c^5*d^5*e^2)*f^4*g \\
&^2 - 4*(a*c^5*d^6*e - a^2*c^4*d^4*e^3)*f^3*g^3 + 2*(3*a^2*c^4*d^5 \\
&*e^2 + 2*a^3*c^3*d^3*e^4)*f^2*g^4 - 2*(2*a^3*c^3*d^4*e^3 + 3*a^4* \\
&c^2*d^2*e^5)*f*g^5 + (a^4*c^2*d^3*e^4 + 2*a^5*c*d*e^6)*g^6)*x^4 + \\
&(c^6*d^6*e*f^6 + 2*c^6*d^7*f^5*g - 6*a^4*c^2*d^3*e^4*f*g^5 - 3*( \\
&2*a*c^5*d^6*e + 3*a^2*c^4*d^4*e^3)*f^4*g^2 + 4*(a^2*c^4*d^5*e^2 + \\
&4*a^3*c^3*d^3*e^4)*f^3*g^3 + (4*a^3*c^3*d^4*e^3 - 9*a^4*c^2*d^2* \\
&e^5)*f^2*g^4 + (2*a^5*c*d^2*e^5 + a^6*e^7)*g^6)*x^3 - (6*a^2*c^4* \\
&d^4*e^3*f^5*g - 2*a^6*e^7*f*g^5 - a^6*d*e^6*g^6 - (c^6*d^7 + 2*a* \\
&c^5*d^5*e^2)*f^6 + (9*a^2*c^4*d^5*e^2 - 4*a^3*c^3*d^3*e^4)*f^4*g^ \\
&2 - 4*(4*a^3*c^3*d^4*e^3 + a^4*c^2*d^2*e^5)*f^3*g^3 + 3*(3*a^4*c^ \\
&2*d^3*e^4 + 2*a^5*c*d*e^6)*f^2*g^4)*x^2 + (2*a^6*d*e^6*f*g^5 + (2 \\
&*a*c^5*d^6*e + a^2*c^4*d^4*e^3)*f^6 - 2*(3*a^2*c^4*d^5*e^2 + 2*a^ \\
&3*c^3*d^3*e^4)*f^5*g + 2*(2*a^3*c^3*d^4*e^3 + 3*a^4*c^2*d^2*e^5)* \\
&f^4*g^2 + 4*(a^4*c^2*d^3*e^4 - a^5*c*d*e^6)*f^3*g^3 - (6*a^5*c*d^ \\
&2*e^5 - a^6*e^7)*f^2*g^4)*x]
\end{aligned}$$


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**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(5/2)/(g\*x+f)\*\*3/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)

[Out] Timed out

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**GIAC/XCAS [A]** time = 13.2964, size = 4, normalized size = 0.01

*sage<sub>0</sub>x*

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(5/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*(g\*x + f

[Out] sage0\*x

$$3.679 \quad \int \frac{(f+gx)^4 \sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}} dx$$

**Optimal.** Leaf size=336

$$\begin{aligned} & \frac{128(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-ae^2g)^3(2ae^2g-cd(5ef-3dg))}{3465c^5d^5e(d+ex)^{3/2}} \\ & + \frac{128g(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-ae^2g)^3}{1155c^4d^4e\sqrt{d+ex}} \\ & + \frac{32(f+gx)^2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-ae^2g)^2}{231c^3d^3(d+ex)^{3/2}} \\ & + \frac{16(f+gx)^3(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-ae^2g)}{99c^2d^2(d+ex)^{3/2}} \\ & + \frac{2(f+gx)^4(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{11cd(d+ex)^{3/2}} \end{aligned}$$

[Out]  $(-128*(c*d*f - a*e*g)^3*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{3/2})/(3465*c^5*d^5*e*(d + e*x)^{3/2}) + (128*g*(c*d*f - a*e*g)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{3/2})/(1155*c^4*d^4*e*\text{Sqrt}[d + e*x]) + (32*(c*d*f - a*e*g)^2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{3/2})/(231*c^3*d^3*(d + e*x)^{3/2}) + (16*(c*d*f - a*e*g)*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{3/2})/(99*c^2*d^2*(d + e*x)^{3/2}) + (2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{3/2})/(11*c*d*(d + e*x)^{3/2})$

**Rubi [A]** time = 1.63289, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\begin{aligned} & \frac{128(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-ae^2g)^3(2ae^2g-cd(5ef-3dg))}{3465c^5d^5e(d+ex)^{3/2}} \\ & + \frac{128g(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-ae^2g)^3}{1155c^4d^4e\sqrt{d+ex}} \\ & + \frac{32(f+gx)^2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-ae^2g)^2}{231c^3d^3(d+ex)^{3/2}} \\ & + \frac{16(f+gx)^3(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-ae^2g)}{99c^2d^2(d+ex)^{3/2}} \\ & + \frac{2(f+gx)^4(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{11cd(d+ex)^{3/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(f+g*x)^4*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/\text{Sqrt}[d + e*x], x]$

[Out]  $(-128*(c*d*f - a*e*g)^3*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{3/2})/(3465*c^5*d^5*e*(d + e*x)^{3/2}) + (128*g*(c*d*f - a*e*g)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{3/2})/(1155*c^4*d^4*e*\text{Sqrt}[d + e*x]) + (32*(c*d*f - a*e*g)^2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{3/2})/(231*c^3*d^3*(d + e*x)^{3/2}) + (16*(c*d*f - a*e*g)*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{3/2})/(99*c^2*d^2*(d + e*x)^{3/2}) + (2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{3/2})/(11*c*d*(d + e*x)^{3/2})$

**Rubi in Sympy [A]** time = 132.075, size = 330, normalized size = 0.98

$$\frac{2(f+gx)^4(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}{11cd(d+ex)^{\frac{3}{2}}}$$

$$-\frac{16(f+gx)^3(aeg-cdf)(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}{99c^2d^2(d+ex)^{\frac{3}{2}}}$$

$$+\frac{32(f+gx)^2(aeg-cdf)^2(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}{231c^3d^3(d+ex)^{\frac{3}{2}}}$$

$$-\frac{128g(aeg-cdf)^3(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}{1155c^4d^4e\sqrt{d+ex}}$$

$$+\frac{128(aeg-cdf)^3(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}(2ae^2g+3cd^2g-5cdef)}{3465c^5d^5e(d+ex)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x+f)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**`

[Out] `2*(f+g*x)**4*(a*d*e+c*d*e*x**2+x*(a*e**2+c*d**2))**(3/2)/(11*c*d*(d+e*x)**(3/2))-16*(f+g*x)**3*(a*e*g-c*d*f)*(a*d*e+c*d*e*x**2+x*(a*e**2+c*d**2))**(3/2)/(99*c**2*d**2*(d+e*x)**(3/2))+32*(f+g*x)**2*(a*e*g-c*d*f)**2*(a*d*e+c*d*e*x**2+x*(a*e**2+c*d**2))**(3/2)/(231*c**3*d**3*(d+e*x)**(3/2))-128*g*(a*e*g-c*d*f)**3*(a*d*e+c*d*e*x**2+x*(a*e**2+c*d**2))**(3/2)/(1155*c**4*d**4*e*sqrt(d+e*x))+128*(a*e*g-c*d*f)**3*(a*d*e+c*d*e*x**2+x*(a*e**2+c*d**2))**(3/2)*(2*a*e**2*g+3*c*d**2*g-5*c*d*e*f)/(3465*c**5*d**5*e*(d+e*x)**(3/2))`

**Mathematica [A]** time = 0.255427, size = 195, normalized size = 0.58

$$\frac{2((d+ex)(ae+cdx))^{3/2}(128a^4e^4g^4-64a^3cde^3g^3(11f+3gx)+48a^2c^2d^2e^2g^2(33f^2+22fgx+5g^2x^2)-8ac^3d^3eg(231f^3+3465c^5d^5(d+ex)^{3/2}))}{3465c^5d^5(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

[In] `Integrate[((f+g*x)^4*Sqrt[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])/Sqrt[d+e*x]`

[Out] `(2*((a*e+c*d*x)*(d+e*x))^(3/2)*(128*a^4*e^4*g^4-64*a^3*c*d*e^3*g^3*(11*f+3*g*x)+48*a^2*c^2*d^2*e^2*g^2*(33*f^2+22*f*g*x+5*g^2*x^2)-8*a*c^3*d^3*e*g*(231*f^3+297*f^2*g*x+165*f*g^2*x^2+35*g^3*x^3)+c^4*d^4*(1155*f^4+2772*f^3*g*x+2970*f^2*g^2*x^2+1540*f*g^3*x^3+315*g^4*x^4)))/(3465*c^5*d^5*(d+e*x)^(3/2))`

**Maple [A]** time = 0.012, size = 283, normalized size = 0.8

$$(2cdx+2ae)(315g^4x^4c^4d^4-280ac^3d^3eg^4x^3+1540c^4d^4fg^3x^3+240a^2c^2d^2e^2g^4x^2-1320ac^3d^3efg^3x^2+2970c^4d^4f^2g^2x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)`

[Out] `2/3465*(c*d*x+a*e)*(315*c^4*d^4*g^4*x^4-280*a*c^3*d^3*e*g^4*x^3+1540*c^4*d^4*f*g^3*x^3+240*a^2*c^2*d^2*e^2*g^4*x^2-1320*a*c^3*d^3*e*f*g^3*x^2+2970*c^4*d^4*f^2*g^2*x+1056*a`



$$\begin{aligned} &^2*c^2*d^2*e^2*f*g^3*x-2376*a*c^3*d^3*e*f^2*g^2*x+2772*c^4*d^4*f^3 \\ &^3*g*x+128*a^4*e^4*g^4-704*a^3*c*d*e^3*f*g^3+1584*a^2*c^2*d^2*e^2* \\ &f^2*g^2-1848*a*c^3*d^3*e*f^3*g+1155*c^4*d^4*f^4)*(c*d*e*x^2+a*e^2 \\ &*x+c*d^2*x+a*d*e)^{(1/2)}/c^5/d^5/(e*x+d)^{(1/2)} \end{aligned}$$

**Maxima [A]** time = 0.759534, size = 432, normalized size = 1.29

$$\begin{aligned} &\frac{2(cdx+ae)^{\frac{3}{2}}f^4}{3cd} + \frac{8(3c^2d^2x^2+acdex-2a^2e^2)\sqrt{cdx+ae}f^3g}{15c^2d^2} \\ &+ \frac{4(15c^3d^3x^3+3ac^2d^2ex^2-4a^2cde^2x+8a^3e^3)\sqrt{cdx+ae}f^2g^2}{35c^3d^3} \\ &+ \frac{8(35c^4d^4x^4+5ac^3d^3ex^3-6a^2c^2d^2e^2x^2+8a^3cde^3x-16a^4e^4)\sqrt{cdx+ae}fg^3}{315c^4d^4} \\ &+ \frac{2(315c^5d^5x^5+35ac^4d^4ex^4-40a^2c^3d^3e^2x^3+48a^3c^2d^2e^3x^2-64a^4cde^4x+128a^5e^5)\sqrt{cdx+ae}g^4}{3465c^5d^5} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2+a\*d\*e+(c\*d^2+a\*e^2)\*x)\*(g\*x+f)^4/sqrt(e\*x+d),x

[Out]  $\frac{2}{3}*(c*d*x+a*e)^{(3/2)}*f^4/(c*d) + \frac{8}{15}*(3*c^2*d^2*x^2+a*c*d*e*x-2*a^2*e^2)*\sqrt{c*d*x+a*e}*f^3*g/(c^2*d^2) + \frac{4}{35}*(15*c^3*d^3*x^3+3*a*c^2*d^2*e*x^2-4*a^2*c*d*e^2*x+8*a^3*e^3)*\sqrt{c*d*x+a*e}*f^2*g^2/(c^3*d^3) + \frac{8}{315}*(35*c^4*d^4*x^4+5*a*c^3*d^3*e*x^3-6*a^2*c^2*d^2*e^2*x^2+8*a^3*c*d*e^3*x-16*a^4*e^4)*\sqrt{c*d*x+a*e}*f*g^3/(c^4*d^4) + \frac{2}{3465}*(315*c^5*d^5*x^5+35*a*c^4*d^4*e*x^4-40*a^2*c^3*d^3*e^2*x^3+48*a^3*c^2*d^2*e^3*x^2-64*a^4*c*d*e^4*x+128*a^5*e^5)*\sqrt{c*d*x+a*e}*g^4/(c^5*d^5)$

**Fricas [A]** time = 0.274664, size = 1127, normalized size = 3.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2+a\*d\*e+(c\*d^2+a\*e^2)\*x)\*(g\*x+f)^4/sqrt(e\*x+d),x

[Out]  $\frac{2}{3465}*(315*c^6*d^6*e*g^4*x^7+1155*a^2*c^4*d^5*e^2*f^4-1848*a^3*c^3*d^4*e^3*f^3*g+1584*a^4*c^2*d^3*e^4*f^2*g^2-704*a^5*c*d^2*e^5*f*g^3+128*a^6*d*e^6*g^4+35*(44*c^6*d^6*e*f^2*g^3+(9*c^6*d^7+10*a*c^5*d^5*e^2)*g^4)*x^6+5*(594*c^6*d^6*e*f^2*g^2+44*(7*c^6*d^7+8*a*c^5*d^5*e^2)*f^2*g^3+(70*a*c^5*d^6*e-a^2*c^4*d^4*e^3)*g^4)*x^5+(2772*c^6*d^6*e*f^3*g+594*(5*c^6*d^7+6*a*c^5*d^5*e^2)*f^2*g^2+44*(40*a*c^5*d^6*e-a^2*c^4*d^4*e^3)*f*g^3-(5*a^2*c^4*d^5*e^2-8*a^3*c^3*d^3*e^4)*g^4)*x^4+(1155*c^6*d^6*e*f^4+924*(3*c^6*d^7+4*a*c^5*d^5*e^2)*f^3*g+198*(18*a*c^5*d^6*e-a^2*c^4*d^4*e^3)*f^2*g^2-44*(a^2*c^4*d^5*e^2-2*a^3*c^3*d^3*e^4)*f*g^3+8*(a^3*c^3*d^4*e^3-2*a^4*c^2*d^2*e^5)*g^4)*x^3+(1155*(c^6*d^7+2*a*c^5*d^5*e^2)*f^4+924*(4*a*c^5*d^6*e-a^2*c^4*d^4*e^3)*f^3*g-198*(a^2*c^4*d^5*e^2-4*a^3*c^3*d^3*e^4)*f^2*g^2+88*(a^3*c^3*d^4*e^3-4*a^4*c^2*d^2*e^5)*f*g^3-16*(a^4*c^2*d^3*e^4-4*a^5*c*d*e^6)*g^4)*x^2+(1155*(2*a*c^5*d^6*e+a^2*c^4*d^4*e^3)*f^4-924*(a^2*c^4*d^5*e^2+2*a^3*c^3*d^3*e^4)*f^3*g+792*(a^3*c^3*d^4*e^3+2*a^4*c^2*d^2*e^5)*f^2*g^2-352*(a^4*c^2*d^3*e^4+2*a^5*c*d*e^6)*f*g^3+64*(a^5*c*d^2*e^5+2*a^6*e^7)*g^4)*x)/(sqrt(c*d*e*x^2+a*d*e+(c*d^2+a*e^2)*x)*sqrt(e*x+d)*c^5*d^5)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)**4*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/2)`

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^4/sqrt(e*x + d), x`

[Out] Timed out

$$3.680 \quad \int \frac{(f+gx)^3 \sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}} dx$$

**Optimal.** Leaf size=269

$$\begin{aligned} & -\frac{16(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)^2(2ae^2g-cd(5ef-3dg))}{315c^4d^4e(d+ex)^{3/2}} \\ & + \frac{16g(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)^2}{105c^3d^3e\sqrt{d+ex}} \\ & + \frac{4(f+gx)^2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)}{21c^2d^2(d+ex)^{3/2}} \\ & + \frac{2(f+gx)^3(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{9cd(d+ex)^{3/2}} \end{aligned}$$

[Out]  $(-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/(315*c^4*d^4*e*(d + e*x)^{(3/2)}) + (16*g*(c*d*f - a*e*g)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/(105*c^3*d^3*e*sqrt[d + e*x]) + (4*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(21*c^2*d^2*(d + e*x)^{(3/2)}) + (2*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(9*c*d*(d + e*x)^{(3/2)})$

**Rubi [A]** time = 1.10868, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\begin{aligned} & -\frac{16(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)^2(2ae^2g-cd(5ef-3dg))}{315c^4d^4e(d+ex)^{3/2}} \\ & + \frac{16g(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)^2}{105c^3d^3e\sqrt{d+ex}} \\ & + \frac{4(f+gx)^2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)}{21c^2d^2(d+ex)^{3/2}} \\ & + \frac{2(f+gx)^3(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{9cd(d+ex)^{3/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(f+g*x)^3*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/\text{Sqrt}[d + e*x], x]$

[Out]  $(-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/(315*c^4*d^4*e*(d + e*x)^{(3/2)}) + (16*g*(c*d*f - a*e*g)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/(105*c^3*d^3*e*sqrt[d + e*x]) + (4*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(21*c^2*d^2*(d + e*x)^{(3/2)}) + (2*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(9*c*d*(d + e*x)^{(3/2)})$

**Rubi in Sympy [A]** time = 95.0646, size = 264, normalized size = 0.98

$$\frac{2(f+gx)^3(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}{9cd(d+ex)^{\frac{3}{2}}}$$

$$-\frac{4(f+gx)^2(aeg-cdf)(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}{21c^2d^2(d+ex)^{\frac{3}{2}}}$$

$$+\frac{16g(aeg-cdf)^2(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}{105c^3d^3e\sqrt{d+ex}}$$

$$-\frac{16(aeg-cdf)^2(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}(2ae^2g+3cd^2g-5cdef)}{315c^4d^4e(d+ex)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x+f)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**`

[Out] `2*(f+g*x)**3*(a*d*e+c*d*e*x**2+x*(a*e**2+c*d**2))**(3/2)/(9*c*d*(d+e*x)**(3/2))-4*(f+g*x)**2*(a*e*g-c*d*f)*(a*d*e+c*d*e*x**2+x*(a*e**2+c*d**2))**(3/2)/(21*c**2*d**2*(d+e*x)**(3/2))+16*g*(a*e*g-c*d*f)**2*(a*d*e+c*d*e*x**2+x*(a*e**2+c*d**2))**(3/2)/(105*c**3*d**3*e*sqrt(d+e*x))-16*(a*e*g-c*d*f)**2*(a*d*e+c*d*e*x**2+x*(a*e**2+c*d**2))**(3/2)*(2*a*e**2*g+3*c*d**2*g-5*c*d*e*f)/(315*c**4*d**4*e*(d+e*x)**(3/2))`

**Mathematica [A]** time = 0.214077, size = 136, normalized size = 0.51

$$\frac{2((d+ex)(ae+cdx))^{3/2}(-16a^3e^3g^3+24a^2cde^2g^2(3f+gx)-6ac^2d^2eg(21f^2+18fgx+5g^2x^2)+c^3d^3(105f^3+189f^2gx+135fd^2x^2+35g^3x^3))}{315c^4d^4(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

[In] `Integrate[((f+g*x)^3*sqrt[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])/sqrt[d+e*x],x]`

[Out] `(2*((a*e+c*d*x)*(d+e*x))^(3/2)*(-16*a^3*e^3*g^3+24*a^2*c*d*e^2*g^2*(3*f+g*x)-6*a*c^2*d^2*e*g*(21*f^2+18*f*g*x+5*g^2*x^2)+c^3*d^3*(105*f^3+189*f^2*g*x+135*f*g^2*x^2+35*g^3*x^3)))/(315*c^4*d^4*(d+e*x)^(3/2))`

**Maple [A]** time = 0.012, size = 188, normalized size = 0.7

$$\frac{(2cdx+2ae)(-35g^3x^3c^3d^3+30ac^2d^2eg^3x^2-135c^3d^3fg^2x^2-24a^2cde^2g^3x+108ac^2d^2efg^2x-189c^3d^3f^2gx+16a^3e^3g^3)}{315c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)`

[Out] `-2/315*(c*d*x+a*e)*(-35*c^3*d^3*g^3*x^3+30*a*c^2*d^2*e*g^3*x^2-135*c^3*d^3*f*g^2*x^2-24*a^2*c*d*e^2*g^3*x+108*a*c^2*d^2*e*f*g^2*x-189*c^3*d^3*f^2*g*x+16*a^3*e^3*g^3-72*a^2*c*d*e^2*f*g^2+126*a*c^2*d^2*e*f^2*g-105*c^3*d^3*f^3)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/c^4/d^4/(e*x+d)^(1/2)`

**Maxima [A]** time = 0.747628, size = 294, normalized size = 1.09

$$\frac{2(cdx + ae)^{\frac{3}{2}}f^3}{3cd} + \frac{2(3c^2d^2x^2 + acdex - 2a^2e^2)\sqrt{cdx + aef^2}g}{5c^2d^2}$$

$$+ \frac{2(15c^3d^3x^3 + 3ac^2d^2ex^2 - 4a^2cde^2x + 8a^3e^3)\sqrt{cdx + aefg^2}}{35c^3d^3}$$

$$+ \frac{2(35c^4d^4x^4 + 5ac^3d^3ex^3 - 6a^2c^2d^2e^2x^2 + 8a^3cde^3x - 16a^4e^4)\sqrt{cdx + aeg^3}}{315c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^3/sqrt(e\*x + d), x)

[Out] 2/3\*(c\*d\*x + a\*e)^(3/2)\*f^3/(c\*d) + 2/5\*(3\*c^2\*d^2\*x^2 + a\*c\*d\*e\*x - 2\*a^2\*e^2)\*sqrt(c\*d\*x + a\*e)\*f^2\*g/(c^2\*d^2) + 2/35\*(15\*c^3\*d^3\*x^3 + 3\*a\*c^2\*d^2\*e\*x^2 - 4\*a^2\*c\*d\*e^2\*x + 8\*a^3\*e^3)\*sqrt(c\*d\*x + a\*e)\*f\*g^2/(c^3\*d^3) + 2/315\*(35\*c^4\*d^4\*x^4 + 5\*a\*c^3\*d^3\*e\*x^3 - 6\*a^2\*c^2\*d^2\*e^2\*x^2 + 8\*a^3\*c\*d\*e^3\*x - 16\*a^4\*e^4)\*sqrt(c\*d\*x + a\*e)\*g^3/(c^4\*d^4)

**Fricas [A]** time = 0.276498, size = 799, normalized size = 2.97

$$\frac{2(35c^5d^5eg^3x^6 + 105a^2c^3d^4e^2f^3 - 126a^3c^2d^3e^3f^2g + 72a^4cd^2e^4fg^2 - 16a^5de^5g^3 + 5(27c^5d^5efg^2 + (7c^5d^6 + 8ac^4d^4e^2)g^3))}{315c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^3/sqrt(e\*x + d), x)

[Out] 2/315\*(35\*c^5\*d^5\*e\*g^3\*x^6 + 105\*a^2\*c^3\*d^4\*e^2\*f^3 - 126\*a^3\*c^2\*d^3\*e^3\*f^2\*g + 72\*a^4\*c\*d^2\*e^4\*f\*g^2 - 16\*a^5\*d\*e^5\*g^3 + 5\*(27\*c^5\*d^5\*e\*f\*g^2 + (7\*c^5\*d^6 + 8\*a\*c^4\*d^4\*e^2)\*g^3))\*x^5 + (189\*c^5\*d^5\*e\*f^2\*g + 27\*(5\*c^5\*d^6 + 6\*a\*c^4\*d^4\*e^2)\*f\*g^2 + (40\*a\*c^4\*d^5\*e - a^2\*c^3\*d^3\*e^3)\*g^3)\*x^4 + (105\*c^5\*d^5\*e\*f^3 + 63\*(3\*c^5\*d^6 + 4\*a\*c^4\*d^4\*e^2)\*f^2\*g + 9\*(18\*a\*c^4\*d^5\*e - a^2\*c^3\*d^3\*e^3)\*f\*g^2 - (a^2\*c^3\*d^4\*e^2 - 2\*a^3\*c^2\*d^2\*e^4)\*g^3)\*x^3 + (105\*(c^5\*d^6 + 2\*a\*c^4\*d^4\*e^2)\*f^3 + 63\*(4\*a\*c^4\*d^5\*e - a^2\*c^3\*d^3\*e^3)\*f^2\*g - 9\*(a^2\*c^3\*d^4\*e^2 - 4\*a^3\*c^2\*d^2\*e^4)\*f\*g^2 + 2\*(a^3\*c^2\*d^3\*e^3 - 4\*a^4\*c\*d\*e^5)\*g^3)\*x^2 + (105\*(2\*a\*c^4\*d^5\*e + a^2\*c^3\*d^3\*e^3)\*f^3 - 63\*(a^2\*c^3\*d^4\*e^2 + 2\*a^3\*c^2\*d^2\*e^4)\*f^2\*g + 36\*(a^3\*c^2\*d^3\*e^3 + 2\*a^4\*c\*d\*e^5)\*f\*g^2 - 8\*(a^4\*c\*d^2\*e^4 + 2\*a^5\*e^6)\*g^3)\*x)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*c^4\*d^4)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*3\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/(e\*x+d)\*\*(1/2), x)

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^3/sqrt(e*x + d), x
```

```
[Out] Timed out
```

$$3.681 \quad \int \frac{(f+gx)^2 \sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}} dx$$

**Optimal.** Leaf size=200

$$\frac{8(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)(2ae^2g-cd(5ef-3dg))}{105c^3d^3e(d+ex)^{3/2}} + \frac{8g(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)}{35c^2d^2e\sqrt{d+ex}} + \frac{2(f+gx)^2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{7cd(d+ex)^{3/2}}$$

[Out]  $(-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{3/2})/(105*c^3*d^3*e*(d + e*x)^{3/2}) + (8*g*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{3/2})/(35*c^2*d^2*e*\text{Sqrt}[d + e*x]) + (2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{3/2})/(7*c*d*(d + e*x)^{3/2})$

**Rubi [A]** time = 0.68347, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{8(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)(2ae^2g-cd(5ef-3dg))}{105c^3d^3e(d+ex)^{3/2}} + \frac{8g(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)}{35c^2d^2e\sqrt{d+ex}} + \frac{2(f+gx)^2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{7cd(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(f+g*x)^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/\text{Sqrt}[d + e*x], x]$

[Out]  $(-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{3/2})/(105*c^3*d^3*e*(d + e*x)^{3/2}) + (8*g*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{3/2})/(35*c^2*d^2*e*\text{Sqrt}[d + e*x]) + (2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{3/2})/(7*c*d*(d + e*x)^{3/2})$

**Rubi in Sympy [A]** time = 61.6233, size = 196, normalized size = 0.98

$$\frac{2(f+gx)^2(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}{7cd(d+ex)^{\frac{3}{2}}} - \frac{8g(aeg-cdf)(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}{35c^2d^2e\sqrt{d+ex}} + \frac{8(aeg-cdf)(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}(2ae^2g+3cd^2g-5cdef)}{105c^3d^3e(d+ex)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((g*x+f)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**$

[Out]  $2*(f + g*x)**2*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2)/(7*c*d*(d + e*x)**(3/2)) - 8*g*(a*e*g - c*d*f)*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2)/(35*c**2*d**2*e*\text{sqrt}(d + e*x)) + 8*(a*e*g - c*d*f)*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2)*(2*a*e**2*g + 3*c*d**2*g - 5*c*d*e*f)/(105*c**3*d**3*e*(d + e*x)**(3/2))$

**Mathematica [A]** time = 0.135204, size = 90, normalized size = 0.45

$$\frac{2((d+ex)(ae+cdx))^{3/2}(8a^2e^2g^2-4acdeg(7f+3gx)+c^2d^2(35f^2+42fgx+15g^2x^2))}{105c^3d^3(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((f + g\*x)^2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/Sqrt[d + e\*x]

[Out] (2\*((a\*e + c\*d\*x)\*(d + e\*x))^(3/2)\*(8\*a^2\*e^2\*g^2 - 4\*a\*c\*d\*e\*g\*(7\*f + 3\*g\*x) + c^2\*d^2\*(35\*f^2 + 42\*f\*g\*x + 15\*g^2\*x^2)))/(105\*c^3\*d^3\*(d + e\*x)^(3/2))

**Maple [A]** time = 0.012, size = 116, normalized size = 0.6

$$\frac{(2cdx + 2ae)(15g^2x^2c^2d^2 - 12acdeg^2x + 42c^2d^2fgx + 8a^2e^2g^2 - 28acdefg + 35f^2c^2d^2)}{105c^3d^3} \sqrt{cdex^2 + ae^2x + cd^2x + ade}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^2\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)/(e\*x+d)^(1/2), x)

[Out] 2/105\*(c\*d\*x+a\*e)\*(15\*c^2\*d^2\*g^2\*x^2-12\*a\*c\*d\*e\*g^2\*x+42\*c^2\*d^2\*f\*g\*x+8\*a^2\*e^2\*g^2-28\*a\*c\*d\*e\*f\*g+35\*c^2\*d^2\*f^2)\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)/c^3/d^3/(e\*x+d)^(1/2)

**Maxima [A]** time = 0.733543, size = 180, normalized size = 0.9

$$\frac{2(cdx + ae)^{\frac{3}{2}}f^2}{3cd} + \frac{4(3c^2d^2x^2 + acdex - 2a^2e^2)\sqrt{cdx + aefg}}{15c^2d^2} + \frac{2(15c^3d^3x^3 + 3ac^2d^2ex^2 - 4a^2cde^2x + 8a^3e^3)\sqrt{cdx + aeg^2}}{105c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^2/sqrt(e\*x + d), x)

[Out] 2/3\*(c\*d\*x + a\*e)^(3/2)\*f^2/(c\*d) + 4/15\*(3\*c^2\*d^2\*x^2 + a\*c\*d\*e\*x - 2\*a^2\*e^2)\*sqrt(c\*d\*x + a\*e)\*f\*g/(c^2\*d^2) + 2/105\*(15\*c^3\*d^3\*x^3 + 3\*a\*c^2\*d^2\*e\*x^2 - 4\*a^2\*c\*d\*e^2\*x + 8\*a^3\*e^3)\*sqrt(c\*d\*x + a\*e)\*g^2/(c^3\*d^3)

**Fricas [A]** time = 0.270867, size = 522, normalized size = 2.61

$$\frac{2(15c^4d^4eg^2x^5 + 35a^2c^2d^3e^2f^2 - 28a^3cd^2e^3fg + 8a^4de^4g^2 + 3(14c^4d^4efg + (5c^4d^5 + 6ac^3d^3e^2)g^2)x^4 + (35c^4d^4ef^2 + 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^2/sqrt(e\*x + d), x)

[Out] 2/105\*(15\*c^4\*d^4\*e\*g^2\*x^5 + 35\*a^2\*c^2\*d^3\*e^2\*f^2 - 28\*a^3\*c\*d^2\*e^3\*f\*g + 8\*a^4\*d^4\*e^4\*g^2 + 3\*(14\*c^4\*d^4\*e\*f\*g + (5\*c^4\*d^5 + 6\*a\*c^3\*d^3\*e^2)\*g^2)\*x^4 + (35\*c^4\*d^4\*e\*f^2 + 4\*a\*c^3\*d^3\*e^2)\*f\*g + (18\*a\*c^3\*d^4\*e - a^2\*c^2\*d^2\*e^3)\*g^2)\*x^3 + (35\*(c^4\*d^5 + 2\*a\*c^3\*d^3\*e^2)\*f^2 + 14\*(4\*a\*c^3\*d^4\*e - a^2\*c^2\*d^2\*e^3)\*f\*g - (a^2\*c^2\*d^3\*e^2 - 4\*a^3\*c\*d^4\*e^4)\*g^2)\*x^2 + (35\*(2\*a\*c^3\*d^4\*e + a^2\*c^2\*d^2\*e^3)\*f^2 - 14\*(a^2\*c^2\*d^3\*e^2 + 2\*a^3\*c\*d^4\*e^4)\*f\*g + 4\*(a^3\*c\*d^2\*e^3 + 2\*a^4\*e^5)\*g^2)\*x)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*c^3\*d^3)



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**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}(f+gx)^2}{\sqrt{d+ex}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*2\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/(e\*x+d)\*\*(1/2)

[Out] Integral(sqrt((d + e\*x)\*(a\*e + c\*d\*x))\*(f + g\*x)\*\*2/sqrt(d + e\*x), x)

---

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^2/sqrt(e\*x + d), x

[Out] Timed out

$$3.682 \quad \int \frac{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}} dx$$

**Optimal.** Leaf size=125

$$\frac{2g(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{5cde\sqrt{d+ex}} - \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(2ae^2g-cd(5ef-3dg))}{15c^2d^2e(d+ex)^{3/2}}$$

[Out]  $(-2*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/(15*c^2*d^2*e*(d + e*x)^{(3/2)}) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/(5*c*d*e*\text{Sqrt}[d + e*x])$

**Rubi [A]** time = 0.315894, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 44,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{2g(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{5cde\sqrt{d+ex}} - \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(2ae^2g-cd(5ef-3dg))}{15c^2d^2e(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(f + g*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/\text{Sqrt}[d + e*x], x]$

[Out]  $(-2*(2*a*e^2*g - c*d*(5*e*f - 3*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/(15*c^2*d^2*e*(d + e*x)^{(3/2)}) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/(5*c*d*e*\text{Sqrt}[d + e*x])$

**Rubi in Sympy [A]** time = 35.5716, size = 121, normalized size = 0.97

$$\frac{2g(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}{5cde\sqrt{d+ex}} - \frac{2(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}(2ae^2g+3cd^2g-5cdef)}{15c^2d^2e(d+ex)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)**(1/2)/(e*x+d)**(1/2), x)$

[Out]  $2*g*(a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2))**(3/2)/(5*c*d*e*\text{sqrt}(d + e*x)) - 2*(a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2))**(3/2)*(2*a*e^2*g + 3*c*d^2*g - 5*c*d*e*f)/(15*c^2*d^2*e*(d + e*x)**(3/2))$

**Mathematica [A]** time = 0.068662, size = 54, normalized size = 0.43

$$\frac{2((d+ex)(ae+cdx))^{3/2}(cd(5f+3gx)-2aeg)}{15c^2d^2(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(f + g*x)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/\text{Sqrt}[d + e*x], x]$

[Out]  $(2*((a*e + c*d*x)*(d + e*x))^{3/2}*(-2*a*e*g + c*d*(5*f + 3*g*x)))/(15*c^2*d^2*(d + e*x)^{3/2})$

**Maple [A]** time = 0.008, size = 67, normalized size = 0.5

$$-\frac{(2cdx + 2ae)(-3xcdg + 2aeg - 5cdf)\sqrt{cdex^2 + ae^2x + cd^2x + ade}}{15c^2d^2} \frac{1}{\sqrt{ex + d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)/(e\*x+d)^(1/2), x)

[Out] -2/15\*(c\*d\*x+a\*e)\*(-3\*c\*d\*g\*x+2\*a\*e\*g-5\*c\*d\*f)\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)/c^2/d^2/(e\*x+d)^(1/2)

**Maxima [A]** time = 0.720249, size = 88, normalized size = 0.7

$$\frac{2(cdx + ae)^{\frac{3}{2}}f}{3cd} + \frac{2(3c^2d^2x^2 + acdex - 2a^2e^2)\sqrt{cdx + aeg}}{15c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)/sqrt(e\*x + d), x, a

[Out] 2/3\*(c\*d\*x + a\*e)^(3/2)\*f/(c\*d) + 2/15\*(3\*c^2\*d^2\*x^2 + a\*c\*d\*e\*x - 2\*a^2\*e^2)\*sqrt(c\*d\*x + a\*e)\*g/(c^2\*d^2)

**Fricas [A]** time = 0.270489, size = 293, normalized size = 2.34

$$\frac{2(3c^3d^3egx^4 + 5a^2cd^2e^2f - 2a^3de^3g + (5c^3d^3ef + (3c^3d^4 + 4ac^2d^2e^2)g)x^3 + (5(c^3d^4 + 2ac^2d^2e^2)f + (4ac^2d^3e - a^2cde^2)g)x^2 + (5c^3d^4 + 2ac^2d^2e^2)f + (4ac^2d^3e - a^2cde^2)g)x}{15\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{ex + d}dc^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)/sqrt(e\*x + d), x, a

[Out] 2/15\*(3\*c^3\*d^3\*e\*g\*x^4 + 5\*a^2\*c\*d^2\*e^2\*f - 2\*a^3\*d\*e^3\*g + (5\*c^3\*d^3\*e\*f + (3\*c^3\*d^4 + 4\*a\*c^2\*d^2\*e^2)\*g)\*x^3 + (5\*(c^3\*d^4 + 2\*a\*c^2\*d^2\*e^2)\*f + (4\*a\*c^2\*d^3\*e - a^2\*c\*d\*e^2)\*g)\*x^2 + (5\*(2\*a\*c^2\*d^3\*e + a^2\*c\*d\*e^3)\*f - (a^2\*c\*d^2\*e^2 + 2\*a^3\*e^4)\*g)\*x)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*c^2\*d^2)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(d + ex)(ae + cdx)}(f + gx)}{\sqrt{d + ex}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/(e\*x+d)\*\*(1/2), x

[Out] Integral(sqrt((d + e\*x)\*(a\*e + c\*d\*x))\*(f + g\*x)/sqrt(d + e\*x), x)

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)/sqrt(e*x + d), x, a`

[Out] Timed out

$$3.683 \quad \int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}} dx$$

**Optimal.** Leaf size=48

$$\frac{2(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{3cd(d + ex)^{3/2}}$$

[Out] (2\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2))/(3\*c\*d\*(d + e\*x)^(3/2))

**Rubi [A]** time = 0.0695717, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$

$$\frac{2(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{3cd(d + ex)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/Sqrt[d + e\*x], x]

[Out] (2\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2))/(3\*c\*d\*(d + e\*x)^(3/2))

**Rubi in Sympy [A]** time = 17.4176, size = 42, normalized size = 0.88

$$\frac{2(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{3}{2}}}{3cd(d + ex)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/(e\*x+d)\*\*(1/2), x)

[Out] 2\*(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*\*(3/2)/(3\*c\*d\*(d + e\*x)\*\*(3/2))

**Mathematica [A]** time = 0.0373404, size = 37, normalized size = 0.77

$$\frac{2((d + ex)(ae + cdex))^{3/2}}{3cd(d + ex)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/Sqrt[d + e\*x], x]

[Out] (2\*((a\*e + c\*d\*x)\*(d + e\*x))^(3/2))/(3\*c\*d\*(d + e\*x)^(3/2))

**Maple [A]** time = 0.003, size = 50, normalized size = 1.

$$\frac{2cdx + 2ae}{3cd} \sqrt{cdex^2 + ae^2x + cd^2x + ade} \frac{1}{\sqrt{ex + d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)`

[Out]  $2/3*(c*d*x+a*e)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)/c/d/(e*x+d)^(1/2)$

**Maxima [A]** time = 0.714996, size = 24, normalized size = 0.5

$$\frac{2(cdx + ae)^{\frac{3}{2}}}{3cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/sqrt(e*x + d),x, algorithm=`

[Out]  $2/3*(c*d*x + a*e)^(3/2)/(c*d)$

**Fricas [A]** time = 0.266347, size = 135, normalized size = 2.81

$$\frac{2(c^2d^2ex^3 + a^2de^2 + (c^2d^3 + 2acde^2)x^2 + (2acd^2e + a^2e^3)x)}{3\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{ex + dcd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/sqrt(e*x + d),x, algorithm=`

[Out]  $2/3*(c^2*d^2*e*x^3 + a^2*d*e^2 + (c^2*d^3 + 2*a*c*d*e^2)*x^2 + (2*a*c*d^2*e + a^2*e^3)*x)/(\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x}*\sqrt{e*x + d}*c*d)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{\sqrt{d+ex}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**(1/2),x)`

[Out] `Integral(sqrt((d + e*x)*(a*e + c*d*x))/sqrt(d + e*x), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/sqrt(e*x + d),x, algorithm=`

[Out] `integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/sqrt(e*x + d), x)`

$$3.684 \quad \int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)} dx$$

**Optimal.** Leaf size=124

$$\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}} - \frac{2\sqrt{cdf-ae^2g} \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-ae^2g}}\right)}{g^{3/2}}$$

[Out] (2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(g\*Sqrt[d + e\*x]) - (2\*Sqrt[c\*d\*f - a\*e\*g]\*ArcTan[(Sqrt[g]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(Sqrt[c\*d\*f - a\*e\*g]\*Sqrt[d + e\*x])])/g^(3/2)

**Rubi [A]** time = 0.594616, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}} - \frac{2\sqrt{cdf-ae^2g} \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-ae^2g}}\right)}{g^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(Sqrt[d + e\*x]\*(f + g\*x)), x]

[Out] (2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(g\*Sqrt[d + e\*x]) - (2\*Sqrt[c\*d\*f - a\*e\*g]\*ArcTan[(Sqrt[g]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(Sqrt[c\*d\*f - a\*e\*g]\*Sqrt[d + e\*x])])/g^(3/2)

**Rubi in Sympy [A]** time = 56.4437, size = 117, normalized size = 0.94

$$\frac{2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{g\sqrt{d+ex}} - \frac{2\sqrt{ae^2g-cdf} \operatorname{atanh}\left(\frac{\sqrt{g}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}\sqrt{ae^2g-cdf}}\right)}{g^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/(g\*x+f)/(e\*x+d)\*\*(1/2), x)

[Out] 2\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(g\*sqrt(d + e\*x)) - 2\*sqrt(a\*e\*g - c\*d\*f)\*atanh(sqrt(g)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(sqrt(d + e\*x)\*sqrt(a\*e\*g - c\*d\*f)))/g\*\*(3/2)

**Mathematica [A]** time = 0.132342, size = 114, normalized size = 0.92

$$\frac{2\sqrt{d+ex}\sqrt{ae+cdx}\left(\sqrt{g}\sqrt{ae+cdx}-\sqrt{ae^2g-cdf}\operatorname{tanh}^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{ae^2g-cdf}}\right)\right)}{g^{3/2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(Sqrt[d + e\*x]\*(f + g\*x)), x]

[Out]  $(2\sqrt{a^2e + c^2d^2x} \sqrt{d + ex} (\sqrt{g} \sqrt{a^2e + c^2d^2x} - \sqrt{-(c^2d^2f) + a^2e^2g} \operatorname{ArcTanh}(\frac{\sqrt{g} \sqrt{a^2e + c^2d^2x}}{\sqrt{-(c^2d^2f) + a^2e^2g}})) / (g^{3/2} \sqrt{(a^2e + c^2d^2x)(d + ex)})$

**Maple [A]** time = 0.026, size = 153, normalized size = 1.2

$$-2 \frac{\sqrt{cdex^2 + ae^2x + cd^2x + ade}}{\sqrt{ex + d}\sqrt{cdx + aeg}\sqrt{(aeg - cdf)g}} \left( \operatorname{Artanh} \left( \frac{g\sqrt{cdx + ae}}{\sqrt{(aeg - cdf)g}} \right) aeg - \operatorname{Artanh} \left( \frac{g\sqrt{cdx + ae}}{\sqrt{(aeg - cdf)g}} \right) cdf - \sqrt{cdx + ae}\sqrt{(aeg - cdf)g} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{int}((a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{1/2} / (g^2x + f) / (e^2x + d)^{1/2}, x)$

[Out]  $-2^*(c^2d^2e^2x^2 + a^2e^2x + c^2d^2x + a^2d^2e)^{1/2} * (\operatorname{arctanh}(g^2(c^2d^2x + a^2e)^{1/2} / ((a^2e^2g - c^2d^2f)g)^{1/2}) * a^2e^2g - \operatorname{arctanh}(g^2(c^2d^2x + a^2e)^{1/2} / ((a^2e^2g - c^2d^2f)g)^{1/2}) * c^2d^2f - (c^2d^2x + a^2e)^{1/2} * ((a^2e^2g - c^2d^2f)g)^{1/2}) / (e^2x + d)^{1/2} / (c^2d^2x + a^2e)^{1/2} / g / ((a^2e^2g - c^2d^2f)g)^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}(\sqrt{c^2d^2e^2x^2 + a^2d^2e + (c^2d^2 + a^2e^2)x} / (\sqrt{e^2x + d} (g^2x + f)), x)$

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.282393, size = 1, normalized size = 0.01

$$\frac{2cdex^2 + 2ade + \sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{ex + d} \sqrt{-\frac{cdf - aeg}{g}} \log\left(-\frac{cdex^2 - cd^2f + 2adeg - 2\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{ex + d} \sqrt{-\frac{cdf - aeg}{g}}}{egx^2 + df + (ef + dg)x}\right)}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{ex + d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{integrate}(\sqrt{c^2d^2e^2x^2 + a^2d^2e + (c^2d^2 + a^2e^2)x} / (\sqrt{e^2x + d} (g^2x + f)), x)$

[Out]  $[(2^2c^2d^2e^2x^2 + 2^2a^2d^2e + \sqrt{c^2d^2e^2x^2 + a^2d^2e + (c^2d^2 + a^2e^2)x}) * \sqrt{e^2x + d} * \sqrt{-(c^2d^2f - a^2e^2g)/g} * \log(-\frac{c^2d^2e^2g^2x^2 - c^2d^2f + 2^2a^2d^2e^2g - 2^2\sqrt{c^2d^2e^2x^2 + a^2d^2e + (c^2d^2 + a^2e^2)x} * \sqrt{e^2x + d} * g * \sqrt{-(c^2d^2f - a^2e^2g)/g}}{(e^2g^2x^2 + d^2f + (e^2f + d^2g)x)}) + 2^2(c^2d^2 + a^2e^2)x] / (\sqrt{c^2d^2e^2x^2 + a^2d^2e + (c^2d^2 + a^2e^2)x} * \sqrt{e^2x + d} * g), 2^2(c^2d^2e^2x^2 + a^2d^2e - \sqrt{c^2d^2e^2x^2 + a^2d^2e + (c^2d^2 + a^2e^2)x} * \sqrt{e^2x + d} * \sqrt{(c^2d^2f - a^2e^2g)/g}) * \operatorname{arctan}(-\frac{\sqrt{c^2d^2e^2x^2 + a^2d^2e + (c^2d^2 + a^2e^2)x} * (c^2d^2f - a^2e^2g) * \sqrt{e^2x + d}}{(c^2d^2e^2g^2x^2 + a^2d^2e^2g + (c^2d^2 + a^2e^2)g^2x) * \sqrt{(c^2d^2f - a^2e^2g)/g}})) + (c^2d^2 + a^2e^2)x] / (\sqrt{c^2d^2e^2x^2 + a^2d^2e + (c^2d^2 + a^2e^2)x} * \sqrt{e^2x + d} * g]$



**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{\sqrt{d+ex}(f+gx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/(g\*x+f)/(e\*x+d)\*\*(1/2), x

[Out] Integral(sqrt((d + e\*x)\*(a\*e + c\*d\*x))/(sqrt(d + e\*x)\*(f + g\*x)), x)

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(e\*x + d)\*(g\*x + f)), x

[Out] Timed out

$$3.685 \quad \int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^2} dx$$

**Optimal.** Leaf size=132

$$\frac{cd \tan^{-1} \left( \frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}} \right)}{g^{3/2}\sqrt{cdf-aeg}} - \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)}$$

[Out]  $-(\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*\text{Sqrt}[d + e*x]*(f + g*x))) + (c*d*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x]))/(g^{3/2}*\text{Sqrt}[c*d*f - a*e*g])$

**Rubi [A]** time = 0.546814, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{cd \tan^{-1} \left( \frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}} \right)}{g^{3/2}\sqrt{cdf-aeg}} - \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}(f+gx)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(\text{Sqrt}[d + e*x]*(f + g*x)^2), x]$

[Out]  $-(\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(g*\text{Sqrt}[d + e*x]*(f + g*x))) + (c*d*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x]))/(g^{3/2}*\text{Sqrt}[c*d*f - a*e*g])$

**Rubi in Sympy [A]** time = 56.2263, size = 124, normalized size = 0.94

$$\frac{cd \operatorname{atanh} \left( \frac{\sqrt{g}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}\sqrt{aeg-cdf}} \right)}{g^{3/2}\sqrt{aeg-cdf}} - \frac{\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{g\sqrt{d+ex}(f+gx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**2/(e*x+d)**2)$

[Out]  $-c*d*\operatorname{atanh}(\text{sqrt}(g)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/(\text{sqrt}(d + e*x)*\text{sqrt}(a*e*g - c*d*f))/(g**(3/2)*\text{sqrt}(a*e*g - c*d*f)) - \text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(g*\text{sqrt}(d + e*x)*(f + g*x))$

**Mathematica [A]** time = 0.22079, size = 111, normalized size = 0.84

$$\frac{\sqrt{(d+ex)(ae+cdx)} \left( -\frac{cd \tanh^{-1} \left( \frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{aeg-cdf}} \right)}{\sqrt{aeg-cdf}} - \frac{\sqrt{g}}{f+gx} \right)}{g^{3/2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(Sqrt[d + e\*x]\*(f + g\*x)^2)

[Out] (Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)]\*(-(Sqrt[g]/(f + g\*x)) - (c\*d\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/Sqrt[-(c\*d\*f) + a\*e\*g]])/(Sqrt[-(c\*d\*f) + a\*e\*g]\*Sqrt[a\*e + c\*d\*x])))/(g^(3/2)\*Sqrt[d + e\*x])

**Maple [A]** time = 0.032, size = 161, normalized size = 1.2

$$\frac{1}{g(gx+f)} \left( -\operatorname{Artanh} \left( g\sqrt{cdx+ae} \frac{1}{\sqrt{(aeg-cdf)g}} \right) xcdg - \operatorname{Artanh} \left( g\sqrt{cdx+ae} \frac{1}{\sqrt{(aeg-cdf)g}} \right) cdf - \sqrt{cdx+ae}\sqrt{(aeg-cdf)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)/(g\*x+f)^2/(e\*x+d)^(1/2),x)

[Out] (-arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x\*c\*d\*g-arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*c\*d\*f-(c\*d\*x+a\*e)^(1/2)\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)/(e\*x+d)^(1/2)/(c\*d\*x+a\*e)^(1/2)/g/(g\*x+f)/((a\*e\*g-c\*d\*f)\*g)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(e\*x + d)\*(g\*x + f)^2)

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.28509, size = 1, normalized size = 0.01

$$\frac{\left( cdegx^2 + cd^2f + (cdf + cd^2g)x \right) \log \left( -\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}(cdfg-ae^2)\sqrt{ex+d}+(cdegx^2-cd^2f+2adeg-(cdf-(cd^2+2ae^2)g)x)}{egx^2+df+(ef+dg)x}}{2(eg^2x^2 + dfg + (efg + dg^2)x)\sqrt{-cdfg + aeg^2}} \right) + \sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{cdfg}}{(eg^2x^2 + dfg + (efg + dg^2)x)\sqrt{cdfg - aeg^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(e\*x + d)\*(g\*x + f)^2)

[Out] [1/2\*((c\*d\*e\*g\*x^2 + c\*d^2\*f + (c\*d\*e\*f + c\*d^2\*g)\*x)\*log(-(2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(c\*d\*f\*g - a\*e\*g^2)\*sqrt(e\*x + d) + (c\*d\*e\*g\*x^2 - c\*d^2\*f + 2\*a\*d\*e\*g - (c\*d\*e\*f - (c\*d^2 + 2\*a\*e^2)\*g)\*x)\*sqrt(-c\*d\*f\*g + a\*e\*g^2))/(e\*g\*x^2 + d\*f + (e\*f + d\*g)\*x) - 2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*f\*g + a\*e\*g^2)\*sqrt(e\*x + d))/((e\*g^2\*x^2 + d\*f\*g + (e\*f\*g + d\*g^2)\*x)\*sqrt(-c\*d\*f\*g + a\*e\*g^2)), -((c\*d\*e\*g\*x^2 + c\*d^2\*f + (c\*d\*e\*f + c\*d^2\*g)\*x)\*arctan(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*f\*g - a\*e\*g^2)\*sqrt(e\*x + d)/(c\*d\*e\*g\*x^2 + a

$$d*e*g + (c*d^2 + a*e^2)*g*x)) + \text{sqrt}(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*\text{sqrt}(c*d*f*g - a*e*g^2)*\text{sqrt}(e*x + d))/((e*g^2*x^2 + d*f*g + (e*f*g + d*g^2)*x)*\text{sqrt}(c*d*f*g - a*e*g^2))]$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{\sqrt{d+ex}(f+gx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/(g\*x+f)\*\*2/(e\*x+d)\*\*(1/2

[Out] Integral(sqrt((d + e\*x)\*(a\*e + c\*d\*x))/(sqrt(d + e\*x)\*(f + g\*x)\*\*2), x)

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(e\*x + d)\*(g\*x + f)^2)

[Out] Timed out

$$3.686 \quad \int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^3} dx$$

**Optimal.** Leaf size=207

$$\frac{c^2 d^2 \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{3/2}(cdf-aeg)^{3/2}} + \frac{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)(cdf-aeg)} - \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}$$

[Out]  $-\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*g*\text{Sqrt}[d + e*x]*(f + g*x)^2) + (c*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g*(c*d*f - a*e*g)*\text{Sqrt}[d + e*x]*(f + g*x)) + (c^2*d^2*\text{ArcTan}[\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x])])/(4*g^{3/2}*(c*d*f - a*e*g)^{3/2})$

**Rubi [A]** time = 0.937682, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{c^2 d^2 \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{4g^{3/2}(cdf-aeg)^{3/2}} + \frac{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)(cdf-aeg)} - \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}(f+gx)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(\text{Sqrt}[d + e*x]*(f + g*x)^3), x]$

[Out]  $-\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(2*g*\text{Sqrt}[d + e*x]*(f + g*x)^2) + (c*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g*(c*d*f - a*e*g)*\text{Sqrt}[d + e*x]*(f + g*x)) + (c^2*d^2*\text{ArcTan}[\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x])])/(4*g^{3/2}*(c*d*f - a*e*g)^{3/2})$

**Rubi in Sympy [A]** time = 83.2094, size = 190, normalized size = 0.92

$$\frac{c^2 d^2 \operatorname{atanh}\left(\frac{\sqrt{g}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}\sqrt{aeg-cdf}}\right)}{4g^{3/2}(aeg-cdf)^{3/2}} - \frac{cd\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{4g\sqrt{d+ex}(f+gx)(aeg-cdf)} - \frac{\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{2g\sqrt{d+ex}(f+gx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**3/(e*x+d)**2)$

[Out]  $c**2*d**2*\operatorname{atanh}(\text{sqrt}(g)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/(\text{sqrt}(d + e*x)*\text{sqrt}(a*e*g - c*d*f))/(4*g**(3/2)*(a*e*g - c*d*f)**(3/2)) - c*d*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(4*g*\text{sqrt}(d + e*x)*(f + g*x)*(a*e*g - c*d*f)) - \text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(2*g*\text{sqrt}(d + e*x)*(f + g*x)**2)$

**Mathematica [A]** time = 0.387294, size = 164, normalized size = 0.79

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(c^2d^2(f+gx)^2 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{aeg-cdf}}\right) + \sqrt{g}\sqrt{ae+cdx}\sqrt{aeg-cdf}(cd(f-gx)-2aeg)\right)}{4g^{3/2}(f+gx)^2\sqrt{(d+ex)(ae+cdx)(aeg-cdf)^{3/2}}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(Sqrt[d + e\*x]\*(f + g\*x)^3)

[Out] (Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*(Sqrt[g]\*Sqrt[-(c\*d\*f) + a\*e\*g]\*  
Sqrt[a\*e + c\*d\*x]\*(-2\*a\*e\*g + c\*d\*(f - g\*x)) + c^2\*d^2\*(f + g\*x)^  
2\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/Sqrt[-(c\*d\*f) + a\*e\*g]]))/(  
4\*g^(3/2)\*(-(c\*d\*f) + a\*e\*g)^(3/2)\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)]\*  
(f + g\*x)^2)

**Maple [A]** time = 0.035, size = 285, normalized size = 1.4

$$\frac{1}{(4aeg - 4cdf)g(gx + f)^2} \sqrt{cdex^2 + ae^2x + cd^2x + ade} \left( \text{Artanh} \left( g\sqrt{cdx + ae} \frac{1}{\sqrt{(aeg - cdf)g}} \right) x^2 c^2 d^2 g^2 + 2 \text{Artanh} \left( \frac{g\sqrt{cdx + ae}}{\sqrt{(aeg - cdf)g}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)/(g\*x+f)^3/(e\*x+d)^(1/2), x)

[Out] 1/4\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)\*(arctanh(g\*(c\*d\*x+a\*e)  
)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x^2\*c^2\*d^2\*g^2+2\*arctanh(g\*(c\*d  
\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x\*c^2\*d^2\*f\*g+arctanh(g\*(c  
\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*c^2\*d^2\*f^2-((a\*e\*g-c\*d\*  
f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*x\*c\*d\*g-2\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(  
c\*d\*x+a\*e)^(1/2)\*a\*e\*g+((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*  
c\*d\*f/(e\*x+d)^(1/2)/(c\*d\*x+a\*e)^(1/2)/(a\*e\*g-c\*d\*f)/g/(g\*x+f)^2/  
((a\*e\*g-c\*d\*f)\*g)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(e\*x + d)\*(g\*x + f)^3)

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.28979, size = 1, normalized size = 0.

$$\left[ \frac{2 \sqrt{cdex^2 + ade + (cd^2 + ae^2)x} \sqrt{-cdfg + aeg^2} (cdgx - cdf + 2aeg) \sqrt{ex + d} - (c^2d^2eg^2x^3 + c^2d^3f^2 + (2c^2d^2efg + c^2d^3g^2))}{8(cd^2f^3g - adef^2g^2 + (cdfg^3 - ae^2g^4)x^3 + (2cdf^2g^2 - adeg^4 + (cd^2f^2g^2 + aeg^2)g))} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(e\*x + d)\*(g\*x + f)^3)

[Out] [1/8\*(2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*f\*g  
+ a\*e\*g^2)\*(c\*d\*g\*x - c\*d\*f + 2\*a\*e\*g)\*sqrt(e\*x + d) - (c^2\*d^2\*  
e\*g^2\*x^3 + c^2\*d^3\*f^2 + (2\*c^2\*d^2\*e\*f\*g + c^2\*d^3\*g^2)\*x^2 + (  
c^2\*d^2\*e\*f^2 + 2\*c^2\*d^3\*f\*g)\*x)\*log((2\*sqrt(c\*d\*e\*x^2 + a\*d\*e +  
(c\*d^2 + a\*e^2)\*x)\*(c\*d\*f\*g - a\*e\*g^2)\*sqrt(e\*x + d) - (c\*d\*e\*g  
x^2 - c\*d^2\*f + 2\*a\*d\*e\*g - (c\*d\*e\*f - (c\*d^2 + 2\*a\*e^2)\*g)\*x)\*sq  
rt(-c\*d\*f\*g + a\*e\*g^2))/(e\*g\*x^2 + d\*f + (e\*f + d\*g)\*x))/((c\*d^2  
\*f^3\*g - a\*d\*e\*f^2\*g^2 + (c\*d\*e\*f\*g^3 - a\*e^2\*g^4)\*x^3 + (2\*c\*d\*e  
\*f^2\*g^2 - a\*d\*e\*g^4 + (c\*d^2 - 2\*a\*e^2)\*f\*g^3)\*x^2 + (c\*d\*e\*f^3\*

$$g - 2*a*d*e*f*g^3 + (2*c*d^2 - a*e^2)*f^2*g^2)*x)*\sqrt{-c*d*f*g + a*e*g^2}), 1/4*(\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x})*\sqrt{(c*d*f*g - a*e*g^2)*(c*d*g*x - c*d*f + 2*a*e*g)*\sqrt{e*x + d} - (c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + (2*c^2*d^2*e*f*g + c^2*d^3*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*c^2*d^3*f*g)*x)*\arctan(\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x})*\sqrt{c*d*f*g - a*e*g^2)*\sqrt{e*x + d} / ((c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)))/((c*d^2*f^3*g - a*d*e*f^2*g^2 + (c*d*e*f*g^3 - a*e^2*g^4)*x^3 + (2*c*d*e*f^2*g^2 - a*d*e*g^4 + (c*d^2 - 2*a*e^2)*f*g^3)*x^2 + (c*d*e*f^3*g - 2*a*d*e*f*g^3 + (2*c*d^2 - a*e^2)*f^2*g^2)*x)*\sqrt{c*d*f*g - a*e*g^2})]$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/(g\*x+f)\*\*3/(e\*x+d)\*\*(1/2

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(e\*x + d)\*(g\*x + f)^3)

[Out] Timed out

$$3.687 \quad \int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^4} dx$$

**Optimal.** Leaf size=277

$$\frac{c^3 d^3 \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{3/2}(cdf-aeg)^{5/2}} + \frac{c^2 d^2 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{8g\sqrt{d+ex}(f+gx)(cdf-aeg)^2}$$

$$+ \frac{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{12g\sqrt{d+ex}(f+gx)^2(cdf-aeg)} - \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}$$

[Out]  $-\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(3*g*\text{Sqrt}[d + e*x]*(f + g*x)^3) + (c*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((12*g*(c*d*f - a*e*g)*\text{Sqrt}[d + e*x]*(f + g*x)^2) + (c^2*d^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*g*(c*d*f - a*e*g)^2*\text{Sqrt}[d + e*x]*(f + g*x))) + (c^3*d^3*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x]))/(8*g^(3/2)*(c*d*f - a*e*g)^(5/2))$

**Rubi [A]** time = 1.15375, antiderivative size = 277, normalized size of antiderivative = 1., number of rules used = 5, number of rules used = 4, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{c^3 d^3 \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{8g^{3/2}(cdf-aeg)^{5/2}} + \frac{c^2 d^2 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{8g\sqrt{d+ex}(f+gx)(cdf-aeg)^2}$$

$$+ \frac{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{12g\sqrt{d+ex}(f+gx)^2(cdf-aeg)} - \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3g\sqrt{d+ex}(f+gx)^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(\text{Sqrt}[d + e*x]*(f + g*x)^4), x]$

[Out]  $-\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(3*g*\text{Sqrt}[d + e*x]*(f + g*x)^3) + (c*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((12*g*(c*d*f - a*e*g)*\text{Sqrt}[d + e*x]*(f + g*x)^2) + (c^2*d^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*g*(c*d*f - a*e*g)^2*\text{Sqrt}[d + e*x]*(f + g*x))) + (c^3*d^3*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x]))/(8*g^(3/2)*(c*d*f - a*e*g)^(5/2))$

**Rubi in Sympy [A]** time = 116.688, size = 257, normalized size = 0.93

$$-\frac{c^3 d^3 \operatorname{atanh}\left(\frac{\sqrt{g}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}\sqrt{aeg-cdf}}\right)}{8g^{\frac{3}{2}}(aeg-cdf)^{\frac{5}{2}}} + \frac{c^2 d^2 \sqrt{ade+cdex^2+x(ae^2+cd^2)}}{8g\sqrt{d+ex}(f+gx)(aeg-cdf)^2}$$

$$-\frac{cd\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{12g\sqrt{d+ex}(f+gx)^2(aeg-cdf)} - \frac{\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{3g\sqrt{d+ex}(f+gx)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**4/(e*x+d)**$

[Out]  $-c**3*d**3*\operatorname{atanh}(\text{sqrt}(g)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/(\text{sqrt}(d + e*x)*\text{sqrt}(a*e*g - c*d*f)))/(8*g**(3/2)*(a*e*g - c*d*f)**(5/2)) + c**2*d**2*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(8*g*\text{sqrt}(d + e*x)*(f + g*x)*(a*e*g - c*d*f)**2) - c*d*s$



$$\frac{\sqrt{d+ex} \sqrt{ae+cdx} \left( \frac{3c^2d^2(f+gx)^2 + 2cd(f+gx) - 8}{3g(f+gx)^3} - \frac{c^3d^3 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{aeg-cdf}}\right)}{g^{3/2}\sqrt{ae+cdx}(aeg-cdf)^{5/2}} \right)}{8\sqrt{d+ex}}$$

**Mathematica [A]** time = 0.703858, size = 168, normalized size = 0.61

$$\frac{\sqrt{(d+ex)(ae+cdx)} \left( \frac{3c^2d^2(f+gx)^2 + 2cd(f+gx) - 8}{3g(f+gx)^3} - \frac{c^3d^3 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{aeg-cdf}}\right)}{g^{3/2}\sqrt{ae+cdx}(aeg-cdf)^{5/2}} \right)}{8\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(Sqrt[d + e\*x]\*(f + g\*x)^4)

[Out] (Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)]\*(-8 + (2\*c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g) + (3\*c^2\*d^2\*(f + g\*x)^2)/(c\*d\*f - a\*e\*g)^2)/(3\*g\*(f + g\*x)^3) - (c^3\*d^3\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/Sqrt[-(c\*d\*f) + a\*e\*g]])/(g^(3/2)\*(-(c\*d\*f) + a\*e\*g)^(5/2)\*Sqrt[a\*e + c\*d\*x])/(8\*Sqrt[d + e\*x])

**Maple [A]** time = 0.042, size = 453, normalized size = 1.6

$$-\frac{1}{24(gx+f)^3 g(aeg-cdf)^2} \sqrt{cdex^2 + ae^2x + cd^2x + ade} \left( 3 \operatorname{Artanh} \left( \frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}} \right) x^3 c^3 d^3 g^3 + 9 \operatorname{Artanh} \left( \frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)/(g\*x+f)^4/(e\*x+d)^(1/2),x)

[Out] -1/24\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)\*(3\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x^3\*c^3\*d^3\*g^3+9\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x^2\*c^3\*d^3\*f\*g^2+9\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x\*c^3\*d^3\*f^2\*g+3\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*c^3\*d^3\*f^3-3\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*x^2\*c^2\*d^2\*g^2+2\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*x\*a\*c\*d\*e\*g^2-8\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*x\*c^2\*d^2\*f\*g+8\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*a^2\*e^2\*g^2-14\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*a\*c\*d\*e\*f\*g+3\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*c^2\*d^2\*f^2/(e\*x+d)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2)/(g\*x+f)^3/g/(a\*e\*g-c\*d\*f)^2/(c\*d\*x+a\*e)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(e\*x + d)\*(g\*x + f)^4)

[Out] Exception raised: ValueError

**Ericas [A]** time = 0.302511, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x + f)^4)`

[Out] 
$$\left[ \frac{1}{48} \left( 2 \left( 3c^2d^2g^2x^2 - 3c^2d^2f^2 + 14acd^2efg - 8a^2e^2g^2 + 2(4c^2d^2f^2g - acd^2eg^2)x \right) \sqrt{c^2d^2e^2x^2 + a^2d^2e + (c^2d^2 + a^2e^2)x} \sqrt{-c^2d^2fg + a^2eg^2} \sqrt{ex + d} + 3(c^3d^3e^2g^3x^4 + c^3d^4f^3 + (3c^3d^3e^2fg^2 + c^3d^4g^3)x^3 + 3(c^3d^3e^2f^2g + c^3d^4f^2g^2)x^2 + (c^3d^3e^2f^3 + 3c^3d^4f^2g)x \right) \log\left(-2\sqrt{c^2d^2e^2x^2 + a^2d^2e + (c^2d^2 + a^2e^2)x} (c^2d^2fg - a^2eg^2) \sqrt{ex + d} + (c^2d^2e^2g^2x^2 - c^2d^2f^2 + 2a^2d^2eg - (c^2d^2ef - (c^2d^2 + 2a^2e^2)g)x) \sqrt{-c^2d^2fg + a^2eg^2}\right) / (e^2g^2x^2 + d^2f + (ef + dg)x) \right) / \left( (c^2d^3f^5g - 2a^2c^2d^2e^2f^4g^2 + a^2d^2e^2f^3g^3 + (c^2d^2e^2f^2g^4 - 2a^2c^2d^2e^2f^2g^5 + a^2e^3g^6)x^4 + (3c^2d^2e^2f^3g^3 + a^2d^2e^2g^6 + (c^2d^3 - 6a^2c^2d^2e^2)f^2g^4 - (2a^2c^2d^2e - 3a^2e^3)f^2g^5)x^3 + 3(c^2d^2e^2f^4g^2 + a^2d^2e^2f^2g^5 + (c^2d^3 - 2a^2c^2d^2e^2)f^3g^3 - (2a^2c^2d^2e - a^2e^3)f^2g^4)x^2 + (c^2d^2e^2f^5g + 3a^2d^2e^2f^2g^4 + (3c^2d^3 - 2a^2c^2d^2e^2)f^4g^2 - (6a^2c^2d^2e - a^2e^3)f^3g^3)x \right) \sqrt{-c^2d^2fg + a^2eg^2} \right), \frac{1}{24} \left( (3c^2d^2g^2x^2 - 3c^2d^2f^2 + 14acd^2efg - 8a^2e^2g^2 + 2(4c^2d^2f^2g - acd^2eg^2)x) \sqrt{c^2d^2e^2x^2 + a^2d^2e + (c^2d^2 + a^2e^2)x} \sqrt{c^2d^2fg - a^2eg^2} \sqrt{ex + d} - 3(c^3d^3e^2g^3x^4 + c^3d^4f^3 + (3c^3d^3e^2fg^2 + c^3d^4g^3)x^3 + 3(c^3d^3e^2f^2g + c^3d^4f^2g^2)x^2 + (c^3d^3e^2f^3 + 3c^3d^4f^2g)x) \arctan\left(\sqrt{c^2d^2e^2x^2 + a^2d^2e + (c^2d^2 + a^2e^2)x} \sqrt{c^2d^2fg - a^2eg^2} \sqrt{ex + d} / (c^2d^2e^2g^2x^2 + a^2d^2eg + (c^2d^2 + a^2e^2)gx)\right) / \left( (c^2d^3f^5g - 2a^2c^2d^2e^2f^4g^2 + a^2d^2e^2f^3g^3 + (c^2d^2e^2f^2g^4 - 2a^2c^2d^2e^2f^2g^5 + a^2e^3g^6)x^4 + (3c^2d^2e^2f^3g^3 + a^2d^2e^2g^6 + (c^2d^3 - 6a^2c^2d^2e^2)f^2g^4 - (2a^2c^2d^2e - 3a^2e^3)f^2g^5)x^3 + 3(c^2d^2e^2f^4g^2 + a^2d^2e^2f^2g^5 + (c^2d^3 - 2a^2c^2d^2e^2)f^3g^3 - (2a^2c^2d^2e - a^2e^3)f^2g^4)x^2 + (c^2d^2e^2f^5g + 3a^2d^2e^2f^2g^4 + (3c^2d^3 - 2a^2c^2d^2e^2)f^4g^2 - (6a^2c^2d^2e - a^2e^3)f^3g^3)x \right) \sqrt{c^2d^2fg - a^2eg^2} \right) \right]$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**4/(e*x+d)**(1/2)`

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x + f)^4)`

[Out] Timed out

$$3.688 \quad \int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^5} dx$$

**Optimal.** Leaf size=347

$$\frac{5c^4 d^4 \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{64g^{3/2}(cdf-aeg)^{7/2}} + \frac{5c^3 d^3 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{64g\sqrt{d+ex}(f+gx)(cdf-aeg)^3} + \frac{5c^2 d^2 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{96g\sqrt{d+ex}(f+gx)^2(cdf-aeg)^2} + \frac{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{24g\sqrt{d+ex}(f+gx)^3(cdf-aeg)} - \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}$$

[Out]  $-\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(4*g*\text{Sqrt}[d + e*x]^*(f + g*x)^4) + (c*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(24*g*(c*d*f - a*e*g)*\text{Sqrt}[d + e*x]^*(f + g*x)^3) + (5*c^2*d^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(96*g*(c*d*f - a*e*g)^2*\text{Sqrt}[d + e*x]^*(f + g*x)^2) + (5*c^3*d^3*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*g*(c*d*f - a*e*g)^3*\text{Sqrt}[d + e*x]^*(f + g*x)) + (5*c^4*d^4*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x]))/(64*g^(3/2)*(c*d*f - a*e*g)^(7/2))$

**Rubi [A]** time = 1.60925, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{5c^4 d^4 \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{64g^{3/2}(cdf-aeg)^{7/2}} + \frac{5c^3 d^3 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{64g\sqrt{d+ex}(f+gx)(cdf-aeg)^3} + \frac{5c^2 d^2 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{96g\sqrt{d+ex}(f+gx)^2(cdf-aeg)^2} + \frac{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{24g\sqrt{d+ex}(f+gx)^3(cdf-aeg)} - \frac{\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4g\sqrt{d+ex}(f+gx)^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(\text{Sqrt}[d + e*x]^*(f + g*x)^5), x]$

[Out]  $-\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(4*g*\text{Sqrt}[d + e*x]^*(f + g*x)^4) + (c*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(24*g*(c*d*f - a*e*g)*\text{Sqrt}[d + e*x]^*(f + g*x)^3) + (5*c^2*d^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(96*g*(c*d*f - a*e*g)^2*\text{Sqrt}[d + e*x]^*(f + g*x)^2) + (5*c^3*d^3*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*g*(c*d*f - a*e*g)^3*\text{Sqrt}[d + e*x]^*(f + g*x)) + (5*c^4*d^4*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x]))/(64*g^(3/2)*(c*d*f - a*e*g)^(7/2))$

**Rubi in Sympy [A]** time = 152.208, size = 328, normalized size = 0.95

$$\frac{5c^4 d^4 \operatorname{atanh}\left(\frac{\sqrt{g}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}\sqrt{aeg-cdf}}\right)}{64g^{\frac{3}{2}}(aeg-cdf)^{\frac{7}{2}}} - \frac{5c^3 d^3 \sqrt{ade+cdex^2+x(ae^2+cd^2)}}{64g\sqrt{d+ex}(f+gx)(aeg-cdf)^3} + \frac{5c^2 d^2 \sqrt{ade+cdex^2+x(ae^2+cd^2)}}{96g\sqrt{d+ex}(f+gx)^2(aeg-cdf)^2} - \frac{cd\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{24g\sqrt{d+ex}(f+gx)^3(aeg-cdf)} - \frac{\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{4g\sqrt{d+ex}(f+gx)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**5/(e*x+d)**`

[Out]  $5*c**4*d**4*atanh(\sqrt{g}*\sqrt{a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)})/(\sqrt{d + e*x}*\sqrt{a*e*g - c*d*f}))/ (64*g**(3/2)*(a*e*g - c*d*f)**(7/2)) - 5*c**3*d**3*\sqrt{a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)}/(64*g*\sqrt{d + e*x}*(f + g*x)*(a*e*g - c*d*f)**3) + 5*c**2*d**2*\sqrt{a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)}/(96*g*\sqrt{d + e*x}*(f + g*x)**2*(a*e*g - c*d*f)**2) - c*d*\sqrt{a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)}/(24*g*\sqrt{d + e*x}*(f + g*x)**3*(a*e*g - c*d*f)) - \sqrt{a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)}/(4*g*\sqrt{d + e*x}*(f + g*x)**4)$

**Mathematica [A]** time = 1.04662, size = 195, normalized size = 0.56

$$\frac{\sqrt{(d+ex)(ae+cdx)} \left( \frac{5c^4 d^4 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{aeg-cdf}}\right)}{g^{3/2}\sqrt{ae+cdx}(aeg-cdf)^{7/2}} + \frac{15c^3 d^3 (f+gx)^3}{(cdf-aeg)^3} + \frac{10c^2 d^2 (f+gx)^2}{(cdf-aeg)^2} + \frac{8cdf+gx}{cdf-aeg} - 48 \right)}{64\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(Sqrt[d + e*x]*(f + g*x)^5)`

[Out]  $(\sqrt{(a*e + c*d*x)*(d + e*x)}*((-48 + (8*c*d*(f + g*x))/(c*d*f - a*e*g) + (10*c^2*d^2*(f + g*x)^2)/(c*d*f - a*e*g)^2 + (15*c^3*d^3*(f + g*x)^3)/(c*d*f - a*e*g)^3)/(3*g*(f + g*x)^4) + (5*c^4*d^4*ArcTanh[(\sqrt{g}*\sqrt{a*e + c*d*x})/\sqrt{-(c*d*f) + a*e*g}]))/(g^{3/2}*(-(c*d*f) + a*e*g)^{7/2}*\sqrt{a*e + c*d*x}))/ (64*\sqrt{d + e*x})$

**Maple [B]** time = 0.041, size = 696, normalized size = 2.

$$\frac{1}{192(gx+f)^4 g(aeg-cdf)(a^2e^2g^2-2acdefg+c^2d^2f^2)} \sqrt{cdex^2+ae^2x+cd^2x+ade} \left( 15 \operatorname{Arctanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right) \right) x^4 c^4 d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(g*x+f)^5/(e*x+d)^(1/2),x)`

[Out]  $1/192*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)))*x^4*c^4*d^4*g^4+60*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*x^3*c^4*d^4*f*g^3+90*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*x^2*c^4*d^4*f^2*g^2+60*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*x*c^4*d^4*f^3*g-15*x^3*c^3*d^3*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^4*d^4*f^4+10*x^2*a*c^2*d^2*e*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-55*x^2*c^3*d^3*f*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-8*x*a^2*c*d*e^2*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+36*x*a*c^2*d^2*e*f*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-73*x*c^3*d^3*f^2*g*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-48*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^3*e^3*g^3+136*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c*d*e^2*f*g^2-118*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a*c^2*d^2*e*f^2*g+15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^3*d^3*f^3)/(e*x+d)^(1/2)/((a*e*g-c*d*f)*g)^(1/2)/(g*x+f)^4/g/(a*e*g-c*d*f)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(c*d*x+a*e)^(1/2)$

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**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(e\*x + d)\*(g\*x + f)^5)

[Out] Exception raised: ValueError

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**Fricas [A]** time = 0.313657, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(e\*x + d)\*(g\*x + f)^5)

[Out] 
$$\frac{1}{384} \left( 2 \left( 15c^3d^3g^3x^3 - 15c^3d^3f^3 + 118a^2c^2d^2e^2f^2g - 136a^2c^2d^2e^2fg^2 + 48a^3e^3g^3 + 5(11c^3d^3f^2g^2 - 2a^2c^2d^2e^2fg^3) \right) x^2 + (73c^3d^3f^2g - 36a^2c^2d^2e^2fg^2 + 8a^2c^2d^2e^2g^3) x \right) \sqrt{c^2d^2e^2x^2 + a^2d^2e^2 + (c^2d^2 + a^2e^2)x} \sqrt{-c^2d^2fg + a^2e^2g^2} \sqrt{ex + d} - 15(c^4d^4e^2g^4x^5 + c^4d^4e^2f^4 + (4c^4d^4e^2fg^3 + c^4d^4e^2g^4)x^4 + 2(3c^4d^4e^2f^2g^2 + 2c^4d^4e^2fg^3)x^3 + 2(2c^4d^4e^2f^3g + 3c^4d^4e^2f^2g^2)x^2 + (c^4d^4e^2f^4 + 4c^4d^4e^2fg^3)x) \log\left(\frac{2\sqrt{c^2d^2e^2x^2 + a^2d^2e^2 + (c^2d^2 + a^2e^2)x} \sqrt{c^2d^2fg - a^2e^2g^2} \sqrt{ex + d} - (c^2d^2e^2gx^2 - c^2d^2f + 2a^2d^2e^2g - (c^2d^2e^2f - (c^2d^2 + 2a^2e^2)g)x) \sqrt{-c^2d^2fg + a^2e^2g^2}}{(c^3d^4f^7g - 3a^2c^2d^3e^2f^6g^2 + 3a^2c^2d^2e^2f^5g^3 - a^3d^2e^3f^4g^4 + (c^3d^3e^2f^3g^5 - 3a^2c^2d^2e^2f^2g^6 + 3a^2c^2d^2e^2fg^7 - a^3e^4g^8)x^5 + (4c^3d^3e^2f^4g^4 - a^3d^2e^3g^8 + (c^3d^4 - 12a^2c^2d^2e^2)f^3g^5 - 3(a^2c^2d^3e - 4a^2c^2d^3e)f^2g^6 + (3a^2c^2d^2e^2 - 4a^3e^4)f^2g^7)x^4 + 2(3c^3d^3e^2f^5g^3 - 2a^3d^2e^3fg^7 + (2c^3d^4 - 9a^2c^2d^2e^2)f^4g^4 - 3(2a^2c^2d^3e - 3a^2c^2d^3e)f^3g^5 + 3(2a^2c^2d^2e^2 - a^3e^4)f^2g^6)x^3 + 2(2c^3d^3e^2f^6g^2 - 3a^3d^2e^3f^2g^6 + 3(c^3d^4 - 2a^2c^2d^2e^2)f^5g^3 - 3(3a^2c^2d^3e - 2a^2c^2d^3e)f^4g^4 + (9a^2c^2d^2e^2 - 2a^3e^4)f^3g^5)x^2 + (c^3d^3e^2f^7g - 4a^3d^2e^3f^3g^5 + (4c^3d^4 - 3a^2c^2d^2e^2)f^6g^2 - 3(4a^2c^2d^3e - a^2c^2d^3e)f^5g^3 + (12a^2c^2d^2e^2 - a^3e^4)f^4g^4)x) \sqrt{-c^2d^2fg + a^2e^2g^2} \right), \frac{1}{19} 2 \left( (15c^3d^3g^3x^3 - 15c^3d^3f^3 + 118a^2c^2d^2e^2f^2g - 136a^2c^2d^2e^2fg^2 + 48a^3e^3g^3 + 5(11c^3d^3f^2g^2 - 2a^2c^2d^2e^2fg^3) \right) x^2 + (73c^3d^3f^2g - 36a^2c^2d^2e^2fg^2 + 8a^2c^2d^2e^2g^3) x \right) \sqrt{c^2d^2e^2x^2 + a^2d^2e^2 + (c^2d^2 + a^2e^2)x} \sqrt{c^2d^2fg - a^2e^2g^2} \sqrt{ex + d} - 15(c^4d^4e^2g^4x^5 + c^4d^4e^2f^4 + (4c^4d^4e^2fg^3 + c^4d^4e^2g^4)x^4 + 2(3c^4d^4e^2f^2g^2 + 2c^4d^4e^2fg^3)x^3 + 2(2c^4d^4e^2f^3g + 3c^4d^4e^2f^2g^2)x^2 + (c^4d^4e^2f^4 + 4c^4d^4e^2fg^3)x) \arctan\left(\frac{\sqrt{c^2d^2e^2x^2 + a^2d^2e^2 + (c^2d^2 + a^2e^2)x} \sqrt{c^2d^2fg - a^2e^2g^2} \sqrt{ex + d}}{(c^2d^2e^2gx^2 + a^2d^2e^2g + (c^2d^2 + a^2e^2)gx)}\right) \right) / \left( (c^3d^4f^7g - 3a^2c^2d^3e^2f^6g^2 + 3a^2c^2d^2e^2f^5g^3 - a^3d^2e^3f^4g^4 + (c^3d^3e^2f^3g^5 - 3a^2c^2d^2e^2f^2g^6 + 3a^2c^2d^2e^2fg^7 - a^3e^4g^8)x^5 + (4c^3d^3e^2f^4g^4 - a^3d^2e^3g^8 + (c^3d^4 - 12a^2c^2d^2e^2)f^3g^5 - 3(a^2c^2d^3e - 4a^2c^2d^3e)f^2g^6 + (3a^2c^2d^2e^2 - 4a^3e^4)f^2g^7)x^4 + 2(3c^3d^3e^2f^5g^3 - 2a^3d^2e^3fg^7 + (2c^3d^4 - 9a^2c^2d^2e^2)f^4g^4 - 3(2a^2c^2d^3e - 3a^2c^2d^3e)f^3g^5 + 3(2a^2c^2d^2e^2 - a^3e^4)f^2g^6)x^3 + 2(2c^3d^3e^2f^6g^2 - 3a^3d^2e^3f^2g^6 + 3(c^3d^4 - 2a^2c^2d^2e^2)f^5g^3 - 3(3a^2c^2d^3e - 2a^2c^2d^3e)f^4g^4 + (9a^2c^2d^2e^2 - 2a^3e^4)f^3g^5)x^2 + (c^3d^3e^2f^7g - 4a^3d^2e^3f^3g^5 - 4a^3d^2e^3f^3g^5) \right)$$

$$d^3 e^3 f^3 g^5 + (4c^3 d^4 - 3a^2 c^2 d^2 e^2) f^6 g^2 - 3(4a^2 c^2 d^3 e - a^2 c^2 d^3 e^3) f^5 g^3 + (12a^2 c^2 d^2 e^2 - a^3 e^4) f^4 g^4 x \sqrt{c d f g - a e g^2}]$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/(g\*x+f)\*\*5/(e\*x+d)\*\*(1/2

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(e\*x + d)\*(g\*x + f)^5)

[Out] Timed out

$$3.689 \quad \int \frac{(f+gx)^4 (ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}} dx$$

**Optimal.** Leaf size=336

$$\begin{aligned} & \frac{128 (x (ae^2 + cd^2) + ade + cdex^2)^{5/2} (cdf - aeg)^3 (2ae^2g - cd(7ef - 5dg))}{15015c^5d^5e(d+ex)^{5/2}} \\ & + \frac{128g (x (ae^2 + cd^2) + ade + cdex^2)^{5/2} (cdf - aeg)^3}{3003c^4d^4e(d+ex)^{3/2}} \\ & + \frac{32(f+gx)^2 (x (ae^2 + cd^2) + ade + cdex^2)^{5/2} (cdf - aeg)^2}{429c^3d^3(d+ex)^{5/2}} \\ & + \frac{16(f+gx)^3 (x (ae^2 + cd^2) + ade + cdex^2)^{5/2} (cdf - aeg)}{143c^2d^2(d+ex)^{5/2}} \\ & + \frac{2(f+gx)^4 (x (ae^2 + cd^2) + ade + cdex^2)^{5/2}}{13cd(d+ex)^{5/2}} \end{aligned}$$

[Out]  $(-128*(c*d*f - a*e*g)^3*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}/(15015*c^5*d^5*e*(d + e*x)^{5/2}) + (128*g*(c*d*f - a*e*g)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}/(3003*c^4*d^4*e*(d + e*x)^{3/2}) + (32*(c*d*f - a*e*g)^2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}/(429*c^3*d^3*(d + e*x)^{5/2}) + (16*(c*d*f - a*e*g)*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}/(143*c^2*d^2*(d + e*x)^{5/2}) + (2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}/(13*c*d*(d + e*x)^{5/2})$

**Rubi [A]** time = 1.61184, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\begin{aligned} & \frac{128 (x (ae^2 + cd^2) + ade + cdex^2)^{5/2} (cdf - aeg)^3 (2ae^2g - cd(7ef - 5dg))}{15015c^5d^5e(d+ex)^{5/2}} \\ & + \frac{128g (x (ae^2 + cd^2) + ade + cdex^2)^{5/2} (cdf - aeg)^3}{3003c^4d^4e(d+ex)^{3/2}} \\ & + \frac{32(f+gx)^2 (x (ae^2 + cd^2) + ade + cdex^2)^{5/2} (cdf - aeg)^2}{429c^3d^3(d+ex)^{5/2}} \\ & + \frac{16(f+gx)^3 (x (ae^2 + cd^2) + ade + cdex^2)^{5/2} (cdf - aeg)}{143c^2d^2(d+ex)^{5/2}} \\ & + \frac{2(f+gx)^4 (x (ae^2 + cd^2) + ade + cdex^2)^{5/2}}{13cd(d+ex)^{5/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{3/2}/(d + e*x)^{3/2}, x]$

[Out]  $(-128*(c*d*f - a*e*g)^3*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}/(15015*c^5*d^5*e*(d + e*x)^{5/2}) + (128*g*(c*d*f - a*e*g)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}/(3003*c^4*d^4*e*(d + e*x)^{3/2}) + (32*(c*d*f - a*e*g)^2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}/(429*c^3*d^3*(d + e*x)^{5/2}) + (16*(c*d*f - a*e*g)*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}/(143*c^2*d^2*(d + e*x)^{5/2}) + (2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}/(13*c*d*(d + e*x)^{5/2})$

**Rubi in Sympy [A]** time = 129.65, size = 330, normalized size = 0.98

$$\frac{2(f+gx)^4 (ade + cdex^2 + x(ae^2 + cd^2))^{\frac{5}{2}}}{13cd(d+ex)^{\frac{5}{2}}} - \frac{16(f+gx)^3 (aeg - cdf) (ade + cdex^2 + x(ae^2 + cd^2))^{\frac{5}{2}}}{143c^2d^2(d+ex)^{\frac{5}{2}}} + \frac{32(f+gx)^2 (aeg - cdf)^2 (ade + cdex^2 + x(ae^2 + cd^2))^{\frac{5}{2}}}{429c^3d^3(d+ex)^{\frac{5}{2}}} - \frac{128g(aeg - cdf)^3 (ade + cdex^2 + x(ae^2 + cd^2))^{\frac{5}{2}}}{3003c^4d^4e(d+ex)^{\frac{3}{2}}} + \frac{128(aeg - cdf)^3 (ade + cdex^2 + x(ae^2 + cd^2))^{\frac{5}{2}} (2ae^2g + 5cd^2g - 7cdfef)}{15015c^5d^5e(d+ex)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x+f)**4*(a*d*e+(a**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**`

[Out] `2*(f + g*x)**4*(a*d*e + c*d*e*x**2 + x*(a**2 + c*d**2))**(5/2)/(13*c*d*(d + e*x)**(5/2)) - 16*(f + g*x)**3*(a*e*g - c*d*f)*(a*d*e + c*d*e*x**2 + x*(a**2 + c*d**2))**(5/2)/(143*c**2*d**2*(d + e*x)**(5/2)) + 32*(f + g*x)**2*(a*e*g - c*d*f)**2*(a*d*e + c*d*e*x**2 + x*(a**2 + c*d**2))**(5/2)/(429*c**3*d**3*(d + e*x)**(5/2)) - 128*g*(a*e*g - c*d*f)**3*(a*d*e + c*d*e*x**2 + x*(a**2 + c*d**2))**(5/2)/(3003*c**4*d**4*e*(d + e*x)**(3/2)) + 128*(a*e*g - c*d*f)**3*(a*d*e + c*d*e*x**2 + x*(a**2 + c*d**2))**(5/2)*(2*a**2*g + 5*c*d**2*g - 7*c*d*e*f)/(15015*c**5*d**5*e*(d + e*x)**(5/2))`

**Mathematica [A]** time = 0.334669, size = 195, normalized size = 0.58

$$\frac{2((d+ex)(ae+cdx))^{5/2} (128a^4e^4g^4 - 64a^3cde^3g^3(13f+5gx) + 16a^2c^2d^2e^2g^2(143f^2+130fgx+35g^2x^2) - 8ac^3d^3eg(429f^2+130fgx+35g^2x^2) + 10010c^4d^4f^2g^2 - 5460c^4d^4fg^3x^3 + 1155c^4d^4g^4x^4) + 128a^2c^2d^2e^2g^2(143f^2+130fgx+35g^2x^2) - 8ac^3d^3eg(429f^2+130fgx+35g^2x^2) + 10010c^4d^4f^2g^2 - 5460c^4d^4fg^3x^3 + 1155c^4d^4g^4x^4}{15015c^5d^5e(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] `Integrate[((f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)^(`

[Out] `(2*((a*e + c*d*x)*(d + e*x))^(5/2)*(128*a^4*e^4*g^4 - 64*a^3*c*d*e^3*g^3*(13*f + 5*g*x) + 16*a^2*c^2*d^2*e^2*g^2*(143*f^2 + 130*f*g*x + 35*g^2*x^2) - 8*a*c^3*d^3*e*g*(429*f^2 + 130*f*g*x + 35*g^2*x^2) + 105*g^3*x^3) + c^4*d^4*(3003*f^4 + 8580*f^3*g*x + 10010*f^2*g^2*x^2 + 5460*f*g^3*x^3 + 1155*g^4*x^4))/(15015*c^5*d^5*(d + e*x)^(5/2))`

**Maple [A]** time = 0.013, size = 283, normalized size = 0.8

$$\frac{(2cdx + 2ae)(1155g^4x^4c^4d^4 - 840ac^3d^3eg^4x^3 + 5460c^4d^4fg^3x^3 + 560a^2c^2d^2e^2g^4x^2 - 3640ac^3d^3efg^3x^2 + 10010c^4d^4f^2g^2 - 5460c^4d^4fg^3x^3 + 1155c^4d^4g^4x^4)}{15015c^5d^5e(d+ex)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^4*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2), x)`

[Out] `2/15015*(c*d*x+a*e)*(1155*c^4*d^4*g^4*x^4-840*a*c^3*d^3*e*g^4*x^3+5460*c^4*d^4*f*g^3*x^3+560*a^2*c^2*d^2*e^2*g^4*x^2-3640*a*c^3*d^3`



$$3 * e * f * g^3 * x^2 + 10010 * c^4 * d^4 * f^2 * g^2 * x^2 - 320 * a^3 * c * d * e^3 * g^4 * x + 2080 * a^2 * c^2 * d^2 * e^2 * f * g^3 * x - 5720 * a * c^3 * d^3 * e * f^2 * g^2 * x + 8580 * c^4 * d^4 * f^3 * g * x + 128 * a^4 * e^4 * g^4 - 832 * a^3 * c * d * e^3 * f * g^3 + 2288 * a^2 * c^2 * d^2 * e^2 * f^2 * g^2 - 3432 * a * c^3 * d^3 * e * f^3 * g + 3003 * c^4 * d^4 * f^4) * (c * d * e * x^2 + a * e^2 * x + c * d^2 * x + a * d * e)^{(3/2)} / c^5 / d^5 / (e * x + d)^{(3/2)}$$

**Maxima [A]** time = 0.767313, size = 558, normalized size = 1.66

$$\frac{2(c^2 d^2 x^2 + 2 a c d e x + a^2 e^2) \sqrt{c d x + a e} f^4}{5 c d} + \frac{8(5 c^3 d^3 x^3 + 8 a c^2 d^2 e x^2 + a^2 c d e^2 x - 2 a^3 e^3) \sqrt{c d x + a e} f^3 g}{35 c^2 d^2}$$

$$+ \frac{4(35 c^4 d^4 x^4 + 50 a c^3 d^3 e x^3 + 3 a^2 c^2 d^2 e^2 x^2 - 4 a^3 c d e^3 x + 8 a^4 e^4) \sqrt{c d x + a e} f^2 g^2}{105 c^3 d^3}$$

$$+ \frac{8(105 c^5 d^5 x^5 + 140 a c^4 d^4 e x^4 + 5 a^2 c^3 d^3 e^2 x^3 - 6 a^3 c^2 d^2 e^3 x^2 + 8 a^4 c d e^4 x - 16 a^5 e^5) \sqrt{c d x + a e} f g^3}{1155 c^4 d^4}$$

$$+ \frac{2(1155 c^6 d^6 x^6 + 1470 a c^5 d^5 e x^5 + 35 a^2 c^4 d^4 e^2 x^4 - 40 a^3 c^3 d^3 e^3 x^3 + 48 a^4 c^2 d^2 e^4 x^2 - 64 a^5 c d e^5 x + 128 a^6 e^6) \sqrt{c d x + a e} g^4}{15015 c^5 d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f)^4/(e\*x + d)^(3/2))

[Out]  $\frac{2}{5} * (c^2 * d^2 * x^2 + 2 * a * c * d * e * x + a^2 * e^2) * \text{sqrt}(c * d * x + a * e) * f^4 / (c * d) + \frac{8}{35} * (5 * c^3 * d^3 * x^3 + 8 * a * c^2 * d^2 * e * x^2 + a^2 * c * d * e^2 * x - 2 * a^3 * e^3) * \text{sqrt}(c * d * x + a * e) * f^3 * g / (c^2 * d^2) + \frac{4}{105} * (35 * c^4 * d^4 * x^4 + 50 * a * c^3 * d^3 * e * x^3 + 3 * a^2 * c^2 * d^2 * e^2 * x^2 - 4 * a^3 * c * d * e^3 * x + 8 * a^4 * e^4) * \text{sqrt}(c * d * x + a * e) * f^2 * g^2 / (c^3 * d^3) + \frac{8}{1155} * (105 * c^5 * d^5 * x^5 + 140 * a * c^4 * d^4 * e * x^4 + 5 * a^2 * c^3 * d^3 * e^2 * x^3 - 6 * a^3 * c^2 * d^2 * e^3 * x^2 + 8 * a^4 * c * d * e^4 * x - 16 * a^5 * e^5) * \text{sqrt}(c * d * x + a * e) * f * g^3 / (c^4 * d^4) + \frac{2}{15015} * (1155 * c^6 * d^6 * x^6 + 1470 * a * c^5 * d^5 * e * x^5 + 35 * a^2 * c^4 * d^4 * e^2 * x^4 - 40 * a^3 * c^3 * d^3 * e^3 * x^3 + 48 * a^4 * c^2 * d^2 * e^4 * x^2 - 64 * a^5 * c * d * e^5 * x + 128 * a^6 * e^6) * \text{sqrt}(c * d * x + a * e) * g^4 / (c^5 * d^5)$

**Fricas [A]** time = 0.275302, size = 1368, normalized size = 4.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f)^4/(e\*x + d)^(3/2))

[Out]  $\frac{2}{15015} * (1155 * c^7 * d^7 * e * g^4 * x^8 + 3003 * a^3 * c^4 * d^5 * e^3 * f^4 - 3432 * a^4 * c^3 * d^4 * e^4 * f^3 * g + 2288 * a^5 * c^2 * d^3 * e^5 * f^2 * g^2 - 832 * a^6 * c * d^2 * e^6 * f * g^3 + 128 * a^7 * d * e^7 * g^4 + 105 * (52 * c^7 * d^7 * e * f * g^3 + (11 * c^7 * d^8 + 25 * a * c^6 * d^6 * e^2) * g^4) * x^7 + 35 * (286 * c^7 * d^7 * e * f^2 * g^2 + 52 * (3 * c^7 * d^8 + 7 * a * c^6 * d^6 * e^2) * f * g^3 + (75 * a * c^6 * d^7 * e + 43 * a^2 * c^5 * d^5 * e^3) * g^4) * x^6 + 5 * (1716 * c^7 * d^7 * e * f^3 * g + 286 * (7 * c^7 * d^8 + 17 * a * c^6 * d^6 * e^2) * f^2 * g^2 + 52 * (49 * a * c^6 * d^7 * e + 29 * a^2 * c^5 * d^5 * e^3) * f * g^3 + (301 * a^2 * c^5 * d^6 * e^2 - a^3 * c^4 * d^4 * e^4) * g^4) * x^5 + (3003 * c^7 * d^7 * e * f^4 + 1716 * (5 * c^7 * d^8 + 13 * a * c^6 * d^6 * e^2) * f^3 * g + 286 * (85 * a * c^6 * d^7 * e + 53 * a^2 * c^5 * d^5 * e^3) * f^2 * g^2 + 52 * (145 * a^2 * c^5 * d^6 * e^2 - a^3 * c^4 * d^4 * e^4) * f * g^3 - (5 * a^3 * c^4 * d^5 * e^3 - 8 * a^4 * c^3 * d^3 * e^5) * g^4) * x^4 + (3003 * (c^7 * d^8 + 3 * a * c^6 * d^6 * e^2) * f^4 + 1716 * (13 * a * c^6 * d^7 * e + 9 * a^2 * c^5 * d^5 * e^3) * f^3 * g + 286 * (53 * a^2 * c^5 * d^6 * e^2 - a^3 * c^4 * d^4 * e^4) * f^2 * g^2 - 52 * (a^3 * c^4 * d^5 * e^3 - 2 * a^4 * c^3 * d^3 * e^5) * f * g^3 + 8 * (a^4 * c^3 * d^4 * e^4 - 2 * a^5 * c^2 * d^2 * e^6) * g^4) * x^3 + (9009 * (a * c^6 * d^7 * e + a^2 * c^5 * d^5 * e^3) * f^4 + 1716 * (9 * a^2 * c^5 * d^6 * e^2 - a^3 * c^4 * d^4 * e^4) * f^3 * g - 286 * (a^3 * c^4 * d^5 * e^3 - 4 * a^4 * c^3 * d^3 * e^5) * f^2 * g^2 + 104 * (a^4 * c^3 * d^4 * e^4 - 4 * a^5 * c^2 * d^2 * e^6) * f * g^3 - 16 * (a^5 * c^2 * d^3 * e^5 - 4 * a^6 * c * d * e^7) * g^4) * x^2 + (3003 * (3 * a^2 * c^5 * d^6 * e^2 + a^3 * c^4 * d^4 * e^4) * f^4 - 1716 * (a^3 * c^4 * d^5 * e^3 - 2 * a^4 * c^3 * d^3 * e^5) * f^3 * g + 286 * (85 * a * c^6 * d^7 * e + 53 * a^2 * c^5 * d^5 * e^3) * f^2 * g^2 + 52 * (145 * a^2 * c^5 * d^6 * e^2 - a^3 * c^4 * d^4 * e^4) * f * g^3 - (5 * a^3 * c^4 * d^5 * e^3 - 8 * a^4 * c^3 * d^3 * e^5) * g^4) * x$

$$\begin{aligned} & *e^3 + 2*a^4*c^3*d^3*e^5)*f^3*g + 1144*(a^4*c^3*d^4*e^4 + 2*a^5*c \\ & ^2*d^2*e^6)*f^2*g^2 - 416*(a^5*c^2*d^3*e^5 + 2*a^6*c*d*e^7)*f*g^3 \\ & + 64*(a^6*c*d^2*e^6 + 2*a^7*e^8)*g^4)*x)/(sqrt(c*d*e*x^2 + a*d*e \\ & + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*c^5*d^5) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*4\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/(e\*x+d)\*\*(3/2)

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f)^4/(e\*x + d)^(3/2)

[Out] Timed out

$$3.690 \quad \int \frac{(f+gx)^3 (ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}} dx$$

Optimal. Leaf size=269

$$\begin{aligned} & \frac{16(x(ae^2+cd^2)+ade+cdex^2)^{5/2}(cdf-aeg)^2(2ae^2g-cd(7ef-5dg))}{1155c^4d^4e(d+ex)^{5/2}} \\ & + \frac{16g(x(ae^2+cd^2)+ade+cdex^2)^{5/2}(cdf-aeg)^2}{231c^3d^3e(d+ex)^{3/2}} \\ & + \frac{4(f+gx)^2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}(cdf-aeg)}{33c^2d^2(d+ex)^{5/2}} \\ & + \frac{2(f+gx)^3(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{11cd(d+ex)^{5/2}} \end{aligned}$$

[Out]  $(-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}/(1155*c^4*d^4*e*(d + e*x)^{5/2}) + (16*g*(c*d*f - a*e*g)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}/(231*c^3*d^3*e*(d + e*x)^{3/2}) + (4*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}/(33*c^2*d^2*(d + e*x)^{5/2}) + (2*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}/(11*c*d*(d + e*x)^{5/2})$

Rubi [A] time = 1.10891, antiderivative size = 269, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\begin{aligned} & \frac{16(x(ae^2+cd^2)+ade+cdex^2)^{5/2}(cdf-aeg)^2(2ae^2g-cd(7ef-5dg))}{1155c^4d^4e(d+ex)^{5/2}} \\ & + \frac{16g(x(ae^2+cd^2)+ade+cdex^2)^{5/2}(cdf-aeg)^2}{231c^3d^3e(d+ex)^{3/2}} \\ & + \frac{4(f+gx)^2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}(cdf-aeg)}{33c^2d^2(d+ex)^{5/2}} \\ & + \frac{2(f+gx)^3(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{11cd(d+ex)^{5/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(f+g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{3/2}/(d + e*x)^{3/2}, x]$

[Out]  $(-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}/(1155*c^4*d^4*e*(d + e*x)^{5/2}) + (16*g*(c*d*f - a*e*g)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}/(231*c^3*d^3*e*(d + e*x)^{3/2}) + (4*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}/(33*c^2*d^2*(d + e*x)^{5/2}) + (2*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}/(11*c*d*(d + e*x)^{5/2})$

**Rubi in Sympy [A]** time = 93.7573, size = 264, normalized size = 0.98

$$\frac{2(f+gx)^3(ade+cdex^2+x(ae^2+cd^2))^{\frac{5}{2}}}{11cd(d+ex)^{\frac{5}{2}}}$$

$$-\frac{4(f+gx)^2(aeg-cdf)(ade+cdex^2+x(ae^2+cd^2))^{\frac{5}{2}}}{33c^2d^2(d+ex)^{\frac{5}{2}}}$$

$$+\frac{16g(aeg-cdf)^2(ade+cdex^2+x(ae^2+cd^2))^{\frac{5}{2}}}{231c^3d^3e(d+ex)^{\frac{3}{2}}}$$

$$-\frac{16(aeg-cdf)^2(ade+cdex^2+x(ae^2+cd^2))^{\frac{5}{2}}(2ae^2g+5cd^2g-7cdf)}{1155c^4d^4e(d+ex)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*3\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/(e\*x+d)\*\*

[Out] 2\*(f + g\*x)\*\*3\*(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*\*(5/2)/  
 (11\*c\*d\*(d + e\*x)\*\*(5/2)) - 4\*(f + g\*x)\*\*2\*(a\*e\*g - c\*d\*f)\*(a\*d\*e  
 + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*\*(5/2)/(33\*c\*\*2\*d\*\*2\*(d + e\*  
 x)\*\*(5/2)) + 16\*g\*(a\*e\*g - c\*d\*f)\*\*2\*(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e  
 \*\*2 + c\*d\*\*2))\*\*(5/2)/(231\*c\*\*3\*d\*\*3\*e\*(d + e\*x)\*\*(3/2)) - 16\*(a\*  
 e\*g - c\*d\*f)\*\*2\*(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*\*(5/2)  
 \*(2\*a\*e\*\*2\*g + 5\*c\*d\*\*2\*g - 7\*c\*d\*e\*f)/(1155\*c\*\*4\*d\*\*4\*e\*(d + e\*x  
 )\*\*(5/2))

**Mathematica [A]** time = 0.242918, size = 137, normalized size = 0.51

$$\frac{2((d+ex)(ae+cdx))^{5/2}(-16a^3e^3g^3+8a^2cde^2g^2(11f+5gx)-2ac^2d^2eg(99f^2+110fgx+35g^2x^2)+c^3d^3(231f^3+495f^2g^2))}{1155c^4d^4(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((f + g\*x)^3\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2))/(d + e\*x)^(3/2), x]

[Out] (2\*((a\*e + c\*d\*x)\*(d + e\*x))^(5/2)\*(-16\*a^3\*e^3\*g^3 + 8\*a^2\*c\*d\*e  
 ^2\*g^2\*(11\*f + 5\*g\*x) - 2\*a\*c^2\*d^2\*e\*g\*(99\*f^2 + 110\*f\*g\*x + 35\*  
 g^2\*x^2) + c^3\*d^3\*(231\*f^3 + 495\*f^2\*g\*x + 385\*f\*g^2\*x^2 + 105\*g  
 ^3\*x^3)))/(1155\*c^4\*d^4\*(d + e\*x)^(5/2))

**Maple [A]** time = 0.012, size = 188, normalized size = 0.7

$$\frac{(2cdx+2ae)(-105g^3x^3c^3d^3+70ac^2d^2eg^3x^2-385c^3d^3fg^2x^2-40a^2cde^2g^3x+220ac^2d^2efg^2x-495c^3d^3f^2gx+16a^3)}{1155c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^3\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)/(e\*x+d)^(3/2), x)

[Out] -2/1155\*(c\*d\*x+a\*e)\*(-105\*c^3\*d^3\*g^3\*x^3+70\*a\*c^2\*d^2\*e\*g^3\*x^2-  
 385\*c^3\*d^3\*f\*g^2\*x^2-40\*a^2\*c\*d\*e^2\*g^3\*x+220\*a\*c^2\*d^2\*e\*f\*g^2\*  
 x-495\*c^3\*d^3\*f^2\*g\*x+16\*a^3\*e^3\*g^3-88\*a^2\*c\*d\*e^2\*f\*g^2+198\*a\*c  
 ^2\*d^2\*e\*f^2\*g-231\*c^3\*d^3\*f^3)\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)  
 ^3/2/c^4/d^4/(e\*x+d)^(3/2)

**Maxima [A]** time = 0.743763, size = 397, normalized size = 1.48

$$\frac{2(c^2d^2x^2 + 2acdex + a^2e^2)\sqrt{cdx + aef^3}}{5cd} + \frac{6(5c^3d^3x^3 + 8ac^2d^2ex^2 + a^2cde^2x - 2a^3e^3)\sqrt{cdx + aef^2g}}{35c^2d^2} + \frac{2(35c^4d^4x^4 + 50ac^3d^3ex^3 + 3a^2c^2d^2e^2x^2 - 4a^3cde^3x + 8a^4e^4)\sqrt{cdx + aefg^2}}{105c^3d^3} + \frac{2(105c^5d^5x^5 + 140ac^4d^4ex^4 + 5a^2c^3d^3e^2x^3 - 6a^3c^2d^2e^3x^2 + 8a^4cde^4x - 16a^5e^5)\sqrt{cdx + aeg^3}}{1155c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f)^3/(e\*x + d)^(3/2))

[Out] 2/5\*(c^2\*d^2\*x^2 + 2\*a\*c\*d\*e\*x + a^2\*e^2)\*sqrt(c\*d\*x + a\*e)\*f^3/(c\*d) + 6/35\*(5\*c^3\*d^3\*x^3 + 8\*a\*c^2\*d^2\*e\*x^2 + a^2\*c\*d\*e^2\*x - 2\*a^3\*e^3)\*sqrt(c\*d\*x + a\*e)\*f^2\*g/(c^2\*d^2) + 2/105\*(35\*c^4\*d^4\*x^4 + 50\*a\*c^3\*d^3\*e\*x^3 + 3\*a^2\*c^2\*d^2\*e^2\*x^2 - 4\*a^3\*c\*d\*e^3\*x + 8\*a^4\*e^4)\*sqrt(c\*d\*x + a\*e)\*f\*g^2/(c^3\*d^3) + 2/1155\*(105\*c^5\*d^5\*x^5 + 140\*a\*c^4\*d^4\*e\*x^4 + 5\*a^2\*c^3\*d^3\*e^2\*x^3 - 6\*a^3\*c^2\*d^2\*e^3\*x^2 + 8\*a^4\*c\*d\*e^4\*x - 16\*a^5\*e^5)\*sqrt(c\*d\*x + a\*e)\*g^3/(c^4\*d^4)

**Fricas [A]** time = 0.276334, size = 990, normalized size = 3.68

$$\frac{2(105c^6d^6eg^3x^7 + 231a^3c^3d^4e^3f^3 - 198a^4c^2d^3e^4f^2g + 88a^5cd^2e^5fg^2 - 16a^6de^6g^3 + 35(11c^6d^6efg^2 + (3c^6d^7 + 7ac^5d^5e^2)fg^2))\sqrt{cdx + aef^3}}{1155c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f)^3/(e\*x + d)^(3/2))

[Out] 2/1155\*(105\*c^6\*d^6\*e\*g^3\*x^7 + 231\*a^3\*c^3\*d^4\*e^3\*f^3 - 198\*a^4\*c^2\*d^3\*e^4\*f^2\*g + 88\*a^5\*c\*d^2\*e^5\*f\*g^2 - 16\*a^6\*d\*e^6\*g^3 + 35\*(11\*c^6\*d^6\*e\*f\*g^2 + (3\*c^6\*d^7 + 7\*a\*c^5\*d^5\*e^2)\*g^2)\*x^6 + 5\*(99\*c^6\*d^6\*e\*f^2\*g + 11\*(7\*c^6\*d^7 + 17\*a\*c^5\*d^5\*e^2)\*f\*g^2 + (49\*a\*c^5\*d^6\*e + 29\*a^2\*c^4\*d^4\*e^3)\*g^3)\*x^5 + (231\*c^6\*d^6\*e\*f^3 + 99\*(5\*c^6\*d^7 + 13\*a\*c^5\*d^5\*e^2)\*f^2\*g + 11\*(85\*a\*c^5\*d^6\*e + 53\*a^2\*c^4\*d^4\*e^3)\*f\*g^2 + (145\*a^2\*c^4\*d^5\*e^2 - a^3\*c^3\*d^3\*e^4)\*g^3)\*x^4 + (231\*(c^6\*d^7 + 3\*a\*c^5\*d^5\*e^2)\*f^3 + 99\*(13\*a\*c^5\*d^6\*e + 9\*a^2\*c^4\*d^4\*e^3)\*f^2\*g + 11\*(53\*a^2\*c^4\*d^5\*e^2 - a^3\*c^3\*d^3\*e^4)\*f\*g^2 - (a^3\*c^3\*d^4\*e^3 - 2\*a^4\*c^2\*d^2\*e^5)\*g^3)\*x^3 + (693\*(a\*c^5\*d^6\*e + a^2\*c^4\*d^4\*e^3)\*f^3 + 99\*(9\*a^2\*c^4\*d^5\*e^2 - a^3\*c^3\*d^3\*e^4)\*f^2\*g - 11\*(a^3\*c^3\*d^4\*e^3 - 4\*a^4\*c^2\*d^2\*e^5)\*f\*g^2 + 2\*(a^4\*c^2\*d^3\*e^4 - 4\*a^5\*c\*d\*e^6)\*g^3)\*x^2 + (231\*(3\*a^2\*c^4\*d^5\*e^2 + a^3\*c^3\*d^3\*e^4)\*f^3 - 99\*(a^3\*c^3\*d^4\*e^3 + 2\*a^4\*c^2\*d^2\*e^5)\*f^2\*g + 44\*(a^4\*c^2\*d^3\*e^4 + 2\*a^5\*c\*d\*e^6)\*f\*g^2 - 8\*(a^5\*c\*d^2\*e^5 + 2\*a^6\*e^7)\*g^3)\*x)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*c^4\*d^4)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*3\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/(e\*x+d)\*\*(3/2))

[Out] Timed out

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**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^3/(e*x + d)^(3/2)`

[Out] Timed out

$$3.691 \quad \int \frac{(f+gx)^2 (ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}} dx$$

**Optimal.** Leaf size=200

$$\frac{8(x(ae^2 + cd^2) + ade + cdex^2)^{5/2} (cdf - aeg) (2ae^2g - cd(7ef - 5dg))}{315c^3d^3e(d+ex)^{5/2}} + \frac{8g(x(ae^2 + cd^2) + ade + cdex^2)^{5/2} (cdf - aeg)}{63c^2d^2e(d+ex)^{3/2}} + \frac{2(f+gx)^2 (x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{9cd(d+ex)^{5/2}}$$

[Out]  $(-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}/(315*c^3*d^3*e*(d + e*x)^{5/2}) + (8*g*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}/(63*c^2*d^2*e*(d + e*x)^{3/2}) + (2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}/(9*c*d*(d + e*x)^{5/2})$

**Rubi [A]** time = 0.709869, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{8(x(ae^2 + cd^2) + ade + cdex^2)^{5/2} (cdf - aeg) (2ae^2g - cd(7ef - 5dg))}{315c^3d^3e(d+ex)^{5/2}} + \frac{8g(x(ae^2 + cd^2) + ade + cdex^2)^{5/2} (cdf - aeg)}{63c^2d^2e(d+ex)^{3/2}} + \frac{2(f+gx)^2 (x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{9cd(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{3/2}/(d + e*x)^{3/2}, x]$

[Out]  $(-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}/(315*c^3*d^3*e*(d + e*x)^{5/2}) + (8*g*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}/(63*c^2*d^2*e*(d + e*x)^{3/2}) + (2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}/(9*c*d*(d + e*x)^{5/2})$

**Rubi in Sympy [A]** time = 60.3884, size = 196, normalized size = 0.98

$$\frac{2(f+gx)^2 (ade + cdex^2 + x(ae^2 + cd^2))^{\frac{5}{2}}}{9cd(d+ex)^{\frac{5}{2}}} - \frac{8g(aeg - cdf) (ade + cdex^2 + x(ae^2 + cd^2))^{\frac{5}{2}}}{63c^2d^2e(d+ex)^{\frac{3}{2}}} + \frac{8(aeg - cdf) (ade + cdex^2 + x(ae^2 + cd^2))^{\frac{5}{2}} (2ae^2g + 5cd^2g - 7cdef)}{315c^3d^3e(d+ex)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((g*x+f)**2*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(5/2))$

[Out]  $2*(f + g*x)**2*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))** (5/2)/ (9*c*d*(d + e*x)** (5/2)) - 8*g*(a*e*g - c*d*f)*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))** (5/2)/(63*c**2*d**2*e*(d + e*x)** (3/2)) + 8*(a*e*g - c*d*f)*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))** (5/2)*(2*a*e**2*g + 5*c*d**2*g - 7*c*d*e*f)/(315*c**3*d**3*e*(d + e*x)** (5/2))$

**Mathematica [A]** time = 0.18955, size = 90, normalized size = 0.45

$$\frac{2((d+ex)(ae+cdx))^{5/2} (8a^2e^2g^2 - 4acdeg(9f+5gx) + c^2d^2(63f^2 + 90fgx + 35g^2x^2))}{315c^3d^3(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((f + g\*x)^2\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2))/(d + e\*x)^(3/2)]

[Out] (2\*((a\*e + c\*d\*x)\*(d + e\*x))^(5/2)\*(8\*a^2\*e^2\*g^2 - 4\*a\*c\*d\*e\*g\*(9\*f + 5\*g\*x) + c^2\*d^2\*(63\*f^2 + 90\*f\*g\*x + 35\*g^2\*x^2)))/(315\*c^3\*d^3\*(d + e\*x)^(5/2))

**Maple [A]** time = 0.011, size = 116, normalized size = 0.6

$$\frac{(2cdx + 2ae)(35g^2x^2c^2d^2 - 20acdeg^2x + 90c^2d^2fgx + 8a^2e^2g^2 - 36acdefg + 63f^2c^2d^2)}{315c^3d^3} (cdex^2 + ae^2x + cd^2x + ade)^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^2\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)/(e\*x+d)^(3/2), x)

[Out] 2/315\*(c\*d\*x+a\*e)\*(35\*c^2\*d^2\*g^2\*x^2-20\*a\*c\*d\*e\*g^2\*x+90\*c^2\*d^2\*f\*g\*x+8\*a^2\*e^2\*g^2-36\*a\*c\*d\*e\*f\*g+63\*c^2\*d^2\*f^2)\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(3/2)/c^3/d^3/(e\*x+d)^(3/2)

**Maxima [A]** time = 0.729543, size = 259, normalized size = 1.3

$$\begin{aligned} & \frac{2(c^2d^2x^2 + 2acdex + a^2e^2)\sqrt{cdx + aef^2}}{5cd} \\ & + \frac{4(5c^3d^3x^3 + 8ac^2d^2ex^2 + a^2cde^2x - 2a^3e^3)\sqrt{cdx + aefg}}{35c^2d^2} \\ & + \frac{2(35c^4d^4x^4 + 50ac^3d^3ex^3 + 3a^2c^2d^2e^2x^2 - 4a^3cde^3x + 8a^4e^4)\sqrt{cdx + aeg^2}}{315c^3d^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f)^2/(e\*x + d)^(3/2))

[Out] 2/5\*(c^2\*d^2\*x^2 + 2\*a\*c\*d\*e\*x + a^2\*e^2)\*sqrt(c\*d\*x + a\*e)\*f^2/(c\*d) + 4/35\*(5\*c^3\*d^3\*x^3 + 8\*a\*c^2\*d^2\*e\*x^2 + a^2\*c\*d\*e^2\*x - 2\*a^3\*e^3)\*sqrt(c\*d\*x + a\*e)\*f\*g/(c^2\*d^2) + 2/315\*(35\*c^4\*d^4\*x^4 + 50\*a\*c^3\*d^3\*e\*x^3 + 3\*a^2\*c^2\*d^2\*e^2\*x^2 - 4\*a^3\*c\*d\*e^3\*x + 8\*a^4\*e^4)\*sqrt(c\*d\*x + a\*e)\*g^2/(c^3\*d^3)

**Fricas [A]** time = 0.271004, size = 662, normalized size = 3.31

$$\frac{2(35c^5d^5eg^2x^6 + 63a^3c^2d^3e^3f^2 - 36a^4cd^2e^4fg + 8a^5de^5g^2 + 5(18c^5d^5efg + (7c^5d^6 + 17ac^4d^4e^2)g^2)x^5 + (63c^5d^5ef^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f)^2/(e\*x + d)^(3/2))

[Out] 2/315\*(35\*c^5\*d^5\*e\*g^2\*x^6 + 63\*a^3\*c^2\*d^3\*e^3\*f^2 - 36\*a^4\*c\*d^2\*e^4\*f\*g + 8\*a^5\*d\*e^5\*g^2 + 5\*(18\*c^5\*d^5\*e\*f\*g + (7\*c^5\*d^6 + 17\*a\*c^4\*d^4\*e^2)\*g^2)\*x^5 + (63\*c^5\*d^5\*e\*f^2 + 18\*(5\*c^5\*d^6 + 13\*a\*c^4\*d^4\*e^2)\*f\*g + (85\*a\*c^4\*d^4\*e^2 + 53\*a^2\*c^3\*d^3\*e^3)\*g^2)\*x^4 + (63\*(c^5\*d^6 + 3\*a\*c^4\*d^4\*e^2)\*f^2 + 18\*(13\*a\*c^4\*d^4\*e^2 + 9\*a^2\*c^3\*d^3\*e^3)\*f\*g + (53\*a^2\*c^3\*d^4\*e^2 - a^3\*c^2\*d^2\*e^4)



$$\begin{aligned} & ) * g^2) * x^3 + (189 * (a * c^4 * d^5 * e + a^2 * c^3 * d^3 * e^3) * f^2 + 18 * (9 * a^2 \\ & * c^3 * d^4 * e^2 - a^3 * c^2 * d^2 * e^4) * f * g - (a^3 * c^2 * d^3 * e^3 - 4 * a^4 * c * \\ & d * e^5) * g^2) * x^2 + (63 * (3 * a^2 * c^3 * d^4 * e^2 + a^3 * c^2 * d^2 * e^4) * f^2 - \\ & 18 * (a^3 * c^2 * d^3 * e^3 + 2 * a^4 * c * d * e^5) * f * g + 4 * (a^4 * c * d^2 * e^4 + 2 * \\ & a^5 * e^6) * g^2) * x) / (\text{sqrt}(c * d * e * x^2 + a * d * e + (c * d^2 + a * e^2) * x) * \text{sq} \\ & \text{r}(e * x + d) * c^3 * d^3) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*2\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/(e\*x+d)\*\*(3/2)

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f)^2/(e\*x + d)^(3/2)

[Out] Timed out

$$3.692 \quad \int \frac{(f+gx)(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}} dx$$

**Optimal.** Leaf size=125

$$\frac{2g(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{7cde(d+ex)^{3/2}} - \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}(2ae^2g-cd(7ef-5dg))}{35c^2d^2e(d+ex)^{5/2}}$$

[Out]  $(-2*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}/(35*c^2*d^2*e*(d + e*x)^{(5/2)}) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}/(7*c*d*e*(d + e*x)^{(3/2)})$

**Rubi [A]** time = 0.327352, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 44,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{2g(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{7cde(d+ex)^{3/2}} - \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}(2ae^2g-cd(7ef-5dg))}{35c^2d^2e(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[((f + g\*x)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2))/(d + e\*x)^(3/2), x]

[Out]  $(-2*(2*a*e^2*g - c*d*(7*e*f - 5*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}/(35*c^2*d^2*e*(d + e*x)^{(5/2)}) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}/(7*c*d*e*(d + e*x)^{(3/2)})$

**Rubi in Sympy [A]** time = 34.9547, size = 121, normalized size = 0.97

$$\frac{2g(ade+cdex^2+x(ae^2+cd^2))^{5/2}}{7cde(d+ex)^{3/2}} - \frac{2(ade+cdex^2+x(ae^2+cd^2))^{5/2}(2ae^2g+5cd^2g-7cdef)}{35c^2d^2e(d+ex)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/(e\*x+d)\*\*(3/2), x)

[Out]  $2*g*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))** (5/2)/(7*c*d*e*(d + e*x)** (3/2)) - 2*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))** (5/2)*(2*a*e**2*g + 5*c*d**2*g - 7*c*d*e*f)/(35*c**2*d**2*e*(d + e*x)** (5/2))$

**Mathematica [A]** time = 0.0991003, size = 54, normalized size = 0.43

$$\frac{2((d+ex)(ae+cdx))^{5/2}(cd(7f+5gx)-2aeg)}{35c^2d^2(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((f + g\*x)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2))/(d + e\*x)^(3/2), x]

[Out]  $(2*((a*e + c*d*x)*(d + e*x))^{5/2}*(-2*a*e*g + c*d*(7*f + 5*g*x)))/(35*c^2*d^2*(d + e*x)^{5/2})$

**Maple [A]** time = 0.007, size = 67, normalized size = 0.5

$$-\frac{(2cdx + 2ae)(-5xcdg + 2aeg - 7cdf)}{35c^2d^2} (cdex^2 + ae^2x + cd^2x + ade)^{\frac{3}{2}} (ex + d)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)`

[Out] `-2/35*(c*d*x+a*e)*(-5*c*d*g*x+2*a*e*g-7*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)/c^2/d^2/(e*x+d)^(3/2)`

**Maxima [A]** time = 0.72322, size = 144, normalized size = 1.15

$$\frac{2(c^2d^2x^2 + 2acdex + a^2e^2)\sqrt{cdx + aef}}{5cd} + \frac{2(5c^3d^3x^3 + 8ac^2d^2ex^2 + a^2cde^2x - 2a^3e^3)\sqrt{cdx + aeg}}{35c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)/(e*x + d)^(3/2)`

[Out] `2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*sqrt(c*d*x + a*e)*f/(c*d) + 2/35*(5*c^3*d^3*x^3 + 8*a*c^2*d^2*e*x^2 + a^2*c*d*e^2*x - 2*a^3*e^3)*sqrt(c*d*x + a*e)*g/(c^2*d^2)`

**Fricas [A]** time = 0.267671, size = 382, normalized size = 3.06

$$\frac{2(5c^4d^4egx^5 + 7a^3cd^2e^3f - 2a^4de^4g + (7c^4d^4ef + (5c^4d^5 + 13ac^3d^3e^2)g)x^4 + (7(c^4d^5 + 3ac^3d^3e^2)f + (13ac^3d^4e + 9a^4d^4e^2)g)x^3 + (7(c^4d^5 + 3ac^3d^3e^2)fg + (13ac^3d^4e + 9a^4d^4e^2)g^2)x^2 + (7(c^4d^5 + 3ac^3d^3e^2)fg^2 + (13ac^3d^4e + 9a^4d^4e^2)g^3)x + (7(c^4d^5 + 3ac^3d^3e^2)fg^3 + (13ac^3d^4e + 9a^4d^4e^2)g^4))}{35\sqrt{cdex^2 + ade} + (c^2d^2 + a^2e^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)/(e*x + d)^(3/2)`

[Out] `2/35*(5*c^4*d^4*e*g*x^5 + 7*a^3*c*d^2*e^3*f - 2*a^4*d*e^4*g + (7*c^4*d^4*e*f + (5*c^4*d^5 + 13*a*c^3*d^3*e^2)*g)*x^4 + (7*(c^4*d^5 + 3*a*c^3*d^3*e^2)*f + (13*a*c^3*d^4*e + 9*a^2*c^2*d^2*e^3)*g)*x^3 + (21*(a*c^3*d^4*e + a^2*c^2*d^2*e^3)*f + (9*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*g)*x^2 + (7*(3*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*f - (a^3*c*d^2*e^3 + 2*a^4*e^5)*g)*x)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*c^2*d^2)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2),x)`

[Out] Timed out

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)/(e*x + d)^(3/2)`

[Out] Timed out

$$3.693 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}} dx$$

**Optimal.** Leaf size=48

$$\frac{2(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{5cd(d+ex)^{5/2}}$$

[Out] (2\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2))/(5\*c\*d\*(d + e\*x)^(5/2))

**Rubi [A]** time = 0.0711415, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$

$$\frac{2(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{5cd(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/(d + e\*x)^(3/2), x]

[Out] (2\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2))/(5\*c\*d\*(d + e\*x)^(5/2))

**Rubi in Sympy [A]** time = 16.9086, size = 42, normalized size = 0.88

$$\frac{2(ade + cdex^2 + x(ae^2 + cd^2))^{5/2}}{5cd(d+ex)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/(e\*x+d)\*\*(3/2), x)

[Out] 2\*(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*\* (5/2)/(5\*c\*d\*(d + e\*x)\*\* (5/2))

**Mathematica [A]** time = 0.0463383, size = 37, normalized size = 0.77

$$\frac{2((d+ex)(ae+cdx))^{5/2}}{5cd(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/(d + e\*x)^(3/2), x]

[Out] (2\*((a\*e + c\*d\*x)\*(d + e\*x))^(5/2))/(5\*c\*d\*(d + e\*x)^(5/2))

**Maple [A]** time = 0.006, size = 50, normalized size = 1.

$$\frac{2cdx + 2ae}{5cd} (cdex^2 + ae^2x + cd^2x + ade)^{\frac{3}{2}} (ex + d)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)`

[Out]  $2/5*(c*d*x+a*e)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(3/2)/c/d/(e*x+d)^(3/2)$

**Maxima [A]** time = 0.713501, size = 58, normalized size = 1.21

$$\frac{2(c^2d^2x^2 + 2acdex + a^2e^2)\sqrt{cdx + ae}}{5cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/(e*x + d)^(3/2),x, algorithm="maxima")`

[Out]  $2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*\text{sqrt}(c*d*x + a*e)/(c*d)$

**Fricas [A]** time = 0.269196, size = 178, normalized size = 3.71

$$\frac{2(c^3d^3ex^4 + a^3de^3 + (c^3d^4 + 3ac^2d^2e^2)x^3 + 3(ac^2d^3e + a^2cde^3)x^2 + (3a^2cd^2e^2 + a^3e^4)x)}{5\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{ex + dcd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/(e*x + d)^(3/2),x, algorithm="fricas")`

[Out]  $2/5*(c^3*d^3*e*x^4 + a^3*d*e^3 + (c^3*d^4 + 3*a*c^2*d^2*e^2)*x^3 + 3*(a*c^2*d^3*e + a^2*c*d*e^3)*x^2 + (3*a^2*c*d^2*e^2 + a^3*e^4)*x)/(\text{sqrt}(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*\text{sqrt}(e*x + d)*c*d)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{((d + ex)(ae + cdx))^{\frac{3}{2}}}{(d + ex)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2),x)`

[Out] `Integral(((d + e*x)*(a*e + c*d*x))**(3/2)/(d + e*x)**(3/2), x)`

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/(e*x + d)^(3/2),x, algorithm="giac")`

[Out] Timed out

$$3.694 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)} dx$$

**Optimal.** Leaf size=179

$$\frac{2(cdf - aeg)^{3/2} \tan^{-1} \left( \frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}} \right)}{g^{5/2}} - \frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}{g^2\sqrt{d+ex}} + \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{3g(d+ex)^{3/2}}$$

[Out]  $(-2*(c*d*f - a*e*g)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/ (g^2*\text{Sqrt}[d + e*x]) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(3*g*(d + e*x)^{(3/2)}) + (2*(c*d*f - a*e*g)^{(3/2)}*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x])])/g^{(5/2)}$

**Rubi [A]** time = 0.97726, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{2(cdf - aeg)^{3/2} \tan^{-1} \left( \frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}} \right)}{g^{5/2}} - \frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}{g^2\sqrt{d+ex}} + \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{3g(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/((d + e*x)^{(3/2)}*(f + g*x)), x]$

[Out]  $(-2*(c*d*f - a*e*g)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/ (g^2*\text{Sqrt}[d + e*x]) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(3*g*(d + e*x)^{(3/2)}) + (2*(c*d*f - a*e*g)^{(3/2)}*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x])])/g^{(5/2)}$

**Rubi in Sympy [A]** time = 82.9965, size = 172, normalized size = 0.96

$$\frac{2(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{3}{2}}}{3g(d+ex)^{\frac{3}{2}}} + \frac{2(aeg - cdf)\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{g^2\sqrt{d+ex}} - \frac{2(aeg - cdf)^{\frac{3}{2}} \operatorname{atanh} \left( \frac{\sqrt{g}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}\sqrt{aeg-cdf}} \right)}{g^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x$

[Out]  $2*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2)/(3*g*(d + e*x)**(3/2)) + 2*(a*e*g - c*d*f)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(g**2*\text{sqrt}(d + e*x)) - 2*(a*e*g - c*d*f)**(3/2)*\text{atanh}(\text{sqrt}(g)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/(\text{sqrt}(d + e*x)*\text{sqrt}(a*e*g - c*d*f))/g**(5/2)$

**Mathematica [A]** time = 0.306365, size = 132, normalized size = 0.74

$$\frac{2\sqrt{d+ex}\sqrt{ae+cdx}\left(\sqrt{g}\sqrt{ae+cdx}(4aeg-3cdf+cdgx)-3(aeg-cdf)^{3/2}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{aeg-cdf}}\right)\right)}{3g^{5/2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/((d + e\*x)^(3/2)\*(f + g\*x

[Out] (2\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*(Sqrt[g]\*Sqrt[a\*e + c\*d\*x]\*(-3\*c\*d\*f + 4\*a\*e\*g + c\*d\*g\*x) - 3\*(-(c\*d\*f) + a\*e\*g)^(3/2)\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/Sqrt[-(c\*d\*f) + a\*e\*g]])/(3\*g^(5/2)\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)])

**Maple [A]** time = 0.027, size = 263, normalized size = 1.5

$$-\frac{2}{3g^2}\sqrt{cdex^2 + ae^2x + cd^2x + ade}\left(3\operatorname{Artanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)a^2e^2g^2 - 6\operatorname{Artanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)acdefg + 3\operatorname{Artanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)/(e\*x+d)^(3/2)/(g\*x+f), x)

[Out] -2/3\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)\*(3\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*a^2\*e^2\*g^2-6\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*a\*c\*d\*e\*f\*g+3\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*c^2\*d^2\*f^2-((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*x\*c\*d\*g-4\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*a\*e\*g+3\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*c\*d\*f/(e\*x+d)^(1/2)/(c\*d\*x+a\*e)^(1/2)/g^2/((a\*e\*g-c\*d\*f)\*g)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.282931, size = 1, normalized size = 0.01

$$\frac{2c^2d^2egx^3 - 6acd^2ef + 8a^2de^2g - 3\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(cdf - aeg)\sqrt{ex + d}\sqrt{\frac{cdf - aeg}{g}}\log\left(-\frac{cdex^2 - cd^2f + 2ade}{3\sqrt{cde}}\right)}{3\sqrt{cde}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f



```
[Out] [1/3*(2*c^2*d^2*e*g*x^3 - 6*a*c*d^2*e*f + 8*a^2*d*e^2*g - 3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt(-(c*d*f - a*e*g)/g)*log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g*sqrt(-(c*d*f - a*e*g)/g) - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(3*c^2*d^2*e*f - (c^2*d^3 + 5*a*c*d*e^2)*g)*x^2 - 2*(3*(c^2*d^3 + a*c*d*e^2)*f - (5*a*c*d^2*e + 4*a^2*e^3)*g)*x)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g^2), 2/3*(c^2*d^2*e*g*x^3 - 3*a*c*d^2*e*f + 4*a^2*d*e^2*g + 3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*sqrt(e*x + d)*sqrt((c*d*f - a*e*g)/g)*arctan(-sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f - a*e*g)*sqrt(e*x + d)/((c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)*sqrt((c*d*f - a*e*g)/g))) - (3*c^2*d^2*e*f - (c^2*d^3 + 5*a*c*d*e^2)*g)*x^2 - (3*(c^2*d^3 + a*c*d*e^2)*f - (5*a*c*d^2*e + 4*a^2*e^3)*g)*x)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*g^2)]
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f), x)
```

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}(gx + f)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g*x + f)), x)
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g*x + f)), x)
```

$$3.695 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^2} dx$$

**Optimal.** Leaf size=178

$$\frac{3cd\sqrt{cdf - aeg} \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{5/2}} - \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{g(d+ex)^{3/2}(f+gx)} + \frac{3cd\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{g^2\sqrt{d+ex}}$$

[Out] (3\*c\*d\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(g^2\*Sqrt[d + e\*x]) - (a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/(g\*(d + e\*x)^(3/2)\*(f + g\*x)) - (3\*c\*d\*Sqrt[c\*d\*f - a\*e\*g]\*ArcTan[(Sqrt[g]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(Sqrt[c\*d\*f - a\*e\*g]\*Sqrt[d + e\*x])])/g^(5/2)

**Rubi [A]** time = 0.878574, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{3cd\sqrt{cdf - aeg} \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{5/2}} - \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{g(d+ex)^{3/2}(f+gx)} + \frac{3cd\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{g^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/((d + e\*x)^(3/2)\*(f + g\*x)^2), x

[Out] (3\*c\*d\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(g^2\*Sqrt[d + e\*x]) - (a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/(g\*(d + e\*x)^(3/2)\*(f + g\*x)) - (3\*c\*d\*Sqrt[c\*d\*f - a\*e\*g]\*ArcTan[(Sqrt[g]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(Sqrt[c\*d\*f - a\*e\*g]\*Sqrt[d + e\*x])])/g^(5/2)

**Rubi in Sympy [A]** time = 82.0464, size = 170, normalized size = 0.96

$$\frac{3cd\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{g^2\sqrt{d+ex}} - \frac{3cd\sqrt{aeg - cdf} \operatorname{atanh}\left(\frac{\sqrt{g}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}\sqrt{aeg-cdf}}\right)}{g^{5/2}} - \frac{(ade + cdex^2 + x(ae^2 + cd^2))^{3/2}}{g(d+ex)^{3/2}(f+gx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/(e\*x+d)\*\*(3/2)/(g\*x

[Out] 3\*c\*d\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(g\*\*2\*sqrt(d + e\*x)) - 3\*c\*d\*sqrt(a\*e\*g - c\*d\*f)\*atanh(sqrt(g)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2)))/(sqrt(d + e\*x)\*sqrt(a\*e\*g - c\*d\*f))/g\*\*(5/2) - (a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*\*(3/2)/(g\*(d + e\*x)\*\*(3/2)\*(f + g\*x))

**Mathematica [A]** time = 0.742489, size = 127, normalized size = 0.71

$$\frac{\sqrt{(d+ex)(ae+cdx)} \left( \frac{\sqrt{g}(cd(3f+2gx)-aeg)}{f+gx} - \frac{3cd\sqrt{aeg-cdf} \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{aeg-cdf}}\right)}{\sqrt{ae+cdx}} \right)}{g^{5/2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/((d + e\*x)^(3/2)\*(f + g\*x

[Out] (Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)]\*((Sqrt[g]\*(-(a\*e\*g) + c\*d\*(3\*f + 2\*g\*x)))/(f + g\*x) - (3\*c\*d\*Sqrt[-(c\*d\*f) + a\*e\*g]\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/Sqrt[-(c\*d\*f) + a\*e\*g]])/Sqrt[a\*e + c\*d\*x]))/(g^(5/2)\*Sqrt[d + e\*x])

**Maple [A]** time = 0.035, size = 306, normalized size = 1.7

$$\frac{1}{g^2(gx+f)} \sqrt{cdex^2 + ae^2x + cd^2x + ade} \left( -3 \operatorname{Arctanh} \left( \frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}} \right) xacdeg^2 + 3 \operatorname{Arctanh} \left( \frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}} \right) xc^2d^2fg - 3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)/(e\*x+d)^(3/2)/(g\*x+f)^2,x)

[Out] (c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)/(e\*x+d)^(1/2)\*(-3\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x\*a\*c\*d\*e\*g^2+3\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x\*c^2\*d^2\*f\*g-3\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*a\*c\*d\*e\*f\*g+3\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*c^2\*d^2\*f^2+2\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*x\*c\*d\*g-((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*a\*e\*g+3\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*c\*d\*f)/(c\*d\*x+a\*e)^(1/2)/g^2/(g\*x+f)/((a\*e\*g-c\*d\*f)\*g)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.30806, size = 1, normalized size = 0.01

$$\frac{4c^2d^2egx^3 + 6acd^2ef - 2a^2de^2g + 3\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(cdgx + cdf)\sqrt{ex + d}\sqrt{-\frac{cdf-aeg}{g}} \log\left(-\frac{cdegx^2 - cd^2f + 2aax}{2\sqrt{cdex^2 + a}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f

[Out] [1/2\*(4\*c^2\*d^2\*e\*g\*x^3 + 6\*a\*c\*d^2\*e\*f - 2\*a^2\*d\*e^2\*g + 3\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(c\*d\*g\*x + c\*d\*f)\*sqrt(e\*x + d)\*sqrt(-(c\*d\*f - a\*e\*g)/g)\*log(-(c\*d\*e\*g\*x^2 - c\*d^2\*f + 2\*a\*d\*e\*g - 2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*g\*sqrt(-(c\*d\*f - a\*e\*g)/g) - (c\*d\*e\*f - (c\*d^2 + 2\*a\*e^2)\*g)\*x)/(e\*g\*x^2 + d\*f + (e\*f + d\*g)\*x)) + 2\*(3\*c^2\*d^2\*e\*f + (2\*c^2\*d^3 + a\*c\*d\*e^2)\*g)\*x^2 + 2\*(3\*(c^2\*d^3 + a\*c\*d\*e^2)\*f + (a\*c\*d^2\*e - a^2\*e^3)\*g)\*x)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g^3\*x + f\*g^2)\*sqrt(e\*x + d)), (2\*c^2\*d^2\*e\*g\*x^3 + 3\*a\*c\*d^2\*e\*f - a^2\*d\*e^2\*g - 3\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(c\*d\*g\*x + c\*d\*f)\*sqrt(e\*x + d)\*sqrt((c\*d\*f - a\*e\*g)/g)\*arctan(-sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(c\*d\*f - a\*e\*g)\*sqrt(e\*x + d)/((c\*d\*e\*g\*x^2 + a\*d\*e\*g + (c\*d^2 + a\*e^2)\*g\*x)\*sqrt((c\*d\*f - a\*e\*g)/g))) + (3\*c^2\*d^2\*e\*f + (2\*c^2\*d^3 + a\*c\*d\*e^2)\*g)\*x^2 + (3\*(c^2\*d^3 + a\*c\*d\*e^2)\*f + (a\*c\*d^2\*e - a^2\*e^3)\*g)\*x)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g^3\*x + f\*g^2)\*sqrt(e\*x + d))]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/(e\*x+d)\*\*(3/2)/(g\*x+f)\*\*

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f

[Out] Timed out

$$3.696 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^3} dx$$

**Optimal.** Leaf size=195

$$\frac{3c^2d^2 \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-ae^2}}\right)}{4g^{5/2}\sqrt{cdf-ae^2}} - \frac{3cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)} - \frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}$$

[Out]  $(-3*c*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^2*\text{Sqrt}[d + e*x]*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/(2*g*(d + e*x)^{(3/2)}*(f + g*x)^2) + (3*c^2*d^2*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x]))/(4*g^{(5/2)}*\text{Sqrt}[c*d*f - a*e*g])$

**Rubi [A]** time = 0.876498, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{3c^2d^2 \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-ae^2}}\right)}{4g^{5/2}\sqrt{cdf-ae^2}} - \frac{3cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4g^2\sqrt{d+ex}(f+gx)} - \frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/((d + e*x)^{(3/2)}*(f + g*x)^3), x]$

[Out]  $(-3*c*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^2*\text{Sqrt}[d + e*x]*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/(2*g*(d + e*x)^{(3/2)}*(f + g*x)^2) + (3*c^2*d^2*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x]))/(4*g^{(5/2)}*\text{Sqrt}[c*d*f - a*e*g])$

**Rubi in Sympy [A]** time = 82.3383, size = 187, normalized size = 0.96

$$\frac{3c^2d^2 \operatorname{atanh}\left(\frac{\sqrt{g}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}\sqrt{aeg-cdf}}\right)}{4g^{5/2}\sqrt{aeg-cdf}} - \frac{3cd\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{4g^2\sqrt{d+ex}(f+gx)} - \frac{(ade+cdex^2+x(ae^2+cd^2))^{3/2}}{2g(d+ex)^{3/2}(f+gx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a*d*e+(a*e^{**2}+c*d^{**2})*x+c*d*e*x^{**2})^{**}(3/2)/(e*x+d)^{**}(3/2)/(g*x$

[Out]  $-3*c^{**2}*d^{**2}*\operatorname{atanh}(\text{sqrt}(g)*\text{sqrt}(a*d*e + c*d*e*x^{**2} + x*(a*e^{**2} + c*d^{**2}))/(\text{sqrt}(d + e*x)*\text{sqrt}(a*e*g - c*d*f)))/(4*g^{**}(5/2)*\text{sqrt}(a*e*g - c*d*f)) - 3*c*d*\text{sqrt}(a*d*e + c*d*e*x^{**2} + x*(a*e^{**2} + c*d^{**2}))/((4*g^{**2}*\text{sqrt}(d + e*x)*(f + g*x)) - (a*d*e + c*d*e*x^{**2} + x*(a*e^{**2} + c*d^{**2}))^{**}(3/2)/(2*g*(d + e*x)^{**}(3/2)*(f + g*x)^{**2}))$

**Mathematica [A]** time = 0.38898, size = 135, normalized size = 0.69

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(-\frac{3c^2d^2 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{aeg-cdf}}\right)}{\sqrt{ae+cdx}\sqrt{aeg-cdf}} - \frac{\sqrt{g}(2aeg+cd(3f+5gx))}{(f+gx)^2}\right)}{4g^{5/2}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/((d + e\*x)^(3/2)\*(f + g\*x

[Out] (Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)]\*(-((Sqrt[g]\*(2\*a\*e\*g + c\*d\*(3\*f + 5\*g\*x)))/(f + g\*x)^2) - (3\*c^2\*d^2\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/Sqrt[-(c\*d\*f) + a\*e\*g]])/(Sqrt[-(c\*d\*f) + a\*e\*g]\*Sqrt[a\*e + c\*d\*x])))/(4\*g^(5/2)\*Sqrt[d + e\*x])

**Maple [A]** time = 0.035, size = 276, normalized size = 1.4

$$-\frac{1}{4g^2(gx+f)^2}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(3\operatorname{Artanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)x^2c^2d^2g^2+6\operatorname{Artanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)xc^2d^2fg\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)/(e\*x+d)^(3/2)/(g\*x+f)^3,x)

[Out] -1/4\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)\*(3\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x^2\*c^2\*d^2\*g^2+6\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x\*c^2\*d^2\*f\*g+3\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*c^2\*d^2\*f^2+5\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*x\*c\*d\*g+2\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*a\*e\*g+3\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*c\*d\*f/(e\*x+d)^(1/2)/(c\*d\*x+a\*e)^(1/2)/g^2/(g\*x+f)^2/((a\*e\*g-c\*d\*f)\*g)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.289833, size = 1, normalized size = 0.01

$$\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{-cdfg+aeg^2}(5cdgx+3cdf+2aeg)\sqrt{ex+d}-3(c^2d^2eg^2x^3+c^2d^3f^2+(2c^2d^2efg+c^2d^3g^2))\sqrt{ex+d}}{8(eg^4x^3+df^2g^2+(2efg^3+df^2g^2))\sqrt{ex+d}+3(c^2d^2eg^2x^3+c^2d^3f^2+(2c^2d^2efg+c^2d^3g^2))\sqrt{ex+d}}-\frac{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{cdfg-aeg^2}(5cdgx+3cdf+2aeg)\sqrt{ex+d}+3(c^2d^2eg^2x^3+c^2d^3f^2+(2c^2d^2efg+c^2d^3g^2))\sqrt{ex+d}}{4(eg^4x^3+df^2g^2+(2efg^3+dg^4)x^2+(ef^2g^2+2dfg^3)x)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f

[Out] [-1/8\*(2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*f\*g + a\*e\*g^2)\*(5\*c\*d\*g\*x + 3\*c\*d\*f + 2\*a\*e\*g)\*sqrt(e\*x + d) - 3\*(c^2\*d^2\*e\*g^2\*x^3 + c^2\*d^3\*f^2 + (2\*c^2\*d^2\*e\*f\*g + c^2\*d^3\*g^2)\*

$$\begin{aligned}
& x^2 + (c^2 d^2 e^f + 2 c^2 d^3 f g) x) \log(- (2 \sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x}) (c d f g - a e g^2) \sqrt{e x + d} + (c d e g x^2 - c d^2 f + 2 a d e g - (c d e f - (c d^2 + 2 a e^2) g) x) \sqrt{-c d f g + a e g^2}) / (e g x^2 + d f + (e f + d g) x)) \\
& / ((e g^4 x^3 + d f^2 g^2 + (2 e f g^3 + d g^4) x^2 + (e f^2 g^2 + 2 d f g^3) x) \sqrt{-c d f g + a e g^2}), -1/4 (\sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x}) \sqrt{c d f g - a e g^2} (5 c d g x + 3 c d f + 2 a e g) \sqrt{e x + d} + 3 (c^2 d^2 e g^2 x^3 + c^2 d^3 f^2 + (2 c^2 d^2 e f g + c^2 d^3 g^2) x^2 + (c^2 d^2 e f^2 + 2 c^2 d^3 f g) x) \arctan(\sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x}) \sqrt{c d f g - a e g^2} \sqrt{e x + d} / (c d e g x^2 + a d e g + (c d^2 + a e^2) g x)) / ((e g^4 x^3 + d f^2 g^2 + (2 e f g^3 + d g^4) x^2 + (e f^2 g^2 + 2 d f g^3) x) \sqrt{c d f g - a e g^2})]
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/(e\*x+d)\*\*(3/2)/(g\*x+f)\*\*

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f

[Out] Timed out

$$3.697 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^4} dx$$

**Optimal.** Leaf size=265

$$\frac{c^3 d^3 \tan^{-1} \left( \frac{\sqrt{g} \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{\sqrt{d+ex} \sqrt{cdf - aeg}} \right)}{8g^{5/2} (cdf - aeg)^{3/2}} + \frac{c^2 d^2 \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{8g^2 \sqrt{d+ex} (f+gx) (cdf - aeg)}$$

$$- \frac{cd \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{4g^2 \sqrt{d+ex} (f+gx)^2} - \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{3g(d+ex)^{3/2} (f+gx)^3}$$

[Out]  $-(c*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^2*\text{Sqrt}[d + e*x]*(f + g*x)^2) + (c^2*d^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*g^2*(c*d*f - a*e*g)*\text{Sqrt}[d + e*x]*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/(3*g*(d + e*x)^{(3/2)}*(f + g*x)^3) + (c^3*d^3*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x]))/(8*g^{(5/2)}*(c*d*f - a*e*g)^{(3/2)})$

**Rubi [A]** time = 1.22644, antiderivative size = 265, normalized size of antiderivative = 1., number of rules used = 5, number of rules used = 4, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{c^3 d^3 \tan^{-1} \left( \frac{\sqrt{g} \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{\sqrt{d+ex} \sqrt{cdf - aeg}} \right)}{8g^{5/2} (cdf - aeg)^{3/2}} + \frac{c^2 d^2 \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{8g^2 \sqrt{d+ex} (f+gx) (cdf - aeg)}$$

$$- \frac{cd \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{4g^2 \sqrt{d+ex} (f+gx)^2} - \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{3g(d+ex)^{3/2} (f+gx)^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/((d + e*x)^{(3/2)}*(f + g*x)^4), x]$

[Out]  $-(c*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^2*\text{Sqrt}[d + e*x]*(f + g*x)^2) + (c^2*d^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*g^2*(c*d*f - a*e*g)*\text{Sqrt}[d + e*x]*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/(3*g*(d + e*x)^{(3/2)}*(f + g*x)^3) + (c^3*d^3*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x]))/(8*g^{(5/2)}*(c*d*f - a*e*g)^{(3/2)})$

**Rubi in Sympy [A]** time = 114.174, size = 248, normalized size = 0.94

$$\frac{c^3 d^3 \operatorname{atanh} \left( \frac{\sqrt{g} \sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{\sqrt{d+ex} \sqrt{aeg - cdf}} \right)}{8g^{\frac{5}{2}} (aeg - cdf)^{\frac{3}{2}}} - \frac{c^2 d^2 \sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{8g^2 \sqrt{d+ex} (f+gx) (aeg - cdf)}$$

$$- \frac{cd \sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{4g^2 \sqrt{d+ex} (f+gx)^2} - \frac{(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{3}{2}}}{3g(d+ex)^{\frac{3}{2}} (f+gx)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a*d*e + (a*e**2 + c*d**2)*x + c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x$

[Out]  $c**3*d**3*\operatorname{atanh}(\text{sqrt}(g)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/(\text{sqrt}(d + e*x)*\text{sqrt}(a*e*g - c*d*f))/(8*g** (5/2)*(a*e*g - c*d*f)**(3/2)) - c**2*d**2*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c$



$$\frac{d^{**2}}{8g^{**2}\sqrt{d+ex}}(f+gx)(aeg-cdf) - cd\sqrt{ae+cdx} + cd^2e^2x^2 + x(ae^2+cd^2) / (4g^{**2}\sqrt{d+ex})(f+gx)^2 - (ae+cdx)^2(3/2)(f+gx)^3$$

**Mathematica [A]** time = 0.715133, size = 190, normalized size = 0.72

$$\frac{((d+ex)(ae+cdx))^{3/2} \left( \frac{-8a^2e^2g^2+2acdeg(f-7gx)+c^2d^2(3f^2+8fgx-3g^2x^2)}{3g^2(f+gx)^3(ae+cdx)(aeg-cdf)} + \frac{c^3d^3 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{aeg-cdf}}\right)}{g^{5/2}(ae+cdx)^{3/2}(aeg-cdf)^{3/2}} \right)}{8(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/((d + e\*x)^(3/2)\*(f + g\*x

[Out] (((a\*e + c\*d\*x)\*(d + e\*x))^(3/2)\*((-8\*a^2\*e^2\*g^2 + 2\*a\*c\*d\*e\*g\*(f - 7\*g\*x) + c^2\*d^2\*(3\*f^2 + 8\*f\*g\*x - 3\*g^2\*x^2))/(3\*g^2\*(-(c\*d\*f) + a\*e\*g)\*(a\*e + c\*d\*x)\*(f + g\*x)^3) + (c^3\*d^3\*ArcTanh[Sqrt[g]\*Sqrt[a\*e + c\*d\*x)]/Sqrt[-(c\*d\*f) + a\*e\*g]))/(g^(5/2)\*(-(c\*d\*f) + a\*e\*g)^(3/2)\*(a\*e + c\*d\*x)^(3/2)))/(8\*(d + e\*x)^(3/2))

**Maple [A]** time = 0.037, size = 453, normalized size = 1.7

$$\frac{1}{(24aeg - 24cdf)g^2(gx + f)^3} \sqrt{cdex^2 + ae^2x + cd^2x + ade} \left( 3 \operatorname{Artanh} \left( \frac{g\sqrt{cdx + ae}}{\sqrt{(aeg - cdf)g}} \right) x^3 c^3 d^3 g^3 + 9 \operatorname{Artanh} \left( \frac{g\sqrt{cdx + ae}}{\sqrt{(aeg - cdf)g}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)/(e\*x+d)^(3/2)/(g\*x+f)^4,x)

[Out] 1/24\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)\*(3\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x^3\*c^3\*d^3\*g^3+9\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x^2\*c^3\*d^3\*f\*g^2+9\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x\*c^3\*d^3\*f^2\*g+3\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*c^3\*d^3\*f^3+3\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*x^2\*c^2\*d^2\*g^2-14\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*x\*a\*c\*d\*e\*g^2+8\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*x\*c^2\*d^2\*f\*g-8\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*a^2\*e^2\*g^2+2\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*a\*c\*d\*e\*f\*g+3\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*c^2\*d^2\*f^2)/(e\*x+d)^(1/2)/(c\*d\*x+a\*e)^(1/2)/(a\*e\*g-c\*d\*f)/g^2/(g\*x+f)^3/((a\*e\*g-c\*d\*f)\*g)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.297627, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^{(3/2)} / ((e*x + d)^{(3/2)} * (g*x + f)^{(3/2)}))$

[Out] 
$$\left[ \frac{1}{48} \left( 2 \left( 3c^2d^2g^2x^2 - 3c^2d^2f^2 - 2acde^fg + 8a^2e^2g^2 - 2(4c^2d^2fg - 7acde^g^2)x \right) \sqrt{cde^x^2 + a*d*e + (c*d^2 + a*e^2)*x} \sqrt{-cdf^g + ae^g^2} \sqrt{e*x + d} - 3(c^3d^3e^g^3x^4 + c^3d^4f^3 + (3c^3d^3e^fg^2 + c^3d^4g^3)x^3 + 3(c^3d^3e^f^2g + c^3d^4f^2g^2)x^2 + (c^3d^3e^f^3 + 3c^3d^4f^2g)x) \log\left(\frac{2\sqrt{cde^x^2 + a*d*e + (c*d^2 + a*e^2)*x} (cdf^g - ae^g^2) \sqrt{e*x + d} - (cde^g^x^2 - c*d^2*f + 2a*d*e*g - (c*d^2 + 2a*e^2)*g)x}{-cdf^g + ae^g^2}\right) / (e^g*x^2 + d*f + (e*f + d*g)*x) \right) / \left( (c^4d^2f^4g^2 - a*d^2e^f^3g^3 + (c*d^2e^f^5g^5 - a^2e^2g^6)x^4 + (3c*d^2e^f^2g^4 - a*d^2e^fg^6 + (c*d^2 - 3a^2e^2)*f^2g^5)x^3 + 3(c*d^2e^f^3g^3 - a*d^2e^fg^5 + (c*d^2 - a^2e^2)*f^2g^4)x^2 + (c*d^2e^f^4g^2 - 3a*d^2e^f^2g^4 + (3c*d^2 - a^2e^2)*f^3g^3)x \right) \sqrt{-cdf^g + ae^g^2} \right), \frac{1}{24} \left( (3c^2d^2g^2x^2 - 3c^2d^2f^2 - 2acde^fg + 8a^2e^2g^2 - 2(4c^2d^2fg - 7acde^g^2)x) \sqrt{cde^x^2 + a*d*e + (c*d^2 + a*e^2)*x} \sqrt{cdf^g - ae^g^2} \sqrt{e*x + d} - 3(c^3d^3e^g^3x^4 + c^3d^4f^3 + (3c^3d^3e^fg^2 + c^3d^4g^3)x^3 + 3(c^3d^3e^f^2g + c^3d^4f^2g^2)x^2 + (c^3d^3e^f^3 + 3c^3d^4f^2g)x) \arctan\left(\frac{\sqrt{cde^x^2 + a*d*e + (c*d^2 + a*e^2)*x} \sqrt{cdf^g - ae^g^2} \sqrt{e*x + d}}{(c*d^2e^g^x^2 + a*d^2e^g + (c*d^2 + a*e^2)*g*x)}\right) / \left( (c^4d^2f^4g^2 - a*d^2e^f^3g^3 + (c*d^2e^f^5g^5 - a^2e^2g^6)x^4 + (3c*d^2e^f^2g^4 - a*d^2e^fg^6 + (c*d^2 - 3a^2e^2)*f^2g^5)x^3 + 3(c*d^2e^f^3g^3 - a*d^2e^fg^5 + (c*d^2 - a^2e^2)*f^2g^4)x^2 + (c*d^2e^f^4g^2 - 3a*d^2e^f^2g^4 + (3c*d^2 - a^2e^2)*f^3g^3)x \right) \sqrt{cdf^g - ae^g^2} \right) \right] ]$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a*d*e + (a^2e^2 + c*d^2)*x + c*d*e*x^2)^{(3/2)} / ((e*x + d)^{(3/2)} * (g*x + f)^{(3/2)}))$

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^{(3/2)} / ((e*x + d)^{(3/2)} * (g*x + f)^{(3/2)}))$

[Out] Timed out

$$3.698 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^5} dx$$

**Optimal.** Leaf size=335

$$\frac{3c^4d^4 \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-ae^2}}\right)}{64g^{5/2}(cdf-ae^2)^{5/2}} + \frac{3c^3d^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{64g^2\sqrt{d+ex}(f+gx)(cdf-ae^2)^2}$$

$$+ \frac{c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{32g^2\sqrt{d+ex}(f+gx)^2(cdf-ae^2)} - \frac{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{8g^2\sqrt{d+ex}(f+gx)^3}$$

$$- \frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}$$

[Out]  $-(c*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*g^2*\text{Sqrt}[d + e*x]*(f + g*x)^3) + (c^2*d^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*g^2*(c*d*f - a*e*g)*\text{Sqrt}[d + e*x]*(f + g*x)^2) + (3*c^3*d^3*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*g^2*(c*d*f - a*e*g)^2*\text{Sqrt}[d + e*x]*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/(4*g*(d + e*x)^{(3/2)}*(f + g*x)^4) + (3*c^4*d^4*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x]))/(64*g^{(5/2)}*(c*d*f - a*e*g)^{(5/2)})$

**Rubi [A]** time = 1.50351, antiderivative size = 335, normalized size of antiderivative = 1., number of rules used = 6, number of rules used = 4, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{3c^4d^4 \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-ae^2}}\right)}{64g^{5/2}(cdf-ae^2)^{5/2}} + \frac{3c^3d^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{64g^2\sqrt{d+ex}(f+gx)(cdf-ae^2)^2}$$

$$+ \frac{c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{32g^2\sqrt{d+ex}(f+gx)^2(cdf-ae^2)} - \frac{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{8g^2\sqrt{d+ex}(f+gx)^3}$$

$$- \frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{4g(d+ex)^{3/2}(f+gx)^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/((d + e*x)^{(3/2)}*(f + g*x)^5), x]$

[Out]  $-(c*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*g^2*\text{Sqrt}[d + e*x]*(f + g*x)^3) + (c^2*d^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*g^2*(c*d*f - a*e*g)*\text{Sqrt}[d + e*x]*(f + g*x)^2) + (3*c^3*d^3*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*g^2*(c*d*f - a*e*g)^2*\text{Sqrt}[d + e*x]*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/(4*g*(d + e*x)^{(3/2)}*(f + g*x)^4) + (3*c^4*d^4*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x]))/(64*g^{(5/2)}*(c*d*f - a*e*g)^{(5/2)})$

**Rubi in Sympy [A]** time = 150.577, size = 320, normalized size = 0.96

$$\frac{3c^4d^4 \operatorname{atanh}\left(\frac{\sqrt{g}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}\sqrt{aeg-cdf}}\right)}{64g^{\frac{5}{2}}(aeg-cdf)^{\frac{5}{2}}}$$

$$+ \frac{3c^3d^3\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{64g^2\sqrt{d+ex}(f+gx)(aeg-cdf)^2} - \frac{c^2d^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{32g^2\sqrt{d+ex}(f+gx)^2(aeg-cdf)}$$

$$- \frac{cd\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{8g^2\sqrt{d+ex}(f+gx)^3} - \frac{(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}{4g(d+ex)^{\frac{3}{2}}(f+gx)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x`

[Out] 
$$\frac{-3c^{**4}d^{**4}\operatorname{atanh}(\operatorname{sqrt}(g)\operatorname{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/(\operatorname{sqrt}(d + e*x)\operatorname{sqrt}(a*e*g - c*d*f)))/(64*g^{**5/2}(a*e*g - c*d*f)**(5/2)) + 3*c^{**3}d^{**3}\operatorname{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/(64*g^{**2}\operatorname{sqrt}(d + e*x)*(f + g*x)*(a*e*g - c*d*f)**2) - c^{**2}d^{**2}\operatorname{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/(32*g^{**2}\operatorname{sqrt}(d + e*x)*(f + g*x)**2*(a*e*g - c*d*f)) - c*d*\operatorname{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(8*g^{**2}\operatorname{sqrt}(d + e*x)*(f + g*x)**3) - (a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))^{**3/2}/(4*g*(d + e*x)**(3/2)*(f + g*x)**4)$$

**Mathematica [A]** time = 1.0119, size = 201, normalized size = 0.6

$$\frac{((d + ex)(ae + cdx))^{3/2} \left( \frac{3c^3 d^3 (f+gx)^3 + 2c^2 d^2 (f+gx)^2 + 16(cdf - aeg) - 24cd(f+gx)}{g^2 (f+gx)^4 (ae+cdx)} - \frac{3c^4 d^4 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{aeg-cdf}}\right)}{g^{5/2} (ae+cdx)^{3/2} (aeg-cdf)^{5/2}} \right)}{64(d + ex)^{3/2}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)/((d + e*x)^(3/2)*(f + g*x`

[Out] 
$$\frac{(((a*e + c*d*x)^*(d + e*x))^{3/2})*((16*(c*d*f - a*e*g) - 24*c*d*(f + g*x) + (2*c^2*d^2*(f + g*x)^2)/(c*d*f - a*e*g) + (3*c^3*d^3*(f + g*x)^3)/(c*d*f - a*e*g)^2)/(g^{5/2}*(a*e + c*d*x)^*(f + g*x)^4) - (3*c^4*d^4*\operatorname{ArcTanh}[\operatorname{Sqrt}[g]*\operatorname{Sqrt}[a*e + c*d*x] ]/\operatorname{Sqrt}[-(c*d*f) + a*e*g])/(g^{5/2}*(-(c*d*f) + a*e*g)^{5/2}*(a*e + c*d*x)^{3/2}))}{64*(d + e*x)^{3/2}}$$

**Maple [B]** time = 0.045, size = 665, normalized size = 2.

$$\frac{1}{64(gx + f)^4 g^2 (aeg - cdf)^2} \sqrt{cdex^2 + ae^2x + cd^2x + ade} \left( 3 \operatorname{Artanh}\left(\frac{g\sqrt{cdx + ae}}{\sqrt{(aeg - cdf)g}}\right) x^4 c^4 d^4 g^4 + 12 \operatorname{Artanh}\left(\frac{g\sqrt{cdx + ae}}{\sqrt{(aeg - cdf)g}}\right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2)/(g*x+f)^5,x)`

[Out] 
$$\frac{-1/64*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^{(1/2)}*(3*\operatorname{arctanh}(g*(c*d*x+a*e)^{(1/2)})/((a*e*g-c*d*f)*g)^{(1/2)})*x^4*c^4*d^4*g^4+12*\operatorname{arctanh}(g*(c*d*x+a*e)^{(1/2)})/((a*e*g-c*d*f)*g)^{(1/2)})*x^3*c^4*d^4*f*g^3+18*\operatorname{arctanh}(g*(c*d*x+a*e)^{(1/2)})/((a*e*g-c*d*f)*g)^{(1/2)})*x^2*c^4*d^4*f^2*g^2+12*\operatorname{arctanh}(g*(c*d*x+a*e)^{(1/2)})/((a*e*g-c*d*f)*g)^{(1/2)})*x*c^4*d^4*f^3*g-3*x^3*c^3*d^3*g^3*(c*d*x+a*e)^{(1/2)}*((a*e*g-c*d*f)*g)^{(1/2)}+3*\operatorname{arctanh}(g*(c*d*x+a*e)^{(1/2)})/((a*e*g-c*d*f)*g)^{(1/2)})*c^4*d^4*f^4+2*x^2*a*c^2*d^2*e*g^3*(c*d*x+a*e)^{(1/2)}*((a*e*g-c*d*f)*g)^{(1/2)}-11*x^2*c^3*d^3*f*g^2*(c*d*x+a*e)^{(1/2)}*((a*e*g-c*d*f)*g)^{(1/2)}+24*x*a^2*c*d*e^2*g^3*(c*d*x+a*e)^{(1/2)}*((a*e*g-c*d*f)*g)^{(1/2)}-44*x*a*c^2*d^2*e*f*g^2*(c*d*x+a*e)^{(1/2)}*((a*e*g-c*d*f)*g)^{(1/2)}+11*x*c^3*d^3*f^2*g*(c*d*x+a*e)^{(1/2)}*((a*e*g-c*d*f)*g)^{(1/2)}+16*((a*e*g-c*d*f)*g)^{(1/2)}*(c*d*x+a*e)^{(1/2)}*a^3*e^3*g^3-24*((a*e*g-c*d*f)*g)^{(1/2)}*(c*d*x+a*e)^{(1/2)}*a^2*c*d*e^2*f*g^2+2*((a*e*g-c*d*f)*g)^{(1/2)}*(c*d*x+a*e)^{(1/2)}*a*c^2*d^2*e*f^2*g+3*((a*e*g-c*d*f)*g)^{(1/2)}*(c*d*x+a*e)^{(1/2)}*c^3*d^3*f^3)/(e*x+d)^{(1/2)}/((a*e*g-c*d*f)*g)^{(1/2)}/(g*x+f)^4/g^2/(a*e*g-c*d*f)^2/(c*d*x+a*e)^{(1/2)}$$

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**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f

[Out] Exception raised: ValueError

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**Fricas [A]** time = 0.307154, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f

[Out] [1/128\*(2\*(3\*c^3\*d^3\*g^3\*x^3 - 3\*c^3\*d^3\*f^3 - 2\*a\*c^2\*d^2\*e\*f^2\*g + 24\*a^2\*c\*d\*e^2\*f\*g^2 - 16\*a^3\*e^3\*g^3 + (11\*c^3\*d^3\*f\*g^2 - 2\*a\*c^2\*d^2\*e\*g^3)\*x^2 - (11\*c^3\*d^3\*f^2\*g - 44\*a\*c^2\*d^2\*e\*f\*g^2 + 24\*a^2\*c\*d\*e^2\*g^3)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*f\*g + a\*e\*g^2)\*sqrt(e\*x + d) + 3\*(c^4\*d^4\*e\*g^4\*x^5 + c^4\*d^5\*f^4 + (4\*c^4\*d^4\*e\*f\*g^3 + c^4\*d^5\*g^4)\*x^4 + 2\*(3\*c^4\*d^4\*e\*f^2\*g^2 + 2\*c^4\*d^5\*f\*g^3)\*x^3 + 2\*(2\*c^4\*d^4\*e\*f^3\*g + 3\*c^4\*d^5\*f^2\*g^2)\*x^2 + (c^4\*d^4\*e\*f^4 + 4\*c^4\*d^5\*f^3\*g)\*x)\*log(-(2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(c\*d\*f\*g - a\*e\*g^2)\*sqrt(e\*x + d) + (c\*d\*e\*g\*x^2 - c\*d^2\*f + 2\*a\*d\*e\*g - (c\*d\*e\*f - (c\*d^2 + 2\*a\*e^2)\*g)\*x)\*sqrt(-c\*d\*f\*g + a\*e\*g^2))/(e\*g\*x^2 + d\*f + (e\*f + d\*g)\*x))/((c^2\*d^3\*f^6\*g^2 - 2\*a\*c\*d^2\*e\*f^5\*g^3 + a^2\*d\*e^2\*f^4\*g^4 + (c^2\*d^2\*e\*f^2\*g^6 - 2\*a\*c\*d\*e^2\*f\*g^7 + a^2\*e^3\*g^8)\*x^5 + (4\*c^2\*d^2\*e\*f^3\*g^5 + a^2\*d\*e^2\*g^8 + (c^2\*d^3 - 8\*a\*c\*d\*e^2)\*f^2\*g^6 - 2\*(a\*c\*d^2\*e - 2\*a^2\*e^3)\*f\*g^7)\*x^4 + 2\*(3\*c^2\*d^2\*e\*f^4\*g^4 + 2\*a^2\*d\*e^2\*f\*g^7 + 2\*(c^2\*d^3 - 3\*a\*c\*d\*e^2)\*f^3\*g^5 - (4\*a\*c\*d^2\*e - 3\*a^2\*e^3)\*f^2\*g^6)\*x^3 + 2\*(2\*c^2\*d^2\*e\*f^5\*g^3 + 3\*a^2\*d\*e^2\*f^2\*g^6 + (3\*c^2\*d^3 - 4\*a\*c\*d\*e^2)\*f^4\*g^4 - 2\*(3\*a\*c\*d^2\*e - a^2\*e^3)\*f^3\*g^5)\*x^2 + (c^2\*d^2\*e\*f^6\*g^2 + 4\*a^2\*d\*e^2\*f^3\*g^5 + 2\*(2\*c^2\*d^3 - a\*c\*d\*e^2)\*f^5\*g^3 - (8\*a\*c\*d^2\*e - a^2\*e^3)\*f^4\*g^4)\*x)\*sqrt(-c\*d\*f\*g + a\*e\*g^2), 1/64\*((3\*c^3\*d^3\*g^3\*x^3 - 3\*c^3\*d^3\*f^3 - 2\*a\*c^2\*d^2\*e\*f^2\*g + 24\*a^2\*c\*d\*e^2\*f\*g^2 - 16\*a^3\*e^3\*g^3 + (11\*c^3\*d^3\*f\*g^2 - 2\*a\*c^2\*d^2\*e\*g^3)\*x^2 - (11\*c^3\*d^3\*f^2\*g - 44\*a\*c^2\*d^2\*e\*f\*g^2 + 24\*a^2\*c\*d\*e^2\*g^3)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*f\*g - a\*e\*g^2)\*sqrt(e\*x + d) - 3\*(c^4\*d^4\*e\*g^4\*x^5 + c^4\*d^5\*f^4 + (4\*c^4\*d^4\*e\*f\*g^3 + c^4\*d^5\*g^4)\*x^4 + 2\*(3\*c^4\*d^4\*e\*f^2\*g^2 + 2\*c^4\*d^5\*f\*g^3)\*x^3 + 2\*(2\*c^4\*d^4\*e\*f^3\*g + 3\*c^4\*d^5\*f^2\*g^2)\*x^2 + (c^4\*d^4\*e\*f^4 + 4\*c^4\*d^5\*f^3\*g)\*x)\*arctan(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*f\*g - a\*e\*g^2)\*sqrt(e\*x + d)/(c\*d\*e\*g\*x^2 + a\*d\*e\*g + (c\*d^2 + a\*e^2)\*g\*x))/((c^2\*d^3\*f^6\*g^2 - 2\*a\*c\*d^2\*e\*f^5\*g^3 + a^2\*d\*e^2\*f^4\*g^4 + (c^2\*d^2\*e\*f^2\*g^6 - 2\*a\*c\*d\*e^2\*f\*g^7 + a^2\*e^3\*g^8)\*x^5 + (4\*c^2\*d^2\*e\*f^3\*g^5 + a^2\*d\*e^2\*g^8 + (c^2\*d^3 - 8\*a\*c\*d\*e^2)\*f^2\*g^6 - 2\*(a\*c\*d^2\*e - 2\*a^2\*e^3)\*f\*g^7)\*x^4 + 2\*(3\*c^2\*d^2\*e\*f^4\*g^4 + 2\*a^2\*d\*e^2\*f\*g^7 + 2\*(c^2\*d^3 - 3\*a\*c\*d\*e^2)\*f^3\*g^5 - (4\*a\*c\*d^2\*e - 3\*a^2\*e^3)\*f^2\*g^6)\*x^3 + 2\*(2\*c^2\*d^2\*e\*f^5\*g^3 + 3\*a^2\*d\*e^2\*f^2\*g^6 + (3\*c^2\*d^3 - 4\*a\*c\*d\*e^2)\*f^4\*g^4 - 2\*(3\*a\*c\*d^2\*e - a^2\*e^3)\*f^3\*g^5)\*x^2 + (c^2\*d^2\*e\*f^6\*g^2 + 4\*a^2\*d\*e^2\*f^3\*g^5 + 2\*(2\*c^2\*d^3 - a\*c\*d\*e^2)\*f^5\*g^3 - (8\*a\*c\*d^2\*e - a^2\*e^3)\*f^4\*g^4)\*x)\*sqrt(c\*d\*f\*g - a\*e\*g^2)]

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**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/(e\*x+d)\*\*(3/2)/(g\*x+f)\*\*

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f)

[Out] Timed out

$$3.699 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^6} dx$$

**Optimal.** Leaf size=405

$$\begin{aligned} & \frac{3c^5 d^5 \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{128g^{5/2}(cdf-aeg)^{7/2}} + \frac{3c^4 d^4 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{128g^2 \sqrt{d+ex}(f+gx)(cdf-aeg)^3} \\ & + \frac{c^3 d^3 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{64g^2 \sqrt{d+ex}(f+gx)^2(cdf-aeg)^2} + \frac{c^2 d^2 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{80g^2 \sqrt{d+ex}(f+gx)^3(cdf-aeg)} \\ & - \frac{3cd \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{40g^2 \sqrt{d+ex}(f+gx)^4} - \frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5} \end{aligned}$$

[Out]  $(-3*c*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (40*g^2*\text{Sqrt}[d + e*x]*(f + g*x)^4) + (c^2*d^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (80*g^2*(c*d*f - a*e*g)*\text{Sqrt}[d + e*x]*(f + g*x)^3) + (c^3*d^3*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (64*g^2*(c*d*f - a*e*g)^2*\text{Sqrt}[d + e*x]*(f + g*x)^2) + (3*c^4*d^4*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (128*g^2*(c*d*f - a*e*g)^3*\text{Sqrt}[d + e*x]*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)} / (5*g*(d + e*x)^{(3/2)}*(f + g*x)^5) + (3*c^5*d^5*\text{ArcTan}[\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x])]) / (128*g^{(5/2)}*(c*d*f - a*e*g)^{(7/2)})$

**Rubi [A]** time = 2.08671, antiderivative size = 405, normalized size of antiderivative = 1., number of rules used = 7, number of rules used = 4, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\begin{aligned} & \frac{3c^5 d^5 \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{128g^{5/2}(cdf-aeg)^{7/2}} + \frac{3c^4 d^4 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{128g^2 \sqrt{d+ex}(f+gx)(cdf-aeg)^3} \\ & + \frac{c^3 d^3 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{64g^2 \sqrt{d+ex}(f+gx)^2(cdf-aeg)^2} + \frac{c^2 d^2 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{80g^2 \sqrt{d+ex}(f+gx)^3(cdf-aeg)} \\ & - \frac{3cd \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{40g^2 \sqrt{d+ex}(f+gx)^4} - \frac{(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{5g(d+ex)^{3/2}(f+gx)^5} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)} / ((d + e*x)^{(3/2)}*(f + g*x)^6), x]$

[Out]  $(-3*c*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (40*g^2*\text{Sqrt}[d + e*x]*(f + g*x)^4) + (c^2*d^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (80*g^2*(c*d*f - a*e*g)*\text{Sqrt}[d + e*x]*(f + g*x)^3) + (c^3*d^3*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (64*g^2*(c*d*f - a*e*g)^2*\text{Sqrt}[d + e*x]*(f + g*x)^2) + (3*c^4*d^4*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (128*g^2*(c*d*f - a*e*g)^3*\text{Sqrt}[d + e*x]*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)} / (5*g*(d + e*x)^{(3/2)}*(f + g*x)^5) + (3*c^5*d^5*\text{ArcTan}[\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x])]) / (128*g^{(5/2)}*(c*d*f - a*e*g)^{(7/2)})$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a*d*e + (a*e^2 + c*d^2)*x + c*d*e*x^2)^{(3/2)} / (e*x + d)^{(3/2)} / (g*x$

[Out] Timed out

**Mathematica [A]** time = 1.36176, size = 231, normalized size = 0.57

$$\frac{((d + ex)(ae + cdx))^{3/2} \left( \frac{3c^5 d^5 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{aeg-cdf}}\right)}{g^{5/2}(ae+cdx)^{3/2}(aeg-cdf)^{7/2}} + \frac{\frac{15c^4 d^4 (f+gx)^4}{(cdf-aeg)^3} + \frac{10c^3 d^3 (f+gx)^3}{(cdf-aeg)^2} + \frac{8c^2 d^2 (f+gx)^2}{cdf-aeg} + 128(cdf-aeg) - 176cd(f+gx)}{5g^2(f+gx)^5(ae+cdx)} \right)}{128(d + ex)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/((d + e\*x)^(3/2)\*(f + g\*x

[Out] (((a\*e + c\*d\*x)\*(d + e\*x))^(3/2)\*((128\*(c\*d\*f - a\*e\*g) - 176\*c\*d\*(f + g\*x) + (8\*c^2\*d^2\*(f + g\*x)^2)/(c\*d\*f - a\*e\*g) + (10\*c^3\*d^3\*(f + g\*x)^3)/(c\*d\*f - a\*e\*g)^2 + (15\*c^4\*d^4\*(f + g\*x)^4)/(c\*d\*f - a\*e\*g)^3)/(5\*g^2\*(a\*e + c\*d\*x)\*(f + g\*x)^5) + (3\*c^5\*d^5\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/Sqrt[-(c\*d\*f) + a\*e\*g]])/(g^(5/2)\*(-(c\*d\*f) + a\*e\*g)^(7/2)\*(a\*e + c\*d\*x)^(3/2)))/(128\*(d + e\*x)^(3/2))

**Maple [B]** time = 0.048, size = 955, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)/(e\*x+d)^(3/2)/(g\*x+f)^6,x)

[Out] 1/640\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)\*(15\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x^5\*c^5\*d^5\*g^5+75\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x^4\*c^5\*d^5\*f\*g^4+150\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x^3\*c^5\*d^5\*f^2\*g^3+150\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x^2\*c^5\*d^5\*f^3\*g^2-15\*x^4\*c^4\*d^4\*g^4\*(c\*d\*x+a\*e)^(1/2)\*((a\*e\*g-c\*d\*f)\*g)^(1/2)+75\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x\*c^5\*d^5\*f^4\*g+10\*x^3\*a\*c^3\*d^3\*e\*g^4\*(c\*d\*x+a\*e)^(1/2)\*((a\*e\*g-c\*d\*f)\*g)^(1/2)-70\*x^3\*c^4\*d^4\*f\*g^3\*(c\*d\*x+a\*e)^(1/2)\*((a\*e\*g-c\*d\*f)\*g)^(1/2)+15\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*c^5\*d^5\*f^5-8\*x^2\*a^2\*c^2\*d^2\*e^2\*g^4\*(c\*d\*x+a\*e)^(1/2)\*((a\*e\*g-c\*d\*f)\*g)^(1/2)+46\*x^2\*a\*c^3\*d^3\*e\*f\*g^3\*(c\*d\*x+a\*e)^(1/2)\*((a\*e\*g-c\*d\*f)\*g)^(1/2)-128\*x^2\*c^4\*d^4\*f^2\*g^2\*(c\*d\*x+a\*e)^(1/2)\*((a\*e\*g-c\*d\*f)\*g)^(1/2)-176\*x\*a^3\*c\*d\*e^3\*g^4\*(c\*d\*x+a\*e)^(1/2)\*((a\*e\*g-c\*d\*f)\*g)^(1/2)+512\*x\*a^2\*c^2\*d^2\*e^2\*f\*g^3\*(c\*d\*x+a\*e)^(1/2)\*((a\*e\*g-c\*d\*f)\*g)^(1/2)-466\*x\*a\*c^3\*d^3\*e\*f^2\*g^2\*(c\*d\*x+a\*e)^(1/2)\*((a\*e\*g-c\*d\*f)\*g)^(1/2)+70\*x\*c^4\*d^4\*f^3\*g\*(c\*d\*x+a\*e)^(1/2)\*((a\*e\*g-c\*d\*f)\*g)^(1/2)-128\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*a^4\*e^4\*g^4+336\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*a^3\*c\*d\*e^3\*f\*g^3-248\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*a^2\*c^2\*d^2\*e^2\*f^2\*g^2+10\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*a\*c^3\*d^3\*e\*f^3\*g+15\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*c^4\*d^4\*f^4)/(e\*x+d)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2)/(g\*x+f)^5/g^2/(a\*e\*g-c\*d\*f)/(a^2\*e^2\*g^2-2\*a\*c\*d\*e\*f\*g+c^2\*d^2\*f^2)/(c\*d\*x+a\*e)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.323933, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f

[Out] [1/1280\*(2\*(15\*c^4\*d^4\*g^4\*x^4 - 15\*c^4\*d^4\*f^4 - 10\*a\*c^3\*d^3\*e\*f^3\*g + 248\*a^2\*c^2\*d^2\*e^2\*f^2\*g^2 - 336\*a^3\*c\*d\*e^3\*f\*g^3 + 128\*a^4\*e^4\*g^4 + 10\*(7\*c^4\*d^4\*f\*g^3 - a\*c^3\*d^3\*e\*g^4)\*x^3 + 2\*(64\*c^4\*d^4\*f^2\*g^2 - 23\*a\*c^3\*d^3\*e\*f\*g^3 + 4\*a^2\*c^2\*d^2\*e^2\*g^4)\*x^2 - 2\*(35\*c^4\*d^4\*f^3\*g - 233\*a\*c^3\*d^3\*e\*f^2\*g^2 + 256\*a^2\*c^2\*d^2\*e^2\*f\*g^3 - 88\*a^3\*c\*d\*e^3\*g^4)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*f\*g + a\*e\*g^2)\*sqrt(e\*x + d) - 15\*(c^5\*d^5\*e\*g^5\*x^6 + c^5\*d^6\*f^5 + (5\*c^5\*d^5\*e\*f\*g^4 + c^5\*d^6\*g^5)\*x^5 + 5\*(2\*c^5\*d^5\*e\*f^2\*g^3 + c^5\*d^6\*f\*g^4)\*x^4 + 10\*(c^5\*d^5\*e\*f^3\*g^2 + c^5\*d^6\*f^2\*g^3)\*x^3 + 5\*(c^5\*d^5\*e\*f^4\*g + 2\*c^5\*d^6\*f^3\*g^2)\*x^2 + (c^5\*d^5\*e\*f^5 + 5\*c^5\*d^6\*f^4\*g)\*x)\*log((2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(c\*d\*f\*g - a\*e\*g^2)\*sqrt(e\*x + d) - (c\*d\*e\*g\*x^2 - c\*d^2\*f + 2\*a\*d\*e\*g - (c\*d\*e\*f - (c\*d^2 + 2\*a\*e^2)\*g)\*x)\*sqrt(-c\*d\*f\*g + a\*e\*g^2))/(e\*g\*x^2 + d\*f + (e\*f + d\*g)\*x))/((c^3\*d^4\*f^8\*g^2 - 3\*a\*c^2\*d^3\*e\*f^7\*g^3 + 3\*a^2\*c\*d^2\*e^2\*f^6\*g^4 - a^3\*d\*e^3\*f^5\*g^5 + (c^3\*d^3\*e\*f^3\*g^7 - 3\*a\*c^2\*d^2\*e^2\*f^2\*g^8 + 3\*a^2\*c\*d\*e^3\*f\*g^9 - a^3\*e^4\*g^10)\*x^6 + (5\*c^3\*d^3\*e\*f^4\*g^6 - a^3\*d\*e^3\*g^10 + (c^3\*d^4 - 15\*a\*c^2\*d^2\*e^2)\*f^3\*g^7 - 3\*(a\*c^2\*d^3\*e - 5\*a^2\*c\*d\*e^3)\*f^2\*g^8 + (3\*a^2\*c\*d^2\*e^2 - 5\*a^3\*e^4)\*f\*g^9)\*x^5 + 5\*(2\*c^3\*d^3\*e\*f^5\*g^5 - a^3\*d\*e^3\*f\*g^9 + (c^3\*d^4 - 6\*a\*c^2\*d^2\*e^2)\*f^4\*g^6 - 3\*(a\*c^2\*d^3\*e - 2\*a^2\*c\*d\*e^3)\*f^3\*g^7 + (3\*a^2\*c\*d^2\*e^2 - 2\*a^3\*e^4)\*f^2\*g^8)\*x^4 + 10\*(c^3\*d^3\*e\*f^6\*g^4 - a^3\*d\*e^3\*f^2\*g^8 + (c^3\*d^4 - 3\*a\*c^2\*d^2\*e^2)\*f^5\*g^5 - 3\*(a\*c^2\*d^3\*e - a^2\*c\*d\*e^3)\*f^4\*g^6 + (3\*a^2\*c\*d^2\*e^2 - a^3\*e^4)\*f^3\*g^7)\*x^3 + 5\*(c^3\*d^3\*e\*f^7\*g^3 - 2\*a^3\*d\*e^3\*f^3\*g^7 + (2\*c^3\*d^4 - 3\*a\*c^2\*d^2\*e^2)\*f^6\*g^4 - 3\*(2\*a\*c^2\*d^3\*e - a^2\*c\*d\*e^3)\*f^5\*g^5 + (6\*a^2\*c\*d^2\*e^2 - a^3\*e^4)\*f^4\*g^6)\*x^2 + (c^3\*d^3\*e\*f^8\*g^2 - 5\*a^3\*d\*e^3\*f^4\*g^6 + (5\*c^3\*d^4 - 3\*a\*c^2\*d^2\*e^2)\*f^7\*g^3 - 3\*(5\*a\*c^2\*d^3\*e - a^2\*c\*d\*e^3)\*f^6\*g^4 + (15\*a^2\*c\*d^2\*e^2 - a^3\*e^4)\*f^5\*g^5)\*x)\*sqrt(-c\*d\*f\*g + a\*e\*g^2), 1/640\*((15\*c^4\*d^4\*g^4\*x^4 - 15\*c^4\*d^4\*f^4 - 10\*a\*c^3\*d^3\*e\*f^3\*g + 248\*a^2\*c^2\*d^2\*e^2\*f^2\*g^2 - 336\*a^3\*c\*d\*e^3\*f\*g^3 + 128\*a^4\*e^4\*g^4 + 10\*(7\*c^4\*d^4\*f\*g^3 - a\*c^3\*d^3\*e\*g^4)\*x^3 + 2\*(64\*c^4\*d^4\*f^2\*g^2 - 23\*a\*c^3\*d^3\*e\*f\*g^3 + 4\*a^2\*c^2\*d^2\*e^2\*g^4)\*x^2 - 2\*(35\*c^4\*d^4\*f^3\*g - 233\*a\*c^3\*d^3\*e\*f^2\*g^2 + 256\*a^2\*c^2\*d^2\*e^2\*f\*g^3 - 88\*a^3\*c\*d\*e^3\*g^4)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*f\*g - a\*e\*g^2)\*sqrt(e\*x + d) - 15\*(c^5\*d^5\*e\*g^5\*x^6 + c^5\*d^6\*f^5 + (5\*c^5\*d^5\*e\*f\*g^4 + c^5\*d^6\*g^5)\*x^5 + 5\*(2\*c^5\*d^5\*e\*f^2\*g^3 + c^5\*d^6\*f\*g^4)\*x^4 + 10\*(c^5\*d^5\*e\*f^3\*g^2 + c^5\*d^6\*f^2\*g^3)\*x^3 + 5\*(c^5\*d^5\*e\*f^4\*g + 2\*c^5\*d^6\*f^3\*g^2)\*x^2 + (c^5\*d^5\*e\*f^5 + 5\*c^5\*d^6\*f^4\*g)\*x)\*arctan(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*f\*g - a\*e\*g^2)\*sqrt(e\*x + d)/(c\*d\*e\*g\*x^2 + a\*d\*e\*g + (c\*d^2 + a\*e^2)\*g\*x))/((c^3\*d^4\*f^8\*g^2 - 3\*a\*c^2\*d^3\*e\*f^7\*g^3 + 3\*a^2\*c\*d^2\*e^2\*f^6\*g^4 - a^3\*d\*e^3\*f^5\*g^5 + (c^3\*d^3\*e\*f^3\*g^7 - 3\*a\*c^2\*d^2\*e^2\*f^2\*g^8 + 3\*a^2\*c\*d\*e^3\*f\*g^9 - a^3\*e^4\*g^10)\*x^6 + (5\*c^3\*d^3\*e\*f^4\*g^6 - a^3\*d\*e^3\*g^10 + (c^3\*d^4 - 15\*a\*c^2\*d^2\*e^2)\*f^3\*g^7 - 3\*(a\*c^2\*d^3\*e - 5\*a^2\*c\*d\*e^3)\*f^2\*g^8 + (3\*a^2\*c\*d^2\*e^2 - 5\*a^3\*e^4)\*f\*g^9)\*x^5 + 5\*(2\*c^3\*d^3\*e\*f^5\*g^5 - a^3\*d\*e^3\*f\*g^9 + (c^3\*d^4 - 6\*a\*c^2\*d^2\*e^2)\*f^4\*g^6 - 3\*(a\*c^2\*d^3\*e - 2\*a^2\*c\*d\*e^3)\*f^3\*g^7 + (3\*a^2\*c\*d^2\*e^2 - 2\*a^3\*e^4)\*f^2\*g^8)\*x^4 + 10\*(c^3\*d^3\*e\*f^6\*g^4 - a^3\*d\*e^3\*f^2\*g^8 + (c^3\*d^4 - 3\*a\*c^2\*d^2\*e^2)\*f^5\*g^5 - 3\*(a\*c^2\*d^3\*e - a^2\*c\*d\*e^3)\*f^4\*g^6 + (3\*a^2\*c\*d^2\*e^2 - a^3\*e^4)\*f^3\*g^7)\*x^3 + 5\*(c^3\*d^3\*e\*f^7\*g^3 - 2\*a^3\*d\*e^3\*f^3\*g^7 + (2\*c^3\*d^4 - 3\*a\*c^2\*d^2\*e^2)\*f^6\*g^4 - 3\*(2\*a\*c^2\*d^3\*e - a^2\*c\*d\*e^3)\*f^5\*g^5 + (6\*a^2\*c\*d^2\*e^2 - a^3\*e^4)\*f^4\*g^6)\*x^2 + (c^3\*d^3\*e\*f^8\*g^2 - 5\*a^3\*d\*e^3\*f^4\*g^6 + (5\*c^3\*d^4 - 3\*a\*c^2\*d^2\*e^2)\*f^7\*g^3 - 3\*(5\*a\*c^2\*d^3\*e - a^2\*c\*d\*e^3)\*f^6\*g^4 + (15\*a^2\*c\*d^2\*e^2 - a^3\*e^4)\*f^5\*g^5)\*x)\*sqrt(-c\*d\*f\*g + a\*e\*g^2)

$$- a^2 c d e^3) f^5 g^5 + (6 a^2 c d^2 e^2 - a^3 e^4) f^4 g^6) x^2 + (c^3 d^3 e f^8 g^2 - 5 a^3 d e^3 f^4 g^6 + (5 c^3 d^4 - 3 a^2 c^2 d^2 e^2) f^7 g^3 - 3 (5 a^2 c^2 d^3 e - a^2 c d e^3) f^6 g^4 + (15 a^2 c d^2 e^2 - a^3 e^4) f^5 g^5) x) \sqrt{c d f g - a e g^2}]$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/(e\*x+d)\*\*(3/2)/(g\*x+f)\*\*

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f

[Out] Timed out

$$3.700 \quad \int \frac{(f+gx)^4 (ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}} dx$$

**Optimal.** Leaf size=336

$$\begin{aligned} & \frac{128 (x (ae^2 + cd^2) + ade + cdex^2)^{7/2} (cdf - aeg)^3 (2ae^2g - cd(9ef - 7dg))}{45045c^5d^5e(d+ex)^{7/2}} \\ & + \frac{128g (x (ae^2 + cd^2) + ade + cdex^2)^{7/2} (cdf - aeg)^3}{6435c^4d^4e(d+ex)^{5/2}} \\ & + \frac{32(f+gx)^2 (x (ae^2 + cd^2) + ade + cdex^2)^{7/2} (cdf - aeg)^2}{715c^3d^3(d+ex)^{7/2}} \\ & + \frac{16(f+gx)^3 (x (ae^2 + cd^2) + ade + cdex^2)^{7/2} (cdf - aeg)}{195c^2d^2(d+ex)^{7/2}} \\ & + \frac{2(f+gx)^4 (x (ae^2 + cd^2) + ade + cdex^2)^{7/2}}{15cd(d+ex)^{7/2}} \end{aligned}$$

[Out]  $(-128*(c*d*f - a*e*g)^3*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)}/(45045*c^5*d^5*e*(d + e*x)^{(7/2)}) + (128*g*(c*d*f - a*e*g)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)}/(6435*c^4*d^4*e*(d + e*x)^{(5/2)}) + (32*(c*d*f - a*e*g)^2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)}/(715*c^3*d^3*(d + e*x)^{(7/2)}) + (16*(c*d*f - a*e*g)*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)}/(195*c^2*d^2*(d + e*x)^{(7/2)}) + (2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)}/(15*c*d*(d + e*x)^{(7/2)})$

**Rubi [A]** time = 1.62424, antiderivative size = 336, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\begin{aligned} & \frac{128 (x (ae^2 + cd^2) + ade + cdex^2)^{7/2} (cdf - aeg)^3 (2ae^2g - cd(9ef - 7dg))}{45045c^5d^5e(d+ex)^{7/2}} \\ & + \frac{128g (x (ae^2 + cd^2) + ade + cdex^2)^{7/2} (cdf - aeg)^3}{6435c^4d^4e(d+ex)^{5/2}} \\ & + \frac{32(f+gx)^2 (x (ae^2 + cd^2) + ade + cdex^2)^{7/2} (cdf - aeg)^2}{715c^3d^3(d+ex)^{7/2}} \\ & + \frac{16(f+gx)^3 (x (ae^2 + cd^2) + ade + cdex^2)^{7/2} (cdf - aeg)}{195c^2d^2(d+ex)^{7/2}} \\ & + \frac{2(f+gx)^4 (x (ae^2 + cd^2) + ade + cdex^2)^{7/2}}{15cd(d+ex)^{7/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}/(d + e*x)^{(5/2)}, x]$

[Out]  $(-128*(c*d*f - a*e*g)^3*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)}/(45045*c^5*d^5*e*(d + e*x)^{(7/2)}) + (128*g*(c*d*f - a*e*g)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)}/(6435*c^4*d^4*e*(d + e*x)^{(5/2)}) + (32*(c*d*f - a*e*g)^2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)}/(715*c^3*d^3*(d + e*x)^{(7/2)}) + (16*(c*d*f - a*e*g)*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)}/(195*c^2*d^2*(d + e*x)^{(7/2)}) + (2*(f + g*x)^4*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)}/(15*c*d*(d + e*x)^{(7/2)})$

**Rubi in Sympy [A]** time = 129.653, size = 330, normalized size = 0.98

$$\frac{2(f+gx)^4(ade+cdex^2+x(ae^2+cd^2))^{\frac{7}{2}}}{15cd(d+ex)^{\frac{7}{2}}} - \frac{16(f+gx)^3(aeg-cdf)(ade+cdex^2+x(ae^2+cd^2))^{\frac{7}{2}}}{195c^2d^2(d+ex)^{\frac{7}{2}}} + \frac{32(f+gx)^2(aeg-cdf)^2(ade+cdex^2+x(ae^2+cd^2))^{\frac{7}{2}}}{715c^3d^3(d+ex)^{\frac{7}{2}}} - \frac{128g(aeg-cdf)^3(ade+cdex^2+x(ae^2+cd^2))^{\frac{7}{2}}}{6435c^4d^4e(d+ex)^{\frac{5}{2}}} + \frac{128(aeg-cdf)^3(ade+cdex^2+x(ae^2+cd^2))^{\frac{7}{2}}(2ae^2g+7cd^2g-9cdf)}{45045c^5d^5e(d+ex)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*4\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)/(e\*x+d)\*\*

[Out] 2\*(f+g\*x)\*\*4\*(a\*d\*e+c\*d\*e\*x\*\*2+x\*(a\*e\*\*2+c\*d\*\*2))\*\*(7/2)/(15\*c\*d\*(d+e\*x)\*\*(7/2))-16\*(f+g\*x)\*\*3\*(a\*e\*g-c\*d\*f)\*(a\*d\*e+c\*d\*e\*x\*\*2+x\*(a\*e\*\*2+c\*d\*\*2))\*\*(7/2)/(195\*c\*\*2\*d\*\*2\*(d+e\*x)\*\*(7/2))+32\*(f+g\*x)\*\*2\*(a\*e\*g-c\*d\*f)\*\*2\*(a\*d\*e+c\*d\*e\*x\*\*2+x\*(a\*e\*\*2+c\*d\*\*2))\*\*(7/2)/(715\*c\*\*3\*d\*\*3\*(d+e\*x)\*\*(7/2))-128\*g\*(a\*e\*g-c\*d\*f)\*\*3\*(a\*d\*e+c\*d\*e\*x\*\*2+x\*(a\*e\*\*2+c\*d\*\*2))\*\*(7/2)/(6435\*c\*\*4\*d\*\*4\*e\*(d+e\*x)\*\*(5/2))+128\*(a\*e\*g-c\*d\*f)\*\*3\*(a\*d\*e+c\*d\*e\*x\*\*2+x\*(a\*e\*\*2+c\*d\*\*2))\*\*(7/2)\*(2\*a\*e\*\*2\*g+7\*c\*d\*\*2\*g-9\*c\*d\*e\*f)/(45045\*c\*\*5\*d\*\*5\*e\*(d+e\*x)\*\*(7/2))

**Mathematica [A]** time = 0.353751, size = 205, normalized size = 0.61

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}(128a^4e^4g^4-64a^3cde^3g^3(15f+7gx)+48a^2c^2d^2e^2g^2(65f^2+70fgx+21g^2x^2)-8ac^3d^3eg^3)}{45045c^5d^5\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((f+g\*x)^4\*(a\*d\*e+(c\*d^2+a\*e^2)\*x+c\*d\*e\*x^2)^(5/2))/(d+e\*x)^(5/2), x]

[Out] (2\*(a\*e+c\*d\*x)^3\*Sqrt[(a\*e+c\*d\*x)\*(d+e\*x)]\*(128\*a^4\*e^4\*g^4-64\*a^3\*c\*d\*e^3\*g^3\*(15\*f+7\*g\*x)+48\*a^2\*c^2\*d^2\*e^2\*g^2\*(65\*f^2+70\*f\*g\*x+21\*g^2\*x^2)-8\*a\*c^3\*d^3\*e\*g\*(715\*f^3+1365\*f^2\*g\*x+945\*f\*g^2\*x^2+231\*g^3\*x^3)+c^4\*d^4\*(6435\*f^4+20020\*f^3\*g\*x+24570\*f^2\*g^2\*x^2+13860\*f\*g^3\*x^3+3003\*g^4\*x^4)))/(45045\*c^5\*d^5\*Sqrt[d+e\*x])

**Maple [A]** time = 0.013, size = 283, normalized size = 0.8

$$\frac{(2cdx+2ae)(3003g^4x^4c^4d^4-1848ac^3d^3eg^4x^3+13860c^4d^4fg^3x^3+1008a^2c^2d^2e^2g^4x^2-7560ac^3d^3efg^3x^2+24570c^4d^4eg^4x)}{45045c^5d^5\sqrt{(d+ex)(ae+cdx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^4\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2)/(e\*x+d)^(5/2), x)

[Out] 2/45045\*(c\*d\*x+a\*e)\*(3003\*c^4\*d^4\*g^4\*x^4-1848\*a\*c^3\*d^3\*e\*g^4\*x^3+13860\*c^4\*d^4\*f\*g^3\*x^3+1008\*a^2\*c^2\*d^2\*e^2\*g^4\*x^2-7560\*a\*c^3\*d^3\*efg^3\*x^2+24570\*c^4\*d^4\*eg^4\*x)

$$d^3 e^f g^3 x^2 + 24570 c^4 d^4 f^2 g^2 x^2 - 448 a^3 c^3 d^3 e^3 g^4 x + 3360 a^2 c^2 d^2 e^2 f^3 g^3 x - 10920 a^3 c^3 d^3 e^2 f^2 g^2 x + 20020 c^4 d^4 e^3 f^3 g^3 x + 128 a^4 e^4 g^4 - 960 a^3 c^3 d^3 e^3 f^3 g^3 + 3120 a^2 c^2 d^2 e^2 f^2 g^2 - 5720 a^3 c^3 d^3 e^2 f^2 g^2 + 6435 c^4 d^4 f^4) (c^2 d^2 e^2 x^2 + a^2 e^2 x + c^2 d^2 x + a^2 d^2 e)^{5/2} / c^5 d^5 (e^2 x + d)^{5/2}$$

**Maxima [A]** time = 0.767596, size = 672, normalized size = 2.

$$\frac{2(c^3 d^3 x^3 + 3 a c^2 d^2 e x^2 + 3 a^2 c d e^2 x + a^3 e^3) \sqrt{c d x + a e} f^4}{7 c d} + \frac{8(7 c^4 d^4 x^4 + 19 a c^3 d^3 e x^3 + 15 a^2 c^2 d^2 e^2 x^2 + a^3 c d e^3 x - 2 a^4 e^4) \sqrt{c d x + a e} f^3 g}{63 c^2 d^2} + \frac{4(63 c^5 d^5 x^5 + 161 a c^4 d^4 e x^4 + 113 a^2 c^3 d^3 e^2 x^3 + 3 a^3 c^2 d^2 e^3 x^2 - 4 a^4 c d e^4 x + 8 a^5 e^5) \sqrt{c d x + a e} f^2 g^2}{231 c^3 d^3} + \frac{8(231 c^6 d^6 x^6 + 567 a c^5 d^5 e x^5 + 371 a^2 c^4 d^4 e^2 x^4 + 5 a^3 c^3 d^3 e^3 x^3 - 6 a^4 c^2 d^2 e^4 x^2 + 8 a^5 c d e^5 x - 16 a^6 e^6) \sqrt{c d x + a e} f g^3}{3003 c^4 d^4} + \frac{2(3003 c^7 d^7 x^7 + 7161 a c^6 d^6 e x^6 + 4473 a^2 c^5 d^5 e^2 x^5 + 35 a^3 c^4 d^4 e^3 x^4 - 40 a^4 c^3 d^3 e^4 x^3 + 48 a^5 c^2 d^2 e^5 x^2 - 64 a^6 c d e^6 x + 128 a^7 e^7) \sqrt{c d x + a e} f^4}{45045 c^5 d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*(g\*x + f)^4/(e\*x + d)^(5/2))

[Out]  $\frac{2}{7} (c^3 d^3 x^3 + 3 a^2 c^2 d^2 e x^2 + 3 a^2 c^2 d^2 e^2 x + a^3 e^3) \sqrt{c^2 d^2 x + a^2 e} f^4 / (c^2 d) + \frac{8}{63} (7 c^4 d^4 x^4 + 19 a^2 c^3 d^3 e x^3 + 15 a^2 c^2 d^2 e^2 x^2 + a^3 c^2 d^2 e^3 x - 2 a^4 e^4) \sqrt{c^2 d^2 x + a^2 e} f^3 g / (c^2 d^2) + \frac{4}{231} (63 c^5 d^5 x^5 + 161 a^2 c^4 d^4 e x^4 + 113 a^2 c^3 d^3 e^2 x^3 + 3 a^3 c^2 d^2 e^3 x^2 - 4 a^4 c^2 d^2 e^4 x + 8 a^5 e^5) \sqrt{c^2 d^2 x + a^2 e} f^2 g^2 / (c^2 d^3) + \frac{8}{3003} (231 c^6 d^6 x^6 + 567 a^2 c^5 d^5 e x^5 + 371 a^2 c^4 d^4 e^2 x^4 + 5 a^3 c^3 d^3 e^3 x^3 - 6 a^4 c^2 d^2 e^4 x^2 + 8 a^5 c^2 d^2 e^5 x - 16 a^6 e^6) \sqrt{c^2 d^2 x + a^2 e} f g^3 / (c^2 d^4) + \frac{2}{45045} (3003 c^7 d^7 x^7 + 7161 a^2 c^6 d^6 e x^6 + 4473 a^2 c^5 d^5 e^2 x^5 + 35 a^3 c^4 d^4 e^3 x^4 - 40 a^4 c^3 d^3 e^4 x^3 + 48 a^5 c^2 d^2 e^5 x^2 - 64 a^6 c^2 d^2 e^6 x + 128 a^7 e^7) \sqrt{c^2 d^2 x + a^2 e} f^4 / (c^2 d^5)$

**Fricas [A]** time = 0.287211, size = 1615, normalized size = 4.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*(g\*x + f)^4/(e\*x + d)^(5/2))

[Out]  $\frac{2}{45045} (3003 c^8 d^8 e^3 g^4 x^9 + 6435 a^4 c^4 d^5 e^4 f^4 - 5720 a^5 c^3 d^4 e^5 f^3 g + 3120 a^6 c^2 d^3 e^6 f^2 g^2 - 960 a^7 c^2 d^2 e^7 f^2 g^3 + 128 a^8 d^2 e^8 g^4 + 231 (60 c^8 d^8 e^3 f^3 g^3 + (13 c^8 d^9 + 44 a^2 c^7 d^7 e^2) g^4) x^8 + 42 (585 c^8 d^8 e^2 f^2 g^2 + 30 (11 c^8 d^9 + 38 a^2 c^7 d^7 e^2) f^2 g^3 + (242 a^2 c^7 d^8 e + 277 a^2 c^6 d^6 e^3) g^4) x^7 + 14 (1430 c^8 d^8 e^3 f^3 g + 195 (9 c^8 d^9 + 32 a^2 c^7 d^7 e^2) f^2 g^2 + 60 (57 a^2 c^7 d^8 e + 67 a^2 c^6 d^6 e^3) f^2 g^3 + (831 a^2 c^6 d^7 e^2 + 322 a^3 c^5 d^5 e^4) g^4) x^6 + (6435 c^8 d^8 e^3 f^4 + 2860 (7 c^8 d^9 + 26 a^2 c^7 d^7 e^2) f^3 g + 780 (112 a^2 c^7 d^8 e + 137 a^2 c^6 d^6 e^3) f^2 g^2 + 120 (469 a^2 c^6 d^7 e^2 + 188 a^3 c^5 d^5 e^4) f^2 g^3 + (4508 a^3 c^5 d^6 e^3 - 5 a^4 c^4 d^4 e^5) g^4) x^5 + (6435 (c^8 d^9 + 4 a^2 c^7 d^7 e^2) f^4 + 5720 (13 a^2 c^7 d^8 e + 17 a^2 c^6 d^6 e^3) f^3 g + 780 (137 a^2 c^6 d^7 e^2 + 58 a^3 c^5 d^5 e^4) f^2 g^2 + 60 (376 a^3 c^5 d^6 e^3 - a^4 c^4 d^4 e^5) f^2 g^3 - (5 a^4 c^4 d^4 e^5 - 8 a^5 c^3 d^3 e^6) g^4) x^4 + 2 (6435 (2 a^2 c^7 d^8 e + 3$

$$\begin{aligned} & *a^2*c^6*d^6*e^3)*f^4 + 2860*(17*a^2*c^6*d^7*e^2 + 8*a^3*c^5*d^5* \\ & e^4)*f^3*g + 195*(116*a^3*c^5*d^6*e^3 - a^4*c^4*d^4*e^5)*f^2*g^2 \\ & - 30*(a^4*c^4*d^5*e^4 - 2*a^5*c^3*d^3*e^6)*f*g^3 + 4*(a^5*c^3*d^4 \\ & *e^5 - 2*a^6*c^2*d^2*e^7)*g^4)*x^3 + 2*(6435*(3*a^2*c^6*d^7*e^2 + \\ & 2*a^3*c^5*d^5*e^4)*f^4 + 1430*(16*a^3*c^5*d^6*e^3 - a^4*c^4*d^4* \\ & e^5)*f^3*g - 195*(a^4*c^4*d^5*e^4 - 4*a^5*c^3*d^3*e^6)*f^2*g^2 + \\ & 60*(a^5*c^3*d^4*e^5 - 4*a^6*c^2*d^2*e^7)*f*g^3 - 8*(a^6*c^2*d^3*e \\ & ^6 - 4*a^7*c*d*e^8)*g^4)*x^2 + (6435*(4*a^3*c^5*d^6*e^3 + a^4*c^4 \\ & *d^4*e^5)*f^4 - 2860*(a^4*c^4*d^5*e^4 + 2*a^5*c^3*d^3*e^6)*f^3*g \\ & + 1560*(a^5*c^3*d^4*e^5 + 2*a^6*c^2*d^2*e^7)*f^2*g^2 - 480*(a^6*c \\ & ^2*d^3*e^6 + 2*a^7*c*d*e^8)*f*g^3 + 64*(a^7*c*d^2*e^7 + 2*a^8*e^9 \\ & )*g^4)*x)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + \\ & d)*c^5*d^5) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*4\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)/(e\*x+d)\*\*(5/2)

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*(g\*x + f)^4/(e\*x + d)^(5/2)

[Out] Exception raised: AttributeError

$$3.701 \quad \int \frac{(f+gx)^3 (ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}} dx$$

Optimal. Leaf size=269

$$\begin{aligned} & \frac{16(x(ae^2+cd^2)+ade+cdex^2)^{7/2}(cdf-aeg)^2(2ae^2g-cd(9ef-7dg))}{3003c^4d^4e(d+ex)^{7/2}} \\ & + \frac{16g(x(ae^2+cd^2)+ade+cdex^2)^{7/2}(cdf-aeg)^2}{429c^3d^3e(d+ex)^{5/2}} \\ & + \frac{12(f+gx)^2(x(ae^2+cd^2)+ade+cdex^2)^{7/2}(cdf-aeg)}{143c^2d^2(d+ex)^{7/2}} \\ & + \frac{2(f+gx)^3(x(ae^2+cd^2)+ade+cdex^2)^{7/2}}{13cd(d+ex)^{7/2}} \end{aligned}$$

[Out]  $(-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{7/2}/(3003*c^4*d^4*e*(d + e*x)^{7/2}) + (16*g*(c*d*f - a*e*g)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{7/2}/(429*c^3*d^3*e*(d + e*x)^{5/2}) + (12*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{7/2}/(143*c^2*d^2*(d + e*x)^{7/2}) + (2*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{7/2}/(13*c*d*(d + e*x)^{7/2})$

Rubi [A] time = 1.14544, antiderivative size = 269, normalized size of antiderivative = 1., number of rules used = 4, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\begin{aligned} & \frac{16(x(ae^2+cd^2)+ade+cdex^2)^{7/2}(cdf-aeg)^2(2ae^2g-cd(9ef-7dg))}{3003c^4d^4e(d+ex)^{7/2}} \\ & + \frac{16g(x(ae^2+cd^2)+ade+cdex^2)^{7/2}(cdf-aeg)^2}{429c^3d^3e(d+ex)^{5/2}} \\ & + \frac{12(f+gx)^2(x(ae^2+cd^2)+ade+cdex^2)^{7/2}(cdf-aeg)}{143c^2d^2(d+ex)^{7/2}} \\ & + \frac{2(f+gx)^3(x(ae^2+cd^2)+ade+cdex^2)^{7/2}}{13cd(d+ex)^{7/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(f+g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}/(d + e*x)^{5/2}, x]$

[Out]  $(-16*(c*d*f - a*e*g)^2*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{7/2}/(3003*c^4*d^4*e*(d + e*x)^{7/2}) + (16*g*(c*d*f - a*e*g)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{7/2}/(429*c^3*d^3*e*(d + e*x)^{5/2}) + (12*(c*d*f - a*e*g)*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{7/2}/(143*c^2*d^2*(d + e*x)^{7/2}) + (2*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{7/2}/(13*c*d*(d + e*x)^{7/2})$

**Rubi in Sympy [A]** time = 94.2038, size = 264, normalized size = 0.98

$$\frac{2(f+gx)^3(ade+cdex^2+x(ae^2+cd^2))^{\frac{7}{2}}}{13cd(d+ex)^{\frac{7}{2}}}$$

$$-\frac{12(f+gx)^2(aeg-cdf)(ade+cdex^2+x(ae^2+cd^2))^{\frac{7}{2}}}{143c^2d^2(d+ex)^{\frac{7}{2}}}$$

$$+\frac{16g(aeg-cdf)^2(ade+cdex^2+x(ae^2+cd^2))^{\frac{7}{2}}}{429c^3d^3e(d+ex)^{\frac{5}{2}}}$$

$$-\frac{16(aeg-cdf)^2(ade+cdex^2+x(ae^2+cd^2))^{\frac{7}{2}}(2ae^2g+7cd^2g-9cdf)}{3003c^4d^4e(d+ex)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x+f)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**`

[Out] `2*(f + g*x)**3*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(7/2)/`  
`(13*c*d*(d + e*x)**(7/2)) - 12*(f + g*x)**2*(a*e*g - c*d*f)*(a*d*`  
`e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(7/2)/(143*c**2*d**2*(d +`  
`e*x)**(7/2)) + 16*g*(a*e*g - c*d*f)**2*(a*d*e + c*d*e*x**2 + x*(a`  
`*e**2 + c*d**2))**(7/2)/(429*c**3*d**3*e*(d + e*x)**(5/2)) - 16*(`  
`a*e*g - c*d*f)**2*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(7/`  
`2)*(2*a*e**2*g + 7*c*d**2*g - 9*c*d*e*f)/(3003*c**4*d**4*e*(d + e`  
`*x)**(7/2))`

**Mathematica [A]** time = 0.280091, size = 147, normalized size = 0.55

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}(-16a^3e^3g^3+8a^2cde^2g^2(13f+7gx)-2ac^2d^2eg(143f^2+182fgx+63g^2x^2)+c^3d^3(429fg^2+12g^3x^2))}{3003c^4d^4\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] `Integrate[((f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)^(5/2), x]`

[Out] `(2*(a*e + c*d*x)^3*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-16*a^3*e^3*g^3`  
`+ 8*a^2*c*d*e^2*g^2*(13*f + 7*g*x) - 2*a*c^2*d^2*e*g*(143*f^2 +`  
`182*f*g*x + 63*g^2*x^2) + c^3*d^3*(429*f^3 + 1001*f^2*g*x + 819*f`  
`*g^2*x^2 + 231*g^3*x^3)))/(3003*c^4*d^4*Sqrt[d + e*x])`

**Maple [A]** time = 0.012, size = 188, normalized size = 0.7

$$\frac{(2cdx+2ae)(-231g^3x^3c^3d^3+126ac^2d^2eg^3x^2-819c^3d^3fg^2x^2-56a^2cde^2g^3x+364ac^2d^2efg^2x-1001c^3d^3f^2gx+16c^4d^4)}{3003c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^3*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2), x)`

[Out] `-2/3003*(c*d*x+a*e)*(-231*c^3*d^3*g^3*x^3+126*a*c^2*d^2*e*g^3*x^2`  
`-819*c^3*d^3*f*g^2*x^2-56*a^2*c*d*e^2*g^3*x+364*a*c^2*d^2*e*f*g^2`  
`*x-1001*c^3*d^3*f^2*g*x+16*a^3*e^3*g^3-104*a^2*c*d*e^2*f*g^2+286*`  
`a*c^2*d^2*e*f^2*g-429*c^3*d^3*f^3)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d`  
`*e)^(5/2)/c^4/d^4/(e*x+d)^(5/2)`



**Maxima [A]** time = 0.745638, size = 489, normalized size = 1.82

$$\frac{2(c^3 d^3 x^3 + 3ac^2 d^2 ex^2 + 3a^2 cde^2 x + a^3 e^3) \sqrt{cdx + aef^3}}{7cd} + \frac{2(7c^4 d^4 x^4 + 19ac^3 d^3 ex^3 + 15a^2 c^2 d^2 e^2 x^2 + a^3 cde^3 x - 2a^4 e^4) \sqrt{cdx + aef^2} g}{21c^2 d^2} + \frac{2(63c^5 d^5 x^5 + 161ac^4 d^4 ex^4 + 113a^2 c^3 d^3 e^2 x^3 + 3a^3 c^2 d^2 e^3 x^2 - 4a^4 cde^4 x + 8a^5 e^5) \sqrt{cdx + aef} g^2}{231c^3 d^3} + \frac{2(231c^6 d^6 x^6 + 567ac^5 d^5 ex^5 + 371a^2 c^4 d^4 e^2 x^4 + 5a^3 c^3 d^3 e^3 x^3 - 6a^4 c^2 d^2 e^4 x^2 + 8a^5 cde^5 x - 16a^6 e^6) \sqrt{cdx + aeg^3}}{3003c^4 d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*(g\*x + f)^3/(e\*x + d)^(5/2))

[Out] 2/7\*(c^3\*d^3\*x^3 + 3\*a\*c^2\*d^2\*e\*x^2 + 3\*a^2\*c\*d\*e^2\*x + a^3\*e^3)\*sqrt(c\*d\*x + a\*e)\*f^3/(c\*d) + 2/21\*(7\*c^4\*d^4\*x^4 + 19\*a\*c^3\*d^3\*e\*x^3 + 15\*a^2\*c^2\*d^2\*e^2\*x^2 + a^3\*c\*d\*e^3\*x - 2\*a^4\*e^4)\*sqrt(c\*d\*x + a\*e)\*f^2\*g/(c^2\*d^2) + 2/231\*(63\*c^5\*d^5\*x^5 + 161\*a\*c^4\*d^4\*e\*x^4 + 113\*a^2\*c^3\*d^3\*e^2\*x^3 + 3\*a^3\*c^2\*d^2\*e^3\*x^2 - 4\*a^4\*c\*d\*e^4\*x + 8\*a^5\*e^5)\*sqrt(c\*d\*x + a\*e)\*f\*g^2/(c^3\*d^3) + 2/3003\*(231\*c^6\*d^6\*x^6 + 567\*a\*c^5\*d^5\*e\*x^5 + 371\*a^2\*c^4\*d^4\*e^2\*x^4 + 5\*a^3\*c^3\*d^3\*e^3\*x^3 - 6\*a^4\*c^2\*d^2\*e^4\*x^2 + 8\*a^5\*c\*d\*e^5\*x - 16\*a^6\*e^6)\*sqrt(c\*d\*x + a\*e)\*g^3/(c^4\*d^4)

**Fricas [A]** time = 0.275202, size = 1187, normalized size = 4.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*(g\*x + f)^3/(e\*x + d)^(5/2))

[Out] 2/3003\*(231\*c^7\*d^7\*e\*g^3\*x^8 + 429\*a^4\*c^3\*d^4\*e^4\*f^3 - 286\*a^5\*c^2\*d^3\*e^5\*f^2\*g + 104\*a^6\*c\*d^2\*e^6\*f\*g^2 - 16\*a^7\*d\*e^7\*g^3 + 21\*(39\*c^7\*d^7\*e\*f\*g^2 + (11\*c^7\*d^8 + 38\*a\*c^6\*d^6\*e^2)\*g^3)\*x^7 + 7\*(143\*c^7\*d^7\*e\*f^2\*g + 13\*(9\*c^7\*d^8 + 32\*a\*c^6\*d^6\*e^2)\*f\*g^2 + 2\*(57\*a\*c^6\*d^7\*e + 67\*a^2\*c^5\*d^5\*e^3)\*g^3)\*x^6 + (429\*c^7\*d^7\*e\*f^3 + 143\*(7\*c^7\*d^8 + 26\*a\*c^6\*d^6\*e^2)\*f^2\*g + 26\*(112\*a\*c^6\*d^7\*e + 137\*a^2\*c^5\*d^5\*e^3)\*f\*g^2 + 2\*(469\*a^2\*c^5\*d^6\*e^2 + 188\*a^3\*c^4\*d^4\*e^4)\*g^3)\*x^5 + (429\*(c^7\*d^8 + 4\*a\*c^6\*d^6\*e^2)\*f^3 + 286\*(13\*a\*c^6\*d^7\*e + 17\*a^2\*c^5\*d^5\*e^3)\*f^2\*g + 26\*(137\*a^2\*c^5\*d^6\*e^2 + 58\*a^3\*c^4\*d^4\*e^4)\*f\*g^2 + (376\*a^3\*c^4\*d^5\*e^3 - a^4\*c^3\*d^3\*e^5)\*g^3)\*x^4 + (858\*(2\*a\*c^6\*d^7\*e + 3\*a^2\*c^5\*d^5\*e^3)\*f^3 + 286\*(17\*a^2\*c^5\*d^6\*e^2 + 8\*a^3\*c^4\*d^4\*e^4)\*f^2\*g + 13\*(116\*a^3\*c^4\*d^5\*e^3 - a^4\*c^3\*d^3\*e^5)\*f\*g^2 - (a^4\*c^3\*d^4\*e^4 - 2\*a^5\*c^2\*d^2\*e^6)\*g^3)\*x^3 + (858\*(3\*a^2\*c^5\*d^6\*e^2 + 2\*a^3\*c^4\*d^4\*e^4)\*f^3 + 143\*(16\*a^3\*c^4\*d^5\*e^3 - a^4\*c^3\*d^3\*e^5)\*f^2\*g - 13\*(a^4\*c^3\*d^4\*e^4 - 4\*a^5\*c^2\*d^2\*e^6)\*f\*g^2 + 2\*(a^5\*c^2\*d^3\*e^5 - 4\*a^6\*c\*d\*e^7)\*g^3)\*x^2 + (429\*(4\*a^3\*c^4\*d^5\*e^3 + a^4\*c^3\*d^3\*e^5)\*f^3 - 143\*(a^4\*c^3\*d^4\*e^4 + 2\*a^5\*c^2\*d^2\*e^6)\*f^2\*g + 52\*(a^5\*c^2\*d^3\*e^5 + 2\*a^6\*c\*d\*e^7)\*f\*g^2 - 8\*(a^6\*c^2\*d^2\*e^6 + 2\*a^7\*e^8)\*g^3)\*x)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*c^4\*d^4)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g*x + f)^3/(e*x + d)^(5/2)
```

```
[Out] Exception raised: AttributeError
```

$$3.702 \quad \int \frac{(f+gx)^2 (ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}} dx$$

**Optimal.** Leaf size=200

$$\frac{8(x(ae^2 + cd^2) + ade + cdex^2)^{7/2} (cdf - aeg) (2ae^2g - cd(9ef - 7dg))}{693c^3d^3e(d+ex)^{7/2}} + \frac{8g(x(ae^2 + cd^2) + ade + cdex^2)^{7/2} (cdf - aeg)}{99c^2d^2e(d+ex)^{5/2}} + \frac{2(f+gx)^2 (x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{11cd(d+ex)^{7/2}}$$

[Out]  $(-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{7/2})/(693*c^3*d^3*e*(d + e*x)^{7/2}) + (8*g*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{7/2})/(99*c^2*d^2*e*(d + e*x)^{5/2}) + (2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{7/2})/(11*c*d*(d + e*x)^{7/2})$

**Rubi [A]** time = 0.709366, antiderivative size = 200, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{8(x(ae^2 + cd^2) + ade + cdex^2)^{7/2} (cdf - aeg) (2ae^2g - cd(9ef - 7dg))}{693c^3d^3e(d+ex)^{7/2}} + \frac{8g(x(ae^2 + cd^2) + ade + cdex^2)^{7/2} (cdf - aeg)}{99c^2d^2e(d+ex)^{5/2}} + \frac{2(f+gx)^2 (x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{11cd(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[((f + g\*x)^2\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2))/(d + e\*x)^(5/2), x

[Out]  $(-8*(c*d*f - a*e*g)*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{7/2})/(693*c^3*d^3*e*(d + e*x)^{7/2}) + (8*g*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{7/2})/(99*c^2*d^2*e*(d + e*x)^{5/2}) + (2*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{7/2})/(11*c*d*(d + e*x)^{7/2})$

**Rubi in Sympy [A]** time = 60.5693, size = 196, normalized size = 0.98

$$\frac{2(f+gx)^2 (ade + cdex^2 + x(ae^2 + cd^2))^{\frac{7}{2}}}{11cd(d+ex)^{\frac{7}{2}}} - \frac{8g(aeg - cdf) (ade + cdex^2 + x(ae^2 + cd^2))^{\frac{7}{2}}}{99c^2d^2e(d+ex)^{\frac{5}{2}}} + \frac{8(aeg - cdf) (ade + cdex^2 + x(ae^2 + cd^2))^{\frac{7}{2}} (2ae^2g + 7cd^2g - 9cdef)}{693c^3d^3e(d+ex)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*2\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)/(e\*x+d)\*\*

[Out]  $2*(f + g*x)**2*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))** (7/2)/(11*c*d*(d + e*x)** (7/2)) - 8*g*(a*e*g - c*d*f)*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))** (7/2)/(99*c**2*d**2*e*(d + e*x)** (5/2)) + 8*(a*e*g - c*d*f)*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))** (7/2)*(2*a*e**2*g + 7*c*d**2*g - 9*c*d*e*f)/(693*c**3*d**3*e*(d + e*x)** (7/2))$

**Mathematica [A]** time = 0.208595, size = 100, normalized size = 0.5

$$\frac{2(ae + cdx)^3 \sqrt{(d+ex)(ae + cdx)} (8a^2e^2g^2 - 4acdeg(11f + 7gx) + c^2d^2(99f^2 + 154fgx + 63g^2x^2))}{693c^3d^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Integrate[((f + g\*x)^2\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2))/(d + e\*x)^(5/2)]

[Out] (2\*(a\*e + c\*d\*x)^3\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)]\*(8\*a^2\*e^2\*g^2 - 4\*a\*c\*d\*e\*g\*(11\*f + 7\*g\*x) + c^2\*d^2\*(99\*f^2 + 154\*f\*g\*x + 63\*g^2\*x^2)))/(693\*c^3\*d^3\*Sqrt[d + e\*x])

**Maple [A]** time = 0.011, size = 116, normalized size = 0.6

$$\frac{(2cdx + 2ae)(63g^2x^2c^2d^2 - 28acdeg^2x + 154c^2d^2fgx + 8a^2e^2g^2 - 44acdefg + 99f^2c^2d^2)}{693c^3d^3} (cdex^2 + ae^2x + cd^2x + ade)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^2\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2)/(e\*x+d)^(5/2), x)

[Out] 2/693\*(c\*d\*x+a\*e)\*(63\*c^2\*d^2\*g^2\*x^2-28\*a\*c\*d\*e\*g^2\*x+154\*c^2\*d^2\*f\*g\*x+8\*a^2\*e^2\*g^2-44\*a\*c\*d\*e\*f\*g+99\*c^2\*d^2\*f^2)\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(5/2)/c^3/d^3/(e\*x+d)^(5/2)

**Maxima [A]** time = 0.735522, size = 328, normalized size = 1.64

$$\frac{2(c^3d^3x^3 + 3ac^2d^2ex^2 + 3a^2cde^2x + a^3e^3)\sqrt{cdx + aef^2}}{7cd} + \frac{4(7c^4d^4x^4 + 19ac^3d^3ex^3 + 15a^2c^2d^2e^2x^2 + a^3cde^3x - 2a^4e^4)\sqrt{cdx + aefg}}{63c^2d^2} + \frac{2(63c^5d^5x^5 + 161ac^4d^4ex^4 + 113a^2c^3d^3e^2x^3 + 3a^3c^2d^2e^3x^2 - 4a^4cde^4x + 8a^5e^5)\sqrt{cdx + aeg^2}}{693c^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*(g\*x + f)^2/(e\*x + d)^(5/2))

[Out] 2/7\*(c^3\*d^3\*x^3 + 3\*a\*c^2\*d^2\*e\*x^2 + 3\*a^2\*c\*d\*e^2\*x + a^3\*e^3)\*sqrt(c\*d\*x + a\*e)\*f^2/(c\*d) + 4/63\*(7\*c^4\*d^4\*x^4 + 19\*a\*c^3\*d^3\*e\*x^3 + 15\*a^2\*c^2\*d^2\*e^2\*x^2 + a^3\*c\*d\*e^3\*x - 2\*a^4\*e^4)\*sqrt(c\*d\*x + a\*e)\*f\*g/(c^2\*d^2) + 2/693\*(63\*c^5\*d^5\*x^5 + 161\*a\*c^4\*d^4\*e\*x^4 + 113\*a^2\*c^3\*d^3\*e^2\*x^3 + 3\*a^3\*c^2\*d^2\*e^3\*x^2 - 4\*a^4\*c\*d\*e^4\*x + 8\*a^5\*e^5)\*sqrt(c\*d\*x + a\*e)\*g^2/(c^3\*d^3)

**Fricas [A]** time = 0.278832, size = 809, normalized size = 4.04

$$\frac{2(63c^6d^6eg^2x^7 + 99a^4c^2d^3e^4f^2 - 44a^5cd^2e^5fg + 8a^6de^6g^2 + 7(22c^6d^6efg + (9c^6d^7 + 32ac^5d^5e^2)g^2)x^6 + (99c^6d^6ef^2 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*(g\*x + f)^2/(e\*x + d)^(5/2))

[Out] 2/693\*(63\*c^6\*d^6\*e\*g^2\*x^7 + 99\*a^4\*c^2\*d^3\*e^4\*f^2 - 44\*a^5\*c\*d^2\*e^5\*f\*g + 8\*a^6\*d\*e^6\*g^2 + 7\*(22\*c^6\*d^6\*e\*f\*g + (9\*c^6\*d^7 + 32\*a\*c^5\*d^5\*e^2)\*g^2)\*x^6 + (99\*c^6\*d^6\*e\*f^2 + 22\*(7\*c^6\*d^7 + 26\*a\*c^5\*d^5\*e^2)\*f\*g + 2\*(112\*a\*c^5\*d^6\*e + 137\*a^2\*c^4\*d^4\*e^3)\*g^2)\*x^5 + (99\*(c^6\*d^7 + 4\*a\*c^5\*d^5\*e^2)\*f^2 + 44\*(13\*a\*c^5\*d

$$\begin{aligned} & e^6 + 17a^2c^4d^4e^3)fg + 2(137a^2c^4d^5e^2 + 58a^3c^3d^3e^4)g^2)x^4 + (198(2ac^5d^6e + 3a^2c^4d^4e^3) \\ & f^2 + 44(17a^2c^4d^5e^2 + 8a^3c^3d^3e^4)fg + (116a^3c^3d^4e^3 - a^4c^2d^2e^5)g^2)x^3 + (198(3a^2c^4d^5e^2 \\ & + 2a^3c^3d^3e^4)f^2 + 22(16a^3c^3d^4e^3 - a^4c^2d^2e^5)fg - (a^4c^2d^3e^4 - 4a^5cd^2e^6)g^2)x^2 + (99(4a^3c^3d^4e^3 \\ & + a^4c^2d^2e^5)f^2 - 22(a^4c^2d^3e^4 + 2a^5cd^2e^6)fg + 4(a^5cd^2e^5 + 2a^6e^7)g^2)x) / (\sqrt{cd^2e^2x^2 + ade + (cd^2 + ae^2)x})\sqrt{ex + d}c^3d^3) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*2\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)/(e\*x+d)\*\*(5/2)

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*(g\*x + f)^2/(e\*x + d)^(5/2)

[Out] Exception raised: AttributeError

$$3.703 \quad \int \frac{(f+gx)(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}} dx$$

**Optimal.** Leaf size=125

$$\frac{2g(x(ae^2+cd^2)+ade+cdex^2)^{7/2}}{9cde(d+ex)^{5/2}} - \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{7/2}(2ae^2g-cd(9ef-7dg))}{63c^2d^2e(d+ex)^{7/2}}$$

[Out]  $(-2*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)})/(63*c^2*d^2*e*(d + e*x)^{(7/2)}) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)})/(9*c*d*e*(d + e*x)^{(5/2)})$

**Rubi [A]** time = 0.32954, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 44,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{2g(x(ae^2+cd^2)+ade+cdex^2)^{7/2}}{9cde(d+ex)^{5/2}} - \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{7/2}(2ae^2g-cd(9ef-7dg))}{63c^2d^2e(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[((f + g\*x)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2))/(d + e\*x)^(5/2), x]

[Out]  $(-2*(2*a*e^2*g - c*d*(9*e*f - 7*d*g))*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)})/(63*c^2*d^2*e*(d + e*x)^{(7/2)}) + (2*g*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)})/(9*c*d*e*(d + e*x)^{(5/2)})$

**Rubi in Sympy [A]** time = 35.1427, size = 121, normalized size = 0.97

$$\frac{2g(ade+cdex^2+x(ae^2+cd^2))^{\frac{7}{2}}}{9cde(d+ex)^{\frac{5}{2}}} - \frac{2(ade+cdex^2+x(ae^2+cd^2))^{\frac{7}{2}}(2ae^2g+7cd^2g-9cdef)}{63c^2d^2e(d+ex)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)/(e\*x+d)\*\*(5/2))

[Out]  $2*g*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(7/2)/(9*c*d*e*(d + e*x)**(5/2)) - 2*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(7/2)*(2*a*e**2*g + 7*c*d**2*g - 9*c*d*e*f)/(63*c**2*d**2*e*(d + e*x)**(7/2))$

**Mathematica [A]** time = 0.114176, size = 64, normalized size = 0.51

$$\frac{2(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}(cd(9f+7gx)-2aeg)}{63c^2d^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Integrate[((f + g\*x)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2))/(d + e\*x)^(5/2)]

[Out]  $(2*(a*e + c*d*x)^3*\text{Sqrt}[(a*e + c*d*x)*(d + e*x)]*(-2*a*e*g + c*d*(9*f + 7*g*x)))/(63*c^2*d^2*\text{Sqrt}[d + e*x])$

**Maple [A]** time = 0.007, size = 67, normalized size = 0.5

$$-\frac{(2cdx + 2ae)(-7xcdg + 2aeg - 9cdf)}{63c^2d^2} (cdex^2 + ae^2x + cd^2x + ade)^{\frac{5}{2}} (ex + d)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x)`

[Out] `-2/63*(c*d*x+a*e)*(-7*c*d*g*x+2*a*e*g-9*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)/c^2/d^2/(e*x+d)^(5/2)`

**Maxima [A]** time = 0.717825, size = 190, normalized size = 1.52

$$\frac{2(c^3d^3x^3 + 3ac^2d^2ex^2 + 3a^2cde^2x + a^3e^3)\sqrt{cdx + aef}}{7cd} + \frac{2(7c^4d^4x^4 + 19ac^3d^3ex^3 + 15a^2c^2d^2e^2x^2 + a^3cde^3x - 2a^4e^4)\sqrt{cdx + aeg}}{63c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g*x + f)/(e*x + d)^(5/2)`

[Out] `2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*sqrt(c*d*x + a*e)*f/(c*d) + 2/63*(7*c^4*d^4*x^4 + 19*a*c^3*d^3*e*x^3 + 15*a^2*c^2*d^2*e^2*x^2 + a^3*c*d*e^3*x - 2*a^4*e^4)*sqrt(c*d*x + a*e)*g/(c^2*d^2)`

**Fricas [A]** time = 0.270054, size = 479, normalized size = 3.83

$$\frac{2(7c^5d^5egx^6 + 9a^4cd^2e^4f - 2a^5de^5g + (9c^5d^5ef + (7c^5d^6 + 26ac^4d^4e^2)g)x^5 + (9(c^5d^6 + 4ac^4d^4e^2)f + 2(13ac^4d^5e + 1$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g*x + f)/(e*x + d)^(5/2)`

[Out] `2/63*(7*c^5*d^5*e*g*x^6 + 9*a^4*c*d^2*e^4*f - 2*a^5*d*e^5*g + (9*c^5*d^5*e*f + (7*c^5*d^6 + 26*a*c^4*d^4*e^2)*g)*x^5 + (9*(c^5*d^6 + 4*a*c^4*d^4*e^2)*f + 2*(13*a*c^4*d^5*e + 17*a^2*c^3*d^3*e^3)*g)*x^4 + 2*(9*(2*a*c^4*d^5*e + 3*a^2*c^3*d^3*e^3)*f + (17*a^2*c^3*d^4*e^2 + 8*a^3*c^2*d^2*e^4)*g)*x^3 + (18*(3*a^2*c^3*d^4*e^2 + 2*a^3*c^2*d^2*e^4)*f + (16*a^3*c^2*d^3*e^3 - a^4*c*d*e^5)*g)*x^2 + (9*(4*a^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f - (a^4*c*d^2*e^4 + 2*a^5*e^6)*g)*x)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*c^2*d^2)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x)`

[Out] Timed out

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**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g*x + f)/(e*x + d)^(5/2)`

[Out] Exception raised: AttributeError



$$3.704 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}} dx$$

**Optimal.** Leaf size=48

$$\frac{2(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{7cd(d+ex)^{7/2}}$$

[Out]  $(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)})/(7*c*d*(d + e*x)^{(7/2)})$

**Rubi [A]** time = 0.0723978, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.026$

$$\frac{2(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{7cd(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}/(d + e*x)^{(5/2)}, x]$

[Out]  $(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)})/(7*c*d*(d + e*x)^{(7/2)})$

**Rubi in Sympy [A]** time = 17.404, size = 42, normalized size = 0.88

$$\frac{2(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{7}{2}}}{7cd(d+ex)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2), x)$

[Out]  $2*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(7/2)/(7*c*d*(d + e*x)**(7/2))$

**Mathematica [A]** time = 0.0651428, size = 37, normalized size = 0.77

$$\frac{2((d+ex)(ae+cdx))^{7/2}}{7cd(d+ex)^{7/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}/(d + e*x)^{(5/2)}, x]$

[Out]  $(2*((a*e + c*d*x)*(d + e*x))^{(7/2)})/(7*c*d*(d + e*x)^{(7/2)})$

**Maple [A]** time = 0.007, size = 50, normalized size = 1.

$$\frac{2cdx + 2ae}{7cd} (cdex^2 + ae^2x + cd^2x + ade)^{\frac{5}{2}} (ex + d)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2),x)`

[Out]  $2/7*(c*d*x+a*e)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)/c/d/(e*x+d)^(5/2)$

**Maxima [A]** time = 0.710899, size = 81, normalized size = 1.69

$$\frac{2(c^3d^3x^3 + 3ac^2d^2ex^2 + 3a^2cde^2x + a^3e^3)\sqrt{cdx + ae}}{7cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/(e*x + d)^(5/2),x, algorithm="maxima")`

[Out]  $2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*\sqrt{c*d*x + a*e}/(c*d)$

**Fricas [A]** time = 0.269069, size = 227, normalized size = 4.73

$$\frac{2(c^4d^4ex^5 + a^4de^4 + (c^4d^5 + 4ac^3d^3e^2)x^4 + 2(2ac^3d^4e + 3a^2c^2d^2e^3)x^3 + 2(3a^2c^2d^3e^2 + 2a^3cde^4)x^2 + (4a^3cd^2e^3 + a^4e^5))}{7\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{ex + dcd}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/(e*x + d)^(5/2),x, algorithm="fricas")`

[Out]  $2/7*(c^4*d^4*e*x^5 + a^4*d*e^4 + (c^4*d^5 + 4*a*c^3*d^3*e^2)*x^4 + 2*(2*a*c^3*d^4*e + 3*a^2*c^2*d^2*e^3)*x^3 + 2*(3*a^2*c^2*d^3*e^2 + 2*a^3*c*d*e^4)*x^2 + (4*a^3*c*d^2*e^3 + a^4*e^5)*x)/(\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x}*\sqrt{e*x + d})*c*d$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2),x)`

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/(e*x + d)^(5/2),x, algorithm="giac")`

[Out] Exception raised: AttributeError

$$3.705 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)} dx$$

**Optimal.** Leaf size=236

$$\begin{aligned} & -\frac{2(cdf - aeg)^{5/2} \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{7/2}} + \frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf - aeg)^2}{g^3\sqrt{d+ex}} \\ & -\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf - aeg)}{3g^2(d+ex)^{3/2}} + \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{5g(d+ex)^{5/2}} \end{aligned}$$

[Out]  $(2*(c*d*f - a*e*g)^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (g^3*\text{Sqrt}[d + e*x]) - (2*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) / (3*g^2*(d + e*x)^{(3/2)}) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}) / (5*g*(d + e*x)^{(5/2)}) - (2*(c*d*f - a*e*g)^{(5/2)}*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x])]) / g^{(7/2)}$

**Rubi [A]** time = 1.39717, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\begin{aligned} & -\frac{2(cdf - aeg)^{5/2} \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{7/2}} + \frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf - aeg)^2}{g^3\sqrt{d+ex}} \\ & -\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf - aeg)}{3g^2(d+ex)^{3/2}} + \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{5g(d+ex)^{5/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)} / ((d + e*x)^{(5/2)}*(f + g*x)), x]$

[Out]  $(2*(c*d*f - a*e*g)^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (g^3*\text{Sqrt}[d + e*x]) - (2*(c*d*f - a*e*g)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) / (3*g^2*(d + e*x)^{(3/2)}) + (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}) / (5*g*(d + e*x)^{(5/2)}) - (2*(c*d*f - a*e*g)^{(5/2)}*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x])]) / g^{(7/2)}$

**Rubi in Sympy [A]** time = 110.871, size = 228, normalized size = 0.97

$$\begin{aligned} & \frac{2(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{5}{2}}}{5g(d+ex)^{\frac{5}{2}}} + \frac{2(aeg - cdf)(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{3}{2}}}{3g^2(d+ex)^{\frac{3}{2}}} \\ & + \frac{2(aeg - cdf)^2\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{g^3\sqrt{d+ex}} - \frac{2(aeg - cdf)^{\frac{5}{2}}\text{atanh}\left(\frac{\sqrt{g}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}\sqrt{aeg-cdf}}\right)}{g^{\frac{7}{2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x$

[Out]  $2*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(5/2)/(5*g*(d + e*x)**(5/2)) + 2*(a*e*g - c*d*f)*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2)/(3*g**2*(d + e*x)**(3/2)) + 2*(a*e*g - c*d*f)**2*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(g**3*\text{sqrt}(d + e*x)) - 2*(a*e*g - c*d*f)**(5/2)*\text{atanh}(\text{sqrt}(g)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/(\text{sqrt}(d + e*x)*\text{sqrt}(a*e*g - c*d*f))/g**$

(7/2)

**Mathematica [A]** time = 0.445704, size = 168, normalized size = 0.71

$$\frac{2\sqrt{d+ex}\sqrt{ae+cdx}\left(\sqrt{g}\sqrt{ae+cdx}(23a^2e^2g^2+acdeg(11gx-35f)+c^2d^2(15f^2-5fgx+3g^2x^2))-15(aeg-cdf)^{5/2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{(d+ex)(ae+cdx)}}\right)\right)}{15g^{7/2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2)/((d + e\*x)^(5/2)\*(f + g\*x

[Out] (2\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*(Sqrt[g]\*Sqrt[a\*e + c\*d\*x]\*(23\*a^2\*e^2\*g^2 + a\*c\*d\*e\*g\*(-35\*f + 11\*g\*x) + c^2\*d^2\*(15\*f^2 - 5\*f\*g\*x + 3\*g^2\*x^2)) - 15\*(-(c\*d\*f) + a\*e\*g)^(5/2)\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/Sqrt[-(c\*d\*f) + a\*e\*g]])/(15\*g^(7/2)\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)])

**Maple [B]** time = 0.029, size = 431, normalized size = 1.8

$$-\frac{2}{15g^3}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(15\operatorname{Artanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)a^3e^3g^3-45\operatorname{Artanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)a^2cde^2fg^2+45A$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2)/(e\*x+d)^(5/2)/(g\*x+f), x)

[Out] -2/15\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)\*(15\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*a^3\*e^3\*g^3-45\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*a^2\*c\*d\*e^2\*f\*g^2+45\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*a\*c^2\*d^2\*e\*f^2\*g-15\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*c^3\*d^3\*f^3-3\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*x^2\*c^2\*d^2\*g^2-11\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*x\*a\*c\*d\*e\*g^2+5\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*x\*c^2\*d^2\*f\*g-23\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*a^2\*e^2\*g^2+35\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*a\*c\*d\*e\*f\*g-15\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*c^2\*d^2\*f^2)/(e\*x+d)^(1/2)/(c\*d\*x+a\*e)^(1/2)/g^3/((a\*e\*g-c\*d\*f)\*g)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2)\*(g\*x + f

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.293139, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g*x + f), x)`

[Out] 
$$\begin{aligned} & [1/15*(6*c^3*d^3*e*g^2*x^4 + 30*a*c^2*d^3*e*f^2 - 70*a^2*c*d^2*e^2*f*g + 46*a^3*d^2*e^3*g^2 - 2*(5*c^3*d^3*e*f*g - (3*c^3*d^4 + 14*a*c^2*d^2*e^2)*g^2)*x^3 + 15*(c^2*d^2*f^2 - 2*a*c*d*e*f*g + a^2*e^2*g^2)*\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x}*\sqrt{e*x + d}*\sqrt{-(c*d*f - a*e*g)/g}*\log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x}*\sqrt{e*x + d}*g*\sqrt{-(c*d*f - a*e*g)/g} - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x)) + 2*(15*c^3*d^3*e*f^2 - 5*(c^3*d^4 + 8*a*c^2*d^2*e^2)*f*g + 2*(7*a*c^2*d^3*e + 17*a^2*c*d*e^3)*g^2)*x^2 + 2*(15*(c^3*d^4 + a*c^2*d^2*e^2)*f^2 - 5*(8*a*c^2*d^3*e + 7*a^2*c*d*e^3)*f*g + (34*a^2*c*d^2*e^2 + 23*a^3*e^4)*g^2)*x]/(\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x}*\sqrt{e*x + d}*g^3), 2/15*(3*c^3*d^3*e*g^2*x^4 + 15*a*c^2*d^3*e*f^2 - 35*a^2*c*d^2*e^2*f*g + 23*a^3*d^2*e^3*g^2 - (5*c^3*d^3*e*f*g - (3*c^3*d^4 + 14*a*c^2*d^2*e^2)*g^2)*x^3 - 15*(c^2*d^2*f^2 - 2*a*c*d*e*f*g + a^2*e^2*g^2)*\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x}*\sqrt{e*x + d}*\sqrt{(c*d*f - a*e*g)/g}*\arctan(-\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x}*(c*d*f - a*e*g)*\sqrt{e*x + d}/((c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)*\sqrt{(c*d*f - a*e*g)/g})) + (15*c^3*d^3*e*f^2 - 5*(c^3*d^4 + 8*a*c^2*d^2*e^2)*f*g + 2*(7*a*c^2*d^3*e + 17*a^2*c*d*e^3)*g^2)*x^2 + (15*(c^3*d^4 + a*c^2*d^2*e^2)*f^2 - 5*(8*a*c^2*d^3*e + 7*a^2*c*d*e^3)*f*g + (34*a^2*c*d^2*e^2 + 23*a^3*e^4)*g^2)*x]/(\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x}*\sqrt{e*x + d}*g^3)] \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f), x)`

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g*x + f), x)`

[Out] Exception raised: AttributeError

$$3.706 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^2} dx$$

**Optimal.** Leaf size=235

$$\frac{5cd(cdf - aeg)^{3/2} \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{7/2}} - \frac{5cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf - aeg)}{g^3\sqrt{d+ex}}$$

$$- \frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{g(d+ex)^{5/2}(f+gx)} + \frac{5cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{3g^2(d+ex)^{3/2}}$$

[Out]  $(-5*c*d*(c*d*f - a*e*g)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*\text{Sqrt}[d + e*x]) + (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(3*g^2*(d + e*x)^{(3/2)}) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}/(g*(d + e*x)^{(5/2)}*(f + g*x)) + (5*c*d*(c*d*f - a*e*g)^{(3/2)}*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x]))/g^{(7/2)}$

**Rubi [A]** time = 1.22366, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{5cd(cdf - aeg)^{3/2} \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{7/2}} - \frac{5cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf - aeg)}{g^3\sqrt{d+ex}}$$

$$- \frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{g(d+ex)^{5/2}(f+gx)} + \frac{5cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{3g^2(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}/((d + e*x)^{(5/2)}*(f + g*x)^2), x]$

[Out]  $(-5*c*d*(c*d*f - a*e*g)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^3*\text{Sqrt}[d + e*x]) + (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(3*g^2*(d + e*x)^{(3/2)}) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}/(g*(d + e*x)^{(5/2)}*(f + g*x)) + (5*c*d*(c*d*f - a*e*g)^{(3/2)}*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x]))/g^{(7/2)}$

**Rubi in Sympy [A]** time = 109.327, size = 228, normalized size = 0.97

$$\frac{5cd(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{3}{2}}}{3g^2(d+ex)^{\frac{3}{2}}} + \frac{5cd(aeg - cdf)\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{g^3\sqrt{d+ex}}$$

$$- \frac{5cd(aeg - cdf)^{\frac{3}{2}} \operatorname{atanh}\left(\frac{\sqrt{g}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}\sqrt{aeg-cdf}}\right)}{g^{\frac{7}{2}}} - \frac{(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{5}{2}}}{g(d+ex)^{\frac{5}{2}}(f+gx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x$

[Out]  $5*c*d*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2)/(3*g**2*(d + e*x)**(3/2)) + 5*c*d*(a*e*g - c*d*f)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(g**3*\text{sqrt}(d + e*x)) - 5*c*d*(a*e*g - c*d*f)**(3/2)*\text{atanh}(\text{sqrt}(g)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/(\text{sqrt}(d + e*x)*\text{sqrt}(a*e*g - c*d*f))/g**(7/2) - (a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(5/2)/(g*(d + e*x)**(5/2)*(f + g$

\* x))

**Mathematica [A]** time = 0.695137, size = 161, normalized size = 0.69

$$\frac{((d + ex)(ae + cdx))^{5/2} \left( \frac{-\frac{3(cdf - aeg)^2}{f + gx} - 2cd(6cdf - 7aeg) + 2c^2d^2gx}{3g^3(ae + cdx)^2} - \frac{5cd(aeg - cdf)^{3/2} \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae + cdx}}{\sqrt{aeg - cdf}}\right)}{g^{7/2}(ae + cdx)^{5/2}} \right)}{(d + ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2)/((d + e\*x)^(5/2)\*(f + g\*x

[Out] (((a\*e + c\*d\*x)\*(d + e\*x))^(5/2)\*((-2\*c\*d\*(6\*c\*d\*f - 7\*a\*e\*g) + 2\*c^2\*d^2\*g\*x - (3\*(c\*d\*f - a\*e\*g)^2)/(f + g\*x))/(3\*g^3\*(a\*e + c\*d\*x)^2) - (5\*c\*d\*(-(c\*d\*f) + a\*e\*g)^(3/2)\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/Sqrt[-(c\*d\*f) + a\*e\*g]])/(g^(7/2)\*(a\*e + c\*d\*x)^(5/2)))/(d + e\*x)^(5/2)

**Maple [B]** time = 0.039, size = 523, normalized size = 2.2

$$-\frac{1}{3g^3(gx + f)}\sqrt{cdex^2 + ae^2x + cd^2x + ade} \left( 15 \operatorname{Artanh} \left( \frac{g\sqrt{cdx + ae}}{\sqrt{(aeg - cdf)g}} \right) x a^2 c d e^2 g^3 - 30 \operatorname{Artanh} \left( \frac{g\sqrt{cdx + ae}}{\sqrt{(aeg - cdf)g}} \right) x a c^2 d^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2)/((e\*x+d)^(5/2)/(g\*x+f)^2,x)

[Out] -1/3\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)\*(15\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x\*a^2\*c\*d\*e^2\*g^3-30\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x\*a^2\*c\*d^2\*e\*f\*g^2+15\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x\*c^3\*d^3\*f^2\*g+15\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*a^2\*c\*d\*e^2\*f\*g^2-30\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*a^2\*c^2\*d^2\*e\*f^2\*g+15\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*c^3\*d^3\*f^3-2\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*x^2\*c^2\*d^2\*g^2-14\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*x\*a^2\*c\*d\*e\*g^2+10\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*x\*c^2\*d^2\*f\*g+3\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*a^2\*e^2\*g^2-20\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*a\*c\*d\*e\*f\*g+15\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*c^2\*d^2\*f^2)/(e\*x+d)^(1/2)/(c\*d\*x+a\*e)^(1/2)/g^3/(g\*x+f)/((a\*e\*g-c\*d\*f)\*g)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2)\*(g\*x + f

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.315764, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^{(5/2)} / ((e*x + d)^{(5/2)} * (g*x + f)^{(5/2)})$

[Out]  $[1/6*(4*c^3*d^3*e*g^2*x^4 - 30*a*c^2*d^3*e*f^2 + 40*a^2*c*d^2*e^2*f*g - 6*a^3*d*e^3*g^2 - 4*(5*c^3*d^3*e*f*g - (c^3*d^4 + 8*a*c^2*d^2*e^2)*g^2)*x^3 - 15*(c^2*d^2*f^2 - a*c*d*e*f*g + (c^2*d^2*f*g - a*c*d*e*g^2)*x)*\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x}*\sqrt{e*x + d}*\sqrt{-(c*d*f - a*e*g)/g}*\log(-(c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - 2*\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x}*\sqrt{e*x + d})*g*\sqrt{-(c*d*f - a*e*g)/g} - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)/(e*g*x^2 + d*f + (e*f + d*g)*x) - 2*(15*c^3*d^3*e*f^2 + 10*(c^3*d^4 - a*c^2*d^2*e^2)*f*g - (16*a*c^2*d^3*e + 11*a^2*c*d*e^3)*g^2)*x^2 - 2*(15*(c^3*d^4 + a*c^2*d^2*e^2)*f^2 - 10*(a*c^2*d^3*e + 2*a^2*c*d*e^3)*f*g - (11*a^2*c*d^2*e^2 - 3*a^3*e^4)*g^2)*x) / ((g^4*x + f*g^3)*\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x}*\sqrt{e*x + d}), 1/3*(2*c^3*d^3*e*g^2*x^4 - 15*a*c^2*d^3*e*f^2 + 20*a^2*c*d^2*e^2*f*g - 3*a^3*d*e^3*g^2 - 2*(5*c^3*d^3*e*f*g - (c^3*d^4 + 8*a*c^2*d^2*e^2)*g^2)*x^3 + 15*(c^2*d^2*f^2 - a*c*d*e*f*g + (c^2*d^2*f*g - a*c*d*e*g^2)*x)*\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x}*\sqrt{e*x + d}*\sqrt{((c*d*f - a*e*g)/g)*\arctan(-\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x}*(c*d*f - a*e*g)*\sqrt{e*x + d} / ((c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)*\sqrt{((c*d*f - a*e*g)/g)})} - (15*c^3*d^3*e*f^2 + 10*(c^3*d^4 - a*c^2*d^2*e^2)*f*g - (16*a*c^2*d^3*e + 11*a^2*c*d*e^3)*g^2)*x^2 - (15*(c^3*d^4 + a*c^2*d^2*e^2)*f^2 - 10*(a*c^2*d^3*e + 2*a^2*c*d*e^3)*f*g - (11*a^2*c*d^2*e^2 - 3*a^3*e^4)*g^2)*x) / ((g^4*x + f*g^3)*\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x}*\sqrt{e*x + d})]$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a*d*e + (a*e**2 + c*d**2)*x + c*d*e*x**2)**(5/2) / (e*x + d)**(5/2) / (g*x + f)**(5/2))$

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: AttributeError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^{(5/2)} / ((e*x + d)^{(5/2)} * (g*x + f)^{(5/2)})$

[Out] Exception raised: AttributeError



$$3.707 \quad \int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^3} dx$$

**Optimal.** Leaf size=246

$$\frac{15c^2d^2\sqrt{cdf-ae^2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-ae^2}}\right)}{4g^{7/2}} + \frac{15c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4g^3\sqrt{d+ex}} - \frac{5cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{4g^2(d+ex)^{3/2}(f+gx)} - \frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2}$$

[Out]  $(15*c^2*d^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^3*\text{Sqrt}[d + e*x]) - (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(4*g^2*(d + e*x)^(3/2)*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(2*g*(d + e*x)^(5/2)*(f + g*x)^2) - (15*c^2*d^2*\text{Sqrt}[c*d*f - a*e*g]*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x])])/(4*g^(7/2))$

**Rubi [A]** time = 1.14144, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{15c^2d^2\sqrt{cdf-ae^2}\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-ae^2}}\right)}{4g^{7/2}} + \frac{15c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4g^3\sqrt{d+ex}} - \frac{5cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{4g^2(d+ex)^{3/2}(f+gx)} - \frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x)^3), x]$

[Out]  $(15*c^2*d^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^3*\text{Sqrt}[d + e*x]) - (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(4*g^2*(d + e*x)^(3/2)*(f + g*x)) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/(2*g*(d + e*x)^(5/2)*(f + g*x)^2) - (15*c^2*d^2*\text{Sqrt}[c*d*f - a*e*g]*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x])])/(4*g^(7/2))$

**Rubi in Sympy [A]** time = 108.741, size = 236, normalized size = 0.96

$$\frac{15c^2d^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{4g^3\sqrt{d+ex}} - \frac{15c^2d^2\sqrt{ae^2-cdf}\text{atanh}\left(\frac{\sqrt{g}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}\sqrt{ae^2-cdf}}\right)}{4g^{7/2}} - \frac{5cd(ade+cdex^2+x(ae^2+cd^2))^{3/2}}{4g^2(d+ex)^{3/2}(f+gx)} - \frac{(ade+cdex^2+x(ae^2+cd^2))^{5/2}}{2g(d+ex)^{5/2}(f+gx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x$

[Out]  $15*c**2*d**2*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(4*g**3*\text{sqrt}(d + e*x)) - 15*c**2*d**2*\text{sqrt}(a*e*g - c*d*f)*\text{atanh}(\text{sqrt}(g)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/(\text{sqrt}(d + e*x)*\text{sqrt}(a*e*g - c*d*f))/(4*g**(7/2)) - 5*c*d*(a*d*e + c*d*e*x**2 + x$

$$\frac{(a^2e^2 + c^2d^2)^{3/2} (4g^2(d + ex)^{3/2}(f + gx) - (ade + cde^2x^2 + x(a^2e^2 + c^2d^2))^{5/2})}{(2g(d + ex))^{5/2} (f + gx)^2}$$

**Mathematica [A]** time = 0.737715, size = 178, normalized size = 0.72

$$\frac{((d + ex)(ae + cdx))^{5/2} \left( \frac{-2a^2e^2g^2 - acdeg(5f + 9gx) + c^2d^2(15f^2 + 25fgx + 8g^2x^2)}{g^3(f + gx)^2(ae + cdx)^2} - \frac{15c^2d^2\sqrt{aeg - cdf} \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae + cdx}}{\sqrt{aeg - cdf}}\right)}{g^{7/2}(ae + cdx)^{5/2}} \right)}{4(d + ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2)/((d + e\*x)^(5/2)\*(f + g\*x

[Out] (((a\*e + c\*d\*x)\*(d + e\*x))^(5/2)\*((-2\*a^2\*e^2\*g^2 - a\*c\*d\*e\*g\*(5\*f + 9\*g\*x) + c^2\*d^2\*(15\*f^2 + 25\*f\*g\*x + 8\*g^2\*x^2))/(g^3\*(a\*e + c\*d\*x)^2\*(f + g\*x)^2) - (15\*c^2\*d^2\*sqrt[-(c\*d\*f) + a\*e\*g]\*ArcTanh[(sqrt[g]\*sqrt[a\*e + c\*d\*x])/sqrt[-(c\*d\*f) + a\*e\*g]])/(g^(7/2)\*(a\*e + c\*d\*x)^(5/2)))/(4\*(d + e\*x)^(5/2))

**Maple [B]** time = 0.04, size = 526, normalized size = 2.1

$$-\frac{1}{4g^3(gx + f)^2} \sqrt{cdex^2 + ae^2x + cd^2x + ade} \left( 15 \operatorname{Artanh} \left( \frac{g\sqrt{cdx + ae}}{\sqrt{(aeg - cdf)g}} \right) x^2 ac^2 d^2 eg^3 - 15 \operatorname{Artanh} \left( \frac{g\sqrt{cdx + ae}}{\sqrt{(aeg - cdf)g}} \right) x^2 c^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2)/((e\*x+d)^(5/2)/(g\*x+f)^3,x)

[Out] -1/4\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)\*(15\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x^2\*a\*c^2\*d^2\*e\*g^3-15\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x^2\*c^3\*d^3\*f\*g^2+30\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x\*a\*c^2\*d^2\*e\*f\*g^2-30\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x\*c^3\*d^3\*f^2\*g+15\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*a\*c^2\*d^2\*e\*f^2\*g-15\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*c^3\*d^3\*f^3-8\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*x^2\*c^2\*d^2\*g^2+9\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*x\*a\*c\*d\*e\*g^2-25\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*x\*c^2\*d^2\*f\*g+2\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*a^2\*e^2\*g^2+5\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*a\*c\*d\*e\*f\*g-15\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*c^2\*d^2\*f^2)/((e\*x+d)^(1/2)/(c\*d\*x+a\*e)^(1/2)/g^3/(g\*x+f)^2/((a\*e\*g-c\*d\*f)\*g)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2)\*(g\*x + f

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.568585, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2)\*(g\*x + f)

[Out] [1/8\*(16\*c^3\*d^3\*e\*g^2\*x^4 + 30\*a\*c^2\*d^3\*e\*f^2 - 10\*a^2\*c\*d^2\*e^2\*f\*g - 4\*a^3\*d\*e^3\*g^2 + 2\*(25\*c^3\*d^3\*e\*f\*g + (8\*c^3\*d^4 - a\*c^2\*d^2\*e^2)\*g^2)\*x^3 + 15\*(c^2\*d^2\*g^2\*x^2 + 2\*c^2\*d^2\*f\*g\*x + c^2\*d^2\*f^2)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(-(c\*d\*f - a\*e\*g)/g)\*log(-(c\*d\*e\*g\*x^2 - c\*d^2\*f + 2\*a\*d\*e\*g - 2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*g\*sqrt(-(c\*d\*f - a\*e\*g)/g) - (c\*d\*e\*f - (c\*d^2 + 2\*a\*e^2)\*g)\*x)/(e\*g\*x^2 + d\*f + (e\*f + d\*g)\*x) + 2\*(15\*c^3\*d^3\*e\*f^2 + 5\*(5\*c^3\*d^4 + 4\*a\*c^2\*d^2\*e^2)\*f\*g - (a\*c^2\*d^3\*e + 11\*a^2\*c\*d\*e^3)\*g^2)\*x^2 + 2\*(15\*(c^3\*d^4 + a\*c^2\*d^2\*e^2)\*f^2 + 5\*(4\*a\*c^2\*d^3\*e - a^2\*c\*d\*e^3)\*f\*g - (11\*a^2\*c\*d^2\*e^2 + 2\*a^3\*e^4)\*g^2)\*x)/((g^5\*x^2 + 2\*f\*g^4\*x + f^2\*g^3)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)), 1/4\*(8\*c^3\*d^3\*e\*g^2\*x^4 + 15\*a\*c^2\*d^3\*e\*f^2 - 5\*a^2\*c\*d^2\*e^2\*f\*g - 2\*a^3\*d\*e^3\*g^2 + (25\*c^3\*d^3\*e\*f\*g + (8\*c^3\*d^4 - a\*c^2\*d^2\*e^2)\*g^2)\*x^3 - 15\*(c^2\*d^2\*g^2\*x^2 + 2\*c^2\*d^2\*f\*g\*x + c^2\*d^2\*f^2)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt((c\*d\*f - a\*e\*g)/g)\*arctan(-sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(c\*d\*f - a\*e\*g)\*sqrt(e\*x + d)/((c\*d\*e\*g\*x^2 + a\*d\*e\*g + (c\*d^2 + a\*e^2)\*g\*x)\*sqrt((c\*d\*f - a\*e\*g)/g))) + (15\*c^3\*d^3\*e\*f^2 + 5\*(5\*c^3\*d^4 + 4\*a\*c^2\*d^2\*e^2)\*f\*g - (a\*c^2\*d^3\*e + 11\*a^2\*c\*d\*e^3)\*g^2)\*x^2 + (15\*(c^3\*d^4 + a\*c^2\*d^2\*e^2)\*f^2 + 5\*(4\*a\*c^2\*d^3\*e - a^2\*c\*d\*e^3)\*f\*g - (11\*a^2\*c\*d^2\*e^2 + 2\*a^3\*e^4)\*g^2)\*x)/((g^5\*x^2 + 2\*f\*g^4\*x + f^2\*g^3)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d))]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)/(e\*x+d)\*\*(5/2)/(g\*x+f)\*\*

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2)\*(g\*x + f)

[Out] Timed out

$$3.708 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^4} dx$$

**Optimal.** Leaf size=253

$$\frac{5c^3 d^3 \tan^{-1} \left( \frac{\sqrt{g} \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{\sqrt{d+ex} \sqrt{cdf - aeg}} \right)}{8g^{7/2} \sqrt{cdf - aeg}} - \frac{5c^2 d^2 \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{8g^3 \sqrt{d+ex} (f+gx)} \\ - \frac{5cd (x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{12g^2 (d+ex)^{3/2} (f+gx)^2} - \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{3g(d+ex)^{5/2} (f+gx)^3}$$

[Out]  $(-5*c^2*d^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (8*g^3*\text{Sqrt}[d + e*x]*(f + g*x)) - (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) / (12*g^2*(d + e*x)^{(3/2)}*(f + g*x)^2) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)} / (3*g*(d + e*x)^{(5/2)}*(f + g*x)^3) + (5*c^3*d^3*\text{ArcTan}[\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x]) / (8*g^{(7/2)}*\text{Sqrt}[c*d*f - a*e*g])$

**Rubi [A]** time = 1.13212, antiderivative size = 253, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{5c^3 d^3 \tan^{-1} \left( \frac{\sqrt{g} \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{\sqrt{d+ex} \sqrt{cdf - aeg}} \right)}{8g^{7/2} \sqrt{cdf - aeg}} - \frac{5c^2 d^2 \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{8g^3 \sqrt{d+ex} (f+gx)} \\ - \frac{5cd (x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{12g^2 (d+ex)^{3/2} (f+gx)^2} - \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{3g(d+ex)^{5/2} (f+gx)^3}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)} / ((d + e*x)^{(5/2)}*(f + g*x)^4), x]$

[Out]  $(-5*c^2*d^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (8*g^3*\text{Sqrt}[d + e*x]*(f + g*x)) - (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}) / (12*g^2*(d + e*x)^{(3/2)}*(f + g*x)^2) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)} / (3*g*(d + e*x)^{(5/2)}*(f + g*x)^3) + (5*c^3*d^3*\text{ArcTan}[\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x]) / (8*g^{(7/2)}*\text{Sqrt}[c*d*f - a*e*g])$

**Rubi in Sympy [A]** time = 107.933, size = 245, normalized size = 0.97

$$\frac{5c^3 d^3 \operatorname{atanh} \left( \frac{\sqrt{g} \sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{\sqrt{d+ex} \sqrt{aeg - cdf}} \right)}{8g^{7/2} \sqrt{aeg - cdf}} - \frac{5c^2 d^2 \sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{8g^3 \sqrt{d+ex} (f+gx)} \\ - \frac{5cd (ade + cdex^2 + x(ae^2 + cd^2))^{3/2}}{12g^2 (d+ex)^{3/2} (f+gx)^2} - \frac{(ade + cdex^2 + x(ae^2 + cd^2))^{5/2}}{3g(d+ex)^{5/2} (f+gx)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a*d*e + (a*e^2 + c*d^2)*x + c*d*e*x^2)^{(5/2)} / (e*x + d)^{(5/2)} / (g*x$

[Out]  $-5*c^3*d^3*\operatorname{atanh}(\text{sqrt}(g)*\text{sqrt}(a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2))) / (\text{sqrt}(d + e*x)*\text{sqrt}(a*e*g - c*d*f)) / (8*g^{(7/2)}*\text{sqrt}(a*e*g - c*d*f)) - 5*c^2*d^2*\text{sqrt}(a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2)) / (8*g^3*\text{sqrt}(d + e*x)*(f + g*x)) - 5*c*d*(a*d*e + c*d*e$

$$x^2 + x(ae + cd)^{3/2} / (12g^2(d + ex)^{3/2}(f + gx)^2) - (ade + cdex^2 + x(ae + cd)^{5/2}) / (3g(d + ex)^{5/2}(f + gx)^3)$$

**Mathematica [A]** time = 0.744377, size = 181, normalized size = 0.72

$$\frac{((d + ex)(ae + cdx))^{5/2} \left( -\frac{8a^2e^2g^2 + 2acdeg(5f + 13gx) + c^2d^2(15f^2 + 40fgx + 33g^2x^2)}{3g^3(f + gx)^3(ae + cdx)^2} - \frac{5c^3d^3 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae + cdx}}{\sqrt{aeg - cdf}}\right)}{g^{7/2}(ae + cdx)^{5/2}\sqrt{aeg - cdf}} \right)}{8(d + ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2)/((d + e\*x)^(5/2)\*(f + g\*x

[Out] (((a\*e + c\*d\*x)\*(d + e\*x))^(5/2)\*(-(8\*a^2\*e^2\*g^2 + 2\*a\*c\*d\*e\*g\*(5\*f + 13\*g\*x) + c^2\*d^2\*(15\*f^2 + 40\*f\*g\*x + 33\*g^2\*x^2))/(3\*g^3\*(a\*e + c\*d\*x)^2\*(f + g\*x)^3) - (5\*c^3\*d^3\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/Sqrt[-(c\*d\*f) + a\*e\*g]])/(g^(7/2)\*Sqrt[-(c\*d\*f) + a\*e\*g]\*(a\*e + c\*d\*x)^(5/2)))/(8\*(d + e\*x)^(5/2))

**Maple [A]** time = 0.038, size = 441, normalized size = 1.7

$$-\frac{1}{24g^3(gx + f)^3} \sqrt{cdex^2 + ae^2x + cd^2x + ade} \left( 15 \operatorname{Artanh} \left( \frac{g\sqrt{cdx + ae}}{\sqrt{(aeg - cdf)g}} \right) x^3 c^3 d^3 g^3 + 45 \operatorname{Artanh} \left( \frac{g\sqrt{cdx + ae}}{\sqrt{(aeg - cdf)g}} \right) x^2 c^3 d^3 g^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2)/((e\*x+d)^(5/2)/(g\*x+f)^4,x)

[Out] -1/24\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)\*(15\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x^3\*c^3\*d^3\*g^3+45\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x^2\*c^3\*d^3\*f\*g^2+45\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x\*c^3\*d^3\*f^2\*g+15\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*c^3\*d^3\*f^3+33\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*x^2\*c^2\*d^2\*g^2+26\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*x\*a\*c\*d\*e\*g^2+40\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*x\*c^2\*d^2\*f\*g+8\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*a^2\*e^2\*g^2+10\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*a\*c\*d\*e\*f\*g+15\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*c^2\*d^2\*f^2)/((e\*x+d)^(1/2)/(c\*d\*x+a\*e)^(1/2)/g^3/(g\*x+f)^3/((a\*e\*g-c\*d\*f)\*g)^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2)\*(g\*x + f

[Out] Exception raised: ValueError

**Ericas [A]** time = 0.297206, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g*x + f`

[Out] 
$$\begin{aligned} & [-1/48*(2*(33*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 + 10*a*c*d*e*f*g + \\ & 8*a^2*e^2*g^2 + 2*(20*c^2*d^2*f*g + 13*a*c*d*e*g^2)*x)*\sqrt{c*d* \\ & e*x^2 + a*d*e + (c*d^2 + a*e^2)*x}*\sqrt{-c*d*f*g + a*e*g^2}*\sqrt{ \\ & e*x + d} - 15*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f*g \\ & ^2 + c^3*d^4*g^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 + \\ & (c^3*d^3*e*f^3 + 3*c^3*d^4*f^2*g)*x)*\log(-(2*\sqrt{c*d*e*x^2 + a* \\ & d*e + (c*d^2 + a*e^2)*x}*(c*d*f*g - a*e*g^2)*\sqrt{e*x + d} + (c*d \\ & *e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)* \\ & x)*\sqrt{-c*d*f*g + a*e*g^2})/(e*g*x^2 + d*f + (e*f + d*g)*x))/(( \\ & e*g^6*x^4 + d*f^3*g^3 + (3*e*f*g^5 + d*g^6)*x^3 + 3*(e*f^2*g^4 + \\ & d*f*g^5)*x^2 + (e*f^3*g^3 + 3*d*f^2*g^4)*x)*\sqrt{-c*d*f*g + a*e*g \\ & ^2}), -1/24*((33*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 + 10*a*c*d*e*f* \\ & g + 8*a^2*e^2*g^2 + 2*(20*c^2*d^2*f*g + 13*a*c*d*e*g^2)*x)*\sqrt{c \\ & *d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x}*\sqrt{c*d*f*g - a*e*g^2}*\sqrt{ \\ & e*x + d} + 15*(c^3*d^3*e*g^3*x^4 + c^3*d^4*f^3 + (3*c^3*d^3*e*f \\ & *g^2 + c^3*d^4*g^3)*x^3 + 3*(c^3*d^3*e*f^2*g + c^3*d^4*f*g^2)*x^2 \\ & + (c^3*d^3*e*f^3 + 3*c^3*d^4*f^2*g)*x)*\arctan(\sqrt{c*d*e*x^2 + a \\ & *d*e + (c*d^2 + a*e^2)*x}*\sqrt{c*d*f*g - a*e*g^2}*\sqrt{e*x + d}/( \\ & c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x))/((e*g^6*x^4 + d*f^ \\ & 3*g^3 + (3*e*f*g^5 + d*g^6)*x^3 + 3*(e*f^2*g^4 + d*f*g^5)*x^2 + ( \\ & e*f^3*g^3 + 3*d*f^2*g^4)*x)*\sqrt{c*d*f*g - a*e*g^2})] \end{aligned}$$

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**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f)**`

[Out] Timed out

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**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g*x + f`

[Out] Timed out

$$3.709 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^5} dx$$

**Optimal.** Leaf size=323

$$\begin{aligned} & \frac{5c^4 d^4 \tan^{-1} \left( \frac{\sqrt{g} \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{\sqrt{d+ex} \sqrt{cdf - aeg}} \right)}{64g^{7/2} (cdf - aeg)^{3/2}} + \frac{5c^3 d^3 \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{64g^3 \sqrt{d+ex} (f+gx) (cdf - aeg)} \\ & - \frac{5c^2 d^2 \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{32g^3 \sqrt{d+ex} (f+gx)^2} \\ & - \frac{5cd (x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{24g^2 (d+ex)^{3/2} (f+gx)^3} - \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{4g(d+ex)^{5/2} (f+gx)^4} \end{aligned}$$

[Out]  $(-5*c^2*d^2*sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (32*g^3*sqrt[d + e*x]*(f + g*x)^2) + (5*c^3*d^3*sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (64*g^3*(c*d*f - a*e*g)*sqrt[d + e*x]*(f + g*x)) - (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) / (24*g^2*(d + e*x)^(3/2)*(f + g*x)^3) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2) / (4*g*(d + e*x)^(5/2)*(f + g*x)^4) + (5*c^4*d^4*ArcTan[(sqrt[g]*sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (sqrt[c*d*f - a*e*g]*sqrt[d + e*x])]) / (64*g^(7/2)*(c*d*f - a*e*g)^(3/2))$

**Rubi [A]** time = 1.62911, antiderivative size = 323, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\begin{aligned} & \frac{5c^4 d^4 \tan^{-1} \left( \frac{\sqrt{g} \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{\sqrt{d+ex} \sqrt{cdf - aeg}} \right)}{64g^{7/2} (cdf - aeg)^{3/2}} + \frac{5c^3 d^3 \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{64g^3 \sqrt{d+ex} (f+gx) (cdf - aeg)} \\ & - \frac{5c^2 d^2 \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{32g^3 \sqrt{d+ex} (f+gx)^2} \\ & - \frac{5cd (x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{24g^2 (d+ex)^{3/2} (f+gx)^3} - \frac{(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{4g(d+ex)^{5/2} (f+gx)^4} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2) / ((d + e*x)^(5/2) * (f + g*x)^5), x$

[Out]  $(-5*c^2*d^2*sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (32*g^3*sqrt[d + e*x]*(f + g*x)^2) + (5*c^3*d^3*sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (64*g^3*(c*d*f - a*e*g)*sqrt[d + e*x]*(f + g*x)) - (5*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2)) / (24*g^2*(d + e*x)^(3/2)*(f + g*x)^3) - (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2) / (4*g*(d + e*x)^(5/2)*(f + g*x)^4) + (5*c^4*d^4*ArcTan[(sqrt[g]*sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (sqrt[c*d*f - a*e*g]*sqrt[d + e*x])]) / (64*g^(7/2)*(c*d*f - a*e*g)^(3/2))$

**Rubi in Sympy [A]** time = 144.115, size = 311, normalized size = 0.96

$$\begin{aligned} & \frac{5c^4 d^4 \operatorname{atanh} \left( \frac{\sqrt{g} \sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{\sqrt{d+ex} \sqrt{aeg - cdf}} \right)}{64g^{7/2} (aeg - cdf)^{3/2}} - \frac{5c^3 d^3 \sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{64g^3 \sqrt{d+ex} (f+gx) (aeg - cdf)} \\ & - \frac{5c^2 d^2 \sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{32g^3 \sqrt{d+ex} (f+gx)^2} \\ & - \frac{5cd (ade + cdex^2 + x(ae^2 + cd^2))^{3/2}}{24g^2 (d+ex)^{3/2} (f+gx)^3} - \frac{(ade + cdex^2 + x(ae^2 + cd^2))^{5/2}}{4g(d+ex)^{5/2} (f+gx)^4} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x`

[Out]  $5*c**4*d**4*atanh(sqrt(g)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/(sqrt(d + e*x)*sqrt(a*e*g - c*d*f))/(64*g**(7/2)*(a*e*g - c*d*f)**(3/2)) - 5*c**3*d**3*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(64*g**3*sqrt(d + e*x)*(f + g*x)*(a*e*g - c*d*f)) - 5*c**2*d**2*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(32*g**3*sqrt(d + e*x)*(f + g*x)**2) - 5*c*d*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2)/(24*g**2*(d + e*x)**(3/2)*(f + g*x)**3) - (a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(5/2)/(4*g*(d + e*x)**(5/2)*(f + g*x)**4)$

**Mathematica [A]** time = 1.04262, size = 204, normalized size = 0.63

$$\frac{((d + ex)(ae + cdx))^{5/2} \left( \frac{5c^4 d^4 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{aeg-cdf}}\right)}{g^{7/2}(ae+cdx)^{5/2}(aeg-cdf)^{3/2}} + \frac{\frac{15c^3 d^3 (f+gx)^3}{cdf-aeg} + 136cd(f+gx)(cdf-aeg) - 48(cdf-aeg)^2 - 118c^2 d^2 (f+gx)^2}{3g^3(f+gx)^4(ae+cdx)^2} \right)}{64(d + ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x`

[Out]  $((a*e + c*d*x)*(d + e*x))^{5/2} * ((-48*(c*d*f - a*e*g)^2 + 136*c*d*(c*d*f - a*e*g)*(f + g*x) - 118*c^2*d^2*(f + g*x)^2 + (15*c^3*d^3*(f + g*x)^3)/(c*d*f - a*e*g))/(3*g^3*(a*e + c*d*x)^2*(f + g*x)^4) + (5*c^4*d^4*ArcTanH[Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[-(c*d*f) + a*e*g])/(g^{7/2}*(-(c*d*f) + a*e*g)^{3/2}*(a*e + c*d*x)^{5/2})))/(64*(d + e*x)^{5/2})$

**Maple [B]** time = 0.041, size = 665, normalized size = 2.1

$$\frac{1}{192 g^3 (aeg - cdf) (gx + f)^4} \sqrt{cdex^2 + ae^2x + cd^2x + ade} \left( 15 \operatorname{Artanh} \left( \frac{g\sqrt{cdx + ae}}{\sqrt{(aeg - cdf)g}} \right) x^4 c^4 d^4 g^4 + 60 \operatorname{Artanh} \left( \frac{g\sqrt{cdx + ae}}{\sqrt{(aeg - cdf)g}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^5,x)`

[Out]  $1/192*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^{(1/2)}*(15*arctanh(g*(c*d*x+a*e)^{(1/2)}/((a*e*g-c*d*f)*g)^{(1/2)})*x^4*c^4*d^4*g^4+60*arctanh(g*(c*d*x+a*e)^{(1/2)}/((a*e*g-c*d*f)*g)^{(1/2)})*x^3*c^4*d^4*f*g^3+90*arctanh(g*(c*d*x+a*e)^{(1/2)}/((a*e*g-c*d*f)*g)^{(1/2)})*x^2*c^4*d^4*f^2*g^2+60*arctanh(g*(c*d*x+a*e)^{(1/2)}/((a*e*g-c*d*f)*g)^{(1/2)})*x*c^4*d^4*f^3*g-15*x^3*c^3*d^3*g^3*(c*d*x+a*e)^{(1/2)}*((a*e*g-c*d*f)*g)^{(1/2)}+15*arctanh(g*(c*d*x+a*e)^{(1/2)}/((a*e*g-c*d*f)*g)^{(1/2)})*c^4*d^4*f^4-118*x^2*a*c^2*d^2*e*g^3*(c*d*x+a*e)^{(1/2)}*((a*e*g-c*d*f)*g)^{(1/2)}+73*x^2*c^3*d^3*f*g^2*(c*d*x+a*e)^{(1/2)}*((a*e*g-c*d*f)*g)^{(1/2)}-136*x*a^2*c*d*e^2*g^3*(c*d*x+a*e)^{(1/2)}*((a*e*g-c*d*f)*g)^{(1/2)}+36*x*a*c^2*d^2*e*f*g^2*(c*d*x+a*e)^{(1/2)}*((a*e*g-c*d*f)*g)^{(1/2)}+55*x*c^3*d^3*f^2*g*(c*d*x+a*e)^{(1/2)}*((a*e*g-c*d*f)*g)^{(1/2)}-48*((a*e*g-c*d*f)*g)^{(1/2)}*(c*d*x+a*e)^{(1/2)}*a^3*e^3*g^3+8*((a*e*g-c*d*f)*g)^{(1/2)}*(c*d*x+a*e)^{(1/2)}*a^2*c*d*e^2*f*g^2+10*((a*e*g-c*d*f)*g)^{(1/2)}*(c*d*x+a*e)^{(1/2)}*a*c^2*d^2*e*f^2*g+15*((a*e*g-c*d*f)*g)^{(1/2)}*(c*d*x+a*e)^{(1/2)}*c^3*d^3*f^3)/(e*x+d)^{(1/2)}/(c*d*x+a*e)^{(1/2)}/g^3/(a*e*g-c*d*f)/(g*x+f)^4/((a*e*g-c*d*f)*g)^{(1/2)}$



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**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2)\*(g\*x + f

[Out] Exception raised: ValueError

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**Fricas [A]** time = 0.304298, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2)\*(g\*x + f

[Out] [1/384\*(2\*(15\*c^3\*d^3\*g^3\*x^3 - 15\*c^3\*d^3\*f^3 - 10\*a\*c^2\*d^2\*e\*f  
^2\*g - 8\*a^2\*c\*d\*e^2\*f\*g^2 + 48\*a^3\*e^3\*g^3 - (73\*c^3\*d^3\*f\*g^2 -  
118\*a\*c^2\*d^2\*e\*g^3)\*x^2 - (55\*c^3\*d^3\*f^2\*g + 36\*a\*c^2\*d^2\*e\*f\*  
g^2 - 136\*a^2\*c\*d\*e^2\*g^3)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a  
\*e^2)\*x)\*sqrt(-c\*d\*f\*g + a\*e\*g^2)\*sqrt(e\*x + d) - 15\*(c^4\*d^4\*e\*g  
^4\*x^5 + c^4\*d^5\*f^4 + (4\*c^4\*d^4\*e\*f\*g^3 + c^4\*d^5\*g^4)\*x^4 + 2\*  
(3\*c^4\*d^4\*e\*f^2\*g^2 + 2\*c^4\*d^5\*f\*g^3)\*x^3 + 2\*(2\*c^4\*d^4\*e\*f^3\*  
g + 3\*c^4\*d^5\*f^2\*g^2)\*x^2 + (c^4\*d^4\*e\*f^4 + 4\*c^4\*d^5\*f^3\*g)\*x)  
\*log((2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(c\*d\*f\*g - a\*  
e\*g^2)\*sqrt(e\*x + d) - (c\*d\*e\*g\*x^2 - c\*d^2\*f + 2\*a\*d\*e\*g - (c\*d\*  
e\*f - (c\*d^2 + 2\*a\*e^2)\*g)\*x)\*sqrt(-c\*d\*f\*g + a\*e\*g^2))/(e\*g\*x^2  
+ d\*f + (e\*f + d\*g)\*x)))/((c\*d^2\*f^5\*g^3 - a\*d\*e\*f^4\*g^4 + (c\*d\*e  
\*f\*g^7 - a\*e^2\*g^8)\*x^5 + (4\*c\*d\*e\*f^2\*g^6 - a\*d\*e\*g^8 + (c\*d^2 -  
4\*a\*e^2)\*f\*g^7)\*x^4 + 2\*(3\*c\*d\*e\*f^3\*g^5 - 2\*a\*d\*e\*f\*g^7 + (2\*c\*  
d^2 - 3\*a\*e^2)\*f^2\*g^6)\*x^3 + 2\*(2\*c\*d\*e\*f^4\*g^4 - 3\*a\*d\*e\*f^2\*g^6  
+ (3\*c\*d^2 - 2\*a\*e^2)\*f^3\*g^5)\*x^2 + (c\*d\*e\*f^5\*g^3 - 4\*a\*d\*e\*f  
^3\*g^5 + (4\*c\*d^2 - a\*e^2)\*f^4\*g^4)\*x)\*sqrt(-c\*d\*f\*g + a\*e\*g^2)),  
1/192\*((15\*c^3\*d^3\*g^3\*x^3 - 15\*c^3\*d^3\*f^3 - 10\*a\*c^2\*d^2\*e\*f^2  
\*g - 8\*a^2\*c\*d\*e^2\*f\*g^2 + 48\*a^3\*e^3\*g^3 - (73\*c^3\*d^3\*f\*g^2 - 1  
18\*a\*c^2\*d^2\*e\*g^3)\*x^2 - (55\*c^3\*d^3\*f^2\*g + 36\*a\*c^2\*d^2\*e\*f\*g^2  
- 136\*a^2\*c\*d\*e^2\*g^3)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e  
^2)\*x)\*sqrt(c\*d\*f\*g - a\*e\*g^2)\*sqrt(e\*x + d) - 15\*(c^4\*d^4\*e\*g^4\*  
x^5 + c^4\*d^5\*f^4 + (4\*c^4\*d^4\*e\*f\*g^3 + c^4\*d^5\*g^4)\*x^4 + 2\*(3\*  
c^4\*d^4\*e\*f^2\*g^2 + 2\*c^4\*d^5\*f\*g^3)\*x^3 + 2\*(2\*c^4\*d^4\*e\*f^3\*g +  
3\*c^4\*d^5\*f^2\*g^2)\*x^2 + (c^4\*d^4\*e\*f^4 + 4\*c^4\*d^5\*f^3\*g)\*x)\*ar  
ctan(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*f\*g - a  
\*e\*g^2)\*sqrt(e\*x + d)/(c\*d\*e\*g\*x^2 + a\*d\*e\*g + (c\*d^2 + a\*e^2)\*g\*  
x)))/((c\*d^2\*f^5\*g^3 - a\*d\*e\*f^4\*g^4 + (c\*d\*e\*f\*g^7 - a\*e^2\*g^8)\*  
x^5 + (4\*c\*d\*e\*f^2\*g^6 - a\*d\*e\*g^8 + (c\*d^2 - 4\*a\*e^2)\*f\*g^7)\*x^4  
+ 2\*(3\*c\*d\*e\*f^3\*g^5 - 2\*a\*d\*e\*f\*g^7 + (2\*c\*d^2 - 3\*a\*e^2)\*f^2\*g  
^6)\*x^3 + 2\*(2\*c\*d\*e\*f^4\*g^4 - 3\*a\*d\*e\*f^2\*g^6 + (3\*c\*d^2 - 2\*a\*e  
^2)\*f^3\*g^5)\*x^2 + (c\*d\*e\*f^5\*g^3 - 4\*a\*d\*e\*f^3\*g^5 + (4\*c\*d^2 -  
a\*e^2)\*f^4\*g^4)\*x)\*sqrt(c\*d\*f\*g - a\*e\*g^2))]

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**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)/(e\*x+d)\*\*(5/2)/(g\*x+f)\*\*

[Out] Timed out

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**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g*x + f`

[Out] Timed out

$$3.710 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^6} dx$$

**Optimal.** Leaf size=393

$$\frac{3c^5 d^5 \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{128g^{7/2}(cdf-aeg)^{5/2}} + \frac{3c^4 d^4 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{128g^3 \sqrt{d+ex}(f+gx)(cdf-aeg)^2}$$

$$+ \frac{c^3 d^3 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{64g^3 \sqrt{d+ex}(f+gx)^2(cdf-aeg)} - \frac{c^2 d^2 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{16g^3 \sqrt{d+ex}(f+gx)^3}$$

$$- \frac{cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{8g^2(d+ex)^{3/2}(f+gx)^4} - \frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}$$

[Out]  $-(c^2 d^2 \sqrt{a^* d^* e + (c^* d^2 + a^* e^2) * x + c^* d^* e^* x^2}) / (16 * g^3 * \text{Sqrt}[d + e^* x] * (f + g^* x)^3) + (c^3 d^3 \sqrt{a^* d^* e + (c^* d^2 + a^* e^2) * x + c^* d^* e^* x^2}) / (64 * g^3 * (c^* d^* f - a^* e^* g) * \text{Sqrt}[d + e^* x] * (f + g^* x)^2) + (3 * c^4 d^4 \sqrt{a^* d^* e + (c^* d^2 + a^* e^2) * x + c^* d^* e^* x^2}) / (128 * g^3 * (c^* d^* f - a^* e^* g)^2 * \text{Sqrt}[d + e^* x] * (f + g^* x)) - (c^* d^* (a^* d^* e + (c^* d^2 + a^* e^2) * x + c^* d^* e^* x^2)^{(3/2)}) / (8 * g^2 * (d + e^* x)^{(3/2)} * (f + g^* x)^4) - (a^* d^* e + (c^* d^2 + a^* e^2) * x + c^* d^* e^* x^2)^{(5/2)} / (5 * g^* (d + e^* x)^{(5/2)} * (f + g^* x)^5) + (3 * c^5 d^5 * \text{ArcTan}[(\text{Sqrt}[g] * \text{Sqrt}[a^* d^* e + (c^* d^2 + a^* e^2) * x + c^* d^* e^* x^2]) / (\text{Sqrt}[c^* d^* f - a^* e^* g] * \text{Sqrt}[d + e^* x])]) / (128 * g^{(7/2)} * (c^* d^* f - a^* e^* g)^{(5/2)})$

**Rubi [A]** time = 1.83707, antiderivative size = 393, normalized size of antiderivative = 1., number of rules used = 7, number of rules used = 4, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{3c^5 d^5 \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{128g^{7/2}(cdf-aeg)^{5/2}} + \frac{3c^4 d^4 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{128g^3 \sqrt{d+ex}(f+gx)(cdf-aeg)^2}$$

$$+ \frac{c^3 d^3 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{64g^3 \sqrt{d+ex}(f+gx)^2(cdf-aeg)} - \frac{c^2 d^2 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{16g^3 \sqrt{d+ex}(f+gx)^3}$$

$$- \frac{cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{8g^2(d+ex)^{3/2}(f+gx)^4} - \frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^5}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a^* d^* e + (c^* d^2 + a^* e^2) * x + c^* d^* e^* x^2)^{(5/2)} / ((d + e^* x)^{(5/2)} * (f + g^* x)^6), x]$

[Out]  $-(c^2 d^2 \sqrt{a^* d^* e + (c^* d^2 + a^* e^2) * x + c^* d^* e^* x^2}) / (16 * g^3 * \text{Sqrt}[d + e^* x] * (f + g^* x)^3) + (c^3 d^3 \sqrt{a^* d^* e + (c^* d^2 + a^* e^2) * x + c^* d^* e^* x^2}) / (64 * g^3 * (c^* d^* f - a^* e^* g) * \text{Sqrt}[d + e^* x] * (f + g^* x)^2) + (3 * c^4 d^4 \sqrt{a^* d^* e + (c^* d^2 + a^* e^2) * x + c^* d^* e^* x^2}) / (128 * g^3 * (c^* d^* f - a^* e^* g)^2 * \text{Sqrt}[d + e^* x] * (f + g^* x)) - (c^* d^* (a^* d^* e + (c^* d^2 + a^* e^2) * x + c^* d^* e^* x^2)^{(3/2)}) / (8 * g^2 * (d + e^* x)^{(3/2)} * (f + g^* x)^4) - (a^* d^* e + (c^* d^2 + a^* e^2) * x + c^* d^* e^* x^2)^{(5/2)} / (5 * g^* (d + e^* x)^{(5/2)} * (f + g^* x)^5) + (3 * c^5 d^5 * \text{ArcTan}[(\text{Sqrt}[g] * \text{Sqrt}[a^* d^* e + (c^* d^2 + a^* e^2) * x + c^* d^* e^* x^2]) / (\text{Sqrt}[c^* d^* f - a^* e^* g] * \text{Sqrt}[d + e^* x])]) / (128 * g^{(7/2)} * (c^* d^* f - a^* e^* g)^{(5/2)})$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a^* d^* e + (a^* e^* * 2 + c^* d^* * 2) * x + c^* d^* e^* x^2)^{(5/2)} / (e^* x + d)^{(5/2)} / (g^* x$

[Out] Timed out

**Mathematica [A]** time = 1.42825, size = 231, normalized size = 0.59

$$\frac{((d + ex)(ae + cdx))^{5/2} \left( \frac{15c^4 d^4 (f+gx)^4}{(cdf-ae g)^2} + \frac{10c^3 d^3 (f+gx)^3}{cdf-ae g} + 336cd(f+gx)(cdf-ae g) - 128(cdf-ae g)^2 - 248c^2 d^2 (f+gx)^2 \right) - \frac{3c^5 d^5 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{ae g-cdf}}\right)}{g^{7/2}(ae+cdx)^{5/2}(ae g-cdf)^{5/2}}}{128(d + ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2)/((d + e\*x)^(5/2)\*(f + g\*x

[Out] (((a\*e + c\*d\*x)\*(d + e\*x))^(5/2)\*((-128\*(c\*d\*f - a\*e\*g)^2 + 336\*c\*d\*(c\*d\*f - a\*e\*g)\*(f + g\*x) - 248\*c^2\*d^2\*(f + g\*x)^2 + (10\*c^3\*d^3\*(f + g\*x)^3)/(c\*d\*f - a\*e\*g) + (15\*c^4\*d^4\*(f + g\*x)^4)/(c\*d\*f - a\*e\*g)^2)/(5\*g^3\*(a\*e + c\*d\*x)^2\*(f + g\*x)^5) - (3\*c^5\*d^5\*ArcTanh[Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/Sqrt[-(c\*d\*f) + a\*e\*g]]/(g^(7/2)\*(-(c\*d\*f) + a\*e\*g)^(5/2)\*(a\*e + c\*d\*x)^(5/2)))/(128\*(d + e\*x)^(5/2))

**Maple [B]** time = 0.049, size = 924, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2)/(e\*x+d)^(5/2)/(g\*x+f)^6,x)

[Out] -1/640\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)\*(15\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x^5\*c^5\*d^5\*g^5+75\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x^4\*c^5\*d^5\*f\*g^4+150\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x^3\*c^5\*d^5\*f^2\*g^3+150\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x^2\*c^5\*d^5\*f^3\*g^2-15\*x^4\*c^4\*d^4\*g^4\*(c\*d\*x+a\*e)^(1/2)\*((a\*e\*g-c\*d\*f)\*g)^(1/2)+75\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x\*c^5\*d^5\*f^4\*g+10\*x^3\*a\*c^3\*d^3\*e\*g^4\*(c\*d\*x+a\*e)^(1/2)\*((a\*e\*g-c\*d\*f)\*g)^(1/2)-70\*x^3\*c^4\*d^4\*f\*g^3\*(c\*d\*x+a\*e)^(1/2)\*((a\*e\*g-c\*d\*f)\*g)^(1/2)+15\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*c^5\*d^5\*f^5+248\*x^2\*a^2\*c^2\*d^2\*e^2\*g^4\*(c\*d\*x+a\*e)^(1/2)\*((a\*e\*g-c\*d\*f)\*g)^(1/2)-466\*x^2\*a\*c^3\*d^3\*e\*f\*g^3\*(c\*d\*x+a\*e)^(1/2)\*((a\*e\*g-c\*d\*f)\*g)^(1/2)+128\*x^2\*c^4\*d^4\*f^2\*g^2\*(c\*d\*x+a\*e)^(1/2)\*((a\*e\*g-c\*d\*f)\*g)^(1/2)+336\*x\*a^3\*c\*d\*e^3\*g^4\*(c\*d\*x+a\*e)^(1/2)\*((a\*e\*g-c\*d\*f)\*g)^(1/2)-512\*x\*a^2\*c^2\*d^2\*e^2\*f\*g^3\*(c\*d\*x+a\*e)^(1/2)\*((a\*e\*g-c\*d\*f)\*g)^(1/2)+46\*x\*a\*c^3\*d^3\*e\*f^2\*g^2\*(c\*d\*x+a\*e)^(1/2)\*((a\*e\*g-c\*d\*f)\*g)^(1/2)+70\*x\*c^4\*d^4\*f^3\*g\*(c\*d\*x+a\*e)^(1/2)\*((a\*e\*g-c\*d\*f)\*g)^(1/2)+128\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*a^4\*e^4\*g^4-176\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*a^3\*c\*d\*e^3\*f\*g^3+8\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*a^2\*c^2\*d^2\*e^2\*f^2\*g^2+10\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*a\*c^3\*d^3\*e\*f^3\*g+15\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*c^4\*d^4\*f^4)/(e\*x+d)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2)/(g\*x+f)^5/g^3/(a\*e\*g-c\*d\*f)^2/(c\*d\*x+a\*e)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2)\*(g\*x + f

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.31882, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2)\*(g\*x + f

[Out] [1/1280\*(2\*(15\*c^4\*d^4\*g^4\*x^4 - 15\*c^4\*d^4\*f^4 - 10\*a\*c^3\*d^3\*e\*f^3\*g - 8\*a^2\*c^2\*d^2\*e^2\*f^2\*g^2 + 176\*a^3\*c\*d\*e^3\*f\*g^3 - 128\*a^4\*e^4\*g^4 + 10\*(7\*c^4\*d^4\*f\*g^3 - a\*c^3\*d^3\*e\*g^4)\*x^3 - 2\*(64\*c^4\*d^4\*f^2\*g^2 - 233\*a\*c^3\*d^3\*e\*f\*g^3 + 124\*a^2\*c^2\*d^2\*e^2\*g^4)\*x^2 - 2\*(35\*c^4\*d^4\*f^3\*g + 23\*a\*c^3\*d^3\*e\*f^2\*g^2 - 256\*a^2\*c^2\*d^2\*e^2\*f\*g^3 + 168\*a^3\*c\*d\*e^3\*g^4)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*f\*g + a\*e\*g^2)\*sqrt(e\*x + d) + 15\*(c^5\*d^5\*e\*g^5\*x^6 + c^5\*d^6\*f^5 + (5\*c^5\*d^5\*e\*f\*g^4 + c^5\*d^6\*g^5)\*x^5 + 5\*(2\*c^5\*d^5\*e\*f^2\*g^3 + c^5\*d^6\*f^2\*g^4)\*x^4 + 10\*(c^5\*d^5\*e\*f^3\*g^2 + c^5\*d^6\*f^2\*g^3)\*x^3 + 5\*(c^5\*d^5\*e\*f^4\*g + 2\*c^5\*d^6\*f^3\*g^2)\*x^2 + (c^5\*d^5\*e\*f^5 + 5\*c^5\*d^6\*f^4\*g)\*x)\*log(-(2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(c\*d\*f\*g - a\*e\*g^2)\*sqrt(e\*x + d) + (c\*d\*e\*g\*x^2 - c\*d^2\*f + 2\*a\*d\*e\*g - (c\*d\*e\*f - (c\*d^2 + 2\*a\*e^2)\*g)\*x)\*sqrt(-c\*d\*f\*g + a\*e\*g^2))/(e\*g\*x^2 + d\*f + (e\*f + d\*g)\*x)))/((c^2\*d^3\*f^7\*g^3 - 2\*a\*c\*d^2\*e\*f^6\*g^4 + a^2\*d^2\*e^2\*f^5\*g^5 + (c^2\*d^2\*e\*f^2\*g^8 - 2\*a\*c\*d^2\*e^2\*f^2\*g^9 + a^2\*d^2\*e^3\*g^10)\*x^6 + (5\*c^2\*d^2\*e\*f^3\*g^7 + a^2\*d^2\*e^2\*g^10 + (c^2\*d^3 - 10\*a\*c\*d^2\*e^2)\*f^2\*g^8 - (2\*a\*c\*d^2\*e - 5\*a^2\*e^3)\*f^2\*g^9)\*x^5 + 5\*(2\*c^2\*d^2\*e\*f^4\*g^6 + a^2\*d^2\*e^2\*f^2\*g^9 + (c^2\*d^3 - 4\*a\*c\*d^2\*e^2)\*f^3\*g^7 - 2\*(a\*c\*d^2\*e - a^2\*e^3)\*f^2\*g^8)\*x^4 + 10\*(c^2\*d^2\*e\*f^5\*g^5 + a^2\*d^2\*e^2\*f^2\*g^8 + (c^2\*d^3 - 2\*a\*c\*d^2\*e^2)\*f^4\*g^6 - (2\*a\*c\*d^2\*e - a^2\*e^3)\*f^3\*g^7)\*x^3 + 5\*(c^2\*d^2\*e\*f^6\*g^4 + 2\*a^2\*d^2\*e^2\*f^3\*g^7 + 2\*(c^2\*d^3 - a\*c\*d^2\*e^2)\*f^5\*g^5 - (4\*a\*c\*d^2\*e - a^2\*e^3)\*f^4\*g^6)\*x^2 + (c^2\*d^2\*e\*f^7\*g^3 + 5\*a^2\*d^2\*e^2\*f^4\*g^6 + (5\*c^2\*d^3 - 2\*a\*c\*d^2\*e^2)\*f^6\*g^4 - (10\*a\*c\*d^2\*e - a^2\*e^3)\*f^5\*g^5)\*x)\*sqrt(-c\*d\*f\*g + a\*e\*g^2), 1/640\*((15\*c^4\*d^4\*g^4\*x^4 - 15\*c^4\*d^4\*f^4 - 10\*a\*c^3\*d^3\*e\*f^3\*g - 8\*a^2\*c^2\*d^2\*e^2\*f^2\*g^2 + 176\*a^3\*c\*d\*e^3\*f\*g^3 - 128\*a^4\*e^4\*g^4 + 10\*(7\*c^4\*d^4\*f\*g^3 - a\*c^3\*d^3\*e\*g^4)\*x^3 - 2\*(64\*c^4\*d^4\*f^2\*g^2 - 233\*a\*c^3\*d^3\*e\*f\*g^3 + 124\*a^2\*c^2\*d^2\*e^2\*g^4)\*x^2 - 2\*(35\*c^4\*d^4\*f^3\*g + 23\*a\*c^3\*d^3\*e\*f^2\*g^2 - 256\*a^2\*c^2\*d^2\*e^2\*f\*g^3 + 168\*a^3\*c\*d\*e^3\*g^4)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*f\*g - a\*e\*g^2)\*sqrt(e\*x + d) - 15\*(c^5\*d^5\*e\*g^5\*x^6 + c^5\*d^6\*f^5 + (5\*c^5\*d^5\*e\*f\*g^4 + c^5\*d^6\*g^5)\*x^5 + 5\*(2\*c^5\*d^5\*e\*f^2\*g^3 + c^5\*d^6\*f^2\*g^4)\*x^4 + 10\*(c^5\*d^5\*e\*f^3\*g^2 + c^5\*d^6\*f^2\*g^3)\*x^3 + 5\*(c^5\*d^5\*e\*f^4\*g + 2\*c^5\*d^6\*f^3\*g^2)\*x^2 + (c^5\*d^5\*e\*f^5 + 5\*c^5\*d^6\*f^4\*g)\*x)\*arctan(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*f\*g - a\*e\*g^2)\*sqrt(e\*x + d)/(c\*d\*e\*g\*x^2 + a\*d\*e\*g + (c\*d^2 + a\*e^2)\*g\*x)))/((c^2\*d^3\*f^7\*g^3 - 2\*a\*c\*d^2\*e\*f^6\*g^4 + a^2\*d^2\*e^2\*f^5\*g^5 + (c^2\*d^2\*e\*f^2\*g^8 - 2\*a\*c\*d^2\*e^2\*f^2\*g^9 + a^2\*d^2\*e^3\*g^10)\*x^6 + (5\*c^2\*d^2\*e\*f^3\*g^7 + a^2\*d^2\*e^2\*g^10 + (c^2\*d^3 - 10\*a\*c\*d^2\*e^2)\*f^2\*g^8 - (2\*a\*c\*d^2\*e - 5\*a^2\*e^3)\*f^2\*g^9)\*x^5 + 5\*(2\*c^2\*d^2\*e\*f^4\*g^6 + a^2\*d^2\*e^2\*f^2\*g^9 + (c^2\*d^3 - 4\*a\*c\*d^2\*e^2)\*f^3\*g^7 - 2\*(a\*c\*d^2\*e - a^2\*e^3)\*f^2\*g^8)\*x^4 + 10\*(c^2\*d^2\*e\*f^5\*g^5 + a^2\*d^2\*e^2\*f^2\*g^8 + (c^2\*d^3 - 2\*a\*c\*d^2\*e^2)\*f^4\*g^6 - (2\*a\*c\*d^2\*e - a^2\*e^3)\*f^3\*g^7)\*x^3 + 5\*(c^2\*d^2\*e\*f^6\*g^4 + 2\*a^2\*d^2\*e^2\*f^3\*g^7 + 2\*(c^2\*d^3 - a\*c\*d^2\*e^2)\*f^5\*g^5 - (4\*a\*c\*d^2\*e - a^2\*e^3)\*f^4\*g^6)\*x^2 + (c^2\*d^2\*e\*f^7\*g^3 + 5\*a^2\*d^2\*e^2\*f^4\*g^6 + (5\*c^2\*d^3 - 2\*a\*c\*d^2\*e^2)\*f^6\*g^4 - (10\*a\*c\*d^2\*e - a^2\*e^3)\*f^5\*g^5)\*x)\*sqrt(c\*d\*f\*g - a\*e\*g^2)]]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)/(e\*x+d)\*\*(5/2)/(g\*x+f)\*\*

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2)\*(g\*x + f)

[Out] Timed out

$$3.711 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^7} dx$$

**Optimal.** Leaf size=463

$$\begin{aligned} & \frac{5c^6 d^6 \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-ae^2}}\right)}{512g^{7/2}(cdf-ae^2)^{7/2}} + \frac{5c^5 d^5 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{512g^3 \sqrt{d+ex}(f+gx)(cdf-ae^2)^3} \\ & + \frac{5c^4 d^4 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{768g^3 \sqrt{d+ex}(f+gx)^2(cdf-ae^2)^2} \\ & + \frac{c^3 d^3 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{192g^3 \sqrt{d+ex}(f+gx)^3(cdf-ae^2)} - \frac{c^2 d^2 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{32g^3 \sqrt{d+ex}(f+gx)^4} \\ & - \frac{cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^5} - \frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6} \end{aligned}$$

[Out]  $-(c^2 d^2 \sqrt{x(ae^2+cd^2)+ade+cdex^2}) / (32 g^3 \sqrt{d+ex} (f+gx)^4) + (c^3 d^3 \sqrt{x(ae^2+cd^2)+ade+cdex^2}) / (192 g^3 (cdf-ae^2) \sqrt{d+ex} (f+gx)^3) + (5 c^4 d^4 \sqrt{x(ae^2+cd^2)+ade+cdex^2}) / (768 g^3 (cdf-ae^2)^2 \sqrt{d+ex} (f+gx)^2) + (5 c^5 d^5 \sqrt{x(ae^2+cd^2)+ade+cdex^2}) / (512 g^3 (cdf-ae^2)^3 \sqrt{d+ex} (f+gx)) - (c^2 d^2 \sqrt{x(ae^2+cd^2)+ade+cdex^2}) / (12 g^2 (d+ex)^{3/2} (f+gx)^5) - (c^2 d^2 \sqrt{x(ae^2+cd^2)+ade+cdex^2}) / (6 g (d+ex)^{5/2} (f+gx)^6) + (5 c^6 d^6 \operatorname{ArcTan}[\sqrt{g} \sqrt{x(ae^2+cd^2)+ade+cdex^2} / (\sqrt{d+ex} \sqrt{cdf-ae^2})]) / (512 g^7 (cdf-ae^2)^{7/2})$

**Rubi [A]** time = 2.41647, antiderivative size = 463, normalized size of antiderivative = 1., number of rules used = 8, number of rules used = 4, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\begin{aligned} & \frac{5c^6 d^6 \tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-ae^2}}\right)}{512g^{7/2}(cdf-ae^2)^{7/2}} + \frac{5c^5 d^5 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{512g^3 \sqrt{d+ex}(f+gx)(cdf-ae^2)^3} \\ & + \frac{5c^4 d^4 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{768g^3 \sqrt{d+ex}(f+gx)^2(cdf-ae^2)^2} \\ & + \frac{c^3 d^3 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{192g^3 \sqrt{d+ex}(f+gx)^3(cdf-ae^2)} - \frac{c^2 d^2 \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{32g^3 \sqrt{d+ex}(f+gx)^4} \\ & - \frac{cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{12g^2(d+ex)^{3/2}(f+gx)^5} - \frac{(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{6g(d+ex)^{5/2}(f+gx)^6} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[(a^2 d^2 e + (c^2 d^2 + a^2 e^2)x + c^2 d^2 e x^2)^{5/2} / ((d+ex)^{5/2} (f+gx)^7), x]$

[Out]  $-(c^2 d^2 \sqrt{x(ae^2+cd^2)+ade+cdex^2}) / (32 g^3 \sqrt{d+ex} (f+gx)^4) + (c^3 d^3 \sqrt{x(ae^2+cd^2)+ade+cdex^2}) / (192 g^3 (cdf-ae^2) \sqrt{d+ex} (f+gx)^3) + (5 c^4 d^4 \sqrt{x(ae^2+cd^2)+ade+cdex^2}) / (768 g^3 (cdf-ae^2)^2 \sqrt{d+ex} (f+gx)^2) + (5 c^5 d^5 \sqrt{x(ae^2+cd^2)+ade+cdex^2}) / (512 g^3 (cdf-ae^2)^3 \sqrt{d+ex} (f+gx)) - (c^2 d^2 \sqrt{x(ae^2+cd^2)+ade+cdex^2}) / (12 g^2 (d+ex)^{3/2} (f+gx)^5) - (c^2 d^2 \sqrt{x(ae^2+cd^2)+ade+cdex^2}) / (6 g (d+ex)^{5/2} (f+gx)^6) + (5 c^6 d^6 \operatorname{ArcTan}[\sqrt{g} \sqrt{x(ae^2+cd^2)+ade+cdex^2} / (\sqrt{d+ex} \sqrt{cdf-ae^2})]) / (512 g^7 (cdf-ae^2)^{7/2})$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x`

[Out] Timed out

**Mathematica [A]** time = 1.7946, size = 258, normalized size = 0.56

$$\frac{((d + ex)(ae + cdx))^{5/2} \left( \frac{5c^6 d^6 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{aeg-cdf}}\right)}{g^{7/2}(ae+cdx)^{5/2}(aeg-cdf)^{7/2}} + \frac{\frac{15c^5 d^5 (f+gx)^5}{(cdf-aeg)^3} + \frac{10c^4 d^4 (f+gx)^4}{(cdf-aeg)^2} + \frac{8c^3 d^3 (f+gx)^3}{cdf-aeg} + 640cd(f+gx)(cdf-aeg) - 256(cdf-aeg)^2 - 432c^2}{3g^3(f+gx)^6(ae+cdx)^2} \right)}{512(d + ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x`

[Out] `((a*e + c*d*x)*(d + e*x))^(5/2)*((-256*(c*d*f - a*e*g)^2 + 640*c*d*(c*d*f - a*e*g)*(f + g*x) - 432*c^2*d^2*(f + g*x)^2 + (8*c^3*d^3*(f + g*x)^3)/(c*d*f - a*e*g) + (10*c^4*d^4*(f + g*x)^4)/(c*d*f - a*e*g)^2 + (15*c^5*d^5*(f + g*x)^5)/(c*d*f - a*e*g)^3)/(3*g^3*(a*e + c*d*x)^2*(f + g*x)^6) + (5*c^6*d^6*ArcTanh[Sqrt[g]*Sqrt[a*e + c*d*x])/Sqrt[-(c*d*f) + a*e*g]]/(g^(7/2)*(-(c*d*f) + a*e*g)^(7/2)*(a*e + c*d*x)^(5/2)))/(512*(d + e*x)^(5/2))`

**Maple [B]** time = 0.049, size = 1261, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^7,x)`

[Out] `1/1536*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(10*x^4*a*c^4*d^4*e*g^5*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c^6*d^6*f^6+300*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*x^3*c^6*d^6*f^3*g^3+15*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*x^6*c^6*d^6*g^6+1272*x^2*a^2*c^3*d^3*e^2*f*g^4*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-1188*x^2*a^2*c^4*d^4*e*f^2*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+1696*x^3*c^2*d^2*e^3*f*g^4*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-1272*x^2*c^3*d^3*e^2*f^2*g^3*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+56*x^3*a^2*c^4*d^4*e*f^3*g^2*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+56*x^3*a^2*c^4*d^4*e*f^3*g^4*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-256*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^5*e^5*g^5+15*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*c^5*d^5*f^5-8*x^3*a^2*c^3*d^3*e^2*g^5*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-432*x^2*a^3*c^2*d^2*e^3*g^5*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)-640*x^2*a^4*c*d*e^4*g^5*(c*d*x+a*e)^(1/2)*((a*e*g-c*d*f)*g)^(1/2)+640*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^4*c*d*e^4*f*g^4-432*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^3*c^2*d^2*e^3*f^2*g^3+8*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c^3*d^3*e^2*f^3*g^2+10*((a*e*g-c*d*f)*g)^(1/2)*(c*d*x+a*e)^(1/2)*a^2*c^4*d^4*e*f^4*g+225*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*x^2*c^6*d^6*f^4*g^2+90*arctanh(g*(c*d*x+a*e)^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*x*c^6*d^6*f^5*g+90*arctanh(g`



$$\frac{(c^2 d x + a^2 e)^{1/2} \operatorname{arctanh}\left(\frac{(c^2 d x + a^2 e)^{1/2}}{(a^2 e g - c^2 d f)^{1/2}}\right) x^5 c^6 d^6 f^5 g^{225} + 198 x^2 c^5 d^5 f^3 g^2 (c^2 d x + a^2 e)^{1/2} (a^2 e g - c^2 d f)^{1/2} + 85 x c^5 d^5 f^4 g (c^2 d x + a^2 e)^{1/2} (a^2 e g - c^2 d f)^{1/2} - 85 x^4 c^5 d^5 f^2 g^4 (c^2 d x + a^2 e)^{1/2} (a^2 e g - c^2 d f)^{1/2} - 198 x^3 c^5 d^5 f^2 g^3 (c^2 d x + a^2 e)^{1/2} (a^2 e g - c^2 d f)^{1/2}}{(e x + d)^{1/2} (a^2 e g - c^2 d f)^{1/2} (g x + f)^6 / g^3 (a^2 e g - c^2 d f)^2} \frac{1}{(a^2 e^2 g^2 - 2 a^2 c^2 d e f + c^2 d^2 f^2)^{1/2} (c^2 d x + a^2 e)^{1/2}}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2)\*(g\*x + f

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.340575, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2)\*(g\*x + f

[Out] [1/3072\*(2\*(15\*c^5\*d^5\*g^5\*x^5 - 15\*c^5\*d^5\*f^5 - 10\*a\*c^4\*d^4\*e\*f^4\*g - 8\*a^2\*c^3\*d^3\*e^2\*f^3\*g^2 + 432\*a^3\*c^2\*d^2\*e^3\*f^2\*g^3 - 640\*a^4\*c\*d\*e^4\*f\*g^4 + 256\*a^5\*e^5\*g^5 + 5\*(17\*c^5\*d^5\*f\*g^4 - 2\*a\*c^4\*d^4\*e\*g^5)\*x^4 + 2\*(99\*c^5\*d^5\*f^2\*g^3 - 28\*a\*c^4\*d^4\*e\*f\*g^4 + 4\*a^2\*c^3\*d^3\*e^2\*g^5)\*x^3 - 6\*(33\*c^5\*d^5\*f^3\*g^2 - 198\*a\*c^4\*d^4\*e\*f^2\*g^3 + 212\*a^2\*c^3\*d^3\*e^2\*f\*g^4 - 72\*a^3\*c^2\*d^2\*e^3\*g^5)\*x^2 - (85\*c^5\*d^5\*f^4\*g + 56\*a\*c^4\*d^4\*e\*f^3\*g^2 - 1272\*a^2\*c^3\*d^3\*e^2\*f^2\*g^3 + 1696\*a^3\*c^2\*d^2\*e^3\*f\*g^4 - 640\*a^4\*c\*d\*e^4\*g^5)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*f\*g + a\*e\*g^2)\*sqrt(e\*x + d) - 15\*(c^6\*d^6\*e\*g^6\*x^7 + c^6\*d^7\*f^6 + (6\*c^6\*d^6\*e\*f\*g^5 + c^6\*d^7\*g^6)\*x^6 + 3\*(5\*c^6\*d^6\*e\*f^2\*g^4 + 2\*c^6\*d^7\*f\*g^5)\*x^5 + 5\*(4\*c^6\*d^6\*e\*f^3\*g^3 + 3\*c^6\*d^7\*f^2\*g^4)\*x^4 + 5\*(3\*c^6\*d^6\*e\*f^4\*g^2 + 4\*c^6\*d^7\*f^3\*g^3)\*x^3 + 3\*(2\*c^6\*d^6\*e\*f^5\*g + 5\*c^6\*d^7\*f^4\*g^2)\*x^2 + (c^6\*d^6\*e\*f^6 + 6\*c^6\*d^7\*f^5\*g)\*x)\*log((2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(c\*d\*f\*g - a\*e\*g^2)\*sqrt(e\*x + d) - (c\*d\*e\*g\*x^2 - c\*d^2\*f + 2\*a\*d\*e\*g - (c\*d\*e\*f - (c\*d^2 + 2\*a\*e^2)\*g)\*x)\*sqrt(-c\*d\*f\*g + a\*e\*g^2))/(e\*g\*x^2 + d\*f + (e\*f + d\*g)\*x))/((c^3\*d^4\*f^9\*g^3 - 3\*a\*c^2\*d^3\*e\*f^8\*g^4 + 3\*a^2\*c\*d^2\*e^2\*f^7\*g^5 - a^3\*d\*e^3\*f^6\*g^6 + (c^3\*d^3\*e\*f^3\*g^9 - 3\*a\*c^2\*d^2\*e^2\*f^2\*g^10 + 3\*a^2\*c\*d\*e^3\*f\*g^11 - a^3\*e^4\*g^12)\*x^7 + (6\*c^3\*d^3\*e\*f^4\*g^8 - a^3\*d\*e^3\*g^12 + (c^3\*d^4 - 18\*a\*c^2\*d^2\*e^2)\*f^3\*g^9 - 3\*(a\*c^2\*d^3\*e - 6\*a^2\*c\*d\*e^3)\*f^2\*g^10 + 3\*(a^2\*c\*d^2\*e^2 - 2\*a^3\*e^4)\*f\*g^11)\*x^6 + 3\*(5\*c^3\*d^3\*e\*f^5\*g^7 - 2\*a^3\*d\*e^3\*f\*g^11 + (2\*c^3\*d^4 - 15\*a\*c^2\*d^2\*e^2)\*f^4\*g^8 - 3\*(2\*a\*c^2\*d^3\*e - 5\*a^2\*c\*d\*e^3)\*f^3\*g^9 + (6\*a^2\*c\*d^2\*e^2 - 5\*a^3\*e^4)\*f^2\*g^10)\*x^5 + 5\*(4\*c^3\*d^3\*e\*f^6\*g^6 - 3\*a^3\*d\*e^3\*f^2\*g^10 + 3\*(c^3\*d^4 - 4\*a\*c^2\*d^2\*e^2)\*f^5\*g^7 - 3\*(3\*a\*c^2\*d^3\*e - 4\*a^2\*c\*d\*e^3)\*f^4\*g^8 + (9\*a^2\*c\*d^2\*e^2 - 4\*a^3\*e^4)\*f^3\*g^9)\*x^4 + 5\*(3\*c^3\*d^3\*e\*f^7\*g^5 - 4\*a^3\*d\*e^3\*f^3\*g^9 + (4\*c^3\*d^4 - 9\*a\*c^2\*d^2\*e^2)\*f^6\*g^6 - 3\*(4\*a\*c^2\*d^3\*e - 3\*a^2\*c\*d\*e^3)\*f^5\*g^7 + 3\*(4\*a^2\*c\*d^2\*e^2 - a^3\*e^4)\*f^4\*g^8)\*x^3 + 3\*(2\*c^3\*d^3\*e\*f^8\*g^4 - 5\*a^3\*d\*e^3\*f^4\*g^8 + (5\*c^3\*d^4 - 6\*a\*c^2\*d^2\*e^2)\*f^7\*g^5 - 3\*(5\*a\*c^2\*d^3\*e - 2\*a^2\*c\*d\*e^3)\*f^6\*g^6 + (15\*a^2\*c\*d^2\*e^2 - 2\*a^3\*e^4)\*f^5\*g^7)\*x^2 + (c^3\*d^3\*e\*f^9\*g^3 - 6\*a^3\*d\*e^3\*f^5\*g^7 + 3\*(2\*c^3\*d^4 - a\*c^2\*d^2\*e^2)\*

$$f^8 g^4 - 3(6a^2 c^2 d^3 e - a^2 c d^2 e^3) f^7 g^5 + (18a^2 c^2 d^2 e^2 - a^3 e^4) f^6 g^6) x) \sqrt{-c d f g + a e g^2}), 1/1536((15c^5 d^5 g^5 x^5 - 15c^5 d^5 f^5 - 10a^2 c^4 d^4 e f^4 g - 8a^2 c^3 d^3 e^2 f^3 g^2 + 432a^3 c^2 d^2 e^3 f^2 g^3 - 640a^4 c^2 d^2 e^4 f g^4 + 256a^5 e^5 g^5 + 5(17c^5 d^5 f^2 g^4 - 2a^2 c^4 d^4 e g^5) x^4 + 2(99c^5 d^5 f^2 g^3 - 28a^2 c^4 d^4 e f^2 g^4 + 4a^2 c^3 d^3 e^2 g^5) x^3 - 6(33c^5 d^5 f^3 g^2 - 198a^2 c^4 d^4 e f^3 g^2 - 212a^2 c^3 d^3 e^2 f^2 g^4 - 72a^3 c^2 d^2 e^3 g^5) x^2 - (85c^5 d^5 f^4 g + 56a^2 c^4 d^4 e f^3 g^2 - 1272a^2 c^3 d^3 e^2 f^2 g^3 + 1696a^3 c^2 d^2 e^3 f^2 g^4 - 640a^4 c^2 d^2 e^4 g^5) x) \sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x} \sqrt{c d f g - a e g^2} \sqrt{e x + d} - 15(c^6 d^6 e g^6 x^7 + c^6 d^7 f^6 + (6c^6 d^6 e f^2 g^5 + c^6 d^7 g^6) x^6 + 3(5c^6 d^6 e f^2 g^4 + 2c^6 d^7 f^2 g^5) x^5 + 5(4c^6 d^6 e f^3 g^3 + 3c^6 d^7 f^2 g^4) x^4 + 5(3c^6 d^6 e f^4 g^2 + 4c^6 d^7 f^3 g^3) x^3 + 3(2c^6 d^6 e f^5 g + 5c^6 d^7 f^4 g^2) x^2 + (c^6 d^6 e f^6 + 6c^6 d^7 f^5 g) x) \arctan(\sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x} \sqrt{c d f g - a e g^2}) \sqrt{e x + d} / (c d e g x^2 + a d e g + (c d^2 + a e^2) g x)) / ((c^3 d^4 f^9 g^3 - 3a^2 c^2 d^3 e f^8 g^4 + 3a^2 c^2 d^2 e^2 f^7 g^5 - a^3 d^2 e^3 f^6 g^6 + (c^3 d^3 e f^3 g^9 - 3a^2 c^2 d^2 e^2 f^2 g^10 + 3a^2 c^2 d^2 e^3 f^2 g^11 - a^3 e^4 g^12) x^7 + (6c^3 d^3 e f^4 g^8 - a^3 d^2 e^3 g^12 + (c^3 d^4 - 18a^2 c^2 d^2 e^2) f^3 g^9 - 3(a^2 c^2 d^3 e - 6a^2 c^2 d^3 e) f^2 g^10 + 3(a^2 c^2 d^2 e^2 - 2a^3 e^4) f^2 g^11) x^6 + 3(5c^3 d^3 e f^5 g^7 - 2a^3 d^2 e^3 f^2 g^11 + (2c^3 d^4 - 15a^2 c^2 d^2 e^2) f^4 g^8 - 3(2a^2 c^2 d^3 e - 5a^2 c^2 d^3 e) f^3 g^9 + (6a^2 c^2 d^2 e^2 - 5a^3 e^4) f^2 g^10) x^5 + 5(4c^3 d^3 e f^6 g^6 - 3a^3 d^2 e^3 f^2 g^10 + 3(c^3 d^4 - 4a^2 c^2 d^2 e^2) f^5 g^7 - 3(3a^2 c^2 d^3 e - 4a^2 c^2 d^3 e) f^4 g^8 + (9a^2 c^2 d^2 e^2 - 4a^3 e^4) f^3 g^9) x^4 + 5(3c^3 d^3 e f^7 g^5 - 4a^3 d^2 e^3 f^3 g^9 + (4c^3 d^4 - 9a^2 c^2 d^2 e^2) f^6 g^6 - 3(4a^2 c^2 d^3 e - 3a^2 c^2 d^3 e) f^5 g^7 + 3(4a^2 c^2 d^2 e^2 - a^3 e^4) f^4 g^8) x^3 + 3(2c^3 d^3 e f^8 g^4 - 5a^3 d^2 e^3 f^4 g^8 + (5c^3 d^4 - 6a^2 c^2 d^2 e^2) f^7 g^5 - 3(5a^2 c^2 d^3 e - 2a^2 c^2 d^3 e) f^6 g^6 + (15a^2 c^2 d^2 e^2 - 2a^3 e^4) f^5 g^7) x^2 + (c^3 d^3 e f^9 g^3 - 6a^3 d^2 e^3 f^5 g^7 + 3(2c^3 d^4 - a^2 c^2 d^2 e^2) f^8 g^4 - 3(6a^2 c^2 d^3 e - a^2 c^2 d^3 e) f^7 g^5 + (18a^2 c^2 d^2 e^2 - a^3 e^4) f^6 g^6) x) \sqrt{c d f g - a e g^2})]$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)/(e\*x+d)\*\*(5/2)/(g\*x+f)\*\*

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2+a\*d\*e+(c\*d^2+a\*e^2)\*x)^(5/2)/((e\*x+d)^(5/2)\*(g\*x+f)

[Out] Timed out

$$3.712 \quad \int \frac{\sqrt{d+ex}(f+gx)^{5/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=313

$$\frac{5\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^3 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{7/2}d^{7/2}\sqrt{g}\sqrt{x}(ae^2+cd^2)+ade+cdex^2} + \frac{5\sqrt{f+gx}\sqrt{x}(ae^2+cd^2)+ade+cdex^2(cdf-aeg)^2}{8c^3d^3\sqrt{d+ex}} + \frac{5(f+gx)^{3/2}\sqrt{x}(ae^2+cd^2)+ade+cdex^2(cdf-aeg)}{12c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\sqrt{x}(ae^2+cd^2)+ade+cdex^2}{3cd\sqrt{d+ex}}$$

[Out]  $(5*(c*d*f - a*e*g)^2*\text{Sqrt}[f + g*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*c^3*d^3*\text{Sqrt}[d + e*x]) + (5*(c*d*f - a*e*g)*(f + g*x)^{(3/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*c^2*d^2*\text{Sqrt}[d + e*x]) + ((f + g*x)^{(5/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*\text{Sqrt}[d + e*x]) + (5*(c*d*f - a*e*g)^3*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x])]/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x]))/(8*c^{(7/2)}*d^{(7/2)}*\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi [A]** time = 1.4674, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{5\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^3 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{7/2}d^{7/2}\sqrt{g}\sqrt{x}(ae^2+cd^2)+ade+cdex^2} + \frac{5\sqrt{f+gx}\sqrt{x}(ae^2+cd^2)+ade+cdex^2(cdf-aeg)^2}{8c^3d^3\sqrt{d+ex}} + \frac{5(f+gx)^{3/2}\sqrt{x}(ae^2+cd^2)+ade+cdex^2(cdf-aeg)}{12c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\sqrt{x}(ae^2+cd^2)+ade+cdex^2}{3cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Sqrt}[d + e*x]*(f + g*x)^{(5/2)})/\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2], x$

[Out]  $(5*(c*d*f - a*e*g)^2*\text{Sqrt}[f + g*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*c^3*d^3*\text{Sqrt}[d + e*x]) + (5*(c*d*f - a*e*g)*(f + g*x)^{(3/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(12*c^2*d^2*\text{Sqrt}[d + e*x]) + ((f + g*x)^{(5/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d*\text{Sqrt}[d + e*x]) + (5*(c*d*f - a*e*g)^3*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x])]/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x]))/(8*c^{(7/2)}*d^{(7/2)}*\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi in Sympy [A]** time = 121.9, size = 301, normalized size = 0.96

$$\frac{(f+gx)^{5/2}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{3cd\sqrt{d+ex}} - \frac{5(f+gx)^{3/2}(aeg-cdf)\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{12c^2d^2\sqrt{d+ex}} + \frac{5\sqrt{f+gx}(aeg-cdf)^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{8c^3d^3\sqrt{d+ex}} - \frac{5(aeg-cdf)^3\sqrt{ade+cdex^2+x(ae^2+cd^2)}\text{atanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)}{8c^{7/2}d^{7/2}\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x+f)**(5/2)*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2), x)`

[Out] 
$$\frac{(f + g*x)^{5/2} \sqrt{a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2)}}{3*c*d*\sqrt{d + e*x}} - \frac{5*(f + g*x)^{3/2}*(a*e*g - c*d*f)*\sqrt{a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2)}}{12*c^2*d^2*\sqrt{d + e*x}} + \frac{5*\sqrt{f + g*x}*(a*e*g - c*d*f)^2*\sqrt{a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2)}}{8*c^3*d^3*\sqrt{d + e*x}} - \frac{5*(a*e*g - c*d*f)^3*\sqrt{a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2)}*\operatorname{atanh}(\sqrt{c}*\sqrt{d}*\sqrt{f + g*x}/(\sqrt{g}*\sqrt{a*e + c*d*x}))}{8*c^7/2*d^7/2*\sqrt{g}*\sqrt{d + e*x}*\sqrt{a*e + c*d*x}}$$

**Mathematica [A]** time = 0.386525, size = 205, normalized size = 0.65

$$\frac{\sqrt{d + ex} \left( \frac{2\sqrt{f+gx}(ae+cdx)(15a^2e^2g^2-10acdeg(4f+gx)+c^2d^2(33f^2+26fgx+8g^2x^2))}{3c^3d^3} + \frac{5\sqrt{ae+cdx}(cdf-aeg)^3 \log\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}\sqrt{ae+cdx+aeg+cdx}}{c^{7/2}d^{7/2}\sqrt{g}}\right)}{c^{7/2}d^{7/2}\sqrt{g}} \right)}{16\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] `Integrate[(Sqrt[d + e*x]^(5/2))/(Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]), x]`

[Out] 
$$\frac{(\sqrt{d + e*x})^5 \left( (2*(a*e + c*d*x))^2 \sqrt{f + g*x} \left( 15*a^2*e^2*g^2 - 10*a*c*d*e*g*(4*f + g*x) + c^2*d^2*(33*f^2 + 26*f*g*x + 8*g^2*x^2) \right) \right)}{3*c^3*d^3} + \frac{5*(c*d*f - a*e*g)^3 \sqrt{a*e + c*d*x} \operatorname{Log}[a*e*g + 2*\sqrt{c}*\sqrt{d}*\sqrt{g}*\sqrt{a*e + c*d*x}]}{c^7/2*d^7/2*\sqrt{g}}$$

**Maple [A]** time = 0.052, size = 511, normalized size = 1.6

$$-\frac{1}{48c^3d^3} \sqrt{gx+f} \sqrt{cdex^2 + ae^2x + cd^2x + ade} \left( 15 \ln \left( \frac{2xcdg + aeg + cdf + 2\sqrt{(gx+f)(cdx+ae)}\sqrt{dgc}}{\sqrt{dgc}} \right) a^3e^3g^3 - 45 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^(5/2)*(e*x+d)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2), x)`

[Out] 
$$-\frac{1}{48} (g*x+f)^{1/2} (c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^{1/2} \left( 15*1n(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^{1/2}*(d*g*c)^{1/2}))/((d*g*c)^{1/2})*a^3*e^3*g^3-45*\ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^{1/2}*(d*g*c)^{1/2}))/((d*g*c)^{1/2})*a^2*e^2*g^2*f*c*d+45*\ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^{1/2}*(d*g*c)^{1/2}))/((d*g*c)^{1/2})*a*e*g*f^2*c^2*d^2-15*\ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^{1/2}*(d*g*c)^{1/2}))*f^3*c^3*d^3-16*x^2*c^2*d^2*((g*x+f)*(c*d*x+a*e))^{1/2}*(d*g*c)^{1/2}+20*g^2*((g*x+f)*(c*d*x+a*e))^{1/2}*(d*g*c)^{1/2}-52*g*((g*x+f)*(c*d*x+a*e))^{1/2}*x*f*c^2*d^2*(d*g*c)^{1/2}-30*((g*x+f)*(c*d*x+a*e))^{1/2}*a^2*e^2*g^2*(d*g*c)^{1/2}+80*((g*x+f)*(c*d*x+a*e))^{1/2}*a*e*f*g*c*d*(d*g*c)^{1/2}-66*((g*x+f)*(c*d*x+a*e))^{1/2}*f^2*c^2*d^2*(d*g*c)^{1/2} \right) / ((g*x+f)*(c*d*x+a*e))^{1/2}/c^3/d^3/(d*g*c)^{1/2}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*(g\*x + f)^(5/2)/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

[Out] Exception raised: ValueError

**Fricas** [A] time = 1.05696, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*(g\*x + f)^(5/2)/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

[Out] [1/96\*(4\*(8\*c^2\*d^2\*g^2\*x^2 + 33\*c^2\*d^2\*f^2 - 40\*a\*c\*d\*e\*f\*g + 15\*a^2\*e^2\*g^2 + 2\*(13\*c^2\*d^2\*f\*g - 5\*a\*c\*d\*e\*g^2)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f) - 15\*(c^3\*d^4\*f^3 - 3\*a\*c^2\*d^3\*e\*f^2\*g + 3\*a^2\*c\*d^2\*e^2\*f\*g^2 - a^3\*d\*e^3\*g^3 + (c^3\*d^3\*e\*f^3 - 3\*a\*c^2\*d^2\*e^2\*f^2\*g + 3\*a^2\*c\*d\*e^3\*f\*g^2 - a^3\*e^4\*g^3)\*x)\*log((4\*(2\*c^2\*d^2\*g^2\*x + c^2\*d^2\*f\*g + a\*c\*d\*e\*g^2)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f) - (8\*c^2\*d^2\*e\*g^2\*x^3 + c^2\*d^3\*f^2 + 6\*a\*c\*d^2\*e\*f\*g + a^2\*d\*e^2\*g^2 + 8\*(c^2\*d^2\*e\*f\*g + (c^2\*d^3 + a\*c\*d\*e^2)\*g^2)\*x^2 + (c^2\*d^2\*e\*f^2 + 2\*(4\*c^2\*d^3 + 3\*a\*c\*d\*e^2)\*f\*g + (8\*a\*c\*d^2\*e + a^2\*e^3)\*g^2)\*x)\*sqrt(c\*d\*g))/(e\*x + d)))/((c^3\*d^3\*e\*x + c^3\*d^4)\*sqrt(c\*d\*g)), 1/48\*(2\*(8\*c^2\*d^2\*g^2\*x^2 + 33\*c^2\*d^2\*f^2 - 40\*a\*c\*d\*e\*f\*g + 15\*a^2\*e^2\*g^2 + 2\*(13\*c^2\*d^2\*f\*g - 5\*a\*c\*d\*e\*g^2)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f) + 15\*(c^3\*d^4\*f^3 - 3\*a\*c^2\*d^3\*e\*f^2\*g + 3\*a^2\*c\*d^2\*e^2\*f\*g^2 - a^3\*d\*e^3\*g^3 + (c^3\*d^3\*e\*f^3 - 3\*a\*c^2\*d^2\*e^2\*f^2\*g + 3\*a^2\*c\*d\*e^3\*f\*g^2 - a^3\*e^4\*g^3)\*x)\*arctan(2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f)/(2\*c\*d\*e\*g\*x^2 + c\*d^2\*f + a\*d\*e\*g + (c\*d\*e\*f + (2\*c\*d^2 + a\*e^2)\*g)\*x)))/((c^3\*d^3\*e\*x + c^3\*d^4)\*sqrt(-c\*d\*g))]

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*(5/2)\*(e\*x+d)\*\*(1/2)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2), x)

[Out] Timed out

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}(gx+f)^{\frac{5}{2}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*(g\*x + f)^(5/2)/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

[Out] integrate(sqrt(e\*x + d)\*(g\*x + f)^(5/2)/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

$$3.713 \quad \int \frac{\sqrt{d+ex}(f+gx)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=244

$$\frac{3\sqrt{d+ex}\sqrt{ae+cdx}(cdf-ae^2)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4c^{5/2}d^{5/2}\sqrt{g}\sqrt{x}(ae^2+cd^2)+ade+cdex^2} + \frac{3\sqrt{f+gx}\sqrt{x}(ae^2+cd^2)+ade+cdex^2(cdf-ae^2)}{4c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{x}(ae^2+cd^2)+ade+cdex^2}{2cd\sqrt{d+ex}}$$

[Out] (3\*(c\*d\*f - a\*e\*g)\*Sqrt[f + g\*x]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(4\*c^2\*d^2\*Sqrt[d + e\*x]) + ((f + g\*x)^(3/2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(2\*c\*d\*Sqrt[d + e\*x]) + (3\*(c\*d\*f - a\*e\*g)^2\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/(Sqrt[c]\*Sqrt[d]\*Sqrt[f + g\*x])])/(4\*c^(5/2)\*d^(5/2)\*Sqrt[g]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])

**Rubi [A]** time = 1.03494, antiderivative size = 244, normalized size of antiderivative = 1., number of rules used = 5, number of rules used = 4, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{3\sqrt{d+ex}\sqrt{ae+cdx}(cdf-ae^2)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4c^{5/2}d^{5/2}\sqrt{g}\sqrt{x}(ae^2+cd^2)+ade+cdex^2} + \frac{3\sqrt{f+gx}\sqrt{x}(ae^2+cd^2)+ade+cdex^2(cdf-ae^2)}{4c^2d^2\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{x}(ae^2+cd^2)+ade+cdex^2}{2cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[d + e\*x] \* (f + g\*x)^(3/2))/Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2], x

[Out] (3\*(c\*d\*f - a\*e\*g)\*Sqrt[f + g\*x]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(4\*c^2\*d^2\*Sqrt[d + e\*x]) + ((f + g\*x)^(3/2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(2\*c\*d\*Sqrt[d + e\*x]) + (3\*(c\*d\*f - a\*e\*g)^2\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/(Sqrt[c]\*Sqrt[d]\*Sqrt[f + g\*x])])/(4\*c^(5/2)\*d^(5/2)\*Sqrt[g]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])

**Rubi in Sympy [A]** time = 89.1391, size = 233, normalized size = 0.95

$$\frac{(f+gx)^{\frac{3}{2}}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{2cd\sqrt{d+ex}} - \frac{3\sqrt{f+gx}(aeg-cdf)\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{4c^2d^2\sqrt{d+ex}} + \frac{3(aeg-cdf)^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}\operatorname{atanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)}{4c^{\frac{5}{2}}d^{\frac{5}{2}}\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*(3/2)\*(e\*x+d)\*\*(1/2)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2), x)

[Out] (f + g\*x)\*\*(3/2)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(2\*c\*d\*sqrt(d + e\*x)) - 3\*sqrt(f + g\*x)\*(a\*e\*g - c\*d\*f)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(4\*c\*\*2\*d\*\*2\*sqrt(d + e\*x)) + 3\*(a\*e\*g - c\*d\*f)\*\*2\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*atanh(sqrt(c)\*sqrt(d)\*sqrt(f + g\*x)/(sqrt(g)\*sqrt(a\*e + c\*d\*x)))/(4\*c\*\*(5/2)\*d\*\*(5/2)\*sqrt(g)\*sqrt(d + e\*x)\*sqrt(a\*e + c\*d\*x))

)

**Mathematica [A]** time = 0.246514, size = 168, normalized size = 0.69

$$\frac{\sqrt{d+ex} \left( \frac{3\sqrt{ae+cdx}(cdf-ae g)^2 \log\left(2\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}\sqrt{ae+cdx+ae g+cd(f+2gx)}\right)}{c^{5/2}d^{5/2}\sqrt{g}} + \frac{2\sqrt{f+gx}(ae+cdx)(cd(5f+2gx)-3ae g)}{c^2d^2} \right)}{8\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[d + e\*x] \* (f + g\*x)^(3/2))/Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2], x]

[Out] (Sqrt[d + e\*x] \* ((2\*(a\*e + c\*d\*x)\*Sqrt[f + g\*x] \* (-3\*a\*e\*g + c\*d\*(5\*f + 2\*g\*x)))/(c^2\*d^2) + (3\*(c\*d\*f - a\*e\*g)^2\*Sqrt[a\*e + c\*d\*x]\*Log[a\*e\*g + 2\*Sqrt[c]\*Sqrt[d]\*Sqrt[g]\*Sqrt[a\*e + c\*d\*x]\*Sqrt[f + g\*x] + c\*d\*(f + 2\*g\*x)]/(c^(5/2)\*d^(5/2)\*Sqrt[g]))/(8\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)])

**Maple [A]** time = 0.035, size = 328, normalized size = 1.3

$$\frac{1}{8c^2d^2}\sqrt{gx+f}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(3\ln\left(\frac{2xcdg+ae g+cdf+2\sqrt{(gx+f)(cdx+ae)}\sqrt{dgc}}{\sqrt{dgc}}\right)a^2e^2g^2-6\ln\left(\frac{2\sqrt{(gx+f)(cdx+ae)}\sqrt{dgc}}{\sqrt{dgc}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^(3/2)\*(e\*x+d)^(1/2)/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2), x)

[Out] 1/8\*(g\*x+f)^(1/2)\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)\*(3\*ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*(d\*g\*c)^(1/2)))/(d\*g\*c)^(1/2))\*a^2\*e^2\*g^2-6\*ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*(d\*g\*c)^(1/2)))/(d\*g\*c)^(1/2))\*a\*e\*g\*f\*c\*d+3\*ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*(d\*g\*c)^(1/2)))/(d\*g\*c)^(1/2))\*f^2\*c^2\*d^2+4\*g\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*x\*c\*d\*(d\*g\*c)^(1/2)-6\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*a\*e\*g\*(d\*g\*c)^(1/2)+10\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*f\*c\*d\*(d\*g\*c)^(1/2))/(e\*x+d)^(1/2)/((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)/c^2/d^2/(d\*g\*c)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d) \* (g\*x + f)^(3/2)/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.951054, size = 1, normalized size = 0.

$$\frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2cdgx+5cdf-3ae g)\sqrt{cdg}\sqrt{ex+d}\sqrt{gx+f}+3(c^2d^3f^2-2acd^2efg+a^2de^2g^2+(c^2d^2e^2g^2+2cd^2efg+ade^2g^2))\sqrt{cdg}\sqrt{ex+d}\sqrt{gx+f}}{8\sqrt{(d+ex)(ae+cdx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(e*x + d)*(g*x + f)^(3/2)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)`

[Out] `[1/16*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + 5*c*d*f - 3*a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a^2*e^3*g^2)*x)*log(-(4*(2*c^2*d^2*g^2*x + c^2*d^2*f*g + a*c*d*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + (8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)*sqrt(c*d*g))/(e*x + d))/((c^2*d^2*e*x + c^2*d^3)*sqrt(c*d*g)), 1/8*(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x + 5*c*d*f - 3*a*e*g)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a^2*e^3*g^2)*x)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/((c^2*d^2*e*x + c^2*d^3)*sqrt(-c*d*g))]`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)**(3/2)*(e*x+d)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2))`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}(gx+f)^{\frac{3}{2}}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(e*x + d)*(g*x + f)^(3/2)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)`

[Out] `integrate(sqrt(e*x + d)*(g*x + f)^(3/2)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x), x)`



$$3.714 \quad \int \frac{\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=169

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf - aeg) \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} + \frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

[Out] (Sqrt[f + g\*x]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(c\*d\*Sqrt[d + e\*x]) + ((c\*d\*f - a\*e\*g)\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/(Sqrt[c]\*Sqrt[d]\*Sqrt[f + g\*x])])/(c^(3/2)\*d^(3/2)\*Sqrt[g]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])

**Rubi [A]** time = 0.654544, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf - aeg) \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} + \frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[d + e\*x]\*Sqrt[f + g\*x])/Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2], x]

[Out] (Sqrt[f + g\*x]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(c\*d\*Sqrt[d + e\*x]) + ((c\*d\*f - a\*e\*g)\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/(Sqrt[c]\*Sqrt[d]\*Sqrt[f + g\*x])])/(c^(3/2)\*d^(3/2)\*Sqrt[g]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])

**Rubi in Sympy [A]** time = 60.7833, size = 160, normalized size = 0.95

$$\frac{\sqrt{f+gx}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{cd\sqrt{d+ex}} - \frac{(aeg - cdf)\sqrt{ade+cdex^2+x(ae^2+cd^2)} \operatorname{atanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{\frac{3}{2}}d^{\frac{3}{2}}\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*(1/2)\*(e\*x+d)\*\*(1/2)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2), x)

[Out] sqrt(f + g\*x)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(c\*d\*sqrt(d + e\*x)) - (a\*e\*g - c\*d\*f)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*atanh(sqrt(g)\*sqrt(a\*e + c\*d\*x)/(sqrt(c)\*sqrt(d)\*sqrt(f + g\*x)))/(c\*\*(3/2)\*d\*\*(3/2)\*sqrt(g)\*sqrt(d + e\*x)\*sqrt(a\*e + c\*d\*x))

**Mathematica [A]** time = 0.228691, size = 157, normalized size = 0.93

$$\frac{\sqrt{d+ex}\left(2\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}(ae+cdx) + \sqrt{ae+cdx}(cdf - aeg) \log\left(2\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}\sqrt{ae+cdx} + aeg + cd(f+2gx)\right)\right)}{2c^{3/2}d^{3/2}\sqrt{g}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[d + e\*x]\*Sqrt[f + g\*x])/Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2], x]

[Out] (Sqrt[d + e\*x]\*(2\*Sqrt[c]\*Sqrt[d]\*Sqrt[g]\*(a\*e + c\*d\*x)\*Sqrt[f + g\*x] + (c\*d\*f - a\*e\*g)\*Sqrt[a\*e + c\*d\*x]\*Log[a\*e\*g + 2\*Sqrt[c]\*Sqrt[d]\*Sqrt[g]\*Sqrt[a\*e + c\*d\*x]\*Sqrt[f + g\*x] + c\*d\*(f + 2\*g\*x)))/(2\*c^(3/2)\*d^(3/2)\*Sqrt[g]\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)])

**Maple [A]** time = 0.029, size = 201, normalized size = 1.2

$$-\frac{1}{2cd}\sqrt{gx+f}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(\ln\left(\frac{1}{2}\left(2xcdg+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\sqrt{dgc}\right)\frac{1}{\sqrt{dgc}}\right)aeg-\ln\left(\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^(1/2)\*(e\*x+d)^(1/2)/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2), x)

[Out] -1/2\*(g\*x+f)^(1/2)\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)\*(ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*(d\*g\*c)^(1/2))/(d\*g\*c)^(1/2))\*a\*e\*g-ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*(d\*g\*c)^(1/2))/(d\*g\*c)^(1/2))\*c\*d\*f-2\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*(d\*g\*c)^(1/2))/(e\*x+d)^(1/2)/((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)/c/d/(d\*g\*c)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*sqrt(g\*x + f)/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.920057, size = 1, normalized size = 0.01

$$\left[ \frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{cdg}\sqrt{ex+d}\sqrt{gx+f}-(cd^2f-ade g+(cdf-ae^2g)x)\log\left(\frac{4(2c^2d^2g^2x+c^2d^2fg+acdeg^2)\sqrt{cdex^2+ade+(cd^2+ae^2)x}}{(c*d*e*x^2+a*d*e+(c*d^2+a*e^2)*x)}\right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*sqrt(g\*x + f)/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

[Out] [1/4\*(4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f) - (c\*d^2\*f - a\*d\*e\*g + (c\*d\*e\*f - a\*e^2\*g)\*x)\*log((4\*(2\*c^2\*d^2\*g^2\*x + c^2\*d^2\*f\*g + a\*c\*d\*e\*g^2)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f) - (8\*c^2\*d^2\*e\*g^2\*x^3 + c^2\*d^3\*f^2 + 6\*a\*c\*d^2\*e\*f\*g + a^2\*d^2\*e^2\*g^2 + 8\*(c^2\*d^2\*e\*f\*g + (c^2\*d^3 + a\*c\*d\*e^2)\*g^2)\*x^2 + (c^2\*d^2\*e\*f^2 + 2\*(4\*c^2\*d^3 + 3\*a\*c\*d\*e^2)\*f\*g + (8\*a\*c\*d^2\*e + a^2\*e^3)\*g^2)\*x)\*sqrt(c\*d\*g))/(e\*x + d))/((c\*d\*e\*x + c\*d^2)\*sqrt(c\*d\*g)), 1/2\*(2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f) + (c\*d^2\*f - a\*d\*e\*g + (c\*d\*e\*f - a\*e^2\*g)\*x)\*arctan(2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f)/(2\*c\*d\*e\*g\*x^2 + c

$$d^2 f + a d e g + (c d e f + (2 c d^2 + a e^2) g x) / ((c d e x + c d^2) \sqrt{-c d g})$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex} \sqrt{f+gx}}{\sqrt{(d+ex)(ae+cdx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*(1/2)\*(e\*x+d)\*\*(1/2)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*

[Out] Integral(sqrt(d + e\*x)\*sqrt(f + g\*x)/sqrt((d + e\*x)\*(a\*e + c\*d\*x)), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d} \sqrt{gx+f}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*sqrt(g\*x + f)/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)

[Out] integrate(sqrt(e\*x + d)\*sqrt(g\*x + f)/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

$$3.715 \quad \int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=105

$$\frac{2\sqrt{d+ex}\sqrt{ae+cdx} \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

[Out] (2\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/(Sqrt[c]\*Sqrt[d]\*Sqrt[f + g\*x])])/(Sqrt[c]\*Sqrt[d]\*Sqrt[g]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])

**Rubi [A]** time = 0.322933, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{2\sqrt{d+ex}\sqrt{ae+cdx} \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e\*x]/(Sqrt[f + g\*x]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]), x]

[Out] (2\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/(Sqrt[c]\*Sqrt[d]\*Sqrt[f + g\*x])])/(Sqrt[c]\*Sqrt[d]\*Sqrt[g]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])

**Rubi in Sympy [A]** time = 38.1734, size = 102, normalized size = 0.97

$$\frac{2\sqrt{ade+cdex^2+x(ae^2+cd^2)} \operatorname{atanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)}{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*(1/2)/(g\*x+f)\*\*(1/2)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2))

[Out] 2\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*atanh(sqrt(c)\*sqrt(d)\*sqrt(f + g\*x)/(sqrt(g)\*sqrt(a\*e + c\*d\*x)))/(sqrt(c)\*sqrt(d)\*sqrt(g)\*sqrt(d + e\*x)\*sqrt(a\*e + c\*d\*x))

**Mathematica [A]** time = 0.0905625, size = 109, normalized size = 1.04

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx} \log\left(2\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}\sqrt{ae+cdx} + aeg + cdf + 2cdgx\right)}{\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e\*x]/(Sqrt[f + g\*x]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])

[Out] (Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*Log[c\*d\*f + a\*e\*g + 2\*c\*d\*g\*x + 2\*Sqrt[c]\*Sqrt[d]\*Sqrt[g]\*Sqrt[a\*e + c\*d\*x]\*Sqrt[f + g\*x])/(Sqrt

[c]\*Sqrt[d]\*Sqrt[g]\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)]

**Maple [A]** time = 0.038, size = 112, normalized size = 1.1

$$1\sqrt{gx+f}\sqrt{cdex^2+ae^2x+cd^2x+ade}\ln\left(\frac{1}{2}\left(2xcdg+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\sqrt{dgc}\right)\frac{1}{\sqrt{dgc}}\right)\frac{1}{\sqrt{ex+d}}\frac{1}{\sqrt{dgc}}\frac{1}{\sqrt{(g^2x+ae^2)(d+ex)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(1/2)/(g\*x+f)^(1/2)/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2), x)

[Out] 1/(e\*x+d)^(1/2)\*(g\*x+f)^(1/2)\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)\*ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*(d\*g\*c)^(1/2))/(d\*g\*c)^(1/2))/(d\*g\*c)^(1/2)/((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(g\*x + f)

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.895129, size = 1, normalized size = 0.01

$$\left[ \frac{\log\left(-\frac{4(2c^2d^2g^2x+c^2d^2fg+acdeg^2)\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}\sqrt{gx+f}+(8c^2d^2eg^2x^3+c^2d^3f^2+6acd^2efg+a^2de^2g^2+8(c^2d^2efg+(c^2d^3+acde^2)g^2))\sqrt{ex+d}}{ex+d}\right)}{2\sqrt{cdg}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(g\*x + f)

[Out] [1/2\*log(-(4\*(2\*c^2\*d^2\*g^2\*x + c^2\*d^2\*f\*g + a\*c\*d\*e\*g^2)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f) + (8\*c^2\*d^2\*e\*g^2\*x^3 + c^2\*d^3\*f^2 + 6\*a\*c\*d^2\*e\*f\*g + a^2\*d\*e^2\*g^2 + 8\*(c^2\*d^2\*e\*f\*g + (c^2\*d^3 + a\*c\*d\*e^2)\*g^2)\*x^2 + (c^2\*d^2\*e\*f^2 + 2\*(4\*c^2\*d^3 + 3\*a\*c\*d\*e^2)\*f\*g + (8\*a\*c\*d^2\*e + a^2\*e^3)\*g^2)\*x)\*sqrt(c\*d\*g))/(e\*x + d))/sqrt(c\*d\*g), arctan(2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f)/(2\*c\*d\*e\*g\*x^2 + c\*d^2\*f + a\*d\*e\*g + (c\*d\*e\*f + (2\*c\*d^2 + a\*e^2)\*g)\*x))/sqrt(-c\*d\*g)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}}{\sqrt{(d+ex)(ae+cdx)}\sqrt{f+gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(1/2)/(g\*x+f)\*\*(1/2)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*

[Out] Integral(sqrt(d + e\*x)/(sqrt((d + e\*x)\*(a\*e + c\*d\*x))\*sqrt(f + g\*x)), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{gx+f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(g\*x + f

[Out] integrate(sqrt(e\*x + d)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(g\*x + f)), x)

$$3.716 \quad \int \frac{\sqrt{d+ex}}{(f+gx)^{3/2} \sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=61

$$\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)}$$

[Out] (2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/((c\*d\*f - a\*e\*g)\*Sqrt[d + e\*x]\*Sqrt[f + g\*x])

**Rubi [A]** time = 0.254093, antiderivative size = 61, normalized size of antiderivative = 1., number of rules used = 1, number of rules used = 1, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$

$$\frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e\*x]/((f + g\*x)^(3/2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]), x

[Out] (2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/((c\*d\*f - a\*e\*g)\*Sqrt[d + e\*x]\*Sqrt[f + g\*x])

**Rubi in Sympy [A]** time = 22.1253, size = 58, normalized size = 0.95

$$\frac{2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}\sqrt{f+gx}(aeg-cdf)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*(1/2)/(g\*x+f)\*\*(3/2)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2), x

[Out] -2\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(sqrt(d + e\*x)\*sqrt(f + g\*x)\*(a\*e\*g - c\*d\*f))

**Mathematica [A]** time = 0.0919078, size = 50, normalized size = 0.82

$$\frac{2\sqrt{(d+ex)(ae+cdx)}}{\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e\*x]/((f + g\*x)^(3/2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]), x

[Out] (2\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)])/((c\*d\*f - a\*e\*g)\*Sqrt[d + e\*x]\*Sqrt[f + g\*x])

**Maple [A]** time = 0.01, size = 63, normalized size = 1.

$$-2 \frac{(cdx + ae)\sqrt{ex + d}}{\sqrt{gx + f}(aeg - cdf)\sqrt{cdex^2 + ae^2x + cd^2x + ade}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((e^x+d)^{(1/2)}/(g^x+f)^{(3/2)}/(a^d e+(a^e e^2+c^d)^x+c^d e^x)^{(1/2)}, x)$

[Out]  $-2/(g^x+f)^{(1/2)}*(c^d x+a^e)/(a^e g-c^d f)*(e^x+d)^{(1/2)}/(c^d e^x^2+a^e e^2 x+c^d)^2 x+a^d e)^{(1/2)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\sqrt{e^x+d}/(\sqrt{c^d e^x^2+a^d e+(c^d e^2+a^e)^x})*(g^x+f)^3)$

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.281747, size = 154, normalized size = 2.52

$$\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}\sqrt{gx+f}}{cd^2f^2-ade fg+(cdfg-ae^2g^2)x^2+(cdf^2-ade g^2+(cd^2-ae^2)fg)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\sqrt{e^x+d}/(\sqrt{c^d e^x^2+a^d e+(c^d e^2+a^e)^x})*(g^x+f)^3)$

[Out]  $2*\sqrt{c^d e^x^2+a^d e+(c^d e^2+a^e)^x}*\sqrt{e^x+d}*\sqrt{g^x+f}/(c^d e^2 f^2-a^d e^e f^2 g+(c^d e^e f^2 g-a^e e^2 g^2)^x^2+(c^d e^e f^2-a^d e^e g^2+(c^d e^2-a^e)^e f^2 g)^x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^x+d)^{(1/2)}/(g^x+f)^{(3/2)}/(a^d e+(a^e e^2+c^d)^x+c^d e^x)^{(1/2)}, x)$

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\sqrt{e^x+d}/(\sqrt{c^d e^x^2+a^d e+(c^d e^2+a^e)^x})*(g^x+f)^3)$

[Out]  $\text{integrate}(\sqrt{e^x+d}/(\sqrt{c^d e^x^2+a^d e+(c^d e^2+a^e)^x})*(g^x+f)^{(3/2)}, x)$



$$3.717 \quad \int \frac{\sqrt{d+ex}}{(f+gx)^{5/2} \sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=129

$$\frac{4cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2} + \frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}$$

[Out] (2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(3\*(c\*d\*f - a\*e\*g)\*Sqrt[d + e\*x]\*(f + g\*x)^(3/2)) + (4\*c\*d\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(3\*(c\*d\*f - a\*e\*g)^2\*Sqrt[d + e\*x]\*Sqrt[f + g\*x])

**Rubi [A]** time = 0.515515, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{4cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2} + \frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e\*x]/((f + g\*x)^(5/2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]), x

[Out] (2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(3\*(c\*d\*f - a\*e\*g)\*Sqrt[d + e\*x]\*(f + g\*x)^(3/2)) + (4\*c\*d\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(3\*(c\*d\*f - a\*e\*g)^2\*Sqrt[d + e\*x]\*Sqrt[f + g\*x])

**Rubi in Sympy [A]** time = 43.6975, size = 122, normalized size = 0.95

$$\frac{4cd\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{3\sqrt{d+ex}\sqrt{f+gx}(aeg-cdf)^2} - \frac{2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{3\sqrt{d+ex}(f+gx)^{3/2}(aeg-cdf)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*(1/2)/(g\*x+f)\*\*(5/2)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*

[Out] 4\*c\*d\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(3\*sqrt(d + e\*x)\*sqrt(f + g\*x)\*(a\*e\*g - c\*d\*f)\*\*2) - 2\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(3\*sqrt(d + e\*x)\*(f + g\*x)\*\*(3/2)\*(a\*e\*g - c\*d\*f))

**Mathematica [A]** time = 0.145734, size = 69, normalized size = 0.53

$$\frac{2\sqrt{(d+ex)(ae+cdx)}(cd(3f+2gx)-aeg)}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e\*x]/((f + g\*x)^(5/2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x

[Out] (2\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)]\*(-(a\*e\*g) + c\*d\*(3\*f + 2\*g\*x)))/(3\*(c\*d\*f - a\*e\*g)^2\*Sqrt[d + e\*x]\*(f + g\*x)^(3/2))

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**Maple [A]** time = 0.011, size = 98, normalized size = 0.8

$$\frac{(2cdx + 2ae)(-2xcdg + aeg - 3cdf)\sqrt{ex + d}(gx + f)^{-\frac{3}{2}}}{3a^2e^2g^2 - 6acdefg + 3c^2d^2f^2} \frac{1}{\sqrt{cdex^2 + ae^2x + cd^2x + ade}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(1/2)/(g\*x+f)^(5/2)/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2), x)

[Out] -2/3\*(c\*d\*x+a\*e)\*(-2\*c\*d\*g\*x+a\*e\*g-3\*c\*d\*f)\*(e\*x+d)^(1/2)/(g\*x+f)^(3/2)/(a^2\*e^2\*g^2-2\*a\*c\*d\*e\*f\*g+c^2\*d^2\*f^2)/(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex + d}}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(gx + f)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^(5/2)), x)

[Out] integrate(sqrt(e\*x + d)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^(5/2)), x)

---

**Fricas [A]** time = 0.284762, size = 389, normalized size = 3.02

$$\frac{2\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(2cdgx + 3cdf)}{3(c^2d^3f^4 - 2acd^2ef^3g + a^2de^2f^2g^2 + (c^2d^2ef^2g^2 - 2acde^2fg^3 + a^2e^3g^4)x^3 + (2c^2d^2ef^3g + a^2de^2g^4 + (c^2d^3 - 4acde^2)f^2g^2))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^(5/2)), x)

[Out] 2/3\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(2\*c\*d\*g\*x + 3\*c\*d\*f - a\*e\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f)/(c^2\*d^3\*f^4 - 2\*a\*c\*d^2\*e\*f^3\*g + a^2\*d^2\*e^2\*f^2\*g^2 + (c^2\*d^2\*e\*f^2\*g^2 - 2\*a\*c\*d^2\*e\*f^2\*g^2 - 2\*a\*c\*d^2\*e^2\*f\*g^3 + a^2\*e^3\*g^4)\*x^3 + (2\*c^2\*d^2\*e\*f^3\*g + a^2\*d^2\*e^2\*g^4 + (c^2\*d^3 - 4\*a\*c\*d^2\*e^2)\*f^2\*g^2) - 2\*(a\*c\*d^2\*e - a^2\*e^3)\*f\*g^3)\*x^2 + (c^2\*d^2\*e\*f^4 + 2\*a^2\*d^2\*e^2\*f\*g^3 + 2\*(c^2\*d^3 - a\*c\*d^2\*e^2)\*f^3\*g - (4\*a\*c\*d^2\*e - a^2\*e^3)\*f^2\*g^2)\*x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(1/2)/(g\*x+f)\*\*(5/2)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2), x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^(5/2)), x)

[Out] integrate(sqrt(e\*x + d)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^(5/2)), x)

$$3.718 \quad \int \frac{\sqrt{d+ex}}{(f+gx)^{7/2} \sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=198

$$\frac{16c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{15\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^3} + \frac{8cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{15\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^2} + \frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)}$$

[Out] (2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(5\*(c\*d\*f - a\*e\*g)\*Sqrt[d + e\*x]\*(f + g\*x)^(5/2)) + (8\*c\*d\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(15\*(c\*d\*f - a\*e\*g)^2\*Sqrt[d + e\*x]\*(f + g\*x)^(3/2)) + (16\*c^2\*d^2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(15\*(c\*d\*f - a\*e\*g)^3\*Sqrt[d + e\*x]\*Sqrt[f + g\*x])

**Rubi [A]** time = 0.795988, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{16c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{15\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^3} + \frac{8cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{15\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^2} + \frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e\*x]/((f + g\*x)^(7/2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]), x

[Out] (2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(5\*(c\*d\*f - a\*e\*g)\*Sqrt[d + e\*x]\*(f + g\*x)^(5/2)) + (8\*c\*d\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(15\*(c\*d\*f - a\*e\*g)^2\*Sqrt[d + e\*x]\*(f + g\*x)^(3/2)) + (16\*c^2\*d^2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(15\*(c\*d\*f - a\*e\*g)^3\*Sqrt[d + e\*x]\*Sqrt[f + g\*x])

**Rubi in Sympy [A]** time = 70.0242, size = 190, normalized size = 0.96

$$\frac{16c^2d^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{15\sqrt{d+ex}\sqrt{f+gx}(aeg-cdf)^3} + \frac{8cd\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{15\sqrt{d+ex}(f+gx)^{3/2}(aeg-cdf)^2} - \frac{2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{5\sqrt{d+ex}(f+gx)^{5/2}(aeg-cdf)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*(1/2)/(g\*x+f)\*\*(7/2)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x

[Out] -16\*c\*\*2\*d\*\*2\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(15\*sqrt(d + e\*x)\*sqrt(f + g\*x)\*(a\*e\*g - c\*d\*f)\*\*3) + 8\*c\*d\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(15\*sqrt(d + e\*x)\*(f + g\*x)\*\*(3/2)\*(a\*e\*g - c\*d\*f)\*\*2) - 2\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(5\*sqrt(d + e\*x)\*(f + g\*x)\*\*(5/2)\*(a\*e\*g - c\*d\*f))

**Mathematica [A]** time = 0.254817, size = 105, normalized size = 0.53

$$\frac{2\sqrt{(d+ex)(ae+cdx)}(3a^2e^2g^2-2acdeg(5f+2gx)+c^2d^2(15f^2+20fgx+8g^2x^2))}{15\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e\*x]/((f + g\*x)^(7/2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x

[Out]  $(2\sqrt{(a^2e + c^2d^2x)(d + ex)})(3a^2e^2g^2 - 2ac^2d^2e^2g(5f + 2gx) + c^2d^2(15f^2 + 20fgx + 8g^2x^2)) / (15(c^2d^2f - a^2e^2g)^3 \sqrt{(d + ex)(f + gx)^{5/2}})$

**Maple [A]** time = 0.013, size = 169, normalized size = 0.9

$$\frac{(2cdx + 2ae)(8c^2d^2g^2x^2 - 4acdeg^2x + 20c^2d^2fgx + 3a^2e^2g^2 - 10acdefg + 15c^2d^2f^2)}{15a^3e^3g^3 - 45a^2cde^2fg^2 + 45ac^2d^2ef^2g - 15c^3d^3f^3} \sqrt{ex + d} (gx + f)^{-\frac{5}{2}} \frac{1}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((e^x+d)^{(1/2)}/(g^x+f)^{(7/2)}/(a^d*e+(a^e^2+c^d^2)*x+c^d*e^x^2)^{(1/2)}, x)$

[Out]  $-2/15*(c^d*x+a^e)*(8*c^2*d^2*g^2*x^2-4*a*c^d*e^g^2*x+20*c^2*d^2*f*g^2*x+3*a^2*e^2*g^2-10*acdefg+15*c^2*d^2*f^2)*(e^x+d)^{(1/2)}/(g^x+f)^{(5/2)}/(a^3*e^3*g^3-3*a^2*c^d*e^2*f*g^2+3*a*c^2*d^2*e*f^2*g-c^3*d^3*f^3)/(c^d*e^x^2+a^e^2*x+c^d^2*x+a^d*e)^{(1/2)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex + d}}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(gx + f)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\text{sqrt}(e^x + d)/(\text{sqrt}(c^d*e^x^2 + a^d*e + (c^d^2 + a^e^2)*x)*(g^x + f)^{(7/2)}), x)$

[Out]  $\text{integrate}(\text{sqrt}(e^x + d)/(\text{sqrt}(c^d*e^x^2 + a^d*e + (c^d^2 + a^e^2)*x)*(g^x + f)^{(7/2)}), x)$

**Fricas [A]** time = 0.289814, size = 772, normalized size = 3.9

$$15(c^3d^4f^6 - 3ac^2d^3ef^5g + 3a^2cd^2e^2f^4g^2 - a^3de^3f^3g^3 + (c^3d^3ef^3g^3 - 3ac^2d^2e^2f^2g^4 + 3a^2cde^3fg^5 - a^3e^4g^6)x^4 + (3c^3d^3ef^3g^3 - 3ac^2d^2e^2f^2g^4 + 3a^2cde^3fg^5 - a^3e^4g^6)x^3 + \dots)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\text{sqrt}(e^x + d)/(\text{sqrt}(c^d*e^x^2 + a^d*e + (c^d^2 + a^e^2)*x)*(g^x + f)^{(7/2)}), x)$

[Out]  $2/15*(8*c^2*d^2*g^2*x^2 + 15*c^2*d^2*f^2 - 10*a*c^d*e^f*g + 3*a^2*e^2*g^2 + 4*(5*c^2*d^2*f*g - a*c^d*e^g^2)*x)*\text{sqrt}(c^d*e^x^2 + a^d*e + (c^d^2 + a^e^2)*x)*\text{sqrt}(e^x + d)*\text{sqrt}(g^x + f)/(c^3*d^4*f^6 - 3*a^2*c^d^3*e^f^5*g + 3*a^2*c^d^2*e^2*f^4*g^2 - a^3*d^3*e^3*f^3*g^3 + (c^3*d^3*e^f^3*g^3 - 3*a^2*c^d^2*e^2*f^2*g^4 + 3*a^2*c^d*e^3*f^2*g^5 - a^3*d^3*e^4*g^6)*x^4 + (3*c^3*d^3*e^f^3*g^3 - 3*a^2*c^d^2*e^2*f^2*g^4 + 3*a^2*c^d*e^3*f^2*g^5 - a^3*d^3*e^4*g^6)*x^3 + \dots)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(1/2)/(g\*x+f)\*\*(7/2)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x+d)/(sqrt(c\*d\*e\*x^2+a\*d\*e+(c\*d^2+a\*e^2)\*x)\*(g\*x+f)^(7

[Out] integrate(sqrt(e\*x+d)/(sqrt(c\*d\*e\*x^2+a\*d\*e+(c\*d^2+a\*e^2)\*x)\*(g\*x+f)^(7/2)), x)

$$3.719 \quad \int \frac{\sqrt{d+ex}}{(f+gx)^{9/2} \sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=267

$$\frac{32c^3d^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{35\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^4} + \frac{16c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{35\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^3}$$

$$+ \frac{12cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{35\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)^2} + \frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{7\sqrt{d+ex}(f+gx)^{7/2}(cdf-aeg)}$$

[Out] (2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(7\*(c\*d\*f - a\*e\*g)\*Sqrt[d + e\*x]\*(f + g\*x)^(7/2)) + (12\*c\*d\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(35\*(c\*d\*f - a\*e\*g)^2\*Sqrt[d + e\*x]\*(f + g\*x)^(5/2)) + (16\*c^2\*d^2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(35\*(c\*d\*f - a\*e\*g)^3\*Sqrt[d + e\*x]\*(f + g\*x)^(3/2)) + (32\*c^3\*d^3\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(35\*(c\*d\*f - a\*e\*g)^4\*Sqrt[d + e\*x]\*Sqrt[f + g\*x])

**Rubi [A]** time = 1.09972, antiderivative size = 267, normalized size of antiderivative = 1., number of rules used = 4, number of rules used = 2, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{32c^3d^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{35\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^4} + \frac{16c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{35\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^3}$$

$$+ \frac{12cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{35\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)^2} + \frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{7\sqrt{d+ex}(f+gx)^{7/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e\*x]/((f + g\*x)^(9/2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]), x

[Out] (2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(7\*(c\*d\*f - a\*e\*g)\*Sqrt[d + e\*x]\*(f + g\*x)^(7/2)) + (12\*c\*d\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(35\*(c\*d\*f - a\*e\*g)^2\*Sqrt[d + e\*x]\*(f + g\*x)^(5/2)) + (16\*c^2\*d^2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(35\*(c\*d\*f - a\*e\*g)^3\*Sqrt[d + e\*x]\*(f + g\*x)^(3/2)) + (32\*c^3\*d^3\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(35\*(c\*d\*f - a\*e\*g)^4\*Sqrt[d + e\*x]\*Sqrt[f + g\*x])

**Rubi in Sympy [A]** time = 99.9555, size = 258, normalized size = 0.97

$$\frac{32c^3d^3\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{35\sqrt{d+ex}\sqrt{f+gx}(aeg-cdf)^4} - \frac{16c^2d^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{35\sqrt{d+ex}(f+gx)^{\frac{3}{2}}(aeg-cdf)^3}$$

$$+ \frac{12cd\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{35\sqrt{d+ex}(f+gx)^{\frac{5}{2}}(aeg-cdf)^2} - \frac{2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{7\sqrt{d+ex}(f+gx)^{\frac{7}{2}}(aeg-cdf)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*(1/2)/(g\*x+f)\*\*(9/2)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*

[Out] 32\*c\*\*3\*d\*\*3\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(35\*sqr  
t(d + e\*x)\*sqrt(f + g\*x)\*(a\*e\*g - c\*d\*f)\*\*4) - 16\*c\*\*2\*d\*\*2\*sq  
r(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(35\*sqrt(d + e\*x)\*(f  
+ g\*x)\*\*(3/2)\*(a\*e\*g - c\*d\*f)\*\*3) + 12\*c\*d\*sqrt(a\*d\*e + c\*d\*e\*x\*\*  
2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(35\*sqrt(d + e\*x)\*(f + g\*x)\*\*(5/2)\*(a\*e  
g - c\*d\*f)\*\*2) - 2\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))

$$/((7*\sqrt{d + e*x})*(f + g*x)**(7/2)*(a*e*g - c*d*f))$$

**Mathematica [A]** time = 0.367312, size = 128, normalized size = 0.48

$$\frac{2\sqrt{(d+ex)(ae+cdx)}(8c^2d^2(f+gx)^2(cdf-aeg)+6cd(f+gx)(cdf-aeg)^2+5(cdf-aeg)^3+16c^3d^3(f+gx)^3)}{35\sqrt{d+ex}(f+gx)^{7/2}(cdf-aeg)^4}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e\*x]/((f + g\*x)^(9/2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x

[Out] (2\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)]\*(5\*(c\*d\*f - a\*e\*g)^3 + 6\*c\*d\*(c\*d\*f - a\*e\*g)^2\*(f + g\*x) + 8\*c^2\*d^2\*(c\*d\*f - a\*e\*g)\*(f + g\*x)^2 + 16\*c^3\*d^3\*(f + g\*x)^3))/(35\*(c\*d\*f - a\*e\*g)^4\*Sqrt[d + e\*x]\*(f + g\*x)^(7/2))

**Maple [A]** time = 0.017, size = 260, normalized size = 1.

$$\frac{(2cdx + 2ae)(-16c^3d^3g^3x^3 + 8ac^2d^2eg^3x^2 - 56c^3d^3fg^2x^2 - 6a^2cde^2g^3x + 28ac^2d^2efg^2x - 70c^3d^3f^2gx + 5a^3e^3g^3 - 235g^4e^4a^4 - 140cdg^3fe^3a^3 + 210c^2d^2g^2f^2e^2a^2 - 140c^3d^3gf^3ea + 35c^4d^4f^4)}{35g^4e^4a^4 - 140cdg^3fe^3a^3 + 210c^2d^2g^2f^2e^2a^2 - 140c^3d^3gf^3ea + 35c^4d^4f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(1/2)/(g\*x+f)^(9/2)/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2), x)

[Out] -2/35\*(c\*d\*x+a\*e)\*(-16\*c^3\*d^3\*g^3\*x^3+8\*a\*c^2\*d^2\*e\*g^3\*x^2-56\*c^3\*d^3\*f\*g^2\*x+28\*a\*c^2\*d^2\*e\*f\*g^2\*x-70\*c^3\*d^3\*f^2\*g\*x+5\*a^3\*e^3\*g^3-235\*g^4\*e^4\*a^4-140\*cdg^3fe^3a^3+210\*c^2\*d^2\*g^2\*f^2\*e^2\*a^2-140\*c^3\*d^3\*gf^3ea+35\*c^4\*d^4\*f^4)/(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^(9/2)), x)

[Out] integrate(sqrt(e\*x + d)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^(9/2)), x)

**Fricas [A]** time = 0.297505, size = 1287, normalized size = 4.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^(9/2)), x)



```
[Out] 2/35*(16*c^3*d^3*g^3*x^3 + 35*c^3*d^3*f^3 - 35*a*c^2*d^2*e*f^2*g
+ 21*a^2*c*d*e^2*f*g^2 - 5*a^3*e^3*g^3 + 8*(7*c^3*d^3*f*g^2 - a*c
^2*d^2*e*g^3)*x^2 + 2*(35*c^3*d^3*f^2*g - 14*a*c^2*d^2*e*f*g^2 +
3*a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
*sqrt(e*x + d)*sqrt(g*x + f)/(c^4*d^5*f^8 - 4*a*c^3*d^4*e*f^7*g +
6*a^2*c^2*d^3*e^2*f^6*g^2 - 4*a^3*c*d^2*e^3*f^5*g^3 + a^4*d*e^4*
f^4*g^4 + (c^4*d^4*e*f^4*g^4 - 4*a*c^3*d^3*e^2*f^3*g^5 + 6*a^2*c^
2*d^2*e^3*f^2*g^6 - 4*a^3*c*d*e^4*f*g^7 + a^4*e^5*g^8)*x^5 + (4*c
^4*d^4*e*f^5*g^3 + a^4*d*e^4*g^8 + (c^4*d^5 - 16*a*c^3*d^3*e^2)*f
^4*g^4 - 4*(a*c^3*d^4*e - 6*a^2*c^2*d^2*e^3)*f^3*g^5 + 2*(3*a^2*c
^2*d^3*e^2 - 8*a^3*c*d*e^4)*f^2*g^6 - 4*(a^3*c*d^2*e^3 - a^4*e^5)
*f*g^7)*x^4 + 2*(3*c^4*d^4*e*f^6*g^2 + 2*a^4*d*e^4*f*g^7 + 2*(c^4
*d^5 - 6*a*c^3*d^3*e^2)*f^5*g^3 - 2*(4*a*c^3*d^4*e - 9*a^2*c^2*d^
2*e^3)*f^4*g^4 + 12*(a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^3*g^5 - (8*
a^3*c*d^2*e^3 - 3*a^4*e^5)*f^2*g^6)*x^3 + 2*(2*c^4*d^4*e*f^7*g +
3*a^4*d*e^4*f^2*g^6 + (3*c^4*d^5 - 8*a*c^3*d^3*e^2)*f^6*g^2 - 12*
(a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^5*g^3 + 2*(9*a^2*c^2*d^3*e^2 -
4*a^3*c*d*e^4)*f^4*g^4 - 2*(6*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^5)*x
^2 + (c^4*d^4*e*f^8 + 4*a^4*d*e^4*f^3*g^5 + 4*(c^4*d^5 - a*c^3*d^
3*e^2)*f^7*g - 2*(8*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^6*g^2 + 4*
(6*a^2*c^2*d^3*e^2 - a^3*c*d*e^4)*f^5*g^3 - (16*a^3*c*d^2*e^3 - a
^4*e^5)*f^4*g^4)*x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(1/2)/(g*x+f)**(9/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
```

```
[Out] Timed out
```

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}(gx+f)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^(9
```

```
[Out] integrate(sqrt(e*x + d)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)
*x)*(g*x + f)^(9/2)), x)
```

$$3.720 \quad \int \frac{(d+ex)^{3/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx$$

**Optimal.** Leaf size=301

$$\frac{15\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^2 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4c^{7/2}d^{7/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} + \frac{15g\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}{4c^3d^3\sqrt{d+ex}} + \frac{5g(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2c^2d^2\sqrt{d+ex}} - \frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

[Out]  $(-2*\text{Sqrt}[d + e*x]*(f + g*x)^{(5/2)})/(c*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (15*g*(c*d*f - a*e*g)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c^3*d^3*\text{Sqrt}[d + e*x]) + (5*g*(f + g*x)^{(3/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*c^2*d^2*\text{Sqrt}[d + e*x]) + (15*\text{Sqrt}[g]*(c*d*f - a*e*g)^2*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x])]/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x]))/(4*c^{(7/2)}*d^{(7/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi [A]** time = 1.32171, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$

$$\frac{15\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^2 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4c^{7/2}d^{7/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} + \frac{15g\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}{4c^3d^3\sqrt{d+ex}} + \frac{5g(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2c^2d^2\sqrt{d+ex}} - \frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^{(3/2)}*(f + g*x)^{(5/2)})/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}$

[Out]  $(-2*\text{Sqrt}[d + e*x]*(f + g*x)^{(5/2)})/(c*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (15*g*(c*d*f - a*e*g)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*c^3*d^3*\text{Sqrt}[d + e*x]) + (5*g*(f + g*x)^{(3/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(2*c^2*d^2*\text{Sqrt}[d + e*x]) + (15*\text{Sqrt}[g]*(c*d*f - a*e*g)^2*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x])]/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x]))/(4*c^{(7/2)}*d^{(7/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi in Sympy [A]** time = 120.841, size = 292, normalized size = 0.97

$$-\frac{2\sqrt{d+ex}(f+gx)^{5/2}}{cd\sqrt{ade+cdex^2+x(ae^2+cd^2)}} + \frac{5g(f+gx)^{3/2}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{2c^2d^2\sqrt{d+ex}} - \frac{15g\sqrt{f+gx}(aeg-cdf)\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{4c^3d^3\sqrt{d+ex}} + \frac{15\sqrt{g}(aeg-cdf)^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}\text{atanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)}{4c^{7/2}d^{7/2}\sqrt{d+ex}\sqrt{ae+cdx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**(3/2)*(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)`

[Out] 
$$\begin{aligned} & -2\sqrt{d+ex}(f+gx)^{5/2}/(c^2d\sqrt{a^2de+c^2d^2ex^2+x^2(a^2e^2+c^2d^2)}) + 5g^2(f+gx)^{3/2}\sqrt{a^2de+c^2d^2ex^2+x^2(a^2e^2+c^2d^2)}/(2c^2d^2\sqrt{d+ex}) \\ & - 15g^2\sqrt{d+ex}(f+gx)(a^2eg-c^2df)\sqrt{a^2de+c^2d^2ex^2+x^2(a^2e^2+c^2d^2)}/(4c^3d^3\sqrt{d+ex}) \\ & + 15\sqrt{g}(a^2eg-c^2df)^2\sqrt{a^2de+c^2d^2ex^2+x^2(a^2e^2+c^2d^2)}\operatorname{atanh}(\sqrt{c}\sqrt{d}\sqrt{f+gx}/(\sqrt{g}\sqrt{a^2e+c^2d^2x})) \\ & / (4c^{7/2}d^{7/2}\sqrt{d+ex}\sqrt{a^2e+c^2d^2x}) \end{aligned}$$

**Mathematica [A]** time = 0.44741, size = 199, normalized size = 0.66

$$\frac{\sqrt{d+ex} \left( 2\sqrt{c}\sqrt{d}\sqrt{f+gx} (-15a^2e^2g^2 - 5acdeg(gx-5f) + c^2d^2(-8f^2 + 9fgx + 2g^2x^2)) + 15\sqrt{g}\sqrt{ae+cdx}(cdf - aeg)^2 \log\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{f+gx} + \sqrt{d+ex}}{2\sqrt{c}\sqrt{d}\sqrt{f+gx} - \sqrt{d+ex}}\right) \right)}{8c^{7/2}d^{7/2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] `Integrate[((d + e*x)^(3/2) * (f + g*x)^(5/2)) / (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2), x]`

[Out] 
$$\begin{aligned} & (\operatorname{Sqrt}[d+ex] * (2 * \operatorname{Sqrt}[c] * \operatorname{Sqrt}[d] * \operatorname{Sqrt}[f+gx] * (-15 * a^2 * e^2 * g^2 - 5 * a * c * d * e * g * (-5 * f + gx) + c^2 * d^2 * (-8 * f^2 + 9 * f * gx + 2 * g^2 * x^2)) \\ & + 15 * \operatorname{Sqrt}[g] * (c * d * f - a * e * g)^2 * \operatorname{Sqrt}[a * e + c * d * x] * \operatorname{Log}[a * e * g + 2 * \operatorname{Sqrt}[c] * \operatorname{Sqrt}[d] * \operatorname{Sqrt}[g] * \operatorname{Sqrt}[a * e + c * d * x] * \operatorname{Sqrt}[f + g * x] + c * d * (f + 2 * g * x)])) \\ & ) / (8 * c^{7/2} * d^{7/2} * \operatorname{Sqrt}[(a * e + c * d * x) * (d + e * x)]) \end{aligned}$$

**Maple [B]** time = 0.048, size = 648, normalized size = 2.2

$$\frac{1}{(8cdx + 8ae)c^3d^3} \left( 15 \ln \left( \frac{1}{2} \frac{2xcdg + aeg + cdf + 2\sqrt{(gx+f)(cdx+ae)}\sqrt{dgc}}{\sqrt{dgc}} \right) xa^2cde^2g^3 - 30 \ln \left( \frac{1}{2} \frac{2xcdg + aeg + cdf + 2\sqrt{(gx+f)(cdx+ae)}\sqrt{dgc}}{\sqrt{dgc}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^(3/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2), x)`

[Out] 
$$\begin{aligned} & 1/8 * (15 * \ln(1/2 * (2 * x * c * d * g + a * e * g + c * d * f + 2 * ((g * x + f) * (c * d * x + a * e))^{1/2}) * (d * g * c)^{1/2}) / (d * g * c)^{1/2}) * x * a^2 * c * d * e^2 * g^3 - 30 * \ln(1/2 * (2 * x * c * d * g + a * e * g + c * d * f + 2 * ((g * x + f) * (c * d * x + a * e))^{1/2}) * (d * g * c)^{1/2}) / (d * g * c)^{1/2}) * x * a^2 * c^2 * d^2 * e * f * g^2 + 15 * \ln(1/2 * (2 * x * c * d * g + a * e * g + c * d * f + 2 * ((g * x + f) * (c * d * x + a * e))^{1/2}) * (d * g * c)^{1/2}) / (d * g * c)^{1/2}) * x * c^3 * d^3 * f^2 * g + 15 * \ln(1/2 * (2 * x * c * d * g + a * e * g + c * d * f + 2 * ((g * x + f) * (c * d * x + a * e))^{1/2}) * (d * g * c)^{1/2}) / (d * g * c)^{1/2}) * a^3 * e^3 * g^3 - 30 * \ln(1/2 * (2 * x * c * d * g + a * e * g + c * d * f + 2 * ((g * x + f) * (c * d * x + a * e))^{1/2}) * (d * g * c)^{1/2}) / (d * g * c)^{1/2}) * a^2 * e^2 * g^2 * f * c * d + 15 * \ln(1/2 * (2 * x * c * d * g + a * e * g + c * d * f + 2 * ((g * x + f) * (c * d * x + a * e))^{1/2}) * (d * g * c)^{1/2}) / (d * g * c)^{1/2}) * a * e * g * f^2 * c^2 * d^2 + 4 * x^2 * c^2 * d^2 * g^2 * ((g * x + f) * (c * d * x + a * e))^{1/2} * (d * g * c)^{1/2} - 10 * g^2 * ((g * x + f) * (c * d * x + a * e))^{1/2} * x * a * e * c * d * (d * g * c)^{1/2} + 18 * g * ((g * x + f) * (c * d * x + a * e))^{1/2} * x * f * c^2 * d^2 * (d * g * c)^{1/2} - 30 * ((g * x + f) * (c * d * x + a * e))^{1/2} * a^2 * e^2 * g^2 * (d * g * c)^{1/2} + 50 * ((g * x + f) * (c * d * x + a * e))^{1/2} * a * e * f * g * c * d * (d * g * c)^{1/2} - 16 * ((g * x + f) * (c * d * x + a * e))^{1/2} * f^2 * c^2 * d^2 * (d * g * c)^{1/2}) * (c * d * e * x^2 + a * e^2 * x + c * d^2 * x + a * d * e)^{1/2} * (g * x + f)^{1/2} / (((g * x + f) * (c * d * x + a * e))^{1/2} / (d * g * c)^{1/2}) / (c * d * x + a * e) / c^3 / d^3 / (e * x + d)^{1/2} \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^x + d)^{(3/2)} * (g^x + f)^{(5/2)} / (c*d*e^x + a*d*e + (c*d^2 + a*e^2)*x)$

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.959072, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^x + d)^{(3/2)} * (g^x + f)^{(5/2)} / (c*d*e^x + a*d*e + (c*d^2 + a*e^2)*x)$

[Out] 
$$\left[ \frac{1}{16} (4 (2 c^2 d^2 g^2 x^2 - 8 c^2 d^2 f^2 + 25 a c d e f g - 15 a^2 e^2 g^2 + (9 c^2 d^2 f g - 5 a c d e g^2) x) \sqrt{c d e^x + a d e + (c d^2 + a e^2) x} \sqrt{e^x + d} \sqrt{g^x + f} + 15 (a c^2 d^3 e f^2 - 2 a^2 c d^2 e^2 f g + a^3 d e^3 g^2 + (c^3 d^3 e f^2 - 2 a c^2 d^2 e^2 f g + a^2 c d e^3 g^2) x^2 + ((c^3 d^4 + a c^2 d^2 e^2) f^2 - 2 (a c^2 d^3 e + a^2 c d e^3) f g + (a^2 c d^2 e^2 + a^3 e^4) g^2) x) \sqrt{g/(c d)} \log(- (8 c^2 d^2 e g^2 x^3 + c^2 d^3 f^2 + 6 a c d^2 e f g + a^2 d e^2 g^2 + 8 (c^2 d^2 e f g + (c^2 d^3 + a c d e^2) g^2) x^2 + 4 (2 c^2 d^2 g x + c^2 d^2 f + a c d e g) \sqrt{c d e^x + a d e + (c d^2 + a e^2) x} \sqrt{e^x + d} \sqrt{g^x + f} \sqrt{g/(c d)} + (c^2 d^2 e f^2 + 2 (4 c^2 d^3 + 3 a c d e^2) f g + (8 a c d^2 e + a^2 e^3) g^2) x) / (e^x + d)) / (c^4 d^4 e^x + a c^3 d^4 e + (c^4 d^5 + a c^3 d^3 e^2) x), \frac{1}{8} (2 (2 c^2 d^2 g^2 x^2 - 8 c^2 d^2 f^2 + 25 a c d e f g - 15 a^2 e^2 g^2 + (9 c^2 d^2 f g - 5 a c d e g^2) x) \sqrt{c d e^x + a d e + (c d^2 + a e^2) x} \sqrt{e^x + d} \sqrt{g^x + f} + 15 (a c^2 d^3 e f^2 - 2 a^2 c d^2 e^2 f g + a^3 d e^3 g^2 + (c^3 d^3 e f^2 - 2 a c^2 d^2 e^2 f g + a^2 c d e^3 g^2) x^2 + ((c^3 d^4 + a c^2 d^2 e^2) f^2 - 2 (a c^2 d^3 e + a^2 c d e^3) f g + (a^2 c d^2 e^2 + a^3 e^4) g^2) x) \sqrt{-g/(c d)} \arctan(2 \sqrt{c d e^x + a d e + (c d^2 + a e^2) x} \sqrt{e^x + d} \sqrt{g^x + f} g / ((2 c d e^x + c d^2 f + a d e g + (c d e f + (2 c d^2 + a e^2) g) x) \sqrt{-g/(c d)})) / (c^4 d^4 e^x + a c^3 d^4 e + (c^4 d^5 + a c^3 d^3 e^2) x) \right]$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^x+d)^{(3/2)} * (g^x+f)^{(5/2)} / (a*d*e+(a*e**2+c*d**2)*x+c*d*e^x**2)**2)$

[Out] Timed out

**GIAC/XCAS [A]** time = 0.755865, size = 4, normalized size = 0.01

$sage_0 x$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^x + d)^{(3/2)} * (g^x + f)^{(5/2)} / (c*d*e^x + a*d*e + (c*d^2 + a*e^2)*x)$

[Out]  $sage_0 x$

$$3.721 \quad \int \frac{(d+ex)^{3/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx$$

**Optimal.** Leaf size=227

$$\frac{3\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{5/2}d^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} + \frac{3g\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{c^2d^2\sqrt{d+ex}} - \frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

[Out]  $(-2*\text{Sqrt}[d + e*x]*(f + g*x)^{(3/2)})/(c*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (3*g*\text{Sqrt}[f + g*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c^2*d^2*\text{Sqrt}[d + e*x]) + (3*\text{Sqrt}[g]*(c*d*f - a*e*g)*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x])]/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x]))/(c^{(5/2)}*d^{(5/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi [A]** time = 0.963205, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$

$$\frac{3\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{5/2}d^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} + \frac{3g\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{c^2d^2\sqrt{d+ex}} - \frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^{(3/2)}*(f + g*x)^{(3/2)}/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}$

[Out]  $(-2*\text{Sqrt}[d + e*x]*(f + g*x)^{(3/2)})/(c*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (3*g*\text{Sqrt}[f + g*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(c^2*d^2*\text{Sqrt}[d + e*x]) + (3*\text{Sqrt}[g]*(c*d*f - a*e*g)*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x])]/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x]))/(c^{(5/2)}*d^{(5/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi in Sympy [A]** time = 88.0488, size = 219, normalized size = 0.96

$$-\frac{2\sqrt{d+ex}(f+gx)^{3/2}}{cd\sqrt{ade+cdex^2+x(ae^2+cd^2)}} + \frac{3g\sqrt{f+gx}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{c^2d^2\sqrt{d+ex}} - \frac{3\sqrt{g}(aeg-cdf)\sqrt{ade+cdex^2+x(ae^2+cd^2)}\text{atanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{5/2}d^{5/2}\sqrt{d+ex}\sqrt{ae+cdx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)^{(3/2)}*(g*x+f)^{(3/2)}/(a*d*e+(a*e^{**2}+c*d^{**2})*x+c*d*e*x^{**2})$

[Out]  $-2*\text{sqrt}(d + e*x)*(f + g*x)^{(3/2)}/(c*d*\text{sqrt}(a*d*e + c*d*e*x^{**2} + x*(a*e^{**2} + c*d^{**2}))) + 3*g*\text{sqrt}(f + g*x)*\text{sqrt}(a*d*e + c*d*e*x^{**2} + x*(a*e^{**2} + c*d^{**2}))/((c^{**2}*d^{**2}*\text{sqrt}(d + e*x)) - 3*\text{sqrt}(g)*(a*e*g - c*d*f)*\text{sqrt}(a*d*e + c*d*e*x^{**2} + x*(a*e^{**2} + c*d^{**2}))*\text{atanh}(\text{sqrt}(g)*\text{sqrt}(a*e + c*d*x)/(\text{sqrt}(c)*\text{sqrt}(d)*\text{sqrt}(f + g*x)))/((c^{**2}*$

$$5/2 * d^{5/2} * \sqrt{d + e * x} * \sqrt{a * e + c * d * x}$$

**Mathematica [A]** time = 0.289269, size = 161, normalized size = 0.71

$$\frac{\sqrt{d + ex} \left( 2\sqrt{c}\sqrt{d}\sqrt{f + gx}(3aeg - 2cdf + cdgx) + 3\sqrt{g}\sqrt{ae + cdx}(cdf - aeg) \log \left( 2\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f + gx}\sqrt{ae + cdx} + aeg + cd \right) \right)}{2c^{5/2}d^{5/2}\sqrt{(d + ex)(ae + cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^(3/2) \* (f + g\*x)^(3/2)) / (a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2), x]

[Out] (Sqrt[d + e\*x] \* (2\*Sqrt[c] \* Sqrt[d] \* Sqrt[f + g\*x] \* (-2\*c\*d\*f + 3\*a\*e\*g + c\*d\*g\*x) + 3\*Sqrt[g] \* (c\*d\*f - a\*e\*g) \* Sqrt[a\*e + c\*d\*x] \* Log[a\*e\*g + 2\*Sqrt[c] \* Sqrt[d] \* Sqrt[g] \* Sqrt[a\*e + c\*d\*x] \* Sqrt[f + g\*x] + c\*d\*(f + 2\*g\*x)])) / (2\*c^(5/2) \* d^(5/2) \* Sqrt[(a\*e + c\*d\*x) \* (d + e\*x)])

**Maple [B]** time = 0.039, size = 396, normalized size = 1.7

$$-\frac{1}{(2cdx + 2ae)c^2d^2} \left( 3 \ln \left( \frac{1}{2} \frac{2xcdg + aeg + cdf + 2\sqrt{(gx + f)(cdx + ae)}\sqrt{dgc}}{\sqrt{dgc}} \right) xacdeg^2 - 3 \ln \left( \frac{1}{2} \frac{2xcdg + aeg + cdf + 2\sqrt{(gx + f)(cdx + ae)}\sqrt{dgc}}{\sqrt{dgc}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(3/2) \* (g\*x+f)^(3/2) / (a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2), x)

[Out] -1/2\*(3\*ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*((g\*x+f)\*(c\*d\*x+a\*e)))^(1/2)\*(d\*g\*c)^(1/2))/(d\*g\*c)^(1/2))\*x\*a\*c\*d\*e\*g^2-3\*ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*((g\*x+f)\*(c\*d\*x+a\*e)))^(1/2)\*(d\*g\*c)^(1/2))/(d\*g\*c)^(1/2))\*x^2\*d^2\*f\*g+3\*ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*((g\*x+f)\*(c\*d\*x+a\*e)))^(1/2)\*(d\*g\*c)^(1/2))/(d\*g\*c)^(1/2))\*a^2\*e^2\*g^2-3\*ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*((g\*x+f)\*(c\*d\*x+a\*e)))^(1/2)\*(d\*g\*c)^(1/2))/(d\*g\*c)^(1/2))\*a\*e\*g\*f\*c\*d-2\*g\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*(d\*g\*c)^(1/2))-6\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*a\*e\*g\*(d\*g\*c)^(1/2)+4\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*f\*c\*d\*(d\*g\*c)^(1/2))\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)\*(g\*x+f)^(1/2)/((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)/(d\*g\*c)^(1/2)/(c\*d\*x+a\*e)/c^2/d^2/(e\*x+d)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2) \* (g\*x + f)^(3/2) / (c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.935521, size = 1, normalized size = 0.

$$\left[ \frac{4\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(cdgx - 2cdf + 3aeg)\sqrt{ex + d}\sqrt{gx + f} - 3(acd^2ef - a^2de^2g + (c^2d^2ef - acde^2g)x^2 + ((c^2d^2ef - acde^2g)x + aeg + cd)\sqrt{ex + d})}{2c^{5/2}d^{5/2}\sqrt{(d + ex)(ae + cdx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^x + d)^{(3/2)} (g^x + f)^{(3/2)} / (c^d e^x + a^d e + (c^d + a^e)^x)$

[Out]  $[1/4*(4*\sqrt{c^d e^x + a^d e + (c^d + a^e)^x}*(c^d g^x - 2*c^d f + 3*a^e g)*\sqrt{e^x + d}*\sqrt{g^x + f} - 3*(a^c d^2 e^f - a^2 d^2 e^2 g + (c^2 d^2 e^f - a^c d^2 e^2 g)^x + ((c^2 d^3 + a^c d^e)^f - (a^c d^2 e + a^2 e^3)^g)^x)*\sqrt{g/(c^d)}*\log(-(8*c^2 d^2 e^g^2 x^3 + c^2 d^3 f^2 + 6*a^c d^2 e^f g + a^2 d^2 e^2 g^2 + 8*(c^2 d^2 e^f g + (c^2 d^3 + a^c d^e)^g^2)^x - 4*(2*c^2 d^2 g^x + c^2 d^2 f + a^c d^e g)*\sqrt{c^d e^x + a^d e + (c^d + a^e)^x})*\sqrt{e^x + d}*\sqrt{g^x + f}*\sqrt{g/(c^d)} + (c^2 d^2 e^f^2 + 2*(4*c^2 d^3 + 3*a^c d^e)^f g + (8*a^c d^2 e + a^2 e^3)^g^2)^x)/(e^x + d)))/(c^3 d^3 e^x + a^c d^3 e + (c^3 d^4 + a^c d^2 e^2)^x), 1/2*(2*\sqrt{c^d e^x + a^d e + (c^d + a^e)^x}*(c^d g^x - 2*c^d f + 3*a^e g)*\sqrt{e^x + d}*\sqrt{g^x + f} + 3*(a^c d^2 e^f - a^2 d^2 e^2 g + (c^2 d^2 e^f - a^c d^2 e^2 g)^x + ((c^2 d^3 + a^c d^e)^f - (a^c d^2 e + a^2 e^3)^g)^x)*\sqrt{-g/(c^d)})*\arctan(2*\sqrt{c^d e^x + a^d e + (c^d + a^e)^x}*\sqrt{e^x + d}*\sqrt{g^x + f})*g/((2*c^d e^g^2 x^2 + c^d f^2 + a^d e^g + (c^d e^f + (2*c^d + a^e)^g)^x)*\sqrt{-g/(c^d)})))/(c^3 d^3 e^x + a^c d^3 e + (c^3 d^4 + a^c d^2 e^2)^x)]$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^x + d)^{(3/2)} (g^x + f)^{(3/2)} / (a^d e + (a^e + c^d)^2 x + c^d e^x)^2)$

[Out] Timed out

**GIAC/XCAS [A]** time = 0.694245, size = 4, normalized size = 0.02

$\text{sage}_0 x$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^x + d)^{(3/2)} (g^x + f)^{(3/2)} / (c^d e^x + a^d e + (c^d + a^e)^x)$

[Out]  $\text{sage}_0 x$

$$3.722 \quad \int \frac{(d+ex)^{3/2} \sqrt{f+gx}}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx$$

**Optimal.** Leaf size=161

$$\frac{2\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx} \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

[Out]  $(-2*\text{Sqrt}[d + e*x]*\text{Sqrt}[f + g*x])/(c*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (2*\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x])/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x])])/(c^{(3/2)}*d^{(3/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi [A]** time = 0.617123, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{2\sqrt{g}\sqrt{d+ex}\sqrt{ae+cdx} \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{3/2}d^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^{(3/2)}*\text{Sqrt}[f + g*x]/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}$

[Out]  $(-2*\text{Sqrt}[d + e*x]*\text{Sqrt}[f + g*x])/(c*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) + (2*\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x])/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x])])/(c^{(3/2)}*d^{(3/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi in Sympy [A]** time = 62.4654, size = 153, normalized size = 0.95

$$-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{cd\sqrt{ade+cdex^2+x(ae^2+cd^2)}} + \frac{2\sqrt{g}\sqrt{ade+cdex^2+x(ae^2+cd^2)} \operatorname{atanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)}{c^{3/2}d^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)^{(3/2)}*(g*x+f)^{(1/2)}/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)$

[Out]  $-2*\text{sqrt}(d + e*x)*\text{sqrt}(f + g*x)/(c*d*\text{sqrt}(a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2))) + 2*\text{sqrt}(g)*\text{sqrt}(a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2))*\text{atanh}(\text{sqrt}(c)*\text{sqrt}(d)*\text{sqrt}(f + g*x)/(\text{sqrt}(g)*\text{sqrt}(a*e + c*d*x)))/(c^{(3/2)}*d^{(3/2)}*\text{sqrt}(d + e*x)*\text{sqrt}(a*e + c*d*x))$

**Mathematica [A]** time = 0.189023, size = 131, normalized size = 0.81

$$\frac{\sqrt{d+ex}\left(\sqrt{g}\sqrt{ae+cdx}\log\left(2\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}\sqrt{ae+cdx}+aeg+cd(f+2gx)\right)-2\sqrt{c}\sqrt{d}\sqrt{f+gx}\right)}{c^{3/2}d^{3/2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(d + e*x)^{(3/2)}*\text{Sqrt}[f + g*x]/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)$



[Out]  $(\sqrt{d + e^*x} * (-2 * \sqrt{c} * \sqrt{d} * \sqrt{f + g^*x}) + \sqrt{g} * \sqrt{a * e + c^*d^*x} * \text{Log}[a^*e^*g + 2 * \sqrt{c} * \sqrt{d} * \sqrt{g} * \sqrt{a^*e + c^*d^*x}] * \sqrt{f + g^*x} + c^*d * (f + 2 * g^*x)) / (c^{(3/2)} * d^{(3/2)} * \sqrt{(a^*e + c^*d^*x) * (d + e^*x)})$

**Maple [A]** time = 0.034, size = 210, normalized size = 1.3

$$\frac{1}{(cdx + ae)dc} \sqrt{gx + f} \sqrt{cdex^2 + ae^2x + cd^2x + ade} \left( \ln \left( \frac{1}{2} \left( 2xcdg + aeg + cdf + 2\sqrt{(gx + f)(cdx + ae)}\sqrt{dgc} \right) \frac{1}{\sqrt{dgc}} \right) \right) xcdg$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((e^*x+d)^{(3/2)} * (g^*x+f)^{(1/2)} / (a^*d^*e + (a^*e^2 + c^*d^2)^*x + c^*d^*e^*x^2)^{(3/2)}, x)$

[Out]  $(g^*x+f)^{(1/2)} * (c^*d^*e^*x^2 + a^*e^2^*x + c^*d^2^*x + a^*d^*e)^{(1/2)} * (\ln(1/2 * (2^*x^*c^*d^*g + a^*e^*g + c^*d^*f + 2 * ((g^*x+f) * (c^*d^*x + a^*e))^{(1/2)} * (d^*g^*c)^{(1/2)})) / (d^*g^*c)^{(1/2)}) * x^*c^*d^*g + \ln(1/2 * (2^*x^*c^*d^*g + a^*e^*g + c^*d^*f + 2 * ((g^*x+f) * (c^*d^*x + a^*e))^{(1/2)} * (d^*g^*c)^{(1/2)})) / (d^*g^*c)^{(1/2)}) * a^*e^*g - 2 * ((g^*x+f) * (c^*d^*x + a^*e))^{(1/2)} * (d^*g^*c)^{(1/2)} / (d^*g^*c)^{(1/2)} / (c^*d^*x + a^*e) / ((g^*x + f) * (c^*d^*x + a^*e))^{(1/2)} / d / c / (e^*x + d)^{(1/2)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^*x + d)^{(3/2)} * \text{sqrt}(g^*x + f) / (c^*d^*e^*x^2 + a^*d^*e + (c^*d^2 + a^*e^2)^*x)^{(3/2)}$

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.898196, size = 1, normalized size = 0.01

$$\left[ \frac{(cdex^2 + ade + (cd^2 + ae^2)x) \sqrt{\frac{g}{cd}} \log \left( -\frac{8c^2d^2eg^2x^3 + c^2d^3f^2 + 6acd^2efg + a^2de^2g^2 + 8(c^2d^2efg + (c^2d^3 + acde^2)g^2)x^2 + 4(2c^2d^2gx + c^2d^2f + acd^2e^2g^2)x + 2(c^2d^2ex^2 + ade^2)}{2(c^2d^2ex^2 + ade^2)} \right)}{2(c^2d^2ex^2 + ade^2)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^*x + d)^{(3/2)} * \text{sqrt}(g^*x + f) / (c^*d^*e^*x^2 + a^*d^*e + (c^*d^2 + a^*e^2)^*x)^{(3/2)}$

[Out]  $[1/2 * ((c^*d^*e^*x^2 + a^*d^*e + (c^*d^2 + a^*e^2)^*x) * \text{sqrt}(g/(c^*d)) * \log(- (8 * c^2 * d^2 * e * g^2 * x^3 + c^2 * d^3 * f^2 + 6 * a * c * d^2 * e * f * g + a^2 * d * e^2 * g^2 * x^2 + 4 * (2 * c^2 * d^2 * g * x + c^2 * d^2 * f + a * c * d^2 * e^2 * g^2) * x + 2 * (c^2 * d^2 * e * x^2 + a * d * e))^{(1/2)} * (d * g * c)^{(1/2)})) / (d * g * c)^{(1/2)}) * x * c * d * g + \ln(1/2 * (2 * x * c * d * g + a * e * g + c * d * f + 2 * ((g * x + f) * (c * d * x + a * e))^{(1/2)} * (d * g * c)^{(1/2)})) / (d * g * c)^{(1/2)}) * a * e * g - 2 * ((g * x + f) * (c * d * x + a * e))^{(1/2)} * (d * g * c)^{(1/2)} / (d * g * c)^{(1/2)} / (c * d * x + a * e) / ((g * x + f) * (c * d * x + a * e))^{(1/2)} / d / c / (e * x + d)^{(1/2)}$

$$f)) / (c^2 d^2 e^x x^2 + a c d^2 e + (c^2 d^3 + a c d e^2) x)]$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(3/2)\*(g\*x+f)\*\*(1/2)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*

[Out] Timed out

**GIAC/XCAS [A]** time = 0.632856, size = 4, normalized size = 0.02

$sage_0 x$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*sqrt(g\*x + f)/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(

[Out] sage0\*x

$$3.723 \quad \int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx$$

**Optimal.** Leaf size=61

$$-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}$$

[Out]  $(-2*\text{Sqrt}[d + e*x]*\text{Sqrt}[f + g*x])/((c*d*f - a*e*g)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi [A]** time = 0.252123, antiderivative size = 61, normalized size of antiderivative = 1., number of rules used = 1, number of rules used = 1, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$

$$-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^{(3/2)}/(\text{Sqrt}[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})]$

[Out]  $(-2*\text{Sqrt}[d + e*x]*\text{Sqrt}[f + g*x])/((c*d*f - a*e*g)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi in Sympy [A]** time = 21.7531, size = 56, normalized size = 0.92

$$\frac{2\sqrt{d+ex}\sqrt{f+gx}}{(aeg-cdf)\sqrt{ade+cdex^2+x(ae^2+cd^2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)^{(3/2)}/(g*x+f)^{(1/2)}/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2))$

[Out]  $2*\text{sqrt}(d + e*x)*\text{sqrt}(f + g*x)/((a*e*g - c*d*f)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))$

**Mathematica [A]** time = 0.0779741, size = 50, normalized size = 0.82

$$-\frac{2\sqrt{d+ex}\sqrt{f+gx}}{\sqrt{(d+ex)(ae+cdx)}(cdf-aeg)}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(d + e*x)^{(3/2)}/(\text{Sqrt}[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})]$

[Out]  $(-2*\text{Sqrt}[d + e*x]*\text{Sqrt}[f + g*x])/((c*d*f - a*e*g)*\text{Sqrt}[(a*e + c*d*x)*(d + e*x)])$

**Maple [A]** time = 0.01, size = 63, normalized size = 1.

$$2\frac{\sqrt{gx+f}(cdx+ae)(ex+d)^{3/2}}{(aeg-cdf)(cdex^2+ae^2x+cd^2x+ade)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((e^x+d)^{(3/2)}/(g^x+f)^{(1/2)}/(a^d e+(a^e e^2+c^d e^2)^x+c^d e^x)^{(3/2)}, x)$

[Out]  $2*(g^x+f)^{(1/2)}*(c^d x+a^e)/(a^e g-c^d f)*(e^x+d)^{(3/2)}/(c^d e^x^2+a^e e^2 x+c^d e^2 x+a^d e)^{(3/2)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^x + d)^{(3/2)}/((c^d e^x^2 + a^d e + (c^d e^2 + a^e e^2)^x)^{(3/2)} \sqrt{g^x + f})$

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.283882, size = 169, normalized size = 2.77

$$\frac{2\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{ex + d}\sqrt{gx + f}}{acd^2ef - a^2de^2g + (c^2d^2ef - acde^2g)x^2 + ((c^2d^3 + acde^2)f - (acd^2e + a^2e^3)g)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^x + d)^{(3/2)}/((c^d e^x^2 + a^d e + (c^d e^2 + a^e e^2)^x)^{(3/2)} \sqrt{g^x + f})$

[Out]  $-2*\sqrt{c^d e^x^2 + a^d e + (c^d e^2 + a^e e^2)^x}*\sqrt{e^x + d}*\sqrt{g^x + f}/(a^c d^2 e^f - a^2 d^2 e^2 g + (c^2 d^2 e^f - a^c d^2 e^2 g)^x^2 + ((c^2 d^3 + a^c d^2 e^2)^f - (a^c d^2 e + a^2 e^3)^g)^x$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^x+d)^{(3/2)}/(g^x+f)^{(1/2)}/(a^d e+(a^e e^2+c^d e^2)^x+c^d e^x)^{(3/2)}, x)$

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^{\frac{3}{2}}}{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}} \sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^x + d)^{(3/2)}/((c^d e^x^2 + a^d e + (c^d e^2 + a^e e^2)^x)^{(3/2)} \sqrt{g^x + f})$

[Out]  $\text{integrate}((e^x + d)^{(3/2)}/((c^d e^x^2 + a^d e + (c^d e^2 + a^e e^2)^x)^{(3/2)} \sqrt{g^x + f}), x)$

$$3.724 \quad \int \frac{(d+ex)^{3/2}}{(f+gx)^{3/2}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx$$

**Optimal.** Leaf size=124

$$\frac{4g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2} - \frac{2\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}$$

[Out]  $(-2*\text{Sqrt}[d + e*x])/((c*d*f - a*e*g)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (4*g*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d*f - a*e*g)^2*\text{Sqrt}[d + e*x]*\text{Sqrt}[f + g*x])$

**Rubi [A]** time = 0.522323, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{4g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^2} - \frac{2\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^{(3/2)}/((f + g*x)^{(3/2)}*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}$

[Out]  $(-2*\text{Sqrt}[d + e*x])/((c*d*f - a*e*g)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (4*g*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/((c*d*f - a*e*g)^2*\text{Sqrt}[d + e*x]*\text{Sqrt}[f + g*x])$

**Rubi in Sympy [A]** time = 44.3451, size = 117, normalized size = 0.94

$$-\frac{4g\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}\sqrt{f+gx}(aeg-cdf)^2} + \frac{2\sqrt{d+ex}}{\sqrt{f+gx}(aeg-cdf)\sqrt{ade+cdex^2+x(ae^2+cd^2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)^{(3/2)}/(g*x+f)^{(3/2)}/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)$

[Out]  $-4*g*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(\text{sqrt}(d + e*x)*\text{sqrt}(f + g*x)*(a*e*g - c*d*f)**2) + 2*\text{sqrt}(d + e*x)/(\text{sqrt}(f + g*x)*(a*e*g - c*d*f)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))$

**Mathematica [A]** time = 0.142898, size = 64, normalized size = 0.52

$$-\frac{2\sqrt{d+ex}(aeg+cd(f+2gx))}{\sqrt{f+gx}\sqrt{(d+ex)(ae+cdx)}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(d + e*x)^{(3/2)}/((f + g*x)^{(3/2)}*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)$

[Out]  $(-2*\text{Sqrt}[d + e*x]*(a*e*g + c*d*(f + 2*g*x)))/((c*d*f - a*e*g)^2*\text{Sqrt}[(a*e + c*d*x)*(d + e*x)]*\text{Sqrt}[f + g*x])$

**Maple [A]** time = 0.009, size = 97, normalized size = 0.8

$$-2 \frac{(2xcdg + aeg + cdf)(cdx + ae)(ex + d)^{3/2}}{\sqrt{gx + f}(a^2e^2g^2 - 2acdefg + c^2d^2f^2)(cdex^2 + ae^2x + cd^2x + ade)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(3/2)/(g\*x+f)^(3/2)/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2),x)

[Out] -2\*(c\*d\*x+a\*e)\*(2\*c\*d\*g\*x+a\*e\*g+c\*d\*f)\*(e\*x+d)^(3/2)/(g\*x+f)^(1/2)/(a^2\*e^2\*g^2-2\*a\*c\*d\*e\*f\*g+c^2\*d^2\*f^2)/(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(3/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^{3/2}}{(cdex^2 + ade + (cd^2 + ae^2)x)^{3/2}(gx + f)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f)^(3/2)),x)

[Out] integrate((e\*x + d)^(3/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f)^(3/2)), x)

**Fricas [A]** time = 0.29023, size = 439, normalized size = 3.54

$$\frac{2\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{ac^2d^3ef^3 - 2a^2cd^2e^2f^2g + a^3de^3fg^2 + (c^3d^3ef^2g - 2ac^2d^2e^2fg^2 + a^2cde^3g^3)x^3 + (c^3d^3ef^3 + (c^3d^4 - ac^2d^2e^2)f^2g - (2ac^2d^3ef^2 + a^3d^3e^2f^2g^2)x^2 + (c^3d^4e^2f^2g + a^3de^3fg^2)x + (c^3d^4e^3f^2g + a^3d^3e^2f^2g^2)x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f)^(3/2)),x)

[Out] -2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(2\*c\*d\*g\*x + c\*d\*f + a\*e\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f)/(a\*c^2\*d^3\*e\*f^3 - 2\*a^2\*c\*d^2\*e^2\*f^2\*g + a^3\*d^3\*e^3\*f^2\*g^2 + (c^3\*d^3\*e\*f^2\*g - 2\*a\*c^2\*d^2\*e^2\*f\*g^2 + a^2\*c\*d^3\*e^3\*f^2\*g^2)\*x^3 + (c^3\*d^3\*e\*f^3 + (c^3\*d^4 - a\*c^2\*d^2\*e^2)\*f^2\*g - (2\*a\*c^2\*d^3\*e\*f^2 + a^3\*d^3\*e^2\*f^2\*g^2)x^2 + (a^3\*d^3\*e^3\*f^2\*g + (c^3\*d^4 + a\*c^2\*d^2\*e^2)\*f^2\*g - (a\*c^2\*d^3\*e\*f^2 + 2\*a^2\*c\*d^3\*e^3)\*f^2\*g - (a^2\*c\*d^2\*e^2 + a^3\*e^4)\*g^3)\*x^2 + (a^3\*d^3\*e^3\*f^3 + (c^3\*d^4 + a\*c^2\*d^2\*e^2)\*f^3 - (a\*c^2\*d^3\*e + 2\*a^2\*c\*d^3\*e^3)\*f^2\*g - (a^2\*c\*d^2\*e^2 + a^3\*e^4)\*f^2\*g^2)\*x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(3/2)/(g\*x+f)\*\*(3/2)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2),x)

[Out] Timed out

GIAC/XCAS [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f

[Out] Exception raised: NotImplementedError

$$3.725 \quad \int \frac{(d+ex)^{3/2}}{(f+gx)^{5/2}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx$$

**Optimal.** Leaf size=192

$$\frac{16cdg\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^3} - \frac{8g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^2} - \frac{2\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}$$

[Out]  $(-2*\text{Sqrt}[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^{(3/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (8*g*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)^2*\text{Sqrt}[d + e*x]*(f + g*x)^{(3/2)}) - (16*c*d*g*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)^3*\text{Sqrt}[d + e*x]*\text{Sqrt}[f + g*x])$

**Rubi [A]** time = 0.793229, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{16cdg\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^3} - \frac{8g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^2} - \frac{2\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^{(3/2)} / ((f + g*x)^{(5/2)} * (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}$

[Out]  $(-2*\text{Sqrt}[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^{(3/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (8*g*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)^2*\text{Sqrt}[d + e*x]*(f + g*x)^{(3/2)}) - (16*c*d*g*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*(c*d*f - a*e*g)^3*\text{Sqrt}[d + e*x]*\text{Sqrt}[f + g*x])$

**Rubi in Sympy [A]** time = 69.3041, size = 185, normalized size = 0.96

$$\frac{16cdg\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{3\sqrt{d+ex}\sqrt{f+gx}(aeg-cdf)^3} - \frac{8g\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{3\sqrt{d+ex}(f+gx)^{3/2}(aeg-cdf)^2} + \frac{2\sqrt{d+ex}}{(f+gx)^{3/2}(aeg-cdf)\sqrt{ade+cdex^2+x(ae^2+cd^2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)^{(3/2)} / (g*x+f)^{(5/2)} / (a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)$

[Out]  $16*c*d*g*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)) / (3*\text{sqrt}(d + e*x)*\text{sqrt}(f + g*x)*(a*e*g - c*d*f)**3) - 8*g*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)) / (3*\text{sqrt}(d + e*x)*(f + g*x)**(3/2)*(a*e*g - c*d*f)**2) + 2*\text{sqrt}(d + e*x) / ((f + g*x)**(3/2)*(a*e*g - c*d*f)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))$

**Mathematica [A]** time = 0.231669, size = 105, normalized size = 0.55

$$\frac{2\sqrt{d+ex}(-a^2e^2g^2+2acdeg(3f+2gx)+c^2d^2(3f^2+12fgx+8g^2x^2))}{3(f+gx)^{3/2}\sqrt{(d+ex)(ae+cdx)}(cdf-aeg)^3}$$



Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^(3/2)/((f + g\*x)^(5/2)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2

[Out] (-2\*Sqrt[d + e\*x]\*(-(a^2\*e^2\*g^2) + 2\*a\*c\*d\*e\*g\*(3\*f + 2\*g\*x) + c^2\*d^2\*(3\*f^2 + 12\*f\*g\*x + 8\*g^2\*x^2)))/(3\*(c\*d\*f - a\*e\*g)^3\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)]\*(f + g\*x)^(3/2))

**Maple [A]** time = 0.014, size = 168, normalized size = 0.9

$$\frac{(2cdx + 2ae)(-8c^2d^2g^2x^2 - 4acdeg^2x - 12c^2d^2fgx + a^2e^2g^2 - 6acdefg - 3c^2d^2f^2)}{3a^3e^3g^3 - 9a^2cde^2fg^2 + 9ac^2d^2ef^2g - 3c^3d^3f^3}(ex + d)^{\frac{3}{2}}(gx + f)^{-\frac{3}{2}}(cdex^2 + ae^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(3/2)/(g\*x+f)^(5/2)/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2), x)

[Out] -2/3\*(c\*d\*x+a\*e)\*(-8\*c^2\*d^2\*g^2\*x^2-4\*a\*c\*d\*e\*g^2\*x-12\*c^2\*d^2\*f\*g\*x+a^2\*e^2\*g^2-6\*a\*c\*d\*e\*f\*g-3\*c^2\*d^2\*f^2)\*(e\*x+d)^(3/2)/(g\*x+f)^(3/2)/(a^3\*e^3\*g^3-3\*a^2\*c\*d\*e^2\*f\*g^2+3\*a\*c^2\*d^2\*e\*f^2\*g-c^3\*d^3\*f^3)/(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(3/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^{\frac{3}{2}}}{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}(gx + f)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f

[Out] integrate((e\*x + d)^(3/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f)^(5/2)), x)

**Fricas [A]** time = 0.298727, size = 876, normalized size = 4.56

$$\frac{3(ac^3d^4ef^5 - 3a^2c^2d^3e^2f^4g + 3a^3cd^2e^3f^3g^2 - a^4d^4e^2f^2g^3 + (c^4d^4ef^3g^2 - 3ac^3d^3e^2f^2g^3 + 3a^2c^2d^2e^3fg^4 - a^3cde^4g^5)x^4 -$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f

[Out] -2/3\*(8\*c^2\*d^2\*g^2\*x^2 + 3\*c^2\*d^2\*f^2 + 6\*a\*c\*d\*e\*f\*g - a^2\*e^2\*g^2 + 4\*(3\*c^2\*d^2\*f\*g + a\*c\*d\*e\*g^2)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f)/(a\*c^3\*d^4\*e\*f^5 - 3\*a^2\*c^2\*d^3\*e^2\*f^4\*g + 3\*a^3\*c\*d^2\*e^3\*f^3\*g^2 - a^4\*d^4\*e^2\*f^2\*g^3 + (c^4\*d^4\*e\*f^3\*g^2 - 3\*a\*c^3\*d^3\*e^2\*f^2\*g^3 + 3\*a^2\*c^2\*d^2\*e^3\*f\*g^4 - a^3\*c\*d\*e^4\*g^5)\*x^4 + (2\*c^4\*d^4\*e\*f^4\*g + (c^4\*d^5 - 5\*a\*c^3\*d^3\*e^2)\*f^3\*g^2 - 3\*(a\*c^3\*d^4\*e - a^2\*c^2\*d^2\*e^3)\*f^2\*g^3 + (3\*a^2\*c^2\*d^3\*e^2 + a^3\*c\*d\*e^4)\*f\*g^4 - (a^3\*c\*d^2\*e^3 + a^4\*e^5)\*g^5)\*x^3 + (c^4\*d^4\*e\*f^5 - a^4\*d^4\*e^4\*g^5 + (2\*c^4\*d^5 - a\*c^3\*d^3\*e^2)\*f^4\*g - (5\*a\*c^3\*d^4\*e + 3\*a^2\*c^2\*d^2\*e^3)\*f^3\*g^2 + (3\*a^2\*c^2\*d^3\*e^2 + 5\*a^3\*c\*d\*e^4)\*f^2\*g^3 + (a^3\*c\*d^2\*e^3 - 2\*a^4\*e^5)\*f\*g^4)\*x^2 - (2\*a^4\*d^4\*e^4\*f\*g^4 - (c^4\*d^5

$$+ a^3 c^3 d^3 e^2 f^5 + (a^3 c^3 d^4 e + 3 a^2 c^2 d^2 e^3) f^4 g + 3 (a^2 c^2 d^3 e^2 - a^3 c d e^4) f^3 g^2 - (5 a^3 c d^2 e^3 - a^4 e^5) f^2 g^3 x$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(3/2)/(g\*x+f)\*\*(5/2)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f

[Out] Exception raised: NotImplementedError

$$3.726 \quad \int \frac{(d+ex)^{3/2}}{(f+gx)^{7/2}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx$$

**Optimal.** Leaf size=262

$$\frac{32c^2d^2g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^4} - \frac{16cdg\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^3} - \frac{12g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)^2} - \frac{2\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}$$

[Out]  $(-2*\text{Sqrt}[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^{(5/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (12*g*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*(c*d*f - a*e*g)^2*\text{Sqrt}[d + e*x]*(f + g*x)^{(5/2)}) - (16*c*d*g*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*(c*d*f - a*e*g)^3*\text{Sqrt}[d + e*x]*(f + g*x)^{(3/2)}) - (32*c^2*d^2*g*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*(c*d*f - a*e*g)^4*\text{Sqrt}[d + e*x]*\text{Sqrt}[f + g*x])$

**Rubi [A]** time = 1.09274, antiderivative size = 262, normalized size of antiderivative = 1., number of rules used = 4, number of rules used = 3, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{32c^2d^2g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^4} - \frac{16cdg\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^3} - \frac{12g\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{5\sqrt{d+ex}(f+gx)^{5/2}(cdf-aeg)^2} - \frac{2\sqrt{d+ex}}{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^{(3/2)}/((f + g*x)^{(7/2)}*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}$

[Out]  $(-2*\text{Sqrt}[d + e*x])/((c*d*f - a*e*g)*(f + g*x)^{(5/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) - (12*g*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*(c*d*f - a*e*g)^2*\text{Sqrt}[d + e*x]*(f + g*x)^{(5/2)}) - (16*c*d*g*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*(c*d*f - a*e*g)^3*\text{Sqrt}[d + e*x]*(f + g*x)^{(3/2)}) - (32*c^2*d^2*g*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*(c*d*f - a*e*g)^4*\text{Sqrt}[d + e*x]*\text{Sqrt}[f + g*x])$

**Rubi in Sympy [A]** time = 98.6059, size = 255, normalized size = 0.97

$$\frac{32c^2d^2g\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{5\sqrt{d+ex}\sqrt{f+gx}(aeg-cdf)^4} + \frac{16cdg\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{5\sqrt{d+ex}(f+gx)^{3/2}(aeg-cdf)^3} - \frac{12g\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{5\sqrt{d+ex}(f+gx)^{5/2}(aeg-cdf)^2} + \frac{2\sqrt{d+ex}}{(f+gx)^{5/2}(aeg-cdf)\sqrt{ade+cdex^2+x(ae^2+cd^2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)**(3/2)/(g*x+f)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)$

[Out]  $-32*c**2*d**2*g*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(5*\text{sqrt}(d + e*x)*\text{sqrt}(f + g*x)*(a*e*g - c*d*f)**4) + 16*c*d*g*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(5*\text{sqrt}(d + e*x)*(f + g*x)**(3/2)*(a*e*g - c*d*f)**3) - 12*g*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(5*\text{sqrt}(d + e*x)*(f + g*x)**(5/2)*(a*e*g - c*d*f)**2) + 2*\text{sqrt}(d + e*x)/((f + g*x)**(5/2)*(a*e*g - c*d*f)*\text{sqrt}$

$$(a^*d^*e + c^*d^*e^*x^{**2} + x^*(a^*e^{**2} + c^*d^{**2}))$$

**Mathematica [A]** time = 0.339974, size = 150, normalized size = 0.57

$$\frac{2\sqrt{d+ex}(a^3e^3g^3 - a^2cde^2g^2(5f+2gx) + ac^2d^2eg(15f^2+20fgx+8g^2x^2) + c^3d^3(5f^3+30f^2gx+40fg^2x^2+16g^3x^3))}{5(f+gx)^{5/2}\sqrt{(d+ex)(ae+cdx)(cdf-ae g)^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^(3/2)/((f + g\*x)^(7/2)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)

[Out] (-2\*Sqrt[d + e\*x]\*(a^3\*e^3\*g^3 - a^2\*c\*d\*e^2\*g^2\*(5\*f + 2\*g\*x) + a\*c^2\*d^2\*e\*g\*(15\*f^2 + 20\*f\*g\*x + 8\*g^2\*x^2) + c^3\*d^3\*(5\*f^3 + 30\*f^2\*g\*x + 40\*f\*g^2\*x^2 + 16\*g^3\*x^3)))/(5\*(c\*d\*f - a\*e\*g)^4\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)]\*(f + g\*x)^(5/2))

**Maple [A]** time = 0.017, size = 259, normalized size = 1.

$$\frac{(2cdx + 2ae)(16c^3d^3g^3x^3 + 8ac^2d^2eg^3x^2 + 40c^3d^3fg^2x^2 - 2a^2cde^2g^3x + 20ac^2d^2efg^2x + 30c^3d^3f^2gx + a^3e^3g^3 - 5a^2c^3d^3f^2g^2x + a^3e^3g^3 - 5a^2c^3d^3f^2g^2x)}{5g^4e^4a^4 - 20cdg^3fe^3a^3 + 30c^2d^2g^2f^2e^2a^2 - 20c^3d^3gf^3ea + 5c^4d^4f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(3/2)/(g\*x+f)^(7/2)/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2), x)

[Out] -2/5\*(c\*d\*x+a\*e)\*(16\*c^3\*d^3\*g^3\*x^3+8\*a\*c^2\*d^2\*e\*g^3\*x^2+40\*c^3\*d^3\*f\*g^2\*x^2-2\*a^2\*c\*d\*e^2\*g^3\*x+20\*a\*c^2\*d^2\*e\*f\*g^2\*x+30\*c^3\*d^3\*f^2\*g\*x+a^3\*e^3\*g^3-5\*a^2\*c\*d\*e^2\*f\*g^2+15\*a\*c^2\*d^2\*e\*f^2\*g+5\*c^3\*d^3\*f^3)\*(e\*x+d)^(3/2)/(g\*x+f)^(5/2)/(a^4\*e^4\*g^4-4\*a^3\*c\*d\*e^3\*f\*g^3+6\*a^2\*c^2\*d^2\*e^2\*f^2\*g^2-4\*a\*c^3\*d^3\*e\*f^3\*g+c^4\*d^4\*f^4)/(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(3/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^{\frac{3}{2}}}{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}(gx + f)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f)^(7/2)), x)

[Out] integrate((e\*x + d)^(3/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f)^(7/2)), x)

**Ericas [A]** time = 0.318316, size = 1434, normalized size = 5.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f)^(7/2)), x)

```
[Out] -2/5*(16*c^3*d^3*g^3*x^3 + 5*c^3*d^3*f^3 + 15*a*c^2*d^2*e*f^2*g -
5*a^2*c*d*e^2*f*g^2 + a^3*e^3*g^3 + 8*(5*c^3*d^3*f*g^2 + a*c^2*d
^2*e*g^3)*x^2 + 2*(15*c^3*d^3*f^2*g + 10*a*c^2*d^2*e*f*g^2 - a^2*
c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(
e*x + d)*sqrt(g*x + f)/(a*c^4*d^5*e*f^7 - 4*a^2*c^3*d^4*e^2*f^6*g
+ 6*a^3*c^2*d^3*e^3*f^5*g^2 - 4*a^4*c*d^2*e^4*f^4*g^3 + a^5*d*e^
5*f^3*g^4 + (c^5*d^5*e*f^4*g^3 - 4*a*c^4*d^4*e^2*f^3*g^4 + 6*a^2*
c^3*d^3*e^3*f^2*g^5 - 4*a^3*c^2*d^2*e^4*f*g^6 + a^4*c*d*e^5*g^7)*
x^5 + (3*c^5*d^5*e*f^5*g^2 + (c^5*d^6 - 11*a*c^4*d^4*e^2)*f^4*g^3
- 2*(2*a*c^4*d^5*e - 7*a^2*c^3*d^3*e^3)*f^3*g^4 + 6*(a^2*c^3*d^4
*e^2 - a^3*c^2*d^2*e^4)*f^2*g^5 - (4*a^3*c^2*d^3*e^3 + a^4*c*d*e^
5)*f*g^6 + (a^4*c*d^2*e^4 + a^5*e^6)*g^7)*x^4 + (3*c^5*d^5*e*f^6*
g + a^5*d*e^5*g^7 + 3*(c^5*d^6 - 3*a*c^4*d^4*e^2)*f^5*g^2 - (11*a
*c^4*d^5*e - 6*a^2*c^3*d^3*e^3)*f^4*g^3 + 2*(7*a^2*c^3*d^4*e^2 +
3*a^3*c^2*d^2*e^4)*f^3*g^4 - 3*(2*a^3*c^2*d^3*e^3 + 3*a^4*c*d*e^5
)*f^2*g^5 - (a^4*c*d^2*e^4 - 3*a^5*e^6)*f*g^6)*x^3 + (c^5*d^5*e*f
^7 + 3*a^5*d*e^5*f*g^6 + (3*c^5*d^6 - a*c^4*d^4*e^2)*f^6*g - 3*(3
*a*c^4*d^5*e + 2*a^2*c^3*d^3*e^3)*f^5*g^2 + 2*(3*a^2*c^3*d^4*e^2
+ 7*a^3*c^2*d^2*e^4)*f^4*g^3 + (6*a^3*c^2*d^3*e^3 - 11*a^4*c*d*e^
5)*f^3*g^4 - 3*(3*a^4*c*d^2*e^4 - a^5*e^6)*f^2*g^5)*x^2 + (3*a^5*
d*e^5*f^2*g^5 + (c^5*d^6 + a*c^4*d^4*e^2)*f^7 - (a*c^4*d^5*e + 4*
a^2*c^3*d^3*e^3)*f^6*g - 6*(a^2*c^3*d^4*e^2 - a^3*c^2*d^2*e^4)*f^
5*g^2 + 2*(7*a^3*c^2*d^3*e^3 - 2*a^4*c*d*e^5)*f^4*g^3 - (11*a^4*c
*d^2*e^4 - a^5*e^6)*f^3*g^4)*x)
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**(7/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**
```

```
[Out] Timed out
```

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^(3/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f
```

```
[Out] Exception raised: NotImplementedError
```

$$3.727 \quad \int \frac{(d+ex)^{5/2}(f+gx)^{5/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}} dx$$

**Optimal.** Leaf size=289

$$\frac{5g^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}(cdf - aeg) \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{7/2}d^{7/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} + \frac{5g^2\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{c^3d^3\sqrt{d+ex}} - \frac{10g\sqrt{d+ex}(f+gx)^{3/2}}{3c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

[Out]  $(-2*(d+e*x)^{(3/2)}*(f+g*x)^{(5/2)})/(3*c*d*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(3/2)} - (10*g*\text{Sqrt}[d+e*x]*(f+g*x)^{(3/2)})/(3*c^2*d^2*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2]) + (5*g^2*\text{Sqrt}[f+g*x]*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])/(c^3*d^3*\text{Sqrt}[d+e*x]) + (5*g^{(3/2)}*(c*d*f-a*e*g)*\text{Sqrt}[a*e+c*d*x]*\text{Sqrt}[d+e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e+c*d*x])/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f+g*x])])/(c^{(7/2)}*d^{(7/2)}*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])$

**Rubi [A]** time = 1.25042, antiderivative size = 289, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$

$$\frac{5g^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}(cdf - aeg) \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{7/2}d^{7/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} + \frac{5g^2\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{c^3d^3\sqrt{d+ex}} - \frac{10g\sqrt{d+ex}(f+gx)^{3/2}}{3c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{2(d+ex)^{3/2}(f+gx)^{5/2}}{3cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d+e*x)^{(5/2)}*(f+g*x)^{(5/2)})/(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(5/2)}$

[Out]  $(-2*(d+e*x)^{(3/2)}*(f+g*x)^{(5/2)})/(3*c*d*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(3/2)} - (10*g*\text{Sqrt}[d+e*x]*(f+g*x)^{(3/2)})/(3*c^2*d^2*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2]) + (5*g^2*\text{Sqrt}[f+g*x]*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])/(c^3*d^3*\text{Sqrt}[d+e*x]) + (5*g^{(3/2)}*(c*d*f-a*e*g)*\text{Sqrt}[a*e+c*d*x]*\text{Sqrt}[d+e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e+c*d*x])/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f+g*x])])/(c^{(7/2)}*d^{(7/2)}*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])$

**Rubi in Sympy [A]** time = 118.211, size = 280, normalized size = 0.97

$$-\frac{2(d+ex)^{\frac{3}{2}}(f+gx)^{\frac{5}{2}}}{3cd(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}} - \frac{10g\sqrt{d+ex}(f+gx)^{\frac{3}{2}}}{3c^2d^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}} + \frac{5g^2\sqrt{f+gx}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{c^3d^3\sqrt{d+ex}} - \frac{5g^{\frac{3}{2}}(aeg-cdf)\sqrt{ade+cdex^2+x(ae^2+cd^2)}\text{atanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)}{c^{\frac{7}{2}}d^{\frac{7}{2}}\sqrt{d+ex}\sqrt{ae+cdx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**(5/2)*(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)`

[Out] 
$$-2*(d + e*x)**(3/2)*(f + g*x)**(5/2)/(3*c*d*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2)) - 10*g*\sqrt{d + e*x}*(f + g*x)**(3/2)/(3*c**2*d**2*\sqrt{a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)}) + 5*g**2*\sqrt{f + g*x}*\sqrt{a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)}/(c**3*d**3*\sqrt{d + e*x}) - 5*g**(3/2)*(a*e*g - c*d*f)*\sqrt{a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)}*\operatorname{atanh}(\sqrt{c}*\sqrt{d}*\sqrt{f + g*x}/(\sqrt{g}*\sqrt{a*e + c*d*x})))/(c**(7/2)*d**(7/2)*\sqrt{d + e*x}*\sqrt{a*e + c*d*x})$$

**Mathematica [A]** time = 0.549688, size = 202, normalized size = 0.7

$$\frac{(d + ex)^{5/2} \left( \frac{5g^{3/2}(ae+cdx)^{5/2}(cdf-ae) \log\left( \frac{2\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}\sqrt{ae+cdx+ae+cd(f+2gx)}}{c^{7/2}d^{7/2}} \right) - \frac{2\sqrt{f+gx}(ae+cdx)(14g(ae+cdx)(cdf-ae)+2(cdf-ae)g^2)}{3c^3d^3}}{2((d + ex)(ae + cdx))^{5/2}} \right)}{2((d + ex)(ae + cdx))^{5/2}}$$

Antiderivative was successfully verified.

[In] `Integrate[((d + e*x)^(5/2)*(f + g*x)^(5/2))/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)`

[Out] 
$$\frac{((d + e*x)^{5/2} * ((-2*(a*e + c*d*x)*\sqrt{f + g*x})^{2*(c*d*f - a*e*g)^2 + 14*g*(c*d*f - a*e*g)*(a*e + c*d*x) - 3*g^2*(a*e + c*d*x)^2}) / (3*c^3*d^3) + (5*g^{3/2}*(c*d*f - a*e*g)*(a*e + c*d*x)^{5/2}) * \operatorname{Log}[a*e*g + 2*\sqrt{c}*\sqrt{d}*\sqrt{g}*\sqrt{a*e + c*d*x}*\sqrt{f + g*x} + c*d*(f + 2*g*x)] / (c^{7/2}*d^{7/2})}{2*((a*e + c*d*x)*(d + e*x))^{5/2}}$$

**Maple [B]** time = 0.043, size = 652, normalized size = 2.3

$$-\frac{1}{6(cdx + ae)^2 c^3 d^3} \left( 15 \ln \left( \frac{2xcdg + aeg + cdf + 2\sqrt{(gx + f)(cdx + ae)}\sqrt{dgc}}{\sqrt{dgc}} \right) x^2 ac^2 d^2 eg^3 - 15 \ln \left( \frac{2xcdg + aeg}{\sqrt{dgc}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^(5/2)*(g*x+f)^(5/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2), x)`

[Out] 
$$-1/6*(15*\ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^{1/2}*(d*g*c)^{1/2}))/((d*g*c)^{1/2})*x^2*a*c^2*d^2*e*g^3-15*\ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^{1/2}*(d*g*c)^{1/2}))/((d*g*c)^{1/2})*x^2*c^3*d^3*f*g^2+30*\ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^{1/2}*(d*g*c)^{1/2}))/((d*g*c)^{1/2})*x*a^2*c*d*e^2*g^3-30*\ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^{1/2}*(d*g*c)^{1/2}))/((d*g*c)^{1/2})*x*a*c^2*d^2*e*f*g^2+15*\ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^{1/2}*(d*g*c)^{1/2}))/((d*g*c)^{1/2})*a^3*e^3*g^3-15*\ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^{1/2}*(d*g*c)^{1/2}))/((d*g*c)^{1/2})*a^2*e^2*g^2*f*c*d-6*x^2*c^2*d^2*g^2*((g*x+f)*(c*d*x+a*e))^{1/2}*(d*g*c)^{1/2}-40*g^2*((g*x+f)*(c*d*x+a*e))^{1/2})*x*a*e*c*d*(d*g*c)^{1/2}+28*g*((g*x+f)*(c*d*x+a*e))^{1/2})*x*f*c^2*d^2*(d*g*c)^{1/2}-30*((g*x+f)*(c*d*x+a*e))^{1/2})*a^2*e^2*g^2*(d*g*c)^{1/2}+20*((g*x+f)*(c*d*x+a*e))^{1/2})*a*e*f*g*c*d*(d*g*c)^{1/2}+4*((g*x+f)*(c*d*x+a*e))^{1/2})*f^2*c^2*d^2*(d*g*c)^{1/2}*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^{1/2}*(g*x+f)^{1/2}/(((g*x+f)*(c*d*x+a*e))^{1/2}/(d*g*c)^{1/2})/(c*d*x+a*e)^2/c^3/d^3/(e*x+d)^{1/2}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^{\frac{5}{2}}(gx + f)^{\frac{5}{2}}}{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(5/2)\*(g\*x + f)^(5/2)/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)

[Out] integrate((e\*x + d)^(5/2)\*(g\*x + f)^(5/2)/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2), x)

**Fricas [A]** time = 0.964019, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(5/2)\*(g\*x + f)^(5/2)/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)

[Out] [1/12\*(4\*(3\*c^2\*d^2\*g^2\*x^2 - 2\*c^2\*d^2\*f^2 - 10\*a\*c\*d\*e\*f\*g + 15\*a^2\*e^2\*g^2 - 2\*(7\*c^2\*d^2\*f\*g - 10\*a\*c\*d\*e\*g^2)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f) - 15\*(a^2\*c\*d^2\*e^2\*f\*g - a^3\*d\*e^3\*g^2 + (c^3\*d^3\*e\*f\*g - a\*c^2\*d^2\*e^2\*g^2)\*x^3 + ((c^3\*d^4 + 2\*a\*c^2\*d^2\*e^2)\*f\*g - (a\*c^2\*d^3\*e + 2\*a^2\*c\*d\*e^3)\*g^2)\*x^2 + ((2\*a\*c^2\*d^3\*e + a^2\*c\*d\*e^3)\*f\*g - (2\*a^2\*c\*d^2\*e^2 + a^3\*e^4)\*g^2)\*x)\*sqrt(g/(c\*d))\*log(-(8\*c^2\*d^2\*e\*g^2\*x^3 + c^2\*d^3\*f^2 + 6\*a\*c\*d^2\*e\*f\*g + a^2\*d\*e^2\*g^2 + 8\*(c^2\*d^2\*e\*f\*g + (c^2\*d^3 + a\*c\*d\*e^2)\*g^2)\*x^2 - 4\*(2\*c^2\*d^2\*g\*x + c^2\*d^2\*f + a\*c\*d\*e\*g)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f)\*sqrt(g/(c\*d)) + (c^2\*d^2\*e\*f^2 + 2\*(4\*c^2\*d^3 + 3\*a\*c\*d\*e^2)\*f\*g + (8\*a\*c\*d^2\*e + a^2\*e^3)\*g^2)\*x)/(e\*x + d)))/(c^5\*d^5\*e\*x^3 + a^2\*c^3\*d^4\*e^2 + (c^5\*d^6 + 2\*a\*c^4\*d^4\*e^2)\*x^2 + (2\*a\*c^4\*d^5\*e + a^2\*c^3\*d^3\*e^3)\*x), 1/6\*(2\*(3\*c^2\*d^2\*g^2\*x^2 - 2\*c^2\*d^2\*f^2 - 10\*a\*c\*d\*e\*f\*g + 15\*a^2\*e^2\*g^2 - 2\*(7\*c^2\*d^2\*f\*g - 10\*a\*c\*d\*e\*g^2)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f) + 15\*(a^2\*c\*d^2\*e^2\*f\*g - a^3\*d\*e^3\*g^2 + (c^3\*d^3\*e\*f\*g - a\*c^2\*d^2\*e^2\*g^2)\*x^3 + (c^3\*d^4 + 2\*a\*c^2\*d^2\*e^2)\*f\*g - (a\*c^2\*d^3\*e + 2\*a^2\*c\*d\*e^3)\*g^2)\*x^2 + ((2\*a\*c^2\*d^3\*e + a^2\*c\*d\*e^3)\*f\*g - (2\*a^2\*c\*d^2\*e^2 + a^3\*e^4)\*g^2)\*x)\*sqrt(-g/(c\*d))\*arctan(2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f)\*g/((2\*c\*d\*e\*g\*x^2 + c\*d^2\*f + a\*d\*e\*g + (c\*d\*e\*f + (2\*c\*d^2 + a\*e^2)\*g)\*x)\*sqrt(-g/(c\*d)))))/(c^5\*d^5\*e\*x^3 + a^2\*c^3\*d^4\*e^2 + (c^5\*d^6 + 2\*a\*c^4\*d^4\*e^2)\*x^2 + (2\*a\*c^4\*d^5\*e + a^2\*c^3\*d^3\*e^3)\*x)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(5/2)\*(g\*x+f)\*\*(5/2)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*

[Out] Timed out

**GIAC/XCAS [A]** time = 1.02622, size = 4, normalized size = 0.01

sage<sub>0</sub>x



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^(5/2)*(g*x + f)^(5/2)/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)
```

```
[Out] sage0*x
```

$$3.728 \quad \int \frac{(d+ex)^{5/2}(f+gx)^{3/2}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}} dx$$

**Optimal.** Leaf size=219

$$\frac{2g^{3/2}\sqrt{d+ex}\sqrt{ae+cdx} \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{5/2}d^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{2g\sqrt{d+ex}\sqrt{f+gx}}{c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

[Out]  $(-2*(d+e*x)^{(3/2)}*(f+g*x)^{(3/2)})/(3*c*d*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(3/2)} - (2*g*\text{Sqrt}[d+e*x]*\text{Sqrt}[f+g*x])/(c^2*d^2*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2]) + (2*g^{(3/2)}*\text{Sqrt}[a*e+c*d*x]*\text{Sqrt}[d+e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e+c*d*x])]/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f+g*x]))/(c^{(5/2)}*d^{(5/2)}*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])$

**Rubi [A]** time = 0.886983, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{2g^{3/2}\sqrt{d+ex}\sqrt{ae+cdx} \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{5/2}d^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{2g\sqrt{d+ex}\sqrt{f+gx}}{c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d+e*x)^{(5/2)}*(f+g*x)^{(3/2)}/(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(5/2)}$

[Out]  $(-2*(d+e*x)^{(3/2)}*(f+g*x)^{(3/2)})/(3*c*d*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(3/2)} - (2*g*\text{Sqrt}[d+e*x]*\text{Sqrt}[f+g*x])/(c^2*d^2*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2]) + (2*g^{(3/2)}*\text{Sqrt}[a*e+c*d*x]*\text{Sqrt}[d+e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e+c*d*x])]/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f+g*x]))/(c^{(5/2)}*d^{(5/2)}*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])$

**Rubi in Sympy [A]** time = 89.2581, size = 211, normalized size = 0.96

$$-\frac{2(d+ex)^{\frac{3}{2}}(f+gx)^{\frac{3}{2}}}{3cd(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}} - \frac{2g\sqrt{d+ex}\sqrt{f+gx}}{c^2d^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}} + \frac{2g^{\frac{3}{2}}\sqrt{ade+cdex^2+x(ae^2+cd^2)} \operatorname{atanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{c^{\frac{5}{2}}d^{\frac{5}{2}}\sqrt{d+ex}\sqrt{ae+cdx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)**(5/2)*(g*x+f)**(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)$

[Out]  $-2*(d+e*x)**(3/2)*(f+g*x)**(3/2)/(3*c*d*(a*d*e+c*d*e*x**2+x*(a*e**2+c*d**2))** (3/2)) - 2*g*\text{sqrt}(d+e*x)*\text{sqrt}(f+g*x)/(c**2*d**2*\text{sqrt}(a*d*e+c*d*e*x**2+x*(a*e**2+c*d**2))) + 2*g** (3/2)*\text{sqrt}(a*d*e+c*d*e*x**2+x*(a*e**2+c*d**2))*\text{atanh}(\text{sqrt}(g)*\text{sqrt}(a*e+c*d*x)/(\text{sqrt}(c)*\text{sqrt}(d)*\text{sqrt}(f+g*x)))/(c**(5/2)*d**$

$$*(5/2)*\text{sqrt}(d + e*x)*\text{sqrt}(a*e + c*d*x))$$

**Mathematica [A]** time = 0.36269, size = 150, normalized size = 0.68

$$\frac{(d + ex)^{3/2} \left( 3g^{3/2}(ae + cdx)^{3/2} \log \left( 2\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f + gx}\sqrt{ae + cdx} + aeg + cd(f + 2gx) \right) - 2\sqrt{c}\sqrt{d}\sqrt{f + gx}(3aeg + cd(f + 2gx)) \right)}{3c^{5/2}d^{5/2}((d + ex)(ae + cdx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^(5/2)\*(f + g\*x)^(3/2))/(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2), x]

[Out] ((d + e\*x)^(3/2)\*(-2\*Sqrt[c]\*Sqrt[d]\*Sqrt[f + g\*x]\*(3\*a\*e\*g + c\*d\*(f + 4\*g\*x)) + 3\*g^(3/2)\*(a\*e + c\*d\*x)^(3/2)\*Log[a\*e\*g + 2\*Sqrt[c]\*Sqrt[d]\*Sqrt[g]\*Sqrt[a\*e + c\*d\*x]\*Sqrt[f + g\*x] + c\*d\*(f + 2\*g\*x)))/(3\*c^(5/2)\*d^(5/2)\*((a\*e + c\*d\*x)\*(d + e\*x))^(3/2))

**Maple [A]** time = 0.042, size = 343, normalized size = 1.6

$$\frac{1}{3(cdx + ae)^2 d^2 c^2} \sqrt{gx + f} \sqrt{cdex^2 + ae^2x + cd^2x + ade} \left( 3 \ln \left( \frac{1}{2} \frac{2xcdg + aeg + cdf + 2\sqrt{(gx + f)(cdx + ae)}\sqrt{dgc}}{\sqrt{dgc}} \right) \right) x^2 c^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(5/2)\*(g\*x+f)^(3/2)/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2), x)

[Out] 1/3\*(g\*x+f)^(1/2)\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)\*(3\*ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*(d\*g\*c)^(1/2)))/(d\*g\*c)^(1/2))\*x^2\*c^2\*d^2\*g^2+6\*ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*(d\*g\*c)^(1/2)))/(d\*g\*c)^(1/2))\*x\*a\*c\*d\*e\*g^2+3\*ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*(d\*g\*c)^(1/2)))/(d\*g\*c)^(1/2))\*a^2\*e^2\*g^2-8\*g\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*x\*c\*d\*(d\*g\*c)^(1/2)-6\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*a\*e\*g\*(d\*g\*c)^(1/2)-2\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*f\*c\*d\*(d\*g\*c)^(1/2))/(d\*g\*c)^(1/2)/(c\*d\*x+a\*e)^2/((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)/d^2/c^2/(e\*x+d)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^{\frac{5}{2}}(gx + f)^{\frac{3}{2}}}{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(5/2)\*(g\*x + f)^(3/2)/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

[Out] integrate((e\*x + d)^(5/2)\*(g\*x + f)^(3/2)/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2), x)

**Fricas [A]** time = 0.924163, size = 1, normalized size = 0.

$$\frac{4\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(4cdgx + cdf + 3aeg)\sqrt{ex + d}\sqrt{gx + f} - 3(c^2d^2egx^3 + a^2de^2g + (c^2d^3 + 2acde^2)gx^2 + (2c^3d^4 + a^2c^2d^3e^2 + (c^4d^5 + 2ac^3d^3e^2)x^2 + (2ac^3d^4e^2 + a^2c^2d^3e^2)x + a^2c^3d^4e^2))\sqrt{ex + d}\sqrt{gx + f}}{3(c^4d^4ex^3 + a^2c^2d^3e^2 + (c^4d^5 + 2ac^3d^3e^2)x^2 + (2ac^3d^4e^2 + a^2c^2d^3e^2)x + a^2c^3d^4e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(5/2)\*(g\*x + f)^(3/2)/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)

[Out] [-1/6\*(4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(4\*c\*d\*g\*x + c\*d\*f + 3\*a\*e\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f) - 3\*(c^2\*d^2\*e\*g\*x^3 + a^2\*d\*e^2\*g + (c^2\*d^3 + 2\*a\*c\*d^2\*e + a^2\*e^3)\*g\*x)\*sqrt(g/(c\*d))\*log(-(8\*c^2\*d^2\*e\*g^2\*x^3 + c^2\*d^3\*f^2 + 6\*a\*c\*d^2\*e\*f\*g + a^2\*d\*e^2\*g^2 + 8\*(c^2\*d^2\*e\*f\*g + (c^2\*d^3 + a\*c\*d^2\*e^2)\*g^2)\*x^2 + 4\*(2\*c^2\*d^2\*g\*x + c^2\*d^2\*f + a\*c\*d\*e\*g)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f)\*sqrt(g/(c\*d)) + (c^2\*d^2\*e\*f^2 + 2\*(4\*c^2\*d^3 + 3\*a\*c\*d^2\*e^2)\*f\*g + (8\*a\*c\*d^2\*e + a^2\*e^3)\*g^2)\*x)/(e\*x + d)))/(c^4\*d^4\*e\*x^3 + a^2\*c^2\*d^3\*e^2 + (c^4\*d^5 + 2\*a\*c^3\*d^3\*e^2)\*x^2 + (2\*a\*c^3\*d^4\*e + a^2\*c^2\*d^2\*e^3)\*x), -1/3\*(2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(4\*c\*d\*g\*x + c\*d\*f + 3\*a\*e\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f) - 3\*(c^2\*d^2\*e\*g\*x^3 + a^2\*d\*e^2\*g + (c^2\*d^3 + 2\*a\*c\*d^2\*e^2)\*g\*x^2 + (2\*a\*c\*d^2\*e + a^2\*e^3)\*g\*x)\*sqrt(-g/(c\*d))\*arctan(2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f)\*g/((2\*c\*d\*e\*g\*x^2 + c\*d^2\*f + a\*d\*e\*g + (c\*d\*e\*f + (2\*c\*d^2 + a\*e^2)\*g)\*x)\*sqrt(-g/(c\*d)))))/(c^4\*d^4\*e\*x^3 + a^2\*c^2\*d^3\*e^2 + (c^4\*d^5 + 2\*a\*c^3\*d^3\*e^2)\*x^2 + (2\*a\*c^3\*d^4\*e + a^2\*c^2\*d^2\*e^3)\*x)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(5/2)\*(g\*x+f)\*\*(3/2)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*

[Out] Timed out

**GIAC/XCAS [A]** time = 0.85577, size = 4, normalized size = 0.02

sage<sub>0</sub>x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(5/2)\*(g\*x + f)^(3/2)/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)

[Out] sage<sub>0</sub>\*x

$$3.729 \quad \int \frac{(d+ex)^{5/2} \sqrt{f+gx}}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}} dx$$

**Optimal.** Leaf size=63

$$\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)}$$

[Out]  $(-2*(d+e*x)^{(3/2)}*(f+g*x)^{(3/2)})/(3*(c*d*f-a*e*g)*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(3/2)})$

**Rubi [A]** time = 0.2517, antiderivative size = 63, normalized size of antiderivative = 1., number of rules used = 1, number of rules used = 1, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$

$$\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d+e*x)^{(5/2)}*\text{Sqrt}[f+g*x]/(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(5/2)}$

[Out]  $(-2*(d+e*x)^{(3/2)}*(f+g*x)^{(3/2)})/(3*(c*d*f-a*e*g)*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(3/2)})$

**Rubi in Sympy [A]** time = 21.7428, size = 58, normalized size = 0.92

$$\frac{2(d+ex)^{\frac{3}{2}}(f+gx)^{\frac{3}{2}}}{3(aeg-cdf)(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)^{(5/2)}*(g*x+f)^{(1/2)}/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)$

[Out]  $2*(d+e*x)^{(3/2)}*(f+g*x)^{(3/2)}/(3*(a*e*g-c*d*f)*(a*d*e+c*d*e*x^2+x*(a*e^2+c*d^2)))^{(3/2)}$

**Mathematica [A]** time = 0.0955412, size = 52, normalized size = 0.83

$$\frac{2(d+ex)^{3/2}(f+gx)^{3/2}}{3((d+ex)(ae+cdx))^{3/2}(aeg-cdf)}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(d+e*x)^{(5/2)}*\text{Sqrt}[f+g*x]/(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)$

[Out]  $(2*(d+e*x)^{(3/2)}*(f+g*x)^{(3/2)})/(3*(-(c*d*f)+a*e*g)*((a*e+c*d*x)*(d+e*x))^{(3/2)})$

**Maple [A]** time = 0.01, size = 63, normalized size = 1.

$$\frac{2cdx+2ae}{3aeg-3cdf}(gx+f)^{\frac{3}{2}}(ex+d)^{\frac{5}{2}}(cdex^2+ae^2x+cd^2x+ade)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((e^*x+d)^{(5/2)}*(g^*x+f)^{(1/2)}/(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(5/2)},x)$

[Out]  $2/3*(g^*x+f)^{(3/2)}*(c^*d^*x+a^*e)/(a^*e^*g-c^*d^*f)^*(e^*x+d)^{(5/2)}/(c^*d^*e^*x^2+a^*e^2*x+c^*d^2*x+a^*d^*e)^{(5/2)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex+d)^{\frac{5}{2}}\sqrt{gx+f}}{(cdex^2+ade+(cd^2+ae^2)x)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^*x+d)^{(5/2)}*\text{sqrt}(g^*x+f)/(c^*d^*e^*x^2+a^*d^*e+(c^*d^2+a^*e^2)^*x)^{(5/2)},x)$

[Out]  $\text{integrate}((e^*x+d)^{(5/2)}*\text{sqrt}(g^*x+f)/(c^*d^*e^*x^2+a^*d^*e+(c^*d^2+a^*e^2)^*x)^{(5/2)},x)$

**Fricas [A]** time = 0.289095, size = 261, normalized size = 4.14

$$\frac{2\sqrt{cdex^2+ade+(cd^2+ae^2)x}\sqrt{ex+d}(gx+f)^{\frac{3}{2}}}{3(a^2cd^2e^2f-a^3de^3g+(c^3d^3ef-ac^2d^2e^2g)x^3+((c^3d^4+2ac^2d^2e^2)f-(ac^2d^3e+2a^2cde^3)g)x^2+((2ac^2d^3e+a^2cde^3)f-2a^2c^2d^2e^2g)x+a^3d^4e^2f)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^*x+d)^{(5/2)}*\text{sqrt}(g^*x+f)/(c^*d^*e^*x^2+a^*d^*e+(c^*d^2+a^*e^2)^*x)^{(5/2)},x)$

[Out]  $-2/3*\text{sqrt}(c^*d^*e^*x^2+a^*d^*e+(c^*d^2+a^*e^2)^*x)*\text{sqrt}(e^*x+d)*(g^*x+f)^{(3/2)}/(a^2*c^*d^2*e^2*f-a^3*d^3*e^3*g+(c^3*d^3*e^3*f-a^2*c^*d^2*e^2*g)^*x^3+((c^3*d^4+2*a^*c^2*d^2*e^2)*f-(a^2*c^2*d^3*e+2*a^2*c^*d^2*e^3)*g)^*x^2+((2*a^*c^2*d^3*e+a^2*c^*d^2*e^3)*f-(2*a^2*c^*d^2*e^2+a^3*e^4)*g)^*x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^*x+d)^{(5/2)}*(g^*x+f)^{(1/2)}/(a^*d^*e+(a^*e^2+c^*d^2)^*x+c^*d^*e^*x^2)^{(5/2)},x)$

[Out] Timed out

**GIAC/XCAS [A]** time = 0.736344, size = 4, normalized size = 0.06

$sage_0x$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^*x+d)^{(5/2)}*\text{sqrt}(g^*x+f)/(c^*d^*e^*x^2+a^*d^*e+(c^*d^2+a^*e^2)^*x)^{(5/2)},x)$

[Out]  $sage_0*x$

$$3.730 \quad \int \frac{(d+ex)^{5/2}}{\sqrt{f+gx}(ade+(cd^2+ae^2)x+cdex^2)^{5/2}} dx$$

**Optimal.** Leaf size=128

$$\frac{4g\sqrt{d+ex}\sqrt{f+gx}}{3\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2} - \frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)}$$

[Out]  $(-2*(d+e*x)^{(3/2)}*\text{Sqrt}[f+g*x])/(3*(c*d*f-a*e*g)*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(3/2)})+(4*g*\text{Sqrt}[d+e*x]*\text{Sqrt}[f+g*x])/(3*(c*d*f-a*e*g)^2*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])$

**Rubi [A]** time = 0.532058, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{4g\sqrt{d+ex}\sqrt{f+gx}}{3\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2} - \frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d+e*x)^{(5/2)}/(\text{Sqrt}[f+g*x]*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(5/2)})]$

[Out]  $(-2*(d+e*x)^{(3/2)}*\text{Sqrt}[f+g*x])/(3*(c*d*f-a*e*g)*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(3/2)})+(4*g*\text{Sqrt}[d+e*x]*\text{Sqrt}[f+g*x])/(3*(c*d*f-a*e*g)^2*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])$

**Rubi in Sympy [A]** time = 44.197, size = 121, normalized size = 0.95

$$\frac{4g\sqrt{d+ex}\sqrt{f+gx}}{3(aeg-cdf)^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}} + \frac{2(d+ex)^{3/2}\sqrt{f+gx}}{3(aeg-cdf)(ade+cdex^2+x(ae^2+cd^2))^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)^{(5/2)}/(g*x+f)^{(1/2)}/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)$

[Out]  $4*g*\text{sqrt}(d+e*x)*\text{sqrt}(f+g*x)/(3*(a*e*g-c*d*f)**2*\text{sqrt}(a*d*e+c*d*e*x**2+x*(a*e**2+c*d**2)))+(2*(d+e*x)**(3/2)*\text{sqrt}(f+g*x))/(3*(a*e*g-c*d*f)*(a*d*e+c*d*e*x**2+x*(a*e**2+c*d**2))** (3/2))$

**Mathematica [A]** time = 0.144077, size = 68, normalized size = 0.53

$$\frac{2(d+ex)^{3/2}\sqrt{f+gx}(3aeg-cd(f-2gx))}{3((d+ex)(ae+cdx))^{3/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(d+e*x)^{(5/2)}/(\text{Sqrt}[f+g*x]*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(5/2)})]$

[Out]  $(2*(d+e*x)^{(3/2)}*\text{Sqrt}[f+g*x]*(3*a*e*g-c*d*(f-2*g*x)))/(3*(c*d*f-a*e*g)^2*((a*e+c*d*x)*(d+e*x))^{(3/2)})$

---

**Maple [A]** time = 0.011, size = 99, normalized size = 0.8

$$\frac{(2cdx + 2ae)(2xcdg + 3aeg - cdf)}{3a^2e^2g^2 - 6acdefg + 3c^2d^2f^2} \sqrt{gx + f} (ex + d)^{\frac{5}{2}} (cdex^2 + ae^2x + cd^2x + ade)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^(5/2)/(g*x+f)^(1/2)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2), x)`

[Out] `2/3*(g*x+f)^(1/2)*(c*d*x+a*e)*(2*c*d*g*x+3*a*e*g-c*d*f)*(e*x+d)^(5/2)/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(5/2)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^{\frac{5}{2}}}{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}} \sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*sqrt(g*x + f)), x)`

[Out] `integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*sqrt(g*x + f)), x)`

---

**Fricas [A]** time = 0.292028, size = 429, normalized size = 3.35

$$\frac{2\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(2cdg + 3aeg - cdf)}{3(a^2c^2d^3e^2f^2 - 2a^3cd^2e^3fg + a^4de^4g^2 + (c^4d^4ef^2 - 2ac^3d^3e^2fg + a^2c^2d^2e^3g^2)x^3 + ((c^4d^5 + 2ac^3d^3e^2)f^2 - 2(ac^3d^4e + 2a^2c^2d^3e^2)f)g + (a^4d^5 + 2ac^3d^4e + 2a^2c^2d^3e^2)g^2)x^2 + ((c^4d^5 + 2ac^3d^3e^2)f^2 - 2(ac^3d^4e + 2a^2c^2d^3e^2)f)g + (a^4d^5 + 2ac^3d^4e + 2a^2c^2d^3e^2)g^2)x}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*sqrt(g*x + f)), x)`

[Out] `2/3*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x - c*d*f + 3*a*e*g)*sqrt(e*x + d)*sqrt(g*x + f)/(a^2*c^2*d^3*e^2*f^2 - 2*a^3*c*d^2*e^3*f*g + a^4*d*e^4*g^2 + (c^4*d^4*e*f^2 - 2*a*c^3*d^3*e^2*f*g + a^2*c^2*d^2*e^3*g^2)*x^3 + ((c^4*d^5 + 2*a*c^3*d^3*e^2)*f^2 - 2*(a*c^3*d^4*e + 2*a^2*c^2*d^3*e^2)*f*g + (a^2*c^2*d^3*e^2 + 2*a^3*c*d^2*e^3)*g^2)*x^2 + ((2*a*c^3*d^4*e + a^2*c^2*d^3*e^2)*f^2 - 2*(2*a^2*c^2*d^3*e^2 + a^3*c*d^2*e^3)*f*g + (2*a^3*c*d^2*e^3 + a^4*e^4)*g^2)*x)`

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**(5/2)/(g*x+f)**(1/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2), x)`

[Out] Timed out



---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(5/2)/((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*sqrt(g*x`

[Out] Exception raised: NotImplementedError

$$3.731 \quad \int \frac{(d+ex)^{5/2}}{(f+gx)^{3/2}(ade+(cd^2+ae^2)x+cdex^2)^{5/2}} dx$$

**Optimal.** Leaf size=194

$$\frac{16g^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^3} + \frac{8g\sqrt{d+ex}}{3\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2} - \frac{2(d+ex)^{3/2}}{3\sqrt{f+gx}(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)}$$

[Out]  $(-2*(d+e*x)^{(3/2)})/(3*(c*d*f-a*e*g)*\text{Sqrt}[f+g*x]*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(3/2)})+(8*g*\text{Sqrt}[d+e*x])/(3*(c*d*f-a*e*g)^2*\text{Sqrt}[f+g*x]*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])+(16*g^2*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])/(3*(c*d*f-a*e*g)^3*\text{Sqrt}[d+e*x]*\text{Sqrt}[f+g*x])$

**Rubi [A]** time = 0.793548, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{16g^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^3} + \frac{8g\sqrt{d+ex}}{3\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2} - \frac{2(d+ex)^{3/2}}{3\sqrt{f+gx}(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d+e*x)^{(5/2)/((f+g*x)^{(3/2)*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(5/2)}}$

[Out]  $(-2*(d+e*x)^{(3/2)})/(3*(c*d*f-a*e*g)*\text{Sqrt}[f+g*x]*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(3/2)})+(8*g*\text{Sqrt}[d+e*x])/(3*(c*d*f-a*e*g)^2*\text{Sqrt}[f+g*x]*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])+(16*g^2*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])/(3*(c*d*f-a*e*g)^3*\text{Sqrt}[d+e*x]*\text{Sqrt}[f+g*x])$

**Rubi in Sympy [A]** time = 70.2718, size = 185, normalized size = 0.95

$$-\frac{16g^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{3\sqrt{d+ex}\sqrt{f+gx}(aeg-cdf)^3} + \frac{8g\sqrt{d+ex}}{3\sqrt{f+gx}(aeg-cdf)^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}} + \frac{2(d+ex)^{\frac{3}{2}}}{3\sqrt{f+gx}(aeg-cdf)(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)^{(5/2)/(g*x+f)^{(3/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2}}$

[Out]  $-16*g**2*\text{sqrt}(a*d*e+c*d*e*x**2+x*(a*e**2+c*d**2))/(3*\text{sqrt}(d+e*x)*\text{sqrt}(f+g*x)*(a*e*g-c*d*f)**3)+8*g*\text{sqrt}(d+e*x)/(3*\text{sqrt}(f+g*x)*(a*e*g-c*d*f)**2*\text{sqrt}(a*d*e+c*d*e*x**2+x*(a*e**2+c*d**2))) + 2*(d+e*x)**(3/2)/(3*\text{sqrt}(f+g*x)*(a*e*g-c*d*f)*(a*d*e+c*d*e*x**2+x*(a*e**2+c*d**2))**(3/2))$

**Mathematica [A]** time = 0.232661, size = 103, normalized size = 0.53

$$\frac{2(d+ex)^{3/2}(3a^2e^2g^2+6acdeg(f+2gx)+c^2d^2(-f^2+4fgx+8g^2x^2))}{3\sqrt{f+gx}((d+ex)(ae+cdx))^{3/2}(cdf-aeg)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^(5/2)/((f + g\*x)^(3/2)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2

[Out] (2\*(d + e\*x)^(3/2)\*(3\*a^2\*e^2\*g^2 + 6\*a\*c\*d\*e\*g\*(f + 2\*g\*x) + c^2\*d^2\*(-f^2 + 4\*f\*g\*x + 8\*g^2\*x^2)))/(3\*(c\*d\*f - a\*e\*g)^3\*((a\*e + c\*d\*x)\*(d + e\*x))^(3/2)\*Sqrt[f + g\*x])

**Maple [A]** time = 0.013, size = 169, normalized size = 0.9

$$\frac{(2cdx + 2ae)(8c^2d^2g^2x^2 + 12acdeg^2x + 4c^2d^2fgx + 3a^2e^2g^2 + 6acdefg - c^2d^2f^2)}{3a^3e^3g^3 - 9a^2cde^2fg^2 + 9ac^2d^2ef^2g - 3c^3d^3f^3} (ex + d)^{\frac{5}{2}} \frac{1}{\sqrt{gx + f}} (cdex^2 + ae^2x +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(5/2)/(g\*x+f)^(3/2)/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2), x)

[Out] -2/3\*(c\*d\*x+a\*e)\*(8\*c^2\*d^2\*g^2\*x^2+12\*a\*c\*d\*e\*g^2\*x+4\*c^2\*d^2\*f^2\*g\*x+3\*a^2\*e^2\*g^2+6\*a\*c\*d\*e\*f\*g-c^2\*d^2\*f^2)\*(e\*x+d)^(5/2)/(g\*x+f)^(1/2)/(a^3\*e^3\*g^3-3\*a^2\*c\*d\*e^2\*f\*g^2+3\*a\*c^2\*d^2\*e\*f^2\*g-c^3\*d^3\*f^3)/(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(5/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^{\frac{5}{2}}}{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}(gx + f)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(5/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*(g\*x + f

[Out] integrate((e\*x + d)^(5/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*(g\*x + f)^(3/2)), x)

**Fricas [A]** time = 0.305374, size = 900, normalized size = 4.64

$$3(a^2c^3d^4e^2f^4 - 3a^3c^2d^3e^3f^3g + 3a^4cd^2e^4f^2g^2 - a^5de^5fg^3 + (c^5d^5ef^3g - 3ac^4d^4e^2f^2g^2 + 3a^2c^3d^3e^3fg^3 - a^3c^2d^2e^4g^4)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(5/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*(g\*x + f

[Out] 2/3\*(8\*c^2\*d^2\*g^2\*x^2 - c^2\*d^2\*f^2 + 6\*a\*c\*d\*e\*f\*g + 3\*a^2\*e^2\*g^2 + 4\*(c^2\*d^2\*f\*g + 3\*a\*c\*d\*e\*g^2)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f)/(a^2\*c^3\*d^4\*e^2\*f^4 - 3\*a^3\*c^2\*d^3\*e^3\*f^3\*g + 3\*a^4\*c\*d^2\*e^4\*f^2\*g^2 - a^5\*d\*e^5\*f\*g^3 + (c^5\*d^5\*e\*f^3\*g - 3\*a\*c^4\*d^4\*e^2\*f^2\*g^2 + 3\*a^2\*c^3\*d^3\*e^3\*f\*g^3 - a^3\*c^2\*d^2\*e^4\*g^4)\*x^4 + (c^5\*d^5\*e\*f^4 + (c^5\*d^6 - a\*c^4\*d^4\*e^2)\*f^3\*g - 3\*(a\*c^4\*d^5\*e + a^2\*c^3\*d^3\*e^3)\*f^2\*g^2 + (3\*a^2\*c^3\*d^4\*e^2 + 5\*a^3\*c^2\*d^2\*e^4)\*f\*g^3 - (a^3\*c^2\*d^3\*e^3 + 2\*a^4\*c\*d\*e^5)\*g^4)\*x^3 + ((c^5\*d^6 + 2\*a\*c^4\*d^4\*e^2)\*f^4 - (a\*c^4\*d^5\*e + 5\*a^2\*c^3\*d^3\*e^3)\*f^3\*g - 3\*(a^2\*c^3\*d^4\*e^2 - a^3\*c^2\*d^2\*e^4)\*f^2\*g^2 + (5\*a^3\*c^2\*d^3\*e^3 + a^4\*c\*d\*e^5)\*f\*g^3 - (2\*a^4\*c\*d^2\*e^4 + a^5\*e^6)\*g^4)\*x^2 - (a^5\*d\*e^5\*g^4 -

$$(2*a*c^4*d^5*e + a^2*c^3*d^3*e^3)*f^4 + (5*a^2*c^3*d^4*e^2 + 3*a^3*c^2*d^2*e^4)*f^3*g - 3*(a^3*c^2*d^3*e^3 + a^4*c*d*e^5)*f^2*g^2 - (a^4*c*d^2*e^4 - a^5*e^6)*f*g^3)*x$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(5/2)/(g\*x+f)\*\*(3/2)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(5/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*(g\*x + f

[Out] Exception raised: NotImplementedError

$$3.732 \quad \int \frac{(d+ex)^{5/2}}{(f+gx)^{5/2}(ade+(cd^2+ae^2)x+cdex^2)^{5/2}} dx$$

**Optimal.** Leaf size=260

$$\frac{32cdg^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^4} + \frac{16g^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^3}$$

$$+ \frac{4g\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2}$$

$$- \frac{2(d+ex)^{3/2}}{3(f+gx)^{3/2}(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)}$$

[Out]  $(-2*(d+e*x)^{(3/2)})/(3*(c*d*f-a*e*g)*(f+g*x)^{(3/2)}*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(3/2)})+(4*g*\text{Sqrt}[d+e*x])/((c*d*f-a*e*g)^2*(f+g*x)^{(3/2)}*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])+(16*g^2*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])/(3*(c*d*f-a*e*g)^3*\text{Sqrt}[d+e*x]*(f+g*x)^{(3/2)})+(32*c*d*g^2*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])/(3*(c*d*f-a*e*g)^4*\text{Sqrt}[d+e*x]*\text{Sqrt}[f+g*x])$

**Rubi [A]** time = 1.09818, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$

$$\frac{32cdg^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}\sqrt{f+gx}(cdf-aeg)^4} + \frac{16g^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{3\sqrt{d+ex}(f+gx)^{3/2}(cdf-aeg)^3}$$

$$+ \frac{4g\sqrt{d+ex}}{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2}$$

$$- \frac{2(d+ex)^{3/2}}{3(f+gx)^{3/2}(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d+e*x)^{(5/2)}]/((f+g*x)^{(5/2)}*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(5/2)}$

[Out]  $(-2*(d+e*x)^{(3/2)})/(3*(c*d*f-a*e*g)*(f+g*x)^{(3/2)}*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{(3/2)})+(4*g*\text{Sqrt}[d+e*x])/((c*d*f-a*e*g)^2*(f+g*x)^{(3/2)}*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])+(16*g^2*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])/(3*(c*d*f-a*e*g)^3*\text{Sqrt}[d+e*x]*(f+g*x)^{(3/2)})+(32*c*d*g^2*\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2])/(3*(c*d*f-a*e*g)^4*\text{Sqrt}[d+e*x]*\text{Sqrt}[f+g*x])$

**Rubi in Sympy [A]** time = 99.3271, size = 252, normalized size = 0.97

$$\frac{32cdg^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{3\sqrt{d+ex}\sqrt{f+gx}(aeg-cdf)^4} - \frac{16g^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{3\sqrt{d+ex}(f+gx)^{3/2}(aeg-cdf)^3}$$

$$+ \frac{4g\sqrt{d+ex}}{(f+gx)^{3/2}(aeg-cdf)^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}$$

$$+ \frac{2(d+ex)^{3/2}}{3(f+gx)^{3/2}(aeg-cdf)(ade+cdex^2+x(ae^2+cd^2))^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)^{(5/2)}/(g*x+f)^{(5/2)}/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x^2)$

[Out]  $32 \cdot c \cdot d \cdot g^{**2} \cdot \sqrt{a \cdot d \cdot e + c \cdot d \cdot e \cdot x^{**2} + x \cdot (a \cdot e^{**2} + c \cdot d^{**2})} / (3 \cdot \sqrt{t(d + e \cdot x) \cdot \sqrt{f + g \cdot x} \cdot (a \cdot e \cdot g - c \cdot d \cdot f)^{**4}} - 16 \cdot g^{**2} \cdot \sqrt{a \cdot d \cdot e + c \cdot d \cdot e \cdot x^{**2} + x \cdot (a \cdot e^{**2} + c \cdot d^{**2})} / (3 \cdot \sqrt{t(d + e \cdot x) \cdot (f + g \cdot x)^{**3/2}} \cdot (a \cdot e \cdot g - c \cdot d \cdot f)^{**3}) + 4 \cdot g \cdot \sqrt{t(d + e \cdot x) \cdot (f + g \cdot x)^{**3/2}} \cdot (a \cdot e \cdot g - c \cdot d \cdot f)^{**2} \cdot \sqrt{a \cdot d \cdot e + c \cdot d \cdot e \cdot x^{**2} + x \cdot (a \cdot e^{**2} + c \cdot d^{**2})} + 2 \cdot (d + e \cdot x)^{**3/2} / (3 \cdot (f + g \cdot x)^{**3/2} \cdot (a \cdot e \cdot g - c \cdot d \cdot f) \cdot (a \cdot d \cdot e + c \cdot d \cdot e \cdot x^{**2} + x \cdot (a \cdot e^{**2} + c \cdot d^{**2}))^{**3/2})$

**Mathematica [A]** time = 0.320296, size = 152, normalized size = 0.58

$$\frac{2(d + ex)^{3/2} (-a^3 e^3 g^3 + 3a^2 c d e^2 g^2 (3f + 2gx) + 3ac^2 d^2 e g (3f^2 + 12fgx + 8g^2 x^2) + c^3 d^3 (-f^3 + 6f^2 gx + 24fg^2 x^2 + 16g^3 x^3))}{3(f + gx)^{3/2} ((d + ex)(ae + cdx))^{3/2} (cdf - aeg)^4}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^(5/2)/((f + g\*x)^(5/2)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)

[Out]  $(2 \cdot (d + e \cdot x)^{3/2} \cdot (-a^3 \cdot e^3 \cdot g^3) + 3 \cdot a^2 \cdot c \cdot d \cdot e^2 \cdot g^2 \cdot (3 \cdot f + 2 \cdot g \cdot x) + 3 \cdot a \cdot c^2 \cdot d^2 \cdot e \cdot g \cdot (3 \cdot f^2 + 12 \cdot f \cdot g \cdot x + 8 \cdot g^2 \cdot x^2) + c^3 \cdot d^3 \cdot (-f^3 + 6 \cdot f^2 \cdot g \cdot x + 24 \cdot f \cdot g^2 \cdot x^2 + 16 \cdot g^3 \cdot x^3)) / (3 \cdot (c \cdot d \cdot f - a \cdot e \cdot g) \cdot (a \cdot d \cdot e + c \cdot d \cdot e \cdot x + c \cdot d^2 \cdot x^2 + a \cdot e^2 \cdot x^3)) / (3 \cdot (c \cdot d \cdot f - a \cdot e \cdot g) \cdot (a \cdot d \cdot e + c \cdot d \cdot e \cdot x + c \cdot d^2 \cdot x^2 + a \cdot e^2 \cdot x^3))^{3/2} \cdot (f + g \cdot x)^{3/2}$

**Maple [A]** time = 0.015, size = 258, normalized size = 1.

$$\frac{(2cdx + 2ae)(-16c^3d^3g^3x^3 - 24ac^2d^2eg^3x^2 - 24c^3d^3fg^2x^2 - 6a^2cde^2g^3x - 36ac^2d^2efg^2x - 6c^3d^3f^2gx + a^3e^3g^3 - 9a^3e^3g^3 - 9a^3e^3g^3)}{3g^4e^4a^4 - 12cdg^3fe^3a^3 + 18c^2d^2g^2f^2e^2a^2 - 12c^3d^3gf^3ea + 3c^4d^4f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(5/2)/(g\*x+f)^(5/2)/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2), x)

[Out]  $-2/3 \cdot (c \cdot d \cdot x + a \cdot e) \cdot (-16 \cdot c^3 \cdot d^3 \cdot g^3 \cdot x^3 - 24 \cdot a \cdot c^2 \cdot d^2 \cdot e \cdot g^3 \cdot x^2 - 24 \cdot c^3 \cdot d^3 \cdot f \cdot g^2 \cdot x^2 - 6 \cdot a^2 \cdot c \cdot d \cdot e^2 \cdot g^3 \cdot x - 36 \cdot a \cdot c^2 \cdot d^2 \cdot e \cdot f \cdot g^2 \cdot x - 6 \cdot c^3 \cdot d^3 \cdot f^2 \cdot g \cdot x + a^3 \cdot e^3 \cdot g^3 - 9 \cdot a^3 \cdot e^3 \cdot g^3 - 9 \cdot a^3 \cdot e^3 \cdot g^3) \cdot (e \cdot x + d)^{5/2} / (g \cdot x + f)^{5/2} / (a^4 \cdot e^4 \cdot g^4 - 4 \cdot a^3 \cdot c \cdot d \cdot e \cdot g^3 \cdot f + 6 \cdot a^2 \cdot c^2 \cdot d^2 \cdot e^2 \cdot f^2 \cdot g^2 - 4 \cdot a \cdot c^3 \cdot d^3 \cdot e \cdot f^3 \cdot g + c^4 \cdot d^4 \cdot f^4) / (c \cdot d \cdot e \cdot x^2 + a \cdot e^2 \cdot x + c \cdot d^2 \cdot x + a \cdot d \cdot e)^{5/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^{\frac{5}{2}}}{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}(gx + f)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(5/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*(g\*x + f)^(5/2), x)

[Out] integrate((e\*x + d)^(5/2)/((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*(g\*x + f)^(5/2)), x)

**Fricas [A]** time = 0.321584, size = 1438, normalized size = 5.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^x + d)^{5/2}/((c*d*e^x + a*d*e + (c*d^2 + a*e^2)*x)^{5/2})*(g*x + f)$

[Out]  $\frac{2}{3}*(16*c^3*d^3*g^3*x^3 - c^3*d^3*f^3 + 9*a*c^2*d^2*e*f^2*g + 9*a^2*c*d*e^2*f*g^2 - a^3*e^3*g^3 + 24*(c^3*d^3*f*g^2 + a*c^2*d^2*e*g^3)*x^2 + 6*(c^3*d^3*f^2*g + 6*a*c^2*d^2*e*f*g^2 + a^2*c*d*e^2*g^3)*x)*\sqrt{c*d*e^x + a*d*e + (c*d^2 + a*e^2)*x}*\sqrt{e^x + d}*\sqrt{g*x + f}/(a^2*c^4*d^5*e^2*f^6 - 4*a^3*c^3*d^4*e^3*f^5*g + 6*a^4*c^2*d^3*e^4*f^4*g^2 - 4*a^5*c*d^2*e^5*f^3*g^3 + a^6*d*e^6*f^2*g^4 + (c^6*d^6*e*f^4*g^2 - 4*a*c^5*d^5*e^2*f^3*g^3 + 6*a^2*c^4*d^4*e^3*f^2*g^4 - 4*a^3*c^3*d^3*e^4*f*g^5 + a^4*c^2*d^2*e^5*g^6)*x^5 + (2*c^6*d^6*e*f^5*g + (c^6*d^7 - 6*a*c^5*d^5*e^2)*f^4*g^2 - 4*(a*c^5*d^6*e - a^2*c^4*d^4*e^3)*f^3*g^3 + 2*(3*a^2*c^4*d^5*e^2 + 2*a^3*c^3*d^3*e^4)*f^2*g^4 - 2*(2*a^3*c^3*d^4*e^3 + 3*a^4*c^2*d^2*e^5)*f*g^5 + (a^4*c^2*d^3*e^4 + 2*a^5*c*d*e^6)*g^6)*x^4 + (c^6*d^6*e*f^6 + 2*c^6*d^7*f^5*g - 6*a^4*c^2*d^3*e^4*f^5*g^2 - 3*(2*a^5*d^6*e + 3*a^2*c^4*d^4*e^3)*f^4*g^2 + 4*(a^2*c^4*d^5*e^2 + 4*a^3*c^3*d^3*e^4)*f^3*g^3 + (4*a^3*c^3*d^4*e^3 - 9*a^4*c^2*d^2*e^5)*f^2*g^4 + (2*a^5*c*d^2*e^5 + a^6*e^7)*g^6)*x^3 - (6*a^2*c^4*d^4*e^3*f^5*g - 2*a^6*e^7*f^5*g^2 - a^6*d*e^6*g^6 - (c^6*d^7 + 2*a*c^5*d^5*e^2)*f^6 + (9*a^2*c^4*d^5*e^2 - 4*a^3*c^3*d^3*e^4)*f^4*g^2 - 4*(4*a^3*c^3*d^4*e^3 + a^4*c^2*d^2*e^5)*f^3*g^3 + 3*(3*a^4*c^2*d^3*e^4 + 2*a^5*c*d*e^6)*f^2*g^4)*x^2 + (2*a^6*d*e^6*f^5*g^2 + (2*a^5*d^6*e + a^2*c^4*d^4*e^3)*f^6 - 2*(3*a^2*c^4*d^5*e^2 + 2*a^3*c^3*d^3*e^4)*f^5*g + 2*(2*a^3*c^3*d^4*e^3 + 3*a^4*c^2*d^2*e^5)*f^4*g^2 + 4*(a^4*c^2*d^3*e^4 - a^5*c*d*e^6)*f^3*g^3 - (6*a^5*c*d^2*e^5 - a^6*e^7)*f^2*g^4)*x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^x+d)**(5/2)/(g*x+f)**(5/2)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**2)$

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^x + d)^{5/2}/((c*d*e^x + a*d*e + (c*d^2 + a*e^2)*x)^{5/2})*(g*x + f)$

[Out] Exception raised: NotImplementedError

$$3.733 \quad \int \frac{(f+gx)^{5/2} \sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}} dx$$

**Optimal.** Leaf size=385

$$\begin{aligned} & \frac{5\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^4 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{7/2}d^{7/2}g^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \\ & - \frac{5\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^3}{64c^3d^3g\sqrt{d+ex}} \\ & - \frac{5(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2}{96c^2d^2g\sqrt{d+ex}} \\ & + \frac{(f+gx)^{7/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4g\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(\frac{ae}{cd}-\frac{f}{g}\right)}{24\sqrt{d+ex}} \end{aligned}$$

[Out]  $(-5*(c*d*f - a*e*g)^3*\text{Sqrt}[f + g*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*c^{7/2}*d^{7/2}*g^{3/2}*\text{Sqrt}[d + e*x]) - (5*(c*d*f - a*e*g)^2*(f + g*x)^{(3/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(96*c^{7/2}*d^{7/2}*g^{3/2}*\text{Sqrt}[d + e*x]) + (((a*e)/(c*d) - f/g)*(f + g*x)^{(5/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(24*\text{Sqrt}[d + e*x]) + ((f + g*x)^{(7/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g*\text{Sqrt}[d + e*x]) - (5*(c*d*f - a*e*g)^4*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x])]/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x]))/(64*c^{7/2}*d^{7/2}*g^{3/2}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi [A]** time = 1.9036, antiderivative size = 385, normalized size of antiderivative = 1., number of rules used = 7, number of rules used = 5, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$

$$\begin{aligned} & \frac{5\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^4 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{7/2}d^{7/2}g^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \\ & - \frac{5\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^3}{64c^3d^3g\sqrt{d+ex}} \\ & - \frac{5(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2}{96c^2d^2g\sqrt{d+ex}} \\ & + \frac{(f+gx)^{7/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{4g\sqrt{d+ex}} + \frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(\frac{ae}{cd}-\frac{f}{g}\right)}{24\sqrt{d+ex}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[((f + g*x)^{(5/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/ \text{Sqrt}[d + e*x], x$

[Out]  $(-5*(c*d*f - a*e*g)^3*\text{Sqrt}[f + g*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*c^{7/2}*d^{7/2}*g^{3/2}*\text{Sqrt}[d + e*x]) - (5*(c*d*f - a*e*g)^2*(f + g*x)^{(3/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(96*c^{7/2}*d^{7/2}*g^{3/2}*\text{Sqrt}[d + e*x]) + (((a*e)/(c*d) - f/g)*(f + g*x)^{(5/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(24*\text{Sqrt}[d + e*x]) + ((f + g*x)^{(7/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g*\text{Sqrt}[d + e*x]) - (5*(c*d*f - a*e*g)^4*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x])]/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x]))/(64*c^{7/2}*d^{7/2}*g^{3/2}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$



**Rubi in Sympy [A]** time = 157.767, size = 364, normalized size = 0.95

$$\frac{(f+gx)^{\frac{5}{2}}\left(-\frac{ae}{24cd}+\frac{f}{24g}\right)\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}}+\frac{(f+gx)^{\frac{7}{2}}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{4g\sqrt{d+ex}}$$

$$-\frac{5(f+gx)^{\frac{3}{2}}(aeg-cdf)^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{96c^2d^2g\sqrt{d+ex}}$$

$$+\frac{5\sqrt{f+gx}(aeg-cdf)^3\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{64c^3d^3g\sqrt{d+ex}}$$

$$-\frac{5(aeg-cdf)^4\sqrt{ade+cdex^2+x(ae^2+cd^2)}\operatorname{atanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)}{64c^{\frac{7}{2}}d^{\frac{7}{2}}g^{\frac{3}{2}}\sqrt{d+ex}\sqrt{ae+cdx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x+f)**(5/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x`

[Out] `-(f + g*x)**(5/2)*(-a*e/(24*c*d) + f/(24*g))*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/sqrt(d + e*x) + (f + g*x)**(7/2)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(4*g*sqrt(d + e*x)) - 5*(f + g*x)**(3/2)*(a*e*g - c*d*f)**2*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(96*c**2*d**2*g*sqrt(d + e*x)) + 5*sqrt(f + g*x)*(a*e*g - c*d*f)**3*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(64*c**3*d**3*g*sqrt(d + e*x)) - 5*(a*e*g - c*d*f)**4*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))*atanh(sqrt(c)*sqrt(d)*sqrt(f + g*x)/(sqrt(g)*sqrt(a*e + c*d*x)))/(64*c**(7/2)*d**(7/2)*g**(3/2)*sqrt(d + e*x)*sqrt(a*e + c*d*x))`

**Mathematica [A]** time = 0.409775, size = 247, normalized size = 0.64

$$\frac{\sqrt{(d+ex)(ae+cdx)}\left(\frac{2\sqrt{f+gx}(15a^3e^3g^3-5a^2cde^2g^2(11f+2gx)+ac^2d^2eg(73f^2+36fgx+8g^2x^2))+c^3d^3(15f^3+118f^2gx+136fg^2x^2+48g^3x^3)}{3c^3d^3g}\right)}{128\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] `Integrate[((f + g*x)^(5/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d +`

[Out] `(Sqrt[(a*e + c*d*x)*(d + e*x)]*((2*Sqrt[f + g*x]*(15*a^3*e^3*g^3 - 5*a^2*c*d*e^2*g^2*(11*f + 2*g*x) + a*c^2*d^2*e*g*(73*f^2 + 36*f*g*x + 8*g^2*x^2) + c^3*d^3*(15*f^3 + 118*f^2*g*x + 136*f*g^2*x^2 + 48*g^3*x^3)))/(3*c^3*d^3*g) - (5*(c*d*f - a*e*g)^4*Log[a*e*g + 2*Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x]*Sqrt[f + g*x] + c*d*(f + 2*g*x)))/(c^(7/2)*d^(7/2)*g^(3/2)*Sqrt[a*e + c*d*x])))/(128*Sqrt[d + e*x])`

**Maple [B]** time = 0.032, size = 870, normalized size = 2.3

$$-\frac{1}{384gc^3d^3}\sqrt{gx+f}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(-96x^3c^3d^3g^3\sqrt{dgc}\sqrt{cdgx^2+aegx+cdfx+ae^2f}+15g^4\ln\left(\frac{2xcdg+}{1/2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^(5/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)`

[Out] `-1/384*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(-96*x^3*c^3*d^3*g^3*(d*g*c)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)`

$$\begin{aligned} & (1/2)+15*g^4*\ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c \\ & *d*f*x+a*e*f)^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a^4*e^4-60*g^3* \\ & a^3*e^3*f*\ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d \\ & f*x+a*e*f)^(1/2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*c*d+90*f^2*g^2*\ln( \\ & 1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1 \\ & /2)*(d*g*c)^(1/2))/(d*g*c)^(1/2))*a^2*e^2*c^2*d^2-60*f^3*\ln(1/2*( \\ & 2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*( \\ & d*g*c)^(1/2))/(d*g*c)^(1/2))*a*e*g*c^3*d^3+15*f^4*d^4*c^4*\ln(1/2* \\ & (2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)* \\ & (d*g*c)^(1/2))/(d*g*c)^(1/2))-16*x^2*a*c^2*d^2*e*g^3*(d*g*c)^(1/2) \\ & )*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)-272*x^2*c^3*d^3*f*g^2*( \\ & d*g*c)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)+20*g^3*(c*d* \\ & g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*a^2*e^2*(d*g*c)^(1/2)*c*d-72 \\ & *g^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*a*e*f*(d*g*c)^(1/2) \\ & )*c^2*d^2-236*f^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*g*(d* \\ & g*c)^(1/2)*c^3*d^3-30*g^3*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2) \\ & *a^3*e^3*(d*g*c)^(1/2)+110*g^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1 \\ & /2)*a^2*e^2*f*(d*g*c)^(1/2)*c*d-146*f^2*(c*d*g*x^2+a*e*g*x+c*d* \\ & f*x+a*e*f)^(1/2)*a*e*g*(d*g*c)^(1/2)*c^2*d^2-30*f^3*(c*d*g*x^2+a* \\ & e*g*x+c*d*f*x+a*e*f)^(1/2)*(d*g*c)^(1/2)*c^3*d^3)/(e*x+d)^(1/2)/( \\ & c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)/g/(d*g*c)^(1/2)/c^3/d^3 \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^(5/2)/sqrt(e\*x +

[Out] Exception raised: ValueError

**Erics [A]** time = 1.24216, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^(5/2)/sqrt(e\*x +

[Out] 
$$\begin{aligned} & [1/768*(4*(48*c^3*d^3*g^3*x^3 + 15*c^3*d^3*f^3 + 73*a*c^2*d^2*e*f \\ & ^2*g - 55*a^2*c*d*e^2*f*g^2 + 15*a^3*e^3*g^3 + 8*(17*c^3*d^3*f*g^2 \\ & + a*c^2*d^2*e*g^3)*x^2 + 2*(59*c^3*d^3*f^2*g + 18*a*c^2*d^2*e*f \\ & *g^2 - 5*a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a* \\ & e^2)*x)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 15*(c^4*d^5*f^4 \\ & - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f^2*g^2 - 4*a^3*c*d^2* \\ & e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a*c^3*d^3*e^2*f^3* \\ & g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3 + a^4*e^5*g^4 \\ & )*x)*log((4*(2*c^2*d^2*g^2*x + c^2*d^2*f*g + a*c*d*e*g^2)*sqrt(c* \\ & d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) \\ & - (8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2 \\ & *g^2 + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2* \\ & d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2* \\ & e^3)*g^2)*x)*sqrt(c*d*g))/(e*x + d)))/((c^3*d^3*e*g*x + c^3*d^4*g \\ & )*sqrt(c*d*g)), 1/384*(2*(48*c^3*d^3*g^3*x^3 + 15*c^3*d^3*f^3 + 7 \\ & 3*a*c^2*d^2*e*f^2*g - 55*a^2*c*d*e^2*f*g^2 + 15*a^3*e^3*g^3 + 8*( \\ & 17*c^3*d^3*f*g^2 + a*c^2*d^2*e*g^3)*x^2 + 2*(59*c^3*d^3*f^2*g + 1 \\ & 8*a*c^2*d^2*e*f*g^2 - 5*a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2 + a*d* \\ & e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) - \\ & 15*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f^2*g^2 \\ & - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a* \\ & c^3*d^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g \\ & ^3 + a^4*e^5*g^4)*x)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a \end{aligned}$$

```
*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2
+ c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x))/((c^3*
d^3*e*g*x + c^3*d^4*g)*sqrt(-c*d*g))]
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*(5/2)\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/(e\*x+d)\*\*

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(gx + f)^{\frac{5}{2}}}{\sqrt{ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^(5/2)/sqrt(e\*x +

[Out] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^(5/2)/sqrt(e\*x + d), x)

$$3.734 \quad \int \frac{(f+gx)^{3/2} \sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}} dx$$

**Optimal.** Leaf size=313

$$\frac{\sqrt{d+ex} \sqrt{ae+cdx} (cdf - aeg)^3 \tanh^{-1} \left( \frac{\sqrt{g} \sqrt{ae+cdx}}{\sqrt{c} \sqrt{d} \sqrt{f+gx}} \right)}{8c^{5/2} d^{5/2} g^{3/2} \sqrt{x(ae^2 + cd^2) + ade + cdex^2}} - \frac{\sqrt{f+gx} \sqrt{x(ae^2 + cd^2) + ade + cdex^2} (cdf - aeg)^2}{8c^2 d^2 g \sqrt{d+ex}} + \frac{(f+gx)^{5/2} \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{3g \sqrt{d+ex}} + \frac{(f+gx)^{3/2} \sqrt{x(ae^2 + cd^2) + ade + cdex^2} \left( \frac{ae}{cd} - \frac{f}{g} \right)}{12 \sqrt{d+ex}}$$

[Out]  $-\left((c^*d^*f - a^*e^*g)^2 * \text{Sqrt}[f + g^*x] * \text{Sqrt}[a^*d^*e + (c^*d^2 + a^*e^2)^*x + c^*d^*e^*x^2]\right) / \left(8^*c^2 * d^2 * g^3 * \text{Sqrt}[d + e^*x]\right) + \left(\left(\frac{a^*e}{c^*d} - \frac{f}{g}\right)^* (f + g^*x)^{(3/2)} * \text{Sqrt}[a^*d^*e + (c^*d^2 + a^*e^2)^*x + c^*d^*e^*x^2]\right) / \left(12^* \text{Sqrt}[d + e^*x]\right) + \left(\left(f + g^*x\right)^{(5/2)} * \text{Sqrt}[a^*d^*e + (c^*d^2 + a^*e^2)^*x + c^*d^*e^*x^2]\right) / \left(3^*g^3 * \text{Sqrt}[d + e^*x]\right) - \left(\left(c^*d^*f - a^*e^*g\right)^3 * \text{Sqrt}[a^*e + c^*d^*x] * \text{Sqrt}[d + e^*x] * \text{ArcTanh}\left[\frac{\text{Sqrt}[g] * \text{Sqrt}[a^*e + c^*d^*x]}{\text{Sqrt}[c] * \text{Sqrt}[d] * \text{Sqrt}[f + g^*x]}\right]\right) / \left(8^*c^2 * d^2 * g^3 * \text{Sqrt}[a^*d^*e + (c^*d^2 + a^*e^2)^*x + c^*d^*e^*x^2]\right)$

**Rubi [A]** time = 1.43124, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$

$$\frac{\sqrt{d+ex} \sqrt{ae+cdx} (cdf - aeg)^3 \tanh^{-1} \left( \frac{\sqrt{g} \sqrt{ae+cdx}}{\sqrt{c} \sqrt{d} \sqrt{f+gx}} \right)}{8c^{5/2} d^{5/2} g^{3/2} \sqrt{x(ae^2 + cd^2) + ade + cdex^2}} - \frac{\sqrt{f+gx} \sqrt{x(ae^2 + cd^2) + ade + cdex^2} (cdf - aeg)^2}{8c^2 d^2 g \sqrt{d+ex}} + \frac{(f+gx)^{5/2} \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{3g \sqrt{d+ex}} + \frac{(f+gx)^{3/2} \sqrt{x(ae^2 + cd^2) + ade + cdex^2} \left( \frac{ae}{cd} - \frac{f}{g} \right)}{12 \sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}\left[\left(\left(f + g^*x\right)^{(3/2)} * \text{Sqrt}[a^*d^*e + (c^*d^2 + a^*e^2)^*x + c^*d^*e^*x^2]\right) / \text{Sqrt}[d + e^*x], x\right]$

[Out]  $-\left((c^*d^*f - a^*e^*g)^2 * \text{Sqrt}[f + g^*x] * \text{Sqrt}[a^*d^*e + (c^*d^2 + a^*e^2)^*x + c^*d^*e^*x^2]\right) / \left(8^*c^2 * d^2 * g^3 * \text{Sqrt}[d + e^*x]\right) + \left(\left(\frac{a^*e}{c^*d} - \frac{f}{g}\right)^* (f + g^*x)^{(3/2)} * \text{Sqrt}[a^*d^*e + (c^*d^2 + a^*e^2)^*x + c^*d^*e^*x^2]\right) / \left(12^* \text{Sqrt}[d + e^*x]\right) + \left(\left(f + g^*x\right)^{(5/2)} * \text{Sqrt}[a^*d^*e + (c^*d^2 + a^*e^2)^*x + c^*d^*e^*x^2]\right) / \left(3^*g^3 * \text{Sqrt}[d + e^*x]\right) - \left(\left(c^*d^*f - a^*e^*g\right)^3 * \text{Sqrt}[a^*e + c^*d^*x] * \text{Sqrt}[d + e^*x] * \text{ArcTanh}\left[\frac{\text{Sqrt}[g] * \text{Sqrt}[a^*e + c^*d^*x]}{\text{Sqrt}[c] * \text{Sqrt}[d] * \text{Sqrt}[f + g^*x]}\right]\right) / \left(8^*c^2 * d^2 * g^3 * \text{Sqrt}[a^*d^*e + (c^*d^2 + a^*e^2)^*x + c^*d^*e^*x^2]\right)$

**Rubi in Sympy [A]** time = 120.556, size = 291, normalized size = 0.93

$$\frac{(f+gx)^{3/2} \left(-\frac{ae}{12cd} + \frac{f}{12g}\right) \sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}} + \frac{(f+gx)^{5/2} \sqrt{ade+cdex^2+x(ae^2+cd^2)}}{3g \sqrt{d+ex}} - \frac{\sqrt{f+gx} (aeg - cdf)^2 \sqrt{ade+cdex^2+x(ae^2+cd^2)}}{8c^2 d^2 g \sqrt{d+ex}} + \frac{(aeg - cdf)^3 \sqrt{ade+cdex^2+x(ae^2+cd^2)} \operatorname{atanh}\left(\frac{\sqrt{g} \sqrt{ae+cdx}}{\sqrt{c} \sqrt{d} \sqrt{f+gx}}\right)}{8c^{5/2} d^{5/2} g^{3/2} \sqrt{d+ex} \sqrt{ae+cdx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x+f)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x`

[Out]  $-(f + g*x)^{(3/2)}*(-a*e/(12*c*d) + f/(12*g))*\sqrt{a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2)}/\sqrt{d + e*x} + (f + g*x)^{(5/2)}*\sqrt{a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2)}/(3*g*\sqrt{d + e*x}) - \sqrt{f + g*x}*(a*e*g - c*d*f)^2*\sqrt{a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2)}/(8*c^2*d^2*g*\sqrt{d + e*x}) + (a*e*g - c*d*f)^3*\sqrt{a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2)}*\operatorname{atanh}(\sqrt{g}*\sqrt{a*e + c*d*x})/(\sqrt{c}*\sqrt{d}*\sqrt{f + g*x})/(8*c^{5/2}*d^{5/2})*g^{3/2}*\sqrt{d + e*x}*\sqrt{a*e + c*d*x}$

**Mathematica [A]** time = 0.291477, size = 200, normalized size = 0.64

$$\frac{\sqrt{(d+ex)(ae+cdx)} \left( \frac{2\sqrt{f+gx}(-3a^2e^2g^2+2acdeg(4f+gx)+c^2d^2(3f^2+14fgx+8g^2x^2))}{3c^2d^2g} - \frac{(cdf-aeg)^3 \log\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}\sqrt{ae+cdx}+aeg+cdf+2g^2x}{c^{5/2}d^{5/2}g^{3/2}\sqrt{ae+cdx}}\right)}{c^{5/2}d^{5/2}g^{3/2}\sqrt{ae+cdx}} \right)}{16\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] `Integrate[((f + g*x)^(3/2)*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/Sqrt[d +`

[Out]  $(\sqrt{a*e + c*d*x}*(d + e*x))^2*(2*\sqrt{f + g*x}*(-3*a^2*e^2*g^2 + 2*a*c*d*e*g*(4*f + g*x) + c^2*d^2*(3*f^2 + 14*f*g*x + 8*g^2*x^2)))/(3*c^2*d^2*g) - ((c*d*f - a*e*g)^3*\operatorname{Log}[a*e*g + 2*\sqrt{c}*\sqrt{d}*\sqrt{f + g*x} + c*d*(f + 2*g*x)])/c^5*d^5*g^3*\sqrt{a*e + c*d*x})/(16*\sqrt{d + e*x})$

**Maple [B]** time = 0.027, size = 602, normalized size = 1.9

$$\frac{1}{48gd^2c^2}\sqrt{gx+f}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(3g^3\ln\left(\frac{1}{2}\frac{2xcdg+aeg+cdf+2\sqrt{cdgx^2+aegx+cdfx+af\sqrt{dgc}}}{\sqrt{dgc}}\right)\right)a^3e^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2)/(e*x+d)^(1/2),x)`

[Out]  $1/48*(g*x+f)^{(1/2)}*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^{(1/2)}*(3*g^3*\ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*(d*g*c)^{(1/2)})/(d*g*c)^{(1/2)})^3*a^3*e^3-9*g^2*\ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*(d*g*c)^{(1/2)})/(d*g*c)^{(1/2)})^2*a^2*e^2*f*c*d+9*g*\ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*(d*g*c)^{(1/2)})/(d*g*c)^{(1/2)})^2*a*e*f^2*c^2*d^2-3*\ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*(d*g*c)^{(1/2)})/(d*g*c)^{(1/2)})^2*f^3*c^3*d^3+16*x^2*c^2*d^2*g^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*(d*g*c)^{(1/2)}+4*g^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*x*a*e*c*d*(d*g*c)^{(1/2)}+28*g*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*x*f*c^2*d^2*(d*g*c)^{(1/2)}-6*g^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*a^2*e^2*(d*g*c)^{(1/2)}+16*a*c*d*e*f*g*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*(d*g*c)^{(1/2)}+6*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*f^2*c^2*d^2*(d*g*c)^{(1/2)})/(e*x+d)^(1/2)/(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}/g/d^2/c^2/(d*g*c)^{(1/2)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^(3/2)/sqrt(e*x +`

[Out] Exception raised: ValueError

**Fricas** [A] time = 1.04889, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^(3/2)/sqrt(e*x +`

[Out] 
$$\left[ \frac{1}{96} \left( 4 \left( 8c^2d^2g^2x^2 + 3c^2d^2f^2 + 8acd^2efg - 3a^2e^2g^2 + 2(7c^2d^2fg + acd^2eg^2)x \right) \sqrt{cde^2x^2 + ade + (cd^2 + ae^2)x} \sqrt{cd^2g} \sqrt{ex + d} \sqrt{gx + f} - 3(c^3d^4f^3 - 3ac^2d^3ef^2g + 3a^2cd^2e^2fg^2 - a^3d^3e^3g^3 + (c^3d^3ef^3 - 3ac^2d^2e^2f^2g + 3a^2cd^2e^3fg^2 - a^3e^4g^3)x \right) \log(-4(2c^2d^2g^2x + c^2d^2fg + acd^2eg^2) \sqrt{cde^2x^2 + ade + (cd^2 + ae^2)x} \sqrt{ex + d} \sqrt{gx + f} + (8c^2d^2e^2g^2x^3 + c^2d^3f^2 + 6acd^2efg + a^2de^2g^2 + 8(c^2d^2efg + (c^2d^3 + acd^2e^2)g^2)x^2 + (c^2d^2ef^2 + 2(4c^2d^3 + 3acd^2e^2)fg + (8acd^2e + a^2e^3)g^2)x) \sqrt{cd^2g}) / (ex + d) \right) / ((c^2d^2egx + c^2d^3g) \sqrt{cd^2g}), \frac{1}{48} (2(8c^2d^2g^2x^2 + 3c^2d^2f^2 + 8acd^2efg - 3a^2e^2g^2 + 2(7c^2d^2fg + acd^2eg^2)x) \sqrt{cde^2x^2 + ade + (cd^2 + ae^2)x} \sqrt{-cd^2g} \sqrt{ex + d} \sqrt{gx + f} - 3(c^3d^4f^3 - 3ac^2d^3ef^2g + 3a^2cd^2e^2fg^2 - a^3d^3e^3g^3 + (c^3d^3ef^3 - 3ac^2d^2e^2f^2g + 3a^2cd^2e^3fg^2 - a^3e^4g^3)x) \arctan(2\sqrt{cde^2x^2 + ade + (cd^2 + ae^2)x} \sqrt{-cd^2g} \sqrt{ex + d} \sqrt{gx + f} / (2cd^2egx^2 + cd^2f + ade^2g + (cd^2ef + (2cd^2 + ae^2)g)x)) / ((c^2d^2egx + c^2d^3g) \sqrt{-cd^2g}) \right]$$

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**`

[Out] Timed out

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(gx + f)^{\frac{3}{2}}}{\sqrt{ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^(3/2)/sqrt(e*x +`

[Out] `integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^(3/2)/sqrt(e*x + d), x)`

$$3.735 \quad \int \frac{\sqrt{f+gx}\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}} dx$$

**Optimal.** Leaf size=241

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^2 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4c^{3/2}d^{3/2}g^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} + \frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}} + \frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(\frac{ae}{cd}-\frac{f}{g}\right)}{4\sqrt{d+ex}}$$

[Out] (((a\*e)/(c\*d) - f/g)\*Sqrt[f + g\*x]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(4\*Sqrt[d + e\*x]) + ((f + g\*x)^(3/2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(2\*g\*Sqrt[d + e\*x]) - ((c\*d\*f - a\*e\*g)^2\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/(Sqrt[c]\*Sqrt[d]\*Sqrt[f + g\*x])])/(4\*c^(3/2)\*d^(3/2)\*g^(3/2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])

**Rubi [A]** time = 1.00641, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^2 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4c^{3/2}d^{3/2}g^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} + \frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{2g\sqrt{d+ex}} + \frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(\frac{ae}{cd}-\frac{f}{g}\right)}{4\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[f + g\*x]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/Sqrt[d + e\*x], x]

[Out] (((a\*e)/(c\*d) - f/g)\*Sqrt[f + g\*x]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(4\*Sqrt[d + e\*x]) + ((f + g\*x)^(3/2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(2\*g\*Sqrt[d + e\*x]) - ((c\*d\*f - a\*e\*g)^2\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/(Sqrt[c]\*Sqrt[d]\*Sqrt[f + g\*x])])/(4\*c^(3/2)\*d^(3/2)\*g^(3/2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])

**Rubi in Sympy [A]** time = 88.2925, size = 223, normalized size = 0.93

$$\frac{\sqrt{f+gx}\left(-\frac{ae}{4cd}+\frac{f}{4g}\right)\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}} + \frac{(f+gx)^{3/2}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{2g\sqrt{d+ex}} - \frac{(aeg-cdf)^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}\operatorname{atanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)}{4c^{3/2}d^{3/2}g^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*(1/2)\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/(e\*x

[Out] -sqrt(f + g\*x)\*(-a\*e/(4\*c\*d) + f/(4\*g))\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/sqrt(d + e\*x) + (f + g\*x)\*\*(3/2)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(2\*g\*sqrt(d + e\*x)) - (a\*e\*g - c\*d\*f)\*\*2\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*atanh

$$\frac{(\sqrt{c} \sqrt{d} \sqrt{f + g x}) / (\sqrt{g} \sqrt{a e + c d x})}{(4 c^{3/2} d^{3/2} g^{3/2} \sqrt{d + e x} \sqrt{a e + c d x})}$$

**Mathematica [A]** time = 0.270697, size = 178, normalized size = 0.74

$$\frac{\sqrt{d + e x} \sqrt{a e + c d x} \left( 2 \sqrt{c} \sqrt{d} \sqrt{g} \sqrt{f + g x} \sqrt{a e + c d x} (a e g + c d (f + 2 g x)) - (c d f - a e g)^2 \log \left( 2 \sqrt{c} \sqrt{d} \sqrt{g} \sqrt{f + g x} \sqrt{a e + c d x} + a \sqrt{d + e x} \sqrt{a e + c d x} \right) \right)}{8 c^{3/2} d^{3/2} g^{3/2} \sqrt{(d + e x)(a e + c d x)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[f + g\*x]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/Sqrt[d + e\*x], x]

[Out] (Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*(2\*Sqrt[c]\*Sqrt[d]\*Sqrt[g]\*Sqrt[a\*e + c\*d\*x]\*Sqrt[f + g\*x]\*(a\*e\*g + c\*d\*(f + 2\*g\*x)) - (c\*d\*f - a\*e\*g)^2\*Log[a\*e\*g + 2\*Sqrt[c]\*Sqrt[d]\*Sqrt[g]\*Sqrt[a\*e + c\*d\*x]\*Sqrt[f + g\*x] + c\*d\*(f + 2\*g\*x)))/(8\*c^(3/2)\*d^(3/2)\*g^(3/2)\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)])

**Maple [A]** time = 0.023, size = 385, normalized size = 1.6

$$-\frac{1}{8 d g c} \sqrt{g x + f} \sqrt{c d e x^2 + a e^2 x + c d^2 x + a d e} \left( g^2 \ln \left( \frac{1}{2} \left( 2 x c d g + a e g + c d f + 2 \sqrt{c d g x^2 + a e g x + c d f x + a e f} \sqrt{d g c} \right) \frac{1}{\sqrt{d g c}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^(1/2)\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)/(e\*x+d)^(1/2), x)

[Out] -1/8\*(g\*x+f)^(1/2)\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)\*(g^2\*ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*(c\*d\*g\*x^2+a\*e\*g\*x+c\*d\*f\*x+a\*e\*f)^(1/2)\*(d\*g\*c)^(1/2))/(d\*g\*c)^(1/2))\*a^2\*e^2-2\*g\*ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*(c\*d\*g\*x^2+a\*e\*g\*x+c\*d\*f\*x+a\*e\*f)^(1/2)\*(d\*g\*c)^(1/2))/(d\*g\*c)^(1/2))\*a\*e\*f\*c\*d+ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*(c\*d\*g\*x^2+a\*e\*g\*x+c\*d\*f\*x+a\*e\*f)^(1/2)\*(d\*g\*c)^(1/2))/(d\*g\*c)^(1/2))\*f^2\*c^2\*d^2-4\*g\*(c\*d\*g\*x^2+a\*e\*g\*x+c\*d\*f\*x+a\*e\*f)^(1/2)\*x\*c\*d\*(d\*g\*c)^(1/2)-2\*g\*(c\*d\*g\*x^2+a\*e\*g\*x+c\*d\*f\*x+a\*e\*f)^(1/2)\*a\*e\*(d\*g\*c)^(1/2)-2\*(c\*d\*g\*x^2+a\*e\*g\*x+c\*d\*f\*x+a\*e\*f)^(1/2)\*f\*c\*d\*(d\*g\*c)^(1/2))/(e\*x+d)^(1/2)/(c\*d\*g\*x^2+a\*e\*g\*x+c\*d\*f\*x+a\*e\*f)^(1/2)/d/g/c/(d\*g\*c)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(g\*x + f)/sqrt(e\*x + d), x)

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.944766, size = 1, normalized size = 0.

$$\left[ \frac{4 \sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x} (2 c d g x + c d f + a e g) \sqrt{c d g} \sqrt{e x + d} \sqrt{g x + f} + (c^2 d^3 f^2 - 2 a c d^2 e f g + a^2 d e^2 g^2 + (c^2 d^2 e f^2 - 2 a c d e f g + a^2 d e^2 g^2)) \sqrt{d + e x} \sqrt{a e + c d x}}{\dots} \right]$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g*x + f)/sqrt(e*x + d)`

[Out] 
$$\left[ \frac{1}{16} \left( 4 \sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x} \right) \left( 2*c*d*g*x + c*d*f + a*e*g \right) \sqrt{c*d*g} \sqrt{e*x + d} \sqrt{g*x + f} + (c^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a^2*e^3*g^2)*x) \log\left(\frac{4*(2*c^2*d^2*g^2*x + c^2*d^2*f*g + a*c*d*e*g^2) \sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x} \sqrt{e*x + d} \sqrt{g*x + f} - (8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x) \sqrt{c*d*g}}{(e*x + d)}\right) / \left( (c*d*e*g*x + c*d^2*g) \sqrt{c*d*g} \right), \frac{1}{8} \left( 2 \sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x} \right) \left( 2*c*d*g*x + c*d*f + a*e*g \right) \sqrt{-c*d*g} \sqrt{e*x + d} \sqrt{g*x + f} - (c^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a^2*e^3*g^2)*x) \arctan\left(\frac{2 \sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x} \sqrt{-c*d*g} \sqrt{e*x + d} \sqrt{g*x + f}}{(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)}\right) / \left( (c*d*e*g*x + c*d^2*g) \sqrt{-c*d*g} \right) \right]$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(e*x+d)**`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x} \sqrt{gx + f}}{\sqrt{ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g*x + f)/sqrt(e*x + d)`

[Out] `integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(g*x + f)/sqrt(e*x + d), x)`

$$3.736 \quad \int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=167

$$\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}} - \frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}g^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

[Out] (Sqrt[f + g\*x]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(g\*Sqrt[d + e\*x]) - ((c\*d\*f - a\*e\*g)\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/(Sqrt[c]\*Sqrt[d]\*Sqrt[f + g\*x])])/(Sqrt[c]\*Sqrt[d]\*g^(3/2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])

**Rubi [A]** time = 0.641921, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}} - \frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{\sqrt{c}\sqrt{d}g^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(Sqrt[d + e\*x]\*Sqrt[f + g\*x]), x]

[Out] (Sqrt[f + g\*x]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(g\*Sqrt[d + e\*x]) - ((c\*d\*f - a\*e\*g)\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/(Sqrt[c]\*Sqrt[d]\*Sqrt[f + g\*x])])/(Sqrt[c]\*Sqrt[d]\*g^(3/2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])

**Rubi in Sympy [A]** time = 61.3028, size = 158, normalized size = 0.95

$$\frac{\sqrt{f+gx}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{g\sqrt{d+ex}} + \frac{(aeg-cdf)\sqrt{ade+cdex^2+x(ae^2+cd^2)}\operatorname{atanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)}{\sqrt{c}\sqrt{d}g^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/(g\*x+f)\*\*(1/2)/(e\*x

[Out] sqrt(f + g\*x)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(g\*sqrtd + e\*x)) + (a\*e\*g - c\*d\*f)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*atanh(sqrt(c)\*sqrt(d)\*sqrt(f + g\*x)/(sqrt(g)\*sqrt(a\*e + c\*d\*x)))/(sqrt(c)\*sqrt(d)\*g\*\*(3/2)\*sqrt(d + e\*x)\*sqrt(a\*e + c\*d\*x))

**Mathematica [A]** time = 0.193509, size = 163, normalized size = 0.98

$$\frac{\sqrt{(d+ex)(ae+cdx)}(aeg-cdf)\log\left(2\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}\sqrt{ae+cdx}+aeg+cdf+2cdgx\right)}{2\sqrt{c}\sqrt{d}g^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}} + \frac{\sqrt{f+gx}\sqrt{(d+ex)(ae+cdx)}}{g\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(Sqrt[d + e\*x]\*Sqrt[f + g\*x

[Out] (Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)]\*Sqrt[f + g\*x])/(g\*Sqrt[d + e\*x]) + ((-(c\*d\*f) + a\*e\*g)\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)]\*Log[c\*d\*f + a\*e\*g + 2\*c\*d\*g\*x + 2\*Sqrt[c]\*Sqrt[d]\*Sqrt[g]\*Sqrt[a\*e + c\*d\*x]\*Sqrt[f + g\*x]))/(2\*Sqrt[c]\*Sqrt[d]\*g^(3/2)\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x])

**Maple [A]** time = 0.027, size = 198, normalized size = 1.2

$$\frac{1}{2g} \sqrt{cdex^2 + ae^2x + cd^2x + ade} \sqrt{gx + f} \left( \ln \left( \frac{1}{2} (2xcdg + aeg + cdf + 2\sqrt{(gx + f)(cdx + ae)}\sqrt{dgc}) \frac{1}{\sqrt{dgc}} \right) aeg - \ln \left( \frac{1}{2} (2$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)/(g\*x+f)^(1/2)/(e\*x+d)^(1/2), x)

[Out] 1/2\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)\*(g\*x+f)^(1/2)/(e\*x+d)^(1/2)\*(ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*(d\*g\*c)^(1/2)))/(d\*g\*c)^(1/2))\*a\*e\*g-ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*(d\*g\*c)^(1/2)))/(d\*g\*c)^(1/2))\*c\*d\*f+2\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*(d\*g\*c)^(1/2))/((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)/g/(d\*g\*c)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(e\*x + d)\*sqrt(g\*x + f

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.931028, size = 1, normalized size = 0.01

$$\left[ \frac{4 \sqrt{cdex^2 + ade + (cd^2 + ae^2)x} \sqrt{cdg} \sqrt{ex + d} \sqrt{gx + f} - (cd^2f - adeg + (cdf - ae^2g)x) \log \left( -\frac{4(2c^2d^2g^2x + c^2d^2fg + acdeg^2) \sqrt{e^2x + d}}{2g} \right)}{1} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(e\*x + d)\*sqrt(g\*x + f

[Out] [1/4\*(4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f) - (c\*d^2\*f - a\*d\*e\*g + (c\*d\*e\*f - a\*e^2\*g)\*x)\*log(-(4\*(2\*c^2\*d^2\*g^2\*x + c^2\*d^2\*f\*g + a\*c\*d\*e\*g^2)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f) + (8\*c^2\*d^2\*e\*g^2\*x^3 + c^2\*d^3\*f^2 + 6\*a\*c\*d^2\*e\*f\*g + a^2\*d\*e^2\*g^2 + 8\*(c^2\*d^2\*e\*f\*g + (c^2\*d^3 + a\*c\*d\*e^2)\*g^2)\*x^2 + (c^2\*d^2\*e\*f^2 + 2\*(4\*c^2\*d^3 + 3\*a\*c\*d\*e^2)\*f\*g + (8\*a\*c\*d^2\*e + a^2\*e^3)\*g^2)\*x)\*sqrt(c\*d\*g))/(e\*x + d)))/(sqrt(c\*d\*g)\*(e\*g\*x + d\*g)), 1/2\*(2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*

$$d*g)*\sqrt{e*x + d)*\sqrt{g*x + f) - (c*d^2*f - a*d*e*g + (c*d*e*f - a*e^2*g)*x)*\arctan(2*\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x})*\sqrt{-c*d*g)*\sqrt{e*x + d)*\sqrt{g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/(\sqrt{-c*d*g)*(e*g*x + d*g))}]$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{\sqrt{d+ex}\sqrt{f+gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/(g\*x+f)\*\*(1/2)/(e\*x+d)\*\*

[Out] Integral(sqrt((d + e\*x)\*(a\*e + c\*d\*x))/(sqrt(d + e\*x)\*sqrt(f + g\*x)), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(e\*x + d)\*sqrt(g\*x + f)

[Out] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(e\*x + d)\*sqrt(g\*x + f)), x)

$$3.737 \quad \int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{3/2}} dx$$

**Optimal.** Leaf size=158

$$\frac{2\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{ae+cdx} \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}$$

[Out]  $(-2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*\text{Sqrt}[d + e*x] * \text{Sqrt}[f + g*x]) + (2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x])/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x])])/(g^{(3/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi [A]** time = 0.609772, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{2\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{ae+cdx} \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g\sqrt{d+ex}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(\text{Sqrt}[d + e*x]*(f + g*x)^{(3/2)}), x]$

[Out]  $(-2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g*\text{Sqrt}[d + e*x] * \text{Sqrt}[f + g*x]) + (2*\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x])/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x])])/(g^{(3/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi in Sympy [A]** time = 60.8591, size = 151, normalized size = 0.96

$$\frac{2\sqrt{c}\sqrt{d}\sqrt{ade+cdex^2+x(ae^2+cd^2)} \operatorname{atanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)}{g^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}} - \frac{2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{g\sqrt{d+ex}\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(3/2)/(e*x$

[Out]  $2*\text{sqrt}(c)*\text{sqrt}(d)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))*\text{atanh}(\text{sqrt}(c)*\text{sqrt}(d)*\text{sqrt}(f + g*x)/(\text{sqrt}(g)*\text{sqrt}(a*e + c*d*x)))/(g**(3/2)*\text{sqrt}(d + e*x)*\text{sqrt}(a*e + c*d*x)) - 2*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(g*\text{sqrt}(d + e*x)*\text{sqrt}(f + g*x))$

**Mathematica [A]** time = 0.189418, size = 147, normalized size = 0.93

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx} \left( \sqrt{c}\sqrt{d}\sqrt{f+gx} \log\left(2\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}\sqrt{ae+cdx} + aeg + cd(f+2gx)\right) - 2\sqrt{g}\sqrt{ae+cdx} \right)}{g^{3/2}\sqrt{f+gx}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(\text{Sqrt}[d + e*x]*(f + g*x)^{(3/2)}), x]$

[Out]  $(\sqrt{a^2e + c^2d^2x} \sqrt{d + ex} (-2\sqrt{g} \sqrt{a^2e + c^2d^2x} + \sqrt{c} \sqrt{d} \sqrt{f + gx}) \text{Log}[a^2e^2g + 2\sqrt{c} \sqrt{d} \sqrt{g} \sqrt{a^2e + c^2d^2x} \sqrt{f + gx}] + c^2d^2(f + 2gx)) / (g^{3/2}) \sqrt{(a^2e + c^2d^2x)(d + ex)} \sqrt{f + gx}$

**Maple [A]** time = 0.037, size = 197, normalized size = 1.3

$$\frac{1}{g} \sqrt{cdex^2 + ae^2x + cd^2x + ade} \left( \ln \left( \frac{1}{2} (2xcdg + aeg + cdf + 2\sqrt{(gx+f)(cdx+ae)}\sqrt{dgc}) \frac{1}{\sqrt{dgc}} \right) xcdg + \ln \left( \frac{1}{2} (2xcdg + aeg + cdf + 2\sqrt{(gx+f)(cdx+ae)}\sqrt{dgc}) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a^2d^2e + (a^2e^2 + c^2d^2)x + c^2d^2e^2x^2)^{1/2} / (g^2x + f)^{3/2} / (e^2x + d)^{1/2}, x)$

[Out]  $(c^2d^2e^2x^2 + a^2e^2x + c^2d^2x + a^2d^2e)^{1/2} (\ln(1/2(2x^2c^2d^2g + a^2e^2g + c^2d^2f + 2((g^2x + f)(c^2d^2x + a^2e))^{1/2}(d^2g^2c)^{1/2}) / (d^2g^2c)^{1/2}) + x^2c^2d^2g + \ln(1/2(2x^2c^2d^2g + a^2e^2g + c^2d^2f + 2((g^2x + f)(c^2d^2x + a^2e))^{1/2}(d^2g^2c)^{1/2}) / (d^2g^2c)^{1/2}) + c^2d^2f - 2((g^2x + f)(c^2d^2x + a^2e))^{1/2}(d^2g^2c)^{1/2} / (d^2g^2c)^{1/2} / ((g^2x + f)(c^2d^2x + a^2e))^{1/2} / g / (e^2x + d)^{1/2} / (g^2x + f)^{1/2})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\sqrt{c^2d^2e^2x^2 + a^2d^2e + (c^2d^2 + a^2e^2)x} / (\sqrt{e^2x + d}) (g^2x + f)^3)$

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.902083, size = 1, normalized size = 0.01

$$\left[ \frac{(egx^2 + df + (ef + dg)x) \sqrt{\frac{cd}{g}} \log \left( -\frac{8c^2d^2eg^2x^3 + c^2d^3f^2 + 6acd^2efg + a^2de^2g^2 + 4(2cdg^2x + cdfg + aeg^2) \sqrt{cdex^2 + ade + (cd^2 + ae^2)x} \sqrt{ex + d} \sqrt{gx + f}}{ex + d} \right)}{2(eg^2x^2 + d)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\sqrt{c^2d^2e^2x^2 + a^2d^2e + (c^2d^2 + a^2e^2)x} / (\sqrt{e^2x + d}) (g^2x + f)^3)$

[Out]  $[1/2((e^2g^2x^2 + d^2f + (e^2f + d^2g)x) \sqrt{c^2d/g}) \log(-8c^2d^2e^2g^2x^3 + c^2d^3f^2 + 6a^2c^2d^2e^2f^2g + a^2d^2e^2g^2 + 4(2c^2d^2g^2x + c^2d^2f^2g + a^2e^2g^2) \sqrt{c^2d^2e^2x^2 + a^2d^2e + (c^2d^2 + a^2e^2)x} \sqrt{e^2x + d} \sqrt{g^2x + f}) \sqrt{c^2d/g} + 8(c^2d^2e^2f^2g + (c^2d^3 + a^2c^2d^2e^2)g^2)x^2 + (c^2d^2e^2f^2 + 2(4c^2d^3 + 3a^2c^2d^2e^2)f^2g + (8a^2c^2d^2e + a^2e^3)g^2)x) / (e^2x + d) - 4\sqrt{c^2d^2e^2x^2 + a^2d^2e + (c^2d^2 + a^2e^2)x} \sqrt{e^2x + d} \sqrt{g^2x + f} / (e^2g^2x^2 + d^2f + (e^2f + d^2g)x), ((e^2g^2x^2 + d^2f + (e^2f + d^2g)x) \sqrt{-c^2d/g}) \arctan(2\sqrt{c^2d^2e^2x^2 + a^2d^2e + (c^2d^2 + a^2e^2)x} \sqrt{e^2x + d} \sqrt{g^2x + f}) \sqrt{c^2d} / ((2c^2d^2e^2g^2x^2 + c^2d^2f + a^2d^2e + (c^2d^2e^2f + (2c^2d^2 + a^2e^2)g)x) \sqrt{-c^2d/g}) - 2\sqrt{c^2d^2e^2x^2 + a^2d^2e + (c^2d^2 + a^2e^2)x} \sqrt{e^2x + d} \sqrt{g^2x + f} / (e^2g^2x^2 + d^2f + (e^2f + d^2g)x)$

x)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{(d+ex)(ae+cdx)}}{\sqrt{d+ex}(f+gx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/(g\*x+f)\*\*(3/2)/(e\*x+d)\*\*

[Out] Integral(sqrt((d + e\*x)\*(a\*e + c\*d\*x))/(sqrt(d + e\*x)\*(f + g\*x)\*\*(3/2)), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex+d}(gx+f)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(e\*x + d)\*(g\*x + f)^(3/2)), x)

[Out] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(e\*x + d)\*(g\*x + f)^(3/2)), x)

$$3.738 \quad \int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{5/2}} dx$$

**Optimal.** Leaf size=63

$$\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{3(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)}$$

[Out]  $(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(3*(c*d*f - a*e*g)*(d + e*x)^{(3/2)}*(f + g*x)^{(3/2)})$

**Rubi [A]** time = 0.255467, antiderivative size = 63, normalized size of antiderivative = 1., number of rules used = 1, number of rules used = 1, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$

$$\frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{3(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(Sqrt[d + e\*x]\*(f + g\*x)^(5/2)), x

[Out]  $(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(3*(c*d*f - a*e*g)*(d + e*x)^{(3/2)}*(f + g*x)^{(3/2)})$

**Rubi in Sympy [A]** time = 21.854, size = 60, normalized size = 0.95

$$\frac{2(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}{3(d+ex)^{\frac{3}{2}}(f+gx)^{\frac{3}{2}}(aeg-cdf)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/(g\*x+f)\*\*(5/2)/(e\*x

[Out]  $-2*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2)/(3*(d + e*x)**(3/2)*(f + g*x)**(3/2)*(a*e*g - c*d*f))$

**Mathematica [A]** time = 0.0912768, size = 52, normalized size = 0.83

$$\frac{2((d+ex)(ae+cdx))^{3/2}}{3(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(Sqrt[d + e\*x]\*(f + g\*x)^(5/2)), x

[Out]  $(2*((a*e + c*d*x)*(d + e*x))^{(3/2)})/(3*(c*d*f - a*e*g)*(d + e*x)^{(3/2)}*(f + g*x)^{(3/2)})$

**Maple [A]** time = 0.01, size = 63, normalized size = 1.

$$-\frac{2cdx+2ae}{3aeg-3cdf}\sqrt{cdex^2+ae^2x+cd^2x+ade}(gx+f)^{-\frac{3}{2}}\frac{1}{\sqrt{ex+d}}$$



Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)}/(g*x+f)^{(5/2)}/(e*x+d)^{(1/2)}, x)$

[Out]  $-2/3/(g*x+f)^{(3/2)}*(c*d*x+a*e)/(a*e*g-c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^{(1/2)}/(e*x+d)^{(1/2)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}(gx + f)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\text{sqrt}(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(\text{sqrt}(e*x + d)*(g*x + f)^{(5/2)}), x)$

[Out]  $\text{integrate}(\text{sqrt}(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(\text{sqrt}(e*x + d)*(g*x + f)^{(5/2)}), x)$

**Fricas [A]** time = 0.285737, size = 228, normalized size = 3.62

$$\frac{2\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(cdx + ae)\sqrt{ex + d}\sqrt{gx + f}}{3(cd^2f^3 - adef^2g + cdefg^2 - ae^2g^3)x^3 + (2cdef^2g - adeg^3 + (cd^2 - 2ae^2)fg^2)x^2 + (cdf^3 - 2adefg^2 + (2cd^2 - ae^2)f^2g^2)x + cdf^3 - adef^2g + cdefg^2 - ae^2g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\text{sqrt}(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(\text{sqrt}(e*x + d)*(g*x + f)^{(5/2)}), x)$

[Out]  $2/3*\text{sqrt}(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*x + a*e)*\text{sqrt}(e*x + d)*\text{sqrt}(g*x + f)/(c*d^2*f^3 - a*d*e*f^2*g + (c*d*e*f*g^2 - a*e^2*g^3)*x^3 + (2*c*d*e*f^2*g - a*d*e*g^3 + (c*d^2 - 2*a*e^2)*f*g^2)*x^2 + (c*d*e*f^3 - 2*a*d*e*f*g^2 + (2*c*d^2 - a*e^2)*f^2*g^2)*x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(5/2)/(e*x+d)**(1/2), x)$

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}(gx + f)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\text{sqrt}(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(\text{sqrt}(e*x + d)*(g*x + f)^{(5/2)}), x)$

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x +  
d)*(g*x + f)^(5/2)), x)
```

$$3.739 \quad \int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{7/2}} dx$$

**Optimal.** Leaf size=129

$$\frac{4cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{15(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^2} + \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{5(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)}$$

[Out]  $(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(5*(c*d*f - a*e*g)*(d + e*x)^{(3/2)}*(f + g*x)^{(5/2)}) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(15*(c*d*f - a*e*g)^2*(d + e*x)^{(3/2)}*(f + g*x)^{(3/2)})$

**Rubi [A]** time = 0.528434, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{4cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{15(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^2} + \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{5(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(Sqrt[d + e\*x]\*(f + g\*x)^(7/2)), x

[Out]  $(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(5*(c*d*f - a*e*g)*(d + e*x)^{(3/2)}*(f + g*x)^{(5/2)}) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(15*(c*d*f - a*e*g)^2*(d + e*x)^{(3/2)}*(f + g*x)^{(3/2)})$

**Rubi in Sympy [A]** time = 43.6312, size = 122, normalized size = 0.95

$$\frac{4cd(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}{15(d+ex)^{\frac{3}{2}}(f+gx)^{\frac{3}{2}}(aeg-cdf)^2} - \frac{2(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}{5(d+ex)^{\frac{3}{2}}(f+gx)^{\frac{5}{2}}(aeg-cdf)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/(g\*x+f)\*\*(7/2)/(e\*x

[Out]  $4*c*d*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2)/(15*(d + e*x)**(3/2)*(f + g*x)**(3/2)*(a*e*g - c*d*f)**2) - 2*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2)/(5*(d + e*x)**(3/2)*(f + g*x)**(5/2)*(a*e*g - c*d*f))$

**Mathematica [A]** time = 0.145187, size = 69, normalized size = 0.53

$$\frac{2((d+ex)(ae+cdx))^{3/2}(cd(5f+2gx)-3aeg)}{15(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(Sqrt[d + e\*x]\*(f + g\*x)^(7

[Out]  $(2*((a*e + c*d*x)*(d + e*x))^{(3/2)}*(-3*a*e*g + c*d*(5*f + 2*g*x)))/(15*(c*d*f - a*e*g)^2*(d + e*x)^{(3/2)}*(f + g*x)^{(5/2)})$



[Out] Timed out

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}(gx + f)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(e\*x + d)\*(g\*x + f)^(7/2)), x)

[Out] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(e\*x + d)\*(g\*x + f)^(7/2)), x)

$$3.740 \quad \int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{9/2}} dx$$

**Optimal.** Leaf size=198

$$\frac{16c^2d^2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{105(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^3} + \frac{8cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{35(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)^2} + \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{7(d+ex)^{3/2}(f+gx)^{7/2}(cdf-aeg)}$$

[Out]  $(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(7*(c*d*f - a*e*g)*(d + e*x)^{(3/2)}*(f + g*x)^{(7/2)}) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(35*(c*d*f - a*e*g)^2*(d + e*x)^{(3/2)}*(f + g*x)^{(5/2)}) + (16*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(105*(c*d*f - a*e*g)^3*(d + e*x)^{(3/2)}*(f + g*x)^{(3/2)})$

**Rubi [A]** time = 0.816242, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{16c^2d^2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{105(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^3} + \frac{8cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{35(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)^2} + \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{7(d+ex)^{3/2}(f+gx)^{7/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(\text{Sqrt}[d + e*x]*(f + g*x)^{(9/2)})], x$

[Out]  $(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(7*(c*d*f - a*e*g)*(d + e*x)^{(3/2)}*(f + g*x)^{(7/2)}) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(35*(c*d*f - a*e*g)^2*(d + e*x)^{(3/2)}*(f + g*x)^{(5/2)}) + (16*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(105*(c*d*f - a*e*g)^3*(d + e*x)^{(3/2)}*(f + g*x)^{(3/2)})$

**Rubi in Sympy [A]** time = 69.2863, size = 190, normalized size = 0.96

$$-\frac{16c^2d^2(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}{105(d+ex)^{\frac{3}{2}}(f+gx)^{\frac{3}{2}}(aeg-cdf)^3} + \frac{8cd(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}{35(d+ex)^{\frac{3}{2}}(f+gx)^{\frac{5}{2}}(aeg-cdf)^2} - \frac{2(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}{7(d+ex)^{\frac{3}{2}}(f+gx)^{\frac{7}{2}}(aeg-cdf)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(9/2)/(e*x$

[Out]  $-16*c**2*d**2*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2)/(105*(d + e*x)**(3/2)*(f + g*x)**(3/2)*(a*e*g - c*d*f)**3) + 8*c*d*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2)/(35*(d + e*x)**(3/2)*(f + g*x)**(5/2)*(a*e*g - c*d*f)**2) - 2*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2)/(7*(d + e*x)**(3/2)*(f + g*x)**(7/2)*(a*e*g - c*d*f))$

**Mathematica [A]** time = 0.212353, size = 105, normalized size = 0.53

$$\frac{2((d+ex)(ae+cdx))^{3/2} (15a^2e^2g^2 - 6acdeg(7f+2gx) + c^2d^2(35f^2 + 28fgx + 8g^2x^2))}{105(d+ex)^{3/2}(f+gx)^{7/2}(cdf - aeg)^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(Sqrt[d + e\*x]\*(f + g\*x)^(9/2))]

[Out] (2\*((a\*e + c\*d\*x)\*(d + e\*x))^(3/2)\*(15\*a^2\*e^2\*g^2 - 6\*a\*c\*d\*e\*g\*(7\*f + 2\*g\*x) + c^2\*d^2\*(35\*f^2 + 28\*f\*g\*x + 8\*g^2\*x^2)))/(105\*(c\*d\*f - a\*e\*g)^3\*(d + e\*x)^(3/2)\*(f + g\*x)^(7/2))

**Maple [A]** time = 0.013, size = 169, normalized size = 0.9

$$\frac{(2cdx + 2ae)(8c^2d^2g^2x^2 - 12acdeg^2x + 28c^2d^2fgx + 15a^2e^2g^2 - 42acdefg + 35c^2d^2f^2)\sqrt{cdex^2 + ae^2x + cd^2x + ade}}{105a^3e^3g^3 - 315a^2cde^2fg^2 + 315ac^2d^2ef^2g - 105c^3d^3f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)/(g\*x+f)^(9/2)/(e\*x+d)^(1/2), x)

[Out] -2/105\*(c\*d\*x+a\*e)\*(8\*c^2\*d^2\*g^2\*x^2-12\*a\*c\*d\*e\*g^2\*x+28\*c^2\*d^2\*f\*g\*x+15\*a^2\*e^2\*g^2-42\*a\*c\*d\*e\*f\*g+35\*c^2\*d^2\*f^2)\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)/(g\*x+f)^(7/2)/(a^3\*e^3\*g^3-3\*a^2\*c\*d\*e^2\*f\*g+3\*a\*c^2\*d^2\*e\*f^2g-c^3\*d^3\*f^3)/(e\*x+d)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}(gx + f)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(e\*x + d)\*(g\*x + f)^(9/2)))

[Out] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(e\*x + d)\*(g\*x + f)^(9/2)), x)

**Fricas [A]** time = 0.297231, size = 1010, normalized size = 5.1

$$105(c^3d^4f^7 - 3ac^2d^3ef^6g + 3a^2cd^2e^2f^5g^2 - a^3de^3f^4g^3 + (c^3d^3ef^3g^4 - 3ac^2d^2e^2f^2g^5 + 3a^2cde^3fg^6 - a^3e^4g^7)x^5 + (4c^3d^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(e\*x + d)\*(g\*x + f)^(9/2)))

[Out] 2/105\*(8\*c^3\*d^3\*g^2\*x^3 + 35\*a\*c^2\*d^2\*e\*f^2 - 42\*a^2\*c\*d\*e^2\*f\*g + 15\*a^3\*e^3\*g^2 + 4\*(7\*c^3\*d^3\*f\*g - a\*c^2\*d^2\*e\*g^2)\*x^2 + (35\*c^3\*d^3\*f^2 - 14\*a\*c^2\*d^2\*e\*f\*g + 3\*a^2\*c\*d\*e^2\*g^2)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f)/(c^3\*d^4\*f^7 - 3\*a\*c^2\*d^3\*e\*f^6\*g + 3\*a^2\*c\*d^2\*e^2\*f^5\*g^2 - a^3\*d^3\*e^3\*f^4\*g^3 + (c^3\*d^3\*e^3\*f^3\*g^4 - 3\*a\*c^2\*d^2\*e^2\*f^2\*g^5 + 3\*a^2\*c\*d^2\*e^2\*f^2\*g^5 - a^3\*d^3\*e^4\*g^7)x^5 + (4\*c^3\*d^3

$$3*a^2*c*d*e^3*f*g^6 - a^3*e^4*g^7)*x^5 + (4*c^3*d^3*e*f^4*g^3 - a^3*d*e^3*g^7 + (c^3*d^4 - 12*a*c^2*d^2*e^2)*f^3*g^4 - 3*(a*c^2*d^3*e - 4*a^2*c*d*e^3)*f^2*g^5 + (3*a^2*c*d^2*e^2 - 4*a^3*e^4)*f*g^6)*x^4 + 2*(3*c^3*d^3*e*f^5*g^2 - 2*a^3*d*e^3*f*g^6 + (2*c^3*d^4 - 9*a*c^2*d^2*e^2)*f^4*g^3 - 3*(2*a*c^2*d^3*e - 3*a^2*c*d*e^3)*f^3*g^4 + 3*(2*a^2*c*d^2*e^2 - a^3*e^4)*f^2*g^5)*x^3 + 2*(2*c^3*d^3*e*f^6*g - 3*a^3*d*e^3*f^2*g^5 + 3*(c^3*d^4 - 2*a*c^2*d^2*e^2)*f^5*g^2 - 3*(3*a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^4*g^3 + (9*a^2*c*d^2*e^2 - 2*a^3*e^4)*f^3*g^4)*x^2 + (c^3*d^3*e*f^7 - 4*a^3*d*e^3*f^3*g^4 + (4*c^3*d^4 - 3*a*c^2*d^2*e^2)*f^6*g - 3*(4*a*c^2*d^3*e - a^2*c*d*e^3)*f^5*g^2 + (12*a^2*c*d^2*e^2 - a^3*e^4)*f^4*g^3)*x$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/(g\*x+f)\*\*(9/2)/(e\*x+d)\*\*

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}(gx + f)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(e\*x + d)\*(g\*x + f)^(9/2)), x)

[Out] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(e\*x + d)\*(g\*x + f)^(9/2)), x)



$$3.741 \quad \int \frac{\sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}(f+gx)^{11/2}} dx$$

**Optimal.** Leaf size=267

$$\frac{32c^3d^3(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{315(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^4} + \frac{16c^2d^2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{105(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)^3}$$

$$+ \frac{4cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{21(d+ex)^{3/2}(f+gx)^{7/2}(cdf-aeg)^2} + \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{9(d+ex)^{3/2}(f+gx)^{9/2}(cdf-aeg)}$$

[Out]  $(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(9*(c*d*f - a*e*g)*(d + e*x)^{(3/2)}*(f + g*x)^{(9/2)}) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(21*(c*d*f - a*e*g)^2*(d + e*x)^{(3/2)}*(f + g*x)^{(7/2)}) + (16*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(105*(c*d*f - a*e*g)^3*(d + e*x)^{(3/2)}*(f + g*x)^{(5/2)}) + (32*c^3*d^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(315*(c*d*f - a*e*g)^4*(d + e*x)^{(3/2)}*(f + g*x)^{(3/2)})$

**Rubi [A]** time = 1.09989, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{32c^3d^3(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{315(d+ex)^{3/2}(f+gx)^{3/2}(cdf-aeg)^4} + \frac{16c^2d^2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{105(d+ex)^{3/2}(f+gx)^{5/2}(cdf-aeg)^3}$$

$$+ \frac{4cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{21(d+ex)^{3/2}(f+gx)^{7/2}(cdf-aeg)^2} + \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{9(d+ex)^{3/2}(f+gx)^{9/2}(cdf-aeg)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]/(\text{Sqrt}[d + e*x]*(f + g*x)^{(11/2)})]$

[Out]  $(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(9*(c*d*f - a*e*g)*(d + e*x)^{(3/2)}*(f + g*x)^{(9/2)}) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(21*(c*d*f - a*e*g)^2*(d + e*x)^{(3/2)}*(f + g*x)^{(7/2)}) + (16*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(105*(c*d*f - a*e*g)^3*(d + e*x)^{(3/2)}*(f + g*x)^{(5/2)}) + (32*c^3*d^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(315*(c*d*f - a*e*g)^4*(d + e*x)^{(3/2)}*(f + g*x)^{(3/2)})$

**Rubi in Sympy [A]** time = 99.016, size = 258, normalized size = 0.97

$$\frac{32c^3d^3(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}{315(d+ex)^{\frac{3}{2}}(f+gx)^{\frac{3}{2}}(aeg-cdf)^4} - \frac{16c^2d^2(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}{105(d+ex)^{\frac{3}{2}}(f+gx)^{\frac{5}{2}}(aeg-cdf)^3}$$

$$+ \frac{4cd(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}{21(d+ex)^{\frac{3}{2}}(f+gx)^{\frac{7}{2}}(aeg-cdf)^2} - \frac{2(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}{9(d+ex)^{\frac{3}{2}}(f+gx)^{\frac{9}{2}}(aeg-cdf)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(11/2)/(e**$

[Out]  $32*c**3*d**3*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))** (3/2)/(315*(d + e*x)** (3/2)*(f + g*x)** (3/2)*(a*e*g - c*d*f)**4) - 16*c**2*d**2*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))** (3/2)/(105*(d + e*x)** (3/2)*(f + g*x)** (5/2)*(a*e*g - c*d*f)**3) + 4*c*d*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))** (3/2)/(21*(d + e*x)** (3/2)*(f + g*x)** (7/2)*(a*e*g - c*d*f)**2) - 2*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))** (3/2)/(9*(d + e*x)** (3/2)*(f + g*x)** (9/2)*(a$

\*e\*g - c\*d\*f))

**Mathematica [A]** time = 0.355628, size = 152, normalized size = 0.57

$$\frac{2((d + ex)(ae + cdx))^{3/2} (-35a^3e^3g^3 + 15a^2cde^2g^2(9f + 2gx) - 3ac^2d^2eg(63f^2 + 36fgx + 8g^2x^2) + c^3d^3(105f^3 + 126f^2gx))}{315(d + ex)^{3/2}(f + gx)^{9/2}(cdf - aeg)^4}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(Sqrt[d + e\*x]\*(f + g\*x)^(1

[Out] (2\*((a\*e + c\*d\*x)\*(d + e\*x))^(3/2)\*(-35\*a^3\*e^3\*g^3 + 15\*a^2\*c\*d\*e^2\*g^2\*(9\*f + 2\*g\*x) - 3\*a\*c^2\*d^2\*e\*g\*(63\*f^2 + 36\*f\*g\*x + 8\*g^2\*x^2) + c^3\*d^3\*(105\*f^3 + 126\*f^2\*g\*x + 72\*f\*g^2\*x^2 + 16\*g^3\*x^3)))/(315\*(c\*d\*f - a\*e\*g)^4\*(d + e\*x)^(3/2)\*(f + g\*x)^(9/2))

**Maple [A]** time = 0.017, size = 260, normalized size = 1.

$$\frac{(2cdx + 2ae)(-16c^3d^3g^3x^3 + 24ac^2d^2eg^3x^2 - 72c^3d^3fg^2x^2 - 30a^2cde^2g^3x + 108ac^2d^2efg^2x - 126c^3d^3f^2gx + 35a^3e^3g^3)}{315g^4e^4a^4 - 1260cdg^3fe^3a^3 + 1890c^2d^2g^2f^2e^2a^2 - 1260c^3d^3gf^3ea + 315c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)/(g\*x+f)^(11/2)/(e\*x+d)^(1/2), x)

[Out] -2/315\*(c\*d\*x+a\*e)\*(-16\*c^3\*d^3\*g^3\*x^3+24\*a\*c^2\*d^2\*e\*g^3\*x^2-72\*c^3\*d^3\*f\*g^2\*x^2-30\*a^2\*c\*d\*e^2\*g^3\*x+108\*a\*c^2\*d^2\*e\*f\*g^2\*x-126\*c^3\*d^3\*f^2\*g\*x+35\*a^3\*e^3\*g^3-135\*a^2\*c\*d\*e^2\*f\*g^2+189\*a\*c^2\*d^2\*e\*f^2\*g-105\*c^3\*d^3\*f^3)\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)/(g\*x+f)^(9/2)/(a^4\*e^4\*g^4-4\*a^3\*c\*d\*e^3\*f\*g^3+6\*a^2\*c^2\*d^2\*e^2\*f^2\*g^2-4\*a\*c^3\*d^3\*e\*f^3\*g+c^4\*d^4\*f^4)/(e\*x+d)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}(gx + f)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(e\*x + d)\*(g\*x + f)^(11/2)), x)

[Out] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(e\*x + d)\*(g\*x + f)^(11/2)), x)

**Fricas [A]** time = 0.304827, size = 1592, normalized size = 5.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)/(sqrt(e\*x + d)\*(g\*x + f)^(11/2)), x)

```
[Out] 2/315*(16*c^4*d^4*g^3*x^4 + 105*a*c^3*d^3*e*f^3 - 189*a^2*c^2*d^2
*e^2*f^2*g + 135*a^3*c*d*e^3*f*g^2 - 35*a^4*e^4*g^3 + 8*(9*c^4*d^4
*f*g^2 - a*c^3*d^3*e*g^3)*x^3 + 6*(21*c^4*d^4*f^2*g - 6*a*c^3*d^3
*e*f*g^2 + a^2*c^2*d^2*e^2*g^3)*x^2 + (105*c^4*d^4*f^3 - 63*a*c^3
*d^3*e*f^2*g + 27*a^2*c^2*d^2*e^2*f*g^2 - 5*a^3*c*d*e^3*g^3)*x)*
sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*
x + f)/(c^4*d^5*f^9 - 4*a*c^3*d^4*e*f^8*g + 6*a^2*c^2*d^3*e^2*f^7
*g^2 - 4*a^3*c*d^2*e^3*f^6*g^3 + a^4*d*e^4*f^5*g^4 + (c^4*d^4*e*f
^4*g^5 - 4*a*c^3*d^3*e^2*f^3*g^6 + 6*a^2*c^2*d^2*e^3*f^2*g^7 - 4*
a^3*c*d*e^4*f*g^8 + a^4*e^5*g^9)*x^6 + (5*c^4*d^4*e*f^5*g^4 + a^4
*d*e^4*g^9 + (c^4*d^5 - 20*a*c^3*d^3*e^2)*f^4*g^5 - 2*(2*a*c^3*d^4
*e - 15*a^2*c^2*d^2*e^3)*f^3*g^6 + 2*(3*a^2*c^2*d^3*e^2 - 10*a^3
*c*d*e^4)*f^2*g^7 - (4*a^3*c*d^2*e^3 - 5*a^4*e^5)*f*g^8)*x^5 + 5*
(2*c^4*d^4*e*f^6*g^3 + a^4*d*e^4*f*g^8 + (c^4*d^5 - 8*a*c^3*d^3*e
^2)*f^5*g^4 - 4*(a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^4*g^5 + 2*(3*
a^2*c^2*d^3*e^2 - 4*a^3*c*d*e^4)*f^3*g^6 - 2*(2*a^3*c*d^2*e^3 - a
^4*e^5)*f^2*g^7)*x^4 + 10*(c^4*d^4*e*f^7*g^2 + a^4*d*e^4*f^2*g^7
+ (c^4*d^5 - 4*a*c^3*d^3*e^2)*f^6*g^3 - 2*(2*a*c^3*d^4*e - 3*a^2*
c^2*d^2*e^3)*f^5*g^4 + 2*(3*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^4*
g^5 - (4*a^3*c*d^2*e^3 - a^4*e^5)*f^3*g^6)*x^3 + 5*(c^4*d^4*e*f^8
*g + 2*a^4*d*e^4*f^3*g^6 + 2*(c^4*d^5 - 2*a*c^3*d^3*e^2)*f^7*g^2
- 2*(4*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^6*g^3 + 4*(3*a^2*c^2*d^3
*e^2 - a^3*c*d*e^4)*f^5*g^4 - (8*a^3*c*d^2*e^3 - a^4*e^5)*f^4*g^5
)*x^2 + (c^4*d^4*e*f^9 + 5*a^4*d*e^4*f^4*g^5 + (5*c^4*d^5 - 4*a*
c^3*d^3*e^2)*f^8*g - 2*(10*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^7*g
^2 + 2*(15*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^6*g^3 - (20*a^3*c*d
^2*e^3 - a^4*e^5)*f^5*g^4)*x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)/(g*x+f)**(11/2)/(e*x+d)**
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{\sqrt{ex + d}(gx + f)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x + f)^(11/2)), x)
```

```
[Out] integrate(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)/(sqrt(e*x + d)*(g*x + f)^(11/2)), x)
```

$$3.742 \quad \int \frac{(f+gx)^{3/2}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}} dx$$

Optimal. Leaf size=382

$$\begin{aligned} & \frac{3\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^4 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{e}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{5/2}d^{5/2}g^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \\ & + \frac{3\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^3}{64c^2d^2g^2\sqrt{d+ex}} \\ & + \frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2}{32cdg^2\sqrt{d+ex}} \\ & - \frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}{8g^2\sqrt{d+ex}} \\ & + \frac{(f+gx)^{5/2}(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{4g(d+ex)^{3/2}} \end{aligned}$$

[Out] (3\*(c\*d\*f - a\*e\*g)^3\*Sqrt[f + g\*x]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/((64\*c^2\*d^2\*g^2\*Sqrt[d + e\*x]) + ((c\*d\*f - a\*e\*g)^2\*(f + g\*x)^(3/2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]))/(32\*c\*d\*g^2\*Sqrt[d + e\*x]) - ((c\*d\*f - a\*e\*g)\*(f + g\*x)^(5/2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]))/(8\*g^2\*Sqrt[d + e\*x]) + ((f + g\*x)^(5/2)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2))/(4\*g\*(d + e\*x)^(3/2)) + (3\*(c\*d\*f - a\*e\*g)^4\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/(Sqrt[c]\*Sqrt[d]\*Sqrt[f + g\*x])])/(64\*c^(5/2)\*d^(5/2)\*g^(5/2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])

Rubi [A] time = 1.87287, antiderivative size = 382, normalized size of antiderivative = 1., number of rules used = 7, number of rules used = 5, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$

$$\begin{aligned} & \frac{3\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^4 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{e}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{5/2}d^{5/2}g^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \\ & + \frac{3\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^3}{64c^2d^2g^2\sqrt{d+ex}} \\ & + \frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2}{32cdg^2\sqrt{d+ex}} \\ & - \frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}{8g^2\sqrt{d+ex}} \\ & + \frac{(f+gx)^{5/2}(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{4g(d+ex)^{3/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[((f + g\*x)^(3/2)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2))/(d + e\*x)^(3/2)]

[Out] (3\*(c\*d\*f - a\*e\*g)^3\*Sqrt[f + g\*x]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/((64\*c^2\*d^2\*g^2\*Sqrt[d + e\*x]) + ((c\*d\*f - a\*e\*g)^2\*(f + g\*x)^(3/2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]))/(32\*c\*d\*g^2\*Sqrt[d + e\*x]) - ((c\*d\*f - a\*e\*g)\*(f + g\*x)^(5/2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]))/(8\*g^2\*Sqrt[d + e\*x]) + ((f + g\*x)^(5/2)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2))/(4\*g\*(d + e\*x)^(3/2)) + (3\*(c\*d\*f - a\*e\*g)^4\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/(Sqrt[c]\*Sqrt[d]\*Sqrt[f + g\*x])])/(64\*c^(5/2)\*d^(5/2)\*g^(5/2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])

**Rubi in Sympy [A]** time = 155.836, size = 364, normalized size = 0.95

$$\frac{(f+gx)^{\frac{5}{2}}(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}{4g(d+ex)^{\frac{3}{2}}} + \frac{(f+gx)^{\frac{5}{2}}(aeg-cdf)\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{8g^2\sqrt{d+ex}}$$

$$+ \frac{(f+gx)^{\frac{3}{2}}(aeg-cdf)^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{32cdg^2\sqrt{d+ex}}$$

$$- \frac{3\sqrt{f+gx}(aeg-cdf)^3\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{64c^2d^2g^2\sqrt{d+ex}}$$

$$+ \frac{3(aeg-cdf)^4\sqrt{ade+cdex^2+x(ae^2+cd^2)}\operatorname{atanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{\frac{5}{2}}d^{\frac{5}{2}}g^{\frac{5}{2}}\sqrt{d+ex}\sqrt{ae+cdx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x+f)**(3/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x`

[Out] `(f + g*x)**(5/2)*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2)/(4*g*(d + e*x)**(3/2)) + (f + g*x)**(5/2)*(a*e*g - c*d*f)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(8*g**2*sqrt(d + e*x)) + (f + g*x)**(3/2)*(a*e*g - c*d*f)**2*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(32*c*d*g**2*sqrt(d + e*x)) - 3*sqrt(f + g*x)*(a*e*g - c*d*f)**3*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(64*c**2*d**2*g**2*sqrt(d + e*x)) + 3*(a*e*g - c*d*f)**4*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))*atanh(sqrt(g)*sqrt(a*e + c*d*x)/(sqrt(c)*sqrt(d)*sqrt(f + g*x)))/(64*c**(5/2)*d**(5/2)*g**(5/2)*sqrt(d + e*x)*sqrt(a*e + c*d*x))`

**Mathematica [A]** time = 0.650017, size = 254, normalized size = 0.66

$$\frac{((d+ex)(ae+cdx))^{3/2} \left( \frac{2\sqrt{f+gx}(-3a^3e^3g^3+a^2cde^2g^2(11f+2gx)+ac^2d^2eg(11f^2+44fgx+24g^2x^2)+c^3d^3(-3f^3+2f^2gx+24fg^2x^2+16g^3x^3))}{c^2d^2g^2(ae+cdx)} \right)}{128(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

[In] `Integrate[((f + g*x)^(3/2)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x`

[Out] `((a*e + c*d*x)*(d + e*x))^(3/2)*((2*sqrt(f + g*x)*(-3*a^3*e^3*g^3 + a^2*c*d*e^2*g^2*(11*f + 2*g*x) + a*c^2*d^2*e*g*(11*f^2 + 44*f*g*x + 24*g^2*x^2) + c^3*d^3*(-3*f^3 + 2*f^2*g*x + 24*f*g^2*x^2 + 16*g^3*x^3)))/(c^2*d^2*g^2*(a*e + c*d*x)) + (3*(c*d*f - a*e*g)^4*Log[a*e*g + 2*sqrt(c)*sqrt(d)*sqrt(g)*sqrt(a*e + c*d*x)*sqrt(f + g*x) + c*d*(f + 2*g*x)]/(c^(5/2)*d^(5/2)*g^(5/2)*(a*e + c*d*x)^(3/2)))/(128*(d + e*x)^(3/2))`

**Maple [B]** time = 0.028, size = 870, normalized size = 2.3

$$\frac{1}{128g^2c^2d^2}\sqrt{gx+f}\sqrt{cdex^2+ae^2x+cd^2x+ade} \left( 32x^3c^3d^3g^3\sqrt{dgc}\sqrt{cdgx^2+aegx+cdfx+ae^2f} + 3g^4 \ln \left( \frac{2xcdg+ae^2f}{\dots} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^(3/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2), x)`

```
[Out] 1/128*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(32*x
^3*c^3*d^3*g^3*(d*g*c)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1
/2)+3*g^4*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*
f*x+a*e*f)^(1/2)*(d*g*c)^(1/2)))/(d*g*c)^(1/2))*a^4*e^4-12*g^3*a^3
*e^3*f*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x
+a*e*f)^(1/2)*(d*g*c)^(1/2)))/(d*g*c)^(1/2))*c*d+18*f^2*g^2*ln(1/2
*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)
*(d*g*c)^(1/2)))/(d*g*c)^(1/2))*a^2*e^2*c^2*d^2-12*f^3*ln(1/2*(2*x
*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(d*g
*c)^(1/2)))/(d*g*c)^(1/2))*a*e*g*c^3*d^3+3*f^4*d^4*c^4*ln(1/2*(2*x
*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(d*g
*c)^(1/2)))/(d*g*c)^(1/2))+48*x^2*a*c^2*d^2*e*g^3*(d*g*c)^(1/2)*(c
*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)+48*x^2*c^3*d^3*f*g^2*(d*g*c
)^(1/2)*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)+4*g^3*(c*d*g*x^2+
a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*a^2*e^2*(d*g*c)^(1/2)*c*d+88*g^2*(
c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*a*e*f*(d*g*c)^(1/2)*c^2*
d^2+4*f^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*x*g*(d*g*c)^(1/
2)*c^3*d^3-6*g^3*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^3*e^3*
(d*g*c)^(1/2)+22*g^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*a^2*
e^2*f*(d*g*c)^(1/2)*c*d+22*f^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(
1/2)*a*e*g*(d*g*c)^(1/2)*c^2*d^2-6*f^3*(c*d*g*x^2+a*e*g*x+c*d*f*
x+a*e*f)^(1/2)*(d*g*c)^(1/2)*c^3*d^3)/(e*x+d)^(1/2)/(c*d*g*x^2+a*
e*g*x+c*d*f*x+a*e*f)^(1/2)/g^2/(d*g*c)^(1/2)/c^2/d^2
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^(3/2)/(e*x + d)
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [A]** time = 1.27811, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)*(g*x + f)^(3/2)/(e*x + d)
```

```
[Out] [1/256*(4*(16*c^3*d^3*g^3*x^3 - 3*c^3*d^3*f^3 + 11*a*c^2*d^2*e*f^
2*g + 11*a^2*c*d*e^2*f*g^2 - 3*a^3*e^3*g^3 + 24*(c^3*d^3*f*g^2 +
a*c^2*d^2*e*g^3)*x^2 + 2*(c^3*d^3*f^2*g + 22*a*c^2*d^2*e*f*g^2 +
a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*s
qrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^4*d^5*f^4 - 4*a*c^3
*d^4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3
+ a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a*c^3*d^3*e^2*f^3*g + 6*a^2*
c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3 + a^4*e^5*g^4)*x)*log(-
(4*(2*c^2*d^2*g^2*x + c^2*d^2*f*g + a*c*d*e*g^2)*sqrt(c*d*e*x^2 +
a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + (8*c^2*
d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8
*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2
+ 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)
*x)*sqrt(c*d*g))/(e*x + d)))/((c^2*d^2*e*g^2*x + c^2*d^3*g^2)*sqr
t(c*d*g)), 1/128*(2*(16*c^3*d^3*g^3*x^3 - 3*c^3*d^3*f^3 + 11*a*c^
2*d^2*e*f^2*g + 11*a^2*c*d*e^2*f*g^2 - 3*a^3*e^3*g^3 + 24*(c^3*d^
3*f*g^2 + a*c^2*d^2*e*g^3)*x^2 + 2*(c^3*d^3*f^2*g + 22*a*c^2*d^2*
e*f*g^2 + a^2*c*d*e^2*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a
*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^4*d^5*f^
4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f^2*g^2 - 4*a^3*c*d^2
*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a*c^3*d^3*e^2*f^3
```

$$*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3 + a^4*e^5*g^4)*x)*\arctan(2*\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x}*\sqrt{-c*d*g}*\sqrt{e*x + d}*\sqrt{g*x + f}/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x))/((c^2*d^2*e*g^2*x + c^2*d^3*g^2)*\sqrt{-c*d*g})]$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*(3/2)\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/(e\*x+d)\*\*

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f)^(3/2)/(e\*x + d)

[Out] Exception raised: NotImplementedError

$$3.743 \quad \int \frac{\sqrt{f+gx}(ade+(cd^2+ae^2)x+cdex^2)^{3/2}}{(d+ex)^{3/2}} dx$$

**Optimal.** Leaf size=310

$$\begin{aligned} & \frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^3 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{3/2}d^{3/2}g^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \\ & + \frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2}{8cdg^2\sqrt{d+ex}} \\ & - \frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}{4g^2\sqrt{d+ex}} \\ & + \frac{(f+gx)^{3/2}(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{3g(d+ex)^{3/2}} \end{aligned}$$

[Out]  $((c*d*f - a*e*g)^2*\text{Sqrt}[f + g*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*c*d*g^2*\text{Sqrt}[d + e*x]) - ((c*d*f - a*e*g)*(f + g*x)^{(3/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^2*\text{Sqrt}[d + e*x]) + ((f + g*x)^{(3/2)}*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(3*g*(d + e*x)^{(3/2)}) + ((c*d*f - a*e*g)^3*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x])]/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x]))/(8*c^{(3/2)}*d^{(3/2)}*g^{(5/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi [A]** time = 1.41012, antiderivative size = 310, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$

$$\begin{aligned} & \frac{\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^3 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8c^{3/2}d^{3/2}g^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \\ & + \frac{\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2}{8cdg^2\sqrt{d+ex}} \\ & - \frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}{4g^2\sqrt{d+ex}} \\ & + \frac{(f+gx)^{3/2}(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{3g(d+ex)^{3/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Sqrt}[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(d + e*x)^{(3/2)}]$

[Out]  $((c*d*f - a*e*g)^2*\text{Sqrt}[f + g*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*c*d*g^2*\text{Sqrt}[d + e*x]) - ((c*d*f - a*e*g)*(f + g*x)^{(3/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^2*\text{Sqrt}[d + e*x]) + ((f + g*x)^{(3/2)}*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(3*g*(d + e*x)^{(3/2)}) + ((c*d*f - a*e*g)^3*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x])]/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x]))/(8*c^{(3/2)}*d^{(3/2)}*g^{(5/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$



**Rubi in Sympy [A]** time = 120.398, size = 291, normalized size = 0.94

$$\frac{(f+gx)^{\frac{3}{2}}(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}{3g(d+ex)^{\frac{3}{2}}} + \frac{(f+gx)^{\frac{3}{2}}(aeg-cdf)\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{4g^2\sqrt{d+ex}}$$

$$+ \frac{\sqrt{f+gx}(aeg-cdf)^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{8cdg^2\sqrt{d+ex}}$$

$$- \frac{(aeg-cdf)^3\sqrt{ade+cdex^2+x(ae^2+cd^2)}\operatorname{atanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)}{8c^{\frac{3}{2}}d^{\frac{3}{2}}g^{\frac{5}{2}}\sqrt{d+ex}\sqrt{ae+cdx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x+f)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x`

[Out] `(f + g*x)**(3/2)*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))** (3/2)/(3*g*(d + e*x)**(3/2)) + (f + g*x)**(3/2)*(a*e*g - c*d*f)*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(4*g**2*sqrt(d + e*x)) + sqrt(f + g*x)*(a*e*g - c*d*f)**2*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(8*c*d*g**2*sqrt(d + e*x)) - (a*e*g - c*d*f)**3*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))*atanh(sqrt(c)*sqrt(d)*sqrt(f + g*x)/(sqrt(g)*sqrt(a*e + c*d*x)))/(8*c**(3/2)*d**(3/2)*g**(5/2)*sqrt(d + e*x)*sqrt(a*e + c*d*x))`

**Mathematica [A]** time = 0.463889, size = 210, normalized size = 0.68

$$\frac{((d+ex)(ae+cdx))^{3/2} \left( \frac{2\sqrt{f+gx}(3a^2e^2g^2+2acdeg(4f+7gx)+c^2d^2(-3f^2+2fgx+8g^2x^2))}{3cdg^2(ae+cdx)} + \frac{(cdf-aeg)^3 \log\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}\sqrt{ae+cdx}+aeg+cd(f+2d+ex)}{c^{3/2}d^{3/2}g^{5/2}(ae+cdx)^{3/2}}\right)}{c^{3/2}d^{3/2}g^{5/2}(ae+cdx)^{3/2}} \right)}{16(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

[In] `Integrate[(Sqrt[f + g*x] * (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(3/2))/(d + e*x)`

[Out] `((a*e + c*d*x)*(d + e*x))^(3/2)*((2*Sqrt[f + g*x]*(3*a^2*e^2*g^2 + 2*a*c*d*e*g*(4*f + 7*g*x) + c^2*d^2*(-3*f^2 + 2*f*g*x + 8*g^2*x^2)))/(3*c*d*g^2*(a*e + c*d*x)) + ((c*d*f - a*e*g)^3*Log[a*e*g + 2*Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x]*Sqrt[f + g*x] + c*d*(f + 2*g*x)]/(c^(3/2)*d^(3/2)*g^(5/2)*(a*e + c*d*x)^(3/2)))/(16*(d + e*x)^(3/2))`

**Maple [B]** time = 0.028, size = 602, normalized size = 1.9

$$-\frac{1}{48cdg^2}\sqrt{gx+f}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(3g^3\ln\left(\frac{1}{2}\frac{2xcdg+aeg+cdf+2\sqrt{cdgx^2+aegx+cdfx+ae}\sqrt{dgc}}{\sqrt{dgc}}\right)\right)a^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^(1/2)*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(3/2)/(e*x+d)^(3/2),x)`

[Out] `-1/48*(g*x+f)^(1/2)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(3*g^3*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(d*g*c)^(1/2)))/(d*g*c)^(1/2))*a^3*e^3-9*g^2*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(d*g*c)^(1/2)))/(d*g*c)^(1/2))*a^2*e^2*f*c*d+9*g*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^(1/2)*(d*g*c)^(1/2)))/(d*g*c)^(1/2))*a*e*f^2*c^2*d^2-3*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*`

$$\frac{(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*(d*g*c)^{(1/2)}}{(d*g*c)^{(1/2)}}*f^3*c^3*d^3-16*x^2*c^2*d^2*g^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*(d*g*c)^{(1/2)}-28*g^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*x*a*e*c*d*(d*g*c)^{(1/2)}-4*g*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*x*f*c^2*d^2*(d*g*c)^{(1/2)}-6*g^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*a^2*e^2*(d*g*c)^{(1/2)}-16*a*c*d*e*f*g*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*(d*g*c)^{(1/2)}+6*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*f^2*c^2*d^2*(d*g*c)^{(1/2)}}{(e*x+d)^{(1/2)}}/(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}/c/d/(d*g*c)^{(1/2)}/g^2$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*sqrt(g\*x + f)/(e\*x + d)^(1/2))

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.04997, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*sqrt(g\*x + f)/(e\*x + d)^(1/2))

[Out] [1/96\*(4\*(8\*c^2\*d^2\*g^2\*x^2 - 3\*c^2\*d^2\*f^2 + 8\*a\*c\*d\*e\*f\*g + 3\*a^2\*e^2\*g^2 + 2\*(c^2\*d^2\*f\*g + 7\*a\*c\*d\*e\*g^2)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f) - 3\*(c^3\*d^4\*f^3 - 3\*a\*c^2\*d^3\*e\*f^2\*g + 3\*a^2\*c\*d^2\*e^2\*f\*g^2 - a^3\*d\*e^3\*g^3 + (c^3\*d^3\*e\*f^3 - 3\*a\*c^2\*d^2\*e^2\*f^2\*g + 3\*a^2\*c\*d\*e^3\*f\*g^2 - a^3\*e^4\*g^3)\*x)\*log((4\*(2\*c^2\*d^2\*g^2\*x + c^2\*d^2\*f\*g + a\*c\*d\*e\*g^2)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f) - (8\*c^2\*d^2\*e\*g^2\*x^3 + c^2\*d^3\*f^2 + 6\*a\*c\*d^2\*e\*f\*g + a^2\*d\*e^2\*g^2 + 8\*(c^2\*d^2\*e\*f\*g + (c^2\*d^3 + a\*c\*d\*e^2)\*g^2)\*x^2 + (c^2\*d^2\*e\*f^2 + 2\*(4\*c^2\*d^3 + 3\*a\*c\*d\*e^2)\*f\*g + (8\*a\*c\*d^2\*e + a^2\*e^3)\*g^2)\*x)\*sqrt(c\*d\*g))/(e\*x + d)))/((c\*d\*e\*g^2\*x + c\*d^2\*g^2)\*sqrt(c\*d\*g)), 1/48\*(2\*(8\*c^2\*d^2\*g^2\*x^2 - 3\*c^2\*d^2\*f^2 + 8\*a\*c\*d\*e\*f\*g + 3\*a^2\*e^2\*g^2 + 2\*(c^2\*d^2\*f\*g + 7\*a\*c\*d\*e\*g^2)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f) + 3\*(c^3\*d^4\*f^3 - 3\*a\*c^2\*d^3\*e\*f^2\*g + 3\*a^2\*c\*d^2\*e^2\*f\*g^2 - a^3\*d\*e^3\*g^3 + (c^3\*d^3\*e\*f^3 - 3\*a\*c^2\*d^2\*e^2\*f^2\*g + 3\*a^2\*c\*d\*e^3\*f\*g^2 - a^3\*e^4\*g^3)\*x)\*arctan(2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(-c\*d\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f)/(2\*c\*d\*e\*g\*x^2 + c\*d^2\*f + a\*d\*e\*g + (c\*d\*e\*f + (2\*c\*d^2 + a\*e^2)\*g)\*x)))/((c\*d\*e\*g^2\*x + c\*d^2\*g^2)\*sqrt(-c\*d\*g))]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*(1/2)\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/(e\*x+d)\*\*(1/2))

[Out] Timed out

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}} \sqrt{gx + f}}{(ex + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*sqrt(g\*x + f)/(e\*x + d)^(3/2), x)

[Out] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*sqrt(g\*x + f)/(e\*x + d)^(3/2), x)

$$3.744 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=238

$$\frac{3\sqrt{d+ex}\sqrt{ae+cdx}(cdf - aeg)^2 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4\sqrt{c}\sqrt{d}g^{5/2}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}} - \frac{3\sqrt{f+gx}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}(cdf - aeg)}{4g^2\sqrt{d+ex}} + \frac{\sqrt{f+gx}(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{2g(d+ex)^{3/2}}$$

[Out]  $(-3*(c*d*f - a*e*g)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^2*\text{Sqrt}[d + e*x]) + (\text{Sqrt}[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(2*g*(d + e*x)^{(3/2)}) + (3*(c*d*f - a*e*g)^2*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x])]/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x]))/(4*\text{Sqrt}[c]*\text{Sqrt}[d]*g^{(5/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi [A]** time = 1.01067, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{3\sqrt{d+ex}\sqrt{ae+cdx}(cdf - aeg)^2 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4\sqrt{c}\sqrt{d}g^{5/2}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}} - \frac{3\sqrt{f+gx}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}(cdf - aeg)}{4g^2\sqrt{d+ex}} + \frac{\sqrt{f+gx}(x(ae^2 + cd^2) + ade + cdex^2)^{3/2}}{2g(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/((d + e*x)^{(3/2)}*\text{Sqrt}[f + g*x])]$

[Out]  $(-3*(c*d*f - a*e*g)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^2*\text{Sqrt}[d + e*x]) + (\text{Sqrt}[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(2*g*(d + e*x)^{(3/2)}) + (3*(c*d*f - a*e*g)^2*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x])]/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x]))/(4*\text{Sqrt}[c]*\text{Sqrt}[d]*g^{(5/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi in Sympy [A]** time = 89.2677, size = 228, normalized size = 0.96

$$\frac{\sqrt{f+gx}(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{3}{2}}}{2g(d+ex)^{\frac{3}{2}}} + \frac{3\sqrt{f+gx}(aeg - cdf)\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{4g^2\sqrt{d+ex}} + \frac{3(aeg - cdf)^2\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}\text{atanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)}{4\sqrt{c}\sqrt{d}g^{\frac{5}{2}}\sqrt{d+ex}\sqrt{ae+cdx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x$

[Out]  $\text{sqrt}(f + g*x)*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2)/((2*g*(d + e*x))**(3/2)) + 3*\text{sqrt}(f + g*x)*(a*e*g - c*d*f)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(4*g**2*\text{sqrt}(d + e*x)) + 3*(a*e*g - c*d*f)**2*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))*\text{atanh}(\text{sqrt}(c)*\text{sqrt}(d)*\text{sqrt}(f + g*x)/(\text{sqrt}(g)*\text{sqrt}(a*e + c*d*x)))$

$$/(4*\sqrt{c})*\sqrt{d}*g^{(5/2)}*\sqrt{d+e*x}*\sqrt{a*e+c*d*x})$$

**Mathematica [A]** time = 0.415013, size = 181, normalized size = 0.76

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(2\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}\sqrt{ae+cdx}(5aeg+cd(2gx-3f))+3(cdf-aeg)^2\log\left(2\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}\sqrt{ae+cdx}\right)\right)}{8\sqrt{c}\sqrt{d}g^{5/2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/((d + e\*x)^(3/2)\*Sqrt[f +

[Out] (Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*(2\*Sqrt[c]\*Sqrt[d]\*Sqrt[g]\*Sqrt[a\*e + c\*d\*x]\*Sqrt[f + g\*x]\*(5\*a\*e\*g + c\*d\*(-3\*f + 2\*g\*x)) + 3\*(c\*d\*f - a\*e\*g)^2\*Log[a\*e\*g + 2\*Sqrt[c]\*Sqrt[d]\*Sqrt[g]\*Sqrt[a\*e + c\*d\*x]\*Sqrt[f + g\*x] + c\*d\*(f + 2\*g\*x)))/(8\*Sqrt[c]\*Sqrt[d]\*g^(5/2)\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)])

**Maple [A]** time = 0.033, size = 325, normalized size = 1.4

$$\frac{1}{8g^2}\sqrt{cdex^2+ae^2x+cd^2x+ade}\sqrt{gx+f}\left(3\ln\left(\frac{1}{2}\frac{2xcdg+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\sqrt{dgc}}{\sqrt{dgc}}\right)a^2e^2g^2-6\ln\left(\frac{1}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)/(e\*x+d)^(3/2)/(g\*x+f)^(1/2),x)

[Out] 1/8\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)\*(g\*x+f)^(1/2)\*(3\*ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*(d\*g\*c)^(1/2)))/(d\*g\*c)^(1/2))\*a^2\*e^2\*g^2-6\*ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*(d\*g\*c)^(1/2)))/(d\*g\*c)^(1/2))\*a\*e\*g\*f\*c\*d+3\*ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*(d\*g\*c)^(1/2)))/(d\*g\*c)^(1/2))\*f^2\*c^2\*d^2+4\*g\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*x\*c\*d\*(d\*g\*c)^(1/2)+10\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*a\*e\*g\*(d\*g\*c)^(1/2)-6\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*f\*c\*d\*(d\*g\*c)^(1/2))/(e\*x+d)^(1/2)/((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)/g^2/(d\*g\*c)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*sqrt(g\*x

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.951537, size = 1, normalized size = 0.

$$\left[ \frac{4\sqrt{cdex^2+ade+(cd^2+ae^2)x}(2cdgx-3cdf+5aeg)\sqrt{cdg}\sqrt{ex+d}\sqrt{gx+f}+3(c^2d^3f^2-2acd^2efg+a^2de^2g^2+(c^2d^2e$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*sqrt(g*x`

[Out] `[1/16*(4*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x - 3*c*d*f + 5*a*e*g)*sqrt(c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a^2*e^3*g^2)*x)*log(-(4*(2*c^2*d^2*g^2*x + c^2*d^2*f*g + a*c*d*e*g^2)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f) + (8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)*sqrt(c*d*g))/(e*x + d)))/((e*g^2*x + d*g^2)*sqrt(c*d*g)), 1/8*(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(2*c*d*g*x - 3*c*d*f + 5*a*e*g)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f) + 3*(c^2*d^3*f^2 - 2*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + (c^2*d^2*e*f^2 - 2*a*c*d*e^2*f*g + a^2*e^3*g^2)*x)*arctan(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*g)*sqrt(e*x + d)*sqrt(g*x + f)/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/((e*g^2*x + d*g^2)*sqrt(-c*d*g))]`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f)**`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*sqrt(g*x`

[Out] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*sqrt(g*x + f)), x)`

$$3.745 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{3/2}} dx$$

**Optimal.** Leaf size=222

$$\frac{3\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} + \frac{3cd\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g^2\sqrt{d+ex}} - \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}$$

[Out] (3\*c\*d\*Sqrt[f + g\*x]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]) / (g^2\*Sqrt[d + e\*x]) - (2\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)) / (g\*(d + e\*x)^(3/2)\*Sqrt[f + g\*x]) - (3\*Sqrt[c]\*Sqrt[d]\*(c\*d\*f - a\*e\*g)\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x]) / (Sqrt[c]\*Sqrt[d]\*Sqrt[f + g\*x])]) / (g^(5/2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])

**Rubi [A]** time = 0.934953, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$

$$\frac{3\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} + \frac{3cd\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g^2\sqrt{d+ex}} - \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{g(d+ex)^{3/2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

[In] Int[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2) / ((d + e\*x)^(3/2) \* (f + g\*x)^(3/2))]

[Out] (3\*c\*d\*Sqrt[f + g\*x]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]) / (g^2\*Sqrt[d + e\*x]) - (2\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)) / (g\*(d + e\*x)^(3/2)\*Sqrt[f + g\*x]) - (3\*Sqrt[c]\*Sqrt[d]\*(c\*d\*f - a\*e\*g)\*Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x]) / (Sqrt[c]\*Sqrt[d]\*Sqrt[f + g\*x])]) / (g^(5/2)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])

**Rubi in Sympy [A]** time = 87.7019, size = 216, normalized size = 0.97

$$\frac{3\sqrt{c}\sqrt{d}(aeg - cdf)\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}\operatorname{atanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)}{g^{\frac{5}{2}}\sqrt{d+ex}\sqrt{ae+cdx}} + \frac{3cd\sqrt{f+gx}\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{g^2\sqrt{d+ex}} - \frac{2(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{3}{2}}}{g(d+ex)^{\frac{3}{2}}\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/(e\*x+d)\*\*(3/2)/(g\*x

[Out] 3\*sqrt(c)\*sqrt(d)\*(a\*e\*g - c\*d\*f)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*atanh(sqrt(c)\*sqrt(d)\*sqrt(f + g\*x)/(sqrt(g)\*sqrt(a\*e + c\*d\*x)))/(g\*\*(5/2)\*sqrt(d + e\*x)\*sqrt(a\*e + c\*d\*x)) + 3\*c\*d\*sqrt(f + g\*x)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(g\*\*2\*sqrt(d + e\*x)) - 2\*(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))

$(d + ex)^{3/2} / (g(d + ex)^{3/2} \sqrt{f + gx})$

**Mathematica [A]** time = 0.57936, size = 164, normalized size = 0.74

$$\frac{((d + ex)(ae + cdx))^{3/2} \left( \frac{2cd(3f+gx)-2aeg}{g^2\sqrt{f+gx}(ae+cdx)} - \frac{3\sqrt{c}\sqrt{d}(cdf-aeg)\log\left(2\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}\sqrt{ae+cdx}+aeg+cd(f+2gx)\right)}{g^{5/2}(ae+cdx)^{3/2}} \right)}{2(d + ex)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/((d + e\*x)^(3/2)\*(f + g\*x

[Out] (((a\*e + c\*d\*x)\*(d + e\*x))^(3/2)\*((2\*(-2\*a\*e\*g + c\*d\*(3\*f + g\*x)))/(g^2\*(a\*e + c\*d\*x)\*Sqrt[f + g\*x]) - (3\*Sqrt[c]\*Sqrt[d]\*(c\*d\*f - a\*e\*g)\*Log[a\*e\*g + 2\*Sqrt[c]\*Sqrt[d]\*Sqrt[g]\*Sqrt[a\*e + c\*d\*x]\*Sqrt[f + g\*x] + c\*d\*(f + 2\*g\*x)))/(g^(5/2)\*(a\*e + c\*d\*x)^(3/2)))/(2\*(d + e\*x)^(3/2))

**Maple [B]** time = 0.037, size = 383, normalized size = 1.7

$$\frac{1}{2g^2} \left( 3 \ln \left( \frac{1}{2} \frac{2xcdg + aeg + cdf + 2\sqrt{(gx + f)(cdx + ae)}\sqrt{dgc}}{\sqrt{dgc}} \right) xacdeg^2 - 3 \ln \left( \frac{1}{2} \frac{2xcdg + aeg + cdf + 2\sqrt{(gx + f)(cdx + ae)}\sqrt{dgc}}{\sqrt{dgc}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)/((e\*x+d)^(3/2)/(g\*x+f)^(3/2)), x)

[Out] 1/2\*(3\*ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*(d\*g\*c)^(1/2)))/(d\*g\*c)^(1/2))\*x\*a\*c\*d\*e\*g^2-3\*ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*(d\*g\*c)^(1/2)))/(d\*g\*c)^(1/2))\*x\*c^2\*d^2\*f\*g+3\*ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*(d\*g\*c)^(1/2)))/(d\*g\*c)^(1/2))\*a\*e\*g\*f\*c\*d-3\*ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*(d\*g\*c)^(1/2)))/(d\*g\*c)^(1/2))\*f^2\*c^2\*d^2+2\*g\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*(d\*g\*c)^(1/2))\*x\*c\*d\*(d\*g\*c)^(1/2)-4\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*a\*e\*g\*(d\*g\*c)^(1/2)+6\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*f\*c\*d\*(d\*g\*c)^(1/2))\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)/((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)/(d\*g\*c)^(1/2)/g^2/(g\*x+f)^(1/2)/(e\*x+d)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f

[Out] Exception raised: ValueError



**Fricas [A]** time = 0.941302, size = 1, normalized size = 0.

$$\left[ \frac{4 \sqrt{cdex^2 + ade + (cd^2 + ae^2)x} (cdgx + 3cdf - 2aeg) \sqrt{ex + d} \sqrt{gx + f} - 3 (cd^2 f^2 - adefg + (cdfg - ae^2 g^2) x^2 + (cdf^2}{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f

[Out] [1/4\*(4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(c\*d\*g\*x + 3\*c\*d\*f - 2\*a\*e\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f) - 3\*(c\*d^2\*f^2 - a\*d\*e\*f\*g + (c\*d\*e\*f\*g - a\*e^2\*g^2)\*x^2 + (c\*d\*e\*f^2 - a\*d\*e\*g^2 + (c\*d^2 - a\*e^2)\*f\*g)\*x)\*sqrt(c\*d/g)\*log(-(8\*c^2\*d^2\*e\*g^2\*x^3 + c^2\*d^3\*f^2 + 6\*a\*c\*d^2\*e\*f\*g + a^2\*d\*e^2\*g^2 + 4\*(2\*c\*d\*g^2\*x + c\*d\*f\*g + a\*e\*g^2)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f)\*sqrt(c\*d/g) + 8\*(c^2\*d^2\*e\*f\*g + (c^2\*d^3 + a\*c\*d\*e^2)\*g^2)\*x^2 + (c^2\*d^2\*e\*f^2 + 2\*(4\*c^2\*d^3 + 3\*a\*c\*d\*e^2)\*f\*g + (8\*a\*c\*d^2\*e + a^2\*e^3)\*g^2)\*x)/(e\*x + d)))/(e\*g^3\*x^2 + d\*f\*g^2 + (e\*f\*g^2 + d\*g^3)\*x), 1/2\*(2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(c\*d\*g\*x + 3\*c\*d\*f - 2\*a\*e\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f) - 3\*(c\*d^2\*f^2 - a\*d\*e\*f\*g + (c\*d\*e\*f\*g - a\*e^2\*g^2)\*x^2 + (c\*d\*e\*f^2 - a\*d\*e\*g^2 + (c\*d^2 - a\*e^2)\*f\*g)\*x)\*sqrt(-c\*d/g)\*arctan(2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f)\*c\*d/((2\*c\*d\*e\*g\*x^2 + c\*d^2\*f + a\*d\*e\*g + (c\*d\*e\*f + (2\*c\*d^2 + a\*e^2)\*g)\*x)\*sqrt(-c\*d/g)))/(e\*g^3\*x^2 + d\*f\*g^2 + (e\*f\*g^2 + d\*g^3)\*x)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/((e\*x+d)\*\*(3/2)/(g\*x+f)\*\*

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}(gx + f)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f

[Out] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f)^(3/2)), x)

$$3.746 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{5/2}} dx$$

**Optimal.** Leaf size=214

$$\frac{2c^{3/2}d^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{2cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g^2\sqrt{d+ex}\sqrt{f+gx}} - \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}$$

[Out]  $(-2*c*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*\text{Sqrt}[d + e*x]*\text{Sqrt}[f + g*x]) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(3*g*(d + e*x)^{(3/2)}*(f + g*x)^{(3/2)}) + (2*c^{(3/2)}*d^{(3/2)}*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x])]/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x]))/(g^{(5/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi [A]** time = 0.880688, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{2c^{3/2}d^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{2cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g^2\sqrt{d+ex}\sqrt{f+gx}} - \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/((d + e*x)^{(3/2)}*(f + g*x)^{(5/2)})]$

[Out]  $(-2*c*d*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(g^2*\text{Sqrt}[d + e*x]*\text{Sqrt}[f + g*x]) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(3*g*(d + e*x)^{(3/2)}*(f + g*x)^{(3/2)}) + (2*c^{(3/2)}*d^{(3/2)}*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x])]/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x]))/(g^{(5/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi in Sympy [A]** time = 84.9134, size = 207, normalized size = 0.97

$$\frac{2c^{3/2}d^{3/2}\sqrt{ade+cdex^2+x(ae^2+cd^2)}\text{atanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{5/2}\sqrt{d+ex}\sqrt{ae+cdx}} - \frac{2cd\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{g^2\sqrt{d+ex}\sqrt{f+gx}} - \frac{2(ade+cdex^2+x(ae^2+cd^2))^{3/2}}{3g(d+ex)^{3/2}(f+gx)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x$

[Out]  $2*c^{(3/2)}*d^{(3/2)}*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))*\text{atanh}(\text{sqrt}(g)*\text{sqrt}(a*e + c*d*x)/(\text{sqrt}(c)*\text{sqrt}(d)*\text{sqrt}(f + g*x)))/(g^{(5/2)}*\text{sqrt}(d + e*x)*\text{sqrt}(a*e + c*d*x)) - 2*c*d*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(g**2*\text{sqrt}(d + e*x)*\text{sqrt}(f + g*x)) - 2*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))^{(3/2)}/(3*g*($

$$d + e^*x)^{(3/2)} * (f + g^*x)^{(3/2)}$$

**Mathematica [A]** time = 0.587753, size = 167, normalized size = 0.78

$$\frac{\sqrt{d+ex}\sqrt{ae+cdx}\left(3c^{3/2}d^{3/2}(f+gx)^{3/2}\log\left(2\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}\sqrt{ae+cdx}+aeg+cd(f+2gx)\right)-2\sqrt{g}\sqrt{ae+cdx}(aeg+cd(f+2gx))\right)}{3g^{5/2}(f+gx)^{3/2}\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/((d + e\*x)^(3/2)\*(f + g\*x)^(5/2)), x]

[Out] (Sqrt[a\*e + c\*d\*x]\*Sqrt[d + e\*x]\*(-2\*Sqrt[g]\*Sqrt[a\*e + c\*d\*x]\*(a\*e\*g + c\*d\*(3\*f + 4\*g\*x)) + 3\*c^(3/2)\*d^(3/2)\*(f + g\*x)^(3/2)\*Log[a\*e\*g + 2\*Sqrt[c]\*Sqrt[d]\*Sqrt[g]\*Sqrt[a\*e + c\*d\*x]\*Sqrt[f + g\*x] + c\*d\*(f + 2\*g\*x)))/(3\*g^(5/2)\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)]\*(f + g\*x)^(3/2))

**Maple [A]** time = 0.042, size = 331, normalized size = 1.6

$$\frac{1}{3g^2}\sqrt{cdex^2 + ae^2x + cd^2x + ade}\left(3\ln\left(\frac{2xcdg + aeg + cdf + 2\sqrt{(gx+f)(cdx+ae)}\sqrt{dgc}}{\sqrt{dgc}}\right)x^2c^2d^2g^2 + 6\ln\left(\frac{2xcdg + aeg + cdf + 2\sqrt{(gx+f)(cdx+ae)}\sqrt{dgc}}{\sqrt{dgc}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)/(e\*x+d)^(3/2)/(g\*x+f)^(5/2), x)

[Out] 1/3\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)\*(3\*ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*(d\*g\*c)^(1/2)))/(d\*g\*c)^(1/2))^2\*x^2\*c^2\*d^2\*g^2+6\*ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*(d\*g\*c)^(1/2)))/(d\*g\*c)^(1/2))^2\*x\*c^2\*d^2\*f\*g+3\*ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*(d\*g\*c)^(1/2)))/(d\*g\*c)^(1/2))\*f^2\*c^2\*d^2-8\*g\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*x\*c\*d\*(d\*g\*c)^(1/2)-2\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*a\*e\*g\*(d\*g\*c)^(1/2)-6\*((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)\*f\*c\*d\*(d\*g\*c)^(1/2))/(d\*g\*c)^(1/2)/((g\*x+f)\*(c\*d\*x+a\*e))^(1/2)/g^2/(g\*x+f)^(3/2)/(e\*x+d)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}(gx + f)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f)^(5/2)), x)

[Out] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f)^(5/2)), x)

**Fricas [A]** time = 0.909244, size = 1, normalized size = 0.

$$\frac{4\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(4cdgx + 3cdf + aeg)\sqrt{ex + d}\sqrt{gx + f} - 3(cdeg^2x^3 + cd^2f^2 + (2cdefg + cd^2g^2)x^2 + (cd^2eg^2x^3 + df^2g^2 + (2efg^3 + dg^4)x^2 + (ef^2g^2 + 2df^2g^2)x + cd^2eg^2))}{3(eg^4x^3 + df^2g^2 + (2efg^3 + dg^4)x^2 + (ef^2g^2 + 2df^2g^2)x + cd^2eg^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f)^(3/2))

[Out] [-1/6\*(4\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(4\*c\*d\*g\*x + 3\*c\*d\*f + a\*e\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f) - 3\*(c\*d\*e\*g^2\*x^3 + c\*d^2\*f^2 + (2\*c\*d\*e\*f\*g + c\*d^2\*g^2)\*x^2 + (c\*d\*e\*f^2 + 2\*c\*d^2\*f\*g)\*x)\*sqrt(c\*d/g)\*log(-8\*c^2\*d^2\*e\*g^2\*x^3 + c^2\*d^3\*f^2 + 6\*a\*c\*d^2\*e\*f\*g + a^2\*d^2\*e^2\*g^2 + 4\*(2\*c\*d\*g^2\*x + c\*d\*f\*g + a\*e\*g^2)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f)\*sqrt(c\*d/g) + 8\*(c^2\*d^2\*e\*f\*g + (c^2\*d^3 + a\*c\*d\*e^2)\*g^2)\*x^2 + (c^2\*d^2\*e\*f^2 + 2\*(4\*c^2\*d^3 + 3\*a\*c\*d\*e^2)\*f\*g + (8\*a\*c\*d^2\*e + a^2\*e^3)\*g^2)\*x)/(e\*x + d))/((e\*g^4\*x^3 + d\*f^2\*g^2 + (2\*e\*f\*g^3 + d\*g^4)\*x^2 + (e\*f^2\*g^2 + 2\*d\*f\*g^3)\*x), -1/3\*(2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(4\*c\*d\*g\*x + 3\*c\*d\*f + a\*e\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f) - 3\*(c\*d\*e\*g^2\*x^3 + c\*d^2\*f^2 + (2\*c\*d\*e\*f\*g + c\*d^2\*g^2)\*x^2 + (c\*d\*e\*f^2 + 2\*c\*d^2\*f\*g)\*x)\*sqrt(-c\*d/g)\*arctan(2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f)\*c\*d/((2\*c\*d\*e\*g\*x^2 + c\*d^2\*f + a\*d\*e\*g + (c\*d\*e\*f + (2\*c\*d^2 + a\*e^2)\*g)\*x)\*sqrt(-c\*d/g)))/(e\*g^4\*x^3 + d\*f^2\*g^2 + (2\*e\*f\*g^3 + d\*g^4)\*x^2 + (e\*f^2\*g^2 + 2\*d\*f\*g^3)\*x)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/(e\*x+d)\*\*(3/2)/(g\*x+f)\*\*(3/2))

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}(gx + f)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f)^(3/2))

[Out] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f)^(5/2)), x)

$$3.747 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{7/2}} dx$$

**Optimal.** Leaf size=63

$$\frac{2(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{5(d+ex)^{5/2}(f+gx)^{5/2}(cdf - aeg)}$$

[Out]  $(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(5*(c*d*f - a*e*g)*(d + e*x)^{(5/2)*(f + g*x)^{(5/2)})}$

**Rubi [A]** time = 0.254732, antiderivative size = 63, normalized size of antiderivative = 1., number of rules used = 1, number of rules used = 1, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$

$$\frac{2(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{5(d+ex)^{5/2}(f+gx)^{5/2}(cdf - aeg)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/((d + e*x)^{(3/2)*(f + g*x)^{(7/2)})}$

[Out]  $(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(5*(c*d*f - a*e*g)*(d + e*x)^{(5/2)*(f + g*x)^{(5/2)})}$

**Rubi in Sympy [A]** time = 21.6618, size = 60, normalized size = 0.95

$$\frac{2(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{5}{2}}}{5(d+ex)^{\frac{5}{2}}(f+gx)^{\frac{5}{2}}(aeg - cdf)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x$

[Out]  $-2*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(5/2)/(5*(d + e*x)**(5/2)*(f + g*x)**(5/2)*(a*e*g - c*d*f))$

**Mathematica [A]** time = 0.169936, size = 52, normalized size = 0.83

$$\frac{2((d+ex)(ae+cdx))^{5/2}}{5(d+ex)^{5/2}(f+gx)^{5/2}(cdf - aeg)}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/((d + e*x)^{(3/2)*(f + g*x}$

[Out]  $(2*((a*e + c*d*x)*(d + e*x))^{(5/2)})/(5*(c*d*f - a*e*g)*(d + e*x)^{(5/2)*(f + g*x)^{(5/2)})}$

**Maple [A]** time = 0.01, size = 63, normalized size = 1.

$$-\frac{2cdx + 2ae}{5aeg - 5cdf} (cdex^2 + ae^2x + cd^2x + ade)^{\frac{3}{2}} (gx + f)^{-\frac{5}{2}} (ex + d)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(3/2)}/(e*x+d)^{(3/2)}/(g*x+f)^{(7/2)}, x)$

[Out]  $-2/5/(g*x+f)^{(5/2)}*(c*d*x+a*e)/(a*e*g-c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^{(3/2)}/(e*x+d)^{(3/2)}$

**Maxima [A]** time = 1.06745, size = 308, normalized size = 4.89

$$\frac{2(c^2d^2x^2 + 2acdex + a^2e^2)\sqrt{cdx + ae}(ex + d)\sqrt{gx + f}}{5(cd^2f^4 - adef^3g + (cdfg^3 - ae^2g^4)x^4 - ((3e^2fg^3 + deg^4)a - (3def^2g^2 + d^2fg^3)c)x^3 - 3((e^2f^2g^2 + defg^3)a - (def^3g +$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^{(3/2)}/((e*x + d)^{(3/2)}*(g*x + f$

[Out]  $2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*\text{sqrt}(c*d*x + a*e)*(e*x + d)*\text{sqrt}(g*x + f)/(c*d^2*f^4 - a*d*e*f^3*g + (c*d*e*f*g^3 - a*e^2*g^4)*x^4 - ((3*e^2*f*g^3 + d*e*g^4)*a - (3*d*e*f^2*g^2 + d^2*f*g^3)*c)*x^3 - 3*((e^2*f^2*g^2 + d*e*f*g^3)*a - (d*e*f^3*g + d^2*f^2*g^2)*c)*x^2 - ((e^2*f^3*g + 3*d*e*f^2*g^2)*a - (d*e*f^4 + 3*d^2*f^3*g)*c)*x)$

**Ericas [A]** time = 0.289347, size = 313, normalized size = 4.97

$$\frac{2(c^2d^2x^2 + 2acdex + a^2e^2)\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{ex + d}\sqrt{gx + f}}{5(cd^2f^4 - adef^3g + (cdfg^3 - ae^2g^4)x^4 + (3cdf^2g^2 - adeg^4 + (cd^2 - 3ae^2)fg^3)x^3 + 3(cdf^3g - adefg^3 + (cd^2 - ae^2)f^2$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^{(3/2)}/((e*x + d)^{(3/2)}*(g*x + f$

[Out]  $2/5*(c^2*d^2*x^2 + 2*a*c*d*e*x + a^2*e^2)*\text{sqrt}(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*\text{sqrt}(e*x + d)*\text{sqrt}(g*x + f)/(c*d^2*f^4 - a*d*e*f^3*g + (c*d*e*f*g^3 - a*e^2*g^4)*x^4 + (3*c*d*e*f^2*g^2 - a*d*e*g^4 + (c*d^2 - 3*a*e^2)*f*g^3)*x^3 + 3*(c*d*e*f^3*g - a*d*e*f^2*g^3 + (c*d^2 - a*e^2)*f^2*g^2)*x^2 + (c*d*e*f^4 - 3*a*d*e*f^2*g^2 + (3*c*d^2 - a*e^2)*f^3*g)*x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f)**$

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}(gx + f)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g*x + f
```

```
[Out] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g*x + f)^(7/2)), x)
```

$$3.748 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{9/2}} dx$$

**Optimal.** Leaf size=129

$$\frac{4cd(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{35(d+ex)^{5/2}(f+gx)^{5/2}(cdf - aeg)^2} + \frac{2(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{7(d+ex)^{5/2}(f+gx)^{7/2}(cdf - aeg)}$$

[Out]  $(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(7*(c*d*f - a*e*g)*(d + e*x)^{(5/2)}*(f + g*x)^{(7/2)}) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(35*(c*d*f - a*e*g)^2*(d + e*x)^{(5/2)}*(f + g*x)^{(5/2)})$

**Rubi [A]** time = 0.525762, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{4cd(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{35(d+ex)^{5/2}(f+gx)^{5/2}(cdf - aeg)^2} + \frac{2(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{7(d+ex)^{5/2}(f+gx)^{7/2}(cdf - aeg)}$$

Antiderivative was successfully verified.

[In] Int[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/((d + e\*x)^(3/2)\*(f + g\*x)^(9/2))

[Out]  $(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(7*(c*d*f - a*e*g)*(d + e*x)^{(5/2)}*(f + g*x)^{(7/2)}) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(35*(c*d*f - a*e*g)^2*(d + e*x)^{(5/2)}*(f + g*x)^{(5/2)})$

**Rubi in Sympy [A]** time = 43.1757, size = 122, normalized size = 0.95

$$\frac{4cd(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{5}{2}}}{35(d+ex)^{\frac{5}{2}}(f+gx)^{\frac{5}{2}}(aeg - cdf)^2} - \frac{2(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{5}{2}}}{7(d+ex)^{\frac{5}{2}}(f+gx)^{\frac{7}{2}}(aeg - cdf)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/(e\*x+d)\*\*(3/2)/(g\*x

[Out]  $4*c*d*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(5/2)/(35*(d + e*x)**(5/2)*(f + g*x)**(5/2)*(a*e*g - c*d*f)**2) - 2*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(5/2)/(7*(d + e*x)**(5/2)*(f + g*x)**(7/2)*(a*e*g - c*d*f))$

**Mathematica [A]** time = 0.206252, size = 69, normalized size = 0.53

$$\frac{2((d+ex)(ae+cdx))^{5/2}(cd(7f+2gx) - 5aeg)}{35(d+ex)^{5/2}(f+gx)^{7/2}(cdf - aeg)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/((d + e\*x)^(3/2)\*(f + g\*x

[Out]  $(2*((a*e + c*d*x)*(d + e*x))^{(5/2)}*(-5*a*e*g + c*d*(7*f + 2*g*x)))/(35*(c*d*f - a*e*g)^2*(d + e*x)^{(5/2)}*(f + g*x)^{(7/2)})$



**Maple [A]** time = 0.012, size = 99, normalized size = 0.8

$$\frac{(2cdx + 2ae)(-2xcdg + 5aeg - 7cdf)}{35a^2e^2g^2 - 70acdefg + 35c^2d^2f^2} (cdex^2 + ae^2x + cd^2x + ade)^{\frac{3}{2}} (gx + f)^{-\frac{7}{2}} (ex + d)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(3/2)}/(e*x+d)^{(3/2)}/(g*x+f)^{(9/2)}, x)$

[Out]  $-2/35*(c*d*x+a*e)*(-2*c*d*g*x+5*a*e*g-7*c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^{(3/2)}/(g*x+f)^{(7/2)}/(a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2)/(e*x+d)^{(3/2)}$

**Maxima [A]** time = 1.20462, size = 707, normalized size = 5.48

$$35(c^2d^3f^6 - 2acd^2ef^5g + a^2de^2f^4g^2 + (c^2d^2ef^2g^4 - 2acde^2fg^5 + a^2e^3g^6)x^5 + ((4e^3fg^5 + de^2g^6)a^2 - 2(4de^2f^2g^4 + d^2efg^3))x^4 + (4e^3fg^5 + de^2g^6)a^2 - 2(4de^2f^2g^4 + d^2efg^3))x^3 + (4e^3fg^5 + de^2g^6)a^2 - 2(4de^2f^2g^4 + d^2efg^3))x^2 + (4e^3fg^5 + de^2g^6)a^2 - 2(4de^2f^2g^4 + d^2efg^3))x + (4e^3fg^5 + de^2g^6)a^2 - 2(4de^2f^2g^4 + d^2efg^3))$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^{(3/2)}/((e*x + d)^{(3/2)}*(g*x + f)^{(9/2)}), x)$

[Out]  $2/35*(2*c^3*d^3*g*x^3 + 7*a^2*c*d*e^2*f - 5*a^3*e^3*g + (7*c^3*d^3*f - a*c^2*d^2*e*g)*x^2 + 2*(7*a*c^2*d^2*e*f - 4*a^2*c*d*e^2*g)*x)*\text{sqrt}(c*d*x + a*e)*(e*x + d)*\text{sqrt}(g*x + f)/(c^2*d^3*f^6 - 2*a*c*d^2*e*f^5*g + a^2*d^2*e^2*f^4*g^2 + (c^2*d^2*e*f^2*g^4 - 2*a*c*d^2*e*f^2*g^4 + a^2*d^2*e^2*f^4*g^2)*x^5 + ((4*e^3*f*g^5 + d^2*e^2*g^6)*a^2 - 2*(4*d^2*e^2*f^2*g^4 + d^2*e^2*f^2*g^4)*a^2 + (4*d^2*e^2*f^2*g^4 + d^2*e^2*f^2*g^4)*a^2 - 2*(3*d^2*e^2*f^2*g^4 + 2*d^2*e^2*f^2*g^4)*a^2 + (3*d^2*e^2*f^2*g^4 + 2*d^2*e^2*f^2*g^4)*a^2 - 2*(2*d^2*e^2*f^2*g^4 + 3*d^2*e^2*f^2*g^4)*a^2 + (2*d^2*e^2*f^2*g^4 + 3*d^2*e^2*f^2*g^4)*a^2 - 2*(d^2*e^2*f^2*g^4 + 4*d^2*e^2*f^2*g^4)*a^2 + (d^2*e^2*f^2*g^4 + 4*d^2*e^2*f^2*g^4)*a^2)*x)$

**Fricas [A]** time = 0.296334, size = 710, normalized size = 5.5

$$35(c^2d^3f^6 - 2acd^2ef^5g + a^2de^2f^4g^2 + (c^2d^2ef^2g^4 - 2acde^2fg^5 + a^2e^3g^6)x^5 + (4c^2d^2ef^3g^3 + a^2de^2g^6 + (c^2d^3 - 8acde^2)f^2)x^4 + (4e^3fg^5 + de^2g^6)a^2 - 2(4de^2f^2g^4 + d^2efg^3))x^3 + (4e^3fg^5 + de^2g^6)a^2 - 2(4de^2f^2g^4 + d^2efg^3))x^2 + (4e^3fg^5 + de^2g^6)a^2 - 2(4de^2f^2g^4 + d^2efg^3))x + (4e^3fg^5 + de^2g^6)a^2 - 2(4de^2f^2g^4 + d^2efg^3))$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^{(3/2)}/((e*x + d)^{(3/2)}*(g*x + f)^{(9/2)}), x)$

[Out]  $2/35*(2*c^3*d^3*g*x^3 + 7*a^2*c*d*e^2*f - 5*a^3*e^3*g + (7*c^3*d^3*f - a*c^2*d^2*e*g)*x^2 + 2*(7*a*c^2*d^2*e*f - 4*a^2*c*d*e^2*g)*x)*\text{sqrt}(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*\text{sqrt}(e*x + d)*\text{sqrt}(g*x + f)/(c^2*d^3*f^6 - 2*a*c*d^2*e*f^5*g + a^2*d^2*e^2*f^4*g^2 + (c^2*d^2*e*f^2*g^4 - 2*a*c*d^2*e*f^2*g^4 + a^2*d^2*e^2*f^4*g^2)*x^5 + (4*c^2*d^2*e*f^2*g^4 + a^2*d^2*e^2*f^4*g^2 + (c^2*d^3 - 8*a*c*d^2*e^2)*f^2*g^4 - 2*(a*c*d^2*e - 2*a^2*e^3)*f^2*g^4)*x^4 + 2*(3*c^2*d^2*e*f^4*g^2 + 2*a^2*d^2*e^2*f^2*g^4 + 2*(c^2*d^3 - 3*a*c*d^2*e^2)*f^3*g^3 - (4*a*c*d^2*e - 3*a^2*e^3)*f^2*g^4)*x^3 + 2*(2*c^2*d^2*e*f^5*g + 3*a^2*d^2*e^2*f^2*g^4 + (3*c^2*d^3 - 4*a*c*d^2*e^2)*f^4*g^2 - 2*(3*a*c*d^2*e - a^2*e^3)*f^3*g^3)*x^2 + (c^2*d^2*e*f^6 + 4*a^2*d^2*e^2*f^3*g^3 + 2*(2*c^2*d^3 - a*c*d^2*e^2)*f^5*g - (8*a*c*d^2*e - a^2*e^3)*f^4*g^2)*x)$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/(e\*x+d)\*\*(3/2)/(g\*x+f)\*\*

[Out] Timed out

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}(gx + f)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f)

[Out] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f)^(9/2)), x)

$$3.749 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{11/2}} dx$$

**Optimal.** Leaf size=198

$$\frac{16c^2d^2(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{315(d+ex)^{5/2}(f+gx)^{5/2}(cdf - aeg)^3} + \frac{8cd(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{63(d+ex)^{5/2}(f+gx)^{7/2}(cdf - aeg)^2} + \frac{2(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{9(d+ex)^{5/2}(f+gx)^{9/2}(cdf - aeg)}$$

[Out]  $(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(9*(c*d*f - a*e*g)*(d + e*x)^{(5/2)}*(f + g*x)^{(9/2)}) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(63*(c*d*f - a*e*g)^2*(d + e*x)^{(5/2)}*(f + g*x)^{(7/2)}) + (16*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(315*(c*d*f - a*e*g)^3*(d + e*x)^{(5/2)}*(f + g*x)^{(5/2)})$

**Rubi [A]** time = 0.814011, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{16c^2d^2(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{315(d+ex)^{5/2}(f+gx)^{5/2}(cdf - aeg)^3} + \frac{8cd(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{63(d+ex)^{5/2}(f+gx)^{7/2}(cdf - aeg)^2} + \frac{2(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{9(d+ex)^{5/2}(f+gx)^{9/2}(cdf - aeg)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/((d + e*x)^{(3/2)}*(f + g*x)^{(11/2)})]$

[Out]  $(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(9*(c*d*f - a*e*g)*(d + e*x)^{(5/2)}*(f + g*x)^{(9/2)}) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(63*(c*d*f - a*e*g)^2*(d + e*x)^{(5/2)}*(f + g*x)^{(7/2)}) + (16*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(315*(c*d*f - a*e*g)^3*(d + e*x)^{(5/2)}*(f + g*x)^{(5/2)})$

**Rubi in Sympy [A]** time = 68.8613, size = 190, normalized size = 0.96

$$\frac{16c^2d^2(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{5}{2}}}{315(d+ex)^{\frac{5}{2}}(f+gx)^{\frac{5}{2}}(aeg - cdf)^3} + \frac{8cd(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{5}{2}}}{63(d+ex)^{\frac{5}{2}}(f+gx)^{\frac{7}{2}}(aeg - cdf)^2} - \frac{2(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{5}{2}}}{9(d+ex)^{\frac{5}{2}}(f+gx)^{\frac{9}{2}}(aeg - cdf)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x$

[Out]  $-16*c**2*d**2*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(5/2)/(315*(d + e*x)**(5/2)*(f + g*x)**(5/2)*(a*e*g - c*d*f)**3) + 8*c*d*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(5/2)/(63*(d + e*x)**(5/2)*(f + g*x)**(7/2)*(a*e*g - c*d*f)**2) - 2*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(5/2)/(9*(d + e*x)**(5/2)*(f + g*x)**(9/2)*(a*e*g - c*d*f))$

**Mathematica [A]** time = 0.284453, size = 105, normalized size = 0.53

$$\frac{2((d+ex)(ae+cdx))^{5/2} (35a^2e^2g^2 - 10acdeg(9f+2gx) + c^2d^2(63f^2 + 36fgx + 8g^2x^2))}{315(d+ex)^{5/2}(f+gx)^{9/2}(cdf - aeg)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/((d + e\*x)^(3/2)\*(f + g\*x

[Out] (2\*((a\*e + c\*d\*x)\*(d + e\*x))^(5/2)\*(35\*a^2\*e^2\*g^2 - 10\*a\*c\*d\*e\*g\*(9\*f + 2\*g\*x) + c^2\*d^2\*(63\*f^2 + 36\*f\*g\*x + 8\*g^2\*x^2)))/(315\*(c\*d\*f - a\*e\*g)^3\*(d + e\*x)^(5/2)\*(f + g\*x)^(9/2))

**Maple [A]** time = 0.014, size = 169, normalized size = 0.9

$$\frac{(2cdx + 2ae)(8c^2d^2g^2x^2 - 20acdeg^2x + 36c^2d^2fgx + 35a^2e^2g^2 - 90acdefg + 63c^2d^2f^2)}{315a^3e^3g^3 - 945a^2cde^2fg^2 + 945ac^2d^2ef^2g - 315c^3d^3f^3} (cdex^2 + ae^2x + cd^2x + ade)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)/(e\*x+d)^(3/2)/(g\*x+f)^(11/2), x)

[Out] -2/315\*(c\*d\*x+a\*e)\*(8\*c^2\*d^2\*g^2\*x^2-20\*a\*c\*d\*e\*g^2\*x+36\*c^2\*d^2\*f\*g\*x+35\*a^2\*e^2\*g^2-90\*a\*c\*d\*e\*f\*g+63\*c^2\*d^2\*f^2)\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(3/2)/(g\*x+f)^(9/2)/(a^3\*e^3\*g^3-3\*a^2\*c\*d\*e^2\*f\*g^2+3\*a\*c^2\*d^2\*e\*f^2\*g-c^3\*d^3\*f^3)/(e\*x+d)^(3/2)

**Maxima [A]** time = 1.40244, size = 1245, normalized size = 6.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f

[Out] 2/315\*(8\*c^4\*d^4\*g^2\*x^4 + 63\*a^2\*c^2\*d^2\*e^2\*f^2 - 90\*a^3\*c\*d\*e^3\*f\*g + 35\*a^4\*e^4\*g^2 + 4\*(9\*c^4\*d^4\*f\*g - a\*c^3\*d^3\*e\*g^2)\*x^3 + 3\*(21\*c^4\*d^4\*f^2 - 6\*a\*c^3\*d^3\*e\*f\*g + a^2\*c^2\*d^2\*e^2\*g^2)\*x^2 + 2\*(63\*a\*c^3\*d^3\*e\*f^2 - 72\*a^2\*c^2\*d^2\*e^2\*f\*g + 25\*a^3\*c\*d\*e^3\*g^2)\*x)\*sqrt(c\*d\*x + a\*e)\*(e\*x + d)\*sqrt(g\*x + f)/(c^3\*d^4\*f^8 - 3\*a\*c^2\*d^3\*e\*f^7\*g + 3\*a^2\*c\*d^2\*e^2\*f^6\*g^2 - a^3\*d\*e^3\*f^5\*g^3 + (c^3\*d^3\*e\*f^3\*g^5 - 3\*a\*c^2\*d^2\*e^2\*f^2\*g^6 + 3\*a^2\*c\*d\*e^3\*f\*g^7 - a^3\*e^4\*g^8)\*x^6 - ((5\*e^4\*f^7\*g^7 + d\*e^3\*g^8)\*a^3 - 3\*(5\*d^2\*e^3\*f^2\*g^6 + d^2\*e^2\*f^3\*g^7)\*a^2\*c + 3\*(5\*d^2\*e^2\*f^3\*g^5 + d^3\*e\*f^2\*g^6)\*a\*c^2 - (5\*d^3\*e\*f^4\*g^4 + d^4\*f^3\*g^5)\*c^3)\*x^5 - 5\*((2\*e^4\*f^2\*g^6 + d^2\*e^3\*f^3\*g^7)\*a^3 - 3\*(2\*d^2\*e^3\*f^3\*g^5 + d^2\*e^2\*f^2\*g^6)\*a^2\*c + 3\*(2\*d^2\*e^2\*f^4\*g^4 + d^3\*e\*f^3\*g^5)\*a\*c^2 - (2\*d^3\*e\*f^5\*g^3 + d^4\*f^4\*g^4)\*c^3)\*x^4 - 10\*((e^4\*f^3\*g^5 + d^3\*e^3\*f^2\*g^6)\*a^3 - 3\*(d^2\*e^3\*f^4\*g^4 + d^2\*e^2\*f^3\*g^5)\*a^2\*c + 3\*(d^2\*e^2\*f^5\*g^3 + d^3\*e\*f^4\*g^4)\*a\*c^2 - (d^3\*e\*f^6\*g^2 + d^4\*f^5\*g^3)\*c^3)\*x^3 - 5\*((e^4\*f^4\*g^4 + 2\*d^2\*e^3\*f^3\*g^5)\*a^3 - 3\*(d^3\*e^3\*f^5\*g^3 + 2\*d^2\*e^2\*f^4\*g^4)\*a^2\*c + 3\*(d^2\*e^2\*f^6\*g^2 + 2\*d^3\*e\*f^5\*g^3)\*a\*c^2 - (d^3\*e\*f^7\*g + 2\*d^4\*f^6\*g^2)\*c^3)\*x^2 - ((e^4\*f^5\*g^3 + 5\*d^2\*e^3\*f^4\*g^4)\*a^3 - 3\*(d^2\*e^3\*f^6\*g^2 + 5\*d^2\*e^2\*f^5\*g^3)\*a^2\*c + 3\*(d^2\*e^2\*f^7\*g + 5\*d^3\*e\*f^6\*g^2)\*a\*c^2 - (d^3\*e\*f^8 + 5\*d^4\*f^7\*g)\*c^3)\*x)

**Fricas [A]** time = 0.306683, size = 1239, normalized size = 6.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^{(3/2)} / ((e*x + d)^{(3/2)} * (g*x + f)^{(3/2)})$

[Out]  $\frac{2}{315} * (8 * c^4 * d^4 * g^2 * x^4 + 63 * a^2 * c^2 * d^2 * e^2 * f^2 - 90 * a^3 * c * d * e^3 * f * g + 35 * a^4 * e^4 * g^2 + 4 * (9 * c^4 * d^4 * f * g - a * c^3 * d^3 * e * g^2) * x^3 + 3 * (21 * c^4 * d^4 * f^2 - 6 * a * c^3 * d^3 * e * f * g + a^2 * c^2 * d^2 * e^2 * g^2) * x^2 + 2 * (63 * a * c^3 * d^3 * e * f^2 - 72 * a^2 * c^2 * d^2 * e^2 * f * g + 25 * a^3 * c * d * e^3 * g^2) * x) * \sqrt{c * d * e * x^2 + a * d * e + (c * d^2 + a * e^2) * x} * \sqrt{e * x + d} * \sqrt{g * x + f} / (c^3 * d^4 * f^8 - 3 * a * c^2 * d^3 * e * f^7 * g + 3 * a^2 * c * d^2 * e^2 * f^6 * g^2 - a^3 * d * e^3 * f^5 * g^3 + (c^3 * d^3 * e * f^3 * g^5 - 3 * a * c^2 * d^2 * e^2 * f^2 * g^6 + 3 * a^2 * c * d * e^3 * f * g^7 - a^3 * e^4 * g^8) * x^6 + (5 * c^3 * d^3 * e * f^4 * g^4 - a^3 * d * e^3 * g^8 + (c^3 * d^4 - 15 * a * c^2 * d^2 * e^2) * f^3 * g^5 - 3 * (a * c^2 * d^3 * e - 5 * a^2 * c * d * e^3) * f^2 * g^6 + (3 * a^2 * c * d^2 * e^2 - 5 * a^3 * e^4) * f * g^7) * x^5 + 5 * (2 * c^3 * d^3 * e * f^5 * g^3 - a^3 * d * e^3 * f * g^7 + (c^3 * d^4 - 6 * a * c^2 * d^2 * e^2) * f^4 * g^4 - 3 * (a * c^2 * d^3 * e - 2 * a^2 * c * d * e^3) * f^3 * g^5 + (3 * a^2 * c * d^2 * e^2 - 2 * a^3 * e^4) * f^2 * g^6) * x^4 + 10 * (c^3 * d^3 * e * f^6 * g^2 - a^3 * d * e^3 * f^2 * g^6 + (c^3 * d^4 - 3 * a * c^2 * d^2 * e^2) * f^5 * g^3 - 3 * (a * c^2 * d^3 * e - a^2 * c * d * e^3) * f^4 * g^4 + (3 * a^2 * c * d^2 * e^2 - a^3 * e^4) * f^3 * g^5) * x^3 + 5 * (c^3 * d^3 * e * f^7 * g - 2 * a^3 * d * e^3 * f^3 * g^5 + (2 * c^3 * d^4 - 3 * a * c^2 * d^2 * e^2) * f^6 * g^2 - 3 * (2 * a * c^2 * d^3 * e - a^2 * c * d * e^3) * f^5 * g^3 + (6 * a^2 * c * d^2 * e^2 - a^3 * e^4) * f^4 * g^4) * x^2 + (c^3 * d^3 * e * f^8 - 5 * a^3 * d * e^3 * f^4 * g^4 + (5 * c^3 * d^4 - 3 * a * c^2 * d^2 * e^2) * f^7 * g - 3 * (5 * a * c^2 * d^3 * e - a^2 * c * d * e^3) * f^6 * g^2 + (15 * a^2 * c * d^2 * e^2 - a^3 * e^4) * f^5 * g^3) * x)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/((e*x+d)**(3/2)/(g*x+f)**(3/2))$

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^{(3/2)} / ((e*x + d)^{(3/2)} * (g*x + f)^{(3/2)})$

[Out] Exception raised: NotImplementedError

$$3.750 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}(f+gx)^{13/2}} dx$$

**Optimal.** Leaf size=267

$$\frac{32c^3d^3(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{1155(d+ex)^{5/2}(f+gx)^{5/2}(cdf - aeg)^4} + \frac{16c^2d^2(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{231(d+ex)^{5/2}(f+gx)^{7/2}(cdf - aeg)^3} + \frac{4cd(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{33(d+ex)^{5/2}(f+gx)^{9/2}(cdf - aeg)^2} + \frac{2(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{11(d+ex)^{5/2}(f+gx)^{11/2}(cdf - aeg)}$$

[Out]  $(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(11*(c*d*f - a*e*g)*(d + e*x)^{(5/2)}*(f + g*x)^{(11/2)}) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(33*(c*d*f - a*e*g)^2*(d + e*x)^{(5/2)}*(f + g*x)^{(9/2)}) + (16*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(231*(c*d*f - a*e*g)^3*(d + e*x)^{(5/2)}*(f + g*x)^{(7/2)}) + (32*c^3*d^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(1155*(c*d*f - a*e*g)^4*(d + e*x)^{(5/2)}*(f + g*x)^{(5/2)})$

**Rubi [A]** time = 1.11342, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{32c^3d^3(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{1155(d+ex)^{5/2}(f+gx)^{5/2}(cdf - aeg)^4} + \frac{16c^2d^2(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{231(d+ex)^{5/2}(f+gx)^{7/2}(cdf - aeg)^3} + \frac{4cd(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{33(d+ex)^{5/2}(f+gx)^{9/2}(cdf - aeg)^2} + \frac{2(x(ae^2 + cd^2) + ade + cdex^2)^{5/2}}{11(d+ex)^{5/2}(f+gx)^{11/2}(cdf - aeg)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)}/((d + e*x)^{(3/2)}*(f + g*x)^{(13/2)})]$

[Out]  $(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(11*(c*d*f - a*e*g)*(d + e*x)^{(5/2)}*(f + g*x)^{(11/2)}) + (4*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(33*(c*d*f - a*e*g)^2*(d + e*x)^{(5/2)}*(f + g*x)^{(9/2)}) + (16*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(231*(c*d*f - a*e*g)^3*(d + e*x)^{(5/2)}*(f + g*x)^{(7/2)}) + (32*c^3*d^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(1155*(c*d*f - a*e*g)^4*(d + e*x)^{(5/2)}*(f + g*x)^{(5/2)})$

**Rubi in Sympy [A]** time = 98.0574, size = 258, normalized size = 0.97

$$\frac{32c^3d^3(ade + cdex^2 + x(ae^2 + cd^2))^{5/2}}{1155(d+ex)^{5/2}(f+gx)^{5/2}(aeg - cdf)^4} - \frac{16c^2d^2(ade + cdex^2 + x(ae^2 + cd^2))^{5/2}}{231(d+ex)^{5/2}(f+gx)^{7/2}(aeg - cdf)^3} + \frac{4cd(ade + cdex^2 + x(ae^2 + cd^2))^{5/2}}{33(d+ex)^{5/2}(f+gx)^{9/2}(aeg - cdf)^2} - \frac{2(ade + cdex^2 + x(ae^2 + cd^2))^{5/2}}{11(d+ex)^{5/2}(f+gx)^{11/2}(aeg - cdf)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x$

[Out]  $32*c**3*d**3*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))** (5/2)/(1155*(d + e*x)** (5/2)*(f + g*x)** (5/2)*(a*e*g - c*d*f)**4) - 16*c**2*d**2*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))** (5/2)/(231*(d + e*x)** (5/2)*(f + g*x)** (7/2)*(a*e*g - c*d*f)**3) + 4*c*d*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))** (5/2)/(33*(d + e*x)** (5/2)*(f + g*x)** (9/2)*(a*e*g - c*d*f)**2) - 2*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))** (5/2)/(11*(d + e*x)** (5/2)*(f + g*x)** (11/2))$

\*(a\*e\*g - c\*d\*f))

**Mathematica [A]** time = 0.38676, size = 152, normalized size = 0.57

$$\frac{2((d+ex)(ae+cdx))^{5/2}(-105a^3e^3g^3+35a^2cde^2g^2(11f+2gx)-5ac^2d^2eg(99f^2+44fgx+8g^2x^2)+c^3d^3(231f^3+198f^2g))}{1155(d+ex)^{5/2}(f+gx)^{11/2}(cdf-aeg)^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)/((d + e\*x)^(3/2)\*(f + g\*x

[Out] (2\*((a\*e + c\*d\*x)\*(d + e\*x))^(5/2)\*(-105\*a^3\*e^3\*g^3 + 35\*a^2\*c\*d\*e^2\*g^2\*(11\*f + 2\*g\*x) - 5\*a\*c^2\*d^2\*e\*g\*(99\*f^2 + 44\*f\*g\*x + 8\*g^2\*x^2) + c^3\*d^3\*(231\*f^3 + 198\*f^2\*g\*x + 88\*f\*g^2\*x^2 + 16\*g^3\*x^3)))/(1155\*(c\*d\*f - a\*e\*g)^4\*(d + e\*x)^(5/2)\*(f + g\*x)^(11/2))

**Maple [A]** time = 0.017, size = 260, normalized size = 1.

$$\frac{(2cdx+2ae)(-16c^3d^3g^3x^3+40ac^2d^2eg^3x^2-88c^3d^3fg^2x^2-70a^2cde^2g^3x+220ac^2d^2efg^2x-198c^3d^3f^2gx+105a^3e^3g^3)}{1155g^4e^4a^4-4620cdg^3fe^3a^3+6930c^2d^2g^2f^2e^2a^2-4620c^3d^3gf^3ea+1155a^4e^4g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)/(e\*x+d)^(3/2)/(g\*x+f)^(13/2), x)

[Out] -2/1155\*(c\*d\*x+a\*e)\*(-16\*c^3\*d^3\*g^3\*x^3+40\*a\*c^2\*d^2\*e\*g^3\*x^2-88\*c^3\*d^3\*f\*g^2\*x^2-70\*a^2\*c\*d\*e^2\*g^3\*x+220\*a\*c^2\*d^2\*e\*f\*g^2\*x-198\*c^3\*d^3\*f^2\*g\*x+105\*a^3\*e^3\*g^3-385\*a^2\*c\*d\*e^2\*f\*g^2+495\*a\*c^2\*d^2\*e\*f^2\*g-231\*c^3\*d^3\*f^3)\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(3/2)/(g\*x+f)^(11/2)/(a^4\*e^4\*g^4-4\*a^3\*c\*d\*e^3\*f\*g^3+6\*a^2\*c^2\*d^2\*e^2\*f^2\*g^2-4\*a\*c^3\*d^3\*e\*f^3\*g+c^4\*d^4\*f^4)/(e\*x+d)^(3/2)

**Maxima [A]** time = 1.68852, size = 1928, normalized size = 7.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f

[Out] 2/1155\*(16\*c^5\*d^5\*g^3\*x^5 + 231\*a^2\*c^3\*d^3\*e^2\*f^3 - 495\*a^3\*c^4\*d^2\*e^3\*f^2\*g + 385\*a^4\*c\*d\*e^4\*f\*g^2 - 105\*a^5\*e^5\*g^3 + 8\*(11\*c^5\*d^5\*f\*g^2 - a\*c^4\*d^4\*e\*g^3)\*x^4 + 2\*(99\*c^5\*d^5\*f^2\*g - 22\*a\*c^4\*d^4\*e\*f\*g^2 + 3\*a^2\*c^3\*d^3\*e^2\*g^3)\*x^3 + (231\*c^5\*d^5\*f^3 - 99\*a\*c^4\*d^4\*e\*f^2\*g + 33\*a^2\*c^3\*d^3\*e^2\*f\*g^2 - 5\*a^3\*c^2\*d^2\*e^3\*g^3)\*x^2 + 2\*(231\*a\*c^4\*d^4\*e\*f^3 - 396\*a^2\*c^3\*d^3\*e^2\*f^2\*g + 275\*a^3\*c^2\*d^2\*e^3\*f\*g^2 - 70\*a^4\*c\*d\*e^4\*g^3)\*x)\*sqrt(c\*d\*x + a\*e)\*(e\*x + d)\*sqrt(g\*x + f)/(c^4\*d^5\*f^10 - 4\*a\*c^3\*d^4\*e\*f^9\*g + 6\*a^2\*c^2\*d^3\*e^2\*f^8\*g^2 - 4\*a^3\*c\*d^2\*e^3\*f^7\*g^3 + a^4\*d^2\*e^4\*f^6\*g^4 + (c^4\*d^4\*e\*f^4\*g^6 - 4\*a\*c^3\*d^3\*e^2\*f^3\*g^7 + 6\*a^2\*c^2\*d^2\*e^3\*f^2\*g^8 - 4\*a^3\*c\*d\*e^4\*f\*g^9 + a^4\*e^5\*g^10)\*x^7 + ((6\*e^5\*f\*g^9 + d\*e^4\*g^10)\*a^4 - 4\*(6\*d^2\*e^4\*f^2\*g^8 + d^2\*e^3\*f\*g^9)\*a^3\*c + 6\*(6\*d^2\*e^3\*f^3\*g^7 + d^3\*e^2\*f^2\*g^8)\*a^2\*c^2 - 4\*(6\*d^3\*e^2\*f^4\*g^6 + d^4\*e\*f^3\*g^7)\*a\*c^3 + (6\*d^4\*e\*f^5\*g^5 + d^5\*f^4\*g^6)\*c^4)\*x^6 + 3\*((5\*e^5\*f^2\*g^8 + 2\*d^2\*e^4\*f\*g^9)\*a^4 - 4\*(5\*d^2\*e^4\*f^3\*g^7 + 2\*d^2\*e^3\*f^2\*g^8)\*a^3\*c + 6\*(5\*d^2\*e^3\*f^4\*g^6 + 2\*d^3\*e^2\*f^3\*g^7)\*a^2\*c^2 - 4\*(5\*d^3\*e^2\*f^5\*g^5 + 2\*d^4\*e

$$\begin{aligned}
& *f^4*g^6)*a^3c^3 + (5*d^4*e*f^6*g^4 + 2*d^5*f^5*g^5)*c^4)*x^5 + 5* \\
& ((4*e^5*f^3*g^7 + 3*d^4*e^4*f^2*g^8)*a^4 - 4*(4*d^4*e^4*f^4*g^6 + 3*d \\
& ^2*e^3*f^3*g^7)*a^3*c + 6*(4*d^2*e^3*f^5*g^5 + 3*d^3*e^2*f^4*g^6) \\
& *a^2*c^2 - 4*(4*d^3*e^2*f^6*g^4 + 3*d^4*e*f^5*g^5)*a^3c^3 + (4*d^4 \\
& *e*f^7*g^3 + 3*d^5*f^6*g^4)*c^4)*x^4 + 5*((3*e^5*f^4*g^6 + 4*d^4 \\
& *e^4*f^3*g^7)*a^4 - 4*(3*d^4*e^4*f^5*g^5 + 4*d^2*e^3*f^4*g^6)*a^3*c + \\
& 6*(3*d^2*e^3*f^6*g^4 + 4*d^3*e^2*f^5*g^5)*a^2*c^2 - 4*(3*d^3*e^2* \\
& f^7*g^3 + 4*d^4*e*f^6*g^4)*a^3c^3 + (3*d^4*e*f^8*g^2 + 4*d^5*f^7*g \\
& ^3)*c^4)*x^3 + 3*((2*e^5*f^5*g^5 + 5*d^4*e^4*f^4*g^6)*a^4 - 4*(2*d^ \\
& e^4*f^6*g^4 + 5*d^2*e^3*f^5*g^5)*a^3*c + 6*(2*d^2*e^3*f^7*g^3 + 5 \\
& *d^3*e^2*f^6*g^4)*a^2*c^2 - 4*(2*d^3*e^2*f^8*g^2 + 5*d^4*e*f^7*g^ \\
& 3)*a^3c^3 + (2*d^4*e*f^9*g + 5*d^5*f^8*g^2)*c^4)*x^2 + ((e^5*f^6*g \\
& ^4 + 6*d^4*e^4*f^5*g^5)*a^4 - 4*(d^4*e^4*f^7*g^3 + 6*d^2*e^3*f^6*g^4) \\
& *a^3c^3 + 6*(d^2*e^3*f^8*g^2 + 6*d^3*e^2*f^7*g^3)*a^2*c^2 - 4*(d^3 \\
& *e^2*f^9*g + 6*d^4*e*f^8*g^2)*a^3c^3 + (d^4*e*f^10 + 6*d^5*f^9*g) \\
& *c^4)*x)
\end{aligned}$$

**Ericas [A]** time = 0.317606, size = 1917, normalized size = 7.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)/((e\*x + d)^(3/2)\*(g\*x + f

[Out] 2/1155\*(16\*c^5\*d^5\*g^3\*x^5 + 231\*a^2\*c^3\*d^3\*e^2\*f^3 - 495\*a^3\*c^4  
2\*d^2\*e^3\*f^2\*g + 385\*a^4\*c\*d^4\*e^4\*f^2\*g^2 - 105\*a^5\*e^5\*g^3 + 8\*(11  
\*c^5\*d^5\*f^2\*g^2 - a\*c^4\*d^4\*e^3\*g^3)\*x^4 + 2\*(99\*c^5\*d^5\*f^2\*g - 22\*  
a\*c^4\*d^4\*e\*f^2\*g^2 + 3\*a^2\*c^3\*d^3\*e^2\*g^3)\*x^3 + (231\*c^5\*d^5\*f^3  
- 99\*a\*c^4\*d^4\*e\*f^2\*g + 33\*a^2\*c^3\*d^3\*e^2\*f^2\*g^2 - 5\*a^3\*c^2\*d^2  
e^3\*g^3)\*x^2 + 2\*(231\*a\*c^4\*d^4\*e\*f^3 - 396\*a^2\*c^3\*d^3\*e^2\*f^2  
\*g + 275\*a^3\*c^2\*d^2\*e^3\*f^2\*g^2 - 70\*a^4\*c\*d^4\*e^4\*g^3)\*x)\*sqrt(c\*d\*  
e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f)/(c  
^4\*d^5\*f^10 - 4\*a\*c^3\*d^4\*e\*f^9\*g + 6\*a^2\*c^2\*d^3\*e^2\*f^8\*g^2 - 4  
\*a^3\*c\*d^2\*e^3\*f^7\*g^3 + a^4\*d^4\*e^4\*f^6\*g^4 + (c^4\*d^4\*e\*f^4\*g^6 -  
4\*a\*c^3\*d^3\*e^2\*f^3\*g^7 + 6\*a^2\*c^2\*d^2\*e^3\*f^2\*g^8 - 4\*a^3\*c\*d^4  
e^4\*f^2\*g^9 + a^4\*e^5\*g^10)\*x^7 + (6\*c^4\*d^4\*e\*f^5\*g^5 + a^4\*d^4\*e^4\*  
g^10 + (c^4\*d^5 - 24\*a\*c^3\*d^3\*e^2)\*f^4\*g^6 - 4\*(a\*c^3\*d^4\*e - 9\*  
a^2\*c^2\*d^2\*e^3)\*f^3\*g^7 + 6\*(a^2\*c^2\*d^3\*e^2 - 4\*a^3\*c\*d^4\*e^4)\*f^2  
g^8 - 2\*(2\*a^3\*c\*d^2\*e^3 - 3\*a^4\*e^5)\*f^2\*g^9)\*x^6 + 3\*(5\*c^4\*d^4  
\*e\*f^6\*g^4 + 2\*a^4\*d^4\*e^4\*f^2\*g^9 + 2\*(c^4\*d^5 - 10\*a\*c^3\*d^3\*e^2)\*f  
^5\*g^5 - 2\*(4\*a\*c^3\*d^4\*e - 15\*a^2\*c^2\*d^2\*e^3)\*f^4\*g^6 + 4\*(3\*a^2  
c^2\*d^3\*e^2 - 5\*a^3\*c\*d^4\*e^4)\*f^3\*g^7 - (8\*a^3\*c\*d^2\*e^3 - 5\*a^4  
e^5)\*f^2\*g^8)\*x^5 + 5\*(4\*c^4\*d^4\*e\*f^7\*g^3 + 3\*a^4\*d^4\*e^4\*f^2\*g^8  
+ (3\*c^4\*d^5 - 16\*a\*c^3\*d^3\*e^2)\*f^6\*g^4 - 12\*(a\*c^3\*d^4\*e - 2\*a  
^2\*c^2\*d^2\*e^3)\*f^5\*g^5 + 2\*(9\*a^2\*c^2\*d^3\*e^2 - 8\*a^3\*c\*d^4\*e^4)\*f  
^4\*g^6 - 4\*(3\*a^3\*c\*d^2\*e^3 - a^4\*e^5)\*f^3\*g^7)\*x^4 + 5\*(3\*c^4\*d^4  
e\*f^8\*g^2 + 4\*a^4\*d^4\*e^4\*f^3\*g^7 + 4\*(c^4\*d^5 - 3\*a\*c^3\*d^3\*e^2)  
\*f^7\*g^3 - 2\*(8\*a\*c^3\*d^4\*e - 9\*a^2\*c^2\*d^2\*e^3)\*f^6\*g^4 + 12\*(2\*  
a^2\*c^2\*d^3\*e^2 - a^3\*c\*d^4\*e^4)\*f^5\*g^5 - (16\*a^3\*c\*d^2\*e^3 - 3\*a^4  
e^5)\*f^4\*g^6)\*x^3 + 3\*(2\*c^4\*d^4\*e\*f^9\*g + 5\*a^4\*d^4\*e^4\*f^4\*g^6  
+ (5\*c^4\*d^5 - 8\*a\*c^3\*d^3\*e^2)\*f^8\*g^2 - 4\*(5\*a\*c^3\*d^4\*e - 3\*a^2  
c^2\*d^2\*e^3)\*f^7\*g^3 + 2\*(15\*a^2\*c^2\*d^3\*e^2 - 4\*a^3\*c\*d^4\*e^4)\*f  
^6\*g^4 - 2\*(10\*a^3\*c\*d^2\*e^3 - a^4\*e^5)\*f^5\*g^5)\*x^2 + (c^4\*d^4\*e  
\*f^10 + 6\*a^4\*d^4\*e^4\*f^5\*g^5 + 2\*(3\*c^4\*d^5 - 2\*a\*c^3\*d^3\*e^2)\*f^9  
\*g - 6\*(4\*a\*c^3\*d^4\*e - a^2\*c^2\*d^2\*e^3)\*f^8\*g^2 + 4\*(9\*a^2\*c^2\*d  
^3\*e^2 - a^3\*c\*d^4\*e^4)\*f^7\*g^3 - (24\*a^3\*c\*d^2\*e^3 - a^4\*e^5)\*f^6\*  
g^4)\*x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.



[In] `integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(3/2)/(e*x+d)**(3/2)/(g*x+f)**`

[Out] Timed out

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**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)/((e*x + d)^(3/2)*(g*x + f`

[Out] Exception raised: NotImplementedError

$$3.751 \quad \int \frac{(f+gx)^{3/2}(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}} dx$$

**Optimal.** Leaf size=448

$$\begin{aligned} & \frac{3\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^5 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{128c^{5/2}d^{5/2}g^{7/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \\ & - \frac{3\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^4}{128c^2d^2g^3\sqrt{d+ex}} \\ & - \frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^3}{64cdg^3\sqrt{d+ex}} \\ & + \frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2}{16g^3\sqrt{d+ex}} \\ & - \frac{(f+gx)^{5/2}(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)}{8g^2(d+ex)^{3/2}} \\ & + \frac{(f+gx)^{5/2}(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{5g(d+ex)^{5/2}} \end{aligned}$$

[Out]  $(-3*(c*d*f - a*e*g)^4*\text{Sqrt}[f + g*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(128*c^2*d^2*g^3*\text{Sqrt}[d + e*x]) - ((c*d*f - a*e*g)^3*(f + g*x)^{(3/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*c*d*g^3*\text{Sqrt}[d + e*x]) + ((c*d*f - a*e*g)^2*(f + g*x)^{(5/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(16*g^3*\text{Sqrt}[d + e*x]) - ((c*d*f - a*e*g)*(f + g*x)^{(5/2)}*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(8*g^2*(d + e*x)^{(3/2)}) + ((f + g*x)^{(5/2)}*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(5*g*(d + e*x)^{(5/2)}) - (3*(c*d*f - a*e*g)^5*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x])]/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x]))/(128*c^{(5/2)}*d^{(5/2)}*g^{(7/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi [A]** time = 2.33505, antiderivative size = 448, normalized size of antiderivative = 1., number of rules used = 8, number of rules used = 5, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$

$$\begin{aligned} & \frac{3\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^5 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{128c^{5/2}d^{5/2}g^{7/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \\ & - \frac{3\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^4}{128c^2d^2g^3\sqrt{d+ex}} \\ & - \frac{(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^3}{64cdg^3\sqrt{d+ex}} \\ & + \frac{(f+gx)^{5/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2}{16g^3\sqrt{d+ex}} \\ & - \frac{(f+gx)^{5/2}(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)}{8g^2(d+ex)^{3/2}} \\ & + \frac{(f+gx)^{5/2}(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{5g(d+ex)^{5/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(f + g*x)^{(3/2)}*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}]/(d + e*x)^{(5/2)}$

[Out]  $(-3*(c*d*f - a*e*g)^4*\text{Sqrt}[f + g*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(128*c^2*d^2*g^3*\text{Sqrt}[d + e*x]) - ((c*d*f - a*e*g)^3*(f + g*x)^{(3/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*c*d*g^3*\text{Sqrt}[d + e*x]) + ((c*d*f - a*e*g)^2*(f + g*x)^{(5/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(16*g^3*\text{Sqrt}[d + e*x]) - ((c*d*f - a*e*g)*(f + g*x)^{(5/2)}*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(8*g^2*(d + e*x)^{(3/2)}) + ((f + g*x)^{(5/2)}*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(5*g*(d + e*x)^{(5/2)}) - (3*(c*d*f - a*e*g)^5*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x])]/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x]))/(128*c^{(5/2)}*d^{(5/2)}*g^{(7/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

$$(64*c*d*g^3*\text{Sqrt}[d + e*x]) + ((c*d*f - a*e*g)^2*(f + g*x)^{(5/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(16*g^3*\text{Sqrt}[d + e*x]) - ((c*d*f - a*e*g)*(f + g*x)^{(5/2)}*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(8*g^2*(d + e*x)^{(3/2)}) + ((f + g*x)^{(5/2)}*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(5*g*(d + e*x)^{(5/2)}) - (3*(c*d*f - a*e*g)^5*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x])/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x])])/(128*c^{(5/2)}*d^{(5/2)}*g^{(7/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*(3/2)\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)/(e\*x

[Out] Timed out

**Mathematica [A]** time = 1.11265, size = 315, normalized size = 0.7

$$\frac{((d + ex)(ae + cdx))^{5/2} \left( \frac{2\sqrt{f+gx}(-15a^4e^4g^4+10a^3cde^3g^3(7f+gx)+2a^2c^2d^2e^2g^2(64f^2+233fgx+124g^2x^2))+2ac^3d^3eg(-35f^3+23f^2gx+256fg^2x^2+168g^3x^3)}{5c^2d^2g^3(ae+cdx)^2} \right)}{256(d + ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((f + g\*x)^(3/2)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2))/(d + e\*x

[Out] (((a\*e + c\*d\*x)\*(d + e\*x))^(5/2))\*((2\*Sqrt[f + g\*x]\*(-15\*a^4\*e^4\*g^4 + 10\*a^3\*c\*d\*e^3\*g^3\*(7\*f + g\*x) + 2\*a^2\*c^2\*d^2\*e^2\*g^2\*(64\*f^2 + 233\*f\*g\*x + 124\*g^2\*x^2) + 2\*a\*c^3\*d^3\*e\*g\*(-35\*f^3 + 23\*f^2\*g\*x + 256\*f\*g^2\*x^2 + 168\*g^3\*x^3) + c^4\*d^4\*(15\*f^4 - 10\*f^3\*g\*x + 8\*f^2\*g^2\*x^2 + 176\*f\*g^3\*x^3 + 128\*g^4\*x^4)))/(5\*c^2\*d^2\*g^3\*(a\*e + c\*d\*x)^2) - (3\*(c\*d\*f - a\*e\*g)^5\*Log[a\*e\*g + 2\*Sqrt[c]\*Sqrt[d]\*Sqrt[g]\*Sqrt[a\*e + c\*d\*x]\*Sqrt[f + g\*x] + c\*d\*(f + 2\*g\*x)))/(c^(5/2)\*d^(5/2)\*g^(7/2)\*(a\*e + c\*d\*x)^(5/2)))/(256\*(d + e\*x)^(5/2))

**Maple [B]** time = 0.035, size = 1191, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^(3/2)\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2)/(e\*x+d)^(5/2), x)

[Out] 1/1280\*(g\*x+f)^(1/2)\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)\*(256\*x^4\*c^4\*d^4\*g^4\*(c\*d\*g\*x^2+a\*e\*g\*x+c\*d\*f\*x+a\*e\*f)^(1/2)\*(d\*g\*c)^(1/2)+672\*x^3\*a\*c^3\*d^3\*e\*g^4\*(c\*d\*g\*x^2+a\*e\*g\*x+c\*d\*f\*x+a\*e\*f)^(1/2)\*(d\*g\*c)^(1/2)+352\*x^3\*c^4\*d^4\*f\*g^3\*(c\*d\*g\*x^2+a\*e\*g\*x+c\*d\*f\*x+a\*e\*f)^(1/2)\*(d\*g\*c)^(1/2)+15\*g^5\*ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*(c\*d\*g\*x^2+a\*e\*g\*x+c\*d\*f\*x+a\*e\*f)^(1/2)\*(d\*g\*c)^(1/2)))/(d\*g\*c)^(1/2))\*a^5\*e^5-75\*g^4\*ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*(c\*d\*g\*x^2+a\*e\*g\*x+c\*d\*f\*x+a\*e\*f)^(1/2)\*(d\*g\*c)^(1/2)))/(d\*g\*c)^(1/2))\*a^4\*e^4\*f\*d\*c+150\*ln(1/2\*(2\*x\*c\*d\*g+a\*e\*g+c\*d\*f+2\*(c\*d\*g\*x^2+a\*e\*g\*x

$$\frac{c^2 d^2 f^2 x + a^2 e^2 f^2}{(d^2 g^2 c)^{1/2}} \frac{(d^2 g^2 c)^{1/2}}{(d^2 g^2 c)^{1/2}} a^3 e^3 f^2 d^2 c^2 g^3 - 150 d^3 c^3 \ln\left(\frac{1}{2} (2^2 x^2 c^2 d^2 g + a^2 e^2 g + c^2 d^2 f + 2^2 (c^2 d^2 g^2 x^2 + a^2 e^2 g^2 x + c^2 d^2 f^2 x + a^2 e^2 f^2)^{1/2} (d^2 g^2 c)^{1/2})\right) \frac{(d^2 g^2 c)^{1/2}}{(d^2 g^2 c)^{1/2}} a^2 e^2 f^3 g^2 + 75 d^4 c^4 \ln\left(\frac{1}{2} (2^2 x^2 c^2 d^2 g + a^2 e^2 g + c^2 d^2 f + 2^2 (c^2 d^2 g^2 x^2 + a^2 e^2 g^2 x + c^2 d^2 f^2 x + a^2 e^2 f^2)^{1/2} (d^2 g^2 c)^{1/2})\right) \frac{(d^2 g^2 c)^{1/2}}{(d^2 g^2 c)^{1/2}} f^4 a^2 e^2 g - 15 d^5 c^5 \ln\left(\frac{1}{2} (2^2 x^2 c^2 d^2 g + a^2 e^2 g + c^2 d^2 f + 2^2 (c^2 d^2 g^2 x^2 + a^2 e^2 g^2 x + c^2 d^2 f^2 x + a^2 e^2 f^2)^{1/2} (d^2 g^2 c)^{1/2})\right) \frac{(d^2 g^2 c)^{1/2}}{(d^2 g^2 c)^{1/2}} f^5 + 496 x^2 a^2 c^2 d^2 e^2 g^4 (c^2 d^2 g^2 x^2 + a^2 e^2 g^2 x + c^2 d^2 f^2 x + a^2 e^2 f^2)^{1/2} (d^2 g^2 c)^{1/2} + 1024 x^2 a^2 c^3 d^3 e^2 f^2 g^3 (c^2 d^2 g^2 x^2 + a^2 e^2 g^2 x + c^2 d^2 f^2 x + a^2 e^2 f^2)^{1/2} (d^2 g^2 c)^{1/2} + 16 x^2 c^4 d^4 f^2 g^2 (c^2 d^2 g^2 x^2 + a^2 e^2 g^2 x + c^2 d^2 f^2 x + a^2 e^2 f^2)^{1/2} (d^2 g^2 c)^{1/2} + 20 (c^2 d^2 g^2 x^2 + a^2 e^2 g^2 x + c^2 d^2 f^2 x + a^2 e^2 f^2)^{1/2} x^2 a^3 e^3 g^4 d^2 c (d^2 g^2 c)^{1/2} + 932 (c^2 d^2 g^2 x^2 + a^2 e^2 g^2 x + c^2 d^2 f^2 x + a^2 e^2 f^2)^{1/2} x^2 a^2 e^2 f^2 d^2 c^2 (d^2 g^2 c)^{1/2} g^3 + 92 d^3 c^3 (c^2 d^2 g^2 x^2 + a^2 e^2 g^2 x + c^2 d^2 f^2 x + a^2 e^2 f^2)^{1/2} x^2 a^2 e^2 f^2 (d^2 g^2 c)^{1/2} g^2 - 20 (c^2 d^2 g^2 x^2 + a^2 e^2 g^2 x + c^2 d^2 f^2 x + a^2 e^2 f^2)^{1/2} x^2 f^3 c^4 d^4 (d^2 g^2 c)^{1/2} g - 30 (c^2 d^2 g^2 x^2 + a^2 e^2 g^2 x + c^2 d^2 f^2 x + a^2 e^2 f^2)^{1/2} a^4 e^4 g^4 (d^2 g^2 c)^{1/2} + 140 (c^2 d^2 g^2 x^2 + a^2 e^2 g^2 x + c^2 d^2 f^2 x + a^2 e^2 f^2)^{1/2} a^3 e^3 f^2 d^2 c (d^2 g^2 c)^{1/2} g^3 + 256 a^2 c^2 d^2 e^2 f^2 g^2 (c^2 d^2 g^2 x^2 + a^2 e^2 g^2 x + c^2 d^2 f^2 x + a^2 e^2 f^2)^{1/2} (d^2 g^2 c)^{1/2} - 140 d^3 c^3 (c^2 d^2 g^2 x^2 + a^2 e^2 g^2 x + c^2 d^2 f^2 x + a^2 e^2 f^2)^{1/2} a^2 e^2 f^3 (d^2 g^2 c)^{1/2} g + 30 (c^2 d^2 g^2 x^2 + a^2 e^2 g^2 x + c^2 d^2 f^2 x + a^2 e^2 f^2)^{1/2} f^4 c^4 d^4 (d^2 g^2 c)^{1/2} \frac{(d^2 g^2 c)^{1/2}}{(e^2 x + d)^{1/2}} \frac{(d^2 g^2 c)^{1/2}}{(c^2 d^2 g^2 x^2 + a^2 e^2 g^2 x + c^2 d^2 f^2 x + a^2 e^2 f^2)^{1/2}} \frac{(d^2 g^2 c)^{1/2}}{d^2/c^2/(d^2 g^2 c)^{1/2}} \frac{(d^2 g^2 c)^{1/2}}{g^3}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*(g\*x + f)^(3/2)/(e\*x + d)

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.70762, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*(g\*x + f)^(3/2)/(e\*x + d)

[Out] [1/2560\*(4\*(128\*c^4\*d^4\*g^4\*x^4 + 15\*c^4\*d^4\*f^4 - 70\*a\*c^3\*d^3\*e\*f^3\*g + 128\*a^2\*c^2\*d^2\*e^2\*f^2\*g^2 + 70\*a^3\*c\*d\*e^3\*f\*g^3 - 15\*a^4\*e^4\*g^4 + 16\*(11\*c^4\*d^4\*f\*g^3 + 21\*a\*c^3\*d^3\*e\*g^4)\*x^3 + 8\*(c^4\*d^4\*f^2\*g^2 + 64\*a\*c^3\*d^3\*e\*f\*g^3 + 31\*a^2\*c^2\*d^2\*e^2\*g^4)\*x^2 - 2\*(5\*c^4\*d^4\*f^3\*g - 23\*a\*c^3\*d^3\*e\*f^2\*g^2 - 233\*a^2\*c^2\*d^2\*e^2\*f\*g^3 - 5\*a^3\*c\*d\*e^3\*g^4)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f) - 15\*(c^5\*d^6\*f^5 - 5\*a\*c^4\*d^5\*e\*f^4\*g + 10\*a^2\*c^3\*d^4\*e^2\*f^3\*g^2 - 10\*a^3\*c^2\*d^3\*e^3\*f^2\*g^3 + 5\*a^4\*c\*d^2\*e^4\*f\*g^4 - a^5\*d\*e^5\*g^5 + (c^5\*d^5\*e\*f^5 - 5\*a\*c^4\*d^4\*e^2\*f^4\*g + 10\*a^2\*c^3\*d^3\*e^3\*f^3\*g^2 - 10\*a^3\*c^2\*d^2\*e^4\*f^2\*g^3 + 5\*a^4\*c\*d^2\*e^5\*f\*g^4 - a^5\*e^6\*g^5)\*x)\*log(-(4\*(2\*c^2\*d^2\*g^2\*x + c^2\*d^2\*f\*g + a\*c\*d\*e\*g^2)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f) + (8\*c^2\*d^2\*e\*g^2\*x^3 + c^2\*d^3\*f^2 + 6\*a\*c\*d^2\*e\*f\*g + a^2\*d^2\*e^2\*g^2 + 8\*(c^2\*d^2\*e\*f\*g + (c^2\*d^3 + a\*c\*d^2\*e^2)\*g^2)\*x^2 + (c^2\*d^2\*e\*f^2 + 2\*(4\*c^2\*d^3 + 3\*a\*c\*d^2\*e^2)\*f\*g + (8\*a\*c\*d^2\*e + a^2\*e^3)\*g^2)\*x)\*sqrt(c\*d\*g))/(e\*x + d)))/((c^2\*d^2\*e\*g^3\*x + c^2\*d^3\*g^3)\*sqrt(c\*d\*g)), 1/1280\*(2\*(128\*c^4\*d^4\*g^4\*x^4 + 15\*c^4\*d^4\*f^4 - 70\*a\*c^3\*d^3\*e\*f^3\*g + 128\*a^2\*c^2\*d^2\*e^2\*f^2\*g^2 + 70\*a^3\*c\*d\*e^3\*f\*g^3 - 15\*a^4\*e^4\*g^4 + 16\*(11\*c^4\*d^4\*f\*g^3 + 21\*a\*c^3\*d^3\*e\*g^4)\*x^3 + 8\*(c^4\*d^4\*f^2\*g^2 + 64\*a\*c^3\*d^3\*e\*f\*g^3 +

$$31*a^2*c^2*d^2*e^2*g^4)*x^2 - 2*(5*c^4*d^4*f^3*g - 23*a*c^3*d^3*e*f^2*g^2 - 233*a^2*c^2*d^2*e^2*f*g^3 - 5*a^3*c*d*e^3*g^4)*x)*\sqrt{(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*\sqrt{-c*d*g)*\sqrt{e*x + d)*\sqrt{g*x + f)} - 15*(c^5*d^6*f^5 - 5*a*c^4*d^5*e*f^4*g + 10*a^2*c^3*d^4*e^2*f^3*g^2 - 10*a^3*c^2*d^3*e^3*f^2*g^3 + 5*a^4*c*d^2*e^4*f*g^4 - a^5*d*e^5*g^5 + (c^5*d^5*e*f^5 - 5*a*c^4*d^4*e^2*f^4*g + 10*a^2*c^3*d^3*e^3*f^3*g^2 - 10*a^3*c^2*d^2*e^4*f^2*g^3 + 5*a^4*c*d*e^5*f*g^4 - a^5*e^6*g^5)*x)*\arctan(2*\sqrt{(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*\sqrt{-c*d*g)*\sqrt{e*x + d)*\sqrt{g*x + f)})/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x)))/((c^2*d^2*e*g^3*x + c^2*d^3*g^3)*\sqrt{-c*d*g})]$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*(3/2)\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)/(e\*x+d)\*\*

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*(g\*x + f)^(3/2)/(e\*x + d)

[Out] Exception raised: NotImplementedError

$$3.752 \quad \int \frac{\sqrt{f+gx}(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}} dx$$

Optimal. Leaf size=376

$$\begin{aligned} & \frac{5\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^4 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{3/2}d^{3/2}g^{7/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \\ & - \frac{5\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^3}{64cdg^3\sqrt{d+ex}} \\ & + \frac{5(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2}{32g^3\sqrt{d+ex}} \\ & - \frac{5(f+gx)^{3/2}(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)}{24g^2(d+ex)^{3/2}} \\ & + \frac{(f+gx)^{3/2}(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{4g(d+ex)^{5/2}} \end{aligned}$$

[Out]  $(-5*(c*d*f - a*e*g)^3*\text{Sqrt}[f + g*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*c*d*g^3*\text{Sqrt}[d + e*x]) + (5*(c*d*f - a*e*g)^2*(f + g*x)^{(3/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*g^3*\text{Sqrt}[d + e*x]) - (5*(c*d*f - a*e*g)*(f + g*x)^{(3/2)}*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(24*g^2*(d + e*x)^{(3/2)}) + ((f + g*x)^{(3/2)}*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(4*g*(d + e*x)^{(5/2)}) - (5*(c*d*f - a*e*g)^4*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x])]/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x]))/(64*c^{(3/2)}*d^{(3/2)}*g^{(7/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

Rubi [A] time = 1.84073, antiderivative size = 376, normalized size of antiderivative = 1., number of rules used = 7, number of rules used = 5, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$

$$\begin{aligned} & \frac{5\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^4 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{64c^{3/2}d^{3/2}g^{7/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \\ & - \frac{5\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^3}{64cdg^3\sqrt{d+ex}} \\ & + \frac{5(f+gx)^{3/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2}{32g^3\sqrt{d+ex}} \\ & - \frac{5(f+gx)^{3/2}(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)}{24g^2(d+ex)^{3/2}} \\ & + \frac{(f+gx)^{3/2}(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{4g(d+ex)^{5/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Sqrt}[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(d + e*x)^{(5/2)}]$

[Out]  $(-5*(c*d*f - a*e*g)^3*\text{Sqrt}[f + g*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(64*c*d*g^3*\text{Sqrt}[d + e*x]) + (5*(c*d*f - a*e*g)^2*(f + g*x)^{(3/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(32*g^3*\text{Sqrt}[d + e*x]) - (5*(c*d*f - a*e*g)*(f + g*x)^{(3/2)}*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(24*g^2*(d + e*x)^{(3/2)}) + ((f + g*x)^{(3/2)}*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(4*g*(d + e*x)^{(5/2)}) - (5*(c*d*f - a*e*g)^4*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x])]/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x]))/(64*c^{(3/2)}*d^{(3/2)}*g^{(7/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi in Sympy [A]** time = 153.646, size = 360, normalized size = 0.96

$$\frac{(f+gx)^{\frac{3}{2}}(ade+cdex^2+x(ae^2+cd^2))^{\frac{5}{2}}}{4g(d+ex)^{\frac{5}{2}}} + \frac{5(f+gx)^{\frac{3}{2}}(aeg-cdf)(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}{24g^2(d+ex)^{\frac{3}{2}}} + \frac{5(f+gx)^{\frac{3}{2}}(aeg-cdf)^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{32g^3\sqrt{d+ex}} + \frac{5\sqrt{f+gx}(aeg-cdf)^3\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{64cdg^3\sqrt{d+ex}} - \frac{5(aeg-cdf)^4\sqrt{ade+cdex^2+x(ae^2+cd^2)}\operatorname{atanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)}{64c^{\frac{3}{2}}d^{\frac{3}{2}}g^{\frac{7}{2}}\sqrt{d+ex}\sqrt{ae+cdx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x+f)**(1/2)*(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x`

[Out] `(f + g*x)**(3/2)*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(5/2)/(4*g*(d + e*x)**(5/2)) + 5*(f + g*x)**(3/2)*(a*e*g - c*d*f)*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2)/(24*g**2*(d + e*x)**(3/2)) + 5*(f + g*x)**(3/2)*(a*e*g - c*d*f)**2*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(32*g**3*sqrt(d + e*x)) + 5*sqrt(f + g*x)*(a*e*g - c*d*f)**3*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(64*c*d*g**3*sqrt(d + e*x)) - 5*(a*e*g - c*d*f)**4*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))*atanh(sqrt(c)*sqrt(d)*sqrt(f + g*x)/(sqrt(g)*sqrt(a*e + c*d*x)))/(64*c**(3/2)*d**(3/2)*g**(7/2)*sqrt(d + e*x)*sqrt(a*e + c*d*x))`

**Mathematica [A]** time = 0.685829, size = 256, normalized size = 0.68

$$\frac{((d+ex)(ae+cdx))^{5/2} \left( \frac{2\sqrt{f+gx}(15a^3e^3g^3+a^2cde^2g^2(73f+118gx)+ae^2d^2eg(-55f^2+36fgx+136g^2x^2))+c^3d^3(15f^3-10f^2gx+8fg^2x^2+48g^3x^3)}{3cdg^3(ae+cdx)^2} - \frac{5(cdg^3(ae+cdx)^2)}{128(d+ex)^{5/2}} \right)}{128(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] `Integrate[(Sqrt[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2))/(d + e*x)`

[Out] `((a*e + c*d*x)*(d + e*x))^(5/2)*((2*Sqrt[f + g*x]*(15*a^3*e^3*g^3 + a^2*c*d*e^2*g^2*(73*f + 118*g*x) + a*c^2*d^2*e*g*(-55*f^2 + 36*f*g*x + 136*g^2*x^2) + c^3*d^3*(15*f^3 - 10*f^2*g*x + 8*f*g^2*x^2 + 48*g^3*x^3)))/(3*c*d*g^3*(a*e + c*d*x)^2) - (5*(c*d*f - a*e*g)^4*Log[a*e*g + 2*Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x]*Sqrt[f + g*x] + c*d*(f + 2*g*x)))/(c^(3/2)*d^(3/2)*g^(7/2)*(a*e + c*d*x)^(5/2)))/(128*(d + e*x)^(5/2))`

**Maple [B]** time = 0.03, size = 870, normalized size = 2.3

$$-\frac{1}{384g^3cd}\sqrt{gx+f}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(-96x^3c^3d^3g^3\sqrt{dgc}\sqrt{cdgx^2+aegx+cdfx+ae^2f}+15g^4\ln\left(\frac{1}{2}\sqrt{\frac{2xcdg+ae^2f}{cdex^2+ae^2x+cd^2x+ade}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((g*x+f)^{(1/2)}*(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(5/2)}/(e*x+d)^{(5/2)}, x)$

[Out]  $-1/384*(g*x+f)^{(1/2)}*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^{(1/2)}*(-96*x^3*c^3*d^3*g^3*(d*g*c)^{(1/2)}*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}+15*g^4*\ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*(d*g*c)^{(1/2)})/(d*g*c)^{(1/2)})*a^4*e^4-60*g^3*a^3*e^3*f*\ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*(d*g*c)^{(1/2)})/(d*g*c)^{(1/2)})*c*d+90*f^2*g^2*\ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*(d*g*c)^{(1/2)})/(d*g*c)^{(1/2)})*a^2*e^2*c^2*d^2-60*f^3*\ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*(d*g*c)^{(1/2)})/(d*g*c)^{(1/2)})*a*e*g*c^3*d^3+15*f^4*d^4*c^4*\ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*(d*g*c)^{(1/2)})/(d*g*c)^{(1/2)})-272*x^2*a*c^2*d^2*e*g^3*(d*g*c)^{(1/2)}*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}-16*x^2*c^3*d^3*f*g^2*(d*g*c)^{(1/2)}*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}-236*g^3*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*x*a^2*e^2*(d*g*c)^{(1/2)}*c*d-72*g^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*x*a*e*f*(d*g*c)^{(1/2)}*c^2*d^2+20*f^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*x*g*(d*g*c)^{(1/2)}*c^3*d^3-30*g^3*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*a^3*e^3*(d*g*c)^{(1/2)}-146*g^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*a^2*e^2*f*(d*g*c)^{(1/2)}*c*d+110*f^2*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*a*e*g*(d*g*c)^{(1/2)}*c^2*d^2-30*f^3*(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}*(d*g*c)^{(1/2)}*c^3*d^3)/(e*x+d)^{(1/2)}/(c*d*g*x^2+a*e*g*x+c*d*f*x+a*e*f)^{(1/2)}/g^3/c/d/(d*g*c)^{(1/2)}$

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**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^{(5/2)}*\text{sqrt}(g*x + f)/(e*x + d)^{(5/2)}, x)$

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 1.24665, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^{(5/2)}*\text{sqrt}(g*x + f)/(e*x + d)^{(5/2)}, x)$

[Out]  $[1/768*(4*(48*c^3*d^3*g^3*x^3 + 15*c^3*d^3*f^3 - 55*a*c^2*d^2*e*f^2*g + 73*a^2*c*d*e^2*f*g^2 + 15*a^3*e^3*g^3 + 8*(c^3*d^3*f*g^2 + 17*a*c^2*d^2*e*g^3)*x^2 - 2*(5*c^3*d^3*f^2*g - 18*a*c^2*d^2*e*f*g^2 - 59*a^2*c*d*e^2*g^3)*x)*\text{sqrt}(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*\text{sqrt}(c*d*g)*\text{sqrt}(e*x + d)*\text{sqrt}(g*x + f) + 15*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a*c^3*d^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3 + a^4*e^5*g^4)*x)*\log((4*(2*c^2*d^2*g^2*x + c^2*d^2*f*g + a*c*d*e*g^2)*\text{sqrt}(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*\text{sqrt}(e*x + d)*\text{sqrt}(g*x + f) - (8*c^2*d^2*e*g^2*x^3 + c^2*d^3*f^2 + 6*a*c*d^2*e*f*g + a^2*d*e^2*g^2 + 8*(c^2*d^2*e*f*g + (c^2*d^3 + a*c*d*e^2)*g^2)*x^2 + (c^2*d^2*e*f^2 + 2*(4*c^2*d^3 + 3*a*c*d*e^2)*f*g + (8*a*c*d^2*e + a^2*e^3)*g^2)*x)*\text{sqrt}(c*d*g))/(e*x + d)))/((c*d*e*g^3*x + c*d^2*g^3)*\text{sqrt}(c*d*g)), 1/384*(2*(48*c^3*d^3*g^3*x^3 + 15*c^3*d^3*f^3 - 55*a*c^2*d^2*e*f^2*g + 73*a^2*c*d*e^2*f*g^2 + 15*a^3*e^3*g^3 + 8*(c^3*d^3*f*g^2 + 17*a*c^2*d^2*e*g^3)*x^2 - 2*(5*c^3*d^3*f^2*g - 18*a*c^2*d^2*e*f*g^2 - 59*a^2*c*d*e^2*g^3)*x)*\text{sqrt}(c*d*e*x^2 + a*d*e$



$$+ (c*d^2 + a*e^2)*x)*\sqrt{-c*d*g}*\sqrt{e*x + d}*\sqrt{g*x + f} - 1$$

$$5*(c^4*d^5*f^4 - 4*a*c^3*d^4*e*f^3*g + 6*a^2*c^2*d^3*e^2*f^2*g^2 - 4*a^3*c*d^2*e^3*f*g^3 + a^4*d*e^4*g^4 + (c^4*d^4*e*f^4 - 4*a*c^3*d^3*e^2*f^3*g + 6*a^2*c^2*d^2*e^3*f^2*g^2 - 4*a^3*c*d*e^4*f*g^3 + a^4*e^5*g^4)*x)*\arctan(2*\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x}*\sqrt{-c*d*g}*\sqrt{e*x + d}*\sqrt{g*x + f}/(2*c*d*e*g*x^2 + c*d^2*f + a*d*e*g + (c*d*e*f + (2*c*d^2 + a*e^2)*g)*x))/((c*d*e*g^3*x + c*d^2*g^3)*\sqrt{-c*d*g})]$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*(1/2)\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)/(e\*x+d)\*\*

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*sqrt(g\*x + f)/(e\*x + d)^(

[Out] Exception raised: NotImplementedError

$$3.753 \quad \int \frac{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=304

$$\begin{aligned} & \frac{5\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^3 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8\sqrt{c}\sqrt{d}g^{7/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \\ & + \frac{5\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2}{8g^3\sqrt{d+ex}} \\ & - \frac{5\sqrt{f+gx}(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)}{12g^2(d+ex)^{3/2}} \\ & + \frac{\sqrt{f+gx}(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{3g(d+ex)^{5/2}} \end{aligned}$$

[Out]  $(5*(c*d*f - a*e*g)^2*\text{Sqrt}[f + g*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*g^3*\text{Sqrt}[d + e*x]) - (5*(c*d*f - a*e*g)*\text{Sqrt}[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(12*g^2*(d + e*x)^{(3/2)}) + (\text{Sqrt}[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(3*g*(d + e*x)^{(5/2)}) - (5*(c*d*f - a*e*g)^3*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x])]/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x]))/(8*\text{Sqrt}[c]*\text{Sqrt}[d]*g^{(7/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi [A]** time = 1.39882, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\begin{aligned} & \frac{5\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^3 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{8\sqrt{c}\sqrt{d}g^{7/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \\ & + \frac{5\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2}{8g^3\sqrt{d+ex}} \\ & - \frac{5\sqrt{f+gx}(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)}{12g^2(d+ex)^{3/2}} \\ & + \frac{\sqrt{f+gx}(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{3g(d+ex)^{5/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}/((d + e*x)^{(5/2)}*\text{Sqrt}[f + g*x])$

[Out]  $(5*(c*d*f - a*e*g)^2*\text{Sqrt}[f + g*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(8*g^3*\text{Sqrt}[d + e*x]) - (5*(c*d*f - a*e*g)*\text{Sqrt}[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(12*g^2*(d + e*x)^{(3/2)}) + (\text{Sqrt}[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(3*g*(d + e*x)^{(5/2)}) - (5*(c*d*f - a*e*g)^3*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x])]/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x]))/(8*\text{Sqrt}[c]*\text{Sqrt}[d]*g^{(7/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi in Sympy [A]** time = 118.3, size = 292, normalized size = 0.96

$$\frac{\sqrt{f+gx}(ade+cdex^2+x(ae^2+cd^2))^{\frac{5}{2}}}{3g(d+ex)^{\frac{5}{2}}} + \frac{5\sqrt{f+gx}(aeg-cdf)(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}{12g^2(d+ex)^{\frac{3}{2}}}$$

$$+ \frac{5\sqrt{f+gx}(aeg-cdf)^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{8g^3\sqrt{d+ex}}$$

$$+ \frac{5(aeg-cdf)^3\sqrt{ade+cdex^2+x(ae^2+cd^2)}\operatorname{atanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)}{8\sqrt{c}\sqrt{d}g^{\frac{7}{2}}\sqrt{d+ex}\sqrt{ae+cdx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x`

[Out] `sqrt(f + g*x)*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(5/2)/((3*g*(d + e*x)**(5/2)) + 5*sqrt(f + g*x)*(a*e*g - c*d*f)*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(3/2)/(12*g**2*(d + e*x)**(3/2)) + 5*sqrt(f + g*x)*(a*e*g - c*d*f)**2*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(8*g**3*sqrt(d + e*x)) + 5*(a*e*g - c*d*f)**3*sqrt(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))*atanh(sqrt(c)*sqrt(d)*sqrt(f + g*x)/(sqrt(g)*sqrt(a*e + c*d*x)))/(8*sqrt(c)*sqrt(d)*g**(7/2)*sqrt(d + e*x)*sqrt(a*e + c*d*x))`

**Mathematica [A]** time = 0.475106, size = 205, normalized size = 0.67

$$\frac{((d+ex)(ae+cdx))^{5/2} \left( \frac{2\sqrt{f+gx}(33a^2e^2g^2+2acdeg(13gx-20f)+c^2d^2(15f^2-10fgx+8g^2x^2))}{3g^3(ae+cdx)^2} + \frac{5(aeg-cdf)^3 \log\left(\frac{2\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}\sqrt{ae+cdx}+aeg+cdx}{\sqrt{c}\sqrt{d}g^{7/2}(ae+cdx)^{5/2}}\right)}{\sqrt{c}\sqrt{d}g^{7/2}(ae+cdx)^{5/2}} \right)}{16(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*Sqrt[f +`

[Out] `((a*e + c*d*x)*(d + e*x))^(5/2)*((2*Sqrt[f + g*x]*(33*a^2*e^2*g^2 + 2*a*c*d*e*g*(-20*f + 13*g*x) + c^2*d^2*(15*f^2 - 10*f*g*x + 8*g^2*x^2)))/(3*g^3*(a*e + c*d*x)^2) + (5*(-(c*d*f) + a*e*g)^3*Log[a*e*g + 2*Sqrt[c]*Sqrt[d]*Sqrt[g]*Sqrt[a*e + c*d*x]*Sqrt[f + g*x] + c*d*(f + 2*g*x)]/(Sqrt[c]*Sqrt[d]*g^(7/2)*(a*e + c*d*x)^(5/2))))/(16*(d + e*x)^(5/2))`

**Maple [A]** time = 0.038, size = 508, normalized size = 1.7

$$\frac{1}{48g^3}\sqrt{cdex^2+ae^2x+cd^2x+ade}\sqrt{gx+f}\left(15\ln\left(\frac{2xcdg+aeg+cdf+2\sqrt{(gx+f)(cdx+ae)}\sqrt{dgc}}{\sqrt{dgc}}\right)a^3e^3g^3-45\ln\left(\frac{2\sqrt{(gx+f)(cdx+ae)}\sqrt{dgc}}{\sqrt{dgc}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(1/2),x)`

[Out] `1/48*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(g*x+f)^(1/2)*(15*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)))/(d*g*c)^(1/2))*a^3*e^3*g^3-45*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)))/(d*g*c)^(1/2))*a^2*e^2*g^2*f*c*d+45*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)))/(d*g*c)^(1/2))*a*e*g*f^2*c^2*d^2-15*ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e))^(1/2)*(d*g*c)^(1/2)))/(d*g*c)^(1/2))*a*d*e`

$$\begin{aligned} & c^{(1/2)}) / (d^*g^*c)^{(1/2)}) * f^3 * c^3 * d^3 + 16 * x^2 * c^2 * d^2 * g^2 * ((g^*x+f)^* \\ & (c^*d^*x+a^*e))^{(1/2)} * (d^*g^*c)^{(1/2)} + 52 * g^2 * ((g^*x+f)^* (c^*d^*x+a^*e))^{(1/2)} \\ & * x^*a^*e^*c^*d^* (d^*g^*c)^{(1/2)} - 20 * g * ((g^*x+f)^* (c^*d^*x+a^*e))^{(1/2)} * x^*f^*c \\ & ^2 * d^2 * (d^*g^*c)^{(1/2)} + 66 * ((g^*x+f)^* (c^*d^*x+a^*e))^{(1/2)} * a^2 * e^2 * g^2 * ( \\ & d^*g^*c)^{(1/2)} - 80 * ((g^*x+f)^* (c^*d^*x+a^*e))^{(1/2)} * a^*e^*f^*g^*c^*d^* (d^*g^*c)^{(1/2)} \\ & + 30 * ((g^*x+f)^* (c^*d^*x+a^*e))^{(1/2)} * f^2 * c^2 * d^2 * (d^*g^*c)^{(1/2)}) / (e \\ & *x+d)^{(1/2)} / ((g^*x+f)^* (c^*d^*x+a^*e))^{(1/2)} / g^3 / (d^*g^*c)^{(1/2)} \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2)\*sqrt(g\*x

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.0569, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2)\*sqrt(g\*x

[Out] [1/96\*(4\*(8\*c^2\*d^2\*g^2\*x^2 + 15\*c^2\*d^2\*f^2 - 40\*a\*c\*d\*e\*f\*g + 3  
3\*a^2\*e^2\*g^2 - 2\*(5\*c^2\*d^2\*f\*g - 13\*a\*c\*d\*e\*g^2)\*x)\*sqrt(c\*d\*e\*  
x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(c\*d\*g)\*sqrt(e\*x + d)\*sqrt(g  
\*x + f) - 15\*(c^3\*d^4\*f^3 - 3\*a\*c^2\*d^3\*e\*f^2\*g + 3\*a^2\*c\*d^2\*e^2  
\*f\*g^2 - a^3\*d^e^3\*g^3 + (c^3\*d^3\*e\*f^3 - 3\*a\*c^2\*d^2\*e^2\*f^2\*g +  
3\*a^2\*c\*d^e^3\*f\*g^2 - a^3\*e^4\*g^3)\*x)\*log(-(4\*(2\*c^2\*d^2\*g^2\*x +  
c^2\*d^2\*f\*g + a\*c\*d\*e\*g^2)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e  
^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f) + (8\*c^2\*d^2\*e\*g^2\*x^3 + c^2\*d  
^3\*f^2 + 6\*a\*c\*d^2\*e\*f\*g + a^2\*d^e^2\*g^2 + 8\*(c^2\*d^2\*e\*f\*g + (c^  
2\*d^3 + a\*c\*d^e^2)\*g^2)\*x^2 + (c^2\*d^2\*e\*f^2 + 2\*(4\*c^2\*d^3 + 3\*a  
\*c\*d^e^2)\*f\*g + (8\*a\*c\*d^2\*e + a^2\*e^3)\*g^2)\*x)\*sqrt(c\*d\*g))/(e\*x  
+ d)))/((e\*g^3\*x + d\*g^3)\*sqrt(c\*d\*g)), 1/48\*(2\*(8\*c^2\*d^2\*g^2\*x  
^2 + 15\*c^2\*d^2\*f^2 - 40\*a\*c\*d^e\*f\*g + 33\*a^2\*e^2\*g^2 - 2\*(5\*c^2\*d  
^2\*f\*g - 13\*a\*c\*d^e\*g^2)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*  
e^2)\*x)\*sqrt(-c\*d\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f) - 15\*(c^3\*d^4\*f^  
3 - 3\*a\*c^2\*d^3\*e\*f^2\*g + 3\*a^2\*c\*d^2\*e^2\*f\*g^2 - a^3\*d^e^3\*g^3 +  
(c^3\*d^3\*e\*f^3 - 3\*a\*c^2\*d^2\*e^2\*f^2\*g + 3\*a^2\*c\*d^e^3\*f\*g^2 - a  
^3\*e^4\*g^3)\*x)\*arctan(2\*sqrt(c\*d^e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*  
x)\*sqrt(-c\*d\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f)/(2\*c\*d^e\*g\*x^2 + c\*d^  
2\*f + a\*d^e\*g + (c\*d^e\*f + (2\*c\*d^2 + a\*e^2)\*g)\*x)))/((e\*g^3\*x +  
d\*g^3)\*sqrt(-c\*d\*g))]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d^e+(a^e\*\*2+c\*d\*\*2)\*x+c\*d^e\*x\*\*2)\*\*(5/2)/(e\*x+d)\*\*(5/2)/(g\*x+f)\*\*

[Out] Timed out

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*sqrt(g*x`

[Out] Exception raised: NotImplementedError

$$3.754 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{3/2}} dx$$

**Optimal.** Leaf size=294

$$\frac{15\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^2 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4g^{7/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{15cd\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}{4g^3\sqrt{d+ex}} + \frac{5cd\sqrt{f+gx}(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{2g^2(d+ex)^{3/2}} - \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}}$$

[Out]  $(-15*c*d*(c*d*f - a*e*g)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^3*\text{Sqrt}[d + e*x]) + (5*c*d*\text{Sqrt}[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(2*g^2*(d + e*x)^{(3/2)}) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(g*(d + e*x)^{(5/2)}*\text{Sqrt}[f + g*x]) + (15*\text{Sqrt}[c]*\text{Sqrt}[d]*(c*d*f - a*e*g)^2*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x])]/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x]))/(4*g^{(7/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi [A]** time = 1.30457, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$

$$\frac{15\sqrt{c}\sqrt{d}\sqrt{d+ex}\sqrt{ae+cdx}(cdf-aeg)^2 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{4g^{7/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{15cd\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}{4g^3\sqrt{d+ex}} + \frac{5cd\sqrt{f+gx}(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{2g^2(d+ex)^{3/2}} - \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}/((d + e*x)^{(5/2)}*(f + g*x)^{(3/2)})]$

[Out]  $(-15*c*d*(c*d*f - a*e*g)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(4*g^3*\text{Sqrt}[d + e*x]) + (5*c*d*\text{Sqrt}[f + g*x]*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)})/(2*g^2*(d + e*x)^{(3/2)}) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)})/(g*(d + e*x)^{(5/2)}*\text{Sqrt}[f + g*x]) + (15*\text{Sqrt}[c]*\text{Sqrt}[d]*(c*d*f - a*e*g)^2*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x])]/(\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x]))/(4*g^{(7/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi in Sympy [A]** time = 117.598, size = 287, normalized size = 0.98

$$\frac{15\sqrt{c}\sqrt{d}(aeg-cdf)^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}\text{atanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)}{4g^{7/2}\sqrt{d+ex}\sqrt{ae+cdx}} + \frac{5cd\sqrt{f+gx}(ade+cdex^2+x(ae^2+cd^2))^{3/2}}{2g^2(d+ex)^{3/2}} + \frac{15cd\sqrt{f+gx}(aeg-cdf)\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{4g^3\sqrt{d+ex}} - \frac{2(ade+cdex^2+x(ae^2+cd^2))^{5/2}}{g(d+ex)^{5/2}\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x`

[Out]  $15\sqrt{c}\sqrt{d}(a^*e^*g - c^*d^*f)^{**2}\sqrt{a^*d^*e + c^*d^*e^*x^{**2} + x^*(a^*e^{**2} + c^*d^{**2})}*\operatorname{atanh}(\sqrt{c}\sqrt{d}\sqrt{f + g^*x})/(\sqrt{g}*\sqrt{a^*e + c^*d^*x})/(4^*g^{**}(7/2)^*\sqrt{d + e^*x})^*\sqrt{a^*e + c^*d^*x}) + 5^*c^*d^*\sqrt{f + g^*x}*(a^*d^*e + c^*d^*e^*x^{**2} + x^*(a^*e^{**2} + c^*d^{**2}))^*(3/2)/(2^*g^{**2}(d + e^*x)^{(3/2)}) + 15^*c^*d^*\sqrt{f + g^*x}*(a^*e^*g - c^*d^*f)^*\sqrt{a^*d^*e + c^*d^*e^*x^{**2} + x^*(a^*e^{**2} + c^*d^{**2})}/(4^*g^{**3}\sqrt{t(d + e^*x)}) - 2^*(a^*d^*e + c^*d^*e^*x^{**2} + x^*(a^*e^{**2} + c^*d^{**2}))^{**}(5/2)/(g^*(d + e^*x)^{(5/2)}\sqrt{f + g^*x})$

**Mathematica [A]** time = 0.596191, size = 202, normalized size = 0.69

$$\frac{((d + ex)(ae + cdx))^{5/2} \left( \frac{2(-8a^2e^2g^2 + acdeg(25f + 9gx) + c^2d^2(-15f^2 - 5fgx + 2g^2x^2))}{g^3\sqrt{f+gx}(ae+cdx)^2} + \frac{15\sqrt{c}\sqrt{d}(cdf - aeg)^2 \log\left(2\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}\sqrt{ae+cdx} + aeg + cd\right)}{g^{7/2}(ae+cdx)^{5/2}} \right)}{8(d + ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x`

[Out]  $((a^*e + c^*d^*x)^*(d + e^*x))^{(5/2)}*((2^*(-8^*a^2*e^2*g^2 + a^*c^*d^*e^*g^*(25^*f + 9^*g^*x) + c^2*d^2*(-15^*f^2 - 5^*f^*g^*x + 2^*g^2*x^2)))/(g^3*(a^*e + c^*d^*x)^2*\operatorname{Sqrt}[f + g^*x]) + (15^*\operatorname{Sqrt}[c]^*\operatorname{Sqrt}[d]^*(c^*d^*f - a^*e^*g)^2*\operatorname{Log}[a^*e^*g + 2^*\operatorname{Sqrt}[c]^*\operatorname{Sqrt}[d]^*\operatorname{Sqrt}[g]^*\operatorname{Sqrt}[a^*e + c^*d^*x]^*\operatorname{Sqrt}[f + g^*x] + c^*d^*(f + 2^*g^*x)])/(g^{(7/2)}*(a^*e + c^*d^*x)^{(5/2)})))/(8^*(d + e^*x)^{(5/2)})$

**Maple [B]** time = 0.042, size = 635, normalized size = 2.2

$$\frac{1}{8g^3} \left( 15 \ln \left( \frac{1}{2} \frac{2xcdg + aeg + cdf + 2\sqrt{(gx+f)(cdx+ae)}\sqrt{dgc}}{\sqrt{dgc}} \right) x a^2 c d e^2 g^3 - 30 \ln \left( \frac{1}{2} \frac{2xcdg + aeg + cdf + 2\sqrt{(gx+f)(cdx+ae)}\sqrt{dgc}}{\sqrt{dgc}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(3/2), x)`

[Out]  $1/8*(15*\ln(1/2*(2*x*c*d*g+a^*e^*g+c^*d^*f+2*((g^*x+f)^*(c^*d^*x+a^*e)))^{(1/2)}*(d^*g^*c)^{(1/2)})/(d^*g^*c)^{(1/2)})^*x^*a^2*c^*d^*e^2*g^3-30*\ln(1/2*(2*x*c*d*g+a^*e^*g+c^*d^*f+2*((g^*x+f)^*(c^*d^*x+a^*e)))^{(1/2)}*(d^*g^*c)^{(1/2)})/(d^*g^*c)^{(1/2)})^*x^*a^2*c^2*d^2*e^*f*g^2+15*\ln(1/2*(2*x*c*d*g+a^*e^*g+c^*d^*f+2*((g^*x+f)^*(c^*d^*x+a^*e)))^{(1/2)}*(d^*g^*c)^{(1/2)})/(d^*g^*c)^{(1/2)})^*x^*c^3*d^3*f^2*g+15*\ln(1/2*(2*x*c*d*g+a^*e^*g+c^*d^*f+2*((g^*x+f)^*(c^*d^*x+a^*e)))^{(1/2)}*(d^*g^*c)^{(1/2)})/(d^*g^*c)^{(1/2)})^*a^2*e^2*g^2*f*c^*d-30*\ln(1/2*(2*x*c*d*g+a^*e^*g+c^*d^*f+2*((g^*x+f)^*(c^*d^*x+a^*e)))^{(1/2)}*(d^*g^*c)^{(1/2)})/(d^*g^*c)^{(1/2)})^*a^*e^*g^*f^2*c^2*d^2+15*\ln(1/2*(2*x*c*d*g+a^*e^*g+c^*d^*f+2*((g^*x+f)^*(c^*d^*x+a^*e)))^{(1/2)}*(d^*g^*c)^{(1/2)})/(d^*g^*c)^{(1/2)})^*f^3*c^3*d^3+4^*x^2*c^2*d^2*g^2*((g^*x+f)^*(c^*d^*x+a^*e))^{(1/2)}*(d^*g^*c)^{(1/2)}+18^*g^2*((g^*x+f)^*(c^*d^*x+a^*e))^{(1/2)}*x^*a^*e^*c^*d^*(d^*g^*c)^{(1/2)}-10^*g*((g^*x+f)^*(c^*d^*x+a^*e))^{(1/2)}*x^*f^*c^2*d^2*(d^*g^*c)^{(1/2)}-16*((g^*x+f)^*(c^*d^*x+a^*e))^{(1/2)}*a^2*e^2*g^2*(d^*g^*c)^{(1/2)}+50*((g^*x+f)^*(c^*d^*x+a^*e))^{(1/2)}*a^*e^*f^*g^*c^*d^*(d^*g^*c)^{(1/2)}-30*((g^*x+f)^*(c^*d^*x+a^*e))^{(1/2)}*f^2*c^2*d^2*(d^*g^*c)^{(1/2)}*(c^*d^*e^*x^2+a^*e^2*x+c^*d^2*x+a^*d^*e)^{(1/2)}/((g^*x+f)^*(c^*d^*x+a^*e))^{(1/2)}/(d^*g^*c)^{(1/2)}/g^3/(g^*x+f)^{(1/2)}/(e^*x+d)^{(1/2)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g*x + f`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.953888, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g*x + f`

[Out] 
$$\left[ \frac{1}{16} \left( 4 \left( 2c^2d^2g^2x^2 - 15c^2d^2f^2 + 25acde^fg - 8a^2e^2g^2 - (5c^2d^2fg - 9acde^g^2)x \right) \sqrt{cde^x^2 + ade + (cd^2 + ae^2)x} \sqrt{ex + d} \sqrt{gx + f} + 15 \left( c^2d^3f^3 - 2acde^f^2g + a^2de^2fg^2 + (c^2d^2ef^2g - 2acde^2fg^2 + a^2e^3g^3)x^2 + (c^2d^2ef^3 + a^2de^2g^3 + (c^2d^3 - 2acde^2)f^2g - (2acde^2e - a^2e^3)f^2g^2 \right) x \right) \sqrt{cd/g} \log\left(-\frac{8c^2d^2e^g^2x^3 + c^2d^3f^2 + 6acde^fg + a^2de^2g^2 + 4(2cdg^2x + cdfg + ae^g^2)\sqrt{cde^x^2 + ade + (cd^2 + ae^2)x}\sqrt{ex + d}\sqrt{gx + f}\sqrt{cd/g} + 8(c^2d^2ef^2g + (c^2d^3 + acde^2)g^2)x^2 + (c^2d^2ef^2 + 2(4c^2d^3 + 3acde^2)fg + (8acde^2e + a^2e^3)g^2)x}{(ex + d)}\right) / (e^g^4x^2 + df^g^3 + (ef^g^3 + dg^4)x), \frac{1}{8} \left( 2 \left( 2c^2d^2g^2x^2 - 15c^2d^2f^2 + 25acde^fg - 8a^2e^2g^2 - (5c^2d^2fg - 9acde^g^2)x \right) \sqrt{cde^x^2 + ade + (cd^2 + ae^2)x} \sqrt{ex + d} \sqrt{gx + f} + 15 \left( c^2d^3f^3 - 2acde^f^2g + a^2de^2fg^2 + (c^2d^2ef^2g - 2acde^2fg^2 + a^2e^3g^3)x^2 + (c^2d^2ef^3 + a^2de^2g^3 + (c^2d^3 - 2acde^2)f^2g - (2acde^2e - a^2e^3)f^2g^2 \right) x \right) \sqrt{-cd/g} \arctan\left(2\sqrt{cde^x^2 + ade + (cd^2 + ae^2)x}\sqrt{ex + d}\sqrt{gx + f}\sqrt{cd/g} / ((2cde^g^2x^2 + cd^2f + ade^g + (cde^f + (2cd^2 + ae^2)g)x)\sqrt{-cd/g})\right) / (e^g^4x^2 + df^g^3 + (ef^g^3 + dg^4)x) \right]$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f)**`

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g*x + f
```

```
[Out] Exception raised: NotImplementedError
```

$$3.755 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{5/2}} dx$$

**Optimal.** Leaf size=284

$$\begin{aligned} & \frac{5c^{3/2}d^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}(cdf - aeg) \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{7/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \\ & + \frac{5c^2d^2\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g^3\sqrt{d+ex}} \\ & - \frac{10cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{3g^2(d+ex)^{3/2}\sqrt{f+gx}} - \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}} \end{aligned}$$

[Out]  $(5*c^2*d^2*\sqrt{f+g*x}*\sqrt{a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2})/(g^3*\sqrt{d+e*x}) - (10*c*d*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{3/2})/(3*g^2*(d+e*x)^{3/2}*\sqrt{f+g*x}) - (2*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{5/2})/(3*g*(d+e*x)^{5/2}*(f+g*x)^{3/2}) - (5*c^{3/2}*d^{3/2}*\sqrt{d+ex}*\sqrt{ae+cdx}(cdf - aeg)*\sqrt{f+gx})/(g^{7/2}*\sqrt{x(ae^2+cd^2)+ade+cdex^2}) - (5*c^2*d^2*\sqrt{f+gx}*\sqrt{x(ae^2+cd^2)+ade+cdex^2})/(g^3*\sqrt{d+ex}) - (10*cd*(x(ae^2+cd^2)+ade+cdex^2)^{3/2})/(3*g^2*(d+ex)^{3/2}*\sqrt{f+gx}) - (2*(x(ae^2+cd^2)+ade+cdex^2)^{5/2})/(3*g*(d+ex)^{5/2}*(f+gx)^{3/2})$

**Rubi [A]** time = 1.22759, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.104$

$$\begin{aligned} & \frac{5c^{3/2}d^{3/2}\sqrt{d+ex}\sqrt{ae+cdx}(cdf - aeg) \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{7/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} \\ & + \frac{5c^2d^2\sqrt{f+gx}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g^3\sqrt{d+ex}} \\ & - \frac{10cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{3g^2(d+ex)^{3/2}\sqrt{f+gx}} - \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{3g(d+ex)^{5/2}(f+gx)^{3/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{5/2}/((d+e*x)^{5/2}*(f+g*x)^{5/2})]$

[Out]  $(5*c^2*d^2*\sqrt{f+g*x}*\sqrt{a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2})/(g^3*\sqrt{d+e*x}) - (10*c*d*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{3/2})/(3*g^2*(d+e*x)^{3/2}*\sqrt{f+g*x}) - (2*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^{5/2})/(3*g*(d+e*x)^{5/2}*(f+g*x)^{3/2}) - (5*c^{3/2}*d^{3/2}*\sqrt{d+ex}*\sqrt{ae+cdx}(cdf - aeg)*\sqrt{f+gx})/(g^{7/2}*\sqrt{x(ae^2+cd^2)+ade+cdex^2}) - (5*c^2*d^2*\sqrt{f+gx}*\sqrt{x(ae^2+cd^2)+ade+cdex^2})/(g^3*\sqrt{d+ex}) - (10*cd*(x(ae^2+cd^2)+ade+cdex^2)^{3/2})/(3*g^2*(d+ex)^{3/2}*\sqrt{f+gx}) - (2*(x(ae^2+cd^2)+ade+cdex^2)^{5/2})/(3*g*(d+ex)^{5/2}*(f+gx)^{3/2})$

**Rubi in Sympy [A]** time = 112.763, size = 277, normalized size = 0.98

$$\begin{aligned} & \frac{5c^{\frac{3}{2}}d^{\frac{3}{2}}(aeg - cdf)\sqrt{ade+cdex^2+x(ae^2+cd^2)} \operatorname{atanh}\left(\frac{\sqrt{c}\sqrt{d}\sqrt{f+gx}}{\sqrt{g}\sqrt{ae+cdx}}\right)}{g^{\frac{7}{2}}\sqrt{d+ex}\sqrt{ae+cdx}} \\ & + \frac{5c^2d^2\sqrt{f+gx}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{g^3\sqrt{d+ex}} \\ & - \frac{10cd(ade+cdex^2+x(ae^2+cd^2))^{\frac{3}{2}}}{3g^2(d+ex)^{\frac{3}{2}}\sqrt{f+gx}} - \frac{2(ade+cdex^2+x(ae^2+cd^2))^{\frac{5}{2}}}{3g(d+ex)^{\frac{5}{2}}(f+gx)^{\frac{3}{2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x`

[Out]  $5*c^{3/2}*d^{3/2}*(a*e*g - c*d*f)*\sqrt{a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2)}*\operatorname{atanh}(\sqrt{c}*\sqrt{d}*\sqrt{f+g*x}/(\sqrt{g}*\sqrt{a*e + c*d*x}))/((g^{7/2}*\sqrt{d+e*x}*\sqrt{a*e + c*d*x}) + 5*c^2*d^2*\sqrt{f+g*x}*\sqrt{a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2)})/(g^3*\sqrt{d+e*x}) - 10*c*d*(a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2))^{3/2}/(3*g^2*(d+e*x)^{3/2}*\sqrt{f+g*x}) - 2*(a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2))^{5/2}/(3*g*(d+e*x)^{5/2}*(f+g*x)^{3/2})$

**Mathematica [A]** time = 0.667672, size = 204, normalized size = 0.72

$$\frac{((d+ex)(ae+cdx))^{5/2} \left( \frac{-4a^2e^2g^2-4acdeg(5f+7gx)+2c^2d^2(15f^2+20fgx+3g^2x^2)}{3g^3(f+gx)^{3/2}(ae+cdx)^2} - \frac{5c^{3/2}d^{3/2}(cdf-aeg)\log\left(2\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}\sqrt{ae+cdx}+aeg+cd(f+gx)\right)}{g^{7/2}(ae+cdx)^{5/2}} \right)}{2(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] `Integrate[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x`

[Out]  $((a*e + c*d*x)*(d + e*x))^{5/2} * ((-4*a^2*e^2*g^2 - 4*a*c*d*e*g*(5*f + 7*g*x) + 2*c^2*d^2*(15*f^2 + 20*f*g*x + 3*g^2*x^2))/(3*g^3*(a*e + c*d*x)^2*(f + g*x)^{3/2}) - (5*c^{3/2}*d^{3/2}*(c*d*f - a*e*g)*\operatorname{Log}[a*e*g + 2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[d]*\operatorname{Sqrt}[g]*\operatorname{Sqrt}[a*e + c*d*x]*\operatorname{Sqrt}[f + g*x] + c*d*(f + 2*g*x)])/(g^{7/2}*(a*e + c*d*x)^{5/2})))/(2*(d + e*x)^{5/2})$

**Maple [B]** time = 0.039, size = 638, normalized size = 2.3

$$\frac{1}{6g^3} \left( 15 \ln \left( \frac{1}{2} \frac{2xcdg + aeg + cdf + 2\sqrt{(gx+f)(cdx+ae)}\sqrt{dgc}}{\sqrt{dgc}} \right) x^2 ac^2 d^2 eg^3 - 15 \ln \left( \frac{1}{2} \frac{2xcdg + aeg + cdf + 2\sqrt{(gx+f)(cdx+ae)}\sqrt{dgc}}{\sqrt{dgc}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(5/2)/(e*x+d)^(5/2)/(g*x+f)^(5/2), x)`

[Out]  $1/6*(15*\ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e)))^{1/2}*(d*g*c)^{1/2}))/((d*g*c)^{1/2})*x^2*a*c^2*d^2*e*g^3-15*\ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e)))^{1/2}*(d*g*c)^{1/2}))/((d*g*c)^{1/2})*x^2*c^3*d^3*f*g^2+30*\ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e)))^{1/2}*(d*g*c)^{1/2}))/((d*g*c)^{1/2})*x*a*c^2*d^2*e*f*g^2-30*\ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e)))^{1/2}*(d*g*c)^{1/2}))/((d*g*c)^{1/2})*x*c^3*d^3*f^2*g+15*\ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e)))^{1/2}*(d*g*c)^{1/2}))/((d*g*c)^{1/2})*a*e*g*f^2*c^2*d^2-15*\ln(1/2*(2*x*c*d*g+a*e*g+c*d*f+2*((g*x+f)*(c*d*x+a*e)))^{1/2}*(d*g*c)^{1/2}))/((d*g*c)^{1/2})*f^3*c^3*d^3+6*x^2*c^2*d^2*g^2*((g*x+f)*(c*d*x+a*e))^{1/2}*(d*g*c)^{1/2}-28*g^2*((g*x+f)*(c*d*x+a*e))^{1/2})*x*a*e*c*d*(d*g*c)^{1/2}+40*g*((g*x+f)*(c*d*x+a*e))^{1/2})*x*f*c^2*d^2*(d*g*c)^{1/2}-4*((g*x+f)*(c*d*x+a*e))^{1/2})*a^2*e^2*g^2*(d*g*c)^{1/2}-20*((g*x+f)*(c*d*x+a*e))^{1/2})*a*e*f*g*c*d*(d*g*c)^{1/2}+30*((g*x+f)*(c*d*x+a*e))^{1/2})*f^2*c^2*d^2*(d*g*c)^{1/2}*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^{1/2}))/((g*x+f)*(c*d*x+a*e))^{1/2}/(d*g*c)^{1/2}/g^3/(g*x+f)^{3/2}/(e*x+d)^{1/2})$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}(gx + f)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2)\*(g\*x + f)^(5/2)), x)

[Out] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2)\*(g\*x + f)^(5/2)), x)

**Fricas [A]** time = 0.940909, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2)\*(g\*x + f)^(5/2)), x)

[Out] [1/12\*(4\*(3\*c^2\*d^2\*g^2\*x^2 + 15\*c^2\*d^2\*f^2 - 10\*a\*c\*d\*e\*f\*g - 2\*a^2\*e^2\*g^2 + 2\*(10\*c^2\*d^2\*f\*g - 7\*a\*c\*d\*e\*g^2)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f) - 15\*(c^2\*d^3\*f^3 - a\*c\*d^2\*e\*f^2\*g + (c^2\*d^2\*e\*f\*g^2 - a\*c\*d\*e^2\*g^3)\*x^3 + (2\*c^2\*d^2\*e\*f^2\*g - a\*c\*d^2\*e\*g^3 + (c^2\*d^3 - 2\*a\*c\*d\*e^2)\*f\*g^2)\*x^2 + (c^2\*d^2\*e\*f^3 - 2\*a\*c\*d^2\*e\*f\*g^2 + (2\*c^2\*d^3 - a\*c\*d\*e^2)\*f^2\*g)\*x)\*sqrt(c\*d/g)\*log(-(8\*c^2\*d^2\*e\*g^2\*x^3 + c^2\*d^3\*f^2 + 6\*a\*c\*d^2\*e\*f\*g + a^2\*d\*e^2\*g^2 + 4\*(2\*c\*d\*g^2\*x + c\*d\*f\*g + a\*e\*g^2)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f)\*sqrt(c\*d/g) + 8\*(c^2\*d^2\*e\*f\*g + (c^2\*d^3 + a\*c\*d\*e^2)\*g^2)\*x^2 + (c^2\*d^2\*e\*f^2 + 2\*(4\*c^2\*d^3 + 3\*a\*c\*d\*e^2)\*f\*g + (8\*a\*c\*d^2\*e + a^2\*e^3)\*g^2)\*x)/(e\*x + d)))/(e\*g^5\*x^3 + d\*f^2\*g^3 + (2\*e\*f\*g^4 + d\*g^5)\*x^2 + (e\*f^2\*g^3 + 2\*d\*f\*g^4)\*x), 1/6\*(2\*(3\*c^2\*d^2\*g^2\*x^2 + 15\*c^2\*d^2\*f^2 - 10\*a\*c\*d\*e\*f\*g - 2\*a^2\*e^2\*g^2 + 2\*(10\*c^2\*d^2\*f\*g - 7\*a\*c\*d\*e\*g^2)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f) - 15\*(c^2\*d^3\*f^3 - a\*c\*d^2\*e\*f^2\*g + (c^2\*d^2\*e\*f\*g^2 - a\*c\*d\*e^2\*g^3)\*x^3 + (2\*c^2\*d^2\*e\*f^2\*g - a\*c\*d^2\*e\*g^3 + (c^2\*d^3 - 2\*a\*c\*d\*e^2)\*f\*g^2)\*x^2 + (c^2\*d^2\*e\*f^3 - 2\*a\*c\*d^2\*e\*f\*g^2 + (2\*c^2\*d^3 - a\*c\*d\*e^2)\*f^2\*g)\*x)\*sqrt(-c\*d/g)\*arctan(2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f)\*c\*d/((2\*c\*d\*e\*g\*x^2 + c\*d^2\*f + a\*d\*e\*g + (c\*d\*e\*f + (2\*c\*d^2 + a\*e^2)\*g)\*x)\*sqrt(-c\*d/g)))/(e\*g^5\*x^3 + d\*f^2\*g^3 + (2\*e\*f\*g^4 + d\*g^5)\*x^2 + (e\*f^2\*g^3 + 2\*d\*f\*g^4)\*x)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)/(e\*x+d)\*\*(5/2)/(g\*x+f)\*\*(5/2)), x)

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g*x + f
```

```
[Out] Exception raised: NotImplementedError
```

$$3.756 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{7/2}} dx$$

**Optimal.** Leaf size=274

$$\frac{2c^{5/2}d^{5/2}\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{7/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{2c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g^3\sqrt{d+ex}\sqrt{f+gx}} - \frac{2cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}} - \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}$$

[Out]  $(-2*c^2*d^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (g^3*\text{Sqrt}[d + e*x]*\text{Sqrt}[f + g*x]) - (2*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)} / (3*g^2*(d + e*x)^{(3/2)}*(f + g*x)^{(3/2)}) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)} / (5*g*(d + e*x)^{(5/2)}*(f + g*x)^{(5/2)}) + (2*c^{(5/2)}*d^{(5/2)}*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x]) / (\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x])]) / (g^{(7/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi [A]** time = 1.18096, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{2c^{5/2}d^{5/2}\sqrt{d+ex}\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{7/2}\sqrt{x(ae^2+cd^2)+ade+cdex^2}} - \frac{2c^2d^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{g^3\sqrt{d+ex}\sqrt{f+gx}} - \frac{2cd(x(ae^2+cd^2)+ade+cdex^2)^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}} - \frac{2(x(ae^2+cd^2)+ade+cdex^2)^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)} / ((d + e*x)^{(5/2)}*(f + g*x)^{(7/2)})]$

[Out]  $(-2*c^2*d^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]) / (g^3*\text{Sqrt}[d + e*x]*\text{Sqrt}[f + g*x]) - (2*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(3/2)} / (3*g^2*(d + e*x)^{(3/2)}*(f + g*x)^{(3/2)}) - (2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)} / (5*g*(d + e*x)^{(5/2)}*(f + g*x)^{(5/2)}) + (2*c^{(5/2)}*d^{(5/2)}*\text{Sqrt}[a*e + c*d*x]*\text{Sqrt}[d + e*x]*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[a*e + c*d*x]) / (\text{Sqrt}[c]*\text{Sqrt}[d]*\text{Sqrt}[f + g*x])]) / (g^{(7/2)}*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])$

**Rubi in Sympy [A]** time = 109.725, size = 267, normalized size = 0.97

$$\frac{2c^{5/2}d^{5/2}\sqrt{ade+cdex^2+x(ae^2+cd^2)}\text{atanh}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{c}\sqrt{d}\sqrt{f+gx}}\right)}{g^{7/2}\sqrt{d+ex}\sqrt{ae+cdx}} - \frac{2c^2d^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{g^3\sqrt{d+ex}\sqrt{f+gx}} - \frac{2cd(ade+cdex^2+x(ae^2+cd^2))^{3/2}}{3g^2(d+ex)^{3/2}(f+gx)^{3/2}} - \frac{2(ade+cdex^2+x(ae^2+cd^2))^{5/2}}{5g(d+ex)^{5/2}(f+gx)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x$

[Out]  $2*c^{(5/2)}*d^{(5/2)}*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))*\text{atanh}(\text{sqrt}(g)*\text{sqrt}(a*e + c*d*x)/(\text{sqrt}(c)*\text{sqrt}(d)*\text{sqrt}(f + g*x)))/(g^{(7/2)}*\text{sqrt}(d + e*x)*\text{sqrt}(a*e + c*d*x)) - 2*c^{(5/2)}*d^{(5/2)}*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(g^{(7/2)}*\text{sqrt}(d + e*x)*\text{sqrt}(f + g*x)) - 2*cd*(ade + cdex^2 + x*(ae^2 + cd^2))^{(3/2)}/(3*g^2*(d + ex)^{(3/2)}*(f + gx)^{(3/2)}) - 2*(ade + cdex^2 + x*(ae^2 + cd^2))^{(5/2)}/(5*g*(d + ex)^{(5/2)}*(f + gx)^{(5/2)})$

$$(f + gx) - 2cd(a^2d^2e + c^2d^2e^2x^2 + x(a^2e^2 + c^2d^2))^{3/2} / (3g^2(d + ex)^{3/2}(f + gx)^{3/2}) - 2(a^2d^2e + c^2d^2e^2x^2 + x(a^2e^2 + c^2d^2))^{5/2} / (5g^2(d + ex)^{5/2}(f + gx)^{5/2})$$

**Mathematica [A]** time = 0.694894, size = 185, normalized size = 0.68

$$\frac{((d + ex)(ae + cdx))^{5/2} \left( \frac{c^{5/2}d^{5/2} \log(2\sqrt{c}\sqrt{d}\sqrt{g}\sqrt{f+gx}\sqrt{ae+cdx+ae+cd(f+2gx)})}{g^{7/2}(ae+cdx)^{5/2}} - \frac{2(-11cdf+gx)(cdf-ae)+3(cdf-ae)^2+23c^2d^2(f+gx)^2}{15g^3(f+gx)^{5/2}(ae+cdx)^2} \right)}{(d + ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a^2d^2e + (c^2d^2 + a^2e^2)x + c^2d^2e^2x^2)^(5/2)/((d + ex)^(5/2)\*(f + gx)

[Out] (((a^2e + c^2d^2x)^(5/2)\*(d + ex)^(5/2)\*((-2\*(3\*(c^2d^2f - a^2e^2g)^2 - 11\*c^2d^2(c^2d^2f - a^2e^2g)\*(f + gx) + 23\*c^2d^2d^2\*(f + gx)^2))/(15\*g^3\*(a^2e + c^2d^2x)^2\*(f + gx)^(5/2)) + (c^(5/2)\*d^(5/2)\*Log[a^2e^2g + 2\*Sqrt[c]\*Sqrt[d]\*Sqrt[g]\*Sqrt[a^2e + c^2d^2x]\*Sqrt[f + gx] + c^2d^2(f + 2\*g\*x)))/(g^(7/2)\*(a^2e + c^2d^2x)^(5/2)))/(d + ex)^(5/2)

**Maple [B]** time = 0.046, size = 511, normalized size = 1.9

$$\frac{1}{15g^3} \sqrt{cdex^2 + ae^2x + cd^2x + ade} \left( 15 \ln \left( \frac{2xcdg + aeg + cdf + 2\sqrt{(gx+f)(cdx+ae)}\sqrt{dgc}}{\sqrt{dgc}} \right) x^3 c^3 d^3 g^3 + 45 \ln \left( \frac{2}{1/2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a^2d^2e+(a^2e^2+c^2d^2)\*x+c^2d^2e^2x^2)^(5/2)/((e\*x+d)^(5/2)/(g\*x+f)^(7/2)), x)

[Out] 1/15\*(c^2d^2e^2x^2+a^2e^2x+c^2d^2x+a^2d^2e)^(1/2)\*(15\*ln(1/2\*(2\*x\*c^2d^2g+a^2e^2g+c^2d^2f+2\*((g\*x+f)\*(c^2d^2x+a^2e))^(1/2)\*(d\*g\*c)^(1/2)))/(d\*g\*c)^(1/2))\*x^3\*c^3\*d^3\*g^3+45\*ln(1/2\*(2\*x\*c^2d^2g+a^2e^2g+c^2d^2f+2\*((g\*x+f)\*(c^2d^2x+a^2e))^(1/2)\*(d\*g\*c)^(1/2)))/(d\*g\*c)^(1/2))\*x^2\*c^3\*d^3\*f^2\*g+15\*ln(1/2\*(2\*x\*c^2d^2g+a^2e^2g+c^2d^2f+2\*((g\*x+f)\*(c^2d^2x+a^2e))^(1/2)\*(d\*g\*c)^(1/2)))/(d\*g\*c)^(1/2))\*x\*c^3\*d^3\*f^2\*g+15\*ln(1/2\*(2\*x\*c^2d^2g+a^2e^2g+c^2d^2f+2\*((g\*x+f)\*(c^2d^2x+a^2e))^(1/2)\*(d\*g\*c)^(1/2)))/(d\*g\*c)^(1/2))\*f^3\*c^3\*d^3-46\*x^2\*c^2\*d^2\*g^2\*((g\*x+f)\*(c^2d^2x+a^2e))^(1/2)\*(d\*g\*c)^(1/2)-22\*g^2\*((g\*x+f)\*(c^2d^2x+a^2e))^(1/2)\*x\*a^2e^2\*c^2d^2\*(d\*g\*c)^(1/2)-70\*g\*((g\*x+f)\*(c^2d^2x+a^2e))^(1/2)\*x\*f\*c^2\*d^2\*(d\*g\*c)^(1/2)-6\*((g\*x+f)\*(c^2d^2x+a^2e))^(1/2)\*a^2e^2\*g^2\*(d\*g\*c)^(1/2)-10\*((g\*x+f)\*(c^2d^2x+a^2e))^(1/2)\*a^2e\*f\*g\*c\*d\*(d\*g\*c)^(1/2)-30\*((g\*x+f)\*(c^2d^2x+a^2e))^(1/2)\*f^2\*c^2\*d^2\*(d\*g\*c)^(1/2))/((g\*x+f)\*(c^2d^2x+a^2e))^(1/2)/(d\*g\*c)^(1/2)/g^3/(g\*x+f)^(5/2)/(e\*x+d)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{5/2}}{(ex + d)^{5/2}(gx + f)^{7/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c^2d^2e^2x^2 + a^2d^2e + (c^2d^2 + a^2e^2)x)^(5/2)/((e\*x + d)^(5/2)\*(g\*x + f)^(7/2)), x)

[Out] integrate((c^2d^2e^2x^2 + a^2d^2e + (c^2d^2 + a^2e^2)x)^(5/2)/((e\*x + d)^(5/2)\*(g\*x + f)^(7/2)), x)

---

**Fricas [A]** time = 0.916216, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2)\*(g\*x + f)

[Out] [-1/30\*(4\*(23\*c^2\*d^2\*g^2\*x^2 + 15\*c^2\*d^2\*f^2 + 5\*a\*c\*d\*e\*f\*g + 3\*a^2\*e^2\*g^2 + (35\*c^2\*d^2\*f\*g + 11\*a\*c\*d\*e\*g^2)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f) - 15\*(c^2\*d^2\*e\*g^3\*x^4 + c^2\*d^3\*f^3 + (3\*c^2\*d^2\*e\*f\*g^2 + c^2\*d^3\*g^3)\*x^3 + 3\*(c^2\*d^2\*e\*f^2\*g + c^2\*d^3\*f\*g^2)\*x^2 + (c^2\*d^2\*e\*f^3 + 3\*c^2\*d^3\*f^2\*g)\*x)\*sqrt(c\*d/g)\*log(-(8\*c^2\*d^2\*e\*g^2\*x^3 + c^2\*d^3\*f^2 + 6\*a\*c\*d^2\*e\*f\*g + a^2\*d\*e^2\*g^2 + 4\*(2\*c\*d\*g^2\*x + c\*d\*f\*g + a\*e\*g^2)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f)\*sqrt(c\*d/g) + 8\*(c^2\*d^2\*e\*f\*g + (c^2\*d^3 + a\*c\*d\*e^2)\*g^2)\*x^2 + (c^2\*d^2\*e\*f^2 + 2\*(4\*c^2\*d^3 + 3\*a\*c\*d\*e^2)\*f\*g + (8\*a\*c\*d^2\*e + a^2\*e^3)\*g^2)\*x)/(e\*x + d)))/(e\*g^6\*x^4 + d\*f^3\*g^3 + (3\*e\*f\*g^5 + d\*g^6)\*x^3 + 3\*(e\*f^2\*g^4 + d\*f\*g^5)\*x^2 + (e\*f^3\*g^3 + 3\*d\*f^2\*g^4)\*x), -1/15\*(2\*(23\*c^2\*d^2\*g^2\*x^2 + 15\*c^2\*d^2\*f^2 + 5\*a\*c\*d\*e\*f\*g + 3\*a^2\*e^2\*g^2 + (35\*c^2\*d^2\*f\*g + 11\*a\*c\*d\*e\*g^2)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f) - 15\*(c^2\*d^2\*e\*g^3\*x^4 + c^2\*d^3\*f^3 + (3\*c^2\*d^2\*e\*f\*g^2 + c^2\*d^3\*g^3)\*x^3 + 3\*(c^2\*d^2\*e\*f^2\*g + c^2\*d^3\*f\*g^2)\*x^2 + (c^2\*d^2\*e\*f^3 + 3\*c^2\*d^3\*f^2\*g)\*x)\*sqrt(-c\*d/g)\*arctan(2\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f)\*c\*d/((2\*c\*d\*e\*g\*x^2 + c\*d^2\*f + a\*d\*e\*g + (c\*d\*e\*f + (2\*c\*d^2 + a\*e^2)\*g)\*x)\*sqrt(-c\*d/g)))/(e\*g^6\*x^4 + d\*f^3\*g^3 + (3\*e\*f\*g^5 + d\*g^6)\*x^3 + 3\*(e\*f^2\*g^4 + d\*f\*g^5)\*x^2 + (e\*f^3\*g^3 + 3\*d\*f^2\*g^4)\*x)]

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**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)/((e\*x+d)\*\*(5/2)/(g\*x+f)\*\*

[Out] Timed out

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**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2)\*(g\*x + f)

[Out] Exception raised: NotImplementedError



$$3.757 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{9/2}} dx$$

**Optimal.** Leaf size=63

$$\frac{2(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{7(d+ex)^{7/2}(f+gx)^{7/2}(cdf - aeg)}$$

[Out]  $(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(7*(c*d*f - a*e*g)*(d + e*x)^(7/2)*(f + g*x)^(7/2))$

**Rubi [A]** time = 0.253086, antiderivative size = 63, normalized size of antiderivative = 1., number of rules used = 1, number of rules used = 1, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$

$$\frac{2(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{7(d+ex)^{7/2}(f+gx)^{7/2}(cdf - aeg)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x)^(9/2))$

[Out]  $(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(7/2))/(7*(c*d*f - a*e*g)*(d + e*x)^(7/2)*(f + g*x)^(7/2))$

**Rubi in Sympy [A]** time = 21.8255, size = 60, normalized size = 0.95

$$\frac{2(ade + cdex^2 + x(ae^2 + cd^2))^{7/2}}{7(d+ex)^{7/2}(f+gx)^{7/2}(aeg - cdf)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x$

[Out]  $-2*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(7/2)/(7*(d + e*x)**(7/2)*(f + g*x)**(7/2)*(a*e*g - c*d*f))$

**Mathematica [A]** time = 0.21193, size = 62, normalized size = 0.98

$$\frac{2(ae + cdx)^3 \sqrt{(d+ex)(ae+cdx)}}{7\sqrt{d+ex}(f+gx)^{7/2}(cdf - aeg)}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^(5/2)/((d + e*x)^(5/2)*(f + g*x$

[Out]  $(2*(a*e + c*d*x)^3*\text{Sqrt}[(a*e + c*d*x)*(d + e*x)])/(7*(c*d*f - a*e*g)*\text{Sqrt}[d + e*x]*(f + g*x)^(7/2))$

**Maple [A]** time = 0.01, size = 63, normalized size = 1.

$$-\frac{2cdx + 2ae}{7aeg - 7cdf} (cdex^2 + ae^2x + cd^2x + ade)^{\frac{5}{2}} (gx + f)^{-\frac{7}{2}} (ex + d)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(5/2)}/(e*x+d)^{(5/2)}/(g*x+f)^{(9/2)}, x)$

[Out]  $-2/7/(g*x+f)^{(7/2)}*(c*d*x+a*e)/(a*e*g-c*d*f)*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^{(5/2)}/(e*x+d)^{(5/2)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}(gx + f)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^{(5/2)}/((e*x + d)^{(5/2)}*(g*x + f)^{(9/2)}), x)$

[Out]  $\text{integrate}((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^{(5/2)}/((e*x + d)^{(5/2)}*(g*x + f)^{(9/2)}), x)$

**Fricas [A]** time = 0.294534, size = 404, normalized size = 6.41

$$\frac{2(c^3d^3x^3 + 3ac^2d^2ex^2 + 3a^2cde^2x + a^3e^3)\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}{7(cd^2f^5 - adef^4g + (cdfg^4 - ae^2g^5)x^5 + (4cdf^2g^3 - adeg^5 + (cd^2 - 4ae^2)fg^4)x^4 + 2(3cdf^3g^2 - 2adefg^4 + (2cd^2 - 3ae^2)fg^3)x^3 + 2(2c^2d^2ef^4g - 3a^2d^2ef^3g^2 + (3c^2d^2 - 2a^2e^2)fg^3)x^2 + (c^2d^2ef^5 - 4a^2d^2ef^4g^2 + (4c^2d^2 - a^2e^2)fg^4)x + a^3e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^{(5/2)}/((e*x + d)^{(5/2)}*(g*x + f)^{(9/2)}), x)$

[Out]  $2/7*(c^3*d^3*x^3 + 3*a*c^2*d^2*e*x^2 + 3*a^2*c*d*e^2*x + a^3*e^3)*\sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x}*\sqrt{e*x + d}*\sqrt{(g*x + f)}/(c*d^2*f^5 - a*d*e*f^4*g + (c*d^2*e*f^3*g^4 - a*e^2*g^5)*x^5 + (4*c*d^2*e*f^2*g^3 - a*d*e*g^5 + (c*d^2 - 4*a*e^2)*f*g^4)*x^4 + 2*(3*c*d^2*e*f^3*g^2 - 2*a*d^2*e*f^2*g^3 + (2*c*d^2 - 3*a*e^2)*f^2*g^3)*x^3 + 2*(2*c^2*d^2*e*f^4*g - 3*a^2*d^2*e*f^3*g^2 + (3*c^2*d^2 - 2*a^2*e^2)*f^3*g^2)*x^2 + (c^2*d^2*e*f^5 - 4*a^2*d^2*e*f^4*g^2 + (4*c^2*d^2 - a^2*e^2)*f^4*g)*x$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(5/2)}/(e*x+d)^{(5/2)}/(g*x+f)^{(9/2)}, x)$

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g*x + f
```

```
[Out] Exception raised: NotImplementedError
```

$$3.758 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{11/2}} dx$$

**Optimal.** Leaf size=129

$$\frac{4cd(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{63(d+ex)^{7/2}(f+gx)^{7/2}(cdf - aeg)^2} + \frac{2(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{9(d+ex)^{7/2}(f+gx)^{9/2}(cdf - aeg)}$$

[Out] (2\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(7/2))/(9\*(c\*d\*f - a\*e\*g)\*(d + e\*x)^(7/2)\*(f + g\*x)^(9/2)) + (4\*c\*d\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(7/2))/(63\*(c\*d\*f - a\*e\*g)^2\*(d + e\*x)^(7/2)\*(f + g\*x)^(7/2))

**Rubi [A]** time = 0.528807, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{4cd(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{63(d+ex)^{7/2}(f+gx)^{7/2}(cdf - aeg)^2} + \frac{2(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{9(d+ex)^{7/2}(f+gx)^{9/2}(cdf - aeg)}$$

Antiderivative was successfully verified.

[In] Int[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2)/((d + e\*x)^(5/2)\*(f + g\*x)^(11/2)) dx

[Out] (2\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(7/2))/(9\*(c\*d\*f - a\*e\*g)\*(d + e\*x)^(7/2)\*(f + g\*x)^(9/2)) + (4\*c\*d\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(7/2))/(63\*(c\*d\*f - a\*e\*g)^2\*(d + e\*x)^(7/2)\*(f + g\*x)^(7/2))

**Rubi in Sympy [A]** time = 43.1642, size = 122, normalized size = 0.95

$$\frac{4cd(ade + cdex^2 + x(ae^2 + cd^2))^{7/2}}{63(d+ex)^{7/2}(f+gx)^{7/2}(aeg - cdf)^2} - \frac{2(ade + cdex^2 + x(ae^2 + cd^2))^{7/2}}{9(d+ex)^{7/2}(f+gx)^{9/2}(aeg - cdf)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)/(e\*x+d)\*\*(5/2)/(g\*x

[Out] 4\*c\*d\*(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*\*(7/2)/(63\*(d + e\*x)\*\*(7/2)\*(f + g\*x)\*\*(7/2)\*(a\*e\*g - c\*d\*f)\*\*2) - 2\*(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*\*(7/2)/(9\*(d + e\*x)\*\*(7/2)\*(f + g\*x)\*\*(9/2)\*(a\*e\*g - c\*d\*f))

**Mathematica [A]** time = 0.265054, size = 79, normalized size = 0.61

$$\frac{2(ae + cdx)^3 \sqrt{(d+ex)(ae+cdx)}(cd(9f+2gx) - 7aeg)}{63\sqrt{d+ex}(f+gx)^{9/2}(cdf - aeg)^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2)/((d + e\*x)^(5/2)\*(f + g\*x)

[Out] (2\*(a\*e + c\*d\*x)^3\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)]\*(-7\*a\*e\*g + c\*d\*(9\*f + 2\*g\*x)))/(63\*(c\*d\*f - a\*e\*g)^2\*Sqrt[d + e\*x]\*(f + g\*x)^(9/2))

2))

---

**Maple [A]** time = 0.011, size = 99, normalized size = 0.8

$$-\frac{(2cdx + 2ae)(-2xcdg + 7aeg - 9cdf)}{63a^2e^2g^2 - 126acdefg + 63c^2d^2f^2} (cdex^2 + ae^2x + cd^2x + ade)^{\frac{5}{2}} (gx + f)^{-\frac{9}{2}} (ex + d)^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2)/(e\*x+d)^(5/2)/(g\*x+f)^(11/2), x)

[Out]  $-\frac{2}{63} (c*d*x+a*e) * (-2*c*d*g*x+7*a*e*g-9*c*d*f) * (c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^{\frac{5}{2}} / (g*x+f)^{\frac{9}{2}} / (a^2*e^2*g^2-2*a*c*d*e*f*g+c^2*d^2*f^2) / (e*x+d)^{\frac{5}{2}}$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}(gx + f)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2))\*(g\*x + f)^(11/2), x)

[Out] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2))\*(g\*x + f)^(11/2), x)

---

**Fricas [A]** time = 0.303034, size = 863, normalized size = 6.69

$$63(c^2d^3f^7 - 2acd^2ef^6g + a^2de^2f^5g^2 + (c^2d^2ef^2g^5 - 2acde^2fg^6 + a^2e^3g^7)x^6 + (5c^2d^2ef^3g^4 + a^2de^2g^7 + (c^2d^3 - 10acde^2)g^5)x^5 + (5c^2d^2ef^4g^3 + a^2de^2g^7 + (c^2d^3 - 10acde^2)g^5)x^4 + (5c^2d^2ef^5g^2 + a^2de^2g^7 + (c^2d^3 - 10acde^2)g^5)x^3 + (5c^2d^2ef^6g + a^2de^2g^7 + (c^2d^3 - 10acde^2)g^5)x^2 + (5c^2d^2ef^7 + a^2de^2g^7 + (c^2d^3 - 10acde^2)g^5)x + (5c^2d^2ef^8 + a^2de^2g^7 + (c^2d^3 - 10acde^2)g^5))x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2))\*(g\*x + f)^(11/2), x)

[Out]  $\frac{2}{63} (2*c^4*d^4*g*x^4 + 9*a^3*c*d*e^3*f - 7*a^4*e^4*g + (9*c^4*d^4*f - a*c^3*d^3*e*g)*x^3 + 3*(9*a^3*c^3*d^3*e*f - 5*a^2*c^2*d^2*e^2*g)*x^2 + (27*a^2*c^2*d^2*e^2*f - 19*a^3*c*d*e^3*g)*x) * \sqrt{c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x} * \sqrt{e*x + d} * \sqrt{g*x + f} / (c^2*d^3*f^7 - 2*a*c*d^2*e*f^6*g + a^2*d^2*e^2*f^5*g^2 + (c^2*d^2*e*f^3*g^4 + a^2*d^2*e^2*f^4*g^3 + (c^2*d^3 - 10*a*c*d^2*e - 5*a^2*e^3)*f^5*g^2 - 2*a*c*d^2*e - a^2*e^3)*f^4*g^3 - 2*(a*c*d^2*e - a^2*e^3)*f^3*g^4 - 2*(a*c*d^2*e - a^2*e^3)*f^2*g^5 - 2*a*c*d^2*e*f^6*g + a^2*d^2*e^2*f^5*g^2 + (c^2*d^2*e*f^3*g^4 + a^2*d^2*e^2*f^4*g^3 + (c^2*d^3 - 10*a*c*d^2*e - 5*a^2*e^3)*f^5*g^2 - 2*a*c*d^2*e - a^2*e^3)*f^4*g^3 - 2*(a*c*d^2*e - a^2*e^3)*f^3*g^4 - 2*(a*c*d^2*e - a^2*e^3)*f^2*g^5 - 2*a*c*d^2*e*f^6*g + 2*a^2*d^2*e^2*f^5*g^2 + 2*(c^2*d^3 - a*c*d^2*e)*f^5*g^2 - (4*a*c*d^2*e - a^2*e^3)*f^4*g^3 + (c^2*d^2*e*f^3*g^4 + 5*a^2*d^2*e^2*f^4*g^3 + (5*c^2*d^3 - 2*a*c*d^2*e)*f^5*g^2 - 2*a*c*d^2*e - a^2*e^3)*f^6*g - (10*a*c*d^2*e - a^2*e^3)*f^5*g^2)*x$

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f)**`

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g*x + f`

[Out] Exception raised: NotImplementedError

$$3.759 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{13/2}} dx$$

**Optimal.** Leaf size=198

$$\frac{16c^2d^2(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{693(d+ex)^{7/2}(f+gx)^{7/2}(cdf - aeg)^3} + \frac{8cd(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{99(d+ex)^{7/2}(f+gx)^{9/2}(cdf - aeg)^2} + \frac{2(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{11(d+ex)^{7/2}(f+gx)^{11/2}(cdf - aeg)}$$

[Out]  $(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)}) / (11*(c*d*f - a*e*g)^*(d + e*x)^{(7/2)}*(f + g*x)^{(11/2)}) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)}) / (99*(c*d*f - a*e*g)^2*(d + e*x)^{(7/2)}*(f + g*x)^{(9/2)}) + (16*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)}) / (693*(c*d*f - a*e*g)^3*(d + e*x)^{(7/2)}*(f + g*x)^{(7/2)})$

**Rubi [A]** time = 0.809628, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{16c^2d^2(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{693(d+ex)^{7/2}(f+gx)^{7/2}(cdf - aeg)^3} + \frac{8cd(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{99(d+ex)^{7/2}(f+gx)^{9/2}(cdf - aeg)^2} + \frac{2(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{11(d+ex)^{7/2}(f+gx)^{11/2}(cdf - aeg)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)} / ((d + e*x)^{(5/2)}*(f + g*x)^{(13/2)})]$

[Out]  $(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)}) / (11*(c*d*f - a*e*g)^*(d + e*x)^{(7/2)}*(f + g*x)^{(11/2)}) + (8*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)}) / (99*(c*d*f - a*e*g)^2*(d + e*x)^{(7/2)}*(f + g*x)^{(9/2)}) + (16*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)}) / (693*(c*d*f - a*e*g)^3*(d + e*x)^{(7/2)}*(f + g*x)^{(7/2)})$

**Rubi in Sympy [A]** time = 68.8351, size = 190, normalized size = 0.96

$$\frac{16c^2d^2(ade + cdex^2 + x(ae^2 + cd^2))^{7/2}}{693(d+ex)^{7/2}(f+gx)^{7/2}(aeg - cdf)^3} + \frac{8cd(ade + cdex^2 + x(ae^2 + cd^2))^{7/2}}{99(d+ex)^{7/2}(f+gx)^{9/2}(aeg - cdf)^2} - \frac{2(ade + cdex^2 + x(ae^2 + cd^2))^{7/2}}{11(d+ex)^{7/2}(f+gx)^{11/2}(aeg - cdf)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a*d*e + (a*e^2 + c*d^2)*x + c*d*e*x^2)^{(5/2)} / ((d + e*x)^{(5/2)}*(f + g*x)^{(13/2)}))$

[Out]  $-16*c^2*d^2*(a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2))^{(7/2)} / (693*(d + e*x)^{(7/2)}*(f + g*x)^{(7/2)}*(a*e*g - c*d*f)^3) + 8*c*d*(a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2))^{(7/2)} / (99*(d + e*x)^{(7/2)}*(f + g*x)^{(9/2)}*(a*e*g - c*d*f)^2) - 2*(a*d*e + c*d*e*x^2 + x*(a*e^2 + c*d^2))^{(7/2)} / (11*(d + e*x)^{(7/2)}*(f + g*x)^{(11/2)}*(a*e*g - c*d*f))$

**Mathematica [A]** time = 0.337855, size = 115, normalized size = 0.58

$$\frac{2(ae + cdx)^3 \sqrt{(d + ex)(ae + cdx)} (63a^2e^2g^2 - 14acdeg(11f + 2gx) + c^2d^2(99f^2 + 44fgx + 8g^2x^2))}{693\sqrt{d + ex}(f + gx)^{11/2}(cdf - aeg)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2)/((d + e\*x)^(5/2)\*(f + g\*x

[Out] (2\*(a\*e + c\*d\*x)^3\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)]\*(63\*a^2\*e^2\*g^2 - 14\*a\*c\*d\*e\*g\*(11\*f + 2\*g\*x) + c^2\*d^2\*(99\*f^2 + 44\*f\*g\*x + 8\*g^2\*x^2)))/(693\*(c\*d\*f - a\*e\*g)^3\*Sqrt[d + e\*x]\*(f + g\*x)^(11/2))

**Maple [A]** time = 0.013, size = 169, normalized size = 0.9

$$\frac{(2cdx + 2ae)(8c^2d^2g^2x^2 - 28acdeg^2x + 44c^2d^2fgx + 63a^2e^2g^2 - 154acdefg + 99c^2d^2f^2)}{693a^3e^3g^3 - 2079a^2cde^2fg^2 + 2079ac^2d^2ef^2g - 693c^3d^3f^3} (cdex^2 + ae^2x + cd^2x + ade$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2)/(e\*x+d)^(5/2)/(g\*x+f)^(13/2), x)

[Out] -2/693\*(c\*d\*x+a\*e)\*(8\*c^2\*d^2\*g^2\*x^2-28\*a\*c\*d\*e\*g^2\*x+44\*c^2\*d^2\*f\*g\*x+63\*a^2\*e^2\*g^2-154\*a\*c\*d\*e\*f\*g+99\*c^2\*d^2\*f^2)\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(5/2)/(g\*x+f)^(11/2)/(a^3\*e^3\*g^3-3\*a^2\*c\*d\*e^2\*f\*g+3\*a\*c^2\*d^2\*e\*f^2\*g-c^3\*d^3\*f^3)/(e\*x+d)^(5/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}(gx + f)^{\frac{13}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2)\*(g\*x + f

[Out] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2)\*(g\*x + f)^(13/2)), x)

**Fricas [A]** time = 0.313847, size = 1486, normalized size = 7.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2)\*(g\*x + f

[Out] 2/693\*(8\*c^5\*d^5\*g^2\*x^5 + 99\*a^3\*c^2\*d^2\*e^3\*f^2 - 154\*a^4\*c\*d\*e^4\*f\*g + 63\*a^5\*e^5\*g^2 + 4\*(11\*c^5\*d^5\*f\*g - a\*c^4\*d^4\*e\*g^2)\*x^4 + (99\*c^5\*d^5\*f^2 - 22\*a\*c^4\*d^4\*e\*f\*g + 3\*a^2\*c^3\*d^3\*e^2\*g^2)\*x^3 + (297\*a\*c^4\*d^4\*e\*f^2 - 330\*a^2\*c^3\*d^3\*e^2\*f\*g + 113\*a^3\*c^2\*d^2\*e^3\*g^2)\*x^2 + (297\*a^2\*c^3\*d^3\*e^2\*f^2 - 418\*a^3\*c^2\*d^2\*e^3\*f\*g + 161\*a^4\*c\*d\*e^4\*g^2)\*x)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*sqrt(g\*x + f)/(c^3\*d^4\*f^9 - 3\*a\*c^2\*d



$$\begin{aligned}
& ^3e^*f^8*g + 3*a^2*c*d^2*e^2*f^7*g^2 - a^3*d*e^3*f^6*g^3 + (c^3*d \\
& ^3*e^*f^3*g^6 - 3*a*c^2*d^2*e^2*f^2*g^7 + 3*a^2*c*d*e^3*f*g^8 - a^ \\
& ^3*e^4*g^9)*x^7 + (6*c^3*d^3*e*f^4*g^5 - a^3*d*e^3*g^9 + (c^3*d^4 \\
& - 18*a*c^2*d^2*e^2)*f^3*g^6 - 3*(a*c^2*d^3*e - 6*a^2*c*d*e^3)*f^2 \\
& *g^7 + 3*(a^2*c*d^2*e^2 - 2*a^3*e^4)*f*g^8)*x^6 + 3*(5*c^3*d^3*e* \\
& f^5*g^4 - 2*a^3*d*e^3*f*g^8 + (2*c^3*d^4 - 15*a*c^2*d^2*e^2)*f^4* \\
& g^5 - 3*(2*a*c^2*d^3*e - 5*a^2*c*d*e^3)*f^3*g^6 + (6*a^2*c*d^2*e^2 \\
& - 5*a^3*e^4)*f^2*g^7)*x^5 + 5*(4*c^3*d^3*e*f^6*g^3 - 3*a^3*d*e^ \\
& ^3*f^2*g^7 + 3*(c^3*d^4 - 4*a*c^2*d^2*e^2)*f^5*g^4 - 3*(3*a*c^2*d^ \\
& ^3*e - 4*a^2*c*d*e^3)*f^4*g^5 + (9*a^2*c*d^2*e^2 - 4*a^3*e^4)*f^3* \\
& g^6)*x^4 + 5*(3*c^3*d^3*e*f^7*g^2 - 4*a^3*d*e^3*f^3*g^6 + (4*c^3* \\
& d^4 - 9*a*c^2*d^2*e^2)*f^6*g^3 - 3*(4*a*c^2*d^3*e - 3*a^2*c*d*e^3 \\
& )*f^5*g^4 + 3*(4*a^2*c*d^2*e^2 - a^3*e^4)*f^4*g^5)*x^3 + 3*(2*c^3 \\
& *d^3*e*f^8*g - 5*a^3*d*e^3*f^4*g^5 + (5*c^3*d^4 - 6*a*c^2*d^2*e^2 \\
& )*f^7*g^2 - 3*(5*a*c^2*d^3*e - 2*a^2*c*d*e^3)*f^6*g^3 + (15*a^2*c \\
& *d^2*e^2 - 2*a^3*e^4)*f^5*g^4)*x^2 + (c^3*d^3*e*f^9 - 6*a^3*d*e^3 \\
& *f^5*g^4 + 3*(2*c^3*d^4 - a*c^2*d^2*e^2)*f^8*g - 3*(6*a*c^2*d^3*e \\
& - a^2*c*d*e^3)*f^7*g^2 + (18*a^2*c*d^2*e^2 - a^3*e^4)*f^6*g^3)*x \\
& )
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)/(e\*x+d)\*\*(5/2)/(g\*x+f)\*\*

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2)\*(g\*x + f

[Out] Exception raised: NotImplementedError

$$3.760 \quad \int \frac{(ade + (cd^2 + ae^2)x + cdex^2)^{5/2}}{(d+ex)^{5/2}(f+gx)^{15/2}} dx$$

**Optimal.** Leaf size=267

$$\frac{32c^3d^3(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{3003(d+ex)^{7/2}(f+gx)^{7/2}(cdf - aeg)^4} + \frac{16c^2d^2(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{429(d+ex)^{7/2}(f+gx)^{9/2}(cdf - aeg)^3} \\ + \frac{12cd(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{143(d+ex)^{7/2}(f+gx)^{11/2}(cdf - aeg)^2} + \frac{2(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{13(d+ex)^{7/2}(f+gx)^{13/2}(cdf - aeg)}$$

[Out]  $(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)})/(13*(c*d*f - a*e*g)*(d + e*x)^{(7/2)}*(f + g*x)^{(13/2)}) + (12*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)})/(143*(c*d*f - a*e*g)^2*(d + e*x)^{(7/2)}*(f + g*x)^{(11/2)}) + (16*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)})/(429*(c*d*f - a*e*g)^3*(d + e*x)^{(7/2)}*(f + g*x)^{(9/2)}) + (32*c^3*d^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)})/(3003*(c*d*f - a*e*g)^4*(d + e*x)^{(7/2)}*(f + g*x)^{(7/2)})$

**Rubi [A]** time = 1.12172, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 48,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$

$$\frac{32c^3d^3(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{3003(d+ex)^{7/2}(f+gx)^{7/2}(cdf - aeg)^4} + \frac{16c^2d^2(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{429(d+ex)^{7/2}(f+gx)^{9/2}(cdf - aeg)^3} \\ + \frac{12cd(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{143(d+ex)^{7/2}(f+gx)^{11/2}(cdf - aeg)^2} + \frac{2(x(ae^2 + cd^2) + ade + cdex^2)^{7/2}}{13(d+ex)^{7/2}(f+gx)^{13/2}(cdf - aeg)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(5/2)}/((d + e*x)^{(5/2)}*(f + g*x)^{(15/2)})]$

[Out]  $(2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)})/(13*(c*d*f - a*e*g)*(d + e*x)^{(7/2)}*(f + g*x)^{(13/2)}) + (12*c*d*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)})/(143*(c*d*f - a*e*g)^2*(d + e*x)^{(7/2)}*(f + g*x)^{(11/2)}) + (16*c^2*d^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)})/(429*(c*d*f - a*e*g)^3*(d + e*x)^{(7/2)}*(f + g*x)^{(9/2)}) + (32*c^3*d^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(7/2)})/(3003*(c*d*f - a*e*g)^4*(d + e*x)^{(7/2)}*(f + g*x)^{(7/2)})$

**Rubi in Sympy [A]** time = 98.5494, size = 258, normalized size = 0.97

$$\frac{32c^3d^3(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{7}{2}}}{3003(d+ex)^{\frac{7}{2}}(f+gx)^{\frac{7}{2}}(aeg - cdf)^4} - \frac{16c^2d^2(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{7}{2}}}{429(d+ex)^{\frac{7}{2}}(f+gx)^{\frac{9}{2}}(aeg - cdf)^3} \\ + \frac{12cd(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{7}{2}}}{143(d+ex)^{\frac{7}{2}}(f+gx)^{\frac{11}{2}}(aeg - cdf)^2} - \frac{2(ade + cdex^2 + x(ae^2 + cd^2))^{\frac{7}{2}}}{13(d+ex)^{\frac{7}{2}}(f+gx)^{\frac{13}{2}}(aeg - cdf)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x$

[Out]  $32*c**3*d**3*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(7/2)/(3003*(d + e*x)**(7/2)*(f + g*x)**(7/2)*(a*e*g - c*d*f)**4) - 16*c**2*d**2*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(7/2)/(429*(d + e*x)**(7/2)*(f + g*x)**(9/2)*(a*e*g - c*d*f)**3) + 12*c*d*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(7/2)/(143*(d + e*x)**(7/2)*(f + g*x)**(11/2)*(a*e*g - c*d*f)**2) - 2*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(7/2)/(13*(d + e*x)**(7/2)*(f + g*x)**(13$

$$/2) * (a * e * g - c * d * f))$$

**Mathematica [A]** time = 0.405057, size = 162, normalized size = 0.61

$$\frac{2(ae + cdx)^3 \sqrt{(d + ex)(ae + cdx)} (-231a^3 e^3 g^3 + 63a^2 c d e^2 g^2 (13f + 2gx) - 7ac^2 d^2 e g (143f^2 + 52fgx + 8g^2 x^2) + c^3 d^3 (429f^3 + 52fgx + 8g^2 x^2))}{3003 \sqrt{d + ex} (f + gx)^{13/2} (cdf - aeg)^4}$$

Antiderivative was successfully verified.

[In] Integrate[(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2)/((d + e\*x)^(5/2)\*(f + g\*x

[Out] (2\*(a\*e + c\*d\*x)^3\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)]\*(-231\*a^3\*e^3\*g^3 + 63\*a^2\*c\*d\*e^2\*g^2\*(13\*f + 2\*g\*x) - 7\*a\*c^2\*d^2\*e\*g\*(143\*f^2 + 52\*f\*g\*x + 8\*g^2\*x^2) + c^3\*d^3\*(429\*f^3 + 286\*f^2\*g\*x + 104\*f\*g^2\*x^2 + 16\*g^3\*x^3)))/(3003\*(c\*d\*f - a\*e\*g)^4\*Sqrt[d + e\*x]\*(f + g\*x)^(13/2))

**Maple [A]** time = 0.017, size = 260, normalized size = 1.

$$\frac{(2cdx + 2ae)(-16c^3d^3g^3x^3 + 56ac^2d^2eg^3x^2 - 104c^3d^3fg^2x^2 - 126a^2cde^2g^3x + 364ac^2d^2efg^2x - 286c^3d^3f^2gx + 231a^3d^3f^2g^2x^2)}{3003g^4e^4a^4 - 12012cdg^3fe^3a^3 + 18018c^2d^2g^2f^2e^2a^2 - 12012c^3d^3gf^3ea + 3003d^4f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2)/((e\*x+d)^(5/2)/(g\*x+f)^(15/2)), x)

[Out] -2/3003\*(c\*d\*x+a\*e)\*(-16\*c^3\*d^3\*g^3\*x^3+56\*a\*c^2\*d^2\*e\*g^3\*x^2-104\*c^3\*d^3\*f\*g^2\*x^2-126\*a^2\*c\*d\*e^2\*g^3\*x+364\*a\*c^2\*d^2\*e\*f\*g^2\*x-286\*c^3\*d^3\*f^2\*g\*x+231\*a^3\*e^3\*g^3-819\*a^2\*c\*d\*e^2\*f\*g^2+1001\*a\*c^2\*d^2\*e\*f^2\*g-429\*c^3\*d^3\*f^3)\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(5/2)/(g\*x+f)^(13/2)/(a^4\*e^4\*g^4-4\*a^3\*c\*d\*e^3\*f\*g^3+6\*a^2\*c^2\*d^2\*e^2\*f^2\*g^2-4\*a\*c^3\*d^3\*e\*f^3\*g+c^4\*d^4\*f^4)/(e\*x+d)^(5/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}}{(ex + d)^{\frac{5}{2}}(gx + f)^{\frac{15}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2)\*(g\*x + f)^(15/2)), x)

[Out] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2)\*(g\*x + f)^(15/2)), x)

**Fricas [A]** time = 0.338476, size = 2225, normalized size = 8.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)/((e\*x + d)^(5/2)\*(g\*x + f)^(15/2)), x)

```
[Out] 2/3003*(16*c^6*d^6*g^3*x^6 + 429*a^3*c^3*d^3*e^3*f^3 - 1001*a^4*c^2*d^2*e^4*f^2*g + 819*a^5*c*d*e^5*f*g^2 - 231*a^6*e^6*g^3 + 8*(13*c^6*d^6*f*g^2 - a*c^5*d^5*e*g^3)*x^5 + 2*(143*c^6*d^6*f^2*g - 26*a*c^5*d^5*e*f*g^2 + 3*a^2*c^4*d^4*e^2*g^3)*x^4 + (429*c^6*d^6*f^3 - 143*a*c^5*d^5*e*f^2*g + 39*a^2*c^4*d^4*e^2*f*g^2 - 5*a^3*c^3*d^3*e^3*g^3)*x^3 + (1287*a*c^5*d^5*e*f^3 - 2145*a^2*c^4*d^4*e^2*f^2*g + 1469*a^3*c^3*d^3*e^3*f*g^2 - 371*a^4*c^2*d^2*e^4*g^3)*x^2 + (1287*a^2*c^4*d^4*e^2*f^3 - 2717*a^3*c^3*d^3*e^3*f^2*g + 2093*a^4*c^2*d^2*e^4*f*g^2 - 567*a^5*c*d*e^5*g^3)*x)*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*sqrt(g*x + f)/(c^4*d^5*f^11 - 4*a*c^3*d^4*e*f^10*g + 6*a^2*c^2*d^3*e^2*f^9*g^2 - 4*a^3*c*d^2*e^3*f^8*g^3 + a^4*d*e^4*f^7*g^4 + (c^4*d^4*e*f^4*g^7 - 4*a*c^3*d^3*e^2*f^3*g^8 + 6*a^2*c^2*d^2*e^3*f^2*g^9 - 4*a^3*c*d*e^4*f*g^10 + a^4*e^5*g^11)*x^8 + (7*c^4*d^4*e*f^5*g^6 + a^4*d*e^4*g^11 + (c^4*d^5 - 28*a*c^3*d^3*e^2)*f^4*g^7 - 2*(2*a*c^3*d^4*e - 21*a^2*c^2*d^2*e^3)*f^3*g^8 + 2*(3*a^2*c^2*d^3*e^2 - 14*a^3*c*d*e^4)*f^2*g^9 - (4*a^3*c*d^2*e^3 - 7*a^4*e^5)*f*g^10)*x^7 + 7*(3*c^4*d^4*e*f^6*g^5 + a^4*d*e^4*f*g^10 + (c^4*d^5 - 12*a*c^3*d^3*e^2)*f^5*g^6 - 2*(2*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^4*g^7 + 6*(a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^3*g^8 - (4*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^2*g^9)*x^6 + 7*(5*c^4*d^4*e*f^7*g^4 + 3*a^4*d*e^4*f^2*g^9 + (3*c^4*d^5 - 20*a*c^3*d^3*e^2)*f^6*g^5 - 6*(2*a*c^3*d^4*e - 5*a^2*c^2*d^2*e^3)*f^5*g^6 + 2*(9*a^2*c^2*d^3*e^2 - 10*a^3*c*d*e^4)*f^4*g^7 - (12*a^3*c*d^2*e^3 - 5*a^4*e^5)*f^3*g^8)*x^5 + 35*(c^4*d^4*e*f^8*g^3 + a^4*d*e^4*f^3*g^8 + (c^4*d^5 - 4*a*c^3*d^3*e^2)*f^7*g^4 - 2*(2*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^6*g^5 + 2*(3*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^5*g^6 - (4*a^3*c*d^2*e^3 - a^4*e^5)*f^4*g^7)*x^4 + 7*(3*c^4*d^4*e*f^9*g^2 + 5*a^4*d*e^4*f^4*g^7 + (5*c^4*d^5 - 12*a*c^3*d^3*e^2)*f^8*g^3 - 2*(10*a*c^3*d^4*e - 9*a^2*c^2*d^2*e^3)*f^7*g^4 + 6*(5*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^6*g^5 - (20*a^3*c*d^2*e^3 - 3*a^4*e^5)*f^5*g^6)*x^3 + 7*(c^4*d^4*e*f^10*g + 3*a^4*d*e^4*f^5*g^6 + (3*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^9*g^2 - 6*(2*a*c^3*d^4*e - a^2*c^2*d^2*e^3)*f^8*g^3 + 2*(9*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^7*g^4 - (12*a^3*c*d^2*e^3 - a^4*e^5)*f^6*g^5)*x^2 + (c^4*d^4*e*f^11 + 7*a^4*d*e^4*f^6*g^5 + (7*c^4*d^5 - 4*a*c^3*d^3*e^2)*f^10*g - 2*(14*a*c^3*d^4*e - 3*a^2*c^2*d^2*e^3)*f^9*g^2 + 2*(21*a^2*c^2*d^3*e^2 - 2*a^3*c*d*e^4)*f^8*g^3 - (28*a^3*c*d^2*e^3 - a^4*e^5)*f^7*g^4)*x)
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(5/2)/(e*x+d)**(5/2)/(g*x+f)**
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)/((e*x + d)^(5/2)*(g*x + f
```

```
[Out] Exception raised: NotImplementedError
```

$$3.761 \quad \int \frac{(d+ex)^{5/2}(f+gx)^n}{(ade+(cd^2+ae^2)x+cdex^2)^{5/2}} dx$$

**Optimal.** Leaf size=104

$$\frac{(d+ex)^{5/2}(f+gx)^{n+1}(ae+cdx) {}_2F_1\left(1, n-\frac{1}{2}; n+2; \frac{cd(f+gx)}{cdf-ae g}\right)}{(n+1)(x(ae^2+cd^2)+ade+cdex^2)^{5/2}(cdf-ae g)}$$

[Out] -(((a\*e + c\*d\*x)\*(d + e\*x)^(5/2)\*(f + g\*x)^(1 + n)\*Hypergeometric2F1[1, -1/2 + n, 2 + n, (c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g)]/(c\*d\*f - a\*e\*g)\*(1 + n)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2)))

**Rubi [A]** time = 0.369578, antiderivative size = 122, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{2\sqrt{d+ex}(f+gx)^n \left(\frac{cd(f+gx)}{cdf-ae g}\right)^{-n} {}_2F_1\left(-\frac{3}{2}, -n; -\frac{1}{2}; -\frac{g(ae+cdx)}{cdf-ae g}\right)}{3cd(ae+cdx)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^(5/2)\*(f + g\*x)^n)/(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2), x

[Out] (-2\*sqrt[d + e\*x]\*(f + g\*x)^n\*Hypergeometric2F1[-3/2, -n, -1/2, -(g\*(a\*e + c\*d\*x))/(c\*d\*f - a\*e\*g)]/(3\*c\*d\*(a\*e + c\*d\*x)\*((c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g))^n\*sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])

**Rubi in Sympy [A]** time = 61.6644, size = 114, normalized size = 1.1

$$\frac{2 \left(\frac{cd(-f-gx)}{ae g-cdf}\right)^{-n} (f+gx)^n \sqrt{ade+cdex^2+x(ae^2+cd^2)} {}_2F_1\left(-n, -\frac{3}{2}; -\frac{1}{2}; \frac{g(ae+cdx)}{ae g-cdf}\right)}{3cd\sqrt{d+ex}(ae+cdx)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*(5/2)\*(g\*x+f)\*\*n/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*

[Out] -2\*(c\*d\*(-f - g\*x)/(a\*e\*g - c\*d\*f))\*\*(-n)\*(f + g\*x)\*\*n\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*hyper((-n, -3/2), (-1/2, ), g\*(a\*e + c\*d\*x)/(a\*e\*g - c\*d\*f))/(3\*c\*d\*sqrt(d + e\*x)\*(a\*e + c\*d\*x)\*\*2)

**Mathematica [A]** time = 0.208506, size = 100, normalized size = 0.96

$$\frac{2(d+ex)^{3/2}(f+gx)^n \left(\frac{cd(f+gx)}{cdf-ae g}\right)^{-n} {}_2F_1\left(-\frac{3}{2}, -n; -\frac{1}{2}; \frac{g(ae+cdx)}{ae g-cdf}\right)}{3cd((d+ex)(ae+cdx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^(5/2)\*(f + g\*x)^n)/(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2), x

[Out] (-2\*(d + e\*x)^(3/2)\*(f + g\*x)^n\*Hypergeometric2F1[-3/2, -n, -1/2, (g\*(a\*e + c\*d\*x))/(-c\*d\*f + a\*e\*g)]/(3\*c\*d\*((a\*e + c\*d\*x)\*(d

$$+ e^*x))^{(3/2)} * ((c*d*(f + g*x))/(c*d*f - a*e*g))^{n}$$

**Maple [F]** time = 0.157, size = 0, normalized size = 0.

$$\int (gx + f)^n (ex + d)^{\frac{5}{2}} (ade + (ae^2 + cd^2)x + cdex^2)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(5/2)\*(g\*x+f)^n/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2),x)

[Out] int((e\*x+d)^(5/2)\*(g\*x+f)^n/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^{\frac{5}{2}}(gx + f)^n}{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(5/2)\*(g\*x + f)^n/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2), x)

[Out] integrate((e\*x + d)^(5/2)\*(g\*x + f)^n/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex + d}(gx + f)^n}{(c^2d^2x^2 + 2acdex + a^2e^2)\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(5/2)\*(g\*x + f)^n/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2), x)

[Out] integral(sqrt(e\*x + d)\*(g\*x + f)^n/((c^2\*d^2\*x^2 + 2\*a\*c\*d\*e\*x + a^2\*e^2)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(5/2)\*(g\*x+f)\*\*n/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2), x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.805773, size = 4, normalized size = 0.04

sage<sub>0</sub>x

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^(5/2)*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)
```

```
[Out] sage0*x
```

$$3.762 \quad \int \frac{(d+ex)^{3/2}(f+gx)^n}{(ade+(cd^2+ae^2)x+cdex^2)^{3/2}} dx$$

**Optimal.** Leaf size=104

$$\frac{(d+ex)^{3/2}(f+gx)^{n+1}(ae+cdx) {}_2F_1\left(1, n+\frac{1}{2}; n+2; \frac{cd(f+gx)}{cdf-aeg}\right)}{(n+1)(x(ae^2+cd^2)+ade+cdex^2)^{3/2}(cdf-aeg)}$$

[Out] -(((a\*e + c\*d\*x)\*(d + e\*x)^(3/2)\*(f + g\*x)^(1 + n)\*Hypergeometric2F1[1, 1/2 + n, 2 + n, (c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g)])/((c\*d\*f - a\*e\*g)^(1 + n)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)))

**Rubi [A]** time = 0.353044, antiderivative size = 110, normalized size of antiderivative = 1.06, number of steps used = 3, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{2\sqrt{d+ex}(f+gx)^n \left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n} {}_2F_1\left(-\frac{1}{2}, -n; \frac{1}{2}; -\frac{g(ae+cdx)}{cdf-aeg}\right)}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^(3/2)\*(f + g\*x)^n)/(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2), x

[Out] (-2\*Sqrt[d + e\*x]\*(f + g\*x)^n\*Hypergeometric2F1[-1/2, -n, 1/2, -(g\*(a\*e + c\*d\*x))/(c\*d\*f - a\*e\*g)])/(c\*d\*((c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g))^n\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])

**Rubi in Sympy [A]** time = 61.5166, size = 109, normalized size = 1.05

$$\frac{2 \left(\frac{cd(-f-gx)}{aeg-cdf}\right)^{-n} (f+gx)^n \sqrt{ade+cdex^2+x(ae^2+cd^2)} {}_2F_1\left(-n, -\frac{1}{2}; \frac{1}{2}; \frac{g(ae+cdx)}{aeg-cdf}\right)}{cd\sqrt{d+ex}(ae+cdx)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*(3/2)\*(g\*x+f)\*\*n/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*

[Out] -2\*(c\*d\*(-f - g\*x)/(a\*e\*g - c\*d\*f))\*\*(-n)\*(f + g\*x)\*\*n\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*hyper((-n, -1/2), (1/2, ), g\*(a\*e + c\*d\*x)/(a\*e\*g - c\*d\*f))/(c\*d\*sqrt(d + e\*x)\*(a\*e + c\*d\*x))

**Mathematica [A]** time = 0.0975129, size = 98, normalized size = 0.94

$$\frac{2\sqrt{d+ex}(f+gx)^n \left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n} {}_2F_1\left(-\frac{1}{2}, -n; \frac{1}{2}; \frac{g(ae+cdx)}{aeg-cdf}\right)}{cd\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^(3/2)\*(f + g\*x)^n)/(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2), x

[Out] (-2\*Sqrt[d + e\*x]\*(f + g\*x)^n\*Hypergeometric2F1[-1/2, -n, 1/2, (g\*(a\*e + c\*d\*x))/(-(c\*d\*f) + a\*e\*g)])/(c\*d\*sqrt[(a\*e + c\*d\*x)\*(d +



$$e^*x)]^*((c*d*(f + g*x))/(c*d*f - a*e*g))^n)$$

**Maple [F]** time = 0.126, size = 0, normalized size = 0.

$$\int (gx + f)^n (ex + d)^{\frac{3}{2}} (ade + (ae^2 + cd^2)x + cdex^2)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(3/2)\*(g\*x+f)^n/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2), x)

[Out] int((e\*x+d)^(3/2)\*(g\*x+f)^n/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^{\frac{3}{2}}(gx + f)^n}{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*(g\*x + f)^n/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2), x)

[Out] integrate((e\*x + d)^(3/2)\*(g\*x + f)^n/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex + d}(gx + f)^n}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(cdx + ae)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*(g\*x + f)^n/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2), x)

[Out] integral(sqrt(e\*x + d)\*(g\*x + f)^n/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(c\*d\*x + a\*e)), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(3/2)\*(g\*x+f)\*\*n/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2), x)

[Out] Timed out

**GIAC/XCAS [A]** time = 1.064, size = 4, normalized size = 0.04

sage<sub>0</sub>x

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^(3/2)*(g*x + f)^n/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(3/2)
```

```
[Out] sage0*x
```

$$3.763 \quad \int \frac{\sqrt{d+ex}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=104

$$\frac{\sqrt{d+ex}(f+gx)^{n+1}(ae+cdx) {}_2F_1\left(1, n+\frac{3}{2}; n+2; \frac{cd(f+gx)}{cdf-aeg}\right)}{(n+1)\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)}$$

[Out] -(((a\*e + c\*d\*x)\*Sqrt[d + e\*x]\*(f + g\*x)^(1 + n)\*Hypergeometric2F1[1, 3/2 + n, 2 + n, (c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g)])/((c\*d\*f - a\*e\*g)\*(1 + n)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]))

**Rubi [A]** time = 0.34536, antiderivative size = 118, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{2\sqrt{d+ex}(f+gx)^n(ae+cdx)\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n} {}_2F_1\left(\frac{1}{2}, -n; \frac{3}{2}; -\frac{g(ae+cdx)}{cdf-aeg}\right)}{cd\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[d + e\*x]\*(f + g\*x)^n)/Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2], x]

[Out] (2\*(a\*e + c\*d\*x)\*Sqrt[d + e\*x]\*(f + g\*x)^n\*Hypergeometric2F1[1/2, -n, 3/2, -((g\*(a\*e + c\*d\*x))/(c\*d\*f - a\*e\*g))]/(c\*d\*((c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g))^n\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]))

**Rubi in Sympy [A]** time = 61.8758, size = 97, normalized size = 0.93

$$\frac{2\left(\frac{cd(-f-gx)}{aeg-cdf}\right)^{-n}(f+gx)^n\sqrt{ade+cdex^2+x(ae^2+cd^2)} {}_2F_1\left(-n, \frac{1}{2}; \frac{3}{2}; \frac{g(ae+cdx)}{aeg-cdf}\right)}{cd\sqrt{d+ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*(1/2)\*(g\*x+f)\*\*n/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*

[Out] 2\*(c\*d\*(-f - g\*x)/(a\*e\*g - c\*d\*f))\*\*(-n)\*(f + g\*x)\*\*n\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*hyper((-n, 1/2), (3/2, ), g\*(a\*e + c\*d\*x)/(a\*e\*g - c\*d\*f))/(c\*d\*sqrt(d + e\*x))

**Mathematica [A]** time = 0.0949758, size = 98, normalized size = 0.94

$$\frac{2(f+gx)^n\sqrt{(d+ex)(ae+cdx)}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n} {}_2F_1\left(\frac{1}{2}, -n; \frac{3}{2}; \frac{g(ae+cdx)}{aeg-cdf}\right)}{cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[d + e\*x]\*(f + g\*x)^n)/Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]

[Out] (2\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)]\*(f + g\*x)^n\*Hypergeometric2F1[1/2, -n, 3/2, (g\*(a\*e + c\*d\*x))/(-(c\*d\*f) + a\*e\*g)]/(c\*d\*Sqrt[d +

$$e^*x)^*((c*d*(f + g*x))/(c*d*f - a*e*g))^n)$$

**Maple [F]** time = 0.11, size = 0, normalized size = 0.

$$\int (gx + f)^n \sqrt{ex + d} \frac{1}{\sqrt{ade + (ae^2 + cd^2)x + cdex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(1/2)\*(g\*x+f)^n/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2), x)

[Out] int((e\*x+d)^(1/2)\*(g\*x+f)^n/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex + d}(gx + f)^n}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*(g\*x + f)^n/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

[Out] integrate(sqrt(e\*x + d)\*(g\*x + f)^n/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{ex + d}(gx + f)^n}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*(g\*x + f)^n/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

[Out] integral(sqrt(e\*x + d)\*(g\*x + f)^n/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(1/2)\*(g\*x+f)\*\*n/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)

[Out] Timed out

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}(gx+f)^n}{\sqrt{cdex^2+ade+(cd^2+ae^2)x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)\*(g\*x + f)^n/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x

[Out] integrate(sqrt(e\*x + d)\*(g\*x + f)^n/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

$$3.764 \quad \int \frac{(f+gx)^n \sqrt{ade+(cd^2+ae^2)x+cdex^2}}{\sqrt{d+ex}} dx$$

**Optimal.** Leaf size=104

$$\frac{(f+gx)^{n+1}(ae+cdx)\sqrt{x(ae^2+cd^2)+ade+cdex^2} {}_2F_1\left(1, n+\frac{5}{2}; n+2; \frac{cd(f+gx)}{cdf-aeg}\right)}{(n+1)\sqrt{d+ex}(cdf-aeg)}$$

[Out] -(((a\*e + c\*d\*x)\*(f + g\*x)^(1 + n)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])\*Hypergeometric2F1[1, 5/2 + n, 2 + n, (c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g)]/(c\*d\*f - a\*e\*g))/((c\*d\*f - a\*e\*g)\*(1 + n)\*Sqrt[d + e\*x])

**Rubi [A]** time = 0.339298, antiderivative size = 120, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{2(f+gx)^n(ae+cdx)\sqrt{x(ae^2+cd^2)+ade+cdex^2} \left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n} {}_2F_1\left(\frac{3}{2}, -n; \frac{5}{2}; -\frac{g(ae+cdx)}{cdf-aeg}\right)}{3cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[((f + g\*x)^n\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/Sqrt[d + e\*x], x]

[Out] (2\*(a\*e + c\*d\*x)\*(f + g\*x)^n\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])\*Hypergeometric2F1[3/2, -n, 5/2, -(g\*(a\*e + c\*d\*x))/(c\*d\*f - a\*e\*g)]/(3\*c\*d\*Sqrt[d + e\*x]\*((c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g))^n)

**Rubi in Sympy [A]** time = 62.0315, size = 107, normalized size = 1.03

$$\frac{2\left(\frac{cd(-f-gx)}{aeg-cdf}\right)^{-n} (f+gx)^n (ae+cdx)\sqrt{ade+cdex^2+x(ae^2+cd^2)} {}_2F_1\left(-n, \frac{3}{2}; \frac{5}{2}; \frac{g(ae+cdx)}{aeg-cdf}\right)}{3cd\sqrt{d+ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*n\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/(e\*x+d)\*\*

[Out] 2\*(c\*d\*(-f - g\*x)/(a\*e\*g - c\*d\*f))\*\*(-n)\*(f + g\*x)\*\*n\*(a\*e + c\*d\*x)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*hyper((-n, 3/2), (5/2, ), g\*(a\*e + c\*d\*x)/(a\*e\*g - c\*d\*f))/(3\*c\*d\*sqrt(d + e\*x))

**Mathematica [A]** time = 0.124929, size = 100, normalized size = 0.96

$$\frac{2(f+gx)^n((d+ex)(ae+cdx))^{3/2} \left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n} {}_2F_1\left(\frac{3}{2}, -n; \frac{5}{2}; \frac{g(ae+cdx)}{aeg-cdf}\right)}{3cd(d+ex)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((f + g\*x)^n\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/Sqrt[d + e\*x]

[Out] (2\*((a\*e + c\*d\*x)\*(d + e\*x))^(3/2)\*(f + g\*x)^n\*Hypergeometric2F1[3/2, -n, 5/2, (g\*(a\*e + c\*d\*x))/(-(c\*d\*f) + a\*e\*g)]/(3\*c\*d\*(d +

$$e^x)^{3/2} * ((c*d*(f + g*x))/(c*d*f - a*e*g))^n$$

**Maple [F]** time = 0.112, size = 0, normalized size = 0.

$$\int (gx + f)^n \sqrt{ade + (ae^2 + cd^2)x + cdex^2} \frac{1}{\sqrt{ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^n\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)/(e\*x+d)^(1/2), x)

[Out] int((g\*x+f)^n\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2)/(e\*x+d)^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(gx + f)^n}{\sqrt{ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^n/sqrt(e\*x + d), x)

[Out] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^n/sqrt(e\*x + d), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(gx + f)^n}{\sqrt{ex + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^n/sqrt(e\*x + d), x)

[Out] integral(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^n/sqrt(e\*x + d), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*n\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)/(e\*x+d)\*\*(1/2), x)

[Out] Timed out

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(gx + f)^n}{\sqrt{ex + d}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^n/sqrt(e\*x + d), x

[Out] integrate(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^n/sqrt(e\*x + d), x)



$$3.765 \quad \int \frac{(f+gx)^n (ade + (cd^2 + ae^2)x + cdex^2)^{3/2}}{(d+ex)^{3/2}} dx$$

**Optimal.** Leaf size=104

$$\frac{(f+gx)^{n+1} (ae+cdx) (x(ae^2+cd^2)+ade+cdex^2)^{3/2} {}_2F_1\left(1, n+\frac{7}{2}; n+2; \frac{cd(f+gx)}{cdf-aeg}\right)}{(n+1)(d+ex)^{3/2}(cdf-aeg)}$$

[Out] -(((a\*e + c\*d\*x)\*(f + g\*x)^(1 + n)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2)\*Hypergeometric2F1[1, 7/2 + n, 2 + n, (c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g)])/(c\*d\*f - a\*e\*g))\*((c\*d\*f - a\*e\*g)^(1 + n)\*(d + e\*x)^(3/2)))

**Rubi [A]** time = 0.341928, antiderivative size = 122, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{2(f+gx)^n (ae+cdx)^2 \sqrt{x(ae^2+cd^2)+ade+cdex^2} \left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n} {}_2F_1\left(\frac{5}{2}, -n; \frac{7}{2}; -\frac{g(ae+cdx)}{cdf-aeg}\right)}{5cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[((f + g\*x)^n\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2))/(d + e\*x)^(3/2), x

[Out] (2\*(a\*e + c\*d\*x)^2\*(f + g\*x)^n\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]\*Hypergeometric2F1[5/2, -n, 7/2, -(g\*(a\*e + c\*d\*x))/(c\*d\*f - a\*e\*g)])/(5\*c\*d\*Sqrt[d + e\*x]\*((c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g))^n)

**Rubi in Sympy [A]** time = 61.4463, size = 109, normalized size = 1.05

$$\frac{2 \left(\frac{cd(-f-gx)}{aeg-cdf}\right)^{-n} (f+gx)^n (ae+cdx)^2 \sqrt{ade+cdex^2+x(ae^2+cd^2)} {}_2F_1\left(-n, \frac{5}{2} \middle| \frac{g(ae+cdx)}{aeg-cdf}\right)}{5cd\sqrt{d+ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*n\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/(e\*x+d)\*

[Out] 2\*(c\*d\*(-f - g\*x)/(a\*e\*g - c\*d\*f))\*\*(-n)\*(f + g\*x)\*\*n\*(a\*e + c\*d\*x)\*\*2\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*hyper((-n, 5/2), (7/2, ), g\*(a\*e + c\*d\*x)/(a\*e\*g - c\*d\*f))/(5\*c\*d\*sqrt(d + e\*x))

**Mathematica [A]** time = 0.201332, size = 100, normalized size = 0.96

$$\frac{2(f+gx)^n((d+ex)(ae+cdx))^{5/2} \left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n} {}_2F_1\left(\frac{5}{2}, -n; \frac{7}{2}; \frac{g(ae+cdx)}{aeg-cdf}\right)}{5cd(d+ex)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[((f + g\*x)^n\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(3/2))/(d + e\*x)^(5/2), x

[Out] (2\*((a\*e + c\*d\*x)\*(d + e\*x))^(5/2)\*(f + g\*x)^n\*Hypergeometric2F1[5/2, -n, 7/2, (g\*(a\*e + c\*d\*x))/(-(c\*d\*f) + a\*e\*g)])/(5\*c\*d\*(d +

$$e^*x)^{(5/2)}*((c*d*(f + g*x))/(c*d*f - a*e*g))^{n)}$$

**Maple [F]** time = 0.172, size = 0, normalized size = 0.

$$\int (gx + f)^n (ade + (ae^2 + cd^2)x + cdex^2)^{\frac{3}{2}} (ex + d)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^n\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)/(e\*x+d)^(3/2), x)

[Out] int((g\*x+f)^n\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(3/2)/(e\*x+d)^(3/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}(gx + f)^n}{(ex + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f)^n/(e\*x + d)^(3/2), x)

[Out] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f)^n/(e\*x + d)^(3/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(cdx + ae)(gx + f)^n}{\sqrt{ex + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f)^n/(e\*x + d)^(3/2), x)

[Out] integral(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(c\*d\*x + a\*e)^n/sqrt(e\*x + d), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*n\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(3/2)/(e\*x+d)\*\*(3/2), x)

[Out] Timed out

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{3}{2}}(gx + f)^n}{(ex + d)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f)^n/(e\*x + d)^(3/2), x)

[Out] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(3/2)\*(g\*x + f)^n/(e\*x + d)^(3/2), x)

$$3.766 \quad \int \frac{(f+gx)^n (ade+(cd^2+ae^2)x+cdex^2)^{5/2}}{(d+ex)^{5/2}} dx$$

**Optimal.** Leaf size=104

$$\frac{(f+gx)^{n+1}(ae+cdx)(x(ae^2+cd^2)+ade+cdex^2)^{5/2} {}_2F_1\left(1, n+\frac{9}{2}; n+2; \frac{cd(f+gx)}{cdf-aeg}\right)}{(n+1)(d+ex)^{5/2}(cdf-aeg)}$$

[Out] -(((a\*e + c\*d\*x)\*(f + g\*x)^(1 + n)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2)\*Hypergeometric2F1[1, 9/2 + n, 2 + n, (c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g)])/(c\*d\*f - a\*e\*g))\*((c\*d\*f - a\*e\*g)^(1 + n)\*(d + e\*x)^(5/2)))

**Rubi [A]** time = 0.348124, antiderivative size = 122, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{2(f+gx)^n(ae+cdx)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n} {}_2F_1\left(\frac{7}{2}, -n; \frac{9}{2}; -\frac{g(ae+cdx)}{cdf-aeg}\right)}{7cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[((f + g\*x)^n\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2))/(d + e\*x)^(5/2), x

[Out] (2\*(a\*e + c\*d\*x)^3\*(f + g\*x)^n\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]\*Hypergeometric2F1[7/2, -n, 9/2, -(g\*(a\*e + c\*d\*x))/(c\*d\*f - a\*e\*g)])/(7\*c\*d\*Sqrt[d + e\*x]\*((c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g))^n)

**Rubi in SymPy [A]** time = 61.5188, size = 109, normalized size = 1.05

$$\frac{2\left(\frac{cd(-f-gx)}{aeg-cdf}\right)^{-n}(f+gx)^n(ae+cdx)^3\sqrt{ade+cdex^2+x(ae^2+cd^2)}{}_2F_1\left(-n, \frac{7}{2}; \frac{9}{2}; \frac{g(ae+cdx)}{aeg-cdf}\right)}{7cd\sqrt{d+ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*n\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)/(e\*x+d)\*\*

[Out] 2\*(c\*d\*(-f - g\*x)/(a\*e\*g - c\*d\*f))\*\*(-n)\*(f + g\*x)\*\*n\*(a\*e + c\*d\*x)\*\*3\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*hyper((-n, 7/2), (9/2, ), g\*(a\*e + c\*d\*x)/(a\*e\*g - c\*d\*f))/(7\*c\*d\*sqrt(d + e\*x))

**Mathematica [A]** time = 0.15229, size = 110, normalized size = 1.06

$$\frac{2(f+gx)^n(ae+cdx)^3\sqrt{(d+ex)(ae+cdx)}\left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n} {}_2F_1\left(\frac{7}{2}, -n; \frac{9}{2}; \frac{g(ae+cdx)}{aeg-cdf}\right)}{7cd\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Integrate[((f + g\*x)^n\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(5/2))/(d + e\*x)^(5/2), x

[Out] (2\*(a\*e + c\*d\*x)^3\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)]\*(f + g\*x)^n\*Hypergeometric2F1[7/2, -n, 9/2, (g\*(a\*e + c\*d\*x))/(-(c\*d\*f) + a\*e\*g)]

)/(7\*c\*d\*Sqrt[d + e\*x]\*((c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g))^n)

**Maple [F]** time = 0.107, size = 0, normalized size = 0.

$$\int (gx + f)^n (ade + (ae^2 + cd^2)x + cdex^2)^{\frac{5}{2}} (ex + d)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^n\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2)/(e\*x+d)^(5/2), x)

[Out] int((g\*x+f)^n\*(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(5/2)/(e\*x+d)^(5/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{\frac{5}{2}}(gx + f)^n}{(ex + d)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*(g\*x + f)^n/(e\*x + d)^(5/2), x)

[Out] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*(g\*x + f)^n/(e\*x + d)^(5/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(c^2d^2x^2 + 2acdex + a^2e^2)\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(gx + f)^n}{\sqrt{ex + d}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(5/2)\*(g\*x + f)^n/(e\*x + d)^(5/2), x)

[Out] integral((c^2\*d^2\*x^2 + 2\*a\*c\*d\*e\*x + a^2\*e^2)\*sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^n/sqrt(e\*x + d), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*n\*(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(5/2)/(e\*x+d)\*\*(5/2), x)

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^(5/2)*(g*x + f)^n/(e*x + d)^(5/2)
```

```
[Out] Exception raised: NotImplementedError
```

$$3.767 \quad \int (d + ex)^m (f + gx)^n (ade + (cd^2 + ae^2)x + cdex^2)^{-m} dx$$

**Optimal.** Leaf size=103

$$\frac{(d + ex)^m (f + gx)^{n+1} (ae + cdx) (x (ae^2 + cd^2) + ade + cdex^2)^{-m} {}_2F_1\left(1, -m + n + 2; n + 2; \frac{cd(f+gx)}{cdf-aeg}\right)}{(n + 1)(cdf - aeg)}$$

[Out] -(((a\*e + c\*d\*x)\*(d + e\*x)^m\*(f + g\*x)^(1 + n)\*Hypergeometric2F1[1, 2 - m + n, 2 + n, (c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g)])/((c\*d\*f - a\*e\*g)\*(1 + n)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m))

**Rubi [A]** time = 0.200102, antiderivative size = 107, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, integrand size = 44,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$

$$\frac{(d + ex)^m (f + gx)^{n+1} (x (ae^2 + cd^2) + ade + cdex^2)^{-m} \left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m {}_2F_1\left(m, n + 1; n + 2; \frac{cd(f+gx)}{cdf-aeg}\right)}{g(n + 1)}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^m\*(f + g\*x)^n)/(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m, x]

[Out] (((-(g\*(a\*e + c\*d\*x))/(c\*d\*f - a\*e\*g)))^m\*(d + e\*x)^m\*(f + g\*x)^(1 + n)\*Hypergeometric2F1[m, 1 + n, 2 + n, (c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g)])/(g\*(1 + n)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m)

**Rubi in Sympy [A]** time = 47.7694, size = 94, normalized size = 0.91

$$\frac{\left(\frac{g(ae+cdx)}{aeg-cdf}\right)^m (d + ex)^m (f + gx)^{n+1} (ade + cdex^2 + x (ae^2 + cd^2))^{-m} {}_2F_1\left(m, n + 1; n + 2; \frac{cd(-f-gx)}{aeg-cdf}\right)}{g(n + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*m\*(g\*x+f)\*\*n/((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*m)

[Out] (g\*(a\*e + c\*d\*x)/(a\*e\*g - c\*d\*f))\*\*m\*(d + e\*x)\*\*m\*(f + g\*x)\*\*(n + 1)\*(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*\*(-m)\*hyper((m, n + 1), (n + 2, ), c\*d\*(-f - g\*x)/(a\*e\*g - c\*d\*f))/(g\*(n + 1))

**Mathematica [A]** time = 0.196123, size = 95, normalized size = 0.92

$$\frac{(d + ex)^m (f + gx)^{n+1} ((d + ex)(ae + cdx))^{-m} \left(\frac{g(ae+cdx)}{aeg-cdf}\right)^m {}_2F_1\left(m, n + 1; n + 2; \frac{cd(f+gx)}{cdf-aeg}\right)}{g(n + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^m\*(f + g\*x)^n)/(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m, x]

[Out] (((g\*(a\*e + c\*d\*x))/(-(c\*d\*f) + a\*e\*g))^m\*(d + e\*x)^m\*(f + g\*x)^(1 + n)\*Hypergeometric2F1[m, 1 + n, 2 + n, (c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g)])/(g\*(1 + n)\*((a\*e + c\*d\*x)\*(d + e\*x))^m)

---

**Maple [F]** time = 0.24, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^m (gx + f)^n}{(ade + (ae^2 + cd^2)x + cdex^2)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^m\*(g\*x+f)^n/((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^m), x)

[Out] int((e\*x+d)^m\*(g\*x+f)^n/((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^m), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (cdex^2 + ade + (cd^2 + ae^2)x)^{-m} (ex + d)^m (gx + f)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^m\*(g\*x + f)^n/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m, x, alg

[Out] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(-m)\*(e\*x + d)^m\*(g\*x + f)^n, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex + d)^m (gx + f)^n}{(cdex^2 + ade + (cd^2 + ae^2)x)^m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^m\*(g\*x + f)^n/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m, x, alg

[Out] integral((e\*x + d)^m\*(g\*x + f)^n/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*m\*(g\*x+f)\*\*n/((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*m), x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^m (gx + f)^n}{(cdex^2 + ade + (cd^2 + ae^2)x)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((e\*x + d)^m\*(g\*x + f)^n/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m, x, alg

[Out] integrate((e\*x + d)^m\*(g\*x + f)^n/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m, x)

$$3.768 \quad \int (d + ex)^m (f + gx)^3 (ade + (cd^2 + ae^2)x + cdex^2)^{-m} dx$$

**Optimal.** Leaf size=343

$$\begin{aligned} & - \frac{6(d + ex)^{m-1}(cdf - aeg)^2 (x (ae^2 + cd^2) + ade + cdex^2)^{1-m} (ae^2g + cd(dg(1 - m) - ef(2 - m)))}{c^4d^4e(1 - m)(2 - m)(3 - m)(4 - m)} \\ & + \frac{6g(d + ex)^m(cdf - aeg)^2 (x (ae^2 + cd^2) + ade + cdex^2)^{1-m}}{c^3d^3e(2 - m)(3 - m)(4 - m)} \\ & + \frac{3(f + gx)^2(d + ex)^{m-1}(cdf - aeg) (x (ae^2 + cd^2) + ade + cdex^2)^{1-m}}{c^2d^2(3 - m)(4 - m)} \\ & + \frac{(f + gx)^3(d + ex)^{m-1} (x (ae^2 + cd^2) + ade + cdex^2)^{1-m}}{cd(4 - m)} \end{aligned}$$

[Out]  $(-6*(c*d*f - a*e*g)^2*(a*e^2*g + c*d*(d*g*(1 - m) - e*f*(2 - m))) * (d + e*x)^{-1 + m} * (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{1 - m}) / (c^4*d^4*e*(1 - m)*(2 - m)*(3 - m)*(4 - m)) + (6*g*(c*d*f - a*e*g)^2*(d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{1 - m}) / (c^3*d^3*e*(2 - m)*(3 - m)*(4 - m)) + (3*(c*d*f - a*e*g)*(d + e*x)^{-1 + m}*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{1 - m}) / (c^2*d^2*(3 - m)*(4 - m)) + ((d + e*x)^{-1 + m}*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{1 - m}) / (c*d*(4 - m))$

**Rubi [A]** time = 0.931671, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 44,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$

$$\begin{aligned} & - \frac{6(d + ex)^{m-1}(cdf - aeg)^2 (x (ae^2 + cd^2) + ade + cdex^2)^{1-m} (ae^2g + cd(dg(1 - m) - ef(2 - m)))}{c^4d^4e(1 - m)(2 - m)(3 - m)(4 - m)} \\ & + \frac{6g(d + ex)^m(cdf - aeg)^2 (x (ae^2 + cd^2) + ade + cdex^2)^{1-m}}{c^3d^3e(2 - m)(3 - m)(4 - m)} \\ & + \frac{3(f + gx)^2(d + ex)^{m-1}(cdf - aeg) (x (ae^2 + cd^2) + ade + cdex^2)^{1-m}}{c^2d^2(3 - m)(4 - m)} \\ & + \frac{(f + gx)^3(d + ex)^{m-1} (x (ae^2 + cd^2) + ade + cdex^2)^{1-m}}{cd(4 - m)} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^m*(f + g*x)^3/(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m, x]$

[Out]  $(-6*(c*d*f - a*e*g)^2*(a*e^2*g + c*d*(d*g*(1 - m) - e*f*(2 - m))) * (d + e*x)^{-1 + m} * (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{1 - m}) / (c^4*d^4*e*(1 - m)*(2 - m)*(3 - m)*(4 - m)) + (6*g*(c*d*f - a*e*g)^2*(d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{1 - m}) / (c^3*d^3*e*(2 - m)*(3 - m)*(4 - m)) + (3*(c*d*f - a*e*g)*(d + e*x)^{-1 + m}*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{1 - m}) / (c^2*d^2*(3 - m)*(4 - m)) + ((d + e*x)^{-1 + m}*(f + g*x)^3*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{1 - m}) / (c*d*(4 - m))$

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)**m*(g*x+f)**3/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m)$

[Out] Timed out

**Mathematica [A]** time = 0.3017, size = 233, normalized size = 0.68

$$\frac{(d+ex)^{m-1}((d+ex)(ae+cdx))^{1-m}(6a^3e^3g^3+6a^2cde^2g^2(f(m-4)+g(m-1)x)+3ac^2d^2eg(f^2(m^2-7m+12)+2fg(m^2-7m+12)+2fg(m^2-7m+12)+2fg(m^2-7m+12))}{(d+ex)^{m-1}((d+ex)(ae+cdx))^{1-m}(6a^3e^3g^3+6a^2cde^2g^2(f(m-4)+g(m-1)x)+3ac^2d^2eg(f^2(m^2-7m+12)+2fg(m^2-7m+12)+2fg(m^2-7m+12)+2fg(m^2-7m+12))}$$

Antiderivative was successfully verified.

[In] Integrate[(((d + e\*x)^m\*(f + g\*x)^3)/(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m, x]

[Out] -(((d + e\*x)^(-1 + m)\*((a\*e + c\*d\*x)\*(d + e\*x))^(1 - m)\*(6\*a^3\*e^3\*g^3 + 6\*a^2\*c\*d\*e^2\*g^2\*(f\*(-4 + m) + g\*(-1 + m)\*x) + 3\*a\*c^2\*d^2\*e\*g\*(f^2\*(12 - 7\*m + m^2) + 2\*f\*g\*(4 - 5\*m + m^2)\*x + g^2\*(2 - 3\*m + m^2)\*x^2) + c^3\*d^3\*(f^3\*(-24 + 26\*m - 9\*m^2 + m^3) + 3\*f^2\*g\*(-12 + 19\*m - 8\*m^2 + m^3)\*x + 3\*f\*g^2\*(-8 + 14\*m - 7\*m^2 + m^3)\*x^2 + g^3\*(-6 + 11\*m - 6\*m^2 + m^3)\*x^3)))/(c^4\*d^4\*(-4 + m)\*(-3 + m)\*(-2 + m)\*(-1 + m)))

**Maple [A]** time = 0.015, size = 527, normalized size = 1.5

$$\frac{(ex+d)^m(c^3d^3g^3m^3x^3+3c^3d^3fg^2m^3x^2-6c^3d^3g^3m^2x^3+3ac^2d^2eg^3m^2x^2+3c^3d^3f^2gm^3x-21c^3d^3fg^2m^2x^2+11c^3d^3g^3m^2x^2-6c^3d^3g^3m^2x^3+3ac^2d^2eg^3m^2x^2+3c^3d^3f^2gm^3x-21c^3d^3fg^2m^2x^2+11c^3d^3g^3m^2x^2)}{(ex+d)^m(c^3d^3g^3m^3x^3+3c^3d^3fg^2m^3x^2-6c^3d^3g^3m^2x^3+3ac^2d^2eg^3m^2x^2+3c^3d^3f^2gm^3x-21c^3d^3fg^2m^2x^2+11c^3d^3g^3m^2x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^m\*(g\*x+f)^3/((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^m), x)

[Out] -(e\*x+d)^m\*(c^3\*d^3\*g^3\*m^3\*x^3+3\*c^3\*d^3\*f\*g^2\*m^3\*x^2-6\*c^3\*d^3\*g^3\*m^2\*x^3+3\*a\*c^2\*d^2\*e\*g^3\*m^2\*x^2+3\*c^3\*d^3\*f^2\*g\*m^3\*x-21\*c^3\*d^3\*f\*g^2\*m^2\*x^2+11\*c^3\*d^3\*g^3\*m^2\*x^2+6\*a\*c^2\*d^2\*e\*f\*g^2\*m^2\*x-9\*a\*c^2\*d^2\*e\*g^3\*m^2\*x^2+c^3\*d^3\*f^3\*m^3-24\*c^3\*d^3\*f^2\*g\*m^2\*x+42\*c^3\*d^3\*f\*g^2\*m^2\*x^2-6\*c^3\*d^3\*g^3\*x^3+6\*a^2\*c\*d\*e^2\*g^3\*m^2\*x+3\*a\*c^2\*d^2\*e\*f^2\*g\*m^2-30\*a\*c^2\*d^2\*e\*f\*g^2\*m^2\*x+6\*a\*c^2\*d^2\*e\*g^3\*x^2-9\*c^3\*d^3\*f^3\*m^2+57\*c^3\*d^3\*f^2\*g\*m^2\*x-24\*c^3\*d^3\*f\*g^2\*x^2+6\*a^2\*c\*d\*e^2\*f\*g^2\*m-6\*a^2\*c\*d\*e^2\*g^3\*x-21\*a\*c^2\*d^2\*e\*f^2\*g\*m+24\*a\*c^2\*d^2\*e\*f\*g^2\*x+26\*c^3\*d^3\*f^3\*m-36\*c^3\*d^3\*f^2\*g\*x+6\*a^3\*e^3\*g^3-24\*a^2\*c\*d\*e^2\*f\*g^2+36\*a\*c^2\*d^2\*e\*f^2\*g-24\*c^3\*d^3\*f^3)\*(c\*d\*x+a\*e)/((c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^m)/c^4/d^4/(m^4-10\*m^3+35\*m^2-50\*m+24)

**Maxima [A]** time = 0.7568, size = 447, normalized size = 1.3

$$\frac{(cdx+ae)(cdx+ae)^{-m}f^3}{cd(m-1)} - \frac{3(c^2d^2(m-1)x^2+acdemx+a^2e^2)(cdx+ae)^{-m}f^2g}{(m^2-3m+2)c^2d^2} - \frac{3((m^2-3m+2)c^3d^3x^3+(m^2-m)ac^2d^2ex^2+2a^2cde^2mx+2a^3e^3)(cdx+ae)^{-m}fg^2}{(m^3-6m^2+11m-6)c^3d^3} - \frac{((m^3-6m^2+11m-6)c^4d^4x^4+(m^3-3m^2+2m)ac^3d^3ex^3+3(m^2-m)a^2c^2d^2e^2x^2+6a^3cde^3mx+6a^4e^4)(cdx+ae)^{-m}}{(m^4-10m^3+35m^2-50m+24)c^4d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^3\*(e\*x + d)^m/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m, x, alg

[Out] -(c\*d\*x + a\*e)\*(c\*d\*x + a\*e)^(-m)\*f^3/(c\*d\*(m - 1)) - 3\*(c^2\*d^2\*(m - 1)\*x^2 + a\*c\*d\*e\*m\*x + a^2\*e^2)\*(c\*d\*x + a\*e)^(-m)\*f^2\*g/((m

$$\begin{aligned} & ^2 - 3m + 2) * c^2 * d^2) - 3 * ((m^2 - 3m + 2) * c^3 * d^3 * x^3 + (m^2 - \\ & m) * a * c^2 * d^2 * e * x^2 + 2 * a^2 * c * d * e^2 * m * x + 2 * a^3 * e^3) * (c * d * x + a * e) \\ & ^{-m} * f * g^2 / ((m^3 - 6 * m^2 + 11 * m - 6) * c^3 * d^3) - ((m^3 - 6 * m^2 + \\ & 11 * m - 6) * c^4 * d^4 * x^4 + (m^3 - 3 * m^2 + 2 * m) * a * c^3 * d^3 * e * x^3 + 3 * ( \\ & m^2 - m) * a^2 * c^2 * d^2 * e^2 * x^2 + 6 * a^3 * c * d * e^3 * m * x + 6 * a^4 * e^4) * (c * \\ & d * x + a * e)^{-m} * g^3 / ((m^4 - 10 * m^3 + 35 * m^2 - 50 * m + 24) * c^4 * d^4) \end{aligned}$$

**Fricas [A]** time = 0.287794, size = 952, normalized size = 2.78

$$(ac^3d^3ef^3m^3 - 24ac^3d^3ef^3 + 36a^2c^2d^2e^2f^2g - 24a^3cde^3fg^2 + 6a^4e^4g^3 + (c^4d^4g^3m^3 - 6c^4d^4g^3m^2 + 11c^4d^4g^3m - 6c^4d^4g^3m^0))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^3\*(e\*x + d)^m/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m, x, alg

[Out] 
$$\begin{aligned} & -(a * c^3 * d^3 * e * f^3 * m^3 - 24 * a * c^3 * d^3 * e * f^3 + 36 * a^2 * c^2 * d^2 * e^2 * f \\ & ^2 * g - 24 * a^3 * c * d * e^3 * f * g^2 + 6 * a^4 * e^4 * g^3 + (c^4 * d^4 * g^3 * m^3 - \\ & 6 * c^4 * d^4 * g^3 * m^2 + 11 * c^4 * d^4 * g^3 * m - 6 * c^4 * d^4 * g^3) * x^4 - (24 * c \\ & ^4 * d^4 * f * g^2 - (3 * c^4 * d^4 * f * g^2 + a * c^3 * d^3 * e * g^3) * m^3 + 3 * (7 * c^4 \\ & * d^4 * f * g^2 + a * c^3 * d^3 * e * g^3) * m^2 - 2 * (21 * c^4 * d^4 * f * g^2 + a * c^3 * d \\ & ^3 * e * g^3) * m) * x^3 - 3 * (3 * a * c^3 * d^3 * e * f^3 - a^2 * c^2 * d^2 * e^2 * f^2 * g) * \\ & m^2 - 3 * (12 * c^4 * d^4 * f^2 * g - (c^4 * d^4 * f^2 * g + a * c^3 * d^3 * e * f * g^2) * m \\ & ^3 + (8 * c^4 * d^4 * f^2 * g + 5 * a * c^3 * d^3 * e * f * g^2 - a^2 * c^2 * d^2 * e^2 * g^3 \\ & ) * m^2 - (19 * c^4 * d^4 * f^2 * g + 4 * a * c^3 * d^3 * e * f * g^2 - a^2 * c^2 * d^2 * e^2 \\ & * g^3) * m) * x^2 + (26 * a * c^3 * d^3 * e * f^3 - 21 * a^2 * c^2 * d^2 * e^2 * f^2 * g + 6 \\ & * a^3 * c * d * e^3 * f * g^2) * m - (24 * c^4 * d^4 * f^3 - (c^4 * d^4 * f^3 + 3 * a * c^3 * \\ & d^3 * e * f^2 * g) * m^3 + 3 * (3 * c^4 * d^4 * f^3 + 7 * a * c^3 * d^3 * e * f^2 * g - 2 * a^2 \\ & * c^2 * d^2 * e^2 * f * g^2) * m^2 - 2 * (13 * c^4 * d^4 * f^3 + 18 * a * c^3 * d^3 * e * f^2 * \\ & g - 12 * a^2 * c^2 * d^2 * e^2 * f * g^2 + 3 * a^3 * c * d * e^3 * g^3) * m) * x) * (e * x + d) \\ & ^m / ((c^4 * d^4 * m^4 - 10 * c^4 * d^4 * m^3 + 35 * c^4 * d^4 * m^2 - 50 * c^4 * d^4 * m \\ & + 24 * c^4 * d^4) * (c * d * e * x^2 + a * d * e + (c * d^2 + a * e^2) * x)^m) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*m\*(g\*x+f)\*\*3/((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*m), x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.276912, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^3\*(e\*x + d)^m/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m, x, alg

[Out] Done

$$3.769 \quad \int (d + ex)^m (f + gx)^2 (ade + (cd^2 + ae^2)x + cdex^2)^{-m} dx$$

**Optimal.** Leaf size=246

$$\frac{2(d + ex)^{m-1}(cdf - aeg)(x(ae^2 + cd^2) + ade + cdex^2)^{1-m}(ae^2g + cd(dg(1 - m) - ef(2 - m)))}{c^3d^3e(1 - m)(2 - m)(3 - m)} + \frac{2g(d + ex)^m(cdf - aeg)(x(ae^2 + cd^2) + ade + cdex^2)^{1-m}}{c^2d^2e(2 - m)(3 - m)} + \frac{(f + gx)^2(d + ex)^{m-1}(x(ae^2 + cd^2) + ade + cdex^2)^{1-m}}{cd(3 - m)}$$

[Out]  $(-2*(c*d*f - a*e*g)*(a*e^2*g + c*d*(d*g*(1 - m) - e*f*(2 - m)))*(d + e*x)^{(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(1 - m)}})/(c^3*d^3*e*(1 - m)*(2 - m)*(3 - m)) + (2*g*(c*d*f - a*e*g)*(d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(1 - m)}}/(c^2*d^2*e*(2 - m)*(3 - m)) + ((d + e*x)^{(m - 1)}*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(1 - m)}}/(c*d*(3 - m))$

**Rubi [A]** time = 0.50225, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 44,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$

$$\frac{2(d + ex)^{m-1}(cdf - aeg)(x(ae^2 + cd^2) + ade + cdex^2)^{1-m}(ae^2g + cd(dg(1 - m) - ef(2 - m)))}{c^3d^3e(1 - m)(2 - m)(3 - m)} + \frac{2g(d + ex)^m(cdf - aeg)(x(ae^2 + cd^2) + ade + cdex^2)^{1-m}}{c^2d^2e(2 - m)(3 - m)} + \frac{(f + gx)^2(d + ex)^{m-1}(x(ae^2 + cd^2) + ade + cdex^2)^{1-m}}{cd(3 - m)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(d + e*x)^m*(f + g*x)^2}{(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m}, x]$

[Out]  $(-2*(c*d*f - a*e*g)*(a*e^2*g + c*d*(d*g*(1 - m) - e*f*(2 - m)))*(d + e*x)^{(-1 + m)*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(1 - m)}})/(c^3*d^3*e*(1 - m)*(2 - m)*(3 - m)) + (2*g*(c*d*f - a*e*g)*(d + e*x)^m*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(1 - m)}}/(c^2*d^2*e*(2 - m)*(3 - m)) + ((d + e*x)^{(m - 1)}*(f + g*x)^2*(a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(1 - m)}}/(c*d*(3 - m))$

**Rubi in Sympy [A]** time = 111.687, size = 223, normalized size = 0.91

$$\frac{(d + ex)^{m-1}(f + gx)^2(ade + cdex^2 + x(ae^2 + cd^2))^{-m+1}}{cd(-m + 3)} - \frac{2g(d + ex)^m(aeg - cdf)(ade + cdex^2 + x(ae^2 + cd^2))^{-m+1}}{c^2d^2e(m^2 - 5m + 6)} + \frac{2(d + ex)^{m-1}(aeg - cdf)(ade + cdex^2 + x(ae^2 + cd^2))^{-m+1}(ae^2g - cd^2gm + cd^2g + cdefm - 2cdef)}{c^3d^3e(-m + 1)(-m + 2)(-m + 3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)**m*(g*x+f)**2/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m)$

[Out]  $(d + e*x)**(m - 1)*(f + g*x)**2*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(-m + 1)/(c*d*(-m + 3)) - 2*g*(d + e*x)**m*(a*e*g - c*d*f)*(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(-m + 1)/(c**2*d$



Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x + f)^2*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m, x, alg`

[Out] 
$$-(a^2c^2d^2ef^2m^2 + 6a^2c^2d^2ef^2 - 6a^2cd^2efg + 2a^3e^3g^2 + (c^3d^3g^2m^2 - 3c^3d^3g^2m + 2c^3d^3g^2) * x^3 + (6c^3d^3f^2g + (2c^3d^3f^2g + a^2c^2d^2efg^2) * m^2 - (8c^3d^3f^2g + a^2c^2d^2efg^2) * m) * x^2 - (5a^2c^2d^2ef^2 - 2a^2cd^2efg) * m + (6c^3d^3f^2 + (c^3d^3f^2 + 2a^2c^2d^2efg) * m^2 - (5c^3d^3f^2 + 6a^2c^2d^2efg - 2a^2cd^2efg^2) * m) * x) * (e*x + d)^m / ((c^3d^3m^3 - 6c^3d^3m^2 + 11c^3d^3m - 6c^3d^3) * (c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**m*(g*x+f)**2/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m), x)`

[Out] Timed out

**GIAC/XCAS [A]** time = 0.275164, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x + f)^2*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m, x, alg`

[Out] Done

$$3.770 \quad \int (d + ex)^m (f + gx) (ade + (cd^2 + ae^2)x + cdex^2)^{-m} dx$$

**Optimal.** Leaf size=150

$$\frac{g(d + ex)^m (x (ae^2 + cd^2) + ade + cdex^2)^{1-m}}{cde(2 - m)} - \frac{(d + ex)^{m-1} (x (ae^2 + cd^2) + ade + cdex^2)^{1-m} (ae^2g + cd(dg(1 - m) - ef(2 - m)))}{c^2d^2e(1 - m)(2 - m)}$$

[Out] -(((a\*e^2\*g + c\*d\*(d\*g\*(1 - m) - e\*f\*(2 - m)))\*(d + e\*x)^(-1 + m) \* (a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(1 - m))/(c^2\*d^2\*e\*(1 - m)\*(2 - m))) + (g\*(d + e\*x)^m\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(1 - m))/(c\*d\*e\*(2 - m))

**Rubi [A]** time = 0.231958, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 42,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$

$$\frac{g(d + ex)^m (x (ae^2 + cd^2) + ade + cdex^2)^{1-m}}{cde(2 - m)} - \frac{(d + ex)^{m-1} (x (ae^2 + cd^2) + ade + cdex^2)^{1-m} (ae^2g + cd(dg(1 - m) - ef(2 - m)))}{c^2d^2e(1 - m)(2 - m)}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^m\*(f + g\*x))/(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m, x]

[Out] -(((a\*e^2\*g + c\*d\*(d\*g\*(1 - m) - e\*f\*(2 - m)))\*(d + e\*x)^(-1 + m) \* (a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(1 - m))/(c^2\*d^2\*e\*(1 - m)\*(2 - m))) + (g\*(d + e\*x)^m\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^(1 - m))/(c\*d\*e\*(2 - m))

**Rubi in Sympy [A]** time = 59.7488, size = 136, normalized size = 0.91

$$\frac{g(d + ex)^m (ade + cdex^2 + x (ae^2 + cd^2))^{-m+1}}{cde(-m + 2)} - \frac{(d + ex)^{m-1} (ade + cdex^2 + x (ae^2 + cd^2))^{-m+1} (ae^2g - cd^2gm + cd^2g + cdefm - 2cdf)}{c^2d^2e(-m + 1)(-m + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*m\*(g\*x+f)/((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*m), x)

[Out] g\*(d + e\*x)\*\*m\*(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*\*(-m + 1)/(c\*d\*e\*(-m + 2)) - (d + e\*x)\*\*(m - 1)\*(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*\*(-m + 1)\*(a\*e\*\*2\*g - c\*d\*\*2\*g\*m + c\*d\*\*2\*g + c\*d\*e\*f\*m - 2\*c\*d\*e\*f)/(c\*\*2\*d\*\*2\*e\*(-m + 1)\*(-m + 2))

**Mathematica [A]** time = 0.0912918, size = 67, normalized size = 0.45

$$\frac{(d + ex)^{m-1} ((d + ex)(ae + cdx))^{1-m} (aeg + cd(f(m - 2) + g(m - 1)x))}{c^2d^2(m - 2)(m - 1)}$$

Antiderivative was successfully verified.



[In] Integrate[((d + e\*x)^m\*(f + g\*x))/(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m, x]

[Out] -(((d + e\*x)^(-1 + m)\*((a\*e + c\*d\*x)\*(d + e\*x))^(1 - m)\*(a\*e\*g + c\*d\*(f\*(-2 + m) + g\*(-1 + m)\*x)))/(c^2\*d^2\*(-2 + m)\*(-1 + m)))

**Maple [A]** time = 0.007, size = 89, normalized size = 0.6

$$\frac{(ex + d)^m (cdgmx + cdfm - xcdg + aeg - 2cdf)(cdx + ae)}{(cdex^2 + ae^2x + cd^2x + ade)^m c^2d^2(m^2 - 3m + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^m\*(g\*x+f)/((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^m), x)

[Out] -(e\*x+d)^m\*(c\*d\*g\*m\*x+c\*d\*f\*m-c\*d\*g\*x+a\*e\*g-2\*c\*d\*f)\*(c\*d\*x+a\*e)/((c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^m)/c^2/d^2/(m^2-3\*m+2)

**Maxima [A]** time = 0.742372, size = 127, normalized size = 0.85

$$\frac{(cdx + ae)(cdx + ae)^{-m} f}{cd(m - 1)} - \frac{(c^2d^2(m - 1)x^2 + acdemx + a^2e^2)(cdx + ae)^{-m} g}{(m^2 - 3m + 2)c^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)\*(e\*x + d)^m/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m, x, algo

[Out] -(c\*d\*x + a\*e)\*(c\*d\*x + a\*e)^(-m)\*f/(c\*d\*(m - 1)) - (c^2\*d^2\*(m - 1)\*x^2 + a\*c\*d\*e\*m\*x + a^2\*e^2)\*(c\*d\*x + a\*e)^(-m)\*g/((m^2 - 3\*m + 2)\*c^2\*d^2)

**Fricas [A]** time = 0.285293, size = 196, normalized size = 1.31

$$\frac{(acdefm - 2acdef + a^2e^2g + (c^2d^2gm - c^2d^2g)x^2 - (2c^2d^2f - (c^2d^2f + acdeg)m)x)(ex + d)^m}{(c^2d^2m^2 - 3c^2d^2m + 2c^2d^2)(cdex^2 + ade + (cd^2 + ae^2)x)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)\*(e\*x + d)^m/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m, x, algo

[Out] -(a\*c\*d\*e\*f\*m - 2\*a\*c\*d\*e\*f + a^2\*e^2\*g + (c^2\*d^2\*g\*m - c^2\*d^2\*g)\*x^2 - (2\*c^2\*d^2\*f - (c^2\*d^2\*f + a\*c\*d\*e\*g)\*m)\*x)\*(e\*x + d)^m/((c^2\*d^2\*m^2 - 3\*c^2\*d^2\*m + 2\*c^2\*d^2)\*(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*m\*(g\*x+f)/((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*m), x)

[Out] Timed out

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**GIAC/XCAS [A]** time = 0.277623, size = 298, normalized size = 1.99

$$\frac{c^2 d^2 g m x^2 e^{(-m \ln(c d x + a e))} + c^2 d^2 f m x e^{(-m \ln(c d x + a e))} - c^2 d^2 g x^2 e^{(-m \ln(c d x + a e))} + a c d g m x e^{(-m \ln(c d x + a e) + 1)} - 2 c^2 d^2 f x e^{(-m \ln(c d x + a e) + 1)}}{c^2 d^2 m^2 - 3 c^2 d^2 m + 2 c^2 d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)\*(e\*x + d)^m/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m, x, algo

[Out] 
$$\frac{-(c^2 d^2 g^m x^2 e^{(-m \ln(c d x + a e))} + c^2 d^2 f^m x e^{(-m \ln(c d x + a e))} - c^2 d^2 g^m x^2 e^{(-m \ln(c d x + a e))} + a^m c^2 d^2 g^m x e^{(-m \ln(c d x + a e) + 1)} - 2^m c^2 d^2 f^m x e^{(-m \ln(c d x + a e))} + a^m c^2 d^2 f^m e^{(-m \ln(c d x + a e) + 1)} - 2^m a^m c^2 d^2 f^m e^{(-m \ln(c d x + a e) + 1)} + a^2 g^m e^{(-m \ln(c d x + a e) + 2)})}{c^2 d^2 m^2 - 3 c^2 d^2 m + 2 c^2 d^2}$$

$$3.771 \quad \int (d + ex)^m (ade + (cd^2 + ae^2)x + cdex^2)^{-m} dx$$

**Optimal.** Leaf size=54

$$\frac{(d + ex)^{m-1} (x (ae^2 + cd^2) + ade + cdex^2)^{1-m}}{cd(1 - m)}$$

[Out]  $((d + e*x)^{(-1 + m)} * (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(1 - m)}) / (c*d*(1 - m))$

**Rubi [A]** time = 0.0421882, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 37,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.027$

$$\frac{(d + ex)^{m-1} (x (ae^2 + cd^2) + ade + cdex^2)^{1-m}}{cd(1 - m)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^m / (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m, x]$

[Out]  $((d + e*x)^{(-1 + m)} * (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(1 - m)}) / (c*d*(1 - m))$

**Rubi in Sympy [A]** time = 18.3126, size = 42, normalized size = 0.78

$$\frac{(d + ex)^{m-1} (ade + cdex^2 + x (ae^2 + cd^2))^{-m+1}}{cd(-m + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)**m / ((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m), x)$

[Out]  $(d + e*x)**(m - 1) * (a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(-m + 1) / (c*d*(-m + 1))$

**Mathematica [A]** time = 0.0312307, size = 42, normalized size = 0.78

$$\frac{(d + ex)^{m-1} ((d + ex)(ae + cdx))^{1-m}}{cd(m - 1)}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(d + e*x)^m / (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m, x]$

[Out]  $-(((d + e*x)^{(-1 + m)} * ((a*e + c*d*x)*(d + e*x))^{(1 - m)}) / (c*d*(-1 + m)))$

**Maple [A]** time = 0.002, size = 57, normalized size = 1.1

$$\frac{(cdx + ae)(ex + d)^m}{cd(-1 + m)(cdex^2 + ae^2x + cd^2x + ade)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m),x)`

[Out]  $-(c*d*x+a*e)/c/d/(-1+m)*(e*x+d)^m/((c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^m)$

**Maxima [A]** time = 0.73719, size = 45, normalized size = 0.83

$$-\frac{(cdx+ae)(cdx+ae)^{-m}}{cd(m-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^m/(c*d*e*x^2+a*d*e+(c*d^2+a*e^2)*x)^m,x,algorithm="maxima")`

[Out]  $-(c*d*x+a*e)*(c*d*x+a*e)^{-m}/(c*d*(m-1))$

**Fricas [A]** time = 0.280819, size = 77, normalized size = 1.43

$$-\frac{(cdx+ae)(ex+d)^m}{(cdm-cd)(cdex^2+ade+(cd^2+ae^2)x)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^m/(c*d*e*x^2+a*d*e+(c*d^2+a*e^2)*x)^m,x,algorithm="fricas")`

[Out]  $-(c*d*x+a*e)*(e*x+d)^m/((c*d*m-c*d)*(c*d*e*x^2+a*d*e+(c*d^2+a*e^2)*x)^m)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**m/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m),x)`

[Out] Timed out

**GIAC/XCAS [A]** time = 0.276246, size = 68, normalized size = 1.26

$$-\frac{cdxe^{(-m\ln(cdx+ae))}+ae^{(-m\ln(cdx+ae)+1)}}{cdm-cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)^m/(c*d*e*x^2+a*d*e+(c*d^2+a*e^2)*x)^m,x,algorithm="giac")`

[Out]  $-(c*d*x*e^{(-m*\ln(c*d*x+a*e))}+a*e^{(-m*\ln(c*d*x+a*e)+1)})/(c*d*m-c*d)$

$$3.772 \quad \int \frac{(d+ex)^m (ade+(cd^2+ae^2)x+cdex^2)^{-m}}{f+gx} dx$$

**Optimal.** Leaf size=99

$$\frac{(d+ex)^m (ae+cdx) (x(ae^2+cd^2)+ade+cdex^2)^{-m} {}_2F_1\left(1, 1-m; 2-m; -\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)}$$

[Out]  $((a^*e + c^*d^*x)^*(d + e^*x)^m \text{Hypergeometric2F1}[1, 1 - m, 2 - m, -((g^*(a^*e + c^*d^*x))/(c^*d^*f - a^*e^*g))]) / ((c^*d^*f - a^*e^*g)^*(1 - m)^*(a^*d^*e + (c^*d^2 + a^*e^2)^*x + c^*d^*e^*x^2)^m)$

**Rubi [A]** time = 0.167592, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 44,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{(d+ex)^m (ae+cdx) (x(ae^2+cd^2)+ade+cdex^2)^{-m} {}_2F_1\left(1, 1-m; 2-m; -\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^m/((f + g\*x)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m), x]

[Out]  $((a^*e + c^*d^*x)^*(d + e^*x)^m \text{Hypergeometric2F1}[1, 1 - m, 2 - m, -((g^*(a^*e + c^*d^*x))/(c^*d^*f - a^*e^*g))]) / ((c^*d^*f - a^*e^*g)^*(1 - m)^*(a^*d^*e + (c^*d^2 + a^*e^2)^*x + c^*d^*e^*x^2)^m)$

**Rubi in Sympy [A]** time = 35.3771, size = 95, normalized size = 0.96

$$\frac{(d+ex)^m (ae+cdx)^m (ae+cdx)^{-m+1} (ade+cdex^2+x(ae^2+cd^2))^{-m} {}_2F_1\left(1, -m+1; -m+2; \frac{g(ae+cdx)}{aeg-cdf}\right)}{(-m+1)(aeg-cdf)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*m/(g\*x+f)/((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*m), x)

[Out]  $-(d + e^*x)^m (a^*e + c^*d^*x)^m (a^*e + c^*d^*x)^{-m+1} (a^*d^*e + c^*d^*e^*x^2 + x^*(a^*e^2 + c^*d^2))^{-m} \text{hyper}((1, -m+1), (-m+2, ), g^*(a^*e + c^*d^*x)/(a^*e^*g - c^*d^*f))/((-m+1)^*(a^*e^*g - c^*d^*f))$

**Mathematica [A]** time = 0.0953098, size = 87, normalized size = 0.88

$$\frac{(d+ex)^m ((d+ex)(ae+cdx))^{-m} \left(\frac{aeg+cdgx}{cdf+cdgx}\right)^m {}_2F_1\left(m, m; m+1; \frac{cdf-aeg}{cdf+cdgx}\right)}{gm}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^m/((f + g\*x)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m), x]

[Out]  $-(((d + e^*x)^m ((a^*e^*g + c^*d^*g^*x)/(c^*d^*f + c^*d^*g^*x))^m \text{Hypergeometric2F1}[m, m, 1 + m, (c^*d^*f - a^*e^*g)/(c^*d^*f + c^*d^*g^*x)]) / (g^*m ((a^*e + c^*d^*x)^*(d + e^*x))^m))$

---

**Maple [F]** time = 0.134, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^m}{(gx + f)(ade + (ae^2 + cd^2)x + cdex^2)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^m/(g\*x+f)/((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^m), x)

[Out] int((e\*x+d)^m/(g\*x+f)/((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^m), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{-m}(ex + d)^m}{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^m/((g\*x + f)\*(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m), x, alg

[Out] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(-m)\*(e\*x + d)^m/(g\*x + f), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex + d)^m}{(gx + f)(cdex^2 + ade + (cd^2 + ae^2)x)^m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^m/((g\*x + f)\*(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m), x, alg

[Out] integral((e\*x + d)^m/((g\*x + f)\*(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*m/(g\*x+f)/((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*m), x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^m}{(gx + f)(cdex^2 + ade + (cd^2 + ae^2)x)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^m/((g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m), x, alg`

[Out] `integrate((e*x + d)^m/((g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m), x)`

$$3.773 \quad \int \frac{(d+ex)^m (ade+(cd^2+ae^2)x+cdex^2)^{-m}}{(f+gx)^2} dx$$

**Optimal.** Leaf size=101

$$\frac{cd(d+ex)^m (ae+cdx) (x(ae^2+cd^2)+ade+cdex^2)^{-m} {}_2F_1\left(2, 1-m; 2-m; -\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)^2}$$

[Out] (c\*d\*(a\*e + c\*d\*x)\*(d + e\*x)^m\*Hypergeometric2F1[2, 1 - m, 2 - m, -((g\*(a\*e + c\*d\*x))/(c\*d\*f - a\*e\*g))])/((c\*d\*f - a\*e\*g)^2\*(1 - m)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m)

**Rubi [A]** time = 0.157541, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 44,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{cd(d+ex)^m (ae+cdx) (x(ae^2+cd^2)+ade+cdex^2)^{-m} {}_2F_1\left(2, 1-m; 2-m; -\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)^2}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^m/((f + g\*x)^2\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m), x]

[Out] (c\*d\*(a\*e + c\*d\*x)\*(d + e\*x)^m\*Hypergeometric2F1[2, 1 - m, 2 - m, -((g\*(a\*e + c\*d\*x))/(c\*d\*f - a\*e\*g))])/((c\*d\*f - a\*e\*g)^2\*(1 - m)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m)

**Rubi in Sympy [A]** time = 35.4745, size = 99, normalized size = 0.98

$$\frac{cd(d+ex)^m (ae+cdx)^m (ae+cdx)^{-m+1} (ade+cdex^2+x(ae^2+cd^2))^{-m} {}_2F_1\left(2, -m+1; -m+2; \frac{g(ae+cdx)}{aeg-cdf}\right)}{(-m+1)(aeg-cdf)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*m/(g\*x+f)\*\*2/((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*m)

[Out] c\*d\*(d + e\*x)\*\*m\*(a\*e + c\*d\*x)\*\*m\*(a\*e + c\*d\*x)\*\*(-m + 1)\*(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*\*(-m)\*hyper((2, -m + 1), (-m + 2, ), g\*(a\*e + c\*d\*x)/(a\*e\*g - c\*d\*f))/((-m + 1)\*(a\*e\*g - c\*d\*f)\*\*2)

**Mathematica [A]** time = 0.0789036, size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m (ade+(cd^2+ae^2)x+cdex^2)^{-m}}{(f+gx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(d + e\*x)^m/((f + g\*x)^2\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m), x]

[Out] Integrate[(d + e\*x)^m/((f + g\*x)^2\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m), x]



---

**Maple [F]** time = 0.139, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^m}{(gx + f)^2 (ade + (ae^2 + cd^2)x + cdex^2)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^m/(g\*x+f)^2/((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^m), x)

[Out] int((e\*x+d)^m/(g\*x+f)^2/((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^m), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{-m}(ex + d)^m}{(gx + f)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^m/((g\*x + f)^2\*(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m), x, a

[Out] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(-m)\*(e\*x + d)^m/(g\*x + f)^2, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex + d)^m}{(g^2x^2 + 2fgx + f^2)(cdex^2 + ade + (cd^2 + ae^2)x)^m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^m/((g\*x + f)^2\*(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m), x, a

[Out] integral((e\*x + d)^m/((g^2\*x^2 + 2\*f\*g\*x + f^2)\*(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*m/(g\*x+f)\*\*2/((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*m), x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^m}{(gx + f)^2 (cdex^2 + ade + (cd^2 + ae^2)x)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^m/((g*x + f)^2*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m), x, a
```

```
[Out] integrate((e*x + d)^m/((g*x + f)^2*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m), x)
```

$$3.774 \quad \int \frac{(d+ex)^m (ade+(cd^2+ae^2)x+cdex^2)^{-m}}{(f+gx)^3} dx$$

**Optimal.** Leaf size=105

$$\frac{c^2 d^2 (d+ex)^m (ae+cdx) (x(ae^2+cd^2)+ade+cdex^2)^{-m} {}_2F_1\left(3, 1-m; 2-m; -\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)^3}$$

[Out] (c^2\*d^2\*(a\*e + c\*d\*x)\*(d + e\*x)^m\*Hypergeometric2F1[3, 1 - m, 2 - m, -((g\*(a\*e + c\*d\*x))/(c\*d\*f - a\*e\*g))])/((c\*d\*f - a\*e\*g)^3\*(1 - m)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m)

**Rubi [A]** time = 0.172661, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 44,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$

$$\frac{c^2 d^2 (d+ex)^m (ae+cdx) (x(ae^2+cd^2)+ade+cdex^2)^{-m} {}_2F_1\left(3, 1-m; 2-m; -\frac{g(ae+cdx)}{cdf-aeg}\right)}{(1-m)(cdf-aeg)^3}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^m/((f + g\*x)^3\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m), x]

[Out] (c^2\*d^2\*(a\*e + c\*d\*x)\*(d + e\*x)^m\*Hypergeometric2F1[3, 1 - m, 2 - m, -((g\*(a\*e + c\*d\*x))/(c\*d\*f - a\*e\*g))])/((c\*d\*f - a\*e\*g)^3\*(1 - m)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m)

**Rubi in Sympy [A]** time = 39.0232, size = 104, normalized size = 0.99

$$\frac{c^2 d^2 (d+ex)^m (ae+cdx)^m (ae+cdx)^{-m+1} (ade+cdex^2+x(ae^2+cd^2))^{-m} {}_2F_1\left(3, -m+1; -m+2; \frac{g(ae+cdx)}{aeg-cdf}\right)}{(-m+1)(aeg-cdf)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*m/(g\*x+f)\*\*3/((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*m)

[Out] -c\*\*2\*d\*\*2\*(d + e\*x)\*\*m\*(a\*e + c\*d\*x)\*\*m\*(a\*e + c\*d\*x)\*\*(-m + 1)\*(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*\*(-m)\*hyper((3, -m + 1), (-m + 2, ), g\*(a\*e + c\*d\*x)/(a\*e\*g - c\*d\*f))/((-m + 1)\*(a\*e\*g - c\*d\*f)\*\*3)

**Mathematica [A]** time = 0.0974697, size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m (ade+(cd^2+ae^2)x+cdex^2)^{-m}}{(f+gx)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[(d + e\*x)^m/((f + g\*x)^3\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m), x]

[Out] Integrate[(d + e\*x)^m/((f + g\*x)^3\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m), x]

---

**Maple [F]** time = 0.156, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^m}{(gx + f)^3 (ade + (ae^2 + cd^2)x + cdex^2)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^m/(g\*x+f)^3/((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^m), x)

[Out] int((e\*x+d)^m/(g\*x+f)^3/((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^m), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{-m}(ex + d)^m}{(gx + f)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^m/((g\*x + f)^3\*(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m), x, a

[Out] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(-m)\*(e\*x + d)^m/(g\*x + f)^3, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex + d)^m}{(g^3x^3 + 3fg^2x^2 + 3f^2gx + f^3)(cdex^2 + ade + (cd^2 + ae^2)x)^m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^m/((g\*x + f)^3\*(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m), x, a

[Out] integral((e\*x + d)^m/((g^3\*x^3 + 3\*f\*g^2\*x^2 + 3\*f^2\*g\*x + f^3)\*(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*m/(g\*x+f)\*\*3/((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*m), x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^m}{(gx + f)^3 (cdex^2 + ade + (cd^2 + ae^2)x)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^m/((g*x + f)^3*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m), x, a`

[Out] `integrate((e*x + d)^m/((g*x + f)^3*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m), x)`

$$3.775 \quad \int (d+ex)^m (f+gx)^{3/2} (ade + (cd^2 + ae^2)x + cdex^2)^{-m} dx$$

**Optimal.** Leaf size=105

$$\frac{2(f+gx)^{5/2}(d+ex)^m (x(ae^2+cd^2) + ade + cdex^2)^{-m} \left(-\frac{g(ae+cdx)}{cdf-ae^2}\right)^m {}_2F_1\left(\frac{5}{2}, m; \frac{7}{2}; \frac{cd(f+gx)}{cdf-ae^2}\right)}{5g}$$

[Out] (2\*(-((g\*(a\*e + c\*d\*x))/(c\*d\*f - a\*e\*g)))^m\*(d + e\*x)^m\*(f + g\*x)^(5/2)\*Hypergeometric2F1[5/2, m, 7/2, (c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g)]/(5\*g\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m)

**Rubi [A]** time = 0.21392, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{2(f+gx)^{5/2}(d+ex)^m (x(ae^2+cd^2) + ade + cdex^2)^{-m} \left(-\frac{g(ae+cdx)}{cdf-ae^2}\right)^m {}_2F_1\left(\frac{5}{2}, m; \frac{7}{2}; \frac{cd(f+gx)}{cdf-ae^2}\right)}{5g}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^m\*(f + g\*x)^(3/2))/(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m, x]

[Out] (2\*(-((g\*(a\*e + c\*d\*x))/(c\*d\*f - a\*e\*g)))^m\*(d + e\*x)^m\*(f + g\*x)^(5/2)\*Hypergeometric2F1[5/2, m, 7/2, (c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g)]/(5\*g\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m)

**Rubi in Sympy [A]** time = 46.4216, size = 94, normalized size = 0.9

$$\frac{2\left(\frac{g(ae+cdx)}{aeg-cdf}\right)^m (d+ex)^m (f+gx)^{\frac{5}{2}} (ade + cdex^2 + x(ae^2 + cd^2))^{-m} {}_2F_1\left(m, \frac{5}{2} \middle| \frac{cd(-f-gx)}{aeg-cdf}\right)}{5g}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*m\*(g\*x+f)\*\*(3/2)/((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)

[Out] 2\*(g\*(a\*e + c\*d\*x)/(a\*e\*g - c\*d\*f))\*\*m\*(d + e\*x)\*\*m\*(f + g\*x)\*\*(5/2)\*(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*\*(-m)\*hyper((m, 5/2), (7/2, ), c\*d\*(-f - g\*x)/(a\*e\*g - c\*d\*f))/(5\*g)

**Mathematica [C]** time = 0.62115, size = 234, normalized size = 2.23

$$\frac{f\sqrt{f+gx}(d+ex)^m((d+ex)(ae+cdx))^{-m} \left(\frac{9aeg^2x^2F_1\left(2,m,-\frac{1}{2};3,-\frac{cdx}{ae},-\frac{gx}{f}\right)}{6aefF_1\left(2,m,-\frac{1}{2};3,-\frac{cdx}{ae},-\frac{gx}{f}\right)+aegx^2F_1\left(3,m,\frac{1}{2};4,-\frac{cdx}{ae},-\frac{gx}{f}\right)-2cdfmxF_1\left(3,m+1,-\frac{1}{2};4,-\frac{cdx}{ae},-\frac{gx}{f}\right)}\right)}{3g}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((d + e\*x)^m\*(f + g\*x)^(3/2))/(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m

[Out] (f\*(d + e\*x)^m\*Sqrt[f + g\*x]\*((9\*a\*e\*g^2\*x^2\*AppellF1[2, m, -1/2, 3, -((c\*d\*x)/(a\*e)), -((g\*x)/f)])/(6\*a\*e\*f\*AppellF1[2, m, -1/2, 3, -((c\*d\*x)/(a\*e)), -((g\*x)/f)] + a\*e\*g\*x\*AppellF1[3, m, 1/2, 4,

$$-\left(\frac{c^*d^*x}{a^*e}\right), -\left(\frac{g^*x}{f}\right)] - 2^*c^*d^*f^*m^*x^*AppellF1[3, 1 + m, -1/2, 4, -\left(\frac{c^*d^*x}{a^*e}\right), -\left(\frac{g^*x}{f}\right)] + 2^*\left(\frac{g^*(a^*e + c^*d^*x)}{-(c^*d^*f) + a^*e^*g}\right)^m^*(f + g^*x)^*Hypergeometric2F1[3/2, m, 5/2, (c^*d^*(f + g^*x))/(c^*d^*f - a^*e^*g)]\right)/(3^*g^*((a^*e + c^*d^*x)^*(d + e^*x))^m)$$

**Maple [F]** time = 0.112, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^m}{(ade + (ae^2 + cd^2)x + cdex^2)^m} (gx + f)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^m\*(g\*x+f)^(3/2)/((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^m), x)

[Out] int((e\*x+d)^m\*(g\*x+f)^(3/2)/((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^m), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (gx + f)^{\frac{3}{2}} (cdex^2 + ade + (cd^2 + ae^2)x)^{-m} (ex + d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^(3/2)\*(e\*x + d)^m/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m, x)

[Out] integrate((g\*x + f)^(3/2)\*(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(-m)\*(e\*x + d)^m, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(gx + f)^{\frac{3}{2}}(ex + d)^m}{(cdex^2 + ade + (cd^2 + ae^2)x)^m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^(3/2)\*(e\*x + d)^m/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m, x)

[Out] integral((g\*x + f)^(3/2)\*(e\*x + d)^m/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*m\*(g\*x+f)\*\*(3/2)/((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*m),

[Out] Timed out

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx + f)^{\frac{3}{2}}(ex + d)^m}{(cdex^2 + ade + (cd^2 + ae^2)x)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^(3/2)\*(e\*x + d)^m/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m, x

[Out] integrate((g\*x + f)^(3/2)\*(e\*x + d)^m/(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m, x)



$$3.776 \quad \int (d + ex)^m \sqrt{f + gx} (ade + (cd^2 + ae^2)x + cdex^2)^{-m} dx$$

**Optimal.** Leaf size=105

$$\frac{2(f + gx)^{3/2}(d + ex)^m (x(ae^2 + cd^2) + ade + cdex^2)^{-m} \left(-\frac{g(ae+cdx)}{cdf-ae g}\right)^m {}_2F_1\left(\frac{3}{2}, m; \frac{5}{2}; \frac{cd(f+gx)}{cdf-ae g}\right)}{3g}$$

[Out] (2\*((g\*(a\*e + c\*d\*x))/(c\*d\*f - a\*e\*g)))^m\*(d + e\*x)^m\*(f + g\*x)^(3/2)\*Hypergeometric2F1[3/2, m, 5/2, (c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g)]/(3\*g\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m)

**Rubi [A]** time = 0.207663, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{2(f + gx)^{3/2}(d + ex)^m (x(ae^2 + cd^2) + ade + cdex^2)^{-m} \left(-\frac{g(ae+cdx)}{cdf-ae g}\right)^m {}_2F_1\left(\frac{3}{2}, m; \frac{5}{2}; \frac{cd(f+gx)}{cdf-ae g}\right)}{3g}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^m\*Sqrt[f + g\*x])/(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m, x]

[Out] (2\*((g\*(a\*e + c\*d\*x))/(c\*d\*f - a\*e\*g)))^m\*(d + e\*x)^m\*(f + g\*x)^(3/2)\*Hypergeometric2F1[3/2, m, 5/2, (c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g)]/(3\*g\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m)

**Rubi in Sympy [A]** time = 46.5318, size = 94, normalized size = 0.9

$$\frac{2\left(\frac{g(ae+cdx)}{ae g-cdf}\right)^m (d + ex)^m (f + gx)^{\frac{3}{2}} (ade + cdex^2 + x(ae^2 + cd^2))^{-m} {}_2F_1\left(m, \frac{3}{2} \middle| \frac{cd(-f-gx)}{ae g-cdf}\right)}{3g}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*m\*(g\*x+f)\*\*(1/2)/((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)

[Out] 2\*(g\*(a\*e + c\*d\*x)/(a\*e\*g - c\*d\*f))\*\*m\*(d + e\*x)\*\*m\*(f + g\*x)\*\*(3/2)\*(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*\*(-m)\*hyper((m, 3/2), (5/2, ), c\*d\*(-f - g\*x)/(a\*e\*g - c\*d\*f))/(3\*g)

**Mathematica [A]** time = 0.103981, size = 93, normalized size = 0.89

$$\frac{2(f + gx)^{3/2}(d + ex)^m((d + ex)(ae + cdx))^{-m} \left(\frac{g(ae+cdx)}{ae g-cdf}\right)^m {}_2F_1\left(\frac{3}{2}, m; \frac{5}{2}; \frac{cd(f+gx)}{cdf-ae g}\right)}{3g}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^m\*Sqrt[f + g\*x])/(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m, x

[Out] (2\*((g\*(a\*e + c\*d\*x))/(-(c\*d\*f) + a\*e\*g)))^m\*(d + e\*x)^m\*(f + g\*x)^(3/2)\*Hypergeometric2F1[3/2, m, 5/2, (c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g)]/(3\*g\*((a\*e + c\*d\*x)\*(d + e\*x))^m)

---

**Maple [F]** time = 0.114, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^m}{(ade + (ae^2 + cd^2)x + cdex^2)^m} \sqrt{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^m\*(g\*x+f)^(1/2)/((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^m), x)

[Out] int((e\*x+d)^m\*(g\*x+f)^(1/2)/((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^m), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{gx + f} (cdex^2 + ade + (cd^2 + ae^2)x)^{-m} (ex + d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(g\*x + f) \* (e\*x + d)^m / (c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m, x, a

[Out] integrate(sqrt(g\*x + f) \* (c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(  
-m) \* (e\*x + d)^m, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{\sqrt{gx + f} (ex + d)^m}{(cdex^2 + ade + (cd^2 + ae^2)x)^m}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(g\*x + f) \* (e\*x + d)^m / (c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m, x, a

[Out] integral(sqrt(g\*x + f) \* (e\*x + d)^m / (c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*m\*(g\*x+f)\*\*(1/2)/((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*m),

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx + f} (ex + d)^m}{(cdex^2 + ade + (cd^2 + ae^2)x)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(g*x + f)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m, x, a
```

```
[Out] integrate(sqrt(g*x + f)*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m, x)
```

$$3.777 \quad \int \frac{(d+ex)^m (ade+(cd^2+ae^2)x+cdex^2)^{-m}}{\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=103

$$\frac{2\sqrt{f+gx}(d+ex)^m (x(ae^2+cd^2)+ade+cdex^2)^{-m} \left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m {}_2F_1\left(\frac{1}{2}, m; \frac{3}{2}; \frac{cd(f+gx)}{cdf-aeg}\right)}{g}$$

[Out] (2\*(-((g\*(a\*e + c\*d\*x))/(c\*d\*f - a\*e\*g)))^m\*(d + e\*x)^m\*Sqrt[f + g\*x]\*Hypergeometric2F1[1/2, m, 3/2, (c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g)]/(g\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m)

**Rubi [A]** time = 0.204133, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{2\sqrt{f+gx}(d+ex)^m (x(ae^2+cd^2)+ade+cdex^2)^{-m} \left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m {}_2F_1\left(\frac{1}{2}, m; \frac{3}{2}; \frac{cd(f+gx)}{cdf-aeg}\right)}{g}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^m/(Sqrt[f + g\*x]\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m), x]

[Out] (2\*(-((g\*(a\*e + c\*d\*x))/(c\*d\*f - a\*e\*g)))^m\*(d + e\*x)^m\*Sqrt[f + g\*x]\*Hypergeometric2F1[1/2, m, 3/2, (c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g)]/(g\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m)

**Rubi in Sympy [A]** time = 47.0762, size = 92, normalized size = 0.89

$$\frac{2 \left(\frac{g(ae+cdx)}{aeg-cdf}\right)^m (d+ex)^m \sqrt{f+gx} (ade+cdex^2+x(ae^2+cd^2))^{-m} {}_2F_1\left(m, \frac{1}{2} \middle| \frac{cd(-f-gx)}{aeg-cdf}\right)}{g}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*m/(g\*x+f)\*\*(1/2)/((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)

[Out] 2\*(g\*(a\*e + c\*d\*x)/(a\*e\*g - c\*d\*f))\*\*m\*(d + e\*x)\*\*m\*sqrt(f + g\*x)\*(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*\*(-m)\*hyper((m, 1/2), (3/2, ), c\*d\*(-f - g\*x)/(a\*e\*g - c\*d\*f))/g

**Mathematica [A]** time = 0.0923525, size = 91, normalized size = 0.88

$$\frac{2\sqrt{f+gx}(d+ex)^m((d+ex)(ae+cdx))^{-m} \left(\frac{g(ae+cdx)}{aeg-cdf}\right)^m {}_2F_1\left(\frac{1}{2}, m; \frac{3}{2}; \frac{cd(f+gx)}{cdf-aeg}\right)}{g}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^m/(Sqrt[f + g\*x]\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m), x

[Out] (2\*((g\*(a\*e + c\*d\*x))/(-(c\*d\*f) + a\*e\*g))^m\*(d + e\*x)^m\*Sqrt[f + g\*x]\*Hypergeometric2F1[1/2, m, 3/2, (c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g)]/(g\*((a\*e + c\*d\*x)\*(d + e\*x))^m)

---

**Maple [F]** time = 0.115, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^m}{(ade + (ae^2 + cd^2)x + cdex^2)^m} \frac{1}{\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^m/(g\*x+f)^(1/2)/((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^m), x)

[Out] int((e\*x+d)^m/(g\*x+f)^(1/2)/((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^m), x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{-m}(ex + d)^m}{\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^m/(sqrt(g\*x + f)\*(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m), x)

[Out] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(-m)\*(e\*x + d)^m/sqrt(g\*x + f), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex + d)^m}{\sqrt{gx + f}(cdex^2 + ade + (cd^2 + ae^2)x)^m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^m/(sqrt(g\*x + f)\*(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m), x)

[Out] integral((e\*x + d)^m/(sqrt(g\*x + f)\*(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*m/(g\*x+f)\*\*(1/2)/((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*m),

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^m}{\sqrt{gx + f}(cdex^2 + ade + (cd^2 + ae^2)x)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^m/(sqrt(g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m), x
```

```
[Out] integrate((e*x + d)^m/(sqrt(g*x + f)*(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m), x)
```

$$3.778 \quad \int \frac{(d+ex)^m (ade+(cd^2+ae^2)x+cdex^2)^{-m}}{(f+gx)^{3/2}} dx$$

**Optimal.** Leaf size=103

$$\frac{2(d+ex)^m (x(ae^2+cd^2)+ade+cdex^2)^{-m} \left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m {}_2F_1\left(-\frac{1}{2}, m; \frac{1}{2}; \frac{cd(f+gx)}{cdf-aeg}\right)}{g\sqrt{f+gx}}$$

[Out] (-2\*(-((g\*(a\*e + c\*d\*x))/(c\*d\*f - a\*e\*g)))^m\*(d + e\*x)^m\*Hypergeometric2F1[-1/2, m, 1/2, (c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g)]/(g\*Sqrt[f + g\*x]\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m)

**Rubi [A]** time = 0.204386, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{2(d+ex)^m (x(ae^2+cd^2)+ade+cdex^2)^{-m} \left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m {}_2F_1\left(-\frac{1}{2}, m; \frac{1}{2}; \frac{cd(f+gx)}{cdf-aeg}\right)}{g\sqrt{f+gx}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^m/((f + g\*x)^(3/2)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m), x]

[Out] (-2\*(-((g\*(a\*e + c\*d\*x))/(c\*d\*f - a\*e\*g)))^m\*(d + e\*x)^m\*Hypergeometric2F1[-1/2, m, 1/2, (c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g)]/(g\*Sqrt[f + g\*x]\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m)

**Rubi in Sympy [A]** time = 46.4418, size = 95, normalized size = 0.92

$$\frac{2\left(\frac{g(ae+cdx)}{aeg-cdf}\right)^m (d+ex)^m (ade+cdex^2+x(ae^2+cd^2))^{-m} {}_2F_1\left(m, -\frac{1}{2} \middle| \frac{cd(-f-gx)}{aeg-cdf}\right)}{g\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*m/(g\*x+f)\*\*(3/2)/((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)

[Out] -2\*(g\*(a\*e + c\*d\*x)/(a\*e\*g - c\*d\*f))\*\*m\*(d + e\*x)\*\*m\*(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*\*(-m)\*hyper((m, -1/2), (1/2, ), c\*d\*(-f - g\*x)/(a\*e\*g - c\*d\*f))/(g\*sqrt(f + g\*x))

**Mathematica [A]** time = 0.0874882, size = 91, normalized size = 0.88

$$\frac{2(d+ex)^m((d+ex)(ae+cdx))^{-m} \left(\frac{g(ae+cdx)}{aeg-cdf}\right)^m {}_2F_1\left(-\frac{1}{2}, m; \frac{1}{2}; \frac{cd(f+gx)}{cdf-aeg}\right)}{g\sqrt{f+gx}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^m/((f + g\*x)^(3/2)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m)

[Out] (-2\*((g\*(a\*e + c\*d\*x))/(-(c\*d\*f) + a\*e\*g))^m\*(d + e\*x)^m\*Hypergeometric2F1[-1/2, m, 1/2, (c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g)]/(g\*((a\*

$e + c*d*x)^*(d + e*x))^m*\text{Sqrt}[f + g*x]$

**Maple [F]** time = 0.114, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^m}{(ade + (ae^2 + cd^2)x + cdex^2)^m} (gx + f)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^m/(g\*x+f)^(3/2)/((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^m), x)

[Out] int((e\*x+d)^m/(g\*x+f)^(3/2)/((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^m), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{-m}(ex + d)^m}{(gx + f)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^m/((g\*x + f)^(3/2)\*(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m)

[Out] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(-m)\*(e\*x + d)^m/(g\*x + f)^(3/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex + d)^m}{(gx + f)^{\frac{3}{2}}(cdex^2 + ade + (cd^2 + ae^2)x)^m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^m/((g\*x + f)^(3/2)\*(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m)

[Out] integral((e\*x + d)^m/((g\*x + f)^(3/2)\*(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*m/(g\*x+f)\*\*(3/2)/((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*m),

[Out] Timed out



GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^m}{(gx + f)^{\frac{3}{2}}(cdex^2 + ade + (cd^2 + ae^2)x)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^m/((g\*x + f)^(3/2)\*(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m)

[Out] integrate((e\*x + d)^m/((g\*x + f)^(3/2)\*(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m), x)

$$3.779 \quad \int \frac{(d+ex)^m (ade+(cd^2+ae^2)x+cdex^2)^{-m}}{(f+gx)^{5/2}} dx$$

**Optimal.** Leaf size=105

$$\frac{2(d+ex)^m (x(ae^2+cd^2)+ade+cdex^2)^{-m} \left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m {}_2F_1\left(-\frac{3}{2}, m; -\frac{1}{2}; \frac{cd(f+gx)}{cdf-aeg}\right)}{3g(f+gx)^{3/2}}$$

[Out]  $(-2*((g*(a*e+c*d*x))/(c*d*f-a*e*g)))^m*(d+e*x)^m*\text{Hypergeometric2F1}[-3/2, m, -1/2, (c*d*(f+g*x))/(c*d*f-a*e*g)]/(3*g*(f+g*x)^(3/2)*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^m)$

**Rubi [A]** time = 0.207044, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{2(d+ex)^m (x(ae^2+cd^2)+ade+cdex^2)^{-m} \left(-\frac{g(ae+cdx)}{cdf-aeg}\right)^m {}_2F_1\left(-\frac{3}{2}, m; -\frac{1}{2}; \frac{cd(f+gx)}{cdf-aeg}\right)}{3g(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^m/((f + g\*x)^(5/2)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m), x]

[Out]  $(-2*((g*(a*e+c*d*x))/(c*d*f-a*e*g)))^m*(d+e*x)^m*\text{Hypergeometric2F1}[-3/2, m, -1/2, (c*d*(f+g*x))/(c*d*f-a*e*g)]/(3*g*(f+g*x)^(3/2)*(a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2)^m)$

**Rubi in Sympy [A]** time = 46.551, size = 99, normalized size = 0.94

$$\frac{2\left(\frac{g(ae+cdx)}{aeg-cdf}\right)^m (d+ex)^m (ade+cdex^2+x(ae^2+cd^2))^{-m} {}_2F_1\left(m, -\frac{3}{2}; -\frac{1}{2}; \frac{cd(-f-gx)}{aeg-cdf}\right)}{3g(f+gx)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*m/(g\*x+f)\*\*(5/2)/((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)

[Out]  $-2*(g*(a*e+c*d*x)/(a*e*g-c*d*f))**m*(d+e*x)**m*(a*d*e+c*d*e*x**2+x*(a*e**2+c*d**2))**(-m)*\text{hyper}((m, -3/2), (-1/2, ), c*d*(-f-g*x)/(a*e*g-c*d*f))/(3*g*(f+g*x)**(3/2))$

**Mathematica [A]** time = 0.104664, size = 93, normalized size = 0.89

$$\frac{2(d+ex)^m((d+ex)(ae+cdx))^{-m} \left(\frac{g(ae+cdx)}{aeg-cdf}\right)^m {}_2F_1\left(-\frac{3}{2}, m; -\frac{1}{2}; \frac{cd(f+gx)}{cdf-aeg}\right)}{3g(f+gx)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^m/((f + g\*x)^(5/2)\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m)

[Out]  $(-2*((g*(a*e+c*d*x))/(-(c*d*f)+a*e*g))^m*(d+e*x)^m*\text{Hypergeometric2F1}[-3/2, m, -1/2, (c*d*(f+g*x))/(c*d*f-a*e*g)]/(3*g*($

$$(a^*e + c^*d^*x)^*(d + e^*x)^m*(f + g^*x)^{(3/2)}$$

**Maple [F]** time = 0.115, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^m}{(ade + (ae^2 + cd^2)x + cdex^2)^m} (gx + f)^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^m/(g\*x+f)^(5/2)/((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^m), x)

[Out] int((e\*x+d)^m/(g\*x+f)^(5/2)/((a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^m), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cdex^2 + ade + (cd^2 + ae^2)x)^{-m}(ex + d)^m}{(gx + f)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^m/((g\*x + f)^(5/2)\*(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m)

[Out] integrate((c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^(-m)\*(e\*x + d)^m/(g\*x + f)^(5/2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex + d)^m}{(g^2x^2 + 2fgx + f^2)\sqrt{gx + f}(cdex^2 + ade + (cd^2 + ae^2)x)^m}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^m/((g\*x + f)^(5/2)\*(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m)

[Out] integral((e\*x + d)^m/((g^2\*x^2 + 2\*f\*g\*x + f^2)\*sqrt(g\*x + f)\*(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*m/(g\*x+f)\*\*(5/2)/((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*m),

[Out] Timed out

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^m}{(gx + f)^{\frac{5}{2}}(cdex^2 + ade + (cd^2 + ae^2)x)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^m/((g\*x + f)^(5/2)\*(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m)

[Out] integrate((e\*x + d)^m/((g\*x + f)^(5/2)\*(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)^m), x)

$$3.780 \quad \int (ae+cdx)^n (d+ex)^m (ade + (cd^2 + ae^2)x + cdex^2)^{-m} dx$$

**Optimal.** Leaf size=65

$$\frac{(d+ex)^{m-1} (x(ae^2+cd^2) + ade + cdex^2)^{1-m} (ae+cdx)^n}{cd(-m+n+1)}$$

[Out]  $((a*e + c*d*x)^n * (d + e*x)^{(-1 + m)} * (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(1 - m)}) / (c*d*(1 - m + n))$

**Rubi [A]** time = 0.122533, antiderivative size = 65, normalized size of antiderivative = 1., number of rules used = 1, number of rules used = 1, integrand size = 47,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.021$

$$\frac{(d+ex)^{m-1} (x(ae^2+cd^2) + ade + cdex^2)^{1-m} (ae+cdx)^n}{cd(-m+n+1)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a*e + c*d*x)^n * (d + e*x)^m / (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m, x]$

[Out]  $((a*e + c*d*x)^n * (d + e*x)^{(-1 + m)} * (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^{(1 - m)}) / (c*d*(1 - m + n))$

**Rubi in Sympy [A]** time = 40.4363, size = 54, normalized size = 0.83

$$\frac{(d+ex)^{m-1} (ae+cdx)^n (ade + cdex^2 + x(ae^2 + cd^2))^{-m+1}}{cd(-m+n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((c*d*x+a*e)**n*(e*x+d)**m/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2))$

[Out]  $(d + e*x)**(m - 1) * (a*e + c*d*x)**n * (a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))**(-m + 1) / (c*d*(-m + n + 1))$

**Mathematica [A]** time = 0.0458331, size = 53, normalized size = 0.82

$$\frac{(d+ex)^m ((d+ex)(ae+cdx))^{-m} (ae+cdx)^{n+1}}{-cdm + cdn + cd}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(a*e + c*d*x)^n * (d + e*x)^m / (a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2)^m]$

[Out]  $((a*e + c*d*x)^{(1 + n)} * (d + e*x)^m) / ((c*d - c*d*m + c*d*n) * ((a*e + c*d*x) * (d + e*x))^m)$

**Maple [A]** time = 0.006, size = 64, normalized size = 1.

$$\frac{(cdx + ae)^{1+n} (ex + d)^m}{cd(-1 + m - n)(cdex^2 + ae^2x + cd^2x + ade)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((c*d*x+a*e)^n*(e*x+d)^m/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m), x)$

[Out]  $-(c*d*x+a*e)^{(1+n)}/c/d/(-1+m-n)*(e*x+d)^m/((c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^m)$

**Maxima [A]** time = 0.709393, size = 66, normalized size = 1.02

$$\frac{(cdx + ae)e^{(-m \log(cdx+ae)+n \log(cdx+ae))}}{cd(m - n - 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*d*x + a*e)^n*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m, x)$

[Out]  $-(c*d*x + a*e)*e^{(-m*\log(c*d*x + a*e) + n*\log(c*d*x + a*e))}/(c*d*(m - n - 1))$

**Fricas [A]** time = 0.280404, size = 89, normalized size = 1.37

$$\frac{(cdx + ae)(cdx + ae)^n(ex + d)^m e^{(-m \log(cdx+ae)-m \log(ex+d))}}{cdm - cdm - cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*d*x + a*e)^n*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m, x)$

[Out]  $-(c*d*x + a*e)*(c*d*x + a*e)^n*(e*x + d)^m*e^{(-m*\log(c*d*x + a*e) - m*\log(e*x + d))}/(c*d*m - c*d*n - c*d)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*d*x+a*e)**n*(e*x+d)**m/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m), x)$

[Out] Timed out

**GIAC/XCAS [A]** time = 0.278258, size = 108, normalized size = 1.66

$$\frac{cdxe^{(-m \ln(cdx+ae)+n \ln(cdx+ae))} + ae^{(-m \ln(cdx+ae)+n \ln(cdx+ae)+1)}}{cdm - cdm - cd}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*d*x + a*e)^n*(e*x + d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m, x)$

[Out]  $-(c*d*x*e^{(-m*\ln(c*d*x + a*e) + n*\ln(c*d*x + a*e))} + a*e^{(-m*\ln(c*d*x + a*e) + n*\ln(c*d*x + a*e) + 1)})/(c*d*m - c*d*n - c*d)$

$$3.781 \quad \int (d+ex)^m (cd^2eg - e(cd^2 + ae^2)g - cde^2gx)^{-1+m} (ade + (cd^2 + ae^2)x + cde^2x^2)^m dx$$

**Optimal.** Leaf size=78

$$\frac{(d+ex)^m (x(ae^2 + cd^2) + ade + cdex^2)^{-m} \log(ae + cdx) (-ae^3g - cde^2gx)^m}{cde^2g}$$

[Out] -(((d + e\*x)^m\*(-(a\*e^3\*g) - c\*d\*e^2\*g\*x)^m\*Log[a\*e + c\*d\*x])/(c\*d\*e^2\*g\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m))

**Rubi [A]** time = 0.326491, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 73,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.041$

$$\frac{(d+ex)^m (x(ae^2 + cd^2) + ade + cdex^2)^{-m} \log(ae + cdx) (-ae^3g - cde^2gx)^m}{cde^2g}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^m\*(c\*d^2\*e\*g - e\*(c\*d^2 + a\*e^2)\*g - c\*d\*e^2\*g\*x)^(-1 + m)]/(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m, x]

[Out] -(((d + e\*x)^m\*(-(a\*e^3\*g) - c\*d\*e^2\*g\*x)^m\*Log[a\*e + c\*d\*x])/(c\*d\*e^2\*g\*(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m))

**Rubi in Sympy [A]** time = 174.401, size = 73, normalized size = 0.94

$$\frac{(d+ex)^m (-ae^3g - cde^2gx)^m (ade + cdex^2 + x(ae^2 + cd^2))^{-m} \log(ae + cdx)}{cde^2g}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*m\*(c\*d\*\*2\*e\*g-e\*(a\*e\*\*2+c\*d\*\*2)\*g-c\*d\*e\*\*2\*g\*x)\*\*(-1+m)/((a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*m), x)

[Out] -(d + e\*x)\*\*m\*(-a\*e\*\*3\*g - c\*d\*e\*\*2\*g\*x)\*\*m\*(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))\*\*(-m)\*log(a\*e + c\*d\*x)/(c\*d\*e\*\*2\*g)

**Mathematica [A]** time = 0.0552863, size = 64, normalized size = 0.82

$$\frac{(d+ex)^m((d+ex)(ae+cdx))^{-m} \log(ae+cdx) (-e^2g(ae+cdx))^m}{cde^2g}$$

Antiderivative was successfully verified.

[In] Integrate[(((d + e\*x)^m\*(c\*d^2\*e\*g - e\*(c\*d^2 + a\*e^2)\*g - c\*d\*e^2\*g\*x)^(-1 + m)))/(a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2)^m, x]

[Out] -((((-(e^2\*g\*(a\*e + c\*d\*x)))^m\*(d + e\*x)^m\*Log[a\*e + c\*d\*x])/(c\*d\*e^2\*g\*((a\*e + c\*d\*x)\*(d + e\*x))^m))

**Maple [F]** time = 0.311, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m (cd^2eg - e(ae^2 + cd^2)g - cde^2gx)^{-1+m}}{(ade + (ae^2 + cd^2)x + cdex^2)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m), x)`

[Out] `int((e*x+d)^m*(c*d^2*e*g-e*(a*e^2+c*d^2)*g-c*d*e^2*g*x)^(-1+m)/((a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^m), x)`

**Maxima [A]** time = 0.720535, size = 43, normalized size = 0.55

$$-\frac{e^{2m-2}(-g)^m \log(cdx + ae)}{cdg}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-c*d*e^2*g*x + c*d^2*e*g - (c*d^2 + a*e^2)*e*g)^(m-1)*(e*x+d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m, x)`

[Out] `-e^(2*m-2)*(-g)^m*log(c*d*x+a*e)/(c*d*g)`

**Fricas [A]** time = 0.274066, size = 47, normalized size = 0.6

$$-\frac{\log(cdx + ae)}{cde^2g\left(-\frac{1}{e^2g}\right)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-c*d*e^2*g*x + c*d^2*e*g - (c*d^2 + a*e^2)*e*g)^(m-1)*(e*x+d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m, x)`

[Out] `-log(c*d*x+a*e)/(c*d*e^2*g*(-1/(e^2*g))^m)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**m*(c*d**2*e*g-e*(a*e**2+c*d**2)*g-c*d*e**2*g*x)**(-1+m)/((a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**m), x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(-cde^2gx + cd^2eg - (cd^2 + ae^2)eg)^{m-1}(ex + d)^m}{(cdex^2 + ade + (cd^2 + ae^2)x)^m} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-c*d*e^2*g*x + c*d^2*e*g - (c*d^2 + a*e^2)*e*g)^(m-1)*(e*x+d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m, x)`

[Out] `integrate((-c*d*e^2*g*x + c*d^2*e*g - (c*d^2 + a*e^2)*e*g)^(m-1)*(e*x+d)^m/(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)^m, x)`



$$3.782 \quad \int \frac{(d+ex)^{3/2}(f+gx)^n}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=213

$$\frac{\sqrt{d+ex}(f+gx)^{n+1}(ae+cdx)(2ae^2g(n+1)+cd(ef-dg(2n+3))) {}_2F_1\left(1, n+\frac{3}{2}; n+2; \frac{cd(f+gx)}{cdf-aeg}\right)}{cdg(n+1)(2n+3)\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)} + \frac{2e(f+gx)^{n+1}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cdg(2n+3)\sqrt{d+ex}}$$

[Out] (2\*e\*(f+g\*x)^(1+n)\*Sqrt[a\*d\*e+(c\*d^2+a\*e^2)\*x+c\*d\*e\*x^2])/(c\*d\*g\*(3+2\*n)\*Sqrt[d+e\*x]) + ((2\*a\*e^2\*g\*(1+n)+c\*d\*(e\*f-d\*g\*(3+2\*n)))\*(a\*e+c\*d\*x)\*Sqrt[d+e\*x]\*(f+g\*x)^(1+n))\*Hypergeometric2F1[1, 3/2+n, 2+n, (c\*d\*(f+g\*x))/(c\*d\*f-a\*e\*g)]/(c\*d\*g\*(c\*d\*f-a\*e\*g)\*(1+n)\*(3+2\*n)\*Sqrt[a\*d\*e+(c\*d^2+a\*e^2)\*x+c\*d\*e\*x^2])

**Rubi [A]** time = 0.773666, antiderivative size = 222, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{2e(f+gx)^{n+1}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cdg(2n+3)\sqrt{d+ex}} - \frac{2\sqrt{d+ex}(f+gx)^n(ae+cdx)(2ae^2g(n+1)+cd(ef-dg(2n+3))) \left(\frac{cd(f+gx)}{cdf-aeg}\right)^{-n} {}_2F_1\left(\frac{1}{2}, -n; \frac{3}{2}; -\frac{g(ae+cdx)}{cdf-aeg}\right)}{c^2d^2g(2n+3)\sqrt{x(ae^2+cd^2)+ade+cdex^2}}$$

Antiderivative was successfully verified.

[In] Int[((d+e\*x)^(3/2)\*(f+g\*x)^n)/Sqrt[a\*d\*e+(c\*d^2+a\*e^2)\*x+c\*d\*e\*x^2], x]

[Out] (2\*e\*(f+g\*x)^(1+n)\*Sqrt[a\*d\*e+(c\*d^2+a\*e^2)\*x+c\*d\*e\*x^2])/(c\*d\*g\*(3+2\*n)\*Sqrt[d+e\*x]) - (2\*(2\*a\*e^2\*g\*(1+n)+c\*d\*(e\*f-d\*g\*(3+2\*n)))\*(a\*e+c\*d\*x)\*Sqrt[d+e\*x]\*(f+g\*x)^n\*Hypergeometric2F1[1/2, -n, 3/2, -((g\*(a\*e+c\*d\*x))/(c\*d\*f-a\*e\*g))])/(c^2\*d^2\*g\*(3+2\*n)\*((c\*d\*(f+g\*x))/(c\*d\*f-a\*e\*g))^n\*Sqrt[a\*d\*e+(c\*d^2+a\*e^2)\*x+c\*d\*e\*x^2])

**Rubi in Sympy [A]** time = 89.9942, size = 202, normalized size = 0.95

$$\frac{2e(f+gx)^{n+1}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{cdg\sqrt{d+ex}(2n+3)} - \frac{2\left(\frac{cd(-f-gx)}{aeg-cdf}\right)^{-n}(f+gx)^n\sqrt{ade+cdex^2+x(ae^2+cd^2)}(-cd^2g(4n+5)+cdf+2g(n+1)(ae^2+cd^2)) {}_2F_1\left(-n, \frac{1}{2}; \frac{3}{2}; \frac{g(ae+cdx)}{aeg-cdf}\right)}{c^2d^2g\sqrt{d+ex}(2n+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*(3/2)\*(g\*x+f)\*\*n/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2), x)

[Out] 2\*e\*(f+g\*x)\*\*(n+1)\*sqrt(a\*d\*e+c\*d\*e\*x\*\*2+x\*(a\*e\*\*2+c\*d\*\*2))/(c\*d\*g\*sqrt(d+e\*x)\*(2\*n+3)) - 2\*(c\*d\*(-f-g\*x)/(a\*e\*g-c\*d\*f))\*\*(-n)\*(f+g\*x)\*\*n\*sqrt(a\*d\*e+c\*d\*e\*x\*\*2+x\*(a\*e\*\*2+c\*d\*\*2))\*(-c\*d\*\*2\*g\*(4\*n+5)+c\*d\*e\*f+2\*g\*(n+1)\*(a\*e\*\*2+c\*d\*\*2))\*hyper((-n, 1/2), (3/2, ), g\*(a\*e+c\*d\*x)/(a\*e\*g-c\*d\*f))/(c\*\*2\*d\*\*2\*g\*sqrt(d+e\*x)\*(2\*n+3))

**Mathematica [A]** time = 0.28842, size = 157, normalized size = 0.74

$$\frac{2(f+gx)^n \sqrt{(d+ex)(ae+cdx)} \left(\frac{cd(f+gx)}{cdf-ae^2}\right)^{-n} \left(3(cd^2-ae^2) {}_2F_1\left(\frac{1}{2}, -n; \frac{3}{2}; \frac{g(ae+cdx)}{aeg-cdf}\right) + e(ae+cdx) {}_2F_1\left(\frac{3}{2}, -n; \frac{5}{2}; \frac{g(ae+cdx)}{aeg-cdf}\right)\right)}{3c^2d^2\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^(3/2) \* (f + g\*x)^n) / Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2], x]

[Out] (2\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)]\*(f + g\*x)^n\*(3\*(c\*d^2 - a\*e^2)\*Hypergeometric2F1[1/2, -n, 3/2, (g\*(a\*e + c\*d\*x))/(-(c\*d\*f) + a\*e\*g)] + e\*(a\*e + c\*d\*x)\*Hypergeometric2F1[3/2, -n, 5/2, (g\*(a\*e + c\*d\*x))/(-(c\*d\*f) + a\*e\*g)])) / (3\*c^2\*d^2\*Sqrt[d + e\*x]\*((c\*d\*(f + g\*x))/(c\*d\*f - a\*e\*g))^n)

**Maple [F]** time = 0.112, size = 0, normalized size = 0.

$$\int (gx + f)^n (ex + d)^{\frac{3}{2}} \frac{1}{\sqrt{ade + (ae^2 + cd^2)x + cdex^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(3/2)\*(g\*x+f)^n/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2), x)

[Out] int((e\*x+d)^(3/2)\*(g\*x+f)^n/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^{\frac{3}{2}}(gx + f)^n}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*(g\*x + f)^n/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

[Out] integrate((e\*x + d)^(3/2)\*(g\*x + f)^n/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex + d)^{\frac{3}{2}}(gx + f)^n}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*(g\*x + f)^n/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

[Out] integral((e\*x + d)^(3/2)\*(g\*x + f)^n/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(3/2)\*(g\*x+f)\*\*n/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)

[Out] Timed out

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^{\frac{3}{2}}(gx + f)^n}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*(g\*x + f)^n/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)

[Out] integrate((e\*x + d)^(3/2)\*(g\*x + f)^n/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

$$3.783 \quad \int \frac{(d+ex)^{3/2}(f+gx)^4}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=501

$$\begin{aligned} & \frac{128\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^3(10ae^2g+cd(ef-11dg))(2ae^2g-cd(3ef-dg))}{3465c^6d^6eg\sqrt{d+ex}} \\ & - \frac{128\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^3(10ae^2g+cd(ef-11dg))}{3465c^5d^5e} \\ & - \frac{32(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2(10ae^2g+cd(ef-11dg))}{1155c^4d^4g\sqrt{d+ex}} \\ & - \frac{16(f+gx)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)(10ae^2g+cd(ef-11dg))}{693c^3d^3g\sqrt{d+ex}} \\ & - \frac{2(f+gx)^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}(10ae^2g+cd(ef-11dg))}{99c^2d^2g\sqrt{d+ex}} \\ & + \frac{2e(f+gx)^5\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{11cdg\sqrt{d+ex}} \end{aligned}$$

[Out] (128\*(c\*d\*f - a\*e\*g)^3\*(10\*a\*e^2\*g + c\*d\*(e\*f - 11\*d\*g))\*(2\*a\*e^2\*g - c\*d\*(3\*e\*f - d\*g))\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(3465\*c^6\*d^6\*e\*g\*Sqrt[d + e\*x]) - (128\*(c\*d\*f - a\*e\*g)^3\*(10\*a\*e^2\*g + c\*d\*(e\*f - 11\*d\*g))\*Sqrt[d + e\*x]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(3465\*c^5\*d^5\*e) - (32\*(c\*d\*f - a\*e\*g)^2\*(10\*a\*e^2\*g + c\*d\*(e\*f - 11\*d\*g))\*(f + g\*x)^2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(1155\*c^4\*d^4\*g\*Sqrt[d + e\*x]) - (16\*(c\*d\*f - a\*e\*g)\*(10\*a\*e^2\*g + c\*d\*(e\*f - 11\*d\*g))\*(f + g\*x)^3\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(693\*c^3\*d^3\*g\*Sqrt[d + e\*x]) - (2\*(10\*a\*e^2\*g + c\*d\*(e\*f - 11\*d\*g))\*(f + g\*x)^4\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(99\*c^2\*d^2\*g\*Sqrt[d + e\*x]) + (2\*e\*(f + g\*x)^5\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(11\*c\*d\*g\*Sqrt[d + e\*x])

**Rubi [A]** time = 2.27652, antiderivative size = 501, normalized size of antiderivative = 1., number of rules used = 6, number of rules used = 4, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\begin{aligned} & \frac{128\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^3(10ae^2g+cd(ef-11dg))(2ae^2g-cd(3ef-dg))}{3465c^6d^6eg\sqrt{d+ex}} \\ & - \frac{128\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^3(10ae^2g+cd(ef-11dg))}{3465c^5d^5e} \\ & - \frac{32(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2(10ae^2g+cd(ef-11dg))}{1155c^4d^4g\sqrt{d+ex}} \\ & - \frac{16(f+gx)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)(10ae^2g+cd(ef-11dg))}{693c^3d^3g\sqrt{d+ex}} \\ & - \frac{2(f+gx)^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}(10ae^2g+cd(ef-11dg))}{99c^2d^2g\sqrt{d+ex}} \\ & + \frac{2e(f+gx)^5\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{11cdg\sqrt{d+ex}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^(3/2)\*(f + g\*x)^4)/Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2], x]

[Out] (128\*(c\*d\*f - a\*e\*g)^3\*(10\*a\*e^2\*g + c\*d\*(e\*f - 11\*d\*g))\*(2\*a\*e^2\*g - c\*d\*(3\*e\*f - d\*g))\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(3465\*c^6\*d^6\*e\*g\*Sqrt[d + e\*x]) - (128\*(c\*d\*f - a\*e\*g)^3\*(10\*a\*e^2\*g + c\*d\*(e\*f - 11\*d\*g))\*Sqrt[d + e\*x]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(3465\*c^5\*d^5\*e) - (32\*(c\*d\*f - a\*e\*g)^2\*(10\*a\*e^2\*g + c\*d\*(e\*f - 11\*d\*g))\*(f + g\*x)^2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(1155\*c^4\*d^4\*g\*Sqrt[d + e\*x]) - (16\*(c\*d\*f - a\*e\*g)\*(10\*a\*e^2\*g + c\*d\*(e\*f - 11\*d\*g))\*(f + g\*x)^3\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(693\*c^3\*d^3\*g\*Sqrt[d + e\*x]) - (2\*(10\*a\*e^2\*g + c\*d\*(e\*f - 11\*d\*g))\*(f + g\*x)^4\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]/(99\*c^2\*d^2\*g\*Sqrt[d + e\*x]) + (2\*e\*(f + g\*x)^5\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(11\*c\*d\*g\*Sqrt[d + e\*x])

$$\begin{aligned} &+ a^*e^2)^*x + c^*d^*e^*x^2])/(3465^*c^5*d^5*e) - (32^*(c^*d^*f - a^*e^*g)^2 \\ &*(10^*a^*e^2*g + c^*d^*(e^*f - 11^*d^*g))^*(f + g^*x)^2*\text{Sqrt}[a^*d^*e + (c^*d^2 \\ &+ a^*e^2)^*x + c^*d^*e^*x^2])/(1155^*c^4*d^4*g^*\text{Sqrt}[d + e^*x]) - (16^*( \\ &c^*d^*f - a^*e^*g)^*(10^*a^*e^2*g + c^*d^*(e^*f - 11^*d^*g))^*(f + g^*x)^3*\text{Sqrt} \\ &[a^*d^*e + (c^*d^2 + a^*e^2)^*x + c^*d^*e^*x^2])/(693^*c^3*d^3*g^*\text{Sqrt}[d + \\ &e^*x]) - (2^*(10^*a^*e^2*g + c^*d^*(e^*f - 11^*d^*g))^*(f + g^*x)^4*\text{Sqrt}[a^*d \\ &^*e + (c^*d^2 + a^*e^2)^*x + c^*d^*e^*x^2])/(99^*c^2*d^2*g^*\text{Sqrt}[d + e^*x]) \\ &+ (2^*e^*(f + g^*x)^5*\text{Sqrt}[a^*d^*e + (c^*d^2 + a^*e^2)^*x + c^*d^*e^*x^2])/( \\ &(11^*c^*d^*g^*\text{Sqrt}[d + e^*x]) \end{aligned}$$

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**(3/2)*(g*x+f)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)*`

[Out] Timed out

**Mathematica [A]** time = 0.58769, size = 380, normalized size = 0.76

$$\frac{2\sqrt{(d+ex)(ae+cdx)}(-1280a^5e^6g^4+128a^4cde^4g^3(11dg+44ef+5egx)-32a^3c^2d^2e^3g^2(22dg(9f+gx)+e(297f^2+88fg$$

Antiderivative was successfully verified.

[In] `Integrate[((d + e*x)^(3/2)*(f + g*x)^4)/Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^`

[Out] `(2*Sqrt[(a*e + c*d*x)*(d + e*x)]*(-1280*a^5*e^6*g^4 + 128*a^4*c*d^*e^4*g^3*(44*e*f + 11*d*g + 5*e*g*x) - 32*a^3*c^2*d^2*e^3*g^2*(22*d*g*(9*f + g*x) + e*(297*f^2 + 88*f*g*x + 15*g^2*x^2)) + 16*a^2*c^3*d^3*e^2*g*(33*d*g*(21*f^2 + 6*f*g*x + g^2*x^2) + e*(462*f^3 + 297*f^2*g*x + 132*f*g^2*x^2 + 25*g^3*x^3)) - 2*a*c^4*d^4*e*(44*d*g*(105*f^3 + 63*f^2*g*x + 27*f*g^2*x^2 + 5*g^3*x^3) + e*(1155*f^4 + 1848*f^3*g*x + 1782*f^2*g^2*x^2 + 880*f*g^3*x^3 + 175*g^4*x^4)) + c^5*d^5*(11*d*(315*f^4 + 420*f^3*g*x + 378*f^2*g^2*x^2 + 180*f*g^3*x^3 + 35*g^4*x^4) + e*x*(1155*f^4 + 2772*f^3*g*x + 2970*f^2*g^2*x^2 + 1540*f*g^3*x^3 + 315*g^4*x^4)))/(3465*c^6*d^6*Sqrt[d + e*x])`

**Maple [A]** time = 0.014, size = 641, normalized size = 1.3

$$\frac{(2cdx + 2ae)(-315eg^4x^5c^5d^5 + 350ac^4d^4e^2g^4x^4 - 385c^5d^6g^4x^4 - 1540c^5d^5efg^3x^4 - 400a^2c^3d^3e^3g^4x^3 + 440ac^4d^5eg^4x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^(3/2)*(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)`

[Out] `-2/3465*(c*d*x+a*e)*(-315*c^5*d^5*e*g^4*x^5+350*a*c^4*d^4*e^2*g^4*x^4-385*c^5*d^6*g^4*x^4-1540*c^5*d^5*e*f*g^3*x^4-400*a^2*c^3*d^3*e^3*g^4*x^3+440*a*c^4*d^5*e*g^4*x^3+1760*a*c^4*d^4*e^2*f*g^3*x^3-1980*c^5*d^6*f*g^3*x^3-2970*c^5*d^5*e*f^2*g^2*x^3+480*a^3*c^2*d^2*e^4*g^4*x^2-528*a^2*c^3*d^4*e^2*g^4*x^2-2112*a^2*c^3*d^3*e^3*f*g^3*x^2+2376*a*c^4*d^5*e*f*g^3*x^2+3564*a*c^4*d^4*e^2*f^2*g^2*x^2-4158*c^5*d^6*f^2*g^2*x^2-2772*c^5*d^5*e*f^3*g*x^2-640*a^4*c*d*e^`



$$\begin{aligned} & *c^6*d^7*e - a*c^5*d^5*e^3)*f^2*g^2 + 44*(45*c^6*d^8 - 14*a*c^5*d \\ & ^6*e^2 + 8*a^2*c^4*d^4*e^4)*f*g^3 - (55*a*c^5*d^7*e - 138*a^2*c^4 \\ & *d^5*e^3 + 80*a^3*c^3*d^3*e^5)*g^4)*x^4 + (1155*c^6*d^6*e^2*f^4 + \\ & 924*(8*c^6*d^7*e - a*c^5*d^5*e^3)*f^3*g + 198*(21*c^6*d^8 - 10*a \\ & *c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4)*f^2*g^2 - 44*(9*a*c^5*d^7*e - 2 \\ & 6*a^2*c^4*d^5*e^3 + 16*a^3*c^3*d^3*e^5)*f*g^3 + 8*(11*a^2*c^4*d^6 \\ & *e^2 - 32*a^3*c^3*d^4*e^4 + 20*a^4*c^2*d^2*e^6)*g^4)*x^3 + (1155* \\ & (4*c^6*d^7*e - a*c^5*d^5*e^3)*f^4 + 924*(5*c^6*d^8 - 6*a*c^5*d^6* \\ & e^2 + 4*a^2*c^4*d^4*e^4)*f^3*g - 198*(7*a*c^5*d^7*e - 34*a^2*c^4* \\ & d^5*e^3 + 24*a^3*c^3*d^3*e^5)*f^2*g^2 + 88*(9*a^2*c^4*d^6*e^2 - 4 \\ & 4*a^3*c^3*d^4*e^4 + 32*a^4*c^2*d^2*e^6)*f*g^3 - 16*(11*a^3*c^3*d^4 \\ & 5*e^3 - 54*a^4*c^2*d^3*e^5 + 40*a^5*c*d*e^7)*g^4)*x^2 + (1155*(3* \\ & c^6*d^8 + 2*a*c^5*d^6*e^2 - 2*a^2*c^4*d^4*e^4)*f^4 - 924*(5*a*c^5 \\ & *d^7*e + 6*a^2*c^4*d^5*e^3 - 8*a^3*c^3*d^3*e^5)*f^3*g + 792*(7*a^2 \\ & *c^4*d^6*e^2 + 8*a^3*c^3*d^4*e^4 - 12*a^4*c^2*d^2*e^6)*f^2*g^2 - \\ & 352*(9*a^3*c^3*d^5*e^3 + 10*a^4*c^2*d^3*e^5 - 16*a^5*c*d*e^7)*f* \\ & g^3 + 64*(11*a^4*c^2*d^4*e^4 + 12*a^5*c*d^2*e^6 - 20*a^6*e^8)*g^4 \\ & )*x)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*c \\ & ^6*d^6) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(3/2)\*(g\*x+f)\*\*4/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.414998, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*(g\*x + f)^4/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)

[Out] Done

$$3.784 \quad \int \frac{(d+ex)^{3/2}(f+gx)^3}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=412

$$\begin{aligned} & \frac{16\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2(8ae^2g+cd(ef-9dg))(2ae^2g-cd(3ef-dg))}{315c^5d^5eg\sqrt{d+ex}} \\ & - \frac{16\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2(8ae^2g+cd(ef-9dg))}{315c^4d^4e} \\ & - \frac{4(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)(8ae^2g+cd(ef-9dg))}{105c^3d^3g\sqrt{d+ex}} \\ & - \frac{2(f+gx)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}(8ae^2g+cd(ef-9dg))}{63c^2d^2g\sqrt{d+ex}} \\ & + \frac{2e(f+gx)^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{9cdg\sqrt{d+ex}} \end{aligned}$$

[Out] (16\*(c\*d\*f - a\*e\*g)^2\*(8\*a\*e^2\*g + c\*d\*(e\*f - 9\*d\*g))\*(2\*a\*e^2\*g - c\*d\*(3\*e\*f - d\*g))\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]) / (315\*c^5\*d^5\*e\*g\*Sqrt[d + e\*x]) - (16\*(c\*d\*f - a\*e\*g)^2\*(8\*a\*e^2\*g + c\*d\*(e\*f - 9\*d\*g))\*Sqrt[d + e\*x]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]) / (315\*c^4\*d^4\*e) - (4\*(c\*d\*f - a\*e\*g)\*(8\*a\*e^2\*g + c\*d\*(e\*f - 9\*d\*g))\*(f + g\*x)^2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]) / (105\*c^3\*d^3\*g\*Sqrt[d + e\*x]) - (2\*(8\*a\*e^2\*g + c\*d\*(e\*f - 9\*d\*g))\*(f + g\*x)^3\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]) / (63\*c^2\*d^2\*g\*Sqrt[d + e\*x]) + (2\*e\*(f + g\*x)^4\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]) / (9\*c\*d\*g\*Sqrt[d + e\*x])

**Rubi [A]** time = 1.6817, antiderivative size = 412, normalized size of antiderivative = 1., number of rules used = 5, number of rules used = 4, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\begin{aligned} & \frac{16\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2(8ae^2g+cd(ef-9dg))(2ae^2g-cd(3ef-dg))}{315c^5d^5eg\sqrt{d+ex}} \\ & - \frac{16\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)^2(8ae^2g+cd(ef-9dg))}{315c^4d^4e} \\ & - \frac{4(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)(8ae^2g+cd(ef-9dg))}{105c^3d^3g\sqrt{d+ex}} \\ & - \frac{2(f+gx)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}(8ae^2g+cd(ef-9dg))}{63c^2d^2g\sqrt{d+ex}} \\ & + \frac{2e(f+gx)^4\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{9cdg\sqrt{d+ex}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(((d + e\*x)^(3/2)\*(f + g\*x)^3)/Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2], x]

[Out] (16\*(c\*d\*f - a\*e\*g)^2\*(8\*a\*e^2\*g + c\*d\*(e\*f - 9\*d\*g))\*(2\*a\*e^2\*g - c\*d\*(3\*e\*f - d\*g))\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]) / (315\*c^5\*d^5\*e\*g\*Sqrt[d + e\*x]) - (16\*(c\*d\*f - a\*e\*g)^2\*(8\*a\*e^2\*g + c\*d\*(e\*f - 9\*d\*g))\*Sqrt[d + e\*x]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]) / (315\*c^4\*d^4\*e) - (4\*(c\*d\*f - a\*e\*g)\*(8\*a\*e^2\*g + c\*d\*(e\*f - 9\*d\*g))\*(f + g\*x)^2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]) / (105\*c^3\*d^3\*g\*Sqrt[d + e\*x]) - (2\*(8\*a\*e^2\*g + c\*d\*(e\*f - 9\*d\*g))\*(f + g\*x)^3\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]) / (63\*c^2\*d^2\*g\*Sqrt[d + e\*x]) + (2\*e\*(f + g\*x)^4\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]) / (9\*c\*d\*g\*Sqrt[d + e\*x])



**Rubi in Sympy [A]** time = 139.16, size = 418, normalized size = 1.01

$$\frac{2e(f+gx)^4 \sqrt{ade+cdex^2+x(ae^2+cd^2)}}{9cdg\sqrt{d+ex}} - \frac{2(f+gx)^3 \sqrt{ade+cdex^2+x(ae^2+cd^2)}(8ae^2g-9cd^2g+cdef)}{63c^2d^2g\sqrt{d+ex}} + \frac{4(f+gx)^2(aeg-cdf)\sqrt{ade+cdex^2+x(ae^2+cd^2)}(8ae^2g-9cd^2g+cdef)}{105c^3d^3g\sqrt{d+ex}} - \frac{16\sqrt{d+ex}(aeg-cdf)^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}(8ae^2g-9cd^2g+cdef)}{315c^4d^4e} + \frac{16(aeg-cdf)^2\sqrt{ade+cdex^2+x(ae^2+cd^2)}(2ae^2g+cd^2g-3cdef)(8ae^2g-9cd^2g+cdef)}{315c^5d^5eg\sqrt{d+ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**(3/2)*(g*x+f)**3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2))`

[Out]  $2*e*(f+g*x)**4*\sqrt{a*d*e+c*d*e*x**2+x*(a*e**2+c*d**2)} / (9*c*d*g*\sqrt{d+e*x}) - 2*(f+g*x)**3*\sqrt{a*d*e+c*d*e*x**2+x*(a*e**2+c*d**2)}*(8*a*e**2*g-9*c*d**2*g+c*d*e*f) / (63*c**2*d**2*g*\sqrt{d+e*x}) + 4*(f+g*x)**2*(a*e*g-c*d*f)*\sqrt{a*d*e+c*d*e*x**2+x*(a*e**2+c*d**2)}*(8*a*e**2*g-9*c*d**2*g+c*d*e*f) / (105*c**3*d**3*g*\sqrt{d+e*x}) - 16*\sqrt{d+e*x}*(a*e*g-c*d*f)**2*\sqrt{a*d*e+c*d*e*x**2+x*(a*e**2+c*d**2)}*(8*a*e**2*g-9*c*d**2*g+c*d*e*f) / (315*c**4*d**4*e) + 16*(a*e*g-c*d*f)**2*\sqrt{a*d*e+c*d*e*x**2+x*(a*e**2+c*d**2)}*(2*a*e**2*g+c*d**2*g-3*c*d*e*f)*(8*a*e**2*g-9*c*d**2*g+c*d*e*f) / (315*c**5*d**5*e*g*\sqrt{d+e*x})$

**Mathematica [A]** time = 0.429614, size = 264, normalized size = 0.64

$$\frac{2\sqrt{(d+ex)(ae+cdx)}(128a^4e^5g^3-16a^3cde^3g^2(9dg+27ef+4egx)+24a^2c^2d^2e^2g(3dg(7f+gx)+e(21f^2+9fgx+2g^2x^2))}{}$$

Antiderivative was successfully verified.

[In] `Integrate[((d+e*x)^(3/2)*(f+g*x)^3)/Sqrt[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2]]`

[Out]  $(2*\sqrt{(a*e+c*d*x)*(d+e*x)}*(128*a^4*e^5*g^3-16*a^3*c*d*e^3*g^2*(27*e*f+9*d*g+4*e*g*x)+24*a^2*c^2*d^2*e^2*g*(3*d*g*(7*f+g*x)+e*(21*f^2+9*f*g*x+2*g^2*x^2)))-2*a*c^3*d^3*e*(9*d*g*(35*f^2+14*f*g*x+3*g^2*x^2)+e*(105*f^3+126*f^2*g*x+81*f*g^2*x^2+20*g^3*x^3))+c^4*d^4*(9*d*(35*f^3+35*f^2*g*x+21*f*g^2*x^2+5*g^3*x^3)+e*x*(105*f^3+189*f^2*g*x+135*f*g^2*x^2+35*g^3*x^3)))/(315*c^5*d^5*\sqrt{d+e*x})$

**Maple [A]** time = 0.013, size = 425, normalized size = 1.

$$\frac{(2cdx+2ae)(35eg^3x^4c^4d^4-40ac^3d^3e^2g^3x^3+45c^4d^5g^3x^3+135c^4d^4efg^2x^3+48a^2c^2d^2e^3g^3x^2-54ac^3d^4eg^3x^2-162ac^3d^4eg^3x^2)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^(3/2)*(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)`

[Out]  $2/315*(c*d*x+a*e)*(35*c^4*d^4*e*g^3*x^4-40*a*c^3*d^3*e^2*g^3*x^3+45*c^4*d^5*g^3*x^3+135*c^4*d^4*e*f*g^2*x^3+48*a^2*c^2*d^2*e^3*g^3*x^2-54*ac^3d^4eg^3x^2-162ac^3d^4eg^3x^2)$

$$\begin{aligned} & *x^2 - 54*a*c^3*d^4*e*g^3*x^2 - 162*a*c^3*d^3*e^2*f*g^2*x^2 + 189*c^4*d \\ & ^5*f*g^2*x^2 + 189*c^4*d^4*e*f^2*g*x^2 - 64*a^3*c*d*e^4*g^3*x + 72*a^2* \\ & c^2*d^3*e^2*g^3*x + 216*a^2*c^2*d^2*e^3*f*g^2*x - 252*a*c^3*d^4*e*f*g \\ & ^2*x - 252*a*c^3*d^3*e^2*f^2*g*x + 315*c^4*d^5*f^2*g*x + 105*c^4*d^4*e* \\ & f^3*x + 128*a^4*e^5*g^3 - 144*a^3*c*d^2*e^3*g^3 - 432*a^3*c*d*e^4*f*g^2 \\ & + 504*a^2*c^2*d^3*e^2*f*g^2 + 504*a^2*c^2*d^2*e^3*f^2*g - 630*a*c^3*d^4 \\ & ^4*e*f^2*g - 210*a*c^3*d^3*e^2*f^3 + 315*c^4*d^5*f^3) * (e*x+d)^(1/2)/c^4 \\ & 5/d^5/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2) \end{aligned}$$

**Maxima [A]** time = 0.755974, size = 653, normalized size = 1.58

$$\begin{aligned} & \frac{2(c^2d^2ex^2 + 3acd^2e - 2a^2e^3 + (3c^2d^3 - acde^2)x)f^3}{3\sqrt{cdx + aec^2d^2}} \\ & + \frac{2(3c^3d^3ex^3 - 10a^2cd^2e^2 + 8a^3e^4 + (5c^3d^4 - ac^2d^2e^2)x^2 - (5ac^2d^3e - 4a^2cde^3)x)f^2g}{5\sqrt{cdx + aec^3d^3}} \\ & + \frac{2(15c^4d^4ex^4 + 56a^3cd^2e^3 - 48a^4e^5 + 3(7c^4d^5 - ac^3d^3e^2)x^3 - (7ac^3d^4e - 6a^2c^2d^2e^3)x^2 + 4(7a^2c^2d^3e^2 - 6a^3cde^4)x)f}{35\sqrt{cdx + aec^4d^4}} \\ & + \frac{2(35c^5d^5ex^5 - 144a^4cd^2e^4 + 128a^5e^6 + 5(9c^5d^6 - ac^4d^4e^2)x^4 - (9ac^4d^5e - 8a^2c^3d^3e^3)x^3 + 2(9a^2c^3d^4e^2 - 8a^3c^2d^2e^4))}{315\sqrt{cdx + aec^5d^5}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*(g\*x + f)^3/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)

[Out]  $\frac{2}{3}*(c^2*d^2*e*x^2 + 3*a*c*d^2*e - 2*a^2*e^3 + (3*c^2*d^3 - a*c*d^4*e^2)*x)*f^3/(sqrt(c*d*x + a*e)*c^2*d^2) + \frac{2}{5}*(3*c^3*d^3*e*x^3 - 10*a^2*c*d^2*e^2 + 8*a^3*e^4 + (5*c^3*d^4 - a*c^2*d^2*e^2)*x^2 - (5*a*c^2*d^3*e - 4*a^2*c*d*e^3)*x)*f^2*g/(sqrt(c*d*x + a*e)*c^3*d^3) + \frac{2}{35}*(15*c^4*d^4*e*x^4 + 56*a^3*c*d^2*e^3 - 48*a^4*e^5 + 3*(7*c^4*d^5 - a*c^3*d^3*e^2)*x^3 - (7*a*c^3*d^4*e - 6*a^2*c^2*d^2*e^3)*x^2 + 4*(7*a^2*c^2*d^3*e^2 - 6*a^3*cde^4)*x)*f/(sqrt(c*d*x + a*e)*c^4*d^4) + \frac{2}{315}*(35*c^5*d^5*e*x^5 - 144*a^4*c*d^2*e^4 + 128*a^5*e^6 + 5*(9*c^5*d^6 - a*c^4*d^4*e^2)*x^4 - (9*a*c^4*d^5*e - 8*a^2*c^3*d^3*e^3)*x^3 + 2*(9*a^2*c^3*d^4*e^2 - 8*a^3*c^2*d^2*e^4)*x^2 - 8*(9*a^3*c^2*d^3*e^3 - 8*a^4*c*d*e^5)*x)*g^3/(sqrt(c*d*x + a*e)*c^5*d^5)$

**Fricas [A]** time = 0.27576, size = 1061, normalized size = 2.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*(g\*x + f)^3/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)

[Out]  $\frac{2}{315}*(35*c^5*d^5*e^2*g^3*x^6 + 5*(27*c^5*d^5*e^2*f*g^2 + (16*c^5*d^6*e - a*c^4*d^4*e^3)*g^3)*x^5 + (189*c^5*d^5*e^2*f^2*g + 27*(12*c^5*d^6*e - a*c^4*d^4*e^3)*f*g^2 + (45*c^5*d^7 - 14*a*c^4*d^5*e^2 + 8*a^2*c^3*d^3*e^4)*g^3)*x^4 + 105*(3*a*c^4*d^6*e - 2*a^2*c^3*d^4*e^3)*f^3 - 126*(5*a^2*c^3*d^5*e^2 - 4*a^3*c^2*d^3*e^4)*f^2*g + 72*(7*a^3*c^2*d^4*e^3 - 6*a^4*c*d^2*e^5)*f*g^2 - 16*(9*a^4*c*d^3*e^4 - 8*a^5*d^2*e^6)*g^3 + (105*c^5*d^5*e^2*f^3 + 63*(8*c^5*d^6*e - a*c^4*d^4*e^3)*f^2*g + 9*(21*c^5*d^7 - 10*a*c^4*d^5*e^2 + 6*a^2*c^3*d^3*e^4)*f*g^2 - (9*a*c^4*d^6*e - 26*a^2*c^3*d^4*e^3 + 16*a^3*c^2*d^2*e^5)*g^3)*x^3 + (105*(4*c^5*d^6*e - a*c^4*d^4*e^3)*f^3 + 63*(5*c^5*d^7 - 6*a*c^4*d^5*e^2 + 4*a^2*c^3*d^3*e^4)*f^2*g - 9*(7*a*c^4*d^6*e - 34*a^2*c^3*d^4*e^3 + 24*a^3*c^2*d^2*e^5)*f*g^2 + 2*(9*a^2*c^3*d^5*e^2 - 44*a^3*c^2*d^3*e^4 + 32*a^4*c*d^2*e^6)*g^3)*x^2 + (105*(3*c^5*d^7 + 2*a*c^4*d^5*e^2 - 2*a^2*c^3*d^3*e^4)*f^3 - 63*(5*a*c^4*d^6*e + 6*a^2*c^3*d^4*e^3 - 8*a^3*c^2*d^2*e^5)*f^2*g + 36*(7*a^2*c^3*d^5*e^2 + 8*a^3*c^2*d^3*e^4 - 12*a^4*c*d^2*e^6))$

$$\frac{(f^2 g^2 - 8(9a^3 c^2 d^4 e^3 + 10a^4 c d^2 e^5 - 16a^5 e^7) g^3) x}{(\sqrt{c d e x^2 + a d e + (c d^2 + a e^2) x}) \sqrt{e x + d} c^5 d^5}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(3/2)\*(g\*x+f)\*\*3/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.39072, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*(g\*x + f)^3/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)

[Out] Done

$$3.785 \quad \int \frac{(d+ex)^{3/2}(f+gx)^2}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=321

$$\begin{aligned} & \frac{8\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)(6ae^2g+cd(ef-7dg))(2ae^2g-cd(3ef-dg))}{105c^4d^4eg\sqrt{d+ex}} \\ & - \frac{8\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)(6ae^2g+cd(ef-7dg))}{105c^3d^3e} \\ & - \frac{2(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(6ae^2g+cd(ef-7dg))}{35c^2d^2g\sqrt{d+ex}} \\ & + \frac{2e(f+gx)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{7cdg\sqrt{d+ex}} \end{aligned}$$

[Out]  $(8*(c*d*f - a*e*g)*(6*a*e^2*g + c*d*(e*f - 7*d*g))*(2*a*e^2*g - c*d*(3*e*f - d*g))*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(105*c^4*d^4*e*g*\text{Sqrt}[d + e*x]) - (8*(c*d*f - a*e*g)*(6*a*e^2*g + c*d*(e*f - 7*d*g))*\text{Sqrt}[d + e*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(105*c^3*d^3*e) - (2*(6*a*e^2*g + c*d*(e*f - 7*d*g))*(f + g*x)^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(35*c^2*d^2*g*\text{Sqrt}[d + e*x]) + (2*e*(f + g*x)^3*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(7*c*d*g*\text{Sqrt}[d + e*x])$

**Rubi [A]** time = 1.19965, antiderivative size = 321, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\begin{aligned} & \frac{8\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)(6ae^2g+cd(ef-7dg))(2ae^2g-cd(3ef-dg))}{105c^4d^4eg\sqrt{d+ex}} \\ & - \frac{8\sqrt{d+ex}\sqrt{x(ae^2+cd^2)+ade+cdex^2}(cdf-aeg)(6ae^2g+cd(ef-7dg))}{105c^3d^3e} \\ & - \frac{2(f+gx)^2\sqrt{x(ae^2+cd^2)+ade+cdex^2}(6ae^2g+cd(ef-7dg))}{35c^2d^2g\sqrt{d+ex}} \\ & + \frac{2e(f+gx)^3\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{7cdg\sqrt{d+ex}} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\frac{(d+e*x)^{3/2}*(f+g*x)^2}{\text{Sqrt}[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2]},x]$

[Out]  $(8*(c*d*f - a*e*g)*(6*a*e^2*g + c*d*(e*f - 7*d*g))*(2*a*e^2*g - c*d*(3*e*f - d*g))*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(105*c^4*d^4*e*g*\text{Sqrt}[d + e*x]) - (8*(c*d*f - a*e*g)*(6*a*e^2*g + c*d*(e*f - 7*d*g))*\text{Sqrt}[d + e*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(105*c^3*d^3*e) - (2*(6*a*e^2*g + c*d*(e*f - 7*d*g))*(f + g*x)^2*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(35*c^2*d^2*g*\text{Sqrt}[d + e*x]) + (2*e*(f + g*x)^3*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(7*c*d*g*\text{Sqrt}[d + e*x])$

**Rubi in Sympy [A]** time = 95.1423, size = 325, normalized size = 1.01

$$\frac{2e(f+gx)^3 \sqrt{ade+cdex^2+x(ae^2+cd^2)}}{7cdg\sqrt{d+ex}} - \frac{2(f+gx)^2 \sqrt{ade+cdex^2+x(ae^2+cd^2)}(6ae^2g-7cd^2g+cdef)}{35c^2d^2g\sqrt{d+ex}} + \frac{8\sqrt{d+ex}(aeg-cdf)\sqrt{ade+cdex^2+x(ae^2+cd^2)}(6ae^2g-7cd^2g+cdef)}{105c^3d^3e} - \frac{8(aeg-cdf)\sqrt{ade+cdex^2+x(ae^2+cd^2)}(2ae^2g+cd^2g-3cdef)(6ae^2g-7cd^2g+cdef)}{105c^4d^4eg\sqrt{d+ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**(3/2)*(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2))`

[Out]  $2*e*(f+g*x)**3*\text{sqrt}(a*d*e+c*d*e*x**2+x*(a*e**2+c*d**2))/(7*c*d*g*\text{sqrt}(d+e*x))-2*(f+g*x)**2*\text{sqrt}(a*d*e+c*d*e*x**2+x*(a*e**2+c*d**2))*(6*a*e**2*g-7*c*d**2*g+c*d*e*f)/(35*c**2*d**2*g*\text{sqrt}(d+e*x))+8*\text{sqrt}(d+e*x)*(a*e*g-c*d*f)*\text{sqrt}(a*d*e+c*d*e*x**2+x*(a*e**2+c*d**2))*(6*a*e**2*g-7*c*d**2*g+c*d*e*f)/(105*c**3*d**3*e)-8*(a*e*g-c*d*f)*\text{sqrt}(a*d*e+c*d*e*x**2+x*(a*e**2+c*d**2))*(2*a*e**2*g+c*d**2*g-3*c*d*e*f)*(6*a*e**2*g-7*c*d**2*g+c*d*e*f)/(105*c**4*d**4*e*g*\text{sqrt}(d+e*x))$

**Mathematica [A]** time = 0.270622, size = 169, normalized size = 0.53

$$\frac{2\sqrt{(d+ex)(ae+cdx)}(-48a^3e^4g^2+8a^2cde^2g(7dg+14ef+3egx)-2ac^2d^2e(14dg(5f+gx)+e(35f^2+28fgx+9g^2x^2)))+105c^4d^4\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] `Integrate[((d+e*x)^(3/2)*(f+g*x)^2)/Sqrt[a*d*e+(c*d^2+a*e^2)*x+c*d*e*x^2]]`

[Out]  $(2*\text{Sqrt}[(a*e+c*d*x)*(d+e*x)]*(-48*a^3*e^4*g^2+8*a^2*c*d*e^2*g*(14*e*f+7*d*g+3*e*g*x)-2*a*c^2*d^2*e*(14*d*g*(5*f+g*x)+e*(35*f^2+28*f*g*x+9*g^2*x^2))+c^3*d^3*(7*d*(15*f^2+10*f*g*x+3*g^2*x^2)+e*x*(35*f^2+42*f*g*x+15*g^2*x^2)))/(105*c^4*d^4*\text{Sqrt}[d+e*x])$

**Maple [A]** time = 0.013, size = 255, normalized size = 0.8

$$\frac{(2cdx+2ae)(-15eg^2x^3c^3d^3+18ac^2d^2e^2g^2x^2-21c^3d^4g^2x^2-42c^3d^3efgx^2-24a^2cde^3g^2x+28ac^2d^3eg^2x+56ac^2d^2e^2g^2x^2)+105c^4d^4\sqrt{d+ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^(3/2)*(g*x+f)^2/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)`

[Out]  $-2/105*(c*d*x+a*e)*(-15*c^3*d^3*e*g^2*x^3+18*a*c^2*d^2*e^2*g^2*x^2-21*c^3*d^4*g^2*x^2-42*c^3*d^3*e*f*g*x^2-24*a^2*c*d*e^3*g^2*x+28*a*c^2*d^3*e*g^2*x+56*a*c^2*d^2*e^2*f*g*x-70*c^3*d^4*f*g*x-35*c^3*d^3*e*f^2*x+48*a^3*e^4*g^2-56*a^2*c*d^2*e^2*g^2-112*a^2*c*d*e^3*f*g+140*a*c^2*d^3*e*f*g+70*a*c^2*d^2*e^2*f^2-105*c^3*d^4*f^2)*(e*x+d)^(1/2)/c^4/d^4/(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)$

---

**Maxima [A]** time = 0.736747, size = 417, normalized size = 1.3

$$\frac{2(c^2d^2ex^2 + 3acd^2e - 2a^2e^3 + (3c^2d^3 - acde^2)x)f^2}{3\sqrt{cdx + aec^2d^2}} + \frac{4(3c^3d^3ex^3 - 10a^2cd^2e^2 + 8a^3e^4 + (5c^3d^4 - ac^2d^2e^2)x^2 - (5ac^2d^3e - 4a^2cde^3)x)fg}{15\sqrt{cdx + aec^3d^3}} + \frac{2(15c^4d^4ex^4 + 56a^3cd^2e^3 - 48a^4e^5 + 3(7c^4d^5 - ac^3d^3e^2)x^3 - (7ac^3d^4e - 6a^2c^2d^2e^3)x^2 + 4(7a^2c^2d^3e^2 - 6a^3cde^4)x)g}{105\sqrt{cdx + aec^4d^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*(g\*x + f)^2/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)

[Out] 2/3\*(c^2\*d^2\*e\*x^2 + 3\*a\*c\*d^2\*e - 2\*a^2\*e^3 + (3\*c^2\*d^3 - a\*c\*d\*e^2)\*x)\*f^2/(sqrt(c\*d\*x + a\*e)\*c^2\*d^2) + 4/15\*(3\*c^3\*d^3\*e\*x^3 - 10\*a^2\*c\*d^2\*e^2 + 8\*a^3\*e^4 + (5\*c^3\*d^4 - a\*c^2\*d^2\*e^2)\*x^2 - (5\*a\*c^2\*d^3\*e - 4\*a^2\*c\*d\*e^3)\*x)\*f\*g/(sqrt(c\*d\*x + a\*e)\*c^3\*d^3) + 2/105\*(15\*c^4\*d^4\*e\*x^4 + 56\*a^3\*c\*d^2\*e^3 - 48\*a^4\*e^5 + 3\*(7\*c^4\*d^5 - a\*c^3\*d^3\*e^2)\*x^3 - (7\*a\*c^3\*d^4\*e - 6\*a^2\*c^2\*d^2\*e^3)\*x^2 + 4\*(7\*a^2\*c^2\*d^3\*e^2 - 6\*a^3\*c\*d\*e^4)\*x)\*g^2/(sqrt(c\*d\*x + a\*e)\*c^4\*d^4)

---

**Fricas [A]** time = 0.270736, size = 689, normalized size = 2.15

$$\frac{2(15c^4d^4e^2g^2x^5 + 3(14c^4d^4e^2fg + (12c^4d^5e - ac^3d^3e^3)g^2)x^4 + (35c^4d^4e^2f^2 + 14(8c^4d^5e - ac^3d^3e^3)fg + (21c^4d^6 - 10c^4d^5e - a^2c^3d^3e^3)f^2)x^3 + 35(3a^2c^3d^5e - 2a^2c^2d^3e^3)f^2 - 28(5a^2c^2d^4e^2 - 4a^3c^2d^2e^4)f^2 + 8(7a^3c^2d^3e^3 - 6a^4d^2e^5)g^2 + (35(4c^4d^5e - a^2c^3d^3e^3)f^2 + 14(5c^4d^6 - 6a^2c^3d^4e^2 + 4a^2c^2d^2e^4)f^2 - 7a^2c^3d^5e - 34a^2c^2d^3e^3 + 24a^3c^2d^2e^5)g^2)x^2 + (35(3c^4d^6 + 2a^2c^3d^4e^2 - 2a^2c^2d^2e^4)f^2 - 14(5a^2c^3d^5e + 6a^2c^2d^3e^3 - 8a^3c^2d^2e^5)f^2 + 4(7a^2c^2d^4e^2 + 8a^3c^2d^2e^4 - 12a^4e^6)g^2)*x)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(e*x + d)*c^4*d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*(g\*x + f)^2/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)

[Out] 2/105\*(15\*c^4\*d^4\*e^2\*g^2\*x^5 + 3\*(14\*c^4\*d^4\*e^2\*f\*g + (12\*c^4\*d^5\*e - a\*c^3\*d^3\*e^3)\*g^2)\*x^4 + (35\*c^4\*d^4\*e^2\*f^2 + 14\*(8\*c^4\*d^5\*e - a\*c^3\*d^3\*e^3)\*f\*g + (21\*c^4\*d^6 - 10\*a\*c^3\*d^4\*e^2 + 6\*a^2\*c^2\*d^2\*e^4)\*g^2)\*x^3 + 35\*(3\*a^2\*c^3\*d^5\*e - 2\*a^2\*c^2\*d^3\*e^3)\*f^2 - 28\*(5\*a^2\*c^2\*d^4\*e^2 - 4\*a^3\*c^2\*d^2\*e^4)\*f^2 + 8\*(7\*a^3\*c^2\*d^3\*e^3 - 6\*a^4\*d^2\*e^5)\*g^2 + (35\*(4\*c^4\*d^5\*e - a^2\*c^3\*d^3\*e^3)\*f^2 + 14\*(5\*c^4\*d^6 - 6\*a^2\*c^3\*d^4\*e^2 + 4\*a^2\*c^2\*d^2\*e^4)\*f^2 - 7\*a^2\*c^3\*d^5\*e - 34\*a^2\*c^2\*d^3\*e^3 + 24\*a^3\*c^2\*d^2\*e^5)\*g^2)\*x^2 + (35\*(3\*c^4\*d^6 + 2\*a^2\*c^3\*d^4\*e^2 - 2\*a^2\*c^2\*d^2\*e^4)\*f^2 - 14\*(5\*a^2\*c^3\*d^5\*e + 6\*a^2\*c^2\*d^3\*e^3 - 8\*a^3\*c^2\*d^2\*e^5)\*f^2 + 4\*(7\*a^2\*c^2\*d^4\*e^2 + 8\*a^3\*c^2\*d^2\*e^4 - 12\*a^4\*e^6)\*g^2)\*x)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*c^4\*d^4)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(3/2)\*(g\*x+f)\*\*2/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)

[Out] Timed out

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GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^{\frac{3}{2}}(gx + f)^2}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*(g\*x + f)^2/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)

[Out] integrate((e\*x + d)^(3/2)\*(g\*x + f)^2/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

$$3.786 \quad \int \frac{(d+ex)^{3/2}(f+gx)}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=209

$$\begin{aligned} & -\frac{4(cd^2 - ae^2) \sqrt{x(ae^2 + cd^2) + ade + cdex^2} (4ae^2g - cd(5ef - dg))}{15c^3d^3e\sqrt{d+ex}} \\ & -\frac{2\sqrt{d+ex}\sqrt{x(ae^2 + cd^2) + ade + cdex^2} (4ae^2g - cd(5ef - dg))}{15c^2d^2e} \\ & +\frac{2g(d+ex)^{3/2}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{5cde} \end{aligned}$$

[Out]  $(-4*(c*d^2 - a*e^2)*(4*a*e^2*g - c*d*(5*e*f - d*g))*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15*c^3*d^3*e*\text{Sqrt}[d + e*x]) - (2*(4*a*e^2*g - c*d*(5*e*f - d*g))*\text{Sqrt}[d + e*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15*c^2*d^2*e) + (2*g*(d + e*x)^(3/2)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c*d*e)$

**Rubi [A]** time = 0.543832, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 44,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.068$

$$\begin{aligned} & -\frac{4(cd^2 - ae^2) \sqrt{x(ae^2 + cd^2) + ade + cdex^2} (4ae^2g - cd(5ef - dg))}{15c^3d^3e\sqrt{d+ex}} \\ & -\frac{2\sqrt{d+ex}\sqrt{x(ae^2 + cd^2) + ade + cdex^2} (4ae^2g - cd(5ef - dg))}{15c^2d^2e} \\ & +\frac{2g(d+ex)^{3/2}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{5cde} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^(3/2)*(f + g*x)/\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2], x]$

[Out]  $(-4*(c*d^2 - a*e^2)*(4*a*e^2*g - c*d*(5*e*f - d*g))*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15*c^3*d^3*e*\text{Sqrt}[d + e*x]) - (2*(4*a*e^2*g - c*d*(5*e*f - d*g))*\text{Sqrt}[d + e*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(15*c^2*d^2*e) + (2*g*(d + e*x)^(3/2)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(5*c*d*e)$

**Rubi in Sympy [A]** time = 54.7406, size = 202, normalized size = 0.97

$$\begin{aligned} & \frac{2g(d+ex)^{3/2}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{5cde} \\ & -\frac{2\sqrt{d+ex}\sqrt{ade+cdex^2+x(ae^2+cd^2)}(4ae^2g+cd^2g-5cdef)}{15c^2d^2e} \\ & +\frac{4(ae^2-cd^2)\sqrt{ade+cdex^2+x(ae^2+cd^2)}(4ae^2g+cd^2g-5cdef)}{15c^3d^3e\sqrt{d+ex}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)**(3/2)*(g*x+f)/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1$

[Out]  $2*g*(d + e*x)**(3/2)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))/(5*c*d*e) - 2*\text{sqrt}(d + e*x)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))*(4*a*e**2*g + c*d**2*g - 5*c*d*e*f)/(15*c**2*d**2*e) + 4*(a*e**2 - c*d**2)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))*(4*a*e**2*g + c*d**2*g - 5*c*d*e*f)/(15*c**3*d**3*e*\text{sqrt}(d +$



$e^*x))$

**Mathematica [A]** time = 0.130275, size = 96, normalized size = 0.46

$$\frac{2\sqrt{(d+ex)(ae+cdx)}(8a^2e^3g-2acde(5dg+5ef+2egx)+c^2d^2(5d(3f+gx)+ex(5f+3gx)))}{15c^3d^3\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)^(3/2)\*(f + g\*x))/Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]

[Out] (2\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)]\*(8\*a^2\*e^3\*g - 2\*a\*c\*d\*e\*(5\*e\*f + 5\*d\*g + 2\*e\*g\*x) + c^2\*d^2\*(5\*d\*(3\*f + g\*x) + e\*x\*(5\*f + 3\*g\*x)))/(15\*c^3\*d^3\*Sqrt[d + e\*x])

**Maple [A]** time = 0.006, size = 131, normalized size = 0.6

$$\frac{(2cdx + 2ae)(3egx^2c^2d^2 - 4acde^2gx + 5c^2d^3gx + 5c^2d^2efx + 8a^2e^3g - 10acd^2eg - 10acde^2f + 15d^3fc^2)}{15c^3d^3}\sqrt{ex+d}\sqrt{cde}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(3/2)\*(g\*x+f)/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2),x)

[Out] 2/15\*(c\*d\*x+a\*e)\*(3\*c^2\*d^2\*e\*g\*x^2-4\*a\*c\*d\*e^2\*g\*x+5\*c^2\*d^3\*g\*x+5\*c^2\*d^2\*e\*f\*x+8\*a^2\*e^3\*g-10\*a\*c\*d^2\*e\*g-10\*a\*c\*d\*e^2\*f+15\*c^2\*d^3\*f)\*(e\*x+d)^(1/2)/c^3/d^3/(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)

**Maxima [A]** time = 0.72474, size = 227, normalized size = 1.09

$$\frac{2(c^2d^2ex^2 + 3acd^2e - 2a^2e^3 + (3c^2d^3 - acde^2)x)f}{3\sqrt{cdx + aec^2d^2}} + \frac{2(3c^3d^3ex^3 - 10a^2cd^2e^2 + 8a^3e^4 + (5c^3d^4 - ac^2d^2e^2)x^2 - (5ac^2d^3e - 4a^2cde^3)x)g}{15\sqrt{cdx + aec^3d^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*(g\*x + f)/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

[Out] 2/3\*(c^2\*d^2\*e\*x^2 + 3\*a\*c\*d^2\*e - 2\*a^2\*e^3 + (3\*c^2\*d^3 - a\*c\*d\*e^2)\*x)\*f/(sqrt(c\*d\*x + a\*e)\*c^2\*d^2) + 2/15\*(3\*c^3\*d^3\*e\*x^3 - 10\*a^2\*c\*d^2\*e^2 + 8\*a^3\*e^4 + (5\*c^3\*d^4 - a\*c^2\*d^2\*e^2)\*x^2 - (5\*a\*c^2\*d^3\*e - 4\*a^2\*c\*d\*e^3)\*x)\*g/(sqrt(c\*d\*x + a\*e)\*c^3\*d^3)

**Fricas [A]** time = 0.269422, size = 383, normalized size = 1.83

$$\frac{2(3c^3d^3e^2gx^4 + (5c^3d^3e^2f + (8c^3d^4e - ac^2d^2e^3)g)x^3 + (5(4c^3d^4e - ac^2d^2e^3)f + (5c^3d^5 - 6ac^2d^3e^2 + 4a^2cde^4)g)x^2 + 5c^3d^3e^2f)}{15\sqrt{cdex^2 + ade}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*(g\*x + f)/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x

[Out] 2/15\*(3\*c^3\*d^3\*e^2\*g\*x^4 + (5\*c^3\*d^3\*e^2\*f + (8\*c^3\*d^4\*e - a\*c^2\*d^2\*e^3)\*g)\*x^3 + (5\*(4\*c^3\*d^4\*e - a\*c^2\*d^2\*e^3)\*f + (5\*c^3\*d^5 - 6\*a\*c^2\*d^3\*e^2 + 4\*a^2\*c\*d\*e^4)\*g)\*x^2 + 5\*(3\*a\*c^2\*d^4\*e - 2\*a^2\*c\*d^2\*e^3)\*f - 2\*(5\*a^2\*c\*d^3\*e^2 - 4\*a^3\*d\*e^4)\*g + (5\*(3\*c^3\*d^5 + 2\*a\*c^2\*d^3\*e^2 - 2\*a^2\*c\*d\*e^4)\*f - (5\*a\*c^2\*d^4\*e + 6\*a^2\*c\*d^2\*e^3 - 8\*a^3\*e^5)\*g)\*x)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*c^3\*d^3)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(3/2)\*(g\*x+f)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2), x

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^{\frac{3}{2}}(gx + f)}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)\*(g\*x + f)/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x

[Out] integrate((e\*x + d)^(3/2)\*(g\*x + f)/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x), x)

$$3.787 \quad \int \frac{(d+ex)^{3/2}}{\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=109

$$\frac{4(cd^2 - ae^2)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{3c^2d^2\sqrt{d + ex}} + \frac{2\sqrt{d + ex}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{3cd}$$

[Out]  $(4*(c*d^2 - a*e^2)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^2*d^2*\text{Sqrt}[d + e*x]) + (2*\text{Sqrt}[d + e*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d)$

**Rubi [A]** time = 0.173833, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 39,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.051$

$$\frac{4(cd^2 - ae^2)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{3c^2d^2\sqrt{d + ex}} + \frac{2\sqrt{d + ex}\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{3cd}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^{(3/2)}/\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2], x]$

[Out]  $(4*(c*d^2 - a*e^2)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c^2*d^2*\text{Sqrt}[d + e*x]) + (2*\text{Sqrt}[d + e*x]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])/(3*c*d)$

**Rubi in Sympy [A]** time = 32.3623, size = 100, normalized size = 0.92

$$\frac{2\sqrt{d + ex}\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{3cd} - \frac{4(ae^2 - cd^2)\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{3c^2d^2\sqrt{d + ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)^{(3/2)}/(a*d*e+(a*e^{**2}+c*d^{**2})*x+c*d*e*x^{**2})^{(1/2)}, x)$

[Out]  $2*\text{sqrt}(d + e*x)*\text{sqrt}(a*d*e + c*d*e*x^{**2} + x*(a*e^{**2} + c*d^{**2}))/ (3*c*d) - 4*(a*e^{**2} - c*d^{**2})*\text{sqrt}(a*d*e + c*d*e*x^{**2} + x*(a*e^{**2} + c*d^{**2}))/ (3*c^{**2}*d^{**2}*\text{sqrt}(d + e*x))$

**Mathematica [A]** time = 0.0561004, size = 54, normalized size = 0.5

$$\frac{2\sqrt{(d + ex)(ae + cdx)}(cd(3d + ex) - 2ae^2)}{3c^2d^2\sqrt{d + ex}}$$

Antiderivative was successfully verified.

[In]  $\text{Integrate}[(d + e*x)^{(3/2)}/\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2], x]$

[Out]  $(2*\text{Sqrt}[(a*e + c*d*x)*(d + e*x)]*(-2*a*e^2 + c*d*(3*d + e*x)))/(3*c^2*d^2*\text{Sqrt}[d + e*x])$

**Maple [A]** time = 0.005, size = 69, normalized size = 0.6

$$-\frac{(2cdx + 2ae)(-cdex + 2ae^2 - 3cd^2)}{3c^2d^2} \sqrt{ex + d} \frac{1}{\sqrt{cdex^2 + ae^2x + cd^2x + ade}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(3/2)/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2),x)

[Out] -2/3\*(c\*d\*x+a\*e)\*(-c\*d\*e\*x+2\*a\*e^2-3\*c\*d^2)\*(e\*x+d)^(1/2)/c^2/d^2/(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)

**Maxima [A]** time = 0.720509, size = 88, normalized size = 0.81

$$\frac{2(c^2d^2ex^2 + 3acd^2e - 2a^2e^3 + (3c^2d^3 - acde^2)x)}{3\sqrt{cdx + aec^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x),x, algorithm

[Out] 2/3\*(c^2\*d^2\*e\*x^2 + 3\*a\*c\*d^2\*e - 2\*a^2\*e^3 + (3\*c^2\*d^3 - a\*c\*d\*e^2)\*x)/(sqrt(c\*d\*x + a\*e)\*c^2\*d^2)

**Fricas [A]** time = 0.263745, size = 167, normalized size = 1.53

$$\frac{2(c^2d^2e^2x^3 + 3acd^3e - 2a^2de^3 + (4c^2d^3e - acde^3)x^2 + (3c^2d^4 + 2acd^2e^2 - 2a^2e^4)x)}{3\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{ex + dc^2d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)/sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x),x, algorithm

[Out] 2/3\*(c^2\*d^2\*e^2\*x^3 + 3\*a\*c\*d^3\*e - 2\*a^2\*d\*e^3 + (4\*c^2\*d^3\*e - a\*c\*d\*e^3)\*x^2 + (3\*c^2\*d^4 + 2\*a\*c\*d^2\*e^2 - 2\*a^2\*e^4)\*x)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*sqrt(e\*x + d)\*c^2\*d^2)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(3/2)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2),x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^{\frac{3}{2}}}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^(3/2)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x), x, algorithm="default")
```

```
[Out] integrate((e*x + d)^(3/2)/sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x), x)
```

$$3.788 \quad \int \frac{(d+ex)^{3/2}}{(f+gx)\sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=139

$$\frac{2e\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cdg\sqrt{d+ex}} - \frac{2(ef-dg)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}\sqrt{cdf-aeg}}$$

[Out] (2\*e\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(c\*d\*g\*Sqrt[d + e\*x]) - (2\*(e\*f - d\*g)\*ArcTan[(Sqrt[g]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(Sqrt[c\*d\*f - a\*e\*g]\*Sqrt[d + e\*x])])/(g^(3/2)\*Sqrt[c\*d\*f - a\*e\*g])

**Rubi [A]** time = 0.639139, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{2e\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{cdg\sqrt{d+ex}} - \frac{2(ef-dg)\tan^{-1}\left(\frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}}\right)}{g^{3/2}\sqrt{cdf-aeg}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^(3/2)/((f + g\*x)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]), x]

[Out] (2\*e\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(c\*d\*g\*Sqrt[d + e\*x]) - (2\*(e\*f - d\*g)\*ArcTan[(Sqrt[g]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(Sqrt[c\*d\*f - a\*e\*g]\*Sqrt[d + e\*x])])/(g^(3/2)\*Sqrt[c\*d\*f - a\*e\*g])

**Rubi in Sympy [A]** time = 58.1885, size = 129, normalized size = 0.93

$$-\frac{2(dg-ef)\operatorname{atanh}\left(\frac{\sqrt{g}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}\sqrt{aeg-cdf}}\right)}{g^{3/2}\sqrt{aeg-cdf}} + \frac{2e\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{cdg\sqrt{d+ex}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*(3/2)/(g\*x+f)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1

[Out] -2\*(d\*g - e\*f)\*atanh(sqrt(g)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2)))/(sqrt(d + e\*x)\*sqrt(a\*e\*g - c\*d\*f))/(g\*\*(3/2)\*sqrt(a\*e\*g - c\*d\*f)) + 2\*e\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(c\*d\*g\*sqrt(d + e\*x))

**Mathematica [A]** time = 0.329823, size = 141, normalized size = 1.01

$$\frac{2\sqrt{d+ex}\left(e\sqrt{g}(ae+cdx)\sqrt{aeg-cdf}-cd(dg-ef)\sqrt{ae+cdx}\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{aeg-cdf}}\right)\right)}{cdg^{3/2}\sqrt{(d+ex)(ae+cdx)}\sqrt{aeg-cdf}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^(3/2)/((f + g\*x)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])

```
[Out] (2*Sqrt[d + e*x]*(e*Sqrt[g]*Sqrt[-(c*d*f) + a*e*g]*(a*e + c*d*x)
- c*d*(-(e*f) + d*g)*Sqrt[a*e + c*d*x]*ArcTanh[(Sqrt[g]*Sqrt[a*e
+ c*d*x])/Sqrt[-(c*d*f) + a*e*g]])/(c*d*g^(3/2)*Sqrt[-(c*d*f) +
a*e*g]*Sqrt[(a*e + c*d*x)*(d + e*x)])
```

**Maple [A]** time = 0.029, size = 163, normalized size = 1.2

$$-2 \frac{\sqrt{cdex^2 + ae^2x + cd^2x + ade}}{\sqrt{ex + d}\sqrt{cdx + aecdg}\sqrt{(aeg - cdf)g}} \left( \operatorname{Artanh} \left( \frac{g\sqrt{cdx + ae}}{\sqrt{(aeg - cdf)g}} \right) cd^2g - \operatorname{Artanh} \left( \frac{g\sqrt{cdx + ae}}{\sqrt{(aeg - cdf)g}} \right) cdef - \sqrt{cdx + aee} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^(3/2)/(g*x+f)/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2), x)
```

```
[Out] -2*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^(1/2)*(arctanh(g*(c*d*x+a*e)
^(1/2)/((a*e*g-c*d*f)*g)^(1/2))*c*d^2*g-arctanh(g*(c*d*x+a*e)^(1/
2)/((a*e*g-c*d*f)*g)^(1/2))*c*d*e*f-(c*d*x+a*e)^(1/2)*e*((a*e*g-c
*d*f)*g)^(1/2))/(e*x+d)^(1/2)/(c*d*x+a*e)^(1/2)/c/d/g/((a*e*g-c*d
*f)*g)^(1/2)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f))
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 0.285065, size = 1, normalized size = 0.01

$$\left[ \frac{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(cdef - cd^2g) \sqrt{ex + d} \log \left( -\frac{2\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(cdfg - aeg^2) \sqrt{ex + d} + (cdegx^2 - cd^2f + 2adeg - ce^2g)x}{egx^2 + df + (ef + dg)x} \right)}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x} \sqrt{-cdfg + aeg^2} \sqrt{ex + d}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f))
```

```
[Out] [-(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*e*f - c*d^2*g)
)*sqrt(e*x + d)*log(-(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*
x)*(c*d*f*g - a*e*g^2)*sqrt(e*x + d) + (c*d*e*g*x^2 - c*d^2*f + 2
*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)*sqrt(-c*d*f*g + a*e
*g^2))/(e*g*x^2 + d*f + (e*f + d*g)*x)) - 2*(c*d*e^2*x^2 + a*d*e^
2 + (c*d^2*e + a*e^3)*x)*sqrt(-c*d*f*g + a*e*g^2))/(sqrt(c*d*e*x^
2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*g + a*e*g^2)*sqrt(e*x
+ d)*c*d*g), 2*(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d
e*f - c*d^2*g)*sqrt(e*x + d)*arctan(sqrt(c*d*e*x^2 + a*d*e + (c*d
^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g*x^2
+ a*d*e*g + (c*d^2 + a*e^2)*g*x)) + (c*d*e^2*x^2 + a*d*e^2 + (c
d^2*e + a*e^3)*x)*sqrt(c*d*f*g - a*e*g^2))/(sqrt(c*d*e*x^2 + a*d
e + (c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)*c*d
g)]
```

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(3/2)/(g\*x+f)/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2), x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^{\frac{3}{2}}}{\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}(gx + f)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f))

[Out] integrate((e\*x + d)^(3/2)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)), x)



$$3.789 \quad \int \frac{(d+ex)^{3/2}}{(f+gx)^2 \sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=170

$$\frac{(2ae^2g - cd(dg + ef)) \tan^{-1} \left( \frac{\sqrt{g} \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex} \sqrt{cdf-ae^2}} \right)}{g^{3/2}(cdf - aeg)^{3/2}} - \frac{(ef - dg) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{g \sqrt{d + ex} (f + gx)(cdf - aeg)}$$

[Out] -(((e\*f - d\*g)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(g\*(c\*d\*f - a\*e\*g)\*Sqrt[d + e\*x]\*(f + g\*x))) - ((2\*a\*e^2\*g - c\*d\*(e\*f + d\*g))\*ArcTan[(Sqrt[g]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(Sqrt[c\*d\*f - a\*e\*g]\*Sqrt[d + e\*x])])/(g^(3/2)\*(c\*d\*f - a\*e\*g)^(3/2))

**Rubi [A]** time = 0.725651, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$

$$\frac{(2ae^2g - cd(dg + ef)) \tan^{-1} \left( \frac{\sqrt{g} \sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex} \sqrt{cdf-ae^2}} \right)}{g^{3/2}(cdf - aeg)^{3/2}} - \frac{(ef - dg) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{g \sqrt{d + ex} (f + gx)(cdf - aeg)}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^(3/2)/((f + g\*x)^2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]), x]

[Out] -(((e\*f - d\*g)\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(g\*(c\*d\*f - a\*e\*g)\*Sqrt[d + e\*x]\*(f + g\*x))) - ((2\*a\*e^2\*g - c\*d\*(e\*f + d\*g))\*ArcTan[(Sqrt[g]\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2])/(Sqrt[c\*d\*f - a\*e\*g]\*Sqrt[d + e\*x])])/(g^(3/2)\*(c\*d\*f - a\*e\*g)^(3/2))

**Rubi in Sympy [A]** time = 65.4651, size = 160, normalized size = 0.94

$$\frac{(dg - ef) \sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{g \sqrt{d + ex} (f + gx)(aeg - cdf)} - \frac{(2ae^2g - cd^2g - cdef) \operatorname{atanh} \left( \frac{\sqrt{g} \sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex} \sqrt{aeg-cdf}} \right)}{g^{3/2} (aeg - cdf)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*(3/2)/(g\*x+f)\*\*2/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2), x)

[Out] -(d\*g - e\*f)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2))/(g\*sqrt(d + e\*x)\*(f + g\*x)\*(a\*e\*g - c\*d\*f)) - (2\*a\*e\*\*2\*g - c\*d\*\*2\*g - c\*d\*e\*f)\*atanh(sqrt(g)\*sqrt(a\*d\*e + c\*d\*e\*x\*\*2 + x\*(a\*e\*\*2 + c\*d\*\*2)))/(sqrt(d + e\*x)\*sqrt(a\*e\*g - c\*d\*f))/(g\*\*(3/2)\*(a\*e\*g - c\*d\*f)\*\*(3/2))

**Mathematica [A]** time = 0.487303, size = 153, normalized size = 0.9

$$\frac{\sqrt{d + ex} \left( \frac{\sqrt{ae+cdx}(cd(dg+ef)-2ae^2g) \tanh^{-1} \left( \frac{\sqrt{g} \sqrt{ae+cdx}}{\sqrt{aeg-cdf}} \right)}{(aeg-cdf)^{3/2}} + \frac{\sqrt{g}(ef-dg)(ae+cdx)}{(f+gx)(aeg-cdf)} \right)}{g^{3/2} \sqrt{(d + ex)(ae + cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^(3/2)/((f + g\*x)^2\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2

[Out] (Sqrt[d + e\*x]\*((Sqrt[g]\*(e\*f - d\*g)\*(a\*e + c\*d\*x))/((-c\*d\*f) + a\*e\*g)\*(f + g\*x)) + ((-2\*a\*e^2\*g + c\*d\*(e\*f + d\*g))\*Sqrt[a\*e + c\*d\*x]\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/Sqrt[-(c\*d\*f) + a\*e\*g]])/((-c\*d\*f) + a\*e\*g)^(3/2))/(g^(3/2)\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)])

**Maple [B]** time = 0.036, size = 347, normalized size = 2.

$$\frac{1}{(aeg - cdf)g(gx + f)} \left( -2 \operatorname{Artanh} \left( \frac{g\sqrt{cdx + ae}}{\sqrt{(aeg - cdf)g}} \right) xae^2g^2 + \operatorname{Artanh} \left( g\sqrt{cdx + ae} \frac{1}{\sqrt{(aeg - cdf)g}} \right) xcd^2g^2 + \operatorname{Artanh} \left( g\sqrt{cdx + ae} \right) xcd^2g^2 + \operatorname{Artanh} \left( g\sqrt{cdx + ae} \right) xcd^2g^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(3/2)/(g\*x+f)^2/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2), x)

[Out] (-2\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x\*a\*e^2\*g^2+arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x\*c\*d^2\*g^2+arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x\*c\*d\*e\*f\*g-2\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*a\*e^2\*f\*g+arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*c\*d^2\*f\*g+arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*c\*d\*e\*f^2-((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*d\*g+((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)\*e\*f/(e\*x+d)^(1/2)\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)/(c\*d\*x+a\*e)^(1/2)/(a\*e\*g-c\*d\*f)/g/(g\*x+f)/((a\*e\*g-c\*d\*f)\*g)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^2

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.290165, size = 1, normalized size = 0.01

$$\frac{2\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{-cdfg + aeg^2}(ef - dg)\sqrt{ex + d} - (cd^2ef^2 + (cd^3 - 2ade^2)fg + (cde^2fg + (cd^2e - 2ae^3)g^2))\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{cdfg - aeg^2}(ef - dg)\sqrt{ex + d} + (cd^2ef^2 + (cd^3 - 2ade^2)fg + (cde^2fg + (cd^2e - 2ae^3)g^2))\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{cdfg - aeg^2}(ef - dg)\sqrt{ex + d}}{(cd^2f^2g - adefg^2 + (cdefg^2 - ae^2g^3)x^2 + (cdef^2g - adefg^2 + (cde^2fg + (cd^2e - 2ae^3)g^2))\sqrt{cdex^2 + ade + (cd^2 + ae^2)x}\sqrt{cdfg - aeg^2}(ef - dg)\sqrt{ex + d}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^2

```
[Out] [-1/2*(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*sqrt(-c*d*f*
g + a*e*g^2)*(e*f - d*g)*sqrt(e*x + d) - (c*d^2*e*f^2 + (c*d^3 -
2*a*d*e^2)*f*g + (c*d*e^2*f*g + (c*d^2*e - 2*a*e^3)*g^2)*x^2 + (c
*d*e^2*f^2 + 2*(c*d^2*e - a*e^3)*f*g + (c*d^3 - 2*a*d*e^2)*g^2)*x
)*log(-(2*sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(c*d*f*g -
a*e*g^2)*sqrt(e*x + d) + (c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*
d*e*f - (c*d^2 + 2*a*e^2)*g)*x)*sqrt(-c*d*f*g + a*e*g^2))/(e*g*x^
2 + d*f + (e*f + d*g)*x)))/((c*d^2*f^2*g - a*d*e*f*g^2 + (c*d*e*f
*g^2 - a*e^2*g^3)*x^2 + (c*d*e*f^2*g - a*d*e*g^3 + (c*d^2 - a*e^2
)*f*g^2)*x)*sqrt(-c*d*f*g + a*e*g^2)), -(sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*(e*f - d*g)*sqrt(e*x
+ d) + (c*d^2*e*f^2 + (c*d^3 - 2*a*d*e^2)*f*g + (c*d*e^2*f*g + (c
*d^2*e - 2*a*e^3)*g^2)*x^2 + (c*d*e^2*f^2 + 2*(c*d^2*e - a*e^3)*f
*g + (c*d^3 - 2*a*d*e^2)*g^2)*x)*arctan(sqrt(c*d*e*x^2 + a*d*e +
(c*d^2 + a*e^2)*x)*sqrt(c*d*f*g - a*e*g^2)*sqrt(e*x + d)/(c*d*e*g
*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x)))/((c*d^2*f^2*g - a*d*e*f*g
^2 + (c*d*e*f*g^2 - a*e^2*g^3)*x^2 + (c*d*e*f^2*g - a*d*e*g^3 + (
c*d^2 - a*e^2)*f*g^2)*x)*sqrt(c*d*f*g - a*e*g^2))]
```

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)/(g*x+f)**2/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)**(1/2)
```

```
[Out] Timed out
```

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^(1/2))
```

```
[Out] Timed out
```

$$3.790 \quad \int \frac{(d+ex)^{3/2}}{(f+gx)^3 \sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=261

$$\frac{cd(4ae^2g - cd(3dg + ef)) \tan^{-1} \left( \frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}} \right)}{4g^{3/2}(cdf - aeg)^{5/2}} - \frac{(ef - dg)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{2g\sqrt{d + ex}(f + gx)^2(cdf - aeg)} - \frac{\sqrt{x(ae^2 + cd^2) + ade + cdex^2} (4ae^2g - cd(3dg + ef))}{4g\sqrt{d + ex}(f + gx)(cdf - aeg)^2}$$

[Out]  $-\left((e*f - d*g)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]\right)/(2*g*(c*d*f - a*e*g)*\text{Sqrt}[d + e*x]*(f + g*x)^2) - \left(\left(4*a*e^2*g - c*d*(e*f + 3*d*g)\right)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]\right)/(4*g*(c*d*f - a*e*g)^2*\text{Sqrt}[d + e*x]*(f + g*x)) - (c*d*(4*a*e^2*g - c*d*(e*f + 3*d*g))*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x]))/(4*g^{(3/2)}*(c*d*f - a*e*g)^{(5/2)})$

**Rubi [A]** time = 1.07144, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{cd(4ae^2g - cd(3dg + ef)) \tan^{-1} \left( \frac{\sqrt{g}\sqrt{x(ae^2+cd^2)+ade+cdex^2}}{\sqrt{d+ex}\sqrt{cdf-aeg}} \right)}{4g^{3/2}(cdf - aeg)^{5/2}} - \frac{(ef - dg)\sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{2g\sqrt{d + ex}(f + gx)^2(cdf - aeg)} - \frac{\sqrt{x(ae^2 + cd^2) + ade + cdex^2} (4ae^2g - cd(3dg + ef))}{4g\sqrt{d + ex}(f + gx)(cdf - aeg)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^{(3/2)}/((f + g*x)^3*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]), x]$

[Out]  $-\left((e*f - d*g)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]\right)/(2*g*(c*d*f - a*e*g)*\text{Sqrt}[d + e*x]*(f + g*x)^2) - \left(\left(4*a*e^2*g - c*d*(e*f + 3*d*g)\right)*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]\right)/(4*g*(c*d*f - a*e*g)^2*\text{Sqrt}[d + e*x]*(f + g*x)) - (c*d*(4*a*e^2*g - c*d*(e*f + 3*d*g))*\text{ArcTan}[(\text{Sqrt}[g]*\text{Sqrt}[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2])]/(\text{Sqrt}[c*d*f - a*e*g]*\text{Sqrt}[d + e*x]))/(4*g^{(3/2)}*(c*d*f - a*e*g)^{(5/2)})$

**Rubi in Sympy [A]** time = 97.9371, size = 250, normalized size = 0.96

$$\frac{cd(4ae^2g - 3cd^2g - cdef) \operatorname{atanh} \left( \frac{\sqrt{g}\sqrt{ade+cdex^2+x(ae^2+cd^2)}}{\sqrt{d+ex}\sqrt{aeg-cdf}} \right)}{4g^{\frac{3}{2}}(aeg - cdf)^{\frac{5}{2}}} - \frac{\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}(4ae^2g - 3cd^2g - cdef)}{4g\sqrt{d + ex}(f + gx)(aeg - cdf)^2} - \frac{(dg - ef)\sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{2g\sqrt{d + ex}(f + gx)^2(aeg - cdf)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)^{(3/2)}/(g*x+f)^3/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2), x)$

[Out]  $c*d*(4*a*e**2*g - 3*c*d**2*g - c*d*e*f)*\operatorname{atanh}(\text{sqrt}(g)*\text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2)))/(\text{sqrt}(d + e*x)*\text{sqrt}(a*e*g - c*d*f)))/(4*g** (3/2)*(a*e*g - c*d*f)** (5/2)) - \text{sqrt}(a*d*e + c*d*e*x**2 + x*(a*e**2 + c*d**2))*(4*a*e**2*g - 3*c*d**2*g - c*d*e*f)/(\text{sqrt}(d + e*x)*\text{sqrt}(aeg - cdf)** 2) - (dg - ef)*\text{sqrt}(ade + cdex^2 + x(ae^2 + cd^2))/(2*g*\text{sqrt}(d + ex)*(f + gx)** 2*(aeg - cdf))$

$$4g\sqrt{d+ex}(f+gx)(ae^2g-cd^2f)^2 - (dg-ef)\sqrt{d+ex} \\ t(ad^2e+c^2d^2e^2x^2+x(ad^2e+c^2d^2e^2))/(2g\sqrt{d+ex}(f+gx)^2(ae^2g-cd^2f))$$

**Mathematica [A]** time = 0.978982, size = 193, normalized size = 0.74

$$\frac{\sqrt{d+ex} \left( \frac{(ae+cdx)(cd(dg(5f+3gx)+ef(gx-f))-2aeg(dg+e(f+2gx)))}{g(f+gx)^2(cdf-aeg)^2} - \frac{cd\sqrt{ae+cdx}(cd(3dg+ef)-4ae^2g) \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{ae+cdx}}{\sqrt{aeg-cdf}}\right)}{g^{3/2}(aeg-cdf)^{5/2}} \right)}{4\sqrt{(d+ex)(ae+cdx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^(3/2)/((f + g\*x)^3\*Sqrt[a\*d\*e + (c\*d^2 + a\*e^2)\*x + c\*d\*e\*x^2]

[Out] (Sqrt[d + e\*x]\*(((a\*e + c\*d\*x)\*(-2\*a\*e\*g\*(d\*g + e\*(f + 2\*g\*x)) + c\*d\*(e\*f\*(-f + g\*x) + d\*g\*(5\*f + 3\*g\*x))))/(g\*(c\*d\*f - a\*e\*g)^2\*(f + g\*x)^2) - (c\*d\*(-4\*a\*e^2\*g + c\*d\*(e\*f + 3\*d\*g))\*Sqrt[a\*e + c\*d\*x]\*ArcTanh[(Sqrt[g]\*Sqrt[a\*e + c\*d\*x])/Sqrt[-(c\*d\*f) + a\*e\*g]])/(g^(3/2)\*(-(c\*d\*f) + a\*e\*g)^(5/2)))/(4\*Sqrt[(a\*e + c\*d\*x)\*(d + e\*x)])

**Maple [B]** time = 0.05, size = 673, normalized size = 2.6

$$\frac{1}{4(gx+f)^2g(aeg-cdf)^2}\sqrt{cdex^2+ae^2x+cd^2x+ade}\left(4\operatorname{Artanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)x^2acde^2g^3-3\operatorname{Artanh}\left(\frac{g\sqrt{cdx+ae}}{\sqrt{(aeg-cdf)g}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(3/2)/(g\*x+f)^3/(a\*d\*e+(a\*e^2+c\*d^2)\*x+c\*d\*e\*x^2)^(1/2), x)

[Out] 1/4\*(c\*d\*e\*x^2+a\*e^2\*x+c\*d^2\*x+a\*d\*e)^(1/2)\*(4\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x^2\*a\*c\*d\*e^2\*g^3-3\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x^2\*c^2\*d^3\*g^3-arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x^2\*c^2\*d^2\*e\*f\*g^2+8\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x\*a\*c\*d\*e^2\*f\*g^2-6\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x\*c^2\*d^3\*f\*g^2-2\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*x\*c^2\*d^2\*e\*f^2\*g+4\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*a\*c\*d\*e^2\*f^2\*g-3\*arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*c^2\*d^3\*f^2\*g-arctanh(g\*(c\*d\*x+a\*e)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2))\*c^2\*d^2\*e\*f^3-4\*x\*a\*e^2\*g^2\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)+3\*x\*c\*d^2\*g^2\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)+x\*c\*d\*e\*f\*g\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)-2\*a\*d\*e\*g^2\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)-2\*a\*e^2\*f\*g\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)+5\*c\*d^2\*f\*g\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2)-c\*d\*e\*f^2\*((a\*e\*g-c\*d\*f)\*g)^(1/2)\*(c\*d\*x+a\*e)^(1/2))/(e\*x+d)^(1/2)/((a\*e\*g-c\*d\*f)\*g)^(1/2)/(g\*x+f)^2/g/(a\*e\*g-c\*d\*f)^2/(c\*d\*x+a\*e)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)/(sqrt(c\*d\*e\*x^2 + a\*d\*e + (c\*d^2 + a\*e^2)\*x)\*(g\*x + f)^(1/2)), x)

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.297056, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^x + d)^{3/2}/(\sqrt{c*d*e^x^2 + a*d*e + (c*d^2 + a*e^2)*x})*(g*x + f)^{\wedge})$

[Out] 
$$\begin{aligned} & [-1/8*(2*(c*d*e*f^2 + 2*a*d*e*g^2 - (5*c*d^2 - 2*a*e^2)*f*g - (c*d*e*f*g + (3*c*d^2 - 4*a*e^2)*g^2)*x)*\sqrt{c*d*e^x^2 + a*d*e + (c*d^2 + a*e^2)*x}*\sqrt{-c*d*f*g + a*e*g^2}*\sqrt{e^x + d} + (c^2*d^3*e*f^3 + (3*c^2*d^4 - 4*a*c*d^2*e^2)*f^2*g + (c^2*d^2*e^2*f*g^2 + (3*c^2*d^3*e - 4*a*c*d*e^3)*g^3)*x^3 + (2*c^2*d^2*e^2*f^2*g + (7*c^2*d^3*e - 8*a*c*d*e^3)*f*g^2 + (3*c^2*d^4 - 4*a*c*d^2*e^2)*g^3)*x^2 + (c^2*d^2*e^2*f^3 + (5*c^2*d^3*e - 4*a*c*d*e^3)*f^2*g + 2*(3*c^2*d^4 - 4*a*c*d^2*e^2)*f*g^2)*x)*\log((2*\sqrt{c*d*e^x^2 + a*d*e + (c*d^2 + a*e^2)*x}*(c*d*f*g - a*e*g^2)*\sqrt{e^x + d} - (c*d*e*g*x^2 - c*d^2*f + 2*a*d*e*g - (c*d*e*f - (c*d^2 + 2*a*e^2)*g)*x)*\sqrt{-c*d*f*g + a*e*g^2}))/((c^2*d^3*f^4*g - 2*a*c*d^2*e*f^3*g^2 + a^2*d*e^2*f^2*g^3 + (c^2*d^2*e*f^2*g^3 - 2*a*c*d*e^2*f*g^4 + a^2*e^3*g^5)*x^3 + (2*c^2*d^2*e*f^3*g^2 + a^2*d*e^2*g^5 + (c^2*d^3 - 4*a*c*d*e^2)*f^2*g^3 - 2*(a*c*d^2*e - a^2*e^3)*f*g^4)*x^2 + (c^2*d^2*e*f^4*g + 2*a^2*d*e^2*f*g^4 + 2*(c^2*d^3 - a*c*d*e^2)*f^3*g^2 - (4*a*c*d^2*e - a^2*e^3)*f^2*g^3)*x)*\sqrt{-c*d*f*g + a*e*g^2}), -1/4*((c*d*e*f^2 + 2*a*d*e*g^2 - (5*c*d^2 - 2*a*e^2)*f*g - (c*d*e*f*g + (3*c*d^2 - 4*a*e^2)*g^2)*x)*\sqrt{c*d*e^x^2 + a*d*e + (c*d^2 + a*e^2)*x}*\sqrt{c*d*f*g - a*e*g^2}*\sqrt{e^x + d} + (c^2*d^3*e*f^3 + (3*c^2*d^4 - 4*a*c*d^2*e^2)*f^2*g + (c^2*d^2*e^2*f*g^2 + (3*c^2*d^3*e - 4*a*c*d*e^3)*g^3)*x^3 + (2*c^2*d^2*e^2*f^2*g + (7*c^2*d^3*e - 8*a*c*d*e^3)*f*g^2 + (3*c^2*d^4 - 4*a*c*d^2*e^2)*g^3)*x^2 + (c^2*d^2*e^2*f^3 + (5*c^2*d^3*e - 4*a*c*d*e^3)*f^2*g + 2*(3*c^2*d^4 - 4*a*c*d^2*e^2)*f*g^2)*x)*\arctan(\sqrt{c*d*e^x^2 + a*d*e + (c*d^2 + a*e^2)*x}*\sqrt{c*d*f*g - a*e*g^2}*\sqrt{e^x + d}/(c*d*e*g*x^2 + a*d*e*g + (c*d^2 + a*e^2)*g*x))/((c^2*d^3*f^4*g - 2*a*c*d^2*e*f^3*g^2 + a^2*d*e^2*f^2*g^3 + (c^2*d^2*e*f^2*g^3 - 2*a*c*d*e^2*f*g^4 + a^2*e^3*g^5)*x^3 + (2*c^2*d^2*e*f^3*g^2 + a^2*d*e^2*g^5 + (c^2*d^3 - 4*a*c*d*e^2)*f^2*g^3 - 2*(a*c*d^2*e - a^2*e^3)*f*g^4)*x^2 + (c^2*d^2*e*f^4*g + 2*a^2*d*e^2*f*g^4 + 2*(c^2*d^3 - a*c*d*e^2)*f^3*g^2 - (4*a*c*d^2*e - a^2*e^3)*f^2*g^3)*x)*\sqrt{c*d*f*g - a*e*g^2}]] \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((e^x+d)^{(3/2)}/(g*x+f)^3/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^{(1/2)}$

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^(3/2)/(sqrt(c*d*e*x^2 + a*d*e + (c*d^2 + a*e^2)*x)*(g*x + f)^(1/2))
```

```
[Out] Timed out
```

$$3.791 \quad \int \frac{(d+ex)^{3/2}}{(f+gx)^4 \sqrt{ade+(cd^2+ae^2)x+cdex^2}} dx$$

**Optimal.** Leaf size=351

$$\frac{c^2 d^2 (6ae^2 g - cd(5dg + ef)) \tan^{-1} \left( \frac{\sqrt{g} \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{\sqrt{d+ex} \sqrt{cdf - aeg}} \right)}{8g^{3/2} (cdf - aeg)^{7/2}} - \frac{cd \sqrt{x(ae^2 + cd^2) + ade + cdex^2} (6ae^2 g - cd(5dg + ef))}{8g \sqrt{d+ex} (f+gx) (cdf - aeg)^3} - \frac{\sqrt{x(ae^2 + cd^2) + ade + cdex^2} (6ae^2 g - cd(5dg + ef))}{12g \sqrt{d+ex} (f+gx)^2 (cdf - aeg)^2} - \frac{(ef - dg) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{3g \sqrt{d+ex} (f+gx)^3 (cdf - aeg)}$$

[Out]  $-\left((e^*f - d^*g) * \text{Sqrt}[a^*d^*e + (c^*d^{\wedge}2 + a^*e^{\wedge}2)^*x + c^*d^*e^*x^{\wedge}2]\right) / \left(3^*g^* \left(c^*d^*f - a^*e^*g\right) * \text{Sqrt}[d + e^*x]^*(f + g^*x)^{\wedge}3 - \left(\left(6^*a^*e^{\wedge}2^*g - c^*d^*(e^*f + 5^*d^*g)\right) * \text{Sqrt}[a^*d^*e + (c^*d^{\wedge}2 + a^*e^{\wedge}2)^*x + c^*d^*e^*x^{\wedge}2]\right) / \left(12^*g^*(c^*d^*f - a^*e^*g)^{\wedge}2 * \text{Sqrt}[d + e^*x]^*(f + g^*x)^{\wedge}2 - (c^*d^*(6^*a^*e^{\wedge}2^*g - c^*d^*(e^*f + 5^*d^*g)) * \text{Sqrt}[a^*d^*e + (c^*d^{\wedge}2 + a^*e^{\wedge}2)^*x + c^*d^*e^*x^{\wedge}2]\right) / \left(8^*g^*(c^*d^*f - a^*e^*g)^{\wedge}3 * \text{Sqrt}[d + e^*x]^*(f + g^*x) - (c^{\wedge}2^*d^{\wedge}2^*(6^*a^*e^{\wedge}2^*g - c^*d^*(e^*f + 5^*d^*g)) * \text{ArcTan}[\left(\text{Sqrt}[g]^*\text{Sqrt}[a^*d^*e + (c^*d^{\wedge}2 + a^*e^{\wedge}2)^*x + c^*d^*e^*x^{\wedge}2]\right) / \left(\text{Sqrt}[c^*d^*f - a^*e^*g]^*\text{Sqrt}[d + e^*x]\right)]\right) / \left(8^*g^{\wedge}(3/2)^*(c^*d^*f - a^*e^*g)^{\wedge}(7/2)\right)$

**Rubi [A]** time = 1.65225, antiderivative size = 351, normalized size of antiderivative = 1., number of rules used = 5, number of rules used = 4, integrand size = 46,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$

$$\frac{c^2 d^2 (6ae^2 g - cd(5dg + ef)) \tan^{-1} \left( \frac{\sqrt{g} \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{\sqrt{d+ex} \sqrt{cdf - aeg}} \right)}{8g^{3/2} (cdf - aeg)^{7/2}} - \frac{cd \sqrt{x(ae^2 + cd^2) + ade + cdex^2} (6ae^2 g - cd(5dg + ef))}{8g \sqrt{d+ex} (f+gx) (cdf - aeg)^3} - \frac{\sqrt{x(ae^2 + cd^2) + ade + cdex^2} (6ae^2 g - cd(5dg + ef))}{12g \sqrt{d+ex} (f+gx)^2 (cdf - aeg)^2} - \frac{(ef - dg) \sqrt{x(ae^2 + cd^2) + ade + cdex^2}}{3g \sqrt{d+ex} (f+gx)^3 (cdf - aeg)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e^*x)^{\wedge}(3/2) / ((f + g^*x)^{\wedge}4 * \text{Sqrt}[a^*d^*e + (c^*d^{\wedge}2 + a^*e^{\wedge}2)^*x + c^*d^*e^*x^{\wedge}2]), x]$

[Out]  $-\left((e^*f - d^*g) * \text{Sqrt}[a^*d^*e + (c^*d^{\wedge}2 + a^*e^{\wedge}2)^*x + c^*d^*e^*x^{\wedge}2]\right) / \left(3^*g^* \left(c^*d^*f - a^*e^*g\right) * \text{Sqrt}[d + e^*x]^*(f + g^*x)^{\wedge}3 - \left(\left(6^*a^*e^{\wedge}2^*g - c^*d^*(e^*f + 5^*d^*g)\right) * \text{Sqrt}[a^*d^*e + (c^*d^{\wedge}2 + a^*e^{\wedge}2)^*x + c^*d^*e^*x^{\wedge}2]\right) / \left(12^*g^*(c^*d^*f - a^*e^*g)^{\wedge}2 * \text{Sqrt}[d + e^*x]^*(f + g^*x)^{\wedge}2 - (c^*d^*(6^*a^*e^{\wedge}2^*g - c^*d^*(e^*f + 5^*d^*g)) * \text{Sqrt}[a^*d^*e + (c^*d^{\wedge}2 + a^*e^{\wedge}2)^*x + c^*d^*e^*x^{\wedge}2]\right) / \left(8^*g^*(c^*d^*f - a^*e^*g)^{\wedge}3 * \text{Sqrt}[d + e^*x]^*(f + g^*x) - (c^{\wedge}2^*d^{\wedge}2^*(6^*a^*e^{\wedge}2^*g - c^*d^*(e^*f + 5^*d^*g)) * \text{ArcTan}[\left(\text{Sqrt}[g]^*\text{Sqrt}[a^*d^*e + (c^*d^{\wedge}2 + a^*e^{\wedge}2)^*x + c^*d^*e^*x^{\wedge}2]\right) / \left(\text{Sqrt}[c^*d^*f - a^*e^*g]^*\text{Sqrt}[d + e^*x]\right)]\right) / \left(8^*g^{\wedge}(3/2)^*(c^*d^*f - a^*e^*g)^{\wedge}(7/2)\right)$

**Rubi in Sympy [A]** time = 136.285, size = 340, normalized size = 0.97

$$\frac{c^2 d^2 (6ae^2 g - 5cd^2 g - cdef) \operatorname{atanh} \left( \frac{\sqrt{g} \sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{\sqrt{d+ex} \sqrt{aeg - cdf}} \right)}{8g^{\frac{3}{2}} (aeg - cdf)^{\frac{7}{2}}} + \frac{cd \sqrt{ade + cdex^2 + x(ae^2 + cd^2)} (6ae^2 g - 5cd^2 g - cdef)}{8g \sqrt{d+ex} (f+gx) (aeg - cdf)^3} - \frac{\sqrt{ade + cdex^2 + x(ae^2 + cd^2)} (6ae^2 g - 5cd^2 g - cdef)}{12g \sqrt{d+ex} (f+gx)^2 (aeg - cdf)^2} - \frac{(dg - ef) \sqrt{ade + cdex^2 + x(ae^2 + cd^2)}}{3g \sqrt{d+ex} (f+gx)^3 (aeg - cdf)}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**(3/2)/(g*x+f)**4/(a*d*e+(a*e**2+c*d**2)*x+c*d*e*x**2)*`

[Out] 
$$-c^{**2}d^{**2}(6*a*e^{**2}g - 5*c*d^{**2}g - c*d*e*f)*\operatorname{atanh}(\operatorname{sqrt}(g)*\operatorname{sqrt}(a*d*e + c*d*e*x^{**2} + x*(a*e^{**2} + c*d^{**2}))/(\operatorname{sqrt}(d + e*x)*\operatorname{sqrt}(a*e*g - c*d*f)))/(8*g^{**}(3/2)*(a*e*g - c*d*f)^{(7/2)}) + c*d*\operatorname{sqrt}(a*d*e + c*d*e*x^{**2} + x*(a*e^{**2} + c*d^{**2}))* (6*a*e^{**2}g - 5*c*d^{**2}g - c*d*e*f)/(8*g*\operatorname{sqrt}(d + e*x)*(f + g*x)*(a*e*g - c*d*f)^{**3}) - \operatorname{sqrt}(a*d*e + c*d*e*x^{**2} + x*(a*e^{**2} + c*d^{**2}))* (6*a*e^{**2}g - 5*c*d^{**2}g - c*d*e*f)/(12*g*\operatorname{sqrt}(d + e*x)*(f + g*x)^{**2}*(a*e*g - c*d*f)^{**2}) - (d*g - e*f)*\operatorname{sqrt}(a*d*e + c*d*e*x^{**2} + x*(a*e^{**2} + c*d^{**2}))/ (3*g*\operatorname{sqrt}(d + e*x)*(f + g*x)^{**3}*(a*e*g - c*d*f))$$

**Mathematica [A]** time = 1.70025, size = 245, normalized size = 0.7

$$\sqrt{d+ex} \left( \frac{c^2 d^2 \sqrt{ae+cdx} (cd(5dg+ef) - 6ae^2 g) \tanh^{-1} \left( \frac{\sqrt{g} \sqrt{ae+cdx}}{\sqrt{aeg-cdf}} \right)}{g^{3/2} (aeg-cdf)^{7/2}} + \frac{(ae+cdx)(-3cd(f+gx)^2 (cd(5dg+ef) - 6ae^2 g) + 2(f+gx)(aeg-cdf)(cd(5dg+ef) - 6ae^2 g) + 3g(f+gx)^3 (aeg-cdf)^3)}{3g(f+gx)^3 (aeg-cdf)^3} \right) / 8\sqrt{(d+ex)(ae+cdx)}$$

Antiderivative was successfully verified.

[In] `Integrate[(d + e*x)^(3/2)/((f + g*x)^4*Sqrt[a*d*e + (c*d^2 + a*e^2)*x + c*d*e*x^2]`

[Out] 
$$(\operatorname{Sqrt}[d + e*x]*((a*e + c*d*x)^*(8*(e*f - d*g)*(c*d*f - a*e*g)^2 + 2*(-(c*d*f) + a*e*g)*(-6*a*e^2*g + c*d*(e*f + 5*d*g))*(f + g*x) - 3*c*d*(-6*a*e^2*g + c*d*(e*f + 5*d*g))*(f + g*x)^2)))/(3*g*(-(c*d*f) + a*e*g)^3*(f + g*x)^3) + (c^2*d^2*(-6*a*e^2*g + c*d*(e*f + 5*d*g))*\operatorname{Sqrt}[a*e + c*d*x]*\operatorname{ArcTanh}[(\operatorname{Sqrt}[g]*\operatorname{Sqrt}[a*e + c*d*x])/(\operatorname{Sqrt}[-(c*d*f) + a*e*g])]/(g^{(3/2)}*(-(c*d*f) + a*e*g)^{(7/2)}))/ (8*\operatorname{Sqrt}[(a*e + c*d*x)*(d + e*x)])$$

**Maple [B]** time = 0.051, size = 1142, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^(3/2)/(g*x+f)^4/(a*d*e+(a*e^2+c*d^2)*x+c*d*e*x^2)^(1/2),x)`

[Out] 
$$-1/24*(c*d*e*x^2+a*e^2*x+c*d^2*x+a*d*e)^{(1/2)}*(15*x^2*c^2*d^3*g^3*(c*d*x+a*e)^{(1/2)}*((a*e*g-c*d*f)*g)^{(1/2)}-18*x^2*a*c*d*e^2*g^3*(c*d*x+a*e)^{(1/2)}*((a*e*g-c*d*f)*g)^{(1/2)}-15*\operatorname{arctanh}(g*(c*d*x+a*e)^{(1/2)})/((a*e*g-c*d*f)*g)^{(1/2)})*x^3*c^3*d^4*g^4+18*\operatorname{arctanh}(g*(c*d*x+a*e)^{(1/2)})/((a*e*g-c*d*f)*g)^{(1/2)})*x^3*a*c^2*d^2*e^2*g^4-3*\operatorname{arctanh}(g*(c*d*x+a*e)^{(1/2)})/((a*e*g-c*d*f)*g)^{(1/2)})*x^3*c^3*d^3*e*f*g^3-9*\operatorname{arctanh}(g*(c*d*x+a*e)^{(1/2)})/((a*e*g-c*d*f)*g)^{(1/2)})*x^2*c^3*d^3*e*f^2*g^2-9*\operatorname{arctanh}(g*(c*d*x+a*e)^{(1/2)})/((a*e*g-c*d*f)*g)^{(1/2)})*x*c^3*d^3*e*f^3*g+18*\operatorname{arctanh}(g*(c*d*x+a*e)^{(1/2)})/((a*e*g-c*d*f)*g)^{(1/2)})*a*c^2*d^2*e^2*f^3*g+40*x*c^2*d^3*f*g^2*(c*d*x+a*e)^{(1/2)}*((a*e*g-c*d*f)*g)^{(1/2)}-50*x*a*c*d*e^2*f*g^2*(c*d*x+a*e)^{(1/2)}*((a*e*g-c*d*f)*g)^{(1/2)}-16*a*c*d*e^2*f^2*g*(c*d*x+a*e)^{(1/2)}*((a*e*g-c*d*f)*g)^{(1/2)}+54*\operatorname{arctanh}(g*(c*d*x+a*e)^{(1/2)})/((a*e*g-c*d*f)*g)^{(1/2)})*x^2*a*c^2*d^2*e^2*f^3*g+54*\operatorname{arctanh}(g*(c*d*x+a*e)^{(1/2)})/((a*e*g-c*d*f)*g)^{(1/2)})*x*a*c^2*d^2*e^2*f^2*g^2+12*x*a^2*e^3*g^3*(c*d*x+a*e)^{(1/2)}*((a*e*g-c*d*f)*g)^{(1/2)}+8*a^2*d*e^2*g^3*(c*d*x+a*e)^{(1/2)}*((a*e*g-c*d*f)*g)^{(1/2)}+4*a^2*e^3*f*g^2*(c*d*x+a*e)^{(1/2)}*((a*e*g-c*d*f)*g)^{(1/2)}+33*c^2*d^3*f^2*g*(c*d*x+a*e)^{(1/2)}*((a*e*g-c*d*f)*g)^{(1/2)}-3*c^2*d^2*e*f^3*(c*d*x+a*e)^{(1/2)}*((a*e*g-c*d*f)*g)^{(1/2)}-45*\operatorname{arctanh}(g*(c*d*x+a*e)^{(1/2)})/((a*e*g-c$$



$$\begin{aligned} &^4e - 9a^2c^2d^2e^3)fg^3 + (5c^3d^5 - 6a^2c^2d^3e^2)g^4 \\ &)^x^3 + 3(c^3d^3e^2f^3g + 6(c^3d^4e - a^2c^2d^2e^3)f^2g^2 + (5c^3d^5 - 6a^2c^2d^3e^2)fg^3)x^2 + (c^3d^3e^2f^4 \\ &+ 2(4c^3d^4e - 3a^2c^2d^2e^3)f^3g + 3(5c^3d^5 - 6a^2c^2d^3e^2)f^2g^2)x) \arctan(\sqrt{c^2d^2e^2x^2 + a^2d^2e + (c^2d^2 + a^2e^2)x}) \sqrt{c^2d^2fg - a^2e^2g^2} \sqrt{ex + d}) / (c^2d^2e^2g^2x^2 + a^2d^2e^2g + (c^2d^2 + a^2e^2)g^2x)) / ((c^3d^4f^6g - 3a^2c^2d^3e^2f^5g^2 + 3a^2c^2d^2e^2f^4g^3 - a^3d^2e^3f^3g^4 + (c^3d^3e^2f^3g^4 - 3a^2c^2d^2e^2f^2g^5 + 3a^2c^2d^2e^3fg^6 - a^3e^4g^7)x^4 + (3c^3d^3e^2f^4g^3 - a^3d^2e^3g^7 + (c^3d^4 - 9a^2c^2d^2e^2)f^3g^4 - 3(a^2c^2d^3e - 3a^2c^2d^2e^3)f^2g^5 + 3(a^2c^2d^2e^2 - a^3e^4)f^2g^6)x^3 + 3(c^3d^3e^2f^5g^2 - a^3d^2e^3fg^6 + (c^3d^4 - 3a^2c^2d^2e^2)f^4g^3 - 3(a^2c^2d^3e - a^2c^2d^2e^3)f^3g^4 + (3a^2c^2d^2e^2 - a^3e^4)f^2g^5)x^2 + (c^3d^3e^2f^6g - 3a^3d^2e^3f^2g^5 + 3(c^3d^4 - a^2c^2d^2e^2)f^5g^2 - 3(3a^2c^2d^3e - a^2c^2d^2e^3)f^4g^3 + (9a^2c^2d^2e^2 - a^3e^4)f^3g^4)x) \sqrt{c^2d^2fg - a^2e^2g^2})] \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(3/2)/(g\*x+f)\*\*4/(a\*d\*e+(a\*e\*\*2+c\*d\*\*2)\*x+c\*d\*e\*x\*\*2)\*\*(1/2)

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^(3/2)/(sqrt(c^2\*d^2\*e^2\*x^2 + a^2\*d^2\*e + (c^2\*d^2 + a^2\*e^2)\*x) \* (g\*x + f)^(1/2)

[Out] Timed out

$$3.792 \quad \int \frac{(a+bx+cx^2)^3}{\sqrt{1-dx}\sqrt{1+dx}} dx$$

**Optimal.** Leaf size=324

$$\begin{aligned} & - \frac{b\sqrt{1-d^2x^2}(45a^2d^4 + 60acd^2 + 10b^2d^2 + 24c^2)}{15d^6} \\ & - \frac{x\sqrt{1-d^2x^2}(24a^2cd^4 + 24ab^2d^4 + 18ac^2d^2 + 18b^2cd^2 + 5c^3)}{16d^6} \\ & + \frac{\sin^{-1}(dx)(16a^3d^6 + 24a^2cd^4 + 24ab^2d^4 + 18ac^2d^2 + 18b^2cd^2 + 5c^3)}{16d^7} \\ & - \frac{bx^2\sqrt{1-d^2x^2}(30acd^2 + 5b^2d^2 + 12c^2)}{15d^4} \\ & - \frac{cx^3\sqrt{1-d^2x^2}(18acd^2 + 18b^2d^2 + 5c^2)}{24d^4} - \frac{3bc^2x^4\sqrt{1-d^2x^2}}{5d^2} - \frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2} \end{aligned}$$

[Out]  $-(b*(24*c^2 + 10*b^2*d^2 + 60*a*c*d^2 + 45*a^2*d^4)*\text{Sqrt}[1 - d^2*x^2])/(15*d^6) - ((5*c^3 + 18*b^2*c*d^2 + 18*a*c^2*d^2 + 24*a*b^2*d^4 + 24*a^2*c*d^4)*x*\text{Sqrt}[1 - d^2*x^2])/(16*d^6) - (b*(12*c^2 + 5*b^2*d^2 + 30*a*c*d^2)*x^2*\text{Sqrt}[1 - d^2*x^2])/(15*d^4) - (c*(5*c^2 + 18*b^2*d^2 + 18*a*c*d^2)*x^3*\text{Sqrt}[1 - d^2*x^2])/(24*d^4) - (3*b*c^2*x^4*\text{Sqrt}[1 - d^2*x^2])/(5*d^2) - (c^3*x^5*\text{Sqrt}[1 - d^2*x^2])/(6*d^2) + ((5*c^3 + 18*b^2*c*d^2 + 18*a*c^2*d^2 + 24*a*b^2*d^4 + 24*a^2*c*d^4 + 16*a^3*d^6)*\text{ArcSin}[d*x])/(16*d^7)$

**Rubi [A]** time = 1.4199, antiderivative size = 324, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\begin{aligned} & - \frac{b\sqrt{1-d^2x^2}(45a^2d^4 + 60acd^2 + 10b^2d^2 + 24c^2)}{15d^6} \\ & - \frac{x\sqrt{1-d^2x^2}(24a^2cd^4 + 24ab^2d^4 + 18ac^2d^2 + 18b^2cd^2 + 5c^3)}{16d^6} \\ & + \frac{\sin^{-1}(dx)(16a^3d^6 + 24a^2cd^4 + 24ab^2d^4 + 18ac^2d^2 + 18b^2cd^2 + 5c^3)}{16d^7} \\ & - \frac{bx^2\sqrt{1-d^2x^2}(30acd^2 + 5b^2d^2 + 12c^2)}{15d^4} \\ & - \frac{cx^3\sqrt{1-d^2x^2}(18acd^2 + 18b^2d^2 + 5c^2)}{24d^4} - \frac{3bc^2x^4\sqrt{1-d^2x^2}}{5d^2} - \frac{c^3x^5\sqrt{1-d^2x^2}}{6d^2} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x + c*x^2)^3/(\text{Sqrt}[1 - d*x]*\text{Sqrt}[1 + d*x]),x]$

[Out]  $-(b*(24*c^2 + 10*b^2*d^2 + 60*a*c*d^2 + 45*a^2*d^4)*\text{Sqrt}[1 - d^2*x^2])/(15*d^6) - ((5*c^3 + 18*b^2*c*d^2 + 18*a*c^2*d^2 + 24*a*b^2*d^4 + 24*a^2*c*d^4)*x*\text{Sqrt}[1 - d^2*x^2])/(16*d^6) - (b*(12*c^2 + 5*b^2*d^2 + 30*a*c*d^2)*x^2*\text{Sqrt}[1 - d^2*x^2])/(15*d^4) - (c*(5*c^2 + 18*b^2*d^2 + 18*a*c*d^2)*x^3*\text{Sqrt}[1 - d^2*x^2])/(24*d^4) - (3*b*c^2*x^4*\text{Sqrt}[1 - d^2*x^2])/(5*d^2) - (c^3*x^5*\text{Sqrt}[1 - d^2*x^2])/(6*d^2) + ((5*c^3 + 18*b^2*c*d^2 + 18*a*c^2*d^2 + 24*a*b^2*d^4 + 24*a^2*c*d^4 + 16*a^3*d^6)*\text{ArcSin}[d*x])/(16*d^7)$

**Rubi in Sympy [A]** time = 169.637, size = 309, normalized size = 0.95

$$\frac{(8b + 5cx)\sqrt{-d^2x^2 + 1}(a + bx + cx^2)^2}{30d^2} + \frac{(16a^3d^6 + 24a^2cd^4 + 24ab^2d^4 + 18ac^2d^2 + 18b^2cd^2 + 5c^3)\operatorname{asin}(dx)}{16d^7} - \frac{(3b(86acd^2 + 2b^2d^2 + 71c^2) + 3cx(6b^2d^2 + 25c(2ad^2 + c)))\sqrt{-d^2x^2 + 1}(a + bx + cx^2)}{360cd^4} - \frac{(6b(242a^2cd^4 - 2ab^2d^4 + 409ac^2d^2 + 80b^2cd^2 + 192c^3) + x(660a^2c^2d^4 + 144ab^2cd^4 + 660ac^3d^2 - 12b^4d^4 + 384b^2c^2d^2 + 225c^3))\sqrt{-d^2x^2 + 1}}{720cd^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x**2+b*x+a)**3/(-d*x+1)**(1/2)/(d*x+1)**(1/2), x)`

[Out] `-(8*b + 5*c*x)*sqrt(-d**2*x**2 + 1)*(a + b*x + c*x**2)**2/(30*d**2) + (16*a**3*d**6 + 24*a**2*c*d**4 + 24*a*b**2*d**4 + 18*a*c**2*d**2 + 18*b**2*c*d**2 + 5*c**3)*asin(d*x)/(16*d**7) - (3*b*(86*a*c*d**2 + 2*b**2*d**2 + 71*c**2) + 3*c*x*(6*b**2*d**2 + 25*c*(2*a*d**2 + c)))*sqrt(-d**2*x**2 + 1)*(a + b*x + c*x**2)/(360*c*d**4) - (6*b*(242*a**2*c*d**4 - 2*a*b**2*d**4 + 409*a*c**2*d**2 + 80*b**2*c*d**2 + 192*c**3) + x*(660*a**2*c**2*d**4 + 144*a*b**2*c*d**4 + 660*a*c**3*d**2 - 12*b**4*d**4 + 384*b**2*c**2*d**2 + 225*c**3))*sqrt(-d**2*x**2 + 1)/(720*c*d**6)`

**Mathematica [A]** time = 0.364603, size = 229, normalized size = 0.71

$$15 \sin^{-1}(dx) (16a^3d^6 + 24a^2cd^4 + 24ab^2d^4 + 18ac^2d^2 + 18b^2cd^2 + 5c^3) - d\sqrt{1 - d^2x^2} (48b(15a^2d^4 + 10acd^2(d^2x^2 + 2) + c^2$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x + c*x^2)^3/(Sqrt[1 - d*x]*Sqrt[1 + d*x]), x]`

[Out] `(-(d*Sqrt[1 - d^2*x^2])*(80*b^3*d^2*(2 + d^2*x^2) + 90*b^2*d^2*x*(4*a*d^2 + c*(3 + 2*d^2*x^2)) + 48*b*(15*a^2*d^4 + 10*a*c*d^2*(2 + d^2*x^2) + c^2*(8 + 4*d^2*x^2 + 3*d^4*x^4)) + 5*c*x*(72*a^2*d^4 + 18*a*c*d^2*(3 + 2*d^2*x^2) + c^2*(15 + 10*d^2*x^2 + 8*d^4*x^4))) + 15*(5*c^3 + 18*b^2*c*d^2 + 18*a*c^2*d^2 + 24*a*b^2*d^4 + 24*a^2*c*d^4 + 16*a^3*d^6)*ArcSin[d*x]/(240*d^7)`

**Maple [C]** time = 0.084, size = 602, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)^3/(-d*x+1)^(1/2)/(d*x+1)^(1/2), x)`

[Out] `-1/240*(-d*x+1)^(1/2)*(d*x+1)^(1/2)*(40*csgn(d)*x^5*c^3*d^5*(-d^2*x^2+1)^(1/2)+144*csgn(d)*x^4*b*c^2*d^5*(-d^2*x^2+1)^(1/2)+180*csgn(d)*x^3*a*c^2*d^5*(-d^2*x^2+1)^(1/2)+180*csgn(d)*x^3*b^2*c*d^5*(-d^2*x^2+1)^(1/2)+480*csgn(d)*x^2*a*b*c*d^5*(-d^2*x^2+1)^(1/2)+80*csgn(d)*x^2*b^3*d^5*(-d^2*x^2+1)^(1/2)+50*c^3*x^3*(-d^2*x^2+1)^(1/2)*d^3*csgn(d)+360*x*(-d^2*x^2+1)^(1/2)*a^2*c*d^5*csgn(d)+360*x*(-d^2*x^2+1)^(1/2)*a*b^2*d^5*csgn(d)+192*csgn(d)*d^3*(-d^2*x^2+1)^(1/2)*x^2*b*c^2+720*(-d^2*x^2+1)^(1/2)*a^2*b*d^5*csgn(d)-240*arctan(csgn(d)*d*x/(-d^2*x^2+1)^(1/2))*a^3*d^6+270*x*(-d^2*x^2+1)^(1/2)*a*c^2*d^3*csgn(d)+270*x*(-d^2*x^2+1)^(1/2)*b^2*c*d^3*csgn(d)`

) + 960 \* (-d^2 \* x^2 + 1)^(1/2) \* a \* b \* c \* d^3 \* csgn(d) + 160 \* (-d^2 \* x^2 + 1)^(1/2) \* b^3 \* d^3 \* csgn(d) - 360 \* arctan(csgn(d) \* d \* x / (-d^2 \* x^2 + 1)^(1/2)) \* a^2 \* c \* d^4 - 360 \* arctan(csgn(d) \* d \* x / (-d^2 \* x^2 + 1)^(1/2)) \* a \* b^2 \* d^4 + 75 \* x \* (-d^2 \* x^2 + 1)^(1/2) \* c^3 \* csgn(d) \* d + 384 \* (-d^2 \* x^2 + 1)^(1/2) \* b \* c^2 \* csgn(d) \* d - 270 \* arctan(csgn(d) \* d \* x / (-d^2 \* x^2 + 1)^(1/2)) \* a \* c^2 \* d^2 - 270 \* arctan(csgn(d) \* d \* x / (-d^2 \* x^2 + 1)^(1/2)) \* b^2 \* c \* d^2 - 75 \* arctan(csgn(d) \* d \* x / (-d^2 \* x^2 + 1)^(1/2)) \* c^3 \* csgn(d) / (-d^2 \* x^2 + 1)^(1/2) / d^7

**Maxima [A]** time = 0.794152, size = 554, normalized size = 1.71

$$\begin{aligned} & -\frac{\sqrt{-d^2x^2+1}c^3x^5}{6d^2} - \frac{3\sqrt{-d^2x^2+1}bc^2x^4}{5d^2} + \frac{a^3\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{\sqrt{d^2}} - \frac{5\sqrt{-d^2x^2+1}c^3x^3}{24d^4} \\ & - \frac{3\sqrt{-d^2x^2+1}(b^2c+ac^2)x^3}{4d^2} - \frac{3\sqrt{-d^2x^2+1}a^2b}{d^2} - \frac{4\sqrt{-d^2x^2+1}bc^2x^2}{5d^4} \\ & - \frac{\sqrt{-d^2x^2+1}(b^3+6abc)x^2}{3d^2} - \frac{3\sqrt{-d^2x^2+1}(ab^2+a^2c)x}{2d^2} + \frac{3(ab^2+a^2c)\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{2\sqrt{d^2}d^2} \\ & - \frac{5\sqrt{-d^2x^2+1}c^3x}{16d^6} - \frac{9\sqrt{-d^2x^2+1}(b^2c+ac^2)x}{8d^4} - \frac{8\sqrt{-d^2x^2+1}bc^2}{5d^6} \\ & - \frac{2\sqrt{-d^2x^2+1}(b^3+6abc)}{3d^4} + \frac{5c^3\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{16\sqrt{d^2}d^6} + \frac{9(b^2c+ac^2)\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{8\sqrt{d^2}d^4} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)^3/(sqrt(d\*x + 1)\*sqrt(-d\*x + 1)),x, algorithm="maxima

[Out] -1/6\*sqrt(-d^2\*x^2 + 1)\*c^3\*x^5/d^2 - 3/5\*sqrt(-d^2\*x^2 + 1)\*b\*c^2\*x^4/d^2 + a^3\*arcsin(d^2\*x/sqrt(d^2))/sqrt(d^2) - 5/24\*sqrt(-d^2\*x^2 + 1)\*c^3\*x^3/d^4 - 3/4\*sqrt(-d^2\*x^2 + 1)\*(b^2\*c + a\*c^2)\*x^3/d^2 - 3\*sqrt(-d^2\*x^2 + 1)\*a^2\*b/d^2 - 4/5\*sqrt(-d^2\*x^2 + 1)\*b\*c^2\*x^2/d^4 - 1/3\*sqrt(-d^2\*x^2 + 1)\*(b^3 + 6\*a\*b\*c)\*x^2/d^2 - 3/2\*sqrt(-d^2\*x^2 + 1)\*(a\*b^2 + a^2\*c)\*x/d^2 + 3/2\*(a\*b^2 + a^2\*c)\*arcsin(d^2\*x/sqrt(d^2))/(sqrt(d^2)\*d^2) - 5/16\*sqrt(-d^2\*x^2 + 1)\*c^3\*x/d^6 - 9/8\*sqrt(-d^2\*x^2 + 1)\*(b^2\*c + a\*c^2)\*x/d^4 - 8/5\*sqrt(-d^2\*x^2 + 1)\*b\*c^2/d^6 - 2/3\*sqrt(-d^2\*x^2 + 1)\*(b^3 + 6\*a\*b\*c)/d^4 + 5/16\*c^3\*arcsin(d^2\*x/sqrt(d^2))/(sqrt(d^2)\*d^6) + 9/8\*(b^2\*c + a\*c^2)\*arcsin(d^2\*x/sqrt(d^2))/(sqrt(d^2)\*d^4)

**Fricas [A]** time = 0.289888, size = 1593, normalized size = 4.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)^3/(sqrt(d\*x + 1)\*sqrt(-d\*x + 1)),x, algorithm="fricas

[Out] 1/240\*(240\*c^3\*d^11\*x^11 + 864\*b\*c^2\*d^11\*x^10 + 11520\*a^2\*b\*d^7\*x^2 - 20\*(61\*c^3\*d^9 - 54\*(b^2\*c + a\*c^2)\*d^11)\*x^9 - 480\*(9\*b\*c^2\*d^9 - (b^3 + 6\*a\*b\*c)\*d^11)\*x^8 + 30\*(72\*(a\*b^2 + a^2\*c)\*d^11 + 37\*c^3\*d^7 - 174\*(b^2\*c + a\*c^2)\*d^9)\*x^7 + 80\*(45\*a^2\*b\*d^11 + 48\*b\*c^2\*d^7 - 28\*(b^3 + 6\*a\*b\*c)\*d^9)\*x^6 - 30\*(456\*(a\*b^2 + a^2\*c)\*d^9 + 31\*c^3\*d^5 - 42\*(b^2\*c + a\*c^2)\*d^7)\*x^5 - 960\*(15\*a^2\*b\*d^9 - 2\*(b^3 + 6\*a\*b\*c)\*d^7)\*x^4 + 640\*(36\*(a\*b^2 + a^2\*c)\*d^7 + 5\*c^3\*d^3 + 18\*(b^2\*c + a\*c^2)\*d^5)\*x^3 - (40\*c^3\*d^11\*x^11 + 144\*b\*c^2\*d^11\*x^10 + 11520\*a^2\*b\*d^7\*x^2 - 10\*(67\*c^3\*d^9 - 18\*(b^2\*c + a\*c^2)\*d^11)\*x^9 - 80\*(30\*b\*c^2\*d^9 - (b^3 + 6\*a\*b\*c)\*d^11)\*x^8 + 15\*(24\*(a\*b^2 + a^2\*c)\*d^11 + 73\*c^3\*d^7 - 198\*(b^2\*c + a\*c^2)\*d^9)\*x^7 + 80\*(9\*a^2\*b\*d^11 + 48\*b\*c^2\*d^7 - 16\*(b^3 + 6\*a\*b\*c)\*d^9)\*x^6 - 10\*(648\*(a\*b^2 + a^2\*c)\*d^9 + 23\*c^3\*d^5 - 378\*(b

$$\begin{aligned} & ^2*c + a*c^2)*d^7)*x^5 - 960*(9*a^2*b*d^9 - 2*(b^3 + 6*a*b*c)*d^7) \\ & )*x^4 + 80*(216*(a*b^2 + a^2*c)*d^7 + 25*c^3*d^3 + 90*(b^2*c + a* \\ & c^2)*d^5)*x^3 - 480*(24*(a*b^2 + a^2*c)*d^5 + 5*c^3*d + 18*(b^2*c \\ & + a*c^2)*d^3)*x)*\sqrt{d*x + 1}*\sqrt{-d*x + 1} - 480*(24*(a*b^2 + \\ & a^2*c)*d^5 + 5*c^3*d + 18*(b^2*c + a*c^2)*d^3)*x + 30*(512*a^3*d \\ & ^6 - (16*a^3*d^12 + 24*(a*b^2 + a^2*c)*d^10 + 5*c^3*d^6 + 18*(b^2 \\ & *c + a*c^2)*d^8)*x^6 + 768*(a*b^2 + a^2*c)*d^4 + 18*(16*a^3*d^10 \\ & + 24*(a*b^2 + a^2*c)*d^8 + 5*c^3*d^4 + 18*(b^2*c + a*c^2)*d^6)*x^4 \\ & + 160*c^3 + 576*(b^2*c + a*c^2)*d^2 - 48*(16*a^3*d^8 + 24*(a*b^2 \\ & + a^2*c)*d^6 + 5*c^3*d^2 + 18*(b^2*c + a*c^2)*d^4)*x^2 - 2*(256 \\ & *a^3*d^6 + 384*(a*b^2 + a^2*c)*d^4 + 3*(16*a^3*d^10 + 24*(a*b^2 + \\ & a^2*c)*d^8 + 5*c^3*d^4 + 18*(b^2*c + a*c^2)*d^6)*x^4 + 80*c^3 + \\ & 288*(b^2*c + a*c^2)*d^2 - 16*(16*a^3*d^8 + 24*(a*b^2 + a^2*c)*d^6 \\ & + 5*c^3*d^2 + 18*(b^2*c + a*c^2)*d^4)*x^2)*\sqrt{d*x + 1}*\sqrt{-d \\ & *x + 1})*\arctan((\sqrt{d*x + 1})*\sqrt{-d*x + 1} - 1)/(d*x))/d^{13} \\ & x^6 - 18*d^{11}*x^4 + 48*d^9*x^2 - 32*d^7 + 2*(3*d^{11}*x^4 - 16*d^9* \\ & x^2 + 16*d^7)*\sqrt{d*x + 1}*\sqrt{-d*x + 1}) \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*\*3/(-d\*x+1)\*\*(1/2)/(d\*x+1)\*\*(1/2),x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.31835, size = 518, normalized size = 1.6

$$(720 a^2 b d^{41} - 360 a b^2 d^{40} - 360 a^2 c d^{40} + 240 b^3 d^{39} + 1440 a b c d^{39} - 450 b^2 c d^{38} - 450 a c^2 d^{38} + 720 b c^2 d^{37} - 165 c^3 d^{36} + (3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)^3/(sqrt(d\*x + 1)\*sqrt(-d\*x + 1)),x, algorithm="giac")

[Out] 
$$\begin{aligned} & -1/21626880*((720*a^2*b*d^41 - 360*a*b^2*d^40 - 360*a^2*c*d^40 + \\ & 240*b^3*d^39 + 1440*a*b*c*d^39 - 450*b^2*c*d^38 - 450*a*c^2*d^38 \\ & + 720*b*c^2*d^37 - 165*c^3*d^36 + (360*a*b^2*d^40 + 360*a^2*c*d^4 \\ & 0 - 160*b^3*d^39 - 960*a*b*c*d^39 + 810*b^2*c*d^38 + 810*a*c^2*d^ \\ & 38 - 960*b*c^2*d^37 + 425*c^3*d^36 + 2*(40*b^3*d^39 + 240*a*b*c*d \\ & ^39 - 270*b^2*c*d^38 - 270*a*c^2*d^38 + 528*b*c^2*d^37 - 275*c^3* \\ & d^36 + (90*b^2*c*d^38 + 90*a*c^2*d^38 - 288*b*c^2*d^37 + 225*c^3* \\ & d^36 + 4*(5*(d*x + 1)*c^3*d^36 + 18*b*c^2*d^37 - 25*c^3*d^36)*(d* \\ & x + 1))*(d*x + 1))*(d*x + 1))*\sqrt{d*x + 1}*\sqrt{-d*x \\ & + 1} - 30*(16*a^3*d^42 + 24*a*b^2*d^40 + 24*a^2*c*d^40 + 18*b^2*c \\ & *d^38 + 18*a*c^2*d^38 + 5*c^3*d^36)*\arcsin(1/2*\sqrt{2}*\sqrt{d*x + \\ & 1}))/d \end{aligned}$$

$$3.793 \quad \int \frac{(a+bx+cx^2)^2}{\sqrt{1-dx}\sqrt{1+dx}} dx$$

**Optimal.** Leaf size=166

$$\frac{\sin^{-1}(dx) (8a^2d^4 + 8acd^2 + 4b^2d^2 + 3c^2)}{8d^5} - \frac{x\sqrt{1-d^2x^2} \left( c \left( 8a + \frac{3c}{d^2} \right) + 4b^2 \right)}{8d^2}$$

$$- \frac{2b\sqrt{1-d^2x^2} (3ad^2 + 2c)}{3d^4} - \frac{2bcx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2}$$

[Out]  $(-2*b*(2*c + 3*a*d^2)*\text{Sqrt}[1 - d^2*x^2])/(3*d^4) - ((4*b^2 + c*(8*a + (3*c)/d^2))*x*\text{Sqrt}[1 - d^2*x^2])/(8*d^2) - (2*b*c*x^2*\text{Sqrt}[1 - d^2*x^2])/(3*d^2) - (c^2*x^3*\text{Sqrt}[1 - d^2*x^2])/(4*d^2) + ((3*c^2 + 4*b^2*d^2 + 8*a*c*d^2 + 8*a^2*d^4)*\text{ArcSin}[d*x])/(8*d^5)$

**Rubi [A]** time = 0.535479, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{\sin^{-1}(dx) (8a^2d^4 + 8acd^2 + 4b^2d^2 + 3c^2)}{8d^5} - \frac{x\sqrt{1-d^2x^2} \left( c \left( 8a + \frac{3c}{d^2} \right) + 4b^2 \right)}{8d^2}$$

$$- \frac{2b\sqrt{1-d^2x^2} (3ad^2 + 2c)}{3d^4} - \frac{2bcx^2\sqrt{1-d^2x^2}}{3d^2} - \frac{c^2x^3\sqrt{1-d^2x^2}}{4d^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)^2/(Sqrt[1 - d\*x]\*Sqrt[1 + d\*x]), x]

[Out]  $(-2*b*(2*c + 3*a*d^2)*\text{Sqrt}[1 - d^2*x^2])/(3*d^4) - ((4*b^2 + c*(8*a + (3*c)/d^2))*x*\text{Sqrt}[1 - d^2*x^2])/(8*d^2) - (2*b*c*x^2*\text{Sqrt}[1 - d^2*x^2])/(3*d^2) - (c^2*x^3*\text{Sqrt}[1 - d^2*x^2])/(4*d^2) + ((3*c^2 + 4*b^2*d^2 + 8*a*c*d^2 + 8*a^2*d^4)*\text{ArcSin}[d*x])/(8*d^5)$

**Rubi in Sympy [A]** time = 44.7715, size = 128, normalized size = 0.77

$$\frac{(5b + 3cx)\sqrt{-d^2x^2 + 1}(a + bx + cx^2)}{12d^2}$$

$$- \frac{(2b(19ad^2 + 16c) + x(2b^2d^2 + 9c(2ad^2 + c)))\sqrt{-d^2x^2 + 1}}{24d^4}$$

$$+ \frac{(8a^2d^4 + 8acd^2 + 4b^2d^2 + 3c^2)\text{asin}(dx)}{8d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+b\*x+a)\*\*2/(-d\*x+1)\*\*(1/2)/(d\*x+1)\*\*(1/2), x)

[Out]  $-(5*b + 3*c*x)*\text{sqrt}(-d**2*x**2 + 1)*(a + b*x + c*x**2)/(12*d**2) - (2*b*(19*a*d**2 + 16*c) + x*(2*b**2*d**2 + 9*c*(2*a*d**2 + c)))*\text{sqrt}(-d**2*x**2 + 1)/(24*d**4) + (8*a**2*d**4 + 8*a*c*d**2 + 4*b**2*d**2 + 3*c**2)*\text{asin}(d*x)/(8*d**5)$

**Mathematica [A]** time = 0.171887, size = 114, normalized size = 0.69

$$\frac{3 \sin^{-1}(dx) (8a^2d^4 + 8acd^2 + 4b^2d^2 + 3c^2) - d\sqrt{1-d^2x^2} (16b(3ad^2 + cd^2x^2 + 2c) + 3cx(8ad^2 + 2cd^2x^2 + 3c) + 12b^2d^2x)}{24d^5}$$



Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)^2/(Sqrt[1 - d\*x]\*Sqrt[1 + d\*x]),x]

[Out]  $(-(d*\text{Sqrt}[1 - d^2*x^2])*(12*b^2*d^2*x + 16*b*(2*c + 3*a*d^2 + c*d^2*x^2) + 3*c*x*(3*c + 8*a*d^2 + 2*c*d^2*x^2))) + 3*(3*c^2 + 4*b^2*d^2 + 8*a*c*d^2 + 8*a^2*d^4)*\text{ArcSin}[d*x])/(24*d^5)$

**Maple [C]** time = 0.033, size = 291, normalized size = 1.8

$$-\frac{\text{csgn}(d)}{24d^5}\sqrt{-dx+1}\sqrt{dx+1}\left(6\text{csgn}(d)x^3c^2d^3\sqrt{-d^2x^2+1}+16\text{csgn}(d)x^2bcd^3\sqrt{-d^2x^2+1}+24acx\sqrt{-d^2x^2+1}d^3\text{csgn}(d)+\dots\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)^2/(-d\*x+1)^(1/2)/(d\*x+1)^(1/2),x)

[Out]  $-1/24*(-d*x+1)^{(1/2)}*(d*x+1)^{(1/2)}*(6*\text{csgn}(d)*x^3*c^2*d^3*(-d^2*x^2+1)^{(1/2)}+16*\text{csgn}(d)*x^2*b*c*d^3*(-d^2*x^2+1)^{(1/2)}+24*a*c*x*(-d^2*x^2+1)^{(1/2)}*d^3*\text{csgn}(d)+12*b^2*x*(-d^2*x^2+1)^{(1/2)}*d^3*\text{csgn}(d)+48*(-d^2*x^2+1)^{(1/2)}*a*b*d^3*\text{csgn}(d)-24*\arctan(\text{csgn}(d)*d*x/(-d^2*x^2+1)^{(1/2)})*a^2*d^4+9*c^2*x*(-d^2*x^2+1)^{(1/2)}*\text{csgn}(d)*d+3*2*(-d^2*x^2+1)^{(1/2)}*b*c*\text{csgn}(d)*d-24*a*c*\arctan(\text{csgn}(d)*d*x/(-d^2*x^2+1)^{(1/2)})*d^2-12*b^2*\arctan(\text{csgn}(d)*d*x/(-d^2*x^2+1)^{(1/2)})*d^2-9*c^2*\arctan(\text{csgn}(d)*d*x/(-d^2*x^2+1)^{(1/2)})*\text{csgn}(d)/(-d^2*x^2+1)^{(1/2)})/d^5$

**Maxima [A]** time = 0.779709, size = 275, normalized size = 1.66

$$-\frac{\sqrt{-d^2x^2+1}c^2x^3}{4d^2}-\frac{2\sqrt{-d^2x^2+1}bcx^2}{3d^2}+\frac{a^2\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{\sqrt{d^2}}-\frac{2\sqrt{-d^2x^2+1}ab}{d^2}-\frac{\sqrt{-d^2x^2+1}(b^2+2ac)x}{2d^2}-\frac{3\sqrt{-d^2x^2+1}c^2x}{8d^4}+\frac{(b^2+2ac)\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{2\sqrt{d^2}d^2}-\frac{4\sqrt{-d^2x^2+1}bc}{3d^4}+\frac{3c^2\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{8\sqrt{d^2}d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)^2/(sqrt(d\*x + 1)\*sqrt(-d\*x + 1)),x, algorithm="maxima")

[Out]  $-1/4*\text{sqrt}(-d^2*x^2 + 1)*c^2*x^3/d^2 - 2/3*\text{sqrt}(-d^2*x^2 + 1)*b*c*x^2/d^2 + a^2*\arcsin(d^2*x/\text{sqrt}(d^2))/\text{sqrt}(d^2) - 2*\text{sqrt}(-d^2*x^2 + 1)*a*b/d^2 - 1/2*\text{sqrt}(-d^2*x^2 + 1)*(b^2 + 2*a*c)*x/d^2 - 3/8*\text{sqrt}(-d^2*x^2 + 1)*c^2*x/d^4 + 1/2*(b^2 + 2*a*c)*\arcsin(d^2*x/\text{sqrt}(d^2))/(\text{sqrt}(d^2)*d^2) - 4/3*\text{sqrt}(-d^2*x^2 + 1)*b*c/d^4 + 3/8*c^2*\arcsin(d^2*x/\text{sqrt}(d^2))/(\text{sqrt}(d^2)*d^4)$

**Fricas [A]** time = 0.280622, size = 717, normalized size = 4.32

$$24c^2d^7x^7 + 64bcd^7x^6 - 192abd^5x^5 + 12(4(b^2 + 2ac)d^7 - 3c^2d^5)x^5 + 48(3abd^7 - 2bcd^5)x^4 - 12(12(b^2 + 2ac)d^5 + 5c^2d^3)x^3 + \dots$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)^2/(sqrt(d\*x + 1)\*sqrt(-d\*x + 1)),x, algorithm="fricas

[Out]  $\frac{1}{24} \cdot (24 \cdot c^2 \cdot d^7 \cdot x^7 + 64 \cdot b \cdot c \cdot d^7 \cdot x^6 - 192 \cdot a \cdot b \cdot d^5 \cdot x^2 + 12 \cdot (4 \cdot (b^2 + 2 \cdot a \cdot c) \cdot d^7 - 3 \cdot c^2 \cdot d^5) \cdot x^5 + 48 \cdot (3 \cdot a \cdot b \cdot d^7 - 2 \cdot b \cdot c \cdot d^5) \cdot x^4 - 12 \cdot (12 \cdot (b^2 + 2 \cdot a \cdot c) \cdot d^5 + 5 \cdot c^2 \cdot d^3) \cdot x^3 - (6 \cdot c^2 \cdot d^7 \cdot x^7 + 16 \cdot b \cdot c \cdot d^7 \cdot x^6 - 192 \cdot a \cdot b \cdot d^5 \cdot x^2 + 3 \cdot (4 \cdot (b^2 + 2 \cdot a \cdot c) \cdot d^7 - 13 \cdot c^2 \cdot d^5) \cdot x^5 + 48 \cdot (a \cdot b \cdot d^7 - 2 \cdot b \cdot c \cdot d^5) \cdot x^4 - 24 \cdot (4 \cdot (b^2 + 2 \cdot a \cdot c) \cdot d^5 + c^2 \cdot d^3) \cdot x^3 + 24 \cdot (4 \cdot (b^2 + 2 \cdot a \cdot c) \cdot d^3 + 3 \cdot c^2 \cdot d) \cdot x) \cdot \sqrt{d \cdot x + 1} \cdot \sqrt{-d \cdot x + 1} + 24 \cdot (4 \cdot (b^2 + 2 \cdot a \cdot c) \cdot d^3 + 3 \cdot c^2 \cdot d) \cdot x - 6 \cdot (64 \cdot a^2 \cdot d^4 + (8 \cdot a^2 \cdot d^8 + 4 \cdot (b^2 + 2 \cdot a \cdot c) \cdot d^6 + 3 \cdot c^2 \cdot d^4) \cdot x^4 + 32 \cdot (b^2 + 2 \cdot a \cdot c) \cdot d^2 - 8 \cdot (8 \cdot a^2 \cdot d^6 + 4 \cdot (b^2 + 2 \cdot a \cdot c) \cdot d^4 + 3 \cdot c^2 \cdot d^2) \cdot x^2 - 4 \cdot (16 \cdot a^2 \cdot d^4 + 8 \cdot (b^2 + 2 \cdot a \cdot c) \cdot d^2 - (8 \cdot a^2 \cdot d^6 + 4 \cdot (b^2 + 2 \cdot a \cdot c) \cdot d^4 + 3 \cdot c^2 \cdot d^2) \cdot x^2 + 6 \cdot c^2) \cdot \sqrt{d \cdot x + 1} \cdot \sqrt{-d \cdot x + 1} + 24 \cdot c^2) \cdot \arctan(\frac{\sqrt{d \cdot x + 1} \cdot \sqrt{-d \cdot x + 1} - 1}{d \cdot x}) / (d^9 \cdot x^4 - 8 \cdot d^7 \cdot x^2 + 8 \cdot d^5 + 4 \cdot (d^7 \cdot x^2 - 2 \cdot d^5) \cdot \sqrt{d \cdot x + 1} \cdot \sqrt{-d \cdot x + 1}))$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*\*2/(-d\*x+1)\*\*(1/2)/(d\*x+1)\*\*(1/2),x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.292303, size = 243, normalized size = 1.46

$(48 \cdot a \cdot b \cdot d^{19} - 12 \cdot b^2 \cdot d^{18} - 24 \cdot a \cdot c \cdot d^{18} + 48 \cdot b \cdot c \cdot d^{17} - 15 \cdot c^2 \cdot d^{16} + (12 \cdot b^2 \cdot d^{18} + 24 \cdot a \cdot c \cdot d^{18} - 32 \cdot b \cdot c \cdot d^{17} + 27 \cdot c^2 \cdot d^{16} + 2 \cdot (3 \cdot (d \cdot x + 1) \cdot c^2 \cdot d^{16} + 8$

34406

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)^2/(sqrt(d\*x + 1)\*sqrt(-d\*x + 1)),x, algorithm="giac")

[Out]  $-\frac{1}{344064} \cdot ((48 \cdot a \cdot b \cdot d^{19} - 12 \cdot b^2 \cdot d^{18} - 24 \cdot a \cdot c \cdot d^{18} + 48 \cdot b \cdot c \cdot d^{17} - 15 \cdot c^2 \cdot d^{16} + (12 \cdot b^2 \cdot d^{18} + 24 \cdot a \cdot c \cdot d^{18} - 32 \cdot b \cdot c \cdot d^{17} + 27 \cdot c^2 \cdot d^{16} + 2 \cdot (3 \cdot (d \cdot x + 1) \cdot c^2 \cdot d^{16} + 8 \cdot b \cdot c \cdot d^{17} - 9 \cdot c^2 \cdot d^{16}) \cdot (d \cdot x + 1)) \cdot (d \cdot x + 1)) \cdot \sqrt{d \cdot x + 1} \cdot \sqrt{-d \cdot x + 1} - 6 \cdot (8 \cdot a^2 \cdot d^{20} + 4 \cdot b^2 \cdot d^{18} + 8 \cdot a \cdot c \cdot d^{18} + 3 \cdot c^2 \cdot d^{16}) \cdot \arcsin(\frac{1}{2} \cdot \sqrt{2} \cdot \sqrt{d \cdot x + 1})) / d$

$$3.794 \quad \int \frac{a+bx+cx^2}{\sqrt{1-dx}\sqrt{1+dx}} dx$$

**Optimal.** Leaf size=63

$$\frac{(2ad^2 + c) \sin^{-1}(dx)}{2d^3} - \frac{b\sqrt{1-d^2x^2}}{d^2} - \frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

[Out]  $-\left(\frac{b\sqrt{1-d^2x^2}}{d^2}\right) - \frac{(c+2ad^2)\text{ArcSin}[dx]}{2d^3} + \frac{cx\sqrt{1-d^2x^2}}{2d^2}$

**Rubi [A]** time = 0.13165, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{(2ad^2 + c) \sin^{-1}(dx)}{2d^3} - \frac{b\sqrt{1-d^2x^2}}{d^2} - \frac{cx\sqrt{1-d^2x^2}}{2d^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)/(Sqrt[1 - d\*x]\*Sqrt[1 + d\*x]), x]

[Out]  $-\left(\frac{b\sqrt{1-d^2x^2}}{d^2}\right) - \frac{(c+2ad^2)\text{ArcSin}[dx]}{2d^3} + \frac{cx\sqrt{1-d^2x^2}}{2d^2}$

**Rubi in Sympy [A]** time = 16.3064, size = 41, normalized size = 0.65

$$-\frac{(2b+cx)\sqrt{-d^2x^2+1}}{2d^2} + \frac{(2ad^2+c)\text{asin}(dx)}{2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+b\*x+a)/(-d\*x+1)\*\*(1/2)/(d\*x+1)\*\*(1/2), x)

[Out]  $-\frac{(2b+cx)\sqrt{-d^2x^2+1}}{2d^2} + \frac{(2ad^2+c)\text{asin}(dx)}{2d^3}$

**Mathematica [A]** time = 0.0606096, size = 45, normalized size = 0.71

$$\frac{(2ad^2 + c) \sin^{-1}(dx) - d\sqrt{1-d^2x^2}(2b+cx)}{2d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)/(Sqrt[1 - d\*x]\*Sqrt[1 + d\*x]), x]

[Out]  $\frac{-(d(2b+cx)\sqrt{1-d^2x^2}) + (c+2ad^2)\text{ArcSin}[dx]}{2d^3}$

**Maple [C]** time = 0.023, size = 117, normalized size = 1.9

$$-\frac{c\text{sgn}(d)}{2d^3}\sqrt{-dx+1}\sqrt{dx+1}\left(cx\sqrt{-d^2x^2+1}c\text{sgn}(d)d+2\sqrt{-d^2x^2+1}bc\text{sgn}(d)d-2\arctan\left(\frac{c\text{sgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)\right)ad^2-c\arctan\left(\frac{c\text{sgn}(d)dx}{\sqrt{-d^2x^2+1}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)`

[Out]  $-1/2*(-d*x+1)^{(1/2)}*(d*x+1)^{(1/2)}*(c*x*(-d^2*x^2+1)^{(1/2)}*\text{csgn}(d) + d+2*(-d^2*x^2+1)^{(1/2)}*b*\text{csgn}(d)*d-2*\arctan(\text{csgn}(d)*d*x/(-d^2*x^2+1)^{(1/2)})*a*d^2-c*\arctan(\text{csgn}(d)*d*x/(-d^2*x^2+1)^{(1/2)})*\text{csgn}(d)/(-d^2*x^2+1)^{(1/2)}/d^3$

**Maxima [A]** time = 0.767701, size = 105, normalized size = 1.67

$$\frac{a \arcsin\left(\frac{d^2 x}{\sqrt{d^2}}\right)}{\sqrt{d^2}} - \frac{\sqrt{-d^2 x^2 + 1} c x}{2 d^2} - \frac{\sqrt{-d^2 x^2 + 1} b}{d^2} + \frac{c \arcsin\left(\frac{d^2 x}{\sqrt{d^2}}\right)}{2 \sqrt{d^2} d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)/(sqrt(d*x + 1)*sqrt(-d*x + 1)),x, algorithm="maxima")`

[Out]  $a*\arcsin(d^2*x/\sqrt{d^2})/\sqrt{d^2} - 1/2*\sqrt{-d^2*x^2 + 1}*c*x/d^2 - \sqrt{-d^2*x^2 + 1}*b/d^2 + 1/2*c*\arcsin(d^2*x/\sqrt{d^2})/(\sqrt{d^2}*d^2)$

**Fricas [A]** time = 0.272735, size = 244, normalized size = 3.87

$$\frac{2 c d^3 x^3 + 2 b d^3 x^2 - 2 c d x - (c d^3 x^3 + 2 b d^3 x^2 - 2 c d x) \sqrt{d x + 1} \sqrt{-d x + 1} + 2 \left(4 a d^2 - (2 a d^4 + c d^2) x^2 - 2 (2 a d^2 + c) \sqrt{d x + 1}\right)}{2 \left(d^5 x^2 + 2 \sqrt{d x + 1} \sqrt{-d x + 1} d^3 - 2 d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)/(sqrt(d*x + 1)*sqrt(-d*x + 1)),x, algorithm="fricas")`

[Out]  $1/2*(2*c*d^3*x^3 + 2*b*d^3*x^2 - 2*c*d*x - (c*d^3*x^3 + 2*b*d^3*x^2 - 2*c*d*x)*\sqrt{d*x + 1}*\sqrt{-d*x + 1} + 2*(4*a*d^2 - (2*a*d^4 + c*d^2)*x^2 - 2*(2*a*d^2 + c)*\arctan((\sqrt{d*x + 1}*\sqrt{-d*x + 1} - 1)/(d*x)))/(d^5*x^2 + 2*\sqrt{d*x + 1}*\sqrt{-d*x + 1}*d^3 - 2*d^3)$

**Sympy [A]** time = 55.1506, size = 282, normalized size = 4.48

$$\begin{aligned} & \frac{iaG_{6,6}^{6,2} \left( \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 0 \end{matrix} \middle| \frac{1}{d^2 x^2} \right) + aG_{6,6}^{2,6} \left( \begin{matrix} -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 1 \\ -\frac{1}{4}, \frac{1}{4} \end{matrix} \middle| \frac{e^{-2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d} \\ & + \frac{ibG_{6,6}^{6,2} \left( \begin{matrix} -\frac{1}{4}, \frac{1}{4} \\ -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, 0 \end{matrix} \middle| \frac{1}{d^2 x^2} \right) + bG_{6,6}^{2,6} \left( \begin{matrix} -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 1 \\ -\frac{3}{4}, -\frac{1}{4} \end{matrix} \middle| \frac{e^{-2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^2} \\ & + \frac{icG_{6,6}^{6,2} \left( \begin{matrix} -\frac{3}{4}, -\frac{1}{4} \\ -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, 0 \end{matrix} \middle| \frac{1}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^3} \\ & + \frac{cG_{6,6}^{2,6} \left( \begin{matrix} -\frac{3}{2}, -\frac{5}{4}, -1, -\frac{3}{4}, -\frac{1}{2}, 1 \\ -\frac{5}{4}, -\frac{3}{4} \end{matrix} \middle| \frac{e^{-2i\pi}}{d^2 x^2} \right)}{4\pi^{\frac{3}{2}} d^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)/(-d\*x+1)\*\*(1/2)/(d\*x+1)\*\*(1/2),x)

[Out] -I\*a\*meijerg(((1/4, 3/4), (1/2, 1/2, 1, 1)), ((0, 1/4, 1/2, 3/4, 1, 0), ()), 1/(d\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*d) + a\*meijerg(((1/2, -1/4, 0, 1/4, 1/2, 1), ()), ((-1/4, 1/4), (-1/2, 0, 0, 0)), exp\_polar(-2\*I\*pi)/(d\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*d) - I\*b\*meijerg(((1/4, 1/4), (0, 0, 1/2, 1)), ((-1/2, -1/4, 0, 1/4, 1/2, 0), ()), 1/(d\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*d\*\*2) - b\*meijerg(((1, -3/4, -1/2, -1/4, 0, 1), ()), ((-3/4, -1/4), (-1, -1/2, -1/2, 0)), exp\_polar(-2\*I\*pi)/(d\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*d\*\*2) - I\*c\*meijerg(((3/4, -1/4), (-1/2, -1/2, 0, 1)), ((-1, -3/4, -1/2, -1/4, 0, 0), ()), 1/(d\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*d\*\*3) + c\*meijerg(((3/2, -5/4, -1, -3/4, -1/2, 1), ()), ((-5/4, -3/4), (-3/2, -1, -1, 0)), exp\_polar(-2\*I\*pi)/(d\*\*2\*x\*\*2))/(4\*pi\*\*(3/2)\*d\*\*3)

**GIAC/XCAS [A]** time = 0.285598, size = 97, normalized size = 1.54

$$\frac{((dx + 1)cd^4 + 2bd^5 - cd^4)\sqrt{dx + 1}\sqrt{-dx + 1} - 2(2ad^6 + cd^4)\arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx + 1}\right)}{192d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/(sqrt(d\*x + 1)\*sqrt(-d\*x + 1)),x, algorithm="giac")

[Out] -1/192\*(((d\*x + 1)\*c\*d^4 + 2\*b\*d^5 - c\*d^4)\*sqrt(d\*x + 1)\*sqrt(-d\*x + 1) - 2\*(2\*a\*d^6 + c\*d^4)\*arcsin(1/2\*sqrt(2)\*sqrt(d\*x + 1)))/d

$$3.795 \quad \int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}(a+bx+cx^2)} dx$$

**Optimal.** Leaf size=282

$$\frac{\sqrt{2c} \tanh^{-1}\left(\frac{d^2x(\sqrt{b^2-4ac}+b)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2(\sqrt{b^2-4ac}+b)+2acd^2+2c^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-bd^2(\sqrt{b^2-4ac}+b)+2acd^2+2c^2}} - \frac{\sqrt{2c} \tanh^{-1}\left(\frac{d^2x(b-\sqrt{b^2-4ac})+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2(b-\sqrt{b^2-4ac})+2acd^2+2c^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-bd^2(b-\sqrt{b^2-4ac})+2acd^2+2c^2}}$$

[Out]  $-\left(\frac{\text{Sqrt}[2]*c*\text{ArcTanh}[(2*c+(b-\text{Sqrt}[b^2-4*a*c]))*d^2*x]}{\text{Sqrt}[2]*\text{Sqrt}[2*c^2+2*a*c*d^2-b*(b-\text{Sqrt}[b^2-4*a*c])*d^2]*\text{Sqrt}[1-d^2*x^2]}\right)/\left(\text{Sqrt}[b^2-4*a*c]*\text{Sqrt}[2*c^2+2*a*c*d^2-b*(b-\text{Sqrt}[b^2-4*a*c])*d^2]\right) + \left(\frac{\text{Sqrt}[2]*c*\text{ArcTanh}[(2*c+(b+\text{Sqrt}[b^2-4*a*c]))*d^2*x]}{\text{Sqrt}[2]*\text{Sqrt}[2*c^2+2*a*c*d^2-b*(b+\text{Sqrt}[b^2-4*a*c])*d^2]*\text{Sqrt}[1-d^2*x^2]}\right)/\left(\text{Sqrt}[b^2-4*a*c]*\text{Sqrt}[2*c^2+2*a*c*d^2-b*(b+\text{Sqrt}[b^2-4*a*c])*d^2]\right)$

**Rubi [A]** time = 0.993301, antiderivative size = 282, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$

$$\frac{\sqrt{2c} \tanh^{-1}\left(\frac{d^2x(\sqrt{b^2-4ac}+b)+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2(\sqrt{b^2-4ac}+b)+2acd^2+2c^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-bd^2(\sqrt{b^2-4ac}+b)+2acd^2+2c^2}} - \frac{\sqrt{2c} \tanh^{-1}\left(\frac{d^2x(b-\sqrt{b^2-4ac})+2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2(b-\sqrt{b^2-4ac})+2acd^2+2c^2}}\right)}{\sqrt{b^2-4ac}\sqrt{-bd^2(b-\sqrt{b^2-4ac})+2acd^2+2c^2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[1/(\text{Sqrt}[1-d*x]*\text{Sqrt}[1+d*x]*(a+b*x+c*x^2)),x]$

[Out]  $-\left(\frac{\text{Sqrt}[2]*c*\text{ArcTanh}[(2*c+(b-\text{Sqrt}[b^2-4*a*c]))*d^2*x]}{\text{Sqrt}[2]*\text{Sqrt}[2*c^2+2*a*c*d^2-b*(b-\text{Sqrt}[b^2-4*a*c])*d^2]*\text{Sqrt}[1-d^2*x^2]}\right)/\left(\text{Sqrt}[b^2-4*a*c]*\text{Sqrt}[2*c^2+2*a*c*d^2-b*(b-\text{Sqrt}[b^2-4*a*c])*d^2]\right) + \left(\frac{\text{Sqrt}[2]*c*\text{ArcTanh}[(2*c+(b+\text{Sqrt}[b^2-4*a*c]))*d^2*x]}{\text{Sqrt}[2]*\text{Sqrt}[2*c^2+2*a*c*d^2-b*(b+\text{Sqrt}[b^2-4*a*c])*d^2]*\text{Sqrt}[1-d^2*x^2]}\right)/\left(\text{Sqrt}[b^2-4*a*c]*\text{Sqrt}[2*c^2+2*a*c*d^2-b*(b+\text{Sqrt}[b^2-4*a*c])*d^2]\right)$

**Rubi in Sympy [A]** time = 78.9749, size = 284, normalized size = 1.01

$$\frac{\sqrt{2c} \operatorname{atanh}\left(\frac{\sqrt{2}(2c+d^2x(b-\sqrt{-4ac+b^2}))}{2\sqrt{-d^2x^2+1}\sqrt{2acd^2-b^2d^2+bd^2\sqrt{-4ac+b^2}+2c^2}}\right)}{\sqrt{-4ac+b^2}\sqrt{2acd^2-b^2d^2+bd^2\sqrt{-4ac+b^2}+2c^2}} + \frac{\sqrt{2c} \operatorname{atanh}\left(\frac{\sqrt{2}(2c+d^2x(b+\sqrt{-4ac+b^2}))}{2\sqrt{-d^2x^2+1}\sqrt{2acd^2-b^2d^2-bd^2\sqrt{-4ac+b^2}+2c^2}}\right)}{\sqrt{-4ac+b^2}\sqrt{2acd^2-b^2d^2-bd^2\sqrt{-4ac+b^2}+2c^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}(1/(c*x^2+b*x+a)/(-d*x+1)^{(1/2)}/(d*x+1)^{(1/2)},x)$

[Out]  $-\text{sqrt}(2)*c*\operatorname{atanh}(\text{sqrt}(2)*(2*c+d^2*x*(b-\text{sqrt}(-4*a*c+b^2))))/(2*\text{sqrt}(-d^2*x^2+1)*\text{sqrt}(2*a*c*d^2-b^2*d^2+b*d^2*\text{sqrt}(-4*a*c+b^2)+2*c^2))/(\text{sqrt}(-4*a*c+b^2)*\text{sqrt}(2*a*c*d^2-b^2*d^2+b*d^2*\text{sqrt}(-4*a*c+b^2)+2*c^2))+\text{sqrt}(2)*c*\operatorname{atanh}(\text{sqrt}(2)*(2*c+d^2*x*(b+\text{sqrt}(-4*a*c+b^2))))/(2*\text{sqrt}(-d^2*x^2+1)*\text{sqrt}(2*a*c*d^2-b^2*d^2-b*d^2*\text{sqrt}(-4*a*c+b^2)+2*c^2))/(\text{sqrt}(-4*a*c+b^2)*\text{sqrt}(2*a*c*d^2-b^2*d^2-b*d^2*\text{sqrt}(-4*a*c+b^2)+2*c^2))$

**Mathematica [A]** time = 1.15617, size = 455, normalized size = 1.61

$$\sqrt{2}c \left( -\sqrt{-bd^2 \left( \sqrt{b^2 - 4ac} + b \right) + 2acd^2 + 2c^2 \log \left( -\sqrt{1 - d^2x^2} \sqrt{2bd^2 \left( \sqrt{b^2 - 4ac} - b \right) + 4acd^2 + 4c^2 + d^2x\sqrt{b^2 - 4ac} - b} \right)} \right)$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[1 - d\*x]\*Sqrt[1 + d\*x]\*(a + b\*x + c\*x^2)),x]

[Out] (Sqrt[2]\*c\*(Sqrt[2\*c^2 + 2\*a\*c\*d^2 - b\*(b + Sqrt[b^2 - 4\*a\*c])\*d^2]\*Log[-b + Sqrt[b^2 - 4\*a\*c] - 2\*c\*x] - Sqrt[2\*c^2 + 2\*a\*c\*d^2 + b\*(-b + Sqrt[b^2 - 4\*a\*c])\*d^2]\*Log[b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x] - Sqrt[2\*c^2 + 2\*a\*c\*d^2 - b\*(b + Sqrt[b^2 - 4\*a\*c])\*d^2]\*Log[-2\*c - b\*d^2\*x + Sqrt[b^2 - 4\*a\*c]\*d^2\*x - Sqrt[4\*c^2 + 4\*a\*c\*d^2 + 2\*b\*(-b + Sqrt[b^2 - 4\*a\*c])\*d^2]\*Sqrt[1 - d^2\*x^2]] + Sqrt[2\*c^2 + 2\*a\*c\*d^2 + b\*(-b + Sqrt[b^2 - 4\*a\*c])\*d^2]\*Log[2\*c + b\*d^2\*x + Sqrt[b^2 - 4\*a\*c]\*d^2\*x + Sqrt[4\*c^2 + 4\*a\*c\*d^2 - 2\*b\*(b + Sqrt[b^2 - 4\*a\*c])\*d^2]\*Sqrt[1 - d^2\*x^2]]))/(Sqrt[b^2 - 4\*a\*c]\*Sqrt[2\*c^2 + 2\*a\*c\*d^2 + b\*(-b + Sqrt[b^2 - 4\*a\*c])\*d^2]\*Sqrt[2\*c^2 + 2\*a\*c\*d^2 - b\*(b + Sqrt[b^2 - 4\*a\*c])\*d^2])

**Maple [C]** time = 0.14, size = 1759, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(c\*x^2+b\*x+a)/(-d\*x+1)^(1/2)/(d\*x+1)^(1/2),x)

[Out]  $32 \cdot (-d \cdot x + 1)^{1/2} \cdot (d \cdot x + 1)^{1/2} \cdot \text{csgn}(d)^2 \cdot c^2 \cdot (\ln(2 \cdot ((-4 \cdot a \cdot c + b^2)^{1/2} \cdot x \cdot d^2 + x \cdot b \cdot d^2 + (-d^2 \cdot x^2 + 1)^{1/2}) \cdot (-b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} - 2 \cdot a \cdot c + b^2) \cdot (2 \cdot a^2 \cdot d^2 + b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} + 2 \cdot a \cdot c - b^2) / a^2 / c^2)^{1/2} \cdot c + 2 \cdot c) / (b + 2 \cdot c \cdot x + (-4 \cdot a \cdot c + b^2)^{1/2})) \cdot a^2 \cdot d^4 \cdot (-b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} + 2 \cdot a \cdot c - b^2) \cdot (-2 \cdot a^2 \cdot d^2 + b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} - 2 \cdot a \cdot c + b^2) / a^2 / c^2)^{1/2} - \ln(2 \cdot ((-4 \cdot a \cdot c + b^2)^{1/2} \cdot x \cdot d^2 + x \cdot b \cdot d^2 + (-d^2 \cdot x^2 + 1)^{1/2}) \cdot (-b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} + 2 \cdot a \cdot c - b^2) \cdot (-2 \cdot a^2 \cdot d^2 + b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} - 2 \cdot a \cdot c + b^2) / a^2 / c^2)^{1/2} \cdot c + 2 \cdot c) / (2 \cdot c \cdot x - (-4 \cdot a \cdot c + b^2)^{1/2} + b) \cdot a^2 \cdot d^4 \cdot (-b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} - 2 \cdot a \cdot c + b^2) \cdot (2 \cdot a^2 \cdot d^2 + b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} + 2 \cdot a \cdot c - b^2) / a^2 / c^2)^{1/2} + 2 \cdot \ln(2 \cdot ((-4 \cdot a \cdot c + b^2)^{1/2} \cdot x \cdot d^2 + x \cdot b \cdot d^2 + (-d^2 \cdot x^2 + 1)^{1/2}) \cdot (-b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} - 2 \cdot a \cdot c + b^2) \cdot (2 \cdot a^2 \cdot d^2 + b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} + 2 \cdot a \cdot c - b^2) / a^2 / c^2)^{1/2} \cdot c + 2 \cdot c) / (b + 2 \cdot c \cdot x + (-4 \cdot a \cdot c + b^2)^{1/2})) \cdot a \cdot c \cdot d^2 \cdot (-b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} + 2 \cdot a \cdot c - b^2) \cdot (-2 \cdot a^2 \cdot d^2 + b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} - 2 \cdot a \cdot c + b^2) / a^2 / c^2)^{1/2} - \ln(2 \cdot ((-4 \cdot a \cdot c + b^2)^{1/2} \cdot x \cdot d^2 + x \cdot b \cdot d^2 + (-d^2 \cdot x^2 + 1)^{1/2}) \cdot (-b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} - 2 \cdot a \cdot c + b^2) \cdot (2 \cdot a^2 \cdot d^2 + b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} + 2 \cdot a \cdot c - b^2) / a^2 / c^2)^{1/2} \cdot c + 2 \cdot c) / (2 \cdot c \cdot x - (-4 \cdot a \cdot c + b^2)^{1/2} + b) \cdot a \cdot c \cdot d^2 \cdot (-b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} - 2 \cdot a \cdot c + b^2) \cdot (2 \cdot a^2 \cdot d^2 + b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} + 2 \cdot a \cdot c - b^2) / a^2 / c^2)^{1/2} + \ln(2 \cdot ((-4 \cdot a \cdot c + b^2)^{1/2} \cdot x \cdot d^2 + x \cdot b \cdot d^2 + (-d^2 \cdot x^2 + 1)^{1/2}) \cdot (-b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} + 2 \cdot a \cdot c - b^2) \cdot (-2 \cdot a^2 \cdot d^2 + b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} - 2 \cdot a \cdot c + b^2) / a^2 / c^2)^{1/2} \cdot c + 2 \cdot c) / (b + 2 \cdot c \cdot x + (-4 \cdot a \cdot c + b^2)^{1/2})) \cdot b^2 \cdot d^2 \cdot (-b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} + 2 \cdot a \cdot c - b^2) \cdot (-2 \cdot a^2 \cdot d^2 + b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} - 2 \cdot a \cdot c + b^2) / a^2 / c^2)^{1/2} - 2 \cdot \ln(2 \cdot ((-4 \cdot a \cdot c + b^2)^{1/2} \cdot x \cdot d^2 + x \cdot b \cdot d^2 + (-d^2 \cdot x^2 + 1)^{1/2}) \cdot (-b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} + 2 \cdot a \cdot c - b^2) \cdot (-2 \cdot a^2 \cdot d^2 + b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} - 2 \cdot a \cdot c + b^2) / a^2 / c^2)^{1/2} \cdot c + 2 \cdot c) / (2 \cdot c \cdot x - (-4 \cdot a \cdot c + b^2)^{1/2} + b) \cdot a \cdot c \cdot d^2 \cdot (-b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} - 2 \cdot a \cdot c + b^2) \cdot (2 \cdot a^2 \cdot d^2 + b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} + 2 \cdot a \cdot c - b^2) / a^2 / c^2)^{1/2} + \ln(2 \cdot ((-4 \cdot a \cdot c + b^2)^{1/2} \cdot x \cdot d^2 + x \cdot b \cdot d^2 + (-d^2 \cdot x^2 + 1)^{1/2}) \cdot (-b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} - 2 \cdot a \cdot c + b^2) \cdot (-2 \cdot a^2 \cdot d^2 + b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} + 2 \cdot a \cdot c - b^2) / a^2 / c^2)^{1/2} \cdot c + 2 \cdot c) / (b + 2 \cdot c \cdot x + (-4 \cdot a \cdot c + b^2)^{1/2})) \cdot c^2 \cdot (-b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} + 2 \cdot a \cdot c - b^2) \cdot (-2 \cdot a^2 \cdot d^2 + b \cdot (-4 \cdot a \cdot c + b^2)^{1/2} - 2 \cdot a \cdot c + b^2) / a^2 / c^2)^{1/2} -$

$$\ln(2^* (-(-4^* a^* c + b^2)^{(1/2)} * x^2 d^2 + x^2 b^* d^2 + (-d^2 * x^2 + 1)^{(1/2)} * (- (b^* (-4^* a^* c + b^2)^{(1/2)} + 2^* a^* c - b^2) * (-2^* a^2 * d^2 + b^* (-4^* a^* c + b^2)^{(1/2)} - 2^* a^* c + b^2) / a^2 / c^2)^{(1/2)} * c + 2^* c) / (2^* c * x - (-4^* a^* c + b^2)^{(1/2)} + b)) * c^2 * (- (b^* (-4^* a^* c + b^2)^{(1/2)} - 2^* a^* c + b^2) * (2^* a^2 * d^2 + b^* (-4^* a^* c + b^2)^{(1/2)} + 2^* a^* c - b^2) / a^2 / c^2)^{(1/2)} / (-d^2 * x^2 + 1)^{(1/2)} / (b^* d - d^* (-4^* a^* c + b^2)^{(1/2)} + 2^* c) / (d^* (-4^* a^* c + b^2)^{(1/2)} + b^* d + 2^* c) / (b^* d - d^* (-4^* a^* c + b^2)^{(1/2)} - 2^* c) / (d^* (-4^* a^* c + b^2)^{(1/2)} + b^* d - 2^* c) / (-4^* a^* c + b^2)^{(1/2)} / (- (b^* (-4^* a^* c + b^2)^{(1/2)} + 2^* a^* c - b^2) * (-2^* a^2 * d^2 + b^* (-4^* a^* c + b^2)^{(1/2)} - 2^* a^* c + b^2) / a^2 / c^2)^{(1/2)} / (- (b^* (-4^* a^* c + b^2)^{(1/2)} - 2^* a^* c + b^2) * (2^* a^2 * d^2 + b^* (-4^* a^* c + b^2)^{(1/2)} + 2^* a^* c - b^2) / a^2 / c^2)^{(1/2)})$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx + a)\sqrt{dx + 1}\sqrt{-dx + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + b\*x + a)\*sqrt(d\*x + 1)\*sqrt(-d\*x + 1)),x, algorithm="maxima")

[Out] integrate(1/((c\*x^2 + b\*x + a)\*sqrt(d\*x + 1)\*sqrt(-d\*x + 1)), x)

**Fricas [A]** time = 0.424823, size = 5823, normalized size = 20.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + b\*x + a)\*sqrt(d\*x + 1)\*sqrt(-d\*x + 1)),x, algorithm="fricas")

[Out] 1/2\*sqrt(2)\*sqrt(-((b^2 - 2\*a\*c)\*d^2 - 2\*c^2 - ((a^2\*b^2 - 4\*a^3\*c)\*d^4 + b^2\*c^2 - 4\*a\*c^3 - (b^4 - 6\*a\*b^2\*c + 8\*a^2\*c^2)\*d^2))\*sqrt(b^2\*d^4/((a^4\*b^2 - 4\*a^5\*c)\*d^8 - 2\*(a^2\*b^4 - 6\*a^3\*b^2\*c + 8\*a^4\*c^2)\*d^6 + b^2\*c^4 - 4\*a\*c^5 + (b^6 - 8\*a\*b^4\*c + 22\*a^2\*b^2\*c^2 - 24\*a^3\*c^3)\*d^4 - 2\*(b^4\*c^2 - 6\*a\*b^2\*c^3 + 8\*a^2\*c^4)\*d^2)))/((a^2\*b^2 - 4\*a^3\*c)\*d^4 + b^2\*c^2 - 4\*a\*c^3 - (b^4 - 6\*a\*b^2\*c + 8\*a^2\*c^2)\*d^2))\*log((4\*sqrt(d\*x + 1)\*sqrt(-d\*x + 1)\*a\*b\*c\*d^2 - 2\*b^2\*c\*d^2\*x - 4\*a\*b\*c\*d^2 + 2\*(b^2\*c^3 - 4\*a\*c^4 + (a^2\*b^2\*c - 4\*a^3\*c^2)\*d^4 - (b^4\*c - 6\*a\*b^2\*c^2 + 8\*a^2\*c^3)\*d^2))\*sqrt(b^2\*d^4/((a^4\*b^2 - 4\*a^5\*c)\*d^8 - 2\*(a^2\*b^4 - 6\*a^3\*b^2\*c + 8\*a^4\*c^2)\*d^6 + b^2\*c^4 - 4\*a\*c^5 + (b^6 - 8\*a\*b^4\*c + 22\*a^2\*b^2\*c^2 - 24\*a^3\*c^3)\*d^4 - 2\*(b^4\*c^2 - 6\*a\*b^2\*c^3 + 8\*a^2\*c^4)\*d^2))\*x + sqrt(2)\*((a^3\*b^3 - 4\*a^4\*b\*c)\*d^6 - b^3\*c^3 + 4\*a\*b\*c^4 - (a\*b^5 - 5\*a^2\*b^3\*c + 4\*a^3\*b\*c^2)\*d^4 + (b^5\*c - 5\*a\*b^3\*c^2 + 4\*a^2\*b\*c^3)\*d^2))\*sqrt(b^2\*d^4/((a^4\*b^2 - 4\*a^5\*c)\*d^8 - 2\*(a^2\*b^4 - 6\*a^3\*b^2\*c + 8\*a^4\*c^2)\*d^6 + b^2\*c^4 - 4\*a\*c^5 + (b^6 - 8\*a\*b^4\*c + 22\*a^2\*b^2\*c^2 - 24\*a^3\*c^3)\*d^4 - 2\*(b^4\*c^2 - 6\*a\*b^2\*c^3 + 8\*a^2\*c^4)\*d^2))\*x + ((a\*b^3 - 4\*a^2\*b\*c)\*d^4 + (b^3\*c - 4\*a\*b\*c^2)\*d^2))\*x)\*sqrt(-((b^2 - 2\*a\*c)\*d^2 - 2\*c^2 - ((a^2\*b^2 - 4\*a^3\*c)\*d^4 + b^2\*c^2 - 4\*a\*c^3 - (b^4 - 6\*a\*b^2\*c + 8\*a^2\*c^2)\*d^2))\*sqrt(b^2\*d^4/((a^4\*b^2 - 4\*a^5\*c)\*d^8 - 2\*(a^2\*b^4 - 6\*a^3\*b^2\*c + 8\*a^4\*c^2)\*d^6 + b^2\*c^4 - 4\*a\*c^5 + (b^6 - 8\*a\*b^4\*c + 22\*a^2\*b^2\*c^2 - 24\*a^3\*c^3)\*d^4 - 2\*(b^4\*c^2 - 6\*a\*b^2\*c^3 + 8\*a^2\*c^4)\*d^2)))/((a^2\*b^2 - 4\*a^3\*c)\*d^4 + b^2\*c^2 - 4\*a\*c^3 - (b^4 - 6\*a\*b^2\*c + 8\*a^2\*c^2)\*d^2))/x) - 1/2\*sqrt(2)\*sqrt(-((b^2 - 2\*a\*c)\*d^2 - 2\*c^2 - ((a^2\*b^2 - 4\*a^3\*c)\*d^4 + b^2\*c^2 - 4\*a\*c^3 - (b^4 - 6\*a\*b^2\*c + 8\*a^2\*c^2)\*d^2))\*sqrt(b^2\*d^4/((a^4\*b^2 - 4\*a^5\*c)\*d^8 - 2\*(a^2\*b^4 - 6\*a^3\*b^2\*c + 8\*a^4\*c^2)\*d^6 + b^2\*c^4 - 4\*a\*c^5 + (b^6 - 8\*a\*b^4\*c + 22\*a^2\*b^2\*c^2 - 24\*a^3\*c^3)\*d^4 - 2\*(b^4\*c^2 - 6\*a\*b^2\*c^3 + 8\*a^2\*c^4)\*d^2)))/((a^2\*b^2 - 4\*a^3\*c)\*d^4 + b^2\*c^2 - 4\*a\*c^3 - (b^4 - 6\*a\*b^2\*c + 8\*a^2\*c^2)\*d^2))\*log((4\*sqrt(d\*x + 1)\*sqrt(-d\*x + 1)\*a\*b\*c\*d^2 - 2\*b^2\*c\*d^2\*x - 4\*a\*b\*c\*d^2 + 2\*(b^2\*c^3 - 4\*a\*c^4 + (a^2\*b^2\*c - 4\*a^3\*c^2)\*d^4 - (b^4\*c - 6\*a\*b^2\*c^2 + 8\*a^2\*c^3)\*d^2))\*d



$$\begin{aligned}
&^4 - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d^2)*\sqrt{b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)}*x - \sqrt{2}*((a^3*b^3 - 4*a^4*b*c)*d^6 - b^3*c^3 + 4*a*b*c^4 - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^4 + (b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^2)*\sqrt{b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)}*x + ((a*b^3 - 4*a^2*b*c)*d^4 + (b^3*c - 4*a*b*c^2)*d^2)*x*\sqrt{-((b^2 - 2*a*c)*d^2 - 2*c^2 - ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*\sqrt{b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)}}/(a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2))/x) - 1/2*\sqrt{2}*\sqrt{-((b^2 - 2*a*c)*d^2 - 2*c^2 + ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*\sqrt{b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)}}/(a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2))*\log((4*\sqrt{d*x + 1})*\sqrt{-d*x + 1})*a*b*c*d^2 - 2*b^2*c*d^2*x - 4*a*b*c*d^2 - 2*(b^2*c^3 - 4*a*c^4 + (a^2*b^2*c - 4*a^3*c^2)*d^4 - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d^2)*\sqrt{b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)}*x + \sqrt{2}*(((a^3*b^3 - 4*a^4*b*c)*d^6 - b^3*c^3 + 4*a*b*c^4 - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^4 + (b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^2)*\sqrt{b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)}*x - ((a*b^3 - 4*a^2*b*c)*d^4 + (b^3*c - 4*a*b*c^2)*d^2)*x)*\sqrt{-((b^2 - 2*a*c)*d^2 - 2*c^2 + ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*\sqrt{b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)}}/(a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2))/x) + 1/2*\sqrt{2}*\sqrt{-((b^2 - 2*a*c)*d^2 - 2*c^2 + ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*\sqrt{b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)}}/(a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2))*\log((4*\sqrt{d*x + 1})*\sqrt{-d*x + 1})*a*b*c*d^2 - 2*b^2*c*d^2*x - 4*a*b*c*d^2 - 2*(b^2*c^3 - 4*a*c^4 + (a^2*b^2*c - 4*a^3*c^2)*d^4 - (b^4*c - 6*a*b^2*c^2 + 8*a^2*c^3)*d^2)*\sqrt{b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)}*x - \sqrt{2}*(((a^3*b^3 - 4*a^4*b*c)*d^6 - b^3*c^3 + 4*a*b*c^4 - (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^4 + (b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d^2)*\sqrt{b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)}*x - ((a*b^3 - 4*a^2*b*c)*d^4 + (b^3*c - 4*a*b*c^2)*d^2)*x)*\sqrt{-((b^2 - 2*a*c)*d^2 - 2*c^2 + ((a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2)*\sqrt{b^2*d^4/((a^4*b^2 - 4*a^5*c)*d^8 - 2*(a^2*b^4 - 6*a^3*b^2*c + 8*a^4*c^2)*d^6 + b^2*c^4 - 4*a*c^5 + (b^6 - 8*a*b^4*c + 22*a^2*b^2*c^2 - 24*a^3*c^3)*d^4 - 2*(b^4*c^2 - 6*a*b^2*c^3 + 8*a^2*c^4)*d^2)}}/(a^2*b^2 - 4*a^3*c)*d^4 + b^2*c^2 - 4*a*c^3 - (b^4 - 6*a*b^2*c + 8*a^2*c^2)*d^2))/x)
\end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-dx+1}\sqrt{dx+1}(a+bx+cx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x\*\*2+b\*x+a)/(-d\*x+1)\*\*(1/2)/(d\*x+1)\*\*(1/2),x)

[Out] Integral(1/(sqrt(-d\*x + 1)\*sqrt(d\*x + 1)\*(a + b\*x + c\*x\*\*2)), x)

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + b\*x + a)\*sqrt(d\*x + 1)\*sqrt(-d\*x + 1)),x, algorithm="giac")

[Out] Timed out

$$3.796 \quad \int \frac{1}{\sqrt{1-dx}\sqrt{1+dx}(a+bx+cx^2)^2} dx$$

**Optimal.** Leaf size=571

$$\frac{c \left( -cd^2 \left( -8a^2d^2 - b\sqrt{b^2 - 4ac} + 5b^2 \right) - abd^4 \left( \sqrt{b^2 - 4ac} + b \right) + 12ac^2d^2 + 4c^3 \right) \tanh^{-1} \left( \frac{d^2x(b - \sqrt{b^2 - 4ac}) + 2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2(b - \sqrt{b^2 - 4ac}) + 2acd^2 + 2c^2}} \right)}{\sqrt{2}(b^2 - 4ac)^{3/2} \sqrt{-bd^2(b - \sqrt{b^2 - 4ac}) + 2acd^2 + 2c^2} (b^2d^2 - (ad^2 + c)^2)}$$

$$+ \frac{c \left( -4cd^2 (b^2 - 2a^2d^2) - bd^2 \left( \sqrt{b^2 - 4ac} + b \right) (c - ad^2) - 2ab^2d^4 + 12ac^2d^2 + 4c^3 \right) \tanh^{-1} \left( \frac{d^2x(\sqrt{b^2 - 4ac} + b) + 2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2(\sqrt{b^2 - 4ac} + b) + 2acd^2 + 2c^2}} \right)}{\sqrt{2}(b^2 - 4ac)^{3/2} \sqrt{-bd^2(\sqrt{b^2 - 4ac} + b) + 2acd^2 + 2c^2} (b^2d^2 - (ad^2 + c)^2)}$$

$$- \frac{\sqrt{1-d^2x^2} (b (b^2d^2 - c (3ad^2 + c)) - cx (2acd^2 - b^2d^2 + 2c^2))}{(b^2 - 4ac) (b^2d^2 - (ad^2 + c)^2) (a + bx + cx^2)}$$

[Out] -(((b\*(b^2\*d^2 - c\*(c + 3\*a\*d^2)) - c\*(2\*c^2 - b^2\*d^2 + 2\*a\*c\*d^2)\*x)\*Sqrt[1 - d^2\*x^2])/((b^2 - 4\*a\*c)\*(b^2\*d^2 - (c + a\*d^2)^2)\*(a + b\*x + c\*x^2))) - (c\*(4\*c^3 + 12\*a\*c^2\*d^2 - a\*b\*(b + Sqrt[b^2 - 4\*a\*c])\*d^4 - c\*d^2\*(5\*b^2 - b\*Sqrt[b^2 - 4\*a\*c] - 8\*a^2\*d^2))\*ArcTanh[(2\*c + (b - Sqrt[b^2 - 4\*a\*c])\*d^2\*x)/(Sqrt[2]\*Sqrt[2\*c^2 + 2\*a\*c\*d^2 - b\*(b - Sqrt[b^2 - 4\*a\*c])\*d^2]\*Sqrt[1 - d^2\*x^2]])]/(Sqrt[2]\*(b^2 - 4\*a\*c)^(3/2)\*Sqrt[2\*c^2 + 2\*a\*c\*d^2 - b\*(b - Sqrt[b^2 - 4\*a\*c])\*d^2]\*(b^2\*d^2 - (c + a\*d^2)^2)) + (c\*(4\*c^3 + 12\*a\*c^2\*d^2 - 2\*a\*b^2\*d^4 - b\*(b + Sqrt[b^2 - 4\*a\*c])\*d^2\*(c - a\*d^2) - 4\*c\*d^2\*(b^2 - 2\*a^2\*d^2))\*ArcTanh[(2\*c + (b + Sqrt[b^2 - 4\*a\*c])\*d^2\*x)/(Sqrt[2]\*Sqrt[2\*c^2 + 2\*a\*c\*d^2 - b\*(b + Sqrt[b^2 - 4\*a\*c])\*d^2]\*Sqrt[1 - d^2\*x^2]])]/(Sqrt[2]\*(b^2 - 4\*a\*c)^(3/2)\*Sqrt[2\*c^2 + 2\*a\*c\*d^2 - b\*(b + Sqrt[b^2 - 4\*a\*c])\*d^2]\*(b^2\*d^2 - (c + a\*d^2)^2))

**Rubi [A]** time = 10.4805, antiderivative size = 571, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$

$$\frac{c \left( -cd^2 \left( -8a^2d^2 - b\sqrt{b^2 - 4ac} + 5b^2 \right) - abd^4 \left( \sqrt{b^2 - 4ac} + b \right) + 12ac^2d^2 + 4c^3 \right) \tanh^{-1} \left( \frac{d^2x(b - \sqrt{b^2 - 4ac}) + 2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2(b - \sqrt{b^2 - 4ac}) + 2acd^2 + 2c^2}} \right)}{\sqrt{2}(b^2 - 4ac)^{3/2} \sqrt{-bd^2(b - \sqrt{b^2 - 4ac}) + 2acd^2 + 2c^2} (b^2d^2 - (ad^2 + c)^2)}$$

$$+ \frac{c \left( -4cd^2 (b^2 - 2a^2d^2) - bd^2 \left( \sqrt{b^2 - 4ac} + b \right) (c - ad^2) - 2ab^2d^4 + 12ac^2d^2 + 4c^3 \right) \tanh^{-1} \left( \frac{d^2x(\sqrt{b^2 - 4ac} + b) + 2c}{\sqrt{2}\sqrt{1-d^2x^2}\sqrt{-bd^2(\sqrt{b^2 - 4ac} + b) + 2acd^2 + 2c^2}} \right)}{\sqrt{2}(b^2 - 4ac)^{3/2} \sqrt{-bd^2(\sqrt{b^2 - 4ac} + b) + 2acd^2 + 2c^2} (b^2d^2 - (ad^2 + c)^2)}$$

$$- \frac{\sqrt{1-d^2x^2} (b (b^2d^2 - c (3ad^2 + c)) - cx (2acd^2 - b^2d^2 + 2c^2))}{(b^2 - 4ac) (b^2d^2 - (ad^2 + c)^2) (a + bx + cx^2)}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[1 - d\*x]\*Sqrt[1 + d\*x]\*(a + b\*x + c\*x^2)^2), x]

[Out] -(((b\*(b^2\*d^2 - c\*(c + 3\*a\*d^2)) - c\*(2\*c^2 - b^2\*d^2 + 2\*a\*c\*d^2)\*x)\*Sqrt[1 - d^2\*x^2])/((b^2 - 4\*a\*c)\*(b^2\*d^2 - (c + a\*d^2)^2)\*(a + b\*x + c\*x^2))) - (c\*(4\*c^3 + 12\*a\*c^2\*d^2 - a\*b\*(b + Sqrt[b^2 - 4\*a\*c])\*d^4 - c\*d^2\*(5\*b^2 - b\*Sqrt[b^2 - 4\*a\*c] - 8\*a^2\*d^2))\*ArcTanh[(2\*c + (b - Sqrt[b^2 - 4\*a\*c])\*d^2\*x)/(Sqrt[2]\*Sqrt[2\*c^2 + 2\*a\*c\*d^2 - b\*(b - Sqrt[b^2 - 4\*a\*c])\*d^2]\*Sqrt[1 - d^2\*x^2]])]/(Sqrt[2]\*(b^2 - 4\*a\*c)^(3/2)\*Sqrt[2\*c^2 + 2\*a\*c\*d^2 - b\*(b - Sqrt[b^2 - 4\*a\*c])\*d^2]\*(b^2\*d^2 - (c + a\*d^2)^2)) + (c\*(4\*c^3 + 12\*a\*c^2\*d^2 - 2\*a\*b^2\*d^4 - b\*(b + Sqrt[b^2 - 4\*a\*c])\*d^2\*(c - a\*d^2) - 4\*c\*d^2\*(b^2 - 2\*a^2\*d^2))\*ArcTanh[(2\*c + (b + Sqrt[b^2 - 4\*a\*c])\*d^2\*x)/(Sqrt[2]\*Sqrt[2\*c^2 + 2\*a\*c\*d^2 - b\*(b + Sqrt[b^2 - 4\*a\*c])\*d^2]\*Sqrt[1 - d^2\*x^2]])]/(Sqrt[2]\*(b^2 - 4\*a\*c)^(3/2)\*Sqrt[2\*c^2 + 2\*a\*c\*d^2 - b\*(b + Sqrt[b^2 - 4\*a\*c])\*d^2]\*(b^2\*d^2 - (c + a\*d^2)^2))

$$\left. \right) \left. \right) / (\text{Sqrt}[2] * (b^2 - 4*a*c)^{(3/2)} * \text{Sqrt}[2*c^2 + 2*a*c*d^2 - b*(b - \text{Sqrt}[b^2 - 4*a*c]) * d^2] * (b^2*d^2 - (c + a*d^2)^2)) + (c*(4*c^3 + 12*a*c^2*d^2 - 2*a*b^2*d^4 - b*(b + \text{Sqrt}[b^2 - 4*a*c]) * d^2 * (c - a*d^2) - 4*c*d^2*(b^2 - 2*a^2*d^2)) * \text{ArcTanh}[(2*c + (b + \text{Sqrt}[b^2 - 4*a*c]) * d^2 * x) / (\text{Sqrt}[2] * \text{Sqrt}[2*c^2 + 2*a*c*d^2 - b*(b + \text{Sqrt}[b^2 - 4*a*c]) * d^2] * \text{Sqrt}[1 - d^2*x^2])]) / (\text{Sqrt}[2] * (b^2 - 4*a*c)^{(3/2)} * \text{Sqrt}[2*c^2 + 2*a*c*d^2 - b*(b + \text{Sqrt}[b^2 - 4*a*c]) * d^2] * (b^2*d^2 - (c + a*d^2)^2))$$

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(c*x**2+b*x+a)**2/(-d*x+1)**(1/2)/(d*x+1)**(1/2),x)`

[Out] Timed out

**Mathematica [A]** time = 4.03246, size = 800, normalized size = 1.4

$$-\frac{2\sqrt{1-d^2x^2}(d^2b^3+cd^2xb^2-c(3ad^2+c)b-2c^2(ad^2+c)x)}{(b^2-4ac)(a+x(b+cx))} + \frac{c(-ab(b+\sqrt{b^2-4ac})d^4+12ac^2d^2+c(-5b^2+\sqrt{b^2-4ac}b+8a^2d^2)d^2+4c^3)\log(-b-2cx+\sqrt{b^2-4ac})}{(b^2-4ac)^{3/2}\sqrt{c^2+ad^2c+\frac{1}{2}b(\sqrt{b^2-4ac}-b)}d^2}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(\text{Sqrt}[1 - d*x]*\text{Sqrt}[1 + d*x]*(a + b*x + c*x^2)^2),x]`

[Out]  $-\left(-2*(b^3*d^2 - b*c*(c + 3*a*d^2) + b^2*c*d^2*x - 2*c^2*(c + a*d^2)*x)*\text{Sqrt}[1 - d^2*x^2]\right) / \left((b^2 - 4*a*c)*(a + x*(b + c*x))\right) + (c*(4*c^3 + 12*a*c^2*d^2 - a*b*(b + \text{Sqrt}[b^2 - 4*a*c]) * d^4 + c*d^2*(-5*b^2 + b*\text{Sqrt}[b^2 - 4*a*c] + 8*a^2*d^2)) * \text{Log}[-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x]) / \left((b^2 - 4*a*c)^{(3/2)} * \text{Sqrt}[c^2 + a*c*d^2 + (b*(-b + \text{Sqrt}[b^2 - 4*a*c]) * d^2) / 2]\right) + (c*(-4*c^3 - 12*a*c^2*d^2 + a*b*(b - \text{Sqrt}[b^2 - 4*a*c]) * d^4 + c*d^2*(5*b^2 + b*\text{Sqrt}[b^2 - 4*a*c] - 8*a^2*d^2)) * \text{Log}[b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x]) / \left((b^2 - 4*a*c)^{(3/2)} * \text{Sqrt}[c^2 + a*c*d^2 - (b*(b + \text{Sqrt}[b^2 - 4*a*c]) * d^2) / 2]\right) - (c*(4*c^3 + 12*a*c^2*d^2 - a*b*(b + \text{Sqrt}[b^2 - 4*a*c]) * d^4 + c*d^2*(-5*b^2 + b*\text{Sqrt}[b^2 - 4*a*c] + 8*a^2*d^2)) * \text{Log}[-2*c - b*d^2*x + \text{Sqrt}[b^2 - 4*a*c] * d^2 * x - \text{Sqrt}[4*c^2 + 4*a*c*d^2 + 2*b*(-b + \text{Sqrt}[b^2 - 4*a*c]) * d^2] * \text{Sqrt}[1 - d^2*x^2])]) / \left((b^2 - 4*a*c)^{(3/2)} * \text{Sqrt}[c^2 + a*c*d^2 + (b*(-b + \text{Sqrt}[b^2 - 4*a*c]) * d^2) / 2]\right) + (c*(4*c^3 + 12*a*c^2*d^2 + a*b*(-b + \text{Sqrt}[b^2 - 4*a*c]) * d^4 + c*d^2*(-5*b^2 - b*\text{Sqrt}[b^2 - 4*a*c] + 8*a^2*d^2)) * \text{Log}[2*c + b*d^2*x + \text{Sqrt}[b^2 - 4*a*c] * d^2 * x + \text{Sqrt}[4*c^2 + 4*a*c*d^2 - 2*b*(b + \text{Sqrt}[b^2 - 4*a*c]) * d^2] * \text{Sqrt}[1 - d^2*x^2])]) / \left((b^2 - 4*a*c)^{(3/2)} * \text{Sqrt}[c^2 + a*c*d^2 - (b*(b + \text{Sqrt}[b^2 - 4*a*c]) * d^2) / 2]\right) / (2*(c^2 - b^2*d^2 + 2*a*c*d^2 + a^2*d^4))$

**Maple [C]** time = 0.77, size = 41837, normalized size = 73.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(c*x^2+b*x+a)^2/(-d*x+1)^(1/2)/(d*x+1)^(1/2),x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx + a)^2 \sqrt{dx + 1} \sqrt{-dx + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + b\*x + a)^2\*sqrt(d\*x + 1)\*sqrt(-d\*x + 1)),x, algorithm="maxi

[Out] integrate(1/((c\*x^2 + b\*x + a)^2\*sqrt(d\*x + 1)\*sqrt(-d\*x + 1)), x  
)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + b\*x + a)^2\*sqrt(d\*x + 1)\*sqrt(-d\*x + 1)),x, algorithm="fric

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(c\*x\*\*2+b\*x+a)\*\*2/(-d\*x+1)\*\*(1/2)/(d\*x+1)\*\*(1/2),x)

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + b\*x + a)^2\*sqrt(d\*x + 1)\*sqrt(-d\*x + 1)),x, algorithm="giac

[Out] Timed out

$$3.797 \quad \int \frac{(a+bx+cx^2)^3}{(1-dx)^{3/2}(1+dx)^{3/2}} dx$$

**Optimal.** Leaf size=276

$$\begin{aligned} & \frac{x(ad^2 + c)(a^2d^4 + 2acd^2 + 3b^2d^2 + c^2) + bd^4 \left(3a^2 + \frac{6ac}{d^2} + \frac{b^2}{d^2} + \frac{3c^2}{d^4}\right)}{d^6\sqrt{1-d^2x^2}} \\ & - \frac{3\sin^{-1}(dx)(8a^2cd^4 + 8ab^2d^4 + 12ac^2d^2 + 12b^2cd^2 + 5c^3)}{8d^7} \\ & + \frac{cx\sqrt{1-d^2x^2}(12acd^2 + 12b^2d^2 + 7c^2)}{8d^6} \\ & + \frac{b\sqrt{1-d^2x^2}(6acd^2 + b^2d^2 + 5c^2)}{d^6} + \frac{bc^2x^2\sqrt{1-d^2x^2}}{d^4} + \frac{c^3x^3\sqrt{1-d^2x^2}}{4d^4} \end{aligned}$$

[Out] (b\*(3\*a^2 + (3\*c^2)/d^4 + b^2/d^2 + (6\*a\*c)/d^2)\*d^4 + (c + a\*d^2)\*(c^2 + 3\*b^2\*d^2 + 2\*a\*c\*d^2 + a^2\*d^4)\*x)/(d^6\*sqrt[1 - d^2\*x^2]) + (b\*(5\*c^2 + b^2\*d^2 + 6\*a\*c\*d^2)\*sqrt[1 - d^2\*x^2])/d^6 + (c\*(7\*c^2 + 12\*b^2\*d^2 + 12\*a\*c\*d^2)\*x\*sqrt[1 - d^2\*x^2])/(8\*d^6) + (b\*c^2\*x^2\*sqrt[1 - d^2\*x^2])/d^4 + (c^3\*x^3\*sqrt[1 - d^2\*x^2])/(4\*d^4) - (3\*(5\*c^3 + 12\*b^2\*c\*d^2 + 12\*a\*c^2\*d^2 + 8\*a\*b^2\*d^4 + 8\*a^2\*c\*d^4)\*ArcSin[d\*x])/(8\*d^7)

**Rubi [A]** time = 1.02247, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$

$$\begin{aligned} & \frac{x(ad^2 + c)(a^2d^4 + 2acd^2 + 3b^2d^2 + c^2) + b(3a^2d^4 + 6acd^2 + b^2d^2 + 3c^2)}{d^6\sqrt{1-d^2x^2}} \\ & - \frac{3\sin^{-1}(dx)(8a^2cd^4 + 8ab^2d^4 + 12ac^2d^2 + 12b^2cd^2 + 5c^3)}{8d^7} \\ & + \frac{cx\sqrt{1-d^2x^2}(12acd^2 + 12b^2d^2 + 7c^2)}{8d^6} \\ & + \frac{b\sqrt{1-d^2x^2}(6acd^2 + b^2d^2 + 5c^2)}{d^6} + \frac{bc^2x^2\sqrt{1-d^2x^2}}{d^4} + \frac{c^3x^3\sqrt{1-d^2x^2}}{4d^4} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)^3/((1 - d\*x)^(3/2)\*(1 + d\*x)^(3/2)), x]

[Out] (b\*(3\*c^2 + b^2\*d^2 + 6\*a\*c\*d^2 + 3\*a^2\*d^4) + (c + a\*d^2)\*(c^2 + 3\*b^2\*d^2 + 2\*a\*c\*d^2 + a^2\*d^4)\*x)/(d^6\*sqrt[1 - d^2\*x^2]) + (b\*(5\*c^2 + b^2\*d^2 + 6\*a\*c\*d^2)\*sqrt[1 - d^2\*x^2])/d^6 + (c\*(7\*c^2 + 12\*b^2\*d^2 + 12\*a\*c\*d^2)\*x\*sqrt[1 - d^2\*x^2])/(8\*d^6) + (b\*c^2\*x^2\*sqrt[1 - d^2\*x^2])/d^4 + (c^3\*x^3\*sqrt[1 - d^2\*x^2])/(4\*d^4) - (3\*(5\*c^3 + 12\*b^2\*c\*d^2 + 12\*a\*c^2\*d^2 + 8\*a\*b^2\*d^4 + 8\*a^2\*c\*d^4)\*ArcSin[d\*x])/(8\*d^7)

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+b\*x+a)\*\*3/(-d\*x+1)\*\*(3/2)/(d\*x+1)\*\*(3/2), x)

[Out] Timed out

**Mathematica [A]** time = 0.976828, size = 226, normalized size = 0.82

$$3 \sin^{-1}(dx) (8a^2cd^4 + 8ab^2d^4 + 12ac^2d^2 + 12b^2cd^2 + 5c^3) + \frac{d(8b(-3a^2d^4+6acd^2(d^2x^2-2)+c^2(d^4x^4+4d^2x^2-8))+x(-8a^3d^6-24a^2cd^4+12acd^3+8a^2b^2d^4+8ab^2d^4+12ac^2d^2+12b^2cd^2+5c^3))}{8d^7}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)^3/((1 - d\*x)^(3/2)\*(1 + d\*x)^(3/2)), x]

[Out] -((d\*(8\*b^3\*d^2\*(-2 + d^2\*x^2) + 12\*b^2\*d^2\*x\*(-2\*a\*d^2 + c\*(-3 + d^2\*x^2)) + 8\*b\*(-3\*a^2\*d^4 + 6\*a\*c\*d^2\*(-2 + d^2\*x^2) + c^2\*(-8 + 4\*d^2\*x^2 + d^4\*x^4)) + x\*(-24\*a^2\*c\*d^4 - 8\*a^3\*d^6 + 12\*a\*c^2\*d^2\*d^2\*(-3 + d^2\*x^2) + c^3\*(-15 + 5\*d^2\*x^2 + 2\*d^4\*x^4)))/Sqrt[1 - d^2\*x^2] + 3\*(5\*c^3 + 12\*b^2\*c\*d^2 + 12\*a\*c^2\*d^2 + 8\*a\*b^2\*d^4 + 8\*a^2\*c\*d^4)\*ArcSin[d\*x])/(8\*d^7)

**Maple [C]** time = 0.053, size = 755, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)^3/(-d\*x+1)^(3/2)/(d\*x+1)^(3/2), x)

[Out] 1/8\*(-d\*x+1)^(1/2)\*(-15\*arctan(csgn(d)\*d\*x/(-d^2\*x^2+1)^(1/2))\*x^2\*c^3\*d^2-16\*(-d^2\*x^2+1)^(1/2)\*b^3\*d^3\*csgn(d)+24\*arctan(csgn(d)\*d\*x/(-d^2\*x^2+1)^(1/2))\*a^2\*c\*d^4+24\*arctan(csgn(d)\*d\*x/(-d^2\*x^2+1)^(1/2))\*a\*b^2\*d^4+36\*arctan(csgn(d)\*d\*x/(-d^2\*x^2+1)^(1/2))\*a\*c^2\*d^2+36\*arctan(csgn(d)\*d\*x/(-d^2\*x^2+1)^(1/2))\*b^2\*c\*d^2+15\*arctan(csgn(d)\*d\*x/(-d^2\*x^2+1)^(1/2))\*c^3-24\*arctan(csgn(d)\*d\*x/(-d^2\*x^2+1)^(1/2))\*x^2\*a\*b^2\*d^6-8\*(-d^2\*x^2+1)^(1/2)\*csgn(d)\*d^7\*x\*a^3-36\*arctan(csgn(d)\*d\*x/(-d^2\*x^2+1)^(1/2))\*x^2\*a\*c^2\*d^4-36\*arctan(csgn(d)\*d\*x/(-d^2\*x^2+1)^(1/2))\*x^2\*b^2\*c\*d^4+2\*csgn(d)\*x^5\*c^3\*d^5\*(-d^2\*x^2+1)^(1/2)-24\*arctan(csgn(d)\*d\*x/(-d^2\*x^2+1)^(1/2))\*x^2\*a^2\*c\*d^6+8\*csgn(d)\*x^4\*b\*c^2\*d^5\*(-d^2\*x^2+1)^(1/2)+12\*csgn(d)\*x^3\*b^2\*c\*d^5\*(-d^2\*x^2+1)^(1/2)-24\*x\*(-d^2\*x^2+1)^(1/2)\*a^2\*c\*d^5\*csgn(d)-24\*x\*(-d^2\*x^2+1)^(1/2)\*a\*b^2\*d^5\*csgn(d)+48\*csgn(d)\*x^2\*a\*b\*c\*d^5\*(-d^2\*x^2+1)^(1/2)+32\*csgn(d)\*d^3\*(-d^2\*x^2+1)^(1/2)\*x^2\*b\*c^2-36\*x\*(-d^2\*x^2+1)^(1/2)\*a\*c^2\*d^3\*csgn(d)-36\*x\*(-d^2\*x^2+1)^(1/2)\*b^2\*c\*d^3\*csgn(d)-96\*(-d^2\*x^2+1)^(1/2)\*a\*b\*c\*d^3\*csgn(d)+8\*csgn(d)\*x^2\*b^3\*d^5\*(-d^2\*x^2+1)^(1/2)+5\*c^3\*x^3\*(-d^2\*x^2+1)^(1/2)\*d^3\*csgn(d)-24\*(-d^2\*x^2+1)^(1/2)\*a^2\*b\*d^5\*csgn(d)-15\*x\*(-d^2\*x^2+1)^(1/2)\*c^3\*csgn(d)\*d-64\*(-d^2\*x^2+1)^(1/2)\*b\*c^2\*csgn(d)\*d)\*csgn(d)/(d\*x-1)/(-d^2\*x^2+1)^(1/2)/d^7/(d\*x+1)^(1/2)

**Maxima [A]** time = 0.775117, size = 549, normalized size = 1.99

$$\begin{aligned} & -\frac{c^3x^5}{4\sqrt{-d^2x^2+1d^2}} - \frac{bc^2x^4}{\sqrt{-d^2x^2+1d^2}} + \frac{a^3x}{\sqrt{-d^2x^2+1}} - \frac{5c^3x^3}{8\sqrt{-d^2x^2+1d^4}} \\ & - \frac{3(b^2c+ac^2)x^3}{2\sqrt{-d^2x^2+1d^2}} + \frac{3a^2b}{\sqrt{-d^2x^2+1d^2}} - \frac{4bc^2x^2}{\sqrt{-d^2x^2+1d^4}} - \frac{(b^3+6abc)x^2}{\sqrt{-d^2x^2+1d^2}} \\ & + \frac{3(ab^2+a^2c)x}{\sqrt{-d^2x^2+1d^2}} - \frac{3(ab^2+a^2c)\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{\sqrt{d^2d^2}} + \frac{15c^3x}{8\sqrt{-d^2x^2+1d^6}} + \frac{9(b^2c+ac^2)x}{2\sqrt{-d^2x^2+1d^4}} \\ & - \frac{15c^3\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{8\sqrt{d^2d^6}} - \frac{9(b^2c+ac^2)\arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{2\sqrt{d^2d^4}} + \frac{8bc^2}{\sqrt{-d^2x^2+1d^6}} + \frac{2(b^3+6abc)}{\sqrt{-d^2x^2+1d^4}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)^3/((d\*x + 1)^(3/2)\*(-d\*x + 1)^(3/2)),x, algorithm="ma

[Out] 
$$-1/4*c^3*x^5/(sqrt(-d^2*x^2 + 1)*d^2) - b*c^2*x^4/(sqrt(-d^2*x^2 + 1)*d^2) + a^3*x/sqrt(-d^2*x^2 + 1) - 5/8*c^3*x^3/(sqrt(-d^2*x^2 + 1)*d^4) - 3/2*(b^2*c + a*c^2)*x^3/(sqrt(-d^2*x^2 + 1)*d^2) + 3*a^2*b/(sqrt(-d^2*x^2 + 1)*d^2) - 4*b*c^2*x^2/(sqrt(-d^2*x^2 + 1)*d^4) - (b^3 + 6*a*b*c)*x^2/(sqrt(-d^2*x^2 + 1)*d^2) + 3*(a*b^2 + a^2*c)*x/(sqrt(-d^2*x^2 + 1)*d^2) - 3*(a*b^2 + a^2*c)*arcsin(d^2*x/sqrt(d^2))/(sqrt(d^2)*d^2) + 15/8*c^3*x/(sqrt(-d^2*x^2 + 1)*d^6) + 9/2*(b^2*c + a*c^2)*x/(sqrt(-d^2*x^2 + 1)*d^4) - 15/8*c^3*arcsin(d^2*x/sqrt(d^2))/(sqrt(d^2)*d^6) - 9/2*(b^2*c + a*c^2)*arcsin(d^2*x/sqrt(d^2))/(sqrt(d^2)*d^4) + 8*b*c^2/(sqrt(-d^2*x^2 + 1)*d^6) + 2*(b^3 + 6*a*b*c)/(sqrt(-d^2*x^2 + 1)*d^4)$$

**Fricas [A]** time = 0.30135, size = 1362, normalized size = 4.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)^3/((d\*x + 1)^(3/2)\*(-d\*x + 1)^(3/2)),x, algorithm="fr

[Out] 
$$-1/8*(10*c^3*d^9*x^9 + 40*b*c^2*d^9*x^8 - 192*a^2*b*d^7*x^2 - 15*(c^3*d^7 - 4*(b^2*c + a*c^2)*d^9)*x^7 - 8*(3*a^2*b*d^11 + 8*b*c^2*d^7 - 3*(b^3 + 6*a*b*c)*d^9)*x^6 - (40*a^3*d^11 + 120*(a*b^2 + a^2*c)*d^9 + 143*c^3*d^5 + 420*(b^2*c + a*c^2)*d^7)*x^5 + 32*(6*a^2*b*d^9 - (b^3 + 6*a*b*c)*d^7)*x^4 + 4*(40*a^3*d^9 + 120*(a*b^2 + a^2*c)*d^7 + 95*c^3*d^3 + 228*(b^2*c + a*c^2)*d^5)*x^3 - (2*c^3*d^9*x^9 + 8*b*c^2*d^9*x^8 - 192*a^2*b*d^7*x^2 - (19*c^3*d^7 - 12*(b^2*c + a*c^2)*d^9)*x^7 - 8*(8*b*c^2*d^7 - (b^3 + 6*a*b*c)*d^9)*x^6 - (8*a^3*d^11 + 24*(a*b^2 + a^2*c)*d^9 + 43*c^3*d^5 + 180*(b^2*c + a*c^2)*d^7)*x^5 + 32*(3*a^2*b*d^9 - (b^3 + 6*a*b*c)*d^7)*x^4 + 4*(24*a^3*d^9 + 72*(a*b^2 + a^2*c)*d^7 + 65*c^3*d^3 + 156*(b^2*c + a*c^2)*d^5)*x^3 - 16*(8*a^3*d^7 + 24*(a*b^2 + a^2*c)*d^5 + 15*c^3*d + 36*(b^2*c + a*c^2)*d^3)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) - 16*(8*a^3*d^7 + 24*(a*b^2 + a^2*c)*d^5 + 15*c^3*d + 36*(b^2*c + a*c^2)*d^3)*x - 6*((8*(a*b^2 + a^2*c)*d^10 + 5*c^3*d^6 + 12*(b^2*c + a*c^2)*d^8)*x^6 - 128*(a*b^2 + a^2*c)*d^4 - 13*(8*(a*b^2 + a^2*c)*d^8 + 5*c^3*d^4 + 12*(b^2*c + a*c^2)*d^6)*x^4 - 80*c^3 - 192*(b^2*c + a*c^2)*d^2 + 28*(8*(a*b^2 + a^2*c)*d^6 + 5*c^3*d^2 + 12*(b^2*c + a*c^2)*d^4)*x^2 + (128*(a*b^2 + a^2*c)*d^4 + 5*(8*(a*b^2 + a^2*c)*d^8 + 5*c^3*d^4 + 12*(b^2*c + a*c^2)*d^6)*x^4 + 80*c^3 + 192*(b^2*c + a*c^2)*d^2 - 20*(8*(a*b^2 + a^2*c)*d^6 + 5*c^3*d^2 + 12*(b^2*c + a*c^2)*d^4)*x^2)*sqrt(d*x + 1)*sqrt(-d*x + 1))*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x))/(d^13*x^6 - 13*d^11*x^4 + 28*d^9*x^2 - 16*d^7 + (5*d^11*x^4 - 20*d^9*x^2 + 16*d^7)*sqrt(d*x + 1)*sqrt(-d*x + 1))$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx + cx^2)^3}{(-dx + 1)^{\frac{3}{2}}(dx + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*\*3/(-d\*x+1)\*\*(3/2)/(d\*x+1)\*\*(3/2),x)

[Out] Integral((a + b\*x + c\*x\*\*2)\*\*3/((-d\*x + 1)\*\*(3/2)\*(d\*x + 1)\*\*(3/2)), x)



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**GIAC/XCAS [A]** time = 0.343008, size = 961, normalized size = 3.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)^3/((d\*x + 1)^(3/2)\*(-d\*x + 1)^(3/2)),x, algorithm="giac")

[Out] 
$$-1/86016*(8*a*b^2*d^39 + 8*a^2*c*d^39 + 12*b^2*c*d^37 + 12*a*c^2*d^37 + 5*c^3*d^35)*\arcsin(1/2*\sqrt{2}*\sqrt{d*x + 1}) - 1/516096*(4*a^3*d^41 + 12*a^2*b*d^40 + 12*a*b^2*d^39 + 12*a^2*c*d^39 + 20*b^3*d^38 + 120*a*b*c*d^38 - 12*b^2*c*d^37 - 12*a*c^2*d^37 + 108*b*c^2*d^36 - 14*c^3*d^35 - (8*b^3*d^38 + 48*a*b*c*d^38 - 36*b^2*c*d^37 - 36*a*c^2*d^37 + 80*b*c^2*d^36 - 35*c^3*d^35 + (12*b^2*c*d^37 + 12*a*c^2*d^37 - 32*b*c^2*d^36 + 25*c^3*d^35 + 2*((d*x + 1)*c^3*d^35 + 4*b*c^2*d^36 - 5*c^3*d^35)*(d*x + 1))*(d*x + 1))*(d*x + 1))*\sqrt{d*x + 1}*\sqrt{-d*x + 1}/(d*x - 1) + 1/4*(a^3*d^6*(\sqrt{2} - \sqrt{-d*x + 1})/\sqrt{d*x + 1} - 3*a^2*b*d^5*(\sqrt{2} - \sqrt{-d*x + 1})/\sqrt{d*x + 1} + 3*a*b^2*d^4*(\sqrt{2} - \sqrt{-d*x + 1})/\sqrt{d*x + 1} + 3*a^2*c*d^4*(\sqrt{2} - \sqrt{-d*x + 1})/\sqrt{d*x + 1} - b^3*d^3*(\sqrt{2} - \sqrt{-d*x + 1})/\sqrt{d*x + 1} - 6*a*b*c*d^3*(\sqrt{2} - \sqrt{-d*x + 1})/\sqrt{d*x + 1} + 3*b^2*c*d^2*(\sqrt{2} - \sqrt{-d*x + 1})/\sqrt{d*x + 1} + 3*a*c^2*d^2*(\sqrt{2} - \sqrt{-d*x + 1})/\sqrt{d*x + 1} - 3*b*c^2*d*(\sqrt{2} - \sqrt{-d*x + 1})/\sqrt{d*x + 1} + c^3*(\sqrt{2} - \sqrt{-d*x + 1})/\sqrt{d*x + 1})/d^7 - 1/4*(a^3*d^6 - 3*a^2*b*d^5 + 3*a*b^2*d^4 + 3*a^2*c*d^4 - b^3*d^3 - 6*a*b*c*d^3 + 3*b^2*c*d^2 + 3*a*c^2*d^2 - 3*b*c^2*d + c^3)*\sqrt{d*x + 1}/(d^7*(\sqrt{2} - \sqrt{-d*x + 1}))$$

$$3.798 \quad \int \frac{(a+bx+cx^2)^2}{(1-dx)^{3/2}(1+dx)^{3/2}} dx$$

**Optimal.** Leaf size=138

$$\frac{x(a^2d^4 + 2acd^2 + b^2d^2 + c^2) + 2bd^2\left(a + \frac{c}{d^2}\right)}{d^4\sqrt{1-d^2x^2}} - \frac{\sin^{-1}(dx)(4acd^2 + 2b^2d^2 + 3c^2)}{2d^5} + \frac{2bc\sqrt{1-d^2x^2}}{d^4} + \frac{c^2x\sqrt{1-d^2x^2}}{2d^4}$$

[Out] (2\*b\*(a + c/d^2)\*d^2 + (c^2 + b^2\*d^2 + 2\*a\*c\*d^2 + a^2\*d^4)\*x)/(d^4\*Sqrt[1 - d^2\*x^2]) + (2\*b\*c\*Sqrt[1 - d^2\*x^2])/d^4 + (c^2\*x\*Sqrt[1 - d^2\*x^2])/(2\*d^4) - ((3\*c^2 + 2\*b^2\*d^2 + 4\*a\*c\*d^2)\*ArcSin[d\*x])/(2\*d^5)

**Rubi [A]** time = 0.32729, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$

$$\frac{x(a^2d^4 + 2acd^2 + b^2d^2 + c^2) + 2b(ad^2 + c)}{d^4\sqrt{1-d^2x^2}} - \frac{\sin^{-1}(dx)(4acd^2 + 2b^2d^2 + 3c^2)}{2d^5} + \frac{2bc\sqrt{1-d^2x^2}}{d^4} + \frac{c^2x\sqrt{1-d^2x^2}}{2d^4}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)^2/((1 - d\*x)^(3/2)\*(1 + d\*x)^(3/2)), x]

[Out] (2\*b\*(c + a\*d^2) + (c^2 + b^2\*d^2 + 2\*a\*c\*d^2 + a^2\*d^4)\*x)/(d^4\*Sqrt[1 - d^2\*x^2]) + (2\*b\*c\*Sqrt[1 - d^2\*x^2])/d^4 + (c^2\*x\*Sqrt[1 - d^2\*x^2])/(2\*d^4) - ((3\*c^2 + 2\*b^2\*d^2 + 4\*a\*c\*d^2)\*ArcSin[d\*x])/(2\*d^5)

**Rubi in Sympy [A]** time = 97.2099, size = 155, normalized size = 1.12

$$\frac{2bcx^2\sqrt{-d^2x^2+1}}{d^2} + \frac{c^2x^3\sqrt{-d^2x^2+1}}{d^2} + \frac{x(a+bx+cx^2)^2}{\sqrt{-d^2x^2+1}} + \frac{(96b(ad^2+2c) + x(96acd^2+48b^2d^2+72c^2))\sqrt{-d^2x^2+1}}{48d^4} - \frac{(4acd^2+2b^2d^2+3c^2)\operatorname{asin}(dx)}{2d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+b\*x+a)\*\*2/(-d\*x+1)\*\*(3/2)/(d\*x+1)\*\*(3/2), x)

[Out] 2\*b\*c\*x\*\*2\*sqrt(-d\*\*2\*x\*\*2 + 1)/d\*\*2 + c\*\*2\*x\*\*3\*sqrt(-d\*\*2\*x\*\*2 + 1)/d\*\*2 + x\*(a + b\*x + c\*x\*\*2)\*\*2/sqrt(-d\*\*2\*x\*\*2 + 1) + (96\*b\*(a\*d\*\*2 + 2\*c) + x\*(96\*a\*c\*d\*\*2 + 48\*b\*\*2\*d\*\*2 + 72\*c\*\*2))\*sqrt(-d\*\*2\*x\*\*2 + 1)/(48\*d\*\*4) - (4\*a\*c\*d\*\*2 + 2\*b\*\*2\*d\*\*2 + 3\*c\*\*2)\*asin(d\*x)/(2\*d\*\*5)

**Mathematica [A]** time = 0.261702, size = 114, normalized size = 0.83

$$\frac{d(x(2a^2d^4+4acd^2+c^2(3-d^2x^2))+4b(ad^2+c(2-d^2x^2))+2b^2d^2x)}{\sqrt{1-d^2x^2}} - \frac{\sin^{-1}(dx)(4acd^2 + 2b^2d^2 + 3c^2)}{2d^5}$$



[Out] 
$$-1/2*(3*c^2*d^5*x^5 + 8*a*b*d^5*x^2 - 4*(a*b*d^7 - b*c*d^5)*x^4 - (6*a^2*d^7 + 6*(b^2 + 2*a*c)*d^5 + 13*c^2*d^3)*x^3 - (c^2*d^5*x^5 + 4*b*c*d^5*x^4 + 8*a*b*d^5*x^2 - (2*a^2*d^7 + 2*(b^2 + 2*a*c)*d^5 + 7*c^2*d^3)*x^3 + 4*(2*a^2*d^5 + 2*(b^2 + 2*a*c)*d^3 + 3*c^2*d)*x)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 4*(2*a^2*d^5 + 2*(b^2 + 2*a*c)*d^3 + 3*c^2*d)*x - 2*((2*(b^2 + 2*a*c)*d^6 + 3*c^2*d^4)*x^4 + 8*(b^2 + 2*a*c)*d^2 - 5*(2*(b^2 + 2*a*c)*d^4 + 3*c^2*d^2)*x^2 - (8*(b^2 + 2*a*c)*d^2 - 3*(2*(b^2 + 2*a*c)*d^4 + 3*c^2*d^2)*x^2 + 12*c^2)*sqrt(d*x + 1)*sqrt(-d*x + 1) + 12*c^2)*arctan((sqrt(d*x + 1)*sqrt(-d*x + 1) - 1)/(d*x))/(d^9*x^4 - 5*d^7*x^2 + 4*d^5 + (3*d^7*x^2 - 4*d^5)*sqrt(d*x + 1)*sqrt(-d*x + 1))$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx + cx^2)^2}{(-dx + 1)^{\frac{3}{2}}(dx + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+b*x+a)**2/(-d*x+1)**(3/2)/(d*x+1)**(3/2),x)`

[Out] `Integral((a + b*x + c*x**2)**2/((-d*x + 1)**(3/2)*(d*x + 1)**(3/2)), x)`

**GIAC/XCAS [A]** time = 0.3037, size = 509, normalized size = 3.69

$$-\frac{1}{384} (2b^2d^{17} + 4acd^{17} + 3c^2d^{15}) \arcsin\left(\frac{1}{2} \sqrt{2}\sqrt{dx+1}\right) - \frac{(a^2d^{19} + 2abd^{18} + b^2d^{17} + 2acd^{17} + 10bcd^{16} - c^2d^{15} - ((dx+1)c^2d^{15} + 4bcd^{16} - 3c^2d^{15})(dx+1))\sqrt{dx+1}\sqrt{-dx+1}}{768(dx-1)} + \frac{a^2d^4(\sqrt{2}-\sqrt{-dx+1})}{\sqrt{dx+1}} - \frac{2abd^3(\sqrt{2}-\sqrt{-dx+1})}{\sqrt{dx+1}} + \frac{b^2d^2(\sqrt{2}-\sqrt{-dx+1})}{\sqrt{dx+1}} + \frac{2acd^2(\sqrt{2}-\sqrt{-dx+1})}{\sqrt{dx+1}} - \frac{2bcd(\sqrt{2}-\sqrt{-dx+1})}{\sqrt{dx+1}} + \frac{c^2(\sqrt{2}-\sqrt{-dx+1})}{\sqrt{dx+1}} - \frac{(a^2d^4 - 2abd^3 + b^2d^2 + 2acd^2 - 2bcd + c^2)\sqrt{dx+1}}{4d^5(\sqrt{2}-\sqrt{-dx+1})}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^2/((d*x + 1)^(3/2)*(-d*x + 1)^(3/2)),x, algorithm="giac")`

[Out] 
$$-1/384*(2*b^2*d^17 + 4*a*c*d^17 + 3*c^2*d^15)*arcsin(1/2*sqrt(2)*sqrt(d*x + 1)) - 1/768*(a^2*d^19 + 2*a*b*d^18 + b^2*d^17 + 2*a*c*d^17 + 10*b*c*d^16 - c^2*d^15 - ((d*x + 1)*c^2*d^15 + 4*b*c*d^16 - 3*c^2*d^15)*(d*x + 1))*sqrt(d*x + 1)*sqrt(-d*x + 1)/(d*x - 1) + 1/4*(a^2*d^4*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - 2*a*b*d^3*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + b^2*d^2*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + 2*a*c*d^2*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) - 2*b*c*d*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1) + c^2*(sqrt(2) - sqrt(-d*x + 1))/sqrt(d*x + 1))/d^5 - 1/4*(a^2*d^4 - 2*a*b*d^3 + b^2*d^2 + 2*a*c*d^2 - 2*b*c*d + c^2)*sqrt(d*x + 1)/(d^5*(sqrt(2) - sqrt(-d*x + 1)))$$

$$3.799 \quad \int \frac{a+bx+cx^2}{(1-dx)^{3/2}(1+dx)^{3/2}} dx$$

**Optimal.** Leaf size=40

$$\frac{x(ad^2+c)+b}{d^2\sqrt{1-d^2x^2}} - \frac{c\sin^{-1}(dx)}{d^3}$$

[Out] (b + (c + a\*d^2)\*x)/(d^2\*sqrt[1 - d^2\*x^2]) - (c\*ArcSin[d\*x])/d^3

**Rubi [A]** time = 0.101324, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 30,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$

$$\frac{x(ad^2+c)+b}{d^2\sqrt{1-d^2x^2}} - \frac{c\sin^{-1}(dx)}{d^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)/((1 - d\*x)^(3/2)\*(1 + d\*x)^(3/2)), x]

[Out] (b + (c + a\*d^2)\*x)/(d^2\*sqrt[1 - d^2\*x^2]) - (c\*ArcSin[d\*x])/d^3

**Rubi in Sympy [A]** time = 15.8444, size = 34, normalized size = 0.85

$$-\frac{c\operatorname{asin}(dx)}{d^3} + \frac{b+x(ad^2+c)}{d^2\sqrt{-d^2x^2+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+b\*x+a)/(-d\*x+1)\*\*(3/2)/(d\*x+1)\*\*(3/2), x)

[Out] -c\*asin(d\*x)/d\*\*3 + (b + x\*(a\*d\*\*2 + c))/(d\*\*2\*sqrt(-d\*\*2\*x\*\*2 + 1))

**Mathematica [A]** time = 0.123465, size = 39, normalized size = 0.98

$$\frac{\frac{d(x(ad^2+c)+b)}{\sqrt{1-d^2x^2}} - c\sin^{-1}(dx)}{d^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)/((1 - d\*x)^(3/2)\*(1 + d\*x)^(3/2)), x]

[Out] ((d\*(b + (c + a\*d^2)\*x))/sqrt[1 - d^2\*x^2] - c\*ArcSin[d\*x])/d^3

**Maple [C]** time = 0.026, size = 147, normalized size = 3.7

$$\frac{\operatorname{csgn}(d)}{(dx-1)d^3} \left( -\sqrt{-d^2x^2+1} \operatorname{csgn}(d) d^3 x a - \arctan \left( \operatorname{csgn}(d) dx \frac{1}{\sqrt{-d^2x^2+1}} \right) x^2 c d^2 - cx \sqrt{-d^2x^2+1} \operatorname{csgn}(d) d - \sqrt{-d^2x^2+1} b \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)/(-d*x+1)^(3/2)/(d*x+1)^(3/2),x)`

[Out]  $(-(-d^2x^2+1)^{1/2} \operatorname{csgn}(d) d^3 x^2 a - \arctan(\operatorname{csgn}(d) d x / (-d^2x^2+1)^{1/2})) x^2 c d^2 - c x (-d^2x^2+1)^{1/2} \operatorname{csgn}(d) d - (-d^2x^2+1)^{1/2} b \operatorname{csgn}(d) d + c \arctan(\operatorname{csgn}(d) d x / (-d^2x^2+1)^{1/2})) (-d^2x^2+1)^{1/2} \operatorname{csgn}(d) / (d x - 1) / (-d^2x^2+1)^{1/2} / d^3 / (d x + 1)^{1/2}$

**Maxima [A]** time = 0.775644, size = 99, normalized size = 2.48

$$\frac{ax}{\sqrt{-d^2x^2+1}} + \frac{cx}{\sqrt{-d^2x^2+1d^2}} - \frac{c \arcsin\left(\frac{d^2x}{\sqrt{d^2}}\right)}{\sqrt{d^2d^2}} + \frac{b}{\sqrt{-d^2x^2+1d^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)/((d*x + 1)^(3/2)*(-d*x + 1)^(3/2)),x, algorithm="maxi`

[Out]  $a x / \sqrt{-d^2x^2 + 1} + c x / (\sqrt{-d^2x^2 + 1} d^2) - c \arcsin(d^2 x / \sqrt{d^2}) / (\sqrt{d^2} d^2) + b / (\sqrt{-d^2x^2 + 1} d^2)$

**Fricas [A]** time = 0.28183, size = 188, normalized size = 4.7

$$\frac{bd^3x^2 - (ad^3 + cd) \sqrt{dx + 1} \sqrt{-dx + 1} x + (ad^3 + cd) x + 2 \left( cd^2x^2 + \sqrt{dx + 1} \sqrt{-dx + 1} c - c \right) \arctan\left(\frac{\sqrt{dx+1}\sqrt{-dx+1}-1}{dx}\right)}{d^5x^2 + \sqrt{dx + 1} \sqrt{-dx + 1} d^3 - d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)/((d*x + 1)^(3/2)*(-d*x + 1)^(3/2)),x, algorithm="fric`

[Out]  $(b d^3 x^2 - (a d^3 + c d) \sqrt{d x + 1} \sqrt{-d x + 1} x + (a d^3 + c d) x + 2 (c d^2 x^2 + \sqrt{d x + 1} \sqrt{-d x + 1} c - c) \arctan((\sqrt{d x + 1} \sqrt{-d x + 1} - 1) / (d x))) / (d^5 x^2 + \sqrt{d x + 1} \sqrt{-d x + 1} d^3 - d^3)$

**Sympy [A]** time = 91.9823, size = 255, normalized size = 6.38

$$a \left( \frac{i G_{6,6}^{5,3} \left( \begin{array}{c|c} \frac{3}{4}, \frac{5}{4}, 1 & \frac{1}{2}, \frac{3}{2}, 2 \\ \hline \frac{1}{d^2 x^2} \end{array} \right)}{2\pi^{\frac{3}{2}} d} + \frac{G_{6,6}^{2,6} \left( \begin{array}{c|c} -\frac{1}{2}, 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1 & \\ \hline \frac{1}{4}, \frac{3}{4} & -\frac{1}{2}, 0, 1, 0 \\ \hline & \frac{e^{-2i\pi}}{d^2 x^2} \end{array} \right)}{2\pi^{\frac{3}{2}} d} \right) + b \left( \frac{i G_{6,6}^{5,3} \left( \begin{array}{c|c} \frac{1}{4}, \frac{3}{4}, 1 & 0, 1, \frac{3}{2} \\ \hline \frac{1}{d^2 x^2} \end{array} \right)}{2\pi^{\frac{3}{2}} d^2} - \frac{G_{6,6}^{2,6} \left( \begin{array}{c|c} -1, -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4}, 1 & \\ \hline -\frac{1}{4}, \frac{1}{4} & -1, -\frac{1}{2}, \frac{1}{2}, 0 \\ \hline & \frac{e^{-2i\pi}}{d^2 x^2} \end{array} \right)}{2\pi^{\frac{3}{2}} d^2} \right) + c \left( \frac{i G_{6,6}^{6,2} \left( \begin{array}{c|c} -\frac{1}{4}, \frac{1}{4} & -\frac{1}{2}, \frac{1}{2}, 1, 1 \\ \hline \frac{1}{d^2 x^2} \end{array} \right)}{2\pi^{\frac{3}{2}} d^3} + \frac{G_{6,6}^{2,6} \left( \begin{array}{c|c} -\frac{3}{2}, -1, -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 1 & \\ \hline -\frac{3}{4}, -\frac{1}{4} & -\frac{3}{2}, -1, 0, 0 \\ \hline & \frac{e^{-2i\pi}}{d^2 x^2} \end{array} \right)}{2\pi^{\frac{3}{2}} d^3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)/(-d\*x+1)\*\*(3/2)/(d\*x+1)\*\*(3/2),x)

[Out] a\*(-I\*meijerg(((3/4, 5/4, 1), (1/2, 3/2, 2)), ((3/4, 1, 5/4, 3/2, 2), (0,)), 1/(d\*\*2\*x\*\*2))/(2\*pi\*\*(3/2)\*d) + meijerg(((1/2, 0, 1/4, 1/2, 3/4, 1), ()), ((1/4, 3/4), (-1/2, 0, 1, 0)), exp\_polar(-2\*I\*pi)/(d\*\*2\*x\*\*2))/(2\*pi\*\*(3/2)\*d) + b\*(-I\*meijerg(((1/4, 3/4, 1), (0, 1, 3/2)), ((1/4, 1/2, 3/4, 1, 3/2), (0,)), 1/(d\*\*2\*x\*\*2))/(2\*pi\*\*(3/2)\*d\*\*2) - meijerg(((1, -1/2, -1/4, 0, 1/4, 1), ()), ((1/4, 1/4), (-1, -1/2, 1/2, 0)), exp\_polar(-2\*I\*pi)/(d\*\*2\*x\*\*2))/(2\*pi\*\*(3/2)\*d\*\*2) + c\*(I\*meijerg(((1/4, 1/4), (-1/2, 1/2, 1, 1)), ((1/4, 0, 1/4, 1/2, 1, 0), ()), 1/(d\*\*2\*x\*\*2))/(2\*pi\*\*(3/2)\*d\*\*3) + meijerg(((3/2, -1, -3/4, -1/2, -1/4, 1), ()), ((3/4, -1/4), (-3/2, -1, 0, 0)), exp\_polar(-2\*I\*pi)/(d\*\*2\*x\*\*2))/(2\*pi\*\*(3/2)\*d\*\*3))

**GIAC/XCAS [A]** time = 0.293489, size = 246, normalized size = 6.15

$$-\frac{2c \arcsin\left(\frac{1}{2}\sqrt{2}\sqrt{dx+1}\right)}{d^3} + \frac{ad^2(\sqrt{2-\sqrt{-dx+1}})}{\sqrt{dx+1}} - \frac{bd(\sqrt{2-\sqrt{-dx+1}})}{\sqrt{dx+1}} + \frac{c(\sqrt{2-\sqrt{-dx+1}})}{\sqrt{dx+1}}$$

$$-\frac{(ad^2 - bd + c)\sqrt{dx+1}}{4d^3(\sqrt{2-\sqrt{-dx+1}})} - \frac{(ad^5 + bd^4 + cd^3)\sqrt{dx+1}\sqrt{-dx+1}}{2(dx-1)d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/((d\*x + 1)^(3/2)\*(-d\*x + 1)^(3/2)),x, algorithm="giac")

[Out] -2\*c\*arcsin(1/2\*sqrt(2)\*sqrt(d\*x + 1))/d^3 + 1/4\*(a\*d^2\*(sqrt(2) - sqrt(-d\*x + 1))/sqrt(d\*x + 1) - b\*d\*(sqrt(2) - sqrt(-d\*x + 1))/sqrt(d\*x + 1) + c\*(sqrt(2) - sqrt(-d\*x + 1))/sqrt(d\*x + 1))/d^3 - 1/4\*(a\*d^2 - b\*d + c)\*sqrt(d\*x + 1)/(d^3\*(sqrt(2) - sqrt(-d\*x + 1))) - 1/2\*(a\*d^5 + b\*d^4 + c\*d^3)\*sqrt(d\*x + 1)\*sqrt(-d\*x + 1)/((d\*x - 1)\*d^6)

$$3.800 \quad \int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}(a+bx+cx^2)} dx$$

**Optimal.** Leaf size=443

$$\frac{c \left( -bd^2 \left( \sqrt{b^2 - 4ac} + b \right) + 2acd^2 + 2c^2 \right) \tanh^{-1} \left( \frac{d^2 x \left( b - \sqrt{b^2 - 4ac} \right) + 2c}{\sqrt{2} \sqrt{1-d^2 x^2} \sqrt{-bd^2 \left( b - \sqrt{b^2 - 4ac} \right) + 2acd^2 + 2c^2}} \right)}{\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{-bd^2 \left( b - \sqrt{b^2 - 4ac} \right) + 2acd^2 + 2c^2} \left( b^2 d^2 - (ad^2 + c)^2 \right)} \\ - \frac{c \left( -bd^2 \left( b - \sqrt{b^2 - 4ac} \right) + 2acd^2 + 2c^2 \right) \tanh^{-1} \left( \frac{d^2 x \left( \sqrt{b^2 - 4ac} + b \right) + 2c}{\sqrt{2} \sqrt{1-d^2 x^2} \sqrt{-bd^2 \left( \sqrt{b^2 - 4ac} + b \right) + 2acd^2 + 2c^2}} \right)}{\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{-bd^2 \left( \sqrt{b^2 - 4ac} + b \right) + 2acd^2 + 2c^2} \left( b^2 d^2 - (ad^2 + c)^2 \right)} \\ + \frac{d^2 \left( b - x \left( ad^2 + c \right) \right)}{\sqrt{1 - d^2 x^2} \left( b^2 d^2 - (ad^2 + c)^2 \right)}$$

[Out] (d^2\*(b - (c + a\*d^2)\*x))/((b^2\*d^2 - (c + a\*d^2)^2)\*Sqrt[1 - d^2\*x^2]) + (c\*(2\*c^2 + 2\*a\*c\*d^2 - b\*(b + Sqrt[b^2 - 4\*a\*c])\*d^2)\*ArcTanh[(2\*c + (b - Sqrt[b^2 - 4\*a\*c])\*d^2\*x)/(Sqrt[2]\*Sqrt[2\*c^2 + 2\*a\*c\*d^2 - b\*(b - Sqrt[b^2 - 4\*a\*c])\*d^2]\*Sqrt[1 - d^2\*x^2])])/(Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[2\*c^2 + 2\*a\*c\*d^2 - b\*(b - Sqrt[b^2 - 4\*a\*c])\*d^2]\*(b^2\*d^2 - (c + a\*d^2)^2)) - (c\*(2\*c^2 + 2\*a\*c\*d^2 - b\*(b - Sqrt[b^2 - 4\*a\*c])\*d^2)\*ArcTanh[(2\*c + (b + Sqrt[b^2 - 4\*a\*c])\*d^2\*x)/(Sqrt[2]\*Sqrt[2\*c^2 + 2\*a\*c\*d^2 - b\*(b + Sqrt[b^2 - 4\*a\*c])\*d^2]\*Sqrt[1 - d^2\*x^2])])/(Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[2\*c^2 + 2\*a\*c\*d^2 - b\*(b + Sqrt[b^2 - 4\*a\*c])\*d^2]\*(b^2\*d^2 - (c + a\*d^2)^2))

**Rubi [A]** time = 3.21421, antiderivative size = 443, normalized size of antiderivative = 1., number of rules used = 7, number of rules used = 5, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$

$$\frac{c \left( -bd^2 \left( \sqrt{b^2 - 4ac} + b \right) + 2acd^2 + 2c^2 \right) \tanh^{-1} \left( \frac{d^2 x \left( b - \sqrt{b^2 - 4ac} \right) + 2c}{\sqrt{2} \sqrt{1-d^2 x^2} \sqrt{-bd^2 \left( b - \sqrt{b^2 - 4ac} \right) + 2acd^2 + 2c^2}} \right)}{\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{-bd^2 \left( b - \sqrt{b^2 - 4ac} \right) + 2acd^2 + 2c^2} \left( b^2 d^2 - (ad^2 + c)^2 \right)} \\ - \frac{c \left( -bd^2 \left( b - \sqrt{b^2 - 4ac} \right) + 2acd^2 + 2c^2 \right) \tanh^{-1} \left( \frac{d^2 x \left( \sqrt{b^2 - 4ac} + b \right) + 2c}{\sqrt{2} \sqrt{1-d^2 x^2} \sqrt{-bd^2 \left( \sqrt{b^2 - 4ac} + b \right) + 2acd^2 + 2c^2}} \right)}{\sqrt{2} \sqrt{b^2 - 4ac} \sqrt{-bd^2 \left( \sqrt{b^2 - 4ac} + b \right) + 2acd^2 + 2c^2} \left( b^2 d^2 - (ad^2 + c)^2 \right)} \\ + \frac{d^2 \left( b - x \left( ad^2 + c \right) \right)}{\sqrt{1 - d^2 x^2} \left( b^2 d^2 - (ad^2 + c)^2 \right)}$$

Antiderivative was successfully verified.

[In] Int[1/((1 - d\*x)^(3/2)\*(1 + d\*x)^(3/2)\*(a + b\*x + c\*x^2)), x]

[Out] (d^2\*(b - (c + a\*d^2)\*x))/((b^2\*d^2 - (c + a\*d^2)^2)\*Sqrt[1 - d^2\*x^2]) + (c\*(2\*c^2 + 2\*a\*c\*d^2 - b\*(b + Sqrt[b^2 - 4\*a\*c])\*d^2)\*ArcTanh[(2\*c + (b - Sqrt[b^2 - 4\*a\*c])\*d^2\*x)/(Sqrt[2]\*Sqrt[2\*c^2 + 2\*a\*c\*d^2 - b\*(b - Sqrt[b^2 - 4\*a\*c])\*d^2]\*Sqrt[1 - d^2\*x^2])])/(Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[2\*c^2 + 2\*a\*c\*d^2 - b\*(b - Sqrt[b^2 - 4\*a\*c])\*d^2]\*(b^2\*d^2 - (c + a\*d^2)^2)) - (c\*(2\*c^2 + 2\*a\*c\*d^2 - b\*(b - Sqrt[b^2 - 4\*a\*c])\*d^2)\*ArcTanh[(2\*c + (b + Sqrt[b^2 - 4\*a\*c])\*d^2\*x)/(Sqrt[2]\*Sqrt[2\*c^2 + 2\*a\*c\*d^2 - b\*(b + Sqrt[b^2 - 4\*a\*c])\*d^2]\*Sqrt[1 - d^2\*x^2])])/(Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[2\*c^2 + 2\*a\*c\*d^2 - b\*(b + Sqrt[b^2 - 4\*a\*c])\*d^2]\*(b^2\*d^2 - (c + a\*d^2)^2))



$$\int \frac{1}{(cx^2 + bx + a)(dx + 1)^{3/2}(-dx + 1)^{3/2}} dx$$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(-d*x+1)**(3/2)/(d*x+1)**(3/2)/(c*x**2+b*x+a),x)`

[Out] Timed out

**Mathematica [A]** time = 1.60469, size = 658, normalized size = 1.49

$$c\sqrt{2-2d^2x^2} \left( -bd^2 \left( \sqrt{b^2-4ac} + b \right) + 2acd^2 + 2c^2 \right)^{3/2} \log \left( \sqrt{b^2-4ac} - b - 2cx \right) - c\sqrt{2-2d^2x^2} \left( bd^2 \left( \sqrt{b^2-4ac} - b \right) + \dots \right)$$

Antiderivative was successfully verified.

[In] `Integrate[1/((1 - d*x)^(3/2)*(1 + d*x)^(3/2)*(a + b*x + c*x^2)),x]`

[Out] 
$$\frac{-2\sqrt{b^2-4ac}d^2\sqrt{2c^2+2acd^2+b(-b+\sqrt{b^2-4ac})d^2}\sqrt{2c^2+2acd^2-b(b+\sqrt{b^2-4ac})d^2}(b-(c+ad^2)x)+c(2c^2+2acd^2-b(b+\sqrt{b^2-4ac})d^2)^{3/2}\sqrt{2-2d^2x^2}\log[-b+\sqrt{b^2-4ac}-2cx]-c(2c^2+2acd^2+b(-b+\sqrt{b^2-4ac})d^2)^{3/2}\sqrt{2-2d^2x^2}\log[b+\sqrt{b^2-4ac}+2cx]-c(2c^2+2acd^2-b(b+\sqrt{b^2-4ac})d^2)^{3/2}\sqrt{2-2d^2x^2}\log[-2c-bd^2x+\sqrt{b^2-4ac}d^2x-\sqrt{4c^2+4acd^2+2b(-b+\sqrt{b^2-4ac})d^2}]\sqrt{1-d^2x^2}+c(2c^2+2acd^2+b(-b+\sqrt{b^2-4ac})d^2)^{3/2}\sqrt{2-2d^2x^2}\log[2c+bd^2x+\sqrt{b^2-4ac}d^2x+\sqrt{4c^2+4acd^2-2b(b+\sqrt{b^2-4ac})d^2}]\sqrt{1-d^2x^2}}{(2\sqrt{b^2-4ac}d^2\sqrt{2c^2+2acd^2+b(-b+\sqrt{b^2-4ac})d^2}\sqrt{2c^2+2acd^2-b(b+\sqrt{b^2-4ac})d^2}(c^2-b^2d^2+2acd^2+a^2d^4)\sqrt{1-d^2x^2})}$$

**Maple [C]** time = 0.13, size = 11142, normalized size = 25.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a),x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx + a)(dx + 1)^{3/2}(-dx + 1)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((c*x^2 + b*x + a)*(d*x + 1)^(3/2)*(-d*x + 1)^(3/2)),x, algorithm="ma`

[Out] `integrate(1/((c*x^2 + b*x + a)*(d*x + 1)^(3/2)*(-d*x + 1)^(3/2)),  
x)`

**Fricas** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((c*x^2 + b*x + a)*(d*x + 1)^(3/2)*(-d*x + 1)^(3/2)),x, algorithm="fr`

[Out] Timed out

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(-d*x+1)**(3/2)/(d*x+1)**(3/2)/(c*x**2+b*x+a),x)`

[Out] Timed out

**GIAC/XCAS** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((c*x^2 + b*x + a)*(d*x + 1)^(3/2)*(-d*x + 1)^(3/2)),x, algorithm="gi`

[Out] Timed out

$$3.801 \quad \int \frac{1}{(1-dx)^{3/2}(1+dx)^{3/2}(a+bx+cx^2)^2} dx$$

**Optimal.** Leaf size=939

$$\frac{(b(3ab^2d^4 - 11a^2cd^4 - 10ac^2d^2 + 2b^2cd^2 + c^3) - (2c^4 - d^2(b^2 + 6a^2d^2))c^2 - (4a^3d^6 + 6ab^2d^4)c + b^2d^4(2b^2 + a^2d^2))x}{(b^2 - 4ac)(ad^2 - bd + c)^2(ad^2 + bd + c)^2\sqrt{1 - d^2x^2}}$$

$$+ \frac{c(3ab^3(b + \sqrt{b^2 - 4ac})d^6 - 2ac^2(7b^2 + 5\sqrt{b^2 - 4ac}b - 8a^2d^2)d^4 + bc(2b^3 + 2\sqrt{b^2 - 4ac}b^2 - 17a^2d^2b - 11a^2\sqrt{b^2 - 4ac}))}{\sqrt{2}(b^2 - 4ac)^{3/2}\sqrt{2c^2 + 2ad^2c - b(b - \sqrt{b^2 - 4ac})}}$$

$$+ \frac{b(b^2d^2 - c(3ad^2 + c)) - c(2c^2 + 2ad^2c - b^2d^2)x}{(b^2 - 4ac)(b^2d^2 - (ad^2 + c)^2)(cx^2 + bx + a)\sqrt{1 - d^2x^2}}$$

$$+ \frac{c(b(b + \sqrt{b^2 - 4ac})d^4(3ab^2d^4 - 11a^2cd^4 - 10ac^2d^2 + 2b^2cd^2 + c^3) - 2(3ab^4d^8 + 2b^2c(b^2 - 7a^2d^2)d^6 + 12ac^4d^4 + 2c^5d^2))}{\sqrt{2}(b^2 - 4ac)^{3/2}\sqrt{2c^2 + 2ad^2c - b(b + \sqrt{b^2 - 4ac})}d^2}$$

[Out]  $-\left(\frac{d^2(b(c^3 + 2b^2c^2d^2 - 10a^2c^2d^2 + 3a^2b^2d^4 - 11a^2c^2d^4) - (2c^4 + b^2d^4(2b^2 + a^2d^2))c^2 - c^2d^2(b^2 + 6a^2d^2)) - c(6a^3b^2d^4 + 4a^3d^6)}{(b^2 - 4ac)(c - b^2d + a^2d^2)^2(c + b^2d + a^2d^2)^2\sqrt{1 - d^2x^2}}\right) - \frac{b(b^2d^2 - c(3ad^2 + c)) - c(2c^2 + 2ad^2c - b^2d^2)x}{(b^2 - 4ac)(b^2d^2 - (ad^2 + c)^2)(cx^2 + bx + a)\sqrt{1 - d^2x^2}}$

**Rubi [A]** time = 18.1343, antiderivative size = 938, normalized size of antiderivative = 1., number of rules used = 8, number of rules used = 6, integrand size = 32,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$

$$\frac{(b(3ab^2d^4 - 11a^2cd^4 - 10ac^2d^2 + 2b^2cd^2 + c^3) - (2c^4 - d^2(b^2 + 6a^2d^2))c^2 - (4a^3d^6 + 6ab^2d^4)c + b^2d^4(2b^2 + a^2d^2))x}{(b^2 - 4ac)(c - d(b - ad))^2(c + d(b + ad))^2\sqrt{1 - d^2x^2}}$$

$$+ \frac{c(3ab^3(b + \sqrt{b^2 - 4ac})d^6 - 2ac^2(7b^2 + 5\sqrt{b^2 - 4ac}b - 8a^2d^2)d^4 + bc(2b^3 + 2\sqrt{b^2 - 4ac}b^2 - 17a^2d^2b - 11a^2\sqrt{b^2 - 4ac}))}{\sqrt{2}(b^2 - 4ac)^{3/2}\sqrt{2c^2 + 2ad^2c - b(b - \sqrt{b^2 - 4ac})}}$$

$$+ \frac{b(b^2d^2 - c(3ad^2 + c)) - c(2c^2 + 2ad^2c - b^2d^2)x}{(b^2 - 4ac)(b^2d^2 - (ad^2 + c)^2)(cx^2 + bx + a)\sqrt{1 - d^2x^2}}$$

$$+ \frac{c(6ab^4d^8 + 4b^2c(b^2 - 7a^2d^2)d^6 + 24ac^4d^4 - b(b + \sqrt{b^2 - 4ac})(3ab^2d^4 - 11a^2cd^4 - 10ac^2d^2 + 2b^2cd^2 + c^3)d^4 + 4c^5d^2)}{\sqrt{2}(b^2 - 4ac)^{3/2}\sqrt{2c^2 + 2ad^2c - b(b + \sqrt{b^2 - 4ac})}d^2}$$

Warning: Unable to verify antiderivative.

[In] Int[1/((1 - d\*x)^(3/2)\*(1 + d\*x)^(3/2)\*(a + b\*x + c\*x^2)^2), x]

[Out] 
$$-\left(\frac{d^2(b(c^3 + 2b^2c^2d^2 - 10ac^2d^2 + 3ab^2d^4 - 11a^2c^2d^4) - (2c^4 + b^2d^4(2b^2 + a^2d^2) - c^2d^2(b^2 + 6a^2d^2) - c(6ab^2d^4 + 4a^3d^6))x)}{(b^2 - 4ac)(c - d(b - ad))^2(c + d(b + ad))^2\sqrt{1 - d^2x^2}}\right) - \frac{b(b^2d^2 - c^2(c + 3ad^2) - c(2c^2 - b^2d^2 + 2ac^2d^2)x)}{(b^2 - 4ac)(b^2d^2 - (c + ad^2)^2)(a + bx + cx^2)\sqrt{1 - d^2x^2}} + \frac{c(4c^5 + 24ac^4d^2 + 3ab^3(b + \sqrt{b^2 - 4ac})d^6 - c^3d^2(9b^2 - b\sqrt{b^2 - 4ac}) - 36a^2d^2) - 2ac^2d^4(7b^2 + 5b\sqrt{b^2 - 4ac}) - 8a^2d^2 + b^2cd^4(2b^3 + 2b^2\sqrt{b^2 - 4ac}) - 17a^2bd^2 - 11a^2\sqrt{b^2 - 4ac}d^2)}{(2c + (b - \sqrt{b^2 - 4ac})d^2x)(\sqrt{2}\sqrt{2c^2 + 2ac^2d^2 - b(b - \sqrt{b^2 - 4ac})d^2})\sqrt{1 - d^2x^2}} + \frac{c(4c^5d^2 + 24ac^4d^4 + 6ab^4d^8 + 4b^2c^2d^6(b^2 - 7a^2d^2) - b(b + \sqrt{b^2 - 4ac})d^4(c^3 + 2b^2c^2d^2 - 10ac^2d^2 + 3ab^2d^4 - 11a^2c^2d^4) - 4c^3(2b^2d^4 - 9a^2d^6) - 8c^2(3ab^2d^6 - 2a^3d^8))}{(2c + (b + \sqrt{b^2 - 4ac})d^2x)(\sqrt{2}\sqrt{2c^2 + 2ac^2d^2 - b(b + \sqrt{b^2 - 4ac})d^2})\sqrt{1 - d^2x^2}} + \frac{c(3ab^4d^6 + 16a^3c^2d^6 - 17a^2b^2cd^6 + 3ab^3\sqrt{b^2 - 4ac}d^6 - 11a^2bc\sqrt{b^2 - 4ac}d^6 + 36a^2c^3d^4 - 14ab^2c^2d^4 + 2b^4cd^4 - 10abc^2\sqrt{2}d^4 - 10a^2c^2d^4)}{\sqrt{2}(b^2 - 4ac)^{3/2}\sqrt{2c^2 + 2ad^2c - b^2d^2 + b\sqrt{b^2 - 4ac}}}$$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(-d\*x+1)\*\*(3/2)/(d\*x+1)\*\*(3/2)/(c\*x\*\*2+b\*x+a)\*\*2, x)

[Out] Timed out

**Mathematica [A]** time = 6.66337, size = 1656, normalized size = 1.76

$$\frac{\sqrt{1 - d^2x^2} \left( \frac{-d^4b^5 - cd^4xb^4 + 5acd^4b^3 + 2c^2d^2b^3 + 4ac^2d^4xb^2 + 2c^3d^2xb^2 - c^4b - 5a^2c^2d^4b - 6ac^3d^2b - 2c^5x - 2a^2c^3d^4x - a^2xd^8 + 2abd^6 - b^2xd^6 - 2acxd^6 + 2bcd^4 - c^2xd^4}{(b^2 - 4ac)(-a^2d^4 + b^2d^2 - 2acd^2 - c^2)^2(cx^2 + bx + a)} \right) + \frac{c(3ab^4d^6 + 16a^3c^2d^6 - 17a^2b^2cd^6 + 3ab^3\sqrt{b^2 - 4acd^6} - 11a^2bc\sqrt{b^2 - 4acd^6} + 36a^2c^3d^4 - 14ab^2c^2d^4 + 2b^4cd^4 - 10abc^2\sqrt{2}d^4 - 10a^2c^2d^4)}{\sqrt{2}(b^2 - 4ac)^{3/2}\sqrt{2c^2 + 2ad^2c - b^2d^2 + b\sqrt{b^2 - 4ac}}}}{\sqrt{2}(b^2 - 4ac)^{3/2}\sqrt{2c^2 + 2ad^2c - b^2d^2 - b\sqrt{b^2 - 4ac}}}$$

$$+ \frac{c(-3ab^4d^6 - 16a^3c^2d^6 + 17a^2b^2cd^6 + 3ab^3\sqrt{b^2 - 4acd^6} - 11a^2bc\sqrt{b^2 - 4acd^6} - 36a^2c^3d^4 + 14ab^2c^2d^4 - 2b^4cd^4 - 10abc^2\sqrt{2}d^4 - 10a^2c^2d^4)}{\sqrt{2}(b^2 - 4ac)^{3/2}\sqrt{2c^2 + 2ad^2c - b^2d^2 - b\sqrt{b^2 - 4ac}}}$$

$$+ \frac{c(3ab^4d^6 + 16a^3c^2d^6 - 17a^2b^2cd^6 + 3ab^3\sqrt{b^2 - 4acd^6} - 11a^2bc\sqrt{b^2 - 4acd^6} + 36a^2c^3d^4 - 14ab^2c^2d^4 + 2b^4cd^4 - 10abc^2\sqrt{2}d^4 - 10a^2c^2d^4)}{\sqrt{2}(b^2 - 4ac)^{3/2}\sqrt{2c^2 + 2ad^2c - b^2d^2 - b\sqrt{b^2 - 4ac}}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((1 - d\*x)^(3/2)\*(1 + d\*x)^(3/2)\*(a + b\*x + c\*x^2)^2), x]

```
[Out] Sqrt[1 - d^2*x^2]*((- (b*c^4) + 2*b^3*c^2*d^2 - 6*a*b*c^3*d^2 - b^5*d^4 + 5*a*b^3*c*d^4 - 5*a^2*b*c^2*d^4 - 2*c^5*x + 2*b^2*c^3*d^2*x - 4*a*c^4*d^2*x - b^4*c*d^4*x + 4*a*b^2*c^2*d^4*x - 2*a^2*c^3*d^4*x)/((b^2 - 4*a*c)*(-c^2 + b^2*d^2 - 2*a*c*d^2 - a^2*d^4)^2*(a + b*x + c*x^2)) + (2*b*c*d^4 + 2*a*b*d^6 - c^2*d^4*x - b^2*d^6*x - 2*a*c*d^6*x - a^2*d^8*x)/((c^2 - b^2*d^2 + 2*a*c*d^2 + a^2*d^4)^2*(-1 + d^2*x^2)) - (c*(4*c^5 - 9*b^2*c^3*d^2 + 24*a*c^4*d^2 + b*c^3*Sqrt[b^2 - 4*a*c]*d^2 + 2*b^4*c*d^4 - 14*a*b^2*c^2*d^4 + 36*a^2*c^3*d^4 + 2*b^3*c*Sqrt[b^2 - 4*a*c]*d^4 - 10*a*b*c^2*Sqrt[b^2 - 4*a*c]*d^4 + 3*a*b^4*d^6 - 17*a^2*b^2*c*d^6 + 16*a^3*c^2*d^6 + 3*a*b^3*Sqrt[b^2 - 4*a*c]*d^6 - 11*a^2*b*c*Sqrt[b^2 - 4*a*c]*d^6)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x])/((Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[2*c^2 - b^2*d^2 + 2*a*c*d^2 + b*Sqrt[b^2 - 4*a*c]*d^2]*(-c^2 + b^2*d^2 - 2*a*c*d^2 - a^2*d^4)^2) - (c*(-4*c^5 + 9*b^2*c^3*d^2 - 24*a*c^4*d^2 + b*c^3*Sqrt[b^2 - 4*a*c]*d^2 - 2*b^4*c*d^4 + 14*a*b^2*c^2*d^4 - 36*a^2*c^3*d^4 + 2*b^3*c*Sqrt[b^2 - 4*a*c]*d^4 - 10*a*b*c^2*Sqrt[b^2 - 4*a*c]*d^4 - 3*a*b^4*d^6 + 17*a^2*b^2*c*d^6 - 16*a^3*c^2*d^6 + 3*a*b^3*Sqrt[b^2 - 4*a*c]*d^6 - 11*a^2*b*c*Sqrt[b^2 - 4*a*c]*d^6)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x])/((Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[2*c^2 - b^2*d^2 + 2*a*c*d^2 - b*Sqrt[b^2 - 4*a*c]*d^2]*(-c^2 + b^2*d^2 - 2*a*c*d^2 - a^2*d^4)^2) + (c*(-4*c^5 + 9*b^2*c^3*d^2 - 24*a*c^4*d^2 + b*c^3*Sqrt[b^2 - 4*a*c]*d^2 - 2*b^4*c*d^4 + 14*a*b^2*c^2*d^4 - 36*a^2*c^3*d^4 + 2*b^3*c*Sqrt[b^2 - 4*a*c]*d^4 - 10*a*b*c^2*Sqrt[b^2 - 4*a*c]*d^4 - 3*a*b^4*d^6 + 17*a^2*b^2*c*d^6 - 16*a^3*c^2*d^6 + 3*a*b^3*Sqrt[b^2 - 4*a*c]*d^6 - 11*a^2*b*c*Sqrt[b^2 - 4*a*c]*d^6)*Log[2*c + b*d^2*x + Sqrt[b^2 - 4*a*c]*d^2*x + Sqrt[2]*Sqrt[2*c^2 - b^2*d^2 + 2*a*c*d^2 - b*Sqrt[b^2 - 4*a*c]*d^2]*Sqrt[1 - d^2*x^2])]/((Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[2*c^2 - b^2*d^2 + 2*a*c*d^2 - b*Sqrt[b^2 - 4*a*c]*d^2]*(-c^2 + b^2*d^2 - 2*a*c*d^2 - a^2*d^4)^2) + (c*(4*c^5 - 9*b^2*c^3*d^2 + 24*a*c^4*d^2 + b*c^3*Sqrt[b^2 - 4*a*c]*d^2 + 2*b^4*c*d^4 - 14*a*b^2*c^2*d^4 + 36*a^2*c^3*d^4 + 2*b^3*c*Sqrt[b^2 - 4*a*c]*d^4 - 10*a*b*c^2*Sqrt[b^2 - 4*a*c]*d^4 + 3*a*b^4*d^6 - 17*a^2*b^2*c*d^6 + 16*a^3*c^2*d^6 + 3*a*b^3*Sqrt[b^2 - 4*a*c]*d^6 - 11*a^2*b*c*Sqrt[b^2 - 4*a*c]*d^6)*Log[-2*c - b*d^2*x + Sqrt[b^2 - 4*a*c]*d^2*x - Sqrt[2]*Sqrt[2*c^2 - b^2*d^2 + 2*a*c*d^2 + b*Sqrt[b^2 - 4*a*c]*d^2]*Sqrt[1 - d^2*x^2])]/((Sqrt[2]*(b^2 - 4*a*c)^(3/2)*Sqrt[2*c^2 - b^2*d^2 + 2*a*c*d^2 + b*Sqrt[b^2 - 4*a*c]*d^2]*(-c^2 + b^2*d^2 - 2*a*c*d^2 - a^2*d^4)^2)
```

---

**Maple [C]** time = 3.458, size = 108974, normalized size = 116.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-d*x+1)^(3/2)/(d*x+1)^(3/2)/(c*x^2+b*x+a)^2,x)
```

```
[Out] result too large to display
```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx + a)^2(dx + 1)^{\frac{3}{2}}(-dx + 1)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^2 + b*x + a)^2*(d*x + 1)^(3/2)*(-d*x + 1)^(3/2)),x, algorithm="
```

```
[Out] integrate(1/((c*x^2 + b*x + a)^2*(d*x + 1)^(3/2)*(-d*x + 1)^(3/2)), x)
```

---

**Fricas** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + b\*x + a)^2\*(d\*x + 1)^(3/2)\*(-d\*x + 1)^(3/2)),x, algorithm="

[Out] Timed out

---

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(-d\*x+1)\*\*(3/2)/(d\*x+1)\*\*(3/2)/(c\*x\*\*2+b\*x+a)\*\*2,x)

[Out] Timed out

---

**GIAC/XCAS** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + b\*x + a)^2\*(d\*x + 1)^(3/2)\*(-d\*x + 1)^(3/2)),x, algorithm="

[Out] Timed out

$$3.802 \quad \int (1 - ex)^m (1 + ex)^m (a + cx^2)^p dx$$

**Optimal.** Leaf size=54

$$x (a + cx^2)^p \left( \frac{cx^2}{a} + 1 \right)^{-p} F_1 \left( \frac{1}{2}; -p, -m; \frac{3}{2}; -\frac{cx^2}{a}, e^2 x^2 \right)$$

[Out]  $(x*(a + c*x^2)^p*AppellF1[1/2, -p, -m, 3/2, -((c*x^2)/a), e^2*x^2])/(1 + (c*x^2)/a)^p$

**Rubi [A]** time = 0.113934, antiderivative size = 54, normalized size of antiderivative = 1., number of rules used = 3, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$x (a + cx^2)^p \left( \frac{cx^2}{a} + 1 \right)^{-p} F_1 \left( \frac{1}{2}; -p, -m; \frac{3}{2}; -\frac{cx^2}{a}, e^2 x^2 \right)$$

Antiderivative was successfully verified.

[In] Int[(1 - e\*x)^m\*(1 + e\*x)^m\*(a + c\*x^2)^p, x]

[Out]  $(x*(a + c*x^2)^p*AppellF1[1/2, -p, -m, 3/2, -((c*x^2)/a), e^2*x^2])/(1 + (c*x^2)/a)^p$

**Rubi in Sympy [A]** time = 23.4026, size = 41, normalized size = 0.76

$$x \left( 1 + \frac{cx^2}{a} \right)^{-p} (a + cx^2)^p \text{appellf1} \left( \frac{1}{2}, -m, -p, \frac{3}{2}, e^2 x^2, -\frac{cx^2}{a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*x+1)\*\*m\*(e\*x+1)\*\*m\*(c\*x\*\*2+a)\*\*p, x)

[Out]  $x*(1 + c*x**2/a)**(-p)*(a + c*x**2)**p*appellf1(1/2, -m, -p, 3/2, e**2*x**2, -c*x**2/a)$

**Mathematica [B]** time = 0.35557, size = 167, normalized size = 3.09

$$\frac{3ax (1 - e^2 x^2)^m (a + cx^2)^p F_1 \left( \frac{1}{2}; -p, -m; \frac{3}{2}; -\frac{cx^2}{a}, e^2 x^2 \right)}{2x^2 \left( cpF_1 \left( \frac{3}{2}; 1 - p, -m; \frac{5}{2}; -\frac{cx^2}{a}, e^2 x^2 \right) - ae^2 mF_1 \left( \frac{3}{2}; -p, 1 - m; \frac{5}{2}; -\frac{cx^2}{a}, e^2 x^2 \right) \right) + 3aF_1 \left( \frac{1}{2}; -p, -m; \frac{3}{2}; -\frac{cx^2}{a}, e^2 x^2 \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(1 - e\*x)^m\*(1 + e\*x)^m\*(a + c\*x^2)^p, x]

[Out]  $(3*a*x*(a + c*x^2)^p*(1 - e^2*x^2)^m*AppellF1[1/2, -p, -m, 3/2, -((c*x^2)/a), e^2*x^2])/(3*a*AppellF1[1/2, -p, -m, 3/2, -((c*x^2)/a), e^2*x^2] + 2*x^2*(c*p*AppellF1[3/2, 1 - p, -m, 5/2, -((c*x^2)/a), e^2*x^2] - a*e^2*m*AppellF1[3/2, -p, 1 - m, 5/2, -((c*x^2)/a), e^2*x^2]))$

**Maple [F]** time = 0.226, size = 0, normalized size = 0.

$$\int (-ex + 1)^m (ex + 1)^m (cx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-e\*x+1)^m\*(e\*x+1)^m\*(c\*x^2+a)^p,x)

[Out] int((-e\*x+1)^m\*(e\*x+1)^m\*(c\*x^2+a)^p,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (cx^2 + a)^p (ex + 1)^m (-ex + 1)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)^p\*(e\*x + 1)^m\*(-e\*x + 1)^m,x, algorithm="maxima")

[Out] integrate((c\*x^2 + a)^p\*(e\*x + 1)^m\*(-e\*x + 1)^m, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((cx^2 + a)^p (ex + 1)^m (-ex + 1)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)^p\*(e\*x + 1)^m\*(-e\*x + 1)^m,x, algorithm="fricas")

[Out] integral((c\*x^2 + a)^p\*(e\*x + 1)^m\*(-e\*x + 1)^m, x)

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-e\*x+1)\*\*m\*(e\*x+1)\*\*m\*(c\*x\*\*2+a)\*\*p,x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (cx^2 + a)^p (ex + 1)^m (-ex + 1)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + a)^p\*(e\*x + 1)^m\*(-e\*x + 1)^m,x, algorithm="giac")

[Out] integrate((c\*x^2 + a)^p\*(e\*x + 1)^m\*(-e\*x + 1)^m, x)



### 3.803 $\int (d - ex)^m (d + ex)^m (a + cx^2)^p dx$

**Optimal.** Leaf size=89

$$x (a + cx^2)^p \left( \frac{cx^2}{a} + 1 \right)^{-p} (d - ex)^m (d + ex)^m \left( 1 - \frac{e^2 x^2}{d^2} \right)^{-m} F_1 \left( \frac{1}{2}; -p, -m; \frac{3}{2}; -\frac{cx^2}{a}, \frac{e^2 x^2}{d^2} \right)$$

[Out] (x\*(d - e\*x)^m\*(d + e\*x)^m\*(a + c\*x^2)^p\*AppellF1[1/2, -p, -m, 3/2, -((c\*x^2)/a), (e^2\*x^2)/d^2])/((1 + (c\*x^2)/a)^p\*(1 - (e^2\*x^2)/d^2)^m)

**Rubi [A]** time = 0.177625, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$x (a + cx^2)^p \left( \frac{cx^2}{a} + 1 \right)^{-p} (d - ex)^m (d + ex)^m \left( 1 - \frac{e^2 x^2}{d^2} \right)^{-m} F_1 \left( \frac{1}{2}; -p, -m; \frac{3}{2}; -\frac{cx^2}{a}, \frac{e^2 x^2}{d^2} \right)$$

Antiderivative was successfully verified.

[In] Int[(d - e\*x)^m\*(d + e\*x)^m\*(a + c\*x^2)^p,x]

[Out] (x\*(d - e\*x)^m\*(d + e\*x)^m\*(a + c\*x^2)^p\*AppellF1[1/2, -p, -m, 3/2, -((c\*x^2)/a), (e^2\*x^2)/d^2])/((1 + (c\*x^2)/a)^p\*(1 - (e^2\*x^2)/d^2)^m)

**Rubi in Sympy [A]** time = 41.34, size = 71, normalized size = 0.8

$$x \left( 1 + \frac{cx^2}{a} \right)^{-p} \left( 1 - \frac{e^2 x^2}{d^2} \right)^{-m} (a + cx^2)^p (d - ex)^m (d + ex)^m \text{appellf}_1 \left( \frac{1}{2}, -m, -p, \frac{3}{2}, \frac{e^2 x^2}{d^2}, -\frac{cx^2}{a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-e\*x+d)\*\*m\*(e\*x+d)\*\*m\*(c\*x\*\*2+a)\*\*p,x)

[Out] x\*(1 + c\*x\*\*2/a)\*\*(-p)\*(1 - e\*\*2\*x\*\*2/d\*\*2)\*\*(-m)\*(a + c\*x\*\*2)\*\*p\*(d - e\*x)\*\*m\*(d + e\*x)\*\*m\*appellf1(1/2, -m, -p, 3/2, e\*\*2\*x\*\*2/d\*\*2, -c\*x\*\*2/a)

**Mathematica [A]** time = 0.138309, size = 0, normalized size = 0.

$$\int (d - ex)^m (d + ex)^m (a + cx^2)^p dx$$

Verification is Not applicable to the result.

[In] Integrate[(d - e\*x)^m\*(d + e\*x)^m\*(a + c\*x^2)^p,x]

[Out] Integrate[(d - e\*x)^m\*(d + e\*x)^m\*(a + c\*x^2)^p, x]

**Maple [F]** time = 0.219, size = 0, normalized size = 0.

$$\int (-ex + d)^m (ex + d)^m (cx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x)`

[Out] `int((-e*x+d)^m*(e*x+d)^m*(c*x^2+a)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (cx^2 + a)^p (ex + d)^m (-ex + d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)^p*(e*x + d)^m*(-e*x + d)^m,x, algorithm="maxima")`

[Out] `integrate((c*x^2 + a)^p*(e*x + d)^m*(-e*x + d)^m, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((cx^2 + a)^p (ex + d)^m (-ex + d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)^p*(e*x + d)^m*(-e*x + d)^m,x, algorithm="fricas")`

[Out] `integral((c*x^2 + a)^p*(e*x + d)^m*(-e*x + d)^m, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e*x+d)**m*(e*x+d)**m*(c*x**2+a)**p,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (cx^2 + a)^p (ex + d)^m (-ex + d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + a)^p*(e*x + d)^m*(-e*x + d)^m,x, algorithm="giac")`

[Out] `integrate((c*x^2 + a)^p*(e*x + d)^m*(-e*x + d)^m, x)`

$$3.804 \quad \int (d + ex)^m (df - efx)^m (a + cx^2)^p dx$$

**Optimal.** Leaf size=92

$$x (a + cx^2)^p \left( \frac{cx^2}{a} + 1 \right)^{-p} (d + ex)^m \left( 1 - \frac{e^2 x^2}{d^2} \right)^{-m} (df - efx)^m F_1 \left( \frac{1}{2}; -p, -m; \frac{3}{2}; -\frac{cx^2}{a}, \frac{e^2 x^2}{d^2} \right)$$

[Out] (x\*(d + e\*x)^m\*(d\*f - e\*f\*x)^m\*(a + c\*x^2)^p\*AppellF1[1/2, -p, -m, 3/2, -((c\*x^2)/a), (e^2\*x^2)/d^2])/((1 + (c\*x^2)/a)^p\*(1 - (e^2\*x^2)/d^2)^m)

**Rubi [A]** time = 0.193132, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$

$$x (a + cx^2)^p \left( \frac{cx^2}{a} + 1 \right)^{-p} (d + ex)^m \left( 1 - \frac{e^2 x^2}{d^2} \right)^{-m} (df - efx)^m F_1 \left( \frac{1}{2}; -p, -m; \frac{3}{2}; -\frac{cx^2}{a}, \frac{e^2 x^2}{d^2} \right)$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^m\*(d\*f - e\*f\*x)^m\*(a + c\*x^2)^p, x]

[Out] (x\*(d + e\*x)^m\*(d\*f - e\*f\*x)^m\*(a + c\*x^2)^p\*AppellF1[1/2, -p, -m, 3/2, -((c\*x^2)/a), (e^2\*x^2)/d^2])/((1 + (c\*x^2)/a)^p\*(1 - (e^2\*x^2)/d^2)^m)

**Rubi in Sympy [A]** time = 48.8009, size = 75, normalized size = 0.82

$$x \left( 1 + \frac{cx^2}{a} \right)^{-p} \left( 1 - \frac{e^2 x^2}{d^2} \right)^{-m} (a + cx^2)^p (d + ex)^m (df - efx)^m \text{appellf}_1 \left( \frac{1}{2}, -m, -p, \frac{3}{2}, \frac{e^2 x^2}{d^2}, -\frac{cx^2}{a} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*m\*(-e\*f\*x+d\*f)\*\*m\*(c\*x\*\*2+a)\*\*p, x)

[Out] x\*(1 + c\*x\*\*2/a)\*\*(-p)\*(1 - e\*\*2\*x\*\*2/d\*\*2)\*\*(-m)\*(a + c\*x\*\*2)\*\*p\*(d + e\*x)\*\*m\*(d\*f - e\*f\*x)\*\*m\*appellf1(1/2, -m, -p, 3/2, e\*\*2\*x\*\*2/d\*\*2, -c\*x\*\*2/a)

**Mathematica [A]** time = 0.116997, size = 0, normalized size = 0.

$$\int (d + ex)^m (df - efx)^m (a + cx^2)^p dx$$

Verification is Not applicable to the result.

[In] Integrate[(d + e\*x)^m\*(d\*f - e\*f\*x)^m\*(a + c\*x^2)^p, x]

[Out] Integrate[(d + e\*x)^m\*(d\*f - e\*f\*x)^m\*(a + c\*x^2)^p, x]

**Maple [F]** time = 0.23, size = 0, normalized size = 0.

$$\int (ex + d)^m (-efx + df)^m (cx^2 + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x)`

[Out] `int((e*x+d)^m*(-e*f*x+d*f)^m*(c*x^2+a)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (-efx + df)^m (cx^2 + a)^p (ex + d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e*f*x + d*f)^m*(c*x^2 + a)^p*(e*x + d)^m,x, algorithm="maxima")`

[Out] `integrate((-e*f*x + d*f)^m*(c*x^2 + a)^p*(e*x + d)^m, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((-efx + df)^m (cx^2 + a)^p (ex + d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e*f*x + d*f)^m*(c*x^2 + a)^p*(e*x + d)^m,x, algorithm="fricas")`

[Out] `integral((-e*f*x + d*f)^m*(c*x^2 + a)^p*(e*x + d)^m, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**m*(-e*f*x+d*f)**m*(c*x**2+a)**p,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (-efx + df)^m (cx^2 + a)^p (ex + d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((-e*f*x + d*f)^m*(c*x^2 + a)^p*(e*x + d)^m,x, algorithm="giac")`

[Out] `integrate((-e*f*x + d*f)^m*(c*x^2 + a)^p*(e*x + d)^m, x)`

### 3.805 $\int (d + ex)^3 (f + gx)^n (a + 2cdx + cex^2) dx$

**Optimal.** Leaf size=275

$$\begin{aligned} & \frac{(ef - dg)^2 (f + gx)^{n+2} (3aeg^2 + c(2d^2g^2 - 10defg + 5e^2f^2))}{g^6(n+2)} \\ & - \frac{e(ef - dg)(f + gx)^{n+3} (3aeg^2 + c(7d^2g^2 - 20defg + 10e^2f^2))}{g^6(n+3)} \\ & + \frac{e^2(f + gx)^{n+4} (aeg^2 + c(9d^2g^2 - 20defg + 10e^2f^2))}{g^6(n+4)} \\ & - \frac{(ef - dg)^3 (f + gx)^{n+1} (ag^2 + cf(ef - 2dg))}{g^6(n+1)} - \frac{5ce^3(ef - dg)(f + gx)^{n+5}}{g^6(n+5)} + \frac{ce^4(f + gx)^{n+6}}{g^6(n+6)} \end{aligned}$$

[Out] -(((e\*f - d\*g)^3\*(a\*g^2 + c\*f\*(e\*f - 2\*d\*g))\*(f + g\*x)^(1 + n))/(g^6\*(1 + n))) + ((e\*f - d\*g)^2\*(3\*a\*e\*g^2 + c\*(5\*e^2\*f^2 - 10\*d\*e\*f\*g + 2\*d^2\*g^2))\*(f + g\*x)^(2 + n))/(g^6\*(2 + n)) - (e\*(e\*f - d\*g)\*(3\*a\*e\*g^2 + c\*(10\*e^2\*f^2 - 20\*d\*e\*f\*g + 7\*d^2\*g^2))\*(f + g\*x)^(3 + n))/(g^6\*(3 + n)) + (e^2\*(a\*e\*g^2 + c\*(10\*e^2\*f^2 - 20\*d\*e\*f\*g + 9\*d^2\*g^2))\*(f + g\*x)^(4 + n))/(g^6\*(4 + n)) - (5\*c\*e^3\*(e\*f - d\*g)\*(f + g\*x)^(5 + n))/(g^6\*(5 + n)) + (c\*e^4\*(f + g\*x)^(6 + n))/(g^6\*(6 + n))

**Rubi [A]** time = 0.568992, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$

$$\begin{aligned} & \frac{(ef - dg)^2 (f + gx)^{n+2} (3aeg^2 + c(2d^2g^2 - 10defg + 5e^2f^2))}{g^6(n+2)} \\ & - \frac{e(ef - dg)(f + gx)^{n+3} (3aeg^2 + c(7d^2g^2 - 20defg + 10e^2f^2))}{g^6(n+3)} \\ & + \frac{e^2(f + gx)^{n+4} (aeg^2 + c(9d^2g^2 - 20defg + 10e^2f^2))}{g^6(n+4)} \\ & - \frac{(ef - dg)^3 (f + gx)^{n+1} (ag^2 + cf(ef - 2dg))}{g^6(n+1)} - \frac{5ce^3(ef - dg)(f + gx)^{n+5}}{g^6(n+5)} + \frac{ce^4(f + gx)^{n+6}}{g^6(n+6)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^3\*(f + g\*x)^n\*(a + 2\*c\*d\*x + c\*e\*x^2), x]

[Out] -(((e\*f - d\*g)^3\*(a\*g^2 + c\*f\*(e\*f - 2\*d\*g))\*(f + g\*x)^(1 + n))/(g^6\*(1 + n))) + ((e\*f - d\*g)^2\*(3\*a\*e\*g^2 + c\*(5\*e^2\*f^2 - 10\*d\*e\*f\*g + 2\*d^2\*g^2))\*(f + g\*x)^(2 + n))/(g^6\*(2 + n)) - (e\*(e\*f - d\*g)\*(3\*a\*e\*g^2 + c\*(10\*e^2\*f^2 - 20\*d\*e\*f\*g + 7\*d^2\*g^2))\*(f + g\*x)^(3 + n))/(g^6\*(3 + n)) + (e^2\*(a\*e\*g^2 + c\*(10\*e^2\*f^2 - 20\*d\*e\*f\*g + 9\*d^2\*g^2))\*(f + g\*x)^(4 + n))/(g^6\*(4 + n)) - (5\*c\*e^3\*(e\*f - d\*g)\*(f + g\*x)^(5 + n))/(g^6\*(5 + n)) + (c\*e^4\*(f + g\*x)^(6 + n))/(g^6\*(6 + n))

**Rubi in Sympy [A]** time = 118.36, size = 274, normalized size = 1.

$$\begin{aligned} & \frac{ce^4(f + gx)^{n+6}}{g^6(n+6)} + \frac{5ce^3(f + gx)^{n+5}(dg - ef)}{g^6(n+5)} + \frac{e^2(f + gx)^{n+4}(aeg^2 + 9cd^2g^2 - 20cdefg + 10ce^2f^2)}{g^6(n+4)} \\ & + \frac{e(f + gx)^{n+3}(dg - ef)(3aeg^2 + 7cd^2g^2 - 20cdefg + 10ce^2f^2)}{g^6(n+3)} \\ & + \frac{(f + gx)^{n+1}(dg - ef)^3(ag^2 - 2cdfg + cef^2)}{g^6(n+1)} \\ & + \frac{(f + gx)^{n+2}(dg - ef)^2(3aeg^2 + 2cd^2g^2 - 10cdefg + 5ce^2f^2)}{g^6(n+2)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**3*(g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)`

[Out]  $c e^{4x} (f + g x)^{n+6} / (g^6 (n+6)) + 5 c e^{3x} (f + g x)^{n+5} (d g - e f) / (g^6 (n+5)) + e^{2x} (f + g x)^{n+4} (a e g^2 + 9 c d^2 g^2 - 20 c d e f g + 10 c e^2 f^2) / (g^6 (n+4)) + e (f + g x)^{n+3} (d g - e f) (3 a e g^2 + 7 c d^2 g^2 - 20 c d e f g + 10 c e^2 f^2) / (g^6 (n+3)) + (f + g x)^{n+1} (d g - e f)^3 (a g^2 - 2 c d f g + c e f^2) / (g^6 (n+1)) + (f + g x)^{n+2} (d g - e f)^2 (3 a e g^2 + 2 c d^2 g^2 - 10 c d e f g + 5 c e^2 f^2) / (g^6 (n+2))$

**Mathematica [B]** time = 1.36352, size = 577, normalized size = 2.1

$(f + gx)^{n+1} (ag^2 (n^2 + 11n + 30) (d^3 g^3 (n^3 + 9n^2 + 26n + 24) + 3d^2 eg^2 (n^2 + 7n + 12) (g(n+1)x - f) + 3de^2 g(n+4) (2f^2 -$

Antiderivative was successfully verified.

[In] `Integrate[(d + e*x)^3*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2),x]`

[Out]  $((f + g x)^{1+n} (a g^2 (30 + 11 n + n^2) (d^3 g^3 (24 + 26 n + 9 n^2 + n^3) + 3 d^2 e g^2 (12 + 7 n + n^2) (-f + g (1 + n) x) + 3 d e^2 g (4 + n) (2 f^2 - 2 f g (1 + n) x + g^2 (2 + 3 n + n^2) x^2) + e^3 (-6 f^3 + 6 f^2 g (1 + n) x - 3 f g^2 (2 + 3 n + n^2) x^2 + g^3 (6 + 11 n + 6 n^2 + n^3) x^3)) + c (2 d^4 g^4 (360 + 342 n + 119 n^2 + 18 n^3 + n^4) (-f + g (1 + n) x) + 7 d^3 e g^3 (120 + 74 n + 15 n^2 + n^3) (2 f^2 - 2 f g (1 + n) x + g^2 (2 + 3 n + n^2) x^2) + 9 d^2 e^2 g^2 (30 + 11 n + n^2) (-6 f^3 + 6 f^2 g (1 + n) x - 3 f g^2 (2 + 3 n + n^2) x^2 + g^3 (6 + 11 n + 6 n^2 + n^3) x^3) + 5 d e^3 g (6 + n) (24 f^4 - 24 f^3 g (1 + n) x + 12 f^2 g^2 (2 + 3 n + n^2) x^2 - 4 f g^3 (6 + 11 n + 6 n^2 + n^3) x^3 + g^4 (24 + 50 n + 35 n^2 + 10 n^3 + n^4) x^4) - e^4 (120 f^5 - 120 f^4 g (1 + n) x + 60 f^3 g^2 (2 + 3 n + n^2) x^2 - 20 f^2 g^3 (6 + 11 n + 6 n^2 + n^3) x^3 + 5 f g^4 (24 + 50 n + 35 n^2 + 10 n^3 + n^4) x^4 - g^5 (120 + 274 n + 225 n^2 + 85 n^3 + 15 n^4 + n^5) x^5)) / (g^6 (1 + n) (2 + n) (3 + n) (4 + n) (5 + n) (6 + n))$

**Maple [B]** time = 0.019, size = 2017, normalized size = 7.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^3*(g*x+f)^n*(c*e*x^2+2*c*d*x+a),x)`

[Out]  $(g x + f)^{1+n} (c e^4 g^5 n^5 x^5 + 5 c d e^3 g^5 n^5 x^4 + 15 c e^4 g^5 n^4 x^5 + 9 c d^2 e^2 g^5 n^5 x^3 + 80 c d e^3 g^5 n^4 x^4 - 5 c e^4 f g^4 n^4 x^4 + 85 c e^4 g^5 n^3 x^5 + a e^3 g^5 n^5 x^3 + 7 c d^3 e g^5 n^5 x^2 + 153 c d^2 e^2 g^5 n^4 x^3 - 20 c d e^3 f g^4 n^4 x^3 + 475 c d e^3 g^5 n^3 x^4 - 50 c e^4 f g^4 n^3 x^4 + 225 c e^4 g^5 n^2 x^5 + 3 a d e^2 g^5 n^5 x^2 + 17 a e^3 g^5 n^4 x^3 + 2 c d^4 g^5 n^5 x + 126 c d^3 e g^5 n^4 x^2 - 27 c d^2 e^2 f g^4 n^4 x^2 + 963 c d^2 e^2 g^5 n^3 x^3 - 240 c d e^3 f g^4 n^3 x^3 + 1300 c d e^3 g^5 n^2 x^4 + 20 c e^4 f^2 g^3 n^3 x^3 - 175 c e^4 f g^4 n^2 x^4 + 274 c e^4 g^5 n x^5 + 3 a d^2 e g^5 n^5 x + 54 a d e^2 g^5 n^4 x^2 - 3 a e^3 f g^4 n^4 x^2 + 107 a e^3 g^5 n^3 x^3 + 38 c d^4 g^5 n^4 x - 14 c d^3 e f g^4 n^4 x + 847 c d^3 e g^5 n^3 x^2 - 378 c d^2 e^2 f g^4 n^3 x^2 + 2763 c d^2 e^2 g^5 n^2 x^3 + 60 c d e^3 f^2 g^3 n^3 x^2 - 940 c d e^3 f g^4 n^2 x^3 +$

$$1620*c*d^e^3*g^5*n^x^4+120*c^e^4*f^2*g^3*n^2*x^3-250*c^e^4*f*g^4*n^x^4+120*c^e^4*g^5*x^5+a*d^3*g^5*n^5+57*a*d^2*e^g^5*n^4*x-6*a*d^e^2*f*g^4*n^4*x+363*a*d^e^2*g^5*n^3*x^2-42*a^e^3*f*g^4*n^3*x^2+307*a^e^3*g^5*n^2*x^3-2*c^d^4*f*g^4*n^4+274*c^d^4*g^5*n^3*x-224*c^d^3*e^f*g^4*n^3*x+2604*c^d^3*e^g^5*n^2*x^2+54*c^d^2*e^2*f^2*g^3*n^3*x-1755*c^d^2*e^2*f*g^4*n^2*x^2+3564*c^d^2*e^2*g^5*n^x^3+540*c^d^e^3*f^2*g^3*n^2*x^2-1440*c^d^e^3*f*g^4*n^x^3+720*c^d^e^3*g^5*x^4-60*c^e^4*f^3*g^2*n^2*x^2+220*c^e^4*f^2*g^3*n^x^3-120*c^e^4*f*g^4*x^4+20*a^d^3*g^5*n^4-3*a^d^2*e^f*g^4*n^4+411*a^d^2*e^g^5*n^3*x-96*a^d^e^2*f*g^4*n^3*x+1116*a^d^e^2*g^5*n^2*x^2+6*a^e^3*f^2*g^3*n^3*x-195*a^e^3*f*g^4*n^2*x^2+396*a^e^3*g^5*n^x^3-36*c^d^4*f*g^4*n^3+922*c^d^4*g^5*n^2*x+14*c^d^3*e^f^2*g^3*n^3-1246*c^d^3*e^f*g^4*n^2*x+3556*c^d^3*e^g^5*n^x^2+648*c^d^2*e^2*f^2*g^3*n^2*x-3024*c^d^2*e^2*f*g^4*n^x^2+1620*c^d^2*e^2*g^5*x^3-120*c^d^e^3*f^3*g^2*n^2*x+1200*c^d^e^3*f^2*g^3*n^x^2-720*c^d^e^3*f*g^4*x^3-180*c^e^4*f^3*g^2*n^x^2+120*c^e^4*f^2*g^3*x^3+155*a^d^3*g^5*n^3-54*a^d^2*e^f*g^4*n^3+1383*a^d^2*e^g^5*n^2*x+6*a^d^e^2*f^2*g^3*n^3-534*a^d^e^2*f*g^4*n^2*x+1524*a^d^e^2*g^5*n^x^2+72*a^e^3*f^2*g^3*n^2*x-336*a^e^3*f*g^4*n^x^2+180*a^e^3*g^5*x^3-238*c^d^4*f*g^4*n^2+1404*c^d^4*g^5*n^x+210*c^d^3*e^f^2*g^3*n^2-2716*c^d^3*e^f*g^4*n^x+1680*c^d^3*e^g^5*x^2-54*c^d^2*e^2*f^3*g^2*n^2+2214*c^d^2*e^2*f^2*g^3*n^x-1620*c^d^2*e^2*f*g^4*x^2-840*c^d^e^3*f^3*g^2*n^x+720*c^d^e^3*f^2*g^3*x^2+120*c^e^4*f^4*g^2*n^x-120*c^e^4*f^3*g^2*x^2+580*a^d^3*g^5*n^2-357*a^d^2*e^f*g^4*n^2+2106*a^d^2*e^g^5*n^x+90*a^d^e^2*f^2*g^3*n^2-164*a^d^e^2*f*g^4*n^x+720*a^d^e^2*g^5*x^2-6*a^e^3*f^3*g^2*n^2+246*a^e^3*f^2*g^3*n^x-180*a^e^3*f*g^4*x^2-684*c^d^4*f*g^4*n+720*c^d^4*g^5*x+1036*c^d^3*e^f^2*g^3*n-1680*c^d^3*e^f*g^4*x-594*c^d^2*e^2*f^3*g^2*n+1620*c^d^2*e^2*f^2*g^3*x+120*c^d^e^3*f^4*g^2*n-720*c^d^e^3*f^3*g^2*x+120*c^e^4*f^4*g^2*x+1044*a^d^3*g^5*n-1026*a^d^2*e^f*g^4*n+1080*a^d^2*e^g^5*x+444*a^d^e^2*f^2*g^3*n-720*a^d^e^2*f*g^4*x-66*a^e^3*f^3*g^2*n+180*a^e^3*f^2*g^3*x-720*c^d^4*f*g^4+1680*c^d^3*e^f^2*g^3-1620*c^d^2*e^2*f^3*g^2+720*c^d^e^3*f^4*g-120*c^e^4*f^5+720*a^d^3*g^5-1080*a^d^2*e^f*g^4+720*a^d^e^2*f^2*g^3-180*a^e^3*f^3*g^2)/g^6/(n^6+21*n^5+175*n^4+735*n^3+1624*n^2+1764*n+720)$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*e\*x^2 + 2\*c\*d\*x + a)\*(e\*x + d)^3\*(g\*x + f)^n,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.313283, size = 2743, normalized size = 9.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*e\*x^2 + 2\*c\*d\*x + a)\*(e\*x + d)^3\*(g\*x + f)^n,x, algorithm="fricas")

[Out] (a\*d^3\*f\*g^5\*n^5 - 120\*c^e^4\*f^6 + 720\*c^d^e^3\*f^5\*g + 720\*a^d^3\*f\*g^5 - 180\*(9\*c^d^2\*e^2 + a^e^3)\*f^4\*g^2 + 240\*(7\*c^d^3\*e + 3\*a^d^e^2)\*f^3\*g^3 - 360\*(2\*c^d^4 + 3\*a^d^2\*e)\*f^2\*g^4 + (c^e^4\*g^6\*n^5 + 15\*c^e^4\*g^6\*n^4 + 85\*c^e^4\*g^6\*n^3 + 225\*c^e^4\*g^6\*n^2 + 274\*c^e^4\*g^6\*n + 120\*c^e^4\*g^6)\*x^6 + (720\*c^d^e^3\*g^6 + (c^e^4\*f\*g^5 + 5\*c^d^e^3\*g^6)\*n^5 + 10\*(c^e^4\*f\*g^5 + 8\*c^d^e^3\*g^6)\*n^4 + 5\*(7\*c^e^4\*f\*g^5 + 95\*c^d^e^3\*g^6)\*n^3 + 50\*(c^e^4\*f\*g^5 + 26\*c^d^e^3\*g^6)\*n^2 + 12\*(2\*c^e^4\*f\*g^5 + 135\*c^d^e^3\*g^6)\*n)\*x^5 + (20\*a^d^3\*f\*g^5 - (2\*c^d^4 + 3\*a^d^2\*e)\*f^2\*g^4)\*n^4 + (180\*(9\*c^d^2\*e^2 + a^e^3)\*g^6 + (5\*c^d^e^3\*f\*g^5 + (9\*c^d^2\*e^2 + a^e^3)\*g^6

$$\begin{aligned}
& ) * n^5 - (5 * c * e^4 * f^2 * g^4 - 60 * c * d * e^3 * f * g^5 - 17 * (9 * c * d^2 * e^2 + a * e^3) * g^6) * n^4 - (30 * c * e^4 * f^2 * g^4 - 235 * c * d * e^3 * f * g^5 - 107 * (9 * c * d^2 * e^2 + a * e^3) * g^6) * n^3 - (55 * c * e^4 * f^2 * g^4 - 360 * c * d * e^3 * f * g^5 - 307 * (9 * c * d^2 * e^2 + a * e^3) * g^6) * n^2 - 6 * (5 * c * e^4 * f^2 * g^4 - 30 * c * d * e^3 * f * g^5 - 66 * (9 * c * d^2 * e^2 + a * e^3) * g^6) * n * x^4 + (155 * a * d^3 * f * g^5 + 2 * (7 * c * d^3 * e + 3 * a * d * e^2) * f^3 * g^3 - 18 * (2 * c * d^4 + 3 * a * d^2 * e) * f^2 * g^4) * n^3 + (240 * (7 * c * d^3 * e + 3 * a * d * e^2) * g^6 + ((9 * c * d^2 * e^2 + a * e^3) * f * g^5 + (7 * c * d^3 * e + 3 * a * d * e^2) * g^6) * n^5 - 2 * (10 * c * d * e^3 * f^2 * g^4 - 7 * (9 * c * d^2 * e^2 + a * e^3) * f * g^5 - 9 * (7 * c * d^3 * e + 3 * a * d * e^2) * g^6) * n^4 + (20 * c * e^4 * f^3 * g^3 - 180 * c * d * e^3 * f^2 * g^4 + 65 * (9 * c * d^2 * e^2 + a * e^3) * f * g^5 + 121 * (7 * c * d^3 * e + 3 * a * d * e^2) * g^6) * n^3 + 4 * (15 * c * e^4 * f^3 * g^3 - 100 * c * d * e^3 * f^2 * g^4 + 28 * (9 * c * d^2 * e^2 + a * e^3) * f * g^5 + 93 * (7 * c * d^3 * e + 3 * a * d * e^2) * g^6) * n^2 + 4 * (10 * c * e^4 * f^3 * g^3 - 60 * c * d * e^3 * f^2 * g^4 + 15 * (9 * c * d^2 * e^2 + a * e^3) * f * g^5 + 127 * (7 * c * d^3 * e + 3 * a * d * e^2) * g^6) * n * x^3 + (580 * a * d^3 * f * g^5 - 6 * (9 * c * d^2 * e^2 + a * e^3) * f^4 * g^2 + 30 * (7 * c * d^3 * e + 3 * a * d * e^2) * f^3 * g^3 - 119 * (2 * c * d^4 + 3 * a * d^2 * e) * f^2 * g^4) * n^2 + (360 * (2 * c * d^4 + 3 * a * d^2 * e) * g^6 + ((7 * c * d^3 * e + 3 * a * d * e^2) * f * g^5 + (2 * c * d^4 + 3 * a * d^2 * e) * g^6) * n^5 - (3 * (9 * c * d^2 * e^2 + a * e^3) * f^2 * g^4 - 16 * (7 * c * d^3 * e + 3 * a * d * e^2) * f * g^5 - 19 * (2 * c * d^4 + 3 * a * d^2 * e) * g^6) * n^4 + (60 * c * d * e^3 * f^3 * g^3 - 36 * (9 * c * d^2 * e^2 + a * e^3) * f^2 * g^4 + 89 * (7 * c * d^3 * e + 3 * a * d * e^2) * f * g^5 + 137 * (2 * c * d^4 + 3 * a * d^2 * e) * g^6) * n^3 - (60 * c * e^4 * f^4 * g^2 - 420 * c * d * e^3 * f^3 * g^3 + 123 * (9 * c * d^2 * e^2 + a * e^3) * f^2 * g^4 - 194 * (7 * c * d^3 * e + 3 * a * d * e^2) * f * g^5 - 461 * (2 * c * d^4 + 3 * a * d^2 * e) * g^6) * n^2 - 6 * (10 * c * e^4 * f^4 * g^2 - 60 * c * d * e^3 * f^3 * g^3 + 15 * (9 * c * d^2 * e^2 + a * e^3) * f^2 * g^4 - 20 * (7 * c * d^3 * e + 3 * a * d * e^2) * f * g^5 - 117 * (2 * c * d^4 + 3 * a * d^2 * e) * g^6) * n * x^2 + 2 * (60 * c * d * e^3 * f^5 * g + 522 * a * d^3 * f * g^5 - 33 * (9 * c * d^2 * e^2 + a * e^3) * f^4 * g^2 + 74 * (7 * c * d^3 * e + 3 * a * d * e^2) * f^3 * g^3 - 171 * (2 * c * d^4 + 3 * a * d^2 * e) * f^2 * g^4) * n + (720 * a * d^3 * g^6 + (a * d^3 * g^6 + (2 * c * d^4 + 3 * a * d^2 * e) * f * g^5) * n^5 + 2 * (10 * a * d^3 * g^6 - (7 * c * d^3 * e + 3 * a * d * e^2) * f^2 * g^4 + 9 * (2 * c * d^4 + 3 * a * d^2 * e) * f * g^5) * n^4 + (155 * a * d^3 * g^6 + 6 * (9 * c * d^2 * e^2 + a * e^3) * f^3 * g^3 - 30 * (7 * c * d^3 * e + 3 * a * d * e^2) * f^2 * g^4 + 119 * (2 * c * d^4 + 3 * a * d^2 * e) * f * g^5) * n^3 - 2 * (60 * c * d * e^3 * f^4 * g^2 - 290 * a * d^3 * g^6 - 33 * (9 * c * d^2 * e^2 + a * e^3) * f^3 * g^3 + 74 * (7 * c * d^3 * e + 3 * a * d * e^2) * f^2 * g^4 - 171 * (2 * c * d^4 + 3 * a * d^2 * e) * f * g^5) * n^2 + 12 * (10 * c * e^4 * f^5 * g - 60 * c * d * e^3 * f^4 * g^2 + 87 * a * d^3 * g^6 + 15 * (9 * c * d^2 * e^2 + a * e^3) * f^3 * g^3 - 20 * (7 * c * d^3 * e + 3 * a * d * e^2) * f^2 * g^4 + 30 * (2 * c * d^4 + 3 * a * d^2 * e) * f * g^5) * n * x) * (g * x + f)^n / (g^6 * n^6 + 21 * g^6 * n^5 + 175 * g^6 * n^4 + 735 * g^6 * n^3 + 1624 * g^6 * n^2 + 1764 * g^6 * n + 720 * g^6)
\end{aligned}$$


---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(g\*x+f)\*\*n\*(c\*e\*x\*\*2+2\*c\*d\*x+a),x)

[Out] Timed out

---

**GIAC/XCAS [A]** time = 0.277802, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*e\*x^2 + 2\*c\*d\*x + a)\*(e\*x + d)^3\*(g\*x + f)^n,x, algorithm="giac")

[Out] Done



### 3.806 $\int (d + ex)^2 (f + gx)^n (a + 2cdx + cex^2) dx$

**Optimal.** Leaf size=208

$$\begin{aligned} & - \frac{2(ef - dg)(f + gx)^{n+2} (aeg^2 + c(d^2g^2 - 4defg + 2e^2f^2))}{g^5(n+2)} \\ & + \frac{e(f + gx)^{n+3} (aeg^2 + c(5d^2g^2 - 12defg + 6e^2f^2))}{g^5(n+3)} \\ & + \frac{(ef - dg)^2(f + gx)^{n+1} (ag^2 + cf(ef - 2dg))}{g^5(n+1)} - \frac{4ce^2(ef - dg)(f + gx)^{n+4}}{g^5(n+4)} + \frac{ce^3(f + gx)^{n+5}}{g^5(n+5)} \end{aligned}$$

[Out]  $((e*f - d*g)^2*(a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^5*(1 + n)) - (2*(e*f - d*g)*(a*e*g^2 + c*(2*e^2*f^2 - 4*d*e*f*g + d^2*g^2))*(f + g*x)^(2 + n))/(g^5*(2 + n)) + (e*(a*e*g^2 + c*(6*e^2*f^2 - 12*d*e*f*g + 5*d^2*g^2))*(f + g*x)^(3 + n))/(g^5*(3 + n)) - (4*c*e^2*(e*f - d*g)*(f + g*x)^(4 + n))/(g^5*(4 + n)) + (c*e^3*(f + g*x)^(5 + n))/(g^5*(5 + n))$

**Rubi [A]** time = 0.408108, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.036$

$$\begin{aligned} & - \frac{2(ef - dg)(f + gx)^{n+2} (aeg^2 + c(d^2g^2 - 4defg + 2e^2f^2))}{g^5(n+2)} \\ & + \frac{e(f + gx)^{n+3} (aeg^2 + c(5d^2g^2 - 12defg + 6e^2f^2))}{g^5(n+3)} \\ & + \frac{(ef - dg)^2(f + gx)^{n+1} (ag^2 + cf(ef - 2dg))}{g^5(n+1)} - \frac{4ce^2(ef - dg)(f + gx)^{n+4}}{g^5(n+4)} + \frac{ce^3(f + gx)^{n+5}}{g^5(n+5)} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^2*(f + g*x)^n*(a + 2*c*d*x + c*e*x^2), x]$

[Out]  $((e*f - d*g)^2*(a*g^2 + c*f*(e*f - 2*d*g))*(f + g*x)^(1 + n))/(g^5*(1 + n)) - (2*(e*f - d*g)*(a*e*g^2 + c*(2*e^2*f^2 - 4*d*e*f*g + d^2*g^2))*(f + g*x)^(2 + n))/(g^5*(2 + n)) + (e*(a*e*g^2 + c*(6*e^2*f^2 - 12*d*e*f*g + 5*d^2*g^2))*(f + g*x)^(3 + n))/(g^5*(3 + n)) - (4*c*e^2*(e*f - d*g)*(f + g*x)^(4 + n))/(g^5*(4 + n)) + (c*e^3*(f + g*x)^(5 + n))/(g^5*(5 + n))$

**Rubi in Sympy [A]** time = 87.5296, size = 206, normalized size = 0.99

$$\begin{aligned} & \frac{ce^3(f + gx)^{n+5}}{g^5(n+5)} + \frac{4ce^2(f + gx)^{n+4}(dg - ef)}{g^5(n+4)} + \frac{e(f + gx)^{n+3}(aeg^2 + 5cd^2g^2 - 12cdefg + 6ce^2f^2)}{g^5(n+3)} \\ & + \frac{(f + gx)^{n+1}(dg - ef)^2(ag^2 - 2cdfg + cef^2)}{g^5(n+1)} \\ & + \frac{2(f + gx)^{n+2}(dg - ef)(aeg^2 + cd^2g^2 - 4cdefg + 2ce^2f^2)}{g^5(n+2)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)**2*(g*x+f)**n*(c*e*x**2+2*c*d*x+a), x)$

[Out]  $c*e**3*(f + g*x)**(n + 5)/(g**5*(n + 5)) + 4*c*e**2*(f + g*x)**(n + 4)*(d*g - e*f)/(g**5*(n + 4)) + e*(f + g*x)**(n + 3)*(a*e*g**2 + 5*c*d**2*g**2 - 12*c*d*e*f*g + 6*c*e**2*f**2)/(g**5*(n + 3)) + (f + g*x)**(n + 1)*(d*g - e*f)**2*(a*g**2 - 2*c*d*f*g + c*e*f**2$

$$\frac{(f + gx)^{n+1} (ag^2 (n^2 + 9n + 20) (d^2 g^2 (n^2 + 5n + 6) + 2deg(n+3)(g(n+1)x - f) + e^2 (2f^2 - 2fg(n+1)x + g^2 (n^2 + 3n + 2)))}{(g^{5(n+1)} + 2(f + gx)^{n+2} (dg - ef) (ae^2 g^2 + cd^2 g^2 - 4cdefg + 2ce^2 f^2)) / (g^{5(n+2)})}$$

**Mathematica [A]** time = 0.562852, size = 348, normalized size = 1.67

$$(f + gx)^{n+1} (ag^2 (n^2 + 9n + 20) (d^2 g^2 (n^2 + 5n + 6) + 2deg(n+3)(g(n+1)x - f) + e^2 (2f^2 - 2fg(n+1)x + g^2 (n^2 + 3n + 2)))$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^2\*(f + g\*x)^n\*(a + 2\*c\*d\*x + c\*e\*x^2), x]

[Out] ((f + g\*x)^(1 + n)\*(a\*g^2\*(20 + 9\*n + n^2)\*(d^2\*g^2\*(6 + 5\*n + n^2) + 2\*d\*e\*g\*(3 + n)\*(-f + g\*(1 + n)\*x) + e^2\*(2\*f^2 - 2\*f\*g\*(1 + n)\*x + g^2\*(2 + 3\*n + n^2)\*x^2)) + c\*(2\*d^3\*g^3\*(60 + 47\*n + 12\*n^2 + n^3)\*(-f + g\*(1 + n)\*x) + 5\*d^2\*e\*g^2\*(20 + 9\*n + n^2)\*(2\*f^2 - 2\*f\*g\*(1 + n)\*x + g^2\*(2 + 3\*n + n^2)\*x^2) + 4\*d\*e^2\*g\*(5 + n)\*(-6\*f^3 + 6\*f^2\*g\*(1 + n)\*x - 3\*f\*g^2\*(2 + 3\*n + n^2)\*x^2 + g^3\*(6 + 11\*n + 6\*n^2 + n^3)\*x^3) + e^3\*(24\*f^4 - 24\*f^3\*g\*(1 + n)\*x + 12\*f^2\*g^2\*(2 + 3\*n + n^2)\*x^2 - 4\*f\*g^3\*(6 + 11\*n + 6\*n^2 + n^3)\*x^3 + g^4\*(24 + 50\*n + 35\*n^2 + 10\*n^3 + n^4)\*x^4)))/(g^5\*(1 + n)\*(2 + n)\*(3 + n)\*(4 + n)\*(5 + n))

**Maple [B]** time = 0.015, size = 1048, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^2\*(g\*x+f)^n\*(c\*e\*x^2+2\*c\*d\*x+a), x)

[Out] (g\*x+f)^(1+n)\*(c\*e^3\*g^4\*n^4\*x^4+4\*c\*d\*e^2\*g^4\*n^4\*x^3+10\*c\*e^3\*g^4\*n^3\*x^2+5\*c\*d^2\*e\*g^4\*n^4\*x^2+44\*c\*d\*e^2\*g^4\*n^3\*x^2-4\*c\*e^3\*f\*g^3\*n^3\*x^3+35\*c\*e^3\*g^4\*n^2\*x^4+a\*e^2\*g^4\*n^4\*x^2+2\*c\*d^3\*g^4\*n^4\*x+60\*c\*d^2\*e\*g^4\*n^3\*x^2-12\*c\*d\*e^2\*f\*g^3\*n^3\*x^2+164\*c\*d\*e^2\*g^4\*n^2\*x^3-24\*c\*e^3\*f\*g^3\*n^2\*x^3+50\*c\*e^3\*g^4\*n\*x^4+2\*a\*d\*e\*g^4\*n^4\*x+12\*a\*e^2\*g^4\*n^3\*x^2+26\*c\*d^3\*g^4\*n^3\*x-10\*c\*d^2\*e\*f\*g^3\*n^3\*x+245\*c\*d^2\*e\*g^4\*n^2\*x^2-96\*c\*d\*e^2\*f\*g^3\*n^2\*x^2+244\*c\*d\*e^2\*g^4\*n\*x^3+12\*c\*e^3\*f^2\*g^2\*n^2\*x^2-44\*c\*e^3\*f\*g^3\*n\*x^3+24\*c\*e^3\*g^4\*x^4+a\*d^2\*g^4\*n^4+26\*a\*d\*e\*g^4\*n^3\*x-2\*a\*e^2\*f\*g^3\*n^3\*x+49\*a\*e^2\*g^4\*n^2\*x^2-2\*c\*d^3\*f\*g^3\*n^3+118\*c\*d^3\*g^4\*n^2\*x-100\*c\*d^2\*e\*f\*g^3\*n^2\*x+390\*c\*d^2\*e\*g^4\*n\*x^2+24\*c\*d\*e^2\*f^2\*g^2\*n^2\*x-204\*c\*d\*e^2\*f\*g^3\*n\*x^2+120\*c\*d\*e^2\*g^4\*x^3+36\*c\*e^3\*f^2\*g^2\*n\*x^2-24\*c\*e^3\*f\*g^3\*x^3+14\*a\*d^2\*g^4\*n^3-2\*a\*d\*e\*f\*g^3\*n^3+118\*a\*d\*e\*g^4\*n^2\*x-20\*a\*e^2\*f\*g^3\*n^2\*x+78\*a\*e^2\*g^4\*n\*x^2-24\*c\*d^3\*f\*g^3\*n^2+214\*c\*d^3\*g^4\*n\*x+10\*c\*d^2\*e\*f^2\*g^2\*n^2-290\*c\*d^2\*e\*f\*g^3\*n\*x+200\*c\*d^2\*e\*g^4\*x^2+144\*c\*d\*e^2\*f^2\*g^2\*n\*x-120\*c\*d\*e^2\*f\*g^3\*x^2-24\*c\*e^3\*f^3\*g\*n\*x+24\*c\*e^3\*f^2\*g^2\*x^2+71\*a\*d^2\*g^4\*n^2-24\*a\*d\*e\*f\*g^3\*n^2+214\*a\*d\*e\*g^4\*n\*x+2\*a\*e^2\*f^2\*g^2\*n^2-58\*a\*e^2\*f\*g^3\*n\*x+40\*a\*e^2\*g^4\*x^2-94\*c\*d^3\*f\*g^3\*n+120\*c\*d^3\*g^4\*x+90\*c\*d^2\*e\*f^2\*g^2\*n-200\*c\*d^2\*e\*f\*g^3\*x-24\*c\*d\*e^2\*f^3\*g\*n+120\*c\*d\*e^2\*f^2\*g^2\*x-24\*c\*e^3\*f^3\*g\*x+154\*a\*d^2\*g^4\*n-94\*a\*d\*e\*f\*g^3\*n+120\*a\*d\*e\*g^4\*x+18\*a\*e^2\*f^2\*g^2\*n-40\*a\*e^2\*f\*g^3\*x-120\*c\*d^3\*f\*g^3+200\*c\*d^2\*e\*f^2\*g^2-120\*c\*d\*e^2\*f^3\*g+24\*c\*e^3\*f^4+120\*a\*d^2\*g^4-120\*a\*d\*e\*f\*g^3+40\*a\*e^2\*f^2\*g^2)/g^5/(n^5+15\*n^4+85\*n^3+225\*n^2+274\*n+120)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e*x^2 + 2*c*d*x + a)*(e*x + d)^2*(g*x + f)^n,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 0.306948, size = 1515, normalized size = 7.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e*x^2 + 2*c*d*x + a)*(e*x + d)^2*(g*x + f)^n,x, algorithm="fricas")
```

```
[Out] (a*d^2*f*g^4*n^4 + 24*c*e^3*f^5 - 120*c*d*e^2*f^4*g + 120*a*d^2*f
*g^4 + 40*(5*c*d^2*e + a*e^2)*f^3*g^2 - 120*(c*d^3 + a*d*e)*f^2*g
^3 + (c*e^3*g^5*n^4 + 10*c*e^3*g^5*n^3 + 35*c*e^3*g^5*n^2 + 50*c
e^3*g^5*n + 24*c*e^3*g^5)*x^5 + (120*c*d*e^2*g^5 + (c*e^3*f*g^4 +
4*c*d*e^2*g^5)*n^4 + 2*(3*c*e^3*f*g^4 + 22*c*d*e^2*g^5)*n^3 + (1
1*c*e^3*f*g^4 + 164*c*d*e^2*g^5)*n^2 + 2*(3*c*e^3*f*g^4 + 122*c*d
*e^2*g^5)*n)*x^4 + 2*(7*a*d^2*f*g^4 - (c*d^3 + a*d*e)*f^2*g^3)*n^
3 + (40*(5*c*d^2*e + a*e^2)*g^5 + (4*c*d*e^2*f*g^4 + (5*c*d^2*e +
a*e^2)*g^5)*n^4 - 4*(c*e^3*f^2*g^3 - 8*c*d*e^2*f*g^4 - 3*(5*c*d^
2*e + a*e^2)*g^5)*n^3 - (12*c*e^3*f^2*g^3 - 68*c*d*e^2*f*g^4 - 49
*(5*c*d^2*e + a*e^2)*g^5)*n^2 - 2*(4*c*e^3*f^2*g^3 - 20*c*d*e^2*f
*g^4 - 39*(5*c*d^2*e + a*e^2)*g^5)*n)*x^3 + (71*a*d^2*f*g^4 + 2*(
5*c*d^2*e + a*e^2)*f^3*g^2 - 24*(c*d^3 + a*d*e)*f^2*g^3)*n^2 + (1
20*(c*d^3 + a*d*e)*g^5 + ((5*c*d^2*e + a*e^2)*f*g^4 + 2*(c*d^3 +
a*d*e)*g^5)*n^4 - 2*(6*c*d*e^2*f^2*g^3 - 5*(5*c*d^2*e + a*e^2)*f
g^4 - 13*(c*d^3 + a*d*e)*g^5)*n^3 + (12*c*e^3*f^3*g^2 - 72*c*d*e^
2*f^2*g^3 + 29*(5*c*d^2*e + a*e^2)*f*g^4 + 118*(c*d^3 + a*d*e)*g^
5)*n^2 + 2*(6*c*e^3*f^3*g^2 - 30*c*d*e^2*f^2*g^3 + 10*(5*c*d^2*e
+ a*e^2)*f*g^4 + 107*(c*d^3 + a*d*e)*g^5)*n)*x^2 - 2*(12*c*d*e^2
f^4*g - 77*a*d^2*f*g^4 - 9*(5*c*d^2*e + a*e^2)*f^3*g^2 + 47*(c*d^
3 + a*d*e)*f^2*g^3)*n + (120*a*d^2*g^5 + (a*d^2*g^5 + 2*(c*d^3 +
a*d*e)*f*g^4)*n^4 + 2*(7*a*d^2*g^5 - (5*c*d^2*e + a*e^2)*f^2*g^3
+ 12*(c*d^3 + a*d*e)*f*g^4)*n^3 + (24*c*d*e^2*f^3*g^2 + 71*a*d^2
g^5 - 18*(5*c*d^2*e + a*e^2)*f^2*g^3 + 94*(c*d^3 + a*d*e)*f*g^4)*
n^2 - 2*(12*c*e^3*f^4*g - 60*c*d*e^2*f^3*g^2 - 77*a*d^2*g^5 + 20
*(5*c*d^2*e + a*e^2)*f^2*g^3 - 60*(c*d^3 + a*d*e)*f*g^4)*n)*x*(g
*x + f)^n/(g^5*n^5 + 15*g^5*n^4 + 85*g^5*n^3 + 225*g^5*n^2 + 274*g
^5*n + 120*g^5)
```

**Sympy [A]** time = 34.9644, size = 11924, normalized size = 57.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**2*(g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)
```

```
[Out] Piecewise((f**n*(a*d**2*x + a*d*e*x**2 + a*e**2*x**3/3 + c*d**3*x
**2 + 5*c*d**2*e*x**3/3 + c*d*e**2*x**4 + c*e**3*x**5/5), Eq(g, 0
)), (-3*a*d**2*g**4/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7
*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 2*a*d*e*f*g**3/(12*f**4
g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g
**9*x**4) - 8*a*d*e*g**4*x/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f
**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - a*e**2*f**2*g**2/
(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x
**3 + 12*g**9*x**4) - 4*a*e**2*f*g**3*x/(12*f**4*g**5 + 48*f**3*g
**6*x + 72*f**2*g**7*x**2 + 48*f*g**8*x**3 + 12*g**9*x**4) - 6*a
e**2*g**4*x**2/(12*f**4*g**5 + 48*f**3*g**6*x + 72*f**2*g**7*x**2
```

$$\begin{aligned}
& + 48f^2g^8x^3 + 12g^9x^4) - 2cd^3f^3g^3/(12f^4g^5 \\
& + 48f^3g^6x + 72f^2g^7x^2 + 48f^2g^8x^3 + 12g^9x^4) - 8cd^3g^4x/(12f^4g^5 + 48f^3g^6x + 72f^2g^7x^2 + 48f^2g^8x^3 + 12g^9x^4) - 5cd^2e^2f^2g^2/(12f^4g^5 + 48f^3g^6x + 72f^2g^7x^2 + 48f^2g^8x^3 + 12g^9x^4) - 20cd^2e^2f^3g^3x/(12f^4g^5 + 48f^3g^6x + 72f^2g^7x^2 + 48f^2g^8x^3 + 12g^9x^4) - 30cd^2e^2g^4x^2/(12f^4g^5 + 48f^3g^6x + 72f^2g^7x^2 + 48f^2g^8x^3 + 12g^9x^4) - 12cd^2e^2f^3g/(12f^4g^5 + 48f^3g^6x + 72f^2g^7x^2 + 48f^2g^8x^3 + 12g^9x^4) - 48cd^2e^2f^2g^2x/(12f^4g^5 + 48f^3g^6x + 72f^2g^7x^2 + 48f^2g^8x^3 + 12g^9x^4) - 72cd^2e^2f^3g^3x^2/(12f^4g^5 + 48f^3g^6x + 72f^2g^7x^2 + 48f^2g^8x^3 + 12g^9x^4) - 48cd^2e^2g^4x^3/(12f^4g^5 + 48f^3g^6x + 72f^2g^7x^2 + 48f^2g^8x^3 + 12g^9x^4) + 12ce^3f^4\log(f/g + x)/(12f^4g^5 + 48f^3g^6x + 72f^2g^7x^2 + 48f^2g^8x^3 + 12g^9x^4) + 25ce^3f^4/(12f^4g^5 + 48f^3g^6x + 72f^2g^7x^2 + 48f^2g^8x^3 + 12g^9x^4) + 48ce^3f^3gx\log(f/g + x)/(12f^4g^5 + 48f^3g^6x + 72f^2g^7x^2 + 48f^2g^8x^3 + 12g^9x^4) + 88ce^3f^3gx/(12f^4g^5 + 48f^3g^6x + 72f^2g^7x^2 + 48f^2g^8x^3 + 12g^9x^4) + 72ce^3f^2g^2x^2\log(f/g + x)/(12f^4g^5 + 48f^3g^6x + 72f^2g^7x^2 + 48f^2g^8x^3 + 12g^9x^4) + 108ce^3f^2g^2x^2/(12f^4g^5 + 48f^3g^6x + 72f^2g^7x^2 + 48f^2g^8x^3 + 12g^9x^4) + 48ce^3f^3g^3x^3\log(f/g + x)/(12f^4g^5 + 48f^3g^6x + 72f^2g^7x^2 + 48f^2g^8x^3 + 12g^9x^4) + 48ce^3f^3g^3x^3/(12f^4g^5 + 48f^3g^6x + 72f^2g^7x^2 + 48f^2g^8x^3 + 12g^9x^4) + 12ce^3g^4x^4\log(f/g + x)/(12f^4g^5 + 48f^3g^6x + 72f^2g^7x^2 + 48f^2g^8x^3 + 12g^9x^4), \\
& \text{Eq}(n, -5)), (-ad^2f^2g^4/(3f^5g^5 + 9f^4g^6x + 9f^3g^7x^2 + 3f^2g^8x^3) + 3ade^2f^5x^2/(3f^5g^5 + 9f^4g^6x + 9f^3g^7x^2 + 3f^2g^8x^3) + ade^2g^6x^3/(3f^5g^5 + 9f^4g^6x + 9f^3g^7x^2 + 3f^2g^8x^3) + ae^2f^5g^5x^3/(3f^5g^5 + 9f^4g^6x + 9f^3g^7x^2 + 3f^2g^8x^3) + 3cd^3f^6x^3/(3f^5g^5 + 9f^4g^6x + 9f^3g^7x^2 + 3f^2g^8x^3) + 5cd^2e^2f^5g^5x^3/(3f^5g^5 + 9f^4g^6x + 9f^3g^7x^2 + 3f^2g^8x^3) + 12cd^2e^2f^5g^5\log(f/g + x)/(3f^5g^5 + 9f^4g^6x + 9f^3g^7x^2 + 3f^2g^8x^3) + 4cd^2e^2f^5g/(3f^5g^5 + 9f^4g^6x + 9f^3g^7x^2 + 3f^2g^8x^3) + 36cd^2e^2f^4g^2x\log(f/g + x)/(3f^5g^5 + 9f^4g^6x + 9f^3g^7x^2 + 3f^2g^8x^3) + 36cd^2e^2f^3g^3x^2\log(f/g + x)/(3f^5g^5 + 9f^4g^6x + 9f^3g^7x^2 + 3f^2g^8x^3) - 18cd^2e^2f^3g^3x^2/(3f^5g^5 + 9f^4g^6x + 9f^3g^7x^2 + 3f^2g^8x^3) + 12cd^2e^2f^2g^4x^3\log(f/g + x)/(3f^5g^5 + 9f^4g^6x + 9f^3g^7x^2 + 3f^2g^8x^3) - 18cd^2e^2f^2g^4x^3/(3f^5g^5 + 9f^4g^6x + 9f^3g^7x^2 + 3f^2g^8x^3) - 12ce^3f^6\log(f/g + x)/(3f^5g^5 + 9f^4g^6x + 9f^3g^7x^2 + 3f^2g^8x^3) - 4ce^3f^6/(3f^5g^5 + 9f^4g^6x + 9f^3g^7x^2 + 3f^2g^8x^3) - 36ce^3f^5gx\log(f/g + x)/(3f^5g^5 + 9f^4g^6x + 9f^3g^7x^2 + 3f^2g^8x^3) - 36ce^3f^4g^2x^2\log(f/g + x)/(3f^5g^5 + 9f^4g^6x + 9f^3g^7x^2 + 3f^2g^8x^3) + 18ce^3f^4g^2x^2/(3f^5g^5 + 9f^4g^6x + 9f^3g^7x^2 + 3f^2g^8x^3) - 12ce^3f^3g^3x^3\log(f/g + x)/(3f^5g^5 + 9f^4g^6x + 9f^3g^7x^2 + 3f^2g^8x^3) + 18ce^3f^3g^3x^3/(3f^5g^5 + 9f^4g^6x + 9f^3g^7x^2 + 3f^2g^8x^3) + 3ce^3f^2g^4x^4/(3f^5g^5 + 9f^4g^6x + 9f^3g^7x^2 + 3f^2g^8x^3), \text{Eq}(n, -4)), (-ad^2f^2g^4/(2f^3g^5 + 4f^2g^6x + 2f^2g^7x^2) + 2ade^2g^5x^2/(2f^3g^5 + 4f^2g^6x + 2f^2g^7x^2) + 2ae^2f^3g^2\log(f/g + x)/(2f^3g^5 + 4f^2g^6x + 2f^2g^7x^2) + ae^2f^3g^2/(2f^3g^5 + 4f^2g^6x + 2f^2g^7x^2) + 4ae^2f^2g^3x\log(f/g + x)/(2f^3g^5 + 4f^2g^6x + 2f^2g^7x^2) + 2ae^2f^2g^3x^2\log(f/g + x)/(2f^3g^5 + 4f^2g^6x + 2f^2g^7x^2) - 2ae^2f^2g^4x^2/(2f^3g^5 + 4f^2g^6x + 2f^2g^7x^2) + 2cd^3g^5
\end{aligned}$$

$$\begin{aligned}
& x^{**2}/(2*f^{**3}*g^{**5} + 4*f^{**2}*g^{**6}*x + 2*f*g^{**7}*x^{**2}) + 10*c*d^{**2}*e* \\
& f^{**3}*g^{**2}*\log(f/g + x)/(2*f^{**3}*g^{**5} + 4*f^{**2}*g^{**6}*x + 2*f*g^{**7}*x^{**2}) + 5*c*d^{**2}*e*f^{**3}*g^{**2}/(2*f^{**3}*g^{**5} + 4*f^{**2}*g^{**6}*x + 2*f*g^{**7}*x^{**2}) + 20*c*d^{**2}*e*f^{**2}*g^{**3}*x*\log(f/g + x)/(2*f^{**3}*g^{**5} + 4*f^{**2}*g^{**6}*x + 2*f*g^{**7}*x^{**2}) + 10*c*d^{**2}*e*f*g^{**4}*x^{**2}*\log(f/g + x)/(2*f^{**3}*g^{**5} + 4*f^{**2}*g^{**6}*x + 2*f*g^{**7}*x^{**2}) - 10*c*d^{**2}*e*f*g^{**4}*x^{**2}/(2*f^{**3}*g^{**5} + 4*f^{**2}*g^{**6}*x + 2*f*g^{**7}*x^{**2}) - 24*c*d*e^{**2}*f^{**4}*g*\log(f/g + x)/(2*f^{**3}*g^{**5} + 4*f^{**2}*g^{**6}*x + 2*f*g^{**7}*x^{**2}) - 12*c*d*e^{**2}*f^{**4}*g/(2*f^{**3}*g^{**5} + 4*f^{**2}*g^{**6}*x + 2*f*g^{**7}*x^{**2}) - 48*c*d*e^{**2}*f^{**3}*g^{**2}*x*\log(f/g + x)/(2*f^{**3}*g^{**5} + 4*f^{**2}*g^{**6}*x + 2*f*g^{**7}*x^{**2}) - 24*c*d*e^{**2}*f^{**2}*g^{**3}*x^{**2}*\log(f/g + x)/(2*f^{**3}*g^{**5} + 4*f^{**2}*g^{**6}*x + 2*f*g^{**7}*x^{**2}) + 24*c*d*e^{**2}*f^{**2}*g^{**3}*x^{**2}/(2*f^{**3}*g^{**5} + 4*f^{**2}*g^{**6}*x + 2*f*g^{**7}*x^{**2}) + 8*c*d*e^{**2}*f*g^{**4}*x^{**3}/(2*f^{**3}*g^{**5} + 4*f^{**2}*g^{**6}*x + 2*f*g^{**7}*x^{**2}) + 12*c*e^{**3}*f^{**5}*\log(f/g + x)/(2*f^{**3}*g^{**5} + 4*f^{**2}*g^{**6}*x + 2*f*g^{**7}*x^{**2}) + 6*c*e^{**3}*f^{**5}/(2*f^{**3}*g^{**5} + 4*f^{**2}*g^{**6}*x + 2*f*g^{**7}*x^{**2}) + 24*c*e^{**3}*f^{**4}*g*x*\log(f/g + x)/(2*f^{**3}*g^{**5} + 4*f^{**2}*g^{**6}*x + 2*f*g^{**7}*x^{**2}) + 12*c*e^{**3}*f^{**3}*g^{**2}*x^{**2}*\log(f/g + x)/(2*f^{**3}*g^{**5} + 4*f^{**2}*g^{**6}*x + 2*f*g^{**7}*x^{**2}) - 12*c*e^{**3}*f^{**3}*g^{**2}*x^{**2}/(2*f^{**3}*g^{**5} + 4*f^{**2}*g^{**6}*x + 2*f*g^{**7}*x^{**2}) - 4*c*e^{**3}*f^{**2}*g^{**3}*x^{**3}/(2*f^{**3}*g^{**5} + 4*f^{**2}*g^{**6}*x + 2*f*g^{**7}*x^{**2}) + c*e^{**3}*f*g^{**4}*x^{**4}/(2*f^{**3}*g^{**5} + 4*f^{**2}*g^{**6}*x + 2*f*g^{**7}*x^{**2}), Eq(n, -3), (-3*a*d^{**2}*g^{**4}/(3*f*g^{**5} + 3*g^{**6}*x) + 6*a*d*e*f*g^{**3}*\log(f/g + x)/(3*f*g^{**5} + 3*g^{**6}*x) + 6*a*d*e*f*g^{**3}/(3*f*g^{**5} + 3*g^{**6}*x) + 6*a*d*e*g^{**4}*x*\log(f/g + x)/(3*f*g^{**5} + 3*g^{**6}*x) - 6*a*e^{**2}*f^{**2}*g^{**2}*\log(f/g + x)/(3*f*g^{**5} + 3*g^{**6}*x) - 6*a*e^{**2}*f^{**2}*g^{**2}/(3*f*g^{**5} + 3*g^{**6}*x) - 6*a*e^{**2}*f*g^{**3}*x*\log(f/g + x)/(3*f*g^{**5} + 3*g^{**6}*x) + 3*a*e^{**2}*g^{**4}*x^{**2}/(3*f*g^{**5} + 3*g^{**6}*x) + 6*c*d^{**3}*f*g^{**3}*\log(f/g + x)/(3*f*g^{**5} + 3*g^{**6}*x) + 6*c*d^{**3}*f*g^{**3}/(3*f*g^{**5} + 3*g^{**6}*x) + 6*c*d^{**3}*g^{**4}*x*\log(f/g + x)/(3*f*g^{**5} + 3*g^{**6}*x) - 30*c*d^{**2}*e*f^{**2}*g^{**2}*\log(f/g + x)/(3*f*g^{**5} + 3*g^{**6}*x) - 30*c*d^{**2}*e*f*g^{**3}*x*\log(f/g + x)/(3*f*g^{**5} + 3*g^{**6}*x) + 15*c*d^{**2}*e*g^{**4}*x^{**2}/(3*f*g^{**5} + 3*g^{**6}*x) + 36*c*d*e^{**2}*f^{**3}*g*\log(f/g + x)/(3*f*g^{**5} + 3*g^{**6}*x) + 36*c*d*e^{**2}*f^{**3}*g/(3*f*g^{**5} + 3*g^{**6}*x) + 36*c*d*e^{**2}*f^{**2}*g^{**2}*x*\log(f/g + x)/(3*f*g^{**5} + 3*g^{**6}*x) - 18*c*d*e^{**2}*f*g^{**3}*x^{**2}/(3*f*g^{**5} + 3*g^{**6}*x) + 6*c*d*e^{**2}*g^{**4}*x^{**3}/(3*f*g^{**5} + 3*g^{**6}*x) - 12*c*e^{**3}*f^{**4}*\log(f/g + x)/(3*f*g^{**5} + 3*g^{**6}*x) - 12*c*e^{**3}*f^{**3}*g*x*\log(f/g + x)/(3*f*g^{**5} + 3*g^{**6}*x) + 6*c*e^{**3}*f^{**2}*g^{**2}*x^{**2}/(3*f*g^{**5} + 3*g^{**6}*x) - 2*c*e^{**3}*f*g^{**3}*x^{**3}/(3*f*g^{**5} + 3*g^{**6}*x) + c*e^{**3}*g^{**4}*x^{**4}/(3*f*g^{**5} + 3*g^{**6}*x), Eq(n, -2), (a*d^{**2}*\log(f/g + x)/g - 2*a*d*e*f*\log(f/g + x)/g^{**2} + 2*a*d*e*x/g + a*e^{**2}*f^{**2}*\log(f/g + x)/g^{**3} - a*e^{**2}*f*x/g^{**2} + a*e^{**2}*x^{**2}/(2*g) - 2*c*d^{**3}*f*\log(f/g + x)/g^{**2} + 2*c*d^{**3}*x/g + 5*c*d^{**2}*e*f^{**2}*\log(f/g + x)/g^{**3} - 5*c*d^{**2}*e*f*x/g^{**2} + 5*c*d^{**2}*e*x^{**2}/(2*g) - 4*c*d*e^{**2}*f^{**3}*\log(f/g + x)/g^{**4} + 4*c*d*e^{**2}*f^{**2}*x/g^{**3} - 2*c*d*e^{**2}*f*x^{**2}/g^{**2} + 4*c*d*e^{**2}*x^{**3}/(3*g) + c*e^{**3}*f^{**4}*\log(f/g + x)/g^{**5} - c*e^{**3}*f^{**3}*x/g^{**4} + c*e^{**3}*f^{**2}*x^{**2}/(2*g^{**3}) - c*e^{**3}*f*x^{**3}/(3*g^{**2}) + c*e^{**3}*x^{**4}/(4*g), Eq(n, -1)), (a*d^{**2}*f*g^{**4}*n^{**4}*(f + g*x)^n/(g^{**5}*n^{**5} + 15*g^{**5}*n^{**4} + 85*g^{**5}*n^{**3} + 225*g^{**5}*n^{**2} + 274*g^{**5}*n + 120*g^{**5}) + 14*a*d^{**2}*f*g^{**4}*n^{**3}*(f + g*x)^n/(g^{**5}*n^{**5} + 15*g^{**5}*n^{**4} + 85*g^{**5}*n^{**3} + 225*g^{**5}*n^{**2} + 274*g^{**5}*n + 120*g^{**5}) + 71*a*d^{**2}*f*g^{**4}*n^{**2}*(f + g*x)^n/(g^{**5}*n^{**5} + 15*g^{**5}*n^{**4} + 85*g^{**5}*n^{**3} + 225*g^{**5}*n^{**2} + 274*g^{**5}*n + 120*g^{**5}) + 154*a*d^{**2}*f*g^{**4}*n*(f + g*x)^n/(g^{**5}*n^{**5} + 15*g^{**5}*n^{**4} + 85*g^{**5}*n^{**3} + 225*g^{**5}*n^{**2} + 274*g^{**5}*n + 120*g^{**5}) + 120*a*d^{**2}*f*g^{**4}*(f + g*x)^n/(g^{**5}*n^{**5} + 15*g^{**5}*n^{**4} + 85*g^{**5}*n^{**3} + 225*g^{**5}*n^{**2} + 274*g^{**5}*n + 120*g^{**5}) + a*d^{**2}*g^{**5}*n^{**4}*x*(f + g*x)^n/(g^{**5}*n^{**5} + 15*g^{**5}*n^{**4} + 85*g^{**5}*n^{**3} + 225*g^{**5}*n^{**2} + 274*g^{**5}*n + 120*g^{**5}) + 14*a*d^{**2}*g^{**5}*n^{**3}*x*(f + g*x)^n/(g^{**5}*n^{**5} + 15*g^{**5}*n^{**4} + 85*g^{**5}*n^{**3} + 225*g^{**5}*n^{**2} + 274*g^{**5}*n + 120*g^{**5}) + 71*a*d^{**2}*g^{**5}*n^{**2}*x*(f + g*x)^n/(g^{**5}*n^{**5} + 15*g^{**5}*n^{**4} + 85*g^{**5}*n^{**3} + 225*g^{**5}*n^{**2} + 274*g^{**5}*n + 120*g^{**5}) + 154*a*d^{**2}*g^{**5}*n*x*(f + g*x)^n/(g^{**5}*n^{**5} + 15*g^{**5}*n^{**4} + 85*g^{**5}*n^{**3} + 225*g^{**5}*n^{**2} + 274*g^{**5}*n + 120*g^{**5}) + 120*a*d^{**2}*g^{**5}*x*(f + g*x)^n/(g^{**5}*n^{**5} + 15*g^{**5}*n^{**4} + 85*g^{**5}*n^{**3} + 225*g^{**5}*n^{**2} + 274*g^{**5}*n + 120*g^{**5}) - 2*a*d*e*f^{**2}*g^{**3}*n^{**3}*(f + g*x)^n/(g^{**5}*n^{**5} + 15*g^{**5}*n^{**4} + 85*g^{**5}*n^{**3} + 225*g^{**5}*n^{**2} + 274*g^{**5}*n + 120*g^{**5}) - 24*a*d*e*f^{**2}*g^{**3}*n^{**2}*(f + g*x)^n/(g^{**5}*n^{**5} + 15*g^{**5}*n^{**4} + 85*g^{**5}*n^{**3} + 225*g^{**5}*n^{**2} + 274*g^{**5}*n + 120*g^{**5}) - 94*a*d*e*f^{**2}*g^{**3}*n*(f + g*x)^n
\end{aligned}$$





```

3*f*g**4*n**4*x**4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**
5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 6*c*e**3*f*g**
4*n**3*x**4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3
+ 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 11*c*e**3*f*g**4*n**2
*x**4*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225
*g**5*n**2 + 274*g**5*n + 120*g**5) + 6*c*e**3*f*g**4*n*x**4*(f +
g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2
+ 274*g**5*n + 120*g**5) + c*e**3*g**5*n**4*x**5*(f + g*x)**n/(g
**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5
*n + 120*g**5) + 10*c*e**3*g**5*n**3*x**5*(f + g*x)**n/(g**5*n**5
+ 15*g**5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120
*g**5) + 35*c*e**3*g**5*n**2*x**5*(f + g*x)**n/(g**5*n**5 + 15*g**
5*n**4 + 85*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) +
50*c*e**3*g**5*n*x**5*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 8
5*g**5*n**3 + 225*g**5*n**2 + 274*g**5*n + 120*g**5) + 24*c*e**3*
g**5*x**5*(f + g*x)**n/(g**5*n**5 + 15*g**5*n**4 + 85*g**5*n**3 +
225*g**5*n**2 + 274*g**5*n + 120*g**5), True))

```

**GIAC/XCAS [A]** time = 0.269699, size = 1, normalized size = 0.

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*e\*x^2 + 2\*c\*d\*x + a)\*(e\*x + d)^2\*(g\*x + f)^n,x, algorithm="giac")

[Out] Done



### 3.807 $\int (d + ex)(f + gx)^n (a + 2cdx + cex^2) dx$

**Optimal.** Leaf size=146

$$\frac{(f + gx)^{n+2} (aeg^2 + c(2d^2g^2 - 6defg + 3e^2f^2))}{g^4(n+2)} - \frac{(ef - dg)(f + gx)^{n+1} (ag^2 + cf(ef - 2dg))}{g^4(n+1)}$$

$$- \frac{3ce(ef - dg)(f + gx)^{n+3}}{g^4(n+3)} + \frac{ce^2(f + gx)^{n+4}}{g^4(n+4)}$$

[Out] -(((e\*f - d\*g)\*(a\*g^2 + c\*f\*(e\*f - 2\*d\*g))\*(f + g\*x)^(1 + n))/(g^4\*(1 + n))) + ((a\*e\*g^2 + c\*(3\*e^2\*f^2 - 6\*d\*e\*f\*g + 2\*d^2\*g^2))\*(f + g\*x)^(2 + n))/(g^4\*(2 + n)) - (3\*c\*e\*(e\*f - d\*g)\*(f + g\*x)^(3 + n))/(g^4\*(3 + n)) + (c\*e^2\*(f + g\*x)^(4 + n))/(g^4\*(4 + n))

**Rubi [A]** time = 0.258078, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 26,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.038$

$$\frac{(f + gx)^{n+2} (aeg^2 + c(2d^2g^2 - 6defg + 3e^2f^2))}{g^4(n+2)} - \frac{(ef - dg)(f + gx)^{n+1} (ag^2 + cf(ef - 2dg))}{g^4(n+1)}$$

$$- \frac{3ce(ef - dg)(f + gx)^{n+3}}{g^4(n+3)} + \frac{ce^2(f + gx)^{n+4}}{g^4(n+4)}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)\*(f + g\*x)^n\*(a + 2\*c\*d\*x + c\*e\*x^2), x]

[Out] -(((e\*f - d\*g)\*(a\*g^2 + c\*f\*(e\*f - 2\*d\*g))\*(f + g\*x)^(1 + n))/(g^4\*(1 + n))) + ((a\*e\*g^2 + c\*(3\*e^2\*f^2 - 6\*d\*e\*f\*g + 2\*d^2\*g^2))\*(f + g\*x)^(2 + n))/(g^4\*(2 + n)) - (3\*c\*e\*(e\*f - d\*g)\*(f + g\*x)^(3 + n))/(g^4\*(3 + n)) + (c\*e^2\*(f + g\*x)^(4 + n))/(g^4\*(4 + n))

**Rubi in Sympy [A]** time = 54.5194, size = 141, normalized size = 0.97

$$\frac{ce^2(f + gx)^{n+4}}{g^4(n+4)} + \frac{3ce(f + gx)^{n+3}(dg - ef)}{g^4(n+3)} + \frac{(f + gx)^{n+1}(dg - ef)(ag^2 - 2cdfg + cf^2)}{g^4(n+1)}$$

$$+ \frac{(f + gx)^{n+2}(aeg^2 + 2cd^2g^2 - 6cdefg + 3ce^2f^2)}{g^4(n+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*(g\*x+f)\*\*n\*(c\*e\*x\*\*2+2\*c\*d\*x+a), x)

[Out] c\*e\*\*2\*(f + g\*x)\*\*(n + 4)/(g\*\*4\*(n + 4)) + 3\*c\*e\*(f + g\*x)\*\*(n + 3)\*(d\*g - e\*f)/(g\*\*4\*(n + 3)) + (f + g\*x)\*\*(n + 1)\*(d\*g - e\*f)\*(a\*g\*\*2 - 2\*c\*d\*f\*g + c\*e\*f\*\*2)/(g\*\*4\*(n + 1)) + (f + g\*x)\*\*(n + 2)\*(a\*e\*g\*\*2 + 2\*c\*d\*\*2\*g\*\*2 - 6\*c\*d\*e\*f\*g + 3\*c\*e\*\*2\*f\*\*2)/(g\*\*4\*(n + 2))

**Mathematica [A]** time = 0.307104, size = 190, normalized size = 1.3

$$\frac{(f + gx)^{n+1} (ag^2 (n^2 + 7n + 12) (dg(n + 2) - ef + eg(n + 1)x) + c(2d^2g^2 (n^2 + 7n + 12) (g(n + 1)x - f) + 3deg(n + 4) (2f^2 - g^4(n + 1)(n + 2)))}{g^4(n + 1)(n + 2)}$$

Antiderivative was successfully verified.



$$a^*e^*g^4)^*n)^*x^2 + (6^*c^*d^*e^*f^3^*g + 26^*a^*d^*f^*g^3 - 7^*(2^*c^*d^2 + a^*e^*)^*f^2^*g^2)^*n + (24^*a^*d^*g^4 + (a^*d^*g^4 + (2^*c^*d^2 + a^*e^*)^*f^*g^3)^*n^3 - (6^*c^*d^*e^*f^2^*g^2 - 9^*a^*d^*g^4 - 7^*(2^*c^*d^2 + a^*e^*)^*f^*g^3)^*n^2 + 2^*(3^*c^*e^2^*f^3^*g - 12^*c^*d^*e^*f^2^*g^2 + 13^*a^*d^*g^4 + 6^*(2^*c^*d^2 + a^*e^*)^*f^*g^3)^*n)^*x)^*(g^*x + f)^n/(g^4^n^4 + 10^*g^4^n^3 + 35^*g^4^n^2 + 50^*g^4^n + 24^*g^4)$$

**Sympy [A]** time = 14.9814, size = 4935, normalized size = 33.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(g\*x+f)\*\*n\*(c\*e\*x\*\*2+2\*c\*d\*x+a),x)

[Out] Piecewise(((f\*\*n\*(a\*d\*x + a\*e\*x\*\*2/2 + c\*d\*\*2\*x\*\*2 + c\*d\*e\*x\*\*3 + c\*e\*\*2\*x\*\*4/4), Eq(g, 0)), (-2\*a\*d\*f\*\*2\*g\*\*3/(6\*f\*\*5\*g\*\*4 + 18\*f\*\*4\*g\*\*5\*x + 18\*f\*\*3\*g\*\*6\*x\*\*2 + 6\*f\*\*2\*g\*\*7\*x\*\*3) + 3\*a\*e\*f\*g\*\*4\*x\*\*2/(6\*f\*\*5\*g\*\*4 + 18\*f\*\*4\*g\*\*5\*x + 18\*f\*\*3\*g\*\*6\*x\*\*2 + 6\*f\*\*2\*g\*\*7\*x\*\*3) + a\*e\*g\*\*5\*x\*\*3/(6\*f\*\*5\*g\*\*4 + 18\*f\*\*4\*g\*\*5\*x + 18\*f\*\*3\*g\*\*6\*x\*\*2 + 6\*f\*\*2\*g\*\*7\*x\*\*3) + 6\*c\*d\*\*2\*f\*g\*\*4\*x\*\*2/(6\*f\*\*5\*g\*\*4 + 18\*f\*\*4\*g\*\*5\*x + 18\*f\*\*3\*g\*\*6\*x\*\*2 + 6\*f\*\*2\*g\*\*7\*x\*\*3) + 2\*c\*d\*\*2\*g\*\*5\*x\*\*3/(6\*f\*\*5\*g\*\*4 + 18\*f\*\*4\*g\*\*5\*x + 18\*f\*\*3\*g\*\*6\*x\*\*2 + 6\*f\*\*2\*g\*\*7\*x\*\*3) + 6\*c\*d\*e\*f\*g\*\*4\*x\*\*3/(6\*f\*\*5\*g\*\*4 + 18\*f\*\*4\*g\*\*5\*x + 18\*f\*\*3\*g\*\*6\*x\*\*2 + 6\*f\*\*2\*g\*\*7\*x\*\*3) + 6\*c\*e\*\*2\*f\*\*5\*log(f/g + x)/(6\*f\*\*5\*g\*\*4 + 18\*f\*\*4\*g\*\*5\*x + 18\*f\*\*3\*g\*\*6\*x\*\*2 + 6\*f\*\*2\*g\*\*7\*x\*\*3) + 2\*c\*e\*\*2\*f\*\*5/(6\*f\*\*5\*g\*\*4 + 18\*f\*\*4\*g\*\*5\*x + 18\*f\*\*3\*g\*\*6\*x\*\*2 + 6\*f\*\*2\*g\*\*7\*x\*\*3) + 18\*c\*e\*\*2\*f\*\*4\*g\*x\*log(f/g + x)/(6\*f\*\*5\*g\*\*4 + 18\*f\*\*4\*g\*\*5\*x + 18\*f\*\*3\*g\*\*6\*x\*\*2 + 6\*f\*\*2\*g\*\*7\*x\*\*3) + 18\*c\*e\*\*2\*f\*\*3\*g\*\*2\*x\*\*2\*log(f/g + x)/(6\*f\*\*5\*g\*\*4 + 18\*f\*\*4\*g\*\*5\*x + 18\*f\*\*3\*g\*\*6\*x\*\*2 + 6\*f\*\*2\*g\*\*7\*x\*\*3) - 9\*c\*e\*\*2\*f\*\*3\*g\*\*2\*x\*\*2/(6\*f\*\*5\*g\*\*4 + 18\*f\*\*4\*g\*\*5\*x + 18\*f\*\*3\*g\*\*6\*x\*\*2 + 6\*f\*\*2\*g\*\*7\*x\*\*3) + 6\*c\*e\*\*2\*f\*\*2\*g\*\*3\*x\*\*3\*log(f/g + x)/(6\*f\*\*5\*g\*\*4 + 18\*f\*\*4\*g\*\*5\*x + 18\*f\*\*3\*g\*\*6\*x\*\*2 + 6\*f\*\*2\*g\*\*7\*x\*\*3) - 9\*c\*e\*\*2\*f\*\*2\*g\*\*3\*x\*\*3/(6\*f\*\*5\*g\*\*4 + 18\*f\*\*4\*g\*\*5\*x + 18\*f\*\*3\*g\*\*6\*x\*\*2 + 6\*f\*\*2\*g\*\*7\*x\*\*3), Eq(n, -4)), (-a\*d\*f\*g\*\*3/(2\*f\*\*3\*g\*\*4 + 4\*f\*\*2\*g\*\*5\*x + 2\*f\*g\*\*6\*x\*\*2) + a\*e\*g\*\*4\*x\*\*2/(2\*f\*\*3\*g\*\*4 + 4\*f\*\*2\*g\*\*5\*x + 2\*f\*g\*\*6\*x\*\*2) + 2\*c\*d\*\*2\*g\*\*4\*x\*\*2/(2\*f\*\*3\*g\*\*4 + 4\*f\*\*2\*g\*\*5\*x + 2\*f\*g\*\*6\*x\*\*2) + 6\*c\*d\*e\*f\*\*3\*g\*log(f/g + x)/(2\*f\*\*3\*g\*\*4 + 4\*f\*\*2\*g\*\*5\*x + 2\*f\*g\*\*6\*x\*\*2) + 3\*c\*d\*e\*f\*\*3\*g/(2\*f\*\*3\*g\*\*4 + 4\*f\*\*2\*g\*\*5\*x + 2\*f\*g\*\*6\*x\*\*2) + 12\*c\*d\*e\*f\*\*2\*g\*\*2\*x\*log(f/g + x)/(2\*f\*\*3\*g\*\*4 + 4\*f\*\*2\*g\*\*5\*x + 2\*f\*g\*\*6\*x\*\*2) + 6\*c\*d\*e\*f\*g\*\*3\*x\*\*2\*log(f/g + x)/(2\*f\*\*3\*g\*\*4 + 4\*f\*\*2\*g\*\*5\*x + 2\*f\*g\*\*6\*x\*\*2) - 6\*c\*d\*e\*f\*g\*\*3\*x\*\*2/(2\*f\*\*3\*g\*\*4 + 4\*f\*\*2\*g\*\*5\*x + 2\*f\*g\*\*6\*x\*\*2) - 6\*c\*e\*\*2\*f\*\*4\*log(f/g + x)/(2\*f\*\*3\*g\*\*4 + 4\*f\*\*2\*g\*\*5\*x + 2\*f\*g\*\*6\*x\*\*2) - 3\*c\*e\*\*2\*f\*\*4/(2\*f\*\*3\*g\*\*4 + 4\*f\*\*2\*g\*\*5\*x + 2\*f\*g\*\*6\*x\*\*2) - 12\*c\*e\*\*2\*f\*\*3\*g\*x\*log(f/g + x)/(2\*f\*\*3\*g\*\*4 + 4\*f\*\*2\*g\*\*5\*x + 2\*f\*g\*\*6\*x\*\*2) - 6\*c\*e\*\*2\*f\*\*2\*g\*\*2\*x\*\*2\*log(f/g + x)/(2\*f\*\*3\*g\*\*4 + 4\*f\*\*2\*g\*\*5\*x + 2\*f\*g\*\*6\*x\*\*2) + 6\*c\*e\*\*2\*f\*\*2\*g\*\*2\*x\*\*2/(2\*f\*\*3\*g\*\*4 + 4\*f\*\*2\*g\*\*5\*x + 2\*f\*g\*\*6\*x\*\*2) + 2\*c\*e\*\*2\*f\*g\*\*3\*x\*\*3/(2\*f\*\*3\*g\*\*4 + 4\*f\*\*2\*g\*\*5\*x + 2\*f\*g\*\*6\*x\*\*2), Eq(n, -3)), (-2\*a\*d\*g\*\*3/(2\*f\*g\*\*4 + 2\*g\*\*5\*x) + 2\*a\*e\*f\*g\*\*2\*log(f/g + x)/(2\*f\*g\*\*4 + 2\*g\*\*5\*x) + 2\*a\*e\*f\*g\*\*2/(2\*f\*g\*\*4 + 2\*g\*\*5\*x) + 2\*a\*e\*g\*\*3\*x\*log(f/g + x)/(2\*f\*g\*\*4 + 2\*g\*\*5\*x) + 4\*c\*d\*\*2\*f\*g\*\*2\*log(f/g + x)/(2\*f\*g\*\*4 + 2\*g\*\*5\*x) + 4\*c\*d\*\*2\*f\*g\*\*2/(2\*f\*g\*\*4 + 2\*g\*\*5\*x) + 4\*c\*d\*\*2\*g\*\*3\*x\*log(f/g + x)/(2\*f\*g\*\*4 + 2\*g\*\*5\*x) - 12\*c\*d\*e\*f\*\*2\*g\*log(f/g + x)/(2\*f\*g\*\*4 + 2\*g\*\*5\*x) - 12\*c\*d\*e\*f\*\*2\*g/(2\*f\*g\*\*4 + 2\*g\*\*5\*x) - 12\*c\*d\*e\*f\*\*2\*x\*log(f/g + x)/(2\*f\*g\*\*4 + 2\*g\*\*5\*x) + 6\*c\*d\*e\*g\*\*3\*x\*\*2/(2\*f\*g\*\*4 + 2\*g\*\*5\*x) + 6\*c\*e\*\*2\*f\*\*3\*log(f/g + x)/(2\*f\*g\*\*4 + 2\*g\*\*5\*x) + 6\*c\*e\*\*2\*f\*\*3/(2\*f\*g\*\*4 + 2\*g\*\*5\*x) + 6\*c\*e\*\*2\*f\*\*2\*g\*x\*log(f/g + x)/(2\*f\*g\*\*4 + 2\*g\*\*5\*x) - 3\*c\*e\*\*2\*f\*\*2\*g\*\*2\*x\*\*2/(2\*f\*g\*\*4 + 2\*g\*\*5\*x) + c\*e\*\*2\*g\*\*3\*x\*\*3/(2\*f\*g\*\*4 + 2\*g\*\*5\*x), Eq(n, -2)), (a\*d\*log(f/g + x)/g - a\*e\*f\*log(f/g + x)/g\*\*2 + a\*e\*x/g - 2\*c\*d\*\*2\*f\*log(f/g + x)/g\*\*2 + 2\*c\*d\*\*2\*x/g + 3\*c\*d\*e\*f\*\*2\*log(f/g + x)/g\*\*3 - 3\*c\*d\*e\*f\*x/g\*\*2 + 3\*c\*d\*e\*x\*\*2/(2\*g) - c\*e\*\*2\*f\*\*3\*log(f/g + x)/g\*\*4 + c\*e\*\*2\*f\*\*2\*x/g\*\*3 - c\*e\*\*2\*f\*x\*\*2/(2\*g\*\*2) + c\*e\*\*2\*x\*\*3/(3

$$\begin{aligned}
&g), \text{Eq}(n, -1)), (a^d f^3 g^{3n^3} (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 9a^d f^3 g^{3n^2} \\
&(f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 26a^d f^3 g^{3n} (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 24a^d f^3 g^3 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + a^d g^{4n^3} x (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 9a^d g^{4n^2} x (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 26a^d g^{4n} x (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 24a^d g^4 x (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) - a^e \\
&f^2 g^{2n^2} (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) - 7a^e f^2 g^{2n} (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) - 12a^e f^2 g^{2n} (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + a^e f^3 g^{3n^3} x (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 7a^e \\
&f^3 g^{3n^2} x (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 12a^e f^3 g^{3n} x (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + a^e \\
&g^{4n^3} x^2 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 8a^e g^{4n^2} x^2 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 1 \\
&9a^e g^{4n} x^2 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 12a^e g^4 x^2 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) - 2 \\
&c^d f^2 g^{2n^2} (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) - 14c^d f^2 g^{2n} (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) - 24c^d f^2 g^{2n} (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 2c^d f^3 g^{3n^3} x (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 14c^d f^3 g^{3n^2} x (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 24c^d f^3 g^{3n} x (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 2c^d f^4 g^{4n^4} x^2 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 38c^d f^4 g^{4n^3} x^2 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 24c^d f^4 g^{4n^2} x^2 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 6c^d f^4 g^{4n} x^2 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 24c^d f^4 g^4 x^2 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) - 6c^d e f^2 g^{2n^2} x (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) - 24c^d e f^2 g^{2n} x (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 3c^d e f^3 g^{3n^3} x^2 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 15c^d e f^3 g^{3n^2} x^2 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 12c^d e f^3 g^{3n} x^2 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 3c^d e f^4 g^{4n^4} x^3 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 21c^d e f^4 g^{4n^3} x^3 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 42c^d e f^4 g^{4n^2} x^3 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 24c^d e f^4 g^{4n} x^3 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) - 6c^e f^2 f^4 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 6c^e f^3 g^{3n^3} x (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) - 3c^e f^2 g^{2n^2} x^2 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) - 3c^e f^2 g^{2n} x^2 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + c^e f^3 g^{3n^3} x^3 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 3c^e f^2 g^{2n^2} x^3 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + 2c^e f^3 g^{3n^3} x^3 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + c^e f^4 g^{4n^4} x^3 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + c^e f^4 g^{4n^3} x^3 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + c^e f^4 g^{4n^2} x^3 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + c^e f^4 g^{4n} x^3 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4) + c^e f^4 g^4 x^3 (f + g^x)^n / (g^{4n^4} + 10g^{4n^3} + 35g^{4n^2} + 50g^{4n} + 24g^4)
\end{aligned}$$

```

n + 24*g**4) + 6*c*e**2*g**4*n**2*x**4*(f + g*x)**n/(g**4*n**4 +
10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4) + 11*c*e**2*g*
*4*n*x**4*(f + g*x)**n/(g**4*n**4 + 10*g**4*n**3 + 35*g**4*n**2 +
50*g**4*n + 24*g**4) + 6*c*e**2*g**4*x**4*(f + g*x)**n/(g**4*n**
4 + 10*g**4*n**3 + 35*g**4*n**2 + 50*g**4*n + 24*g**4), True))

```

**GIAC/XCAS [A]** time = 0.266955, size = 1, normalized size = 0.01

*Done*

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e*x^2 + 2*c*d*x + a)*(e*x + d)*(g*x + f)^n,x, algorithm="giac")
```

```
[Out] Done
```

### 3.808 $\int (f + gx)^n (a + 2cdx + cex^2) dx$

**Optimal.** Leaf size=84

$$\frac{(f + gx)^{n+1} (ag^2 + cf(ef - 2dg))}{g^3(n+1)} - \frac{2c(ef - dg)(f + gx)^{n+2}}{g^3(n+2)} + \frac{ce(f + gx)^{n+3}}{g^3(n+3)}$$

[Out]  $((a * g^2 + c * f * (e * f - 2 * d * g)) * (f + g * x)^{(1 + n)}) / (g^3 * (1 + n)) - (2 * c * (e * f - d * g) * (f + g * x)^{(2 + n)}) / (g^3 * (2 + n)) + (c * e * (f + g * x)^{(3 + n)}) / (g^3 * (3 + n))$

**Rubi [A]** time = 0.121457, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$

$$\frac{(f + gx)^{n+1} (ag^2 + cf(ef - 2dg))}{g^3(n+1)} - \frac{2c(ef - dg)(f + gx)^{n+2}}{g^3(n+2)} + \frac{ce(f + gx)^{n+3}}{g^3(n+3)}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)^n\*(a + 2\*c\*d\*x + c\*e\*x^2), x]

[Out]  $((a * g^2 + c * f * (e * f - 2 * d * g)) * (f + g * x)^{(1 + n)}) / (g^3 * (1 + n)) - (2 * c * (e * f - d * g) * (f + g * x)^{(2 + n)}) / (g^3 * (2 + n)) + (c * e * (f + g * x)^{(3 + n)}) / (g^3 * (3 + n))$

**Rubi in Sympy [A]** time = 25.7388, size = 78, normalized size = 0.93

$$\frac{ce(f + gx)^{n+3}}{g^3(n+3)} + \frac{2c(f + gx)^{n+2}(dg - ef)}{g^3(n+2)} + \frac{(f + gx)^{n+1}(ag^2 - 2cdfg + cef^2)}{g^3(n+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*n\*(c\*e\*x\*\*2+2\*c\*d\*x+a), x)

[Out]  $c * e * (f + g * x)^{(n + 3)} / (g^3 * (n + 3)) + 2 * c * (f + g * x)^{(n + 2)} * (d * g - e * f) / (g^3 * (n + 2)) + (f + g * x)^{(n + 1)} * (a * g^2 - 2 * c * d * f * g + c * e * f^2) / (g^3 * (n + 1))$

**Mathematica [A]** time = 0.115003, size = 92, normalized size = 1.1

$$\frac{(f + gx)^{n+1} (ag^2 (n^2 + 5n + 6) + 2cdg(n+3)(g(n+1)x - f) + ce (2f^2 - 2fg(n+1)x + g^2 (n^2 + 3n + 2) x^2))}{g^3(n+1)(n+2)(n+3)}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^n\*(a + 2\*c\*d\*x + c\*e\*x^2), x]

[Out]  $((f + g * x)^{(1 + n)} * (a * g^2 * (6 + 5 * n + n^2) + 2 * c * d * g * (3 + n) * (-f + g * (1 + n) * x) + c * e * (2 * f^2 - 2 * f * g * (1 + n) * x + g^2 * (2 + 3 * n + n^2) * x^2))) / (g^3 * (1 + n) * (2 + n) * (3 + n))$

**Maple [A]** time = 0.007, size = 147, normalized size = 1.8

$$\frac{(gx + f)^{1+n} (ceg^2n^2x^2 + 2cdg^2n^2x + 3ceg^2nx^2 + 8cdg^2nx - 2cefgnx + 2cex^2g^2 + ag^2n^2 - 2cdfgn + 6cdg^2x - 2cefgx + \dots)}{g^3(n^3 + 6n^2 + 11n + 6)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^n*(c*e*x^2+2*c*d*x+a), x)`

[Out] `(g*x+f)^(1+n)*(c*e*g^2*n^2*x^2+2*c*d*g^2*n^2*x+3*c*e*g^2*n*x^2+8*c*d*g^2*n*x-2*c*e*f*g*n*x+2*c*e*g^2*x^2+a*g^2*n^2-2*c*d*f*g*n+6*c*d*g^2*x-2*c*e*f*g*x+5*a*g^2*n-6*c*d*f*g+2*c*e*f^2+6*a*g^2)/g^3/(n^3+6*n^2+11*n+6)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.294048, size = 294, normalized size = 3.5

$$\frac{(afg^2n^2 + 2cef^3 - 6cdf^2g + 6afg^2 + (ceg^3n^2 + 3ceg^3n + 2ceg^3)x^3 + (6cdg^3 + (cefg^2 + 2cdg^3)n^2 + (cefg^2 + 8cdg^3)n) \dots)}{g^3n^3 + 6g^3n^2 + 11g^3n + \dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n,x, algorithm="fricas")`

[Out] `(a*f*g^2*n^2 + 2*c*e*f^3 - 6*c*d*f^2*g + 6*a*f*g^2 + (c*e*g^3*n^2 + 3*c*e*g^3*n + 2*c*e*g^3)*x^3 + (6*c*d*g^3 + (c*e*f*g^2 + 2*c*d*g^3)*n^2 + (c*e*f*g^2 + 8*c*d*g^3)*n)*x^2 - (2*c*d*f^2*g - 5*a*f*g^2)*n + (6*a*g^3 + (2*c*d*f*g^2 + a*g^3)*n^2 - (2*c*e*f^2*g - 6*c*d*f*g^2 - 5*a*g^3)*n)*x*(g*x + f)^n/(g^3*n^3 + 6*g^3*n^2 + 11*g^3*n + 6*g^3)`

**Sympy [A]** time = 6.11464, size = 1530, normalized size = 18.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)**n*(c*e*x**2+2*c*d*x+a), x)`

[Out] `Piecewise((f**n*(a*x + c*d*x**2 + c*e*x**3/3), Eq(g, 0)), (-a*f*g**2/(2*f**3*g**3 + 4*f**2*g**4*x + 2*f*g**5*x**2) + 2*c*d*g**3*x**2/(2*f**3*g**3 + 4*f**2*g**4*x + 2*f*g**5*x**2) + 2*c*e*f**3*log(f/g + x)/(2*f**3*g**3 + 4*f**2*g**4*x + 2*f*g**5*x**2) + c*e*f**3/(2*f**3*g**3 + 4*f**2*g**4*x + 2*f*g**5*x**2) + 4*c*e*f**2*g*x*log(f/g + x)/(2*f**3*g**3 + 4*f**2*g**4*x + 2*f*g**5*x**2) + 2*c*e*f*g**2*x**2*log(f/g + x)/(2*f**3*g**3 + 4*f**2*g**4*x + 2*f*g**5*x**2))`

```

5*x**2) - 2*c*e*f*g**2*x**2/(2*f**3*g**3 + 4*f**2*g**4*x + 2*f*g
*5*x**2), Eq(n, -3)), (a*g**3*x/(f**2*g**3 + f*g**4*x) + 2*c*d*f
*2*g*log(f/g + x)/(f**2*g**3 + f*g**4*x) + 2*c*d*f*g**2*x*log(f/g
+ x)/(f**2*g**3 + f*g**4*x) - 2*c*d*f*g**2*x/(f**2*g**3 + f*g**4
*x) - 2*c*e*f**3*log(f/g + x)/(f**2*g**3 + f*g**4*x) - 2*c*e*f**2
*g*x*log(f/g + x)/(f**2*g**3 + f*g**4*x) + 2*c*e*f**2*g*x/(f**2*g
**3 + f*g**4*x) + c*e*f*g**2*x**2/(f**2*g**3 + f*g**4*x), Eq(n, -
2)), (a*log(f/g + x)/g - 2*c*d*f*log(f/g + x)/g**2 + 2*c*d*x/g +
c*e*f**2*log(f/g + x)/g**3 - c*e*f*x/g**2 + c*e*x**2/(2*g), Eq(n,
-1)), (a*f*g**2*n**2*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*
g**3*n + 6*g**3) + 5*a*f*g**2*n*(f + g*x)**n/(g**3*n**3 + 6*g**3*
n**2 + 11*g**3*n + 6*g**3) + 6*a*f*g**2*(f + g*x)**n/(g**3*n**3 +
6*g**3*n**2 + 11*g**3*n + 6*g**3) + a*g**3*n**2*x*(f + g*x)**n/(
g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + 5*a*g**3*n*x*(f +
g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + 6*a*g**
3*x*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) -
2*c*d*f**2*g*n*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n
+ 6*g**3) - 6*c*d*f**2*g*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 +
11*g**3*n + 6*g**3) + 2*c*d*f*g**2*n**2*x*(f + g*x)**n/(g**3*n**
3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + 6*c*d*f*g**2*n*x*(f + g*x
)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + 2*c*d*g**3*
n**2*x**2*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g
**3) + 8*c*d*g**3*n*x**2*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 +
11*g**3*n + 6*g**3) + 6*c*d*g**3*x**2*(f + g*x)**n/(g**3*n**3 + 6
*g**3*n**2 + 11*g**3*n + 6*g**3) + 2*c*e*f**3*(f + g*x)**n/(g**3*
n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) - 2*c*e*f**2*g*n*x*(f +
g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + c*e*f*g
**2*n**2*x**2*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n +
6*g**3) + c*e*f*g**2*n*x**2*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2
+ 11*g**3*n + 6*g**3) + c*e*g**3*n**2*x**3*(f + g*x)**n/(g**3*n*
**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + 3*c*e*g**3*n*x**3*(f + g
*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3) + 2*c*e*g**
3*x**3*(f + g*x)**n/(g**3*n**3 + 6*g**3*n**2 + 11*g**3*n + 6*g**3
), True))

```

---

**GIAC/XCAS [A]** time = 0.267371, size = 558, normalized size = 6.64

$$cg^3n^2x^3e^{(n\ln(gx+f)+1)} + 2cdg^3n^2x^2e^{(n\ln(gx+f))} + cfdg^2n^2x^2e^{(n\ln(gx+f)+1)} + 3cg^3nx^3e^{(n\ln(gx+f)+1)} + 2cdfg^2n^2xe^{(n\ln(gx+f))} + 8cd$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*e\*x^2 + 2\*c\*d\*x + a)\*(g\*x + f)^n,x, algorithm="giac")

[Out] (c\*g^3\*n^2\*x^3\*e^(n\*ln(g\*x + f) + 1) + 2\*c\*d\*g^3\*n^2\*x^2\*e^(n\*ln(g\*x + f)) + c\*f\*g^2\*n^2\*x^2\*e^(n\*ln(g\*x + f) + 1) + 3\*c\*g^3\*n\*x^3\*e^(n\*ln(g\*x + f) + 1) + 2\*c\*d\*f\*g^2\*n^2\*x\*e^(n\*ln(g\*x + f)) + 8\*c\*d\*g^3\*n\*x^2\*e^(n\*ln(g\*x + f)) + c\*f\*g^2\*n\*x^2\*e^(n\*ln(g\*x + f) + 1) + 2\*c\*g^3\*x^3\*e^(n\*ln(g\*x + f) + 1) + 6\*c\*d\*f\*g^2\*n\*x\*e^(n\*ln(g\*x + f)) + a\*g^3\*n^2\*x\*e^(n\*ln(g\*x + f)) + 6\*c\*d\*g^3\*x^2\*e^(n\*ln(g\*x + f)) - 2\*c\*f^2\*g\*n\*x\*e^(n\*ln(g\*x + f) + 1) - 2\*c\*d\*f^2\*g\*n\*e^(n\*ln(g\*x + f)) + a\*f\*g^2\*n^2\*e^(n\*ln(g\*x + f)) + 5\*a\*g^3\*n\*x\*e^(n\*ln(g\*x + f)) - 6\*c\*d\*f^2\*g\*e^(n\*ln(g\*x + f)) + 5\*a\*f\*g^2\*n\*e^(n\*ln(g\*x + f)) + 6\*a\*g^3\*x\*e^(n\*ln(g\*x + f)) + 2\*c\*f^3\*e^(n\*ln(g\*x + f) + 1) + 6\*a\*f\*g^2\*e^(n\*ln(g\*x + f)))/(g^3\*n^3 + 6\*g^3\*n^2 + 11\*g^3\*n + 6\*g^3)



$$3.809 \quad \int \frac{(f+gx)^n (a+2cdx+cex^2)}{d+ex} dx$$

**Optimal.** Leaf size=114

$$\frac{(cd^2 - ae)(f+gx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{e(f+gx)}{ef-dg}\right)}{e(n+1)(ef-dg)} - \frac{c(ef-dg)(f+gx)^{n+1}}{eg^2(n+1)} + \frac{c(f+gx)^{n+2}}{g^2(n+2)}$$

[Out]  $-\left(\frac{c(e^*f - d^*g)(f + g^*x)^{(1+n)}}{e^*g^{2^*(1+n)}}\right) + \frac{c(f + g^*x)^{(2+n)}}{g^{2^*(2+n)}} + \left(\frac{c(d^{*2} - a^*e)(f + g^*x)^{(1+n)} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{e^*(f + g^*x)}{e^*f - d^*g}\right]}{e^*(e^*f - d^*g)^{(1+n)}}\right)$

**Rubi [A]** time = 0.302347, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$

$$\frac{(cd^2 - ae)(f+gx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{e(f+gx)}{ef-dg}\right)}{e(n+1)(ef-dg)} - \frac{c(ef-dg)(f+gx)^{n+1}}{eg^2(n+1)} + \frac{c(f+gx)^{n+2}}{g^2(n+2)}$$

Antiderivative was successfully verified.

[In] Int[((f + g\*x)^n\*(a + 2\*c\*d\*x + c\*e\*x^2))/(d + e\*x), x]

[Out]  $-\left(\frac{c(e^*f - d^*g)(f + g^*x)^{(1+n)}}{e^*g^{2^*(1+n)}}\right) + \frac{c(f + g^*x)^{(2+n)}}{g^{2^*(2+n)}} + \left(\frac{c(d^{*2} - a^*e)(f + g^*x)^{(1+n)} \text{Hypergeometric2F1}\left[1, 1+n, 2+n, \frac{e^*(f + g^*x)}{e^*f - d^*g}\right]}{e^*(e^*f - d^*g)^{(1+n)}}\right)$

**Rubi in Sympy [A]** time = 35.3139, size = 90, normalized size = 0.79

$$\frac{c(f+gx)^{n+2}}{g^2(n+2)} + \frac{c(f+gx)^{n+1}(dg-ef)}{eg^2(n+1)} + \frac{(f+gx)^{n+1}(ae-cd^2) {}_2F_1\left(1, n+1; n+2; \frac{e(-f-gx)}{dg-ef}\right)}{e(n+1)(dg-ef)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*n\*(c\*e\*x\*\*2+2\*c\*d\*x+a)/(e\*x+d), x)

[Out]  $c(f + g^*x)^{(n+2)}/(g^{*2^*(n+2)}) + c(f + g^*x)^{(n+1)}(d^*g - e^*f)/(e^*g^{*2^*(n+1)}) + (f + g^*x)^{(n+1)}(a^*e - c^*d^{*2}) \text{hyper}\left(1, n+1, (n+2, ), e^*(-f - g^*x)/(d^*g - e^*f)\right)/(e^*(n+1)(d^*g - e^*f))$

**Mathematica [A]** time = 0.557975, size = 125, normalized size = 1.1

$$\frac{(f+gx)^n \left( \frac{(f+gx)(cd^2-ae) {}_2F_1\left(1, n+1; n+2; \frac{e(f+gx)}{ef-dg}\right)}{e(ef-dg)} + \frac{c \left( \frac{dg(f+gx)}{e} + \frac{f^2 \left( \frac{gx}{f} + 1 \right)^{-n-1} + fg n x + g^2(n+1)x^2}{n+2} \right)}{g^2} \right)}{n+1}$$

Antiderivative was successfully verified.

[In] Integrate[(((f + g\*x)^n\*(a + 2\*c\*d\*x + c\*e\*x^2))/(d + e\*x), x]

[Out] ((f + g\*x)^n\*((c\*((d\*g\*(f + g\*x))/e + (f\*g\*n\*x + g^2\*(1 + n)\*x^2 + f^2\*(-1 + (1 + (g\*x)/f)^(-n)))/(2 + n)))/g^2 + ((c\*d^2 - a\*e)\*(f + g\*x)\*Hypergeometric2F1[1, 1 + n, 2 + n, (e\*(f + g\*x))/(e\*f - d\*g)])/(e\*(e\*f - d\*g)))/(1 + n)

**Maple** [F] time = 0.065, size = 0, normalized size = 0.

$$\int \frac{(gx + f)^n (cex^2 + 2cdx + a)}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^n\*(c\*e\*x^2+2\*c\*d\*x+a)/(e\*x+d), x)

[Out] int((g\*x+f)^n\*(c\*e\*x^2+2\*c\*d\*x+a)/(e\*x+d), x)

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cex^2 + 2cdx + a)(gx + f)^n}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*e\*x^2 + 2\*c\*d\*x + a)\*(g\*x + f)^n/(e\*x + d), x, algorithm="maxima")

[Out] integrate((c\*e\*x^2 + 2\*c\*d\*x + a)\*(g\*x + f)^n/(e\*x + d), x)

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cex^2 + 2cdx + a)(gx + f)^n}{ex + d}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*e\*x^2 + 2\*c\*d\*x + a)\*(g\*x + f)^n/(e\*x + d), x, algorithm="fricas")

[Out] integral((c\*e\*x^2 + 2\*c\*d\*x + a)\*(g\*x + f)^n/(e\*x + d), x)

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(f + gx)^n (a + 2cdx + cex^2)}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*n\*(c\*e\*x\*\*2+2\*c\*d\*x+a)/(e\*x+d), x)

[Out] Integral((f + g\*x)\*\*n\*(a + 2\*c\*d\*x + c\*e\*x\*\*2)/(d + e\*x), x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cex^2 + 2cdx + a)(gx + f)^n}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*e\*x^2 + 2\*c\*d\*x + a)\*(g\*x + f)^n/(e\*x + d),x, algorithm="giac")

[Out] integrate((c\*e\*x^2 + 2\*c\*d\*x + a)\*(g\*x + f)^n/(e\*x + d), x)

$$3.810 \quad \int \frac{(f+gx)^n (a+2cdx+cex^2)}{(d+ex)^2} dx$$

**Optimal.** Leaf size=88

$$\frac{c(f+gx)^{n+1}}{eg(n+1)} - \frac{g(cd^2 - ae)(f+gx)^{n+1} {}_2F_1\left(2, n+1; n+2; \frac{e(f+gx)}{ef-dg}\right)}{e(n+1)(ef-dg)^2}$$

[Out] (c\*(f + g\*x)^(1 + n))/(e\*g\*(1 + n)) - ((c\*d^2 - a\*e)\*g\*(f + g\*x)^(1 + n)\*Hypergeometric2F1[2, 1 + n, 2 + n, (e\*(f + g\*x))/(e\*f - d\*g)])/(e\*(e\*f - d\*g)^2\*(1 + n))

**Rubi [A]** time = 0.195228, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$

$$\frac{c(f+gx)^{n+1}}{eg(n+1)} - \frac{g(cd^2 - ae)(f+gx)^{n+1} {}_2F_1\left(2, n+1; n+2; \frac{e(f+gx)}{ef-dg}\right)}{e(n+1)(ef-dg)^2}$$

Antiderivative was successfully verified.

[In] Int[((f + g\*x)^n\*(a + 2\*c\*d\*x + c\*e\*x^2))/(d + e\*x)^2, x]

[Out] (c\*(f + g\*x)^(1 + n))/(e\*g\*(1 + n)) - ((c\*d^2 - a\*e)\*g\*(f + g\*x)^(1 + n)\*Hypergeometric2F1[2, 1 + n, 2 + n, (e\*(f + g\*x))/(e\*f - d\*g)])/(e\*(e\*f - d\*g)^2\*(1 + n))

**Rubi in Sympy [A]** time = 29.1235, size = 68, normalized size = 0.77

$$\frac{c(f+gx)^{n+1}}{eg(n+1)} + \frac{g(f+gx)^{n+1} (ae - cd^2) {}_2F_1\left(2, n+1; n+2; \frac{e(-f-gx)}{dg-ef}\right)}{e(n+1)(dg-ef)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*n\*(c\*e\*x\*\*2+2\*c\*d\*x+a)/(e\*x+d)\*\*2, x)

[Out] c\*(f + g\*x)\*\*(n + 1)/(e\*g\*(n + 1)) + g\*(f + g\*x)\*\*(n + 1)\*(a\*e - c\*d\*\*2)\*hyper((2, n + 1), (n + 2, ), e\*(-f - g\*x)/(d\*g - e\*f))/(e\*(n + 1)\*(d\*g - e\*f)\*\*2)

**Mathematica [A]** time = 0.18044, size = 0, normalized size = 0.

$$\int \frac{(f+gx)^n (a+2cdx+cex^2)}{(d+ex)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[((f + g\*x)^n\*(a + 2\*c\*d\*x + c\*e\*x^2))/(d + e\*x)^2, x]

[Out] Integrate[((f + g\*x)^n\*(a + 2\*c\*d\*x + c\*e\*x^2))/(d + e\*x)^2, x]

**Maple [F]** time = 0.076, size = 0, normalized size = 0.

$$\int \frac{(gx + f)^n (cex^2 + 2cdx + a)}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^n\*(c\*e\*x^2+2\*c\*d\*x+a)/(e\*x+d)^2,x)

[Out] int((g\*x+f)^n\*(c\*e\*x^2+2\*c\*d\*x+a)/(e\*x+d)^2,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cex^2 + 2cdx + a)(gx + f)^n}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*e\*x^2 + 2\*c\*d\*x + a)\*(g\*x + f)^n/(e\*x + d)^2,x, algorithm="maxima")

[Out] integrate((c\*e\*x^2 + 2\*c\*d\*x + a)\*(g\*x + f)^n/(e\*x + d)^2, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cex^2 + 2cdx + a)(gx + f)^n}{e^2x^2 + 2dex + d^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*e\*x^2 + 2\*c\*d\*x + a)\*(g\*x + f)^n/(e\*x + d)^2,x, algorithm="fricas")

[Out] integral((c\*e\*x^2 + 2\*c\*d\*x + a)\*(g\*x + f)^n/(e^2\*x^2 + 2\*d\*e\*x + d^2), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(f + gx)^n (a + 2cdx + cex^2)}{(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*n\*(c\*e\*x\*\*2+2\*c\*d\*x+a)/(e\*x+d)\*\*2,x)

[Out] Integral((f + g\*x)\*\*n\*(a + 2\*c\*d\*x + c\*e\*x\*\*2)/(d + e\*x)\*\*2, x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cex^2 + 2cdx + a)(gx + f)^n}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d)^2,x, algorithm="giac")
```

```
[Out] integrate((c*e*x^2 + 2*c*d*x + a)*(g*x + f)^n/(e*x + d)^2, x)
```

$$3.811 \quad \int \frac{(f+gx)^n (a+2cdx+cex^2)}{(d+ex)^3} dx$$

**Optimal.** Leaf size=193

$$\frac{(f+gx)^{n+1} (aeg^2(1-n)n - c(d^2g^2(-n^2+n+2) - 4defg + 2e^2f^2)) {}_2F_1\left(1, n+1; n+2; \frac{e(f+gx)}{ef-dg}\right)}{2e(n+1)(ef-dg)^3} - \frac{g(1-n)(cd^2-ae)(f+gx)^{n+1}}{2e(d+ex)(ef-dg)^2} - \frac{\left(a - \frac{cd^2}{e}\right)(f+gx)^{n+1}}{2(d+ex)^2(ef-dg)}$$

[Out] -((a - (c\*d^2)/e)\*(f + g\*x)^(1 + n))/(2\*(e\*f - d\*g)\*(d + e\*x)^2) - ((c\*d^2 - a\*e)\*g\*(1 - n)\*(f + g\*x)^(1 + n))/(2\*e\*(e\*f - d\*g)^2\*(d + e\*x)) + ((a\*e\*g^2\*(1 - n)\*n - c\*(2\*e^2\*f^2 - 4\*d\*e\*f\*g + d^2\*g^2\*(2 + n - n^2)))\*(f + g\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (e\*(f + g\*x))/(e\*f - d\*g)])/((2\*e\*(e\*f - d\*g)^3\*(1 + n))

**Rubi [A]** time = 0.468345, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$

$$\frac{(f+gx)^{n+1} (aeg^2(1-n)n - c(d^2g^2(-n^2+n+2) - 4defg + 2e^2f^2)) {}_2F_1\left(1, n+1; n+2; \frac{e(f+gx)}{ef-dg}\right)}{2e(n+1)(ef-dg)^3} - \frac{g(1-n)(cd^2-ae)(f+gx)^{n+1}}{2e(d+ex)(ef-dg)^2} - \frac{\left(a - \frac{cd^2}{e}\right)(f+gx)^{n+1}}{2(d+ex)^2(ef-dg)}$$

Antiderivative was successfully verified.

[In] Int[((f + g\*x)^n\*(a + 2\*c\*d\*x + c\*e\*x^2))/(d + e\*x)^3, x]

[Out] -((a - (c\*d^2)/e)\*(f + g\*x)^(1 + n))/(2\*(e\*f - d\*g)\*(d + e\*x)^2) - ((c\*d^2 - a\*e)\*g\*(1 - n)\*(f + g\*x)^(1 + n))/(2\*e\*(e\*f - d\*g)^2\*(d + e\*x)) + ((a\*e\*g^2\*(1 - n)\*n - c\*(2\*e^2\*f^2 - 4\*d\*e\*f\*g + d^2\*g^2\*(2 + n - n^2)))\*(f + g\*x)^(1 + n)\*Hypergeometric2F1[1, 1 + n, 2 + n, (e\*(f + g\*x))/(e\*f - d\*g)])/((2\*e\*(e\*f - d\*g)^3\*(1 + n))

**Rubi in Sympy [A]** time = 37.2965, size = 95, normalized size = 0.49

$$\frac{c(f+gx)^{n+1} {}_2F_1\left(1, n+1; n+2; \frac{e(-f-gx)}{dg-ef}\right)}{e(n+1)(dg-ef)} + \frac{g^2(f+gx)^{n+1} (ae - cd^2) {}_2F_1\left(3, n+1; n+2; \frac{e(-f-gx)}{dg-ef}\right)}{e(n+1)(dg-ef)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*n\*(c\*e\*x\*\*2+2\*c\*d\*x+a)/(e\*x+d)\*\*3, x)

[Out] c\*(f + g\*x)\*\*(n + 1)\*hyper((1, n + 1), (n + 2, ), e\*(-f - g\*x)/(d\*g - e\*f))/(e\*(n + 1)\*(d\*g - e\*f)) + g\*\*2\*(f + g\*x)\*\*(n + 1)\*(a\*e - c\*d\*\*2)\*hyper((3, n + 1), (n + 2, ), e\*(-f - g\*x)/(d\*g - e\*f))/(e\*(n + 1)\*(d\*g - e\*f)\*\*3)

**Mathematica [A]** time = 0.232944, size = 0, normalized size = 0.

$$\int \frac{(f+gx)^n (a+2cdx+cex^2)}{(d+ex)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[((f + g\*x)^n\*(a + 2\*c\*d\*x + c\*e\*x^2))/(d + e\*x)^3, x]

[Out] Integrate[((f + g\*x)^n\*(a + 2\*c\*d\*x + c\*e\*x^2))/(d + e\*x)^3, x]

**Maple** [F] time = 0.109, size = 0, normalized size = 0.

$$\int \frac{(gx + f)^n (cex^2 + 2cdx + a)}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^n\*(c\*e\*x^2+2\*c\*d\*x+a)/(e\*x+d)^3, x)

[Out] int((g\*x+f)^n\*(c\*e\*x^2+2\*c\*d\*x+a)/(e\*x+d)^3, x)

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cex^2 + 2cdx + a)(gx + f)^n}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*e\*x^2 + 2\*c\*d\*x + a)\*(g\*x + f)^n/(e\*x + d)^3, x, algorithm="maxima")

[Out] integrate((c\*e\*x^2 + 2\*c\*d\*x + a)\*(g\*x + f)^n/(e\*x + d)^3, x)

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cex^2 + 2cdx + a)(gx + f)^n}{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*e\*x^2 + 2\*c\*d\*x + a)\*(g\*x + f)^n/(e\*x + d)^3, x, algorithm="fricas")

[Out] integral((c\*e\*x^2 + 2\*c\*d\*x + a)\*(g\*x + f)^n/(e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3), x)

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(f + gx)^n (a + 2cdx + cex^2)}{(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*n\*(c\*e\*x\*\*2+2\*c\*d\*x+a)/(e\*x+d)\*\*3, x)

[Out] Integral((f + g\*x)\*\*n\*(a + 2\*c\*d\*x + c\*e\*x\*\*2)/(d + e\*x)\*\*3, x)



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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cex^2 + 2cdx + a)(gx + f)^n}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*e\*x^2 + 2\*c\*d\*x + a)\*(g\*x + f)^n/(e\*x + d)^3,x, algorithm="giac")

[Out] integrate((c\*e\*x^2 + 2\*c\*d\*x + a)\*(g\*x + f)^n/(e\*x + d)^3, x)

$$3.812 \quad \int \frac{(f+gx)^n (a+2cdx+cex^2)}{(d+ex)^4} dx$$

**Optimal.** Leaf size=197

$$\frac{g(f+gx)^{n+1} (aeg^2 (n^2 - 3n + 2) + c (d^2g^2 (-n^2 + 3n + 4) - 12defg + 6e^2f^2)) {}_2F_1\left(2, n+1; n+2; \frac{e(f+gx)}{ef-dg}\right)}{6e(n+1)(ef-dg)^4} - \frac{g(2-n)(cd^2-ae)(f+gx)^{n+1}}{6e(d+ex)^2(ef-dg)^2} - \frac{\left(a - \frac{cd^2}{e}\right)(f+gx)^{n+1}}{3(d+ex)^3(ef-dg)}$$

[Out]  $-\left(\frac{a - (c \cdot d^2)/e}{e}\right) \cdot (f + g \cdot x)^{(1+n)} / (3 \cdot (e \cdot f - d \cdot g) \cdot (d + e \cdot x)^3) - \left(\frac{(c \cdot d^2 - a \cdot e) \cdot g \cdot (2-n) \cdot (f + g \cdot x)^{(1+n)}}{6 \cdot e \cdot (e \cdot f - d \cdot g)^2 \cdot (d + e \cdot x)^2} + \frac{g \cdot (a \cdot e \cdot g^2 \cdot (2 - 3 \cdot n + n^2) + c \cdot (6 \cdot e^2 \cdot f^2 - 12 \cdot d \cdot e \cdot f \cdot g + d^2 \cdot g^2 \cdot (4 + 3 \cdot n - n^2))) \cdot (f + g \cdot x)^{(1+n)} \cdot \text{Hypergeometric2F1}\left[2, 1+n, 2+n, \frac{e \cdot (f + g \cdot x)}{e \cdot f - d \cdot g}\right]}{6 \cdot e \cdot (e \cdot f - d \cdot g)^4 \cdot (1+n)}\right)$

**Rubi [A]** time = 0.513851, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$

$$\frac{g(f+gx)^{n+1} (aeg^2 (n^2 - 3n + 2) + c (d^2g^2 (-n^2 + 3n + 4) - 12defg + 6e^2f^2)) {}_2F_1\left(2, n+1; n+2; \frac{e(f+gx)}{ef-dg}\right)}{6e(n+1)(ef-dg)^4} - \frac{g(2-n)(cd^2-ae)(f+gx)^{n+1}}{6e(d+ex)^2(ef-dg)^2} - \frac{\left(a - \frac{cd^2}{e}\right)(f+gx)^{n+1}}{3(d+ex)^3(ef-dg)}$$

Antiderivative was successfully verified.

[In] Int[((f + g\*x)^n\*(a + 2\*c\*d\*x + c\*e\*x^2))/(d + e\*x)^4, x]

[Out]  $-\left(\frac{a - (c \cdot d^2)/e}{e}\right) \cdot (f + g \cdot x)^{(1+n)} / (3 \cdot (e \cdot f - d \cdot g) \cdot (d + e \cdot x)^3) - \left(\frac{(c \cdot d^2 - a \cdot e) \cdot g \cdot (2-n) \cdot (f + g \cdot x)^{(1+n)}}{6 \cdot e \cdot (e \cdot f - d \cdot g)^2 \cdot (d + e \cdot x)^2} + \frac{g \cdot (a \cdot e \cdot g^2 \cdot (2 - 3 \cdot n + n^2) + c \cdot (6 \cdot e^2 \cdot f^2 - 12 \cdot d \cdot e \cdot f \cdot g + d^2 \cdot g^2 \cdot (4 + 3 \cdot n - n^2))) \cdot (f + g \cdot x)^{(1+n)} \cdot \text{Hypergeometric2F1}\left[2, 1+n, 2+n, \frac{e \cdot (f + g \cdot x)}{e \cdot f - d \cdot g}\right]}{6 \cdot e \cdot (e \cdot f - d \cdot g)^4 \cdot (1+n)}\right)$

**Rubi in Sympy [A]** time = 38.8343, size = 99, normalized size = 0.5

$$\frac{cg(f+gx)^{n+1} {}_2F_1\left(2, n+1; n+2; \frac{e(-f-gx)}{dg-ef}\right)}{e(n+1)(dg-ef)^2} + \frac{g^3(f+gx)^{n+1} (ae - cd^2) {}_2F_1\left(4, n+1; n+2; \frac{e(-f-gx)}{dg-ef}\right)}{e(n+1)(dg-ef)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*n\*(c\*e\*x\*\*2+2\*c\*d\*x+a)/(e\*x+d)\*\*4, x)

[Out]  $c \cdot g \cdot (f + g \cdot x)^{(n+1)} \cdot \text{hyper}\left(\left(2, n+1, \right), (n+2, ), e \cdot (-f - g \cdot x) / (d \cdot g - e \cdot f)\right) / (e \cdot (n+1) \cdot (d \cdot g - e \cdot f)^2) + g^3 \cdot (f + g \cdot x)^{(n+1)} \cdot (a \cdot e - c \cdot d^2) \cdot \text{hyper}\left(\left(4, n+1, \right), (n+2, ), e \cdot (-f - g \cdot x) / (d \cdot g - e \cdot f)\right) / (e \cdot (n+1) \cdot (d \cdot g - e \cdot f)^4)$

**Mathematica [A]** time = 0.283431, size = 0, normalized size = 0.

$$\int \frac{(f+gx)^n (a+2cdx+cex^2)}{(d+ex)^4} dx$$

Verification is Not applicable to the result.

[In] Integrate[((f + g\*x)^n\*(a + 2\*c\*d\*x + c\*e\*x^2))/(d + e\*x)^4, x]

[Out] Integrate[((f + g\*x)^n\*(a + 2\*c\*d\*x + c\*e\*x^2))/(d + e\*x)^4, x]

**Maple** [F] time = 0.161, size = 0, normalized size = 0.

$$\int \frac{(gx + f)^n (cex^2 + 2cdx + a)}{(ex + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^n\*(c\*e\*x^2+2\*c\*d\*x+a)/(e\*x+d)^4, x)

[Out] int((g\*x+f)^n\*(c\*e\*x^2+2\*c\*d\*x+a)/(e\*x+d)^4, x)

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cex^2 + 2cdx + a)(gx + f)^n}{(ex + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*e\*x^2 + 2\*c\*d\*x + a)\*(g\*x + f)^n/(e\*x + d)^4, x, algorithm="maxima")

[Out] integrate((c\*e\*x^2 + 2\*c\*d\*x + a)\*(g\*x + f)^n/(e\*x + d)^4, x)

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cex^2 + 2cdx + a)(gx + f)^n}{e^4x^4 + 4de^3x^3 + 6d^2e^2x^2 + 4d^3ex + d^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*e\*x^2 + 2\*c\*d\*x + a)\*(g\*x + f)^n/(e\*x + d)^4, x, algorithm="fricas")

[Out] integral((c\*e\*x^2 + 2\*c\*d\*x + a)\*(g\*x + f)^n/(e^4\*x^4 + 4\*d\*e^3\*x^3 + 6\*d^2\*e^2\*x^2 + 4\*d^3\*e\*x + d^4), x)

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(f + gx)^n (a + 2cdx + cex^2)}{(d + ex)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*n\*(c\*e\*x\*\*2+2\*c\*d\*x+a)/(e\*x+d)\*\*4, x)

[Out] Integral((f + g\*x)\*\*n\*(a + 2\*c\*d\*x + c\*e\*x\*\*2)/(d + e\*x)\*\*4, x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cex^2 + 2cdx + a)(gx + f)^n}{(ex + d)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*e\*x^2 + 2\*c\*d\*x + a)\*(g\*x + f)^n/(e\*x + d)^4,x, algorithm="giac")

[Out] integrate((c\*e\*x^2 + 2\*c\*d\*x + a)\*(g\*x + f)^n/(e\*x + d)^4, x)

### 3.813 $\int (d + ex)^m (f + gx)^n (a + 2cdx + cex^2) dx$

**Optimal.** Leaf size=231

$$\frac{(d + ex)^{m+1} (f + gx)^n \left( \frac{e(f+gx)}{ef-dg} \right)^{-n} (g(m+n+2)(aeg(m+n+3) - cd(dg(n+1) + ef(m+2))) + c(m+2)(ef - dg)(dg(n+1) + e^2g^2(m+1)(m+n+2)(m+n+3))}{e^2g^2(m+1)(m+n+2)(m+n+3)} - \frac{c(m+2)(ef - dg)(d + ex)^{m+1} (f + gx)^{n+1}}{eg^2(m+n+2)(m+n+3)} + \frac{c(d + ex)^{m+2} (f + gx)^{n+1}}{eg(m+n+3)}$$

[Out]  $-\left(\left(c^*(e*f - d*g)^*(2 + m)^*(d + e*x)^{(1 + m)}*(f + g*x)^{(1 + n)}\right)/\left(e^*g^{2*(2 + m + n)}*(3 + m + n)\right)\right) + \left(c^*(d + e*x)^{(2 + m)}*(f + g*x)^{(1 + n)}\right)/\left(e^*g^*(3 + m + n)\right) + \left(\left(c^*(e*f - d*g)^*(2 + m)^*(e*f*(1 + m) + d*g*(1 + n)) + g^*(2 + m + n)^*(a*e*g^*(3 + m + n) - c*d*(e*f*(2 + m) + d*g*(1 + n)))\right)^*(d + e*x)^{(1 + m)}*(f + g*x)^n*\text{Hypergeometric2F1}\left[1 + m, -n, 2 + m, -\left(\frac{g*(d + e*x)}{(e*f - d*g)}\right)\right]/\left(e^{2*m}g^{2*(1 + m)}*(2 + m + n)^*(3 + m + n)^*\left(\frac{e*(f + g*x)}{(e*f - d*g)}\right)^n\right)$

**Rubi [A]** time = 0.605679, antiderivative size = 227, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, integrand size = 28,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$

$$\frac{(d + ex)^{m+1} (f + gx)^n \left( \frac{e(f+gx)}{ef-dg} \right)^{-n} \left( aeg(m+n+3) + \frac{c(m+2)(ef-dg)(dg(n+1)+ef(m+1))}{g(m+n+2)} - cd(dg(n+1) + ef(m+2)) \right) {}_2F_1\left(m+1, -n, 2+m, -\frac{g(d+ex)}{dg-ef}\right)}{e^2g^2(m+1)(m+n+3)} - \frac{c(m+2)(ef - dg)(d + ex)^{m+1} (f + gx)^{n+1}}{eg^2(m+n+2)(m+n+3)} + \frac{c(d + ex)^{m+2} (f + gx)^{n+1}}{eg(m+n+3)}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^m\*(f + g\*x)^n\*(a + 2\*c\*d\*x + c\*e\*x^2), x]

[Out]  $-\left(\left(c^*(e*f - d*g)^*(2 + m)^*(d + e*x)^{(1 + m)}*(f + g*x)^{(1 + n)}\right)/\left(e^*g^{2*(2 + m + n)}*(3 + m + n)\right)\right) + \left(c^*(d + e*x)^{(2 + m)}*(f + g*x)^{(1 + n)}\right)/\left(e^*g^*(3 + m + n)\right) + \left(\left(a*e*g^*(3 + m + n) + \left(c^*(e*f - d*g)^*(2 + m)^*(e*f*(1 + m) + d*g*(1 + n))\right)/\left(g^*(2 + m + n) - c*d*(e*f*(2 + m) + d*g*(1 + n))\right)\right)^*(d + e*x)^{(1 + m)}*(f + g*x)^n*\text{Hypergeometric2F1}\left[1 + m, -n, 2 + m, -\left(\frac{g*(d + e*x)}{(e*f - d*g)}\right)\right]/\left(e^{2*m}g^*(1 + m)^*(3 + m + n)^*\left(\frac{e*(f + g*x)}{(e*f - d*g)}\right)^n\right)$

**Rubi in Sympy [A]** time = 104.94, size = 231, normalized size = 1.

$$\frac{c \left( \frac{e(-f-gx)}{dg-ef} \right)^{-n} (d + ex)^{m+1} (f + gx)^n (dg - ef)^2 {}_2F_1\left(-n - 2, m + 1, m + 2, \frac{g(d+ex)}{dg-ef}\right)}{e^2g^2(m+1)} - \frac{2c \left( \frac{e(-f-gx)}{dg-ef} \right)^{-n} (d + ex)^{m+1} (f + gx)^n (dg - ef)^2 {}_2F_1\left(-n - 1, m + 1, m + 2, \frac{g(d+ex)}{dg-ef}\right)}{e^2g^2(m+1)} + \frac{\left( \frac{e(-f-gx)}{dg-ef} \right)^{-n} (d + ex)^{m+1} (f + gx)^n (ag^2 - 2cdfg + cef^2) {}_2F_1\left(-n, m + 1, m + 2, \frac{g(d+ex)}{dg-ef}\right)}{eg^2(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*m\*(g\*x+f)\*\*n\*(c\*e\*x\*\*2+2\*c\*d\*x+a), x)

[Out]  $c^*(e*(-f - g*x)/(d*g - e*f))^{n+1}*(-n)^*(d + e*x)^{m+1}*(f + g*x)^{n+1}*(d*g - e*f)^{2*n}*\text{hyper}\left((-n - 2, m + 1), (m + 2, ), g*(d + e*x)/(d*g - e*f)\right)$

$$\frac{g - e^*f)}{(e^{**2}g^{**2}(m + 1)) - 2*c*(e*(-f - g*x)/(d*g - e*f))^{**}(-n)*(d + e*x)^{(m + 1)*(f + g*x)^{**}n*(d*g - e*f)^{**2}*hyper((-n - 1, m + 1), (m + 2, ), g*(d + e*x)/(d*g - e*f))/(e^{**2}g^{**2}(m + 1)) + (e*(-f - g*x)/(d*g - e*f))^{**}(-n)*(d + e*x)^{(m + 1)*(f + g*x)^{**}n*(a*g^{**2} - 2*c*d*f*g + c*e*f^{**2})*hyper((-n, m + 1), (m + 2, ), g*(d + e*x)/(d*g - e*f))/(e*g^{**2}(m + 1))}$$

**Mathematica [C]** time = 1.30129, size = 328, normalized size = 1.42

$$\frac{1}{3}(d + ex)^m(f + gx)^n \left( \frac{3a(f + gx) \left( \frac{g(d+ex)}{dg-ef} \right)^{-m} {}_2F_1 \left( -m, n + 1; n + 2; \frac{e(f+gx)}{ef-dg} \right)}{g(n + 1)} \right. \\ \left. + \frac{9cd^2fx^2F_1 \left( 2; -m, -n; 3; -\frac{ex}{d}, -\frac{gx}{f} \right)}{3dfF_1 \left( 2; -m, -n; 3; -\frac{ex}{d}, -\frac{gx}{f} \right) + efmxF_1 \left( 3; 1 - m, -n; 4; -\frac{ex}{d}, -\frac{gx}{f} \right) + dgnxF_1 \left( 3; -m, 1 - n; 4; -\frac{ex}{d}, -\frac{gx}{f} \right)} \right. \\ \left. + \frac{4cdefx^3F_1 \left( 3; -m, -n; 4; -\frac{ex}{d}, -\frac{gx}{f} \right)}{4dfF_1 \left( 3; -m, -n; 4; -\frac{ex}{d}, -\frac{gx}{f} \right) + efmxF_1 \left( 4; 1 - m, -n; 5; -\frac{ex}{d}, -\frac{gx}{f} \right) + dgnxF_1 \left( 4; -m, 1 - n; 5; -\frac{ex}{d}, -\frac{gx}{f} \right)} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(d + e\*x)^m\*(f + g\*x)^n\*(a + 2\*c\*d\*x + c\*e\*x^2), x]

[Out] ((d + e\*x)^m\*(f + g\*x)^n\*((9\*c\*d^2\*f\*x^2\*AppellF1[2, -m, -n, 3, -((e\*x)/d), -((g\*x)/f)]/(3\*d\*f\*AppellF1[2, -m, -n, 3, -((e\*x)/d), -((g\*x)/f)] + e\*f\*m\*x\*AppellF1[3, 1 - m, -n, 4, -((e\*x)/d), -((g\*x)/f)] + d\*g\*n\*x\*AppellF1[3, -m, 1 - n, 4, -((e\*x)/d), -((g\*x)/f)]) + (4\*c\*d\*e\*f\*x^3\*AppellF1[3, -m, -n, 4, -((e\*x)/d), -((g\*x)/f)]/(4\*d\*f\*AppellF1[3, -m, -n, 4, -((e\*x)/d), -((g\*x)/f)] + e\*f\*m\*x\*AppellF1[4, 1 - m, -n, 5, -((e\*x)/d), -((g\*x)/f)] + d\*g\*n\*x\*AppellF1[4, -m, 1 - n, 5, -((e\*x)/d), -((g\*x)/f)]) + (3\*a\*(f + g\*x)\*Hypergeometric2F1[-m, 1 + n, 2 + n, (e\*(f + g\*x))/(e\*f - d\*g)]/(g\*(1 + n)\*((g\*(d + e\*x))/(-e\*f) + d\*g))^m))/3

**Maple [F]** time = 0.089, size = 0, normalized size = 0.

$$\int (ex + d)^m (gx + f)^n (cex^2 + 2cdx + a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^m\*(g\*x+f)^n\*(c\*e\*x^2+2\*c\*d\*x+a), x)

[Out] int((e\*x+d)^m\*(g\*x+f)^n\*(c\*e\*x^2+2\*c\*d\*x+a), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (cex^2 + 2cdx + a)(ex + d)^m(gx + f)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*e\*x^2 + 2\*c\*d\*x + a)\*(e\*x + d)^m\*(g\*x + f)^n, x, algorithm="maxima")

[Out] integrate((c\*e\*x^2 + 2\*c\*d\*x + a)\*(e\*x + d)^m\*(g\*x + f)^n, x)

---

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((cex^2 + 2cdx + a)(ex + d)^m(gx + f)^n, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*e*x^2 + 2*c*d*x + a)*(e*x + d)^m*(g*x + f)^n,x, algorithm="fricas")`

[Out] `integral((c*e*x^2 + 2*c*d*x + a)*(e*x + d)^m*(g*x + f)^n, x)`

---

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**m*(g*x+f)**n*(c*e*x**2+2*c*d*x+a),x)`

[Out] Timed out

---

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int (cex^2 + 2cdx + a)(ex + d)^m(gx + f)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*e*x^2 + 2*c*d*x + a)*(e*x + d)^m*(g*x + f)^n,x, algorithm="giac")`

[Out] `integrate((c*e*x^2 + 2*c*d*x + a)*(e*x + d)^m*(g*x + f)^n, x)`

$$3.814 \quad \int \frac{a+bx+cx^2}{(d+ex)(f+gx)} dx$$

**Optimal.** Leaf size=83

$$\frac{\log(d+ex)(ae^2 - bde + cd^2)}{e^2(ef - dg)} - \frac{\log(f+gx)(ag^2 - bfg + cf^2)}{g^2(ef - dg)} + \frac{cx}{eg}$$

[Out] (c\*x)/(e\*g) + ((c\*d^2 - b\*d\*e + a\*e^2)\*Log[d + e\*x])/(e^2\*(e\*f - d\*g)) - ((c\*f^2 - b\*f\*g + a\*g^2)\*Log[f + g\*x])/(g^2\*(e\*f - d\*g))

**Rubi [A]** time = 0.204558, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$\frac{\log(d+ex)(ae^2 - bde + cd^2)}{e^2(ef - dg)} - \frac{\log(f+gx)(ag^2 - bfg + cf^2)}{g^2(ef - dg)} + \frac{cx}{eg}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)/((d + e\*x)\*(f + g\*x)), x]

[Out] (c\*x)/(e\*g) + ((c\*d^2 - b\*d\*e + a\*e^2)\*Log[d + e\*x])/(e^2\*(e\*f - d\*g)) - ((c\*f^2 - b\*f\*g + a\*g^2)\*Log[f + g\*x])/(g^2\*(e\*f - d\*g))

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{(ag^2 - bfg + cf^2) \log(f + gx)}{g^2(dg - ef)} + \frac{\int c dx}{eg} - \frac{(ae^2 - bde + cd^2) \log(d + ex)}{e^2(dg - ef)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+b\*x+a)/(e\*x+d)/(g\*x+f), x)

[Out] (a\*g\*\*2 - b\*f\*g + c\*f\*\*2)\*log(f + g\*x)/(g\*\*2\*(d\*g - e\*f)) + Integral(c, x)/(e\*g) - (a\*e\*\*2 - b\*d\*e + c\*d\*\*2)\*log(d + e\*x)/(e\*\*2\*(d\*g - e\*f))

**Mathematica [A]** time = 0.0956298, size = 85, normalized size = 1.02

$$-\frac{\log(d+ex)(-ae^2 + bde - cd^2)}{e^2(ef - dg)} - \frac{\log(f+gx)(ag^2 - bfg + cf^2)}{g^2(ef - dg)} + \frac{cx}{eg}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)/((d + e\*x)\*(f + g\*x)), x]

[Out] (c\*x)/(e\*g) - ((-(c\*d^2) + b\*d\*e - a\*e^2)\*Log[d + e\*x])/(e^2\*(e\*f - d\*g)) - ((c\*f^2 - b\*f\*g + a\*g^2)\*Log[f + g\*x])/(g^2\*(e\*f - d\*g))

**Maple [A]** time = 0.011, size = 142, normalized size = 1.7

$$\frac{cx}{eg} + \frac{\ln(gx+f)a}{dg-ef} - \frac{\ln(gx+f)bf}{(dg-ef)g} + \frac{\ln(gx+f)cf^2}{g^2(dg-ef)} - \frac{\ln(ex+d)a}{dg-ef} + \frac{\ln(ex+d)bd}{(dg-ef)e} - \frac{\ln(ex+d)cd^2}{(dg-ef)e^2}$$



Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)/(e*x+d)/(g*x+f),x)`

[Out]  $c*x/e/g+1/(d*g-e*f)*\ln(g*x+f)*a-1/g/(d*g-e*f)*\ln(g*x+f)*b*f+1/g^2/(d*g-e*f)*\ln(g*x+f)*c*f^2-1/(d*g-e*f)*\ln(e*x+d)*a+1/(d*g-e*f)/e*\ln(e*x+d)*b*d-1/(d*g-e*f)/e^2*\ln(e*x+d)*c*d^2$

**Maxima [A]** time = 0.692818, size = 117, normalized size = 1.41

$$\frac{(cd^2 - bde + ae^2) \log(ex + d)}{e^3 f - de^2 g} - \frac{(cf^2 - bfg + ag^2) \log(gx + f)}{efg^2 - dg^3} + \frac{cx}{eg}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)/((e*x + d)*(g*x + f)),x, algorithm="maxima")`

[Out]  $(c*d^2 - b*d*e + a*e^2)*\log(e*x + d)/(e^3*f - d*e^2*g) - (c*f^2 - b*f*g + a*g^2)*\log(g*x + f)/(e*f*g^2 - d*g^3) + c*x/(e*g)$

**Fricas [A]** time = 0.295032, size = 134, normalized size = 1.61

$$\frac{(cd^2 - bde + ae^2)g^2 \log(ex + d) + (ce^2fg - cdeg^2)x - (ce^2f^2 - be^2fg + ae^2g^2) \log(gx + f)}{e^3fg^2 - de^2g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)/((e*x + d)*(g*x + f)),x, algorithm="fricas")`

[Out]  $((c*d^2 - b*d*e + a*e^2)*g^2*\log(e*x + d) + (c*e^2*f*g - c*d*e*g^2)*x - (c*e^2*f^2 - b*e^2*f*g + a*e^2*g^2)*\log(g*x + f))/(e^3*f*g^2 - d*e^2*g^3)$

**Sympy [A]** time = 15.4255, size = 420, normalized size = 5.06

$\frac{cx}{eg}$

$$\frac{(ag^2 - bfg + cf^2) \log\left(x + \frac{adeg^2 + ae^2fg - 2bdefg + cd^2fg + cdef^2 - \frac{d^2eg(ag^2 - bfg + cf^2)}{dg - ef} + \frac{2de^2f(ag^2 - bfg + cf^2)}{dg - ef} - \frac{e^3f^2(ag^2 - bfg + cf^2)}{g(dg - ef)}}{2ae^2g^2 - bdeg^2 - be^2fg + cd^2g^2 + ce^2f^2}\right)}{g^2(dg - ef)} + \frac{(ae^2 - bde + cd^2) \log\left(x + \frac{adeg^2 + ae^2fg - 2bdefg + cd^2fg + cdef^2 + \frac{d^2g^3(ae^2 - bde + cd^2)}{e(dg - ef)} - \frac{2dfg^2(ae^2 - bde + cd^2)}{dg - ef} + \frac{ef^2g(ae^2 - bde + cd^2)}{dg - ef}}{2ae^2g^2 - bdeg^2 - be^2fg + cd^2g^2 + ce^2f^2}\right)}{e^2(dg - ef)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+b*x+a)/(e*x+d)/(g*x+f),x)`

[Out]  $c*x/(e*g) + (a*g**2 - b*f*g + c*f**2)*\log(x + (a*d*e*g**2 + a*e**2*f*g - 2*b*d*e*f*g + c*d**2*f*g + c*d*e*f**2 - d**2*e*g*(a*g**2 - b*f*g + c*f**2))/(d*g - e*f) + 2*d*e**2*f*(a*g**2 - b*f*g + c*f**2)/(d*g - e*f) - e**3*f**2*(a*g**2 - b*f*g + c*f**2)/(g*(d*g - e*f)))/(2*a*e**2*g**2 - b*d*e*g**2 - b*e**2*f*g + c*d**2*g**2 + c$

$$\frac{e^{2f}}{g^2(dg - ef)} - (ae^2 - bde + cd^2) \log(x + \frac{ad^2e^2g^2 + ae^2fg - 2bde^2fg + cd^2fg + cde^2f^2 + d^2g^3(ae^2 - bde + cd^2)}{e(dg - ef)} - 2d^2fg^2(ae^2 - bde + cd^2)/(dg - ef) + e^2fg^2(ae^2 - bde + cd^2)/(dg - ef))/(2ae^2g^2 - bde^2g^2 - be^2fg + cd^2g^2 + ce^2f^2)/(e^2(dg - ef))$$

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/((e\*x + d)\*(g\*x + f)),x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.815 \quad \int \frac{(a+bx+cx^2)^2}{(d+ex)(f+gx)} dx$$

**Optimal.** Leaf size=184

$$\frac{x(-2ceg(-aeg+bdg+bef)+b^2e^2g^2+c^2(d^2g^2+defg+e^2f^2))}{e^3g^3} + \frac{\log(d+ex)(ae^2-bde+cd^2)^2}{e^4(ef-dg)}$$

$$- \frac{\log(f+gx)(ag^2-bfg+cf^2)^2}{g^4(ef-dg)} - \frac{cx^2(-2beg+cdg+cef)}{2e^2g^2} + \frac{c^2x^3}{3eg}$$

[Out] ((b^2\*e^2\*g^2 - 2\*c\*e\*g\*(b\*e\*f + b\*d\*g - a\*e\*g) + c^2\*(e^2\*f^2 + d\*e\*f\*g + d^2\*g^2))\*x)/(e^3\*g^3) - (c\*(c\*e\*f + c\*d\*g - 2\*b\*e\*g)\*x^2)/(2\*e^2\*g^2) + (c^2\*x^3)/(3\*e\*g) + ((c\*d^2 - b\*d\*e + a\*e^2)^2\*Log[d + e\*x])/(e^4\*(e\*f - d\*g)) - ((c\*f^2 - b\*f\*g + a\*g^2)^2\*Log[f + g\*x])/(g^4\*(e\*f - d\*g))

**Rubi [A]** time = 0.603069, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$

$$\frac{x(-2ceg(-aeg+bdg+bef)+b^2e^2g^2+c^2(d^2g^2+defg+e^2f^2))}{e^3g^3} + \frac{\log(d+ex)(ae^2-bde+cd^2)^2}{e^4(ef-dg)}$$

$$- \frac{\log(f+gx)(ag^2-bfg+cf^2)^2}{g^4(ef-dg)} - \frac{cx^2(-2beg+cdg+cef)}{2e^2g^2} + \frac{c^2x^3}{3eg}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)^2/((d + e\*x)\*(f + g\*x)), x]

[Out] ((b^2\*e^2\*g^2 - 2\*c\*e\*g\*(b\*e\*f + b\*d\*g - a\*e\*g) + c^2\*(e^2\*f^2 + d\*e\*f\*g + d^2\*g^2))\*x)/(e^3\*g^3) - (c\*(c\*e\*f + c\*d\*g - 2\*b\*e\*g)\*x^2)/(2\*e^2\*g^2) + (c^2\*x^3)/(3\*e\*g) + ((c\*d^2 - b\*d\*e + a\*e^2)^2\*Log[d + e\*x])/(e^4\*(e\*f - d\*g)) - ((c\*f^2 - b\*f\*g + a\*g^2)^2\*Log[f + g\*x])/(g^4\*(e\*f - d\*g))

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{c^2x^3}{3eg} + \frac{c(2beg - cdg - cef) \int x dx}{e^2g^2}$$

$$+ \frac{(2ace^2g^2 + b^2e^2g^2 - 2bcdeg^2 - 2bce^2fg + c^2d^2g^2 + c^2defg + c^2e^2f^2) \int \frac{1}{e^3} dx}{g^3}$$

$$+ \frac{(ag^2 - bfg + cf^2)^2 \log(f + gx)}{g^4(dg - ef)} - \frac{(ae^2 - bde + cd^2)^2 \log(d + ex)}{e^4(dg - ef)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+b\*x+a)\*\*2/(e\*x+d)/(g\*x+f), x)

[Out] c\*\*2\*x\*\*3/(3\*e\*g) + c\*(2\*b\*e\*g - c\*d\*g - c\*e\*f)\*Integral(x, x)/(e\*\*2\*g\*\*2) + (2\*a\*c\*e\*\*2\*g\*\*2 + b\*\*2\*e\*\*2\*g\*\*2 - 2\*b\*c\*d\*e\*g\*\*2 - 2\*b\*c\*e\*\*2\*f\*g + c\*\*2\*d\*\*2\*g\*\*2 + c\*\*2\*d\*e\*f\*g + c\*\*2\*e\*\*2\*f\*\*2)\*Integral(e\*\*(-3), x)/g\*\*3 + (a\*g\*\*2 - b\*f\*g + c\*f\*\*2)\*\*2\*log(f + g\*x)/(g\*\*4\*(d\*g - e\*f)) - (a\*e\*\*2 - b\*d\*e + c\*d\*\*2)\*\*2\*log(d + e\*x)/(e\*\*4\*(d\*g - e\*f))

**Mathematica [A]** time = 0.272458, size = 177, normalized size = 0.96

$$\frac{egx(dg - ef)(6ceg(2aeg + b(-2dg - 2ef + egx)) + 6b^2e^2g^2 + c^2(6d^2g^2 - 3deg(gx - 2f) + e^2(6f^2 - 3fgx + 2g^2x^2)))}{6e^4g^4(ef - dg)}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)^2/((d + e\*x)\*(f + g\*x)), x]

[Out] 
$$-(e^*g^*(-(e^*f) + d^*g))^*x^*(6^*b^{\wedge}2^*e^{\wedge}2^*g^{\wedge}2 + 6^*c^*e^*g^*(2^*a^*e^*g + b^*(-2^*e^*f - 2^*d^*g + e^*g^*x)) + c^{\wedge}2^*(6^*d^{\wedge}2^*g^{\wedge}2 - 3^*d^*e^*g^*(-2^*f + g^*x) + e^{\wedge}2^*(6^*f^{\wedge}2 - 3^*f^*g^*x + 2^*g^{\wedge}2^*x^{\wedge}2))) - 6^*(c^*d^{\wedge}2 + e^*(-(b^*d) + a^*e))^{\wedge}2^*g^{\wedge}4^*\text{Log}[d + e^*x] + 6^*e^{\wedge}4^*(c^*f^{\wedge}2 + g^*(-(b^*f) + a^*g))^{\wedge}2^*\text{Log}[f + g^*x])/(6^*e^{\wedge}4^*g^{\wedge}4^*(e^*f - d^*g))$$

**Maple [B]** time = 0.015, size = 444, normalized size = 2.4

$$\begin{aligned} & \frac{c^2x^3}{3eg} + \frac{bx^2c}{eg} - \frac{x^2c^2d}{2e^2g} - \frac{x^2c^2f}{2eg^2} + 2\frac{acx}{eg} + \frac{b^2x}{eg} - 2\frac{bcdx}{e^2g} - 2\frac{bcfx}{eg^2} + \frac{c^2d^2x}{e^3g} + \frac{c^2dfx}{e^2g^2} \\ & + \frac{c^2f^2x}{eg^3} + \frac{\ln(gx+f)a^2}{dg-ef} - 2\frac{\ln(gx+f)abf}{(dg-ef)g} + 2\frac{\ln(gx+f)acf^2}{g^2(dg-ef)} + \frac{\ln(gx+f)b^2f^2}{g^2(dg-ef)} \\ & - 2\frac{\ln(gx+f)bcf^3}{g^3(dg-ef)} + \frac{\ln(gx+f)c^2f^4}{g^4(dg-ef)} - \frac{\ln(ex+d)a^2}{dg-ef} + 2\frac{\ln(ex+d)abd}{(dg-ef)e} \\ & - 2\frac{\ln(ex+d)acd^2}{e^2(dg-ef)} - \frac{\ln(ex+d)b^2d^2}{e^2(dg-ef)} + 2\frac{\ln(ex+d)bcd^3}{e^3(dg-ef)} - \frac{\ln(ex+d)c^2d^4}{e^4(dg-ef)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)^2/(e\*x+d)/(g\*x+f), x)

[Out] 
$$\frac{1}{3}c^{\wedge}2^*x^{\wedge}3/e/g+1/e/g^*x^{\wedge}2^*b^*c-1/2/e^{\wedge}2/g^*x^{\wedge}2^*c^{\wedge}2^*d-1/2/e/g^{\wedge}2^*x^{\wedge}2^*c^{\wedge}2^*f+2/e/g^*a^*c^*x+1/e/g^*b^{\wedge}2^*x-2/e^{\wedge}2/g^*b^*c^*d^*x-2/e/g^{\wedge}2^*b^*c^*f^*x+1/e^{\wedge}3/g^*c^{\wedge}2^*d^{\wedge}2^*x+1/e^{\wedge}2/g^{\wedge}2^*c^{\wedge}2^*d^*f^*x+1/e/g^{\wedge}3^*c^{\wedge}2^*f^{\wedge}2^*x+1/(d^*g-e^*f)^*1n(g^*x+f)^*a^{\wedge}2-2/g/(d^*g-e^*f)^*ln(g^*x+f)^*a^*b^*f+2/g^{\wedge}2/(d^*g-e^*f)^*ln(g^*x+f)^*a^*c^*f^{\wedge}2+1/g^{\wedge}2/(d^*g-e^*f)^*ln(g^*x+f)^*b^{\wedge}2^*f^{\wedge}2-2/g^{\wedge}3/(d^*g-e^*f)^*ln(g^*x+f)^*b^*c^*f^{\wedge}3+1/g^{\wedge}4/(d^*g-e^*f)^*ln(g^*x+f)^*c^{\wedge}2^*f^{\wedge}4-1/(d^*g-e^*f)^*ln(e^*x+d)^*a^{\wedge}2+2/e/(d^*g-e^*f)^*ln(e^*x+d)^*a^*b^*d-2/e^{\wedge}2/(d^*g-e^*f)^*ln(e^*x+d)^*a^*c^*d^{\wedge}2-1/e^{\wedge}2/(d^*g-e^*f)^*ln(e^*x+d)^*b^{\wedge}2^*d^{\wedge}2+2/e^{\wedge}3/(d^*g-e^*f)^*ln(e^*x+d)^*b^*c^*d^{\wedge}3-1/e^{\wedge}4/(d^*g-e^*f)^*ln(e^*x+d)^*c^{\wedge}2^*d^{\wedge}4$$

**Maxima [A]** time = 0.696315, size = 344, normalized size = 1.87

$$\begin{aligned} & \frac{(c^2d^4 - 2bcd^3e - 2abde^3 + a^2e^4 + (b^2 + 2ac)d^2e^2) \log(ex + d)}{e^5f - de^4g} \\ & - \frac{(c^2f^4 - 2bcf^3g - 2abfg^3 + a^2g^4 + (b^2 + 2ac)f^2g^2) \log(gx + f)}{efg^4 - dg^5} \\ & + \frac{2c^2e^2g^2x^3 - 3(c^2e^2fg + (c^2de - 2bce^2)g^2)x^2 + 6(c^2e^2f^2 + (c^2de - 2bce^2)fg + (c^2d^2 - 2bcde + (b^2 + 2ac)e^2)g^2)x}{6e^3g^3} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)^2/((e\*x + d)\*(g\*x + f)), x, algorithm="maxima")

[Out] 
$$(c^{\wedge}2^*d^{\wedge}4 - 2^*b^*c^*d^{\wedge}3^*e - 2^*a^*b^*d^*e^{\wedge}3 + a^{\wedge}2^*e^{\wedge}4 + (b^{\wedge}2 + 2^*a^*c)^*d^{\wedge}2^*e^{\wedge}2)^*\text{log}(e^*x + d)/(e^{\wedge}5^*f - d^*e^{\wedge}4^*g) - (c^{\wedge}2^*f^{\wedge}4 - 2^*b^*c^*f^{\wedge}3^*g - 2^*a^*b^*f^*g^{\wedge}3 + a^{\wedge}2^*g^{\wedge}4 + (b^{\wedge}2 + 2^*a^*c)^*f^{\wedge}2^*g^{\wedge}2)^*\text{log}(g^*x + f)/(e^*f^*g^{\wedge}4 - d^*g^{\wedge}5) + 1/6^*(2^*c^{\wedge}2^*e^{\wedge}2^*g^{\wedge}2^*x^{\wedge}3 - 3^*(c^{\wedge}2^*e^{\wedge}2^*f^*g + (c^{\wedge}2^*d^2 - 2^*bcde + (b^{\wedge}2 + 2^*ac)e^2)^*g^2)x$$

$$- 2*b*c*e^2)*g^2)*x^2 + 6*(c^2*e^2*f^2 + (c^2*d*e - 2*b*c*e^2)*f$$

$$*g + (c^2*d^2 - 2*b*c*d*e + (b^2 + 2*a*c)*e^2)*g^2)*x)/(e^3*g^3)$$

**Fricas [A]** time = 0.599989, size = 423, normalized size = 2.3

$$6(c^2d^4 - 2bcd^3e - 2abde^3 + a^2e^4 + (b^2 + 2ac)d^2e^2)g^4 \log(ex + d) + 2(c^2e^4fg^3 - c^2de^3g^4)x^3 - 3(c^2e^4f^2g^2 - 2bce^4fg^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)^2/((e\*x + d)\*(g\*x + f)),x, algorithm="fricas")

[Out] 1/6\*(6\*(c^2\*d^4 - 2\*b\*c\*d^3\*e - 2\*a\*b\*d^2\*e^3 + a^2\*e^4 + (b^2 + 2\*a\*c)\*d^2\*e^2)\*g^4\*log(e\*x + d) + 2\*(c^2\*e^4\*f^2\*g^3 - c^2\*d^2\*e^3\*g^4)\*x^3 - 3\*(c^2\*e^4\*f^2\*g^2 - 2\*b\*c\*d^2\*e^3\*g^4)\*x^2 + 6\*(c^2\*e^4\*f^3\*g - 2\*b\*c\*e^4\*f^2\*g^2 + (b^2 + 2\*a\*c)\*e^4\*f^2\*g^3 - (c^2\*d^3\*e - 2\*b\*c\*d^2\*e^2 + (b^2 + 2\*a\*c)\*d^2\*e^3)\*g^4)\*x - 6\*(c^2\*e^4\*f^4 - 2\*b\*c\*e^4\*f^3\*g - 2\*a\*b\*e^4\*f^2\*g^3 + a^2\*e^4\*g^4 + (b^2 + 2\*a\*c)\*e^4\*f^2\*g^2)\*log(g\*x + f))/(e^5\*f^4 - d^2\*e^4\*g^5)

**Sympy [A]** time = 117.993, size = 989, normalized size = 5.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*\*2/(e\*x+d)/(g\*x+f),x)

[Out] c\*\*2\*x\*\*3/(3\*e\*g) + (a\*g\*\*2 - b\*f\*g + c\*f\*\*2)\*\*2\*log(x + (a\*\*2\*d\*e\*\*3\*g\*\*4 + a\*\*2\*e\*\*4\*f\*g\*\*3 - 4\*a\*b\*d\*e\*\*3\*f\*g\*\*3 + 2\*a\*c\*d\*\*2\*e\*\*2\*f\*g\*\*3 + 2\*a\*c\*d\*e\*\*3\*f\*\*2\*g\*\*2 + b\*\*2\*d\*\*2\*e\*\*2\*f\*g\*\*3 + b\*\*2\*d\*e\*\*3\*f\*\*2\*g\*\*2 - 2\*b\*c\*d\*\*3\*e\*f\*g\*\*3 - 2\*b\*c\*d\*e\*\*3\*f\*\*3\*g + c\*\*2\*d\*\*4\*f\*g\*\*3 + c\*\*2\*d\*e\*\*3\*f\*\*4 - d\*\*2\*e\*\*3\*g\*(a\*g\*\*2 - b\*f\*g + c\*f\*\*2)\*\*2/(d\*g - e\*f) + 2\*d\*e\*\*4\*f\*(a\*g\*\*2 - b\*f\*g + c\*f\*\*2)\*\*2/(d\*g - e\*f) - e\*\*5\*f\*\*2\*(a\*g\*\*2 - b\*f\*g + c\*f\*\*2)\*\*2/(g\*(d\*g - e\*f)))/(2\*a\*\*2\*e\*\*4\*g\*\*4 - 2\*a\*b\*d\*e\*\*3\*g\*\*4 - 2\*a\*b\*e\*\*4\*f\*g\*\*3 + 2\*a\*c\*d\*\*2\*e\*\*2\*g\*\*4 + 2\*a\*c\*e\*\*4\*f\*\*2\*g\*\*2 + b\*\*2\*d\*\*2\*e\*\*2\*g\*\*4 + b\*\*2\*e\*\*4\*f\*\*2\*g\*\*2 - 2\*b\*c\*d\*\*3\*e\*g\*\*4 - 2\*b\*c\*e\*\*4\*f\*\*3\*g + c\*\*2\*d\*\*4\*g\*\*4 + c\*\*2\*e\*\*4\*f\*\*4))/(g\*\*4\*(d\*g - e\*f)) + x\*\*2\*(2\*b\*c\*e\*g - c\*\*2\*d\*g - c\*\*2\*e\*f)/(2\*e\*\*2\*g\*\*2) + x\*(2\*a\*c\*e\*\*2\*g\*\*2 + b\*\*2\*e\*\*2\*g\*\*2 - 2\*b\*c\*d\*e\*g\*\*2 - 2\*b\*c\*e\*\*2\*f\*g + c\*\*2\*d\*\*2\*g\*\*2 + c\*\*2\*d\*e\*f\*g + c\*\*2\*e\*\*2\*f\*\*2)/(e\*\*3\*g\*\*3) - (a\*e\*\*2 - b\*d\*e + c\*d\*\*2)\*\*2\*log(x + (a\*\*2\*d\*e\*\*3\*g\*\*4 + a\*\*2\*e\*\*4\*f\*g\*\*3 - 4\*a\*b\*d\*e\*\*3\*f\*g\*\*3 + 2\*a\*c\*d\*\*2\*e\*\*2\*f\*g\*\*3 + 2\*a\*c\*d\*e\*\*3\*f\*\*2\*g\*\*2 + b\*\*2\*d\*\*2\*e\*\*2\*f\*g\*\*3 + b\*\*2\*d\*e\*\*3\*f\*\*2\*g\*\*2 - 2\*b\*c\*d\*\*3\*e\*f\*g\*\*3 - 2\*b\*c\*d\*e\*\*3\*f\*\*3\*g + c\*\*2\*d\*\*4\*f\*g\*\*3 + c\*\*2\*d\*e\*\*3\*f\*\*4 + d\*\*2\*g\*\*5\*(a\*e\*\*2 - b\*d\*e + c\*d\*\*2)\*\*2/(e\*(d\*g - e\*f)) - 2\*d\*f\*g\*\*4\*(a\*e\*\*2 - b\*d\*e + c\*d\*\*2)\*\*2/(d\*g - e\*f) + e\*f\*\*2\*g\*\*3\*(a\*e\*\*2 - b\*d\*e + c\*d\*\*2)\*\*2/(d\*g - e\*f))/(2\*a\*\*2\*e\*\*4\*g\*\*4 - 2\*a\*b\*d\*e\*\*3\*g\*\*4 - 2\*a\*b\*e\*\*4\*f\*g\*\*3 + 2\*a\*c\*d\*\*2\*e\*\*2\*g\*\*4 + 2\*a\*c\*e\*\*4\*f\*\*2\*g\*\*2 + b\*\*2\*d\*\*2\*e\*\*2\*g\*\*4 + b\*\*2\*e\*\*4\*f\*\*2\*g\*\*2 - 2\*b\*c\*d\*\*3\*e\*g\*\*4 - 2\*b\*c\*e\*\*4\*f\*\*3\*g + c\*\*2\*d\*\*4\*g\*\*4 + c\*\*2\*e\*\*4\*f\*\*4))/(e\*\*4\*(d\*g - e\*f))

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)^2/((e*x + d)*(g*x + f)),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.816 \quad \int \frac{(a+bx+cx^2)^3}{(d+ex)(f+gx)} dx$$

**Optimal.** Leaf size=531

$$\begin{aligned} & x(-3ce^2g^2(a^2e^2g^2 - 2abeg(dg + ef) + b^2(d^2g^2 + defg + e^2f^2)) + b^2e^3g^3(-3aeg + bdg + bef) - 3c^2eg(aeg(d^2g^2 + defg + e^2f^2) - b^2e^3g^3)) \\ & + \frac{x^2(-3c^2eg(aeg(dg + ef) - b(d^2g^2 + defg + e^2f^2)) - 3bce^2g^2(-2aeg + bdg + bef) + b^3e^3g^3 + c^3(-(d^3g^3 + d^2efg^2 + de^2fg^2) - b^2e^3g^3))}{2e^4g^4} \\ & + \frac{cx^3(-3ceg(-aeg + bdg + bef) + 3b^2e^2g^2 + c^2(d^2g^2 + defg + e^2f^2))}{3e^3g^3} \\ & + \frac{\log(d+ex)(ae^2 - bde + cd^2)^3}{e^6(ef - dg)} - \frac{\log(f+gx)(ag^2 - bfg + cf^2)^3}{g^6(ef - dg)} - \frac{c^2x^4(-3beg + cdg + cef)}{4e^2g^2} + \frac{c^3x^5}{5eg} \end{aligned}$$

[Out] -(((b^2\*e^3\*g^3\*(b\*e\*f + b\*d\*g - 3\*a\*e\*g) - c^3\*(e^4\*f^4 + d\*e^3\*f^3\*g + d^2\*e^2\*f^2\*g^2 + d^3\*e\*f\*g^3 + d^4\*g^4) - 3\*c\*e^2\*g^2\*(a^2\*e^2\*g^2 - 2\*a\*b\*e\*g\*(e\*f + d\*g) + b^2\*(e^2\*f^2 + d\*e\*f\*g + d^2\*g^2)) - 3\*c^2\*e\*g\*(a\*e\*g\*(e^2\*f^2 + d\*e\*f\*g + d^2\*g^2) - b\*(e^3\*f^3 + d\*e^2\*f^2\*g + d^2\*e\*f\*g^2 + d^3\*g^3))) \* x)/(e^5\*g^5)) + ((b^3\*e^3\*g^3 - 3\*b\*c\*e^2\*g^2\*(b\*e\*f + b\*d\*g - 2\*a\*e\*g) - c^3\*(e^3\*f^3 + d\*e^2\*f^2\*g + d^2\*e\*f\*g^2 + d^3\*g^3) - 3\*c^2\*e\*g\*(a\*e\*g\*(e\*f + d\*g) - b\*(e^2\*f^2 + d\*e\*f\*g + d^2\*g^2))) \* x^2)/(2\*e^4\*g^4) + (c\*(3\*b^2\*e^2\*g^2 - 3\*c\*e\*g\*(b\*e\*f + b\*d\*g - a\*e\*g) + c^2\*(e^2\*f^2 + d\*e\*f\*g + d^2\*g^2)) \* x^3)/(3\*e^3\*g^3) - (c^2\*(c\*e\*f + c\*d\*g - 3\*b\*e\*g) \* x^4)/(4\*e^2\*g^2) + (c^3\*x^5)/(5\*e\*g) + ((c\*d^2 - b\*d\*e + a\*e^2)^3 \* Log[d + e\*x])/(e^6\*(e\*f - d\*g)) - ((c\*f^2 - b\*f\*g + a\*g^2)^3 \* Log[f + g\*x])/(g^6\*(e\*f - d\*g))

**Rubi [A]** time = 2.8496, antiderivative size = 531, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$

$$\begin{aligned} & x(-3ce^2g^2(a^2e^2g^2 - 2abeg(dg + ef) + b^2(d^2g^2 + defg + e^2f^2)) + b^2e^3g^3(-3aeg + bdg + bef) - 3c^2eg(aeg(d^2g^2 + defg + e^2f^2) - b^2e^3g^3)) \\ & + \frac{x^2(-3c^2eg(aeg(dg + ef) - b(d^2g^2 + defg + e^2f^2)) - 3bce^2g^2(-2aeg + bdg + bef) + b^3e^3g^3 + c^3(-(d^3g^3 + d^2efg^2 + de^2fg^2) - b^2e^3g^3))}{2e^4g^4} \\ & + \frac{cx^3(-3ceg(-aeg + bdg + bef) + 3b^2e^2g^2 + c^2(d^2g^2 + defg + e^2f^2))}{3e^3g^3} \\ & + \frac{\log(d+ex)(ae^2 - bde + cd^2)^3}{e^6(ef - dg)} - \frac{\log(f+gx)(ag^2 - bfg + cf^2)^3}{g^6(ef - dg)} - \frac{c^2x^4(-3beg + cdg + cef)}{4e^2g^2} + \frac{c^3x^5}{5eg} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)^3/((d + e\*x)\*(f + g\*x)), x]

[Out] -(((b^2\*e^3\*g^3\*(b\*e\*f + b\*d\*g - 3\*a\*e\*g) - c^3\*(e^4\*f^4 + d\*e^3\*f^3\*g + d^2\*e^2\*f^2\*g^2 + d^3\*e\*f\*g^3 + d^4\*g^4) - 3\*c\*e^2\*g^2\*(a^2\*e^2\*g^2 - 2\*a\*b\*e\*g\*(e\*f + d\*g) + b^2\*(e^2\*f^2 + d\*e\*f\*g + d^2\*g^2)) - 3\*c^2\*e\*g\*(a\*e\*g\*(e^2\*f^2 + d\*e\*f\*g + d^2\*g^2) - b\*(e^3\*f^3 + d\*e^2\*f^2\*g + d^2\*e\*f\*g^2 + d^3\*g^3))) \* x)/(e^5\*g^5)) + ((b^3\*e^3\*g^3 - 3\*b\*c\*e^2\*g^2\*(b\*e\*f + b\*d\*g - 2\*a\*e\*g) - c^3\*(e^3\*f^3 + d\*e^2\*f^2\*g + d^2\*e\*f\*g^2 + d^3\*g^3) - 3\*c^2\*e\*g\*(a\*e\*g\*(e\*f + d\*g) - b\*(e^2\*f^2 + d\*e\*f\*g + d^2\*g^2))) \* x^2)/(2\*e^4\*g^4) + (c\*(3\*b^2\*e^2\*g^2 - 3\*c\*e\*g\*(b\*e\*f + b\*d\*g - a\*e\*g) + c^2\*(e^2\*f^2 + d\*e\*f\*g + d^2\*g^2)) \* x^3)/(3\*e^3\*g^3) - (c^2\*(c\*e\*f + c\*d\*g - 3\*b\*e\*g) \* x^4)/(4\*e^2\*g^2) + (c^3\*x^5)/(5\*e\*g) + ((c\*d^2 - b\*d\*e + a\*e^2)^3 \* Log[d + e\*x])/(e^6\*(e\*f - d\*g)) - ((c\*f^2 - b\*f\*g + a\*g^2)^3 \* Log[f + g\*x])/(g^6\*(e\*f - d\*g))

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x**2+b*x+a)**3/(e*x+d)/(g*x+f),x)`

[Out] Timed out

**Mathematica [A]** time = 1.13836, size = 476, normalized size = 0.9

$$\frac{egx(-30ce^2g^2(ef-dg)(6a^2e^2g^2+6abeg(-2dg-2ef+egx))+b^2(6d^2g^2-3deg(gx-2f))+e^2(6f^2-3fgx+2g^2x^2))}{1}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x + c*x^2)^3/((d + e*x)*(f + g*x)),x]`

[Out] 
$$-(e^*g^*x^*(-30*b^2*e^3*g^3*(e*f - d*g)*(6*a*e*g + b*(-2*e*f - 2*d*g + e*g*x)) + c^3*(60*d^5*g^5 - 30*d^4*e*g^5*x + 20*d^3*e^2*g^5*x^2 - 15*d^2*e^3*g^5*x^3 + 12*d*e^4*g^5*x^4 + e^5*f*(-60*f^4 + 30*f^3*g*x - 20*f^2*g^2*x^2 + 15*f*g^3*x^3 - 12*g^4*x^4)) - 30*c*e^2*g^2*(e*f - d*g)*(6*a^2*e^2*g^2 + 6*a*b*e*g*(-2*e*f - 2*d*g + e*g*x) + b^2*(6*d^2*g^2 - 3*d*e*g*(-2*f + g*x) + e^2*(6*f^2 - 3*f*g*x + 2*g^2*x^2))) + 15*c^2*e*g*(-2*a*e*g*(e*f - d*g)*(6*d^2*g^2 - 3*d*e*g*(-2*f + g*x) + e^2*(6*f^2 - 3*f*g*x + 2*g^2*x^2)) + b*(-12*d^4*g^4 + 6*d^3*e*g^4*x - 4*d^2*e^2*g^4*x^2 + 3*d*e^3*g^4*x^3 + e^4*f*(12*f^3 - 6*f^2*g*x + 4*f*g^2*x^2 - 3*g^3*x^3))) - 60*(c*d^2 + e*(-(b*d) + a*e))^3*g^6*Log[d + e*x] + 60*e^6*(c*f^2 + g*(-(b*f) + a*g))^3*Log[f + g*x])/(60*e^6*g^6*(e*f - d*g))$$

**Maple [B]** time = 0.024, size = 1232, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)^3/(e*x+d)/(g*x+f),x)`

[Out] 
$$\frac{1}{5}c^3x^5/e/g+1/2/e/g^2x^2b^3+3/e/g^2x^2a^2b^2c-3/2/e^2/g^2x^2a^2c^2d-3/g/(d^2g-ef)*\ln(gx+f)*a^2b^2f+3/2/e^2/g^2x^2b^2c^2d^2f-6/e^2/g^2a^2b^2c^2d^2x+3/g^2/(d^2g-ef)*\ln(gx+f)*a^2c^2f^2+3/g^2/(d^2g-ef)*\ln(gx+f)*a^2b^2f^2+3/g^4/(d^2g-ef)*\ln(gx+f)*a^2c^2f^4+3/g^4/(d^2g-ef)*\ln(gx+f)*b^2c^2f^4-3/g^5/(d^2g-ef)*\ln(gx+f)*b^2c^2f^5-1/(d^2g-ef)*\ln(e^2x+d)*a^3+1/(d^2g-ef)*\ln(gx+f)*a^3-1/e^2/g^2b^3d^2x-1/e/g^2b^3f^2x-1/g^3/(d^2g-ef)*\ln(gx+f)*b^3f^3+1/g^6/(d^2g-ef)*\ln(gx+f)*c^3f^6+1/e^3/(d^2g-ef)*\ln(e^2x+d)*b^3d^3-1/e^6/(d^2g-ef)*\ln(e^2x+d)*c^3d^6+1/e/g^2x^3a^2c^2+1/e/g^2x^3b^2c^2+3/4/e/g^2x^4b^2c^2-1/4/e^2/g^2x^4c^3d-1/4/e/g^2x^4c^3f+1/3/e^3/g^2x^3c^3d^2+1/3/e/g^3x^3c^3f^2-1/2/e^4/g^2x^2c^3d^3-3/2/e/g^2x^2a^2c^2f-3/2/e^2/g^2x^2b^2c^2d-3/2/e/g^2x^2b^2c^2f+3/2/e^3/g^2x^2b^2c^2d^2+3/e/(d^2g-ef)*\ln(e^2x+d)*a^2b^2d-3/e^2/(d^2g-ef)*\ln(e^2x+d)*a^2c^2d^2-3/e^2/(d^2g-ef)*\ln(e^2x+d)*a^2b^2d^2-3/e^4/(d^2g-ef)*\ln(e^2x+d)*a^2c^2d^4-3/e^4/(d^2g-ef)*\ln(e^2x+d)*b^2c^2d^4+3/e^5/(d^2g-ef)*\ln(e^2x+d)*b^2c^2d^5+3/2/e/g^3x^2b^2c^2f^2-1/2/e^3/g^2x^2c^3d^2f-1/2/e^2/g^3x^2c^3d^2f^2+3/e^3/g^2a^2c^2d^2x+3/e/g^3a^2c^2f^2x+3/e^3/g^2b^2c^2d^2x+3/e/g^3b^2c^2f^2x-3/e^4/g^2b^2c^2d^3x-3/e/g^4b^2c^2f^3x+1/e^4/g^2c^3d^3f^2x+1/e^3/g^3c^3d^2f^2x+1/e^2/g^4c^3d^3f^3x-1/e^2/g^2x^3b^2c^2d-1/e/g^2x^3$$



$$b^2 c^2 f + 1/3/e^2/g^2 x^3 c^3 d f - 6/e/g^2 a^2 b^2 c^2 f x + 3/e^2/g^2 a^2 c^2 d^2 f x + 3/e^2/g^2 b^2 c^2 d^2 f x - 3/e^3/g^2 b^2 c^2 d^2 f x - 3/e^2/g^3 b^2 c^2 d^2 f^2 x - 6/g^3/(d^2 g - e^2 f) \ln(g^2 x + f) a^2 b^2 c^2 f^3 + 6/e^3/(d^2 g - e^2 f) \ln(e^2 x + d) a^2 b^2 c^2 d^3 + 3/e/g^2 a^2 c^2 x + 3/e/g^2 a^2 b^2 x - 1/2/e/g^4 x^2 c^3 f^3 + 1/e^5/g^2 c^3 d^4 x + 1/e/g^5 c^3 f^4 x$$

**Maxima [A]** time = 0.702825, size = 973, normalized size = 1.83

$$\frac{(c^3 d^6 - 3 b c^2 d^5 e - 3 a^2 b d e^5 + a^3 e^6 + 3 (b^2 c + a c^2) d^4 e^2 - (b^3 + 6 a b c) d^3 e^3 + 3 (a b^2 + a^2 c) d^2 e^4) \log(e x + d)}{e^7 f - d e^6 g} - \frac{(c^3 f^6 - 3 b c^2 f^5 g - 3 a^2 b f g^5 + a^3 g^6 + 3 (b^2 c + a c^2) f^4 g^2 - (b^3 + 6 a b c) f^3 g^3 + 3 (a b^2 + a^2 c) f^2 g^4) \log(g x + f)}{e f g^6 - d g^7} + \frac{12 c^3 e^4 g^4 x^5 - 15 (c^3 e^4 f g^3 + (c^3 d e^3 - 3 b c^2 e^4) g^4) x^4 + 20 (c^3 e^4 f^2 g^2 + (c^3 d e^3 - 3 b c^2 e^4) f g^3 + (c^3 d^2 e^2 - 3 b c^2 d e^3 + 3 (b^2 c + a c^2) d e^4) g^4) x^3 - 30 (c^3 e^4 f^3 g + (c^3 d e^3 - 3 b c^2 e^4) f^2 g^2 + (c^3 d^2 e^2 - 3 b c^2 d e^3 + 3 (b^2 c + a c^2) d e^4) f g^3 + (c^3 d^3 e - 3 b^2 c^2 d^2 e^2 + 3 (b^2 c + a c^2) d^2 e^3 - (b^3 + 6 a^2 b^2 c) e^4) g^4) x^2 + 60 (c^3 e^4 f^4 + (c^3 d e^3 - 3 b^2 c^2 e^4) f^3 g + (c^3 d^2 e^2 - 3 b^2 c^2 d e^3 + 3 (b^2 c + a c^2) d e^4) f^2 g^2 + (c^3 d^3 e - 3 b^2 c^2 d^2 e^2 + 3 (b^2 c + a c^2) d^2 e^3 - (b^3 + 6 a^2 b^2 c) e^4) f g^3 + (c^3 d^4 - 3 b^2 c^2 d^3 e + 3 (b^2 c + a c^2) d^2 e^2 - (b^3 + 6 a^2 b^2 c) d e^3 + 3 (a^2 b^2 + a^2 c) e^4) g^4) x}{e^5 g^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)^3/((e\*x + d)\*(g\*x + f)),x, algorithm="maxima")

[Out] (c^3\*d^6 - 3\*b^2\*c^2\*d^5\*e - 3\*a^2\*b\*d^4\*e^5 + a^3\*e^6 + 3\*(b^2\*c + a^2\*c)\*d^4\*e^2 - (b^3 + 6\*a\*b\*c)\*d^3\*e^3 + 3\*(a\*b^2 + a^2\*c)\*d^2\*e^4)\*log(e\*x + d)/(e^7\*f - d\*e^6\*g) - (c^3\*f^6 - 3\*b^2\*c^2\*f^5\*g - 3\*a^2\*b\*f^4\*g^5 + a^3\*g^6 + 3\*(b^2\*c + a^2\*c)\*f^4\*g^2 - (b^3 + 6\*a\*b\*c)\*f^3\*g^3 + 3\*(a\*b^2 + a^2\*c)\*f^2\*g^4)\*log(g\*x + f)/(e\*f\*g^6 - d\*g^7) + 1/60\*(12\*c^3\*e^4\*g^4\*x^5 - 15\*(c^3\*e^4\*f\*g^3 + (c^3\*d\*e^3 - 3\*b^2\*c^2\*d^2\*e^2 + 3\*(b^2\*c + a^2\*c)\*d^2\*e^3 - (b^3 + 6\*a^2\*b^2\*c)\*e^4)\*g^4)\*x^4 + 20\*(c^3\*e^4\*f^2\*g^2 + (c^3\*d\*e^3 - 3\*b^2\*c^2\*d^2\*e^2 + 3\*(b^2\*c + a^2\*c)\*d^2\*e^3 - (b^3 + 6\*a^2\*b^2\*c)\*e^4)\*f\*g^3 + (c^3\*d^2\*e^2 - 3\*b^2\*c^2\*d\*e^3 + 3\*(b^2\*c + a^2\*c)\*d^2\*e^4)g^4)\*x^3 - 30\*(c^3\*e^4\*f^3\*g + (c^3\*d\*e^3 - 3\*b^2\*c^2\*d^2\*e^2 + 3\*(b^2\*c + a^2\*c)\*d^2\*e^3 - (b^3 + 6\*a^2\*b^2\*c)\*e^4)\*f^2\*g^2 + (c^3\*d^3\*e - 3\*b^2\*c^2\*d^2\*e^2 + 3\*(b^2\*c + a^2\*c)\*d^2\*e^3 - (b^3 + 6\*a^2\*b^2\*c)\*e^4)\*f\*g^3 + (c^3\*d^4 - 3\*b^2\*c^2\*d^3\*e + 3\*(b^2\*c + a^2\*c)\*d^2\*e^2 - (b^3 + 6\*a^2\*b^2\*c)\*d\*e^3 + 3\*(a^2\*b^2 + a^2\*c)\*e^4)\*g^4)\*x)/(e^5\*g^5)

**Fricas [A]** time = 4.41785, size = 994, normalized size = 1.87

$$60 (c^3 d^6 - 3 b c^2 d^5 e - 3 a^2 b d e^5 + a^3 e^6 + 3 (b^2 c + a c^2) d^4 e^2 - (b^3 + 6 a b c) d^3 e^3 + 3 (a b^2 + a^2 c) d^2 e^4) g^6 \log(e x + d) + 12 (c^3 e^6$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)^3/((e\*x + d)\*(g\*x + f)),x, algorithm="fricas")

[Out] 1/60\*(60\*(c^3\*d^6 - 3\*b^2\*c^2\*d^5\*e - 3\*a^2\*b\*d^4\*e^5 + a^3\*e^6 + 3\*(b^2\*c + a^2\*c)\*d^4\*e^2 - (b^3 + 6\*a\*b\*c)\*d^3\*e^3 + 3\*(a\*b^2 + a^2\*c)\*d^2\*e^4)\*g^6\*log(e\*x + d) + 12\*(c^3\*e^6\*f^5\*g^5 - c^3\*d^5\*e^5\*g^6)\*x^5 - 15\*(c^3\*e^6\*f^4\*g^4 - 3\*b^2\*c^2\*e^6\*f^3\*g^5 - (c^3\*d^5\*e^4 - 3\*b^2\*c^2\*d^4\*e^5)\*g^6)\*x^4 + 20\*(c^3\*e^6\*f^3\*g^3 - 3\*b^2\*c^2\*e^6\*f^2\*g^4 + 3\*(b^2\*c + a^2\*c)\*e^6\*f^2\*g^5 - (c^3\*d^5\*e^3 - 3\*b^2\*c^2\*d^4\*e^4 + 3\*(b^2\*c + a^2\*c)\*d^4\*e^5)\*g^6)\*x^3 - 30\*(c^3\*e^6\*f^4\*g^2 - 3\*b^2\*c^2\*e^6\*f^3\*g^3 + 3\*(b^2\*c + a^2\*c)\*e^6\*f^2\*g^4 - (b^3 + 6\*a\*b\*c)\*e^6\*f^2\*g^5 - (c^3\*d^5\*e^2 - 3\*b^2\*c^2\*d^4\*e^3 + 3\*(b^2\*c + a^2\*c)\*d^4\*e^4 - (b^3 + 6\*a^2\*b^2\*c)\*d^4\*e^5)\*g^6)\*x^2 + 60\*(c^3\*e^6\*f^5\*g - 3\*b^2\*c^2\*e^6\*f^4\*g^2 + 3\*(b^2\*c + a^2\*c)\*e^6\*f^3\*g^3 - (b^3 + 6\*a\*b\*c)\*e^6\*f^2\*g^4 + 3\*(a\*b^2 + a^2\*c)\*e^6\*f^2\*g^5 - (c^3\*d^5\*e - 3\*b^2\*c^2\*d^4\*e^2 + 3\*(b^2\*c + a^2\*c)\*d^4\*e^3 - (b^3 + 6\*a^2\*b^2\*c)\*d^4\*e^4 + 3\*(a\*b^2 + a^2\*c)\*d^4\*e^5)\*g^6)\*x - 60\*(c^3\*e^6\*f^6 - 3\*b^2\*c^2\*e^6\*f^5\*g - 3\*a^2\*b^2\*e^6\*f^4\*g^5 + a^3\*e^6\*g^6 + 3\*(b^2\*c + a^2\*c)\*e^6\*

$$\frac{f^4 g^2 - (b^3 + 6 a b c) e^6 f^3 g^3 + 3 (a b^2 + a^2 c) e^6 f^2 g^4 \log(g x + f)}{(e^7 f g^6 - d e^6 g^7)}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*\*3/(e\*x+d)/(g\*x+f),x)

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)^3/((e\*x + d)\*(g\*x + f)),x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.817 \quad \int \frac{1}{(d+ex)(f+gx)(a+bx+cx^2)} dx$$

**Optimal.** Leaf size=246

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right) (-c(2aeg + bdg + bef) + b^2eg + 2c^2df)}{\sqrt{b^2 - 4ac} (ae^2 - bde + cd^2) (cf^2 - g(bf - ag))} - \frac{\log(a + bx + cx^2) (-beg + cdg + cef)}{2(ae^2 - bde + cd^2)(cf^2 - g(bf - ag))} + \frac{e^2 \log(d + ex)}{(ef - dg)(ae^2 - bde + cd^2)} - \frac{g^2 \log(f + gx)}{(ef - dg)(ag^2 - bfg + cf^2)}$$

[Out] -(((2\*c^2\*d\*f + b^2\*e\*g - c\*(b\*e\*f + b\*d\*g + 2\*a\*e\*g))\*ArcTanh[(b + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]])/(Sqrt[b^2 - 4\*a\*c]\*(c\*d^2 - b\*d\*e + a\*e^2)\*(c\*f^2 - g\*(b\*f - a\*g)))) + (e^2\*Log[d + e\*x])/((c\*d^2 - b\*d\*e + a\*e^2)\*(e\*f - d\*g)) - (g^2\*Log[f + g\*x])/((e\*f - d\*g)\*(c\*f^2 - b\*f\*g + a\*g^2)) - ((c\*e\*f + c\*d\*g - b\*e\*g)\*Log[a + b\*x + c\*x^2])/((2\*(c\*d^2 - b\*d\*e + a\*e^2)\*(c\*f^2 - g\*(b\*f - a\*g))))

**Rubi [A]** time = 0.923651, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right) (-c(2aeg + bdg + bef) + b^2eg + 2c^2df)}{\sqrt{b^2 - 4ac} (ae^2 - bde + cd^2) (cf^2 - g(bf - ag))} - \frac{\log(a + bx + cx^2) (-beg + cdg + cef)}{2(ae^2 - bde + cd^2)(cf^2 - g(bf - ag))} + \frac{e^2 \log(d + ex)}{(ef - dg)(ae^2 - bde + cd^2)} - \frac{g^2 \log(f + gx)}{(ef - dg)(ag^2 - bfg + cf^2)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)\*(f + g\*x)\*(a + b\*x + c\*x^2)), x]

[Out] -(((2\*c^2\*d\*f + b^2\*e\*g - c\*(b\*e\*f + b\*d\*g + 2\*a\*e\*g))\*ArcTanh[(b + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]])/(Sqrt[b^2 - 4\*a\*c]\*(c\*d^2 - b\*d\*e + a\*e^2)\*(c\*f^2 - g\*(b\*f - a\*g)))) + (e^2\*Log[d + e\*x])/((c\*d^2 - b\*d\*e + a\*e^2)\*(e\*f - d\*g)) - (g^2\*Log[f + g\*x])/((e\*f - d\*g)\*(c\*f^2 - b\*f\*g + a\*g^2)) - ((c\*e\*f + c\*d\*g - b\*e\*g)\*Log[a + b\*x + c\*x^2])/((2\*(c\*d^2 - b\*d\*e + a\*e^2)\*(c\*f^2 - g\*(b\*f - a\*g))))

**Rubi in Sympy [F-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(e\*x+d)/(g\*x+f)/(c\*x\*\*2+b\*x+a), x)

[Out] Timed out

**Mathematica [A]** time = 0.653152, size = 246, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right) (-c(2aeg + bdg + bef) + b^2eg + 2c^2df)}{\sqrt{4ac - b^2} (e(ae - bd) + cd^2) (g(ag - bf) + cf^2)} + \frac{e^2 \log(d + ex)}{(ef - dg)(e(ae - bd) + cd^2)} - \frac{\log(a + x(b + cx))(-beg + cdg + cef)}{2(e(ae - bd) + cd^2)(g(ag - bf) + cf^2)} - \frac{g^2 \log(f + gx)}{(ef - dg)(g(ag - bf) + cf^2)}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x)\*(f + g\*x)\*(a + b\*x + c\*x^2)),x]

[Out] 
$$\frac{((2c^2d^2f + b^2e^2g - c(b^2ef + b^2dg + 2a^2eg)) \operatorname{ArcTan}\left[\frac{b + 2cx}{\sqrt{-b^2 + 4ac}}\right]) / (\sqrt{-b^2 + 4ac} (cd^2 + e(-bd + a^2e)) (cf^2 + g(-bf + ag))) + (e^2 \operatorname{Log}[d + ex]) / ((cd^2 + e(-bd + a^2e)) (ef - dg)) - (g^2 \operatorname{Log}[f + gx]) / ((ef - dg) (cf^2 + g(-bf + ag))) - ((c^2ef + cd^2g - b^2eg) \operatorname{Log}[a + x(b + cx)]) / (2(cd^2 + e(-bd + a^2e)) (cf^2 + g(-bf + ag)))$$

**Maple [B]** time = 0.015, size = 606, normalized size = 2.5

$$\begin{aligned} & \frac{g^2 \ln(gx + f)}{(dg - ef)(ag^2 - bfg + cf^2)} + \frac{\ln(cx^2 + bx + a) beg}{(2ae^2 - 2bde + 2cd^2)(ag^2 - bfg + cf^2)} \\ & - \frac{c \ln(cx^2 + bx + a) dg}{(2ae^2 - 2bde + 2cd^2)(ag^2 - bfg + cf^2)} - \frac{c \ln(cx^2 + bx + a) ef}{(2ae^2 - 2bde + 2cd^2)(ag^2 - bfg + cf^2)} \\ & - 2 \frac{aceg}{(ae^2 - bde + cd^2)(ag^2 - bfg + cf^2) \sqrt{4ac - b^2}} \operatorname{arctan}\left(\frac{2cx + b}{\sqrt{4ac - b^2}}\right) \\ & + \frac{b^2eg}{(ae^2 - bde + cd^2)(ag^2 - bfg + cf^2)} \operatorname{arctan}\left((2cx + b) \frac{1}{\sqrt{4ac - b^2}}\right) \frac{1}{\sqrt{4ac - b^2}} \\ & - \frac{bcdg}{(ae^2 - bde + cd^2)(ag^2 - bfg + cf^2)} \operatorname{arctan}\left((2cx + b) \frac{1}{\sqrt{4ac - b^2}}\right) \frac{1}{\sqrt{4ac - b^2}} \\ & - \frac{bcef}{(ae^2 - bde + cd^2)(ag^2 - bfg + cf^2)} \operatorname{arctan}\left((2cx + b) \frac{1}{\sqrt{4ac - b^2}}\right) \frac{1}{\sqrt{4ac - b^2}} \\ & + 2 \frac{c^2df}{(ae^2 - bde + cd^2)(ag^2 - bfg + cf^2) \sqrt{4ac - b^2}} \operatorname{arctan}\left(\frac{2cx + b}{\sqrt{4ac - b^2}}\right) \\ & - \frac{e^2 \ln(ex + d)}{(dg - ef)(ae^2 - bde + cd^2)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x+d)/(g\*x+f)/(c\*x^2+b\*x+a),x)

[Out] 
$$\frac{g^2}{(d^2g - e^2f)} / (a^2g^2 - b^2f^2g + c^2f^2) \ln(gx + f) + 1/2 / (a^2e^2 - b^2d^2e + c^2d^2) / (a^2g^2 - b^2f^2g + c^2f^2) \ln(cx^2 + bx + a) + b^2e^2g - 1/2 / (a^2e^2 - b^2d^2e + c^2d^2) / (a^2g^2 - b^2f^2g + c^2f^2) \ln(cx^2 + bx + a) + d^2g - 1/2 / (a^2e^2 - b^2d^2e + c^2d^2) / (a^2g^2 - b^2f^2g + c^2f^2) \ln(cx^2 + bx + a) + e^2f - 2 / (a^2e^2 - b^2d^2e + c^2d^2) / (a^2g^2 - b^2f^2g + c^2f^2) / (4a^2c - b^2)^{1/2} \operatorname{arctan}\left(\frac{2cx + b}{(4a^2c - b^2)^{1/2}}\right) + a^2c^2e^2g + 1 / (a^2e^2 - b^2d^2e + c^2d^2) / (a^2g^2 - b^2f^2g + c^2f^2) / (4a^2c - b^2)^{1/2} \operatorname{arctan}\left(\frac{2cx + b}{(4a^2c - b^2)^{1/2}}\right) + b^2e^2g - 1 / (a^2e^2 - b^2d^2e + c^2d^2) / (a^2g^2 - b^2f^2g + c^2f^2) / (4a^2c - b^2)^{1/2} \operatorname{arctan}\left(\frac{2cx + b}{(4a^2c - b^2)^{1/2}}\right) + b^2c^2e^2f + 2 / (a^2e^2 - b^2d^2e + c^2d^2) / (a^2g^2 - b^2f^2g + c^2f^2) / (4a^2c - b^2)^{1/2} \operatorname{arctan}\left(\frac{2cx + b}{(4a^2c - b^2)^{1/2}}\right) + c^2d^2f - e^2 / (d^2g - e^2f) / (a^2e^2 - b^2d^2e + c^2d^2) \ln(ex + d)$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + b\*x + a)\*(e\*x + d)\*(g\*x + f)),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((c*x^2 + b*x + a)*(e*x + d)*(g*x + f)),x, algorithm="fricas")`

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x+d)/(g*x+f)/(c*x**2+b*x+a), x)`

[Out] Timed out

---

**GIAC/XCAS [A]** time = 0.268243, size = 529, normalized size = 2.15

$$\frac{g^3 \ln(|gx + f|)}{cdf^2g^2 - bdfg^3 + adg^4 - cf^3ge + bf^2g^2e - afg^3e} - \frac{(cdg + cfe - bge) \ln(cx^2 + bx + a)}{2(c^2d^2f^2 - bcd^2fg + acd^2g^2 - bcd^2f^2e + b^2dfge - abdg^2e + acf^2e^2 - abfge^2 + a^2g^2e^2)} - \frac{e^3 \ln(|xe + d|)}{cd^3ge - cd^2fe^2 - bd^2ge^2 + bdf^3e + adge^3 - afe^4} + \frac{(2c^2df - bcdg - bcfe + b^2ge - 2acge) \arctan\left(\frac{2cx+b}{\sqrt{-b^2+4ac}}\right)}{(c^2d^2f^2 - bcd^2fg + acd^2g^2 - bcd^2f^2e + b^2dfge - abdg^2e + acf^2e^2 - abfge^2 + a^2g^2e^2)\sqrt{-b^2 + 4ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((c*x^2 + b*x + a)*(e*x + d)*(g*x + f)),x, algorithm="giac")`

[Out] `g^3*ln(abs(g*x + f))/(c*d*f^2*g^2 - b*d*f*g^3 + a*d*g^4 - c*f^3*g*e + b*f^2*g^2*e - a*f*g^3*e) - 1/2*(c*d*g + c*f*e - b*g*e)*ln(c*x^2 + b*x + a)/(c^2*d^2*f^2 - b*c*d^2*f*g + a*c*d^2*g^2 - b*c*d*f^2*e + b^2*d*f*g*e - a*b*d*g^2*e + a*c*f^2*e^2 - a*b*f*g*e^2 + a^2*g^2*e^2) - e^3*ln(abs(x*e + d))/(c*d^3*g*e - c*d^2*f*e^2 - b*d^2*g*e^2 + b*d*f*e^3 + a*d*g*e^3 - a*f*e^4) + (2*c^2*d*f - b*c*d*g - b*c*f*e + b^2*g*e - 2*a*c*g*e)*arctan((2*c*x + b)/sqrt(-b^2 + 4*a*c))/((c^2*d^2*f^2 - b*c*d^2*f*g + a*c*d^2*g^2 - b*c*d*f^2*e + b^2*d*f*g*e - a*b*d*g^2*e + a*c*f^2*e^2 - a*b*f*g*e^2 + a^2*g^2*e^2)*sqrt(-b^2 + 4*a*c))`

$$3.818 \quad \int \frac{1}{(d+ex)(f+gx)(a+bx+cx^2)^2} dx$$

**Optimal.** Leaf size=644

$$\begin{aligned} & \frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right) (2ceg(a^2e^2g^2 + abeg(dg+ef) - b^2(dg+ef)^2) + b^2e^2g^2(-2aeg + bdg + bef) - c^2(4ade^2fg^2 - b(d^3g^3 + \\ & \sqrt{b^2-4ac}(ae^2 - bde + cd^2)^2(cf^2 - g(bf-ag))^2 \\ & + \frac{2c \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right) (-c(2aeg + bdg + bef) + b^2eg + 2c^2df)}{(b^2-4ac)^{3/2}(ae^2 - bde + cd^2)(cf^2 - g(bf-ag))} \\ & - \frac{cx(-c(2aeg + bdg + bef) + b^2eg + 2c^2df) + bc(cdf - 3aeg) + 2ac^2(dg+ef) + b^3eg - b^2c(dg+ef)}{(b^2-4ac)(a+bx+cx^2)(ae^2 - bde + cd^2)(cf^2 - g(bf-ag))} \\ & - \frac{\log(a+bx+cx^2)(-beg + cdg + cef)(eg(2aeg - b(dg+ef)) + c(d^2g^2 + e^2f^2))}{2(ae^2 - bde + cd^2)^2(cf^2 - g(bf-ag))^2} \\ & + \frac{e^4 \log(d+ex)}{(ef-dg)(ae^2 - bde + cd^2)^2} - \frac{g^4 \log(f+gx)}{(ef-dg)(ag^2 - bfg + cf^2)^2} \end{aligned}$$

$$\begin{aligned} [\text{Out}] & -((b^3e^*g - b^2c^*(e^*f + d^*g) + 2*a*c^2*(e^*f + d^*g) + b*c^*(c*d*f \\ & - 3*a*e^*g) + c*(2*c^2*d*f + b^2*e^*g - c*(b*e^*f + b*d^*g + 2*a*e^*g \\ & ))*x)/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(b*f - a^* \\ & g))*(a + b*x + c*x^2))) + (2*c*(2*c^2*d*f + b^2*e^*g - c*(b*e^*f + \\ & b*d^*g + 2*a*e^*g))*\text{ArcTanh}[(b + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]])/((b^2 - \\ & 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(b*f - a^*g))) + \\ & ((b^2*e^2*g^2*(b*e^*f + b*d^*g - 2*a*e^*g) - 2*c^3*d*f*(e^2*f^2 + d^* \\ & e^*f*g + d^2*g^2) + 2*c*e^*g*(a^2*e^2*g^2 + a*b*e^*g*(e^*f + d^*g) - b \\ & ^2*(e^*f + d^*g)^2) - c^2*(4*a*d*e^2*f*g^2 - b*(e^3*f^3 + 5*d*e^2*f \\ & ^2*g + 5*d^2*e*f*g^2 + d^3*g^3)))*\text{ArcTanh}[(b + 2*c*x)/\text{Sqrt}[b^2 - \\ & 4*a*c]])/(\text{Sqrt}[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)^2*(c*f^2 - g^* \\ & (b*f - a^*g))^2) + (e^4*\text{Log}[d + e*x])/((c*d^2 - b*d*e + a*e^2)^2*( \\ & e^*f - d^*g)) - (g^4*\text{Log}[f + g*x])/((e^*f - d^*g)*(c*f^2 - b*f*g + a^* \\ & g^2)^2) - ((c*e^*f + c*d^*g - b*e^*g)*(c*(e^2*f^2 + d^2*g^2) + e^*g*( \\ & 2*a*e^*g - b*(e^*f + d^*g)))*\text{Log}[a + b*x + c*x^2])/((2*(c*d^2 - b*d^*e \\ & + a*e^2)^2*(c*f^2 - g*(b*f - a^*g))^2) \end{aligned}$$

**Rubi [A]** time = 5.93765, antiderivative size = 644, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$

$$\begin{aligned} & \frac{\tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right) (2ceg(a^2e^2g^2 + abeg(dg+ef) - b^2(dg+ef)^2) + b^2e^2g^2(-2aeg + bdg + bef) - c^2(4ade^2fg^2 - b(d^3g^3 + \\ & \sqrt{b^2-4ac}(ae^2 - bde + cd^2)^2(cf^2 - g(bf-ag))^2 \\ & + \frac{2c \tanh^{-1}\left(\frac{b+2cx}{\sqrt{b^2-4ac}}\right) (-c(2aeg + bdg + bef) + b^2eg + 2c^2df)}{(b^2-4ac)^{3/2}(ae^2 - bde + cd^2)(cf^2 - g(bf-ag))} \\ & - \frac{cx(-c(2aeg + bdg + bef) + b^2eg + 2c^2df) + bc(cdf - 3aeg) + 2ac^2(dg+ef) + b^3eg - b^2c(dg+ef)}{(b^2-4ac)(a+bx+cx^2)(ae^2 - bde + cd^2)(cf^2 - g(bf-ag))} \\ & - \frac{\log(a+bx+cx^2)(-beg + cdg + cef)(eg(2aeg - b(dg+ef)) + c(d^2g^2 + e^2f^2))}{2(ae^2 - bde + cd^2)^2(cf^2 - g(bf-ag))^2} \\ & + \frac{e^4 \log(d+ex)}{(ef-dg)(ae^2 - bde + cd^2)^2} - \frac{g^4 \log(f+gx)}{(ef-dg)(ag^2 - bfg + cf^2)^2} \end{aligned}$$

Antiderivative was successfully verified.

$$[\text{In}] \quad \text{Int}[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^2), x]$$

$$\begin{aligned} [\text{Out}] & -((b^3e^*g - b^2c^*(e^*f + d^*g) + 2*a*c^2*(e^*f + d^*g) + b*c^*(c*d*f \\ & - 3*a*e^*g) + c*(2*c^2*d*f + b^2*e^*g - c*(b*e^*f + b*d^*g + 2*a*e^*g \\ & ))*x)/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(b*f - a^* \\ & g))*(a + b*x + c*x^2))) + (2*c*(2*c^2*d*f + b^2*e^*g - c*(b*e^*f + \\ & b*d^*g + 2*a*e^*g))*\text{ArcTanh}[(b + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]])/((b^2 - \\ & 4*a*c)^(3/2)*(c*d^2 - b*d*e + a*e^2)*(c*f^2 - g*(b*f - a^*g))) + \end{aligned}$$

$$\begin{aligned} & ((b^2 e^2 g^2 (b e f + b d g - 2 a e g) - 2 c^3 d f (e^2 f^2 + d e f g + d^2 g^2) + 2 c e g (a^2 e^2 g^2 + a b e g (e f + d g) - b^2 (e f + d g)^2) - c^2 (4 a d e^2 f g^2 - b (e^3 f^3 + 5 d e^2 f^2 g + 5 d^2 e f g^2 + d^3 g^3))) \operatorname{ArcTan}[(b + 2 c x) / \sqrt{b^2 - 4 a c}] / (\sqrt{b^2 - 4 a c} (c d^2 - b d e + a e^2)^2 (c f^2 - g (b f - a g))^2) + (e^4 \operatorname{Log}[d + e x]) / ((c d^2 - b d e + a e^2)^2 (e f - d g)) - (g^4 \operatorname{Log}[f + g x]) / ((e f - d g) (c f^2 - b f g + a g^2)^2) - ((c e f + c d g - b e g) (c (e^2 f^2 + d^2 g^2) + e g (2 a e g - b (e f + d g)))) \operatorname{Log}[a + b x + c x^2] / (2 (c d^2 - b d e + a e^2)^2 (c f^2 - g (b f - a g))^2) \end{aligned}$$

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(e*x+d)/(g*x+f)/(c*x**2+b*x+a)**2,x)`

[Out] Timed out

**Mathematica [A]** time = 5.47894, size = 710, normalized size = 1.1

$$\begin{aligned} & \tan^{-1}\left(\frac{b+2cx}{\sqrt{4ac-b^2}}\right) (-2c^3 (2a^2 eg (d^2 g^2 - 5defg + e^2 f^2) + ab (3d^3 g^3 + 11d^2 efg^2 + 11de^2 f^2 g + 3e^3 f^3) - 4b^2 d^2 e f^2 g) - 2b^2 ce) \\ & + \frac{bc(3aeg + c(-df + dgx + efx)) - 2c^2(adg + ae(f - gx) + cdfx) + b^3(-e)g + b^2c(dg + e(f - gx))}{(b^2 - 4ac)(a + x(b + cx))(e(bd - ae) - cd^2)(g(bf - ag) - cf^2)} \\ & + \frac{e^4 \log(d + ex)}{(ef - dg)(e(ae - bd) + cd^2)^2} \\ & - \frac{\log(a + x(b + cx))(-beg + cdg + cef)(eg(2aeg - b(dg + ef)) + c(d^2 g^2 + e^2 f^2))}{2(e(ae - bd) + cd^2)^2(g(ag - bf) + cf^2)^2} \\ & - \frac{g^4 \log(f + gx)}{(ef - dg)(g(ag - bf) + cf^2)^2} \end{aligned}$$

Antiderivative was successfully verified.

[In] `Integrate[1/((d + e*x)*(f + g*x)*(a + b*x + c*x^2)^2),x]`

$$\begin{aligned} & [Out] (- (b^3 e g) + b^2 c (d g + e (f - g x)) - 2 c^2 (a d g + c d f x + a e (f - g x)) + b c (3 a e g + c (-d f) + e f x + d g x)) / ((b^2 - 4 a c) (- (c d^2) + e (b d - a e)) (- (c f^2) + g (b f - a g)) (a + x (b + c x))) + ((4 c^5 d^3 f^3 + b^4 e^2 g^2 (b e f + b d g - 2 a e g) - 2 b^2 c e g (-6 a^2 e^2 g^2 + 2 a b e g (e f + d g) + b^2 (e^2 f^2 + d e f g + d^2 g^2)) + 2 c^4 d f (-3 b d f (e f + d g) + 2 a (3 e^2 f^2 + d e f g + 3 d^2 g^2)) + c^2 (-12 a^3 e^3 g^3 - 6 a^2 b e^2 g^2 (e f + d g) + 12 a b^2 e g (e^2 f^2 + d e f g + d^2 g^2) + b^3 (e^3 f^3 + d e^2 f^2 g + d^2 e f g^2 + d^3 g^3)) - 2 c^3 (-4 b^2 d^2 e f^2 g + 2 a^2 e g (e^2 f^2 - 5 d e f g + d^2 g^2) + a b (3 e^3 f^3 + 11 d e^2 f^2 g + 11 d^2 e f g^2 + 3 d^3 g^3))) \operatorname{ArcTan}[(b + 2 c x) / \sqrt{-b^2 + 4 a c}] / ((-b^2 + 4 a c)^{3/2} (c d^2 + e (-b d) + a e))^2 (c f^2 + g (-b f) + a g))^2) + (e^4 \operatorname{Log}[d + e x]) / ((c d^2 + e (-b d) + a e))^2 (e f - d g)) - (g^4 \operatorname{Log}[f + g x]) / ((e f - d g) (c f^2 + g (-b f) + a g))^2) - ((c e f + c d g - b e g) (c (e^2 f^2 + d^2 g^2) + e g (2 a e g - b (e f + d g)))) \operatorname{Log}[a + x (b + c x)] / (2 (c d^2 + e (-b d) + a e))^2 (c f^2 + g (-b f) + a g))^2) \end{aligned}$$

**Maple [B]** time = 0.082, size = 11490, normalized size = 17.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x+d)/(g*x+f)/(c*x^2+b*x+a)^2,x)`

[Out] result too large to display

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((c*x^2 + b*x + a)^2*(e*x + d)*(g*x + f)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((c*x^2 + b*x + a)^2*(e*x + d)*(g*x + f)),x, algorithm="fricas")`

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x+d)/(g*x+f)/(c*x**2+b*x+a)**2,x)`

[Out] Timed out

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((c*x^2 + b*x + a)^2*(e*x + d)*(g*x + f)),x, algorithm="giac")`

[Out] Exception raised: NotImplementedError



$$3.819 \quad \int \frac{(d+ex)^3(a+bx+cx^2)}{\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=287

$$\begin{aligned} & - \frac{2e(f+gx)^{7/2}(eg(-aeg-3bdg+4bef) - c(3d^2g^2 - 12defg + 10e^2f^2))}{7g^6} \\ & + \frac{2(f+gx)^{5/2}(ef-dg)(3eg(-aeg-bdg+2bef) - c(d^2g^2 - 8defg + 10e^2f^2))}{5g^6} \\ & - \frac{2\sqrt{f+gx}(ef-dg)^3(ag^2 - bfg + cf^2)}{g^6} \\ & + \frac{2(f+gx)^{3/2}(ef-dg)^2(cf(5ef-2dg) - g(-3aeg-bdg+4bef))}{3g^6} \\ & - \frac{2e^2(f+gx)^{9/2}(-beg-3cdg+5cef)}{9g^6} + \frac{2ce^3(f+gx)^{11/2}}{11g^6} \end{aligned}$$

[Out]  $(-2*(e*f - d*g)^3*(c*f^2 - b*f*g + a*g^2)*\text{Sqrt}[f + g*x])/g^6 + (2*(e*f - d*g)^2*(c*f*(5*e*f - 2*d*g) - g*(4*b*e*f - b*d*g - 3*a*e*g))*(f + g*x)^{(3/2)}/(3*g^6) + (2*(e*f - d*g)*(3*e*g*(2*b*e*f - b*d*g - a*e*g) - c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^{(5/2)}/(5*g^6) - (2*e*(e*g*(4*b*e*f - 3*b*d*g - a*e*g) - c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^{(7/2)}/(7*g^6) - (2*e^2*(5*c*e*f - 3*c*d*g - b*e*g)*(f + g*x)^{(9/2)}/(9*g^6) + (2*c*e^3*(f + g*x)^{(11/2)}/(11*g^6))$

**Rubi [A]** time = 1.02091, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$

$$\begin{aligned} & - \frac{2e(f+gx)^{7/2}(eg(-aeg-3bdg+4bef) - c(3d^2g^2 - 12defg + 10e^2f^2))}{7g^6} \\ & + \frac{2(f+gx)^{5/2}(ef-dg)(3eg(-aeg-bdg+2bef) - c(d^2g^2 - 8defg + 10e^2f^2))}{5g^6} \\ & - \frac{2\sqrt{f+gx}(ef-dg)^3(ag^2 - bfg + cf^2)}{g^6} \\ & + \frac{2(f+gx)^{3/2}(ef-dg)^2(cf(5ef-2dg) - g(-3aeg-bdg+4bef))}{3g^6} \\ & - \frac{2e^2(f+gx)^{9/2}(-beg-3cdg+5cef)}{9g^6} + \frac{2ce^3(f+gx)^{11/2}}{11g^6} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(a + b\*x + c\*x^2))/Sqrt[f + g\*x], x]

[Out]  $(-2*(e*f - d*g)^3*(c*f^2 - b*f*g + a*g^2)*\text{Sqrt}[f + g*x])/g^6 + (2*(e*f - d*g)^2*(c*f*(5*e*f - 2*d*g) - g*(4*b*e*f - b*d*g - 3*a*e*g))*(f + g*x)^{(3/2)}/(3*g^6) + (2*(e*f - d*g)*(3*e*g*(2*b*e*f - b*d*g - a*e*g) - c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^{(5/2)}/(5*g^6) - (2*e*(e*g*(4*b*e*f - 3*b*d*g - a*e*g) - c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^{(7/2)}/(7*g^6) - (2*e^2*(5*c*e*f - 3*c*d*g - b*e*g)*(f + g*x)^{(9/2)}/(9*g^6) + (2*c*e^3*(f + g*x)^{(11/2)}/(11*g^6))$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**3*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)`

[Out] Timed out

**Mathematica [A]** time = 1.04055, size = 412, normalized size = 1.44

$$2\sqrt{f+gx} (11g (9ag (35d^3g^3 + 35d^2eg^2(gx - 2f) + 7de^2g (8f^2 - 4fgx + 3g^2x^2) + e^3 (-16f^3 + 8f^2gx - 6fg^2x^2 + 5g^3x^3)))$$

Antiderivative was successfully verified.

[In] `Integrate[((d + e*x)^3*(a + b*x + c*x^2))/Sqrt[f + g*x],x]`

$$\begin{aligned} & [Out] (2*\text{Sqrt}[f + g*x]*(c*(231*d^3*g^3*(8*f^2 - 4*f*g*x + 3*g^2*x^2) + \\ & 297*d^2*e*g^2*(-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2 + 5*g^3*x^3) + 3 \\ & 3*d*e^2*g*(128*f^4 - 64*f^3*g*x + 48*f^2*g^2*x^2 - 40*f*g^3*x^3 + \\ & 35*g^4*x^4) - 5*e^3*(256*f^5 - 128*f^4*g*x + 96*f^3*g^2*x^2 - 80 \\ & *f^2*g^3*x^3 + 70*f*g^4*x^4 - 63*g^5*x^5)) + 11*g*(9*a*g*(35*d^3* \\ & g^3 + 35*d^2*e*g^2*(-2*f + g*x) + 7*d*e^2*g*(8*f^2 - 4*f*g*x + 3* \\ & g^2*x^2) + e^3*(-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2 + 5*g^3*x^3)) + \\ & b*(105*d^3*g^3*(-2*f + g*x) + 63*d^2*e*g^2*(8*f^2 - 4*f*g*x + 3* \\ & g^2*x^2) + 27*d*e^2*g*(-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2 + 5*g^3* \\ & x^3) + e^3*(128*f^4 - 64*f^3*g*x + 48*f^2*g^2*x^2 - 40*f*g^3*x^3 \\ & + 35*g^4*x^4))))/(3465*g^6) \end{aligned}$$

**Maple [B]** time = 0.011, size = 540, normalized size = 1.9

$$630 e^3 c x^5 g^5 + 770 b e^3 g^5 x^4 + 2310 c d e^2 g^5 x^4 - 700 c e^3 f g^4 x^4 + 990 a e^3 g^5 x^3 + 2970 b d e^2 g^5 x^3 - 880 b e^3 f g^4 x^3 + 2970 c d e g^5 x^3$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(1/2),x)`

$$\begin{aligned} & [Out] 2/3465*(g*x+f)^(1/2)*(315*c*e^3*g^5*x^5+385*b*e^3*g^5*x^4+1155*c* \\ & d*e^2*g^5*x^4-350*c*e^3*f*g^4*x^4+495*a*e^3*g^5*x^3+1485*b*d*e^2* \\ & g^5*x^3-440*b*e^3*f*g^4*x^3+1485*c*d^2*e*g^5*x^3-1320*c*d*e^2*f*g \\ & ^4*x^3+400*c*e^3*f^2*g^3*x^3+2079*a*d*e^2*g^5*x^2-594*a*e^3*f*g^4 \\ & *x^2+2079*b*d^2*e*g^5*x^2-1782*b*d*e^2*f*g^4*x^2+528*b*e^3*f^2*g^4 \\ & ^3*x^2+693*c*d^3*g^5*x^2-1782*c*d^2*e*f*g^4*x^2+1584*c*d*e^2*f^2*g \\ & ^3*x^2-480*c*e^3*f^3*g^2*x^2+3465*a*d^2*e*g^5*x-2772*a*d*e^2*f*g^4 \\ & ^4*x+792*a*e^3*f^2*g^3*x+1155*b*d^3*g^5*x-2772*b*d^2*e*f*g^4*x+237 \\ & 6*b*d*e^2*f^2*g^3*x-704*b*e^3*f^3*g^2*x-924*c*d^3*f*g^4*x+2376*c* \\ & d^2*e*f^2*g^3*x-2112*c*d*e^2*f^3*g^2*x+640*c*e^3*f^4*g*x+3465*a*d \\ & ^3*g^5-6930*a*d^2*e*f*g^4+5544*a*d*e^2*f^2*g^3-1584*a*e^3*f^3*g^2 \\ & -2310*b*d^3*f*g^4+5544*b*d^2*e*f^2*g^3-4752*b*d*e^2*f^3*g^2+1408* \\ & b*e^3*f^4*g+1848*c*d^3*f^2*g^3-4752*c*d^2*e*f^3*g^2+4224*c*d*e^2* \\ & f^4*g-1280*c*e^3*f^5)/g^6 \end{aligned}$$

**Maxima [A]** time = 0.698721, size = 579, normalized size = 2.02

$$2 \left( 315 (gx + f)^{\frac{11}{2}} ce^3 - 385 (5 ce^3 f - (3 cde^2 + be^3) g) (gx + f)^{\frac{9}{2}} + 495 (10 ce^3 f^2 - 4 (3 cde^2 + be^3) fg + (3 cd^2 e + 3 bde^2 + a$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)*(e*x + d)^3/sqrt(g*x + f),x, algorithm="maxima")`

```
[Out] 2/3465*(315*(g*x + f)^(11/2)*c*e^3 - 385*(5*c*e^3*f - (3*c*d*e^2
+ b*e^3)*g)*(g*x + f)^(9/2) + 495*(10*c*e^3*f^2 - 4*(3*c*d*e^2 +
b*e^3)*f*g + (3*c*d^2*e + 3*b*d*e^2 + a*e^3)*g^2)*(g*x + f)^(7/2)
- 693*(10*c*e^3*f^3 - 6*(3*c*d*e^2 + b*e^3)*f^2*g + 3*(3*c*d^2*e
+ 3*b*d*e^2 + a*e^3)*f*g^2 - (c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*g^3
)*(g*x + f)^(5/2) + 1155*(5*c*e^3*f^4 - 4*(3*c*d*e^2 + b*e^3)*f^3
*g + 3*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*f^2*g^2 - 2*(c*d^3 + 3*b*d
^2*e + 3*a*d*e^2)*f*g^3 + (b*d^3 + 3*a*d^2*e)*g^4)*(g*x + f)^(3/2
) - 3465*(c*e^3*f^5 - a*d^3*g^5 - (3*c*d*e^2 + b*e^3)*f^4*g + (3*
c*d^2*e + 3*b*d*e^2 + a*e^3)*f^3*g^2 - (c*d^3 + 3*b*d^2*e + 3*a*d
*e^2)*f^2*g^3 + (b*d^3 + 3*a*d^2*e)*f*g^4)*sqrt(g*x + f)/g^6
```

---

**Fricas [A]** time = 0.292644, size = 579, normalized size = 2.02

$$2(315ce^3g^5x^5 - 1280ce^3f^5 + 3465ad^3g^5 + 1408(3cde^2 + be^3)f^4g - 1584(3cd^2e + 3bde^2 + ae^3)f^3g^2 + 1848(cd^3 + 3bd^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)*(e*x + d)^3/sqrt(g*x + f),x, algorithm="fricas")
```

```
[Out] 2/3465*(315*c*e^3*g^5*x^5 - 1280*c*e^3*f^5 + 3465*a*d^3*g^5 + 140
8*(3*c*d*e^2 + b*e^3)*f^4*g - 1584*(3*c*d^2*e + 3*b*d*e^2 + a*e^3
)*f^3*g^2 + 1848*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*f^2*g^3 - 2310*(
b*d^3 + 3*a*d^2*e)*f*g^4 - 35*(10*c*e^3*f*g^4 - 11*(3*c*d*e^2 + b
*e^3)*g^5)*x^4 + 5*(80*c*e^3*f^2*g^3 - 88*(3*c*d*e^2 + b*e^3)*f*g
^4 + 99*(3*c*d^2*e + 3*b*d*e^2 + a*e^3)*g^5)*x^3 - 3*(160*c*e^3*f
^3*g^2 - 176*(3*c*d*e^2 + b*e^3)*f^2*g^3 + 198*(3*c*d^2*e + 3*b*d
*e^2 + a*e^3)*f*g^4 - 231*(c*d^3 + 3*b*d^2*e + 3*a*d*e^2)*g^5)*x^
2 + (640*c*e^3*f^4*g - 704*(3*c*d*e^2 + b*e^3)*f^3*g^2 + 792*(3*c
*d^2*e + 3*b*d*e^2 + a*e^3)*f^2*g^3 - 924*(c*d^3 + 3*b*d^2*e + 3*
a*d*e^2)*f*g^4 + 1155*(b*d^3 + 3*a*d^2*e)*g^5)*x)*sqrt(g*x + f)/g
^6
```

---

**Sympy [A]** time = 110.025, size = 1544, normalized size = 5.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**3*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)
```

```
[Out] Piecewise((- (2*a*d**3*f/sqrt(f + g*x) + 2*a*d**3*(-f/sqrt(f + g*x)
) - sqrt(f + g*x)) + 6*a*d**2*e*f*(-f/sqrt(f + g*x) - sqrt(f + g*
x))/g + 6*a*d**2*e*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f +
g*x)**(3/2)/3)/g + 6*a*d*e**2*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f
+ g*x) - (f + g*x)**(3/2)/3)/g**2 + 6*a*d*e**2*(-f**3/sqrt(f + g
*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2
)/5)/g**2 + 2*a*e**3*f*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x
) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**3 + 2*a*e**3*(f**
4/sqrt(f + g*x) + 4*f**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2)
+ 4*f*(f + g*x)**(5/2)/5 - (f + g*x)**(7/2)/7)/g**3 + 2*b*d**3*f*
(-f/sqrt(f + g*x) - sqrt(f + g*x))/g + 2*b*d**3*(f**2/sqrt(f + g*
x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g + 6*b*d**2*e*f*(f*
**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 +
6*b*d**2*e*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f +
g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 + 6*b*d*e**2*f*(-f**3/sqrt
(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)
** (5/2)/5)/g**3 + 6*b*d*e**2*(f**4/sqrt(f + g*x) + 4*f**3*sqrt(f
+ g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (f +
g*x)**(7/2)/7)/g**3 + 2*b*e**3*f*(f**4/sqrt(f + g*x) + 4*f**3*sq
rt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 - (
```

```

f + g*x)**(7/2)/7)/g**4 + 2*b*e**3*(-f**5/sqrt(f + g*x) - 5*f**4*
sqrt(f + g*x) + 10*f**3*(f + g*x)**(3/2)/3 - 2*f**2*(f + g*x)**(5
/2) + 5*f*(f + g*x)**(7/2)/7 - (f + g*x)**(9/2)/9)/g**4 + 2*c*d**
3*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)
/g**2 + 2*c*d**3*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*
(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2 + 6*c*d**2*e*f*(-f**3
/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f +
g*x)**(5/2)/5)/g**3 + 6*c*d**2*e*(f**4/sqrt(f + g*x) + 4*f**3*sq
rt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5/2)/5 -
(f + g*x)**(7/2)/7)/g**3 + 6*c*d*e**2*f*(f**4/sqrt(f + g*x) + 4*f
**3*sqrt(f + g*x) - 2*f**2*(f + g*x)**(3/2) + 4*f*(f + g*x)**(5/2
)/5 - (f + g*x)**(7/2)/7)/g**4 + 6*c*d*e**2*(-f**5/sqrt(f + g*x)
- 5*f**4*sqrt(f + g*x) + 10*f**3*(f + g*x)**(3/2)/3 - 2*f**2*(f +
g*x)**(5/2) + 5*f*(f + g*x)**(7/2)/7 - (f + g*x)**(9/2)/9)/g**4
+ 2*c*e**3*f*(-f**5/sqrt(f + g*x) - 5*f**4*sqrt(f + g*x) + 10*f**
3*(f + g*x)**(3/2)/3 - 2*f**2*(f + g*x)**(5/2) + 5*f*(f + g*x)**(
7/2)/7 - (f + g*x)**(9/2)/9)/g**5 + 2*c*e**3*(f**6/sqrt(f + g*x)
+ 6*f**5*sqrt(f + g*x) - 5*f**4*(f + g*x)**(3/2) + 4*f**3*(f + g*
x)**(5/2) - 15*f**2*(f + g*x)**(7/2)/7 + 2*f*(f + g*x)**(9/2)/3 -
(f + g*x)**(11/2)/11)/g**5/g, Ne(g, 0)), ((a*d**3*x + c*e**3*x*
*6/6 + x**5*(b*e**3 + 3*c*d*e**2)/5 + x**4*(a*e**3 + 3*b*d*e**2 +
3*c*d**2*e)/4 + x**3*(3*a*d*e**2 + 3*b*d**2*e + c*d**3)/3 + x**2
*(3*a*d**2*e + b*d**3)/2)/sqrt(f), True))

```

**GIAC/XCAS [A]** time = 0.267718, size = 1, normalized size = 0.

*Done*

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)*(e*x + d)^3/sqrt(g*x + f),x, algorithm="giac")
```

```
[Out] Done
```

$$3.820 \quad \int \frac{(d+ex)^2(a+bx+cx^2)}{\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=212

$$\begin{aligned} & \frac{2(f+gx)^{5/2}(eg(-aeg-2bdg+3bef)-c(d^2g^2-6defg+6e^2f^2))}{5g^5} \\ & + \frac{2\sqrt{f+gx}(ef-dg)^2(ag^2-bfg+cf^2)}{g^5} \\ & - \frac{2(f+gx)^{3/2}(ef-dg)(2cf(2ef-dg)-g(-2aeg-bdg+3bef))}{3g^5} \\ & - \frac{2e(f+gx)^{7/2}(-beg-2cdg+4cef)}{7g^5} + \frac{2ce^2(f+gx)^{9/2}}{9g^5} \end{aligned}$$

[Out]  $(2*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*\text{Sqrt}[f + g*x])/g^5 - (2*(e*f - d*g)*(2*c*f*(2*e*f - d*g) - g*(3*b*e*f - b*d*g - 2*a*e*g))*(f + g*x)^{(3/2)})/(3*g^5) - (2*(e*g*(3*b*e*f - 2*b*d*g - a*e*g) - c*(6*e^2*f^2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^{(5/2)})/(5*g^5) - (2*e*(4*c*e*f - 2*c*d*g - b*e*g)*(f + g*x)^{(7/2)})/(7*g^5) + (2*c*e^2*(f + g*x)^{(9/2)})/(9*g^5)$

**Rubi [A]** time = 0.692864, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$

$$\begin{aligned} & \frac{2(f+gx)^{5/2}(eg(-aeg-2bdg+3bef)-c(d^2g^2-6defg+6e^2f^2))}{5g^5} \\ & + \frac{2\sqrt{f+gx}(ef-dg)^2(ag^2-bfg+cf^2)}{g^5} \\ & - \frac{2(f+gx)^{3/2}(ef-dg)(2cf(2ef-dg)-g(-2aeg-bdg+3bef))}{3g^5} \\ & - \frac{2e(f+gx)^{7/2}(-beg-2cdg+4cef)}{7g^5} + \frac{2ce^2(f+gx)^{9/2}}{9g^5} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}(((d + e*x)^2*(a + b*x + c*x^2))/\text{Sqrt}[f + g*x], x)$

[Out]  $(2*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*\text{Sqrt}[f + g*x])/g^5 - (2*(e*f - d*g)*(2*c*f*(2*e*f - d*g) - g*(3*b*e*f - b*d*g - 2*a*e*g))*(f + g*x)^{(3/2)})/(3*g^5) - (2*(e*g*(3*b*e*f - 2*b*d*g - a*e*g) - c*(6*e^2*f^2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^{(5/2)})/(5*g^5) - (2*e*(4*c*e*f - 2*c*d*g - b*e*g)*(f + g*x)^{(7/2)})/(7*g^5) + (2*c*e^2*(f + g*x)^{(9/2)})/(9*g^5)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\begin{aligned} & \frac{2ce^2(f+gx)^{\frac{9}{2}}}{9g^5} + \frac{2e(f+gx)^{\frac{7}{2}}(beg+2cdg-4cef)}{7g^5} + \frac{2(dg-ef)^2(ag^2-bfg+cf^2)}{g} \int \sqrt{f+gx} \frac{1}{g^4} dx \\ & + \frac{2(f+gx)^{\frac{5}{2}}(ae^2g^2+2bdeg^2-3be^2fg+cd^2g^2-6cdefg+6ce^2f^2)}{5g^5} \\ & + \frac{2(f+gx)^{\frac{3}{2}}(dg-ef)(2aeg^2+bdg^2-3befg-2cdfg+4cef^2)}{3g^5} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**2*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)`

[Out]  $2*c*e**2*(f + g*x)**(9/2)/(9*g**5) + 2*e*(f + g*x)**(7/2)*(b*e*g + 2*c*d*g - 4*c*e*f)/(7*g**5) + 2*(d*g - e*f)**2*(a*g**2 - b*f*g + c*f**2)*Integral(g**(-4), (x, sqrt(f + g*x)))/g + 2*(f + g*x)**(5/2)*(a*e**2*g**2 + 2*b*d*e*g**2 - 3*b*e**2*f*g + c*d**2*g**2 - 6*c*d*e*f*g + 6*c*e**2*f**2)/(5*g**5) + 2*(f + g*x)**(3/2)*(d*g - e*f)*(2*a*e*g**2 + b*d*g**2 - 3*b*e*f*g - 2*c*d*f*g + 4*c*e*f**2)/(3*g**5)$

**Mathematica [A]** time = 0.330655, size = 256, normalized size = 1.21

$2\sqrt{f+gx}(3g(7ag(15d^2g^2+10deg(gx-2f))+e^2(8f^2-4fgx+3g^2x^2))+b(35d^2g^2(gx-2f)+14deg(8f^2-4fgx+3g^2x^2)))$

Antiderivative was successfully verified.

[In] `Integrate[((d + e*x)^2*(a + b*x + c*x^2))/Sqrt[f + g*x],x]`

[Out]  $(2*\text{Sqrt}[f + g*x]*(c*(21*d^2*g^2*(8*f^2 - 4*f*g*x + 3*g^2*x^2) + 18*d*e*g*(-16*f^3 + 8*f^2*g*x - 6*f*g^2*x^2 + 5*g^3*x^3) + e^2*(12*8*f^4 - 64*f^3*g*x + 48*f^2*g^2*x^2 - 40*f*g^3*x^3 + 35*g^4*x^4)) + 3*g*(7*a*g*(15*d^2*g^2 + 10*d*e*g*(-2*f + g*x) + e^2*(8*f^2 - 4*f*g*x + 3*g^2*x^2)) + b*(35*d^2*g^2*(-2*f + g*x) + 14*d*e*g*(8*f^2 - 4*f*g*x + 3*g^2*x^2) - 3*e^2*(16*f^3 - 8*f^2*g*x + 6*f*g^2*x^2 - 5*g^3*x^3)))))/(315*g^5)$

**Maple [A]** time = 0.01, size = 315, normalized size = 1.5

$70e^2cx^4g^4 + 90be^2g^4x^3 + 180cdeg^4x^3 - 80ce^2fg^3x^3 + 126ae^2g^4x^2 + 252bdeg^4x^2 - 108be^2fg^3x^2 + 126cd^2g^4x^2 - 216cde^2fg^3x^2$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(1/2),x)`

[Out]  $2/315*(g*x+f)^(1/2)*(35*c*e^2*g^4*x^4+45*b*e^2*g^4*x^3+90*c*d*e*g^4*x^3-40*c*e^2*f*g^3*x^3+63*a*e^2*g^4*x^2+126*b*d*e*g^4*x^2-54*b*e^2*f*g^3*x^2+63*c*d^2*g^4*x^2-108*c*d*e*f*g^3*x^2+48*c*e^2*f^2*g^2*x^2+210*a*d*e*g^4*x-84*a*e^2*f*g^3*x+105*b*d^2*g^4*x-168*b*d*e*f*g^3*x+72*b*e^2*f^2*g^2*x-84*c*d^2*f*g^3*x+144*c*d*e*f^2*g^2*x-64*c*e^2*f^3*g*x+315*a*d^2*g^4-420*a*d*e*f*g^3+168*a*e^2*f^2*g^2-210*b*d^2*f*g^3+336*b*d*e*f^2*g^2-144*b*e^2*f^3*g+168*c*d^2*f^2*g^2-288*c*d*e*f^3*g+128*c*e^2*f^4)/g^5$

**Maxima [A]** time = 0.696518, size = 352, normalized size = 1.66

$2\left(35(gx+f)^{\frac{9}{2}}ce^2 - 45(4ce^2f - (2cde + be^2)g)(gx+f)^{\frac{7}{2}} + 63(6ce^2f^2 - 3(2cde + be^2)fg + (cd^2 + 2bde + ae^2)g^2)(gx+f)^{\frac{5}{2}}\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)*(e*x + d)^2/sqrt(g*x + f),x, algorithm="maxima")`

[Out]  $2/315*(35*(g*x + f)^(9/2)*c*e^2 - 45*(4*c*e^2*f - (2*c*d*e + b*e^2)*g)*(g*x + f)^(7/2) + 63*(6*c*e^2*f^2 - 3*(2*c*d*e + b*e^2)*f*g$

$$+ (c^2d^2 + 2bd^2e + a^2e^2)g^2)(gx + f)^{5/2} - 105(4c^2e^2f^3 - 3(2c^2d^2e + b^2e^2)f^2g + 2(c^2d^2 + 2bd^2e + a^2e^2)f^2g^2 - (b^2d^2 + 2a^2d^2e)g^3)(gx + f)^{3/2} + 315(c^2e^2f^4 + a^2d^2g^4 - (2c^2d^2e + b^2e^2)f^3g + (c^2d^2 + 2bd^2e + a^2e^2)f^2g^2 - (b^2d^2 + 2a^2d^2e)f^2g^3)\sqrt{gx + f}/g^5$$

**Fricas [A]** time = 0.276862, size = 351, normalized size = 1.66

$$2(35ce^2g^4x^4 + 128ce^2f^4 + 315ad^2g^4 - 144(2cde + be^2)f^3g + 168(cd^2 + 2bde + ae^2)f^2g^2 - 210(bd^2 + 2ade)fg^3 - 5(8c^2d^2e + b^2e^2)f^3g^2 + 168(c^2d^2 + 2bd^2e + a^2e^2)f^2g^3 - 210(b^2d^2 + 2a^2d^2e)fg^4 - 105(c^2d^2 + 2bd^2e + a^2e^2)f^2g^4 - 105(b^2d^2 + 2a^2d^2e)f^2g^4)\sqrt{gx + f}/g^5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)^2/sqrt(g\*x + f), x, algorithm="fricas")

[Out] 2/315\*(35\*c^2\*e^2\*g^4\*x^4 + 128\*c^2\*e^2\*f^4 + 315\*a\*d^2\*g^4 - 144\*(2\*c^2\*d^2\*e + b^2\*e^2)\*f^3\*g + 168\*(c^2\*d^2 + 2\*b\*d^2\*e + a^2\*e^2)\*f^2\*g^2 - 210\*(b^2\*d^2 + 2\*a^2\*d^2\*e)\*f^2\*g^3 - 5\*(8\*c^2\*e^2\*f^3\*g^2 - 9\*(2\*c^2\*d^2\*e + b^2\*e^2)\*g^4)\*x^3 + 3\*(16\*c^2\*e^2\*f^2\*g^2 - 18\*(2\*c^2\*d^2\*e + b^2\*e^2)\*f^2\*g^3 + 21\*(c^2\*d^2 + 2\*b\*d^2\*e + a^2\*e^2)\*g^4)\*x^2 - (64\*c^2\*e^2\*f^3\*g - 72\*(2\*c^2\*d^2\*e + b^2\*e^2)\*f^2\*g^2 + 84\*(c^2\*d^2 + 2\*b\*d^2\*e + a^2\*e^2)\*f^2\*g^3 - 105\*(b^2\*d^2 + 2\*a^2\*d^2\*e)\*g^4)\*x)\*sqrt(g\*x + f)/g^5

**Sympy [A]** time = 64.9989, size = 1001, normalized size = 4.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2\*(c\*x\*\*2+b\*x+a)/(g\*x+f)\*\*(1/2), x)

[Out] Piecewise((-2\*a\*d\*\*2\*f/sqrt(f + g\*x) + 2\*a\*d\*\*2\*(-f/sqrt(f + g\*x) - sqrt(f + g\*x))/g + 4\*a\*d\*e\*f\*(-f/sqrt(f + g\*x) - sqrt(f + g\*x))/g + 4\*a\*d\*e\*(f\*\*2/sqrt(f + g\*x) + 2\*f\*sqrt(f + g\*x) - (f + g\*x)\*\*(3/2)/3)/g + 2\*a\*e\*\*2\*f\*(f\*\*2/sqrt(f + g\*x) + 2\*f\*sqrt(f + g\*x) - (f + g\*x)\*\*(3/2)/3)/g\*\*2 + 2\*a\*e\*\*2\*(-f\*\*3/sqrt(f + g\*x) - 3\*f\*\*2\*sqrt(f + g\*x) + f\*(f + g\*x)\*\*(3/2) - (f + g\*x)\*\*(5/2)/5)/g\*\*2 + 2\*b\*d\*\*2\*f\*(-f/sqrt(f + g\*x) - sqrt(f + g\*x))/g + 2\*b\*d\*\*2\*(f\*\*2/sqrt(f + g\*x) + 2\*f\*sqrt(f + g\*x) - (f + g\*x)\*\*(3/2)/3)/g + 4\*b\*d\*e\*f\*(f\*\*2/sqrt(f + g\*x) + 2\*f\*sqrt(f + g\*x) - (f + g\*x)\*\*(3/2)/3)/g\*\*2 + 4\*b\*d\*e\*(-f\*\*3/sqrt(f + g\*x) - 3\*f\*\*2\*sqrt(f + g\*x) + f\*(f + g\*x)\*\*(3/2) - (f + g\*x)\*\*(5/2)/5)/g\*\*2 + 2\*b\*e\*\*2\*f\*(-f\*\*3/sqrt(f + g\*x) - 3\*f\*\*2\*sqrt(f + g\*x) + f\*(f + g\*x)\*\*(3/2) - (f + g\*x)\*\*(5/2)/5)/g\*\*3 + 2\*b\*e\*\*2\*(f\*\*4/sqrt(f + g\*x) + 4\*f\*\*3\*sqrt(f + g\*x) - 2\*f\*\*2\*(f + g\*x)\*\*(3/2) + 4\*f\*(f + g\*x)\*\*(5/2)/5 - (f + g\*x)\*\*(7/2)/7)/g\*\*3 + 2\*c\*d\*\*2\*f\*(f\*\*2/sqrt(f + g\*x) + 2\*f\*sqrt(f + g\*x) - (f + g\*x)\*\*(3/2)/3)/g\*\*2 + 2\*c\*d\*\*2\*(-f\*\*3/sqrt(f + g\*x) - 3\*f\*\*2\*sqrt(f + g\*x) + f\*(f + g\*x)\*\*(3/2) - (f + g\*x)\*\*(5/2)/5)/g\*\*2 + 4\*c\*d\*e\*f\*(-f\*\*3/sqrt(f + g\*x) - 3\*f\*\*2\*sqrt(f + g\*x) + f\*(f + g\*x)\*\*(3/2) - (f + g\*x)\*\*(5/2)/5)/g\*\*3 + 4\*c\*d\*e\*(f\*\*4/sqrt(f + g\*x) + 4\*f\*\*3\*sqrt(f + g\*x) - 2\*f\*\*2\*(f + g\*x)\*\*(3/2) + 4\*f\*(f + g\*x)\*\*(5/2)/5 - (f + g\*x)\*\*(7/2)/7)/g\*\*3 + 2\*c\*e\*\*2\*f\*(f\*\*4/sqrt(f + g\*x) + 4\*f\*\*3\*sqrt(f + g\*x) - 2\*f\*\*2\*(f + g\*x)\*\*(3/2) + 4\*f\*(f + g\*x)\*\*(5/2)/5 - (f + g\*x)\*\*(7/2)/7)/g\*\*4 + 2\*c\*e\*\*2\*(-f\*\*5/sqrt(f + g\*x) - 5\*f\*\*4\*sqrt(f + g\*x) + 10\*f\*\*3\*(f + g\*x)\*\*(3/2)/3 - 2\*f\*\*2\*(f + g\*x)\*\*(5/2) + 5\*f\*(f + g\*x)\*\*(7/2)/7 - (f + g\*x)\*\*(9/2)/9)/g\*\*4)/g, Ne(g, 0)), ((a\*d\*\*2\*x + c\*e\*\*2\*x\*\*5/5 + x\*\*4\*(b\*e\*\*2 + 2\*c\*d\*e)/4 + x\*\*3\*(a\*e\*\*2 + 2\*b\*d\*e + c\*d\*\*2)/3 + x\*\*2\*(2\*a\*d\*e + b\*d\*\*2)/2)/sqrt(f), True))

**GIAC/XCAS [A]** time = 0.266469, size = 579, normalized size = 2.73

$$2 \left( 315 \sqrt{gx + f} ad^2 + \frac{105 \left( (gx+f)^{\frac{3}{2}} - 3 \sqrt{gx+ff} \right) bd^2}{g} + \frac{210 \left( (gx+f)^{\frac{3}{2}} - 3 \sqrt{gx+ff} \right) ade}{g} + \frac{21 \left( 3(gx+f)^{\frac{5}{2}} g^8 - 10(gx+f)^{\frac{3}{2}} f g^8 + 15 \sqrt{gx+ff} g^8 \right) cd^2}{g^{10}} + \dots \right)$$


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Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)^2/sqrt(g\*x + f),x, algorithm="giac")

[Out]  $\frac{2}{315} \left( 315 \sqrt{gx + f} a^2 d^2 + 105 \left( (gx + f)^{3/2} - 3 \sqrt{gx + f} \right) a d^2 e/g + 210 \left( (gx + f)^{3/2} - 3 \sqrt{gx + f} \right) a d^2 e/g + 21 \left( 3 (gx + f)^{5/2} g^8 - 10 (gx + f)^{3/2} f g^8 + 15 \sqrt{gx + f} f^2 g^8 \right) c^2 d^2 / g^{10} + 42 \left( 3 (gx + f)^{5/2} g^8 - 10 (gx + f)^{3/2} f g^8 + 15 \sqrt{gx + f} f^2 g^8 \right) b^2 d^2 e / g^{10} + 21 \left( 3 (gx + f)^{5/2} g^8 - 10 (gx + f)^{3/2} f g^8 + 15 \sqrt{gx + f} f^2 g^8 \right) a^2 e^2 / g^{10} + 18 \left( 5 (gx + f)^{7/2} g^{18} - 21 (gx + f)^{5/2} f g^{18} + 35 (gx + f)^{3/2} f^2 g^{18} - 35 \sqrt{gx + f} f^3 g^{18} \right) c^2 d^2 e / g^{21} + 9 \left( 5 (gx + f)^{7/2} g^{18} - 21 (gx + f)^{5/2} f g^{18} + 35 (gx + f)^{3/2} f^2 g^{18} - 35 \sqrt{gx + f} f^3 g^{18} \right) b^2 e^2 / g^{21} + \left( 35 (gx + f)^{9/2} g^{32} - 180 (gx + f)^{7/2} f g^{32} + 378 (gx + f)^{5/2} f^2 g^{32} - 420 (gx + f)^{3/2} f^3 g^{32} + 315 \sqrt{gx + f} f^4 g^{32} \right) c^2 e^2 / g^{36} \right) / g$



$$3.821 \quad \int \frac{(d+ex)(a+bx+cx^2)}{\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=137

$$-\frac{2\sqrt{f+gx}(ef-dg)(ag^2-bfg+cf^2)}{g^4} + \frac{2(f+gx)^{3/2}(cf(3ef-2dg)-g(-aeg-bdg+2bef))}{3g^4} \\ - \frac{2(f+gx)^{5/2}(-beg-cdg+3cef)}{5g^4} + \frac{2ce(f+gx)^{7/2}}{7g^4}$$

[Out]  $(-2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*\text{Sqrt}[f + g*x])/g^4 + (2*(c*f*(3*e*f - 2*d*g) - g*(2*b*e*f - b*d*g - a*e*g))*(f + g*x)^{(3/2)})/(3*g^4) - (2*(3*c*e*f - c*d*g - b*e*g)*(f + g*x)^{(5/2)})/(5*g^4) + (2*c*e*(f + g*x)^{(7/2)})/(7*g^4)$

**Rubi [A]** time = 0.250471, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$-\frac{2\sqrt{f+gx}(ef-dg)(ag^2-bfg+cf^2)}{g^4} + \frac{2(f+gx)^{3/2}(cf(3ef-2dg)-g(-aeg-bdg+2bef))}{3g^4} \\ - \frac{2(f+gx)^{5/2}(-beg-cdg+3cef)}{5g^4} + \frac{2ce(f+gx)^{7/2}}{7g^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)*(a + b*x + c*x^2)/\text{Sqrt}[f + g*x], x]$

[Out]  $(-2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*\text{Sqrt}[f + g*x])/g^4 + (2*(c*f*(3*e*f - 2*d*g) - g*(2*b*e*f - b*d*g - a*e*g))*(f + g*x)^{(3/2)})/(3*g^4) - (2*(3*c*e*f - c*d*g - b*e*g)*(f + g*x)^{(5/2)})/(5*g^4) + (2*c*e*(f + g*x)^{(7/2)})/(7*g^4)$

**Rubi in Sympy [A]** time = 38.1175, size = 139, normalized size = 1.01

$$\frac{2ce(f+gx)^{7/2}}{7g^4} + \frac{2(f+gx)^{5/2}(beg+cdg-3cef)}{5g^4} \\ + \frac{2(f+gx)^{3/2}(aeg^2+bdg^2-2befg-2cdfg+3cef^2)}{3g^4} + \frac{2\sqrt{f+gx}(dg-ef)(ag^2-bfg+cf^2)}{g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^{(1/2)}, x)$

[Out]  $2*c*e*(f + g*x)^{(7/2)}/(7*g^4) + 2*(f + g*x)^{(5/2)}*(b*e*g + c*d*g - 3*c*e*f)/(5*g^4) + 2*(f + g*x)^{(3/2)}*(a*e*g^2 + b*d*g^2 - 2*b*e*f*g - 2*c*d*f*g + 3*c*e*f^2)/(3*g^4) + 2*\text{sqrt}(f + g*x)*(d*g - e*f)*(a*g^2 - b*f*g + c*f^2)/g^4$

**Mathematica [A]** time = 0.191967, size = 131, normalized size = 0.96

$$\frac{2\sqrt{f+gx}(7g(5ag(3dg-2ef+egx)+5bdg(gx-2f))+be(8f^2-4fgx+3g^2x^2))+c(7dg(8f^2-4fgx+3g^2x^2)-3e(105g^4))}{105g^4}$$

Antiderivative was successfully verified.

[In] Integrate[((d + e\*x)\*(a + b\*x + c\*x^2))/Sqrt[f + g\*x],x]

[Out] (2\*Sqrt[f + g\*x]\*(7\*g\*(5\*b\*d\*g\*(-2\*f + g\*x) + 5\*a\*g\*(-2\*e\*f + 3\*d\*g + e\*g\*x) + b\*e\*(8\*f^2 - 4\*f\*g\*x + 3\*g^2\*x^2)) + c\*(7\*d\*g\*(8\*f^2 - 4\*f\*g\*x + 3\*g^2\*x^2) - 3\*e\*(16\*f^3 - 8\*f^2\*g\*x + 6\*f\*g^2\*x^2 - 5\*g^3\*x^3))))/(105\*g^4)

**Maple [A]** time = 0.006, size = 144, normalized size = 1.1

$$\frac{30 c e x^3 g^3 + 42 b e g^3 x^2 + 42 c d g^3 x^2 - 36 c e f g^2 x^2 + 70 a e g^3 x + 70 b d g^3 x - 56 b e f g^2 x - 56 c d f g^2 x + 48 c e f^2 g x + 210 a d g^3 - 105 g^4}{105 g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)\*(c\*x^2+b\*x+a)/(g\*x+f)^(1/2),x)

[Out] 2/105\*(g\*x+f)^(1/2)\*(15\*c\*e\*g^3\*x^3+21\*b\*e\*g^3\*x^2+21\*c\*d\*g^3\*x^2-18\*c\*e\*f\*g^2\*x^2+35\*a\*e\*g^3\*x+35\*b\*d\*g^3\*x-28\*b\*e\*f\*g^2\*x-28\*c\*d\*f\*g^2\*x+24\*c\*e\*f^2\*g\*x+105\*a\*d\*g^3-70\*a\*e\*f\*g^2-70\*b\*d\*f\*g^2+56\*b\*e\*f^2\*g+56\*c\*d\*f^2\*g-48\*c\*e\*f^3)/g^4

**Maxima [A]** time = 0.69127, size = 174, normalized size = 1.27

$$\frac{2 \left( 15 (g x + f)^{\frac{7}{2}} c e - 21 (3 c e f - (c d + b e) g) (g x + f)^{\frac{5}{2}} + 35 (3 c e f^2 - 2 (c d + b e) f g + (b d + a e) g^2) (g x + f)^{\frac{3}{2}} - 105 (c e f^3 - a a) \right)}{105 g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)/sqrt(g\*x + f),x, algorithm="maxima")

[Out] 2/105\*(15\*(g\*x + f)^(7/2)\*c\*e - 21\*(3\*c\*e\*f - (c\*d + b\*e)\*g)\*(g\*x + f)^(5/2) + 35\*(3\*c\*e\*f^2 - 2\*(c\*d + b\*e)\*f\*g + (b\*d + a\*e)\*g^2)\*(g\*x + f)^(3/2) - 105\*(c\*e\*f^3 - a\*d\*g^3 - (c\*d + b\*e)\*f^2\*g + (b\*d + a\*e)\*f\*g^2)\*sqrt(g\*x + f))/g^4

**Fricas [A]** time = 0.270547, size = 169, normalized size = 1.23

$$\frac{2 (15 c e g^3 x^3 - 48 c e f^3 + 105 a d g^3 + 56 (c d + b e) f^2 g - 70 (b d + a e) f g^2 - 3 (6 c e f g^2 - 7 (c d + b e) g^3) x^2 + (24 c e f^2 g - 28 (c d + b e) f g^2))}{105 g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)/sqrt(g\*x + f),x, algorithm="fricas")

[Out] 2/105\*(15\*c\*e\*g^3\*x^3 - 48\*c\*e\*f^3 + 105\*a\*d\*g^3 + 56\*(c\*d + b\*e)\*f^2\*g - 70\*(b\*d + a\*e)\*f\*g^2 - 3\*(6\*c\*e\*f\*g^2 - 7\*(c\*d + b\*e)\*g^3)\*x^2 + (24\*c\*e\*f^2\*g - 28\*(c\*d + b\*e)\*f\*g^2 + 35\*(b\*d + a\*e)\*g^3)\*x)\*sqrt(g\*x + f)/g^4

**Sympy [A]** time = 32.7412, size = 549, normalized size = 4.01

$$\left[ \frac{\frac{2adf}{\sqrt{f+gx}} + 2ad\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right) + \frac{2aef\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right)}{g} + \frac{2ae\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g} + \frac{2bdf\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right)}{g} + \frac{2bd\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g} + \frac{2bef\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right)}{g}}{\frac{adx + \frac{cex^4}{4} + \frac{x^3(be+cd)}{3} + \frac{x^2(ae+bd)}{2}}{\sqrt{f}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(c\*x\*\*2+b\*x+a)/(g\*x+f)\*\*(1/2),x)

[Out] Piecewise((- (2\*a\*d\*f/sqrt(f + g\*x) + 2\*a\*d\*(-f/sqrt(f + g\*x) - sqrt(f + g\*x)) + 2\*a\*e\*f\*(-f/sqrt(f + g\*x) - sqrt(f + g\*x))/g + 2\*a\*e\*(f\*\*2/sqrt(f + g\*x) + 2\*f\*sqrt(f + g\*x) - (f + g\*x)\*\*(3/2)/3)/g + 2\*b\*d\*f\*(-f/sqrt(f + g\*x) - sqrt(f + g\*x))/g + 2\*b\*d\*(f\*\*2/sqrt(f + g\*x) + 2\*f\*sqrt(f + g\*x) - (f + g\*x)\*\*(3/2)/3)/g + 2\*b\*e\*f\*(f\*\*2/sqrt(f + g\*x) + 2\*f\*sqrt(f + g\*x) - (f + g\*x)\*\*(3/2)/3)/g\*\*2 + 2\*b\*e\*(-f\*\*3/sqrt(f + g\*x) - 3\*f\*\*2\*sqrt(f + g\*x) + f\*(f + g\*x)\*\*(3/2) - (f + g\*x)\*\*(5/2)/5)/g\*\*2 + 2\*c\*d\*f\*(f\*\*2/sqrt(f + g\*x) + 2\*f\*sqrt(f + g\*x) - (f + g\*x)\*\*(3/2)/3)/g\*\*2 + 2\*c\*d\*(-f\*\*3/sqrt(f + g\*x) - 3\*f\*\*2\*sqrt(f + g\*x) + f\*(f + g\*x)\*\*(3/2) - (f + g\*x)\*\*(5/2)/5)/g\*\*2 + 2\*c\*e\*f\*(-f\*\*3/sqrt(f + g\*x) - 3\*f\*\*2\*sqrt(f + g\*x) + f\*(f + g\*x)\*\*(3/2) - (f + g\*x)\*\*(5/2)/5)/g\*\*3 + 2\*c\*e\*(f\*\*4/sqrt(f + g\*x) + 4\*f\*\*3\*sqrt(f + g\*x) - 2\*f\*\*2\*(f + g\*x)\*\*(3/2) + 4\*f\*(f + g\*x)\*\*(5/2)/5 - (f + g\*x)\*\*(7/2)/7)/g\*\*3)/g, Ne(g, 0)), ((a\*d\*x + c\*e\*x\*\*4/4 + x\*\*3\*(b\*e + c\*d)/3 + x\*\*2\*(a\*e + b\*d)/2)/sqrt(f), True))

**GIAC/XCAS [A]** time = 0.263025, size = 309, normalized size = 2.26

$$2 \left( 105 \sqrt{gx + f} ad + \frac{35 (gx+f)^{\frac{3}{2}} - 3 \sqrt{gx+ff} bd}{g} + \frac{35 (gx+f)^{\frac{3}{2}} - 3 \sqrt{gx+ff} ae}{g} + \frac{7 (3 (gx+f)^{\frac{5}{2}} g^8 - 10 (gx+f)^{\frac{3}{2}} f g^8 + 15 \sqrt{gx+ff} f^2 g^8) cd}{g^{10}} + \frac{7 (3 (gx+f)^{\frac{5}{2}} g^8 - 10 (gx+f)^{\frac{3}{2}} f g^8 + 15 \sqrt{gx+ff} f^2 g^8) cd}{g^{10}} \right)$$

105 g

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)/sqrt(g\*x + f),x, algorithm="giac")

[Out] 2/105\*(105\*sqrt(g\*x + f)\*a\*d + 35\*((g\*x + f)^(3/2) - 3\*sqrt(g\*x + f)\*f)\*b\*d/g + 35\*((g\*x + f)^(3/2) - 3\*sqrt(g\*x + f)\*f)\*a\*e/g + 7\*(3\*(g\*x + f)^(5/2)\*g^8 - 10\*(g\*x + f)^(3/2)\*f\*g^8 + 15\*sqrt(g\*x + f)\*f^2\*g^8)\*c\*d/g^10 + 7\*(3\*(g\*x + f)^(5/2)\*g^8 - 10\*(g\*x + f)^(3/2)\*f\*g^8 + 15\*sqrt(g\*x + f)\*f^2\*g^8)\*b\*e/g^10 + 3\*(5\*(g\*x + f)^(7/2)\*g^18 - 21\*(g\*x + f)^(5/2)\*f\*g^18 + 35\*(g\*x + f)^(3/2)\*f^2\*g^18 - 35\*sqrt(g\*x + f)\*f^3\*g^18)\*c\*e/g^21)/g

$$3.822 \quad \int \frac{a+bx+cx^2}{\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=73

$$\frac{2\sqrt{f+gx}(ag^2 - bfg + cf^2)}{g^3} - \frac{2(f+gx)^{3/2}(2cf - bg)}{3g^3} + \frac{2c(f+gx)^{5/2}}{5g^3}$$

[Out]  $(2*(c*f^2 - b*f*g + a*g^2)*\text{Sqrt}[f + g*x])/g^3 - (2*(2*c*f - b*g)*(f + g*x)^{(3/2)})/(3*g^3) + (2*c*(f + g*x)^{(5/2)})/(5*g^3)$

**Rubi [A]** time = 0.0894436, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$\frac{2\sqrt{f+gx}(ag^2 - bfg + cf^2)}{g^3} - \frac{2(f+gx)^{3/2}(2cf - bg)}{3g^3} + \frac{2c(f+gx)^{5/2}}{5g^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)/Sqrt[f + g\*x], x]

[Out]  $(2*(c*f^2 - b*f*g + a*g^2)*\text{Sqrt}[f + g*x])/g^3 - (2*(2*c*f - b*g)*(f + g*x)^{(3/2)})/(3*g^3) + (2*c*(f + g*x)^{(5/2)})/(5*g^3)$

**Rubi in Sympy [A]** time = 14.4367, size = 68, normalized size = 0.93

$$\frac{2c(f+gx)^{5/2}}{5g^3} + \frac{2(f+gx)^{3/2}(bg - 2cf)}{3g^3} + \frac{2\sqrt{f+gx}(ag^2 - bfg + cf^2)}{g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+b\*x+a)/(g\*x+f)\*\*(1/2), x)

[Out]  $2*c*(f + g*x)**(5/2)/(5*g**3) + 2*(f + g*x)**(3/2)*(b*g - 2*c*f)/(3*g**3) + 2*\text{sqrt}(f + g*x)*(a*g**2 - b*f*g + c*f**2)/g**3$

**Mathematica [A]** time = 0.0491225, size = 54, normalized size = 0.74

$$\frac{2\sqrt{f+gx}(5g(3ag - 2bf + bgx) + c(8f^2 - 4fgx + 3g^2x^2))}{15g^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)/Sqrt[f + g\*x], x]

[Out]  $(2*\text{Sqrt}[f + g*x]*(5*g*(-2*b*f + 3*a*g + b*g*x) + c*(8*f^2 - 4*f*g*x + 3*g^2*x^2)))/(15*g^3)$

**Maple [A]** time = 0.006, size = 53, normalized size = 0.7

$$\frac{6cx^2g^2 + 10bg^2x - 8cfgx + 30ag^2 - 20bfg + 16cf^2}{15g^3} \sqrt{gx + f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)/(g*x+f)^(1/2),x)`

[Out]  $2/15*(g*x+f)^{(1/2)}*(3*c*g^2*x^2+5*b*g^2*x-4*c*f*g*x+15*a*g^2-10*b*f*g+8*c*f^2)/g^3$

**Maxima [A]** time = 0.688993, size = 104, normalized size = 1.42

$$\frac{2 \left( 15 \sqrt{gx+f} a + \frac{5 \left( (gx+f)^{\frac{3}{2}} - 3 \sqrt{gx+ff} \right) b}{g} + \frac{\left( 3 (gx+f)^{\frac{5}{2}} - 10 (gx+f)^{\frac{3}{2}} f + 15 \sqrt{gx+ff^2} \right) c}{g^2} \right)}{15g}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)/sqrt(g*x + f),x, algorithm="maxima")`

[Out]  $2/15*(15*\sqrt{g*x+f}*a + 5*((g*x+f)^{(3/2)} - 3*\sqrt{g*x+f})*f)*b/g + (3*(g*x+f)^{(5/2)} - 10*(g*x+f)^{(3/2)}*f + 15*\sqrt{g*x+f}*f^2)*c/g^2)/g$

**Fricas [A]** time = 0.269355, size = 73, normalized size = 1.

$$\frac{2(3cg^2x^2 + 8cf^2 - 10bfg + 15ag^2 - (4cfg - 5bg^2)x)\sqrt{gx+f}}{15g^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)/sqrt(g*x + f),x, algorithm="fricas")`

[Out]  $2/15*(3*c*g^2*x^2 + 8*c*f^2 - 10*b*f*g + 15*a*g^2 - (4*c*f*g - 5*b*g^2)*x)*\sqrt{g*x+f}/g^3$

**Sympy [A]** time = 7.70675, size = 223, normalized size = 3.05

$$\left\{ \frac{\frac{2af}{\sqrt{f+gx}} + 2a\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right) + \frac{2bf\left(-\frac{f}{\sqrt{f+gx}} - \sqrt{f+gx}\right)}{g} + \frac{2b\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g} + \frac{2cf\left(\frac{f^2}{\sqrt{f+gx}} + 2f\sqrt{f+gx} - \frac{(f+gx)^{\frac{3}{2}}}{3}\right)}{g^2} + \frac{2c\left(-\frac{f^3}{\sqrt{f+gx}} - 3f^2\sqrt{f+gx} + f(f+gx)^{\frac{3}{2}} - (f+gx)^{\frac{5}{2}}\right)}{g^2}}{ax + \frac{bx^2}{2} + \frac{cx^3}{3}}, \frac{ax + \frac{bx^2}{2} + \frac{cx^3}{3}}{\sqrt{f}} \right\}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+b*x+a)/(g*x+f)**(1/2),x)`

[Out] `Piecewise((- (2*a*f/sqrt(f + g*x) + 2*a*(-f/sqrt(f + g*x) - sqrt(f + g*x)) + 2*b*f*(-f/sqrt(f + g*x) - sqrt(f + g*x))/g + 2*b*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g + 2*c*f*(f**2/sqrt(f + g*x) + 2*f*sqrt(f + g*x) - (f + g*x)**(3/2)/3)/g**2 + 2*c*(-f**3/sqrt(f + g*x) - 3*f**2*sqrt(f + g*x) + f*(f + g*x)**(3/2) - (f + g*x)**(5/2)/5)/g**2)/g, Ne(g, 0)), ((a*x + b*x**2/2 + c*x**3/3)/sqrt(f), True))`

**GIAC/XCAS [A]** time = 0.264774, size = 116, normalized size = 1.59

$$\frac{2 \left( 15 \sqrt{gx+f} a + \frac{5 \left( (gx+f)^{\frac{3}{2}} - 3 \sqrt{gx+ff} \right) b}{g} + \frac{\left( 3 (gx+f)^{\frac{5}{2}} g^8 - 10 (gx+f)^{\frac{3}{2}} f g^8 + 15 \sqrt{gx+ff} f^2 g^8 \right) c}{g^{10}} \right)}{15g}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/sqrt(g\*x + f),x, algorithm="giac")

[Out] 2/15\*(15\*sqrt(g\*x + f)\*a + 5\*((g\*x + f)^(3/2) - 3\*sqrt(g\*x + f)\*f)\*b/g + (3\*(g\*x + f)^(5/2)\*g^8 - 10\*(g\*x + f)^(3/2)\*f\*g^8 + 15\*sqrt(g\*x + f)\*f^2\*g^8)\*c/g^10)/g

$$3.823 \quad \int \frac{a+bx+cx^2}{(d+ex)\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=116

$$-\frac{2(ae^2 - bde + cd^2) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}} + \frac{2\sqrt{f+gx}(beg - c(dg + ef))}{e^2g^2} + \frac{2c(f+gx)^{3/2}}{3eg^2}$$

[Out]  $(2*(b*e*g - c*(e*f + d*g))*Sqrt[f + g*x])/(e^2*g^2) + (2*c*(f + g*x)^(3/2))/(3*e*g^2) - (2*(c*d^2 - b*d*e + a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(5/2)*Sqrt[e*f - d*g])$

**Rubi [A]** time = 0.337769, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$-\frac{2(ae^2 - bde + cd^2) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}} + \frac{2\sqrt{f+gx}(beg - c(dg + ef))}{e^2g^2} + \frac{2c(f+gx)^{3/2}}{3eg^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)/((d + e\*x)\*Sqrt[f + g\*x]), x]

[Out]  $(2*(b*e*g - c*(e*f + d*g))*Sqrt[f + g*x])/(e^2*g^2) + (2*c*(f + g*x)^(3/2))/(3*e*g^2) - (2*(c*d^2 - b*d*e + a*e^2)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])/Sqrt[e*f - d*g]])/(e^(5/2)*Sqrt[e*f - d*g])$

**Rubi in Sympy [A]** time = 80.0663, size = 107, normalized size = 0.92

$$\frac{2c(f+gx)^{3/2}}{3eg^2} + \frac{2\sqrt{f+gx}(beg - cdg - cef)}{e^2g^2} + \frac{2(ae^2 - bde + cd^2) \operatorname{atan}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{dg-ef}}\right)}{e^{5/2}\sqrt{dg-ef}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+b\*x+a)/(e\*x+d)/(g\*x+f)\*\*(1/2), x)

[Out]  $2*c*(f + g*x)^(3/2)/(3*e*g^2) + 2*\sqrt{f + g*x}*(b*e*g - c*d*g - c*e*f)/(e^2*g^2) + 2*(a*e^2 - b*d*e + c*d^2)*\operatorname{atan}(\sqrt{e}*\sqrt{f + g*x}/\sqrt{d*g - e*f})/(e^(5/2)*\sqrt{d*g - e*f})$

**Mathematica [A]** time = 0.158345, size = 104, normalized size = 0.9

$$\frac{2\sqrt{f+gx}(3beg + c(-3dg - 2ef + egx))}{3e^2g^2} - \frac{2(e(ae - bd) + cd^2) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{5/2}\sqrt{ef-dg}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)/((d + e\*x)\*Sqrt[f + g\*x]), x]

[Out]  $(2*Sqrt[f + g*x]*(3*b*e*g + c*(-2*e*f - 3*d*g + e*g*x)))/(3*e^2*g^2) - (2*(c*d^2 + e*(-b*d) + a*e)*ArcTanh[(Sqrt[e]*Sqrt[f + g*x])])/(e^(5/2)*Sqrt[e*f - d*g])$

]/Sqrt[e\*f - d\*g]]/(e^(5/2)\*Sqrt[e\*f - d\*g])

**Maple [A]** time = 0.014, size = 189, normalized size = 1.6

$$\frac{2c}{3eg^2}(gx+f)^{\frac{3}{2}} + 2\frac{b\sqrt{gx+f}}{eg} - 2\frac{cd\sqrt{gx+f}}{ge^2} - 2\frac{cf\sqrt{gx+f}}{eg^2} + 2\frac{a}{\sqrt{(dg-ef)e}} \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{(dg-ef)e}}\right) - 2\frac{bd}{e\sqrt{(dg-ef)e}} \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{(dg-ef)e}}\right) + 2\frac{cd^2}{e^2\sqrt{(dg-ef)e}} \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{(dg-ef)e}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)/(e\*x+d)/(g\*x+f)^(1/2),x)

[Out] 2/3\*c\*(g\*x+f)^(3/2)/e/g^2+2/g/e\*b\*(g\*x+f)^(1/2)-2/g\*c/e^2\*d\*(g\*x+f)^(1/2)-2/g^2\*c/e\*f\*(g\*x+f)^(1/2)+2/((d\*g-e\*f)\*e)^(1/2)\*arctan((g\*x+f)^(1/2)\*e/((d\*g-e\*f)\*e)^(1/2))\*a-2/e/((d\*g-e\*f)\*e)^(1/2)\*arctan((g\*x+f)^(1/2)\*e/((d\*g-e\*f)\*e)^(1/2))\*b\*d+2/e^2/((d\*g-e\*f)\*e)^(1/2)\*arctan((g\*x+f)^(1/2)\*e/((d\*g-e\*f)\*e)^(1/2))\*c\*d^2

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/((e\*x + d)\*sqrt(g\*x + f)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.284794, size = 1, normalized size = 0.01

$$\left[ \frac{3(cd^2 - bde + ae^2)g^2 \log\left(\frac{\sqrt{e^2f - deg}(egx + 2ef - dg) - 2(e^2f - deg)\sqrt{gx+f}}{ex+d}\right) + 2(cegx - 2cef - 3(cd - be)g)\sqrt{e^2f - deg}\sqrt{gx+f}}{3\sqrt{e^2f - de}ge^2g^2}, \frac{2\left(3(cd^2 - bde + ae^2)g^2 \arctan\left(-\frac{ef-dg}{\sqrt{-e^2f+deg}\sqrt{gx+f}}\right) - (cegx - 2cef - 3(cd - be)g)\sqrt{-e^2f + deg}\sqrt{gx+f}\right)}{3\sqrt{-e^2f + de}ge^2g^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/((e\*x + d)\*sqrt(g\*x + f)),x, algorithm="fricas")

[Out] [1/3\*(3\*(c\*d^2 - b\*d\*e + a\*e^2)\*g^2\*log((sqrt(e^2\*f - d\*e\*g)\*(e\*g\*x + 2\*e\*f - d\*g) - 2\*(e^2\*f - d\*e\*g)\*sqrt(g\*x + f))/(e\*x + d)) + 2\*(c\*e\*g\*x - 2\*c\*e\*f - 3\*(c\*d - b\*e)\*g)\*sqrt(e^2\*f - d\*e\*g)\*sqrt(g\*x + f)/(sqrt(e^2\*f - d\*e\*g)\*e^2\*g^2), -2/3\*(3\*(c\*d^2 - b\*d\*e + a\*e^2)\*g^2\*arctan(-(e\*f - d\*g)/(sqrt(-e^2\*f + d\*e\*g)\*sqrt(g\*x + f))) - (c\*e\*g\*x - 2\*c\*e\*f - 3\*(c\*d - b\*e)\*g)\*sqrt(-e^2\*f + d\*e\*g)\*sqrt(g\*x + f))/(sqrt(-e^2\*f + d\*e\*g)\*e^2\*g^2)]



**Sympy [A]** time = 30.853, size = 257, normalized size = 2.22

$$\frac{2c(f+gx)^{\frac{3}{2}}}{3eg^2} + \frac{2\sqrt{f+gx}(beg-cdg-cef)}{e^2g^2} - \frac{2(ae^2 - bde + cd^2)}{e^2} \left( \begin{array}{l} \frac{\operatorname{atan}\left(\frac{1}{\sqrt{\frac{e}{dg-ef}}\sqrt{f+gx}}\right)}{\sqrt{\frac{e}{dg-ef}}(dg-ef)} \quad \text{for } \frac{e}{dg-ef} > 0 \\ -\frac{\operatorname{acoth}\left(\frac{1}{\sqrt{-\frac{e}{dg-ef}}\sqrt{f+gx}}\right)}{\sqrt{-\frac{e}{dg-ef}}(dg-ef)} \quad \text{for } \frac{1}{f+gx} > -\frac{e}{dg-ef} \wedge \frac{e}{dg-ef} < 0 \\ -\frac{\operatorname{atanh}\left(\frac{1}{\sqrt{-\frac{e}{dg-ef}}\sqrt{f+gx}}\right)}{\sqrt{-\frac{e}{dg-ef}}(dg-ef)} \quad \text{for } \frac{e}{dg-ef} < 0 \wedge \frac{1}{f+gx} < -\frac{e}{dg-ef} \end{array} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)/(e\*x+d)/(g\*x+f)\*\*(1/2),x)

[Out] 2\*c\*(f + g\*x)\*\*(3/2)/(3\*e\*g\*\*2) - 2\*(a\*e\*\*2 - b\*d\*e + c\*d\*\*2)\*Pie  
 cewise((atan(1/(sqrt(e/(d\*g - e\*f))\*sqrt(f + g\*x)))/(sqrt(e/(d\*g  
 - e\*f))\*(d\*g - e\*f)), e/(d\*g - e\*f) > 0), (-acoth(1/(sqrt(-e/(d\*g  
 - e\*f))\*sqrt(f + g\*x)))/(sqrt(-e/(d\*g - e\*f))\*(d\*g - e\*f)), (e/(  
 d\*g - e\*f) < 0) & (1/(f + g\*x) > -e/(d\*g - e\*f))), (-atanh(1/(sqr  
 t(-e/(d\*g - e\*f))\*sqrt(f + g\*x)))/(sqrt(-e/(d\*g - e\*f))\*(d\*g - e\*  
 f)), (e/(d\*g - e\*f) < 0) & (1/(f + g\*x) < -e/(d\*g - e\*f))))/e\*\*2  
 + 2\*sqrt(f + g\*x)\*(b\*e\*g - c\*d\*g - c\*e\*f)/(e\*\*2\*g\*\*2)

**GIAC/XCAS [A]** time = 0.265168, size = 173, normalized size = 1.49

$$\frac{2(cd^2 - bde + ae^2) \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{dge-fe^2}}\right) e^{-2}}{\sqrt{dge - fe^2}} - \frac{2\left(3\sqrt{gx+fc}dg^5e - (gx+f)^{\frac{3}{2}}cg^4e^2 + 3\sqrt{gx+fc}fg^4e^2 - 3\sqrt{gx+fb}g^5e^2\right) e^{-3}}{3g^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/((e\*x + d)\*sqrt(g\*x + f)),x, algorithm="giac")

[Out] 2\*(c\*d^2 - b\*d\*e + a\*e^2)\*arctan(sqrt(g\*x + f)\*e/sqrt(d\*g\*e - f\*e  
 ^2))\*e^(-2)/sqrt(d\*g\*e - f\*e^2) - 2/3\*(3\*sqrt(g\*x + f)\*c\*d\*g^5\*e  
 - (g\*x + f)^(3/2)\*c\*g^4\*e^2 + 3\*sqrt(g\*x + f)\*c\*f\*g^4\*e^2 - 3\*sqr  
 t(g\*x + f)\*b\*g^5\*e^2)\*e^(-3)/g^6

$$3.824 \quad \int \frac{a+bx+cx^2}{(d+ex)^2\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=140

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(cd(4ef-3dg)-e(-aeg-bdg+2bef))}{e^{5/2}(ef-dg)^{3/2}} - \frac{\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{(d+ex)(ef-dg)} + \frac{2c\sqrt{f+gx}}{e^2g}$$

[Out] (2\*c\*Sqrt[f + g\*x])/(e^2\*g) - ((a + (d\*(c\*d - b\*e))/e^2)\*Sqrt[f + g\*x])/((e\*f - d\*g)\*(d + e\*x)) + ((c\*d\*(4\*e\*f - 3\*d\*g) - e\*(2\*b\*e\*f - b\*d\*g - a\*e\*g))\*ArcTanh[(Sqrt[e]\*Sqrt[f + g\*x])/Sqrt[e\*f - d\*g]])/(e^(5/2)\*(e\*f - d\*g)^(3/2))

**Rubi [A]** time = 0.602624, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(cd(4ef-3dg)-e(-aeg-bdg+2bef))}{e^{5/2}(ef-dg)^{3/2}} - \frac{\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{(d+ex)(ef-dg)} + \frac{2c\sqrt{f+gx}}{e^2g}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)/((d + e\*x)^2\*Sqrt[f + g\*x]), x]

[Out] (2\*c\*Sqrt[f + g\*x])/(e^2\*g) - ((a + (d\*(c\*d - b\*e))/e^2)\*Sqrt[f + g\*x])/((e\*f - d\*g)\*(d + e\*x)) + ((c\*d\*(4\*e\*f - 3\*d\*g) - e\*(2\*b\*e\*f - b\*d\*g - a\*e\*g))\*ArcTanh[(Sqrt[e]\*Sqrt[f + g\*x])/Sqrt[e\*f - d\*g]])/(e^(5/2)\*(e\*f - d\*g)^(3/2))

**Rubi in Sympy [A]** time = 128.764, size = 144, normalized size = 1.03

$$\frac{2c\sqrt{f+gx}}{e^2g} + \frac{g\sqrt{f+gx}(ae^2 - bde + cd^2)}{e^2(dg - ef)(dg - ef + e(f + gx))} + \frac{(ae^2g + bdeg - 2be^2f - 3cd^2g + 4cdef) \operatorname{atan}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{dg-ef}}\right)}{e^{5/2}(dg - ef)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+b\*x+a)/(e\*x+d)\*\*2/(g\*x+f)\*\*(1/2), x)

[Out] 2\*c\*sqrt(f + g\*x)/(e\*\*2\*g) + g\*sqrt(f + g\*x)\*(a\*e\*\*2 - b\*d\*e + c\*d\*\*2)/(e\*\*2\*(d\*g - e\*f)\*(d\*g - e\*f + e\*(f + g\*x))) + (a\*e\*\*2\*g + b\*d\*e\*g - 2\*b\*e\*\*2\*f - 3\*c\*d\*\*2\*g + 4\*c\*d\*e\*f)\*atan(sqrt(e)\*sqrt(f + g\*x)/sqrt(d\*g - e\*f))/(e\*\*(5/2)\*(d\*g - e\*f)\*\*(3/2))

**Mathematica [A]** time = 0.448304, size = 133, normalized size = 0.95

$$\frac{\sqrt{f+gx}\left(\frac{e(ae-bd)+cd^2}{(d+ex)(dg-ef)} + \frac{2c}{g}\right)}{e^2} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(e(-aeg-bdg+2bef)+cd(3dg-4ef))}{e^{5/2}(ef-dg)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)/((d + e\*x)^2\*Sqrt[f + g\*x]), x]

[Out]  $(\sqrt{f + gx} * ((2*c)/g + (c*d^2 + e*(-b*d) + a*e)) / ((-(e*f) + d*g) * (d + e*x))) / e^2 - ((c*d*(-4*e*f + 3*d*g) + e*(2*b*e*f - b*d*g - a*e*g)) * \text{ArcTanh}[\sqrt{e} * \sqrt{f + gx}] / \sqrt{e*f - d*g}) / (e^{5/2} * (e*f - d*g)^{3/2})$

**Maple [B]** time = 0.022, size = 371, normalized size = 2.7

$$\begin{aligned}
& 2 \frac{c\sqrt{gx+f}}{e^2g} + \frac{ag}{(dg-ef)(egx+dg)}\sqrt{gx+f} - \frac{bdg}{(dg-ef)e(egx+dg)}\sqrt{gx+f} \\
& + \frac{cd^2g}{e^2(dg-ef)(egx+dg)}\sqrt{gx+f} + \frac{ag}{dg-ef} \arctan\left(e\sqrt{gx+f} \frac{1}{\sqrt{(dg-ef)e}}\right) \frac{1}{\sqrt{(dg-ef)e}} \\
& + \frac{bdg}{(dg-ef)e} \arctan\left(e\sqrt{gx+f} \frac{1}{\sqrt{(dg-ef)e}}\right) \frac{1}{\sqrt{(dg-ef)e}} \\
& - 2 \frac{bf}{(dg-ef)\sqrt{(dg-ef)e}} \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{(dg-ef)e}}\right) \\
& - 3 \frac{cd^2g}{e^2(dg-ef)\sqrt{(dg-ef)e}} \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{(dg-ef)e}}\right) \\
& + 4 \frac{cdf}{(dg-ef)e\sqrt{(dg-ef)e}} \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{(dg-ef)e}}\right)
\end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)/(e*x+d)^2/(g*x+f)^(1/2),x)`

[Out]  $2*c*(g*x+f)^{1/2}/e^2/g+g/(d*g-e*f)*(g*x+f)^{1/2}/(e*g*x+d*g)^{a-g}/e/(d*g-e*f)*(g*x+f)^{1/2}/(e*g*x+d*g)^{b*d+g}/e^2/(d*g-e*f)*(g*x+f)^{1/2}/(e*g*x+d*g)^{c*d^2+g}/(d*g-e*f)/((d*g-e*f)*e)^{1/2}*\arctan((g*x+f)^{1/2}*e/((d*g-e*f)*e)^{1/2})*a+g/e/(d*g-e*f)/((d*g-e*f)*e)^{1/2}*\arctan((g*x+f)^{1/2}*e/((d*g-e*f)*e)^{1/2})*b*d-2/(d*g-e*f)/((d*g-e*f)*e)^{1/2}*\arctan((g*x+f)^{1/2}*e/((d*g-e*f)*e)^{1/2})*b*f-3*g/e^2/(d*g-e*f)/((d*g-e*f)*e)^{1/2}*\arctan((g*x+f)^{1/2}*e/((d*g-e*f)*e)^{1/2})*c*d^2+4/e/(d*g-e*f)/((d*g-e*f)*e)^{1/2}*\arctan((g*x+f)^{1/2}*e/((d*g-e*f)*e)^{1/2})*c*d*f$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)/((e*x + d)^2*sqrt(g*x + f)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.29409, size = 1, normalized size = 0.01

$$\left[ \frac{2(2cdef - (3cd^2 - bde + ae^2)g + 2(ce^2f - cdeg)x)\sqrt{e^2f - deg}\sqrt{gx+f} - (2(2cd^2e - bde^2)fg - (3cd^3 - bd^2e - ade^2))}{2(de^3fg - d^2e^2g^2 + (e^4fg - de^3g^2))} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/((e\*x + d)^2\*sqrt(g\*x + f)),x, algorithm="fricas")

[Out] [1/2\*(2\*(2\*c\*d\*e\*f - (3\*c\*d^2 - b\*d\*e + a\*e^2)\*g + 2\*(c\*e^2\*f - c\*d\*e\*g)\*x)\*sqrt(e^2\*f - d\*e\*g)\*sqrt(g\*x + f) - (2\*(2\*c\*d^2\*e - b\*d\*e^2)\*f\*g - (3\*c\*d^3 - b\*d^2\*e - a\*d\*e^2)\*g^2 + (2\*(2\*c\*d\*e^2 - b\*e^3)\*f\*g - (3\*c\*d^2\*e - b\*d\*e^2 - a\*e^3)\*g^2)\*x)\*log((sqrt(e^2\*f - d\*e\*g)\*(e\*g\*x + 2\*e\*f - d\*g) - 2\*(e^2\*f - d\*e\*g)\*sqrt(g\*x + f))/(e\*x + d)))/((d\*e^3\*f\*g - d^2\*e^2\*g^2 + (e^4\*f\*g - d\*e^3\*g^2)\*x)\*sqrt(e^2\*f - d\*e\*g)), ((2\*c\*d\*e\*f - (3\*c\*d^2 - b\*d\*e + a\*e^2)\*g + 2\*(c\*e^2\*f - c\*d\*e\*g)\*x)\*sqrt(-e^2\*f + d\*e\*g)\*sqrt(g\*x + f) + (2\*(2\*c\*d^2\*e - b\*d\*e^2)\*f\*g - (3\*c\*d^3 - b\*d^2\*e - a\*d\*e^2)\*g^2 + (2\*(2\*c\*d\*e^2 - b\*e^3)\*f\*g - (3\*c\*d^2\*e - b\*d\*e^2 - a\*e^3)\*g^2)\*x)\*arctan(-(e\*f - d\*g)/(sqrt(-e^2\*f + d\*e\*g)\*sqrt(g\*x + f)))/((d\*e^3\*f\*g - d^2\*e^2\*g^2 + (e^4\*f\*g - d\*e^3\*g^2)\*x)\*sqrt(-e^2\*f + d\*e\*g))]

**Sympy** [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)/(e\*x+d)\*\*2/(g\*x+f)\*\*(1/2),x)

[Out] Exception raised: ValueError

**GIAC/XCAS** [A] time = 0.266678, size = 236, normalized size = 1.69

$$\frac{2\sqrt{gx+fe}(-2)}{g} - \frac{(3cd^2g - 4cdf e - bdge + 2bfe^2 - age^2) \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{dge-fe^2}}\right)}{(dge^2 - fe^3)\sqrt{dge - fe^2}} + \frac{\sqrt{gx+fe}cd^2g - \sqrt{gx+fe}bdge + \sqrt{gx+fe}age^2}{(dge^2 - fe^3)(dg + (gx+f)e - fe)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/((e\*x + d)^2\*sqrt(g\*x + f)),x, algorithm="giac")

[Out] 2\*sqrt(g\*x + f)\*c\*e^(-2)/g - (3\*c\*d^2\*g - 4\*c\*d\*f\*e - b\*d\*g\*e + 2\*b\*f\*e^2 - a\*g\*e^2)\*arctan(sqrt(g\*x + f)\*e/sqrt(d\*g\*e - f\*e^2))/((d\*g\*e^2 - f\*e^3)\*sqrt(d\*g\*e - f\*e^2)) + (sqrt(g\*x + f)\*c\*d^2\*g - sqrt(g\*x + f)\*b\*d\*g\*e + sqrt(g\*x + f)\*a\*g\*e^2)/((d\*g\*e^2 - f\*e^3)\*(d\*g + (g\*x + f)\*e - f\*e))

$$3.825 \quad \int \frac{a+bx+cx^2}{(d+ex)^3\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=206

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right) (eg(-3aeg - bdg + 4bef) - c(3d^2g^2 - 8defg + 8e^2f^2))}{4e^{5/2}(ef - dg)^{5/2}} - \frac{\sqrt{f+gx}\left(a + \frac{d(cd-be)}{e^2}\right)}{2(d+ex)^2(ef-dg)} + \frac{\sqrt{f+gx}(cd(8ef - 5dg) - e(-3aeg - bdg + 4bef))}{4e^2(d+ex)(ef-dg)^2}$$

[Out]  $-\left(\left(a + \frac{d(c*d - b*e)}{e^2}\right) \sqrt{f + g*x}\right) / \left(2*(e*f - d*g)*(d + e*x)^2\right) + \left(\left(c*d*(8*e*f - 5*d*g) - e*(4*b*e*f - b*d*g - 3*a*e*g)\right) \sqrt{f + g*x}\right) / \left(4*e^2*(e*f - d*g)^2*(d + e*x)\right) + \left(\left(e*g*(4*b*e*f - b*d*g - 3*a*e*g) - c*(8*e^2*f^2 - 8*d*e*f*g + 3*d^2*g^2)\right) \operatorname{ArcTanh}\left[\frac{\sqrt{e}*\sqrt{f + g*x}}{\sqrt{e*f - d*g}}\right]\right) / \left(4*e^{5/2}*(e*f - d*g)^{5/2}\right)$

**Rubi [A]** time = 0.883772, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right) (eg(-3aeg - bdg + 4bef) - c(3d^2g^2 - 8defg + 8e^2f^2))}{4e^{5/2}(ef - dg)^{5/2}} - \frac{\sqrt{f+gx}\left(a + \frac{d(cd-be)}{e^2}\right)}{2(d+ex)^2(ef-dg)} + \frac{\sqrt{f+gx}(cd(8ef - 5dg) - e(-3aeg - bdg + 4bef))}{4e^2(d+ex)(ef-dg)^2}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\frac{a + b*x + c*x^2}{(d + e*x)^3 \sqrt{f + g*x}}, x\right]$

[Out]  $-\left(\left(a + \frac{d(c*d - b*e)}{e^2}\right) \sqrt{f + g*x}\right) / \left(2*(e*f - d*g)*(d + e*x)^2\right) + \left(\left(c*d*(8*e*f - 5*d*g) - e*(4*b*e*f - b*d*g - 3*a*e*g)\right) \sqrt{f + g*x}\right) / \left(4*e^2*(e*f - d*g)^2*(d + e*x)\right) + \left(\left(e*g*(4*b*e*f - b*d*g - 3*a*e*g) - c*(8*e^2*f^2 - 8*d*e*f*g + 3*d^2*g^2)\right) \operatorname{ArcTanh}\left[\frac{\sqrt{e}*\sqrt{f + g*x}}{\sqrt{e*f - d*g}}\right]\right) / \left(4*e^{5/2}*(e*f - d*g)^{5/2}\right)$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\operatorname{rubi\_integrate}\left(\frac{c*x^2 + b*x + a}{(e*x + d)^3 (g*x + f)^{1/2}}, x\right)$

[Out] Timed out

**Mathematica [A]** time = 0.638908, size = 191, normalized size = 0.93

$$\frac{\sqrt{f+gx} (2(ef - dg) (eae - bd) + cd^2) + (d + ex)(e(-3aeg - bdg + 4bef) + cd(5dg - 8ef))}{4e^2(d + ex)^2(ef - dg)^2} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right) (eg(3aeg + bdg - 4bef) + c(3d^2g^2 - 8defg + 8e^2f^2))}{4e^{5/2}(ef - dg)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)/((d + e\*x)^3\*Sqrt[f + g\*x]),x]

[Out] 
$$-(\text{Sqrt}[f + g*x] * (2 * (c*d^2 + e * (-b*d) + a*e)) * (e*f - d*g) + (c*d * (-8 * e*f + 5 * d*g) + e * (4 * b * e*f - b * d * g - 3 * a * e * g)) * (d + e * x)) / (4 * e^2 * (e*f - d*g)^2 * (d + e * x)^2) - ((e * g * (-4 * b * e*f + b * d * g + 3 * a * e * g) + c * (8 * e^2 * f^2 - 8 * d * e * f * g + 3 * d^2 * g^2)) * \text{ArcTanh}[\text{Sqrt}[e] * \text{Sqrt}[f + g*x]) / \text{Sqrt}[e * f - d * g]]) / (4 * e^{5/2} * (e*f - d*g)^{5/2})$$

**Maple [B]** time = 0.026, size = 538, normalized size = 2.6

$$2 \frac{1}{(e(gx+f) + dg - ef)^2} \left( \frac{1}{8} \frac{g(3ae^2g + bdeg - 4be^2f - 5cd^2g + 8decf)(gx+f)^{3/2}}{e(d^2g^2 - 2defg + e^2f^2)} + \frac{1}{8} \frac{(5ae^2g - bdeg - 4be^2f - 3cd^2g + 8decf)(gx+f)^{3/2}}{e^2(dg - ef)} \right) \\ + \frac{3ag^2}{4d^2g^2 - 8defg + 4e^2f^2} \arctan\left(e\sqrt{gx+f} \frac{1}{\sqrt{(dg-ef)e}}\right) \frac{1}{\sqrt{(dg-ef)e}} \\ + \frac{bdg^2}{(4d^2g^2 - 8defg + 4e^2f^2)e} \arctan\left(e\sqrt{gx+f} \frac{1}{\sqrt{(dg-ef)e}}\right) \frac{1}{\sqrt{(dg-ef)e}} \\ - \frac{bfg}{d^2g^2 - 2defg + e^2f^2} \arctan\left(e\sqrt{gx+f} \frac{1}{\sqrt{(dg-ef)e}}\right) \frac{1}{\sqrt{(dg-ef)e}} \\ + \frac{3cd^2g^2}{(4d^2g^2 - 8defg + 4e^2f^2)e^2} \arctan\left(e\sqrt{gx+f} \frac{1}{\sqrt{(dg-ef)e}}\right) \frac{1}{\sqrt{(dg-ef)e}} \\ - 2 \frac{cdfg}{e(d^2g^2 - 2defg + e^2f^2)\sqrt{(dg-ef)e}} \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{(dg-ef)e}}\right) \\ + 2 \frac{cf^2}{(d^2g^2 - 2defg + e^2f^2)\sqrt{(dg-ef)e}} \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{(dg-ef)e}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)/(e\*x+d)^3/(g\*x+f)^(1/2),x)

[Out] 
$$2 * (1/8 * g * (3 * a * e^2 * g + b * d * e * g - 4 * b * e^2 * f - 5 * c * d^2 * g + 8 * c * d * e * f) / e / (d^2 * g^2 - 2 * d * e * f * g + e^2 * f^2) * (g * x + f)^{3/2} + 1/8 * (5 * a * e^2 * g - b * d * e * g - 4 * b * e^2 * f - 3 * c * d^2 * g + 8 * c * d * e * f) / e^2 * g / (d * g - e * f) * (g * x + f)^{3/2}) / (e * (g * x + f) + d * g - e * f)^2 + 3/4 / (d^2 * g^2 - 2 * d * e * f * g + e^2 * f^2) / ((d * g - e * f) * e)^{1/2} * \arctan((g * x + f)^{1/2} * e / ((d * g - e * f) * e)^{1/2}) * a * g^2 + 1/4 / (d^2 * g^2 - 2 * d * e * f * g + e^2 * f^2) / e / ((d * g - e * f) * e)^{1/2} * \arctan((g * x + f)^{1/2} * e / ((d * g - e * f) * e)^{1/2}) * b * d * g^2 - 1 / (d^2 * g^2 - 2 * d * e * f * g + e^2 * f^2) / ((d * g - e * f) * e)^{1/2} * \arctan((g * x + f)^{1/2} * e / ((d * g - e * f) * e)^{1/2}) * b * f * g + 3/4 / (d^2 * g^2 - 2 * d * e * f * g + e^2 * f^2) / e^2 / ((d * g - e * f) * e)^{1/2} * \arctan((g * x + f)^{1/2} * e / ((d * g - e * f) * e)^{1/2}) * c * d^2 * g^2 - 2 / (d^2 * g^2 - 2 * d * e * f * g + e^2 * f^2) / e / ((d * g - e * f) * e)^{1/2} * \arctan((g * x + f)^{1/2} * e / ((d * g - e * f) * e)^{1/2}) * c * d * f * g + 2 / (d^2 * g^2 - 2 * d * e * f * g + e^2 * f^2) / ((d * g - e * f) * e)^{1/2} * \arctan((g * x + f)^{1/2} * e / ((d * g - e * f) * e)^{1/2}) * c * f^2$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/((e\*x + d)^3\*sqrt(g\*x + f)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.294196, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/((e\*x + d)^3\*sqrt(g\*x + f)),x, algorithm="fricas")

[Out] [1/8\*(2\*sqrt(e^2\*f - d\*e\*g)\*(2\*(3\*c\*d^2\*e - b\*d\*e^2 - a\*e^3)\*f - (3\*c\*d^3 + b\*d^2\*e - 5\*a\*d\*e^2)\*g + (4\*(2\*c\*d\*e^2 - b\*e^3)\*f - (5\*c\*d^2\*e - b\*d\*e^2 - 3\*a\*e^3)\*g)\*x)\*sqrt(g\*x + f) + (8\*c\*d^2\*e^2\*f^2 - 4\*(2\*c\*d^3\*e + b\*d^2\*e^2)\*f\*g + (3\*c\*d^4 + b\*d^3\*e + 3\*a\*d^2\*e^2)\*g^2 + (8\*c\*e^4\*f^2 - 4\*(2\*c\*d\*e^3 + b\*e^4)\*f\*g + (3\*c\*d^2\*e^2 + b\*d\*e^3 + 3\*a\*e^4)\*g^2)\*x^2 + 2\*(8\*c\*d\*e^3\*f^2 - 4\*(2\*c\*d^2\*e^2 + b\*d\*e^3)\*f\*g + (3\*c\*d^3\*e + b\*d^2\*e^2 + 3\*a\*d\*e^3)\*g^2)\*x)\*log((sqrt(e^2\*f - d\*e\*g)\*(e\*g\*x + 2\*e\*f - d\*g) - 2\*(e^2\*f - d\*e\*g)\*sqrt(g\*x + f))/(e\*x + d))/((d^2\*e^4\*f^2 - 2\*d^3\*e^3\*f\*g + d^4\*e^2\*g^2 + (e^6\*f^2 - 2\*d\*e^5\*f\*g + d^2\*e^4\*g^2)\*x^2 + 2\*(d\*e^5\*f^2 - 2\*d^2\*e^4\*f\*g + d^3\*e^3\*g^2)\*x)\*sqrt(e^2\*f - d\*e\*g)), 1/4\*(sqrt(-e^2\*f + d\*e\*g)\*(2\*(3\*c\*d^2\*e - b\*d\*e^2 - a\*e^3)\*f - (3\*c\*d^3 + b\*d^2\*e - 5\*a\*d\*e^2)\*g + (4\*(2\*c\*d\*e^2 - b\*e^3)\*f - (5\*c\*d^2\*e - b\*d\*e^2 - 3\*a\*e^3)\*g)\*x)\*sqrt(g\*x + f) - (8\*c\*d^2\*e^2\*f^2 - 4\*(2\*c\*d^3\*e + b\*d^2\*e^2)\*f\*g + (3\*c\*d^4 + b\*d^3\*e + 3\*a\*d^2\*e^2)\*g^2 + (8\*c\*e^4\*f^2 - 4\*(2\*c\*d\*e^3 + b\*e^4)\*f\*g + (3\*c\*d^2\*e^2 + b\*d\*e^3 + 3\*a\*e^4)\*g^2)\*x^2 + 2\*(8\*c\*d\*e^3\*f^2 - 4\*(2\*c\*d^2\*e^2 + b\*d\*e^3)\*f\*g + (3\*c\*d^3\*e + b\*d^2\*e^2 + 3\*a\*d\*e^3)\*g^2)\*x)\*arctan(-(e\*f - d\*g)/(sqrt(-e^2\*f + d\*e\*g)\*sqrt(g\*x + f)))/((d^2\*e^4\*f^2 - 2\*d^3\*e^3\*f\*g + d^4\*e^2\*g^2 + (e^6\*f^2 - 2\*d\*e^5\*f\*g + d^2\*e^4\*g^2)\*x^2 + 2\*(d\*e^5\*f^2 - 2\*d^2\*e^4\*f\*g + d^3\*e^3\*g^2)\*x)\*sqrt(-e^2\*f + d\*e\*g))]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)/(e\*x+d)\*\*3/(g\*x+f)\*\*(1/2),x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.273824, size = 504, normalized size = 2.45

$$\frac{(3cd^2g^2 - 8cdfge + bdg^2e + 8cf^2e^2 - 4bfge^2 + 3ag^2e^2) \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{dge-fe^2}}\right)}{4(d^2g^2e^2 - 2dfge^3 + f^2e^4)\sqrt{dge - fe^2}}$$

$$\frac{3\sqrt{gx+f}cd^3g^3 + 5(gx+f)^{\frac{3}{2}}cd^2g^2e - 11\sqrt{gx+f}cd^2fg^2e + \sqrt{gx+f}bd^2g^3e - 8(gx+f)^{\frac{3}{2}}cdfge^2 + 8\sqrt{gx+f}cdf^2ge^2 - 4(d^2g^2e^2 - 2dfge^3 + f^2e^4)\sqrt{dge - fe^2}}{4(d^2g^2e^2 - 2dfge^3 + f^2e^4)\sqrt{dge - fe^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/((e\*x + d)^3\*sqrt(g\*x + f)),x, algorithm="giac")

[Out] 1/4\*(3\*c\*d^2\*g^2 - 8\*c\*d\*f\*g\*e + b\*d\*g^2\*e + 8\*c\*f^2\*e^2 - 4\*b\*f\*g\*e^2 + 3\*a\*g^2\*e^2)\*arctan(sqrt(g\*x + f)\*e/sqrt(d\*g\*e - f\*e^2))/((d^2\*g^2\*e^2 - 2\*d\*f\*g\*e^3 + f^2\*e^4)\*sqrt(d\*g\*e - f\*e^2)) - 1/4\*(3\*sqrt(g\*x + f)\*c\*d^3\*g^3 + 5\*(g\*x + f)^(3/2)\*c\*d^2\*g^2\*e - 11\*

$$\frac{\sqrt{g^*x + f} * c^*d^2 * f^*g^2 * e + \sqrt{g^*x + f} * b^*d^2 * g^3 * e - 8 * (g^*x + f)^{(3/2)} * c^*d * f^*g^*e^2 + 8 * \sqrt{g^*x + f} * c^*d * f^2 * g^*e^2 - (g^*x + f)^{(3/2)} * b^*d * g^2 * e^2 + 3 * \sqrt{g^*x + f} * b^*d * f^*g^2 * e^2 - 5 * \sqrt{g^*x + f} * a^*d * g^3 * e^2 + 4 * (g^*x + f)^{(3/2)} * b^*f^*g^*e^3 - 4 * \sqrt{g^*x + f} * b^*f^2 * g^*e^3 - 3 * (g^*x + f)^{(3/2)} * a^*g^2 * e^3 + 5 * \sqrt{g^*x + f} * a^*f^*g^2 * e^3}{((d^2 * g^2 * e^2 - 2 * d * f * g * e^3 + f^2 * e^4) * (d * g + (g^*x + f) * e - f * e)^2)}$$



$$3.826 \quad \int \frac{(d+ex)^3(a+bx+cx^2)}{(f+gx)^{3/2}} dx$$

**Optimal.** Leaf size=285

$$\begin{aligned} & -\frac{2e(f+gx)^{5/2}(eg(-aeg-3bdg+4bef)-c(3d^2g^2-12defg+10e^2f^2))}{5g^6} \\ & +\frac{2(f+gx)^{3/2}(ef-dg)(3eg(-aeg-bdg+2bef)-c(d^2g^2-8defg+10e^2f^2))}{3g^6} \\ & +\frac{2(ef-dg)^3(ag^2-bfg+cf^2)}{g^6\sqrt{f+gx}} \\ & +\frac{2\sqrt{f+gx}(ef-dg)^2(cf(5ef-2dg)-g(-3aeg-bdg+4bef))}{g^6} \\ & -\frac{2e^2(f+gx)^{7/2}(-beg-3cdg+5cef)}{7g^6} +\frac{2ce^3(f+gx)^{9/2}}{9g^6} \end{aligned}$$

[Out]  $(2*(e*f - d*g)^3*(c*f^2 - b*f*g + a*g^2))/(g^6*\text{Sqrt}[f + g*x]) + (2*(e*f - d*g)^2*(c*f*(5*e*f - 2*d*g) - g*(4*b*e*f - b*d*g - 3*a*e*g))*\text{Sqrt}[f + g*x])/g^6 + (2*(e*f - d*g)*(3*e*g*(2*b*e*f - b*d*g - a*e*g) - c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^{(3/2)})/(3*g^6) - (2*e*(e*g*(4*b*e*f - 3*b*d*g - a*e*g) - c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^{(5/2)})/(5*g^6) - (2*e^2*(5*c*e*f - 3*c*d*g - b*e*g)*(f + g*x)^{(7/2)})/(7*g^6) + (2*c*e^3*(f + g*x)^{(9/2)})/(9*g^6)$

**Rubi [A]** time = 1.00847, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$

$$\begin{aligned} & -\frac{2e(f+gx)^{5/2}(eg(-aeg-3bdg+4bef)-c(3d^2g^2-12defg+10e^2f^2))}{5g^6} \\ & +\frac{2(f+gx)^{3/2}(ef-dg)(3eg(-aeg-bdg+2bef)-c(d^2g^2-8defg+10e^2f^2))}{3g^6} \\ & +\frac{2(ef-dg)^3(ag^2-bfg+cf^2)}{g^6\sqrt{f+gx}} \\ & +\frac{2\sqrt{f+gx}(ef-dg)^2(cf(5ef-2dg)-g(-3aeg-bdg+4bef))}{g^6} \\ & -\frac{2e^2(f+gx)^{7/2}(-beg-3cdg+5cef)}{7g^6} +\frac{2ce^3(f+gx)^{9/2}}{9g^6} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*(a + b\*x + c\*x^2))/(f + g\*x)^(3/2), x]

[Out]  $(2*(e*f - d*g)^3*(c*f^2 - b*f*g + a*g^2))/(g^6*\text{Sqrt}[f + g*x]) + (2*(e*f - d*g)^2*(c*f*(5*e*f - 2*d*g) - g*(4*b*e*f - b*d*g - 3*a*e*g))*\text{Sqrt}[f + g*x])/g^6 + (2*(e*f - d*g)*(3*e*g*(2*b*e*f - b*d*g - a*e*g) - c*(10*e^2*f^2 - 8*d*e*f*g + d^2*g^2))*(f + g*x)^{(3/2)})/(3*g^6) - (2*e*(e*g*(4*b*e*f - 3*b*d*g - a*e*g) - c*(10*e^2*f^2 - 12*d*e*f*g + 3*d^2*g^2))*(f + g*x)^{(5/2)})/(5*g^6) - (2*e^2*(5*c*e*f - 3*c*d*g - b*e*g)*(f + g*x)^{(7/2)})/(7*g^6) + (2*c*e^3*(f + g*x)^{(9/2)})/(9*g^6)$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**3*(c*x**2+b*x+a)/(g*x+f)**(3/2),x)`

[Out] Timed out

**Mathematica [A]** time = 0.752229, size = 406, normalized size = 1.42

$$2(9g(7ag(-5d^3g^3 + 15d^2eg^2(2f + gx) + 5de^2g(-8f^2 - 4fgx + g^2x^2) + e^3(16f^3 + 8f^2gx - 2fg^2x^2 + g^3x^3)) + b(35d^3g^3($$

Antiderivative was successfully verified.

[In] `Integrate[((d + e*x)^3*(a + b*x + c*x^2))/(f + g*x)^(3/2),x]`

[Out]  $(2*(c*(105*d^3*g^3*(-8*f^2 - 4*f*g*x + g^2*x^2) + 189*d^2*e*g^2*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3) + 27*d*e^2*g*(-128*f^4 - 64*f^3*g*x + 16*f^2*g^2*x^2 - 8*f*g^3*x^3 + 5*g^4*x^4) + 5*e^3*(256*f^5 + 128*f^4*g*x - 32*f^3*g^2*x^2 + 16*f^2*g^3*x^3 - 10*f*g^4*x^4 + 7*g^5*x^5)) + 9*g*(7*a*g*(-5*d^3*g^3 + 15*d^2*e*g^2*(2*f + g*x) + 5*d*e^2*g*(-8*f^2 - 4*f*g*x + g^2*x^2) + e^3*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3)) + b*(35*d^3*g^3*(2*f + g*x) + 35*d^2*e*g^2*(-8*f^2 - 4*f*g*x + g^2*x^2) + 21*d*e^2*g*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3) + e^3*(-128*f^4 - 64*f^3*g*x + 16*f^2*g^2*x^2 - 8*f*g^3*x^3 + 5*g^4*x^4)))))/(315*g^6*sqrt[f + g*x])$

**Maple [B]** time = 0.01, size = 540, normalized size = 1.9

$$-70e^3cx^5g^5 - 90be^3g^5x^4 - 270cde^2g^5x^4 + 100ce^3fg^4x^4 - 126ae^3g^5x^3 - 378bde^2g^5x^3 + 144be^3fg^4x^3 - 378cd^2eg^5x^3 +$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^3*(c*x^2+b*x+a)/(g*x+f)^(3/2),x)`

[Out]  $-2/315/(g*x+f)^{1/2}*(-35*c*e^3*g^5*x^5-45*b*e^3*g^5*x^4-135*c*d*e^2*g^5*x^4+50*c*e^3*f*g^4*x^4-63*a*e^3*g^5*x^3-189*b*d*e^2*g^5*x^3+72*b*e^3*f*g^4*x^3-189*c*d^2*e*g^5*x^3+216*c*d*e^2*f*g^4*x^3-80*c*e^3*f^2*g^3*x^3-315*a*d*e^2*g^5*x^2+126*a*e^3*f*g^4*x^2-315*b*d^2*e*g^5*x^2+378*b*d*e^2*f*g^4*x^2-144*b*e^3*f^2*g^3*x^2-105*c*d^3*g^5*x^2+378*c*d^2*e*f*g^4*x^2-432*c*d*e^2*f^2*g^3*x^2+160*c*e^3*f^3*g^2*x^2-945*a*d^2*e*g^5*x+1260*a*d*e^2*f*g^4*x-504*a*e^3*f^2*g^3*x-315*b*d^3*g^5*x+1260*b*d^2*e*f*g^4*x-1512*b*d*e^2*f^2*g^3*x+576*b*e^3*f^3*g^2*x+420*c*d^3*f*g^4*x-1512*c*d^2*e*f^2*g^3*x+1728*c*d*e^2*f^3*g^2*x-640*c*e^3*f^4*g*x+315*a*d^3*g^5-1890*a*d^2*e*f*g^4+2520*a*d*e^2*f^2*g^3-1008*a*e^3*f^3*g^2-630*b*d^3*f*g^4+2520*b*d^2*e*f^2*g^3-3024*b*d*e^2*f^3*g^2+1152*b*e^3*f^4*g+840*c*d^3*f^2*g^3-3024*c*d^2*e*f^3*g^2+3456*c*d*e^2*f^4*g-1280*c*e^3*f^5)/g^6$

**Maxima [A]** time = 0.69667, size = 590, normalized size = 2.07

$$2\left(\frac{35(gx+f)^{\frac{9}{2}}ce^3-45(5ce^3f-(3cde^2+be^3)g)(gx+f)^{\frac{7}{2}}+63(10ce^3f^2-4(3cde^2+be^3)fg+(3cd^2e+3bde^2+ae^3)g^2)(gx+f)^{\frac{5}{2}}-105(10ce^3f^3-6(3cde^2+be^3)fg^2+3cd^2e^2+3bde^2+ae^3)g^2}{(gx+f)^{\frac{5}{2}}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)^3/(g\*x + f)^(3/2),x, algorithm="maxima")

[Out]  $\frac{2}{315} \left( (35(gx + f)^{9/2} c^3 e^3 - 45(5c^3 e^3 f - (3cd^2 e^2 + b^3 e^3)g) (gx + f)^{7/2} + 63(10c^3 e^3 f^2 - 4(3cd^2 e^2 + b^3 e^3)f^2 g + (3cd^2 e^2 + 3b^3 d^2 e^2 + a^3 e^3)g^2) (gx + f)^{5/2} - 105(10c^3 e^3 f^3 - 6(3cd^2 e^2 + b^3 e^3)f^2 g + 3(3cd^2 e^2 + 3b^3 d^2 e^2 + a^3 e^3)f^2 g^2 - (cd^3 + 3b^3 d^2 e^2 + 3a^3 d^2 e^2)g^3) (gx + f)^{3/2} + 315(5c^3 e^3 f^4 - 4(3cd^2 e^2 + b^3 e^3)f^3 g + 3(3cd^2 e^2 + 3b^3 d^2 e^2 + a^3 e^3)f^2 g^2 - 2(cd^3 + 3b^3 d^2 e^2 + 3a^3 d^2 e^2)f^2 g^3 + (b^3 d^3 + 3a^3 d^2 e^2)g^4) \sqrt{gx + f} \right) / g^5 + 315(c^3 e^3 f^5 - a^3 d^3 g^5 - (3cd^2 e^2 + b^3 e^3)f^4 g + (3cd^2 e^2 + 3b^3 d^2 e^2 + a^3 e^3)f^3 g^2 - (cd^3 + 3b^3 d^2 e^2 + 3a^3 d^2 e^2)f^2 g^3 + (b^3 d^3 + 3a^3 d^2 e^2)f^2 g^4) / (\sqrt{gx + f} g^5) / g$

**Fricas [A]** time = 0.27232, size = 578, normalized size = 2.03

$$\frac{2(35ce^3g^5x^5 + 1280ce^3f^5 - 315ad^3g^5 - 1152(3cde^2 + be^3)f^4g + 1008(3cd^2e + 3bde^2 + ae^3)f^3g^2 - 840(cd^3 + 3bd^2e + 3ad^2e^2))}{(gx + f)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)^3/(g\*x + f)^(3/2),x, algorithm="fricas")

[Out]  $\frac{2}{315} (35c^3e^3g^5x^5 + 1280c^3e^3f^5 - 315a^3d^3g^5 - 1152(3cd^2e^2 + b^3e^3)f^4g + 1008(3cd^2e + 3bde^2 + ae^3)f^3g^2 - 840(cd^3 + 3bd^2e + 3ad^2e^2)) \sqrt{gx + f} + (80c^3e^3f^2g^3 - 72(3cd^2e^2 + b^3e^3)f^2g^4 + 63(3cd^2e^2 + 3b^3d^2e^2 + a^3e^3)g^5)x^4 + (160c^3e^3f^3g^4 - 144(3cd^2e^2 + b^3e^3)f^2g^3 + 126(3cd^2e^2 + 3b^3d^2e^2 + a^3e^3)f^2g^4 - 105(cd^3 + 3b^3d^2e^2 + 3a^3d^2e^2)g^5)x^3 + (640c^3e^3f^4g - 576(3cd^2e^2 + b^3e^3)f^3g^2 + 504(3cd^2e^2 + 3b^3d^2e^2 + a^3e^3)f^2g^3 - 420(cd^3 + 3b^3d^2e^2 + 3a^3d^2e^2)f^2g^4 + 315(b^3d^3 + 3a^3d^2e^2)g^5)x) / (\sqrt{gx + f} g^6)$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^3 (a + bx + cx^2)}{(f + gx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(c\*x\*\*2+b\*x+a)/(g\*x+f)\*\*(3/2),x)

[Out] Integral((d + e\*x)\*\*3\*(a + b\*x + c\*x\*\*2)/(f + g\*x)\*\*(3/2), x)

**GIAC/XCAS [A]** time = 0.276629, size = 903, normalized size = 3.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)^3/(g\*x + f)^(3/2),x, algorithm="giac")

[Out]  $-2(cd^3f^2g^3 - b^3d^3f^2g^4 + a^3d^3g^5 - 3cd^2f^2g^3e + 3b^3d^2f^2g^3e - 3a^3d^2f^2g^4e + 3cd^2f^4g^2e^2 - 3b^3d^2f^4g^2e^2 - 3a^3d^2f^4g^2e^2) \sqrt{gx + f} + (3cd^2f^2g^3e - 3b^3d^2f^2g^3e - 3a^3d^2f^2g^4e + 3cd^2f^4g^2e^2 - 3b^3d^2f^4g^2e^2 - 3a^3d^2f^4g^2e^2) \sqrt{gx + f} / g^5$

$$\begin{aligned}
& 3g^2e^2 + 3adf^2g^3e^2 - cf^5e^3 + bf^4g^3e^3 - af^3g^2e^3) / (\sqrt{gx+f}g^6) + 2/315(105(gx+f)^{3/2}cd^3g^5 \\
& 51 - 630\sqrt{gx+f}c^d^3f^g^51 + 315\sqrt{gx+f}b^d^3g^5 \\
& 2 + 189(gx+f)^{5/2}c^d^2g^50e - 945(gx+f)^{3/2}c^d^2f^g^50e + 2835\sqrt{gx+f}c^d^2f^2g^50e + 315(gx+f)^{3/2} \\
& b^d^2g^51e - 1890\sqrt{gx+f}b^d^2f^g^51e + 945\sqrt{gx+f}a^d^2g^52e + 135(gx+f)^{7/2}c^d^g^49e^2 - 756(gx+f)^{5/2}c^d^f^g^49e^2 + 1890(gx+f)^{3/2}c^d^f^2g^49e^2 - 3780\sqrt{gx+f}c^d^f^3g^49e^2 + 189(gx+f)^{5/2}b^d^g^50e^2 - 945(gx+f)^{3/2}b^d^f^g^50e^2 + 2835\sqrt{gx+f}b^d^f^2g^50e^2 + 315(gx+f)^{3/2}a^d^g^51e^2 - 1890\sqrt{gx+f}a^d^f^g^51e^2 + 35(gx+f)^{9/2}c^g^48e^3 - 225(gx+f)^{7/2}c^f^g^48e^3 + 630(gx+f)^{5/2}c^f^2g^48e^3 - 1050(gx+f)^{3/2}c^f^3g^48e^3 + 1575\sqrt{gx+f}c^f^4g^48e^3 + 45(gx+f)^{7/2}b^g^49e^3 - 252(gx+f)^{5/2}b^f^g^49e^3 + 630(gx+f)^{3/2}b^f^2g^49e^3 - 1260\sqrt{gx+f}b^f^3g^49e^3 + 63(gx+f)^{5/2}a^g^50e^3 - 315(gx+f)^{3/2}a^f^g^50e^3 + 945\sqrt{gx+f}a^f^2g^50e^3) / g^54
\end{aligned}$$

$$3.827 \quad \int \frac{(d+ex)^2(a+bx+cx^2)}{(f+gx)^{3/2}} dx$$

**Optimal.** Leaf size=210

$$\frac{2(f+gx)^{3/2}(eg(-aeg-2bdg+3bef)-c(d^2g^2-6defg+6e^2f^2))}{3g^5} - \frac{2(e^2f-dg)^2(ag^2-bfg+cf^2)}{g^5\sqrt{f+gx}} - \frac{2\sqrt{f+gx}(ef-dg)(2cf(2ef-dg)-g(-2aeg-bdg+3bef))}{g^5} - \frac{2e(f+gx)^{5/2}(-beg-2cdg+4cef)}{5g^5} + \frac{2ce^2(f+gx)^{7/2}}{7g^5}$$

[Out]  $(-2*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2))/(g^5*\text{Sqrt}[f + g*x]) - (2*(e*f - d*g)*(2*c*f*(2*e*f - d*g) - g*(3*b*e*f - b*d*g - 2*a*e*g))*\text{Sqrt}[f + g*x])/g^5 - (2*(e*g*(3*b*e*f - 2*b*d*g - a*e*g) - c*(6*e^2*f^2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^{(3/2)})/(3*g^5) - (2*e*(4*c*e*f - 2*c*d*g - b*e*g)*(f + g*x)^{(5/2)})/(5*g^5) + (2*c*e^2*(f + g*x)^{(7/2)})/(7*g^5)$

**Rubi [A]** time = 0.708561, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$

$$\frac{2(f+gx)^{3/2}(eg(-aeg-2bdg+3bef)-c(d^2g^2-6defg+6e^2f^2))}{3g^5} - \frac{2(e^2f-dg)^2(ag^2-bfg+cf^2)}{g^5\sqrt{f+gx}} - \frac{2\sqrt{f+gx}(ef-dg)(2cf(2ef-dg)-g(-2aeg-bdg+3bef))}{g^5} - \frac{2e(f+gx)^{5/2}(-beg-2cdg+4cef)}{5g^5} + \frac{2ce^2(f+gx)^{7/2}}{7g^5}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^2\*(a + b\*x + c\*x^2))/(f + g\*x)^(3/2), x]

[Out]  $(-2*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2))/(g^5*\text{Sqrt}[f + g*x]) - (2*(e*f - d*g)*(2*c*f*(2*e*f - d*g) - g*(3*b*e*f - b*d*g - 2*a*e*g))*\text{Sqrt}[f + g*x])/g^5 - (2*(e*g*(3*b*e*f - 2*b*d*g - a*e*g) - c*(6*e^2*f^2 - 6*d*e*f*g + d^2*g^2))*(f + g*x)^{(3/2)})/(3*g^5) - (2*e*(4*c*e*f - 2*c*d*g - b*e*g)*(f + g*x)^{(5/2)})/(5*g^5) + (2*c*e^2*(f + g*x)^{(7/2)})/(7*g^5)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2ce^2(f+gx)^{\frac{7}{2}}}{7g^5} + \frac{2e(f+gx)^{\frac{5}{2}}(beg+2cdg-4cef)}{5g^5} + \frac{2(dg-ef)(2aeg^2+bdg^2-3befg-2cdfg+4cef^2)\int^{\sqrt{f+gx}}\frac{1}{g^4}dx}{g} + \frac{2(f+gx)^{\frac{3}{2}}(ae^2g^2+2bdeg^2-3be^2fg+cd^2g^2-6cdefg+6ce^2f^2)}{3g^5} - \frac{2(dg-ef)^2(ag^2-bfg+cf^2)}{g^5\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*2\*(c\*x\*\*2+b\*x+a)/(g\*x+f)\*\*(3/2), x)

```
[Out] 2*c*e**2*(f + g*x)**(7/2)/(7*g**5) + 2*e*(f + g*x)**(5/2)*(b*e*g
+ 2*c*d*g - 4*c*e*f)/(5*g**5) + 2*(d*g - e*f)*(2*a*e*g**2 + b*d*g
**2 - 3*b*e*f*g - 2*c*d*f*g + 4*c*e*f**2)*Integral(g**(-4), (x, s
qrt(f + g*x)))/g + 2*(f + g*x)**(3/2)*(a*e**2*g**2 + 2*b*d*e*g**2
- 3*b*e**2*f*g + c*d**2*g**2 - 6*c*d*e*f*g + 6*c*e**2*f**2)/(3*g
**5) - 2*(d*g - e*f)**2*(a*g**2 - b*f*g + c*f**2)/(g**5*sqrt(f +
g*x))
```

**Mathematica [A]** time = 0.342102, size = 252, normalized size = 1.2

$$\frac{2(7g(5ag(-3d^2g^2 + 6deg(2f + gx)) + e^2(-8f^2 - 4fgx + g^2x^2)) + b(15d^2g^2(2f + gx) + 10deg(-8f^2 - 4fgx + g^2x^2) + 3e^2(-8f^2 - 4fgx + g^2x^2))}{g^5\sqrt{f + gx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^2*(a + b*x + c*x^2))/(f + g*x)^(3/2), x]
```

```
[Out] (2*(c*(35*d^2*g^2*(-8*f^2 - 4*f*g*x + g^2*x^2) + 42*d*e*g*(16*f^3
+ 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3) - 3*e^2*(128*f^4 + 64*f^3*g
*x - 16*f^2*g^2*x^2 + 8*f*g^3*x^3 - 5*g^4*x^4)) + 7*g*(5*a*g*(-3*
d^2*g^2 + 6*d*e*g*(2*f + g*x) + e^2*(-8*f^2 - 4*f*g*x + g^2*x^2))
+ b*(15*d^2*g^2*(2*f + g*x) + 10*d*e*g*(-8*f^2 - 4*f*g*x + g^2*x
^2) + 3*e^2*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3))))/(105
*g^5*Sqrt[f + g*x])
```

**Maple [A]** time = 0.011, size = 315, normalized size = 1.5

$$\frac{-30e^2cx^4g^4 - 42be^2g^4x^3 - 84cdeg^4x^3 + 48ce^2fg^3x^3 - 70ae^2g^4x^2 - 140bdeg^4x^2 + 84be^2fg^3x^2 - 70cd^2g^4x^2 + 168cde^2fg^3x^2 - 105d^2g^4x^2 + 105d^2g^4x^2}{g^5\sqrt{f + gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^2*(c*x^2+b*x+a)/(g*x+f)^(3/2), x)
```

```
[Out] -2/105/(g*x+f)^(1/2)*(-15*c*e^2*g^4*x^4-21*b*e^2*g^4*x^3-42*c*d*e
*g^4*x^3+24*c*e^2*f*g^3*x^3-35*a*e^2*g^4*x^2-70*b*d*e*g^4*x^2+42*
b*e^2*f*g^3*x^2-35*c*d^2*g^4*x^2+84*c*d*e*f*g^3*x^2-48*c*e^2*f^2*
g^2*x^2-210*a*d*e*g^4*x+140*a*e^2*f*g^3*x-105*b*d^2*g^4*x+280*b*d
*e*f*g^3*x-168*b*e^2*f^2*g^2*x+140*c*d^2*f*g^3*x-336*c*d*e*f^2*g^
2*x+192*c*e^2*f^3*g*x+105*a*d^2*g^4-420*a*d*e*f*g^3+280*a*e^2*f^2
*g^2-210*b*d^2*f*g^3+560*b*d*e*f^2*g^2-336*b*e^2*f^3*g+280*c*d^2*
f^2*g^2-672*c*d*e*f^3*g+384*c*e^2*f^4)/g^5
```

**Maxima [A]** time = 0.698885, size = 363, normalized size = 1.73

$$\frac{2\left(\frac{15(gx+f)^{\frac{7}{2}}ce^2-21(4ce^2f-(2cde+be^2)g)(gx+f)^{\frac{5}{2}}+35(6ce^2f^2-3(2cde+be^2)fg+(cd^2+2bde+ae^2)g^2)(gx+f)^{\frac{3}{2}}-105(4ce^2f^3-3(2cde+be^2)f^2g+2(cd^2+2bde+ae^2)g^2)}{g^4}\right)}{105g}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)*(e*x + d)^2/(g*x + f)^(3/2), x, algorithm="maxima")
```

```
[Out] 2/105*((15*(g*x + f)^(7/2)*c*e^2 - 21*(4*c*e^2*f - (2*c*d*e + b*e
^2)*g)*(g*x + f)^(5/2) + 35*(6*c*e^2*f^2 - 3*(2*c*d*e + b*e^2)*f*
g + (c*d^2 + 2*b*d*e + a*e^2)*g^2)*(g*x + f)^(3/2) - 105*(4*c*e^2
*f^3 - 3*(2*c*d*e + b*e^2)*f^2*g + 2*(c*d^2 + 2*b*d*e + a*e^2)*f*
```

$$\frac{g^2 - (b^2 d^2 + 2 a d^2 e) g^3 \sqrt{g x + f}}{g^4} - \frac{105 (c^2 e^2 f^4 + a^2 d^2 g^4 - (2 c^2 d^2 e + b^2 e^2) f^3 g + (c^2 d^2 + 2 b^2 d^2 e + a^2 e^2) f^2 g^2 - (b^2 d^2 + 2 a d^2 e) f g^3) / (\sqrt{g x + f} g^4)}{g}$$

**Fricas [A]** time = 0.27179, size = 350, normalized size = 1.67

$$\frac{2 (15 c e^2 g^4 x^4 - 384 c e^2 f^4 - 105 a d^2 g^4 + 336 (2 c d e + b e^2) f^3 g - 280 (c d^2 + 2 b d e + a e^2) f^2 g^2 + 210 (b d^2 + 2 a d e) f g^3 - 3 (8$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)^2/(g\*x + f)^(3/2),x, algorithm="fricas")

[Out]  $\frac{2/105 (15 c^2 e^2 g^4 x^4 - 384 c^2 e^2 f^4 - 105 a^2 d^2 g^4 + 336 (2 c^2 d^2 e + b^2 e^2) f^3 g - 280 (c^2 d^2 + 2 b^2 d^2 e + a^2 e^2) f^2 g^2 + 210 (b^2 d^2 + 2 a d^2 e) f g^3 - 3 (8 c^2 e^2 f^3 g^3 - 7 (2 c^2 d^2 e + b^2 e^2) g^4) x^3 + (48 c^2 e^2 f^2 g^2 - 42 (2 c^2 d^2 e + b^2 e^2) f g^3 + 35 (c^2 d^2 + 2 b^2 d^2 e + a^2 e^2) g^4) x^2 - (192 c^2 e^2 f^3 g - 168 (2 c^2 d^2 e + b^2 e^2) f^2 g^2 + 140 (c^2 d^2 + 2 b^2 d^2 e + a^2 e^2) f g^3 - 105 (b^2 d^2 + 2 a d^2 e) g^4) x}{(\sqrt{g x + f} g^5)}$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^2 (a + bx + cx^2)}{(f + gx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2\*(c\*x\*\*2+b\*x+a)/(g\*x+f)\*\*(3/2),x)

[Out] Integral((d + e\*x)\*\*2\*(a + b\*x + c\*x\*\*2)/(f + g\*x)\*\*(3/2), x)

**GIAC/XCAS [A]** time = 0.272074, size = 545, normalized size = 2.6

$$\frac{2 (c d^2 f^2 g^2 - b d^2 f g^3 + a d^2 g^4 - 2 c d f^3 g e + 2 b d f^2 g^2 e - 2 a d f g^3 e + c f^4 e^2 - b f^3 g e^2 + a f^2 g^2 e^2)}{\sqrt{g x + f} g^5} + \frac{2 \left( 35 (g x + f)^{\frac{3}{2}} c d^2 g^{32} - 210 \sqrt{g x + f} c d^2 f g^{32} + 105 \sqrt{g x + f} b d^2 g^{33} + 42 (g x + f)^{\frac{5}{2}} c d g^{31} e - 210 (g x + f)^{\frac{3}{2}} c d f g^{31} e + 630 \sqrt{g x + f} c d^2 f^2 g^{31} e + 70 (g x + f)^{\frac{3}{2}} b^2 d^2 g^3 2^2 e - 420 \sqrt{g x + f} b^2 d^2 f^2 g^{32} e + 210 \sqrt{g x + f} a^2 d^2 g^{33} e + 15 (g x + f)^{\frac{7}{2}} c^2 g^{30} e^2 - 84 (g x + f)^{\frac{5}{2}} c^2 f^2 g^{30} e^2 + 210 (g x + f)^{\frac{3}{2}} c^2 f^2 g^{30} e^2 - 420 \sqrt{g x + f} c^2 f^3 g^{30} e^2 + 21 (g x + f)^{\frac{5}{2}} b^2 g^{31} e^2 - 105 (g x + f)^{\frac{3}{2}} b^2 f^2 g^{31} e^2 + 315 \sqrt{g x + f} b^2 f^2 g^{31} e^2 + 35 (g x + f)^{\frac{3}{2}} a^2 g^{32} e^2 - 210 \sqrt{g x + f} a^2 f^2 g^{32} e^2 \right)}{g^{35}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)^2/(g\*x + f)^(3/2),x, algorithm="giac")

[Out]  $-2 (c^2 d^2 f^2 g^2 - b^2 d^2 f^2 g^3 + a^2 d^2 g^4 - 2 c^2 d^2 f^3 g^3 e + 2 b^2 d^2 f^2 g^2 e - 2 a^2 d^2 f^2 g^3 e + c^2 f^4 e^2 - b^2 f^3 g^3 e^2 + a^2 f^2 g^2 e^2) / (\sqrt{g x + f} g^5) + 2/105 (35 (g x + f)^{\frac{3}{2}} c^2 d^2 g^{32} - 210 \sqrt{g x + f} c^2 d^2 f g^{32} + 105 \sqrt{g x + f} b^2 d^2 g^{33} + 42 (g x + f)^{\frac{5}{2}} c d g^{31} e - 210 (g x + f)^{\frac{3}{2}} c d f g^{31} e + 630 \sqrt{g x + f} c d^2 f^2 g^{31} e + 70 (g x + f)^{\frac{3}{2}} b^2 d^2 g^3 2^2 e - 420 \sqrt{g x + f} b^2 d^2 f^2 g^{32} e + 210 \sqrt{g x + f} a^2 d^2 g^{33} e + 15 (g x + f)^{\frac{7}{2}} c^2 g^{30} e^2 - 84 (g x + f)^{\frac{5}{2}} c^2 f^2 g^{30} e^2 + 210 (g x + f)^{\frac{3}{2}} c^2 f^2 g^{30} e^2 - 420 \sqrt{g x + f} c^2 f^3 g^{30} e^2 + 21 (g x + f)^{\frac{5}{2}} b^2 g^{31} e^2 - 105 (g x + f)^{\frac{3}{2}} b^2 f^2 g^{31} e^2 + 315 \sqrt{g x + f} b^2 f^2 g^{31} e^2 + 35 (g x + f)^{\frac{3}{2}} a^2 g^{32} e^2 - 210 \sqrt{g x + f} a^2 f^2 g^{32} e^2) / g^{35}$

$$3.828 \quad \int \frac{(d+ex)(a+bx+cx^2)}{(f+gx)^{3/2}} dx$$

**Optimal.** Leaf size=135

$$\frac{2(ef-dg)(ag^2-bfg+cf^2)}{g^4\sqrt{f+gx}} + \frac{2\sqrt{f+gx}(cf(3ef-2dg)-g(-aeg-bdg+2bef))}{g^4} - \frac{2(f+gx)^{3/2}(-beg-cdg+3cef)}{3g^4} + \frac{2ce(f+gx)^{5/2}}{5g^4}$$

[Out]  $(2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2))/(g^4*\text{Sqrt}[f + g*x]) + (2*(c*f*(3*e*f - 2*d*g) - g*(2*b*e*f - b*d*g - a*e*g))*\text{Sqrt}[f + g*x])/g^4 - (2*(3*c*e*f - c*d*g - b*e*g)*(f + g*x)^{(3/2)})/(3*g^4) + (2*c*e*(f + g*x)^{(5/2)})/(5*g^4)$

**Rubi [A]** time = 0.238885, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$\frac{2(ef-dg)(ag^2-bfg+cf^2)}{g^4\sqrt{f+gx}} + \frac{2\sqrt{f+gx}(cf(3ef-2dg)-g(-aeg-bdg+2bef))}{g^4} - \frac{2(f+gx)^{3/2}(-beg-cdg+3cef)}{3g^4} + \frac{2ce(f+gx)^{5/2}}{5g^4}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)*(a + b*x + c*x^2)/(f + g*x)^{(3/2)}, x]$

[Out]  $(2*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2))/(g^4*\text{Sqrt}[f + g*x]) + (2*(c*f*(3*e*f - 2*d*g) - g*(2*b*e*f - b*d*g - a*e*g))*\text{Sqrt}[f + g*x])/g^4 - (2*(3*c*e*f - c*d*g - b*e*g)*(f + g*x)^{(3/2)})/(3*g^4) + (2*c*e*(f + g*x)^{(5/2)})/(5*g^4)$

**Rubi in Sympy [A]** time = 38.0806, size = 138, normalized size = 1.02

$$\frac{2ce(f+gx)^{5/2}}{5g^4} + \frac{2(f+gx)^{3/2}(beg+cdg-3cef)}{3g^4} + \frac{2\sqrt{f+gx}(aeg^2+bdg^2-2befg-2cdfg+3cef^2)}{g^4} - \frac{2(dg-ef)(ag^2-bfg+cf^2)}{g^4\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)*(c*x^2+b*x+a)/(g*x+f)^{(3/2)}, x)$

[Out]  $2*c*e*(f + g*x)^{(5/2)}/(5*g^4) + 2*(f + g*x)^{(3/2)}*(b*e*g + c*d*g - 3*c*e*f)/(3*g^4) + 2*\text{sqrt}(f + g*x)*(a*e*g^2 + b*d*g^2 - 2*b*e*f*g - 2*c*d*f*g + 3*c*e*f^2)/g^4 - 2*(d*g - e*f)*(a*g^2 - b*f*g + c*f^2)/(g^4*\text{sqrt}(f + g*x))$

**Mathematica [A]** time = 0.159507, size = 128, normalized size = 0.95

$$\frac{2(5g(3ag(-dg+2ef+egx)+3bdg(2f+gx)+be(-8f^2-4fgx+g^2x^2))+c(5dg(-8f^2-4fgx+g^2x^2))+3e(16f^3+8fg^2))}{15g^4\sqrt{f+gx}}$$

Antiderivative was successfully verified.



[In] Integrate[((d + e\*x)\*(a + b\*x + c\*x^2))/(f + g\*x)^(3/2), x]

[Out]  $(2*(5*g*(3*b*d*g*(2*f + g*x) + 3*a*g*(2*e*f - d*g + e*g*x) + b*e*(-8*f^2 - 4*f*g*x + g^2*x^2)) + c*(5*d*g*(-8*f^2 - 4*f*g*x + g^2*x^2) + 3*e*(16*f^3 + 8*f^2*g*x - 2*f*g^2*x^2 + g^3*x^3)))/(15*g^4*\sqrt{f + g*x})$

**Maple [A]** time = 0.008, size = 144, normalized size = 1.1

$$\frac{-6 c e x^3 g^3 - 10 b e g^3 x^2 - 10 c d g^3 x^2 + 12 c e f g^2 x^2 - 30 a e g^3 x - 30 b d g^3 x + 40 b e f g^2 x + 40 c d f g^2 x - 48 c e f^2 g x + 30 a d g^3}{15 g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)\*(c\*x^2+b\*x+a)/(g\*x+f)^(3/2), x)

[Out]  $-2/15/(g*x+f)^{1/2}*(-3*c*e*g^3*x^3-5*b*e*g^3*x^2-5*c*d*g^3*x^2+6*c*e*f*g^2*x^2-15*a*e*g^3*x-15*b*d*g^3*x+20*b*e*f*g^2*x+20*c*d*f*g^2*x-24*c*e*f^2*g*x+15*a*d*g^3-30*a*e*f*g^2-30*b*d*f*g^2+40*b*e*f^2*g+40*c*d*f^2*g-48*c*e*f^3)/g^4$

**Maxima [A]** time = 0.695653, size = 185, normalized size = 1.37

$$\frac{2 \left( \frac{3(gx+f)^{\frac{5}{2}}ce-5(3cef-(cd+be)g)(gx+f)^{\frac{3}{2}}+15(3cef^2-2(cd+be)fg+(bd+ae)g^2)\sqrt{gx+f}}{g^3} + \frac{15(cef^3-adg^3-(cd+be)f^2g+(bd+ae)fg^2)}{\sqrt{gx+f}g^3} \right)}{15g}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)/(g\*x + f)^(3/2), x, algorithm="maxima")

[Out]  $2/15*((3*(g*x + f)^{5/2}*c*e - 5*(3*c*e*f - (c*d + b*e)*g)*(g*x + f)^{3/2} + 15*(3*c*e*f^2 - 2*(c*d + b*e)*f*g + (b*d + a*e)*g^2)*\sqrt{g*x + f})/g^3 + 15*(c*e*f^3 - a*d*g^3 - (c*d + b*e)*f^2*g + (b*d + a*e)*f*g^2)/(\sqrt{g*x + f}*g^3)/g$

**Fricas [A]** time = 0.269217, size = 169, normalized size = 1.25

$$\frac{2(3ceg^3x^3 + 48cef^3 - 15adg^3 - 40(cd + be)f^2g + 30(bd + ae)fg^2 - (6cef^2g - 5(cd + be)g^3)x^2 + (24cef^2g - 20(cd + be)fg^2 - 15\sqrt{gx + f}g^4))}{15\sqrt{gx + f}g^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)/(g\*x + f)^(3/2), x, algorithm="fricas")

[Out]  $2/15*(3*c*e*g^3*x^3 + 48*c*e*f^3 - 15*a*d*g^3 - 40*(c*d + b*e)*f^2*g + 30*(b*d + a*e)*f*g^2 - (6*c*e*f^2g - 5*(c*d + b*e)*g^3)*x^2 + (24*c*e*f^2g - 20*(c*d + b*e)*f*g^2 + 15*(b*d + a*e)*g^3)*x)/(\sqrt{g*x + f}*g^4)$

**Sympy [A]** time = 26.1302, size = 2720, normalized size = 20.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*(c\*x\*\*2+b\*x+a)/(g\*x+f)\*\*(3/2),x)

[Out]  $-2*a*d/(g*\sqrt{f+g*x}) + a*e*\text{Piecewise}((4*f/(g**2*\sqrt{f+g*x}) + 2*x/(g*\sqrt{f+g*x})), \text{Ne}(g, 0)), (x**2/(2*f**(3/2))), \text{True})) + b*d*\text{Piecewise}((4*f/(g**2*\sqrt{f+g*x}) + 2*x/(g*\sqrt{f+g*x})), \text{Ne}(g, 0)), (x**2/(2*f**(3/2))), \text{True})) + b*e*(-16*f**(19/2)*\sqrt{(1+g*x/f)/(3*f**8*g**3+9*f**7*g**4*x+9*f**6*g**5*x**2+3*f**5*g**6*x**3)} + 16*f**(19/2)/(3*f**8*g**3+9*f**7*g**4*x+9*f**6*g**5*x**2+3*f**5*g**6*x**3) - 40*f**(17/2)*g*x*\sqrt{(1+g*x/f)/(3*f**8*g**3+9*f**7*g**4*x+9*f**6*g**5*x**2+3*f**5*g**6*x**3)} + 48*f**(17/2)*g*x/(3*f**8*g**3+9*f**7*g**4*x+9*f**6*g**5*x**2+3*f**5*g**6*x**3) - 30*f**(15/2)*g**2*x**2*\sqrt{(1+g*x/f)/(3*f**8*g**3+9*f**7*g**4*x+9*f**6*g**5*x**2+3*f**5*g**6*x**3)} + 48*f**(15/2)*g**2*x**2/(3*f**8*g**3+9*f**7*g**4*x+9*f**6*g**5*x**2+3*f**5*g**6*x**3) - 4*f**(13/2)*g**3*x**3*\sqrt{(1+g*x/f)/(3*f**8*g**3+9*f**7*g**4*x+9*f**6*g**5*x**2+3*f**5*g**6*x**3)} + 16*f**(13/2)*g**3*x**3/(3*f**8*g**3+9*f**7*g**4*x+9*f**6*g**5*x**2+3*f**5*g**6*x**3) + 2*f**(11/2)*g**4*x**4*\sqrt{(1+g*x/f)/(3*f**8*g**3+9*f**7*g**4*x+9*f**6*g**5*x**2+3*f**5*g**6*x**3)}) + c*d*(-16*f**(19/2)*\sqrt{(1+g*x/f)/(3*f**8*g**3+9*f**7*g**4*x+9*f**6*g**5*x**2+3*f**5*g**6*x**3)} + 16*f**(19/2)/(3*f**8*g**3+9*f**7*g**4*x+9*f**6*g**5*x**2+3*f**5*g**6*x**3) - 40*f**(17/2)*g*x*\sqrt{(1+g*x/f)/(3*f**8*g**3+9*f**7*g**4*x+9*f**6*g**5*x**2+3*f**5*g**6*x**3)} + 48*f**(17/2)*g*x/(3*f**8*g**3+9*f**7*g**4*x+9*f**6*g**5*x**2+3*f**5*g**6*x**3) - 30*f**(15/2)*g**2*x**2*\sqrt{(1+g*x/f)/(3*f**8*g**3+9*f**7*g**4*x+9*f**6*g**5*x**2+3*f**5*g**6*x**3)} + 48*f**(15/2)*g**2*x**2/(3*f**8*g**3+9*f**7*g**4*x+9*f**6*g**5*x**2+3*f**5*g**6*x**3) - 4*f**(13/2)*g**3*x**3*\sqrt{(1+g*x/f)/(3*f**8*g**3+9*f**7*g**4*x+9*f**6*g**5*x**2+3*f**5*g**6*x**3)} + 16*f**(13/2)*g**3*x**3/(3*f**8*g**3+9*f**7*g**4*x+9*f**6*g**5*x**2+3*f**5*g**6*x**3) + 2*f**(11/2)*g**4*x**4*\sqrt{(1+g*x/f)/(3*f**8*g**3+9*f**7*g**4*x+9*f**6*g**5*x**2+3*f**5*g**6*x**3)}) + c*e*(32*f**(45/2)*\sqrt{(1+g*x/f)/(5*f**20*g**4+30*f**19*g**5*x+75*f**18*g**6*x**2+100*f**17*g**7*x**3+75*f**16*g**8*x**4+30*f**15*g**9*x**5+5*f**14*g**10*x**6)} - 32*f**(45/2)/(5*f**20*g**4+30*f**19*g**5*x+75*f**18*g**6*x**2+100*f**17*g**7*x**3+75*f**16*g**8*x**4+30*f**15*g**9*x**5+5*f**14*g**10*x**6)} + 176*f**(43/2)*g*x*\sqrt{(1+g*x/f)/(5*f**20*g**4+30*f**19*g**5*x+75*f**18*g**6*x**2+100*f**17*g**7*x**3+75*f**16*g**8*x**4+30*f**15*g**9*x**5+5*f**14*g**10*x**6)} - 192*f**(43/2)*g*x/(5*f**20*g**4+30*f**19*g**5*x+75*f**18*g**6*x**2+100*f**17*g**7*x**3+75*f**16*g**8*x**4+30*f**15*g**9*x**5+5*f**14*g**10*x**6)} + 396*f**(41/2)*g**2*x**2*\sqrt{(1+g*x/f)/(5*f**20*g**4+30*f**19*g**5*x+75*f**18*g**6*x**2+100*f**17*g**7*x**3+75*f**16*g**8*x**4+30*f**15*g**9*x**5+5*f**14*g**10*x**6)} - 480*f**(41/2)*g**2*x**2/(5*f**20*g**4+30*f**19*g**5*x+75*f**18*g**6*x**2+100*f**17*g**7*x**3+75*f**16*g**8*x**4+30*f**15*g**9*x**5+5*f**14*g**10*x**6)} + 462*f**(39/2)*g**3*x**3*\sqrt{(1+g*x/f)/(5*f**20*g**4+30*f**19*g**5*x+75*f**18*g**6*x**2+100*f**17*g**7*x**3+75*f**16*g**8*x**4+30*f**15*g**9*x**5+5*f**14*g**10*x**6)} - 640*f**(39/2)*g**3*x**3/(5*f**20*g**4+30*f**19*g**5*x+75*f**18*g**6*x**2+100*f**17*g**7*x**3+75*f**16*g**8*x**4+30*f**15*g**9*x**5+5*f**14*g**10*x**6)} + 290*f**(37/2)*g**4*x**4*\sqrt{(1+g*x/f)/(5*f**20*g**4+30*f**19*g**5*x+75*f**18*g**6*x**2+100*f**17*g**7*x**3+75*f**16*g**8*x**4+30*f**15*g**9*x**5+5*f**14*g**10*x**6)} - 480*f**(37/2)*g**4*x**4/(5*f**20*g**4+30*f**19*g**5*x+75*f**18*g**6*x**2+100*f**17*g**7*x**3+75*f**16*g**8*x**4+30*f**15*g**9*x**5+5*f**14*g**10*x**6)} + 92*f**(35/2)*g**5*x**5*\sqrt{(1+g*x/f)/(5*f**20*g**4+30*f**19*g**5*x+75*f**18*g**6*x**2+100*f**17*g**7*x**3+75*f**16*g**8*x**4+30*f**15*g**9*x**5+5*f**14*g**10*x**6)} - 192*f**(35/2)*g**5*x**5/(5*f**20*g**4+30*f**19*g**5*x+75*f**18*g**6*x**2+100*f**17*g**7*x**3+75*f**16*g**8*x**4+30*f**15*g**9*x**5+5*f**14*g**10*x**6)} + 16*f**(33/2)*g**6*x**6*\sqrt{(1+g*x/f)/(5*f**20*g**4+30*f**19*g**5*x+75*f**18*g**6*x**2+100*f**17*g**7*x**3+75*f**16*g**8*x**4+30*f**15*g**9*x**5+5*f**14*g**10*x**6)} - 32*f**(33/2)*g**6*x**6/(5*f**20*g**4+30*f**19*g**5*x+75*f**18*g**6*x**2+100*f**17*g**7*x**3+75*f**16*g**8*x**4+30*f**15*g**9*x**5+5*f**14*g**10*x**6)}$

$$8*x^{**4} + 30*f^{**15}*g^{**9}*x^{**5} + 5*f^{**14}*g^{**10}*x^{**6}) + 6*f^{**}(31/2)*g^{**7}*x^{**7}*sqrt(1 + g*x/f)/(5*f^{**20}*g^{**4} + 30*f^{**19}*g^{**5}*x + 75*f^{**18}*g^{**6}*x^{**2} + 100*f^{**17}*g^{**7}*x^{**3} + 75*f^{**16}*g^{**8}*x^{**4} + 30*f^{**15}*g^{**9}*x^{**5} + 5*f^{**14}*g^{**10}*x^{**6}) + 2*f^{**}(29/2)*g^{**8}*x^{**8}*sqrt(1 + g*x/f)/(5*f^{**20}*g^{**4} + 30*f^{**19}*g^{**5}*x + 75*f^{**18}*g^{**6}*x^{**2} + 100*f^{**17}*g^{**7}*x^{**3} + 75*f^{**16}*g^{**8}*x^{**4} + 30*f^{**15}*g^{**9}*x^{**5} + 5*f^{**14}*g^{**10}*x^{**6}))$$

**GIAC/XCAS [A]** time = 0.270537, size = 275, normalized size = 2.04

$$\frac{2(cdf^2g - bdfg^2 + adg^3 - cf^3e + bf^2ge - afg^2e)}{\sqrt{gx + fg^4}} + \frac{2\left(5(gx + f)^{\frac{3}{2}}cdg^{17} - 30\sqrt{gx + f}cdfg^{17} + 15\sqrt{gx + f}bdg^{18} + 3(gx + f)^{\frac{5}{2}}cg^{16}e - 15(gx + f)^{\frac{3}{2}}cfg^{16}e + 45\sqrt{gx + f}cf^2g^{16}e\right)}{15g^{20}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)/(g\*x + f)^(3/2),x, algorithm="giac")

[Out] -2\*(c\*d\*f^2\*g - b\*d\*f\*g^2 + a\*d\*g^3 - c\*f^3\*e + b\*f^2\*g\*e - a\*f\*g^2\*e)/(sqrt(g\*x + f)\*g^4) + 2/15\*(5\*(g\*x + f)^(3/2)\*c\*d\*g^17 - 30\*sqrt(g\*x + f)\*c\*d\*f\*g^17 + 15\*sqrt(g\*x + f)\*b\*d\*g^18 + 3\*(g\*x + f)^(5/2)\*c\*g^16\*e - 15\*(g\*x + f)^(3/2)\*c\*f\*g^16\*e + 45\*sqrt(g\*x + f)\*c\*f^2\*g^16\*e + 5\*(g\*x + f)^(3/2)\*b\*g^17\*e - 30\*sqrt(g\*x + f)\*b\*f\*g^17\*e + 15\*sqrt(g\*x + f)\*a\*g^18\*e)/g^20

$$3.829 \quad \int \frac{a+bx+cx^2}{(f+gx)^{3/2}} dx$$

**Optimal.** Leaf size=71

$$-\frac{2(ag^2 - bfg + cf^2)}{g^3\sqrt{f+gx}} - \frac{2\sqrt{f+gx}(2cf - bg)}{g^3} + \frac{2c(f+gx)^{3/2}}{3g^3}$$

[Out]  $(-2*(c*f^2 - b*f*g + a*g^2))/(g^3*\text{Sqrt}[f + g*x]) - (2*(2*c*f - b*g)*\text{Sqrt}[f + g*x])/g^3 + (2*c*(f + g*x)^{(3/2)})/(3*g^3)$

**Rubi [A]** time = 0.0884247, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.05$

$$-\frac{2(ag^2 - bfg + cf^2)}{g^3\sqrt{f+gx}} - \frac{2\sqrt{f+gx}(2cf - bg)}{g^3} + \frac{2c(f+gx)^{3/2}}{3g^3}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)/(f + g\*x)^(3/2), x]

[Out]  $(-2*(c*f^2 - b*f*g + a*g^2))/(g^3*\text{Sqrt}[f + g*x]) - (2*(2*c*f - b*g)*\text{Sqrt}[f + g*x])/g^3 + (2*c*(f + g*x)^{(3/2)})/(3*g^3)$

**Rubi in Sympy [A]** time = 14.5306, size = 70, normalized size = 0.99

$$\frac{2c(f+gx)^{3/2}}{3g^3} + \frac{2\sqrt{f+gx}(bg - 2cf)}{g^3} - \frac{2(ag^2 - bfg + cf^2)}{g^3\sqrt{f+gx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+b\*x+a)/(g\*x+f)\*\*(3/2), x)

[Out]  $2*c*(f + g*x)**(3/2)/(3*g**3) + 2*\text{sqrt}(f + g*x)*(b*g - 2*c*f)/g**3 - 2*(a*g**2 - b*f*g + c*f**2)/(g**3*\text{sqrt}(f + g*x))$

**Mathematica [A]** time = 0.0538819, size = 54, normalized size = 0.76

$$\frac{6g(-ag + 2bf + bgx) + 2c(-8f^2 - 4fgx + g^2x^2)}{3g^3\sqrt{f+gx}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)/(f + g\*x)^(3/2), x]

[Out]  $(6*g*(2*b*f - a*g + b*g*x) + 2*c*(-8*f^2 - 4*f*g*x + g^2*x^2))/(3*g^3*\text{Sqrt}[f + g*x])$

**Maple [A]** time = 0.005, size = 53, normalized size = 0.8

$$-\frac{-2cx^2g^2 - 6bg^2x + 8cfgx + 6ag^2 - 12bfg + 16cf^2}{3g^3} \frac{1}{\sqrt{gx+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)/(g*x+f)^(3/2),x)`

[Out]  $-2/3/(g*x+f)^{(1/2)}*(-c*g^2*x^2-3*b*g^2*x+4*c*f*g*x+3*a*g^2-6*b*f*g+8*c*f^2)/g^3$

**Maxima [A]** time = 0.684049, size = 89, normalized size = 1.25

$$\frac{2 \left( \frac{(gx+f)^{\frac{3}{2}}c - 3(2cf - bg)\sqrt{gx+f}}{g^2} - \frac{3(cf^2 - bfg + ag^2)}{\sqrt{gx+f}g^2} \right)}{3g}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)/(g*x + f)^(3/2),x, algorithm="maxima")`

[Out]  $2/3*((g*x + f)^{(3/2)}*c - 3*(2*c*f - b*g)*\text{sqrt}(g*x + f))/g^2 - 3*(c*f^2 - b*f*g + a*g^2)/(\text{sqrt}(g*x + f)*g^2)/g$

**Fricas [A]** time = 0.269362, size = 72, normalized size = 1.01

$$\frac{2(cg^2x^2 - 8cf^2 + 6bfg - 3ag^2 - (4cfg - 3bg^2)x)}{3\sqrt{gx + fg^3}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)/(g*x + f)^(3/2),x, algorithm="fricas")`

[Out]  $2/3*(c*g^2*x^2 - 8*c*f^2 + 6*b*f*g - 3*a*g^2 - (4*c*f*g - 3*b*g^2)*x)/(\text{sqrt}(g*x + f)*g^3)$

**Sympy [A]** time = 11.0852, size = 590, normalized size = 8.31

$$\begin{aligned} & -\frac{2a}{g\sqrt{f+gx}} + b \left( \begin{cases} \frac{4f}{g^2\sqrt{f+gx}} + \frac{2x}{g\sqrt{f+gx}} & \text{for } g \neq 0 \\ \frac{x^2}{2f^{\frac{3}{2}}} & \text{otherwise} \end{cases} \right) + c \left( -\frac{16f^{\frac{19}{2}}\sqrt{1+\frac{gx}{f}}}{3f^8g^3 + 9f^7g^4x + 9f^6g^5x^2 + 3f^5g^6x^3} \right. \\ & + \frac{16f^{\frac{19}{2}}}{3f^8g^3 + 9f^7g^4x + 9f^6g^5x^2 + 3f^5g^6x^3} - \frac{40f^{\frac{17}{2}}gx\sqrt{1+\frac{gx}{f}}}{3f^8g^3 + 9f^7g^4x + 9f^6g^5x^2 + 3f^5g^6x^3} \\ & + \frac{48f^{\frac{17}{2}}gx}{3f^8g^3 + 9f^7g^4x + 9f^6g^5x^2 + 3f^5g^6x^3} - \frac{30f^{\frac{15}{2}}g^2x^2\sqrt{1+\frac{gx}{f}}}{3f^8g^3 + 9f^7g^4x + 9f^6g^5x^2 + 3f^5g^6x^3} \\ & + \frac{48f^{\frac{15}{2}}g^2x^2}{3f^8g^3 + 9f^7g^4x + 9f^6g^5x^2 + 3f^5g^6x^3} - \frac{4f^{\frac{13}{2}}g^3x^3\sqrt{1+\frac{gx}{f}}}{3f^8g^3 + 9f^7g^4x + 9f^6g^5x^2 + 3f^5g^6x^3} \\ & \left. + \frac{16f^{\frac{13}{2}}g^3x^3}{3f^8g^3 + 9f^7g^4x + 9f^6g^5x^2 + 3f^5g^6x^3} + \frac{2f^{\frac{11}{2}}g^4x^4\sqrt{1+\frac{gx}{f}}}{3f^8g^3 + 9f^7g^4x + 9f^6g^5x^2 + 3f^5g^6x^3} \right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+b*x+a)/(g*x+f)**(3/2),x)`

```
[Out] -2*a/(g*sqrt(f + g*x)) + b*Piecewise((4*f/(g**2*sqrt(f + g*x)) +
2*x/(g*sqrt(f + g*x)), Ne(g, 0)), (x**2/(2*f**(3/2)), True)) + c*
(-16*f**(19/2)*sqrt(1 + g*x/f)/(3*f**8*g**3 + 9*f**7*g**4*x + 9*f
**6*g**5*x**2 + 3*f**5*g**6*x**3) + 16*f**(19/2)/(3*f**8*g**3 + 9
*f**7*g**4*x + 9*f**6*g**5*x**2 + 3*f**5*g**6*x**3) - 40*f**(17/2
)*g*x*sqrt(1 + g*x/f)/(3*f**8*g**3 + 9*f**7*g**4*x + 9*f**6*g**5
*x**2 + 3*f**5*g**6*x**3) + 48*f**(17/2)*g*x/(3*f**8*g**3 + 9*f**7
*g**4*x + 9*f**6*g**5*x**2 + 3*f**5*g**6*x**3) - 30*f**(15/2)*g**
2*x**2*sqrt(1 + g*x/f)/(3*f**8*g**3 + 9*f**7*g**4*x + 9*f**6*g**5
*x**2 + 3*f**5*g**6*x**3) + 48*f**(15/2)*g**2*x**2/(3*f**8*g**3 +
9*f**7*g**4*x + 9*f**6*g**5*x**2 + 3*f**5*g**6*x**3) - 4*f**(13/
2)*g**3*x**3*sqrt(1 + g*x/f)/(3*f**8*g**3 + 9*f**7*g**4*x + 9*f**
6*g**5*x**2 + 3*f**5*g**6*x**3) + 16*f**(13/2)*g**3*x**3/(3*f**8
*g**3 + 9*f**7*g**4*x + 9*f**6*g**5*x**2 + 3*f**5*g**6*x**3) + 2*f
**(11/2)*g**4*x**4*sqrt(1 + g*x/f)/(3*f**8*g**3 + 9*f**7*g**4*x +
9*f**6*g**5*x**2 + 3*f**5*g**6*x**3))
```

**GIAC/XCAS [A]** time = 0.264497, size = 100, normalized size = 1.41

$$-\frac{2(cf^2 - bfg + ag^2)}{\sqrt{gx + fg^3}} + \frac{2\left((gx + f)^{\frac{3}{2}}cg^6 - 6\sqrt{gx + f}cfg^6 + 3\sqrt{gx + f}bg^7\right)}{3g^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)/(g*x + f)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(c*f^2 - b*f*g + a*g^2)/(sqrt(g*x + f)*g^3) + 2/3*((g*x + f)^(
3/2)*c*g^6 - 6*sqrt(g*x + f)*c*f*g^6 + 3*sqrt(g*x + f)*b*g^7)/g^9
```

$$3.830 \quad \int \frac{a+bx+cx^2}{(d+ex)(f+gx)^{3/2}} dx$$

**Optimal.** Leaf size=122

$$-\frac{2(ae^2 - bde + cd^2) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}} + \frac{2(ag^2 - bfg + cf^2)}{g^2\sqrt{f+gx}(ef-dg)} + \frac{2c\sqrt{f+gx}}{eg^2}$$

[Out] (2\*(c\*f^2 - b\*f\*g + a\*g^2))/(g^2\*(e\*f - d\*g)\*Sqrt[f + g\*x]) + (2\*c\*Sqrt[f + g\*x])/(e\*g^2) - (2\*(c\*d^2 - b\*d\*e + a\*e^2)\*ArcTanh[(Sqrt[e]\*Sqrt[f + g\*x])/Sqrt[e\*f - d\*g]])/(e^(3/2)\*(e\*f - d\*g)^(3/2))

**Rubi [A]** time = 0.482186, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$-\frac{2(ae^2 - bde + cd^2) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}} + \frac{2(ag^2 - bfg + cf^2)}{g^2\sqrt{f+gx}(ef-dg)} + \frac{2c\sqrt{f+gx}}{eg^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)/((d + e\*x)\*(f + g\*x)^(3/2)), x]

[Out] (2\*(c\*f^2 - b\*f\*g + a\*g^2))/(g^2\*(e\*f - d\*g)\*Sqrt[f + g\*x]) + (2\*c\*Sqrt[f + g\*x])/(e\*g^2) - (2\*(c\*d^2 - b\*d\*e + a\*e^2)\*ArcTanh[(Sqrt[e]\*Sqrt[f + g\*x])/Sqrt[e\*f - d\*g]])/(e^(3/2)\*(e\*f - d\*g)^(3/2))

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\frac{2(ag^2 - bfg + cf^2)}{g^2\sqrt{f+gx}(dg-ef)} + \frac{2\int\sqrt{f+gx} c dx}{eg^2} - \frac{2(ae^2 - bde + cd^2) \operatorname{atan}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{dg-ef}}\right)}{e^{3/2}(dg-ef)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+b\*x+a)/(e\*x+d)/(g\*x+f)\*\*(3/2), x)

[Out] -2\*(a\*g\*\*2 - b\*f\*g + c\*f\*\*2)/(g\*\*2\*sqrt(f + g\*x)\*(d\*g - e\*f)) + 2\*Integral(c, (x, sqrt(f + g\*x)))/(e\*g\*\*2) - 2\*(a\*e\*\*2 - b\*d\*e + c\*d\*\*2)\*atan(sqrt(e)\*sqrt(f + g\*x)/sqrt(d\*g - e\*f))/(e\*\*(3/2)\*(d\*g - e\*f)\*\*(3/2))

**Mathematica [A]** time = 0.406366, size = 118, normalized size = 0.97

$$\frac{2\sqrt{f+gx}\left(\frac{g(ag-bf)+cf^2}{(f+gx)(ef-dg)} + \frac{c}{e}\right)}{g^2} - \frac{2(e(ae-bd) + cd^2) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)}{e^{3/2}(ef-dg)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)/((d + e\*x)\*(f + g\*x)^(3/2)), x]

[Out]  $(2\sqrt{f+gx})^2(c/e + (c^2f^2 + g^2(-bf) + a^2g))/((e^2f - d^2g)(f + gx)))/g^2 - (2(c^2d^2 + e^2(-bd) + a^2e))\text{ArcTanh}[\sqrt{e}\sqrt{f+gx}]/\sqrt{e^2f - d^2g}]/(e^{3/2}(e^2f - d^2g)^{3/2})$

**Maple [B]** time = 0.019, size = 237, normalized size = 1.9

$$2\frac{c\sqrt{gx+f}}{eg^2} - 2\frac{a}{(dg-ef)\sqrt{gx+f}} + 2\frac{bf}{(dg-ef)g\sqrt{gx+f}} - 2\frac{cf^2}{g^2(dg-ef)\sqrt{gx+f}} - 2\frac{ae}{(dg-ef)\sqrt{(dg-ef)e}} \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{(dg-ef)e}}\right) + 2\frac{bd}{(dg-ef)\sqrt{(dg-ef)e}} \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{(dg-ef)e}}\right) - 2\frac{cd^2}{(dg-ef)e\sqrt{(dg-ef)e}} \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{(dg-ef)e}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)/(e*x+d)/(g*x+f)^(3/2), x)`

[Out]  $2c(g^2x+f)^{1/2}/e/g^2 - 2(d^2g - e^2f)/(g^2x+f)^{1/2} * a + 2/g/(d^2g - e^2f)/(g^2x+f)^{1/2} * b^2f - 2/g^2/(d^2g - e^2f)/(g^2x+f)^{1/2} * c^2f^2 - 2/(d^2g - e^2f) * e/((d^2g - e^2f)^2e)^{1/2} * \arctan((g^2x+f)^{1/2} * e/((d^2g - e^2f)^2e)^{1/2}) * a + 2/(d^2g - e^2f)/((d^2g - e^2f)^2e)^{1/2} * \arctan((g^2x+f)^{1/2} * e/((d^2g - e^2f)^2e)^{1/2}) * b^2d - 2/(d^2g - e^2f)/e/((d^2g - e^2f)^2e)^{1/2} * \arctan((g^2x+f)^{1/2} * e/((d^2g - e^2f)^2e)^{1/2}) * c^2d^2$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)/((e*x + d)*(g*x + f)^(3/2)), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.283976, size = 1, normalized size = 0.01

$$\frac{(cd^2 - bde + ae^2)\sqrt{gx+f}g^2 \log\left(\frac{\sqrt{e^2f - deg}(egx + 2ef - dg) + 2(e^2f - deg)\sqrt{gx+f}}{ex+d}\right) - 2(2cef^2 + aeg^2 - (cd + be)fg + (cefg - cdg^2)x)\sqrt{-e^2f + deg}\sqrt{gx+f}}{(e^2fg^2 - deg^3)\sqrt{e^2f - deg}\sqrt{gx+f}} - 2\left((cd^2 - bde + ae^2)\sqrt{gx+f}g^2 \arctan\left(-\frac{ef - dg}{\sqrt{-e^2f + deg}\sqrt{gx+f}}\right) - (2cef^2 + aeg^2 - (cd + be)fg + (cefg - cdg^2)x)\sqrt{-e^2f + deg}\sqrt{gx+f}\right)/(e^2fg^2 - deg^3)\sqrt{-e^2f + deg}\sqrt{gx+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)/((e*x + d)*(g*x + f)^(3/2)), x, algorithm="fricas")`

[Out]  $[-((c^2d^2 - b^2d^2e + a^2e^2)\sqrt{g^2x+f}) * g^2 * \log((\sqrt{e^2f - d^2g}) * (e^2g^2x + 2e^2f - d^2g) + 2(e^2f - d^2g)\sqrt{g^2x+f})]/(e^2fg^2 - deg^3)\sqrt{-e^2f + deg}\sqrt{gx+f}$



$x + d)) - 2*(2*c*e*f^2 + a*e*g^2 - (c*d + b*e)*f*g + (c*e*f*g - c*d*g^2)*x)*\sqrt{e^2*f - d*e*g})/((e^2*f*g^2 - d*e*g^3)*\sqrt{e^2*f - d*e*g})*\sqrt{g*x + f}), -2*((c*d^2 - b*d*e + a*e^2)*\sqrt{g*x + f})*g^2*\arctan(-(e*f - d*g)/(\sqrt{-e^2*f + d*e*g})*\sqrt{g*x + f})) - (2*c*e*f^2 + a*e*g^2 - (c*d + b*e)*f*g + (c*e*f*g - c*d*g^2)*x)*\sqrt{-e^2*f + d*e*g})/((e^2*f*g^2 - d*e*g^3)*\sqrt{-e^2*f + d*e*g})*\sqrt{g*x + f})]$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + bx + cx^2}{(d + ex)(f + gx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)/(e\*x+d)/(g\*x+f)\*\*(3/2),x)

[Out] Integral((a + b\*x + c\*x\*\*2)/((d + e\*x)\*(f + g\*x)\*\*(3/2)), x)

**GIAC/XCAS [A]** time = 0.268861, size = 151, normalized size = 1.24

$$-\frac{2(cd^2 - bde + ae^2) \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{dge-fe^2}}\right)}{(dge - fe^2)^{\frac{3}{2}}} + \frac{2\sqrt{gx+fe}ce^{(-1)}}{g^2} - \frac{2(cf^2 - bfg + ag^2)}{(dg^3 - fg^2e)\sqrt{gx+f}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/((e\*x + d)\*(g\*x + f)^(3/2)),x, algorithm="giac")

[Out] -2\*(c\*d^2 - b\*d\*e + a\*e^2)\*arctan(sqrt(g\*x + f)\*e/sqrt(d\*g\*e - f\*e^2))/((d\*g\*e - f\*e^2)^(3/2) + 2\*sqrt(g\*x + f)\*c\*e^(-1)/g^2 - 2\*(c\*f^2 - b\*f\*g + a\*g^2)/((d\*g^3 - f\*g^2\*e)\*sqrt(g\*x + f))

$$3.831 \quad \int \frac{a+bx+cx^2}{(d+ex)^2(f+gx)^{3/2}} dx$$

**Optimal.** Leaf size=165

$$\frac{\sqrt{f+gx}(ae^2 - bde + cd^2)}{e(d+ex)(ef-dg)^2} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(cd(4ef-dg) - e(-3aeg + bdg + 2bef))}{e^{3/2}(ef-dg)^{5/2}} - \frac{2(ag^2 - bfg + cf^2)}{g\sqrt{f+gx}(ef-dg)^2}$$

[Out]  $(-2*(c*f^2 - b*f*g + a*g^2))/(g*(e*f - d*g)^2*\text{Sqrt}[f + g*x]) - ((c*d^2 - b*d*e + a*e^2)*\text{Sqrt}[f + g*x])/(e*(e*f - d*g)^2*(d + e*x)) + ((c*d*(4*e*f - d*g) - e*(2*b*e*f + b*d*g - 3*a*e*g))*\text{ArcTanh}[(\text{Sqrt}[e]*\text{Sqrt}[f + g*x])/\text{Sqrt}[e*f - d*g]])/(e^{(3/2)}*(e*f - d*g)^{(5/2)})$

**Rubi [A]** time = 0.773315, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$\frac{\sqrt{f+gx}(ae^2 - bde + cd^2)}{e(d+ex)(ef-dg)^2} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(cd(4ef-dg) - e(-3aeg + bdg + 2bef))}{e^{3/2}(ef-dg)^{5/2}} - \frac{2(ag^2 - bfg + cf^2)}{g\sqrt{f+gx}(ef-dg)^2}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)/((d + e\*x)^2\*(f + g\*x)^(3/2)), x]

[Out]  $(-2*(c*f^2 - b*f*g + a*g^2))/(g*(e*f - d*g)^2*\text{Sqrt}[f + g*x]) - ((c*d^2 - b*d*e + a*e^2)*\text{Sqrt}[f + g*x])/(e*(e*f - d*g)^2*(d + e*x)) + ((c*d*(4*e*f - d*g) - e*(2*b*e*f + b*d*g - 3*a*e*g))*\text{ArcTanh}[(\text{Sqrt}[e]*\text{Sqrt}[f + g*x])/\text{Sqrt}[e*f - d*g]])/(e^{(3/2)}*(e*f - d*g)^{(5/2)})$

**Rubi in Sympy [F-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+b\*x+a)/(e\*x+d)\*\*2/(g\*x+f)\*\*(3/2), x)

[Out] Timed out

**Mathematica [A]** time = 0.877007, size = 155, normalized size = 0.94

$$\frac{\sqrt{f+gx}\left(\frac{ae^2-bde+cd^2}{de+e^2x} + \frac{2(g(ag-bf)+cf^2)}{g(f+gx)}\right)}{(ef-dg)^2} - \frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(e(-3aeg + bdg + 2bef) + cd(dg - 4ef))}{e^{3/2}(ef-dg)^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)/((d + e\*x)^2\*(f + g\*x)^(3/2)),x]

[Out]  $-\left(\frac{\sqrt{f+gx} \left( (c^2d^2 - b^2de + a^2e^2)/(de + e^2x) + (2(c^2f^2 + g^2(-bf) + ag))/(g(f+gx)) \right)}{(ef - dg)^2} - \frac{(c^2d^2(-4ef + dg) + e(2b^2ef + b^2dg - 3a^2eg)) \operatorname{ArcTanh}[\sqrt{e} \operatorname{Sqrt}[f+gx]]}{e^{3/2}(ef - dg)^{5/2}}\right)$

**Maple [B]** time = 0.03, size = 418, normalized size = 2.5

$$\begin{aligned} & -2 \frac{ag}{(dg-ef)^2 \sqrt{gx+f}} + 2 \frac{bf}{(dg-ef)^2 \sqrt{gx+f}} - 2 \frac{cf^2}{g(dg-ef)^2 \sqrt{gx+f}} \\ & - \frac{aeg}{(dg-ef)^2 (egx+dg)} \sqrt{gx+f} + \frac{bdg}{(dg-ef)^2 (egx+dg)} \sqrt{gx+f} \\ & - \frac{cd^2g}{(dg-ef)^2 e(egx+dg)} \sqrt{gx+f} - 3 \frac{aeg}{(dg-ef)^2 \sqrt{(dg-ef)e}} \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{(dg-ef)e}}\right) \\ & + \frac{bdg}{(dg-ef)^2} \arctan\left(e\sqrt{gx+f} \frac{1}{\sqrt{(dg-ef)e}}\right) \frac{1}{\sqrt{(dg-ef)e}} \\ & + 2 \frac{bef}{(dg-ef)^2 \sqrt{(dg-ef)e}} \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{(dg-ef)e}}\right) \\ & + \frac{cd^2g}{(dg-ef)^2 e} \arctan\left(e\sqrt{gx+f} \frac{1}{\sqrt{(dg-ef)e}}\right) \frac{1}{\sqrt{(dg-ef)e}} \\ & - 4 \frac{cdf}{(dg-ef)^2 \sqrt{(dg-ef)e}} \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{(dg-ef)e}}\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)/(e\*x+d)^2/(g\*x+f)^(3/2),x)

[Out]  $-2g/(dg-ef)^2/(g*x+f)^{1/2} + a/(dg-ef)^2/(g*x+f)^{1/2} + bf - 2/g/(dg-ef)^2/(g*x+f)^{1/2} + c^2f^2 - g/(dg-ef)^2e/(g*x+f)^{1/2} / (e^2g*x+d^2g) + a/(dg-ef)^2/(g*x+f)^{1/2} / (e^2g*x+d^2g) + b^2d - g/(dg-ef)^2/e/(g*x+f)^{1/2} / (e^2g*x+d^2g) + c^2d^2 - 3g/(dg-ef)^2e / ((dg-ef)^2e)^{1/2} + \arctan((g*x+f)^{1/2}e/((dg-ef)^2e)^{1/2}) + a/(dg-ef)^2 / ((dg-ef)^2e)^{1/2} + \arctan((g*x+f)^{1/2}e/((dg-ef)^2e)^{1/2}) + b^2d + 2/(dg-ef)^2e / ((dg-ef)^2e)^{1/2} + \arctan((g*x+f)^{1/2}e/((dg-ef)^2e)^{1/2}) + b^2f + g/(dg-ef)^2/e / ((dg-ef)^2e)^{1/2} + \arctan((g*x+f)^{1/2}e/((dg-ef)^2e)^{1/2}) + c^2d^2 - 4/(dg-ef)^2 / ((dg-ef)^2e)^{1/2} + \arctan((g*x+f)^{1/2}e/((dg-ef)^2e)^{1/2}) + d^2c^2f$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/((e\*x + d)^2\*(g\*x + f)^(3/2)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.291994, size = 1, normalized size = 0.01

$$\left[ \frac{(2(2cd^2e - bde^2)fg - (cd^3 + bd^2e - 3ade^2)g^2 + (2(2cde^2 - be^3)fg - (cd^2e + bde^2 - 3ae^3)g^2)x)\sqrt{gx+f}\log\left(\frac{\sqrt{e^2f-d}}{\sqrt{d^3f^2g - 2d^2e^2fg^2 + d^3eg^3 + \dots}}\right)}{2(de^3f^2g - 2d^2e^2fg^2 + d^3eg^3 + \dots)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/((e\*x + d)^2\*(g\*x + f)^(3/2)),x, algorithm="fricas")

[Out] [1/2\*((2\*(2\*c\*d^2\*e - b\*d\*e^2)\*f\*g - (c\*d^3 + b\*d^2\*e - 3\*a\*d\*e^2)\*g^2 + (2\*(2\*c\*d\*e^2 - b\*e^3)\*f\*g - (c\*d^2\*e + b\*d\*e^2 - 3\*a\*e^3)\*g^2)\*x)\*sqrt(g\*x + f)\*log((sqrt(e^2\*f - d\*e\*g)\*(e\*g\*x + 2\*e\*f - d\*g) + 2\*(e^2\*f - d\*e\*g)\*sqrt(g\*x + f))/(e\*x + d)) - 2\*(2\*c\*d\*e\*f^2 + 2\*a\*d\*e\*g^2 + (c\*d^2 - 3\*b\*d\*e + a\*e^2)\*f\*g + (2\*c\*e^2\*f^2 - 2\*b\*e^2\*f\*g + (c\*d^2 - b\*d\*e + 3\*a\*e^2)\*g^2)\*x)\*sqrt(e^2\*f - d\*e\*g)/((d\*e^3\*f^2\*g - 2\*d^2\*e^2\*f\*g^2 + d^3\*e\*g^3 + (e^4\*f^2\*g - 2\*d\*e^3\*f\*g^2 + d^2\*e^2\*g^3)\*x)\*sqrt(e^2\*f - d\*e\*g)\*sqrt(g\*x + f)), ((2\*(2\*c\*d^2\*e - b\*d\*e^2)\*f\*g - (c\*d^3 + b\*d^2\*e - 3\*a\*d\*e^2)\*g^2 + (2\*(2\*c\*d\*e^2 - b\*e^3)\*f\*g - (c\*d^2\*e + b\*d\*e^2 - 3\*a\*e^3)\*g^2)\*x)\*sqrt(g\*x + f)\*arctan(-(e\*f - d\*g)/(sqrt(-e^2\*f + d\*e\*g)\*sqrt(g\*x + f))) - (2\*c\*d\*e\*f^2 + 2\*a\*d\*e\*g^2 + (c\*d^2 - 3\*b\*d\*e + a\*e^2)\*f\*g + (2\*c\*e^2\*f^2 - 2\*b\*e^2\*f\*g + (c\*d^2 - b\*d\*e + 3\*a\*e^2)\*g^2)\*x)\*sqrt(-e^2\*f + d\*e\*g)/((d\*e^3\*f^2\*g - 2\*d^2\*e^2\*f\*g^2 + d^3\*e\*g^3 + (e^4\*f^2\*g - 2\*d\*e^3\*f\*g^2 + d^2\*e^2\*g^3)\*x)\*sqrt(-e^2\*f + d\*e\*g)\*sqrt(g\*x + f))]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)/(e\*x+d)\*\*2/(g\*x+f)\*\*(3/2),x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.277342, size = 381, normalized size = 2.31

$$\frac{(cd^2g - 4cdf e + bdge + 2bfe^2 - 3age^2) \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{dge-fe^2}}\right)}{(d^2g^2e - 2dfge^2 + f^2e^3)\sqrt{dge - fe^2}} \frac{(gx + f)cd^2g^2 + 2cdf^2ge - (gx + f)bdg^2e - 2bdfg^2e + 2adg^3e + 2(gx + f)cf^2e^2 - 2cf^3e^2 - 2(gx + f)bfg e^2 + 2bf^2ge}{(d^2g^3e - 2dfg^2e^2 + f^2ge^3)\left(\sqrt{gx + fdg} + (gx + f)^{\frac{3}{2}}e - \sqrt{gx + ffe}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/((e\*x + d)^2\*(g\*x + f)^(3/2)),x, algorithm="giac")

[Out] (c\*d^2\*g - 4\*c\*d\*f\*e + b\*d\*g\*e + 2\*b\*f\*e^2 - 3\*a\*g\*e^2)\*arctan(sqrt(g\*x + f)\*e/sqrt(d\*g\*e - f\*e^2))/((d^2\*g^2\*e - 2\*d\*f\*g\*e^2 + f^2\*e^3)\*sqrt(d\*g\*e - f\*e^2)) - ((g\*x + f)\*c\*d^2\*g^2 + 2\*c\*d\*f^2\*g\*e - (g\*x + f)\*b\*d\*g^2\*e - 2\*b\*d\*f\*g^2\*e + 2\*a\*d\*g^3\*e + 2\*(g\*x + f)\*c\*f^2\*e^2 - 2\*c\*f^3\*e^2 - 2\*(g\*x + f)\*b\*f\*g^2\*e + 2\*b\*f^2\*g\*e^2 + 3\*(g\*x + f)\*a\*g^2\*e^2 - 2\*a\*f\*g^2\*e^2)/((d^2\*g^3\*e - 2\*d\*f\*g^2\*e^2 + f^2\*g^2\*e^3)\*(sqrt(g\*x + f)\*d\*g + (g\*x + f)^(3/2)\*e - sqrt(g\*x + f)\*f\*e))

$$3.832 \quad \int \frac{a+bx+cx^2}{(d+ex)^3(f+gx)^{3/2}} dx$$

**Optimal.** Leaf size=247

$$\begin{aligned} & \frac{\sqrt{f+gx}(ae^2 - bde + cd^2)}{2e(d+ex)^2(ef-dg)^2} \\ & + \frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(3eg(-5aeg + bdg + 4bef) - c(-d^2g^2 + 8defg + 8e^2f^2))}{4e^{3/2}(ef-dg)^{7/2}} \\ & + \frac{2(ag^2 - bfg + cf^2)}{\sqrt{f+gx}(ef-dg)^3} + \frac{\sqrt{f+gx}(cd(8ef-dg) - e(-7aeg + 3bdg + 4bef))}{4e(d+ex)(ef-dg)^3} \end{aligned}$$

[Out] (2\*(c\*f^2 - b\*f\*g + a\*g^2))/((e\*f - d\*g)^3\*Sqrt[f + g\*x]) - ((c\*d^2 - b\*d\*e + a\*e^2)\*Sqrt[f + g\*x])/(2\*e\*(e\*f - d\*g)^2\*(d + e\*x)^2) + ((c\*d\*(8\*e\*f - d\*g) - e\*(4\*b\*e\*f + 3\*b\*d\*g - 7\*a\*e\*g))\*Sqrt[f + g\*x])/(4\*e\*(e\*f - d\*g)^3\*(d + e\*x)) + ((3\*e\*g\*(4\*b\*e\*f + b\*d\*g - 5\*a\*e\*g) - c\*(8\*e^2\*f^2 + 8\*d\*e\*f\*g - d^2\*g^2))\*ArcTanh[(Sqrt[e]\*Sqrt[f + g\*x])/Sqrt[e\*f - d\*g]])/(4\*e^(3/2)\*(e\*f - d\*g)^(7/2))

**Rubi [A]** time = 1.34319, antiderivative size = 247, normalized size of antiderivative = 1., number of rules used = 5, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\begin{aligned} & \frac{\sqrt{f+gx}(ae^2 - bde + cd^2)}{2e(d+ex)^2(ef-dg)^2} \\ & + \frac{\tanh^{-1}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}}\right)(3eg(-5aeg + bdg + 4bef) - c(-d^2g^2 + 8defg + 8e^2f^2))}{4e^{3/2}(ef-dg)^{7/2}} \\ & + \frac{2(ag^2 - bfg + cf^2)}{\sqrt{f+gx}(ef-dg)^3} + \frac{\sqrt{f+gx}(cd(8ef-dg) - e(-7aeg + 3bdg + 4bef))}{4e(d+ex)(ef-dg)^3} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)/((d + e\*x)^3\*(f + g\*x)^(3/2)), x]

[Out] (2\*(c\*f^2 - b\*f\*g + a\*g^2))/((e\*f - d\*g)^3\*Sqrt[f + g\*x]) - ((c\*d^2 - b\*d\*e + a\*e^2)\*Sqrt[f + g\*x])/(2\*e\*(e\*f - d\*g)^2\*(d + e\*x)^2) + ((c\*d\*(8\*e\*f - d\*g) - e\*(4\*b\*e\*f + 3\*b\*d\*g - 7\*a\*e\*g))\*Sqrt[f + g\*x])/(4\*e\*(e\*f - d\*g)^3\*(d + e\*x)) + ((3\*e\*g\*(4\*b\*e\*f + b\*d\*g - 5\*a\*e\*g) - c\*(8\*e^2\*f^2 + 8\*d\*e\*f\*g - d^2\*g^2))\*ArcTanh[(Sqrt[e]\*Sqrt[f + g\*x])/Sqrt[e\*f - d\*g]])/(4\*e^(3/2)\*(e\*f - d\*g)^(7/2))

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+b\*x+a)/(e\*x+d)\*\*3/(g\*x+f)\*\*(3/2), x)

[Out] Timed out

**Mathematica [A]** time = 2.16204, size = 223, normalized size = 0.9

$$\frac{1}{4} \left( \frac{\sqrt{f+gx} \left( \frac{2(dg-ef)(e(ae-bd)+cd^2)}{e(d+ex)^2} + \frac{e(7aeg-3bdg-4bef)+cd(8ef-dg)}{e(d+ex)} + \frac{8(g(ag-bf)+cf^2)}{f+gx} \right)}{(ef-dg)^3} - \frac{\tanh^{-1} \left( \frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{ef-dg}} \right) (3eg(5aeg-b(dg+4ef))+c(-d^2g^2+8defg+8e^2f^2))}{e^{3/2}(ef-dg)^{7/2}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)/((d + e\*x)^3\*(f + g\*x)^(3/2)),x]

[Out] ((Sqrt[f + g\*x]\*((2\*(c\*d^2 + e\*(-b\*d) + a\*e))\*(-(e\*f) + d\*g))/(e\*(d + e\*x)^2) + (c\*d\*(8\*e\*f - d\*g) + e\*(-4\*b\*e\*f - 3\*b\*d\*g + 7\*a\*e\*g))/(e\*(d + e\*x)) + (8\*(c\*f^2 + g\*(-b\*f) + a\*g))/(f + g\*x))/((e\*f - d\*g)^3 - ((c\*(8\*e^2\*f^2 + 8\*d\*e\*f\*g - d^2\*g^2) + 3\*e\*g\*(5\*a\*e\*g - b\*(4\*e\*f + d\*g)))\*ArcTanh[Sqrt[e]\*Sqrt[f + g\*x])/Sqrt[e\*f - d\*g]))/(e^(3/2)\*(e\*f - d\*g)^(7/2))/4

**Maple [B]** time = 0.034, size = 847, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)/(e\*x+d)^3/(g\*x+f)^(3/2),x)

[Out] -2/(d\*g-e\*f)^3/(g\*x+f)^(1/2)\*a\*g^2+2/(d\*g-e\*f)^3/(g\*x+f)^(1/2)\*b\*f\*g-2/(d\*g-e\*f)^3/(g\*x+f)^(1/2)\*c\*f^2-7/4/(d\*g-e\*f)^3/(e\*g\*x+d\*g)^2\*(g\*x+f)^(3/2)\*a\*e^2\*g^2+3/4/(d\*g-e\*f)^3/(e\*g\*x+d\*g)^2\*(g\*x+f)^(3/2)\*b\*d\*e\*g^2+1/(d\*g-e\*f)^3/(e\*g\*x+d\*g)^2\*(g\*x+f)^(3/2)\*b\*e^2\*f\*g+1/4/(d\*g-e\*f)^3/(e\*g\*x+d\*g)^2\*(g\*x+f)^(3/2)\*c\*d^2\*g^2-2/(d\*g-e\*f)^3/(e\*g\*x+d\*g)^2\*(g\*x+f)^(3/2)\*c\*d\*e\*f\*g-9/4/(d\*g-e\*f)^3/(e\*g\*x+d\*g)^2\*g^3\*e\*(g\*x+f)^(1/2)\*a\*d+9/4/(d\*g-e\*f)^3/(e\*g\*x+d\*g)^2\*g^2\*e^2\*(g\*x+f)^(1/2)\*a\*f+5/4/(d\*g-e\*f)^3/(e\*g\*x+d\*g)^2\*g^3\*(g\*x+f)^(1/2)\*b\*d^2-1/4/(d\*g-e\*f)^3/(e\*g\*x+d\*g)^2\*g^2\*e\*(g\*x+f)^(1/2)\*b\*f^2-1/4/(d\*g-e\*f)^3/(e\*g\*x+d\*g)^2\*g^3/e\*(g\*x+f)^(1/2)\*c\*d^3-7/4/(d\*g-e\*f)^3/(e\*g\*x+d\*g)^2\*g^2\*(g\*x+f)^(1/2)\*c\*d^2\*f+2/(d\*g-e\*f)^3/(e\*g\*x+d\*g)^2\*g\*e\*(g\*x+f)^(1/2)\*c\*d\*f^2-15/4/(d\*g-e\*f)^3\*e/((d\*g-e\*f)\*e)^(1/2)\*arctan((g\*x+f)^(1/2)\*e/((d\*g-e\*f)\*e)^(1/2))\*a\*g^2+3/4/(d\*g-e\*f)^3/((d\*g-e\*f)\*e)^(1/2)\*arctan((g\*x+f)^(1/2)\*e/((d\*g-e\*f)\*e)^(1/2))\*b\*d\*g^2+3/(d\*g-e\*f)^3\*e/((d\*g-e\*f)\*e)^(1/2)\*arctan((g\*x+f)^(1/2)\*e/((d\*g-e\*f)\*e)^(1/2))\*b\*f\*g+1/4/(d\*g-e\*f)^3/e/((d\*g-e\*f)\*e)^(1/2)\*arctan((g\*x+f)^(1/2)\*e/((d\*g-e\*f)\*e)^(1/2))\*c\*d^2\*g^2-2/(d\*g-e\*f)^3/((d\*g-e\*f)\*e)^(1/2)\*arctan((g\*x+f)^(1/2)\*e/((d\*g-e\*f)\*e)^(1/2))\*c\*d\*f\*g-2/(d\*g-e\*f)^3\*e/((d\*g-e\*f)\*e)^(1/2)\*arctan((g\*x+f)^(1/2)\*e/((d\*g-e\*f)\*e)^(1/2))\*c\*f^2

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/((e\*x + d)^3\*(g\*x + f)^(3/2)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas** [A] time = 0.30137, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/((e\*x + d)^3\*(g\*x + f)^(3/2)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/8*((8*c*d^2*e^2*f^2 + 4*(2*c*d^3*e - 3*b*d^2*e^2)*f*g - (c*d^4 + 3*b*d^3*e - 15*a*d^2*e^2)*g^2 + (8*c*e^4*f^2 + 4*(2*c*d*e^3 - 3*b*e^4)*f*g - (c*d^2*e^2 + 3*b*d*e^3 - 15*a*e^4)*g^2)*x^2 + 2*(8*c*d*e^3*f^2 + 4*(2*c*d^2*e^2 - 3*b*d*e^3)*f*g - (c*d^3*e + 3*b*d^2*e^2 - 15*a*d*e^3)*g^2)*x)*\sqrt{g*x + f}*\log((\sqrt{e^2*f - d*e*g})*(e*g*x + 2*e*f - d*g) + 2*(e^2*f - d*e*g)*\sqrt{g*x + f})/(e*x + d)) - 2*(8*a*d^2*e*g^2 + 2*(7*c*d^2*e - b*d*e^2 - a*e^3)*f^2 + (c*d^3 - 13*b*d^2*e + 9*a*d*e^2)*f*g + (8*c*e^3*f^2 + 4*(2*c*d*e^2 - 3*b*e^3)*f*g - (c*d^2*e + 3*b*d*e^2 - 15*a*e^3)*g^2)*x^2 + (4*(6*c*d*e^2 - b*e^3)*f^2 + (5*c*d^2*e - 21*b*d*e^2 + 5*a*e^3)*f*g + (c*d^3 - 5*b*d^2*e + 25*a*d*e^2)*g^2)*x)*\sqrt{e^2*f - d*e*g}) / ((d^2*e^4*f^3 - 3*d^3*e^3*f^2*g + 3*d^4*e^2*f*g^2 - d^5*e*g^3 + (e^6*f^3 - 3*d*e^5*f^2*g + 3*d^2*e^4*f*g^2 - d^3*e^3*g^3)*x^2 + 2*(d*e^5*f^3 - 3*d^2*e^4*f^2*g + 3*d^3*e^3*f*g^2 - d^4*e^2*g^3)*x)*\sqrt{e^2*f - d*e*g}*\sqrt{g*x + f}), -1/4*((8*c*d^2*e^2*f^2 + 4*(2*c*d^3*e - 3*b*d^2*e^2)*f*g - (c*d^4 + 3*b*d^3*e - 15*a*d^2*e^2)*g^2 + (8*c*e^4*f^2 + 4*(2*c*d*e^3 - 3*b*e^4)*f*g - (c*d^2*e^2 + 3*b*d*e^3 - 15*a*e^4)*g^2)*x^2 + 2*(8*c*d*e^3*f^2 + 4*(2*c*d^2*e^2 - 3*b*d*e^3)*f*g - (c*d^3*e + 3*b*d^2*e^2 - 15*a*d*e^3)*g^2)*x)*\sqrt{g*x + f}*\arctan(-(e*f - d*g)/(\sqrt{-e^2*f + d*e*g}*\sqrt{g*x + f})) - (8*a*d^2*e*g^2 + 2*(7*c*d^2*e - b*d*e^2 - a*e^3)*f^2 + (c*d^3 - 13*b*d^2*e + 9*a*d*e^2)*f*g + (8*c*e^3*f^2 + 4*(2*c*d*e^2 - 3*b*e^3)*f*g - (c*d^2*e + 3*b*d*e^2 - 15*a*e^3)*g^2)*x^2 + (4*(6*c*d*e^2 - b*e^3)*f^2 + (5*c*d^2*e - 21*b*d*e^2 + 5*a*e^3)*f*g + (c*d^3 - 5*b*d^2*e + 25*a*d*e^2)*g^2)*x)*\sqrt{-e^2*f + d*e*g}) / ((d^2*e^4*f^3 - 3*d^3*e^3*f^2*g + 3*d^4*e^2*f*g^2 - d^5*e*g^3 + (e^6*f^3 - 3*d*e^5*f^2*g + 3*d^2*e^4*f*g^2 - d^3*e^3*g^3)*x^2 + 2*(d*e^5*f^3 - 3*d^2*e^4*f^2*g + 3*d^3*e^3*f*g^2 - d^4*e^2*g^3)*x)*\sqrt{-e^2*f + d*e*g}*\sqrt{g*x + f})] \end{aligned}$$

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)/(e\*x+d)\*\*3/(g\*x+f)\*\*(3/2),x)

[Out] Timed out

**GIAC/XCAS** [A] time = 0.2786, size = 624, normalized size = 2.53

$$\frac{(cd^2g^2 - 8cdfge + 3bdg^2e - 8cf^2e^2 + 12bfge^2 - 15ag^2e^2) \arctan\left(\frac{\sqrt{gx+fe}}{\sqrt{dge-fe^2}}\right)}{4(d^3g^3e - 3d^2fg^2e^2 + 3df^2ge^3 - f^3e^4)\sqrt{dge - fe^2} - 2(cf^2 - bfg + ag^2)} - \frac{(d^3g^3 - 3d^2fg^2e + 3df^2ge^2 - f^3e^3)\sqrt{gx + f}}{\sqrt{gx + f}cd^3g^3 - (gx + f)^{\frac{3}{2}}cd^2g^2e + 7\sqrt{gx + f}cd^2fg^2e - 5\sqrt{gx + f}bd^2g^3e + 8(gx + f)^{\frac{3}{2}}cdfge^2 - 8\sqrt{gx + f}cd^2f^2ge^2 - 3} + \frac{4(d^3g^3e - 3d^2fg^2e^2 + 3df^2ge^3 - f^3e^4)\sqrt{dge - fe^2}}{4(d^3g^3e - 3d^2fg^2e^2 + 3df^2ge^3 - f^3e^4)\sqrt{dge - fe^2} - 2(cf^2 - bfg + ag^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/((e\*x + d)^3\*(g\*x + f)^(3/2)),x, algorithm="giac")

[Out] 
$$\frac{1}{4} \frac{(c d^2 g^2 - 8 c d f g e + 3 b^2 d g^2 e - 8 c f^2 e^2 + 12 b f g e^2 - 15 a g^2 e^2) \arctan\left(\frac{\sqrt{g x + f} e}{\sqrt{d g e - f e^2}}\right) + ((d^3 g^3 e - 3 d^2 f g^2 e^2 + 3 d f^2 g e^3 - f^3 e^4) \sqrt{d g e - f e^2}) - 2(c f^2 - b f g + a g^2) \sqrt{g x + f} - \frac{1}{4}(\sqrt{g x + f})^3 (c d^3 g^3 - (g x + f)^{3/2} c d^2 g^2 e + 7 \sqrt{g x + f} c d^2 f g^2 e - 5 \sqrt{g x + f} b d^2 g^3 e + 8 (g x + f)^{3/2} c d f g^2 e^2 - 8 \sqrt{g x + f} c d f^2 g^2 e^2 - 3 (g x + f)^{3/2} b d g^2 e^2 + \sqrt{g x + f} b d f g^2 e^2 + 9 \sqrt{g x + f} a d g^3 e^2 - 4 (g x + f)^{3/2} b f g^2 e^3 + 4 \sqrt{g x + f} b f^2 g^2 e^3 + 7 (g x + f)^{3/2} a g^2 e^3 - 9 \sqrt{g x + f} a f g^2 e^3)}{(d^3 g^3 e - 3 d^2 f g^2 e^2 + 3 d f^2 g e^3 - f^3 e^4) (d g + (g x + f) e - f e)^2}$$



$$3.833 \quad \int \frac{\sqrt{-1+x}\sqrt{1+x}}{1+x-x^2} dx$$

**Optimal.** Leaf size=91

$$\sqrt{\frac{2}{5}}(\sqrt{5}-1) \tan^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{\sqrt{5}-2}\sqrt{x-1}}\right) - \cosh^{-1}(x) + \sqrt{\frac{2}{5}}(1+\sqrt{5}) \tanh^{-1}\left(\frac{\sqrt{x+1}}{\sqrt{2+\sqrt{5}}\sqrt{x-1}}\right)$$

[Out] -ArcCosh[x] + Sqrt[(2\*(-1 + Sqrt[5]))/5]\*ArcTan[Sqrt[1 + x]/(Sqrt[-2 + Sqrt[5]]\*Sqrt[-1 + x])] + Sqrt[(2\*(1 + Sqrt[5]))/5]\*ArcTanh[Sqrt[1 + x]/(Sqrt[2 + Sqrt[5]]\*Sqrt[-1 + x])]

**Rubi [B]** time = 0.297751, antiderivative size = 191, normalized size of antiderivative = 2.1, number of steps used = 9, number of rules used = 7, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.28$

$$\frac{\sqrt{\frac{1}{10}}(\sqrt{5}-1)\sqrt{x-1}\sqrt{x+1} \tan^{-1}\left(\frac{2-(1-\sqrt{5})x}{\sqrt{2(\sqrt{5}-1)}\sqrt{x^2-1}}\right) - \frac{\sqrt{x-1}\sqrt{x+1} \tanh^{-1}\left(\frac{x}{\sqrt{x^2-1}}\right)}{\sqrt{x^2-1}}}{\sqrt{x^2-1}} - \frac{\sqrt{\frac{1}{10}}(1+\sqrt{5})\sqrt{x-1}\sqrt{x+1} \tanh^{-1}\left(\frac{2-(1+\sqrt{5})x}{\sqrt{2(1+\sqrt{5})}\sqrt{x^2-1}}\right)}{\sqrt{x^2-1}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[-1 + x]\*Sqrt[1 + x])/(1 + x - x^2), x]

[Out] (Sqrt[(-1 + Sqrt[5])/10]\*Sqrt[-1 + x]\*Sqrt[1 + x]\*ArcTan[(2 - (1 - Sqrt[5])\*x)/(Sqrt[2\*(-1 + Sqrt[5])]\*Sqrt[-1 + x^2])])/Sqrt[-1 + x^2] - (Sqrt[-1 + x]\*Sqrt[1 + x]\*ArcTanh[x/Sqrt[-1 + x^2]])/Sqrt[-1 + x^2] - (Sqrt[(1 + Sqrt[5])/10]\*Sqrt[-1 + x]\*Sqrt[1 + x]\*ArcTanh[(2 - (1 + Sqrt[5])\*x)/(Sqrt[2\*(1 + Sqrt[5])]\*Sqrt[-1 + x^2])])/Sqrt[-1 + x^2]

**Rubi in Sympy [A]** time = 33.4144, size = 187, normalized size = 2.05

$$\frac{\sqrt{10}\sqrt{-1+\sqrt{5}}\sqrt{x-1}\sqrt{x+1} \operatorname{atan}\left(\frac{\sqrt{2}(x(-1+\sqrt{5})+2)}{2\sqrt{-1+\sqrt{5}}\sqrt{x^2-1}}\right) - \frac{\sqrt{x-1}\sqrt{x+1} \operatorname{atanh}\left(\frac{x}{\sqrt{x^2-1}}\right)}{\sqrt{x^2-1}}}{10\sqrt{x^2-1}} - \frac{\sqrt{10}\sqrt{1+\sqrt{5}}\sqrt{x-1}\sqrt{x+1} \operatorname{atanh}\left(\frac{\sqrt{2}(x(-\sqrt{5}-1)+2)}{2\sqrt{1+\sqrt{5}}\sqrt{x^2-1}}\right)}{10\sqrt{x^2-1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((-1+x)\*\*(1/2)\*(1+x)\*\*(1/2)/(-x\*\*2+x+1), x)

[Out] sqrt(10)\*sqrt(-1 + sqrt(5))\*sqrt(x - 1)\*sqrt(x + 1)\*atan(sqrt(2)\*(x\*(-1 + sqrt(5)) + 2)/(2\*sqrt(-1 + sqrt(5))\*sqrt(x\*\*2 - 1)))/(10\*sqrt(x\*\*2 - 1)) - sqrt(x - 1)\*sqrt(x + 1)\*atanh(x/sqrt(x\*\*2 - 1))/sqrt(x\*\*2 - 1) - sqrt(10)\*sqrt(1 + sqrt(5))\*sqrt(x - 1)\*sqrt(x + 1)\*atanh(sqrt(2)\*(x\*(-sqrt(5) - 1) + 2)/(2\*sqrt(1 + sqrt(5))\*sqrt(x\*\*2 - 1)))/(10\*sqrt(x\*\*2 - 1))

**Mathematica [A]** time = 0.775611, size = 143, normalized size = 1.57

$$\frac{-2 \sinh^{-1}\left(\frac{\sqrt{x-1}}{\sqrt{2}}\right) \sqrt{\frac{2}{x-1} + 1} \sqrt{x-1} \left( (\sqrt{5}-3) \sqrt{2+\sqrt{5}} \tan^{-1}\left(\sqrt{2+\sqrt{5}} \sqrt{\frac{2}{x-1} + 1}\right) - \sqrt{\sqrt{5}-2} (3+\sqrt{5}) \tanh^{-1}\left(\sqrt{\sqrt{5}-2} \sqrt{\frac{2}{x-1} + 1}\right) \right)}{\sqrt{5}\sqrt{x+1}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[-1 + x]\*Sqrt[1 + x])/(1 + x - x^2), x]

[Out] -2\*ArcSinh[Sqrt[-1 + x]/Sqrt[2]] - (Sqrt[1 + 2/(-1 + x)]\*Sqrt[-1 + x]\*((-3 + Sqrt[5])\*Sqrt[2 + Sqrt[5]]\*ArcTan[Sqrt[2 + Sqrt[5]]\*Sqrt[1 + 2/(-1 + x)]] - Sqrt[-2 + Sqrt[5]]\*(3 + Sqrt[5])\*ArcTanh[Sqrt[-2 + Sqrt[5]]\*Sqrt[1 + 2/(-1 + x)]]))/(Sqrt[5]\*Sqrt[1 + x])

**Maple [B]** time = 0.094, size = 226, normalized size = 2.5

$$-\frac{1}{5\sqrt{2}\sqrt{5+2}\sqrt{-2+2}\sqrt{5}}\sqrt{-1+x}\sqrt{1+x}\left(\arctan\left(\frac{x\sqrt{5}-x+2}{\sqrt{-2+2}\sqrt{5}}\frac{1}{\sqrt{x^2-1}}\right)\sqrt{5}\sqrt{2}\sqrt{5+2}+5\ln\left(x+\sqrt{x^2-1}\right)\sqrt{2}\sqrt{5+2}\sqrt{-2+2}\sqrt{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((-1+x)^(1/2)\*(1+x)^(1/2)/(-x^2+x+1), x)

[Out] -1/5\*(-1+x)^(1/2)\*(1+x)^(1/2)\*(arctan((x\*5^(1/2)-x+2)/(-2+2\*5^(1/2)))^(1/2)/(x^2-1)^(1/2))\*5^(1/2)\*(2\*5^(1/2)+2)^(1/2)+5\*ln(x+(x^2-1)^(1/2))\*(2\*5^(1/2)+2)^(1/2)\*(-2+2\*5^(1/2))^(1/2)-arctanh((x\*5^(1/2)+x-2)/(2\*5^(1/2)+2)^(1/2)/(x^2-1)^(1/2))\*5^(1/2)\*(-2+2\*5^(1/2))^(1/2)-5\*arctan((x\*5^(1/2)-x+2)/(-2+2\*5^(1/2))^(1/2)/(x^2-1)^(1/2))\*(2\*5^(1/2)+2)^(1/2)-5\*arctanh((x\*5^(1/2)+x-2)/(2\*5^(1/2)+2)^(1/2)/(x^2-1)^(1/2))\*(-2+2\*5^(1/2))^(1/2)/(x^2-1)^(1/2)/(2\*5^(1/2)+2)^(1/2)/(-2+2\*5^(1/2))^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{x+1}\sqrt{x-1}}{x^2-x-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-sqrt(x + 1)\*sqrt(x - 1)/(x^2 - x - 1), x, algorithm="maxima")

[Out] -integrate(sqrt(x + 1)\*sqrt(x - 1)/(x^2 - x - 1), x)

**Fricas [A]** time = 0.296825, size = 308, normalized size = 3.38

$$\begin{aligned} & \frac{2}{5} \sqrt{2} \sqrt{-\sqrt{5}(\sqrt{5}-5)} \arctan \left( \frac{\sqrt{5}\sqrt{2}\sqrt{-\sqrt{5}(\sqrt{5}-5)}}{5 \left( 2\sqrt{x+1}\sqrt{x-1} - 2x - \sqrt{5} + \sqrt{-4(2x+\sqrt{5}-1)\sqrt{x+1}\sqrt{x-1} + 8x^2 + 4\sqrt{5}x - 4x + 1} \right)} \right) \\ & + \frac{1}{10} \sqrt{2} \sqrt{\sqrt{5}(\sqrt{5}+5)} \log \left( \frac{1}{5} \sqrt{5}\sqrt{2}\sqrt{\sqrt{5}(\sqrt{5}+5)} + 2\sqrt{x+1}\sqrt{x-1} - 2x + \sqrt{5} + 1 \right) \\ & - \frac{1}{10} \sqrt{2} \sqrt{\sqrt{5}(\sqrt{5}+5)} \log \left( -\frac{1}{5} \sqrt{5}\sqrt{2}\sqrt{\sqrt{5}(\sqrt{5}+5)} + 2\sqrt{x+1}\sqrt{x-1} - 2x + \sqrt{5} + 1 \right) \\ & + \log(\sqrt{x+1}\sqrt{x-1} - x) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-sqrt(x + 1)\*sqrt(x - 1)/(x^2 - x - 1), x, algorithm="fricas")

[Out] 2/5\*sqrt(2)\*sqrt(-sqrt(5)\*(sqrt(5) - 5))\*arctan(1/5\*sqrt(5)\*sqrt(2)\*sqrt(-sqrt(5)\*(sqrt(5) - 5)))/(2\*sqrt(x + 1)\*sqrt(x - 1) - 2\*x - sqrt(5) + sqrt(-4\*(2\*x + sqrt(5) - 1)\*sqrt(x + 1)\*sqrt(x - 1) + 8\*x^2 + 4\*sqrt(5)\*x - 4\*x) + 1) + 1/10\*sqrt(2)\*sqrt(sqrt(5)\*(sqrt(5) + 5))\*log(1/5\*sqrt(5)\*sqrt(2)\*sqrt(sqrt(5)\*(sqrt(5) + 5)) + 2\*sqrt(x + 1)\*sqrt(x - 1) - 2\*x + sqrt(5) + 1) - 1/10\*sqrt(2)\*sqrt(sqrt(5)\*(sqrt(5) + 5))\*log(-1/5\*sqrt(5)\*sqrt(2)\*sqrt(sqrt(5)\*(sqrt(5) + 5)) + 2\*sqrt(x + 1)\*sqrt(x - 1) - 2\*x + sqrt(5) + 1) + log(sqrt(x + 1)\*sqrt(x - 1) - x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sqrt{x-1}\sqrt{x+1}}{x^2-x-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((-1+x)\*\*(1/2)\*(1+x)\*\*(1/2)/(-x\*\*2+x+1), x)

[Out] -Integral(sqrt(x - 1)\*sqrt(x + 1)/(x\*\*2 - x - 1), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int -\frac{\sqrt{x+1}\sqrt{x-1}}{x^2-x-1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(-sqrt(x + 1)\*sqrt(x - 1)/(x^2 - x - 1), x, algorithm="giac")

[Out] integrate(-sqrt(x + 1)\*sqrt(x - 1)/(x^2 - x - 1), x)

$$3.834 \quad \int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=164

$$\frac{\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right) (4eg(2aeg - b(dg + ef)) + c(3d^2g^2 + 2defg + 3e^2f^2))}{4e^{5/2}g^{5/2}} - \frac{\sqrt{d+ex}\sqrt{f+gx}(-4beg + 5cdg + 3cef)}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

[Out]  $-\left(\left(3c^2ef + 5c^2dg - 4b^2eg\right)\sqrt{d+ex}\sqrt{f+gx}\right)/\left(4e^{5/2}g^{5/2}\right) + \left(c^2(d+ex)^{3/2}\sqrt{f+gx}\right)/\left(2e^2g\right) + \left(\left(c^2(3e^2f^2 + 2d^2efg + 3d^2g^2) + 4e^2g^2(2ae^2g - b^2(e^2f + d^2g))\right)\operatorname{ArcTanh}\left[\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right]\right)/\left(4e^{5/2}g^{5/2}\right)$

**Rubi [A]** time = 0.413231, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right) (4eg(2aeg - b(dg + ef)) + c(3d^2g^2 + 2defg + 3e^2f^2))}{4e^{5/2}g^{5/2}} - \frac{\sqrt{d+ex}\sqrt{f+gx}(-4beg + 5cdg + 3cef)}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)/(Sqrt[d + e\*x]\*Sqrt[f + g\*x]),x]

[Out]  $-\left(\left(3c^2ef + 5c^2dg - 4b^2eg\right)\sqrt{d+ex}\sqrt{f+gx}\right)/\left(4e^{5/2}g^{5/2}\right) + \left(c^2(d+ex)^{3/2}\sqrt{f+gx}\right)/\left(2e^2g\right) + \left(\left(c^2(3e^2f^2 + 2d^2efg + 3d^2g^2) + 4e^2g^2(2ae^2g - b^2(e^2f + d^2g))\right)\operatorname{ArcTanh}\left[\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right]\right)/\left(4e^{5/2}g^{5/2}\right)$

**Rubi in Sympy [A]** time = 37.9118, size = 199, normalized size = 1.21

$$\frac{b\sqrt{d+ex}\sqrt{f+gx}}{eg} + \frac{cx\sqrt{d+ex}\sqrt{f+gx}}{2eg} - \frac{3c\sqrt{d+ex}\sqrt{f+gx}(dg+ef)}{4e^2g^2} - \frac{c(4defg - 3(dg+ef)^2) \operatorname{atanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{4e^{5/2}g^{5/2}} - \frac{2\left(-aeg + \frac{b(dg+ef)}{2}\right) \operatorname{atanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{e^{3/2}g^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+b\*x+a)/(e\*x+d)\*\*(1/2)/(g\*x+f)\*\*(1/2),x)

[Out]  $b\sqrt{d+ex}\sqrt{f+gx}/(e^2g) + c^2x\sqrt{d+ex}\sqrt{f+gx}/(2e^2g) - 3c^2\sqrt{d+ex}\sqrt{f+gx}(dg+ef)/(4e^2g^2) - c^2(4d^2efg - 3(dg+ef)^2)\operatorname{atanh}(\sqrt{e}\sqrt{f+gx}/(\sqrt{g}\sqrt{d+ex}))/\left(4e^{5/2}g^{5/2}\right) - 2(-aeg + \frac{b(dg+ef)}{2})\operatorname{atanh}(\sqrt{e}\sqrt{f+gx}/(\sqrt{g}\sqrt{d+ex}))/\left(e^{3/2}g^{3/2}\right)$

**Mathematica [A]** time = 0.207739, size = 154, normalized size = 0.94

$$\frac{\log\left(2\sqrt{e}\sqrt{g}\sqrt{d+ex}\sqrt{f+gx}+dg+ef+2egx\right)\left(4eg(2aeg-b(dg+ef))+c(3d^2g^2+2defg+3e^2f^2)\right)}{8e^{5/2}g^{5/2}} + \frac{\sqrt{d+ex}\sqrt{f+gx}(4beg+c(-3dg-3ef+2egx))}{4e^2g^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)/(Sqrt[d + e\*x]\*Sqrt[f + g\*x]),x]

[Out] (Sqrt[d + e\*x]\*Sqrt[f + g\*x]\*(4\*b\*e\*g + c\*(-3\*e\*f - 3\*d\*g + 2\*e\*g\*x)))/(4\*e^2\*g^2) + ((c\*(3\*e^2\*f^2 + 2\*d\*e\*f\*g + 3\*d^2\*g^2) + 4\*e\*g\*(2\*a\*e\*g - b\*(e\*f + d\*g)))\*Log[e\*f + d\*g + 2\*e\*g\*x + 2\*Sqrt[e]\*Sqrt[g]\*Sqrt[d + e\*x]\*Sqrt[f + g\*x]])/(8\*e^(5/2)\*g^(5/2))

**Maple [B]** time = 0.032, size = 425, normalized size = 2.6

$$\frac{1}{8e^2g^2} \left( 8 \ln \left( \frac{1}{2} \frac{2egx + 2\sqrt{(ex+d)(gx+f)}\sqrt{eg} + dg + ef}{\sqrt{eg}} \right) ae^2g^2 - 4 \ln \left( \frac{1}{2} \frac{2egx + 2\sqrt{(ex+d)(gx+f)}\sqrt{eg} + dg + ef}{\sqrt{eg}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)/(e\*x+d)^(1/2)/(g\*x+f)^(1/2),x)

[Out] 1/8\*(8\*ln(1/2\*(2\*e\*g\*x+2\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+d\*g+e\*f)/(e\*g)^(1/2))\*a\*e^2\*g^2-4\*ln(1/2\*(2\*e\*g\*x+2\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+d\*g+e\*f)/(e\*g)^(1/2))\*b\*d\*e\*g^2-4\*ln(1/2\*(2\*e\*g\*x+2\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+d\*g+e\*f)/(e\*g)^(1/2))\*b\*e^2\*f\*g+3\*ln(1/2\*(2\*e\*g\*x+2\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+d\*g+e\*f)/(e\*g)^(1/2))\*c\*d^2\*g^2+2\*c\*ln(1/2\*(2\*e\*g\*x+2\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+d\*g+e\*f)/(e\*g)^(1/2))\*d\*f\*e\*g+3\*ln(1/2\*(2\*e\*g\*x+2\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+d\*g+e\*f)/(e\*g)^(1/2))\*c\*e^2\*f^2+4\*c\*((e\*x+d)\*(g\*x+f))^(1/2)\*x\*e\*g\*(e\*g)^(1/2)+8\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)\*b\*e\*g-6\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)\*c\*d\*g-6\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)\*c\*e\*f\*(e\*x+d)^(1/2)\*(g\*x+f)^(1/2)/(e\*g)^(1/2)/g^2/e^2/((e\*x+d)\*(g\*x+f))^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/(sqrt(e\*x + d)\*sqrt(g\*x + f)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.47362, size = 1, normalized size = 0.01

$$\left[ \frac{4(2cegx - 3cef - (3cd - 4be)g)\sqrt{eg}\sqrt{ex+d}\sqrt{gx+f} + (3ce^2f^2 + 2(cde - 2be^2)fg + (3cd^2 - 4bde + 8ae^2)g^2) \log(4)}{16\sqrt{eg}e^2g^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/(sqrt(e\*x + d)\*sqrt(g\*x + f)),x, algorithm="fricas")

[Out] [1/16\*(4\*(2\*c\*e\*g\*x - 3\*c\*e\*f - (3\*c\*d - 4\*b\*e)\*g)\*sqrt(e\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f) + (3\*c\*e^2\*f^2 + 2\*(c\*d\*e - 2\*b\*e^2)\*f\*g + (3\*c\*d^2 - 4\*b\*d\*e + 8\*a\*e^2)\*g^2)\*log(4\*(2\*e^2\*g^2\*x + e^2\*f\*g + d\*e\*g^2)\*sqrt(e\*x + d)\*sqrt(g\*x + f) + (8\*e^2\*g^2\*x^2 + e^2\*f^2 + 6\*d\*e\*f\*g + d^2\*g^2 + 8\*(e^2\*f\*g + d\*e\*g^2)\*x)\*sqrt(e\*g)))/(sqrt(e\*g)\*e^2\*g^2), 1/8\*(2\*(2\*c\*e\*g\*x - 3\*c\*e\*f - (3\*c\*d - 4\*b\*e)\*g)\*sqrt(-e\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f) + (3\*c\*e^2\*f^2 + 2\*(c\*d\*e - 2\*b\*e^2)\*f\*g + (3\*c\*d^2 - 4\*b\*d\*e + 8\*a\*e^2)\*g^2)\*arctan(1/2\*(2\*e\*g\*x + e\*f + d\*g)\*sqrt(-e\*g)/(sqrt(e\*x + d)\*sqrt(g\*x + f)\*e\*g)))/(sqrt(-e\*g)\*e^2\*g^2)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + bx + cx^2}{\sqrt{d + ex}\sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)/(e\*x+d)\*\*(1/2)/(g\*x+f)\*\*(1/2),x)

[Out] Integral((a + b\*x + c\*x\*\*2)/(sqrt(d + e\*x)\*sqrt(f + g\*x)), x)

**GIAC/XCAS [A]** time = 0.287946, size = 242, normalized size = 1.48

$$\frac{\frac{1}{4}\sqrt{(xe+d)ge-dge+fe^2}\sqrt{xe+d}\left(\frac{2(xe+d)ce^{(-3)}}{g}-\frac{(5cdg^2e^5+3cfge^6-4bg^2e^6)e^{(-8)}}{g^3}\right)}{(3cd^2g^2+2cdfge-4bdg^2e+3cf^2e^2-4bfge^2+8ag^2e^2)e^{(-\frac{5}{2})}\ln\left(\left|-\sqrt{xe+d}\sqrt{ge}^{\frac{1}{2}}+\sqrt{(xe+d)ge-dge+fe^2}\right|\right)}}{4g^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/(sqrt(e\*x + d)\*sqrt(g\*x + f)),x, algorithm="giac")

[Out] 1/4\*sqrt((x\*e + d)\*g\*e - d\*g\*e + f\*e^2)\*sqrt(x\*e + d)\*(2\*(x\*e + d)\*c\*e^(-3)/g - (5\*c\*d\*g^2\*e^5 + 3\*c\*f\*g\*e^6 - 4\*b\*g^2\*e^6)\*e^(-8)/g^3) - 1/4\*(3\*c\*d^2\*g^2 + 2\*c\*d\*f\*g\*e - 4\*b\*d\*g^2\*e + 3\*c\*f^2\*e^2 - 4\*b\*f\*g\*e^2 + 8\*a\*g^2\*e^2)\*e^(-5/2)\*ln(abs(-sqrt(x\*e + d)\*sqrt(g)\*e^(1/2) + sqrt((x\*e + d)\*g\*e - d\*g\*e + f\*e^2)))/g^(5/2)

$$3.835 \quad \int \frac{(d+ex)^{3/2}(a+bx+cx^2)}{\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=333

$$\begin{aligned} & \frac{\sqrt{d+ex}\sqrt{f+gx}(ef-dg)(8eg(6aeg-b(dg+5ef))+c(3d^2g^2+10defg+35e^2f^2))}{64e^2g^4} \\ & + \frac{(d+ex)^{3/2}\sqrt{f+gx}(8eg(6aeg-b(dg+5ef))+c(3d^2g^2+10defg+35e^2f^2))}{96e^2g^3} \\ & + \frac{(ef-dg)^2 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)(8eg(6aeg-b(dg+5ef))+c(3d^2g^2+10defg+35e^2f^2))}{64e^{5/2}g^{9/2}} \\ & - \frac{(d+ex)^{5/2}\sqrt{f+gx}(-8beg+9cdg+7cef)}{24e^2g^2} + \frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g} \end{aligned}$$

[Out]  $-\left((e^*f - d^*g) * (c * (35 * e^2 * f^2 + 10 * d * e * f * g + 3 * d^2 * g^2) + 8 * e * g * (6 * a * e * g - b * (5 * e * f + d * g))) * \text{Sqrt}[d + e * x] * \text{Sqrt}[f + g * x]\right) / (64 * e^2 * g^4) + \left((c * (35 * e^2 * f^2 + 10 * d * e * f * g + 3 * d^2 * g^2) + 8 * e * g * (6 * a * e * g - b * (5 * e * f + d * g))) * (d + e * x)^{(3/2)} * \text{Sqrt}[f + g * x]\right) / (96 * e^2 * g^3) - \left((7 * c * e * f + 9 * c * d * g - 8 * b * e * g) * (d + e * x)^{(5/2)} * \text{Sqrt}[f + g * x]\right) / (24 * e^2 * g^2) + \left(c * (d + e * x)^{(7/2)} * \text{Sqrt}[f + g * x]\right) / (4 * e^2 * g) + \left((e^*f - d^*g)^2 * (c * (35 * e^2 * f^2 + 10 * d * e * f * g + 3 * d^2 * g^2) + 8 * e * g * (6 * a * e * g - b * (5 * e * f + d * g))) * \text{ArcTanh}[\left(\frac{\text{Sqrt}[g] * \text{Sqrt}[d + e * x]}{\text{Sqrt}[e] * \text{Sqrt}[f + g * x]}\right)]\right) / (64 * e^{5/2} * g^{9/2})$

**Rubi [A]** time = 0.788135, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$\begin{aligned} & \frac{\sqrt{d+ex}\sqrt{f+gx}(ef-dg)(8eg(6aeg-b(dg+5ef))+c(3d^2g^2+10defg+35e^2f^2))}{64e^2g^4} \\ & + \frac{(d+ex)^{3/2}\sqrt{f+gx}(8eg(6aeg-b(dg+5ef))+c(3d^2g^2+10defg+35e^2f^2))}{96e^2g^3} \\ & + \frac{(ef-dg)^2 \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)(8eg(6aeg-b(dg+5ef))+c(3d^2g^2+10defg+35e^2f^2))}{64e^{5/2}g^{9/2}} \\ & - \frac{(d+ex)^{5/2}\sqrt{f+gx}(-8beg+9cdg+7cef)}{24e^2g^2} + \frac{c(d+ex)^{7/2}\sqrt{f+gx}}{4e^2g} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\left((d + e * x)^{(3/2)} * (a + b * x + c * x^2)\right) / \text{Sqrt}[f + g * x], x]$

[Out]  $-\left((e^*f - d^*g) * (c * (35 * e^2 * f^2 + 10 * d * e * f * g + 3 * d^2 * g^2) + 8 * e * g * (6 * a * e * g - b * (5 * e * f + d * g))) * \text{Sqrt}[d + e * x] * \text{Sqrt}[f + g * x]\right) / (64 * e^2 * g^4) + \left((c * (35 * e^2 * f^2 + 10 * d * e * f * g + 3 * d^2 * g^2) + 8 * e * g * (6 * a * e * g - b * (5 * e * f + d * g))) * (d + e * x)^{(3/2)} * \text{Sqrt}[f + g * x]\right) / (96 * e^2 * g^3) - \left((7 * c * e * f + 9 * c * d * g - 8 * b * e * g) * (d + e * x)^{(5/2)} * \text{Sqrt}[f + g * x]\right) / (24 * e^2 * g^2) + \left(c * (d + e * x)^{(7/2)} * \text{Sqrt}[f + g * x]\right) / (4 * e^2 * g) + \left((e^*f - d^*g)^2 * (c * (35 * e^2 * f^2 + 10 * d * e * f * g + 3 * d^2 * g^2) + 8 * e * g * (6 * a * e * g - b * (5 * e * f + d * g))) * \text{ArcTanh}[\left(\frac{\text{Sqrt}[g] * \text{Sqrt}[d + e * x]}{\text{Sqrt}[e] * \text{Sqrt}[f + g * x]}\right)]\right) / (64 * e^{5/2} * g^{9/2})$

**Rubi in Sympy [A]** time = 74.4476, size = 434, normalized size = 1.3

$$\begin{aligned} & \frac{b(d+ex)^{\frac{5}{2}}\sqrt{f+gx}}{3eg} + \frac{cx(d+ex)^{\frac{5}{2}}\sqrt{f+gx}}{4eg} - \frac{c(d+ex)^{\frac{5}{2}}\sqrt{f+gx}(3dg+7ef)}{24e^2g^2} \\ & + \frac{c(d+ex)^{\frac{3}{2}}\sqrt{f+gx}(3d^2g^2+10defg+35e^2f^2)}{96e^2g^3} \\ & + \frac{c\sqrt{d+ex}\sqrt{f+gx}(dg-ef)(3d^2g^2+10defg+35e^2f^2)}{64e^2g^4} \\ & + \frac{c(dg-ef)^2(3d^2g^2+10defg+35e^2f^2)\operatorname{atanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{64e^{\frac{5}{2}}g^{\frac{9}{2}}} \\ & + \frac{(d+ex)^{\frac{3}{2}}\sqrt{f+gx}(6aeg-bdg-5bef)}{12eg^2} + \frac{\sqrt{d+ex}\sqrt{f+gx}(dg-ef)(6aeg-bdg-5bef)}{8eg^3} \\ & + \frac{(dg-ef)^2(6aeg-bdg-5bef)\operatorname{atanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{8e^{\frac{3}{2}}g^{\frac{7}{2}}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**(3/2)*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)`

[Out] `b*(d+e*x)**(5/2)*sqrt(f+g*x)/(3*e*g) + c*x*(d+e*x)**(5/2)*sqrt(f+g*x)/(4*e*g) - c*(d+e*x)**(5/2)*sqrt(f+g*x)*(3*d*g+7*e*f)/(24*e**2*g**2) + c*(d+e*x)**(3/2)*sqrt(f+g*x)*(3*d**2*g**2+10*d*e*f*g+35*e**2*f**2)/(96*e**2*g**3) + c*sqrt(d+e*x)*sqrt(f+g*x)*(d*g-e*f)*(3*d**2*g**2+10*d*e*f*g+35*e**2*f**2)/(64*e**2*g**4) + c*(d*g-e*f)**2*(3*d**2*g**2+10*d*e*f*g+35*e**2*f**2)*atanh(sqrt(e)*sqrt(f+g*x)/(sqrt(g)*sqrt(d+e*x)))/(64*e**(5/2)*g**(9/2)) + (d+e*x)**(3/2)*sqrt(f+g*x)*(6*a*e*g-b*d*g-5*b*e*f)/(12*e*g**2) + sqrt(d+e*x)*sqrt(f+g*x)*(d*g-e*f)*(6*a*e*g-b*d*g-5*b*e*f)/(8*e*g**3) + (d*g-e*f)**2*(6*a*e*g-b*d*g-5*b*e*f)*atanh(sqrt(e)*sqrt(f+g*x)/(sqrt(g)*sqrt(d+e*x)))/(8*e**(3/2)*g**(7/2))`

**Mathematica [A]** time = 0.573291, size = 306, normalized size = 0.92

$$\frac{3(ef-dg)^2 \log\left(2\sqrt{e}\sqrt{g}\sqrt{d+ex}\sqrt{f+gx}+dg+ef+2egx\right) (8eg(6aeg-b(dg+5ef))+c(3d^2g^2+10defg+35e^2f^2))-2}{}$$

Antiderivative was successfully verified.

[In] `Integrate[((d+e*x)^(3/2)*(a+b*x+c*x^2))/Sqrt[f+g*x],x]`

[Out] `(-2*Sqrt[e]*Sqrt[g]*Sqrt[d+e*x]*Sqrt[f+g*x]*(c*(9*d^3*g^3+3*d^2*e*g^2*(5*f-2*g*x)+d*e^2*g*(-145*f^2+92*f*g*x-72*g^2*x^2)+e^3*(105*f^3-70*f^2*g*x+56*f*g^2*x^2-48*g^3*x^3))-8*e*g*(6*a*e*g*(-3*e*f+5*d*g+2*e*g*x)+b*(3*d^2*g^2+2*d*e*g*(-11*f+7*g*x)+e^2*(15*f^2-10*f*g*x+8*g^2*x^2))))+3*(e*f-d*g)^2*(c*(35*e^2*f^2+10*d*e*f*g+3*d^2*g^2)+8*e*g*(6*a*e*g-b*(5*e*f+d*g)))*Log[e*f+d*g+2*e*g*x+2*Sqrt[e]*Sqrt[g]*Sqrt[d+e*x]*Sqrt[f+g*x]]/(384*e^(5/2)*g^(9/2))`

**Maple [B]** time = 0.041, size = 1207, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.



[In]  $\text{int}((e^x+d)^{3/2} * (c*x^2+b*x+a)/(g*x+f)^{1/2}, x)$

[Out]  $\frac{1}{384} (e^x+d)^{1/2} (g*x+f)^{1/2} (-184 ((e^x+d) (g*x+f))^{1/2} x^2 d^2 c^2 f^2 g^2 (e^g)^{1/2} e^2 - 112 x^2 c^2 e^3 f^2 g^2 ((e^x+d) (g*x+f))^{1/2} (e^g)^{1/2} + 224 ((e^x+d) (g*x+f))^{1/2} x^2 d^2 b^2 g^3 (e^g)^{1/2} e^2 - 160 ((e^x+d) (g*x+f))^{1/2} x^2 f^2 c^2 e^3 g^2 (e^g)^{1/2} + 12 ((e^x+d) (g*x+f))^{1/2} x^2 d^2 c^2 g^3 (e^g)^{1/2} e + 140 ((e^x+d) (g*x+f))^{1/2} x^2 f^2 c^2 e^3 g^2 (e^g)^{1/2} - 30 ((e^x+d) (g*x+f))^{1/2} d^2 c^2 f^2 g^2 (e^g)^{1/2} e - 352 ((e^x+d) (g*x+f))^{1/2} b^2 d^2 f^2 g^2 (e^g)^{1/2} e^2 + 290 ((e^x+d) (g*x+f))^{1/2} c^2 d^2 f^2 g^2 (e^g)^{1/2} e^2 + 9 \ln(1/2 (2 e^g x + 2 ((e^x+d) (g*x+f))^{1/2} (e^g)^{1/2} + d^2 g + e^2 f) / (e^g)^{1/2}) d^4 c^2 g^4 + 105 \ln(1/2 (2 e^g x + 2 ((e^x+d) (g*x+f))^{1/2} (e^g)^{1/2} + d^2 g + e^2 f) / (e^g)^{1/2}) e^4 f^4 c - 288 \ln(1/2 (2 e^g x + 2 ((e^x+d) (g*x+f))^{1/2} (e^g)^{1/2} + d^2 g + e^2 f) / (e^g)^{1/2}) d^2 a^2 e^3 f^2 g^3 + 144 \ln(1/2 (2 e^g x + 2 ((e^x+d) (g*x+f))^{1/2} (e^g)^{1/2} + d^2 g + e^2 f) / (e^g)^{1/2}) d^2 a^2 g^4 e^2 + 144 x^2 c^2 d^2 e^2 g^3 ((e^x+d) (g*x+f))^{1/2} (e^g)^{1/2} - 18 ((e^x+d) (g*x+f))^{1/2} d^3 c^2 g^3 (e^g)^{1/2} + 144 \ln(1/2 (2 e^g x + 2 ((e^x+d) (g*x+f))^{1/2} (e^g)^{1/2} + d^2 g + e^2 f) / (e^g)^{1/2}) e^4 f^2 a^2 g^2 - 120 \ln(1/2 (2 e^g x + 2 ((e^x+d) (g*x+f))^{1/2} (e^g)^{1/2} + d^2 g + e^2 f) / (e^g)^{1/2}) e^4 f^3 b^2 g - 210 ((e^x+d) (g*x+f))^{1/2} c^2 e^3 f^3 (e^g)^{1/2} - 24 \ln(1/2 (2 e^g x + 2 ((e^x+d) (g*x+f))^{1/2} (e^g)^{1/2} + d^2 g + e^2 f) / (e^g)^{1/2}) d^3 b^2 g^4 e + 96 x^3 c^2 e^3 g^3 ((e^x+d) (g*x+f))^{1/2} (e^g)^{1/2} + 128 x^2 b^2 e^3 g^3 ((e^x+d) (g*x+f))^{1/2} (e^g)^{1/2} - 72 \ln(1/2 (2 e^g x + 2 ((e^x+d) (g*x+f))^{1/2} (e^g)^{1/2} + d^2 g + e^2 f) / (e^g)^{1/2}) d^2 b^2 f^2 g^3 e^2 + 216 \ln(1/2 (2 e^g x + 2 ((e^x+d) (g*x+f))^{1/2} (e^g)^{1/2} + d^2 g + e^2 f) / (e^g)^{1/2}) d^2 b^2 e^3 f^2 g^2 + 54 \ln(1/2 (2 e^g x + 2 ((e^x+d) (g*x+f))^{1/2} (e^g)^{1/2} + d^2 g + e^2 f) / (e^g)^{1/2}) d^2 c^2 f^2 g^2 e^2 - 180 \ln(1/2 (2 e^g x + 2 ((e^x+d) (g*x+f))^{1/2} (e^g)^{1/2} + d^2 g + e^2 f) / (e^g)^{1/2}) d^2 c^2 e^3 f^3 g + 480 ((e^x+d) (g*x+f))^{1/2} a^2 d^2 g^3 (e^g)^{1/2} e^2 - 288 ((e^x+d) (g*x+f))^{1/2} a^2 e^3 f^2 g^2 (e^g)^{1/2} + 240 ((e^x+d) (g*x+f))^{1/2} b^2 e^3 f^2 g^2 (e^g)^{1/2} + 192 a^2 e^3 ((e^x+d) (g*x+f))^{1/2} x^2 g^3 (e^g)^{1/2} + 12 \ln(1/2 (2 e^g x + 2 ((e^x+d) (g*x+f))^{1/2} (e^g)^{1/2} + d^2 g + e^2 f) / (e^g)^{1/2}) d^3 c^2 f^2 g^3 e + 48 ((e^x+d) (g*x+f))^{1/2} d^2 b^2 g^3 (e^g)^{1/2} e) / ((e^x+d) (g*x+f))^{1/2} / g^4 / (e^g)^{1/2} / e^2$

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**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*x^2 + b*x + a)*(e^x + d)^{3/2}/\text{sqrt}(g*x + f), x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

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**Fricas [A]** time = 1.8049, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((c*x^2 + b*x + a)*(e^x + d)^{3/2}/\text{sqrt}(g*x + f), x, \text{algorithm}="fricas")$

[Out]  $\frac{1}{768} (4 (48 c^2 e^3 g^3 x^3 - 105 c^2 e^3 f^3 + 5 (29 c^2 d e^2 + 24 b^2 e^3) f^2 g - (15 c^2 d^2 e + 176 b^2 d e^2 + 144 a^2 e^3) f^2 g^2 - 3 (3 c^2 d^3 - 8 b^2 d^2 e - 80 a^2 d e^2) g^3 - 8 (7 c^2 e^3 f^2 g^2 - (9 c^2 d^2 e^2 + 8 b^2 e^3) g^3) x^2 + 2 (35 c^2 e^3 f^2 g - 2 (23 c^2 d e^2 + 20 b^2 e^3) f^2 g^2 + (3 c^2 d^2 e + 56 b^2 d e^2 + 48 a^2 e^3) g^3) x) \text{sqrt}(e^g) \text{sqrt}(e^x + d) \text{sqrt}(g*x + f) + 3 (35 c^2 e^4 f^4 - 20 (3 c^2 d e^3 + 2 b^2 e^4) f^3 g + 6 (3 c^2 d^2 e^2 + 12 b^2 d e^3 + 8 a^2 e^4) f^2 g$

$$\begin{aligned} &^2 + 4*(c*d^3*e - 6*b*d^2*e^2 - 24*a*d*e^3)*f*g^3 + (3*c*d^4 - 8* \\ &b*d^3*e + 48*a*d^2*e^2)*g^4)*\log(4*(2*e^2*g^2*x + e^2*f*g + d*e*g \\ &^2)*\sqrt{e*x + d}*\sqrt{g*x + f} + (8*e^2*g^2*x^2 + e^2*f^2 + 6*d* \\ &e*f*g + d^2*g^2 + 8*(e^2*f*g + d*e*g^2)*x)*\sqrt{e*g}))/(\sqrt{e*g} \\ &*e^2*g^4), 1/384*(2*(48*c*e^3*g^3*x^3 - 105*c*e^3*f^3 + 5*(29*c*d \\ &*e^2 + 24*b*e^3)*f^2*g - (15*c*d^2*e + 176*b*d*e^2 + 144*a*e^3)*f \\ &*g^2 - 3*(3*c*d^3 - 8*b*d^2*e - 80*a*d*e^2)*g^3 - 8*(7*c*e^3*f*g^2 \\ &- (9*c*d*e^2 + 8*b*e^3)*g^3)*x^2 + 2*(35*c*e^3*f^2*g - 2*(23*c* \\ &d*e^2 + 20*b*e^3)*f*g^2 + (3*c*d^2*e + 56*b*d*e^2 + 48*a*e^3)*g^3 \\ &)*x)*\sqrt{-e*g}*\sqrt{e*x + d}*\sqrt{g*x + f} + 3*(35*c*e^4*f^4 - 2 \\ &0*(3*c*d*e^3 + 2*b*e^4)*f^3*g + 6*(3*c*d^2*e^2 + 12*b*d*e^3 + 8*a \\ &*e^4)*f^2*g^2 + 4*(c*d^3*e - 6*b*d^2*e^2 - 24*a*d*e^3)*f*g^3 + (3 \\ &*c*d^4 - 8*b*d^3*e + 48*a*d^2*e^2)*g^4)*\arctan(1/2*(2*e*g*x + e*f \\ &+ d*g)*\sqrt{-e*g}/(\sqrt{e*x + d}*\sqrt{g*x + f}*e*g))/(\sqrt{-e*g} \\ &)*e^2*g^4)] \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(3/2)\*(c\*x\*\*2+b\*x+a)/(g\*x+f)\*\*(1/2),x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.299809, size = 605, normalized size = 1.82

$$\frac{1}{192} \sqrt{(xe+d)ge - dge + fe^2} \left( 2 \left( 4(xe+d) \left( \frac{6(xe+d)ce^{(-3)}}{g} - \frac{(9cdg^6e^6 + 7cfdg^5e^7 - 8bg^6e^7)e^{(-9)}}{g^7} \right) \right) + \frac{(3cd^2g^6e^6 + 10cdfg^5e^7 - 8bd^3g^4e^4 + 18cd^2f^2g^2e^2 - 24bd^2fg^3e^2 + 48ad^2g^4e^2 - 60cdf^3ge^3 + 72bdf^2g^2e^3 - 96adf^3e^3 + 35c^2f^4e^4 - 40b^2f^3ge^4 + 48a^2f^2g^2e^4)*e^{(-5/2)} \ln(\text{abs}(-\sqrt{xe+d})\sqrt{g})e^{(1/2)} + \sqrt{(xe+d)ge - dge + fe^2})}{64g^{\frac{9}{2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)^(3/2)/sqrt(g\*x + f),x, algorithm="giac")

[Out] 1/192\*sqrt((x\*e + d)\*g\*e - d\*g\*e + f\*e^2)\*(2\*(4\*(x\*e + d)\*(6\*(x\*e + d)\*c\*e^(-3)/g - (9\*c\*d\*g^6\*e^6 + 7\*c\*f\*g^5\*e^7 - 8\*b\*g^6\*e^7)\*e^(-9)/g^7) + (3\*c\*d^2\*g^6\*e^6 + 10\*c\*d\*f\*g^5\*e^7 - 8\*b\*d\*g^6\*e^7 + 35\*c\*f^2\*g^4\*e^8 - 40\*b\*f\*g^5\*e^8 + 48\*a\*g^6\*e^8)\*e^(-9)/g^7)\*(x\*e + d) + 3\*(3\*c\*d^3\*g^6\*e^6 + 7\*c\*d^2\*f\*g^5\*e^7 - 8\*b\*d^2\*g^6\*e^7 + 25\*c\*d\*f^2\*g^4\*e^8 - 32\*b\*d\*f\*g^5\*e^8 + 48\*a\*d\*g^6\*e^8 - 35\*c\*f^3\*g^3\*e^9 + 40\*b\*f^2\*g^4\*e^9 - 48\*a\*f\*g^5\*e^9)\*e^(-9)/g^7)\*sqrt(x\*e + d) - 1/64\*(3\*c\*d^4\*g^4 + 4\*c\*d^3\*f\*g^3\*e - 8\*b\*d^3\*g^4\*e + 18\*c\*d^2\*f^2\*g^2\*e^2 - 24\*b\*d^2\*f\*g^3\*e^2 + 48\*a\*d^2\*g^4\*e^2 - 60\*c\*d\*f^3\*g^3\*e^3 + 72\*b\*d\*f^2\*g^2\*e^3 - 96\*a\*d\*f^3\*e^3 + 35\*c\*f^4\*e^4 - 40\*b\*f^3\*g^3\*e^4 + 48\*a\*f^2\*g^2\*e^4)\*e^(-5/2)\*ln(abs(-sqrt(x\*e + d)\*sqrt(g)\*e^(1/2) + sqrt((x\*e + d)\*g\*e - d\*g\*e + f\*e^2)))/g^(9/2)

$$3.836 \quad \int \frac{\sqrt{d+ex}(a+bx+cx^2)}{\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=246

$$\frac{\sqrt{d+ex}\sqrt{f+gx}(2eg(4aeg-b(dg+3ef))+c(d^2g^2+2defg+5e^2f^2))}{8e^2g^3} \\ - \frac{(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)(2eg(4aeg-b(dg+3ef))+c(d^2g^2+2defg+5e^2f^2))}{8e^{5/2}g^{7/2}} \\ - \frac{(d+ex)^{3/2}\sqrt{f+gx}(-6beg+7cdg+5cef)}{12e^2g^2} + \frac{c(d+ex)^{5/2}\sqrt{f+gx}}{3e^2g}$$

[Out]  $((c*(5*e^2*f^2 + 2*d*e*f*g + d^2*g^2) + 2*e*g*(4*a*e*g - b*(3*e*f + d*g))) * \text{Sqrt}[d + e*x] * \text{Sqrt}[f + g*x]) / (8*e^2*g^3) - ((5*c*e*f + 7*c*d*g - 6*b*e*g) * (d + e*x)^{(3/2)} * \text{Sqrt}[f + g*x]) / (12*e^2*g^2) + (c*(d + e*x)^{(5/2)} * \text{Sqrt}[f + g*x]) / (3*e^2*g) - ((e*f - d*g) * (c*(5*e^2*f^2 + 2*d*e*f*g + d^2*g^2) + 2*e*g*(4*a*e*g - b*(3*e*f + d*g))) * \text{ArcTanh}[(\text{Sqrt}[g] * \text{Sqrt}[d + e*x]) / (\text{Sqrt}[e] * \text{Sqrt}[f + g*x])]) / (8*e^{5/2} * g^{7/2})$

**Rubi [A]** time = 0.584502, antiderivative size = 246, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$\frac{\sqrt{d+ex}\sqrt{f+gx}(2eg(4aeg-b(dg+3ef))+c(d^2g^2+2defg+5e^2f^2))}{8e^2g^3} \\ - \frac{(ef-dg)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)(2eg(4aeg-b(dg+3ef))+c(d^2g^2+2defg+5e^2f^2))}{8e^{5/2}g^{7/2}} \\ - \frac{(d+ex)^{3/2}\sqrt{f+gx}(-6beg+7cdg+5cef)}{12e^2g^2} + \frac{c(d+ex)^{5/2}\sqrt{f+gx}}{3e^2g}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Sqrt}[d + e*x] * (a + b*x + c*x^2)) / \text{Sqrt}[f + g*x], x]$

[Out]  $((c*(5*e^2*f^2 + 2*d*e*f*g + d^2*g^2) + 2*e*g*(4*a*e*g - b*(3*e*f + d*g))) * \text{Sqrt}[d + e*x] * \text{Sqrt}[f + g*x]) / (8*e^2*g^3) - ((5*c*e*f + 7*c*d*g - 6*b*e*g) * (d + e*x)^{(3/2)} * \text{Sqrt}[f + g*x]) / (12*e^2*g^2) + (c*(d + e*x)^{(5/2)} * \text{Sqrt}[f + g*x]) / (3*e^2*g) - ((e*f - d*g) * (c*(5*e^2*f^2 + 2*d*e*f*g + d^2*g^2) + 2*e*g*(4*a*e*g - b*(3*e*f + d*g))) * \text{ArcTanh}[(\text{Sqrt}[g] * \text{Sqrt}[d + e*x]) / (\text{Sqrt}[e] * \text{Sqrt}[f + g*x])]) / (8*e^{5/2} * g^{7/2})$

**Rubi in Sympy [A]** time = 51.9142, size = 318, normalized size = 1.29

$$\frac{b(d+ex)^{3/2}\sqrt{f+gx}}{2eg} + \frac{cx(d+ex)^{3/2}\sqrt{f+gx}}{3eg} - \frac{c(d+ex)^{3/2}\sqrt{f+gx}(3dg+5ef)}{12e^2g^2} \\ + \frac{c\sqrt{d+ex}\sqrt{f+gx}(d^2g^2+2defg+5e^2f^2)}{8e^2g^3} \\ + \frac{c(dg-ef)(d^2g^2+2defg+5e^2f^2)\operatorname{atanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{8e^{5/2}g^{7/2}} \\ + \frac{\sqrt{d+ex}\sqrt{f+gx}(4aeg-bdg-3bef)}{4eg^2} + \frac{(dg-ef)(4aeg-bdg-3bef)\operatorname{atanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{4e^{3/2}g^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**(1/2)*(c*x**2+b*x+a)/(g*x+f)**(1/2),x)`

[Out]  $b(d + ex)^{3/2} \sqrt{f + gx} / (2eg) + c x (d + ex)^{3/2} \sqrt{f + gx} / (3e^2 g) - c (d + ex)^{3/2} \sqrt{f + gx} (3d^2 g + 5e^2 f) / (12e^2 g^2) + c \sqrt{d + ex} \sqrt{f + gx} (d^2 g^2 + 2d^2 e f g + 5e^2 f^2) / (8e^2 g^3) + c (d g - e f) (d^2 g^2 + 2d^2 e f g + 5e^2 f^2) \operatorname{atanh}(\sqrt{e} \sqrt{f + gx} / (\sqrt{g} \sqrt{d + ex})) / (8e^{5/2} g^{7/2}) + \sqrt{d + ex} \sqrt{f + gx} (4a e g - b d g - 3b e f) / (4e^2 g^2) + (d g - e f) (4a e g - b d g - 3b e f) \operatorname{atanh}(\sqrt{e} \sqrt{f + gx} / (\sqrt{g} \sqrt{d + ex})) / (4e^{3/2} g^{5/2})$

**Mathematica [A]** time = 0.297278, size = 212, normalized size = 0.86

$$\frac{\sqrt{d+ex}\sqrt{f+gx}(6eg(4aeg+b(dg-3ef+2egx))+c(-3d^2g^2+2deg(gx-2f)+e^2(15f^2-10fgx+8g^2x^2)))}{24e^2g^3} \\ - \frac{(ef-dg)\log\left(2\sqrt{e}\sqrt{g}\sqrt{d+ex}\sqrt{f+gx}+dg+ef+2egx\right)(2eg(4aeg-b(dg+3ef))+c(d^2g^2+2defg+5e^2f^2))}{16e^{5/2}g^{7/2}}$$

Antiderivative was successfully verified.

[In] `Integrate[(Sqrt[d + e*x]*(a + b*x + c*x^2))/Sqrt[f + g*x],x]`

[Out]  $(\sqrt{d + ex} \sqrt{f + gx} (6e^2 g (4a e g + b(-3e f + d g + 2e^2 g x)) + c(-3d^2 g^2 + 2d^2 e g(-2f + gx) + e^2(15f^2 - 10f g x + 8g^2 x^2))) / (24e^2 g^3) - ((e f - d g) (c(5e^2 f^2 + 2d^2 e f g + d^2 g^2) + 2e^2 g(4a e g - b(3e f + d g))) \operatorname{Log}[e f + d g + 2e^2 g x + 2\sqrt{e} \sqrt{g} \sqrt{d + ex} \sqrt{f + gx}]) / (16e^{5/2} g^{7/2}))$

**Maple [B]** time = 0.03, size = 763, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^(1/2)*(c*x^2+b*x+a)/(g*x+f)^(1/2),x)`

[Out]  $1/48 (e x + d)^{1/2} (g x + f)^{1/2} (16 x^2 c e^2 g^2 ((e x + d) (g x + f))^{1/2} (e g)^{1/2} + 24 \ln(1/2 (2 e^2 g x + 2 ((e x + d) (g x + f))^{1/2} (e g)^{1/2} + d g + e f) / (e g)^{1/2}) d^2 a g^3 e^2 - 24 \ln(1/2 (2 e^2 g x + 2 ((e x + d) (g x + f))^{1/2} (e g)^{1/2} + d g + e f) / (e g)^{1/2}) e^3 f a g^2 - 6 b \ln(1/2 (2 e^2 g x + 2 ((e x + d) (g x + f))^{1/2} (e g)^{1/2} + d g + e f) / (e g)^{1/2}) d^2 g^3 e - 12 \ln(1/2 (2 e^2 g x + 2 ((e x + d) (g x + f))^{1/2} (e g)^{1/2} + d g + e f) / (e g)^{1/2}) d^2 b f g^2 e^2 + 18 \ln(1/2 (2 e^2 g x + 2 ((e x + d) (g x + f))^{1/2} (e g)^{1/2} + d g + e f) / (e g)^{1/2}) e^3 f^2 b g + 3 c \ln(1/2 (2 e^2 g x + 2 ((e x + d) (g x + f))^{1/2} (e g)^{1/2} + d g + e f) / (e g)^{1/2}) d^3 g^3 + 3 c \ln(1/2 (2 e^2 g x + 2 ((e x + d) (g x + f))^{1/2} (e g)^{1/2} + d g + e f) / (e g)^{1/2}) d^2 f g^2 e + 9 \ln(1/2 (2 e^2 g x + 2 ((e x + d) (g x + f))^{1/2} (e g)^{1/2} + d g + e f) / (e g)^{1/2}) d^2 c f^2 g e^2 - 15 \ln(1/2 (2 e^2 g x + 2 ((e x + d) (g x + f))^{1/2} (e g)^{1/2} + d g + e f) / (e g)^{1/2}) e^3 f^3 c + 24 b ((e x + d) (g x + f))^{1/2} x g^2 e^2 (e g)^{1/2} + 4 c ((e x + d) (g x + f))^{1/2} x d g^2 e (e g)^{1/2} - 20 c ((e x + d) (g x + f))^{1/2} x f g e^2 (e g)^{1/2} + 48 ((e x + d) (g x + f))^{1/2} a g^2 e^2 (e g)^{1/2} + 12 b ((e x + d) (g x + f))^{1/2} d g^2 e (e g)^{1/2} - 36 ((e x + d) (g x + f))^{1/2} b f g e^2 (e g)^{1/2} - 6 c ((e x + d) (g x + f))^{1/2} d^2 g^2 (e g)^{1/2} - 8 c ((e x + d) (g x + f))^{1/2} d f g e (e g)^{1/2} + 3$

$$0 * ((e * x + d) * (g * x + f))^{(1/2)} * c * f^2 * e^2 * (e * g)^{(1/2)} / (((e * x + d) * (g * x + f))^{(1/2)} / g^3 / e^2 / (e * g)^{(1/2)})$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*sqrt(e\*x + d)/sqrt(g\*x + f), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.37137, size = 1, normalized size = 0.

$$\left[ \frac{4(8ce^2g^2x^2 + 15ce^2f^2 - 2(2cde + 9be^2)fg - 3(cd^2 - 2bde - 8ae^2)g^2 - 2(5ce^2fg - (cde + 6be^2)g^2)x)\sqrt{eg}\sqrt{ex + d}}{\sqrt{eg}\sqrt{ex + d}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*sqrt(e\*x + d)/sqrt(g\*x + f), x, algorithm="fricas")

[Out] [1/96\*(4\*(8\*c\*e^2\*g^2\*x^2 + 15\*c\*e^2\*f^2 - 2\*(2\*c\*d\*e + 9\*b\*e^2)\*f\*g - 3\*(c\*d^2 - 2\*b\*d\*e - 8\*a\*e^2)\*g^2 - 2\*(5\*c\*e^2\*f\*g - (c\*d\*e + 6\*b\*e^2)\*g^2)\*x)\*sqrt(e\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f) - 3\*(5\*c\*e^3\*f^3 - 3\*(c\*d\*e^2 + 2\*b\*e^3)\*f^2\*g - (c\*d^2\*e - 4\*b\*d\*e^2 - 8\*a\*e^3)\*f\*g^2 - (c\*d^3 - 2\*b\*d^2\*e + 8\*a\*d\*e^2)\*g^3)\*log(4\*(2\*e^2\*g^2\*x + e^2\*f\*g + d\*e\*g^2)\*sqrt(e\*x + d)\*sqrt(g\*x + f) + (8\*e^2\*g^2\*x^2 + e^2\*f^2 + 6\*d\*e\*f\*g + d^2\*g^2 + 8\*(e^2\*f\*g + d\*e\*g^2)\*x)\*sqrt(e\*g))/(sqrt(e\*g)\*e^2\*g^3), 1/48\*(2\*(8\*c\*e^2\*g^2\*x^2 + 15\*c\*e^2\*f^2 - 2\*(2\*c\*d\*e + 9\*b\*e^2)\*f\*g - 3\*(c\*d^2 - 2\*b\*d\*e - 8\*a\*e^2)\*g^2 - 2\*(5\*c\*e^2\*f\*g - (c\*d\*e + 6\*b\*e^2)\*g^2)\*x)\*sqrt(-e\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f) - 3\*(5\*c\*e^3\*f^3 - 3\*(c\*d\*e^2 + 2\*b\*e^3)\*f^2\*g - (c\*d^2\*e - 4\*b\*d\*e^2 - 8\*a\*e^3)\*f\*g^2 - (c\*d^3 - 2\*b\*d^2\*e + 8\*a\*d\*e^2)\*g^3)\*arctan(1/2\*(2\*e\*g\*x + e\*f + d\*g)\*sqrt(-e\*g)/(sqrt(e\*x + d)\*sqrt(g\*x + f)\*e\*g))/(sqrt(-e\*g)\*e^2\*g^3)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(1/2)\*(c\*x\*\*2+b\*x+a)/(g\*x+f)\*\*(1/2), x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.290715, size = 393, normalized size = 1.6

$$\frac{1}{24} \sqrt{(xe + d)ge - dge + fe^2} \left( 2(xe + d) \left( \frac{4(xe + d)ce^{(-3)}}{g} - \frac{(7cdg^4e^6 + 5cfdg^3e^7 - 6bg^4e^7)e^{(-9)}}{g^5} \right) + \frac{3(cd^2g^4e^6 + 2cdfg^3e^7 - (cd^3g^3 + cd^2fg^2e - 2bd^2g^3e + 3cdf^2ge^2 - 4bdfg^2e^2 + 8adg^3e^2 - 5cf^3e^3 + 6bf^2ge^3 - 8afg^2e^3)e^{(-\frac{5}{2})} \ln \left( \left| -\sqrt{xe + d} \sqrt{g} \right| \right)}{8g^{\frac{7}{2}}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)*sqrt(e*x + d)/sqrt(g*x + f),x, algorithm="giac")
```

```
[Out] 1/24*sqrt((x*e + d)*g*e - d*g*e + f*e^2)*(2*(x*e + d)*(4*(x*e + d)
)*c*e^(-3)/g - (7*c*d*g^4*e^6 + 5*c*f*g^3*e^7 - 6*b*g^4*e^7)*e^(-
9)/g^5) + 3*(c*d^2*g^4*e^6 + 2*c*d*f*g^3*e^7 - 2*b*d*g^4*e^7 + 5*
c*f^2*g^2*e^8 - 6*b*f*g^3*e^8 + 8*a*g^4*e^8)*e^(-9)/g^5)*sqrt(x*e
+ d) - 1/8*(c*d^3*g^3 + c*d^2*f*g^2*e - 2*b*d^2*g^3*e + 3*c*d*f^
2*g*e^2 - 4*b*d*f*g^2*e^2 + 8*a*d*g^3*e^2 - 5*c*f^3*e^3 + 6*b*f^2
*g*e^3 - 8*a*f*g^2*e^3)*e^(-5/2)*ln(abs(-sqrt(x*e + d)*sqrt(g)*e^
(1/2) + sqrt((x*e + d)*g*e - d*g*e + f*e^2)))/g^(7/2)
```

$$3.837 \quad \int \frac{a+bx+cx^2}{\sqrt{d+ex}\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=164

$$\frac{\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right) (4eg(2aeg - b(dg + ef)) + c(3d^2g^2 + 2defg + 3e^2f^2))}{4e^{5/2}g^{5/2}} - \frac{\sqrt{d+ex}\sqrt{f+gx}(-4beg + 5cdg + 3cef)}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

[Out]  $-\left(\left(3c^2ef + 5c^2dg - 4b^2eg\right)\sqrt{d+ex}\sqrt{f+gx}\right)/\left(4e^{5/2}g^{5/2}\right) + \left(c^2(d+ex)^{3/2}\sqrt{f+gx}\right)/\left(2e^2g\right) + \left(\left(c^2(3e^2f^2 + 2d^2efg + 3d^2g^2) + 4e^2g(2ae^2g - b^2(e^2f + d^2g))\right)\operatorname{ArcTanh}\left[\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right]\right)/\left(4e^{5/2}g^{5/2}\right)$

**Rubi [A]** time = 0.376869, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right) (4eg(2aeg - b(dg + ef)) + c(3d^2g^2 + 2defg + 3e^2f^2))}{4e^{5/2}g^{5/2}} - \frac{\sqrt{d+ex}\sqrt{f+gx}(-4beg + 5cdg + 3cef)}{4e^2g^2} + \frac{c(d+ex)^{3/2}\sqrt{f+gx}}{2e^2g}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)/(Sqrt[d + e\*x]\*Sqrt[f + g\*x]), x]

[Out]  $-\left(\left(3c^2ef + 5c^2dg - 4b^2eg\right)\sqrt{d+ex}\sqrt{f+gx}\right)/\left(4e^{5/2}g^{5/2}\right) + \left(c^2(d+ex)^{3/2}\sqrt{f+gx}\right)/\left(2e^2g\right) + \left(\left(c^2(3e^2f^2 + 2d^2efg + 3d^2g^2) + 4e^2g(2ae^2g - b^2(e^2f + d^2g))\right)\operatorname{ArcTanh}\left[\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right]\right)/\left(4e^{5/2}g^{5/2}\right)$

**Rubi in Sympy [A]** time = 38.9625, size = 199, normalized size = 1.21

$$\frac{b\sqrt{d+ex}\sqrt{f+gx}}{eg} + \frac{cx\sqrt{d+ex}\sqrt{f+gx}}{2eg} - \frac{3c\sqrt{d+ex}\sqrt{f+gx}(dg+ef)}{4e^2g^2} - \frac{c(4defg - 3(dg+ef)^2) \operatorname{atanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{4e^{5/2}g^{5/2}} - \frac{2\left(-aeg + \frac{b(dg+ef)}{2}\right) \operatorname{atanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{e^{3/2}g^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+b\*x+a)/(e\*x+d)\*\*(1/2)/(g\*x+f)\*\*(1/2), x)

[Out]  $b\sqrt{d+ex}\sqrt{f+gx}/(e^2g) + c^2x\sqrt{d+ex}\sqrt{f+gx}/(2e^2g) - 3c^2\sqrt{d+ex}\sqrt{f+gx}(dg+ef)/(4e^{5/2}g^{5/2}) - c^2(4d^2efg - 3(dg+ef)^2)\operatorname{atanh}(\sqrt{e}\sqrt{f+gx}/(\sqrt{g}\sqrt{d+ex}))/\left(4e^{5/2}g^{5/2}\right) - 2(-aeg + \frac{b(dg+ef)}{2})\operatorname{atanh}(\sqrt{e}\sqrt{f+gx}/(\sqrt{g}\sqrt{d+ex}))/\left(e^{3/2}g^{3/2}\right)$

**Mathematica [A]** time = 0.173714, size = 154, normalized size = 0.94

$$\frac{\log\left(2\sqrt{e}\sqrt{g}\sqrt{d+ex}\sqrt{f+gx}+dg+ef+2egx\right)\left(4eg(2aeg-b(dg+ef))+c(3d^2g^2+2defg+3e^2f^2)\right)}{8e^{5/2}g^{5/2}} + \frac{\sqrt{d+ex}\sqrt{f+gx}(4beg+c(-3dg-3ef+2egx))}{4e^2g^2}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)/(Sqrt[d + e\*x]\*Sqrt[f + g\*x]),x]

[Out] (Sqrt[d + e\*x]\*Sqrt[f + g\*x]\*(4\*b\*e\*g + c\*(-3\*e\*f - 3\*d\*g + 2\*e\*g\*x)))/(4\*e^2\*g^2) + ((c\*(3\*e^2\*f^2 + 2\*d\*e\*f\*g + 3\*d^2\*g^2) + 4\*e\*g\*(2\*a\*e\*g - b\*(e\*f + d\*g)))\*Log[e\*f + d\*g + 2\*e\*g\*x + 2\*Sqrt[e]\*Sqrt[g]\*Sqrt[d + e\*x]\*Sqrt[f + g\*x]])/(8\*e^(5/2)\*g^(5/2))

**Maple [B]** time = 0., size = 425, normalized size = 2.6

$$\frac{1}{8g^2e^2} \left( 8 \ln \left( \frac{1}{2} \frac{2egx + 2\sqrt{(ex+d)(gx+f)}\sqrt{eg} + dg + ef}{\sqrt{eg}} \right) ae^2g^2 - 4 \ln \left( \frac{1}{2} \frac{2egx + 2\sqrt{(ex+d)(gx+f)}\sqrt{eg} + dg + ef}{\sqrt{eg}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)/(e\*x+d)^(1/2)/(g\*x+f)^(1/2),x)

[Out] 1/8\*(8\*ln(1/2\*(2\*e\*g\*x+2\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+d\*g+e\*f)/(e\*g)^(1/2))\*a\*e^2\*g^2-4\*ln(1/2\*(2\*e\*g\*x+2\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+d\*g+e\*f)/(e\*g)^(1/2))\*b\*d\*e\*g^2-4\*ln(1/2\*(2\*e\*g\*x+2\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+d\*g+e\*f)/(e\*g)^(1/2))\*b\*e^2\*f\*g+3\*ln(1/2\*(2\*e\*g\*x+2\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+d\*g+e\*f)/(e\*g)^(1/2))\*c\*d^2\*g^2+2\*c\*ln(1/2\*(2\*e\*g\*x+2\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+d\*g+e\*f)/(e\*g)^(1/2))\*d\*f\*e\*g+3\*ln(1/2\*(2\*e\*g\*x+2\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+d\*g+e\*f)/(e\*g)^(1/2))\*c\*e^2\*f^2+4\*c\*((e\*x+d)\*(g\*x+f))^(1/2)\*x\*e\*g\*(e\*g)^(1/2)+8\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)\*b\*e\*g-6\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)\*c\*d\*g-6\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)\*c\*e\*f\*(e\*x+d)^(1/2)\*(g\*x+f)^(1/2)/(e\*g)^(1/2)/g^2/e^2/((e\*x+d)\*(g\*x+f))^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/(sqrt(e\*x + d)\*sqrt(g\*x + f)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.49738, size = 1, normalized size = 0.01

$$\left[ \frac{4(2cegx - 3cef - (3cd - 4be)g)\sqrt{eg}\sqrt{ex+d}\sqrt{gx+f} + (3ce^2f^2 + 2(cde - 2be^2)fg + (3cd^2 - 4bde + 8ae^2)g^2) \log\left(4\sqrt{eg}\sqrt{ex+d}\sqrt{gx+f}\right)}{16\sqrt{eg}e^2g^2} \right]$$



Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/(sqrt(e\*x + d)\*sqrt(g\*x + f)),x, algorithm="fricas")

[Out] [1/16\*(4\*(2\*c\*e\*g\*x - 3\*c\*e\*f - (3\*c\*d - 4\*b\*e)\*g)\*sqrt(e\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f) + (3\*c\*e^2\*f^2 + 2\*(c\*d\*e - 2\*b\*e^2)\*f\*g + (3\*c\*d^2 - 4\*b\*d\*e + 8\*a\*e^2)\*g^2)\*log(4\*(2\*e^2\*g^2\*x + e^2\*f\*g + d\*e\*g^2)\*sqrt(e\*x + d)\*sqrt(g\*x + f) + (8\*e^2\*g^2\*x^2 + e^2\*f^2 + 6\*d\*e\*f\*g + d^2\*g^2 + 8\*(e^2\*f\*g + d\*e\*g^2)\*x)\*sqrt(e\*g)))/(sqrt(e\*g)\*e^2\*g^2), 1/8\*(2\*(2\*c\*e\*g\*x - 3\*c\*e\*f - (3\*c\*d - 4\*b\*e)\*g)\*sqrt(-e\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f) + (3\*c\*e^2\*f^2 + 2\*(c\*d\*e - 2\*b\*e^2)\*f\*g + (3\*c\*d^2 - 4\*b\*d\*e + 8\*a\*e^2)\*g^2)\*arctan(1/2\*(2\*e\*g\*x + e\*f + d\*g)\*sqrt(-e\*g)/(sqrt(e\*x + d)\*sqrt(g\*x + f)\*e\*g)))/(sqrt(-e\*g)\*e^2\*g^2)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + bx + cx^2}{\sqrt{d + ex}\sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)/(e\*x+d)\*\*(1/2)/(g\*x+f)\*\*(1/2),x)

[Out] Integral((a + b\*x + c\*x\*\*2)/(sqrt(d + e\*x)\*sqrt(f + g\*x)), x)

**GIAC/XCAS [A]** time = 0.287325, size = 242, normalized size = 1.48

$$\frac{\frac{1}{4}\sqrt{(xe + d)ge - dge + fe^2}\sqrt{xe + d}\left(\frac{2(xe + d)ce^{(-3)}}{g} - \frac{(5cdg^2e^5 + 3cfge^6 - 4bg^2e^6)e^{(-8)}}{g^3}\right)}{(3cd^2g^2 + 2cdfge - 4bdg^2e + 3cf^2e^2 - 4bfge^2 + 8ag^2e^2)e^{(-\frac{5}{2})}\ln\left(\left|-\sqrt{xe + d}\sqrt{ge}^{\frac{1}{2}} + \sqrt{(xe + d)ge - dge + fe^2}\right|\right)}}{4g^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/(sqrt(e\*x + d)\*sqrt(g\*x + f)),x, algorithm="giac")

[Out] 1/4\*sqrt((x\*e + d)\*g\*e - d\*g\*e + f\*e^2)\*sqrt(x\*e + d)\*(2\*(x\*e + d)\*c\*e^(-3)/g - (5\*c\*d\*g^2\*e^5 + 3\*c\*f\*g\*e^6 - 4\*b\*g^2\*e^6)\*e^(-8)/g^3) - 1/4\*(3\*c\*d^2\*g^2 + 2\*c\*d\*f\*g\*e - 4\*b\*d\*g^2\*e + 3\*c\*f^2\*e^2 - 4\*b\*f\*g\*e^2 + 8\*a\*g^2\*e^2)\*e^(-5/2)\*ln(abs(-sqrt(x\*e + d)\*sqrt(g)\*e^(1/2) + sqrt((x\*e + d)\*g\*e - d\*g\*e + f\*e^2)))/g^(5/2)

$$3.838 \quad \int \frac{a+bx+cx^2}{(d+ex)^{3/2}\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=129

$$\frac{2\sqrt{f+gx}\left(a + \frac{d(cd-be)}{e^2}\right)}{\sqrt{d+ex}(ef-dg)} - \frac{(-2beg+3cdg+cef)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{e^{5/2}g^{3/2}} + \frac{c\sqrt{d+ex}\sqrt{f+gx}}{e^2g}$$

[Out]  $(-2*(a + (d*(c*d - b*e))/e^2)*\text{Sqrt}[f + g*x])/((e*f - d*g)*\text{Sqrt}[d + e*x]) + (c*\text{Sqrt}[d + e*x]*\text{Sqrt}[f + g*x])/(e^2*g) - ((c*e*f + 3*c*d*g - 2*b*e*g)*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[d + e*x])]/(\text{Sqrt}[e]*\text{Sqrt}[f + g*x]))/(e^{(5/2)}*g^{(3/2)})$

**Rubi [A]** time = 0.332583, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$

$$\frac{2\sqrt{f+gx}\left(a + \frac{d(cd-be)}{e^2}\right)}{\sqrt{d+ex}(ef-dg)} - \frac{(-2beg+3cdg+cef)\tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{e^{5/2}g^{3/2}} + \frac{c\sqrt{d+ex}\sqrt{f+gx}}{e^2g}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x + c*x^2)/((d + e*x)^{(3/2)}*\text{Sqrt}[f + g*x]), x]$

[Out]  $(-2*(a + (d*(c*d - b*e))/e^2)*\text{Sqrt}[f + g*x])/((e*f - d*g)*\text{Sqrt}[d + e*x]) + (c*\text{Sqrt}[d + e*x]*\text{Sqrt}[f + g*x])/(e^2*g) - ((c*e*f + 3*c*d*g - 2*b*e*g)*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[d + e*x])]/(\text{Sqrt}[e]*\text{Sqrt}[f + g*x]))/(e^{(5/2)}*g^{(3/2)})$

**Rubi in Sympy [A]** time = 40.8442, size = 182, normalized size = 1.41

$$\frac{2b \operatorname{atanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{e^{\frac{3}{2}}\sqrt{g}} + \frac{2cd^2\sqrt{f+gx}}{e^2\sqrt{d+ex}(dg-ef)} + \frac{c\sqrt{d+ex}\sqrt{f+gx}}{e^2g}$$

$$- \frac{c(3dg+ef)\operatorname{atanh}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{e^{\frac{5}{2}}g^{\frac{3}{2}}} + \frac{2\sqrt{f+gx}(ae-bd)}{e\sqrt{d+ex}(dg-ef)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((c*x**2+b*x+a)/(e*x+d)**(3/2)/(g*x+f)**(1/2), x)$

[Out]  $2*b*\operatorname{atanh}(\text{sqrt}(g)*\text{sqrt}(d + e*x)/(\text{sqrt}(e)*\text{sqrt}(f + g*x)))/(e**(3/2)*\text{sqrt}(g)) + 2*c*d**2*\text{sqrt}(f + g*x)/(e**2*\text{sqrt}(d + e*x)*(d*g - e*f)) + c*\text{sqrt}(d + e*x)*\text{sqrt}(f + g*x)/(e**2*g) - c*(3*d*g + e*f)*\operatorname{atanh}(\text{sqrt}(g)*\text{sqrt}(d + e*x)/(\text{sqrt}(e)*\text{sqrt}(f + g*x)))/(e**(5/2)*g**(3/2)) + 2*\text{sqrt}(f + g*x)*(a*e - b*d)/(e*\text{sqrt}(d + e*x)*(d*g - e*f))$

**Mathematica [A]** time = 0.341927, size = 136, normalized size = 1.05

$$\frac{\sqrt{d+ex}\sqrt{f+gx}\left(\frac{2(e(ae-bd)+cd^2)}{(d+ex)(dg-ef)} + \frac{c}{g}\right)}{e^2}$$

$$- \frac{(-2beg+3cdg+cef)\log\left(2\sqrt{e}\sqrt{g}\sqrt{d+ex}\sqrt{f+gx}+dg+ef+2egx\right)}{2e^{5/2}g^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)/((d + e*x)^(3/2)*Sqrt[f + g*x]),x]
```

```
[Out] (Sqrt[d + e*x]*Sqrt[f + g*x]*(c/g + (2*(c*d^2 + e*(-b*d) + a*e))
)/((-e*f) + d*g)*(d + e*x)))/e^2 - ((c*e*f + 3*c*d*g - 2*b*e*g)
*Log[e*f + d*g + 2*e*g*x + 2*Sqrt[e]*Sqrt[g]*Sqrt[d + e*x]*Sqrt[f
+ g*x]))/(2*e^(5/2)*g^(3/2))
```

**Maple [B]** time = 0.037, size = 697, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)/(e*x+d)^(3/2)/(g*x+f)^(1/2),x)
```

```
[Out] 1/2*(g*x+f)^(1/2)*(2*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e
*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*x*b*d*e^2*g^2-2*ln(1/2*(2*e*g*x+2
*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*x*b*e^
3*f*g-3*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g
+e*f)/(e*g)^(1/2))*x*c*d^2*e*g^2+2*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*
x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*x*c*d*e^2*f*g+ln(1/
2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(
1/2))*x*c*e^3*f^2+2*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*
g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d^2*e*g^2-2*ln(1/2*(2*e*g*x+2*((
e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*b*d*e^2*f
*g-3*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*
f)/(e*g)^(1/2))*c*d^3*g^2+2*ln(1/2*(2*e*g*x+2*((e*x+d)*(g*x+f))^(
1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d^2*e*f*g+ln(1/2*(2*e*g*
x+2*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+d*g+e*f)/(e*g)^(1/2))*c*d
*e^2*f^2+2*x*c*d*e*g*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)-2*x*c*e^
2*f*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)+4*a*e^2*g*((e*x+d)*(g*x+f
))^(1/2)*(e*g)^(1/2)-4*b*d*e*g*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2
))+6*c*d^2*g*((e*x+d)*(g*x+f))^(1/2)*(e*g)^(1/2)-2*c*d*e*f*((e*x+d
)*(g*x+f))^(1/2)*(e*g)^(1/2))/g/(e*g)^(1/2)/(d*g-e*f)/((e*x+d)*(g
*x+f))^(1/2)/e^2/(e*x+d)^(1/2)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)/((e*x + d)^(3/2)*sqrt(g*x + f)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

**Fricas [A]** time = 1.6326, size = 1, normalized size = 0.01

$$\frac{4(cdef - (3cd^2 - 2bde + 2ae^2)g + (ce^2f - cdeg)x)\sqrt{eg}\sqrt{ex+d}\sqrt{gx+f} - (cde^2f^2 + 2(cd^2e - bde^2)fg - (3cd^3 - 2bde^2)g^2)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)/((e*x + d)^(3/2)*sqrt(g*x + f)),x, algorithm="fricas")
```

```
[Out] [1/4*(4*(c*d*e*f - (3*c*d^2 - 2*b*d*e + 2*a*e^2)*g + (c*e^2*f - c*d*e*g)*x)*sqrt(e*g)*sqrt(e*x + d)*sqrt(g*x + f) - (c*d*e^2*f^2 + 2*(c*d^2*e - b*d*e^2)*f*g - (3*c*d^3 - 2*b*d^2*e)*g^2 + (c*e^3*f^2 + 2*(c*d*e^2 - b*e^3)*f*g - (3*c*d^2*e - 2*b*d*e^2)*g^2)*x)*log(4*(2*e^2*g^2*x + e^2*f*g + d*e*g^2)*sqrt(e*x + d)*sqrt(g*x + f) + (8*e^2*g^2*x^2 + e^2*f^2 + 6*d*e*f*g + d^2*g^2 + 8*(e^2*f*g + d*e*g^2)*x)*sqrt(e*g)))/((d*e^3*f*g - d^2*e^2*g^2 + (e^4*f*g - d*e^3*g^2)*x)*sqrt(e*g)), 1/2*(2*(c*d*e*f - (3*c*d^2 - 2*b*d*e + 2*a*e^2)*g + (c*e^2*f - c*d*e*g)*x)*sqrt(-e*g)*sqrt(e*x + d)*sqrt(g*x + f) - (c*d*e^2*f^2 + 2*(c*d^2*e - b*d*e^2)*f*g - (3*c*d^3 - 2*b*d^2*e)*g^2 + (c*e^3*f^2 + 2*(c*d*e^2 - b*e^3)*f*g - (3*c*d^2*e - 2*b*d*e^2)*g^2)*x)*arctan(1/2*(2*e*g*x + e*f + d*g)*sqrt(-e*g)/(sqrt(e*x + d)*sqrt(g*x + f)*e*g)))/((d*e^3*f*g - d^2*e^2*g^2 + (e^4*f*g - d*e^3*g^2)*x)*sqrt(-e*g))]
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + bx + cx^2}{(d + ex)^{\frac{3}{2}} \sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)/(e*x+d)**(3/2)/(g*x+f)**(1/2),x)
```

```
[Out] Integral((a + b*x + c*x**2)/((d + e*x)**(3/2)*sqrt(f + g*x)), x)
```

**GIAC/XCAS [A]** time = 0.591771, size = 4, normalized size = 0.03

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)/((e*x + d)^(3/2)*sqrt(g*x + f)),x, algorithm="giac")
```

```
[Out] sage0*x
```

$$3.839 \quad \int \frac{a+bx+cx^2}{(d+ex)^{5/2}\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=160

$$\frac{2\sqrt{f+gx}(c(6def-4d^2g)-e(-2aeg-bdg+3bef))}{3e^2\sqrt{d+ex}(ef-dg)^2} - \frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{3(d+ex)^{3/2}(ef-dg)} + \frac{2c \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{e^{5/2}\sqrt{g}}$$

[Out]  $(-2*(a+(d*(c*d-b*e))/e^2)*\text{Sqrt}[f+g*x])/(3*(e*f-d*g)*(d+e*x)^{(3/2)})+(2*(c*(6*d*e*f-4*d^2*g)-e*(3*b*e*f-b*d*g-2*a*e*g))*\text{Sqrt}[f+g*x])/(3*e^{2*(e*f-d*g)^2}*\text{Sqrt}[d+e*x])+(2*c*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[d+e*x])/(\text{Sqrt}[e]*\text{Sqrt}[f+g*x])])/(e^{(5/2)*\text{Sqrt}[g]})$

**Rubi [A]** time = 0.426206, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$

$$-\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{3(d+ex)^{3/2}(ef-dg)} + \frac{2\sqrt{f+gx}(2cd(3ef-2dg)-e(-2aeg-bdg+3bef))}{3e^2\sqrt{d+ex}(ef-dg)^2} + \frac{2c \tanh^{-1}\left(\frac{\sqrt{g}\sqrt{d+ex}}{\sqrt{e}\sqrt{f+gx}}\right)}{e^{5/2}\sqrt{g}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a+b*x+c*x^2)/((d+e*x)^{(5/2)}*\text{Sqrt}[f+g*x]),x]$

[Out]  $(-2*(a+(d*(c*d-b*e))/e^2)*\text{Sqrt}[f+g*x])/(3*(e*f-d*g)*(d+e*x)^{(3/2)})+(2*(2*c*d*(3*e*f-2*d*g)-e*(3*b*e*f-b*d*g-2*a*e*g))*\text{Sqrt}[f+g*x])/(3*e^{2*(e*f-d*g)^2}*\text{Sqrt}[d+e*x])+(2*c*\text{ArcTanh}[(\text{Sqrt}[g]*\text{Sqrt}[d+e*x])/(\text{Sqrt}[e]*\text{Sqrt}[f+g*x])])/(e^{(5/2)*\text{Sqrt}[g]})$

**Rubi in Sympy [A]** time = 48.2009, size = 207, normalized size = 1.29

$$\frac{2cd^2\sqrt{f+gx}}{3e^2(d+ex)^{\frac{3}{2}}(dg-ef)} - \frac{4cd\sqrt{f+gx}(2dg-3ef)}{3e^2\sqrt{d+ex}(dg-ef)^2} + \frac{2c \operatorname{atanh}\left(\frac{\sqrt{e}\sqrt{f+gx}}{\sqrt{g}\sqrt{d+ex}}\right)}{e^{\frac{5}{2}}\sqrt{g}} + \frac{2\sqrt{f+gx}(2aeg+bdg-3bef)}{3e\sqrt{d+ex}(dg-ef)^2} + \frac{2\sqrt{f+gx}(ae-bd)}{3e(d+ex)^{\frac{3}{2}}(dg-ef)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((c*x**2+b*x+a)/(e*x+d)**(5/2)/(g*x+f)**(1/2),x)$

[Out]  $2*c*d**2*\text{sqrt}(f+g*x)/(3*e**2*(d+e*x)**(3/2)*(d*g-e*f))-4*c*d*\text{sqrt}(f+g*x)*(2*d*g-3*e*f)/(3*e**2*\text{sqrt}(d+e*x)*(d*g-e*f)**2)+2*c*\text{atanh}(\text{sqrt}(e)*\text{sqrt}(f+g*x)/(\text{sqrt}(g)*\text{sqrt}(d+e*x)))/(e**(5/2)*\text{sqrt}(g))+2*\text{sqrt}(f+g*x)*(2*a*e*g+b*d*g-3*b*e*f)/(3*e*\text{sqrt}(d+e*x)*(d*g-e*f)**2)+2*\text{sqrt}(f+g*x)*(a*e-b*d)/(3*e*(d+e*x)**(3/2)*(d*g-e*f))$

**Mathematica [A]** time = 0.346789, size = 157, normalized size = 0.98

$$\frac{2\sqrt{f+gx}(e^2(a(3dg-ef+2egx)+b(-2df+dgx-3efx))+cd(-3d^2g+de(5f-4gx)+6e^2fx))}{3e^2(d+ex)^{3/2}(ef-dg)^2} + \frac{c \log\left(2\sqrt{e}\sqrt{g}\sqrt{d+ex}\sqrt{f+gx}+dg+ef+2egx\right)}{e^{5/2}\sqrt{g}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)/((d + e\*x)^(5/2)\*Sqrt[f + g\*x]),x]

[Out] (2\*Sqrt[f + g\*x]\*(c\*d\*(-3\*d^2\*g + 6\*e^2\*f\*x + d\*e\*(5\*f - 4\*g\*x)) + e^2\*(b\*(-2\*d\*f - 3\*e\*f\*x + d\*g\*x) + a\*(-(e\*f) + 3\*d\*g + 2\*e\*g\*x))))/(3\*e^2\*(e\*f - d\*g)^2\*(d + e\*x)^(3/2)) + (c\*Log[e\*f + d\*g + 2\*e\*g\*x + 2\*Sqrt[e]\*Sqrt[g]\*Sqrt[d + e\*x]\*Sqrt[f + g\*x]])/(e^(5/2)\*Sqrt[g])

**Maple [B]** time = 0.038, size = 773, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)/(e\*x+d)^(5/2)/(g\*x+f)^(1/2),x)

[Out] 1/3\*(g\*x+f)^(1/2)\*(3\*ln(1/2\*(2\*e\*g\*x+2\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+d\*g+e\*f)/(e\*g)^(1/2))\*x^2\*c\*d^2\*e^2\*g^2-6\*ln(1/2\*(2\*e\*g\*x+2\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+d\*g+e\*f)/(e\*g)^(1/2))\*x^2\*c\*d\*e^3\*f\*g+3\*ln(1/2\*(2\*e\*g\*x+2\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+d\*g+e\*f)/(e\*g)^(1/2))\*x^2\*c\*e^4\*f^2+6\*ln(1/2\*(2\*e\*g\*x+2\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+d\*g+e\*f)/(e\*g)^(1/2))\*x\*c\*d^3\*e\*g^2-12\*ln(1/2\*(2\*e\*g\*x+2\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+d\*g+e\*f)/(e\*g)^(1/2))\*x\*c\*d^2\*e^2\*f\*g+6\*ln(1/2\*(2\*e\*g\*x+2\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+d\*g+e\*f)/(e\*g)^(1/2))\*x\*c\*d\*e^3\*f^2+3\*ln(1/2\*(2\*e\*g\*x+2\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+d\*g+e\*f)/(e\*g)^(1/2))\*c\*d^4\*g^2-6\*ln(1/2\*(2\*e\*g\*x+2\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+d\*g+e\*f)/(e\*g)^(1/2))\*c\*d^3\*e\*f\*g+3\*ln(1/2\*(2\*e\*g\*x+2\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+d\*g+e\*f)/(e\*g)^(1/2))\*c\*d^2\*e^2\*f^2+4\*x\*a\*e^3\*g\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+2\*x\*b\*d\*e^2\*g\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)-6\*x\*b\*e^3\*f\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)-8\*x\*c\*d^2\*e\*g\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+12\*x\*c\*d\*e^2\*f\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+6\*a\*d\*e^2\*g\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)-2\*a\*e^3\*f\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)-4\*b\*d\*e^2\*f\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)-6\*c\*d^3\*g\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2)+10\*c\*d^2\*e\*f\*((e\*x+d)\*(g\*x+f))^(1/2)\*(e\*g)^(1/2))/(e\*g)^(1/2)/(d\*g-e\*f)^2/(e\*x+d)\*(g\*x+f)^(1/2)/e^2/(e\*x+d)^(3/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/((e\*x + d)^(5/2)\*sqrt(g\*x + f)),x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 1.62119, size = 1, normalized size = 0.01

$$\left[ \frac{4((5cd^2e - 2bde^2 - ae^3)f - 3(cd^3 - ade^2)g + (3(2cde^2 - be^3)f - (4cd^2e - bde^2 - 2ae^3)g)x)\sqrt{eg}\sqrt{ex+d}\sqrt{gx+f} + \dots}{6(d^2e^2g^2x^2 + 2(d^2e^4f^2 - 2c^2d^2e^2f^2g + c^2d^3e^2g^2)x)\sqrt{e^2g}} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/((e\*x + d)^(5/2)\*sqrt(g\*x + f)),x, algorithm="fricas")

[Out] [1/6\*(4\*((5\*c\*d^2\*e - 2\*b\*d\*e^2 - a\*e^3)\*f - 3\*(c\*d^3 - a\*d\*e^2)\*g + (3\*(2\*c\*d^2\*e - b\*e^3)\*f - (4\*c\*d^2\*e - b\*d\*e^2 - 2\*a\*e^3)\*g)\*x)\*sqrt(e\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f) + 3\*(c\*d^2\*e^2\*f^2 - 2\*c\*d^3\*e\*f\*g + c\*d^4\*g^2 + (c\*e^4\*f^2 - 2\*c\*d\*e^3\*f\*g + c\*d^2\*e^2\*g^2)\*x^2 + 2\*(c\*d\*e^3\*f^2 - 2\*c\*d^2\*e^2\*f\*g + c\*d^3\*e\*g^2)\*x)\*log(4\*(2\*e^2\*g^2\*x + e^2\*f\*g + d\*e\*g^2)\*sqrt(e\*x + d)\*sqrt(g\*x + f) + (8\*e^2\*g^2\*x^2 + e^2\*f^2 + 6\*d\*e\*f\*g + d^2\*g^2 + 8\*(e^2\*f\*g + d\*e\*g^2)\*x)\*sqrt(e\*g)))/((d^2\*e^4\*f^2 - 2\*d^3\*e^3\*f\*g + d^4\*e^2\*g^2 + (e^6\*f^2 - 2\*d\*e^5\*f\*g + d^2\*e^4\*g^2)\*x^2 + 2\*(d\*e^5\*f^2 - 2\*d^2\*e^4\*f\*g + d^3\*e^3\*g^2)\*x)\*sqrt(e\*g)), 1/3\*(2\*((5\*c\*d^2\*e - 2\*b\*d\*e^2 - a\*e^3)\*f - 3\*(c\*d^3 - a\*d\*e^2)\*g + (3\*(2\*c\*d^2\*e - b\*e^3)\*f - (4\*c\*d^2\*e - b\*d\*e^2 - 2\*a\*e^3)\*g)\*x)\*sqrt(-e\*g)\*sqrt(e\*x + d)\*sqrt(g\*x + f) + 3\*(c\*d^2\*e^2\*f^2 - 2\*c\*d^3\*e\*f\*g + c\*d^4\*g^2 + (c\*e^4\*f^2 - 2\*c\*d\*e^3\*f\*g + c\*d^2\*e^2\*g^2)\*x^2 + 2\*(c\*d\*e^3\*f^2 - 2\*c\*d^2\*e^2\*f\*g + c\*d^3\*e\*g^2)\*x)\*arctan(1/2\*(2\*e\*g\*x + e\*f + d\*g)\*sqrt(-e\*g)/(sqrt(e\*x + d)\*sqrt(g\*x + f)\*e\*g)))/((d^2\*e^4\*f^2 - 2\*d^3\*e^3\*f\*g + d^4\*e^2\*g^2 + (e^6\*f^2 - 2\*d\*e^5\*f\*g + d^2\*e^4\*g^2)\*x^2 + 2\*(d\*e^5\*f^2 - 2\*d^2\*e^4\*f\*g + d^3\*e^3\*g^2)\*x)\*sqrt(-e\*g))]

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{a + bx + cx^2}{(d + ex)^{\frac{5}{2}} \sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)/(e\*x+d)\*\*(5/2)/(g\*x+f)\*\*(1/2),x)

[Out] Integral((a + b\*x + c\*x\*\*2)/((d + e\*x)\*\*(5/2)\*sqrt(f + g\*x)), x)

---

**GIAC/XCAS [A]** time = 0.600819, size = 4, normalized size = 0.02

sage<sub>0</sub>x

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/((e\*x + d)^(5/2)\*sqrt(g\*x + f)),x, algorithm="giac")

[Out] sage<sub>0</sub>\*x

$$3.840 \quad \int \frac{a+bx+cx^2}{(d+ex)^{7/2}\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=198

$$\frac{2\sqrt{f+gx}(2eg(-4aeg-bdg+5bef)-c(3d^2g^2-10defg+15e^2f^2))}{15e^2\sqrt{d+ex}(ef-dg)^3} - \frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{5(d+ex)^{5/2}(ef-dg)} + \frac{2\sqrt{f+gx}(2cd(5ef-3dg)-e(-4aeg-bdg+5bef))}{15e^2(d+ex)^{3/2}(ef-dg)^2}$$

[Out]  $(-2*(a+(d*(c*d-b*e))/e^2)*\text{Sqrt}[f+g*x])/(5*(e*f-d*g)*(d+e*x)^{(5/2)}) + (2*(2*c*d*(5*e*f-3*d*g)-e*(5*b*e*f-b*d*g-4*a*e*g))*\text{Sqrt}[f+g*x])/(15*e^2*(e*f-d*g)^2*(d+e*x)^{(3/2)}) + (2*(2*e*g*(5*b*e*f-b*d*g-4*a*e*g)-c*(15*e^2*f^2-10*d*e*f*g+3*d^2*g^2))*\text{Sqrt}[f+g*x])/(15*e^2*(e*f-d*g)^3*\text{Sqrt}[d+e*x])$

**Rubi [A]** time = 0.513001, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$\frac{2\sqrt{f+gx}(2eg(-4aeg-bdg+5bef)-c(3d^2g^2-10defg+15e^2f^2))}{15e^2\sqrt{d+ex}(ef-dg)^3} - \frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{5(d+ex)^{5/2}(ef-dg)} + \frac{2\sqrt{f+gx}(2cd(5ef-3dg)-e(-4aeg-bdg+5bef))}{15e^2(d+ex)^{3/2}(ef-dg)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a+b*x+c*x^2)/((d+e*x)^{(7/2)}*\text{Sqrt}[f+g*x]),x]$

[Out]  $(-2*(a+(d*(c*d-b*e))/e^2)*\text{Sqrt}[f+g*x])/(5*(e*f-d*g)*(d+e*x)^{(5/2)}) + (2*(2*c*d*(5*e*f-3*d*g)-e*(5*b*e*f-b*d*g-4*a*e*g))*\text{Sqrt}[f+g*x])/(15*e^2*(e*f-d*g)^2*(d+e*x)^{(3/2)}) + (2*(2*e*g*(5*b*e*f-b*d*g-4*a*e*g)-c*(15*e^2*f^2-10*d*e*f*g+3*d^2*g^2))*\text{Sqrt}[f+g*x])/(15*e^2*(e*f-d*g)^3*\text{Sqrt}[d+e*x])$

**Rubi in Sympy [A]** time = 62.4999, size = 275, normalized size = 1.39

$$\frac{2cd^2\sqrt{f+gx}}{5e^2(d+ex)^{5/2}(dg-ef)} - \frac{4cd\sqrt{f+gx}(3dg-5ef)}{15e^2(d+ex)^{3/2}(dg-ef)^2} + \frac{2c\sqrt{f+gx}(3d^2g^2-10defg+15e^2f^2)}{15e^2\sqrt{d+ex}(dg-ef)^3} + \frac{4g\sqrt{f+gx}(4aeg+bdg-5bef)}{15e\sqrt{d+ex}(dg-ef)^3} + \frac{2\sqrt{f+gx}(4aeg+bdg-5bef)}{15e(d+ex)^{3/2}(dg-ef)^2} + \frac{2\sqrt{f+gx}(ae-bd)}{5e(d+ex)^{5/2}(dg-ef)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((c*x**2+b*x+a)/(e*x+d)**(7/2)/(g*x+f)**(1/2),x)$

[Out]  $2*c*d**2*\text{sqrt}(f+g*x)/(5*e**2*(d+e*x)**(5/2)*(d*g-e*f)) - 4*c*d*\text{sqrt}(f+g*x)*(3*d*g-5*e*f)/(15*e**2*(d+e*x)**(3/2)*(d*g-e*f)**2) + 2*c*\text{sqrt}(f+g*x)*(3*d**2*g**2-10*d*e*f*g+15*e**2*f**2)/(15*e**2*\text{sqrt}(d+e*x)*(d*g-e*f)**3) + 4*g*\text{sqrt}(f+g*x)*(4*a*e*g+b*d*g-5*b*e*f)/(15*e*\text{sqrt}(d+e*x)*(d*g-e*f)**3) + 2*\text{sqrt}(f+g*x)*(4*a*e*g+b*d*g-5*b*e*f)/(15*e*(d+e*x)**(3/2)*(d*g-e*f)**2) + 2*\text{sqrt}(f+g*x)*(a*e-b*d)/(5*e*(d+e*x)**(5/2)*(d*g-e*f))$



**Mathematica [A]** time = 0.474128, size = 178, normalized size = 0.9

$$\frac{2\sqrt{f+gx}(a(15d^2g^2-10deg(f-2gx)+e^2(3f^2-4fgx+8g^2x^2))+b(5d^2g(gx-2f)+2de(f^2-13fgx+g^2x^2))+5e^2f)}{15(d+ex)^{5/2}(dg-ef)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)/((d + e\*x)^(7/2)\*Sqrt[f + g\*x]),x]

[Out] (2\*Sqrt[f + g\*x]\*(b\*(5\*e^2\*f\*x\*(f - 2\*g\*x) + 5\*d^2\*g\*(-2\*f + g\*x) + 2\*d\*e\*(f^2 - 13\*f\*g\*x + g^2\*x^2)) + c\*(15\*e^2\*f^2\*x^2 + 10\*d\*e\*f\*x\*(2\*f - g\*x) + d^2\*(8\*f^2 - 4\*f\*g\*x + 3\*g^2\*x^2)) + a\*(15\*d^2\*g^2 - 10\*d\*e\*g\*(f - 2\*g\*x) + e^2\*(3\*f^2 - 4\*f\*g\*x + 8\*g^2\*x^2)))/(15\*(-(e\*f) + d\*g)^3\*(d + e\*x)^(5/2))

**Maple [A]** time = 0.012, size = 238, normalized size = 1.2

$$\frac{16ae^2g^2x^2 + 4bdeg^2x^2 - 20be^2fgx^2 + 6cd^2g^2x^2 - 20cdfg^2x^2 + 30ce^2f^2x^2 + 40adeg^2x - 8ae^2fgx + 10bd^2g^2x - 52bde^2fgx}{15g^3d^3 - 45d^2efg^2 + 45de^2fg^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)/(e\*x+d)^(7/2)/(g\*x+f)^(1/2),x)

[Out] 2/15\*(g\*x+f)^(1/2)\*(8\*a\*e^2\*g^2\*x^2+2\*b\*d\*e\*g^2\*x^2-10\*b\*e^2\*f\*g\*x^2+3\*c\*d^2\*g^2\*x^2-10\*c\*d\*e\*f\*g\*x^2+15\*c\*e^2\*f^2\*x^2+20\*a\*d\*e\*g^2\*x-4\*a\*e^2\*f\*g\*x+5\*b\*d^2\*g^2\*x-26\*b\*d\*e\*f\*g\*x+5\*b\*e^2\*f^2\*x-4\*c\*d^2\*f\*g\*x+20\*c\*d\*e\*f^2\*x+15\*a\*d^2\*g^2-10\*a\*d\*e\*f\*g+3\*a\*e^2\*f^2-10\*b\*d^2\*f\*g+2\*b\*d\*e\*f^2+8\*c\*d^2\*f^2)/(e\*x+d)^(5/2)/(d^3\*g^3-3\*d^2\*e\*f\*g^2+3\*d\*e^2\*f^2-g-e^3\*f^3)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/((e\*x + d)^(7/2)\*sqrt(g\*x + f)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.59129, size = 477, normalized size = 2.41

$$\frac{2(15ad^2g^2 + (8cd^2 + 2bde + 3ae^2)f^2 - 10(bd^2 + ade)fg + (15ce^2f^2 - 10(cde + be^2)fg + (3cd^2 + 2bde + 8ae^2)g^2)x}{15(d^3e^3f^3 - 3d^4e^2f^2g + 3d^5efg^2 - d^6g^3 + (e^6f^3 - 3d^5f^2g + 3d^2e^4fg^2 - d^3e^3g^3)x^3 + 3(de^5f^3 - 3d^2e^4fg^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/((e\*x + d)^(7/2)\*sqrt(g\*x + f)),x, algorithm="fricas")

[Out] -2/15\*(15\*a\*d^2\*g^2 + (8\*c\*d^2 + 2\*b\*d\*e + 3\*a\*e^2)\*f^2 - 10\*(b\*d^2 + a\*d\*e)\*f\*g + (15\*c\*e^2\*f^2 - 10\*(c\*d\*e + b\*e^2)\*f\*g + (3\*c\*d^2 + 2\*b\*d\*e + 8\*a\*e^2)\*g^2)\*x^2 + (5\*(4\*c\*d\*e + b\*e^2)\*f^2 - 2\*(

$$2*c*d^2 + 13*b*d*e + 2*a*e^2)*f*g + 5*(b*d^2 + 4*a*d*e)*g^2)*x)*s$$

$$\text{qrt}(e*x + d)*\text{sqrt}(g*x + f)/(d^3*e^3*f^3 - 3*d^4*e^2*f^2*g + 3*d^5$$

$$*e*f*g^2 - d^6*g^3 + (e^6*f^3 - 3*d*e^5*f^2*g + 3*d^2*e^4*f*g^2 -$$

$$d^3*e^3*g^3)*x^3 + 3*(d*e^5*f^3 - 3*d^2*e^4*f^2*g + 3*d^3*e^3*f*$$

$$g^2 - d^4*e^2*g^3)*x^2 + 3*(d^2*e^4*f^3 - 3*d^3*e^3*f^2*g + 3*d^4$$

$$*e^2*f*g^2 - d^5*e*g^3)*x)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)/(e\*x+d)\*\*(7/2)/(g\*x+f)\*\*(1/2),x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.387331, size = 1, normalized size = 0.01

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/((e\*x + d)^(7/2)\*sqrt(g\*x + f)),x, algorithm="giac")

[Out] Done

$$3.841 \quad \int \frac{a+bx+cx^2}{(d+ex)^{9/2}\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=281

$$\begin{aligned} & -\frac{4g\sqrt{f+gx}(4eg(-6aeg-bdg+7bef)-c(3d^2g^2-14defg+35e^2f^2))}{105e^2\sqrt{d+ex}(ef-dg)^4} \\ & +\frac{2\sqrt{f+gx}(4eg(-6aeg-bdg+7bef)-c(3d^2g^2-14defg+35e^2f^2))}{105e^2(d+ex)^{3/2}(ef-dg)^3} \\ & -\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{7(d+ex)^{7/2}(ef-dg)} + \frac{2\sqrt{f+gx}(2cd(7ef-4dg)-e(-6aeg-bdg+7bef))}{35e^2(d+ex)^{5/2}(ef-dg)^2} \end{aligned}$$

[Out]  $(-2*(a+(d*(c*d-b*e))/e^2)*\text{Sqrt}[f+g*x])/(7*(e*f-d*g)^*(d+e*x)^{(7/2)})+(2*(2*c*d*(7*e*f-4*d*g)-e*(7*b*e*f-b*d*g-6*a*e*g))*\text{Sqrt}[f+g*x]/(35*e^2*(e*f-d*g)^2*(d+e*x)^{(5/2)})+(2*(4*e*g*(7*b*e*f-b*d*g-6*a*e*g)-c*(35*e^2*f^2-14*d*e*f*g+3*d^2*g^2))*\text{Sqrt}[f+g*x]/(105*e^2*(e*f-d*g)^3*(d+e*x)^{(3/2)})-(4*g*(4*e*g*(7*b*e*f-b*d*g-6*a*e*g)-c*(35*e^2*f^2-14*d*e*f*g+3*d^2*g^2))*\text{Sqrt}[f+g*x]/(105*e^2*(e*f-d*g)^4*\text{Sqrt}[d+e*x])$

**Rubi [A]** time = 0.699531, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$

$$\begin{aligned} & -\frac{4g\sqrt{f+gx}(4eg(-6aeg-bdg+7bef)-c(3d^2g^2-14defg+35e^2f^2))}{105e^2\sqrt{d+ex}(ef-dg)^4} \\ & +\frac{2\sqrt{f+gx}(4eg(-6aeg-bdg+7bef)-c(3d^2g^2-14defg+35e^2f^2))}{105e^2(d+ex)^{3/2}(ef-dg)^3} \\ & -\frac{2\sqrt{f+gx}\left(a+\frac{d(cd-be)}{e^2}\right)}{7(d+ex)^{7/2}(ef-dg)} + \frac{2\sqrt{f+gx}(2cd(7ef-4dg)-e(-6aeg-bdg+7bef))}{35e^2(d+ex)^{5/2}(ef-dg)^2} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a+b*x+c*x^2)/((d+e*x)^{(9/2)}*\text{Sqrt}[f+g*x]),x]$

[Out]  $(-2*(a+(d*(c*d-b*e))/e^2)*\text{Sqrt}[f+g*x])/(7*(e*f-d*g)^*(d+e*x)^{(7/2)})+(2*(2*c*d*(7*e*f-4*d*g)-e*(7*b*e*f-b*d*g-6*a*e*g))*\text{Sqrt}[f+g*x]/(35*e^2*(e*f-d*g)^2*(d+e*x)^{(5/2)})+(2*(4*e*g*(7*b*e*f-b*d*g-6*a*e*g)-c*(35*e^2*f^2-14*d*e*f*g+3*d^2*g^2))*\text{Sqrt}[f+g*x]/(105*e^2*(e*f-d*g)^3*(d+e*x)^{(3/2)})-(4*g*(4*e*g*(7*b*e*f-b*d*g-6*a*e*g)-c*(35*e^2*f^2-14*d*e*f*g+3*d^2*g^2))*\text{Sqrt}[f+g*x]/(105*e^2*(e*f-d*g)^4*\text{Sqrt}[d+e*x])$

**Rubi in Sympy [A]** time = 88.0822, size = 389, normalized size = 1.38

$$\begin{aligned} & \frac{2cd^2\sqrt{f+gx}}{7e^2(d+ex)^{\frac{7}{2}}(dg-ef)} - \frac{4cd\sqrt{f+gx}(4dg-7ef)}{35e^2(d+ex)^{\frac{5}{2}}(dg-ef)^2} + \frac{4cg\sqrt{f+gx}(3d^2g^2-14defg+35e^2f^2)}{105e^2\sqrt{d+ex}(dg-ef)^4} \\ & + \frac{2c\sqrt{f+gx}(3d^2g^2-14defg+35e^2f^2)}{105e^2(d+ex)^{\frac{3}{2}}(dg-ef)^3} + \frac{16g^2\sqrt{f+gx}(6aeg+bdg-7bef)}{105e\sqrt{d+ex}(dg-ef)^4} \\ & + \frac{8g\sqrt{f+gx}(6aeg+bdg-7bef)}{105e(d+ex)^{\frac{3}{2}}(dg-ef)^3} + \frac{2\sqrt{f+gx}(6aeg+bdg-7bef)}{35e(d+ex)^{\frac{5}{2}}(dg-ef)^2} + \frac{2\sqrt{f+gx}(ae-bd)}{7e(d+ex)^{\frac{7}{2}}(dg-ef)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x**2+b*x+a)/(e*x+d)**(9/2)/(g*x+f)**(1/2),x)`

[Out]  $2*c*d**2*\sqrt{f+g*x}/(7*e**2*(d+e*x)**(7/2)*(d*g-e*f)) - 4*c*d*\sqrt{f+g*x}*(4*d*g-7*e*f)/(35*e**2*(d+e*x)**(5/2)*(d*g-e*f)**2) + 4*c*g*\sqrt{f+g*x}*(3*d**2*g**2-14*d*e*f*g+35*e**2*f**2)/(105*e**2*\sqrt{d+e*x}*(d*g-e*f)**4) + 2*c*\sqrt{f+g*x}*(3*d**2*g**2-14*d*e*f*g+35*e**2*f**2)/(105*e**2*(d+e*x)**(3/2)*(d*g-e*f)**3) + 16*g**2*\sqrt{f+g*x}*(6*a*e*g+b*d*g-7*b*e*f)/(105*e*\sqrt{d+e*x}*(d*g-e*f)**4) + 8*g*\sqrt{f+g*x}*(6*a*e*g+b*d*g-7*b*e*f)/(105*e*(d+e*x)**(3/2)*(d*g-e*f)**3) + 2*\sqrt{f+g*x}*(6*a*e*g+b*d*g-7*b*e*f)/(35*e*(d+e*x)**(5/2)*(d*g-e*f)**2) + 2*\sqrt{f+g*x}*(a*e-b*d)/(7*e*(d+e*x)**(7/2)*(d*g-e*f))$

**Mathematica [A]** time = 1.69695, size = 230, normalized size = 0.82

$$\frac{2\sqrt{f+gx}(2g(d+ex)^3(4eg(6aeg+bdg-7bef)+c(3d^2g^2-14defg+35e^2f^2))-(d+ex)^2(ef-dg)(4eg(6aeg+bdg-7bef)+35e^2f^2))}{105e^2}$$

Antiderivative was successfully verified.

[In] `Integrate[(a+b*x+c*x^2)/((d+e*x)^(9/2)*Sqrt[f+g*x]),x]`

[Out]  $(2*\sqrt{f+g*x}*(15*(c*d^2+e*(-b*d)+a*e))*(-(e*f)+d*g)^3 - 3*(e*f-d*g)^2*(2*c*d*(-7*e*f+4*d*g)+e*(7*b*e*f-b*d*g-6*a*e*g))*(d+e*x) - (e*f-d*g)*(4*e*g*(-7*b*e*f+b*d*g+6*a*e*g)+c*(35*e^2*f^2-14*d*e*f*g+3*d^2*g^2))*(d+e*x)^2 + 2*g*(4*e*g*(-7*b*e*f+b*d*g+6*a*e*g)+c*(35*e^2*f^2-14*d*e*f*g+3*d^2*g^2))*(d+e*x)^3)/(105*e^2*(e*f-d*g)^4*(d+e*x)^(7/2))$

**Maple [A]** time = 0.014, size = 468, normalized size = 1.7

$$\frac{96ae^3g^3x^3 + 16bde^2g^3x^3 - 112be^3fg^2x^3 + 12cd^2eg^3x^3 - 56cde^2fg^2x^3 + 140ce^3f^2gx^3 + 336ade^2g^3x^2 - 48ae^3fg^2x^2 + 50ae^3fg^2x^2 - 48ae^3fg^2x^2 + 50ae^3fg^2x^2}{105e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)/(e*x+d)^(9/2)/(g*x+f)^(1/2),x)`

[Out]  $2/105*(g*x+f)^(1/2)*(48*a*e^3*g^3*x^3+8*b*d*e^2*g^3*x^3-56*b*e^3*f*g^2*x^3+6*c*d^2*e*g^3*x^3-28*c*d*e^2*f*g^2*x^3+70*c*e^3*f^2*g*x^3+168*a*d*e^2*g^3*x^2-24*a*e^3*f*g^2*x^2+28*b*d^2*e*g^3*x^2-200*b*d*e^2*f*g^2*x^2+28*b*e^3*f^2*g*x^2+21*c*d^3*g^3*x^2-101*c*d^2*e*f*g^2*x^2+259*c*d*e^2*f^2*g*x^2-35*c*e^3*f^3*x^2+210*a*d^2*e*g^3*x-84*a*d*e^2*f*g^2*x+18*a*e^3*f^2*g*x+35*b*d^3*g^3*x-259*b*d^2*e*f*g^2*x+101*b*d*e^2*f^2*g*x-21*b*e^3*f^3*x-28*c*d^3*f*g^2*x+200*c*d^2*e*f^2*g*x-28*c*d*e^2*f^3*x+105*a*d^3*g^3-105*a*d^2*e*f*g^2+63*a*d*e^2*f^2*g-15*a*e^3*f^3-70*b*d^3*f*g^2+28*b*d^2*e*f^2*g-6*b*d*e^2*f^3+56*c*d^3*f^2*g-8*c*d^2*e*f^3)/(e*x+d)^(7/2)/(d^4*g^4-4*d^3*e*f*g^3+6*d^2*e^2*f^2*g^2-4*d*e^3*f^3*g+e^4*f^4)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/((e\*x + d)^(9/2)\*sqrt(g\*x + f)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.45997, size = 865, normalized size = 3.08

$$\frac{2(105ad^3g^3 - (8cd^2e + 6bde^2 + 15ae^3)f^3 + 7(8cd^3 + 4bd^2e + 9ade^2)f^2g - 35(2bd^3 + 3ad^2e)fg^2 + 2(35ce^3f^2g - 14cd^3e^2f^2g^2 - 4d^7efg^3 + d^8g^4 + (e^8f^4 - 4d^4e^4f^4 - 4d^5e^3f^3g + 6d^6e^2f^2g^2 - 4d^7efg^3 + d^8g^4 + (e^8f^4 - 4d^4e^4f^4 - 4d^5e^3f^3g + 6d^6e^2f^2g^2 - 4d^7efg^3 + d^8g^4 + (e^8f^4 - 4d^4e^4f^4 - 4d^5e^3f^3g + 6d^6e^2f^2g^2 - 4d^7efg^3 + d^8g^4 + \dots))\dots)}{105(d^4e^4f^4 - 4d^5e^3f^3g + 6d^6e^2f^2g^2 - 4d^7efg^3 + d^8g^4 + (e^8f^4 - 4d^4e^4f^4 - 4d^5e^3f^3g + 6d^6e^2f^2g^2 - 4d^7efg^3 + d^8g^4 + \dots))\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/((e\*x + d)^(9/2)\*sqrt(g\*x + f)),x, algorithm="fricas")

[Out] 
$$\frac{2}{105} \cdot (105 \cdot a \cdot d^3 \cdot g^3 - (8 \cdot c \cdot d^2 \cdot e + 6 \cdot b \cdot d \cdot e^2 + 15 \cdot a \cdot e^3) \cdot f^3 + 7 \cdot (8 \cdot c \cdot d^3 + 4 \cdot b \cdot d^2 \cdot e + 9 \cdot a \cdot d \cdot e^2) \cdot f^2 \cdot g - 35 \cdot (2 \cdot b \cdot d^3 + 3 \cdot a \cdot d^2 \cdot e) \cdot f \cdot g^2 + 2 \cdot (35 \cdot c \cdot e^3 \cdot f^2 \cdot g - 14 \cdot c \cdot d^3 \cdot e^2 \cdot f^2 \cdot g^2 - 4 \cdot d^7 \cdot e \cdot f \cdot g^3 + d^8 \cdot g^4 + (e^8 \cdot f^4 - 4 \cdot d^4 \cdot e^4 \cdot f^4 - 4 \cdot d^5 \cdot e^3 \cdot f^3 \cdot g + 6 \cdot d^6 \cdot e^2 \cdot f^2 \cdot g^2 - 4 \cdot d^7 \cdot e \cdot f \cdot g^3 + d^8 \cdot g^4 + \dots)) \dots) \cdot \sqrt{e \cdot x + d} \cdot \sqrt{g \cdot x + f} / (d^4 \cdot e^4 \cdot f^4 - 4 \cdot d^5 \cdot e^3 \cdot f^3 \cdot g + 6 \cdot d^6 \cdot e^2 \cdot f^2 \cdot g^2 - 4 \cdot d^7 \cdot e \cdot f \cdot g^3 + d^8 \cdot g^4 + (e^8 \cdot f^4 - 4 \cdot d^4 \cdot e^4 \cdot f^4 - 4 \cdot d^5 \cdot e^3 \cdot f^3 \cdot g + 6 \cdot d^6 \cdot e^2 \cdot f^2 \cdot g^2 - 4 \cdot d^7 \cdot e \cdot f \cdot g^3 + d^8 \cdot g^4 + \dots)) \dots)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)/(e\*x+d)\*\*(9/2)/(g\*x+f)\*\*(1/2),x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.482085, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)/((e\*x + d)^(9/2)\*sqrt(g\*x + f)),x, algorithm="giac")

[Out] Done

$$3.842 \quad \int \frac{\sqrt{d+ex}(a+bx+cx^2)}{(e+fx)^{3/2}} dx$$

**Optimal.** Leaf size=249

$$\frac{\sqrt{d+ex}\sqrt{e+fx}(4ef(-2aef-bdf+3be^2)-c(-d^2f^2-6de^2f+15e^4))}{4ef^3(e^2-df)} \\ - \frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{e}\sqrt{e+fx}}\right)(4ef(-2aef-bdf+3be^2)-c(-d^2f^2-6de^2f+15e^4))}{4e^{3/2}f^{7/2}} \\ + \frac{2(d+ex)^{3/2}\left(a+\frac{e(ce-bf)}{f^2}\right)}{(e^2-df)\sqrt{e+fx}} + \frac{c(d+ex)^{3/2}\sqrt{e+fx}}{2ef^2}$$

[Out] (2\*(a+(e\*(c\*e-b\*f))/f^2)\*(d+e\*x)^(3/2))/((e^2-d\*f)\*Sqrt[e+f\*x]) + ((4\*e\*f\*(3\*b\*e^2-b\*d\*f-2\*a\*e\*f)-c\*(15\*e^4-6\*d\*e^2\*f-d^2\*f^2))\*Sqrt[d+e\*x]\*Sqrt[e+f\*x])/(4\*e\*f^3\*(e^2-d\*f)) + (c\*(d+e\*x)^(3/2)\*Sqrt[e+f\*x])/(2\*e\*f^2) - ((4\*e\*f\*(3\*b\*e^2-b\*d\*f-2\*a\*e\*f)-c\*(15\*e^4-6\*d\*e^2\*f-d^2\*f^2))\*ArcTan h[(Sqrt[f]\*Sqrt[d+e\*x])/(Sqrt[e]\*Sqrt[e+f\*x])])/(4\*e^(3/2)\*f^(7/2))

**Rubi [A]** time = 0.606924, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$\frac{\sqrt{d+ex}\sqrt{e+fx}(4ef(-2aef-bdf+3be^2)-c(-d^2f^2-6de^2f+15e^4))}{4ef^3(e^2-df)} \\ - \frac{\tanh^{-1}\left(\frac{\sqrt{f}\sqrt{d+ex}}{\sqrt{e}\sqrt{e+fx}}\right)(4ef(-2aef-bdf+3be^2)-c(-d^2f^2-6de^2f+15e^4))}{4e^{3/2}f^{7/2}} \\ + \frac{2(d+ex)^{3/2}\left(a+\frac{e(ce-bf)}{f^2}\right)}{(e^2-df)\sqrt{e+fx}} + \frac{c(d+ex)^{3/2}\sqrt{e+fx}}{2ef^2}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[d+e\*x]\*(a+b\*x+c\*x^2))/(e+f\*x)^(3/2),x]

[Out] (2\*(a+(e\*(c\*e-b\*f))/f^2)\*(d+e\*x)^(3/2))/((e^2-d\*f)\*Sqrt[e+f\*x]) + ((4\*e\*f\*(3\*b\*e^2-b\*d\*f-2\*a\*e\*f)-c\*(15\*e^4-6\*d\*e^2\*f-d^2\*f^2))\*Sqrt[d+e\*x]\*Sqrt[e+f\*x])/(4\*e\*f^3\*(e^2-d\*f)) + (c\*(d+e\*x)^(3/2)\*Sqrt[e+f\*x])/(2\*e\*f^2) - ((4\*e\*f\*(3\*b\*e^2-b\*d\*f-2\*a\*e\*f)-c\*(15\*e^4-6\*d\*e^2\*f-d^2\*f^2))\*ArcTan h[(Sqrt[f]\*Sqrt[d+e\*x])/(Sqrt[e]\*Sqrt[e+f\*x])])/(4\*e^(3/2)\*f^(7/2))

**Rubi in Sympy [A]** time = 69.5464, size = 311, normalized size = 1.25

$$\frac{2ce^2(d+ex)^{\frac{3}{2}}}{f^2\sqrt{e+fx}(-df+e^2)} + \frac{c(d+ex)^{\frac{3}{2}}\sqrt{e+fx}}{2ef^2} - \frac{c\sqrt{d+ex}\sqrt{e+fx}(-d^2f^2-6de^2f+15e^4)}{4ef^3(-df+e^2)} \\ + \frac{c(-d^2f^2-6de^2f+15e^4)\operatorname{atanh}\left(\frac{\sqrt{e}\sqrt{e+fx}}{\sqrt{f}\sqrt{d+ex}}\right)}{4e^{\frac{3}{2}}f^{\frac{7}{2}}} + \frac{2(d+ex)^{\frac{3}{2}}(af-be)}{f\sqrt{e+fx}(-df+e^2)} \\ + \frac{2\sqrt{d+ex}\sqrt{e+fx}\left(-aef+\frac{b(-df+3e^2)}{2}\right)}{f^2(-df+e^2)} - \frac{(-2aef-bdf+3be^2)\operatorname{atanh}\left(\frac{\sqrt{e}\sqrt{e+fx}}{\sqrt{f}\sqrt{d+ex}}\right)}{\sqrt{e}f^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**(1/2)*(c*x**2+b*x+a)/(f*x+e)**(3/2),x)`

[Out]  $2*c*e^{**2}(d + e*x)^{(3/2)}/(f^{**2}\sqrt{e + f*x}*(-d*f + e^{**2})) + c*(d + e*x)^{(3/2)}\sqrt{e + f*x}/(2*e*f^{**2}) - c*\sqrt{d + e*x}\sqrt{e + f*x}*(-d^{**2}*f^{**2} - 6*d*e^{**2}*f + 15*e^{**4})/(4*e*f^{**3}*(-d*f + e^{**2})) + c*(-d^{**2}*f^{**2} - 6*d*e^{**2}*f + 15*e^{**4})*\operatorname{atanh}(\sqrt{e}*\sqrt{e + f*x})/(\sqrt{f}*\sqrt{d + e*x})/(4*e^{**3/2}*f^{**7/2}) + 2*(d + e*x)^{(3/2)}*(a*f - b*e)/(f*\sqrt{e + f*x}*(-d*f + e^{**2})) + 2*\sqrt{d + e*x}\sqrt{e + f*x}*(-a*e*f + b*(-d*f + 3*e^{**2})/2)/(f^{**2}*(-d*f + e^{**2})) - (-2*a*e*f - b*d*f + 3*b*e^{**2})*\operatorname{atanh}(\sqrt{e}*\sqrt{e + f*x})/(\sqrt{f}*\sqrt{d + e*x})/(sqrt(e)*f^{**5/2})$

**Mathematica [A]** time = 0.264183, size = 180, normalized size = 0.72

$$\frac{\log\left(2\sqrt{e}\sqrt{f}\sqrt{d+ex}\sqrt{e+fx}+df+e^2+2efx\right)\left(4ef(2aef+ddf-3be^2)+c(-d^2f^2-6de^2f+15e^4)\right)}{8e^{3/2}f^{7/2}} + \frac{\sqrt{d+ex}\left(4ef(-2af+3be+bfx)+c\left(ef(d+2fx^2)+df^2x-15e^3-5e^2fx\right)\right)}{4ef^3\sqrt{e+fx}}$$

Antiderivative was successfully verified.

[In] `Integrate[(Sqrt[d + e*x]*(a + b*x + c*x^2))/(e + f*x)^(3/2),x]`

[Out]  $(\sqrt{d + e*x}(4*e*f*(3*b*e - 2*a*f + b*f*x) + c*(-15*e^3 - 5*e^2*f*x + d*f^2*x + e*f*(d + 2*f*x^2)))/((4*e*f^3*\sqrt{e + f*x}) + ((4*e*f*(-3*b*e^2 + b*d*f + 2*a*e*f) + c*(15*e^4 - 6*d*e^2*f - d^2*f^2))*\operatorname{Log}[e^2 + d*f + 2*e*f*x + 2*\sqrt{e}*\sqrt{f}*\sqrt{d + e*x}]*\sqrt{e + f*x}]/(8*e^{3/2}*f^{7/2}))$

**Maple [B]** time = 0.048, size = 834, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^(1/2)*(c*x^2+b*x+a)/(f*x+e)^(3/2),x)`

[Out]  $1/8*(e*x+d)^{(1/2)}*(8*\ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^{1/2}*(e*f)^{(1/2)+d*f+e^2})/(e*f)^{(1/2)})*x*a*e^2*f^3+4*\ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^{1/2}*(e*f)^{(1/2)+d*f+e^2})/(e*f)^{(1/2)})*x*b*d*e*f^3-12*\ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^{1/2}*(e*f)^{(1/2)+d*f+e^2})/(e*f)^{(1/2)})*x*b*e^3*f^2-\ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^{1/2}*(e*f)^{(1/2)+d*f+e^2})/(e*f)^{(1/2)})*x*c*d^2*f^3-6*\ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^{1/2}*(e*f)^{(1/2)+d*f+e^2})/(e*f)^{(1/2)})*x*c*d^2*f^2+15*\ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^{1/2}*(e*f)^{(1/2)+d*f+e^2})/(e*f)^{(1/2)})*x*c*e^4*f+4*x^2*c*e*f^2*((e*x+d)*(f*x+e))^{1/2}*(e*f)^{(1/2)+8*\ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^{1/2}*(e*f)^{(1/2)+d*f+e^2})/(e*f)^{(1/2)})*a*e^3*f^2+4*\ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^{1/2}*(e*f)^{(1/2)+d*f+e^2})/(e*f)^{(1/2)})*b*d*e^2*f^2-12*\ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^{1/2}*(e*f)^{(1/2)+d*f+e^2})/(e*f)^{(1/2)})*b*e^4*f-\ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^{1/2}*(e*f)^{(1/2)+d*f+e^2})/(e*f)^{(1/2)})*c*d^2*e*f^2-6*\ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^{1/2}*(e*f)^{(1/2)+d*f+e^2})/(e*f)^{(1/2)})*c*d^2*f+15*\ln(1/2*(2*e*f*x+2*((e*x+d)*(f*x+e))^{1/2}*(e*f)^{(1/2)+d*f+e^2})/(e*f)^{(1/2)})*c*e^5+8*x*b*e*f^2*((e*x+d)*(f*x+e))^{1/2}*(e*f)^{(1/2)+2*x*c*d*f^2*((e*x+d)*(f*x+e))^{1/2}*(e*f)^{(1/2)}-10*x*c*e^2*f*((e*x+d)*(f*x+e))^{1/2}*(e*f)^{(1/2)}-16*a*e*f^2*((e*x+d)*(f*x+e))^{1/2}*(e*f)^{(1/2)}+24*b*e^2*f*((e*x+d)*(f*x+e))^{1/2}*(e$

$$f^{1/2} + 2cd^2e^2f^2((ex+d)(fx+e))^{1/2} * (ef)^{1/2} - 30c^2e^3((ex+d)(fx+e))^{1/2} * (ef)^{1/2} / (ef)^{1/2} / e / ((ex+d)(fx+e))^{1/2} / f^3 / (fx+e)^{1/2}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*sqrt(e\*x + d)/(f\*x + e)^(3/2), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.715297, size = 1, normalized size = 0.

$$\frac{4(2cef^2x^2 - 15ce^3 - 8aef^2 + (cde + 12be^2)f - (5ce^2f - (cd + 4be)f^2)x)\sqrt{ef}\sqrt{ex+d}\sqrt{fx+e} + (15ce^5 - (cd^2e - 4bde^2 - 8a^2e^3)f^2 - 6(c^2d^2e^3 + 2b^2e^4)f + (15c^2e^4f - (c^2d^2 - 4b^2d^2e - 8a^2e^2)f^2 - 6(c^2d^2e^2 + 2b^2e^3)f^2)x)\log(4(2e^2f^2x^2 + e^3f + d^2e^2f^2)\sqrt{ex+d}\sqrt{fx+e} + (8e^2f^2x^2 + e^4 + 6d^2e^2f + d^2f^2 + 8(e^3f + d^2e^2f^2)x)\sqrt{ef}))}{(ef^4x + e^2f^3)\sqrt{ef}}, \frac{1}{8}(2(2c^2e^2f^2x^2 - 15c^2e^3 - 8a^2e^2f^2 + (c^2d^2e + 12b^2e^2)f - (5c^2e^2f - (cd + 4be)f^2)x)\sqrt{-ef}\sqrt{ex+d}\sqrt{fx+e} + (15c^2e^5 - (c^2d^2e - 4b^2d^2e - 8a^2e^3)f^2 - 6(c^2d^2e^3 + 2b^2e^4)f + (15c^2e^4f - (c^2d^2 - 4b^2d^2e - 8a^2e^2)f^2 - 6(c^2d^2e^2 + 2b^2e^3)f^2)x)\arctan(1/2(2e^2fx + e^2 + d^2f)\sqrt{-ef}/(\sqrt{ex+d}\sqrt{fx+e})*ef)))/((ef^4x + e^2f^3)\sqrt{-ef})]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*sqrt(e\*x + d)/(f\*x + e)^(3/2), x, algorithm="fricas")

[Out] [1/16\*(4\*(2\*c^2\*e^2\*f^2\*x^2 - 15\*c^2\*e^3 - 8\*a^2\*e^2\*f^2 + (c^2\*d^2\*e + 12\*b^2\*e^2)\*f - (5\*c^2\*e^2\*f - (c^2\*d + 4\*b^2\*e)\*f^2)\*x)\*sqrt(e\*f)\*sqrt(e\*x + d)\*sqrt(f\*x + e) + (15\*c^2\*e^5 - (c^2\*d^2\*e - 4\*b^2\*d^2\*e^2 - 8\*a^2\*e^3)\*f^2 - 6\*(c^2\*d^2\*e^3 + 2\*b^2\*e^4)\*f + (15\*c^2\*e^4\*f - (c^2\*d^2 - 4\*b^2\*d^2\*e - 8\*a^2\*e^2)\*f^2 - 6\*(c^2\*d^2\*e^2 + 2\*b^2\*e^3)\*f^2)\*x)\*log(4\*(2\*e^2\*f^2\*x^2 + e^3\*f + d^2\*e^2\*f^2)\*sqrt(e\*x + d)\*sqrt(f\*x + e) + (8\*e^2\*f^2\*x^2 + e^4 + 6\*d^2\*e^2\*f + d^2\*f^2 + 8\*(e^3\*f + d^2\*e^2\*f^2)\*x)\*sqrt(e\*f)))/((e\*f^4\*x + e^2\*f^3)\*sqrt(e\*f)), 1/8\*(2\*(2\*c^2\*e^2\*f^2\*x^2 - 15\*c^2\*e^3 - 8\*a^2\*e^2\*f^2 + (c^2\*d^2\*e + 12\*b^2\*e^2)\*f - (5\*c^2\*e^2\*f - (c^2\*d + 4\*b^2\*e)\*f^2)\*x)\*sqrt(-e\*f)\*sqrt(e\*x + d)\*sqrt(f\*x + e) + (15\*c^2\*e^5 - (c^2\*d^2\*e - 4\*b^2\*d^2\*e^2 - 8\*a^2\*e^3)\*f^2 - 6\*(c^2\*d^2\*e^3 + 2\*b^2\*e^4)\*f + (15\*c^2\*e^4\*f - (c^2\*d^2 - 4\*b^2\*d^2\*e - 8\*a^2\*e^2)\*f^2 - 6\*(c^2\*d^2\*e^2 + 2\*b^2\*e^3)\*f^2)\*x)\*arctan(1/2\*(2\*e^2\*f\*x + e^2 + d^2\*f)\*sqrt(-e\*f)/(sqrt(e\*x + d)\*sqrt(f\*x + e)\*e\*f)))/((e\*f^4\*x + e^2\*f^3)\*sqrt(-e\*f))]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}(a+bx+cx^2)}{(e+fx)^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(1/2)\*(c\*x\*\*2+b\*x+a)/(f\*x+e)\*\*(3/2), x)

[Out] Integral(sqrt(d + e\*x)\*(a + b\*x + c\*x\*\*2)/(e + f\*x)\*\*(3/2), x)



**GIAC/XCAS [A]** time = 0.300739, size = 320, normalized size = 1.29

$$\frac{\left((xe + d)\left(\frac{2(xe+d)ce^{(-1)}}{f} - \frac{(3cdf^4e^2 - 4bf^4e^3 + 5cf^3e^4)e^{(-3)}}{f^5}\right) + \frac{(cd^2f^4e^2 - 4bdf^4e^3 + 6cdf^3e^4 - 8af^4e^4 + 12bf^3e^5 - 15cf^2e^6)e^{(-3)}}{f^5}\right)\sqrt{xe + d}}{4\sqrt{(xe + d)fe - dfe + e^3}} + \frac{(cd^2f^2 - 4bdf^2e + 6cdf^2e^2 - 8af^2e^2 + 12bfe^3 - 15ce^4)e^{(-\frac{3}{2})}\ln\left(\left|-\sqrt{xe + d}\sqrt{fe^{\frac{1}{2}}} + \sqrt{(xe + d)fe - dfe + e^3}\right|\right)}{4f^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*sqrt(e\*x + d)/(f\*x + e)^(3/2), x, algorithm="giac")

[Out] 1/4\*((x\*e + d)\*(2\*(x\*e + d)\*c\*e^(-1)/f - (3\*c\*d\*f^4\*e^2 - 4\*b\*f^4\*e^3 + 5\*c\*f^3\*e^4)\*e^(-3)/f^5) + (c\*d^2\*f^4\*e^2 - 4\*b\*d\*f^4\*e^3 + 6\*c\*d\*f^3\*e^4 - 8\*a\*f^4\*e^4 + 12\*b\*f^3\*e^5 - 15\*c\*f^2\*e^6)\*e^(-3)/f^5)\*sqrt(x\*e + d)/sqrt((x\*e + d)\*f\*e - d\*f\*e + e^3) + 1/4\*(c\*d^2\*f^2 - 4\*b\*d\*f^2\*e + 6\*c\*d\*f^2\*e^2 - 8\*a\*f^2\*e^2 + 12\*b\*f^2\*e^3 - 15\*c\*f^2\*e^4)\*e^(-3/2)\*ln(abs(-sqrt(x\*e + d)\*sqrt(f)\*e^(1/2) + sqrt((x\*e + d)\*f\*e - d\*f\*e + e^3)))/f^(7/2)

$$3.843 \quad \int \frac{(d+ex)^{3/2}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}} dx$$

Optimal. Leaf size=240

$$\begin{aligned} & \frac{(bd - ae)^2 (35a^2e^2 - 90abde + 73b^2d^2) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{8b^{9/2}\sqrt{e}} \\ & + \frac{\sqrt{a+bx}\sqrt{d+ex}(bd - ae) (35a^2e^2 - 90abde + 73b^2d^2)}{8b^4} \\ & + \frac{\sqrt{a+bx}(d+ex)^{3/2} (35a^2e^2 - 90abde + 73b^2d^2)}{12b^3} \\ & + \frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2} + \frac{\sqrt{a+bx}(d+ex)^{5/2}(17bd - 13ae)}{3b^2} \end{aligned}$$

[Out] ((b\*d - a\*e)\*(73\*b^2\*d^2 - 90\*a\*b\*d\*e + 35\*a^2\*e^2)\*Sqrt[a + b\*x] \* Sqrt[d + e\*x])/(8\*b^4) + ((73\*b^2\*d^2 - 90\*a\*b\*d\*e + 35\*a^2\*e^2) \* Sqrt[a + b\*x]\*(d + e\*x)^(3/2))/(12\*b^3) + ((17\*b\*d - 13\*a\*e)\*Sqrt[a + b\*x]\*(d + e\*x)^(5/2))/(3\*b^2) + (2\*e\*(a + b\*x)^(3/2)\*(d + e\*x)^(5/2))/b^2 + ((b\*d - a\*e)^2\*(73\*b^2\*d^2 - 90\*a\*b\*d\*e + 35\*a^2\*e^2)\*ArcTanh[(Sqrt[e]\*Sqrt[a + b\*x])/(Sqrt[b]\*Sqrt[d + e\*x])])/(8\*b^(9/2)\*Sqrt[e])

**Rubi [A]** time = 0.485785, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$

$$\begin{aligned} & \frac{(bd - ae)^2 (35a^2e^2 - 90abde + 73b^2d^2) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{8b^{9/2}\sqrt{e}} \\ & + \frac{\sqrt{a+bx}\sqrt{d+ex}(bd - ae) (35a^2e^2 - 90abde + 73b^2d^2)}{8b^4} \\ & + \frac{\sqrt{a+bx}(d+ex)^{3/2} (35a^2e^2 - 90abde + 73b^2d^2)}{12b^3} \\ & + \frac{2e(a+bx)^{3/2}(d+ex)^{5/2}}{b^2} + \frac{\sqrt{a+bx}(d+ex)^{5/2}(17bd - 13ae)}{3b^2} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^(3/2)\*(15\*d^2 + 20\*d\*e\*x + 8\*e^2\*x^2))/Sqrt[a + b\*x], x]

[Out] ((b\*d - a\*e)\*(73\*b^2\*d^2 - 90\*a\*b\*d\*e + 35\*a^2\*e^2)\*Sqrt[a + b\*x] \* Sqrt[d + e\*x])/(8\*b^4) + ((73\*b^2\*d^2 - 90\*a\*b\*d\*e + 35\*a^2\*e^2) \* Sqrt[a + b\*x]\*(d + e\*x)^(3/2))/(12\*b^3) + ((17\*b\*d - 13\*a\*e)\*Sqrt[a + b\*x]\*(d + e\*x)^(5/2))/(3\*b^2) + (2\*e\*(a + b\*x)^(3/2)\*(d + e\*x)^(5/2))/b^2 + ((b\*d - a\*e)^2\*(73\*b^2\*d^2 - 90\*a\*b\*d\*e + 35\*a^2\*e^2)\*ArcTanh[(Sqrt[e]\*Sqrt[a + b\*x])/(Sqrt[b]\*Sqrt[d + e\*x])])/(8\*b^(9/2)\*Sqrt[e])

**Rubi in Sympy [A]** time = 75.985, size = 396, normalized size = 1.65

$$\begin{aligned} & \frac{20d\sqrt{a+bx}(d+ex)^{\frac{5}{2}}}{3b} + \frac{2ex\sqrt{a+bx}(d+ex)^{\frac{5}{2}}}{b} - \frac{5d\sqrt{a+bx}(d+ex)^{\frac{3}{2}}(10ae-7bd)}{6b^2} \\ & - \frac{\sqrt{a+bx}(d+ex)^{\frac{5}{2}}(7ae+3bd)}{3b^2} + \frac{5d\sqrt{a+bx}\sqrt{d+ex}(ae-bd)(10ae-7bd)}{4b^3} \\ & + \frac{\sqrt{a+bx}(d+ex)^{\frac{3}{2}}(35a^2e^2+10abde+3b^2d^2)}{12b^3} \\ & - \frac{\sqrt{a+bx}\sqrt{d+ex}(ae-bd)(35a^2e^2+10abde+3b^2d^2)}{8b^4} \\ & - \frac{5d(ae-bd)^2(10ae-7bd)\operatorname{atanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{4b^{\frac{7}{2}}\sqrt{e}} \\ & + \frac{(ae-bd)^2(35a^2e^2+10abde+3b^2d^2)\operatorname{atanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{8b^{\frac{9}{2}}\sqrt{e}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**(3/2)*(8*e**2*x**2+20*d*e*x+15*d**2)/(b*x+a)**(1/2),x)`

[Out]  $20*d*\sqrt{a+b*x}*(d+e*x)**(5/2)/(3*b) + 2*e*x*\sqrt{a+b*x}*(d+e*x)**(5/2)/b - 5*d*\sqrt{a+b*x}*(d+e*x)**(3/2)*(10*a*e - 7*b*d)/(6*b**2) - \sqrt{a+b*x}*(d+e*x)**(5/2)*(7*a*e + 3*b*d)/(3*b**2) + 5*d*\sqrt{a+b*x}*\sqrt{d+e*x}*(a*e - b*d)*(10*a*e - 7*b*d)/(4*b**3) + \sqrt{a+b*x}*(d+e*x)**(3/2)*(35*a**2*e**2 + 10*a*b*d*e + 3*b**2*d**2)/(12*b**3) - \sqrt{a+b*x}*\sqrt{d+e*x}*(a*e - b*d)*(35*a**2*e**2 + 10*a*b*d*e + 3*b**2*d**2)/(8*b**4) - 5*d*(a*e - b*d)**2*(10*a*e - 7*b*d)*\operatorname{atanh}(\sqrt{e}*\sqrt{a+b*x}/(\sqrt{b}*\sqrt{d+e*x}))/ (4*b**(7/2)*\sqrt{e}) + (a*e - b*d)**2*(35*a**2*e**2 + 10*a*b*d*e + 3*b**2*d**2)*\operatorname{atanh}(\sqrt{e}*\sqrt{a+b*x}/(\sqrt{b}*\sqrt{d+e*x}))/ (8*b**(9/2)*\sqrt{e})$

**Mathematica [A]** time = 0.223334, size = 202, normalized size = 0.84

$$\begin{aligned} & \frac{(35a^2e^2 - 90abde + 73b^2d^2)(bd - ae)^2 \log\left(2\sqrt{b}\sqrt{e}\sqrt{a+bx}\sqrt{d+ex} + ae + bd + 2bex\right)}{16b^{9/2}\sqrt{e}} \\ & + \frac{\sqrt{a+bx}\sqrt{d+ex}(-105a^3e^3 + 5a^2be^2(89d + 14ex) - ab^2e(725d^2 + 292dex + 56e^2x^2) + b^3(501d^3 + 466d^2ex + 232de^2x^2 + 48e^3x^3))}{24b^4} \end{aligned}$$

Antiderivative was successfully verified.

[In] `Integrate[((d + e*x)^(3/2)*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/Sqrt[a + b*x],x]`

[Out]  $(\sqrt{a+b*x}*\sqrt{d+e*x}*(-105*a^3*e^3 + 5*a^2*b*e^2*(89*d + 14*e*x) - a*b^2*e*(725*d^2 + 292*d*e*x + 56*e^2*x^2) + b^3*(501*d^3 + 466*d^2*e*x + 232*d*e^2*x^2 + 48*e^3*x^3)))/(24*b^4) + ((b*d - a*e)^2*(73*b^2*d^2 - 90*a*b*d*e + 35*a^2*e^2)*\operatorname{Log}[b*d + a*e + 2*b*e*x + 2*\sqrt{b}*\sqrt{e}*\sqrt{a+b*x}*\sqrt{d+e*x}])/(16*b^(9/2)*\sqrt{e})$

**Maple [B]** time = 0.049, size = 571, normalized size = 2.4

$$\frac{1}{48b^4}\sqrt{ex+d}\sqrt{bx+a}\left(96x^3b^3e^3\sqrt{(bx+a)(ex+d)}\sqrt{be} - 112x^2ab^2e^3\sqrt{(bx+a)(ex+d)}\sqrt{be} + 464x^2b^3de^2\sqrt{(bx+a)(ex+d)}\sqrt{be} - 112x^2ab^2e^3\sqrt{(bx+a)(ex+d)}\sqrt{be} + 464x^2b^3de^2\sqrt{(bx+a)(ex+d)}\sqrt{be} - 112x^2ab^2e^3\sqrt{(bx+a)(ex+d)}\sqrt{be} + 464x^2b^3de^2\sqrt{(bx+a)(ex+d)}\sqrt{be}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((e^x+d)^{3/2} * (8*e^2*x^2+20*d*e^x+15*d^2)/(b*x+a)^{1/2}, x)$

[Out]  $\frac{1}{48} (e^x+d)^{1/2} (b^3 x^3 + a^3 e^3) ((b^2 x + a) (e^x + d))^{1/2} (b^2 e)^{1/2} - 112 x^2 a^2 b^2 e^3 ((b^2 x + a) (e^x + d))^{1/2} (b^2 e)^{1/2} + 464 x^2 b^3 d^2 e^2 ((b^2 x + a) (e^x + d))^{1/2} (b^2 e)^{1/2} + 105 \ln(1/2 * (2 b^2 e^x + 2 ((b^2 x + a) (e^x + d))^{1/2} (b^2 e)^{1/2} + a e + b d) / (b^2 e)^{1/2}) a^4 e^4 - 480 \ln(1/2 * (2 b^2 e^x + 2 ((b^2 x + a) (e^x + d))^{1/2} (b^2 e)^{1/2} + a e + b d) / (b^2 e)^{1/2}) a^3 e^3 d^2 b + 864 \ln(1/2 * (2 b^2 e^x + 2 ((b^2 x + a) (e^x + d))^{1/2} (b^2 e)^{1/2} + a e + b d) / (b^2 e)^{1/2}) a^2 e^2 d^2 b^2 - 708 \ln(1/2 * (2 b^2 e^x + 2 ((b^2 x + a) (e^x + d))^{1/2} (b^2 e)^{1/2} + a e + b d) / (b^2 e)^{1/2}) a^2 e^2 d^3 b^3 + 219 \ln(1/2 * (2 b^2 e^x + 2 ((b^2 x + a) (e^x + d))^{1/2} (b^2 e)^{1/2} + a e + b d) / (b^2 e)^{1/2}) d^4 b^4 + 140 e^3 ((b^2 x + a) (e^x + d))^{1/2} x a^2 b (b^2 e)^{1/2} - 584 e^2 ((b^2 x + a) (e^x + d))^{1/2} x a d b^2 (b^2 e)^{1/2} + 932 e ((b^2 x + a) (e^x + d))^{1/2} x d^2 b^3 (b^2 e)^{1/2} - 210 ((b^2 x + a) (e^x + d))^{1/2} a^3 e^3 (b^2 e)^{1/2} + 890 ((b^2 x + a) (e^x + d))^{1/2} a^2 d^2 e^2 b (b^2 e)^{1/2} - 1450 ((b^2 x + a) (e^x + d))^{1/2} a d^2 e b^2 (b^2 e)^{1/2} + 1002 ((b^2 x + a) (e^x + d))^{1/2} d^3 b^3 (b^2 e)^{1/2} / ((b^2 x + a) (e^x + d))^{1/2} / b^4 / (b^2 e)^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((8*e^2*x^2 + 20*d*e^x + 15*d^2)*(e^x + d)^{3/2}/\text{sqrt}(b*x + a), x, \text{algorithm})$

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.343964, size = 1, normalized size = 0.

$$\left[ \frac{4(48b^3e^3x^3 + 501b^3d^3 - 725ab^2d^2e + 445a^2bde^2 - 105a^3e^3 + 8(29b^3de^2 - 7ab^2e^3)x^2 + 2(233b^3d^2e - 146ab^2de^2 + 35a^2b^2d^2e^2 - 160a^3b^2d^2e^2 + 35a^4e^4) \log(4(2b^2e^2x + b^2d^2e + ab^2e^2) \sqrt{b^2x + a} \sqrt{e^x + d} + 3(73b^4d^4 - 236a^2b^3d^3e + 288a^2b^2d^2e^2 - 160a^3b^2d^2e^2 + 35a^4e^4) \log(4(2b^2e^2x + b^2d^2e + ab^2e^2) \sqrt{b^2x + a} \sqrt{e^x + d} + (8b^2e^2x^2 + b^2d^2 + 6ab^2d^2e + a^2e^2 + 8(b^2d^2e + ab^2e^2)x) \sqrt{b^2x + a} \sqrt{e^x + d} + 3(73b^4d^4 - 236a^2b^3d^3e + 288a^2b^2d^2e^2 - 160a^3b^2d^2e^2 + 35a^4e^4) \arctan(1/2(2b^2e^2x + b^2d + a) \sqrt{-b^2e}) / (\sqrt{b^2x + a} \sqrt{e^x + d} b^2e)) / (\sqrt{-b^2e} b^4)}{1/96(4(48b^3e^3x^3 + 501b^3d^3 - 725ab^2d^2e + 445a^2bde^2 - 105a^3e^3 + 8(29b^3de^2 - 7ab^2e^3)x^2 + 2(233b^3d^2e - 146ab^2de^2 + 35a^2b^2d^2e^2 - 160a^3b^2d^2e^2 + 35a^4e^4) \log(4(2b^2e^2x + b^2d^2e + ab^2e^2) \sqrt{b^2x + a} \sqrt{e^x + d} + 3(73b^4d^4 - 236a^2b^3d^3e + 288a^2b^2d^2e^2 - 160a^3b^2d^2e^2 + 35a^4e^4) \log(4(2b^2e^2x + b^2d^2e + ab^2e^2) \sqrt{b^2x + a} \sqrt{e^x + d} + (8b^2e^2x^2 + b^2d^2 + 6ab^2d^2e + a^2e^2 + 8(b^2d^2e + ab^2e^2)x) \sqrt{b^2x + a} \sqrt{e^x + d} + 3(73b^4d^4 - 236a^2b^3d^3e + 288a^2b^2d^2e^2 - 160a^3b^2d^2e^2 + 35a^4e^4) \arctan(1/2(2b^2e^2x + b^2d + a) \sqrt{-b^2e}) / (\sqrt{b^2x + a} \sqrt{e^x + d} b^2e)) / (\sqrt{-b^2e} b^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((8*e^2*x^2 + 20*d*e^x + 15*d^2)*(e^x + d)^{3/2}/\text{sqrt}(b*x + a), x, \text{algorithm})$

[Out]  $\left[ \frac{1}{96} (4(48b^3e^3x^3 + 501b^3d^3 - 725ab^2d^2e + 445a^2bde^2 - 105a^3e^3 + 8(29b^3de^2 - 7ab^2e^3)x^2 + 2(233b^3d^2e - 146ab^2de^2 + 35a^2b^2d^2e^2 - 160a^3b^2d^2e^2 + 35a^4e^4) \log(4(2b^2e^2x + b^2d^2e + ab^2e^2) \sqrt{b^2x + a} \sqrt{e^x + d} + 3(73b^4d^4 - 236a^2b^3d^3e + 288a^2b^2d^2e^2 - 160a^3b^2d^2e^2 + 35a^4e^4) \log(4(2b^2e^2x + b^2d^2e + ab^2e^2) \sqrt{b^2x + a} \sqrt{e^x + d} + (8b^2e^2x^2 + b^2d^2 + 6ab^2d^2e + a^2e^2 + 8(b^2d^2e + ab^2e^2)x) \sqrt{b^2x + a} \sqrt{e^x + d} + 3(73b^4d^4 - 236a^2b^3d^3e + 288a^2b^2d^2e^2 - 160a^3b^2d^2e^2 + 35a^4e^4) \arctan(1/2(2b^2e^2x + b^2d + a) \sqrt{-b^2e}) / (\sqrt{b^2x + a} \sqrt{e^x + d} b^2e)) / (\sqrt{-b^2e} b^4)$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)**(3/2)*(8*e**2*x**2+20*d*e*x+15*d**2)/(b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [A]** time = 0.380714, size = 1, normalized size = 0.

*Done*

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((8*e^2*x^2 + 20*d*e*x + 15*d^2)*(e*x + d)^(3/2)/sqrt(b*x + a),x, algorith
```

```
[Out] Done
```

$$3.844 \quad \int \frac{\sqrt{d+ex}(15d^2+20dex+8e^2x^2)}{\sqrt{a+bx}} dx$$

**Optimal.** Leaf size=176

$$\begin{aligned} & \frac{(bd - ae)(5a^2e^2 - 13abde + 11b^2d^2) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{7/2}\sqrt{e}} \\ & + \frac{\sqrt{a+bx}\sqrt{d+ex}(5a^2e^2 - 13abde + 11b^2d^2)}{b^3} \\ & + \frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2} + \frac{2\sqrt{a+bx}(d+ex)^{3/2}(4bd - 3ae)}{b^2} \end{aligned}$$

[Out]  $((11*b^2*d^2 - 13*a*b*d*e + 5*a^2*e^2)*\text{Sqrt}[a + b*x]*\text{Sqrt}[d + e*x])/b^3 + (2*(4*b*d - 3*a*e)*\text{Sqrt}[a + b*x]*(d + e*x)^{(3/2)})/b^2 + (8*e*(a + b*x)^{(3/2)}*(d + e*x)^{(3/2)})/(3*b^2) + ((b*d - a*e)*(11*b^2*d^2 - 13*a*b*d*e + 5*a^2*e^2)*\text{ArcTanh}[(\text{Sqrt}[e]*\text{Sqrt}[a + b*x])/(\text{Sqrt}[b]*\text{Sqrt}[d + e*x])])/(b^{(7/2)}*\text{Sqrt}[e])$

**Rubi [A]** time = 0.372731, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$

$$\begin{aligned} & \frac{(bd - ae)(5a^2e^2 - 13abde + 11b^2d^2) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{7/2}\sqrt{e}} \\ & + \frac{\sqrt{a+bx}\sqrt{d+ex}(5a^2e^2 - 13abde + 11b^2d^2)}{b^3} \\ & + \frac{8e(a+bx)^{3/2}(d+ex)^{3/2}}{3b^2} + \frac{2\sqrt{a+bx}(d+ex)^{3/2}(4bd - 3ae)}{b^2} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(\text{Sqrt}[d + e*x]*(15*d^2 + 20*d*e*x + 8*e^2*x^2))/\text{Sqrt}[a + b*x], x]$

[Out]  $((11*b^2*d^2 - 13*a*b*d*e + 5*a^2*e^2)*\text{Sqrt}[a + b*x]*\text{Sqrt}[d + e*x])/b^3 + (2*(4*b*d - 3*a*e)*\text{Sqrt}[a + b*x]*(d + e*x)^{(3/2)})/b^2 + (8*e*(a + b*x)^{(3/2)}*(d + e*x)^{(3/2)})/(3*b^2) + ((b*d - a*e)*(11*b^2*d^2 - 13*a*b*d*e + 5*a^2*e^2)*\text{ArcTanh}[(\text{Sqrt}[e]*\text{Sqrt}[a + b*x])/(\text{Sqrt}[b]*\text{Sqrt}[d + e*x])])/(b^{(7/2)}*\text{Sqrt}[e])$

**Rubi in Sympy [A]** time = 54.633, size = 287, normalized size = 1.63

$$\begin{aligned} & \frac{10d\sqrt{a+bx}(d+ex)^{\frac{3}{2}}}{b} + \frac{8ex\sqrt{a+bx}(d+ex)^{\frac{3}{2}}}{3b} - \frac{5d\sqrt{a+bx}\sqrt{d+ex}(3ae - 2bd)}{b^2} \\ & - \frac{2\sqrt{a+bx}(d+ex)^{\frac{3}{2}}(5ae + 3bd)}{3b^2} + \frac{\sqrt{a+bx}\sqrt{d+ex}(5a^2e^2 + 2abde + b^2d^2)}{b^3} \\ & + \frac{5d(ae - bd)(3ae - 2bd) \operatorname{atanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{\frac{5}{2}}\sqrt{e}} - \frac{(ae - bd)(5a^2e^2 + 2abde + b^2d^2) \operatorname{atanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{\frac{7}{2}}\sqrt{e}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)^{(1/2)}*(8*e**2*x**2+20*d*e*x+15*d**2)/(b*x+a)^{(1/2)}, x)$

[Out]  $10*d*\text{sqrt}(a + b*x)*(d + e*x)^{(3/2)}/b + 8*e*x*\text{sqrt}(a + b*x)*(d + e*x)^{(3/2)}/(3*b) - 5*d*\text{sqrt}(a + b*x)*\text{sqrt}(d + e*x)*(3*a*e - 2*b*d)/b**2 - 2*\text{sqrt}(a + b*x)*(d + e*x)^{(3/2)}*(5*a*e + 3*b*d)/(3*b**2) + \text{sqrt}(a + b*x)*\text{sqrt}(d + e*x)*(5*a**2*e**2 + 2*a*b*d*e + b**2$

$$\frac{d^2}{b^3} + 5d \frac{(a^2e - b^2d)(3ae - 2bd) \operatorname{atanh}(\sqrt{e}\sqrt{a+bx})}{(\sqrt{b}\sqrt{d+ex})} - \frac{(a^2e - b^2d)(5a^2e^2 + 2abde + b^2d^2) \operatorname{atanh}(\sqrt{e}\sqrt{a+bx})}{(\sqrt{b}\sqrt{d+ex})} - \frac{(a^2e - b^2d)(5a^2e^2 + 2abde + b^2d^2) \operatorname{atanh}(\sqrt{e}\sqrt{a+bx})}{(\sqrt{b}\sqrt{d+ex})} - \frac{(a^2e - b^2d)(5a^2e^2 + 2abde + b^2d^2) \operatorname{atanh}(\sqrt{e}\sqrt{a+bx})}{(\sqrt{b}\sqrt{d+ex})} - \frac{(a^2e - b^2d)(5a^2e^2 + 2abde + b^2d^2) \operatorname{atanh}(\sqrt{e}\sqrt{a+bx})}{(\sqrt{b}\sqrt{d+ex})}$$

**Mathematica [A]** time = 0.14925, size = 159, normalized size = 0.9

$$\frac{(bd - ae)(5a^2e^2 - 13abde + 11b^2d^2) \log\left(2\sqrt{b}\sqrt{e}\sqrt{a+bx}\sqrt{d+ex} + ae + bd + 2bex\right)}{2b^{7/2}\sqrt{e}} + \frac{\sqrt{a+bx}\sqrt{d+ex}(15a^2e^2 - abe(49d + 10ex) + b^2(57d^2 + 32dex + 8e^2x^2))}{3b^3}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[d + e\*x]\*(15\*d^2 + 20\*d\*e\*x + 8\*e^2\*x^2))/Sqrt[a + b\*x], x]

[Out] (Sqrt[a + b\*x]\*Sqrt[d + e\*x]\*(15\*a^2\*e^2 - a\*b\*e\*(49\*d + 10\*e\*x) + b^2\*(57\*d^2 + 32\*d\*e\*x + 8\*e^2\*x^2)))/(3\*b^3) + ((b\*d - a\*e)\*(1 + b^2\*d^2 - 13\*a\*b\*d\*e + 5\*a^2\*e^2)\*Log[b\*d + a\*e + 2\*b\*e\*x + 2\*Sqrt[b]\*Sqrt[e]\*Sqrt[a + b\*x]\*Sqrt[d + e\*x]])/(2\*b^(7/2)\*Sqrt[e])

**Maple [B]** time = 0.03, size = 392, normalized size = 2.2

$$-\frac{1}{6b^3}\sqrt{ex+d}\sqrt{bx+a}\left(-16x^2b^2e^2\sqrt{(bx+a)(ex+d)}\sqrt{be} + 15\ln\left(\frac{1}{2}\frac{2bex + 2\sqrt{(bx+a)(ex+d)}\sqrt{be} + ae + bd}{\sqrt{be}}\right)\right)a^3e^3 - 5$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^(1/2)\*(8\*e^2\*x^2+20\*d\*e\*x+15\*d^2)/(b\*x+a)^(1/2), x)

[Out] -1/6\*(e\*x+d)^(1/2)\*(b\*x+a)^(1/2)\*(-16\*x^2\*b^2\*e^2\*((b\*x+a)\*(e\*x+d))^(1/2)\*(b\*e)^(1/2)+15\*ln(1/2\*(2\*b\*e\*x+2\*((b\*x+a)\*(e\*x+d))^(1/2)\*(b\*e)^(1/2)+a\*e+b\*d)/(b\*e)^(1/2))\*a^3\*e^3-54\*ln(1/2\*(2\*b\*e\*x+2\*((b\*x+a)\*(e\*x+d))^(1/2)\*(b\*e)^(1/2)+a\*e+b\*d)/(b\*e)^(1/2))\*a^2\*e^2\*d\*b+72\*ln(1/2\*(2\*b\*e\*x+2\*((b\*x+a)\*(e\*x+d))^(1/2)\*(b\*e)^(1/2)+a\*e+b\*d)/(b\*e)^(1/2))\*a\*e\*d^2\*b^2-33\*ln(1/2\*(2\*b\*e\*x+2\*((b\*x+a)\*(e\*x+d))^(1/2)\*(b\*e)^(1/2)+a\*e+b\*d)/(b\*e)^(1/2))\*d^3\*b^3+20\*e^2\*a\*((b\*x+a)\*(e\*x+d))^(1/2)\*x\*b\*(b\*e)^(1/2)-64\*e\*d\*((b\*x+a)\*(e\*x+d))^(1/2)\*x\*b^2\*(b\*e)^(1/2)-30\*((b\*x+a)\*(e\*x+d))^(1/2)\*a^2\*e^2\*(b\*e)^(1/2)+98\*((b\*x+a)\*(e\*x+d))^(1/2)\*a\*d\*e\*b\*(b\*e)^(1/2)-114\*((b\*x+a)\*(e\*x+d))^(1/2)\*d^2\*b^2\*(b\*e)^(1/2))/((b\*x+a)\*(e\*x+d))^(1/2)/b^3/(b\*e)^(1/2)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8\*e^2\*x^2 + 20\*d\*e\*x + 15\*d^2)\*sqrt(e\*x + d)/sqrt(b\*x + a), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.313925, size = 1, normalized size = 0.01

$$\frac{4(8b^2e^2x^2 + 57b^2d^2 - 49abde + 15a^2e^2 + 2(16b^2de - 5abe^2)x)\sqrt{be}\sqrt{bx+a}\sqrt{ex+d} - 3(11b^3d^3 - 24ab^2d^2e + 18a^2bd^2e^2 - 5a^3d^3e^3)}{12\sqrt{b^2d + (bx+a)be - a^2b^2e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8\*e^2\*x^2 + 20\*d\*e\*x + 15\*d^2)\*sqrt(e\*x + d)/sqrt(b\*x + a),x, algorithm="fricas")

[Out] [1/12\*(4\*(8\*b^2\*e^2\*x^2 + 57\*b^2\*d^2 - 49\*a\*b\*d\*e + 15\*a^2\*e^2 + 2\*(16\*b^2\*d\*e - 5\*a\*b\*e^2)\*x)\*sqrt(b\*e)\*sqrt(b\*x + a)\*sqrt(e\*x + d) - 3\*(11\*b^3\*d^3 - 24\*a\*b^2\*d^2\*e + 18\*a^2\*b\*d\*e^2 - 5\*a^3\*e^3)\*log(-4\*(2\*b^2\*e^2\*x + b^2\*d\*e + a\*b\*e^2)\*sqrt(b\*x + a)\*sqrt(e\*x + d) + (8\*b^2\*e^2\*x^2 + b^2\*d^2 + 6\*a\*b\*d\*e + a^2\*e^2 + 8\*(b^2\*d\*e + a\*b\*e^2)\*x)\*sqrt(b\*e))/(sqrt(b\*e)\*b^3), 1/6\*(2\*(8\*b^2\*e^2\*x^2 + 57\*b^2\*d^2 - 49\*a\*b\*d\*e + 15\*a^2\*e^2 + 2\*(16\*b^2\*d\*e - 5\*a\*b\*e^2)\*x)\*sqrt(-b\*e)\*sqrt(b\*x + a)\*sqrt(e\*x + d) + 3\*(11\*b^3\*d^3 - 24\*a\*b^2\*d^2\*e + 18\*a^2\*b\*d\*e^2 - 5\*a^3\*e^3)\*arctan(1/2\*(2\*b\*e\*x + b\*d + a\*e)\*sqrt(-b\*e)/(sqrt(b\*x + a)\*sqrt(e\*x + d)\*b\*e))/(sqrt(-b\*e)\*b^3)]

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(1/2)\*(8\*e\*\*2\*x\*\*2+20\*d\*e\*x+15\*d\*\*2)/(b\*x+a)\*\*(1/2),x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.332364, size = 602, normalized size = 3.42

$$\frac{180 \left( \frac{(b^2d - abe)e^{-\frac{1}{2}} \ln \left( \frac{-\sqrt{bx+a}\sqrt{be}^{\frac{1}{2}} + \sqrt{b^2d + (bx+a)be - abe}}{\sqrt{b}} \right)}{\sqrt{b}} - \sqrt{b^2d + (bx+a)be - abe} \sqrt{bx+a} \right) d^2 |b|}{b^2} - 4 \left( \sqrt{b^2d + (bx+a)be - abe} \sqrt{bx+a} \left( 2(bx+a) \left( \frac{4(bx+a)}{b^2} + \frac{b}{b^2} \right) \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8\*e^2\*x^2 + 20\*d\*e\*x + 15\*d^2)\*sqrt(e\*x + d)/sqrt(b\*x + a),x, algorithm="giac")

[Out] -1/12\*(180\*((b^2\*d - a\*b\*e)\*e^(-1/2)\*ln(abs(-sqrt(b\*x + a)\*sqrt(b^2\*d + (b\*x + a)\*b\*e - a\*b\*e))/sqrt(b) - sqrt(b^2\*d + (b\*x + a)\*b\*e - a\*b\*e)\*sqrt(b\*x + a))/sqrt(b) - 4\*(sqrt(b^2\*d + (b\*x + a)\*b\*e - a\*b\*e)\*sqrt(b\*x + a)\*(2\*(b\*x + a)\*(4\*(b\*x + a)/b^2 + (b^6\*d\*e^3 - 13\*a\*b^5\*e^4)\*e^(-4)/b^7) - 3\*(b^7\*d^2\*e^2 + 2\*a\*b^6\*d\*e^3 - 11\*a^2\*b^5\*e^4)\*e^(-4)/b^7) - 3\*(b^3\*d^3 + a\*b^2\*d^2\*e + 3\*a^2\*b\*d\*e^2 - 5\*a^3\*e^3)\*e^(-5/2)\*ln(abs(-sqrt(b\*x + a)\*sqrt(b)\*e^(1/2) + sqrt(b^2\*d + (b\*x + a)\*b\*e - a\*b\*e))/b^(3/2)))\*abs(b)\*e^2/b^2 - 5\*(sqrt(b^2\*d + (b\*x + a)\*b\*e - a\*b\*e)\*sqrt(b\*x + a)\*(2\*(b\*x + a)\*e^(-2)/b^4 + (b\*d\*e - 5\*a\*e^2)\*e^(-4)/b^4) + (b^2\*d^2 + 2\*a\*b\*d\*e - 3\*a^2\*e^2)\*e^(-7/2)\*ln(abs(-sqrt(b\*x + a)\*sqrt(b)\*e^(1/2) + sqrt(b^2\*d + (b\*x + a)\*b\*e - a\*b\*e))/b^(7/2))\*d\*abs(b)\*e/b^3)/b



$$3.845 \quad \int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}\sqrt{d+ex}} dx$$

**Optimal.** Leaf size=122

$$\frac{2(3a^2e^2 - 8abde + 8b^2d^2) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{5/2}\sqrt{e}} + \frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2} + \frac{2\sqrt{a+bx}\sqrt{d+ex}(7bd - 5ae)}{b^2}$$

[Out]  $(2*(7*b*d - 5*a*e)*\text{Sqrt}[a + b*x]*\text{Sqrt}[d + e*x])/b^2 + (4*e*(a + b*x)^{(3/2)}*\text{Sqrt}[d + e*x])/b^2 + (2*(8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)*\text{ArcTanh}[(\text{Sqrt}[e]*\text{Sqrt}[a + b*x])/(\text{Sqrt}[b]*\text{Sqrt}[d + e*x])])/(b^{(5/2)}*\text{Sqrt}[e])$

**Rubi [A]** time = 0.287447, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{2(3a^2e^2 - 8abde + 8b^2d^2) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{5/2}\sqrt{e}} + \frac{4e(a+bx)^{3/2}\sqrt{d+ex}}{b^2} + \frac{2\sqrt{a+bx}\sqrt{d+ex}(7bd - 5ae)}{b^2}$$

Antiderivative was successfully verified.

[In] Int[(15\*d^2 + 20\*d\*e\*x + 8\*e^2\*x^2)/(Sqrt[a + b\*x]\*Sqrt[d + e\*x]), x]

[Out]  $(2*(7*b*d - 5*a*e)*\text{Sqrt}[a + b*x]*\text{Sqrt}[d + e*x])/b^2 + (4*e*(a + b*x)^{(3/2)}*\text{Sqrt}[d + e*x])/b^2 + (2*(8*b^2*d^2 - 8*a*b*d*e + 3*a^2*e^2)*\text{ArcTanh}[(\text{Sqrt}[e]*\text{Sqrt}[a + b*x])/(\text{Sqrt}[b]*\text{Sqrt}[d + e*x])])/(b^{(5/2)}*\text{Sqrt}[e])$

**Rubi in Sympy [A]** time = 43.0614, size = 184, normalized size = 1.51

$$\frac{20d\sqrt{a+bx}\sqrt{d+ex}}{b} + \frac{4ex\sqrt{a+bx}\sqrt{d+ex}}{b} - \frac{6\sqrt{a+bx}\sqrt{d+ex}(ae+bd)}{b^2} - \frac{10d(2ae-bd) \operatorname{atanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{e}\sqrt{a+bx}}\right)}{b^{3/2}\sqrt{e}} - \frac{8\left(abde - \frac{3(ae+bd)^2}{4}\right) \operatorname{atanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{e}\sqrt{a+bx}}\right)}{b^{5/2}\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((8\*e\*\*2\*x\*\*2+20\*d\*e\*x+15\*d\*\*2)/(e\*x+d)\*\*(1/2)/(b\*x+a)\*\*(1/2), x)

[Out]  $20*d*\text{sqrt}(a + b*x)*\text{sqrt}(d + e*x)/b + 4*e*x*\text{sqrt}(a + b*x)*\text{sqrt}(d + e*x)/b - 6*\text{sqrt}(a + b*x)*\text{sqrt}(d + e*x)*(a*e + b*d)/b^2 - 10*d*(2*a*e - b*d)*\text{atanh}(\text{sqrt}(b)*\text{sqrt}(d + e*x)/(\text{sqrt}(e)*\text{sqrt}(a + b*x)))/b^{(3/2)}*\text{sqrt}(e) - 8*(a*b*d*e - 3*(a*e + b*d)**2/4)*\text{atanh}(\text{sqrt}(b)*\text{sqrt}(d + e*x)/(\text{sqrt}(e)*\text{sqrt}(a + b*x)))/b^{(5/2)}*\text{sqrt}(e)$

**Mathematica [A]** time = 0.108814, size = 115, normalized size = 0.94

$$\frac{(3a^2e^2 - 8abde + 8b^2d^2) \log\left(2\sqrt{b}\sqrt{e}\sqrt{a+bx}\sqrt{d+ex} + ae + bd + 2bex\right)}{b^{5/2}\sqrt{e}} + \frac{2\sqrt{a+bx}\sqrt{d+ex}(-3ae + 7bd + 2bex)}{b^2}$$

Antiderivative was successfully verified.

[In] Integrate[(15\*d^2 + 20\*d\*e\*x + 8\*e^2\*x^2)/(Sqrt[a + b\*x]\*Sqrt[d + e\*x]),x]

[Out] (2\*Sqrt[a + b\*x]\*Sqrt[d + e\*x]\*(7\*b\*d - 3\*a\*e + 2\*b\*e\*x))/b^2 + ((8\*b^2\*d^2 - 8\*a\*b\*d\*e + 3\*a^2\*e^2)\*Log[b\*d + a\*e + 2\*b\*e\*x + 2\*Sqrt[b]\*Sqrt[e]\*Sqrt[a + b\*x]\*Sqrt[d + e\*x])/(b^(5/2)\*Sqrt[e])

**Maple [B]** time = 0.035, size = 247, normalized size = 2.

$$\frac{1}{b^2} \left( 3 \ln \left( \frac{1}{2} \frac{2 b e x + 2 \sqrt{(b x + a)(e x + d)} \sqrt{b e} + a e + b d}{\sqrt{b e}} \right) a^2 e^2 - 8 \ln \left( \frac{1}{2} \frac{2 b e x + 2 \sqrt{(b x + a)(e x + d)} \sqrt{b e} + a e + b d}{\sqrt{b e}} \right) a b d \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8\*e^2\*x^2+20\*d\*e\*x+15\*d^2)/(e\*x+d)^(1/2)/(b\*x+a)^(1/2),x)

[Out] (3\*ln(1/2\*(2\*b\*e\*x+2\*((b\*x+a)\*(e\*x+d))^(1/2)\*(b\*e)^(1/2)+a\*e+b\*d)/(b\*e)^(1/2))\*a^2\*e^2-8\*ln(1/2\*(2\*b\*e\*x+2\*((b\*x+a)\*(e\*x+d))^(1/2)\*(b\*e)^(1/2)+a\*e+b\*d)/(b\*e)^(1/2))\*a\*b\*d\*e+8\*ln(1/2\*(2\*b\*e\*x+2\*((b\*x+a)\*(e\*x+d))^(1/2)\*(b\*e)^(1/2)+a\*e+b\*d)/(b\*e)^(1/2))\*b^2\*d^2+4\*e\*((b\*x+a)\*(e\*x+d))^(1/2)\*x\*b\*(b\*e)^(1/2)-6\*((b\*x+a)\*(e\*x+d))^(1/2)\*(b\*e)^(1/2)\*a\*e+14\*((b\*x+a)\*(e\*x+d))^(1/2)\*(b\*e)^(1/2)\*b\*d\*(e\*x+d)^(1/2)\*(b\*x+a)^(1/2)/(b\*e)^(1/2)/b^2/((b\*x+a)\*(e\*x+d))^(1/2))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8\*e^2\*x^2 + 20\*d\*e\*x + 15\*d^2)/(sqrt(b\*x + a)\*sqrt(e\*x + d)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.317592, size = 1, normalized size = 0.01

$$\left[ \frac{4(2 b e x + 7 b d - 3 a e) \sqrt{b e} \sqrt{b x + a} \sqrt{e x + d} + (8 b^2 d^2 - 8 a b d e + 3 a^2 e^2) \log \left( 4 (2 b^2 e^2 x + b^2 d e + a b e^2) \sqrt{b x + a} \sqrt{e x + d} + (8 b^2 d^2 - 8 a b d e + 3 a^2 e^2) \right)}{2 \sqrt{b e} b^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8\*e^2\*x^2 + 20\*d\*e\*x + 15\*d^2)/(sqrt(b\*x + a)\*sqrt(e\*x + d)),x, algorithm="fricas")

[Out] [1/2\*(4\*(2\*b\*e\*x + 7\*b\*d - 3\*a\*e)\*sqrt(b\*e)\*sqrt(b\*x + a)\*sqrt(e\*x + d) + (8\*b^2\*d^2 - 8\*a\*b\*d\*e + 3\*a^2\*e^2)\*log(4\*(2\*b^2\*e^2\*x + b^2\*d\*e + a\*b\*e^2)\*sqrt(b\*x + a)\*sqrt(e\*x + d) + (8\*b^2\*d^2 - 8\*a\*b\*d\*e + 3\*a^2\*e^2)\*x)\*sqrt(b\*e))/(sqrt(b\*e)\*b^2), (2\*(2\*b\*e\*x + 7\*b\*d - 3\*a\*e)\*sqrt(-b\*e)\*sqrt(b\*x + a)\*sqrt(e\*x + d) + (8\*b^2\*d^2 - 8\*a\*b\*d\*e + 3\*a^2\*e^2)\*arctan(1/2\*(2\*b\*e\*x + b\*d + a\*e)\*sqrt(-b\*e)/(sqrt(b\*x + a)\*sqrt(e\*x + d)\*b\*e)))/(sqrt(-b\*e)\*b^2)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a + bx}\sqrt{d + ex}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8\*e\*\*2\*x\*\*2+20\*d\*e\*x+15\*d\*\*2)/(e\*x+d)\*\*(1/2)/(b\*x+a)\*\*(1/2),x)

[Out] Integral((15\*d\*\*2 + 20\*d\*e\*x + 8\*e\*\*2\*x\*\*2)/(sqrt(a + b\*x)\*sqrt(d + e\*x)), x)

**GIAC/XCAS [A]** time = 0.285379, size = 196, normalized size = 1.61

$$2 \left( \sqrt{b^2d + (bx + a)be - abe}\sqrt{bx + a} \left( \frac{2(bx+a)e}{b^3} + \frac{(7b^6de^2 - 5ab^5e^3)e^{(-2)}}{b^8} \right) - \frac{(8b^2d^2 - 8abde + 3a^2e^2)e^{(-\frac{1}{2})} \ln\left( \frac{-\sqrt{bx+a}\sqrt{be}^{\frac{1}{2}} + \sqrt{b^2d + (bx+a)be}}{b^{\frac{5}{2}}} \right)}{b^{\frac{5}{2}}} \right) |b|$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8\*e^2\*x^2 + 20\*d\*e\*x + 15\*d^2)/(sqrt(b\*x + a)\*sqrt(e\*x + d)),x, algorithm="giac")

[Out] 2\*(sqrt(b^2\*d + (b\*x + a)\*b\*e - a\*b\*e)\*sqrt(b\*x + a)\*(2\*(b\*x + a)\*e/b^3 + (7\*b^6\*d\*e^2 - 5\*a\*b^5\*e^3)\*e^(-2)/b^8) - (8\*b^2\*d^2 - 8\*a\*b\*d\*e + 3\*a^2\*e^2)\*e^(-1/2)\*ln(abs(-sqrt(b\*x + a)\*sqrt(b)\*e^(1/2) + sqrt(b^2\*d + (b\*x + a)\*b\*e - a\*b\*e)))/b^(5/2))\*b/abs(b)

$$3.846 \quad \int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}(d+ex)^{3/2}} dx$$

**Optimal.** Leaf size=108

$$\frac{8(2bd - ae) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{3/2}\sqrt{e}} + \frac{6d^2\sqrt{a+bx}}{\sqrt{d+ex}(bd - ae)} + \frac{8\sqrt{a+bx}\sqrt{d+ex}}{b}$$

[Out] (6\*d^2\*Sqrt[a + b\*x])/((b\*d - a\*e)\*Sqrt[d + e\*x]) + (8\*Sqrt[a + b\*x]\*Sqrt[d + e\*x])/b + (8\*(2\*b\*d - a\*e)\*ArcTanh[(Sqrt[e]\*Sqrt[a + b\*x])/(Sqrt[b]\*Sqrt[d + e\*x])])/(b^(3/2)\*Sqrt[e])

**Rubi [A]** time = 0.272959, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{8(2bd - ae) \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{b^{3/2}\sqrt{e}} + \frac{6d^2\sqrt{a+bx}}{\sqrt{d+ex}(bd - ae)} + \frac{8\sqrt{a+bx}\sqrt{d+ex}}{b}$$

Antiderivative was successfully verified.

[In] Int[(15\*d^2 + 20\*d\*e\*x + 8\*e^2\*x^2)/(Sqrt[a + b\*x]\*(d + e\*x)^(3/2)), x]

[Out] (6\*d^2\*Sqrt[a + b\*x])/((b\*d - a\*e)\*Sqrt[d + e\*x]) + (8\*Sqrt[a + b\*x]\*Sqrt[d + e\*x])/b + (8\*(2\*b\*d - a\*e)\*ArcTanh[(Sqrt[e]\*Sqrt[a + b\*x])/(Sqrt[b]\*Sqrt[d + e\*x])])/(b^(3/2)\*Sqrt[e])

**Rubi in Sympy [A]** time = 45.332, size = 139, normalized size = 1.29

$$-\frac{6d^2\sqrt{a+bx}}{\sqrt{d+ex}(ae - bd)} + \frac{8\sqrt{a+bx}\sqrt{d+ex}}{b} + \frac{40d \operatorname{atanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{e}\sqrt{a+bx}}\right)}{\sqrt{b}\sqrt{e}} - \frac{8(ae + 3bd) \operatorname{atanh}\left(\frac{\sqrt{b}\sqrt{d+ex}}{\sqrt{e}\sqrt{a+bx}}\right)}{b^{3/2}\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((8\*e\*\*2\*x\*\*2+20\*d\*e\*x+15\*d\*\*2)/(e\*x+d)\*\*(3/2)/(b\*x+a)\*\*(1/2), x)

[Out] -6\*d\*\*2\*sqrt(a + b\*x)/(sqrt(d + e\*x)\*(a\*e - b\*d)) + 8\*sqrt(a + b\*x)\*sqrt(d + e\*x)/b + 40\*d\*atanh(sqrt(b)\*sqrt(d + e\*x)/(sqrt(e)\*sqrt(a + b\*x)))/(sqrt(b)\*sqrt(e)) - 8\*(a\*e + 3\*b\*d)\*atanh(sqrt(b)\*sqrt(d + e\*x)/(sqrt(e)\*sqrt(a + b\*x)))/(b\*\*(3/2)\*sqrt(e))

**Mathematica [A]** time = 0.206155, size = 112, normalized size = 1.04

$$\frac{4(2bd - ae) \log\left(2\sqrt{b}\sqrt{e}\sqrt{a+bx}\sqrt{d+ex} + ae + bd + 2bex\right)}{b^{3/2}\sqrt{e}} + \sqrt{a+bx}\sqrt{d+ex} \left(\frac{8}{b} - \frac{6d^2}{(d+ex)(ae - bd)}\right)$$

Antiderivative was successfully verified.

[In] Integrate[(15\*d^2 + 20\*d\*e\*x + 8\*e^2\*x^2)/(Sqrt[a + b\*x]\*(d + e\*x)^(3/2)), x]

[Out] Sqrt[a + b\*x]\*Sqrt[d + e\*x]\*(8/b - (6\*d^2)/((-b\*d) + a\*e)\*(d + e\*x)) + (4\*(2\*b\*d - a\*e)\*Log[b\*d + a\*e + 2\*b\*e\*x + 2\*Sqrt[b]\*Sqrt

$[e] \cdot \text{Sqrt}[a + b \cdot x] \cdot \text{Sqrt}[d + e \cdot x]] / (b^{3/2} \cdot \text{Sqrt}[e])$

**Maple [B]** time = 0.049, size = 438, normalized size = 4.1

$$-2 \frac{\sqrt{bx+a}}{\sqrt{beb}(ae-bd)\sqrt{(bx+a)(ex+d)}\sqrt{ex+d}} \left( 2 \ln \left( \frac{1}{2} \frac{2bex + 2\sqrt{(bx+a)(ex+d)}\sqrt{be} + ae + bd}{\sqrt{be}} \right) \right) xa^2e^3 - 6 \ln \left( \frac{1}{2} \frac{2be}{\sqrt{be}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8\*e^2\*x^2+20\*d\*e\*x+15\*d^2)/(e\*x+d)^(3/2)/(b\*x+a)^(1/2),x)

[Out]  $-2 \cdot (b \cdot x + a)^{1/2} \cdot (2 \cdot \ln(1/2 \cdot (2 \cdot b \cdot e \cdot x + 2 \cdot ((b \cdot x + a) \cdot (e \cdot x + d))^{1/2} \cdot (b \cdot e)^{1/2} + a \cdot e + b \cdot d) / (b \cdot e)^{1/2})) \cdot x \cdot a^2 \cdot e^3 - 6 \cdot \ln(1/2 \cdot (2 \cdot b \cdot e \cdot x + 2 \cdot ((b \cdot x + a) \cdot (e \cdot x + d))^{1/2} \cdot (b \cdot e)^{1/2} + a \cdot e + b \cdot d) / (b \cdot e)^{1/2})) \cdot x \cdot a \cdot b \cdot d \cdot e^2 + 4 \cdot \ln(1/2 \cdot (2 \cdot b \cdot e \cdot x + 2 \cdot ((b \cdot x + a) \cdot (e \cdot x + d))^{1/2} \cdot (b \cdot e)^{1/2} + a \cdot e + b \cdot d) / (b \cdot e)^{1/2})) \cdot x \cdot b^2 \cdot d^2 \cdot e + 2 \cdot \ln(1/2 \cdot (2 \cdot b \cdot e \cdot x + 2 \cdot ((b \cdot x + a) \cdot (e \cdot x + d))^{1/2} \cdot (b \cdot e)^{1/2} + a \cdot e + b \cdot d) / (b \cdot e)^{1/2})) \cdot a^2 \cdot d \cdot e^2 - 6 \cdot \ln(1/2 \cdot (2 \cdot b \cdot e \cdot x + 2 \cdot ((b \cdot x + a) \cdot (e \cdot x + d))^{1/2} \cdot (b \cdot e)^{1/2} + a \cdot e + b \cdot d) / (b \cdot e)^{1/2})) \cdot a \cdot b \cdot d^2 \cdot e + 4 \cdot \ln(1/2 \cdot (2 \cdot b \cdot e \cdot x + 2 \cdot ((b \cdot x + a) \cdot (e \cdot x + d))^{1/2} \cdot (b \cdot e)^{1/2} + a \cdot e + b \cdot d) / (b \cdot e)^{1/2})) \cdot b^2 \cdot d^3 - 4 \cdot x \cdot a \cdot e^2 \cdot ((b \cdot x + a) \cdot (e \cdot x + d))^{1/2} \cdot (b \cdot e)^{1/2} + 4 \cdot x \cdot b \cdot d \cdot e \cdot ((b \cdot x + a) \cdot (e \cdot x + d))^{1/2} \cdot (b \cdot e)^{1/2} - 4 \cdot a \cdot d \cdot e \cdot ((b \cdot x + a) \cdot (e \cdot x + d))^{1/2} \cdot (b \cdot e)^{1/2} + 7 \cdot b \cdot d^2 \cdot ((b \cdot x + a) \cdot (e \cdot x + d))^{1/2} \cdot (b \cdot e)^{1/2}) / (b \cdot e)^{1/2} / b / (a \cdot e - b \cdot d) / ((b \cdot x + a) \cdot (e \cdot x + d))^{1/2} / (e \cdot x + d)^{1/2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8\*e^2\*x^2 + 20\*d\*e\*x + 15\*d^2)/(sqrt(b\*x + a)\*(e\*x + d)^(3/2)),x, algo

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.391309, size = 1, normalized size = 0.01

$$\left[ \frac{2 \left( (7bd^2 - 4ade + 4(bde - ae^2)x) \sqrt{be} \sqrt{bx+a} \sqrt{ex+d} - (2b^2d^3 - 3abd^2e + a^2de^2 + (2b^2d^2e - 3abde^2 + a^2e^3)x) \log \left( \frac{(b^2d^2 - abde + (b^2de - abde^2)x)}{\dots} \right) \right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8\*e^2\*x^2 + 20\*d\*e\*x + 15\*d^2)/(sqrt(b\*x + a)\*(e\*x + d)^(3/2)),x, algo

[Out]  $[2 \cdot ((7 \cdot b \cdot d^2 - 4 \cdot a \cdot d \cdot e + 4 \cdot (b \cdot d \cdot e - a \cdot e^2) \cdot x) \cdot \text{sqrt}(b \cdot e) \cdot \text{sqrt}(b \cdot x + a) \cdot \text{sqrt}(e \cdot x + d) - (2 \cdot b^2 \cdot d^3 - 3 \cdot a \cdot b \cdot d^2 \cdot e + a^2 \cdot d \cdot e^2 + (2 \cdot b^2 \cdot d^2 \cdot e - 3 \cdot abde^2 + a^2 \cdot e^3) \cdot x) \cdot \log(-4 \cdot (2 \cdot b^2 \cdot d^2 \cdot e^2 \cdot x + b^2 \cdot d \cdot e + a \cdot b \cdot e^2) \cdot \text{sqrt}(b \cdot x + a) \cdot \text{sqrt}(e \cdot x + d) + (8 \cdot b^2 \cdot d^2 \cdot e^2 \cdot x^2 + b^2 \cdot d^2 \cdot e^2 + 6 \cdot a \cdot b \cdot d \cdot e + a^2 \cdot e^2 + 8 \cdot (b^2 \cdot d \cdot e + a \cdot b \cdot e^2) \cdot x) \cdot \text{sqrt}(b \cdot e))) / ((b^2 \cdot d^2 \cdot e^2 - a \cdot b \cdot d \cdot e + (b^2 \cdot d \cdot e - a \cdot b \cdot e^2) \cdot x) \cdot \text{sqrt}(b \cdot e)), 2 \cdot ((7 \cdot b \cdot d^2 - 4 \cdot a \cdot d \cdot e + 4 \cdot (b \cdot d \cdot e - a \cdot e^2) \cdot x) \cdot \text{sqrt}(-b \cdot e) \cdot \text{sqrt}(b \cdot x + a) \cdot \text{sqrt}(e \cdot x + d) + 2 \cdot (2 \cdot b^2 \cdot d^3 - 3 \cdot a \cdot b \cdot d^2 \cdot e + a^2 \cdot d \cdot e^2 + (2 \cdot b^2 \cdot d^2 \cdot e - 3 \cdot a \cdot b \cdot d \cdot e^2 + a^2 \cdot e^3) \cdot x) \cdot \arctan(1/2 \cdot (2 \cdot b \cdot e \cdot x + b \cdot d + a \cdot e) \cdot \text{sqrt}(-b \cdot e) / (\text{sqrt}(b \cdot x + a) \cdot \text{sqrt}(e \cdot x + d) \cdot b \cdot e))) / ((b^2 \cdot d^2 \cdot e^2 - a \cdot b \cdot d \cdot e + (b^2 \cdot d \cdot e - a \cdot b \cdot e^2) \cdot x) \cdot \text{sqrt}(-b \cdot e))]$

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**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a + bx}(d + ex)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8\*e\*\*2\*x\*\*2+20\*d\*e\*x+15\*d\*\*2)/(e\*x+d)\*\*(3/2)/(b\*x+a)\*\*(1/2),x)

[Out] Integral((15\*d\*\*2 + 20\*d\*e\*x + 8\*e\*\*2\*x\*\*2)/(sqrt(a + b\*x)\*(d + e\*x)\*\*(3/2)), x)

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**GIAC/XCAS [A]** time = 0.29906, size = 261, normalized size = 2.42

$$\frac{8(2bd - ae)e^{(-\frac{1}{2})} \ln\left(\left|-\sqrt{bx+a}\sqrt{be}^{\frac{1}{2}} + \sqrt{b^2d + (bx+a)be - abe}\right|\right)}{\sqrt{b}|b|} + \frac{2\sqrt{bx+a}\left(\frac{4(b^3de^3 - ab^2e^4)(bx+a)}{b^3d|b|e^2 - ab^2|b|e^3} + \frac{7b^4d^2e^2 - 8ab^3de^3 + 4a^2b^2e^4}{b^3d|b|e^2 - ab^2|b|e^3}\right)}{\sqrt{b^2d + (bx+a)be - abe}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8\*e^2\*x^2 + 20\*d\*e\*x + 15\*d^2)/(sqrt(b\*x + a)\*(e\*x + d)^(3/2)),x, algo

[Out] -8\*(2\*b\*d - a\*e)\*e^(-1/2)\*ln(abs(-sqrt(b\*x + a)\*sqrt(b)\*e^(1/2) + sqrt(b^2\*d + (b\*x + a)\*b\*e - a\*b\*e)))/(sqrt(b)\*abs(b)) + 2\*sqrt(b\*x + a)\*(4\*(b^3\*d\*e^3 - a\*b^2\*e^4)\*(b\*x + a)/(b^3\*d\*abs(b)\*e^2 - a\*b^2\*abs(b)\*e^3) + (7\*b^4\*d^2\*e^2 - 8\*a\*b^3\*d\*e^3 + 4\*a^2\*b^2\*e^4)/(b^3\*d\*abs(b)\*e^2 - a\*b^2\*abs(b)\*e^3))/sqrt(b^2\*d + (b\*x + a)\*b\*e - a\*b\*e)

$$3.847 \quad \int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}(d+ex)^{5/2}} dx$$

**Optimal.** Leaf size=116

$$\frac{2d^2\sqrt{a+bx}}{(d+ex)^{3/2}(bd-ae)} + \frac{4d\sqrt{a+bx}(3bd-2ae)}{\sqrt{d+ex}(bd-ae)^2} + \frac{16 \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{\sqrt{b}\sqrt{e}}$$

[Out] (2\*d^2\*Sqrt[a + b\*x])/((b\*d - a\*e)\*(d + e\*x)^(3/2)) + (4\*d\*(3\*b\*d - 2\*a\*e)\*Sqrt[a + b\*x])/((b\*d - a\*e)^2\*Sqrt[d + e\*x]) + (16\*ArcTanh[(Sqrt[e]\*Sqrt[a + b\*x])/(Sqrt[b]\*Sqrt[d + e\*x])])/(Sqrt[b]\*Sqrt[e])

**Rubi [A]** time = 0.277939, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{2d^2\sqrt{a+bx}}{(d+ex)^{3/2}(bd-ae)} + \frac{4d\sqrt{a+bx}(3bd-2ae)}{\sqrt{d+ex}(bd-ae)^2} + \frac{16 \tanh^{-1}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{\sqrt{b}\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Int[(15\*d^2 + 20\*d\*e\*x + 8\*e^2\*x^2)/(Sqrt[a + b\*x]\*(d + e\*x)^(5/2)), x]

[Out] (2\*d^2\*Sqrt[a + b\*x])/((b\*d - a\*e)\*(d + e\*x)^(3/2)) + (4\*d\*(3\*b\*d - 2\*a\*e)\*Sqrt[a + b\*x])/((b\*d - a\*e)^2\*Sqrt[d + e\*x]) + (16\*ArcTanh[(Sqrt[e]\*Sqrt[a + b\*x])/(Sqrt[b]\*Sqrt[d + e\*x])])/(Sqrt[b]\*Sqrt[e])

**Rubi in Sympy [A]** time = 49.1699, size = 150, normalized size = 1.29

$$-\frac{2d^2\sqrt{a+bx}}{(d+ex)^{3/2}(ae-bd)} + \frac{32d\sqrt{a+bx}(3ae-2bd)}{3\sqrt{d+ex}(ae-bd)^2} - \frac{20d\sqrt{a+bx}(6ae-5bd)}{3\sqrt{d+ex}(ae-bd)^2} + \frac{16 \operatorname{atanh}\left(\frac{\sqrt{e}\sqrt{a+bx}}{\sqrt{b}\sqrt{d+ex}}\right)}{\sqrt{b}\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((8\*e\*\*2\*x\*\*2+20\*d\*e\*x+15\*d\*\*2)/(e\*x+d)\*\*(5/2)/(b\*x+a)\*\*(1/2), x)

[Out] -2\*d\*\*2\*sqrt(a + b\*x)/((d + e\*x)\*\*(3/2)\*(a\*e - b\*d)) + 32\*d\*sqrt(a + b\*x)\*(3\*a\*e - 2\*b\*d)/(3\*sqrt(d + e\*x)\*(a\*e - b\*d)\*\*2) - 20\*d\*sqrt(a + b\*x)\*(6\*a\*e - 5\*b\*d)/(3\*sqrt(d + e\*x)\*(a\*e - b\*d)\*\*2) + 16\*atanh(sqrt(e)\*sqrt(a + b\*x)/(sqrt(b)\*sqrt(d + e\*x)))/(sqrt(b)\*sqrt(e))

**Mathematica [A]** time = 0.240835, size = 111, normalized size = 0.96

$$\frac{2d\sqrt{a+bx}(bd(7d+6ex)-ae(5d+4ex))}{(d+ex)^{3/2}(bd-ae)^2} + \frac{8 \log\left(2\sqrt{b}\sqrt{e}\sqrt{a+bx}\sqrt{d+ex} + ae + bd + 2bex\right)}{\sqrt{b}\sqrt{e}}$$

Antiderivative was successfully verified.

[In] Integrate[(15\*d^2 + 20\*d\*e\*x + 8\*e^2\*x^2)/(Sqrt[a + b\*x]\*(d + e\*x)^(5/2)), x]

[Out]  $(2*d*\sqrt{a + b*x}*(-(a*e*(5*d + 4*e*x)) + b*d*(7*d + 6*e*x)))/((b*d - a*e)^2*(d + e*x)^{(3/2)}) + (8*\text{Log}[b*d + a*e + 2*b*e*x + 2*\text{Sqrt}[b]*\text{Sqrt}[e]*\text{Sqrt}[a + b*x]*\text{Sqrt}[d + e*x]])/(\text{Sqrt}[b]*\text{Sqrt}[e])$

**Maple [B]** time = 0.04, size = 601, normalized size = 5.2

$$2 \frac{\sqrt{bx+a}}{\sqrt{be}(ae-bd)^2 \sqrt{(bx+a)(ex+d)}(ex+d)^{3/2}} \left( 4 \ln \left( \frac{2bex+2\sqrt{(bx+a)(ex+d)}\sqrt{be}+ae+bd}{\sqrt{be}} \right) x^2 a^2 e^4 - 8 \ln \left( \frac{2}{1/2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(5/2)/(b*x+a)^(1/2),x)`

[Out]  $2*(b*x+a)^{(1/2)}*(4*\ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^{(1/2)}*(b*e)^{(1/2)}+a*e+b*d)/(b*e)^{(1/2)})*x^2*a^2*e^4-8*\ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^{(1/2)}*(b*e)^{(1/2)}+a*e+b*d)/(b*e)^{(1/2)})*x^2*a*b*d*e^3+4*\ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^{(1/2)}*(b*e)^{(1/2)}+a*e+b*d)/(b*e)^{(1/2)})*x^2*b^2*d^2*e^2+8*\ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^{(1/2)}*(b*e)^{(1/2)}+a*e+b*d)/(b*e)^{(1/2)})*x*a^2*d^3-16*\ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^{(1/2)}*(b*e)^{(1/2)}+a*e+b*d)/(b*e)^{(1/2)})*x*a*b*d^2*e^2+8*\ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^{(1/2)}*(b*e)^{(1/2)}+a*e+b*d)/(b*e)^{(1/2)})*x*b^2*d^3*e+4*\ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^{(1/2)}*(b*e)^{(1/2)}+a*e+b*d)/(b*e)^{(1/2)})*a^2*d^2*e^2-8*\ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^{(1/2)}*(b*e)^{(1/2)}+a*e+b*d)/(b*e)^{(1/2)})*a*b*d^3*e+4*\ln(1/2*(2*b*e*x+2*((b*x+a)*(e*x+d))^{(1/2)}*(b*e)^{(1/2)}+a*e+b*d)/(b*e)^{(1/2)})*b^2*d^4-4*x*a*d^2*e*((b*x+a)*(e*x+d))^{(1/2)}*(b*e)^{(1/2)}-5*a*d^2*e*((b*x+a)*(e*x+d))^{(1/2)}*(b*e)^{(1/2)}+7*b*d^3*((b*x+a)*(e*x+d))^{(1/2)}*(b*e)^{(1/2)})/(b*e)^{(1/2)}/(a*e-b*d)^2/((b*x+a)*(e*x+d))^{(1/2)}/(e*x+d)^{(3/2)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((8*e^2*x^2 + 20*d*e*x + 15*d^2)/(sqrt(b*x + a)*(e*x + d)^(5/2)),x, algo`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.450986, size = 1, normalized size = 0.01

$$\left[ \frac{2 \left( (7bd^3 - 5ad^2e + 2(3bd^2e - 2ade^2)x) \sqrt{be} \sqrt{bx+a} \sqrt{ex+d} + 2(b^2d^4 - 2abd^3e + a^2d^2e^2 + (b^2d^2e^2 - 2abde^3 + a^2e^4)x \right)}{(b^2d^4 - 2abd^3e + a^2d^2e^2 + (b^2d^2e^2 - 2abde^3 + a^2e^4)x} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((8*e^2*x^2 + 20*d*e*x + 15*d^2)/(sqrt(b*x + a)*(e*x + d)^(5/2)),x, algo`

[Out]  $[2*((7*b*d^3 - 5*a*d^2*e + 2*(3*b*d^2*e - 2*a*d*e^2)*x)*\text{sqrt}(b*e)*\text{sqrt}(b*x + a)*\text{sqrt}(e*x + d) + 2*(b^2*d^4 - 2*a*b*d^3*e + a^2*d^2*e^2 + (b^2*d^2*e^2 - 2*a*b*d^3*e + a^2*e^4)*x^2 + 2*(b^2*d^3*e - 2*a*b*d^2*e^2 + a^2*d*e^3)*x)*\text{log}(4*(2*b^2*e^2*x + b^2*d*e + a*b*e^2)*\text{sqrt}(b*x + a)*\text{sqrt}(e*x + d) + (8*b^2*e^2*x^2 + b^2*d^2 + 6*$



$$\frac{a^2 b^2 d^2 e + a^2 e^2 + 8(b^2 d^2 e + a b^2 e^2) x \sqrt{b^2 e}}{(b^2 d^4 - 2 a b^2 d^3 e + a^2 d^2 e^2 + (b^2 d^2 e^2 - 2 a b^2 d^3 e + a^2 e^4) x^2 + 2(b^2 d^3 e - 2 a b^2 d^2 e^2 + a^2 d e^3) x) \sqrt{b^2 e}}, 2((7 b^2 d^3 - 5 a d^2 e + 2(3 b^2 d^2 e - 2 a d e^2) x) \sqrt{-b^2 e} \sqrt{b^2 x + a} \sqrt{e^2 x + d} + 4(b^2 d^4 - 2 a b^2 d^3 e + a^2 d^2 e^2 + (b^2 d^2 e^2 - 2 a b^2 d^3 e + a^2 e^4) x^2 + 2(b^2 d^3 e - 2 a b^2 d^2 e^2 + a^2 d e^3) x) \arctan(1/2(2 b^2 e x + b^2 d + a e) \sqrt{-b^2 e} / (\sqrt{b^2 x + a} \sqrt{e^2 x + d} b^2 e)) / ((b^2 d^4 - 2 a b^2 d^3 e + a^2 d^2 e^2 + (b^2 d^2 e^2 - 2 a b^2 d^3 e + a^2 e^4) x^2 + 2(b^2 d^3 e - 2 a b^2 d^2 e^2 + a^2 d e^3) x) \sqrt{-b^2 e})]$$

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**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a + bx}(d + ex)^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8\*e\*\*2\*x\*\*2+20\*d\*e\*x+15\*d\*\*2)/(e\*x+d)\*\*(5/2)/(b\*x+a)\*\*(1/2),x)

[Out] Integral((15\*d\*\*2 + 20\*d\*e\*x + 8\*e\*\*2\*x\*\*2)/(sqrt(a + b\*x)\*(d + e\*x)\*\*(5/2)), x)

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**GIAC/XCAS [A]** time = 0.313123, size = 294, normalized size = 2.53

$$-\frac{16 \sqrt{be}^{-\frac{1}{2}} \ln \left( \left| -\sqrt{bx+a} \sqrt{be}^{\frac{1}{2}} + \sqrt{b^2d + (bx+a)be - abe} \right| \right)}{|b|} + \frac{2 \sqrt{bx+a} \left( \frac{2(3b^6d^2e^2 - 2ab^5de^3)(bx+a)}{b^4d^2|b|e - 2ab^3d|b|e^2 + a^2b^2|b|e^3} + \frac{7b^7d^3e - 11ab^6d^2e^2 + 4a^2b^5de^3}{b^4d^2|b|e - 2ab^3d|b|e^2 + a^2b^2|b|e^3} \right)}{(b^2d + (bx+a)be - abe)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8\*e^2\*x^2 + 20\*d\*e\*x + 15\*d^2)/(sqrt(b\*x + a)\*(e\*x + d)^(5/2)),x, algo

[Out] -16\*sqrt(b)\*e^(-1/2)\*ln(abs(-sqrt(b\*x + a)\*sqrt(b)\*e^(1/2) + sqrt(b^2\*d + (b\*x + a)\*b\*e - a\*b\*e))/abs(b) + 2\*sqrt(b\*x + a)\*(2\*(3\*b^6\*d^2\*e^2 - 2\*a\*b^5\*d\*e^3)\*(b\*x + a)/(b^4\*d^2\*abs(b)\*e - 2\*a\*b^3\*d\*abs(b)\*e^2 + a^2\*b^2\*abs(b)\*e^3) + (7\*b^7\*d^3\*e - 11\*a\*b^6\*d^2\*e^2 + 4\*a^2\*b^5\*d\*e^3)/(b^4\*d^2\*abs(b)\*e - 2\*a\*b^3\*d\*abs(b)\*e^2 + a^2\*b^2\*abs(b)\*e^3))/(b^2\*d + (b\*x + a)\*b\*e - a\*b\*e)^(3/2)

$$3.848 \quad \int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}(d+ex)^{7/2}} dx$$

**Optimal.** Leaf size=133

$$\frac{16\sqrt{a+bx}(15a^2e^2 - 35abde + 23b^2d^2)}{15\sqrt{d+ex}(bd-ae)^3} + \frac{6d^2\sqrt{a+bx}}{5(d+ex)^{5/2}(bd-ae)} + \frac{8d\sqrt{a+bx}(8bd-5ae)}{15(d+ex)^{3/2}(bd-ae)^2}$$

[Out] (6\*d^2\*Sqrt[a + b\*x])/(5\*(b\*d - a\*e)\*(d + e\*x)^(5/2)) + (8\*d\*(8\*b\*d - 5\*a\*e)\*Sqrt[a + b\*x])/(15\*(b\*d - a\*e)^2\*(d + e\*x)^(3/2)) + (16\*(23\*b^2\*d^2 - 35\*a\*b\*d\*e + 15\*a^2\*e^2)\*Sqrt[a + b\*x])/(15\*(b\*d - a\*e)^3\*Sqrt[d + e\*x])

**Rubi [A]** time = 0.322472, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$

$$\frac{16\sqrt{a+bx}(15a^2e^2 - 35abde + 23b^2d^2)}{15\sqrt{d+ex}(bd-ae)^3} + \frac{6d^2\sqrt{a+bx}}{5(d+ex)^{5/2}(bd-ae)} + \frac{8d\sqrt{a+bx}(8bd-5ae)}{15(d+ex)^{3/2}(bd-ae)^2}$$

Antiderivative was successfully verified.

[In] Int[(15\*d^2 + 20\*d\*e\*x + 8\*e^2\*x^2)/(Sqrt[a + b\*x]\*(d + e\*x)^(7/2)), x]

[Out] (6\*d^2\*Sqrt[a + b\*x])/(5\*(b\*d - a\*e)\*(d + e\*x)^(5/2)) + (8\*d\*(8\*b\*d - 5\*a\*e)\*Sqrt[a + b\*x])/(15\*(b\*d - a\*e)^2\*(d + e\*x)^(3/2)) + (16\*(23\*b^2\*d^2 - 35\*a\*b\*d\*e + 15\*a^2\*e^2)\*Sqrt[a + b\*x])/(15\*(b\*d - a\*e)^3\*Sqrt[d + e\*x])

**Rubi in Sympy [A]** time = 58.1368, size = 207, normalized size = 1.56

$$\frac{16bd\sqrt{a+bx}(5ae-4bd)}{3\sqrt{d+ex}(ae-bd)^3} - \frac{6d^2\sqrt{a+bx}}{5(d+ex)^{5/2}(ae-bd)} - \frac{8d\sqrt{a+bx}(5ae-4bd)}{3(d+ex)^{3/2}(ae-bd)^2} + \frac{32d\sqrt{a+bx}(5ae-3bd)}{15(d+ex)^{3/2}(ae-bd)^2} - \frac{16\sqrt{a+bx}(15a^2e^2-10abde+3b^2d^2)}{15\sqrt{d+ex}(ae-bd)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((8\*e\*\*2\*x\*\*2+20\*d\*e\*x+15\*d\*\*2)/(e\*x+d)\*\*(7/2)/(b\*x+a)\*\*(1/2), x)

[Out] 16\*b\*d\*sqrt(a + b\*x)\*(5\*a\*e - 4\*b\*d)/(3\*sqrt(d + e\*x)\*(a\*e - b\*d)\*\*3) - 6\*d\*\*2\*sqrt(a + b\*x)/(5\*(d + e\*x)\*\*(5/2)\*(a\*e - b\*d)) - 8\*d\*sqrt(a + b\*x)\*(5\*a\*e - 4\*b\*d)/(3\*(d + e\*x)\*\*(3/2)\*(a\*e - b\*d)\*\*2) + 32\*d\*sqrt(a + b\*x)\*(5\*a\*e - 3\*b\*d)/(15\*(d + e\*x)\*\*(3/2)\*(a\*e - b\*d)\*\*2) - 16\*sqrt(a + b\*x)\*(15\*a\*\*2\*e\*\*2 - 10\*a\*b\*d\*e + 3\*b\*\*2\*d\*\*2)/(15\*sqrt(d + e\*x)\*(a\*e - b\*d)\*\*3)

**Mathematica [A]** time = 0.17149, size = 110, normalized size = 0.83

$$\frac{2\sqrt{a+bx}(a^2e^2(149d^2 + 260dex + 120e^2x^2) - 2abde(175d^2 + 306dex + 140e^2x^2) + b^2d^2(225d^2 + 400dex + 184e^2x^2))}{15(d+ex)^{5/2}(bd-ae)^3}$$

Antiderivative was successfully verified.

[In] Integrate[(15\*d^2 + 20\*d\*e\*x + 8\*e^2\*x^2)/(Sqrt[a + b\*x]\*(d + e\*x)^(7/2)), x]

[Out]  $(2\sqrt{a+bx} \cdot (a^2e^2(149d^2 + 260de^2x + 120e^2x^2) - 2a^2b^2d^2e^2(175d^2 + 306de^2x + 140e^2x^2) + b^2d^2(225d^2 + 400de^2x + 184e^2x^2)))/(15(bd - ae)^3(d + ex)^{5/2})$

**Maple [A]** time = 0.014, size = 150, normalized size = 1.1

$$\frac{240a^2e^4x^2 - 560abde^3x^2 + 368b^2d^2e^2x^2 + 520a^2de^3x - 1224abd^2e^2x + 800b^2d^3ex + 298a^2d^2e^2 - 700abd^3e + 450b^2d^4}{15a^3e^3 - 45a^2bde^2 + 45ab^2d^2e - 15b^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((8*e^2*x^2+20*d*e*x+15*d^2)/(e*x+d)^(7/2)/(b*x+a)^(1/2),x)`

[Out]  $-2/15(bx+a)^{1/2} \cdot (120a^2e^4x^2 - 280a^2b^2d^2e^3x^2 + 184b^2d^2e^2x^2 + 260a^2d^2e^3x - 612a^2b^2d^2e^2x + 400b^2d^3e^2x + 149a^2d^2e^2 - 350a^2b^2d^3e + 225b^2d^4)/(e^2x+d)^{5/2}/(a^3e^3 - 3a^2b^2d^2e + 3ab^2d^3e - b^3d^3)$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((8*e^2*x^2 + 20*d*e*x + 15*d^2)/(sqrt(b*x + a)*(e*x + d)^(7/2)),x, algo`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.408424, size = 396, normalized size = 2.98

$$\frac{2(225b^2d^4 - 350abd^3e + 149a^2d^2e^2 + 8(23b^2d^2e^2 - 35abde^3 + 15a^2e^4)x^2 + 4(100b^2d^3e - 153abd^3e^2 - 15a^2d^2e^2 + 65a^2d^2e^3)x) \cdot \sqrt{bx+a} \cdot \sqrt{ex+d}}{15(b^3d^6 - 3ab^2d^5e + 3a^2bd^4e^2 - a^3d^3e^3 + (b^3d^3e^3 - 3ab^2d^2e^4 + 3a^2bde^5 - a^3e^6)x^3 + 3(b^3d^4e^2 - 3ab^2d^3e^3 + 3a^2bd^2e^4 - 3a^2b^2d^2e^3 + 3a^2bd^2e^4 - a^3d^2e^4)x^2 + 3(b^3d^4e^2 - 3ab^2d^3e^3 + 3a^2bd^2e^4 - 3a^2b^2d^2e^3 + 3a^2bd^2e^4 - a^3d^2e^4)x + 3(b^3d^4e^2 - 3ab^2d^3e^3 + 3a^2bd^2e^4 - 3a^2b^2d^2e^3 + 3a^2bd^2e^4 - a^3d^2e^4))}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((8*e^2*x^2 + 20*d*e*x + 15*d^2)/(sqrt(b*x + a)*(e*x + d)^(7/2)),x, algo`

[Out]  $2/15 \cdot (225b^2d^4 - 350abd^3e + 149a^2d^2e^2 + 8(23b^2d^2e^2 - 35abde^3 + 15a^2e^4)x^2 + 4(100b^2d^3e - 153abd^3e^2 - 15a^2d^2e^2 + 65a^2d^2e^3)x) \cdot \sqrt{bx+a} \cdot \sqrt{ex+d} / (b^3d^6 - 3ab^2d^5e + 3a^2bd^4e^2 - a^3d^3e^3 + (b^3d^3e^3 - 3ab^2d^2e^4 + 3a^2bde^5 - a^3e^6)x^3 + 3(b^3d^4e^2 - 3ab^2d^3e^3 + 3a^2bd^2e^4 - 3a^2b^2d^2e^3 + 3a^2bd^2e^4 - a^3d^2e^4)x^2 + 3(b^3d^4e^2 - 3ab^2d^3e^3 + 3a^2bd^2e^4 - 3a^2b^2d^2e^3 + 3a^2bd^2e^4 - a^3d^2e^4)x + 3(b^3d^4e^2 - 3ab^2d^3e^3 + 3a^2bd^2e^4 - 3a^2b^2d^2e^3 + 3a^2bd^2e^4 - a^3d^2e^4))$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(7/2)/(b*x+a)**(1/2),x)`

[Out] Timed out

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**GIAC/XCAS [A]** time = 0.31914, size = 455, normalized size = 3.42

$$2 \left( 4(bx + a) \left( \frac{2(23b^8d^2e^4 - 35ab^7de^5 + 15a^2b^6e^6)(bx+a)}{b^5d^3|b|e^2 - 3ab^4d^2|b|e^3 + 3a^2b^3d|b|e^4 - a^3b^2|b|e^5} + \frac{5(20b^9d^3e^3 - 49ab^8d^2e^4 + 41a^2b^7de^5 - 12a^3b^6e^6)}{b^5d^3|b|e^2 - 3ab^4d^2|b|e^3 + 3a^2b^3d|b|e^4 - a^3b^2|b|e^5} \right) + \frac{15(15b^{10}d^4e^2 - 50ab^9d^3e^3)}{b^5d^3|b|e^2 - 3ab^4d^2|b|e^3 + 3a^2b^3d|b|e^4 - a^3b^2|b|e^5} \right) \frac{1}{15(b^2d + (bx + a)be - a^2e)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8\*e^2\*x^2 + 20\*d\*e\*x + 15\*d^2)/(sqrt(b\*x + a)\*(e\*x + d)^(7/2)),x, algo

[Out] 2/15\*(4\*(b\*x + a)\*(2\*(23\*b^8\*d^2\*e^4 - 35\*a\*b^7\*d\*e^5 + 15\*a^2\*b^6\*e^6)\*(b\*x + a)/(b^5\*d^3\*abs(b)\*e^2 - 3\*a\*b^4\*d^2\*abs(b)\*e^3 + 3\*a^2\*b^3\*d\*abs(b)\*e^4 - a^3\*b^2\*abs(b)\*e^5) + 5\*(20\*b^9\*d^3\*e^3 - 49\*a\*b^8\*d^2\*e^4 + 41\*a^2\*b^7\*d\*e^5 - 12\*a^3\*b^6\*e^6)/(b^5\*d^3\*abs(b)\*e^2 - 3\*a\*b^4\*d^2\*abs(b)\*e^3 + 3\*a^2\*b^3\*d\*abs(b)\*e^4 - a^3\*b^2\*abs(b)\*e^5)) + 15\*(15\*b^10\*d^4\*e^2 - 50\*a\*b^9\*d^3\*e^3 + 63\*a^2\*b^8\*d^2\*e^4 - 36\*a^3\*b^7\*d\*e^5 + 8\*a^4\*b^6\*e^6)/(b^5\*d^3\*abs(b)\*e^2 - 3\*a\*b^4\*d^2\*abs(b)\*e^3 + 3\*a^2\*b^3\*d\*abs(b)\*e^4 - a^3\*b^2\*abs(b)\*e^5))\*sqrt(b\*x + a)/(b^2\*d + (b\*x + a)\*b\*e - a\*b\*e)^(5/2)

$$3.849 \quad \int \frac{15d^2 + 20dex + 8e^2x^2}{\sqrt{a+bx}(d+ex)^{9/2}} dx$$

**Optimal.** Leaf size=189

$$\frac{32b\sqrt{a+bx}(35a^2e^2 - 84abde + 58b^2d^2)}{105\sqrt{d+ex}(bd-ae)^4} + \frac{16\sqrt{a+bx}(35a^2e^2 - 84abde + 58b^2d^2)}{105(d+ex)^{3/2}(bd-ae)^3}$$

$$+ \frac{6d^2\sqrt{a+bx}}{7(d+ex)^{7/2}(bd-ae)} + \frac{4d\sqrt{a+bx}(23bd - 14ae)}{35(d+ex)^{5/2}(bd-ae)^2}$$

[Out]  $(6*d^2*\text{Sqrt}[a + b*x])/(7*(b*d - a*e)*(d + e*x)^{(7/2)}) + (4*d*(23*b*d - 14*a*e)*\text{Sqrt}[a + b*x])/(35*(b*d - a*e)^2*(d + e*x)^{(5/2)}) + (16*(58*b^2*d^2 - 84*a*b*d*e + 35*a^2*e^2)*\text{Sqrt}[a + b*x])/(105*(b*d - a*e)^3*(d + e*x)^{(3/2)}) + (32*b*(58*b^2*d^2 - 84*a*b*d*e + 35*a^2*e^2)*\text{Sqrt}[a + b*x])/(105*(b*d - a*e)^4*\text{Sqrt}[d + e*x])$

**Rubi [A]** time = 0.448451, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 38,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$

$$\frac{32b\sqrt{a+bx}(35a^2e^2 - 84abde + 58b^2d^2)}{105\sqrt{d+ex}(bd-ae)^4} + \frac{16\sqrt{a+bx}(35a^2e^2 - 84abde + 58b^2d^2)}{105(d+ex)^{3/2}(bd-ae)^3}$$

$$+ \frac{6d^2\sqrt{a+bx}}{7(d+ex)^{7/2}(bd-ae)} + \frac{4d\sqrt{a+bx}(23bd - 14ae)}{35(d+ex)^{5/2}(bd-ae)^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(15*d^2 + 20*d*e*x + 8*e^2*x^2)/(\text{Sqrt}[a + b*x]*(d + e*x)^{(9/2)}), x]$

[Out]  $(6*d^2*\text{Sqrt}[a + b*x])/(7*(b*d - a*e)*(d + e*x)^{(7/2)}) + (4*d*(23*b*d - 14*a*e)*\text{Sqrt}[a + b*x])/(35*(b*d - a*e)^2*(d + e*x)^{(5/2)}) + (16*(58*b^2*d^2 - 84*a*b*d*e + 35*a^2*e^2)*\text{Sqrt}[a + b*x])/(105*(b*d - a*e)^3*(d + e*x)^{(3/2)}) + (32*b*(58*b^2*d^2 - 84*a*b*d*e + 35*a^2*e^2)*\text{Sqrt}[a + b*x])/(105*(b*d - a*e)^4*\text{Sqrt}[d + e*x])$

**Rubi in Sympy [A]** time = 80.9068, size = 308, normalized size = 1.63

$$\frac{32b^2d\sqrt{a+bx}(14ae - 11bd)}{21\sqrt{d+ex}(ae - bd)^4} + \frac{16bd\sqrt{a+bx}(14ae - 11bd)}{21(d+ex)^{3/2}(ae - bd)^3}$$

$$+ \frac{32b\sqrt{a+bx}(35a^2e^2 - 14abde + 3b^2d^2)}{105\sqrt{d+ex}(ae - bd)^4} - \frac{6d^2\sqrt{a+bx}}{7(d+ex)^{7/2}(ae - bd)} + \frac{32d\sqrt{a+bx}(7ae - 4bd)}{35(d+ex)^{5/2}(ae - bd)^2}$$

$$- \frac{4d\sqrt{a+bx}(14ae - 11bd)}{7(d+ex)^{5/2}(ae - bd)^2} - \frac{16\sqrt{a+bx}(35a^2e^2 - 14abde + 3b^2d^2)}{105(d+ex)^{3/2}(ae - bd)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((8*e**2*x**2+20*d*e*x+15*d**2)/(e*x+d)**(9/2)/(b*x+a)**(1/2), x)$

[Out]  $-32*b**2*d*\text{sqrt}(a + b*x)*(14*a*e - 11*b*d)/(21*\text{sqrt}(d + e*x)*(a*e - b*d)**4) + 16*b*d*\text{sqrt}(a + b*x)*(14*a*e - 11*b*d)/(21*(d + e*x)**(3/2)*(a*e - b*d)**3) + 32*b*\text{sqrt}(a + b*x)*(35*a**2*e**2 - 14*a*b*d*e + 3*b**2*d**2)/(105*\text{sqrt}(d + e*x)*(a*e - b*d)**4) - 6*d**2*\text{sqrt}(a + b*x)/(7*(d + e*x)**(7/2)*(a*e - b*d)) + 32*d*\text{sqrt}(a + b*x)*(7*a*e - 4*b*d)/(35*(d + e*x)**(5/2)*(a*e - b*d)**2) - 4*d*\text{sqrt}(a + b*x)*(14*a*e - 11*b*d)/(7*(d + e*x)**(5/2)*(a*e - b*d)**2) - 16*\text{sqrt}(a + b*x)*(35*a**2*e**2 - 14*a*b*d*e + 3*b**2*d**2)/(105*(d + e*x)**(3/2)*(a*e - b*d)**3)$

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**Mathematica [A]** time = 0.36553, size = 148, normalized size = 0.78

$$\frac{2\sqrt{a+bx} (8(d+ex)^2 (35a^2e^2 - 84abde + 58b^2d^2) (bd - ae) + 16b(d+ex)^3 (35a^2e^2 - 84abde + 58b^2d^2) + 45d^2(bd - ae)^3 + 105(d+ex)^{7/2}(bd - ae)^4}{105(d+ex)^{7/2}(bd - ae)^4}$$

Antiderivative was successfully verified.

[In] Integrate[(15\*d^2 + 20\*d\*e\*x + 8\*e^2\*x^2)/(Sqrt[a + b\*x]\*(d + e\*x)^(9/2)), x]

[Out] (2\*Sqrt[a + b\*x]\*(45\*d^2\*(b\*d - a\*e)^3 + 6\*d\*(23\*b\*d - 14\*a\*e)\*(b\*d - a\*e)^2\*(d + e\*x) + 8\*(b\*d - a\*e)\*(58\*b^2\*d^2 - 84\*a\*b\*d\*e + 35\*a^2\*e^2)\*(d + e\*x)^2 + 16\*b\*(58\*b^2\*d^2 - 84\*a\*b\*d\*e + 35\*a^2\*e^2)\*(d + e\*x)^3))/(105\*(b\*d - a\*e)^4\*(d + e\*x)^(7/2))

---

**Maple [A]** time = 0.017, size = 248, normalized size = 1.3

$$\frac{-1120 a^2 b e^5 x^3 + 2688 a b^2 d e^4 x^3 - 1856 b^3 d^2 e^3 x^3 + 560 a^3 e^5 x^2 - 5264 a^2 b d e^4 x^2 + 10336 a b^2 d^2 e^3 x^2 - 6496 b^3 d^3 e^2 x^2 + 1280 a^4 e^5 x - 5264 a^3 b d e^4 x + 10336 a^2 b^2 d^2 e^3 x - 6496 a b^3 d^3 e^2 x + 1280 a^4 e^5}{105 e^4 a^4 - 420 b e^3 d a^3 + 630 a^2 b^2 d^2 e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((8\*e^2\*x^2+20\*d\*e\*x+15\*d^2)/(e\*x+d)^(9/2)/(b\*x+a)^(1/2), x)

[Out] -2/105\*(b\*x+a)^(1/2)\*(-560\*a^2\*b\*e^5\*x^3+1344\*a\*b^2\*d\*e^4\*x^3-928\*b^3\*d^2\*e^3\*x^3+280\*a^3\*e^5\*x^2-2632\*a^2\*b\*d\*e^4\*x^2+5168\*a\*b^2\*d^2\*e^3\*x^2-3248\*b^3\*d^3\*e^2\*x^2+644\*a^3\*d\*e^4\*x-3890\*a^2\*b\*d^2\*e^3\*x+6664\*a\*b^2\*d^3\*e^2\*x-3850\*b^3\*d^4\*e\*x+409\*a^3\*d^2\*e^3-1953\*a^2\*b\*d^3\*e^2+2975\*a\*b^2\*d^4\*e-1575\*b^3\*d^5)/(e\*x+d)^(7/2)/(a^4\*e^4-4\*a^3\*b\*d\*e^3+6\*a^2\*b^2\*d^2\*e^2-4\*a\*b^3\*d^3\*e+b^4\*d^4)

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**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8\*e^2\*x^2 + 20\*d\*e\*x + 15\*d^2)/(sqrt(b\*x + a)\*(e\*x + d)^(9/2)), x, algo

[Out] Exception raised: ValueError

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**Fricas [A]** time = 0.781525, size = 657, normalized size = 3.48

$$\frac{2(1575 b^3 d^5 - 2975 a b^2 d^4 e + 1953 a^2 b d^3 e^2 - 409 a^3 d^2 e^3 + 16(58 b^3 d^2 e^3 - 84 a b^2 d e^4 + 35 a^2 b e^5) x^3 + 8(406 b^3 d^3 e^2 - 646 a b^2 d^2 e^3 + 329 a^2 b d e^4 - 35 a^3 e^5) x^2 + 2(1925 b^3 d^4 e - 3332 a b^2 d^3 e^2 + 1945 a^3 d^2 e^3 - 5264 a^2 b d e^4 + 10336 a b^2 d^2 e^3 - 6496 b^3 d^3 e^2 + 1280 a^4 e^5) x - 5264 a^3 b d e^4 + 10336 a^2 b^2 d^2 e^3 - 6496 a b^3 d^3 e^2 + 1280 a^4 e^5}{105(b^4 d^8 - 4 a b^3 d^7 e + 6 a^2 b^2 d^6 e^2 - 4 a^3 b d^5 e^3 + a^4 d^4 e^4 + (b^4 d^4 e^4 - 4 a b^3 d^3 e^5 + 6 a^2 b^2 d^2 e^6 - 4 a^3 b d e^7 + a^4 e^8) x^4 + 4(b^4 d^5 e^3 - 4 a b^3 d^4 e^4 + 6 a^2 b^2 d^3 e^5 - 4 a^3 b d^2 e^6 + a^4 d e^7 - a^5 e^8) x^3 + 6 a^2 b^2 d^2 e^6 - 4 a^3 b d e^7 + a^4 e^8) x^4 + 4(b^4 d^5 e^3 - 4 a b^3 d^4 e^4 + 6 a^2 b^2 d^3 e^5 - 4 a^3 b d^2 e^6 + a^4 d e^7 - a^5 e^8) x^3 + 6 a^2 b^2 d^2 e^6 - 4 a^3 b d e^7 + a^4 e^8}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8\*e^2\*x^2 + 20\*d\*e\*x + 15\*d^2)/(sqrt(b\*x + a)\*(e\*x + d)^(9/2)), x, algo

[Out] 2/105\*(1575\*b^3\*d^5 - 2975\*a\*b^2\*d^4\*e + 1953\*a^2\*b\*d^3\*e^2 - 409\*a^3\*d^2\*e^3 + 16\*(58\*b^3\*d^2\*e^3 - 84\*a\*b^2\*d\*e^4 + 35\*a^2\*b\*e^5)\*x^3 + 8\*(406\*b^3\*d^3\*e^2 - 646\*a\*b^2\*d^2\*e^3 + 329\*a^2\*b\*d\*e^4 - 35\*a^3\*e^5)\*x^2 + 2\*(1925\*b^3\*d^4\*e - 3332\*a\*b^2\*d^3\*e^2 + 1945

$$*a^2*b*d^2*e^3 - 322*a^3*d*e^4)*x)*\sqrt{b*x + a)*\sqrt{e*x + d)/(b^4*d^8 - 4*a*b^3*d^7*e + 6*a^2*b^2*d^6*e^2 - 4*a^3*b*d^5*e^3 + a^4*d^4*e^4 + (b^4*d^4*e^4 - 4*a*b^3*d^3*e^5 + 6*a^2*b^2*d^2*e^6 - 4*a^3*b*d*e^7 + a^4*e^8)*x^4 + 4*(b^4*d^5*e^3 - 4*a*b^3*d^4*e^4 + 6*a^2*b^2*d^3*e^5 - 4*a^3*b*d^2*e^6 + a^4*d*e^7)*x^3 + 6*(b^4*d^6*e^2 - 4*a*b^3*d^5*e^3 + 6*a^2*b^2*d^4*e^4 - 4*a^3*b*d^3*e^5 + a^4*d^2*e^6)*x^2 + 4*(b^4*d^7*e - 4*a*b^3*d^6*e^2 + 6*a^2*b^2*d^5*e^3 - 4*a^3*b*d^4*e^4 + a^4*d^3*e^5)*x)$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8\*e\*\*2\*x\*\*2+20\*d\*e\*x+15\*d\*\*2)/(e\*x+d)\*\*(9/2)/(b\*x+a)\*\*(1/2),x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.362292, size = 707, normalized size = 3.74

$$2 \left( 2 \left( 4 (bx + a) \left( \frac{2 (58 b^{10} d^2 e^6 - 84 a b^9 d e^7 + 35 a^2 b^8 e^8) (bx + a)}{b^6 d^4 |b| e^3 - 4 a b^5 d^3 |b| e^4 + 6 a^2 b^4 d^2 |b| e^5 - 4 a^3 b^3 d |b| e^6 + a^4 b^2 |b| e^7} + \frac{7 (58 b^{11} d^3 e^5 - 142 a b^{10} d^2 e^6 + 119 a^2 b^9 d e^7 - 35 a^3 b^8 e^8)}{b^6 d^4 |b| e^3 - 4 a b^5 d^3 |b| e^4 + 6 a^2 b^4 d^2 |b| e^5 - 4 a^3 b^3 d |b| e^6 + a^4 b^2 |b| e^7} \right) \right)$$

10.

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((8\*e^2\*x^2 + 20\*d\*e\*x + 15\*d^2)/(sqrt(b\*x + a)\*(e\*x + d)^(9/2)),x, algo

[Out]  $2/105*(2*(4*(b*x + a)*(2*(58*b^10*d^2*e^6 - 84*a*b^9*d*e^7 + 35*a^2*b^8*e^8)*(b*x + a)/(b^6*d^4*abs(b)*e^3 - 4*a*b^5*d^3*abs(b)*e^4 + 6*a^2*b^4*d^2*abs(b)*e^5 - 4*a^3*b^3*d*abs(b)*e^6 + a^4*b^2*abs(b)*e^7) + 7*(58*b^11*d^3*e^5 - 142*a*b^10*d^2*e^6 + 119*a^2*b^9*d*e^7 - 35*a^3*b^8*e^8)/(b^6*d^4*abs(b)*e^3 - 4*a*b^5*d^3*abs(b)*e^4 + 6*a^2*b^4*d^2*abs(b)*e^5 - 4*a^3*b^3*d*abs(b)*e^6 + a^4*b^2*abs(b)*e^7)) + 35*(55*b^12*d^4*e^4 - 188*a*b^11*d^3*e^5 + 243*a^2*b^10*d^2*e^6 - 142*a^3*b^9*d*e^7 + 32*a^4*b^8*e^8)/(b^6*d^4*abs(b)*e^3 - 4*a*b^5*d^3*abs(b)*e^4 + 6*a^2*b^4*d^2*abs(b)*e^5 - 4*a^3*b^3*d*abs(b)*e^6 + a^4*b^2*abs(b)*e^7))*sqrt(b*x + a)/(b^2*d + (b*x + a)*b*e - a*b*e)^(7/2)$

$$3.850 \quad \int \frac{(d+ex)^{3/2}}{\sqrt{f+gx}(a+bx+cx^2)} dx$$

**Optimal.** Leaf size=417

$$\frac{2 \left( \frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}} + e(2cd - be) \right) \tanh^{-1} \left( \frac{\sqrt{d+ex} \sqrt{2cf-g(b-\sqrt{b^2-4ac})}}{\sqrt{f+gx} \sqrt{2cd-e(b-\sqrt{b^2-4ac})}} \right)}{c \sqrt{2cd - e(b - \sqrt{b^2 - 4ac})} \sqrt{2cf - g(b - \sqrt{b^2 - 4ac})}} - \frac{2 \left( e(2cd - be) - \frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}} \right) \tanh^{-1} \left( \frac{\sqrt{d+ex} \sqrt{2cf-g(\sqrt{b^2-4ac}+b)}}{\sqrt{f+gx} \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} \right)}{c \sqrt{2cd - e(\sqrt{b^2 - 4ac} + b)} \sqrt{2cf - g(\sqrt{b^2 - 4ac} + b)}} + \frac{2e^{3/2} \tanh^{-1} \left( \frac{\sqrt{g} \sqrt{d+ex}}{\sqrt{e} \sqrt{f+gx}} \right)}{c \sqrt{g}}$$

[Out] (2\*e^(3/2)\*ArcTanh[(Sqrt[g]\*Sqrt[d + e\*x])/(Sqrt[e]\*Sqrt[f + g\*x])])/(c\*Sqrt[g]) - (2\*(e\*(2\*c\*d - b\*e) + (2\*c^2\*d^2 + b^2\*e^2 - 2\*c\*e\*(b\*d + a\*e))/Sqrt[b^2 - 4\*a\*c])\*ArcTanh[(Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g]\*Sqrt[d + e\*x])/(Sqrt[2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e]\*Sqrt[f + g\*x])])/(c\*Sqrt[2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e]\*Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g]) - (2\*(e\*(2\*c\*d - b\*e) - (2\*c^2\*d^2 + b^2\*e^2 - 2\*c\*e\*(b\*d + a\*e))/Sqrt[b^2 - 4\*a\*c])\*ArcTanh[(Sqrt[2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g]\*Sqrt[d + e\*x])/(Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e]\*Sqrt[f + g\*x])])/(c\*Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e]\*Sqrt[2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g])

**Rubi [A]** time = 5.75305, antiderivative size = 417, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$

$$\frac{2 \left( \frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}} + e(2cd - be) \right) \tanh^{-1} \left( \frac{\sqrt{d+ex} \sqrt{2cf-g(b-\sqrt{b^2-4ac})}}{\sqrt{f+gx} \sqrt{2cd-e(b-\sqrt{b^2-4ac})}} \right)}{c \sqrt{2cd - e(b - \sqrt{b^2 - 4ac})} \sqrt{2cf - g(b - \sqrt{b^2 - 4ac})}} - \frac{2 \left( e(2cd - be) - \frac{-2ce(ae+bd)+b^2e^2+2c^2d^2}{\sqrt{b^2-4ac}} \right) \tanh^{-1} \left( \frac{\sqrt{d+ex} \sqrt{2cf-g(\sqrt{b^2-4ac}+b)}}{\sqrt{f+gx} \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} \right)}{c \sqrt{2cd - e(\sqrt{b^2 - 4ac} + b)} \sqrt{2cf - g(\sqrt{b^2 - 4ac} + b)}} + \frac{2e^{3/2} \tanh^{-1} \left( \frac{\sqrt{g} \sqrt{d+ex}}{\sqrt{e} \sqrt{f+gx}} \right)}{c \sqrt{g}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^(3/2)/(Sqrt[f + g\*x]\*(a + b\*x + c\*x^2)), x]

[Out] (2\*e^(3/2)\*ArcTanh[(Sqrt[g]\*Sqrt[d + e\*x])/(Sqrt[e]\*Sqrt[f + g\*x])])/(c\*Sqrt[g]) - (2\*(e\*(2\*c\*d - b\*e) + (2\*c^2\*d^2 + b^2\*e^2 - 2\*c\*e\*(b\*d + a\*e))/Sqrt[b^2 - 4\*a\*c])\*ArcTanh[(Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g]\*Sqrt[d + e\*x])/(Sqrt[2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e]\*Sqrt[f + g\*x])])/(c\*Sqrt[2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e]\*Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g]) - (2\*(e\*(2\*c\*d - b\*e) - (2\*c^2\*d^2 + b^2\*e^2 - 2\*c\*e\*(b\*d + a\*e))/Sqrt[b^2 - 4\*a\*c])\*ArcTanh[(Sqrt[2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g]\*Sqrt[d + e\*x])/(Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e]\*Sqrt[f + g\*x])])/(c\*Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e]\*Sqrt[2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g])



) \* ArcTanh[(Sqrt[2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g]\*Sqrt[d + e\*x]) / (Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e]\*Sqrt[f + g\*x])] / (c\*Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e]\*Sqrt[2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g])

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*(3/2)/(c\*x\*\*2+b\*x+a)/(g\*x+f)\*\*(1/2),x)

[Out] Timed out

**Mathematica [B]** time = 6.21184, size = 1319, normalized size = 3.16

$$\frac{\log\left(ef + dg + 2egx + 2\sqrt{e}\sqrt{g}\sqrt{d + ex}\sqrt{f + gx}\right) e^{3/2}}{c\sqrt{g}}$$

$$-\frac{\left(-2c^2d^2 + 2bcde - 2c\sqrt{b^2 - 4ace}d - b^2e^2 + 2ace^2 + b\sqrt{b^2 - 4ace^2}\right) \log\left(-b - 2cx + \sqrt{b^2 - 4ac}\right)}{\sqrt{2c}\sqrt{b^2 - 4ac}\sqrt{egb^2 - cefb - cdgb - \sqrt{b^2 - 4ace}gb + 2c^2df + c\sqrt{b^2 - 4ace}f + c\sqrt{b^2 - 4ac}dg - 2aceg}}$$

$$-\frac{\left(2c^2d^2 - 2bcde - 2c\sqrt{b^2 - 4ace}d + b^2e^2 - 2ace^2 + b\sqrt{b^2 - 4ace^2}\right) \log\left(b + 2cx + \sqrt{b^2 - 4ac}\right)}{\sqrt{2c}\sqrt{b^2 - 4ac}\sqrt{egb^2 - cefb - cdgb + \sqrt{b^2 - 4ace}gb + 2c^2df - c\sqrt{b^2 - 4ace}f - c\sqrt{b^2 - 4ac}dg - 2aceg}}$$

$$\left(-2c^2d^2 + 2bcde - 2c\sqrt{b^2 - 4ace}d - b^2e^2 + 2ace^2 + b\sqrt{b^2 - 4ace^2}\right) \log\left(efb^2 + dgb^2 + 2egxb^2 - \sqrt{b^2 - 4ace}fb - \sqrt{b^2 - 4ac}dg - 2aceg\right)$$

$$+\frac{\left(2c^2d^2 - 2bcde - 2c\sqrt{b^2 - 4ace}d + b^2e^2 - 2ace^2 + b\sqrt{b^2 - 4ace^2}\right) \log\left(-efb^2 - dgb^2 - 2egxb^2 - \sqrt{b^2 - 4ace}fb - \sqrt{b^2 - 4ac}dg - 2aceg\right)}{\sqrt{2c}\sqrt{b^2 - 4ac}\sqrt{egb^2 - cefb - cdgb - \sqrt{b^2 - 4ace}gb + 2c^2df + c\sqrt{b^2 - 4ace}f + c\sqrt{b^2 - 4ac}dg - 2aceg}}$$

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^(3/2)/(Sqrt[f + g\*x]\*(a + b\*x + c\*x^2)),x]

[Out] -((( -2\*c^2\*d^2 + 2\*b\*c\*d\*e - 2\*c\*Sqrt[b^2 - 4\*a\*c]\*d\*e - b^2\*e^2 + 2\*a\*c\*e^2 + b\*Sqrt[b^2 - 4\*a\*c]\*e^2)\*Log[-b + Sqrt[b^2 - 4\*a\*c] - 2\*c\*x]) / (Sqrt[2]\*c\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[2\*c^2\*d\*f - b\*c\*e\*f + c\*Sqrt[b^2 - 4\*a\*c]\*e\*f - b\*c\*d\*g + c\*Sqrt[b^2 - 4\*a\*c]\*d\*g + b^2\*e\*g - 2\*a\*c\*e\*g - b\*Sqrt[b^2 - 4\*a\*c]\*e\*g)) - ((2\*c^2\*d^2 - 2\*b\*c\*d\*e - 2\*c\*Sqrt[b^2 - 4\*a\*c]\*d\*e + b^2\*e^2 - 2\*a\*c\*e^2 + b\*Sqrt[b^2 - 4\*a\*c]\*e^2)\*Log[b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x]) / (Sqrt[2]\*c\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[2\*c^2\*d\*f - b\*c\*e\*f - c\*Sqrt[b^2 - 4\*a\*c]\*e\*f - b\*c\*d\*g - c\*Sqrt[b^2 - 4\*a\*c]\*d\*g + b^2\*e\*g - 2\*a\*c\*e\*g + b\*Sqrt[b^2 - 4\*a\*c]\*e\*g)) + (e^(3/2)\*Log[e\*f + d\*g + 2\*e\*g\*x + 2\*Sqrt[e]\*Sqrt[g]\*Sqrt[d + e\*x]\*Sqrt[f + g\*x]]) / (c\*Sqrt[g]) + (( -2\*c^2\*d^2 + 2\*b\*c\*d\*e - 2\*c\*Sqrt[b^2 - 4\*a\*c]\*d\*e - b^2\*e^2 + 2\*a\*c\*e^2 + b\*Sqrt[b^2 - 4\*a\*c]\*e^2)\*Log[4\*c\*Sqrt[b^2 - 4\*a\*c]\*d\*f + b^2\*e\*f - 4\*a\*c\*e\*f - b\*Sqrt[b^2 - 4\*a\*c]\*e\*f + b^2\*d\*g - 4\*a\*c\*d\*g - b\*Sqrt[b^2 - 4\*a\*c]\*d\*g + 2\*c\*Sqrt[b^2 - 4\*a\*c]\*e\*f\*x + 2\*c\*Sqrt[b^2 - 4\*a\*c]\*d\*g\*x + 2\*b^2\*e\*g\*x - 8\*a\*c\*e\*g\*x - 2\*b\*Sqrt[b^2 - 4\*a\*c]\*e\*g\*x + 2\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[2\*c^2\*d\*f - b\*c\*e\*f + c\*Sqrt[b^2 - 4\*a\*c]\*e\*f - b\*c\*d\*g + c\*Sqrt[b^2 - 4\*a\*c]\*d\*g + b^2\*e\*g - 2\*a\*c\*e\*g - b\*Sqrt[b^2 - 4\*a\*c]\*e\*g]\*Sqrt[d + e\*x]\*Sqrt[f + g\*x]]) / (Sqrt[2]\*c\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[2\*c^2\*d\*f - b\*c\*e\*f + c\*Sqrt[b^2 - 4\*a\*c]\*e\*f - b\*c\*d\*g + c\*Sqrt[b^2 - 4\*a\*c]

$$\begin{aligned}
& *d*g + b^2*e*g - 2*a*c*e*g - b*\text{Sqrt}[b^2 - 4*a*c]*e*g]) + ((2*c^2* \\
& d^2 - 2*b*c*d*e - 2*c*\text{Sqrt}[b^2 - 4*a*c]*d*e + b^2*e^2 - 2*a*c*e^2 \\
& + b*\text{Sqrt}[b^2 - 4*a*c]*e^2)*\text{Log}[4*c*\text{Sqrt}[b^2 - 4*a*c]*d*f - b^2*e \\
& *f + 4*a*c*e*f - b*\text{Sqrt}[b^2 - 4*a*c]*e*f - b^2*d*g + 4*a*c*d*g - \\
& b*\text{Sqrt}[b^2 - 4*a*c]*d*g + 2*c*\text{Sqrt}[b^2 - 4*a*c]*e*f*x + 2*c*\text{Sqrt}[ \\
& b^2 - 4*a*c]*d*g*x - 2*b^2*e*g*x + 8*a*c*e*g*x - 2*b*\text{Sqrt}[b^2 - 4 \\
& *a*c]*e*g*x + 2*\text{Sqrt}[2]*\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c^2*d*f - b*c*e \\
& f - c*\text{Sqrt}[b^2 - 4*a*c]*e*f - b*c*d*g - c*\text{Sqrt}[b^2 - 4*a*c]*d*g + \\
& b^2*e*g - 2*a*c*e*g + b*\text{Sqrt}[b^2 - 4*a*c]*e*g]*\text{Sqrt}[d + e*x]*\text{Sqr} \\
& \text{t}[f + g*x]])/(\text{Sqrt}[2]*c*\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c^2*d*f - b*c*e \\
& f - c*\text{Sqrt}[b^2 - 4*a*c]*e*f - b*c*d*g - c*\text{Sqrt}[b^2 - 4*a*c]*d*g + \\
& b^2*e*g - 2*a*c*e*g + b*\text{Sqrt}[b^2 - 4*a*c]*e*g])
\end{aligned}$$

**Maple [B]** time = 0.174, size = 11688, normalized size = 28.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^{\frac{3}{2}}}{(cx^2 + bx + a)\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*sqrt(g*x + f)),x, algorithm="maxima")`

[Out] `integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*sqrt(g*x + f)), x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*sqrt(g*x + f)),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**(3/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)`

[Out] Timed out

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**GIAC/XCAS [A]** time = 0.636525, size = 4, normalized size = 0.01

*sage<sub>0</sub>x*

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^(3/2)/((c*x^2 + b*x + a)*sqrt(g*x + f)),x, algorithm="giac")`

[Out] `sage0x`

$$3.851 \quad \int \frac{\sqrt{d+ex}}{\sqrt{f+gx}(a+bx+cx^2)} dx$$

**Optimal.** Leaf size=285

$$\frac{2\sqrt{2cd - e(\sqrt{b^2 - 4ac} + b)} \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g(\sqrt{b^2-4ac}+b)}}{\sqrt{f+gx}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}\right)}{\sqrt{b^2 - 4ac}\sqrt{2cf - g(\sqrt{b^2 - 4ac} + b)}} - \frac{2\sqrt{2cd - e(b - \sqrt{b^2 - 4ac})} \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g(b-\sqrt{b^2-4ac})}}{\sqrt{f+gx}\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}\right)}{\sqrt{b^2 - 4ac}\sqrt{2cf - g(b - \sqrt{b^2 - 4ac})}}$$

[Out]  $(-2*\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]*\text{ArcTanh}[(\text{Sqrt}[2*c*f - (b - \text{Sqrt}[b^2 - 4*a*c])*g]*\text{Sqrt}[d + e*x])/(\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]*\text{Sqrt}[f + g*x])]) / (\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*f - (b - \text{Sqrt}[b^2 - 4*a*c])*g]) + (2*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]*\text{ArcTanh}[(\text{Sqrt}[2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g]*\text{Sqrt}[d + e*x])/(\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]*\text{Sqrt}[f + g*x])]) / (\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g])$

**Rubi [A]** time = 1.19717, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$

$$\frac{2\sqrt{2cd - e(\sqrt{b^2 - 4ac} + b)} \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g(\sqrt{b^2-4ac}+b)}}{\sqrt{f+gx}\sqrt{2cd-e(\sqrt{b^2-4ac}+b)}}\right)}{\sqrt{b^2 - 4ac}\sqrt{2cf - g(\sqrt{b^2 - 4ac} + b)}} - \frac{2\sqrt{2cd - e(b - \sqrt{b^2 - 4ac})} \tanh^{-1}\left(\frac{\sqrt{d+ex}\sqrt{2cf-g(b-\sqrt{b^2-4ac})}}{\sqrt{f+gx}\sqrt{2cd-e(b-\sqrt{b^2-4ac})}}\right)}{\sqrt{b^2 - 4ac}\sqrt{2cf - g(b - \sqrt{b^2 - 4ac})}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[d + e*x]/(\text{Sqrt}[f + g*x]*(a + b*x + c*x^2)), x]$

[Out]  $(-2*\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]*\text{ArcTanh}[(\text{Sqrt}[2*c*f - (b - \text{Sqrt}[b^2 - 4*a*c])*g]*\text{Sqrt}[d + e*x])/(\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c])*e]*\text{Sqrt}[f + g*x])]) / (\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*f - (b - \text{Sqrt}[b^2 - 4*a*c])*g]) + (2*\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]*\text{ArcTanh}[(\text{Sqrt}[2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g]*\text{Sqrt}[d + e*x])/(\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c])*e]*\text{Sqrt}[f + g*x])]) / (\text{Sqrt}[b^2 - 4*a*c]*\text{Sqrt}[2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g])$

**Rubi in Sympy [A]** time = 178.437, size = 267, normalized size = 0.94

$$\frac{2\sqrt{be - 2cd - e\sqrt{-4ac + b^2}} \operatorname{atanh}\left(\frac{\sqrt{d+ex}\sqrt{bg-2cf-g\sqrt{-4ac+b^2}}}{\sqrt{f+gx}\sqrt{be-2cd-e\sqrt{-4ac+b^2}}}\right)}{\sqrt{-4ac + b^2}\sqrt{bg - 2cf - g\sqrt{-4ac + b^2}}} + \frac{2\sqrt{be - 2cd + e\sqrt{-4ac + b^2}} \operatorname{atanh}\left(\frac{\sqrt{d+ex}\sqrt{bg-2cf+g\sqrt{-4ac+b^2}}}{\sqrt{f+gx}\sqrt{be-2cd+e\sqrt{-4ac+b^2}}}\right)}{\sqrt{-4ac + b^2}\sqrt{bg - 2cf + g\sqrt{-4ac + b^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**(1/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)`

[Out] `-2*sqrt(b*e - 2*c*d - e*sqrt(-4*a*c + b**2))*atanh(sqrt(d + e*x)*sqrt(b*g - 2*c*f - g*sqrt(-4*a*c + b**2)))/(sqrt(f + g*x)*sqrt(b*e - 2*c*d - e*sqrt(-4*a*c + b**2)))/(sqrt(-4*a*c + b**2)*sqrt(b*g - 2*c*f - g*sqrt(-4*a*c + b**2))) + 2*sqrt(b*e - 2*c*d + e*sqrt(-4*a*c + b**2))*atanh(sqrt(d + e*x)*sqrt(b*g - 2*c*f + g*sqrt(-4*a*c + b**2)))/(sqrt(f + g*x)*sqrt(b*e - 2*c*d + e*sqrt(-4*a*c + b**2)))/(sqrt(-4*a*c + b**2)*sqrt(b*g - 2*c*f + g*sqrt(-4*a*c + b**2)))`

**Mathematica [B]** time = 4.75224, size = 925, normalized size = 3.25

$$\frac{(2cd + (\sqrt{b^2 - 4ac} - b)e) \log(-b - 2cx + \sqrt{b^2 - 4ac})}{\sqrt{2dfc^2 + (\sqrt{b^2 - 4ac}ef + \sqrt{b^2 - 4ac}dg - 2aeg - b(ef + dg))}c + b(\sqrt{b^2 - 4ac})eg} - \frac{(2cd - (b + \sqrt{b^2 - 4ac})e) \log(b + 2cx + \sqrt{b^2 - 4ac})}{\sqrt{2dfc^2 - (bef + \sqrt{b^2 - 4ac}ef + bdg + \sqrt{b^2 - 4ac}dg + 2aeg)}c + b(\sqrt{b^2 - 4ac})eg}$$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[d + e*x]/(Sqrt[f + g*x]*(a + b*x + c*x^2)),x]`

[Out] `((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*Log[-b + Sqrt[b^2 - 4*a*c] - 2*c*x])/Sqrt[2*c^2*d*f + b*(b - Sqrt[b^2 - 4*a*c])*e*g + c*(Sqrt[b^2 - 4*a*c]*e*f + Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g - b*(e*f + d*g))] - ((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x])/Sqrt[2*c^2*d*f + b*(b + Sqrt[b^2 - 4*a*c])*e*g - c*(b*e*f + Sqrt[b^2 - 4*a*c]*e*f + b*d*g + Sqrt[b^2 - 4*a*c]*d*g + 2*a*e*g)] - ((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*Log[2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d*f + b*(b - Sqrt[b^2 - 4*a*c])*e*g + c*(Sqrt[b^2 - 4*a*c]*e*f + Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g - b*(e*f + d*g))]*Sqrt[d + e*x]*Sqrt[f + g*x] + b^2*(d*g + e*(f + 2*g*x)) - b*Sqrt[b^2 - 4*a*c]*(d*g + e*(f + 2*g*x)) + 2*c*(Sqrt[b^2 - 4*a*c]*e*f*x - 2*a*e*(f + 2*g*x) + d*(2*Sqrt[b^2 - 4*a*c]*f - 2*a*g + Sqrt[b^2 - 4*a*c]*g*x))]/Sqrt[2*c^2*d*f + b*(b - Sqrt[b^2 - 4*a*c])*e*g + c*(Sqrt[b^2 - 4*a*c]*e*f + Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g - b*(e*f + d*g))] + ((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*Log[2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c^2*d*f + b*(b + Sqrt[b^2 - 4*a*c])*e*g - c*(b*e*f + Sqrt[b^2 - 4*a*c]*e*f + b*d*g + Sqrt[b^2 - 4*a*c]*d*g + 2*a*e*g)]*Sqrt[d + e*x]*Sqrt[f + g*x] - b^2*(d*g + e*(f + 2*g*x)) - b*Sqrt[b^2 - 4*a*c]*(d*g + e*(f + 2*g*x)) + 2*c*(Sqrt[b^2 - 4*a*c]*e*f*x + 2*a*e*(f + 2*g*x) + d*(2*Sqrt[b^2 - 4*a*c]*f + 2*a*g + Sqrt[b^2 - 4*a*c]*g*x))]/Sqrt[2*c^2*d*f + b*(b + Sqrt[b^2 - 4*a*c])*e*g - c*(b*e*f + Sqrt[b^2 - 4*a*c]*e*f + b*d*g + Sqrt[b^2 - 4*a*c]*d*g + 2*a*e*g)]/(Sqrt[2]*Sqrt[b^2 - 4*a*c])`

**Maple [B]** time = 0.06, size = 5484, normalized size = 19.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{(cx^2+bx+a)\sqrt{gx+f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(e*x+d)/((c*x^2+b*x+a)*sqrt(g*x+f)),x,algorithm="maxima")`

[Out] `integrate(sqrt(e*x+d)/((c*x^2+b*x+a)*sqrt(g*x+f)),x)`

**Fricas [A]** time = 40.9607, size = 6036, normalized size = 21.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(e*x+d)/((c*x^2+b*x+a)*sqrt(g*x+f)),x,algorithm="fricas")`

[Out]  $\frac{1}{4} \sqrt{2} \sqrt{((2cd - be) f - (bd - 2ae) g + ((b^2c - 4a^2c^2) f^2 - (b^3 - 4ab^2c) f g + (ab^2 - 4a^2c^3) g^2) \sqrt{(e^2 f^2 - 2d e f g + d^2 g^2) / ((b^2c^2 - 4a^2c^3) f^4 - 2(b^3c - 4ab^2c^2) f^3 g + (b^4 - 2ab^2c - 8a^2c^2) f^2 g^2 - 2(a^2b^2 - 4a^3c) f g^3 + (a^2b^2 - 4a^3c) g^4))} / ((b^2c - 4a^2c^2) f^2 - (b^3 - 4ab^2c) f g + (ab^2 - 4a^2c^3) g^2) \log(-2bd^2 f g - 2ad^2 g^2 - 2(bde - ae^2) f^2 + \sqrt{2} ((b^2 - 4ac) e f^2 - (b^2 - 4ac) d f g + ((b^3c - 4ab^2c^2) f^3 - (b^4 - 2ab^2c - 8a^2c^2) f^2 g + 3(ab^3 - 4a^2b^2c) f g^2 - 2(a^2b^2 - 4a^3c) g^3) \sqrt{(e^2 f^2 - 2d e f g + d^2 g^2) / ((b^2c^2 - 4a^2c^3) f^4 - 2(b^3c - 4ab^2c^2) f^3 g + (b^4 - 2ab^2c - 8a^2c^2) f^2 g^2 - 2(a^2b^2 - 4a^3c) f g^3 + (a^2b^2 - 4a^3c) g^4))} \sqrt{ex+d} \sqrt{gx+f} \sqrt{((2cd - be) f - (bd - 2ae) g + ((b^2c - 4a^2c^2) f^2 - (b^3 - 4ab^2c) f g + (ab^2 - 4a^2c^3) g^2) \sqrt{(e^2 f^2 - 2d e f g + d^2 g^2) / ((b^2c^2 - 4a^2c^3) f^4 - 2(b^3c - 4ab^2c^2) f^3 g + (b^4 - 2ab^2c - 8a^2c^2) f^2 g^2 - 2(a^2b^2 - 4a^3c) f g^3 + (a^2b^2 - 4a^3c) g^4))} / ((b^2c - 4a^2c^2) f^2 - (b^3 - 4ab^2c) f g + (ab^2 - 4a^2c^3) g^2) - (be^2 f^2 - 4ae^2 f g - (bd^2 - 4ad^2 e) g^2) x - (2(b^2c - 4a^2c^2) d f^3 - 2(b^3 - 4ab^2c) d f^2 g + 2(ab^2 - 4a^2c^2) d f g^2 + ((b^2c - 4a^2c^2) e f^3 + (ab^2 - 4a^2c^2) d g^3 + ((b^2c - 4a^2c^2) d - (b^3 - 4ab^2c) e) f^2 g - ((b^3 - 4ab^2c) d - (ab^2 - 4a^2c^2) e) f g^2) x) \sqrt{(e^2 f^2 - 2d e f g + d^2 g^2) / ((b^2c^2 - 4a^2c^3) f^4 - 2(b^3c - 4ab^2c^2) f^3 g + (b^4 - 2ab^2c - 8a^2c^2) f^2 g^2 - 2(a^2b^2 - 4a^3c) f g^3 + (a^2b^2 - 4a^3c) g^4))} / x - \frac{1}{4} \sqrt{2} \sqrt{((2cd - be) f - (bd - 2ae) g + ((b^2c - 4a^2c^2) f^2 - (b^3 - 4ab^2c) f g + (ab^2 - 4a^2c^3) g^2) \sqrt{(e^2 f^2 - 2d e f g + d^2 g^2) / ((b^2c^2 - 4a^2c^3) f^4 - 2(b^3c - 4ab^2c^2) f^3 g + (b^4 - 2ab^2c - 8a^2c^2) f^2 g^2 - 2(a^2b^2 - 4a^3c) f g^3 + (a^2b^2 - 4a^3c) g^4))} / x$



$$\frac{b^3c - 4ab^2c^2)f^3g + (b^4 - 2ab^2c - 8a^2c^2)f^2g^2 - 2(ab^3 - 4a^2bc)f^3g + (a^2b^2 - 4a^3c)g^4}{x}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*(1/2)/(c\*x\*\*2+b\*x+a)/(g\*x+f)\*\*(1/2),x)

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(e\*x + d)/((c\*x^2 + b\*x + a)\*sqrt(g\*x + f)),x, algorithm="giac")

[Out] Exception raised: RuntimeError



$$3.852 \quad \int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}(a+bx+cx^2)} dx$$

**Optimal.** Leaf size=287

$$\frac{4c \tanh^{-1} \left( \frac{\sqrt{d+ex}\sqrt{2cf-g(\sqrt{b^2-4ac+b})}}{\sqrt{f+gx}\sqrt{2cd-e(\sqrt{b^2-4ac+b})}} \right)}{\sqrt{b^2-4ac}\sqrt{2cd-e(\sqrt{b^2-4ac+b})}\sqrt{2cf-g(\sqrt{b^2-4ac+b})}} - \frac{4c \tanh^{-1} \left( \frac{\sqrt{d+ex}\sqrt{2cf-g(b-\sqrt{b^2-4ac})}}{\sqrt{f+gx}\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} \right)}{\sqrt{b^2-4ac}\sqrt{2cd-e(b-\sqrt{b^2-4ac})}\sqrt{2cf-g(b-\sqrt{b^2-4ac})}}$$

[Out]  $(-4*c*ArcTanh[(Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])]*g)*Sqrt[d + e*x])/(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x]))/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]) + (4*c*ArcTanh[(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x]))/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g])$

**Rubi [A]** time = 1.05461, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$

$$\frac{4c \tanh^{-1} \left( \frac{\sqrt{d+ex}\sqrt{2cf-g(\sqrt{b^2-4ac+b})}}{\sqrt{f+gx}\sqrt{2cd-e(\sqrt{b^2-4ac+b})}} \right)}{\sqrt{b^2-4ac}\sqrt{2cd-e(\sqrt{b^2-4ac+b})}\sqrt{2cf-g(\sqrt{b^2-4ac+b})}} - \frac{4c \tanh^{-1} \left( \frac{\sqrt{d+ex}\sqrt{2cf-g(b-\sqrt{b^2-4ac})}}{\sqrt{f+gx}\sqrt{2cd-e(b-\sqrt{b^2-4ac})}} \right)}{\sqrt{b^2-4ac}\sqrt{2cd-e(b-\sqrt{b^2-4ac})}\sqrt{2cf-g(b-\sqrt{b^2-4ac})}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[d + e\*x]\*Sqrt[f + g\*x]\*(a + b\*x + c\*x^2)),x]

[Out]  $(-4*c*ArcTanh[(Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x]))/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]) + (4*c*ArcTanh[(Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g]*Sqrt[d + e*x])/(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[f + g*x]))/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*Sqrt[2*c*f - (b + Sqrt[b^2 - 4*a*c])*g])$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(e*x+d)**(1/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)`

[Out] Timed out

**Mathematica [B]** time = 6.23863, size = 1000, normalized size = 3.48

$$\frac{\sqrt{2c} \log\left(-b - 2cx + \sqrt{b^2 - 4ac}\right)}{\sqrt{b^2 - 4ac} \sqrt{egb^2 - cefb - cdgb - \sqrt{b^2 - 4ac}egb + 2c^2df + c\sqrt{b^2 - 4ac}ef + c\sqrt{b^2 - 4ac}dg - 2aceg}} \frac{\sqrt{2c} \log\left(b + 2cx + \sqrt{b^2 - 4ac}\right)}{\sqrt{b^2 - 4ac} \sqrt{egb^2 - cefb - cdgb + \sqrt{b^2 - 4ac}egb + 2c^2df - c\sqrt{b^2 - 4ac}ef - c\sqrt{b^2 - 4ac}dg - 2aceg}}$$

$$\frac{\sqrt{2c} \log\left(efb^2 + dgb^2 + 2egxb^2 - \sqrt{b^2 - 4ac}efb - \sqrt{b^2 - 4ac}dgb - 2\sqrt{b^2 - 4ac}egxb + 4c\sqrt{b^2 - 4ac}df - 4acef - 4acdgb - \sqrt{b^2 - 4ac} \sqrt{egb^2 - cefb}\right)}{\sqrt{2c} \log\left(-efb^2 - dgb^2 - 2egxb^2 - \sqrt{b^2 - 4ac}efb - \sqrt{b^2 - 4ac}dgb - 2\sqrt{b^2 - 4ac}egxb + 4c\sqrt{b^2 - 4ac}df + 4acef + 4acdgb + \sqrt{b^2 - 4ac} \sqrt{egb^2 - cefb}\right)}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]`

[Out]  $(\sqrt{2}c \operatorname{Log}[-b + \sqrt{b^2 - 4ac} - 2cx]) / (\sqrt{b^2 - 4ac} \sqrt{2c^2d^2f - b^2c^2ef + c\sqrt{b^2 - 4ac}ef - b^2cd^2g + c\sqrt{b^2 - 4ac}d^2g + b^2e^2g - 2ac^2e^2g - b\sqrt{b^2 - 4ac}e^2g}) - (\sqrt{2}c \operatorname{Log}[b + \sqrt{b^2 - 4ac} + 2cx]) / (\sqrt{b^2 - 4ac} \sqrt{2c^2d^2f - b^2c^2ef - c\sqrt{b^2 - 4ac}ef - b^2cd^2g - c\sqrt{b^2 - 4ac}d^2g + b^2e^2g - 2ac^2e^2g + b\sqrt{b^2 - 4ac}e^2g}) - (\sqrt{2}c \operatorname{Log}[4c\sqrt{b^2 - 4ac}d^2f + b^2e^2f - 4ac^2e^2f - b\sqrt{b^2 - 4ac}e^2f + b^2d^2g - 4ac^2d^2g - b\sqrt{b^2 - 4ac}d^2g + 2c\sqrt{b^2 - 4ac}e^2fx + 2c\sqrt{b^2 - 4ac}d^2gx + 2b^2e^2gx - 8ac^2e^2gx - 2b\sqrt{b^2 - 4ac}e^2gx + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{2c^2d^2f - b^2c^2ef + c\sqrt{b^2 - 4ac}ef - b^2cd^2g + c\sqrt{b^2 - 4ac}d^2g + b^2e^2g - 2ac^2e^2g - b\sqrt{b^2 - 4ac}e^2g]\sqrt{d + e*x}]\sqrt{f + g*x}) / (\sqrt{b^2 - 4ac} \sqrt{2c^2d^2f - b^2c^2ef + c\sqrt{b^2 - 4ac}ef - b^2cd^2g + c\sqrt{b^2 - 4ac}d^2g + b^2e^2g - 2ac^2e^2g - b\sqrt{b^2 - 4ac}e^2g}) + (\sqrt{2}c \operatorname{Log}[4c\sqrt{b^2 - 4ac}d^2f - b^2e^2f + 4ac^2e^2f - b\sqrt{b^2 - 4ac}e^2f - b^2d^2g + 4ac^2d^2g - b\sqrt{b^2 - 4ac}d^2g + 2c\sqrt{b^2 - 4ac}e^2fx + 2c\sqrt{b^2 - 4ac}d^2gx - 2b^2e^2gx + 8ac^2e^2gx - 2b\sqrt{b^2 - 4ac}e^2gx + 2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{2c^2d^2f - b^2c^2ef - c\sqrt{b^2 - 4ac}ef - b^2cd^2g - c\sqrt{b^2 - 4ac}d^2g + b^2e^2g - 2ac^2e^2g + b\sqrt{b^2 - 4ac}e^2g]\sqrt{d + e*x}]\sqrt{f + g*x}) / (\sqrt{b^2 - 4ac} \sqrt{2c^2d^2f - b^2c^2ef - c\sqrt{b^2 - 4ac}ef - b^2cd^2g - c\sqrt{b^2 - 4ac}d^2g + b^2e^2g - 2ac^2e^2g + b\sqrt{b^2 - 4ac}e^2g})$

**Maple [B]** time = 0.072, size = 5507, normalized size = 19.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x+d)^(1/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx + a)\sqrt{ex + d}\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + b\*x + a)\*sqrt(e\*x + d)\*sqrt(g\*x + f)),x, algorithm="maxima")

[Out] integrate(1/((c\*x^2 + b\*x + a)\*sqrt(e\*x + d)\*sqrt(g\*x + f)), x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + b\*x + a)\*sqrt(e\*x + d)\*sqrt(g\*x + f)),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d + ex}\sqrt{f + gx}(a + bx + cx^2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x+d)\*\*(1/2)/(c\*x\*\*2+b\*x+a)/(g\*x+f)\*\*(1/2),x)

[Out] Integral(1/(sqrt(d + e\*x)\*sqrt(f + g\*x)\*(a + b\*x + c\*x\*\*2)), x)

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + b\*x + a)\*sqrt(e\*x + d)\*sqrt(g\*x + f)),x, algorithm="giac")

[Out] Exception raised: RuntimeError

$$3.853 \quad \int \frac{1}{(d+ex)^{3/2} \sqrt{f+gx} (a+bx+cx^2)} dx$$

**Optimal.** Leaf size=429

$$\frac{8c^2 \tanh^{-1} \left( \frac{\sqrt{d+ex} \sqrt{2cf-g(b-\sqrt{b^2-4ac})}}{\sqrt{f+gx} \sqrt{2cd-e(b-\sqrt{b^2-4ac})}} \right)}{\sqrt{b^2-4ac} \left( 2cd-e(b-\sqrt{b^2-4ac}) \right)^{3/2} \sqrt{2cf-g(b-\sqrt{b^2-4ac})}} + \frac{8c^2 \tanh^{-1} \left( \frac{\sqrt{d+ex} \sqrt{2cf-g(\sqrt{b^2-4ac}+b)}}{\sqrt{f+gx} \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} \right)}{\sqrt{b^2-4ac} \left( 2cd-e(\sqrt{b^2-4ac}+b) \right)^{3/2} \sqrt{2cf-g(\sqrt{b^2-4ac}+b)}} + \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\sqrt{d+ex}(ef-dg) \left( 2cd-e(b-\sqrt{b^2-4ac}) \right)} - \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\sqrt{d+ex}(ef-dg) \left( 2cd-e(\sqrt{b^2-4ac}+b) \right)}$$

[Out]  $(4*c*e*\text{Sqrt}[f + g*x]) / (\text{Sqrt}[b^2 - 4*a*c] * (2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c]) * e) * (e*f - d*g) * \text{Sqrt}[d + e*x]) - (4*c*e*\text{Sqrt}[f + g*x]) / (\text{Sqrt}[b^2 - 4*a*c] * (2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]) * e) * (e*f - d*g) * \text{Sqrt}[d + e*x]) - (8*c^2*\text{ArcTanh}[(\text{Sqrt}[2*c*f - (b - \text{Sqrt}[b^2 - 4*a*c]) * g] * \text{Sqrt}[d + e*x]) / (\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c]) * e] * \text{Sqrt}[f + g*x])]) / (\text{Sqrt}[b^2 - 4*a*c] * (2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c]) * e)^(3/2) * \text{Sqrt}[2*c*f - (b - \text{Sqrt}[b^2 - 4*a*c]) * g]) + (8*c^2*\text{ArcTanh}[(\text{Sqrt}[2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c]) * g] * \text{Sqrt}[d + e*x]) / (\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]) * e] * \text{Sqrt}[f + g*x])]) / (\text{Sqrt}[b^2 - 4*a*c] * (2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]) * e)^(3/2) * \text{Sqrt}[2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c]) * g])$

**Rubi [A]** time = 3.35416, antiderivative size = 429, normalized size of antiderivative = 1., number of rules used = 8, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$

$$\frac{8c^2 \tanh^{-1} \left( \frac{\sqrt{d+ex} \sqrt{2cf-g(b-\sqrt{b^2-4ac})}}{\sqrt{f+gx} \sqrt{2cd-e(b-\sqrt{b^2-4ac})}} \right)}{\sqrt{b^2-4ac} \left( 2cd-e(b-\sqrt{b^2-4ac}) \right)^{3/2} \sqrt{2cf-g(b-\sqrt{b^2-4ac})}} + \frac{8c^2 \tanh^{-1} \left( \frac{\sqrt{d+ex} \sqrt{2cf-g(\sqrt{b^2-4ac}+b)}}{\sqrt{f+gx} \sqrt{2cd-e(\sqrt{b^2-4ac}+b)}} \right)}{\sqrt{b^2-4ac} \left( 2cd-e(\sqrt{b^2-4ac}+b) \right)^{3/2} \sqrt{2cf-g(\sqrt{b^2-4ac}+b)}} + \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\sqrt{d+ex}(ef-dg) \left( 2cd-e(b-\sqrt{b^2-4ac}) \right)} + \frac{4ce\sqrt{f+gx}}{\sqrt{b^2-4ac}\sqrt{d+ex}(ef-dg) \left( 2cd-e(\sqrt{b^2-4ac}+b) \right)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)^(3/2)\*Sqrt[f + g\*x]\*(a + b\*x + c\*x^2)),x]

[Out]  $(4*c*e*\text{Sqrt}[f + g*x]) / (\text{Sqrt}[b^2 - 4*a*c] * (2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c]) * e) * (e*f - d*g) * \text{Sqrt}[d + e*x]) - (4*c*e*\text{Sqrt}[f + g*x]) / (\text{Sqrt}[b^2 - 4*a*c] * (2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]) * e) * (e*f - d*g) * \text{Sqrt}[d + e*x]) - (8*c^2*\text{ArcTanh}[(\text{Sqrt}[2*c*f - (b - \text{Sqrt}[b^2 - 4*a*c]) * g] * \text{Sqrt}[d + e*x]) / (\text{Sqrt}[2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c]) * e] * \text{Sqrt}[f + g*x])]) / (\text{Sqrt}[b^2 - 4*a*c] * (2*c*d - (b - \text{Sqrt}[b^2 - 4*a*c]) * e)^(3/2) * \text{Sqrt}[2*c*f - (b - \text{Sqrt}[b^2 - 4*a*c]) * g]) + (8*c^2*\text{ArcTanh}[(\text{Sqrt}[2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c]) * g] * \text{Sqrt}[d + e*x]) / (\text{Sqrt}[2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]) * e] * \text{Sqrt}[f + g*x])]) / (\text{Sqrt}[b^2 - 4*a*c] * (2*c*d - (b + \text{Sqrt}[b^2 - 4*a*c]) * e)^(3/2) * \text{Sqrt}[2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c]) * g])$

$$\begin{aligned} & \sqrt{b^2 - 4ac} (2cd - (b + \sqrt{b^2 - 4ac})e) (ef - dg) \sqrt{d + ex} \\ & - (8c^2 \operatorname{ArcTanh}[\frac{\sqrt{2cf - (b - \sqrt{b^2 - 4ac})}g}{\sqrt{2cd - (b - \sqrt{b^2 - 4ac})}e} \sqrt{f + gx}]) / (\sqrt{b^2 - 4ac} (2cd - (b - \sqrt{b^2 - 4ac})e)^{3/2} \sqrt{2cf - (b - \sqrt{b^2 - 4ac})}g) \\ & + (8c^2 \operatorname{ArcTanh}[\frac{\sqrt{2cf - (b + \sqrt{b^2 - 4ac})}g}{\sqrt{2cd - (b + \sqrt{b^2 - 4ac})}e} \sqrt{f + gx}]) / (\sqrt{b^2 - 4ac} (2cd - (b + \sqrt{b^2 - 4ac})e)^{3/2} \sqrt{2cf - (b + \sqrt{b^2 - 4ac})}g) \end{aligned}$$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(e*x+d)**(3/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)`

[Out] Timed out

**Mathematica [B]** time = 2.84465, size = 1011, normalized size = 2.36

$$\begin{aligned} & \frac{2\sqrt{f+gxe^2}}{(cd^2 + e(ae - bd))(ef - dg)\sqrt{d+ex}} \\ & + \frac{c \left( (b + \sqrt{b^2 - 4ac})e - 2cd \right) \log \left( -b - 2cx + \sqrt{b^2 - 4ac} \right)}{\sqrt{2}\sqrt{b^2 - 4ac}(e(bd - ae) - cd^2) \sqrt{2dfc^2 + (\sqrt{b^2 - 4ac}ef + \sqrt{b^2 - 4ac}dg - 2aeg - b(ef + dg))} c + b (b - \sqrt{b^2 - 4ac}) eg} \\ & + \frac{c \left( 2cd + (\sqrt{b^2 - 4ac} - b) e \right) \log \left( b + 2cx + \sqrt{b^2 - 4ac} \right)}{\sqrt{2}\sqrt{b^2 - 4ac}(e(bd - ae) - cd^2) \sqrt{2dfc^2 - (bef + \sqrt{b^2 - 4ac}ef + bdg + \sqrt{b^2 - 4ac}dg + 2aeg)} c + b (b + \sqrt{b^2 - 4ac}) eg} \\ & + \frac{c \left( 2cd + (\sqrt{b^2 - 4ac} - b) e \right) \log \left( bef + \sqrt{b^2 - 4ac}ef + bdg + \sqrt{b^2 - 4ac}dg + 2begx + 2\sqrt{b^2 - 4ac}egx - 2c(2df + exf + c) \right)}{\sqrt{2}\sqrt{b^2 - 4ac}(e(bd - ae) - cd^2) \sqrt{2dfc^2 - (bef + \sqrt{b^2 - 4ac}ef + bdg + \sqrt{b^2 - 4ac}dg + 2aeg)} c + b (b + \sqrt{b^2 - 4ac}) eg} \\ & + \frac{c \left( (b + \sqrt{b^2 - 4ac})e - 2cd \right) \log \left( -bef + \sqrt{b^2 - 4ac}ef - bdg + \sqrt{b^2 - 4ac}dg - 2begx + 2\sqrt{b^2 - 4ac}egx + 2c(2df + exf + c) \right)}{\sqrt{2}\sqrt{b^2 - 4ac}(e(bd - ae) - cd^2) \sqrt{2dfc^2 + (\sqrt{b^2 - 4ac}ef + \sqrt{b^2 - 4ac}dg - 2aeg - b(ef + dg))} c + b (b - \sqrt{b^2 - 4ac}) eg} \end{aligned}$$

Antiderivative was successfully verified.

[In] `Integrate[1/((d + e*x)^(3/2)*Sqrt[f + g*x]*(a + b*x + c*x^2)),x]`

[Out] 
$$\begin{aligned} & (-2e^2 \sqrt{f + gx}) / ((c^2 d^2 + e^2 (-(b^2 d) + a^2 e)) (ef - dg) \sqrt{d + ex}) \\ & + (c^2 (-2cd + (b + \sqrt{b^2 - 4ac})e) \operatorname{Log}[-b + \sqrt{b^2 - 4ac} - 2cx]) / (\sqrt{2} \sqrt{b^2 - 4ac} (-(c^2 d^2) + e^2 (b^2 d - a^2 e)) \sqrt{2c^2 d^2 f + b^2 (b - \sqrt{b^2 - 4ac})e^2 g + c^2 (\sqrt{b^2 - 4ac}ef + \sqrt{b^2 - 4ac}dg - 2aeg - b(ef + dg))}) \\ & + (c^2 (2cd + (-b + \sqrt{b^2 - 4ac})e) \operatorname{Log}[b + \sqrt{b^2 - 4ac} + 2cx]) / (\sqrt{2} \sqrt{b^2 - 4ac} (-(c^2 d^2) + e^2 (b^2 d - a^2 e)) \sqrt{2c^2 d^2 f + b^2 (b + \sqrt{b^2 - 4ac})e^2 g - c^2 (b^2 ef + \sqrt{b^2 - 4ac}ef + b^2 dg + \sqrt{b^2 - 4ac}dg + 2aeg)}) \\ & - (c^2 (2cd + (-b + \sqrt{b^2 - 4ac})e) \operatorname{Log}[b^2 ef + \sqrt{b^2 - 4ac}ef + b^2 dg + \sqrt{b^2 - 4ac}dg + 2b^2 egx + 2\sqrt{b^2 - 4ac}egx - 2c(2df + exf + c)]) \sqrt{d + ex} \sqrt{f + gx} - 2 \end{aligned}$$

$$*c*(2*d*f + e*f*x + d*g*x)]/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*(-(c*d^2) + e*(b*d - a*e))*Sqrt[2*c^2*d*f + b*(b + Sqrt[b^2 - 4*a*c])*e*g - c*(b*e*f + Sqrt[b^2 - 4*a*c]*e*f + b*d*g + Sqrt[b^2 - 4*a*c]*d*g + 2*a*e*g)]) - (c*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*Log[-(b*e*f) + Sqrt[b^2 - 4*a*c]*e*f - b*d*g + Sqrt[b^2 - 4*a*c]*d*g - 2*b*e*g*x + 2*Sqrt[b^2 - 4*a*c]*e*g*x + 2*Sqrt[2]*Sqrt[2*c^2*d*f + b*(b - Sqrt[b^2 - 4*a*c])*e*g + c*(Sqrt[b^2 - 4*a*c]*e*f + Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g - b*(e*f + d*g))])*Sqrt[d + e*x]*Sqrt[f + g*x] + 2*c*(2*d*f + e*f*x + d*g*x)]/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*(-(c*d^2) + e*(b*d - a*e))*Sqrt[2*c^2*d*f + b*(b - Sqrt[b^2 - 4*a*c])*e*g + c*(Sqrt[b^2 - 4*a*c]*e*f + Sqrt[b^2 - 4*a*c]*d*g - 2*a*e*g - b*(e*f + d*g)))]$$

**Maple [B]** time = 0.266, size = 47349, normalized size = 110.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x+d)^(3/2)/(c*x^2+b*x+a)/(g*x+f)^(1/2),x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx + a)(ex + d)^{\frac{3}{2}}\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((c*x^2 + b*x + a)*(e*x + d)^(3/2)*sqrt(g*x + f)),x, algorithm="maxima")`

[Out] `integrate(1/((c*x^2 + b*x + a)*(e*x + d)^(3/2)*sqrt(g*x + f)), x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((c*x^2 + b*x + a)*(e*x + d)^(3/2)*sqrt(g*x + f)),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x+d)**(3/2)/(c*x**2+b*x+a)/(g*x+f)**(1/2),x)`

[Out] Timed out

---

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((c*x^2 + b*x + a)*(e*x + d)^(3/2)*sqrt(g*x + f)),x, algorithm="giac"`

[Out] Timed out

$$3.854 \quad \int \frac{(f+gx)^3 \sqrt{a+bx+cx^2}}{d+ex} dx$$

**Optimal.** Leaf size=532

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) \left(4ce(2cd-be)(-4cdg^2(aeg-2bdg+6bef)+5b^2deg^3+16c^2e^2f^3) - 2g\left(-2ce(bd-ae) - \frac{b^2e^2}{2} + \frac{128c^{7/2}e^5}{64c^3e^4}\right)\right)}{\sqrt{a+bx+cx^2} (2ceg(-4ceg(aeg-2bdg+6bef)+5b^2e^2g^2+16c^2(d^2g^2-3defg+3e^2f^2)) - 4bce^2g^2(aeg-2bdg+6bef))} + \frac{g^2(a+bx+cx^2)^{3/2}(-5beg-14cdg+24cef)}{24c^2e^2} + \frac{(ef-dg)^3\sqrt{ae^2-bde+cd^2} \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^5} + \frac{g^3(d+ex)(a+bx+cx^2)^{3/2}}{4ce^2}$$

[Out]  $((5*b^3*e^3*g^3 + 64*c^3*(e*f - d*g)^3 - 4*b*c*e^2*g^2*(6*b*e*f - 2*b*d*g + a*e*g) + 16*b*c^2*e*g*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2) + 2*c*e*g*(5*b^2*e^2*g^2 - 4*c*e*g*(6*b*e*f - 2*b*d*g + a*e*g) + 16*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*x)*\text{Sqrt}[a + b*x + c*x^2]) / (64*c^3*e^4) + (g^2*(24*c*e*f - 14*c*d*g - 5*b*e*g)*(a + b*x + c*x^2)^{(3/2)}) / (24*c^2*e^2) + (g^3*(d + e*x)*(a + b*x + c*x^2)^{(3/2)}) / (4*c*e^2) - ((4*c*e*(2*c*d - b*e)*(16*c^2*e^2*f^3 + 5*b^2*d*e*g^3 - 4*c*d*g^2*(6*b*e*f - 2*b*d*g + a*e*g)) - 2*(4*c^2*d^2 - (b^2*e^2)/2 - 2*c*e*(b*d - a*e))*g*(5*b^2*e^2*g^2 - 4*c*e*g*(6*b*e*f - 2*b*d*g + a*e*g) + 16*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])]) / (128*c^{(7/2)}*e^5) + (\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*(e*f - d*g)^3*\text{ArcTanh}[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*\text{Sqrt}[a + b*x + c*x^2])]) / e^5$

**Rubi [A]** time = 3.34148, antiderivative size = 532, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) \left(4ce(2cd-be)(-4cdg^2(aeg-2bdg+6bef)+5b^2deg^3+16c^2e^2f^3) - 2g\left(-2ce(bd-ae) - \frac{b^2e^2}{2} + \frac{128c^{7/2}e^5}{64c^3e^4}\right)\right)}{\sqrt{a+bx+cx^2} (2ceg(-4ceg(aeg-2bdg+6bef)+5b^2e^2g^2+16c^2(d^2g^2-3defg+3e^2f^2)) - 4bce^2g^2(aeg-2bdg+6bef))} + \frac{g^2(a+bx+cx^2)^{3/2}(-5beg-14cdg+24cef)}{24c^2e^2} + \frac{(ef-dg)^3\sqrt{ae^2-bde+cd^2} \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^5} + \frac{g^3(d+ex)(a+bx+cx^2)^{3/2}}{4ce^2}$$

Antiderivative was successfully verified.

[In] Int[((f + g\*x)^3\*Sqrt[a + b\*x + c\*x^2])/(d + e\*x), x]

[Out]  $((5*b^3*e^3*g^3 + 64*c^3*(e*f - d*g)^3 - 4*b*c*e^2*g^2*(6*b*e*f - 2*b*d*g + a*e*g) + 16*b*c^2*e*g*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2) + 2*c*e*g*(5*b^2*e^2*g^2 - 4*c*e*g*(6*b*e*f - 2*b*d*g + a*e*g) + 16*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*x)*\text{Sqrt}[a + b*x + c*x^2]) / (64*c^3*e^4) + (g^2*(24*c*e*f - 14*c*d*g - 5*b*e*g)*(a + b*x + c*x^2)^{(3/2)}) / (24*c^2*e^2) + (g^3*(d + e*x)*(a + b*x + c*x^2)^{(3/2)}) / (4*c*e^2) - ((4*c*e*(2*c*d - b*e)*(16*c^2*e^2*f^3 + 5*b^2*d*e*g^3 - 4*c*d*g^2*(6*b*e*f - 2*b*d*g + a*e*g)) - 2*(4*c^2*d^2 - (b^2*e^2)/2 - 2*c*e*(b*d - a*e))*g*(5*b^2*e^2*g^2 - 4*c*e*g*(6*b*e*f - 2*b*d*g + a*e*g) + 16*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])]) / (128*c^{(7/2)}*e^5) + (\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*(e*f - d*g)^3*\text{ArcTanh}[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*\text{Sqrt}[a + b*x + c*x^2])]) / e^5$



**Rubi in Sympy [A]** time = 153.488, size = 578, normalized size = 1.09

$$\begin{aligned}
 & -\frac{5bg^3(a+bx+cx^2)^{\frac{3}{2}}}{24c^2e} + \frac{bg^2(b+2cx)(dg-3ef)\sqrt{a+bx+cx^2}}{8c^2e^2} \\
 & -\frac{bg^2(-4ac+b^2)(dg-3ef)\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{\frac{5}{2}}e^2} - \frac{(dg-ef)^3\sqrt{a+bx+cx^2}}{e^4} \\
 & + \frac{(dg-ef)^3\sqrt{ae^2-bde+cd^2}\operatorname{atanh}\left(\frac{2ae-bd+x(be-2cd)}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^5} + \frac{g^3x(a+bx+cx^2)^{\frac{3}{2}}}{4ce} \\
 & - \frac{g^2(dg-3ef)(a+bx+cx^2)^{\frac{3}{2}}}{3ce^2} + \frac{g(b+2cx)\sqrt{a+bx+cx^2}(d^2g^2-3defg+3e^2f^2)}{4ce^3} \\
 & + \frac{g^3(b+2cx)(-4ac+5b^2)\sqrt{a+bx+cx^2}}{64c^3e} - \frac{(be-2cd)(dg-ef)^3\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}e^5} \\
 & - \frac{g(-4ac+b^2)(d^2g^2-3defg+3e^2f^2)\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{\frac{3}{2}}e^3} \\
 & - \frac{g^3(-4ac+b^2)(-4ac+5b^2)\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{128c^{\frac{7}{2}}e}
 \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x+f)**3*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)`

[Out] 
$$\begin{aligned}
 & -5*b*g**3*(a+b*x+c*x**2)**(3/2)/(24*c**2*e) + b*g**2*(b+2*c*x) \\
 & *(d*g-3*e*f)*\operatorname{sqrt}(a+b*x+c*x**2)/(8*c**2*e**2) - b*g**2*(-4*a*c+b**2) \\
 & *(d*g-3*e*f)*\operatorname{atanh}((b+2*c*x)/(2*\operatorname{sqrt}(c)*\operatorname{sqrt}(a+b*x+c*x**2))) \\
 & /(16*c**(5/2)*e**2) - (d*g-e*f)**3*\operatorname{sqrt}(a+b*x+c*x**2)/e**4 \\
 & + (d*g-e*f)**3*\operatorname{sqrt}(a*e**2-b*d*e+c*d**2)*\operatorname{atanh}((2*a*e-b*d+x*(b*e-2*c*d))/(2*\operatorname{sqrt}(a+b*x+c*x**2)*\operatorname{sqrt}(a*e**2-b*d*e+c*d**2)))/e**5 \\
 & + g**3*x*(a+b*x+c*x**2)**(3/2)/(4*c*e) - g**2*(d*g-3*e*f)*(a+b*x+c*x**2)**(3/2)/(3*c*e**2) \\
 & + g*(b+2*c*x)*\operatorname{sqrt}(a+b*x+c*x**2)*(d**2*g**2-3*d*e*f*g+3*e**2*f**2)/(4*c*e**3) \\
 & + g**3*(b+2*c*x)*(-4*a*c+5*b**2)*\operatorname{sqrt}(a+b*x+c*x**2)/(64*c**3*e) - (b*e-2*c*d)*(d*g-e*f)**3 \\
 & *\operatorname{atanh}((b+2*c*x)/(2*\operatorname{sqrt}(c)*\operatorname{sqrt}(a+b*x+c*x**2)))/(2*\operatorname{sqrt}(c)*e**5) \\
 & - g*(-4*a*c+b**2)*(d**2*g**2-3*d*e*f*g+3*e**2*f**2)*\operatorname{atanh}((b+2*c*x)/(2*\operatorname{sqrt}(c)*\operatorname{sqrt}(a+b*x+c*x**2)))/(8*c**(3/2)*e**3) \\
 & - g**3*(-4*a*c+b**2)*(-4*a*c+5*b**2)*\operatorname{atanh}((b+2*c*x)/(2*\operatorname{sqrt}(c)*\operatorname{sqrt}(a+b*x+c*x**2)))/(128*c**(7/2)*e)
 \end{aligned}$$

**Mathematica [A]** time = 1.92324, size = 553, normalized size = 1.04

$$\frac{3\log\left(2\sqrt{c}\sqrt{a+x(b+cx)+b+2cx}\right)(-16c^2e^2g(a^2e^2g^2+2abeg(3ef-dg)+b^2(d^2g^2-3defg+3e^2f^2))+8b^2ce^3g^2(3aeg-bdg+3bef)+64c^3e(aeg(d^2g^2-3defg+3e^2f^2))}{c^{7/2}}$$

Antiderivative was successfully verified.

[In] `Integrate[((f+g*x)^3*Sqrt[a+b*x+c*x^2])/(d+e*x),x]`

[Out] 
$$\begin{aligned}
 & ((2*e*\operatorname{Sqrt}[a+x*(b+c*x)])*(15*b^3*e^3*g^3-2*b*c*e^2*g^2*(26*a \\
 & *e*g+b*(36*e*f-12*d*g+5*e*g*x))+16*c^3*(-12*d^3*g^3+6*d \\
 & ^2*e*g^2*(6*f+g*x)-2*d*e^2*g*(18*f^2+9*f*g*x+2*g^2*x^2)+ \\
 & 3*e^3*(4*f^3+6*f^2*g*x+4*f*g^2*x^2+g^3*x^3))+8*c^2*e*g*( \\
 & a*e*g*(-8*d*g+3*e*(8*f+g*x))+b*(6*d^2*g^2-2*d*e*g*(9*f+ \\
 & g*x)+e^2*(18*f^2+6*f*g*x+g^2*x^2))))/c^3+384*\operatorname{Sqrt}[c*d^2 \\
 & +e*(-b*d)+a*e]*(e*f-d*g)^3*\operatorname{Log}[d+e*x]+(3*(-5*b^4*e^4*g \\
 & ^3+128*c^4*d*(-e*f)+d*g)^3+8*b^2*c*e^3*g^2*(3*b*e*f-b*d*
 \end{aligned}$$

$$g + 3*a*e*g) - 16*c^2*e^2*g*(a^2*e^2*g^2 + 2*a*b*e*g*(3*e*f - d*g) + b^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)) + 64*c^3*e*(b*(e*f - d*g)^3 + a*e*g*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]]/c^(7/2) + 384*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*(-(e*f) + d*g)^3*Log[-(b*d) + 2*a*e - 2*c*d*x + b*e*x + 2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)]]/(384*e^5)$$

**Maple [B]** time = 0.048, size = 3941, normalized size = 7.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((g*x+f)^3*(c*x^2+b*x+a)^{(1/2)}/(e*x+d), x)$

[Out] 
$$\begin{aligned} & -1/e^4*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*d^3*g^3+3/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*\ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x+d/e))*b*d^3*f*g^2-3/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*\ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x+d/e))*b*d^2*f^2*g-3/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*\ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x+d/e))*c*d^4*f*g^2+3/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*\ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x+d/e))*c*d^3*f^2*g+1/4*g^3/e^2*b/c^(3/2)*\ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*d-3/4*g^2/e*b/c^(3/2)*\ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a*f+3/2/e^3*\ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+(x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/c^(1/2))*b*d^2*f*g^2-3/2/e^2*\ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+(x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/c^(1/2))*b*d*f^2*g-3/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*\ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x+d/e))*a*d^2*f*g^2+3/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*\ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x+d/e))*a*d*f^2*g-3/4*g^2/e^2*d*f/c*(c*x^2+b*x+a)^(1/2))*b-3/2*g^2/e^2*d*f/c^(1/2)*\ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*a+3/8*g^2/e^2*d*f/c^(3/2)*\ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*b^2+1/4*g^3/e^2*b/c*(c*x^2+b*x+a)^(1/2))*x*d-3/4*g^2/e*b/c*(c*x^2+b*x+a)^(1/2))*x*f+g^2/e*(c*x^2+b*x+a)^(3/2)/c*f-5/128*g^3/e*b^4/c^(7/2)*\ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+1/2/e*\ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+(x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/c^(1/2))*b*f^3+1/e^5*\ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+(x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/c^(1/2))*d^4*g^3-1/e^2*\ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+(x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*\ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x+d/e))*a*f^3-1/8*g^3/e/c^(3/2))*a^2*\ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))+3/2*g/e*f^2*(c*x^2+b*x+a)^(1/2))*x+1/2*g^3/e^3*d^2*(c*x^2+b*x+a)^(1/2))*x-1/3*g^3/e^2*(c*x^2+b*x+a)^(3/2)/c*d+3/e^3*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*d^2*f*g^2-3/e^2*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*d*f^2*g+1/4*g^3/e*x*(c*x^2+b*x+a)^(3/2)/c-5/24*g^3/e*b/c^2*(c*x^2+b*x+a)^(3/2))+5/64*g^3/e*b^3/c^3*(c*x^2+b*x+a)^(1/2))-1/8*g^3/e/c*a*(c*x^2+b*x+a)^(1/2))*x+3/16*g^2/e*b^3/c^(5/2)*\ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f-1/16*g^3/e^2*b^3/c^(5/2)*\ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x+a)^(1/2))*f$$

$$\begin{aligned} & (x+a)^{1/2}) * d + 1/8 * g^3 / e^2 * b^2 / c^2 * (c * x^2 + b * x + a)^{1/2} * d - 3/8 * g^2 / e \\ & * b^2 / c^2 * (c * x^2 + b * x + a)^{1/2} * f + 3/16 * g^3 / e * b^2 / c^{5/2} * \ln((1/2 * b + c \\ & * x) / c^{1/2} + (c * x^2 + b * x + a)^{1/2}) * a + 5/32 * g^3 / e * b^2 / c^2 * (c * x^2 + b * x + \\ & a)^{1/2} * x + 1/4 * g^3 / e^3 * d^2 / c * (c * x^2 + b * x + a)^{1/2} * b + 1/2 * g^3 / e^3 * d^2 \\ & / c^{1/2} * \ln((1/2 * b + c * x) / c^{1/2} + (c * x^2 + b * x + a)^{1/2}) * a - 1/8 * g^3 / e \\ & ^3 * d^2 / c^{3/2} * \ln((1/2 * b + c * x) / c^{1/2} + (c * x^2 + b * x + a)^{1/2}) * b^2 + 3/ \\ & 4 * g / e * f^2 / c * (c * x^2 + b * x + a)^{1/2} * b + 3/2 * g / e * f^2 / c^{1/2} * \ln((1/2 * b + c \\ & * x) / c^{1/2} + (c * x^2 + b * x + a)^{1/2}) * a + 1 / e * ((x + d / e)^2 * c + (b * e - 2 * c * d) / e \\ & * (x + d / e) + (a * e^2 - b * d * e + c * d^2) / e^2)^{1/2} * f^3 - 3/8 * g / e * f^2 / c^{3/2} * \ln \\ & ((1/2 * b + c * x) / c^{1/2} + (c * x^2 + b * x + a)^{1/2}) * b^2 - 1/16 * g^3 / e / c^2 * a * ( \\ & c * x^2 + b * x + a)^{1/2} * b - 3/2 * g^2 / e^2 * d * f * (c * x^2 + b * x + a)^{1/2} * x - 1/2 / e^4 \\ & * \ln((1/2 * (b * e - 2 * c * d) / e + c * (x + d / e)) / c^{1/2} + ((x + d / e)^2 * c + (b * e - 2 * c * \\ & d) / e * (x + d / e) + (a * e^2 - b * d * e + c * d^2) / e^2)^{1/2}) / c^{1/2} * b * d^3 * g^3 - 3/ \\ & e^4 * \ln((1/2 * (b * e - 2 * c * d) / e + c * (x + d / e)) / c^{1/2} + ((x + d / e)^2 * c + (b * e - 2 * \\ & c * d) / e * (x + d / e) + (a * e^2 - b * d * e + c * d^2) / e^2)^{1/2}) * c^{1/2} * d^3 * f * g^2 + \\ & 3 / e^3 * \ln((1/2 * (b * e - 2 * c * d) / e + c * (x + d / e)) / c^{1/2} + ((x + d / e)^2 * c + (b * e - \\ & 2 * c * d) / e * (x + d / e) + (a * e^2 - b * d * e + c * d^2) / e^2)^{1/2}) * c^{1/2} * d^2 * f^2 * \\ & g + 1 / e^4 / ((a * e^2 - b * d * e + c * d^2) / e^2)^{1/2} * \ln((2 * (a * e^2 - b * d * e + c * d^2) \\ & / e^2 + (b * e - 2 * c * d) / e * (x + d / e) + 2 * ((a * e^2 - b * d * e + c * d^2) / e^2)^{1/2}) * ((x + \\ & d / e)^2 * c + (b * e - 2 * c * d) / e * (x + d / e) + (a * e^2 - b * d * e + c * d^2) / e^2)^{1/2}) / (x \\ & + d / e) * a * d^3 * g^3 - 1 / e^5 / ((a * e^2 - b * d * e + c * d^2) / e^2)^{1/2} * \ln((2 * (a * e \\ & ^2 - b * d * e + c * d^2) / e^2 + (b * e - 2 * c * d) / e * (x + d / e) + 2 * ((a * e^2 - b * d * e + c * d^2) / \\ & e^2)^{1/2}) * ((x + d / e)^2 * c + (b * e - 2 * c * d) / e * (x + d / e) + (a * e^2 - b * d * e + c * d^2) / \\ & e^2)^{1/2}) / (x + d / e) * b * d^4 * g^3 + 1 / e^2 / ((a * e^2 - b * d * e + c * d^2) / e^2)^{1/2} * \ln((2 * (a * e \\ & ^2 - b * d * e + c * d^2) / e^2 + (b * e - 2 * c * d) / e * (x + d / e) + 2 * ((a * e^2 - b * d * e + c * d^2) / \\ & e^2)^{1/2}) * ((x + d / e)^2 * c + (b * e - 2 * c * d) / e * (x + d / e) + (a * e^2 - b * d * e + c * d^2) / \\ & e^2)^{1/2}) / (x + d / e) * b * d * f^3 + 1 / e^6 / ((a * e^2 - b * d * e + c * d^2) / e^2)^{1/2} * \ln((2 * (a * e^2 - b * d * e + c * d^2) / \\ & e^2 + (b * e - 2 * c * d) / e * (x + d / e) + 2 * ((a * e^2 - b * d * e + c * d^2) / e^2)^{1/2}) * ((x + d / e)^2 * c + (b * e - 2 * c * d) / e * \\ & (x + d / e) + (a * e^2 - b * d * e + c * d^2) / e^2)^{1/2}) / (x + d / e) * c * d^5 * g^3 - 1 / e^3 \\ & / ((a * e^2 - b * d * e + c * d^2) / e^2)^{1/2} * \ln((2 * (a * e^2 - b * d * e + c * d^2) / e^2 + (b * \\ & e - 2 * c * d) / e * (x + d / e) + 2 * ((a * e^2 - b * d * e + c * d^2) / e^2)^{1/2}) * ((x + d / e)^2 * \\ & c + (b * e - 2 * c * d) / e * (x + d / e) + (a * e^2 - b * d * e + c * d^2) / e^2)^{1/2}) / (x + d / e) * \\ & c * d^2 * f^3 \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)\*(g\*x + f)^3/(e\*x + d), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)\*(g\*x + f)^3/(e\*x + d), x, algorithm="fricas")

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(f + gx)^3 \sqrt{a + bx + cx^2}}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)
```

```
[Out] Integral((f + g*x)**3*sqrt(a + b*x + c*x**2)/(d + e*x), x)
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)^3/(e*x + d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.855 \quad \int \frac{(f+gx)^2 \sqrt{a+bx+cx^2}}{d+ex} dx$$

**Optimal.** Leaf size=325

$$\frac{\sqrt{a+bx+cx^2} (b^2 e^2 g^2 - 2ceg x(-beg - 2cdg + 4cef) - 2bceg(2ef - dg) - 8c^2(ef - dg)^2)}{8c^2 e^3} + \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (g(-4ce(bd - ae) - b^2 e^2 + 8c^2 d^2) (-beg - 2cdg + 4cef) - 4ce(2cd - be)(2cef^2 - bdg^2))}{16c^{5/2} e^4} + \frac{(ef - dg)^2 \sqrt{ae^2 - bde + cd^2} \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^4} + \frac{g^2 (a + bx + cx^2)^{3/2}}{3ce}$$

[Out]  $-\left((b^2 e^2 g^2 - 8c^2 (ef - dg)^2 - 2bceg(2ef - dg) - 2c^2 e^2 g^2 (4c^2 e^2 f - 2c^2 d^2 g - b^2 e^2 g) x) \sqrt{a + bx + cx^2}\right) / (8c^2 e^3) + (g^2 (a + bx + cx^2)^{3/2}) / (3c^2 e) + \left(\left(8c^2 d^2 - b^2 e^2 - 4c^2 e (bd - ae)\right) g^2 (4c^2 e^2 f - 2c^2 d^2 g - b^2 e^2 g) - 4c^2 e^2 (2c^2 d - b^2 e) (2c^2 e^2 f^2 - b^2 d^2 g^2)\right) \operatorname{ArcTanh}\left[\frac{(b + 2cx) \sqrt{c}}{2\sqrt{a + bx + cx^2}}\right] / (16c^{5/2} e^4) + \left(\sqrt{ae^2 - bde + cd^2} \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right) + (\sqrt{c^2 d^2 - b^2 d^2 e + a^2 e^2}) (ef - dg)^2 \operatorname{ArcTanh}\left[\frac{(bd - 2ae + (2c^2 d - b^2 e) x)}{2\sqrt{a + bx + cx^2}}\right] \sqrt{a + bx + cx^2}\right) / e^4$

**Rubi [A]** time = 1.47167, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$

$$\frac{\sqrt{a+bx+cx^2} (b^2 e^2 g^2 - 2ceg x(-beg - 2cdg + 4cef) - 2bceg(2ef - dg) - 8c^2(ef - dg)^2)}{8c^2 e^3} + \frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (g(-4ce(bd - ae) - b^2 e^2 + 8c^2 d^2) (-beg - 2cdg + 4cef) - 4ce(2cd - be)(2cef^2 - bdg^2))}{16c^{5/2} e^4} + \frac{(ef - dg)^2 \sqrt{ae^2 - bde + cd^2} \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^4} + \frac{g^2 (a + bx + cx^2)^{3/2}}{3ce}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}\left[\frac{(f + gx)^2 \sqrt{a + bx + cx^2}}{d + ex}, x\right]$

[Out]  $-\left((b^2 e^2 g^2 - 8c^2 (ef - dg)^2 - 2bceg(2ef - dg) - 2c^2 e^2 g^2 (4c^2 e^2 f - 2c^2 d^2 g - b^2 e^2 g) x) \sqrt{a + bx + cx^2}\right) / (8c^2 e^3) + (g^2 (a + bx + cx^2)^{3/2}) / (3c^2 e) + \left(\left(8c^2 d^2 - b^2 e^2 - 4c^2 e (bd - ae)\right) g^2 (4c^2 e^2 f - 2c^2 d^2 g - b^2 e^2 g) - 4c^2 e^2 (2c^2 d - b^2 e) (2c^2 e^2 f^2 - b^2 d^2 g^2)\right) \operatorname{ArcTanh}\left[\frac{(b + 2cx) \sqrt{c}}{2\sqrt{a + bx + cx^2}}\right] / (16c^{5/2} e^4) + \left(\sqrt{ae^2 - bde + cd^2} \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right) + (\sqrt{c^2 d^2 - b^2 d^2 e + a^2 e^2}) (ef - dg)^2 \operatorname{ArcTanh}\left[\frac{(bd - 2ae + (2c^2 d - b^2 e) x)}{2\sqrt{a + bx + cx^2}}\right] \sqrt{a + bx + cx^2}\right) / e^4$

**Rubi in Sympy [A]** time = 113.029, size = 366, normalized size = 1.13

$$\frac{bg^2 (b + 2cx) \sqrt{a + bx + cx^2}}{8c^2 e} + \frac{bg^2 (-4ac + b^2) \operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{5/2} e} + \frac{(dg - ef)^2 \sqrt{a + bx + cx^2}}{e^3} - \frac{(dg - ef)^2 \sqrt{ae^2 - bde + cd^2} \operatorname{atanh}\left(\frac{2ae-bd+x(be-2cd)}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^4} + \frac{g^2 (a + bx + cx^2)^{3/2}}{3ce} - \frac{g(b + 2cx)(dg - 2ef) \sqrt{a + bx + cx^2}}{4ce^2} + \frac{(be - 2cd)(dg - ef)^2 \operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c} e^4} + \frac{g(-4ac + b^2)(dg - 2ef) \operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2} e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x+f)**2*(c*x**2+b*x+a)**(1/2)/(e*x+d),x)`

[Out]  $-b^2 g^2 (b + 2cx) \sqrt{a + bx + cx^2} / (8c^2 e) + b^2 g^2 (-4ac + b^2) \operatorname{atanh}((b + 2cx) / (2\sqrt{c} \sqrt{a + bx + cx^2})) / (16c^{5/2} e) + (dg - ef)^2 \sqrt{a + bx + cx^2} / e^3 - (dg - ef)^2 \sqrt{a^2 e^2 - b^2 d^2 + c^2 d^2} \operatorname{atanh}((2ae - b^2 d + x(b^2 e - 2cd)) / (2\sqrt{a + bx + cx^2} \sqrt{a^2 e^2 - b^2 d^2 + c^2 d^2})) / e^4 + g^2 (a + bx + cx^2)^{3/2} / (3c^2 e) - g(b + 2cx) (dg - 2ef) \sqrt{a + bx + cx^2} / (4c^2 e^2) + (b^2 e - 2cd) (dg - ef)^2 \operatorname{atanh}((b + 2cx) / (2\sqrt{c} \sqrt{a + bx + cx^2})) / (2\sqrt{c} e^4) + g(-4ac + b^2) (dg - 2ef) \operatorname{atanh}((b + 2cx) / (2\sqrt{c} \sqrt{a + bx + cx^2})) / (8c^{3/2} e^2)$

**Mathematica [A]** time = 0.795448, size = 356, normalized size = 1.1

$$\frac{3 \log\left(2\sqrt{c}\sqrt{a+x(b+cx)+b+2cx}\right) (-8c^2 e(aeg(2ef-dg)+b(ef-dg)^2)+2bce^2 g(2aeg-bdg+2bef)-b^3 e^3 g^2+16c^3 d(ef-dg)^2)}{c^{5/2}} + \frac{2e\sqrt{a+x(b+cx)}(2ceg(4aeg+b$$

Antiderivative was successfully verified.

[In] `Integrate[((f + g*x)^2*Sqrt[a + b*x + c*x^2])/(d + e*x),x]`

[Out]  $((2e \sqrt{a + x(b + cx)})^2 (-3b^2 e^2 g^2 + 2c^2 e^2 g^2 + 4a^2 e^2 g + b^2 (6ef - 3d^2 g + e^2 g^2 x) + 4c^2 (6d^2 g^2 - 3d^2 e^2 g^2 (4f + gx) + 2e^2 (3f^2 + 3fg^2 x + g^2 x^2))) / c^2 + 48 \sqrt{c^2 d^2 + e^2 (-bd + ae)} (ef - dg)^2 \operatorname{Log}[d + ex] - (3(-b^3 e^3 g^2) + 16c^3 d^2 (ef - dg)^2 + 2b^2 c^2 e^2 g^2 (2b^2 ef - b^2 dg + 2a^2 e^2 g) - 8c^2 e^2 (b^2 (ef - dg)^2 + a^2 e^2 (2ef - dg))) \operatorname{Log}[b + 2cx + 2\sqrt{c} \sqrt{a + x(b + cx)}]) / c^{5/2} - 48 \sqrt{c^2 d^2 + e^2 (-bd + ae)} (ef - dg)^2 \operatorname{Log}[-bd + 2ae - 2cd^2 x + b^2 e^2 x + 2\sqrt{c^2 d^2 + e^2 (-bd + ae)} \sqrt{a + x(b + cx)}]) / (48 e^4)$

**Maple [B]** time = 0.022, size = 2602, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^2*(c*x^2+b*x+a)^(1/2)/(e*x+d),x)`

[Out]  $1/3 g^2 (c^2 x^2 + b^2 x + a)^{3/2} / c e + 1/e^3 ((x+d/e)^2 c + (b^2 e - 2cd) / e^2 (x+d/e) + (a^2 e^2 - b^2 d^2 + c^2 d^2) / e^2)^{1/2} d^2 g^2 + 1/e^2 ((x+d/e)^2 c + (b^2 e - 2cd) / e^2 (x+d/e) + (a^2 e^2 - b^2 d^2 + c^2 d^2) / e^2)^{1/2} f^2 - 1/e^3 ((a^2 e^2 - b^2 d^2 + c^2 d^2) / e^2)^{1/2} \ln((2(a^2 e^2 - b^2 d^2 + c^2 d^2) / e^2 + (b^2 e - 2cd) / e^2 (x+d/e) + 2((a^2 e^2 - b^2 d^2 + c^2 d^2) / e^2)^{1/2} ((x+d/e)^2 c + (b^2 e - 2cd) / e^2 (x+d/e) + (a^2 e^2 - b^2 d^2 + c^2 d^2) / e^2)^{1/2}) / (x+d/e)) a^2 d^2 g^2 + 1/e^4 ((a^2 e^2 - b^2 d^2 + c^2 d^2) / e^2)^{1/2} \ln((2(a^2 e^2 - b^2 d^2 + c^2 d^2) / e^2 + (b^2 e - 2cd) / e^2 (x+d/e) + 2((a^2 e^2 - b^2 d^2 + c^2 d^2) / e^2)^{1/2} ((x+d/e)^2 c + (b^2 e - 2cd) / e^2 (x+d/e) + (a^2 e^2 - b^2 d^2 + c^2 d^2) / e^2)^{1/2}) / (x+d/e)) b^2 d^3 g^2 + 1/e^2 ((a^2 e^2 - b^2 d^2 + c^2 d^2) / e^2)^{1/2} \ln((2(a^2 e^2 - b^2 d^2 + c^2 d^2) / e^2 + (b^2 e - 2cd) / e^2 (x+d/e) + 2((a^2 e^2 - b^2 d^2 + c^2 d^2) / e^2)^{1/2} ((x+d/e)^2 c + (b^2 e - 2cd) / e^2 (x+d/e) + (a^2 e^2 - b^2 d^2 + c^2 d^2) / e^2)^{1/2}) / (x+d/e)) b^2 d^2 f^2 - 1/e^5 ((a^2 e^2 - b^2 d^2 + c^2 d^2) / e^2)^{1/2} \ln((2(a^2 e^2 - b^2 d^2 + c^2 d^2) / e^2 + (b^2 e - 2cd) / e^2 (x+d/e) + 2((a^2 e^2 - b^2 d^2 + c^2 d^2) / e^2)^{1/2} ((x+d/e)^2 c + (b^2 e - 2cd) / e^2 (x+d/e) + (a^2 e^2 - b^2 d^2 + c^2 d^2) / e^2)^{1/2}) / (x+d/e)) c^2 d^4 g^2 - 1/e^3 ((a^2 e^2 - b^2$

$$\begin{aligned}
& d^*e+c^*d^2)/e^2)^{(1/2)} * \ln((2^*(a^*e^2-b^*d^*e+c^*d^2)/e^2+(b^*e-2^*c^*d)/e \\
& *(x+d/e)+2^*((a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c+(b^*e-2^*c^* \\
& d)/e*(x+d/e)+(a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)})/(x+d/e))^*c^*d^2*f^2-1 \\
& /2^*g^2/e^2*d^*(c^*x^2+b^*x+a)^{(1/2)}*x+g/e*f*(c^*x^2+b^*x+a)^{(1/2)}*x-2/ \\
& e^2^*((x+d/e)^2*c+(b^*e-2^*c^*d)/e*(x+d/e)+(a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)} \\
& *d^*f^*g+1/2/e*\ln((1/2^*(b^*e-2^*c^*d)/e+c^*(x+d/e))/c^{(1/2)}+((x+d/e) \\
& )^2*c+(b^*e-2^*c^*d)/e*(x+d/e)+(a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)})/c^{(1/2)} \\
& *b^*f^2-1/e^4*\ln((1/2^*(b^*e-2^*c^*d)/e+c^*(x+d/e))/c^{(1/2)}+((x+d/e)^2 \\
& *c+(b^*e-2^*c^*d)/e*(x+d/e)+(a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)})^*c^{(1/2)} \\
& *d^3*g^2-1/e^2*\ln((1/2^*(b^*e-2^*c^*d)/e+c^*(x+d/e))/c^{(1/2)}+((x+d/e)^2 \\
& *c+(b^*e-2^*c^*d)/e*(x+d/e)+(a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)})^*c^{(1/2)} \\
& *d^*f^2-1/e/((a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)}*\ln((2^*(a^*e^2-b^*d^*e+c^*d^2) \\
& ^2)/e^2+(b^*e-2^*c^*d)/e*(x+d/e)+2^*((a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)}*( \\
& (x+d/e)^2*c+(b^*e-2^*c^*d)/e*(x+d/e)+(a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)}) \\
& /(x+d/e))^*a^*f^2+1/16^*g^2/e^*b^3/c^{(5/2)}*\ln((1/2^*b+c^*x)/c^{(1/2)}+(c^* \\
& x^2+b^*x+a)^{(1/2)})-1/e^2*\ln((1/2^*(b^*e-2^*c^*d)/e+c^*(x+d/e))/c^{(1/2)}+ \\
& ((x+d/e)^2*c+(b^*e-2^*c^*d)/e*(x+d/e)+(a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)} \\
& )/c^{(1/2)}*b^*d^*f^*g+2/e^2/((a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)}*\ln((2^*(a^* \\
& e^2-b^*d^*e+c^*d^2)/e^2+(b^*e-2^*c^*d)/e*(x+d/e)+2^*((a^*e^2-b^*d^*e+c^*d^2) \\
& /e^2)^{(1/2)}*((x+d/e)^2*c+(b^*e-2^*c^*d)/e*(x+d/e)+(a^*e^2-b^*d^*e+c^*d^2) \\
& )/e^2)^{(1/2)})/(x+d/e))^*a^*d^*f^*g-2/e^3/((a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)} \\
& *\ln((2^*(a^*e^2-b^*d^*e+c^*d^2)/e^2+(b^*e-2^*c^*d)/e*(x+d/e)+2^*((a^*e^2 \\
& -b^*d^*e+c^*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c+(b^*e-2^*c^*d)/e*(x+d/e)+(a^*e^2 \\
& -b^*d^*e+c^*d^2)/e^2)^{(1/2)})/(x+d/e))^*b^*d^2*f^*g+2/e^4/((a^*e^2-b^*d^*e \\
& +c^*d^2)/e^2)^{(1/2)}*\ln((2^*(a^*e^2-b^*d^*e+c^*d^2)/e^2+(b^*e-2^*c^*d)/e*(x \\
& +d/e)+2^*((a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c+(b^*e-2^*c^*d)/ \\
& e*(x+d/e)+(a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)})/(x+d/e))^*c^*d^3*f^*g-1/8^* \\
& g^2/e^*b^2/c^2*(c^*x^2+b^*x+a)^{(1/2)}-1/4^*g^2/e^*b/c*(c^*x^2+b^*x+a)^{(1/2)} \\
& *x-1/4^*g^2/e^*b/c^{(3/2)}*\ln((1/2^*b+c^*x)/c^{(1/2)}+(c^*x^2+b^*x+a)^{(1/2)}) \\
& )^*a-1/4^*g^2/e^2*d/c*(c^*x^2+b^*x+a)^{(1/2)}*b-1/2^*g^2/e^2*d/c^{(1/2)} \\
& *\ln((1/2^*b+c^*x)/c^{(1/2)}+(c^*x^2+b^*x+a)^{(1/2)})^*a+1/8^*g^2/e^2*d/c^{(3/2)} \\
& *\ln((1/2^*b+c^*x)/c^{(1/2)}+(c^*x^2+b^*x+a)^{(1/2)})^*b^2+1/2^*g/e^*f/c*(c^* \\
& x^2+b^*x+a)^{(1/2)}*b+g/e^*f/c^{(1/2)}*\ln((1/2^*b+c^*x)/c^{(1/2)}+(c^*x^2+b^* \\
& x+a)^{(1/2)})^*a-1/4^*g/e^*f/c^{(3/2)}*\ln((1/2^*b+c^*x)/c^{(1/2)}+(c^*x^2+b^* \\
& x+a)^{(1/2)})^*b^2+1/2/e^3*\ln((1/2^*(b^*e-2^*c^*d)/e+c^*(x+d/e))/c^{(1/2)} \\
& +((x+d/e)^2*c+(b^*e-2^*c^*d)/e*(x+d/e)+(a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)} \\
& )/c^{(1/2)}*b^*d^2*g^2+2/e^3*\ln((1/2^*(b^*e-2^*c^*d)/e+c^*(x+d/e))/c^{(1/2)} \\
& +((x+d/e)^2*c+(b^*e-2^*c^*d)/e*(x+d/e)+(a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)} \\
& )^*c^{(1/2)}*d^2*f^*g
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)\*(g\*x + f)^2/(e\*x + d), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)\*(g\*x + f)^2/(e\*x + d), x, algorithm="fricas")

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(f + gx)^2 \sqrt{a + bx + cx^2}}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*2\*(c\*x\*\*2+b\*x+a)\*\*(1/2)/(e\*x+d),x)

[Out] Integral((f + g\*x)\*\*2\*sqrt(a + b\*x + c\*x\*\*2)/(d + e\*x), x)

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)\*(g\*x + f)^2/(e\*x + d),x, algorithm="giac")

[Out] Exception raised: TypeError



$$3.856 \quad \int \frac{(f+gx)\sqrt{a+bx+cx^2}}{d+ex} dx$$

**Optimal.** Leaf size=219

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (-4ce(aeg - bdg + bef) + b^2e^2g + 8c^2d(ef - dg))}{8c^{3/2}e^3} + \frac{(ef - dg)\sqrt{ae^2 - bde + cd^2} \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^3} + \frac{\sqrt{a+bx+cx^2}(beg - 4cdg + 4cef + 2cegx)}{4ce^2}$$

[Out]  $((4*c*e*f - 4*c*d*g + b*e*g + 2*c*e*g*x)*\text{Sqrt}[a + b*x + c*x^2])/((4*c*e^2) - ((b^2*e^2*g + 8*c^2*d*(e*f - d*g) - 4*c*e*(b*e*f - b*d*g + a*e*g))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])]))/(8*c^{3/2}*e^3) + (\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*(e*f - d*g)*\text{ArcTanh}[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*\text{Sqrt}[a + b*x + c*x^2])])/e^3$

**Rubi [A]** time = 0.712034, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (-4ce(aeg - bdg + bef) + b^2e^2g + 8c^2d(ef - dg))}{8c^{3/2}e^3} + \frac{(ef - dg)\sqrt{ae^2 - bde + cd^2} \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^3} + \frac{\sqrt{a+bx+cx^2}(beg - 4cdg + 4cef + 2cegx)}{4ce^2}$$

Antiderivative was successfully verified.

[In] Int[((f + g\*x)\*Sqrt[a + b\*x + c\*x^2])/(d + e\*x), x]

[Out]  $((4*c*e*f - 4*c*d*g + b*e*g + 2*c*e*g*x)*\text{Sqrt}[a + b*x + c*x^2])/((4*c*e^2) - ((b^2*e^2*g + 8*c^2*d*(e*f - d*g) - 4*c*e*(b*e*f - b*d*g + a*e*g))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])]))/(8*c^{3/2}*e^3) + (\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*(e*f - d*g)*\text{ArcTanh}[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*\text{Sqrt}[a + b*x + c*x^2])])/e^3$

**Rubi in Sympy [A]** time = 101.168, size = 211, normalized size = 0.96

$$\frac{(dg - ef)\sqrt{ae^2 - bde + cd^2} \operatorname{atanh}\left(\frac{2ae - bd + x(be - 2cd)}{2\sqrt{a+bx+cx^2}\sqrt{ae^2 - bde + cd^2}}\right)}{e^3} + \frac{\sqrt{a+bx+cx^2}\left(2cef + cegx + \frac{g(be - 4cd)}{2}\right)}{2ce^2} - \frac{(-4cef(be - 2cd) + g(b^2e^2 - 8c^2d^2 - 4ce(ae - bd))) \operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{3/2}e^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*(c\*x\*\*2+b\*x+a)\*\*(1/2)/(e\*x+d), x)

[Out]  $(d*g - e*f)*\text{sqrt}(a*e**2 - b*d*e + c*d**2)*\text{atanh}((2*a*e - b*d + x*(b*e - 2*c*d))/(2*\text{sqrt}(a + b*x + c*x**2)*\text{sqrt}(a*e**2 - b*d*e + c$



$2 * f$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)/(e*x + d), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)/(e*x + d), x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(f + gx)\sqrt{a + bx + cx^2}}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)*(c*x**2+b*x+a)**(1/2)/(e*x+d), x)`

[Out] `Integral((f + g*x)*sqrt(a + b*x + c*x**2)/(d + e*x), x)`

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)/(e*x + d), x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.857 \quad \int \frac{\sqrt{a+bx+cx^2}}{d+ex} dx$$

**Optimal.** Leaf size=152

$$\frac{\sqrt{ae^2 - bde + cd^2} \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2} - \frac{(2cd - be) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{ce^2}} + \frac{\sqrt{a+bx+cx^2}}{e}$$

[Out] Sqrt[a + b\*x + c\*x^2]/e - ((2\*c\*d - b\*e)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(2\*Sqrt[c]\*e^2) + (Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*ArcTanh[(b\*d - 2\*a\*e + (2\*c\*d - b\*e)\*x)/(2\*Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*Sqrt[a + b\*x + c\*x^2])])/e^2

**Rubi [A]** time = 0.336723, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$

$$\frac{\sqrt{ae^2 - bde + cd^2} \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2} - \frac{(2cd - be) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{ce^2}} + \frac{\sqrt{a+bx+cx^2}}{e}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x + c\*x^2]/(d + e\*x), x]

[Out] Sqrt[a + b\*x + c\*x^2]/e - ((2\*c\*d - b\*e)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(2\*Sqrt[c]\*e^2) + (Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*ArcTanh[(b\*d - 2\*a\*e + (2\*c\*d - b\*e)\*x)/(2\*Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*Sqrt[a + b\*x + c\*x^2])])/e^2

**Rubi in Sympy [A]** time = 54.4867, size = 138, normalized size = 0.91

$$\frac{\sqrt{a+bx+cx^2}}{e} - \frac{\sqrt{ae^2 - bde + cd^2} \operatorname{atanh}\left(\frac{2ae-bd+x(be-2cd)}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2} + \frac{(be - 2cd) \operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{ce^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+b\*x+a)\*\*(1/2)/(e\*x+d), x)

[Out] sqrt(a + b\*x + c\*x\*\*2)/e - sqrt(a\*e\*\*2 - b\*d\*e + c\*d\*\*2)\*atanh((2\*a\*e - b\*d + x\*(b\*e - 2\*c\*d))/(2\*sqrt(a + b\*x + c\*x\*\*2)\*sqrt(a\*e\*\*2 - b\*d\*e + c\*d\*\*2)))/e\*\*2 + (b\*e - 2\*c\*d)\*atanh((b + 2\*c\*x)/(2\*sqrt(c)\*sqrt(a + b\*x + c\*x\*\*2)))/(2\*sqrt(c)\*e\*\*2)

**Mathematica [A]** time = 0.555587, size = 169, normalized size = 1.11

$$\frac{2 \log(d + ex) \sqrt{e(ae - bd) + cd^2} - 2 \sqrt{e(ae - bd) + cd^2} \log\left(2\sqrt{a + x(b + cx)} \sqrt{e(ae - bd) + cd^2} + 2ae - bd + bex - 2cdx\right) + \dots}{2e^2}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b\*x + c\*x^2]/(d + e\*x), x]

[Out] (2\*e\*Sqrt[a + x\*(b + c\*x)] + 2\*Sqrt[c\*d^2 + e\*(-(b\*d) + a\*e)]\*Log[d + e\*x] + ((-2\*c\*d + b\*e)\*Log[b + 2\*c\*x + 2\*Sqrt[c]\*Sqrt[a + x\*

$$\frac{(b + c^*x)]])/\text{Sqrt}[c] - 2*\text{Sqrt}[c*d^2 + e*(-(b*d) + a*e)]*\text{Log}[-(b*d) + 2*a*e - 2*c*d*x + b*e*x + 2*\text{Sqrt}[c*d^2 + e*(-(b*d) + a*e)]*\text{Sqrt}[a + x*(b + c*x)]])/(2*e^2)$$

**Maple [B]** time = 0.007, size = 715, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)^(1/2)/(e*x+d), x)`

[Out]  $\frac{1}{e} \left( \frac{(x+d/e)^2 c + (b e - 2 c^2 d)/e}{(x+d/e) + (a e^2 - b^2 d^2 + c^2 d^2)/e^2} \right)^{1/2} + \frac{1}{2} \frac{1}{e} \ln \left( \frac{(1/2 (b e - 2 c^2 d)/e + c (x+d/e))/c^{1/2} + ((x+d/e)^2 c + (b e - 2 c^2 d)/e (x+d/e) + (a e^2 - b^2 d^2 + c^2 d^2)/e^2)^{1/2}}{c^{1/2}} \right) \frac{b-1}{e^2} \ln \left( \frac{(1/2 (b e - 2 c^2 d)/e + c (x+d/e))/c^{1/2} + ((x+d/e)^2 c + (b e - 2 c^2 d)/e (x+d/e) + (a e^2 - b^2 d^2 + c^2 d^2)/e^2)^{1/2}}{c^{1/2}} \right) \frac{d-1}{e} \left( \frac{a e^2 - b^2 d^2 + c^2 d^2}{e^2} \right)^{1/2} \ln \left( \frac{2 (a e^2 - b^2 d^2 + c^2 d^2)/e^2 + (b e - 2 c^2 d)/e (x+d/e) + 2 ((a e^2 - b^2 d^2 + c^2 d^2)/e^2)^{1/2} ((x+d/e)^2 c + (b e - 2 c^2 d)/e (x+d/e) + (a e^2 - b^2 d^2 + c^2 d^2)/e^2)^{1/2}}{(x+d/e)} \right) \frac{a+1}{e^2} \left( \frac{a e^2 - b^2 d^2 + c^2 d^2}{e^2} \right)^{1/2} \ln \left( \frac{2 (a e^2 - b^2 d^2 + c^2 d^2)/e^2 + (b e - 2 c^2 d)/e (x+d/e) + 2 ((a e^2 - b^2 d^2 + c^2 d^2)/e^2)^{1/2} ((x+d/e)^2 c + (b e - 2 c^2 d)/e (x+d/e) + (a e^2 - b^2 d^2 + c^2 d^2)/e^2)^{1/2}}{(x+d/e)} \right) \frac{b d - 1}{e^3} \left( \frac{a e^2 - b^2 d^2 + c^2 d^2}{e^2} \right)^{1/2} \ln \left( \frac{2 (a e^2 - b^2 d^2 + c^2 d^2)/e^2 + (b e - 2 c^2 d)/e (x+d/e) + 2 ((a e^2 - b^2 d^2 + c^2 d^2)/e^2)^{1/2} ((x+d/e)^2 c + (b e - 2 c^2 d)/e (x+d/e) + (a e^2 - b^2 d^2 + c^2 d^2)/e^2)^{1/2}}{(x+d/e)} \right) \frac{c^2 d^2}{(x+d/e)}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)/(e*x + d), x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 1.34467, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)/(e*x + d), x, algorithm="fricas")`

[Out]  $\frac{1}{4} (4 \sqrt{c x^2 + b x + a} \sqrt{c} e - (2 c d - b^2 e) \log(-4 (2 c^2 x + b^2 c) \sqrt{c x^2 + b x + a} - (8 c^2 x^2 + 8 b^2 c x + b^2 + 4 a^2 c) \sqrt{c})) + 2 \sqrt{c d^2 - b^2 d^2 e + a^2 e^2} \sqrt{c} \log((8 a^2 b^2 d^2 e - 8 a^2 e^2 - (b^2 + 4 a^2 c) d^2 - (8 c^2 d^2 - 8 b^2 c^2 d^2 e + (b^2 + 4 a^2 c) e^2) x^2 - 4 \sqrt{c d^2 - b^2 d^2 e + a^2 e^2} \sqrt{c x^2 + b x + a}) (b d - 2 a^2 e + (2 c d - b^2 e) x) - 2 (4 b^2 c^2 d^2 + 4 a^2 b^2 e^2 - (3 b^2 + 4 a^2 c) d^2 e) x) / (e^2 x^2 + 2 d^2 e x + d^2)) / (\sqrt{c} e^2), \frac{1}{2} (2 \sqrt{c x^2 + b x + a} \sqrt{-c} e - (2 c d - b^2 e) \arctan(1/2 (2 c x + b) \sqrt{-c} / (\sqrt{c x^2 + b x + a} c)) + \sqrt{c d^2 - b^2 d^2 e + a^2 e^2} \sqrt{-c} \log((8 a^2 b^2 d^2 e - 8 a^2 e^2 - (b^2 + 4 a^2 c) d^2 - (8 c^2 d^2 - 8 b^2 c^2 d^2 e + (b^2 + 4 a^2 c) e^2) x^2 - 4 \sqrt{c d^2 - b^2 d^2 e + a^2 e^2} \sqrt{c x^2 + b x + a}) (b d - 2$

```
*a*e + (2*c*d - b*e)*x) - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a
*c)*d*e)*x)/(e^2*x^2 + 2*d*e*x + d^2))/(sqrt(-c)*e^2), 1/4*(4*sq
rt(c*x^2 + b*x + a)*sqrt(c)*e - 4*sqrt(-c*d^2 + b*d*e - a*e^2)*sq
rt(c)*arctan(-1/2*(b*d - 2*a*e + (2*c*d - b*e)*x)/(sqrt(-c*d^2 +
b*d*e - a*e^2)*sqrt(c*x^2 + b*x + a))) - (2*c*d - b*e)*log(-4*(2*
c^2*x + b*c)*sqrt(c*x^2 + b*x + a) - (8*c^2*x^2 + 8*b*c*x + b^2 +
4*a*c)*sqrt(c)))/(sqrt(c)*e^2), 1/2*(2*sqrt(c*x^2 + b*x + a)*sq
rt(-c)*e - 2*sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(-c)*arctan(-1/2*(b*
d - 2*a*e + (2*c*d - b*e)*x)/(sqrt(-c*d^2 + b*d*e - a*e^2)*sqrt(c
*x^2 + b*x + a))) - (2*c*d - b*e)*arctan(1/2*(2*c*x + b)*sqrt(-c)
/(sqrt(c*x^2 + b*x + a)*c)))/(sqrt(-c)*e^2)]
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*\*(1/2)/(e\*x+d),x)

[Out] Integral(sqrt(a + b\*x + c\*x\*\*2)/(d + e\*x), x)

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)/(e\*x + d),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.858 \quad \int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)} dx$$

**Optimal.** Leaf size=228

$$\frac{\sqrt{ae^2 - bde + cd^2} \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e(ef - dg)} - \frac{\sqrt{ag^2 - bfg + cf^2} \tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef - dg)} + \frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{eg}$$

[Out] (Sqrt[c]\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(e\*g) + (Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*ArcTanh[(b\*d - 2\*a\*e + (2\*c\*d - b\*e)\*x)/(2\*Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*Sqrt[a + b\*x + c\*x^2])])/(e\*(e\*f - d\*g)) - (Sqrt[c\*f^2 - b\*f\*g + a\*g^2]\*ArcTanh[(b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)/(2\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2]\*Sqrt[a + b\*x + c\*x^2])])/(g\*(e\*f - d\*g))

**Rubi [A]** time = 0.683496, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$\frac{\sqrt{ae^2 - bde + cd^2} \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e(ef - dg)} - \frac{\sqrt{ag^2 - bfg + cf^2} \tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef - dg)} + \frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{eg}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x + c\*x^2]/((d + e\*x)\*(f + g\*x)), x]

[Out] (Sqrt[c]\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(e\*g) + (Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*ArcTanh[(b\*d - 2\*a\*e + (2\*c\*d - b\*e)\*x)/(2\*Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*Sqrt[a + b\*x + c\*x^2])])/(e\*(e\*f - d\*g)) - (Sqrt[c\*f^2 - b\*f\*g + a\*g^2]\*ArcTanh[(b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)/(2\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2]\*Sqrt[a + b\*x + c\*x^2])])/(g\*(e\*f - d\*g))

**Rubi in Sympy [A]** time = 96.7341, size = 199, normalized size = 0.87

$$\frac{\sqrt{c} \operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{eg} - \frac{\sqrt{ag^2 - bfg + cf^2} \operatorname{atanh}\left(\frac{2ag-bf+x(bg-2cf)}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(dg - ef)} + \frac{\sqrt{ae^2 - bde + cd^2} \operatorname{atanh}\left(\frac{2ae-bd+x(be-2cd)}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e(dg - ef)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+b\*x+a)\*\*(1/2)/(e\*x+d)/(g\*x+f), x)

[Out] sqrt(c)\*atanh((b + 2\*c\*x)/(2\*sqrt(c)\*sqrt(a + b\*x + c\*x\*\*2)))/(e\*g) - sqrt(a\*g\*\*2 - b\*f\*g + c\*f\*\*2)\*atanh((2\*a\*g - b\*f + x\*(b\*g - 2\*c\*f))/(2\*sqrt(a + b\*x + c\*x\*\*2)\*sqrt(a\*g\*\*2 - b\*f\*g + c\*f\*\*2)))/(g\*(d\*g - e\*f)) + sqrt(a\*e\*\*2 - b\*d\*e + c\*d\*\*2)\*atanh((2\*a\*e - b\*d + x\*(b\*e - 2\*c\*d))/(2\*sqrt(a + b\*x + c\*x\*\*2)\*sqrt(a\*e\*\*2 - b\*d\*e + c\*d\*\*2)))/(e\*(d\*g - e\*f))

---

**Mathematica [A]** time = 0.653289, size = 299, normalized size = 1.31

$$-g\sqrt{ae^2 - bde + cd^2} \log\left(2\sqrt{a + x(b + cx)}\sqrt{ae^2 - bde + cd^2} + 2ae - bd + bex - 2cdx\right) + g \log(d + ex)\sqrt{e(ae - bd) + cd^2} - \dots$$


---

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b\*x + c\*x^2]/((d + e\*x)\*(f + g\*x)),x]

[Out] (Sqrt[c\*d^2 + e\*(-(b\*d) + a\*e)]\*g\*Log[d + e\*x] - e\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2]\*Log[f + g\*x] + Sqrt[c]\*e\*f\*Log[b + 2\*c\*x + 2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)]] - Sqrt[c]\*d\*g\*Log[b + 2\*c\*x + 2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)]] - Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*g\*Log[-(b\*d) + 2\*a\*e - 2\*c\*d\*x + b\*e\*x + 2\*Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*Sqrt[a + x\*(b + c\*x)]] + e\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2]\*Log[-(b\*f) + 2\*a\*g - 2\*c\*f\*x + b\*g\*x + 2\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2]\*Sqrt[a + x\*(b + c\*x)]])/(e\*g\*(e\*f - d\*g))

---

**Maple [B]** time = 0.037, size = 1529, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)^(1/2)/(e\*x+d)/(g\*x+f),x)

[Out] 1/(d\*g-e\*f)\*((x+f/g)^2\*c+(b\*g-2\*c\*f)/g\*(x+f/g)+(a\*g^2-b\*f\*g+c\*f^2)/g^2)^(1/2)+1/2/(d\*g-e\*f)\*ln((1/2\*(b\*g-2\*c\*f)/g+c\*(x+f/g))/c^(1/2))+((x+f/g)^2\*c+(b\*g-2\*c\*f)/g\*(x+f/g)+(a\*g^2-b\*f\*g+c\*f^2)/g^2)^(1/2)/c^(1/2)\*b-1/(d\*g-e\*f)/g\*ln((1/2\*(b\*g-2\*c\*f)/g+c\*(x+f/g))/c^(1/2))+((x+f/g)^2\*c+(b\*g-2\*c\*f)/g\*(x+f/g)+(a\*g^2-b\*f\*g+c\*f^2)/g^2)^(1/2)\*c^(1/2)\*f-1/(d\*g-e\*f)/((a\*g^2-b\*f\*g+c\*f^2)/g^2)^(1/2)\*ln((2\*(a\*g^2-b\*f\*g+c\*f^2)/g^2+(b\*g-2\*c\*f)/g\*(x+f/g)+2\*((a\*g^2-b\*f\*g+c\*f^2)/g^2)^(1/2))\*((x+f/g)^2\*c+(b\*g-2\*c\*f)/g\*(x+f/g)+(a\*g^2-b\*f\*g+c\*f^2)/g^2)^(1/2))/(x+f/g)\*a+1/(d\*g-e\*f)/g/((a\*g^2-b\*f\*g+c\*f^2)/g^2)^(1/2)\*ln((2\*(a\*g^2-b\*f\*g+c\*f^2)/g^2+(b\*g-2\*c\*f)/g\*(x+f/g)+2\*((a\*g^2-b\*f\*g+c\*f^2)/g^2)^(1/2))\*((x+f/g)^2\*c+(b\*g-2\*c\*f)/g\*(x+f/g)+(a\*g^2-b\*f\*g+c\*f^2)/g^2)^(1/2))/(x+f/g)\*b\*f-1/(d\*g-e\*f)/g^2/((a\*g^2-b\*f\*g+c\*f^2)/g^2)^(1/2)\*ln((2\*(a\*g^2-b\*f\*g+c\*f^2)/g^2+(b\*g-2\*c\*f)/g\*(x+f/g)+2\*((a\*g^2-b\*f\*g+c\*f^2)/g^2)^(1/2))\*((x+f/g)^2\*c+(b\*g-2\*c\*f)/g\*(x+f/g)+(a\*g^2-b\*f\*g+c\*f^2)/g^2)^(1/2))/(x+f/g)\*c\*f^2-1/(d\*g-e\*f)\*((x+d/e)^2\*c+(b\*e-2\*c\*d)/e\*(x+d/e)+(a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)-1/2/(d\*g-e\*f)\*ln((1/2\*(b\*e-2\*c\*d)/e+c\*(x+d/e))/c^(1/2))+((x+d/e)^2\*c+(b\*e-2\*c\*d)/e\*(x+d/e)+(a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2))/c^(1/2)\*b+1/(d\*g-e\*f)/e\*ln((1/2\*(b\*e-2\*c\*d)/e+c\*(x+d/e))/c^(1/2))+((x+d/e)^2\*c+(b\*e-2\*c\*d)/e\*(x+d/e)+(a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2))\*c^(1/2)\*d+1/(d\*g-e\*f)/((a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2-b\*d\*e+c\*d^2)/e^2+(b\*e-2\*c\*d)/e\*(x+d/e)+2\*((a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2))\*((x+d/e)^2\*c+(b\*e-2\*c\*d)/e\*(x+d/e)+(a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2))/(x+d/e))\*a-1/(d\*g-e\*f)/e/((a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2-b\*d\*e+c\*d^2)/e^2+(b\*e-2\*c\*d)/e\*(x+d/e)+2\*((a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2))\*((x+d/e)^2\*c+(b\*e-2\*c\*d)/e\*(x+d/e)+(a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2))/(x+d/e))\*b\*d+1/(d\*g-e\*f)/e^2/((a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2-b\*d\*e+c\*d^2)/e^2+(b\*e-2\*c\*d)/e\*(x+d/e)+2\*((a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2))\*((x+d/e)^2\*c+(b\*e-2\*c\*d)/e\*(x+d/e)+(a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2))/(x+d/e))\*c\*d^2

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}}{(d + ex)(f + gx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)*(f + g*x)), x)
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.859 \quad \int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^2} dx$$

**Optimal.** Leaf size=490

$$\frac{\sqrt{ae^2 - bde + cd^2} \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef - dg)^2} - \frac{e\sqrt{ag^2 - bfg + cf^2} \tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef - dg)^2} + \frac{(2cf - bg) \tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2g(ef - dg)\sqrt{ag^2 - bfg + cf^2}} + \frac{\sqrt{a + bx + cx^2}}{(f + gx)(ef - dg)} - \frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{g(ef - dg)} - \frac{(2cd - be) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}(ef - dg)^2} + \frac{e(2cf - bg) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}g(ef - dg)^2}$$

[Out] Sqrt[a + b\*x + c\*x^2]/((e\*f - d\*g)\*(f + g\*x)) - ((2\*c\*d - b\*e)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(2\*Sqrt[c]\*(e\*f - d\*g)^2) + (e\*(2\*c\*f - b\*g)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(2\*Sqrt[c]\*g\*(e\*f - d\*g)^2) - (Sqrt[c]\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(g\*(e\*f - d\*g)) + (Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*ArcTanh[(b\*d - 2\*a\*e + (2\*c\*d - b\*e)\*x)/(2\*Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*Sqrt[a + b\*x + c\*x^2])])/(e\*f - d\*g)^2 + ((2\*c\*f - b\*g)\*ArcTanh[(b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)/(2\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2]\*Sqrt[a + b\*x + c\*x^2])])/(2\*g\*(e\*f - d\*g)\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2]) - (e\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2]\*ArcTanh[(b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)/(2\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2]\*Sqrt[a + b\*x + c\*x^2])])/(g\*(e\*f - d\*g)^2)

**Rubi [A]** time = 1.51042, antiderivative size = 490, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 7, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$

$$\frac{\sqrt{ae^2 - bde + cd^2} \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef - dg)^2} - \frac{e\sqrt{ag^2 - bfg + cf^2} \tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef - dg)^2} + \frac{(2cf - bg) \tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2g(ef - dg)\sqrt{ag^2 - bfg + cf^2}} + \frac{\sqrt{a + bx + cx^2}}{(f + gx)(ef - dg)} - \frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{g(ef - dg)} - \frac{(2cd - be) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}(ef - dg)^2} + \frac{e(2cf - bg) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}g(ef - dg)^2}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x + c\*x^2]/((d + e\*x)\*(f + g\*x)^2), x]

[Out] Sqrt[a + b\*x + c\*x^2]/((e\*f - d\*g)\*(f + g\*x)) - ((2\*c\*d - b\*e)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(2\*Sqrt[c]\*(e\*f - d\*g)^2) + (e\*(2\*c\*f - b\*g)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(2\*Sqrt[c]\*g\*(e\*f - d\*g)^2) - (Sqrt[c]\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(g\*(e\*f - d\*g)) + (Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*ArcTanh[(b\*d - 2\*a\*e + (2\*c\*d - b\*e)\*x)/(2\*Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*Sqrt[a + b\*x + c\*x^2])])/(e\*f - d\*g)^2 + ((2\*c\*f - b\*g)\*ArcTanh[(b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)/(2\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2]\*Sqrt[a + b\*x + c\*x^2])])/(2\*g\*(e\*f - d\*g)\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2]) - (e\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2]\*ArcTanh[(b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)/(2\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2]\*Sqrt[a + b\*x + c\*x^2])])/(g\*(e\*f - d\*g)^2)

$$\sqrt{a^2 - b^2 f g + a^2 g^2} \sqrt{a + b x + c x^2} / (g^2 (e f - d g)^2)$$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**2,x)`

[Out] Timed out

**Mathematica [A]** time = 0.553012, size = 294, normalized size = 0.6

$$\frac{2 \log(d + ex) \sqrt{e(ae - bd) + cd^2} - 2 \sqrt{e(ae - bd) + cd^2} \log\left(2\sqrt{a + x(b + cx)} \sqrt{e(ae - bd) + cd^2} + 2ae - bd + bex - 2cdx\right) - \frac{1}{2} \log\left(\frac{2(e f - d g)^2}{\dots}\right)}{2(e f - d g)^2}$$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^2),x]`

[Out] 
$$\frac{((2*(e*f - d*g)*\sqrt{a + x*(b + c*x)})/(f + g*x) + 2*\sqrt{c*d^2 + e*(-(b*d) + a*e)}*\text{Log}[d + e*x] - ((2*c*d*f + 2*a*e*g - b*(e*f + d*g))*\text{Log}[f + g*x])/ \sqrt{c*f^2 + g*(-(b*f) + a*g)} - 2*\sqrt{c*d^2 + e*(-(b*d) + a*e)}*\text{Log}[-(b*d) + 2*a*e - 2*c*d*x + b*e*x + 2*\sqrt{c*d^2 + e*(-(b*d) + a*e)}*\sqrt{a + x*(b + c*x)}]) + ((2*c*d*f + 2*a*e*g - b*(e*f + d*g))*\text{Log}[-(b*f) + 2*a*g - 2*c*f*x + b*g*x + 2*\sqrt{c*f^2 + g*(-(b*f) + a*g)}*\sqrt{a + x*(b + c*x)}])/\sqrt{c*f^2 + g*(-(b*f) + a*g)}}{(2*(e*f - d*g)^2)}$$

**Maple [B]** time = 0.029, size = 3162, normalized size = 6.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^2,x)`

[Out] 
$$\frac{1/(d*g - e*f)^2 * e * ((x+d/e)^2 * c + (b*e - 2*c*d)/e * (x+d/e) + (a*e^2 - b*d*e + c*d^2)/e^2)^{1/2} + 1/2/(d*g - e*f)^2 * e * \ln((1/2 * (b*e - 2*c*d)/e + c * (x+d/e))/c^{1/2} + ((x+d/e)^2 * c + (b*e - 2*c*d)/e * (x+d/e) + (a*e^2 - b*d*e + c*d^2)/e^2)^{1/2})/c^{1/2} * b - 1/(d*g - e*f)^2 * \ln((1/2 * (b*e - 2*c*d)/e + c * (x+d/e))/c^{1/2} + ((x+d/e)^2 * c + (b*e - 2*c*d)/e * (x+d/e) + (a*e^2 - b*d*e + c*d^2)/e^2)^{1/2}) * c^{1/2} * d - 1/(d*g - e*f)^2 * e / ((a*e^2 - b*d*e + c*d^2)/e^2)^{1/2} * \ln((2 * (a*e^2 - b*d*e + c*d^2)/e^2 + (b*e - 2*c*d)/e * (x+d/e) + 2 * ((a*e^2 - b*d*e + c*d^2)/e^2)^{1/2}) / ((x+d/e)^2 * c + (b*e - 2*c*d)/e * (x+d/e) + (a*e^2 - b*d*e + c*d^2)/e^2)^{1/2}) / (x+d/e) * a + 1/(d*g - e*f)^2 / ((a*e^2 - b*d*e + c*d^2)/e^2)^{1/2} * \ln((2 * (a*e^2 - b*d*e + c*d^2)/e^2 + (b*e - 2*c*d)/e * (x+d/e) + 2 * ((a*e^2 - b*d*e + c*d^2)/e^2)^{1/2}) / ((x+d/e)^2 * c + (b*e - 2*c*d)/e * (x+d/e) + (a*e^2 - b*d*e + c*d^2)/e^2)^{1/2}) / (x+d/e) * b * d - 1/(d*g - e*f)^2 * e / ((a*e^2 - b*d*e + c*d^2)/e^2)^{1/2} * \ln((2 * (a*e^2 - b*d*e + c*d^2)/e^2 + (b*e - 2*c*d)/e * (x+d/e) + 2 * ((a*e^2 - b*d*e + c*d^2)/e^2)^{1/2}) * ((x+d/e)^2 * c + (b*e - 2*c*d)/e * (x+d/e) + (a*e^2 - b*d*e + c*d^2)/e^2)^{1/2}) / (x+d/e) * c * d^2 - g/(d*g - e*f) / (a*g^2 - b*f*g + c*f^2) / (x+f/g) * ((x+f/g)^2 * c + (b*g - 2*c*f)/g * (x+f/g) + (a*g^2 - b*f*g + c*f^2)/g^2)^{3/2} + g/(d*g - e*f) / (a*g^2 - b*f*g + c*f^2) * ((x+f/g)^2 * c + (b*g - 2*c*f)/g * (x+f/g) + (a*g^2 -$$

$$\begin{aligned}
& b^*f^*g+c^*f^2)/g^2)^{(1/2)}*b-1/(d^*g-e^*f)/(a^*g^2-b^*f^*g+c^*f^2)^*((x+f/g) \\
& )^2*c+(b^*g-2^*c^*f)/g^*(x+f/g)+(a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)}*c^*f-1/ \\
& (d^*g-e^*f)/(a^*g^2-b^*f^*g+c^*f^2)^*\ln((1/2^*(b^*g-2^*c^*f)/g+c^*(x+f/g))/c^ \\
& (1/2)+(x+f/g)^2*c+(b^*g-2^*c^*f)/g^*(x+f/g)+(a^*g^2-b^*f^*g+c^*f^2)/g^2) \\
& ^{(1/2)})^*c^{(1/2)}*f*b+1/g/(d^*g-e^*f)/(a^*g^2-b^*f^*g+c^*f^2)^*\ln((1/2^*(b^* \\
& g-2^*c^*f)/g+c^*(x+f/g))/c^{(1/2)+(x+f/g)^2*c+(b^*g-2^*c^*f)/g^*(x+f/g)+ \\
& (a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)})^*c^{(3/2)}*f^2-1/2^*g/(d^*g-e^*f)/(a^*g^2- \\
& b^*f^*g+c^*f^2)/((a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)}*\ln((2^*(a^*g^2-b^*f^*g \\
& +c^*f^2)/g^2+(b^*g-2^*c^*f)/g^*(x+f/g)+2^*((a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/ \\
& 2)}*((x+f/g)^2*c+(b^*g-2^*c^*f)/g^*(x+f/g)+(a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1 \\
& /2)))/(x+f/g))^*a*b+1/(d^*g-e^*f)/(a^*g^2-b^*f^*g+c^*f^2)/((a^*g^2-b^*f^*g+c \\
& ^*f^2)/g^2)^{(1/2)}*\ln((2^*(a^*g^2-b^*f^*g+c^*f^2)/g^2+(b^*g-2^*c^*f)/g^*(x+f \\
& /g)+2^*((a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)}*((x+f/g)^2*c+(b^*g-2^*c^*f)/g^* \\
& (x+f/g)+(a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)))/(x+f/g))^*a^*c^*f+1/2/(d^*g-e \\
& ^*f)/(a^*g^2-b^*f^*g+c^*f^2)/((a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)}*\ln((2^*(a^* \\
& g^2-b^*f^*g+c^*f^2)/g^2+(b^*g-2^*c^*f)/g^*(x+f/g)+2^*((a^*g^2-b^*f^*g+c^*f^2) \\
& /g^2)^{(1/2)}*((x+f/g)^2*c+(b^*g-2^*c^*f)/g^*(x+f/g)+(a^*g^2-b^*f^*g+c^*f^2) \\
& )/g^2)^{(1/2)))/(x+f/g))^*b^2*f-3/2^*g/(d^*g-e^*f)/(a^*g^2-b^*f^*g+c^*f^2)/ \\
& ((a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)}*\ln((2^*(a^*g^2-b^*f^*g+c^*f^2)/g^2+(b^* \\
& g-2^*c^*f)/g^*(x+f/g)+2^*((a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)}*((x+f/g)^2*c \\
& +(b^*g-2^*c^*f)/g^*(x+f/g)+(a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)))/(x+f/g))^*b \\
& ^*f^2*c+1/g^2/(d^*g-e^*f)/(a^*g^2-b^*f^*g+c^*f^2)/((a^*g^2-b^*f^*g+c^*f^2)/g \\
& ^2)^{(1/2)}*\ln((2^*(a^*g^2-b^*f^*g+c^*f^2)/g^2+(b^*g-2^*c^*f)/g^*(x+f/g)+2^*( \\
& (a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)}*((x+f/g)^2*c+(b^*g-2^*c^*f)/g^*(x+f/g) \\
& +(a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)))/(x+f/g))^*c^2*f^3+g/(d^*g-e^*f)^*c/( \\
& a^*g^2-b^*f^*g+c^*f^2)^*((x+f/g)^2*c+(b^*g-2^*c^*f)/g^*(x+f/g)+(a^*g^2-b^*f^* \\
& g+c^*f^2)/g^2)^{(1/2)}*x+g/(d^*g-e^*f)^*c^{(1/2)}/(a^*g^2-b^*f^*g+c^*f^2)^*\ln( \\
& (1/2^*(b^*g-2^*c^*f)/g+c^*(x+f/g))/c^{(1/2)+(x+f/g)^2*c+(b^*g-2^*c^*f)/g^* \\
& (x+f/g)+(a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)})^*a-1/(d^*g-e^*f)^2*e^*((x+f/g) \\
& )^2*c+(b^*g-2^*c^*f)/g^*(x+f/g)+(a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)}-1/2/(d \\
& ^*g-e^*f)^2*e^*\ln((1/2^*(b^*g-2^*c^*f)/g+c^*(x+f/g))/c^{(1/2)+(x+f/g)^2*c \\
& +(b^*g-2^*c^*f)/g^*(x+f/g)+(a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)))/c^{(1/2)}*b+ \\
& 1/(d^*g-e^*f)^2*e/g^*\ln((1/2^*(b^*g-2^*c^*f)/g+c^*(x+f/g))/c^{(1/2)+(x+f/ \\
& g)^2*c+(b^*g-2^*c^*f)/g^*(x+f/g)+(a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)})^*c^{(1 \\
& /2)}*f+1/(d^*g-e^*f)^2*e/((a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)}*\ln((2^*(a^*g^ \\
& 2-b^*f^*g+c^*f^2)/g^2+(b^*g-2^*c^*f)/g^*(x+f/g)+2^*((a^*g^2-b^*f^*g+c^*f^2)/g \\
& ^2)^{(1/2)}*((x+f/g)^2*c+(b^*g-2^*c^*f)/g^*(x+f/g)+(a^*g^2-b^*f^*g+c^*f^2)/ \\
& g^2)^{(1/2)))/(x+f/g))^*a-1/(d^*g-e^*f)^2*e/g/((a^*g^2-b^*f^*g+c^*f^2)/g^2) \\
& ^{(1/2)}*\ln((2^*(a^*g^2-b^*f^*g+c^*f^2)/g^2+(b^*g-2^*c^*f)/g^*(x+f/g)+2^*((a \\
& ^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)}*((x+f/g)^2*c+(b^*g-2^*c^*f)/g^*(x+f/g)+( \\
& a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)))/(x+f/g))^*b*f+1/(d^*g-e^*f)^2*e/g^2/( \\
& (a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)}*\ln((2^*(a^*g^2-b^*f^*g+c^*f^2)/g^2+(b^*g \\
& -2^*c^*f)/g^*(x+f/g)+2^*((a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)}*((x+f/g)^2*c+ \\
& (b^*g-2^*c^*f)/g^*(x+f/g)+(a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)))/(x+f/g))^*c^ \\
& f^2
\end{aligned}$$


---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{(ex + d)(gx + f)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)/((e\*x + d)\*(g\*x + f)^2), x, algorithm="maxima")

[Out] integrate(sqrt(c\*x^2 + b\*x + a)/((e\*x + d)\*(g\*x + f)^2), x)

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)^2),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**2,x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS** [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.860 \quad \int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^3} dx$$

**Optimal.** Leaf size=673

$$\begin{aligned} & \frac{g(b^2 - 4ac) \tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{8(ef-dg)(ag^2-bfg+cf^2)^{3/2}} \\ & + \frac{e\sqrt{ae^2-bde+cd^2} \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^3} \\ & - \frac{e^2\sqrt{ag^2-bfg+cf^2} \tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef-dg)^3} + \frac{e^2(2cf-bg) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}g(ef-dg)^3} \\ & - \frac{g\sqrt{a+bx+cx^2}(-2ag+x(2cf-bg)+bf)}{4(f+gx)^2(ef-dg)(ag^2-bfg+cf^2)} + \frac{e(2cf-bg) \tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2g(ef-dg)^2\sqrt{ag^2-bfg+cf^2}} \\ & + \frac{e\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)^2} - \frac{\sqrt{ce} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^2} - \frac{e(2cd-be) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}(ef-dg)^3} \end{aligned}$$

[Out] (e\*Sqrt[a + b\*x + c\*x^2])/((e\*f - d\*g)^2\*(f + g\*x)) - (g\*(b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*Sqrt[a + b\*x + c\*x^2])/(4\*(e\*f - d\*g)\*(c\*f^2 - b\*f\*g + a\*g^2)\*(f + g\*x)^2) - (e\*(2\*c\*d - b\*e)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(2\*Sqrt[c]\*(e\*f - d\*g)^3) + (e^2\*(2\*c\*f - b\*g)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(2\*Sqrt[c]\*g\*(e\*f - d\*g)^3) - (Sqrt[c]\*e\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(g\*(e\*f - d\*g)^2) + (e\*Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*ArcTanh[(b\*d - 2\*a\*e + (2\*c\*d - b\*e)\*x)/(2\*Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*Sqrt[a + b\*x + c\*x^2])])/(e\*f - d\*g)^3 + ((b^2 - 4\*a\*c)\*g\*ArcTanh[(b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)/(2\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2]\*Sqrt[a + b\*x + c\*x^2])])/(8\*(e\*f - d\*g)\*(c\*f^2 - b\*f\*g + a\*g^2)^(3/2)) + (e\*(2\*c\*f - b\*g)\*ArcTanh[(b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)/(2\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2]\*Sqrt[a + b\*x + c\*x^2])])/(2\*g\*(e\*f - d\*g)^2\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2]) - (e^2\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2]\*ArcTanh[(b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)/(2\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2]\*Sqrt[a + b\*x + c\*x^2])])/(g\*(e\*f - d\*g)^3)

**Rubi [A]** time = 2.08197, antiderivative size = 673, normalized size of antiderivative = 1., number of rules used = 23, number of rules used = 8, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$

$$\begin{aligned} & \frac{g(b^2 - 4ac) \tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{8(ef-dg)(ag^2-bfg+cf^2)^{3/2}} \\ & + \frac{e\sqrt{ae^2-bde+cd^2} \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^3} \\ & - \frac{e^2\sqrt{ag^2-bfg+cf^2} \tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g(ef-dg)^3} + \frac{e^2(2cf-bg) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}g(ef-dg)^3} \\ & - \frac{g\sqrt{a+bx+cx^2}(-2ag+x(2cf-bg)+bf)}{4(f+gx)^2(ef-dg)(ag^2-bfg+cf^2)} + \frac{e(2cf-bg) \tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2g(ef-dg)^2\sqrt{ag^2-bfg+cf^2}} \\ & + \frac{e\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)^2} - \frac{\sqrt{ce} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{g(ef-dg)^2} - \frac{e(2cd-be) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{c}(ef-dg)^3} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x + c\*x^2]/((d + e\*x)\*(f + g\*x)^3), x]

[Out] 
$$\frac{(e\sqrt{a+bx+cx^2})/((ef-dg)^2(f+gx)) - (g(bf-2ag+(2cf-bg)x)\sqrt{a+bx+cx^2})/(4(ef-dg)(cf^2-bfg+ag^2)(f+gx)^2) - (e(2cd-be)\operatorname{ArcTanh}((b+2cx)/(2\sqrt{c}\sqrt{a+bx+cx^2}))) / (2\sqrt{c}(ef-dg)^3) + (e^2(2cf-bg)\operatorname{ArcTanh}((b+2cx)/(2\sqrt{c}\sqrt{a+bx+cx^2}))) / (2\sqrt{c}g(ef-dg)^3) - (\sqrt{c}e\operatorname{ArcTanh}((b+2cx)/(2\sqrt{c}\sqrt{a+bx+cx^2}))) / (g(ef-dg)^2) + (e\sqrt{cd^2-bde+ae^2}\operatorname{ArcTanh}((bd-2ae+(2cd-be)x)/(2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx+cx^2}))) / (ef-dg)^3 + ((b^2-4ac)g\operatorname{ArcTanh}((bf-2ag+(2cf-bg)x)/(2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}))) / (8(ef-dg)(cf^2-bfg+ag^2)^{3/2}) + (e(2cf-bg)\operatorname{ArcTanh}((bf-2ag+(2cf-bg)x)/(2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}))) / (2g(ef-dg)^2\sqrt{cf^2-bfg+ag^2}) - (e^2\sqrt{cf^2-bfg+ag^2}\operatorname{ArcTanh}((bf-2ag+(2cf-bg)x)/(2\sqrt{cf^2-bfg+ag^2}\sqrt{a+bx+cx^2}))) / (g(ef-dg)^3)$$

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+b\*x+a)\*\*(1/2)/(e\*x+d)/(g\*x+f)\*\*3, x)

[Out] Timed out

**Mathematica [A]** time = 2.99405, size = 523, normalized size = 0.78

$$\frac{-\log(f+gx)(g(8a^2e^2g^2-4abeg(dg+3ef)+b^2(-d^2g^2+6defg+3e^2f^2))+4acg(d^2g^2+3e^2f^2)-4bcef^2(3dg+ef)+8c^2def^3)}{(g(ag-bf)+cf^2)^{3/2}} + \frac{g(8a^2e^2g^2-4abeg(dg+3ef)+b^2(-d^2g^2+6defg+3e^2f^2))+4acg(d^2g^2+3e^2f^2)-4bcef^2(3dg+ef)+8c^2def^3}{(g(ag-bf)+cf^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[a + b\*x + c\*x^2]/((d + e\*x)\*(f + g\*x)^3), x]

[Out] 
$$\frac{((2(ef-dg)\sqrt{a+x(b+cx)})^2(2ef-2dg+(2cf(e^2f+dg)-g(3be^2f+bdg-4ae^2g))(f+gx))/(cf^2+g^2(-bf+ag))) / (f+gx)^2 + 8e\sqrt{cd^2+e(-bd+ae)}\operatorname{Log}[d+ex] - ((8c^2de^2f^3-4b^2ce^2f^2(ef+3dg))+4acg(3e^2f^2+d^2g^2)+g(8a^2e^2g^2-4abeg(dg+3ef)+b^2(-d^2g^2+6defg+3e^2f^2)))\operatorname{Log}[f+gx]) / (cf^2+g^2(-bf+ag))^{3/2} - 8e\sqrt{cd^2+e(-bd+ae)}\operatorname{Log}[-(bd)+2ae-2cdx+be^2x+2\sqrt{cd^2+e(-bd+ae)}]\sqrt{a+x(b+cx)} + ((8c^2de^2f^3-4b^2ce^2f^2(ef+3dg))+4acg(3e^2f^2+d^2g^2)+g(8a^2e^2g^2-4abeg(dg+3ef)+b^2(-d^2g^2+6defg+3e^2f^2)))\operatorname{Log}[-(bf)+2ag-2cfx+bgx+2\sqrt{cf^2+g^2(-bf+ag)}]\sqrt{a+x(b+cx)} / (cf^2+g^2(-bf+ag))^{3/2}}{(8(ef-dg)^3)}$$

**Maple [B]** time = 0.032, size = 6714, normalized size = 10.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^3,x)`

[Out] result too large to display

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{(ex + d)(gx + f)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)^3),x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)^3), x)`

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)^3),x, algorithm="fricas")`

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**3,x)`

[Out] Timed out

---

**GIAC/XCAS [A]** time = 1.60089, size = 4, normalized size = 0.01

$sage_0x$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)^3),x, algorithm="giac")`

[Out] `sage0*x`



$$3.861 \quad \int \frac{\sqrt{a+bx+cx^2}}{(d+ex)(f+gx)^4} dx$$

**Optimal.** Leaf size=933

$$\begin{aligned} & \frac{(2cf - bg) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right) e^3}{2\sqrt{cg}(ef - dg)^4} \\ & - \frac{\sqrt{cf^2 - bgf + ag^2} \tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right) e^3}{g(ef - dg)^4} \\ & - \frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right) e^2}{g(ef - dg)^3} - \frac{(2cd - be) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right) e^2}{2\sqrt{c}(ef - dg)^4} \\ & + \frac{\sqrt{cd^2 - bed + ae^2} \tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\sqrt{cx^2+bx+a}}\right) e^2}{(ef - dg)^4} \\ & + \frac{(2cf - bg) \tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right) e^2}{2g(ef - dg)^3\sqrt{cf^2 - bgf + ag^2}} + \frac{\sqrt{cx^2 + bx + ae^2}}{(ef - dg)^3(f + gx)} \\ & + \frac{(b^2 - 4ac) g \tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right) e}{8(ef - dg)^2(cf^2 - bgf + ag^2)^{3/2}} \\ & - \frac{g(bf - 2ag + (2cf - bg)x)\sqrt{cx^2 + bx + ae}}{4(ef - dg)^2(cf^2 - bgf + ag^2)(f + gx)^2} + \frac{g^2(cx^2 + bx + a)^{3/2}}{3(ef - dg)(cf^2 - bgf + ag^2)(f + gx)^3} \\ & + \frac{(b^2 - 4ac) g(2cf - bg) \tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)}{16(ef - dg)(cf^2 - bgf + ag^2)^{5/2}} \\ & - \frac{g(2cf - bg)(bf - 2ag + (2cf - bg)x)\sqrt{cx^2 + bx + ae}}{8(ef - dg)(cf^2 - bgf + ag^2)^2(f + gx)^2} \end{aligned}$$

[Out] (e^2\*Sqrt[a + b\*x + c\*x^2])/((e\*f - d\*g)^3\*(f + g\*x)) - (g\*(2\*c\*f - b\*g)\*(b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*Sqrt[a + b\*x + c\*x^2])/(8\*(e\*f - d\*g)\*(c\*f^2 - b\*f\*g + a\*g^2)^2\*(f + g\*x)^2) - (e\*g\*(b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)\*Sqrt[a + b\*x + c\*x^2])/(4\*(e\*f - d\*g)^2\*(c\*f^2 - b\*f\*g + a\*g^2)\*(f + g\*x)^2) + (g^2\*(a + b\*x + c\*x^2)^(3/2))/(3\*(e\*f - d\*g)\*(c\*f^2 - b\*f\*g + a\*g^2)\*(f + g\*x)^3) - (e^2\*(2\*c\*d - b\*e)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(2\*Sqrt[c]\*(e\*f - d\*g)^4) + (e^3\*(2\*c\*f - b\*g)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(2\*Sqrt[c]\*g\*(e\*f - d\*g)^4) - (Sqrt[c]\*e^2\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(g\*(e\*f - d\*g)^3) + (e^2\*Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*ArcTanh[(b\*d - 2\*a\*e + (2\*c\*d - b\*e)\*x)/(2\*Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*Sqrt[a + b\*x + c\*x^2])])/(e\*f - d\*g)^4 + ((b^2 - 4\*a\*c)\*g\*(2\*c\*f - b\*g)\*ArcTanh[(b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)/(2\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2]\*Sqrt[a + b\*x + c\*x^2])])/(16\*(e\*f - d\*g)\*(c\*f^2 - b\*f\*g + a\*g^2)^(5/2)) + ((b^2 - 4\*a\*c)\*e\*g\*ArcTanh[(b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)/(2\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2]\*Sqrt[a + b\*x + c\*x^2])])/(8\*(e\*f - d\*g)^2\*(c\*f^2 - b\*f\*g + a\*g^2)^(3/2)) + (e^2\*(2\*c\*f - b\*g)\*ArcTanh[(b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)/(2\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2]\*Sqrt[a + b\*x + c\*x^2])])/(2\*g\*(e\*f - d\*g)^3\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2]) - (e^3\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2]\*ArcTanh[(b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)/(2\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2]\*Sqrt[a + b\*x + c\*x^2])])/(g\*(e\*f - d\*g)^4)

**Rubi [A]** time = 2.99643, antiderivative size = 933, normalized size of antiderivative = 1., number of

steps used = 27, number of rules used = 9, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.31$

$$\begin{aligned}
& \frac{(2cf - bg) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right) e^3}{2\sqrt{c}g(ef - dg)^4} \\
& - \frac{\sqrt{cf^2 - bgf + ag^2} \tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right) e^3}{g(ef - dg)^4} \\
& - \frac{\sqrt{c} \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right) e^2}{g(ef - dg)^3} - \frac{(2cd - be) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right) e^2}{2\sqrt{c}(ef - dg)^4} \\
& + \frac{\sqrt{cd^2 - bed + ae^2} \tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\sqrt{cx^2+bx+a}}\right) e^2}{(ef - dg)^4} \\
& + \frac{(2cf - bg) \tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right) e^2}{2g(ef - dg)^3\sqrt{cf^2 - bgf + ag^2}} + \frac{\sqrt{cx^2 + bx + ae^2}}{(ef - dg)^3(f + gx)} \\
& + \frac{(b^2 - 4ac) g \tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right) e}{8(ef - dg)^2(cf^2 - bgf + ag^2)^{3/2}} \\
& - \frac{g(bf - 2ag + (2cf - bg)x)\sqrt{cx^2 + bx + ae}}{4(ef - dg)^2(cf^2 - bgf + ag^2)(f + gx)^2} + \frac{g^2(cx^2 + bx + a)^{3/2}}{3(ef - dg)(cf^2 - bgf + ag^2)(f + gx)^3} \\
& + \frac{(b^2 - 4ac) g(2cf - bg) \tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)}{16(ef - dg)(cf^2 - bgf + ag^2)^{5/2}} \\
& - \frac{g(2cf - bg)(bf - 2ag + (2cf - bg)x)\sqrt{cx^2 + bx + ae}}{8(ef - dg)(cf^2 - bgf + ag^2)^2(f + gx)^2}
\end{aligned}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x + c\*x^2]/((d + e\*x)\*(f + g\*x)^4), x]

[Out]  $(e^{2*\text{Sqrt}[a + b*x + c*x^2]})/((e*f - d*g)^3*(f + g*x)) - (g*(2*c*f - b*g)*(b*f - 2*a*g + (2*c*f - b*g)*x)*\text{Sqrt}[a + b*x + c*x^2])/(8*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)^2*(f + g*x)^2) - (e*g*(b*f - 2*a*g + (2*c*f - b*g)*x)*\text{Sqrt}[a + b*x + c*x^2])/(4*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*(f + g*x)^2) + (g^2*(a + b*x + c*x^2)^{(3/2)})/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g*x)^3) - (e^{2*(2*c*d - b*e)}*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(2*\text{Sqrt}[c]*(e*f - d*g)^4) + (e^{3*(2*c*f - b*g)}*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(2*\text{Sqrt}[c]*g*(e*f - d*g)^4) - (\text{Sqrt}[c]*e^{2*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])})/(g*(e*f - d*g)^3) + (e^{2*\text{Sqrt}[c*d^2 - b*d*e + a*e^2]}*\text{ArcTanh}[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*\text{Sqrt}[a + b*x + c*x^2])])/(e*f - d*g)^4 + ((b^2 - 4*a*c)*g*(2*c*f - b*g)*\text{ArcTanh}[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*\text{Sqrt}[c*f^2 - b*f*g + a*g^2]*\text{Sqrt}[a + b*x + c*x^2])])/(16*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)^{(5/2)}) + ((b^2 - 4*a*c)*e*g*\text{ArcTanh}[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*\text{Sqrt}[c*f^2 - b*f*g + a*g^2]*\text{Sqrt}[a + b*x + c*x^2])])/(8*(e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)^{(3/2)}) + (e^{2*(2*c*f - b*g)}*\text{ArcTanh}[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*\text{Sqrt}[c*f^2 - b*f*g + a*g^2]*\text{Sqrt}[a + b*x + c*x^2])])/(2*g*(e*f - d*g)^3*\text{Sqrt}[c*f^2 - b*f*g + a*g^2]) - (e^{3*\text{Sqrt}[c*f^2 - b*f*g + a*g^2]}*\text{ArcTanh}[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*\text{Sqrt}[c*f^2 - b*f*g + a*g^2]*\text{Sqrt}[a + b*x + c*x^2])])/(g*(e*f - d*g)^4)$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**4,x)`

[Out] Timed out

**Mathematica [A]** time = 6.93499, size = 1421, normalized size = 1.52

$$\frac{\sqrt{cd^2 - bed + ae^2} \sqrt{a + x(b + cx)} \log(d + ex) e^2}{(ef - dg)^4 \sqrt{cx^2 + bx + a}}$$

$$+ \frac{\sqrt{cd^2 - bed + ae^2} \sqrt{a + x(b + cx)} \log\left(-bd - 2cxd + 2ae + bex + 2\sqrt{cd^2 - bed + ae^2} \sqrt{cx^2 + bx + a}\right) e^2}{(ef - dg)^4 \sqrt{cx^2 + bx + a}}$$

$$+ \frac{\sqrt{a + x(b + cx)} \left( \frac{4cef^2 + 2cdgf - 5begf - bdg^2 + 6aeg^2}{12(ef - dg)^2 (cf^2 - bgf + ag^2) (f + gx)^2} \right)}{24(ef - dg)^3 (cf^2 - bgf + ag^2)^2 (f + gx)}$$

$$+ \frac{8c^2e^2f^4 - 26bce^2gf^3 + 20c^2degf^3 - 4c^2d^2g^2f^2 + 15b^2e^2g^2f^2 + 44ace^2g^2f^2 - 26bcdeg^2f^2 + 4bcd^2g^3f - 42abe^2g^3f + 12b^2cde^2g^3f - 32ac^2deg^3f^2 + 12b^2ce^3gf^4 - 40bc^2de^2gf^4 + 5b^3e^3g^2f^3 + 60abce^3g^2f^3 + 8ac^2de^2g^2f^3 - 42b^2cde^2g^2f^3 - 16c^3de^2f^5 - 32ac^2e^3gf^4 - 12b^2ce^3gf^4 + 40bc^2de^2gf^4 + 5b^3e^3g^2f^3 + 60abce^3g^2f^3 + 8ac^2de^2g^2f^3 - 42b^2cde^2g^2f^3}{3(dg - ef)(f + gx)^3}$$

$$+ \frac{1}{3(dg - ef)(f + gx)^3}$$

$$+ \frac{(8bc^2e^3f^5 - 16c^3de^2f^5 - 32ac^2e^3gf^4 - 12b^2ce^3gf^4 + 40bc^2de^2gf^4 + 5b^3e^3g^2f^3 + 60abce^3g^2f^3 + 8ac^2de^2g^2f^3 - 42b^2cde^2g^2f^3 - 16c^3de^2f^5 - 32ac^2e^3gf^4 - 12b^2ce^3gf^4 + 40bc^2de^2gf^4 + 5b^3e^3g^2f^3 + 60abce^3g^2f^3 + 8ac^2de^2g^2f^3 - 42b^2cde^2g^2f^3)}{3(dg - ef)(f + gx)^3}$$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*(f + g*x)^4),x]`

$$\text{[Out] } \frac{\sqrt{a + x(b + cx)} \left( -\frac{1}{(3(-ef) + dg)(f + gx)^3} + \frac{4c^2e^2f^2 + 2c^2d^2fg - 5b^2e^2fg - b^2d^2g^2 + 6a^2e^2g^2}{12(ef - dg)^2 (cf^2 - bgf + ag^2) (f + gx)^2} + \frac{8c^2e^2f^4 + 20c^2d^2e^2f^3g - 26b^2c^2e^2f^3g - 4c^2d^2e^2f^2g^2 - 26b^2c^2d^2e^2f^2g^2 + 15b^2e^2f^2g^2 + 44a^2c^2e^2f^2g^2 + 4b^2c^2d^2e^2fg^3 + 12b^2d^2e^2fg^3 - 4a^2c^2d^2e^2fg^3 - 42a^2b^2e^2fg^3 - 3b^2d^2e^2fg^4 + 8a^2c^2d^2e^2fg^4 - 6a^2b^2d^2e^2fg^4 + 24a^2e^2fg^4}{24(ef - dg)^3 (cf^2 - bgf + ag^2)^2 (f + gx)} + \frac{e^2 \sqrt{cd^2 - bed + ae^2} \sqrt{a + x(b + cx)} \log(d + ex)}{(ef - dg)^4 \sqrt{cx^2 + bx + a}} + \frac{(-16c^3d^2e^2f^5 + 8b^2c^2e^3f^5 + 40b^2c^2d^2e^2f^4g - 12b^2c^2e^3f^4g - 32a^2c^2e^3f^4g - 42b^2c^2d^2e^2f^3g^2 + 8a^2c^2d^2e^2f^3g^2 + 5b^3e^3f^3g^2 + 60abce^3f^3g^2 + 8ac^2de^2f^3g^2 - 42b^2cde^2f^3g^2 - 16c^3de^2f^5 - 32ac^2e^3fg^4 - 12b^2ce^3fg^4 + 40bc^2de^2fg^4 + 5b^3e^3g^2f^3 + 60abce^3g^2f^3 + 8ac^2de^2g^2f^3 - 42b^2cde^2g^2f^3 - 16c^3de^2f^5 - 32ac^2e^3fg^4 - 12b^2ce^3fg^4 + 40bc^2de^2fg^4 + 5b^3e^3g^2f^3 + 60abce^3g^2f^3 + 8ac^2de^2g^2f^3 - 42b^2cde^2g^2f^3)}{3(dg - ef)(f + gx)^3} \right)}{(d + ex)(f + gx)^4}$$

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**Maple [B]** time = 0.039, size = 11995, normalized size = 12.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^4,x)`

[Out] result too large to display

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{(ex + d)(gx + f)^4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)^4),x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)^4), x)`

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)^4),x, algorithm="fricas")`

[Out] Exception raised: TypeError

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**4,x)`

[Out] Timed out

---

**GIAC/XCAS [A]** time = 14.2537, size = 4, normalized size = 0.

*sage<sub>0</sub>x*

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)*(g*x + f)^4),x, algorithm="giac")`

[Out] *sage<sub>0</sub>x*

$$3.862 \quad \int \frac{(f+gx)^3(a+bx+cx^2)^{3/2}}{d+ex} dx$$

Optimal. Leaf size=1098

result too large to display

[Out]  $-\left((3*(7*b^5*e^5*g^3 - 512*c^5*d^2*(e*f - d*g)^3 + 128*c^4*e*(5*b*d - 4*a*e)*(e*f - d*g)^3 - 4*b^3*c*e^4*g^2*(9*b*e*f - 3*b*d*g + 8*a*e*g) + 8*b*c^2*e^3*g*(2*a^2*e^2*g^2 + 6*a*b*e*g*(3*e*f - d*g) + 3*b^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)) - 32*b*c^3*e^2*(2*b*(e*f - d*g)^3 + 3*a*e*g*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)) + 2*c*e*(8*c*e*(2*c*d - b*e)*(24*c^2*e^2*f^3 + 7*b^2*d*e*g^3 - 4*c*d*g^2*(9*b*e*f - 3*b*d*g + a*e*g)) - 2*(8*c^2*d^2 - 4*b*c*d*e - (3*b^2*e^2)/2 + 6*a*c*e^2)*g*(7*b^2*e^2*g^2 - 4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)))*x)*\text{Sqrt}[a + b*x + c*x^2])/(1536*c^4*e^6) + ((7*b^3*e^3*g^3 + 64*c^3*(e*f - d*g)^3 - 4*b*c*e^2*g^2*(9*b*e*f - 3*b*d*g + a*e*g) + 24*b*c^2*e*g*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2) + 2*c*e*g*(7*b^2*e^2*g^2 - 4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*x)*(a + b*x + c*x^2)^{(3/2)})/(192*c^3*e^4) + (g^2*(36*c*e*f - 22*c*d*g - 7*b*e*g)*(a + b*x + c*x^2)^{(5/2)})/(60*c^2*e^2) + (g^3*(d + e*x)*(a + b*x + c*x^2)^{(5/2)})/(6*c*e^2) + ((4*c*e*(2*c*d - b*e)*(8*c*e*(b*d - 2*a*e)*(24*c^2*e^2*f^3 + 7*b^2*d*e*g^3 - 4*c*d*g^2*(9*b*e*f - 3*b*d*g + a*e*g)) - d*(8*b*c*d - 3*b^2*e - 4*a*c*e)*g*(7*b^2*e^2*g^2 - 4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))) - 2*(4*c^2*d^2 - (b^2*e^2)/2 - 2*c*e*(b*d - a*e))*(8*c*e*(2*c*d - b*e)*(24*c^2*e^2*f^3 + 7*b^2*d*e*g^3 - 4*c*d*g^2*(9*b*e*f - 3*b*d*g + a*e*g)) - 2*(8*c^2*d^2 - 4*b*c*d*e - (3*b^2*e^2)/2 + 6*a*c*e^2)*g*(7*b^2*e^2*g^2 - 4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(3072*c^(9/2)*e^7) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*(e*f - d*g)^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])]/e^7$

Rubi [A] time = 8.09025, antiderivative size = 1098, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$

$$\frac{(d+ex)(cx^2+bx+a)^{5/2}g^3}{6ce^2} + \frac{(36cef-22cdg-7beg)(cx^2+bx+a)^{5/2}g^2}{60c^2e^2}$$

$$+ \frac{(7b^3e^3g^3 - 4bce^2(9bef - 3bdg + aeg)g^2 + 24bc^2e(3e^2f^2 - 3degf + d^2g^2)g + 2ce(24(3e^2f^2 - 3degf + d^2g^2)c^2 - 4eg(9b^2d - 4ace)g^2))}{192c^3e^4}$$

$$+ \frac{(4ce(2cd - be)(8ce(bd - 2ae)(24c^2e^2f^3 + 7b^2deg^3 - 4cdg^2(9bef - 3bdg + aeg)) - d(-3eb^2 + 8cdb - 4ace)g(24(3e^2f^2 - 3degf + d^2g^2)c^2 - 4eg(9b^2d - 4ace)g^2))}{192c^3e^4}$$

$$+ \frac{(cd^2 - bed + ae^2)^{3/2}(ef - dg)^3 \tanh^{-1}\left(\frac{bd - 2ae + (2cd - be)x}{2\sqrt{cd^2 - bed + ae^2}\sqrt{cx^2 + bx + a}}\right)}{e^7}$$

$$+ \frac{(3(-512d^2(ef - dg)^3c^5 + 128e(5bd - 4ae)(ef - dg)^3c^4 - 32be^2(2b(ef - dg)^3 + 3aeg(3e^2f^2 - 3degf + d^2g^2))c^3 + 8be^3(3e^2f^2 - 3degf + d^2g^2))}{192c^3e^4}$$

Antiderivative was successfully verified.

[In] Int[((f + g\*x)^3\*(a + b\*x + c\*x^2)^(3/2))/(d + e\*x), x]

[Out]  $-\left((3*(7*b^5*e^5*g^3 - 512*c^5*d^2*(e*f - d*g)^3 + 128*c^4*e*(5*b*d - 4*a*e)*(e*f - d*g)^3 - 4*b^3*c*e^4*g^2*(9*b*e*f - 3*b*d*g + 8*a*e*g) + 8*b*c^2*e^3*g*(2*a^2*e^2*g^2 + 6*a*b*e*g*(3*e*f - d*g) + 3*b^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)) - 32*b*c^3*e^2*(2*b*(e*f - d*g)^3 + 3*a*e*g*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)) + 2*c*e*(8*c*e*(2*c*d - b*e)*(24*c^2*e^2*f^3 + 7*b^2*d*e*g^3 - 4*c*d*g^2*(9*b*e*f - 3*b*d*g + a*e*g)) - 2*(8*c^2*d^2 - 4*b*c*d*e - (3*b^2*e^2)/2 + 6*a*c*e^2)*g*(7*b^2*e^2*g^2 - 4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)))*x)*\text{Sqrt}[a + b*x + c*x^2])/(1536*c^4*e^6) + ((7*b^3*e^3*g^3 + 64*c^3*(e*f - d*g)^3 - 4*b*c*e^2*g^2*(9*b*e*f - 3*b*d*g + a*e*g) + 24*b*c^2*e*g*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2) + 2*c*e*g*(7*b^2*e^2*g^2 - 4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*x)*(a + b*x + c*x^2)^{(3/2)})/(192*c^3*e^4) + (g^2*(36*c*e*f - 22*c*d*g - 7*b*e*g)*(a + b*x + c*x^2)^{(5/2)})/(60*c^2*e^2) + (g^3*(d + e*x)*(a + b*x + c*x^2)^{(5/2)})/(6*c*e^2) + ((4*c*e*(2*c*d - b*e)*(8*c*e*(b*d - 2*a*e)*(24*c^2*e^2*f^3 + 7*b^2*d*e*g^3 - 4*c*d*g^2*(9*b*e*f - 3*b*d*g + a*e*g)) - d*(8*b*c*d - 3*b^2*e - 4*a*c*e)*g*(7*b^2*e^2*g^2 - 4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))) - 2*(4*c^2*d^2 - (b^2*e^2)/2 - 2*c*e*(b*d - a*e))*(8*c*e*(2*c*d - b*e)*(24*c^2*e^2*f^3 + 7*b^2*d*e*g^3 - 4*c*d*g^2*(9*b*e*f - 3*b*d*g + a*e*g)) - 2*(8*c^2*d^2 - 4*b*c*d*e - (3*b^2*e^2)/2 + 6*a*c*e^2)*g*(7*b^2*e^2*g^2 - 4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))))*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(3072*c^(9/2)*e^7) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*(e*f - d*g)^3*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])]/e^7$

$$\begin{aligned}
& + b*x + c*x^2] / (1536*c^4*e^6) + ((7*b^3*e^3*g^3 + 64*c^3*(e*f - d*g)^3 - 4*b*c*e^2*g^2*(9*b*e*f - 3*b*d*g + a*e*g) + 24*b*c^2*e*g*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2) + 2*c*e*g*(7*b^2*e^2*g^2 - 4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*x) * (a + b*x + c*x^2)^(3/2) / (192*c^3*e^4) + (g^2*(36*c*e*f - 22*c*d*g - 7*b*e*g) * (a + b*x + c*x^2)^(5/2) / (60*c^2*e^2) + (g^3*(d + e*x) * (a + b*x + c*x^2)^(5/2) / (6*c*e^2) + ((4*c*e*(2*c*d - b*e) * (8*c*e*(b*d - 2*a*e) * (24*c^2*e^2*f^3 + 7*b^2*d*e*g^3 - 4*c*d*g^2*(9*b*e*f - 3*b*d*g + a*e*g)) - d*(8*b*c*d - 3*b^2*e - 4*a*c*e) * g*(7*b^2*e^2*g^2 - 4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))) - 2*(4*c^2*d^2 - (b^2*e^2)/2 - 2*c*e*(b*d - a*e)) * (8*c*e*(2*c*d - b*e) * (24*c^2*e^2*f^3 + 7*b^2*d*e*g^3 - 4*c*d*g^2*(9*b*e*f - 3*b*d*g + a*e*g)) - 2*(8*c^2*d^2 - 4*b*c*d*e - (3*b^2*e^2)/2 + 6*a*c*e^2) * g*(7*b^2*e^2*g^2 - 4*c*e*g*(9*b*e*f - 3*b*d*g + a*e*g) + 24*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2)))) * ArcTanh[(b + 2*c*x) / (2*sqrt[c]*sqrt[a + b*x + c*x^2])]) / (3072*c^(9/2)*e^7) + ((c*d^2 - b*d*e + a*e^2)^(3/2) * (e*f - d*g)^3 * ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x) / (2*sqrt[c*d^2 - b*d*e + a*e^2]*sqrt[a + b*x + c*x^2])]) / e^7
\end{aligned}$$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x+f)**3*(c*x**2+b*x+a)**(3/2)/(e*x+d),x)`

[Out] Timed out

**Mathematica [A]** time = 6.70757, size = 1968, normalized size = 1.79

result too large to display

Antiderivative was successfully verified.

[In] `Integrate[((f + g*x)^3*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]`

[Out] 
$$\begin{aligned}
& (((7680*c^5*d^2*e^3*f^3 - 9600*b*c^4*d*e^4*f^3 + 960*b^2*c^3*e^5*f^3 + 10240*a*c^4*e^5*f^3 - 23040*c^5*d^3*e^2*f^2*g + 28800*b*c^4*d^2*e^3*f^2*g - 2880*b^2*c^3*d*e^4*f^2*g - 30720*a*c^4*d*e^4*f^2*g - 1080*b^3*c^2*e^5*f^2*g + 7200*a*b*c^3*e^5*f^2*g + 23040*c^5*d^4*e*f*g^2 - 28800*b*c^4*d^3*e^2*f*g^2 + 2880*b^2*c^3*d^2*e^3*f*g^2 + 30720*a*c^4*d^2*e^3*f*g^2 + 1080*b^3*c^2*d*e^4*f*g^2 - 7200*a*b*c^3*d*e^4*f*g^2 + 540*b^4*c*e^5*f*g^2 - 3600*a*b^2*c^2*e^5*f*g^2 + 4608*a^2*c^3*e^5*f*g^2 - 7680*c^5*d^5*g^3 + 9600*b*c^4*d^4*e*g^3 - 960*b^2*c^3*d^3*e^2*g^3 - 10240*a*c^4*d^3*e^2*g^3 - 360*b^3*c^2*d^2*e^3*g^3 + 2400*a*b*c^3*d^2*e^3*g^3 - 180*b^4*c*d*e^4*g^3 + 1200*a*b^2*c^2*d*e^4*g^3 - 1536*a^2*c^3*d*e^4*g^3 - 105*b^5*e^5*g^3 + 760*a*b^3*c*e^5*g^3 - 1296*a^2*b*c^2*e^5*g^3) / (7680*c^4*e^6) + ((-1920*c^4*d*e^3*f^3 + 2240*b*c^3*e^4*f^3 + 5760*c^4*d^2*e^2*f^2*g - 6720*b*c^3*d*e^3*f^2*g + 360*b^2*c^2*e^4*f^2*g + 7200*a*c^3*e^4*f^2*g - 5760*c^4*d^3*e*f*g^2 + 6720*b*c^3*d^2*e^2*f*g^2 - 360*b^2*c^2*d*e^3*f*g^2 - 7200*a*c^3*d*e^3*f*g^2 - 180*b^3*c*e^4*f*g^2 + 1008*a*b*c^2*e^4*f*g^2 + 1920*c^4*d^4*g^3 - 2240*b*c^3*d^3*e*g^3 + 120*b^2*c^2*d^2*e^2*g^3 + 2400*a*c^3*d^2*e^2*g^3 + 60*b^3*c*d*e^3*g^3 - 336*a*b*c^2*d*e^3*g^3 + 35*b^4*e^4*g^3 - 216*a*b^2*c*e^4*g^3 + 240*a^2*c^2*e^4*g^3)*x) / (3840*c^3*e^5) + ((320*c^3*e^3*f^3 - 960*c^3*d*e^2*f^2*g + 1080*b*c^2*e^3*f^2*g + 960*c^3*d^2*e*f*g^2 - 1080*b*c^2*d*e^2*f*g^2 + 36*b^2*c*e^3*f*g^2 + 1152*a*c^2*e^3*f*g^2 - 320*c^3*d^3*g^3 + 360*b*c^2*d^2*e*g^3 - 12*b^2*c*d*e^2*g^3 - 384*a*c^2*d*e^2*g^3 - 7*b^3*e^3*g^3 + 36*a*b*c*e^3*g^3)*x^2) / (960*c^2*e^4) + (g*(360*c^2*e^2*f^2 - 360*c^2*d*e
\end{aligned}$$

$$\begin{aligned}
& f^2g + 396bc^2e^2fg + 120c^2d^2g^2 - 132b^2c^2d^2e^2g^2 + 3b^2e^2g^2 + 140a^2c^2e^2g^2)x^3)/(480c^2e^3) + (g^2(36c^2ef - 12c^2d^2g + 13b^2e^2g)x^4)/(60e^2) + (c^2g^3x^5)/(6e)) \cdot (a + x(b + cx))^{3/2}) / (a + bx + cx^2) + ((c^2d^2 - b^2d^2e + a^2e^2)^{3/2} \cdot (ef - d^2g)^3 \cdot (a + x(b + cx))^{3/2} \cdot \text{Log}[d + ex]) / (e^7(a + bx + cx^2)^{3/2}) + (((c^2d^3f^3)/e^4) - (3a^2c^2d^2f^3)/(2e^2) + (3a^2b^2f^3)/(4e) + (3c^2d^4f^2g)/e^5 - (9a^2b^2d^2f^2g)/(4e^2) + (9a^2f^2g)/(8e) + (9b^4f^2g)/(128c^2e) - (3c^2d^5f^2g^2)/e^6 - (9b^2d^3f^2g^2)/(8e^4) + (9a^2b^2d^2f^2g^2)/(4e^3) - (9b^4d^2f^2g^2)/(128c^2e^2) + (9a^2b^3f^2g^2)/(32c^2e) + (c^2d^6g^3)/e^7 - (3b^2c^2d^5g^3)/(2e^6) + (3b^2d^4g^3)/(8e^5) - (3a^2b^2d^3g^3)/(4e^4) + (b^3d^3g^3)/(16c^2e^4) + (3b^4d^2g^3)/(128c^2e^3) + (3b^5d^2g^3)/(256c^3e^2) - (3a^2b^3d^2g^3)/(32c^2e^2) + (7b^6g^3)/(1024c^4e) + (9a^2b^2g^3)/(64c^2e) - (3b^4g^2(3b^2f + 5a^2g))/(256c^3e) + (3(b^2c^2d^2f^3 + 3a^2c^2d^2f^2g))/(2e^3) - ((b^2f + a^2g) \cdot (b^2f^2 + 8a^2b^2fg + a^2g^2))/(16c^2e) - (3(b^2d^2f^3 + 3a^2d^2f^2g^2))/(8e^2) - (9(b^2c^2d^3f^2g + a^2c^2d^3f^2g^2))/(2e^4) + (3(b^3d^2f^2g + 3a^2b^2d^2f^2g^2 + a^2b^2d^2g^3))/(16c^2e^2) + (3(3b^2d^2f^2g + a^2d^2g^3))/(8e^3) - (3(b^3d^2f^2g^2 + a^2b^2d^2g^3))/(16c^2e^3) + (3(3b^2c^2d^4f^2g^2 + a^2c^2d^4g^3))/(2e^5)) \cdot (a + x(b + cx))^{3/2} \cdot \text{Log}[b + 2cx + 2\sqrt{c}\sqrt{a + bx + cx^2}]] / (\sqrt{c} \cdot (a + bx + cx^2)^{3/2}) - ((c^2d^2 - b^2d^2e + a^2e^2)^{3/2} \cdot (ef - d^2g)^3 \cdot (a + x(b + cx))^{3/2} \cdot \text{Log}[-(b^2d) + 2ae - 2c^2d^2x + b^2e^2x + 2\sqrt{c^2d^2 - b^2d^2e + a^2e^2}\sqrt{a + bx + cx^2}]) / (e^7(a + bx + cx^2)^{3/2})
\end{aligned}$$

**Maple [B]** time = 0.042, size = 10058, normalized size = 9.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^3\*(c\*x^2+b\*x+a)^(3/2)/(e\*x+d),x)

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)^(3/2)\*(g\*x + f)^3/(e\*x + d),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)^(3/2)\*(g\*x + f)^3/(e\*x + d),x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**3*(c*x**2+b*x+a)**(3/2)/(e*x+d),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)^(3/2)*(g*x + f)^3/(e*x + d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```



$$3.863 \quad \int \frac{(f+gx)^2(a+bx+cx^2)^{3/2}}{d+ex} dx$$

**Optimal.** Leaf size=662

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (96c^3e^2(-a^2e^2g(2ef-dg) - 2abe(ef-dg)^2 + b^2d(ef-dg)^2) + 16bc^2e^3(3a^2e^2g^2 + 3abeg(2ef - 256c^{7/2}e^2))}{(a+bx+cx^2)^{3/2}(3b^2e^2g^2 - 6ceg(-beg - 2cdg + 4cef) - 6bceg(2ef-dg) - 16c^2(ef-dg)^2)} + \frac{\sqrt{a+bx+cx^2}(2cex(g(-4ce(2bd-3ae) - 3b^2e^2 + 16c^2d^2)(-beg - 2cdg + 4cef) - 8ce(2cd-be)(2cef^2 - bdg^2)) - 6b^2e^2g^2)}{48c^2e^3}}{e^6} + \frac{g^2(a+bx+cx^2)^{5/2}}{5ce}$$

[Out]  $((3*b^4*e^4*g^2 + 128*c^4*d^2*(e*f - d*g)^2 - 32*c^3*e*(5*b*d - 4*a*e)*(e*f - d*g)^2 - 6*b^2*c*e^3*g*(2*b*e*f - b*d*g + 2*a*e*g) + 8*b*c^2*e^2*(2*b*(e*f - d*g)^2 + 3*a*e*g*(2*e*f - d*g)) + 2*c*e*((16*c^2*d^2 - 3*b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*g*(4*c*e*f - 2*c*d*g - b*e*g) - 8*c*e*(2*c*d - b*e)*(2*c*e*f^2 - b*d*g^2))*x)*\text{Sqrt}[a + b*x + c*x^2]/(128*c^3*e^5) - ((3*b^2*e^2*g^2 - 16*c^2*(e*f - d*g)^2 - 6*b*c*e*g*(2*e*f - d*g) - 6*c*e*g*(4*c*e*f - 2*c*d*g - b*e*g))*x)*(a + b*x + c*x^2)^{(3/2)}/(48*c^2*e^3) + (g^2*(a + b*x + c*x^2)^{(5/2)})/(5*c*e) - ((3*b^5*e^5*g^2 + 256*c^5*d^3*(e*f - d*g)^2 - 384*c^4*d*e*(b*d - a*e)*(e*f - d*g)^2 - 6*b^3*c*e^4*g*(2*b*e*f - b*d*g + 4*a*e*g) + 16*b*c^2*e^3*(3*a^2*e^2*g^2 + b^2*(e*f - d*g)^2 + 3*a*b*e*g*(2*e*f - d*g)) + 96*c^3*e^2*(b^2*d*(e*f - d*g)^2 - 2*a*b*e*(e*f - d*g)^2 - a^2*e^2*g*(2*e*f - d*g)))*\text{ArcTan}h[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])]/(256*c^{(7/2)}*e^6) + ((c*d^2 - b*d*e + a*e^2)^{(3/2)}*(e*f - d*g)^2*\text{ArcTan}h[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*\text{Sqrt}[a + b*x + c*x^2])])/e^6$

**Rubi [A]** time = 3.46123, antiderivative size = 662, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (96c^3e^2(-a^2e^2g(2ef-dg) - 2abe(ef-dg)^2 + b^2d(ef-dg)^2) + 16bc^2e^3(3a^2e^2g^2 + 3abeg(2ef - 256c^{7/2}e^2))}{(a+bx+cx^2)^{3/2}(3b^2e^2g^2 - 6ceg(-beg - 2cdg + 4cef) - 6bceg(2ef-dg) - 16c^2(ef-dg)^2)} + \frac{\sqrt{a+bx+cx^2}(2cex(g(-4ce(2bd-3ae) - 3b^2e^2 + 16c^2d^2)(-beg - 2cdg + 4cef) - 8ce(2cd-be)(2cef^2 - bdg^2)) - 6b^2e^2g^2)}{48c^2e^3}}{e^6} + \frac{g^2(a+bx+cx^2)^{5/2}}{5ce}$$

Antiderivative was successfully verified.

[In] Int[((f + g\*x)^2\*(a + b\*x + c\*x^2)^(3/2))/(d + e\*x), x]

[Out]  $((3*b^4*e^4*g^2 + 128*c^4*d^2*(e*f - d*g)^2 - 32*c^3*e*(5*b*d - 4*a*e)*(e*f - d*g)^2 - 6*b^2*c*e^3*g*(2*b*e*f - b*d*g + 2*a*e*g) + 8*b*c^2*e^2*(2*b*(e*f - d*g)^2 + 3*a*e*g*(2*e*f - d*g)) + 2*c*e*((16*c^2*d^2 - 3*b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*g*(4*c*e*f - 2*c*d*g - b*e*g) - 8*c*e*(2*c*d - b*e)*(2*c*e*f^2 - b*d*g^2))*x)*\text{Sqrt}[a + b*x + c*x^2]/(128*c^3*e^5) - ((3*b^2*e^2*g^2 - 16*c^2*(e*f - d*g)^2 - 6*b*c*e*g*(2*e*f - d*g) - 6*c*e*g*(4*c*e*f - 2*c*d*g - b*e*g))*x)*(a + b*x + c*x^2)^{(3/2)}/(48*c^2*e^3) + (g^2*(a + b*x + c*x^2)^{(5/2)})/(5*c*e) - ((3*b^5*e^5*g^2 + 256*c^5*d^3*(e*f - d*g)^2 - 384*c^4*d*e*(b*d - a*e)*(e*f - d*g)^2 - 6*b^3*c*e^4*g*(2*b*e*f - b*d*g + 4*a*e*g) + 16*b*c^2*e^3*(3*a^2*e^2*g^2 + b^2*(e*f - d*g)^2 + 3*a*b*e*g*(2*e*f - d*g)) + 96*c^3*e^2*(b^2*d*(e*f -$

$$\frac{d^2g^2 - 2ab^2e^2(e^2f - d^2g)^2 - a^2e^2g^2(2e^2f - d^2g)}{h\left[\frac{b + 2cx}{2\sqrt{c}\sqrt{a + bx + cx^2}}\right]} \frac{1}{(256c^{7/2}e^6 + ((c^2d^2 - b^2d^2e + a^2e^2)^{3/2}(e^2f - d^2g)^2 \operatorname{ArcTanh}\left[\frac{bd - 2ae + (2cd - b^2e)x}{2\sqrt{c^2d^2 - b^2d^2e + a^2e^2}}\sqrt{a + bx + cx^2}\right])})/e^6$$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x+f)**2*(c*x**2+b*x+a)**(3/2)/(e*x+d), x)`

[Out] Timed out

**Mathematica [A]** time = 3.61516, size = 740, normalized size = 1.12

$$2e\sqrt{a+bx+cx^2}(12c^2e^2(32a^2e^2g^2+2abeg(-25dg+50ef+7egx)+b^2(20d^2g^2-5deg(8f+gx)+2e^2(10f^2+5fgx+g^2x^2)))-30b^2ce^3g(10aeg+b(-3dg+6ef+egx))+16$$

Antiderivative was successfully verified.

[In] `Integrate[((f + g*x)^2*(a + b*x + c*x^2)^(3/2))/(d + e*x), x]`

[Out]  $((2e\sqrt{a + x(b + cx)})^2(45b^4e^4g^2 - 30b^2c^2e^3g^2(10a^2e^2g + b(6e^2f - 3d^2g + e^2g^2x)) + 32c^4(60d^4g^2 - 30d^3e^2g^2(4f + gx) + 20d^2e^2(3f^2 + 3fgx + g^2x^2) - 5d^2e^3x(6f^2 + 8fgx + 3g^2x^2) + 2e^4x^2(10f^2 + 15fgx + 6g^2x^2)) + 12c^2e^2(32a^2e^2g^2 + 2ab^2e^2g^2(50e^2f - 25d^2g + 7e^2g^2x) + b^2(20d^2g^2 - 5d^2e^2g^2(8f + gx) + 2e^2(10f^2 + 5fgx + g^2x^2))) + 16c^3e^2(a^2e^2(160d^2g^2 - 5d^2e^2g^2(64f + 15g^2x) + 2e^2(80f^2 + 75fgx + 24g^2x^2)) + b^2(-150d^3g^2 + 10d^2e^2g^2(30f + 7g^2x) - 5d^2e^2(30f^2 + 28fgx + 9g^2x^2) + e^3x(70f^2 + 90fgx + 33g^2x^2))))/c^3 + 3840(c^2d^2 + e^2(-bd + ae))^{3/2}(e^2f - d^2g)^2 \operatorname{Log}[d + e*x] - (15(3b^5e^5g^2 + 256c^5d^3(e^2f - d^2g)^2 - 384c^4d^2e^2(bd - ae)(e^2f - d^2g)^2 - 6b^3c^2e^4g^2(2b^2e^2f - bd^2g + 4a^2e^2g) + 16b^2c^2e^3(3a^2e^2g^2 + b^2(e^2f - d^2g)^2 + 3a^2b^2e^2g^2(2e^2f - d^2g) + 96c^3e^2(b^2d^2(e^2f - d^2g)^2 - 2a^2b^2e^2(e^2f - d^2g)^2 + a^2e^2g^2(-2e^2f + d^2g))) \operatorname{Log}[b + 2cx + 2\sqrt{c}\sqrt{a + bx + cx^2}])/c^{7/2} - 3840(c^2d^2 + e^2(-bd + ae))^{3/2}(e^2f - d^2g)^2 \operatorname{Log}[-(bd) + 2ae - 2cdx + b^2e^2x + 2\sqrt{c^2d^2 + e^2(-bd + ae)}\sqrt{a + bx + cx^2}]/(3840e^6)$

**Maple [B]** time = 0.026, size = 6860, normalized size = 10.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^2*(c*x^2+b*x+a)^(3/2)/(e*x+d), x)`

[Out] result too large to display

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^(3/2)*(g*x + f)^2/(e*x + d), x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^(3/2)*(g*x + f)^2/(e*x + d), x, algorithm="fricas")`

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)**2*(c*x**2+b*x+a)**(3/2)/(e*x+d), x)`

[Out] Timed out

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^(3/2)*(g*x + f)^2/(e*x + d), x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.864 \quad \int \frac{(f+gx)(a+bx+cx^2)^{3/2}}{d+ex} dx$$

**Optimal.** Leaf size=441

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (48c^2e^2 (a^2e^2g + 2abe(ef - dg) + b^2(-d)(ef - dg)) - 8b^2ce^3(3aeg - bdg + bef) + 192c^3de(bd - ae))}{128c^{5/2}e^5} \\ + \frac{\sqrt{a+bx+cx^2} (2cex (-4ce(3aeg - 2bdg + 2bef) + 3b^2e^2g + 16c^2d(ef - dg)) + 16c^2e(5bd - 4ae)(ef - dg) - 4bce^2(3aeg - ae^2))}{64c^2e^4} \\ + \frac{(ef - dg) (ae^2 - bde + cd^2)^{3/2} \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^5} \\ + \frac{(a+bx+cx^2)^{3/2} (3beg - 8cdg + 8cef + 6ceg)}{24ce^2}$$

[Out]  $-\left((3*b^3*e^3*g - 64*c^3*d^2*(e*f - d*g) + 16*c^2*e*(5*b*d - 4*a*e)\right)*(e*f - d*g) - 4*b*c*e^2*(2*b*e*f - 2*b*d*g + 3*a*e*g) + 2*c*e*(3*b^2*e^2*g + 16*c^2*d*(e*f - d*g) - 4*c*e*(2*b*e*f - 2*b*d*g + 3*a*e*g))*x*\text{Sqrt}[a + b*x + c*x^2]/(64*c^2*e^4) + ((8*c*e*f - 8*c*d*g + 3*b*e*g + 6*c*e*g*x)*(a + b*x + c*x^2)^{(3/2)})/(24*c*e^2) + ((3*b^4*e^4*g - 128*c^4*d^3*(e*f - d*g) + 192*c^3*d*e*(b*d - a*e))*(e*f - d*g) - 8*b^2*c*e^3*(b*e*f - b*d*g + 3*a*e*g) + 48*c^2*e^2*(a^2*e^2*g - b^2*d*(e*f - d*g) + 2*a*b*e*(e*f - d*g)))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])]/(128*c^{(5/2)}*e^5) + ((c*d^2 - b*d*e + a*e^2)^{(3/2)}*(e*f - d*g)*\text{ArcTanh}[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*\text{Sqrt}[a + b*x + c*x^2])])/e^5$

**Rubi [A]** time = 1.68833, antiderivative size = 441, normalized size of antiderivative = 1., number of rules used = 7, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (48c^2e^2 (a^2e^2g + 2abe(ef - dg) + b^2(-d)(ef - dg)) - 8b^2ce^3(3aeg - bdg + bef) + 192c^3de(bd - ae))}{128c^{5/2}e^5} \\ + \frac{\sqrt{a+bx+cx^2} (2cex (-4ce(3aeg - 2bdg + 2bef) + 3b^2e^2g + 16c^2d(ef - dg)) + 16c^2e(5bd - 4ae)(ef - dg) - 4bce^2(3aeg - ae^2))}{64c^2e^4} \\ + \frac{(ef - dg) (ae^2 - bde + cd^2)^{3/2} \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^5} \\ + \frac{(a+bx+cx^2)^{3/2} (3beg - 8cdg + 8cef + 6ceg)}{24ce^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}(((f + g*x)*(a + b*x + c*x^2)^{(3/2)})/(d + e*x), x)$

[Out]  $-\left((3*b^3*e^3*g - 64*c^3*d^2*(e*f - d*g) + 16*c^2*e*(5*b*d - 4*a*e)\right)*(e*f - d*g) - 4*b*c*e^2*(2*b*e*f - 2*b*d*g + 3*a*e*g) + 2*c*e*(3*b^2*e^2*g + 16*c^2*d*(e*f - d*g) - 4*c*e*(2*b*e*f - 2*b*d*g + 3*a*e*g))*x*\text{Sqrt}[a + b*x + c*x^2]/(64*c^2*e^4) + ((8*c*e*f - 8*c*d*g + 3*b*e*g + 6*c*e*g*x)*(a + b*x + c*x^2)^{(3/2)})/(24*c*e^2) + ((3*b^4*e^4*g - 128*c^4*d^3*(e*f - d*g) + 192*c^3*d*e*(b*d - a*e))*(e*f - d*g) - 8*b^2*c*e^3*(b*e*f - b*d*g + 3*a*e*g) + 48*c^2*e^2*(a^2*e^2*g - b^2*d*(e*f - d*g) + 2*a*b*e*(e*f - d*g)))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])]/(128*c^{(5/2)}*e^5) + ((c*d^2 - b*d*e + a*e^2)^{(3/2)}*(e*f - d*g)*\text{ArcTanh}[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*\text{Sqrt}[a + b*x + c*x^2])])/e^5$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x+f)*(c*x^2+b*x+a)^(3/2)/(e*x+d),x)`

[Out] Timed out

**Mathematica [A]** time = 1.42208, size = 449, normalized size = 1.02

$$\frac{3 \log\left(2\sqrt{c}\sqrt{a+x(b+cx)+b+2cx}\right) (48c^2e^2(a^2e^2g+2abe(ef-dg)+b^2d(dg-ef))-8b^2ce^3(3aeg-bdg+bef)-192c^3de(bd-ae)(dg-ef)+3b^4e^4g+128c^4d^3(dg-ef))}{c^{5/2}}$$

Antiderivative was successfully verified.

[In] `Integrate[((f + g*x)*(a + b*x + c*x^2)^(3/2))/(d + e*x),x]`

[Out] 
$$\begin{aligned} & ((2^*e*\text{Sqrt}[a + x*(b + c*x)]*(-9*b^3*e^3*g - 16*c^3*(12*d^3*g - 6*d^2*e*(2*f + g*x) + 2*d*e^2*x*(3*f + 2*g*x) - e^3*x^2*(4*f + 3*g*x)) + 6*b*c*e^2*(10*a*e*g + b*(4*e*f - 4*d*g + e*g*x)) + 8*c^2*e*(a*e*(32*e*f - 32*d*g + 15*e*g*x) + b*(30*d^2*g - 2*d*e*(15*f + 7*g*x) + e^2*x*(14*f + 9*g*x))))/c^2 + 384*(c*d^2 + e*(-(b*d) + a*e))^{3/2}*(e*f - d*g)*\text{Log}[d + e*x] + (3*(3*b^4*e^4*g + 128*c^4*d^3*(-(e*f) + d*g) - 192*c^3*d*e*(b*d - a*e)*(-(e*f) + d*g) - 8*b^2*c*e^3*(b*e*f - b*d*g + 3*a*e*g) + 48*c^2*e^2*(a^2*e^2*g + 2*a*b*e*(e*f - d*g) + b^2*d*(-(e*f) + d*g)))*\text{Log}[b + 2*c*x + 2*\text{Sqrt}[c]*\text{Sqrt}[a + x*(b + c*x)]])/c^{5/2} + 384*(c*d^2 + e*(-(b*d) + a*e))^{3/2}*(-(e*f) + d*g)*\text{Log}[-(b*d) + 2*a*e - 2*c*d*x + b*e*x + 2*\text{Sqrt}[c*d^2 + e*(-(b*d) + a*e)]*\text{Sqrt}[a + x*(b + c*x)]]/(384*e^5) \end{aligned}$$

**Maple [B]** time = 0.014, size = 4188, normalized size = 9.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)*(c*x^2+b*x+a)^(3/2)/(e*x+d),x)`

[Out] 
$$\begin{aligned} & 1/e^3*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{1/2}*c*d^2*f-1/e^4*\ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^{1/2}+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{1/2})*c^{3/2}*d^3*f-1/e^4*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{1/2}*c*d^3*g-1/16/e/c^{3/2}*\ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^{1/2}+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{1/2})*b^3*f+1/4/e*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{1/2}*x*b*f-5/4/e^2*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{1/2}*b*d*f+3/128/e*g/c^{5/2}*\ln((1/2*b+c*x)/c^{1/2}+(c*x^2+b*x+a)^{1/2})*b^4+1/4/e*g*(c*x^2+b*x+a)^{3/2}*x-1/3/e^2*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{3/2}*d*g+1/e*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{1/2}*a*f+3/8/e*g*(c*x^2+b*x+a)^{1/2}*x*a+1/8/e*g/c*(c*x^2+b*x+a)^{3/2}*b+1/e^5*\ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^{1/2}+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{1/2})*c^{3/2}*d^4*g-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^{1/2}*\ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^{1/2})*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{1/2}))/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{3/2}*f+1/8/e/c*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{1/2}*b^2*f+5/4/e^3*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{1/2}*b*d^2*g-3/64/e*g/c^2*(c*x^2+b*x+a)^{1/2} \end{aligned}$$



```
[In] integrate((c*x^2 + b*x + a)^(3/2)*(g*x + f)/(e*x + d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)^(3/2)*(g*x + f)/(e*x + d),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)*(c*x**2+b*x+a)**(3/2)/(e*x+d),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS** [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)^(3/2)*(g*x + f)/(e*x + d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.865 \quad \int \frac{(a+bx+cx^2)^{3/2}}{d+ex} dx$$

**Optimal.** Leaf size=252

$$\frac{\sqrt{a+bx+cx^2}(-2ce(5bd-4ae)+b^2e^2-2cex(2cd-be)+8c^2d^2)}{8ce^3} - \frac{(2cd-be)(-4ce(2bd-3ae)-b^2e^2+8c^2d^2)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}e^4} + \frac{(ae^2-bde+cd^2)^{3/2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^4} + \frac{(a+bx+cx^2)^{3/2}}{3e}$$

[Out]  $((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*\text{Sqrt}[a + b*x + c*x^2])/(8*c*e^3) + (a + b*x + c*x^2)^{(3/2)}/(3*e) - ((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(16*c^{3/2}*e^4) + ((c*d^2 - b*d*e + a*e^2)^{(3/2})*\text{ArcTanh}[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*\text{Sqrt}[a + b*x + c*x^2])]) / e^4$

**Rubi [A]** time = 0.760348, antiderivative size = 252, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$

$$\frac{\sqrt{a+bx+cx^2}(-2ce(5bd-4ae)+b^2e^2-2cex(2cd-be)+8c^2d^2)}{8ce^3} - \frac{(2cd-be)(-4ce(2bd-3ae)-b^2e^2+8c^2d^2)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}e^4} + \frac{(ae^2-bde+cd^2)^{3/2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^4} + \frac{(a+bx+cx^2)^{3/2}}{3e}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)^(3/2)/(d + e\*x), x]

[Out]  $((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*\text{Sqrt}[a + b*x + c*x^2])/(8*c*e^3) + (a + b*x + c*x^2)^{(3/2)}/(3*e) - ((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(16*c^{3/2}*e^4) + ((c*d^2 - b*d*e + a*e^2)^{(3/2})*\text{ArcTanh}[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*\text{Sqrt}[a + b*x + c*x^2])]) / e^4$

**Rubi in Sympy [A]** time = 119.964, size = 240, normalized size = 0.95

$$\frac{(a+bx+cx^2)^{3/2}}{3e} - \frac{(ae^2-bde+cd^2)^{3/2}\text{atanh}\left(\frac{2ae-bd+x(be-2cd)}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^4} + \frac{\sqrt{a+bx+cx^2}\left(4ace^2 + \frac{b^2e^2}{2} - 5bcde + 4c^2d^2 + cex(be-2cd)\right)}{4ce^3} - \frac{(be-2cd)(-12ace^2 + b^2e^2 + 8bcde - 8c^2d^2)\text{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}e^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+b\*x+a)\*\*(3/2)/(e\*x+d), x)



```
[Out] (a + b*x + c*x**2)**(3/2)/(3*e) - (a*e**2 - b*d*e + c*d**2)**(3/2)
)*atanh((2*a*e - b*d + x*(b*e - 2*c*d))/(2*sqrt(a + b*x + c*x**2)
*sqrt(a*e**2 - b*d*e + c*d**2)))/e**4 + sqrt(a + b*x + c*x**2)*(4
*a*c*e**2 + b**2*e**2/2 - 5*b*c*d*e + 4*c**2*d**2 + c*e*x*(b*e -
2*c*d))/(4*c*e**3) - (b*e - 2*c*d)*(-12*a*c*e**2 + b**2*e**2 + 8*
b*c*d*e - 8*c**2*d**2)*atanh((b + 2*c*x)/(2*sqrt(c)*sqrt(a + b*x
+ c*x**2)))/(16*c**(3/2)*e**4)
```

**Mathematica [A]** time = 0.411648, size = 255, normalized size = 1.01

$$\frac{2e\sqrt{a+x(b+cx)}(2ce(16ae-15bd+7bex)+3b^2e^2+4c^2(6d^2-3dex+2e^2x^2))}{c} - \frac{3(2cd-be)(4ce(3ae-2bd)-b^2e^2+8c^2d^2)\log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)}{c^{3/2}} + 48\log$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x + c*x^2)^(3/2)/(d + e*x), x]
```

```
[Out] ((2*e*Sqrt[a + x*(b + c*x)]*(3*b^2*e^2 + 2*c*e*(-15*b*d + 16*a*e
+ 7*b*e*x) + 4*c^2*(6*d^2 - 3*d*e*x + 2*e^2*x^2)))/c + 48*(c*d^2
+ e*(-(b*d) + a*e))^(3/2)*Log[d + e*x] - (3*(2*c*d - b*e)*(8*c^2*
d^2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*a*e))*Log[b + 2*c*x + 2*Sqrt[c]
*Sqrt[a + x*(b + c*x)]])/c^(3/2) - 48*(c*d^2 + e*(-(b*d) + a*e))^(
3/2)*Log[-(b*d) + 2*a*e - 2*c*d*x + b*e*x + 2*Sqrt[c*d^2 + e*(-(
b*d) + a*e)]*Sqrt[a + x*(b + c*x)])/ (48*e^4)
```

**Maple [B]** time = 0.008, size = 1946, normalized size = 7.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(3/2)/(e*x+d), x)
```

```
[Out] 1/3/e*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)
^(3/2)+1/4/e*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)
/e^2)^(1/2)*x*b-1/2/e^2*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e
^2-b*d*e+c*d^2)/e^2)^(1/2)*x*c*d+1/8/e/c*((x+d/e)^2*c+(b*e-2*c*d)
/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b^2-5/4/e^2*((x+d/e)^2*
c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*b*d+3/4/e/
c^(1/2)*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+((x+d/e)^2*c+(b*
e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*a*b-3/2/e^2*ln
((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e
*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d*a-1/16/e/c^(3/
2)*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+((x+d/e)^2*c+(b*e-2*c
*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*b^3-3/8/e^2*ln((1/2
*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d
/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b^2*d+1/e*((x+d/e)^2*
c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*a+1/e^3*((
x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*c
*d^2+3/2/e^3*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+((x+d/e)^2*
c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d
^2*b-1/e^4*ln((1/2*(b*e-2*c*d)/e+c*(x+d/e))/c^(1/2)+((x+d/e)^2*c+
(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(3/2)*d^3
-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^
2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*((x+d/e)
)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x+d/
e))*a^2+2/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+
c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/
2))/(x+d/e))*a*b*d-2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a
*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)
)/e^2)^(1/2))*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)
```

$$\frac{2}{e^2} \sqrt{\frac{1}{2}} \sqrt{x+d/e} \cdot a \cdot c \cdot d^2 - \frac{1}{e^3} \sqrt{\frac{(a \cdot e^2 - b \cdot d \cdot e + c \cdot d^2)}{e^2}} \sqrt{\frac{1}{2}} \ln\left(\frac{2 \cdot (a \cdot e^2 - b \cdot d \cdot e + c \cdot d^2)}{e^2 + (b \cdot e - 2 \cdot c \cdot d)/e \cdot (x+d/e)} + 2 \cdot \frac{(a \cdot e^2 - b \cdot d \cdot e + c \cdot d^2)}{e^2} \sqrt{\frac{1}{2}} \sqrt{\frac{(x+d/e)^2 \cdot c + (b \cdot e - 2 \cdot c \cdot d)/e \cdot (x+d/e)}{(a \cdot e^2 - b \cdot d \cdot e + c \cdot d^2)} \sqrt{\frac{1}{2}} \sqrt{\frac{1}{2}} \sqrt{x+d/e}}\right) \cdot b^2 \cdot d^2 + \frac{2}{e^4} \sqrt{\frac{(a \cdot e^2 - b \cdot d \cdot e + c \cdot d^2)}{e^2}} \sqrt{\frac{1}{2}} \ln\left(\frac{2 \cdot (a \cdot e^2 - b \cdot d \cdot e + c \cdot d^2)}{e^2 + (b \cdot e - 2 \cdot c \cdot d)/e \cdot (x+d/e)} + 2 \cdot \frac{(a \cdot e^2 - b \cdot d \cdot e + c \cdot d^2)}{e^2} \sqrt{\frac{1}{2}} \sqrt{\frac{(x+d/e)^2 \cdot c + (b \cdot e - 2 \cdot c \cdot d)/e \cdot (x+d/e)}{(a \cdot e^2 - b \cdot d \cdot e + c \cdot d^2)} \sqrt{\frac{1}{2}} \sqrt{\frac{1}{2}} \sqrt{x+d/e}}\right) \cdot b \cdot d^3 \cdot c - \frac{1}{e^5} \sqrt{\frac{(a \cdot e^2 - b \cdot d \cdot e + c \cdot d^2)}{e^2}} \sqrt{\frac{1}{2}} \ln\left(\frac{2 \cdot (a \cdot e^2 - b \cdot d \cdot e + c \cdot d^2)}{e^2 + (b \cdot e - 2 \cdot c \cdot d)/e \cdot (x+d/e)} + 2 \cdot \frac{(a \cdot e^2 - b \cdot d \cdot e + c \cdot d^2)}{e^2} \sqrt{\frac{1}{2}} \sqrt{\frac{(x+d/e)^2 \cdot c + (b \cdot e - 2 \cdot c \cdot d)/e \cdot (x+d/e)}{(a \cdot e^2 - b \cdot d \cdot e + c \cdot d^2)} \sqrt{\frac{1}{2}} \sqrt{\frac{1}{2}} \sqrt{x+d/e}}\right) \cdot c \cdot d^2 \cdot d^4$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)^(3/2)/(e\*x + d), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)^(3/2)/(e\*x + d), x, algorithm="fricas")

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx + cx^2)^{\frac{3}{2}}}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*\*(3/2)/(e\*x+d), x)

[Out] Integral((a + b\*x + c\*x\*\*2)\*\*(3/2)/(d + e\*x), x)

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)^(3/2)/(e\*x + d), x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.866 \quad \int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)} dx$$

**Optimal.** Leaf size=491

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (-4cg(3bef^2 - ag(3ef - dg)) + bg^2(-4aeg + bdg + 3bef) + 8c^2ef^3)}{8\sqrt{ceg^3}(ef - dg)} + \frac{\sqrt{a+bx+cx^2}(ae^2 - bde + cd^2)}{e^2(ef - dg)} + \frac{(ae^2 - bde + cd^2)^{3/2} \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^3(ef - dg)} - \frac{(2cd - be)(ae^2 - bde + cd^2) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{ce^3}(ef - dg)} - \frac{\sqrt{a+bx+cx^2}(-g(-4aeg - bdg + 5bef) - 2cgx(ef - dg) + 4cef^2)}{4eg^2(ef - dg)} - \frac{(ag^2 - bfg + cf^2)^{3/2} \tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g^3(ef - dg)}$$

[Out]  $((c*d^2 - b*d*e + a*e^2)*\text{Sqrt}[a + b*x + c*x^2])/(e^2*(e*f - d*g)) - ((4*c*e*f^2 - g*(5*b*e*f - b*d*g - 4*a*e*g) - 2*c*g*(e*f - d*g)*x)*\text{Sqrt}[a + b*x + c*x^2])/(4*e*g^2*(e*f - d*g)) - ((2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(2*\text{Sqrt}[c]*e^3*(e*f - d*g)) + ((8*c^2*e*f^3 + b*g^2*(3*b*e*f + b*d*g - 4*a*e*g) - 4*c*g*(3*b*e*f^2 - a*g*(3*e*f - d*g)))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(8*\text{Sqrt}[c]*e*g^3*(e*f - d*g)) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*\text{ArcTanh}[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*\text{Sqrt}[a + b*x + c*x^2])])/(e^3*(e*f - d*g)) - ((c*f^2 - b*f*g + a*g^2)^(3/2)*\text{ArcTanh}[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*\text{Sqrt}[c*f^2 - b*f*g + a*g^2]*\text{Sqrt}[a + b*x + c*x^2])])/(g^3*(e*f - d*g))$

**Rubi [A]** time = 1.76046, antiderivative size = 491, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$

$$\frac{\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (-4cg(3bef^2 - ag(3ef - dg)) + bg^2(-4aeg + bdg + 3bef) + 8c^2ef^3)}{8\sqrt{ceg^3}(ef - dg)} + \frac{\sqrt{a+bx+cx^2}(ae^2 - bde + cd^2)}{e^2(ef - dg)} + \frac{(ae^2 - bde + cd^2)^{3/2} \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^3(ef - dg)} - \frac{(2cd - be)(ae^2 - bde + cd^2) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2\sqrt{ce^3}(ef - dg)} - \frac{\sqrt{a+bx+cx^2}(-g(-4aeg - bdg + 5bef) - 2cgx(ef - dg) + 4cef^2)}{4eg^2(ef - dg)} - \frac{(ag^2 - bfg + cf^2)^{3/2} \tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g^3(ef - dg)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)), x]$

[Out]  $((c*d^2 - b*d*e + a*e^2)*\text{Sqrt}[a + b*x + c*x^2])/(e^2*(e*f - d*g)) - ((4*c*e*f^2 - g*(5*b*e*f - b*d*g - 4*a*e*g) - 2*c*g*(e*f - d*g)*x)*\text{Sqrt}[a + b*x + c*x^2])/(4*e*g^2*(e*f - d*g)) - ((2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(2*\text{Sqrt}[c]*e^3*(e*f - d*g)) + ((8*c^2*e*f^3 + b*g^2*(3*b*e*f + b*d*g - 4*a*e*g) - 4*c*g*(3*b*e*f^2 - a*g*(3*e*f - d*g)))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/($

$$8\sqrt{c}e^g g^3(e^f - d^g) + ((c^2 d^2 - b^2 d^2 e + a^2 e^2)^{3/2} \operatorname{ArcTanh}[(b^2 d - 2^2 a^2 e + (2^2 c^2 d - b^2 e)^2 x)/(2\sqrt{c^2 d^2 - b^2 d^2 e + a^2 e^2}] \sqrt{a + b^2 x + c^2 x^2}))/e^3(e^f - d^g) - ((c^2 f^2 - b^2 f^2 g + a^2 g^2)^{3/2} \operatorname{ArcTanh}[(b^2 f - 2^2 a^2 g + (2^2 c^2 f - b^2 g)^2 x)/(2\sqrt{c^2 f^2 - b^2 f^2 g + a^2 g^2}] \sqrt{a + b^2 x + c^2 x^2}))/g^3(e^f - d^g)$$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)/(g*x+f),x)`

[Out] Timed out

**Mathematica [A]** time = 0.849698, size = 410, normalized size = 0.84

$$(ef - dg) \log\left(2\sqrt{c}\sqrt{a + x(b + cx)} + b + 2cx\right) (-12ceg(-aeg + bdg + bef) + 3b^2e^2g^2 + 8c^2(d^2g^2 + defg + e^2f^2)) + 8\sqrt{cg^3}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)),x]`

$$[Out] (2\sqrt{c}e^g(e^f - d^g)\sqrt{a + x(b + c^2x)}(5b^2e^g + c^2(-4e^f - 4d^2g + 2e^2g^2x)) + 8\sqrt{c}(c^2d^2 + e^2(-b^2d + a^2e))^{3/2}g^3\operatorname{Log}[d + e^2x] - 8\sqrt{c}e^3(c^2f^2 + g^2(-b^2f + a^2g))^{3/2}\operatorname{Log}[f + g^2x] + (e^f - d^2g)(3b^2e^2g^2 - 12c^2e^2g(b^2e^f + b^2d^2g - a^2e^2g) + 8c^2(e^2f^2 + d^2e^2fg + d^2g^2))\operatorname{Log}[b + 2c^2x + 2\sqrt{c}\sqrt{a + x(b + c^2x)}] - 8\sqrt{c}(c^2d^2 + e^2(-b^2d + a^2e))^{3/2}g^3\operatorname{Log}[-(b^2d) + 2a^2e - 2c^2d^2x + b^2e^2x + 2\sqrt{c}\sqrt{c^2d^2 + e^2(-b^2d + a^2e)}]\sqrt{a + x(b + c^2x)} + 8\sqrt{c}e^3(c^2f^2 + g^2(-b^2f + a^2g))^{3/2}\operatorname{Log}[-(b^2f) + 2a^2g - 2c^2f^2x + b^2g^2x + 2\sqrt{c}\sqrt{c^2f^2 + g^2(-b^2f + a^2g)}]\sqrt{a + x(b + c^2x)}))/ (8\sqrt{c}e^3g^3(e^f - d^2g))$$

**Maple [B]** time = 0.025, size = 4226, normalized size = 8.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f),x)`

$$[Out] -1/4/(d^2g - e^2f) * ((x+d/e)^2 * c + (b^2e - 2^2c^2d)/e * (x+d/e) + (a^2e^2 - b^2d^2e + c^2d^2)/e^2)^{1/2} * x^b - 1/8/(d^2g - e^2f)/c * ((x+d/e)^2 * c + (b^2e - 2^2c^2d)/e * (x+d/e) + (a^2e^2 - b^2d^2e + c^2d^2)/e^2)^{1/2} * b^2 + 1/16/(d^2g - e^2f)/c^{3/2} * \ln((1/2 * (b^2e - 2^2c^2d)/e + c^2 * (x+d/e))/c^{1/2} + ((x+d/e)^2 * c + (b^2e - 2^2c^2d)/e * (x+d/e) + (a^2e^2 - b^2d^2e + c^2d^2)/e^2)^{1/2}) * b^3 + 1/(d^2g - e^2f) * ((a^2e^2 - b^2d^2e + c^2d^2)/e^2)^{1/2} * \ln((2 * (a^2e^2 - b^2d^2e + c^2d^2)/e^2 + (b^2e - 2^2c^2d)/e * (x+d/e) + 2 * ((a^2e^2 - b^2d^2e + c^2d^2)/e^2)^{1/2} * ((x+d/e)^2 * c + (b^2e - 2^2c^2d)/e * (x+d/e) + (a^2e^2 - b^2d^2e + c^2d^2)/e^2)^{1/2})/(x+d/e)) * a^2 + 1/4/(d^2g - e^2f) * ((x+f/g)^2 * c + (b^2g - 2^2c^2f)/g * (x+f/g) + (a^2g^2 - b^2f^2g + c^2f^2)/g^2)^{1/2} * x^b + 1/8/(d^2g - e^2f)/c * ((x+f/g)^2 * c + (b^2g - 2^2c^2f)/g * (x+f/g) + (a^2g^2 - b^2f^2g + c^2f^2)/g^2)^{1/2} * b^2 - 1/16/(d^2g - e^2f)/c^{3/2} * \ln((1/2 * (b^2g - 2^2c^2f)/g + c^2 * (x+f/g))/c^{1/2} + ((x+f/g)^2 * c + (b^2g - 2^2c^2f)/g * (x+f/g) + (a^2g^2 - b^2f^2g + c^2f^2)/g^2)^{1/2})$$

$$\begin{aligned}
& f/g)+(a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)}*b^3-1/(d^*g-e^*f)/((a^*g^2-b^*f^* \\
& g+c^*f^2)/g^2)^{(1/2)}*\ln((2^*(a^*g^2-b^*f^*g+c^*f^2)/g^2+(b^*g-2^*c^*f)/g^*( \\
& x+f/g)+2^*((a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)}*((x+f/g)^2*c+(b^*g-2^*c^*f) \\
& /g^*(x+f/g)+(a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)})/(x+f/g))^*a^2-1/3/(d^*g- \\
& e^*f)^*((x+d/e)^2*c+(b^*e-2^*c^*d)/e^*(x+d/e)+(a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(3/2)} \\
& +1/3/(d^*g-e^*f)^*((x+f/g)^2*c+(b^*g-2^*c^*f)/g^*(x+f/g)+(a^*g^2-b^*f^* \\
& *g+c^*f^2)/g^2)^{(3/2)}-1/(d^*g-e^*f)^*((x+d/e)^2*c+(b^*e-2^*c^*d)/e^*(x+d/ \\
& e)+(a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)}*a+1/(d^*g-e^*f)^*((x+f/g)^2*c+(b^*g \\
& -2^*c^*f)/g^*(x+f/g)+(a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)}*a-3/4/(d^*g-e^*f)/ \\
& c^{(1/2)}*\ln((1/2^*(b^*e-2^*c^*d)/e+c^*(x+d/e))/c^{(1/2)}+((x+d/e)^2*c+(b^* \\
& e-2^*c^*d)/e^*(x+d/e)+(a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)})*a*b-2/(d^*g-e^*f \\
& )/e^3/((a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)}*\ln((2^*(a^*e^2-b^*d^*e+c^*d^2)/e \\
& ^2+(b^*e-2^*c^*d)/e^*(x+d/e)+2^*((a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)}*((x+d/ \\
& e)^2*c+(b^*e-2^*c^*d)/e^*(x+d/e)+(a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)})/(x+d \\
& /e))^*b^d^3*c-2/(d^*g-e^*f)/g^2/((a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)}*\ln(( \\
& 2^*(a^*g^2-b^*f^*g+c^*f^2)/g^2+(b^*g-2^*c^*f)/g^*(x+f/g)+2^*((a^*g^2-b^*f^*g+c^* \\
& *f^2)/g^2)^{(1/2)}*((x+f/g)^2*c+(b^*g-2^*c^*f)/g^*(x+f/g)+(a^*g^2-b^*f^*g+ \\
& c^*f^2)/g^2)^{(1/2)})/(x+f/g))^*a^*c^*f^2+2/(d^*g-e^*f)/g^3/((a^*g^2-b^*f^*g \\
& +c^*f^2)/g^2)^{(1/2)}*\ln((2^*(a^*g^2-b^*f^*g+c^*f^2)/g^2+(b^*g-2^*c^*f)/g^*(x \\
& +f/g)+2^*((a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)}*((x+f/g)^2*c+(b^*g-2^*c^*f)/ \\
& g^*(x+f/g)+(a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)})/(x+f/g))^*b^*f^3*c+2/(d^*g \\
& -e^*f)/g/((a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)}*\ln((2^*(a^*g^2-b^*f^*g+c^*f^2) \\
& /g^2+(b^*g-2^*c^*f)/g^*(x+f/g)+2^*((a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)}*((x+ \\
& f/g)^2*c+(b^*g-2^*c^*f)/g^*(x+f/g)+(a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)})/(x \\
& +f/g))^*a^*b^*f-2/(d^*g-e^*f)/e/((a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)}*\ln((2^* \\
& (a^*e^2-b^*d^*e+c^*d^2)/e^2+(b^*e-2^*c^*d)/e^*(x+d/e)+2^*((a^*e^2-b^*d^*e+c^*d \\
& ^2)/e^2)^{(1/2)}*((x+d/e)^2*c+(b^*e-2^*c^*d)/e^*(x+d/e)+(a^*e^2-b^*d^*e+c^* \\
& d^2)/e^2)^{(1/2)})/(x+d/e))^*a^*b^*d+2/(d^*g-e^*f)/e^2/((a^*e^2-b^*d^*e+c^*d \\
& ^2)/e^2)^{(1/2)}*\ln((2^*(a^*e^2-b^*d^*e+c^*d^2)/e^2+(b^*e-2^*c^*d)/e^*(x+d/e \\
& )+2^*((a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c+(b^*e-2^*c^*d)/e^*(x \\
& +d/e)+(a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)})/(x+d/e))^*a^*c^*d^2-5/4/(d^*g-e \\
& *f)/g^*((x+f/g)^2*c+(b^*g-2^*c^*f)/g^*(x+f/g)+(a^*g^2-b^*f^*g+c^*f^2)/g^2) \\
& ^{(1/2)}*b^*f+3/4/(d^*g-e^*f)/c^{(1/2)}*\ln((1/2^*(b^*g-2^*c^*f)/g+c^*(x+f/g)) \\
& /c^{(1/2)}+((x+f/g)^2*c+(b^*g-2^*c^*f)/g^*(x+f/g)+(a^*g^2-b^*f^*g+c^*f^2)/g \\
& ^2)^{(1/2)})^*a^*b+1/(d^*g-e^*f)/g^2*((x+f/g)^2*c+(b^*g-2^*c^*f)/g^*(x+f/g) \\
& +(a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)}*c^*f^2-1/(d^*g-e^*f)/g^3*\ln((1/2^*(b^* \\
& g-2^*c^*f)/g+c^*(x+f/g))/c^{(1/2)}+((x+f/g)^2*c+(b^*g-2^*c^*f)/g^*(x+f/g)+ \\
& (a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)})*c^{(3/2)}*f^3+5/4/(d^*g-e^*f)/e^*((x+d \\
& /e)^2*c+(b^*e-2^*c^*d)/e^*(x+d/e)+(a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)}*b^*d- \\
& 1/(d^*g-e^*f)/e^2*((x+d/e)^2*c+(b^*e-2^*c^*d)/e^*(x+d/e)+(a^*e^2-b^*d^*e+c^* \\
& *d^2)/e^2)^{(1/2)}*c^*d^2+1/(d^*g-e^*f)/e^3*\ln((1/2^*(b^*e-2^*c^*d)/e+c^*(x \\
& +d/e))/c^{(1/2)}+((x+d/e)^2*c+(b^*e-2^*c^*d)/e^*(x+d/e)+(a^*e^2-b^*d^*e+c^* \\
& d^2)/e^2)^{(1/2)})*c^{(3/2)}*d^3+3/2/(d^*g-e^*f)/e^*\ln((1/2^*(b^*e-2^*c^*d)/ \\
& e+c^*(x+d/e))/c^{(1/2)}+((x+d/e)^2*c+(b^*e-2^*c^*d)/e^*(x+d/e)+(a^*e^2-b^* \\
& d^*e+c^*d^2)/e^2)^{(1/2)})*c^{(1/2)}*d^*a+3/8/(d^*g-e^*f)/e^*\ln((1/2^*(b^*e-2 \\
& *c^*d)/e+c^*(x+d/e))/c^{(1/2)}+((x+d/e)^2*c+(b^*e-2^*c^*d)/e^*(x+d/e)+(a^* \\
& e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)})/c^{(1/2)}*b^2*d-3/2/(d^*g-e^*f)/e^2*\ln(( \\
& 1/2^*(b^*e-2^*c^*d)/e+c^*(x+d/e))/c^{(1/2)}+((x+d/e)^2*c+(b^*e-2^*c^*d)/e^*( \\
& x+d/e)+(a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)})*c^{(1/2)}*d^2*b-1/2/(d^*g-e^*f \\
& )/g^*((x+f/g)^2*c+(b^*g-2^*c^*f)/g^*(x+f/g)+(a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)} \\
& *x^*c^*f-3/2/(d^*g-e^*f)/g*\ln((1/2^*(b^*g-2^*c^*f)/g+c^*(x+f/g))/c^{(1/2)} \\
& +((x+f/g)^2*c+(b^*g-2^*c^*f)/g^*(x+f/g)+(a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)} \\
& )^*c^{(1/2)}*f^*a-3/8/(d^*g-e^*f)/g^*\ln((1/2^*(b^*g-2^*c^*f)/g+c^*(x+f/g)) \\
& /c^{(1/2)}+((x+f/g)^2*c+(b^*g-2^*c^*f)/g^*(x+f/g)+(a^*g^2-b^*f^*g+c^*f^2)/g \\
& ^2)^{(1/2)})/c^{(1/2)}*b^2*f+3/2/(d^*g-e^*f)/g^2*\ln((1/2^*(b^*g-2^*c^*f)/g+ \\
& c^*(x+f/g))/c^{(1/2)}+((x+f/g)^2*c+(b^*g-2^*c^*f)/g^*(x+f/g)+(a^*g^2-b^*f^* \\
& g+c^*f^2)/g^2)^{(1/2)})*c^{(1/2)}*f^2*b-1/(d^*g-e^*f)/g^2/((a^*g^2-b^*f^*g+ \\
& c^*f^2)/g^2)^{(1/2)}*\ln((2^*(a^*g^2-b^*f^*g+c^*f^2)/g^2+(b^*g-2^*c^*f)/g^*(x+ \\
& f/g)+2^*((a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)}*((x+f/g)^2*c+(b^*g-2^*c^*f)/g \\
& ^*(x+f/g)+(a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)})/(x+f/g))^*b^2*f^2-1/(d^*g- \\
& e^*f)/g^4/((a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)}*\ln((2^*(a^*g^2-b^*f^*g+c^*f^2) \\
& )/g^2+(b^*g-2^*c^*f)/g^*(x+f/g)+2^*((a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)}*((x \\
& +f/g)^2*c+(b^*g-2^*c^*f)/g^*(x+f/g)+(a^*g^2-b^*f^*g+c^*f^2)/g^2)^{(1/2)})/( \\
& x+f/g))^*c^2*f^4+1/(d^*g-e^*f)/e^2/((a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)}*1 \\
& n((2^*(a^*e^2-b^*d^*e+c^*d^2)/e^2+(b^*e-2^*c^*d)/e^*(x+d/e)+2^*((a^*e^2-b^*d^* \\
& e+c^*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c+(b^*e-2^*c^*d)/e^*(x+d/e)+(a^*e^2-b^*d^* \\
& *e+c^*d^2)/e^2)^{(1/2)})/(x+d/e))^*b^2*d^2+1/(d^*g-e^*f)/e^4/((a^*e^2-b^* \\
& d^*e+c^*d^2)/e^2)^{(1/2)}*\ln((2^*(a^*e^2-b^*d^*e+c^*d^2)/e^2+(b^*e-2^*c^*d)/e \\
& ^*(x+d/e)+2^*((a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c+(b^*e-2^*c^* \\
& d)/e^*(x+d/e)+(a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)})/(x+d/e))^*c^2*d^4+1/2 \\
& /d^*g-e^*f)/e^*((x+d/e)^2*c+(b^*e-2^*c^*d)/e^*(x+d/e)+(a^*e^2-b^*d^*e+c^*d^2) \\
& /e^2)^{(1/2)}*x^*c^*d
\end{aligned}$$

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^(3/2)/((e*x + d)*(g*x + f)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^(3/2)/((e*x + d)*(g*x + f)),x, algorithm="fricas")`

[Out] Timed out

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(a + bx + cx^2)^{\frac{3}{2}}}{(d + ex)(f + gx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)/(g*x+f),x)`

[Out] `Integral((a + b*x + c*x**2)**(3/2)/((d + e*x)*(f + g*x)), x)`

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^(3/2)/((e*x + d)*(g*x + f)),x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.867 \quad \int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^2} dx$$

Optimal. Leaf size=787

$$\begin{aligned} & \frac{\sqrt{a+bx+cx^2}(-2ce(5bd-4ae)+b^2e^2-2cex(2cd-be)+8c^2d^2)}{8ce(ef-dg)^2} \\ & - \frac{e\sqrt{a+bx+cx^2}(-2cg(5bf-4ag)+b^2g^2-2cgx(2cf-bg)+8c^2f^2)}{8cg^2(ef-dg)^2} \\ & - \frac{3(-4cg(2bf-ag)+b^2g^2+8c^2f^2)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}g^3(ef-dg)} \\ & - \frac{(2cd-be)(-4ce(2bd-3ae)-b^2e^2+8c^2d^2)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}e^2(ef-dg)^2} \\ & + \frac{e(2cf-bg)(-4cg(2bf-3ag)-b^2g^2+8c^2f^2)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}g^3(ef-dg)^2} \\ & + \frac{(ae^2-bde+cd^2)^{3/2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2(ef-dg)^2} \\ & - \frac{e(ag^2-bfg+cf^2)^{3/2}\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g^3(ef-dg)^2} \\ & + \frac{3(2cf-bg)\sqrt{ag^2-bfg+cf^2}\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2g^3(ef-dg)} \\ & + \frac{3\sqrt{a+bx+cx^2}(-3bg+4cf-2cgx)}{4g^2(ef-dg)} + \frac{(a+bx+cx^2)^{3/2}}{(f+gx)(ef-dg)} \end{aligned}$$

[Out]  $((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*\text{Sqrt}[a + b*x + c*x^2])/((8*c*e*(e*f - d*g)^2) + (3*(4*c*f - 3*b*g - 2*c*g*x)*\text{Sqrt}[a + b*x + c*x^2]))/(4*g^2*(e*f - d*g)) - (e*(8*c^2*f^2 + b^2*g^2 - 2*c*g*(5*b*f - 4*a*g) - 2*c*g*(2*c*f - b*g)*x)*\text{Sqrt}[a + b*x + c*x^2])/((8*c*g^2*(e*f - d*g)^2) + (a + b*x + c*x^2)^(3/2))/((e*f - d*g)*(f + g*x)) - ((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(16*c^(3/2)*e^2*(e*f - d*g)^2) + (e*(2*c*f - b*g)*(8*c^2*f^2 - b^2*g^2 - 4*c*g*(2*b*f - 3*a*g))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(16*c^(3/2)*g^3*(e*f - d*g)^2) - (3*(8*c^2*f^2 + b^2*g^2 - 4*c*g*(2*b*f - a*g))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(8*\text{Sqrt}[c]*g^3*(e*f - d*g)) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*\text{ArcTanh}[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*\text{Sqrt}[a + b*x + c*x^2])])/(e^2*(e*f - d*g)^2) + (3*(2*c*f - b*g)*\text{Sqrt}[c*f^2 - b*f*g + a*g^2]*\text{ArcTanh}[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*\text{Sqrt}[c*f^2 - b*f*g + a*g^2]*\text{Sqrt}[a + b*x + c*x^2])])/(2*g^3*(e*f - d*g)) - (e*(c*f^2 - b*f*g + a*g^2)^(3/2)*\text{ArcTanh}[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*\text{Sqrt}[c*f^2 - b*f*g + a*g^2]*\text{Sqrt}[a + b*x + c*x^2])])/(g^3*(e*f - d*g)^2)$

Rubi [A] time = 3.08525, antiderivative size = 787, normalized size of antiderivative = 1., number of

steps used = 23, number of rules used = 8, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$

$$\begin{aligned} & \frac{\sqrt{a+bx+cx^2}(-2ce(5bd-4ae)+b^2e^2-2cex(2cd-be)+8c^2d^2)}{8ce(ef-dg)^2} \\ & - \frac{e\sqrt{a+bx+cx^2}(-2cg(5bf-4ag)+b^2g^2-2cgx(2cf-bg)+8c^2f^2)}{8cg^2(ef-dg)^2} \\ & - \frac{3(-4cg(2bf-ag)+b^2g^2+8c^2f^2)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8\sqrt{c}g^3(ef-dg)} \\ & - \frac{(2cd-be)(-4ce(2bd-3ae)-b^2e^2+8c^2d^2)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}e^2(ef-dg)^2} \\ & + \frac{e(2cf-bg)(-4cg(2bf-3ag)-b^2g^2+8c^2f^2)\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{3/2}g^3(ef-dg)^2} \\ & + \frac{(ae^2-bde+cd^2)^{3/2}\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2(ef-dg)^2} \\ & - \frac{e(ag^2-bfg+cf^2)^{3/2}\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{g^3(ef-dg)^2} \\ & + \frac{3(2cf-bg)\sqrt{ag^2-bfg+cf^2}\tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2g^3(ef-dg)} \\ & + \frac{3\sqrt{a+bx+cx^2}(-3bg+4cf-2cgx)}{4g^2(ef-dg)} + \frac{(a+bx+cx^2)^{3/2}}{(f+gx)(ef-dg)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)^(3/2)/((d + e\*x)\*(f + g\*x)^2), x]

[Out]  $((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*\text{Sqrt}[a + b*x + c*x^2])/((8*c*e*(e*f - d*g)^2 + (3*(4*c*f - 3*b*g - 2*c*g*x)*\text{Sqrt}[a + b*x + c*x^2])/(4*g^2*(e*f - d*g)) - (e*(8*c^2*f^2 + b^2*g^2 - 2*c*g*(5*b*f - 4*a*g) - 2*c*g*(2*c*f - b*g)*x)*\text{Sqrt}[a + b*x + c*x^2])/(8*c*g^2*(e*f - d*g)^2) + (a + b*x + c*x^2)^(3/2)/((e*f - d*g)*(f + g*x)) - ((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(16*c^(3/2)*e^2*(e*f - d*g)^2) + (e*(2*c*f - b*g)*(8*c^2*f^2 - b^2*g^2 - 4*c*g*(2*b*f - 3*a*g))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(16*c^(3/2)*g^3*(e*f - d*g)^2) - (3*(8*c^2*f^2 + b^2*g^2 - 4*c*g*(2*b*f - a*g))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(8*\text{Sqrt}[c]*g^3*(e*f - d*g)) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*\text{ArcTanh}[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*\text{Sqrt}[a + b*x + c*x^2])])/(e^2*(e*f - d*g)^2) + (3*(2*c*f - b*g)*\text{Sqrt}[c*f^2 - b*f*g + a*g^2]*\text{ArcTanh}[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*\text{Sqrt}[c*f^2 - b*f*g + a*g^2]*\text{Sqrt}[a + b*x + c*x^2])])/(2*g^3*(e*f - d*g)) - (e*(c*f^2 - b*f*g + a*g^2)^(3/2)*\text{ArcTanh}[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*\text{Sqrt}[c*f^2 - b*f*g + a*g^2]*\text{Sqrt}[a + b*x + c*x^2])])/(g^3*(e*f - d*g)^2)$

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+b\*x+a)\*\*(3/2)/(e\*x+d)/(g\*x+f)\*\*2,x)

[Out] Timed out



---

**Mathematica [A]** time = 1.75953, size = 441, normalized size = 0.56

$$\frac{1}{2} \left( \frac{2 \log(d + ex) (e(ae - bd) + cd^2)^{3/2}}{e^2 (ef - dg)^2} \right. \\ - \frac{2 (e(ae - bd) + cd^2)^{3/2} \log \left( 2\sqrt{a + x(b + cx)} \sqrt{e(ae - bd) + cd^2} + 2ae - bd + bex - 2cdx \right)}{e^2 (ef - dg)^2} \\ - \frac{\sqrt{c} \log \left( 2\sqrt{c} \sqrt{a + x(b + cx)} + b + 2cx \right) (-3beg + 2cdg + 4cef)}{e^2 g^3} \\ + \frac{\log(f + gx) \sqrt{g(ag - bf) + cf^2} (g(-2aeg + 3bdg - bef) + 2cf(2ef - 3dg))}{g^3 (ef - dg)^2} \\ + \frac{\sqrt{g(ag - bf) + cf^2} (g(2aeg - 3bdg + bef) + c(6dfg - 4ef^2)) \log \left( 2\sqrt{a + x(b + cx)} \sqrt{g(ag - bf) + cf^2} + 2ag - bf + bgx \right)}{g^3 (ef - dg)^2} \\ \left. + \frac{2\sqrt{a + x(b + cx)} \left( \frac{g(ag - bf) + cf^2}{(f + gx)(ef - dg)} + \frac{c}{e} \right)}{g^2} \right)$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)^(3/2)/((d + e\*x)\*(f + g\*x)^2), x]

[Out] ((2\*Sqrt[a + x\*(b + c\*x)]\*(c/e + (c\*f^2 + g\*(-(b\*f) + a\*g)))/((e\*f - d\*g)\*(f + g\*x)))/g^2 + (2\*(c\*d^2 + e\*(-(b\*d) + a\*e))^(3/2)\*Log[d + e\*x])/(e^2\*(e\*f - d\*g)^2) + (Sqrt[c\*f^2 + g\*(-(b\*f) + a\*g)]\*(2\*c\*f\*(2\*e\*f - 3\*d\*g) + g\*(-(b\*e\*f) + 3\*b\*d\*g - 2\*a\*e\*g))\*Log[f + g\*x])/(g^3\*(e\*f - d\*g)^2) - (Sqrt[c]\*(4\*c\*e\*f + 2\*c\*d\*g - 3\*b\*e\*g)\*Log[b + 2\*c\*x + 2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)]])/(e^2\*g^3) - (2\*(c\*d^2 + e\*(-(b\*d) + a\*e))^(3/2)\*Log[-(b\*d) + 2\*a\*e - 2\*c\*d\*x + b\*e\*x + 2\*Sqrt[c\*d^2 + e\*(-(b\*d) + a\*e)]\*Sqrt[a + x\*(b + c\*x)]])/(e^2\*(e\*f - d\*g)^2) + (Sqrt[c\*f^2 + g\*(-(b\*f) + a\*g)]\*(g\*(b\*e\*f - 3\*b\*d\*g + 2\*a\*e\*g) + c\*(-4\*e\*f^2 + 6\*d\*f\*g))\*Log[-(b\*f) + 2\*a\*g - 2\*c\*f\*x + b\*g\*x + 2\*Sqrt[c\*f^2 + g\*(-(b\*f) + a\*g)]\*Sqrt[a + x\*(b + c\*x)]])/(g^3\*(e\*f - d\*g)^2)/2

---

**Maple [B]** time = 0.032, size = 7959, normalized size = 10.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)^(3/2)/(e\*x+d)/(g\*x+f)^2, x)

[Out] result too large to display

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)^{\frac{3}{2}}}{(ex + d)(gx + f)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)^(3/2)/((e\*x + d)\*(g\*x + f)^2), x, algorithm="maxima")

[Out] `integrate((c*x^2 + b*x + a)^(3/2)/((e*x + d)*(g*x + f)^2), x)`

---

**Fricas** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^(3/2)/((e*x + d)*(g*x + f)^2), x, algorithm="fricas")`

[Out] Timed out

---

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)/(g*x+f)**2, x)`

[Out] Timed out

---

**GIAC/XCAS** [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^(3/2)/((e*x + d)*(g*x + f)^2), x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.868 \quad \int \frac{(a+bx+cx^2)^{3/2}}{(d+ex)(f+gx)^3} dx$$

Optimal. Leaf size=1066

$$\begin{aligned} & \frac{(2cf - bg)(8c^2f^2 - b^2g^2 - 4cg(2bf - 3ag)) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right) e^2}{16c^{3/2}g^3(ef - dg)^3} \\ & - \frac{(cf^2 - bgf + ag^2)^{3/2} \tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right) e^2}{g^3(ef - dg)^3} \\ & - \frac{(8c^2f^2 + b^2g^2 - 2cg(5bf - 4ag) - 2cg(2cf - bg)x) \sqrt{cx^2 + bx + ae^2}}{8cg^2(ef - dg)^3} \\ & + \frac{(cx^2 + bx + a)^{3/2} e}{(ef - dg)^2(f + gx)} - \frac{3(8c^2f^2 + b^2g^2 - 4cg(2bf - ag)) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right) e}{8\sqrt{c}g^3(ef - dg)^2} \\ & + \frac{3(2cf - bg)\sqrt{cf^2 - bgf + ag^2} \tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right) e}{2g^3(ef - dg)^2} \\ & + \frac{3(4cf - 3bg - 2cgx)\sqrt{cx^2 + bx + ae}}{4g^2(ef - dg)^2} + \frac{(cx^2 + bx + a)^{3/2}}{2(ef - dg)(f + gx)^2} \\ & + \frac{3\sqrt{c}(2cf - bg) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)}{2g^3(ef - dg)} \\ & - \frac{3(8c^2f^2 + b^2g^2 - 4cg(2bf - ag)) \tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)}{8g^3(ef - dg)\sqrt{cf^2 - bgf + ag^2}} \\ & + \frac{(8c^2d^2 + b^2e^2 - 2ce(5bd - 4ae) - 2ce(2cd - be)x) \sqrt{cx^2 + bx + a}}{8c(ef - dg)^3} \\ & - \frac{3(4cf - bg + 2cgx)\sqrt{cx^2 + bx + a}}{4g^2(ef - dg)(f + gx)} \\ & - \frac{(2cd - be)(8c^2d^2 - b^2e^2 - 4ce(2bd - 3ae)) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)}{16c^{3/2}(ef - dg)^3e} \\ & + \frac{(cd^2 - bed + ae^2)^{3/2} \tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\sqrt{cx^2+bx+a}}\right)}{(ef - dg)^3e} \end{aligned}$$

[Out]  $((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*\text{Sqrt}[a + b*x + c*x^2])/((8*c*(e*f - d*g)^3) + (3*e*(4*c*f - 3*b*g - 2*c*g*x)*\text{Sqrt}[a + b*x + c*x^2])/((4*g^2*(e*f - d*g)^2) - (3*(4*c*f - b*g + 2*c*g*x)*\text{Sqrt}[a + b*x + c*x^2])/((4*g^2*(e*f - d*g)*(f + g*x)) - (e^2*(8*c^2*f^2 + b^2*g^2 - 2*c*g*(5*b*f - 4*a*g) - 2*c*g*(2*c*f - b*g)*x)*\text{Sqrt}[a + b*x + c*x^2])/((8*c*g^2*(e*f - d*g)^3) + (a + b*x + c*x^2)^(3/2)/((2*(e*f - d*g)*(f + g*x)^2) + (e*(a + b*x + c*x^2)^(3/2))/((e*f - d*g)^2*(f + g*x)) - ((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])]))/(16*c^(3/2)*e*(e*f - d*g)^3) + (3*\text{Sqrt}[c]*(2*c*f - b*g)*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])]))/(2*g^3*(e*f - d*g)) + (e^2*(2*c*f - b*g)*(8*c^2*f^2 - b^2*g^2 - 4*c*g*(2*b*f - 3*a*g))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])]))/(16*c^(3/2)*g^3*(e*f - d*g)^3) - (3*e*(8*c^2*f^2 + b^2*g^2 - 4*c*g*(2*b*f - a*g))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])]))/(8*\text{Sqrt}[c]*g^3*(e*f - d*g)^2) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*\text{ArcTanh}[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*\text{Sqrt}[a + b*x + c*x^2])]))/(e*(e*f - d*g)^3) + (3*e*(2*c*f - b*g)*\text{Sqrt}[c*f^2 - b*f*g + a*g^2]*\text{ArcTanh}[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*\text{Sqrt}[c*f^2 - b*f*g + a*g^2]*\text{Sqrt}[a + b*x + c*x^2])]))/(2*g^3*(e*f - d*g)^2) - (e^2*(c*f^2 - b*f*g + a*g^2)^(3/2)*\text{ArcTanh}[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*\text{Sqrt}[c*f^2 - b*f*g + a*g^2]*\text{Sqrt}[a + b*x + c*x^2])]))/(g^3*(e*f - d*g)^3) - (3*(8*c^2*f^2 + b^2*g^2 - 4*c*g*(2*b*f - a*g))*\text{ArcTanh}[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*\text{Sqrt}[c*f^2 - b*f*g + a*g^2]*\text{Sqrt}[a + b*x + c*x^2])]))/(8*g^3*(e*f - d*g)*\text{Sqrt}[c*f^2$

$$2 - b*f*g + a*g^2])$$

**Rubi [A]** time = 4.02009, antiderivative size = 1066, normalized size of antiderivative = 1., number of steps used = 30, number of rules used = 9, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.31$

$$\begin{aligned} & \frac{(2cf - bg)(8c^2f^2 - b^2g^2 - 4cg(2bf - 3ag)) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right) e^2}{16c^{3/2}g^3(ef - dg)^3} \\ & - \frac{(cf^2 - bgf + ag^2)^{3/2} \tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right) e^2}{g^3(ef - dg)^3} \\ & - \frac{(8c^2f^2 + b^2g^2 - 2cg(5bf - 4ag) - 2cg(2cf - bg)x) \sqrt{cx^2 + bx + ae^2}}{8cg^2(ef - dg)^3} \\ & + \frac{(cx^2 + bx + a)^{3/2} e}{(ef - dg)^2(f + gx)} - \frac{3(8c^2f^2 + b^2g^2 - 4cg(2bf - ag)) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right) e}{8\sqrt{c}g^3(ef - dg)^2} \\ & + \frac{3(2cf - bg)\sqrt{cf^2 - bgf + ag^2} \tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right) e}{2g^3(ef - dg)^2} \\ & + \frac{3(4cf - 3bg - 2cgx)\sqrt{cx^2 + bx + ae}}{4g^2(ef - dg)^2} + \frac{(cx^2 + bx + a)^{3/2}}{2(ef - dg)(f + gx)^2} \\ & + \frac{3\sqrt{c}(2cf - bg) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)}{2g^3(ef - dg)} \\ & - \frac{3(8c^2f^2 + b^2g^2 - 4cg(2bf - ag)) \tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)}{8g^3(ef - dg)\sqrt{cf^2 - bgf + ag^2}} \\ & + \frac{(8c^2d^2 + b^2e^2 - 2ce(5bd - 4ae) - 2ce(2cd - be)x) \sqrt{cx^2 + bx + a}}{8c(ef - dg)^3} \\ & - \frac{3(4cf - bg + 2cgx)\sqrt{cx^2 + bx + a}}{4g^2(ef - dg)(f + gx)} \\ & - \frac{(2cd - be)(8c^2d^2 - b^2e^2 - 4ce(2bd - 3ae)) \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)}{16c^{3/2}(ef - dg)^3e} \\ & + \frac{(cd^2 - bed + ae^2)^{3/2} \tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\sqrt{cx^2+bx+a}}\right)}{(ef - dg)^3e} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)^(3/2)/((d + e\*x)\*(f + g\*x)^3), x]

[Out]  $((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*\text{Sqrt}[a + b*x + c*x^2])/((8*c*(e*f - d*g)^3) + (3*e*(4*c*f - 3*b*g - 2*c*g*x)*\text{Sqrt}[a + b*x + c*x^2])/((4*g^2*(e*f - d*g)^2) - (3*(4*c*f - b*g + 2*c*g*x)*\text{Sqrt}[a + b*x + c*x^2])/((4*g^2*(e*f - d*g)*(f + g*x)) - (e^2*(8*c^2*f^2 + b^2*g^2 - 2*c*g*(5*b*f - 4*a*g) - 2*c*g*(2*c*f - b*g)*x)*\text{Sqrt}[a + b*x + c*x^2])/((8*c*g^2*(e*f - d*g)^3) + (a + b*x + c*x^2)^(3/2)/((2*(e*f - d*g)*(f + g*x)^2) + (e*(a + b*x + c*x^2)^(3/2))/((e*f - d*g)^2*(f + g*x)) - ((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(16*c^(3/2)*e*(e*f - d*g)^3) + (3*\text{Sqrt}[c]*(2*c*f - b*g)*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(2*g^3*(e*f - d*g)) + (e^2*(2*c*f - b*g)*(8*c^2*f^2 - b^2*g^2 - 4*c*g*(2*b*f - 3*a*g))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(16*c^(3/2)*g^3*(e*f - d*g)^3) - (3*e*(8*c^2*f^2 + b^2*g^2 - 4*c*g*(2*b*f - a*g))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(8*\text{Sqrt}[c]*g^3*(e*f - d*g)^2) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*\text{ArcTanh}[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*\text{Sqrt}[a + b*x + c*x^2]))/(e*(e*f - d*g)^3) + (3*e*(2*c*f - b*g)*\text{Sqrt}[c*f^2 - b*f*$

$$g + a*g^2]*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])]/(2*g^3*(e*f - d*g)^2) - (e^2*(c*f^2 - b*f*g + a*g^2)^(3/2)*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(g^3*(e*f - d*g)^3) - (3*(8*c^2*f^2 + b^2*g^2 - 4*c*g*(2*b*f - a*g))*ArcTanh[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*Sqrt[c*f^2 - b*f*g + a*g^2]*Sqrt[a + b*x + c*x^2])])/(8*g^3*(e*f - d*g)*Sqrt[c*f^2 - b*f*g + a*g^2])$$

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)/(g*x+f)**3,x)`

[Out] Timed out

**Mathematica [A]** time = 2.11643, size = 690, normalized size = 0.65

$$\frac{\log(f + gx) (g^2 (8a^2e^2g^2 - 4abeg(3dg + ef) + b^2 (3d^2g^2 + 6defg - e^2f^2)) + 4cg (ag (3d^2g^2 + e^2f^2) - bf (6d^2g^2 - 3defg + e^2f^2)))}{8g^3(ef - dg)^3\sqrt{g(ag - bf) + cf^2}} + \frac{(g^2 (8a^2e^2g^2 - 4abeg(3dg + ef) + b^2 (3d^2g^2 + 6defg - e^2f^2)) + 4cg (ag (3d^2g^2 + e^2f^2) - bf (6d^2g^2 - 3defg + e^2f^2)))}{8g^3(ef - dg)^3\sqrt{g(ag - bf)}} + \frac{c^{3/2} \log\left(2\sqrt{c}\sqrt{a + x(b + cx)} + b + 2cx\right)}{eg^3} + \frac{\log(d + ex) (e(ae - bd) + cd^2)^{3/2}}{e(ef - dg)^3} - \frac{(e(ae - bd) + cd^2)^{3/2} \log\left(2\sqrt{a + x(b + cx)}\sqrt{e(ae - bd) + cd^2} + 2ae - bd + bex - 2cdx\right)}{e(ef - dg)^3} + \frac{\sqrt{a + x(b + cx)}(g(2ag(-dg + 3ef + 2egx) - b(dg(3f + 5gx) + ef(f - gx))) + 2cf(dg(4f + 5gx) - ef(2f + 3gx)))}{4g^2(f + gx)^2(ef - dg)^2}$$

Antiderivative was successfully verified.

[In] `Integrate[(a + b*x + c*x^2)^(3/2)/((d + e*x)*(f + g*x)^3),x]`

$$[Out] (\text{Sqrt}[a + x*(b + c*x)]*(2*c*f*(-(e*f*(2*f + 3*g*x)) + d*g*(4*f + 5*g*x)) + g*(2*a*g*(3*e*f - d*g + 2*e*g*x) - b*(e*f*(f - g*x) + d*g*(3*f + 5*g*x))))/(4*g^2*(e*f - d*g)^2*(f + g*x)^2) + ((c*d^2 + e*(-(b*d) + a*e))^(3/2)*Log[d + e*x])/(e*(e*f - d*g)^3) - ((8*c^2*f^2*(e^2*f^2 - 3*d*e*f*g + 3*d^2*g^2) + g^2*(8*a^2*e^2*g^2 - 4*a*b*e*g*(e*f + 3*d*g) + b^2*(-(e^2*f^2) + 6*d*e*f*g + 3*d^2*g^2)) + 4*c*g*(a*g*(e^2*f^2 + 3*d^2*g^2) - b*f*(e^2*f^2 - 3*d*e*f*g + 6*d^2*g^2))) * Log[f + g*x])/(8*g^3*(e*f - d*g)^3*Sqrt[c*f^2 + g*(-(b*f) + a*g)]) + (c^(3/2)*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)])]/(e*g^3) - ((c*d^2 + e*(-(b*d) + a*e))^(3/2)*Log[-(b*d) + 2*a*e - 2*c*d*x + b*e*x + 2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])]/(e*(e*f - d*g)^3) + ((8*c^2*f^2*(e^2*f^2 - 3*d*e*f*g + 3*d^2*g^2) + g^2*(8*a^2*e^2*g^2 - 4*a*b*e*g*(e*f + 3*d*g) + b^2*(-(e^2*f^2) + 6*d*e*f*g + 3*d^2*g^2)) + 4*c*g*(a*g*(e^2*f^2 + 3*d^2*g^2) - b*f*(e^2*f^2 - 3*d*e*f*g + 6*d^2*g^2))) * Log[-(b*f) + 2*a*g - 2*c*f*x + b*g*x + 2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]]*Sqrt[a + x*(b + c*x)])/(8*g^3*(e*f - d*g)^3*Sqrt[c*f^2 + g*(-(b*f) + a*g)])$$

**Maple [B]** time = 0.04, size = 15927, normalized size = 14.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)^(3/2)/(e*x+d)/(g*x+f)^3,x)`

[Out] result too large to display

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)^{\frac{3}{2}}}{(ex + d)(gx + f)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^(3/2)/((e*x + d)*(g*x + f)^3),x, algorithm="maxima")`

[Out] `integrate((c*x^2 + b*x + a)^(3/2)/((e*x + d)*(g*x + f)^3), x)`

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^(3/2)/((e*x + d)*(g*x + f)^3),x, algorithm="fricas")`

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+b*x+a)**(3/2)/(e*x+d)/(g*x+f)**3,x)`

[Out] Timed out

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^(3/2)/((e*x + d)*(g*x + f)^3),x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.869 \quad \int \frac{(a+bx+cx^2)^{5/2}}{(d+ex)(f+gx)} dx$$

**Optimal.** Leaf size=886

$$\frac{\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\sqrt{cx^2+bx+a}}\right)(cd^2-bed+ae^2)^{5/2}}{e^5(ef-dg)} + \frac{(cx^2+bx+a)^{3/2}(cd^2-bed+ae^2)}{3e^2(ef-dg)}$$

$$- \frac{(2cd-be)(8c^2d^2-b^2e^2-4ce(2bd-3ae))\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)(cd^2-bed+ae^2)}{16c^{3/2}e^5(ef-dg)}$$

$$+ \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{cx^2+bx+a}(cd^2-bed+ae^2)}{8ce^4(ef-dg)}$$

$$- \frac{(8cef^2-g(11bef-3bdg-8aeg)-6cg(ef-dg)x)(cx^2+bx+a)^{3/2}}{24eg^2(ef-dg)}$$

$$+ \frac{(128c^4ef^5-320c^3eg(bf-ag)f^3-b^3g^4(5bef+3bdg-8aeg)+48c^2g^2(5b^2ef^3-10abegf^2+a^2g^2(5ef-dg))-8bcg^3(5ef-dg))}{128c^{3/2}eg^5(ef-dg)}$$

$$- \frac{(cf^2-bgf+ag^2)^{5/2}\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)}{g^5(ef-dg)}$$

$$- \frac{(64c^3ef^4-16c^2eg(9bf-8ag)f^2-b^2g^3(5bef+3bdg-8aeg)+4cg^2(22b^2ef^2+16a^2eg^2-3abg(13ef-dg))-2cg(16c^2ef^2-b^2g^2(5ef-dg)))}{64ceg^4(ef-dg)}$$

[Out]  $((c*d^2 - b*d*e + a*e^2)*(8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x)*\text{Sqrt}[a + b*x + c*x^2])/(8*c^2*e^4*(e*f - d*g)) - ((64*c^3*e*f^4 - 16*c^2*e*f^2*g*(9*b*f - 8*a*g) - b^2*g^3*(5*b*e*f + 3*b*d*g - 8*a*e*g) + 4*c*g^2*(22*b^2*e*f^2 + 16*a^2*e*g^2 - 3*a*b*g*(13*e*f - d*g)) - 2*c*g*(16*c^2*e*f^3 + b*g^2*(5*b*e*f + 3*b*d*g - 8*a*e*g) - 4*c*g*(6*b*e*f^2 - a*g*(7*e*f - 3*d*g))) * x) * \text{Sqrt}[a + b*x + c*x^2]) / ((64*c^2*e*g^4*(e*f - d*g)) + ((c*d^2 - b*d*e + a*e^2)*(a + b*x + c*x^2)^(3/2)) / (3*e^2*(e*f - d*g)) - ((8*c^2*e*f^2 - g*(11*b*e*f - 3*b*d*g - 8*a*e*g) - 6*c*g*(e*f - d*g)*x)*(a + b*x + c*x^2)^(3/2)) / (24*e*g^2*(e*f - d*g)) - ((2*c*d - b*e)*(c*d^2 - b*d*e + a*e^2)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])]) / (16*c^(3/2)*e^5*(e*f - d*g)) + ((128*c^4*e*f^5 - 320*c^3*e*f^3*g*(b*f - a*g) - b^3*g^4*(5*b*e*f + 3*b*d*g - 8*a*e*g) + 48*c^2*g^2*(5*b^2*e*f^3 - 10*a*b*e*f^2*g + a^2*g^2*(5*e*f - d*g)) - 8*b*c*g^3*(5*b^2*e*f^2 + 12*a^2*e*g^2 - 3*a*b*g*(5*e*f + d*g))) * \text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])]) / (128*c^(3/2)*e*g^5*(e*f - d*g)) + ((c*d^2 - b*d*e + a*e^2)^(5/2)*\text{ArcTanh}[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*\text{Sqrt}[a + b*x + c*x^2])]) / (e^5*(e*f - d*g)) - ((c*f^2 - b*f*g + a*g^2)^(5/2)*\text{ArcTanh}[(b*f - 2*a*g + (2*c*f - b*g)*x)/(2*\text{Sqrt}[c*f^2 - b*f*g + a*g^2]*\text{Sqrt}[a + b*x + c*x^2])]) / (g^5*(e*f - d*g))$

**Rubi [A]** time = 3.88809, antiderivative size = 886, normalized size of antiderivative = 1., number of

steps used = 15, number of rules used = 7, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$

$$\frac{\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\sqrt{cx^2+bx+a}}\right)(cd^2-bed+ae^2)^{5/2}}{e^5(e^5-dg)} + \frac{(cx^2+bx+a)^{3/2}(cd^2-bed+ae^2)}{3e^2(e^5-dg)}$$

$$- \frac{(2cd-be)(8c^2d^2-b^2e^2-4ce(2bd-3ae))\tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{cx^2+bx+a}}\right)(cd^2-bed+ae^2)}{16c^{3/2}e^5(e^5-dg)}$$

$$+ \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x)\sqrt{cx^2+bx+a}(cd^2-bed+ae^2)}{8ce^4(e^5-dg)}$$

$$- \frac{(8cef^2-g(11bef-3bdg-8aeg)-6cg(e^5-dg)x)(cx^2+bx+a)^{3/2}}{24eg^2(e^5-dg)}$$

$$+ \frac{(128c^4ef^5-320c^3eg(bf-ag)f^3-b^3g^4(5bef+3bdg-8aeg)+48c^2g^2(5b^2ef^3-10abegf^2+a^2g^2(5ef-dg))-8bcg^3(5ef-dg))}{128c^{3/2}eg^5(e^5-dg)}$$

$$- \frac{(cf^2-bgf+ag^2)^{5/2}\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)}{g^5(e^5-dg)}$$

$$- \frac{(64c^3ef^4-16c^2eg(9bf-8ag)f^2-b^2g^3(5bef+3bdg-8aeg)+4cg^2(22b^2ef^2+16a^2eg^2-3abg(13ef-dg))-2cg(16c^2ef^2-b^2g^2(5ef-dg)))}{64ceg^4(e^5-dg)}$$

Antiderivative was successfully verified.

[In] Int[(a + b\*x + c\*x^2)^(5/2)/((d + e\*x)\*(f + g\*x)), x]

[Out] ((c\*d^2 - b\*d\*e + a\*e^2)\*(8\*c^2\*d^2 + b^2\*e^2 - 2\*c\*e\*(5\*b\*d - 4\*a\*e) - 2\*c\*e\*(2\*c\*d - b\*e)\*x)\*Sqrt[a + b\*x + c\*x^2])/((8\*c^2\*d^2 + b^2\*e^2 - 2\*c\*e\*(5\*b\*d - 4\*a\*e) - 2\*c\*e\*(2\*c\*d - b\*e)\*x)\*Sqrt[a + b\*x + c\*x^2]) - ((64\*c^3\*e\*f^4 - 16\*c^2\*e\*f^2\*g\*(9\*b\*f - 8\*a\*g) - b^2\*g^3\*(5\*b\*e\*f + 3\*b\*d\*g - 8\*a\*e\*g) + 4\*c\*g^2\*(22\*b^2\*e\*f^2 + 16\*a^2\*e\*g^2 - 3\*a\*b\*g\*(13\*e\*f - d\*g)) - 2\*c\*g\*(16\*c^2\*e\*f^3 + b\*g^2\*(5\*b\*e\*f + 3\*b\*d\*g - 8\*a\*e\*g)) - 4\*c\*g\*(6\*b\*e\*f^2 - a\*g\*(7\*e\*f - 3\*d\*g)))\*x)\*Sqrt[a + b\*x + c\*x^2])/((64\*c^2\*e\*g^4\*(e\*f - d\*g) + ((c\*d^2 - b\*d\*e + a\*e^2)\*(a + b\*x + c\*x^2)^(3/2))/(3\*e^2\*(e\*f - d\*g)) - ((8\*c\*e\*f^2 - g\*(11\*b\*e\*f - 3\*b\*d\*g - 8\*a\*e\*g) - 6\*c\*g\*(e\*f - d\*g)\*x)\*(a + b\*x + c\*x^2)^(3/2))/(24\*e\*g^2\*(e\*f - d\*g)) - ((2\*c\*d - b\*e)\*(c\*d^2 - b\*d\*e + a\*e^2)\*(8\*c^2\*d^2 - b^2\*e^2 - 4\*c\*e\*(2\*b\*d - 3\*a\*e))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(16\*c^(3/2)\*e^5\*(e\*f - d\*g)) + ((128\*c^4\*e\*f^5 - 320\*c^3\*e\*f^3\*g\*(b\*f - a\*g) - b^3\*g^4\*(5\*b\*e\*f + 3\*b\*d\*g - 8\*a\*e\*g) + 48\*c^2\*g^2\*(5\*b^2\*e\*f^3 - 10\*a\*b\*e\*f^2\*g + a^2\*g^2\*(5\*e\*f - d\*g)) - 8\*b\*c\*g^3\*(5\*b^2\*e\*f^2 + 12\*a^2\*e\*g^2 - 3\*a\*b\*g\*(5\*e\*f + d\*g)))\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(128\*c^(3/2)\*e\*g^5\*(e\*f - d\*g)) + ((c\*d^2 - b\*d\*e + a\*e^2)^(5/2)\*ArcTanh[(b\*d - 2\*a\*e + (2\*c\*d - b\*e)\*x)/(2\*Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*Sqrt[a + b\*x + c\*x^2])])/(e^5\*(e\*f - d\*g)) - ((c\*f^2 - b\*f\*g + a\*g^2)^(5/2)\*ArcTanh[(b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)/(2\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2]\*Sqrt[a + b\*x + c\*x^2])])/(g^5\*(e\*f - d\*g))

**Rubi in Sympy [F-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+b\*x+a)\*\*(5/2)/(e\*x+d)/(g\*x+f), x)

[Out] Timed out



**Mathematica [A]** time = 3.08084, size = 749, normalized size = 0.85

$$\begin{aligned} & \log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)\left(240c^2e^2g^2\left(a^2e^2g^2-2abeg(dg+ef)+b^2\left(d^2g^2+defg+e^2f^2\right)\right)-40b^2ce^3g^3(-3aeg+\right. \\ & \left.+\sqrt{a+x(b+cx)}\left(8c^2eg\left(aeg(-56dg-56ef+27egx)+b\left(54d^2g^2+2deg(27f-13gx)+e^2\left(54f^2-26fgx+17g^2x^2\right)\right)\right)\right)+2\right. \\ & \left.+\frac{\log(d+ex)\left(e(ae-bd)+cd^2\right)^{5/2}}{e^5(ef-dg)}\right. \\ & \left.-\frac{\left(e(ae-bd)+cd^2\right)^{5/2}\log\left(2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}+2ae-bd+bex-2cfx\right)}{e^5(ef-dg)}\right. \\ & \left.+\frac{\log(f+gx)\left(g(ag-bf)+cf^2\right)^{5/2}}{g^5(dg-ef)}\right. \\ & \left.+\frac{\left(g(ag-bf)+cf^2\right)^{5/2}\log\left(2\sqrt{a+x(b+cx)}\sqrt{g(ag-bf)+cf^2}+2ag-bf+bgx-2cfx\right)}{g^5(ef-dg)}\right) \end{aligned}$$

Antiderivative was successfully verified.

[In] Integrate[(a + b\*x + c\*x^2)^(5/2)/((d + e\*x)\*(f + g\*x)),x]

[Out] (Sqrt[a + x\*(b + c\*x)]\*(15\*b^3\*e^3\*g^3 + 2\*b\*c\*e^2\*g^2\*(278\*a\*e\*g + b\*(-132\*e\*f - 132\*d\*g + 59\*e\*g\*x)) - 16\*c^3\*(12\*d^3\*g^3 - 6\*d^2\*e\*g^2\*(-2\*f + g\*x) + 2\*d\*e^2\*g\*(6\*f^2 - 3\*f\*g\*x + 2\*g^2\*x^2) + e^3\*(12\*f^3 - 6\*f^2\*g\*x + 4\*f\*g^2\*x^2 - 3\*g^3\*x^3)) + 8\*c^2\*e\*g\*(a\*e\*g\*(-56\*e\*f - 56\*d\*g + 27\*e\*g\*x) + b\*(54\*d^2\*g^2 + 2\*d\*e\*g\*(27\*f - 13\*g\*x) + e^2\*(54\*f^2 - 26\*f\*g\*x + 17\*g^2\*x^2)))))/(192\*c^4\*g^4) + ((c\*d^2 + e\*(-b\*d) + a\*e)^(5/2)\*Log[d + e\*x])/(e^5\*(e\*f - d\*g)) + ((c\*f^2 + g\*(-b\*f) + a\*g)^(5/2)\*Log[f + g\*x])/(g^5\*(-(e\*f) + d\*g)) + ((-5\*b^4\*e^4\*g^4 - 40\*b^2\*c\*e^3\*g^3\*(b\*e\*f + b\*d\*g - 3\*a\*e\*g) + 128\*c^4\*(e^4\*f^4 + d\*e^3\*f^3\*g + d^2\*e^2\*f^2\*g^2 + d^3\*e\*f\*g^3 + d^4\*g^4) + 240\*c^2\*e^2\*g^2\*(a^2\*e^2\*g^2 - 2\*a\*b\*e\*g\*(e\*f + d\*g) + b^2\*(e^2\*f^2 + d\*e\*f\*g + d^2\*g^2)) - 320\*c^3\*e\*g\*(-(a\*e\*g\*(e^2\*f^2 + d\*e\*f\*g + d^2\*g^2)) + b\*(e^3\*f^3 + d\*e^2\*f^2\*g + d^2\*e\*f\*g^2 + d^3\*g^3)))\*Log[b + 2\*c\*x + 2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)])/(128\*c^(3/2)\*e^5\*g^5) - ((c\*d^2 + e\*(-b\*d) + a\*e)^(5/2)\*Log[-(b\*d) + 2\*a\*e - 2\*c\*d\*x + b\*e\*x + 2\*Sqrt[c\*d^2 + e\*(-b\*d) + a\*e])\*Sqrt[a + x\*(b + c\*x)])/(e^5\*(e\*f - d\*g)) + ((c\*f^2 + g\*(-b\*f) + a\*g)^(5/2)\*Log[-(b\*f) + 2\*a\*g - 2\*c\*f\*x + b\*g\*x + 2\*Sqrt[c\*f^2 + g\*(-b\*f) + a\*g])\*Sqrt[a + x\*(b + c\*x)])/(g^5\*(e\*f - d\*g))

**Maple [B]** time = 0.034, size = 9052, normalized size = 10.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c\*x^2+b\*x+a)^(5/2)/(e\*x+d)/(g\*x+f),x)

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)^(5/2)/((e*x + d)*(g*x + f)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

---

**Fricas** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)^(5/2)/((e*x + d)*(g*x + f)),x, algorithm="fricas")
```

```
[Out] Timed out
```

---

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(5/2)/(e*x+d)/(g*x+f),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS** [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)^(5/2)/((e*x + d)*(g*x + f)),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.870 \quad \int \frac{(f+gx)^4}{(d+ex)\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=431

$$\frac{g \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (8c^2eg (aeg(4ef - dg) + b (d^2g^2 - 4defg + 6e^2f^2)) - 6bce^2g^2(2aeg - bdg + 4bef) + 5b^3e^3g^3 - 16c^{7/2}e^4}{g^2\sqrt{a+bx+cx^2} (-4ceg(4aeg - 7bdg + 18bef) + 15b^2e^2g^2 + 4c^2 (11d^2g^2 - 36defg + 36e^2f^2))} + \frac{g^3(d+ex)\sqrt{a+bx+cx^2}(-5beg - 14cdg + 24cef)}{24c^3e^3} + \frac{g^3(d+ex)\sqrt{a+bx+cx^2}(-5beg - 14cdg + 24cef)}{12c^2e^3} + \frac{(ef - dg)^4 \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^4\sqrt{ae^2-bde+cd^2}} + \frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3}$$

[Out]  $(g^2*(15*b^2*e^2*g^2 - 4*c*e*g*(18*b*e*f - 7*b*d*g + 4*a*e*g) + 4*c^2*(36*e^2*f^2 - 36*d*e*f*g + 11*d^2*g^2))*\text{Sqrt}[a + b*x + c*x^2]/(24*c^3*e^3) + (g^3*(24*c*e*f - 14*c*d*g - 5*b*e*g)*(d + e*x)*\text{Sqrt}[a + b*x + c*x^2])/((12*c^2*e^3) + (g^4*(d + e*x)^2*\text{Sqrt}[a + b*x + c*x^2]))/(3*c*e^3) - (g*(5*b^3*e^3*g^3 - 6*b*c*e^2*g^2*(4*b*e*f - b*d*g + 2*a*e*g) - 16*c^3*(4*e^3*f^3 - 6*d*e^2*f^2*g + 4*d^2*e*f*g^2 - d^3*g^3) + 8*c^2*e*g*(a*e*g*(4*e*f - d*g) + b*(6*e^2*f^2 - 4*d*e*f*g + d^2*g^2)))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2]))/(16*c^(7/2)*e^4) + ((e*f - d*g)^4*\text{ArcTanh}[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*\text{Sqrt}[a + b*x + c*x^2]))/(e^4*\text{Sqrt}[c*d^2 - b*d*e + a*e^2])$

**Rubi [A]** time = 2.30441, antiderivative size = 431, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$\frac{g \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (8c^2eg (aeg(4ef - dg) + b (d^2g^2 - 4defg + 6e^2f^2)) - 6bce^2g^2(2aeg - bdg + 4bef) + 5b^3e^3g^3 - 16c^{7/2}e^4}{g^2\sqrt{a+bx+cx^2} (-4ceg(4aeg - 7bdg + 18bef) + 15b^2e^2g^2 + 4c^2 (11d^2g^2 - 36defg + 36e^2f^2))} + \frac{g^3(d+ex)\sqrt{a+bx+cx^2}(-5beg - 14cdg + 24cef)}{24c^3e^3} + \frac{g^3(d+ex)\sqrt{a+bx+cx^2}(-5beg - 14cdg + 24cef)}{12c^2e^3} + \frac{(ef - dg)^4 \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^4\sqrt{ae^2-bde+cd^2}} + \frac{g^4(d+ex)^2\sqrt{a+bx+cx^2}}{3ce^3}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)^4/((d + e\*x)\*Sqrt[a + b\*x + c\*x^2]),x]

[Out]  $(g^2*(15*b^2*e^2*g^2 - 4*c*e*g*(18*b*e*f - 7*b*d*g + 4*a*e*g) + 4*c^2*(36*e^2*f^2 - 36*d*e*f*g + 11*d^2*g^2))*\text{Sqrt}[a + b*x + c*x^2]/(24*c^3*e^3) + (g^3*(24*c*e*f - 14*c*d*g - 5*b*e*g)*(d + e*x)*\text{Sqrt}[a + b*x + c*x^2])/((12*c^2*e^3) + (g^4*(d + e*x)^2*\text{Sqrt}[a + b*x + c*x^2]))/(3*c*e^3) - (g*(5*b^3*e^3*g^3 - 6*b*c*e^2*g^2*(4*b*e*f - b*d*g + 2*a*e*g) - 16*c^3*(4*e^3*f^3 - 6*d*e^2*f^2*g + 4*d^2*e*f*g^2 - d^3*g^3) + 8*c^2*e*g*(a*e*g*(4*e*f - d*g) + b*(6*e^2*f^2 - 4*d*e*f*g + d^2*g^2)))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2]))/(16*c^(7/2)*e^4) + ((e*f - d*g)^4*\text{ArcTanh}[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*\text{Sqrt}[a + b*x + c*x^2]))/(e^4*\text{Sqrt}[c*d^2 - b*d*e + a*e^2])$

**Rubi in Sympy [A]** time = 135.431, size = 524, normalized size = 1.22

$$\begin{aligned} & \frac{3bg^3(dg - 4ef)\sqrt{a + bx + cx^2}}{4c^2e^2} - \frac{bg^2(d^2g^2 - 4defg + 6e^2f^2)\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{\frac{3}{2}}e^3} \\ & - \frac{bg^4(-12ac + 5b^2)\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{16c^{\frac{7}{2}}e} - \frac{(dg - ef)^4\operatorname{atanh}\left(\frac{2ae-bd+x(be-2cd)}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^4\sqrt{ae^2 - bde + cd^2}} \\ & + \frac{g^4x^2\sqrt{a + bx + cx^2}}{3ce} - \frac{g^3x(dg - 4ef)\sqrt{a + bx + cx^2}}{2ce^2} \\ & + \frac{g^2\sqrt{a + bx + cx^2}(d^2g^2 - 4defg + 6e^2f^2)}{ce^3} + \frac{g^4\sqrt{a + bx + cx^2}\left(-4ac + \frac{15b^2}{4} - \frac{5bcx}{2}\right)}{6c^3e} \\ & - \frac{g(dg - 2ef)(d^2g^2 - 2defg + 2e^2f^2)\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ce^4}} \\ & - \frac{g^3(-4ac + 3b^2)(dg - 4ef)\operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{\frac{5}{2}}e^2} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x+f)**4/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)`

[Out]  $3*b*g**3*(d*g - 4*e*f)*\operatorname{sqrt}(a + b*x + c*x**2)/(4*c**2*e**2) - b*g**2*(d**2*g**2 - 4*d*e*f*g + 6*e**2*f**2)*\operatorname{atanh}((b + 2*c*x)/(2*\operatorname{sqrt}(c)*\operatorname{sqrt}(a + b*x + c*x**2)))/(2*c**(3/2)*e**3) - b*g**4*(-12*a*c + 5*b**2)*\operatorname{atanh}((b + 2*c*x)/(2*\operatorname{sqrt}(c)*\operatorname{sqrt}(a + b*x + c*x**2)))/(16*c**(7/2)*e) - (d*g - e*f)**4*\operatorname{atanh}((2*a*e - b*d + x*(b*e - 2*c*d))/(2*\operatorname{sqrt}(a + b*x + c*x**2)*\operatorname{sqrt}(a*e**2 - b*d*e + c*d**2)))/(e**4*\operatorname{sqrt}(a*e**2 - b*d*e + c*d**2)) + g**4*x**2*\operatorname{sqrt}(a + b*x + c*x**2)/(3*c*e) - g**3*x*(d*g - 4*e*f)*\operatorname{sqrt}(a + b*x + c*x**2)/(2*c*e**2) + g**2*\operatorname{sqrt}(a + b*x + c*x**2)*(d**2*g**2 - 4*d*e*f*g + 6*e**2*f**2)/(c*e**3) + g**4*\operatorname{sqrt}(a + b*x + c*x**2)*(-4*a*c + 15*b**2/4 - 5*b*c*x/2)/(6*c**3*e) - g*(d*g - 2*e*f)*(d**2*g**2 - 2*d*e*f*g + 2*e**2*f**2)*\operatorname{atanh}((b + 2*c*x)/(2*\operatorname{sqrt}(c)*\operatorname{sqrt}(a + b*x + c*x**2)))/(sqrt(c)*e**4) - g**3*(-4*a*c + 3*b**2)*(d*g - 4*e*f)*\operatorname{atanh}((b + 2*c*x)/(2*\operatorname{sqrt}(c)*\operatorname{sqrt}(a + b*x + c*x**2)))/(8*c**(5/2)*e**2)$

**Mathematica [A]** time = 1.61875, size = 402, normalized size = 0.93

$$\frac{3g \log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)\left(-8c^2eg(aeg(4ef-dg)+b(d^2g^2-4defg+6e^2f^2))+6bce^2g^2(2aeg-bdg+4bef)-5b^3e^3g^3+16c^3(-d^3g^3+4d^2efg^2-6de^2f^2g+4e^3f^3)\right)}{c^{7/2}}$$

Antiderivative was successfully verified.

[In] `Integrate[(f + g*x)^4/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]`

[Out]  $((2*e*g^2*\operatorname{Sqrt}[a + x*(b + c*x)]*(15*b^2*e^2*g^2 - 2*c*e*g*(8*a*e*g + b*(36*e*f - 9*d*g + 5*e*g*x)) + 4*c^2*(6*d^2*g^2 - 3*d*e*g*(8*f + g*x) + 2*e^2*(18*f^2 + 6*f*g*x + g^2*x^2)))/c^3 + (48*(e*f - d*g)^4*\operatorname{Log}[d + e*x])/Sqrt[c*d^2 + e*(-(b*d) + a*e)] + (3*g*(-5*b^3*e^3*g^3 + 6*b*c*e^2*g^2*(4*b*e*f - b*d*g + 2*a*e*g) + 16*c^3*(4*e^3*f^3 - 6*d*e^2*f^2*g + 4*d^2*e*f*g^2 - d^3*g^3) - 8*c^2*e*g*(a*e*g*(4*e*f - d*g) + b*(6*e^2*f^2 - 4*d*e*f*g + d^2*g^2)))*\operatorname{Log}[b + 2*c*x + 2*\operatorname{Sqrt}[c]*\operatorname{Sqrt}[a + x*(b + c*x)])/c^{(7/2)} - (48*(e*f - d*g)^4*\operatorname{Log}[-(b*d) + 2*a*e - 2*c*d*x + b*e*x + 2*\operatorname{Sqrt}[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])/Sqrt[c*d^2 + e*(-(b*d) + a*e)]/(48*e^4)$

**Maple [B]** time = 0.035, size = 1597, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^{(1/2)}, x)$

[Out] 
$$\begin{aligned} & -1/e/((a^2e^2-b^2d^2+c^2d^2)/e^2)^{(1/2)} * \ln((2*(a^2e^2-b^2d^2+c^2d^2)/e^2 + \\ & 2*(b^2e-2*c^2d)/e*(x+d/e)+2*((a^2e^2-b^2d^2+c^2d^2)/e^2)^{(1/2)}*((x+d/e) \\ & )^2*c+(b^2e-2*c^2d)/e*(x+d/e)+(a^2e^2-b^2d^2+c^2d^2)/e^2)^{(1/2)}/(x+d/ \\ & e))^f^4+6*g^2/e/c*(c*x^2+b*x+a)^{(1/2)}*f^2+1/3*g^4/e*x^2/c*(c*x^2+ \\ & b*x+a)^{(1/2)}+5/8*g^4/e*b^2/c^3*(c*x^2+b*x+a)^{(1/2)}-5/16*g^4/e*b^3 \\ & /c^{(7/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})-2/3*g^4/e/c^2 \\ & *a*(c*x^2+b*x+a)^{(1/2)}-g^4/e^4*d^3*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2 \\ & +b*x+a)^{(1/2)})/c^{(1/2)}+4*g/e*f^3*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b* \\ & x+a)^{(1/2)})/c^{(1/2)}-1/e^5/((a^2e^2-b^2d^2+c^2d^2)/e^2)^{(1/2)}*\ln((2*( \\ & a^2e^2-b^2d^2+c^2d^2)/e^2+(b^2e-2*c^2d)/e*(x+d/e)+2*((a^2e^2-b^2d^2+c^2d^2) \\ & )/e^2)^{(1/2)}*((x+d/e)^2*c+(b^2e-2*c^2d)/e*(x+d/e)+(a^2e^2-b^2d^2+c^2d^2) \\ & )/e^2)^{(1/2)}/(x+d/e))^d^4*g^4+g^4/e^3/c*(c*x^2+b*x+a)^{(1/2)}*d^2+2*g^3/e^2*b/c^{(3/2)} \\ & *\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)}) *d*f-2*g^3/e*a/c^{(3/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)}) \\ & ) *f-4*g^3/e^2/c*(c*x^2+b*x+a)^{(1/2)} *d*f-1/2*g^4/e^3*b/c^{(3/2)}*\ln( \\ & (1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)}) *d^2-3*g^2/e*b/c^{(3/2)}*\ln( \\ & ((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)}) *f^2-1/2*g^4/e^2*x/c*(c* \\ & x^2+b*x+a)^{(1/2)} *d+2*g^3/e*x/c*(c*x^2+b*x+a)^{(1/2)} *f+3/4*g^4/e^2* \\ & b/c^2*(c*x^2+b*x+a)^{(1/2)} *d-3*g^3/e*b/c^2*(c*x^2+b*x+a)^{(1/2)} *f-3 \\ & /8*g^4/e^2*b^2/c^{(5/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)}) \\ & ) *d+3/2*g^3/e*b^2/c^{(5/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)}) \\ & ) *f-6/e^3/((a^2e^2-b^2d^2+c^2d^2)/e^2)^{(1/2)}*\ln((2*(a^2e^2-b^2d^2+c^2d^2) \\ & )/e^2+(b^2e-2*c^2d)/e*(x+d/e)+2*((a^2e^2-b^2d^2+c^2d^2)/e^2)^{(1/2)} \\ & )*((x+d/e)^2*c+(b^2e-2*c^2d)/e*(x+d/e)+(a^2e^2-b^2d^2+c^2d^2)/e^2)^{(1/2)}/(x+d/e))^d^2*f^2*g^2+4/e^2/((a^2e^2-b^2d^2+c^2d^2)/e^2)^{(1/2)}*\ln( \\ & (2*(a^2e^2-b^2d^2+c^2d^2)/e^2+(b^2e-2*c^2d)/e*(x+d/e)+2*((a^2e^2-b^2d^2+c^2d^2) \\ & )/e^2)^{(1/2)}*((x+d/e)^2*c+(b^2e-2*c^2d)/e*(x+d/e)+(a^2e^2-b^2d^2+c^2d^2) \\ & )/e^2)^{(1/2)}/(x+d/e))^d*f^3*g^4/e^4/((a^2e^2-b^2d^2+c^2d^2)/e^2)^{(1/2)}*\ln((2*(a^2e^2-b^2d^2+c^2d^2) \\ & )/e^2+(b^2e-2*c^2d)/e*(x+d/e)+2*((a^2e^2-b^2d^2+c^2d^2)/e^2)^{(1/2)}*((x+d/e)^2*c+(b^2e-2*c^2d)/e*(x+d/e) \\ & )+(a^2e^2-b^2d^2+c^2d^2)/e^2)^{(1/2)}/(x+d/e))^d^3*f^2*g^3-5/12*g^4/e*b/c^2*x*(c*x^2+b*x+a)^{(1/2)}+3/4*g^4/e*b/c^{(5/2)}*a*\ln((1/2*b+c*x)/c^{(1/2)} \\ & (1/2)+(c*x^2+b*x+a)^{(1/2)})-6*g^2/e^2*d*f^2*\ln((1/2*b+c*x)/c^{(1/2)} \\ & +(c*x^2+b*x+a)^{(1/2)})/c^{(1/2)}+4*g^3/e^3*d^2*f*\ln((1/2*b+c*x)/c^{(1/2)} \\ & +(c*x^2+b*x+a)^{(1/2)})/c^{(1/2)}+1/2*g^4/e^2*a/c^{(3/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)}) *d \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((g*x + f)^4/(\text{sqrt}(c*x^2 + b*x + a)*(e*x + d)), x, \text{algorithm}="maxima")$

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}((g*x + f)^4/(\text{sqrt}(c*x^2 + b*x + a)*(e*x + d)), x, \text{algorithm}="fricas")$

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(f + gx)^4}{(d + ex) \sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)**4/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)`

[Out] `Integral((f + g*x)**4/((d + e*x)*sqrt(a + b*x + c*x**2)), x)`

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x + f)^4/(sqrt(c*x^2 + b*x + a)*(e*x + d)),x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.871 \quad \int \frac{(f+gx)^3}{(d+ex)\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=270

$$\frac{g \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (-4ceg(aeg - bdg + 3bef) + 3b^2e^2g^2 + 8c^2(d^2g^2 - 3defg + 3e^2f^2))}{8c^{5/2}e^3} + \frac{3g^2\sqrt{a+bx+cx^2}(-beg - 2cdg + 4cef)}{4c^2e^2} + \frac{(ef - dg)^3 \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^3\sqrt{ae^2-bde+cd^2}} + \frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2}$$

[Out]  $(3*g^2*(4*c*e*f - 2*c*d*g - b*e*g)*\text{Sqrt}[a + b*x + c*x^2])/(4*c^2*e^2) + (g^3*(d + e*x)*\text{Sqrt}[a + b*x + c*x^2])/(2*c*e^2) + (g*(3*b^2*e^2*g^2 - 4*c*e*g*(3*b*e*f - b*d*g + a*e*g) + 8*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(8*c^{5/2}*e^3) + ((e*f - d*g)^3*\text{ArcTanh}[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*\text{Sqrt}[a + b*x + c*x^2])])/(e^3*\text{Sqrt}[c*d^2 - b*d*e + a*e^2])$

**Rubi [A]** time = 1.239, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$\frac{g \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (-4ceg(aeg - bdg + 3bef) + 3b^2e^2g^2 + 8c^2(d^2g^2 - 3defg + 3e^2f^2))}{8c^{5/2}e^3} + \frac{3g^2\sqrt{a+bx+cx^2}(-beg - 2cdg + 4cef)}{4c^2e^2} + \frac{(ef - dg)^3 \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^3\sqrt{ae^2-bde+cd^2}} + \frac{g^3(d+ex)\sqrt{a+bx+cx^2}}{2ce^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(f + g*x)^3/((d + e*x)*\text{Sqrt}[a + b*x + c*x^2]), x]$

[Out]  $(3*g^2*(4*c*e*f - 2*c*d*g - b*e*g)*\text{Sqrt}[a + b*x + c*x^2])/(4*c^2*e^2) + (g^3*(d + e*x)*\text{Sqrt}[a + b*x + c*x^2])/(2*c*e^2) + (g*(3*b^2*e^2*g^2 - 4*c*e*g*(3*b*e*f - b*d*g + a*e*g) + 8*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2])])/(8*c^{5/2}*e^3) + ((e*f - d*g)^3*\text{ArcTanh}[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*\text{Sqrt}[a + b*x + c*x^2])])/(e^3*\text{Sqrt}[c*d^2 - b*d*e + a*e^2])$

**Rubi in Sympy [A]** time = 94.1935, size = 332, normalized size = 1.23

$$-\frac{3bg^3\sqrt{a+bx+cx^2}}{4c^2e} + \frac{bg^2(dg - 3ef) \operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{\frac{3}{2}}e^2} + \frac{(dg - ef)^3 \operatorname{atanh}\left(\frac{2ae-bd+x(be-2cd)}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^3\sqrt{ae^2-bde+cd^2}} + \frac{g^3x\sqrt{a+bx+cx^2}}{2ce} - \frac{g^2(dg - 3ef)\sqrt{a+bx+cx^2}}{ce^2} + \frac{g(d^2g^2 - 3defg + 3e^2f^2) \operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ce^3}} + \frac{g^3(-4ac + 3b^2) \operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{8c^{\frac{5}{2}}e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x+f)**3/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)`

[Out] 
$$\begin{aligned} & -3*b*g**3*\sqrt{a+b*x+c*x**2}/(4*c**2*e) + b*g**2*(d*g - 3*e*f) \\ & * \operatorname{atanh}((b+2*c*x)/(2*\sqrt{c}*\sqrt{a+b*x+c*x**2}))/ (2*c**(3/2)*e**2) + (d*g - e*f)**3* \operatorname{atanh}((2*a*e - b*d + x*(b*e - 2*c*d))/ \\ & (2*\sqrt{a+b*x+c*x**2})*\sqrt{a**2 - b*d*e + c*d**2}))/ (e**3*\sqrt{a**2 - b*d*e + c*d**2}) + g**3*x*\sqrt{a+b*x+c*x**2}/(2*c \\ & *e) - g**2*(d*g - 3*e*f)*\sqrt{a+b*x+c*x**2}/(c*e**2) + g*(d**2*g**2 - 3*d*e*f*g + 3*e**2*f**2)* \operatorname{atanh}((b+2*c*x)/(2*\sqrt{c})*\sqrt{a+b*x+c*x**2}))/ \\ & (\sqrt{c}*e**3) + g**3*(-4*a*c + 3*b**2)* \operatorname{atanh}((b+2*c*x)/(2*\sqrt{c})*\sqrt{a+b*x+c*x**2}))/ (8*c**(5/2)*e) \end{aligned}$$

**Mathematica [A]** time = 0.633402, size = 269, normalized size = 1.

$$\frac{g \log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)\left(-4ceg(aeg-bdg+3bef)+3b^2e^2g^2+8c^2(d^2g^2-3defg+3e^2f^2)\right)}{c^{5/2}} + \frac{2eg^2\sqrt{a+x(b+cx)}(2c(-2dg+6ef+egx)-3beg)}{c^2} + \frac{8(ef-dg)}{\sqrt{e(ae-d^2g^2)}}}{8e^3}$$

Antiderivative was successfully verified.

[In] `Integrate[(f + g*x)^3/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]`

[Out] 
$$\begin{aligned} & ((2*e*g^2*\sqrt{a+x*(b+c*x)}*(-3*b*e*g + 2*c*(6*e*f - 2*d*g + e*g*x)))/c^2 + (8*(e*f - d*g)^3*\operatorname{Log}[d + e*x])/ \sqrt{c*d^2 + e*(-(b*d) + a*e)} \\ & + (g*(3*b^2*e^2*g^2 - 4*c*e*g*(3*b*e*f - b*d*g + a*e*g) + 8*c^2*(3*e^2*f^2 - 3*d*e*f*g + d^2*g^2))* \operatorname{Log}[b + 2*c*x + 2*\sqrt{c}*\sqrt{a+x*(b+c*x)}])/c^{5/2} + (8*(-(e*f) + d*g)^3*\operatorname{Log}[-(b*d) + 2*a*e - 2*c*d*x + b*e*x + 2*\sqrt{c*d^2 + e*(-(b*d) + a*e)}])/ \sqrt{a+x*(b+c*x)} \\ & )/ \sqrt{c*d^2 + e*(-(b*d) + a*e)} \end{aligned}$$

**Maple [B]** time = 0.022, size = 1007, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)`

[Out] 
$$\begin{aligned} & g^3/e^3*d^2*\ln((1/2*b+c*x)/c^{1/2}+(c*x^2+b*x+a)^{1/2})/c^{1/2} - g^3/e^2/c*(c*x^2+b*x+a)^{1/2}*d+3*g^2/e*c*(c*x^2+b*x+a)^{1/2}*f+1/2*g^3/e^2*b/c^{3/2}*\ln((1/2*b+c*x)/c^{1/2}+(c*x^2+b*x+a)^{1/2})*d \\ & -3/2*g^2/e*b/c^{3/2}*\ln((1/2*b+c*x)/c^{1/2}+(c*x^2+b*x+a)^{1/2})*f+1/2*g^3/e*x/c*(c*x^2+b*x+a)^{1/2}-3/4*g^3/e*b/c^2*(c*x^2+b*x+a)^{1/2}+3/8*g^3/e*b^2/c^{5/2}*\ln((1/2*b+c*x)/c^{1/2}+(c*x^2+b*x+a)^{1/2})-1/2*g^3/e*a/c^{3/2}*\ln((1/2*b+c*x)/c^{1/2}+(c*x^2+b*x+a)^{1/2})+3*g/e*f^2*\ln((1/2*b+c*x)/c^{1/2}+(c*x^2+b*x+a)^{1/2})/c^{1/2}-3*g^2/e^2*d*f*\ln((1/2*b+c*x)/c^{1/2}+(c*x^2+b*x+a)^{1/2})/c^{1/2}+1/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^{1/2}*\ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^{1/2}*(x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{1/2})/(x+d/e)*d^3*g^3-3/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^{1/2}*\ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^{1/2}*(x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{1/2})/(x+d/e)*d^2*f*g^2+3/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^{1/2}*\ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^{1/2}*(x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{1/2})/(x+d/e)*d*f^2*g-1/e/((a*e^2-b*d*e+c*d^2)/e^2)^{1/2}*\ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^{1/2}*(x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{1/2})/(x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{1/2} \end{aligned}$$



$$(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^{1/2})/(x+d/e)*f^3$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^3/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^3/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(f + gx)^3}{(d + ex) \sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*3/(e\*x+d)/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral((f + g\*x)\*\*3/((d + e\*x)\*sqrt(a + b\*x + c\*x\*\*2)), x)

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^3/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.872 \quad \int \frac{(f+gx)^2}{(d+ex)\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=176

$$\frac{g \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (-beg - 2cdg + 4cef)}{2c^{3/2}e^2} + \frac{(ef - dg)^2 \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2\sqrt{ae^2 - bde + cd^2}} + \frac{g^2\sqrt{a + bx + cx^2}}{ce}$$

[Out] (g^2\*Sqrt[a + b\*x + c\*x^2])/(c\*e) + (g\*(4\*c\*e\*f - 2\*c\*d\*g - b\*e\*g)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(2\*c^(3/2)\*e^2) + ((e\*f - d\*g)^2\*ArcTanh[(b\*d - 2\*a\*e + (2\*c\*d - b\*e)\*x)/(2\*Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*Sqrt[a + b\*x + c\*x^2])])/(e^2\*Sqrt[c\*d^2 - b\*d\*e + a\*e^2])

**Rubi [A]** time = 0.591174, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$\frac{g \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right) (-beg - 2cdg + 4cef)}{2c^{3/2}e^2} + \frac{(ef - dg)^2 \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2\sqrt{ae^2 - bde + cd^2}} + \frac{g^2\sqrt{a + bx + cx^2}}{ce}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)^2/((d + e\*x)\*Sqrt[a + b\*x + c\*x^2]), x]

[Out] (g^2\*Sqrt[a + b\*x + c\*x^2])/(c\*e) + (g\*(4\*c\*e\*f - 2\*c\*d\*g - b\*e\*g)\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2])])/(2\*c^(3/2)\*e^2) + ((e\*f - d\*g)^2\*ArcTanh[(b\*d - 2\*a\*e + (2\*c\*d - b\*e)\*x)/(2\*Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*Sqrt[a + b\*x + c\*x^2])])/(e^2\*Sqrt[c\*d^2 - b\*d\*e + a\*e^2])

**Rubi in Sympy [A]** time = 59.6637, size = 194, normalized size = 1.1

$$\frac{bg^2 \operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{2c^{\frac{3}{2}}e} - \frac{(dg - ef)^2 \operatorname{atanh}\left(\frac{2ae-bd+x(be-2cd)}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2\sqrt{ae^2 - bde + cd^2}} + \frac{g^2\sqrt{a + bx + cx^2}}{ce} - \frac{g(dg - 2ef) \operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{\sqrt{ce^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*2/(e\*x+d)/(c\*x\*\*2+b\*x+a)\*\*(1/2), x)

[Out] -b\*g\*\*2\*atanh((b + 2\*c\*x)/(2\*sqrt(c)\*sqrt(a + b\*x + c\*x\*\*2)))/(2\*c\*\*(3/2)\*e) - (d\*g - e\*f)\*\*2\*atanh((2\*a\*e - b\*d + x\*(b\*e - 2\*c\*d))/(2\*sqrt(a + b\*x + c\*x\*\*2)\*sqrt(a\*e\*\*2 - b\*d\*e + c\*d\*\*2)))/(e\*\*2\*sqrt(a\*e\*\*2 - b\*d\*e + c\*d\*\*2)) + g\*\*2\*sqrt(a + b\*x + c\*x\*\*2)/(c\*e) - g\*(d\*g - 2\*e\*f)\*atanh((b + 2\*c\*x)/(2\*sqrt(c)\*sqrt(a + b\*x + c\*x\*\*2)))/(sqrt(c)\*e\*\*2)

**Mathematica [A]** time = 0.349091, size = 204, normalized size = 1.16

$$\frac{-\frac{g \log\left(2\sqrt{c}\sqrt{a+x(b+cx)}+b+2cx\right)(beg+2cdg-4cef)}{c^{3/2}} + \frac{2(ef-dg)^2 \log(d+ex)}{\sqrt{e(ae-bd)+cd^2}} - \frac{2(ef-dg)^2 \log\left(2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}+2ae-bd+bex-2cdx\right)}{\sqrt{e(ae-bd)+cd^2}}}{2e^2} + \frac{2efg}{e^2}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^2/((d + e\*x)\*Sqrt[a + b\*x + c\*x^2]), x]

[Out] ((2\*e\*g^2\*Sqrt[a + x\*(b + c\*x)])/c + (2\*(e\*f - d\*g)^2\*Log[d + e\*x])/Sqrt[c\*d^2 + e\*(-(b\*d) + a\*e)] - (g\*(-4\*c\*e\*f + 2\*c\*d\*g + b\*e\*g)\*Log[b + 2\*c\*x + 2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)]])/c^(3/2) - (2\*(e\*f - d\*g)^2\*Log[-(b\*d) + 2\*a\*e - 2\*c\*d\*x + b\*e\*x + 2\*Sqrt[c\*d^2 + e\*(-(b\*d) + a\*e)]\*Sqrt[a + x\*(b + c\*x)]])/Sqrt[c\*d^2 + e\*(-(b\*d) + a\*e)]/(2\*e^2)

**Maple [B]** time = 0.019, size = 613, normalized size = 3.5

$$\begin{aligned} & \frac{g^2}{ce} \sqrt{cx^2 + bx + a} - \frac{g^2 b}{2e} \ln\left(1 + \frac{b}{2} + cx\right) \frac{1}{\sqrt{c}} + \sqrt{cx^2 + bx + a} c^{-\frac{3}{2}} \\ & - \frac{g^2 d}{e^2} \ln\left(1 + \frac{b}{2} + cx\right) \frac{1}{\sqrt{c}} + \sqrt{cx^2 + bx + a} \frac{1}{\sqrt{c}} + 2 \frac{fg}{e\sqrt{c}} \ln\left(\frac{b/2 + cx}{\sqrt{c}} + \sqrt{cx^2 + bx + a}\right) \\ & - \frac{d^2 g^2}{e^3} \ln\left(1 + \left(2 \frac{ae^2 - bde + cd^2}{e^2} + \frac{be - 2cd}{e} \left(x + \frac{d}{e}\right) + 2 \sqrt{\frac{ae^2 - bde + cd^2}{e^2}} \sqrt{\left(x + \frac{d}{e}\right)^2 c + \frac{be - 2cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 - bde}{e^2}}\right)\right) \\ & + 2 \frac{dfg}{e^2} \ln\left(1 + \left(2 \frac{ae^2 - bde + cd^2}{e^2} + \frac{be - 2cd}{e} \left(x + \frac{d}{e}\right) + 2 \sqrt{\frac{ae^2 - bde + cd^2}{e^2}} \sqrt{\left(x + \frac{d}{e}\right)^2 c + \frac{be - 2cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 - bde}{e^2}}\right)\right) \\ & - \frac{f^2}{e} \ln\left(1 + \left(2 \frac{ae^2 - bde + cd^2}{e^2} + \frac{be - 2cd}{e} \left(x + \frac{d}{e}\right) + 2 \sqrt{\frac{ae^2 - bde + cd^2}{e^2}} \sqrt{\left(x + \frac{d}{e}\right)^2 c + \frac{be - 2cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 - bde}{e^2}}\right)\right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^2/(e\*x+d)/(c\*x^2+b\*x+a)^(1/2), x)

[Out] g^2\*(c\*x^2+b\*x+a)^(1/2)/c/e-1/2\*g^2/e\*b/c^(3/2)\*ln((1/2\*b+c\*x)/c^(1/2)+(c\*x^2+b\*x+a)^(1/2))-g^2/e^2\*d\*ln((1/2\*b+c\*x)/c^(1/2)+(c\*x^2+b\*x+a)^(1/2))/c^(1/2)+2\*g/e\*f\*ln((1/2\*b+c\*x)/c^(1/2)+(c\*x^2+b\*x+a)^(1/2))/c^(1/2)-1/e^3/((a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2-b\*d\*e+c\*d^2)/e^2+(b\*e-2\*c\*d)/e\*(x+d/e)+2\*((a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*((x+d/e)^2\*c+(b\*e-2\*c\*d)/e\*(x+d/e)+(a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2))/(x+d/e)\*d^2\*g^2+2/e^2/((a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2-b\*d\*e+c\*d^2)/e^2+(b\*e-2\*c\*d)/e\*(x+d/e)+2\*((a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*((x+d/e)^2\*c+(b\*e-2\*c\*d)/e\*(x+d/e)+(a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2))/(x+d/e)\*d\*f\*g-1/e/((a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2-b\*d\*e+c\*d^2)/e^2+(b\*e-2\*c\*d)/e\*(x+d/e)+2\*((a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*((x+d/e)^2\*c+(b\*e-2\*c\*d)/e\*(x+d/e)+(a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2))/(x+d/e)\*f^2

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^2/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^2/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(f + gx)^2}{(d + ex) \sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*2/(e\*x+d)/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral((f + g\*x)\*\*2/((d + e\*x)\*sqrt(a + b\*x + c\*x\*\*2)), x)

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^2/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.873 \quad \int \frac{f+gx}{(d+ex)\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=131

$$\frac{(ef - dg) \tanh^{-1} \left( \frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}} \right)}{e\sqrt{ae^2 - bde + cd^2}} + \frac{g \tanh^{-1} \left( \frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}} \right)}{\sqrt{ce}}$$

[Out] (g\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2]))/(Sqrt[c]\*e) + ((e\*f - d\*g)\*ArcTanh[(b\*d - 2\*a\*e + (2\*c\*d - b\*e)\*x]/(2\*Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*Sqrt[a + b\*x + c\*x^2]))/(e\*Sqrt[c\*d^2 - b\*d\*e + a\*e^2])

**Rubi [A]** time = 0.235787, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$\frac{(ef - dg) \tanh^{-1} \left( \frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}} \right)}{e\sqrt{ae^2 - bde + cd^2}} + \frac{g \tanh^{-1} \left( \frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}} \right)}{\sqrt{ce}}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)/((d + e\*x)\*Sqrt[a + b\*x + c\*x^2]), x]

[Out] (g\*ArcTanh[(b + 2\*c\*x)/(2\*Sqrt[c]\*Sqrt[a + b\*x + c\*x^2]))/(Sqrt[c]\*e) + ((e\*f - d\*g)\*ArcTanh[(b\*d - 2\*a\*e + (2\*c\*d - b\*e)\*x]/(2\*Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*Sqrt[a + b\*x + c\*x^2]))/(e\*Sqrt[c\*d^2 - b\*d\*e + a\*e^2])

**Rubi in Sympy [A]** time = 36.4225, size = 117, normalized size = 0.89

$$\frac{(dg - ef) \operatorname{atanh} \left( \frac{2ae-bd+x(be-2cd)}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}} \right)}{e\sqrt{ae^2 - bde + cd^2}} + \frac{g \operatorname{atanh} \left( \frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}} \right)}{\sqrt{ce}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)/(e\*x+d)/(c\*x\*\*2+b\*x+a)\*\*(1/2), x)

[Out] (d\*g - e\*f)\*atanh((2\*a\*e - b\*d + x\*(b\*e - 2\*c\*d))/(2\*sqrt(a + b\*x + c\*x\*\*2)\*sqrt(a\*e\*\*2 - b\*d\*e + c\*d\*\*2)))/(e\*sqrt(a\*e\*\*2 - b\*d\*e + c\*d\*\*2)) + g\*atanh((b + 2\*c\*x)/(2\*sqrt(c)\*sqrt(a + b\*x + c\*x\*\*2)))/(sqrt(c)\*e)

**Mathematica [A]** time = 0.319102, size = 167, normalized size = 1.27

$$\frac{\sqrt{c}(dg - ef) \log \left( 2\sqrt{a + x(b + cx)}\sqrt{ae^2 - bde + cd^2} + 2ae - bd + bex - 2cdx \right) + g\sqrt{ae^2 - bde + cd^2} \log \left( 2\sqrt{c}\sqrt{a + x(b + cx)} \right)}{\sqrt{ce}\sqrt{e(ae - bd) + cd^2}}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)/((d + e\*x)\*Sqrt[a + b\*x + c\*x^2]), x]

[Out] (Sqrt[c]\*(e\*f - d\*g)\*Log[d + e\*x] + Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*g\*Log[b + 2\*c\*x + 2\*Sqrt[c]\*Sqrt[a + x\*(b + c\*x)]] + Sqrt[c]\*(-(e\*

f) + d\*g)\*Log[-(b\*d) + 2\*a\*e - 2\*c\*d\*x + b\*e\*x + 2\*Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*Sqrt[a + x\*(b + c\*x))]/(Sqrt[c]\*e\*Sqrt[c\*d^2 + e\*(-(b\*d) + a\*e))])

**Maple [B]** time = 0.011, size = 349, normalized size = 2.7

$$\frac{g}{e} \ln \left( 1 \left( \frac{b}{2} + cx \right) \frac{1}{\sqrt{c}} + \sqrt{cx^2 + bx + a} \right) \frac{1}{\sqrt{c}}$$

$$+ \frac{dg}{e^2} \ln \left( 1 \left( 2 \frac{ae^2 - bde + cd^2}{e^2} + \frac{be - 2cd}{e} \left( x + \frac{d}{e} \right) + 2 \sqrt{\frac{ae^2 - bde + cd^2}{e^2}} \sqrt{\left( x + \frac{d}{e} \right)^2 c + \frac{be - 2cd}{e} \left( x + \frac{d}{e} \right) + \frac{ae^2 - bde + cd^2}{e^2}} \right) \right)$$

$$- \frac{f}{e} \ln \left( 1 \left( 2 \frac{ae^2 - bde + cd^2}{e^2} + \frac{be - 2cd}{e} \left( x + \frac{d}{e} \right) + 2 \sqrt{\frac{ae^2 - bde + cd^2}{e^2}} \sqrt{\left( x + \frac{d}{e} \right)^2 c + \frac{be - 2cd}{e} \left( x + \frac{d}{e} \right) + \frac{ae^2 - bde + cd^2}{e^2}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)/(e\*x+d)/(c\*x^2+b\*x+a)^(1/2),x)

[Out] 1/e\*g\*ln((1/2\*b\*c\*x)/c^(1/2)+(c\*x^2+b\*x+a)^(1/2))/c^(1/2)+1/e^2/((a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2-b\*d\*e+c\*d^2)/e^2+(b\*e-2\*c\*d)/e\*(x+d/e)+2\*((a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*((x+d/e)^2\*c+(b\*e-2\*c\*d)/e\*(x+d/e)+(a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2))/(x+d/e))\*d\*g-1/e/((a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2-b\*d\*e+c\*d^2)/e^2+(b\*e-2\*c\*d)/e\*(x+d/e)+2\*((a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*((x+d/e)^2\*c+(b\*e-2\*c\*d)/e\*(x+d/e)+(a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2))/(x+d/e))\*f

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 28.5568, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)),x, algorithm="fricas")

[Out] [1/2\*(sqrt(c\*d^2 - b\*d\*e + a\*e^2)\*g\*log(-4\*(2\*c^2\*x + b\*c)\*sqrt(c\*x^2 + b\*x + a) - (8\*c^2\*x^2 + 8\*b\*c\*x + b^2 + 4\*a\*c)\*sqrt(c)) - (e\*f - d\*g)\*sqrt(c)\*log(((8\*a\*b\*d\*e - 8\*a^2\*e^2 - (b^2 + 4\*a\*c)\*d^2 - (8\*c^2\*d^2 - 8\*b\*c\*d\*e + (b^2 + 4\*a\*c)\*e^2)\*x^2 - 2\*(4\*b\*c\*d^2 + 4\*a\*b\*e^2 - (3\*b^2 + 4\*a\*c)\*d\*e)\*x)\*sqrt(c\*d^2 - b\*d\*e + a\*e^2) + 4\*(b\*c\*d^3 + 3\*a\*b\*d\*e^2 - 2\*a^2\*e^3 - (b^2 + 2\*a\*c)\*d^2\*e + (2\*c^2\*d^3 - 3\*b\*c\*d^2\*e - a\*b\*e^3 + (b^2 + 2\*a\*c)\*d^2\*e^2)\*x)\*sqrt(c\*x^2 + b\*x + a))/(e^2\*x^2 + 2\*d\*e\*x + d^2)))/(sqrt(c\*d^2 - b\*d\*e + a\*e^2)\*sqrt(c)\*e), -1/2\*(2\*(e\*f - d\*g)\*sqrt(c)\*arctan(-1/2\*sqrt(-c\*d^2 + b\*d\*e - a\*e^2)\*(b\*d - 2\*a\*e + (2\*c\*d - b\*e)\*x)/((c

```

d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a))) - sqrt(-c*d^2 + b*d*
e - a*e^2)*g*log(-4*(2*c^2*x + b*c)*sqrt(c*x^2 + b*x + a) - (8*c^
2*x^2 + 8*b*c*x + b^2 + 4*a*c)*sqrt(c)))/(sqrt(-c*d^2 + b*d*e - a
*e^2)*sqrt(c)*e), 1/2*(2*sqrt(c*d^2 - b*d*e + a*e^2)*g*arctan(1/2
*(2*c*x + b)*sqrt(-c)/(sqrt(c*x^2 + b*x + a)*c)) - (e*f - d*g)*sq
rt(-c)*log(((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d
^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 2*(4*b*c*d^2 + 4*a*b*e^
2 - (3*b^2 + 4*a*c)*d*e)*x)*sqrt(c*d^2 - b*d*e + a*e^2) + 4*(b*c*
d^3 + 3*a*b*d*e^2 - 2*a^2*e^3 - (b^2 + 2*a*c)*d^2*e + (2*c^2*d^3
- 3*b*c*d^2*e - a*b*e^3 + (b^2 + 2*a*c)*d*e^2)*x)*sqrt(c*x^2 + b*
x + a))/(e^2*x^2 + 2*d*e*x + d^2)))/(sqrt(c*d^2 - b*d*e + a*e^2)*
sqrt(-c)*e), -((e*f - d*g)*sqrt(-c)*arctan(-1/2*sqrt(-c*d^2 + b*d
*e - a*e^2)*(b*d - 2*a*e + (2*c*d - b*e)*x)/((c*d^2 - b*d*e + a*e
^2)*sqrt(c*x^2 + b*x + a))) - sqrt(-c*d^2 + b*d*e - a*e^2)*g*arct
an(1/2*(2*c*x + b)*sqrt(-c)/(sqrt(c*x^2 + b*x + a)*c)))/(sqrt(-c*
d^2 + b*d*e - a*e^2)*sqrt(-c)*e)]

```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{f + gx}{(d + ex)\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)/(e\*x+d)/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral((f + g\*x)/((d + e\*x)\*sqrt(a + b\*x + c\*x\*\*2)), x)

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.874 \quad \int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=79

$$\frac{\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\sqrt{ae^2-bde+cd^2}}$$

[Out] ArcTanh[(b\*d - 2\*a\*e + (2\*c\*d - b\*e)\*x)/(2\*Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*Sqrt[a + b\*x + c\*x^2])/Sqrt[c\*d^2 - b\*d\*e + a\*e^2]

**Rubi [A]** time = 0.0897159, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{\tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)\*Sqrt[a + b\*x + c\*x^2]),x]

[Out] ArcTanh[(b\*d - 2\*a\*e + (2\*c\*d - b\*e)\*x)/(2\*Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*Sqrt[a + b\*x + c\*x^2])/Sqrt[c\*d^2 - b\*d\*e + a\*e^2]

**Rubi in Sympy [A]** time = 17.3157, size = 73, normalized size = 0.92

$$-\frac{\operatorname{atanh}\left(\frac{2ae-bd+x(be-2cd)}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{\sqrt{ae^2-bde+cd^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(e\*x+d)/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] -atanh((2\*a\*e - b\*d + x\*(b\*e - 2\*c\*d))/(2\*sqrt(a + b\*x + c\*x\*\*2)\*sqrt(a\*e\*\*2 - b\*d\*e + c\*d\*\*2)))/sqrt(a\*e\*\*2 - b\*d\*e + c\*d\*\*2)

**Mathematica [A]** time = 0.0758974, size = 84, normalized size = 1.06

$$\frac{\log(d+ex) - \log\left(2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2} + 2ae - bd + bex - 2cdx\right)}{\sqrt{e(ae-bd)+cd^2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x)\*Sqrt[a + b\*x + c\*x^2]),x]

[Out] (Log[d + e\*x] - Log[-(b\*d) + 2\*a\*e - 2\*c\*d\*x + b\*e\*x + 2\*Sqrt[c\*d^2 + e\*(-(b\*d) + a\*e)]\*Sqrt[a + x\*(b + c\*x)]])/Sqrt[c\*d^2 + e\*(-(b\*d) + a\*e)]



**Maple [B]** time = 0.007, size = 157, normalized size = 2.

$$-\frac{1}{e} \ln \left( 1 \left( 2 \frac{ae^2 - bde + cd^2}{e^2} + \frac{be - 2cd}{e} \left( x + \frac{d}{e} \right) + 2 \sqrt{\frac{ae^2 - bde + cd^2}{e^2}} \sqrt{\left( x + \frac{d}{e} \right)^2 c + \frac{be - 2cd}{e} \left( x + \frac{d}{e} \right) + \frac{ae^2 - bde + cd^2}{e^2}} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x+d)/(c\*x^2+b\*x+a)^(1/2), x)

[Out] -1/e/((a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2-b\*d\*e+c\*d^2)/e^2+2\*(b\*e-2\*c\*d)/e\*(x+d/e)+2\*((a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*((x+d/e)^2\*c+(b\*e-2\*c\*d)/e\*(x+d/e)+(a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2))/(x+d/e))

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.356442, size = 1, normalized size = 0.01

$$\left[ \frac{\log \left( \frac{(8abde - 8a^2e^2 - (b^2 + 4ac)d^2 - (8c^2d^2 - 8bcde + (b^2 + 4ac)e^2)x^2 - 2(4bcd^2 + 4abe^2 - (3b^2 + 4ac)de)x)\sqrt{cd^2 - bde + ae^2} - 4(bcd^3 + 3abde^2 - 2a^2e^3 - (b^2 + 4ac)d^2e)}{e^2x^2 + 2dex + d^2} \right)}{2\sqrt{cd^2 - bde + ae^2}} \right]$$

$$- \frac{\arctan \left( \frac{\sqrt{-cd^2 + bde - ae^2}(bd - 2ae + (2cd - be)x)}{2(cd^2 - bde + ae^2)\sqrt{cx^2 + bx + a}} \right)}{\sqrt{-cd^2 + bde - ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)), x, algorithm="fricas")

[Out] [1/2\*log(((8\*a\*b\*d\*e - 8\*a^2\*e^2 - (b^2 + 4\*a\*c)\*d^2 - (8\*c^2\*d^2 - 8\*b\*c\*d\*e + (b^2 + 4\*a\*c)\*e^2)\*x^2 - 2\*(4\*b\*c\*d^2 + 4\*a\*b\*e^2 - (3\*b^2 + 4\*a\*c)\*d\*e)\*x)\*sqrt(c\*d^2 - b\*d\*e + a\*e^2) - 4\*(b\*c\*d^3 + 3\*a\*b\*d\*e^2 - 2\*a^2\*e^3 - (b^2 + 4\*a\*c)\*d^2\*e)/(e^2\*x^2 + 2\*d\*e\*x + d^2))/sqrt(c\*d^2 - b\*d\*e + a\*e^2), -arctan(-1/2\*sqrt(-c\*d^2 + b\*d\*e - a\*e^2)\*(b\*d - 2\*a\*e + (2\*c\*d - b\*e)\*x)/((c\*d^2 - b\*d\*e + a\*e^2)\*sqrt(c\*x^2 + b\*x + a)))/sqrt(-c\*d^2 + b\*d\*e - a\*e^2)]

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex)\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x+d)/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral(1/((d + e\*x)\*sqrt(a + b\*x + c\*x\*\*2)), x)

**GIAC/XCAS [A]** time = 0.274868, size = 97, normalized size = 1.23

$$\frac{2 \arctan\left(-\frac{(\sqrt{c}x - \sqrt{cx^2 + bx + a})e + \sqrt{cd}}{\sqrt{-cd^2 + bde - ae^2}}\right)}{\sqrt{-cd^2 + bde - ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)),x, algorithm="giac")

[Out] 2\*arctan(-((sqrt(c)\*x - sqrt(c\*x^2 + b\*x + a))\*e + sqrt(c)\*d)/sqrt(-c\*d^2 + b\*d\*e - a\*e^2))/sqrt(-c\*d^2 + b\*d\*e - a\*e^2)

$$3.875 \quad \int \frac{1}{(d+ex)(f+gx)\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=182

$$\frac{e \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)\sqrt{ae^2-bde+cd^2}} - \frac{g \tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)\sqrt{ag^2-bfg+cf^2}}$$

[Out] (e\*ArcTanh[(b\*d - 2\*a\*e + (2\*c\*d - b\*e)\*x)/(2\*Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*Sqrt[a + b\*x + c\*x^2]))/(Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*(e\*f - d\*g)) - (g\*ArcTanh[(b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)/(2\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2]\*Sqrt[a + b\*x + c\*x^2]))/((e\*f - d\*g)\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2])

**Rubi [A]** time = 0.492819, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$

$$\frac{e \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)\sqrt{ae^2-bde+cd^2}} - \frac{g \tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)\sqrt{ag^2-bfg+cf^2}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)\*(f + g\*x)\*Sqrt[a + b\*x + c\*x^2]), x]

[Out] (e\*ArcTanh[(b\*d - 2\*a\*e + (2\*c\*d - b\*e)\*x)/(2\*Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*Sqrt[a + b\*x + c\*x^2]))/(Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*(e\*f - d\*g)) - (g\*ArcTanh[(b\*f - 2\*a\*g + (2\*c\*f - b\*g)\*x)/(2\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2]\*Sqrt[a + b\*x + c\*x^2]))/((e\*f - d\*g)\*Sqrt[c\*f^2 - b\*f\*g + a\*g^2])

**Rubi in Sympy [A]** time = 64.1461, size = 162, normalized size = 0.89

$$\frac{e \operatorname{atanh}\left(\frac{2ae-bd+x(be-2cd)}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(dg-ef)\sqrt{ae^2-bde+cd^2}} - \frac{g \operatorname{atanh}\left(\frac{2ag-bf+x(bg-2cf)}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(dg-ef)\sqrt{ag^2-bfg+cf^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(e\*x+d)/(g\*x+f)/(c\*x\*\*2+b\*x+a)\*\*(1/2), x)

[Out] e\*atanh((2\*a\*e - b\*d + x\*(b\*e - 2\*c\*d))/(2\*sqrt(a + b\*x + c\*x\*\*2)\*sqrt(a\*e\*\*2 - b\*d\*e + c\*d\*\*2)))/((d\*g - e\*f)\*sqrt(a\*e\*\*2 - b\*d\*e + c\*d\*\*2)) - g\*atanh((2\*a\*g - b\*f + x\*(b\*g - 2\*c\*f))/(2\*sqrt(a + b\*x + c\*x\*\*2)\*sqrt(a\*g\*\*2 - b\*f\*g + c\*f\*\*2)))/((d\*g - e\*f)\*sqrt(a\*g\*\*2 - b\*f\*g + c\*f\*\*2))

**Mathematica [A]** time = 0.867588, size = 222, normalized size = 1.22

$$\frac{e \log\left(2\sqrt{a+x(b+cx)}\sqrt{ae^2-bde+cd^2}+2ae-bd+bex-2cdx\right)}{\sqrt{ae^2-bde+cd^2}} - \frac{e \log(d+ex)}{\sqrt{e(ae-bd)+cd^2}} + \frac{g \log(f+gx)}{\sqrt{ag^2-bfg+cf^2}} - \frac{g \log\left(2\sqrt{a+x(b+cx)}\sqrt{ag^2-bfg+cf^2}+2ag-bf+bgx-\right)}{\sqrt{ag^2-bfg+cf^2}}$$

$$dg - ef$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x)\*(f + g\*x)\*Sqrt[a + b\*x + c\*x^2]),x]

[Out] 
$$\begin{aligned} & -\left(\frac{e \operatorname{Log}[d + e x]}{\operatorname{Sqrt}[c d^2 + e(-b d + a e)]}\right) + \left(\frac{g \operatorname{Log}[f + g x]}{\operatorname{Sqrt}[c f^2 - b f g + a g^2]} + \frac{e \operatorname{Log}[-(b d) + 2 a e - 2 c d x + b e x + 2 \operatorname{Sqrt}[c d^2 - b d e + a e^2] \operatorname{Sqrt}[a + x(b + c x)]]}{\operatorname{Sqrt}[c d^2 - b d e + a e^2]} - \frac{g \operatorname{Log}[-(b f) + 2 a g - 2 c f x + b g x + 2 \operatorname{Sqrt}[c f^2 - b f g + a g^2] \operatorname{Sqrt}[a + x(b + c x)]]}{\operatorname{Sqrt}[c f^2 - b f g + a g^2]}\right) / (-e f + d g) \end{aligned}$$

**Maple [A]** time = 0.022, size = 327, normalized size = 1.8

$$\begin{aligned} & -\frac{1}{dg - ef} \ln \left( 1 \left( 2 \frac{ag^2 - bfg + cf^2}{g^2} + \frac{bg - 2cf}{g} \left( x + \frac{f}{g} \right) + 2 \sqrt{\frac{ag^2 - bfg + cf^2}{g^2}} \sqrt{\left( x + \frac{f}{g} \right)^2 c + \frac{bg - 2cf}{g} \left( x + \frac{f}{g} \right) + \frac{ag^2}{g^2}} \right) \right. \\ & \left. + \frac{1}{dg - ef} \ln \left( 1 \left( 2 \frac{ae^2 - bde + cd^2}{e^2} + \frac{be - 2cd}{e} \left( x + \frac{d}{e} \right) + 2 \sqrt{\frac{ae^2 - bde + cd^2}{e^2}} \sqrt{\left( x + \frac{d}{e} \right)^2 c + \frac{be - 2cd}{e} \left( x + \frac{d}{e} \right) + \frac{ae^2 - bde + cd^2}{e^2}} \right) \right) \right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x+d)/(g\*x+f)/(c\*x^2+b\*x+a)^(1/2),x)

[Out] 
$$\begin{aligned} & -1/(d g - e f) / \left( (a g^2 - b f g + c f^2) / g^2 \right)^{1/2} \ln \left( \frac{2(a g^2 - b f g + c f^2) / g^2 + (b g - 2 c f) / g \left( x + \frac{f}{g} \right) + 2 \sqrt{\frac{a g^2 - b f g + c f^2}{g^2}} \sqrt{\left( x + \frac{f}{g} \right)^2 c + \frac{b g - 2 c f}{g} \left( x + \frac{f}{g} \right) + \frac{a g^2}{g^2}}}{(a g^2 - b f g + c f^2) / g^2} \right)^{1/2} \\ & + 1/(d g - e f) / \left( (a e^2 - b d e + c d^2) / e^2 \right)^{1/2} \ln \left( \frac{2(a e^2 - b d e + c d^2) / e^2 + (b e - 2 c d) / e \left( x + \frac{d}{e} \right) + 2 \sqrt{\frac{a e^2 - b d e + c d^2}{e^2}} \sqrt{\left( x + \frac{d}{e} \right)^2 c + \frac{b e - 2 c d}{e} \left( x + \frac{d}{e} \right) + \frac{a e^2 - b d e + c d^2}{e^2}}}{(a e^2 - b d e + c d^2) / e^2} \right)^{1/2} \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)\*(g\*x + f)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 124.009, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)\*(g\*x + f)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/2 * (\operatorname{sqrt}(c f^2 - b f g + a g^2) * e \operatorname{log}(((8 a b d e - 8 a^2 e^2 - (b^2 + 4 a^2 c) d^2 - (8 c^2 d^2 - 8 b^2 c d e + (b^2 + 4 a^2 c) e^2) * x^2 - 2(4 b^2 c d^2 + 4 a^2 b e^2 - (3 b^2 + 4 a^2 c) d e) x) * \operatorname{sqrt}(c d^2 - b d e + a e^2) + 4(b^2 c d^3 + 3 a^2 b d e^2 - 2 a^2 e^3 - (b^2 + 2 a^2 c) d^2 e + (2 c^2 d^3 - 3 b^2 c d^2 e - a b e^3 + (b^2 + 2 a^2 c) d e^2) x) * \operatorname{sqrt}(c x^2 + b x + a)) / (e^2 x^2 + 2 d e x + d^2)) \\ & + \operatorname{sqrt}(c d^2 - b d e + a e^2) * g \operatorname{log}(((8 a b f g - 8 a^2 g^2 - (b^2 + 4 a^2 c) f^2 - (8 c^2 f^2 - 8 b^2 c f g + (b^2 + 4 a^2 c) g^2) * x^2 \end{aligned}$$

$$\begin{aligned}
& - 2*(4*b*c*f^2 + 4*a*b*g^2 - (3*b^2 + 4*a*c)*f*g)*x)*\sqrt{c*f^2 - b*f*g + a*g^2} - 4*(b*c*f^3 + 3*a*b*f*g^2 - 2*a^2*g^3 - (b^2 + 2*a*c)*f^2*g + (2*c^2*f^3 - 3*b*c*f^2*g - a*b*g^3 + (b^2 + 2*a*c)*f*g^2)*x)*\sqrt{c*x^2 + b*x + a})/(g^2*x^2 + 2*f*g*x + f^2))/(\sqrt{c*d^2 - b*d*e + a*e^2})*\sqrt{c*f^2 - b*f*g + a*g^2}*(e*f - d*g)) \\
& , 1/2*(2*\sqrt{c*d^2 - b*d*e + a*e^2})*g*\arctan(-1/2*\sqrt{-c*f^2 + b*f*g - a*g^2}*(b*f - 2*a*g + (2*c*f - b*g)*x)/((c*f^2 - b*f*g + a*g^2)*\sqrt{c*x^2 + b*x + a})) - \sqrt{-c*f^2 + b*f*g - a*g^2}*e*\log(((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)*\sqrt{c*d^2 - b*d*e + a*e^2} + 4*(b*c*d^3 + 3*a*b*d*e^2 - 2*a^2*e^3 - (b^2 + 2*a*c)*d^2*e + (2*c^2*d^3 - 3*b*c*d^2*e - a*b*e^3 + (b^2 + 2*a*c)*d*e^2)*x)*\sqrt{c*x^2 + b*x + a}))/((e^2*x^2 + 2*d*e*x + d^2)))/(\sqrt{c*d^2 - b*d*e + a*e^2})*\sqrt{-c*f^2 + b*f*g - a*g^2}*(e*f - d*g)), -1/2*(2*\sqrt{c*f^2 - b*f*g + a*g^2})*e*\arctan(-1/2*\sqrt{-c*d^2 + b*d*e - a*e^2}*(b*d - 2*a*e + (2*c*d - b*e)*x)/((c*d^2 - b*d*e + a*e^2)*\sqrt{c*x^2 + b*x + a})) + \sqrt{-c*d^2 + b*d*e - a*e^2})*g*\log(((8*a*b*f*g - 8*a^2*g^2 - (b^2 + 4*a*c)*f^2 - (8*c^2*f^2 - 8*b*c*f*g + (b^2 + 4*a*c)*g^2)*x^2 - 2*(4*b*c*f^2 + 4*a*b*g^2 - (3*b^2 + 4*a*c)*f*g)*x)*\sqrt{c*f^2 - b*f*g + a*g^2} - 4*(b*c*f^3 + 3*a*b*f*g^2 - 2*a^2*g^3 - (b^2 + 2*a*c)*f^2*g + (2*c^2*f^3 - 3*b*c*f^2*g - a*b*g^3 + (b^2 + 2*a*c)*f*g^2)*x)*\sqrt{c*x^2 + b*x + a}))/((g^2*x^2 + 2*f*g*x + f^2)))/(\sqrt{-c*d^2 + b*d*e - a*e^2})*\sqrt{c*f^2 - b*f*g + a*g^2}*(e*f - d*g)), -(\sqrt{-c*f^2 + b*f*g - a*g^2})*e*\arctan(-1/2*\sqrt{-c*d^2 + b*d*e - a*e^2}*(b*d - 2*a*e + (2*c*d - b*e)*x)/((c*d^2 - b*d*e + a*e^2)*\sqrt{c*x^2 + b*x + a})) - \sqrt{-c*d^2 + b*d*e - a*e^2})*g*\arctan(-1/2*\sqrt{-c*f^2 + b*f*g - a*g^2}*(b*f - 2*a*g + (2*c*f - b*g)*x)/((c*f^2 - b*f*g + a*g^2)*\sqrt{c*x^2 + b*x + a}))/(\sqrt{-c*d^2 + b*d*e - a*e^2})*\sqrt{-c*f^2 + b*f*g - a*g^2}*(e*f - d*g))]
\end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex)(f + gx)\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x+d)/(g\*x+f)/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral(1/((d + e\*x)\*(f + g\*x)\*sqrt(a + b\*x + c\*x\*\*2)), x)

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)\*(g\*x + f)),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.876 \quad \int \frac{1}{(d+ex)(f+gx)^2 \sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=340

$$\frac{e^2 \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^2\sqrt{ae^2-bde+cd^2}} + \frac{g^2\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)(ag^2-bfg+cf^2)}$$

$$- \frac{eg \tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)^2\sqrt{ag^2-bfg+cf^2}} - \frac{g(2cf-bg) \tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2(ef-dg)(ag^2-bfg+cf^2)^{3/2}}$$

[Out]  $(g^2 \sqrt{a + b^*x + c^*x^2}) / ((e^*f - d^*g)^*(c^*f^2 - b^*f^*g + a^*g^2)^*(f + g^*x)) + (e^2 \operatorname{ArcTanh}[(b^*d - 2^*a^*e + (2^*c^*d - b^*e)^*x) / (2^*\sqrt{c^*d^2 - b^*d^*e + a^*e^2})^*\sqrt{a + b^*x + c^*x^2}]) / (\sqrt{c^*d^2 - b^*d^*e + a^*e^2})^*(e^*f - d^*g)^2) - (g^*(2^*c^*f - b^*g)^*\operatorname{ArcTanh}[(b^*f - 2^*a^*g + (2^*c^*f - b^*g)^*x) / (2^*\sqrt{c^*f^2 - b^*f^*g + a^*g^2})^*\sqrt{a + b^*x + c^*x^2}]) / (2^*(e^*f - d^*g)^*(c^*f^2 - b^*f^*g + a^*g^2)^{3/2}) - (e^*g^*\operatorname{ArcTanh}[(b^*f - 2^*a^*g + (2^*c^*f - b^*g)^*x) / (2^*\sqrt{c^*f^2 - b^*f^*g + a^*g^2})^*\sqrt{a + b^*x + c^*x^2}]) / ((e^*f - d^*g)^2 \sqrt{c^*f^2 - b^*f^*g + a^*g^2})$

**Rubi [A]** time = 0.902698, antiderivative size = 340, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$

$$\frac{e^2 \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^2\sqrt{ae^2-bde+cd^2}} + \frac{g^2\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)(ag^2-bfg+cf^2)}$$

$$- \frac{eg \tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)^2\sqrt{ag^2-bfg+cf^2}} - \frac{g(2cf-bg) \tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2(ef-dg)(ag^2-bfg+cf^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)\*(f + g\*x)^2\*sqrt[a + b\*x + c\*x^2]), x]

[Out]  $(g^2 \sqrt{a + b^*x + c^*x^2}) / ((e^*f - d^*g)^*(c^*f^2 - b^*f^*g + a^*g^2)^*(f + g^*x)) + (e^2 \operatorname{ArcTanh}[(b^*d - 2^*a^*e + (2^*c^*d - b^*e)^*x) / (2^*\sqrt{c^*d^2 - b^*d^*e + a^*e^2})^*\sqrt{a + b^*x + c^*x^2}]) / (\sqrt{c^*d^2 - b^*d^*e + a^*e^2})^*(e^*f - d^*g)^2) - (g^*(2^*c^*f - b^*g)^*\operatorname{ArcTanh}[(b^*f - 2^*a^*g + (2^*c^*f - b^*g)^*x) / (2^*\sqrt{c^*f^2 - b^*f^*g + a^*g^2})^*\sqrt{a + b^*x + c^*x^2}]) / (2^*(e^*f - d^*g)^*(c^*f^2 - b^*f^*g + a^*g^2)^{3/2}) - (e^*g^*\operatorname{ArcTanh}[(b^*f - 2^*a^*g + (2^*c^*f - b^*g)^*x) / (2^*\sqrt{c^*f^2 - b^*f^*g + a^*g^2})^*\sqrt{a + b^*x + c^*x^2}]) / ((e^*f - d^*g)^2 \sqrt{c^*f^2 - b^*f^*g + a^*g^2})$

**Rubi in Sympy [A]** time = 101.24, size = 304, normalized size = 0.89

$$\frac{e^2 \operatorname{atanh}\left(\frac{2ae-bd+x(be-2cd)}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(dg-ef)^2\sqrt{ae^2-bde+cd^2}} + \frac{eg \operatorname{atanh}\left(\frac{2ag-bf+x(bg-2cf)}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(dg-ef)^2\sqrt{ag^2-bfg+cf^2}}$$

$$- \frac{g^2\sqrt{a+bx+cx^2}}{(f+gx)(dg-ef)(ag^2-bfg+cf^2)} + \frac{g(bg-2cf) \operatorname{atanh}\left(\frac{2ag-bf+x(bg-2cf)}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2(dg-ef)(ag^2-bfg+cf^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(e\*x+d)/(g\*x+f)\*\*2/(c\*x\*\*2+b\*x+a)\*\*(1/2), x)

```
[Out] -e**2*atanh((2*a*e - b*d + x*(b*e - 2*c*d))/(2*sqrt(a + b*x + c*x
**2)*sqrt(a*e**2 - b*d*e + c*d**2)))/((d*g - e*f)**2*sqrt(a*e**2
- b*d*e + c*d**2)) + e*g*atanh((2*a*g - b*f + x*(b*g - 2*c*f))/(2
*sqrt(a + b*x + c*x**2)*sqrt(a*g**2 - b*f*g + c*f**2)))/((d*g - e
*f)**2*sqrt(a*g**2 - b*f*g + c*f**2)) - g**2*sqrt(a + b*x + c*x**
2)/((f + g*x)*(d*g - e*f)*(a*g**2 - b*f*g + c*f**2)) + g*(b*g - 2
*c*f)*atanh((2*a*g - b*f + x*(b*g - 2*c*f))/(2*sqrt(a + b*x + c*x
**2)*sqrt(a*g**2 - b*f*g + c*f**2)))/(2*(d*g - e*f)*(a*g**2 - b*f
*g + c*f**2)**(3/2))
```

**Mathematica [A]** time = 1.58469, size = 343, normalized size = 1.01

$$\frac{2e^2 \log(d+ex)}{\sqrt{e(ae-bd)+cd^2}} - \frac{2e^2 \log\left(2\sqrt{a+x(b+cx)}\sqrt{e(ae-bd)+cd^2}+2ae-bd+bex-2cdx\right)}{\sqrt{e(ae-bd)+cd^2}} + \frac{2g^2\sqrt{a+x(b+cx)}(ef-dg)}{(f+gx)(g(ag-bf)+cf^2)} - \frac{g \log(f+gx)(g(2aeg+bdg-3bef)+2cf(2ef-dg)^2)}{(g(ag-bf)+cf^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x)*(f + g*x)^2*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] ((2*g^2*(e*f - d*g)*Sqrt[a + x*(b + c*x)])/((c*f^2 + g*(-(b*f) +
a*g))*(f + g*x)) + (2*e^2*Log[d + e*x])/Sqrt[c*d^2 + e*(-(b*d) +
a*e)] - (g*(2*c*f*(2*e*f - d*g) + g*(-3*b*e*f + b*d*g + 2*a*e*g))
*Log[f + g*x])/((c*f^2 + g*(-(b*f) + a*g))^(3/2)) - (2*e^2*Log[-(b*
d) + 2*a*e - 2*c*d*x + b*e*x + 2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*S
qrt[a + x*(b + c*x)]])/Sqrt[c*d^2 + e*(-(b*d) + a*e)] + (g*(2*c*f
*(2*e*f - d*g) + g*(-3*b*e*f + b*d*g + 2*a*e*g))*Log[-(b*f) + 2*a
*g - 2*c*f*x + b*g*x + 2*Sqrt[c*f^2 + g*(-(b*f) + a*g)]*Sqrt[a +
x*(b + c*x)]])/((c*f^2 + g*(-(b*f) + a*g))^(3/2))/(2*(e*f - d*g)^2
)
```

**Maple [B]** time = 0.026, size = 788, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x)
```

```
[Out] -e/(d*g-e*f)^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e
+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/
2)*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1
/2))/(x+d/e))-g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/(x+f/g)*((x+f/g)^2*
c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2)+1/2*g/(d*g
-e*f)/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(
a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^
2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f
^2)/g^2)^(1/2))/(x+f/g))*b-1/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/((a*g^
2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*
f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*((x+f/g)^2*c+(b*g-
2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))/(x+f/g))*c*f+e/(
d*g-e*f)^2/((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*ln((2*(a*g^2-b*f*g+c*f
^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^(1/2)*
((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^(1/2))
/(x+f/g))
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}(ex + d)(gx + f)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^2),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^2), x)`

**Fricas [A]** time = 9.735, size = 1, normalized size = 0.

$$\left[ \frac{e^2 \log \left( \frac{(8 abde - 8 a^2 e^2 - (b^2 + 4 ac) d^2 - (8 c^2 d^2 - 8 bcde + (b^2 + 4 ac) e^2) x^2 - 2 (4 bcd^2 + 4 abe^2 - (3 b^2 + 4 ac) de) x) \sqrt{cd^2 - bde + ae^2} - 4 (bcd^3 + 3 abde^2 - 2 a^2 e^3 - e^2 x^2 + 2 dex + d^2)}{e^2 x^2 + 2 dex + d^2} \right)}{2 (e^2 f^2 - 2 defg + d^2 g^2) \sqrt{cd^2 - bde + ae^2}} \right]$$

$$\frac{e^2 \arctan \left( -\frac{\sqrt{-cd^2 + bde - ae^2} (bd - 2ae + (2cd - be)x)}{2(cd^2 - bde + ae^2) \sqrt{cx^2 + bx + a}} \right)}{(e^2 f^2 - 2 defg + d^2 g^2) \sqrt{-cd^2 + bde - ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^2),x, algorithm="fricas")`

[Out] `[1/2*e^2*log(((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)*sqrt(c*d^2 - b*d*e + a*e^2) - 4*(b*c*d^3 + 3*a*b*d*e^2 - 2*a^2*e^3 - (b^2 + 2*a*c)*d^2*e + (2*c^2*d^3 - 3*b*c*d^2*e - a*b*e^3 + (b^2 + 2*a*c)*d*e^2)*x)*sqrt(c*x^2 + b*x + a))/(e^2*x^2 + 2*d*e*x + d^2))/((e^2*f^2 - 2*d*e*f*g + d^2*g^2)*sqrt(c*d^2 - b*d*e + a*e^2)), -e^2*arctan(-1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*(b*d - 2*a*e + (2*c*d - b*e)*x)/((c*d^2 - b*d*e + a*e^2)*sqrt(c*x^2 + b*x + a)))/((e^2*f^2 - 2*d*e*f*g + d^2*g^2)*sqrt(-c*d^2 + b*d*e - a*e^2))]`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex)(f + gx)^2 \sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x+d)/(g*x+f)**2/(c*x**2+b*x+a)**(1/2),x)`

[Out] `Integral(1/((d + e*x)*(f + g*x)**2*sqrt(a + b*x + c*x**2)), x)`

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^2),x, algorithm="giac")`

[Out] Exception raised: TypeError



$$3.877 \quad \int \frac{1}{(d+ex)(f+gx)^3 \sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=587

$$\frac{g(-4cg(ag+2bf)+3b^2g^2+8c^2f^2) \tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{8(ef-dg)(ag^2-bfg+cf^2)^{5/2}} + \frac{e^3 \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^3\sqrt{ae^2-bde+cd^2}} - \frac{e^2g \tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)^3\sqrt{ag^2-bfg+cf^2}} + \frac{eg^2\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)^2(ag^2-bfg+cf^2)} + \frac{3g^2\sqrt{a+bx+cx^2}(2cf-bg)}{4(f+gx)(ef-dg)(ag^2-bfg+cf^2)^2} + \frac{g^2\sqrt{a+bx+cx^2}}{2(f+gx)^2(ef-dg)(ag^2-bfg+cf^2)} - \frac{eg(2cf-bg) \tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2(ef-dg)^2(ag^2-bfg+cf^2)^{3/2}}$$

[Out]  $(g^2 \sqrt{a+bx+cx^2}) / (2(e^2 f - d^2 g) (c^2 f^2 - b^2 f g + a^2 g^2) (f + g^2 x)^2) + (3g^2 (2c^2 f - b^2 g) \sqrt{a+bx+cx^2}) / (4(e^2 f - d^2 g) (c^2 f^2 - b^2 f g + a^2 g^2)^2 (f + g^2 x)) + (e^2 g^2 \sqrt{a+bx+cx^2}) / ((e^2 f - d^2 g)^2 (c^2 f^2 - b^2 f g + a^2 g^2) (f + g^2 x)) + (e^3 \operatorname{ArcTanh}[(b^2 d - 2a^2 e + (2c^2 d - b^2 e) x) / (2 \sqrt{c^2 d^2 - b^2 d e + a^2 e^2} \sqrt{a+bx+cx^2})]) / (\sqrt{c^2 d^2 - b^2 d e + a^2 e^2} (e^2 f - d^2 g)^3) - (e^2 g (2c^2 f - b^2 g) \operatorname{ArcTanh}[(b^2 f - 2a^2 g + (2c^2 f - b^2 g) x) / (2 \sqrt{c^2 f^2 - b^2 f g + a^2 g^2} \sqrt{a+bx+cx^2})]) / (2(e^2 f - d^2 g)^2 (c^2 f^2 - b^2 f g + a^2 g^2)^{3/2}) - (e^2 g \operatorname{ArcTanh}[(b^2 f - 2a^2 g + (2c^2 f - b^2 g) x) / (2 \sqrt{c^2 f^2 - b^2 f g + a^2 g^2} \sqrt{a+bx+cx^2})]) / ((e^2 f - d^2 g)^3 \sqrt{c^2 f^2 - b^2 f g + a^2 g^2}) - (g^2 (8c^2 f^2 + 3b^2 g^2 - 4c^2 g (2b^2 f + a^2 g)) \operatorname{ArcTanh}[(b^2 f - 2a^2 g + (2c^2 f - b^2 g) x) / (2 \sqrt{c^2 f^2 - b^2 f g + a^2 g^2} \sqrt{a+bx+cx^2})]) / (8(e^2 f - d^2 g) (c^2 f^2 - b^2 f g + a^2 g^2)^{5/2})$

**Rubi [A]** time = 1.85603, antiderivative size = 587, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$

$$\frac{g(-4cg(ag+2bf)+3b^2g^2+8c^2f^2) \tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{8(ef-dg)(ag^2-bfg+cf^2)^{5/2}} + \frac{e^3 \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^3\sqrt{ae^2-bde+cd^2}} - \frac{e^2g \tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)^3\sqrt{ag^2-bfg+cf^2}} + \frac{eg^2\sqrt{a+bx+cx^2}}{(f+gx)(ef-dg)^2(ag^2-bfg+cf^2)} + \frac{3g^2\sqrt{a+bx+cx^2}(2cf-bg)}{4(f+gx)(ef-dg)(ag^2-bfg+cf^2)^2} + \frac{g^2\sqrt{a+bx+cx^2}}{2(f+gx)^2(ef-dg)(ag^2-bfg+cf^2)} - \frac{eg(2cf-bg) \tanh^{-1}\left(\frac{-2ag+x(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2(ef-dg)^2(ag^2-bfg+cf^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\operatorname{Int}[1/((d+e*x)*(f+g*x)^3*\sqrt{a+b*x+c*x^2}),x]$

[Out]  $(g^2 \sqrt{a+bx+cx^2}) / (2(e^2 f - d^2 g) (c^2 f^2 - b^2 f g + a^2 g^2) (f + g^2 x)^2) + (3g^2 (2c^2 f - b^2 g) \sqrt{a+bx+cx^2}) / (4(e^2 f - d^2 g) (c^2 f^2 - b^2 f g + a^2 g^2)^2 (f + g^2 x)) + (e^2 g^2 \sqrt{a+bx+cx^2}) / ((e^2 f - d^2 g)^2 (c^2 f^2 - b^2 f g + a^2 g^2) (f + g^2 x)) + (e^3 \operatorname{ArcTanh}[(b^2 d - 2a^2 e + (2c^2 d - b^2 e) x) / (2 \sqrt{c^2 d^2 - b^2 d e + a^2 e^2} \sqrt{a+bx+cx^2})]) / (\sqrt{c^2 d^2 - b^2 d e + a^2 e^2} (e^2 f - d^2 g)^3) - (e^2 g (2c^2 f - b^2 g) \operatorname{ArcTanh}[(b^2 f - 2a^2 g + (2c^2 f - b^2 g) x) / (2 \sqrt{c^2 f^2 - b^2 f g + a^2 g^2} \sqrt{a+bx+cx^2})]) / (2(e^2 f - d^2 g)^2 (c^2 f^2 - b^2 f g + a^2 g^2)^{3/2}) - (e^2 g \operatorname{ArcTanh}[(b^2 f - 2a^2 g + (2c^2 f - b^2 g) x) / (2 \sqrt{c^2 f^2 - b^2 f g + a^2 g^2} \sqrt{a+bx+cx^2})]) / ((e^2 f - d^2 g)^3 \sqrt{c^2 f^2 - b^2 f g + a^2 g^2}) - (g^2 (8c^2 f^2 + 3b^2 g^2 - 4c^2 g (2b^2 f + a^2 g)) \operatorname{ArcTanh}[(b^2 f - 2a^2 g + (2c^2 f - b^2 g) x) / (2 \sqrt{c^2 f^2 - b^2 f g + a^2 g^2} \sqrt{a+bx+cx^2})]) / (8(e^2 f - d^2 g) (c^2 f^2 - b^2 f g + a^2 g^2)^{5/2})$

$$\frac{(f - bg)x / (2\sqrt{cf^2 - bfg + ag^2}) \sqrt{a + bx + cx^2}}{((ef - dg)^2 (cf^2 - bfg + ag^2)^{3/2}) - (e^2 g \operatorname{ArcTanh}[(bf - 2ag + (2cf - bg)x) / (2\sqrt{cf^2 - bfg + ag^2})] \sqrt{a + bx + cx^2})} - \frac{(g(8c^2 f^2 + 3b^2 g^2 - 4c^2 g(2bf + ag)) \operatorname{ArcTanh}[(bf - 2ag + (2cf - bg)x) / (2\sqrt{cf^2 - bfg + ag^2})] \sqrt{a + bx + cx^2})}{(8(ef - dg)(cf^2 - bfg + ag^2)^{5/2})}$$

**Rubi in Sympy [F-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(e*x+d)/(g*x+f)**3/(c*x**2+b*x+a)**(1/2),x)`

[Out] Timed out

**Mathematica [A]** time = 6.9293, size = 835, normalized size = 1.42

$$\frac{\sqrt{cx^2 + bx + a} \log(d + ex) e^3}{\sqrt{cd^2 - bed + ae^2} (ef - dg)^3 \sqrt{a + x(b + cx)}} - \frac{\sqrt{cx^2 + bx + a} \log(-bd - 2cxd + 2ae + bex + 2\sqrt{cd^2 - bed + ae^2} \sqrt{cx^2 + bx + a}) e^3}{\sqrt{cd^2 - bed + ae^2} (ef - dg)^3 \sqrt{a + x(b + cx)}} + \frac{(cx^2 + bx + a) \left( \frac{(10cef^2 - 6cdgf - 7begf + 3bdg^2 + 4aeg^2) g^2}{4(ef - dg)^2 (cf^2 - bgf + ag^2)^2 (f + gx)} + \frac{g^2}{2(ef - dg)(cf^2 - bgf + ag^2)(f + gx)^2} \right)}{\sqrt{a + x(b + cx)}} + \frac{g(24c^2 e^2 f^4 - 36bce^2 g f^3 - 24c^2 deg f^3 + 8c^2 d^2 g^2 f^2 + 15b^2 e^2 g^2 f^2 + 20ace^2 g^2 f^2 + 28bcdeg^2 f^2 - 8bcd^2 g^3 f - 20abe^2 g^3 f - 8(dg - ef)^3 (cf^2 - bgf + ag^2)^{5/2} \sqrt{a + x(b + cx)})}{g(24c^2 e^2 f^4 - 36bce^2 g f^3 - 24c^2 deg f^3 + 8c^2 d^2 g^2 f^2 + 15b^2 e^2 g^2 f^2 + 20ace^2 g^2 f^2 + 28bcdeg^2 f^2 - 8bcd^2 g^3 f - 20abe^2 g^3 f - 8(dg - ef)^3 (cf^2 - bgf + ag^2)^{5/2} \sqrt{a + x(b + cx)}}$$

Antiderivative was successfully verified.

[In] `Integrate[1/((d + e*x)*(f + g*x)^3*Sqrt[a + b*x + c*x^2]),x]`

[Out]  $((a + bx + cx^2) (g^2 / (2(ef - dg)(cf^2 - bfg + ag^2)) (f + gx)^2) + (g^2 (10c^2 e^2 f^2 - 6c^2 d^2 f^2 g - 7b^2 e^2 f^2 g + 3b^2 d^2 g^2 + 4a^2 e^2 g^2)) / (4(ef - dg)^2 (cf^2 - bfg + ag^2)^2 (f + gx))) / \sqrt{a + x(b + cx)} + (e^3 \sqrt{a + bx + cx^2} \operatorname{Log}[d + ex]) / (\sqrt{cd^2 - b^2 d^2 e + a^2 e^2} (ef - dg)^3 \sqrt{a + x(b + cx)}) + (g(24c^2 e^2 f^4 - 24c^2 d^2 e^2 f^3 g - 36b^2 c^2 e^2 f^3 g + 8c^2 d^2 e^2 f^2 g^2 + 28b^2 c^2 d^2 e^2 f^2 g^2 + 15b^2 e^2 f^2 g^2 + 20a^2 c^2 e^2 f^2 g^2 - 8b^2 c^2 d^2 e^2 f^2 g^3 - 10b^2 d^2 e^2 f^2 g^3 - 20a^2 b^2 e^2 f^2 g^3 + 3b^2 d^2 e^2 f^2 g^4 - 4a^2 c^2 d^2 e^2 f^2 g^4 + 4a^2 b^2 d^2 e^2 f^2 g^4 + 8a^2 e^2 f^2 g^4) \sqrt{a + bx + cx^2} \operatorname{Log}[f + gx]) / (8(-ef + dg)^3 (cf^2 - bfg + ag^2)^{5/2} \sqrt{a + x(b + cx)}) - (e^3 \sqrt{a + bx + cx^2} \operatorname{Log}[-(b^2 d) + 2a^2 e - 2c^2 d^2 x + b^2 e^2 x + 2\sqrt{cd^2 - b^2 d^2 e + a^2 e^2} \sqrt{a + bx + cx^2}]) / (\sqrt{cd^2 - b^2 d^2 e + a^2 e^2} (ef - dg)^3 \sqrt{a + x(b + cx)}) - (g(24c^2 e^2 f^4 - 24c^2 d^2 e^2 f^3 g - 36b^2 c^2 e^2 f^3 g + 8c^2 d^2 e^2 f^2 g^2 + 28b^2 c^2 d^2 e^2 f^2 g^2 + 15b^2 e^2 f^2 g^2 + 20a^2 c^2 e^2 f^2 g^2 - 8b^2 c^2 d^2 e^2 f^2 g^3 - 10b^2 d^2 e^2 f^2 g^3 - 20a^2 b^2 e^2 f^2 g^3 + 3b^2 d^2 e^2 f^2 g^4 - 4a^2 c^2 d^2 e^2 f^2 g^4 + 4a^2 b^2 d^2 e^2 f^2 g^4 + 8a^2 e^2 f^2 g^4) \sqrt{a + bx + cx^2} \operatorname{Log}[-(b^2 f) + 2a^2 g - 2c^2 f^2 x + b^2 g^2 x + 2\sqrt{cf^2 - b^2 f^2 g + ag^2} \sqrt{a + bx + cx^2}]) / (8(-ef + dg)^3 (cf^2 - bfg + ag^2)^{5/2} \sqrt{a + x(b + cx)})$

$$*f*g + a*g^2)^{(5/2)}*\text{Sqrt}[a + x*(b + c*x)]$$

**Maple [B]** time = 0.03, size = 1817, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(1/2), x)`

[Out] 
$$\begin{aligned} & -1/2/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/(x+f/g)^2*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{(1/2)}+3/4*g^2/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{(1/2)}*b-3/2*g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{(1/2)}*c*f-3/8*g^2/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/((a*g^2-b*f*g+c*f^2)/g^2)^{(1/2)}*\ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^{(1/2)}*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{(1/2)})/(x+f/g)^2*g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/((a*g^2-b*f*g+c*f^2)/g^2)^{(1/2)}*\ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^{(1/2)}*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{(1/2)})/(x+f/g)^2*b*c*f-3/2/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/((a*g^2-b*f*g+c*f^2)/g^2)^{(1/2)}*\ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^{(1/2)}*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{(1/2)})/(x+f/g)^2*c^2*f^2+1/2/(d*g-e*f)*c/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2)^{(1/2)}*\ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^{(1/2)}*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{(1/2)})/(x+f/g)^2+e^2/(d*g-e*f)^3/((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*\ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x+d/e))+g*e/(d*g-e*f)^2/(a*g^2-b*f*g+c*f^2)/(x+f/g)*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{(1/2)}-1/2*g*e/(d*g-e*f)^2/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2)^{(1/2)}*\ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^{(1/2)}*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{(1/2)})/(x+f/g)^2*b+e/(d*g-e*f)^2/(a*g^2-b*f*g+c*f^2)/((a*g^2-b*f*g+c*f^2)/g^2)^{(1/2)}*\ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^{(1/2)}*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{(1/2)})/(x+f/g)^2*c*f$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}(ex + d)(gx + f)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^3), x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^3), x)`

**Ericas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^3),x, algorithm="fricas")
```

[Out] Timed out

---

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x+d)/(g*x+f)**3/(c*x**2+b*x+a)**(1/2),x)
```

[Out] Timed out

---

**GIAC/XCAS** [A] time = 1.47485, size = 4, normalized size = 0.01

$sage_0x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^3),x, algorithm="giac")
```

[Out]  $sage_0x$

$$3.878 \quad \int \frac{(f+gx)^4}{(d+ex)(a+bx+cx^2)^{3/2}} dx$$

**Optimal.** Leaf size=496

$$\frac{2(-b^2(a^2eg^4 + 4acdfg^3 + c^2ef^4) + x(2c^2g^2(a^2(-g)(4ef - dg) - 3abf(ef - 2dg) + 3b^2df^2) - bcg^3(-3a^2eg - 4ab(ef - dg) + 3b^2df^2) - bcg^3(-3a^2eg - 4ab(ef - dg) + 3b^2df^2))}{2c^{5/2}e^2} + \frac{g^3 \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-3beg - 2cdg + 8cef)}{2c^{5/2}e^2} + \frac{g^4\sqrt{a+bx+cx^2}}{c^2e} + \frac{(ef - dg)^4 \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2(ae^2 - bde + cd^2)^{3/2}}$$

[Out]  $(-2*(a*b^3*d*g^4 - b^2*(c^2*e*f^4 + 4*a*c*d*f*g^3 + a^2*e*g^4) + 2*a*c*(a^2*e*g^4 + c^2*f^3*(e*f - 4*d*g) - 2*a*c*f*g^2*(3*e*f - 2*d*g)) + b*c*(c^2*d*f^4 + a^2*g^3*(4*e*f - 3*d*g) + 2*a*c*f^2*g*(2*e*f + 3*d*g)) + (2*c^4*d*f^4 + b^3*(b*d - a*e)*g^4 - b*c*g^3*(4*b^2*d*f - 3*a^2*e*g - 4*a*b*(e*f - d*g)) + 2*c^2*g^2*(3*b^2*d*f^2 - 3*a*b*f*(e*f - 2*d*g) - a^2*g*(4*e*f - d*g)) + c^3*f^2*(4*a*g*(2*e*f - 3*d*g) - b*f*(e*f + 4*d*g)))*x)/(c^2*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (g^4*Sqrt[a + b*x + c*x^2])/(c^2*e) + (g^3*(8*c*e*f - 2*c*d*g - 3*b*e*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(5/2)*e^2) + ((e*f - d*g)^4*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e^2*(c*d^2 - b*d*e + a*e^2)^(3/2))$

**Rubi [A]** time = 2.32804, antiderivative size = 496, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$

$$\frac{2(-b^2(a^2eg^4 + 4acdfg^3 + c^2ef^4) + x(2c^2g^2(a^2(-g)(4ef - dg) - 3abf(ef - 2dg) + 3b^2df^2) - bcg^3(-3a^2eg - 4ab(ef - dg) + 3b^2df^2) - bcg^3(-3a^2eg - 4ab(ef - dg) + 3b^2df^2))}{2c^{5/2}e^2} + \frac{g^3 \tanh^{-1}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)(-3beg - 2cdg + 8cef)}{2c^{5/2}e^2} + \frac{g^4\sqrt{a+bx+cx^2}}{c^2e} + \frac{(ef - dg)^4 \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e^2(ae^2 - bde + cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)^4/((d + e\*x)\*(a + b\*x + c\*x^2)^(3/2)), x]

[Out]  $(-2*(a*b^3*d*g^4 - b^2*(c^2*e*f^4 + 4*a*c*d*f*g^3 + a^2*e*g^4) + 2*a*c*(a^2*e*g^4 + c^2*f^3*(e*f - 4*d*g) - 2*a*c*f*g^2*(3*e*f - 2*d*g)) + b*c*(c^2*d*f^4 + a^2*g^3*(4*e*f - 3*d*g) + 2*a*c*f^2*g*(2*e*f + 3*d*g)) + (2*c^4*d*f^4 + b^3*(b*d - a*e)*g^4 - b*c*g^3*(4*b^2*d*f - 3*a^2*e*g - 4*a*b*(e*f - d*g)) + 2*c^2*g^2*(3*b^2*d*f^2 - 3*a*b*f*(e*f - 2*d*g) - a^2*g*(4*e*f - d*g)) + c^3*f^2*(4*a*g*(2*e*f - 3*d*g) - b*f*(e*f + 4*d*g)))*x)/(c^2*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x + c*x^2]) + (g^4*Sqrt[a + b*x + c*x^2])/(c^2*e) + (g^3*(8*c*e*f - 2*c*d*g - 3*b*e*g)*ArcTanh[(b + 2*c*x)/(2*Sqrt[c]*Sqrt[a + b*x + c*x^2])])/(2*c^(5/2)*e^2) + ((e*f - d*g)^4*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x + c*x^2])])/(e^2*(c*d^2 - b*d*e + a*e^2)^(3/2))$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x+f)**4/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)`

[Out] Timed out

**Mathematica [A]** time = 5.97108, size = 587, normalized size = 1.18

$$\frac{2e(b^2(3a^2e^2g^4+acg^3(d^2g+4de(2f+3gx)+e^2x(gx-8f))+c^2(d^2g^4x^2-12def^2g^2x+2e^2f^4))-2bc(a^2eg^3(-5dg+4ef+5egx)+2acg(d^2g^3x+deg(3f^2+6fgx-g^2x$$

Antiderivative was successfully verified.

[In] `Integrate[(f + g*x)^4/((d + e*x)*(a + b*x + c*x^2)^(3/2)),x]`

[Out] 
$$\begin{aligned} &((-2*e*(-3*b^4*d*e*g^4*x + b^3*g^3*(3*a*e*g*(-d + e*x) + c*d*x*(8 \\ &*e*f + d*g - e*g*x)) + b^2*(3*a^2*e^2*g^4 + c^2*(2*e^2*f^4 - 12*d \\ &*e*f^2*g^2*x + d^2*g^4*x^2) + a*c*g^3*(d^2*g + e^2*x*(-8*f + g*x) \\ &+ 4*d*e*(2*f + 3*g*x))) - 2*b*c*(a^2*e*g^3*(4*e*f - 5*d*g + 5*e \\ &g*x) + c^2*e*f^3*(-(e*f*x) + d*(f - 4*g*x)) + 2*a*c*g*(d^2*g^3*x \\ &+ e^2*f^2*(2*f - 3*g*x) + d*e*g*(3*f^2 + 6*f*g*x - g^2*x^2))) - 4 \\ &*c*(2*a^3*e^2*g^4 + c^3*d*e*f^4*x + a*c^2*(d^2*g^4*x^2 - 2*d*e*f^2 \\ &g*(2*f + 3*g*x) + e^2*f^3*(f + 4*g*x)) + a^2*c*g^2*(d^2*g^2 + d \\ &*e*g*(4*f + g*x) + e^2*(-6*f^2 - 4*f*g*x + g^2*x^2))))/(c^2*(b^2 \\ &- 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*Sqrt[a + x*(b + c*x)]) + (2* \\ &(e*f - d*g)^4*Log[d + e*x])/(c*d^2 + e*(-(b*d) + a*e))^(3/2) + (g \\ &^3*(8*c*e*f - 2*c*d*g - 3*b*e*g)*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a \\ &+ x*(b + c*x)])]/c^(5/2) - (2*(e*f - d*g)^4*Log[-(b*d) + 2*a*e - \\ &2*c*d*x + b*e*x + 2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b \\ &+ c*x)])/(c*d^2 + e*(-(b*d) + a*e))^(3/2))/(2*e^2) \end{aligned}$$

**Maple [B]** time = 0.035, size = 4453, normalized size = 9.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^4/(e*x+d)/(c*x^2+b*x+a)^(3/2),x)`

[Out] 
$$\begin{aligned} &e/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b \\ &*d*e+c*d^2)/e^2)^{(1/2)}*f^4-e/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c \\ &d^2)/e^2)^{(1/2)}*\ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/ \\ &e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c+(b*e-2*c*d)/e*( \\ &x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x+d/e))*f^4-12/e/(a*e^2-b \\ &*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2 \\ &-b*d*e+c*d^2)/e^2)^{(1/2)}*x*b*c*d^2*f^2*g^2+8/e^2/(a*e^2-b*d*e+c*d \\ &^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c \\ &*d^2)/e^2)^{(1/2)}*x*b*c*d^3*f*g^3+8*g^3/e^2*b/(4*a*c-b^2)/(c*x^2+b \\ &*x+a)^{(1/2)}*x*d*f+4*g^3/e^2*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)} \\ &*d*f+4*g^3/e*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*f+4*g^4/e/c* \\ &a*b/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x+16*g^3/e^3*d^2*f/(4*a*c-b^2 \\ &)/(c*x^2+b*x+a)^{(1/2)}*c*x-g^4/e^2*b^2/c/(4*a*c-b^2)/(c*x^2+b*x+a) \\ &^{(1/2)}*x*d+2/e^4/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b* \\ &e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*b*c*d^5*g^4+4/e \\ &^4/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+ \\ &d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*x*c^2*d^5*g^4+4/e^2/(a*e^2-b* \\ &d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2- \\ &b*d*e+c*d^2)/e^2)^{(1/2)}*b^2*d^3*f*g^3-6/e/(a*e^2-b*d*e+c*d^2)/(4* \\ &a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e \\ &^2)^{(1/2)}*b^2*d^2*f^2*g^2-4/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e \end{aligned}$$

$$\begin{aligned}
& -2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*d*f^3*g-6*g^2/e/ \\
& c/(c*x^2+b*x+a)^{(1/2)}*f^2+g^4/e*x^2/c/(c*x^2+b*x+a)^{(1/2)}-g^4/e^2 \\
& /c^{(3/2)}*\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})^*d+4*g^3/e/c^{(3/2)} \\
& *\ln((1/2*b+c*x)/c^{(1/2)}+(c*x^2+b*x+a)^{(1/2)})^*f-3/4*g^4/e*b^2 \\
& /c^3/(c*x^2+b*x+a)^{(1/2)}-3/2*g^4/e*b/c^{(5/2)}*\ln((1/2*b+c*x)/c^{(1/2)} \\
& +(c*x^2+b*x+a)^{(1/2)})+2*g^4/e/c^2*a/(c*x^2+b*x+a)^{(1/2)}-g^4/e^3 \\
& /c/(c*x^2+b*x+a)^{(1/2)}*d^2-24*g^2/e^2*d*f^2/(4*a*c-b^2)/(c*x^2+b* \\
& x+a)^{(1/2)}*c*x+1/e^3/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*d) \\
& /e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*d^4*g^4-2*e/(a*e^2-b*d* \\
& e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b* \\
& d*e+c*d^2)/e^2)^{(1/2)}*x*b*c*f^4+8/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2) \\
& /((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)} \\
& )^*x*b*c*d*f^3*g-2/e^3/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2* \\
& c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*x*b*c*d^4* \\
& g^4-16/e^3/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c* \\
& d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*x*c^2*d^4*f*g^3+24/e^2 \\
& /((a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d \\
& /e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*x*c^2*d^3*f^2*g^2-16/e/(a*e^2- \\
& b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^ \\
& 2-b*d*e+c*d^2)/e^2)^{(1/2)}*x*c^2*d^2*f^3*g-8/e^3/(a*e^2-b*d*e+c*d^ \\
& 2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c* \\
& d^2)/e^2)^{(1/2)}*b*c*d^4*f*g^3+12/e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b \\
& ^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)} \\
& )^*b*c*d^3*f^2*g^2-8/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e) \\
& ^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*b*c*d^2 \\
& *f^3*g+8*g^3/e^3*d^2*f/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*b+16*g/e*f \\
& ^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*c*x-12*g^2/e^2*d*f^2/(4*a*c-b^ \\
& 2)/(c*x^2+b*x+a)^{(1/2)}*b-4*g^4/e^4*d^3/(4*a*c-b^2)/(c*x^2+b*x+a)^ \\
& (1/2)*c*x-1/2*g^4/e^2*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*d+2 \\
& *g^3/e*b^3/c^2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*f-3/2*g^4/e*b^3/c^ \\
& 2/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x+2*g^4/e/c^2*a*b^2/(4*a*c-b^2) \\
& /((c*x^2+b*x+a)^{(1/2)}-2*g^4/e^3*b/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}* \\
& x*d^2-12*g^2/e*b/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*x*f^2-1/e^3/(a*e \\
& ^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a \\
& *e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*b^2*d^4*g^4+2/(a*e^2-b*d*e+c*d^2)/(4 \\
& *a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/ \\
& e^2)^{(1/2)}*b*c*d*f^4+4/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2 \\
& *c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*b^2*d*f^3 \\
& *g+4/e^2/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*\ln(( \\
& 2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c \\
& *d^2)/e^2)^{(1/2)}*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+ \\
& c*d^2)/e^2)^{(1/2)})/(x+d/e))^*d^3*f*g^3-6/e/(a*e^2-b*d*e+c*d^2)/((a \\
& *e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*\ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2 \\
& *c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c+(b \\
& *e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x+d/e))^*d^2* \\
& f^2*g^2+4/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d \\
& )/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*x*c^2*d*f^4-g^4/e^3*b^ \\
& 2/c/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*d^2-6*g^2/e*b^2/c/(4*a*c-b^2) \\
& /((c*x^2+b*x+a)^{(1/2)}*f^2-3/4*g^4/e*b^4/c^3/(4*a*c-b^2)/(c*x^2+b*x \\
& +a)^{(1/2)}+g^4/e^2*x/c/(c*x^2+b*x+a)^{(1/2)}*d-4*g^3/e*x/c/(c*x^2+b* \\
& x+a)^{(1/2)}*f-1/2*g^4/e^2*b/c^2/(c*x^2+b*x+a)^{(1/2)}*d+2*g^3/e*b/c^ \\
& 2/(c*x^2+b*x+a)^{(1/2)}*f+4*g^3/e^2/c/(c*x^2+b*x+a)^{(1/2)}*d*f+8*g/e \\
& *f^3/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)}*b-2*g^4/e^4*d^3/(4*a*c-b^2)/ \\
& (c*x^2+b*x+a)^{(1/2)}*b+3/2*g^4/e*b/c^2*x/(c*x^2+b*x+a)^{(1/2)}-e/(a* \\
& e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+( \\
& a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*b^2*f^4-1/e^3/(a*e^2-b*d*e+c*d^2)/ \\
& ((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*\ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e \\
& -2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c+ \\
& (b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x+d/e))^*d^4 \\
& *g^4-4/e^2/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e) \\
& )+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*d^3*f*g^3+6/e/(a*e^2-b*d*e+c*d^2) \\
& /((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)} \\
& )^*d^2*f^2*g^2+4/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)} \\
& )^*\ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2 \\
& -b*d*e+c*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^ \\
& 2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x+d/e))^*d*f^3*g
\end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x + f)^4/((c*x^2 + b*x + a)^(3/2)*(e*x + d)),x, algorithm="maxima")`

[Out] Exception raised: ValueError

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x + f)^4/((c*x^2 + b*x + a)^(3/2)*(e*x + d)),x, algorithm="fricas")`

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)**4/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)`

[Out] Timed out

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x + f)^4/((c*x^2 + b*x + a)^(3/2)*(e*x + d)),x, algorithm="giac")`

[Out] Exception raised: TypeError



$$3.879 \quad \int \frac{(f+gx)^3}{(d+ex)(a+bx+cx^2)^{3/2}} dx$$

**Optimal.** Leaf size=357

$$\frac{2(-x(cg^2(-2a^2eg + 3abdg - 3abef + 3b^2df) - b^2g^3(bd - ae) + c^2f(6ag(ef - dg) - bf(3dg + ef)) + 2c^3df^3) - b(a^2eg^3 + c(b^2 - 4ac)\sqrt{a+bx+cx^2}(ae^2 - bde + cd^2))}{c^3/2e} + \frac{(ef - dg)^3 \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e(ae^2 - bde + cd^2)^{3/2}}$$

[Out]  $(2*(b^2*(c*e*f^3 + a*d*g^3) - 2*a*c*(c*f^2*(e*f - 3*d*g) - a*g^2*(3*e*f - d*g)) - b*(c^2*d*f^3 + a^2*e*g^3 + 3*a*c*f*g*(e*f + d*g)) - (2*c^3*d*f^3 - b^2*(b*d - a*e)*g^3 + c*g^2*(3*b^2*d*f - 3*a*b*e*f + 3*a*b*d*g - 2*a^2*e*g) + c^2*f*(6*a*g*(e*f - d*g) - b*f*(e*f + 3*d*g)))*x)/(c*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*\text{Sqrt}[a + b*x + c*x^2]) + (g^3*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2]))/(c^{3/2}*e) + ((e*f - d*g)^3*\text{ArcTanh}[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*\text{Sqrt}[a + b*x + c*x^2]))/(e*(c*d^2 - b*d*e + a*e^2)^{3/2}))$

**Rubi [A]** time = 1.1792, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$\frac{2(-x(cg^2(-2a^2eg - 3ab(ef - dg) + 3b^2df) - b^2g^3(bd - ae) + c^2f(6ag(ef - dg) - bf(3dg + ef)) + 2c^3df^3) - b(a^2eg^3 + c(b^2 - 4ac)\sqrt{a+bx+cx^2}(ae^2 - bde + cd^2))}{c^3/2e} + \frac{(ef - dg)^3 \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e(ae^2 - bde + cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(f + g*x)^3/((d + e*x)*(a + b*x + c*x^2)^{(3/2)}), x]$

[Out]  $(2*(b^2*(c*e*f^3 + a*d*g^3) - 2*a*c*(c*f^2*(e*f - 3*d*g) - a*g^2*(3*e*f - d*g)) - b*(c^2*d*f^3 + a^2*e*g^3 + 3*a*c*f*g*(e*f + d*g)) - (2*c^3*d*f^3 - b^2*(b*d - a*e)*g^3 + c*g^2*(3*b^2*d*f - 2*a^2*e*g - 3*a*b*(e*f - d*g)) + c^2*f*(6*a*g*(e*f - d*g) - b*f*(e*f + 3*d*g)))*x)/(c*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*\text{Sqrt}[a + b*x + c*x^2]) + (g^3*\text{ArcTanh}[(b + 2*c*x)/(2*\text{Sqrt}[c]*\text{Sqrt}[a + b*x + c*x^2]))/(c^{3/2}*e) + ((e*f - d*g)^3*\text{ArcTanh}[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*\text{Sqrt}[a + b*x + c*x^2]))/(e*(c*d^2 - b*d*e + a*e^2)^{3/2}))$

**Rubi in Sympy [A]** time = 143.946, size = 376, normalized size = 1.05

$$\frac{2bg^3\sqrt{a+bx+cx^2}}{ce(-4ac+b^2)} + \frac{2g^3x(2a+bx)}{e(-4ac+b^2)\sqrt{a+bx+cx^2}} + \frac{(dg-ef)^3 \operatorname{atanh}\left(\frac{2ae-bd+x(be-2cd)}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{e(ae^2-bde+cd^2)^{3/2}}$$

$$- \frac{g^2(4a+2bx)(dg-3ef)}{e^2(-4ac+b^2)\sqrt{a+bx+cx^2}} - \frac{g(2b+4cx)(d^2g^2-3defg+3e^2f^2)}{e^3(-4ac+b^2)\sqrt{a+bx+cx^2}}$$

$$- \frac{2(dg-ef)^3(-2ace+b^2e-bcd+cx(be-2cd))}{e^3(-4ac+b^2)\sqrt{a+bx+cx^2}(ae^2-bde+cd^2)} + \frac{g^3 \operatorname{atanh}\left(\frac{b+2cx}{2\sqrt{c}\sqrt{a+bx+cx^2}}\right)}{c^{3/2}e}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((g*x+f)^3/(e*x+d)/(c*x^2+b*x+a)^{(3/2)}, x)$

[Out]  $-2*b*g^{3*sqrt(a + b*x + c*x^2)}/(c*e^{(-4*a*c + b^2)}) + 2*g^{3*x}*(2*a + b*x)/(e^{(-4*a*c + b^2)*sqrt(a + b*x + c*x^2)}) + (d*g - e*f)^{3*atanh((2*a*e - b*d + x*(b*e - 2*c*d))/(2*sqrt(a + b*x + c*x^2)*sqrt(a^2 - b*d*e + c*d^2)))/(e^{(a^2 - b*d*e + c*d^2)^{(3/2)}}) - g^{2*(4*a + 2*b*x)}*(d*g - 3*e*f)/(e^{2*(-4*a*c + b^2)*sqrt(a + b*x + c*x^2)}) - g^{2*b + 4*c*x}*(d^2*g^2 - 3*d^2*e*f*g + 3*e^2*f^2)/(e^{3*(-4*a*c + b^2)*sqrt(a + b*x + c*x^2)}) - 2*(d*g - e*f)^{3*(-2*a*c*e + b^2*e - b*c*d + c*x*(b*e - 2*c*d)))/(e^{3*(-4*a*c + b^2)*sqrt(a + b*x + c*x^2)}*(a^2 - b*d*e + c*d^2)) + g^{3*atanh((b + 2*c*x)/(2*sqrt(c)*sqrt(a + b*x + c*x^2)))/(c^{(3/2)*e}}$

**Mathematica [A]** time = 1.58641, size = 373, normalized size = 1.04

$$\frac{2(b(a^2eg^3 + 3acg(dg(f + gx) + ef(f - gx)) + c^2f^2(df - 3gx) - efx)) + 2c(a^2g^2(dg - e(3f + gx)) + acf(ef(f + 3gx) - 3c(4ac - b^2)\sqrt{a + x(b + cx)}(e(ae - bd) + cd^2))}{c^3 \log\left(\frac{2\sqrt{c}\sqrt{a + x(b + cx)} + b + 2cx}{c^{3/2}e}\right) + \frac{(ef - dg)^3 \log(d + ex)}{e(e(ae - bd) + cd^2)^{3/2}} + \frac{(dg - ef)^3 \log\left(\frac{2\sqrt{a + x(b + cx)}\sqrt{e(ae - bd) + cd^2} + 2ae - bd + bex - 2cdx}{e(e(ae - bd) + cd^2)^{3/2}}\right)}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^3/((d + e\*x)\*(a + b\*x + c\*x^2)^(3/2)), x]

[Out]  $(2*(-(b^3*d*g^3*x) + b^2*(a*g^3*(-d + e*x) + c*(-(e*f^3) + 3*d*f*g^2*x)) + b*(a^2*e*g^3 + c^2*f^2*(-(e*f*x) + d*(f - 3*g*x)) + 3*a*c*g*(e*f*(f - g*x) + d*g*(f + g*x))) + 2*c*(c^2*d*f^3*x + a^2*g^2*(d*g - e*(3*f + g*x)) + a*c*f*(-3*d*g*(f + g*x) + e*f*(f + 3*g*x))))/(c*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[a + x*(b + c*x)]) + ((e*f - d*g)^3*Log[d + e*x])/(e*(c*d^2 + e*(-(b*d) + a*e))^(3/2)) + (g^3*Log[b + 2*c*x + 2*Sqrt[c]*Sqrt[a + x*(b + c*x)]])/(c^(3/2)*e) + ((-(e*f) + d*g)^3*Log[-(b*d) + 2*a*e - 2*c*d*x + b*e*x + 2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)]])/(e*(c*d^2 + e*(-(b*d) + a*e))^(3/2))$

**Maple [B]** time = 0.022, size = 3127, normalized size = 8.8

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^3/(e\*x+d)/(c\*x^2+b\*x+a)^(3/2), x)

[Out]  $2*g^3/e^3*d^2/(4*a*c - b^2)/(c*x^2 + b*x + a)^{(1/2)}*b + 1/2*g^3/e*b^3/c^2/(4*a*c - b^2)/(c*x^2 + b*x + a)^{(1/2)} + e/(a^2 - b*d*e + c*d^2)/((x + d/e)^2*c + (b*e - 2*c*d)/e*(x + d/e) + (a^2 - b*d*e + c*d^2)/e^2)^{(1/2)}*f^3 + g^3/e/c^{(3/2)}*ln((1/2*b + c*x)/c^{(1/2)} + (c*x^2 + b*x + a)^{(1/2)}) + 2/e^2/(a^2 - b*d*e + c*d^2)/(4*a*c - b^2)/((x + d/e)^2*c + (b*e - 2*c*d)/e*(x + d/e) + (a^2 - b*d*e + c*d^2)/e^2)^{(1/2)}*x*b*c*d^3*g^3 + 6/(a^2 - b*d*e + c*d^2)/(4*a*c - b^2)/((x + d/e)^2*c + (b*e - 2*c*d)/e*(x + d/e) + (a^2 - b*d*e + c*d^2)/e^2)^{(1/2)}*x^2*c^2*d^3*f^2*g^2 - 12/e/(a^2 - b*d*e + c*d^2)/(4*a*c - b^2)/((x + d/e)^2*c + (b*e - 2*c*d)/e*(x + d/e) + (a^2 - b*d*e + c*d^2)/e^2)^{(1/2)}*x^2*d^2*f^2*g + 6/e^2/(a^2 - b*d*e + c*d^2)/(4*a*c - b^2)/((x + d/e)^2*c + (b*e - 2*c*d)/e*(x + d/e) + (a^2 - b*d*e + c*d^2)/e^2)^{(1/2)}*b*c*d^3*f^2*g^2 - 6/e/(a^2 - b*d*e + c*d^2)/(4*a*c - b^2)/((x + d/e)^2*c + (b*e - 2*c*d)/e*(x + d/e) + (a^2 - b*d*e + c*d^2)/e^2)^{(1/2)}*b*c*d^2*f^2*g - 3/(a^2 - b*d*e + c*d^2)/((x + d/e)^2*c + (b*e - 2*c*d)/e*(x + d/e) + (a^2 - b*d*e + c*d^2)/e^2)^{(1/2)}$

$$\begin{aligned} & ) * d * f^2 * g + 6 * g / e * f^2 / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(1/2)} * b + 1 / e^2 / (a * e^2 \\ & - b * d * e + c * d^2) / ((a * e^2 - b * d * e + c * d^2) / e^2)^{(1/2)} * \ln((2 * (a * e^2 - b * d * e \\ & + c * d^2) / e^2 + (b * e - 2 * c * d) / e * (x + d / e) + 2 * ((a * e^2 - b * d * e + c * d^2) / e^2)^{(1/2)} \\ & * ((x + d / e)^2 * c + (b * e - 2 * c * d) / e * (x + d / e) + (a * e^2 - b * d * e + c * d^2) / e^2)^{(1/2)}) / (x + d / e) \\ & ) * d^3 * g^3 + 3 / (a * e^2 - b * d * e + c * d^2) / ((a * e^2 - b * d * e + c * d^2) / e^2)^{(1/2)} * \ln((2 * (a * e^2 - b * d * e + c * d^2) / e^2 + (b * e - 2 * c * d) / e * (x + d / e) + 2 * ((a * e^2 - b * d * e + c * d^2) / e^2)^{(1/2)} * ((x + d / e)^2 * c + (b * e - 2 * c * d) / e * (x + d / e) + (a * e^2 - b * d * e + c * d^2) / e^2)^{(1/2)}) / (x + d / e) \\ & ) * d^2 * f^2 * g - e / (a * e^2 - b * d * e + c * d^2) / (4 * a * c - b^2) / ((x + d / e)^2 * c + (b * e - 2 * c * d) / e * (x + d / e) + (a * e^2 - b * d * e + c * d^2) / e^2)^{(1/2)} * b^2 * f^3 + 3 / e / (a * e^2 - b * d * e + c * d^2) / ((x + d / e)^2 * c + (b * e - 2 * c * d) / e * (x + d / e) + (a * e^2 - b * d * e + c * d^2) / e^2)^{(1/2)} * d^2 * f * g^2 - 6 / e / (a * e^2 - b * d * e + c * d^2) / (4 * a * c - b^2) / ((x + d / e)^2 * c + (b * e - 2 * c * d) / e * (x + d / e) + (a * e^2 - b * d * e + c * d^2) / e^2)^{(1/2)} * x * b * c * d^2 * f * g^2 - 3 / e / (a * e^2 - b * d * e + c * d^2) / ((a * e^2 - b * d * e + c * d^2) / e^2)^{(1/2)} * \ln((2 * (a * e^2 - b * d * e + c * d^2) / e^2 + (b * e - 2 * c * d) / e * (x + d / e) + 2 * ((a * e^2 - b * d * e + c * d^2) / e^2)^{(1/2)} * ((x + d / e)^2 * c + (b * e - 2 * c * d) / e * (x + d / e) + (a * e^2 - b * d * e + c * d^2) / e^2)^{(1/2)}) / (x + d / e) \\ & ) * d^2 * f * g^2 + 4 / (a * e^2 - b * d * e + c * d^2) / (4 * a * c - b^2) / ((x + d / e)^2 * c + (b * e - 2 * c * d) / e * (x + d / e) + (a * e^2 - b * d * e + c * d^2) / e^2)^{(1/2)} * x * c^2 * d * f^3 + 1 / e^2 / (a * e^2 - b * d * e + c * d^2) / (4 * a * c - b^2) / ((x + d / e)^2 * c + (b * e - 2 * c * d) / e * (x + d / e) + (a * e^2 - b * d * e + c * d^2) / e^2)^{(1/2)} * b^2 * d^3 * g^3 + 12 * g / e * f^2 / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(1/2)} * c * x - 6 * g^2 / e^2 * d * f / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(1/2)} * b + g^3 / e^2 * b^2 / c / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(1/2)} * d - 3 * g^2 / e * b^2 / c / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(1/2)} * f + g^3 / e * b^2 / c / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(1/2)} * x + 1 / 2 * g^3 / e * b / c^2 / (c * x^2 + b * x + a)^{(1/2)} + g^3 / e^2 / c / (c * x^2 + b * x + a)^{(1/2)} * d - 3 * g^2 / e / c / (c * x^2 + b * x + a)^{(1/2)} * f - g^3 / e * x / c / (c * x^2 + b * x + a)^{(1/2)} - e / (a * e^2 - b * d * e + c * d^2) / ((a * e^2 - b * d * e + c * d^2) / e^2)^{(1/2)} * \ln((2 * (a * e^2 - b * d * e + c * d^2) / e^2 + (b * e - 2 * c * d) / e * (x + d / e) + 2 * ((a * e^2 - b * d * e + c * d^2) / e^2)^{(1/2)} * ((x + d / e)^2 * c + (b * e - 2 * c * d) / e * (x + d / e) + (a * e^2 - b * d * e + c * d^2) / e^2)^{(1/2)}) / (x + d / e) \\ & ) * f^3 - 1 / e^2 / (a * e^2 - b * d * e + c * d^2) / ((x + d / e)^2 * c + (b * e - 2 * c * d) / e * (x + d / e) + (a * e^2 - b * d * e + c * d^2) / e^2)^{(1/2)} * d^3 * g^3 - 2 * e / (a * e^2 - b * d * e + c * d^2) / (4 * a * c - b^2) / ((x + d / e)^2 * c + (b * e - 2 * c * d) / e * (x + d / e) + (a * e^2 - b * d * e + c * d^2) / e^2)^{(1/2)} * x * b * c * f^3 - 4 / e^3 / (a * e^2 - b * d * e + c * d^2) / (4 * a * c - b^2) / ((x + d / e)^2 * c + (b * e - 2 * c * d) / e * (x + d / e) + (a * e^2 - b * d * e + c * d^2) / e^2)^{(1/2)} * x * c^2 * d^4 * g^3 - 3 / e / (a * e^2 - b * d * e + c * d^2) / (4 * a * c - b^2) / ((x + d / e)^2 * c + (b * e - 2 * c * d) / e * (x + d / e) + (a * e^2 - b * d * e + c * d^2) / e^2)^{(1/2)} * b^2 * d^2 * f * g^2 - 2 / e^3 / (a * e^2 - b * d * e + c * d^2) / (4 * a * c - b^2) / ((x + d / e)^2 * c + (b * e - 2 * c * d) / e * (x + d / e) + (a * e^2 - b * d * e + c * d^2) / e^2)^{(1/2)} * b * c * d^4 * g^3 + 4 * g^3 / e^3 * d^2 / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(1/2)} * c * x + 2 * g^3 / e^2 * b / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(1/2)} * x * d - 6 * g^2 / e * b / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(1/2)} * x * f + 3 / (a * e^2 - b * d * e + c * d^2) / (4 * a * c - b^2) / ((x + d / e)^2 * c + (b * e - 2 * c * d) / e * (x + d / e) + (a * e^2 - b * d * e + c * d^2) / e^2)^{(1/2)} * b^2 * d * f^2 * g + 2 / (a * e^2 - b * d * e + c * d^2) / (4 * a * c - b^2) / ((x + d / e)^2 * c + (b * e - 2 * c * d) / e * (x + d / e) + (a * e^2 - b * d * e + c * d^2) / e^2)^{(1/2)} * b * c * d * f^3 - 12 * g^2 / e^2 * d * f / (4 * a * c - b^2) / (c * x^2 + b * x + a)^{(1/2)} * c * x \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^3/((c\*x^2 + b\*x + a)^(3/2)\*(e\*x + d)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^3/((c\*x^2 + b\*x + a)^(3/2)\*(e\*x + d)),x, algorithm="fricas")

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)**3/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)`

[Out] Timed out

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x + f)^3/((c*x^2 + b*x + a)^(3/2)*(e*x + d)),x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.880 \quad \int \frac{(f+gx)^2}{(d+ex)(a+bx+cx^2)^{3/2}} dx$$

**Optimal.** Leaf size=240

$$\frac{2(-x(c(2ag(2ef-dg)-bf(2dg+ef))+bg^2(bd-ae)+2c^2df^2)-b(ag(dg+2ef)+cdf^2)+2a(aeg^2-cf(ef-2dg))}{(b^2-4ac)\sqrt{a+bx+cx^2}(ae^2-bde+cd^2)} + \frac{(ef-dg)^2 \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ae^2-bde+cd^2)^{3/2}}$$

[Out] (2\*(b^2\*e\*f^2 + 2\*a\*(a\*e\*g^2 - c\*f\*(e\*f - 2\*d\*g)) - b\*(c\*d\*f^2 + a\*g\*(2\*e\*f + d\*g)) - (2\*c^2\*d\*f^2 + b\*(b\*d - a\*e)\*g^2 + c\*(2\*a\*g\*(2\*e\*f - d\*g) - b\*f\*(e\*f + 2\*d\*g)))\*x)/((b^2 - 4\*a\*c)\*(c\*d^2 - b\*d\*e + a\*e^2)\*Sqrt[a + b\*x + c\*x^2]) + ((e\*f - d\*g)^2\*ArcTanh[(b\*d - 2\*a\*e + (2\*c\*d - b\*e)\*x)/(2\*Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*Sqrt[a + b\*x + c\*x^2])])/(c\*d^2 - b\*d\*e + a\*e^2)^(3/2)

**Rubi [A]** time = 0.655042, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$

$$\frac{2(-x(bg^2(bd-ae)+2acg(2ef-dg)-bcf(2dg+ef))+2c^2df^2)-b(ag(dg+2ef)+cdf^2)+2a(aeg^2-cf(ef-2dg))}{(b^2-4ac)\sqrt{a+bx+cx^2}(ae^2-bde+cd^2)} + \frac{(ef-dg)^2 \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ae^2-bde+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)^2/((d + e\*x)\*(a + b\*x + c\*x^2)^(3/2)), x]

[Out] (2\*(b^2\*e\*f^2 + 2\*a\*(a\*e\*g^2 - c\*f\*(e\*f - 2\*d\*g)) - b\*(c\*d\*f^2 + a\*g\*(2\*e\*f + d\*g)) - (2\*c^2\*d\*f^2 + b\*(b\*d - a\*e)\*g^2 + 2\*a\*c\*g\*(2\*e\*f - d\*g) - b\*c\*f\*(e\*f + 2\*d\*g))\*x)/((b^2 - 4\*a\*c)\*(c\*d^2 - b\*d\*e + a\*e^2)\*Sqrt[a + b\*x + c\*x^2]) + ((e\*f - d\*g)^2\*ArcTanh[(b\*d - 2\*a\*e + (2\*c\*d - b\*e)\*x)/(2\*Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*Sqrt[a + b\*x + c\*x^2])])/(c\*d^2 - b\*d\*e + a\*e^2)^(3/2)

**Rubi in Sympy [A]** time = 103.567, size = 240, normalized size = 1.

$$-\frac{(dg-ef)^2 \operatorname{atanh}\left(\frac{2ae-bd+x(be-2cd)}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ae^2-bde+cd^2)^{3/2}} + \frac{g^2(4a+2bx)}{e(-4ac+b^2)\sqrt{a+bx+cx^2}} + \frac{g(2b+4cx)(dg-2ef)}{e^2(-4ac+b^2)\sqrt{a+bx+cx^2}} + \frac{2(dg-ef)^2(-2ace+b^2e-bcd+cx(be-2cd))}{e^2(-4ac+b^2)\sqrt{a+bx+cx^2}(ae^2-bde+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*2/(e\*x+d)/(c\*x\*\*2+b\*x+a)\*\*(3/2), x)

[Out] -(d\*g - e\*f)\*\*2\*atanh((2\*a\*e - b\*d + x\*(b\*e - 2\*c\*d))/(2\*sqrt(a + b\*x + c\*x\*\*2)\*sqrt(a\*e\*\*2 - b\*d\*e + c\*d\*\*2)))/(a\*e\*\*2 - b\*d\*e + c\*d\*\*2)\*\*(3/2) + g\*\*2\*(4\*a + 2\*b\*x)/(e\*(-4\*a\*c + b\*\*2)\*sqrt(a + b\*x + c\*x\*\*2)) + g\*(2\*b + 4\*c\*x)\*(d\*g - 2\*e\*f)/(e\*\*2\*(-4\*a\*c + b\*\*2)\*sqrt(a + b\*x + c\*x\*\*2)) + 2\*(d\*g - e\*f)\*\*2\*(-2\*a\*c\*e + b\*\*2\*e - b\*c\*d + c\*x\*(b\*e - 2\*c\*d))/(e\*\*2\*(-4\*a\*c + b\*\*2)\*sqrt(a + b\*x + c\*x\*\*2)\*(a\*e\*\*2 - b\*d\*e + c\*d\*\*2))

**Mathematica [A]** time = 0.987066, size = 265, normalized size = 1.1

$$\frac{2(-2a^2eg^2 + abg(dg + 2ef - egx) - 2acd(2f + gx) + 2acef(f + 2gx) + b^2(dg^2x - ef^2) + bcf(d(f - 2gx) - ef x) + 2c^2d)}{(b^2 - 4ac)\sqrt{a + x(b + cx)}(e(bd - ae) - cd^2)}$$

$$+ \frac{(ef - dg)^2 \log(d + ex)}{(e(ae - bd) + cd^2)^{3/2}} - \frac{(ef - dg)^2 \log\left(2\sqrt{a + x(b + cx)}\sqrt{e(ae - bd) + cd^2} + 2ae - bd + bex - 2cdx\right)}{(e(ae - bd) + cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)^2/((d + e\*x)\*(a + b\*x + c\*x^2)^(3/2)),x]

[Out] (2\*(-2\*a^2\*e\*g^2 + 2\*c^2\*d\*f^2\*x - 2\*a\*c\*d\*g\*(2\*f + g\*x) + 2\*a\*c\*e\*f\*(f + 2\*g\*x) + a\*b\*g\*(2\*e\*f + d\*g - e\*g\*x) + b^2\*(-(e\*f^2) + d\*g^2\*x) + b\*c\*f\*(-(e\*f\*x) + d\*(f - 2\*g\*x)))/((b^2 - 4\*a\*c)\*(-(c\*d^2) + e\*(b\*d - a\*e))\*Sqrt[a + x\*(b + c\*x)]) + ((e\*f - d\*g)^2\*Log[d + e\*x]/(c\*d^2 + e\*(-(b\*d) + a\*e))^(3/2) - ((e\*f - d\*g)^2\*Log[-(b\*d) + 2\*a\*e - 2\*c\*d\*x + b\*e\*x + 2\*Sqrt[c\*d^2 + e\*(-(b\*d) + a\*e)]]\*Sqrt[a + x\*(b + c\*x)])/(c\*d^2 + e\*(-(b\*d) + a\*e))^(3/2))

**Maple [B]** time = 0.021, size = 2123, normalized size = 8.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^2/(e\*x+d)/(c\*x^2+b\*x+a)^(3/2),x)

[Out] -g^2/e/c/(c\*x^2+b\*x+a)^(1/2)+e/(a\*e^2-b\*d\*e+c\*d^2)/((x+d/e)^2\*c+(b\*e-2\*c\*d)/e\*(x+d/e)+(a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*f^2-2\*g^2/e\*b/(4\*a\*c-b^2)/(c\*x^2+b\*x+a)^(1/2)\*x-g^2/e\*b^2/c/(4\*a\*c-b^2)/(c\*x^2+b\*x+a)^(1/2)-2\*g^2/e^2\*d/(4\*a\*c-b^2)/(c\*x^2+b\*x+a)^(1/2)\*b+4\*g/e\*f/(4\*a\*c-b^2)/(c\*x^2+b\*x+a)^(1/2)\*b+2/(a\*e^2-b\*d\*e+c\*d^2)/((a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2-b\*d\*e+c\*d^2)/e^2+(b\*e-2\*c\*d)/e\*(x+d/e)+2\*((a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*((x+d/e)^2\*c+(b\*e-2\*c\*d)/e\*(x+d/e)+(a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2))/(x+d/e)\*d\*f\*g-e/(a\*e^2-b\*d\*e+c\*d^2)/(4\*a\*c-b^2)/((x+d/e)^2\*c+(b\*e-2\*c\*d)/e\*(x+d/e)+(a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*b^2\*f^2+2/(a\*e^2-b\*d\*e+c\*d^2)/(4\*a\*c-b^2)/((x+d/e)^2\*c+(b\*e-2\*c\*d)/e\*(x+d/e)+(a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*b^2\*d\*f\*g+2/(a\*e^2-b\*d\*e+c\*d^2)/(4\*a\*c-b^2)/((x+d/e)^2\*c+(b\*e-2\*c\*d)/e\*(x+d/e)+(a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*b\*c\*d\*f^2-4\*g^2/e^2\*d/(4\*a\*c-b^2)/(c\*x^2+b\*x+a)^(1/2)\*c\*x+8\*g/e\*f/(4\*a\*c-b^2)/(c\*x^2+b\*x+a)^(1/2)\*c\*x+4/(a\*e^2-b\*d\*e+c\*d^2)/(4\*a\*c-b^2)/((x+d/e)^2\*c+(b\*e-2\*c\*d)/e\*(x+d/e)+(a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*x\*c^2\*d\*f^2-1/e/(a\*e^2-b\*d\*e+c\*d^2)/(4\*a\*c-b^2)/((x+d/e)^2\*c+(b\*e-2\*c\*d)/e\*(x+d/e)+(a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*b^2\*d^2\*g^2-e/(a\*e^2-b\*d\*e+c\*d^2)/((a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2-b\*d\*e+c\*d^2)/e^2+(b\*e-2\*c\*d)/e\*(x+d/e)+2\*((a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*((x+d/e)^2\*c+(b\*e-2\*c\*d)/e\*(x+d/e)+(a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2))/(x+d/e))\*f^2-2/(a\*e^2-b\*d\*e+c\*d^2)/((x+d/e)^2\*c+(b\*e-2\*c\*d)/e\*(x+d/e)+(a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*d\*f\*g+1/e/(a\*e^2-b\*d\*e+c\*d^2)/((x+d/e)^2\*c+(b\*e-2\*c\*d)/e\*(x+d/e)+(a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*d^2\*g^2-1/e/(a\*e^2-b\*d\*e+c\*d^2)/((a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*ln((2\*(a\*e^2-b\*d\*e+c\*d^2)/e^2+(b\*e-2\*c\*d)/e\*(x+d/e)+2\*((a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*((x+d/e)^2\*c+(b\*e-2\*c\*d)/e\*(x+d/e)+(a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2))/(x+d/e))\*d^2\*g^2+4/e^2/(a\*e^2-b\*d\*e+c\*d^2)/(4\*a\*c-b^2)/((x+d/e)^2\*c+(b\*e-2\*c\*d)/e\*(x+d/e)+(a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*x\*c^2\*d^3\*g^2+2/e^2/(a\*e^2-b\*d\*e+c\*d^2)/(4\*a\*c-b^2)/((x+d/e)^2\*c+(b\*e-2\*c\*d)/e\*(x+d/e)+(a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*x\*b\*c\*d\*f\*g-2/e/(a\*e^2-b\*d\*e+c\*d^2)/(4\*a\*c-b^2)/((x+d/e)^2\*c+(b\*e-2\*c\*d)/e\*(x+d/e)+(a\*e^2-b\*d\*e+c\*d^2)/e^2)^(1/2)\*x\*b\*c\*d^2\*g^2-8/e/(a\*e^2

$$-b^*d^*e+c^*d^2)/((4^*a^*c-b^2)/((x+d/e)^2*c+(b^*e-2^*c^*d)/e^*(x+d/e)+(a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)}*x^*c^2*d^2*f^*g-4/e/(a^*e^2-b^*d^*e+c^*d^2)/(4^*a^*c-b^2)/((x+d/e)^2*c+(b^*e-2^*c^*d)/e^*(x+d/e)+(a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)}*b^*c^*d^2*f^*g-2^*e/(a^*e^2-b^*d^*e+c^*d^2)/(4^*a^*c-b^2)/((x+d/e)^2*c+(b^*e-2^*c^*d)/e^*(x+d/e)+(a^*e^2-b^*d^*e+c^*d^2)/e^2)^{(1/2)}*x^*b^*c^*f^2$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^2/((c\*x^2 + b\*x + a)^(3/2)\*(e\*x + d)),x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 2.03328, size = 1, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^2/((c\*x^2 + b\*x + a)^(3/2)\*(e\*x + d)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/2*(4*\sqrt{c*d^2 - b*d*e + a*e^2})*((b*c*d - (b^2 - 2*a*c)*e)*f^2 - 2*(2*a*c*d - a*b*e)*f*g + (a*b*d - 2*a^2*e)*g^2 + ((2*c^2*d - b*c*e)*f^2 - 2*(b*c*d - 2*a*c*e)*f*g - (a*b*e - (b^2 - 2*a*c)*d)*g^2)*x]*\sqrt{c*x^2 + b*x + a} - ((a*b^2 - 4*a^2*c)*e^2*f^2 - 2*(a*b^2 - 4*a^2*c)*d*e*f*g + (a*b^2 - 4*a^2*c)*d^2*g^2 + ((b^2*c - 4*a*c^2)*e^2*f^2 - 2*(b^2*c - 4*a*c^2)*d*e*f*g + (b^2*c - 4*a*c^2)*d^2*g^2)*x^2 + ((b^3 - 4*a*b*c)*e^2*f^2 - 2*(b^3 - 4*a*b*c)*d*e*f*g + (b^3 - 4*a*b*c)*d^2*g^2)*x]*\log(((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)*\sqrt{c*d^2 - b*d*e + a*e^2}) - 4*(b*c*d^3 + 3*a*b*d*e^2 - 2*a^2*e^3 - (b^2 + 2*a*c)*d^2*e + (2*c^2*d^3 - 3*b*c*d^2*e - a*b*e^3 + (b^2 + 2*a*c)*d*e^2)*x)*\sqrt{c*x^2 + b*x + a})/(e^2*x^2 + 2*d*e*x + d^2))/(((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2 + ((b^2*c^2 - 4*a*c^3)*d^2 - (b^3*c - 4*a*b*c^2)*d*e + (a*b^2*c - 4*a^2*c^2)*e^2)*x^2 + ((b^3*c - 4*a*b*c^2)*d^2 - (b^4 - 4*a*b^2*c)*d*e + (a*b^3 - 4*a^2*b*c)*e^2)*x)*\sqrt{c*d^2 - b*d*e + a*e^2}), -(2*\sqrt{-c*d^2 + b*d*e - a*e^2})*((b*c*d - (b^2 - 2*a*c)*e)*f^2 - 2*(2*a*c*d - a*b*e)*f*g + (a*b*d - 2*a^2*e)*g^2 + ((2*c^2*d - b*c*e)*f^2 - 2*(b*c*d - 2*a*c*e)*f*g - (a*b*e - (b^2 - 2*a*c)*d)*g^2)*x)*\sqrt{c*x^2 + b*x + a} + ((a*b^2 - 4*a^2*c)*e^2*f^2 - 2*(a*b^2 - 4*a^2*c)*d*e*f*g + (a*b^2 - 4*a^2*c)*d^2*g^2 + ((b^2*c - 4*a*c^2)*e^2*f^2 - 2*(b^2*c - 4*a*c^2)*d*e*f*g + (b^2*c - 4*a*c^2)*d^2*g^2)*x^2 + ((b^3 - 4*a*b*c)*e^2*f^2 - 2*(b^3 - 4*a*b*c)*d*e*f*g + (b^3 - 4*a*b*c)*d^2*g^2)*x]*\arctan(-1/2*\sqrt{-c*d^2 + b*d*e - a*e^2})*(b*d - 2*a*e + (2*c*d - b*e)*x)/((c*d^2 - b*d*e + a*e^2)*\sqrt{c*x^2 + b*x + a}))/(((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2 + ((b^2*c^2 - 4*a*c^3)*d^2 - (b^3*c - 4*a*b*c^2)*d*e + (a*b^2*c - 4*a^2*c^2)*e^2)*x^2 + ((b^3*c - 4*a*b*c^2)*d^2 - (b^4 - 4*a*b^2*c)*d*e + (a*b^3 - 4*a^2*b*c)*e^2)*x)*\sqrt{-c*d^2 + b*d*e - a*e^2})] \end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**2/(e*x+d)/(c*x**2+b*x+a)**(3/2),x)
```

[Out] Timed out

**GIAC/XCAS [A]** time = 0.282558, size = 1022, normalized size = 4.26

$$2 \left( \frac{(2c^3d^3f^2 - 2bc^2d^3fg + b^2cd^3g^2 - 2ac^2d^3g^2 - 3bc^2d^2f^2e + 2b^2cd^2fge + 4ac^2d^2fge - b^3d^2g^2e + abcd^2g^2e + b^2cdf^2e^2 + 2ac^2df^2e^2 - 6abcdfge^2 + 2ab^2dg^2e^2 - b^2c^2d^4 - 4ac^3d^4 - 2b^3cd^3e + 8abc^2d^3e + b^4d^2e^2 - 2ab^2cd^2e^2 - 8a^2c^2d^2e^2 - 2ab^3de^3 + 8a^2bcde^3 + a^2b^2e^4 - 4a^3e^4)}{b^2c^2d^4 - 4ac^3d^4 - 2b^3cd^3e + 8abc^2d^3e + b^4d^2e^2 - 2ab^2cd^2e^2 - 8a^2c^2d^2e^2 - 2ab^3de^3 + 8a^2bcde^3 + a^2b^2e^4 - 4a^3e^4} \right) + \frac{2(d^2g^2 - 2dfge + f^2e^2) \arctan\left(-\frac{(\sqrt{cx} - \sqrt{cx^2 + bx + a})e + \sqrt{cd}}{\sqrt{-cd^2 + bde - ae^2}}\right)}{(cd^2 - bde + ae^2)\sqrt{-cd^2 + bde - ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x + f)^2/((c*x^2 + b*x + a)^(3/2)*(e*x + d)),x, algorithm="giac")
```

$$-2 * ((2 * c^3 * d^3 * f^2 - 2 * b * c^2 * d^3 * f * g + b^2 * c * d^3 * g^2 - 2 * a * c^2 * d^3 * g^2 - 3 * b * c^2 * d^2 * f^2 * e + 2 * b^2 * c * d^2 * f * g * e + 4 * a * c^2 * d^2 * f * g * e - b^3 * d^2 * g^2 * e + a * b * c * d^2 * g^2 * e + b^2 * c * d * f^2 * e^2 + 2 * a * c^2 * d * f^2 * e^2 - 6 * a * b * c * d * f * g * e^2 + 2 * a * b^2 * d * g^2 * e^2 - 2 * a^2 * c * d * g^2 * e^2 - 6 * a * b * c * d * f * g * e^2 + 2 * a * b^2 * d * g^2 * e^2 - 2 * a^2 * c * d * g^2 * e^2 - a * b * c * f^2 * e^3 + 4 * a^2 * c * f * g * e^3 - a^2 * b * g^2 * e^3) * x / (b^2 * c^2 * d^4 - 4 * a * c^3 * d^4 - 2 * b^3 * c * d^3 * e + 8 * a * b * c^2 * d^3 * e + b^4 * d^2 * e^2 - 2 * a * b^2 * c * d^2 * e^2 - 8 * a^2 * c^2 * d^2 * e^2 - 2 * a * b^3 * d * e^3 + 8 * a^2 * b * c * d * e^3 + a^2 * b^2 * e^4 - 4 * a^3 * c * e^4) + (b * c^2 * d^3 * f^2 - 4 * a * c^2 * d^3 * f * g + a * b * c * d^3 * g^2 - 2 * b^2 * c * d^2 * f^2 * e + 2 * a * c^2 * d^2 * f^2 * e + 6 * a * b * c * d^2 * f * g * e - a * b^2 * d^2 * g^2 * e - 2 * a^2 * c * d^2 * g^2 * e + b^3 * d * f^2 * e^2 - a * b * c * d * f^2 * e^2 - 2 * a * b^2 * d * f * g * e^2 - 4 * a^2 * c * d * f * g * e^2 + 3 * a^2 * b * d * g^2 * e^2 - a * b^2 * f^2 * e^3 + 2 * a^2 * c * f^2 * e^3 + 2 * a^2 * b * f * g * e^3 - 2 * a^3 * g^2 * e^3) / (b^2 * c^2 * d^4 - 4 * a * c^3 * d^4 - 2 * b^3 * c * d^3 * e + 8 * a * b * c^2 * d^3 * e + b^4 * d^2 * e^2 - 2 * a * b^2 * c * d^2 * e^2 - 8 * a^2 * c^2 * d^2 * e^2 - 2 * a * b^3 * d * e^3 + 8 * a^2 * b * c * d * e^3 + a^2 * b^2 * e^4 - 4 * a^3 * c * e^4) / sqrt(c * x^2 + b * x + a) + 2 * (d^2 * g^2 - 2 * d * f * g * e + f^2 * e^2) * arctan(-((sqrt(c) * x - sqrt(c * x^2 + b * x + a)) * e + sqrt(c) * d) / sqrt(-c * d^2 + b * d * e - a * e^2)) / ((c * d^2 - b * d * e + a * e^2) * sqrt(-c * d^2 + b * d * e - a * e^2))$$



$$3.881 \quad \int \frac{f+gx}{(d+ex)(a+bx+cx^2)^{3/2}} dx$$

**Optimal.** Leaf size=187

$$\frac{e(f-dg) \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ae^2-bde+cd^2)^{3/2}} - \frac{2(cx(2aeg-b(dg+ef)+2cdf)+abeg-2acd g+2acef+b^2(-e)f+bcdf)}{(b^2-4ac)\sqrt{a+bx+cx^2}(ae^2-bde+cd^2)}$$

[Out]  $(-2*(b*c*d*f - b^2*e*f + 2*a*c*e*f - 2*a*c*d*g + a*b*e*g + c*(2*c*d*f + 2*a*e*g - b*(e*f + d*g))*x))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*\text{Sqrt}[a + b*x + c*x^2]) + (e*(e*f - d*g)*\text{ArcTanh}[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*\text{Sqrt}[a + b*x + c*x^2]]))/(c*d^2 - b*d*e + a*e^2)^{(3/2)}$

**Rubi [A]** time = 0.326243, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$\frac{e(f-dg) \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ae^2-bde+cd^2)^{3/2}} - \frac{2(cx(2aeg-b(dg+ef)+2cdf)+abeg-2acd g+2acef+b^2(-e)f+bcdf)}{(b^2-4ac)\sqrt{a+bx+cx^2}(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(f + g*x)/((d + e*x)*(a + b*x + c*x^2)^{(3/2)}), x]$

[Out]  $(-2*(b*c*d*f - b^2*e*f + 2*a*c*e*f - 2*a*c*d*g + a*b*e*g + c*(2*c*d*f + 2*a*e*g - b*(e*f + d*g))*x))/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*\text{Sqrt}[a + b*x + c*x^2]) + (e*(e*f - d*g)*\text{ArcTanh}[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*\text{Sqrt}[a + b*x + c*x^2]]))/(c*d^2 - b*d*e + a*e^2)^{(3/2)}$

**Rubi in Sympy [A]** time = 55.5514, size = 180, normalized size = 0.96

$$\frac{e(dg-ef) \operatorname{atanh}\left(\frac{2ae-bd+x(be-2cd)}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ae^2-bde+cd^2)^{3/2}} + \frac{2(-ag(be-2cd)-cx(2aeg-bdg-bef+2cdf)+f(-2ace+b^2e-bcd))}{(-4ac+b^2)\sqrt{a+bx+cx^2}(ae^2-bde+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((g*x+f)/(e*x+d)/(c*x^2+b*x+a)^{(3/2)}, x)$

[Out]  $e*(d*g - e*f)*\operatorname{atanh}((2*a*e - b*d + x*(b*e - 2*c*d))/(2*\text{sqrt}(a + b*x + c*x^2)*\text{sqrt}(a*e^2 - b*d*e + c*d^2)))/(a*e^2 - b*d*e + c*d^2)^{(3/2)} + 2*(-a*g*(b*e - 2*c*d) - c*x*(2*a*e*g - b*d*g - b*e*f + 2*c*d*f) + f*(-2*a*c*e + b^2*e - b*c*d))/((-4*a*c + b^2)*\text{sqrt}(a + b*x + c*x^2)*(a*e^2 - b*d*e + c*d^2))$

**Mathematica [A]** time = 0.546335, size = 217, normalized size = 1.16

$$\frac{2b(aeg + cd(f - gx) - cefx) + 4c(-adg + ae(f + gx) + cdfx) - 2b^2ef}{(b^2 - 4ac)\sqrt{a + x(b + cx)}(e(bd - ae) - cd^2)} + \frac{e(ef - dg)\log(d + ex)}{(e(ae - bd) + cd^2)^{3/2}}$$

$$+ \frac{e(dg - ef)\log\left(2\sqrt{a + x(b + cx)}\sqrt{e(ae - bd) + cd^2} + 2ae - bd + bex - 2cdx\right)}{(e(ae - bd) + cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(f + g\*x)/((d + e\*x)\*(a + b\*x + c\*x^2)^(3/2)), x]

[Out]  $(-2*b^2*e*f + 2*b*(a*e*g - c*e*f*x + c*d*(f - g*x)) + 4*c*(-(a*d*g) + c*d*f*x + a*e*(f + g*x)))/((b^2 - 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*Sqrt[a + x*(b + c*x)]) + (e*(e*f - d*g)*Log[d + e*x])/(c*d^2 + e*(-(b*d) + a*e))^{3/2} + (e*(-(e*f) + d*g)*Log[-(b*d) + 2*a*e - 2*c*d*x + b*e*x + 2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + x*(b + c*x)])/(c*d^2 + e*(-(b*d) + a*e))^{3/2}$

**Maple [B]** time = 0.014, size = 1261, normalized size = 6.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)/(e\*x+d)/(c\*x^2+b\*x+a)^(3/2), x)

[Out]  $2/e*g*(2*c*x+b)/(4*a*c-b^2)/(c*x^2+b*x+a)^{(1/2)} - 1/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)} * d*g + e/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)} * f + 2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)} * x*b*c*d*g - 2*e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)} * x*b*c*f - 4/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)} * x*c^2*d^2*g + 4/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)} * x*c^2*d*f + 1/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)} * b^2*d*g - e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)} * b^2*f - 2/e/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)} * b*c*d^2*g + 2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)} * b*c*d*f + 1/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)} * ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x+d/e) * d*g - e/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)} * ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)}*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{(1/2)})/(x+d/e) * f$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)/((c\*x^2 + b\*x + a)^(3/2)\*(e\*x + d)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas** [A] time = 2.01024, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)/((c\*x^2 + b\*x + a)^(3/2)\*(e\*x + d)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/2*(4*\sqrt{c*d^2 - b*d*e + a*e^2})*\sqrt{c*x^2 + b*x + a}*((b*c*d - (b^2 - 2*a*c)*e)*f - (2*a*c*d - a*b*e)*g + ((2*c^2*d - b*c*e)*f - (b*c*d - 2*a*c*e)*g)*x) + ((a*b^2 - 4*a^2*c)*e^2*f - (a*b^2 - 4*a^2*c)*d*e*g + ((b^2*c - 4*a*c^2)*e^2*f - (b^2*c - 4*a*c^2)*d*e*g)*x^2 + ((b^3 - 4*a*b*c)*e^2*f - (b^3 - 4*a*b*c)*d*e*g)*x) * \log(((8*a*b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + (b^2 + 4*a*c)*e^2)*x^2 - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a*c)*d*e)*x)*\sqrt{c*d^2 - b*d*e + a*e^2}) + 4*(b*c*d^3 + 3*a*b*d*e^2 - 2*a^2*e^3 - (b^2 + 2*a*c)*d^2*e + (2*c^2*d^3 - 3*b*c*d^2*e - a*b*e^3 + (b^2 + 2*a*c)*d*e^2)*x)*\sqrt{c*x^2 + b*x + a})/(e^2*x^2 + 2*d*e*x + d^2))/(((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2 + ((b^2*c^2 - 4*a*c^3)*d^2 - (b^3*c - 4*a*b*c^2)*d*e + (a*b^2*c - 4*a^2*c^2)*e^2)*x^2 + ((b^3*c - 4*a*b*c^2)*d^2 - (b^4 - 4*a*b^2*c)*d*e + (a*b^3 - 4*a^2*b*c)*e^2)*x)*\sqrt{c*d^2 - b*d*e + a*e^2}), -(2*\sqrt{-c*d^2 + b*d*e - a*e^2})*\sqrt{c*x^2 + b*x + a}*((b*c*d - (b^2 - 2*a*c)*e)*f - (2*a*c*d - a*b*e)*g + ((2*c^2*d - b*c*e)*f - (b*c*d - 2*a*c*e)*g)*x) + ((a*b^2 - 4*a^2*c)*e^2*f - (a*b^2 - 4*a^2*c)*d*e*g + ((b^2*c - 4*a*c^2)*e^2*f - (b^2*c - 4*a*c^2)*d*e*g)*x^2 + ((b^3 - 4*a*b*c)*e^2*f - (b^3 - 4*a*b*c)*d*e*g)*x)*\arctan(-1/2*\sqrt{-c*d^2 + b*d*e - a*e^2}*(b*d - 2*a*e + (2*c*d - b*e)*x)/((c*d^2 - b*d*e + a*e^2)*\sqrt{c*x^2 + b*x + a})))/(((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2 + ((b^2*c^2 - 4*a*c^3)*d^2 - (b^3*c - 4*a*b*c^2)*d*e + (a*b^2*c - 4*a^2*c^2)*e^2)*x^2 + ((b^3*c - 4*a*b*c^2)*d^2 - (b^4 - 4*a*b^2*c)*d*e + (a*b^3 - 4*a^2*b*c)*e^2)*x)*\sqrt{-c*d^2 + b*d*e - a*e^2}]] \end{aligned}$$

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)/(e\*x+d)/(c\*x\*\*2+b\*x+a)\*\*(3/2),x)

[Out] Timed out

**GIAC/XCAS** [A] time = 0.282408, size = 767, normalized size = 4.1

$$\frac{2 \left( \frac{(2c^3d^3f - bc^2d^3g - 3bc^2d^2fe + b^2cd^2ge + 2ac^2d^2ge + b^2cdf e^2 + 2ac^2dfe^2 - 3abcdge^2 - abcfe^3 + 2a^2cge^3)x}{b^2c^2d^4 - 4ac^3d^4 - 2b^3cd^3e + 8abc^2d^3e + b^4d^2e^2 - 2ab^2cd^2e^2 - 8a^2c^2d^2e^2 - 2ab^3de^3 + 8a^2bcd e^3 + a^2b^2e^4 - 4a^3ce^4} + \frac{bc^2d^3f - 2ac^2d^3g - 2b^2cd^2fe + 2ac^2d^2ge + b^2cdf e^2 + 2ac^2dfe^2 - 3abcdge^2 - abcfe^3 + 2a^2cge^3}{b^2c^2d^4 - 4ac^3d^4 - 2b^3cd^3e + 8abc^2d^3e + b^4d^2e^2 - 2ab^2cd^2e^2 - 8a^2c^2d^2e^2 - 2ab^3de^3 + 8a^2bcd e^3 + a^2b^2e^4 - 4a^3ce^4} \right) \sqrt{cx^2 + bx + a}}{(cd^2 - bde + ae^2)\sqrt{-cd^2 + bde - ae^2}} \arctan \left( -\frac{(\sqrt{cx - \sqrt{cx^2 + bx + a}})e + \sqrt{cd}}{\sqrt{-cd^2 + bde - ae^2}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)/((c\*x^2 + b\*x + a)^(3/2)\*(e\*x + d)),x, algorithm="giac")

[Out] 
$$-2 \cdot \left( (2c^3d^3f - b^2c^2d^3g - 3b^2c^2d^2fe + b^2cd^2ge + 2ac^2d^2ge + b^2cdf^2e^2 + 2ac^2d^2fe^2 - 3ab^2cd^2ge^2 - ab^2cf^2e^3 + 2a^2c^2ge^3) \cdot x + (b^2c^2d^4 - 4ac^3d^4 - 2b^3cd^3e + 8ab^2cd^3e + b^4d^2e^2 - 2ab^2cd^2e^2 - 8a^2c^2d^2e^2 - 2ab^3d^2e^3 + 8a^2b^2cd^2e^3 + a^2b^2e^4 - 4a^3ce^4) + (b^2c^2d^3f - 2ac^2d^3g - 2b^2cd^2fe + 2ac^2d^2fe + 3ab^2cd^2ge + b^3d^2fe^2 - ab^2cdf^2e^2 - ab^2d^2ge^2 - 2a^2cd^2ge^2 - ab^2fe^3 + 2a^2cf^2e^3 + a^2b^2ge^3) \right) / (b^2c^2d^4 - 4ac^3d^4 - 2b^3cd^3e + 8ab^2cd^3e + b^4d^2e^2 - 2ab^2cd^2e^2 - 8a^2c^2d^2e^2 - 2ab^3d^2e^3 + 8a^2b^2cd^2e^3 + a^2b^2e^4 - 4a^3ce^4) / \sqrt{cx^2 + bx + a} - 2 \cdot (d^2ge - f^2e^2) \cdot \arctan\left(\frac{(\sqrt{c})x - \sqrt{cx^2 + bx + a}}{e + \sqrt{c}d} \cdot \frac{d}{\sqrt{-cd^2 + bde - ae^2}}\right) / ((cd^2 - bde + ae^2) \sqrt{-cd^2 + bde - ae^2})$$

$$3.882 \quad \int \frac{1}{(d+ex)(a+bx+cx^2)^{3/2}} dx$$

**Optimal.** Leaf size=155

$$\frac{e^2 \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ae^2-bde+cd^2)^{3/2}} - \frac{2(2ace+b^2(-e)+cx(2cd-be)+bcd)}{(b^2-4ac)\sqrt{a+bx+cx^2}(ae^2-bde+cd^2)}$$

[Out]  $(-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c) * (c*d^2 - b*d*e + a*e^2)*\text{Sqrt}[a + b*x + c*x^2]) + (e^2*\text{ArcTanh}[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*\text{Sqrt}[a + b*x + c*x^2]])/(c*d^2 - b*d*e + a*e^2)^{(3/2)}$

**Rubi [A]** time = 0.238787, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$

$$\frac{e^2 \tanh^{-1}\left(\frac{-2ae+x(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ae^2-bde+cd^2)^{3/2}} - \frac{2(2ace+b^2(-e)+cx(2cd-be)+bcd)}{(b^2-4ac)\sqrt{a+bx+cx^2}(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)\*(a + b\*x + c\*x^2)^(3/2)), x]

[Out]  $(-2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x))/((b^2 - 4*a*c) * (c*d^2 - b*d*e + a*e^2)*\text{Sqrt}[a + b*x + c*x^2]) + (e^2*\text{ArcTanh}[(b*d - 2*a*e + (2*c*d - b*e)*x)/(2*\text{Sqrt}[c*d^2 - b*d*e + a*e^2]*\text{Sqrt}[a + b*x + c*x^2]])/(c*d^2 - b*d*e + a*e^2)^{(3/2)}$

**Rubi in Sympy [A]** time = 45.9805, size = 146, normalized size = 0.94

$$-\frac{e^2 \operatorname{atanh}\left(\frac{2ae-bd+x(be-2cd)}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ae^2-bde+cd^2)^{3/2}} + \frac{2(-2ace+b^2e-bcd+cx(be-2cd))}{(-4ac+b^2)\sqrt{a+bx+cx^2}(ae^2-bde+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(e\*x+d)/(c\*x\*\*2+b\*x+a)\*\*(3/2), x)

[Out]  $-e^{**2}*\operatorname{atanh}((2*a*e - b*d + x*(b*e - 2*c*d))/(2*\text{sqrt}(a + b*x + c*x**2)*\text{sqrt}(a*e**2 - b*d*e + c*d**2)))/(a*e**2 - b*d*e + c*d**2)**(3/2) + 2*(-2*a*c*e + b**2*e - b*c*d + c*x*(b*e - 2*c*d))/((-4*a*c + b**2)*\text{sqrt}(a + b*x + c*x**2)*(a*e**2 - b*d*e + c*d**2))$

**Mathematica [A]** time = 0.699076, size = 207, normalized size = 1.34

$$\frac{e^2 (b^2 - 4ac) \sqrt{a + x(b + cx)} \log\left(2\sqrt{a + x(b + cx)}\sqrt{e(ae - bd) + cd^2} + 2ae - bd + bex - 2cdx\right) + 2\sqrt{e(ae - bd) + cd^2} (2c(ae - bd) + b^2)}{(4ac - b^2)\sqrt{a + x(b + cx)}(e(ae - bd) + cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x)\*(a + b\*x + c\*x^2)^(3/2)), x]

[Out]  $(2*\text{Sqrt}[c*d^2 + e*(-(b*d) + a*e)]*(-(b^2*e) + 2*c*(a*e + c*d*x) + b*c*(d - e*x)) - (b^2 - 4*a*c)*e^2*\text{Sqrt}[a + x*(b + c*x)]*\text{Log}[d +$

$$e^x] + (b^2 - 4ac) e^2 \sqrt{a + x(b + cx)} \operatorname{Log}[-(bd) + 2a e - 2c^2 dx + b^2 e^2 x + 2\sqrt{c^2 d^2 + e^2(-(bd) + a^2 e)}] \sqrt{a + x(b + cx)}] / ((-b^2 + 4ac)(c^2 d^2 + e^2(-(bd) + a^2 e))^{3/2}) \sqrt{a + x(b + cx)}$$

**Maple [B]** time = 0.009, size = 603, normalized size = 3.9

$$\begin{aligned} & \frac{e}{ae^2 - bde + cd^2} \frac{1}{\sqrt{\left(x + \frac{d}{e}\right)^2 c + \frac{be-2cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2-bde+cd^2}{e^2}}} \\ & - 2 \frac{bexc}{(ae^2 - bde + cd^2)(4ac - b^2)} \frac{1}{\sqrt{\left(x + \frac{d}{e}\right)^2 c + \frac{be-2cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2-bde+cd^2}{e^2}}} \\ & + 4 \frac{xc^2d}{(ae^2 - bde + cd^2)(4ac - b^2)} \frac{1}{\sqrt{\left(x + \frac{d}{e}\right)^2 c + \frac{be-2cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2-bde+cd^2}{e^2}}} \\ & - \frac{b^2e}{(ae^2 - bde + cd^2)(4ac - b^2)} \frac{1}{\sqrt{\left(x + \frac{d}{e}\right)^2 c + \frac{be-2cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2-bde+cd^2}{e^2}}} \\ & + 2 \frac{bcd}{(ae^2 - bde + cd^2)(4ac - b^2)} \frac{1}{\sqrt{\left(x + \frac{d}{e}\right)^2 c + \frac{be-2cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2-bde+cd^2}{e^2}}} \\ & - \frac{e}{ae^2 - bde + cd^2} \ln \left( 1 \left( 2 \frac{ae^2 - bde + cd^2}{e^2} + \frac{be - 2cd}{e} \left(x + \frac{d}{e}\right) + 2 \sqrt{\frac{ae^2 - bde + cd^2}{e^2}} \sqrt{\left(x + \frac{d}{e}\right)^2 c + \frac{be - 2cd}{e} \left(x + \frac{d}{e}\right) + \frac{ae^2 - bde + cd^2}{e^2}} \right) \right) \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x+d)/(c\*x^2+b\*x+a)^(3/2), x)

[Out]  $e/(a^2e^2-b^2d^2e+c^2d^2)/((x+d/e)^2c+(b^2e-2c^2d)/e^2(x+d/e)+(a^2e^2-b^2d^2e+c^2d^2)/e^2)^{1/2}-2e/(a^2e^2-b^2d^2e+c^2d^2)/(4a^2c-b^2)/((x+d/e)^2c+(b^2e-2c^2d)/e^2(x+d/e)+(a^2e^2-b^2d^2e+c^2d^2)/e^2)^{1/2}x^2bc+4/(a^2e^2-b^2d^2e+c^2d^2)/(4a^2c-b^2)/((x+d/e)^2c+(b^2e-2c^2d)/e^2(x+d/e)+(a^2e^2-b^2d^2e+c^2d^2)/e^2)^{1/2}x^2c^2d-e/(a^2e^2-b^2d^2e+c^2d^2)/(4a^2c-b^2)/((x+d/e)^2c+(b^2e-2c^2d)/e^2(x+d/e)+(a^2e^2-b^2d^2e+c^2d^2)/e^2)^{1/2}b^2+2/(a^2e^2-b^2d^2e+c^2d^2)/(4a^2c-b^2)/((x+d/e)^2c+(b^2e-2c^2d)/e^2(x+d/e)+(a^2e^2-b^2d^2e+c^2d^2)/e^2)^{1/2}b^2cd-e/(a^2e^2-b^2d^2e+c^2d^2)/((a^2e^2-b^2d^2e+c^2d^2)/e^2)^{1/2} \ln((2(a^2e^2-b^2d^2e+c^2d^2)/e^2+(b^2e-2c^2d)/e^2(x+d/e)+2((a^2e^2-b^2d^2e+c^2d^2)/e^2)^{1/2}((x+d/e)^2c+(b^2e-2c^2d)/e^2(x+d/e)+(a^2e^2-b^2d^2e+c^2d^2)/e^2)^{1/2})/(x+d/e))$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + b\*x + a)^(3/2)\*(e\*x + d)), x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.662652, size = 1, normalized size = 0.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + b\*x + a)^(3/2)\*(e\*x + d)),x, algorithm="fricas")

[Out] 
$$\begin{aligned} & [-1/2*(4*(b*c*d - (b^2 - 2*a*c)*e + (2*c^2*d - b*c*e)*x)*\sqrt{c*d} \\ & \wedge 2 - b*d*e + a*e^2)*\sqrt{c*x^2 + b*x + a} - ((b^2*c - 4*a*c^2)*e^2 \\ & *x^2 + (b^3 - 4*a*b*c)*e^2*x + (a*b^2 - 4*a^2*c)*e^2)*\log(((8*a* \\ & b*d*e - 8*a^2*e^2 - (b^2 + 4*a*c)*d^2 - (8*c^2*d^2 - 8*b*c*d*e + \\ & (b^2 + 4*a*c)*e^2)*x^2 - 2*(4*b*c*d^2 + 4*a*b*e^2 - (3*b^2 + 4*a* \\ & c)*d*e)*x)*\sqrt{c*d^2 - b*d*e + a*e^2} - 4*(b*c*d^3 + 3*a*b*d*e^2 \\ & - 2*a^2*e^3 - (b^2 + 2*a*c)*d^2*e + (2*c^2*d^3 - 3*b*c*d^2*e - a* \\ & b*e^3 + (b^2 + 2*a*c)*d*e^2)*x)*\sqrt{c*x^2 + b*x + a})/(e^2*x^2 \\ & + 2*d*e*x + d^2))/(((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b* \\ & c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2 + ((b^2*c^2 - 4*a*c^3)*d^2 - (b \\ & ^3*c - 4*a*b*c^2)*d*e + (a*b^2*c - 4*a^2*c^2)*e^2)*x^2 + ((b^3*c \\ & - 4*a*b*c^2)*d^2 - (b^4 - 4*a*b^2*c)*d*e + (a*b^3 - 4*a^2*b*c)*e^2 \\ & *x)*\sqrt{c*d^2 - b*d*e + a*e^2}), -(2*(b*c*d - (b^2 - 2*a*c)*e \\ & + (2*c^2*d - b*c*e)*x)*\sqrt{-c*d^2 + b*d*e - a*e^2}*\sqrt{c*x^2 + \\ & b*x + a} + ((b^2*c - 4*a*c^2)*e^2*x^2 + (b^3 - 4*a*b*c)*e^2*x + ( \\ & a*b^2 - 4*a^2*c)*e^2)*\arctan(-1/2*\sqrt{-c*d^2 + b*d*e - a*e^2}*(b \\ & *d - 2*a*e + (2*c*d - b*e)*x)/((c*d^2 - b*d*e + a*e^2)*\sqrt{c*x^2 \\ & + b*x + a}))/(((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)* \\ & d*e + (a^2*b^2 - 4*a^3*c)*e^2 + ((b^2*c^2 - 4*a*c^3)*d^2 - (b^3*c \\ & - 4*a*b*c^2)*d*e + (a*b^2*c - 4*a^2*c^2)*e^2)*x^2 + ((b^3*c - 4* \\ & a*b*c^2)*d^2 - (b^4 - 4*a*b^2*c)*d*e + (a*b^3 - 4*a^2*b*c)*e^2)*x \\ & )*\sqrt{-c*d^2 + b*d*e - a*e^2}]] \end{aligned}$$

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex)(a + bx + cx^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x+d)/(c\*x\*\*2+b\*x+a)\*\*(3/2),x)

[Out] Integral(1/((d + e\*x)\*(a + b\*x + c\*x\*\*2)\*\*(3/2)), x)

**GIAC/XCAS [A]** time = 0.280779, size = 603, normalized size = 3.89

$$\begin{aligned} & 2 \left( \frac{(2c^3d^3 - 3bc^2d^2e + b^2cde^2 + 2ac^2de^2 - abc^3)x}{b^2c^2d^4 - 4ac^3d^4 - 2b^3cd^3e + 8abc^2d^3e + b^4d^2e^2 - 2ab^2cd^2e^2 - 8a^2c^2d^2e^2 - 2ab^3de^3 + 8a^2bcde^3 + a^2b^2e^4 - 4a^3ce^4} + \frac{bc^2d^3 - 2b^2cd^3e + 8abc^2d^3e + b^4d^2e^2 - 2ab^2cd^2e^2 - 8a^2c^2d^2e^2 - 2ab^3de^3 + 8a^2bcde^3 + a^2b^2e^4 - 4a^3ce^4}{b^2c^2d^4 - 4ac^3d^4 - 2b^3cd^3e + 8abc^2d^3e + b^4d^2e^2 - 2ab^2cd^2e^2 - 8a^2c^2d^2e^2 - 2ab^3de^3 + 8a^2bcde^3 + a^2b^2e^4 - 4a^3ce^4} \right) \sqrt{cx^2 + bx + a} \\ & + \frac{2 \arctan \left( -\frac{(\sqrt{cx - \sqrt{cx^2 + bx + a}})e + \sqrt{cd}}{\sqrt{-cd^2 + bde - ae^2}} \right) e^2}{(cd^2 - bde + ae^2)\sqrt{-cd^2 + bde - ae^2}} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + b\*x + a)^(3/2)\*(e\*x + d)),x, algorithm="giac")

[Out] 
$$\begin{aligned} & -2*((2*c^3*d^3 - 3*b*c^2*d^2*e + b^2*c*d*e^2 + 2*a*c^2*d*e^2 - a* \\ & b*c*e^3)*x/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2* \\ & d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2* \\ & a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c \\ & ^2*d^3 - 2*b^2*c*d^2*e + 2*a*c^2*d^2*e + b^3*d*e^2 - a*b*c*d*e^2 \\ & - a*b^2*e^3 + 2*a^2*c*e^3)/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d \\ & ^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2* \\ & c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a \\ & ^3*c*e^4))/\sqrt{c*x^2 + b*x + a} + 2*\arctan(-((\sqrt{c})*x - \sqrt{c} \end{aligned}$$

$$\frac{(x^2 + bx + a)e + \sqrt{c}d/\sqrt{-cd^2 + bde - ae^2})e^2}{((cd^2 - bde + ae^2)\sqrt{-cd^2 + bde - ae^2})}$$



$$3.883 \quad \int \frac{1}{(d+ex)(f+gx)(a+bx+cx^2)^{3/2}} dx$$

**Optimal.** Leaf size=352

$$\begin{aligned} & -\frac{2e(2ace + b^2(-e) + cx(2cd - be) + bcd)}{(b^2 - 4ac)\sqrt{a + bx + cx^2}(ef - dg)(ae^2 - bde + cd^2)} \\ & + \frac{2g(2acg + b^2(-g) + cx(2cf - bg) + bcf)}{(b^2 - 4ac)\sqrt{a + bx + cx^2}(ef - dg)(ag^2 - bfg + cf^2)} \\ & + \frac{e^3 \tanh^{-1}\left(\frac{-2ae + x(2cd - be) + bd}{2\sqrt{a + bx + cx^2}\sqrt{ae^2 - bde + cd^2}}\right)}{(ef - dg)(ae^2 - bde + cd^2)^{3/2}} - \frac{g^3 \tanh^{-1}\left(\frac{-2ag + x(2cf - bg) + bf}{2\sqrt{a + bx + cx^2}\sqrt{ag^2 - bfg + cf^2}}\right)}{(ef - dg)(ag^2 - bfg + cf^2)^{3/2}} \end{aligned}$$

[Out]  $(-2 * e * (b * c * d - b^2 * e + 2 * a * c * e + c * (2 * c * d - b * e) * x)) / ((b^2 - 4 * a * c) * (c * d^2 - b * d * e + a * e^2) * (e * f - d * g) * \text{Sqrt}[a + b * x + c * x^2]) + (2 * g * (b * c * f - b^2 * g + 2 * a * c * g + c * (2 * c * f - b * g) * x)) / ((b^2 - 4 * a * c) * (e * f - d * g) * (c * f^2 - b * f * g + a * g^2) * \text{Sqrt}[a + b * x + c * x^2]) + (e^3 * \text{ArcTanh}[(b * d - 2 * a * e + (2 * c * d - b * e) * x) / (2 * \text{Sqrt}[c * d^2 - b * d * e + a * e^2] * \text{Sqrt}[a + b * x + c * x^2])]) / ((c * d^2 - b * d * e + a * e^2)^{(3/2)} * (e * f - d * g)) - (g^3 * \text{ArcTanh}[(b * f - 2 * a * g + (2 * c * f - b * g) * x) / (2 * \text{Sqrt}[c * f^2 - b * f * g + a * g^2] * \text{Sqrt}[a + b * x + c * x^2])]) / ((e * f - d * g) * (c * f^2 - b * f * g + a * g^2)^{(3/2)})$

**Rubi [A]** time = 1.01051, antiderivative size = 352, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$\begin{aligned} & -\frac{2e(2ace + b^2(-e) + cx(2cd - be) + bcd)}{(b^2 - 4ac)\sqrt{a + bx + cx^2}(ef - dg)(ae^2 - bde + cd^2)} \\ & + \frac{2g(2acg + b^2(-g) + cx(2cf - bg) + bcf)}{(b^2 - 4ac)\sqrt{a + bx + cx^2}(ef - dg)(ag^2 - bfg + cf^2)} \\ & + \frac{e^3 \tanh^{-1}\left(\frac{-2ae + x(2cd - be) + bd}{2\sqrt{a + bx + cx^2}\sqrt{ae^2 - bde + cd^2}}\right)}{(ef - dg)(ae^2 - bde + cd^2)^{3/2}} - \frac{g^3 \tanh^{-1}\left(\frac{-2ag + x(2cf - bg) + bf}{2\sqrt{a + bx + cx^2}\sqrt{ag^2 - bfg + cf^2}}\right)}{(ef - dg)(ag^2 - bfg + cf^2)^{3/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)\*(f + g\*x)\*(a + b\*x + c\*x^2)^(3/2)), x]

[Out]  $(-2 * e * (b * c * d - b^2 * e + 2 * a * c * e + c * (2 * c * d - b * e) * x)) / ((b^2 - 4 * a * c) * (c * d^2 - b * d * e + a * e^2) * (e * f - d * g) * \text{Sqrt}[a + b * x + c * x^2]) + (2 * g * (b * c * f - b^2 * g + 2 * a * c * g + c * (2 * c * f - b * g) * x)) / ((b^2 - 4 * a * c) * (e * f - d * g) * (c * f^2 - b * f * g + a * g^2) * \text{Sqrt}[a + b * x + c * x^2]) + (e^3 * \text{ArcTanh}[(b * d - 2 * a * e + (2 * c * d - b * e) * x) / (2 * \text{Sqrt}[c * d^2 - b * d * e + a * e^2] * \text{Sqrt}[a + b * x + c * x^2])]) / ((c * d^2 - b * d * e + a * e^2)^{(3/2)} * (e * f - d * g)) - (g^3 * \text{ArcTanh}[(b * f - 2 * a * g + (2 * c * f - b * g) * x) / (2 * \text{Sqrt}[c * f^2 - b * f * g + a * g^2] * \text{Sqrt}[a + b * x + c * x^2])]) / ((e * f - d * g) * (c * f^2 - b * f * g + a * g^2)^{(3/2)})$

**Rubi in Sympy [A]** time = 152.818, size = 318, normalized size = 0.9

$$\begin{aligned} & \frac{e^3 \operatorname{atanh}\left(\frac{2ae - bd + x(be - 2cd)}{2\sqrt{a + bx + cx^2}\sqrt{ae^2 - bde + cd^2}}\right)}{(dg - ef)(ae^2 - bde + cd^2)^{3/2}} - \frac{2e(-2ace + b^2e - bcd + cx(be - 2cd))}{(-4ac + b^2)(dg - ef)\sqrt{a + bx + cx^2}(ae^2 - bde + cd^2)} \\ & - \frac{g^3 \operatorname{atanh}\left(\frac{2ag - bf + x(bg - 2cf)}{2\sqrt{a + bx + cx^2}\sqrt{ag^2 - bfg + cf^2}}\right)}{(dg - ef)(ag^2 - bfg + cf^2)^{3/2}} + \frac{2g(-2acg + b^2g - bcf + cx(bg - 2cf))}{(-4ac + b^2)(dg - ef)\sqrt{a + bx + cx^2}(ag^2 - bfg + cf^2)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.



$$\frac{2/(d^*g-e^*f)^*e^{\wedge}2/(a^*e^{\wedge}2-b^*d^*e+c^*d^{\wedge}2)/(4^*a^*c-b^{\wedge}2)/((x+d/e)^{\wedge}2*c+(b^*e-2^*c^*d)/e^*(x+d/e)+(a^*e^{\wedge}2-b^*d^*e+c^*d^{\wedge}2)/e^{\wedge}2)^{\wedge}(1/2)*x^*b^*c-4/(d^*g-e^*f)^*e/(a^*e^{\wedge}2-b^*d^*e+c^*d^{\wedge}2)/(4^*a^*c-b^{\wedge}2)/((x+d/e)^{\wedge}2*c+(b^*e-2^*c^*d)/e^*(x+d/e)+(a^*e^{\wedge}2-b^*d^*e+c^*d^{\wedge}2)/e^{\wedge}2)^{\wedge}(1/2)*x^*c^{\wedge}2*d+1/(d^*g-e^*f)^*e^{\wedge}2/(a^*e^{\wedge}2-b^*d^*e+c^*d^{\wedge}2)/(4^*a^*c-b^{\wedge}2)/((x+d/e)^{\wedge}2*c+(b^*e-2^*c^*d)/e^*(x+d/e)+(a^*e^{\wedge}2-b^*d^*e+c^*d^{\wedge}2)/e^{\wedge}2)^{\wedge}(1/2)*b^{\wedge}2-2/(d^*g-e^*f)^*e/(a^*e^{\wedge}2-b^*d^*e+c^*d^{\wedge}2)/(4^*a^*c-b^{\wedge}2)/((x+d/e)^{\wedge}2*c+(b^*e-2^*c^*d)/e^*(x+d/e)+(a^*e^{\wedge}2-b^*d^*e+c^*d^{\wedge}2)/e^{\wedge}2)^{\wedge}(1/2)*b^*c*d+1/(d^*g-e^*f)/(a^*e^{\wedge}2-b^*d^*e+c^*d^{\wedge}2)^*e^{\wedge}2/((a^*e^{\wedge}2-b^*d^*e+c^*d^{\wedge}2)/e^{\wedge}2)^{\wedge}(1/2)*\ln((2^*(a^*e^{\wedge}2-b^*d^*e+c^*d^{\wedge}2)/e^{\wedge}2+(b^*e-2^*c^*d)/e^*(x+d/e)+2^*((a^*e^{\wedge}2-b^*d^*e+c^*d^{\wedge}2)/e^{\wedge}2)^{\wedge}(1/2)*((x+d/e)^{\wedge}2*c+(b^*e-2^*c^*d)/e^*(x+d/e)+(a^*e^{\wedge}2-b^*d^*e+c^*d^{\wedge}2)/e^{\wedge}2)^{\wedge}(1/2)))/(x+d/e)}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx + a)^{\frac{3}{2}}(ex + d)(gx + f)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + b\*x + a)^(3/2)\*(e\*x + d)\*(g\*x + f)),x, algorithm="maxima")

[Out] integrate(1/((c\*x^2 + b\*x + a)^(3/2)\*(e\*x + d)\*(g\*x + f)), x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + b\*x + a)^(3/2)\*(e\*x + d)\*(g\*x + f)),x, algorithm="fricas")

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex)(f + gx)(a + bx + cx^2)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x+d)/(g\*x+f)/(c\*x\*\*2+b\*x+a)\*\*(3/2),x)

[Out] Integral(1/((d + e\*x)\*(f + g\*x)\*(a + b\*x + c\*x\*\*2)\*\*(3/2)), x)

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/((c\*x^2 + b\*x + a)^(3/2)\*(e\*x + d)\*(g\*x + f)),x, algorithm="giac")

[Out] Exception raised: TypeError

$$3.884 \quad \int \frac{1}{(d+ex)(f+gx)^2(a+bx+cx^2)^{3/2}} dx$$

**Optimal.** Leaf size=642

$$\begin{aligned} & \frac{g^2 \sqrt{a+bx+cx^2} (-4cg(2ag+bf) + 3b^2g^2 + 4c^2f^2)}{(b^2-4ac)(f+gx)(ef-dg)(ag^2-bfg+cf^2)^2} \\ & - \frac{2e^2(2ace+b^2(-e)+cx(2cd-be)+bcd)}{(b^2-4ac)\sqrt{a+bx+cx^2}(ef-dg)^2(ae^2-bde+cd^2)} \\ & + \frac{2eg(2acg+b^2(-g)+cx(2cf-bg)+bcf)}{(b^2-4ac)\sqrt{a+bx+cx^2}(ef-dg)^2(ag^2-bfg+cf^2)} \\ & + \frac{2g(2acg+b^2(-g)+cx(2cf-bg)+bcf)}{(b^2-4ac)(f+gx)\sqrt{a+bx+cx^2}(ef-dg)(ag^2-bfg+cf^2)} \\ & + \frac{e^4 \tanh^{-1}\left(\frac{-2ae+cx(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^2(ae^2-bde+cd^2)^{3/2}} - \frac{eg^3 \tanh^{-1}\left(\frac{-2ag+cx(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)^2(ag^2-bfg+cf^2)^{3/2}} \\ & - \frac{3g^3(2cf-bg) \tanh^{-1}\left(\frac{-2ag+cx(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2(ef-dg)(ag^2-bfg+cf^2)^{5/2}} \end{aligned}$$

[Out]  $(-2^*e^2*(b^*c*d - b^2*e + 2^*a^*c*e + c*(2^*c*d - b^*e)*x))/((b^2 - 4^*a^*c)^*(c^*d^2 - b^*d*e + a^*e^2)*(e^*f - d^*g)^2*\text{Sqrt}[a + b^*x + c^*x^2])$   
 $+ (2^*e^*g*(b^*c*f - b^2*g + 2^*a^*c*g + c*(2^*c*f - b^*g)*x))/((b^2 - 4^*a^*c)^*(e^*f - d^*g)^2*(c^*f^2 - b^*f*g + a^*g^2)*\text{Sqrt}[a + b^*x + c^*x^2])$   
 $+ (2^*g*(b^*c*f - b^2*g + 2^*a^*c*g + c*(2^*c*f - b^*g)*x))/((b^2 - 4^*a^*c)^*(e^*f - d^*g)*(c^*f^2 - b^*f*g + a^*g^2)*(f + g^*x)*\text{Sqrt}[a + b^*x + c^*x^2])$   
 $+ (g^2*(4^*c^2*f^2 + 3^*b^2*g^2 - 4^*c^*g*(b^*f + 2^*a^*g))*\text{Sqrt}[a + b^*x + c^*x^2])/((b^2 - 4^*a^*c)^*(e^*f - d^*g)*(c^*f^2 - b^*f*g + a^*g^2)^2*(f + g^*x))$   
 $+ (e^4*\text{ArcTanh}[(b^*d - 2^*a^*e + (2^*c*d - b^*e)*x)/(2^*\text{Sqrt}[c^*d^2 - b^*d*e + a^*e^2]*\text{Sqrt}[a + b^*x + c^*x^2]])/((c^*d^2 - b^*d*e + a^*e^2)^(3/2)*(e^*f - d^*g)^2 - (3^*g^3*(2^*c*f - b^*g)*\text{ArcTanh}[(b^*f - 2^*a^*g + (2^*c*f - b^*g)*x)/(2^*\text{Sqrt}[c^*f^2 - b^*f*g + a^*g^2]*\text{Sqrt}[a + b^*x + c^*x^2]])/(2^*(e^*f - d^*g)*(c^*f^2 - b^*f*g + a^*g^2)^(5/2)) - (e^*g^3*\text{ArcTanh}[(b^*f - 2^*a^*g + (2^*c*f - b^*g)*x)/(2^*\text{Sqrt}[c^*f^2 - b^*f*g + a^*g^2]*\text{Sqrt}[a + b^*x + c^*x^2]])/((e^*f - d^*g)^2*(c^*f^2 - b^*f*g + a^*g^2)^(3/2))$

**Rubi [A]** time = 2.02871, antiderivative size = 642, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$

$$\begin{aligned} & \frac{g^2 \sqrt{a+bx+cx^2} (-4cg(2ag+bf) + 3b^2g^2 + 4c^2f^2)}{(b^2-4ac)(f+gx)(ef-dg)(ag^2-bfg+cf^2)^2} \\ & - \frac{2e^2(2ace+b^2(-e)+cx(2cd-be)+bcd)}{(b^2-4ac)\sqrt{a+bx+cx^2}(ef-dg)^2(ae^2-bde+cd^2)} \\ & + \frac{2eg(2acg+b^2(-g)+cx(2cf-bg)+bcf)}{(b^2-4ac)\sqrt{a+bx+cx^2}(ef-dg)^2(ag^2-bfg+cf^2)} \\ & + \frac{2g(2acg+b^2(-g)+cx(2cf-bg)+bcf)}{(b^2-4ac)(f+gx)\sqrt{a+bx+cx^2}(ef-dg)(ag^2-bfg+cf^2)} \\ & + \frac{e^4 \tanh^{-1}\left(\frac{-2ae+cx(2cd-be)+bd}{2\sqrt{a+bx+cx^2}\sqrt{ae^2-bde+cd^2}}\right)}{(ef-dg)^2(ae^2-bde+cd^2)^{3/2}} - \frac{eg^3 \tanh^{-1}\left(\frac{-2ag+cx(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{(ef-dg)^2(ag^2-bfg+cf^2)^{3/2}} \\ & - \frac{3g^3(2cf-bg) \tanh^{-1}\left(\frac{-2ag+cx(2cf-bg)+bf}{2\sqrt{a+bx+cx^2}\sqrt{ag^2-bfg+cf^2}}\right)}{2(ef-dg)(ag^2-bfg+cf^2)^{5/2}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)\*(f + g\*x)^2\*(a + b\*x + c\*x^2)^(3/2)),x]

[Out] 
$$\frac{(-2e^2(bcd - b^2e + 2ace + c(2cd - be)x))/((b^2 - 4ac)^2(c^2d^2 - b^2d^2e + a^2e^2)(ef - dg)^2\sqrt{a + bx + cx^2}) + (2e^2g(bcf - b^2g + 2acg + c(2cf - bg)x))/((b^2 - 4ac)^2(ef - dg)^2(c^2f^2 - b^2fg + a^2g^2)\sqrt{a + bx + cx^2}) + (2g^2(bcf - b^2g + 2acg + c(2cf - bg)x))/((b^2 - 4ac)^2(ef - dg)(c^2f^2 - b^2fg + a^2g^2)(f + gx)\sqrt{a + bx + cx^2}) + (g^2(4c^2f^2 + 3b^2g^2 - 4c^2g(bf + 2ag))\sqrt{a + bx + cx^2})/((b^2 - 4ac)^2(ef - dg)(c^2f^2 - b^2fg + a^2g^2)^2(f + gx)) + (e^4\text{ArcTanh}[(bd - 2ae + (2cd - be)x)/(2\sqrt{c^2d^2 - b^2d^2e + a^2e^2}\sqrt{a + bx + cx^2})])/(c^2d^2 - b^2d^2e + a^2e^2)^{3/2}(ef - dg)^2 - (3g^3(2cf - bg)\text{ArcTanh}[(bf - 2ag + (2cf - bg)x)/(2\sqrt{c^2f^2 - b^2fg + a^2g^2}\sqrt{a + bx + cx^2})])/(2(ef - dg)(c^2f^2 - b^2fg + a^2g^2)^{5/2}) - (e^3g^3\text{ArcTanh}[(bf - 2ag + (2cf - bg)x)/(2\sqrt{c^2f^2 - b^2fg + a^2g^2}\sqrt{a + bx + cx^2})])/(2(ef - dg)^2(c^2f^2 - b^2fg + a^2g^2)^{3/2})$$

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(e\*x+d)/(g\*x+f)\*\*2/(c\*x\*\*2+b\*x+a)\*\*(3/2),x)

[Out] Timed out

**Mathematica [A]** time = 9.43007, size = 575, normalized size = 0.9

$$\frac{\sqrt{a + x(b + cx)} \left( \frac{g^4}{(f + gx)(ef - dg)} - \frac{2(2c^2(a^2eg^2 + ac(dg(gx - 2f) - ef(f - 2gx)) - c^2df^2x) + b^2c(-4aeg^2 + cdg(2f - gx) + cef(f - 2gx)) + bc^2(3ag(dg + 2ef - eg^2) - b^2d^2e + a^2e^2))}{(b^2 - 4ac)(a + x(b + cx))(e(bd - ae) - cd^2)} \right)}{(g(ag - bf) + cf^2)^2} + \frac{e^4 \log(d + ex)}{(ef - dg)^2 (e(ae - bd) + cd^2)^{3/2}} - \frac{e^4 \log\left(2\sqrt{a + x(b + cx)}\sqrt{e(ae - bd) + cd^2} + 2ae - bd + bex - 2cdx\right)}{(ef - dg)^2 (e(ae - bd) + cd^2)^{3/2}} - \frac{g^3 \log(f + gx)(g(2aeg + 3bdg - 5bef) + 2cf(4ef - 3dg))}{2(ef - dg)^2 (g(ag - bf) + cf^2)^{5/2}} + \frac{g^3(g(2aeg + 3bdg - 5bef) + 2cf(4ef - 3dg)) \log\left(2\sqrt{a + x(b + cx)}\sqrt{g(ag - bf) + cf^2} + 2ag - bf + bgx - 2cfx\right)}{2(ef - dg)^2 (g(ag - bf) + cf^2)^{5/2}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[1/((d + e\*x)\*(f + g\*x)^2\*(a + b\*x + c\*x^2)^(3/2)),x]

[Out] 
$$\frac{(\sqrt{a + x(b + cx)})^2 (g^4 / ((ef - dg)(f + gx)) - (2(b^4 e^2 g^2 + b^3 c^2 g^2 (-2ef - dg + e^2 gx) + b^2 c^2 (-4ae^2 g^2 + ce^2 f^2 (f - 2gx) + c^2 d^2 g^2 (2f - gx)) + b^2 c^2 (c^2 f^2 (-df) + e^2 f^2 x + 2d^2 g^2 x) + 3a^2 g^2 (2ef + dg - e^2 gx) + 2c^2 (a^2 e^2 g^2 - c^2 d^2 f^2 x + ac^2 (-ef^2 (f - 2gx)) + d^2 g^2 (-2f + gx)))))) / ((b^2 - 4ac)^2 (-c^2 d^2 + e^2 (bd - ae)) (a + x(b + cx))) / (c^2 f^2 + g^2 (-bf + ag))^2 + (e^4 \text{Log}[d + ex]) / ((c^2 d^2 + e^2 (-bd + ae))^{3/2} (ef - dg)^2) - (g^3 (2c^2 f^2 (4e^2 f - 3d^2 g) + g^2 (-5b^2 e^2 f + 3b^2 d^2 g + 2a^2 e^2 g)) \text{Log}[f + gx]) / (2(ef - dg)^2 (c^2 f^2 + g^2 (-bf + ag))^2) - (e^4 \text{Log}[-(bd) + 2ae - 2cdx + b^2 e^2 x + 2\sqrt{c^2 d^2 + e^2 (-bd + ae)} \sqrt{a + x(b + cx)}]) / ((c^2 d^2 + e^2 (-bd + ae))^{3/2} (ef - dg)^2) + (g^3 (2c^2 f^2 (4e^2 f - 3d^2 g) + g^2 (-5b^2 e^2 f + 3b^2 d^2 g + 2a^2 e^2 g)) \text{Log}[-(bf) + 2ag]) / (2(ef - dg)^2 (g(ag - bf) + cf^2)^{5/2})$$

$$- 2*c*f*x + b*g*x + 2*\text{Sqrt}[c*f^2 + g*(-(b*f) + a*g)]*\text{Sqrt}[a + x*(b + c*x)]/(2*(e*f - d*g)^2*(c*f^2 + g*(-(b*f) + a*g))^(5/2))$$

**Maple [B]** time = 0.034, size = 2807, normalized size = 4.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}(1/(e*x+d)/(g*x+f)^2/(c*x^2+b*x+a)^(3/2), x)$

[Out] 
$$\begin{aligned} & -3*g^2/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/((a*g^2-b*f*g+c*f^2)/g^2)^{1/2} \\ & * \ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^{1/2}*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{1/2})/(x+f/g) \\ & * c*f-8*g/(d*g-e*f)*c^2/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{1/2} \\ & * x-4*g/(d*g-e*f)*c/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{1/2} \\ & * b+1/(d*g-e*f)^2*e^3/(a*e^2-b*d*e+c*d^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{1/2} \\ & +4/(d*g-e*f)^2*e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{1/2} \\ & * x*c^2*d+2/(d*g-e*f)^2*e^2/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{1/2} \\ & * b*c*d+3*g^3/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{1/2} \\ & * x*b^2*c+12*g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{1/2} \\ & * x*c^3*f^2-6*g^2/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{1/2} \\ & * b^2*c*f+6*g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{1/2} \\ & * b*c^2*f^2-g/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)/(x+f/g)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{1/2} \\ & -1/(d*g-e*f)^2*e^3/(a*e^2-b*d*e+c*d^2)/((a*e^2-b*d*e+c*d^2)/e^2)^{1/2} \\ & * \ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^{1/2}*((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{1/2})/(x+d/e) \\ & +2/(d*g-e*f)^2*e*g^2/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{1/2} \\ & * x*b*c-4/(d*g-e*f)^2*e*g/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{1/2} \\ & * x*c^2*f-2/(d*g-e*f)^2*e*g/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{1/2} \\ & * b*c^2*f+1/(d*g-e*f)^2*e*g^2/(a*g^2-b*f*g+c*f^2)/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{1/2} \\ & * b^2-2/(d*g-e*f)^2*e^3/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{1/2} \\ & * x*b*c-3/2*g^3/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{1/2} \\ & * b-1/(d*g-e*f)^2*e/(a*g^2-b*f*g+c*f^2)*g^2/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{1/2} \\ & -1/(d*g-e*f)^2*e^3/(a*e^2-b*d*e+c*d^2)/(4*a*c-b^2)/((x+d/e)^2*c+(b*e-2*c*d)/e*(x+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^{1/2} \\ & * b^2+3/2*g^3/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/(4*a*c-b^2)/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{1/2} \\ & * b^3+3/2*g^3/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/((a*g^2-b*f*g+c*f^2)/g^2)^{1/2} \\ & * \ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^{1/2}*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{1/2})/(x+f/g) \\ & * b+3*g^2/(d*g-e*f)/(a*g^2-b*f*g+c*f^2)^2/((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{1/2} \\ & * c*f+1/(d*g-e*f)^2*e/(a*g^2-b*f*g+c*f^2)*g^2/((a*g^2-b*f*g+c*f^2)/g^2)^{1/2} \\ & * \ln((2*(a*g^2-b*f*g+c*f^2)/g^2+(b*g-2*c*f)/g*(x+f/g)+2*((a*g^2-b*f*g+c*f^2)/g^2)^{1/2}*((x+f/g)^2*c+(b*g-2*c*f)/g*(x+f/g)+(a*g^2-b*f*g+c*f^2)/g^2)^{1/2})/(x+f/g) \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx + a)^{\frac{3}{2}}(ex + d)(gx + f)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)*(g*x + f)^2),x, algorithm="maxima")`

[Out] `integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)*(g*x + f)^2), x)`

---

**Fricas [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)*(g*x + f)^2),x, algorithm="fricas")`

[Out] Exception raised: TypeError

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x+d)/(g*x+f)**2/(c*x**2+b*x+a)**(3/2),x)`

[Out] Timed out

---

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)*(g*x + f)^2),x, algorithm="giac")`

[Out] Exception raised: TypeError

$$3.885 \quad \int \frac{1}{(d+ex)(f+gx)^3(a+bx+cx^2)^{3/2}} dx$$

**Optimal.** Leaf size=1064

$$\begin{aligned} & \frac{\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\sqrt{cx^2+bx+a}}\right) e^5}{(cd^2-bed+ae^2)^{3/2}(ef-dg)^3} \\ & - \frac{2(-eb^2+cdb+2ace+c(2cd-be)x) e^3}{(b^2-4ac)(cd^2-bed+ae^2)(ef-dg)^3\sqrt{cx^2+bx+a}} \\ & - \frac{g^3 \tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right) e^2}{(ef-dg)^3(cf^2-bgf+ag^2)^{3/2}} \\ & + \frac{2g(-gb^2+cfb+2acg+c(2cf-bg)x) e^2}{(b^2-4ac)(ef-dg)^3(cf^2-bgf+ag^2)\sqrt{cx^2+bx+a}} \\ & - \frac{3g^3(2cf-bg) \tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right) e}{2(ef-dg)^2(cf^2-bgf+ag^2)^{5/2}} \\ & + \frac{g^2(4c^2f^2+3b^2g^2-4cg(bf+2ag))\sqrt{cx^2+bx+ae}}{(b^2-4ac)(ef-dg)^2(cf^2-bgf+ag^2)^2(f+gx)} \\ & + \frac{2g(-gb^2+cfb+2acg+c(2cf-bg)x) e}{(b^2-4ac)(ef-dg)^2(cf^2-bgf+ag^2)(f+gx)\sqrt{cx^2+bx+a}} \\ & - \frac{3g^3(16c^2f^2+5b^2g^2-4cg(4bf+ag)) \tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)}{8(ef-dg)(cf^2-bgf+ag^2)^{7/2}} \\ & + \frac{g^2(2cf-bg)(8c^2f^2+15b^2g^2-4cg(2bf+13ag))\sqrt{cx^2+bx+a}}{4(b^2-4ac)(ef-dg)(cf^2-bgf+ag^2)^3(f+gx)} \\ & + \frac{g^2(8c^2f^2+5b^2g^2-4cg(2bf+3ag))\sqrt{cx^2+bx+a}}{2(b^2-4ac)(ef-dg)(cf^2-bgf+ag^2)^2(f+gx)^2} \\ & + \frac{2g(-gb^2+cfb+2acg+c(2cf-bg)x)}{(b^2-4ac)(ef-dg)(cf^2-bgf+ag^2)(f+gx)^2\sqrt{cx^2+bx+a}} \end{aligned}$$

[Out]  $(-2^*e^3*(b^*c*d - b^2*e + 2^*a*c*e + c*(2^*c*d - b^*e)*x))/((b^2 - 4^*a^*c)^*(c^*d^2 - b^*d*e + a^*e^2)^*(e^*f - d^*g)^3*\text{Sqrt}[a + b^*x + c^*x^2])$   
 $+ (2^*e^2*g*(b^*c*f - b^2*g + 2^*a*c*g + c*(2^*c*f - b^*g)*x))/((b^2 - 4^*a^*c)^*(e^*f - d^*g)^3*(c^*f^2 - b^*f*g + a^*g^2)*\text{Sqrt}[a + b^*x + c^*x^2])$   
 $+ (2^*g*(b^*c*f - b^2*g + 2^*a*c*g + c*(2^*c*f - b^*g)*x))/((b^2 - 4^*a^*c)^*(e^*f - d^*g)^2*(c^*f^2 - b^*f*g + a^*g^2)*(f + g^*x)^2*\text{Sqrt}[a + b^*x + c^*x^2])$   
 $+ (2^*e*g*(b^*c*f - b^2*g + 2^*a*c*g + c*(2^*c*f - b^*g)*x))/((b^2 - 4^*a^*c)^*(e^*f - d^*g)^2*(c^*f^2 - b^*f*g + a^*g^2)*(f + g^*x)*\text{Sqrt}[a + b^*x + c^*x^2])$   
 $+ (g^2*(8^*c^2*f^2 + 5^*b^2*g^2 - 4^*c*g*(2^*b*f + 13^*a*g))*\text{Sqrt}[a + b^*x + c^*x^2])/((b^2 - 4^*a^*c)^*(e^*f - d^*g)^2*(c^*f^2 - b^*f*g + a^*g^2)^2*(f + g^*x)^2)$   
 $+ (2^*g*(b^*c*f - b^2*g + 2^*a*c*g + c*(2^*c*f - b^*g)*x))/((b^2 - 4^*a^*c)^*(e^*f - d^*g)^2*(c^*f^2 - b^*f*g + a^*g^2)*(f + g^*x)^2*\text{Sqrt}[a + b^*x + c^*x^2])$   
 $+ (3^*g^3*(16^*c^2*f^2 + 5^*b^2*g^2 - 4^*c*g*(4^*b*f + a^*g))*\text{Sqrt}[a + b^*x + c^*x^2])/((b^2 - 4^*a^*c)^*(e^*f - d^*g)^2*(c^*f^2 - b^*f*g + a^*g^2)^{7/2})$   
 $+ (g^2*(2^*c*f - b^*g)*(8^*c^2*f^2 + 15^*b^2*g^2 - 4^*c*g*(2^*b*f + 13^*a*g))*\text{Sqrt}[a + b^*x + c^*x^2])/((b^2 - 4^*a^*c)^*(e^*f - d^*g)^3*(c^*f^2 - b^*f*g + a^*g^2)^3*(f + g^*x))$   
 $+ (g^2*(8^*c^2*f^2 + 5^*b^2*g^2 - 4^*c*g*(2^*b*f + 3^*a*g))*\text{Sqrt}[a + b^*x + c^*x^2])/((b^2 - 4^*a^*c)^*(e^*f - d^*g)^2*(c^*f^2 - b^*f*g + a^*g^2)^2*(f + g^*x)^2)$   
 $+ (2^*g*(b^*c*f - b^2*g + 2^*a*c*g + c*(2^*c*f - b^*g)*x))/((b^2 - 4^*a^*c)^*(e^*f - d^*g)^2*(c^*f^2 - b^*f*g + a^*g^2)*(f + g^*x)^2*\text{Sqrt}[a + b^*x + c^*x^2])$



**Rubi [A]** time = 5.06606, antiderivative size = 1064, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 7, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$

$$\frac{\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x}{2\sqrt{cd^2-bed+ae^2}\sqrt{cx^2+bx+a}}\right) e^5}{(cd^2 - bed + ae^2)^{3/2} (ef - dg)^3} - \frac{2(-eb^2 + cdb + 2ace + c(2cd - be)x) e^3}{(b^2 - 4ac)(cd^2 - bed + ae^2)(ef - dg)^3 \sqrt{cx^2 + bx + a}}$$

$$- \frac{g^3 \tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right) e^2}{(ef - dg)^3 (cf^2 - bgf + ag^2)^{3/2}} + \frac{2g(-gb^2 + cfb + 2acg + c(2cf - bg)x) e^2}{(b^2 - 4ac)(ef - dg)^3 (cf^2 - bgf + ag^2) \sqrt{cx^2 + bx + a}}$$

$$- \frac{3g^3(2cf - bg) \tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right) e}{2(ef - dg)^2 (cf^2 - bgf + ag^2)^{5/2}} + \frac{g^2(4c^2f^2 + 3b^2g^2 - 4cg(bf + 2ag)) \sqrt{cx^2 + bx + a}}{(b^2 - 4ac)(ef - dg)^2 (cf^2 - bgf + ag^2)^2 (f + gx)}$$

$$+ \frac{2g(-gb^2 + cfb + 2acg + c(2cf - bg)x) e}{(b^2 - 4ac)(ef - dg)^2 (cf^2 - bgf + ag^2) (f + gx) \sqrt{cx^2 + bx + a}} - \frac{3g^3(16c^2f^2 + 5b^2g^2 - 4cg(4bf + ag)) \tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right)}{8(ef - dg)(cf^2 - bgf + ag^2)^{7/2}}$$

$$+ \frac{g^2(2cf - bg)(8c^2f^2 + 15b^2g^2 - 4cg(2bf + 13ag)) \sqrt{cx^2 + bx + a}}{4(b^2 - 4ac)(ef - dg)(cf^2 - bgf + ag^2)^3 (f + gx)}$$

$$+ \frac{g^2(8c^2f^2 + 5b^2g^2 - 4cg(2bf + 3ag)) \sqrt{cx^2 + bx + a}}{2(b^2 - 4ac)(ef - dg)(cf^2 - bgf + ag^2)^2 (f + gx)^2}$$

$$+ \frac{2g(-gb^2 + cfb + 2acg + c(2cf - bg)x)}{(b^2 - 4ac)(ef - dg)(cf^2 - bgf + ag^2) (f + gx)^2 \sqrt{cx^2 + bx + a}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)\*(f + g\*x)^3\*(a + b\*x + c\*x^2)^(3/2)),x]

[Out]  $(-2e^3(bcd - b^2e + 2ace + c(2cd - be)x))/((b^2 - 4ac)^3(c^2d^2 - b^2d^2e + a^2e^2)(ef - dg)^3\sqrt{a + bx + cx^2})$   
 $+ (2e^2g(bcf - b^2g + 2acg + c(2cf - bg)x))/((b^2 - 4ac)^3(ef - dg)^3(c^2f^2 - b^2f^2g + ag^2)\sqrt{a + bx + cx^2})$   
 $+ (2g(bcf - b^2g + 2acg + c(2cf - bg)x))/((b^2 - 4ac)^3(ef - dg)(c^2f^2 - b^2f^2g + ag^2)(f + gx)^2\sqrt{a + bx + cx^2})$   
 $+ (2e^2g(bcf - b^2g + 2acg + c(2cf - bg)x))/((b^2 - 4ac)^3(ef - dg)^2(c^2f^2 - b^2f^2g + ag^2)(f + gx)\sqrt{a + bx + cx^2})$   
 $+ (g^2(8c^2f^2 + 15b^2g^2 - 4cg(2bf + 13ag))\sqrt{a + bx + cx^2})/((b^2 - 4ac)^3(ef - dg)(c^2f^2 - b^2f^2g + ag^2)^3(f + gx))$   
 $+ (e^2g^2(4c^2f^2 + 5b^2g^2 - 4cg(2bf + 3ag))\sqrt{a + bx + cx^2})/((b^2 - 4ac)^2(ef - dg)(cf^2 - bgf + ag^2)^2(f + gx)^2)$   
 $+ (2g(-gb^2 + cfb + 2acg + c(2cf - bg)x))/((b^2 - 4ac)(ef - dg)(cf^2 - bgf + ag^2)(f + gx)^2\sqrt{a + bx + cx^2})$   
 $+ (3e^3(16c^2f^2 + 5b^2g^2 - 4cg(4bf + ag))\tanh^{-1}\left(\frac{bf-2ag+(2cf-bg)x}{2\sqrt{cf^2-bgf+ag^2}\sqrt{cx^2+bx+a}}\right))/((b^2 - 4ac)(ef - dg)(cf^2 - bgf + ag^2)^{7/2})$   
 $+ (g^2(2cf - bg)(8c^2f^2 + 15b^2g^2 - 4cg(2bf + 13ag))\sqrt{a + bx + cx^2})/((b^2 - 4ac)(ef - dg)(cf^2 - bgf + ag^2)^3(f + gx))$   
 $+ (g^2(8c^2f^2 + 5b^2g^2 - 4cg(2bf + 3ag))\sqrt{a + bx + cx^2})/((b^2 - 4ac)(ef - dg)(cf^2 - bgf + ag^2)^2(f + gx)^2)$   
 $+ (2g(-gb^2 + cfb + 2acg + c(2cf - bg)x))/((b^2 - 4ac)(ef - dg)(cf^2 - bgf + ag^2)(f + gx)^2\sqrt{a + bx + cx^2})$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(e*x+d)/(g*x+f)**3/(c*x**2+b*x+a)**(3/2),x)`

[Out] Timed out

**Mathematica [A]** time = 14.4658, size = 1288, normalized size = 1.21

$$\frac{(cx^2 + bx + a)^{3/2} \log(d + ex)e^5}{(cd^2 - bed + ae^2)^{3/2} (ef - dg)^3 (a + x(b + cx))^{3/2}}$$

$$\frac{(cx^2 + bx + a)^{3/2} \log\left(-bd - 2cxd + 2ae + bex + 2\sqrt{cd^2 - bed + ae^2}\sqrt{cx^2 + bx + a}\right) e^5}{(cd^2 - bed + ae^2)^{3/2} (ef - dg)^3 (a + x(b + cx))^{3/2}}$$

$$+ \frac{(cx^2 + bx + a)^2 \left( \frac{(18cef^2 - 14cdgf - 11begf + 7bdg^2 + 4aeg^2)g^4}{4(ef - dg)^2(cf^2 - bgf + ag^2)^3(f + gx)} + \frac{g^4}{2(ef - dg)(cf^2 - bgf + ag^2)^2(f + gx)^2} + \frac{2(eg^3b^5 - cdg^3b^4 - 3cef g^2b^4 + ceg^3xb^4 - 5aceg^3)}{8(dg - ef)^3(cf^2 - bgf + ag^2)^{7/2}(a + x(b + cx))^{3/2}} \right)}{g^3(80c^2e^2f^4 - 100bce^2gf^3 - 120c^2degf^3 + 48c^2d^2g^2f^2 + 35b^2e^2g^2f^2 + 28ace^2g^2f^2 + 132bcdeg^2f^2 - 48bcd^2g^3f - 28abed^2g^3) + 8(dg - ef)^3(cf^2 - bgf + ag^2)^{7/2}(a + x(b + cx))^{3/2}}$$

$$+ \frac{g^3(80c^2e^2f^4 - 100bce^2gf^3 - 120c^2degf^3 + 48c^2d^2g^2f^2 + 35b^2e^2g^2f^2 + 28ace^2g^2f^2 + 132bcdeg^2f^2 - 48bcd^2g^3f - 28abed^2g^3)}{8(dg - ef)^3(cf^2 - bgf + ag^2)^{7/2}(a + x(b + cx))^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] `Integrate[1/((d + e*x)*(f + g*x)^3*(a + b*x + c*x^2)^(3/2)),x]`

$$\frac{\left( (a + b*x + c*x^2)^2 * (g^4 / (2 * (e*f - d*g) * (c*f^2 - b*f*g + a*g^2)) ^ 2 * (f + g*x)^2) + (g^4 * (18*c*e*f^2 - 14*c*d*f*g - 11*b*e*f*g + 7*b*d*g^2 + 4*a*e*g^2)) / (4 * (e*f - d*g)^2 * (c*f^2 - b*f*g + a*g^2)^3 * (f + g*x)) + (2 * (b*c^4*d*f^3 - b^2*c^3*e*f^3 + 2*a*c^4*e*f^3 - 3*b^2*c^3*d*f^2*g + 6*a*c^4*d*f^2*g + 3*b^3*c^2*e*f^2*g - 9*a*b*c^3*e*f^2*g + 3*b^3*c^2*d*f*g^2 - 9*a*b*c^3*d*f*g^2 - 3*b^4*c*e*f*g^2 + 12*a*b^2*c^2*e*f*g^2 - 6*a^2*c^3*e*f*g^2 - b^4*c*d*g^3 + 4*a*b^2*c^2*d*g^3 - 2*a^2*c^3*d*g^3 + b^5*e*g^3 - 5*a*b^3*c*e*g^3 + 5*a^2*b*c^2*e*g^3 + 2*c^5*d*f^3*x - b*c^4*e*f^3*x - 3*b*c^4*d*f^2*g*x + 3*b^2*c^3*e*f^2*g*x - 6*a*c^4*e*f^2*g*x + 3*b^2*c^3*d*f*g^2*x - 6*a*c^4*d*f*g^2*x - 3*b^3*c^2*e*f*g^2*x + 9*a*b*c^3*e*f*g^2*x - b^3*c^2*d*g^3*x + 3*a*b*c^3*d*g^3*x + b^4*c*e*g^3*x - 4*a*b^2*c^2*e*g^3*x + 2*a^2*c^3*e*g^3*x) / ((-b^2 + 4*a*c) * (c*d^2 - b*d*e + a*e^2) * (c*f^2 - b*f*g + a*g^2)^3 * (a + b*x + c*x^2)) / (a + x*(b + c*x))^(3/2) + (e^5 * (a + b*x + c*x^2)^(3/2) * Log[d + e*x]) / ((c*d^2 - b*d*e + a*e^2)^(3/2) * (e*f - d*g)^3 * (a + x*(b + c*x))^(3/2)) + (g^3 * (80*c^2*e^2*f^4 - 120*c^2*d*e*f^3*g - 100*b*c*e^2*f^3*g + 48*c^2*d^2*f^2*g^2 + 132*b*c*d*e*f^2*g^2 + 35*b^2*e^2*f^2*g^2 + 28*a*c*e^2*f^2*g^2 - 48*b*c*d^2*f*g^3 - 42*b^2*d*e*f*g^3 - 28*a*b*e^2*f*g^3 + 15*b^2*d^2*g^4 - 12*a*c*d^2*g^4 + 12*a*b*d*e*g^4 + 8*a^2*e^2*g^4) * (a + b*x + c*x^2)^(3/2) * Log[f + g*x]) / (8 * (-e*f + d*g)^3 * (c*f^2 - b*f*g + a*g^2)^(7/2) * (a + x*(b + c*x))^(3/2)) - (e^5 * (a + b*x + c*x^2)^(3/2) * Log[-(b*d) + 2*a*e - 2*c*d*x + b*e*x + 2*sqrt[c*d^2 - b*d*e + a*e^2] * sqrt[a + b*x + c*x^2]]) / ((c*d^2 - b*d*e + a*e^2)^(3/2) * (e*f - d*g)^3 * (a + x*(b + c*x))^(3/2)) - (g^3 * (80*c^2*e^2*f^4 - 120*c^2*d*e*f^3*g - 100*b*c*e^2*f^3*g + 48*c^2*d^2*f^2*g^2 + 132*b*c*d*e*f^2*g^2 + 35*b^2*e^2*f^2*g^2 + 28*a*c*e^2*f^2*g^2 - 48*b*c*d^2*f*g^3 - 42*b^2*d*e*f*g^3 - 28*a*b*e^2*f*g^3 + 15*b^2*d^2*g^4 - 12*a*c*d^2*g^4 + 12*a*b*d*e*g^4 + 8*a^2*e^2*g^4) * (a + b*x + c*x^2)^(3/2) * Log[-(b*f) + 2*a*g - 2*c*f*x + b*g*x + 2*sqrt[c*f^2 - b*f*g + a*g^2] * sqrt[a + b*x + c*x^2]]) / (8 * (-$$

$$(e*f) + d*g)^3*(c*f^2 - b*f*g + a*g^2)^{(7/2)}*(a + x*(b + c*x))^{(3/2)}$$

**Maple [B]** time = 0.054, size = 5459, normalized size = 5.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x+d)/(g*x+f)^3/(c*x^2+b*x+a)^(3/2), x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(cx^2 + bx + a)^{\frac{3}{2}}(ex + d)(gx + f)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)*(g*x + f)^3), x, algorithm="maxima")`

[Out] `integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)*(g*x + f)^3), x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)*(g*x + f)^3), x, algorithm="fricas")`

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x+d)/(g*x+f)**3/(c*x**2+b*x+a)**(3/2), x)`

[Out] Timed out

**GIAC/XCAS [F(-2)]** time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/((c*x^2 + b*x + a)^(3/2)*(e*x + d)*(g*x + f)^3),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

$$3.886 \quad \int (d + ex)^3 \sqrt{f + gx} \sqrt{a + bx + cx^2} dx$$

**Optimal.** Leaf size=1551

result too large to display

```
[Out] (-2*(64*b^4*e^4*g^4 + 4*b^2*c*e^3*g^3*(7*b*e*f - 66*b*d*g - 69*a*
e*g) + c^4*(187*e^4*f^4 - 732*d*e^3*f^3*g + 1098*d^2*e^2*f^2*g^2
- 798*d^3*e*f*g^3 + 315*d^4*g^4) + 3*c^2*e^2*g^2*(50*a^2*e^2*g^2
- a*b*e*g*(29*e*f - 297*d*g) + 3*b^2*(e^2*f^2 - 11*d*e*f*g + 44*d
^2*g^2)) - c^3*e*g*(6*a*e*g*(2*e^2*f^2 - 33*d*e*f*g + 165*d^2*g^2
) + b*(8*e^3*f^3 - 99*d^2*e*f*g^2 + 231*d^3*g^3))*Sqrt[f + g*x]*
Sqrt[a + b*x + c*x^2])/(3465*c^4*e*g^4) + (2*(d + e*x)^4*Sqrt[f +
g*x]*Sqrt[a + b*x + c*x^2])/(11*e) + (2*(48*b^3*e^3*g^3 + b*c*e^2
g^2*(67*b*e*f - 198*b*d*g - 157*a*e*g) + c^3*(233*e^3*f^3 - 843
*d*e^2*f^2*g + 1107*d^2*e*f*g^2 - 567*d^3*g^3) - c^2*e*g*(2*a*e*g
*(74*e*f - 231*d*g) - 3*b*(24*e^2*f^2 - 88*d*e*f*g + 99*d^2*g^2))
)*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(3465*c^3*g^4) - (2*e*(8
*b^2*e^2*g^2 + c*e*g*(19*b*e*f - 33*b*d*g - 18*a*e*g) + c^2*(29*e
^2*f^2 - 96*d*e*f*g + 81*d^2*g^2))*(f + g*x)^(5/2)*Sqrt[a + b*x +
c*x^2])/(693*c^2*g^4) + (2*e^2*(c*e*f - 3*c*d*g + b*e*g)*(f + g*
x)^(7/2)*Sqrt[a + b*x + c*x^2])/(99*c*g^4) + (Sqrt[2]*Sqrt[b^2 -
4*a*c]*(128*b^5*e^3*g^5 - 8*b^3*c*e^2*g^4*(7*b*e*f + 66*b*d*g + 8
7*a*e*g) + 2*c^5*f^2*(64*e^3*f^3 - 264*d*e^2*f^2*g + 396*d^2*e*f*
g^2 - 231*d^3*g^3) + b*c^2*e*g^3*(771*a^2*e^2*g^2 + 6*a*b*e*g*(43
*e*f + 396*d*g) - b^2*(37*e^2*f^2 - 264*d*e*f*g - 792*d^2*g^2)) -
c^4*g*(b*f*(56*e^3*f^3 - 264*d*e^2*f^2*g + 495*d^2*e*f*g^2 - 462
*d^3*g^3) - 18*a*g*(6*e^3*f^3 - 33*d*e^2*f^2*g + 88*d^2*e*f*g^2 +
77*d^3*g^3)) - c^3*g^2*(6*a^2*e^2*g^2*(26*e*f + 231*d*g) - 9*a*b
*e*g*(15*e^2*f^2 - 110*d*e*f*g - 319*d^2*g^2) + b^2*(37*e^3*f^3 -
198*d*e^2*f^2*g + 495*d^2*e*f*g^2 + 462*d^3*g^3))*Sqrt[f + g*x]
*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (
-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]/(34
65*c^5*g^5*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)
]*Sqrt[a + b*x + c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*f^2 -
b*f*g + a*g^2)*(64*b^4*e^3*g^4 + 4*b^2*c*e^2*g^3*(7*b*e*f - 66*b*
d*g - 69*a*e*g) - 2*c^4*f*(64*e^3*f^3 - 264*d*e^2*f^2*g + 396*d^2
*e*f*g^2 - 231*d^3*g^3) + 3*c^2*e*g^2*(50*a^2*e^2*g^2 - a*b*e*g*(
29*e*f - 297*d*g) + 3*b^2*(e^2*f^2 - 11*d*e*f*g + 44*d^2*g^2)) -
c^3*g*(6*a*e*g*(2*e^2*f^2 - 33*d*e*f*g + 165*d^2*g^2) + b*(8*e^3*
f^3 - 99*d^2*e*f*g^2 + 231*d^3*g^3))*Sqrt[(c*(f + g*x))/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sq
rt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b +
Sqrt[b^2 - 4*a*c])*g)]/(3465*c^5*g^5*Sqrt[f + g*x]*Sqrt[a + b*x
+ c*x^2])
```

**Rubi [A]** time = 17.5545, antiderivative size = 1551, normalized size of antiderivative = 1., number



$$\frac{\sqrt{(b^2 - 4ac)g}}{(3465c^5g^5\sqrt{f+gx}\sqrt{a+bx+cx^2})}$$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**3*(g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2),x)`

[Out] Timed out

**Mathematica [C]** time = 22.3229, size = 32331, normalized size = 20.85

Result too large to show

Warning: Unable to verify antiderivative.

[In] `Integrate[(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]`

[Out] Result too large to show

**Maple [B]** time = 0.239, size = 32647, normalized size = 21.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^3*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + bx + a}(ex + d)^3 \sqrt{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^3*sqrt(g*x + f),x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^3*sqrt(g*x + f), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3\right)\sqrt{cx^2 + bx + a}\sqrt{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)^3\*sqrt(g\*x + f),x, algorithm="fricas")

[Out] integral((e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3)\*sqrt(c\*x^2 + b\*x + a)\*sqrt(g\*x + f), x)

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int (d + ex)^3 \sqrt{f + gx} \sqrt{a + bx + cx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(g\*x+f)\*\*(1/2)\*(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral((d + e\*x)\*\*3\*sqrt(f + g\*x)\*sqrt(a + b\*x + c\*x\*\*2), x)

**GIAC/XCAS** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)^3\*sqrt(g\*x + f),x, algorithm="giac")

[Out] Timed out



$$3.887 \quad \int (d + ex)^2 \sqrt{f + gx} \sqrt{a + bx + cx^2} dx$$

**Optimal.** Leaf size=1015

$$\frac{2\sqrt{f + gx}\sqrt{cx^2 + bx + a}(d + ex)^3}{9e}$$

$$2\sqrt{2}\sqrt{b^2 - 4ac} (f^2 (8e^2 f^2 - 24degf + 21d^2 g^2) c^4 + g (3ag (3e^2 f^2 - 16degf - 21d^2 g^2) - bf (4e^2 f^2 - 15degf + 21d^2 g^2))$$

$$2\sqrt{2}\sqrt{b^2 - 4ac} (cf^2 - bgf + ag^2) (-2f (8e^2 f^2 - 24degf + 21d^2 g^2) c^3 - 3g^2(2ae(ef - 10dg) + bd(2ef - 7dg))c^2 + 3beg^2$$

$$315c^4 g^4 \sqrt{f + gx} \sqrt{cx^2 + bx + a}$$

$$+ \frac{2e(cef - 3cdg + beg)(f + gx)^{5/2} \sqrt{cx^2 + bx + a}}{63cg^3}$$

$$+ \frac{4((8e^2 f^2 - 24degf + 21d^2 g^2) c^2 + eg(4bef - 9bdg - 7aeg)c + 3b^2 e^2 g^2) (f + gx)^{3/2} \sqrt{cx^2 + bx + a}}{315c^2 g^3}$$

$$+ \frac{2((19e^3 f^3 - 57de^2 g f^2 + 63d^2 e g^2 f - 35d^3 g^3) c^3 - 3eg^2(2ae(ef - 10dg) + bd(2ef - 7dg))c^2 + 3be^2 g^2(bef - 8bdg - 9aeg))}{315c^3 eg^3}$$

```
[Out] (2*(8*b^3*e^3*g^3 + 3*b*c*e^2*g^2*(b*e*f - 8*b*d*g - 9*a*e*g) + c
^3*(19*e^3*f^3 - 57*d*e^2*f^2*g + 63*d^2*e*f*g^2 - 35*d^3*g^3) -
3*c^2*e*g^2*(2*a*e*(e*f - 10*d*g) + b*d*(2*e*f - 7*d*g)))*Sqrt[f
+ g*x]*Sqrt[a + b*x + c*x^2])/(315*c^3*e*g^3) + (2*(d + e*x)^3*Sq
rt[f + g*x]*Sqrt[a + b*x + c*x^2])/(9*e) - (4*(3*b^2*e^2*g^2 + c*
e*g*(4*b*e*f - 9*b*d*g - 7*a*e*g) + c^2*(8*e^2*f^2 - 24*d*e*f*g +
21*d^2*g^2))*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2])/(315*c^2*g^3
) + (2*e*(c*e*f - 3*c*d*g + b*e*g)*(f + g*x)^(5/2)*Sqrt[a + b*x +
c*x^2])/(63*c*g^3) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(8*b^4*e^2*g^4
- 4*b^2*c*e*g^3*(b*e*f + 6*b*d*g + 9*a*e*g) + c^4*f^2*(8*e^2*f^2
- 24*d*e*f*g + 21*d^2*g^2) + 3*c^2*g^2*(7*a^2*e^2*g^2 + a*b*e*g*
(5*e*f + 29*d*g) - b^2*(e^2*f^2 - 5*d*e*f*g - 7*d^2*g^2)) + c^3*g
*(3*a*g*(3*e^2*f^2 - 16*d*e*f*g - 21*d^2*g^2) - b*f*(4*e^2*f^2 -
15*d*e*f*g + 21*d^2*g^2)))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x
^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(
2*c*f - (b + Sqrt[b^2 - 4*a*c])*g))/(315*c^4*g^4*Sqrt[(c*(f + g*
x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) -
(2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*(8*b^3*e^2*
g^3 + 3*b*c*e*g^2*(b*e*f - 8*b*d*g - 9*a*e*g) - 2*c^3*f*(8*e^2*f^
2 - 24*d*e*f*g + 21*d^2*g^2) - 3*c^2*g^2*(2*a*e*(e*f - 10*d*g) +
b*d*(2*e*f - 7*d*g)))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 -
4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellipti
cF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]
/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g))/(315*c^4*g^4*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])]
```

**Rubi [A]** time = 8.27682, antiderivative size = 1015, normalized size of antiderivative = 1., number

of steps used = 9, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+bx+a}(d+ex)^3}{9e}$$

$$2\sqrt{2}\sqrt{b^2-4ac}(f^2(8e^2f^2-24degf+21d^2g^2)c^4+g(3ag(3e^2f^2-16degf-21d^2g^2)-bf(4e^2f^2-15degf+21d^2g^2))$$

$$2\sqrt{2}\sqrt{b^2-4ac}(cf^2-bgf+ag^2)(-2f(8e^2f^2-24degf+21d^2g^2)c^3-3g^2(2ae(ef-10dg)+bd(2ef-7dg))c^2+3beg^2)$$

$$\frac{2e(cef-3cdg+beg)(f+gx)^{5/2}\sqrt{cx^2+bx+a}}{63cg^3} + \frac{4((8e^2f^2-24degf+21d^2g^2)c^2+eg(4bef-9bdg-7aeg)c+3b^2e^2g^2)(f+gx)^{3/2}\sqrt{cx^2+bx+a}}{315c^2g^3} + \frac{2((19e^3f^3-57de^2gf^2+63d^2eg^2f-35d^3g^3)c^3-3eg^2(2ae(ef-10dg)+bd(2ef-7dg))c^2+3be^2g^2(bef-8bdg-9aeg))}{315c^3eg^3}$$

Warning: Unable to verify antiderivative.

[In] Int[(d + e\*x)^2\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2], x]

[Out] (2\*(8\*b^3\*e^3\*g^3 + 3\*b\*c\*e^2\*g^2\*(b\*e\*f - 8\*b\*d\*g - 9\*a\*e\*g) + c^3\*(19\*e^3\*f^3 - 57\*d\*e^2\*f^2\*g + 63\*d^2\*e\*f\*g^2 - 35\*d^3\*g^3) - 3\*c^2\*e\*g^2\*(2\*a\*e\*(e\*f - 10\*d\*g) + b\*d\*(2\*e\*f - 7\*d\*g)))\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])/(315\*c^3\*e\*g^3) + (2\*(d + e\*x)^3\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])/(9\*e) - (4\*(3\*b^2\*e^2\*g^2 + c\*e\*g\*(4\*b\*e\*f - 9\*b\*d\*g - 7\*a\*e\*g) + c^2\*(8\*e^2\*f^2 - 24\*d\*e\*f\*g + 21\*d^2\*g^2))\*(f + g\*x)^(3/2)\*Sqrt[a + b\*x + c\*x^2])/(315\*c^2\*g^3) + (2\*e\*(c\*e\*f - 3\*c\*d\*g + b\*e\*g)\*(f + g\*x)^(5/2)\*Sqrt[a + b\*x + c\*x^2])/(63\*c\*g^3) - (2\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*(8\*b^4\*e^2\*g^4 - 4\*b^2\*c\*e\*g^3\*(b\*e\*f + 6\*b\*d\*g + 9\*a\*e\*g) + c^4\*f^2\*(8\*e^2\*f^2 - 24\*d\*e\*f\*g + 21\*d^2\*g^2) + 3\*c^2\*g^2\*(7\*a^2\*e^2\*g^2 + a\*b\*e\*g\*(5\*e\*f + 29\*d\*g) - b^2\*(e^2\*f^2 - 5\*d\*e\*f\*g - 7\*d^2\*g^2)) + c^3\*g\*(3\*a\*g\*(3\*e^2\*f^2 - 16\*d\*e\*f\*g - 21\*d^2\*g^2) - b\*f\*(4\*e^2\*f^2 - 15\*d\*e\*f\*g + 21\*d^2\*g^2)))\*Sqrt[f + g\*x]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c])/(b^2 - 4\*a\*c)]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(315\*c^4\*g^4\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[a + b\*x + c\*x^2]) - (2\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*(c\*f^2 - b\*f\*g + a\*g^2)\*(8\*b^3\*e^2\*g^3 + 3\*b\*c\*e\*g^2\*(b\*e\*f - 8\*b\*d\*g - 9\*a\*e\*g) - 2\*c^3\*f\*(8\*e^2\*f^2 - 24\*d\*e\*f\*g + 21\*d^2\*g^2) - 3\*c^2\*g^2\*(2\*a\*e\*(e\*f - 10\*d\*g) + b\*d\*(2\*e\*f - 7\*d\*g)))\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c])/(b^2 - 4\*a\*c)]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(315\*c^4\*g^4\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*2\*(g\*x+f)\*\*(1/2)\*(c\*x\*\*2+b\*x+a)\*\*(1/2), x)

[Out] Timed out

---

**Mathematica [C]** time = 18.2186, size = 15781, normalized size = 15.55

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(d + e\*x)^2\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2],x]

[Out] Result too large to show

---

**Maple [B]** time = 0.078, size = 20224, normalized size = 19.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^2\*(g\*x+f)^(1/2)\*(c\*x^2+b\*x+a)^(1/2),x)

[Out] result too large to display

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + bx + a}(ex + d)^2 \sqrt{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)^2\*sqrt(g\*x + f),x, algorithm="maxima")

[Out] integrate(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)^2\*sqrt(g\*x + f), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(e^2x^2 + 2dex + d^2\right)\sqrt{cx^2 + bx + a}\sqrt{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)^2\*sqrt(g\*x + f),x, algorithm="fricas")

[Out] integral((e^2\*x^2 + 2\*d\*e\*x + d^2)\*sqrt(c\*x^2 + b\*x + a)\*sqrt(g\*x + f), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (d + ex)^2 \sqrt{f + gx} \sqrt{a + bx + cx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2\*(g\*x+f)\*\*(1/2)\*(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out]  $\text{Integral}((d + e*x)**2*\text{sqrt}(f + g*x)*\text{sqrt}(a + b*x + c*x**2), x)$

---

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^2*sqrt(g*x + f),x, algorithm="giac")`

[Out] Timed out

### 3.888 $\int (d + ex)\sqrt{f + gx}\sqrt{a + bx + cx^2} dx$

**Optimal.** Leaf size=652

$$\frac{2\sqrt{f + gx}\sqrt{a + bx + cx^2}(-cg(-5aeg + 7bdg + 2bef) + 4b^2eg^2 - 3cgx(-4beg + 7cdg + cef) + c^2f(4ef - 7dg))}{105c^2g^2}$$

$$+ \frac{2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ag^2 - bfg + cf^2)\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}(cg(-10aeg - 7bdg + bef) + 4b^2eg^2 - 2c^2f(4ef - 7dg))}{105c^3g^3\sqrt{f + gx}\sqrt{a + bx + cx^2}}$$

$$+ \frac{\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{f + gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}((-3cg(bf - 2ag) - 2b^2g^2 + 8c^2f^2)(-4beg + 7cdg + cef) - 5cg(2cf - bg)(7cdf - e))}{105c^3g^3\sqrt{a + bx + cx^2}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}}$$

$$+ \frac{2e\sqrt{f + gx}(a + bx + cx^2)^{3/2}}{7c}$$

[Out]  $(-2*\text{Sqrt}[f + g*x])*(4*b^2*e*g^2 + c^2*f*(4*e*f - 7*d*g) - c*g*(2*b*e*f + 7*b*d*g - 5*a*e*g) - 3*c*g*(c*e*f + 7*c*d*g - 4*b*e*g)*x)*\text{Sqrt}[a + b*x + c*x^2]/(105*c^2*g^2) + (2*e*\text{Sqrt}[f + g*x]*(a + b*x + c*x^2)^{(3/2)})/(7*c) + (\text{Sqrt}[2]*\text{Sqrt}[b^2 - 4*a*c]*((c*e*f + 7*c*d*g - 4*b*e*g)*(8*c^2*f^2 - 2*b^2*g^2 - 3*c*g*(b*f - 2*a*g)) - 5*c*g*(2*c*f - b*g)*(7*c*d*f - e*(3*b*f + a*g)))*\text{Sqrt}[f + g*x]*\text{Sqrt}[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (-2*\text{Sqrt}[b^2 - 4*a*c]*g)/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]/(105*c^3*g^3*\text{Sqrt}[(c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]*\text{Sqrt}[a + b*x + c*x^2]) + (2*\text{Sqrt}[2]*\text{Sqrt}[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*(4*b^2*e*g^2 - 2*c^2*f*(4*e*f - 7*d*g) + c*g*(b*e*f - 7*b*d*g - 10*a*e*g))*\text{Sqrt}[(c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]*\text{Sqrt}[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (-2*\text{Sqrt}[b^2 - 4*a*c]*g)/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]/(105*c^3*g^3*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + b*x + c*x^2])$

**Rubi [A]** time = 2.47642, antiderivative size = 652, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$

$$\frac{2\sqrt{f + gx}\sqrt{a + bx + cx^2}(-cg(-5aeg + 7bdg + 2bef) + 4b^2eg^2 - 3cgx(-4beg + 7cdg + cef) + c^2f(4ef - 7dg))}{105c^2g^2}$$

$$+ \frac{2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ag^2 - bfg + cf^2)\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}(cg(-10aeg - 7bdg + bef) + 4b^2eg^2 - 2c^2f(4ef - 7dg))}{105c^3g^3\sqrt{f + gx}\sqrt{a + bx + cx^2}}$$

$$+ \frac{\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{f + gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}((-3cg(bf - 2ag) - 2b^2g^2 + 8c^2f^2)(-4beg + 7cdg + cef) - 5cg(2cf - bg)(7cdf - e))}{105c^3g^3\sqrt{a + bx + cx^2}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}}$$

$$+ \frac{2e\sqrt{f + gx}(a + bx + cx^2)^{3/2}}{7c}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + b*x + c*x^2], x]$

[Out]  $(-2*\text{Sqrt}[f + g*x])*(4*b^2*e*g^2 + c^2*f*(4*e*f - 7*d*g) - c*g*(2*b*e*f + 7*b*d*g - 5*a*e*g) - 3*c*g*(c*e*f + 7*c*d*g - 4*b*e*g)*x)*\text{Sqrt}[a + b*x + c*x^2]$

$$\begin{aligned} & \text{Sqrt}[a + b*x + c*x^2]/(105*c^2*g^2) + (2*e*\text{Sqrt}[f + g*x]*(a + b*x + c*x^2)^{(3/2)})/(7*c) + (\text{Sqrt}[2]*\text{Sqrt}[b^2 - 4*a*c]*((c*e*f + 7*c*d*g - 4*b*e*g)*(8*c^2*f^2 - 2*b^2*g^2 - 3*c*g*(b*f - 2*a*g)) - 5*c*g*(2*c*f - b*g)*(7*c*d*f - e*(3*b*f + a*g)))*\text{Sqrt}[f + g*x]*\text{Sqrt}[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (-2*\text{Sqrt}[b^2 - 4*a*c]*g)/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)))/(105*c^3*g^3*\text{Sqrt}[(c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]*\text{Sqrt}[a + b*x + c*x^2]) + (2*\text{Sqrt}[2]*\text{Sqrt}[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*(4*b^2*e*g^2 - 2*c^2*f*(4*e*f - 7*d*g) + c*g*(b*e*f - 7*b*d*g - 10*a*e*g))*\text{Sqrt}[(c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]*\text{Sqrt}[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (-2*\text{Sqrt}[b^2 - 4*a*c]*g)/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)))/(105*c^3*g^3*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + b*x + c*x^2]) \end{aligned}$$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)*(g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2),x)`

[Out] Timed out

**Mathematica [C]** time = 15.3236, size = 8432, normalized size = 12.93

Result too large to show

Antiderivative was successfully verified.

[In] `Integrate[(d + e*x)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2],x]`

[Out] Result too large to show

**Maple [B]** time = 0.051, size = 10711, normalized size = 16.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2),x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + bx + a}(ex + d)\sqrt{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x + f),x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x + f), x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^2 + bx + a}(ex + d)\sqrt{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x + f), x, algorithm="fricas")`

[Out] `integral(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x + f), x)`

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int (d + ex)\sqrt{f + gx}\sqrt{a + bx + cx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2), x)`

[Out] `Integral((d + e*x)*sqrt(f + g*x)*sqrt(a + b*x + c*x**2), x)`

**GIAC/XCAS** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x + f), x, algorithm="giac")`

[Out] Timed out

### 3.889 $\int \sqrt{f + gx} \sqrt{a + bx + cx^2} dx$

**Optimal.** Leaf size=513

$$\frac{2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cf - bg)(ag^2 - bfg + cf^2)\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right) - \frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}}{15c^2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$\frac{2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(-cg(3ag+bf)+b^2g^2+c^2f^2)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right) - \frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}}{15c^2g^2\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}}$$

$$+ \frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g} - \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}(2cf-bg)}{15cg}$$

[Out]  $(-2*(2*c*f - b*g)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + b*x + c*x^2])/(15*c*g) + (2*(f + g*x)^{(3/2)}*\text{Sqrt}[a + b*x + c*x^2])/(5*g) - (2*\text{Sqrt}[2]*\text{Sqrt}[b^2 - 4*a*c]*(c^2*f^2 + b^2*g^2 - c*g*(b*f + 3*a*g))*\text{Sqrt}[f + g*x]*\text{Sqrt}[-(c*(a + b*x + c*x^2))/(b^2 - 4*a*c)])*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (-2*\text{Sqrt}[b^2 - 4*a*c]*g)/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]/(15*c^2*g^2*\text{Sqrt}[(c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)])*\text{Sqrt}[a + b*x + c*x^2]) + (2*\text{Sqrt}[2]*\text{Sqrt}[b^2 - 4*a*c]*(2*c*f - b*g)*(c*f^2 - b*f*g + a*g^2)*\text{Sqrt}[(c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)])*\text{Sqrt}[-(c*(a + b*x + c*x^2))/(b^2 - 4*a*c)]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (-2*\text{Sqrt}[b^2 - 4*a*c]*g)/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]/(15*c^2*g^2*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + b*x + c*x^2])$

**Rubi [A]** time = 1.33651, antiderivative size = 513, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.25$

$$\frac{2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cf - bg)(ag^2 - bfg + cf^2)\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right) - \frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}}{15c^2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$\frac{2\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(-cg(3ag+bf)+b^2g^2+c^2f^2)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right) - \frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}}{15c^2g^2\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}}$$

$$+ \frac{2(f+gx)^{3/2}\sqrt{a+bx+cx^2}}{5g} - \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}(2cf-bg)}{15cg}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[\text{Sqrt}[f + g*x]*\text{Sqrt}[a + b*x + c*x^2], x]$

[Out]  $(-2*(2*c*f - b*g)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + b*x + c*x^2])/(15*c*g) + (2*(f + g*x)^{(3/2)}*\text{Sqrt}[a + b*x + c*x^2])/(5*g) - (2*\text{Sqrt}[2]*\text{Sqrt}[b^2 - 4*a*c]*(c^2*f^2 + b^2*g^2 - c*g*(b*f + 3*a*g))*\text{Sqrt}[f + g*x]*\text{Sqrt}[-(c*(a + b*x + c*x^2))/(b^2 - 4*a*c)])*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (-2*\text{Sqrt}[b^2 - 4*a*c]*g)/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]/(15*c^2*g^2*\text{Sqrt}[(c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)])*\text{Sqrt}[a + b*x + c*x^2]) + (2*\text{Sqrt}[2]*\text{Sqrt}[b^2 - 4*a*c]*(2*c*f - b*g)*(c*f^2 - b*f*g + a*g^2)*\text{Sqrt}[(c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)])*\text{Sqrt}[-(c*(a + b*x + c*x^2))/(b^2 - 4*a*c)]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x)/\text{Sqrt}[b^2 -$





```

rt[b^2*g^2 - 4*a*c*g^2)))]/Sqrt[f + g*x]], (2*c*f - b*g - Sqrt[b
^2*g^2 - 4*a*c*g^2))/(2*c*f - b*g + Sqrt[b^2*g^2 - 4*a*c*g^2]))
)/(Sqrt[2]*(c*f^2 - b*f*g + a*g^2)*Sqrt[-((c*f^2 - b*f*g + a*g^2)/
(2*c*f - b*g - Sqrt[b^2*g^2 - 4*a*c*g^2]))]*Sqrt[c + (c*f^2 - b*f
*g + a*g^2)/(f + g*x)^2 + (-2*c*f + b*g)/(f + g*x))] - ((3*I)*a*c
*g^2*(2*c*f - b*g + Sqrt[b^2*g^2 - 4*a*c*g^2))*Sqrt[1 - (2*(c*f^2
- b*f*g + a*g^2))/((2*c*f - b*g - Sqrt[b^2*g^2 - 4*a*c*g^2))*(f
+ g*x)]]*Sqrt[1 - (2*(c*f^2 - b*f*g + a*g^2))/((2*c*f - b*g + Sqr
t[b^2*g^2 - 4*a*c*g^2))*(f + g*x)]]*(EllipticE[I*ArcSinh[(Sqrt[2]
*Sqrt[-((c*f^2 - b*f*g + a*g^2)/(2*c*f - b*g - Sqrt[b^2*g^2 - 4*a
*c*g^2])))]]/Sqrt[f + g*x]], (2*c*f - b*g - Sqrt[b^2*g^2 - 4*a*c*g
^2))/(2*c*f - b*g + Sqrt[b^2*g^2 - 4*a*c*g^2])) - EllipticF[I*Arc
Sinh[(Sqrt[2]*Sqrt[-((c*f^2 - b*f*g + a*g^2)/(2*c*f - b*g - Sqrt[
b^2*g^2 - 4*a*c*g^2])))]]/Sqrt[f + g*x]], (2*c*f - b*g - Sqrt[b^2*
g^2 - 4*a*c*g^2))/(2*c*f - b*g + Sqrt[b^2*g^2 - 4*a*c*g^2])))]/(S
qrt[2]*(c*f^2 - b*f*g + a*g^2)*Sqrt[-((c*f^2 - b*f*g + a*g^2)/(2*
c*f - b*g - Sqrt[b^2*g^2 - 4*a*c*g^2]))]*Sqrt[c + (c*f^2 - b*f*g
+ a*g^2)/(f + g*x)^2 + (-2*c*f + b*g)/(f + g*x))] + (I*Sqrt[2]*c^
2*f*Sqrt[1 - (2*(c*f^2 - b*f*g + a*g^2))/((2*c*f - b*g - Sqrt[b^2
*g^2 - 4*a*c*g^2))*(f + g*x)]]*Sqrt[1 - (2*(c*f^2 - b*f*g + a*g^2
))/((2*c*f - b*g + Sqrt[b^2*g^2 - 4*a*c*g^2))*(f + g*x)]]*Ellipti
cF[I*ArcSinh[(Sqrt[2]*Sqrt[-((c*f^2 - b*f*g + a*g^2)/(2*c*f - b*g
- Sqrt[b^2*g^2 - 4*a*c*g^2])))]]/Sqrt[f + g*x]], (2*c*f - b*g - S
qrt[b^2*g^2 - 4*a*c*g^2))/(2*c*f - b*g + Sqrt[b^2*g^2 - 4*a*c*g^2
]])]/(Sqrt[-((c*f^2 - b*f*g + a*g^2)/(2*c*f - b*g - Sqrt[b^2*g^2
- 4*a*c*g^2]))]*Sqrt[c + (c*f^2 - b*f*g + a*g^2)/(f + g*x)^2 + (-
2*c*f + b*g)/(f + g*x))] - (I*b*c*g*Sqrt[1 - (2*(c*f^2 - b*f*g +
a*g^2))/((2*c*f - b*g - Sqrt[b^2*g^2 - 4*a*c*g^2))*(f + g*x)]]*Sq
rt[1 - (2*(c*f^2 - b*f*g + a*g^2))/((2*c*f - b*g + Sqrt[b^2*g^2 -
4*a*c*g^2))*(f + g*x)]]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[-((c*f
^2 - b*f*g + a*g^2)/(2*c*f - b*g - Sqrt[b^2*g^2 - 4*a*c*g^2])))]]/
Sqrt[f + g*x]], (2*c*f - b*g - Sqrt[b^2*g^2 - 4*a*c*g^2))/(2*c*f
- b*g + Sqrt[b^2*g^2 - 4*a*c*g^2])))]/(Sqrt[2]*Sqrt[-((c*f^2 - b*f
*g + a*g^2)/(2*c*f - b*g - Sqrt[b^2*g^2 - 4*a*c*g^2]))]*Sqrt[c +
(c*f^2 - b*f*g + a*g^2)/(f + g*x)^2 + (-2*c*f + b*g)/(f + g*x)))]
)/(c*Sqrt[((f + g*x)^2*(c*(-1 + f/(f + g*x))^2 + (g*(b - (b*f)/(f
+ g*x) + (a*g)/(f + g*x)))/(f + g*x)))/g^2)))]/(15*c*g^3*Sqrt[a
+ b*x + c*x^2])

```

**Maple [B]** time = 0.035, size = 4356, normalized size = 8.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^(1/2)\*(c\*x^2+b\*x+a)^(1/2),x)

```

[Out] 1/15*(g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/c^2*(2*x*b^2*c*f*g^3+12*2^
(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-2*
c*x+(-4*a*c+b^2)^(1/2)-b)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)
*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)
)^(1/2)*EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2
*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*
a*c+b^2)^(1/2)))^(1/2)*a*c^2*f^2*g^2-3*2^(1/2)*(-(g*x+f)*c/(g*(-
4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-2*c*x+(-4*a*c+b^2)^(1/2)-
b)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^
2)^(1/2))/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticF(2^(1/
2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a
*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)
)*b^2*c*f^2*g^2-8*2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2
*c*f))^(1/2)*(g*(-2*c*x+(-4*a*c+b^2)^(1/2)-b)/(2*c*f-b*g+g*(-4*a*c
+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2))/(g*(-4*a*c+b^
2)^(1/2)+b*g-2*c*f))^(1/2)*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a
*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f
)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*a*c^2*f^2*g^2+8*2^(1/2)
)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)*(g*(-2*c*x+
(-4*a*c+b^2)^(1/2)-b)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*

```

$$\begin{aligned}
& (b+2^*c^*x+(-4^*a^*c+b^2)^{(1/2)})/(g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)} \\
& \text{EllipticE}(2^{(1/2)} * (- (g^*x+f) * c / (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f) \\
& ))^{(1/2)}, (- (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f) / (2^*c^*f-b^*g+g^*(-4^*a^*c+ \\
& b^2)^{(1/2)}))^{(1/2)}) * b^2 * c^*f^2 * g^2 - 8^*2^{(1/2)} * (- (g^*x+f) * c / (g^*(-4^*a^* \\
& c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)} * (g^*(-2^*c^*x+(-4^*a^*c+b^2)^{(1/2)}-b) / ( \\
& 2^*c^*f-b^*g+g^*(-4^*a^*c+b^2)^{(1/2)}))^{(1/2)} * (g^*(b+2^*c^*x+(-4^*a^*c+b^2)^{(1/2)} \\
& )) / (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)} * \text{EllipticE}(2^{(1/2)} * ( \\
& - (g^*x+f) * c / (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}, (- (g^*(-4^*a^*c+b \\
& ^2)^{(1/2)}+b^*g-2^*c^*f) / (2^*c^*f-b^*g+g^*(-4^*a^*c+b^2)^{(1/2)}))^{(1/2)}) * b^*c \\
& ^2 * f^3 * g + 2^{(1/2)} * (- (g^*x+f) * c / (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)} \\
& * (g^*(-2^*c^*x+(-4^*a^*c+b^2)^{(1/2)}-b) / (2^*c^*f-b^*g+g^*(-4^*a^*c+b^2)^{(1/2)}))^{(1/2)} \\
& ))^{(1/2)} * (g^*(b+2^*c^*x+(-4^*a^*c+b^2)^{(1/2)})) / (g^*(-4^*a^*c+b^2)^{(1/2)} \\
& +b^*g-2^*c^*f))^{(1/2)} * \text{EllipticF}(2^{(1/2)} * (- (g^*x+f) * c / (g^*(-4^*a^*c+b^2)^{(1/2)} \\
& ))^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}, (- (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f) / (2^*c^*f \\
& -b^*g+g^*(-4^*a^*c+b^2)^{(1/2)}))^{(1/2)}) * (-4^*a^*c+b^2)^{(1/2)} * a^*b^*g^4 - 2^{(1/2)} \\
& * (- (g^*x+f) * c / (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)} * (g^*(-2^*c^* \\
& x+(-4^*a^*c+b^2)^{(1/2)}-b) / (2^*c^*f-b^*g+g^*(-4^*a^*c+b^2)^{(1/2)}))^{(1/2)} * \\
& (g^*(b+2^*c^*x+(-4^*a^*c+b^2)^{(1/2)})) / (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f)) \\
& ^{(1/2)} * \text{EllipticF}(2^{(1/2)} * (- (g^*x+f) * c / (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^* \\
& c^*f))^{(1/2)}, (- (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f) / (2^*c^*f-b^*g+g^*(-4^*a^* \\
& c+b^2)^{(1/2)}))^{(1/2)}) * (-4^*a^*c+b^2)^{(1/2)} * b^2 * f^*g^3 + 6^*x^2 * a^*c^2 * g \\
& ^4 + 2^*x^2 * b^2 * c^*g^4 + 2^*x^2 * c^3 * f^2 * g^2 + 8^*x^3 * c^3 * f^*g^3 + 8^*x^3 * b^*c^2 * \\
& g^4 + 2^*a^*c^2 * f^2 * g^2 + 12^*2^{(1/2)} * (- (g^*x+f) * c / (g^*(-4^*a^*c+b^2)^{(1/2)}+ \\
& b^*g-2^*c^*f))^{(1/2)} * (g^*(-2^*c^*x+(-4^*a^*c+b^2)^{(1/2)}-b) / (2^*c^*f-b^*g+g^*( \\
& -4^*a^*c+b^2)^{(1/2)}))^{(1/2)} * (g^*(b+2^*c^*x+(-4^*a^*c+b^2)^{(1/2)})) / (g^*(-4^* \\
& a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)} * \text{EllipticF}(2^{(1/2)} * (- (g^*x+f) * c / (g^* \\
& (-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}, (- (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g \\
& -2^*c^*f) / (2^*c^*f-b^*g+g^*(-4^*a^*c+b^2)^{(1/2)}))^{(1/2)}) * a^2 * c^*g^4 + 4^*2^{(1/2)} \\
& * (- (g^*x+f) * c / (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)} * (g^*(-2^*c^* \\
& x+(-4^*a^*c+b^2)^{(1/2)}-b) / (2^*c^*f-b^*g+g^*(-4^*a^*c+b^2)^{(1/2)}))^{(1/2)} * ( \\
& g^*(b+2^*c^*x+(-4^*a^*c+b^2)^{(1/2)})) / (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)} \\
& * \text{EllipticE}(2^{(1/2)} * (- (g^*x+f) * c / (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^* \\
& f))^{(1/2)}, (- (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f) / (2^*c^*f-b^*g+g^*(-4^*a^* \\
& c+b^2)^{(1/2)}))^{(1/2)}) * c^3 * f^4 + 2^*x^*b^*c^2 * f^2 * g^2 + 10^*x^2 * b^*c^2 * f^*g^3 \\
& + 2^*x^*a^*b^*c^*g^4 + 8^*x^*a^*c^2 * f^*g^3 + 6^*x^4 * c^3 * g^4 - 12^*2^{(1/2)} * (- (g^*x+f) \\
& ) * c / (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)} * (g^*(-2^*c^*x+(-4^*a^*c+b^ \\
& 2)^{(1/2)}-b) / (2^*c^*f-b^*g+g^*(-4^*a^*c+b^2)^{(1/2)}))^{(1/2)} * (g^*(b+2^*c^*x+ \\
& -4^*a^*c+b^2)^{(1/2)})) / (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)} * \text{Elliptic} \\
& \text{F}(2^{(1/2)} * (- (g^*x+f) * c / (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}, ( \\
& - (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f) / (2^*c^*f-b^*g+g^*(-4^*a^*c+b^2)^{(1/2)} \\
& ))^{(1/2)}) * a^*b^*c^*f^*g^3 + 8^*2^{(1/2)} * (- (g^*x+f) * c / (g^*(-4^*a^*c+b^2)^{(1/2)} \\
& +b^*g-2^*c^*f))^{(1/2)} * (g^*(-2^*c^*x+(-4^*a^*c+b^2)^{(1/2)}-b) / (2^*c^*f-b^*g+g^* \\
& (-4^*a^*c+b^2)^{(1/2)}))^{(1/2)} * (g^*(b+2^*c^*x+(-4^*a^*c+b^2)^{(1/2)})) / (g^*(-4^* \\
& a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)} * \text{EllipticE}(2^{(1/2)} * (- (g^*x+f) * c / ( \\
& g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}, (- (g^*(-4^*a^*c+b^2)^{(1/2)}+b^* \\
& g-2^*c^*f) / (2^*c^*f-b^*g+g^*(-4^*a^*c+b^2)^{(1/2)}))^{(1/2)}) * a^*b^*c^*f^*g^3 + 3^*2^{(1/2)} \\
& * (- (g^*x+f) * c / (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)} * (g^*(-2^* \\
& c^*x+(-4^*a^*c+b^2)^{(1/2)}-b) / (2^*c^*f-b^*g+g^*(-4^*a^*c+b^2)^{(1/2)}))^{(1/2)} \\
& * (g^*(b+2^*c^*x+(-4^*a^*c+b^2)^{(1/2)})) / (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f) \\
& ))^{(1/2)} * \text{EllipticF}(2^{(1/2)} * (- (g^*x+f) * c / (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g- \\
& 2^*c^*f))^{(1/2)}, (- (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f) / (2^*c^*f-b^*g+g^*(-4^* \\
& a^*c+b^2)^{(1/2)}))^{(1/2)}) * (-4^*a^*c+b^2)^{(1/2)} * b^*c^*f^2 * g^2 - 2^*2^{(1/2)} \\
& * (- (g^*x+f) * c / (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)} * (g^*(-2^*c^*x+ \\
& -4^*a^*c+b^2)^{(1/2)}-b) / (2^*c^*f-b^*g+g^*(-4^*a^*c+b^2)^{(1/2)}))^{(1/2)} * (g^*( \\
& b+2^*c^*x+(-4^*a^*c+b^2)^{(1/2)})) / (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)} \\
& * \text{EllipticF}(2^{(1/2)} * (- (g^*x+f) * c / (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f) \\
& ))^{(1/2)}, (- (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f) / (2^*c^*f-b^*g+g^*(-4^*a^*c+b^2)^{(1/2)} \\
& ))^{(1/2)}) * a^*b^2 * g^4 + 3^*2^{(1/2)} * (- (g^*x+f) * c / (g^*(-4^*a^*c+b^2)^{(1/2)}+ \\
& b^*g-2^*c^*f))^{(1/2)} * (g^*(-2^*c^*x+(-4^*a^*c+b^2)^{(1/2)}-b) / (2^*c^*f-b^*g+g^*( \\
& -4^*a^*c+b^2)^{(1/2)}))^{(1/2)} * (g^*(b+2^*c^*x+(-4^*a^*c+b^2)^{(1/2)})) / (g^*(-4^* \\
& a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)} * \text{EllipticF}(2^{(1/2)} * (- (g^*x+f) * c / (g^* \\
& (-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}, (- (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g \\
& -2^*c^*f) / (2^*c^*f-b^*g+g^*(-4^*a^*c+b^2)^{(1/2)}))^{(1/2)}) * b^3 * f^*g^3 - 12^*2^{(1/2)} \\
& * (- (g^*x+f) * c / (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)} * (g^*(-2^*c^* \\
& x+(-4^*a^*c+b^2)^{(1/2)}-b) / (2^*c^*f-b^*g+g^*(-4^*a^*c+b^2)^{(1/2)}))^{(1/2)} * \\
& (g^*(b+2^*c^*x+(-4^*a^*c+b^2)^{(1/2)})) / (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))
\end{aligned}$$

$$\begin{aligned} & \wedge(1/2)*\text{EllipticE}(2^{(1/2)}*(-(g*x+f)*c/(g*(-4*a*c+b^2)^{(1/2)}+b*g-2*c*f))^{(1/2)}, \\ & (-(g*(-4*a*c+b^2)^{(1/2)}+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^{(1/2)}))^{(1/2)})^2*c*g^4+4*2^{(1/2)}*(-(g*x+f)*c/(g*(-4*a*c+b^2)^{(1/2)}+b*g-2*c*f))^{(1/2)} \\ & *(g*(-2*c*x+(-4*a*c+b^2)^{(1/2)}-b)/(2*c*f-b*g+g*(-4*a*c+b^2)^{(1/2)}))^{(1/2)}*(g*(b+2*c*x+(-4*a*c+b^2)^{(1/2)})/(g*(-4*a*c+b^2)^{(1/2)}+b*g-2*c*f))^{(1/2)} \\ & *\text{EllipticE}(2^{(1/2)}*(-(g*x+f)*c/(g*(-4*a*c+b^2)^{(1/2)}+b*g-2*c*f))^{(1/2)}, \\ & (-(g*(-4*a*c+b^2)^{(1/2)}+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^{(1/2)}))^{(1/2)})^2*a*b^2*g^4-4*2^{(1/2)}*(-(g*x+f)*c/(g*(-4*a*c+b^2)^{(1/2)}+b*g-2*c*f))^{(1/2)} \\ & *(g*(-2*c*x+(-4*a*c+b^2)^{(1/2)}-b)/(2*c*f-b*g+g*(-4*a*c+b^2)^{(1/2)}))^{(1/2)} \\ & *(g*(b+2*c*x+(-4*a*c+b^2)^{(1/2)})/(g*(-4*a*c+b^2)^{(1/2)}+b*g-2*c*f))^{(1/2)} \\ & *\text{EllipticE}(2^{(1/2)}*(-(g*x+f)*c/(g*(-4*a*c+b^2)^{(1/2)}+b*g-2*c*f))^{(1/2)}, \\ & (-(g*(-4*a*c+b^2)^{(1/2)}+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^{(1/2)}))^{(1/2)})^2*b^3*f*g^3+2*a*b*c*f*g^3-2*2^{(1/2)} \\ & *(-(g*x+f)*c/(g*(-4*a*c+b^2)^{(1/2)}+b*g-2*c*f))^{(1/2)}*(g*(-2*c*x+(-4*a*c+b^2)^{(1/2)}-b)/(2*c*f-b*g+g*(-4*a*c+b^2)^{(1/2)}))^{(1/2)} \\ & *(g*(b+2*c*x+(-4*a*c+b^2)^{(1/2)})/(g*(-4*a*c+b^2)^{(1/2)}+b*g-2*c*f))^{(1/2)} \\ & *\text{EllipticF}(2^{(1/2)}*(-(g*x+f)*c/(g*(-4*a*c+b^2)^{(1/2)}+b*g-2*c*f))^{(1/2)}, \\ & (-(g*(-4*a*c+b^2)^{(1/2)}+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^{(1/2)}))^{(1/2)})^2*(-4*a*c+b^2)^{(1/2)}*c^2*f^3*g)/(c*g*x^3+b*g*x^2+c*f*x^2+a*g*x+b*f*x+a*f)/g^3 \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + bx + a} \sqrt{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)\*sqrt(g\*x + f), x, algorithm="maxima")

[Out] integrate(sqrt(c\*x^2 + b\*x + a)\*sqrt(g\*x + f), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^2 + bx + a} \sqrt{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)\*sqrt(g\*x + f), x, algorithm="fricas")

[Out] integral(sqrt(c\*x^2 + b\*x + a)\*sqrt(g\*x + f), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{f + gx} \sqrt{a + bx + cx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*(1/2)\*(c\*x\*\*2+b\*x+a)\*\*(1/2), x)

[Out] Integral(sqrt(f + g\*x)\*sqrt(a + b\*x + c\*x\*\*2), x)

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.890 \quad \int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex} dx$$

**Optimal.** Leaf size=764

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}(eg(2aeg-3bdg+bef)+c(3d^2g^2-e^2f^2))F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right)}{3ce^3g\sqrt{f+gx}\sqrt{a+x(b+cx)}} \\ + \frac{\sqrt{2}(ae^2-bde+cd^2)\sqrt{2cf-g(b-\sqrt{b^2-4ac})}\sqrt{1-\frac{2c(f+gx)}{2cf-g(b-\sqrt{b^2-4ac})}}\sqrt{1-\frac{2c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}\left(\frac{e(2cf-bg+\sqrt{b^2-4ac}g)}{2c(ef-dg)}\right)}{\sqrt{ce^3}\sqrt{a+bx+cx^2}} \\ + \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(beg-3cdg+cef)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right)}{3ce^2g\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}} \\ + \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3e}$$

[Out] (2\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])/(3\*e) + (Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*(c\*e\*f - 3\*c\*d\*g + b\*e\*g)\*Sqrt[f + g\*x]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))])\*(EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(3\*c\*e^2\*g\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[a + b\*x + c\*x^2]) + (2\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*(e\*g\*(b\*e\*f - 3\*b\*d\*g + 2\*a\*e\*g) + c\*(-(e^2\*f^2) + 3\*d^2\*g^2))\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[(c\*(a + x\*(b + c\*x)))/(-b^2 + 4\*a\*c)])\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (2\*Sqrt[b^2 - 4\*a\*c]\*g)/(-2\*c\*f + (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(3\*c\*e^3\*g\*Sqrt[f + g\*x]\*Sqrt[a + x\*(b + c\*x)]) - (Sqrt[2]\*(c\*d^2 - b\*d\*e + a\*e^2)\*Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g]\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)])\*EllipticPi[(e\*(2\*c\*f - b\*g + Sqrt[b^2 - 4\*a\*c]\*g))/(2\*c\*(e\*f - d\*g)), ArcSin[(Sqrt[2]\*Sqrt[c]\*Sqrt[f + g\*x])/Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g]], (b - Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)/(b + Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)]/(Sqrt[c]\*e^3\*Sqrt[a + b\*x + c\*x^2])

**Rubi [A]** time = 8.97128, antiderivative size = 969, normalized size of antiderivative = 1.27, number

of steps used = 15, number of rules used = 10, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$

$$\frac{\sqrt{2}\sqrt{b^2-4ac}(cef-3cdg+beg)\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{3ce^2g\sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\sqrt{cx^2+bx+a}}$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}f(cef-3cdg+beg)\sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{3ce^2g\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}(3cd(ef-dg)-e(2bef-3bdg+2aeg))\sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{3ce^3\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$

$$\frac{\sqrt{2}(cd^2-bed+ae^2)\sqrt{2cf-(b-\sqrt{b^2-4ac})}g\sqrt{1-\frac{2c(f+gx)}{2cf-(b-\sqrt{b^2-4ac})g}}\sqrt{1-\frac{2c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\left(\frac{e(2cf-bg+\sqrt{b^2-4ac}g)}{2c(ef-dg)}\right); \sin}{\sqrt{ce^3}\sqrt{cx^2+bx+a}}$$

$$+\frac{2\sqrt{f+gx}\sqrt{cx^2+bx+a}}{3e}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])/(d + e\*x), x]

[Out] (2\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])/(3\*e) + (Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*(c\*e\*f - 3\*c\*d\*g + b\*e\*g)\*Sqrt[f + g\*x]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(3\*c\*e^2\*g\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[a + b\*x + c\*x^2]) - (2\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*f\*(c\*e\*f - 3\*c\*d\*g + b\*e\*g)\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(3\*c\*e^2\*g\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]) - (2\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*(3\*c\*d\*(e\*f - d\*g) - e\*(2\*b\*e\*f - 3\*b\*d\*g + 2\*a\*e\*g))\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(3\*c\*e^3\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]) - (Sqrt[2]\*(c\*d^2 - b\*d\*e + a\*e^2)\*Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g]\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*EllipticPi[(e\*(2\*c\*f - b\*g + Sqrt[b^2 - 4\*a\*c]\*g))/(2\*c\*(e\*f - d\*g)), ArcSin[(Sqrt[2]\*Sqrt[c]\*Sqrt[f + g\*x])/Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g]], (b - Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)/(b + Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)]/(Sqrt[c]\*e^3\*Sqrt[a + b\*x + c\*x^2])

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{d+ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*(1/2)\*(c\*x\*\*2+b\*x+a)\*\*(1/2)/(e\*x+d), x)

[Out]  $\text{Integral}(\sqrt{f + g*x}*\sqrt{a + b*x + c*x**2}/(d + e*x), x)$

**Mathematica [C]** time = 17.4723, size = 35245, normalized size = 46.13

Result too large to show

Warning: Unable to verify antiderivative.

[In]  $\text{Integrate}[(\text{Sqrt}[f + g*x]*\text{Sqrt}[a + b*x + c*x^2])/(d + e*x), x]$

[Out] Result too large to show

**Maple [B]** time = 0.06, size = 6812, normalized size = 8.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((g*x+f)^{(1/2)}*(c*x^2+b*x+a)^{(1/2)}/(e*x+d), x)$

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}\sqrt{gx + f}}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\sqrt{c*x^2 + b*x + a}*\sqrt{g*x + f}/(e*x + d), x, \text{algorithm}=\text{"maxima"})$

[Out]  $\text{integrate}(\sqrt{c*x^2 + b*x + a}*\sqrt{g*x + f}/(e*x + d), x)$

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\sqrt{c*x^2 + b*x + a}*\sqrt{g*x + f}/(e*x + d), x, \text{algorithm}=\text{"fricas"})$

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{f + gx}\sqrt{a + bx + cx^2}}{d + ex} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] integrate((g\*x+f)\*\*(1/2)\*(c\*x\*\*2+b\*x+a)\*\*(1/2)/(e\*x+d),x)

[Out] Integral(sqrt(f + g\*x)\*sqrt(a + b\*x + c\*x\*\*2)/(d + e\*x), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}\sqrt{gx + f}}{ex + d} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)\*sqrt(g\*x + f)/(e\*x + d),x, algorithm="giac")

[Out] integrate(sqrt(c\*x^2 + b\*x + a)\*sqrt(g\*x + f)/(e\*x + d), x)

$$3.891 \quad \int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^2} dx$$

**Optimal.** Leaf size=743

$$\frac{\sqrt{2cf-g}\left(b-\sqrt{b^2-4ac}\right)\sqrt{1-\frac{2c(f+gx)}{2cf-g(b-\sqrt{b^2-4ac})}}\sqrt{1-\frac{2c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}(cd(2ef-3dg)-e(aeg-2bdg+bef))\left(\frac{e(2cf-bg)}{2c}\right)}{\sqrt{2}\sqrt{ce^3}\sqrt{a+bx+cx^2}(ef-dg)} + \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}(2beg-c(3dg+ef))F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right)\left|\frac{2\sqrt{b^2-4ac}g}{(b+\sqrt{b^2-4ac})g-2cf}\right.}{ce^3\sqrt{f+gx}\sqrt{a+x(b+cx)}} + \frac{3\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right)\left|\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right.}{\sqrt{2}e^2\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}} - \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{e(d+ex)}$$

```
[Out] -((Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(e*(d + e*x))) + (3*Sqrt[b^2 - 4*a*c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]/(Sqrt[2]*e^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(2*b*e*g - c*(e*f + 3*d*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (2*Sqrt[b^2 - 4*a*c]*g)/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]/(c*e^3*Sqrt[f + g*x]*Sqrt[a + x*(b + c*x)]) + (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)]/(Sqrt[2]*Sqrt[c]*e^3*(e*f - d*g)*Sqrt[a + b*x + c*x^2])
```

**Rubi [A]** time = 7.75098, antiderivative size = 934, normalized size of antiderivative = 1.26, number

of steps used = 15, number of rules used = 10, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$

$$\frac{3\sqrt{b^2 - 4ac}\sqrt{f + gx}\sqrt{-\frac{c(cx^2 + bx + a)}{b^2 - 4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{\sqrt{2}e^2\sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\sqrt{cx^2 + bx + a}}$$

$$-\frac{3\sqrt{2}\sqrt{b^2 - 4ac}f\sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{e^2\sqrt{f + gx}\sqrt{cx^2 + bx + a}}$$

$$+\frac{\sqrt{2}\sqrt{b^2 - 4ac}(2cef - 3cdg + 2beg)\sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{ce^3\sqrt{f + gx}\sqrt{cx^2 + bx + a}}$$

$$+\frac{\sqrt{2cf - (b - \sqrt{b^2 - 4ac})}g(cd(2ef - 3dg) - e(bef - 2bdg + aeg))\sqrt{1 - \frac{2c(f+gx)}{2cf-(b-\sqrt{b^2-4ac})g}}\sqrt{1 - \frac{2c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}}{\sqrt{2}\sqrt{ce^3(ef - dg)}\sqrt{cx^2 + bx + a}}$$

$$-\frac{\sqrt{f + gx}\sqrt{cx^2 + bx + a}}{e(d + ex)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])/(d + e\*x)^2, x]

[Out] -((Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])/(e\*(d + e\*x))) + (3\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[f + g\*x]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(Sqrt[2]\*e^2\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[a + b\*x + c\*x^2]) - (3\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*f\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(e^2\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]) + (Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*(2\*c\*e\*f - 3\*c\*d\*g + 2\*b\*e\*g)\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(c\*e^3\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]) + (Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g]\*(c\*d\*(2\*e\*f - 3\*d\*g) - e\*(b\*e\*f - 2\*b\*d\*g + a\*e\*g))\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*EllipticPi[(e\*(2\*c\*f - b\*g + Sqrt[b^2 - 4\*a\*c]\*g))/(2\*c\*(e\*f - d\*g)], ArcSin[(Sqrt[2]\*Sqrt[c]\*Sqrt[f + g\*x])/Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g]], (b - Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)/(b + Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)]/(Sqrt[2]\*Sqrt[c]\*e^3\*(e\*f - d\*g)\*Sqrt[a + b\*x + c\*x^2])

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{f + gx}\sqrt{a + bx + cx^2}}{(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*(1/2)\*(c\*x\*\*2+b\*x+a)\*\*(1/2)/(e\*x+d)\*\*2, x)

[Out] Integral(sqrt(f + g\*x)\*sqrt(a + b\*x + c\*x\*\*2)/(d + e\*x)\*\*2, x)

**Mathematica [C]** time = 15.3883, size = 16573, normalized size = 22.31

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])/(d + e\*x)^2,x]

[Out] Result too large to show

**Maple [B]** time = 0.08, size = 16696, normalized size = 22.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^(1/2)\*(c\*x^2+b\*x+a)^(1/2)/(e\*x+d)^2,x)

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}\sqrt{gx + f}}{(ex + d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)\*sqrt(g\*x + f)/(e\*x + d)^2,x, algorithm="maxima")

[Out] integrate(sqrt(c\*x^2 + b\*x + a)\*sqrt(g\*x + f)/(e\*x + d)^2, x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)\*sqrt(g\*x + f)/(e\*x + d)^2,x, algorithm="fricas")

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{f + gx}\sqrt{a + bx + cx^2}}{(d + ex)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**2,x)
```

```
[Out] Integral(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)/(d + e*x)**2, x)
```

---

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(e*x + d)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.892 \quad \int \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)^3} dx$$

**Optimal.** Leaf size=1034

$$\frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)(cd(2ef-3dg)-e(bef-2bdg+aeq))}{4\sqrt{2}e^2(cd^2-bed+ae^2)(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\sqrt{cx^2+bx+a}} + \frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}(cd(2ef-3dg)-e(bef-2bdg+aeq))}{4e(cd^2-bed+ae^2)(ef-dg)(d+ex)}$$

$$\frac{\sqrt{b^2-4ac}(bef+4bdg-5aeg)-cd(2ef+3dg)\sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{b^2-4ac}}{(b+\sqrt{b^2-4ac})g}\right)}{2\sqrt{2}e^3(cd^2+e(ae-bd))\sqrt{f+gx}\sqrt{a+x(b+cx)}}$$

$$\frac{\sqrt{2cf-bg+\sqrt{b^2-4ac}g}(b^2f^2e^4+a^2g^2e^4-2ac(2e^2f^2-6degf+3d^2g^2)e^2-2bg(afe^3+cd^2(3ef-2dg))e+c^2d^3g(4e^3+cd^2(3ef-2dg)))}{4\sqrt{2}\sqrt{ce^3}(cd^2+e(ae-bd))}$$

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2e(d+ex)^2}$$

```
[Out] -(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(2*e*(d + e*x)^2) + ((c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(4*e*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*(d + e*x)) - (Sqrt[b^2 - 4*a*c]*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]/(4*Sqrt[2]*e^2*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[b^2 - 4*a*c]*(-c*d*(2*e*f + 3*d*g) + e*(b*e*f + 4*b*d*g - 5*a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (2*Sqrt[b^2 - 4*a*c]*g)/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]/(2*Sqrt[2]*e^3*(c*d^2 + e*(-b*d) + a*e))*Sqrt[f + g*x]*Sqrt[a + x*(b + c*x)]) + (Sqrt[2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g]*(b^2*e^4*f^2 + a^2*e^4*g^2 + c^2*d^3*g*(4*e*f - 3*d*g) - 2*a*c*e^2*(2*e^2*f^2 - 6*d*e*f*g + 3*d^2*g^2) - 2*b*e*g*(a*e^3*f + c*d^2*(3*e*f - 2*d*g)))*Sqrt[(g*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[(g*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(2*c*e*f - b*e*g + Sqrt[b^2 - 4*a*c]*e*g)/(2*c*e*f - 2*c*d*g), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g]], (2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]/(4*Sqrt[2]*Sqrt[c]*e^3*(c*d^2 + e*(-b*d) + a*e))*(e*f - d*g)^2*Sqrt[a + x*(b + c*x)])
```

**Rubi [A]** time = 18.6669, antiderivative size = 1705, normalized size of antiderivative = 1.65, number of steps used = 25, number of rules used = 11, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$

result too large to display

Warning: Unable to verify antiderivative.

[In] Int[(Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])/(d + e\*x)^3, x]

```
[Out] -(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(2*e*(d + e*x)^2) + ((c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(4*e*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*(d +
```

$$\begin{aligned}
& e^x) - (\text{Sqrt}[b^2 - 4*a*c]*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g)) * \text{Sqrt}[f + g*x] * \text{Sqrt}[ -((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))] * \text{EllipticE}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (-2*\text{Sqrt}[b^2 - 4*a*c]*g)/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]/(4*\text{Sqrt}[2]*e^2*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*\text{Sqrt}[(c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)] * \text{Sqrt}[a + b*x + c*x^2]) + (3*\text{Sqrt}[b^2 - 4*a*c]*g*\text{Sqrt}[(c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)] * \text{Sqrt}[ -((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))] * \text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (-2*\text{Sqrt}[b^2 - 4*a*c]*g)/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]/( \text{Sqrt}[2]*e^3*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + b*x + c*x^2]) + (\text{Sqrt}[b^2 - 4*a*c]*f*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g)) * \text{Sqrt}[(c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)] * \text{Sqrt}[ -((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))] * \text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (-2*\text{Sqrt}[b^2 - 4*a*c]*g)/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]/(2*\text{Sqrt}[2]*e^2*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + b*x + c*x^2]) - (\text{Sqrt}[b^2 - 4*a*c]*d*g*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g)) * \text{Sqrt}[(c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)] * \text{Sqrt}[ -((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))] * \text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (-2*\text{Sqrt}[b^2 - 4*a*c]*g)/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]/(2*\text{Sqrt}[2]*e^3*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + b*x + c*x^2]) - (\text{Sqrt}[2*c*f - (b - \text{Sqrt}[b^2 - 4*a*c])*g]*(c*e*f - 3*c*d*g + b*e*g)*\text{Sqrt}[1 - (2*c*(f + g*x))/(2*c*f - (b - \text{Sqrt}[b^2 - 4*a*c])*g)] * \text{Sqrt}[1 - (2*c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)] * \text{EllipticPi}[(e*(2*c*f - b*g + \text{Sqrt}[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), \text{ArcSin}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[f + g*x])/ \text{Sqrt}[2*c*f - (b - \text{Sqrt}[b^2 - 4*a*c])*g]], (b - \text{Sqrt}[b^2 - 4*a*c] - (2*c*f)/g)/(b + \text{Sqrt}[b^2 - 4*a*c] - (2*c*f)/g)]/( \text{Sqrt}[2]*\text{Sqrt}[c]*e^3*(e*f - d*g)*\text{Sqrt}[a + b*x + c*x^2]) + (\text{Sqrt}[2*c*f - (b - \text{Sqrt}[b^2 - 4*a*c])*g]*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))^2*\text{Sqrt}[1 - (2*c*(f + g*x))/(2*c*f - (b - \text{Sqrt}[b^2 - 4*a*c])*g)] * \text{Sqrt}[1 - (2*c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)] * \text{EllipticPi}[(e*(2*c*f - b*g + \text{Sqrt}[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), \text{ArcSin}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[f + g*x])/ \text{Sqrt}[2*c*f - (b - \text{Sqrt}[b^2 - 4*a*c])*g]], (b - \text{Sqrt}[b^2 - 4*a*c] - (2*c*f)/g)/(b + \text{Sqrt}[b^2 - 4*a*c] - (2*c*f)/g)]/(4*\text{Sqrt}[2]*\text{Sqrt}[c]*e^3*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^2*\text{Sqrt}[a + b*x + c*x^2])
\end{aligned}$$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{f + gx}\sqrt{a + bx + cx^2}}{(d + ex)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**3,x)`

[Out] `Integral(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)/(d + e*x)**3, x)`

**Mathematica [C]** time = 19.8898, size = 33765, normalized size = 32.65

Result too large to show

Warning: Unable to verify antiderivative.

[In] `Integrate[(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(d + e*x)^3,x]`

[Out] Result too large to show

---

**Maple [B]** time = 0.146, size = 55360, normalized size = 53.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^(1/2)*(c*x^2+b*x+a)^(1/2)/(e*x+d)^3,x)`

[Out] result too large to display

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}\sqrt{gx + f}}{(ex + d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(e*x + d)^3,x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(e*x + d)^3, x)`

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(e*x + d)^3,x, algorithm="fricas")`

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)**(1/2)*(c*x**2+b*x+a)**(1/2)/(e*x+d)**3,x)`

[Out] Timed out

---

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)/(e*x + d)^3,x, algorithm="giac")`

[Out] Timed out



$$3.893 \quad \int \frac{(d+ex)^3 \sqrt{a+bx+cx^2}}{\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=1098

result too large to display

[Out]  $(2*(8*b^3*e^3*g^3 + 3*b*c*e^2*g^2*(5*b*e*f - 12*b*d*g - 9*a*e*g) - c^3*(152*e^3*f^3 - 408*d*e^2*f^2*g + 336*d^2*e*f*g^2 - 70*d^3*g^3) - 3*c^2*e*g*(6*a*e*g*(2*e*f - 5*d*g) - b*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2)))*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + b*x + c*x^2])/(315*c^3*g^4) + (2*(d + e*x)^3*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + b*x + c*x^2])/(9*g) - (2*e*(6*b^2*e^2*g^2 + c*e*g*(17*b*e*f - 27*b*d*g - 14*a*e*g) - 2*c^2*(64*e^2*f^2 - 111*d*e*f*g + 42*d^2*g^2))*(f + g*x)^{(3/2)}*\text{Sqrt}[a + b*x + c*x^2])/(315*c^2*g^4) - (2*e^2*(8*c*e*f - 6*c*d*g - b*e*g)*(f + g*x)^{(5/2)}*\text{Sqrt}[a + b*x + c*x^2])/(63*c*g^4) - (\text{Sqrt}[2]*\text{Sqrt}[b^2 - 4*a*c]*(16*b^4*e^3*g^4 + 8*b^2*c*e^2*g^3*(2*b*e*f - 9*b*d*g - 9*a*e*g) - 2*c^4*f*(64*e^3*f^3 - 216*d*e^2*f^2*g + 252*d^2*e*f*g^2 - 105*d^3*g^3) + 3*c^2*e*g^2*(14*a^2*e^2*g^2 - a*b*e*g*(19*e*f - 87*d*g) + b^2*(7*e^2*f^2 - 27*d*e*f*g + 42*d^2*g^2)) - c^3*g*(6*a*e*g*(10*e^2*f^2 - 39*d*e*f*g + 63*d^2*g^2) - b*(40*e^3*f^3 - 144*d*e^2*f^2*g + 189*d^2*e*f*g^2 - 105*d^3*g^3)))*\text{Sqrt}[f + g*x]*\text{Sqrt}[-(c*(a + b*x + c*x^2))/(b^2 - 4*a*c)]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (-2*\text{Sqrt}[b^2 - 4*a*c]*g)/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]/(315*c^4*g^5*\text{Sqrt}[(c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]*\text{Sqrt}[a + b*x + c*x^2]) - (2*\text{Sqrt}[2]*\text{Sqrt}[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*(8*b^3*e^3*g^3 + 3*b*c*e^2*g^2*(5*b*e*f - 12*b*d*g - 9*a*e*g) + 2*c^3*(64*e^3*f^3 - 216*d*e^2*f^2*g + 252*d^2*e*f*g^2 - 105*d^3*g^3) - 3*c^2*e*g*(6*a*e*g*(2*e*f - 5*d*g) - b*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2)))*\text{Sqrt}[(c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]*\text{Sqrt}[-(c*(a + b*x + c*x^2))/(b^2 - 4*a*c)]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (-2*\text{Sqrt}[b^2 - 4*a*c]*g)/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]/(315*c^4*g^5*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + b*x + c*x^2])$

**Rubi [A]** time = 10.5093, antiderivative size = 1098, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$

$$\frac{2\sqrt{f+gx}\sqrt{cx^2+bx+a}(d+ex)^3}{9g}$$

$$\sqrt{2}\sqrt{b^2-4ac}(-2f(64e^3f^3-216de^2gf^2+252d^2eg^2f-105d^3g^3)c^4-g(6aeg(10e^2f^2-39degf+63d^2g^2)-b(40e^3f^3$$

$$2\sqrt{2}\sqrt{b^2-4ac}(cf^2-bgf+ag^2)(2(64e^3f^3-216de^2gf^2+252d^2eg^2f-105d^3g^3)c^3-3eg(6aeg(2ef-5dg)-b(8e^2f^2$$

$$\frac{2e^2(8cef-6cdg-beg)(f+gx)^{5/2}\sqrt{cx^2+bx+a}}{63cg^4}$$

$$\frac{2e(-2(64e^2f^2-111degf+42d^2g^2)c^2+eg(17bef-27bdg-14aeg)c+6b^2e^2g^2)(f+gx)^{3/2}\sqrt{cx^2+bx+a}}{315c^2g^4}$$

$$+\frac{2(-152e^3f^3-408de^2gf^2+336d^2eg^2f-70d^3g^3)c^3-3eg(6aeg(2ef-5dg)-b(8e^2f^2-24degf+21d^2g^2))c^2+3be^2}{315c^3g^4}$$

Warning: Unable to verify antiderivative.

[In] Int[((d + e\*x)^3\*Sqrt[a + b\*x + c\*x^2])/Sqrt[f + g\*x], x]

```
[Out] (2*(8*b^3*e^3*g^3 + 3*b*c*e^2*g^2*(5*b*e*f - 12*b*d*g - 9*a*e*g)
- c^3*(152*e^3*f^3 - 408*d*e^2*f^2*g + 336*d^2*e*f*g^2 - 70*d^3*g
^3) - 3*c^2*e*g*(6*a*e*g*(2*e*f - 5*d*g) - b*(8*e^2*f^2 - 24*d*e*
f*g + 21*d^2*g^2)))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(315*c^3
*g^4) + (2*(d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(9*g)
- (2*e*(6*b^2*e^2*g^2 + c*e*g*(17*b*e*f - 27*b*d*g - 14*a*e*g) -
2*c^2*(64*e^2*f^2 - 111*d*e*f*g + 42*d^2*g^2))*(f + g*x)^(3/2)*S
qrt[a + b*x + c*x^2])/(315*c^2*g^4) - (2*e^2*(8*c*e*f - 6*c*d*g -
b*e*g)*(f + g*x)^(5/2)*Sqrt[a + b*x + c*x^2])/(63*c*g^4) - (Sqrt
[2]*Sqrt[b^2 - 4*a*c]*(16*b^4*e^3*g^4 + 8*b^2*c*e^2*g^3*(2*b*e*f
- 9*b*d*g - 9*a*e*g) - 2*c^4*f*(64*e^3*f^3 - 216*d*e^2*f^2*g + 25
2*d^2*e*f*g^2 - 105*d^3*g^3) + 3*c^2*e*g^2*(14*a^2*e^2*g^2 - a*b*
e*g*(19*e*f - 87*d*g) + b^2*(7*e^2*f^2 - 27*d*e*f*g + 42*d^2*g^2)
) - c^3*g*(6*a*e*g*(10*e^2*f^2 - 39*d*e*f*g + 63*d^2*g^2) - b*(40
*e^3*f^3 - 144*d*e^2*f^2*g + 189*d^2*e*f*g^2 - 105*d^3*g^3)))*Sqr
t[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE
[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]) + 2*c*x]/Sqrt[b^2 - 4*a*c]]/S
qrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c]
)*g)]/(315*c^4*g^5*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4
*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*
(c*f^2 - b*f*g + a*g^2)*(8*b^3*e^3*g^3 + 3*b*c*e^2*g^2*(5*b*e*f -
12*b*d*g - 9*a*e*g) + 2*c^3*(64*e^3*f^3 - 216*d*e^2*f^2*g + 252*
d^2*e*f*g^2 - 105*d^3*g^3) - 3*c^2*e*g*(6*a*e*g*(2*e*f - 5*d*g) -
b*(8*e^2*f^2 - 24*d*e*f*g + 21*d^2*g^2)))*Sqrt[(c*(f + g*x))/(2*
c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b
^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]) + 2*c*
x]/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)]/(315*c^4*g^5*Sqrt[f + g*x]*Sqrt[a +
b*x + c*x^2])
```

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((e*x+d)**3*(c*x**2+b*x+a)**(1/2)/(g*x+f)**(1/2),x)
```

```
[Out] Timed out
```

**Mathematica [C]** time = 18.6184, size = 17771, normalized size = 16.18

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((d + e*x)^3*Sqrt[a + b*x + c*x^2])/Sqrt[f + g*x],x]
```

```
[Out] Result too large to show
```

**Maple [B]** time = 0.102, size = 22215, normalized size = 20.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^3*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x)
```

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}(ex + d)^3}{\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)^3/sqrt(g\*x + f), x, algorithm="maxima")

[Out] integrate(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)^3/sqrt(g\*x + f), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)\sqrt{cx^2 + bx + a}}{\sqrt{gx + f}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)^3/sqrt(g\*x + f), x, algorithm="fricas")

[Out] integral((e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3)\*sqrt(c\*x^2 + b\*x + a)/sqrt(g\*x + f), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(c\*x\*\*2+b\*x+a)\*\*(1/2)/(g\*x+f)\*\*(1/2), x)

[Out] Timed out

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)^3/sqrt(g\*x + f), x, algorithm="giac")

[Out] Timed out

$$3.894 \quad \int \frac{(d+ex)^2 \sqrt{a+bx+cx^2}}{\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=755

$$\frac{4\sqrt{f+gx}\sqrt{a+bx+cx^2} (ceg(-5aeg - 7bdg + 4bef) + 2b^2e^2g^2 + c^2 (- (10d^2g^2 - 34defg + 21e^2f^2)))}{105c^2g^3}$$

$$+ \frac{4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}} (ag^2 - bfg + cf^2) \sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}} (ceg(-5aeg - 7bdg + 4bef) + 2b^2e^2g^2 + c^2 (35d^2g^2 - 34defg + 21e^2f^2))}{105c^3g^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$+ \frac{\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}} (-c^2g (2aeg(13ef - 42dg) - b (35d^2g^2 - 42defg + 16e^2f^2)) + bceg^2(-29aeg - 28bdg + 16e^2f^2))}{105c^3g^4\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}}$$

$$- \frac{2e(f+gx)^{3/2}\sqrt{a+bx+cx^2}(-beg - 4cdg + 6cef)}{35cg^3} + \frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7g}$$

[Out]  $(-4*(2*b^2*e^2*g^2 + c*e*g*(4*b*e*f - 7*b*d*g - 5*a*e*g) - c^2*(2*1*e^2*f^2 - 34*d*e*f*g + 10*d^2*g^2))*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + b*x + c*x^2])/(105*c^2*g^3) + (2*(d + e*x)^2*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + b*x + c*x^2])/(7*g) - (2*e*(6*c*e*f - 4*c*d*g - b*e*g)*(f + g*x)^(3/2)*\text{Sqrt}[a + b*x + c*x^2])/(35*c*g^3) + (\text{Sqrt}[2]*\text{Sqrt}[b^2 - 4*a*c])*(8*b^3*e^2*g^3 + b*c*e*g^2*(9*b*e*f - 28*b*d*g - 29*a*e*g) - 2*c^3*f*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g^2) - c^2*g*(2*a*e*g*(13*e*f - 42*d*g) - b*(16*e^2*f^2 - 42*d*e*f*g + 35*d^2*g^2)))*\text{Sqrt}[f + g*x]*\text{Sqrt}[-(c*(a + b*x + c*x^2))/(b^2 - 4*a*c)]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (-2*\text{Sqrt}[b^2 - 4*a*c]*g)/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]/(105*c^3*g^4*\text{Sqrt}[(c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]*\text{Sqrt}[a + b*x + c*x^2]) + (4*\text{Sqrt}[2]*\text{Sqrt}[b^2 - 4*a*c]*(c*f^2 - b*f*g + a*g^2)*(2*b^2*e^2*g^2 + c*e*g*(4*b*e*f - 7*b*d*g - 5*a*e*g) + c^2*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g^2))*\text{Sqrt}[(c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]*\text{Sqrt}[-(c*(a + b*x + c*x^2))/(b^2 - 4*a*c)]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (-2*\text{Sqrt}[b^2 - 4*a*c]*g)/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]/(105*c^3*g^4*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + b*x + c*x^2])$

**Rubi [A]** time = 3.97356, antiderivative size = 755, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$

$$\frac{4\sqrt{f+gx}\sqrt{a+bx+cx^2} (ceg(-5aeg - 7bdg + 4bef) + 2b^2e^2g^2 + c^2 (- (10d^2g^2 - 34defg + 21e^2f^2)))}{105c^2g^3}$$

$$+ \frac{4\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}} (ag^2 - bfg + cf^2) \sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}} (ceg(-5aeg - 7bdg + 4bef) + 2b^2e^2g^2 + c^2 (35d^2g^2 - 34defg + 21e^2f^2))}{105c^3g^4\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$+ \frac{\sqrt{2}\sqrt{b^2 - 4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}} (-c^2g (2aeg(13ef - 42dg) - b (35d^2g^2 - 42defg + 16e^2f^2)) + bceg^2(-29aeg - 28bdg + 16e^2f^2))}{105c^3g^4\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}}$$

$$- \frac{2e(f+gx)^{3/2}\sqrt{a+bx+cx^2}(-beg - 4cdg + 6cef)}{35cg^3} + \frac{2(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7g}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^2\*Sqrt[a + b\*x + c\*x^2])/Sqrt[f + g\*x],x]

[Out] 
$$\begin{aligned} & (-4*(2*b^2*e^2*g^2 + c*e*g*(4*b*e*f - 7*b*d*g - 5*a*e*g) - c^2*(2 \\ & 1*e^2*f^2 - 34*d*e*f*g + 10*d^2*g^2))*Sqrt[f + g*x]*Sqrt[a + b*x \\ & + c*x^2])/(105*c^2*g^3) + (2*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b \\ & *x + c*x^2])/(7*g) - (2*e*(6*c*e*f - 4*c*d*g - b*e*g)*(f + g*x)^( \\ & 3/2)*Sqrt[a + b*x + c*x^2])/(35*c*g^3) + (Sqrt[2]*Sqrt[b^2 - 4*a \\ & c]*(8*b^3*e^2*g^3 + b*c*e*g^2*(9*b*e*f - 28*b*d*g - 29*a*e*g) - 2 \\ & *c^3*f*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g^2) - c^2*g*(2*a*e*g*(1 \\ & 3*e*f - 42*d*g) - b*(16*e^2*f^2 - 42*d*e*f*g + 35*d^2*g^2)))*Sqrt \\ & [f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ \\ & ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqr \\ & t[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c]) \\ & *g)]/(105*c^3*g^4*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4* \\ & a*c])*g)]*Sqrt[a + b*x + c*x^2]) + (4*Sqrt[2]*Sqrt[b^2 - 4*a*c]*( \\ & c*f^2 - b*f*g + a*g^2)*(2*b^2*e^2*g^2 + c*e*g*(4*b*e*f - 7*b*d*g \\ & - 5*a*e*g) + c^2*(24*e^2*f^2 - 56*d*e*f*g + 35*d^2*g^2))*Sqrt[(c* \\ & (f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b* \\ & x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - \\ & 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a* \\ & c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]/(105*c^3*g^4*Sqrt[f + \\ & g*x]*Sqrt[a + b*x + c*x^2]) \end{aligned}$$

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*2\*(c\*x\*\*2+b\*x+a)\*\*(1/2)/(g\*x+f)\*\*(1/2),x)

[Out] Timed out

**Mathematica [C]** time = 15.9825, size = 10030, normalized size = 13.28

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((d + e\*x)^2\*Sqrt[a + b\*x + c\*x^2])/Sqrt[f + g\*x],x]

[Out] Result too large to show

**Maple [B]** time = 0.063, size = 12923, normalized size = 17.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^2\*(c\*x^2+b\*x+a)^(1/2)/(g\*x+f)^(1/2),x)

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}(ex + d)^2}{\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)^2/sqrt(g\*x + f),x, algorithm="maxima")

[Out] integrate(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)^2/sqrt(g\*x + f), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e^2x^2 + 2dex + d^2)\sqrt{cx^2 + bx + a}}{\sqrt{gx + f}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)^2/sqrt(g\*x + f),x, algorithm="fricas")

[Out] integral((e^2\*x^2 + 2\*d\*e\*x + d^2)\*sqrt(c\*x^2 + b\*x + a)/sqrt(g\*x + f), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^2 \sqrt{a + bx + cx^2}}{\sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2\*(c\*x\*\*2+b\*x+a)\*\*(1/2)/(g\*x+f)\*\*(1/2),x)

[Out] Integral((d + e\*x)\*\*2\*sqrt(a + b\*x + c\*x\*\*2)/sqrt(f + g\*x), x)

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)^2/sqrt(g\*x + f),x, algorithm="giac")

[Out] Timed out

$$3.895 \quad \int \frac{(d+ex)\sqrt{a+bx+cx^2}}{\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=519

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ag^2-bfg+cf^2)\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}(beg-10cdg+8cef)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right)}{15c^2g^3\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(cg(-6aeg-5bdg+3bef)+2b^2eg^2-2c^2f(4ef-5dg))E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right)}{15c^2g^3\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}} - \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}(-beg-5cdg+4cef-3ceg)}{15cg^2}$$

[Out]  $(-2*\text{Sqrt}[f + g*x]*(4*c*e*f - 5*c*d*g - b*e*g - 3*c*e*g*x)*\text{Sqrt}[a + b*x + c*x^2])/(15*c*g^2) - (\text{Sqrt}[2]*\text{Sqrt}[b^2 - 4*a*c]*(2*b^2*e*g^2 - 2*c^2*f*(4*e*f - 5*d*g) + c*g*(3*b*e*f - 5*b*d*g - 6*a*e*g)))*\text{Sqrt}[f + g*x]*\text{Sqrt}[-(c*(a + b*x + c*x^2))/(b^2 - 4*a*c)]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (-2*\text{Sqrt}[b^2 - 4*a*c]*g)/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c]*g))]/(15*c^2*g^3*\text{Sqrt}[(c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c]*g))]*\text{Sqrt}[a + b*x + c*x^2]) - (2*\text{Sqrt}[2]*\text{Sqrt}[b^2 - 4*a*c]*(8*c*e*f - 10*c*d*g + b*e*g)*(c*f^2 - b*f*g + a*g^2)*\text{Sqrt}[(c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c]*g)]*\text{Sqrt}[-(c*(a + b*x + c*x^2))/(b^2 - 4*a*c)])*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (-2*\text{Sqrt}[b^2 - 4*a*c]*g)/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c]*g))]/(15*c^2*g^3*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + b*x + c*x^2])$

**Rubi [A]** time = 1.34783, antiderivative size = 519, normalized size of antiderivative = 1., number of rules used = 6, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ag^2-bfg+cf^2)\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}(beg-10cdg+8cef)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right)}{15c^2g^3\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(cg(-6aeg-5bdg+3bef)+2b^2eg^2-2c^2f(4ef-5dg))E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right)}{15c^2g^3\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}} - \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}(-beg-5cdg+4cef-3ceg)}{15cg^2}$$

Antiderivative was successfully verified.

[In]  $\text{Int}(((d + e*x)*\text{Sqrt}[a + b*x + c*x^2])/ \text{Sqrt}[f + g*x], x)$

[Out]  $(-2*\text{Sqrt}[f + g*x]*(4*c*e*f - 5*c*d*g - b*e*g - 3*c*e*g*x)*\text{Sqrt}[a + b*x + c*x^2])/(15*c*g^2) - (\text{Sqrt}[2]*\text{Sqrt}[b^2 - 4*a*c]*(2*b^2*e*g^2 - 2*c^2*f*(4*e*f - 5*d*g) + c*g*(3*b*e*f - 5*b*d*g - 6*a*e*g)))*\text{Sqrt}[f + g*x]*\text{Sqrt}[-(c*(a + b*x + c*x^2))/(b^2 - 4*a*c)]*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (-2*\text{Sqrt}[b^2 - 4*a*c]*g)/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c]*g))]/(15*c^2*g^3*\text{Sqrt}[(c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c]*g))]*\text{Sqrt}[a + b*x + c*x^2]) - (2*\text{Sqrt}[2]*\text{Sqrt}[b^2 - 4*a*c]*(8*c*e*f - 10*c*d*g + b*e*g)*(c*f^2 - b*f*g + a*g^2)*\text{Sqrt}[(c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c]*g)]*\text{Sqrt}[-(c*(a + b*x + c*x^2))/(b^2 - 4*a*c)])*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (-2*\text{Sqrt}[b^2 - 4*a*c]*g)/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c]*g))]/(15*c^2*g^3*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + b*x + c*x^2])$







$$\frac{c^2 f^2 - b f g + a g^2}{(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2})} \sqrt{c + \frac{c^2 f^2 - b f g + a g^2}{(f + g x)^2} + \frac{-2 c f + b g}{(f + g x)}} \left( \frac{c \sqrt{((f + g x)^2 (c(-1 + f/(f + g x))^2 + (g(b - (b f)/(f + g x) + (a g)/(f + g x)))/g^2))}}{(15 c^2 g^4 \sqrt{a + b x + c x^2})} \right)$$

**Maple [B]** time = 0.045, size = 6207, normalized size = 12.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)*(c*x^2+b*x+a)^(1/2)/(g*x+f)^(1/2),x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}(ex + d)}{\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)/sqrt(g*x + f),x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)/sqrt(g*x + f), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx + a}(ex + d)}{\sqrt{gx + f}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)/sqrt(g*x + f),x, algorithm="fricas")`

[Out] `integral(sqrt(c*x^2 + b*x + a)*(e*x + d)/sqrt(g*x + f), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex) \sqrt{a + bx + cx^2}}{\sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)*(c*x**2+b*x+a)**(1/2)/(g*x+f)**(1/2),x)`

[Out] `Integral((d + e*x)*sqrt(a + b*x + c*x**2)/sqrt(f + g*x), x)`

GIAC/XCAS [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)/sqrt(g*x + f),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.896 \quad \int \frac{\sqrt{a+bx+cx^2}}{\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=444

$$\frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ag^2-bfg+cf^2)\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{3cg^2\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cf-bg)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{3cg^2\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}} + \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3g}$$

[Out] (2\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])/(3\*g) - (Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*(2\*c\*f - b\*g)\*Sqrt[f + g\*x]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))])\*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(3\*c\*g^2\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)])\*Sqrt[a + b\*x + c\*x^2] + (4\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*(c\*f^2 - b\*f\*g + a\*g^2)\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)])\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))])\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(3\*c\*g^2\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])

**Rubi [A]** time = 0.962545, antiderivative size = 444, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$

$$\frac{4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ag^2-bfg+cf^2)\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{3cg^2\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(2cf-bg)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{3cg^2\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}} + \frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3g}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x + c\*x^2]/Sqrt[f + g\*x], x]

[Out] (2\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])/(3\*g) - (Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*(2\*c\*f - b\*g)\*Sqrt[f + g\*x]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))])\*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(3\*c\*g^2\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)])\*Sqrt[a + b\*x + c\*x^2] + (4\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*(c\*f^2 - b\*f\*g + a\*g^2)\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)])\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))])\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(3\*c\*g^2\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])

$$b*x + c*x^2])$$

**Rubi in Sympy [A]** time = 157.082, size = 418, normalized size = 0.94

$$\frac{2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3g} + \frac{4\sqrt{2}\sqrt{\frac{c(-f-gx)}{bg-2cf+g\sqrt{-4ac+b^2}}}\sqrt{\frac{c(a+bx+cx^2)}{4ac-b^2}}\sqrt{-4ac+b^2}(ag^2-bfg+cf^2)F\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2}\right)\right)\frac{2g\sqrt{-4ac+b^2}}{bg-2cf+g\sqrt{-4ac+b^2}}}{3cg^2\sqrt{f+gx}\sqrt{a+bx+cx^2}} + \frac{\sqrt{2}\sqrt{\frac{c(a+bx+cx^2)}{4ac-b^2}}\sqrt{f+gx}\sqrt{-4ac+b^2}(bg-2cf)E\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2}\right)\right)\frac{2g\sqrt{-4ac+b^2}}{bg-2cf+g\sqrt{-4ac+b^2}}}{3cg^2\sqrt{\frac{c(-f-gx)}{bg-2cf+g\sqrt{-4ac+b^2}}}\sqrt{a+bx+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x**2+b*x+a)**(1/2)/(g*x+f)**(1/2),x)`

[Out]  $2*\sqrt{f+g*x}*\sqrt{a+b*x+c*x**2}/(3*g) + 4*\sqrt{2}*\sqrt{c*(-f-g*x)/(b*g-2*c*f+g*\sqrt{-4*a*c+b**2})}*\sqrt{c*(a+b*x+c*x**2)/(4*a*c-b**2)}*\sqrt{-4*a*c+b**2}*(a*g**2-b*f*g+c*f**2)*\operatorname{elliptic\_f}(\operatorname{asin}(\sqrt{2}*\sqrt{(b+2*c*x+\sqrt{-4*a*c+b**2})/\sqrt{-4*a*c+b**2}}/2), 2*g*\sqrt{-4*a*c+b**2}/(b*g-2*c*f+g*\sqrt{-4*a*c+b**2}))/((3*c*g**2*\sqrt{f+g*x}*\sqrt{a+b*x+c*x**2}) + \sqrt{2}*\sqrt{c*(a+b*x+c*x**2)/(4*a*c-b**2)}*\sqrt{f+g*x}*\sqrt{-4*a*c+b**2}*(b*g-2*c*f)*\operatorname{elliptic\_e}(\operatorname{asin}(\sqrt{2}*\sqrt{(b+2*c*x+\sqrt{-4*a*c+b**2})/\sqrt{-4*a*c+b**2}}/2), 2*g*\sqrt{-4*a*c+b**2}/(b*g-2*c*f+g*\sqrt{-4*a*c+b**2}))/((3*c*g**2*\sqrt{c*(-f-g*x)/(b*g-2*c*f+g*\sqrt{-4*a*c+b**2})}*\sqrt{a+b*x+c*x**2}))$

**Mathematica [C]** time = 11.8593, size = 591, normalized size = 1.33

$$2 \left( \frac{i(f+gx) \sqrt{1 - \frac{2(g(ag-bf)+cf^2)}{(f+gx)(\sqrt{g^2(b^2-4ac)-bg+2cf}})}}{\sqrt{g^2(b^2-4ac)+bg-2cf}} + 1 \right) \left( -2cf\sqrt{g^2(b^2-4ac)+bg\sqrt{g^2(b^2-4ac)+4acg^2-b^2g^2}} F \left( i \sinh^{-1} \left( \frac{\sqrt{2} \sqrt{\frac{cf^2-bg+cf}{-2cf+bg+\sqrt{g^2(b^2-4ac)+4acg^2-b^2g^2}}}}{\sqrt{f+g}}$$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[a + b*x + c*x^2]/Sqrt[f + g*x],x]`

[Out]  $(2*((g^2*(-2*c*f+b*g)*(a+x*(b+c*x)))/(c*\sqrt{f+g*x}) + g^2*\sqrt{f+g*x}*(a+x*(b+c*x)) + ((I/2)*(f+g*x)*\sqrt{1-(2*(c*f^2+g*(-b*f)+a*g))}/((2*c*f-b*g+\sqrt{(b^2-4*a*c)*g^2})*(f+g*x)))*\sqrt{1+(2*(c*f^2+g*(-b*f)+a*g))}/((-2*c*f+b*g+\sqrt{(b^2-4*a*c)*g^2})*(f+g*x)))*(-((-2*c*f+b*g)*(2*c*f-b*g+\sqrt{(b^2-4*a*c)*g^2})*\operatorname{EllipticE}[I*\operatorname{ArcSinh}[(\sqrt{2}*\sqrt{(c*f^2-b*f*g+a*g^2)})/(-2*c*f+b*g+\sqrt{(b^2-4*a*c)*g^2})*\sqrt{f+g*x}], -((-2*c*f+b*g+\sqrt{(b^2-4*a*c)*g^2}))/((2*c*f-b*g+\sqrt{(b^2-4*a*c)*g^2}))]) + (-b^2*g^2+4*a*c*g^2-2*c*f*\sqrt{(b^2-4*a*c)*g^2}+b*g*\sqrt{(b^2-4*a*c)*g^2}))/((3*c*g**2*\sqrt{c*(-f-g*x)/(b*g-2*c*f+g*\sqrt{-4*a*c+b**2})}*\sqrt{a+b*x+c*x**2}))$

$$\wedge 2]) * \text{EllipticF}[I * \text{ArcSinh}[(\text{Sqrt}[2] * \text{Sqrt}[(c * f^2 - b * f * g + a * g^2) / (-2 * c * f + b * g + \text{Sqrt}[(b^2 - 4 * a * c) * g^2])]) / \text{Sqrt}[f + g * x]], -((-2 * c * f + b * g + \text{Sqrt}[(b^2 - 4 * a * c) * g^2]) / (2 * c * f - b * g + \text{Sqrt}[(b^2 - 4 * a * c) * g^2])))] / (\text{Sqrt}[2] * c * \text{Sqrt}[(c * f^2 + g * (-b * f) + a * g)) / (-2 * c * f + b * g + \text{Sqrt}[(b^2 - 4 * a * c) * g^2])]) / (3 * g^3 * \text{Sqrt}[a + x * (b + c * x)])$$

**Maple [B]** time = 0.032, size = 1854, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((c * x^2 + b * x + a)^{1/2} / (g * x + f)^{1/2}, x)$

[Out] 
$$\begin{aligned} & -2/3 * (c * x^2 + b * x + a)^{1/2} * (g * x + f)^{1/2} / c * (2^{1/2} * (-g * x + f) * c / (g * \\ & (-4 * a * c + b^2)^{1/2} + b * g - 2 * c * f))^{1/2} * (g * (-2 * c * x + (-4 * a * c + b^2)^{1/2} \\ & ) - b) / (2 * c * f - b * g + g * (-4 * a * c + b^2)^{1/2})^{1/2} * (g * (b + 2 * c * x + (-4 * a * c + \\ & b^2)^{1/2})) / (g * (-4 * a * c + b^2)^{1/2} + b * g - 2 * c * f))^{1/2} * \text{EllipticF}(2^{1/2} * \\ & (-g * x + f) * c / (g * (-4 * a * c + b^2)^{1/2} + b * g - 2 * c * f))^{1/2}, (-g * (-4 * \\ & a * c + b^2)^{1/2} + b * g - 2 * c * f) / (2 * c * f - b * g + g * (-4 * a * c + b^2)^{1/2}))^{1/2} \\ & ) * (-4 * a * c + b^2)^{1/2} * a * g^3 - 2^{1/2} * (-g * x + f) * c / (g * (-4 * a * c + b^2)^{1/2} \\ & ) + b * g - 2 * c * f))^{1/2} * (g * (-2 * c * x + (-4 * a * c + b^2)^{1/2}) - b) / (2 * c * f - b * \\ & g + g * (-4 * a * c + b^2)^{1/2}))^{1/2} * (g * (b + 2 * c * x + (-4 * a * c + b^2)^{1/2})) / (g * \\ & (-4 * a * c + b^2)^{1/2} + b * g - 2 * c * f))^{1/2} * \text{EllipticF}(2^{1/2} * (-g * x + f) \\ & * c / (g * (-4 * a * c + b^2)^{1/2} + b * g - 2 * c * f))^{1/2}, (-g * (-4 * a * c + b^2)^{1/2} \\ & ) + b * g - 2 * c * f) / (2 * c * f - b * g + g * (-4 * a * c + b^2)^{1/2}))^{1/2} * (-4 * a * c + b^2 \\ & )^{1/2} * b * f * g^2 + 2^{1/2} * (-g * x + f) * c / (g * (-4 * a * c + b^2)^{1/2} + b * g - 2 * c * \\ & f))^{1/2} * (g * (-2 * c * x + (-4 * a * c + b^2)^{1/2}) - b) / (2 * c * f - b * g + g * (-4 * a * c + \\ & b^2)^{1/2}))^{1/2} * (g * (b + 2 * c * x + (-4 * a * c + b^2)^{1/2})) / (g * (-4 * a * c + b^2 \\ & )^{1/2} + b * g - 2 * c * f))^{1/2} * \text{EllipticF}(2^{1/2} * (-g * x + f) * c / (g * (-4 * a * \\ & c + b^2)^{1/2} + b * g - 2 * c * f))^{1/2}, (-g * (-4 * a * c + b^2)^{1/2} + b * g - 2 * c * f) \\ & / (2 * c * f - b * g + g * (-4 * a * c + b^2)^{1/2}))^{1/2} * (-4 * a * c + b^2)^{1/2} * c * f^2 \\ & * g + 2^{1/2} * (-g * x + f) * c / (g * (-4 * a * c + b^2)^{1/2} + b * g - 2 * c * f))^{1/2} * ( \\ & g * (-2 * c * x + (-4 * a * c + b^2)^{1/2}) - b) / (2 * c * f - b * g + g * (-4 * a * c + b^2)^{1/2})) \\ & )^{1/2} * (g * (b + 2 * c * x + (-4 * a * c + b^2)^{1/2})) / (g * (-4 * a * c + b^2)^{1/2} + b * g - \\ & 2 * c * f))^{1/2} * \text{EllipticE}(2^{1/2} * (-g * x + f) * c / (g * (-4 * a * c + b^2)^{1/2} \\ & ) + b * g - 2 * c * f))^{1/2}, (-g * (-4 * a * c + b^2)^{1/2} + b * g - 2 * c * f) / (2 * c * f - b * g + \\ & g * (-4 * a * c + b^2)^{1/2}))^{1/2} * a * b * g^3 - 2 * 2^{1/2} * (-g * x + f) * c / (g * (-4 * \\ & a * c + b^2)^{1/2} + b * g - 2 * c * f))^{1/2} * (g * (-2 * c * x + (-4 * a * c + b^2)^{1/2}) - \\ & b) / (2 * c * f - b * g + g * (-4 * a * c + b^2)^{1/2}))^{1/2} * (g * (b + 2 * c * x + (-4 * a * c + b^2 \\ & )^{1/2})) / (g * (-4 * a * c + b^2)^{1/2} + b * g - 2 * c * f))^{1/2} * \text{EllipticE}(2^{1/2} * (-g * x + f) \\ & * c / (g * (-4 * a * c + b^2)^{1/2} + b * g - 2 * c * f))^{1/2}, (-g * (-4 * a * \\ & c + b^2)^{1/2} + b * g - 2 * c * f) / (2 * c * f - b * g + g * (-4 * a * c + b^2)^{1/2}))^{1/2} \\ & ) * a * c * f * g^2 - 2^{1/2} * (-g * x + f) * c / (g * (-4 * a * c + b^2)^{1/2} + b * g - 2 * c * f))^{1/2} \\ & ) * (g * (-2 * c * x + (-4 * a * c + b^2)^{1/2}) - b) / (2 * c * f - b * g + g * (-4 * a * c + b^2)^{1/2}) \\ & )^{1/2} * (g * (b + 2 * c * x + (-4 * a * c + b^2)^{1/2})) / (g * (-4 * a * c + b^2)^{1/2} + b * g - \\ & 2 * c * f))^{1/2} * \text{EllipticE}(2^{1/2} * (-g * x + f) * c / (g * (-4 * a * c + b^2)^{1/2} \\ & ) + b * g - 2 * c * f))^{1/2}, (-g * (-4 * a * c + b^2)^{1/2} + b * g - 2 * c * f) / (2 * c * \\ & f - b * g + g * (-4 * a * c + b^2)^{1/2}))^{1/2} * b^2 * f * g^2 + 3 * 2^{1/2} * (-g * x + f) \\ & ) * c / (g * (-4 * a * c + b^2)^{1/2} + b * g - 2 * c * f))^{1/2} * (g * (-2 * c * x + (-4 * a * c + b^2 \\ & )^{1/2}) - b) / (2 * c * f - b * g + g * (-4 * a * c + b^2)^{1/2}))^{1/2} * (g * (b + 2 * c * x + (-4 * a * c + b^2 \\ & )^{1/2})) / (g * (-4 * a * c + b^2)^{1/2} + b * g - 2 * c * f))^{1/2} * \text{EllipticE}(2^{1/2} * (-g * x + f) * c / (g * (-4 * a * c + b^2)^{1/2} \\ & ) + b * g - 2 * c * f))^{1/2}, (-g * (-4 * a * c + b^2)^{1/2} + b * g - 2 * c * f) / (2 * c * f - b * g + \\ & g * (-4 * a * c + b^2)^{1/2}))^{1/2} * a * b * c * f^2 * g - 2 * 2^{1/2} * (-g * x + f) * c / (g * (-4 * a * c + b^2)^{1/2} + b * \\ & g - 2 * c * f))^{1/2} * (g * (-2 * c * x + (-4 * a * c + b^2)^{1/2}) - b) / (2 * c * f - b * g + g * (-4 * a * c + b^2)^{1/2}) \\ & )^{1/2} * (g * (b + 2 * c * x + (-4 * a * c + b^2)^{1/2})) / (g * (-4 * a * c + b^2)^{1/2} + b * g - 2 * c * f))^{1/2} * \text{EllipticE}(2^{1/2} * (-g * x + f) * c / (g * (-4 * a * c + b^2)^{1/2} \\ & ) + b * g - 2 * c * f))^{1/2}, (-g * (-4 * a * c + b^2)^{1/2} + b * g - 2 * c * f) / (2 * c * f - b * g + \\ & g * (-4 * a * c + b^2)^{1/2}))^{1/2} * c^2 * f^3 - x^3 * c^2 * g^3 - x^2 * b * c * g^3 - x^2 * c^2 * f * g^2 - x * a * c * g^3 - x * b * c * f * g^2 - a * c * f * g^2) / (c * \\ & g * x^3 + b * g * x^2 + c * f * x^2 + a * g * x + b * f * x + a * f) / g^3 \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)/sqrt(g\*x + f),x, algorithm="maxima")

[Out] integrate(sqrt(c\*x^2 + b\*x + a)/sqrt(g\*x + f), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx + a}}{\sqrt{gx + f}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)/sqrt(g\*x + f),x, algorithm="fricas")

[Out] integral(sqrt(c\*x^2 + b\*x + a)/sqrt(g\*x + f), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}}{\sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*\*(1/2)/(g\*x+f)\*\*(1/2), x)

[Out] Integral(sqrt(a + b\*x + c\*x\*\*2)/sqrt(f + g\*x), x)

---

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)/sqrt(g\*x + f),x, algorithm="giac")

[Out] Timed out

$$3.897 \quad \int \frac{\sqrt{a+bx+cx^2}}{(d+ex)\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=700

$$\frac{\sqrt{2}(ae^2 - bde + cd^2) \sqrt{2cf - g(b - \sqrt{b^2 - 4ac})} \sqrt{1 - \frac{2c(f+gx)}{2cf-g(b-\sqrt{b^2-4ac})}} \sqrt{1 - \frac{2c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}} \left( \frac{e(2cf-bg+\sqrt{b^2-4ac}g)}{2c(ef-dg)}; \sin^{-1} \left( \frac{\sqrt{b+2cx+\sqrt{b^2-4ac}}}{\sqrt{2}} \right) \right)}{2\sqrt{2}\sqrt{b^2-4ac} \sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}} \sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}} (-beg + cdg + cef) F \left( \sin^{-1} \left( \frac{\sqrt{b+2cx+\sqrt{b^2-4ac}}}{\sqrt{2}} \right) \middle| -\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g} \right)} + \frac{\sqrt{2}\sqrt{b^2-4ac} \sqrt{f+gx} \sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}} E \left( \sin^{-1} \left( \frac{\sqrt{b+2cx+\sqrt{b^2-4ac}}}{\sqrt{2}} \right) \middle| -\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g} \right)}{eg\sqrt{a+bx+cx^2} \sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}}$$

[Out] (Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[f + g\*x]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(e\*g\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[a + b\*x + c\*x^2]) - (2\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*(c\*e\*f + c\*d\*g - b\*e\*g)\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(c\*e^2\*g\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]) - (Sqrt[2]\*(c\*d^2 - b\*d\*e + a\*e^2)\*Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g]\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*EllipticPi[(e\*(2\*c\*f - b\*g + Sqrt[b^2 - 4\*a\*c]\*g))/(2\*c\*(e\*f - d\*g)), ArcSin[(Sqrt[2]\*Sqrt[c]\*Sqrt[f + g\*x])/Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g]], (b - Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)/(b + Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)]/(Sqrt[c]\*e^2\*(e\*f - d\*g)\*Sqrt[a + b\*x + c\*x^2])

**Rubi [A]** time = 5.3968, antiderivative size = 700, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.29$

$$\frac{\sqrt{2}(ae^2 - bde + cd^2) \sqrt{2cf - g(b - \sqrt{b^2 - 4ac})} \sqrt{1 - \frac{2c(f+gx)}{2cf-g(b-\sqrt{b^2-4ac})}} \sqrt{1 - \frac{2c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}} \left( \frac{e(2cf-bg+\sqrt{b^2-4ac}g)}{2c(ef-dg)}; \sin^{-1} \left( \frac{\sqrt{b+2cx+\sqrt{b^2-4ac}}}{\sqrt{2}} \right) \right)}{2\sqrt{2}\sqrt{b^2-4ac} \sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}} \sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}} (-beg + cdg + cef) F \left( \sin^{-1} \left( \frac{\sqrt{b+2cx+\sqrt{b^2-4ac}}}{\sqrt{2}} \right) \middle| -\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g} \right)} + \frac{\sqrt{2}\sqrt{b^2-4ac} \sqrt{f+gx} \sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}} E \left( \sin^{-1} \left( \frac{\sqrt{b+2cx+\sqrt{b^2-4ac}}}{\sqrt{2}} \right) \middle| -\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g} \right)}{eg\sqrt{a+bx+cx^2} \sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x + c\*x^2]/((d + e\*x)\*Sqrt[f + g\*x]), x]



```
[Out] (Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x
^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(
2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]/(e*g*Sqrt[(c*(f + g*x))/(2*c
*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt
[2]*Sqrt[b^2 - 4*a*c]*(c*e*f + c*d*g - b*e*g)*Sqrt[(c*(f + g*x))/
(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))
/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2
*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*
f - (b + Sqrt[b^2 - 4*a*c])*g)]/(c*e^2*g*Sqrt[f + g*x]*Sqrt[a +
b*x + c*x^2]) - (Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*Sqrt[2*c*f - (b
- Sqrt[b^2 - 4*a*c])*g]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sq
rt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[
b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*
g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqr
t[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (
2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)]/(Sqrt[c]*e^2*(e*f
- d*g)*Sqrt[a + b*x + c*x^2])
```

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}}{(d + ex)\sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] rubi_integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)*sqrt(f + g*x)), x)
```

**Mathematica [C]** time = 14.2349, size = 16471, normalized size = 23.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)*Sqrt[f + g*x]),x]
```

```
[Out] Result too large to show
```

**Maple [B]** time = 0.047, size = 3126, normalized size = 4.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^2+b*x+a)^(1/2)/(e*x+d)/(g*x+f)^(1/2),x)
```

```
[Out] (2*EllipticE(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)
)^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b
^2)^(1/2)))^(1/2))*c^2*e^2*f^3+EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*
(-4*a*c+b^2)^(1/2)+b*g-2*c*f)^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*
g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-
b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*b*d*e*g^3+2*
EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)^(
1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4
*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2
```



---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{(ex + d)\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)/((e\*x + d)\*sqrt(g\*x + f)),x, algorithm="maxima")

[Out] integrate(sqrt(c\*x^2 + b\*x + a)/((e\*x + d)\*sqrt(g\*x + f)), x)

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)/((e\*x + d)\*sqrt(g\*x + f)),x, algorithm="fricas")

[Out] Timed out

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}}{(d + ex)\sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x\*\*2+b\*x+a)\*\*(1/2)/(e\*x+d)/(g\*x+f)\*\*(1/2),x)

[Out] Integral(sqrt(a + b\*x + c\*x\*\*2)/((d + e\*x)\*sqrt(f + g\*x)), x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{(ex + d)\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)/((e\*x + d)\*sqrt(g\*x + f)),x, algorithm="giac")

[Out] integrate(sqrt(c\*x^2 + b\*x + a)/((e\*x + d)\*sqrt(g\*x + f)), x)

$$3.898 \quad \int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^2\sqrt{f+gx}} dx$$

**Optimal.** Leaf size=736

$$\frac{\sqrt{2cf-g}\left(b-\sqrt{b^2-4ac}\right)\sqrt{1-\frac{2c(f+gx)}{2cf-g(b-\sqrt{b^2-4ac})}}\sqrt{1-\frac{2c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}\left(e^2(bf-ag)-cd(2ef-dg)\right)\left(\frac{e(2cf-bg+\sqrt{b^2-4ac})}{2c(ef-dg)}\right)}{\sqrt{2}\sqrt{ce^2\sqrt{a+bx+cx^2}}(ef-dg)^2} + \frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{\sqrt{2e}\sqrt{a+bx+cx^2}(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}} + \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{b^2-4ac}g}{(b+\sqrt{b^2-4ac})g-2cf}\right)}{e^2\sqrt{f+gx}\sqrt{a+x(b+cx)}} - \frac{\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(d+ex)(ef-dg)}$$

```
[Out] -((Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/((e*f - d*g)*(d + e*x)))
+ (Sqrt[b^2 - 4*a*c]*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(
b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f
- (b + Sqrt[b^2 - 4*a*c])*g)]/(Sqrt[2]*e*(e*f - d*g)*Sqrt[(c*(f
+ g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2
]) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[(c*(f + g*x))/(2*c*f - (b +
Sqrt[b^2 - 4*a*c])*g)]*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]
*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 -
4*a*c]]/Sqrt[2]], (2*Sqrt[b^2 - 4*a*c]*g)/(-2*c*f + (b + Sqrt[b^
2 - 4*a*c])*g)]/(e^2*Sqrt[f + g*x]*Sqrt[a + x*(b + c*x)]) - (Sqr
t[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(e^2*(b*f - a*g) - c*d*(2*e*
f - d*g))*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c
])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])
*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f
- d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b -
Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b +
Sqrt[b^2 - 4*a*c] - (2*c*f)/g)]/(Sqrt[2]*Sqrt[c]*e^2*(e*f - d*g
)^2*Sqrt[a + b*x + c*x^2])
```

**Rubi [A]** time = 7.70742, antiderivative size = 957, normalized size of antiderivative = 1.3, number

of steps used = 15, number of rules used = 10, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$

$$\frac{\sqrt{b^2 - 4ac}\sqrt{f + gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{\sqrt{2}e(ef - dg)\sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\sqrt{cx^2 + bx + a}}$$

$$+ \frac{\sqrt{2}\sqrt{b^2 - 4ac}(2ef - dg)\sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{e^2(ef - dg)\sqrt{f + gx}\sqrt{cx^2 + bx + a}}$$

$$- \frac{\sqrt{2}\sqrt{b^2 - 4ac}f\sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{e(ef - dg)\sqrt{f + gx}\sqrt{cx^2 + bx + a}}$$

$$\frac{\sqrt{2cf - (b - \sqrt{b^2 - 4ac})}g(e^2(bf - ag) - cd(2ef - dg))\sqrt{1 - \frac{2c(f+gx)}{2cf-(b-\sqrt{b^2-4ac})g}}\sqrt{1 - \frac{2c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\left(\frac{e(2cf-bg+\sqrt{b^2-4ac})}{2c(ef-dg)}\right)}{\sqrt{2}\sqrt{ce^2(ef - dg)^2}\sqrt{cx^2 + bx + a}}$$

$$- \frac{\sqrt{f + gx}\sqrt{cx^2 + bx + a}}{(ef - dg)(d + ex)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a + b\*x + c\*x^2]/((d + e\*x)^2\*Sqrt[f + g\*x]),x]

[Out] -((Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])/((e\*f - d\*g)\*(d + e\*x))) + (Sqrt[b^2 - 4\*a\*c]\*Sqrt[f + g\*x]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(Sqrt[2]\*e\*(e\*f - d\*g)\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[a + b\*x + c\*x^2]) - (Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*f\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(e\*(e\*f - d\*g)\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]) + (Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*(2\*e\*f - d\*g)\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(e^2\*(e\*f - d\*g)\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]) - (Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g]\*(e^2\*(b\*f - a\*g) - c\*d\*(2\*e\*f - d\*g))\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*EllipticPi[(e\*(2\*c\*f - b\*g + Sqrt[b^2 - 4\*a\*c]\*g))/(2\*c\*(e\*f - d\*g)), ArcSin[(Sqrt[2]\*Sqrt[c]\*Sqrt[f + g\*x])/Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g]], (b - Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)/(b + Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)]/(Sqrt[2]\*Sqrt[c]\*e^2\*(e\*f - d\*g)^2\*Sqrt[a + b\*x + c\*x^2])

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}}{(d + ex)^2 \sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((c\*x\*\*2+b\*x+a)\*\*(1/2)/(e\*x+d)\*\*2/(g\*x+f)\*\*(1/2),x)

[Out]  $\text{Integral}(\sqrt{a + b*x + c*x**2}/((d + e*x)**2*\sqrt{f + g*x}), x)$

**Mathematica [C]** time = 14.1997, size = 6911, normalized size = 9.39

Result too large to show

Warning: Unable to verify antiderivative.

[In]  $\text{Integrate}[\text{Sqrt}[a + b*x + c*x^2]/((d + e*x)^2*\text{Sqrt}[f + g*x]), x]$

[Out] Result too large to show

**Maple [B]** time = 0.062, size = 13872, normalized size = 18.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((c*x^2+b*x+a)^{(1/2)}/(e*x+d)^2/(g*x+f)^{(1/2}), x)$

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{(ex + d)^2 \sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\sqrt{c*x^2 + b*x + a}/((e*x + d)^2*\sqrt{g*x + f}), x, \text{algorithm}="maxima")$

[Out]  $\text{integrate}(\sqrt{c*x^2 + b*x + a}/((e*x + d)^2*\sqrt{g*x + f}), x)$

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{integrate}(\sqrt{c*x^2 + b*x + a}/((e*x + d)^2*\sqrt{g*x + f}), x, \text{algorithm}="fricas")$

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}}{(d + ex)^2 \sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)**2/(g*x+f)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*x + c*x**2)/((d + e*x)**2*sqrt(f + g*x)), x)
```

---

**GIAC/XCAS** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)^2*sqrt(g*x + f)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.899 \quad \int \frac{\sqrt{a+bx+cx^2}}{(d+ex)^3 \sqrt{f+gx}} dx$$

**Optimal.** Leaf size=1049

$$\frac{\sqrt{b^2-4ac} \sqrt{f+gx} \sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}} E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right) \middle| -\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right) (cd(2ef+dg) - e(bef+2bdg-3aeg))}{4\sqrt{2}e(cd^2-bed+ae^2)(ef-dg)^2 \sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}} \sqrt{cx^2+bx+a}} + \frac{\sqrt{f+gx} \sqrt{cx^2+bx+a} (cd(2ef+dg) - e(bef+2bdg-3aeg))}{4(cd^2-bed+ae^2)(ef-dg)^2(d+ex)}$$

$$\frac{\sqrt{b^2-4ac} ((bf-ag)e^2 + cd(dg-2ef)) \sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}} \sqrt{\frac{c(a+x(b+cx))}{4ac-b^2}} F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right) \middle| \frac{2\sqrt{b^2-4ac}g}{(b+\sqrt{b^2-4ac})g-2cf}\right)}{2\sqrt{2}e^2(cd^2+e(ae-bd))(ef-dg)\sqrt{f+gx}\sqrt{a+x(b+cx)}}$$

$$\frac{\sqrt{2cf-bg+\sqrt{b^2-4ac}g} (3a^2g^2e^4 + b^2f(4dg-ef)e^3 + 2ac(2e^2f^2 - 2degf + 3d^2g^2)e^2 - 2bg(3cfd^2 + ae(ef+2dg))e^2)}{4\sqrt{2}\sqrt{ce^2}(cd^2+e(ae-$$

$$-\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}}{2(ef-dg)(d+ex)^2}$$

```
[Out] -(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(2*(e*f - d*g)*(d + e*x)^2) + ((c*d*(2*e*f + d*g) - e*(b*e*f + 2*b*d*g - 3*a*e*g))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(4*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^2*(d + e*x)) - (Sqrt[b^2 - 4*a*c]*(c*d*(2*e*f + d*g) - e*(b*e*f + 2*b*d*g - 3*a*e*g))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]/(4*Sqrt[2]*e*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^2*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[b^2 - 4*a*c]*(e^2*(b*f - a*g) + c*d*(-2*e*f + d*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (2*Sqrt[b^2 - 4*a*c]*g)/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]/(2*Sqrt[2]*e^2*(c*d^2 + e*(-(b*d) + a*e))*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + x*(b + c*x)]) - (Sqrt[2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g]*(3*a^2*e^4*g^2 + c^2*d^3*g*(4*e*f - d*g) + b^2*e^3*f*(-(e*f) + 4*d*g) + 2*a*c*e^2*(2*e^2*f^2 - 2*d*e*f*g + 3*d^2*g^2) - 2*b*e^2*g*(3*c*d^2*f + a*e*(e*f + 2*d*g)))*Sqrt[(g*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x))/(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[(g*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(2*c*e*f - b*e*g + Sqrt[b^2 - 4*a*c]*e*g)/(2*c*e*f - 2*c*d*g), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g]], (2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]/(4*Sqrt[2]*Sqrt[c]*e^2*(c*d^2 + e*(-(b*d) + a*e))*(e*f - d*g)^3*Sqrt[a + x*(b + c*x)])]
```

**Rubi [A]** time = 18.6864, antiderivative size = 1747, normalized size of antiderivative = 1.67, number of steps used = 25, number of rules used = 11, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[Sqrt[a + b*x + c*x^2]/((d + e*x)^3*Sqrt[f + g*x]),x]
```

```
[Out] -(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(2*(e*f - d*g)*(d + e*x)^2) + ((c*d*(2*e*f + d*g) - e*(b*e*f + 2*b*d*g - 3*a*e*g))*Sqrt[f +
```



$$\begin{aligned}
& g^*x \sqrt{a + b^*x + c^*x^2} / (4^*(c^*d^2 - b^*d^*e + a^*e^2)^*(e^*f - d^*g)^2*(d + e^*x)) - (\sqrt{b^2 - 4^*a^*c}^*(c^*d^*(2^*e^*f + d^*g) - e^*(b^*e^*f + 2^*b^*d^*g - 3^*a^*e^*g))^*\sqrt{f + g^*x}^*\sqrt{-((c^*(a + b^*x + c^*x^2)) / (b^2 - 4^*a^*c))}^*\text{EllipticE}[\text{ArcSin}[\sqrt{(b + \sqrt{b^2 - 4^*a^*c} + 2^*c^*x) / \sqrt{b^2 - 4^*a^*c}}] / \sqrt{2}], (-2^*\sqrt{b^2 - 4^*a^*c}^*g) / (2^*c^*f - (b + \sqrt{b^2 - 4^*a^*c})^*g)] / (4^*\sqrt{2}^*e^*(c^*d^2 - b^*d^*e + a^*e^2)^*(e^*f - d^*g)^2^*\sqrt{(c^*(f + g^*x)) / (2^*c^*f - (b + \sqrt{b^2 - 4^*a^*c})^*g)}^*\sqrt{a + b^*x + c^*x^2}) - (\sqrt{b^2 - 4^*a^*c}^*g^*\sqrt{(c^*(f + g^*x)) / (2^*c^*f - (b + \sqrt{b^2 - 4^*a^*c})^*g)}^*\sqrt{-((c^*(a + b^*x + c^*x^2)) / (b^2 - 4^*a^*c))}^*\text{EllipticF}[\text{ArcSin}[\sqrt{(b + \sqrt{b^2 - 4^*a^*c} + 2^*c^*x) / \sqrt{b^2 - 4^*a^*c}}] / \sqrt{2}], (-2^*\sqrt{b^2 - 4^*a^*c}^*g) / (2^*c^*f - (b + \sqrt{b^2 - 4^*a^*c})^*g)] / (\sqrt{2}^*e^2*(e^*f - d^*g)^*\sqrt{f + g^*x}^*\sqrt{a + b^*x + c^*x^2}) + (\sqrt{b^2 - 4^*a^*c}^*f^*(c^*d^*(2^*e^*f + d^*g) - e^*(b^*e^*f + 2^*b^*d^*g - 3^*a^*e^*g))^*\sqrt{(c^*(f + g^*x)) / (2^*c^*f - (b + \sqrt{b^2 - 4^*a^*c})^*g)}^*\sqrt{-((c^*(a + b^*x + c^*x^2)) / (b^2 - 4^*a^*c))}^*\text{EllipticF}[\text{ArcSin}[\sqrt{(b + \sqrt{b^2 - 4^*a^*c} + 2^*c^*x) / \sqrt{b^2 - 4^*a^*c}}] / \sqrt{2}], (-2^*\sqrt{b^2 - 4^*a^*c}^*g) / (2^*c^*f - (b + \sqrt{b^2 - 4^*a^*c})^*g)] / (2^*\sqrt{2}^*e^*(c^*d^2 - b^*d^*e + a^*e^2)^*(e^*f - d^*g)^2^*\sqrt{f + g^*x}^*\sqrt{a + b^*x + c^*x^2}) - (\sqrt{b^2 - 4^*a^*c}^*d^*g^*(c^*d^*(2^*e^*f + d^*g) - e^*(b^*e^*f + 2^*b^*d^*g - 3^*a^*e^*g))^*\sqrt{(c^*(f + g^*x)) / (2^*c^*f - (b + \sqrt{b^2 - 4^*a^*c})^*g)}^*\sqrt{-((c^*(a + b^*x + c^*x^2)) / (b^2 - 4^*a^*c))}^*\text{EllipticF}[\text{ArcSin}[\sqrt{(b + \sqrt{b^2 - 4^*a^*c} + 2^*c^*x) / \sqrt{b^2 - 4^*a^*c}}] / \sqrt{2}], (-2^*\sqrt{b^2 - 4^*a^*c}^*g) / (2^*c^*f - (b + \sqrt{b^2 - 4^*a^*c})^*g)] / (2^*\sqrt{2}^*e^2*(c^*d^2 - b^*d^*e + a^*e^2)^*(e^*f - d^*g)^2^*\sqrt{f + g^*x}^*\sqrt{a + b^*x + c^*x^2}) - (\sqrt{2^*c^*f - (b - \sqrt{b^2 - 4^*a^*c})^*g}^*(c^*e^*f + c^*d^*g - b^*e^*g)^*\sqrt{1 - (2^*c^*(f + g^*x)) / (2^*c^*f - (b - \sqrt{b^2 - 4^*a^*c})^*g)}^*\sqrt{1 - (2^*c^*(f + g^*x)) / (2^*c^*f - (b + \sqrt{b^2 - 4^*a^*c})^*g)}^*\text{EllipticPi}[(e^*(2^*c^*f - b^*g + \sqrt{b^2 - 4^*a^*c})^*g) / (2^*c^*(e^*f - d^*g)), \text{ArcSin}[(\sqrt{2}^*\sqrt{c}^*\sqrt{f + g^*x}) / \sqrt{2^*c^*f - (b - \sqrt{b^2 - 4^*a^*c})^*g}], (b - \sqrt{b^2 - 4^*a^*c} - (2^*c^*f) / g) / (b + \sqrt{b^2 - 4^*a^*c} - (2^*c^*f) / g)] / (\sqrt{2}^*\sqrt{c}^*e^2*(e^*f - d^*g)^2^*\sqrt{a + b^*x + c^*x^2}) + (\sqrt{2^*c^*f - (b - \sqrt{b^2 - 4^*a^*c})^*g}^*(c^*d^*(2^*e^*f + d^*g) - e^*(b^*e^*f + 2^*b^*d^*g - 3^*a^*e^*g))^*(c^*d^*(2^*e^*f - 3^*d^*g) - e^*(b^*e^*f - 2^*b^*d^*g + a^*e^*g))^*\sqrt{1 - (2^*c^*(f + g^*x)) / (2^*c^*f - (b - \sqrt{b^2 - 4^*a^*c})^*g)}^*\sqrt{1 - (2^*c^*(f + g^*x)) / (2^*c^*f - (b + \sqrt{b^2 - 4^*a^*c})^*g)}^*\text{EllipticPi}[(e^*(2^*c^*f - b^*g + \sqrt{b^2 - 4^*a^*c})^*g) / (2^*c^*(e^*f - d^*g)), \text{ArcSin}[(\sqrt{2}^*\sqrt{c}^*\sqrt{f + g^*x}) / \sqrt{2^*c^*f - (b - \sqrt{b^2 - 4^*a^*c})^*g}], (b - \sqrt{b^2 - 4^*a^*c} - (2^*c^*f) / g) / (b + \sqrt{b^2 - 4^*a^*c} - (2^*c^*f) / g)] / (4^*\sqrt{2}^*\sqrt{c}^*e^2*(c^*d^2 - b^*d^*e + a^*e^2)^*(e^*f - d^*g)^3^*\sqrt{a + b^*x + c^*x^2})
\end{aligned}$$


---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}}{(d + ex)^3 \sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)**3/(g*x+f)**(1/2),x)`

[Out] `Integral(sqrt(a + b*x + c*x**2)/((d + e*x)**3*sqrt(f + g*x)), x)`

---

**Mathematica [C]** time = 19.8126, size = 36617, normalized size = 34.91

Result too large to show

Warning: Unable to verify antiderivative.

[In] `Integrate[Sqrt[a + b*x + c*x^2]/((d + e*x)^3*Sqrt[f + g*x]),x]`

[Out] Result too large to show

**Maple [B]** time = 0.142, size = 57835, normalized size = 55.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((c*x^2+b*x+a)^(1/2)/(e*x+d)^3/(g*x+f)^(1/2), x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}}{(ex + d)^3 \sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)^3*sqrt(g*x + f)), x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)^3*sqrt(g*x + f)), x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)^3*sqrt(g*x + f)), x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{a + bx + cx^2}}{(d + ex)^3 \sqrt{f + gx}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x**2+b*x+a)**(1/2)/(e*x+d)**3/(g*x+f)**(1/2), x)`

[Out] `Integral(sqrt(a + b*x + c*x**2)/((d + e*x)**3*sqrt(f + g*x)), x)`

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(c*x^2 + b*x + a)/((e*x + d)^3*sqrt(g*x + f)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.900 \quad \int \frac{(d+ex)^3 \sqrt{f+gx}}{\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=774

$$\frac{2\sqrt{2e}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ag^2-bfg+cf^2)\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}(ceg(-25aeg-84bdg+13bef)+24b^2e^2g^2+c^2(105c^4g^3\sqrt{f+gx}\sqrt{a+bx+cx^2}+2e\sqrt{f+gx}\sqrt{a+bx+cx^2}(ceg(-25aeg-84bdg+13bef)+24b^2e^2g^2+c^2(-(-90d^2g^2+12defg+7e^2f^2))))}{105c^3g^2} + \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(c^2eg(aeg(189dg+19ef)-b(-210d^2g^2-63defg+9e^2f^2))-8bce^2g^2(13aeg+2105c^4g^3\sqrt{a+bx+cx^2}))}{105c^3g^2} + \frac{2e^2(f+gx)^{3/2}\sqrt{a+bx+cx^2}(-6beg+11cdg+cef)}{35c^2g^2} + \frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7c}$$

[Out] (2\*e\*(24\*b^2\*e^2\*g^2 + c\*e\*g\*(13\*b\*e\*f - 84\*b\*d\*g - 25\*a\*e\*g) - c^2\*(7\*e^2\*f^2 + 12\*d\*e\*f\*g - 90\*d^2\*g^2))\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])/(105\*c^3\*g^2) + (2\*e\*(d + e\*x)^2\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])/(7\*c) + (2\*e^2\*(c\*f + g\*x)^3/2\*Sqrt[a + b\*x + c\*x^2])/(35\*c^2\*g^2) - (Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*(48\*b^3\*e^3\*g^3 - 8\*b\*c\*e^2\*g^2\*(2\*b\*e\*f + 21\*b\*d\*g + 13\*a\*e\*g) - c^3\*(8\*e^3\*f^3 - 42\*d\*e^2\*f^2\*g + 105\*d^2\*e\*f\*g^2 + 105\*d^3\*g^3) + c^2\*e\*g\*(a\*e\*g\*(19\*e\*f + 189\*d\*g) - b\*(9\*e^2\*f^2 - 63\*d\*e\*f\*g - 210\*d^2\*g^2)))\*Sqrt[f + g\*x]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(105\*c^4\*g^3\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[a + b\*x + c\*x^2]) - (2\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*e\*(c\*f^2 - b\*f\*g + a\*g^2)\*(24\*b^2\*e^2\*g^2 + c\*e\*g\*(13\*b\*e\*f - 84\*b\*d\*g - 25\*a\*e\*g) + c^2\*(8\*e^2\*f^2 - 42\*d\*e\*f\*g + 105\*d^2\*g^2))\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(105\*c^4\*g^3\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])

**Rubi [A]** time = 4.46528, antiderivative size = 774, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$

$$\frac{2\sqrt{2e}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ag^2-bfg+cf^2)\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}(ceg(-25aeg-84bdg+13bef)+24b^2e^2g^2+c^2(105c^4g^3\sqrt{f+gx}\sqrt{a+bx+cx^2}+2e\sqrt{f+gx}\sqrt{a+bx+cx^2}(ceg(-25aeg-84bdg+13bef)+24b^2e^2g^2+c^2(-(-90d^2g^2+12defg+7e^2f^2))))}{105c^3g^2} + \frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(c^2eg(aeg(189dg+19ef)-b(-210d^2g^2-63defg+9e^2f^2))-8bce^2g^2(13aeg+2105c^4g^3\sqrt{a+bx+cx^2}))}{105c^3g^2} + \frac{2e^2(f+gx)^{3/2}\sqrt{a+bx+cx^2}(-6beg+11cdg+cef)}{35c^2g^2} + \frac{2e(d+ex)^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{7c}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^3\*Sqrt[f + g\*x])/Sqrt[a + b\*x + c\*x^2],x]

[Out]  $(2*e*(24*b^2*e^2*g^2 + c*e*g*(13*b*e*f - 84*b*d*g - 25*a*e*g) - c^2*(7*e^2*f^2 + 12*d*e*f*g - 90*d^2*g^2))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(105*c^3*g^2) + (2*e*(d + e*x)^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(7*c) + (2*e^2*(c*e*f + 11*c*d*g - 6*b*e*g)*(f + g*x)^{3/2}*Sqrt[a + b*x + c*x^2])/(35*c^2*g^2) - (Sqrt[2]*Sqrt[b^2 - 4*a*c]*(48*b^3*e^3*g^3 - 8*b*c*e^2*g^2*(2*b*e*f + 21*b*d*g + 13*a*e*g) - c^3*(8*e^3*f^3 - 42*d*e^2*f^2*g + 105*d^2*e*f*g^2 + 105*d^3*g^3) + c^2*e*g*(a*e*g*(19*e*f + 189*d*g) - b*(9*e^2*f^2 - 63*d*e*f*g - 210*d^2*g^2)))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g))/(105*c^4*g^3*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*e*(c*f^2 - b*f*g + a*g^2)*(24*b^2*e^2*g^2 + c*e*g*(13*b*e*f - 84*b*d*g - 25*a*e*g) + c^2*(8*e^2*f^2 - 42*d*e*f*g + 105*d^2*g^2))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g))/(105*c^4*g^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*3\*(g\*x+f)\*\*(1/2)/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Timed out

**Mathematica [C]** time = 16.2576, size = 10649, normalized size = 13.76

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[((d + e\*x)^3\*Sqrt[f + g\*x])/Sqrt[a + b\*x + c\*x^2],x]

[Out] Result too large to show

**Maple [B]** time = 0.086, size = 14978, normalized size = 19.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^3\*(g\*x+f)^(1/2)/(c\*x^2+b\*x+a)^(1/2),x)

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^3 \sqrt{gx + f}}{\sqrt{cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*sqrt(g\*x + f)/sqrt(c\*x^2 + b\*x + a), x, algorithm="maxima")

[Out] integrate((e\*x + d)^3\*sqrt(g\*x + f)/sqrt(c\*x^2 + b\*x + a), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e^3x^3 + 3de^2x^2 + 3d^2ex + d^3)\sqrt{gx + f}}{\sqrt{cx^2 + bx + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*sqrt(g\*x + f)/sqrt(c\*x^2 + b\*x + a), x, algorithm="fricas")

[Out] integral((e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3)\*sqrt(g\*x + f)/sqrt(c\*x^2 + b\*x + a), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^3 \sqrt{f + gx}}{\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3\*(g\*x+f)\*\*(1/2)/(c\*x\*\*2+b\*x+a)\*\*(1/2), x)

[Out] Integral((d + e\*x)\*\*3\*sqrt(f + g\*x)/sqrt(a + b\*x + c\*x\*\*2), x)

---

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3\*sqrt(g\*x + f)/sqrt(c\*x^2 + b\*x + a), x, algorithm="giac")

[Out] Timed out

$$3.901 \quad \int \frac{(d+ex)^2 \sqrt{f+gx}}{\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=567

$$\frac{4\sqrt{2e}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ag^2-bfg+cf^2)\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}(2beg-5cdg+cef)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right)-\frac{2\sqrt{2e}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(-ceg(9aeg+20bdg+3bef)+8b^2e^2g^2+c^2(-(-15d^2g^2-10defg+2e^2f^2)))E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right)}{15c^3g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}}{15c^3g^2\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}} + \frac{2e\sqrt{f+gx}\sqrt{a+bx+cx^2}(-4beg+7cdg+cef)}{15c^2g} + \frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5c}$$

[Out] (2\*e\*(c\*e\*f + 7\*c\*d\*g - 4\*b\*e\*g)\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])/(15\*c^2\*g) + (2\*e\*(d + e\*x)\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])/(5\*c) + (Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*(8\*b^2\*e^2\*g^2 - c\*e\*g\*(3\*b\*e\*f + 20\*b\*d\*g + 9\*a\*e\*g) - c^2\*(2\*e^2\*f^2 - 10\*d\*e\*f\*g - 15\*d^2\*g^2))\*Sqrt[f + g\*x]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(15\*c^3\*g^2\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[a + b\*x + c\*x^2]) + (4\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*e\*(c\*e\*f - 5\*c\*d\*g + 2\*b\*e\*g)\*(c\*f^2 - b\*f\*g + a\*g^2)\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(15\*c^3\*g^2\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])

**Rubi [A]** time = 2.04747, antiderivative size = 567, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$

$$\frac{4\sqrt{2e}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ag^2-bfg+cf^2)\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}(2beg-5cdg+cef)F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right)-\frac{2\sqrt{2e}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(-ceg(9aeg+20bdg+3bef)+8b^2e^2g^2+c^2(-(-15d^2g^2-10defg+2e^2f^2)))E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right)}{15c^3g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}}{15c^3g^2\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}} + \frac{2e\sqrt{f+gx}\sqrt{a+bx+cx^2}(-4beg+7cdg+cef)}{15c^2g} + \frac{2e(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5c}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^2\*Sqrt[f + g\*x])/Sqrt[a + b\*x + c\*x^2], x]

[Out] (2\*e\*(c\*e\*f + 7\*c\*d\*g - 4\*b\*e\*g)\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])/(15\*c^2\*g) + (2\*e\*(d + e\*x)\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])/(5\*c) + (Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*(8\*b^2\*e^2\*g^2 - c\*e\*g\*(3\*b\*e\*f + 20\*b\*d\*g + 9\*a\*e\*g) - c^2\*(2\*e^2\*f^2 - 10\*d\*e\*f\*g - 15\*d^2\*g^2))\*Sqrt[f + g\*x]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(15\*c^3\*g^2\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + S

$$\sqrt{b^2 - 4ac}g) \sqrt{a + bx + cx^2} + (4\sqrt{2}\sqrt{b^2 - 4ac})e^{(c^2ef - 5c^2dg + 2b^2eg)(cf^2 - bfg + ag^2)} \sqrt{\frac{c(f + gx)}{2cf - (b + \sqrt{b^2 - 4ac})g}} \sqrt{-\left(\frac{c(a + bx + cx^2)}{b^2 - 4ac}\right)} \operatorname{EllipticF}\left[\operatorname{ArcSin}\left[\frac{\sqrt{b + \sqrt{b^2 - 4ac} + 2cx}}{\sqrt{b^2 - 4ac}}\right]/\sqrt{2}\right], \frac{-2\sqrt{b^2 - 4ac}g}{(2cf - (b + \sqrt{b^2 - 4ac})g)}\right] / (15c^3g^2\sqrt{f + gx}\sqrt{a + bx + cx^2})$$

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**2*(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)`

[Out] Timed out

**Mathematica [C]** time = 14.3334, size = 5536, normalized size = 9.76

Result too large to show

Antiderivative was successfully verified.

[In] `Integrate[(((d + e*x)^2*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2]),x]`

[Out] Result too large to show

**Maple [B]** time = 0.065, size = 8248, normalized size = 14.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^2*(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^2 \sqrt{gx + f}}{\sqrt{cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^2*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a),x, algorithm="maxima")`

[Out] `integrate((e*x + d)^2*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)`



**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e^2x^2 + 2dex + d^2)\sqrt{gx + f}}{\sqrt{cx^2 + bx + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^2*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a),x, algorithm="fricas")`

[Out] `integral((e^2*x^2 + 2*d*e*x + d^2)*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^2 \sqrt{f + gx}}{\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**2*(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2), x)`

[Out] `Integral((d + e*x)**2*sqrt(f + g*x)/sqrt(a + b*x + c*x**2), x)`

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^2*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a),x, algorithm="giac")`

[Out] Timed out

$$3.902 \quad \int \frac{(d+ex)\sqrt{f+gx}}{\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=452

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(-2beg+3cdg+cef)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{3c^2g\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}}$$

$$-\frac{2\sqrt{2}e\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ag^2-bfg+cf^2)\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{3c^2g\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$+\frac{2e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3c}$$

[Out] (2\*e\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])/(3\*c) + (Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*(c\*e\*f + 3\*c\*d\*g - 2\*b\*e\*g)\*Sqrt[f + g\*x]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))])\*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(3\*c^2\*g\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[a + b\*x + c\*x^2]) - (2\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*e\*(c\*f^2 - b\*f\*g + a\*g^2)\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))])\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(3\*c^2\*g\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])

**Rubi [A]** time = 1.10603, antiderivative size = 452, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(-2beg+3cdg+cef)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{3c^2g\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}}$$

$$-\frac{2\sqrt{2}e\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ag^2-bfg+cf^2)\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{3c^2g\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$+\frac{2e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3c}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)\*Sqrt[f + g\*x])/Sqrt[a + b\*x + c\*x^2], x]

[Out] (2\*e\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])/(3\*c) + (Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*(c\*e\*f + 3\*c\*d\*g - 2\*b\*e\*g)\*Sqrt[f + g\*x]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))])\*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(3\*c^2\*g\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[a + b\*x + c\*x^2]) - (2\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*e\*(c\*f^2 - b\*f\*g + a\*g^2)\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))])\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(3\*c^2\*g\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])

$[f + g*x]*\text{Sqrt}[a + b*x + c*x^2])$

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)*(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)`

[Out] Timed out

**Mathematica [C]** time = 10.5844, size = 766, normalized size = 1.69

$$\frac{2e\sqrt{f+gx}(a+bx+cx^2)}{3c\sqrt{a+x(b+cx)}}$$

$$2(f+gx)^{3/2}\sqrt{a+bx+cx^2}\left(\frac{g\left(\frac{ag}{f+gx}-\frac{bf}{f+gx}+b\right)}{f+gx}+c\left(\frac{f}{f+gx}-1\right)^2\right)(-2beg+3cdg+cef)+\frac{i\sqrt{1-\frac{2(g(ag-bf)+cf^2)}{(f+gx)\sqrt{g^2(b^2-4ac)-bg+2cf}}}\sqrt{\frac{2}{(f+gx)}}}{\dots}$$

Antiderivative was successfully verified.

[In] `Integrate[(((d + e*x)*Sqrt[f + g*x])/Sqrt[a + b*x + c*x^2]),x]`

[Out]  $(2*e*\text{Sqrt}[f + g*x]*(a + b*x + c*x^2))/(3*c*\text{Sqrt}[a + x*(b + c*x)]) + (2*(f + g*x)^{(3/2)}*\text{Sqrt}[a + b*x + c*x^2]*((c*e*f + 3*c*d*g - 2*b*e*g)*(c*(-1 + f/(f + g*x))^2 + (g*(b - (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f + g*x)) + ((1/2)*\text{Sqrt}[1 - (2*(c*f^2 + g*(-(b*f) + a*g)))/((2*c*f - b*g + \text{Sqrt}[(b^2 - 4*a*c)*g^2])*(f + g*x))] * \text{Sqrt}[1 + (2*(c*f^2 + g*(-(b*f) + a*g)))/((-2*c*f + b*g + \text{Sqrt}[(b^2 - 4*a*c)*g^2])*(f + g*x))] * ((2*c*f - b*g + \text{Sqrt}[(b^2 - 4*a*c)*g^2]) * (2*b*e*g - c*(e*f + 3*d*g)) * \text{EllipticE}[I*\text{ArcSinh}[(\text{Sqrt}[2]*\text{Sqrt}[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + \text{Sqrt}[(b^2 - 4*a*c)*g^2])]])/\text{Sqrt}[f + g*x]), -(((-2*c*f + b*g + \text{Sqrt}[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + \text{Sqrt}[(b^2 - 4*a*c)*g^2])) + (6*c^2*d*f*g + 2*b*e*g*(b*g - \text{Sqrt}[(b^2 - 4*a*c)*g^2]) + c*(-2*a*e*g^2 - 3*b*g*(e*f + d*g) + \text{Sqrt}[(b^2 - 4*a*c)*g^2]*(e*f + 3*d*g)) * \text{EllipticF}[I*\text{ArcSinh}[(\text{Sqrt}[2]*\text{Sqrt}[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + \text{Sqrt}[(b^2 - 4*a*c)*g^2])]])/\text{Sqrt}[f + g*x]), -(((-2*c*f + b*g + \text{Sqrt}[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + \text{Sqrt}[(b^2 - 4*a*c)*g^2])))/(\text{Sqrt}[2]*\text{Sqrt}[(c*f^2 + g*(-(b*f) + a*g))/(-2*c*f + b*g + \text{Sqrt}[(b^2 - 4*a*c)*g^2])]) * \text{Sqrt}[f + g*x]))/(3*c^2*g^2*\text{Sqrt}[a + x*(b + c*x)]*\text{Sqrt}[(f + g*x)^2*(c*(-1 + f/(f + g*x))^2 + (g*(b - (b*f)/(f + g*x) + (a*g)/(f + g*x)))/(f + g*x)))/g^2)$

**Maple [B]** time = 0.051, size = 3805, normalized size = 8.4

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((e^*x+d)^*(g^*x+f)^{(1/2)}/(c^*x^2+b^*x+a)^{(1/2)}, x)$

[Out] 
$$\begin{aligned} & -1/3*(g^*x+f)^{(1/2)}*(c^*x^2+b^*x+a)^{(1/2)}*(3^*2^{(1/2)}*(-(g^*x+f)^*c/(g^* \\ & (-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}*(g^*(-2^*c^*x+(-4^*a^*c+b^*2)^{(1/2)} \\ & )-b)/(2^*c^*f-b^*g+g^*(-4^*a^*c+b^*2)^{(1/2)))^{(1/2)}*(g^*(b+2^*c^*x+(-4^*a^*c+ \\ & b^*2)^{(1/2)))/(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}*\text{EllipticF}(2^{(1/2)} \\ & *( -(g^*x+f)^*c/(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}, (-(g^*(-4^* \\ & a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f)/(2^*c^*f-b^*g+g^*(-4^*a^*c+b^*2)^{(1/2)))^{(1/2)} \\ & ))^*a^*b^*e^*g^*3-6^*2^{(1/2)}*(-(g^*x+f)^*c/(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^* \\ & f))^{(1/2)}*(g^*(-2^*c^*x+(-4^*a^*c+b^*2)^{(1/2)}-b)/(2^*c^*f-b^*g+g^*(-4^*a^*c+b^* \\ & 2)^{(1/2)))^{(1/2)}*(g^*(b+2^*c^*x+(-4^*a^*c+b^*2)^{(1/2)))/(g^*(-4^*a^*c+b^*2) \\ & )^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}*\text{EllipticF}(2^{(1/2)}*( -(g^*x+f)^*c/(g^*(-4^*a^*c \\ & +b^*2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}, (-(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f)/ \\ & (2^*c^*f-b^*g+g^*(-4^*a^*c+b^*2)^{(1/2)))^{(1/2)})*a^*c^*d^*g^*3-2^{(1/2)}*(-(g^*x \\ & +f)^*c/(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}*(g^*(-2^*c^*x+(-4^*a^*c+ \\ & b^*2)^{(1/2)}-b)/(2^*c^*f-b^*g+g^*(-4^*a^*c+b^*2)^{(1/2)))^{(1/2)}*(g^*(b+2^*c^*x \\ & +(-4^*a^*c+b^*2)^{(1/2)))/(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}*\text{Elli \\ & pticF}(2^{(1/2)}*( -(g^*x+f)^*c/(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)} \\ & , (-(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f)/(2^*c^*f-b^*g+g^*(-4^*a^*c+b^*2)^{(1/ \\ & 2)))^{(1/2)})*(-4^*a^*c+b^*2)^{(1/2)}*a^*e^*g^*3-3^*2^{(1/2)}*(-(g^*x+f)^*c/(g^* \\ & -4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}*(g^*(-2^*c^*x+(-4^*a^*c+b^*2)^{(1/2)} \\ & -b)/(2^*c^*f-b^*g+g^*(-4^*a^*c+b^*2)^{(1/2)))^{(1/2)}*(g^*(b+2^*c^*x+(-4^*a^*c+b^* \\ & 2)^{(1/2)))/(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}*\text{EllipticF}(2^{(1/ \\ & 2)}*( -(g^*x+f)^*c/(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}, (-(g^*(-4^* \\ & a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f)/(2^*c^*f-b^*g+g^*(-4^*a^*c+b^*2)^{(1/2)))^{(1/2)} \\ & )^*b^*2^*e^*f^*g^*2+6^*2^{(1/2)}*(-(g^*x+f)^*c/(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^* \\ & *f))^{(1/2)}*(g^*(-2^*c^*x+(-4^*a^*c+b^*2)^{(1/2)}-b)/(2^*c^*f-b^*g+g^*(-4^*a^*c+ \\ & b^*2)^{(1/2)))^{(1/2)}*(g^*(b+2^*c^*x+(-4^*a^*c+b^*2)^{(1/2)))/(g^*(-4^*a^*c+b^*2) \\ & )^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}*\text{EllipticF}(2^{(1/2)}*( -(g^*x+f)^*c/(g^*(-4^*a^* \\ & c+b^*2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}, (-(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f) \\ & / (2^*c^*f-b^*g+g^*(-4^*a^*c+b^*2)^{(1/2)))^{(1/2)})*b^*c^*d^*f^*g^*2+3^*2^{(1/2)}*( \\ & -(g^*x+f)^*c/(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}*(g^*(-2^*c^*x+(-4^* \\ & a^*c+b^*2)^{(1/2)}-b)/(2^*c^*f-b^*g+g^*(-4^*a^*c+b^*2)^{(1/2)))^{(1/2)}*(g^*(b+ \\ & 2^*c^*x+(-4^*a^*c+b^*2)^{(1/2)))/(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)} \\ & *\text{EllipticF}(2^{(1/2)}*( -(g^*x+f)^*c/(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f))^{(1/ \\ & 2)}, (-(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f)/(2^*c^*f-b^*g+g^*(-4^*a^*c+b^*2) \\ & )^{(1/2)))^{(1/2)})*b^*c^*e^*f^*2^*g+2^{(1/2)}*(-(g^*x+f)^*c/(g^*(-4^*a^*c+b^*2)^{(1/ \\ & 2)}+b^*g-2^*c^*f))^{(1/2)}*(g^*(-2^*c^*x+(-4^*a^*c+b^*2)^{(1/2)}-b)/(2^*c^*f-b \\ & *g+g^*(-4^*a^*c+b^*2)^{(1/2)))^{(1/2)}*(g^*(b+2^*c^*x+(-4^*a^*c+b^*2)^{(1/2)))/( \\ & g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}*\text{EllipticF}(2^{(1/2)}*( -(g^*x+f) \\ & )^*c/(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}, (-(g^*(-4^*a^*c+b^*2)^{(1/ \\ & 2)}+b^*g-2^*c^*f)/(2^*c^*f-b^*g+g^*(-4^*a^*c+b^*2)^{(1/2)))^{(1/2)})*(-4^*a^*c+b^* \\ & 2)^{(1/2)}*b^*e^*f^*g^*2-6^*2^{(1/2)}*(-(g^*x+f)^*c/(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^* \\ & g-2^*c^*f))^{(1/2)}*(g^*(-2^*c^*x+(-4^*a^*c+b^*2)^{(1/2)}-b)/(2^*c^*f-b^*g+g^*(-4^* \\ & a^*c+b^*2)^{(1/2)))^{(1/2)}*(g^*(b+2^*c^*x+(-4^*a^*c+b^*2)^{(1/2)))/(g^*(-4^*a^* \\ & c+b^*2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}*\text{EllipticF}(2^{(1/2)}*( -(g^*x+f)^*c/(g^* \\ & -4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}, (-(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^* \\ & *c^*f)/(2^*c^*f-b^*g+g^*(-4^*a^*c+b^*2)^{(1/2)))^{(1/2)})*c^*2^*d^*f^*2^*g-2^{(1/2)} \\ & )^*(-(g^*x+f)^*c/(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}*(g^*(-2^*c^*x+ \\ & (-4^*a^*c+b^*2)^{(1/2)}-b)/(2^*c^*f-b^*g+g^*(-4^*a^*c+b^*2)^{(1/2)))^{(1/2)}*(g^* \\ & (b+2^*c^*x+(-4^*a^*c+b^*2)^{(1/2)))/(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f))^{(1/ \\ & 2)}*\text{EllipticF}(2^{(1/2)}*( -(g^*x+f)^*c/(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f) \\ & ))^{(1/2)}, (-(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f)/(2^*c^*f-b^*g+g^*(-4^*a^*c+ \\ & b^*2)^{(1/2)))^{(1/2)})*(-4^*a^*c+b^*2)^{(1/2)}*c^*e^*f^*2^*g-4^*2^{(1/2)}*(-(g^*x \\ & +f)^*c/(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}*(g^*(-2^*c^*x+(-4^*a^*c+ \\ & b^*2)^{(1/2)}-b)/(2^*c^*f-b^*g+g^*(-4^*a^*c+b^*2)^{(1/2)))^{(1/2)}*(g^*(b+2^*c^*x \\ & +(-4^*a^*c+b^*2)^{(1/2)))/(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}*\text{Elli \\ & pticE}(2^{(1/2)}*( -(g^*x+f)^*c/(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)} \\ & , (-(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f)/(2^*c^*f-b^*g+g^*(-4^*a^*c+b^*2)^{(1/ \\ & 2)))^{(1/2)})*a^*b^*e^*g^*3+6^*2^{(1/2)}*(-(g^*x+f)^*c/(g^*(-4^*a^*c+b^*2)^{(1/2)} \\ & +b^*g-2^*c^*f))^{(1/2)}*(g^*(-2^*c^*x+(-4^*a^*c+b^*2)^{(1/2)}-b)/(2^*c^*f-b^*g+g^* \\ & (-4^*a^*c+b^*2)^{(1/2)))^{(1/2)}*(g^*(b+2^*c^*x+(-4^*a^*c+b^*2)^{(1/2)))/(g^*(-4^* \\ & a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}*\text{EllipticE}(2^{(1/2)}*( -(g^*x+f)^*c/( \\ & g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}, (-(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^* \\ & g-2^*c^*f)/(2^*c^*f-b^*g+g^*(-4^*a^*c+b^*2)^{(1/2)))^{(1/2)})*a^*c^*d^*g^*3+2^*2^{(1/2)} \\ & )^*(-(g^*x+f)^*c/(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}*(g^*(-2^*c^* \\ & x+(-4^*a^*c+b^*2)^{(1/2)}-b)/(2^*c^*f-b^*g+g^*(-4^*a^*c+b^*2)^{(1/2)))^{(1/2)}*( \\ & g^*(b+2^*c^*x+(-4^*a^*c+b^*2)^{(1/2)))/(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f)) \\ & )^{(1/2)}*\text{EllipticE}(2^{(1/2)}*( -(g^*x+f)^*c/(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^* \\ & c^*f))^{(1/2)}, (-(g^*(-4^*a^*c+b^*2)^{(1/2)}+b^*g-2^*c^*f)/(2^*c^*f-b^*g+g^*(-4^*a^* \\ & *c+b^*2)^{(1/2)))^{(1/2)})*a^*c^*e^*f^*g^*2+4^*2^{(1/2)}*(-(g^*x+f)^*c/(g^*(-4^*a$$

$$\begin{aligned} & (c+b^2)^{1/2} + b^2 g - 2c^2 f)^{1/2} \cdot (g^2(-2cx + (-4ac + b^2)^{1/2}) - b) / \\ & (2c^2 f - b^2 g + g^2(-4ac + b^2)^{1/2})^{1/2} \cdot (g^2(b + 2cx + (-4ac + b^2)^{1/2}) / \\ & (g^2(-4ac + b^2)^{1/2} + b^2 g - 2c^2 f)^{1/2} \cdot \text{EllipticE}(2^{1/2} \cdot \\ & (-g^2 x + f) \cdot c / (g^2(-4ac + b^2)^{1/2} + b^2 g - 2c^2 f)^{1/2}, (-g^2(-4ac + \\ & b^2)^{1/2} + b^2 g - 2c^2 f) / (2c^2 f - b^2 g + g^2(-4ac + b^2)^{1/2}))^{1/2}) \cdot b^2 \\ & 2 \cdot e^2 f^2 g^2 - 6^2 \cdot 2^{1/2} \cdot (-g^2 x + f) \cdot c / (g^2(-4ac + b^2)^{1/2} + b^2 g - 2c^2 f) \\ & )^{1/2} \cdot (g^2(-2cx + (-4ac + b^2)^{1/2}) - b) / (2c^2 f - b^2 g + g^2(-4ac + b^2)^{1/2}) \\ & )^{1/2} \cdot (g^2(b + 2cx + (-4ac + b^2)^{1/2}) / (g^2(-4ac + b^2)^{1/2} + b^2 g - 2c^2 f) \\ & )^{1/2} \cdot \text{EllipticE}(2^{1/2} \cdot (-g^2 x + f) \cdot c / (g^2(-4ac + b^2)^{1/2} + b^2 g - 2c^2 f) \\ & )^{1/2}, (-g^2(-4ac + b^2)^{1/2} + b^2 g - 2c^2 f) / (2c^2 f - b^2 g + g^2(-4ac + b^2)^{1/2}))^{1/2}) \\ & \cdot b^2 \cdot c \cdot d^2 f^2 g^2 - 6^2 \cdot 2^{1/2} \cdot (-g^2 x + f) \cdot c / (g^2(-4ac + b^2)^{1/2} + b^2 g - 2c^2 f) \\ & )^{1/2} \cdot (g^2(-2cx + (-4ac + b^2)^{1/2}) - b) / (2c^2 f - b^2 g + g^2(-4ac + b^2)^{1/2}) \\ & )^{1/2} \cdot (g^2(b + 2cx + (-4ac + b^2)^{1/2}) / (g^2(-4ac + b^2)^{1/2} + b^2 g - 2c^2 f) \\ & )^{1/2} \cdot \text{EllipticE}(2^{1/2} \cdot (-g^2 x + f) \cdot c / (g^2(-4ac + b^2)^{1/2} + b^2 g - 2c^2 f) \\ & )^{1/2}, (-g^2(-4ac + b^2)^{1/2} + b^2 g - 2c^2 f) / (2c^2 f - b^2 g + g^2(-4ac + b^2)^{1/2}))^{1/2}) \\ & \cdot b^2 \cdot c^2 \cdot e^2 f^2 g^2 + 6^2 \cdot 2^{1/2} \cdot (-g^2 x + f) \cdot c / (g^2(-4ac + b^2)^{1/2} + b^2 g - 2c^2 f) \\ & )^{1/2} \cdot (g^2(-2cx + (-4ac + b^2)^{1/2}) - b) / (2c^2 f - b^2 g + g^2(-4ac + b^2)^{1/2}) \\ & )^{1/2} \cdot (g^2(b + 2cx + (-4ac + b^2)^{1/2}) / (g^2(-4ac + b^2)^{1/2} + b^2 g - 2c^2 f) \\ & )^{1/2} \cdot \text{EllipticE}(2^{1/2} \cdot (-g^2 x + f) \cdot c / (g^2(-4ac + b^2)^{1/2} + b^2 g - 2c^2 f) \\ & )^{1/2}, (-g^2(-4ac + b^2)^{1/2} + b^2 g - 2c^2 f) / (2c^2 f - b^2 g + g^2(-4ac + b^2)^{1/2}))^{1/2}) \\ & \cdot c^2 \cdot e^2 f^3 - 2^2 \cdot x^3 \cdot c^2 \cdot e^2 g^3 - 2^2 \cdot x^2 \cdot b^2 \cdot c^2 \cdot e^2 \\ & g^3 - 2^2 \cdot x^2 \cdot c^2 \cdot e^2 f^2 g^2 - 2^2 \cdot x \cdot a^2 \cdot c^2 \cdot e^2 g^3 - 2^2 \cdot x \cdot b^2 \cdot c^2 \cdot e^2 f^2 g^2 - 2^2 \cdot a^2 \cdot c^2 \cdot e^2 f^2 g^2 \\ & ) / c^2 / (c^2 g^2 x^3 + b^2 g^2 x^2 + c^2 f^2 x^2 + a^2 g^2 x + b^2 f^2 x + a^2 f) / g^2 \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)\sqrt{gx + f}}{\sqrt{cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*sqrt(g\*x + f)/sqrt(c\*x^2 + b\*x + a),x, algorithm="maxima")

[Out] integrate((e\*x + d)\*sqrt(g\*x + f)/sqrt(c\*x^2 + b\*x + a), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex + d)\sqrt{gx + f}}{\sqrt{cx^2 + bx + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)\*sqrt(g\*x + f)/sqrt(c\*x^2 + b\*x + a),x, algorithm="fricas")

[Out] integral((e\*x + d)\*sqrt(g\*x + f)/sqrt(c\*x^2 + b\*x + a), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)\sqrt{f + gx}}{\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)*(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)*sqrt(f + g*x)/sqrt(a + b*x + c*x**2), x)
```

---

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)*sqrt(g*x + f)/sqrt(c*x^2 + b*x + a),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.903 \quad \int \frac{\sqrt{f+gx}}{\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=188

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{c\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}$$

[Out] (Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[f + g\*x]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(c\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[a + b\*x + c\*x^2])

**Rubi [A]** time = 0.247494, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{\sqrt{2}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{c\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[f + g\*x]/Sqrt[a + b\*x + c\*x^2], x]

[Out] (Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[f + g\*x]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(c\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[a + b\*x + c\*x^2])

**Rubi in Sympy [A]** time = 42.4952, size = 177, normalized size = 0.94

$$\frac{\sqrt{2}\sqrt{\frac{c(a+bx+cx^2)}{4ac-b^2}}\sqrt{f+gx}\sqrt{-4ac+b^2}E\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2}\right)\middle|\frac{2g\sqrt{-4ac+b^2}}{bg-2cf+g\sqrt{-4ac+b^2}}\right)}{c\sqrt{\frac{c(-f-gx)}{bg-2cf+g\sqrt{-4ac+b^2}}}\sqrt{a+bx+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*(1/2)/(c\*x\*\*2+b\*x+a)\*\*(1/2), x)

[Out] sqrt(2)\*sqrt(c\*(a + b\*x + c\*x\*\*2)/(4\*a\*c - b\*\*2))\*sqrt(f + g\*x)\*sqrt(-4\*a\*c + b\*\*2)\*elliptic\_e(asin(sqrt(2)\*sqrt((b + 2\*c\*x + sqrt(-4\*a\*c + b\*\*2))/sqrt(-4\*a\*c + b\*\*2))/2), 2\*g\*sqrt(-4\*a\*c + b\*\*2)/(b\*g - 2\*c\*f + g\*sqrt(-4\*a\*c + b\*\*2)))/(c\*sqrt(c\*(-f - g\*x)/(b\*g - 2\*c\*f + g\*sqrt(-4\*a\*c + b\*\*2)))\*sqrt(a + b\*x + c\*x\*\*2))

**Mathematica [C]** time = 1.29136, size = 365, normalized size = 1.94

$$i \left( g \left( \sqrt{b^2 - 4ac} - b \right) + 2cf \right) \sqrt{\frac{g(\sqrt{b^2 - 4ac} + b + 2cx)}{g(\sqrt{b^2 - 4ac} + b) - 2cf}} \sqrt{1 - \frac{2c(f+gx)}{g(\sqrt{b^2 - 4ac} + b) + 2cf}} \left( E \left( i \sinh^{-1} \left( \sqrt{2} \sqrt{\frac{c}{(b + \sqrt{b^2 - 4ac})g - 2cf}} \sqrt{f + gx} \right) \right) \right) \frac{2cf}{2cf} \\ \sqrt{2cg} \sqrt{a + x(b + cx)} \sqrt{\frac{c}{g(\sqrt{b^2 - 4ac} + b) - 2cf}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[f + g\*x]/Sqrt[a + b\*x + c\*x^2], x]

[Out] (I\*(2\*c\*f + (-b + Sqrt[b^2 - 4\*a\*c])\*g)\*Sqrt[(g\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/(-2\*c\*f + (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f + (-b + Sqrt[b^2 - 4\*a\*c])\*g)]\*EllipticE[I\*ArcSinh[Sqrt[2]\*Sqrt[c/(-2\*c\*f + (b + Sqrt[b^2 - 4\*a\*c])\*g)]]\*Sqrt[f + g\*x]], (2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)/(2\*c\*f + (-b + Sqrt[b^2 - 4\*a\*c])\*g) - EllipticF[I\*ArcSinh[Sqrt[2]\*Sqrt[c/(-2\*c\*f + (b + Sqrt[b^2 - 4\*a\*c])\*g)]]\*Sqrt[f + g\*x]], (2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)/(2\*c\*f + (-b + Sqrt[b^2 - 4\*a\*c])\*g)))/(Sqrt[2]\*c\*g\*Sqrt[c/(-2\*c\*f + (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[a + x\*(b + c\*x)])

**Maple [B]** time = 0.044, size = 747, normalized size = 4.

$$\frac{\sqrt{2}}{2g(cgx^3 + bgx^2 + cfx^2 + agx + bfx + fa)c^2} \sqrt{gx + f} \sqrt{cx^2 + bx + a} \left( g\sqrt{-4ac + b^2} + bg - 2cf \right) \sqrt{-(gx + f)c \left( g\sqrt{-4ac + b^2} + bg - 2cf \right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((g\*x+f)^(1/2)/(c\*x^2+b\*x+a)^(1/2), x)

[Out] 1/2\*(g\*x+f)^(1/2)\*(c\*x^2+b\*x+a)^(1/2)\*(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f)^2^(1/2)\*(-(g\*x+f)\*c/(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f))^(1/2)\*(g\*(-2\*c\*x+(-4\*a\*c+b^2)^(1/2)-b)/(2\*c\*f-b\*g+g\*(-4\*a\*c+b^2)^(1/2)))^(1/2)\*(g\*(b+2\*c\*x+(-4\*a\*c+b^2)^(1/2))/(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f))^(1/2)\*(EllipticF(2^(1/2)\*(-(g\*x+f)\*c/(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f))^(1/2), (-g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f)/(2\*c\*f-b\*g+g\*(-4\*a\*c+b^2)^(1/2)))^(1/2))\*g\*b-2\*f\*EllipticF(2^(1/2)\*(-(g\*x+f)\*c/(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f))^(1/2), (-g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f)/(2\*c\*f-b\*g+g\*(-4\*a\*c+b^2)^(1/2)))^(1/2))\*c-EllipticF(2^(1/2)\*(-(g\*x+f)\*c/(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f))^(1/2), (-g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f)/(2\*c\*f-b\*g+g\*(-4\*a\*c+b^2)^(1/2)))^(1/2))\*g\*(-4\*a\*c+b^2)^(1/2)-EllipticE(2^(1/2)\*(-(g\*x+f)\*c/(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f))^(1/2), (-g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f)/(2\*c\*f-b\*g+g\*(-4\*a\*c+b^2)^(1/2)))^(1/2))\*b\*g+2\*EllipticE(2^(1/2)\*(-(g\*x+f)\*c/(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f))^(1/2), (-g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f)/(2\*c\*f-b\*g+g\*(-4\*a\*c+b^2)^(1/2)))^(1/2))\*c\*f+EllipticE(2^(1/2)\*(-(g\*x+f)\*c/(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f))^(1/2), (-g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f)/(2\*c\*f-b\*g+g\*(-4\*a\*c+b^2)^(1/2)))^(1/2))\*(-4\*a\*c+b^2)^(1/2)\*g)/g/(c\*g\*x^3+b\*g\*x^2+c\*f\*x^2+a\*g\*x+b\*f\*x+a\*f)/c^2

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx + f}}{\sqrt{cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `integrate(sqrt(g*x + f)/sqrt(c*x^2 + b*x + a),x, algorithm="maxima")`

[Out] `integrate(sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)`

---

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{gx + f}}{\sqrt{cx^2 + bx + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(g*x + f)/sqrt(c*x^2 + b*x + a),x, algorithm="fricas")`

[Out] `integral(sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)`

---

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{f + gx}}{\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2), x)`

[Out] `Integral(sqrt(f + g*x)/sqrt(a + b*x + c*x**2), x)`

---

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx + f}}{\sqrt{cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(g*x + f)/sqrt(c*x^2 + b*x + a),x, algorithm="giac")`

[Out] `integrate(sqrt(g*x + f)/sqrt(c*x^2 + b*x + a), x)`

$$3.904 \quad \int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=467

$$\frac{2\sqrt{2}g\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}F\left(\sin^{-1}\left(\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}\right)\middle|\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}} \\ - \frac{\sqrt{2}\sqrt{2cf-g(b-\sqrt{b^2-4ac})}\sqrt{1-\frac{2c(f+gx)}{2cf-g(b-\sqrt{b^2-4ac})}}\sqrt{1-\frac{2c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}\left(\frac{e(2cf-bg+\sqrt{b^2-4ac}g)}{2c(e f-dg)}\right); \sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{f+gx}}{\sqrt{2cf-(b-\sqrt{b^2-4ac})}}\right)}{\sqrt{ce}\sqrt{a+bx+cx^2}}$$

[Out] (2\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*g\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(c\*e\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]) - (Sqrt[2]\*Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g]\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*EllipticPi[(e\*(2\*c\*f - b\*g + Sqrt[b^2 - 4\*a\*c]\*g))/(2\*c\*(e\*f - d\*g)), ArcSin[(Sqrt[2]\*Sqrt[c]\*Sqrt[f + g\*x])/Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g]], (b - Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)/(b + Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)]/(Sqrt[c]\*e\*Sqrt[a + b\*x + c\*x^2])

**Rubi [A]** time = 4.52003, antiderivative size = 467, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$

$$\frac{2\sqrt{2}g\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}F\left(\sin^{-1}\left(\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}\right)\middle|\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}} \\ - \frac{\sqrt{2}\sqrt{2cf-g(b-\sqrt{b^2-4ac})}\sqrt{1-\frac{2c(f+gx)}{2cf-g(b-\sqrt{b^2-4ac})}}\sqrt{1-\frac{2c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}\left(\frac{e(2cf-bg+\sqrt{b^2-4ac}g)}{2c(e f-dg)}\right); \sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{f+gx}}{\sqrt{2cf-(b-\sqrt{b^2-4ac})}}\right)}{\sqrt{ce}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[f + g\*x]/((d + e\*x)\*Sqrt[a + b\*x + c\*x^2]), x]

[Out] (2\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*g\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(c\*e\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]) - (Sqrt[2]\*Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g]\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*EllipticPi[(e\*(2\*c\*f - b\*g + Sqrt[b^2 - 4\*a\*c]\*g))/(2\*c\*(e\*f - d\*g)), ArcSin[(Sqrt[2]\*Sqrt[c]\*Sqrt[f + g\*x])/Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g]], (b - Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)/(b + Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)]/(Sqrt[c]\*e\*Sqrt[a + b\*x + c\*x^2])

**Rubi in Sympy [A]** time = 148.005, size = 440, normalized size = 0.94

$$\frac{\sqrt{2} \sqrt{\frac{g(-b-2cx+\sqrt{-4ac+b^2})}{-bg+2cf+g\sqrt{-4ac+b^2}}} \sqrt{\frac{g(b+2cx+\sqrt{-4ac+b^2})}{bg-2cf+g\sqrt{-4ac+b^2}}} \left( \frac{e(bg-2cf-g\sqrt{-4ac+b^2})}{2c(dg-ef)}; \operatorname{asin} \left( \sqrt{2} \sqrt{\frac{c}{-bg+2cf+g\sqrt{-4ac+b^2}}} \sqrt{f+gx} \right) \right) \frac{bg-2cf-g\sqrt{-4ac+b^2}}{bg-2cf+g\sqrt{-4ac+b^2}}}{e \sqrt{\frac{c}{-bg+2cf+g\sqrt{-4ac+b^2}}} \sqrt{a+bx+cx^2}} + \frac{2\sqrt{2}g \sqrt{\frac{c(-f-gx)}{bg-2cf+g\sqrt{-4ac+b^2}}} \sqrt{\frac{c(a+bx+cx^2)}{4ac-b^2}} \sqrt{-4ac+b^2} F \left( \operatorname{asin} \left( \frac{\sqrt{2} \sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2} \right) \right) \frac{2g\sqrt{-4ac+b^2}}{bg-2cf+g\sqrt{-4ac+b^2}}}{ce\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x+f)**(1/2)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)`

[Out] 
$$-\sqrt{2} \sqrt{g(-b-2cx+\sqrt{-4ac+b^2})} / (-bg+2cf+g\sqrt{-4ac+b^2}) + g \sqrt{g(b+2cx+\sqrt{-4ac+b^2})} / (bg-2cf+g\sqrt{-4ac+b^2}) + g \sqrt{g(-4ac+b^2)} / (2c(dg-ef)) \operatorname{asin}(\sqrt{2} \sqrt{\frac{c}{-bg+2cf+g\sqrt{-4ac+b^2}}} \sqrt{f+gx}) + (bg-2cf-g\sqrt{-4ac+b^2}) / (bg-2cf+g\sqrt{-4ac+b^2}) \operatorname{asin}(\sqrt{2} \sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}) + 2 \sqrt{2} g \sqrt{\frac{c(-f-gx)}{bg-2cf+g\sqrt{-4ac+b^2}}} \sqrt{\frac{c(a+bx+cx^2)}{4ac-b^2}} \sqrt{-4ac+b^2} F \left( \operatorname{asin} \left( \frac{\sqrt{2} \sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2} \right) \right) \frac{2g\sqrt{-4ac+b^2}}{bg-2cf+g\sqrt{-4ac+b^2}}$$

**Mathematica [C]** time = 1.51468, size = 379, normalized size = 0.81

$$i\sqrt{2} \sqrt{\frac{g(\sqrt{b^2-4ac}+b+2cx)}{g(\sqrt{b^2-4ac}+b)-2cf}} \sqrt{1 - \frac{2c(f+gx)}{g(\sqrt{b^2-4ac}+b)+2cf}} \left( F \left( i \sinh^{-1} \left( \sqrt{2} \sqrt{\frac{c}{(b+\sqrt{b^2-4ac})g-2cf}} \sqrt{f+gx} \right) \right) \frac{2cf-(b+\sqrt{b^2-4ac})g}{2cf+(\sqrt{b^2-4ac}-b)g} - \left( \frac{e(2c\sqrt{a+x(b+cx)})}{g(\sqrt{b^2-4ac}+b)-2cf} \right) \right)$$

Antiderivative was successfully verified.

[In] `Integrate[Sqrt[f + g*x]/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]`

[Out] 
$$(-1) \sqrt{2} \sqrt{g(\sqrt{b^2-4ac}+b+2cx)} / (g(\sqrt{b^2-4ac}+b)-2cf) + g \sqrt{g(\sqrt{b^2-4ac}+b+2cx)} / (g(\sqrt{b^2-4ac}+b)+2cf) + g \sqrt{g(-4ac+b^2)} / (2c(dg-ef)) \operatorname{asin}(\sqrt{2} \sqrt{\frac{c}{(b+\sqrt{b^2-4ac})g-2cf}} \sqrt{f+gx}) + (bg-2cf-g\sqrt{-4ac+b^2}) / (bg-2cf+g\sqrt{-4ac+b^2}) \operatorname{asin}(\sqrt{2} \sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}) + 2 \sqrt{2} g \sqrt{\frac{c(-f-gx)}{bg-2cf+g\sqrt{-4ac+b^2}}} \sqrt{\frac{c(a+bx+cx^2)}{4ac-b^2}} \sqrt{-4ac+b^2} F \left( \operatorname{asin} \left( \frac{\sqrt{2} \sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2} \right) \right) \frac{2g\sqrt{-4ac+b^2}}{bg-2cf+g\sqrt{-4ac+b^2}}$$

**Maple [B]** time = 0.057, size = 834, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^(1/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)`

```
[Out] (-EllipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))
^(1/2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^
2)^(1/2)))^(1/2))*g*(-4*a*c+b^2)^(1/2)-EllipticF(2^(1/2)*(-(g*x+f)
)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),(-(g*(-4*a*c+b^2)^(1/
2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*g*b+2*f*El
lipticF(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/
2),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(
1/2)))^(1/2))*c+EllipticPi(2^(1/2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1
/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d
*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b
^2)^(1/2)))^(1/2))*(-4*a*c+b^2)^(1/2)*g+EllipticPi(2^(1/2)*(-(g*x
+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4*a*c+b^2)
^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f
)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*b*g-2*EllipticPi(2^(1/
2)*(-(g*x+f)*c/(g*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2),1/2*(g*(-4
*a*c+b^2)^(1/2)+b*g-2*c*f)*e/c/(d*g-e*f),(-(g*(-4*a*c+b^2)^(1/2)+
b*g-2*c*f)/(2*c*f-b*g+g*(-4*a*c+b^2)^(1/2)))^(1/2))*c*f*(g*x+f)^(
1/2)*(c*x^2+b*x+a)^(1/2)/e^2*(g*x+f)*c/(g*(-4*a*c+b^2)^(
1/2)+b*g-2*c*f))^(1/2)*(g*(-2*c*x+(-4*a*c+b^2)^(1/2)-b)/(2*c*f-b*
g+g*(-4*a*c+b^2)^(1/2)))^(1/2)*(g*(b+2*c*x+(-4*a*c+b^2)^(1/2)))/(g
*(-4*a*c+b^2)^(1/2)+b*g-2*c*f))^(1/2)/c/(c*g*x^3+b*g*x^2+c*f*x^2+
a*g*x+b*f*x+a*f)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}(ex+d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)),x, algorithm="fricas")
```

```
[Out] Timed out
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{f+gx}}{(d+ex)\sqrt{a+bx+cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x+f)**(1/2)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral(sqrt(f + g*x)/((d + e*x)*sqrt(a + b*x + c*x**2)), x)
```

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx + f}}{\sqrt{cx^2 + bx + a}(ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

$$3.905 \quad \int \frac{\sqrt{f+gx}}{(d+ex)^2 \sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=994

$$\frac{\sqrt{f+gx}\sqrt{cx^2+bx+a}}{(cd^2-bed+ae^2)(d+ex)} + \frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}} E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right) \middle| -\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{\sqrt{2}(cd^2-bed+ae^2)\sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\sqrt{cx^2+bx+a}} + \frac{\sqrt{2}\sqrt{b^2-4ac}f\sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}} F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right) \middle| -\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{(cd^2-bed+ae^2)\sqrt{f+gx}\sqrt{cx^2+bx+a}} + \frac{\sqrt{2}\sqrt{b^2-4ac}dg\sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}} F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right) \middle| -\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{(cd^2-bed+ae^2)\sqrt{f+gx}\sqrt{cx^2+bx+a}} + \frac{\sqrt{2cf-(b-\sqrt{b^2-4ac})}g(e^2(bf-ag)-cd(2ef-dg))\sqrt{1-\frac{2c(f+gx)}{2cf-(b-\sqrt{b^2-4ac})g}}\sqrt{1-\frac{2c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\left(\frac{e(2cf-bg+\sqrt{b^2-4ac})}{2c(ef-dg)}\right)}{\sqrt{2}\sqrt{c}(cd^2-bed+ae^2)(ef-dg)\sqrt{cx^2+bx+a}}$$

[Out]  $-\left(\left(e\sqrt{f+gx}\sqrt{a+bx+cx^2}\right)/\left(\left(c^2d^2-b^2d^2e+a^2e^2\right)\right)\right)\left(d+e^2x\right)+\left(\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\left(\left(c\left(a+bx+cx^2\right)\right)/\left(b^2-4ac\right)\right)}\right)\left(\text{EllipticE}\left[\text{ArcSin}\left[\sqrt{\left(\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}\right)/\sqrt{2}}\right],\left(-2\sqrt{b^2-4ac}g\right)/\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)\right]/\left(\sqrt{2}\left(c^2d^2-b^2d^2e+a^2e^2\right)\sqrt{\frac{c\left(f+gx\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{a+bx+cx^2}\right)-\left(\sqrt{2}\sqrt{b^2-4ac}f\sqrt{\frac{c\left(f+gx\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\left(\left(c\left(a+bx+cx^2\right)\right)/\left(b^2-4ac\right)\right)}\right)\left(\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\left(\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}\right)/\sqrt{2}}\right],\left(-2\sqrt{b^2-4ac}g\right)/\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)\right]/\left(\left(c^2d^2-b^2d^2e+a^2e^2\right)\sqrt{f+gx}\sqrt{a+bx+cx^2}\right)+\left(\sqrt{2}\sqrt{b^2-4ac}dg\sqrt{\frac{c\left(f+gx\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\sqrt{-\left(\left(c\left(a+bx+cx^2\right)\right)/\left(b^2-4ac\right)\right)}\right)\left(\text{EllipticF}\left[\text{ArcSin}\left[\sqrt{\left(\frac{b+\sqrt{b^2-4ac}+2cx}{\sqrt{b^2-4ac}}\right)/\sqrt{2}}\right],\left(-2\sqrt{b^2-4ac}g\right)/\left(2cf-\left(b+\sqrt{b^2-4ac}\right)g\right)\right]/\left(e\left(c^2d^2-b^2d^2e+a^2e^2\right)\sqrt{f+gx}\sqrt{a+bx+cx^2}\right)+\left(\sqrt{2cf-\left(b-\sqrt{b^2-4ac}\right)g}\left(e^2\left(bf-ag\right)-cd\left(2ef-dg\right)\right)\sqrt{1-\frac{2c\left(f+gx\right)}{2cf-\left(b-\sqrt{b^2-4ac}\right)g}}\sqrt{1-\frac{2c\left(f+gx\right)}{2cf-\left(b+\sqrt{b^2-4ac}\right)g}}\left(\frac{e\left(2cf-bg+\sqrt{b^2-4ac}\right)}{2c\left(ef-dg\right)}\right)\right)\right)$

**Rubi [A]** time = 8.64032, antiderivative size = 994, normalized size of antiderivative = 1., number of

steps used = 15, number of rules used = 10, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+bx+ae}}{(cd^2-bed+ae^2)(d+ex)}$$

$$+ \frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}} E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right) \middle| -\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{\sqrt{2}(cd^2-bed+ae^2)\sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\sqrt{cx^2+bx+a}}$$

$$- \frac{\sqrt{2}\sqrt{b^2-4ac}f\sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}} F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right) \middle| -\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{(cd^2-bed+ae^2)\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$

$$+ \frac{\sqrt{2}\sqrt{b^2-4ac}dg\sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}} F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right) \middle| -\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{(cd^2-bed+ae^2)\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$

$$+ \frac{\sqrt{2cf-(b-\sqrt{b^2-4ac})}g(e^2(bf-ag)-cd(2ef-dg))\sqrt{1-\frac{2c(f+gx)}{2cf-(b-\sqrt{b^2-4ac})g}}\sqrt{1-\frac{2c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\left(\frac{e(2cf-bg+\sqrt{b^2-4ac})}{2c(ef-dg)}\right)}{\sqrt{2}\sqrt{c}(cd^2-bed+ae^2)(ef-dg)\sqrt{cx^2+bx+ae}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[f + g\*x]/((d + e\*x)^2\*Sqrt[a + b\*x + c\*x^2]), x]

[Out]  $-\left(\frac{e\sqrt{f+gx}\sqrt{a+bx+cx^2}}{(cd^2-bed+ae^2)(d+ex)}\right) + \left(\frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}} E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right) \middle| -\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{\sqrt{2}(cd^2-bed+ae^2)\sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\sqrt{cx^2+bx+a}}\right) - \left(\frac{\sqrt{2}\sqrt{b^2-4ac}f\sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}} F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right) \middle| -\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{(cd^2-bed+ae^2)\sqrt{f+gx}\sqrt{cx^2+bx+a}}\right) + \left(\frac{\sqrt{2}\sqrt{b^2-4ac}dg\sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}} F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right) \middle| -\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{(cd^2-bed+ae^2)\sqrt{f+gx}\sqrt{cx^2+bx+a}}\right) + \left(\frac{\sqrt{2cf-(b-\sqrt{b^2-4ac})}g(e^2(bf-ag)-cd(2ef-dg))\sqrt{1-\frac{2c(f+gx)}{2cf-(b-\sqrt{b^2-4ac})g}}\sqrt{1-\frac{2c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\left(\frac{e(2cf-bg+\sqrt{b^2-4ac})}{2c(ef-dg)}\right)}{\sqrt{2}\sqrt{c}(cd^2-bed+ae^2)(ef-dg)\sqrt{cx^2+bx+ae}}\right)$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^2\sqrt{a+bx+cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*(1/2)/(e\*x+d)\*\*2/(c\*x\*\*2+b\*x+a)\*\*(1/2), x)

[Out] `Integral(sqrt(f + g*x)/((d + e*x)**2*sqrt(a + b*x + c*x**2)), x)`

**Mathematica [C]** time = 14.9445, size = 18563, normalized size = 18.68

Result too large to show

Warning: Unable to verify antiderivative.

[In] `Integrate[Sqrt[f + g*x]/((d + e*x)^2*Sqrt[a + b*x + c*x^2]), x]`

[Out] Result too large to show

**Maple [B]** time = 0.074, size = 13017, normalized size = 13.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^(1/2)/(e*x+d)^2/(c*x^2+b*x+a)^(1/2), x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}(ex+d)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^2), x, algorithm="maxima")`

[Out] `integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^2), x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^2), x, algorithm="fricas")`

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((g*x+f)**(1/2)/(e*x+d)**2/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Timed out
```

---

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.906 \quad \int \frac{\sqrt{f+gx}}{(d+ex)^3 \sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=1786

result too large to display

```
[Out] -(e*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) - (e*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*e*g))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(4*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*(d + e*x)) + (Sqrt[b^2 - 4*a*c]*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*e*g))*Sqrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g))/(4*Sqrt[2]*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g))/(Sqrt[2]*e*(c*d^2 - b*d*e + a*e^2)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[b^2 - 4*a*c]*f*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g))/(2*Sqrt[2]*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[b^2 - 4*a*c]*d*g*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g))/(2*Sqrt[2]*e*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*e*f - 3*c*d*g + b*e*g)*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g))/(Sqrt[2]*Sqrt[c]*e*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*Sqrt[a + b*x + c*x^2]) - (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*e*g))*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g))/(4*Sqrt[2]*Sqrt[c]*e*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^2*Sqrt[a + b*x + c*x^2])
```

**Rubi [A]** time = 19.5, antiderivative size = 1786, normalized size of antiderivative = 1., number of steps used = 25, number of rules used = 11, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.355$

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[Sqrt[f + g*x]/((d + e*x)^3*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(e*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*e^2)*(d + e*x)^2) - (e*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*e*g))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(4*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*(d + e*x)) + (Sqrt[b^2 - 4*a*c]*(c*d*(6*e
```

```
f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*e*g))*Sqrt[f + g*x]*Sqrt[-(
(c*(a + b*x + c*x^2))/(b^2 - 4*a*c)]*EllipticE[ArcSin[Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[
b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]/(4*Sqrt[2]*
(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*Sqrt[(c*(f + g*x))/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]) - (Sqrt[b^2 -
4*a*c]*g*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]
*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)]*EllipticF[ArcSin[Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (
-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]/(Sq
rt[2]*e*(c*d^2 - b*d*e + a*e^2)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^
2]) - (Sqrt[b^2 - 4*a*c]*f*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*
b*d*g - a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c
])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)]*EllipticF[Arc
Sin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[
2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)
)]/(2*Sqrt[2]*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*Sqrt[f + g*x]
*Sqrt[a + b*x + c*x^2]) + (Sqrt[b^2 - 4*a*c]*d*g*(c*d*(6*e*f - 5*
d*g) - e*(3*b*e*f - 2*b*d*g - a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f -
(b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 -
4*a*c)]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sq
rt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b +
Sqrt[b^2 - 4*a*c])*g)]/(2*Sqrt[2]*e*(c*d^2 - b*d*e + a*e^2)^2*(
e*f - d*g)*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) + (Sqrt[2*c*f - (
b - Sqrt[b^2 - 4*a*c])*g]*(c*e*f - 3*c*d*g + b*e*g)*Sqrt[1 - (2*c
*(f + g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f
+ g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*
f - b*g + Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2
]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]]
, (b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2
*c*f)/g)]/(Sqrt[2]*Sqrt[c]*e*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)
*Sqrt[a + b*x + c*x^2]) - (Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g
]*(c*d*(6*e*f - 5*d*g) - e*(3*b*e*f - 2*b*d*g - a*e*g))*Sqrt[1 - (2*
e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g)]*Sqrt[1 - (2*c*(f + g*x
))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f + g*x))/
(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f - b*g +
Sqrt[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*Sqrt[c]
*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (b - Sq
rt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*f)/g)
)]/(4*Sqrt[2]*Sqrt[c]*e*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^2*Sq
rt[a + b*x + c*x^2])
```

---

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{f+gx}}{(d+ex)^3 \sqrt{a+bx+cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((g\*x+f)\*\*(1/2)/(e\*x+d)\*\*3/(c\*x\*\*2+b\*x+a)\*\*(1/2), x)

[Out] Integral(sqrt(f + g\*x)/((d + e\*x)\*\*3\*sqrt(a + b\*x + c\*x\*\*2)), x)

---

**Mathematica [C]** time = 20.6577, size = 36634, normalized size = 20.51

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[f + g\*x]/((d + e\*x)^3\*Sqrt[a + b\*x + c\*x^2]), x]

[Out] Result too large to show

---

**Maple [B]** time = 0.193, size = 59522, normalized size = 33.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^(1/2)/(e*x+d)^3/(c*x^2+b*x+a)^(1/2), x)`

[Out] result too large to display

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}(ex+d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^3), x, algorithm="maxima")`

[Out] `integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^3), x)`

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^3), x, algorithm="fricas")`

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)**(1/2)/(e*x+d)**3/(c*x**2+b*x+a)**(1/2), x)`

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{gx+f}}{\sqrt{cx^2+bx+a}(ex+d)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^3), x, algorithm="giac")`

[Out] `integrate(sqrt(g*x + f)/(sqrt(c*x^2 + b*x + a)*(e*x + d)^3), x)`

$$3.907 \quad \int \frac{(f+gx)^{3/2}}{(d+ex)\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=675

$$\frac{2\sqrt{2}g\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{ce^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$+\frac{\sqrt{2}(ef-dg)\sqrt{2cf-g(b-\sqrt{b^2-4ac})}\sqrt{1-\frac{2c(f+gx)}{2cf-g(b-\sqrt{b^2-4ac})}}\sqrt{1-\frac{2c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}\left(\frac{e(2cf-bg+\sqrt{b^2-4ac}g)}{2c(ef-dg)}\right);\sin^{-1}\left(\frac{\sqrt{2cf-g(b-\sqrt{b^2-4ac})}}{\sqrt{2cf-g(\sqrt{b^2-4ac}+b)}}\right)}{\sqrt{ce^2}\sqrt{a+bx+cx^2}}$$

$$+\frac{\sqrt{2}g\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}}$$

[Out] (Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*g\*Sqrt[f + g\*x]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(c\*e\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[a + b\*x + c\*x^2]) + (2\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*g\*(e\*f - d\*g)\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(c\*e^2\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]) - (Sqrt[2]\*Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g]\*(e\*f - d\*g)\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*EllipticPi[(e\*(2\*c\*f - b\*g + Sqrt[b^2 - 4\*a\*c]\*g))/(2\*c\*(e\*f - d\*g)), ArcSin[(Sqrt[2]\*Sqrt[c]\*Sqrt[f + g\*x])/Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g]], (b - Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)/(b + Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)]/(Sqrt[c]\*e^2\*Sqrt[a + b\*x + c\*x^2])

**Rubi [A]** time = 4.72547, antiderivative size = 675, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$

$$\frac{2\sqrt{2}g\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{ce^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

$$+\frac{\sqrt{2}(ef-dg)\sqrt{2cf-g(b-\sqrt{b^2-4ac})}\sqrt{1-\frac{2c(f+gx)}{2cf-g(b-\sqrt{b^2-4ac})}}\sqrt{1-\frac{2c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}\left(\frac{e(2cf-bg+\sqrt{b^2-4ac}g)}{2c(ef-dg)}\right);\sin^{-1}\left(\frac{\sqrt{2cf-g(b-\sqrt{b^2-4ac})}}{\sqrt{2cf-g(\sqrt{b^2-4ac}+b)}}\right)}{\sqrt{ce^2}\sqrt{a+bx+cx^2}}$$

$$+\frac{\sqrt{2}g\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{ce\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)^(3/2)/((d + e\*x)\*Sqrt[a + b\*x + c\*x^2]), x]

[Out] (Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*g\*Sqrt[f + g\*x]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c]

$$c] + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (-2*\text{Sqrt}[b^2 - 4*a*c]*g) / ((2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)) / (c*e*\text{Sqrt}[(c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]*\text{Sqrt}[a + b*x + c*x^2]) + (2*\text{Sqrt}[2]*\text{Sqrt}[b^2 - 4*a*c]*g*(e*f - d*g)*\text{Sqrt}[(c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]*\text{Sqrt}[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (-2*\text{Sqrt}[b^2 - 4*a*c]*g)/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)) / (c*e^2*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + b*x + c*x^2]) - (\text{Sqrt}[2]*\text{Sqrt}[2*c*f - (b - \text{Sqrt}[b^2 - 4*a*c])*g]*(e*f - d*g)*\text{Sqrt}[1 - (2*c*(f + g*x))/(2*c*f - (b - \text{Sqrt}[b^2 - 4*a*c])*g)]*\text{Sqrt}[1 - (2*c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]*\text{EllipticPi}[(e*(2*c*f - b*g + \text{Sqrt}[b^2 - 4*a*c])*g)/(2*c*(e*f - d*g)), \text{ArcSin}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[f + g*x])/(\text{Sqrt}[2*c*f - (b - \text{Sqrt}[b^2 - 4*a*c])*g])], (b - \text{Sqrt}[b^2 - 4*a*c] - (2*c*f)/g)/(b + \text{Sqrt}[b^2 - 4*a*c] - (2*c*f)/g)) / (\text{Sqrt}[c]*e^2*\text{Sqrt}[a + b*x + c*x^2])$$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x+f)**(3/2)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)`

[Out] Timed out

**Mathematica [B]** time = 6.36351, size = 2358, normalized size = 3.49

Result too large to show

Warning: Unable to verify antiderivative.

[In] `Integrate[(f + g*x)^(3/2)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]`

[Out]  $(\text{Sqrt}[(f + g*x)*(a + b*x + c*x^2)])*((4*f*g*\text{Sqrt}[(-(-b - \text{Sqrt}[b^2 - 4*a*c]))/(2*c) + x)/(-(-b - \text{Sqrt}[b^2 - 4*a*c]))/(2*c) + (-b + \text{Sqrt}[b^2 - 4*a*c]))/(2*c)))*(-(-b + \text{Sqrt}[b^2 - 4*a*c]))/(2*c) + x)*\text{Sqrt}[(f/g + x)/((-b + \text{Sqrt}[b^2 - 4*a*c]))/(2*c) + f/g]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (2*\text{Sqrt}[b^2 - 4*a*c]*g)/(2*c*f - b*g + \text{Sqrt}[b^2 - 4*a*c]*g)]/(e*\text{Sqrt}[(-(-b + \text{Sqrt}[b^2 - 4*a*c]))/(2*c) + x)/((-b - \text{Sqrt}[b^2 - 4*a*c]))/(2*c) - (-b + \text{Sqrt}[b^2 - 4*a*c]))/(2*c))*\text{Sqrt}[(f + g*x)*(a + b*x + c*x^2)] - (2*d*g^2*\text{Sqrt}[(-(-b - \text{Sqrt}[b^2 - 4*a*c]))/(2*c) + x)/(-(-b - \text{Sqrt}[b^2 - 4*a*c]))/(2*c) + (-b + \text{Sqrt}[b^2 - 4*a*c]))/(2*c))*(-(-b + \text{Sqrt}[b^2 - 4*a*c]))/(2*c) + x)*\text{Sqrt}[(f/g + x)/((-b + \text{Sqrt}[b^2 - 4*a*c]))/(2*c) + f/g]*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (2*\text{Sqrt}[b^2 - 4*a*c]*g)/(2*c*f - b*g + \text{Sqrt}[b^2 - 4*a*c]*g)]/(e^2*\text{Sqrt}[(-(-b + \text{Sqrt}[b^2 - 4*a*c]))/(2*c) + x)/((-b - \text{Sqrt}[b^2 - 4*a*c]))/(2*c) - (-b + \text{Sqrt}[b^2 - 4*a*c]))/(2*c))*\text{Sqrt}[(f + g*x)*(a + b*x + c*x^2)] + (2*\text{Sqrt}[2]*g^2*\text{Sqrt}[(-((c*g*(-(-b - \text{Sqrt}[b^2 - 4*a*c]))/(2*c) + x))/(2*c*f - b*g - \text{Sqrt}[b^2 - 4*a*c]*g))]*(-(-b + \text{Sqrt}[b^2 - 4*a*c]))/(2*c) + x)*\text{Sqrt}[(f/g + x)/((-b + \text{Sqrt}[b^2 - 4*a*c]))/(2*c) + f/g]*(((2*c*f - b*g - \text{Sqrt}[b^2 - 4*a*c])*g)*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[2]*\text{Sqrt}[(c*(f + g*x))/(2*c*f - b*g + \text{Sqrt}[b^2 - 4*a*c]*g)]], (2*c*f - b*g + \text{Sqrt}[b^2 - 4*a*c]*g)/(2*c*f - b*g - \text{Sqrt}[b^2 - 4*a*c]*g)]/(2*c*g) - ((-b - \text{Sqrt}[b^2 - 4*a*c])*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[2]*\text{Sqrt}[(c*(f + g*x))/(2*c*f - b*g + \text{Sqrt}[b^2 - 4*a*c]*g)]], (2*c*f - b*g + \text{Sqrt}[b^2 - 4*a*c]*g)/(2*c*f - b*g - \text{Sqrt}[b^2 - 4*a*c]*g)]/(2*c)))/(e*\text{Sqrt}[(-(-b + \text{Sqrt}[b^2 - 4*a*c]))/(2*c) + x)/((-b - \text{Sqrt}[b^2 - 4*a*c]))/(2*c) - f/g)*\text{Sqrt}[(f + g*x)*(a + b*x + c*x^2)] + (2*(-(-b - \text{Sqrt}[b^2 - 4*a*c]))/(2*c) + (-b + \text{Sqrt}[b^2 - 4*a*c]))/(2*c))*f^2*\text{Sqrt}[(-(((-b - \text{Sqrt}[b^2 - 4*a*c]))/(2*c)$

$$\begin{aligned}
& + x) * (-(-b + \text{Sqrt}[b^2 - 4*a*c]) / (2*c) + x) / (-(-b - \text{Sqrt}[b^2 - 4*a*c]) / (2*c) + (-b + \text{Sqrt}[b^2 - 4*a*c]) / (2*c))^2] * \text{Sqrt}[(f/g + x) / ((-b + \text{Sqrt}[b^2 - 4*a*c]) / (2*c) + f/g)] * \text{EllipticPi}[(2*\text{Sqrt}[b^2 - 4*a*c]*e) / (2*c*d - b*e + \text{Sqrt}[b^2 - 4*a*c]*e), \text{ArcSin}[\text{Sqrt}[(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x) / \text{Sqrt}[b^2 - 4*a*c]] / \text{Sqrt}[2]], (2*\text{Sqrt}[b^2 - 4*a*c]*g) / (2*c*f - b*g + \text{Sqrt}[b^2 - 4*a*c]*g))] / ((-d - ((-b + \text{Sqrt}[b^2 - 4*a*c]*e) / (2*c)) * \text{Sqrt}[(f + g*x)*(a + b*x + c*x^2)]) - (4*(-(-b - \text{Sqrt}[b^2 - 4*a*c]) / (2*c) + (-b + \text{Sqrt}[b^2 - 4*a*c]) / (2*c)) * d*f*g * \text{Sqrt}[(-((-b - \text{Sqrt}[b^2 - 4*a*c]) / (2*c) + x) * (-(-b + \text{Sqrt}[b^2 - 4*a*c]) / (2*c) + x)) / (-(-b - \text{Sqrt}[b^2 - 4*a*c]) / (2*c) + (-b + \text{Sqrt}[b^2 - 4*a*c]) / (2*c))^2] * \text{Sqrt}[(f/g + x) / ((-b + \text{Sqrt}[b^2 - 4*a*c]) / (2*c) + f/g)] * \text{EllipticPi}[(2*\text{Sqrt}[b^2 - 4*a*c]*e) / (2*c*d - b*e + \text{Sqrt}[b^2 - 4*a*c]*e), \text{ArcSin}[\text{Sqrt}[(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x) / \text{Sqrt}[b^2 - 4*a*c]] / \text{Sqrt}[2]], (2*\text{Sqrt}[b^2 - 4*a*c]*g) / (2*c*f - b*g + \text{Sqrt}[b^2 - 4*a*c]*g))] / (e*(-d - ((-b + \text{Sqrt}[b^2 - 4*a*c]*e) / (2*c)) * \text{Sqrt}[(f + g*x)*(a + b*x + c*x^2)])) + (2*(-(-b - \text{Sqrt}[b^2 - 4*a*c]) / (2*c) + (-b + \text{Sqrt}[b^2 - 4*a*c]) / (2*c)) * d^2*g^2 * \text{Sqrt}[(-((-b - \text{Sqrt}[b^2 - 4*a*c]) / (2*c) + x) * (-(-b + \text{Sqrt}[b^2 - 4*a*c]) / (2*c) + x)) / (-(-b - \text{Sqrt}[b^2 - 4*a*c]) / (2*c) + (-b + \text{Sqrt}[b^2 - 4*a*c]) / (2*c))^2] * \text{Sqrt}[(f/g + x) / ((-b + \text{Sqrt}[b^2 - 4*a*c]) / (2*c) + f/g)] * \text{EllipticPi}[(2*\text{Sqrt}[b^2 - 4*a*c]*e) / (2*c*d - b*e + \text{Sqrt}[b^2 - 4*a*c]*e), \text{ArcSin}[\text{Sqrt}[(-b + \text{Sqrt}[b^2 - 4*a*c] - 2*c*x) / \text{Sqrt}[b^2 - 4*a*c]] / \text{Sqrt}[2]], (2*\text{Sqrt}[b^2 - 4*a*c]*g) / (2*c*f - b*g + \text{Sqrt}[b^2 - 4*a*c]*g))] / (e^2*(-d - ((-b + \text{Sqrt}[b^2 - 4*a*c]*e) / (2*c)) * \text{Sqrt}[(f + g*x)*(a + b*x + c*x^2)])) / (\text{Sqrt}[f + g*x] * \text{Sqrt}[a + x*(b + c*x)])
\end{aligned}$$

**Maple [B]** time = 0.057, size = 1879, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{int}((g*x+f)^{3/2}/(e*x+d)/(c*x^2+b*x+a)^{1/2}, x)$

[Out]  $(g*x+f)^{1/2} * (c*x^2+b*x+a)^{1/2} * 2^{1/2} * (- (g*x+f) * c / (g * (-4*a*c + b^2)^{1/2} + b*g - 2*c*f))^{1/2} * (g * (-2*c*x + (-4*a*c + b^2)^{1/2} - b) / (2*c*f - b*g + g * (-4*a*c + b^2)^{1/2}))^{1/2} * (g * (b + 2*c*x + (-4*a*c + b^2)^{1/2}) / (g * (-4*a*c + b^2)^{1/2} + b*g - 2*c*f))^{1/2} / c * (\text{EllipticF}(2^{1/2} * (- (g*x+f) * c / (g * (-4*a*c + b^2)^{1/2} + b*g - 2*c*f))^{1/2}, (- (g * (-4*a*c + b^2)^{1/2} + b*g - 2*c*f) / (2*c*f - b*g + g * (-4*a*c + b^2)^{1/2}))^{1/2}))^{1/2} * (-4*a*c + b^2)^{1/2} * d * g^2 - \text{EllipticF}(2^{1/2} * (- (g*x+f) * c / (g * (-4*a*c + b^2)^{1/2} + b*g - 2*c*f))^{1/2}, (- (g * (-4*a*c + b^2)^{1/2} + b*g - 2*c*f) / (2*c*f - b*g + g * (-4*a*c + b^2)^{1/2}))^{1/2}))^{1/2} * (-4*a*c + b^2)^{1/2} * e * f * g + 2 * \text{EllipticF}(2^{1/2} * (- (g*x+f) * c / (g * (-4*a*c + b^2)^{1/2} + b*g - 2*c*f))^{1/2}, (- (g * (-4*a*c + b^2)^{1/2} + b*g - 2*c*f) / (2*c*f - b*g + g * (-4*a*c + b^2)^{1/2}))^{1/2}))^{1/2} * a * e * g^2 + \text{EllipticF}(2^{1/2} * (- (g*x+f) * c / (g * (-4*a*c + b^2)^{1/2} + b*g - 2*c*f))^{1/2}, (- (g * (-4*a*c + b^2)^{1/2} + b*g - 2*c*f) / (2*c*f - b*g + g * (-4*a*c + b^2)^{1/2}))^{1/2}))^{1/2} * b * d * g^2 - 3 * \text{EllipticF}(2^{1/2} * (- (g*x+f) * c / (g * (-4*a*c + b^2)^{1/2} + b*g - 2*c*f))^{1/2}, (- (g * (-4*a*c + b^2)^{1/2} + b*g - 2*c*f) / (2*c*f - b*g + g * (-4*a*c + b^2)^{1/2}))^{1/2}))^{1/2} * b * e * f * g - 2 * \text{EllipticF}(2^{1/2} * (- (g*x+f) * c / (g * (-4*a*c + b^2)^{1/2} + b*g - 2*c*f))^{1/2}, (- (g * (-4*a*c + b^2)^{1/2} + b*g - 2*c*f) / (2*c*f - b*g + g * (-4*a*c + b^2)^{1/2}))^{1/2}))^{1/2} * c * d * f * g + 4 * \text{EllipticF}(2^{1/2} * (- (g*x+f) * c / (g * (-4*a*c + b^2)^{1/2} + b*g - 2*c*f))^{1/2}, (- (g * (-4*a*c + b^2)^{1/2} + b*g - 2*c*f) / (2*c*f - b*g + g * (-4*a*c + b^2)^{1/2}))^{1/2}))^{1/2} * c * e * f^2 - 2 * \text{EllipticE}(2^{1/2} * (- (g*x+f) * c / (g * (-4*a*c + b^2)^{1/2} + b*g - 2*c*f))^{1/2}, (- (g * (-4*a*c + b^2)^{1/2} + b*g - 2*c*f) / (2*c*f - b*g + g * (-4*a*c + b^2)^{1/2}))^{1/2}))^{1/2} * a * e * g^2 + 2 * \text{EllipticE}(2^{1/2} * (- (g*x+f) * c / (g * (-4*a*c + b^2)^{1/2} + b*g - 2*c*f))^{1/2}, (- (g * (-4*a*c + b^2)^{1/2} + b*g - 2*c*f) / (2*c*f - b*g + g * (-4*a*c + b^2)^{1/2}))^{1/2}))^{1/2} * b * e * f * g - 2 * \text{EllipticE}(2^{1/2} * (- (g*x+f) * c / (g * (-4*a*c + b^2)^{1/2} + b*g - 2*c*f))^{1/2}, (- (g * (-4*a*c + b^2)^{1/2} + b*g - 2*c*f) / (2*c*f - b*g + g * (-4*a*c + b^2)^{1/2}))^{1/2}))^{1/2} * c * e * f^2 - \text{EllipticPi}(2^{1/2} * (- (g*x+f) * c / (g * (-4*a*c + b^2)^{1/2} + b*g - 2*c*f))^{1/2}, 1/2 * (g * (-4*a*c + b^2)^{1/2} + b*g - 2*c*f) * e / c / (d * g - e * f), (- (g * (-4*a*c + b^2)^{1/2} + b*g - 2*c*f) / (2*c*f - b*g + g * (-4*a*c + b^2)^{1/2}))^{1/2}))^{1/2} * (-4*a*c + b^2)^{1/2} * d * g^2 + \text{EllipticPi}(2^{1/2} * (- (g*x+f)$

$$\begin{aligned} & )^*c/(g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}, 1/2^*(g^*(-4^*a^*c+b^2)^{(1/2)}+ \\ & 1/2)+b^*g-2^*c^*f)^*e/c/(d^*g-e^*f), (- (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f)/ \\ & (2^*c^*f-b^*g+g^*(-4^*a^*c+b^2)^{(1/2)}))^{(1/2)})^*(-4^*a^*c+b^2)^{(1/2)}^*e^*f^*g \\ & -\text{EllipticPi}(2^{(1/2)}^*(-(g^*x+f)^*c/(g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f)) \\ & ^{(1/2)}, 1/2^*(g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f)^*e/c/(d^*g-e^*f), (- (g^*(-4^* \\ & 4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f)/(2^*c^*f-b^*g+g^*(-4^*a^*c+b^2)^{(1/2)}))^{(1/2)} \\ & ))^*b^*d^*g^2+\text{EllipticPi}(2^{(1/2)}^*(-(g^*x+f)^*c/(g^*(-4^*a^*c+b^2)^{(1/2)}+ \\ & b^*g-2^*c^*f))^{(1/2)}, 1/2^*(g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f)^*e/c/(d^*g-e \\ & ^*f), (- (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f)/(2^*c^*f-b^*g+g^*(-4^*a^*c+b^2)^{(1/2)} \\ & ^{(1/2)}))^{(1/2)})^*b^*e^*f^*g+2^*\text{EllipticPi}(2^{(1/2)}^*(-(g^*x+f)^*c/(g^*(-4^*a^* \\ & c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}, 1/2^*(g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^* \\ & f)^*e/c/(d^*g-e^*f), (- (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f)/(2^*c^*f-b^*g+g^* \\ & ^*(-4^*a^*c+b^2)^{(1/2)}))^{(1/2)})^*c^*d^*f^*g-2^*\text{EllipticPi}(2^{(1/2)}^*(-(g^*x+f) \\ & )^*c/(g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}, 1/2^*(g^*(-4^*a^*c+b^2)^{(1/2)}+ \\ & 1/2)+b^*g-2^*c^*f)^*e/c/(d^*g-e^*f), (- (g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f)/ \\ & (2^*c^*f-b^*g+g^*(-4^*a^*c+b^2)^{(1/2)}))^{(1/2)})^*c^*e^*f^2)/e^2/(c^*g^*x^3+b^* \\ & g^*x^2+c^*f^*x^2+a^*g^*x+b^*f^*x+a^*f) \end{aligned}$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx + f)^{\frac{3}{2}}}{\sqrt{cx^2 + bx + a}(ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^(3/2)/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)),x, algorithm="maxima"

[Out] integrate((g\*x + f)^(3/2)/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)), x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)^(3/2)/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)),x, algorithm="fricas"

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(f + gx)^{\frac{3}{2}}}{(d + ex)\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x+f)\*\*(3/2)/(e\*x+d)/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral((f + g\*x)\*\*(3/2)/((d + e\*x)\*sqrt(a + b\*x + c\*x\*\*2)), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx + f)^{\frac{3}{2}}}{\sqrt{cx^2 + bx + a}(ex + d)} dx$$



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((g*x + f)^(3/2)/(sqrt(c*x^2 + b*x + a)*(e*x + d)),x, algorithm="giac")
```

```
[Out] integrate((g*x + f)^(3/2)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)
```

$$3.908 \quad \int \frac{(f+gx)^{5/2}}{(d+ex)\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=1138

result too large to display

```
[Out] (2*g^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(3*c*e) + (2*Sqrt[2]*
Sqrt[b^2 - 4*a*c]*g*(2*c*f - b*g)*Sqrt[f + g*x]*Sqrt[-((c*(a + b*
x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 -
4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*
c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]/(3*c^2*e*Sqrt[(c*(f +
g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x + c*x^2]
) + (Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*(e*f - d*g)*Sqrt[f + g*x]*Sqrt[-
((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticE[ArcSin[Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt
[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]/(c*e^2*Sqr
t[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x
+ c*x^2]) + (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*(e*f - d*g)^2*Sqrt[(c
*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b
*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2
- 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a
*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]/(c*e^3*Sqrt[f + g*x]
*Sqrt[a + b*x + c*x^2]) - (2*Sqrt[2]*Sqrt[b^2 - 4*a*c]*g*(c*f^2 -
b*f*g + a*g^2)*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c]
)*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*EllipticF[Arc
Sin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[
2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)
)]/(3*c^2*e*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (Sqrt[2]*Sqrt[
2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]*(e*f - d*g)^2*Sqrt[1 - (2*c*(f
+ g*x))/(2*c*f - (b - Sqrt[b^2 - 4*a*c])*g)]*Sqrt[1 - (2*c*(f +
g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*EllipticPi[(e*(2*c*f -
b*g + Sqrt[b^2 - 4*a*c])*g)/(2*c*(e*f - d*g)), ArcSin[(Sqrt[2]*S
qrt[c]*Sqrt[f + g*x])/Sqrt[2*c*f - (b - Sqrt[b^2 - 4*a*c])*g]], (
b - Sqrt[b^2 - 4*a*c] - (2*c*f)/g)/(b + Sqrt[b^2 - 4*a*c] - (2*c*
f)/g)]/(Sqrt[c]*e^3*Sqrt[a + b*x + c*x^2])
```

---

**Rubi [A]** time = 6.03217, antiderivative size = 1138, normalized size of antiderivative = 1., number

of steps used = 17, number of rules used = 10, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$

$$\begin{aligned}
 & \frac{2\sqrt{f+gx}\sqrt{cx^2+bx+ag^2}}{3ce} \\
 & + \frac{2\sqrt{2}\sqrt{b^2-4ac}(2cf-bg)\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)g}{3c^2e\sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\sqrt{cx^2+bx+a}} \\
 & + \frac{\sqrt{2}\sqrt{b^2-4ac}(ef-dg)\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)g}{ce^2\sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\sqrt{cx^2+bx+a}} \\
 & + \frac{2\sqrt{2}\sqrt{b^2-4ac}(ef-dg)^2\sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)g}{ce^3\sqrt{f+gx}\sqrt{cx^2+bx+a}} \\
 & + \frac{2\sqrt{2}\sqrt{b^2-4ac}(cf^2-bgf+ag^2)\sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)g}{3c^2e\sqrt{f+gx}\sqrt{cx^2+bx+a}} \\
 & + \frac{\sqrt{2}\sqrt{2cf-(b-\sqrt{b^2-4ac})}g(ef-dg)^2\sqrt{1-\frac{2c(f+gx)}{2cf-(b-\sqrt{b^2-4ac})g}}\sqrt{1-\frac{2c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\left(\frac{e^{(2cf-bg+\sqrt{b^2-4ac}g)}}{2c(ef-dg)};\sin^{-1}\left(\frac{1}{\sqrt{2cf-(b-\sqrt{b^2-4ac})g}}\right)\right)}{\sqrt{ce^3}\sqrt{cx^2+bx+a}}
 \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(f + g\*x)^(5/2)/((d + e\*x)\*Sqrt[a + b\*x + c\*x^2]),x]

[Out] (2\*g^2\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])/(3\*c\*e) + (2\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*g\*(2\*c\*f - b\*g)\*Sqrt[f + g\*x]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(3\*c^2\*e\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[a + b\*x + c\*x^2]) + (Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*g\*(e\*f - d\*g)\*Sqrt[f + g\*x]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(c\*e^2\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[a + b\*x + c\*x^2]) + (2\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*g\*(e\*f - d\*g)^2\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(c\*e^3\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]) - (2\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*g\*(c\*f^2 - b\*f\*g + a\*g^2)\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(3\*c^2\*e\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]) - (Sqrt[2]\*Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g]\*(e\*f - d\*g)^2\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*EllipticPi[(e\*(2\*c\*f - b\*g + Sqrt[b^2 - 4\*a\*c]\*g))/(2\*c\*(e\*f - d\*g)], ArcSin[(Sqrt[2]\*Sqrt[c]\*Sqrt[f + g\*x])/Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g]], (b - Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)/(b + Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)]/(Sqrt[c]\*e^3\*Sqrt[a + b\*x + c\*x^2])

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((g*x+f)**(5/2)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)`

[Out] Timed out

**Mathematica [C]** time = 18.56, size = 37137, normalized size = 32.63

Result too large to show

Warning: Unable to verify antiderivative.

[In] `Integrate[(f + g*x)^(5/2)/((d + e*x)*Sqrt[a + b*x + c*x^2]),x]`

[Out] Result too large to show

**Maple [B]** time = 0.068, size = 7464, normalized size = 6.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((g*x+f)^(5/2)/(e*x+d)/(c*x^2+b*x+a)^(1/2),x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx + f)^{\frac{5}{2}}}{\sqrt{cx^2 + bx + a}(ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x + f)^(5/2)/(sqrt(c*x^2 + b*x + a)*(e*x + d)),x, algorithm="maxima")`

[Out] `integrate((g*x + f)^(5/2)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x + f)^(5/2)/(sqrt(c*x^2 + b*x + a)*(e*x + d)),x, algorithm="fricas")`

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x+f)**(5/2)/(e*x+d)/(c*x**2+b*x+a)**(1/2),x)`

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx + f)^{\frac{5}{2}}}{\sqrt{cx^2 + bx + a}(ex + d)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x + f)^(5/2)/(sqrt(c*x^2 + b*x + a)*(e*x + d)),x, algorithm="giac")`

[Out] `integrate((g*x + f)^(5/2)/(sqrt(c*x^2 + b*x + a)*(e*x + d)), x)`

$$3.909 \quad \int \frac{(d+ex)^3}{\sqrt{f+gx}\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=631

$$\frac{\sqrt{2e\sqrt{b^2-4ac}}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ceg(-9aeg-30bdg+7bef)+8b^2e^2g^2+c^2(45d^2g^2-30defg+8e^2f^2))E\left(\sin^{-1}\left(\frac{\sqrt{2e\sqrt{b^2-4ac}}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}}{\sqrt{2cf-g(\sqrt{b^2-4ac}+b)}}\right)}{15c^3g^3\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}}\right)}{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}}(-ce^2g(ag(7ef-15dg)-3bf(ef-5dg))+4be^3g^2(bf-ag)+c^2(-15d^2g^2+45d^2g^2-30defg+8e^2f^2))} + \frac{8e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}(beg-3cdg+cef)}{15c^2g^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5cg} + \frac{15c^3g^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g^2}$$

[Out]  $(-8e^2(c^2ef - 3cdg + b^2eg)\sqrt{f+gx}\sqrt{a+bx+cx^2})/(15c^2g^2) + (2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2})/(5c^2g) + (\text{Sqrt}[2]\sqrt{b^2-4ac})e^2(8b^2e^2g^2 + c^2e^2g(7b^2ef - 30bdg - 9a^2eg) + c^2(8e^2f^2 - 30d^2efg + 45d^2g^2))\sqrt{f+gx}\sqrt{-((c(a+bx+cx^2))/(b^2-4ac))}\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[(b+\text{Sqrt}[b^2-4ac]+2cx)/\text{Sqrt}[b^2-4ac]]/\text{Sqrt}[2]], (-2\text{Sqrt}[b^2-4ac]g)/(2cf-(b+\text{Sqrt}[b^2-4ac]g))]/(15c^3g^3\text{Sqrt}[(c(f+gx))/(2cf-(b+\text{Sqrt}[b^2-4ac]g)]\sqrt{a+bx+cx^2}) - (2\text{Sqrt}[2]\sqrt{b^2-4ac})(4b^2e^3g^2(bf-ag) + c^2(8e^3f^3 - 30d^2e^2f^2g + 45d^2e^2fg^2 - 15d^3g^3) - c^2e^2g(a^2g(7ef - 15dg) - 3bf(ef - 5dg)))\sqrt{(c(f+gx))/(2cf-(b+\text{Sqrt}[b^2-4ac]g))}\sqrt{-((c(a+bx+cx^2))/(b^2-4ac))}\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(b+\text{Sqrt}[b^2-4ac]+2cx)/\text{Sqrt}[b^2-4ac]]/\text{Sqrt}[2]], (-2\text{Sqrt}[b^2-4ac]g)/(2cf-(b+\text{Sqrt}[b^2-4ac]g))]/(15c^3g^3\text{Sqrt}[f+gx]\sqrt{a+bx+cx^2})]$

**Rubi [A]** time = 2.3498, antiderivative size = 631, normalized size of antiderivative = 1., number of rules used = 7, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$

$$\frac{\sqrt{2e\sqrt{b^2-4ac}}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ceg(-9aeg-30bdg+7bef)+8b^2e^2g^2+c^2(45d^2g^2-30defg+8e^2f^2))E\left(\sin^{-1}\left(\frac{\sqrt{2e\sqrt{b^2-4ac}}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}}{\sqrt{2cf-g(\sqrt{b^2-4ac}+b)}}\right)}{15c^3g^3\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}}\right)}{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}}(-ce^2g(ag(7ef-15dg)-3bf(ef-5dg))+4be^3g^2(bf-ag)+c^2(-15d^2g^2+45d^2g^2-30defg+8e^2f^2))} + \frac{8e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}(beg-3cdg+cef)}{15c^2g^2} + \frac{2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}}{5cg} + \frac{15c^3g^3\sqrt{f+gx}\sqrt{a+bx+cx^2}}{15c^2g^2}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^3/(Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]), x]

[Out]  $(-8e^2(c^2ef - 3cdg + b^2eg)\sqrt{f+gx}\sqrt{a+bx+cx^2})/(15c^2g^2) + (2e^2(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2})/(5c^2g) + (\text{Sqrt}[2]\sqrt{b^2-4ac})e^2(8b^2e^2g^2 + c^2e^2g(7b^2ef - 30bdg - 9a^2eg) + c^2(8e^2f^2 - 30d^2efg + 45d^2g^2))\sqrt{f+gx}\sqrt{-((c(a+bx+cx^2))/(b^2-4ac))}\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[(b+\text{Sqrt}[b^2-4ac]+2cx)/\text{Sqrt}[b^2-4ac]]/\text{Sqrt}[2]], (-2\text{Sqrt}[b^2-4ac]g)/(2cf-(b+\text{Sqrt}[b^2-4ac]g))]/(15c^3g^3\text{Sqrt}[f+gx]\sqrt{a+bx+cx^2})]$

$$\frac{1}{\sqrt{b^2 - 4ac}} \frac{1}{\sqrt{2}}, \frac{(-2\sqrt{b^2 - 4ac}g)/(2cf - (b + \sqrt{b^2 - 4ac})g))}{(15c^3g^3\sqrt{(c(f+gx))/(2cf - (b + \sqrt{b^2 - 4ac})g)})\sqrt{a+bx+cx^2}} - (2\sqrt{2}\sqrt{b^2 - 4ac}(4be^3g^2(bf - ag) + c^2(8e^3f^3 - 30d^2e^2f^2g + 45d^2efg^2 - 15d^3g^3) - ce^2g(ag(7ef - 15d^2g) - 3b^2f(ef - 5d^2g))))\sqrt{(c(f+gx))/(2cf - (b + \sqrt{b^2 - 4ac})g)}}\sqrt{-((c(a+bx+cx^2))/(b^2 - 4ac))}\text{EllipticF}[\text{ArcSin}[\sqrt{(b + \sqrt{b^2 - 4ac} + 2cx)/\sqrt{b^2 - 4ac}}]/\sqrt{2}], \frac{(-2\sqrt{b^2 - 4ac}g)/(2cf - (b + \sqrt{b^2 - 4ac})g))}{(15c^3g^3\sqrt{f+gx})\sqrt{a+bx+cx^2}}$$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**3/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2), x)`

[Out] Timed out

**Mathematica [C]** time = 14.6642, size = 12746, normalized size = 20.2

Result too large to show

Antiderivative was successfully verified.

[In] `Integrate[(d + e*x)^3/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]), x]`

[Out] Result too large to show

**Maple [B]** time = 0.079, size = 8755, normalized size = 13.9

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^3/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2), x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^3}{\sqrt{cx^2 + bx + a}\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^3/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x, algorithm="maxima")`

[Out] `integrate((e*x + d)^3/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^3x^3 + 3de^2x^2 + 3d^2ex + d^3}{\sqrt{cx^2 + bx + a}\sqrt{gx + f}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3/(sqrt(c\*x^2 + b\*x + a)\*sqrt(g\*x + f)),x, algorithm="fricas")

[Out] integral((e^3\*x^3 + 3\*d\*e^2\*x^2 + 3\*d^2\*e\*x + d^3)/(sqrt(c\*x^2 + b\*x + a)\*sqrt(g\*x + f)), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^3}{\sqrt{f + gx}\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*3/(g\*x+f)\*\*(1/2)/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral((d + e\*x)\*\*3/(sqrt(f + g\*x)\*sqrt(a + b\*x + c\*x\*\*2)), x)

---

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^3/(sqrt(c\*x^2 + b\*x + a)\*sqrt(g\*x + f)),x, algorithm="giac")

[Out] Timed out



$$3.910 \quad \int \frac{(d+ex)^2}{\sqrt{f+gx}\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=479

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}(e^2g(bf-ag)+c(3d^2g^2-6defg+2e^2f^2))F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right)}{3c^2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{2\sqrt{2}e\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(beg-3cdg+cef)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right) - \frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}}{3c^2g^2\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}} + \frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg}$$

[Out] (2\*e^2\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])/(3\*c\*g) - (2\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*e\*(c\*e\*f - 3\*c\*d\*g + b\*e\*g)\*Sqrt[f + g\*x]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g))/(3\*c^2\*g^2\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[a + b\*x + c\*x^2]) + (2\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*(e^2\*g\*(b\*f - a\*g) + c\*(2\*e^2\*f^2 - 6\*d\*e\*f\*g + 3\*d^2\*g^2))\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g))/(3\*c^2\*g^2\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])

**Rubi [A]** time = 1.54268, antiderivative size = 479, normalized size of antiderivative = 1., number of rules used = 7, number of rules used = 6, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}(e^2g(bf-ag)+c(3d^2g^2-6defg+2e^2f^2))F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right)}{3c^2g^2\sqrt{f+gx}\sqrt{a+bx+cx^2}} - \frac{2\sqrt{2}e\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(beg-3cdg+cef)E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\right) - \frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}}{3c^2g^2\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}} + \frac{2e^2\sqrt{f+gx}\sqrt{a+bx+cx^2}}{3cg}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^2/(Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]), x]

[Out] (2\*e^2\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])/(3\*c\*g) - (2\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*e\*(c\*e\*f - 3\*c\*d\*g + b\*e\*g)\*Sqrt[f + g\*x]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g))/(3\*c^2\*g^2\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[a + b\*x + c\*x^2]) + (2\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*(e^2\*g\*(b\*f - a\*g) + c\*(2\*e^2\*f^2 - 6\*d\*e\*f\*g + 3\*d^2\*g^2))\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g))/(3\*c^2\*g^2\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])

$$\frac{1}{\sqrt{b^2 - 4ac}} \sqrt{2}, \frac{-2\sqrt{b^2 - 4ac}g}{(2cf - (b + \sqrt{b^2 - 4ac})g)} \Big/ (3c^2g^2\sqrt{f + gx}\sqrt{ax + cx^2})$$

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**2/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)`

[Out] Timed out

**Mathematica [C]** time = 13.5581, size = 6194, normalized size = 12.93

Result too large to show

Antiderivative was successfully verified.

[In] `Integrate[(d + e*x)^2/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]`

[Out] Result too large to show

**Maple [B]** time = 0.064, size = 4295, normalized size = 9.

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^2/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)`

[Out] 
$$\begin{aligned} & -\frac{1}{3}(-2x^3c^2e^2g^3-2x^2b^2c^2e^2fg^2+3^2)^{1/2}(-g^2x+f)^c / \\ & (g^2(-4ac+b^2)^{1/2}+b^2g-2cf)^{1/2}(g^2(-2cx+(-4ac+b^2)^{1/2}-b)/(2cf-b^2g+g^2(-4ac+b^2)^{1/2}))^{1/2}(g^2(b+2cx+(-4ac+b^2)^{1/2}))^{1/2} \\ & \text{EllipticF}(2^{1/2}(-g^2x+f)^c/(g^2(-4ac+b^2)^{1/2}+b^2g-2cf)^{1/2},(-g^2(-4ac+b^2)^{1/2}+b^2g-2cf)/(2cf-b^2g+g^2(-4ac+b^2)^{1/2}))^{1/2} \\ & *ab^2e^2g^3+6^2)^{1/2}(-g^2x+f)^c/(g^2(-4ac+b^2)^{1/2}+b^2g-2cf)^{1/2}(g^2(-2cx+(-4ac+b^2)^{1/2}-b)/(2cf-b^2g+g^2(-4ac+b^2)^{1/2}))^{1/2} \\ & (g^2(b+2cx+(-4ac+b^2)^{1/2}))^{1/2}(g^2(b+2cx+(-4ac+b^2)^{1/2}))^{1/2} \text{EllipticF}(2^{1/2}(-g^2x+f)^c/(g^2(-4ac+b^2)^{1/2}+b^2g-2cf)^{1/2}, \\ & (-g^2(-4ac+b^2)^{1/2}+b^2g-2cf)/(2cf-b^2g+g^2(-4ac+b^2)^{1/2}))^{1/2}) * b^2c^2d^2e^2fg^2-4^2)^{1/2} \\ & (-g^2x+f)^c/(g^2(-4ac+b^2)^{1/2}+b^2g-2cf)^{1/2}(g^2(-2cx+(-4ac+b^2)^{1/2}-b)/(2cf-b^2g+g^2(-4ac+b^2)^{1/2}))^{1/2} \\ & (g^2(b+2cx+(-4ac+b^2)^{1/2}))^{1/2} \text{EllipticE}(2^{1/2}(-g^2x+f)^c/(g^2(-4ac+b^2)^{1/2}+b^2g-2cf)^{1/2}, \\ & (-g^2(-4ac+b^2)^{1/2}+b^2g-2cf)/(2cf-b^2g+g^2(-4ac+b^2)^{1/2}))^{1/2}) * c^2e^2f^3-2x^2b^2c^2e^2g^3-2x^2c^2e^2 \\ & f^2g^2-2)^{1/2}(-g^2x+f)^c/(g^2(-4ac+b^2)^{1/2}+b^2g-2cf)^{1/2}(g^2(-2cx+(-4ac+b^2)^{1/2}-b)/(2cf-b^2g+g^2(-4ac+b^2)^{1/2}))^{1/2} \\ & (g^2(b+2cx+(-4ac+b^2)^{1/2}))^{1/2} \text{EllipticF}(2^{1/2}(-g^2x+f)^c/(g^2(-4ac+b^2)^{1/2}+b^2g-2cf)^{1/2}, \\ & (-g^2(-4ac+b^2)^{1/2}+b^2g-2cf)/(2cf-b^2g+g^2(-4ac+b^2)^{1/2}))^{1/2}) * (-4ac+b^2)^{1/2} * a^2e^2g^3+3^2)^{1/2} \\ & (-g^2x+f)^c/(g^2(-4ac+b^2)^{1/2}+b^2g-2cf)^{1/2}(g^2(-2cx+(-4ac+b^2)^{1/2}-b)/(2cf-b^2g+g^2(-4ac+b^2)^{1/2}))^{1/2} \end{aligned}$$



$$-2^*x^*a^*c^*e^2*g^3-2^*a^*c^*e^2*f^*g^2-12^*2^{(1/2)}*(-(g^*x+f)^*c/(g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}*(g^*(-2^*c^*x+(-4^*a^*c+b^2)^{(1/2)}-b)/(2^*c^*f-b^*g+g^*(-4^*a^*c+b^2)^{(1/2)}))^{(1/2)}*(g^*(b+2^*c^*x+(-4^*a^*c+b^2)^{(1/2)}))/(g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}*EllipticE(2^{(1/2)}*(-(g^*x+f)^*c/(g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)},(-g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f)/(2^*c^*f-b^*g+g^*(-4^*a^*c+b^2)^{(1/2)}))^{(1/2)})*b^*c^*d^*e^*f^*g^2-6^*2^{(1/2)}*(-(g^*x+f)^*c/(g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}*(g^*(-2^*c^*x+(-4^*a^*c+b^2)^{(1/2)}-b)/(2^*c^*f-b^*g+g^*(-4^*a^*c+b^2)^{(1/2)}))^{(1/2)}*(g^*(b+2^*c^*x+(-4^*a^*c+b^2)^{(1/2)}))/(g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)}*EllipticF(2^{(1/2)}*(-(g^*x+f)^*c/(g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f))^{(1/2)},(-g^*(-4^*a^*c+b^2)^{(1/2)}+b^*g-2^*c^*f)/(2^*c^*f-b^*g+g^*(-4^*a^*c+b^2)^{(1/2)}))^{(1/2)}*(-4^*a^*c+b^2)^{(1/2)}*c^*d^*e^*f^*g^2*(g^*x+f)^{(1/2)}*(c^*x^2+b^*x+a)^{(1/2)}/c^2/g^3/(c^*g^*x^3+b^*g^*x^2+c^*f^*x^2+a^*g^*x+b^*f^*x+a^*f)$$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^2}{\sqrt{cx^2 + bx + a}\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2/(sqrt(c\*x^2 + b\*x + a)\*sqrt(g\*x + f)),x, algorithm="maxima"

[Out] integrate((e\*x + d)^2/(sqrt(c\*x^2 + b\*x + a)\*sqrt(g\*x + f)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{e^2x^2 + 2dex + d^2}{\sqrt{cx^2 + bx + a}\sqrt{gx + f}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^2/(sqrt(c\*x^2 + b\*x + a)\*sqrt(g\*x + f)),x, algorithm="fricas"

[Out] integral((e^2\*x^2 + 2\*d\*e\*x + d^2)/(sqrt(c\*x^2 + b\*x + a)\*sqrt(g\*x + f)), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^2}{\sqrt{f + gx}\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*2/(g\*x+f)\*\*(1/2)/(c\*x\*\*2+b\*x+a)\*\*(1/2), x)

[Out] Integral((d + e\*x)\*\*2/(sqrt(f + g\*x)\*sqrt(a + b\*x + c\*x\*\*2)), x)

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)^2/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.911 \quad \int \frac{d+ex}{\sqrt{f+gx}\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=393

$$\frac{\sqrt{2e}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}}$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{cg\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

[Out] (Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*e\*Sqrt[f + g\*x]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(c\*g\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[a + b\*x + c\*x^2]) - (2\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*(e\*f - d\*g)\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(c\*g\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]))

**Rubi [A]** time = 0.790712, antiderivative size = 393, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$

$$\frac{\sqrt{2e}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{cg\sqrt{a+bx+cx^2}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}}$$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{cg\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)/(Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]), x]

[Out] (Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*e\*Sqrt[f + g\*x]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(c\*g\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[a + b\*x + c\*x^2]) - (2\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*(e\*f - d\*g)\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(c\*g\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]))

**Rubi in Sympy [A]** time = 116.989, size = 369, normalized size = 0.94

$$\frac{\sqrt{2}e\sqrt{\frac{c(a+bx+cx^2)}{4ac-b^2}}\sqrt{f+gx}\sqrt{-4ac+b^2}E\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2}\right)\middle|\frac{2g\sqrt{-4ac+b^2}}{bg-2cf+g\sqrt{-4ac+b^2}}\right)}{cg\sqrt{\frac{c(-f-gx)}{bg-2cf+g\sqrt{-4ac+b^2}}}\sqrt{a+bx+cx^2}} + \frac{2\sqrt{2}\sqrt{\frac{c(-f-gx)}{bg-2cf+g\sqrt{-4ac+b^2}}}\sqrt{\frac{c(a+bx+cx^2)}{4ac-b^2}}\sqrt{-4ac+b^2}(dg-ef)F\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2}\right)\middle|\frac{2g\sqrt{-4ac+b^2}}{bg-2cf+g\sqrt{-4ac+b^2}}\right)}{cg\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)`

[Out] `sqrt(2)*e*sqrt(c*(a + b*x + c*x**2)/(4*a*c - b**2))*sqrt(f + g*x)*sqrt(-4*a*c + b**2)*elliptic_e(asin(sqrt(2)*sqrt((b + 2*c*x + sqrt(-4*a*c + b**2))/sqrt(-4*a*c + b**2))/2), 2*g*sqrt(-4*a*c + b**2)/(b*g - 2*c*f + g*sqrt(-4*a*c + b**2)))/(c*g*sqrt(c*(-f - g*x)/(b*g - 2*c*f + g*sqrt(-4*a*c + b**2)))*sqrt(a + b*x + c*x**2)) + 2*sqrt(2)*sqrt(c*(-f - g*x)/(b*g - 2*c*f + g*sqrt(-4*a*c + b**2)))*sqrt(c*(a + b*x + c*x**2)/(4*a*c - b**2))*sqrt(-4*a*c + b**2)*(d*g - e*f)*elliptic_f(asin(sqrt(2)*sqrt((b + 2*c*x + sqrt(-4*a*c + b**2))/sqrt(-4*a*c + b**2))/2), 2*g*sqrt(-4*a*c + b**2)/(b*g - 2*c*f + g*sqrt(-4*a*c + b**2)))/(c*g*sqrt(f + g*x)*sqrt(a + b*x + c*x**2))`

**Mathematica [C]** time = 10.247, size = 814, normalized size = 2.07

$$(f+gx)^{3/2} \left( -\frac{4e\sqrt{\frac{cf^2+g(ag-bf)}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}(a+x(b+cx))g^2}{(f+gx)^2} + \frac{i\sqrt{2}e(2cf-bg+\sqrt{(b^2-4ac)g^2})\sqrt{\frac{-2ag^2+2cfxg+b(f-gx)g+\sqrt{(b^2-4ac)g^2}(f+gx)}}{(2cf-bg+\sqrt{(b^2-4ac)g^2})(f+gx)}}\sqrt{\frac{2ag^2-2cfxg+b(f-gx)g+\sqrt{(b^2-4ac)g^2}(f+gx)}}{(-2cf+bg+\sqrt{(b^2-4ac)g^2})(f+gx)}}}{\sqrt{f+gx}} \right)$$

Antiderivative was successfully verified.

[In] `Integrate[(d + e*x)/(Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]`

[Out] `-((f + g*x)^(3/2)*((-4*e*g^2*Sqrt[(c*f^2 + g*(-b*f) + a*g)]/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2]))*(a + x*(b + c*x)))/(f + g*x)^2 + (I*Sqrt[2]*e*(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*Sqrt[(-2*a*g^2 + 2*c*f*g*x + b*g*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 - 2*c*f*g*x + b*g*(-f + g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticE[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]], -((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2]))]/Sqrt[f + g*x] - (I*Sqrt[2]*(2*c*d*g + e*(-b*g) + Sqrt[(b^2 - 4*a*c)*g^2])*Sqrt[(-2*a*g^2 + 2*c*f*g*x + b*g*(f - g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((2*c*f - b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*Sqrt[(2*a*g^2 - 2*c*f*g*x + b*g*(-f + g*x) + Sqrt[(b^2 - 4*a*c)*g^2]*(f + g*x))/((-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])*(f + g*x))]*EllipticF[I*ArcSinh[(Sqrt[2]*Sqrt[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + Sqrt[(b^2 - 4*a*c)*g^2])])/Sqrt[f + g*x]]`

$$\text{rt}[f + g^*x]], -((-2^*c^*f + b^*g + \text{Sqrt}[(b^2 - 4^*a^*c)^*g^2])/(2^*c^*f - b^*g + \text{Sqrt}[(b^2 - 4^*a^*c)^*g^2]))/\text{Sqrt}[f + g^*x]]/(2^*c^*g^2^*\text{Sqrt}[(c^*f^2 + g^*(-b^*f) + a^*g))/(-2^*c^*f + b^*g + \text{Sqrt}[(b^2 - 4^*a^*c)^*g^2])]*\text{Sqrt}[a + x^*(b + c^*x)])$$

**Maple [B]** time = 0.054, size = 1014, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)/(g\*x+f)^(1/2)/(c\*x^2+b\*x+a)^(1/2), x)

[Out] (2\*EllipticF(2^(1/2)\*(-(g\*x+f)\*c/(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f))^^(1/2), (- (g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f)/(2\*c\*f-b\*g+g\*(-4\*a\*c+b^2)^(1/2)))^(1/2))^^(1/2)\*a\*e\*g^2-EllipticF(2^(1/2)\*(-(g\*x+f)\*c/(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f))^^(1/2), (- (g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f)/(2\*c\*f-b\*g+g\*(-4\*a\*c+b^2)^(1/2)))^(1/2))^^(1/2)\*b\*d\*g^2-EllipticF(2^(1/2)\*(-(g\*x+f)\*c/(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f))^^(1/2), (- (g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f)/(2\*c\*f-b\*g+g\*(-4\*a\*c+b^2)^(1/2)))^(1/2))^^(1/2)\*b\*e\*f\*g+2\*EllipticF(2^(1/2)\*(-(g\*x+f)\*c/(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f))^^(1/2), (- (g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f)/(2\*c\*f-b\*g+g\*(-4\*a\*c+b^2)^(1/2)))^(1/2))^^(1/2)\*c\*d\*f\*g-EllipticF(2^(1/2)\*(-(g\*x+f)\*c/(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f))^^(1/2), (- (g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f)/(2\*c\*f-b\*g+g\*(-4\*a\*c+b^2)^(1/2)))^(1/2))^^(1/2)\*(-4\*a\*c+b^2)^(1/2)\*d\*g^2+EllipticF(2^(1/2)\*(-(g\*x+f)\*c/(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f))^^(1/2), (- (g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f)/(2\*c\*f-b\*g+g\*(-4\*a\*c+b^2)^(1/2)))^(1/2))^^(1/2)\*(-4\*a\*c+b^2)^(1/2)\*e\*f\*g-2\*EllipticE(2^(1/2)\*(-(g\*x+f)\*c/(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f))^^(1/2), (- (g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f)/(2\*c\*f-b\*g+g\*(-4\*a\*c+b^2)^(1/2)))^(1/2))^^(1/2)\*a\*e\*g^2+2\*EllipticE(2^(1/2)\*(-(g\*x+f)\*c/(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f))^^(1/2), (- (g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f)/(2\*c\*f-b\*g+g\*(-4\*a\*c+b^2)^(1/2)))^(1/2))^^(1/2)\*b\*e\*f\*g-2\*EllipticE(2^(1/2)\*(-(g\*x+f)\*c/(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f))^^(1/2), (- (g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f)/(2\*c\*f-b\*g+g\*(-4\*a\*c+b^2)^(1/2)))^(1/2))^^(1/2)\*c\*e\*f^2)^^(1/2)\* (g\*(b+2\*c\*x+(-4\*a\*c+b^2)^(1/2))/(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f))^^(1/2)\* (g\*(-2\*c\*x+(-4\*a\*c+b^2)^(1/2)-b)/(2\*c\*f-b\*g+g\*(-4\*a\*c+b^2)^(1/2)))^(1/2)\*2^(1/2)\*(-(g\*x+f)\*c/(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f))^^(1/2)\*(g\*x+f)^(1/2)\*(c\*x^2+b\*x+a)^(1/2)/c/g^2/(c\*g\*x^3+b\*g\*x^2+c\*f\*x^2+a\*g\*x+b\*f\*x+a\*f)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{ex + d}{\sqrt{cx^2 + bx + a}\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)/(sqrt(c\*x^2 + b\*x + a)\*sqrt(g\*x + f)), x, algorithm="maxima")

[Out] integrate((e\*x + d)/(sqrt(c\*x^2 + b\*x + a)\*sqrt(g\*x + f)), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{ex + d}{\sqrt{cx^2 + bx + a}\sqrt{gx + f}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.



```
[In] integrate((e*x + d)/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)),x, algorithm="fricas")
```

```
[Out] integral((e*x + d)/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)
```

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{d + ex}{\sqrt{f + gx}\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x+d)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)
```

```
[Out] Integral((d + e*x)/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)
```

**GIAC/XCAS [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x + d)/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)),x, algorithm="giac")
```

```
[Out] Timed out
```

$$3.912 \quad \int \frac{1}{\sqrt{f+gx}\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=189

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{c\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

[Out] (2\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))])\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(c\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])

**Rubi [A]** time = 0.27869, antiderivative size = 189, normalized size of antiderivative = 1., number of rules used = 2, number of rules used = 2, integrand size = 24,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$

$$\frac{2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{c\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]), x]

[Out] (2\*Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))])\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(c\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])

**Rubi in Sympy [A]** time = 50.6126, size = 178, normalized size = 0.94

$$\frac{2\sqrt{2}\sqrt{\frac{c(-f-gx)}{bg-2cf+g\sqrt{-4ac+b^2}}}\sqrt{\frac{c(a+bx+cx^2)}{4ac-b^2}}\sqrt{-4ac+b^2}F\left(\operatorname{asin}\left(\frac{\sqrt{2}\sqrt{\frac{b+2cx+\sqrt{-4ac+b^2}}{\sqrt{-4ac+b^2}}}}{2}\right)\middle|\frac{2g\sqrt{-4ac+b^2}}{bg-2cf+g\sqrt{-4ac+b^2}}\right)}{c\sqrt{f+gx}\sqrt{a+bx+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(g\*x+f)\*\*(1/2)/(c\*x\*\*2+b\*x+a)\*\*(1/2), x)

[Out] 2\*sqrt(2)\*sqrt(c\*(-f - g\*x)/(b\*g - 2\*c\*f + g\*sqrt(-4\*a\*c + b\*\*2)))\*sqrt(c\*(a + b\*x + c\*x\*\*2)/(4\*a\*c - b\*\*2))\*sqrt(-4\*a\*c + b\*\*2)\*elliptic\_f(asin(sqrt(2)\*sqrt((b + 2\*c\*x + sqrt(-4\*a\*c + b\*\*2))/sqrt(-4\*a\*c + b\*\*2))/2), 2\*g\*sqrt(-4\*a\*c + b\*\*2)/(b\*g - 2\*c\*f + g\*sqrt(-4\*a\*c + b\*\*2)))/(c\*sqrt(f + g\*x)\*sqrt(a + b\*x + c\*x\*\*2))

**Mathematica [C]** time = 1.39202, size = 308, normalized size = 1.63

$$i(f+gx)\sqrt{2-\frac{4(g(ag-bf)+cf^2)}{(f+gx)(\sqrt{g^2(b^2-4ac)}-bg+2cf)}}\sqrt{\frac{2(g(ag-bf)+cf^2)}{(f+gx)(\sqrt{g^2(b^2-4ac)}+bg-2cf)}}+1F\left(i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{cf^2-bgf+ag^2}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}}{\sqrt{f+gx}}\right)\right)\Big|_{-\frac{-2cf+bg+\sqrt{(b^2-4ac)g^2}}{2cf-bg+\sqrt{(b^2-4ac)g^2}}}$$


---


$$g\sqrt{a+x(b+cx)}\sqrt{\frac{g(ag-bf)+cf^2}{\sqrt{g^2(b^2-4ac)}+bg-2cf}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]),x]

[Out] (I\*(f + g\*x)\*Sqrt[2 - (4\*(c\*f^2 + g\*(-(b\*f) + a\*g)))/((2\*c\*f - b\*g + Sqrt[(b^2 - 4\*a\*c)\*g^2])\*(f + g\*x))]\*Sqrt[1 + (2\*(c\*f^2 + g\*(-(b\*f) + a\*g)))/((-2\*c\*f + b\*g + Sqrt[(b^2 - 4\*a\*c)\*g^2])\*(f + g\*x))]\*EllipticF[I\*ArcSinh[(Sqrt[2]\*Sqrt[(c\*f^2 - b\*f\*g + a\*g^2)/(-2\*c\*f + b\*g + Sqrt[(b^2 - 4\*a\*c)\*g^2])])/Sqrt[f + g\*x]], -( (-2\*c\*f + b\*g + Sqrt[(b^2 - 4\*a\*c)\*g^2])/(2\*c\*f - b\*g + Sqrt[(b^2 - 4\*a\*c)\*g^2]) )]/(g\*Sqrt[(c\*f^2 + g\*(-(b\*f) + a\*g)))/(-2\*c\*f + b\*g + Sqrt[(b^2 - 4\*a\*c)\*g^2])]\*Sqrt[a + x\*(b + c\*x)])

**Maple [A]** time = 0.048, size = 287, normalized size = 1.5

$$\frac{\sqrt{2}}{cg(cx^3 + bgx^2 + cfx^2 + agx + bfx + fa)}(-g\sqrt{-4ac + b^2} - bg + 2cf) \operatorname{EllipticF}\left(\sqrt{2}\sqrt{-(gx + f)c(g\sqrt{-4ac + b^2} + bg - 2cf)}, \frac{g\sqrt{-4ac + b^2} + bg - 2cf}{g\sqrt{-4ac + b^2} + bg - 2cf}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(g\*x+f)^(1/2)/(c\*x^2+b\*x+a)^(1/2),x)

[Out] (-g\*(-4\*a\*c+b^2)^(1/2)-b\*g+2\*c\*f)/c\*EllipticF(2^(1/2)\*(-(g\*x+f)\*c/(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f))^(1/2),(-(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f)/(2\*c\*f-b\*g+g\*(-4\*a\*c+b^2)^(1/2)))^(1/2))\* (g\*(b+2\*c\*x+(-4\*a\*c+b^2)^(1/2))/(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f))^(1/2)\*(g\*(-2\*c\*x+(-4\*a\*c+b^2)^(1/2)-b)/(2\*c\*f-b\*g+g\*(-4\*a\*c+b^2)^(1/2)))^(1/2)\*2^(1/2)\*(-(g\*x+f)\*c/(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f))^(1/2)/g\*(g\*x+f)^(1/2)\*(c\*x^2+b\*x+a)^(1/2)/(c\*g\*x^3+b\*g\*x^2+c\*f\*x^2+a\*g\*x+b\*f\*x+a\*f)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*sqrt(g\*x + f)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*sqrt(g\*x + f)), x)

**Ericas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{1}{\sqrt{cx^2 + bx + a}\sqrt{gx + f}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)),x, algorithm="fricas")`

[Out] `integral(1/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{f + gx}\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)`

[Out] `Integral(1/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)),x, algorithm="giac")`

[Out] `integrate(1/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)`

$$3.913 \quad \int \frac{1}{(d+ex)\sqrt{f+gx}\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=280

$$\frac{\sqrt{2}\sqrt{2cf-g}\left(b-\sqrt{b^2-4ac}\right)\sqrt{1-\frac{2c(f+gx)}{2cf-g(b-\sqrt{b^2-4ac})}}\sqrt{1-\frac{2c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}\left(\frac{e(2cf-bg+\sqrt{b^2-4ac}g)}{2c(ef-dg)}\right); \sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{f+gx}}{\sqrt{2cf-(b-\sqrt{b^2-4ac})}}\right)}{\sqrt{c}\sqrt{a+bx+cx^2}(ef-dg)}$$

[Out] -((Sqrt[2]\*Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g])\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*EllipticPi[(e\*(2\*c\*f - b\*g + Sqrt[b^2 - 4\*a\*c])\*g)/(2\*c\*(e\*f - d\*g)), ArcSin[(Sqrt[2]\*Sqrt[c]\*Sqrt[f + g\*x])/Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g]]], (b - Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)/(b + Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)]/(Sqrt[c]\*(e\*f - d\*g)\*Sqrt[a + b\*x + c\*x^2]))

**Rubi [A]** time = 4.03091, antiderivative size = 280, normalized size of antiderivative = 1., number of rules used = 5, number of rules used = 4, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$

$$\frac{\sqrt{2}\sqrt{2cf-g}\left(b-\sqrt{b^2-4ac}\right)\sqrt{1-\frac{2c(f+gx)}{2cf-g(b-\sqrt{b^2-4ac})}}\sqrt{1-\frac{2c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}\left(\frac{e(2cf-bg+\sqrt{b^2-4ac}g)}{2c(ef-dg)}\right); \sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{f+gx}}{\sqrt{2cf-(b-\sqrt{b^2-4ac})}}\right)}{\sqrt{c}\sqrt{a+bx+cx^2}(ef-dg)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]), x]

[Out] -((Sqrt[2]\*Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g])\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*EllipticPi[(e\*(2\*c\*f - b\*g + Sqrt[b^2 - 4\*a\*c])\*g)/(2\*c\*(e\*f - d\*g)), ArcSin[(Sqrt[2]\*Sqrt[c]\*Sqrt[f + g\*x])/Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g]]], (b - Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)/(b + Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)]/(Sqrt[c]\*(e\*f - d\*g)\*Sqrt[a + b\*x + c\*x^2]))

**Rubi in Sympy [A]** time = 32.8874, size = 262, normalized size = 0.94

$$\frac{\sqrt{2}\sqrt{\frac{g(-b-2cx+\sqrt{-4ac+b^2})}{-bg+2cf+g\sqrt{-4ac+b^2}}}\sqrt{\frac{g(b+2cx+\sqrt{-4ac+b^2})}{bg-2cf+g\sqrt{-4ac+b^2}}}\left(\frac{e(bg-2cf-g\sqrt{-4ac+b^2})}{2c(dg-ef)}\right); \operatorname{asin}\left(\sqrt{2}\sqrt{\frac{c}{-bg+2cf+g\sqrt{-4ac+b^2}}}\sqrt{f+gx}\right)\left|\frac{bg-2cf-g\sqrt{-4ac+b^2}}{bg-2cf+g\sqrt{-4ac+b^2}}\right|}{\sqrt{\frac{c}{-bg+2cf+g\sqrt{-4ac+b^2}}}(dg-ef)\sqrt{a+bx+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(e\*x+d)/(g\*x+f)\*\*(1/2)/(c\*x\*\*2+b\*x+a)\*\*(1/2), x)

[Out] sqrt(2)\*sqrt(g\*(-b - 2\*c\*x + sqrt(-4\*a\*c + b\*\*2)))/(-b\*g + 2\*c\*f + g\*sqrt(-4\*a\*c + b\*\*2))\*sqrt(g\*(b + 2\*c\*x + sqrt(-4\*a\*c + b\*\*2)))/(b\*g - 2\*c\*f + g\*sqrt(-4\*a\*c + b\*\*2))\*elliptic\_pi(e\*(b\*g - 2\*c\*f - g\*sqrt(-4\*a\*c + b\*\*2))/(2\*c\*(d\*g - e\*f)), asin(sqrt(2)\*sqrt(c/(-b\*g + 2\*c\*f + g\*sqrt(-4\*a\*c + b\*\*2)))\*sqrt(f + g\*x)), (b\*g - 2\*c\*f - g\*sqrt(-4\*a\*c + b\*\*2))/(b\*g - 2\*c\*f + g\*sqrt(-4\*a\*c + b\*\*2)))/(sqrt(c/(-b\*g + 2\*c\*f + g\*sqrt(-4\*a\*c + b\*\*2)))\*(d\*g - e\*f)\*sqrt(a + b\*x + c\*x\*\*2))

**Mathematica [C]** time = 2.28219, size = 499, normalized size = 1.78

$$i(f+gx)\sqrt{2-\frac{4(g(ag-bf)+cf^2)}{(f+gx)(\sqrt{g^2(b^2-4ac)}-bg+2cf)}}\sqrt{\frac{2(g(ag-bf)+cf^2)}{(f+gx)(\sqrt{g^2(b^2-4ac)}+bg-2cf)}}+1\left(F\left(i\sinh^{-1}\left(\frac{\sqrt{2}\sqrt{\frac{cf^2-bgf+ag^2}{-2cf+bg+\sqrt{(b^2-4ac)g^2}}}}{\sqrt{f+gx}}\right)\right)\right)\Big|_{-\frac{-2cf+bg+\sqrt{(b^2-4ac)g^2}}{2cf-bg+\sqrt{(b^2-4ac)g^2}}}\sqrt{a+x(b+cx)}(dg-ef)\sqrt{\frac{g(ag-bf)+cf^2}{\sqrt{g^2(b^2-4ac)}}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/((d + e\*x)\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]),x]

[Out] (I\*(f + g\*x)\*Sqrt[2 - (4\*(c\*f^2 + g\*(-(b\*f) + a\*g)))/((2\*c\*f - b\*g + Sqrt[(b^2 - 4\*a\*c)\*g^2])\*(f + g\*x))]\*Sqrt[1 + (2\*(c\*f^2 + g\*(-(b\*f) + a\*g)))/((-2\*c\*f + b\*g + Sqrt[(b^2 - 4\*a\*c)\*g^2])\*(f + g\*x))]\*EllipticF[I\*ArcSinh[(Sqrt[2]\*Sqrt[(c\*f^2 - b\*f\*g + a\*g^2)/(-2\*c\*f + b\*g + Sqrt[(b^2 - 4\*a\*c)\*g^2])])/Sqrt[f + g\*x]], -((-2\*c\*f + b\*g + Sqrt[(b^2 - 4\*a\*c)\*g^2])/(2\*c\*f - b\*g + Sqrt[(b^2 - 4\*a\*c)\*g^2]))] - EllipticPi[((e\*f - d\*g)\*(2\*c\*f - b\*g - Sqrt[(b^2 - 4\*a\*c)\*g^2]))/(2\*e\*(c\*f^2 + g\*(-(b\*f) + a\*g))), I\*ArcSinh[(Sqrt[2]\*Sqrt[(c\*f^2 - b\*f\*g + a\*g^2)/(-2\*c\*f + b\*g + Sqrt[(b^2 - 4\*a\*c)\*g^2])])/Sqrt[f + g\*x]], -((-2\*c\*f + b\*g + Sqrt[(b^2 - 4\*a\*c)\*g^2])/(2\*c\*f - b\*g + Sqrt[(b^2 - 4\*a\*c)\*g^2])))]/((-e\*f) + d\*g)\*Sqrt[(c\*f^2 + g\*(-(b\*f) + a\*g))/(-2\*c\*f + b\*g + Sqrt[(b^2 - 4\*a\*c)\*g^2])]\*Sqrt[a + x\*(b + c\*x)])

**Maple [A]** time = 0.057, size = 330, normalized size = 1.2

$$\frac{\sqrt{2}}{(dg-ef)c(cgx^3+bgx^2+cfx^2+agx+bfxf+fa)}\left(-g\sqrt{-4ac+b^2}-bg+2cf\right)EllipticPi\left(\sqrt{2}\sqrt{-(gx+f)c}\left(g\sqrt{-4ac+b^2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x+d)/(g\*x+f)^(1/2)/(c\*x^2+b\*x+a)^(1/2),x)

[Out] (-g\*(-4\*a\*c+b^2)^(1/2)-b\*g+2\*c\*f)\*EllipticPi(2^(1/2)\*(-(g\*x+f)\*c/(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f))^(1/2),1/2\*(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f)\*e/c/(d\*g-e\*f),(-(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f)/(2\*c\*f-b\*g+g\*(-4\*a\*c+b^2)^(1/2)))^(1/2))\* (g\*(b+2\*c\*x+(-4\*a\*c+b^2)^(1/2))/(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f))^(1/2)\*(g\*(-2\*c\*x+(-4\*a\*c+b^2)^(1/2)-b)/(2\*c\*f-b\*g+g\*(-4\*a\*c+b^2)^(1/2)))^(1/2)\*2^(1/2)\*(-(g\*x+f)\*c/(g\*(-4\*a\*c+b^2)^(1/2)+b\*g-2\*c\*f))^(1/2)/c\*(c\*x^2+b\*x+a)^(1/2)\*(g\*x+f)^(1/2)/(d\*g-e\*f)/(c\*g\*x^3+b\*g\*x^2+c\*f\*x^2+a\*g\*x+b\*f\*x+a\*f)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2+bx+a}\sqrt{gx+f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)\*sqrt(g\*x + f)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)\*sqrt(g\*x + f)), x)

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**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x + f)),x, algorithm="fricas"`

[Out] Timed out

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**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex)\sqrt{f + gx}\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x+d)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)`

[Out] `Integral(1/((d + e*x)*sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)`

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**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}(ex + d)\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x + f)),x, algorithm="giac")`

[Out] `integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*sqrt(g*x + f)), x)`

$$3.914 \quad \int \frac{1}{(d+ex)^2 \sqrt{f+gx} \sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=1037

$$\frac{\sqrt{f+gx} \sqrt{cx^2+bx+ae^2}}{(cd^2-bed+ae^2)(ef-dg)(d+ex)} + \frac{\sqrt{b^2-4ac} \sqrt{f+gx} \sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}} E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right) \middle| -\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right) e}{\sqrt{2}(cd^2-bed+ae^2)(ef-dg) \sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}} \sqrt{cx^2+bx+a}} + \frac{\sqrt{2}\sqrt{b^2-4ac} f \sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}} \sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}} F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right) \middle| -\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right) e}{(cd^2-bed+ae^2)(ef-dg) \sqrt{f+gx} \sqrt{cx^2+bx+a}} + \frac{\sqrt{2}\sqrt{b^2-4ac} dg \sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}} \sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}} F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right) \middle| -\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{(cd^2-bed+ae^2)(ef-dg) \sqrt{f+gx} \sqrt{cx^2+bx+a}} + \frac{\sqrt{2cf-(b-\sqrt{b^2-4ac})} g (cd(2ef-3dg) - e(bef-2bdg+ae^2)) \sqrt{1-\frac{2c(f+gx)}{2cf-(b-\sqrt{b^2-4ac})g}} \sqrt{1-\frac{2c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}} \left(\frac{e(2cf-2dg)}{2}\right)}{\sqrt{2}\sqrt{c}(cd^2-bed+ae^2)(ef-dg)^2 \sqrt{cx^2+bx+a}}$$

[Out]  $-\left(\frac{e^2 \sqrt{f+gx} \sqrt{a+bx+cx^2}}{(cd^2-bed+ae^2)(ef-dg)(d+ex)} + \left(\frac{\sqrt{b^2-4ac} e \sqrt{f+gx} \sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}} E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right) \middle| -\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{\sqrt{2}(cd^2-bed+ae^2)(ef-dg) \sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}} \sqrt{cx^2+bx+a}} + \frac{\sqrt{2}\sqrt{b^2-4ac} f \sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}} \sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}} F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right) \middle| -\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right) e}{(cd^2-bed+ae^2)(ef-dg) \sqrt{f+gx} \sqrt{cx^2+bx+a}} + \frac{\sqrt{2}\sqrt{b^2-4ac} dg \sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}} \sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}} F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right) \middle| -\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{(cd^2-bed+ae^2)(ef-dg) \sqrt{f+gx} \sqrt{cx^2+bx+a}} + \frac{\sqrt{2cf-(b-\sqrt{b^2-4ac})} g (cd(2ef-3dg) - e(bef-2bdg+ae^2)) \sqrt{1-\frac{2c(f+gx)}{2cf-(b-\sqrt{b^2-4ac})g}} \sqrt{1-\frac{2c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}} \left(\frac{e(2cf-2dg)}{2}\right)}{\sqrt{2}\sqrt{c}(cd^2-bed+ae^2)(ef-dg)^2 \sqrt{cx^2+bx+a}}\right)$

**Rubi [A]** time = 9.41662, antiderivative size = 1037, normalized size of antiderivative = 1., number



of steps used = 15, number of rules used = 10, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$

$$\frac{\sqrt{f+gx}\sqrt{cx^2+bx+ae^2}}{(cd^2-bed+ae^2)(ef-dg)(d+ex)}$$

$$+\frac{\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)e}{\sqrt{2}(cd^2-bed+ae^2)(ef-dg)\sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\sqrt{cx^2+bx+a}}$$

$$-\frac{\sqrt{2}\sqrt{b^2-4ac}f\sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)e}{(cd^2-bed+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$

$$+\frac{\sqrt{2}\sqrt{b^2-4ac}dg\sqrt{\frac{c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\sqrt{-\frac{c(cx^2+bx+a)}{b^2-4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{(cd^2-bed+ae^2)(ef-dg)\sqrt{f+gx}\sqrt{cx^2+bx+a}}$$

$$-\frac{\sqrt{2cf-(b-\sqrt{b^2-4ac})}g(cd(2ef-3dg)-e(bef-2bdg+aeg))\sqrt{1-\frac{2c(f+gx)}{2cf-(b-\sqrt{b^2-4ac})g}}\sqrt{1-\frac{2c(f+gx)}{2cf-(b+\sqrt{b^2-4ac})g}}\left(\frac{e(2cf-2dg)}{2}\right)}{\sqrt{2}\sqrt{c}(cd^2-bed+ae^2)(ef-dg)^2\sqrt{cx^2+bx+a}}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)^2\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]),x]

[Out] -((e^2\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2])/((c\*d^2 - b\*d\*e + a\*e^2)\*(e\*f - d\*g)\*(d + e\*x))) + (Sqrt[b^2 - 4\*a\*c]\*e\*Sqrt[f + g\*x]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/(Sqrt[2]\*(c\*d^2 - b\*d\*e + a\*e^2)\*(e\*f - d\*g)\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[a + b\*x + c\*x^2]) - (Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*e\*f\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/((c\*d^2 - b\*d\*e + a\*e^2)\*(e\*f - d\*g)\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]) + (Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*d\*g\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]/((c\*d^2 - b\*d\*e + a\*e^2)\*(e\*f - d\*g)\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]) - (Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g]\*(c\*d\*(2\*e\*f - 3\*d\*g) - e\*(b\*e\*f - 2\*b\*d\*g + a\*e\*g))\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g)]\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)]\*EllipticPi[(e\*(2\*c\*f - b\*g + Sqrt[b^2 - 4\*a\*c]\*g))/(2\*c\*(e\*f - d\*g)), ArcSin[(Sqrt[2]\*Sqrt[c]\*Sqrt[f + g\*x])/Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c])\*g]], (b - Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)/(b + Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)]/(Sqrt[2]\*Sqrt[c]\*(c\*d^2 - b\*d\*e + a\*e^2)\*(e\*f - d\*g)^2\*Sqrt[a + b\*x + c\*x^2])

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d+ex)^2 \sqrt{f+gx}\sqrt{a+bx+cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(e\*x+d)\*\*2/(g\*x+f)\*\*(1/2)/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral(1/((d + e\*x)\*\*2\*sqrt(f + g\*x)\*sqrt(a + b\*x + c\*x\*\*2)), x)

**Mathematica [C]** time = 15.2584, size = 10881, normalized size = 10.49

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[1/((d + e\*x)^2\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]), x]

[Out] Result too large to show

**Maple [B]** time = 0.096, size = 14048, normalized size = 13.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x+d)^2/(g\*x+f)^(1/2)/(c\*x^2+b\*x+a)^(1/2), x)

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}(ex + d)^2 \sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)^2\*sqrt(g\*x + f)), x, algorithm="maxima")

[Out] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)^2\*sqrt(g\*x + f)), x)

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)^2\*sqrt(g\*x + f)), x, algorithm="fricas")

[Out] Timed out

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x+d)**2/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}(ex + d)^2 \sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)^2*sqrt(g*x + f)),x, algorithm="giac"`

[Out] `integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)^2*sqrt(g*x + f)), x)`

$$3.915 \quad \int \frac{1}{(d+ex)^3 \sqrt{f+gx} \sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=1114

result too large to display

```
[Out] -(e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*
e^2)*(e*f - d*g)*(d + e*x)^2) - (3*e^2*(c*d*(2*e*f - 3*d*g) - e*(
b*e*f - 2*b*d*g + a*e*g))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(4
*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^2*(d + e*x)) + (3*Sqrt[b^2
- 4*a*c]*e*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*S
qrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellipti
cE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]
/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)]/(4*Sqrt[2]*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^2*Sqrt[
(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x +
c*x^2]) + (Sqrt[b^2 - 4*a*c]*(c*d*(-6*e*f + 7*d*g) + e*(3*b*e*f
- 4*b*d*g + a*e*g))*Sqrt[(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4
*a*c])*g)]*Sqrt[(c*(a + x*(b + c*x)))/(-b^2 + 4*a*c)]*EllipticF[A
rcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqr
t[2]], (2*Sqrt[b^2 - 4*a*c]*g)/(-2*c*f + (b + Sqrt[b^2 - 4*a*c])*
g)]/(2*Sqrt[2]*(c*d^2 + e*(-(b*d) + a*e))^2*(e*f - d*g)*Sqrt[f +
g*x]*Sqrt[a + x*(b + c*x)]) + (Sqrt[2*c*f - b*g + Sqrt[b^2 - 4*a
*c]*g]*(c^2*d^2*(8*e^2*f^2 - 20*d*e*f*g + 15*d^2*g^2) + 2*c*e*(b*
d*(-4*e^2*f^2 + 11*d*e*f*g - 10*d^2*g^2) + a*e*(-2*e^2*f^2 + 2*d*
e*f*g + 3*d^2*g^2)) + e^2*(3*a^2*e^2*g^2 + 2*a*b*e*g*(e*f - 4*d*g
) + b^2*(3*e^2*f^2 - 8*d*e*f*g + 8*d^2*g^2)))*Sqrt[(g*(-b + Sqrt[
b^2 - 4*a*c] - 2*c*x))/(2*c*f + (-b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt
[(g*(b + Sqrt[b^2 - 4*a*c] + 2*c*x))/(-2*c*f + (b + Sqrt[b^2 - 4*
a*c])*g)]*EllipticPi[(2*c*e*f - b*e*g + Sqrt[b^2 - 4*a*c]*e*g)/(2
*c*e*f - 2*c*d*g), ArcSin[(Sqrt[2]*Sqrt[c]*Sqrt[f + g*x])/Sqrt[2*
c*f - b*g + Sqrt[b^2 - 4*a*c]*g]], (2*c*f + (-b + Sqrt[b^2 - 4*a*
c])*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]/(4*Sqrt[2]*Sqrt[c]*(
c*d^2 + e*(-(b*d) + a*e))^2*(-(e*f) + d*g)^3*Sqrt[a + x*(b + c*x)
])]
```

**Rubi [A]** time = 19.821, antiderivative size = 1762, normalized size of antiderivative = 1.58, number of steps used = 25, number of rules used = 10, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.323$

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Int[1/((d + e*x)^3*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]
```

```
[Out] -(e^2*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(2*(c*d^2 - b*d*e + a*
e^2)*(e*f - d*g)*(d + e*x)^2) - (3*e^2*(c*d*(2*e*f - 3*d*g) - e*(
b*e*f - 2*b*d*g + a*e*g))*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2])/(4
*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^2*(d + e*x)) + (3*Sqrt[b^2
- 4*a*c]*e*(c*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*S
qrt[f + g*x]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c))]*Ellipti
cE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2 - 4*a*c]]
/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[b^2 - 4*a*
c])*g)]/(4*Sqrt[2]*(c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)^2*Sqrt[
(c*(f + g*x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[a + b*x +
c*x^2]) - (Sqrt[b^2 - 4*a*c]*g*Sqrt[(c*(f + g*x))/(2*c*f - (b +
Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x^2))/(b^2 - 4*a*c)
)]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x)/Sqrt[b^2
- 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(2*c*f - (b + Sqrt[
b^2 - 4*a*c])*g)]/(Sqrt[2]*(c*d^2 - b*d*e + a*e^2)*(e*f - d*g)*S
qrt[f + g*x]*Sqrt[a + b*x + c*x^2]) - (3*Sqrt[b^2 - 4*a*c]*e*f*(c
*d*(2*e*f - 3*d*g) - e*(b*e*f - 2*b*d*g + a*e*g))*Sqrt[(c*(f + g*
x))/(2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]*Sqrt[-((c*(a + b*x + c*x
^2))/(b^2 - 4*a*c))]*EllipticF[ArcSin[Sqrt[(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x)/Sqrt[b^2 - 4*a*c]]/Sqrt[2]], (-2*Sqrt[b^2 - 4*a*c]*g)/(
2*c*f - (b + Sqrt[b^2 - 4*a*c])*g)]/(2*Sqrt[2]*(c*d^2 - b*d*e +
```

$$\begin{aligned}
& a^2 e^2 (e f - d g)^2 \sqrt{f + g x} \sqrt{a + b x + c x^2} + (3 \sqrt{b^2 - 4 a c} d g (c d (2 e f - 3 d g) - e (b e f - 2 b d g + a e g)) \sqrt{(c (f + g x)) / (2 c f - (b + \sqrt{b^2 - 4 a c}) g)} \\
& \sqrt{-((c (a + b x + c x^2)) / (b^2 - 4 a c))} \text{EllipticF}[\text{ArcSin}[\sqrt{(b + \sqrt{b^2 - 4 a c} + 2 c x) / \sqrt{b^2 - 4 a c}}] / \sqrt{2}], (-2 \sqrt{b^2 - 4 a c} g) / (2 c f - (b + \sqrt{b^2 - 4 a c}) g)) / (2 \sqrt{2} \sqrt{c d^2 - b d e + a e^2} (e f - d g)^2 \sqrt{f + g x} \sqrt{a + b x + c x^2}) \\
& + (\sqrt{2 c f - (b - \sqrt{b^2 - 4 a c}) g} (c e f - 3 c d g + b e g) \sqrt{1 - (2 c (f + g x)) / (2 c f - (b - \sqrt{b^2 - 4 a c}) g)} \\
& \sqrt{1 - (2 c (f + g x)) / (2 c f - (b + \sqrt{b^2 - 4 a c}) g)} \text{EllipticPi}[(e (2 c f - b g + \sqrt{b^2 - 4 a c} g)) / (2 c (e f - d g)), \text{ArcSin}[(\sqrt{2} \sqrt{c} \sqrt{f + g x}) / \sqrt{2 c f - (b - \sqrt{b^2 - 4 a c}) g}], (b - \sqrt{b^2 - 4 a c} - (2 c f) / g) / (b + \sqrt{b^2 - 4 a c} - (2 c f) / g)] / (\sqrt{2} \sqrt{c} (c d^2 - b d e + a e^2) (e f - d g)^2 \sqrt{a + b x + c x^2}) - (3 \sqrt{2 c f - (b - \sqrt{b^2 - 4 a c}) g} (c d (2 e f - 3 d g) - e (b e f - 2 b d g + a e g))^2 \sqrt{1 - (2 c (f + g x)) / (2 c f - (b - \sqrt{b^2 - 4 a c}) g)} \\
& \sqrt{1 - (2 c (f + g x)) / (2 c f - (b + \sqrt{b^2 - 4 a c}) g)} \text{EllipticPi}[(e (2 c f - b g + \sqrt{b^2 - 4 a c} g)) / (2 c (e f - d g)), \text{ArcSin}[(\sqrt{2} \sqrt{c} \sqrt{f + g x}) / \sqrt{2 c f - (b - \sqrt{b^2 - 4 a c}) g}], (b - \sqrt{b^2 - 4 a c} - (2 c f) / g) / (b + \sqrt{b^2 - 4 a c} - (2 c f) / g)] / (4 \sqrt{2} \sqrt{c} (c d^2 - b d e + a e^2) (e f - d g)^3 \sqrt{a + b x + c x^2})
\end{aligned}$$

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex)^3 \sqrt{f + gx} \sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(e\*x+d)\*\*3/(g\*x+f)\*\*(1/2)/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral(1/((d + e\*x)\*\*3\*sqrt(f + g\*x)\*sqrt(a + b\*x + c\*x\*\*2)), x)

**Mathematica [C]** time = 21.9044, size = 40396, normalized size = 36.26

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[1/((d + e\*x)^3\*sqrt(f + g\*x)\*sqrt(a + b\*x + c\*x^2)),x]

[Out] Result too large to show

**Maple [B]** time = 0.314, size = 64947, normalized size = 58.3

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e\*x+d)^3/(g\*x+f)^(1/2)/(c\*x^2+b\*x+a)^(1/2),x)

[Out] result too large to display

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}(ex + d)^3 \sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)^3\*sqrt(g\*x + f)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)^3\*sqrt(g\*x + f)), x)

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)^3\*sqrt(g\*x + f)),x, algorithm="fricas")

[Out] Timed out

---

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x+d)\*\*3/(g\*x+f)\*\*(1/2)/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}(ex + d)^3 \sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)^3\*sqrt(g\*x + f)),x, algorithm="giac")

[Out] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)^3\*sqrt(g\*x + f)), x)

$$3.916 \quad \int \frac{1}{(d+ex)(f+gx)^{3/2}\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=553

$$\frac{\sqrt{2g}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{\sqrt{a+bx+cx^2}(ef-dg)(ag^2-bfg+cf^2)\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}}$$

$$+\frac{\sqrt{2e}\sqrt{2cf-g(b-\sqrt{b^2-4ac})}\sqrt{1-\frac{2c(f+gx)}{2cf-g(b-\sqrt{b^2-4ac})}}\sqrt{1-\frac{2c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}\left(\frac{e(2cf-bg+\sqrt{b^2-4ac}g)}{2c(ef-dg)};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{f+gx}}{\sqrt{2cf-(b-\sqrt{b^2-4ac})g}}\right)\right)}{\sqrt{c}\sqrt{a+bx+cx^2}(ef-dg)^2}$$

$$+\frac{2g^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}(ef-dg)(ag^2-bfg+cf^2)}$$

[Out] (2\*g^2\*Sqrt[a + b\*x + c\*x^2])/((e\*f - d\*g)\*(c\*f^2 - b\*f\*g + a\*g^2)\*Sqrt[f + g\*x]) - (Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*g\*Sqrt[f + g\*x]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c]\*g))]/((e\*f - d\*g)\*(c\*f^2 - b\*f\*g + a\*g^2)\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c]\*g)]\*Sqrt[a + b\*x + c\*x^2]) - (Sqrt[2]\*e\*Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c]\*g)]\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c]\*g)]\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c]\*g)]\*EllipticPi[(e\*(2\*c\*f - b\*g + Sqrt[b^2 - 4\*a\*c]\*g))/(2\*c\*(e\*f - d\*g)), ArcSin[(Sqrt[2]\*Sqrt[c]\*Sqrt[f + g\*x])/Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c]\*g)], (b - Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)/(b + Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)]/(Sqrt[c]\*(e\*f - d\*g)^2\*Sqrt[a + b\*x + c\*x^2]))

**Rubi [A]** time = 4.90045, antiderivative size = 553, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.29$

$$\frac{\sqrt{2g}\sqrt{b^2-4ac}\sqrt{f+gx}\sqrt{-\frac{c(a+bx+cx^2)}{b^2-4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b+2cx+\sqrt{b^2-4ac}}{\sqrt{b^2-4ac}}}}{\sqrt{2}}\right)\middle|-\frac{2\sqrt{b^2-4ac}g}{2cf-(b+\sqrt{b^2-4ac})g}\right)}{\sqrt{a+bx+cx^2}(ef-dg)(ag^2-bfg+cf^2)\sqrt{\frac{c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}}$$

$$+\frac{\sqrt{2e}\sqrt{2cf-g(b-\sqrt{b^2-4ac})}\sqrt{1-\frac{2c(f+gx)}{2cf-g(b-\sqrt{b^2-4ac})}}\sqrt{1-\frac{2c(f+gx)}{2cf-g(\sqrt{b^2-4ac}+b)}}\left(\frac{e(2cf-bg+\sqrt{b^2-4ac}g)}{2c(ef-dg)};\sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{f+gx}}{\sqrt{2cf-(b-\sqrt{b^2-4ac})g}}\right)\right)}{\sqrt{c}\sqrt{a+bx+cx^2}(ef-dg)^2}$$

$$+\frac{2g^2\sqrt{a+bx+cx^2}}{\sqrt{f+gx}(ef-dg)(ag^2-bfg+cf^2)}$$

Antiderivative was successfully verified.

[In] Int[1/((d + e\*x)\*(f + g\*x)^(3/2)\*Sqrt[a + b\*x + c\*x^2]),x]

[Out] (2\*g^2\*Sqrt[a + b\*x + c\*x^2])/((e\*f - d\*g)\*(c\*f^2 - b\*f\*g + a\*g^2)\*Sqrt[f + g\*x]) - (Sqrt[2]\*Sqrt[b^2 - 4\*a\*c]\*g\*Sqrt[f + g\*x]\*Sqrt[-((c\*(a + b\*x + c\*x^2))/(b^2 - 4\*a\*c))]\*EllipticE[ArcSin[Sqrt[(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x)/Sqrt[b^2 - 4\*a\*c]]/Sqrt[2]], (-2\*Sqrt[b^2 - 4\*a\*c]\*g)/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c]\*g))]/((e\*f - d\*g)\*(c\*f^2 - b\*f\*g + a\*g^2)\*Sqrt[(c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c]\*g)]\*Sqrt[a + b\*x + c\*x^2]) - (Sqrt[2]\*e\*Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c]\*g)]\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c]\*g)]\*Sqrt[1 - (2\*c\*(f + g\*x))/(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c]\*g)]\*EllipticPi[(e\*(2\*c\*f - b\*g + Sqrt[b^2 - 4\*a\*c]\*g))/(2\*c\*(e\*f - d\*g)), ArcSin[(Sqrt[2]\*Sqrt[c]\*Sqrt[f + g\*x])/Sqrt[2\*c\*f - (b - Sqrt[b^2 - 4\*a\*c]\*g)], (b - Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)/(b + Sqrt[b^2 - 4\*a\*c] - (2\*c\*f)/g)]/(Sqrt[c]\*(e\*f - d\*g)^2\*Sqrt[a + b\*x + c\*x^2]))

$$+ \text{Sqrt}[b^2 - 4*a*c]*g)]* \text{EllipticPi}[(e*(2*c*f - b*g + \text{Sqrt}[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), \text{ArcSin}[(\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[f + g*x])/ \text{Sqrt}[2*c*f - (b - \text{Sqrt}[b^2 - 4*a*c]*g)], (b - \text{Sqrt}[b^2 - 4*a*c] - (2*c*f)/g)/(b + \text{Sqrt}[b^2 - 4*a*c] - (2*c*f)/g)]/(\text{Sqrt}[c]*(e*f - d*g)^2*\text{Sqrt}[a + b*x + c*x^2])$$

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(e*x+d)/(g*x+f)**(3/2)/(c*x**2+b*x+a)**(1/2), x)`

[Out] Timed out

**Mathematica [C]** time = 11.4314, size = 1061, normalized size = 1.92

$$\frac{2(cx^2 + bx + a)g^2}{(ef - dg)(cf^2 - bgf + ag^2)\sqrt{f + gx}\sqrt{a + x(b + cx)}}$$

$$2(f + gx)^{3/2}\sqrt{cx^2 + bx + a} \left( \frac{cf^2}{(f+gx)^2} - \frac{2cf}{f+gx} - \frac{bgf}{(f+gx)^2} + c - \frac{i \sqrt{1 - \frac{2(cf^2 + g(ag-bf))}{(2cf - bg + \sqrt{(b^2 - 4ac)g^2})(f+gx)}} \sqrt{\frac{4(cf^2 + g(ag-bf))}{(-2cf + bg + \sqrt{(b^2 - 4ac)g^2})(f+gx)}} + 2(ef - dg)(2} \right)$$

Antiderivative was successfully verified.

[In] `Integrate[1/((d + e*x)*(f + g*x)^(3/2)*Sqrt[a + b*x + c*x^2]), x]`

[Out]  $(2*g^2*(a + b*x + c*x^2))/((e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + x*(b + c*x)]) + (2*(f + g*x)^(3/2)*\text{Sqrt}[a + b*x + c*x^2]*(c + (c*f^2)/(f + g*x)^2 - (b*f*g)/(f + g*x)^2 + (a*g^2)/(f + g*x)^2 - (2*c*f)/(f + g*x) + (b*g)/(f + g*x) - ((I/4)*\text{Sqrt}[1 - (2*(c*f^2 + g*(-b*f) + a*g))]/((2*c*f - b*g + \text{Sqrt}[(b^2 - 4*a*c)*g^2])*(f + g*x)))*\text{Sqrt}[2 + (4*(c*f^2 + g*(-b*f) + a*g))]/((-2*c*f + b*g + \text{Sqrt}[(b^2 - 4*a*c)*g^2])*(f + g*x)))*((e*f - d*g)*(2*c*f - b*g + \text{Sqrt}[(b^2 - 4*a*c)*g^2])*(\text{EllipticE}[I*\text{ArcSinh}[(\text{Sqrt}[2]*\text{Sqrt}[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + \text{Sqrt}[(b^2 - 4*a*c)*g^2])])/ \text{Sqrt}[f + g*x]], -((-2*c*f + b*g + \text{Sqrt}[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + \text{Sqrt}[(b^2 - 4*a*c)*g^2])) - \text{EllipticF}[I*\text{ArcSinh}[(\text{Sqrt}[2]*\text{Sqrt}[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + \text{Sqrt}[(b^2 - 4*a*c)*g^2])])/ \text{Sqrt}[f + g*x]], -((-2*c*f + b*g + \text{Sqrt}[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + \text{Sqrt}[(b^2 - 4*a*c)*g^2])) - 2*e*(c*f^2 + g*(-b*f) + a*g)*\text{EllipticF}[I*\text{ArcSinh}[(\text{Sqrt}[2]*\text{Sqrt}[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + \text{Sqrt}[(b^2 - 4*a*c)*g^2])])/ \text{Sqrt}[f + g*x]], -((-2*c*f + b*g + \text{Sqrt}[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + \text{Sqrt}[(b^2 - 4*a*c)*g^2])) + 2*e*(c*f^2 + g*(-b*f) + a*g)*\text{EllipticPi}[((e*f - d*g)*(2*c*f - b*g - \text{Sqrt}[(b^2 - 4*a*c)*g^2])/(2*e*(c*f^2 + g*(-b*f) + a*g))), I*\text{ArcSinh}[(\text{Sqrt}[2]*\text{Sqrt}[(c*f^2 - b*f*g + a*g^2)/(-2*c*f + b*g + \text{Sqrt}[(b^2 - 4*a*c)*g^2])])/ \text{Sqrt}[f + g*x]], -((-2*c*f + b*g + \text{Sqrt}[(b^2 - 4*a*c)*g^2])/(2*c*f - b*g + \text{Sqrt}[(b^2 - 4*a*c)*g^2])))/((e*f - d*g)*\text{Sqrt}[(c*f^2 + g*(-b*f) + a*g)/(-2*c*f + b*g + \text{Sqrt}[(b^2 - 4*a*c)*g^2])]*\text{Sqrt}[f + g*x]))/((-e*f) + d*g)*(c*f^2 - b*f*g + a*g^2)*\text{Sqrt}[a + x*(b$







**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}(ex + d)(gx + f)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)\*(g\*x + f)^(3/2)),x, algorithm="maxima")

[Out] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)\*(g\*x + f)^(3/2)), x)

---

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)\*(g\*x + f)^(3/2)),x, algorithm="fricas")

[Out] Timed out

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex)(f + gx)^{\frac{3}{2}} \sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x+d)/(g\*x+f)\*\*(3/2)/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral(1/((d + e\*x)\*(f + g\*x)\*\*(3/2)\*sqrt(a + b\*x + c\*x\*\*2)), x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}(ex + d)(gx + f)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)\*(g\*x + f)^(3/2)),x, algorithm="giac")

[Out] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)\*(g\*x + f)^(3/2)), x)

$$3.917 \quad \int \frac{1}{(d+ex)(f+gx)^{5/2}\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=1125

result too large to display

[Out]  $(2*g^2*\text{Sqrt}[a + b*x + c*x^2])/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*(f + g*x)^{(3/2)}) + (4*g^2*(2*c*f - b*g)*\text{Sqrt}[a + b*x + c*x^2])/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)^2*\text{Sqrt}[f + g*x]) + (2*e*g^2*\text{Sqrt}[a + b*x + c*x^2])/((e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*\text{Sqrt}[f + g*x]) - (2*\text{Sqrt}[2]*\text{Sqrt}[b^2 - 4*a*c]*g*(2*c*f - b*g)*\text{Sqrt}[f + g*x]*\text{Sqrt}[-(c*(a + b*x + c*x^2))/(b^2 - 4*a*c)])*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (-2*\text{Sqrt}[b^2 - 4*a*c]*g)/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)^2*\text{Sqrt}[(c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]*\text{Sqrt}[a + b*x + c*x^2]) - (\text{Sqrt}[2]*\text{Sqrt}[b^2 - 4*a*c]*e*g*\text{Sqrt}[f + g*x]*\text{Sqrt}[-(c*(a + b*x + c*x^2))/(b^2 - 4*a*c)])*\text{EllipticE}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (-2*\text{Sqrt}[b^2 - 4*a*c]*g)/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]/((e*f - d*g)^2*(c*f^2 - b*f*g + a*g^2)*\text{Sqrt}[(c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]*\text{Sqrt}[a + b*x + c*x^2]) + (2*\text{Sqrt}[2]*\text{Sqrt}[b^2 - 4*a*c]*g*\text{Sqrt}[(c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]*\text{Sqrt}[-(c*(a + b*x + c*x^2))/(b^2 - 4*a*c)])*\text{EllipticF}[\text{ArcSin}[\text{Sqrt}[(b + \text{Sqrt}[b^2 - 4*a*c] + 2*c*x)/\text{Sqrt}[b^2 - 4*a*c]]/\text{Sqrt}[2]], (-2*\text{Sqrt}[b^2 - 4*a*c]*g)/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]/(3*(e*f - d*g)*(c*f^2 - b*f*g + a*g^2)*\text{Sqrt}[f + g*x]*\text{Sqrt}[a + b*x + c*x^2]) - (\text{Sqrt}[2]*e^2*\text{Sqrt}[2*c*f - (b - \text{Sqrt}[b^2 - 4*a*c])*g]*\text{Sqrt}[1 - (2*c*(f + g*x))/(2*c*f - (b - \text{Sqrt}[b^2 - 4*a*c])*g)]*\text{Sqrt}[1 - (2*c*(f + g*x))/(2*c*f - (b + \text{Sqrt}[b^2 - 4*a*c])*g)]*\text{EllipticPi}[(e*(2*c*f - b*g + \text{Sqrt}[b^2 - 4*a*c]*g))/(2*c*(e*f - d*g)), \text{ArcSin}[\text{Sqrt}[2]*\text{Sqrt}[c]*\text{Sqrt}[f + g*x)]/\text{Sqrt}[2*c*f - (b - \text{Sqrt}[b^2 - 4*a*c])*g]], (b - \text{Sqrt}[b^2 - 4*a*c] - (2*c*f)/g)/(b + \text{Sqrt}[b^2 - 4*a*c] - (2*c*f)/g)]/(\text{Sqrt}[c]*(e*f - d*g)^3*\text{Sqrt}[a + b*x + c*x^2])$

**Rubi [A]** time = 7.12339, antiderivative size = 1125, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 12, integrand size = 31,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$

$$\frac{\sqrt{2}\sqrt{2cf - (b - \sqrt{b^2 - 4ac})}g\sqrt{1 - \frac{2c(f+gx)}{2cf - (b - \sqrt{b^2 - 4ac})}g}\sqrt{1 - \frac{2c(f+gx)}{2cf - (b + \sqrt{b^2 - 4ac})}g}\left(\frac{e(2cf - bg + \sqrt{b^2 - 4ac}g)}{2c(e f - dg)}\right); \sin^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{f+gx}}{\sqrt{2cf - (b - \sqrt{b^2 - 4ac})}g}\right)}{\frac{\sqrt{c}(ef - dg)^3\sqrt{cx^2 + bx + a}}{\sqrt{2}\sqrt{b^2 - 4ac}g\sqrt{f + gx}\sqrt{-\frac{c(cx^2 + bx + a)}{b^2 - 4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b + 2cx + \sqrt{b^2 - 4ac}}{b^2 - 4ac}}}{\sqrt{2}}\right)\middle| -\frac{2\sqrt{b^2 - 4ac}g}{2cf - (b + \sqrt{b^2 - 4ac})g}\right)}} + \frac{(ef - dg)^2(cf^2 - bgf + ag^2)\sqrt{\frac{c(f+gx)}{2cf - (b + \sqrt{b^2 - 4ac})g}}\sqrt{cx^2 + bx + a}}{2g^2\sqrt{cx^2 + bx + a}} + \frac{2\sqrt{2}\sqrt{b^2 - 4ac}g(2cf - bg)\sqrt{f + gx}\sqrt{-\frac{c(cx^2 + bx + a)}{b^2 - 4ac}}E\left(\sin^{-1}\left(\frac{\sqrt{\frac{b + 2cx + \sqrt{b^2 - 4ac}}{b^2 - 4ac}}}{\sqrt{2}}\right)\middle| -\frac{2\sqrt{b^2 - 4ac}g}{2cf - (b + \sqrt{b^2 - 4ac})g}\right)}{3(ef - dg)(cf^2 - bgf + ag^2)^2\sqrt{\frac{c(f+gx)}{2cf - (b + \sqrt{b^2 - 4ac})g}}\sqrt{cx^2 + bx + a}} + \frac{2\sqrt{2}\sqrt{b^2 - 4ac}g\sqrt{\frac{c(f+gx)}{2cf - (b + \sqrt{b^2 - 4ac})g}}\sqrt{-\frac{c(cx^2 + bx + a)}{b^2 - 4ac}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{b + 2cx + \sqrt{b^2 - 4ac}}{b^2 - 4ac}}}{\sqrt{2}}\right)\middle| -\frac{2\sqrt{b^2 - 4ac}g}{2cf - (b + \sqrt{b^2 - 4ac})g}\right)}{3(ef - dg)(cf^2 - bgf + ag^2)\sqrt{f + gx}\sqrt{cx^2 + bx + a}} + \frac{4g^2(2cf - bg)\sqrt{cx^2 + bx + a}}{3(ef - dg)(cf^2 - bgf + ag^2)^2\sqrt{f + gx}} + \frac{2g^2\sqrt{cx^2 + bx + a}}{3(ef - dg)(cf^2 - bgf + ag^2)(f + gx)^{3/2}}$$

Warning: Unable to verify antiderivative.

[In] Int[1/((d + e\*x)\*(f + g\*x)^(5/2)\*Sqrt[a + b\*x + c\*x^2]),x]

[Out] 
$$\frac{(2g^2\sqrt{a+bx+cx^2})/(3(e^2f-d^2g)(cf^2-bfg+ag^2)(f+gx)^{3/2}) + (4g^2(2cf-bg)\sqrt{a+bx+cx^2})/(3(e^2f-d^2g)(cf^2-bfg+ag^2)^2\sqrt{f+gx}) + (2e^2g^2\sqrt{a+bx+cx^2})/((e^2f-d^2g)^2(cf^2-bfg+ag^2)\sqrt{f+gx}) - (2\sqrt{2}\sqrt{b^2-4ac}g(2cf-bg)\sqrt{f+gx}\sqrt{-((c(a+bx+cx^2))/(b^2-4ac))})\text{EllipticE}[\text{ArcSin}[\sqrt{(b+\sqrt{b^2-4ac}+2cx)/\sqrt{b^2-4ac}}]/\sqrt{2}], (-2\sqrt{b^2-4ac}g)/(2cf-(b+\sqrt{b^2-4ac})g)]/(3(e^2f-d^2g)(cf^2-bfg+ag^2)^2\sqrt{(c(f+gx)))/(2cf-(b+\sqrt{b^2-4ac})g)}\sqrt{a+bx+cx^2}) - (\sqrt{2}\sqrt{b^2-4ac}e^2g\sqrt{f+gx}\sqrt{-((c(a+bx+cx^2))/(b^2-4ac))})\text{EllipticE}[\text{ArcSin}[\sqrt{(b+\sqrt{b^2-4ac}+2cx)/\sqrt{b^2-4ac}}]/\sqrt{2}], (-2\sqrt{b^2-4ac}g)/(2cf-(b+\sqrt{b^2-4ac})g)]/((e^2f-d^2g)^2(cf^2-bfg+ag^2)\sqrt{(c(f+gx)))/(2cf-(b+\sqrt{b^2-4ac})g)}\sqrt{a+bx+cx^2}) + (2\sqrt{2}\sqrt{b^2-4ac}g\sqrt{((c(f+gx))/(2cf-(b+\sqrt{b^2-4ac})g)}\sqrt{-((c(a+bx+cx^2))/(b^2-4ac))})\text{EllipticF}[\text{ArcSin}[\sqrt{(b+\sqrt{b^2-4ac}+2cx)/\sqrt{b^2-4ac}}]/\sqrt{2}], (-2\sqrt{b^2-4ac}g)/(2cf-(b+\sqrt{b^2-4ac})g)]/(3(e^2f-d^2g)(cf^2-bfg+ag^2)\sqrt{f+gx}\sqrt{a+bx+cx^2}) - (\sqrt{2}e^2\sqrt{2cf-(b-\sqrt{b^2-4ac})g}\sqrt{1-(2c(f+gx))/(2cf-(b-\sqrt{b^2-4ac})g)}\sqrt{1-(2c(f+gx))/(2cf-(b+\sqrt{b^2-4ac})g)}\text{EllipticPi}[(e^2cf-bg+\sqrt{b^2-4ac}g)/(2c(e^2f-d^2g)}, \text{ArcSin}[\sqrt{2}\sqrt{c}\sqrt{f+gx}]/\sqrt{2cf-(b-\sqrt{b^2-4ac})g}], (b-\sqrt{b^2-4ac}-(2cf)/g)/(b+\sqrt{b^2-4ac}-(2cf)/g)]/(\sqrt{c}(e^2f-d^2g)^3\sqrt{a+bx+cx^2})$$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/(e\*x+d)/(g\*x+f)\*\*(5/2)/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Timed out

**Mathematica [C]** time = 18.2661, size = 14762, normalized size = 13.12

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[1/((d + e\*x)\*(f + g\*x)^(5/2)\*Sqrt[a + b\*x + c\*x^2]),x]

[Out] Result too large to show

**Maple [B]** time = 0.202, size = 27601, normalized size = 24.5

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x+d)/(g*x+f)^(5/2)/(c*x^2+b*x+a)^(1/2),x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}(ex + d)(gx + f)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^(5/2)),x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^(5/2)), x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^(5/2)),x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(d + ex)(f + gx)^{\frac{5}{2}} \sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(e*x+d)/(g*x+f)**(5/2)/(c*x**2+b*x+a)**(1/2),x)`

[Out] `Integral(1/((d + e*x)*(f + g*x)**(5/2)*sqrt(a + b*x + c*x**2)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}(ex + d)(gx + f)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^(5/2)),x, algorithm="giac")`

[Out] `integrate(1/(sqrt(c*x^2 + b*x + a)*(e*x + d)*(g*x + f)^(5/2)), x)`

$$3.918 \quad \int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=475

$$\frac{\sqrt{2}(d+ex)\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{2cf-g(\sqrt{b^2-4ac}+b)}\sqrt{\frac{(\sqrt{b^2-4ac}+b+2cx)(ef-dg)}{(d+ex)(2cf-g(\sqrt{b^2-4ac}+b))}}\sqrt{\frac{(x(\sqrt{b^2-4ac}+b)+2a)(ef-dg)}{(d+ex)(f\sqrt{b^2-4ac}-2ag+bf)}}\left(\frac{e(2cf-g(\sqrt{b^2-4ac}+b))}{2cd-e}\right)}{g\sqrt{\frac{2ac}{\sqrt{b^2-4ac}+b}}+cx\sqrt{a+bx+cx^2}\sqrt{2cd-e}\left(\sqrt{b^2-4ac}\right)}$$

[Out] (Sqrt[2]\*Sqrt[2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g]\*Sqrt[b - Sqrt[b^2 - 4\*a\*c] + 2\*c\*x]\*Sqrt[((e\*f - d\*g)\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/((2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)\*(d + e\*x))]\*Sqrt[((e\*f - d\*g)\*(2\*a + (b + Sqrt[b^2 - 4\*a\*c])\*x))/((b\*f + Sqrt[b^2 - 4\*a\*c]\*f - 2\*a\*g)\*(d + e\*x))]\*(d + e\*x)\*EllipticPi[(e\*(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g))/((2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)\*g), ArcSin[(Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e]\*Sqrt[f + g\*x])/(Sqrt[2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g]\*Sqrt[d + e\*x])], ((b\*d + Sqrt[b^2 - 4\*a\*c]\*d - 2\*a\*e)\*(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g))/((2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)\*(b\*f + Sqrt[b^2 - 4\*a\*c]\*f - 2\*a\*g)))]/(Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e]\*g\*Sqrt[(2\*a\*c)/(b + Sqrt[b^2 - 4\*a\*c] + c\*x)\*Sqrt[a + b\*x + c\*x^2])

**Rubi [A]** time = 0.88333, antiderivative size = 475, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.03$

$$\frac{\sqrt{2}(d+ex)\sqrt{-\sqrt{b^2-4ac}+b+2cx}\sqrt{2cf-g(\sqrt{b^2-4ac}+b)}\sqrt{\frac{(\sqrt{b^2-4ac}+b+2cx)(ef-dg)}{(d+ex)(2cf-g(\sqrt{b^2-4ac}+b))}}\sqrt{\frac{(x(\sqrt{b^2-4ac}+b)+2a)(ef-dg)}{(d+ex)(f\sqrt{b^2-4ac}-2ag+bf)}}\left(\frac{e(2cf-g(\sqrt{b^2-4ac}+b))}{2cd-e}\right)}{g\sqrt{\frac{2ac}{\sqrt{b^2-4ac}+b}}+cx\sqrt{a+bx+cx^2}\sqrt{2cd-e}\left(\sqrt{b^2-4ac}\right)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[d + e\*x]/(Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]), x]

[Out] (Sqrt[2]\*Sqrt[2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g]\*Sqrt[b - Sqrt[b^2 - 4\*a\*c] + 2\*c\*x]\*Sqrt[((e\*f - d\*g)\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/((2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g)\*(d + e\*x))]\*Sqrt[((e\*f - d\*g)\*(2\*a + (b + Sqrt[b^2 - 4\*a\*c])\*x))/((b\*f + Sqrt[b^2 - 4\*a\*c]\*f - 2\*a\*g)\*(d + e\*x))]\*(d + e\*x)\*EllipticPi[(e\*(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g))/((2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)\*g), ArcSin[(Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e]\*Sqrt[f + g\*x])/(Sqrt[2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g]\*Sqrt[d + e\*x])], ((b\*d + Sqrt[b^2 - 4\*a\*c]\*d - 2\*a\*e)\*(2\*c\*f - (b + Sqrt[b^2 - 4\*a\*c])\*g))/((2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)\*(b\*f + Sqrt[b^2 - 4\*a\*c]\*f - 2\*a\*g)))]/(Sqrt[2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e]\*g\*Sqrt[(2\*a\*c)/(b + Sqrt[b^2 - 4\*a\*c] + c\*x)\*Sqrt[a + b\*x + c\*x^2])

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{d+ex}}{\sqrt{f+gx}\sqrt{a+bx+cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*(1/2)/(g\*x+f)\*\*(1/2)/(c\*x\*\*2+b\*x+a)\*\*(1/2), x)

[Out] Integral(sqrt(d + e\*x)/(sqrt(f + g\*x)\*sqrt(a + b\*x + c\*x\*\*2)), x)

**Mathematica [B]** time = 13.5702, size = 2493, normalized size = 5.25

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[Sqrt[d + e\*x]/(Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]),x]

[Out] 
$$\begin{aligned} & (-2 \sqrt{f + g x} \sqrt{a + b x + c x^2} \sqrt{\frac{c + (c f^2)}{(f + g x)^2 - (b f g)/(f + g x)^2 + (a g^2)/(f + g x)^2 - (2 c f)/(f + g x) + (b g)/(f + g x)}} (e - (e f)/(f + g x) + (d g)/(f + g x)) \sqrt{d + (f + g x)(e - (e f)/(f + g x))} / g - ((e f \sqrt{-(e/(e f - d g)) + (f + g x)^{-1}}) / (-e/(e f - d g)) + (2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}) / (2 (c f^2 - b f g + a g^2))) \sqrt{-(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2})} / (2 (c f^2 - b f g + a g^2)) + (f + g x)^{-1} / (-2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}) + (2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}) / (2 (c f^2 - b f g + a g^2)) + (2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}) / (2 (c f^2 - b f g + a g^2)) \sqrt{-(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2})} / (2 (c f^2 - b f g + a g^2)) + (f + g x)^{-1} \text{EllipticF}[\text{ArcSin}[\sqrt{(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2})} / \sqrt{2}], (2 \sqrt{(b^2 - 4 a c) g^2} (e f - d g)) / (-2 c d f g + b e f g + b d g^2 - 2 a e g^2 + e f \sqrt{(b^2 - 4 a c) g^2} - d g \sqrt{(b^2 - 4 a c) g^2})] / (\sqrt{-(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2})} / (2 (c f^2 - b f g + a g^2)) + (f + g x)^{-1}) / ((2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}) / (2 (c f^2 - b f g + a g^2)) - (2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}) / (2 (c f^2 - b f g + a g^2))) \sqrt{(c + (c f^2 - b f g + a g^2) / (f + g x)^2 + (-2 c f + b g) / (f + g x)) (e + (-e f) + d g) / (f + g x)} + (d g \sqrt{-(e/(e f - d g)) + (f + g x)^{-1}}) / (-e/(e f - d g)) + (2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}) / (2 (c f^2 - b f g + a g^2)) \sqrt{-(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2})} / (2 (c f^2 - b f g + a g^2)) + (f + g x)^{-1} / (-2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}) + (2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}) / (2 (c f^2 - b f g + a g^2)) \sqrt{-(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2})} / (2 (c f^2 - b f g + a g^2)) + (f + g x)^{-1} \text{EllipticF}[\text{ArcSin}[\sqrt{(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2})} / \sqrt{2}], (2 \sqrt{(b^2 - 4 a c) g^2} (e f - d g)) / (-2 c d f g + b e f g + b d g^2 - 2 a e g^2 + e f \sqrt{(b^2 - 4 a c) g^2} - d g \sqrt{(b^2 - 4 a c) g^2})] / (\sqrt{-(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2})} / (2 (c f^2 - b f g + a g^2)) + (f + g x)^{-1}) / ((2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}) / (2 (c f^2 - b f g + a g^2)) - (2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}) / (2 (c f^2 - b f g + a g^2))) \sqrt{(c + (c f^2 - b f g + a g^2) / (f + g x)^2 + (-2 c f + b g) / (f + g x)) (e + (-e f) + d g) / (f + g x)} - (2 e (c f^2 - b f g + a g^2) (-2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}) / (2 (c f^2 - b f g + a g^2)) + (2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}) / (2 (c f^2 - b f g + a g^2))) \sqrt{-(e/(e f - d g)) + (f + g x)^{-1}} / (-e/(e f - d g)) + (2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}) / (2 (c f^2 - b f g + a g^2)) \sqrt{-(2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2})} / (2 (c f^2 - b f g + a g^2)) + (f + g x)^{-1} / (-2 c f - b g - \sqrt{b^2 g^2 - 4 a c g^2}) + (2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}) / (2 (c f^2 - b f g + a g^2)) \sqrt{-(2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2})} / (2 (c f^2 - b f g + a g^2)) + (f + g x)^{-1} \text{EllipticPi}[(2 \sqrt{(b^2 - 4 a c) g^2}) / (2 c f - b g + \sqrt{(b^2 - 4 a c) g^2}), \text{ArcSin}[\sqrt{(2 c f - b g + \sqrt{(b^2 - 4 a c) g^2})} / \sqrt{2}], (2 \sqrt{(b^2 - 4 a c) g^2} (e f - d g)) / (-2 c d f g + b e f g + b d g^2 - 2 a e g^2 + e f \sqrt{(b^2 - 4 a c) g^2} - d g \sqrt{(b^2 - 4 a c) g^2})] / ((2 c f - b g + \sqrt{b^2 g^2 - 4 a c g^2}) \sqrt{(c + (c f^2 - b f g + a g^2) / (f + g x)^2 + (-2 c f + b g) / (f + g x)) (e + (-e f) + d g) / (f + g x)))] / (g \sqrt{a + x(b + c x)}) (e - (e f) / (f + g x) + (d g) / (f + g x)) \sqrt{((f + g x)^2 (c (-1 +$$





Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**(1/2)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)`

[Out] `Integral(sqrt(d + e*x)/(sqrt(f + g*x)*sqrt(a + b*x + c*x**2)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{ex+d}}{\sqrt{cx^2+bx+a}\sqrt{gx+f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(e*x + d)/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)),x, algorithm="giac"`

[Out] `integrate(sqrt(e*x + d)/(sqrt(c*x^2 + b*x + a)*sqrt(g*x + f)), x)`

$$3.919 \quad \int \frac{1}{\sqrt{d+ex}\sqrt{f+gx}\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=588

$$\frac{(d+ex)\sqrt[3]{cf^2-g(bf-ag)}\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\left(\frac{(f+gx)\sqrt{ae^2-bde+cd^2}}{(d+ex)\sqrt{cf^2-g(bf-ag)}}+1\right)\sqrt{\frac{\frac{(f+gx)^2(ae^2-bde+cd^2)}{(d+ex)^2(cf^2-g(bf-ag))}-\frac{(f+gx)(2aeg-b(dg+ef)+2cdf)+1}{(d+ex)(ag^2-bfg+cf^2)}}{\left(\frac{(f+gx)\sqrt{ae^2-bde+cd^2}}{(d+ex)\sqrt{cf^2-g(bf-ag)}}+1\right)^2}}}{\sqrt{a+bx+cx^2}(ef-dg)\sqrt[4]{ae^2-bde+cd^2}\sqrt{\frac{(f+gx)^2(ae^2-bde+cd^2)}{(d+ex)^2(cf^2-g(bf-ag))}-\frac{(f+gx)}{(d+ex)(ag^2-bfg+cf^2)}}}}$$

[Out] -(((c\*f^2 - g\*(b\*f - a\*g))^(1/4)\*(d + e\*x)\*Sqrt[((e\*f - d\*g)^2\*(a + b\*x + c\*x^2))/((c\*f^2 - b\*f\*g + a\*g^2)\*(d + e\*x)^2)]\*(1 + (Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*(f + g\*x))/(Sqrt[c\*f^2 - g\*(b\*f - a\*g)]\*(d + e\*x))))\*Sqrt[(1 - ((2\*c\*d\*f + 2\*a\*e\*g - b\*(e\*f + d\*g))\*(f + g\*x))/((c\*f^2 - b\*f\*g + a\*g^2)\*(d + e\*x)) + ((c\*d^2 - b\*d\*e + a\*e^2)\*(f + g\*x)^2)/((c\*f^2 - g\*(b\*f - a\*g))\*(d + e\*x)^2)))/(1 + (Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*(f + g\*x))/(Sqrt[c\*f^2 - g\*(b\*f - a\*g)]\*(d + e\*x)))^2]\*EllipticF[2\*ArcTan[((c\*d^2 - b\*d\*e + a\*e^2)^(1/4)\*Sqrt[f + g\*x])/((c\*f^2 - b\*f\*g + a\*g^2)^(1/4)\*Sqrt[d + e\*x])], (2 + (2\*c\*d\*f + 2\*a\*e\*g - b\*(e\*f + d\*g))/(Sqrt[c\*d^2 - b\*d\*e + a\*e^2])\*(e\*f - d\*g)\*Sqrt[a + b\*x + c\*x^2]\*Sqrt[1 - ((2\*c\*d\*f + 2\*a\*e\*g - b\*(e\*f + d\*g))\*(f + g\*x))/((c\*f^2 - b\*f\*g + a\*g^2)\*(d + e\*x)) + ((c\*d^2 - b\*d\*e + a\*e^2)\*(f + g\*x)^2)/((c\*f^2 - g\*(b\*f - a\*g))\*(d + e\*x)^2)))]/4)/((c\*d^2 - b\*d\*e + a\*e^2)^(1/4)\*(e\*f - d\*g)\*Sqrt[a + b\*x + c\*x^2]\*Sqrt[1 - ((2\*c\*d\*f + 2\*a\*e\*g - b\*(e\*f + d\*g))\*(f + g\*x))/((c\*f^2 - b\*f\*g + a\*g^2)\*(d + e\*x)) + ((c\*d^2 - b\*d\*e + a\*e^2)\*(f + g\*x)^2)/((c\*f^2 - g\*(b\*f - a\*g))\*(d + e\*x)^2)])]

**Rubi [A]** time = 2.44815, antiderivative size = 589, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 33,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$

$$\frac{(d+ex)\sqrt[3]{cf^2-g(bf-ag)}\sqrt{\frac{(a+bx+cx^2)(ef-dg)^2}{(d+ex)^2(ag^2-bfg+cf^2)}}\left(\frac{(f+gx)\sqrt{ae^2-bde+cd^2}}{(d+ex)\sqrt{cf^2-g(bf-ag)}}+1\right)\sqrt{\frac{\frac{(f+gx)^2(ae^2-bde+cd^2)}{(d+ex)^2(cf^2-g(bf-ag))}-\frac{(f+gx)(2aeg-b(dg+ef)+2cdf)+1}{(d+ex)(ag^2-bfg+cf^2)}}{\left(\frac{(f+gx)\sqrt{ae^2-bde+cd^2}}{(d+ex)\sqrt{cf^2-g(bf-ag)}}+1\right)^2}}}{\sqrt{a+bx+cx^2}(ef-dg)\sqrt[4]{ae^2-bde+cd^2}\sqrt{\frac{(f+gx)^2(ae^2-bde+cd^2)}{(d+ex)^2(cf^2-g(bf-ag))}-\frac{(f+gx)}{(d+ex)(ag^2-bfg+cf^2)}}}}$$

Warning: Unable to verify antiderivative.

[In] Int[1/(Sqrt[d + e\*x]\*Sqrt[f + g\*x]\*Sqrt[a + b\*x + c\*x^2]),x]

[Out] -(((c\*f^2 - g\*(b\*f - a\*g))^(1/4)\*(d + e\*x)\*Sqrt[((e\*f - d\*g)^2\*(a + b\*x + c\*x^2))/((c\*f^2 - b\*f\*g + a\*g^2)\*(d + e\*x)^2)]\*(1 + (Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*(f + g\*x))/(Sqrt[c\*f^2 - g\*(b\*f - a\*g)]\*(d + e\*x))))\*Sqrt[(1 - ((2\*c\*d\*f + 2\*a\*e\*g - b\*(e\*f + d\*g))\*(f + g\*x))/((c\*f^2 - b\*f\*g + a\*g^2)\*(d + e\*x)) + ((c\*d^2 - b\*d\*e + a\*e^2)\*(f + g\*x)^2)/((c\*f^2 - g\*(b\*f - a\*g))\*(d + e\*x)^2)))/(1 + (Sqrt[c\*d^2 - b\*d\*e + a\*e^2]\*(f + g\*x))/(Sqrt[c\*f^2 - g\*(b\*f - a\*g)]\*(d + e\*x)))^2]\*EllipticF[2\*ArcTan[((c\*d^2 - b\*d\*e + a\*e^2)^(1/4)\*Sqrt[f + g\*x])/((c\*f^2 - b\*f\*g + a\*g^2)^(1/4)\*Sqrt[d + e\*x])], (2 + (2\*c\*d\*f + 2\*a\*e\*g - b\*(e\*f + d\*g))/(Sqrt[c\*d^2 - b\*d\*e + a\*e^2])\*(e\*f - d\*g)\*Sqrt[a + b\*x + c\*x^2]\*Sqrt[1 - ((2\*c\*d\*f + 2\*a\*e\*g - b\*(e\*f + d\*g))\*(f + g\*x))/((c\*f^2 - b\*f\*g + a\*g^2)\*(d + e\*x)) + ((c\*d^2 - b\*d\*e + a\*e^2)\*(f + g\*x)^2)/((c\*f^2 - g\*(b\*f - a\*g))\*(d + e\*x)^2)])]/4)/((c\*d^2 - b\*d\*e + a\*e^2)^(1/4)\*(e\*f - d\*g)\*Sqrt[a + b\*x + c\*x^2]\*Sqrt[1 - ((2\*c\*d\*f + 2\*a\*e\*g - b\*(e\*f + d\*g))\*(f + g\*x))/((c\*f^2 - b\*f\*g + a\*g^2)\*(d + e\*x)) + ((c\*d^2 - b\*d\*e + a\*e^2)\*(f + g\*x)^2)/((c\*f^2 - g\*(b\*f - a\*g))\*(d + e\*x)^2)])]

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate(1/(e*x+d)**(1/2)/(g*x+f)**(1/2)/(c*x**2+b*x+a)**(1/2),x)`

[Out] Timed out

**Mathematica [A]** time = 6.05506, size = 375, normalized size = 0.64

$$2\sqrt{2}e\sqrt{a+x(b+cx)}\sqrt{\frac{e(f+gx)(e(ae-bd)+cd^2)}{(d+ex)(-dg\sqrt{e^2(b^2-4ac)}+ef\sqrt{e^2(b^2-4ac)}-2ae^2g+be(dg+ef)-2cdef)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{2ae^2-2cdxe+b(ex-d)e+\sqrt{(b^2-4ac)}e^2(d+ex)}}{\sqrt{(b^2-4ac)}e^2(d+ex)}}}{\sqrt{2}}}\right)\right)$$

$$\sqrt{d+ex}\sqrt{f+gx}\sqrt{e^2(b^2-4ac)}\sqrt{\frac{(a+x(b+cx))(e(ae-bd)+cd^2)}{(b^2-4ac)(d+ex)^2}}$$

Antiderivative was successfully verified.

[In] `Integrate[1/(Sqrt[d + e*x]*Sqrt[f + g*x]*Sqrt[a + b*x + c*x^2]),x]`

[Out]  $(2\sqrt{2}e\sqrt{a+x(b+cx)}\sqrt{\frac{e(f+gx)(e(ae-bd)+cd^2)}{(d+ex)(-dg\sqrt{e^2(b^2-4ac)}+ef\sqrt{e^2(b^2-4ac)}-2ae^2g+be(dg+ef)-2cdef)}}F\left(\sin^{-1}\left(\frac{\sqrt{\frac{2ae^2-2cdxe+b(ex-d)e+\sqrt{(b^2-4ac)}e^2(d+ex)}}{\sqrt{(b^2-4ac)}e^2(d+ex)}}}{\sqrt{2}}}\right)\right)\sqrt{d+ex}\sqrt{f+gx}\sqrt{e^2(b^2-4ac)}\sqrt{\frac{(a+x(b+cx))(e(ae-bd)+cd^2)}{(b^2-4ac)(d+ex)^2}}$

**Maple [A]** time = 0.114, size = 605, normalized size = 1.

$$4\frac{\left(\sqrt{-4ac+b^2x^2e^2g+be^2gx^2-2ce^2fx^2+2\sqrt{-4ac+b^2}xdeg+2xbdeg-4xcdef+\sqrt{-4ac+b^2}d^2g+bd^2g-2cd^2f}\right)\sqrt{e\sqrt{-4ac+b^2}+be-2cd}(dg-ef)\sqrt{ceg^4+beg^3+cdg^3+cef^3+aegx^2+bdgx^2+befx^2+cdfx^2+adgx^2}}{\left(\sqrt{-4ac+b^2}x^2e^2g+be^2gx^2-2ce^2fx^2+2\sqrt{-4ac+b^2}xdeg+2xbdeg-4xcdef+\sqrt{-4ac+b^2}d^2g+bd^2g-2cd^2f}\right)\sqrt{e\sqrt{-4ac+b^2}+be-2cd}(dg-ef)\sqrt{ceg^4+beg^3+cdg^3+cef^3+aegx^2+bdgx^2+befx^2+cdfx^2+adgx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/(e*x+d)^(1/2)/(g*x+f)^(1/2)/(c*x^2+b*x+a)^(1/2),x)`

[Out]  $4\left(\sqrt{-4ac+b^2}\right)^{1/2}x^2e^2g+be^2gx^2-2ce^2fx^2+2\sqrt{-4ac+b^2}xdeg+2xbdeg-4xcdef+\sqrt{-4ac+b^2}d^2g+bd^2g-2cd^2f\sqrt{e\sqrt{-4ac+b^2}+be-2cd}(dg-ef)\sqrt{ceg^4+beg^3+cdg^3+cef^3+aegx^2+bdgx^2+befx^2+cdfx^2+adgx^2}$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\sqrt{ex + d}\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*sqrt(e\*x + d)\*sqrt(g\*x + f)),x, algorithm="max

[Out] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*sqrt(e\*x + d)\*sqrt(g\*x + f)),  
x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{cx^2 + bx + a}\sqrt{ex + d}\sqrt{gx + f}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*sqrt(e\*x + d)\*sqrt(g\*x + f)),x, algorithm="fri

[Out] integral(1/(sqrt(c\*x^2 + b\*x + a)\*sqrt(e\*x + d)\*sqrt(g\*x + f)), x  
)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{d + ex}\sqrt{f + gx}\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e\*x+d)\*\*(1/2)/(g\*x+f)\*\*(1/2)/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral(1/(sqrt(d + e\*x)\*sqrt(f + g\*x)\*sqrt(a + b\*x + c\*x\*\*2)),  
x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{cx^2 + bx + a}\sqrt{ex + d}\sqrt{gx + f}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*sqrt(e\*x + d)\*sqrt(g\*x + f)),x, algorithm="gia

[Out] integrate(1/(sqrt(c\*x^2 + b\*x + a)\*sqrt(e\*x + d)\*sqrt(g\*x + f)),  
x)

### 3.920 $\int (d + ex)^m (f + gx)^2 (a + bx + cx^2) dx$

**Optimal.** Leaf size=220

$$\begin{aligned} & \frac{(d + ex)^{m+3} (eg(aeg - 3bdg + 2bef) + c(6d^2g^2 - 6defg + e^2f^2))}{e^5(m+3)} \\ & + \frac{(ef - dg)^2(d + ex)^{m+1} (ae^2 - bde + cd^2)}{e^5(m+1)} \\ & - \frac{(ef - dg)(d + ex)^{m+2} (2cd(ef - 2dg) - e(2aeg - 3bdg + bef))}{e^5(m+2)} \\ & + \frac{g(d + ex)^{m+4} (beg - 4cdg + 2cef)}{e^5(m+4)} + \frac{cg^2(d + ex)^{m+5}}{e^5(m+5)} \end{aligned}$$

[Out]  $((c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^2*(d + e*x)^(1 + m))/(e^5*(1 + m)) - ((e*f - d*g)*(2*c*d*(e*f - 2*d*g) - e*(b*e*f - 3*b*d*g + 2*a*e*g))*(d + e*x)^(2 + m))/(e^5*(2 + m)) + ((e*g*(2*b*e*f - 3*b*d*g + a*e*g) + c*(e^2*f^2 - 6*d*e*f*g + 6*d^2*g^2))*(d + e*x)^(3 + m))/(e^5*(3 + m)) + (g*(2*c*e*f - 4*c*d*g + b*e*g)*(d + e*x)^(4 + m))/(e^5*(4 + m)) + (c*g^2*(d + e*x)^(5 + m))/(e^5*(5 + m))$

**Rubi [A]** time = 0.509026, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$\begin{aligned} & \frac{(d + ex)^{m+3} (eg(aeg - 3bdg + 2bef) + c(6d^2g^2 - 6defg + e^2f^2))}{e^5(m+3)} \\ & + \frac{(ef - dg)^2(d + ex)^{m+1} (ae^2 - bde + cd^2)}{e^5(m+1)} \\ & - \frac{(ef - dg)(d + ex)^{m+2} (2cd(ef - 2dg) - e(2aeg - 3bdg + bef))}{e^5(m+2)} \\ & + \frac{g(d + ex)^{m+4} (beg - 4cdg + 2cef)}{e^5(m+4)} + \frac{cg^2(d + ex)^{m+5}}{e^5(m+5)} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2), x]$

[Out]  $((c*d^2 - b*d*e + a*e^2)*(e*f - d*g)^2*(d + e*x)^(1 + m))/(e^5*(1 + m)) - ((e*f - d*g)*(2*c*d*(e*f - 2*d*g) - e*(b*e*f - 3*b*d*g + 2*a*e*g))*(d + e*x)^(2 + m))/(e^5*(2 + m)) + ((e*g*(2*b*e*f - 3*b*d*g + a*e*g) + c*(e^2*f^2 - 6*d*e*f*g + 6*d^2*g^2))*(d + e*x)^(3 + m))/(e^5*(3 + m)) + (g*(2*c*e*f - 4*c*d*g + b*e*g)*(d + e*x)^(4 + m))/(e^5*(4 + m)) + (c*g^2*(d + e*x)^(5 + m))/(e^5*(5 + m))$

**Rubi in Sympy [A]** time = 122.246, size = 230, normalized size = 1.05

$$\begin{aligned} & \frac{cg^2(d + ex)^{m+5}}{e^5(m+5)} + \frac{g(d + ex)^{m+4} (beg - 4cdg + 2cef)}{e^5(m+4)} + \frac{(d + ex)^{m+1} (dg - ef)^2 (ae^2 - bde + cd^2)}{e^5(m+1)} \\ & - \frac{(d + ex)^{m+2} (dg - ef) (2ae^2g - 3bdeg + be^2f + 4cd^2g - 2cdef)}{e^5(m+2)} \\ & + \frac{(d + ex)^{m+3} (ae^2g^2 - 3bdeg^2 + 2be^2fg + 6cd^2g^2 - 6cdefg + ce^2f^2)}{e^5(m+3)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In]  $\text{rubi\_integrate}((e*x+d)**m*(g*x+f)**2*(c*x**2+b*x+a), x)$

[Out]  $c^2 g^2 (d + e^x)^{m+5} / (e^{5(m+5)}) + g (d + e^x)^{m+4} (b^2 e^2 g - 4 c^2 d^2 g + 2 c^2 e^2 f) / (e^{5(m+4)}) + (d + e^x)^{m+1} (d^2 g - e^2 f)^2 (a^2 e^2 - b^2 d^2 e + c^2 d^2) / (e^{5(m+1)}) - (d + e^x)^{m+2} (d^2 g - e^2 f) (2 a^2 e^2 g - 3 b^2 d^2 e^2 g + b^2 e^2 f + 4 c^2 d^2 g - 2 c^2 d^2 e^2 f) / (e^{5(m+2)}) + (d + e^x)^{m+3} (a^2 e^2 g^2 - 3 b^2 d^2 e^2 g^2 + 2 b^2 e^2 f^2 g + 6 c^2 d^2 g^2 - 6 c^2 d^2 e^2 f g + c^2 e^2 f^2) / (e^{5(m+3)})$

**Mathematica [A]** time = 0.886022, size = 440, normalized size = 2.

$(d + ex)^{m+1} (e(m+5) (ae(m+4) (2d^2g^2 - 2deg(f(m+3) + g(m+1)x) + e^2 (f^2 (m^2 + 5m + 6) + 2fg (m^2 + 4m + 3) x + g^2 ($

Antiderivative was successfully verified.

[In] Integrate[(d + e\*x)^m\*(f + g\*x)^2\*(a + b\*x + c\*x^2), x]

[Out]  $((d + e^x)^{(1+m)} (c^2 (24 d^4 g^2 - 12 d^3 e g (f (5+m) + 2 g (1+m)x) + 2 d^2 e^2 (f^2 (20 + 9m + m^2) + 6 f g (5 + 6m + m^2)x + 6 g^2 (2 + 3m + m^2)x^2) - 2 d e^3 (1+m)x (f^2 (20 + 9m + m^2) + 3 f g (10 + 7m + m^2)x + 2 g^2 (6 + 5m + m^2)x^2) + e^4 (2 + 3m + m^2)x^2 (f^2 (20 + 9m + m^2) + 2 f g (15 + 8m + m^2)x + g^2 (12 + 7m + m^2)x^2)) + e^5 (5+m) (a^2 e^2 (4+m) (2 d^2 g^2 - 2 d e^2 g (f (3+m) + g (1+m)x) + e^2 (f^2 (6 + 5m + m^2) + 2 f g (3 + 4m + m^2)x + g^2 (2 + 3m + m^2)x^2)) + b^2 (-6 d^3 g^2 + 2 d^2 e^2 g (2 f (4+m) + 3 g (1+m)x) - d e^2 (f^2 (12 + 7m + m^2) + 4 f g (4 + 5m + m^2)x + 3 g^2 (2 + 3m + m^2)x^2) + e^3 (1+m)x (f^2 (12 + 7m + m^2) + 2 f g (8 + 6m + m^2)x + g^2 (6 + 5m + m^2)x^2)))) / (e^{5(1+m)} (2+m)^3 (3+m)^4 (4+m)^5 (5+m))$

**Maple [B]** time = 0.017, size = 1347, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^m\*(g\*x+f)^2\*(c\*x^2+b\*x+a), x)

[Out]  $(e^x+d)^{(1+m)} (c^2 e^4 g^2 m^4 x^4 + b^2 e^4 g^2 m^4 x^3 + 2 c^2 e^4 f g^2 m^4 x^3 + 10 c^2 e^4 g^2 m^3 x^4 + a^2 e^4 g^2 m^4 x^2 + 2 b^2 e^4 f g^2 m^4 x^2 + 11 b^2 e^4 g^2 m^3 x^3 - 4 c^2 d^2 e^3 g^2 m^3 x^3 + c^2 e^4 f^2 m^4 x^2 + 22 c^2 e^4 f g^2 m^3 x^3 + 35 c^2 e^4 g^2 m^2 x^4 + 2 a^2 e^4 f g^2 m^4 x + 12 a^2 e^4 g^2 m^3 x^2 - 3 b^2 d^2 e^3 g^2 m^3 x^2 + b^2 e^4 f^2 m^4 x + 24 b^2 e^4 f g^2 m^3 x^2 + 41 b^2 e^4 g^2 m^2 x^3 - 6 c^2 d^2 e^3 f g^2 m^3 x^2 - 24 c^2 d^2 e^3 g^2 m^2 x^3 + 12 c^2 e^4 f^2 m^3 x^2 + 82 c^2 e^4 f g^2 m^2 x^3 + 50 c^2 e^4 g^2 m^2 x^4 - 2 a^2 d^2 e^3 g^2 m^3 x + a^2 e^4 f^2 m^4 + 26 a^2 e^4 f g^2 m^3 x + 49 a^2 e^4 g^2 m^2 x^2 - 4 b^2 d^2 e^3 f g^2 m^3 x - 24 b^2 d^2 e^3 g^2 m^2 x^2 + 13 b^2 e^4 f^2 m^3 x + 98 b^2 e^4 f g^2 m^2 x^2 + 61 b^2 e^4 g^2 m^2 x^3 + 12 c^2 d^2 e^2 g^2 m^2 x^2 - 2 c^2 d^2 e^3 f^2 m^3 x - 48 c^2 d^2 e^3 f g^2 m^2 x^2 - 44 c^2 d^2 e^3 g^2 m^2 x^3 + 49 c^2 e^4 f^2 m^2 x^2 + 122 c^2 e^4 f g^2 m^2 x^3 + 24 c^2 e^4 g^2 m^2 x^4 - 2 a^2 d^2 e^3 f g^2 m^3 - 20 a^2 d^2 e^3 g^2 m^2 x + 14 a^2 e^4 f^2 m^3 + 118 a^2 e^4 f g^2 m^2 x + 78 a^2 e^4 g^2 m^2 x^2 + 6 b^2 d^2 e^2 g^2 m^2 x - b^2 d^2 e^3 f^2 m^3 - 40 b^2 d^2 e^3 f g^2 m^2 x - 51 b^2 d^2 e^3 g^2 m^2 x^2 + 59 b^2 e^4 f^2 m^2 x + 156 b^2 e^4 f g^2 m^2 x^2 + 30 b^2 e^4 g^2 m^2 x^3 + 12 c^2 d^2 e^2 f g^2 m^2 x + 36 c^2 d^2 e^2 g^2 m^2 x^2 - 20 c^2 d^2 e^3 f^2 m^2 x - 102 c^2 d^2 e^3 f g^2 m^2 x^2 - 24 c^2 d^2 e^3 g^2 m^2 x^3 + 78 c^2 e^4 f^2 m^2 x^2 + 60 c^2 e^4 f g^2 m^2 x^3 + 2 a^2 d^2 e^2 g^2 m^2 - 24 a^2 d^2 e^3 f g^2 m^2 - 58 a^2 d^2 e^3 g^2 m^2 x + 71 a^2 e^4 f^2 m^2 + 214 a^2 e^4 f g^2 m^2 x + 40 a^2 e^4 g^2 m^2 x^2 + 4 b^2 d^2 e^2 f g^2 m^2 + 36 b^2 d^2 e^2 g^2 m^2 x - 12 b^2 d^2 e^3 f^2 m^2 - 116 b^2 d^2 e^3 f g^2 m^2 x - 30 b^2 d^2 e^3 g^2 m^2 x^2 + 107 b^2 e^4 f^2 m^2 x + 80 b^2 e^4 f g^2 m^2 x^2 - 24 c^2 d^3 e^2 g^2 m^2 x + 2 c^2 d^2 e^2 f^2 m^2 + 72 c^2 d^2 e^2 f g^2 m^2 x + 24 c^2 d^2 e^2 g^2 m^2 x^2 - 58 c^2 d^2 e^3 f^2$

```
*m*x-60*c*d*e^3*f*g*x^2+40*c*e^4*f^2*x^2+18*a*d^2*e^2*g^2*m-94*a*
d*e^3*f*g*m-40*a*d*e^3*g^2*x+154*a*e^4*f^2*m+120*a*e^4*f*g*x-6*b*
d^3*e*g^2*m+36*b*d^2*e^2*f*g*m+30*b*d^2*e^2*g^2*x-47*b*d*e^3*f^2*
m-80*b*d*e^3*f*g*x+60*b*e^4*f^2*x-12*c*d^3*e*f*g*m-24*c*d^3*e*g^2
*x+18*c*d^2*e^2*f^2*m+60*c*d^2*e^2*f*g*x-40*c*d*e^3*f^2*x+40*a*d^
2*e^2*g^2-120*a*d*e^3*f*g+120*a*e^4*f^2-30*b*d^3*e*g^2+80*b*d^2*e
^2*f*g-60*b*d*e^3*f^2+24*c*d^4*g^2-60*c*d^3*e*f*g+40*c*d^2*e^2*f^
2)/e^5/(m^5+15*m^4+85*m^3+225*m^2+274*m+120)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*(g\*x + f)^2\*(e\*x + d)^m,x, algorithm="maxima")

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.313808, size = 1864, normalized size = 8.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*(g\*x + f)^2\*(e\*x + d)^m,x, algorithm="fricas")

```
[Out] (a*d*e^4*f^2*m^4 + (c*e^5*g^2*m^4 + 10*c*e^5*g^2*m^3 + 35*c*e^5*g
^2*m^2 + 50*c*e^5*g^2*m + 24*c*e^5*g^2)*x^5 + (60*c*e^5*f*g + 30*
b*e^5*g^2 + (2*c*e^5*f*g + (c*d*e^4 + b*e^5)*g^2)*m^4 + (22*c*e^5
*f*g + (6*c*d*e^4 + 11*b*e^5)*g^2)*m^3 + (82*c*e^5*f*g + (11*c*d*
e^4 + 41*b*e^5)*g^2)*m^2 + (122*c*e^5*f*g + (6*c*d*e^4 + 61*b*e^5
)*g^2)*m)*x^4 - (2*a*d^2*e^3*f*g + (b*d^2*e^3 - 14*a*d*e^4)*f^2)*
m^3 + (40*c*e^5*f^2 + 80*b*e^5*f*g + 40*a*e^5*g^2 + (c*e^5*f^2 +
2*(c*d*e^4 + b*e^5)*f*g + (b*d*e^4 + a*e^5)*g^2)*m^4 + 4*(3*c*e^5
*f^2 + 2*(2*c*d*e^4 + 3*b*e^5)*f*g - (c*d^2*e^3 - 2*b*d*e^4 - 3*a
*e^5)*g^2)*m^3 + (49*c*e^5*f^2 + 2*(17*c*d*e^4 + 49*b*e^5)*f*g -
(12*c*d^2*e^3 - 17*b*d*e^4 - 49*a*e^5)*g^2)*m^2 + 2*(39*c*e^5*f^2
+ 2*(5*c*d*e^4 + 39*b*e^5)*f*g - (4*c*d^2*e^3 - 5*b*d*e^4 - 39*a
*e^5)*g^2)*m)*x^3 + 20*(2*c*d^3*e^2 - 3*b*d^2*e^3 + 6*a*d*e^4)*f^
2 - 20*(3*c*d^4*e - 4*b*d^3*e^2 + 6*a*d^2*e^3)*f*g + 2*(12*c*d^5
- 15*b*d^4*e + 20*a*d^3*e^2)*g^2 + (2*a*d^3*e^2*g^2 + (2*c*d^3*e^
2 - 12*b*d^2*e^3 + 71*a*d*e^4)*f^2 + 4*(b*d^3*e^2 - 6*a*d^2*e^3)*
f*g)*m^2 + (60*b*e^5*f^2 + 120*a*e^5*f*g + (a*d*e^4*g^2 + (c*d*e^
4 + b*e^5)*f^2 + 2*(b*d*e^4 + a*e^5)*f*g)*m^4 + ((10*c*d*e^4 + 13
*b*e^5)*f^2 - 2*(3*c*d^2*e^3 - 10*b*d*e^4 - 13*a*e^5)*f*g - (3*b*
d^2*e^3 - 10*a*d*e^4)*g^2)*m^3 + ((29*c*d*e^4 + 59*b*e^5)*f^2 - 2
*(18*c*d^2*e^3 - 29*b*d*e^4 - 59*a*e^5)*f*g + (12*c*d^3*e^2 - 18*
b*d^2*e^3 + 29*a*d*e^4)*g^2)*m^2 + ((20*c*d*e^4 + 107*b*e^5)*f^2
- 2*(15*c*d^2*e^3 - 20*b*d*e^4 - 107*a*e^5)*f*g + (12*c*d^3*e^2 -
15*b*d^2*e^3 + 20*a*d*e^4)*g^2)*m)*x^2 + ((18*c*d^3*e^2 - 47*b*d
^2*e^3 + 154*a*d*e^4)*f^2 - 2*(6*c*d^4*e - 18*b*d^3*e^2 + 47*a*d^
2*e^3)*f*g - 6*(b*d^4*e - 3*a*d^3*e^2)*g^2)*m + (120*a*e^5*f^2 +
(2*a*d*e^4*f*g + (b*d*e^4 + a*e^5)*f^2)*m^4 - 2*(a*d^2*e^3*g^2 +
(c*d^2*e^3 - 6*b*d*e^4 - 7*a*e^5)*f^2 + 2*(b*d^2*e^3 - 6*a*d*e^4)
*f*g)*m^3 - ((18*c*d^2*e^3 - 47*b*d*e^4 - 71*a*e^5)*f^2 - 2*(6*c*
d^3*e^2 - 18*b*d^2*e^3 + 47*a*d*e^4)*f*g - 6*(b*d^3*e^2 - 3*a*d^2
*e^3)*g^2)*m^2 - 2*((20*c*d^2*e^3 - 30*b*d*e^4 - 77*a*e^5)*f^2 -
10*(3*c*d^3*e^2 - 4*b*d^2*e^3 + 6*a*d*e^4)*f*g + (12*c*d^4*e - 15
*b*d^3*e^2 + 20*a*d^2*e^3)*g^2)*m)*x*(e*x + d)^m/(e^5*m^5 + 15*e
^5*m^4 + 85*e^5*m^3 + 225*e^5*m^2 + 274*e^5*m + 120*e^5)
```



Sympy [A] time = 52.6749, size = 15734, normalized size = 71.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*m\*(g\*x+f)\*\*2\*(c\*x\*\*2+b\*x+a), x)

[Out] Piecewise((d\*\*m\*(a\*f\*\*2\*x + a\*f\*g\*x\*\*2 + a\*g\*\*2\*x\*\*3/3 + b\*f\*\*2\*x\*\*2/2 + 2\*b\*f\*g\*x\*\*3/3 + b\*g\*\*2\*x\*\*4/4 + c\*f\*\*2\*x\*\*3/3 + c\*f\*g\*x\*\*4/2 + c\*g\*\*2\*x\*\*5/5), Eq(e, 0)), (-a\*d\*\*2\*e\*\*2\*g\*\*2/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) - 2\*a\*d\*e\*\*3\*f\*g/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) - 4\*a\*d\*e\*\*3\*g\*\*2\*x/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) - 3\*a\*e\*\*4\*f\*\*2/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) - 8\*a\*e\*\*4\*f\*g\*x/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) - 6\*a\*e\*\*4\*g\*\*2\*x\*\*2/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) - 3\*b\*d\*\*3\*e\*g\*\*2/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) - 2\*b\*d\*\*2\*e\*\*2\*f\*g/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) - 12\*b\*d\*\*2\*e\*\*2\*g\*\*2\*x/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) - b\*d\*e\*\*3\*f\*\*2/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) - 8\*b\*d\*e\*\*3\*f\*g\*x/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) - 18\*b\*d\*e\*\*3\*g\*\*2\*x\*\*2/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) - 4\*b\*e\*\*4\*f\*\*2\*x/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) - 12\*b\*e\*\*4\*f\*g\*x\*\*2/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) + 25\*c\*d\*\*4\*g\*\*2/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) - 6\*c\*d\*\*3\*e\*f\*g/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) + 48\*c\*d\*\*3\*e\*g\*\*2\*x\*log(d/e + x)/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) + 88\*c\*d\*\*3\*e\*g\*\*2\*x/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) - c\*d\*\*2\*e\*\*2\*f\*\*2/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) - 24\*c\*d\*\*2\*e\*\*2\*f\*g\*x/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) + 72\*c\*d\*\*2\*e\*\*2\*g\*\*2\*x\*\*2\*log(d/e + x)/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) + 108\*c\*d\*\*2\*e\*\*2\*g\*\*2\*x\*\*2/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) - 4\*c\*d\*e\*\*3\*f\*\*2\*x/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) - 36\*c\*d\*e\*\*3\*f\*g\*x\*\*2/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) + 48\*c\*d\*e\*\*3\*g\*\*2\*x\*\*3\*log(d/e + x)/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) + 48\*c\*d\*e\*\*3\*g\*\*2\*x\*\*3/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) - 6\*c\*e\*\*4\*f\*\*2\*x\*\*2/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) - 24\*c\*e\*\*4\*f\*g\*x\*\*3/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4) + 12\*c\*e\*\*4\*g\*\*2\*x\*\*4\*log(d/e + x)/(12\*d\*\*4\*e\*\*5 + 48\*d\*\*3\*e\*\*6\*x + 72\*d\*\*2\*e\*\*7\*x\*\*2 + 48\*d\*e\*\*8\*x\*\*3 + 12\*e\*\*9\*x\*\*4), Eq(m, -5)), (-2\*a\*d\*\*2\*e\*\*4\*f\*\*2/(6\*d\*\*5\*e\*\*5 + 18\*d\*\*4\*e\*\*6\*x + 18\*d\*\*3\*e\*\*7\*x\*\*2 + 6\*d\*\*2\*e\*\*8\*x\*\*3) + 6\*a\*d\*e\*\*5\*f\*g\*x\*\*2/(6\*d\*\*5\*e\*\*5 + 18\*d\*\*4\*e\*\*6\*x + 18\*d\*\*3\*e\*\*7\*x\*\*2 + 6\*d\*\*2\*e\*\*8\*x\*\*3) + 2\*a\*d\*e\*\*5\*g\*\*2\*x\*\*3/(6\*d\*\*5\*e\*\*5 + 18\*d\*\*4\*e\*\*6\*x + 18\*d\*\*3\*e\*\*7\*x\*\*2 + 6\*d\*\*2\*e\*\*8\*x\*\*3) + 2\*a\*e\*\*6\*f\*g\*x\*\*3/(6\*d\*\*5\*e\*\*5 + 18\*d\*\*4\*e\*\*6\*x + 18\*d\*\*3\*e\*\*7\*x\*\*2 + 6\*d\*\*2\*e\*\*8\*x\*\*3) + 6\*b\*d\*\*5\*e\*g\*\*2\*log(d/e + x)/(6\*d\*\*5\*e\*\*5 + 18\*d\*\*4\*e\*\*6\*x + 18\*d\*\*3\*e\*\*7\*x\*\*2 + 6\*d\*\*2\*e\*\*8\*x\*\*3) + 2\*b\*d\*\*5\*e\*g\*\*2/(6\*d\*\*5\*e\*\*5 + 18\*d\*\*4\*e\*\*6\*x + 18\*d\*\*3\*e\*\*7\*x\*\*2 + 6\*d\*\*2\*e\*\*8\*x\*\*3)



$$\begin{aligned}
& *5 + 4*d**2*e**6*x + 2*d*e**7*x**2) + 12*c*d**2*e**3*f*g*x**2/(2* \\
& d**3*e**5 + 4*d**2*e**6*x + 2*d*e**7*x**2) - 4*c*d**2*e**3*g**2*x \\
& **3/(2*d**3*e**5 + 4*d**2*e**6*x + 2*d*e**7*x**2) + 2*c*d*e**4*f* \\
& *2*x**2*log(d/e + x)/(2*d**3*e**5 + 4*d**2*e**6*x + 2*d*e**7*x**2 \\
& ) - 2*c*d*e**4*f**2*x**2/(2*d**3*e**5 + 4*d**2*e**6*x + 2*d*e**7* \\
& x**2) + 4*c*d*e**4*f*g*x**3/(2*d**3*e**5 + 4*d**2*e**6*x + 2*d*e* \\
& **7*x**2) + c*d*e**4*g**2*x**4/(2*d**3*e**5 + 4*d**2*e**6*x + 2*d* \\
& e**7*x**2), Eq(m, -3)), (-12*a*d**2*e**2*g**2*log(d/e + x)/(6*d*e \\
& **5 + 6*e**6*x) - 12*a*d**2*e**2*g**2/(6*d*e**5 + 6*e**6*x) + 12* \\
& a*d*e**3*f*g*log(d/e + x)/(6*d*e**5 + 6*e**6*x) + 12*a*d*e**3*f*g \\
& /(6*d*e**5 + 6*e**6*x) - 12*a*d*e**3*g**2*x*log(d/e + x)/(6*d*e** \\
& 5 + 6*e**6*x) - 6*a*e**4*f**2/(6*d*e**5 + 6*e**6*x) + 12*a*e**4*f \\
& *g*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x) + 6*a*e**4*g**2*x**2/(6*d \\
& *e**5 + 6*e**6*x) + 18*b*d**3*e*g**2*log(d/e + x)/(6*d*e**5 + 6*e \\
& **6*x) + 18*b*d**3*e*g**2/(6*d*e**5 + 6*e**6*x) - 24*b*d**2*e**2* \\
& f*g*log(d/e + x)/(6*d*e**5 + 6*e**6*x) - 24*b*d**2*e**2*f*g/(6*d* \\
& e**5 + 6*e**6*x) + 18*b*d**2*e**2*g**2*x*log(d/e + x)/(6*d*e**5 + \\
& 6*e**6*x) + 6*b*d*e**3*f**2*log(d/e + x)/(6*d*e**5 + 6*e**6*x) + \\
& 6*b*d*e**3*f**2/(6*d*e**5 + 6*e**6*x) - 24*b*d*e**3*f*g*x*log(d/ \\
& e + x)/(6*d*e**5 + 6*e**6*x) - 9*b*d*e**3*g**2*x**2/(6*d*e**5 + 6 \\
& *e**6*x) + 6*b*e**4*f**2*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x) + 1 \\
& 2*b*e**4*f*g*x**2/(6*d*e**5 + 6*e**6*x) + 3*b*e**4*g**2*x**3/(6*d \\
& *e**5 + 6*e**6*x) - 24*c*d**4*g**2*log(d/e + x)/(6*d*e**5 + 6*e** \\
& 6*x) - 24*c*d**4*g**2/(6*d*e**5 + 6*e**6*x) + 36*c*d**3*e*f*g*log \\
& (d/e + x)/(6*d*e**5 + 6*e**6*x) + 36*c*d**3*e*f*g/(6*d*e**5 + 6*e \\
& **6*x) - 24*c*d**3*e*g**2*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x) - \\
& 12*c*d**2*e**2*f**2*log(d/e + x)/(6*d*e**5 + 6*e**6*x) - 12*c*d** \\
& 2*e**2*f**2/(6*d*e**5 + 6*e**6*x) + 36*c*d**2*e**2*f*g*x*log(d/e \\
& + x)/(6*d*e**5 + 6*e**6*x) + 12*c*d**2*e**2*g**2*x**2/(6*d*e**5 + \\
& 6*e**6*x) - 12*c*d*e**3*f**2*x*log(d/e + x)/(6*d*e**5 + 6*e**6*x \\
& ) - 18*c*d*e**3*f*g*x**2/(6*d*e**5 + 6*e**6*x) - 4*c*d*e**3*g**2* \\
& x**3/(6*d*e**5 + 6*e**6*x) + 6*c*e**4*f**2*x**2/(6*d*e**5 + 6*e** \\
& 6*x) + 6*c*e**4*f*g*x**3/(6*d*e**5 + 6*e**6*x) + 2*c*e**4*g**2*x* \\
& **4/(6*d*e**5 + 6*e**6*x), Eq(m, -2)), (a*d**2*g**2*log(d/e + x)/e \\
& **3 - 2*a*d*f*g*log(d/e + x)/e**2 - a*d*g**2*x/e**2 + a*f**2*log( \\
& d/e + x)/e + 2*a*f*g*x/e + a*g**2*x**2/(2*e) - b*d**3*g**2*log(d/ \\
& e + x)/e**4 + 2*b*d**2*f*g*log(d/e + x)/e**3 + b*d**2*g**2*x/e**3 \\
& - b*d*f**2*log(d/e + x)/e**2 - 2*b*d*f*g*x/e**2 - b*d*g**2*x**2/ \\
& (2*e**2) + b*f**2*x/e + b*f*g*x**2/e + b*g**2*x**3/(3*e) + c*d**4 \\
& *g**2*log(d/e + x)/e**5 - 2*c*d**3*f*g*log(d/e + x)/e**4 - c*d**3 \\
& *g**2*x/e**4 + c*d**2*f**2*log(d/e + x)/e**3 + 2*c*d**2*f*g*x/e** \\
& 3 + c*d**2*g**2*x**2/(2*e**3) - c*d*f**2*x/e**2 - c*d*f*g*x**2/e* \\
& **2 - c*d*g**2*x**3/(3*e**2) + c*f**2*x**2/(2*e) + 2*c*f*g*x**3/(3 \\
& *e) + c*g**2*x**4/(4*e), Eq(m, -1)), (2*a*d**3*e**2*g**2*m**2*(d \\
& + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m** \\
& 2 + 274*e**5*m + 120*e**5) + 18*a*d**3*e**2*g**2*m*(d + e*x)**m/( \\
& e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e** \\
& 5*m + 120*e**5) + 40*a*d**3*e**2*g**2*(d + e*x)**m/(e**5*m**5 + 1 \\
& 5*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e** \\
& 5) - 2*a*d**2*e**3*f*g*m**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m** \\
& 4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 24*a* \\
& d**2*e**3*f*g*m**2*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e* \\
& **5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) - 94*a*d**2*e**3 \\
& *f*g*m*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 22 \\
& 5*e**5*m**2 + 274*e**5*m + 120*e**5) - 120*a*d**2*e**3*f*g*(d + e \\
& *x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + \\
& 274*e**5*m + 120*e**5) - 2*a*d**2*e**3*g**2*m**3*x*(d + e*x)**m/ \\
& (e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e* \\
& **5*m + 120*e**5) - 18*a*d**2*e**3*g**2*m**2*x*(d + e*x)**m/(e**5* \\
& m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + \\
& 120*e**5) - 40*a*d**2*e**3*g**2*m*x*(d + e*x)**m/(e**5*m**5 + 15 \\
& *e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5 \\
& ) + a*d*e**4*f**2*m**4*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 8 \\
& 5*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 14*a*d*e** \\
& 4*f**2*m**3*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 \\
& + 225*e**5*m**2 + 274*e**5*m + 120*e**5) + 71*a*d*e**4*f**2*m**2 \\
& *(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5 \\
& *m**2 + 274*e**5*m + 120*e**5) + 154*a*d*e**4*f**2*m*(d + e*x)**m \\
& /(e**5*m**5 + 15*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e \\
& **5*m + 120*e**5) + 120*a*d*e**4*f**2*(d + e*x)**m/(e**5*m**5 + 1 \\
& 5*e**5*m**4 + 85*e**5*m**3 + 225*e**5*m**2 + 274*e**5*m + 120*e** \\
& 5) + 2*a*d*e**4*f*g*m**4*x*(d + e*x)**m/(e**5*m**5 + 15*e**5*m**4
\end{aligned}$$

$$\begin{aligned}
& + 85e^{5m^3} + 225e^{5m^2} + 274e^{5m} + 120e^5) + 24a^d \\
& e^{4f^3g^3x}(d+e^x)^m/(e^{5m^5} + 15e^{5m^4} + 85e^{5m^3} \\
& + 225e^{5m^2} + 274e^{5m} + 120e^5) + 94a^d e^{4f^3g^3} \\
& m^2x(d+e^x)^m/(e^{5m^5} + 15e^{5m^4} + 85e^{5m^3} + 225 \\
& e^{5m^2} + 274e^{5m} + 120e^5) + 120a^d e^{4f^3g^3} x(d+ \\
& e^x)^m/(e^{5m^5} + 15e^{5m^4} + 85e^{5m^3} + 225e^{5m^2} \\
& + 274e^{5m} + 120e^5) + a^d e^{4g^2m^4x^2}(d+e^x)^m/( \\
& e^{5m^5} + 15e^{5m^4} + 85e^{5m^3} + 225e^{5m^2} + 274e^{5m} \\
& + 120e^5) + 10a^d e^{4g^2m^3x^2}(d+e^x)^m/(e^{5m^5} \\
& + 15e^{5m^4} + 85e^{5m^3} + 225e^{5m^2} + 274e^{5m} + \\
& 120e^5) + 29a^d e^{4g^2m^2x^2}(d+e^x)^m/(e^{5m^5} + \\
& 15e^{5m^4} + 85e^{5m^3} + 225e^{5m^2} + 274e^{5m} + 120e^ \\
& ^5) + 20a^d e^{4g^2m^2x^2}(d+e^x)^m/(e^{5m^5} + 15e^{5m^4} \\
& + 85e^{5m^3} + 225e^{5m^2} + 274e^{5m} + 120e^5) + a^e \\
& ^5 f^2m^4x(d+e^x)^m/(e^{5m^5} + 15e^{5m^4} + 85e^{5m^3} \\
& + 225e^{5m^2} + 274e^{5m} + 120e^5) + 14a^e e^{5f^2m^3} \\
& x^3(d+e^x)^m/(e^{5m^5} + 15e^{5m^4} + 85e^{5m^3} + 225e^{5m^2} \\
& + 274e^{5m} + 120e^5) + 71a^e e^{5f^2m^2x}(d+e \\
& ^x)^m/(e^{5m^5} + 15e^{5m^4} + 85e^{5m^3} + 225e^{5m^2} + \\
& 274e^{5m} + 120e^5) + 154a^e e^{5f^2m^2x}(d+e^x)^m/(e^{5m^5} \\
& + 15e^{5m^4} + 85e^{5m^3} + 225e^{5m^2} + 274e^{5m} + \\
& 120e^5) + 120a^e e^{5f^2m^2x}(d+e^x)^m/(e^{5m^5} + 15e^{5m^4} \\
& + 85e^{5m^3} + 225e^{5m^2} + 274e^{5m} + 120e^5) + 2^ \\
& a^e e^{5f^2g^4x^2}(d+e^x)^m/(e^{5m^5} + 15e^{5m^4} + 85e^{5m^3} \\
& + 225e^{5m^2} + 274e^{5m} + 120e^5) + 26a^e e^{5f^2g^3} \\
& x^2(d+e^x)^m/(e^{5m^5} + 15e^{5m^4} + 85e^{5m^3} \\
& + 225e^{5m^2} + 274e^{5m} + 120e^5) + 118a^e e^{5f^2g^2} x^2 \\
& (d+e^x)^m/(e^{5m^5} + 15e^{5m^4} + 85e^{5m^3} + 225e^{5m^2} \\
& + 274e^{5m} + 120e^5) + 214a^e e^{5f^2g^2} x^2(d+e^ \\
& ^x)^m/(e^{5m^5} + 15e^{5m^4} + 85e^{5m^3} + 225e^{5m^2} + \\
& 274e^{5m} + 120e^5) + 120a^e e^{5f^2g^2} x^2(d+e^x)^m/(e^{5m^5} \\
& + 15e^{5m^4} + 85e^{5m^3} + 225e^{5m^2} + 274e^{5m} + \\
& 120e^5) + a^e e^{5g^2m^4x^3}(d+e^x)^m/(e^{5m^5} + 15e^{5m^4} \\
& + 85e^{5m^3} + 225e^{5m^2} + 274e^{5m} + 120e^5) + \\
& 12a^e e^{5g^2m^3x^3}(d+e^x)^m/(e^{5m^5} + 15e^{5m^4} \\
& + 85e^{5m^3} + 225e^{5m^2} + 274e^{5m} + 120e^5) + 49a^e \\
& e^{5g^2m^2x^3}(d+e^x)^m/(e^{5m^5} + 15e^{5m^4} + 85e^{5m^3} \\
& + 225e^{5m^2} + 274e^{5m} + 120e^5) + 78a^e e^{5g^2m^2} \\
& x^3(d+e^x)^m/(e^{5m^5} + 15e^{5m^4} + 85e^{5m^3} + 225 \\
& e^{5m^2} + 274e^{5m} + 120e^5) + 40a^e e^{5g^2x^3}(d+e \\
& ^x)^m/(e^{5m^5} + 15e^{5m^4} + 85e^{5m^3} + 225e^{5m^2} + \\
& 274e^{5m} + 120e^5) - 6b^d e^{4e^2g^2m}(d+e^x)^m/(e^{5m^5} \\
& + 15e^{5m^4} + 85e^{5m^3} + 225e^{5m^2} + 274e^{5m} + 1 \\
& 20e^5) - 30b^d e^{4e^2g^2}(d+e^x)^m/(e^{5m^5} + 15e^{5m^4} \\
& + 85e^{5m^3} + 225e^{5m^2} + 274e^{5m} + 120e^5) + 4b^d \\
& e^{3e^2f^2g^2m^2}(d+e^x)^m/(e^{5m^5} + 15e^{5m^4} + 85e^{5m^3} \\
& + 225e^{5m^2} + 274e^{5m} + 120e^5) + 36b^d e^{3e^2f^2g^2} \\
& (d+e^x)^m/(e^{5m^5} + 15e^{5m^4} + 85e^{5m^3} + 225 \\
& e^{5m^2} + 274e^{5m} + 120e^5) + 80b^d e^{3e^2f^2g^2}(d+e^x \\
& )^m/(e^{5m^5} + 15e^{5m^4} + 85e^{5m^3} + 225e^{5m^2} + 2 \\
& 74e^{5m} + 120e^5) + 6b^d e^{3e^2g^2m^2x^2}(d+e^x)^m/(e \\
& ^{5m^5} + 15e^{5m^4} + 85e^{5m^3} + 225e^{5m^2} + 274e^{5m} \\
& + 120e^5) + 30b^d e^{3e^2g^2m^2x^2}(d+e^x)^m/(e^{5m^5} \\
& + 15e^{5m^4} + 85e^{5m^3} + 225e^{5m^2} + 274e^{5m} + 120 \\
& e^5) - b^d e^{2e^3f^2m^3}(d+e^x)^m/(e^{5m^5} + 15e^{5m^4} \\
& + 85e^{5m^3} + 225e^{5m^2} + 274e^{5m} + 120e^5) - 12^ \\
& b^d e^{2e^3f^2m^2}(d+e^x)^m/(e^{5m^5} + 15e^{5m^4} + 85 \\
& e^{5m^3} + 225e^{5m^2} + 274e^{5m} + 120e^5) - 47b^d e^{2e^3f^2m} \\
& (d+e^x)^m/(e^{5m^5} + 15e^{5m^4} + 85e^{5m^3} \\
& + 225e^{5m^2} + 274e^{5m} + 120e^5) - 60b^d e^{2e^3f^2m^2}(d \\
& +e^x)^m/(e^{5m^5} + 15e^{5m^4} + 85e^{5m^3} + 225e^{5m^2} \\
& + 274e^{5m} + 120e^5) - 4b^d e^{2e^3f^2g^2m^3x^2}(d+e^x)^ \\
& ^m/(e^{5m^5} + 15e^{5m^4} + 85e^{5m^3} + 225e^{5m^2} + 274 \\
& e^{5m} + 120e^5) - 36b^d e^{2e^3f^2g^2m^2x^2}(d+e^x)^m/(e^{5m^5} \\
& + 15e^{5m^4} + 85e^{5m^3} + 225e^{5m^2} + 274e^{5m} \\
& + 120e^5) - 80b^d e^{2e^3f^2g^2m^2x^2}(d+e^x)^m/(e^{5m^5} + 1 \\
& 5e^{5m^4} + 85e^{5m^3} + 225e^{5m^2} + 274e^{5m} + 120e^ \\
& ^5) - 3b^d e^{2e^3g^2m^3x^2}(d+e^x)^m/(e^{5m^5} + 15e^{5m^4} \\
& + 85e^{5m^3} + 225e^{5m^2} + 274e^{5m} + 120e^5) - \\
& 18b^d e^{2e^3g^2m^2x^2}(d+e^x)^m/(e^{5m^5} + 15e^{5m^4} \\
& + 85e^{5m^3} + 225e^{5m^2} + 274e^{5m} + 120e^5) - 15 \\
& b^d e^{2e^3g^2m^2x^2}(d+e^x)^m/(e^{5m^5} + 15e^{5m^4} +
\end{aligned}$$



$$\begin{aligned}
& 4 + 85e^{5m^3} + 225e^{5m^2} + 274e^{5m} + 120e^5) - 18c^* \\
& d^{*2}e^{*3}f^{*2}m^{*2}x^*(d + e^*x)^{**m}/(e^{*5}m^{*5} + 15e^{*5}m^{*4} + 85 \\
& *e^{*5}m^{*3} + 225e^{*5}m^{*2} + 274e^{*5}m + 120e^{*5}) - 40c^*d^{*2}e \\
& **3f^{*2}m^*x^*(d + e^*x)^{**m}/(e^{*5}m^{*5} + 15e^{*5}m^{*4} + 85e^{*5}m^{*3} + 225e^{*5}m^{*2} \\
& + 274e^{*5}m + 120e^{*5}) - 6c^*d^{*2}e^{*3}f^*g^*m^* \\
& *3x^{*2}(d + e^*x)^{**m}/(e^{*5}m^{*5} + 15e^{*5}m^{*4} + 85e^{*5}m^{*3} + 225e^{*5}m^{*2} \\
& + 274e^{*5}m + 120e^{*5}) - 36c^*d^{*2}e^{*3}f^*g^*m^{*2}x \\
& **2(d + e^*x)^{**m}/(e^{*5}m^{*5} + 15e^{*5}m^{*4} + 85e^{*5}m^{*3} + 225e^{*5}m^{*2} \\
& + 274e^{*5}m + 120e^{*5}) - 30c^*d^{*2}e^{*3}f^*g^*m^*x^{*2}(d \\
& + e^*x)^{**m}/(e^{*5}m^{*5} + 15e^{*5}m^{*4} + 85e^{*5}m^{*3} + 225e^{*5}m^{*2} \\
& + 274e^{*5}m + 120e^{*5}) - 4c^*d^{*2}e^{*3}g^{*2}m^{*3}x^{*3}(d + e^* \\
& x)^{**m}/(e^{*5}m^{*5} + 15e^{*5}m^{*4} + 85e^{*5}m^{*3} + 225e^{*5}m^{*2} + \\
& 274e^{*5}m + 120e^{*5}) - 12c^*d^{*2}e^{*3}g^{*2}m^{*2}x^{*3}(d + e^*x)^* \\
& **m/(e^{*5}m^{*5} + 15e^{*5}m^{*4} + 85e^{*5}m^{*3} + 225e^{*5}m^{*2} + 274 \\
& *e^{*5}m + 120e^{*5}) - 8c^*d^{*2}e^{*3}g^{*2}m^*x^{*3}(d + e^*x)^{**m}/(e^{*5} \\
& m^{*5} + 15e^{*5}m^{*4} + 85e^{*5}m^{*3} + 225e^{*5}m^{*2} + 274e^{*5}m \\
& + 120e^{*5}) + c^*d^*e^{*4}f^{*2}m^{*4}x^{*2}(d + e^*x)^{**m}/(e^{*5}m^{*5} + \\
& 15e^{*5}m^{*4} + 85e^{*5}m^{*3} + 225e^{*5}m^{*2} + 274e^{*5}m + 120e^* \\
& *5) + 10c^*d^*e^{*4}f^{*2}m^{*3}x^{*2}(d + e^*x)^{**m}/(e^{*5}m^{*5} + 15e^{*5} \\
& m^{*4} + 85e^{*5}m^{*3} + 225e^{*5}m^{*2} + 274e^{*5}m + 120e^{*5}) + \\
& 29c^*d^*e^{*4}f^{*2}m^{*2}x^{*2}(d + e^*x)^{**m}/(e^{*5}m^{*5} + 15e^{*5}m^{*4} \\
& + 85e^{*5}m^{*3} + 225e^{*5}m^{*2} + 274e^{*5}m + 120e^{*5}) + 20c^*d \\
& *e^{*4}f^{*2}m^*x^{*2}(d + e^*x)^{**m}/(e^{*5}m^{*5} + 15e^{*5}m^{*4} + 85e^{*5} \\
& m^{*3} + 225e^{*5}m^{*2} + 274e^{*5}m + 120e^{*5}) + 2c^*d^*e^{*4}f^*g^* \\
& m^{*4}x^{*3}(d + e^*x)^{**m}/(e^{*5}m^{*5} + 15e^{*5}m^{*4} + 85e^{*5}m^{*3} + \\
& 225e^{*5}m^{*2} + 274e^{*5}m + 120e^{*5}) + 16c^*d^*e^{*4}f^*g^*m^{*3}x^* \\
& *3(d + e^*x)^{**m}/(e^{*5}m^{*5} + 15e^{*5}m^{*4} + 85e^{*5}m^{*3} + 225e^* \\
& *5m^{*2} + 274e^{*5}m + 120e^{*5}) + 34c^*d^*e^{*4}f^*g^*m^{*2}x^{*3}(d + \\
& e^*x)^{**m}/(e^{*5}m^{*5} + 15e^{*5}m^{*4} + 85e^{*5}m^{*3} + 225e^{*5}m^{*2} \\
& + 274e^{*5}m + 120e^{*5}) + 20c^*d^*e^{*4}f^*g^*m^*x^{*3}(d + e^*x)^{**m}/( \\
& e^{*5}m^{*5} + 15e^{*5}m^{*4} + 85e^{*5}m^{*3} + 225e^{*5}m^{*2} + 274e^{*5} \\
& m + 120e^{*5}) + c^*d^*e^{*4}g^{*2}m^{*4}x^{*4}(d + e^*x)^{**m}/(e^{*5}m^{*5} \\
& + 15e^{*5}m^{*4} + 85e^{*5}m^{*3} + 225e^{*5}m^{*2} + 274e^{*5}m + 120 \\
& *e^{*5}) + 6c^*d^*e^{*4}g^{*2}m^{*3}x^{*4}(d + e^*x)^{**m}/(e^{*5}m^{*5} + 15e \\
& **5m^{*4} + 85e^{*5}m^{*3} + 225e^{*5}m^{*2} + 274e^{*5}m + 120e^{*5}) \\
& + 11c^*d^*e^{*4}g^{*2}m^{*2}x^{*4}(d + e^*x)^{**m}/(e^{*5}m^{*5} + 15e^{*5}m^{*4} \\
& + 85e^{*5}m^{*3} + 225e^{*5}m^{*2} + 274e^{*5}m + 120e^{*5}) + 6c^* \\
& d^*e^{*4}g^{*2}m^*x^{*4}(d + e^*x)^{**m}/(e^{*5}m^{*5} + 15e^{*5}m^{*4} + 85e^* \\
& *5m^{*3} + 225e^{*5}m^{*2} + 274e^{*5}m + 120e^{*5}) + c^*e^{*5}f^{*2}m^* \\
& *4x^{*3}(d + e^*x)^{**m}/(e^{*5}m^{*5} + 15e^{*5}m^{*4} + 85e^{*5}m^{*3} + 2 \\
& 25e^{*5}m^{*2} + 274e^{*5}m + 120e^{*5}) + 12c^*e^{*5}f^{*2}m^{*3}x^{*3} \\
& (d + e^*x)^{**m}/(e^{*5}m^{*5} + 15e^{*5}m^{*4} + 85e^{*5}m^{*3} + 225e^{*5} \\
& m^{*2} + 274e^{*5}m + 120e^{*5}) + 49c^*e^{*5}f^{*2}m^{*2}x^{*3}(d + e^*x \\
& )^{**m}/(e^{*5}m^{*5} + 15e^{*5}m^{*4} + 85e^{*5}m^{*3} + 225e^{*5}m^{*2} + 2 \\
& 74e^{*5}m + 120e^{*5}) + 78c^*e^{*5}f^{*2}m^*x^{*3}(d + e^*x)^{**m}/(e^{*5} \\
& m^{*5} + 15e^{*5}m^{*4} + 85e^{*5}m^{*3} + 225e^{*5}m^{*2} + 274e^{*5}m + \\
& 120e^{*5}) + 40c^*e^{*5}f^{*2}x^{*3}(d + e^*x)^{**m}/(e^{*5}m^{*5} + 15e^{*5} \\
& m^{*4} + 85e^{*5}m^{*3} + 225e^{*5}m^{*2} + 274e^{*5}m + 120e^{*5}) + \\
& 2c^*e^{*5}f^*g^*m^{*4}x^{*4}(d + e^*x)^{**m}/(e^{*5}m^{*5} + 15e^{*5}m^{*4} + 8 \\
& 5e^{*5}m^{*3} + 225e^{*5}m^{*2} + 274e^{*5}m + 120e^{*5}) + 22c^*e^{*5} \\
& f^*g^*m^{*3}x^{*4}(d + e^*x)^{**m}/(e^{*5}m^{*5} + 15e^{*5}m^{*4} + 85e^{*5}m^* \\
& *3 + 225e^{*5}m^{*2} + 274e^{*5}m + 120e^{*5}) + 82c^*e^{*5}f^*g^*m^{*2} \\
& x^{*4}(d + e^*x)^{**m}/(e^{*5}m^{*5} + 15e^{*5}m^{*4} + 85e^{*5}m^{*3} + 225e^* \\
& *5m^{*2} + 274e^{*5}m + 120e^{*5}) + 122c^*e^{*5}f^*g^*m^*x^{*4}(d + e \\
& *x)^{**m}/(e^{*5}m^{*5} + 15e^{*5}m^{*4} + 85e^{*5}m^{*3} + 225e^{*5}m^{*2} + \\
& 274e^{*5}m + 120e^{*5}) + 60c^*e^{*5}f^*g^*x^{*4}(d + e^*x)^{**m}/(e^{*5}m \\
& **5 + 15e^{*5}m^{*4} + 85e^{*5}m^{*3} + 225e^{*5}m^{*2} + 274e^{*5}m + \\
& 120e^{*5}) + c^*e^{*5}g^{*2}m^{*4}x^{*5}(d + e^*x)^{**m}/(e^{*5}m^{*5} + 15e^* \\
& *5m^{*4} + 85e^{*5}m^{*3} + 225e^{*5}m^{*2} + 274e^{*5}m + 120e^{*5}) + \\
& 10c^*e^{*5}g^{*2}m^{*3}x^{*5}(d + e^*x)^{**m}/(e^{*5}m^{*5} + 15e^{*5}m^{*4} \\
& + 85e^{*5}m^{*3} + 225e^{*5}m^{*2} + 274e^{*5}m + 120e^{*5}) + 35c^*e^* \\
& *5g^{*2}m^{*2}x^{*5}(d + e^*x)^{**m}/(e^{*5}m^{*5} + 15e^{*5}m^{*4} + 85e^{*5} \\
& m^{*3} + 225e^{*5}m^{*2} + 274e^{*5}m + 120e^{*5}) + 50c^*e^{*5}g^{*2} \\
& m^*x^{*5}(d + e^*x)^{**m}/(e^{*5}m^{*5} + 15e^{*5}m^{*4} + 85e^{*5}m^{*3} + 22 \\
& 5e^{*5}m^{*2} + 274e^{*5}m + 120e^{*5}) + 24c^*e^{*5}g^{*2}x^{*5}(d + e \\
& *x)^{**m}/(e^{*5}m^{*5} + 15e^{*5}m^{*4} + 85e^{*5}m^{*3} + 225e^{*5}m^{*2} + \\
& 274e^{*5}m + 120e^{*5}), True))
\end{aligned}$$

GIAC/XCAS [A] time = 0.269015, size = 1, normalized size = 0.

*Done*

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)*(g*x + f)^2*(e*x + d)^m,x, algorithm="giac")
```

```
[Out] Done
```







$$f - (3*c*d^2*e^2 - 4*b*d*e^3 - 19*a*e^4)*g)*m)*x^2 + 4*(2*c*d^3*e - 3*b*d^2*e^2 + 6*a*d*e^3)*f - 2*(3*c*d^4 - 4*b*d^3*e + 6*a*d^2*e^2)*g + ((2*c*d^3*e - 7*b*d^2*e^2 + 26*a*d*e^3)*f + (2*b*d^3*e - 7*a*d^2*e^2)*g)*m + (24*a*e^4*f + (a*d*e^3*g + (b*d*e^3 + a*e^4)*f)*m^3 - ((2*c*d^2*e^2 - 7*b*d*e^3 - 9*a*e^4)*f + (2*b*d^2*e^2 - 7*a*d*e^3)*g)*m^2 - 2*((4*c*d^2*e^2 - 6*b*d*e^3 - 13*a*e^4)*f - (3*c*d^3*e - 4*b*d^2*e^2 + 6*a*d*e^3)*g)*m)*x)*(e*x + d)^m/(e^4*m^4 + 10*e^4*m^3 + 35*e^4*m^2 + 50*e^4*m + 24*e^4)$$

**Sympy [A]** time = 16.9795, size = 5846, normalized size = 40.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*m\*(g\*x+f)\*(c\*x\*\*2+b\*x+a),x)

[Out] Piecewise(((d\*\*m\*(a\*f\*x + a\*g\*x\*\*2/2 + b\*f\*x\*\*2/2 + b\*g\*x\*\*3/3 + c\*f\*x\*\*3/3 + c\*g\*x\*\*4/4), Eq(e, 0)), (-2\*a\*d\*\*2\*e\*\*3\*f/(6\*d\*\*5\*e\*\*4 + 18\*d\*\*4\*e\*\*5\*x + 18\*d\*\*3\*e\*\*6\*x\*\*2 + 6\*d\*\*2\*e\*\*7\*x\*\*3) + 3\*a\*d\*e\*\*4\*g\*x\*\*2/(6\*d\*\*5\*e\*\*4 + 18\*d\*\*4\*e\*\*5\*x + 18\*d\*\*3\*e\*\*6\*x\*\*2 + 6\*d\*\*2\*e\*\*7\*x\*\*3) + a\*e\*\*5\*g\*x\*\*3/(6\*d\*\*5\*e\*\*4 + 18\*d\*\*4\*e\*\*5\*x + 18\*d\*\*3\*e\*\*6\*x\*\*2 + 6\*d\*\*2\*e\*\*7\*x\*\*3) + 3\*b\*d\*e\*\*4\*f\*x\*\*2/(6\*d\*\*5\*e\*\*4 + 18\*d\*\*4\*e\*\*5\*x + 18\*d\*\*3\*e\*\*6\*x\*\*2 + 6\*d\*\*2\*e\*\*7\*x\*\*3) + 2\*b\*d\*e\*\*4\*g\*x\*\*3/(6\*d\*\*5\*e\*\*4 + 18\*d\*\*4\*e\*\*5\*x + 18\*d\*\*3\*e\*\*6\*x\*\*2 + 6\*d\*\*2\*e\*\*7\*x\*\*3) + b\*e\*\*5\*f\*x\*\*3/(6\*d\*\*5\*e\*\*4 + 18\*d\*\*4\*e\*\*5\*x + 18\*d\*\*3\*e\*\*6\*x\*\*2 + 6\*d\*\*2\*e\*\*7\*x\*\*3) + 6\*c\*d\*\*5\*g\*log(d/e + x)/(6\*d\*\*5\*e\*\*4 + 18\*d\*\*4\*e\*\*5\*x + 18\*d\*\*3\*e\*\*6\*x\*\*2 + 6\*d\*\*2\*e\*\*7\*x\*\*3) + 2\*c\*d\*\*5\*g/(6\*d\*\*5\*e\*\*4 + 18\*d\*\*4\*e\*\*5\*x + 18\*d\*\*3\*e\*\*6\*x\*\*2 + 6\*d\*\*2\*e\*\*7\*x\*\*3) + 18\*c\*d\*\*4\*e\*g\*x\*log(d/e + x)/(6\*d\*\*5\*e\*\*4 + 18\*d\*\*4\*e\*\*5\*x + 18\*d\*\*3\*e\*\*6\*x\*\*2 + 6\*d\*\*2\*e\*\*7\*x\*\*3) + 18\*c\*d\*\*3\*e\*\*2\*g\*x\*\*2\*log(d/e + x)/(6\*d\*\*5\*e\*\*4 + 18\*d\*\*4\*e\*\*5\*x + 18\*d\*\*3\*e\*\*6\*x\*\*2 + 6\*d\*\*2\*e\*\*7\*x\*\*3) - 9\*c\*d\*\*3\*e\*\*2\*g\*x\*\*2/(6\*d\*\*5\*e\*\*4 + 18\*d\*\*4\*e\*\*5\*x + 18\*d\*\*3\*e\*\*6\*x\*\*2 + 6\*d\*\*2\*e\*\*7\*x\*\*3) + 6\*c\*d\*\*2\*e\*\*3\*g\*x\*\*3\*log(d/e + x)/(6\*d\*\*5\*e\*\*4 + 18\*d\*\*4\*e\*\*5\*x + 18\*d\*\*3\*e\*\*6\*x\*\*2 + 6\*d\*\*2\*e\*\*7\*x\*\*3) - 9\*c\*d\*\*2\*e\*\*3\*g\*x\*\*3/(6\*d\*\*5\*e\*\*4 + 18\*d\*\*4\*e\*\*5\*x + 18\*d\*\*3\*e\*\*6\*x\*\*2 + 6\*d\*\*2\*e\*\*7\*x\*\*3) + 2\*c\*d\*e\*\*4\*f\*x\*\*3/(6\*d\*\*5\*e\*\*4 + 18\*d\*\*4\*e\*\*5\*x + 18\*d\*\*3\*e\*\*6\*x\*\*2 + 6\*d\*\*2\*e\*\*7\*x\*\*3), Eq(m, -4)), (-a\*d\*e\*\*3\*f/(2\*d\*\*3\*e\*\*4 + 4\*d\*\*2\*e\*\*5\*x + 2\*d\*e\*\*6\*x\*\*2) + a\*e\*\*4\*g\*x\*\*2/(2\*d\*\*3\*e\*\*4 + 4\*d\*\*2\*e\*\*5\*x + 2\*d\*e\*\*6\*x\*\*2) + 2\*b\*d\*\*3\*e\*g\*log(d/e + x)/(2\*d\*\*3\*e\*\*4 + 4\*d\*\*2\*e\*\*5\*x + 2\*d\*e\*\*6\*x\*\*2) + b\*d\*\*3\*e\*g/(2\*d\*\*3\*e\*\*4 + 4\*d\*\*2\*e\*\*5\*x + 2\*d\*e\*\*6\*x\*\*2) + 4\*b\*d\*\*2\*e\*\*2\*g\*x\*log(d/e + x)/(2\*d\*\*3\*e\*\*4 + 4\*d\*\*2\*e\*\*5\*x + 2\*d\*e\*\*6\*x\*\*2) + 2\*b\*d\*e\*\*3\*g\*x\*\*2\*log(d/e + x)/(2\*d\*\*3\*e\*\*4 + 4\*d\*\*2\*e\*\*5\*x + 2\*d\*e\*\*6\*x\*\*2) - 2\*b\*d\*e\*\*3\*g\*x\*\*2/(2\*d\*\*3\*e\*\*4 + 4\*d\*\*2\*e\*\*5\*x + 2\*d\*e\*\*6\*x\*\*2) + b\*e\*\*4\*f\*x\*\*2/(2\*d\*\*3\*e\*\*4 + 4\*d\*\*2\*e\*\*5\*x + 2\*d\*e\*\*6\*x\*\*2) - 6\*c\*d\*\*4\*g\*log(d/e + x)/(2\*d\*\*3\*e\*\*4 + 4\*d\*\*2\*e\*\*5\*x + 2\*d\*e\*\*6\*x\*\*2) - 3\*c\*d\*\*4\*g/(2\*d\*\*3\*e\*\*4 + 4\*d\*\*2\*e\*\*5\*x + 2\*d\*e\*\*6\*x\*\*2) + 2\*c\*d\*\*3\*e\*f\*log(d/e + x)/(2\*d\*\*3\*e\*\*4 + 4\*d\*\*2\*e\*\*5\*x + 2\*d\*e\*\*6\*x\*\*2) + c\*d\*\*3\*e\*f/(2\*d\*\*3\*e\*\*4 + 4\*d\*\*2\*e\*\*5\*x + 2\*d\*e\*\*6\*x\*\*2) - 12\*c\*d\*\*3\*e\*g\*x\*log(d/e + x)/(2\*d\*\*3\*e\*\*4 + 4\*d\*\*2\*e\*\*5\*x + 2\*d\*e\*\*6\*x\*\*2) + 4\*c\*d\*\*2\*e\*\*2\*f\*x\*log(d/e + x)/(2\*d\*\*3\*e\*\*4 + 4\*d\*\*2\*e\*\*5\*x + 2\*d\*e\*\*6\*x\*\*2) - 6\*c\*d\*\*2\*e\*\*2\*g\*x\*\*2\*log(d/e + x)/(2\*d\*\*3\*e\*\*4 + 4\*d\*\*2\*e\*\*5\*x + 2\*d\*e\*\*6\*x\*\*2) + 6\*c\*d\*\*2\*e\*\*2\*g\*x\*\*2/(2\*d\*\*3\*e\*\*4 + 4\*d\*\*2\*e\*\*5\*x + 2\*d\*e\*\*6\*x\*\*2) + 2\*c\*d\*e\*\*3\*f\*x\*\*2\*log(d/e + x)/(2\*d\*\*3\*e\*\*4 + 4\*d\*\*2\*e\*\*5\*x + 2\*d\*e\*\*6\*x\*\*2) - 2\*c\*d\*e\*\*3\*f\*x\*\*2/(2\*d\*\*3\*e\*\*4 + 4\*d\*\*2\*e\*\*5\*x + 2\*d\*e\*\*6\*x\*\*2) + 2\*c\*d\*e\*\*3\*g\*x\*\*3/(2\*d\*\*3\*e\*\*4 + 4\*d\*\*2\*e\*\*5\*x + 2\*d\*e\*\*6\*x\*\*2), Eq(m, -3)), (2\*a\*d\*e\*\*2\*g\*log(d/e + x)/(2\*d\*\*4 + 2\*e\*\*5\*x) + 2\*a\*d\*e\*\*2\*g/(2\*d\*\*4 + 2\*e\*\*5\*x) - 2\*a\*e\*\*3\*f/(2\*d\*\*4 + 2\*e\*\*5\*x) + 2\*a\*e\*\*3\*g\*x\*log(d/e + x)/(2\*d\*\*4 + 2\*e\*\*5\*x) - 4\*b\*d\*\*2\*e\*g\*log(d/e + x)/(2\*d\*\*4 + 2\*e\*\*5\*x) - 4\*b\*d\*\*2\*e\*g/(2\*d\*\*4 + 2\*e\*\*5\*x) + 2\*b\*d\*e\*\*2\*f\*log(d/e + x)/(2\*d\*\*4 + 2\*e\*\*5\*x) + 2\*b\*d\*e\*\*2\*f/(2\*d\*\*4 + 2\*e\*\*5\*x) - 4\*b\*d\*e\*\*2\*g\*x\*log(d/e + x)/(2\*d\*\*4 + 2\*e\*\*5\*x) + 2\*b\*e\*\*3\*f\*x\*log(d/e + x)/(2\*d\*\*4 + 2\*e\*\*5\*x) + 2\*b\*e\*\*3\*g\*x\*\*2/(2\*d\*\*4 + 2\*e\*\*5\*x) + 6\*c\*d\*\*3\*g\*log

$$\begin{aligned}
& g(d/e + x)/(2*d*e^{**4} + 2*e^{**5}*x) + 6*c*d^{**3}*g/(2*d*e^{**4} + 2*e^{**5}*x) - 4*c*d^{**2}*e*f*\log(d/e + x)/(2*d*e^{**4} + 2*e^{**5}*x) - 4*c*d^{**2}*e \\
& *f/(2*d*e^{**4} + 2*e^{**5}*x) + 6*c*d^{**2}*e*g*x*\log(d/e + x)/(2*d*e^{**4} + 2*e^{**5}*x) - 4*c*d*e^{**2}*f*x*\log(d/e + x)/(2*d*e^{**4} + 2*e^{**5}*x) - \\
& 3*c*d*e^{**2}*g*x^{**2}/(2*d*e^{**4} + 2*e^{**5}*x) + 2*c*e^{**3}*f*x^{**2}/(2*d*e^{**4} + 2*e^{**5}*x) + c*e^{**3}*g*x^{**3}/(2*d*e^{**4} + 2*e^{**5}*x), \text{Eq}(m, -2)) \\
& , (-a*d*g*\log(d/e + x)/e^{**2} + a*f*\log(d/e + x)/e + a*g*x/e + b*d^{**2}*g*\log(d/e + x)/e^{**3} - b*d*f*\log(d/e + x)/e^{**2} - b*d*g*x/e^{**2} + \\
& b*f*x/e + b*g*x^{**2}/(2*e) - c*d^{**3}*g*\log(d/e + x)/e^{**4} + c*d^{**2}*f*\log(d/e + x)/e^{**3} + c*d^{**2}*g*x/e^{**3} - c*d*f*x/e^{**2} - c*d*g*x^{**2}/ \\
& (2*e^{**2}) + c*f*x^{**2}/(2*e) + c*g*x^{**3}/(3*e), \text{Eq}(m, -1)), (-a*d^{**2}*e^{**2}*g*m^{**2}*(d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + 35*e^{**4}*m^{**2} \\
& + 50*e^{**4}*m + 24*e^{**4}) - 7*a*d^{**2}*e^{**2}*g*m*(d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + 35*e^{**4}*m^{**2} + 50*e^{**4}*m + 24*e^{**4}) - 12*a*d^{**2} \\
& *e^{**2}*g*(d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + 35*e^{**4}*m^{**2} + 50*e^{**4}*m + 24*e^{**4}) + a*d*e^{**3}*f*m^{**3}*(d + e*x)^{**m}/(e^{**4}*m^{**4} + \\
& 10*e^{**4}*m^{**3} + 35*e^{**4}*m^{**2} + 50*e^{**4}*m + 24*e^{**4}) + 9*a*d*e^{**3}*f*m^{**2}*(d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + 35*e^{**4}*m^{**2} + 50 \\
& *e^{**4}*m + 24*e^{**4}) + 26*a*d*e^{**3}*f*m*(d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + 35*e^{**4}*m^{**2} + 50*e^{**4}*m + 24*e^{**4}) + 24*a*d*e^{**3}*f* \\
& (d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + 35*e^{**4}*m^{**2} + 50*e^{**4}*m + 24*e^{**4}) + a*d*e^{**3}*g*m^{**3}*x*(d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4} \\
& *m^{**3} + 35*e^{**4}*m^{**2} + 50*e^{**4}*m + 24*e^{**4}) + 7*a*d*e^{**3}*g*m^{**2}*x*(d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + 35*e^{**4}*m^{**2} + 50*e^{**4} \\
& m + 24*e^{**4}) + 12*a*d*e^{**3}*g*m*x*(d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4} \\
& *m^{**3} + 35*e^{**4}*m^{**2} + 50*e^{**4}*m + 24*e^{**4}) + a*e^{**4}*f*m^{**3}*x*(d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + 35*e^{**4}*m^{**2} + 50*e^{**4}*m + \\
& 24*e^{**4}) + 9*a*e^{**4}*f*m^{**2}*x*(d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4}*m \\
& **3 + 35*e^{**4}*m^{**2} + 50*e^{**4}*m + 24*e^{**4}) + 26*a*e^{**4}*f*m*x*(d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + 35*e^{**4}*m^{**2} + 50*e^{**4}*m + 24 \\
& *e^{**4}) + 24*a*e^{**4}*f*x*(d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + 3 \\
& 5*e^{**4}*m^{**2} + 50*e^{**4}*m + 24*e^{**4}) + a*e^{**4}*g*m^{**3}*x^{**2}*(d + e*x) \\
& **m/(e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + 35*e^{**4}*m^{**2} + 50*e^{**4}*m + 24*e^{**4}) + 8*a*e^{**4}*g*m^{**2}*x^{**2}*(d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} \\
& + 35*e^{**4}*m^{**2} + 50*e^{**4}*m + 24*e^{**4}) + 19*a*e^{**4}*g*m*x^{**2}*(d + e \\
& *x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + 35*e^{**4}*m^{**2} + 50*e^{**4}*m + 24* \\
& e^{**4}) + 12*a*e^{**4}*g*x^{**2}*(d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + \\
& 35*e^{**4}*m^{**2} + 50*e^{**4}*m + 24*e^{**4}) + 2*b*d^{**3}*e*g*m*(d + e*x)^{**m} \\
& / (e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + 35*e^{**4}*m^{**2} + 50*e^{**4}*m + 24*e^{**4}) \\
& + 8*b*d^{**3}*e*g*(d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + 35*e^{**4} \\
& *m^{**2} + 50*e^{**4}*m + 24*e^{**4}) - b*d^{**2}*e^{**2}*f*m^{**2}*(d + e*x)^{**m}/(e^{**4} \\
& *m^{**4} + 10*e^{**4}*m^{**3} + 35*e^{**4}*m^{**2} + 50*e^{**4}*m + 24*e^{**4}) - 7* \\
& b*d^{**2}*e^{**2}*f*m*(d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + 35*e^{**4} \\
& *m^{**2} + 50*e^{**4}*m + 24*e^{**4}) - 12*b*d^{**2}*e^{**2}*f*(d + e*x)^{**m}/(e^{**4} \\
& *m^{**4} + 10*e^{**4}*m^{**3} + 35*e^{**4}*m^{**2} + 50*e^{**4}*m + 24*e^{**4}) - 2*b* \\
& d^{**2}*e^{**2}*g*m^{**2}*x*(d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + 35*e^{**4} \\
& *m^{**2} + 50*e^{**4}*m + 24*e^{**4}) - 8*b*d^{**2}*e^{**2}*g*m*x*(d + e*x)^{**m} \\
& / (e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + 35*e^{**4}*m^{**2} + 50*e^{**4}*m + 24*e^{**4}) \\
& + b*d*e^{**3}*f*m^{**3}*x*(d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + 35*e^{**4} \\
& *m^{**2} + 50*e^{**4}*m + 24*e^{**4}) + 7*b*d*e^{**3}*f*m^{**2}*x*(d + e*x)^{**m} \\
& / (e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + 35*e^{**4}*m^{**2} + 50*e^{**4}*m + 24*e^{**4}) \\
& + 12*b*d*e^{**3}*f*m*x*(d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + 35* \\
& e^{**4}*m^{**2} + 50*e^{**4}*m + 24*e^{**4}) + b*d*e^{**3}*g*m^{**3}*x^{**2}*(d + e*x) \\
& **m/(e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + 35*e^{**4}*m^{**2} + 50*e^{**4}*m + 24*e^{**4}) \\
& + 5*b*d*e^{**3}*g*m^{**2}*x^{**2}*(d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} \\
& + 35*e^{**4}*m^{**2} + 50*e^{**4}*m + 24*e^{**4}) + 4*b*d*e^{**3}*g*m*x^{**2}*(d \\
& + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + 35*e^{**4}*m^{**2} + 50*e^{**4}*m + \\
& 24*e^{**4}) + b*e^{**4}*f*m^{**3}*x^{**2}*(d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4}*m \\
& **3 + 35*e^{**4}*m^{**2} + 50*e^{**4}*m + 24*e^{**4}) + 8*b*e^{**4}*f*m^{**2}*x^{**2} \\
& (d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + 35*e^{**4}*m^{**2} + 50*e^{**4}*m \\
& + 24*e^{**4}) + 19*b*e^{**4}*f*m*x^{**2}*(d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4} \\
& *m^{**3} + 35*e^{**4}*m^{**2} + 50*e^{**4}*m + 24*e^{**4}) + 12*b*e^{**4}*f*x^{**2}*( \\
& d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + 35*e^{**4}*m^{**2} + 50*e^{**4}*m \\
& + 24*e^{**4}) + b*e^{**4}*g*m^{**3}*x^{**3}*(d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4} \\
& *m^{**3} + 35*e^{**4}*m^{**2} + 50*e^{**4}*m + 24*e^{**4}) + 7*b*e^{**4}*g*m^{**2}*x^{**3} \\
& *(d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + 35*e^{**4}*m^{**2} + 50*e^{**4} \\
& *m + 24*e^{**4}) + 14*b*e^{**4}*g*m*x^{**3}*(d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e \\
& **4*m^{**3} + 35*e^{**4}*m^{**2} + 50*e^{**4}*m + 24*e^{**4}) + 8*b*e^{**4}*g*x^{**3} \\
& (d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + 35*e^{**4}*m^{**2} + 50*e^{**4}*m \\
& + 24*e^{**4}) - 6*c*d^{**4}*g*(d + e*x)^{**m}/(e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + \\
& 35*e^{**4}*m^{**2} + 50*e^{**4}*m + 24*e^{**4}) + 2*c*d^{**3}*e*f*m*(d + e*x)^{**m} \\
& / (e^{**4}*m^{**4} + 10*e^{**4}*m^{**3} + 35*e^{**4}*m^{**2} + 50*e^{**4}*m + 24*e^{**4})
\end{aligned}$$

```

+ 8*c*d**3*e*f*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*
m**2 + 50*e**4*m + 24*e**4) + 6*c*d**3*e*g*m*x*(d + e*x)**m/(e**4
*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - 2*c*
d**2*e**2*f*m**2*x*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e
**4*m**2 + 50*e**4*m + 24*e**4) - 8*c*d**2*e**2*f*m*x*(d + e*x)**m
/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4)
- 3*c*d**2*e**2*g*m**2*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**
3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) - 3*c*d**2*e**2*g*m*x**2*
(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m
+ 24*e**4) + c*d*e**3*f*m**3*x**2*(d + e*x)**m/(e**4*m**4 + 10*e
**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 5*c*d*e**3*f*m**
2*x**2*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50
*e**4*m + 24*e**4) + 4*c*d*e**3*f*m*x**2*(d + e*x)**m/(e**4*m**4
+ 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + c*d*e**3*g
m**3*x**3*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2
+ 50*e**4*m + 24*e**4) + 3*c*d*e**3*g*m**2*x**3*(d + e*x)**m/(e**
4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 2*c
*d*e**3*g*m*x**3*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4
*m**2 + 50*e**4*m + 24*e**4) + c*e**4*f*m**3*x**3*(d + e*x)**m/(e
**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) + 7
*c*e**4*f*m**2*x**3*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e
**4*m**2 + 50*e**4*m + 24*e**4) + 14*c*e**4*f*m*x**3*(d + e*x)**m
/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4)
+ 8*c*e**4*f*x**3*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e**
4*m**2 + 50*e**4*m + 24*e**4) + c*e**4*g*m**3*x**4*(d + e*x)**m/(
e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4) +
6*c*e**4*g*m**2*x**4*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*
e**4*m**2 + 50*e**4*m + 24*e**4) + 11*c*e**4*g*m*x**4*(d + e*x)**
m/(e**4*m**4 + 10*e**4*m**3 + 35*e**4*m**2 + 50*e**4*m + 24*e**4)
+ 6*c*e**4*g*x**4*(d + e*x)**m/(e**4*m**4 + 10*e**4*m**3 + 35*e
**4*m**2 + 50*e**4*m + 24*e**4), True))

```

**GIAC/XCAS [A]** time = 0.265818, size = 1, normalized size = 0.01

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*(g\*x + f)\*(e\*x + d)^m,x, algorithm="giac")

[Out] Done

$$3.922 \quad \int \frac{(d+ex)^m (a+bx+cx^2)}{f+gx} dx$$

**Optimal.** Leaf size=129

$$\frac{(d+ex)^{m+1} (ag^2 - bfg + cf^2) {}_2F_1\left(1, m+1; m+2; -\frac{g(d+ex)}{ef-dg}\right)}{g^2(m+1)(ef-dg)} - \frac{(d+ex)^{m+1}(-beg + cdg + cef)}{e^2g^2(m+1)} + \frac{c(d+ex)^{m+2}}{e^2g(m+2)}$$

[Out] -(((c\*e\*f + c\*d\*g - b\*e\*g)\*(d + e\*x)^(1 + m))/(e^2\*g^2\*(1 + m))) + (c\*(d + e\*x)^(2 + m))/(e^2\*g\*(2 + m)) + ((c\*f^2 - b\*f\*g + a\*g^2)\*(d + e\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, -(g\*(d + e\*x))/(e\*f - d\*g)])/(g^2\*(e\*f - d\*g)\*(1 + m))

**Rubi [A]** time = 0.399742, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$\frac{(d+ex)^{m+1} (ag^2 - bfg + cf^2) {}_2F_1\left(1, m+1; m+2; -\frac{g(d+ex)}{ef-dg}\right)}{g^2(m+1)(ef-dg)} - \frac{(d+ex)^{m+1}(-beg + cdg + cef)}{e^2g^2(m+1)} + \frac{c(d+ex)^{m+2}}{e^2g(m+2)}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^m\*(a + b\*x + c\*x^2))/(f + g\*x), x]

[Out] -(((c\*e\*f + c\*d\*g - b\*e\*g)\*(d + e\*x)^(1 + m))/(e^2\*g^2\*(1 + m))) + (c\*(d + e\*x)^(2 + m))/(e^2\*g\*(2 + m)) + ((c\*f^2 - b\*f\*g + a\*g^2)\*(d + e\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, -(g\*(d + e\*x))/(e\*f - d\*g)])/(g^2\*(e\*f - d\*g)\*(1 + m))

**Rubi in Sympy [A]** time = 41.7642, size = 107, normalized size = 0.83

$$\frac{c(d+ex)^{m+2}}{e^2g(m+2)} - \frac{(d+ex)^{m+1} (ag^2 - bfg + cf^2) {}_2F_1\left(1, m+1; m+2; \frac{g(d+ex)}{dg-ef}\right)}{g^2(m+1)(dg-ef)} + \frac{(d+ex)^{m+1} (beg - cdg - cef)}{e^2g^2(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*m\*(c\*x\*\*2+b\*x+a)/(g\*x+f), x)

[Out] c\*(d + e\*x)\*\*(m + 2)/(e\*\*2\*g\*(m + 2)) - (d + e\*x)\*\*(m + 1)\*(a\*g\*\*2 - b\*f\*g + c\*f\*\*2)\*hyper((1, m + 1), (m + 2, ), g\*(d + e\*x)/(d\*g - e\*f))/(g\*\*2\*(m + 1)\*(d\*g - e\*f)) + (d + e\*x)\*\*(m + 1)\*(b\*e\*g - c\*d\*g - c\*e\*f)/(e\*\*2\*g\*\*2\*(m + 1))

**Mathematica [A]** time = 1.15942, size = 166, normalized size = 1.29

$$(d+ex)^m \left( \frac{(g(ag-bf)+cf^2) \left(\frac{g(d+ex)}{e(f+gx)}\right)^{-m} {}_2F_1\left(-m, -m; 1-m; \frac{ef-dg}{ef+egx}\right)}{m} + \frac{g(beg(m+2)(d+ex)+c(d^2g\left(\frac{ex}{d}+1\right)^{-m}-1)+de(gmx-f(m+2))+e^2x(g(m+1)x-f(m+2)))}{e^2(m+1)(m+2)} \right)$$

$g^3$

Antiderivative was successfully verified.

[In] Integrate[(((d + e\*x)^m\*(a + b\*x + c\*x^2))/(f + g\*x), x]

[Out] ((d + e\*x)^m\*((g\*(b\*e\*g\*(2 + m)\*(d + e\*x) + c\*(d\*e\*(-(f\*(2 + m)) + g\*m\*x) + e^2\*x\*(-(f\*(2 + m)) + g\*(1 + m)\*x) + d^2\*g\*(-1 + (1 + (e\*x)/d)^(-m)))))/(e^2\*(1 + m)\*(2 + m)) + ((c\*f^2 + g\*(-(b\*f) + a\*g))\*Hypergeometric2F1[-m, -m, 1 - m, (e\*f - d\*g)/(e\*f + e\*g\*x)])/(m\*((g\*(d + e\*x))/(e\*(f + g\*x)))^m))/g^3

**Maple** [F] time = 0.062, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^m (cx^2 + bx + a)}{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^m\*(c\*x^2+b\*x+a)/(g\*x+f), x)

[Out] int((e\*x+d)^m\*(c\*x^2+b\*x+a)/(g\*x+f), x)

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)(ex + d)^m}{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)^m/(g\*x + f), x, algorithm="maxima")

[Out] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)^m/(g\*x + f), x)

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^2 + bx + a)(ex + d)^m}{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)^m/(g\*x + f), x, algorithm="fricas")

[Out] integral((c\*x^2 + b\*x + a)\*(e\*x + d)^m/(g\*x + f), x)

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^m (a + bx + cx^2)}{f + gx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*m\*(c\*x\*\*2+b\*x+a)/(g\*x+f), x)

[Out] Integral((d + e\*x)\*\*m\*(a + b\*x + c\*x\*\*2)/(f + g\*x), x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)(ex + d)^m}{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)^m/(g\*x + f), x, algorithm="giac")

[Out] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)^m/(g\*x + f), x)

$$3.923 \quad \int \frac{(d+ex)^m (a+bx+cx^2)}{(f+gx)^2} dx$$

**Optimal.** Leaf size=157

$$\frac{(d+ex)^{m+1} {}_2F_1\left(1, m+1; m+2; -\frac{g(d+ex)}{ef-dg}\right) (cf(2dg-ef(m+2)) - g(aegm + b(dg-ef(m+1))))}{g^2(m+1)(ef-dg)^2} + \frac{(d+ex)^{m+1} \left(a + \frac{f(cf-bg)}{g^2}\right)}{(f+gx)(ef-dg)} + \frac{c(d+ex)^{m+1}}{eg^2(m+1)}$$

[Out] (c\*(d + e\*x)^(1 + m))/(e\*g^2\*(1 + m)) + ((a + (f\*(c\*f - b\*g))/g^2)\* (d + e\*x)^(1 + m))/((e\*f - d\*g)\*(f + g\*x)) + ((c\*f\*(2\*d\*g - e\*f\*(2 + m)) - g\*(a\*e\*g\*m + b\*(d\*g - e\*f\*(1 + m))))\*(d + e\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, -((g\*(d + e\*x))/(e\*f - d\*g))]/(g^2\*(e\*f - d\*g)^2\*(1 + m))

**Rubi [A]** time = 0.533131, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$\frac{(d+ex)^{m+1} {}_2F_1\left(1, m+1; m+2; -\frac{g(d+ex)}{ef-dg}\right) (g(aegm + bdg - bef(m+1)) - cf(2dg-ef(m+2)))}{g^2(m+1)(ef-dg)^2} + \frac{(d+ex)^{m+1} \left(a + \frac{f(cf-bg)}{g^2}\right)}{(f+gx)(ef-dg)} + \frac{c(d+ex)^{m+1}}{eg^2(m+1)}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^m\*(a + b\*x + c\*x^2))/(f + g\*x)^2, x]

[Out] (c\*(d + e\*x)^(1 + m))/(e\*g^2\*(1 + m)) + ((a + (f\*(c\*f - b\*g))/g^2)\* (d + e\*x)^(1 + m))/((e\*f - d\*g)\*(f + g\*x)) - ((g\*(b\*d\*g + a\*e\*g\*m - b\*e\*f\*(1 + m)) - c\*f\*(2\*d\*g - e\*f\*(2 + m)))\*(d + e\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, -((g\*(d + e\*x))/(e\*f - d\*g))]/(g^2\*(e\*f - d\*g)^2\*(1 + m))

**Rubi in Sympy [A]** time = 42.9209, size = 126, normalized size = 0.8

$$\frac{c(d+ex)^{m+1}}{eg^2(m+1)} + \frac{e(d+ex)^{m+1} (ag^2 - bfg + cf^2) {}_2F_1\left(2, m+1; m+2; \frac{g(d+ex)}{dg-ef}\right)}{g^2(m+1)(dg-ef)^2} - \frac{(d+ex)^{m+1} (bg - 2cf) {}_2F_1\left(1, m+1; m+2; \frac{g(d+ex)}{dg-ef}\right)}{g^2(m+1)(dg-ef)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*m\*(c\*x\*\*2+b\*x+a)/(g\*x+f)\*\*2, x)

[Out] c\*(d + e\*x)\*\*(m + 1)/(e\*g\*\*2\*(m + 1)) + e\*(d + e\*x)\*\*(m + 1)\*(a\*g\*\*2 - b\*f\*g + c\*f\*\*2)\*hyper((2, m + 1), (m + 2, ), g\*(d + e\*x)/(d\*g - e\*f))/(g\*\*2\*(m + 1)\*(d\*g - e\*f)\*\*2) - (d + e\*x)\*\*(m + 1)\*(b\*g - 2\*c\*f)\*hyper((1, m + 1), (m + 2, ), g\*(d + e\*x)/(d\*g - e\*f))/(g\*\*2\*(m + 1)\*(d\*g - e\*f))



**Mathematica [A]** time = 0.210325, size = 0, normalized size = 0.

$$\int \frac{(d + ex)^m (a + bx + cx^2)}{(f + gx)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[((d + e\*x)^m\*(a + b\*x + c\*x^2))/(f + g\*x)^2, x]

[Out] Integrate[((d + e\*x)^m\*(a + b\*x + c\*x^2))/(f + g\*x)^2, x]

---

**Maple [F]** time = 0.083, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^m (cx^2 + bx + a)}{(gx + f)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^m\*(c\*x^2+b\*x+a)/(g\*x+f)^2, x)

[Out] int((e\*x+d)^m\*(c\*x^2+b\*x+a)/(g\*x+f)^2, x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)(ex + d)^m}{(gx + f)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)^m/(g\*x + f)^2, x, algorithm="maxima")

[Out] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)^m/(g\*x + f)^2, x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^2 + bx + a)(ex + d)^m}{g^2x^2 + 2fgx + f^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)^m/(g\*x + f)^2, x, algorithm="fricas")

[Out] integral((c\*x^2 + b\*x + a)\*(e\*x + d)^m/(g^2\*x^2 + 2\*f\*g\*x + f^2), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^m (a + bx + cx^2)}{(f + gx)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*m\*(c\*x\*\*2+b\*x+a)/(g\*x+f)\*\*2,x)

[Out] Integral((d + e\*x)\*\*m\*(a + b\*x + c\*x\*\*2)/(f + g\*x)\*\*2, x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)(ex + d)^m}{(gx + f)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)^m/(g\*x + f)^2,x, algorithm="giac")

[Out] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)^m/(g\*x + f)^2, x)

$$3.924 \quad \int \frac{(d+ex)^m (a+bx+cx^2)}{(f+gx)^3} dx$$

**Optimal.** Leaf size=245

$$\frac{(d+ex)^{m+1} {}_2F_1\left(1, m+1; m+2; -\frac{g(d+ex)}{ef-dg}\right) (c(2d^2g^2 - 4defg(m+1) + e^2f^2(m^2 + 3m + 2)) - egm(aeg(1-m) - b(2dg - ef)))}{2g^2(m+1)(ef-dg)^3} + \frac{(d+ex)^{m+1}(g(aeg(1-m) - b(2dg - ef(m+1))) + cf(4dg - ef(m+3)))}{2g^2(f+gx)(ef-dg)^2} + \frac{(d+ex)^{m+1} \left(a + \frac{f(cf-bg)}{g^2}\right)}{2(f+gx)^2(ef-dg)}$$

[Out]  $((a + (f*(c*f - b*g))/g^2)^*(d + e*x)^(1 + m))/(2*(e*f - d*g)^*(f + g*x)^2) + ((c*f*(4*d*g - e*f*(3 + m)) + g*(a*e*g*(1 - m) - b*(2*d*g - e*f*(1 + m))))*(d + e*x)^(1 + m))/(2*g^2*(e*f - d*g)^2*(f + g*x)) + ((c*(2*d^2*g^2 - 4*d*e*f*g*(1 + m) + e^2*f^2*(2 + 3*m + m^2)) - e*g*m*(a*e*g*(1 - m) - b*(2*d*g - e*f*(1 + m))))*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((g*(d + e*x))/(e*f - d*g))]/(2*g^2*(e*f - d*g)^3*(1 + m))$

**Rubi [A]** time = 0.846946, antiderivative size = 243, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$\frac{(d+ex)^{m+1} {}_2F_1\left(1, m+1; m+2; -\frac{g(d+ex)}{ef-dg}\right) (egm(-aeg(1-m) + 2bdg - bef(m+1)) + c(2d^2g^2 - 4defg(m+1) + e^2f^2(m^2 + 3m + 2)))}{2g^2(m+1)(ef-dg)^3} - \frac{(d+ex)^{m+1}(g(-aeg(1-m) + 2bdg - bef(m+1)) - cf(4dg - ef(m+3)))}{2g^2(f+gx)(ef-dg)^2} + \frac{(d+ex)^{m+1} \left(a + \frac{f(cf-bg)}{g^2}\right)}{2(f+gx)^2(ef-dg)}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^m\*(a + b\*x + c\*x^2))/(f + g\*x)^3, x]

[Out]  $((a + (f*(c*f - b*g))/g^2)^*(d + e*x)^(1 + m))/(2*(e*f - d*g)^*(f + g*x)^2) - ((g*(2*b*d*g - a*e*g*(1 - m) - b*e*f*(1 + m)) - c*f*(4*d*g - e*f*(3 + m))))*(d + e*x)^(1 + m))/(2*g^2*(e*f - d*g)^2*(f + g*x)) + ((e*g*m*(2*b*d*g - a*e*g*(1 - m) - b*e*f*(1 + m)) + c*(2*d^2*g^2 - 4*d*e*f*g*(1 + m) + e^2*f^2*(2 + 3*m + m^2)))*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((g*(d + e*x))/(e*f - d*g))]/(2*g^2*(e*f - d*g)^3*(1 + m))$

**Rubi in Sympy [A]** time = 51.2783, size = 155, normalized size = 0.63

$$\frac{c(d+ex)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{g(d+ex)}{dg-ef}\right)}{g^2(m+1)(dg-ef)} - \frac{e^2(d+ex)^{m+1} (ag^2 - bfg + cf^2) {}_2F_1\left(3, m+1; m+2; \frac{g(d+ex)}{dg-ef}\right)}{g^2(m+1)(dg-ef)^3} + \frac{e(d+ex)^{m+1} (bg - 2cf) {}_2F_1\left(2, m+1; m+2; \frac{g(d+ex)}{dg-ef}\right)}{g^2(m+1)(dg-ef)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*m\*(c\*x\*\*2+b\*x+a)/(g\*x+f)\*\*3, x)

[Out]  $-c*(d + e*x)**(m + 1)*hyper((1, m + 1), (m + 2, ), g*(d + e*x)/(d*g - e*f))/(g**2*(m + 1)*(d*g - e*f)) - e**2*(d + e*x)**(m + 1)*(a*g**2 - b*f*g + c*f**2)*hyper((3, m + 1), (m + 2, ), g*(d + e*x)/(d*g - e*f))/(g**2*(m + 1)*(d*g - e*f)**3)$

$$\frac{d^2 g - e^2 f}{(g^2(m+1)(d^2 g - e^2 f)^3) + e^2(d + e^2 x)^{m+1}(b^2 g - 2c^2 f) \operatorname{hyper}((2, m+1), (m+2, ), g^2(d + e^2 x)/(d^2 g - e^2 f))} / (g^2(m+1)(d^2 g - e^2 f)^2)$$

**Mathematica [A]** time = 0.286995, size = 0, normalized size = 0.

$$\int \frac{(d + ex)^m (a + bx + cx^2)}{(f + gx)^3} dx$$

Verification is Not applicable to the result.

[In] Integrate[((d + e\*x)^m\*(a + b\*x + c\*x^2))/(f + g\*x)^3, x]

[Out] Integrate[((d + e\*x)^m\*(a + b\*x + c\*x^2))/(f + g\*x)^3, x]

**Maple [F]** time = 0.11, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^m (cx^2 + bx + a)}{(gx + f)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^m\*(c\*x^2+b\*x+a)/(g\*x+f)^3, x)

[Out] int((e\*x+d)^m\*(c\*x^2+b\*x+a)/(g\*x+f)^3, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)(ex + d)^m}{(gx + f)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)^m/(g\*x + f)^3, x, algorithm="maxima")

[Out] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)^m/(g\*x + f)^3, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(cx^2 + bx + a)(ex + d)^m}{g^3x^3 + 3fg^2x^2 + 3f^2gx + f^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)^m/(g\*x + f)^3, x, algorithm="fricas")

[Out] integral((c\*x^2 + b\*x + a)\*(e\*x + d)^m/(g^3\*x^3 + 3\*f\*g^2\*x^2 + 3\*f^2\*g\*x + f^3), x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*m\*(c\*x\*\*2+b\*x+a)/(g\*x+f)\*\*3,x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)(ex + d)^m}{(gx + f)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)^m/(g\*x + f)^3,x, algorithm="giac")

[Out] integrate((c\*x^2 + b\*x + a)\*(e\*x + d)^m/(g\*x + f)^3, x)

$$3.925 \quad \int (d + ex)^m (f + gx)^2 (a + bx + cx^2)^2 dx$$

**Optimal.** Leaf size=525

$$\begin{aligned} & \frac{(d + ex)^{m+3} (e^2 (a^2 e^2 g^2 + 2abeg(2ef - 3dg) + b^2 (6d^2 g^2 - 6defg + e^2 f^2)) + 2ce (ae (6d^2 g^2 - 6defg + e^2 f^2) - bd (10d^2 g^2 - 6defg + e^2 f^2))}{e^{7(m+3)}} \\ & + \frac{(d + ex)^{m+5} (2ceg(aeg - 5bdg + 2bef) + b^2 e^2 g^2 + c^2 (15d^2 g^2 - 10defg + e^2 f^2))}{e^{7(m+5)}} \\ & + \frac{2(d + ex)^{m+4} (ce (2aeg(ef - 2dg) + b (10d^2 g^2 - 8defg + e^2 f^2)) + be^2 g(aeg - 2bdg + bef) - 2c^2 d (5d^2 g^2 - 5defg + e^2 f^2))}{e^{7(m+4)}} \\ & + \frac{(ef - dg)^2 (d + ex)^{m+1} (ae^2 - bde + cd^2)^2}{e^{7(m+1)}} \\ & - \frac{2(ef - dg)(d + ex)^{m+2} (ae^2 - bde + cd^2) (cd(2ef - 3dg) - e(aeg - 2bdg + bef))}{e^{7(m+2)}} \\ & + \frac{2cg(d + ex)^{m+6} (beg - 3cdg + cef)}{e^{7(m+6)}} + \frac{c^2 g^2 (d + ex)^{m+7}}{e^{7(m+7)}} \end{aligned}$$

[Out]  $((c^2 d^2 - b^2 d e + a^2 e^2)^2 (e f - d g)^2 (d + e x)^{(1 + m)}) / (e^{7(1 + m)}) - (2(c^2 d^2 - b^2 d e + a^2 e^2) (e f - d g) (c^2 d (2 e^2 f - 3 d^2 g) - e (b^2 e f - 2 b^2 d g + a^2 e g))) (d + e x)^{(2 + m)} / (e^{7(2 + m)}) + ((c^2 d^2 (6 e^2 f^2 - 20 d^2 e f g + 15 d^2 g^2) + e^2 (a^2 e^2 g^2 + 2 a^2 b e g (2 e f - 3 d g) + b^2 (e^2 f^2 - 6 d^2 e f g + 6 d^2 g^2))) + 2 c^2 e (a^2 e (e^2 f^2 - 6 d^2 e f g + 6 d^2 g^2) - b^2 d (3 e^2 f^2 - 12 d^2 e f g + 10 d^2 g^2))) (d + e x)^{(3 + m)} / (e^{7(3 + m)}) + (2(b^2 e^2 g (b^2 e f - 2 b^2 d g + a^2 e g) - 2 c^2 d (e^2 f^2 - 5 d^2 e f g + 5 d^2 g^2) + c^2 e (2 a^2 e g (e f - 2 d g) + b (e^2 f^2 - 8 d^2 e f g + 10 d^2 g^2))) (d + e x)^{(4 + m)} / (e^{7(4 + m)}) + ((b^2 e^2 g^2 + 2 c^2 e g (2 b^2 e f - 5 b^2 d g + a^2 e g) + c^2 (e^2 f^2 - 10 d^2 e f g + 15 d^2 g^2)) (d + e x)^{(5 + m)} / (e^{7(5 + m)}) + (2 c^2 g (c^2 e f - 3 c^2 d g + b^2 e g) (d + e x)^{(6 + m)} / (e^{7(6 + m)}) + (c^2 g^2 (d + e x)^{(7 + m)} / (e^{7(7 + m)}))$

**Rubi [A]** time = 1.93854, antiderivative size = 525, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$

$$\begin{aligned} & \frac{(d + ex)^{m+3} (e^2 (a^2 e^2 g^2 + 2abeg(2ef - 3dg) + b^2 (6d^2 g^2 - 6defg + e^2 f^2)) + 2ce (ae (6d^2 g^2 - 6defg + e^2 f^2) - bd (10d^2 g^2 - 6defg + e^2 f^2))}{e^{7(m+3)}} \\ & + \frac{(d + ex)^{m+5} (2ceg(aeg - 5bdg + 2bef) + b^2 e^2 g^2 + c^2 (15d^2 g^2 - 10defg + e^2 f^2))}{e^{7(m+5)}} \\ & + \frac{2(d + ex)^{m+4} (ce (2aeg(ef - 2dg) + b (10d^2 g^2 - 8defg + e^2 f^2)) + be^2 g(aeg - 2bdg + bef) - 2c^2 d (5d^2 g^2 - 5defg + e^2 f^2))}{e^{7(m+4)}} \\ & + \frac{(ef - dg)^2 (d + ex)^{m+1} (ae^2 - bde + cd^2)^2}{e^{7(m+1)}} \\ & - \frac{2(ef - dg)(d + ex)^{m+2} (ae^2 - bde + cd^2) (cd(2ef - 3dg) - e(aeg - 2bdg + bef))}{e^{7(m+2)}} \\ & + \frac{2cg(d + ex)^{m+6} (beg - 3cdg + cef)}{e^{7(m+6)}} + \frac{c^2 g^2 (d + ex)^{m+7}}{e^{7(m+7)}} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^m\*(f + g\*x)^2\*(a + b\*x + c\*x^2)^2, x]

[Out]  $((c^2 d^2 - b^2 d e + a^2 e^2)^2 (e f - d g)^2 (d + e x)^{(1 + m)}) / (e^{7(1 + m)}) - (2(c^2 d^2 - b^2 d e + a^2 e^2) (e f - d g) (c^2 d (2 e^2 f - 3 d^2 g) - e (b^2 e f - 2 b^2 d g + a^2 e g))) (d + e x)^{(2 + m)} / (e^{7(2 + m)}) + ((c^2 d^2 (6 e^2 f^2 - 20 d^2 e f g + 15 d^2 g^2) + e^2 (a^2 e^2 g^2 + 2 a^2 b e g (2 e f - 3 d g) + b^2 (e^2 f^2 - 6 d^2 e f g + 6 d^2 g^2))) + 2 c^2 e (a^2 e (e^2 f^2 - 6 d^2 e f g + 6 d^2 g^2) - b^2 d (3 e^2 f^2 - 12 d^2 e f g + 10 d^2 g^2))) (d + e x)^{(3 + m)} / (e^{7(3 + m)})$

$$(3 + m) + (2*(b*e^2*g*(b*e*f - 2*b*d*g + a*e*g) - 2*c^2*d*(e^2*f^2 - 5*d*e*f*g + 5*d^2*g^2) + c*e*(2*a*e*g*(e*f - 2*d*g) + b*(e^2*f^2 - 8*d*e*f*g + 10*d^2*g^2)))*(d + e*x)^(4 + m)/(e^7*(4 + m)) + ((b^2*e^2*g^2 + 2*c*e*g*(2*b*e*f - 5*b*d*g + a*e*g) + c^2*(e^2*f^2 - 10*d*e*f*g + 15*d^2*g^2))*(d + e*x)^(5 + m)/(e^7*(5 + m)) + (2*c*g*(c*e*f - 3*c*d*g + b*e*g)*(d + e*x)^(6 + m)/(e^7*(6 + m)) + (c^2*g^2*(d + e*x)^(7 + m)/(e^7*(7 + m)))$$

**Rubi in Sympy [F-1]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+b*x+a)**2,x)`

[Out] Timed out

**Mathematica [B]** time = 4.33454, size = 1263, normalized size = 2.41

$$(d + ex)^{m+1} ((720g^2d^6 - 240eg(f(m+7) + 3g(m+1)x)d^5 + 24e^2((m^2 + 13m + 42)f^2 + 10g(m^2 + 8m + 7)xf + 15g^2(m^2 +$$

Antiderivative was successfully verified.

[In] `Integrate[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^2,x]`

[Out]  $((d + e*x)^{(1 + m)}*(c^2*(720*d^6*g^2 - 240*d^5*e*g*(f*(7 + m) + 3*g*(1 + m)*x) + 24*d^4*e^2*(f^2*(42 + 13*m + m^2) + 10*f*g*(7 + 8*m + m^2)*x + 15*g^2*(2 + 3*m + m^2)*x^2) - 24*d^3*e^3*(1 + m)*x*(f^2*(42 + 13*m + m^2) + 5*f*g*(14 + 9*m + m^2)*x + 5*g^2*(6 + 5*m + m^2)*x^2) + 2*d^2*e^4*(2 + 3*m + m^2)*x^2*(6*f^2*(42 + 13*m + m^2) + 20*f*g*(21 + 10*m + m^2)*x + 15*g^2*(12 + 7*m + m^2)*x^2) - 2*d*e^5*(6 + 11*m + 6*m^2 + m^3)*x^3*(2*f^2*(42 + 13*m + m^2) + 5*f*g*(28 + 11*m + m^2)*x + 3*g^2*(20 + 9*m + m^2)*x^2) + e^6*(24 + 50*m + 35*m^2 + 10*m^3 + m^4)*x^4*(f^2*(42 + 13*m + m^2) + 2*f*g*(35 + 12*m + m^2)*x + g^2*(30 + 11*m + m^2)*x^2)) + e^2*(42 + 13*m + m^2)*(a^2*e^2*(20 + 9*m + m^2)*(2*d^2*g^2 - 2*d*e*g*(f*(3 + m) + g*(1 + m)*x) + e^2*(f^2*(6 + 5*m + m^2) + 2*f*g*(3 + 4*m + m^2)*x + g^2*(2 + 3*m + m^2)*x^2)) + 2*a*b*e*(5 + m)*(-6*d^3*g^2 + 2*d^2*e*g*(2*f*(4 + m) + 3*g*(1 + m)*x) - d*e^2*(f^2*(12 + 7*m + m^2) + 4*f*g*(4 + 5*m + m^2)*x + 3*g^2*(2 + 3*m + m^2)*x^2) + e^3*(1 + m)*x*(f^2*(12 + 7*m + m^2) + 2*f*g*(8 + 6*m + m^2)*x + g^2*(6 + 5*m + m^2)*x^2)) + b^2*(24*d^4*g^2 - 12*d^3*e*g*(f*(5 + m) + 2*g*(1 + m)*x) + 2*d^2*e^2*(f^2*(20 + 9*m + m^2) + 6*f*g*(5 + 6*m + m^2)*x + 6*g^2*(2 + 3*m + m^2)*x^2) - 2*d*e^3*(1 + m)*x*(f^2*(20 + 9*m + m^2) + 3*f*g*(10 + 7*m + m^2)*x + 2*g^2*(6 + 5*m + m^2)*x^2) + e^4*(2 + 3*m + m^2)*x^2*(f^2*(20 + 9*m + m^2) + 2*f*g*(15 + 8*m + m^2)*x + g^2*(12 + 7*m + m^2)*x^2)) + 2*c*e*(7 + m)*(a*e*(6 + m)*(24*d^4*g^2 - 12*d^3*e*g*(f*(5 + m) + 2*g*(1 + m)*x) + 2*d^2*e^2*(f^2*(20 + 9*m + m^2) + 6*f*g*(5 + 6*m + m^2)*x + 6*g^2*(2 + 3*m + m^2)*x^2) - 2*d*e^3*(1 + m)*x*(f^2*(20 + 9*m + m^2) + 3*f*g*(10 + 7*m + m^2)*x + 2*g^2*(6 + 5*m + m^2)*x^2) + e^4*(2 + 3*m + m^2)*x^2*(f^2*(20 + 9*m + m^2) + 2*f*g*(15 + 8*m + m^2)*x + g^2*(12 + 7*m + m^2)*x^2)) + b*(-120*d^5*g^2 + 24*d^4*e*g*(2*f*(6 + m) + 5*g*(1 + m)*x) - 6*d^3*e^2*(f^2*(30 + 11*m + m^2) + 8*f*g*(6 + 7*m + m^2)*x + 10*g^2*(2 + 3*m + m^2)*x^2) + 2*d^2*e^3*(1 + m)*x*(3*f^2*(30 + 11*m + m^2) + 12*f*g*(12 + 8*m + m^2)*x + 10*g^2*(6 + 5*m + m^2)*x^2) - d*e^4*(2 + 3*m + m^2)*x^2*(3*f^2*(30 + 11*m + m^2) + 8*f*g*(18 + 9*m + m^2)*x + 5*g^2*(12 + 7*m + m^2)*x^2) + e^5*(6 + 11*m + 6*m^2 + m^3)*x^3*(f^2*(30 + 11*m + m^2) + 2*f*g*(24 + 10*m + m^2)*x + g^2*(20 + 9*m + m^2)*x^2))))$

$$/(e^{7*(1+m)}(2+m)^{(3+m)}(4+m)^{(5+m)}(6+m)^{(7+m)})$$

**Maple [B]** time = 0.026, size = 5890, normalized size = 11.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^2,x)`

[Out] result too large to display

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^2*(g*x + f)^2*(e*x + d)^m,x, algorithm="maxima")`

[Out] Exception raised: ValueError

**Fricas [A]** time = 0.349421, size = 6408, normalized size = 12.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^2*(g*x + f)^2*(e*x + d)^m,x, algorithm="fricas")`

[Out]  $(a^2*d^e*f^2*m^6 + (c^2*e^7*g^2*m^6 + 21*c^2*e^7*g^2*m^5 + 175*c^2*e^7*g^2*m^4 + 735*c^2*e^7*g^2*m^3 + 1624*c^2*e^7*g^2*m^2 + 1764*c^2*e^7*g^2*m + 720*c^2*e^7*g^2)*x^7 + (1680*c^2*e^7*f*g + 1680*b*c*e^7*g^2 + (2*c^2*e^7*f*g + (c^2*d^e^6 + 2*b*c*e^7)*g^2)*m^6 + (44*c^2*e^7*f*g + (15*c^2*d^e^6 + 44*b*c*e^7)*g^2)*m^5 + 5*(76*c^2*e^7*f*g + (17*c^2*d^e^6 + 76*b*c*e^7)*g^2)*m^4 + 5*(328*c^2*e^7*f*g + (45*c^2*d^e^6 + 328*b*c*e^7)*g^2)*m^3 + 2*(1849*c^2*e^7*f*g + (137*c^2*d^e^6 + 1849*b*c*e^7)*g^2)*m^2 + 4*(1019*c^2*e^7*f*g + (30*c^2*d^e^6 + 1019*b*c*e^7)*g^2)*m)*x^6 - (2*a^2*d^2*e^5*f*g + (2*a*b*d^2*e^5 - 27*a^2*d^e^6)*f^2)*m^5 + (1008*c^2*e^7*f^2 + 4032*b*c*e^7*f*g + 1008*(b^2 + 2*a*c)*e^7*g^2 + (c^2*e^7*f^2 + 2*(c^2*d^e^6 + 2*b*c*e^7)*f*g + (2*b*c*d^e^6 + (b^2 + 2*a*c)*e^7)*g^2)*m^6 + (23*c^2*e^7*f^2 + 2*(17*c^2*d^e^6 + 46*b*c*e^7)*f*g - (6*c^2*d^2*e^5 - 34*b*c*d^e^6 - 23*(b^2 + 2*a*c)*e^7)*g^2)*m^5 + 3*(69*c^2*e^7*f^2 + 2*(35*c^2*d^e^6 + 138*b*c*e^7)*f*g - (20*c^2*d^2*e^5 - 70*b*c*d^e^6 - 69*(b^2 + 2*a*c)*e^7)*g^2)*m^4 + 5*(185*c^2*e^7*f^2 + 2*(59*c^2*d^e^6 + 370*b*c*e^7)*f*g - (42*c^2*d^2*e^5 - 118*b*c*d^e^6 - 185*(b^2 + 2*a*c)*e^7)*g^2)*m^3 + 4*(536*c^2*e^7*f^2 + (187*c^2*d^e^6 + 2144*b*c*e^7)*f*g - (75*c^2*d^2*e^5 - 187*b*c*d^e^6 - 536*(b^2 + 2*a*c)*e^7)*g^2)*m^2 + 12*(201*c^2*e^7*f^2 + 4*(7*c^2*d^e^6 + 201*b*c*e^7)*f*g - (12*c^2*d^2*e^5 - 28*b*c*d^e^6 - 201*(b^2 + 2*a*c)*e^7)*g^2)*m)*x^5 + (2*a^2*d^3*e^4*g^2 - (50*a*b*d^2*e^5 - 295*a^2*d^e^6 - 2*(b^2 + 2*a*c)*d^3*e^4)*f^2 + 2*(4*a*b*d^3*e^4 - 25*a^2*d^2*e^5)*f*g)*m^4 + (2520*b*c*e^7*f^2 + 2520*a*b*e^7*g^2 + 2520*(b^2 + 2*a*c)*e^7*f*g + ((c^2*d^e^6 + 2*b*c*e^7)*f^2 + 2*(2*b*c*d^e^6 + (b^2 + 2*a*c)*e^7)*f*g + (2*a*b*e^7 + (b^2 + 2*a*c)*d^e^6)*g^2)*m^6 + ((19*c^2*d^e^6 + 48*b*c*e^7)*f^2 - 2*(5*c^2*d^2*e^5 - 38*b*c*d^e^6 - 24*(b^2 + 2*a*c)*e$



$$\begin{aligned}
& \wedge 7) * f * g - (10 * b * c * d^2 * e^5 - 48 * a * b * e^7 - 19 * (b^2 + 2 * a * c) * d * e^6) * \\
& g^2) * m^5 + ((131 * c^2 * d^2 * e^6 + 452 * b * c * e^7) * f^2 - 2 * (65 * c^2 * d^2 * e^5 \\
& - 262 * b * c * d * e^6 - 226 * (b^2 + 2 * a * c) * e^7) * f * g + (30 * c^2 * d^3 * e^4 - \\
& 130 * b * c * d^2 * e^5 + 452 * a * b * e^7 + 131 * (b^2 + 2 * a * c) * d * e^6) * g^2) * m^4 \\
& + ((401 * c^2 * d * e^6 + 2112 * b * c * e^7) * f^2 - 2 * (265 * c^2 * d^2 * e^5 - 80 \\
& 2 * b * c * d * e^6 - 1056 * (b^2 + 2 * a * c) * e^7) * f * g + (180 * c^2 * d^3 * e^4 - 53 \\
& 0 * b * c * d^2 * e^5 + 2112 * a * b * e^7 + 401 * (b^2 + 2 * a * c) * d * e^6) * g^2) * m^3 \\
& + 10 * ((54 * c^2 * d * e^6 + 509 * b * c * e^7) * f^2 - (83 * c^2 * d^2 * e^5 - 216 * b * \\
& c * d * e^6 - 509 * (b^2 + 2 * a * c) * e^7) * f * g + (33 * c^2 * d^3 * e^4 - 83 * b * c * d \\
& ^2 * e^5 + 509 * a * b * e^7 + 54 * (b^2 + 2 * a * c) * d * e^6) * g^2) * m^2 + 12 * (3 * ( \\
& 7 * c^2 * d^2 * e^6 + 164 * b * c * e^7) * f^2 - (35 * c^2 * d^2 * e^5 - 84 * b * c * d * e^6 - \\
& 492 * (b^2 + 2 * a * c) * e^7) * f * g + (15 * c^2 * d^3 * e^4 - 35 * b * c * d^2 * e^5 + \\
& 492 * a * b * e^7 + 21 * (b^2 + 2 * a * c) * d * e^6) * g^2) * m) * x^4 - ((12 * b * c * d^4 * \\
& e^3 + 490 * a * b * d^2 * e^5 - 1665 * a^2 * d * e^6 - 44 * (b^2 + 2 * a * c) * d^3 * e^4) \\
& ) * f^2 - 2 * (88 * a * b * d^3 * e^4 - 245 * a^2 * d^2 * e^5 - 6 * (b^2 + 2 * a * c) * d^4 \\
& * e^3) * f * g + 4 * (3 * a * b * d^4 * e^3 - 11 * a^2 * d^3 * e^4) * g^2) * m^3 + (6720 * a \\
& * b * e^7 * f * g + 1680 * a^2 * e^7 * g^2 + 1680 * (b^2 + 2 * a * c) * e^7 * f^2 + ((2 * \\
& b * c * d * e^6 + (b^2 + 2 * a * c) * e^7) * f^2 + 2 * (2 * a * b * e^7 + (b^2 + 2 * a * c) \\
& * d * e^6) * f * g + (2 * a * b * d * e^6 + a^2 * e^7) * g^2) * m^6 - ((4 * c^2 * d^2 * e^5 \\
& - 42 * b * c * d * e^6 - 25 * (b^2 + 2 * a * c) * e^7) * f^2 + 2 * (8 * b * c * d^2 * e^5 - 5 \\
& 0 * a * b * e^7 - 21 * (b^2 + 2 * a * c) * d * e^6) * f * g - (42 * a * b * d * e^6 + 25 * a^2 * \\
& e^7 - 4 * (b^2 + 2 * a * c) * d^2 * e^5) * g^2) * m^5 - ((64 * c^2 * d^2 * e^5 - 326 * \\
& b * c * d * e^6 - 247 * (b^2 + 2 * a * c) * e^7) * f^2 - 2 * (20 * c^2 * d^3 * e^4 - 128 * \\
& b * c * d^2 * e^5 + 494 * a * b * e^7 + 163 * (b^2 + 2 * a * c) * d * e^6) * f * g - (40 * b * \\
& c * d^3 * e^4 + 326 * a * b * d * e^6 + 247 * a^2 * e^7 - 64 * (b^2 + 2 * a * c) * d^2 * e^5) \\
& ) * g^2) * m^4 - ((332 * c^2 * d^2 * e^5 - 1134 * b * c * d * e^6 - 1219 * (b^2 + 2 * \\
& a * c) * e^7) * f^2 - 2 * (200 * c^2 * d^3 * e^4 - 664 * b * c * d^2 * e^5 + 2438 * a * b * e \\
& ^7 + 567 * (b^2 + 2 * a * c) * d * e^6) * f * g + (120 * c^2 * d^4 * e^3 - 400 * b * c * d^3 \\
& * e^4 - 1134 * a * b * d * e^6 - 1219 * a^2 * e^7 + 332 * (b^2 + 2 * a * c) * d^2 * e^5) \\
& ) * g^2) * m^3 - 8 * ((76 * c^2 * d^2 * e^5 - 211 * b * c * d * e^6 - 389 * (b^2 + 2 * a * \\
& c) * e^7) * f^2 - (115 * c^2 * d^3 * e^4 - 304 * b * c * d^2 * e^5 + 1556 * a * b * e^7 + \\
& 211 * (b^2 + 2 * a * c) * d * e^6) * f * g + (45 * c^2 * d^4 * e^3 - 115 * b * c * d^3 * e^4 \\
& - 211 * a * b * d * e^6 - 389 * a^2 * e^7 + 76 * (b^2 + 2 * a * c) * d^2 * e^5) * g^2) * m \\
& ^2 - 4 * ((84 * c^2 * d^2 * e^5 - 210 * b * c * d * e^6 - 949 * (b^2 + 2 * a * c) * e^7) * \\
& f^2 - 2 * (70 * c^2 * d^3 * e^4 - 168 * b * c * d^2 * e^5 + 1898 * a * b * e^7 + 105 * (b \\
& ^2 + 2 * a * c) * d * e^6) * f * g + (60 * c^2 * d^4 * e^3 - 140 * b * c * d^3 * e^4 - 210 * \\
& a * b * d * e^6 - 949 * a^2 * e^7 + 84 * (b^2 + 2 * a * c) * d^2 * e^5) * g^2) * m) * x^3 + \\
& 168 * (6 * c^2 * d^5 * e^2 - 15 * b * c * d^4 * e^3 - 30 * a * b * d^2 * e^5 + 30 * a^2 * d * \\
& e^6 + 10 * (b^2 + 2 * a * c) * d^3 * e^4) * f^2 - 168 * (10 * c^2 * d^6 * e - 24 * b * c * \\
& d^5 * e^2 - 40 * a * b * d^3 * e^4 + 30 * a^2 * d^2 * e^5 + 15 * (b^2 + 2 * a * c) * d^4 * \\
& e^3) * f * g + 24 * (30 * c^2 * d^7 - 70 * b * c * d^6 * e - 105 * a * b * d^4 * e^3 + 70 * a \\
& ^2 * d^3 * e^4 + 42 * (b^2 + 2 * a * c) * d^5 * e^2) * g^2 + 2 * ((12 * c^2 * d^5 * e^2 - \\
& 108 * b * c * d^4 * e^3 - 1175 * a * b * d^2 * e^5 + 2552 * a^2 * d * e^6 + 179 * (b^2 + \\
& 2 * a * c) * d^3 * e^4) * f^2 + (48 * b * c * d^5 * e^2 + 716 * a * b * d^3 * e^4 - 1175 * a \\
& ^2 * d^2 * e^5 - 108 * (b^2 + 2 * a * c) * d^4 * e^3) * f * g - (108 * a * b * d^4 * e^3 - \\
& 179 * a^2 * d^3 * e^4 - 12 * (b^2 + 2 * a * c) * d^5 * e^2) * g^2) * m^2 + (5040 * a * b * \\
& e^7 * f^2 + 5040 * a^2 * e^7 * f * g + (a^2 * d * e^6 * g^2 + (2 * a * b * e^7 + (b^2 + \\
& 2 * a * c) * d * e^6) * f^2 + 2 * (2 * a * b * d * e^6 + a^2 * e^7) * f * g) * m^6 - ((6 * b * c \\
& * d^2 * e^5 - 52 * a * b * e^7 - 23 * (b^2 + 2 * a * c) * d * e^6) * f^2 - 2 * (46 * a * b * d \\
& * e^6 + 26 * a^2 * e^7 - 3 * (b^2 + 2 * a * c) * d^2 * e^5) * f * g + (6 * a * b * d^2 * e^5 \\
& - 23 * a^2 * d * e^6) * g^2) * m^5 + 3 * ((4 * c^2 * d^3 * e^4 - 38 * b * c * d^2 * e^5 + \\
& 180 * a * b * e^7 + 67 * (b^2 + 2 * a * c) * d * e^6) * f^2 + 2 * (8 * b * c * d^3 * e^4 + 13 \\
& 4 * a * b * d * e^6 + 90 * a^2 * e^7 - 19 * (b^2 + 2 * a * c) * d^2 * e^5) * f * g - (38 * a * \\
& b * d^2 * e^5 - 67 * a^2 * d * e^6 - 4 * (b^2 + 2 * a * c) * d^3 * e^4) * g^2) * m^4 + (( \\
& 168 * c^2 * d^3 * e^4 - 750 * b * c * d^2 * e^5 + 2840 * a * b * e^7 + 817 * (b^2 + 2 * a \\
& * c) * d * e^6) * f^2 - 2 * (60 * c^2 * d^4 * e^3 - 336 * b * c * d^3 * e^4 - 1634 * a * b * d \\
& * e^6 - 1420 * a^2 * e^7 + 375 * (b^2 + 2 * a * c) * d^2 * e^5) * f * g - (120 * b * c * d \\
& ^4 * e^3 + 750 * a * b * d^2 * e^5 - 817 * a^2 * d * e^6 - 168 * (b^2 + 2 * a * c) * d^3 * \\
& e^4) * g^2) * m^3 + 2 * ((330 * c^2 * d^3 * e^4 - 951 * b * c * d^2 * e^5 + 3929 * a * b * \\
& e^7 + 739 * (b^2 + 2 * a * c) * d * e^6) * f^2 - (480 * c^2 * d^4 * e^3 - 1320 * b * c * \\
& d^3 * e^4 - 2956 * a * b * d * e^6 - 3929 * a^2 * e^7 + 951 * (b^2 + 2 * a * c) * d^2 * e \\
& ^5) * f * g + (180 * c^2 * d^5 * e^2 - 480 * b * c * d^4 * e^3 - 951 * a * b * d^2 * e^5 + \\
& 739 * a^2 * d * e^6 + 330 * (b^2 + 2 * a * c) * d^3 * e^4) * g^2) * m^2 + 12 * ((42 * c^2 \\
& * d^3 * e^4 - 105 * b * c * d^2 * e^5 + 879 * a * b * e^7 + 70 * (b^2 + 2 * a * c) * d * e^6) \\
& ) * f^2 - (70 * c^2 * d^4 * e^3 - 168 * b * c * d^3 * e^4 - 280 * a * b * d * e^6 - 879 * a \\
& ^2 * e^7 + 105 * (b^2 + 2 * a * c) * d^2 * e^5) * f * g + (30 * c^2 * d^5 * e^2 - 70 * b * \\
& c * d^4 * e^3 - 105 * a * b * d^2 * e^5 + 70 * a^2 * d * e^6 + 42 * (b^2 + 2 * a * c) * d^3 \\
& * e^4) * g^2) * m) * x^2 + 4 * ((78 * c^2 * d^5 * e^2 - 321 * b * c * d^4 * e^3 - 1377 * a \\
& * b * d^2 * e^5 + 2007 * a^2 * d * e^6 + 319 * (b^2 + 2 * a * c) * d^3 * e^4) * f^2 - (6 \\
& 0 * c^2 * d^6 * e - 312 * b * c * d^5 * e^2 - 1276 * a * b * d^3 * e^4 + 1377 * a^2 * d^2 * e \\
& ^5 + 321 * (b^2 + 2 * a * c) * d^4 * e^3) * f * g - (60 * b * c * d^6 * e + 321 * a * b * d^4 \\
& * e^3 - 319 * a^2 * d^3 * e^4 - 78 * (b^2 + 2 * a * c) * d^5 * e^2) * g^2) * m + (5040
\end{aligned}$$

$$\begin{aligned}
& a^2 e^7 f^2 + (2 a^2 d e^6 f g + (2 a b d e^6 + a^2 e^7) f^2) m^6 - (2 a^2 d^2 e^5 g^2 - (50 a b d e^6 + 27 a^2 e^7 - 2 (b^2 + 2 a c) d^2 e^5) f^2 + 2 (4 a b d^2 e^5 - 25 a^2 d e^6) f g) m^5 + \\
& ((12 b^3 c d^3 e^4 + 490 a b d e^6 + 295 a^2 e^7 - 44 (b^2 + 2 a c) d^2 e^5) f^2 - 2 (88 a b d^2 e^5 - 245 a^2 d e^6 - 6 (b^2 + 2 a c) d^3 e^4) f g + 4 (3 a b d^3 e^4 - 11 a^2 d^2 e^5) g^2) m^4 - ((24 c^2 d^4 e^3 - 216 b^3 c d^3 e^4 - 2350 a b d e^6 - 1665 a^2 e^7 + 358 (b^2 + 2 a c) d^2 e^5) f^2 + 2 (48 b^3 c d^4 e^3 + 716 a b d^2 e^5 - 1175 a^2 d e^6 - 108 (b^2 + 2 a c) d^3 e^4) f g - 2 (108 a b d^3 e^4 - 179 a^2 d^2 e^5 - 12 (b^2 + 2 a c) d^4 e^3) g^2) m^3 - 4 ((78 c^2 d^4 e^3 - 321 b^3 c d^3 e^4 - 1377 a b d e^6 - 1276 a^2 e^7 + 319 (b^2 + 2 a c) d^2 e^5) f^2 - (60 c^2 d^5 e^2 - 312 b^3 c d^4 e^3 - 1276 a b d^2 e^5 + 1377 a^2 d e^6 + 321 (b^2 + 2 a c) d^3 e^4) f g - (60 b^3 c d^5 e^2 + 321 a b d^3 e^4 - 319 a^2 d^2 e^5 - 78 (b^2 + 2 a c) d^4 e^3) g^2) m^2 - 12 ((84 c^2 d^4 e^3 - 210 b^3 c d^3 e^4 - 420 a b d e^6 - 669 a^2 e^7 + 140 (b^2 + 2 a c) d^2 e^5) f^2 - 14 (10 c^2 d^5 e^2 - 24 b^3 c d^4 e^3 - 40 a b d^2 e^5 + 30 a^2 d e^6 + 15 (b^2 + 2 a c) d^3 e^4) f g + 2 (30 c^2 d^6 e - 70 b^3 c d^5 e^2 - 105 a b d^3 e^4 + 70 a^2 d^2 e^5 + 42 (b^2 + 2 a c) d^4 e^3) g^2) m) x) (e x + d)^m / (e^7 m^7 + 28 e^7 m^6 + 322 e^7 m^5 + 1960 e^7 m^4 + 6769 e^7 m^3 + 13132 e^7 m^2 + 13068 e^7 m + 5040 e^7)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*m\*(g\*x+f)\*\*2\*(c\*x\*\*2+b\*x+a)\*\*2,x)

[Out] Timed out

**GIAC/XCAS [A]** time = 0.299408, size = 1, normalized size = 0.

Done

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)^2\*(g\*x + f)^2\*(e\*x + d)^m,x, algorithm="giac")

[Out] Done

$$3.926 \quad \int (d + ex)^m (f + gx) (a + bx + cx^2)^2 dx$$

**Optimal.** Leaf size=311

$$\begin{aligned} & \frac{(d + ex)^{m+4} (2ce(aeg - 4bdg + bef) + b^2e^2g - 2c^2d(2ef - 5dg))}{e^6(m + 4)} \\ & + \frac{(d + ex)^{m+3} (2ce(ae(ef - 3dg) - 3bd(ef - 2dg)) + be^2(2aeg - 3bdg + bef) + 2c^2d^2(3ef - 5dg))}{e^6(m + 3)} \\ & + \frac{(ef - dg)(d + ex)^{m+1} (ae^2 - bde + cd^2)^2}{e^6(m + 1)} \\ & - \frac{(d + ex)^{m+2} (ae^2 - bde + cd^2) (cd(4ef - 5dg) - e(aeg - 3bdg + 2bef))}{e^6(m + 2)} \\ & + \frac{c(d + ex)^{m+5} (2beg - 5cdg + cef)}{e^6(m + 5)} + \frac{c^2g(d + ex)^{m+6}}{e^6(m + 6)} \end{aligned}$$

[Out]  $((c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*(d + e*x)^(1 + m))/(e^6*(1 + m)) - ((c*d^2 - b*d*e + a*e^2)*(c*d*(4*e*f - 5*d*g) - e*(2*b*e*f - 3*b*d*g + a*e*g))*(d + e*x)^(2 + m))/(e^6*(2 + m)) + ((2*c^2*d^2*(3*e*f - 5*d*g) + b*e^2*(b*e*f - 3*b*d*g + 2*a*e*g) + 2*c*e*(a*e*(e*f - 3*d*g) - 3*b*d*(e*f - 2*d*g)))*(d + e*x)^(3 + m))/(e^6*(3 + m)) + ((b^2*e^2*g - 2*c^2*d*(2*e*f - 5*d*g) + 2*c*e*(b*e*f - 4*b*d*g + a*e*g))*(d + e*x)^(4 + m))/(e^6*(4 + m)) + (c*(c*e*f - 5*c*d*g + 2*b*e*g)*(d + e*x)^(5 + m))/(e^6*(5 + m)) + (c^2*g*(d + e*x)^(6 + m))/(e^6*(6 + m))$

**Rubi [A]** time = 0.983522, antiderivative size = 311, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.04$

$$\begin{aligned} & \frac{(d + ex)^{m+4} (2ce(aeg - 4bdg + bef) + b^2e^2g - 2c^2d(2ef - 5dg))}{e^6(m + 4)} \\ & + \frac{(d + ex)^{m+3} (2ce(ae(ef - 3dg) - 3bd(ef - 2dg)) + be^2(2aeg - 3bdg + bef) + 2c^2d^2(3ef - 5dg))}{e^6(m + 3)} \\ & + \frac{(ef - dg)(d + ex)^{m+1} (ae^2 - bde + cd^2)^2}{e^6(m + 1)} \\ & - \frac{(d + ex)^{m+2} (ae^2 - bde + cd^2) (cd(4ef - 5dg) - e(aeg - 3bdg + 2bef))}{e^6(m + 2)} \\ & + \frac{c(d + ex)^{m+5} (2beg - 5cdg + cef)}{e^6(m + 5)} + \frac{c^2g(d + ex)^{m+6}}{e^6(m + 6)} \end{aligned}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^2, x]$

[Out]  $((c*d^2 - b*d*e + a*e^2)^2*(e*f - d*g)*(d + e*x)^(1 + m))/(e^6*(1 + m)) - ((c*d^2 - b*d*e + a*e^2)*(c*d*(4*e*f - 5*d*g) - e*(2*b*e*f - 3*b*d*g + a*e*g))*(d + e*x)^(2 + m))/(e^6*(2 + m)) + ((2*c^2*d^2*(3*e*f - 5*d*g) + b*e^2*(b*e*f - 3*b*d*g + 2*a*e*g) + 2*c*e*(a*e*(e*f - 3*d*g) - 3*b*d*(e*f - 2*d*g)))*(d + e*x)^(3 + m))/(e^6*(3 + m)) + ((b^2*e^2*g - 2*c^2*d*(2*e*f - 5*d*g) + 2*c*e*(b*e*f - 4*b*d*g + a*e*g))*(d + e*x)^(4 + m))/(e^6*(4 + m)) + (c*(c*e*f - 5*c*d*g + 2*b*e*g)*(d + e*x)^(5 + m))/(e^6*(5 + m)) + (c^2*g*(d + e*x)^(6 + m))/(e^6*(6 + m))$

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**m*(g*x+f)*(c*x**2+b*x+a)**2,x)`

[Out] Timed out

**Mathematica [A]** time = 1.70122, size = 593, normalized size = 1.91

$(d + ex)^{m+1} (e^2 (m^2 + 11m + 30) (a^2 e^2 (m^2 + 7m + 12) (-dg + ef(m+2) + eg(m+1)x) + 2abe(m+4) (2d^2g - de(f(m+3) -$

Antiderivative was successfully verified.

[In] `Integrate[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^2,x]`

[Out]  $((d + e*x)^{(1+m)} * (-c^2 * (120*d^5*g - 24*d^4*e*(f*(6+m) + 5*g*(1+m)*x) + 12*d^3*e^2*(1+m)*x*(2*f*(6+m) + 5*g*(2+m)*x) - 4*d^2*e^3*(2+3*m+m^2)*x^2*(3*f*(6+m) + 5*g*(3+m)*x) + d*e^4*(6+11*m+6*m^2+m^3)*x^3*(4*f*(6+m) + 5*g*(4+m)*x) - e^5*(24+50*m+35*m^2+10*m^3+m^4)*x^4*(f*(6+m) + g*(5+m)*x))) + e^2*(30+11*m+m^2)*(a^2*e^2*(12+7*m+m^2)*(-d*g + e*f*(2+m) + e*g*(1+m)*x) + 2*a*b*e*(4+m)*(2*d^2*g - d*e*(f*(3+m) + 2*g*(1+m)*x) + e^2*(1+m)*x*(f*(3+m) + g*(2+m)*x)) + b^2*(-6*d^3*g + 2*d^2*e*(f*(4+m) + 3*g*(1+m)*x) - d*e^2*(1+m)*x*(2*f*(4+m) + 3*g*(2+m)*x) + e^3*(2+3*m+m^2)*x^2*(f*(4+m) + g*(3+m)*x))) + 2*c*e*(6+m)*(a*e*(5+m)*(-6*d^3*g + 2*d^2*e*(f*(4+m) + 3*g*(1+m)*x) - d*e^2*(1+m)*x*(2*f*(4+m) + 3*g*(2+m)*x) + e^3*(2+3*m+m^2)*x^2*(f*(4+m) + g*(3+m)*x)) + b*(24*d^4*g - 6*d^3*e*(f*(5+m) + 4*g*(1+m)*x) + 6*d^2*e^2*(1+m)*x*(f*(5+m) + 2*g*(2+m)*x) - d*e^3*(2+3*m+m^2)*x^2*(3*f*(5+m) + 4*g*(3+m)*x) + e^4*(6+11*m+6*m^2+m^3)*x^3*(f*(5+m) + g*(4+m)*x)))))/(e^6*(1+m)*(2+m)*(3+m)*(4+m)*(5+m)*(6+m))$

**Maple [B]** time = 0.019, size = 2563, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^2,x)`

[Out]  $-(e*x+d)^{(1+m)} * (-c^2 * e^5 * g * m^5 * x^5 - 2 * b * c * e^5 * g * m^5 * x^4 - c^2 * e^5 * f * m^5 * x^4 - 15 * c^2 * e^5 * g * m^4 * x^5 - 2 * a * c * e^5 * g * m^5 * x^3 - b^2 * e^5 * g * m^5 * x^4 - 3 - 2 * b * c * e^5 * f * m^5 * x^3 - 32 * b * c * e^5 * g * m^4 * x^4 + 5 * c^2 * d * e^4 * g * m^4 * x^4 - 16 * c^2 * e^5 * f * m^4 * x^4 - 85 * c^2 * e^5 * g * m^3 * x^5 - 2 * a * b * e^5 * g * m^5 * x^2 - 2 * a * c * e^5 * f * m^5 * x^2 - 34 * a * c * e^5 * g * m^4 * x^3 - b^2 * e^5 * f * m^5 * x^2 - 17 * b^2 * e^5 * g * m^4 * x^3 + 8 * b * c * d * e^4 * g * m^4 * x^3 - 34 * b * c * e^5 * f * m^4 * x^3 - 190 * b * c * e^5 * g * m^3 * x^4 + 4 * c^2 * d * e^4 * f * m^4 * x^3 + 50 * c^2 * d * e^4 * g * m^3 * x^4 - 95 * c^2 * e^5 * f * m^3 * x^4 - 225 * c^2 * e^5 * g * m^2 * x^5 - a^2 * e^5 * g * m^5 * x - 2 * a * b * e^5 * f * m^5 * x - 36 * a * b * e^5 * g * m^4 * x^2 + 6 * a * c * d * e^4 * g * m^4 * x^2 - 36 * a * c * e^5 * f * m^4 * x^2 - 214 * a * c * e^5 * g * m^3 * x^3 + 3 * b^2 * d * e^4 * g * m^4 * x^2 - 18 * b^2 * e^5 * f * m^4 * x^2 - 107 * b^2 * e^5 * g * m^3 * x^3 + 6 * b * c * d * e^4 * f * m^4 * x^2 + 96 * b * c * d * e^4 * g * m^3 * x^3 - 214 * b * c * e^5 * f * m^3 * x^3 - 520 * b * c * e^5 * g * m^2 * x^4 - 20 * c^2 * d^2 * e^3 * g * m^3 * x^3 + 48 * c^2 * d * e^4 * f * m^3 * x^3 + 175 * c^2 * d * e^4 * g * m^2 * x^4 - 260 * c^2 * e^5 * f * m^2 * x^4 - 274 * c^2 * e^5 * g * m * x^5 - a^2 * e^5 * f * m^5 - 19 * a^2 * e^5 * g * m^4 * x + 4 * a * b * d * e^4 * g * m^4 * x - 38 * a * b * e^5 * f * m^4 * x - 242 * a * b * e^5 * g * m^3 * x^2 + 4 * a * c * d * e^4 * f * m^4 * x + 84 * a * c * d * e^4 * g * m^3 * x^2 - 242 * a * c * e^5 * f * m^3 * x^2 - 614 * a * c * e^5 * g * m^2 * x^3 + 2 * b^2 * d * e^4 * f * m^4 * x + 42 * b^2 * d * e^4 * g * m^3 * x^2 - 121 * b^2 * e^5 * f * m^3 * x^2 - 307 * b^2 * e^5 * g * m^2 * x^3 - 24 * b * c * d^2 * e^3 * g * m^3 * x^2 + 84 * b * c * d * e^4 * f * m^3 * x^2 + 376 * b * c * d * e^4 * g * m^2 * x^3 - 614 * b * c * e^5 * f * m^2$

```

*x^3-648*b*c*e^5*g*m*x^4-12*c^2*d^2*e^3*f*m^3*x^2-120*c^2*d^2*e^3
*g*m^2*x^3+188*c^2*d*e^4*f*m^2*x^3+250*c^2*d*e^4*g*m*x^4-324*c^2*
e^5*f*m*x^4-120*c^2*e^5*g*x^5+a^2*d*e^4*g*m^4-20*a^2*e^5*f*m^4-13
7*a^2*e^5*g*m^3*x+2*a*b*d*e^4*f*m^4+64*a*b*d*e^4*g*m^3*x-274*a*b*
e^5*f*m^3*x-744*a*b*e^5*g*m^2*x^2-12*a*c*d^2*e^3*g*m^3*x+64*a*c*d
*e^4*f*m^3*x+390*a*c*d*e^4*g*m^2*x^2-744*a*c*e^5*f*m^2*x^2-792*a*
c*e^5*g*m*x^3-6*b^2*d^2*e^3*g*m^3*x+32*b^2*d*e^4*f*m^3*x+195*b^2*
d*e^4*g*m^2*x^2-372*b^2*e^5*f*m^2*x^2-396*b^2*e^5*g*m*x^3-12*b*c*
d^2*e^3*f*m^3*x-216*b*c*d^2*e^3*g*m^2*x^2+390*b*c*d*e^4*f*m^2*x^2
+576*b*c*d*e^4*g*m*x^3-792*b*c*e^5*f*m*x^3-288*b*c*e^5*g*x^4+60*c
^2*d^3*e^2*g*m^2*x^2-108*c^2*d^2*e^3*f*m^2*x^2-220*c^2*d^2*e^3*g*
m*x^3+288*c^2*d*e^4*f*m*x^3+120*c^2*d*e^4*g*x^4-144*c^2*e^5*f*x^4
+18*a^2*d*e^4*g*m^3-155*a^2*e^5*f*m^3-461*a^2*e^5*g*m^2*x-4*a*b*d
^2*e^3*g*m^3+36*a*b*d*e^4*f*m^3+356*a*b*d*e^4*g*m^2*x-922*a*b*e^5
*f*m^2*x-1016*a*b*e^5*g*m*x^2-4*a*c*d^2*e^3*f*m^3-144*a*c*d^2*e^3
*g*m^2*x+356*a*c*d*e^4*f*m^2*x+672*a*c*d*e^4*g*m*x^2-1016*a*c*e^5
*f*m*x^2-360*a*c*e^5*g*x^3-2*b^2*d^2*e^3*f*m^3-72*b^2*d^2*e^3*g*m
^2*x+178*b^2*d*e^4*f*m^2*x+336*b^2*d*e^4*g*m*x^2-508*b^2*e^5*f*m*
x^2-180*b^2*e^5*g*x^3+48*b*c*d^3*e^2*g*m^2*x-144*b*c*d^2*e^3*f*m^
2*x-480*b*c*d^2*e^3*g*m*x^2+672*b*c*d*e^4*f*m*x^2+288*b*c*d*e^4*g
*x^3-360*b*c*e^5*f*x^3+24*c^2*d^3*e^2*f*m^2*x+180*c^2*d^3*e^2*g*m
*x^2-240*c^2*d^2*e^3*f*m*x^2-120*c^2*d^2*e^3*g*x^3+144*c^2*d*e^4*
f*x^3+119*a^2*d*e^4*g*m^2-580*a^2*e^5*f*m^2-702*a^2*e^5*g*m*x-60*
a*b*d^2*e^3*g*m^2+238*a*b*d*e^4*f*m^2+776*a*b*d*e^4*g*m*x-1404*a*
b*e^5*f*m*x-480*a*b*e^5*g*x^2+12*a*c*d^3*e^2*g*m^2-60*a*c*d^2*e^3
*f*m^2-492*a*c*d^2*e^3*g*m*x+776*a*c*d*e^4*f*m*x+360*a*c*d*e^4*g*
x^2-480*a*c*e^5*f*x^2+6*b^2*d^3*e^2*g*m^2-30*b^2*d^2*e^3*f*m^2-24
6*b^2*d^2*e^3*g*m*x+388*b^2*d*e^4*f*m*x+180*b^2*d*e^4*g*x^2-240*b
^2*e^5*f*x^2+12*b*c*d^3*e^2*f*m^2+336*b*c*d^3*e^2*g*m*x-492*b*c*d
^2*e^3*f*m*x-288*b*c*d^2*e^3*g*x^2+360*b*c*d*e^4*f*x^2-120*c^2*d^
4*e*g*m*x+168*c^2*d^3*e^2*f*m*x+120*c^2*d^3*e^2*g*x^2-144*c^2*d^2
*e^3*f*x^2+342*a^2*d*e^4*g*m-1044*a^2*e^5*f*m-360*a^2*e^5*g*x-296
*a*b*d^2*e^3*g*m+684*a*b*d*e^4*f*m+480*a*b*d*e^4*g*x-720*a*b*e^5*
f*x+132*a*c*d^3*e^2*g*m-296*a*c*d^2*e^3*f*m-360*a*c*d^2*e^3*g*x+4
80*a*c*d*e^4*f*x+66*b^2*d^3*e^2*g*m-148*b^2*d^2*e^3*f*m-180*b^2*d
^2*e^3*g*x+240*b^2*d*e^4*f*x-48*b*c*d^4*e*g*m+132*b*c*d^3*e^2*f*m
+288*b*c*d^3*e^2*g*x-360*b*c*d^2*e^3*f*x-24*c^2*d^4*e*f*m-120*c^2
*d^4*e*g*x+144*c^2*d^3*e^2*f*x+360*a^2*d*e^4*g-720*a^2*e^5*f-480*
a*b*d^2*e^3*g+720*a*b*d*e^4*f+360*a*c*d^3*e^2*g-480*a*c*d^2*e^3*f
+180*b^2*d^3*e^2*g-240*b^2*d^2*e^3*f-288*b*c*d^4*e*g+360*b*c*d^3*
e^2*f+120*c^2*d^5*g-144*c^2*d^4*e*f)/e^6/(m^6+21*m^5+175*m^4+735*
m^3+1624*m^2+1764*m+720)

```

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)^2\*(g\*x + f)\*(e\*x + d)^m,x, algorithm="maxima")

[Out] Exception raised: ValueError

---

**Fricas [A]** time = 0.320257, size = 3197, normalized size = 10.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)^2\*(g\*x + f)\*(e\*x + d)^m,x, algorithm="fricas")

[Out] (a^2\*d\*e^5\*f\*m^5 + (c^2\*e^6\*g\*m^5 + 15\*c^2\*e^6\*g\*m^4 + 85\*c^2\*e^6\*
g\*m^3 + 225\*c^2\*e^6\*g\*m^2 + 274\*c^2\*e^6\*g\*m + 120\*c^2\*e^6\*g)\*x^6

$$\begin{aligned}
& + (144*c^2*e^6*f + 288*b*c*e^6*g + (c^2*e^6*f + (c^2*d*e^5 + 2*b*c*e^6)*g)*m^5 + 2*(8*c^2*e^6*f + (5*c^2*d*e^5 + 16*b*c*e^6)*g)*m^4 \\
& + 5*(19*c^2*e^6*f + (7*c^2*d*e^5 + 38*b*c*e^6)*g)*m^3 + 10*(26*c^2*e^6*f + (5*c^2*d*e^5 + 52*b*c*e^6)*g)*m^2 + 12*(27*c^2*e^6*f + 2*(c^2*d*e^5 + 27*b*c*e^6)*g)*m)*x^5 - (a^2*d^2*e^4*g + 2*(a*b*d^2*e^4 - 10*a^2*d*e^5)*f)*m^4 \\
& + (360*b*c*e^6*f + 180*(b^2 + 2*a*c)*e^6*g + ((c^2*d*e^5 + 2*b*c*e^6)*f + (2*b*c*d*e^5 + (b^2 + 2*a*c)*e^6)*g)*m^5 + (2*(6*c^2*d*e^5 + 17*b*c*e^6)*f - (5*c^2*d^2*e^4 - 24*b*c*d*e^5 - 17*(b^2 + 2*a*c)*e^6)*g)*m^4 \\
& + ((47*c^2*d*e^5 + 214*b*c*e^6)*f - (30*c^2*d^2*e^4 - 94*b*c*d*e^5 - 107*(b^2 + 2*a*c)*e^6)*g)*m^3 + (2*(36*c^2*d*e^5 + 307*b*c*e^6)*f - (55*c^2*d^2*e^4 - 144*b*c*d*e^5 - 307*(b^2 + 2*a*c)*e^6)*g)*m^2 \\
& + 6*(6*(c^2*d*e^5 + 22*b*c*e^6)*f - (5*c^2*d^2*e^4 - 12*b*c*d*e^5 - 66*(b^2 + 2*a*c)*e^6)*g)*m)*x^4 - ((36*a*b*d^2*e^4 - 155*a^2*d*e^5 - 2*(b^2 + 2*a*c)*d^3*e^3)*f - 2*(2*a*b*d^3*e^3 - 9*a^2*d^2*e^4)*g)*m^3 \\
& + (480*a*b*e^6*g + 240*(b^2 + 2*a*c)*e^6*f + ((2*b*c*d*e^5 + (b^2 + 2*a*c)*e^6)*f + (2*a*b*e^6 + (b^2 + 2*a*c)*d*e^5)*g)*m^5 - 2*((2*c^2*d^2*e^4 - 14*b*c*d*e^5 - 9*(b^2 + 2*a*c)*e^6)*f + (4*b*c*d^2*e^4 - 18*a*b*e^6 - 7*(b^2 + 2*a*c)*d*e^5)*g)*m^4 \\
& - ((36*c^2*d^2*e^4 - 130*b*c*d*e^5 - 121*(b^2 + 2*a*c)*e^6)*f - (20*c^2*d^3*e^3 - 72*b*c*d^2*e^4 + 242*a*b*e^6 + 65*(b^2 + 2*a*c)*d*e^5)*g)*m^3 - 4*((20*c^2*d^2*e^4 - 56*b*c*d*e^5 - 93*(b^2 + 2*a*c)*e^6)*f - (15*c^2*d^3*e^3 - 40*b*c*d^2*e^4 + 186*a*b*e^6 + 28*(b^2 + 2*a*c)*d*e^5)*g)*m^2 \\
& - 4*((12*c^2*d^2*e^4 - 30*b*c*d*e^5 - 127*(b^2 + 2*a*c)*e^6)*f - (10*c^2*d^3*e^3 - 24*b*c*d^2*e^4 + 254*a*b*e^6 + 15*(b^2 + 2*a*c)*d*e^5)*g)*m)*x^3 - (2*(6*b*c*d^4*e^2 + 119*a*b*d^2*e^4 - 290*a^2*d*e^5 - 15*(b^2 + 2*a*c)*d^3*e^3)*f - (60*a*b*d^3*e^3 - 119*a^2*d^2*e^4 - 6*(b^2 + 2*a*c)*d^4*e^2)*g)*m^2 \\
& + (720*a*b*e^6*f + 360*a^2*e^6*g + ((2*a*b*e^6 + (b^2 + 2*a*c)*d*e^5)*f + (2*a*b*d*e^5 + a^2*e^6)*g)*m^5 - (2*(3*b*c*d^2*e^4 - 19*a*b*e^6 - 8*(b^2 + 2*a*c)*d*e^5)*f - (32*a*b*d^2*e^5 + 19*a^2*e^6 - 3*(b^2 + 2*a*c)*d^2*e^4)*g)*m^4 \\
& + ((12*c^2*d^3*e^3 - 72*b*c*d^2*e^4 + 274*a*b*e^6 + 89*(b^2 + 2*a*c)*d*e^5)*f + (24*b*c*d^3*e^3 + 178*a*b*d^2*e^5 + 137*a^2*e^6 - 36*(b^2 + 2*a*c)*d^2*e^4)*g)*m^3 + (2*(42*c^2*d^3*e^3 - 123*b*c*d^2*e^4 + 461*a*b*e^6 + 97*(b^2 + 2*a*c)*d*e^5)*f - (60*c^2*d^4*e^2 - 168*b*c*d^3*e^3 - 388*a*b*d^2*e^5 - 461*a^2*e^6 + 123*(b^2 + 2*a*c)*d^2*e^4)*g)*m^2 \\
& + 6*(2*(6*c^2*d^3*e^3 - 15*b*c*d^2*e^4 + 117*a*b*e^6 + 10*(b^2 + 2*a*c)*d*e^5)*f - (10*c^2*d^4*e^2 - 24*b*c*d^3*e^3 - 40*a*b*d^2*e^5 - 117*a^2*e^6 + 15*(b^2 + 2*a*c)*d^2*e^4)*g)*m)*x^2 + 24*(6*c^2*d^5*e - 15*b*c*d^4*e^2 - 30*a*b*d^2*e^4 + 30*a^2*d*e^5 + 10*(b^2 + 2*a*c)*d^3*e^3)*f - 12*(10*c^2*d^6 - 24*b*c*d^5*e - 40*a*b*d^3*e^3 + 30*a^2*d^2*e^4 + 15*(b^2 + 2*a*c)*d^4*e^2)*g + 2*(2*(6*c^2*d^5*e - 33*b*c*d^4*e^2 - 171*a*b*d^2*e^4 + 261*a^2*d*e^5 + 37*(b^2 + 2*a*c)*d^3*e^3)*f + (24*b*c*d^5*e + 148*a*b*d^3*e^3 - 171*a^2*d^2*e^4 - 33*(b^2 + 2*a*c)*d^4*e^2)*g)*m + (720*a^2*e^6*f + (a^2*d*e^5*g + (2*a*b*d^2*e^5 + a^2*e^6)*f)*m^5 + 2*((18*a*b*d^2*e^5 + 10*a^2*e^6 - (b^2 + 2*a*c)*d^2*e^4)*f - (2*a*b*d^2*e^4 - 9*a^2*d*e^5)*g)*m^4 + ((12*b*c*d^3*e^3 + 238*a*b*d^2*e^5 + 155*a^2*e^6 - 30*(b^2 + 2*a*c)*d^2*e^4)*f - (60*a*b*d^2*e^4 - 119*a^2*d*e^5 - 6*(b^2 + 2*a*c)*d^3*e^3)*g)*m^3 - 2*(2*(6*c^2*d^4*e^2 - 33*b*c*d^3*e^3 - 171*a*b*d^2*e^5 - 145*a^2*e^6 + 37*(b^2 + 2*a*c)*d^2*e^4)*f + (24*b*c*d^4*e^2 + 148*a*b*d^2*e^4 - 171*a^2*d*e^5 - 33*(b^2 + 2*a*c)*d^3*e^3)*g)*m^2 - 12*((12*c^2*d^4*e^2 - 30*b*c*d^3*e^3 - 60*a*b*d^2*e^5 - 87*a^2*e^6 + 20*(b^2 + 2*a*c)*d^2*e^4)*f - (10*c^2*d^5*e - 24*b*c*d^4*e^2 - 40*a*b*d^2*e^4 + 30*a^2*d*e^5 + 15*(b^2 + 2*a*c)*d^3*e^3)*g)*m)*x*(e*x + d)^m/(e^6*m^6 + 21*e^6*m^5 + 175*e^6*m^4 + 735*e^6*m^3 + 1624*e^6*m^2 + 1764*e^6*m + 720*e^6)
\end{aligned}$$

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*m\*(g\*x+f)\*(c\*x\*\*2+b\*x+a)\*\*2,x)

[Out] Timed out

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**GIAC/XCAS [A]** time = 0.280473, size = 1, normalized size = 0.

*Done*

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^2*(g*x + f)*(e*x + d)^m,x, algorithm="giac")`

[Out] Done

$$3.927 \quad \int \frac{(d+ex)^m (a+bx+cx^2)^2}{f+gx} dx$$

**Optimal.** Leaf size=287

$$\begin{aligned} & \frac{(d+ex)^{m+2} (2ceg(aeg-b(2dg+ef)) + b^2e^2g^2 + c^2(3d^2g^2 + 2defg + e^2f^2))}{e^4g^3(m+2)} \\ & + \frac{(d+ex)^{m+1}(beg-c(dg+ef))(eg(2aeg-b(dg+ef)) + c(d^2g^2 + e^2f^2))}{e^4g^4(m+1)} \\ & + \frac{(d+ex)^{m+1} (ag^2 - bfg + cf^2)^2 {}_2F_1\left(1, m+1; m+2; -\frac{g(d+ex)}{ef-dg}\right)}{g^4(m+1)(ef-dg)} \\ & - \frac{c(d+ex)^{m+3}(-2beg+3cdg+cef)}{e^4g^2(m+3)} + \frac{c^2(d+ex)^{m+4}}{e^4g(m+4)} \end{aligned}$$

[Out]  $((b^*e^*g - c^*(e^*f + d^*g))^*(c^*(e^{\wedge}2*f^{\wedge}2 + d^{\wedge}2*g^{\wedge}2) + e^*g^*(2*a^*e^*g - b^*(e^*f + d^*g))))*(d + e^*x)^{\wedge}(1 + m))/(e^{\wedge}4*g^{\wedge}4*(1 + m)) + ((b^{\wedge}2*e^{\wedge}2*g^{\wedge}2 + c^{\wedge}2*(e^{\wedge}2*f^{\wedge}2 + 2*d^*e^*f*g + 3*d^{\wedge}2*g^{\wedge}2) + 2*c^*e^*g*(a^*e^*g - b^*(e^*f + 2*d^*g))))*(d + e^*x)^{\wedge}(2 + m))/(e^{\wedge}4*g^{\wedge}3*(2 + m)) - (c^*(c^*e^*f + 3*c^*d^*g - 2*b^*e^*g)*(d + e^*x)^{\wedge}(3 + m))/(e^{\wedge}4*g^{\wedge}2*(3 + m)) + (c^{\wedge}2*(d + e^*x)^{\wedge}(4 + m))/(e^{\wedge}4*g^*(4 + m)) + ((c^*f^{\wedge}2 - b^*f*g + a^*g^{\wedge}2)^{\wedge}2*(d + e^*x)^{\wedge}(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((g^*(d + e^*x))/(e^*f - d^*g))])/((g^{\wedge}4*(e^*f - d^*g)^*(1 + m))$

**Rubi [A]** time = 0.925796, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$

$$\begin{aligned} & \frac{(d+ex)^{m+2} (2ceg(aeg-b(2dg+ef)) + b^2e^2g^2 + c^2(3d^2g^2 + 2defg + e^2f^2))}{e^4g^3(m+2)} \\ & + \frac{(d+ex)^{m+1}(beg-c(dg+ef))(eg(2aeg-b(dg+ef)) + c(d^2g^2 + e^2f^2))}{e^4g^4(m+1)} \\ & + \frac{(d+ex)^{m+1} (ag^2 - bfg + cf^2)^2 {}_2F_1\left(1, m+1; m+2; -\frac{g(d+ex)}{ef-dg}\right)}{g^4(m+1)(ef-dg)} \\ & - \frac{c(d+ex)^{m+3}(-2beg+3cdg+cef)}{e^4g^2(m+3)} + \frac{c^2(d+ex)^{m+4}}{e^4g(m+4)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^m\*(a + b\*x + c\*x^2)^2)/(f + g\*x), x]

[Out]  $((b^*e^*g - c^*(e^*f + d^*g))^*(c^*(e^{\wedge}2*f^{\wedge}2 + d^{\wedge}2*g^{\wedge}2) + e^*g^*(2*a^*e^*g - b^*(e^*f + d^*g))))*(d + e^*x)^{\wedge}(1 + m))/(e^{\wedge}4*g^{\wedge}4*(1 + m)) + ((b^{\wedge}2*e^{\wedge}2*g^{\wedge}2 + c^{\wedge}2*(e^{\wedge}2*f^{\wedge}2 + 2*d^*e^*f*g + 3*d^{\wedge}2*g^{\wedge}2) + 2*c^*e^*g*(a^*e^*g - b^*(e^*f + 2*d^*g))))*(d + e^*x)^{\wedge}(2 + m))/(e^{\wedge}4*g^{\wedge}3*(2 + m)) - (c^*(c^*e^*f + 3*c^*d^*g - 2*b^*e^*g)*(d + e^*x)^{\wedge}(3 + m))/(e^{\wedge}4*g^{\wedge}2*(3 + m)) + (c^{\wedge}2*(d + e^*x)^{\wedge}(4 + m))/(e^{\wedge}4*g^*(4 + m)) + ((c^*f^{\wedge}2 - b^*f*g + a^*g^{\wedge}2)^{\wedge}2*(d + e^*x)^{\wedge}(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, -((g^*(d + e^*x))/(e^*f - d^*g))])/((g^{\wedge}4*(e^*f - d^*g)^*(1 + m))$

**Rubi in Sympy [F-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*m\*(c\*x\*\*2+b\*x+a)\*\*2/(g\*x+f), x)



[Out] Timed out

**Mathematica [C]** time = 6.9339, size = 733, normalized size = 2.55

$$\frac{a^2 \left( d + \frac{e(f+gx)}{g} - \frac{ef}{g} \right)^m \left( \frac{g(d-\frac{ef}{g})}{e(f+gx)} + 1 \right)^{-m} {}_2F_1 \left( -m, -m; 1-m; -\frac{(d-\frac{ef}{g})g}{e(f+gx)} \right)}{gm} + \frac{3abdfx^2(d+ex)^m F_1 \left( 2; -m, 1; 3; -\frac{ex}{d}, -\frac{gx}{f} \right)}{(f+gx) \left( 3dfF_1 \left( 2; -m, 1; 3; -\frac{ex}{d}, -\frac{gx}{f} \right) + efmxF_1 \left( 3; 1-m, 1; 4; -\frac{ex}{d}, -\frac{gx}{f} \right) - dgxF_1 \left( 3; -m, 2; 4; -\frac{ex}{d}, -\frac{gx}{f} \right) \right)} + \frac{8acdfx^3(d+ex)^m F_1 \left( 3; -m, 1; 4; -\frac{ex}{d}, -\frac{gx}{f} \right)}{3(f+gx) \left( 4dfF_1 \left( 3; -m, 1; 4; -\frac{ex}{d}, -\frac{gx}{f} \right) + efmxF_1 \left( 4; 1-m, 1; 5; -\frac{ex}{d}, -\frac{gx}{f} \right) - dgxF_1 \left( 4; -m, 2; 5; -\frac{ex}{d}, -\frac{gx}{f} \right) \right)} + \frac{4b^2dfx^3(d+ex)^m F_1 \left( 3; -m, 1; 4; -\frac{ex}{d}, -\frac{gx}{f} \right)}{3(f+gx) \left( 4dfF_1 \left( 3; -m, 1; 4; -\frac{ex}{d}, -\frac{gx}{f} \right) + efmxF_1 \left( 4; 1-m, 1; 5; -\frac{ex}{d}, -\frac{gx}{f} \right) - dgxF_1 \left( 4; -m, 2; 5; -\frac{ex}{d}, -\frac{gx}{f} \right) \right)} + \frac{5bcdfx^4(d+ex)^m F_1 \left( 4; -m, 1; 5; -\frac{ex}{d}, -\frac{gx}{f} \right)}{2(f+gx) \left( 5dfF_1 \left( 4; -m, 1; 5; -\frac{ex}{d}, -\frac{gx}{f} \right) + efmxF_1 \left( 5; 1-m, 1; 6; -\frac{ex}{d}, -\frac{gx}{f} \right) - dgxF_1 \left( 5; -m, 2; 6; -\frac{ex}{d}, -\frac{gx}{f} \right) \right)} + \frac{6c^2dfx^5(d+ex)^m F_1 \left( 5; -m, 1; 6; -\frac{ex}{d}, -\frac{gx}{f} \right)}{5(f+gx) \left( 6dfF_1 \left( 5; -m, 1; 6; -\frac{ex}{d}, -\frac{gx}{f} \right) + efmxF_1 \left( 6; 1-m, 1; 7; -\frac{ex}{d}, -\frac{gx}{f} \right) - dgxF_1 \left( 6; -m, 2; 7; -\frac{ex}{d}, -\frac{gx}{f} \right) \right)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(((d + e\*x)^m\*(a + b\*x + c\*x^2)^2)/(f + g\*x), x]

[Out]  $(3*a*b*d*f*x^2*(d + e*x)^m*AppellF1[2, -m, 1, 3, -((e*x)/d), -((g*x)/f)]/(f + g*x) + e*f*m*x*AppellF1[3, 1 - m, 1, 4, -((e*x)/d), -((g*x)/f)] - d*g*x*AppellF1[3, -m, 2, 4, -((e*x)/d), -((g*x)/f)]) + (4*b^2*d*f*x^3*(d + e*x)^m*AppellF1[3, -m, 1, 4, -((e*x)/d), -((g*x)/f)]/(3*(f + g*x) + e*f*m*x*AppellF1[4, 1 - m, 1, 5, -((e*x)/d), -((g*x)/f)] - d*g*x*AppellF1[4, -m, 2, 5, -((e*x)/d), -((g*x)/f)]) + (8*a*c*d*f*x^3*(d + e*x)^m*AppellF1[3, -m, 1, 4, -((e*x)/d), -((g*x)/f)]/(3*(f + g*x) + e*f*m*x*AppellF1[4, 1 - m, 1, 5, -((e*x)/d), -((g*x)/f)] - d*g*x*AppellF1[4, -m, 2, 5, -((e*x)/d), -((g*x)/f)]) + (5*b*c*d*f*x^4*(d + e*x)^m*AppellF1[4, -m, 1, 5, -((e*x)/d), -((g*x)/f)]/(2*(f + g*x) + e*f*m*x*AppellF1[5, 1 - m, 1, 6, -((e*x)/d), -((g*x)/f)] - d*g*x*AppellF1[5, -m, 2, 6, -((e*x)/d), -((g*x)/f)]) + (6*c^2*d*f*x^5*(d + e*x)^m*AppellF1[5, -m, 1, 6, -((e*x)/d), -((g*x)/f)]/(5*(f + g*x) + e*f*m*x*AppellF1[6, 1 - m, 1, 7, -((e*x)/d), -((g*x)/f)] - d*g*x*AppellF1[6, -m, 2, 7, -((e*x)/d), -((g*x)/f)]) + (a^2*(d - (e*f)/g + (e*(f + g*x))/g)^m*Hypergeometric2F1[-m, -m, 1 - m, -((d - (e*f)/g)*g)/(e*(f + g*x))])/(g*m*(1 + ((d - (e*f)/g)*g)/(e*(f + g*x)))^m)$

**Maple [F]** time = 0.125, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^m (cx^2 + bx + a)^2}{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f),x)`

[Out] `int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)^2 (ex + d)^m}{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f),x, algorithm="maxima")`

[Out] `integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2)(ex + d)^m}{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f),x, algorithm="fricas")`

[Out] `integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*(e*x + d)^m/(g*x + f), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**m*(c*x**2+b*x+a)**2/(g*x+f),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)^2 (ex + d)^m}{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f),x, algorithm="giac")`

[Out] `integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f), x)`

$$3.928 \quad \int \frac{(d+ex)^m (a+bx+cx^2)^2}{(f+gx)^2} dx$$

**Optimal.** Leaf size=298

$$\frac{(d+ex)^{m+1} (2ceg(aeg-b(dg+2ef))+b^2e^2g^2+c^2(d^2g^2+2defg+3e^2f^2))}{e^3g^4(m+1)}$$

$$+ \frac{(d+ex)^{m+1} (ag^2-bfg+cf^2) {}_2F_1\left(1, m+1; m+2; -\frac{g(d+ex)}{ef-dg}\right) (cf(4dg-ef(m+4))-g(aegm+b(2dg-ef(m+2))))}{g^4(m+1)(ef-dg)^2}$$

$$+ \frac{(d+ex)^{m+1} (ag^2-bfg+cf^2)^2}{g^4(f+gx)(ef-dg)} - \frac{2c(d+ex)^{m+2}(-beg+cdg+cef)}{e^3g^3(m+2)} + \frac{c^2(d+ex)^{m+3}}{e^3g^2(m+3)}$$

[Out] ((b^2\*e^2\*g^2 + c^2\*(3\*e^2\*f^2 + 2\*d\*e\*f\*g + d^2\*g^2) + 2\*c\*e\*g\*(a\*e\*g - b\*(2\*e\*f + d\*g)))\*(d + e\*x)^(1 + m))/(e^3\*g^4\*(1 + m)) - (2\*c\*(c\*e\*f + c\*d\*g - b\*e\*g)\*(d + e\*x)^(2 + m))/(e^3\*g^3\*(2 + m)) + (c^2\*(d + e\*x)^(3 + m))/(e^3\*g^2\*(3 + m)) + ((c\*f^2 - b\*f\*g + a\*g^2)^2\*(d + e\*x)^(1 + m))/(g^4\*(e\*f - d\*g)\*(f + g\*x)) + ((c\*f^2 - b\*f\*g + a\*g^2)\*(c\*f\*(4\*d\*g - e\*f\*(4 + m)) - g\*(a\*e\*g\*m + b\*(2\*d\*g - e\*f\*(2 + m))))\*(d + e\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, -((g\*(d + e\*x))/(e\*f - d\*g))]/(g^4\*(e\*f - d\*g)^2\*(1 + m))

**Rubi [A]** time = 2.33824, antiderivative size = 298, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{(d+ex)^{m+1} (2ceg(aeg-b(dg+2ef))+b^2e^2g^2+c^2(d^2g^2+2defg+3e^2f^2))}{e^3g^4(m+1)}$$

$$- \frac{(d+ex)^{m+1} (ag^2-bfg+cf^2) {}_2F_1\left(1, m+1; m+2; -\frac{g(d+ex)}{ef-dg}\right) (g(aegm+2bdg-bef(m+2))-cf(4dg-ef(m+4)))}{g^4(m+1)(ef-dg)^2}$$

$$+ \frac{(d+ex)^{m+1} (ag^2-bfg+cf^2)^2}{g^4(f+gx)(ef-dg)} - \frac{2c(d+ex)^{m+2}(-beg+cdg+cef)}{e^3g^3(m+2)} + \frac{c^2(d+ex)^{m+3}}{e^3g^2(m+3)}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^m\*(a + b\*x + c\*x^2)^2)/(f + g\*x)^2, x]

[Out] ((b^2\*e^2\*g^2 + c^2\*(3\*e^2\*f^2 + 2\*d\*e\*f\*g + d^2\*g^2) + 2\*c\*e\*g\*(a\*e\*g - b\*(2\*e\*f + d\*g)))\*(d + e\*x)^(1 + m))/(e^3\*g^4\*(1 + m)) - (2\*c\*(c\*e\*f + c\*d\*g - b\*e\*g)\*(d + e\*x)^(2 + m))/(e^3\*g^3\*(2 + m)) + (c^2\*(d + e\*x)^(3 + m))/(e^3\*g^2\*(3 + m)) + ((c\*f^2 - b\*f\*g + a\*g^2)^2\*(d + e\*x)^(1 + m))/(g^4\*(e\*f - d\*g)\*(f + g\*x)) - ((c\*f^2 - b\*f\*g + a\*g^2)\*(g\*(2\*b\*d\*g + a\*e\*g\*m - b\*e\*f\*(2 + m)) - c\*f\*(4\*d\*g - e\*f\*(4 + m))))\*(d + e\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, -((g\*(d + e\*x))/(e\*f - d\*g))]/(g^4\*(e\*f - d\*g)^2\*(1 + m))

**Rubi in Sympy [A]** time = 148.022, size = 277, normalized size = 0.93

$$\frac{c^2 (d+ex)^{m+3}}{e^3 g^2 (m+3)} + \frac{2c (d+ex)^{m+2} (beg - cdg - cef)}{e^3 g^3 (m+2)}$$

$$+ \frac{e (d+ex)^{m+1} (ag^2 - bfg + cf^2)^2 {}_2F_1\left(2, m+1 \middle| \frac{g(d+ex)}{dg-ef}\right)}{g^4 (m+1) (dg-ef)^2}$$

$$- \frac{2 (d+ex)^{m+1} (bg - 2cf) (ag^2 - bfg + cf^2) {}_2F_1\left(1, m+1 \middle| \frac{g(d+ex)}{dg-ef}\right)}{g^4 (m+1) (dg-ef)}$$

$$+ \frac{(d+ex)^{m+1} (-2bcdeg^2 - 4bce^2 fg + c^2 d^2 g^2 + 2c^2 defg + 3c^2 e^2 f^2 + e^2 g^2 (2ac + b^2))}{e^3 g^4 (m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**m*(c*x**2+b*x+a)**2/(g*x+f)**2,x)`

[Out] `c**2*(d+e*x)**(m+3)/(e**3*g**2*(m+3)) + 2*c*(d+e*x)**(m+2)*(b*e*g - c*d*g - c*e*f)/(e**3*g**3*(m+2)) + e*(d+e*x)**(m+1)*(a*g**2 - b*f*g + c*f**2)**2*hyper((2, m+1), (m+2, ), g*(d+e*x)/(d*g - e*f))/(g**4*(m+1)*(d*g - e*f)**2) - 2*(d+e*x)**(m+1)*(b*g - 2*c*f)*(a*g**2 - b*f*g + c*f**2)*hyper((1, m+1), (m+2, ), g*(d+e*x)/(d*g - e*f))/(g**4*(m+1)*(d*g - e*f)) + (d+e*x)**(m+1)*(-2*b*c*d*e*g**2 - 4*b*c*e**2*f*g + c**2*d**2*g**2 + 2*c**2*d*e*f*g + 3*c**2*e**2*f**2 + e**2*g**2*(2*a*c + b**2))/(e**3*g**4*(m+1))`

**Mathematica [A]** time = 0.40186, size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m (a+bx+cx^2)^2}{(f+gx)^2} dx$$

Verification is Not applicable to the result.

[In] `Integrate[((d+e*x)^m*(a+b*x+c*x^2)^2)/(f+g*x)^2,x]`

[Out] `Integrate[((d+e*x)^m*(a+b*x+c*x^2)^2)/(f+g*x)^2, x]`

**Maple [F]** time = 0.14, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m (cx^2+bx+a)^2}{(gx+f)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^2,x)`

[Out] `int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2+bx+a)^2(ex+d)^m}{(gx+f)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f)^2,x, algorithm="maxima")`

[Out] `integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f)^2, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2)(ex + d)^m}{g^2x^2 + 2fgx + f^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f)^2,x, algorithm="fricas")`

[Out] `integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*(e*x + d)^m/(g^2*x^2 + 2*f*g*x + f^2), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**m*(c*x**2+b*x+a)**2/(g*x+f)**2,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)^2 (ex + d)^m}{(gx + f)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f)^2,x, algorithm="giac")`

[Out] `integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f)^2, x)`

$$3.929 \quad \int \frac{(d+ex)^m (a+bx+cx^2)^2}{(f+gx)^3} dx$$

**Optimal.** Leaf size=461

$$\frac{(d+ex)^{m+1} {}_2F_1\left(1, m+1; m+2; -\frac{g(d+ex)}{ef-dg}\right) \left(-g^2 (a^2 e^2 g^2 (1-m)m - 2abegm(2dg - ef(m+1))) + b^2 (- (2d^2 g^2 - 4defg(m+1)))\right)}{2g^4 (f+gx)(ef-dg)^2} + \frac{(d+ex)^{m+1} (ag^2 - bfg + cf^2) (gaeq(1-m) - b(4dg - ef(m+3))) + cf(8dg - ef(m+7))}{2g^4 (f+gx)(ef-dg)^2} + \frac{(d+ex)^{m+1} (ag^2 - bfg + cf^2)^2}{2g^4 (f+gx)^2 (ef-dg)} - \frac{c(d+ex)^{m+1} (-2beg + cdg + 3cef)}{e^2 g^4 (m+1)} + \frac{c^2 (d+ex)^{m+2}}{e^2 g^3 (m+2)}$$

[Out]  $-\left(\left(c^*(3*c*e*f + c*d*g - 2*b*e*g)*(d + e*x)^{(1+m)}\right)/\left(e^{2*g^4*(1+m)}\right) + \left(c^{2*(d + e*x)^{(2+m)}\right)/\left(e^{2*g^3*(2+m)}\right) + \left((c*f^2 - b*f*g + a*g^2)^2*(d + e*x)^{(1+m)}\right)/\left(2*g^4*(e*f - d*g)*(f + g*x)^2\right) + \left((c*f^2 - b*f*g + a*g^2)*(c*f*(8*d*g - e*f*(7+m)) + g*(a*e*g*(1-m) - b*(4*d*g - e*f*(3+m)))\right)*(d + e*x)^{(1+m)}\right)/\left(2*g^4*(e*f - d*g)^2*(f + g*x)\right) + \left(\left(c^{2*f^2*(12*d^2*g^2 - 8*d*e*f*g*(3+m) + e^{2*f^2*(12+7*m+m^2)}) - g^2*(a^{2*e^2*g^2*(1-m)*m} - 2*a*b*e*g*m*(2*d*g - e*f*(1+m)) - b^{2*(2*d^2*g^2 - 4*d*e*f*g*(1+m) + e^{2*f^2*(2+3*m+m^2)})}\right) + 2*c*g*(a*g*(2*d^2*g^2 - 4*d*e*f*g*(1+m) + e^{2*f^2*(2+3*m+m^2)}) - b*f*(6*d^2*g^2 - 6*d*e*f*g*(2+m) + e^{2*f^2*(6+5*m+m^2)})\right)*(d + e*x)^{(1+m)}\right)*\text{Hypergeometric2F1}\left[1, 1+m, 2+m, -\left(\frac{g*(d + e*x)}{(e*f - d*g)}\right)\right]/\left(2*g^4*(e*f - d*g)^3*(1+m)\right)$

**Rubi [A]** time = 4.35087, antiderivative size = 461, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$\frac{(d+ex)^{m+1} {}_2F_1\left(1, m+1; m+2; -\frac{g(d+ex)}{ef-dg}\right) \left(-g^2 (a^2 e^2 g^2 (1-m)m - 2abegm(2dg - ef(m+1))) + b^2 (- (2d^2 g^2 - 4defg(m+1)))\right)}{2g^4 (f+gx)(ef-dg)^2} - \frac{(d+ex)^{m+1} (ag^2 - bfg + cf^2) (g(-aeg(1-m) + 4bdg - bef(m+3)) - cf(8dg - ef(m+7)))}{2g^4 (f+gx)(ef-dg)^2} + \frac{(d+ex)^{m+1} (ag^2 - bfg + cf^2)^2}{2g^4 (f+gx)^2 (ef-dg)} - \frac{c(d+ex)^{m+1} (-2beg + cdg + 3cef)}{e^2 g^4 (m+1)} + \frac{c^2 (d+ex)^{m+2}}{e^2 g^3 (m+2)}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^m\*(a + b\*x + c\*x^2)^2)/(f + g\*x)^3, x]

[Out]  $-\left(\left(c^*(3*c*e*f + c*d*g - 2*b*e*g)*(d + e*x)^{(1+m)}\right)/\left(e^{2*g^4*(1+m)}\right) + \left(c^{2*(d + e*x)^{(2+m)}\right)/\left(e^{2*g^3*(2+m)}\right) + \left((c*f^2 - b*f*g + a*g^2)^2*(d + e*x)^{(1+m)}\right)/\left(2*g^4*(e*f - d*g)*(f + g*x)^2\right) - \left((c*f^2 - b*f*g + a*g^2)*(g*(4*b*d*g - a*e*g*(1-m) - b*e*f*(3+m)) - c*f*(8*d*g - e*f*(7+m)))\right)*(d + e*x)^{(1+m)}\right)/\left(2*g^4*(e*f - d*g)^2*(f + g*x)\right) + \left(\left(c^{2*f^2*(12*d^2*g^2 - 8*d*e*f*g*(3+m) + e^{2*f^2*(12+7*m+m^2)}) - g^2*(a^{2*e^2*g^2*(1-m)*m} - 2*a*b*e*g*m*(2*d*g - e*f*(1+m)) - b^{2*(2*d^2*g^2 - 4*d*e*f*g*(1+m) + e^{2*f^2*(2+3*m+m^2)})}\right) + 2*c*g*(a*g*(2*d^2*g^2 - 4*d*e*f*g*(1+m) + e^{2*f^2*(2+3*m+m^2)}) - b*f*(6*d^2*g^2 - 6*d*e*f*g*(2+m) + e^{2*f^2*(6+5*m+m^2)})\right)*(d + e*x)^{(1+m)}\right)*\text{Hypergeometric2F1}\left[1, 1+m, 2+m, -\left(\frac{g*(d + e*x)}{(e*f - d*g)}\right)\right]/\left(2*g^4*(e*f - d*g)^3*(1+m)\right)$

**Rubi in Sympy [A]** time = 112.77, size = 265, normalized size = 0.57

$$\frac{c^2 (d + ex)^{m+2}}{e^2 g^3 (m + 2)} + \frac{c (d + ex)^{m+1} (2beg - cdg - 3cef)}{e^2 g^4 (m + 1)}$$

$$- \frac{e^2 (d + ex)^{m+1} (ag^2 - bfg + cf^2)^2 {}_2F_1\left(\begin{matrix} 3, m + 1 \\ m + 2 \end{matrix} \middle| \frac{g(d+ex)}{dg-ef}\right)}{g^4 (m + 1) (dg - ef)^3}$$

$$+ \frac{2e (d + ex)^{m+1} (bg - 2cf) (ag^2 - bfg + cf^2) {}_2F_1\left(\begin{matrix} 2, m + 1 \\ m + 2 \end{matrix} \middle| \frac{g(d+ex)}{dg-ef}\right)}{g^4 (m + 1) (dg - ef)^2}$$

$$- \frac{(d + ex)^{m+1} (2acg^2 + b^2g^2 - 6bcfg + 6c^2f^2) {}_2F_1\left(\begin{matrix} 1, m + 1 \\ m + 2 \end{matrix} \middle| \frac{g(d+ex)}{dg-ef}\right)}{g^4 (m + 1) (dg - ef)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**m*(c*x**2+b*x+a)**2/(g*x+f)**3,x)`

[Out] `c**2*(d + e*x)**(m + 2)/(e**2*g**3*(m + 2)) + c*(d + e*x)**(m + 1)*(2*b*e*g - c*d*g - 3*c*e*f)/(e**2*g**4*(m + 1)) - e**2*(d + e*x)**(m + 1)*(a*g**2 - b*f*g + c*f**2)**2*hyper((3, m + 1), (m + 2, ), g*(d + e*x)/(d*g - e*f))/(g**4*(m + 1)*(d*g - e*f)**3) + 2*e*(d + e*x)**(m + 1)*(b*g - 2*c*f)*(a*g**2 - b*f*g + c*f**2)*hyper((2, m + 1), (m + 2, ), g*(d + e*x)/(d*g - e*f))/(g**4*(m + 1)*(d*g - e*f)**2) - (d + e*x)**(m + 1)*(2*a*c*g**2 + b**2*g**2 - 6*b*c*f*g + 6*c**2*f**2)*hyper((1, m + 1), (m + 2, ), g*(d + e*x)/(d*g - e*f))/(g**4*(m + 1)*(d*g - e*f))`

**Mathematica [A]** time = 0.693177, size = 0, normalized size = 0.

$$\int \frac{(d + ex)^m (a + bx + cx^2)^2}{(f + gx)^3} dx$$

Verification is Not applicable to the result.

[In] `Integrate[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^3,x]`

[Out] `Integrate[((d + e*x)^m*(a + b*x + c*x^2)^2)/(f + g*x)^3, x]`

**Maple [F]** time = 0.182, size = 0, normalized size = 0.

$$\int \frac{(ex + d)^m (cx^2 + bx + a)^2}{(gx + f)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^3,x)`

[Out] `int((e*x+d)^m*(c*x^2+b*x+a)^2/(g*x+f)^3,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)^2 (ex + d)^m}{(gx + f)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f)^3,x, algorithm="maxima")`

[Out] `integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f)^3, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(c^2x^4 + 2bcx^3 + 2abx + (b^2 + 2ac)x^2 + a^2)(ex + d)^m}{g^3x^3 + 3fg^2x^2 + 3f^2gx + f^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f)^3,x, algorithm="fricas")`

[Out] `integral((c^2*x^4 + 2*b*c*x^3 + 2*a*b*x + (b^2 + 2*a*c)*x^2 + a^2)*(e*x + d)^m/(g^3*x^3 + 3*f*g^2*x^2 + 3*f^2*g*x + f^3), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**m*(c*x**2+b*x+a)**2/(g*x+f)**3,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)^2 (ex + d)^m}{(gx + f)^3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f)^3,x, algorithm="giac")`

[Out] `integrate((c*x^2 + b*x + a)^2*(e*x + d)^m/(g*x + f)^3, x)`



$$3.930 \quad \int \frac{(2+3x)^4(1+4x)^m}{1-5x+3x^2} dx$$

**Optimal.** Leaf size=183

$$\begin{aligned} & \frac{3 \left( 5499 - 1631\sqrt{13} \right) (4x + 1)^{m+1} {}_2F_1 \left( 1, m + 1; m + 2; \frac{3(4x+1)}{13-2\sqrt{13}} \right)}{26 \left( 13 - 2\sqrt{13} \right) (m + 1)} \\ & - \frac{3 \left( 5499 + 1631\sqrt{13} \right) (4x + 1)^{m+1} {}_2F_1 \left( 1, m + 1; m + 2; \frac{3(4x+1)}{13+2\sqrt{13}} \right)}{26 \left( 13 + 2\sqrt{13} \right) (m + 1)} \\ & + \frac{3687(4x + 1)^{m+1}}{64(m + 1)} + \frac{207(4x + 1)^{m+2}}{32(m + 2)} + \frac{27(4x + 1)^{m+3}}{64(m + 3)} \end{aligned}$$

[Out] (3687\*(1 + 4\*x)^(1 + m))/(64\*(1 + m)) + (207\*(1 + 4\*x)^(2 + m))/(32\*(2 + m)) + (27\*(1 + 4\*x)^(3 + m))/(64\*(3 + m)) - (3\*(5499 - 1631\*Sqrt[13])\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 - 2\*Sqrt[13])])/(26\*(13 - 2\*Sqrt[13])\*(1 + m)) - (3\*(5499 + 1631\*Sqrt[13])\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 + 2\*Sqrt[13])])/(26\*(13 + 2\*Sqrt[13])\*(1 + m))

**Rubi [A]** time = 0.433676, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$

$$\begin{aligned} & \frac{3 \left( 5499 - 1631\sqrt{13} \right) (4x + 1)^{m+1} {}_2F_1 \left( 1, m + 1; m + 2; \frac{3(4x+1)}{13-2\sqrt{13}} \right)}{26 \left( 13 - 2\sqrt{13} \right) (m + 1)} \\ & - \frac{3 \left( 5499 + 1631\sqrt{13} \right) (4x + 1)^{m+1} {}_2F_1 \left( 1, m + 1; m + 2; \frac{3(4x+1)}{13+2\sqrt{13}} \right)}{26 \left( 13 + 2\sqrt{13} \right) (m + 1)} \\ & + \frac{3687(4x + 1)^{m+1}}{64(m + 1)} + \frac{207(4x + 1)^{m+2}}{32(m + 2)} + \frac{27(4x + 1)^{m+3}}{64(m + 3)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[((2 + 3\*x)^4\*(1 + 4\*x)^m)/(1 - 5\*x + 3\*x^2), x]

[Out] (3687\*(1 + 4\*x)^(1 + m))/(64\*(1 + m)) + (207\*(1 + 4\*x)^(2 + m))/(32\*(2 + m)) + (27\*(1 + 4\*x)^(3 + m))/(64\*(3 + m)) - (3\*(5499 - 1631\*Sqrt[13])\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 - 2\*Sqrt[13])])/(26\*(13 - 2\*Sqrt[13])\*(1 + m)) - (3\*(5499 + 1631\*Sqrt[13])\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 + 2\*Sqrt[13])])/(26\*(13 + 2\*Sqrt[13])\*(1 + m))

**Rubi in Sympy [A]** time = 26.4807, size = 144, normalized size = 0.79

$$\begin{aligned} & \frac{27(4x + 1)^{m+3}}{64(m + 3)} + \frac{207(4x + 1)^{m+2}}{32(m + 2)} - \frac{\left( -\frac{4893\sqrt{13}}{13} + 1269 \right) (4x + 1)^{m+1} {}_2F_1 \left( 1, m + 1; m + 2; \frac{-12x-3}{-13+2\sqrt{13}} \right)}{\left( -4\sqrt{13} + 26 \right) (m + 1)} \\ & - \frac{\left( 1269 + \frac{4893\sqrt{13}}{13} \right) (4x + 1)^{m+1} {}_2F_1 \left( 1, m + 1; m + 2; \frac{12x+3}{2\sqrt{13}+13} \right)}{\left( 4\sqrt{13} + 26 \right) (m + 1)} + \frac{3687(4x + 1)^{m+1}}{64(m + 1)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((2+3*x)**4*(1+4*x)**m/(3*x**2-5*x+1),x)`

[Out]  $27*(4*x + 1)**(m + 3)/(64*(m + 3)) + 207*(4*x + 1)**(m + 2)/(32*(m + 2)) - (-4893*\sqrt{13}/13 + 1269)*(4*x + 1)**(m + 1)*\text{hyper}((1, m + 1), (m + 2, ), (-12*x - 3)/(-13 + 2*\sqrt{13}))/((-4*\sqrt{13} + 26)*(m + 1)) - (1269 + 4893*\sqrt{13}/13)*(4*x + 1)**(m + 1)*\text{hyper}((1, m + 1), (m + 2, ), (12*x + 3)/(2*\sqrt{13} + 13))/((4*\sqrt{13} + 26)*(m + 1)) + 3687*(4*x + 1)**(m + 1)/(64*(m + 1))$

**Mathematica [B]** time = 1.43798, size = 568, normalized size = 3.1

$$\frac{17 \cdot 2^{m+6} \cdot 3^{-m} (4x+1)^m \left( \left( \frac{4x+1}{6x+\sqrt{13}-5} \right)^{-m} {}_2F_1 \left( -m, -m; 1-m; \frac{-13+2\sqrt{13}}{2(6x+\sqrt{13}-5)} \right) - \left( \frac{4x+1}{-6x+\sqrt{13}+5} \right)^{-m} {}_2F_1 \left( -m, -m; 1-m; \frac{13+2\sqrt{13}}{2(-6x+\sqrt{13}+5)} \right) \right)}{\sqrt{13}m} + \frac{47 \cdot 2^{2m-1} \cdot 3^{2-m} \left( 2x + \frac{1}{2} \right)^m \left( (9 + \sqrt{13}) \left( \frac{4x+1}{-6x+\sqrt{13}+5} \right)^{-m} {}_2F_1 \left( -m, -m; 1-m; \frac{13+2\sqrt{13}}{2(-6x+\sqrt{13}+5)} \right) + (\sqrt{13} - 9) \left( \frac{4x+1}{6x+\sqrt{13}-5} \right)^{-m} {}_2F_1 \left( -m, -m; 1-m; \frac{-13+2\sqrt{13}}{2(6x+\sqrt{13}-5)} \right) \right)}{\sqrt{13}m} + \frac{3^{-m} (4x+1)^m (24m^2(3x+2)^2(4x+1)(-12x-3)^m + 1728x^3(-12x-3)^m + 3456x^2(-12x-3)^m + 12m(216x^3 + 402x^2 + 223x + 12)(-12x-3)^m)}{32(m+1)(m+2)(m+3)} + \frac{3^{2-m} (4x+1)^m (144x^2(-12x-3)^m + 12m(12x^2 + 11x + 2)(-12x-3)^m + 192x(-12x-3)^m + 39(-12x-3)^m + 5^{m+2})}{16(m+1)(m+2)} + \frac{48(4x+1)^{m+1}}{m+1}$$

Antiderivative was successfully verified.

[In] `Integrate[(((2 + 3*x)^4*(1 + 4*x)^m)/(1 - 5*x + 3*x^2),x]`

[Out]  $(48*(1 + 4*x)^{(1 + m)})/(1 + m) + (3^{(2 - m)}*(1 + 4*x)^m*(5^{(2 + m)} + 39*(-3 - 12*x)^m + 192*(-3 - 12*x)^m*x + 144*(-3 - 12*x)^m*x^2 + 12*m*(-3 - 12*x)^m*(2 + 11*x + 12*x^2)))/(16*(1 + m)^*(2 + m)^*(-1 - 4*x)^m) + ((1 + 4*x)^m*(5^{(3 + m)} + 387*(-3 - 12*x)^m + 2304*(-3 - 12*x)^m*x + 3456*(-3 - 12*x)^m*x^2 + 1728*(-3 - 12*x)^m*x^3 + 24*m^2*(-3 - 12*x)^m*(2 + 3*x)^2*(1 + 4*x) + 12*m*(-3 - 12*x)^m*(34 + 223*x + 402*x^2 + 216*x^3)))/(32*3^m*(1 + m)^*(2 + m)^*(3 + m)^*(-1 - 4*x)^m) + (17*2^{(6 + m)}*(1 + 4*x)^m*(-\text{Hypergeometric2F1}[-m, -m, 1 - m, (13 + 2*\sqrt{13})/(2*(5 + \sqrt{13} - 6*x))]/(-((1 + 4*x)/(5 + \sqrt{13} - 6*x)))^m) + \text{Hypergeometric2F1}[-m, -m, 1 - m, (-13 + 2*\sqrt{13})/(2*(-5 + \sqrt{13} + 6*x))]/(((1 + 4*x)/(-5 + \sqrt{13} + 6*x))^m)))/(3^m*\sqrt{13}*m) + (47*2^{(-1 + 2*m)}*3^{(2 - m)}*(1/2 + 2*x)^m*((9 + \sqrt{13})*\text{Hypergeometric2F1}[-m, -m, 1 - m, (13 + 2*\sqrt{13})/(2*(5 + \sqrt{13} - 6*x))]/(-((1 + 4*x)/(5 + \sqrt{13} - 6*x)))^m) + ((-9 + \sqrt{13})*\text{Hypergeometric2F1}[-m, -m, 1 - m, (-13 + 2*\sqrt{13})/(2*(-5 + \sqrt{13} + 6*x))]/(((1 + 4*x)/(-5 + \sqrt{13} + 6*x))^m)))/(\sqrt{13}*m)$

**Maple [F]** time = 0.269, size = 0, normalized size = 0.

$$\int \frac{(2+3x)^4(1+4x)^m}{3x^2-5x+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1),x)`

[Out] `int((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1),x)`

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**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m(3x+2)^4}{3x^2-5x+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x + 1)^m\*(3\*x + 2)^4/(3\*x^2 - 5\*x + 1),x, algorithm="maxima")

[Out] integrate((4\*x + 1)^m\*(3\*x + 2)^4/(3\*x^2 - 5\*x + 1), x)

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**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(81x^4 + 216x^3 + 216x^2 + 96x + 16)(4x+1)^m}{3x^2-5x+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x + 1)^m\*(3\*x + 2)^4/(3\*x^2 - 5\*x + 1),x, algorithm="fricas")

[Out] integral((81\*x^4 + 216\*x^3 + 216\*x^2 + 96\*x + 16)\*(4\*x + 1)^m/(3\*x^2 - 5\*x + 1), x)

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**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x+2)^4(4x+1)^m}{3x^2-5x+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3\*x)\*\*4\*(1+4\*x)\*\*m/(3\*x\*\*2-5\*x+1), x)

[Out] Integral((3\*x + 2)\*\*4\*(4\*x + 1)\*\*m/(3\*x\*\*2 - 5\*x + 1), x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m(3x+2)^4}{3x^2-5x+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x + 1)^m\*(3\*x + 2)^4/(3\*x^2 - 5\*x + 1),x, algorithm="giac")

[Out] integrate((4\*x + 1)^m\*(3\*x + 2)^4/(3\*x^2 - 5\*x + 1), x)

$$3.931 \quad \int \frac{(2+3x)^3(1+4x)^m}{1-5x+3x^2} dx$$

**Optimal.** Leaf size=165

$$\frac{3(416 - 135\sqrt{13})(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13(13-2\sqrt{13})(m+1)} - \frac{3(416 + 135\sqrt{13})(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13+2\sqrt{13}}\right)}{13(13+2\sqrt{13})(m+1)} + \frac{123(4x+1)^{m+1}}{16(m+1)} + \frac{9(4x+1)^{m+2}}{16(m+2)}$$

[Out] (123\*(1+4\*x)^(1+m))/(16\*(1+m)) + (9\*(1+4\*x)^(2+m))/(16\*(2+m)) - (3\*(416-135\*Sqrt[13])\*(1+4\*x)^(1+m)\*Hypergeometric2F1[1, 1+m, 2+m, (3\*(1+4\*x))/(13-2\*Sqrt[13])])/(13\*(13-2\*Sqrt[13])\*(1+m)) - (3\*(416+135\*Sqrt[13])\*(1+4\*x)^(1+m)\*Hypergeometric2F1[1, 1+m, 2+m, (3\*(1+4\*x))/(13+2\*Sqrt[13])])/(13\*(13+2\*Sqrt[13])\*(1+m))

**Rubi [A]** time = 0.304733, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$

$$\frac{3(416 - 135\sqrt{13})(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13(13-2\sqrt{13})(m+1)} - \frac{3(416 + 135\sqrt{13})(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13+2\sqrt{13}}\right)}{13(13+2\sqrt{13})(m+1)} + \frac{123(4x+1)^{m+1}}{16(m+1)} + \frac{9(4x+1)^{m+2}}{16(m+2)}$$

Antiderivative was successfully verified.

[In] Int[((2+3\*x)^3\*(1+4\*x)^m)/(1-5\*x+3\*x^2), x]

[Out] (123\*(1+4\*x)^(1+m))/(16\*(1+m)) + (9\*(1+4\*x)^(2+m))/(16\*(2+m)) - (3\*(416-135\*Sqrt[13])\*(1+4\*x)^(1+m)\*Hypergeometric2F1[1, 1+m, 2+m, (3\*(1+4\*x))/(13-2\*Sqrt[13])])/(13\*(13-2\*Sqrt[13])\*(1+m)) - (3\*(416+135\*Sqrt[13])\*(1+4\*x)^(1+m)\*Hypergeometric2F1[1, 1+m, 2+m, (3\*(1+4\*x))/(13+2\*Sqrt[13])])/(13\*(13+2\*Sqrt[13])\*(1+m))

**Rubi in Sympy [A]** time = 24.1704, size = 129, normalized size = 0.78

$$\frac{9(4x+1)^{m+2}}{16(m+2)} - \frac{\left(-\frac{810\sqrt{13}}{13} + 192\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{-12x-3}{-13+2\sqrt{13}}\right)}{(-4\sqrt{13}+26)(m+1)} - \frac{\left(192 + \frac{810\sqrt{13}}{13}\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{12x+3}{2\sqrt{13}+13}\right)}{(4\sqrt{13}+26)(m+1)} + \frac{123(4x+1)^{m+1}}{16(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((2+3\*x)\*\*3\*(1+4\*x)\*\*m/(3\*x\*\*2-5\*x+1), x)

[Out] 9\*(4\*x+1)\*\*(m+2)/(16\*(m+2)) - (-810\*sqrt(13)/13 + 192)\*(4\*x+1)\*\*(m+1)\*hyper((1, m+1), (m+2, ), (-12\*x-3)/(-13+2\*s

$$\frac{\sqrt{13}}{((-4\sqrt{13} + 26)^{(m+1)} - (192 + 810\sqrt{13})/13)^{(4x+1)^m(m+1)} \operatorname{hyper}((1, m+1), (m+2, ), (12x+3)/(2\sqrt{13} + 13)) / ((4\sqrt{13} + 26)^{(m+1)} + 123(4x+1)^m(m+1) / (16(m+1)))}$$

**Mathematica [B]** time = 0.506035, size = 410, normalized size = 2.48

$$(4x+1)^m \left( 13 2^{m+9} 3^{-m} (m+1)(m+2) \left( -\frac{4x+1}{-6x+\sqrt{13}+5} \right)^{-m} {}_2F_1 \left( -m, -m; 1-m; \frac{13+2\sqrt{13}}{2(-6x+\sqrt{13}+5)} \right) + 5\sqrt{13} 2^{m+4} 3^{-m} (m+1)(m+2) \right)$$

Antiderivative was successfully verified.

[In] Integrate[(((2 + 3\*x)^3\*(1 + 4\*x)^m)/(1 - 5\*x + 3\*x^2), x]

[Out] ((1 + 4\*x)^m\*(507\*m + 312\*m^2 + 1404\*m\*(2 + m) + (325\*m)/(-3/5 - (12\*x)/5)^m + 2496\*m\*x + 1716\*m^2\*x + 5616\*m\*(2 + m)\*x + 1872\*m\*x^2 + 1872\*m^2\*x^2 + (13\*2^(9 + m)\*(1 + m)\*(2 + m)\*Hypergeometric2F1[-m, -m, 1 - m, (13 + 2\*Sqrt[13])/(2\*(5 + Sqrt[13] - 6\*x))])/(3^m\*(-((1 + 4\*x)/(5 + Sqrt[13] - 6\*x)))^m) + (5\*2^(4 + m)\*3^(3 - m)\*Sqrt[13]\*(1 + m)\*(2 + m)\*Hypergeometric2F1[-m, -m, 1 - m, (13 + 2\*Sqrt[13])/(2\*(5 + Sqrt[13] - 6\*x))])/((-(1 + 4\*x)/(5 + Sqrt[13] - 6\*x)))^m + (13\*2^(9 + m)\*(1 + m)\*(2 + m)\*Hypergeometric2F1[-m, -m, 1 - m, (-13 + 2\*Sqrt[13])/(2\*(-5 + Sqrt[13] + 6\*x))])/(3^m\*((1 + 4\*x)/(-5 + Sqrt[13] + 6\*x))^m) - (5\*2^(4 + m)\*3^(3 - m)\*Sqrt[13]\*(1 + m)\*(2 + m)\*Hypergeometric2F1[-m, -m, 1 - m, (-13 + 2\*Sqrt[13])/(2\*(-5 + Sqrt[13] + 6\*x))])/(((1 + 4\*x)/(-5 + Sqrt[13] + 6\*x))^m))/(208\*m\*(1 + m)\*(2 + m))

**Maple [F]** time = 0.163, size = 0, normalized size = 0.

$$\int \frac{(2+3x)^3(1+4x)^m}{3x^2-5x+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2+3\*x)^3\*(1+4\*x)^m/(3\*x^2-5\*x+1), x)

[Out] int((2+3\*x)^3\*(1+4\*x)^m/(3\*x^2-5\*x+1), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m(3x+2)^3}{3x^2-5x+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x + 1)^m\*(3\*x + 2)^3/(3\*x^2 - 5\*x + 1), x, algorithm="maxima")

[Out] integrate((4\*x + 1)^m\*(3\*x + 2)^3/(3\*x^2 - 5\*x + 1), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left( \frac{(27x^3 + 54x^2 + 36x + 8)(4x+1)^m}{3x^2 - 5x + 1}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1),x, algorithm="fricas")`

[Out] `integral((27*x^3 + 54*x^2 + 36*x + 8)*(4*x + 1)^m/(3*x^2 - 5*x + 1), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x + 2)^3 (4x + 1)^m}{3x^2 - 5x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+3*x)**3*(1+4*x)**m/(3*x**2-5*x+1),x)`

[Out] `Integral((3*x + 2)**3*(4*x + 1)**m/(3*x**2 - 5*x + 1), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x + 1)^m (3x + 2)^3}{3x^2 - 5x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1),x, algorithm="giac")`

[Out] `integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1), x)`

$$3.932 \quad \int \frac{(2+3x)^2(1+4x)^m}{1-5x+3x^2} dx$$

Optimal. Leaf size=147

$$\frac{3 \left( 117 - 47\sqrt{13} \right) (4x + 1)^{m+1} {}_2F_1 \left( 1, m + 1; m + 2; \frac{3(4x+1)}{13-2\sqrt{13}} \right)}{26 \left( 13 - 2\sqrt{13} \right) (m + 1)} - \frac{3 \left( 117 + 47\sqrt{13} \right) (4x + 1)^{m+1} {}_2F_1 \left( 1, m + 1; m + 2; \frac{3(4x+1)}{13+2\sqrt{13}} \right)}{26 \left( 13 + 2\sqrt{13} \right) (m + 1)} + \frac{3(4x + 1)^{m+1}}{4(m + 1)}$$

[Out] (3\*(1 + 4\*x)^(1 + m))/(4\*(1 + m)) - (3\*(117 - 47\*Sqrt[13]))\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 - 2\*Sqrt[13])]/(26\*(13 - 2\*Sqrt[13])\*(1 + m)) - (3\*(117 + 47\*Sqrt[13]))\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 + 2\*Sqrt[13])]/(26\*(13 + 2\*Sqrt[13])\*(1 + m))

**Rubi [A]** time = 0.295671, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$

$$\frac{3 \left( 117 - 47\sqrt{13} \right) (4x + 1)^{m+1} {}_2F_1 \left( 1, m + 1; m + 2; \frac{3(4x+1)}{13-2\sqrt{13}} \right)}{26 \left( 13 - 2\sqrt{13} \right) (m + 1)} - \frac{3 \left( 117 + 47\sqrt{13} \right) (4x + 1)^{m+1} {}_2F_1 \left( 1, m + 1; m + 2; \frac{3(4x+1)}{13+2\sqrt{13}} \right)}{26 \left( 13 + 2\sqrt{13} \right) (m + 1)} + \frac{3(4x + 1)^{m+1}}{4(m + 1)}$$

Antiderivative was successfully verified.

[In] Int[((2 + 3\*x)^2\*(1 + 4\*x)^m)/(1 - 5\*x + 3\*x^2), x]

[Out] (3\*(1 + 4\*x)^(1 + m))/(4\*(1 + m)) - (3\*(117 - 47\*Sqrt[13]))\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 - 2\*Sqrt[13])]/(26\*(13 - 2\*Sqrt[13])\*(1 + m)) - (3\*(117 + 47\*Sqrt[13]))\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 + 2\*Sqrt[13])]/(26\*(13 + 2\*Sqrt[13])\*(1 + m))

**Rubi in Sympy [A]** time = 22.4762, size = 114, normalized size = 0.78

$$\frac{\left( -\frac{141\sqrt{13}}{13} + 27 \right) (4x + 1)^{m+1} {}_2F_1 \left( 1, m + 1 \middle| \frac{-12x-3}{-13+2\sqrt{13}} \right)}{\left( -4\sqrt{13} + 26 \right) (m + 1)} - \frac{\left( 27 + \frac{141\sqrt{13}}{13} \right) (4x + 1)^{m+1} {}_2F_1 \left( 1, m + 1 \middle| \frac{12x+3}{2\sqrt{13}+13} \right)}{\left( 4\sqrt{13} + 26 \right) (m + 1)} + \frac{3(4x + 1)^{m+1}}{4(m + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((2+3\*x)\*\*2\*(1+4\*x)\*\*m/(3\*x\*\*2-5\*x+1), x)

[Out] -(-141\*sqrt(13)/13 + 27)\*(4\*x + 1)\*\*(m + 1)\*hyper((1, m + 1), (m + 2, ), (-12\*x - 3)/(-13 + 2\*sqrt(13)))/((-4\*sqrt(13) + 26)\*(m + 1)) - (27 + 141\*sqrt(13)/13)\*(4\*x + 1)\*\*(m + 1)\*hyper((1, m + 1),

$(m + 2, ), (12x + 3)/(2\sqrt{13} + 13))/((4\sqrt{13} + 26)^m(m + 1)) + 3(4x + 1)^m(m + 1)/(4(m + 1))$

**Mathematica [A]** time = 0.833655, size = 188, normalized size = 1.28

$$\frac{3^{-m}(4x + 1)^m \left( (117 + 47\sqrt{13}) 2^{m+1} \left( -\frac{4x+1}{-6x+\sqrt{13}+5} \right)^{-m} {}_2F_1 \left( -m, -m; 1 - m; \frac{13+2\sqrt{13}}{2(-6x+\sqrt{13}+5)} \right) - (47\sqrt{13} - 117) 2^{m+1} \left( \frac{4x+1}{6x+\sqrt{13}-5} \right) \right)}{52m}$$

Antiderivative was successfully verified.

[In] Integrate[((2 + 3\*x)^2\*(1 + 4\*x)^m)/(1 - 5\*x + 3\*x^2), x]

[Out]  $((1 + 4x)^m ((13 \cdot 3^m (1 + m)^m (1 + 4x)) / (1 + m) + (2^{1+m} (117 + 47\sqrt{13}) \text{Hypergeometric2F1}[-m, -m, 1 - m, (13 + 2\sqrt{13}) / (2(5 + \sqrt{13} - 6x))]) / (-((1 + 4x) / (5 + \sqrt{13} - 6x))))^m - (2^{1+m} (-117 + 47\sqrt{13}) \text{Hypergeometric2F1}[-m, -m, 1 - m, (-13 + 2\sqrt{13}) / (2(-5 + \sqrt{13} + 6x))]) / ((1 + 4x) / (-5 + \sqrt{13} + 6x))^m)) / (52 \cdot 3^m m)$

**Maple [F]** time = 0.15, size = 0, normalized size = 0.

$$\int \frac{(2 + 3x)^2 (1 + 4x)^m}{3x^2 - 5x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2+3\*x)^2\*(1+4\*x)^m/(3\*x^2-5\*x+1), x)

[Out] int((2+3\*x)^2\*(1+4\*x)^m/(3\*x^2-5\*x+1), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x + 1)^m (3x + 2)^2}{3x^2 - 5x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x + 1)^m\*(3\*x + 2)^2/(3\*x^2 - 5\*x + 1), x, algorithm="maxima")

[Out] integrate((4\*x + 1)^m\*(3\*x + 2)^2/(3\*x^2 - 5\*x + 1), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{(9x^2 + 12x + 4)(4x + 1)^m}{3x^2 - 5x + 1}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x + 1)^m\*(3\*x + 2)^2/(3\*x^2 - 5\*x + 1), x, algorithm="fricas")

[Out] integral((9\*x^2 + 12\*x + 4)\*(4\*x + 1)^m/(3\*x^2 - 5\*x + 1), x)



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**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x+2)^2 (4x+1)^m}{3x^2-5x+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3\*x)\*\*2\*(1+4\*x)\*\*m/(3\*x\*\*2-5\*x+1), x)

[Out] Integral((3\*x + 2)\*\*2\*(4\*x + 1)\*\*m/(3\*x\*\*2 - 5\*x + 1), x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m (3x+2)^2}{3x^2-5x+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x + 1)^m\*(3\*x + 2)^2/(3\*x^2 - 5\*x + 1), x, algorithm="giac")

[Out] integrate((4\*x + 1)^m\*(3\*x + 2)^2/(3\*x^2 - 5\*x + 1), x)

$$3.933 \quad \int \frac{(2+3x)(1+4x)^m}{1-5x+3x^2} dx$$

**Optimal.** Leaf size=129

$$\frac{3 \left(13 - 9\sqrt{13}\right) (4x + 1)^{m+1} {}_2F_1\left(1, m + 1; m + 2; \frac{3(4x+1)}{13-2\sqrt{13}}\right)}{26 \left(13 - 2\sqrt{13}\right) (m + 1)} - \frac{3 \left(13 + 9\sqrt{13}\right) (4x + 1)^{m+1} {}_2F_1\left(1, m + 1; m + 2; \frac{3(4x+1)}{13+2\sqrt{13}}\right)}{26 \left(13 + 2\sqrt{13}\right) (m + 1)}$$

[Out]  $(-3*(13 - 9*\text{Sqrt}[13])*(1 + 4*x)^(1 + m)*\text{Hypergeometric2F1}[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*\text{Sqrt}[13])])/(26*(13 - 2*\text{Sqrt}[13])*(1 + m)) - (3*(13 + 9*\text{Sqrt}[13])*(1 + 4*x)^(1 + m)*\text{Hypergeometric2F1}[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*\text{Sqrt}[13])])/(26*(13 + 2*\text{Sqrt}[13])*(1 + m))$

**Rubi [A]** time = 0.238715, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$

$$\frac{3 \left(13 - 9\sqrt{13}\right) (4x + 1)^{m+1} {}_2F_1\left(1, m + 1; m + 2; \frac{3(4x+1)}{13-2\sqrt{13}}\right)}{26 \left(13 - 2\sqrt{13}\right) (m + 1)} - \frac{3 \left(13 + 9\sqrt{13}\right) (4x + 1)^{m+1} {}_2F_1\left(1, m + 1; m + 2; \frac{3(4x+1)}{13+2\sqrt{13}}\right)}{26 \left(13 + 2\sqrt{13}\right) (m + 1)}$$

Antiderivative was successfully verified.

[In] Int[((2 + 3\*x)\*(1 + 4\*x)^m)/(1 - 5\*x + 3\*x^2), x]

[Out]  $(-3*(13 - 9*\text{Sqrt}[13])*(1 + 4*x)^(1 + m)*\text{Hypergeometric2F1}[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*\text{Sqrt}[13])])/(26*(13 - 2*\text{Sqrt}[13])*(1 + m)) - (3*(13 + 9*\text{Sqrt}[13])*(1 + 4*x)^(1 + m)*\text{Hypergeometric2F1}[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*\text{Sqrt}[13])])/(26*(13 + 2*\text{Sqrt}[13])*(1 + m))$

**Rubi in Sympy [A]** time = 19.2082, size = 100, normalized size = 0.78

$$\frac{\left(-\frac{27\sqrt{13}}{13} + 3\right) (4x + 1)^{m+1} {}_2F_1\left(\frac{1, m + 1}{m + 2} \left| \frac{-12x-3}{-13+2\sqrt{13}} \right.\right)}{\left(-4\sqrt{13} + 26\right) (m + 1)} - \frac{\left(3 + \frac{27\sqrt{13}}{13}\right) (4x + 1)^{m+1} {}_2F_1\left(\frac{1, m + 1}{m + 2} \left| \frac{12x+3}{2\sqrt{13}+13} \right.\right)}{\left(4\sqrt{13} + 26\right) (m + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((2+3\*x)\*(1+4\*x)\*\*m/(3\*x\*\*2-5\*x+1), x)

[Out]  $-(-27*\text{sqrt}(13)/13 + 3)*(4*x + 1)**(m + 1)*\text{hyper}((1, m + 1), (m + 2, ), (-12*x - 3)/(-13 + 2*\text{sqrt}(13)))/((-4*\text{sqrt}(13) + 26)*(m + 1)) - (3 + 27*\text{sqrt}(13)/13)*(4*x + 1)**(m + 1)*\text{hyper}((1, m + 1), (m + 2, ), (12*x + 3)/(2*\text{sqrt}(13) + 13))/((4*\text{sqrt}(13) + 26)*(m + 1))$

**Mathematica [A]** time = 0.441981, size = 162, normalized size = 1.26

$$\frac{2^{m-1}3^{-m}(4x+1)^m \left( (9+\sqrt{13}) \left( -\frac{4x+1}{-6x+\sqrt{13}+5} \right)^{-m} {}_2F_1 \left( -m, -m; 1-m; \frac{13+2\sqrt{13}}{2(-6x+\sqrt{13}+5)} \right) + (\sqrt{13}-9) \left( \frac{4x+1}{6x+\sqrt{13}-5} \right)^{-m} {}_2F_1 \left( -m, -m; 1-m; \frac{13-2\sqrt{13}}{2(6x+\sqrt{13}-5)} \right) \right)}{\sqrt{13}m}$$

Antiderivative was successfully verified.

[In] Integrate[((2 + 3\*x)\*(1 + 4\*x)^m)/(1 - 5\*x + 3\*x^2), x]

[Out] (2^(-1 + m)\*(1 + 4\*x)^m\*((9 + Sqrt[13])\*Hypergeometric2F1[-m, -m, 1 - m, (13 + 2\*Sqrt[13])/(2\*(5 + Sqrt[13] - 6\*x))])/(-(1 + 4\*x)/(5 + Sqrt[13] - 6\*x))^m + ((-9 + Sqrt[13])\*Hypergeometric2F1[-m, -m, 1 - m, (-13 + 2\*Sqrt[13])/(2\*(-5 + Sqrt[13] + 6\*x))])/((1 + 4\*x)/(-5 + Sqrt[13] + 6\*x))^m)/(3^m\*Sqrt[13]^m)

**Maple [F]** time = 0.144, size = 0, normalized size = 0.

$$\int \frac{(2+3x)(1+4x)^m}{3x^2-5x+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2+3\*x)\*(1+4\*x)^m/(3\*x^2-5\*x+1), x)

[Out] int((2+3\*x)\*(1+4\*x)^m/(3\*x^2-5\*x+1), x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m(3x+2)}{3x^2-5x+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x + 1)^m\*(3\*x + 2)/(3\*x^2 - 5\*x + 1), x, algorithm="maxima")

[Out] integrate((4\*x + 1)^m\*(3\*x + 2)/(3\*x^2 - 5\*x + 1), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{(4x+1)^m(3x+2)}{3x^2-5x+1}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x + 1)^m\*(3\*x + 2)/(3\*x^2 - 5\*x + 1), x, algorithm="fricas")

[Out] integral((4\*x + 1)^m\*(3\*x + 2)/(3\*x^2 - 5\*x + 1), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x+2)(4x+1)^m}{3x^2-5x+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+3*x)*(1+4*x)**m/(3*x**2-5*x+1),x)`

[Out] `Integral((3*x + 2)*(4*x + 1)**m/(3*x**2 - 5*x + 1), x)`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x + 1)^m (3x + 2)}{3x^2 - 5x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x + 1)^m*(3*x + 2)/(3*x^2 - 5*x + 1),x, algorithm="giac")`

[Out] `integrate((4*x + 1)^m*(3*x + 2)/(3*x^2 - 5*x + 1), x)`

$$3.934 \quad \int \frac{(1+4x)^m}{1-5x+3x^2} dx$$

**Optimal.** Leaf size=117

$$\frac{3(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13-2\sqrt{13}}\right)}{\sqrt{13}(13-2\sqrt{13})(m+1)} - \frac{3(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13+2\sqrt{13}}\right)}{\sqrt{13}(13+2\sqrt{13})(m+1)}$$

[Out] (3\*(1+4\*x)^(1+m)\*Hypergeometric2F1[1, 1+m, 2+m, (3\*(1+4\*x))/(13-2\*Sqrt[13])])/(Sqrt[13]\*(13-2\*Sqrt[13])\*(1+m)) - (3\*(1+4\*x)^(1+m)\*Hypergeometric2F1[1, 1+m, 2+m, (3\*(1+4\*x))/(13+2\*Sqrt[13])])/(Sqrt[13]\*(13+2\*Sqrt[13])\*(1+m))

**Rubi [A]** time = 0.244625, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{3(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13-2\sqrt{13}}\right)}{\sqrt{13}(13-2\sqrt{13})(m+1)} - \frac{3(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13+2\sqrt{13}}\right)}{\sqrt{13}(13+2\sqrt{13})(m+1)}$$

Antiderivative was successfully verified.

[In] Int[(1+4\*x)^m/(1-5\*x+3\*x^2), x]

[Out] (3\*(1+4\*x)^(1+m)\*Hypergeometric2F1[1, 1+m, 2+m, (3\*(1+4\*x))/(13-2\*Sqrt[13])])/(Sqrt[13]\*(13-2\*Sqrt[13])\*(1+m)) - (3\*(1+4\*x)^(1+m)\*Hypergeometric2F1[1, 1+m, 2+m, (3\*(1+4\*x))/(13+2\*Sqrt[13])])/(Sqrt[13]\*(13+2\*Sqrt[13])\*(1+m))

**Rubi in Sympy [A]** time = 16.844, size = 95, normalized size = 0.81

$$\frac{6\sqrt{13}(4x+1)^{m+1} {}_2F_1\left(1, m+1 \middle| \frac{-12x-3}{-13+2\sqrt{13}} \right)}{13(4\sqrt{13}+26)(m+1)} - \frac{6\sqrt{13}(4x+1)^{m+1} {}_2F_1\left(1, m+1 \middle| \frac{12x+3}{2\sqrt{13}+13} \right)}{13(4\sqrt{13}+26)(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((1+4\*x)\*\*m/(3\*x\*\*2-5\*x+1), x)

[Out] 6\*sqrt(13)\*(4\*x+1)\*\*(m+1)\*hyper((1, m+1), (m+2, ), (-12\*x-3)/(-13+2\*sqrt(13)))/(13\*(-4\*sqrt(13)+26)\*(m+1)) - 6\*sqrt(13)\*(4\*x+1)\*\*(m+1)\*hyper((1, m+1), (m+2, ), (12\*x+3)/(2\*sqrt(13)+13))/(13\*(4\*sqrt(13)+26)\*(m+1))

**Mathematica [A]** time = 0.236943, size = 144, normalized size = 1.23

$$\frac{\left(\frac{2}{3}\right)^m (4x+1)^m \left( \left( -\frac{4x+1}{-6x+\sqrt{13}+5} \right)^{-m} {}_2F_1\left(-m, -m; 1-m; \frac{13+2\sqrt{13}}{2(-6x+\sqrt{13}+5)}\right) - \left( \frac{4x+1}{6x+\sqrt{13}-5} \right)^{-m} {}_2F_1\left(-m, -m; 1-m; \frac{-13+2\sqrt{13}}{2(6x+\sqrt{13}-5)}\right) \right)}{\sqrt{13}m}$$

Antiderivative was successfully verified.

[In] Integrate[(1+4\*x)^m/(1-5\*x+3\*x^2), x]

[Out]  $\left(\frac{2}{3}\right)^m (1+4x)^m \frac{\text{Hypergeometric2F1}[-m, -m, 1-m, (13+2\sqrt{13})/(2(5+\sqrt{13}-6x))]}{((1+4x)/(5+\sqrt{13}-6x))^m} - \text{Hypergeometric2F1}[-m, -m, 1-m, (-13+2\sqrt{13})/(2(-5+\sqrt{13}+6x))]}{((1+4x)/(-5+\sqrt{13}+6x))^m} \Big/ (\sqrt{13})^m$

---

**Maple [F]** time = 0.144, size = 0, normalized size = 0.

$$\int \frac{(1+4x)^m}{3x^2-5x+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+4*x)^m/(3*x^2-5*x+1), x)`

[Out] `int((1+4*x)^m/(3*x^2-5*x+1), x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{3x^2-5x+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x + 1)^m/(3*x^2 - 5*x + 1), x, algorithm="maxima")`

[Out] `integrate((4*x + 1)^m/(3*x^2 - 5*x + 1), x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(4x+1)^m}{3x^2-5x+1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x + 1)^m/(3*x^2 - 5*x + 1), x, algorithm="fricas")`

[Out] `integral((4*x + 1)^m/(3*x^2 - 5*x + 1), x)`

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{3x^2-5x+1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+4*x)**m/(3*x**2-5*x+1), x)`

[Out] `Integral((4*x + 1)**m/(3*x**2 - 5*x + 1), x)`

---

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x + 1)^m}{3x^2 - 5x + 1} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x + 1)^m/(3*x^2 - 5*x + 1), x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m/(3*x^2 - 5*x + 1), x)
```

$$3.935 \quad \int \frac{(1+4x)^m}{(2+3x)(1-5x+3x^2)} dx$$

Optimal. Leaf size=164

$$\begin{aligned} & \frac{3(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; -\frac{3}{5}(4x+1)\right)}{85(m+1)} \\ & + \frac{3\left(13+9\sqrt{13}\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13-2\sqrt{13}}\right)}{442\left(13-2\sqrt{13}\right)(m+1)} \\ & + \frac{3\left(13-9\sqrt{13}\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13+2\sqrt{13}}\right)}{442\left(13+2\sqrt{13}\right)(m+1)} \end{aligned}$$

[Out] (3\*(1+4\*x)^(1+m)\*Hypergeometric2F1[1, 1+m, 2+m, (-3\*(1+4\*x))/5])/(85\*(1+m)) + (3\*(13+9\*Sqrt[13])\*(1+4\*x)^(1+m)\*Hypergeometric2F1[1, 1+m, 2+m, (3\*(1+4\*x))/(13-2\*Sqrt[13])])/(442\*(13-2\*Sqrt[13])\*(1+m)) + (3\*(13-9\*Sqrt[13])\*(1+4\*x)^(1+m)\*Hypergeometric2F1[1, 1+m, 2+m, (3\*(1+4\*x))/(13+2\*Sqrt[13])])/(442\*(13+2\*Sqrt[13])\*(1+m))

Rubi [A] time = 0.461071, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\begin{aligned} & \frac{3(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; -\frac{3}{5}(4x+1)\right)}{85(m+1)} \\ & + \frac{3\left(13+9\sqrt{13}\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13-2\sqrt{13}}\right)}{442\left(13-2\sqrt{13}\right)(m+1)} \\ & + \frac{3\left(13-9\sqrt{13}\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13+2\sqrt{13}}\right)}{442\left(13+2\sqrt{13}\right)(m+1)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(1+4\*x)^m/((2+3\*x)\*(1-5\*x+3\*x^2)),x]

[Out] (3\*(1+4\*x)^(1+m)\*Hypergeometric2F1[1, 1+m, 2+m, (-3\*(1+4\*x))/5])/(85\*(1+m)) + (3\*(13+9\*Sqrt[13])\*(1+4\*x)^(1+m)\*Hypergeometric2F1[1, 1+m, 2+m, (3\*(1+4\*x))/(13-2\*Sqrt[13])])/(442\*(13-2\*Sqrt[13])\*(1+m)) + (3\*(13-9\*Sqrt[13])\*(1+4\*x)^(1+m)\*Hypergeometric2F1[1, 1+m, 2+m, (3\*(1+4\*x))/(13+2\*Sqrt[13])])/(442\*(13+2\*Sqrt[13])\*(1+m))

Rubi in Sympy [A] time = 33.1572, size = 133, normalized size = 0.81

$$\begin{aligned} & \frac{\left(3 + \frac{27\sqrt{13}}{13}\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{-12x-3}{-13+2\sqrt{13}}\right)}{17\left(-4\sqrt{13}+26\right)(m+1)} \\ & + \frac{\left(-\frac{27\sqrt{13}}{13}+3\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{12x+3}{2\sqrt{13}+13}\right)}{17\left(4\sqrt{13}+26\right)(m+1)} + \frac{3(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; -\frac{12x}{5}-\frac{3}{5}\right)}{85(m+1)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.



[In] `rubi_integrate((1+4*x)**m/(2+3*x)/(3*x**2-5*x+1),x)`

[Out]  $(3 + 27\sqrt{13}/13)(4x + 1)^{m+1} \text{hyper}((1, m + 1), (m + 2, ), (-12x - 3)/(-13 + 2\sqrt{13}))/ (17(-4\sqrt{13} + 26)^{m+1}) + (-27\sqrt{13}/13 + 3)(4x + 1)^{m+1} \text{hyper}((1, m + 1), (m + 2, ), (12x + 3)/(2\sqrt{13} + 13))/ (17(4\sqrt{13} + 26)^{m+1}) + 3(4x + 1)^{m+1} \text{hyper}((1, m + 1), (m + 2, ), -12x/5 - 3/5)/(85^{m+1})$

**Mathematica [A]** time = 0.950001, size = 233, normalized size = 1.42

$$(4x + 1)^m \left( \frac{5 \cdot 3^{-m} \left( -\frac{4x+1}{-6x+\sqrt{13}+5} \right)^{-m} \left( \frac{4x+1}{6x+\sqrt{13}-5} \right)^{-m} \left( (9\sqrt{13}-13) \left( \frac{8x+2}{6x+\sqrt{13}-5} \right)^m {}_2F_1 \left( -m, -m; 1-m; \frac{13+2\sqrt{13}}{2(-6x+\sqrt{13}+5)} \right) - (13+9\sqrt{13}) \left( -\frac{8x+2}{-6x+\sqrt{13}+5} \right)^m {}_2F_1 \left( -m, -m; 1-m; \frac{13+2\sqrt{13}}{2(-6x+\sqrt{13}+5)} \right)}{m} \right)$$

2210

Antiderivative was successfully verified.

[In] `Integrate[(1 + 4*x)^m/((2 + 3*x)*(1 - 5*x + 3*x^2)),x]`

[Out]  $((1 + 4x)^m ((78(1 + 4x) \text{Hypergeometric2F1}[1, 1 + m, 2 + m, (-3(1 + 4x))/5])/(1 + m) + (5((-13 + 9\sqrt{13})^m ((2 + 8x)/(-5 + \sqrt{13} + 6x))^m \text{Hypergeometric2F1}[-m, -m, 1 - m, (13 + 2\sqrt{13}]/(2(5 + \sqrt{13} - 6x))) - (13 + 9\sqrt{13})^m (-(2 + 8x)/(5 + \sqrt{13} - 6x)))^m \text{Hypergeometric2F1}[-m, -m, 1 - m, (-13 + 2\sqrt{13}]/(2(-5 + \sqrt{13} + 6x)))))/(3^m m^m (-(1 + 4x)/(5 + \sqrt{13} - 6x)))^m ((1 + 4x)/(-5 + \sqrt{13} + 6x))^m))/2210$

**Maple [F]** time = 0.171, size = 0, normalized size = 0.

$$\int \frac{(1 + 4x)^m}{(2 + 3x)(3x^2 - 5x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1),x)`

[Out] `int((1+4*x)^m/(2+3*x)/(3*x^2-5*x+1),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x + 1)^m}{(3x^2 - 5x + 1)(3x + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)*(3*x + 2)),x, algorithm="maxima")`

[Out] `integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)*(3*x + 2)), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral} \left( \frac{(4x + 1)^m}{9x^3 - 9x^2 - 7x + 2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)*(3*x + 2)),x, algorithm="fricas")`

[Out] `integral((4*x + 1)^m/(9*x^3 - 9*x^2 - 7*x + 2), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x + 1)^m}{(3x + 2)(3x^2 - 5x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+4*x)**m/(2+3*x)/(3*x**2-5*x+1),x)`

[Out] `Integral((4*x + 1)**m/((3*x + 2)*(3*x**2 - 5*x + 1)), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x + 1)^m}{(3x^2 - 5x + 1)(3x + 2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)*(3*x + 2)),x, algorithm="giac")`

[Out] `integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)*(3*x + 2)), x)`

$$3.936 \quad \int \frac{(1+4x)^m}{(2+3x)^2(1-5x+3x^2)} dx$$

Optimal. Leaf size=199

$$\begin{aligned} & \frac{27(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; -\frac{3}{5}(4x+1)\right)}{1445(m+1)} \\ & + \frac{3\left(117+47\sqrt{13}\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13-2\sqrt{13}}\right)}{7514\left(13-2\sqrt{13}\right)(m+1)} \\ & + \frac{3\left(117-47\sqrt{13}\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13+2\sqrt{13}}\right)}{7514\left(13+2\sqrt{13}\right)(m+1)} \\ & + \frac{12(4x+1)^{m+1} {}_2F_1\left(2, m+1; m+2; -\frac{3}{5}(4x+1)\right)}{425(m+1)} \end{aligned}$$

[Out] (27\*(1+4\*x)^(1+m)\*Hypergeometric2F1[1, 1+m, 2+m, (-3\*(1+4\*x))/5])/(1445\*(1+m)) + (3\*(117+47\*Sqrt[13])\*(1+4\*x)^(1+m)\*Hypergeometric2F1[1, 1+m, 2+m, (3\*(1+4\*x))/(13-2\*Sqrt[13])])/(7514\*(13-2\*Sqrt[13])\*(1+m)) + (3\*(117-47\*Sqrt[13])\*(1+4\*x)^(1+m)\*Hypergeometric2F1[1, 1+m, 2+m, (3\*(1+4\*x))/(13+2\*Sqrt[13])])/(7514\*(13+2\*Sqrt[13])\*(1+m)) + (12\*(1+4\*x)^(1+m)\*Hypergeometric2F1[2, 1+m, 2+m, (-3\*(1+4\*x))/5])/(425\*(1+m))

**Rubi [A]** time = 0.488883, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$\begin{aligned} & \frac{27(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; -\frac{3}{5}(4x+1)\right)}{1445(m+1)} \\ & + \frac{3\left(117+47\sqrt{13}\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13-2\sqrt{13}}\right)}{7514\left(13-2\sqrt{13}\right)(m+1)} \\ & + \frac{3\left(117-47\sqrt{13}\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13+2\sqrt{13}}\right)}{7514\left(13+2\sqrt{13}\right)(m+1)} \\ & + \frac{12(4x+1)^{m+1} {}_2F_1\left(2, m+1; m+2; -\frac{3}{5}(4x+1)\right)}{425(m+1)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(1+4\*x)^m/((2+3\*x)^2\*(1-5\*x+3\*x^2)),x]

[Out] (27\*(1+4\*x)^(1+m)\*Hypergeometric2F1[1, 1+m, 2+m, (-3\*(1+4\*x))/5])/(1445\*(1+m)) + (3\*(117+47\*Sqrt[13])\*(1+4\*x)^(1+m)\*Hypergeometric2F1[1, 1+m, 2+m, (3\*(1+4\*x))/(13-2\*Sqrt[13])])/(7514\*(13-2\*Sqrt[13])\*(1+m)) + (3\*(117-47\*Sqrt[13])\*(1+4\*x)^(1+m)\*Hypergeometric2F1[1, 1+m, 2+m, (3\*(1+4\*x))/(13+2\*Sqrt[13])])/(7514\*(13+2\*Sqrt[13])\*(1+m)) + (12\*(1+4\*x)^(1+m)\*Hypergeometric2F1[2, 1+m, 2+m, (-3\*(1+4\*x))/5])/(425\*(1+m))

**Rubi in Sympy [A]** time = 35.5173, size = 163, normalized size = 0.82

$$\frac{\left(27 + \frac{141\sqrt{13}}{13}\right) (4x + 1)^{m+1} {}_2F_1\left(\begin{matrix} 1, m + 1 \\ m + 2 \end{matrix} \middle| \frac{-12x-3}{-13+2\sqrt{13}}\right)}{289 \left(-4\sqrt{13} + 26\right) (m + 1)} + \frac{\left(-\frac{141\sqrt{13}}{13} + 27\right) (4x + 1)^{m+1} {}_2F_1\left(\begin{matrix} 1, m + 1 \\ m + 2 \end{matrix} \middle| \frac{12x+3}{2\sqrt{13}+13}\right)}{289 \left(4\sqrt{13} + 26\right) (m + 1)} + \frac{27 (4x + 1)^{m+1} {}_2F_1\left(\begin{matrix} 1, m + 1 \\ m + 2 \end{matrix} \middle| -\frac{12x}{5} - \frac{3}{5}\right)}{1445 (m + 1)} + \frac{12 (4x + 1)^{m+1} {}_2F_1\left(\begin{matrix} 2, m + 1 \\ m + 2 \end{matrix} \middle| -\frac{12x}{5} - \frac{3}{5}\right)}{425 (m + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((1+4*x)**m/(2+3*x)**2/(3*x**2-5*x+1),x)`

[Out]  $(27 + 141 \cdot \text{sqrt}(13)/13) \cdot (4 \cdot x + 1)^{(m + 1)} \cdot \text{hyper}((1, m + 1), (m + 2, ), (-12 \cdot x - 3)/(-13 + 2 \cdot \text{sqrt}(13))) / (289 \cdot (-4 \cdot \text{sqrt}(13) + 26) \cdot (m + 1)) + (-141 \cdot \text{sqrt}(13)/13 + 27) \cdot (4 \cdot x + 1)^{(m + 1)} \cdot \text{hyper}((1, m + 1), (m + 2, ), (12 \cdot x + 3)/(2 \cdot \text{sqrt}(13) + 13)) / (289 \cdot (4 \cdot \text{sqrt}(13) + 26) \cdot (m + 1)) + 27 \cdot (4 \cdot x + 1)^{(m + 1)} \cdot \text{hyper}((1, m + 1), (m + 2, ), -12 \cdot x/5 - 3/5) / (1445 \cdot (m + 1)) + 12 \cdot (4 \cdot x + 1)^{(m + 1)} \cdot \text{hyper}((2, m + 1), (m + 2, ), -12 \cdot x/5 - 3/5) / (425 \cdot (m + 1))$

**Mathematica [A]** time = 3.96925, size = 284, normalized size = 1.43

$$(4x + 1)^m \left( \frac{\left(\frac{12x+3}{6x+4}\right)^{-m} \left(47\sqrt{13}-117\right) \left(\frac{4x+1}{3x+2}\right)^m \left(-\frac{4x+1}{-6x+\sqrt{13}+5}\right)^{-m} {}_2F_1\left(-m, -m; 1-m; \frac{13+2\sqrt{13}}{2(-6x+\sqrt{13}+5)}\right) - (117+47\sqrt{13}) \left(\frac{4x+1}{3x+2}\right)^m \left(\frac{4x+1}{6x+\sqrt{13}-5}\right)^{-m} {}_2F_1\left(-m, -m; 1-m; \frac{13-2\sqrt{13}}{2(6x+\sqrt{13}-5)}\right)}{m}$$

7514

Antiderivative was successfully verified.

[In] `Integrate[(1 + 4*x)^m/((2 + 3*x)^2*(1 - 5*x + 3*x^2)),x]`

[Out]  $((1 + 4 \cdot x)^m \cdot ((442 \cdot \text{Hypergeometric2F1}[1 - m, -m, 2 - m, 5/(8 + 12 \cdot x)]) / ((-1 + m) \cdot (2 + 3 \cdot x) \cdot (1 - 5/(8 + 12 \cdot x))^m) + (((-117 + 47 \cdot \text{Sqrt}[13]) \cdot ((1 + 4 \cdot x)/(2 + 3 \cdot x))^m \cdot \text{Hypergeometric2F1}[-m, -m, 1 - m, (13 + 2 \cdot \text{Sqrt}[13])/(2 \cdot (5 + \text{Sqrt}[13] - 6 \cdot x))]) / (-((1 + 4 \cdot x)/(5 + \text{Sqrt}[13] - 6 \cdot x)))^m - ((117 + 47 \cdot \text{Sqrt}[13]) \cdot ((1 + 4 \cdot x)/(2 + 3 \cdot x))^m \cdot \text{Hypergeometric2F1}[-m, -m, 1 - m, (-13 + 2 \cdot \text{Sqrt}[13])/(2 \cdot (-5 + \text{Sqrt}[13] + 6 \cdot x))]) / (((1 + 4 \cdot x)/(-5 + \text{Sqrt}[13] + 6 \cdot x))^m + 117 \cdot 2^{(1 + m)} \cdot \text{Hypergeometric2F1}[-m, -m, 1 - m, 5/(8 + 12 \cdot x)]) / (m \cdot ((3 + 12 \cdot x)/(4 + 6 \cdot x))^m))) / 7514$

**Maple [F]** time = 0.171, size = 0, normalized size = 0.

$$\int \frac{(1 + 4x)^m}{(2 + 3x)^2 (3x^2 - 5x + 1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1),x)`

[Out] `int((1+4*x)^m/(2+3*x)^2/(3*x^2-5*x+1),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{(3x^2-5x+1)(3x+2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x+1)^m/((3*x^2-5*x+1)*(3*x+2)^2),x,algorithm="maxima")`

[Out] `integrate((4*x+1)^m/((3*x^2-5*x+1)*(3*x+2)^2),x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(4x+1)^m}{27x^4-9x^3-39x^2-8x+4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x+1)^m/((3*x^2-5*x+1)*(3*x+2)^2),x,algorithm="fricas")`

[Out] `integral((4*x+1)^m/(27*x^4-9*x^3-39*x^2-8*x+4),x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{(3x+2)^2(3x^2-5x+1)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1+4*x)**m/(2+3*x)**2/(3*x**2-5*x+1),x)`

[Out] `Integral((4*x+1)**m/((3*x+2)**2*(3*x**2-5*x+1)),x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{(3x^2-5x+1)(3x+2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x+1)^m/((3*x^2-5*x+1)*(3*x+2)^2),x,algorithm="giac")`

[Out] `integrate((4*x+1)^m/((3*x^2-5*x+1)*(3*x+2)^2),x)`

$$3.937 \quad \int \frac{(2+3x)^4(1+4x)^m}{(1-5x+3x^2)^2} dx$$

**Optimal.** Leaf size=202

$$\begin{aligned} & \frac{\left(13689 - \sqrt{13} \left(-1570\sqrt{13}m + 4474m + 297\right)\right) (4x + 1)^{m+1} {}_2F_1\left(1, m + 1; m + 2; \frac{3(4x+1)}{13-2\sqrt{13}}\right)}{169 \left(13 - 2\sqrt{13}\right) (m + 1)} \\ & - \frac{\left(\sqrt{13} \left(1570\sqrt{13}m + 4474m + 297\right) + 13689\right) (4x + 1)^{m+1} {}_2F_1\left(1, m + 1; m + 2; \frac{3(4x+1)}{13+2\sqrt{13}}\right)}{169 \left(13 + 2\sqrt{13}\right) (m + 1)} \\ & + \frac{(844 - 2355x)(4x + 1)^{m+1}}{39(3x^2 - 5x + 1)} + \frac{9(4x + 1)^{m+1}}{4(m + 1)} \end{aligned}$$

[Out] (9\*(1 + 4\*x)^(1 + m))/(4\*(1 + m)) + ((844 - 2355\*x)\*(1 + 4\*x)^(1 + m))/(39\*(1 - 5\*x + 3\*x^2)) - ((13689 - Sqrt[13]\*(297 + 4474\*m - 1570\*Sqrt[13]\*m))\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 - 2\*Sqrt[13])])/(169\*(13 - 2\*Sqrt[13])\*(1 + m)) - ((13689 + Sqrt[13]\*(297 + 4474\*m + 1570\*Sqrt[13]\*m))\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 + 2\*Sqrt[13])])/(169\*(13 + 2\*Sqrt[13])\*(1 + m))

**Rubi [A]** time = 0.554124, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\begin{aligned} & \frac{\left(13689 - \sqrt{13} \left(-1570\sqrt{13}m + 4474m + 297\right)\right) (4x + 1)^{m+1} {}_2F_1\left(1, m + 1; m + 2; \frac{3(4x+1)}{13-2\sqrt{13}}\right)}{169 \left(13 - 2\sqrt{13}\right) (m + 1)} \\ & - \frac{\left(\sqrt{13} \left(1570\sqrt{13}m + 4474m + 297\right) + 13689\right) (4x + 1)^{m+1} {}_2F_1\left(1, m + 1; m + 2; \frac{3(4x+1)}{13+2\sqrt{13}}\right)}{169 \left(13 + 2\sqrt{13}\right) (m + 1)} \\ & + \frac{(844 - 2355x)(4x + 1)^{m+1}}{39(3x^2 - 5x + 1)} + \frac{9(4x + 1)^{m+1}}{4(m + 1)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[((2 + 3\*x)^4\*(1 + 4\*x)^m)/(1 - 5\*x + 3\*x^2)^2, x]

[Out] (9\*(1 + 4\*x)^(1 + m))/(4\*(1 + m)) + ((844 - 2355\*x)\*(1 + 4\*x)^(1 + m))/(39\*(1 - 5\*x + 3\*x^2)) - ((13689 - Sqrt[13]\*(297 + 4474\*m - 1570\*Sqrt[13]\*m))\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 - 2\*Sqrt[13])])/(169\*(13 - 2\*Sqrt[13])\*(1 + m)) - ((13689 + Sqrt[13]\*(297 + 4474\*m + 1570\*Sqrt[13]\*m))\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 + 2\*Sqrt[13])])/(169\*(13 + 2\*Sqrt[13])\*(1 + m))

**Rubi in Sympy [A]** time = 58.545, size = 255, normalized size = 1.26

$$\frac{(-30615x + 10972)(4x + 1)^{m+1}}{507(3x^2 - 5x + 1)} - \frac{2(20410m - \sqrt{13}(-4474m + 14679))(4x + 1)^{m+1} {}_2F_1\left(\begin{matrix} 1, m+1 \\ m+2 \end{matrix} \middle| \frac{12x+3}{2\sqrt{13}+13}\right)}{169(4\sqrt{13} + 26)(m+1)} - \frac{2(20410m + \sqrt{13}(-4474m + 14679))(4x + 1)^{m+1} {}_2F_1\left(\begin{matrix} 1, m+1 \\ m+2 \end{matrix} \middle| \frac{-12x-3}{-13+2\sqrt{13}}\right)}{169(-4\sqrt{13} + 26)(m+1)} - \frac{3\left(-\frac{768\sqrt{13}}{13} + 54\right)(4x + 1)^{m+1} {}_2F_1\left(\begin{matrix} 1, m+1 \\ m+2 \end{matrix} \middle| \frac{-12x-3}{-13+2\sqrt{13}}\right)}{(-4\sqrt{13} + 26)(m+1)} - \frac{3\left(54 + \frac{768\sqrt{13}}{13}\right)(4x + 1)^{m+1} {}_2F_1\left(\begin{matrix} 1, m+1 \\ m+2 \end{matrix} \middle| \frac{12x+3}{2\sqrt{13}+13}\right)}{(4\sqrt{13} + 26)(m+1)} + \frac{9(4x + 1)^{m+1}}{4(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((2+3*x)**4*(1+4*x)**m/(3*x**2-5*x+1)**2,x)`

[Out] `(-30615*x + 10972)*(4*x + 1)**(m + 1)/(507*(3*x**2 - 5*x + 1)) - 2*(20410*m - sqrt(13)*(-4474*m + 14679))*(4*x + 1)**(m + 1)*hyper((1, m + 1), (m + 2, ), (12*x + 3)/(2*sqrt(13) + 13))/(169*(4*sqrt(13) + 26)*(m + 1)) - 2*(20410*m + sqrt(13)*(-4474*m + 14679))*(4*x + 1)**(m + 1)*hyper((1, m + 1), (m + 2, ), (-12*x - 3)/(-13 + 2*sqrt(13)))/(169*(-4*sqrt(13) + 26)*(m + 1)) - 3*(-768*sqrt(13)/13 + 54)*(4*x + 1)**(m + 1)*hyper((1, m + 1), (m + 2, ), (-12*x - 3)/(-13 + 2*sqrt(13)))/((-4*sqrt(13) + 26)*(m + 1)) - 3*(54 + 768*sqrt(13)/13)*(4*x + 1)**(m + 1)*hyper((1, m + 1), (m + 2, ), (12*x + 3)/(2*sqrt(13) + 13))/((4*sqrt(13) + 26)*(m + 1)) + 9*(4*x + 1)**(m + 1)/(4*(m + 1))`

**Mathematica [A]** time = 0.20912, size = 0, normalized size = 0.

$$\int \frac{(2+3x)^4(1+4x)^m}{(1-5x+3x^2)^2} dx$$

Verification is Not applicable to the result.

[In] `Integrate[((2 + 3*x)^4*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]`

[Out] `Integrate[((2 + 3*x)^4*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2, x]`

**Maple [F]** time = 0.17, size = 0, normalized size = 0.

$$\int \frac{(2+3x)^4(1+4x)^m}{(3x^2-5x+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1)^2,x)`

[Out] `int((2+3*x)^4*(1+4*x)^m/(3*x^2-5*x+1)^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m(3x+2)^4}{(3x^2-5x+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x + 1)^m*(3*x + 2)^4/(3*x^2 - 5*x + 1)^2,x, algorithm="maxima")`

[Out] `integrate((4*x + 1)^m*(3*x + 2)^4/(3*x^2 - 5*x + 1)^2, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(81x^4 + 216x^3 + 216x^2 + 96x + 16)(4x+1)^m}{9x^4 - 30x^3 + 31x^2 - 10x + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x + 1)^m*(3*x + 2)^4/(3*x^2 - 5*x + 1)^2,x, algorithm="fricas")`

[Out] `integral((81*x^4 + 216*x^3 + 216*x^2 + 96*x + 16)*(4*x + 1)^m/(9*x^4 - 30*x^3 + 31*x^2 - 10*x + 1), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+3*x)**4*(1+4*x)**m/(3*x**2-5*x+1)**2,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m(3x+2)^4}{(3x^2-5x+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x + 1)^m*(3*x + 2)^4/(3*x^2 - 5*x + 1)^2,x, algorithm="giac")`

[Out] `integrate((4*x + 1)^m*(3*x + 2)^4/(3*x^2 - 5*x + 1)^2, x)`



$$3.938 \quad \int \frac{(2+3x)^3(1+4x)^m}{(1-5x+3x^2)^2} dx$$

**Optimal.** Leaf size=181

$$\begin{aligned} & \frac{\left(\sqrt{13}\left(568\sqrt{13}m - 1168m + 1701\right) + 1521\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13-2\sqrt{13}}\right)}{338\left(13-2\sqrt{13}\right)(m+1)} \\ & + \frac{\left(\sqrt{13}(1701-1168m) - 13(568m+117)\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13+2\sqrt{13}}\right)}{338\left(13+2\sqrt{13}\right)(m+1)} \\ & + \frac{(209-426x)(4x+1)^{m+1}}{39(3x^2-5x+1)} \end{aligned}$$

[Out] ((209 - 426\*x)\*(1 + 4\*x)^(1 + m))/(39\*(1 - 5\*x + 3\*x^2)) - ((1521 + Sqrt[13]\*(1701 - 1168\*m + 568\*Sqrt[13]\*m))\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 - 2\*Sqrt[13])])/(338\*(13 - 2\*Sqrt[13])\*(1 + m)) + ((Sqrt[13]\*(1701 - 1168\*m) - 13\*(117 + 568\*m))\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 + 2\*Sqrt[13])])/(338\*(13 + 2\*Sqrt[13])\*(1 + m))

**Rubi [A]** time = 0.552917, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\begin{aligned} & \frac{\left(\sqrt{13}\left(568\sqrt{13}m - 1168m + 1701\right) + 1521\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13-2\sqrt{13}}\right)}{338\left(13-2\sqrt{13}\right)(m+1)} \\ & + \frac{\left(\sqrt{13}(1701-1168m) - 13(568m+117)\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13+2\sqrt{13}}\right)}{338\left(13+2\sqrt{13}\right)(m+1)} \\ & + \frac{(209-426x)(4x+1)^{m+1}}{39(3x^2-5x+1)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[((2 + 3\*x)^3\*(1 + 4\*x)^m)/(1 - 5\*x + 3\*x^2)^2, x]

[Out] ((209 - 426\*x)\*(1 + 4\*x)^(1 + m))/(39\*(1 - 5\*x + 3\*x^2)) - ((1521 + Sqrt[13]\*(1701 - 1168\*m + 568\*Sqrt[13]\*m))\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 - 2\*Sqrt[13])])/(338\*(13 - 2\*Sqrt[13])\*(1 + m)) + ((Sqrt[13]\*(1701 - 1168\*m) - 13\*(117 + 568\*m))\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 + 2\*Sqrt[13])])/(338\*(13 + 2\*Sqrt[13])\*(1 + m))

**Rubi in Sympy [A]** time = 56.6552, size = 240, normalized size = 1.33

$$\frac{(-5538x + 2717)(4x + 1)^{m+1}}{507(3x^2 - 5x + 1)} - \frac{4 \left(1846m - \sqrt{13}(-292m + 1215)\right) (4x + 1)^{m+1} {}_2F_1\left(\begin{matrix} 1, m+1 \\ m+2 \end{matrix} \middle| \frac{12x+3}{2\sqrt{13}+13}\right)}{169 \left(4\sqrt{13} + 26\right) (m+1)} - \frac{4 \left(1846m + \sqrt{13}(-292m + 1215)\right) (4x + 1)^{m+1} {}_2F_1\left(\begin{matrix} 1, m+1 \\ m+2 \end{matrix} \middle| \frac{-12x-3}{-13+2\sqrt{13}}\right)}{169 \left(-4\sqrt{13} + 26\right) (m+1)} - \frac{3 \left(-\frac{81\sqrt{13}}{13} + 3\right) (4x + 1)^{m+1} {}_2F_1\left(\begin{matrix} 1, m+1 \\ m+2 \end{matrix} \middle| \frac{-12x-3}{-13+2\sqrt{13}}\right)}{\left(-4\sqrt{13} + 26\right) (m+1)} - \frac{3 \left(3 + \frac{81\sqrt{13}}{13}\right) (4x + 1)^{m+1} {}_2F_1\left(\begin{matrix} 1, m+1 \\ m+2 \end{matrix} \middle| \frac{12x+3}{2\sqrt{13}+13}\right)}{\left(4\sqrt{13} + 26\right) (m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((2+3*x)**3*(1+4*x)**m/(3*x**2-5*x+1)**2,x)`

[Out] `(-5538*x + 2717)*(4*x + 1)**(m + 1)/(507*(3*x**2 - 5*x + 1)) - 4*(1846*m - sqrt(13)*(-292*m + 1215))*(4*x + 1)**(m + 1)*hyper((1, m + 1), (m + 2, ), (12*x + 3)/(2*sqrt(13) + 13))/(169*(4*sqrt(13) + 26)*(m + 1)) - 4*(1846*m + sqrt(13)*(-292*m + 1215))*(4*x + 1)**(m + 1)*hyper((1, m + 1), (m + 2, ), (-12*x - 3)/(-13 + 2*sqrt(13)))/(169*(-4*sqrt(13) + 26)*(m + 1)) - 3*(-81*sqrt(13)/13 + 3)*(4*x + 1)**(m + 1)*hyper((1, m + 1), (m + 2, ), (-12*x - 3)/(-13 + 2*sqrt(13)))/((-4*sqrt(13) + 26)*(m + 1)) - 3*(3 + 81*sqrt(13)/13)*(4*x + 1)**(m + 1)*hyper((1, m + 1), (m + 2, ), (12*x + 3)/(2*sqrt(13) + 13))/(4*sqrt(13) + 26)*(m + 1))`

**Mathematica [A]** time = 0.173909, size = 0, normalized size = 0.

$$\int \frac{(2 + 3x)^3(1 + 4x)^m}{(1 - 5x + 3x^2)^2} dx$$

Verification is Not applicable to the result.

[In] `Integrate[((2 + 3*x)^3*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]`

[Out] `Integrate[((2 + 3*x)^3*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2, x]`

**Maple [F]** time = 0.176, size = 0, normalized size = 0.

$$\int \frac{(2 + 3x)^3(1 + 4x)^m}{(3x^2 - 5x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1)^2,x)`

[Out] `int((2+3*x)^3*(1+4*x)^m/(3*x^2-5*x+1)^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m(3x+2)^3}{(3x^2-5x+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1)^2,x, algorithm="maxima")`

[Out] `integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1)^2, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(27x^3 + 54x^2 + 36x + 8)(4x + 1)^m}{9x^4 - 30x^3 + 31x^2 - 10x + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1)^2,x, algorithm="fricas")`

[Out] `integral((27*x^3 + 54*x^2 + 36*x + 8)*(4*x + 1)^m/(9*x^4 - 30*x^3 + 31*x^2 - 10*x + 1), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((2+3*x)**3*(1+4*x)**m/(3*x**2-5*x+1)**2,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m(3x+2)^3}{(3x^2-5x+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1)^2,x, algorithm="giac")`

[Out] `integrate((4*x + 1)^m*(3*x + 2)^3/(3*x^2 - 5*x + 1)^2, x)`

$$3.939 \quad \int \frac{(2+3x)^2(1+4x)^m}{(1-5x+3x^2)^2} dx$$

**Optimal.** Leaf size=179

$$\frac{2 \left( 153 - \left( 23 - 29\sqrt{13} \right) m \right) (4x + 1)^{m+1} {}_2F_1 \left( 1, m + 1; m + 2; \frac{3(4x+1)}{13-2\sqrt{13}} \right)}{13\sqrt{13} \left( 13 - 2\sqrt{13} \right) (m + 1)} + \frac{2 \left( 153 - \left( 23 + 29\sqrt{13} \right) m \right) (4x + 1)^{m+1} {}_2F_1 \left( 1, m + 1; m + 2; \frac{3(4x+1)}{13+2\sqrt{13}} \right)}{13\sqrt{13} \left( 13 + 2\sqrt{13} \right) (m + 1)} + \frac{(61 - 87x)(4x + 1)^{m+1}}{39(3x^2 - 5x + 1)}$$

[Out]  $((61 - 87*x) * (1 + 4*x)^(1 + m)) / (39 * (1 - 5*x + 3*x^2)) - (2 * (153 - (23 - 29*sqrt[13])*m) * (1 + 4*x)^(1 + m) * Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*sqrt[13])]) / (13*sqrt[13]*(13 - 2*sqrt[13])*(1 + m)) + (2 * (153 - (23 + 29*sqrt[13])*m) * (1 + 4*x)^(1 + m) * Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*sqrt[13])]) / (13*sqrt[13]*(13 + 2*sqrt[13])*(1 + m))$

**Rubi [A]** time = 0.508101, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{2 \left( 153 - \left( 23 - 29\sqrt{13} \right) m \right) (4x + 1)^{m+1} {}_2F_1 \left( 1, m + 1; m + 2; \frac{3(4x+1)}{13-2\sqrt{13}} \right)}{13\sqrt{13} \left( 13 - 2\sqrt{13} \right) (m + 1)} + \frac{2 \left( 153 - \left( 23 + 29\sqrt{13} \right) m \right) (4x + 1)^{m+1} {}_2F_1 \left( 1, m + 1; m + 2; \frac{3(4x+1)}{13+2\sqrt{13}} \right)}{13\sqrt{13} \left( 13 + 2\sqrt{13} \right) (m + 1)} + \frac{(61 - 87x)(4x + 1)^{m+1}}{39(3x^2 - 5x + 1)}$$

Antiderivative was successfully verified.

[In] Int[((2 + 3\*x)^2\*(1 + 4\*x)^m)/(1 - 5\*x + 3\*x^2)^2, x]

[Out]  $((61 - 87*x) * (1 + 4*x)^(1 + m)) / (39 * (1 - 5*x + 3*x^2)) - (2 * (153 - (23 - 29*sqrt[13])*m) * (1 + 4*x)^(1 + m) * Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 - 2*sqrt[13])]) / (13*sqrt[13]*(13 - 2*sqrt[13])*(1 + m)) + (2 * (153 - (23 + 29*sqrt[13])*m) * (1 + 4*x)^(1 + m) * Hypergeometric2F1[1, 1 + m, 2 + m, (3*(1 + 4*x))/(13 + 2*sqrt[13])]) / (13*sqrt[13]*(13 + 2*sqrt[13])*(1 + m))$

**Rubi in Sympy [A]** time = 53.805, size = 233, normalized size = 1.3

$$\frac{(-1131x + 793)(4x + 1)^{m+1}}{507(3x^2 - 5x + 1)} - \frac{2 \left( 754m - \sqrt{13}(-46m + 423) \right) (4x + 1)^{m+1} {}_2F_1 \left( 1, m + 1; m + 2; \frac{12x+3}{2\sqrt{13}+13} \right)}{169 \left( 4\sqrt{13} + 26 \right) (m + 1)} - \frac{2 \left( 754m + \sqrt{13}(-46m + 423) \right) (4x + 1)^{m+1} {}_2F_1 \left( 1, m + 1; m + 2; \frac{-12x-3}{-13+2\sqrt{13}} \right)}{169 \left( -4\sqrt{13} + 26 \right) (m + 1)} + \frac{18\sqrt{13}(4x + 1)^{m+1} {}_2F_1 \left( 1, m + 1; m + 2; \frac{-12x-3}{-13+2\sqrt{13}} \right)}{13 \left( -4\sqrt{13} + 26 \right) (m + 1)} - \frac{18\sqrt{13}(4x + 1)^{m+1} {}_2F_1 \left( 1, m + 1; m + 2; \frac{12x+3}{2\sqrt{13}+13} \right)}{13 \left( 4\sqrt{13} + 26 \right) (m + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((2+3*x)**2*(1+4*x)**m/(3*x**2-5*x+1)**2,x)`

[Out]  $(-1131x + 793)(4x + 1)^{m+1}/(507(3x^2 - 5x + 1)) - 2(754m - \sqrt{13}(-46m + 423))(4x + 1)^{m+1} \operatorname{hyper}((1, m + 1), (m + 2, ), (12x + 3)/(2\sqrt{13} + 13))/(169(4\sqrt{13} + 26)^{m+1}) - 2(754m + \sqrt{13}(-46m + 423))(4x + 1)^{m+1} \operatorname{hyper}((1, m + 1), (m + 2, ), (-12x - 3)/(-13 + 2\sqrt{13}))/ (169(-4\sqrt{13} + 26)^{m+1}) + 18\sqrt{13}(4x + 1)^{m+1} \operatorname{hyper}((1, m + 1), (m + 2, ), (-12x - 3)/(-13 + 2\sqrt{13}))/ (13(-4\sqrt{13} + 26)^{m+1}) - 18\sqrt{13}(4x + 1)^{m+1} \operatorname{hyper}((1, m + 1), (m + 2, ), (12x + 3)/(2\sqrt{13} + 13))/ (13(4\sqrt{13} + 26)^{m+1})$

**Mathematica [A]** time = 0.159536, size = 0, normalized size = 0.

$$\int \frac{(2+3x)^2(1+4x)^m}{(1-5x+3x^2)^2} dx$$

Verification is Not applicable to the result.

[In] `Integrate[((2 + 3*x)^2*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2,x]`

[Out] `Integrate[((2 + 3*x)^2*(1 + 4*x)^m)/(1 - 5*x + 3*x^2)^2, x]`

**Maple [F]** time = 0.165, size = 0, normalized size = 0.

$$\int \frac{(2+3x)^2(1+4x)^m}{(3x^2-5x+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1)^2,x)`

[Out] `int((2+3*x)^2*(1+4*x)^m/(3*x^2-5*x+1)^2,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m(3x+2)^2}{(3x^2-5x+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((4*x + 1)^m*(3*x + 2)^2/(3*x^2 - 5*x + 1)^2,x, algorithm="maxima")`

[Out] `integrate((4*x + 1)^m*(3*x + 2)^2/(3*x^2 - 5*x + 1)^2, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(9x^2 + 12x + 4)(4x + 1)^m}{9x^4 - 30x^3 + 31x^2 - 10x + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x + 1)^m\*(3\*x + 2)^2/(3\*x^2 - 5\*x + 1)^2,x, algorithm="fricas")

[Out] integral((9\*x^2 + 12\*x + 4)\*(4\*x + 1)^m/(9\*x^4 - 30\*x^3 + 31\*x^2 - 10\*x + 1), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x + 2)^2 (4x + 1)^m}{(3x^2 - 5x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3\*x)\*\*2\*(1+4\*x)\*\*m/(3\*x\*\*2-5\*x+1)\*\*2,x)

[Out] Integral((3\*x + 2)\*\*2\*(4\*x + 1)\*\*m/(3\*x\*\*2 - 5\*x + 1)\*\*2, x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x + 1)^m (3x + 2)^2}{(3x^2 - 5x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x + 1)^m\*(3\*x + 2)^2/(3\*x^2 - 5\*x + 1)^2,x, algorithm="giac")

[Out] integrate((4\*x + 1)^m\*(3\*x + 2)^2/(3\*x^2 - 5\*x + 1)^2, x)

$$3.940 \quad \int \frac{(2+3x)(1+4x)^m}{(1-5x+3x^2)^2} dx$$

**Optimal.** Leaf size=179

$$\begin{aligned} & - \frac{\left(2\left(5+7\sqrt{13}\right)m+81\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)} \\ & + \frac{\left(2\left(5-7\sqrt{13}\right)m+81\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13+2\sqrt{13}}\right)}{13\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)} + \frac{(20-21x)(4x+1)^{m+1}}{39(3x^2-5x+1)} \end{aligned}$$

[Out]  $((20 - 21*x) * (1 + 4*x)^(1 + m)) / (39 * (1 - 5*x + 3*x^2)) - ((81 + 2 * (5 + 7*sqrt[13]) * m) * (1 + 4*x)^(1 + m) * Hypergeometric2F1[1, 1 + m, 2 + m, (3 * (1 + 4*x)) / (13 - 2 * sqrt[13])]) / (13 * sqrt[13] * (13 - 2 * sqrt[13]) * (1 + m)) + ((81 + 2 * (5 - 7*sqrt[13]) * m) * (1 + 4*x)^(1 + m) * Hypergeometric2F1[1, 1 + m, 2 + m, (3 * (1 + 4*x)) / (13 + 2 * sqrt[13])]) / (13 * sqrt[13] * (13 + 2 * sqrt[13]) * (1 + m))$

**Rubi [A]** time = 0.467788, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$\begin{aligned} & - \frac{\left(2\left(5+7\sqrt{13}\right)m+81\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13-2\sqrt{13}}\right)}{13\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)} \\ & + \frac{\left(2\left(5-7\sqrt{13}\right)m+81\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13+2\sqrt{13}}\right)}{13\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)} + \frac{(20-21x)(4x+1)^{m+1}}{39(3x^2-5x+1)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[((2 + 3\*x) \* (1 + 4\*x)^m) / (1 - 5\*x + 3\*x^2)^2, x]

[Out]  $((20 - 21*x) * (1 + 4*x)^(1 + m)) / (39 * (1 - 5*x + 3*x^2)) - ((81 + 2 * (5 + 7*sqrt[13]) * m) * (1 + 4*x)^(1 + m) * Hypergeometric2F1[1, 1 + m, 2 + m, (3 * (1 + 4*x)) / (13 - 2 * sqrt[13])]) / (13 * sqrt[13] * (13 - 2 * sqrt[13]) * (1 + m)) + ((81 + 2 * (5 - 7*sqrt[13]) * m) * (1 + 4*x)^(1 + m) * Hypergeometric2F1[1, 1 + m, 2 + m, (3 * (1 + 4*x)) / (13 + 2 * sqrt[13])]) / (13 * sqrt[13] * (13 + 2 * sqrt[13]) * (1 + m))$

**Rubi in Sympy [A]** time = 34.5946, size = 136, normalized size = 0.76

$$\begin{aligned} & \frac{(-273x+260)(4x+1)^{m+1}}{507(3x^2-5x+1)} - \frac{2\left(182m-\sqrt{13}(10m+81)\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{12x+3}{2\sqrt{13}+13}\right)}{169\left(4\sqrt{13}+26\right)(m+1)} \\ & - \frac{2\left(182m+\sqrt{13}(10m+81)\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{-12x-3}{-13+2\sqrt{13}}\right)}{169\left(-4\sqrt{13}+26\right)(m+1)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((2+3\*x) \* (1+4\*x)\*\*m / (3\*x\*\*2-5\*x+1)\*\*2, x)

[Out]  $(-273*x + 260) * (4*x + 1)**(m + 1) / (507 * (3*x**2 - 5*x + 1)) - 2 * (182*m - sqrt(13) * (10*m + 81)) * (4*x + 1)**(m + 1) * hyper((1, m + 1),$

$(m + 2, ), (12x + 3)/(2\sqrt{13} + 13))/(169(4\sqrt{13} + 26)^*(m + 1)) - 2*(182m + \sqrt{13}*(10m + 81))*(4x + 1)**(m + 1)*\text{hyper}((1, m + 1), (m + 2, ), (-12x - 3)/(-13 + 2\sqrt{13}))/((169*(-4\sqrt{13} + 26)^*(m + 1))$

**Mathematica [A]** time = 0.0949959, size = 0, normalized size = 0.

$$\int \frac{(2 + 3x)(1 + 4x)^m}{(1 - 5x + 3x^2)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[((2 + 3\*x)\*(1 + 4\*x)^m)/(1 - 5\*x + 3\*x^2)^2, x]

[Out] Integrate[((2 + 3\*x)\*(1 + 4\*x)^m)/(1 - 5\*x + 3\*x^2)^2, x]

**Maple [F]** time = 0.157, size = 0, normalized size = 0.

$$\int \frac{(2 + 3x)(1 + 4x)^m}{(3x^2 - 5x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((2+3\*x)\*(1+4\*x)^m/(3\*x^2-5\*x+1)^2, x)

[Out] int((2+3\*x)\*(1+4\*x)^m/(3\*x^2-5\*x+1)^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x + 1)^m(3x + 2)}{(3x^2 - 5x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x + 1)^m\*(3\*x + 2)/(3\*x^2 - 5\*x + 1)^2, x, algorithm="maxima")

[Out] integrate((4\*x + 1)^m\*(3\*x + 2)/(3\*x^2 - 5\*x + 1)^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(4x + 1)^m(3x + 2)}{9x^4 - 30x^3 + 31x^2 - 10x + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x + 1)^m\*(3\*x + 2)/(3\*x^2 - 5\*x + 1)^2, x, algorithm="fricas")

[Out] integral((4\*x + 1)^m\*(3\*x + 2)/(9\*x^4 - 30\*x^3 + 31\*x^2 - 10\*x + 1), x)



**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(3x + 2)(4x + 1)^m}{(3x^2 - 5x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((2+3\*x)\*(1+4\*x)\*\*m/(3\*x\*\*2-5\*x+1)\*\*2,x)

[Out] Integral((3\*x + 2)\*(4\*x + 1)\*\*m/(3\*x\*\*2 - 5\*x + 1)\*\*2, x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x + 1)^m(3x + 2)}{(3x^2 - 5x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x + 1)^m\*(3\*x + 2)/(3\*x^2 - 5\*x + 1)^2,x, algorithm="giac")

[Out] integrate((4\*x + 1)^m\*(3\*x + 2)/(3\*x^2 - 5\*x + 1)^2, x)

$$3.941 \quad \int \frac{(1+4x)^m}{(1-5x+3x^2)^2} dx$$

**Optimal.** Leaf size=177

$$\frac{2 \left( 2 \left( 2 + \sqrt{13} \right) m + 9 \right) (4x + 1)^{m+1} {}_2F_1 \left( 1, m + 1; m + 2; \frac{3(4x+1)}{13-2\sqrt{13}} \right)}{13\sqrt{13} \left( 13 - 2\sqrt{13} \right) (m + 1)} + \frac{2 \left( 2 \left( 2 - \sqrt{13} \right) m + 9 \right) (4x + 1)^{m+1} {}_2F_1 \left( 1, m + 1; m + 2; \frac{3(4x+1)}{13+2\sqrt{13}} \right)}{13\sqrt{13} \left( 13 + 2\sqrt{13} \right) (m + 1)} + \frac{(7 - 6x)(4x + 1)^{m+1}}{39(3x^2 - 5x + 1)}$$

[Out] ((7 - 6\*x)\*(1 + 4\*x)^(1 + m))/(39\*(1 - 5\*x + 3\*x^2)) - (2\*(9 + 2\*(2 + Sqrt[13])\*m)\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 - 2\*Sqrt[13])])/(13\*Sqrt[13]\*(13 - 2\*Sqrt[13])\*(1 + m)) + (2\*(9 + 2\*(2 - Sqrt[13])\*m)\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 + 2\*Sqrt[13])])/(13\*Sqrt[13]\*(13 + 2\*Sqrt[13])\*(1 + m))

**Rubi [A]** time = 0.413095, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.15$

$$\frac{2 \left( 2 \left( 2 + \sqrt{13} \right) m + 9 \right) (4x + 1)^{m+1} {}_2F_1 \left( 1, m + 1; m + 2; \frac{3(4x+1)}{13-2\sqrt{13}} \right)}{13\sqrt{13} \left( 13 - 2\sqrt{13} \right) (m + 1)} + \frac{2 \left( 2 \left( 2 - \sqrt{13} \right) m + 9 \right) (4x + 1)^{m+1} {}_2F_1 \left( 1, m + 1; m + 2; \frac{3(4x+1)}{13+2\sqrt{13}} \right)}{13\sqrt{13} \left( 13 + 2\sqrt{13} \right) (m + 1)} + \frac{(7 - 6x)(4x + 1)^{m+1}}{39(3x^2 - 5x + 1)}$$

Antiderivative was successfully verified.

[In] Int[(1 + 4\*x)^m/(1 - 5\*x + 3\*x^2)^2, x]

[Out] ((7 - 6\*x)\*(1 + 4\*x)^(1 + m))/(39\*(1 - 5\*x + 3\*x^2)) - (2\*(9 + 2\*(2 + Sqrt[13])\*m)\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 - 2\*Sqrt[13])])/(13\*Sqrt[13]\*(13 - 2\*Sqrt[13])\*(1 + m)) + (2\*(9 + 2\*(2 - Sqrt[13])\*m)\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 + 2\*Sqrt[13])])/(13\*Sqrt[13]\*(13 + 2\*Sqrt[13])\*(1 + m))

**Rubi in Sympy [A]** time = 31.8334, size = 136, normalized size = 0.77

$$\frac{(-78x + 91)(4x + 1)^{m+1}}{507(3x^2 - 5x + 1)} - \frac{4 \left( 26m - \sqrt{13}(4m + 9) \right) (4x + 1)^{m+1} {}_2F_1 \left( 1, m + 1; m + 2; \frac{12x+3}{2\sqrt{13}+13} \right)}{169 \left( 4\sqrt{13} + 26 \right) (m + 1)} - \frac{4 \left( 26m + \sqrt{13}(4m + 9) \right) (4x + 1)^{m+1} {}_2F_1 \left( 1, m + 1; m + 2; \frac{-12x-3}{-13+2\sqrt{13}} \right)}{169 \left( -4\sqrt{13} + 26 \right) (m + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((1+4\*x)\*\*m/(3\*x\*\*2-5\*x+1)\*\*2, x)

[Out] (-78\*x + 91)\*(4\*x + 1)\*\*(m + 1)/(507\*(3\*x\*\*2 - 5\*x + 1)) - 4\*(26\*m - sqrt(13)\*(4\*m + 9))\*(4\*x + 1)\*\*(m + 1)\*hyper((1, m + 1), (m +

2, ), (12\*x + 3)/(2\*sqrt(13) + 13))/(169\*(4\*sqrt(13) + 26)\*(m + 1)) - 4\*(26\*m + sqrt(13)\*(4\*m + 9))\*(4\*x + 1)\*\*(m + 1)\*hyper((1, m + 1), (m + 2, ), (-12\*x - 3)/(-13 + 2\*sqrt(13)))/(169\*(-4\*sqrt(13) + 26)\*(m + 1))

**Mathematica [A]** time = 0.0373103, size = 0, normalized size = 0.

$$\int \frac{(1 + 4x)^m}{(1 - 5x + 3x^2)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(1 + 4\*x)^m/(1 - 5\*x + 3\*x^2)^2, x]

[Out] Integrate[(1 + 4\*x)^m/(1 - 5\*x + 3\*x^2)^2, x]

**Maple [F]** time = 0.155, size = 0, normalized size = 0.

$$\int \frac{(1 + 4x)^m}{(3x^2 - 5x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+4\*x)^m/(3\*x^2-5\*x+1)^2, x)

[Out] int((1+4\*x)^m/(3\*x^2-5\*x+1)^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x + 1)^m}{(3x^2 - 5x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x + 1)^m/(3\*x^2 - 5\*x + 1)^2, x, algorithm="maxima")

[Out] integrate((4\*x + 1)^m/(3\*x^2 - 5\*x + 1)^2, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(4x + 1)^m}{9x^4 - 30x^3 + 31x^2 - 10x + 1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x + 1)^m/(3\*x^2 - 5\*x + 1)^2, x, algorithm="fricas")

[Out] integral((4\*x + 1)^m/(9\*x^4 - 30\*x^3 + 31\*x^2 - 10\*x + 1), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x + 1)^m}{(3x^2 - 5x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+4\*x)\*\*m/(3\*x\*\*2-5\*x+1)\*\*2,x)

[Out] Integral((4\*x + 1)\*\*m/(3\*x\*\*2 - 5\*x + 1)\*\*2, x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x + 1)^m}{(3x^2 - 5x + 1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x + 1)^m/(3\*x^2 - 5\*x + 1)^2,x, algorithm="giac")

[Out] integrate((4\*x + 1)^m/(3\*x^2 - 5\*x + 1)^2, x)

$$3.942 \quad \int \frac{(1+4x)^m}{(2+3x)(1-5x+3x^2)^2} dx$$

**Optimal.** Leaf size=340

$$\begin{aligned} & \frac{9(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; -\frac{3}{5}(4x+1)\right)}{1445(m+1)} \\ & - \frac{\left(\left(62+22\sqrt{13}\right)m+81\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13-2\sqrt{13}}\right)}{221\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)} \\ & + \frac{9\left(13+9\sqrt{13}\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13-2\sqrt{13}}\right)}{7514\left(13-2\sqrt{13}\right)(m+1)} \\ & + \frac{\left(\left(62-22\sqrt{13}\right)m+81\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13+2\sqrt{13}}\right)}{221\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)} \\ & + \frac{9\left(13-9\sqrt{13}\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13+2\sqrt{13}}\right)}{7514\left(13+2\sqrt{13}\right)(m+1)} + \frac{(43-33x)(4x+1)^{m+1}}{663(3x^2-5x+1)} \end{aligned}$$

[Out]  $((43 - 33*x) * (1 + 4*x)^(1 + m)) / (663 * (1 - 5*x + 3*x^2)) + (9 * (1 + 4*x)^(1 + m) * \text{Hypergeometric2F1}[1, 1 + m, 2 + m, (-3 * (1 + 4*x)) / 5]) / (1445 * (1 + m)) + (9 * (13 + 9 * \text{Sqrt}[13]) * (1 + 4*x)^(1 + m) * \text{Hypergeometric2F1}[1, 1 + m, 2 + m, (3 * (1 + 4*x)) / (13 - 2 * \text{Sqrt}[13])]) / (7514 * (13 - 2 * \text{Sqrt}[13]) * (1 + m)) - ((81 + (62 + 22 * \text{Sqrt}[13]) * m) * (1 + 4*x)^(1 + m) * \text{Hypergeometric2F1}[1, 1 + m, 2 + m, (3 * (1 + 4*x)) / (13 - 2 * \text{Sqrt}[13])]) / (221 * \text{Sqrt}[13] * (13 - 2 * \text{Sqrt}[13]) * (1 + m)) + (9 * (13 - 9 * \text{Sqrt}[13]) * (1 + 4*x)^(1 + m) * \text{Hypergeometric2F1}[1, 1 + m, 2 + m, (3 * (1 + 4*x)) / (13 + 2 * \text{Sqrt}[13])]) / (7514 * (13 + 2 * \text{Sqrt}[13]) * (1 + m)) + ((81 + (62 - 22 * \text{Sqrt}[13]) * m) * (1 + 4*x)^(1 + m) * \text{Hypergeometric2F1}[1, 1 + m, 2 + m, (3 * (1 + 4*x)) / (13 + 2 * \text{Sqrt}[13])]) / (221 * \text{Sqrt}[13] * (13 + 2 * \text{Sqrt}[13]) * (1 + m))$

**Rubi [A]** time = 1.00982, antiderivative size = 340, normalized size of antiderivative = 1., number of rules used = 12, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$\begin{aligned} & \frac{9(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; -\frac{3}{5}(4x+1)\right)}{1445(m+1)} \\ & - \frac{\left(\left(62+22\sqrt{13}\right)m+81\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13-2\sqrt{13}}\right)}{221\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)} \\ & + \frac{9\left(13+9\sqrt{13}\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13-2\sqrt{13}}\right)}{7514\left(13-2\sqrt{13}\right)(m+1)} \\ & + \frac{\left(\left(62-22\sqrt{13}\right)m+81\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13+2\sqrt{13}}\right)}{221\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)} \\ & + \frac{9\left(13-9\sqrt{13}\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13+2\sqrt{13}}\right)}{7514\left(13+2\sqrt{13}\right)(m+1)} + \frac{(43-33x)(4x+1)^{m+1}}{663(3x^2-5x+1)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(1 + 4\*x)^m / ((2 + 3\*x) \* (1 - 5\*x + 3\*x^2)^2), x]

[Out]  $((43 - 33*x) * (1 + 4*x)^(1 + m)) / (663 * (1 - 5*x + 3*x^2)) + (9 * (1 + 4*x)^(1 + m) * \text{Hypergeometric2F1}[1, 1 + m, 2 + m, (-3 * (1 + 4*x)) / 5]) / (1445 * (1 + m)) + (9 * (13 + 9 * \text{Sqrt}[13]) * (1 + 4*x)^(1 + m) * \text{Hypergeometric2F1}[1, 1 + m, 2 + m, (3 * (1 + 4*x)) / (13 - 2 * \text{Sqrt}[13])]) / (7514 * (13 - 2 * \text{Sqrt}[13]) * (1 + m)) - ((81 + (62 + 22 * \text{Sqrt}[13]) * m) * (1 + 4*x)^(1 + m) * \text{Hypergeometric2F1}[1, 1 + m, 2 + m, (3 * (1 + 4*x)) / (13 - 2 * \text{Sqrt}[13])]) / (221 * \text{Sqrt}[13] * (13 - 2 * \text{Sqrt}[13]) * (1 + m)) + (9 * (13 - 9 * \text{Sqrt}[13]) * (1 + 4*x)^(1 + m) * \text{Hypergeometric2F1}[1, 1 + m, 2 + m, (3 * (1 + 4*x)) / (13 + 2 * \text{Sqrt}[13])]) / (7514 * (13 + 2 * \text{Sqrt}[13]) * (1 + m)) + ((81 + (62 - 22 * \text{Sqrt}[13]) * m) * (1 + 4*x)^(1 + m) * \text{Hypergeometric2F1}[1, 1 + m, 2 + m, (3 * (1 + 4*x)) / (13 + 2 * \text{Sqrt}[13])]) / (221 * \text{Sqrt}[13] * (13 + 2 * \text{Sqrt}[13]) * (1 + m))$

eometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 - 2\*Sqrt[13])]/(7514\*(13 - 2\*Sqrt[13])\*(1 + m)) - ((81 + (62 + 22\*Sqrt[13])\*m)\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 - 2\*Sqrt[13])]/(221\*Sqrt[13]\*(13 - 2\*Sqrt[13])\*(1 + m)) + (9\*(13 - 9\*Sqrt[13])\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 + 2\*Sqrt[13])]/(7514\*(13 + 2\*Sqrt[13])\*(1 + m)) + ((81 + (62 - 22\*Sqrt[13])\*m)\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 + 2\*Sqrt[13])]/(221\*Sqrt[13]\*(13 + 2\*Sqrt[13])\*(1 + m))

**Rubi in Sympy [A]** time = 62.158, size = 274, normalized size = 0.81

$$\begin{aligned} & \frac{(-429x + 559)(4x + 1)^{m+1}}{8619(3x^2 - 5x + 1)} - \frac{2(286m - \sqrt{13}(62m + 81))(4x + 1)^{m+1} {}_2F_1\left(\begin{matrix} 1, m+1 \\ m+2 \end{matrix} \middle| \frac{12x+3}{2\sqrt{13}+13}\right)}{2873(4\sqrt{13} + 26)(m+1)} \\ & - \frac{2(286m + \sqrt{13}(62m + 81))(4x + 1)^{m+1} {}_2F_1\left(\begin{matrix} 1, m+1 \\ m+2 \end{matrix} \middle| \frac{-12x-3}{-13+2\sqrt{13}}\right)}{2873(-4\sqrt{13} + 26)(m+1)} \\ & + \frac{3\left(3 + \frac{27\sqrt{13}}{13}\right)(4x + 1)^{m+1} {}_2F_1\left(\begin{matrix} 1, m+1 \\ m+2 \end{matrix} \middle| \frac{-12x-3}{-13+2\sqrt{13}}\right)}{289(-4\sqrt{13} + 26)(m+1)} \\ & + \frac{3\left(-\frac{27\sqrt{13}}{13} + 3\right)(4x + 1)^{m+1} {}_2F_1\left(\begin{matrix} 1, m+1 \\ m+2 \end{matrix} \middle| \frac{12x+3}{2\sqrt{13}+13}\right)}{289(4\sqrt{13} + 26)(m+1)} + \frac{9(4x + 1)^{m+1} {}_2F_1\left(\begin{matrix} 1, m+1 \\ m+2 \end{matrix} \middle| -\frac{12x}{5} - \frac{3}{5}\right)}{1445(m+1)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((1+4*x)**m/(2+3*x)/(3*x**2-5*x+1)**2,x)`

[Out]  $(-429*x + 559)*(4*x + 1)**(m + 1)/(8619*(3*x**2 - 5*x + 1)) - 2*(286*m - \text{sqrt}(13)*(62*m + 81))*(4*x + 1)**(m + 1)*\text{hyper}((1, m + 1), (m + 2,), (12*x + 3)/(2*\text{sqrt}(13) + 13))/(2873*(4*\text{sqrt}(13) + 26)*(m + 1)) - 2*(286*m + \text{sqrt}(13)*(62*m + 81))*(4*x + 1)**(m + 1)*\text{hyper}((1, m + 1), (m + 2,), (-12*x - 3)/(-13 + 2*\text{sqrt}(13)))/(2873*(-4*\text{sqrt}(13) + 26)*(m + 1)) + 3*(3 + 27*\text{sqrt}(13)/13)*(4*x + 1)**(m + 1)*\text{hyper}((1, m + 1), (m + 2,), (-12*x - 3)/(-13 + 2*\text{sqrt}(13)))/(289*(-4*\text{sqrt}(13) + 26)*(m + 1)) + 3*(-27*\text{sqrt}(13)/13 + 3)*(4*x + 1)**(m + 1)*\text{hyper}((1, m + 1), (m + 2,), (12*x + 3)/(2*\text{sqrt}(13) + 13))/(289*(4*\text{sqrt}(13) + 26)*(m + 1)) + 9*(4*x + 1)**(m + 1)*\text{hyper}((1, m + 1), (m + 2,), -12*x/5 - 3/5)/(1445*(m + 1))$

**Mathematica [A]** time = 0.139731, size = 0, normalized size = 0.

$$\int \frac{(1 + 4x)^m}{(2 + 3x)(1 - 5x + 3x^2)^2} dx$$

Verification is Not applicable to the result.

[In] `Integrate[(1 + 4*x)^m/((2 + 3*x)*(1 - 5*x + 3*x^2)^2),x]`

[Out] `Integrate[(1 + 4*x)^m/((2 + 3*x)*(1 - 5*x + 3*x^2)^2), x]`

**Maple [F]** time = 0.177, size = 0, normalized size = 0.

$$\int \frac{(1+4x)^m}{(2+3x)(3x^2-5x+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+4\*x)^m/(2+3\*x)/(3\*x^2-5\*x+1)^2,x)

[Out] int((1+4\*x)^m/(2+3\*x)/(3\*x^2-5\*x+1)^2,x)

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{(3x^2-5x+1)^2(3x+2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x+1)^m/((3\*x^2-5\*x+1)^2\*(3\*x+2)),x, algorithm="maxima")

[Out] integrate((4\*x+1)^m/((3\*x^2-5\*x+1)^2\*(3\*x+2)), x)

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(4x+1)^m}{27x^5-72x^4+33x^3+32x^2-17x+2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x+1)^m/((3\*x^2-5\*x+1)^2\*(3\*x+2)),x, algorithm="fricas")

[Out] integral((4\*x+1)^m/(27\*x^5-72\*x^4+33\*x^3+32\*x^2-17\*x+2), x)

---

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{(3x+2)(3x^2-5x+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+4\*x)\*\*m/(2+3\*x)/(3\*x\*\*2-5\*x+1)\*\*2,x)

[Out] Integral((4\*x+1)\*\*m/((3\*x+2)\*(3\*x\*\*2-5\*x+1)\*\*2), x)

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{(3x^2-5x+1)^2(3x+2)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)^2*(3*x + 2)),x, algorithm="giac")
```

```
[Out] integrate((4*x + 1)^m/((3*x^2 - 5*x + 1)^2*(3*x + 2)), x)
```



$$3.943 \quad \int \frac{(1+4x)^m}{(2+3x)^2(1-5x+3x^2)^2} dx$$

**Optimal.** Leaf size=376

$$\begin{aligned} & \frac{162(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; -\frac{3}{5}(4x+1)\right)}{24565(m+1)} \\ & - \frac{\left(2\left(211+65\sqrt{13}\right)m+423\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13-2\sqrt{13}}\right)}{3757\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)} \\ & + \frac{9\left(117+64\sqrt{13}\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13-2\sqrt{13}}\right)}{63869\left(13-2\sqrt{13}\right)(m+1)} \\ & + \frac{\left(\left(422-130\sqrt{13}\right)m+423\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13+2\sqrt{13}}\right)}{3757\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)} \\ & + \frac{9\left(117-64\sqrt{13}\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13+2\sqrt{13}}\right)}{63869\left(13+2\sqrt{13}\right)(m+1)} \\ & + \frac{36(4x+1)^{m+1} {}_2F_1\left(2, m+1; m+2; -\frac{3}{5}(4x+1)\right)}{7225(m+1)} + \frac{(268-195x)(4x+1)^{m+1}}{11271(3x^2-5x+1)} \end{aligned}$$

[Out]  $((268 - 195*x) * (1 + 4*x)^(1 + m)) / (11271 * (1 - 5*x + 3*x^2)) + (16 * 2 * (1 + 4*x)^(1 + m) * \text{Hypergeometric2F1}[1, 1 + m, 2 + m, (-3 * (1 + 4*x)) / 5]) / (24565 * (1 + m)) + (9 * (117 + 64 * \text{Sqrt}[13]) * (1 + 4*x)^(1 + m) * \text{Hypergeometric2F1}[1, 1 + m, 2 + m, (3 * (1 + 4*x)) / (13 - 2 * \text{Sqrt}[13])]) / (63869 * (13 - 2 * \text{Sqrt}[13]) * (1 + m)) - ((423 + 2 * (211 + 65 * \text{Sqrt}[13]) * m) * (1 + 4*x)^(1 + m) * \text{Hypergeometric2F1}[1, 1 + m, 2 + m, (3 * (1 + 4*x)) / (13 - 2 * \text{Sqrt}[13])]) / (3757 * \text{Sqrt}[13] * (13 - 2 * \text{Sqrt}[13]) * (1 + m)) + (9 * (117 - 64 * \text{Sqrt}[13]) * (1 + 4*x)^(1 + m) * \text{Hypergeometric2F1}[1, 1 + m, 2 + m, (3 * (1 + 4*x)) / (13 + 2 * \text{Sqrt}[13])]) / (63869 * (13 + 2 * \text{Sqrt}[13]) * (1 + m)) + ((423 + (422 - 130 * \text{Sqrt}[13]) * m) * (1 + 4*x)^(1 + m) * \text{Hypergeometric2F1}[1, 1 + m, 2 + m, (3 * (1 + 4*x)) / (13 + 2 * \text{Sqrt}[13])]) / (3757 * \text{Sqrt}[13] * (13 + 2 * \text{Sqrt}[13]) * (1 + m)) + (36 * (1 + 4*x)^(1 + m) * \text{Hypergeometric2F1}[2, 1 + m, 2 + m, (-3 * (1 + 4*x)) / 5]) / (7225 * (1 + m))$

**Rubi [A]** time = 1.07929, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\begin{aligned} & \frac{162(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; -\frac{3}{5}(4x+1)\right)}{24565(m+1)} \\ & - \frac{\left(2\left(211+65\sqrt{13}\right)m+423\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13-2\sqrt{13}}\right)}{3757\sqrt{13}\left(13-2\sqrt{13}\right)(m+1)} \\ & + \frac{9\left(117+64\sqrt{13}\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13-2\sqrt{13}}\right)}{63869\left(13-2\sqrt{13}\right)(m+1)} \\ & + \frac{\left(\left(422-130\sqrt{13}\right)m+423\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13+2\sqrt{13}}\right)}{3757\sqrt{13}\left(13+2\sqrt{13}\right)(m+1)} \\ & + \frac{9\left(117-64\sqrt{13}\right)(4x+1)^{m+1} {}_2F_1\left(1, m+1; m+2; \frac{3(4x+1)}{13+2\sqrt{13}}\right)}{63869\left(13+2\sqrt{13}\right)(m+1)} \\ & + \frac{36(4x+1)^{m+1} {}_2F_1\left(2, m+1; m+2; -\frac{3}{5}(4x+1)\right)}{7225(m+1)} + \frac{(268-195x)(4x+1)^{m+1}}{11271(3x^2-5x+1)} \end{aligned}$$

Antiderivative was successfully verified.

[In] Int[(1 + 4\*x)^m/((2 + 3\*x)^2\*(1 - 5\*x + 3\*x^2)^2), x]

[Out] ((268 - 195\*x)\*(1 + 4\*x)^(1 + m))/(11271\*(1 - 5\*x + 3\*x^2)) + (16\*2\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (-3\*(1 + 4\*x))/5])/(24565\*(1 + m)) + (9\*(117 + 64\*Sqrt[13])\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 - 2\*Sqrt[13])])/(63869\*(13 - 2\*Sqrt[13])\*(1 + m)) - ((423 + 2\*(211 + 65\*Sqrt[13])\*m)\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 - 2\*Sqrt[13])])/(3757\*Sqrt[13]\*(13 - 2\*Sqrt[13])\*(1 + m)) + (9\*(117 - 64\*Sqrt[13])\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 + 2\*Sqrt[13])])/(63869\*(13 + 2\*Sqrt[13])\*(1 + m)) + ((423 + (422 - 130\*Sqrt[13])\*m)\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[1, 1 + m, 2 + m, (3\*(1 + 4\*x))/(13 + 2\*Sqrt[13])])/(3757\*Sqrt[13]\*(13 + 2\*Sqrt[13])\*(1 + m)) + (36\*(1 + 4\*x)^(1 + m)\*Hypergeometric2F1[2, 1 + m, 2 + m, (-3\*(1 + 4\*x))/5])/(7225\*(1 + m))

**Rubi in Sympy [A]** time = 64.4264, size = 304, normalized size = 0.81

$$\begin{aligned} & \frac{(-2535x + 3484)(4x + 1)^{m+1}}{146523(3x^2 - 5x + 1)} \\ & - \frac{2 \left( 1690m - \sqrt{13}(422m + 423) \right) (4x + 1)^{m+1} {}_2F_1 \left( \begin{matrix} 1, m + 1 \\ m + 2 \end{matrix} \middle| \frac{12x+3}{2\sqrt{13}+13} \right)}{48841 \left( 4\sqrt{13} + 26 \right) (m + 1)} \\ & - \frac{2 \left( 1690m + \sqrt{13}(422m + 423) \right) (4x + 1)^{m+1} {}_2F_1 \left( \begin{matrix} 1, m + 1 \\ m + 2 \end{matrix} \middle| \frac{-12x-3}{-13+2\sqrt{13}} \right)}{48841 \left( -4\sqrt{13} + 26 \right) (m + 1)} \\ & + \frac{3 \left( 54 + \frac{384\sqrt{13}}{13} \right) (4x + 1)^{m+1} {}_2F_1 \left( \begin{matrix} 1, m + 1 \\ m + 2 \end{matrix} \middle| \frac{-12x-3}{-13+2\sqrt{13}} \right)}{4913 \left( -4\sqrt{13} + 26 \right) (m + 1)} \\ & + \frac{3 \left( -\frac{384\sqrt{13}}{13} + 54 \right) (4x + 1)^{m+1} {}_2F_1 \left( \begin{matrix} 1, m + 1 \\ m + 2 \end{matrix} \middle| \frac{12x+3}{2\sqrt{13}+13} \right)}{4913 \left( 4\sqrt{13} + 26 \right) (m + 1)} \\ & + \frac{162(4x + 1)^{m+1} {}_2F_1 \left( \begin{matrix} 1, m + 1 \\ m + 2 \end{matrix} \middle| -\frac{12x}{5} - \frac{3}{5} \right)}{24565(m + 1)} + \frac{36(4x + 1)^{m+1} {}_2F_1 \left( \begin{matrix} 2, m + 1 \\ m + 2 \end{matrix} \middle| -\frac{12x}{5} - \frac{3}{5} \right)}{7225(m + 1)} \end{aligned}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((1+4\*x)\*\*m/(2+3\*x)\*\*2/(3\*x\*\*2-5\*x+1)\*\*2, x)

[Out] (-2535\*x + 3484)\*(4\*x + 1)\*\*(m + 1)/(146523\*(3\*x\*\*2 - 5\*x + 1)) - 2\*(1690\*m - sqrt(13)\*(422\*m + 423))\*(4\*x + 1)\*\*(m + 1)\*hyper((1, m + 1), (m + 2, ), (12\*x + 3)/(2\*sqrt(13) + 13))/(48841\*(4\*sqrt(13) + 26)\*(m + 1)) - 2\*(1690\*m + sqrt(13)\*(422\*m + 423))\*(4\*x + 1)\*\*(m + 1)\*hyper((1, m + 1), (m + 2, ), (-12\*x - 3)/(-13 + 2\*sqrt(13)))/(48841\*(-4\*sqrt(13) + 26)\*(m + 1)) + 3\*(54 + 384\*sqrt(13)/13)\*(4\*x + 1)\*\*(m + 1)\*hyper((1, m + 1), (m + 2, ), (-12\*x - 3)/(-13 + 2\*sqrt(13)))/(4913\*(-4\*sqrt(13) + 26)\*(m + 1)) + 3\*(-384\*sqrt(13)/13 + 54)\*(4\*x + 1)\*\*(m + 1)\*hyper((1, m + 1), (m + 2, ), (12\*x + 3)/(2\*sqrt(13) + 13))/(4913\*(4\*sqrt(13) + 26)\*(m + 1)) + 162\*(4\*x + 1)\*\*(m + 1)\*hyper((1, m + 1), (m + 2, ), -12\*x/5 - 3/5)/(24565\*(m + 1)) + 36\*(4\*x + 1)\*\*(m + 1)\*hyper((2, m + 1), (m + 2, ), -12\*x/5 - 3/5)/(7225\*(m + 1))

**Mathematica [A]** time = 0.142896, size = 0, normalized size = 0.

$$\int \frac{(1+4x)^m}{(2+3x)^2(1-5x+3x^2)^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(1 + 4\*x)^m/((2 + 3\*x)^2\*(1 - 5\*x + 3\*x^2)^2), x]

[Out] Integrate[(1 + 4\*x)^m/((2 + 3\*x)^2\*(1 - 5\*x + 3\*x^2)^2), x]

**Maple [F]** time = 0.18, size = 0, normalized size = 0.

$$\int \frac{(1+4x)^m}{(2+3x)^2(3x^2-5x+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((1+4\*x)^m/(2+3\*x)^2/(3\*x^2-5\*x+1)^2, x)

[Out] int((1+4\*x)^m/(2+3\*x)^2/(3\*x^2-5\*x+1)^2, x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{(3x^2-5x+1)^2(3x+2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x + 1)^m/((3\*x^2 - 5\*x + 1)^2\*(3\*x + 2)^2), x, algorithm="maxima")

[Out] integrate((4\*x + 1)^m/((3\*x^2 - 5\*x + 1)^2\*(3\*x + 2)^2), x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(4x+1)^m}{81x^6 - 162x^5 - 45x^4 + 162x^3 + 13x^2 - 28x + 4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x + 1)^m/((3\*x^2 - 5\*x + 1)^2\*(3\*x + 2)^2), x, algorithm="fricas")

[Out] integral((4\*x + 1)^m/(81\*x^6 - 162\*x^5 - 45\*x^4 + 162\*x^3 + 13\*x^2 - 28\*x + 4), x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x+1)^m}{(3x+2)^2(3x^2-5x+1)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((1+4\*x)\*\*m/(2+3\*x)\*\*2/(3\*x\*\*2-5\*x+1)\*\*2,x)

[Out] Integral((4\*x + 1)\*\*m/((3\*x + 2)\*\*2\*(3\*x\*\*2 - 5\*x + 1)\*\*2), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(4x + 1)^m}{(3x^2 - 5x + 1)^2(3x + 2)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((4\*x + 1)^m/((3\*x^2 - 5\*x + 1)^2\*(3\*x + 2)^2),x, algorithm="giac")

[Out] integrate((4\*x + 1)^m/((3\*x^2 - 5\*x + 1)^2\*(3\*x + 2)^2), x)

$$3.944 \quad \int \frac{(d+ex)^m (a+bx+cx^2)}{(e+fx)^{3/2}} dx$$

**Optimal.** Leaf size=237

$$\frac{2\sqrt{e+fx}(d+ex)^m \left(-\frac{f(d+ex)}{e^2-df}\right)^{-m} {}_2F_1\left(\frac{1}{2}, -m; \frac{3}{2}; \frac{e(e+fx)}{e^2-df}\right) (c(d^2f^2 + 4de^2f(m+1) - 4e^4(m^2 + 3m + 2)) - ef(2m+3)(ae))}{ef^3(2m+3)(e^2-df)} + \frac{2(d+ex)^{m+1} \left(a + \frac{e(ce-bf)}{f^2}\right)}{(e^2-df)\sqrt{e+fx}} + \frac{2c\sqrt{e+fx}(d+ex)^{m+1}}{ef^2(2m+3)}$$

[Out] (2\*(a + (e\*(c\*e - b\*f))/f^2)\*(d + e\*x)^(1 + m))/((e^2 - d\*f)\*Sqrt[e + f\*x]) + (2\*c\*(d + e\*x)^(1 + m)\*Sqrt[e + f\*x])/(e\*f^2\*(3 + 2\*m)) + (2\*(c\*(d^2\*f^2 + 4\*d\*e^2\*f\*(1 + m) - 4\*e^4\*(2 + 3\*m + m^2)) - e\*f\*(3 + 2\*m)\*(a\*e\*f\*(1 + 2\*m) + b\*(d\*f - 2\*e^2\*(1 + m))))\*(d + e\*x)^m\*Sqrt[e + f\*x]\*Hypergeometric2F1[1/2, -m, 3/2, (e\*(e + f\*x))/(e^2 - d\*f)])/(e\*f^3\*(e^2 - d\*f)\*(3 + 2\*m)\*(-(f\*(d + e\*x))/(e^2 - d\*f)))^m)

**Rubi [A]** time = 0.838316, antiderivative size = 230, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$\frac{2\sqrt{e+fx}(d+ex)^m \left(-\frac{f(d+ex)}{e^2-df}\right)^{-m} {}_2F_1\left(\frac{1}{2}, -m; \frac{3}{2}; \frac{e(e+fx)}{e^2-df}\right) (f(aef(2m+1) + bdf - 2be^2(m+1)) - \frac{c(d^2f^2 + 4de^2f(m+1) - 4e^4(m^2 + 3m + 2))}{e(2m+3)})}{f^3(e^2-df)} + \frac{2(d+ex)^{m+1} \left(a + \frac{e(ce-bf)}{f^2}\right)}{(e^2-df)\sqrt{e+fx}} + \frac{2c\sqrt{e+fx}(d+ex)^{m+1}}{ef^2(2m+3)}$$

Antiderivative was successfully verified.

[In] Int[((d + e\*x)^m\*(a + b\*x + c\*x^2))/(e + f\*x)^(3/2), x]

[Out] (2\*(a + (e\*(c\*e - b\*f))/f^2)\*(d + e\*x)^(1 + m))/((e^2 - d\*f)\*Sqrt[e + f\*x]) + (2\*c\*(d + e\*x)^(1 + m)\*Sqrt[e + f\*x])/(e\*f^2\*(3 + 2\*m)) - (2\*(f\*(b\*d\*f - 2\*b\*e^2\*(1 + m) + a\*e\*f\*(1 + 2\*m)) - (c\*(d^2\*f^2 + 4\*d\*e^2\*f\*(1 + m) - 4\*e^4\*(2 + 3\*m + m^2)))/(e\*(3 + 2\*m))))\*(d + e\*x)^m\*Sqrt[e + f\*x]\*Hypergeometric2F1[1/2, -m, 3/2, (e\*(e + f\*x))/(e^2 - d\*f)])/(f^3\*(e^2 - d\*f)\*(-(f\*(d + e\*x))/(e^2 - d\*f)))^m)

**Rubi in Sympy [A]** time = 70.8945, size = 201, normalized size = 0.85

$$\frac{2c \left(\frac{f(d+ex)}{df-e^2}\right)^{-m} (d+ex)^m (e+fx)^{\frac{3}{2}} {}_2F_1\left(-m, \frac{3}{2} \middle| \frac{e(-e-fx)}{df-e^2}\right)}{3f^3} + \frac{2 \left(\frac{f(d+ex)}{df-e^2}\right)^{-m} (d+ex)^m \sqrt{e+fx} (bf - 2ce) {}_2F_1\left(-m, \frac{1}{2} \middle| \frac{e(-e-fx)}{df-e^2}\right)}{f^3} - \frac{2 \left(\frac{f(d+ex)}{df-e^2}\right)^{-m} (d+ex)^m (af^2 - bef + ce^2) {}_2F_1\left(-m, -\frac{1}{2} \middle| \frac{e(-e-fx)}{df-e^2}\right)}{f^3 \sqrt{e+fx}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*m\*(c\*x\*\*2+b\*x+a)/(f\*x+e)\*\*(3/2), x)

```
[Out] 2*c*(f*(d + e*x)/(d*f - e**2))**(-m)*(d + e*x)**m*(e + f*x)**(3/2)
)*hyper((-m, 3/2), (5/2,), e*(-e - f*x)/(d*f - e**2))/(3*f**3) +
2*(f*(d + e*x)/(d*f - e**2))**(-m)*(d + e*x)**m*sqrt(e + f*x)*(b*
f - 2*c*e)*hyper((-m, 1/2), (3/2,), e*(-e - f*x)/(d*f - e**2))/f*
*3 - 2*(f*(d + e*x)/(d*f - e**2))**(-m)*(d + e*x)**m*(a*f**2 - b*
e*f + c*e**2)*hyper((-m, -1/2), (1/2,), e*(-e - f*x)/(d*f - e**2)
)/(f**3*sqrt(e + f*x))
```

**Mathematica [A]** time = 0.28262, size = 171, normalized size = 0.72

$$\frac{2(d+ex)^m \left(\frac{f(d+ex)}{df-e^2}\right)^{-m} \left(-3(f(af-be)+ce^2) {}_2F_1\left(-\frac{1}{2}, -m; \frac{1}{2}; \frac{e(e+fx)}{e^2-df}\right) - (e+fx) \left((6ce-3bf) {}_2F_1\left(\frac{1}{2}, -m; \frac{3}{2}; \frac{e(e+fx)}{e^2-df}\right) - 3f^3\sqrt{e+fx}\right)}{3f^3\sqrt{e+fx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x)^m*(a + b*x + c*x^2))/(e + f*x)^(3/2), x]
```

```
[Out] (2*(d + e*x)^m*(-3*(c*e^2 + f*(-(b*e) + a*f))*Hypergeometric2F1[-
1/2, -m, 1/2, (e*(e + f*x))/(e^2 - d*f)] - (e + f*x)*((6*c*e - 3*
b*f)*Hypergeometric2F1[1/2, -m, 3/2, (e*(e + f*x))/(e^2 - d*f)] -
c*(e + f*x)*Hypergeometric2F1[3/2, -m, 5/2, (e*(e + f*x))/(e^2 -
d*f)])))/(3*f^3*((f*(d + e*x))/(-e^2 + d*f))^m*sqrt[e + f*x])
```

**Maple [F]** time = 0.048, size = 0, normalized size = 0.

$$\int (ex+d)^m (cx^2+bx+a) (fx+e)^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x+d)^m*(c*x^2+b*x+a)/(f*x+e)^(3/2), x)
```

```
[Out] int((e*x+d)^m*(c*x^2+b*x+a)/(f*x+e)^(3/2), x)
```

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2+bx+a)(ex+d)^m}{(fx+e)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^2 + b*x + a)*(e*x + d)^m/(f*x + e)^(3/2), x, algorithm="maxima")
```

```
[Out] integrate((c*x^2 + b*x + a)*(e*x + d)^m/(f*x + e)^(3/2), x)
```

**Ericas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^2+bx+a)(ex+d)^m}{(fx+e)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)*(e*x + d)^m/(f*x + e)^(3/2),x, algorithm="fricas")`

[Out] `integral((c*x^2 + b*x + a)*(e*x + d)^m/(f*x + e)^(3/2), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^m (a + bx + cx^2)}{(e + fx)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**m*(c*x**2+b*x+a)/(f*x+e)**(3/2),x)`

[Out] `Integral((d + e*x)**m*(a + b*x + c*x**2)/(e + f*x)**(3/2), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)(ex + d)^m}{(fx + e)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)*(e*x + d)^m/(f*x + e)^(3/2),x, algorithm="giac")`

[Out] `integrate((c*x^2 + b*x + a)*(e*x + d)^m/(f*x + e)^(3/2), x)`

$$3.945 \quad \int (d + ex)^m (f + gx)^2 \sqrt{a + bx + cx^2} dx$$

**Optimal.** Leaf size=509

$$\frac{\sqrt{a + bx + cx^2} (d + ex)^{m+1} F_1 \left( m + 1; -\frac{1}{2}, -\frac{1}{2}; m + 2; \frac{2c(d+ex)}{2cd - (b - \sqrt{b^2 - 4ac})e}, \frac{2c(d+ex)}{2cd - (b + \sqrt{b^2 - 4ac})e} \right) (eg^2(m+1)(bd - ae) + c(3d^2g^2 - 2ce^3(m+1)(m+4) \sqrt{1 - \frac{2c(d+ex)}{2cd - (b - \sqrt{b^2 - 4ac})e}} \sqrt{1 - \frac{2c(d+ex)}{2cd - (b + \sqrt{b^2 - 4ac})e}}))}{g\sqrt{a + bx + cx^2} (d + ex)^{m+2} (beg(2m+5) + 2c(3dg - 2ef(m+4))) F_1 \left( m + 2; -\frac{1}{2}, -\frac{1}{2}; m + 3; \frac{2c(d+ex)}{2cd - (b - \sqrt{b^2 - 4ac})e}, \frac{2c(d+ex)}{2cd - (b + \sqrt{b^2 - 4ac})e} \right) - 2ce^3(m+2)(m+4) \sqrt{1 - \frac{2c(d+ex)}{2cd - (b - \sqrt{b^2 - 4ac})e}} \sqrt{1 - \frac{2c(d+ex)}{2cd - (b + \sqrt{b^2 - 4ac})e}}}$$

$$+ \frac{g^2 (a + bx + cx^2)^{3/2} (d + ex)^{m+1}}{ce(m+4)}$$

[Out]  $(g^2(d + ex)^{(1+m)}(a + bx + cx^2)^{(3/2)}) / (c^2e^2(4 + m)) + ((b^2d - a^2e)g^2(1 + m) + c(3d^2g^2 + e^2f^2(4 + m) - 2d^2e^2fg^2(4 + m))) (d + ex)^{(1+m)} \text{Sqrt}[a + bx + cx^2] \text{AppellF1}[1 + m, -1/2, -1/2, 2 + m, (2^2c^2(d + ex)) / (2^2c^2d - (b - \text{Sqrt}[b^2 - 4a^2c])^2e), (2^2c^2(d + ex)) / (2^2c^2d - (b + \text{Sqrt}[b^2 - 4a^2c])^2e)] / (c^2e^2(1 + m)^2(4 + m) \text{Sqrt}[1 - (2^2c^2(d + ex)) / (2^2c^2d - (b - \text{Sqrt}[b^2 - 4a^2c])^2e)] \text{Sqrt}[1 - (2^2c^2(d + ex)) / (2^2c^2d - (b + \text{Sqrt}[b^2 - 4a^2c])^2e)] - (g^2(b^2e^2g^2(5 + 2m) + 2^2c^2(3d^2g - 2e^2f^2(4 + m))) (d + ex)^{(2+m)} \text{Sqrt}[a + bx + cx^2] \text{AppellF1}[2 + m, -1/2, -1/2, 3 + m, (2^2c^2(d + ex)) / (2^2c^2d - (b - \text{Sqrt}[b^2 - 4a^2c])^2e), (2^2c^2(d + ex)) / (2^2c^2d - (b + \text{Sqrt}[b^2 - 4a^2c])^2e)] / (2^2c^2e^2(2 + m)^2(4 + m) \text{Sqrt}[1 - (2^2c^2(d + ex)) / (2^2c^2d - (b - \text{Sqrt}[b^2 - 4a^2c])^2e)] \text{Sqrt}[1 - (2^2c^2(d + ex)) / (2^2c^2d - (b + \text{Sqrt}[b^2 - 4a^2c])^2e)]]$

**Rubi [A]** time = 2.36314, antiderivative size = 506, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$

$$\frac{\sqrt{a + bx + cx^2} (d + ex)^{m+1} F_1 \left( m + 1; -\frac{1}{2}, -\frac{1}{2}; m + 2; \frac{2c(d+ex)}{2cd - (b - \sqrt{b^2 - 4ac})e}, \frac{2c(d+ex)}{2cd - (b + \sqrt{b^2 - 4ac})e} \right) (g^2(bd - ae) + \frac{c(3d^2g^2 - 2defg(m+4) + ce^3(m+1)(m+4) \sqrt{1 - \frac{2c(d+ex)}{2cd - (b - \sqrt{b^2 - 4ac})e}} \sqrt{1 - \frac{2c(d+ex)}{2cd - (b + \sqrt{b^2 - 4ac})e}}))}{g\sqrt{a + bx + cx^2} (d + ex)^{m+2} (beg(2m+5) + 6cdg - 4cef(m+4)) F_1 \left( m + 2; -\frac{1}{2}, -\frac{1}{2}; m + 3; \frac{2c(d+ex)}{2cd - (b - \sqrt{b^2 - 4ac})e}, \frac{2c(d+ex)}{2cd - (b + \sqrt{b^2 - 4ac})e} \right) - 2ce^3(m+2)(m+4) \sqrt{1 - \frac{2c(d+ex)}{2cd - (b - \sqrt{b^2 - 4ac})e}} \sqrt{1 - \frac{2c(d+ex)}{2cd - (b + \sqrt{b^2 - 4ac})e}}}$$

$$+ \frac{g^2 (a + bx + cx^2)^{3/2} (d + ex)^{m+1}}{ce(m+4)}$$

Warning: Unable to verify antiderivative.

[In]  $\text{Int}[(d + ex)^m (f + gx)^2 \text{Sqrt}[a + bx + cx^2], x]$

[Out]  $(g^2(d + ex)^{(1+m)}(a + bx + cx^2)^{(3/2)}) / (c^2e^2(4 + m)) + ((b^2d - a^2e)g^2 + (c(3d^2g^2 + e^2f^2(4 + m) - 2d^2e^2fg^2(4 + m))) / (e^2(1 + m))) (d + ex)^{(1+m)} \text{Sqrt}[a + bx + cx^2] \text{AppellF1}[1 + m, -1/2, -1/2, 2 + m, (2^2c^2(d + ex)) / (2^2c^2d - (b - \text{Sqrt}[b^2 - 4a^2c])^2e), (2^2c^2(d + ex)) / (2^2c^2d - (b + \text{Sqrt}[b^2 - 4a^2c])^2e)] / (c^2e^2(4 + m)^2 \text{Sqrt}[1 - (2^2c^2(d + ex)) / (2^2c^2d - (b - \text{Sqrt}[b^2 - 4a^2c])^2e)] \text{Sqrt}[1 - (2^2c^2(d + ex)) / (2^2c^2d - (b + \text{Sqrt}[b^2 - 4a^2c])^2e)] - (g^2(6^2c^2d^2g - 4^2c^2e^2f^2(4 + m) + b^2e^2g^2(5 + 2m)) (d + ex)^{(2+m)} \text{Sqrt}[a + bx + cx^2] \text{AppellF1}[2 + m, -1/2, -1/2, 3 + m, (2^2c^2(d + ex)) / (2^2c^2d - (b - \text{Sqrt}[b^2 - 4a^2c])^2e), (2^2c^2(d + ex)) / (2^2c^2d - (b + \text{Sqrt}[b^2 - 4a^2c])^2e)]]$



, (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]/(2\*c\*e^3\*(2 + m)\*(4 + m)\*Sqrt[1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e)]\*Sqrt[1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)])

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*m\*(g\*x+f)\*\*2\*(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Timed out

**Mathematica [A]** time = 2.12638, size = 0, normalized size = 0.

$$\int (d + ex)^m (f + gx)^2 \sqrt{a + bx + cx^2} dx$$

Verification is Not applicable to the result.

[In] Integrate[(d + e\*x)^m\*(f + g\*x)^2\*Sqrt[a + b\*x + c\*x^2],x]

[Out] Integrate[(d + e\*x)^m\*(f + g\*x)^2\*Sqrt[a + b\*x + c\*x^2], x]

**Maple [F]** time = 0.129, size = 0, normalized size = 0.

$$\int (ex + d)^m (gx + f)^2 \sqrt{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^m\*(g\*x+f)^2\*(c\*x^2+b\*x+a)^(1/2),x)

[Out] int((e\*x+d)^m\*(g\*x+f)^2\*(c\*x^2+b\*x+a)^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + bx + a} (gx + f)^2 (ex + d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)\*(g\*x + f)^2\*(e\*x + d)^m,x, algorithm="maxima")

[Out] integrate(sqrt(c\*x^2 + b\*x + a)\*(g\*x + f)^2\*(e\*x + d)^m, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(g^2x^2 + 2fgx + f^2\right)\sqrt{cx^2 + bx + a}(ex + d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)^2*(e*x + d)^m,x, algorithm="fricas")`

[Out] `integral((g^2*x^2 + 2*f*g*x + f^2)*sqrt(c*x^2 + b*x + a)*(e*x + d)^m, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+b*x+a)**(1/2),x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + bx + a}(gx + f)^2(ex + d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)^2*(e*x + d)^m,x, algorithm="giac")`

[Out] `integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)^2*(e*x + d)^m, x)`

### 3.946 $\int (d + ex)^m (f + gx) \sqrt{a + bx + cx^2} dx$

**Optimal.** Leaf size=388

$$\frac{\sqrt{a + bx + cx^2}(ef - dg)(d + ex)^{m+1} F_1\left(m + 1; -\frac{1}{2}, -\frac{1}{2}; m + 2; \frac{2c(d+ex)}{2cd - (b - \sqrt{b^2 - 4ac})e}, \frac{2c(d+ex)}{2cd - (b + \sqrt{b^2 - 4ac})e}\right)}{e^2(m + 1) \sqrt{1 - \frac{2c(d+ex)}{2cd - e(b - \sqrt{b^2 - 4ac})}} \sqrt{1 - \frac{2c(d+ex)}{2cd - e(\sqrt{b^2 - 4ac} + b)}}} + \frac{g\sqrt{a + bx + cx^2}(d + ex)^{m+2} F_1\left(m + 2; -\frac{1}{2}, -\frac{1}{2}; m + 3; \frac{2c(d+ex)}{2cd - (b - \sqrt{b^2 - 4ac})e}, \frac{2c(d+ex)}{2cd - (b + \sqrt{b^2 - 4ac})e}\right)}{e^2(m + 2) \sqrt{1 - \frac{2c(d+ex)}{2cd - e(b - \sqrt{b^2 - 4ac})}} \sqrt{1 - \frac{2c(d+ex)}{2cd - e(\sqrt{b^2 - 4ac} + b)}}}$$

[Out]  $((e*f - d*g)*(d + e*x)^(1 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/ (e^2*(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]) + (g*(d + e*x)^(2 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[2 + m, -1/2, -1/2, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/ (e^2*(2 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])$

**Rubi [A]** time = 1.40259, antiderivative size = 388, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{\sqrt{a + bx + cx^2}(ef - dg)(d + ex)^{m+1} F_1\left(m + 1; -\frac{1}{2}, -\frac{1}{2}; m + 2; \frac{2c(d+ex)}{2cd - (b - \sqrt{b^2 - 4ac})e}, \frac{2c(d+ex)}{2cd - (b + \sqrt{b^2 - 4ac})e}\right)}{e^2(m + 1) \sqrt{1 - \frac{2c(d+ex)}{2cd - e(b - \sqrt{b^2 - 4ac})}} \sqrt{1 - \frac{2c(d+ex)}{2cd - e(\sqrt{b^2 - 4ac} + b)}}} + \frac{g\sqrt{a + bx + cx^2}(d + ex)^{m+2} F_1\left(m + 2; -\frac{1}{2}, -\frac{1}{2}; m + 3; \frac{2c(d+ex)}{2cd - (b - \sqrt{b^2 - 4ac})e}, \frac{2c(d+ex)}{2cd - (b + \sqrt{b^2 - 4ac})e}\right)}{e^2(m + 2) \sqrt{1 - \frac{2c(d+ex)}{2cd - e(b - \sqrt{b^2 - 4ac})}} \sqrt{1 - \frac{2c(d+ex)}{2cd - e(\sqrt{b^2 - 4ac} + b)}}}$$

Warning: Unable to verify antiderivative.

[In]  $\text{Int}[(d + e*x)^m*(f + g*x)*Sqrt[a + b*x + c*x^2], x]$

[Out]  $((e*f - d*g)*(d + e*x)^(1 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/ (e^2*(1 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]) + (g*(d + e*x)^(2 + m)*Sqrt[a + b*x + c*x^2]*AppellF1[2 + m, -1/2, -1/2, 3 + m, (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e), (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/ (e^2*(2 + m)*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])*Sqrt[1 - (2*c*(d + e*x))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])$

**Rubi in Sympy [A]** time = 93.1527, size = 364, normalized size = 0.94

$$\frac{g(d+ex)^{m+2} \sqrt{a+bx+cx^2} \operatorname{appellf}_1\left(m+2, -\frac{1}{2}, -\frac{1}{2}, m+3, \frac{c(-2d-2ex)}{be-2cd-e\sqrt{-4ac+b^2}}, \frac{c(2d+2ex)}{2cd-e(b+\sqrt{-4ac+b^2})}\right)}{e^2(m+2) \sqrt{\frac{c(-2d-2ex)}{2cd-e(b+\sqrt{-4ac+b^2})} + 1} \sqrt{\frac{c(2d+2ex)}{be-2cd-e\sqrt{-4ac+b^2}} + 1}} \\ \frac{(d+ex)^{m+1} (dg-ef) \sqrt{a+bx+cx^2} \operatorname{appellf}_1\left(m+1, -\frac{1}{2}, -\frac{1}{2}, m+2, \frac{c(-2d-2ex)}{be-2cd-e\sqrt{-4ac+b^2}}, \frac{c(2d+2ex)}{2cd-e(b+\sqrt{-4ac+b^2})}\right)}{e^2(m+1) \sqrt{\frac{c(-2d-2ex)}{2cd-e(b+\sqrt{-4ac+b^2})} + 1} \sqrt{\frac{c(2d+2ex)}{be-2cd-e\sqrt{-4ac+b^2}} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**m*(g*x+f)*(c*x**2+b*x+a)**(1/2),x)`

[Out] `g*(d + e*x)**(m + 2)*sqrt(a + b*x + c*x**2)*appellf1(m + 2, -1/2, -1/2, m + 3, c*(-2*d - 2*e*x)/(b*e - 2*c*d - e*sqrt(-4*a*c + b**2)), c*(2*d + 2*e*x)/(2*c*d - e*(b + sqrt(-4*a*c + b**2))))/(e**2*(m + 2)*sqrt(c*(-2*d - 2*e*x)/(2*c*d - e*(b + sqrt(-4*a*c + b**2))) + 1)*sqrt(c*(2*d + 2*e*x)/(b*e - 2*c*d - e*sqrt(-4*a*c + b**2)) + 1)) - (d + e*x)**(m + 1)*(d*g - e*f)*sqrt(a + b*x + c*x**2)*appellf1(m + 1, -1/2, -1/2, m + 2, c*(-2*d - 2*e*x)/(b*e - 2*c*d - e*sqrt(-4*a*c + b**2)), c*(2*d + 2*e*x)/(2*c*d - e*(b + sqrt(-4*a*c + b**2))))/(e**2*(m + 1)*sqrt(c*(-2*d - 2*e*x)/(2*c*d - e*(b + sqrt(-4*a*c + b**2))) + 1)*sqrt(c*(2*d + 2*e*x)/(b*e - 2*c*d - e*sqrt(-4*a*c + b**2)) + 1))`

**Mathematica [A]** time = 1.57829, size = 0, normalized size = 0.

$$\int (d+ex)^m (f+gx) \sqrt{a+bx+cx^2} dx$$

Verification is Not applicable to the result.

[In] `Integrate[(d + e*x)^m*(f + g*x)*Sqrt[a + b*x + c*x^2],x]`

[Out] `Integrate[(d + e*x)^m*(f + g*x)*Sqrt[a + b*x + c*x^2], x]`

**Maple [F]** time = 0.099, size = 0, normalized size = 0.

$$\int (ex+d)^m (gx+f) \sqrt{cx^2+bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^(1/2),x)`

[Out] `int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^(1/2),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2+bx+a}(gx+f)(ex+d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)*(e*x + d)^m, x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)*(e*x + d)^m, x)`

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^2 + bx + a}(gx + f)(ex + d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)*(e*x + d)^m, x, algorithm="fricas")`

[Out] `integral(sqrt(c*x^2 + b*x + a)*(g*x + f)*(e*x + d)^m, x)`

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int (d + ex)^m (f + gx) \sqrt{a + bx + cx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**m*(g*x+f)*(c*x**2+b*x+a)**(1/2), x)`

[Out] `Integral((d + e*x)**m*(f + g*x)*sqrt(a + b*x + c*x**2), x)`

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + bx + a}(gx + f)(ex + d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)*(e*x + d)^m, x, algorithm="giac")`

[Out] `integrate(sqrt(c*x^2 + b*x + a)*(g*x + f)*(e*x + d)^m, x)`

$$3.947 \quad \int (d + ex)^m \sqrt{a + bx + cx^2} dx$$

**Optimal.** Leaf size=189

$$\frac{\sqrt{a + bx + cx^2}(d + ex)^{m+1} F_1\left(m + 1; -\frac{1}{2}, -\frac{1}{2}; m + 2; \frac{2c(d+ex)}{2cd - (b - \sqrt{b^2 - 4ac})e}, \frac{2c(d+ex)}{2cd - (b + \sqrt{b^2 - 4ac})e}\right)}{e(m + 1) \sqrt{1 - \frac{2c(d+ex)}{2cd - e(b - \sqrt{b^2 - 4ac})}} \sqrt{1 - \frac{2c(d+ex)}{2cd - e(\sqrt{b^2 - 4ac} + b)}}$$

[Out] ((d + e\*x)^(1 + m)\*Sqrt[a + b\*x + c\*x^2]\*AppellF1[1 + m, -1/2, -1/2, 2 + m, (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e), (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]/(e\*(1 + m)\*Sqrt[1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e)]\*Sqrt[1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)])

**Rubi [A]** time = 0.62892, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{\sqrt{a + bx + cx^2}(d + ex)^{m+1} F_1\left(m + 1; -\frac{1}{2}, -\frac{1}{2}; m + 2; \frac{2c(d+ex)}{2cd - (b - \sqrt{b^2 - 4ac})e}, \frac{2c(d+ex)}{2cd - (b + \sqrt{b^2 - 4ac})e}\right)}{e(m + 1) \sqrt{1 - \frac{2c(d+ex)}{2cd - e(b - \sqrt{b^2 - 4ac})}} \sqrt{1 - \frac{2c(d+ex)}{2cd - e(\sqrt{b^2 - 4ac} + b)}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^m\*Sqrt[a + b\*x + c\*x^2], x]

[Out] ((d + e\*x)^(1 + m)\*Sqrt[a + b\*x + c\*x^2]\*AppellF1[1 + m, -1/2, -1/2, 2 + m, (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e), (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]/(e\*(1 + m)\*Sqrt[1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e)]\*Sqrt[1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)])

**Rubi in Sympy [A]** time = 37.1183, size = 175, normalized size = 0.93

$$\frac{(d + ex)^{m+1} \sqrt{a + bx + cx^2} \operatorname{appellf}_1\left(m + 1, -\frac{1}{2}, -\frac{1}{2}, m + 2, \frac{c(-2d - 2ex)}{be - 2cd - e\sqrt{-4ac + b^2}}, \frac{c(2d + 2ex)}{2cd - e(b + \sqrt{-4ac + b^2})}\right)}{e(m + 1) \sqrt{\frac{c(-2d - 2ex)}{2cd - e(b + \sqrt{-4ac + b^2})} + 1} \sqrt{\frac{c(2d + 2ex)}{be - 2cd - e\sqrt{-4ac + b^2}} + 1}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*m\*(c\*x\*\*2+b\*x+a)\*\*(1/2), x)

[Out] (d + e\*x)\*\*(m + 1)\*sqrt(a + b\*x + c\*x\*\*2)\*appellf1(m + 1, -1/2, -1/2, m + 2, c\*(-2\*d - 2\*e\*x)/(b\*e - 2\*c\*d - e\*sqrt(-4\*a\*c + b\*\*2)), c\*(2\*d + 2\*e\*x)/(2\*c\*d - e\*(b + sqrt(-4\*a\*c + b\*\*2))))/(e\*(m + 1)\*sqrt(c\*(-2\*d - 2\*e\*x)/(2\*c\*d - e\*(b + sqrt(-4\*a\*c + b\*\*2)))) + 1)\*sqrt(c\*(2\*d + 2\*e\*x)/(b\*e - 2\*c\*d - e\*sqrt(-4\*a\*c + b\*\*2)) + 1)

**Mathematica [A]** time = 0.0200351, size = 207, normalized size = 1.1

$$\frac{\sqrt{a+x(b+cx)}(d+ex)^{m+1}F_1\left(m+1;-\frac{1}{2},-\frac{1}{2};m+2;\frac{2c(d+ex)}{2cd-(b+\sqrt{b^2-4ac})e},\frac{2c(d+ex)}{2cd+(\sqrt{b^2-4ac}-b)e}\right)}{e^{(m+1)}\sqrt{\frac{e(\sqrt{b^2-4ac}-b-2cx)}{e(\sqrt{b^2-4ac}-b)+2cd}}\sqrt{\frac{e(\sqrt{b^2-4ac}+b+2cx)}{e(\sqrt{b^2-4ac}+b)-2cd}}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(d + e\*x)^m\*Sqrt[a + b\*x + c\*x^2],x]

[Out] ((d + e\*x)^(1 + m)\*Sqrt[a + x\*(b + c\*x)]\*AppellF1[1 + m, -1/2, -1/2, 2 + m, (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e), (2\*c\*(d + e\*x))/(2\*c\*d + (-b + Sqrt[b^2 - 4\*a\*c])\*e)]/(e\*(1 + m)\*Sqrt[(e\*(-b + Sqrt[b^2 - 4\*a\*c] - 2\*c\*x))/(2\*c\*d + (-b + Sqrt[b^2 - 4\*a\*c])\*e)]\*Sqrt[(e\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/(-2\*c\*d + (b + Sqrt[b^2 - 4\*a\*c])\*e)])

**Maple [F]** time = 0.096, size = 0, normalized size = 0.

$$\int (ex + d)^m \sqrt{cx^2 + bx + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^m\*(c\*x^2+b\*x+a)^(1/2),x)

[Out] int((e\*x+d)^m\*(c\*x^2+b\*x+a)^(1/2),x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + bx + a}(ex + d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)^m,x, algorithm="maxima")

[Out] integrate(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)^m, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{cx^2 + bx + a}(ex + d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)^m,x, algorithm="fricas")

[Out] integral(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)^m, x)

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int (d + ex)^m \sqrt{a + bx + cx^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*m\*(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral((d + e\*x)\*\*m\*sqrt(a + b\*x + c\*x\*\*2), x)

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \sqrt{cx^2 + bx + a}(ex + d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)^m,x, algorithm="giac")

[Out] integrate(sqrt(c\*x^2 + b\*x + a)\*(e\*x + d)^m, x)



$$3.948 \quad \int \frac{(d+ex)^m \sqrt{a+bx+cx^2}}{f+gx} dx$$

**Optimal.** Leaf size=32

$$\text{Int} \left( \frac{\sqrt{a+bx+cx^2}(d+ex)^m}{f+gx}, x \right)$$

[Out] Unintegrable[((d + e\*x)^m\*Sqrt[a + b\*x + c\*x^2])/(f + g\*x), x]

**Rubi [A]** time = 0.119672, antiderivative size = 0, normalized size of antiderivative = 0., number of rules used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0$ .

$$\text{Int} \left( \frac{(d+ex)^m \sqrt{a+bx+cx^2}}{f+gx}, x \right)$$

Verification is Not applicable to the result.

[In] Int[((d + e\*x)^m\*Sqrt[a + b\*x + c\*x^2])/(f + g\*x), x]

[Out] Defer[Int][[((d + e\*x)^m\*Sqrt[a + b\*x + c\*x^2])/(f + g\*x), x]

**Rubi in Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m \sqrt{a+bx+cx^2}}{f+gx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*m\*(c\*x\*\*2+b\*x+a)\*\*(1/2)/(g\*x+f), x)

[Out] Integral((d + e\*x)\*\*m\*sqrt(a + b\*x + c\*x\*\*2)/(f + g\*x), x)

**Mathematica [A]** time = 0.0900179, size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m \sqrt{a+bx+cx^2}}{f+gx} dx$$

Verification is Not applicable to the result.

[In] Integrate[((d + e\*x)^m\*Sqrt[a + b\*x + c\*x^2])/(f + g\*x), x]

[Out] Integrate[((d + e\*x)^m\*Sqrt[a + b\*x + c\*x^2])/(f + g\*x), x]

**Maple [A]** time = 0.113, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{gx+f} \sqrt{cx^2+bx+a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x)`

[Out] `int((e*x+d)^m*(c*x^2+b*x+a)^(1/2)/(g*x+f),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}(ex + d)^m}{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f),x, algorithm="maxima")`

[Out] `integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{cx^2 + bx + a}(ex + d)^m}{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f),x, algorithm="fricas")`

[Out] `integral(sqrt(c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f), x)`

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^m \sqrt{a + bx + cx^2}}{f + gx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**m*(c*x**2+b*x+a)**(1/2)/(g*x+f),x)`

[Out] `Integral((d + e*x)**m*sqrt(a + b*x + c*x**2)/(f + g*x), x)`

**GIAC/XCAS [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{cx^2 + bx + a}(ex + d)^m}{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f),x, algorithm="giac")`

[Out] `integrate(sqrt(c*x^2 + b*x + a)*(e*x + d)^m/(g*x + f), x)`

$$3.949 \quad \int \frac{(d+ex)^m(f+gx)^2}{\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=502

$$\frac{(d+ex)^{m+1} \sqrt{1 - \frac{2c(d+ex)}{2cd-e(b-\sqrt{b^2-4ac})}} \sqrt{1 - \frac{2c(d+ex)}{2cd-e(\sqrt{b^2-4ac}+b)}} F_1\left(m+1; \frac{1}{2}, \frac{1}{2}; m+2; \frac{2c(d+ex)}{2cd-(b-\sqrt{b^2-4ac})e}, \frac{2c(d+ex)}{2cd-(b+\sqrt{b^2-4ac})e}\right) (eg^2(b^2d - a^2e)g^2 + c^2(d+ex)^2)}{ce^3(m+1)(m+2)\sqrt{a+bx+cx^2}} + \frac{g(d+ex)^{m+2} \sqrt{1 - \frac{2c(d+ex)}{2cd-e(b-\sqrt{b^2-4ac})}} \sqrt{1 - \frac{2c(d+ex)}{2cd-e(\sqrt{b^2-4ac}+b)}} (beg(2m+3) + c(2dg - 4ef(m+2))) F_1\left(m+2; \frac{1}{2}, \frac{1}{2}; m+3; \frac{2c(d+ex)}{2cd-(b-\sqrt{b^2-4ac})e}, \frac{2c(d+ex)}{2cd-(b+\sqrt{b^2-4ac})e}\right)}{2ce^3(m+2)^2\sqrt{a+bx+cx^2}} + \frac{g^2\sqrt{a+bx+cx^2}(d+ex)^{m+1}}{ce(m+2)}$$

[Out] (g^2\*(d + e\*x)^(1 + m)\*Sqrt[a + b\*x + c\*x^2])/(c\*e\*(2 + m)) + ((e\*(b\*d - a\*e)\*g^2\*(1 + m) + c\*(d^2\*g^2 + e^2\*f^2\*(2 + m) - 2\*d\*e\*f\*g\*(2 + m)))\*(d + e\*x)^(1 + m)\*Sqrt[1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e)]\*Sqrt[1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]\*AppellF1[1 + m, 1/2, 1/2, 2 + m, (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e), (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]/(c\*e^3\*(1 + m)\*(2 + m)\*Sqrt[a + b\*x + c\*x^2]) - (g\*(b\*e\*g\*(3 + 2\*m) + c\*(2\*d\*g - 4\*e\*f\*(2 + m)))\*(d + e\*x)^(2 + m)\*Sqrt[1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e)]\*Sqrt[1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]\*AppellF1[2 + m, 1/2, 1/2, 3 + m, (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e), (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]/(2\*c\*e^3\*(2 + m)^2\*Sqrt[a + b\*x + c\*x^2])

**Rubi [A]** time = 2.10659, antiderivative size = 500, normalized size of antiderivative = 1., number of rules used = 6, number of rules used = 4, integrand size = 29,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$

$$\frac{(d+ex)^{m+1} \sqrt{1 - \frac{2c(d+ex)}{2cd-e(b-\sqrt{b^2-4ac})}} \sqrt{1 - \frac{2c(d+ex)}{2cd-e(\sqrt{b^2-4ac}+b)}} F_1\left(m+1; \frac{1}{2}, \frac{1}{2}; m+2; \frac{2c(d+ex)}{2cd-(b-\sqrt{b^2-4ac})e}, \frac{2c(d+ex)}{2cd-(b+\sqrt{b^2-4ac})e}\right) (g^2(b^2d - a^2e)g^2 + c^2(d+ex)^2)}{ce^2(m+2)\sqrt{a+bx+cx^2}} + \frac{g(d+ex)^{m+2} \sqrt{1 - \frac{2c(d+ex)}{2cd-e(b-\sqrt{b^2-4ac})}} \sqrt{1 - \frac{2c(d+ex)}{2cd-e(\sqrt{b^2-4ac}+b)}} (beg(2m+3) + 2cdg - 4cef(m+2)) F_1\left(m+2; \frac{1}{2}, \frac{1}{2}; m+3; \frac{2c(d+ex)}{2cd-(b-\sqrt{b^2-4ac})e}, \frac{2c(d+ex)}{2cd-(b+\sqrt{b^2-4ac})e}\right)}{2ce^3(m+2)^2\sqrt{a+bx+cx^2}} + \frac{g^2\sqrt{a+bx+cx^2}(d+ex)^{m+1}}{ce(m+2)}$$

Warning: Unable to verify antiderivative.

[In] Int[((d + e\*x)^m\*(f + g\*x)^2)/Sqrt[a + b\*x + c\*x^2], x]

[Out] (g^2\*(d + e\*x)^(1 + m)\*Sqrt[a + b\*x + c\*x^2])/(c\*e\*(2 + m)) + (((b\*d - a\*e)\*g^2 + (c\*(d^2\*g^2 + e^2\*f^2\*(2 + m) - 2\*d\*e\*f\*g\*(2 + m)))/(e\*(1 + m)))\*(d + e\*x)^(1 + m)\*Sqrt[1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e)]\*Sqrt[1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]\*AppellF1[1 + m, 1/2, 1/2, 2 + m, (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e), (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]/(c\*e^2\*(2 + m)\*Sqrt[a + b\*x + c\*x^2]) - (g\*(2\*c\*d\*g - 4\*c\*e\*f\*(2 + m) + b\*e\*g\*(3 + 2\*m))\*(d + e\*x)^(2 + m)\*Sqrt[1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e)]\*Sqrt[1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]\*AppellF1[2 + m, 1/2, 1/2, 3 + m, (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e), (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]/(2\*c\*e^3\*(2 + m)^2\*Sqrt[a + b\*x + c\*x^2])

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**m*(g*x+f)**2/(c*x**2+b*x+a)**(1/2),x)`

[Out] Timed out

**Mathematica [A]** time = 2.13895, size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m (f+gx)^2}{\sqrt{a+bx+cx^2}} dx$$

Verification is Not applicable to the result.

[In] `Integrate[((d + e*x)^m*(f + g*x)^2)/Sqrt[a + b*x + c*x^2],x]`

[Out] `Integrate[((d + e*x)^m*(f + g*x)^2)/Sqrt[a + b*x + c*x^2], x]`

**Maple [F]** time = 0.111, size = 0, normalized size = 0.

$$\int (ex+d)^m (gx+f)^2 \frac{1}{\sqrt{cx^2+bx+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^m*(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x)`

[Out] `int((e*x+d)^m*(g*x+f)^2/(c*x^2+b*x+a)^(1/2),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx+f)^2 (ex+d)^m}{\sqrt{cx^2+bx+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x + f)^2*(e*x + d)^m/sqrt(c*x^2 + b*x + a),x, algorithm="maxima")`

[Out] `integrate((g*x + f)^2*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(g^2x^2 + 2fgx + f^2)(ex+d)^m}{\sqrt{cx^2+bx+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x + f)^2*(e*x + d)^m/sqrt(c*x^2 + b*x + a),x, algorithm="fricas")`

[Out] `integral((g^2*x^2 + 2*f*g*x + f^2)*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x)`

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^m (f + gx)^2}{\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**m*(g*x+f)**2/(c*x**2+b*x+a)**(1/2), x)`

[Out] `Integral((d + e*x)**m*(f + g*x)**2/sqrt(a + b*x + c*x**2), x)`

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx + f)^2 (ex + d)^m}{\sqrt{cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x + f)^2*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x, algorithm="giac")`

[Out] `integrate((g*x + f)^2*(e*x + d)^m/sqrt(c*x^2 + b*x + a), x)`

$$3.950 \quad \int \frac{(d+ex)^m (f+gx)}{\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=388

$$\frac{(ef - dg)(d + ex)^{m+1} \sqrt{1 - \frac{2c(d+ex)}{2cd - e(b - \sqrt{b^2 - 4ac})}} \sqrt{1 - \frac{2c(d+ex)}{2cd - e(\sqrt{b^2 - 4ac} + b)}} F_1 \left( m + 1; \frac{1}{2}, \frac{1}{2}; m + 2; \frac{2c(d+ex)}{2cd - (b - \sqrt{b^2 - 4ac})e}, \frac{2c(d+ex)}{2cd - (b + \sqrt{b^2 - 4ac})e} \right)}{e^2(m+1)\sqrt{a+bx+cx^2}} + \frac{g(d+ex)^{m+2} \sqrt{1 - \frac{2c(d+ex)}{2cd - e(b - \sqrt{b^2 - 4ac})}} \sqrt{1 - \frac{2c(d+ex)}{2cd - e(\sqrt{b^2 - 4ac} + b)}} F_1 \left( m + 2; \frac{1}{2}, \frac{1}{2}; m + 3; \frac{2c(d+ex)}{2cd - (b - \sqrt{b^2 - 4ac})e}, \frac{2c(d+ex)}{2cd - (b + \sqrt{b^2 - 4ac})e} \right)}{e^2(m+2)\sqrt{a+bx+cx^2}}$$

[Out] ((e\*f - d\*g)\*(d + e\*x)^(1 + m)\*Sqrt[1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e)]\*Sqrt[1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]\*AppellF1[1 + m, 1/2, 1/2, 2 + m, (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e), (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]/(e^2\*(1 + m)\*Sqrt[a + b\*x + c\*x^2]) + (g\*(d + e\*x)^(2 + m)\*Sqrt[1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e)]\*Sqrt[1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]\*AppellF1[2 + m, 1/2, 1/2, 3 + m, (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e), (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]/(e^2\*(2 + m)\*Sqrt[a + b\*x + c\*x^2])

**Rubi [A]** time = 1.41648, antiderivative size = 388, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$

$$\frac{(ef - dg)(d + ex)^{m+1} \sqrt{1 - \frac{2c(d+ex)}{2cd - e(b - \sqrt{b^2 - 4ac})}} \sqrt{1 - \frac{2c(d+ex)}{2cd - e(\sqrt{b^2 - 4ac} + b)}} F_1 \left( m + 1; \frac{1}{2}, \frac{1}{2}; m + 2; \frac{2c(d+ex)}{2cd - (b - \sqrt{b^2 - 4ac})e}, \frac{2c(d+ex)}{2cd - (b + \sqrt{b^2 - 4ac})e} \right)}{e^2(m+1)\sqrt{a+bx+cx^2}} + \frac{g(d+ex)^{m+2} \sqrt{1 - \frac{2c(d+ex)}{2cd - e(b - \sqrt{b^2 - 4ac})}} \sqrt{1 - \frac{2c(d+ex)}{2cd - e(\sqrt{b^2 - 4ac} + b)}} F_1 \left( m + 2; \frac{1}{2}, \frac{1}{2}; m + 3; \frac{2c(d+ex)}{2cd - (b - \sqrt{b^2 - 4ac})e}, \frac{2c(d+ex)}{2cd - (b + \sqrt{b^2 - 4ac})e} \right)}{e^2(m+2)\sqrt{a+bx+cx^2}}$$

Warning: Unable to verify antiderivative.

[In] Int[((d + e\*x)^m\*(f + g\*x))/Sqrt[a + b\*x + c\*x^2], x]

[Out] ((e\*f - d\*g)\*(d + e\*x)^(1 + m)\*Sqrt[1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e)]\*Sqrt[1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]\*AppellF1[1 + m, 1/2, 1/2, 2 + m, (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e), (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]/(e^2\*(1 + m)\*Sqrt[a + b\*x + c\*x^2]) + (g\*(d + e\*x)^(2 + m)\*Sqrt[1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e)]\*Sqrt[1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]\*AppellF1[2 + m, 1/2, 1/2, 3 + m, (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e), (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]/(e^2\*(2 + m)\*Sqrt[a + b\*x + c\*x^2])

**Rubi in Sympy [A]** time = 95.7571, size = 357, normalized size = 0.92

$$\frac{g(d+ex)^{m+2} \sqrt{\frac{c(-2d-2ex)}{2cd-e(b+\sqrt{-4ac+b^2})}} + 1 \sqrt{\frac{c(2d+2ex)}{be-2cd-e\sqrt{-4ac+b^2}}} + 1 \operatorname{appellf}_1\left(m+2, \frac{1}{2}, \frac{1}{2}, m+3, \frac{c(-2d-2ex)}{be-2cd-e\sqrt{-4ac+b^2}}, \frac{c(2d+2ex)}{2cd-e(b+\sqrt{-4ac+b^2})}\right)}{e^2(m+2)\sqrt{a+bx+cx^2}} - \frac{(d+ex)^{m+1}(dg-ef) \sqrt{\frac{c(-2d-2ex)}{2cd-e(b+\sqrt{-4ac+b^2})}} + 1 \sqrt{\frac{c(2d+2ex)}{be-2cd-e\sqrt{-4ac+b^2}}} + 1 \operatorname{appellf}_1\left(m+1, \frac{1}{2}, \frac{1}{2}, m+2, \frac{c(-2d-2ex)}{be-2cd-e\sqrt{-4ac+b^2}}, \frac{c(2d+2ex)}{2cd-e(b+\sqrt{-4ac+b^2})}\right)}{e^2(m+1)\sqrt{a+bx+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**m*(g*x+f)/(c*x**2+b*x+a)**(1/2),x)`

[Out]  $g*(d+e*x)^{(m+2)}*\sqrt{c*(-2*d-2*e*x)/(2*c*d-e*(b+\sqrt{-4*a*c+b^2}))}+1)*\sqrt{c*(2*d+2*e*x)/(b*e-2*c*d-e*\sqrt{-4*a*c+b^2})}+1)*\operatorname{appellf}_1(m+2, 1/2, 1/2, m+3, c*(-2*d-2*e*x)/(b*e-2*c*d-e*\sqrt{-4*a*c+b^2}), c*(2*d+2*e*x)/(2*c*d-e*(b+\sqrt{-4*a*c+b^2})))}/(e^{2*(m+2)}*\sqrt{a+b*x+c*x^2}) - (d+e*x)^{(m+1)}*(d*g-e*f)*\sqrt{c*(-2*d-2*e*x)/(2*c*d-e*(b+\sqrt{-4*a*c+b^2}))}+1)*\sqrt{c*(2*d+2*e*x)/(b*e-2*c*d-e*\sqrt{-4*a*c+b^2})}+1)*\operatorname{appellf}_1(m+1, 1/2, 1/2, m+2, c*(-2*d-2*e*x)/(b*e-2*c*d-e*\sqrt{-4*a*c+b^2}), c*(2*d+2*e*x)/(2*c*d-e*(b+\sqrt{-4*a*c+b^2})))}/(e^{2*(m+1)}*\sqrt{a+b*x+c*x^2})$

**Mathematica [A]** time = 1.38842, size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m(f+gx)}{\sqrt{a+bx+cx^2}} dx$$

Verification is Not applicable to the result.

[In] `Integrate[((d+e*x)^m*(f+g*x))/Sqrt[a+b*x+c*x^2],x]`

[Out] `Integrate[((d+e*x)^m*(f+g*x))/Sqrt[a+b*x+c*x^2],x]`

**Maple [F]** time = 0.101, size = 0, normalized size = 0.

$$\int (ex+d)^m(gx+f) \frac{1}{\sqrt{cx^2+bx+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^m*(g*x+f)/(c*x^2+b*x+a)^(1/2),x)`

[Out] `int((e*x+d)^m*(g*x+f)/(c*x^2+b*x+a)^(1/2),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx+f)(ex+d)^m}{\sqrt{cx^2+bx+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)\*(e\*x + d)^m/sqrt(c\*x^2 + b\*x + a),x, algorithm="maxima")

[Out] integrate((g\*x + f)\*(e\*x + d)^m/sqrt(c\*x^2 + b\*x + a), x)

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(gx + f)(ex + d)^m}{\sqrt{cx^2 + bx + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)\*(e\*x + d)^m/sqrt(c\*x^2 + b\*x + a),x, algorithm="fricas")

[Out] integral((g\*x + f)\*(e\*x + d)^m/sqrt(c\*x^2 + b\*x + a), x)

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^m (f + gx)}{\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*m\*(g\*x+f)/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral((d + e\*x)\*\*m\*(f + g\*x)/sqrt(a + b\*x + c\*x\*\*2), x)

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(gx + f)(ex + d)^m}{\sqrt{cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)\*(e\*x + d)^m/sqrt(c\*x^2 + b\*x + a),x, algorithm="giac")

[Out] integrate((g\*x + f)\*(e\*x + d)^m/sqrt(c\*x^2 + b\*x + a), x)



$$3.951 \quad \int \frac{(d+ex)^m}{\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=189

$$\frac{(d+ex)^{m+1} \sqrt{1 - \frac{2c(d+ex)}{2cd-e(b-\sqrt{b^2-4ac})}} \sqrt{1 - \frac{2c(d+ex)}{2cd-e(\sqrt{b^2-4ac}+b)}} F_1\left(m+1; \frac{1}{2}, \frac{1}{2}; m+2; \frac{2c(d+ex)}{2cd-(b-\sqrt{b^2-4ac})e}, \frac{2c(d+ex)}{2cd-(b+\sqrt{b^2-4ac})e}\right)}{e(m+1)\sqrt{a+bx+cx^2}}$$

[Out] ((d + e\*x)^(1 + m)\*Sqrt[1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e)]\*Sqrt[1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]\*AppellF1[1 + m, 1/2, 1/2, 2 + m, (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e), (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]/(e\*(1 + m)\*Sqrt[a + b\*x + c\*x^2])

**Rubi [A]** time = 0.64371, antiderivative size = 189, normalized size of antiderivative = 1., number of rules used = 2, number of rules used = 2, integrand size = 22,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$

$$\frac{(d+ex)^{m+1} \sqrt{1 - \frac{2c(d+ex)}{2cd-e(b-\sqrt{b^2-4ac})}} \sqrt{1 - \frac{2c(d+ex)}{2cd-e(\sqrt{b^2-4ac}+b)}} F_1\left(m+1; \frac{1}{2}, \frac{1}{2}; m+2; \frac{2c(d+ex)}{2cd-(b-\sqrt{b^2-4ac})e}, \frac{2c(d+ex)}{2cd-(b+\sqrt{b^2-4ac})e}\right)}{e(m+1)\sqrt{a+bx+cx^2}}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^m/Sqrt[a + b\*x + c\*x^2], x]

[Out] ((d + e\*x)^(1 + m)\*Sqrt[1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e)]\*Sqrt[1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]\*AppellF1[1 + m, 1/2, 1/2, 2 + m, (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e), (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]/(e\*(1 + m)\*Sqrt[a + b\*x + c\*x^2])

**Rubi in Sympy [A]** time = 37.7253, size = 172, normalized size = 0.91

$$\frac{(d+ex)^{m+1} \sqrt{\frac{c(-2d-2ex)}{2cd-e(b+\sqrt{4ac+b^2})} + 1} \sqrt{\frac{c(2d+2ex)}{be-2cd-e\sqrt{4ac+b^2}} + 1} \text{appellf1}\left(m+1, \frac{1}{2}, \frac{1}{2}, m+2, \frac{c(-2d-2ex)}{be-2cd-e\sqrt{4ac+b^2}}, \frac{c(2d+2ex)}{2cd-e(b+\sqrt{4ac+b^2})}\right)}{e(m+1)\sqrt{a+bx+cx^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*m/(c\*x\*\*2+b\*x+a)\*\*(1/2), x)

[Out] (d + e\*x)\*\*(m + 1)\*sqrt(c\*(-2\*d - 2\*e\*x)/(2\*c\*d - e\*(b + sqrt(-4\*a\*c + b\*\*2))) + 1)\*sqrt(c\*(2\*d + 2\*e\*x)/(b\*e - 2\*c\*d - e\*sqrt(-4\*a\*c + b\*\*2)) + 1)\*appellf1(m + 1, 1/2, 1/2, m + 2, c\*(-2\*d - 2\*e\*x)/(b\*e - 2\*c\*d - e\*sqrt(-4\*a\*c + b\*\*2)), c\*(2\*d + 2\*e\*x)/(2\*c\*d - e\*(b + sqrt(-4\*a\*c + b\*\*2))))/(e\*(m + 1)\*sqrt(a + b\*x + c\*x\*\*2))

**Mathematica [A]** time = 0.02702, size = 207, normalized size = 1.1

$$\frac{(d+ex)^{m+1} \sqrt{\frac{e(\sqrt{b^2-4ac}-b-2cx)}{e(\sqrt{b^2-4ac}-b)+2cd}} \sqrt{\frac{e(\sqrt{b^2-4ac}+b+2cx)}{e(\sqrt{b^2-4ac}+b)-2cd}} F_1\left(m+1; \frac{1}{2}, \frac{1}{2}; m+2; \frac{2c(d+ex)}{2cd-(b+\sqrt{b^2-4ac})e}, \frac{2c(d+ex)}{2cd+(\sqrt{b^2-4ac}-b)e}\right)}{e(m+1)\sqrt{a+x(b+cx)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(d + e\*x)^m/Sqrt[a + b\*x + c\*x^2],x]

[Out] (Sqrt[(e\*(-b + Sqrt[b^2 - 4\*a\*c] - 2\*c\*x))/(2\*c\*d + (-b + Sqrt[b^2 - 4\*a\*c])\*e)]\*Sqrt[(e\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/(-2\*c\*d + (b + Sqrt[b^2 - 4\*a\*c])\*e)]\*(d + e\*x)^(1 + m)\*AppellF1[1 + m, 1/2, 1/2, 2 + m, (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e), (2\*c\*(d + e\*x))/(2\*c\*d + (-b + Sqrt[b^2 - 4\*a\*c])\*e)]/(e\*(1 + m)\*Sqrt[a + x\*(b + c\*x)])

**Maple** [F] time = 0.176, size = 0, normalized size = 0.

$$\int (ex + d)^m \frac{1}{\sqrt{cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^m/(c\*x^2+b\*x+a)^(1/2),x)

[Out] int((e\*x+d)^m/(c\*x^2+b\*x+a)^(1/2),x)

**Maxima** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^m}{\sqrt{cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^m/sqrt(c\*x^2 + b\*x + a),x, algorithm="maxima")

[Out] integrate((e\*x + d)^m/sqrt(c\*x^2 + b\*x + a), x)

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex + d)^m}{\sqrt{cx^2 + bx + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^m/sqrt(c\*x^2 + b\*x + a),x, algorithm="fricas")

[Out] integral((e\*x + d)^m/sqrt(c\*x^2 + b\*x + a), x)

**Sympy** [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^m}{\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*m/(c\*x\*\*2+b\*x+a)\*\*(1/2),x)

[Out] Integral((d + e\*x)\*\*m/sqrt(a + b\*x + c\*x\*\*2), x)

---

GIAC/XCAS [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^m}{\sqrt{cx^2 + bx + a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x + d)^m/sqrt(c\*x^2 + b\*x + a),x, algorithm="giac")

[Out] integrate((e\*x + d)^m/sqrt(c\*x^2 + b\*x + a), x)

$$3.952 \quad \int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}} dx$$

**Optimal.** Leaf size=32

$$\text{Int}\left(\frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}, x\right)$$

[Out] Unintegrable[(d + e\*x)^m/((f + g\*x)\*Sqrt[a + b\*x + c\*x^2]), x]

**Rubi [A]** time = 0.125835, antiderivative size = 0, normalized size of antiderivative = 0., number of rules used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0$ .

$$\text{Int}\left(\frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}}, x\right)$$

Verification is Not applicable to the result.

[In] Int[(d + e\*x)^m/((f + g\*x)\*Sqrt[a + b\*x + c\*x^2]), x]

[Out] Defer[Int][(d + e\*x)^m/((f + g\*x)\*Sqrt[a + b\*x + c\*x^2]), x]

**Rubi in Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*m/(g\*x+f)/(c\*x\*\*2+b\*x+a)\*\*(1/2), x)

[Out] Integral((d + e\*x)\*\*m/((f + g\*x)\*sqrt(a + b\*x + c\*x\*\*2)), x)

**Mathematica [A]** time = 0.101683, size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m}{(f+gx)\sqrt{a+bx+cx^2}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(d + e\*x)^m/((f + g\*x)\*Sqrt[a + b\*x + c\*x^2]), x]

[Out] Integrate[(d + e\*x)^m/((f + g\*x)\*Sqrt[a + b\*x + c\*x^2]), x]

**Maple [A]** time = 0.125, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m}{gx+f} \frac{1}{\sqrt{cx^2+bx+a}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x)`

[Out] `int((e*x+d)^m/(g*x+f)/(c*x^2+b*x+a)^(1/2),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^m}{\sqrt{cx^2 + bx + a}(gx + f)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^m/(sqrt(c*x^2 + b*x + a)*(g*x + f)),x, algorithm="maxima")`

[Out] `integrate((e*x + d)^m/(sqrt(c*x^2 + b*x + a)*(g*x + f)), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(ex + d)^m}{\sqrt{cx^2 + bx + a}(gx + f)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^m/(sqrt(c*x^2 + b*x + a)*(g*x + f)),x, algorithm="fricas")`

[Out] `integral((e*x + d)^m/(sqrt(c*x^2 + b*x + a)*(g*x + f)), x)`

**Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d + ex)^m}{(f + gx)\sqrt{a + bx + cx^2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**m/(g*x+f)/(c*x**2+b*x+a)**(1/2),x)`

[Out] `Integral((d + e*x)**m/((f + g*x)*sqrt(a + b*x + c*x**2)), x)`

**GIAC/XCAS [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(ex + d)^m}{\sqrt{cx^2 + bx + a}(gx + f)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x + d)^m/(sqrt(c*x^2 + b*x + a)*(g*x + f)),x, algorithm="giac")`

[Out] `integrate((e*x + d)^m/(sqrt(c*x^2 + b*x + a)*(g*x + f)), x)`

### 3.953 $\int (d + ex)^m (f + gx)^n (a + bx + cx^2) dx$

**Optimal.** Leaf size=265

$$\frac{(d + ex)^{m+1} (f + gx)^n \left( \frac{e(f+gx)}{ef-dg} \right)^{-n} {}_2F_1 \left( m+1, -n; m+2; -\frac{g(d+ex)}{ef-dg} \right) (g(m+n+2) (ae^2g(m+n+3) - cd(dg(n+1) + ef(m+n+2))) + e^3g^2(m+1)(m+n+2)(m+n+3))}{e^2g^2(m+n+2)(m+n+3)} + \frac{c(d+ex)^{m+2} (f+gx)^{n+1}}{e^2g(m+n+3)}$$

[Out]  $((b * e * g * (3 + m + n) - c * (e * f * (2 + m) + d * g * (4 + m + 2 * n))) * (d + e * x)^{(1 + m)} * (f + g * x)^{(1 + n)}) / (e^{2 * g} * g^{2 * (2 + m + n)} * (3 + m + n)) + (c * (d + e * x)^{(2 + m)} * (f + g * x)^{(1 + n)}) / (e^{2 * g} * g * (3 + m + n)) + ((g * (2 + m + n) * (a * e^{2 * g} * g * (3 + m + n) - c * d * (e * f * (2 + m) + d * g * (1 + n)))) - (e * f * (1 + m) + d * g * (1 + n))) * (b * e * g * (3 + m + n) - c * (e * f * (2 + m) + d * g * (4 + m + 2 * n)))) * (d + e * x)^{(1 + m)} * (f + g * x)^n * \text{Hypergeometric2F1}[1 + m, -n, 2 + m, -((g * (d + e * x)) / (e * f - d * g))] / (e^{3 * g} * g^{2 * (1 + m)} * (2 + m + n) * (3 + m + n) * ((e * (f + g * x)) / (e * f - d * g))^n)$

**Rubi [A]** time = 0.853422, antiderivative size = 263, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$

$$\frac{(d + ex)^{m+1} (f + gx)^n \left( \frac{e(f+gx)}{ef-dg} \right)^{-n} {}_2F_1 \left( m+1, -n; m+2; -\frac{g(d+ex)}{ef-dg} \right) (g(m+n+2) (ae^2g(m+n+3) - cd(dg(n+1) + ef(m+n+2))) + e^3g^2(m+1)(m+n+2)(m+n+3))}{e^2g^2(m+n+2)(m+n+3)} - \frac{(d + ex)^{m+1} (f + gx)^{n+1} (-beg(m+n+3) + cdg(m+2n+4) + cef(m+2))}{e^2g^2(m+n+2)(m+n+3)} + \frac{c(d+ex)^{m+2} (f+gx)^{n+1}}{e^2g(m+n+3)}$$

Antiderivative was successfully verified.

[In]  $\text{Int}[(d + e * x)^m * (f + g * x)^n * (a + b * x + c * x^2), x]$

[Out]  $-(((c * e * f * (2 + m) - b * e * g * (3 + m + n) + c * d * g * (4 + m + 2 * n)) * (d + e * x)^{(1 + m)} * (f + g * x)^{(1 + n)}) / (e^{2 * g} * g^{2 * (2 + m + n)} * (3 + m + n)) + (c * (d + e * x)^{(2 + m)} * (f + g * x)^{(1 + n)}) / (e^{2 * g} * g * (3 + m + n)) + (((e * f * (1 + m) + d * g * (1 + n)) * (c * e * f * (2 + m) - b * e * g * (3 + m + n) + c * d * g * (4 + m + 2 * n)) + g * (2 + m + n) * (a * e^{2 * g} * g * (3 + m + n) - c * d * (e * f * (2 + m) + d * g * (1 + n)))) * (d + e * x)^{(1 + m)} * (f + g * x)^n * \text{Hypergeometric2F1}[1 + m, -n, 2 + m, -((g * (d + e * x)) / (e * f - d * g))] / (e^{3 * g} * g^{2 * (1 + m)} * (2 + m + n) * (3 + m + n) * ((e * (f + g * x)) / (e * f - d * g))^n)$

**Rubi in Sympy [A]** time = 97.6807, size = 230, normalized size = 0.87

$$\frac{c \left( \frac{e(-f-gx)}{dg-ef} \right)^{-n} (d + ex)^{m+1} (f + gx)^n (dg - ef)^2 {}_2F_1 \left( -n - 2, m + 1 \middle| \frac{g(d+ex)}{dg-ef} \right)}{e^3g^2(m+1)} + \frac{\left( \frac{e(-f-gx)}{dg-ef} \right)^{-n} (d + ex)^{m+1} (f + gx)^n (ag^2 - bfg + cf^2) {}_2F_1 \left( -n, m + 1 \middle| \frac{g(d+ex)}{dg-ef} \right)}{eg^2(m+1)} - \frac{\left( \frac{e(-f-gx)}{dg-ef} \right)^{-n} (d + ex)^{m+1} (f + gx)^n (bg - 2cf) (dg - ef) {}_2F_1 \left( -n - 1, m + 1 \middle| \frac{g(d+ex)}{dg-ef} \right)}{e^2g^2(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**m*(g*x+f)**n*(c*x**2+b*x+a),x)`

[Out]  $c*(e*(-f-g*x)/(d*g-e*f))^{**}(-n)*(d+e*x)^{(m+1)}*(f+g*x)^{**}n*(d*g-e*f)^{**}2*\text{hyper}((-n-2, m+1), (m+2, ), g*(d+e*x)/(d*g-e*f))/(e^{**}3*g^{**}2*(m+1)) + (e*(-f-g*x)/(d*g-e*f))^{**}(-n)*(d+e*x)^{(m+1)}*(f+g*x)^{**}n*(a*g^{**}2-b*f*g+c*f^{**}2)*\text{hyper}((-n, m+1), (m+2, ), g*(d+e*x)/(d*g-e*f))/(e^{**}g^{**}2*(m+1)) - (e*(-f-g*x)/(d*g-e*f))^{**}(-n)*(d+e*x)^{(m+1)}*(f+g*x)^{**}n*(b*g-2*c*f)*(d*g-e*f)*\text{hyper}((-n-1, m+1), (m+2, ), g*(d+e*x)/(d*g-e*f))/(e^{**}2*g^{**}2*(m+1))$

**Mathematica [C]** time = 0.926809, size = 327, normalized size = 1.23

$$\frac{1}{3}(d+ex)^m(f+gx)^n \left( \frac{3a(f+gx) \left( \frac{g(d+ex)}{dg-ef} \right)^{-m} {}_2F_1 \left( -m, n+1; n+2; \frac{e(f+gx)}{ef-dg} \right)}{g(n+1)} \right. \\ \left. + \frac{9bdfx^2 {}_2F_1 \left( 2; -m, -n; 3; -\frac{ex}{d}, -\frac{gx}{f} \right)}{6df {}_2F_1 \left( 2; -m, -n; 3; -\frac{ex}{d}, -\frac{gx}{f} \right) + 2efmx {}_2F_1 \left( 3; 1-m, -n; 4; -\frac{ex}{d}, -\frac{gx}{f} \right) + 2dgnx {}_2F_1 \left( 3; -m, 1-n; 4; -\frac{ex}{d}, -\frac{gx}{f} \right)} \right. \\ \left. + \frac{4cdfx^3 {}_2F_1 \left( 3; -m, -n; 4; -\frac{ex}{d}, -\frac{gx}{f} \right)}{4df {}_2F_1 \left( 3; -m, -n; 4; -\frac{ex}{d}, -\frac{gx}{f} \right) + efmx {}_2F_1 \left( 4; 1-m, -n; 5; -\frac{ex}{d}, -\frac{gx}{f} \right) + dgnx {}_2F_1 \left( 4; -m, 1-n; 5; -\frac{ex}{d}, -\frac{gx}{f} \right)} \right)$$

Warning: Unable to verify antiderivative.

[In] `Integrate[(d + e*x)^m*(f + g*x)^n*(a + b*x + c*x^2),x]`

[Out]  $((d+e*x)^m*(f+g*x)^n*((9*b*d*f*x^2*AppellF1[2, -m, -n, 3, -((e*x)/d), -((g*x)/f)]/(6*d*f*AppellF1[2, -m, -n, 3, -((e*x)/d), -((g*x)/f)] + 2*e*f*m*x*AppellF1[3, 1-m, -n, 4, -((e*x)/d), -((g*x)/f)] + 2*d*g*n*x*AppellF1[3, -m, 1-n, 4, -((e*x)/d), -((g*x)/f)] + (4*c*d*f*x^3*AppellF1[3, -m, -n, 4, -((e*x)/d), -((g*x)/f)])/(4*d*f*AppellF1[3, -m, -n, 4, -((e*x)/d), -((g*x)/f)] + e*f*m*x*AppellF1[4, 1-m, -n, 5, -((e*x)/d), -((g*x)/f)] + d*g*n*x*AppellF1[4, -m, 1-n, 5, -((e*x)/d), -((g*x)/f)] + (3*a*(f+g*x)*Hypergeometric2F1[-m, 1+n, 2+n, (e*(f+g*x))/(e*f-d*g)]/(g*(1+n)*((g*(d+e*x))/(-e*f+d*g))^m)))/3$

**Maple [F]** time = 0.089, size = 0, normalized size = 0.

$$\int (ex+d)^m (gx+f)^n (cx^2+bx+a) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^m*(g*x+f)^n*(c*x^2+b*x+a),x)`

[Out] `int((e*x+d)^m*(g*x+f)^n*(c*x^2+b*x+a),x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (cx^2+bx+a)(ex+d)^m(gx+f)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)*(e*x + d)^m*(g*x + f)^n,x, algorithm="maxima")`

[Out] `integrate((c*x^2 + b*x + a)*(e*x + d)^m*(g*x + f)^n, x)`

---

**Fricas** [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((cx^2 + bx + a)(ex + d)^m(gx + f)^n, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)*(e*x + d)^m*(g*x + f)^n,x, algorithm="fricas")`

[Out] `integral((c*x^2 + b*x + a)*(e*x + d)^m*(g*x + f)^n, x)`

---

**Sympy** [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**m*(g*x+f)**n*(c*x**2+b*x+a), x)`

[Out] Timed out

---

**GIAC/XCAS** [F] time = 0., size = 0, normalized size = 0.

$$\int (cx^2 + bx + a)(ex + d)^m(gx + f)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)*(e*x + d)^m*(g*x + f)^n,x, algorithm="giac")`

[Out] `integrate((c*x^2 + b*x + a)*(e*x + d)^m*(g*x + f)^n, x)`



### 3.954 $\int (d + ex)^m (f + gx)^2 (a + bx + cx^2)^p dx$

**Optimal.** Leaf size=525

$$\frac{(d + ex)^{m+1} (a + bx + cx^2)^p \left(1 - \frac{2c(d+ex)}{2cd - e(b - \sqrt{b^2 - 4ac})}\right)^{-p} \left(1 - \frac{2c(d+ex)}{2cd - e(\sqrt{b^2 - 4ac} + b)}\right)^{-p} F_1\left(m + 1; -p, -p; m + 2; \frac{2c(d+ex)}{2cd - (b - \sqrt{b^2 - 4ac})}\right)}{ce^3(m+1)(m+2p+3)}$$

$$\frac{g(d + ex)^{m+2} (a + bx + cx^2)^p \left(1 - \frac{2c(d+ex)}{2cd - e(b - \sqrt{b^2 - 4ac})}\right)^{-p} \left(1 - \frac{2c(d+ex)}{2cd - e(\sqrt{b^2 - 4ac} + b)}\right)^{-p} (beg(m+p+2) + 2c(dg(p+1) - ef(m+1)))}{ce^3(m+2)(m+2p+3)}$$

$$+ \frac{g^2(d + ex)^{m+1} (a + bx + cx^2)^{p+1}}{ce(m+2p+3)}$$

[Out]  $(g^2(d + e^*x)^{(1 + m)}(a + b^*x + c^*x^2)^{(1 + p)}) / (c^*e^*(3 + m + 2^*p)) + ((e^*(b^*d - a^*e)g^2(1 + m) + c^*(2^*d^2g^2(1 + p) + e^2f^2(3 + m + 2^*p) - 2^*d^*e^*f^*g^*(3 + m + 2^*p))) * (d + e^*x)^{(1 + m)}(a + b^*x + c^*x^2)^p \text{AppellF1}[1 + m, -p, -p, 2 + m, (2^*c^*(d + e^*x)) / (2^*c^*d - (b - \text{Sqrt}[b^2 - 4^*a^*c])^*e), (2^*c^*(d + e^*x)) / (2^*c^*d - (b + \text{Sqrt}[b^2 - 4^*a^*c])^*e)]) / (c^*e^3(1 + m)^3(3 + m + 2^*p)^*(1 - (2^*c^*(d + e^*x)) / (2^*c^*d - (b - \text{Sqrt}[b^2 - 4^*a^*c])^*e)))^p (1 - (2^*c^*(d + e^*x)) / (2^*c^*d - (b + \text{Sqrt}[b^2 - 4^*a^*c])^*e))^p - (g^*(b^*e^*g^*(2 + m + p) + 2^*c^*(d^*g^*(1 + p) - e^*f^*(3 + m + 2^*p))) * (d + e^*x)^{(2 + m)}(a + b^*x + c^*x^2)^p \text{AppellF1}[2 + m, -p, -p, 3 + m, (2^*c^*(d + e^*x)) / (2^*c^*d - (b - \text{Sqrt}[b^2 - 4^*a^*c])^*e), (2^*c^*(d + e^*x)) / (2^*c^*d - (b + \text{Sqrt}[b^2 - 4^*a^*c])^*e)]) / (c^*e^3(2 + m)^3(3 + m + 2^*p)^*(1 - (2^*c^*(d + e^*x)) / (2^*c^*d - (b - \text{Sqrt}[b^2 - 4^*a^*c])^*e)))^p (1 - (2^*c^*(d + e^*x)) / (2^*c^*d - (b + \text{Sqrt}[b^2 - 4^*a^*c])^*e))^p$

**Rubi [A]** time = 1.71325, antiderivative size = 523, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$

$$\frac{(d + ex)^{m+1} (a + bx + cx^2)^p \left(1 - \frac{2c(d+ex)}{2cd - e(b - \sqrt{b^2 - 4ac})}\right)^{-p} \left(1 - \frac{2c(d+ex)}{2cd - e(\sqrt{b^2 - 4ac} + b)}\right)^{-p} F_1\left(m + 1; -p, -p; m + 2; \frac{2c(d+ex)}{2cd - (b - \sqrt{b^2 - 4ac})}\right)}{ce^2(m+2p+3)}$$

$$\frac{g(d + ex)^{m+2} (a + bx + cx^2)^p \left(1 - \frac{2c(d+ex)}{2cd - e(b - \sqrt{b^2 - 4ac})}\right)^{-p} \left(1 - \frac{2c(d+ex)}{2cd - e(\sqrt{b^2 - 4ac} + b)}\right)^{-p} (beg(m+p+2) + 2cdg(p+1) - 2cef(m+1))}{ce^3(m+2)(m+2p+3)}$$

$$+ \frac{g^2(d + ex)^{m+1} (a + bx + cx^2)^{p+1}}{ce(m+2p+3)}$$

Warning: Unable to verify antiderivative.

[In]  $\text{Int}[(d + e^*x)^m (f + g^*x)^2 (a + b^*x + c^*x^2)^p, x]$

[Out]  $(g^2(d + e^*x)^{(1 + m)}(a + b^*x + c^*x^2)^{(1 + p)}) / (c^*e^*(3 + m + 2^*p)) + (((b^*d - a^*e)g^2 + (c^*(2^*d^2g^2(1 + p) + e^2f^2(3 + m + 2^*p) - 2^*d^*e^*f^*g^*(3 + m + 2^*p))) / (e^*(1 + m))) * (d + e^*x)^{(1 + m)}(a + b^*x + c^*x^2)^p \text{AppellF1}[1 + m, -p, -p, 2 + m, (2^*c^*(d + e^*x)) / (2^*c^*d - (b - \text{Sqrt}[b^2 - 4^*a^*c])^*e), (2^*c^*(d + e^*x)) / (2^*c^*d - (b + \text{Sqrt}[b^2 - 4^*a^*c])^*e)]) / (c^*e^2(3 + m + 2^*p)^*(1 - (2^*c^*(d + e^*x)) / (2^*c^*d - (b - \text{Sqrt}[b^2 - 4^*a^*c])^*e)))^p (1 - (2^*c^*(d + e^*x)) / (2^*c^*d - (b + \text{Sqrt}[b^2 - 4^*a^*c])^*e))^p - (g^*(2^*c^*d^*g^*(1 + p) + b^*e^*g^*(2 + m + p) - 2^*c^*e^*f^*(3 + m + 2^*p))) * (d + e^*x)^{(2 + m)}(a + b^*x + c^*x^2)^p \text{AppellF1}[2 + m, -p, -p, 3 + m, (2^*c^*(d + e^*x)) / (2^*c^*d - (b - \text{Sqrt}[b^2 - 4^*a^*c])^*e), (2^*c^*(d + e^*x)) / (2^*c^*d - (b + \text{Sqrt}[b^2 - 4^*a^*c])^*e)]) / (c^*e^3(2 + m)^3(3 + m + 2^*p)^*(1 - (2^*c^*(d + e^*x)) / (2^*c^*d - (b - \text{Sqrt}[b^2 - 4^*a^*c])^*e)))^p (1 - (2^*c^*(d + e^*x)) / (2^*c^*d - (b + \text{Sqrt}[b^2 - 4^*a^*c])^*e))^p$

---

**Rubi in Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+b*x+a)**p,x)`

[Out] Timed out

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**Mathematica [A]** time = 2.70802, size = 0, normalized size = 0.

$$\int (d + ex)^m (f + gx)^2 (a + bx + cx^2)^p dx$$

Verification is Not applicable to the result.

[In] `Integrate[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^p,x]`

[Out] `Integrate[(d + e*x)^m*(f + g*x)^2*(a + b*x + c*x^2)^p, x]`

---

**Maple [F]** time = 0.156, size = 0, normalized size = 0.

$$\int (ex + d)^m (gx + f)^2 (cx^2 + bx + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^p,x)`

[Out] `int((e*x+d)^m*(g*x+f)^2*(c*x^2+b*x+a)^p,x)`

---

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (gx + f)^2 (cx^2 + bx + a)^p (ex + d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x + f)^2*(c*x^2 + b*x + a)^p*(e*x + d)^m,x, algorithm="maxima")`

[Out] `integrate((g*x + f)^2*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)`

---

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(g^2x^2 + 2fgx + f^2\right)\left(cx^2 + bx + a\right)^p\left(ex + d\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x + f)^2*(c*x^2 + b*x + a)^p*(e*x + d)^m,x, algorithm="fricas")`

[Out] `integral((g^2*x^2 + 2*f*g*x + f^2)*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**m*(g*x+f)**2*(c*x**2+b*x+a)**p,x)`

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (gx + f)^2 (cx^2 + bx + a)^p (ex + d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x + f)^2*(c*x^2 + b*x + a)^p*(e*x + d)^m,x, algorithm="giac")`

[Out] `integrate((g*x + f)^2*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)`

$$3.955 \quad \int (d + ex)^m (f + gx) (a + bx + cx^2)^p dx$$

**Optimal.** Leaf size=384

$$\frac{(ef - dg)(d + ex)^{m+1} (a + bx + cx^2)^p \left(1 - \frac{2c(d+ex)}{2cd - e(b - \sqrt{b^2 - 4ac})}\right)^{-p} \left(1 - \frac{2c(d+ex)}{2cd - e(\sqrt{b^2 - 4ac} + b)}\right)^{-p} F_1\left(m + 1, -p, -p; m + 2; \frac{2c(d+ex)}{2cd - (b - \sqrt{b^2 - 4ac})}\right)}{e^{2(m+1)}} + \frac{g(d + ex)^{m+2} (a + bx + cx^2)^p \left(1 - \frac{2c(d+ex)}{2cd - e(b - \sqrt{b^2 - 4ac})}\right)^{-p} \left(1 - \frac{2c(d+ex)}{2cd - e(\sqrt{b^2 - 4ac} + b)}\right)^{-p} F_1\left(m + 2, -p, -p; m + 3; \frac{2c(d+ex)}{2cd - (b - \sqrt{b^2 - 4ac})}\right)}{e^{2(m+2)}}$$

[Out] ((e\*f - d\*g)\*(d + e\*x)^(1 + m)\*(a + b\*x + c\*x^2)^p\*AppellF1[1 + m, -p, -p, 2 + m, (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e), (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]/(e^2\*(1 + m)\*(1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e))^p\*(1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e))^p) + (g\*(d + e\*x)^(2 + m)\*(a + b\*x + c\*x^2)^p\*AppellF1[2 + m, -p, -p, 3 + m, (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e), (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]/(e^2\*(2 + m)\*(1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e))^p\*(1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e))^p)

**Rubi [A]** time = 0.883747, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 25,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$

$$\frac{(ef - dg)(d + ex)^{m+1} (a + bx + cx^2)^p \left(1 - \frac{2c(d+ex)}{2cd - e(b - \sqrt{b^2 - 4ac})}\right)^{-p} \left(1 - \frac{2c(d+ex)}{2cd - e(\sqrt{b^2 - 4ac} + b)}\right)^{-p} F_1\left(m + 1, -p, -p; m + 2; \frac{2c(d+ex)}{2cd - (b - \sqrt{b^2 - 4ac})}\right)}{e^{2(m+1)}} + \frac{g(d + ex)^{m+2} (a + bx + cx^2)^p \left(1 - \frac{2c(d+ex)}{2cd - e(b - \sqrt{b^2 - 4ac})}\right)^{-p} \left(1 - \frac{2c(d+ex)}{2cd - e(\sqrt{b^2 - 4ac} + b)}\right)^{-p} F_1\left(m + 2, -p, -p; m + 3; \frac{2c(d+ex)}{2cd - (b - \sqrt{b^2 - 4ac})}\right)}{e^{2(m+2)}}$$

Warning: Unable to verify antiderivative.

[In] Int[(d + e\*x)^m\*(f + g\*x)\*(a + b\*x + c\*x^2)^p,x]

[Out] ((e\*f - d\*g)\*(d + e\*x)^(1 + m)\*(a + b\*x + c\*x^2)^p\*AppellF1[1 + m, -p, -p, 2 + m, (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e), (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]/(e^2\*(1 + m)\*(1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e))^p\*(1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e))^p) + (g\*(d + e\*x)^(2 + m)\*(a + b\*x + c\*x^2)^p\*AppellF1[2 + m, -p, -p, 3 + m, (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e), (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]/(e^2\*(2 + m)\*(1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e))^p\*(1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e))^p)

**Rubi in Sympy [A]** time = 99.0257, size = 347, normalized size = 0.9

$$\frac{g(d + ex)^{m+2} \left(\frac{c(-2d-2ex)}{2cd - e(b + \sqrt{-4ac + b^2})} + 1\right)^{-p} \left(\frac{c(2d+2ex)}{be - 2cd - e\sqrt{-4ac + b^2}} + 1\right)^{-p} (a + bx + cx^2)^p \text{appellf}_1\left(m + 2, -p, -p, m + 3, \frac{c(-2d-2ex)}{be - 2cd - e\sqrt{-4ac + b^2}}\right)}{e^{2(m+2)}} + \frac{(d + ex)^{m+1} (dg - ef) \left(\frac{c(-2d-2ex)}{2cd - e(b + \sqrt{-4ac + b^2})} + 1\right)^{-p} \left(\frac{c(2d+2ex)}{be - 2cd - e\sqrt{-4ac + b^2}} + 1\right)^{-p} (a + bx + cx^2)^p \text{appellf}_1\left(m + 1, -p, -p, m + 2, \frac{c(-2d-2ex)}{be - 2cd - e\sqrt{-4ac + b^2}}\right)}{e^{2(m+1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `rubi_integrate((e*x+d)**m*(g*x+f)*(c*x**2+b*x+a)**p,x)`

[Out] 
$$g*(d + e*x)**(m + 2)*(c*(-2*d - 2*e*x)/(2*c*d - e*(b + \sqrt{-4*a*c + b**2})) + 1)**(-p)*(c*(2*d + 2*e*x)/(b*e - 2*c*d - e*\sqrt{-4*a*c + b**2}) + 1)**(-p)*(a + b*x + c*x**2)**p*\text{appellf1}(m + 2, -p, -p, m + 3, c*(-2*d - 2*e*x)/(b*e - 2*c*d - e*\sqrt{-4*a*c + b**2}), c*(2*d + 2*e*x)/(2*c*d - e*(b + \sqrt{-4*a*c + b**2}))) / (e**2*(m + 2)) - (d + e*x)**(m + 1)*(d*g - e*f)*(c*(-2*d - 2*e*x)/(2*c*d - e*(b + \sqrt{-4*a*c + b**2})) + 1)**(-p)*(c*(2*d + 2*e*x)/(b*e - 2*c*d - e*\sqrt{-4*a*c + b**2}) + 1)**(-p)*(a + b*x + c*x**2)**p*\text{appellf1}(m + 1, -p, -p, m + 2, c*(-2*d - 2*e*x)/(b*e - 2*c*d - e*\sqrt{-4*a*c + b**2}), c*(2*d + 2*e*x)/(2*c*d - e*(b + \sqrt{-4*a*c + b**2}))) / (e**2*(m + 1))$$

**Mathematica [A]** time = 1.99338, size = 0, normalized size = 0.

$$\int (d + ex)^m (f + gx) (a + bx + cx^2)^p dx$$

Verification is Not applicable to the result.

[In] `Integrate[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^p,x]`

[Out] `Integrate[(d + e*x)^m*(f + g*x)*(a + b*x + c*x^2)^p, x]`

**Maple [F]** time = 0.135, size = 0, normalized size = 0.

$$\int (ex + d)^m (gx + f) (cx^2 + bx + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^p,x)`

[Out] `int((e*x+d)^m*(g*x+f)*(c*x^2+b*x+a)^p,x)`

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (gx + f)(cx^2 + bx + a)^p (ex + d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((g*x + f)*(c*x^2 + b*x + a)^p*(e*x + d)^m,x, algorithm="maxima")`

[Out] `integrate((g*x + f)*(c*x^2 + b*x + a)^p*(e*x + d)^m, x)`

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((gx + f)(cx^2 + bx + a)^p (ex + d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)\*(c\*x^2 + b\*x + a)^p\*(e\*x + d)^m,x, algorithm="fricas")

[Out] integral((g\*x + f)\*(c\*x^2 + b\*x + a)^p\*(e\*x + d)^m, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*m\*(g\*x+f)\*(c\*x\*\*2+b\*x+a)\*\*p,x)

[Out] Timed out

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (gx + f)(cx^2 + bx + a)^p (ex + d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((g\*x + f)\*(c\*x^2 + b\*x + a)^p\*(e\*x + d)^m,x, algorithm="giac")

[Out] integrate((g\*x + f)\*(c\*x^2 + b\*x + a)^p\*(e\*x + d)^m, x)

$$3.956 \quad \int (d + ex)^m (a + bx + cx^2)^p dx$$

**Optimal.** Leaf size=187

$$\frac{(d + ex)^{m+1} (a + bx + cx^2)^p \left(1 - \frac{2c(d+ex)}{2cd - e(b - \sqrt{b^2 - 4ac})}\right)^{-p} \left(1 - \frac{2c(d+ex)}{2cd - e(\sqrt{b^2 - 4ac} + b)}\right)^{-p} F_1\left(m + 1; -p, -p; m + 2; \frac{2c(d+ex)}{2cd - (b - \sqrt{b^2 - 4ac})}e\right)}{e(m + 1)}$$

[Out] ((d + e\*x)^(1 + m)\*(a + b\*x + c\*x^2)^p\*AppellF1[1 + m, -p, -p, 2 + m, (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e), (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]/(e\*(1 + m)\*(1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e))^p\*(1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e))^p)

**Rubi [A]** time = 0.379986, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 20,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$

$$\frac{(d + ex)^{m+1} (a + bx + cx^2)^p \left(1 - \frac{2c(d+ex)}{2cd - e(b - \sqrt{b^2 - 4ac})}\right)^{-p} \left(1 - \frac{2c(d+ex)}{2cd - e(\sqrt{b^2 - 4ac} + b)}\right)^{-p} F_1\left(m + 1; -p, -p; m + 2; \frac{2c(d+ex)}{2cd - (b - \sqrt{b^2 - 4ac})}e\right)}{e(m + 1)}$$

Antiderivative was successfully verified.

[In] Int[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p, x]

[Out] ((d + e\*x)^(1 + m)\*(a + b\*x + c\*x^2)^p\*AppellF1[1 + m, -p, -p, 2 + m, (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e), (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e)]/(e\*(1 + m)\*(1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b - Sqrt[b^2 - 4\*a\*c])\*e))^p\*(1 - (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e))^p)

**Rubi in Sympy [A]** time = 38.225, size = 167, normalized size = 0.89

$$\frac{(d + ex)^{m+1} \left(\frac{c(-2d-2ex)}{2cd - e(b + \sqrt{-4ac + b^2})} + 1\right)^{-p} \left(\frac{c(2d+2ex)}{be - 2cd - e\sqrt{-4ac + b^2}} + 1\right)^{-p} (a + bx + cx^2)^p \text{appellf1}\left(m + 1, -p, -p, m + 2, \frac{c(-2d-2ex)}{be - 2cd - e\sqrt{-4ac + b^2}}\right)}{e(m + 1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*m\*(c\*x\*\*2+b\*x+a)\*\*p, x)

[Out] (d + e\*x)\*\*(m + 1)\*(c\*(-2\*d - 2\*e\*x)/(2\*c\*d - e\*(b + sqrt(-4\*a\*c + b\*\*2))) + 1)\*\*(-p)\*(c\*(2\*d + 2\*e\*x)/(b\*e - 2\*c\*d - e\*sqrt(-4\*a\*c + b\*\*2)) + 1)\*\*(-p)\*(a + b\*x + c\*x\*\*2)\*\*p\*appellf1(m + 1, -p, -p, m + 2, c\*(-2\*d - 2\*e\*x)/(b\*e - 2\*c\*d - e\*sqrt(-4\*a\*c + b\*\*2)), c\*(2\*d + 2\*e\*x)/(2\*c\*d - e\*(b + sqrt(-4\*a\*c + b\*\*2))))/(e\*(m + 1))

**Mathematica [A]** time = 0.0820484, size = 205, normalized size = 1.1

$$\frac{(d + ex)^{m+1} (a + x(b + cx))^p \left(\frac{e(\sqrt{b^2 - 4ac} - b - 2cx)}{e(\sqrt{b^2 - 4ac} - b) + 2cd}\right)^{-p} \left(\frac{e(\sqrt{b^2 - 4ac} + b + 2cx)}{e(\sqrt{b^2 - 4ac} + b) - 2cd}\right)^{-p} F_1\left(m + 1; -p, -p; m + 2; \frac{2c(d+ex)}{2cd - (b + \sqrt{b^2 - 4ac})}e, \frac{2c(d+ex)}{2cd + (b - \sqrt{b^2 - 4ac})}e\right)}{e(m + 1)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(d + e\*x)^m\*(a + b\*x + c\*x^2)^p,x]

[Out] ((d + e\*x)^(1 + m)\*(a + x\*(b + c\*x))^p\*AppellF1[1 + m, -p, -p, 2 + m, (2\*c\*(d + e\*x))/(2\*c\*d - (b + Sqrt[b^2 - 4\*a\*c])\*e), (2\*c\*(d + e\*x))/(2\*c\*d + (-b + Sqrt[b^2 - 4\*a\*c])\*e)]/(e\*(1 + m)\*((e\*(-b + Sqrt[b^2 - 4\*a\*c] - 2\*c\*x))/(2\*c\*d + (-b + Sqrt[b^2 - 4\*a\*c])\*e))^p\*((e\*(b + Sqrt[b^2 - 4\*a\*c] + 2\*c\*x))/(-2\*c\*d + (b + Sqrt[b^2 - 4\*a\*c])\*e))^p)

**Maple [F]** time = 0.193, size = 0, normalized size = 0.

$$\int (ex + d)^m (cx^2 + bx + a)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e\*x+d)^m\*(c\*x^2+b\*x+a)^p,x)

[Out] int((e\*x+d)^m\*(c\*x^2+b\*x+a)^p,x)

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int (cx^2 + bx + a)^p (ex + d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)^p\*(e\*x + d)^m,x, algorithm="maxima")

[Out] integrate((c\*x^2 + b\*x + a)^p\*(e\*x + d)^m, x)

**Fricas [F]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((cx^2 + bx + a)^p (ex + d)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c\*x^2 + b\*x + a)^p\*(e\*x + d)^m,x, algorithm="fricas")

[Out] integral((c\*x^2 + b\*x + a)^p\*(e\*x + d)^m, x)

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((e\*x+d)\*\*m\*(c\*x\*\*2+b\*x+a)\*\*p,x)



[Out] Timed out

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int (cx^2 + bx + a)^p (ex + d)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^p*(e*x + d)^m,x, algorithm="giac")`

[Out] `integrate((c*x^2 + b*x + a)^p*(e*x + d)^m, x)`

$$3.957 \quad \int \frac{(d+ex)^m (a+bx+cx^2)^p}{f+gx} dx$$

**Optimal.** Leaf size=30

$$\text{Int} \left( \frac{(d+ex)^m (a+bx+cx^2)^p}{f+gx}, x \right)$$

[Out] Unintegrable[((d + e\*x)^m\*(a + b\*x + c\*x^2)^p)/(f + g\*x), x]

**Rubi [A]** time = 0.0712189, antiderivative size = 0, normalized size of antiderivative = 0., number of rules used = 0, number of rules used = 0, integrand size = 0,  $\frac{\text{number of rules}}{\text{integrand size}} = 0$ .

$$\text{Int} \left( \frac{(d+ex)^m (a+bx+cx^2)^p}{f+gx}, x \right)$$

Verification is Not applicable to the result.

[In] Int[((d + e\*x)^m\*(a + b\*x + c\*x^2)^p)/(f + g\*x), x]

[Out] Defer[Int][[((d + e\*x)^m\*(a + b\*x + c\*x^2)^p)/(f + g\*x), x]

**Rubi in Sympy [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{f+gx} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate((e\*x+d)\*\*m\*(c\*x\*\*2+b\*x+a)\*\*p/(g\*x+f), x)

[Out] Integral((d + e\*x)\*\*m\*(a + b\*x + c\*x\*\*2)\*\*p/(f + g\*x), x)

**Mathematica [A]** time = 0.164058, size = 0, normalized size = 0.

$$\int \frac{(d+ex)^m (a+bx+cx^2)^p}{f+gx} dx$$

Verification is Not applicable to the result.

[In] Integrate[((d + e\*x)^m\*(a + b\*x + c\*x^2)^p)/(f + g\*x), x]

[Out] Integrate[((d + e\*x)^m\*(a + b\*x + c\*x^2)^p)/(f + g\*x), x]

**Maple [A]** time = 0.148, size = 0, normalized size = 0.

$$\int \frac{(ex+d)^m (cx^2+bx+a)^p}{gx+f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x)`

[Out] `int((e*x+d)^m*(c*x^2+b*x+a)^p/(g*x+f),x)`

**Maxima [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)^p (ex + d)^m}{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^p*(e*x + d)^m/(g*x + f),x, algorithm="maxima")`

[Out] `integrate((c*x^2 + b*x + a)^p*(e*x + d)^m/(g*x + f), x)`

**Fricas [A]** time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(cx^2 + bx + a)^p (ex + d)^m}{gx + f}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^p*(e*x + d)^m/(g*x + f),x, algorithm="fricas")`

[Out] `integral((c*x^2 + b*x + a)^p*(e*x + d)^m/(g*x + f), x)`

**Sympy [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((e*x+d)**m*(c*x**2+b*x+a)**p/(g*x+f),x)`

[Out] Timed out

**GIAC/XCAS [A]** time = 0., size = 0, normalized size = 0.

$$\int \frac{(cx^2 + bx + a)^p (ex + d)^m}{gx + f} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((c*x^2 + b*x + a)^p*(e*x + d)^m/(g*x + f),x, algorithm="giac")`

[Out] `integrate((c*x^2 + b*x + a)^p*(e*x + d)^m/(g*x + f), x)`

$$3.958 \quad \int \frac{1}{\sqrt{1 - \frac{1}{c^2 x^2}} x^2 \sqrt{d+ex}} dx$$

**Optimal.** Leaf size=89

$$\frac{2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$

[Out] (-2\*Sqrt[(c\*(d + e\*x))/(c\*d + e)]\*Sqrt[1 - c^2\*x^2]\*EllipticPi[2, ArcSin[Sqrt[1 - c\*x]/Sqrt[2]], (2\*e)/(c\*d + e)]/(Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[d + e\*x])

**Rubi [A]** time = 0.87695, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 27,  $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$

$$\frac{2\sqrt{1-c^2x^2}\sqrt{\frac{c(d+ex)}{cd+e}}\left(2;\sin^{-1}\left(\frac{\sqrt{1-cx}}{\sqrt{2}}\right)\middle|\frac{2e}{cd+e}\right)}{x\sqrt{1-\frac{1}{c^2x^2}}\sqrt{d+ex}}$$

Antiderivative was successfully verified.

[In] Int[1/(Sqrt[1 - 1/(c^2\*x^2)])\*x^2\*Sqrt[d + e\*x]), x]

[Out] (-2\*Sqrt[(c\*(d + e\*x))/(c\*d + e)]\*Sqrt[1 - c^2\*x^2]\*EllipticPi[2, ArcSin[Sqrt[1 - c\*x]/Sqrt[2]], (2\*e)/(c\*d + e)]/(Sqrt[1 - 1/(c^2\*x^2)]\*x\*Sqrt[d + e\*x])

**Rubi in Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\frac{2\sqrt{d+ex}\int\frac{1}{\sqrt{x}}\frac{x^2}{\sqrt{1-\frac{x^4}{c^2}}\sqrt{dx^2+e}}dx}{\sqrt{x}\sqrt{\frac{d}{x}+e}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] rubi\_integrate(1/x\*\*2/(1-1/c\*\*2/x\*\*2)\*\*(1/2)/(e\*x+d)\*\*(1/2), x)

[Out] -2\*sqrt(d + e\*x)\*Integral(x\*\*2/(sqrt(1 - x\*\*4/c\*\*2)\*sqrt(d\*x\*\*2 + e)), (x, 1/sqrt(x)))/(sqrt(x)\*sqrt(d/x + e))

**Mathematica [C]** time = 0.5741, size = 188, normalized size = 2.11

$$\frac{2i(d+ex)\sqrt{\frac{e(cx-1)}{c(d+ex)}}\sqrt{\frac{cex+e}{cd+cex}}\left(F\left(i\sinh^{-1}\left(\frac{\sqrt{-\frac{cd+e}{c}}}{\sqrt{d+ex}}\right)\middle|\frac{cd-e}{cd+e}\right)-\left(\frac{cd}{cd+e};i\sinh^{-1}\left(\frac{\sqrt{-\frac{cd+e}{c}}}{\sqrt{d+ex}}\right)\middle|\frac{cd-e}{cd+e}\right)\right)}{dx\sqrt{1-\frac{1}{c^2x^2}}\sqrt{-\frac{cd+e}{c}}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[1 - 1/(c^2\*x^2)])\*x^2\*Sqrt[d + e\*x]), x]

[Out] ((-2\*I)\*Sqrt[(e\*(-1 + c\*x))/(c\*(d + e\*x))]\*(d + e\*x)\*Sqrt[(e + c\*e\*x)/(c\*d + c\*e\*x)]\*(EllipticF[I\*ArcSinh[Sqrt[-((c\*d + e)/c)]]/Sqr

$t[d + e*x]], (c*d - e)/(c*d + e)] - \text{EllipticPi}[(c*d)/(c*d + e), \text{I} * \text{ArcSinh}[\text{Sqrt}[-((c*d + e)/c)]/\text{Sqrt}[d + e*x]], (c*d - e)/(c*d + e) ])/(d*\text{Sqrt}[-((c*d + e)/c)]*\text{Sqrt}[1 - 1/(c^2*x^2)]*x)$

**Maple [A]** time = 0.165, size = 148, normalized size = 1.7

$$-2 \frac{cd - e}{x\sqrt{ex + dcd}} \text{EllipticPi} \left( \sqrt{\frac{(ex + d)c}{cd - e}}, \frac{cd - e}{cd}, \sqrt{\frac{cd - e}{cd + e}} \right) \sqrt{-\frac{(cx + 1)e}{cd - e}} \sqrt{-\frac{(cx - 1)e}{cd + e}} \sqrt{\frac{(ex + d)c}{cd - e}} \frac{1}{\sqrt{\frac{c^2x^2 - 1}{c^2x^2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(1/x^2/(1-1/c^2/x^2)^(1/2)/(e*x+d)^(1/2), x)`

[Out]  $-2/((c^2*x^2-1)/c^2/x^2)^{(1/2)}/x*(c*d-e)*\text{EllipticPi}(((e*x+d)*c/(c*d-e))^{(1/2)}, (c*d-e)/c/d, ((c*d-e)/(c*d+e))^{(1/2)})*(-(c*x+1)*e/(c*d-e))^{(1/2)}*(-(c*x-1)*e/(c*d+e))^{(1/2)}*((e*x+d)*c/(c*d-e))^{(1/2)}/(e*x+d)^{(1/2)}/c/d$

**Maxima [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ex + dx^2} \sqrt{-\frac{1}{c^2x^2} + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(e*x + d)*x^2*sqrt(-1/(c^2*x^2) + 1)), x, algorithm="maxima")`

[Out] `integrate(1/(sqrt(e*x + d)*x^2*sqrt(-1/(c^2*x^2) + 1)), x)`

**Fricas [F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/(sqrt(e*x + d)*x^2*sqrt(-1/(c^2*x^2) + 1)), x, algorithm="fricas")`

[Out] Timed out

**Sympy [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{x^2 \sqrt{-(-1 + \frac{1}{cx}) (1 + \frac{1}{cx})} \sqrt{d + ex}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(1/x**2/(1-1/c**2/x**2)**(1/2)/(e*x+d)**(1/2), x)`

[Out] `Integral(1/(x**2*sqrt(-(-1 + 1/(c*x))*(1 + 1/(c*x))))*sqrt(d + e*x), x)`

---

**GIAC/XCAS [F]** time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{ex + dx^2} \sqrt{-\frac{1}{c^2 x^2} + 1}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(sqrt(e\*x + d)\*x^2\*sqrt(-1/(c^2\*x^2) + 1)),x, algorithm="giac")

[Out] integrate(1/(sqrt(e\*x + d)\*x^2\*sqrt(-1/(c^2\*x^2) + 1)), x)

## 4 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Mathematica/Rubi followed by one for Maple. The following are links to the source code.

The following are the listing of the above functions.

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)
(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*   is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*   antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
        "B"],
      "C"],
    If[FreeQ[result,Integrate] && FreeQ[result,Int],
      "C",
      "F"]]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)
```





```

# File: GradeAntiderivative.mpl Original version thanks to Albert Rich emailed on 03/21/2017
#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal;

    leaf_count_result:=leafcount(result);
    #do NOT call ExpnType() if leaf size is too large. Recursion problem
    if leaf_count_result > 500000 then
        return "B";
    fi;

    leaf_count_optimal:=leafcount(optimal);

    ExpnType_result:=ExpnType(result);
    ExpnType_optimal:=ExpnType(optimal);
    #This check below actually is not needed, since I only call this grading only for
    #passed integrals. i.e. I check for "F" before calling this.

    if not type(result,freeof('int')) then
        return "F";
    end if;

    if ExpnType_result<=ExpnType_optimal then
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                #both result and optimal complex
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A";
                else
                    return "B";
                end if
            else #result contains complex but optimal is not
                return "C";
            end if
        else # result do not contain complex
            # this assumes optimal do not as well
            if leaf_count_result<=2*leaf_count_optimal then
                return "A";
            else
                return "B";
            end if
        end if
    else #ExpnType(result) > ExpnType(optimal)
        return "C";
    end if
end proc:

```

```

# is_contains_complex(result) takes expressions and returns true if it contains "I"
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1 else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,``^``) then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1 else
        max(2,ExpnType(op(1,expn))) end if else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,``+``) or type(expn,``*``) then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' or op(0,expn)='integrate' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
    9
  end if
end proc:

```

```

ElementaryFunctionQ := proc(func)
  member(func,[exp,log,ln, sin,cos,tan,cot,sec,csc,
    arcsin,arccos,arctan,arccot,arcsec,arccsc,
    sinh,cosh,tanh,coth,sech,csch,
    arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[erf,erfc,erfi,FresnelS,FresnelC,Ei,Ei,Li,Si,Ci,Shi,Chi,
    GAMMA,lnGAMMA,Psi,Zeta,polylog,LambertW,
    EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u) else
    apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```